Computer Algebra Independent Integration Tests

Summer 2023 edition with Rubi V 4.17.3

4-Trig-functions/4.5-Secant/118-4.5.1.2-d-sec-^n-a+b-sec-^m

Nasser M. Abbasi

December 8, 2023 Compiled on December 8, 2023 at 8:57pm

Contents

1 Introduction **2**

2 detailed summary tables of results **20**

3 Listing of integrals **279**

4 Appendix **6668**

CHAPTER 1

Introduction

1.1 Listing of CAS systems tested 3
1.2 Results 4
1.3 Time and leaf size Performance 7
1.4 Performance based on number of rules Rubi used 9
1.5 Performance based on number of steps Rubi used 10
1.6 Solved integrals histogram based on leaf size of result 11
1.7 Solved integrals histogram based on CPU time used 12
1.8 Leaf size vs. CPU time used 13
1.9 list of integrals with no known antiderivative 14
1.10 List of integrals solved by CAS but has no known antiderivative 14
1.11 list of integrals solved by CAS but failed verification 14
1.12 Timing 15
1.13 Verification 15
1.14 Important notes about some of the results 16
1.15 Design of the test system 19

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [879]. This is test number [118].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.3.1 (August 16, 2023) on windows 10.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 13.3.1 on windows 10
3. Maple 2023.1 (July, 12, 2023) on windows 10.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
5. FriCAS 1.3.9 (July 8, 2023) based on sbcl 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
6. Giac/Xcas 1.9.0-57 (June 26, 2023) on Linux via sagemath 10.1 (Aug 20, 2023).
7. Sympy 1.12 (May 10, 2023) Using Python 3.11.3 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	99.89 (878)	0.11 (1)
Mathematica	98.63 (867)	1.37 (12)
Maple	83.62 (735)	16.38 (144)
Fricas	69.28 (609)	30.72 (270)
Mupad	36.75 (323)	63.25 (556)
Maxima	36.75 (323)	63.25 (556)
Giac	27.65 (243)	72.35 (636)
Sympy	5.80 (51)	94.20 (828)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

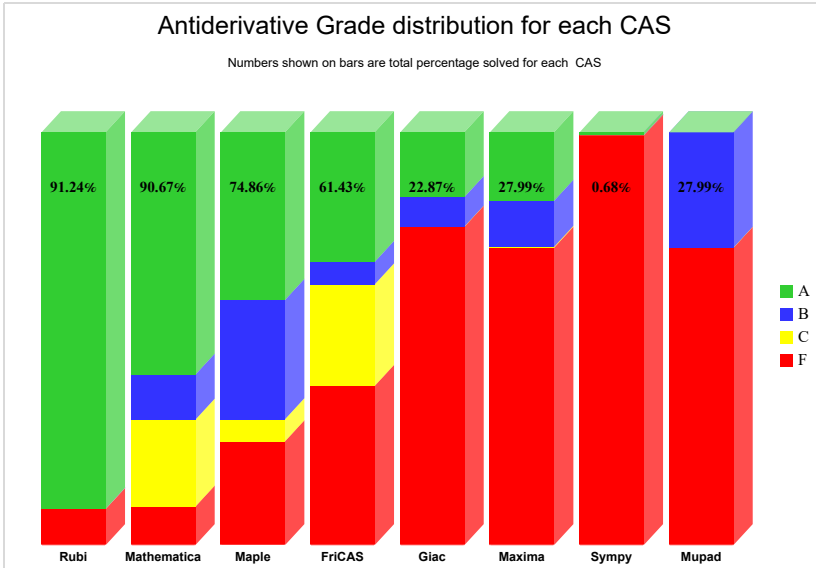
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

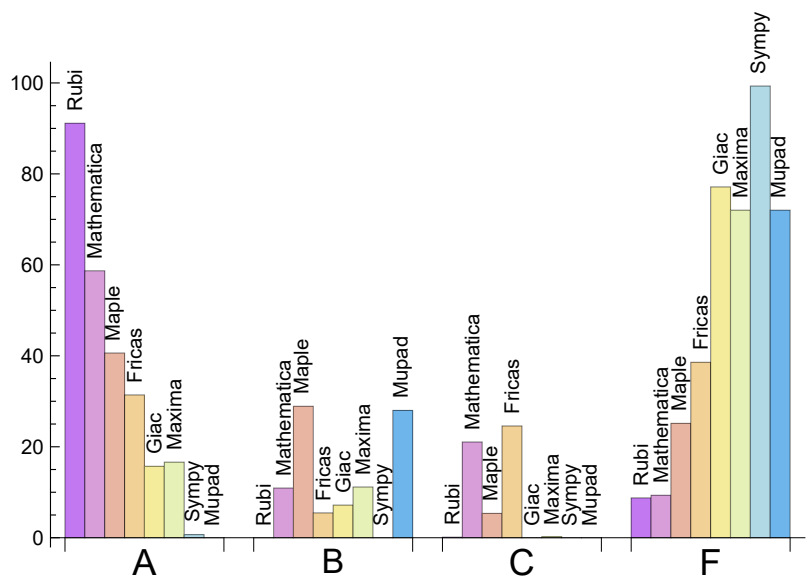
System	% A grade	% B grade	% C grade	% F grade
Rubi	90.899	0.114	0.114	8.874
Mathematica	58.703	10.922	21.047	9.329
Maple	40.614	28.896	5.347	25.142
Fricas	31.399	5.461	24.573	38.567
Maxima	16.610	11.149	0.228	72.014
Giac	15.700	7.167	0.000	77.133
Sympy	0.683	0.000	0.000	99.317
Mupad	0.000	27.986	0.000	72.014

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima

and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	1	100.00	0.00	0.00
Mathematica	12	41.67	58.33	0.00
Maple	144	100.00	0.00	0.00
Fricas	270	66.30	33.70	0.00
Mupad	556	0.00	100.00	0.00
Maxima	556	84.71	8.09	7.19
Giac	636	94.34	5.19	0.47
Sympy	828	68.84	31.16	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Fricas	0.28
Maxima	0.68
Rubi	0.87
Giac	4.11
Mathematica	12.86
Mupad	14.95
Maple	16.96
Sympy	24.33

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Sympy	23.27	1.08	24.00	1.00
Giac	151.36	1.45	106.00	1.27
Rubi	166.03	1.07	140.00	1.02
Fricas	247.31	1.94	175.00	1.63
Mupad	382.40	2.46	88.00	1.08
Maple	583.94	2.70	202.00	1.72
Mathematica	1393.78	6.94	133.00	1.08
Maxima	3913.36	22.64	104.00	1.21

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

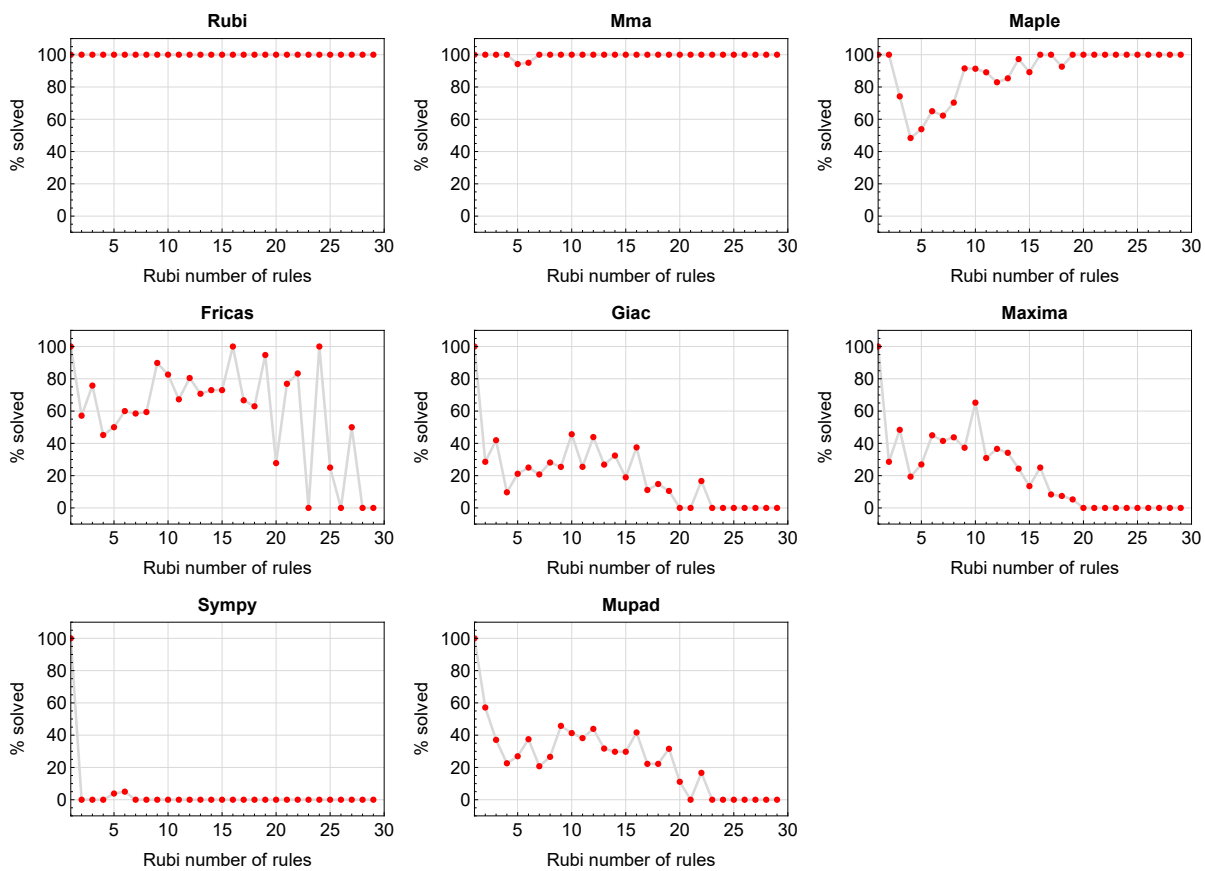


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

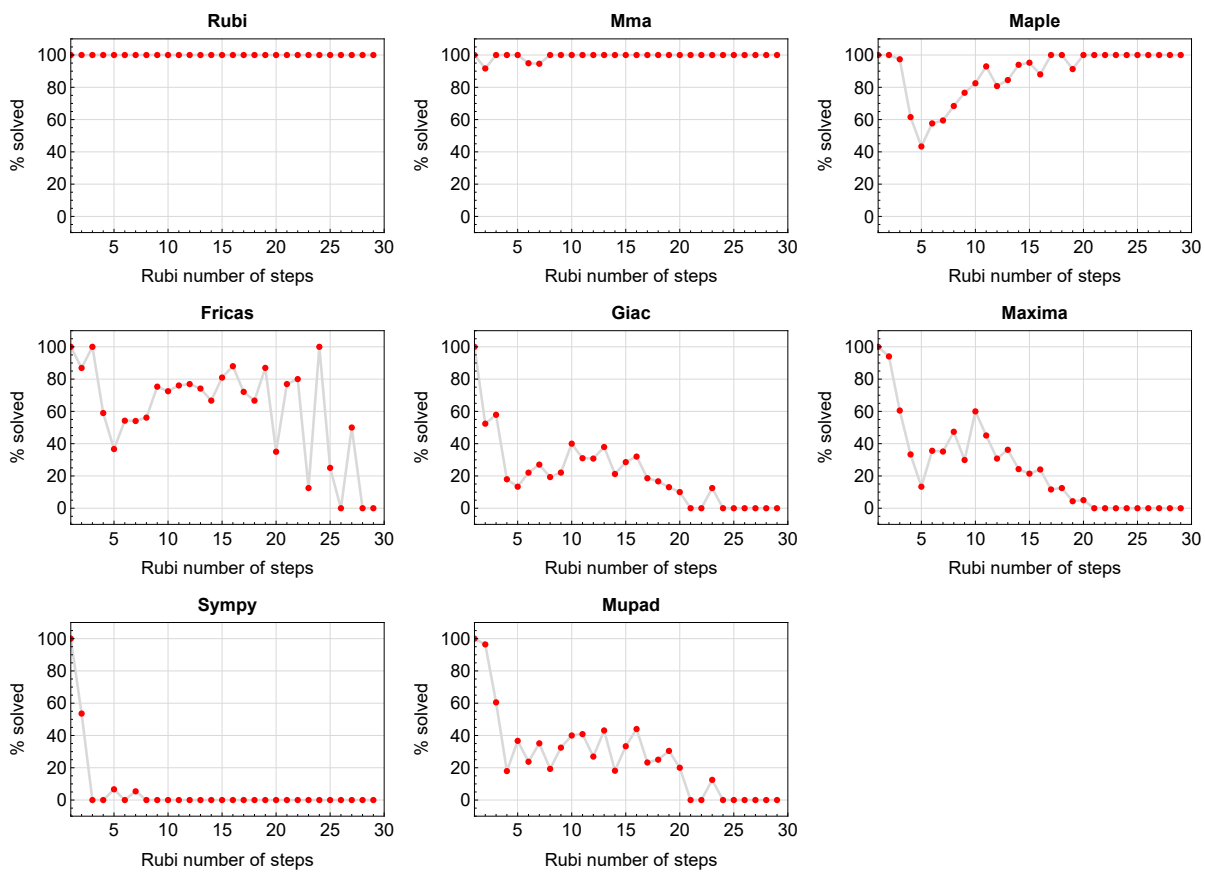


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

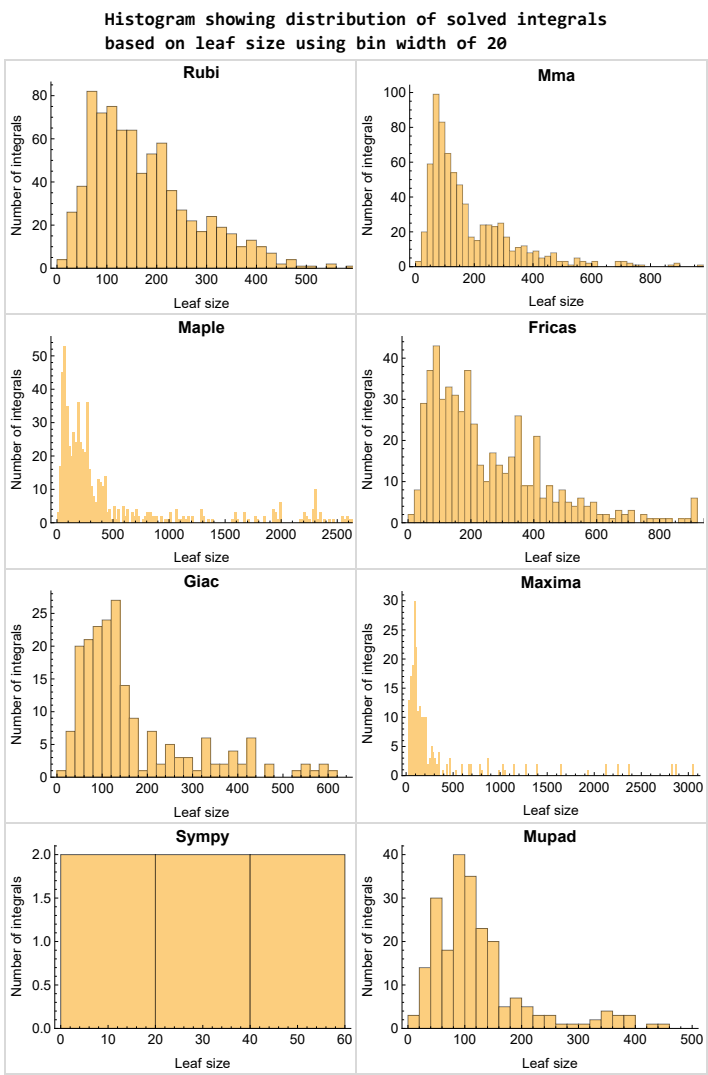


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

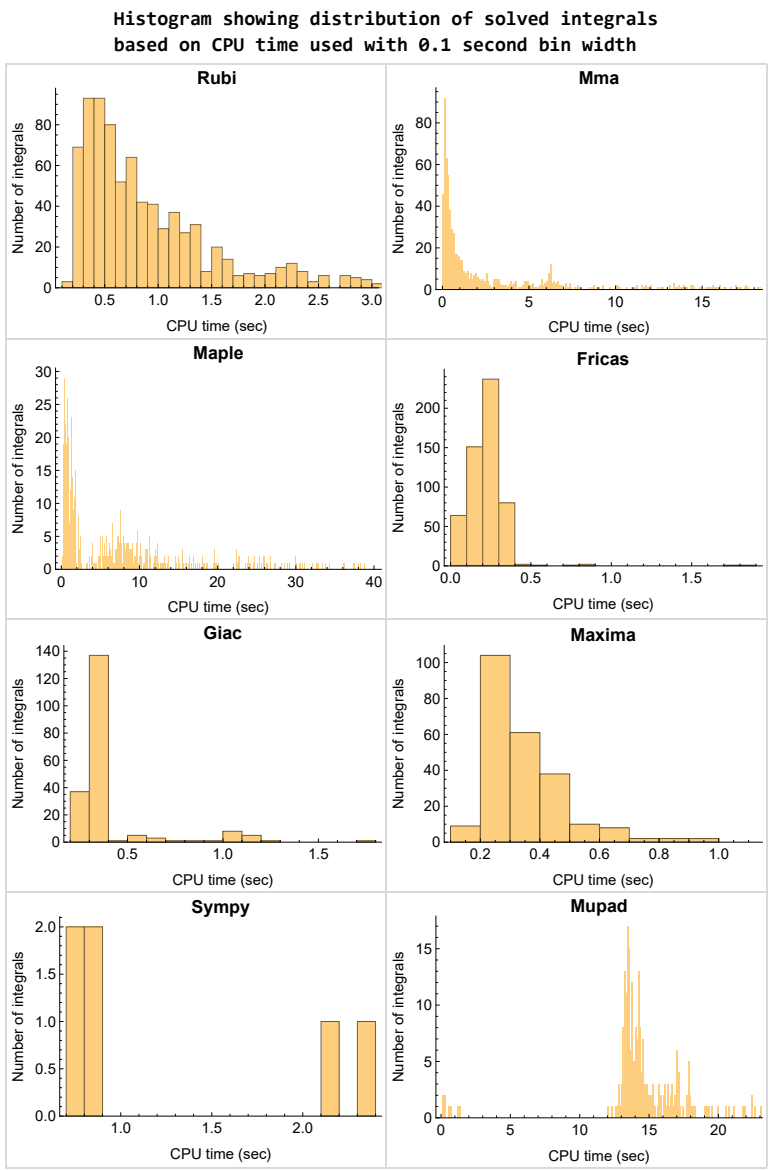


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following gives the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time.

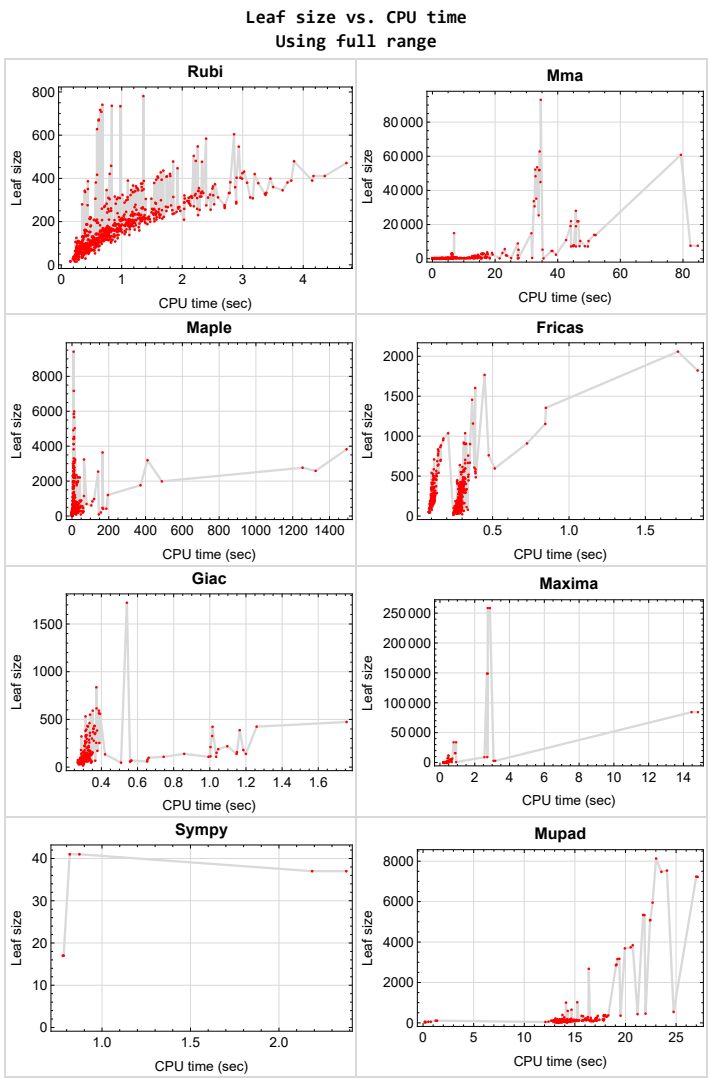


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{686, 691, 693, 698, 703, 705, 707, 712, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 782, 783, 784, 785, 786, 787, 791, 792, 793}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {159, 160, 161, 162, 163, 164, 275, 276, 277, 278, 279, 282, 283}

Mathematica {111, 139, 146, 147, 151, 152, 157, 158, 163, 164, 280, 281, 282, 283, 284, 285, 287, 294, 297, 298, 301, 302, 305, 306, 307, 310, 311, 314, 315, 318, 319, 329, 330, 331, 333, 334, 335, 337, 338, 339, 345, 346, 347, 348, 349, 350, 351, 352, 353, 355, 357, 358, 359, 360, 361, 362, 368, 369, 395, 531, 537, 538, 539, 544, 545, 546, 547, 551, 552, 553, 554, 556, 560, 562, 563, 570, 571,

572, 573, 616, 621, 626, 627, 685, 687, 688, 689, 690, 692, 694, 695, 696, 697, 699, 700, 701, 702, 704, 706, 708, 709, 710, 711, 713, 714, 715, 716, 780, 781, 788, 789, 790, 837, 840, 841, 842, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 859, 860, 861, 867, 868, 869, 870, 872, 873, 874, 878, 879}

Maple {126, 132, 134, 139, 140, 252, 258, 259, 263, 264, 432, 434, 439, 440, 567, 570, 630, 636, 646, 651, 654, 655, 656, 657, 662, 663, 665, 839, 845, 848, 851, 867, 868, 869, 871, 873, 874}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

1.14.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.14.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.14.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

1.14.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

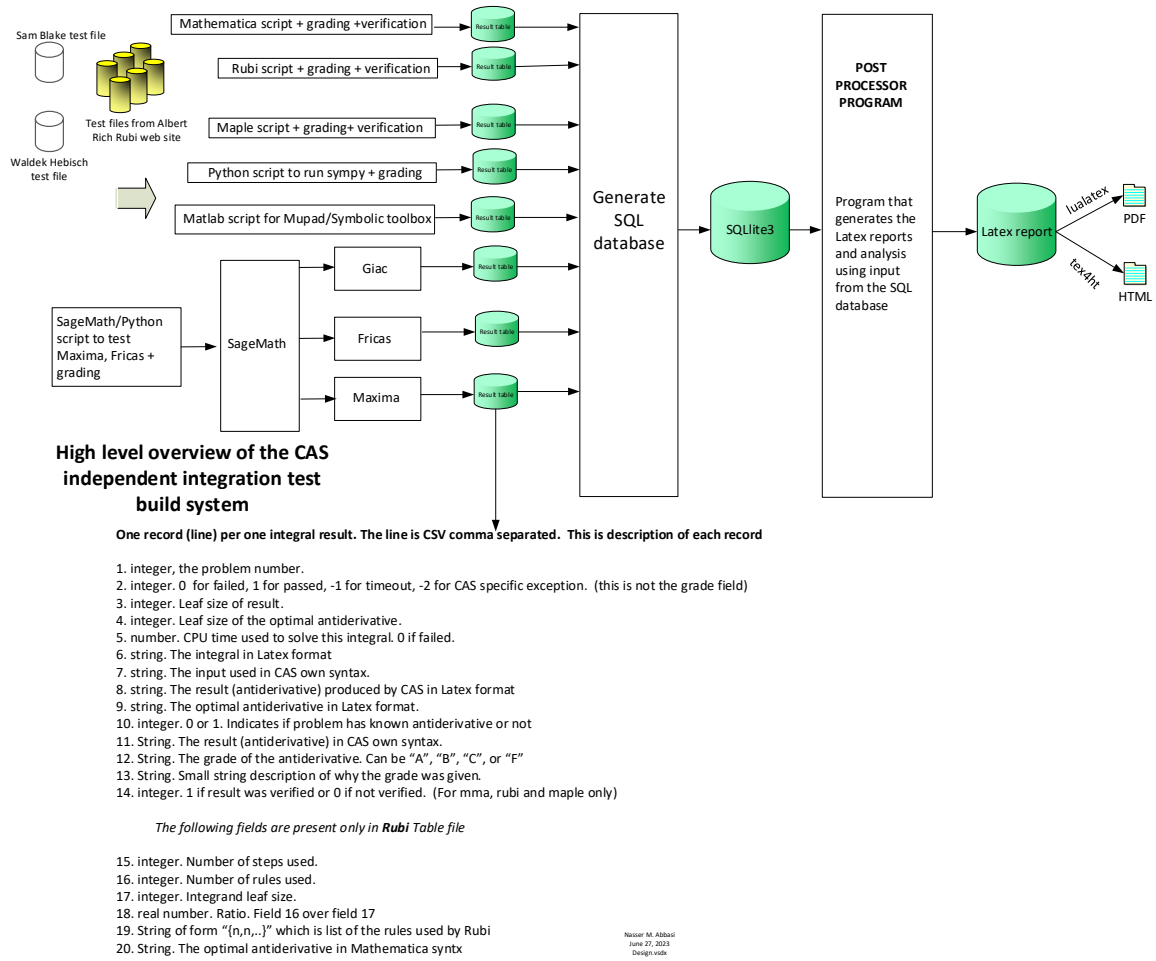
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. This is description of each record

1. integer, the problem number.
2. integer. 0 for failed, 1 for passed, -1 for timeout, -2 for CAS specific exception. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntax.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be "A", "B", "C", or "F"
13. String. Small string description of why the grade was given.
14. integer. 1 if result was verified or 0 if not verified. (For mma, rubi and maple only)

*The following fields are present only in **Rubi** Table file*

15. integer. Number of steps used.
16. integer. Number of rules used.
17. integer. Integrand leaf size.
18. real number. Ratio. Field 16 over field 17
19. String of form "{n,n,..}" which is list of the rules used by Rubi
20. String. The optimal antiderivative in Mathematica syntx

Nasser M. Abbasi
June 27, 2023
Design.vsdx

CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS 21
2.2 Detailed conclusion table per each integral for all CAS systems 32
2.3 Detailed conclusion table specific for Rubi results 252

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi 21
2.1.2 Mma 22
2.1.3 Maple 23
2.1.4 Fricas 25
2.1.5 Maxima 26
2.1.6 Giac 27
2.1.7 Mupad 28
2.1.8 Sympy 30

2.1.1 Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545,

546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 687, 688, 689, 690, 692, 694, 695, 696, 697, 699, 700, 701, 702, 704, 706, 708, 709, 710, 711, 713, 714, 715, 716, 777, 778, 779, 780, 781, 788, 789, 790, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879 }

B grade { 819 }

C grade { 286 }

F normal fail { 854 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.2 Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 112, 116, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 134, 135, 136, 137, 139, 140, 141, 165, 166, 167, 168, 169, 170, 171, 175, 189, 204, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 254, 255, 256, 257, 258, 259, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 286, 288, 289, 290, 291, 292, 293, 295, 296, 299, 300, 303, 304, 308, 309, 312, 313, 316, 317, 322, 326, 328, 341, 342, 343, 344, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 432, 433, 434, 435, 436, 437, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 531, 532, 533, 534, 535, 537, 538, 539, 540, 542, 544, 545, 546, 547, 548, 549, 550, 554, 555, 556,

557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 570, 571, 572, 573, 574, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 617, 618, 619, 620, 621, 622, 623, 624, 625, 627, 629, 630, 631, 632, 633, 636, 637, 638, 639, 644, 645, 646, 649, 650, 651, 652, 653, 656, 657, 658, 659, 660, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 777, 778, 779, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 875, 876, 877 }

B grade { 34, 35, 48, 77, 146, 147, 151, 152, 157, 158, 163, 164, 253, 260, 280, 281, 282, 283, 284, 297, 298, 301, 302, 305, 306, 310, 311, 314, 315, 318, 319, 329, 330, 331, 333, 334, 335, 337, 338, 339, 345, 346, 347, 348, 431, 438, 480, 529, 530, 536, 541, 543, 551, 552, 553, 567, 568, 569, 575, 576, 577, 578, 616, 626, 685, 687, 688, 689, 690, 692, 694, 695, 696, 697, 699, 700, 701, 702, 704, 706, 708, 709, 710, 711, 713, 714, 715, 716, 780, 781, 788, 789, 790, 822, 878, 879 }

C grade { 96, 97, 98, 111, 113, 114, 115, 117, 133, 138, 142, 143, 144, 145, 148, 149, 150, 153, 154, 155, 156, 159, 160, 161, 162, 172, 173, 174, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 272, 273, 274, 275, 276, 277, 278, 279, 285, 287, 294, 307, 320, 321, 325, 327, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 505, 628, 634, 635, 640, 641, 642, 643, 647, 648, 654, 655, 661, 662, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874 }

F normal fail { 323, 324, 332, 336, 340 }

F(-1) timedout fail { 717, 718, 727, 736, 742, 745, 746 }

F(-2) exception fail { }

2.1.3 Maple

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 97, 98, 99, 100, 101, 102, 106, 107, 108, 109, 110, 114, 115, 116, 118, 119, 120, 121, 123, 124, 126, 127, 128, 134, 135, 142, 167, 168, 169, 170, 171, 176, 177, 178, 182, 183, 184, 185, 190, 191, 192, 193, 195, 196, 197, 198, 199, 200, 201, 202, 203, 205, 206, 207, 208, 209, 211, 212, 213, 214, 215, 216, 217, 221, 222, 223, 224, 225, 229, 230, 231, 232, 233, 234, 235, 238, 239, 240, 241, 248, 249, 250, 255, 256, 257, 263, 264, 269, 270, 271, 351, 352, 354, 355, 359, 360, 361, 367, 368, 369, 370, 374, 375, 376, 377, 378, 379, 381, 382, 383, 384, 386,

389, 390, 391, 392, 393, 394, 395, 397, 398, 399, 400, 401, 404, 405, 406, 407, 408, 410, 411, 412, 413, 414, 417, 418, 419, 420, 421, 422, 424, 425, 426, 427, 428, 429, 430, 431, 434, 435, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 534, 558, 559, 581, 582, 583, 584, 585, 589, 590, 592, 596, 598, 599, 604, 605, 606, 607, 608, 611, 612, 650, 678, 679, 794, 795, 798, 799, 806, 813, 819, 820, 858 }

B grade { 94, 95, 96, 103, 104, 105, 111, 112, 113, 117, 122, 125, 129, 130, 131, 132, 133, 136, 137, 138, 139, 140, 141, 165, 166, 172, 173, 174, 175, 179, 180, 181, 186, 187, 188, 189, 194, 204, 210, 218, 219, 220, 226, 227, 228, 236, 237, 243, 244, 245, 246, 247, 251, 252, 253, 254, 258, 259, 260, 261, 262, 265, 266, 267, 268, 353, 356, 357, 358, 362, 363, 364, 365, 366, 371, 372, 373, 380, 385, 387, 388, 396, 402, 403, 409, 415, 416, 423, 432, 433, 436, 437, 438, 439, 440, 531, 532, 533, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 586, 587, 588, 591, 593, 594, 595, 597, 600, 601, 602, 603, 609, 610, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 630, 631, 632, 633, 637, 638, 639, 644, 645, 646, 651, 652, 653, 656, 657, 658, 659, 660, 663, 664, 665, 666, 667, 668, 670, 671, 796, 797, 800, 801, 802, 803, 804, 805, 807, 808, 809, 810, 811, 812, 814, 815, 816, 817, 818, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 842, 843, 844, 848, 849, 850, 855, 856, 857, 862, 863, 864, 865, 866, 869, 870, 871, 872, 873 }

C grade { 20, 41, 82, 530, 628, 629, 634, 635, 636, 640, 641, 642, 643, 647, 648, 649, 654, 655, 661, 662, 669, 672, 673, 674, 675, 676, 677, 680, 681, 682, 683, 684, 840, 841, 845, 846, 847, 851, 852, 853, 854, 859, 860, 861, 867, 868, 874 }

F normal fail { 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 242, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 441, 442, 443, 444, 445, 685, 687, 688, 689, 690, 692, 694, 695, 696, 697, 699, 700, 701, 702, 704, 706, 708, 709, 710, 711, 713, 714, 715, 716, 777, 778, 779, 780, 781, 788, 789, 790, 875, 876, 877, 878, 879 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.4 Fricas

A grade { 1, 2, 3, 6, 7, 8, 9, 10, 11, 12, 13, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 132, 133, 134, 135, 137, 140, 141, 142, 218, 221, 222, 223, 224, 225, 226, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 245, 246, 247, 248, 249, 250, 251, 253, 254, 255, 256, 257, 258, 260, 261, 262, 263, 264, 270, 271, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 446, 447, 448, 451, 452, 453, 454, 455, 456, 457, 458, 459, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 490, 491, 492, 493, 494, 495, 496, 500, 501, 503, 504, 505, 508, 513, 523, 524, 525, 526, 527, 528, 529, 530 }

B grade { 4, 5, 14, 117, 118, 129, 130, 131, 136, 138, 139, 219, 220, 227, 228, 243, 244, 252, 259, 265, 266, 267, 268, 269, 449, 450, 460, 488, 489, 497, 498, 499, 502, 506, 507, 509, 510, 511, 512, 514, 515, 516, 517, 518, 519, 520, 521, 522 }

C grade { 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 630, 631, 632, 633, 637, 638, 639, 644, 645, 646, 650, 651, 652, 653, 656, 657, 658, 659, 660, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 837, 838, 839, 842, 843, 844, 848, 849, 850, 855, 856, 857, 858, 862, 863, 864, 865, 866, 869, 870, 871, 872, 873 }

F normal fail { 143, 144, 145, 148, 149, 150, 153, 154, 155, 156, 159, 160, 161, 162, 272, 273, 274, 275, 276, 277, 278, 279, 284, 286, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 441, 442, 443, 444, 445, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 578, 627, 636, 642, 643, 685, 687, 688, 689, 690, 692, 694, 695, 696, 697, 699, 700, 701, 702, 704, 706, 708, 709, 710, 711, 777, 778, 779, 780, 781, 788, 789, 790, 819, 825, 830, 851, 875, 876, 877, 878, 879 }

F(-1) timedout fail { 146, 147, 151, 152, 157, 158, 163, 164, 280, 281, 282, 283, 285, 287, 551, 577, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 628, 629, 634, 635, 640, 641, 647, 648, 649, 654, 655, 661, 662, 686, 691, 693, 698, 703, 705, 707, 712, 713, 714, 715, 716, 817, 818, 820, 821, 822, 823, 824, 826, 827, 828, 829, 831, 832, 833, 834, 835, 836, 840, 841, 845, 846, 847, 852, 853, 854, 859, 860, 861, 867, 868, 874 }

F(-2) exception fail { }

2.1.5 Maxima

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 43, 45, 46, 47, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 221, 222, 229, 238, 247, 248, 269, 400, 401, 407, 414, 422, 423, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 523, 524, 525, 526, 527, 528, 529, 530 }

B grade { 30, 41, 42, 44, 48, 94, 95, 96, 97, 98, 103, 104, 111, 112, 117, 118, 218, 219, 220, 223, 224, 225, 226, 227, 228, 230, 231, 232, 233, 234, 235, 236, 237, 239, 240, 241, 242, 243, 244, 245, 246, 249, 250, 251, 252, 253, 254, 255, 256, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 270, 271, 398, 399, 402, 403, 404, 405, 406, 408, 409, 410, 411, 412, 413, 415, 416, 417, 418, 419, 420, 421, 424, 425, 426, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440 }

C grade { 123, 142 }

F normal fail { 90, 91, 92, 93, 99, 100, 101, 102, 108, 109, 110, 116, 119, 120, 121, 122, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 203, 204, 205, 206, 207, 208, 214, 215, 217, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 389, 390, 391, 392, 441, 442, 443, 444, 445, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 563, 564, 565, 566, 567, 568, 569, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 618, 619, 620, 626, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660,

661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 687, 688, 689, 690, 692, 694, 695, 696, 697, 699, 700, 701, 702, 704, 706, 708, 709, 710, 711, 713, 714, 715, 716, 777, 778, 779, 780, 781, 788, 789, 790, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 830, 831, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879 }

F(-1) timedout fail { 105, 106, 107, 113, 114, 115, 186, 201, 202, 209, 210, 211, 212, 213, 387, 388, 393, 394, 395, 396, 397, 544, 562, 570, 571, 572, 600, 614, 615, 616, 617, 621, 622, 623, 624, 625, 816, 827, 828, 829, 832, 833, 834, 835, 836 }

F(-2) exception fail { 216, 257, 427, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 627 }

2.1.6 Giac

A grade { 1, 2, 3, 7, 8, 9, 10, 11, 12, 13, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 119, 120, 121, 122, 126, 127, 128, 129, 130, 131, 134, 135, 136, 137, 138, 139, 141, 142, 455, 470, 471, 472, 480, 481, 482, 488, 489, 491, 493, 494, 495, 497, 499, 500, 501, 502, 504, 505, 506, 507, 508, 512, 514, 516, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530 }

B grade { 4, 5, 6, 14, 15, 116, 117, 118, 124, 125, 132, 133, 140, 244, 446, 447, 448, 449, 450, 451, 452, 453, 454, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 473, 474, 475, 476, 477, 478, 479, 483, 484, 485, 486, 487, 490, 492, 496, 498, 503, 509, 510, 511, 513, 515, 517, 518, 519, 520 }

C grade { }

F normal fail { 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 123, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336,

337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 687, 688, 689, 690, 692, 694, 695, 696, 697, 699, 700, 701, 702, 704, 706, 708, 709, 710, 711, 713, 714, 715, 716, 777, 778, 779, 780, 781, 788, 789, 790, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879 }

F(-1) timedout fail { 742, 743, 745, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776 }

F(-2) exception fail { 324, 744, 746 }

2.1.7 Mupad

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 99, 100, 101, 102, 107, 108, 109, 110, 116, 221, 222, 223, 224, 229, 230, 231, 232, 238, 239, 240, 241, 242, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816 }

C grade { }

F normal fail { }

F(-1) timedout fail { 94, 95, 96, 97, 98, 103, 104, 105, 106, 111, 112, 113, 114, 115, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 225, 226, 227, 228, 233, 234, 235, 236, 237, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 687, 688, 689, 690, 692, 694, 695, 696, 697, 699, 700, 701, 702, 704, 706, 708, 709, 710, 711, 713, 714, 715, 716, 777, 778, 779, 780, 781, 788, 789, 790, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879 }

F(-2) exception fail { }

2.1.8 Sympy

A grade { 4, 5, 6, 449, 450, 451 }

B grade { }

C grade { }

F normal fail { 1, 2, 3, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 30, 31, 32, 33, 34, 35, 36, 37, 41, 42, 43, 44, 45, 46, 47, 48, 49, 51, 52, 53, 54, 55, 56, 57, 58, 59, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 98, 99, 100, 101, 102, 103, 104, 108, 109, 110, 111, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 149, 150, 151, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 166, 167, 168, 169, 170, 171, 174, 175, 176, 177, 178, 181, 182, 183, 184, 189, 190, 191, 195, 196, 197, 198, 199, 200, 203, 204, 205, 206, 207, 208, 212, 213, 214, 215, 216, 217, 219, 220, 221, 222, 223, 227, 228, 229, 230, 242, 243, 244, 246, 247, 248, 249, 250, 253, 254, 255, 256, 261, 262, 263, 267, 268, 269, 270, 271, 273, 274, 277, 278, 279, 280, 281, 282, 283, 287, 288, 289, 290, 291, 292, 293, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 348, 349, 350, 354, 355, 356, 363, 364, 365, 371, 372, 373, 375, 376, 377, 378, 382, 383, 384, 385, 390, 391, 392, 401, 402, 403, 409, 410, 421, 422, 423, 424, 428, 429, 430, 441, 442, 443, 444, 445, 446, 447, 448, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 476, 477, 478, 479, 480, 481, 482, 483, 486, 487, 488, 489, 490, 491, 492, 493, 494, 496, 497, 498, 499, 500, 501, 502, 503, 504, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 545, 546, 547, 548, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 580, 581, 582, 583, 584, 585, 588, 589, 590, 591, 592, 595, 596, 597, 598, 603, 604, 605, 608, 609, 610, 611, 612, 613, 616, 617, 618, 619, 620, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 635, 636, 637, 638, 649, 650, 651, 652, 653, 656, 657, 658, 659, 664, 665, 666, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 687, 688, 689, 690, 692, 696, 697, 699, 700, 701, 702, 704, 706, 708, 709, 710, 711, 713, 714, 715, 716, 777, 778, 779, 780, 781, 788, 789, 790, 798, 799, 800, 806, 807, 808, 814, 815, 816, 818, 819, 820, 821, 824, 825, 826, 827, 830, 831, 832, 839, 840, 841, 846, 847, 856, 857, 858, 859, 863, 864, 865, 875, 876, 877, 878, 879 }

F(-1) timedout fail { 28, 29, 38, 39, 40, 50, 60, 97, 105, 106, 107, 112, 113, 114, 115, 148, 152, 165, 172, 173, 179, 180, 185, 186, 187, 188, 192, 193, 194, 201, 202, 209, 210, 211, 218, 224, 225, 226, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 245, 251, 252, 257, 258, 259, 260, 264, 265, 266, 272, 275, 276, 284, 285, 286, 294, 307, 320, 347, 351, 352, 353, 357, 358, 359, 360, 361, 362, 366, 367, 368, 369, 370, 374, 379, 380, 381, 386, 387, 388, 389, 393, 394, 395, 396, 397, 398, 399, 400, 404, 405, 406, 407, 408, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 425, 426, 427,

431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 474, 475, 484, 485, 495, 505, 543, 544, 549, 550, 551, 552, 553, 579, 586, 587, 593, 594, 599, 600, 601, 602, 606, 607, 614, 615, 621, 622, 633, 634, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 654, 655, 660, 661, 662, 663, 667, 668, 694, 695, 717, 718, 719, 726, 727, 728, 729, 730, 731, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 756, 757, 758, 766, 767, 768, 769, 774, 775, 776, 794, 795, 796, 797, 801, 802, 803, 804, 805, 809, 810, 811, 812, 813, 817, 822, 823, 828, 829, 833, 834, 835, 836, 837, 838, 842, 843, 844, 845, 848, 849, 850, 851, 852, 853, 854, 855, 860, 861, 862, 866, 867, 868, 869, 870, 871, 872, 873, 874 }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	90	56	73	95	99	0	110	130
N.S.	1	1.06	0.66	0.86	1.12	1.16	0.00	1.29	1.53
time (sec)	N/A	0.469	0.202	0.836	0.227	0.293	0.000	0.315	16.106

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	64	60	60	70	88	0	96	102
N.S.	1	1.02	0.95	0.95	1.11	1.40	0.00	1.52	1.62
time (sec)	N/A	0.390	0.144	0.607	0.233	0.271	0.000	0.314	14.515

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	48	47	47	58	74	0	80	75
N.S.	1	1.02	1.00	1.00	1.23	1.57	0.00	1.70	1.60
time (sec)	N/A	0.363	0.011	0.482	0.247	0.273	0.000	0.313	13.215

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	24	30	29	60	37	63	47
N.S.	1	1.00	1.00	1.25	1.21	2.50	1.54	2.62	1.96
time (sec)	N/A	0.277	0.008	0.362	0.226	0.257	2.380	0.310	13.484

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	24	23	36	41	49	20
N.S.	1	1.00	1.00	1.50	1.44	2.25	2.56	3.06	1.25
time (sec)	N/A	0.147	0.001	0.142	0.246	0.285	0.872	0.268	13.393

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	26	16	20	17	17	39	15
N.S.	1	1.00	1.73	1.07	1.33	1.13	1.13	2.60	1.00
time (sec)	N/A	0.225	0.008	0.436	0.242	0.245	0.783	0.284	13.223

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	39	32	29	34	29	0	56	50
N.S.	1	1.03	0.84	0.76	0.89	0.76	0.00	1.47	1.32
time (sec)	N/A	0.277	0.053	0.429	0.237	0.272	0.000	0.286	13.764

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	55	57	40	46	42	0	72	55
N.S.	1	1.02	1.06	0.74	0.85	0.78	0.00	1.33	1.02
time (sec)	N/A	0.305	0.066	0.517	0.313	0.242	0.000	0.286	13.255

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	81	73	53	57	53	0	86	79
N.S.	1	1.07	0.96	0.70	0.75	0.70	0.00	1.13	1.04
time (sec)	N/A	0.386	0.105	0.557	0.250	0.271	0.000	0.281	16.835

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	121	68	111	133	124	0	138	170
N.S.	1	0.99	0.56	0.91	1.09	1.02	0.00	1.13	1.39
time (sec)	N/A	0.704	0.631	0.985	0.251	0.271	0.000	0.320	17.891

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	95	58	112	145	111	0	122	141
N.S.	1	0.99	0.60	1.17	1.51	1.16	0.00	1.27	1.47
time (sec)	N/A	0.548	0.428	0.681	0.257	0.277	0.000	0.302	17.006

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	80	74	75	85	96	0	106	112
N.S.	1	1.08	1.00	1.01	1.15	1.30	0.00	1.43	1.51
time (sec)	N/A	0.540	0.285	0.685	0.245	0.268	0.000	0.306	15.224

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	54	70	81	83	0	90	83
N.S.	1	1.00	1.00	1.30	1.50	1.54	0.00	1.67	1.54
time (sec)	N/A	0.398	0.175	0.535	0.260	0.259	0.000	0.316	13.887

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	34	42	41	76	0	79	56
N.S.	1	1.00	1.00	1.24	1.21	2.24	0.00	2.32	1.65
time (sec)	N/A	0.283	0.165	0.351	0.249	0.282	0.000	0.294	13.361

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	47	44	52	53	0	79	33
N.S.	1	1.00	1.38	1.29	1.53	1.56	0.00	2.32	0.97
time (sec)	N/A	0.332	0.009	0.440	0.249	0.265	0.000	0.316	13.427

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	34	31	48	36	0	64	57
N.S.	1	1.00	0.76	0.69	1.07	0.80	0.00	1.42	1.27
time (sec)	N/A	0.353	0.069	0.420	0.274	0.262	0.000	0.315	13.756

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	63	41	42	61	49	0	80	61
N.S.	1	1.11	0.72	0.74	1.07	0.86	0.00	1.40	1.07
time (sec)	N/A	0.438	0.092	0.468	0.279	0.248	0.000	0.297	13.361

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	86	53	55	83	63	0	96	89
N.S.	1	0.99	0.61	0.63	0.95	0.72	0.00	1.10	1.02
time (sec)	N/A	0.447	0.139	0.540	0.238	0.264	0.000	0.319	16.487

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	94	61	64	95	76	0	112	105
N.S.	1	0.91	0.59	0.62	0.92	0.74	0.00	1.09	1.02
time (sec)	N/A	0.577	0.137	0.668	0.265	0.259	0.000	0.320	16.654

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	122	145	179	124	0	138	170
N.S.	1	1.00	1.07	1.27	1.57	1.09	0.00	1.21	1.49
time (sec)	N/A	0.344	0.765	1.103	0.271	0.280	0.000	0.347	17.772

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	99	123	156	111	0	122	141
N.S.	1	1.00	1.06	1.32	1.68	1.19	0.00	1.31	1.52
time (sec)	N/A	0.318	0.482	0.678	0.245	0.259	0.000	0.322	16.314

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	72	94	104	98	0	106	112
N.S.	1	1.00	1.00	1.31	1.44	1.36	0.00	1.47	1.56
time (sec)	N/A	0.273	0.301	0.657	0.246	0.284	0.000	0.327	14.480

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	69	59	80	91	98	0	100	88
N.S.	1	1.05	0.89	1.21	1.38	1.48	0.00	1.52	1.33
time (sec)	N/A	0.408	0.245	0.442	0.253	0.271	0.000	0.304	13.470

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	33	55	64	91	0	80	57
N.S.	1	1.00	0.69	1.15	1.33	1.90	0.00	1.67	1.19
time (sec)	N/A	0.254	0.196	0.539	0.210	0.287	0.000	0.338	13.233

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	81	59	74	65	0	100	88
N.S.	1	1.00	1.37	1.00	1.25	1.10	0.00	1.69	1.49
time (sec)	N/A	0.259	0.292	0.458	0.213	0.275	0.000	0.332	13.280

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	44	42	71	50	0	80	63
N.S.	1	1.00	0.70	0.67	1.13	0.79	0.00	1.27	1.00
time (sec)	N/A	0.271	0.090	0.487	0.225	0.264	0.000	0.322	13.386

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	51	53	94	63	0	96	89
N.S.	1	1.00	0.60	0.62	1.11	0.74	0.00	1.13	1.05
time (sec)	N/A	0.300	0.111	0.543	0.212	0.280	0.000	0.327	16.719

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	63	66	117	76	0	112	105
N.S.	1	1.00	0.60	0.63	1.11	0.72	0.00	1.07	1.00
time (sec)	N/A	0.315	0.150	0.674	0.219	0.266	0.000	0.333	17.127

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	73	75	143	89	0	128	121
N.S.	1	1.00	0.57	0.58	1.11	0.69	0.00	0.99	0.94
time (sec)	N/A	0.340	0.186	0.766	0.213	0.257	0.000	0.328	15.429

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	146	171	270	137	0	154	199
N.S.	1	1.00	1.07	1.26	1.99	1.01	0.00	1.13	1.46
time (sec)	N/A	0.374	5.047	1.482	0.222	0.271	0.000	0.351	17.061

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	125	152	190	124	0	138	170
N.S.	1	1.00	1.13	1.37	1.71	1.12	0.00	1.24	1.53
time (sec)	N/A	0.344	2.553	0.917	0.227	0.284	0.000	0.341	17.799

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	96	133	175	111	0	122	141
N.S.	1	1.00	1.00	1.39	1.82	1.16	0.00	1.27	1.47
time (sec)	N/A	0.306	2.181	0.821	0.216	0.279	0.000	0.337	16.528

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	98	73	89	116	110	0	116	117
N.S.	1	1.08	0.80	0.98	1.27	1.21	0.00	1.27	1.29
time (sec)	N/A	0.571	2.121	0.658	0.211	0.278	0.000	0.291	13.514

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	272	91	110	111	0	129	115
N.S.	1	1.00	3.73	1.25	1.51	1.52	0.00	1.77	1.58
time (sec)	N/A	0.284	5.277	0.635	0.208	0.329	0.000	0.340	13.418

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	241	82	85	105	0	129	117
N.S.	1	1.00	3.30	1.12	1.16	1.44	0.00	1.77	1.60
time (sec)	N/A	0.281	4.816	0.529	0.216	0.300	0.000	0.323	13.430

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	91	70	97	80	0	116	93
N.S.	1	1.00	1.25	0.96	1.33	1.10	0.00	1.59	1.27
time (sec)	N/A	0.291	0.556	0.542	0.206	0.282	0.000	0.320	12.834

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	56	55	104	63	0	96	89
N.S.	1	1.00	0.64	0.63	1.20	0.72	0.00	1.10	1.02
time (sec)	N/A	0.307	0.144	0.618	0.220	0.272	0.000	0.326	16.639

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	63	66	128	76	0	112	105
N.S.	1	1.00	0.62	0.65	1.25	0.75	0.00	1.10	1.03
time (sec)	N/A	0.330	0.137	0.727	0.221	0.261	0.000	0.321	16.863

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	73	75	165	89	0	128	121
N.S.	1	1.00	0.57	0.59	1.30	0.70	0.00	1.01	0.95
time (sec)	N/A	0.355	0.198	0.771	0.208	0.256	0.000	0.316	15.798

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	83	88	187	102	0	144	137
N.S.	1	1.00	0.56	0.60	1.27	0.69	0.00	0.98	0.93
time (sec)	N/A	0.404	0.252	0.817	0.292	0.261	0.000	0.350	15.888

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	156	170	189	314	150	0	170	228
N.S.	1	1.00	1.09	1.21	2.01	0.96	0.00	1.09	1.46
time (sec)	N/A	0.414	5.067	1.456	0.320	0.282	0.000	0.378	17.089

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	102	83	134	205	124	0	114	96
N.S.	1	0.99	0.81	1.30	1.99	1.20	0.00	1.11	0.93
time (sec)	N/A	0.590	0.498	0.423	0.303	0.256	0.000	0.308	12.773

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	87	72	105	162	112	0	101	95
N.S.	1	1.02	0.85	1.24	1.91	1.32	0.00	1.19	1.12
time (sec)	N/A	0.550	0.305	0.378	0.298	0.263	0.000	0.308	13.158

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	42	74	119	97	0	84	67
N.S.	1	1.00	0.82	1.45	2.33	1.90	0.00	1.65	1.31
time (sec)	N/A	0.446	0.268	0.337	0.297	0.291	0.000	0.298	13.282

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	27	46	75	65	0	54	31
N.S.	1	1.00	0.71	1.21	1.97	1.71	0.00	1.42	0.82
time (sec)	N/A	0.342	0.144	0.271	0.290	0.312	0.000	0.293	13.361

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	17	17	23	22	0	16	16
N.S.	1	1.00	0.77	0.77	1.05	1.00	0.00	0.73	0.73
time (sec)	N/A	0.198	0.030	0.235	0.301	0.301	0.000	0.299	13.358

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	58	23	49	37	0	28	23
N.S.	1	1.00	2.00	0.79	1.69	1.28	0.00	0.97	0.79
time (sec)	N/A	0.187	0.296	0.214	0.361	0.282	0.000	0.301	13.170

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	45	89	31	92	46	0	58	66
N.S.	1	1.02	2.02	0.70	2.09	1.05	0.00	1.32	1.50
time (sec)	N/A	0.351	0.818	0.384	0.338	0.288	0.000	0.301	13.188

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	75	117	43	133	57	0	73	89
N.S.	1	1.01	1.58	0.58	1.80	0.77	0.00	0.99	1.20
time (sec)	N/A	0.445	0.880	0.412	0.362	0.292	0.000	0.270	13.283

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	92	143	54	176	70	0	88	70
N.S.	1	0.98	1.52	0.57	1.87	0.74	0.00	0.94	0.74
time (sec)	N/A	0.467	0.999	0.451	0.399	0.302	0.000	0.282	12.873

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	173	67	217	79	0	101	98
N.S.	1	1.00	1.47	0.57	1.84	0.67	0.00	0.86	0.83
time (sec)	N/A	0.561	1.030	0.490	0.414	0.338	0.000	0.305	15.074

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	129	74	120	190	162	0	122	122
N.S.	1	1.05	0.60	0.98	1.54	1.32	0.00	0.99	0.99
time (sec)	N/A	0.836	0.861	0.400	0.359	0.306	0.000	0.313	13.356

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	97	104	92	145	146	0	106	92
N.S.	1	1.09	1.17	1.03	1.63	1.64	0.00	1.19	1.03
time (sec)	N/A	0.677	0.459	0.387	0.408	0.264	0.000	0.295	13.404

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	70	84	62	98	114	0	77	43
N.S.	1	1.06	1.27	0.94	1.48	1.73	0.00	1.17	0.65
time (sec)	N/A	0.510	0.211	0.287	0.264	0.284	0.000	0.309	13.151

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	54	45	31	46	49	0	31	30
N.S.	1	0.98	0.82	0.56	0.84	0.89	0.00	0.56	0.55
time (sec)	N/A	0.311	0.078	0.243	0.244	0.262	0.000	0.282	13.476

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	54	60	31	47	51	0	31	30
N.S.	1	0.98	1.09	0.56	0.85	0.93	0.00	0.56	0.55
time (sec)	N/A	0.302	0.402	0.252	0.246	0.256	0.000	0.277	13.344

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	61	112	36	72	80	0	50	35
N.S.	1	1.07	1.96	0.63	1.26	1.40	0.00	0.88	0.61
time (sec)	N/A	0.398	0.450	0.268	0.365	0.271	0.000	0.278	13.354

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	80	97	55	118	90	0	79	91
N.S.	1	1.11	1.35	0.76	1.64	1.25	0.00	1.10	1.26
time (sec)	N/A	0.561	0.907	0.414	0.373	0.262	0.000	0.285	13.504

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	116	177	66	164	99	0	95	113
N.S.	1	1.05	1.61	0.60	1.49	0.90	0.00	0.86	1.03
time (sec)	N/A	0.662	1.573	0.474	0.362	0.267	0.000	0.293	13.699

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	136	199	77	207	108	0	108	135
N.S.	1	1.10	1.60	0.62	1.67	0.87	0.00	0.87	1.09
time (sec)	N/A	0.716	1.647	0.509	0.357	0.267	0.000	0.312	13.863

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	176	111	135	211	206	0	139	141
N.S.	1	1.09	0.69	0.83	1.30	1.27	0.00	0.86	0.87
time (sec)	N/A	1.111	1.096	0.433	0.274	0.286	0.000	0.320	12.986

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	138	116	105	165	190	0	122	111
N.S.	1	1.08	0.91	0.82	1.29	1.48	0.00	0.95	0.87
time (sec)	N/A	0.938	0.668	0.361	0.281	0.281	0.000	0.339	12.659

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	115	96	75	119	158	0	94	58
N.S.	1	1.10	0.91	0.71	1.13	1.50	0.00	0.90	0.55
time (sec)	N/A	0.753	0.290	0.267	0.341	0.287	0.000	0.334	12.393

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	85	57	45	67	75	0	46	45
N.S.	1	1.02	0.69	0.54	0.81	0.90	0.00	0.55	0.54
time (sec)	N/A	0.468	0.123	0.281	0.275	0.255	0.000	0.302	12.090

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	87	44	31	47	73	0	31	30
N.S.	1	1.05	0.53	0.37	0.57	0.88	0.00	0.37	0.36
time (sec)	N/A	0.431	0.180	0.324	0.268	0.272	0.000	0.291	13.014

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	87	86	45	67	75	0	46	45
N.S.	1	1.05	1.04	0.54	0.81	0.90	0.00	0.55	0.54
time (sec)	N/A	0.414	0.577	0.242	0.284	0.264	0.000	0.303	13.157

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	98	162	51	92	116	0	68	81
N.S.	1	1.11	1.84	0.58	1.05	1.32	0.00	0.77	0.92
time (sec)	N/A	0.580	0.561	0.242	0.355	0.260	0.000	0.304	13.260

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	114	107	66	137	126	0	96	113
N.S.	1	1.11	1.04	0.64	1.33	1.22	0.00	0.93	1.10
time (sec)	N/A	0.767	1.053	0.534	0.356	0.290	0.000	0.321	13.217

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	161	181	77	184	135	0	113	137
N.S.	1	1.10	1.23	0.52	1.25	0.92	0.00	0.77	0.93
time (sec)	N/A	0.929	2.082	0.498	0.428	0.261	0.000	0.310	13.158

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	193	211	121	148	231	250	0	155	160
N.S.	1	1.09	0.63	0.77	1.20	1.30	0.00	0.80	0.83
time (sec)	N/A	1.417	1.719	0.477	0.284	0.295	0.000	0.334	13.460

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	159	183	128	118	186	234	0	139	130
N.S.	1	1.15	0.81	0.74	1.17	1.47	0.00	0.87	0.82
time (sec)	N/A	1.247	1.186	0.424	0.289	0.274	0.000	0.315	13.254

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	151	108	88	139	202	0	110	83
N.S.	1	1.11	0.79	0.65	1.02	1.49	0.00	0.81	0.61
time (sec)	N/A	1.004	0.497	0.311	0.287	0.270	0.000	0.305	13.360

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	126	69	56	87	99	0	59	58
N.S.	1	1.05	0.58	0.47	0.72	0.82	0.00	0.49	0.48
time (sec)	N/A	0.613	0.194	0.252	0.286	0.246	0.000	0.325	13.149

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	118	56	57	87	99	0	59	58
N.S.	1	1.05	0.50	0.51	0.78	0.88	0.00	0.53	0.52
time (sec)	N/A	0.591	0.326	0.282	0.293	0.247	0.000	0.317	13.870

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	120	56	58	87	99	0	59	58
N.S.	1	1.07	0.50	0.52	0.78	0.88	0.00	0.53	0.52
time (sec)	N/A	0.574	1.550	0.325	0.266	0.272	0.000	0.303	13.975

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	120	112	57	87	99	0	59	58
N.S.	1	1.07	1.00	0.51	0.78	0.88	0.00	0.53	0.52
time (sec)	N/A	0.549	2.523	0.279	0.297	0.261	0.000	0.303	13.458

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	126	224	64	112	152	0	83	102
N.S.	1	1.14	2.02	0.58	1.01	1.37	0.00	0.75	0.92
time (sec)	N/A	0.738	2.527	0.263	0.375	0.269	0.000	0.291	13.578

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	150	117	77	158	162	0	112	137
N.S.	1	1.19	0.93	0.61	1.25	1.29	0.00	0.89	1.09
time (sec)	N/A	1.004	2.765	0.541	0.425	0.264	0.000	0.302	13.711

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	176	194	289	88	204	171	0	128	159
N.S.	1	1.10	1.64	0.50	1.16	0.97	0.00	0.73	0.90
time (sec)	N/A	1.219	3.644	0.568	0.337	0.290	0.000	0.327	13.614

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	200	232	140	131	206	278	0	155	149
N.S.	1	1.16	0.70	0.66	1.03	1.39	0.00	0.78	0.74
time (sec)	N/A	1.552	1.824	0.435	0.259	0.276	0.000	0.339	13.759

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	177	205	123	101	159	246	0	126	99
N.S.	1	1.16	0.69	0.57	0.90	1.39	0.00	0.71	0.56
time (sec)	N/A	1.328	0.737	0.318	0.298	0.284	0.000	0.332	13.564

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	159	167	97	69	107	123	0	72	127
N.S.	1	1.05	0.61	0.43	0.67	0.77	0.00	0.45	0.80
time (sec)	N/A	0.819	0.168	0.286	0.250	0.252	0.000	0.333	13.594

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	159	167	64	57	87	123	0	59	58
N.S.	1	1.05	0.40	0.36	0.55	0.77	0.00	0.37	0.36
time (sec)	N/A	0.826	0.409	0.352	0.278	0.260	0.000	0.348	13.625

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	147	64	45	67	123	0	46	45
N.S.	1	1.06	0.46	0.32	0.48	0.88	0.00	0.33	0.32
time (sec)	N/A	0.754	4.113	0.335	0.316	0.269	0.000	0.343	13.384

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	153	66	57	87	123	0	59	58
N.S.	1	1.07	0.46	0.40	0.61	0.86	0.00	0.41	0.41
time (sec)	N/A	0.718	4.926	0.327	0.270	0.270	0.000	0.342	13.264

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	153	138	71	107	123	0	72	127
N.S.	1	1.07	0.97	0.50	0.75	0.86	0.00	0.50	0.89
time (sec)	N/A	0.684	5.423	0.286	0.269	0.255	0.000	0.340	13.025

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	172	280	77	132	188	0	100	125
N.S.	1	1.19	1.94	0.53	0.92	1.31	0.00	0.69	0.87
time (sec)	N/A	0.960	4.873	0.299	0.392	0.266	0.000	0.297	13.439

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	159	191	127	88	178	198	0	129	159
N.S.	1	1.20	0.80	0.55	1.12	1.25	0.00	0.81	1.00
time (sec)	N/A	1.261	6.164	0.554	0.348	0.274	0.000	0.344	13.610

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	215	240	345	98	224	207	0	145	181
N.S.	1	1.12	1.60	0.46	1.04	0.96	0.00	0.67	0.84
time (sec)	N/A	1.519	6.820	0.569	0.325	0.284	0.000	0.338	13.247

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	138	58	72	0	82	0	0	331
N.S.	1	1.13	0.48	0.59	0.00	0.67	0.00	0.00	2.71
time (sec)	N/A	0.678	0.145	0.981	0.000	0.266	0.000	0.000	17.945

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	97	48	59	0	72	0	0	115
N.S.	1	1.13	0.56	0.69	0.00	0.84	0.00	0.00	1.34
time (sec)	N/A	0.512	0.115	0.868	0.000	0.283	0.000	0.000	16.664

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	36	43	0	60	0	0	108
N.S.	1	1.00	0.64	0.77	0.00	1.07	0.00	0.00	1.93
time (sec)	N/A	0.335	0.105	0.865	0.000	0.274	0.000	0.000	1.384

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	29	33	0	41	0	0	41
N.S.	1	1.00	1.12	1.27	0.00	1.58	0.00	0.00	1.58
time (sec)	N/A	0.212	0.074	0.787	0.000	0.264	0.000	0.000	0.187

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	60	76	146	133	0	0	0
N.S.	1	1.00	1.62	2.05	3.95	3.59	0.00	0.00	0.00
time (sec)	N/A	0.196	0.140	0.830	0.384	0.280	0.000	0.000	0.000

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	79	165	791	242	0	0	0
N.S.	1	1.00	1.27	2.66	12.76	3.90	0.00	0.00	0.00
time (sec)	N/A	0.312	0.224	24.225	0.422	0.304	0.000	0.000	0.000

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	B	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	101	47	185	1059	270	0	0	0
N.S.	1	0.99	0.46	1.81	10.38	2.65	0.00	0.00	0.00
time (sec)	N/A	0.475	0.107	22.844	0.474	0.276	0.000	0.000	0.000

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	142	47	201	1921	290	0	0	0
N.S.	1	1.03	0.34	1.46	13.92	2.10	0.00	0.00	0.00
time (sec)	N/A	0.640	0.100	22.689	0.543	0.283	0.000	0.000	0.000

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	174	183	47	217	6638	310	0	0	0
N.S.	1	1.05	0.27	1.25	38.15	1.78	0.00	0.00	0.00
time (sec)	N/A	0.845	0.098	26.775	0.709	0.281	0.000	0.000	0.000

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	182	70	83	0	98	0	0	429
N.S.	1	1.12	0.43	0.51	0.00	0.60	0.00	0.00	2.65
time (sec)	N/A	0.883	0.511	0.868	0.000	0.271	0.000	0.000	21.192

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	131	60	73	0	87	0	0	346
N.S.	1	1.13	0.52	0.63	0.00	0.75	0.00	0.00	2.98
time (sec)	N/A	0.677	0.184	0.831	0.000	0.271	0.000	0.000	17.842

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	91	48	59	0	74	0	0	116
N.S.	1	1.06	0.56	0.69	0.00	0.86	0.00	0.00	1.35
time (sec)	N/A	0.455	0.137	0.805	0.000	0.272	0.000	0.000	17.185

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	38	44	0	61	0	0	111
N.S.	1	1.00	0.64	0.75	0.00	1.03	0.00	0.00	1.88
time (sec)	N/A	0.319	0.097	0.887	0.000	0.267	0.000	0.000	1.246

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	75	160	997	235	0	0	0
N.S.	1	1.00	1.14	2.42	15.11	3.56	0.00	0.00	0.00
time (sec)	N/A	0.292	0.274	1.293	0.416	0.292	0.000	0.000	0.000

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	94	82	168	803	248	0	0	0
N.S.	1	1.45	1.26	2.58	12.35	3.82	0.00	0.00	0.00
time (sec)	N/A	0.459	0.226	23.595	0.440	0.287	0.000	0.000	0.000

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-1)	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	104	108	186	0	278	0	0	0
N.S.	1	0.98	1.02	1.75	0.00	2.62	0.00	0.00	0.00
time (sec)	N/A	0.495	0.383	23.843	0.000	0.281	0.000	0.000	0.000

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-1)	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	145	120	202	0	300	0	0	0
N.S.	1	1.01	0.83	1.40	0.00	2.08	0.00	0.00	0.00
time (sec)	N/A	0.672	0.546	26.059	0.000	0.296	0.000	0.000	0.000

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-1)	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	203	226	80	95	0	121	0	0	542
N.S.	1	1.11	0.39	0.47	0.00	0.60	0.00	0.00	2.67
time (sec)	N/A	1.157	0.228	145.824	0.000	0.279	0.000	0.000	24.757

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	165	70	85	0	108	0	0	456
N.S.	1	1.13	0.48	0.58	0.00	0.74	0.00	0.00	3.12
time (sec)	N/A	0.839	0.478	47.744	0.000	0.270	0.000	0.000	21.984

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	125	60	75	0	95	0	0	349
N.S.	1	1.08	0.52	0.65	0.00	0.82	0.00	0.00	3.01
time (sec)	N/A	0.615	0.189	5.284	0.000	0.264	0.000	0.000	17.263

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	93	50	62	0	82	0	0	146
N.S.	1	1.04	0.56	0.70	0.00	0.92	0.00	0.00	1.64
time (sec)	N/A	0.446	0.101	1.906	0.000	0.271	0.000	0.000	17.419

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	B	A	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	98	102	360	172	1395	310	0	0	0
N.S.	1	1.04	3.67	1.76	14.23	3.16	0.00	0.00	0.00
time (sec)	N/A	0.526	6.448	1.530	0.542	0.294	0.000	0.000	0.000

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	97	82	178	1383	276	0	0	0
N.S.	1	1.03	0.87	1.89	14.71	2.94	0.00	0.00	0.00
time (sec)	N/A	0.492	0.348	32.647	0.545	0.285	0.000	0.000	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-1)	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	108	150	188	0	294	0	0	0
N.S.	1	1.02	1.42	1.77	0.00	2.77	0.00	0.00	0.00
time (sec)	N/A	0.527	0.567	58.659	0.000	0.290	0.000	0.000	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-1)	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	148	151	204	0	320	0	0	0
N.S.	1	1.03	1.05	1.42	0.00	2.22	0.00	0.00	0.00
time (sec)	N/A	0.723	0.839	157.072	0.000	0.290	0.000	0.000	0.000

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-1)	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	182	189	161	220	0	346	0	0	0
N.S.	1	1.04	0.88	1.21	0.00	1.90	0.00	0.00	0.00
time (sec)	N/A	0.914	0.820	0.939	0.000	0.299	0.000	0.000	0.000

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	30	35	0	44	0	57	36
N.S.	1	1.00	1.11	1.30	0.00	1.63	0.00	2.11	1.33
time (sec)	N/A	0.212	0.179	0.855	0.000	0.265	0.000	0.559	13.589

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	B	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	85	73	146	182	0	65	0
N.S.	1	1.00	2.24	1.92	3.84	4.79	0.00	1.71	0.00
time (sec)	N/A	0.205	0.621	0.899	0.423	0.301	0.000	0.563	0.000

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	77	82	791	294	0	134	0
N.S.	1	1.00	1.18	1.26	12.17	4.52	0.00	2.06	0.00
time (sec)	N/A	0.338	0.315	22.382	0.478	0.324	0.000	0.420	0.000

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	154	106	193	0	347	0	150	0
N.S.	1	1.10	0.76	1.38	0.00	2.48	0.00	1.07	0.00
time (sec)	N/A	0.795	0.227	1.220	0.000	0.305	0.000	1.037	0.000

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	112	86	155	0	316	0	113	0
N.S.	1	1.08	0.83	1.49	0.00	3.04	0.00	1.09	0.00
time (sec)	N/A	0.518	0.157	1.052	0.000	0.306	0.000	1.003	0.000

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	83	112	0	262	0	108	0
N.S.	1	1.00	1.14	1.53	0.00	3.59	0.00	1.48	0.00
time (sec)	N/A	0.340	0.088	0.970	0.000	0.299	0.000	0.993	0.000

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	64	93	0	158	0	59	0
N.S.	1	1.00	1.39	2.02	0.00	3.43	0.00	1.28	0.00
time (sec)	N/A	0.221	0.056	0.879	0.000	0.306	0.000	0.653	0.000

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	130	136	698	294	0	0	0
N.S.	1	1.00	1.53	1.60	8.21	3.46	0.00	0.00	0.00
time (sec)	N/A	0.418	0.799	0.862	0.682	0.308	0.000	0.000	0.000

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	109	105	175	0	417	0	326	0
N.S.	1	1.01	0.97	1.62	0.00	3.86	0.00	3.02	0.00
time (sec)	N/A	0.519	0.140	23.940	0.000	0.309	0.000	1.013	0.000

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	159	118	343	0	446	0	423	0
N.S.	1	1.08	0.80	2.33	0.00	3.03	0.00	2.88	0.00
time (sec)	N/A	0.849	0.274	25.804	0.000	0.342	0.000	1.015	0.000

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	183	199	124	215	0	414	0	210	0
N.S.	1	1.09	0.68	1.17	0.00	2.26	0.00	1.15	0.00
time (sec)	N/A	1.065	0.439	1.190	0.000	0.318	0.000	1.004	0.000

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	154	114	193	0	387	0	179	0
N.S.	1	1.06	0.79	1.33	0.00	2.67	0.00	1.23	0.00
time (sec)	N/A	0.787	0.327	0.979	0.000	0.299	0.000	1.185	0.000

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	109	104	145	0	336	0	157	0
N.S.	1	1.04	0.99	1.38	0.00	3.20	0.00	1.50	0.00
time (sec)	N/A	0.525	0.332	0.964	0.000	0.330	0.000	1.150	0.000

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	94	149	0	329	0	108	0
N.S.	1	1.00	1.22	1.94	0.00	4.27	0.00	1.40	0.00
time (sec)	N/A	0.378	0.210	0.847	0.000	0.306	0.000	1.035	0.000

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	93	149	0	327	0	108	0
N.S.	1	1.00	1.21	1.94	0.00	4.25	0.00	1.40	0.00
time (sec)	N/A	0.341	0.120	0.858	0.000	0.327	0.000	0.745	0.000

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	120	181	199	0	491	0	47	0
N.S.	1	1.05	1.59	1.75	0.00	4.31	0.00	0.41	0.00
time (sec)	N/A	0.546	2.615	0.913	0.000	0.340	0.000	0.509	0.000

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	144	152	129	450	0	518	0	387	0
N.S.	1	1.06	0.90	3.12	0.00	3.60	0.00	2.69	0.00
time (sec)	N/A	0.812	0.850	26.794	0.000	0.342	0.000	1.165	0.000

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	185	200	177	511	0	536	0	473	0
N.S.	1	1.08	0.96	2.76	0.00	2.90	0.00	2.56	0.00
time (sec)	N/A	1.133	1.741	22.391	0.000	0.388	0.000	1.757	0.000

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	183	199	135	215	0	455	0	219	0
N.S.	1	1.09	0.74	1.17	0.00	2.49	0.00	1.20	0.00
time (sec)	N/A	1.090	1.353	1.119	0.000	0.306	0.000	1.097	0.000

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	155	125	168	0	404	0	188	0
N.S.	1	1.07	0.86	1.16	0.00	2.79	0.00	1.30	0.00
time (sec)	N/A	0.822	0.868	1.107	0.000	0.299	0.000	1.047	0.000

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	113	116	197	0	403	0	139	0
N.S.	1	1.06	1.08	1.84	0.00	3.77	0.00	1.30	0.00
time (sec)	N/A	0.541	0.677	1.093	0.000	0.313	0.000	1.148	0.000

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	112	115	192	0	399	0	139	0
N.S.	1	1.05	1.07	1.79	0.00	3.73	0.00	1.30	0.00
time (sec)	N/A	0.485	0.663	0.851	0.000	0.300	0.000	1.200	0.000

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	112	52	192	0	403	0	139	0
N.S.	1	1.05	0.49	1.79	0.00	3.77	0.00	1.30	0.00
time (sec)	N/A	0.468	0.068	0.965	0.000	0.325	0.000	0.858	0.000

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	A	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	144	156	211	247	0	585	0	78	0
N.S.	1	1.08	1.47	1.72	0.00	4.06	0.00	0.54	0.00
time (sec)	N/A	0.729	6.453	1.014	0.000	0.388	0.000	0.655	0.000

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	174	190	169	473	0	606	0	424	0
N.S.	1	1.09	0.97	2.72	0.00	3.48	0.00	2.44	0.00
time (sec)	N/A	1.093	1.964	22.851	0.000	0.380	0.000	1.259	0.000

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	62	83	0	161	0	67	0
N.S.	1	1.00	1.29	1.73	0.00	3.35	0.00	1.40	0.00
time (sec)	N/A	0.230	0.192	0.942	0.000	0.316	0.000	0.366	0.000

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	C	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	127	109	698	301	0	69	0
N.S.	1	1.00	1.46	1.25	8.02	3.46	0.00	0.79	0.00
time (sec)	N/A	0.395	0.739	0.886	0.929	0.282	0.000	0.566	0.000

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	383	421	105	0	0	0	0	0	0
N.S.	1	1.10	0.27	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.791	0.281	0.000	0.000	0.000	0.000	0.000	0.000

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	353	381	85	0	0	0	0	0	0
N.S.	1	1.08	0.24	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.586	0.139	0.000	0.000	0.000	0.000	0.000	0.000

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	326	351	66	0	0	0	0	0	0
N.S.	1	1.08	0.20	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.431	0.057	0.000	0.000	0.000	0.000	0.000	0.000

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	77	77	691	0	0	0	0	0	0
N.S.	1	1.00	8.97	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.330	5.215	0.000	0.000	0.000	0.000	0.000	0.000

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	77	77	2700	0	0	0	0	0	0
N.S.	1	1.00	35.06	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.386	16.050	0.000	0.000	0.000	0.000	0.000	0.000

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	413	458	96	0	0	0	0	0	0
N.S.	1	1.11	0.23	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.794	0.367	0.000	0.000	0.000	0.000	0.000	0.000

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	383	417	106	0	0	0	0	0	0
N.S.	1	1.09	0.28	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.575	0.482	0.000	0.000	0.000	0.000	0.000	0.000

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	356	387	66	0	0	0	0	0	0
N.S.	1	1.09	0.19	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.434	0.095	0.000	0.000	0.000	0.000	0.000	0.000

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	86	86	2694	0	0	0	0	0	0
N.S.	1	1.00	31.33	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.329	15.934	0.000	0.000	0.000	0.000	0.000	0.000

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	86	86	2700	0	0	0	0	0	0
N.S.	1	1.00	31.40	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.398	16.010	0.000	0.000	0.000	0.000	0.000	0.000

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	371	390	155	0	0	0	0	0	0
N.S.	1	1.05	0.42	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.047	0.399	0.000	0.000	0.000	0.000	0.000	0.000

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	336	346	95	0	0	0	0	0	0
N.S.	1	1.03	0.28	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.752	0.205	0.000	0.000	0.000	0.000	0.000	0.000

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	306	306	85	0	0	0	0	0	0
N.S.	1	1.00	0.28	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.553	0.120	0.000	0.000	0.000	0.000	0.000	0.000

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	276	276	65	0	0	0	0	0	0
N.S.	1	1.00	0.24	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.414	0.070	0.000	0.000	0.000	0.000	0.000	0.000

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	75	75	718	0	0	0	0	0	0
N.S.	1	1.00	9.57	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.327	4.781	0.000	0.000	0.000	0.000	0.000	0.000

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	75	75	240	0	0	0	0	0	0
N.S.	1	1.00	3.20	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.381	1.896	0.000	0.000	0.000	0.000	0.000	0.000

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	766	781	111	0	0	0	0	0	0
N.S.	1	1.02	0.14	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.370	0.594	0.000	0.000	0.000	0.000	0.000	0.000

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	731	734	98	0	0	0	0	0	0
N.S.	1	1.00	0.13	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.989	0.291	0.000	0.000	0.000	0.000	0.000	0.000

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	731	736	90	0	0	0	0	0	0
N.S.	1	1.01	0.12	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.813	0.344	0.000	0.000	0.000	0.000	0.000	0.000

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	744	741	68	0	0	0	0	0	0
N.S.	1	1.00	0.09	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.693	0.072	0.000	0.000	0.000	0.000	0.000	0.000

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	No	No	N/A	TBD	TBD	TBD	TBD	TBD
size	90	100	3007	0	0	0	0	0	0
N.S.	1	1.11	33.41	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.350	17.580	0.000	0.000	0.000	0.000	0.000	0.000

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	No	No	N/A	TBD	TBD	TBD	TBD	TBD
size	90	100	3011	0	0	0	0	0	0
N.S.	1	1.11	33.46	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.423	16.202	0.000	0.000	0.000	0.000	0.000	0.000

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	153	115	384	0	188	0	0	0
N.S.	1	1.01	0.76	2.54	0.00	1.25	0.00	0.00	0.00
time (sec)	N/A	0.644	0.437	24.592	0.000	0.121	0.000	0.000	0.000

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	125	83	368	0	167	0	0	0
N.S.	1	1.02	0.67	2.99	0.00	1.36	0.00	0.00	0.00
time (sec)	N/A	0.504	0.333	19.537	0.000	0.110	0.000	0.000	0.000

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	98	68	148	0	124	0	0	0
N.S.	1	1.01	0.70	1.53	0.00	1.28	0.00	0.00	0.00
time (sec)	N/A	0.458	0.258	15.668	0.000	0.095	0.000	0.000	0.000

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	49	150	0	107	0	0	0
N.S.	1	1.00	0.65	2.00	0.00	1.43	0.00	0.00	0.00
time (sec)	N/A	0.366	0.187	12.253	0.000	0.097	0.000	0.000	0.000

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	102	73	225	0	125	0	0	0
N.S.	1	1.01	0.72	2.23	0.00	1.24	0.00	0.00	0.00
time (sec)	N/A	0.464	0.257	18.041	0.000	0.099	0.000	0.000	0.000

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	129	93	219	0	145	0	0	0
N.S.	1	1.02	0.73	1.72	0.00	1.14	0.00	0.00	0.00
time (sec)	N/A	0.497	0.316	16.012	0.000	0.099	0.000	0.000	0.000

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	157	103	270	0	156	0	0	0
N.S.	1	1.04	0.68	1.79	0.00	1.03	0.00	0.00	0.00
time (sec)	N/A	0.620	0.458	19.473	0.000	0.102	0.000	0.000	0.000

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	187	187	287	439	0	215	0	0	0
N.S.	1	1.00	1.53	2.35	0.00	1.15	0.00	0.00	0.00
time (sec)	N/A	1.016	3.224	36.247	0.000	0.103	0.000	0.000	0.000

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	159	269	386	0	202	0	0	0
N.S.	1	0.99	1.67	2.40	0.00	1.25	0.00	0.00	0.00
time (sec)	N/A	0.855	2.063	26.277	0.000	0.103	0.000	0.000	0.000

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	264	371	0	179	0	0	0
N.S.	1	1.00	2.02	2.83	0.00	1.37	0.00	0.00	0.00
time (sec)	N/A	0.729	1.712	18.735	0.000	0.097	0.000	0.000	0.000

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	48	185	0	77	0	0	0
N.S.	1	1.00	0.75	2.89	0.00	1.20	0.00	0.00	0.00
time (sec)	N/A	0.427	1.028	14.961	0.000	0.102	0.000	0.000	0.000

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	156	228	0	134	0	0	0
N.S.	1	1.00	1.46	2.13	0.00	1.25	0.00	0.00	0.00
time (sec)	N/A	0.602	1.605	15.163	0.000	0.100	0.000	0.000	0.000

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	136	250	0	157	0	0	0
N.S.	1	1.00	1.01	1.85	0.00	1.16	0.00	0.00	0.00
time (sec)	N/A	0.743	1.997	19.771	0.000	0.107	0.000	0.000	0.000

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	163	149	272	0	170	0	0	0
N.S.	1	1.01	0.93	1.69	0.00	1.06	0.00	0.00	0.00
time (sec)	N/A	0.855	2.328	24.691	0.000	0.110	0.000	0.000	0.000

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	187	187	287	439	0	215	0	0	0
N.S.	1	1.00	1.53	2.35	0.00	1.15	0.00	0.00	0.00
time (sec)	N/A	0.403	3.450	38.063	0.000	0.121	0.000	0.000	0.000

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	157	267	386	0	200	0	0	0
N.S.	1	1.00	1.70	2.46	0.00	1.27	0.00	0.00	0.00
time (sec)	N/A	0.372	3.065	29.166	0.000	0.110	0.000	0.000	0.000

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	187	371	0	179	0	0	0
N.S.	1	1.00	1.43	2.83	0.00	1.37	0.00	0.00	0.00
time (sec)	N/A	0.348	2.535	22.083	0.000	0.107	0.000	0.000	0.000

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	169	172	0	148	0	0	0
N.S.	1	1.00	1.29	1.31	0.00	1.13	0.00	0.00	0.00
time (sec)	N/A	0.337	2.311	20.019	0.000	0.101	0.000	0.000	0.000

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	171	250	0	156	0	0	0
N.S.	1	1.00	1.31	1.91	0.00	1.19	0.00	0.00	0.00
time (sec)	N/A	0.340	2.132	18.329	0.000	0.103	0.000	0.000	0.000

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	146	272	0	170	0	0	0
N.S.	1	1.00	0.91	1.69	0.00	1.06	0.00	0.00	0.00
time (sec)	N/A	0.372	2.452	25.944	0.000	0.111	0.000	0.000	0.000

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	187	187	156	260	0	183	0	0	0
N.S.	1	1.00	0.83	1.39	0.00	0.98	0.00	0.00	0.00
time (sec)	N/A	0.396	3.070	34.230	0.000	0.112	0.000	0.000	0.000

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-1)	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	213	213	289	492	0	228	0	0	0
N.S.	1	1.00	1.36	2.31	0.00	1.07	0.00	0.00	0.00
time (sec)	N/A	0.466	7.608	48.240	0.000	0.117	0.000	0.000	0.000

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	187	187	279	439	0	215	0	0	0
N.S.	1	1.00	1.49	2.35	0.00	1.15	0.00	0.00	0.00
time (sec)	N/A	0.423	5.842	38.788	0.000	0.109	0.000	0.000	0.000

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	286	386	0	202	0	0	0
N.S.	1	1.00	1.78	2.40	0.00	1.25	0.00	0.00	0.00
time (sec)	N/A	0.388	6.152	32.193	0.000	0.113	0.000	0.000	0.000

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	70	292	0	121	0	0	0
N.S.	1	1.00	0.59	2.47	0.00	1.03	0.00	0.00	0.00
time (sec)	N/A	0.359	2.966	24.692	0.000	0.106	0.000	0.000	0.000

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	159	159	184	194	0	162	0	0	0
N.S.	1	1.00	1.16	1.22	0.00	1.02	0.00	0.00	0.00
time (sec)	N/A	0.379	4.029	27.478	0.000	0.117	0.000	0.000	0.000

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	180	272	0	170	0	0	0
N.S.	1	1.00	1.12	1.69	0.00	1.06	0.00	0.00	0.00
time (sec)	N/A	0.384	3.929	25.629	0.000	0.116	0.000	0.000	0.000

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	187	187	156	260	0	183	0	0	0
N.S.	1	1.00	0.83	1.39	0.00	0.98	0.00	0.00	0.00
time (sec)	N/A	0.413	4.217	37.333	0.000	0.115	0.000	0.000	0.000

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	213	213	306	273	0	196	0	0	0
N.S.	1	1.00	1.44	1.28	0.00	0.92	0.00	0.00	0.00
time (sec)	N/A	0.456	6.021	45.271	0.000	0.119	0.000	0.000	0.000

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	164	168	291	413	0	248	0	0	0
N.S.	1	1.02	1.77	2.52	0.00	1.51	0.00	0.00	0.00
time (sec)	N/A	0.714	4.229	10.854	0.000	0.103	0.000	0.000	0.000

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	140	262	253	0	196	0	0	0
N.S.	1	1.03	1.93	1.86	0.00	1.44	0.00	0.00	0.00
time (sec)	N/A	0.675	2.598	7.785	0.000	0.100	0.000	0.000	0.000

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	116	201	200	0	184	0	0	0
N.S.	1	1.05	1.83	1.82	0.00	1.67	0.00	0.00	0.00
time (sec)	N/A	0.571	1.415	3.937	0.000	0.107	0.000	0.000	0.000

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	115	202	198	0	184	0	0	0
N.S.	1	1.05	1.84	1.80	0.00	1.67	0.00	0.00	0.00
time (sec)	N/A	0.530	1.050	5.028	0.000	0.101	0.000	0.000	0.000

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	116	317	199	0	186	0	0	0
N.S.	1	1.04	2.83	1.78	0.00	1.66	0.00	0.00	0.00
time (sec)	N/A	0.548	2.279	5.837	0.000	0.113	0.000	0.000	0.000

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	144	318	215	0	207	0	0	0
N.S.	1	1.03	2.27	1.54	0.00	1.48	0.00	0.00	0.00
time (sec)	N/A	0.703	4.957	6.577	0.000	0.104	0.000	0.000	0.000

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	168	172	347	229	0	217	0	0	0
N.S.	1	1.02	2.07	1.36	0.00	1.29	0.00	0.00	0.00
time (sec)	N/A	0.777	3.343	6.965	0.000	0.110	0.000	0.000	0.000

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-1)	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	202	211	287	413	0	328	0	0	0
N.S.	1	1.04	1.42	2.04	0.00	1.62	0.00	0.00	0.00
time (sec)	N/A	1.036	4.721	16.442	0.000	0.112	0.000	0.000	0.000

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-1)	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	176	182	252	405	0	278	0	0	0
N.S.	1	1.03	1.43	2.30	0.00	1.58	0.00	0.00	0.00
time (sec)	N/A	0.975	2.493	13.238	0.000	0.109	0.000	0.000	0.000

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	159	242	257	0	277	0	0	0
N.S.	1	1.07	1.62	1.72	0.00	1.86	0.00	0.00	0.00
time (sec)	N/A	0.809	2.362	10.570	0.000	0.103	0.000	0.000	0.000

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	98	188	0	150	0	0	0
N.S.	1	1.00	1.27	2.44	0.00	1.95	0.00	0.00	0.00
time (sec)	N/A	0.372	1.473	4.952	0.000	0.098	0.000	0.000	0.000

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	159	239	257	0	277	0	0	0
N.S.	1	1.07	1.60	1.72	0.00	1.86	0.00	0.00	0.00
time (sec)	N/A	0.810	2.558	7.064	0.000	0.098	0.000	0.000	0.000

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	158	260	257	0	277	0	0	0
N.S.	1	1.04	1.71	1.69	0.00	1.82	0.00	0.00	0.00
time (sec)	N/A	0.814	2.281	7.187	0.000	0.121	0.000	0.000	0.000

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	178	187	257	270	0	287	0	0	0
N.S.	1	1.05	1.44	1.52	0.00	1.61	0.00	0.00	0.00
time (sec)	N/A	0.957	2.924	8.135	0.000	0.129	0.000	0.000	0.000

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	200	214	271	283	0	297	0	0	0
N.S.	1	1.07	1.36	1.42	0.00	1.48	0.00	0.00	0.00
time (sec)	N/A	0.999	3.153	7.920	0.000	0.115	0.000	0.000	0.000

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-1)	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	247	262	378	453	0	404	0	0	0
N.S.	1	1.06	1.53	1.83	0.00	1.64	0.00	0.00	0.00
time (sec)	N/A	1.327	6.220	61.116	0.000	0.109	0.000	0.000	0.000

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-1)	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	221	234	371	555	0	354	0	0	0
N.S.	1	1.06	1.68	2.51	0.00	1.60	0.00	0.00	0.00
time (sec)	N/A	1.303	3.118	57.491	0.000	0.113	0.000	0.000	0.000

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-1)	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	195	210	274	268	0	353	0	0	0
N.S.	1	1.08	1.41	1.37	0.00	1.81	0.00	0.00	0.00
time (sec)	N/A	1.111	5.972	55.448	0.000	0.100	0.000	0.000	0.000

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-1)	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	195	210	371	270	0	353	0	0	0
N.S.	1	1.08	1.90	1.38	0.00	1.81	0.00	0.00	0.00
time (sec)	N/A	1.113	2.899	57.171	0.000	0.108	0.000	0.000	0.000

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-1)	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	195	210	371	270	0	353	0	0	0
N.S.	1	1.08	1.90	1.38	0.00	1.81	0.00	0.00	0.00
time (sec)	N/A	1.104	2.920	7.543	0.000	0.104	0.000	0.000	0.000

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	195	210	272	270	0	353	0	0	0
N.S.	1	1.08	1.39	1.38	0.00	1.81	0.00	0.00	0.00
time (sec)	N/A	1.104	6.515	7.914	0.000	0.109	0.000	0.000	0.000

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	195	210	386	270	0	353	0	0	0
N.S.	1	1.08	1.98	1.38	0.00	1.81	0.00	0.00	0.00
time (sec)	N/A	1.132	3.295	8.131	0.000	0.109	0.000	0.000	0.000

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	221	238	285	283	0	363	0	0	0
N.S.	1	1.08	1.29	1.28	0.00	1.64	0.00	0.00	0.00
time (sec)	N/A	1.316	3.874	9.151	0.000	0.114	0.000	0.000	0.000

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	247	266	297	296	0	373	0	0	0
N.S.	1	1.08	1.20	1.20	0.00	1.51	0.00	0.00	0.00
time (sec)	N/A	1.332	4.512	9.508	0.000	0.130	0.000	0.000	0.000

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	115	108	213	1264	356	0	0	0
N.S.	1	0.99	0.93	1.84	10.90	3.07	0.00	0.00	0.00
time (sec)	N/A	0.533	0.395	1.644	0.439	0.293	0.000	0.000	0.000

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	75	183	662	302	0	0	0
N.S.	1	1.00	1.04	2.54	9.19	4.19	0.00	0.00	0.00
time (sec)	N/A	0.371	0.232	1.707	0.394	0.306	0.000	0.000	0.000

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	54	143	241	189	0	0	0
N.S.	1	1.00	1.46	3.86	6.51	5.11	0.00	0.00	0.00
time (sec)	N/A	0.241	0.124	1.849	0.376	0.297	0.000	0.000	0.000

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	39	41	20	49	0	0	53
N.S.	1	1.00	1.08	1.14	0.56	1.36	0.00	0.00	1.47
time (sec)	N/A	0.232	0.098	1.210	0.347	0.279	0.000	0.000	0.513

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	49	51	113	66	0	0	69
N.S.	1	1.00	0.64	0.66	1.47	0.86	0.00	0.00	0.90
time (sec)	N/A	0.380	0.166	1.210	0.369	0.281	0.000	0.000	13.918

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	120	61	67	203	78	0	0	82
N.S.	1	1.04	0.53	0.58	1.77	0.68	0.00	0.00	0.71
time (sec)	N/A	0.524	0.202	1.222	0.390	0.282	0.000	0.000	14.259

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	163	71	72	293	88	0	0	93
N.S.	1	1.07	0.46	0.47	1.92	0.58	0.00	0.00	0.61
time (sec)	N/A	0.692	0.265	1.330	0.375	0.296	0.000	0.000	14.859

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	160	161	136	236	2361	400	0	0	0
N.S.	1	1.01	0.85	1.48	14.76	2.50	0.00	0.00	0.00
time (sec)	N/A	0.722	0.522	1.803	0.494	0.297	0.000	0.000	0.000

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	118	110	206	2244	370	0	0	0
N.S.	1	0.98	0.92	1.72	18.70	3.08	0.00	0.00	0.00
time (sec)	N/A	0.547	0.491	1.570	0.452	0.293	0.000	0.000	0.000

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	75	174	1143	310	0	0	0
N.S.	1	1.00	1.00	2.32	15.24	4.13	0.00	0.00	0.00
time (sec)	N/A	0.390	0.307	1.636	0.392	0.300	0.000	0.000	0.000

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	86	157	274	307	0	0	0
N.S.	1	1.00	1.13	2.07	3.61	4.04	0.00	0.00	0.00
time (sec)	N/A	0.396	0.355	1.572	0.380	0.305	0.000	0.000	0.000

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	50	52	38	69	0	0	70
N.S.	1	1.00	0.63	0.66	0.48	0.87	0.00	0.00	0.89
time (sec)	N/A	0.368	0.237	1.201	0.366	0.293	0.000	0.000	14.368

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	121	60	66	210	80	0	0	81
N.S.	1	1.04	0.52	0.57	1.81	0.69	0.00	0.00	0.70
time (sec)	N/A	0.543	0.295	1.284	0.370	0.276	0.000	0.000	14.552

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	166	72	73	303	92	0	0	94
N.S.	1	1.03	0.45	0.45	1.88	0.57	0.00	0.00	0.58
time (sec)	N/A	0.711	0.383	1.247	0.402	0.290	0.000	0.000	14.824

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	201	209	80	83	396	103	0	0	105
N.S.	1	1.04	0.40	0.41	1.97	0.51	0.00	0.00	0.52
time (sec)	N/A	0.982	0.554	1.208	0.378	0.268	0.000	0.000	15.704

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	200	207	162	260	3860	446	0	0	0
N.S.	1	1.04	0.81	1.30	19.30	2.23	0.00	0.00	0.00
time (sec)	N/A	0.971	0.759	63.247	0.635	0.304	0.000	0.000	0.000

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	160	164	136	230	3469	420	0	0	0
N.S.	1	1.02	0.85	1.44	21.68	2.62	0.00	0.00	0.00
time (sec)	N/A	0.764	0.545	1.881	0.554	0.300	0.000	0.000	0.000

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	122	106	198	2826	386	0	0	0
N.S.	1	1.02	0.88	1.65	23.55	3.22	0.00	0.00	0.00
time (sec)	N/A	0.598	0.476	1.814	3.178	0.293	0.000	0.000	0.000

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	119	91	300	11494	346	0	0	0
N.S.	1	1.06	0.81	2.68	102.62	3.09	0.00	0.00	0.00
time (sec)	N/A	0.591	0.685	1.659	0.486	0.284	0.000	0.000	0.000

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	122	103	300	593	364	0	0	0
N.S.	1	1.03	0.87	2.54	5.03	3.08	0.00	0.00	0.00
time (sec)	N/A	0.598	0.442	1.823	0.413	0.310	0.000	0.000	0.000

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	123	64	70	60	87	0	0	85
N.S.	1	1.03	0.54	0.59	0.50	0.73	0.00	0.00	0.71
time (sec)	N/A	0.535	0.320	1.406	0.358	0.281	0.000	0.000	14.734

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	165	74	75	323	100	0	0	96
N.S.	1	1.06	0.47	0.48	2.07	0.64	0.00	0.00	0.62
time (sec)	N/A	0.722	0.379	1.494	0.377	0.283	0.000	0.000	14.767

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	201	212	80	85	422	113	0	0	107
N.S.	1	1.05	0.40	0.42	2.10	0.56	0.00	0.00	0.53
time (sec)	N/A	0.982	0.595	1.253	0.392	0.278	0.000	0.000	15.741

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	241	255	90	95	521	126	0	0	356
N.S.	1	1.06	0.37	0.39	2.16	0.52	0.00	0.00	1.48
time (sec)	N/A	1.225	0.401	1.190	0.398	0.290	0.000	0.000	19.515

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	A	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	38	38	51	0	121	50	0	0	55
N.S.	1	1.00	1.34	0.00	3.18	1.32	0.00	0.00	1.45
time (sec)	N/A	0.240	0.530	0.000	0.300	0.291	0.000	0.000	0.757

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	54	141	241	189	0	0	0
N.S.	1	1.00	1.46	3.81	6.51	5.11	0.00	0.00	0.00
time (sec)	N/A	0.237	0.155	1.820	0.372	0.281	0.000	0.000	0.000

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	55	130	353	215	0	98	0
N.S.	1	1.00	1.45	3.42	9.29	5.66	0.00	2.58	0.00
time (sec)	N/A	0.256	0.227	1.721	0.408	0.296	0.000	0.662	0.000

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	137	125	235	876	481	0	0	0
N.S.	1	1.07	0.98	1.84	6.84	3.76	0.00	0.00	0.00
time (sec)	N/A	0.733	0.268	1.717	0.406	0.306	0.000	0.000	0.000

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	89	187	476	351	0	0	0
N.S.	1	1.00	0.94	1.97	5.01	3.69	0.00	0.00	0.00
time (sec)	N/A	0.499	0.100	1.864	0.410	0.284	0.000	0.000	0.000

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	75	98	90	160	0	0	0
N.S.	1	1.00	1.34	1.75	1.61	2.86	0.00	0.00	0.00
time (sec)	N/A	0.244	0.059	1.256	0.347	0.278	0.000	0.000	0.000

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	102	100	104	281	0	0	0
N.S.	1	1.00	1.10	1.08	1.12	3.02	0.00	0.00	0.00
time (sec)	N/A	0.382	0.208	1.085	0.372	0.288	0.000	0.000	0.000

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	139	120	172	282	318	0	0	0
N.S.	1	1.06	0.92	1.31	2.15	2.43	0.00	0.00	0.00
time (sec)	N/A	0.604	0.266	1.339	0.390	0.320	0.000	0.000	0.000

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	169	187	117	194	357	342	0	0	0
N.S.	1	1.11	0.69	1.15	2.11	2.02	0.00	0.00	0.00
time (sec)	N/A	0.900	1.159	1.418	0.436	0.283	0.000	0.000	0.000

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	174	182	252	420	4934	579	0	0	0
N.S.	1	1.05	1.45	2.41	28.36	3.33	0.00	0.00	0.00
time (sec)	N/A	1.023	0.626	1.893	0.671	0.311	0.000	0.000	0.000

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	134	140	220	361	2122	559	0	0	0
N.S.	1	1.04	1.64	2.69	15.84	4.17	0.00	0.00	0.00
time (sec)	N/A	0.748	0.533	1.760	0.443	0.314	0.000	0.000	0.000

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	220	170	15721	338	0	0	0
N.S.	1	1.00	2.27	1.75	162.07	3.48	0.00	0.00	0.00
time (sec)	N/A	0.393	0.517	1.281	0.887	0.285	0.000	0.000	0.000

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	120	170	1031	340	0	0	0
N.S.	1	1.00	1.24	1.75	10.63	3.51	0.00	0.00	0.00
time (sec)	N/A	0.402	0.179	1.392	0.517	0.309	0.000	0.000	0.000

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	139	145	201	7176	378	0	0	0
N.S.	1	1.01	1.06	1.47	52.38	2.76	0.00	0.00	0.00
time (sec)	N/A	0.626	0.502	1.434	0.492	0.308	0.000	0.000	0.000

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	177	187	150	247	33960	398	0	0	0
N.S.	1	1.06	0.85	1.40	191.86	2.25	0.00	0.00	0.00
time (sec)	N/A	0.894	0.975	1.290	0.928	0.313	0.000	0.000	0.000

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	217	235	163	263	0	418	0	0	0
N.S.	1	1.08	0.75	1.21	0.00	1.93	0.00	0.00	0.00
time (sec)	N/A	1.164	1.351	1.469	0.000	0.301	0.000	0.000	0.000

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	214	230	340	669	9048	667	0	0	0
N.S.	1	1.07	1.59	3.13	42.28	3.12	0.00	0.00	0.00
time (sec)	N/A	1.368	1.301	1.845	2.721	0.351	0.000	0.000	0.000

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	174	186	308	410	4988	665	0	0	0
N.S.	1	1.07	1.77	2.36	28.67	3.82	0.00	0.00	0.00
time (sec)	N/A	1.025	0.710	1.626	0.600	0.338	0.000	0.000	0.000

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	142	308	304	84332	426	0	0	0
N.S.	1	1.04	2.25	2.22	615.56	3.11	0.00	0.00	0.00
time (sec)	N/A	0.561	0.708	1.255	14.466	0.317	0.000	0.000	0.000

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	142	204	304	2875	422	0	0	0
N.S.	1	1.04	1.49	2.22	20.99	3.08	0.00	0.00	0.00
time (sec)	N/A	0.569	3.411	1.343	0.574	0.302	0.000	0.000	0.000

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	143	146	277	3049	426	0	0	0
N.S.	1	1.04	1.07	2.02	22.26	3.11	0.00	0.00	0.00
time (sec)	N/A	0.629	1.004	1.474	0.635	0.289	0.000	0.000	0.000

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	177	187	186	223	258456	446	0	0	0
N.S.	1	1.06	1.05	1.26	1460.20	2.52	0.00	0.00	0.00
time (sec)	N/A	0.891	1.394	1.439	2.872	0.309	0.000	0.000	0.000

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	217	235	165	361	148823	466	0	0	0
N.S.	1	1.08	0.76	1.66	685.82	2.15	0.00	0.00	0.00
time (sec)	N/A	1.183	2.651	1.395	2.746	0.321	0.000	0.000	0.000

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	132	140	262	1643	337	0	0	0
N.S.	1	1.05	1.11	2.08	13.04	2.67	0.00	0.00	0.00
time (sec)	N/A	0.817	0.435	1.631	0.439	0.301	0.000	0.000	0.000

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	91	111	231	873	297	0	0	0
N.S.	1	1.07	1.31	2.72	10.27	3.49	0.00	0.00	0.00
time (sec)	N/A	0.602	0.357	1.513	0.456	0.291	0.000	0.000	0.000

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	76	182	473	223	0	0	0
N.S.	1	1.00	1.41	3.37	8.76	4.13	0.00	0.00	0.00
time (sec)	N/A	0.406	0.106	1.719	0.429	0.286	0.000	0.000	0.000

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	40	93	87	88	0	0	0
N.S.	1	1.00	1.48	3.44	3.22	3.26	0.00	0.00	0.00
time (sec)	N/A	0.215	0.032	1.290	0.392	0.282	0.000	0.000	0.000

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	90	95	101	144	0	0	0
N.S.	1	1.00	1.45	1.53	1.63	2.32	0.00	0.00	0.00
time (sec)	N/A	0.328	0.299	0.967	0.380	0.296	0.000	0.000	0.000

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	102	118	167	279	163	0	0	0
N.S.	1	1.04	1.20	1.70	2.85	1.66	0.00	0.00	0.00
time (sec)	N/A	0.522	0.246	1.138	0.407	0.283	0.000	0.000	0.000

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	142	122	189	354	174	0	0	0
N.S.	1	1.06	0.91	1.41	2.64	1.30	0.00	0.00	0.00
time (sec)	N/A	0.724	0.299	1.274	0.517	0.284	0.000	0.000	0.000

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	325	337	71	0	0	0	0	0	0
N.S.	1	1.04	0.22	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.407	0.254	0.000	0.000	0.000	0.000	0.000	0.000

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	280	280	71	0	0	0	0	0	0
N.S.	1	1.00	0.25	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.349	0.143	0.000	0.000	0.000	0.000	0.000	0.000

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	326	343	71	0	0	0	0	0	0
N.S.	1	1.05	0.22	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.381	0.159	0.000	0.000	0.000	0.000	0.000	0.000

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F(-1)	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	716	708	71	0	0	0	0	0	0
N.S.	1	0.99	0.10	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.649	0.232	0.000	0.000	0.000	0.000	0.000	0.000

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F(-1)	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	673	668	71	0	0	0	0	0	0
N.S.	1	0.99	0.11	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.606	0.214	0.000	0.000	0.000	0.000	0.000	0.000

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	624	628	71	0	0	0	0	0	0
N.S.	1	1.01	0.11	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.596	0.145	0.000	0.000	0.000	0.000	0.000	0.000

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	662	672	71	0	0	0	0	0	0
N.S.	1	1.02	0.11	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.625	0.158	0.000	0.000	0.000	0.000	0.000	0.000

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	715	717	71	0	0	0	0	0	0
N.S.	1	1.00	0.10	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.640	0.187	0.000	0.000	0.000	0.000	0.000	0.000

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	78	78	760	0	0	0	0	0	0
N.S.	1	1.00	9.74	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.437	7.123	0.000	0.000	0.000	0.000	0.000	0.000

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	76	76	749	0	0	0	0	0	0
N.S.	1	1.00	9.86	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.430	7.804	0.000	0.000	0.000	0.000	0.000	0.000

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	No	No	N/A	TBD	TBD	TBD	TBD	TBD
size	76	64	3346	0	0	0	0	0	0
N.S.	1	0.84	44.03	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.455	19.018	0.000	0.000	0.000	0.000	0.000	0.000

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	No	No	N/A	TBD	TBD	TBD	TBD	TBD
size	78	72	585	0	0	0	0	0	0
N.S.	1	0.92	7.50	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.464	7.845	0.000	0.000	0.000	0.000	0.000	0.000

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	78	78	1982	0	0	0	0	0	0
N.S.	1	1.00	25.41	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.404	23.473	0.000	0.000	0.000	0.000	0.000	0.000

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	79	79	1965	0	0	0	0	0	0
N.S.	1	1.00	24.87	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.385	27.271	0.000	0.000	0.000	0.000	0.000	0.000

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	327	79	274	0	0	0	0	0	0
N.S.	1	0.24	0.84	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.395	9.274	0.000	0.000	0.000	0.000	0.000	0.000

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	80	80	2325	0	0	0	0	0	0
N.S.	1	1.00	29.06	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.424	39.455	0.000	0.000	0.000	0.000	0.000	0.000

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	304	305	208	0	0	0	0	0	0
N.S.	1	1.00	0.68	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.402	5.718	0.000	0.000	0.000	0.000	0.000	0.000

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	230	239	165	0	0	0	0	0	0
N.S.	1	1.04	0.72	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.926	1.500	0.000	0.000	0.000	0.000	0.000	0.000

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	172	175	135	0	0	0	0	0	0
N.S.	1	1.02	0.78	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.754	0.737	0.000	0.000	0.000	0.000	0.000	0.000

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	132	132	106	0	0	0	0	0	0
N.S.	1	1.00	0.80	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.413	0.144	0.000	0.000	0.000	0.000	0.000	0.000

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	174	183	140	0	0	0	0	0	0
N.S.	1	1.05	0.80	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.634	0.799	0.000	0.000	0.000	0.000	0.000	0.000

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	217	221	192	0	0	0	0	0	0
N.S.	1	1.02	0.88	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.962	1.831	0.000	0.000	0.000	0.000	0.000	0.000

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	162	157	398	0	0	0	0	0	0
N.S.	1	0.97	2.46	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.653	31.828	0.000	0.000	0.000	0.000	0.000	0.000

Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	98	98	83	0	0	0	0	0	0
N.S.	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.431	0.449	0.000	0.000	0.000	0.000	0.000	0.000

Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	45	45	45	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.238	0.041	0.000	0.000	0.000	0.000	0.000	0.000

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	59	59	2938	0	0	0	0	0	0
N.S.	1	1.00	49.80	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.295	15.801	0.000	0.000	0.000	0.000	0.000	0.000

Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	62	62	2990	0	0	0	0	0	0
N.S.	1	1.00	48.23	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.289	16.378	0.000	0.000	0.000	0.000	0.000	0.000

Problem 299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	117	117	85	0	0	0	0	0	0
N.S.	1	1.00	0.73	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.444	0.503	0.000	0.000	0.000	0.000	0.000	0.000

Problem 300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	64	64	67	0	0	0	0	0	0
N.S.	1	1.00	1.05	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.237	0.061	0.000	0.000	0.000	0.000	0.000	0.000

Problem 301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	73	73	2951	0	0	0	0	0	0
N.S.	1	1.00	40.42	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.263	6.181	0.000	0.000	0.000	0.000	0.000	0.000

Problem 302	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	73	73	3003	0	0	0	0	0	0
N.S.	1	1.00	41.14	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.275	6.200	0.000	0.000	0.000	0.000	0.000	0.000

Problem 303	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	117	117	85	0	0	0	0	0	0
N.S.	1	1.00	0.73	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.445	0.498	0.000	0.000	0.000	0.000	0.000	0.000

Problem 304	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	64	64	67	0	0	0	0	0	0
N.S.	1	1.00	1.05	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.247	0.066	0.000	0.000	0.000	0.000	0.000	0.000

Problem 305	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	73	73	2951	0	0	0	0	0	0
N.S.	1	1.00	40.42	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.256	6.177	0.000	0.000	0.000	0.000	0.000	0.000

Problem 306	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	73	73	3003	0	0	0	0	0	0
N.S.	1	1.00	41.14	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.271	6.182	0.000	0.000	0.000	0.000	0.000	0.000

Problem 307	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	177	173	400	0	0	0	0	0	0
N.S.	1	0.98	2.26	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.776	3.583	0.000	0.000	0.000	0.000	0.000	0.000

Problem 308	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	108	108	86	0	0	0	0	0	0
N.S.	1	1.00	0.80	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.469	0.445	0.000	0.000	0.000	0.000	0.000	0.000

Problem 309	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	48	48	51	0	0	0	0	0	0
N.S.	1	1.00	1.06	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.263	0.105	0.000	0.000	0.000	0.000	0.000	0.000

Problem 310	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	61	61	2964	0	0	0	0	0	0
N.S.	1	1.00	48.59	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.393	6.213	0.000	0.000	0.000	0.000	0.000	0.000

Problem 311	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	67	67	2992	0	0	0	0	0	0
N.S.	1	1.00	44.66	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.405	6.245	0.000	0.000	0.000	0.000	0.000	0.000

Problem 312	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	130	130	88	0	0	0	0	0	0
N.S.	1	1.00	0.68	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.514	0.505	0.000	0.000	0.000	0.000	0.000	0.000

Problem 313	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	70	70	71	0	0	0	0	0	0
N.S.	1	1.00	1.01	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.275	0.146	0.000	0.000	0.000	0.000	0.000	0.000

Problem 314	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	75	75	2977	0	0	0	0	0	0
N.S.	1	1.00	39.69	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.384	6.234	0.000	0.000	0.000	0.000	0.000	0.000

Problem 315	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	78	78	3005	0	0	0	0	0	0
N.S.	1	1.00	38.53	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.389	6.250	0.000	0.000	0.000	0.000	0.000	0.000

Problem 316	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	130	130	88	0	0	0	0	0	0
N.S.	1	1.00	0.68	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.525	0.477	0.000	0.000	0.000	0.000	0.000	0.000

Problem 317	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	70	70	71	0	0	0	0	0	0
N.S.	1	1.00	1.01	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.272	0.142	0.000	0.000	0.000	0.000	0.000	0.000

Problem 318	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	75	75	2977	0	0	0	0	0	0
N.S.	1	1.00	39.69	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.374	6.229	0.000	0.000	0.000	0.000	0.000	0.000

Problem 319	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	78	78	3005	0	0	0	0	0	0
N.S.	1	1.00	38.53	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.389	6.254	0.000	0.000	0.000	0.000	0.000	0.000

Problem 320	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	178	174	419	0	0	0	0	0	0
N.S.	1	0.98	2.35	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.781	8.956	0.000	0.000	0.000	0.000	0.000	0.000

Problem 321	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	108	108	316	0	0	0	0	0	0
N.S.	1	1.00	2.93	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.491	3.323	0.000	0.000	0.000	0.000	0.000	0.000

Problem 322	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	47	47	50	0	0	0	0	0	0
N.S.	1	1.00	1.06	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.273	0.250	0.000	0.000	0.000	0.000	0.000	0.000

Problem 323	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	58	58	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.423	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 324	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	64	64	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.444	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 325	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	130	130	302	0	0	0	0	0	0
N.S.	1	1.00	2.32	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.519	2.734	0.000	0.000	0.000	0.000	0.000	0.000

Problem 326	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	69	69	68	0	0	0	0	0	0
N.S.	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.272	0.351	0.000	0.000	0.000	0.000	0.000	0.000

Problem 327	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	130	130	316	0	0	0	0	0	0
N.S.	1	1.00	2.43	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.528	1.464	0.000	0.000	0.000	0.000	0.000	0.000

Problem 328	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	69	69	70	0	0	0	0	0	0
N.S.	1	1.00	1.01	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.279	0.316	0.000	0.000	0.000	0.000	0.000	0.000

Problem 329	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	72	72	2246	0	0	0	0	0	0
N.S.	1	1.00	31.19	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.264	14.550	0.000	0.000	0.000	0.000	0.000	0.000

Problem 330	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	89	89	255	0	0	0	0	0	0
N.S.	1	1.00	2.87	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.268	3.222	0.000	0.000	0.000	0.000	0.000	0.000

Problem 331	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	88	88	2248	0	0	0	0	0	0
N.S.	1	1.00	25.55	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.354	6.296	0.000	0.000	0.000	0.000	0.000	0.000

Problem 332	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	90	90	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.383	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 333	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	85	85	2248	0	0	0	0	0	0
N.S.	1	1.00	26.45	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.252	6.242	0.000	0.000	0.000	0.000	0.000	0.000

Problem 334	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	70	70	257	0	0	0	0	0	0
N.S.	1	1.00	3.67	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.254	0.335	0.000	0.000	0.000	0.000	0.000	0.000

Problem 335	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	87	87	2250	0	0	0	0	0	0
N.S.	1	1.00	25.86	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.371	6.308	0.000	0.000	0.000	0.000	0.000	0.000

Problem 336	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	87	87	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.368	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 337	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	79	79	2248	0	0	0	0	0	0
N.S.	1	1.00	28.46	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.261	6.241	0.000	0.000	0.000	0.000	0.000	0.000

Problem 338	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	79	79	257	0	0	0	0	0	0
N.S.	1	1.00	3.25	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.257	0.326	0.000	0.000	0.000	0.000	0.000	0.000

Problem 339	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	95	95	2250	0	0	0	0	0	0
N.S.	1	1.00	23.68	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.370	6.316	0.000	0.000	0.000	0.000	0.000	0.000

Problem 340	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	96	96	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.360	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 341	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	211	209	154	0	0	0	0	0	0
N.S.	1	0.99	0.73	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.026	1.486	0.000	0.000	0.000	0.000	0.000	0.000

Problem 342	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	155	157	123	0	0	0	0	0	0
N.S.	1	1.01	0.79	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.694	0.683	0.000	0.000	0.000	0.000	0.000	0.000

Problem 343	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	107	107	95	0	0	0	0	0	0
N.S.	1	1.00	0.89	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.463	0.227	0.000	0.000	0.000	0.000	0.000	0.000

Problem 344	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	73	73	73	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.321	0.118	0.000	0.000	0.000	0.000	0.000	0.000

Problem 345	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	83	83	711	0	0	0	0	0	0
N.S.	1	1.00	8.57	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.306	7.360	0.000	0.000	0.000	0.000	0.000	0.000

Problem 346	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	84	84	3781	0	0	0	0	0	0
N.S.	1	1.00	45.01	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.346	17.472	0.000	0.000	0.000	0.000	0.000	0.000

Problem 347	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	98	98	2529	0	0	0	0	0	0
N.S.	1	1.00	25.81	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.404	17.668	0.000	0.000	0.000	0.000	0.000	0.000

Problem 348	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	96	96	2225	0	0	0	0	0	0
N.S.	1	1.00	23.18	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.370	16.985	0.000	0.000	0.000	0.000	0.000	0.000

Problem 349	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	96	96	2424	0	0	0	0	0	0
N.S.	1	1.00	25.25	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.399	19.209	0.000	0.000	0.000	0.000	0.000	0.000

Problem 350	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	98	98	3349	0	0	0	0	0	0
N.S.	1	1.00	34.17	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.421	22.329	0.000	0.000	0.000	0.000	0.000	0.000

Problem 351	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	111	117	490	270	0	148	0	0	87
N.S.	1	1.05	4.41	2.43	0.00	1.33	0.00	0.00	0.78
time (sec)	N/A	0.559	6.093	9.436	0.000	0.103	0.000	0.000	13.779

Problem 352	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	87	89	232	219	0	137	0	0	80
N.S.	1	1.02	2.67	2.52	0.00	1.57	0.00	0.00	0.92
time (sec)	N/A	0.456	5.654	6.557	0.000	0.098	0.000	0.000	13.511

Problem 353	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	61	62	222	225	0	125	0	0	53
N.S.	1	1.02	3.64	3.69	0.00	2.05	0.00	0.00	0.87
time (sec)	N/A	0.419	4.887	6.245	0.000	0.094	0.000	0.000	13.745

Problem 354	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	155	150	0	107	0	0	27
N.S.	1	1.00	4.43	4.29	0.00	3.06	0.00	0.00	0.77
time (sec)	N/A	0.343	1.902	4.407	0.000	0.092	0.000	0.000	0.205

Problem 355	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	57	58	209	148	0	156	0	0	60
N.S.	1	1.02	3.67	2.60	0.00	2.74	0.00	0.00	1.05
time (sec)	N/A	0.412	4.980	5.284	0.000	0.089	0.000	0.000	13.268

Problem 356	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	85	444	368	0	175	0	0	87
N.S.	1	1.02	5.35	4.43	0.00	2.11	0.00	0.00	1.05
time (sec)	N/A	0.451	6.128	8.099	0.000	0.097	0.000	0.000	14.233

Problem 357	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	111	113	477	384	0	188	0	0	87
N.S.	1	1.02	4.30	3.46	0.00	1.69	0.00	0.00	0.78
time (sec)	N/A	0.548	6.178	10.769	0.000	0.106	0.000	0.000	13.992

Problem 358	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	135	141	294	437	0	199	0	0	87
N.S.	1	1.04	2.18	3.24	0.00	1.47	0.00	0.00	0.64
time (sec)	N/A	0.581	5.210	12.171	0.000	0.108	0.000	0.000	14.083

Problem 359	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	147	212	548	260	0	175	0	0	136
N.S.	1	1.44	3.73	1.77	0.00	1.19	0.00	0.00	0.93
time (sec)	N/A	1.291	7.192	12.285	0.000	0.102	0.000	0.000	14.277

Problem 360	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	121	184	516	272	0	162	0	0	129
N.S.	1	1.52	4.26	2.25	0.00	1.34	0.00	0.00	1.07
time (sec)	N/A	1.140	6.938	10.937	0.000	0.099	0.000	0.000	13.612

Problem 361	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	95	156	235	250	0	149	0	0	104
N.S.	1	1.64	2.47	2.63	0.00	1.57	0.00	0.00	1.09
time (sec)	N/A	0.978	6.733	8.385	0.000	0.101	0.000	0.000	13.473

Problem 362	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	67	128	224	228	0	134	0	0	59
N.S.	1	1.91	3.34	3.40	0.00	2.00	0.00	0.00	0.88
time (sec)	N/A	0.784	6.231	4.191	0.000	0.104	0.000	0.000	13.529

Problem 363	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	85	112	185	0	97	0	0	82
N.S.	1	1.93	2.55	4.20	0.00	2.20	0.00	0.00	1.86
time (sec)	N/A	0.589	0.422	5.456	0.000	0.098	0.000	0.000	14.285

Problem 364	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	152	111	371	0	187	0	0	109
N.S.	1	1.67	1.22	4.08	0.00	2.05	0.00	0.00	1.20
time (sec)	N/A	0.905	0.366	8.503	0.000	0.100	0.000	0.000	14.353

Problem 365	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	180	114	386	0	202	0	0	114
N.S.	1	1.49	0.94	3.19	0.00	1.67	0.00	0.00	0.94
time (sec)	N/A	1.029	0.484	9.692	0.000	0.107	0.000	0.000	14.150

Problem 366	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	208	114	439	0	215	0	0	114
N.S.	1	1.41	0.78	2.99	0.00	1.46	0.00	0.00	0.78
time (sec)	N/A	1.251	0.636	12.845	0.000	0.097	0.000	0.000	14.355

Problem 367	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	208	548	260	0	175	0	0	206
N.S.	1	1.41	3.73	1.77	0.00	1.19	0.00	0.00	1.40
time (sec)	N/A	0.556	7.341	22.336	0.000	0.107	0.000	0.000	14.624

Problem 368	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	121	182	245	272	0	162	0	0	143
N.S.	1	1.50	2.02	2.25	0.00	1.34	0.00	0.00	1.18
time (sec)	N/A	0.519	5.785	22.638	0.000	0.100	0.000	0.000	14.805

Problem 369	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	91	152	233	250	0	148	0	0	104
N.S.	1	1.67	2.56	2.75	0.00	1.63	0.00	0.00	1.14
time (sec)	N/A	0.467	5.353	19.897	0.000	0.112	0.000	0.000	14.266

Problem 370	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	152	146	172	0	180	0	0	104
N.S.	1	1.67	1.60	1.89	0.00	1.98	0.00	0.00	1.14
time (sec)	N/A	0.470	0.391	6.937	0.000	0.098	0.000	0.000	13.785

Problem 371	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	152	139	371	0	187	0	0	126
N.S.	1	1.67	1.53	4.08	0.00	2.05	0.00	0.00	1.38
time (sec)	N/A	0.464	0.347	8.384	0.000	0.101	0.000	0.000	14.240

Problem 372	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	178	138	386	0	200	0	0	154
N.S.	1	1.52	1.18	3.30	0.00	1.71	0.00	0.00	1.32
time (sec)	N/A	0.485	0.475	10.600	0.000	0.096	0.000	0.000	14.126

Problem 373	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	208	140	439	0	215	0	0	145
N.S.	1	1.41	0.95	2.99	0.00	1.46	0.00	0.00	0.99
time (sec)	N/A	0.523	0.672	12.785	0.000	0.104	0.000	0.000	14.242

Problem 374	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	193	314	229	0	208	0	0	0
N.S.	1	1.51	2.45	1.79	0.00	1.62	0.00	0.00	0.00
time (sec)	N/A	0.868	2.100	8.207	0.000	0.104	0.000	0.000	0.000

Problem 375	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	165	292	215	0	198	0	0	0
N.S.	1	1.65	2.92	2.15	0.00	1.98	0.00	0.00	0.00
time (sec)	N/A	0.822	2.053	6.873	0.000	0.120	0.000	0.000	0.000

Problem 376	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	137	270	199	0	186	0	0	0
N.S.	1	1.90	3.75	2.76	0.00	2.58	0.00	0.00	0.00
time (sec)	N/A	0.716	1.732	5.078	0.000	0.096	0.000	0.000	0.000

Problem 377	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	136	262	198	0	184	0	0	0
N.S.	1	1.94	3.74	2.83	0.00	2.63	0.00	0.00	0.00
time (sec)	N/A	0.692	1.120	4.900	0.000	0.102	0.000	0.000	0.000

Problem 378	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	137	263	200	0	184	0	0	0
N.S.	1	1.96	3.76	2.86	0.00	2.63	0.00	0.00	0.00
time (sec)	N/A	0.701	1.277	3.939	0.000	0.090	0.000	0.000	0.000

Problem 379	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	161	303	253	0	236	0	0	0
N.S.	1	1.68	3.16	2.64	0.00	2.46	0.00	0.00	0.00
time (sec)	N/A	0.818	1.696	6.283	0.000	0.098	0.000	0.000	0.000

Problem 380	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	189	338	413	0	258	0	0	0
N.S.	1	1.52	2.73	3.33	0.00	2.08	0.00	0.00	0.00
time (sec)	N/A	0.881	3.095	9.173	0.000	0.098	0.000	0.000	0.000

Problem 381	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	160	235	148	283	0	288	0	0	0
N.S.	1	1.47	0.92	1.77	0.00	1.80	0.00	0.00	0.00
time (sec)	N/A	1.205	1.015	10.159	0.000	0.129	0.000	0.000	0.000

Problem 382	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	208	136	270	0	278	0	0	0
N.S.	1	1.51	0.99	1.96	0.00	2.01	0.00	0.00	0.00
time (sec)	N/A	1.134	0.665	8.592	0.000	0.107	0.000	0.000	0.000

Problem 383	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	179	128	257	0	268	0	0	0
N.S.	1	1.60	1.14	2.29	0.00	2.39	0.00	0.00	0.00
time (sec)	N/A	0.959	0.529	7.609	0.000	0.101	0.000	0.000	0.000

Problem 384	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	180	114	257	0	268	0	0	0
N.S.	1	1.65	1.05	2.36	0.00	2.46	0.00	0.00	0.00
time (sec)	N/A	0.975	0.392	6.329	0.000	0.104	0.000	0.000	0.000

Problem 385	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	98	71	188	0	150	0	0	0
N.S.	1	1.72	1.25	3.30	0.00	2.63	0.00	0.00	0.00
time (sec)	N/A	0.493	0.306	5.425	0.000	0.101	0.000	0.000	0.000

Problem 386	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	180	312	257	0	268	0	0	0
N.S.	1	1.65	2.86	2.36	0.00	2.46	0.00	0.00	0.00
time (sec)	N/A	0.965	4.431	5.575	0.000	0.095	0.000	0.000	0.000

Problem 387	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-1)	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	203	342	405	0	318	0	0	0
N.S.	1	1.49	2.51	2.98	0.00	2.34	0.00	0.00	0.00
time (sec)	N/A	1.122	6.487	7.252	0.000	0.100	0.000	0.000	0.000

Problem 388	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-1)	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	232	372	413	0	338	0	0	0
N.S.	1	1.43	2.30	2.55	0.00	2.09	0.00	0.00	0.00
time (sec)	N/A	1.200	2.458	11.266	0.000	0.099	0.000	0.000	0.000

Problem 389	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	207	287	175	296	0	364	0	0	0
N.S.	1	1.39	0.85	1.43	0.00	1.76	0.00	0.00	0.00
time (sec)	N/A	1.519	2.219	15.879	0.000	0.118	0.000	0.000	0.000

Problem 390	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	181	259	166	283	0	354	0	0	0
N.S.	1	1.43	0.92	1.56	0.00	1.96	0.00	0.00	0.00
time (sec)	N/A	1.492	1.439	8.740	0.000	0.117	0.000	0.000	0.000

Problem 391	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	231	157	270	0	344	0	0	0
N.S.	1	1.49	1.01	1.74	0.00	2.22	0.00	0.00	0.00
time (sec)	N/A	1.264	1.414	9.358	0.000	0.104	0.000	0.000	0.000

Problem 392	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	231	146	270	0	344	0	0	0
N.S.	1	1.49	0.94	1.74	0.00	2.22	0.00	0.00	0.00
time (sec)	N/A	1.301	1.109	8.440	0.000	0.099	0.000	0.000	0.000

Problem 393	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-1)	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	231	146	270	0	344	0	0	0
N.S.	1	1.49	0.94	1.74	0.00	2.22	0.00	0.00	0.00
time (sec)	N/A	1.248	1.116	7.218	0.000	0.104	0.000	0.000	0.000

Problem 394	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-1)	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	231	139	270	0	344	0	0	0
N.S.	1	1.49	0.90	1.74	0.00	2.22	0.00	0.00	0.00
time (sec)	N/A	1.270	2.395	7.969	0.000	0.100	0.000	0.000	0.000

Problem 395	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-1)	C	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	155	231	603	268	0	344	0	0	0
N.S.	1	1.49	3.89	1.73	0.00	2.22	0.00	0.00	0.00
time (sec)	N/A	1.272	7.229	5.795	0.000	0.104	0.000	0.000	0.000

Problem 396	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-1)	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	181	255	372	555	0	394	0	0	0
N.S.	1	1.41	2.06	3.07	0.00	2.18	0.00	0.00	0.00
time (sec)	N/A	1.531	2.501	9.622	0.000	0.104	0.000	0.000	0.000

Problem 397	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-1)	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	207	283	402	453	0	414	0	0	0
N.S.	1	1.37	1.94	2.19	0.00	2.00	0.00	0.00	0.00
time (sec)	N/A	1.541	2.918	13.145	0.000	0.114	0.000	0.000	0.000

Problem 398	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	184	80	70	293	79	0	0	0
N.S.	1	1.20	0.52	0.46	1.92	0.52	0.00	0.00	0.00
time (sec)	N/A	0.813	0.154	1.768	0.394	0.261	0.000	0.000	0.000

Problem 399	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	141	61	60	203	69	0	0	0
N.S.	1	1.23	0.53	0.52	1.77	0.60	0.00	0.00	0.00
time (sec)	N/A	0.652	0.128	1.807	0.397	0.255	0.000	0.000	0.000

Problem 400	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	98	49	48	113	57	0	0	0
N.S.	1	1.27	0.64	0.62	1.47	0.74	0.00	0.00	0.00
time (sec)	N/A	0.493	0.086	1.790	0.391	0.271	0.000	0.000	0.000

Problem 401	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	39	41	20	49	0	0	0
N.S.	1	1.00	1.08	1.14	0.56	1.36	0.00	0.00	0.00
time (sec)	N/A	0.354	0.075	1.437	0.373	0.261	0.000	0.000	0.000

Problem 402	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	74	135	241	180	0	0	0
N.S.	1	1.00	1.30	2.37	4.23	3.16	0.00	0.00	0.00
time (sec)	N/A	0.368	0.123	2.219	0.393	0.277	0.000	0.000	0.000

Problem 403	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	93	95	173	662	325	0	0	0
N.S.	1	1.01	1.03	1.88	7.20	3.53	0.00	0.00	0.00
time (sec)	N/A	0.510	0.173	2.472	0.420	0.300	0.000	0.000	0.000

Problem 404	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	128	205	1264	355	0	0	0
N.S.	1	1.00	0.94	1.51	9.29	2.61	0.00	0.00	0.00
time (sec)	N/A	0.676	0.283	2.161	0.414	0.291	0.000	0.000	0.000

Problem 405	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	187	72	73	303	84	0	0	0
N.S.	1	1.16	0.45	0.45	1.88	0.52	0.00	0.00	0.00
time (sec)	N/A	0.900	0.233	1.582	0.393	0.297	0.000	0.000	0.000

Problem 406	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	142	60	61	210	72	0	0	0
N.S.	1	1.22	0.52	0.53	1.81	0.62	0.00	0.00	0.00
time (sec)	N/A	0.675	0.160	1.738	0.364	0.255	0.000	0.000	0.000

Problem 407	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	100	50	51	38	61	0	0	0
N.S.	1	1.27	0.63	0.65	0.48	0.77	0.00	0.00	0.00
time (sec)	N/A	0.511	0.125	1.700	0.404	0.269	0.000	0.000	0.000

Problem 408	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	97	81	157	274	298	0	0	0
N.S.	1	1.01	0.84	1.64	2.85	3.10	0.00	0.00	0.00
time (sec)	N/A	0.510	0.113	2.104	0.393	0.296	0.000	0.000	0.000

Problem 409	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	96	92	174	1143	337	0	0	0
N.S.	1	1.01	0.97	1.83	12.03	3.55	0.00	0.00	0.00
time (sec)	N/A	0.532	0.242	2.213	0.404	0.282	0.000	0.000	0.000

Problem 410	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	139	107	206	2244	369	0	0	0
N.S.	1	0.99	0.76	1.47	16.03	2.64	0.00	0.00	0.00
time (sec)	N/A	0.678	0.371	2.112	0.479	0.293	0.000	0.000	0.000

Problem 411	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	180	182	117	236	2361	391	0	0	0
N.S.	1	1.01	0.65	1.31	13.12	2.17	0.00	0.00	0.00
time (sec)	N/A	0.871	0.436	2.391	0.472	0.298	0.000	0.000	0.000

Problem 412	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	201	233	90	85	422	105	0	0	0
N.S.	1	1.16	0.45	0.42	2.10	0.52	0.00	0.00	0.00
time (sec)	N/A	1.106	0.184	1.230	0.390	0.262	0.000	0.000	0.000

Problem 413	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	186	74	75	323	92	0	0	0
N.S.	1	1.19	0.47	0.48	2.07	0.59	0.00	0.00	0.00
time (sec)	N/A	0.847	0.201	0.982	0.387	0.257	0.000	0.000	0.000

Problem 414	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	144	64	65	60	79	0	0	0
N.S.	1	1.21	0.54	0.55	0.50	0.66	0.00	0.00	0.00
time (sec)	N/A	0.689	0.183	1.033	0.373	0.270	0.000	0.000	0.000

Problem 415	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	143	93	306	593	339	0	0	0
N.S.	1	1.04	0.67	2.22	4.30	2.46	0.00	0.00	0.00
time (sec)	N/A	0.732	0.162	2.108	0.401	0.283	0.000	0.000	0.000

Problem 416	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	140	90	322	11494	373	0	0	0
N.S.	1	1.06	0.68	2.44	87.08	2.83	0.00	0.00	0.00
time (sec)	N/A	0.736	0.229	2.160	0.488	0.283	0.000	0.000	0.000

Problem 417	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	143	95	208	2826	385	0	0	0
N.S.	1	1.02	0.68	1.49	20.19	2.75	0.00	0.00	0.00
time (sec)	N/A	0.750	0.455	2.328	3.080	0.283	0.000	0.000	0.000

Problem 418	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	180	185	115	238	3469	411	0	0	0
N.S.	1	1.03	0.64	1.32	19.27	2.28	0.00	0.00	0.00
time (sec)	N/A	0.934	0.506	2.109	0.552	0.291	0.000	0.000	0.000

Problem 419	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	220	228	125	268	3860	437	0	0	0
N.S.	1	1.04	0.57	1.22	17.55	1.99	0.00	0.00	0.00
time (sec)	N/A	1.157	0.623	2.272	0.623	0.309	0.000	0.000	0.000

Problem 420	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	189	208	136	194	357	324	0	0	0
N.S.	1	1.10	0.72	1.03	1.89	1.71	0.00	0.00	0.00
time (sec)	N/A	1.000	0.227	1.653	0.371	0.290	0.000	0.000	0.000

Problem 421	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	160	116	178	282	300	0	0	0
N.S.	1	1.06	0.77	1.18	1.87	1.99	0.00	0.00	0.00
time (sec)	N/A	0.766	0.148	1.871	0.395	0.281	0.000	0.000	0.000

Problem 422	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	114	100	100	104	281	0	0	0
N.S.	1	1.01	0.88	0.88	0.92	2.49	0.00	0.00	0.00
time (sec)	N/A	0.523	0.071	1.431	0.347	0.283	0.000	0.000	0.000

Problem 423	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	76	95	92	90	160	0	0	0
N.S.	1	1.36	1.70	1.64	1.61	2.86	0.00	0.00	0.00
time (sec)	N/A	0.378	0.056	1.868	0.351	0.268	0.000	0.000	0.000

Problem 424	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	116	109	181	476	342	0	0	0
N.S.	1	0.86	0.81	1.34	3.53	2.53	0.00	0.00	0.00
time (sec)	N/A	0.658	0.068	2.129	0.394	0.303	0.000	0.000	0.000

Problem 425	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	168	158	145	222	876	520	0	0	0
N.S.	1	0.94	0.86	1.32	5.21	3.10	0.00	0.00	0.00
time (sec)	N/A	0.910	0.161	2.473	0.409	0.295	0.000	0.000	0.000

Problem 426	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	211	205	178	257	1646	550	0	0	0
N.S.	1	0.97	0.84	1.22	7.80	2.61	0.00	0.00	0.00
time (sec)	N/A	1.188	0.287	2.490	0.429	0.303	0.000	0.000	0.000

Problem 427	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	237	256	152	273	0	400	0	0	0
N.S.	1	1.08	0.64	1.15	0.00	1.69	0.00	0.00	0.00
time (sec)	N/A	1.359	0.708	1.815	0.000	0.304	0.000	0.000	0.000

Problem 428	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	197	208	133	257	33960	380	0	0	0
N.S.	1	1.06	0.68	1.30	172.39	1.93	0.00	0.00	0.00
time (sec)	N/A	1.062	0.488	1.602	0.786	0.279	0.000	0.000	0.000

Problem 429	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	160	138	201	7176	360	0	0	0
N.S.	1	1.02	0.88	1.28	45.71	2.29	0.00	0.00	0.00
time (sec)	N/A	0.781	0.673	1.809	0.433	0.287	0.000	0.000	0.000

Problem 430	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	118	131	164	1031	340	0	0	0
N.S.	1	1.01	1.12	1.40	8.81	2.91	0.00	0.00	0.00
time (sec)	N/A	0.531	0.318	1.795	0.373	0.283	0.000	0.000	0.000

Problem 431	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	118	248	164	15721	338	0	0	0
N.S.	1	1.01	2.12	1.40	134.37	2.89	0.00	0.00	0.00
time (sec)	N/A	0.537	0.648	1.700	0.864	0.275	0.000	0.000	0.000

Problem 432	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	174	161	248	361	2122	550	0	0	0
N.S.	1	0.93	1.43	2.07	12.20	3.16	0.00	0.00	0.00
time (sec)	N/A	0.909	0.572	2.300	0.427	0.292	0.000	0.000	0.000

Problem 433	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	214	203	272	404	4934	614	0	0	0
N.S.	1	0.95	1.27	1.89	23.06	2.87	0.00	0.00	0.00
time (sec)	N/A	1.194	0.468	2.477	0.607	0.302	0.000	0.000	0.000

Problem 434	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	237	256	144	361	148823	448	0	0	0
N.S.	1	1.08	0.61	1.52	627.95	1.89	0.00	0.00	0.00
time (sec)	N/A	1.328	0.785	1.685	2.706	0.268	0.000	0.000	0.000

Problem 435	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	197	208	141	223	258456	428	0	0	0
N.S.	1	1.06	0.72	1.13	1311.96	2.17	0.00	0.00	0.00
time (sec)	N/A	1.048	0.702	1.654	2.749	0.267	0.000	0.000	0.000

Problem 436	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	164	168	277	3049	408	0	0	0
N.S.	1	1.04	1.07	1.76	19.42	2.60	0.00	0.00	0.00
time (sec)	N/A	0.768	0.927	1.558	0.642	0.270	0.000	0.000	0.000

Problem 437	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	163	224	304	2875	408	0	0	0
N.S.	1	1.04	1.43	1.94	18.31	2.60	0.00	0.00	0.00
time (sec)	N/A	0.708	2.682	1.584	0.578	0.277	0.000	0.000	0.000

Problem 438	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	163	328	304	84332	408	0	0	0
N.S.	1	1.04	2.09	1.94	537.15	2.60	0.00	0.00	0.00
time (sec)	N/A	0.706	0.856	1.655	14.835	0.268	0.000	0.000	0.000

Problem 439	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	214	207	328	410	4988	638	0	0	0
N.S.	1	0.97	1.53	1.92	23.31	2.98	0.00	0.00	0.00
time (sec)	N/A	1.193	1.031	2.464	0.615	0.286	0.000	0.000	0.000

Problem 440	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	254	251	360	669	9048	702	0	0	0
N.S.	1	0.99	1.42	2.63	35.62	2.76	0.00	0.00	0.00
time (sec)	N/A	1.527	1.188	2.509	2.557	0.320	0.000	0.000	0.000

Problem 441	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	244	278	155	0	0	0	0	0	0
N.S.	1	1.14	0.64	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.247	0.965	0.000	0.000	0.000	0.000	0.000	0.000

Problem 442	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	179	203	125	0	0	0	0	0	0
N.S.	1	1.13	0.70	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.981	0.391	0.000	0.000	0.000	0.000	0.000	0.000

Problem 443	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	132	137	105	0	0	0	0	0	0
N.S.	1	1.04	0.80	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.442	0.117	0.000	0.000	0.000	0.000	0.000	0.000

Problem 444	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	178	207	165	0	0	0	0	0	0
N.S.	1	1.16	0.93	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.855	0.588	0.000	0.000	0.000	0.000	0.000	0.000

Problem 445	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	215	245	223	0	0	0	0	0	0
N.S.	1	1.14	1.04	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.144	0.882	0.000	0.000	0.000	0.000	0.000	0.000

Problem 446	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	90	59	73	95	99	0	164	152
N.S.	1	1.06	0.69	0.86	1.12	1.16	0.00	1.93	1.79
time (sec)	N/A	0.478	0.185	1.100	0.206	0.277	0.000	0.300	17.067

Problem 447	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	64	60	60	70	88	0	122	109
N.S.	1	1.02	0.95	0.95	1.11	1.40	0.00	1.94	1.73
time (sec)	N/A	0.391	0.112	0.961	0.214	0.281	0.000	0.321	15.392

Problem 448	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	48	47	47	58	74	0	107	85
N.S.	1	1.02	1.00	1.00	1.23	1.57	0.00	2.28	1.81
time (sec)	N/A	0.361	0.011	0.745	0.216	0.274	0.000	0.305	14.160

Problem 449	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	24	30	29	60	37	63	47
N.S.	1	1.00	1.00	1.25	1.21	2.50	1.54	2.62	1.96
time (sec)	N/A	0.279	0.011	0.542	0.205	0.269	2.187	0.299	13.666

Problem 450	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	24	23	36	41	49	57
N.S.	1	1.00	1.00	1.50	1.44	2.25	2.56	3.06	3.56
time (sec)	N/A	0.149	0.002	0.157	0.204	0.279	0.818	0.277	14.546

Problem 451	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	26	16	20	17	17	39	17
N.S.	1	1.00	1.73	1.07	1.33	1.13	1.13	2.60	1.13
time (sec)	N/A	0.227	0.008	0.392	0.197	0.243	0.779	0.282	14.227

Problem 452	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	39	35	32	34	29	0	82	31
N.S.	1	1.03	0.92	0.84	0.89	0.76	0.00	2.16	0.82
time (sec)	N/A	0.283	0.050	0.491	0.207	0.264	0.000	0.298	13.575

Problem 453	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	55	57	44	46	42	0	98	55
N.S.	1	1.02	1.06	0.81	0.85	0.78	0.00	1.81	1.02
time (sec)	N/A	0.323	0.055	0.693	0.201	0.273	0.000	0.283	13.433

Problem 454	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	81	73	57	57	53	0	140	75
N.S.	1	1.07	0.96	0.75	0.75	0.70	0.00	1.84	0.99
time (sec)	N/A	0.383	0.095	0.890	0.203	0.263	0.000	0.288	14.016

Problem 455	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	93	89	69	69	64	0	154	113
N.S.	1	1.01	0.97	0.75	0.75	0.70	0.00	1.67	1.23
time (sec)	N/A	0.386	0.100	0.834	0.214	0.263	0.000	0.305	17.837

Problem 456	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	128	90	110	132	136	0	272	221
N.S.	1	0.95	0.67	0.81	0.98	1.01	0.00	2.01	1.64
time (sec)	N/A	0.728	0.429	1.400	0.200	0.288	0.000	0.309	17.075

Problem 457	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	102	82	111	144	133	0	258	184
N.S.	1	0.93	0.75	1.01	1.31	1.21	0.00	2.35	1.67
time (sec)	N/A	0.564	0.204	1.205	0.208	0.295	0.000	0.334	16.183

Problem 458	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	87	71	74	84	100	0	178	141
N.S.	1	1.09	0.89	0.92	1.05	1.25	0.00	2.22	1.76
time (sec)	N/A	0.550	0.158	1.224	0.203	0.278	0.000	0.307	15.110

Problem 459	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	45	69	80	93	0	129	99
N.S.	1	1.00	0.76	1.17	1.36	1.58	0.00	2.19	1.68
time (sec)	N/A	0.416	0.090	0.805	0.217	0.265	0.000	0.306	13.503

Problem 460	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	32	41	40	74	0	77	181
N.S.	1	1.00	0.97	1.24	1.21	2.24	0.00	2.33	5.48
time (sec)	N/A	0.294	0.071	0.668	0.193	0.262	0.000	0.315	13.496

Problem 461	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	46	43	51	52	0	78	73
N.S.	1	1.00	1.39	1.30	1.55	1.58	0.00	2.36	2.21
time (sec)	N/A	0.335	0.011	0.414	0.200	0.283	0.000	0.294	13.440

Problem 462	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	46	43	47	40	0	96	42
N.S.	1	1.00	0.92	0.86	0.94	0.80	0.00	1.92	0.84
time (sec)	N/A	0.345	0.171	0.389	0.201	0.252	0.000	0.285	13.484

Problem 463	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	66	59	61	60	52	0	153	72
N.S.	1	1.14	1.02	1.05	1.03	0.90	0.00	2.64	1.24
time (sec)	N/A	0.454	0.289	0.639	0.198	0.248	0.000	0.299	13.444

Problem 464	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	93	86	75	82	77	0	224	93
N.S.	1	0.92	0.85	0.74	0.81	0.76	0.00	2.22	0.92
time (sec)	N/A	0.458	0.174	0.880	0.210	0.255	0.000	0.305	13.563

Problem 465	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	113	85	95	94	86	0	247	117
N.S.	1	1.02	0.77	0.86	0.85	0.77	0.00	2.23	1.05
time (sec)	N/A	0.602	0.139	1.172	0.207	0.266	0.000	0.311	13.457

Problem 466	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	189	201	120	148	181	170	0	367	258
N.S.	1	1.06	0.63	0.78	0.96	0.90	0.00	1.94	1.37
time (sec)	N/A	1.176	0.651	1.893	0.206	0.279	0.000	0.354	17.165

Problem 467	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	141	90	125	158	140	0	330	226
N.S.	1	1.08	0.69	0.96	1.22	1.08	0.00	2.54	1.74
time (sec)	N/A	0.870	0.324	1.234	0.209	0.291	0.000	0.336	17.079

Problem 468	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	105	70	96	106	126	0	205	157
N.S.	1	1.06	0.71	0.97	1.07	1.27	0.00	2.07	1.59
time (sec)	N/A	0.652	0.182	1.142	0.202	0.285	0.000	0.325	16.326

Problem 469	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	74	55	82	93	112	0	145	136
N.S.	1	1.01	0.75	1.12	1.27	1.53	0.00	1.99	1.86
time (sec)	N/A	0.261	0.121	0.883	0.196	0.278	0.000	0.294	14.009

Problem 470	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	88	57	66	94	0	131	97
N.S.	1	1.00	1.31	0.85	0.99	1.40	0.00	1.96	1.45
time (sec)	N/A	0.535	0.856	0.791	0.217	0.276	0.000	0.330	13.911

Problem 471	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	80	105	73	76	72	0	137	123
N.S.	1	1.01	1.33	0.92	0.96	0.91	0.00	1.73	1.56
time (sec)	N/A	0.559	0.356	0.542	0.202	0.272	0.000	0.324	13.585

Problem 472	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	102	80	67	73	66	0	170	77
N.S.	1	1.02	0.80	0.67	0.73	0.66	0.00	1.70	0.77
time (sec)	N/A	0.590	0.227	0.671	0.198	0.250	0.000	0.330	13.538

Problem 473	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	117	100	90	95	84	0	297	250
N.S.	1	0.95	0.81	0.73	0.77	0.68	0.00	2.41	2.03
time (sec)	N/A	0.726	0.371	0.853	0.201	0.258	0.000	0.319	16.569

Problem 474	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	160	144	130	113	119	110	0	332	287
N.S.	1	0.90	0.81	0.71	0.74	0.69	0.00	2.08	1.79
time (sec)	N/A	0.746	0.479	1.080	0.213	0.276	0.000	0.317	16.280

Problem 475	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	185	164	159	135	145	132	0	431	350
N.S.	1	0.89	0.86	0.73	0.78	0.71	0.00	2.33	1.89
time (sec)	N/A	0.961	0.504	1.528	0.198	0.277	0.000	0.322	15.667

Problem 476	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	244	258	154	209	275	217	0	592	370
N.S.	1	1.06	0.63	0.86	1.13	0.89	0.00	2.43	1.52
time (sec)	N/A	1.551	0.676	4.206	0.217	0.277	0.000	0.352	18.305

Problem 477	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	179	195	125	162	195	182	0	461	304
N.S.	1	1.09	0.70	0.91	1.09	1.02	0.00	2.58	1.70
time (sec)	N/A	1.180	0.541	3.673	0.212	0.281	0.000	0.334	18.148

Problem 478	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	157	101	147	180	163	0	360	245
N.S.	1	1.08	0.69	1.01	1.23	1.12	0.00	2.47	1.68
time (sec)	N/A	0.950	0.363	1.458	0.203	0.266	0.000	0.337	17.169

Problem 479	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	114	77	109	121	138	0	221	185
N.S.	1	1.07	0.72	1.02	1.13	1.29	0.00	2.07	1.73
time (sec)	N/A	0.425	0.232	1.234	0.212	0.273	0.000	0.329	13.757

Problem 480	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	103	280	96	115	130	0	179	152
N.S.	1	0.99	2.69	0.92	1.11	1.25	0.00	1.72	1.46
time (sec)	N/A	0.795	1.227	0.996	0.196	0.290	0.000	0.330	13.796

Problem 481	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	119	87	90	116	0	170	150
N.S.	1	1.00	1.10	0.81	0.83	1.07	0.00	1.57	1.39
time (sec)	N/A	0.811	0.977	0.901	0.216	0.296	0.000	0.339	14.202

Problem 482	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	116	128	98	102	98	0	212	158
N.S.	1	1.01	1.11	0.85	0.89	0.85	0.00	1.84	1.37
time (sec)	N/A	0.838	0.402	0.765	0.203	0.282	0.000	0.331	13.145

Problem 483	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	153	104	101	109	96	0	318	123
N.S.	1	1.06	0.72	0.70	0.75	0.66	0.00	2.19	0.85
time (sec)	N/A	0.905	0.365	0.902	0.205	0.265	0.000	0.313	12.873

Problem 484	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	173	164	133	125	133	121	0	425	330
N.S.	1	0.95	0.77	0.72	0.77	0.70	0.00	2.46	1.91
time (sec)	N/A	1.084	0.661	1.274	0.206	0.258	0.000	0.326	15.809

Problem 485	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	213	194	156	153	170	150	0	550	214
N.S.	1	0.91	0.73	0.72	0.80	0.70	0.00	2.58	1.00
time (sec)	N/A	1.122	0.745	1.258	0.216	0.288	0.000	0.337	13.078

Problem 486	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	158	170	114	160	198	183	0	380	274
N.S.	1	1.08	0.72	1.01	1.25	1.16	0.00	2.41	1.73
time (sec)	N/A	0.665	0.428	1.662	0.195	0.289	0.000	0.312	13.952

Problem 487	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	174	258	252	0	557	0	286	1021
N.S.	1	1.11	1.64	1.61	0.00	3.55	0.00	1.82	6.50
time (sec)	N/A	1.283	2.045	0.707	0.000	0.387	0.000	0.331	15.234

Problem 488	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	129	238	192	0	485	0	211	1002
N.S.	1	1.08	2.00	1.61	0.00	4.08	0.00	1.77	8.42
time (sec)	N/A	0.873	0.994	0.582	0.000	0.390	0.000	0.331	14.124

Problem 489	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	88	115	123	0	392	0	152	119
N.S.	1	1.04	1.35	1.45	0.00	4.61	0.00	1.79	1.40
time (sec)	N/A	0.577	0.502	0.427	0.000	0.314	0.000	0.344	14.045

Problem 490	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	102	83	0	290	0	120	186
N.S.	1	1.00	1.50	1.22	0.00	4.26	0.00	1.76	2.74
time (sec)	N/A	0.445	0.182	0.419	0.000	0.319	0.000	0.319	13.885

Problem 491	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	48	44	0	185	0	77	40
N.S.	1	1.00	0.98	0.90	0.00	3.78	0.00	1.57	0.82
time (sec)	N/A	0.283	0.031	0.337	0.000	0.256	0.000	0.305	14.056

Problem 492	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	60	65	0	230	0	218	186
N.S.	1	1.00	1.02	1.10	0.00	3.90	0.00	3.69	3.15
time (sec)	N/A	0.274	0.117	0.296	0.000	0.284	0.000	0.309	14.234

Problem 493	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	79	72	97	0	277	0	126	395
N.S.	1	1.04	0.95	1.28	0.00	3.64	0.00	1.66	5.20
time (sec)	N/A	0.387	0.312	0.615	0.000	0.284	0.000	0.306	13.776

Problem 494	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	121	97	138	0	334	0	178	592
N.S.	1	1.10	0.88	1.25	0.00	3.04	0.00	1.62	5.38
time (sec)	N/A	0.790	0.335	0.691	0.000	0.316	0.000	0.325	14.290

Problem 495	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	165	122	179	0	401	0	249	654
N.S.	1	1.11	0.82	1.21	0.00	2.71	0.00	1.68	4.42
time (sec)	N/A	1.117	0.397	0.712	0.000	0.305	0.000	0.311	14.666

Problem 496	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	193	218	153	256	0	482	0	393	2678
N.S.	1	1.13	0.79	1.33	0.00	2.50	0.00	2.04	13.88
time (sec)	N/A	1.589	0.599	0.905	0.000	0.317	0.000	0.316	16.374

Problem 497	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	222	234	285	277	0	909	0	299	3685
N.S.	1	1.05	1.28	1.25	0.00	4.09	0.00	1.35	16.60
time (sec)	N/A	1.561	5.177	0.952	0.000	0.725	0.000	0.348	19.940

Problem 498	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	164	183	162	209	0	760	0	331	3159
N.S.	1	1.12	0.99	1.27	0.00	4.63	0.00	2.02	19.26
time (sec)	N/A	1.089	0.987	0.668	0.000	0.475	0.000	0.313	19.211

Problem 499	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	142	146	166	0	596	0	203	2848
N.S.	1	1.21	1.25	1.42	0.00	5.09	0.00	1.74	24.34
time (sec)	N/A	0.747	0.473	0.521	0.000	0.514	0.000	0.318	19.067

Problem 500	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	96	83	118	0	329	0	150	92
N.S.	1	1.13	0.98	1.39	0.00	3.87	0.00	1.76	1.08
time (sec)	N/A	0.449	0.192	0.352	0.000	0.304	0.000	0.314	14.331

Problem 501	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	97	83	118	0	332	0	150	92
N.S.	1	1.13	0.97	1.37	0.00	3.86	0.00	1.74	1.07
time (sec)	N/A	0.443	0.296	0.364	0.000	0.299	0.000	0.319	13.578

Problem 502	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	134	138	151	0	484	0	179	2886
N.S.	1	1.23	1.27	1.39	0.00	4.44	0.00	1.64	26.48
time (sec)	N/A	0.626	0.376	0.382	0.000	0.308	0.000	0.287	19.115

Problem 503	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	166	172	184	0	565	0	837	3169
N.S.	1	1.14	1.18	1.26	0.00	3.87	0.00	5.73	21.71
time (sec)	N/A	0.935	0.826	0.750	0.000	0.309	0.000	0.372	19.389

Problem 504	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	208	219	144	224	0	660	0	264	3738
N.S.	1	1.05	0.69	1.08	0.00	3.17	0.00	1.27	17.97
time (sec)	N/A	1.408	0.808	0.858	0.000	0.320	0.000	0.318	20.517

Problem 505	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	261	268	176	263	0	757	0	335	3839
N.S.	1	1.03	0.67	1.01	0.00	2.90	0.00	1.28	14.71
time (sec)	N/A	1.917	1.055	1.059	0.000	0.340	0.000	0.317	20.718

Problem 506	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	230	267	205	296	0	1354	0	383	5332
N.S.	1	1.16	0.89	1.29	0.00	5.89	0.00	1.67	23.18
time (sec)	N/A	1.676	3.581	1.150	0.000	0.849	0.000	0.376	21.878

Problem 507	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	188	233	194	255	0	1153	0	347	5078
N.S.	1	1.24	1.03	1.36	0.00	6.13	0.00	1.85	27.01
time (sec)	N/A	1.218	1.100	0.917	0.000	0.844	0.000	0.363	22.431

Problem 508	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	175	113	184	0	594	0	253	204
N.S.	1	1.17	0.76	1.23	0.00	3.99	0.00	1.70	1.37
time (sec)	N/A	0.739	0.349	0.506	0.000	0.320	0.000	0.389	16.036

Problem 509	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	159	115	195	0	565	0	277	210
N.S.	1	1.19	0.86	1.46	0.00	4.22	0.00	2.07	1.57
time (sec)	N/A	0.691	0.388	0.559	0.000	0.311	0.000	0.358	16.038

Problem 510	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	159	115	186	0	595	0	254	204
N.S.	1	1.20	0.86	1.40	0.00	4.47	0.00	1.91	1.53
time (sec)	N/A	0.681	0.588	0.513	0.000	0.320	0.000	0.372	16.170

Problem 511	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	173	217	205	237	0	919	0	322	5090
N.S.	1	1.25	1.18	1.37	0.00	5.31	0.00	1.86	29.42
time (sec)	N/A	0.999	0.779	0.603	0.000	0.311	0.000	0.289	22.444

Problem 512	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	223	259	229	270	0	1037	0	357	5338
N.S.	1	1.16	1.03	1.21	0.00	4.65	0.00	1.60	23.94
time (sec)	N/A	1.531	1.011	1.109	0.000	0.320	0.000	0.367	21.740

Problem 513	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	296	321	199	310	0	1158	0	1723	5950
N.S.	1	1.08	0.67	1.05	0.00	3.91	0.00	5.82	20.10
time (sec)	N/A	2.166	1.842	1.196	0.000	0.372	0.000	0.541	22.678

Problem 514	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	316	373	416	425	0	2058	0	592	7476
N.S.	1	1.18	1.32	1.34	0.00	6.51	0.00	1.87	23.66
time (sec)	N/A	2.471	6.651	1.301	0.000	1.713	0.000	0.386	23.545

Problem 515	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	259	314	250	383	0	1822	0	559	7222
N.S.	1	1.21	0.97	1.48	0.00	7.03	0.00	2.16	27.88
time (sec)	N/A	1.832	3.988	1.501	0.000	1.842	0.000	0.393	27.131

Problem 516	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	222	267	158	285	0	903	0	403	378
N.S.	1	1.20	0.71	1.28	0.00	4.07	0.00	1.82	1.70
time (sec)	N/A	1.183	0.743	0.939	0.000	0.314	0.000	0.357	17.886

Problem 517	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	206	245	165	294	0	902	0	431	380
N.S.	1	1.19	0.80	1.43	0.00	4.38	0.00	2.09	1.84
time (sec)	N/A	1.064	0.946	0.795	0.000	0.323	0.000	0.362	18.016

Problem 518	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	192	233	164	297	0	901	0	431	382
N.S.	1	1.21	0.85	1.55	0.00	4.69	0.00	2.24	1.99
time (sec)	N/A	1.010	1.124	0.835	0.000	0.353	0.000	0.362	17.910

Problem 519	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	184	226	163	284	0	905	0	403	378
N.S.	1	1.23	0.89	1.54	0.00	4.92	0.00	2.19	2.05
time (sec)	N/A	1.004	1.209	0.700	0.000	0.327	0.000	0.355	17.862

Problem 520	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	242	306	268	365	0	1456	0	532	7234
N.S.	1	1.26	1.11	1.51	0.00	6.02	0.00	2.20	29.89
time (sec)	N/A	1.474	1.375	0.634	0.000	0.366	0.000	0.312	26.999

Problem 521	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	299	355	293	399	0	1603	0	564	7534
N.S.	1	1.19	0.98	1.33	0.00	5.36	0.00	1.89	25.20
time (sec)	N/A	2.244	1.694	1.478	0.000	0.386	0.000	0.386	24.100

Problem 522	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	387	431	263	439	0	1767	0	615	8133
N.S.	1	1.11	0.68	1.13	0.00	4.57	0.00	1.59	21.02
time (sec)	N/A	3.002	5.736	2.143	0.000	0.447	0.000	0.373	23.026

Problem 523	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	41	30	32	47	33	0	30	21
N.S.	1	1.32	0.97	1.03	1.52	1.06	0.00	0.97	0.68
time (sec)	N/A	0.238	0.123	0.393	0.278	0.263	0.000	0.274	13.706

Problem 524	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	71	73	58	88	73	0	59	52
N.S.	1	1.27	1.30	1.04	1.57	1.30	0.00	1.05	0.93
time (sec)	N/A	0.453	0.266	0.421	0.284	0.278	0.000	0.283	14.246

Problem 525	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	101	108	74	131	116	0	75	79
N.S.	1	1.25	1.33	0.91	1.62	1.43	0.00	0.93	0.98
time (sec)	N/A	0.603	0.414	0.401	0.277	0.263	0.000	0.307	14.549

Problem 526	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	131	141	87	171	159	0	88	105
N.S.	1	1.24	1.33	0.82	1.61	1.50	0.00	0.83	0.99
time (sec)	N/A	0.789	0.608	0.554	0.289	0.287	0.000	0.310	14.432

Problem 527	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	28	69	39	70	52	0	43	21
N.S.	1	0.40	0.99	0.56	1.00	0.74	0.00	0.61	0.30
time (sec)	N/A	0.242	0.113	0.314	0.280	0.259	0.000	0.304	14.219

Problem 528	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	58	162	76	111	102	0	69	52
N.S.	1	0.61	1.71	0.80	1.17	1.07	0.00	0.73	0.55
time (sec)	N/A	0.454	0.270	0.393	0.271	0.271	0.000	0.291	14.309

Problem 529	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	88	241	106	155	155	0	85	78
N.S.	1	0.73	2.01	0.88	1.29	1.29	0.00	0.71	0.65
time (sec)	N/A	0.624	0.371	0.410	0.284	0.275	0.000	0.313	14.388

Problem 530	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	118	344	132	194	208	0	98	105
N.S.	1	0.81	2.37	0.91	1.34	1.43	0.00	0.68	0.72
time (sec)	N/A	0.795	0.635	0.489	0.289	0.284	0.000	0.302	14.466

Problem 531	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	292	298	366	1982	0	0	0	0	0
N.S.	1	1.02	1.25	6.79	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.077	27.515	11.203	0.000	0.000	0.000	0.000	0.000

Problem 532	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	241	242	293	1127	0	0	0	0	0
N.S.	1	1.00	1.22	4.68	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.779	15.791	9.643	0.000	0.000	0.000	0.000	0.000

Problem 533	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	209	209	208	1063	0	0	0	0	0
N.S.	1	1.00	1.00	5.09	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.560	8.441	8.453	0.000	0.000	0.000	0.000	0.000

Problem 534	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	151	182	0	0	0	0	0
N.S.	1	1.00	1.21	1.46	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.247	1.894	6.027	0.000	0.000	0.000	0.000	0.000

Problem 535	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	330	337	373	1077	0	0	0	0	0
N.S.	1	1.02	1.13	3.26	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.183	10.169	3.801	0.000	0.000	0.000	0.000	0.000

Problem 536	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	396	394	897	1674	0	0	0	0	0
N.S.	1	0.99	2.27	4.23	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.539	17.509	4.853	0.000	0.000	0.000	0.000	0.000

Problem 537	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	405	421	550	3239	0	0	0	0	0
N.S.	1	1.04	1.36	8.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.778	14.074	16.936	0.000	0.000	0.000	0.000	0.000

Problem 538	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	342	350	471	2343	0	0	0	0	0
N.S.	1	1.02	1.38	6.85	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.347	11.789	15.595	0.000	0.000	0.000	0.000	0.000

Problem 539	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	282	288	408	1976	0	0	0	0	0
N.S.	1	1.02	1.45	7.01	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.023	11.238	12.057	0.000	0.000	0.000	0.000	0.000

Problem 540	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	249	250	304	1383	0	0	0	0	0
N.S.	1	1.00	1.22	5.55	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.795	7.495	11.094	0.000	0.000	0.000	0.000	0.000

Problem 541	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	309	309	684	1605	0	0	0	0	0
N.S.	1	1.00	2.21	5.19	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.951	16.623	8.511	0.000	0.000	0.000	0.000	0.000

Problem 542	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	334	341	332	1352	0	0	0	0	0
N.S.	1	1.02	0.99	4.05	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.190	8.669	8.379	0.000	0.000	0.000	0.000	0.000

Problem 543	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	390	394	891	1929	0	0	0	0	0
N.S.	1	1.01	2.28	4.95	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.644	17.207	4.764	0.000	0.000	0.000	0.000	0.000

Problem 544	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-1)	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	463	481	615	3643	0	0	0	0	0
N.S.	1	1.04	1.33	7.87	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.162	14.128	166.318	0.000	0.000	0.000	0.000	0.000

Problem 545	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	399	409	552	3239	0	0	0	0	0
N.S.	1	1.03	1.38	8.12	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.705	13.334	65.476	0.000	0.000	0.000	0.000	0.000

Problem 546	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	333	344	474	2343	0	0	0	0	0
N.S.	1	1.03	1.42	7.04	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.347	11.877	23.996	0.000	0.000	0.000	0.000	0.000

Problem 547	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	296	302	440	2237	0	0	0	0	0
N.S.	1	1.02	1.49	7.56	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.083	13.453	14.612	0.000	0.000	0.000	0.000	0.000

Problem 548	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	352	353	421	1922	0	0	0	0	0
N.S.	1	1.00	1.20	5.46	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.218	10.929	11.477	0.000	0.000	0.000	0.000	0.000

Problem 549	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	353	360	454	2178	0	0	0	0	0
N.S.	1	1.02	1.29	6.17	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.296	17.106	17.648	0.000	0.000	0.000	0.000	0.000

Problem 550	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	399	408	463	2212	0	0	0	0	0
N.S.	1	1.02	1.16	5.54	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.678	13.986	42.594	0.000	0.000	0.000	0.000	0.000

Problem 551	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	460	477	1018	2545	0	0	0	0	0
N.S.	1	1.04	2.21	5.53	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.271	15.258	142.082	0.000	0.000	0.000	0.000	0.000

Problem 552	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	530	547	1274	3192	0	0	0	0	0
N.S.	1	1.03	2.40	6.02	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.876	14.485	410.773	0.000	0.000	0.000	0.000	0.000

Problem 553	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	403	409	873	2777	0	0	0	0	0
N.S.	1	1.01	2.17	6.89	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.572	13.629	13.713	0.000	0.000	0.000	0.000	0.000

Problem 554	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	359	379	463	2343	0	0	0	0	0
N.S.	1	1.06	1.29	6.53	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.508	12.021	17.160	0.000	0.000	0.000	0.000	0.000

Problem 555	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	301	313	365	1983	0	0	0	0	0
N.S.	1	1.04	1.21	6.59	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.075	10.046	13.359	0.000	0.000	0.000	0.000	0.000

Problem 556	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	244	248	341	1128	0	0	0	0	0
N.S.	1	1.02	1.40	4.62	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.760	11.510	12.048	0.000	0.000	0.000	0.000	0.000

Problem 557	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	204	204	238	815	0	0	0	0	0
N.S.	1	1.00	1.17	4.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.566	13.357	8.614	0.000	0.000	0.000	0.000	0.000

Problem 558	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	93	114	0	0	0	0	0
N.S.	1	1.00	0.94	1.15	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.258	1.948	7.241	0.000	0.000	0.000	0.000	0.000

Problem 559	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	138	147	0	0	0	0	0
N.S.	1	1.00	1.30	1.39	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.231	2.109	4.917	0.000	0.000	0.000	0.000	0.000

Problem 560	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	338	346	239	829	0	0	0	0	0
N.S.	1	1.02	0.71	2.45	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.119	5.886	7.385	0.000	0.000	0.000	0.000	0.000

Problem 561	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	401	403	682	1675	0	0	0	0	0
N.S.	1	1.00	1.70	4.18	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.607	12.734	7.235	0.000	0.000	0.000	0.000	0.000

Problem 562	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	399	411	455	3171	0	0	0	0	0
N.S.	1	1.03	1.14	7.95	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.748	11.779	15.945	0.000	0.000	0.000	0.000	0.000

Problem 563	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	325	345	470	2286	0	0	0	0	0
N.S.	1	1.06	1.45	7.03	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.248	11.666	12.332	0.000	0.000	0.000	0.000	0.000

Problem 564	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	257	283	440	1940	0	0	0	0	0
N.S.	1	1.10	1.71	7.55	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.868	13.679	11.222	0.000	0.000	0.000	0.000	0.000

Problem 565	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	237	261	249	1106	0	0	0	0	0
N.S.	1	1.10	1.05	4.67	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.802	7.975	7.382	0.000	0.000	0.000	0.000	0.000

Problem 566	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	236	259	244	1082	0	0	0	0	0
N.S.	1	1.10	1.03	4.58	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.744	7.375	3.961	0.000	0.000	0.000	0.000	0.000

Problem 567	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	347	376	972	1799	0	0	0	0	0
N.S.	1	1.08	2.80	5.18	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.286	16.924	6.357	0.000	0.000	0.000	0.000	0.000

Problem 568	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	396	436	1069	2220	0	0	0	0	0
N.S.	1	1.10	2.70	5.61	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.659	13.573	8.414	0.000	0.000	0.000	0.000	0.000

Problem 569	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	470	504	1330	3126	0	0	0	0	0
N.S.	1	1.07	2.83	6.65	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.136	12.987	8.338	0.000	0.000	0.000	0.000	0.000

Problem 570	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	427	447	578	5026	0	0	0	0	0
N.S.	1	1.05	1.35	11.77	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.908	13.947	14.615	0.000	0.000	0.000	0.000	0.000

Problem 571	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	362	388	556	4450	0	0	0	0	0
N.S.	1	1.07	1.54	12.29	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.310	14.473	11.237	0.000	0.000	0.000	0.000	0.000

Problem 572	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	337	356	503	3303	0	0	0	0	0
N.S.	1	1.06	1.49	9.80	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.213	11.581	9.448	0.000	0.000	0.000	0.000	0.000

Problem 573	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	317	336	486	3001	0	0	0	0	0
N.S.	1	1.06	1.53	9.47	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.150	11.177	4.993	0.000	0.000	0.000	0.000	0.000

Problem 574	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	304	323	360	2259	0	0	0	0	0
N.S.	1	1.06	1.18	7.43	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.097	5.968	6.041	0.000	0.000	0.000	0.000	0.000

Problem 575	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	448	478	1382	4914	0	0	0	0	0
N.S.	1	1.07	3.08	10.97	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.826	12.614	7.322	0.000	0.000	0.000	0.000	0.000

Problem 576	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	510	548	1481	5835	0	0	0	0	0
N.S.	1	1.07	2.90	11.44	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.269	14.824	9.703	0.000	0.000	0.000	0.000	0.000

Problem 577	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	562	604	1730	7167	0	0	0	0	0
N.S.	1	1.07	3.08	12.75	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.841	13.842	9.613	0.000	0.000	0.000	0.000	0.000

Problem 578	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	535	584	1790	9415	0	0	0	0	0
N.S.	1	1.09	3.35	17.60	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.443	13.381	8.581	0.000	0.000	0.000	0.000	0.000

Problem 579	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	153	97	502	0	188	0	0	0
N.S.	1	1.01	0.64	3.32	0.00	1.25	0.00	0.00	0.00
time (sec)	N/A	0.629	0.405	27.290	0.000	0.103	0.000	0.000	0.000

Problem 580	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	125	85	396	0	167	0	0	0
N.S.	1	1.02	0.69	3.22	0.00	1.36	0.00	0.00	0.00
time (sec)	N/A	0.513	0.296	18.205	0.000	0.100	0.000	0.000	0.000

Problem 581	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	98	71	150	0	124	0	0	0
N.S.	1	1.01	0.73	1.55	0.00	1.28	0.00	0.00	0.00
time (sec)	N/A	0.466	0.202	11.886	0.000	0.096	0.000	0.000	0.000

Problem 582	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	52	152	0	107	0	0	0
N.S.	1	1.00	0.69	2.03	0.00	1.43	0.00	0.00	0.00
time (sec)	N/A	0.372	0.232	10.082	0.000	0.089	0.000	0.000	0.000

Problem 583	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	102	76	228	0	125	0	0	0
N.S.	1	1.01	0.75	2.26	0.00	1.24	0.00	0.00	0.00
time (sec)	N/A	0.465	0.252	12.687	0.000	0.096	0.000	0.000	0.000

Problem 584	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	129	88	262	0	145	0	0	0
N.S.	1	1.02	0.69	2.06	0.00	1.14	0.00	0.00	0.00
time (sec)	N/A	0.508	0.372	16.449	0.000	0.099	0.000	0.000	0.000

Problem 585	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	157	99	290	0	156	0	0	0
N.S.	1	1.04	0.66	1.92	0.00	1.03	0.00	0.00	0.00
time (sec)	N/A	0.632	0.556	18.146	0.000	0.106	0.000	0.000	0.000

Problem 586	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	200	194	139	689	0	235	0	0	0
N.S.	1	0.97	0.70	3.44	0.00	1.18	0.00	0.00	0.00
time (sec)	N/A	1.055	0.920	79.159	0.000	0.108	0.000	0.000	0.000

Problem 587	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	175	166	126	633	0	223	0	0	0
N.S.	1	0.95	0.72	3.62	0.00	1.27	0.00	0.00	0.00
time (sec)	N/A	0.892	1.184	33.473	0.000	0.113	0.000	0.000	0.000

Problem 588	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	136	93	513	0	190	0	0	0
N.S.	1	1.01	0.69	3.80	0.00	1.41	0.00	0.00	0.00
time (sec)	N/A	0.765	0.508	22.765	0.000	0.101	0.000	0.000	0.000

Problem 589	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	82	202	0	146	0	0	0
N.S.	1	1.00	0.76	1.87	0.00	1.35	0.00	0.00	0.00
time (sec)	N/A	0.615	0.753	16.613	0.000	0.106	0.000	0.000	0.000

Problem 590	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	87	283	0	147	0	0	0
N.S.	1	1.00	0.78	2.53	0.00	1.31	0.00	0.00	0.00
time (sec)	N/A	0.615	0.609	18.549	0.000	0.096	0.000	0.000	0.000

Problem 591	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	142	100	357	0	170	0	0	0
N.S.	1	1.01	0.71	2.53	0.00	1.21	0.00	0.00	0.00
time (sec)	N/A	0.768	0.845	19.534	0.000	0.108	0.000	0.000	0.000

Problem 592	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	175	170	120	362	0	191	0	0	0
N.S.	1	0.97	0.69	2.07	0.00	1.09	0.00	0.00	0.00
time (sec)	N/A	0.892	1.103	25.866	0.000	0.111	0.000	0.000	0.000

Problem 593	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	234	218	177	820	0	270	0	0	0
N.S.	1	0.93	0.76	3.50	0.00	1.15	0.00	0.00	0.00
time (sec)	N/A	1.306	2.912	45.495	0.000	0.114	0.000	0.000	0.000

Problem 594	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	189	181	134	711	0	244	0	0	0
N.S.	1	0.96	0.71	3.76	0.00	1.29	0.00	0.00	0.00
time (sec)	N/A	1.111	1.677	37.534	0.000	0.104	0.000	0.000	0.000

Problem 595	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	158	159	106	630	0	214	0	0	0
N.S.	1	1.01	0.67	3.99	0.00	1.35	0.00	0.00	0.00
time (sec)	N/A	0.962	1.035	25.322	0.000	0.109	0.000	0.000	0.000

Problem 596	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	166	167	108	303	0	182	0	0	0
N.S.	1	1.01	0.65	1.83	0.00	1.10	0.00	0.00	0.00
time (sec)	N/A	0.953	0.938	24.090	0.000	0.117	0.000	0.000	0.000

Problem 597	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	157	106	412	0	193	0	0	0
N.S.	1	1.01	0.68	2.64	0.00	1.24	0.00	0.00	0.00
time (sec)	N/A	0.960	0.852	22.568	0.000	0.103	0.000	0.000	0.000

Problem 598	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	199	194	132	421	0	216	0	0	0
N.S.	1	0.97	0.66	2.12	0.00	1.09	0.00	0.00	0.00
time (sec)	N/A	1.140	1.267	28.975	0.000	0.113	0.000	0.000	0.000

Problem 599	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	234	222	159	470	0	238	0	0	0
N.S.	1	0.95	0.68	2.01	0.00	1.02	0.00	0.00	0.00
time (sec)	N/A	1.345	1.480	38.278	0.000	0.122	0.000	0.000	0.000

Problem 600	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-1)	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	287	269	256	1147	0	318	0	0	0
N.S.	1	0.94	0.89	4.00	0.00	1.11	0.00	0.00	0.00
time (sec)	N/A	1.700	2.613	64.916	0.000	0.124	0.000	0.000	0.000

Problem 601	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	247	243	168	898	0	289	0	0	0
N.S.	1	0.98	0.68	3.64	0.00	1.17	0.00	0.00	0.00
time (sec)	N/A	1.571	1.971	50.839	0.000	0.113	0.000	0.000	0.000

Problem 602	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	209	213	146	880	0	264	0	0	0
N.S.	1	1.02	0.70	4.21	0.00	1.26	0.00	0.00	0.00
time (sec)	N/A	1.354	2.534	41.953	0.000	0.114	0.000	0.000	0.000

Problem 603	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	208	207	130	777	0	242	0	0	0
N.S.	1	1.00	0.62	3.74	0.00	1.16	0.00	0.00	0.00
time (sec)	N/A	1.320	1.637	30.844	0.000	0.112	0.000	0.000	0.000

Problem 604	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	207	211	138	432	0	215	0	0	0
N.S.	1	1.02	0.67	2.09	0.00	1.04	0.00	0.00	0.00
time (sec)	N/A	1.346	1.229	29.949	0.000	0.118	0.000	0.000	0.000

Problem 605	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	211	219	142	476	0	234	0	0	0
N.S.	1	1.04	0.67	2.26	0.00	1.11	0.00	0.00	0.00
time (sec)	N/A	1.336	1.501	30.487	0.000	0.112	0.000	0.000	0.000

Problem 606	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	245	245	168	529	0	257	0	0	0
N.S.	1	1.00	0.69	2.16	0.00	1.05	0.00	0.00	0.00
time (sec)	N/A	1.570	1.957	36.139	0.000	0.124	0.000	0.000	0.000

Problem 607	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	289	275	199	586	0	286	0	0	0
N.S.	1	0.95	0.69	2.03	0.00	0.99	0.00	0.00	0.00
time (sec)	N/A	1.784	2.471	46.602	0.000	0.135	0.000	0.000	0.000

Problem 608	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	188	201	165	423	0	0	0	0	0
N.S.	1	1.07	0.88	2.25	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.559	35.466	19.590	0.000	0.000	0.000	0.000	0.000

Problem 609	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	116	83	353	0	0	0	0	0
N.S.	1	0.99	0.71	3.02	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.765	15.424	12.997	0.000	0.000	0.000	0.000	0.000

Problem 610	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	63	150	0	0	0	0	0
N.S.	1	1.00	1.29	3.06	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.314	0.308	3.608	0.000	0.000	0.000	0.000	0.000

Problem 611	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	74	47	187	0	0	0	0	0
N.S.	1	0.80	0.51	2.01	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.484	0.379	4.901	0.000	0.000	0.000	0.000	0.000

Problem 612	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	176	226	0	0	0	0	0
N.S.	1	1.00	1.30	1.67	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.880	18.308	7.366	0.000	0.000	0.000	0.000	0.000

Problem 613	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	178	194	552	0	0	0	0	0
N.S.	1	1.03	1.13	3.21	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.210	21.444	7.533	0.000	0.000	0.000	0.000	0.000

Problem 614	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-1)	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	342	325	294	975	0	0	0	0	0
N.S.	1	0.95	0.86	2.85	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.495	5.101	120.415	0.000	0.000	0.000	0.000	0.000

Problem 615	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-1)	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	279	266	351	841	0	0	0	0	0
N.S.	1	0.95	1.26	3.01	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.923	3.963	111.122	0.000	0.000	0.000	0.000	0.000

Problem 616	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	214	212	582	608	0	0	0	0	0
N.S.	1	0.99	2.72	2.84	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.373	6.480	104.881	0.000	0.000	0.000	0.000	0.000

Problem 617	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	208	202	289	707	0	0	0	0	0
N.S.	1	0.97	1.39	3.40	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.276	6.020	11.980	0.000	0.000	0.000	0.000	0.000

Problem 618	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	227	216	251	788	0	0	0	0	0
N.S.	1	0.95	1.11	3.47	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.392	3.047	14.594	0.000	0.000	0.000	0.000	0.000

Problem 619	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	244	232	319	809	0	0	0	0	0
N.S.	1	0.95	1.31	3.32	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.450	4.611	14.766	0.000	0.000	0.000	0.000	0.000

Problem 620	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	304	289	279	1064	0	0	0	0	0
N.S.	1	0.95	0.92	3.50	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.062	5.514	16.943	0.000	0.000	0.000	0.000	0.000

Problem 621	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-1)	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	388	382	721	1987	0	0	0	0	0
N.S.	1	0.98	1.86	5.12	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.795	6.639	488.591	0.000	0.000	0.000	0.000	0.000

Problem 622	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-1)	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	315	318	335	1203	0	0	0	0	0
N.S.	1	1.01	1.06	3.82	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.092	5.637	195.450	0.000	0.000	0.000	0.000	0.000

Problem 623	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	313	310	428	1760	0	0	0	0	0
N.S.	1	0.99	1.37	5.62	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.091	6.264	372.302	0.000	0.000	0.000	0.000	0.000

Problem 624	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	306	307	429	1858	0	0	0	0	0
N.S.	1	1.00	1.40	6.07	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.009	5.710	26.056	0.000	0.000	0.000	0.000	0.000

Problem 625	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	323	321	286	1936	0	0	0	0	0
N.S.	1	0.99	0.89	5.99	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.094	4.798	28.157	0.000	0.000	0.000	0.000	0.000

Problem 626	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	342	342	707	1957	0	0	0	0	0
N.S.	1	1.00	2.07	5.72	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.279	6.668	29.913	0.000	0.000	0.000	0.000	0.000

Problem 627	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	406	403	731	2216	0	0	0	0	0
N.S.	1	0.99	1.80	5.46	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.913	6.860	31.010	0.000	0.000	0.000	0.000	0.000

Problem 628	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	237	254	321	1009	0	0	0	0	0
N.S.	1	1.07	1.35	4.26	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.312	25.121	7.579	0.000	0.000	0.000	0.000	0.000

Problem 629	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	96	251	0	0	0	0	0
N.S.	1	1.00	0.70	1.82	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.114	13.103	6.455	0.000	0.000	0.000	0.000	0.000

Problem 630	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	67	67	67	869	0	355	0	0	0
N.S.	1	1.00	1.00	12.97	0.00	5.30	0.00	0.00	0.00
time (sec)	N/A	0.394	0.244	3.373	0.000	0.108	0.000	0.000	0.000

Problem 631	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	192	193	156	1291	0	415	0	0	0
N.S.	1	1.01	0.81	6.72	0.00	2.16	0.00	0.00	0.00
time (sec)	N/A	1.370	0.626	5.280	0.000	0.110	0.000	0.000	0.000

Problem 632	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	244	253	203	2288	0	461	0	0	0
N.S.	1	1.04	0.83	9.38	0.00	1.89	0.00	0.00	0.00
time (sec)	N/A	1.844	0.907	6.523	0.000	0.116	0.000	0.000	0.000

Problem 633	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	305	322	237	2764	0	501	0	0	0
N.S.	1	1.06	0.78	9.06	0.00	1.64	0.00	0.00	0.00
time (sec)	N/A	2.432	1.192	7.639	0.000	0.134	0.000	0.000	0.000

Problem 634	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	299	308	411	2317	0	0	0	0	0
N.S.	1	1.03	1.37	7.75	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	3.154	4.772	9.780	0.000	0.000	0.000	0.000	0.000

Problem 635	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	249	255	394	1585	0	0	0	0	0
N.S.	1	1.02	1.58	6.37	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.512	14.332	8.887	0.000	0.000	0.000	0.000	0.000

Problem 636	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	209	209	129	1163	0	0	0	0	0
N.S.	1	1.00	0.62	5.56	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.007	27.264	7.580	0.000	0.000	0.000	0.000	0.000

Problem 637	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	187	188	156	1570	0	415	0	0	0
N.S.	1	1.01	0.83	8.40	0.00	2.22	0.00	0.00	0.00
time (sec)	N/A	1.418	0.671	5.296	0.000	0.109	0.000	0.000	0.000

Problem 638	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	240	238	197	2282	0	463	0	0	0
N.S.	1	0.99	0.82	9.51	0.00	1.93	0.00	0.00	0.00
time (sec)	N/A	1.831	1.031	6.448	0.000	0.121	0.000	0.000	0.000

Problem 639	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	303	311	237	2766	0	501	0	0	0
N.S.	1	1.03	0.78	9.13	0.00	1.65	0.00	0.00	0.00
time (sec)	N/A	2.427	1.431	7.747	0.000	0.128	0.000	0.000	0.000

Problem 640	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	369	390	602	3037	0	0	0	0	0
N.S.	1	1.06	1.63	8.23	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	4.075	6.593	12.219	0.000	0.000	0.000	0.000	0.000

Problem 641	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	314	324	560	2602	0	0	0	0	0
N.S.	1	1.03	1.78	8.29	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	3.448	6.531	10.878	0.000	0.000	0.000	0.000	0.000

Problem 642	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	263	270	421	2463	0	0	0	0	0
N.S.	1	1.03	1.60	9.37	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.662	5.973	9.713	0.000	0.000	0.000	0.000	0.000

Problem 643	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	262	263	409	2190	0	0	0	0	0
N.S.	1	1.00	1.56	8.36	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.664	4.272	8.458	0.000	0.000	0.000	0.000	0.000

Problem 644	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	239	250	200	2567	0	462	0	0	0
N.S.	1	1.05	0.84	10.74	0.00	1.93	0.00	0.00	0.00
time (sec)	N/A	1.989	1.157	6.502	0.000	0.114	0.000	0.000	0.000

Problem 645	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	303	312	237	2766	0	501	0	0	0
N.S.	1	1.03	0.78	9.13	0.00	1.65	0.00	0.00	0.00
time (sec)	N/A	2.540	1.618	7.767	0.000	0.133	0.000	0.000	0.000

Problem 646	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	363	378	286	3822	0	541	0	0	0
N.S.	1	1.04	0.79	10.53	0.00	1.49	0.00	0.00	0.00
time (sec)	N/A	3.302	2.213	8.942	0.000	0.146	0.000	0.000	0.000

Problem 647	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	312	312	397	2331	0	0	0	0	0
N.S.	1	1.00	1.27	7.47	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	3.072	4.633	11.009	0.000	0.000	0.000	0.000	0.000

Problem 648	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	246	259	329	1299	0	0	0	0	0
N.S.	1	1.05	1.34	5.28	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.283	15.639	8.688	0.000	0.000	0.000	0.000	0.000

Problem 649	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	68	205	0	0	0	0	0
N.S.	1	1.00	1.00	3.01	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.497	0.305	6.311	0.000	0.000	0.000	0.000	0.000

Problem 650	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	67	141	0	146	0	0	0
N.S.	1	1.00	1.00	2.10	0.00	2.18	0.00	0.00	0.00
time (sec)	N/A	0.410	0.182	5.253	0.000	0.096	0.000	0.000	0.000

Problem 651	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	142	142	96	739	0	355	0	0	0
N.S.	1	1.00	0.68	5.20	0.00	2.50	0.00	0.00	0.00
time (sec)	N/A	1.041	3.310	6.533	0.000	0.103	0.000	0.000	0.000

Problem 652	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	195	199	147	1298	0	415	0	0	0
N.S.	1	1.02	0.75	6.66	0.00	2.13	0.00	0.00	0.00
time (sec)	N/A	1.428	0.696	7.632	0.000	0.107	0.000	0.000	0.000

Problem 653	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	249	261	193	2289	0	464	0	0	0
N.S.	1	1.05	0.78	9.19	0.00	1.86	0.00	0.00	0.00
time (sec)	N/A	1.952	0.808	8.750	0.000	0.123	0.000	0.000	0.000

Problem 654	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	345	360	478	2300	0	0	0	0	0
N.S.	1	1.04	1.39	6.67	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	3.527	3.168	9.750	0.000	0.000	0.000	0.000	0.000

Problem 655	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	206	214	434	1019	0	0	0	0	0
N.S.	1	1.04	2.11	4.95	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.655	4.826	7.572	0.000	0.000	0.000	0.000	0.000

Problem 656	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	126	126	103	587	0	488	0	0	0
N.S.	1	1.00	0.82	4.66	0.00	3.87	0.00	0.00	0.00
time (sec)	N/A	0.640	0.352	3.242	0.000	0.114	0.000	0.000	0.000

Problem 657	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	200	211	156	593	0	525	0	0	0
N.S.	1	1.06	0.78	2.96	0.00	2.62	0.00	0.00	0.00
time (sec)	N/A	1.458	0.647	6.545	0.000	0.109	0.000	0.000	0.000

Problem 658	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	214	228	165	925	0	565	0	0	0
N.S.	1	1.07	0.77	4.32	0.00	2.64	0.00	0.00	0.00
time (sec)	N/A	1.567	0.727	7.877	0.000	0.131	0.000	0.000	0.000

Problem 659	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	289	295	203	1679	0	632	0	0	0
N.S.	1	1.02	0.70	5.81	0.00	2.19	0.00	0.00	0.00
time (sec)	N/A	2.162	0.954	9.093	0.000	0.141	0.000	0.000	0.000

Problem 660	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	360	359	250	2387	0	692	0	0	0
N.S.	1	1.00	0.69	6.63	0.00	1.92	0.00	0.00	0.00
time (sec)	N/A	2.766	1.373	10.073	0.000	0.159	0.000	0.000	0.000

Problem 661	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	458	471	561	5987	0	0	0	0	0
N.S.	1	1.03	1.22	13.07	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	4.740	5.775	11.477	0.000	0.000	0.000	0.000	0.000

Problem 662	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	370	390	487	4415	0	0	0	0	0
N.S.	1	1.05	1.32	11.93	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	3.883	4.169	8.971	0.000	0.000	0.000	0.000	0.000

Problem 663	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	277	295	169	1999	0	688	0	0	0
N.S.	1	1.06	0.61	7.22	0.00	2.48	0.00	0.00	0.00
time (sec)	N/A	2.137	0.901	5.798	0.000	0.138	0.000	0.000	0.000

Problem 664	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	281	288	178	2312	0	759	0	0	0
N.S.	1	1.02	0.63	8.23	0.00	2.70	0.00	0.00	0.00
time (sec)	N/A	2.001	1.000	5.318	0.000	0.143	0.000	0.000	0.000

Problem 665	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	302	314	196	2885	0	811	0	0	0
N.S.	1	1.04	0.65	9.55	0.00	2.69	0.00	0.00	0.00
time (sec)	N/A	2.153	1.149	7.602	0.000	0.146	0.000	0.000	0.000

Problem 666	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	317	332	208	4120	0	863	0	0	0
N.S.	1	1.05	0.66	13.00	0.00	2.72	0.00	0.00	0.00
time (sec)	N/A	2.330	1.319	8.935	0.000	0.157	0.000	0.000	0.000

Problem 667	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	391	399	257	4449	0	949	0	0	0
N.S.	1	1.02	0.66	11.38	0.00	2.43	0.00	0.00	0.00
time (sec)	N/A	2.920	1.724	9.959	0.000	0.176	0.000	0.000	0.000

Problem 668	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	474	479	292	5656	0	1036	0	0	0
N.S.	1	1.01	0.62	11.93	0.00	2.19	0.00	0.00	0.00
time (sec)	N/A	3.782	1.875	11.089	0.000	0.209	0.000	0.000	0.000

Problem 669	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	78	338	0	95	0	0	0
N.S.	1	1.00	0.64	2.77	0.00	0.78	0.00	0.00	0.00
time (sec)	N/A	0.687	0.330	7.101	0.000	0.092	0.000	0.000	0.000

Problem 670	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	68	326	0	92	0	0	0
N.S.	1	1.00	0.62	2.99	0.00	0.84	0.00	0.00	0.00
time (sec)	N/A	0.676	0.286	7.477	0.000	0.084	0.000	0.000	0.000

Problem 671	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	68	343	0	95	0	0	0
N.S.	1	1.00	0.63	3.18	0.00	0.88	0.00	0.00	0.00
time (sec)	N/A	0.898	0.182	7.291	0.000	0.088	0.000	0.000	0.000

Problem 672	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	78	329	0	87	0	0	0
N.S.	1	1.00	0.63	2.67	0.00	0.71	0.00	0.00	0.00
time (sec)	N/A	0.934	0.183	7.489	0.000	0.094	0.000	0.000	0.000

Problem 673	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	81	339	0	108	0	0	0
N.S.	1	1.00	0.64	2.67	0.00	0.85	0.00	0.00	0.00
time (sec)	N/A	0.688	0.320	7.478	0.000	0.092	0.000	0.000	0.000

Problem 674	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	72	339	0	108	0	0	0
N.S.	1	1.00	0.64	3.00	0.00	0.96	0.00	0.00	0.00
time (sec)	N/A	0.709	0.301	7.721	0.000	0.090	0.000	0.000	0.000

Problem 675	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	72	332	0	108	0	0	0
N.S.	1	1.00	0.56	2.57	0.00	0.84	0.00	0.00	0.00
time (sec)	N/A	0.695	0.174	7.570	0.000	0.095	0.000	0.000	0.000

Problem 676	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	81	332	0	108	0	0	0
N.S.	1	1.00	0.70	2.89	0.00	0.94	0.00	0.00	0.00
time (sec)	N/A	0.708	0.197	7.530	0.000	0.092	0.000	0.000	0.000

Problem 677	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	61	123	0	47	0	0	0
N.S.	1	1.00	1.00	2.02	0.00	0.77	0.00	0.00	0.00
time (sec)	N/A	0.302	0.208	4.795	0.000	0.082	0.000	0.000	0.000

Problem 678	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	54	115	0	44	0	0	0
N.S.	1	1.00	1.00	2.13	0.00	0.81	0.00	0.00	0.00
time (sec)	N/A	0.296	0.179	5.719	0.000	0.090	0.000	0.000	0.000

Problem 679	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	54	123	0	47	0	0	0
N.S.	1	1.00	1.00	2.28	0.00	0.87	0.00	0.00	0.00
time (sec)	N/A	0.357	0.131	5.524	0.000	0.086	0.000	0.000	0.000

Problem 680	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	61	119	0	44	0	0	0
N.S.	1	1.00	1.00	1.95	0.00	0.72	0.00	0.00	0.00
time (sec)	N/A	0.371	0.146	5.561	0.000	0.086	0.000	0.000	0.000

Problem 681	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	61	124	0	54	0	0	0
N.S.	1	1.00	1.00	2.03	0.00	0.89	0.00	0.00	0.00
time (sec)	N/A	0.293	0.175	6.109	0.000	0.088	0.000	0.000	0.000

Problem 682	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	54	117	0	54	0	0	0
N.S.	1	1.00	1.00	2.17	0.00	1.00	0.00	0.00	0.00
time (sec)	N/A	0.306	0.190	5.527	0.000	0.084	0.000	0.000	0.000

Problem 683	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	54	114	0	52	0	0	0
N.S.	1	1.00	0.87	1.84	0.00	0.84	0.00	0.00	0.00
time (sec)	N/A	0.293	0.116	5.659	0.000	0.081	0.000	0.000	0.000

Problem 684	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	61	120	0	52	0	0	0
N.S.	1	1.00	1.11	2.18	0.00	0.95	0.00	0.00	0.00
time (sec)	N/A	0.300	0.138	5.828	0.000	0.083	0.000	0.000	0.000

Problem 685	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	105	105	7160	0	0	0	0	0	0
N.S.	1	1.00	68.19	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.312	49.643	0.000	0.000	0.000	0.000	0.000	0.000

Problem 686	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	14	14	16	12	14	0	14	14	16
N.S.	1	1.00	1.14	0.86	1.00	0.00	1.00	1.00	1.14
time (sec)	N/A	0.183	18.049	0.371	0.578	0.000	0.486	0.421	13.707

Problem 687	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	362	376	21877	0	0	0	0	0	0
N.S.	1	1.04	60.43	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.337	46.266	0.000	0.000	0.000	0.000	0.000	0.000

Problem 688	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	305	311	7783	0	0	0	0	0	0
N.S.	1	1.02	25.52	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.926	45.196	0.000	0.000	0.000	0.000	0.000	0.000

Problem 689	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	260	260	10909	0	0	0	0	0	0
N.S.	1	1.00	41.96	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.658	42.666	0.000	0.000	0.000	0.000	0.000	0.000

Problem 690	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	105	105	7142	0	0	0	0	0	0
N.S.	1	1.00	68.02	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.269	45.818	0.000	0.000	0.000	0.000	0.000	0.000

Problem 691	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	14	14	16	12	14	0	14	14	16
N.S.	1	1.00	1.14	0.86	1.00	0.00	1.00	1.00	1.14
time (sec)	N/A	0.181	18.727	0.307	0.537	0.000	0.809	0.445	14.183

Problem 692	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	108	108	7313	0	0	0	0	0	0
N.S.	1	1.00	67.71	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.283	48.677	0.000	0.000	0.000	0.000	0.000	0.000

Problem 693	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	14	14	16	12	14	0	14	14	16
N.S.	1	1.00	1.14	0.86	1.00	0.00	1.00	1.00	1.14
time (sec)	N/A	0.179	100.647	0.310	0.542	0.000	13.307	1.143	15.299

Problem 694	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	412	428	28057	0	0	0	0	0	0
N.S.	1	1.04	68.10	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.640	45.803	0.000	0.000	0.000	0.000	0.000	0.000

Problem 695	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	356	364	21890	0	0	0	0	0	0
N.S.	1	1.02	61.49	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.231	46.600	0.000	0.000	0.000	0.000	0.000	0.000

Problem 696	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	299	304	7809	0	0	0	0	0	0
N.S.	1	1.02	26.12	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.919	45.016	0.000	0.000	0.000	0.000	0.000	0.000

Problem 697	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	108	108	7321	0	0	0	0	0	0
N.S.	1	1.00	67.79	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.272	44.955	0.000	0.000	0.000	0.000	0.000	0.000

Problem 698	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	14	14	16	12	14	0	14	14	16
N.S.	1	1.00	1.14	0.86	1.00	0.00	1.00	1.00	1.14
time (sec)	N/A	0.182	101.674	0.338	0.536	0.000	42.660	1.554	14.670

Problem 699	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	313	324	19015	0	0	0	0	0	0
N.S.	1	1.04	60.75	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.996	45.824	0.000	0.000	0.000	0.000	0.000	0.000

Problem 700	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	265	268	7195	0	0	0	0	0	0
N.S.	1	1.01	27.15	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.710	44.261	0.000	0.000	0.000	0.000	0.000	0.000

Problem 701	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	219	219	1748	0	0	0	0	0	0
N.S.	1	1.00	7.98	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.497	23.817	0.000	0.000	0.000	0.000	0.000	0.000

Problem 702	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	105	105	310	0	0	0	0	0	0
N.S.	1	1.00	2.95	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.277	14.390	0.000	0.000	0.000	0.000	0.000	0.000

Problem 703	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	14	14	16	12	14	0	15	14	16
N.S.	1	1.00	1.14	0.86	1.00	0.00	1.07	1.00	1.14
time (sec)	N/A	0.180	1.574	0.324	0.556	0.000	0.342	0.510	14.266

Problem 704	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	105	105	310	0	0	0	0	0	0
N.S.	1	1.00	2.95	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.282	15.336	0.000	0.000	0.000	0.000	0.000	0.000

Problem 705	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	14	14	16	12	14	0	15	14	16
N.S.	1	1.00	1.14	0.86	1.00	0.00	1.07	1.00	1.14
time (sec)	N/A	0.181	1.727	0.341	0.561	0.000	0.378	0.915	14.437

Problem 706	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	110	110	10343	0	0	0	0	0	0
N.S.	1	1.00	94.03	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.283	47.202	0.000	0.000	0.000	0.000	0.000	0.000

Problem 707	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	14	14	16	12	14	0	15	14	16
N.S.	1	1.00	1.14	0.86	1.00	0.00	1.07	1.00	1.14
time (sec)	N/A	0.186	91.411	0.340	0.571	0.000	1.255	0.882	15.530

Problem 708	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	378	367	21987	0	0	0	0	0	0
N.S.	1	0.97	58.17	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.179	44.234	0.000	0.000	0.000	0.000	0.000	0.000

Problem 709	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	307	304	19126	0	0	0	0	0	0
N.S.	1	0.99	62.30	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.801	44.100	0.000	0.000	0.000	0.000	0.000	0.000

Problem 710	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	289	283	7325	0	0	0	0	0	0
N.S.	1	0.98	25.35	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.736	46.960	0.000	0.000	0.000	0.000	0.000	0.000

Problem 711	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	110	110	10363	0	0	0	0	0	0
N.S.	1	1.00	94.21	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.285	49.842	0.000	0.000	0.000	0.000	0.000	0.000

Problem 712	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	14	14	16	12	14	0	15	14	16
N.S.	1	1.00	1.14	0.86	1.00	0.00	1.07	1.00	1.14
time (sec)	N/A	0.174	95.807	0.353	0.567	0.000	2.158	2.112	15.501

Problem 713	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	174	195	4543	0	0	0	0	0	0
N.S.	1	1.12	26.11	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.602	38.302	0.000	0.000	0.000	0.000	0.000	0.000

Problem 714	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	174	195	4544	0	0	0	0	0	0
N.S.	1	1.12	26.11	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.585	38.041	0.000	0.000	0.000	0.000	0.000	0.000

Problem 715	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	174	195	7542	0	0	0	0	0	0
N.S.	1	1.12	43.34	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.603	84.739	0.000	0.000	0.000	0.000	0.000	0.000

Problem 716	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	174	195	7588	0	0	0	0	0	0
N.S.	1	1.12	43.61	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.598	82.398	0.000	0.000	0.000	0.000	0.000	0.000

Problem 717	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	F(-1)	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	25	0	21	23	23	0	23	27
N.S.	1	1.00	0.00	0.84	0.92	0.92	0.00	0.92	1.08
time (sec)	N/A	0.232	0.000	0.512	0.782	0.362	0.000	1.180	14.957

Problem 718	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	F(-1)	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	25	0	21	23	23	0	23	27
N.S.	1	1.00	0.00	0.84	0.92	0.92	0.00	0.92	1.08
time (sec)	N/A	0.237	0.000	0.475	0.796	0.418	0.000	0.979	14.780

Problem 719	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	25	27	21	23	23	0	23	27
N.S.	1	1.00	1.08	0.84	0.92	0.92	0.00	0.92	1.08
time (sec)	N/A	0.234	105.480	0.523	0.781	0.375	0.000	0.775	14.857

Problem 720	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	25	27	21	23	23	24	23	27
N.S.	1	1.00	1.08	0.84	0.92	0.92	0.96	0.92	1.08
time (sec)	N/A	0.232	135.587	0.467	0.745	1.123	9.412	1.155	14.513

Problem 721	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	25	27	21	23	23	24	23	27
N.S.	1	1.00	1.08	0.84	0.92	0.92	0.96	0.92	1.08
time (sec)	N/A	0.230	22.268	0.466	0.740	0.461	1.102	0.864	14.363

Problem 722	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	25	27	21	23	23	24	23	27
N.S.	1	1.00	1.08	0.84	0.92	0.92	0.96	0.92	1.08
time (sec)	N/A	0.232	89.468	0.501	0.763	1.062	0.669	1.207	14.922

Problem 723	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	25	27	21	23	23	24	23	27
N.S.	1	1.00	1.08	0.84	0.92	0.92	0.96	0.92	1.08
time (sec)	N/A	0.229	100.394	0.492	0.756	0.671	0.689	1.589	13.982

Problem 724	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	25	27	21	23	23	24	23	27
N.S.	1	1.00	1.08	0.84	0.92	0.92	0.96	0.92	1.08
time (sec)	N/A	0.232	88.292	0.481	0.792	1.101	3.984	0.915	15.178

Problem 725	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	25	27	21	23	23	24	23	27
N.S.	1	1.00	1.08	0.84	0.92	0.92	0.96	0.92	1.08
time (sec)	N/A	0.228	146.018	0.481	0.786	0.725	11.370	1.175	14.319

Problem 726	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	25	27	21	23	23	0	23	27
N.S.	1	1.00	1.08	0.84	0.92	0.92	0.00	0.92	1.08
time (sec)	N/A	0.234	162.745	0.463	0.806	1.045	0.000	1.237	16.619

Problem 727	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	F(-1)	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	25	0	21	23	44	0	23	27
N.S.	1	1.00	0.00	0.84	0.92	1.76	0.00	0.92	1.08
time (sec)	N/A	0.231	0.000	0.438	0.804	0.330	0.000	2.721	16.280

Problem 728	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	25	27	21	23	42	0	23	27
N.S.	1	1.00	1.08	0.84	0.92	1.68	0.00	0.92	1.08
time (sec)	N/A	0.235	137.532	0.438	0.803	0.397	0.000	2.342	15.685

Problem 729	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	25	27	21	23	42	0	23	27
N.S.	1	1.00	1.08	0.84	0.92	1.68	0.00	0.92	1.08
time (sec)	N/A	0.229	104.903	0.428	0.775	0.334	0.000	1.662	15.013

Problem 730	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	25	27	21	23	23	0	23	27
N.S.	1	1.00	1.08	0.84	0.92	0.92	0.00	0.92	1.08
time (sec)	N/A	0.235	135.554	0.482	0.739	1.049	0.000	2.308	14.750

Problem 731	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	25	27	21	23	23	0	23	27
N.S.	1	1.00	1.08	0.84	0.92	0.92	0.00	0.92	1.08
time (sec)	N/A	0.228	94.476	0.405	0.735	0.681	0.000	1.667	15.090

Problem 732	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	25	27	21	23	23	24	23	27
N.S.	1	1.00	1.08	0.84	0.92	0.92	0.96	0.92	1.08
time (sec)	N/A	0.229	177.248	0.403	0.751	1.048	21.790	235.741	15.598

Problem 733	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	25	27	21	23	23	24	23	27
N.S.	1	1.00	1.08	0.84	0.92	0.92	0.96	0.92	1.08
time (sec)	N/A	0.233	105.908	0.435	0.760	0.656	13.330	157.945	13.364

Problem 734	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	25	27	21	23	23	24	23	27
N.S.	1	1.00	1.08	0.84	0.92	0.92	0.96	0.92	1.08
time (sec)	N/A	0.230	87.885	0.393	0.796	1.106	32.529	178.249	14.394

Problem 735	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	25	27	21	23	23	24	23	27
N.S.	1	1.00	1.08	0.84	0.92	0.92	0.96	0.92	1.08
time (sec)	N/A	0.236	172.173	0.440	0.778	0.704	78.967	144.137	12.874

Problem 736	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	F(-1)	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	25	0	21	23	23	0	23	27
N.S.	1	1.00	0.00	0.84	0.92	0.92	0.00	0.92	1.08
time (sec)	N/A	0.235	0.000	0.391	0.789	1.051	0.000	147.930	15.191

Problem 737	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	25	27	21	23	60	0	23	27
N.S.	1	1.00	1.08	0.84	0.92	2.40	0.00	0.92	1.08
time (sec)	N/A	0.237	168.432	0.434	2.477	0.344	0.000	4.420	16.964

Problem 738	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	25	27	21	23	58	0	23	27
N.S.	1	1.00	1.08	0.84	0.92	2.32	0.00	0.92	1.08
time (sec)	N/A	0.234	131.812	0.477	2.468	0.412	0.000	4.269	16.403

Problem 739	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	25	27	21	23	58	0	23	27
N.S.	1	1.00	1.08	0.84	0.92	2.32	0.00	0.92	1.08
time (sec)	N/A	0.235	125.057	0.434	2.199	0.339	0.000	2.819	16.404

Problem 740	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	25	27	21	23	49	0	23	27
N.S.	1	1.00	1.08	0.84	0.92	1.96	0.00	0.92	1.08
time (sec)	N/A	0.236	127.237	0.471	2.136	1.071	0.000	4.485	16.005

Problem 741	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	25	27	21	23	49	0	23	27
N.S.	1	1.00	1.08	0.84	0.92	1.96	0.00	0.92	1.08
time (sec)	N/A	0.231	107.663	0.489	2.000	0.711	0.000	2.858	16.203

Problem 742	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	F(-1)	N/A	N/A	N/A	F(-1)	F(-1)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	25	0	21	23	49	0	0	27
N.S.	1	1.00	0.00	0.84	0.92	1.96	0.00	0.00	1.08
time (sec)	N/A	0.240	0.000	0.426	2.079	1.083	0.000	0.000	17.077

Problem 743	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	F(-1)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	25	27	21	23	49	0	0	27
N.S.	1	1.00	1.08	0.84	0.92	1.96	0.00	0.00	1.08
time (sec)	N/A	0.231	159.669	0.415	2.113	0.757	0.000	0.000	14.209

Problem 744	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	F(-2)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	25	27	21	23	49	0	0	27
N.S.	1	1.00	1.08	0.84	0.92	1.96	0.00	0.00	1.08
time (sec)	N/A	0.243	167.847	0.437	2.102	1.056	0.000	0.000	17.023

Problem 745	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	F(-1)	N/A	N/A	N/A	F(-1)	F(-1)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	25	0	21	23	49	0	0	27
N.S.	1	1.00	0.00	0.84	0.92	1.96	0.00	0.00	1.08
time (sec)	N/A	0.238	0.000	0.441	2.108	0.698	0.000	0.000	14.165

Problem 746	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	F(-1)	N/A	N/A	N/A	F(-1)	F(-2)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	25	0	21	23	49	0	0	27
N.S.	1	1.00	0.00	0.84	0.92	1.96	0.00	0.00	1.08
time (sec)	N/A	0.235	0.000	0.452	2.149	1.137	0.000	0.000	17.040

Problem 747	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	F(-1)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	25	27	21	23	23	0	0	27
N.S.	1	1.00	1.08	0.84	0.92	0.92	0.00	0.00	1.08
time (sec)	N/A	0.232	105.062	0.472	0.767	0.332	0.000	0.000	15.588

Problem 748	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	F(-1)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	25	27	21	23	23	0	0	27
N.S.	1	1.00	1.08	0.84	0.92	0.92	0.00	0.00	1.08
time (sec)	N/A	0.230	124.662	0.410	0.778	0.367	0.000	0.000	15.418

Problem 749	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	25	27	21	23	23	24	0	27
N.S.	1	1.00	1.08	0.84	0.92	0.92	0.96	0.00	1.08
time (sec)	N/A	0.236	17.638	0.484	0.805	0.321	123.114	0.000	15.711

Problem 750	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	25	27	21	23	23	24	0	27
N.S.	1	1.00	1.08	0.84	0.92	0.92	0.96	0.00	1.08
time (sec)	N/A	0.233	17.873	0.501	0.747	1.209	2.963	0.000	15.611

Problem 751	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	25	27	21	23	23	24	0	27
N.S.	1	1.00	1.08	0.84	0.92	0.92	0.96	0.00	1.08
time (sec)	N/A	0.232	2.330	0.406	0.744	0.474	1.142	0.000	15.275

Problem 752	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	25	27	21	23	44	26	0	27
N.S.	1	1.00	1.08	0.84	0.92	1.76	1.04	0.00	1.08
time (sec)	N/A	0.233	3.584	0.471	0.762	1.097	0.717	0.000	16.201

Problem 753	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	25	27	21	23	44	26	0	27
N.S.	1	1.00	1.08	0.84	0.92	1.76	1.04	0.00	1.08
time (sec)	N/A	0.226	103.358	0.454	0.764	0.687	1.536	0.000	14.106

Problem 754	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	25	27	21	23	46	26	0	27
N.S.	1	1.00	1.08	0.84	0.92	1.84	1.04	0.00	1.08
time (sec)	N/A	0.229	107.784	0.473	0.809	1.047	12.067	0.000	17.424

Problem 755	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	25	27	21	23	46	26	0	27
N.S.	1	1.00	1.08	0.84	0.92	1.84	1.04	0.00	1.08
time (sec)	N/A	0.227	122.161	0.555	1.302	0.766	28.316	0.000	14.410

Problem 756	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	F(-1)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	25	27	21	23	46	0	0	27
N.S.	1	1.00	1.08	0.84	0.92	1.84	0.00	0.00	1.08
time (sec)	N/A	0.232	138.691	0.468	1.308	1.123	0.000	0.000	17.530

Problem 757	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	F(-1)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	25	27	21	23	51	0	0	27
N.S.	1	1.00	1.08	0.84	0.92	2.04	0.00	0.00	1.08
time (sec)	N/A	0.244	135.060	0.477	0.757	0.367	0.000	0.000	16.591

Problem 758	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	F(-1)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	25	27	21	23	51	0	0	27
N.S.	1	1.00	1.08	0.84	0.92	2.04	0.00	0.00	1.08
time (sec)	N/A	0.237	143.550	0.409	0.766	0.405	0.000	0.000	16.534

Problem 759	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	25	27	21	23	51	24	0	27
N.S.	1	1.00	1.08	0.84	0.92	2.04	0.96	0.00	1.08
time (sec)	N/A	0.231	125.267	0.448	0.774	0.387	137.513	0.000	16.495

Problem 760	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	25	27	21	23	51	24	0	27
N.S.	1	1.00	1.08	0.84	0.92	2.04	0.96	0.00	1.08
time (sec)	N/A	0.237	140.734	0.468	0.737	1.128	10.725	0.000	16.619

Problem 761	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	25	27	21	23	51	24	0	27
N.S.	1	1.00	1.08	0.84	0.92	2.04	0.96	0.00	1.08
time (sec)	N/A	0.235	120.847	0.444	0.743	0.749	3.919	0.000	16.799

Problem 762	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	25	27	21	23	60	26	0	27
N.S.	1	1.00	1.08	0.84	0.92	2.40	1.04	0.00	1.08
time (sec)	N/A	0.237	126.558	0.390	0.760	1.106	5.040	0.000	17.473

Problem 763	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	25	27	21	23	60	26	0	27
N.S.	1	1.00	1.08	0.84	0.92	2.40	1.04	0.00	1.08
time (sec)	N/A	0.236	128.176	0.481	1.279	0.875	12.661	0.000	14.643

Problem 764	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	25	27	21	23	62	26	0	27
N.S.	1	1.00	1.08	0.84	0.92	2.48	1.04	0.00	1.08
time (sec)	N/A	0.237	142.497	0.464	1.296	1.174	72.583	0.000	16.737

Problem 765	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	25	27	21	23	62	26	0	27
N.S.	1	1.00	1.08	0.84	0.92	2.48	1.04	0.00	1.08
time (sec)	N/A	0.238	110.662	0.474	1.307	0.975	166.153	0.000	13.641

Problem 766	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	F(-1)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	25	27	21	23	62	0	0	27
N.S.	1	1.00	1.08	0.84	0.92	2.48	0.00	0.00	1.08
time (sec)	N/A	0.241	147.763	0.432	1.310	1.110	0.000	0.000	18.365

Problem 767	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	F(-1)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	25	27	21	23	67	0	0	27
N.S.	1	1.00	1.08	0.84	0.92	2.68	0.00	0.00	1.08
time (sec)	N/A	0.244	129.791	0.478	0.761	0.451	0.000	0.000	17.151

Problem 768	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	F(-1)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	25	27	21	23	67	0	0	27
N.S.	1	1.00	1.08	0.84	0.92	2.68	0.00	0.00	1.08
time (sec)	N/A	0.242	148.846	0.459	0.780	0.415	0.000	0.000	18.057

Problem 769	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	F(-1)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	25	27	21	23	67	0	0	27
N.S.	1	1.00	1.08	0.84	0.92	2.68	0.00	0.00	1.08
time (sec)	N/A	0.241	126.546	0.403	0.800	0.444	0.000	0.000	17.974

Problem 770	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	25	27	21	23	67	24	0	27
N.S.	1	1.00	1.08	0.84	0.92	2.68	0.96	0.00	1.08
time (sec)	N/A	0.238	144.619	0.481	0.753	1.103	62.853	0.000	19.302

Problem 771	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	25	27	21	23	67	24	0	27
N.S.	1	1.00	1.08	0.84	0.92	2.68	0.96	0.00	1.08
time (sec)	N/A	0.236	131.283	0.477	1.282	1.194	25.263	0.000	19.340

Problem 772	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	25	27	21	23	76	26	0	27
N.S.	1	1.00	1.08	0.84	0.92	3.04	1.04	0.00	1.08
time (sec)	N/A	0.242	148.092	0.395	1.280	1.126	78.157	0.000	17.811

Problem 773	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	25	27	21	23	76	26	0	27
N.S.	1	1.00	1.08	0.84	0.92	3.04	1.04	0.00	1.08
time (sec)	N/A	0.237	135.389	0.476	1.293	1.619	166.166	0.000	14.449

Problem 774	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	F(-1)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	25	27	21	23	78	0	0	27
N.S.	1	1.00	1.08	0.84	0.92	3.12	0.00	0.00	1.08
time (sec)	N/A	0.239	153.892	0.481	1.331	1.143	0.000	0.000	19.245

Problem 775	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	F(-1)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	25	27	21	23	78	0	0	27
N.S.	1	1.00	1.08	0.84	0.92	3.12	0.00	0.00	1.08
time (sec)	N/A	0.239	114.955	0.390	2.278	2.142	0.000	0.000	14.519

Problem 776	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	F(-1)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	25	27	21	23	78	0	0	27
N.S.	1	1.00	1.08	0.84	0.92	3.12	0.00	0.00	1.08
time (sec)	N/A	0.237	160.987	0.406	2.320	1.170	0.000	0.000	19.191

Problem 777	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	251	261	231	0	0	0	0	0	0
N.S.	1	1.04	0.92	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.386	0.928	0.000	0.000	0.000	0.000	0.000	0.000

Problem 778	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	181	180	171	0	0	0	0	0	0
N.S.	1	0.99	0.94	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.787	0.347	0.000	0.000	0.000	0.000	0.000	0.000

Problem 779	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	137	137	107	0	0	0	0	0	0
N.S.	1	1.00	0.78	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.440	0.168	0.000	0.000	0.000	0.000	0.000	0.000

Problem 780	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	192	207	5280	0	0	0	0	0	0
N.S.	1	1.08	27.50	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.697	35.149	0.000	0.000	0.000	0.000	0.000	0.000

Problem 781	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	299	310	13816	0	0	0	0	0	0
N.S.	1	1.04	46.21	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.823	52.095	0.000	0.000	0.000	0.000	0.000	0.000

Problem 782	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	25	27	23	25	25	24	25	29
N.S.	1	1.00	1.08	0.92	1.00	1.00	0.96	1.00	1.16
time (sec)	N/A	0.241	88.891	0.679	1.066	0.283	26.361	0.981	15.766

Problem 783	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	25	27	23	25	25	24	25	29
N.S.	1	1.00	1.08	0.92	1.00	1.00	0.96	1.00	1.16
time (sec)	N/A	0.240	0.934	0.680	1.052	0.287	0.604	0.548	14.915

Problem 784	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	25	27	23	25	25	24	25	29
N.S.	1	1.00	1.08	0.92	1.00	1.00	0.96	1.00	1.16
time (sec)	N/A	0.246	22.050	0.650	1.078	0.275	0.432	0.688	15.531

Problem 785	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	25	27	23	25	53	24	25	29
N.S.	1	1.00	1.08	0.92	1.00	2.12	0.96	1.00	1.16
time (sec)	N/A	0.251	22.405	0.561	1.064	0.313	1.638	1.102	17.778

Problem 786	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	21	21	23	21	23	23	20	23	27
N.S.	1	1.00	1.10	1.00	1.10	1.10	0.95	1.10	1.29
time (sec)	N/A	0.222	8.388	0.617	1.244	0.300	5.622	0.513	15.412

Problem 787	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	23	25	25	22	25	29
N.S.	1	1.00	1.09	1.00	1.09	1.09	0.96	1.09	1.26
time (sec)	N/A	0.230	1.203	0.580	1.248	0.289	5.656	0.544	15.369

Problem 788	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	273	273	8899	0	0	0	0	0	0
N.S.	1	1.00	32.60	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.756	27.249	0.000	0.000	0.000	0.000	0.000	0.000

Problem 789	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	220	220	5564	0	0	0	0	0	0
N.S.	1	1.00	25.29	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.478	23.088	0.000	0.000	0.000	0.000	0.000	0.000

Problem 790	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	103	103	2828	0	0	0	0	0	0
N.S.	1	1.00	27.46	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.261	15.549	0.000	0.000	0.000	0.000	0.000	0.000

Problem 791	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	12	12	14	12	14	14	12	14	16
N.S.	1	1.00	1.17	1.00	1.17	1.17	1.00	1.17	1.33
time (sec)	N/A	0.180	6.649	0.596	1.029	0.261	0.772	0.384	14.079

Problem 792	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	19	19	21	19	21	21	19	21	23
N.S.	1	1.00	1.11	1.00	1.11	1.11	1.00	1.11	1.21
time (sec)	N/A	0.210	13.483	0.563	1.561	0.262	7.936	0.472	14.101

Problem 793	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	21	21	23	21	23	23	20	23	25
N.S.	1	1.00	1.10	1.00	1.10	1.10	0.95	1.10	1.19
time (sec)	N/A	0.218	14.317	1.683	2.221	0.283	37.774	0.490	14.464

Problem 794	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	145	90	318	0	159	0	0	87
N.S.	1	1.07	0.67	2.36	0.00	1.18	0.00	0.00	0.64
time (sec)	N/A	0.591	0.476	12.217	0.000	0.102	0.000	0.000	14.087

Problem 795	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	117	77	290	0	148	0	0	87
N.S.	1	1.05	0.69	2.61	0.00	1.33	0.00	0.00	0.78
time (sec)	N/A	0.561	0.613	10.972	0.000	0.096	0.000	0.000	14.370

Problem 796	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	89	66	262	0	137	0	0	80
N.S.	1	1.02	0.76	3.01	0.00	1.57	0.00	0.00	0.92
time (sec)	N/A	0.448	0.320	9.102	0.000	0.099	0.000	0.000	14.353

Problem 797	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	62	53	228	0	125	0	0	53
N.S.	1	1.02	0.87	3.74	0.00	2.05	0.00	0.00	0.87
time (sec)	N/A	0.424	0.171	7.037	0.000	0.092	0.000	0.000	0.179

Problem 798	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	32	152	0	107	0	0	33
N.S.	1	1.00	0.91	4.34	0.00	3.06	0.00	0.00	0.94
time (sec)	N/A	0.347	0.216	4.544	0.000	0.093	0.000	0.000	0.239

Problem 799	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	58	51	150	0	156	0	0	60
N.S.	1	1.02	0.89	2.63	0.00	2.74	0.00	0.00	1.05
time (sec)	N/A	0.423	0.240	5.725	0.000	0.090	0.000	0.000	14.579

Problem 800	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	85	65	396	0	175	0	0	87
N.S.	1	1.02	0.78	4.77	0.00	2.11	0.00	0.00	1.05
time (sec)	N/A	0.460	0.528	11.237	0.000	0.102	0.000	0.000	14.955

Problem 801	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	113	95	502	0	188	0	0	87
N.S.	1	1.02	0.86	4.52	0.00	1.69	0.00	0.00	0.78
time (sec)	N/A	0.560	0.424	15.048	0.000	0.099	0.000	0.000	15.042

Problem 802	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	160	219	113	398	0	195	0	0	135
N.S.	1	1.37	0.71	2.49	0.00	1.22	0.00	0.00	0.84
time (sec)	N/A	1.282	1.340	30.085	0.000	0.102	0.000	0.000	14.662

Problem 803	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	191	98	362	0	180	0	0	128
N.S.	1	1.41	0.73	2.68	0.00	1.33	0.00	0.00	0.95
time (sec)	N/A	1.181	1.034	32.790	0.000	0.099	0.000	0.000	14.195

Problem 804	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	163	79	357	0	162	0	0	102
N.S.	1	1.61	0.78	3.53	0.00	1.60	0.00	0.00	1.01
time (sec)	N/A	0.946	0.637	28.579	0.000	0.098	0.000	0.000	14.002

Problem 805	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	133	64	283	0	147	0	0	76
N.S.	1	1.85	0.89	3.93	0.00	2.04	0.00	0.00	1.06
time (sec)	N/A	0.823	1.216	7.918	0.000	0.097	0.000	0.000	13.711

Problem 806	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	129	62	202	0	178	0	0	81
N.S.	1	1.90	0.91	2.97	0.00	2.62	0.00	0.00	1.19
time (sec)	N/A	0.795	0.863	8.265	0.000	0.097	0.000	0.000	14.106

Problem 807	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	157	73	513	0	198	0	0	108
N.S.	1	1.65	0.77	5.40	0.00	2.08	0.00	0.00	1.14
time (sec)	N/A	0.939	1.057	12.443	0.000	0.096	0.000	0.000	14.590

Problem 808	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	187	124	633	0	223	0	0	113
N.S.	1	1.39	0.92	4.69	0.00	1.65	0.00	0.00	0.84
time (sec)	N/A	1.086	0.816	18.059	0.000	0.107	0.000	0.000	14.545

Problem 809	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	160	215	142	689	0	235	0	0	113
N.S.	1	1.34	0.89	4.31	0.00	1.47	0.00	0.00	0.71
time (sec)	N/A	1.283	1.111	22.420	0.000	0.109	0.000	0.000	14.707

Problem 810	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	194	243	137	470	0	227	0	0	178
N.S.	1	1.25	0.71	2.42	0.00	1.17	0.00	0.00	0.92
time (sec)	N/A	1.490	1.679	165.945	0.000	0.116	0.000	0.000	14.499

Problem 811	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	159	215	110	421	0	205	0	0	146
N.S.	1	1.35	0.69	2.65	0.00	1.29	0.00	0.00	0.92
time (sec)	N/A	1.331	1.273	186.952	0.000	0.106	0.000	0.000	14.172

Problem 812	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	178	84	412	0	185	0	0	125
N.S.	1	1.53	0.72	3.55	0.00	1.59	0.00	0.00	1.08
time (sec)	N/A	1.102	1.227	169.414	0.000	0.103	0.000	0.000	14.036

Problem 813	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	188	87	303	0	214	0	0	124
N.S.	1	1.49	0.69	2.40	0.00	1.70	0.00	0.00	0.98
time (sec)	N/A	1.164	1.228	8.746	0.000	0.103	0.000	0.000	13.564

Problem 814	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	180	84	630	0	222	0	0	128
N.S.	1	1.53	0.71	5.34	0.00	1.88	0.00	0.00	1.08
time (sec)	N/A	1.133	1.913	13.128	0.000	0.112	0.000	0.000	15.225

Problem 815	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	202	125	711	0	244	0	0	156
N.S.	1	1.36	0.84	4.77	0.00	1.64	0.00	0.00	1.05
time (sec)	N/A	1.318	1.543	17.427	0.000	0.103	0.000	0.000	14.901

Problem 816	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-1)	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	194	239	177	820	0	270	0	0	147
N.S.	1	1.23	0.91	4.23	0.00	1.39	0.00	0.00	0.76
time (sec)	N/A	1.524	1.460	22.425	0.000	0.109	0.000	0.000	15.156

Problem 817	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	245	226	668	0	0	0	0	0
N.S.	1	1.61	1.49	4.39	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.869	12.072	10.119	0.000	0.000	0.000	0.000	0.000

Problem 818	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	199	158	552	0	0	0	0	0
N.S.	1	1.78	1.41	4.93	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.382	1.831	7.418	0.000	0.000	0.000	0.000	0.000

Problem 819	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	B	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	156	81	226	0	0	0	0	0
N.S.	1	2.08	1.08	3.01	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.033	10.679	6.112	0.000	0.000	0.000	0.000	0.000

Problem 820	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	48	187	0	0	0	0	0
N.S.	1	1.00	0.91	3.53	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.557	0.254	4.803	0.000	0.000	0.000	0.000	0.000

Problem 821	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	29	150	0	0	0	0	0
N.S.	1	1.00	1.00	5.17	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.413	0.252	3.993	0.000	0.000	0.000	0.000	0.000

Problem 822	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	137	195	353	0	0	0	0	0
N.S.	1	1.78	2.53	4.58	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.919	3.136	6.529	0.000	0.000	0.000	0.000	0.000

Problem 823	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	222	210	423	0	0	0	0	0
N.S.	1	1.73	1.64	3.30	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.750	4.264	13.703	0.000	0.000	0.000	0.000	0.000

Problem 824	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	244	310	266	1064	0	0	0	0	0
N.S.	1	1.27	1.09	4.36	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.154	2.106	15.538	0.000	0.000	0.000	0.000	0.000

Problem 825	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	184	253	252	809	0	0	0	0	0
N.S.	1	1.38	1.37	4.40	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.676	2.061	15.009	0.000	0.000	0.000	0.000	0.000

Problem 826	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	237	194	788	0	0	0	0	0
N.S.	1	1.42	1.16	4.72	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.519	3.952	13.511	0.000	0.000	0.000	0.000	0.000

Problem 827	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	223	229	707	0	0	0	0	0
N.S.	1	1.51	1.55	4.78	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.452	3.718	11.990	0.000	0.000	0.000	0.000	0.000

Problem 828	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-1)	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	233	239	608	0	0	0	0	0
N.S.	1	1.51	1.55	3.95	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.562	3.807	10.203	0.000	0.000	0.000	0.000	0.000

Problem 829	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-1)	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	219	287	278	841	0	0	0	0	0
N.S.	1	1.31	1.27	3.84	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.080	3.007	16.217	0.000	0.000	0.000	0.000	0.000

Problem 830	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	346	424	353	2216	0	0	0	0	0
N.S.	1	1.23	1.02	6.40	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.998	4.176	33.714	0.000	0.000	0.000	0.000	0.000

Problem 831	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	282	363	311	1957	0	0	0	0	0
N.S.	1	1.29	1.10	6.94	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.498	3.217	30.588	0.000	0.000	0.000	0.000	0.000

Problem 832	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	263	342	286	1936	0	0	0	0	0
N.S.	1	1.30	1.09	7.36	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.281	3.616	26.373	0.000	0.000	0.000	0.000	0.000

Problem 833	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-1)	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	246	328	272	1858	0	0	0	0	0
N.S.	1	1.33	1.11	7.55	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.188	2.320	25.322	0.000	0.000	0.000	0.000	0.000

Problem 834	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-1)	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	253	331	289	1760	0	0	0	0	0
N.S.	1	1.31	1.14	6.96	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.300	3.225	25.125	0.000	0.000	0.000	0.000	0.000

Problem 835	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-1)	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	255	339	297	1203	0	0	0	0	0
N.S.	1	1.33	1.16	4.72	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.311	3.150	15.591	0.000	0.000	0.000	0.000	0.000

Problem 836	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-1)	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	328	403	334	1987	0	0	0	0	0
N.S.	1	1.23	1.02	6.06	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	3.001	4.033	32.898	0.000	0.000	0.000	0.000	0.000

Problem 837	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	244	274	340	2306	0	453	0	0	0
N.S.	1	1.12	1.39	9.45	0.00	1.86	0.00	0.00	0.00
time (sec)	N/A	2.051	10.208	6.674	0.000	0.131	0.000	0.000	0.000

Problem 838	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	192	214	273	1313	0	415	0	0	0
N.S.	1	1.11	1.42	6.84	0.00	2.16	0.00	0.00	0.00
time (sec)	N/A	1.544	10.019	5.962	0.000	0.119	0.000	0.000	0.000

Problem 839	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	67	67	244	869	0	355	0	0	0
N.S.	1	1.00	3.64	12.97	0.00	5.30	0.00	0.00	0.00
time (sec)	N/A	0.522	6.737	3.806	0.000	0.107	0.000	0.000	0.000

Problem 840	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	138	159	14885	245	0	0	0	0	0
N.S.	1	1.15	107.86	1.78	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.344	31.602	7.009	0.000	0.000	0.000	0.000	0.000

Problem 841	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	237	275	33693	995	0	0	0	0	0
N.S.	1	1.16	142.16	4.20	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.442	32.381	8.003	0.000	0.000	0.000	0.000	0.000

Problem 842	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	303	332	383	2766	0	491	0	0	0
N.S.	1	1.10	1.26	9.13	0.00	1.62	0.00	0.00	0.00
time (sec)	N/A	2.725	11.219	8.020	0.000	0.119	0.000	0.000	0.000

Problem 843	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	240	259	344	2302	0	455	0	0	0
N.S.	1	1.08	1.43	9.59	0.00	1.90	0.00	0.00	0.00
time (sec)	N/A	2.026	9.274	6.901	0.000	0.108	0.000	0.000	0.000

Problem 844	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	187	209	284	1594	0	415	0	0	0
N.S.	1	1.12	1.52	8.52	0.00	2.22	0.00	0.00	0.00
time (sec)	N/A	1.598	7.717	5.672	0.000	0.101	0.000	0.000	0.000

Problem 845	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	209	230	25369	1163	0	0	0	0	0
N.S.	1	1.10	121.38	5.56	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.176	33.924	7.984	0.000	0.000	0.000	0.000	0.000

Problem 846	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	249	276	35156	1585	0	0	0	0	0
N.S.	1	1.11	141.19	6.37	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.722	33.139	9.301	0.000	0.000	0.000	0.000	0.000

Problem 847	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	299	329	51904	2317	0	0	0	0	0
N.S.	1	1.10	173.59	7.75	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	3.421	34.020	10.143	0.000	0.000	0.000	0.000	0.000

Problem 848	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	363	399	477	3822	0	531	0	0	0
N.S.	1	1.10	1.31	10.53	0.00	1.46	0.00	0.00	0.00
time (sec)	N/A	3.478	15.611	1492.865	0.000	0.133	0.000	0.000	0.000

Problem 849	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	303	333	419	2766	0	491	0	0	0
N.S.	1	1.10	1.38	9.13	0.00	1.62	0.00	0.00	0.00
time (sec)	N/A	2.777	13.799	1253.641	0.000	0.118	0.000	0.000	0.000

Problem 850	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	239	271	391	2585	0	454	0	0	0
N.S.	1	1.13	1.64	10.82	0.00	1.90	0.00	0.00	0.00
time (sec)	N/A	2.154	13.168	1324.395	0.000	0.109	0.000	0.000	0.000

Problem 851	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	262	284	60796	2588	0	0	0	0	0
N.S.	1	1.08	232.05	9.88	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.919	79.327	9.827	0.000	0.000	0.000	0.000	0.000

Problem 852	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	263	291	44895	2544	0	0	0	0	0
N.S.	1	1.11	170.70	9.67	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.986	34.508	10.743	0.000	0.000	0.000	0.000	0.000

Problem 853	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	314	345	53538	2634	0	0	0	0	0
N.S.	1	1.10	170.50	8.39	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	3.719	33.439	12.012	0.000	0.000	0.000	0.000	0.000

Problem 854	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	C	C	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	N/A	No	Yes	TBD	TBD	TBD	TBD	TBD
size	369	0	62811	3055	0	0	0	0	0
N.S.	1	0.00	170.22	8.28	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	34.246	13.002	0.000	0.000	0.000	0.000	0.000

Problem 855	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	249	282	340	2307	0	456	0	0	0
N.S.	1	1.13	1.37	9.27	0.00	1.83	0.00	0.00	0.00
time (sec)	N/A	2.148	10.190	9.737	0.000	0.130	0.000	0.000	0.000

Problem 856	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	195	220	265	1316	0	415	0	0	0
N.S.	1	1.13	1.36	6.75	0.00	2.13	0.00	0.00	0.00
time (sec)	N/A	1.599	9.664	8.185	0.000	0.103	0.000	0.000	0.000

Problem 857	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	163	216	739	0	355	0	0	0
N.S.	1	1.15	1.52	5.20	0.00	2.50	0.00	0.00	0.00
time (sec)	N/A	1.242	4.938	7.377	0.000	0.098	0.000	0.000	0.000

Problem 858	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	102	135	0	146	0	0	0
N.S.	1	1.00	1.52	2.01	0.00	2.18	0.00	0.00	0.00
time (sec)	N/A	0.518	1.963	6.164	0.000	0.089	0.000	0.000	0.000

Problem 859	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	14986	195	0	0	0	0	0
N.S.	1	1.00	220.38	2.87	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.635	6.902	7.574	0.000	0.000	0.000	0.000	0.000

Problem 860	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	246	280	30674	1281	0	0	0	0	0
N.S.	1	1.14	124.69	5.21	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.466	32.589	9.717	0.000	0.000	0.000	0.000	0.000

Problem 861	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	312	333	51912	2319	0	0	0	0	0
N.S.	1	1.07	166.38	7.43	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	3.365	34.199	12.114	0.000	0.000	0.000	0.000	0.000

Problem 862	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	360	380	419	2421	0	702	0	0	0
N.S.	1	1.06	1.16	6.72	0.00	1.95	0.00	0.00	0.00
time (sec)	N/A	3.019	9.074	10.717	0.000	0.160	0.000	0.000	0.000

Problem 863	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	289	316	382	1711	0	644	0	0	0
N.S.	1	1.09	1.32	5.92	0.00	2.23	0.00	0.00	0.00
time (sec)	N/A	2.263	6.628	10.273	0.000	0.146	0.000	0.000	0.000

Problem 864	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	214	249	330	925	0	585	0	0	0
N.S.	1	1.16	1.54	4.32	0.00	2.73	0.00	0.00	0.00
time (sec)	N/A	1.770	7.196	8.857	0.000	0.115	0.000	0.000	0.000

Problem 865	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	200	232	245	593	0	541	0	0	0
N.S.	1	1.16	1.22	2.96	0.00	2.70	0.00	0.00	0.00
time (sec)	N/A	1.610	6.370	7.520	0.000	0.101	0.000	0.000	0.000

Problem 866	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	147	239	587	0	500	0	0	0
N.S.	1	1.17	1.90	4.66	0.00	3.97	0.00	0.00	0.00
time (sec)	N/A	0.774	6.076	4.344	0.000	0.110	0.000	0.000	0.000

Problem 867	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	206	235	48278	1019	0	0	0	0	0
N.S.	1	1.14	234.36	4.95	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.833	32.909	8.870	0.000	0.000	0.000	0.000	0.000

Problem 868	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	345	381	52199	2300	0	0	0	0	0
N.S.	1	1.10	151.30	6.67	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	3.755	32.827	10.160	0.000	0.000	0.000	0.000	0.000

Problem 869	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	391	420	527	4534	0	971	0	0	0
N.S.	1	1.07	1.35	11.60	0.00	2.48	0.00	0.00	0.00
time (sec)	N/A	3.186	12.413	10.852	0.000	0.178	0.000	0.000	0.000

Problem 870	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	317	353	507	4120	0	884	0	0	0
N.S.	1	1.11	1.60	13.00	0.00	2.79	0.00	0.00	0.00
time (sec)	N/A	2.535	18.246	9.193	0.000	0.162	0.000	0.000	0.000

Problem 871	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	302	335	398	2885	0	835	0	0	0
N.S.	1	1.11	1.32	9.55	0.00	2.76	0.00	0.00	0.00
time (sec)	N/A	2.379	8.734	8.099	0.000	0.143	0.000	0.000	0.000

Problem 872	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	281	309	447	2304	0	782	0	0	0
N.S.	1	1.10	1.59	8.20	0.00	2.78	0.00	0.00	0.00
time (sec)	N/A	2.186	11.967	5.795	0.000	0.144	0.000	0.000	0.000

Problem 873	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	277	316	311	1999	0	709	0	0	0
N.S.	1	1.14	1.12	7.22	0.00	2.56	0.00	0.00	0.00
time (sec)	N/A	2.295	8.709	6.171	0.000	0.117	0.000	0.000	0.000

Problem 874	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	370	411	93062	4415	0	0	0	0	0
N.S.	1	1.11	251.52	11.93	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	3.980	34.585	9.291	0.000	0.000	0.000	0.000	0.000

Problem 875	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	266	299	222	0	0	0	0	0	0
N.S.	1	1.12	0.83	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.643	0.648	0.000	0.000	0.000	0.000	0.000	0.000

Problem 876	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	186	204	161	0	0	0	0	0	0
N.S.	1	1.10	0.87	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.966	0.312	0.000	0.000	0.000	0.000	0.000	0.000

Problem 877	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	132	137	106	0	0	0	0	0	0
N.S.	1	1.04	0.80	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.435	0.097	0.000	0.000	0.000	0.000	0.000	0.000

Problem 878	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	196	209	5216	0	0	0	0	0	0
N.S.	1	1.07	26.61	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.793	27.480	0.000	0.000	0.000	0.000	0.000	0.000

Problem 879	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	309	316	13974	0	0	0	0	0	0
N.S.	1	1.02	45.22	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.874	51.681	0.000	0.000	0.000	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [87] had the largest ratio of [1.33332999999999990]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	11	10	1.06	19	0.526
2	A	9	8	1.02	19	0.421
3	A	9	8	1.02	19	0.421
4	A	7	6	1.00	17	0.353
5	A	1	1	1.00	10	0.100
6	A	5	5	1.00	17	0.294
7	A	6	6	1.03	19	0.316
8	A	8	7	1.02	19	0.368
9	A	10	9	1.07	19	0.474
10	A	13	12	0.99	21	0.571
11	A	11	10	0.99	21	0.476
12	A	11	10	1.08	21	0.476
13	A	9	8	1.00	19	0.421
14	A	7	6	1.00	12	0.500
15	A	7	7	1.00	19	0.368
16	A	6	6	1.00	21	0.286
17	A	10	9	1.11	21	0.429
18	A	10	9	0.99	21	0.429
19	A	14	13	0.91	21	0.619
20	A	3	3	1.00	21	0.143
21	A	3	3	1.00	21	0.143
22	A	3	3	1.00	19	0.158

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
23	A	9	8	1.05	12	0.667
24	A	3	3	1.00	19	0.158
25	A	3	3	1.00	21	0.143
26	A	3	3	1.00	21	0.143
27	A	3	3	1.00	21	0.143
28	A	3	3	1.00	21	0.143
29	A	3	3	1.00	21	0.143
30	A	3	3	1.00	21	0.143
31	A	3	3	1.00	21	0.143
32	A	3	3	1.00	19	0.158
33	A	12	11	1.08	12	0.917
34	A	3	3	1.00	19	0.158
35	A	3	3	1.00	21	0.143
36	A	3	3	1.00	21	0.143
37	A	3	3	1.00	21	0.143
38	A	3	3	1.00	21	0.143
39	A	3	3	1.00	21	0.143
40	A	3	3	1.00	21	0.143
41	A	3	3	1.00	21	0.143
42	A	11	10	0.99	21	0.476
43	A	11	10	1.02	21	0.476
44	A	7	7	1.00	21	0.333
45	A	5	5	1.00	21	0.238
46	A	2	2	1.00	19	0.105
47	A	3	3	1.00	12	0.250
48	A	8	8	1.02	19	0.421
49	A	9	9	1.01	21	0.429
50	A	11	10	0.98	21	0.476
51	A	13	12	1.00	21	0.571
52	A	13	12	1.05	21	0.571
53	A	13	12	1.09	21	0.571
54	A	8	8	1.06	21	0.381
55	A	4	4	0.98	21	0.190
56	A	4	4	0.98	19	0.211

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
57	A	7	7	1.07	12	0.583
58	A	10	10	1.11	19	0.526
59	A	11	11	1.05	21	0.524
60	A	14	13	1.10	21	0.619
61	A	15	14	1.09	21	0.667
62	A	16	15	1.08	21	0.714
63	A	10	10	1.10	21	0.476
64	A	7	7	1.02	21	0.333
65	A	6	6	1.05	21	0.286
66	A	6	6	1.05	19	0.316
67	A	10	10	1.11	12	0.833
68	A	13	13	1.11	19	0.684
69	A	13	13	1.10	21	0.619
70	A	18	17	1.09	21	0.810
71	A	17	16	1.15	21	0.762
72	A	13	13	1.11	21	0.619
73	A	9	9	1.05	21	0.429
74	A	9	9	1.05	21	0.429
75	A	8	8	1.07	21	0.381
76	A	8	8	1.07	19	0.421
77	A	13	13	1.14	12	1.083
78	A	14	14	1.19	19	0.737
79	A	16	16	1.10	21	0.762
80	A	20	19	1.16	21	0.905
81	A	15	15	1.16	21	0.714
82	A	11	11	1.05	21	0.524
83	A	11	11	1.05	21	0.524
84	A	11	11	1.06	21	0.524
85	A	10	10	1.07	21	0.476
86	A	10	10	1.07	19	0.526
87	A	16	16	1.19	12	1.333
88	A	17	17	1.20	19	0.895
89	A	18	18	1.12	21	0.857
90	A	9	9	1.13	23	0.391

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
91	A	7	7	1.13	23	0.304
92	A	4	4	1.00	23	0.174
93	A	2	2	1.00	21	0.095
94	A	4	3	1.00	14	0.214
95	A	6	5	1.00	21	0.238
96	A	8	7	0.99	23	0.304
97	A	10	9	1.03	23	0.391
98	A	12	11	1.05	23	0.478
99	A	12	12	1.12	23	0.522
100	A	9	9	1.13	23	0.391
101	A	6	6	1.06	23	0.261
102	A	4	4	1.00	21	0.190
103	A	7	6	1.00	14	0.429
104	A	9	8	1.45	21	0.381
105	A	9	8	0.98	23	0.348
106	A	11	10	1.01	23	0.435
107	A	14	14	1.11	23	0.609
108	A	11	11	1.13	23	0.478
109	A	8	8	1.08	23	0.348
110	A	6	6	1.04	21	0.286
111	A	10	9	1.04	14	0.643
112	A	9	8	1.03	21	0.381
113	A	9	8	1.02	23	0.348
114	A	11	10	1.03	23	0.435
115	A	13	12	1.04	23	0.522
116	A	2	2	1.00	22	0.091
117	A	4	3	1.00	15	0.200
118	A	6	5	1.00	22	0.227
119	A	11	10	1.10	23	0.435
120	A	9	8	1.08	23	0.348
121	A	6	5	1.00	23	0.217
122	A	4	3	1.00	21	0.143
123	A	8	7	1.00	14	0.500
124	A	9	8	1.01	21	0.381

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
125	A	13	12	1.08	23	0.522
126	A	15	14	1.09	23	0.609
127	A	12	11	1.06	23	0.478
128	A	9	8	1.04	23	0.348
129	A	6	5	1.00	23	0.217
130	A	6	5	1.00	21	0.238
131	A	11	10	1.05	14	0.714
132	A	14	13	1.06	21	0.619
133	A	17	16	1.08	23	0.696
134	A	15	14	1.09	23	0.609
135	A	12	11	1.07	23	0.478
136	A	9	8	1.06	23	0.348
137	A	8	7	1.05	23	0.304
138	A	8	7	1.05	21	0.333
139	A	14	13	1.08	14	0.929
140	A	17	16	1.09	21	0.762
141	A	4	3	1.00	22	0.136
142	A	8	7	1.00	15	0.467
143	A	13	12	1.10	23	0.522
144	A	10	9	1.08	23	0.391
145	A	8	7	1.08	21	0.333
146	A	8	7	1.00	14	0.500
147	A	7	6	1.00	21	0.286
148	A	14	13	1.11	23	0.565
149	A	11	10	1.09	23	0.435
150	A	9	8	1.09	21	0.381
151	A	8	7	1.00	14	0.500
152	A	7	6	1.00	21	0.286
153	A	15	14	1.05	23	0.609
154	A	12	11	1.03	23	0.478
155	A	9	8	1.00	23	0.348
156	A	7	6	1.00	21	0.286
157	A	8	7	1.00	14	0.500
158	A	7	6	1.00	21	0.286

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
159	A	19	18	1.02	23	0.783
160	A	16	15	1.00	23	0.652
161	A	14	13	1.01	23	0.565
162	A	13	12	1.00	21	0.571
163	A	8	7	1.11	14	0.500
164	A	7	6	1.11	21	0.286
165	A	11	11	1.01	21	0.524
166	A	9	9	1.02	21	0.429
167	A	9	9	1.01	21	0.429
168	A	7	7	1.00	21	0.333
169	A	9	9	1.01	21	0.429
170	A	9	9	1.02	21	0.429
171	A	11	11	1.04	21	0.524
172	A	17	17	1.00	23	0.739
173	A	15	15	0.99	23	0.652
174	A	13	13	1.00	23	0.565
175	A	7	7	1.00	23	0.304
176	A	11	11	1.00	23	0.478
177	A	13	13	1.00	23	0.565
178	A	15	15	1.01	23	0.652
179	A	3	3	1.00	23	0.130
180	A	3	3	1.00	23	0.130
181	A	3	3	1.00	23	0.130
182	A	3	3	1.00	23	0.130
183	A	3	3	1.00	23	0.130
184	A	3	3	1.00	23	0.130
185	A	3	3	1.00	23	0.130
186	A	3	3	1.00	23	0.130
187	A	3	3	1.00	23	0.130
188	A	3	3	1.00	23	0.130
189	A	3	3	1.00	23	0.130
190	A	3	3	1.00	23	0.130
191	A	3	3	1.00	23	0.130
192	A	3	3	1.00	23	0.130

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
193	A	3	3	1.00	23	0.130
194	A	12	12	1.02	23	0.522
195	A	12	12	1.03	23	0.522
196	A	10	10	1.05	23	0.435
197	A	9	9	1.05	23	0.391
198	A	10	10	1.04	23	0.435
199	A	12	12	1.03	23	0.522
200	A	12	12	1.02	23	0.522
201	A	15	15	1.04	23	0.652
202	A	14	14	1.03	23	0.609
203	A	13	13	1.07	23	0.565
204	A	7	7	1.00	23	0.304
205	A	13	13	1.07	23	0.565
206	A	12	12	1.04	23	0.522
207	A	15	15	1.05	23	0.652
208	A	14	14	1.07	23	0.609
209	A	17	17	1.06	23	0.739
210	A	17	17	1.06	23	0.739
211	A	15	15	1.08	23	0.652
212	A	16	16	1.08	23	0.696
213	A	15	15	1.08	23	0.652
214	A	16	16	1.08	23	0.696
215	A	15	15	1.08	23	0.652
216	A	17	17	1.08	23	0.739
217	A	18	18	1.08	23	0.783
218	A	8	7	0.99	25	0.280
219	A	6	5	1.00	25	0.200
220	A	4	3	1.00	25	0.120
221	A	2	2	1.00	25	0.080
222	A	4	4	1.00	25	0.160
223	A	6	6	1.04	25	0.240
224	A	8	8	1.07	25	0.320
225	A	11	10	1.01	25	0.400
226	A	9	8	0.98	25	0.320

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
227	A	7	6	1.00	25	0.240
228	A	7	6	1.00	25	0.240
229	A	4	4	1.00	25	0.160
230	A	6	6	1.04	25	0.240
231	A	9	9	1.03	25	0.360
232	A	11	11	1.04	25	0.440
233	A	13	12	1.04	25	0.480
234	A	11	10	1.02	25	0.400
235	A	9	8	1.02	25	0.320
236	A	9	8	1.06	25	0.320
237	A	9	8	1.03	25	0.320
238	A	6	6	1.03	25	0.240
239	A	8	8	1.06	25	0.320
240	A	11	11	1.05	25	0.440
241	A	13	13	1.06	25	0.520
242	A	3	3	1.00	25	0.120
243	A	4	3	1.00	25	0.120
244	A	4	3	1.00	28	0.107
245	A	10	9	1.07	25	0.360
246	A	8	7	1.00	25	0.280
247	A	4	3	1.00	25	0.120
248	A	6	5	1.00	25	0.200
249	A	8	7	1.06	25	0.280
250	A	11	10	1.11	25	0.400
251	A	14	13	1.05	25	0.520
252	A	11	10	1.04	25	0.400
253	A	6	5	1.00	25	0.200
254	A	6	5	1.00	25	0.200
255	A	9	8	1.01	25	0.320
256	A	12	11	1.06	25	0.440
257	A	15	14	1.08	25	0.560
258	A	17	16	1.07	25	0.640
259	A	14	13	1.07	25	0.520
260	A	8	7	1.04	25	0.280

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
261	A	8	7	1.04	25	0.280
262	A	9	8	1.04	25	0.320
263	A	12	11	1.06	25	0.440
264	A	15	14	1.08	25	0.560
265	A	13	12	1.05	23	0.522
266	A	10	9	1.07	23	0.391
267	A	8	7	1.00	23	0.304
268	A	4	3	1.00	23	0.130
269	A	6	5	1.00	23	0.217
270	A	8	7	1.04	23	0.304
271	A	11	10	1.06	23	0.435
272	A	6	5	1.04	27	0.185
273	A	5	4	1.00	27	0.148
274	A	6	5	1.05	27	0.185
275	A	9	8	0.99	27	0.296
276	A	8	7	0.99	27	0.259
277	A	7	6	1.01	27	0.222
278	A	8	7	1.02	27	0.259
279	A	9	8	1.00	27	0.296
280	A	8	7	1.00	27	0.259
281	A	8	7	1.00	27	0.259
282	A	8	7	0.84	27	0.259
283	A	8	7	0.92	27	0.259
284	A	6	5	1.00	25	0.200
285	A	6	5	1.00	25	0.200
286	C	6	5	0.24	25	0.200
287	A	6	5	1.00	25	0.200
288	A	12	12	1.00	21	0.571
289	A	10	10	1.04	21	0.476
290	A	11	11	1.02	21	0.524
291	A	6	6	1.00	19	0.316
292	A	8	8	1.05	21	0.381
293	A	11	11	1.02	21	0.524
294	A	9	8	0.97	21	0.381

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
295	A	8	7	1.00	21	0.333
296	A	4	3	1.00	21	0.143
297	A	5	4	1.00	21	0.190
298	A	5	4	1.00	21	0.190
299	A	8	7	1.00	23	0.304
300	A	4	3	1.00	23	0.130
301	A	4	3	1.00	23	0.130
302	A	4	3	1.00	23	0.130
303	A	8	7	1.00	23	0.304
304	A	4	3	1.00	23	0.130
305	A	4	3	1.00	23	0.130
306	A	4	3	1.00	23	0.130
307	A	9	8	0.98	23	0.348
308	A	8	7	1.00	23	0.304
309	A	4	3	1.00	23	0.130
310	A	7	6	1.00	23	0.261
311	A	7	6	1.00	23	0.261
312	A	9	8	1.00	25	0.320
313	A	5	4	1.00	25	0.160
314	A	6	5	1.00	25	0.200
315	A	6	5	1.00	25	0.200
316	A	9	8	1.00	25	0.320
317	A	5	4	1.00	25	0.160
318	A	6	5	1.00	25	0.200
319	A	6	5	1.00	25	0.200
320	A	9	8	0.98	26	0.308
321	A	8	7	1.00	26	0.269
322	A	4	3	1.00	26	0.115
323	A	7	6	1.00	26	0.231
324	A	7	6	1.00	26	0.231
325	A	9	8	1.00	24	0.333
326	A	5	4	1.00	24	0.167
327	A	9	8	1.00	26	0.308
328	A	5	4	1.00	26	0.154

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
329	A	4	3	1.00	19	0.158
330	A	4	3	1.00	21	0.143
331	A	6	5	1.00	21	0.238
332	A	6	5	1.00	22	0.227
333	A	4	3	1.00	21	0.143
334	A	4	3	1.00	23	0.130
335	A	6	5	1.00	23	0.217
336	A	6	5	1.00	24	0.208
337	A	4	3	1.00	21	0.143
338	A	4	3	1.00	23	0.130
339	A	6	5	1.00	23	0.217
340	A	6	5	1.00	24	0.208
341	A	12	11	0.99	21	0.524
342	A	10	9	1.01	21	0.429
343	A	8	7	1.00	21	0.333
344	A	6	5	1.00	19	0.263
345	A	6	5	1.00	12	0.417
346	A	6	5	1.00	19	0.263
347	A	6	5	1.00	25	0.200
348	A	6	5	1.00	25	0.200
349	A	6	5	1.00	25	0.200
350	A	6	5	1.00	25	0.200
351	A	11	11	1.05	21	0.524
352	A	9	9	1.02	21	0.429
353	A	9	9	1.02	21	0.429
354	A	7	7	1.00	21	0.333
355	A	9	9	1.02	21	0.429
356	A	9	9	1.02	21	0.429
357	A	11	11	1.02	21	0.524
358	A	11	11	1.04	21	0.524
359	A	19	19	1.44	23	0.826
360	A	17	17	1.52	23	0.739
361	A	15	15	1.64	23	0.652
362	A	13	13	1.91	23	0.565

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
363	A	9	9	1.93	23	0.391
364	A	15	15	1.67	23	0.652
365	A	17	17	1.49	23	0.739
366	A	19	19	1.41	23	0.826
367	A	5	5	1.41	23	0.217
368	A	5	5	1.50	23	0.217
369	A	5	5	1.67	23	0.217
370	A	5	5	1.67	23	0.217
371	A	5	5	1.67	23	0.217
372	A	5	5	1.52	23	0.217
373	A	5	5	1.41	23	0.217
374	A	14	14	1.51	23	0.609
375	A	14	14	1.65	23	0.609
376	A	12	12	1.90	23	0.522
377	A	11	11	1.94	23	0.478
378	A	12	12	1.96	23	0.522
379	A	14	14	1.68	23	0.609
380	A	14	14	1.52	23	0.609
381	A	16	16	1.47	23	0.696
382	A	17	17	1.51	23	0.739
383	A	14	14	1.60	23	0.609
384	A	15	15	1.65	23	0.652
385	A	9	9	1.72	23	0.391
386	A	15	15	1.65	23	0.652
387	A	16	16	1.49	23	0.696
388	A	17	17	1.43	23	0.739
389	A	20	20	1.39	23	0.870
390	A	19	19	1.43	23	0.826
391	A	17	17	1.49	23	0.739
392	A	18	18	1.49	23	0.783
393	A	17	17	1.49	23	0.739
394	A	18	18	1.49	23	0.783
395	A	17	17	1.49	23	0.739
396	A	19	19	1.41	23	0.826

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
397	A	19	19	1.37	23	0.826
398	A	10	10	1.20	25	0.400
399	A	8	8	1.23	25	0.320
400	A	6	6	1.27	25	0.240
401	A	4	4	1.00	25	0.160
402	A	6	5	1.00	25	0.200
403	A	8	7	1.01	25	0.280
404	A	10	9	1.00	25	0.360
405	A	11	11	1.16	25	0.440
406	A	8	8	1.22	25	0.320
407	A	6	6	1.27	25	0.240
408	A	9	8	1.01	25	0.320
409	A	9	8	1.01	25	0.320
410	A	11	10	0.99	25	0.400
411	A	13	12	1.01	25	0.480
412	A	13	13	1.16	25	0.520
413	A	10	10	1.19	25	0.400
414	A	8	8	1.21	25	0.320
415	A	11	10	1.04	25	0.400
416	A	11	10	1.06	25	0.400
417	A	11	10	1.02	25	0.400
418	A	13	12	1.03	25	0.480
419	A	15	14	1.04	25	0.560
420	A	13	12	1.10	25	0.480
421	A	10	9	1.06	25	0.360
422	A	8	7	1.01	25	0.280
423	A	6	5	1.36	25	0.200
424	A	10	9	0.86	25	0.360
425	A	12	11	0.94	25	0.440
426	A	15	14	0.97	25	0.560
427	A	17	16	1.08	25	0.640
428	A	14	13	1.06	25	0.520
429	A	11	10	1.02	25	0.400
430	A	8	7	1.01	25	0.280

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
431	A	8	7	1.01	25	0.280
432	A	13	12	0.93	25	0.480
433	A	16	15	0.95	25	0.600
434	A	17	16	1.08	25	0.640
435	A	14	13	1.06	25	0.520
436	A	11	10	1.04	25	0.400
437	A	10	9	1.04	25	0.360
438	A	10	9	1.04	25	0.360
439	A	16	15	0.97	25	0.600
440	A	19	18	0.99	25	0.720
441	A	12	12	1.14	23	0.522
442	A	13	13	1.13	23	0.565
443	A	7	7	1.04	21	0.333
444	A	10	10	1.16	23	0.435
445	A	13	13	1.14	23	0.565
446	A	11	10	1.06	19	0.526
447	A	9	8	1.02	19	0.421
448	A	9	8	1.02	19	0.421
449	A	7	6	1.00	17	0.353
450	A	1	1	1.00	10	0.100
451	A	5	5	1.00	17	0.294
452	A	6	6	1.03	19	0.316
453	A	8	7	1.02	19	0.368
454	A	10	9	1.07	19	0.474
455	A	10	9	1.01	19	0.474
456	A	13	12	0.95	21	0.571
457	A	11	10	0.93	21	0.476
458	A	11	10	1.09	21	0.476
459	A	9	8	1.00	19	0.421
460	A	7	6	1.00	12	0.500
461	A	7	7	1.00	19	0.368
462	A	6	6	1.00	21	0.286
463	A	10	9	1.14	21	0.429
464	A	10	9	0.92	21	0.429

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
465	A	13	12	1.02	21	0.571
466	A	15	14	1.06	21	0.667
467	A	13	12	1.08	21	0.571
468	A	11	10	1.06	19	0.526
469	A	3	3	1.01	12	0.250
470	A	9	9	1.00	19	0.474
471	A	9	9	1.01	21	0.429
472	A	8	8	1.02	21	0.381
473	A	12	11	0.95	21	0.524
474	A	12	11	0.90	21	0.524
475	A	16	15	0.89	21	0.714
476	A	18	17	1.06	21	0.810
477	A	15	14	1.09	21	0.667
478	A	13	12	1.08	19	0.632
479	A	5	5	1.07	12	0.417
480	A	11	11	0.99	19	0.579
481	A	11	11	1.00	21	0.524
482	A	12	12	1.01	21	0.571
483	A	11	11	1.06	21	0.524
484	A	15	14	0.95	21	0.667
485	A	15	14	0.91	21	0.667
486	A	7	7	1.08	12	0.583
487	A	17	16	1.11	21	0.762
488	A	13	12	1.08	21	0.571
489	A	11	10	1.04	21	0.476
490	A	9	8	1.00	21	0.381
491	A	6	5	1.00	19	0.263
492	A	6	5	1.00	12	0.417
493	A	10	9	1.04	19	0.474
494	A	13	12	1.10	21	0.571
495	A	16	15	1.11	21	0.714
496	A	18	17	1.13	21	0.810
497	A	16	15	1.05	21	0.714
498	A	13	12	1.12	21	0.571

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
499	A	12	11	1.21	21	0.524
500	A	10	9	1.13	21	0.429
501	A	10	9	1.13	19	0.474
502	A	11	10	1.23	12	0.833
503	A	13	12	1.14	19	0.632
504	A	15	14	1.05	21	0.667
505	A	19	18	1.03	21	0.857
506	A	16	15	1.16	21	0.714
507	A	13	12	1.24	21	0.571
508	A	13	12	1.17	21	0.571
509	A	12	11	1.19	21	0.524
510	A	13	12	1.20	19	0.632
511	A	14	13	1.25	12	1.083
512	A	17	16	1.16	19	0.842
513	A	19	18	1.08	21	0.857
514	A	19	18	1.18	21	0.857
515	A	16	15	1.21	21	0.714
516	A	13	12	1.20	21	0.571
517	A	15	14	1.19	21	0.667
518	A	15	14	1.21	21	0.667
519	A	15	14	1.23	19	0.737
520	A	17	16	1.26	12	1.333
521	A	20	19	1.19	19	1.000
522	A	23	22	1.11	21	1.048
523	A	4	4	1.32	12	0.333
524	A	8	8	1.27	12	0.667
525	A	10	10	1.25	12	0.833
526	A	13	13	1.24	12	1.083
527	A	6	5	0.40	12	0.417
528	A	11	10	0.61	12	0.833
529	A	14	13	0.73	12	1.083
530	A	17	16	0.81	12	1.333
531	A	11	11	1.02	23	0.478
532	A	7	7	1.00	23	0.304

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
533	A	5	5	1.00	21	0.238
534	A	2	2	1.00	14	0.143
535	A	10	10	1.02	21	0.476
536	A	13	13	0.99	23	0.565
537	A	17	17	1.04	23	0.739
538	A	14	14	1.02	23	0.609
539	A	10	10	1.02	23	0.435
540	A	8	8	1.00	21	0.381
541	A	8	8	1.00	14	0.571
542	A	11	11	1.02	21	0.524
543	A	14	14	1.01	23	0.609
544	A	20	20	1.04	23	0.870
545	A	17	17	1.03	23	0.739
546	A	13	13	1.03	23	0.565
547	A	11	11	1.02	21	0.524
548	A	11	11	1.00	14	0.786
549	A	11	11	1.02	21	0.524
550	A	14	14	1.02	23	0.609
551	A	17	17	1.04	23	0.739
552	A	20	20	1.03	23	0.870
553	A	14	14	1.01	14	1.000
554	A	13	13	1.06	23	0.565
555	A	10	10	1.04	23	0.435
556	A	8	8	1.02	23	0.348
557	A	5	5	1.00	23	0.217
558	A	2	2	1.00	21	0.095
559	A	2	2	1.00	14	0.143
560	A	10	10	1.02	21	0.476
561	A	13	13	1.00	23	0.565
562	A	14	14	1.03	23	0.609
563	A	11	11	1.06	23	0.478
564	A	8	8	1.10	23	0.348
565	A	8	8	1.10	23	0.348
566	A	8	8	1.10	21	0.381

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
567	A	11	11	1.08	14	0.786
568	A	14	14	1.10	21	0.667
569	A	17	17	1.07	23	0.739
570	A	14	14	1.05	23	0.609
571	A	11	11	1.07	23	0.478
572	A	11	11	1.06	23	0.478
573	A	11	11	1.06	23	0.478
574	A	11	11	1.06	21	0.524
575	A	14	14	1.07	14	1.000
576	A	17	17	1.07	21	0.810
577	A	20	20	1.07	23	0.870
578	A	17	17	1.09	14	1.214
579	A	11	11	1.01	21	0.524
580	A	9	9	1.02	21	0.429
581	A	9	9	1.01	21	0.429
582	A	7	7	1.00	21	0.333
583	A	9	9	1.01	21	0.429
584	A	9	9	1.02	21	0.429
585	A	11	11	1.04	21	0.524
586	A	17	17	0.97	23	0.739
587	A	15	15	0.95	23	0.652
588	A	13	13	1.01	23	0.565
589	A	11	11	1.00	23	0.478
590	A	11	11	1.00	23	0.478
591	A	13	13	1.01	23	0.565
592	A	15	15	0.97	23	0.652
593	A	18	18	0.93	23	0.783
594	A	16	16	0.96	23	0.696
595	A	14	14	1.01	23	0.609
596	A	14	14	1.01	23	0.609
597	A	14	14	1.01	23	0.609
598	A	16	16	0.97	23	0.696
599	A	18	18	0.95	23	0.783
600	A	21	21	0.94	23	0.913

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
601	A	19	19	0.98	23	0.826
602	A	17	17	1.02	23	0.739
603	A	17	17	1.00	23	0.739
604	A	17	17	1.02	23	0.739
605	A	17	17	1.04	23	0.739
606	A	19	19	1.00	23	0.826
607	A	21	21	0.95	23	0.913
608	A	18	18	1.07	23	0.783
609	A	11	11	0.99	23	0.478
610	A	4	4	1.00	23	0.174
611	A	7	7	0.80	23	0.304
612	A	12	12	1.00	23	0.522
613	A	15	15	1.03	23	0.652
614	A	21	21	0.95	23	0.913
615	A	18	18	0.95	23	0.783
616	A	15	15	0.99	23	0.652
617	A	15	15	0.97	23	0.652
618	A	15	15	0.95	23	0.652
619	A	15	15	0.95	23	0.652
620	A	18	18	0.95	23	0.783
621	A	21	21	0.98	23	0.913
622	A	18	18	1.01	23	0.783
623	A	18	18	0.99	23	0.783
624	A	18	18	1.00	23	0.783
625	A	18	18	0.99	23	0.783
626	A	18	18	1.00	23	0.783
627	A	21	21	0.99	23	0.913
628	A	23	23	1.07	25	0.920
629	A	13	13	1.00	25	0.520
630	A	6	6	1.00	25	0.240
631	A	15	15	1.01	25	0.600
632	A	18	18	1.04	25	0.720
633	A	21	21	1.06	25	0.840
634	A	26	26	1.03	25	1.040

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
635	A	23	23	1.02	25	0.920
636	A	20	20	1.00	25	0.800
637	A	16	16	1.01	25	0.640
638	A	19	19	0.99	25	0.760
639	A	22	22	1.03	25	0.880
640	A	29	29	1.06	25	1.160
641	A	26	26	1.03	25	1.040
642	A	23	23	1.03	25	0.920
643	A	23	23	1.00	25	0.920
644	A	19	19	1.05	25	0.760
645	A	22	22	1.03	25	0.880
646	A	25	25	1.04	25	1.000
647	A	25	25	1.00	25	1.000
648	A	23	23	1.05	25	0.920
649	A	6	6	1.00	25	0.240
650	A	6	6	1.00	25	0.240
651	A	13	13	1.00	25	0.520
652	A	15	15	1.02	25	0.600
653	A	18	18	1.05	25	0.720
654	A	26	26	1.04	25	1.040
655	A	17	17	1.04	25	0.680
656	A	9	9	1.00	25	0.360
657	A	16	16	1.06	25	0.640
658	A	16	16	1.07	25	0.640
659	A	19	19	1.02	25	0.760
660	A	22	22	1.00	25	0.880
661	A	29	29	1.03	25	1.160
662	A	26	26	1.05	25	1.040
663	A	19	19	1.06	25	0.760
664	A	19	19	1.02	25	0.760
665	A	19	19	1.04	25	0.760
666	A	19	19	1.05	25	0.760
667	A	22	22	1.02	25	0.880
668	A	25	25	1.01	25	1.000

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
669	A	9	9	1.00	25	0.360
670	A	9	9	1.00	25	0.360
671	A	13	13	1.00	25	0.520
672	A	13	13	1.00	25	0.520
673	A	9	9	1.00	25	0.360
674	A	9	9	1.00	25	0.360
675	A	9	9	1.00	25	0.360
676	A	9	9	1.00	25	0.360
677	A	4	4	1.00	25	0.160
678	A	4	4	1.00	25	0.160
679	A	6	6	1.00	25	0.240
680	A	6	6	1.00	25	0.240
681	A	4	4	1.00	25	0.160
682	A	4	4	1.00	25	0.160
683	A	4	4	1.00	25	0.160
684	A	4	4	1.00	25	0.160
685	A	5	4	1.00	21	0.190
686	N/A	2	0	1.00	14	0.000
687	A	16	15	1.04	23	0.652
688	A	13	12	1.02	23	0.522
689	A	9	8	1.00	23	0.348
690	A	5	4	1.00	21	0.190
691	N/A	2	0	1.00	14	0.000
692	A	5	4	1.00	21	0.190
693	N/A	2	0	1.00	14	0.000
694	A	19	18	1.04	23	0.783
695	A	16	15	1.02	23	0.652
696	A	12	11	1.02	23	0.478
697	A	5	4	1.00	21	0.190
698	N/A	2	0	1.00	14	0.000
699	A	13	12	1.04	23	0.522
700	A	10	9	1.01	23	0.391
701	A	7	6	1.00	23	0.261

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
702	A	5	4	1.00	21	0.190
703	N/A	2	0	1.00	14	0.000
704	A	5	4	1.00	21	0.190
705	N/A	2	0	1.00	14	0.000
706	A	5	4	1.00	21	0.190
707	N/A	2	0	1.00	14	0.000
708	A	13	12	0.97	23	0.522
709	A	10	9	0.99	23	0.391
710	A	10	9	0.98	23	0.391
711	A	5	4	1.00	21	0.190
712	N/A	2	0	1.00	14	0.000
713	A	9	8	1.12	23	0.348
714	A	9	8	1.12	23	0.348
715	A	9	8	1.12	23	0.348
716	A	9	8	1.12	23	0.348
717	N/A	2	0	1.00	25	0.000
718	N/A	2	0	1.00	25	0.000
719	N/A	2	0	1.00	25	0.000
720	N/A	2	0	1.00	25	0.000
721	N/A	2	0	1.00	25	0.000
722	N/A	2	0	1.00	25	0.000
723	N/A	2	0	1.00	25	0.000
724	N/A	2	0	1.00	25	0.000
725	N/A	2	0	1.00	25	0.000
726	N/A	2	0	1.00	25	0.000
727	N/A	2	0	1.00	25	0.000
728	N/A	2	0	1.00	25	0.000
729	N/A	2	0	1.00	25	0.000
730	N/A	2	0	1.00	25	0.000
731	N/A	2	0	1.00	25	0.000
732	N/A	2	0	1.00	25	0.000

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
733	N/A	2	0	1.00	25	0.000
734	N/A	2	0	1.00	25	0.000
735	N/A	2	0	1.00	25	0.000
736	N/A	2	0	1.00	25	0.000
737	N/A	2	0	1.00	25	0.000
738	N/A	2	0	1.00	25	0.000
739	N/A	2	0	1.00	25	0.000
740	N/A	2	0	1.00	25	0.000
741	N/A	2	0	1.00	25	0.000
742	N/A	2	0	1.00	25	0.000
743	N/A	2	0	1.00	25	0.000
744	N/A	2	0	1.00	25	0.000
745	N/A	2	0	1.00	25	0.000
746	N/A	2	0	1.00	25	0.000
747	N/A	2	0	1.00	25	0.000
748	N/A	2	0	1.00	25	0.000
749	N/A	2	0	1.00	25	0.000
750	N/A	2	0	1.00	25	0.000
751	N/A	2	0	1.00	25	0.000
752	N/A	2	0	1.00	25	0.000
753	N/A	2	0	1.00	25	0.000
754	N/A	2	0	1.00	25	0.000
755	N/A	2	0	1.00	25	0.000
756	N/A	2	0	1.00	25	0.000
757	N/A	2	0	1.00	25	0.000
758	N/A	2	0	1.00	25	0.000
759	N/A	2	0	1.00	25	0.000
760	N/A	2	0	1.00	25	0.000
761	N/A	2	0	1.00	25	0.000
762	N/A	2	0	1.00	25	0.000

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
763	N/A	2	0	1.00	25	0.000
764	N/A	2	0	1.00	25	0.000
765	N/A	2	0	1.00	25	0.000
766	N/A	2	0	1.00	25	0.000
767	N/A	2	0	1.00	25	0.000
768	N/A	2	0	1.00	25	0.000
769	N/A	2	0	1.00	25	0.000
770	N/A	2	0	1.00	25	0.000
771	N/A	2	0	1.00	25	0.000
772	N/A	2	0	1.00	25	0.000
773	N/A	2	0	1.00	25	0.000
774	N/A	2	0	1.00	25	0.000
775	N/A	2	0	1.00	25	0.000
776	N/A	2	0	1.00	25	0.000
777	A	13	13	1.04	23	0.565
778	A	11	11	0.99	23	0.478
779	A	6	6	1.00	21	0.286
780	A	9	8	1.08	23	0.348
781	A	5	5	1.04	23	0.217
782	N/A	2	0	1.00	25	0.000
783	N/A	2	0	1.00	25	0.000
784	N/A	2	0	1.00	25	0.000
785	N/A	2	0	1.00	25	0.000
786	N/A	2	0	1.00	21	0.000
787	N/A	2	0	1.00	23	0.000
788	A	9	8	1.00	21	0.381
789	A	7	6	1.00	21	0.286
790	A	5	4	1.00	19	0.211
791	N/A	2	0	1.00	12	0.000
792	N/A	2	0	1.00	19	0.000
793	N/A	2	0	1.00	21	0.000

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
794	A	11	11	1.07	21	0.524
795	A	11	11	1.05	21	0.524
796	A	9	9	1.02	21	0.429
797	A	9	9	1.02	21	0.429
798	A	7	7	1.00	21	0.333
799	A	9	9	1.02	21	0.429
800	A	9	9	1.02	21	0.429
801	A	11	11	1.02	21	0.524
802	A	19	19	1.37	23	0.826
803	A	17	17	1.41	23	0.739
804	A	15	15	1.61	23	0.652
805	A	13	13	1.85	23	0.565
806	A	13	13	1.90	23	0.565
807	A	15	15	1.65	23	0.652
808	A	17	17	1.39	23	0.739
809	A	19	19	1.34	23	0.826
810	A	20	20	1.25	23	0.870
811	A	18	18	1.35	23	0.783
812	A	16	16	1.53	23	0.696
813	A	16	16	1.49	23	0.696
814	A	16	16	1.53	23	0.696
815	A	18	18	1.36	23	0.783
816	A	20	20	1.23	23	0.870
817	A	20	20	1.61	23	0.870
818	A	17	17	1.78	23	0.739
819	B	14	14	2.08	23	0.609
820	A	9	9	1.00	23	0.391
821	A	6	6	1.00	23	0.261
822	A	13	13	1.78	23	0.565
823	A	20	20	1.73	23	0.870
824	A	20	20	1.27	23	0.870
825	A	17	17	1.38	23	0.739
826	A	17	17	1.42	23	0.739
827	A	17	17	1.51	23	0.739

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
828	A	17	17	1.51	23	0.739
829	A	20	20	1.31	23	0.870
830	A	23	23	1.23	23	1.000
831	A	20	20	1.29	23	0.870
832	A	20	20	1.30	23	0.870
833	A	20	20	1.33	23	0.870
834	A	20	20	1.31	23	0.870
835	A	20	20	1.33	23	0.870
836	A	23	23	1.23	23	1.000
837	A	20	20	1.12	25	0.800
838	A	17	17	1.11	25	0.680
839	A	8	8	1.00	25	0.320
840	A	15	15	1.15	25	0.600
841	A	25	25	1.16	25	1.000
842	A	24	24	1.10	25	0.960
843	A	21	21	1.08	25	0.840
844	A	18	18	1.12	25	0.720
845	A	22	22	1.10	25	0.880
846	A	25	25	1.11	25	1.000
847	A	28	28	1.10	25	1.120
848	A	27	27	1.10	25	1.080
849	A	24	24	1.10	25	0.960
850	A	21	21	1.13	25	0.840
851	A	25	25	1.08	25	1.000
852	A	25	25	1.11	25	1.000
853	A	28	28	1.10	25	1.120
854	F	0	0	N/A	0.000	N/A
855	A	20	20	1.13	25	0.800
856	A	17	17	1.13	25	0.680
857	A	15	15	1.15	25	0.600
858	A	8	8	1.00	25	0.320
859	A	8	8	1.00	25	0.320
860	A	25	25	1.14	25	1.000

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
861	A	27	27	1.07	25	1.080
862	A	24	24	1.06	25	0.960
863	A	21	21	1.09	25	0.840
864	A	18	18	1.16	25	0.720
865	A	18	18	1.16	25	0.720
866	A	11	11	1.17	25	0.440
867	A	19	19	1.14	25	0.760
868	A	28	28	1.10	25	1.120
869	A	24	24	1.07	25	0.960
870	A	21	21	1.11	25	0.840
871	A	21	21	1.11	25	0.840
872	A	21	21	1.10	25	0.840
873	A	21	21	1.14	25	0.840
874	A	28	28	1.11	25	1.120
875	A	15	15	1.12	23	0.652
876	A	13	13	1.10	23	0.565
877	A	7	7	1.04	21	0.333
878	A	11	10	1.07	23	0.435
879	A	7	7	1.02	23	0.304

CHAPTER 3

Listing of integrals

3.1 $\int \sec^4(c+dx)(a+a\sec(c+dx))\,dx$ 306
3.2 $\int \sec^3(c+dx)(a+a\sec(c+dx))\,dx$ 312
3.3 $\int \sec^2(c+dx)(a+a\sec(c+dx))\,dx$ 318
3.4 $\int \sec(c+dx)(a+a\sec(c+dx))\,dx$ 324
3.5 $\int (a+a\sec(c+dx))\,dx$ 329
3.6 $\int \cos(c+dx)(a+a\sec(c+dx))\,dx$ 333
3.7 $\int \cos^2(c+dx)(a+a\sec(c+dx))\,dx$ 338
3.8 $\int \cos^3(c+dx)(a+a\sec(c+dx))\,dx$ 343
3.9 $\int \cos^4(c+dx)(a+a\sec(c+dx))\,dx$ 349
3.10 $\int \sec^4(c+dx)(a+a\sec(c+dx))^2\,dx$ 355
3.11 $\int \sec^3(c+dx)(a+a\sec(c+dx))^2\,dx$ 362
3.12 $\int \sec^2(c+dx)(a+a\sec(c+dx))^2\,dx$ 369
3.13 $\int \sec(c+dx)(a+a\sec(c+dx))^2\,dx$ 376
3.14 $\int (a+a\sec(c+dx))^2\,dx$ 382
3.15 $\int \cos(c+dx)(a+a\sec(c+dx))^2\,dx$ 387
3.16 $\int \cos^2(c+dx)(a+a\sec(c+dx))^2\,dx$ 393
3.17 $\int \cos^3(c+dx)(a+a\sec(c+dx))^2\,dx$ 398
3.18 $\int \cos^4(c+dx)(a+a\sec(c+dx))^2\,dx$ 404
3.19 $\int \cos^5(c+dx)(a+a\sec(c+dx))^2\,dx$ 410
3.20 $\int \sec^3(c+dx)(a+a\sec(c+dx))^3\,dx$ 417
3.21 $\int \sec^2(c+dx)(a+a\sec(c+dx))^3\,dx$ 423
3.22 $\int \sec(c+dx)(a+a\sec(c+dx))^3\,dx$ 429
3.23 $\int (a+a\sec(c+dx))^3\,dx$ 434
3.24 $\int \cos(c+dx)(a+a\sec(c+dx))^3\,dx$ 440
3.25 $\int \cos^2(c+dx)(a+a\sec(c+dx))^3\,dx$ 445
3.26 $\int \cos^3(c+dx)(a+a\sec(c+dx))^3\,dx$ 450
3.27 $\int \cos^4(c+dx)(a+a\sec(c+dx))^3\,dx$ 455
3.28 $\int \cos^5(c+dx)(a+a\sec(c+dx))^3\,dx$ 460

3.29 $\int \cos^6(c+dx)(a+a\sec(c+dx))^3\,dx$ 465
3.30 $\int \sec^3(c+dx)(a+a\sec(c+dx))^4\,dx$ 471
3.31 $\int \sec^2(c+dx)(a+a\sec(c+dx))^4\,dx$ 478
3.32 $\int \sec(c+dx)(a+a\sec(c+dx))^4\,dx$ 484
3.33 $\int (a+a\sec(c+dx))^4\,dx$ 490
3.34 $\int \cos(c+dx)(a+a\sec(c+dx))^4\,dx$ 497
3.35 $\int \cos^2(c+dx)(a+a\sec(c+dx))^4\,dx$ 503
3.36 $\int \cos^3(c+dx)(a+a\sec(c+dx))^4\,dx$ 509
3.37 $\int \cos^4(c+dx)(a+a\sec(c+dx))^4\,dx$ 514
3.38 $\int \cos^5(c+dx)(a+a\sec(c+dx))^4\,dx$ 519
3.39 $\int \cos^6(c+dx)(a+a\sec(c+dx))^4\,dx$ 524
3.40 $\int \cos^7(c+dx)(a+a\sec(c+dx))^4\,dx$ 530
3.41 $\int \sec^3(c+dx)(a+a\sec(c+dx))^5\,dx$ 536
3.42 $\int \frac{\sec^5(c+dx)}{a+a\sec(c+dx)}\,dx$ 543
3.43 $\int \frac{\sec^4(c+dx)}{a+a\sec(c+dx)}\,dx$ 550
3.44 $\int \frac{\sec^3(c+dx)}{a+a\sec(c+dx)}\,dx$ 557
3.45 $\int \frac{\sec^2(c+dx)}{a+a\sec(c+dx)}\,dx$ 563
3.46 $\int \frac{\sec(c+dx)}{a+a\sec(c+dx)}\,dx$ 568
3.47 $\int \frac{1}{a+a\sec(c+dx)}\,dx$ 572
3.48 $\int \frac{\cos(c+dx)}{a+a\sec(c+dx)}\,dx$ 577
3.49 $\int \frac{\cos^2(c+dx)}{a+a\sec(c+dx)}\,dx$ 583
3.50 $\int \frac{\cos^3(c+dx)}{a+a\sec(c+dx)}\,dx$ 589
3.51 $\int \frac{\cos^4(c+dx)}{a+a\sec(c+dx)}\,dx$ 595
3.52 $\int \frac{\sec^5(c+dx)}{(a+a\sec(c+dx))^2}\,dx$ 602
3.53 $\int \frac{\sec^4(c+dx)}{(a+a\sec(c+dx))^2}\,dx$ 610
3.54 $\int \frac{\sec^3(c+dx)}{(a+a\sec(c+dx))^2}\,dx$ 617
3.55 $\int \frac{\sec^2(c+dx)}{(a+a\sec(c+dx))^2}\,dx$ 623
3.56 $\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^2}\,dx$ 628
3.57 $\int \frac{1}{(a+a\sec(c+dx))^2}\,dx$ 633
3.58 $\int \frac{\cos(c+dx)}{(a+a\sec(c+dx))^2}\,dx$ 639
3.59 $\int \frac{\cos^2(c+dx)}{(a+a\sec(c+dx))^2}\,dx$ 645
3.60 $\int \frac{\cos^3(c+dx)}{(a+a\sec(c+dx))^2}\,dx$ 652
3.61 $\int \frac{\sec^6(c+dx)}{(a+a\sec(c+dx))^3}\,dx$ 660
3.62 $\int \frac{\sec^5(c+dx)}{(a+a\sec(c+dx))^3}\,dx$ 669
3.63 $\int \frac{\sec^4(c+dx)}{(a+a\sec(c+dx))^3}\,dx$ 678
3.64 $\int \frac{\sec^3(c+dx)}{(a+a\sec(c+dx))^3}\,dx$ 685

3.65 $\int \frac{\sec^2(c+dx)}{(a+a\sec(c+dx))^3}\,dx$ 691
3.66 $\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^3}\,dx$ 697
3.67 $\int \frac{1}{(a+a\sec(c+dx))^3}\,dx$ 702
3.68 $\int \frac{\cos(c+dx)}{(a+a\sec(c+dx))^3}\,dx$ 708
3.69 $\int \frac{\cos^2(c+dx)}{(a+a\sec(c+dx))^3}\,dx$ 715
3.70 $\int \frac{\sec^7(c+dx)}{(a+a\sec(c+dx))^4}\,dx$ 723
3.71 $\int \frac{\sec^6(c+dx)}{(a+a\sec(c+dx))^4}\,dx$ 733
3.72 $\int \frac{\sec^5(c+dx)}{(a+a\sec(c+dx))^4}\,dx$ 742
3.73 $\int \frac{\sec^4(c+dx)}{(a+a\sec(c+dx))^4}\,dx$ 750
3.74 $\int \frac{\sec^3(c+dx)}{(a+a\sec(c+dx))^4}\,dx$ 757
3.75 $\int \frac{\sec^2(c+dx)}{(a+a\sec(c+dx))^4}\,dx$ 763
3.76 $\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^4}\,dx$ 769
3.77 $\int \frac{1}{(a+a\sec(c+dx))^4}\,dx$ 775
3.78 $\int \frac{\cos(c+dx)}{(a+a\sec(c+dx))^4}\,dx$ 782
3.79 $\int \frac{\cos^2(c+dx)}{(a+a\sec(c+dx))^4}\,dx$ 790
3.80 $\int \frac{\sec^7(c+dx)}{(a+a\sec(c+dx))^5}\,dx$ 799
3.81 $\int \frac{\sec^6(c+dx)}{(a+a\sec(c+dx))^5}\,dx$ 810
3.82 $\int \frac{\sec^5(c+dx)}{(a+a\sec(c+dx))^5}\,dx$ 819
3.83 $\int \frac{\sec^4(c+dx)}{(a+a\sec(c+dx))^5}\,dx$ 827
3.84 $\int \frac{\sec^3(c+dx)}{(a+a\sec(c+dx))^5}\,dx$ 835
3.85 $\int \frac{\sec^2(c+dx)}{(a+a\sec(c+dx))^5}\,dx$ 842
3.86 $\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^5}\,dx$ 849
3.87 $\int \frac{1}{(a+a\sec(c+dx))^5}\,dx$ 856
3.88 $\int \frac{\cos(c+dx)}{(a+a\sec(c+dx))^5}\,dx$ 864
3.89 $\int \frac{\cos^2(c+dx)}{(a+a\sec(c+dx))^5}\,dx$ 874
3.90 $\int \sec^4(c+dx)\sqrt{a+a\sec(c+dx)}\,dx$ 884
3.91 $\int \sec^3(c+dx)\sqrt{a+a\sec(c+dx)}\,dx$ 891
3.92 $\int \sec^2(c+dx)\sqrt{a+a\sec(c+dx)}\,dx$ 897
3.93 $\int \sec(c+dx)\sqrt{a+a\sec(c+dx)}\,dx$ 903
3.94 $\int \sqrt{a+a\sec(c+dx)}\,dx$ 907
3.95 $\int \cos(c+dx)\sqrt{a+a\sec(c+dx)}\,dx$ 912
3.96 $\int \cos^2(c+dx)\sqrt{a+a\sec(c+dx)}\,dx$ 918
3.97 $\int \cos^3(c+dx)\sqrt{a+a\sec(c+dx)}\,dx$ 925
3.98 $\int \cos^4(c+dx)\sqrt{a+a\sec(c+dx)}\,dx$ 932
3.99 $\int \sec^4(c+dx)(a+a\sec(c+dx))^{3/2}\,dx$ 940

3.100 $\int \sec^3(c+dx)(a+a\sec(c+dx))^{3/2}\,dx$ 948
3.101 $\int \sec^2(c+dx)(a+a\sec(c+dx))^{3/2}\,dx$ 955
3.102 $\int \sec(c+dx)(a+a\sec(c+dx))^{3/2}\,dx$ 961
3.103 $\int (a+a\sec(c+dx))^{3/2}\,dx$ 966
3.104 $\int \cos(c+dx)(a+a\sec(c+dx))^{3/2}\,dx$ 972
3.105 $\int \cos^2(c+dx)(a+a\sec(c+dx))^{3/2}\,dx$ 979
3.106 $\int \cos^3(c+dx)(a+a\sec(c+dx))^{3/2}\,dx$ 985
3.107 $\int \sec^4(c+dx)(a+a\sec(c+dx))^{5/2}\,dx$ 992
3.108 $\int \sec^3(c+dx)(a+a\sec(c+dx))^{5/2}\,dx$ 1001
3.109 $\int \sec^2(c+dx)(a+a\sec(c+dx))^{5/2}\,dx$ 1008
3.110 $\int \sec(c+dx)(a+a\sec(c+dx))^{5/2}\,dx$ 1015
3.111 $\int (a+a\sec(c+dx))^{5/2}\,dx$ 1020
3.112 $\int \cos(c+dx)(a+a\sec(c+dx))^{5/2}\,dx$ 1028
3.113 $\int \cos^2(c+dx)(a+a\sec(c+dx))^{5/2}\,dx$ 1035
3.114 $\int \cos^3(c+dx)(a+a\sec(c+dx))^{5/2}\,dx$ 1042
3.115 $\int \cos^4(c+dx)(a+a\sec(c+dx))^{5/2}\,dx$ 1049
3.116 $\int \sec(c+dx)\sqrt{a-a\sec(c+dx)}\,dx$ 1057
3.117 $\int \sqrt{a-a\sec(c+dx)}\,dx$ 1061
3.118 $\int \cos(c+dx)\sqrt{a-a\sec(c+dx)}\,dx$ 1066
3.119 $\int \frac{\sec^4(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$ 1072
3.120 $\int \frac{\sec^3(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$ 1079
3.121 $\int \frac{\sec^2(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$ 1085
3.122 $\int \frac{\sec(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$ 1090
3.123 $\int \frac{1}{\sqrt{a+a\sec(c+dx)}}\,dx$ 1095
3.124 $\int \frac{\cos(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$ 1102
3.125 $\int \frac{\cos^2(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$ 1109
3.126 $\int \frac{\sec^5(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx$ 1117
3.127 $\int \frac{\sec^4(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx$ 1126
3.128 $\int \frac{\sec^3(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx$ 1133
3.129 $\int \frac{\sec^2(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx$ 1139
3.130 $\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx$ 1144
3.131 $\int \frac{1}{(a+a\sec(c+dx))^{3/2}}\,dx$ 1149
3.132 $\int \frac{\cos(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx$ 1156
3.133 $\int \frac{\cos^2(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx$ 1164
3.134 $\int \frac{\sec^5(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx$ 1173
3.135 $\int \frac{\sec^4(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx$ 1181

3.136 $\int \frac{\sec^3(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx$ 1188
3.137 $\int \frac{\sec^2(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx$ 1194
3.138 $\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx$ 1200
3.139 $\int \frac{1}{(a+a\sec(c+dx))^{5/2}}\,dx$ 1206
3.140 $\int \frac{\cos(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx$ 1213
3.141 $\int \frac{\sec(c+dx)}{\sqrt{a-a\sec(c+dx)}}\,dx$ 1222
3.142 $\int \frac{1}{\sqrt{a-a\sec(c+dx)}}\,dx$ 1227
3.143 $\int \sec^3(c+dx)(a+a\sec(c+dx))^{2/3}\,dx$ 1234
3.144 $\int \sec^2(c+dx)(a+a\sec(c+dx))^{2/3}\,dx$ 1242
3.145 $\int \sec(c+dx)(a+a\sec(c+dx))^{2/3}\,dx$ 1249
3.146 $\int (a+a\sec(c+dx))^{2/3}\,dx$ 1255
3.147 $\int \cos(c+dx)(a+a\sec(c+dx))^{2/3}\,dx$ 1261
3.148 $\int \sec^3(c+dx)(a+a\sec(c+dx))^{5/3}\,dx$ 1267
3.149 $\int \sec^2(c+dx)(a+a\sec(c+dx))^{5/3}\,dx$ 1275
3.150 $\int \sec(c+dx)(a+a\sec(c+dx))^{5/3}\,dx$ 1282
3.151 $\int (a+a\sec(c+dx))^{5/3}\,dx$ 1289
3.152 $\int \cos(c+dx)(a+a\sec(c+dx))^{5/3}\,dx$ 1295
3.153 $\int \frac{\sec^4(c+dx)}{\sqrt[3]{a+a\sec(c+dx)}}\,dx$ 1301
3.154 $\int \frac{\sec^3(c+dx)}{\sqrt[3]{a+a\sec(c+dx)}}\,dx$ 1309
3.155 $\int \frac{\sec^2(c+dx)}{\sqrt[3]{a+a\sec(c+dx)}}\,dx$ 1316
3.156 $\int \frac{\sec(c+dx)}{\sqrt[3]{a+a\sec(c+dx)}}\,dx$ 1322
3.157 $\int \frac{1}{\sqrt[3]{a+a\sec(c+dx)}}\,dx$ 1328
3.158 $\int \frac{\cos(c+dx)}{\sqrt[3]{a+a\sec(c+dx)}}\,dx$ 1334
3.159 $\int \frac{\sec^4(c+dx)}{(a+a\sec(c+dx))^{5/3}}\,dx$ 1339
3.160 $\int \frac{\sec^3(c+dx)}{(a+a\sec(c+dx))^{5/3}}\,dx$ 1351
3.161 $\int \frac{\sec^2(c+dx)}{(a+a\sec(c+dx))^{5/3}}\,dx$ 1361
3.162 $\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^{5/3}}\,dx$ 1371
3.163 $\int \frac{1}{(a+a\sec(c+dx))^{5/3}}\,dx$ 1380
3.164 $\int \frac{\cos(c+dx)}{(a+a\sec(c+dx))^{5/3}}\,dx$ 1386
3.165 $\int \sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))\,dx$ 1392
3.166 $\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))\,dx$ 1399
3.167 $\int \sqrt{\sec(c+dx)}(a+a\sec(c+dx))\,dx$ 1406
3.168 $\int \frac{a+a\sec(c+dx)}{\sqrt{\sec(c+dx)}}\,dx$ 1412

3.169 $\int \frac{a+a\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)}\,dx$ 1418
3.170 $\int \frac{a+a\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)}\,dx$ 1424
3.171 $\int \frac{a+a\sec(c+dx)}{\sec^{\frac{7}{2}}(c+dx)}\,dx$ 1431
3.172 $\int \sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^2\,dx$ 1438
3.173 $\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2\,dx$ 1447
3.174 $\int \sqrt{\sec(c+dx)}(a+a\sec(c+dx))^2\,dx$ 1455
3.175 $\int \frac{(a+a\sec(c+dx))^2}{\sqrt{\sec(c+dx)}}\,dx$ 1463
3.176 $\int \frac{(a+a\sec(c+dx))^2}{\sec^{\frac{3}{2}}(c+dx)}\,dx$ 1469
3.177 $\int \frac{(a+a\sec(c+dx))^2}{\sec^{\frac{5}{2}}(c+dx)}\,dx$ 1475
3.178 $\int \frac{(a+a\sec(c+dx))^2}{\sec^{\frac{7}{2}}(c+dx)}\,dx$ 1483
3.179 $\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3\,dx$ 1491
3.180 $\int \sqrt{\sec(c+dx)}(a+a\sec(c+dx))^3\,dx$ 1497
3.181 $\int \frac{(a+a\sec(c+dx))^3}{\sqrt{\sec(c+dx)}}\,dx$ 1503
3.182 $\int \frac{(a+a\sec(c+dx))^3}{\sec^{\frac{3}{2}}(c+dx)}\,dx$ 1509
3.183 $\int \frac{(a+a\sec(c+dx))^3}{\sec^{\frac{5}{2}}(c+dx)}\,dx$ 1515
3.184 $\int \frac{(a+a\sec(c+dx))^3}{\sec^{\frac{7}{2}}(c+dx)}\,dx$ 1521
3.185 $\int \frac{(a+a\sec(c+dx))^3}{\sec^{\frac{9}{2}}(c+dx)}\,dx$ 1527
3.186 $\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^4\,dx$ 1533
3.187 $\int \sqrt{\sec(c+dx)}(a+a\sec(c+dx))^4\,dx$ 1539
3.188 $\int \frac{(a+a\sec(c+dx))^4}{\sqrt{\sec(c+dx)}}\,dx$ 1545
3.189 $\int \frac{(a+a\sec(c+dx))^4}{\sec^{\frac{3}{2}}(c+dx)}\,dx$ 1551
3.190 $\int \frac{(a+a\sec(c+dx))^4}{\sec^{\frac{5}{2}}(c+dx)}\,dx$ 1556
3.191 $\int \frac{(a+a\sec(c+dx))^4}{\sec^{\frac{7}{2}}(c+dx)}\,dx$ 1562
3.192 $\int \frac{(a+a\sec(c+dx))^4}{\sec^{\frac{9}{2}}(c+dx)}\,dx$ 1568
3.193 $\int \frac{(a+a\sec(c+dx))^4}{\sec^{\frac{11}{2}}(c+dx)}\,dx$ 1574
3.194 $\int \frac{\sec^{\frac{7}{2}}(c+dx)}{a+a\sec(c+dx)}\,dx$ 1580
3.195 $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{a+a\sec(c+dx)}\,dx$ 1588
3.196 $\int \frac{\sec^{\frac{3}{2}}(c+dx)}{a+a\sec(c+dx)}\,dx$ 1595
3.197 $\int \frac{\sqrt{\sec(c+dx)}}{a+a\sec(c+dx)}\,dx$ 1602
3.198 $\int \frac{1}{\sqrt{\sec(c+dx)}(a+a\sec(c+dx))}\,dx$ 1608
3.199 $\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))}\,dx$ 1615

3.200 $\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))}\,dx$ 1623
3.201 $\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a+a\sec(c+dx))^2}\,dx$ 1631
3.202 $\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+a\sec(c+dx))^2}\,dx$ 1640
3.203 $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^2}\,dx$ 1648
3.204 $\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^2}\,dx$ 1656
3.205 $\int \frac{\sqrt{\sec(c+dx)}}{(a+a\sec(c+dx))^2}\,dx$ 1662
3.206 $\int \frac{1}{\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^2}\,dx$ 1670
3.207 $\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx$ 1678
3.208 $\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx$ 1687
3.209 $\int \frac{\sec^{\frac{11}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx$ 1696
3.210 $\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx$ 1706
3.211 $\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx$ 1715
3.212 $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx$ 1724
3.213 $\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx$ 1733
3.214 $\int \frac{\sqrt{\sec(c+dx)}}{(a+a\sec(c+dx))^3}\,dx$ 1742
3.215 $\int \frac{1}{\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^3}\,dx$ 1751
3.216 $\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx$ 1760
3.217 $\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx$ 1769
3.218 $\int \sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\,dx$ 1780
3.219 $\int \sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\,dx$ 1787
3.220 $\int \sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+dx)}\,dx$ 1793
3.221 $\int \frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{\sec(c+dx)}}\,dx$ 1798
3.222 $\int \frac{\sqrt{a+a\sec(c+dx)}}{\sec^{\frac{3}{2}}(c+dx)}\,dx$ 1802
3.223 $\int \frac{\sqrt{a+a\sec(c+dx)}}{\sec^{\frac{5}{2}}(c+dx)}\,dx$ 1807
3.224 $\int \frac{\sqrt{a+a\sec(c+dx)}}{\sec^{\frac{7}{2}}(c+dx)}\,dx$ 1813
3.225 $\int \sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\,dx$ 1819
3.226 $\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\,dx$ 1827
3.227 $\int \sqrt{\sec(c+dx)}(a+a\sec(c+dx))^{3/2}\,dx$ 1834
3.228 $\int \frac{(a+a\sec(c+dx))^{3/2}}{\sqrt{\sec(c+dx)}}\,dx$ 1840
3.229 $\int \frac{(a+a\sec(c+dx))^{3/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx$ 1846
3.230 $\int \frac{(a+a\sec(c+dx))^{3/2}}{\sec^{\frac{5}{2}}(c+dx)}\,dx$ 1851

3.231 $\int \frac{(a+a\sec(c+dx))^{3/2}}{\sec^{\frac{7}{2}}(c+dx)}\,dx$ 1857
3.232 $\int \frac{(a+a\sec(c+dx))^{3/2}}{\sec^{\frac{9}{2}}(c+dx)}\,dx$ 1863
3.233 $\int \sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\,dx$ 1871
3.234 $\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\,dx$ 1879
3.235 $\int \sqrt{\sec(c+dx)}(a+a\sec(c+dx))^{5/2}\,dx$ 1887
3.236 $\int \frac{(a+a\sec(c+dx))^{5/2}}{\sqrt{\sec(c+dx)}}\,dx$ 1894
3.237 $\int \frac{(a+a\sec(c+dx))^{5/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx$ 1901
3.238 $\int \frac{(a+a\sec(c+dx))^{5/2}}{\sec^{\frac{5}{2}}(c+dx)}\,dx$ 1908
3.239 $\int \frac{(a+a\sec(c+dx))^{5/2}}{\sec^{\frac{7}{2}}(c+dx)}\,dx$ 1913
3.240 $\int \frac{(a+a\sec(c+dx))^{5/2}}{\sec^{\frac{9}{2}}(c+dx)}\,dx$ 1919
3.241 $\int \frac{(a+a\sec(c+dx))^{5/2}}{\sec^{\frac{11}{2}}(c+dx)}\,dx$ 1927
3.242 $\int \frac{(a+a\sec(c+dx))^{3/2}}{\sqrt[4]{\sec(c+dx)}}\,dx$ 1935
3.243 $\int \sqrt{\sec(e+fx)}\sqrt{a+a\sec(e+fx)}\,dx$ 1940
3.244 $\int \sqrt{-\sec(e+fx)}\sqrt{a-a\sec(e+fx)}\,dx$ 1945
3.245 $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$ 1951
3.246 $\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$ 1960
3.247 $\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+a\sec(c+dx)}}\,dx$ 1966
3.248 $\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+dx)}}\,dx$ 1971
3.249 $\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\,dx$ 1976
3.250 $\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\,dx$ 1983
3.251 $\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx$ 1991
3.252 $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx$ 2001
3.253 $\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx$ 2009
3.254 $\int \frac{\sqrt{\sec(c+dx)}}{(a+a\sec(c+dx))^{3/2}}\,dx$ 2015
3.255 $\int \frac{1}{\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^{3/2}}\,dx$ 2021
3.256 $\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}\,dx$ 2028
3.257 $\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}\,dx$ 2036
3.258 $\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx$ 2045
3.259 $\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx$ 2056
3.260 $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx$ 2065

3.261 $\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx$ 2072
3.262 $\int \frac{\sqrt{\sec(c+dx)}}{(a+a\sec(c+dx))^{5/2}}\,dx$ 2079
3.263 $\int \frac{1}{\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^{5/2}}\,dx$ 2086
3.264 $\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}}\,dx$ 2094
3.265 $\int \frac{\sec^{\frac{7}{2}}(c+dx)}{\sqrt{1+\sec(c+dx)}}\,dx$ 2103
3.266 $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{\sqrt{1+\sec(c+dx)}}\,dx$ 2112
3.267 $\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{1+\sec(c+dx)}}\,dx$ 2120
3.268 $\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{1+\sec(c+dx)}}\,dx$ 2126
3.269 $\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{1+\sec(c+dx)}}\,dx$ 2131
3.270 $\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)\sqrt{1+\sec(c+dx)}}\,dx$ 2136
3.271 $\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)\sqrt{1+\sec(c+dx)}}\,dx$ 2142
3.272 $\int (e\sec(c+dx))^{4/3}\sqrt{a+a\sec(c+dx)}\,dx$ 2150
3.273 $\int \sqrt[3]{e\sec(c+dx)}\sqrt{a+a\sec(c+dx)}\,dx$ 2156
3.274 $\int \frac{\sqrt{a+a\sec(c+dx)}}{(e\sec(c+dx))^{2/3}}\,dx$ 2161
3.275 $\int (e\sec(c+dx))^{8/3}\sqrt{a+a\sec(c+dx)}\,dx$ 2167
3.276 $\int (e\sec(c+dx))^{5/3}\sqrt{a+a\sec(c+dx)}\,dx$ 2175
3.277 $\int (e\sec(c+dx))^{2/3}\sqrt{a+a\sec(c+dx)}\,dx$ 2183
3.278 $\int \frac{\sqrt{a+a\sec(c+dx)}}{\sqrt[3]{e\sec(c+dx)}}\,dx$ 2190
3.279 $\int \frac{\sqrt{a+a\sec(c+dx)}}{(e\sec(c+dx))^{4/3}}\,dx$ 2198
3.280 $\int \frac{(e\sec(c+dx))^{2/3}}{\sqrt{a+a\sec(c+dx)}}\,dx$ 2207
3.281 $\int \frac{\sqrt[3]{e\sec(c+dx)}}{\sqrt{a+a\sec(c+dx)}}\,dx$ 2213
3.282 $\int \frac{1}{\sqrt[3]{e\sec(c+dx)}\sqrt{a+a\sec(c+dx)}}\,dx$ 2219
3.283 $\int \frac{1}{(e\sec(c+dx))^{2/3}\sqrt{a+a\sec(c+dx)}}\,dx$ 2225
3.284 $\int \sec^{\frac{4}{3}}(c+dx)\sqrt[3]{a+a\sec(c+dx)}\,dx$ 2231
3.285 $\int \sec^{\frac{4}{3}}(c+dx)(a+a\sec(c+dx))^{2/3}\,dx$ 2236
3.286 $\int \sec^{\frac{5}{3}}(c+dx)(a+a\sec(c+dx))^{2/3}\,dx$ 2241
3.287 $\int \frac{(a+a\sec(c+dx))^{4/3}}{\sqrt[3]{\sec(c+dx)}}\,dx$ 2247
3.288 $\int \sec^n(e+fx)(a+a\sec(e+fx))^4\,dx$ 2253
3.289 $\int \sec^n(e+fx)(a+a\sec(e+fx))^3\,dx$ 2261
3.290 $\int \sec^n(e+fx)(a+a\sec(e+fx))^2\,dx$ 2268
3.291 $\int \sec^n(e+fx)(a+a\sec(e+fx))\,dx$ 2275
3.292 $\int \frac{\sec^n(e+fx)}{a+a\sec(e+fx)}\,dx$ 2280

3.293 $\int \frac{\sec^n(e+fx)}{(a+a\sec(e+fx))^2}\,dx$ 2286
3.294 $\int \sec^n(e+fx)(1+\sec(e+fx))^{5/2}\,dx$ 2293
3.295 $\int \sec^n(e+fx)(1+\sec(e+fx))^{3/2}\,dx$ 2299
3.296 $\int \sec^n(e+fx)\sqrt{1+\sec(e+fx)}\,dx$ 2304
3.297 $\int \frac{\sec^n(e+fx)}{\sqrt{1+\sec(e+fx)}}\,dx$ 2308
3.298 $\int \frac{\sec^n(e+fx)}{(1+\sec(e+fx))^{3/2}}\,dx$ 2313
3.299 $\int (-\sec(e+fx))^n(1+\sec(e+fx))^{3/2}\,dx$ 2318
3.300 $\int (-\sec(e+fx))^n\sqrt{1+\sec(e+fx)}\,dx$ 2323
3.301 $\int \frac{(-\sec(e+fx))^n}{\sqrt{1+\sec(e+fx)}}\,dx$ 2327
3.302 $\int \frac{(-\sec(e+fx))^n}{(1+\sec(e+fx))^{3/2}}\,dx$ 2332
3.303 $\int (d\sec(e+fx))^n(1+\sec(e+fx))^{3/2}\,dx$ 2337
3.304 $\int (d\sec(e+fx))^n\sqrt{1+\sec(e+fx)}\,dx$ 2342
3.305 $\int \frac{(d\sec(e+fx))^n}{\sqrt{1+\sec(e+fx)}}\,dx$ 2346
3.306 $\int \frac{(d\sec(e+fx))^n}{(1+\sec(e+fx))^{3/2}}\,dx$ 2351
3.307 $\int \sec^n(e+fx)(a+a\sec(e+fx))^{5/2}\,dx$ 2356
3.308 $\int \sec^n(e+fx)(a+a\sec(e+fx))^{3/2}\,dx$ 2362
3.309 $\int \sec^n(e+fx)\sqrt{a+a\sec(e+fx)}\,dx$ 2367
3.310 $\int \frac{\sec^n(e+fx)}{\sqrt{a+a\sec(e+fx)}}\,dx$ 2371
3.311 $\int \frac{\sec^n(e+fx)}{(a+a\sec(e+fx))^{3/2}}\,dx$ 2377
3.312 $\int (-\sec(e+fx))^n(a+a\sec(e+fx))^{3/2}\,dx$ 2383
3.313 $\int (-\sec(e+fx))^n\sqrt{a+a\sec(e+fx)}\,dx$ 2389
3.314 $\int \frac{(-\sec(e+fx))^n}{\sqrt{a+a\sec(e+fx)}}\,dx$ 2394
3.315 $\int \frac{(-\sec(e+fx))^n}{(a+a\sec(e+fx))^{3/2}}\,dx$ 2400
3.316 $\int (d\sec(e+fx))^n(a+a\sec(e+fx))^{3/2}\,dx$ 2406
3.317 $\int (d\sec(e+fx))^n\sqrt{a+a\sec(e+fx)}\,dx$ 2412
3.318 $\int \frac{(d\sec(e+fx))^n}{\sqrt{a+a\sec(e+fx)}}\,dx$ 2417
3.319 $\int \frac{(d\sec(e+fx))^n}{(a+a\sec(e+fx))^{3/2}}\,dx$ 2422
3.320 $\int (-\sec(e+fx))^n(a-a\sec(e+fx))^{5/2}\,dx$ 2427
3.321 $\int (-\sec(e+fx))^n(a-a\sec(e+fx))^{3/2}\,dx$ 2434
3.322 $\int (-\sec(e+fx))^n\sqrt{a-a\sec(e+fx)}\,dx$ 2440
3.323 $\int \frac{(-\sec(e+fx))^n}{\sqrt{a-a\sec(e+fx)}}\,dx$ 2444
3.324 $\int \frac{(-\sec(e+fx))^n}{(a-a\sec(e+fx))^{3/2}}\,dx$ 2449
3.325 $\int \sec^n(e+fx)(a-a\sec(e+fx))^{3/2}\,dx$ 2454
3.326 $\int \sec^n(e+fx)\sqrt{a-a\sec(e+fx)}\,dx$ 2460
3.327 $\int (d\sec(e+fx))^n(a-a\sec(e+fx))^{3/2}\,dx$ 2465
3.328 $\int (d\sec(e+fx))^n\sqrt{a-a\sec(e+fx)}\,dx$ 2471
3.329 $\int \sec^n(e+fx)(1+\sec(e+fx))^m\,dx$ 2476

3.330 $\int (1-\sec(e+fx))^m \sec^n(e+fx)\,dx$ 2481
3.331 $\int \sec^n(e+fx)(a+a\sec(e+fx))^m\,dx$ 2486
3.332 $\int \sec^n(e+fx)(a-a\sec(e+fx))^m\,dx$ 2491
3.333 $\int (-\sec(e+fx))^n(1+\sec(e+fx))^m\,dx$ 2496
3.334 $\int (1-\sec(e+fx))^m(-\sec(e+fx))^n\,dx$ 2501
3.335 $\int (-\sec(e+fx))^n(a+a\sec(e+fx))^m\,dx$ 2506
3.336 $\int (-\sec(e+fx))^n(a-a\sec(e+fx))^m\,dx$ 2511
3.337 $\int (d\sec(e+fx))^n(1+\sec(e+fx))^m\,dx$ 2516
3.338 $\int (1-\sec(e+fx))^m(d\sec(e+fx))^n\,dx$ 2521
3.339 $\int (d\sec(e+fx))^n(a+a\sec(e+fx))^m\,dx$ 2526
3.340 $\int (d\sec(e+fx))^n(a-a\sec(e+fx))^m\,dx$ 2531
3.341 $\int \sec^4(e+fx)(a+a\sec(e+fx))^m\,dx$ 2536
3.342 $\int \sec^3(e+fx)(a+a\sec(e+fx))^m\,dx$ 2543
3.343 $\int \sec^2(e+fx)(a+a\sec(e+fx))^m\,dx$ 2549
3.344 $\int \sec(e+fx)(a+a\sec(e+fx))^m\,dx$ 2555
3.345 $\int (a+a\sec(e+fx))^m\,dx$ 2560
3.346 $\int \cos(e+fx)(a+a\sec(e+fx))^m\,dx$ 2565
3.347 $\int (d\sec(e+fx))^{3/2}(a+a\sec(e+fx))^m\,dx$ 2570
3.348 $\int \sqrt{d\sec(e+fx)}(a+a\sec(e+fx))^m\,dx$ 2575
3.349 $\int \frac{(a+a\sec(e+fx))^m}{\sqrt{d\sec(e+fx)}}\,dx$ 2580
3.350 $\int \frac{(a+a\sec(e+fx))^m}{(d\sec(e+fx))^{3/2}}\,dx$ 2585
3.351 $\int \cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))\,dx$ 2590
3.352 $\int \cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))\,dx$ 2598
3.353 $\int \cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))\,dx$ 2604
3.354 $\int \sqrt{\cos(c+dx)}(a+a\sec(c+dx))\,dx$ 2610
3.355 $\int \frac{a+a\sec(c+dx)}{\sqrt{\cos(c+dx)}}\,dx$ 2616
3.356 $\int \frac{a+a\sec(c+dx)}{\cos^{\frac{3}{2}}(c+dx)}\,dx$ 2622
3.357 $\int \frac{a+a\sec(c+dx)}{\cos^{\frac{5}{2}}(c+dx)}\,dx$ 2629
3.358 $\int \frac{a+a\sec(c+dx)}{\cos^{\frac{7}{2}}(c+dx)}\,dx$ 2637
3.359 $\int \cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^2\,dx$ 2644
3.360 $\int \cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^2\,dx$ 2653
3.361 $\int \cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^2\,dx$ 2662
3.362 $\int \cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2\,dx$ 2669
3.363 $\int \sqrt{\cos(c+dx)}(a+a\sec(c+dx))^2\,dx$ 2676
3.364 $\int \frac{(a+a\sec(c+dx))^2}{\sqrt{\cos(c+dx)}}\,dx$ 2682
3.365 $\int \frac{(a+a\sec(c+dx))^2}{\cos^{\frac{3}{2}}(c+dx)}\,dx$ 2690
3.366 $\int \frac{(a+a\sec(c+dx))^2}{\cos^{\frac{5}{2}}(c+dx)}\,dx$ 2698

3.367 $\int \cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^3\,dx$ 2706
3.368 $\int \cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^3\,dx$ 2713
3.369 $\int \cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^3\,dx$ 2719
3.370 $\int \cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3\,dx$ 2725
3.371 $\int \sqrt{\cos(c+dx)}(a+a\sec(c+dx))^3\,dx$ 2730
3.372 $\int \frac{(a+a\sec(c+dx))^3}{\sqrt{\cos(c+dx)}}\,dx$ 2736
3.373 $\int \frac{(a+a\sec(c+dx))^3}{\cos^{\frac{3}{2}}(c+dx)}\,dx$ 2742
3.374 $\int \frac{\cos^{\frac{5}{2}}(c+dx)}{a+a\sec(c+dx)}\,dx$ 2748
3.375 $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{a+a\sec(c+dx)}\,dx$ 2756
3.376 $\int \frac{\sqrt{\cos(c+dx)}}{a+a\sec(c+dx)}\,dx$ 2764
3.377 $\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\sec(c+dx))}\,dx$ 2771
3.378 $\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))}\,dx$ 2778
3.379 $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))}\,dx$ 2785
3.380 $\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))}\,dx$ 2793
3.381 $\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^2}\,dx$ 2801
3.382 $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^2}\,dx$ 2810
3.383 $\int \frac{\sqrt{\cos(c+dx)}}{(a+a\sec(c+dx))^2}\,dx$ 2819
3.384 $\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^2}\,dx$ 2827
3.385 $\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx$ 2835
3.386 $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx$ 2841
3.387 $\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx$ 2849
3.388 $\int \frac{1}{\cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx$ 2858
3.389 $\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx$ 2867
3.390 $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx$ 2880
3.391 $\int \frac{\sqrt{\cos(c+dx)}}{(a+a\sec(c+dx))^3}\,dx$ 2890
3.392 $\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^3}\,dx$ 2899
3.393 $\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx$ 2909
3.394 $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx$ 2919
3.395 $\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx$ 2929
3.396 $\int \frac{1}{\cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx$ 2939
3.397 $\int \frac{1}{\cos^{\frac{11}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx$ 2949
3.398 $\int \cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\,dx$ 2959

3.399 $\int \cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\,dx$ 2966
3.400 $\int \cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\,dx$ 2972
3.401 $\int \sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}\,dx$ 2977
3.402 $\int \frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{\cos(c+dx)}}\,dx$ 2982
3.403 $\int \frac{\sqrt{a+a\sec(c+dx)}}{\cos^{\frac{3}{2}}(c+dx)}\,dx$ 2987
3.404 $\int \frac{\sqrt{a+a\sec(c+dx)}}{\cos^{\frac{5}{2}}(c+dx)}\,dx$ 2994
3.405 $\int \cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\,dx$ 3001
3.406 $\int \cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\,dx$ 3008
3.407 $\int \cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\,dx$ 3014
3.408 $\int \sqrt{\cos(c+dx)}(a+a\sec(c+dx))^{3/2}\,dx$ 3019
3.409 $\int \frac{(a+a\sec(c+dx))^{3/2}}{\sqrt{\cos(c+dx)}}\,dx$ 3025
3.410 $\int \frac{(a+a\sec(c+dx))^{3/2}}{\cos^{\frac{3}{2}}(c+dx)}\,dx$ 3032
3.411 $\int \frac{(a+a\sec(c+dx))^{3/2}}{\cos^{\frac{5}{2}}(c+dx)}\,dx$ 3039
3.412 $\int \cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\,dx$ 3047
3.413 $\int \cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\,dx$ 3054
3.414 $\int \cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\,dx$ 3061
3.415 $\int \cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\,dx$ 3067
3.416 $\int \sqrt{\cos(c+dx)}(a+a\sec(c+dx))^{5/2}\,dx$ 3075
3.417 $\int \frac{(a+a\sec(c+dx))^{5/2}}{\sqrt{\cos(c+dx)}}\,dx$ 3083
3.418 $\int \frac{(a+a\sec(c+dx))^{5/2}}{\cos^{\frac{3}{2}}(c+dx)}\,dx$ 3090
3.419 $\int \frac{(a+a\sec(c+dx))^{5/2}}{\cos^{\frac{5}{2}}(c+dx)}\,dx$ 3098
3.420 $\int \frac{\cos^{\frac{5}{2}}(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$ 3107
3.421 $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$ 3115
3.422 $\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+a\sec(c+dx)}}\,dx$ 3122
3.423 $\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}}\,dx$ 3128
3.424 $\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\,dx$ 3133
3.425 $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\,dx$ 3140
3.426 $\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\,dx$ 3149
3.427 $\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx$ 3159
3.428 $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx$ 3169
3.429 $\int \frac{\sqrt{\cos(c+dx)}}{(a+a\sec(c+dx))^{3/2}}\,dx$ 3178
3.430 $\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^{3/2}}\,dx$ 3186

3.431 $\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}\,dx$ . . . 3193
3.432 $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}\,dx$ . . . 3200
3.433 $\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}\,dx$ . . . 3209
3.434 $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx$ . . . 3220
3.435 $\int \frac{\sqrt{\cos(c+dx)}}{(a+a\sec(c+dx))^{5/2}}\,dx$ . . . 3230
3.436 $\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^{5/2}}\,dx$ . . . 3239
3.437 $\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}}\,dx$ . . . 3247
3.438 $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}}\,dx$ . . . 3255
3.439 $\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}}\,dx$ . . . 3263
3.440 $\int \frac{1}{\cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}}\,dx$ . . . 3273
3.441 $\int (d\cos(e+fx))^n(a+a\sec(e+fx))^3\,dx$ . . . 3285
3.442 $\int (d\cos(e+fx))^n(a+a\sec(e+fx))^2\,dx$ . . . 3292
3.443 $\int (d\cos(e+fx))^n(a+a\sec(e+fx))\,dx$ . . . 3299
3.444 $\int \frac{(d\cos(e+fx))^n}{a+a\sec(e+fx)}\,dx$ . . . 3304
3.445 $\int \frac{(d\cos(e+fx))^n}{(a+a\sec(e+fx))^2}\,dx$ . . . 3310
3.446 $\int \sec^4(c+dx)(a+b\sec(c+dx))\,dx$ . . . 3317
3.447 $\int \sec^3(c+dx)(a+b\sec(c+dx))\,dx$ . . . 3323
3.448 $\int \sec^2(c+dx)(a+b\sec(c+dx))\,dx$ . . . 3329
3.449 $\int \sec(c+dx)(a+b\sec(c+dx))\,dx$ . . . 3335
3.450 $\int (a+b\sec(c+dx))\,dx$ . . . 3340
3.451 $\int \cos(c+dx)(a+b\sec(c+dx))\,dx$ . . . 3344
3.452 $\int \cos^2(c+dx)(a+b\sec(c+dx))\,dx$ . . . 3349
3.453 $\int \cos^3(c+dx)(a+b\sec(c+dx))\,dx$ . . . 3354
3.454 $\int \cos^4(c+dx)(a+b\sec(c+dx))\,dx$ . . . 3360
3.455 $\int \cos^5(c+dx)(a+b\sec(c+dx))\,dx$ . . . 3366
3.456 $\int \sec^4(c+dx)(a+b\sec(c+dx))^2\,dx$ . . . 3372
3.457 $\int \sec^3(c+dx)(a+b\sec(c+dx))^2\,dx$ . . . 3380
3.458 $\int \sec^2(c+dx)(a+b\sec(c+dx))^2\,dx$ . . . 3387
3.459 $\int \sec(c+dx)(a+b\sec(c+dx))^2\,dx$ . . . 3394
3.460 $\int (a+b\sec(c+dx))^2\,dx$ . . . 3400
3.461 $\int \cos(c+dx)(a+b\sec(c+dx))^2\,dx$ . . . 3405
3.462 $\int \cos^2(c+dx)(a+b\sec(c+dx))^2\,dx$ . . . 3410
3.463 $\int \cos^3(c+dx)(a+b\sec(c+dx))^2\,dx$ . . . 3415
3.464 $\int \cos^4(c+dx)(a+b\sec(c+dx))^2\,dx$ . . . 3421
3.465 $\int \cos^5(c+dx)(a+b\sec(c+dx))^2\,dx$ . . . 3427
3.466 $\int \sec^3(c+dx)(a+b\sec(c+dx))^3\,dx$ . . . 3434
3.467 $\int \sec^2(c+dx)(a+b\sec(c+dx))^3\,dx$ . . . 3443

3.468 $\int \sec(c+dx)(a+b\sec(c+dx))^3\,dx$ 3452
3.469 $\int (a+b\sec(c+dx))^3\,dx$ 3459
3.470 $\int \cos(c+dx)(a+b\sec(c+dx))^3\,dx$ 3464
3.471 $\int \cos^2(c+dx)(a+b\sec(c+dx))^3\,dx$ 3471
3.472 $\int \cos^3(c+dx)(a+b\sec(c+dx))^3\,dx$ 3478
3.473 $\int \cos^4(c+dx)(a+b\sec(c+dx))^3\,dx$ 3484
3.474 $\int \cos^5(c+dx)(a+b\sec(c+dx))^3\,dx$ 3492
3.475 $\int \cos^6(c+dx)(a+b\sec(c+dx))^3\,dx$ 3500
3.476 $\int \sec^3(c+dx)(a+b\sec(c+dx))^4\,dx$ 3509
3.477 $\int \sec^2(c+dx)(a+b\sec(c+dx))^4\,dx$ 3519
3.478 $\int \sec(c+dx)(a+b\sec(c+dx))^4\,dx$ 3528
3.479 $\int (a+b\sec(c+dx))^4\,dx$ 3537
3.480 $\int \cos(c+dx)(a+b\sec(c+dx))^4\,dx$ 3543
3.481 $\int \cos^2(c+dx)(a+b\sec(c+dx))^4\,dx$ 3551
3.482 $\int \cos^3(c+dx)(a+b\sec(c+dx))^4\,dx$ 3558
3.483 $\int \cos^4(c+dx)(a+b\sec(c+dx))^4\,dx$ 3566
3.484 $\int \cos^5(c+dx)(a+b\sec(c+dx))^4\,dx$ 3574
3.485 $\int \cos^6(c+dx)(a+b\sec(c+dx))^4\,dx$ 3583
3.486 $\int (a+b\sec(c+dx))^5\,dx$ 3593
3.487 $\int \frac{\sec^5(c+dx)}{a+b\sec(c+dx)}\,dx$ 3601
3.488 $\int \frac{\sec^4(c+dx)}{a+b\sec(c+dx)}\,dx$ 3611
3.489 $\int \frac{\sec^3(c+dx)}{a+b\sec(c+dx)}\,dx$ 3619
3.490 $\int \frac{\sec^2(c+dx)}{a+b\sec(c+dx)}\,dx$ 3626
3.491 $\int \frac{\sec(c+dx)}{a+b\sec(c+dx)}\,dx$ 3632
3.492 $\int \frac{1}{a+b\sec(c+dx)}\,dx$ 3637
3.493 $\int \frac{\cos(c+dx)}{a+b\sec(c+dx)}\,dx$ 3643
3.494 $\int \frac{\cos^2(c+dx)}{a+b\sec(c+dx)}\,dx$ 3650
3.495 $\int \frac{\cos^3(c+dx)}{a+b\sec(c+dx)}\,dx$ 3658
3.496 $\int \frac{\cos^4(c+dx)}{a+b\sec(c+dx)}\,dx$ 3667
3.497 $\int \frac{\sec^5(c+dx)}{(a+b\sec(c+dx))^2}\,dx$ 3677
3.498 $\int \frac{\sec^4(c+dx)}{(a+b\sec(c+dx))^2}\,dx$ 3687
3.499 $\int \frac{\sec^3(c+dx)}{(a+b\sec(c+dx))^2}\,dx$ 3696
3.500 $\int \frac{\sec^2(c+dx)}{(a+b\sec(c+dx))^2}\,dx$ 3704
3.501 $\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^2}\,dx$ 3710
3.502 $\int \frac{1}{(a+b\sec(c+dx))^2}\,dx$ 3716
3.503 $\int \frac{\cos(c+dx)}{(a+b\sec(c+dx))^2}\,dx$ 3724
3.504 $\int \frac{\cos^2(c+dx)}{(a+b\sec(c+dx))^2}\,dx$ 3733

3.505 $\int \frac{\cos^3(c+dx)}{(a+b\sec(c+dx))^2}\,dx$ 3742
3.506 $\int \frac{\sec^5(c+dx)}{(a+b\sec(c+dx))^3}\,dx$ 3753
3.507 $\int \frac{\sec^4(c+dx)}{(a+b\sec(c+dx))^3}\,dx$ 3763
3.508 $\int \frac{\sec^3(c+dx)}{(a+b\sec(c+dx))^3}\,dx$ 3772
3.509 $\int \frac{\sec^2(c+dx)}{(a+b\sec(c+dx))^3}\,dx$ 3780
3.510 $\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^3}\,dx$ 3788
3.511 $\int \frac{1}{(a+b\sec(c+dx))^3}\,dx$ 3796
3.512 $\int \frac{\cos(c+dx)}{(a+b\sec(c+dx))^3}\,dx$ 3805
3.513 $\int \frac{\cos^2(c+dx)}{(a+b\sec(c+dx))^3}\,dx$ 3815
3.514 $\int \frac{\sec^6(c+dx)}{(a+b\sec(c+dx))^4}\,dx$ 3827
3.515 $\int \frac{\sec^5(c+dx)}{(a+b\sec(c+dx))^4}\,dx$ 3840
3.516 $\int \frac{\sec^4(c+dx)}{(a+b\sec(c+dx))^4}\,dx$ 3851
3.517 $\int \frac{\sec^3(c+dx)}{(a+b\sec(c+dx))^4}\,dx$ 3860
3.518 $\int \frac{\sec^2(c+dx)}{(a+b\sec(c+dx))^4}\,dx$ 3869
3.519 $\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^4}\,dx$ 3878
3.520 $\int \frac{1}{(a+b\sec(c+dx))^4}\,dx$ 3887
3.521 $\int \frac{\cos(c+dx)}{(a+b\sec(c+dx))^4}\,dx$ 3898
3.522 $\int \frac{\cos^2(c+dx)}{(a+b\sec(c+dx))^4}\,dx$ 3910
3.523 $\int \frac{1}{3+5\sec(c+dx)}\,dx$ 3924
3.524 $\int \frac{1}{(3+5\sec(c+dx))^2}\,dx$ 3929
3.525 $\int \frac{1}{(3+5\sec(c+dx))^3}\,dx$ 3935
3.526 $\int \frac{1}{(3+5\sec(c+dx))^4}\,dx$ 3942
3.527 $\int \frac{1}{5+3\sec(c+dx)}\,dx$ 3950
3.528 $\int \frac{1}{(5+3\sec(c+dx))^2}\,dx$ 3955
3.529 $\int \frac{1}{(5+3\sec(c+dx))^3}\,dx$ 3962
3.530 $\int \frac{1}{(5+3\sec(c+dx))^4}\,dx$ 3970
3.531 $\int \sec^3(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$ 3979
3.532 $\int \sec^2(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$ 3987
3.533 $\int \sec(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$ 3994
3.534 $\int \sqrt{a+b\sec(c+dx)}\,dx$ 4000
3.535 $\int \cos(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$ 4005
3.536 $\int \cos^2(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$ 4013
3.537 $\int \sec^4(c+dx)(a+b\sec(c+dx))^{3/2}\,dx$ 4023
3.538 $\int \sec^3(c+dx)(a+b\sec(c+dx))^{3/2}\,dx$ 4033
3.539 $\int \sec^2(c+dx)(a+b\sec(c+dx))^{3/2}\,dx$ 4042
3.540 $\int \sec(c+dx)(a+b\sec(c+dx))^{3/2}\,dx$ 4050

3.541 $\int (a+b\sec(c+dx))^{3/2}\,dx$ 4057
3.542 $\int \cos(c+dx)(a+b\sec(c+dx))^{3/2}\,dx$ 4065
3.543 $\int \cos^2(c+dx)(a+b\sec(c+dx))^{3/2}\,dx$ 4073
3.544 $\int \sec^4(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$ 4083
3.545 $\int \sec^3(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$ 4094
3.546 $\int \sec^2(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$ 4104
3.547 $\int \sec(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$ 4113
3.548 $\int (a+b\sec(c+dx))^{5/2}\,dx$ 4121
3.549 $\int \cos(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$ 4129
3.550 $\int \cos^2(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$ 4138
3.551 $\int \cos^3(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$ 4147
3.552 $\int \cos^4(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$ 4158
3.553 $\int (a+b\sec(c+dx))^{7/2}\,dx$ 4170
3.554 $\int \frac{\sec^5(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$ 4180
3.555 $\int \frac{\sec^4(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$ 4189
3.556 $\int \frac{\sec^3(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$ 4197
3.557 $\int \frac{\sec^2(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$ 4204
3.558 $\int \frac{\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$ 4211
3.559 $\int \frac{1}{\sqrt{a+b\sec(c+dx)}}\,dx$ 4215
3.560 $\int \frac{\cos(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$ 4220
3.561 $\int \frac{\cos^2(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$ 4228
3.562 $\int \frac{\sec^5(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$ 4237
3.563 $\int \frac{\sec^4(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$ 4246
3.564 $\int \frac{\sec^3(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$ 4255
3.565 $\int \frac{\sec^2(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$ 4263
3.566 $\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$ 4270
3.567 $\int \frac{1}{(a+b\sec(c+dx))^{3/2}}\,dx$ 4277
3.568 $\int \frac{\cos(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$ 4286
3.569 $\int \frac{\cos^2(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$ 4295
3.570 $\int \frac{\sec^5(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$ 4305
3.571 $\int \frac{\sec^4(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$ 4314
3.572 $\int \frac{\sec^3(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$ 4323
3.573 $\int \frac{\sec^2(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$ 4332
3.574 $\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$ 4340
3.575 $\int \frac{1}{(a+b\sec(c+dx))^{5/2}}\,dx$ 4348

3.576 $\int \frac{\cos(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$ 4358
3.577 $\int \frac{\cos^2(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$ 4368
3.578 $\int \frac{1}{(a+b\sec(c+dx))^{7/2}}\,dx$ 4379
3.579 $\int \sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))\,dx$ 4389
3.580 $\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))\,dx$ 4397
3.581 $\int \sqrt{\sec(c+dx)}(a+b\sec(c+dx))\,dx$ 4404
3.582 $\int \frac{a+b\sec(c+dx)}{\sqrt{\sec(c+dx)}}\,dx$ 4410
3.583 $\int \frac{a+b\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)}\,dx$ 4416
3.584 $\int \frac{a+b\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)}\,dx$ 4422
3.585 $\int \frac{a+b\sec(c+dx)}{\sec^{\frac{7}{2}}(c+dx)}\,dx$ 4429
3.586 $\int \sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^2\,dx$ 4436
3.587 $\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2\,dx$ 4445
3.588 $\int \sqrt{\sec(c+dx)}(a+b\sec(c+dx))^2\,dx$ 4454
3.589 $\int \frac{(a+b\sec(c+dx))^2}{\sqrt{\sec(c+dx)}}\,dx$ 4462
3.590 $\int \frac{(a+b\sec(c+dx))^2}{\sec^{\frac{3}{2}}(c+dx)}\,dx$ 4468
3.591 $\int \frac{(a+b\sec(c+dx))^2}{\sec^{\frac{5}{2}}(c+dx)}\,dx$ 4475
3.592 $\int \frac{(a+b\sec(c+dx))^2}{\sec^{\frac{7}{2}}(c+dx)}\,dx$ 4483
3.593 $\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^3\,dx$ 4491
3.594 $\int \sqrt{\sec(c+dx)}(a+b\sec(c+dx))^3\,dx$ 4500
3.595 $\int \frac{(a+b\sec(c+dx))^3}{\sqrt{\sec(c+dx)}}\,dx$ 4509
3.596 $\int \frac{(a+b\sec(c+dx))^3}{\sec^{\frac{3}{2}}(c+dx)}\,dx$ 4518
3.597 $\int \frac{(a+b\sec(c+dx))^3}{\sec^{\frac{5}{2}}(c+dx)}\,dx$ 4526
3.598 $\int \frac{(a+b\sec(c+dx))^3}{\sec^{\frac{7}{2}}(c+dx)}\,dx$ 4534
3.599 $\int \frac{(a+b\sec(c+dx))^3}{\sec^{\frac{9}{2}}(c+dx)}\,dx$ 4543
3.600 $\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^4\,dx$ 4552
3.601 $\int \sqrt{\sec(c+dx)}(a+b\sec(c+dx))^4\,dx$ 4562
3.602 $\int \frac{(a+b\sec(c+dx))^4}{\sqrt{\sec(c+dx)}}\,dx$ 4572
3.603 $\int \frac{(a+b\sec(c+dx))^4}{\sec^{\frac{3}{2}}(c+dx)}\,dx$ 4581
3.604 $\int \frac{(a+b\sec(c+dx))^4}{\sec^{\frac{5}{2}}(c+dx)}\,dx$ 4590
3.605 $\int \frac{(a+b\sec(c+dx))^4}{\sec^{\frac{7}{2}}(c+dx)}\,dx$ 4599
3.606 $\int \frac{(a+b\sec(c+dx))^4}{\sec^{\frac{9}{2}}(c+dx)}\,dx$ 4608
3.607 $\int \frac{(a+b\sec(c+dx))^4}{\sec^{\frac{11}{2}}(c+dx)}\,dx$ 4618

3.608 $\int \frac{\sec^{\frac{7}{2}}(c+dx)}{a+b\sec(c+dx)}\,dx$ 4628
3.609 $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{a+b\sec(c+dx)}\,dx$ 4637
3.610 $\int \frac{\sec^{\frac{3}{2}}(c+dx)}{a+b\sec(c+dx)}\,dx$ 4644
3.611 $\int \frac{\sqrt{\sec(c+dx)}}{a+b\sec(c+dx)}\,dx$ 4649
3.612 $\int \frac{1}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}\,dx$ 4655
3.613 $\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))}\,dx$ 4662
3.614 $\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a+b\sec(c+dx))^2}\,dx$ 4670
3.615 $\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+b\sec(c+dx))^2}\,dx$ 4681
3.616 $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\sec(c+dx))^2}\,dx$ 4691
3.617 $\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^2}\,dx$ 4700
3.618 $\int \frac{\sqrt{\sec(c+dx)}}{(a+b\sec(c+dx))^2}\,dx$ 4709
3.619 $\int \frac{1}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^2}\,dx$ 4718
3.620 $\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2}\,dx$ 4727
3.621 $\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a+b\sec(c+dx))^3}\,dx$ 4737
3.622 $\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+b\sec(c+dx))^3}\,dx$ 4749
3.623 $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\sec(c+dx))^3}\,dx$ 4759
3.624 $\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^3}\,dx$ 4769
3.625 $\int \frac{\sqrt{\sec(c+dx)}}{(a+b\sec(c+dx))^3}\,dx$ 4779
3.626 $\int \frac{1}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^3}\,dx$ 4789
3.627 $\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^3}\,dx$ 4800
3.628 $\int \sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$ 4812
3.629 $\int \sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\,dx$ 4824
3.630 $\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}\,dx$ 4831
3.631 $\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{3}{2}}(c+dx)}\,dx$ 4837
3.632 $\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{5}{2}}(c+dx)}\,dx$ 4846
3.633 $\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{7}{2}}(c+dx)}\,dx$ 4857
3.634 $\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx$ 4869
3.635 $\int \sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}\,dx$ 4883
3.636 $\int \frac{(a+b\sec(c+dx))^{3/2}}{\sqrt{\sec(c+dx)}}\,dx$ 4895
3.637 $\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx$ 4906

3.638 $\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{5}{2}}(c+dx)}\,dx$ 4915
3.639 $\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{7}{2}}(c+dx)}\,dx$ 4926
3.640 $\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$ 4938
3.641 $\int \sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{5/2}\,dx$ 4953
3.642 $\int \frac{(a+b\sec(c+dx))^{5/2}}{\sqrt{\sec(c+dx)}}\,dx$ 4968
3.643 $\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx$ 4980
3.644 $\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{5}{2}}(c+dx)}\,dx$ 4992
3.645 $\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{7}{2}}(c+dx)}\,dx$ 5003
3.646 $\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{9}{2}}(c+dx)}\,dx$ 5015
3.647 $\int \frac{\sec^{\frac{7}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$ 5029
3.648 $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$ 5042
3.649 $\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$ 5054
3.650 $\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}\,dx$ 5059
3.651 $\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}\,dx$ 5064
3.652 $\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx$ 5072
3.653 $\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx$ 5081
3.654 $\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$ 5091
3.655 $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$ 5104
3.656 $\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$ 5114
3.657 $\int \frac{\sqrt{\sec(c+dx)}}{(a+b\sec(c+dx))^{3/2}}\,dx$ 5121
3.658 $\int \frac{1}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}}\,dx$ 5130
3.659 $\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx$ 5139
3.660 $\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx$ 5150
3.661 $\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$ 5162
3.662 $\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$ 5176
3.663 $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$ 5190
3.664 $\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$ 5200
3.665 $\int \frac{\sqrt{\sec(c+dx)}}{(a+b\sec(c+dx))^{5/2}}\,dx$ 5210
3.666 $\int \frac{1}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{5/2}}\,dx$ 5221

3.667 $\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx$ 5232
3.668 $\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx$ 5244
3.669 $\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{2+3\sec(c+dx)}}\,dx$ 5257
3.670 $\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{-2+3\sec(c+dx)}}\,dx$ 5263
3.671 $\int \frac{1}{\sqrt{2-3\sec(c+dx)}\sqrt{\sec(c+dx)}}\,dx$ 5269
3.672 $\int \frac{1}{\sqrt{-2-3\sec(c+dx)}\sqrt{\sec(c+dx)}}\,dx$ 5276
3.673 $\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{3+2\sec(c+dx)}}\,dx$ 5283
3.674 $\int \frac{1}{\sqrt{3-2\sec(c+dx)}\sqrt{\sec(c+dx)}}\,dx$ 5290
3.675 $\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{-3+2\sec(c+dx)}}\,dx$ 5296
3.676 $\int \frac{1}{\sqrt{-3-2\sec(c+dx)}\sqrt{\sec(c+dx)}}\,dx$ 5303
3.677 $\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{2+3\sec(c+dx)}}\,dx$ 5310
3.678 $\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{-2+3\sec(c+dx)}}\,dx$ 5315
3.679 $\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{2-3\sec(c+dx)}}\,dx$ 5320
3.680 $\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{-2-3\sec(c+dx)}}\,dx$ 5325
3.681 $\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{3+2\sec(c+dx)}}\,dx$ 5330
3.682 $\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{3-2\sec(c+dx)}}\,dx$ 5335
3.683 $\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{-3+2\sec(c+dx)}}\,dx$ 5340
3.684 $\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{-3-2\sec(c+dx)}}\,dx$ 5345
3.685 $\int \sec(c+dx)\sqrt[3]{a+b\sec(c+dx)}\,dx$ 5350
3.686 $\int \sqrt[3]{a+b\sec(c+dx)}\,dx$ 5355
3.687 $\int \sec^4(c+dx)(a+b\sec(c+dx))^{2/3}\,dx$ 5359
3.688 $\int \sec^3(c+dx)(a+b\sec(c+dx))^{2/3}\,dx$ 5368
3.689 $\int \sec^2(c+dx)(a+b\sec(c+dx))^{2/3}\,dx$ 5376
3.690 $\int \sec(c+dx)(a+b\sec(c+dx))^{2/3}\,dx$ 5383
3.691 $\int (a+b\sec(c+dx))^{2/3}\,dx$ 5388
3.692 $\int \sec(c+dx)(a+b\sec(c+dx))^{4/3}\,dx$ 5392
3.693 $\int (a+b\sec(c+dx))^{4/3}\,dx$ 5397
3.694 $\int \sec^4(c+dx)(a+b\sec(c+dx))^{5/3}\,dx$ 5401
3.695 $\int \sec^3(c+dx)(a+b\sec(c+dx))^{5/3}\,dx$ 5410
3.696 $\int \sec^2(c+dx)(a+b\sec(c+dx))^{5/3}\,dx$ 5419
3.697 $\int \sec(c+dx)(a+b\sec(c+dx))^{5/3}\,dx$ 5427
3.698 $\int (a+b\sec(c+dx))^{5/3}\,dx$ 5432
3.699 $\int \frac{\sec^4(c+dx)}{\sqrt[3]{a+b\sec(c+dx)}}\,dx$ 5436
3.700 $\int \frac{\sec^3(c+dx)}{\sqrt[3]{a+b\sec(c+dx)}}\,dx$ 5444

3.701 $\int \frac{\sec^2(c+dx)}{\sqrt[3]{a+b\sec(c+dx)}}\,dx$ 5451
3.702 $\int \frac{\sec(c+dx)}{\sqrt[3]{a+b\sec(c+dx)}}\,dx$ 5457
3.703 $\int \frac{1}{\sqrt[3]{a+b\sec(c+dx)}}\,dx$ 5462
3.704 $\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^{2/3}}\,dx$ 5466
3.705 $\int \frac{1}{(a+b\sec(c+dx))^{2/3}}\,dx$ 5471
3.706 $\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^{4/3}}\,dx$ 5475
3.707 $\int \frac{1}{(a+b\sec(c+dx))^{4/3}}\,dx$ 5480
3.708 $\int \frac{\sec^4(c+dx)}{(a+b\sec(c+dx))^{5/3}}\,dx$ 5484
3.709 $\int \frac{\sec^3(c+dx)}{(a+b\sec(c+dx))^{5/3}}\,dx$ 5492
3.710 $\int \frac{\sec^2(c+dx)}{(a+b\sec(c+dx))^{5/3}}\,dx$ 5499
3.711 $\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^{5/3}}\,dx$ 5506
3.712 $\int \frac{1}{(a+b\sec(c+dx))^{5/3}}\,dx$ 5511
3.713 $\int \frac{\sec^{\frac{2}{3}}(c+dx)}{a+b\sec(c+dx)}\,dx$ 5515
3.714 $\int \frac{\sqrt[3]{\sec(c+dx)}}{a+b\sec(c+dx)}\,dx$ 5521
3.715 $\int \frac{1}{\sqrt[3]{\sec(c+dx)}(a+b\sec(c+dx))}\,dx$ 5527
3.716 $\int \frac{1}{\sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))}\,dx$ 5533
3.717 $\int \sec^{\frac{7}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$ 5539
3.718 $\int \sec^{\frac{5}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$ 5543
3.719 $\int \sec^{\frac{4}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$ 5547
3.720 $\int \sec^{\frac{2}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$ 5551
3.721 $\int \sqrt[3]{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\,dx$ 5555
3.722 $\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt[3]{\sec(c+dx)}}\,dx$ 5559
3.723 $\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{2}{3}}(c+dx)}\,dx$ 5563
3.724 $\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{4}{3}}(c+dx)}\,dx$ 5567
3.725 $\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{5}{3}}(c+dx)}\,dx$ 5571
3.726 $\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{7}{3}}(c+dx)}\,dx$ 5575
3.727 $\int \sec^{\frac{7}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx$ 5579
3.728 $\int \sec^{\frac{5}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx$ 5583
3.729 $\int \sec^{\frac{4}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx$ 5587
3.730 $\int \sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx$ 5591
3.731 $\int \sqrt[3]{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}\,dx$ 5595
3.732 $\int \frac{(a+b\sec(c+dx))^{3/2}}{\sqrt[3]{\sec(c+dx)}}\,dx$ 5599

3.733 $\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{2}{3}}(c+dx)}\,dx$ 5603
3.734 $\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{4}{3}}(c+dx)}\,dx$ 5607
3.735 $\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{5}{3}}(c+dx)}\,dx$ 5611
3.736 $\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{7}{3}}(c+dx)}\,dx$ 5615
3.737 $\int \sec^{\frac{7}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$ 5619
3.738 $\int \sec^{\frac{5}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$ 5623
3.739 $\int \sec^{\frac{4}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$ 5627
3.740 $\int \sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$ 5631
3.741 $\int \sqrt[3]{\sec(c+dx)}(a+b\sec(c+dx))^{5/2}\,dx$ 5635
3.742 $\int \frac{(a+b\sec(c+dx))^{5/2}}{\sqrt[3]{\sec(c+dx)}}\,dx$ 5639
3.743 $\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{2}{3}}(c+dx)}\,dx$ 5643
3.744 $\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{4}{3}}(c+dx)}\,dx$ 5647
3.745 $\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{5}{3}}(c+dx)}\,dx$ 5651
3.746 $\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{7}{3}}(c+dx)}\,dx$ 5655
3.747 $\int \frac{\sec^{\frac{7}{3}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$ 5659
3.748 $\int \frac{\sec^{\frac{5}{3}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$ 5663
3.749 $\int \frac{\sec^{\frac{4}{3}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$ 5667
3.750 $\int \frac{\sec^{\frac{2}{3}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$ 5671
3.751 $\int \frac{\sqrt[3]{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}\,dx$ 5675
3.752 $\int \frac{1}{\sqrt[3]{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}\,dx$ 5679
3.753 $\int \frac{1}{\sec^{\frac{2}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx$ 5683
3.754 $\int \frac{1}{\sec^{\frac{4}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx$ 5687
3.755 $\int \frac{1}{\sec^{\frac{5}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx$ 5691
3.756 $\int \frac{1}{\sec^{\frac{7}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx$ 5695
3.757 $\int \frac{\sec^{\frac{7}{3}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$ 5699
3.758 $\int \frac{\sec^{\frac{5}{3}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$ 5703
3.759 $\int \frac{\sec^{\frac{4}{3}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$ 5707
3.760 $\int \frac{\sec^{\frac{2}{3}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$ 5711
3.761 $\int \frac{\sqrt[3]{\sec(c+dx)}}{(a+b\sec(c+dx))^{3/2}}\,dx$ 5715

3.762 $\int \frac{1}{\sqrt[3]{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}}\,dx$ 5719
3.763 $\int \frac{1}{\sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx$ 5723
3.764 $\int \frac{1}{\sec^{\frac{4}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx$ 5727
3.765 $\int \frac{1}{\sec^{\frac{5}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx$ 5731
3.766 $\int \frac{1}{\sec^{\frac{7}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx$ 5735
3.767 $\int \frac{\sec^{\frac{7}{3}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$ 5739
3.768 $\int \frac{\sec^{\frac{5}{3}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$ 5743
3.769 $\int \frac{\sec^{\frac{4}{3}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$ 5747
3.770 $\int \frac{\sec^{\frac{2}{3}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$ 5751
3.771 $\int \frac{\sqrt[3]{\sec(c+dx)}}{(a+b\sec(c+dx))^{5/2}}\,dx$ 5755
3.772 $\int \frac{1}{\sqrt[3]{\sec(c+dx)}(a+b\sec(c+dx))^{5/2}}\,dx$ 5759
3.773 $\int \frac{1}{\sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx$ 5763
3.774 $\int \frac{1}{\sec^{\frac{4}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx$ 5767
3.775 $\int \frac{1}{\sec^{\frac{5}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx$ 5771
3.776 $\int \frac{1}{\sec^{\frac{7}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx$ 5775
3.777 $\int (d\sec(e+fx))^n(a+b\sec(e+fx))^3\,dx$ 5779
3.778 $\int (d\sec(e+fx))^n(a+b\sec(e+fx))^2\,dx$ 5786
3.779 $\int (d\sec(e+fx))^n(a+b\sec(e+fx))\,dx$ 5793
3.780 $\int \frac{(d\sec(e+fx))^n}{a+b\sec(e+fx)}\,dx$ 5798
3.781 $\int \frac{(d\sec(e+fx))^n}{(a+b\sec(e+fx))^2}\,dx$ 5804
3.782 $\int (d\sec(e+fx))^n(a+b\sec(e+fx))^{3/2}\,dx$ 5809
3.783 $\int (d\sec(e+fx))^n\sqrt{a+b\sec(e+fx)}\,dx$ 5813
3.784 $\int \frac{(d\sec(e+fx))^n}{\sqrt{a+b\sec(e+fx)}}\,dx$ 5817
3.785 $\int \frac{(d\sec(e+fx))^n}{(a+b\sec(e+fx))^{3/2}}\,dx$ 5821
3.786 $\int \sec^n(e+fx)(a+b\sec(e+fx))^m\,dx$ 5825
3.787 $\int (d\sec(e+fx))^n(a+b\sec(e+fx))^m\,dx$ 5829
3.788 $\int \sec^3(e+fx)(a+b\sec(e+fx))^m\,dx$ 5833
3.789 $\int \sec^2(e+fx)(a+b\sec(e+fx))^m\,dx$ 5840
3.790 $\int \sec(e+fx)(a+b\sec(e+fx))^m\,dx$ 5846
3.791 $\int (a+b\sec(e+fx))^m\,dx$ 5851
3.792 $\int \cos(e+fx)(a+b\sec(e+fx))^m\,dx$ 5855
3.793 $\int \cos^2(e+fx)(a+b\sec(e+fx))^m\,dx$ 5859
3.794 $\int \cos^{\frac{9}{2}}(c+dx)(a+b\sec(c+dx))\,dx$ 5863
3.795 $\int \cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))\,dx$ 5870

3.796 $\int \cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))\,dx$ 5877
3.797 $\int \cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))\,dx$ 5883
3.798 $\int \sqrt{\cos(c+dx)}(a+b\sec(c+dx))\,dx$ 5889
3.799 $\int \frac{a+b\sec(c+dx)}{\sqrt{\cos(c+dx)}}\,dx$ 5894
3.800 $\int \frac{a+b\sec(c+dx)}{\cos^{\frac{3}{2}}(c+dx)}\,dx$ 5900
3.801 $\int \frac{a+b\sec(c+dx)}{\cos^{\frac{5}{2}}(c+dx)}\,dx$ 5906
3.802 $\int \cos^{\frac{9}{2}}(c+dx)(a+b\sec(c+dx))^2\,dx$ 5913
3.803 $\int \cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^2\,dx$ 5922
3.804 $\int \cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^2\,dx$ 5930
3.805 $\int \cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2\,dx$ 5937
3.806 $\int \sqrt{\cos(c+dx)}(a+b\sec(c+dx))^2\,dx$ 5944
3.807 $\int \frac{(a+b\sec(c+dx))^2}{\sqrt{\cos(c+dx)}}\,dx$ 5951
3.808 $\int \frac{(a+b\sec(c+dx))^2}{\cos^{\frac{3}{2}}(c+dx)}\,dx$ 5959
3.809 $\int \frac{(a+b\sec(c+dx))^2}{\cos^{\frac{5}{2}}(c+dx)}\,dx$ 5967
3.810 $\int \cos^{\frac{9}{2}}(c+dx)(a+b\sec(c+dx))^3\,dx$ 5976
3.811 $\int \cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^3\,dx$ 5985
3.812 $\int \cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^3\,dx$ 5994
3.813 $\int \cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^3\,dx$ 6002
3.814 $\int \sqrt{\cos(c+dx)}(a+b\sec(c+dx))^3\,dx$ 6010
3.815 $\int \frac{(a+b\sec(c+dx))^3}{\sqrt{\cos(c+dx)}}\,dx$ 6018
3.816 $\int \frac{(a+b\sec(c+dx))^3}{\cos^{\frac{3}{2}}(c+dx)}\,dx$ 6027
3.817 $\int \frac{\cos^{\frac{5}{2}}(c+dx)}{a+b\sec(c+dx)}\,dx$ 6036
3.818 $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{a+b\sec(c+dx)}\,dx$ 6046
3.819 $\int \frac{\sqrt{\cos(c+dx)}}{a+b\sec(c+dx)}\,dx$ 6055
3.820 $\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\sec(c+dx))}\,dx$ 6062
3.821 $\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))}\,dx$ 6068
3.822 $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))}\,dx$ 6073
3.823 $\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))}\,dx$ 6080
3.824 $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^2}\,dx$ 6090
3.825 $\int \frac{\sqrt{\cos(c+dx)}}{(a+b\sec(c+dx))^2}\,dx$ 6101
3.826 $\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^2}\,dx$ 6111
3.827 $\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2}\,dx$ 6120
3.828 $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^2}\,dx$ 6129

3.829 $\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^2}\,dx$ 6138
3.830 $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^3}\,dx$ 6149
3.831 $\int \frac{\sqrt{\cos(c+dx)}}{(a+b\sec(c+dx))^3}\,dx$ 6162
3.832 $\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^3}\,dx$ 6173
3.833 $\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^3}\,dx$ 6184
3.834 $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^3}\,dx$ 6195
3.835 $\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^3}\,dx$ 6206
3.836 $\int \frac{1}{\cos^{\frac{9}{2}}(c+dx)(a+b\sec(c+dx))^3}\,dx$ 6217
3.837 $\int \cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$ 6229
3.838 $\int \cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$ 6240
3.839 $\int \sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\,dx$ 6249
3.840 $\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\cos(c+dx)}}\,dx$ 6256
3.841 $\int \frac{\sqrt{a+b\sec(c+dx)}}{\cos^{\frac{3}{2}}(c+dx)}\,dx$ 6264
3.842 $\int \cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx$ 6275
3.843 $\int \cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx$ 6287
3.844 $\int \cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx$ 6298
3.845 $\int \sqrt{\cos(c+dx)}(a+b\sec(c+dx))^{3/2}\,dx$ 6307
3.846 $\int \frac{(a+b\sec(c+dx))^{3/2}}{\sqrt{\cos(c+dx)}}\,dx$ 6317
3.847 $\int \frac{(a+b\sec(c+dx))^{3/2}}{\cos^{\frac{3}{2}}(c+dx)}\,dx$ 6328
3.848 $\int \cos^{\frac{9}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$ 6340
3.849 $\int \cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$ 6353
3.850 $\int \cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$ 6365
3.851 $\int \cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$ 6376
3.852 $\int \sqrt{\cos(c+dx)}(a+b\sec(c+dx))^{5/2}\,dx$ 6387
3.853 $\int \frac{(a+b\sec(c+dx))^{5/2}}{\sqrt{\cos(c+dx)}}\,dx$ 6398
3.854 $\int \frac{(a+b\sec(c+dx))^{5/2}}{\cos^{\frac{3}{2}}(c+dx)}\,dx$ 6410
3.855 $\int \frac{\cos^{\frac{5}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$ 6423
3.856 $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$ 6434
3.857 $\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+b\sec(c+dx)}}\,dx$ 6444
3.858 $\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}\,dx$ 6453
3.859 $\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx$ 6459
3.860 $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx$ 6465
3.861 $\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx$ 6477

3.862 $\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$ 6490
3.863 $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$ 6503
3.864 $\int \frac{\sqrt{\cos(c+dx)}}{(a+b\sec(c+dx))^{3/2}}\,dx$ 6514
3.865 $\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^{3/2}}\,dx$ 6524
3.866 $\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx$ 6534
3.867 $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx$ 6541
3.868 $\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx$ 6551
3.869 $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$ 6564
3.870 $\int \frac{\sqrt{\cos(c+dx)}}{(a+b\sec(c+dx))^{5/2}}\,dx$ 6577
3.871 $\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^{5/2}}\,dx$ 6589
3.872 $\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx$ 6600
3.873 $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx$ 6611
3.874 $\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx$ 6622
3.875 $\int (d\cos(e+fx))^n(a+b\sec(e+fx))^3\,dx$ 6636
3.876 $\int (d\cos(e+fx))^n(a+b\sec(e+fx))^2\,dx$ 6644
3.877 $\int (d\cos(e+fx))^n(a+b\sec(e+fx))\,dx$ 6651
3.878 $\int \frac{(d\cos(e+fx))^n}{a+b\sec(e+fx)}\,dx$ 6656
3.879 $\int \frac{(d\cos(e+fx))^n}{(a+b\sec(e+fx))^2}\,dx$ 6662

3.1 $\int \sec^4(c+dx)(a+a\sec(c+dx))\,dx$

3.1.1 Optimal result . . . 306
3.1.2 Mathematica [A] (verified) . . . 306
3.1.3 Rubi [A] (verified) . . . 307
3.1.4 Maple [A] (verified) . . . 309
3.1.5 Fricas [A] (verification not implemented) . . . 309
3.1.6 Sympy [F] . . . 310
3.1.7 Maxima [A] (verification not implemented) . . . 310
3.1.8 Giac [A] (verification not implemented) . . . 311
3.1.9 Mupad [B] (verification not implemented) . . . 311

3.1.1 Optimal result

Integrand size = 19, antiderivative size = 85

$$\int \sec^4(c+dx)(a+a\sec(c+dx))\,dx = \frac{3a\text{arctanh}(\sin(c+dx))}{8d}+\frac{a\tan(c+dx)}{d}+\frac{3a\sec(c+dx)\tan(c+dx)}{8d}+\frac{a\sec^3(c+dx)\tan(c+dx)}{4d}+\frac{a\tan^3(c+dx)}{3d}$$

output
```
3/8*a*arctanh(sin(d*x+c))/d+a*tan(d*x+c)/d+3/8*a*sec(d*x+c)*tan(d*x+c)/d+1
/4*a*sec(d*x+c)^3*tan(d*x+c)/d+1/3*a*tan(d*x+c)^3/d
```

3.1.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.66

$$\int \sec^4(c+dx)(a+a\sec(c+dx))\,dx = \frac{a(9\text{arctanh}(\sin(c+dx))+\tan(c+dx)\,(9\sec(c+dx)+6\sec^3(c+dx)+8(3+\tan^2(c+dx))))}{24d}$$

input
```
Integrate[Sec[c + d*x]^4*(a + a*Sec[c + d*x]),x]
```

output
```
(a*(9*ArcTanh[Sin[c + d*x]] + Tan[c + d*x]*(9*Sec[c + d*x] + 6*Sec[c + d*x
]^3 + 8*(3 + Tan[c + d*x]^2))))/(24*d)
```

3.1.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.06, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.526, Rules used = {3042, 4274, 3042, 4254, 2009, 4255, 3042, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^4(c+dx)(a\sec(c+dx)+a)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^4\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)dx$$

$$\Big\downarrow \text{4274}$$

$$a\int \sec^5(c+dx)dx+a\int \sec^4(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$a\int \csc\left(c+dx+\frac{\pi}{2}\right)^4dx+a\int \csc\left(c+dx+\frac{\pi}{2}\right)^5dx$$

$$\Big\downarrow \text{4254}$$

$$a\int \csc\left(c+dx+\frac{\pi}{2}\right)^5dx-\frac{a\int\left(\tan^2(c+dx)+1\right)d(-\tan(c+dx))}{d}$$

$$\Big\downarrow \text{2009}$$

$$a\int \csc\left(c+dx+\frac{\pi}{2}\right)^5dx-\frac{a\left(-\frac{1}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{d}$$

$$\Big\downarrow \text{4255}$$

$$a\left(\frac{3}{4}\int \sec^3(c+dx)dx+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)-\frac{a\left(-\frac{1}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{d}$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{3}{4}\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)-\frac{a\left(-\frac{1}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{d}$$

$$\Big\downarrow \text{4255}$$

$$a\left(\frac{3}{4}\left(\frac{1}{2}\int \sec(c+dx)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)-\frac{a\left(-\frac{1}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{d}$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{3}{4}\left(\frac{1}{2}\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)-\frac{a\left(-\frac{1}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{d}$$

$$\Big\downarrow \text{4257}$$

$$a\left(\frac{3}{4}\left(\frac{\text{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)-\frac{a\left(-\frac{1}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{d}$$

input
```
Int[Sec[c + d*x]^4*(a + a*Sec[c + d*x]),x]
```

output
```
-((a*(-Tan[c + d*x] - Tan[c + d*x]^3/3))/d) + a*((Sec[c + d*x]^3*Tan[c + d
*x])/(4*d) + (3*(ArcTanh[Sin[c + d*x]]/(2*d) + (Sec[c + d*x]*Tan[c + d*x])
/(2*d)))/4)
```

3.1.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

3.1.4 Maple [A] (verified)

Time = 0.84 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.86

method	result
derivativedivides	$\frac{a\left(-\left(-\frac{\sec(dx+c)^3}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)-a\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right)\tan(dx+c)}{d}$
default	$\frac{a\left(-\left(-\frac{\sec(dx+c)^3}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)-a\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right)\tan(dx+c)}{d}$
parts	$-\frac{a\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right)\tan(dx+c)}{d}+\frac{a\left(-\left(-\frac{\sec(dx+c)^3}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)}{d}$
norman	$\frac{\frac{13a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d}-\frac{31a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{12d}+\frac{49a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{12d}-\frac{3a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{4d}}{\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^4}-\frac{3a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{8d}+\frac{3a\ln\left(\tan\left(\frac{dx}{2}+\right.\right.}{8d}$
risch	$-\frac{ia\left(9\,e^{7i(dx+c)}+33\,e^{5i(dx+c)}-48\,e^{4i(dx+c)}-33\,e^{3i(dx+c)}-64\,e^{2i(dx+c)}-9\,e^{i(dx+c)}-16\right)}{12d\left(e^{2i(dx+c)}+1\right)^4}-\frac{3a\ln\left(e^{i(dx+c)}-i\right)}{8d}+\frac{3a}{}$
parallelrisch	$\frac{a\left(9(-\cos(4dx+4c)-4\cos(2dx+2c)-3)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+9(3+\cos(4dx+4c)+4\cos(2dx+2c))\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+\right.}{24d(3+\cos(4dx+4c)+4\cos(2dx+2c))}$

input
```
int(sec(d*x+c)^4*(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(a*(-(-1/4*sec(d*x+c)^3-3/8*sec(d*x+c))*tan(d*x+c)+3/8*ln(sec(d*x+c)+t
an(d*x+c)))-a*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c))
```

3.1.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.16

$$\int \sec^4(c+dx)(a+a\sec(c+dx))\,dx$$
$$=\frac{9\,a\cos(dx+c)^4\log(\sin(dx+c)+1)-9\,a\cos(dx+c)^4\log(-\sin(dx+c)+1)+2\left(16\,a\cos(dx+c)^3+\right.}{48\,d\cos(dx+c)^4}$$

input
```
integrate(sec(d*x+c)^4*(a+a*sec(d*x+c)),x, algorithm="fricas")
```

output
```
1/48*(9*a*cos(d*x + c)^4*log(sin(d*x + c) + 1) - 9*a*cos(d*x + c)^4*log(-s
in(d*x + c) + 1) + 2*(16*a*cos(d*x + c)^3 + 9*a*cos(d*x + c)^2 + 8*a*cos(d
*x + c) + 6*a)*sin(d*x + c))/(d*cos(d*x + c)^4)
```

3.1.6 Sympy [F]

$$\int \sec^4(c+dx)(a+a\sec(c+dx))\,dx = a\left(\int \sec^4(c+dx)\,dx + \int \sec^5(c+dx)\,dx\right)$$

input
```
integrate(sec(d*x+c)**4*(a+a*sec(d*x+c)),x)
```

output
```
a*(Integral(sec(c + d*x)**4, x) + Integral(sec(c + d*x)**5, x))
```

3.1.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.12

$$\int \sec^4(c+dx)(a+a\sec(c+dx))\,dx$$
$$= \frac{16\left(\tan(dx+c)^3+3\tan(dx+c)\right)a - 3a\left(\frac{2\left(3\sin(dx+c)^3-5\sin(dx+c)\right)}{\sin(dx+c)^4-2\sin(dx+c)^2+1} - 3\log(\sin(dx+c)+1) + 3\log(\sin\right.}{48d}$$

input
```
integrate(sec(d*x+c)^4*(a+a*sec(d*x+c)),x, algorithm="maxima")
```

output
```
1/48*(16*(tan(d*x + c)^3 + 3*tan(d*x + c))*a - 3*a*(2*(3*sin(d*x + c)^3 -
5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x +
c) + 1) + 3*log(sin(d*x + c) - 1)))/d
```

3.1.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.29

$$\int \sec^4(c+dx)(a+a\sec(c+dx))\,dx$$
$$= \frac{9\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-9\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(9\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-49\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+31\,a\,\mathrm{t}\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-\right.}}{24\,d}$$

input
```
integrate(sec(d*x+c)^4*(a+a*sec(d*x+c)),x, algorithm="giac")
```

output
```
1/24*(9*a*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 9*a*log(abs(tan(1/2*d*x + 1
/2*c) - 1)) - 2*(9*a*tan(1/2*d*x + 1/2*c)^7 - 49*a*tan(1/2*d*x + 1/2*c)^5
+ 31*a*tan(1/2*d*x + 1/2*c)^3 - 39*a*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x +
1/2*c)^2 - 1)^4)/d
```

3.1.9 Mupad [B] (verification not implemented)

Time = 16.11 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.53

$$\int \sec^4(c+dx)(a+a\sec(c+dx))\,dx$$
$$= \frac{-\frac{3\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{4}+\frac{49\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{12}-\frac{31\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{12}+\frac{13\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8-4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}+\frac{3\,a\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{4\,d}$$

input
```
int((a + a/cos(c + d*x))/cos(c + d*x)^4,x)
```

output
```
((13*a*tan(c/2 + (d*x)/2))/4 - (31*a*tan(c/2 + (d*x)/2)^3)/12 + (49*a*tan(
c/2 + (d*x)/2)^5)/12 - (3*a*tan(c/2 + (d*x)/2)^7)/4)/(d*(6*tan(c/2 + (d*x)
/2)^4 - 4*tan(c/2 + (d*x)/2)^2 - 4*tan(c/2 + (d*x)/2)^6 + tan(c/2 + (d*x)/
2)^8 + 1)) + (3*a*atanh(tan(c/2 + (d*x)/2)))/(4*d)
```

3.2 $\int \sec^3(c+dx)(a+a\sec(c+dx))\,dx$

3.2.1 Optimal result . . . 312
3.2.2 Mathematica [A] (verified) . . . 312
3.2.3 Rubi [A] (verified) . . . 313
3.2.4 Maple [A] (verified) . . . 315
3.2.5 Fricas [A] (verification not implemented) . . . 315
3.2.6 Sympy [F] . . . 316
3.2.7 Maxima [A] (verification not implemented) . . . 316
3.2.8 Giac [A] (verification not implemented) . . . 316
3.2.9 Mupad [B] (verification not implemented) . . . 317

3.2.1 Optimal result

Integrand size = 19, antiderivative size = 63

$$\int \sec^3(c+dx)(a+a\sec(c+dx))\,dx = \frac{a\operatorname{arctanh}(\sin(c+dx))}{2d}+\frac{a\tan(c+dx)}{d}+\frac{a\sec(c+dx)\tan(c+dx)}{2d}+\frac{a\tan^3(c+dx)}{3d}$$

output
```
1/2*a*arctanh(sin(d*x+c))/d+a*tan(d*x+c)/d+1/2*a*sec(d*x+c)*tan(d*x+c)/d+1
/3*a*tan(d*x+c)^3/d
```

3.2.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.95

$$\int \sec^3(c+dx)(a+a\sec(c+dx))\,dx = \frac{a\operatorname{arctanh}(\sin(c+dx))}{2d}+\frac{a\sec(c+dx)\tan(c+dx)}{2d}+\frac{a\left(\tan(c+dx)+\frac{1}{3}\tan^3(c+dx)\right)}{d}$$

input
```
Integrate[Sec[c + d*x]^3*(a + a*Sec[c + d*x]),x]
```

output
```
(a*ArcTanh[Sin[c + d*x]])/(2*d) + (a*Sec[c + d*x]*Tan[c + d*x])/(2*d) + (a
*(Tan[c + d*x] + Tan[c + d*x]^3/3))/d
```

3.2.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.02, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {3042, 4274, 3042, 4254, 2009, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^3(c+dx)(a\sec(c+dx)+a)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^3\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)dx$$

$$\Big\downarrow \text{4274}$$

$$a\int \sec^4(c+dx)dx+a\int \sec^3(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$a\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx+a\int \csc\left(c+dx+\frac{\pi}{2}\right)^4dx$$

$$\Big\downarrow \text{4254}$$

$$a\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx-\frac{a\int\left(\tan^2(c+dx)+1\right)d(-\tan(c+dx))}{d}$$

$$\Big\downarrow \text{2009}$$

$$a\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx-\frac{a\left(-\frac{1}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{d}$$

$$\Big\downarrow \text{4255}$$

$$a\left(\frac{1}{2}\int \sec(c+dx)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)-\frac{a\left(-\frac{1}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{d}$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{1}{2}\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)-\frac{a\left(-\frac{1}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{d}$$

$$\Big\downarrow \text{4257}$$

$$a\left(\frac{\operatorname{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)-\frac{a\left(-\frac{1}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{d}$$

input
```
Int[Sec[c + d*x]^3*(a + a*Sec[c + d*x]),x]
```

output
```
a*(ArcTanh[Sin[c + d*x]]/(2*d) + (Sec[c + d*x]*Tan[c + d*x])/(2*d)) - (a*(
-Tan[c + d*x] - Tan[c + d*x]^3/3))/d
```

3.2.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

3.2.4 Maple [A] (verified)

Time = 0.61 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.95

method	result
derivativedivides	$\frac{-a\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right)\tan(dx+c)+a\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)}{d}$
default	$\frac{-a\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right)\tan(dx+c)+a\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)}{d}$
parts	$\frac{a\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)}{d}-\frac{a\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right)\tan(dx+c)}{d}$
risch	$-\frac{ia\left(3\,e^{5i(dx+c)}-12\,e^{2i(dx+c)}-3\,e^{i(dx+c)}-4\right)}{3d\left(e^{2i(dx+c)}+1\right)^3}+\frac{a\ln\left(e^{i(dx+c)}+i\right)}{2d}-\frac{a\ln\left(e^{i(dx+c)}-i\right)}{2d}$
norman	$\frac{-\frac{3a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{4a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3d}-\frac{a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{d}}{\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^3}-\frac{a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2d}+\frac{a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{2d}$
parallelrisch	$-\frac{a\left(9\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\cos(dx+c)-9\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)\cos(dx+c)+3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\cos(3dx+3c)-3\ln\left(\tan\right.\right.}{6d(\cos(3dx+3c)+3\cos(dx+c))}$

input
```
int(sec(d*x+c)^3*(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-a*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c)+a*(1/2*sec(d*x+c)*tan(d*x+c)+1/
2*ln(sec(d*x+c)+tan(d*x+c))))
```

3.2.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.40

$$\int \sec^3(c+dx)(a+a\sec(c+dx))\,dx$$
$$=\frac{3\,a\cos\left(dx+c\right)^3\log\left(\sin\left(dx+c\right)+1\right)-3\,a\cos\left(dx+c\right)^3\log\left(-\sin\left(dx+c\right)+1\right)+2\left(4\,a\cos\left(dx+c\right)^2+\right.}{12\,d\cos\left(dx+c\right)^3}$$

input
```
integrate(sec(d*x+c)^3*(a+a*sec(d*x+c)),x, algorithm="fricas")
```

output
```
1/12*(3*a*cos(d*x + c)^3*log(sin(d*x + c) + 1) - 3*a*cos(d*x + c)^3*log(-s
in(d*x + c) + 1) + 2*(4*a*cos(d*x + c)^2 + 3*a*cos(d*x + c) + 2*a)*sin(d*x
 + c))/(d*cos(d*x + c)^3)
```

3.2.6 Sympy [F]

$$\int \sec^3(c+dx)(a+a\sec(c+dx))\,dx = a\left(\int \sec^3(c+dx)\,dx + \int \sec^4(c+dx)\,dx\right)$$

input
```
integrate(sec(d*x+c)**3*(a+a*sec(d*x+c)),x)
```

output
```
a*(Integral(sec(c + d*x)**3, x) + Integral(sec(c + d*x)**4, x))
```

3.2.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.11

$$\int \sec^3(c+dx)(a+a\sec(c+dx))\,dx = \frac{4\left(\tan\left(dx+c\right)^3+3\,\tan\left(dx+c\right)\right)a-3\,a\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1}-\log\left(\sin\left(dx+c\right)+1\right)+\log\left(\sin\left(dx+c\right)-1\right)\right)}{12\,d}$$

input
```
integrate(sec(d*x+c)^3*(a+a*sec(d*x+c)),x, algorithm="maxima")
```

output
```
1/12*(4*(tan(d*x + c)^3 + 3*tan(d*x + c))*a - 3*a*(2*sin(d*x + c)/(sin(d*x
 + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)))/d
```

3.2.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.52

$$\int \sec^3(c+dx)(a+a\sec(c+dx))\,dx = \frac{3\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(3\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-4\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+9\,a\tan\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^3}}{6\,d}$$

input
```
integrate(sec(d*x+c)^3*(a+a*sec(d*x+c)),x, algorithm="giac")
```

output
```
1/6*(3*a*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*a*log(abs(tan(1/2*d*x + 1/
2*c) - 1)) - 2*(3*a*tan(1/2*d*x + 1/2*c)^5 - 4*a*tan(1/2*d*x + 1/2*c)^3 +
9*a*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^3)/d
```

3.2.9 Mupad [B] (verification not implemented)

Time = 14.52 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.62

$$\int \sec^3(c+dx)(a+a\sec(c+dx))\,dx$$
$$= \frac{a\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d} - \frac{a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5 - \frac{4\,a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{3} + 3\,a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6 - 3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4 + 3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2 - 1\right)}$$

input
```
int((a + a/cos(c + d*x))/cos(c + d*x)^3,x)
```

output
```
(a*atanh(tan(c/2 + (d*x)/2)))/d - (3*a*tan(c/2 + (d*x)/2) - (4*a*tan(c/2 +
 (d*x)/2)^3)/3 + a*tan(c/2 + (d*x)/2)^5)/(d*(3*tan(c/2 + (d*x)/2)^2 - 3*ta
n(c/2 + (d*x)/2)^4 + tan(c/2 + (d*x)/2)^6 - 1))
```

3.3 $\int \sec^2(c+dx)(a+a\sec(c+dx))\,dx$

3.3.1 Optimal result . . . 318
3.3.2 Mathematica [A] (verified) . . . 318
3.3.3 Rubi [A] (verified) . . . 319
3.3.4 Maple [A] (verified) . . . 321
3.3.5 Fricas [A] (verification not implemented) . . . 321
3.3.6 Sympy [F] . . . 322
3.3.7 Maxima [A] (verification not implemented) . . . 322
3.3.8 Giac [A] (verification not implemented) . . . 322
3.3.9 Mupad [B] (verification not implemented) . . . 323

3.3.1 Optimal result

Integrand size = 19, antiderivative size = 47

$$\int \sec^2(c+dx)(a+a\sec(c+dx))\,dx = \frac{a\operatorname{arctanh}(\sin(c+dx))}{2d}+\frac{a\tan(c+dx)}{d}+\frac{a\sec(c+dx)\tan(c+dx)}{2d}$$

output
```
1/2*a*arctanh(sin(d*x+c))/d+a*tan(d*x+c)/d+1/2*a*sec(d*x+c)*tan(d*x+c)/d
```

3.3.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00

$$\int \sec^2(c+dx)(a+a\sec(c+dx))\,dx = \frac{a\operatorname{arctanh}(\sin(c+dx))}{2d}+\frac{a\tan(c+dx)}{d}+\frac{a\sec(c+dx)\tan(c+dx)}{2d}$$

input
```
Integrate[Sec[c + d*x]^2*(a + a*Sec[c + d*x]),x]
```

output
```
(a*ArcTanh[Sin[c + d*x]])/(2*d) + (a*Tan[c + d*x])/d + (a*Sec[c + d*x]*Tan
[c + d*x])/(2*d)
```

3.3.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.02, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {3042, 4274, 3042, 4254, 24, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(c+dx)(a\sec(c+dx)+a)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^2\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)dx$$

$$\Big\downarrow \text{4274}$$

$$a\int \sec^3(c+dx)dx+a\int \sec^2(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$a\int \csc\left(c+dx+\frac{\pi}{2}\right)^2dx+a\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx$$

$$\Big\downarrow \text{4254}$$

$$a\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx-\frac{a\int 1d(-\tan(c+dx))}{d}$$

$$\Big\downarrow \text{24}$$

$$a\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx+\frac{a\tan(c+dx)}{d}$$

$$\Big\downarrow \text{4255}$$

$$a\left(\frac{1}{2}\int \sec(c+dx)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{a\tan(c+dx)}{d}$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{1}{2}\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{a\tan(c+dx)}{d}$$

$$\Big\downarrow \text{4257}$$

$$a\left(\frac{\text{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{a\tan(c+dx)}{d}$$

input
```
Int[Sec[c + d*x]^2*(a + a*Sec[c + d*x]),x]
```

output
```
(a*Tan[c + d*x])/d + a*(ArcTanh[Sin[c + d*x]]/(2*d) + (Sec[c + d*x]*Tan[c
+ d*x])/(2*d))
```

3.3.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

3.3.4 Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00

method	result	s
derivativedivides	$\frac{a\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+a\tan(dx+c)}{d}$	4
default	$\frac{a\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+a\tan(dx+c)}{d}$	4
parts	$\frac{a\tan(dx+c)}{d}+\frac{a\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)}{d}$	4
norman	$\frac{\frac{3a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}-\frac{a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{d}}{\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^2}-\frac{a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2d}+\frac{a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{2d}$	8
parallelrisch	$\frac{\left((-1-\cos(2dx+2c))\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+(1+\cos(2dx+2c))\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+2\sin(dx+c)+2\sin(2dx+2c)\right)a}{2d(1+\cos(2dx+2c))}$	9
risch	$-\frac{ia\left(e^{3i(dx+c)}-2e^{2i(dx+c)}-e^{i(dx+c)}-2\right)}{d\left(e^{2i(dx+c)}+1\right)^2}+\frac{a\ln\left(e^{i(dx+c)}+i\right)}{2d}-\frac{a\ln\left(e^{i(dx+c)}-i\right)}{2d}$	9

input
```
int(sec(d*x+c)^2*(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(a*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c)))+a*tan(d*x
+c))
```

3.3.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.57

$$\int \sec^2(c+dx)(a+a\sec(c+dx))\,dx$$
$$=\frac{a\cos(dx+c)^2\log(\sin(dx+c)+1)-a\cos(dx+c)^2\log(-\sin(dx+c)+1)+2(2a\cos(dx+c)+a)\sin}{4d\cos(dx+c)^2}$$

input
```
integrate(sec(d*x+c)^2*(a+a*sec(d*x+c)),x, algorithm="fricas")
```

output
```
1/4*(a*cos(d*x + c)^2*log(sin(d*x + c) + 1) - a*cos(d*x + c)^2*log(-sin(d*
x + c) + 1) + 2*(2*a*cos(d*x + c) + a)*sin(d*x + c))/(d*cos(d*x + c)^2)
```

3.3.6 Sympy [F]

$$\int \sec^2(c+dx)(a+a\sec(c+dx))\,dx = a\left(\int \sec^2(c+dx)\,dx + \int \sec^3(c+dx)\,dx\right)$$

input
```
integrate(sec(d*x+c)**2*(a+a*sec(d*x+c)),x)
```

output
```
a*(Integral(sec(c + d*x)**2, x) + Integral(sec(c + d*x)**3, x))
```

3.3.7 Maxima [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.23

$$\int \sec^2(c+dx)(a+a\sec(c+dx))\,dx = -\frac{a\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1} - \log(\sin(dx+c)+1) + \log(\sin(dx+c)-1)\right) - 4\,a\tan(dx+c)}{4\,d}$$

input
```
integrate(sec(d*x+c)^2*(a+a*sec(d*x+c)),x, algorithm="maxima")
```

output
```
-1/4*(a*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log
(sin(d*x + c) - 1)) - 4*a*tan(d*x + c))/d
```

3.3.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.70

$$\int \sec^2(c+dx)(a+a\sec(c+dx))\,dx = \frac{a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right) - a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right) - \frac{2\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 - 3\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^2}}{2\,d}$$

input
```
integrate(sec(d*x+c)^2*(a+a*sec(d*x+c)),x, algorithm="giac")
```

output
```
1/2*(a*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - a*log(abs(tan(1/2*d*x + 1/2*c)
 - 1)) - 2*(a*tan(1/2*d*x + 1/2*c)^3 - 3*a*tan(1/2*d*x + 1/2*c))/(tan(1/2*
d*x + 1/2*c)^2 - 1)^2)/d
```

3.3.9 Mupad [B] (verification not implemented)

Time = 13.22 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.60

$$\int \sec^2(c+dx)(a+a\sec(c+dx))\,dx = \frac{3\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)} + \frac{a\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}$$

input
```
int((a + a/cos(c + d*x))/cos(c + d*x)^2,x)
```

output
```
(3*a*tan(c/2 + (d*x)/2) - a*tan(c/2 + (d*x)/2)^3)/(d*(tan(c/2 + (d*x)/2)^4
 - 2*tan(c/2 + (d*x)/2)^2 + 1)) + (a*atanh(tan(c/2 + (d*x)/2)))/d
```

3.4 $\int \sec(c+dx)(a+a\sec(c+dx))\,dx$

3.4.1 Optimal result . . . 324
3.4.2 Mathematica [A] (verified) . . . 324
3.4.3 Rubi [A] (verified) . . . 325
3.4.4 Maple [A] (verified) . . . 326
3.4.5 Fricas [B] (verification not implemented) . . . 327
3.4.6 Sympy [A] (verification not implemented) . . . 327
3.4.7 Maxima [A] (verification not implemented) . . . 328
3.4.8 Giac [B] (verification not implemented) . . . 328
3.4.9 Mupad [B] (verification not implemented) . . . 328

3.4.1 Optimal result

Integrand size = 17, antiderivative size = 24

$$\int \sec(c+dx)(a+a\sec(c+dx))\,dx=\frac{a\operatorname{arctanh}(\sin(c+dx))}{d}+\frac{a\tan(c+dx)}{d}$$

output
```
a*arctanh(sin(d*x+c))/d+a*tan(d*x+c)/d
```

3.4.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \sec(c+dx)(a+a\sec(c+dx))\,dx=\frac{a\operatorname{arctanh}(\sin(c+dx))}{d}+\frac{a\tan(c+dx)}{d}$$

input
```
Integrate[Sec[c + d*x]*(a + a*Sec[c + d*x]),x]
```

output
```
(a*ArcTanh[Sin[c + d*x]])/d + (a*Tan[c + d*x])/d
```

3.4.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {3042, 4274, 3042, 4254, 24, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(c+dx)(a\sec(c+dx)+a)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)dx$$

$$\Big\downarrow \text{4274}$$

$$a\int \sec^2(c+dx)dx+a\int \sec(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$a\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+a\int \csc\left(c+dx+\frac{\pi}{2}\right)^2dx$$

$$\Big\downarrow \text{4254}$$

$$a\int \csc\left(c+dx+\frac{\pi}{2}\right)dx-\frac{a\int 1d(-\tan(c+dx))}{d}$$

$$\Big\downarrow \text{24}$$

$$a\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{a\tan(c+dx)}{d}$$

$$\Big\downarrow \text{4257}$$

$$\frac{a\text{arctanh}(\sin(c+dx))}{d}+\frac{a\tan(c+dx)}{d}$$

input
```
Int[Sec[c + d*x]*(a + a*Sec[c + d*x]),x]
```

output
```
(a*ArcTanh[Sin[c + d*x]])/d + (a*Tan[c + d*x])/d
```

3.4.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

3.4.4 Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.25

method	result	size
derivativedivides	$\frac{a\tan(dx+c)+a\ln(\sec(dx+c)+\tan(dx+c))}{d}$	30
default	$\frac{a\tan(dx+c)+a\ln(\sec(dx+c)+\tan(dx+c))}{d}$	30
parts	$\frac{a\ln(\sec(dx+c)+\tan(dx+c))}{d}+\frac{a\tan(dx+c)}{d}$	32
risch	$\frac{2ia}{d(e^{2i(dx+c)}+1)}+\frac{a\ln(e^{i(dx+c)}+i)}{d}-\frac{a\ln(e^{i(dx+c)}-i)}{d}$	59
parallelrisch	$\frac{a\left(-\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\cos(dx+c)+\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)\cos(dx+c)+\sin(dx+c)\right)}{d\cos(dx+c)}$	60
norman	$-\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)}+\frac{a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{d}-\frac{a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{d}$	67

input
```
int(sec(d*x+c)*(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(a*tan(d*x+c)+a*ln(sec(d*x+c)+tan(d*x+c)))
```

3.4.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 60 vs. $2(24) = 48$.

Time = 0.26 (sec) , antiderivative size = 60, normalized size of antiderivative = 2.50

$$\int \sec(c+dx)(a+a\sec(c+dx))\,dx = \frac{a\cos(dx+c)\log(\sin(dx+c)+1) - a\cos(dx+c)\log(-\sin(dx+c)+1) + 2\,a\sin(dx+c)}{2\,d\cos(dx+c)}$$

input
```
integrate(sec(d*x+c)*(a+a*sec(d*x+c)),x, algorithm="fricas")
```

output
```
1/2*(a*cos(d*x + c)*log(sin(d*x + c) + 1) - a*cos(d*x + c)*log(-sin(d*x +
c) + 1) + 2*a*sin(d*x + c))/(d*cos(d*x + c))
```

3.4.6 Sympy [A] (verification not implemented)

Time = 2.38 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.54

$$\int \sec(c+dx)(a+a\sec(c+dx))\,dx = \begin{cases} \frac{a\log(\tan(c+dx)+\sec(c+dx))+a\tan(c+dx)}{d} & \text{for } d \neq 0 \\ x(a\sec(c)+a)\sec(c) & \text{otherwise} \end{cases}$$

input
```
integrate(sec(d*x+c)*(a+a*sec(d*x+c)),x)
```

output
```
Piecewise(((a*log(tan(c + d*x) + sec(c + d*x)) + a*tan(c + d*x))/d, Ne(d,
0)), (x*(a*sec(c) + a)*sec(c), True))
```

3.4.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.21

$$\int \sec(c+dx)(a+a\sec(c+dx))\,dx = \frac{a\log\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)+a\tan\left(dx+c\right)}{d}$$

input
```
integrate(sec(d*x+c)*(a+a*sec(d*x+c)),x, algorithm="maxima")
```

output
```
(a*log(sec(d*x + c) + tan(d*x + c)) + a*tan(d*x + c))/d
```

3.4.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 63 vs. 2(24) = 48.

Time = 0.31 (sec) , antiderivative size = 63, normalized size of antiderivative = 2.62

$$\int \sec(c+dx)(a+a\sec(c+dx))\,dx = \frac{a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1}}{d}$$

input
```
integrate(sec(d*x+c)*(a+a*sec(d*x+c)),x, algorithm="giac")
```

output
```
(a*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - a*log(abs(tan(1/2*d*x + 1/2*c) - 1
)) - 2*a*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^2 - 1))/d
```

3.4.9 Mupad [B] (verification not implemented)

Time = 13.48 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.96

$$\int \sec(c+dx)(a+a\sec(c+dx))\,dx = \frac{2\,a\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{2\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)}$$

input
```
int((a + a/cos(c + d*x))/cos(c + d*x),x)
```

output
```
(2*a*atanh(tan(c/2 + (d*x)/2)))/d - (2*a*tan(c/2 + (d*x)/2))/(d*(tan(c/2 +
 (d*x)/2)^2 - 1))
```

3.5 $\int (a + a\sec(c + dx))\,dx$

3.5.1 Optimal result . . . 329
3.5.2 Mathematica [A] (verified) . . . 329
3.5.3 Rubi [A] (verified) . . . 330
3.5.4 Maple [A] (verified) . . . 330
3.5.5 Fricas [B] (verification not implemented) . . . 331
3.5.6 Sympy [A] (verification not implemented) . . . 331
3.5.7 Maxima [A] (verification not implemented) . . . 332
3.5.8 Giac [B] (verification not implemented) . . . 332
3.5.9 Mupad [B] (verification not implemented) . . . 332

3.5.1 Optimal result

Integrand size = 10, antiderivative size = 16

$$\int (a + a\sec(c + dx))\,dx = ax + \frac{a\operatorname{arctanh}(\sin(c + dx))}{d}$$

output
```
a*x+a*arctanh(sin(d*x+c))/d
```

3.5.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int (a + a\sec(c + dx))\,dx = ax + \frac{a\operatorname{arctanh}(\sin(c + dx))}{d}$$

input
```
Integrate[a + a*Sec[c + d*x],x]
```

output
```
a*x + (a*ArcTanh[Sin[c + d*x]])/d
```

3.5.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sec(c+dx)+a)\,dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{a\text{arctanh}(\sin(c+dx))}{d}+ax$$

input
```
Int[a + a*Sec[c + d*x],x]
```

output
```
a*x + (a*ArcTanh[Sin[c + d*x]])/d
```

3.5.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.5.4 Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.50

method	result	size
default	$ax+\frac{a\ln(\sec(dx+c)+\tan(dx+c))}{d}$	24
parts	$ax+\frac{a\ln(\sec(dx+c)+\tan(dx+c))}{d}$	24
derivativedivides	$\frac{a(dx+c)+a\ln(\sec(dx+c)+\tan(dx+c))}{d}$	29
parallelrisch	$\frac{a\left(-\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)\right)}{d}+ax$	37
norman	$ax+\frac{a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{d}-\frac{a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{d}$	40
risch	$ax+\frac{a\ln\left(e^{i(dx+c)}+i\right)}{d}-\frac{a\ln\left(e^{i(dx+c)}-i\right)}{d}$	42

input
```
int(a+a*sec(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
a*x+a/d*ln(sec(d*x+c)+tan(d*x+c))
```

3.5.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 36 vs. $2(16) = 32$.

Time = 0.28 (sec) , antiderivative size = 36, normalized size of antiderivative = 2.25

$$\int (a + a\sec(c + dx))\,dx = \frac{2\,adx + a\log\left(\sin\left(dx + c\right) + 1\right) - a\log\left(-\sin\left(dx + c\right) + 1\right)}{2\,d}$$

input
```
integrate(a+a*sec(d*x+c),x, algorithm="fricas")
```

output
```
1/2*(2*a*d*x + a*log(sin(d*x + c) + 1) - a*log(-sin(d*x + c) + 1))/d
```

3.5.6 Sympy [A] (verification not implemented)

Time = 0.87 (sec) , antiderivative size = 41, normalized size of antiderivative = 2.56

$$\int (a + a\sec(c + dx))\,dx = ax + a\left(\begin{cases} \frac{\log\left(\tan\left(c+dx\right)+\sec\left(c+dx\right)\right)}{d} & \text{for}\ d \neq 0 \\ \frac{x\left(\tan\left(c\right)\sec\left(c\right)+\sec^{2}\left(c\right)\right)}{\tan\left(c\right)+\sec\left(c\right)} & \text{otherwise} \end{cases}\right)$$

input
```
integrate(a+a*sec(d*x+c),x)
```

output
```
a*x + a*Piecewise((log(tan(c + d*x) + sec(c + d*x))/d, Ne(d, 0)), (x*(tan(
c)*sec(c) + sec(c)**2)/(tan(c) + sec(c)), True))
```

3.5.7 Maxima [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.44

$$\int (a + a\sec(c + dx))\,dx = ax + \frac{a\log\left(\sec\left(dx + c\right) + \tan\left(dx + c\right)\right)}{d}$$

input
```
integrate(a+a*sec(d*x+c),x, algorithm="maxima")
```

output
```
a*x + a*log(sec(d*x + c) + tan(d*x + c))/d
```

3.5.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 49 vs. 2(16) = 32.

Time = 0.27 (sec) , antiderivative size = 49, normalized size of antiderivative = 3.06

$$\int (a + a\sec(c + dx))\,dx = ax + \frac{a\left(\log\left(\left|\frac{1}{\sin(dx+c)} + \sin\left(dx + c\right) + 2\right|\right) - \log\left(\left|\frac{1}{\sin(dx+c)} + \sin\left(dx + c\right) - 2\right|\right)\right)}{4\,d}$$

input
```
integrate(a+a*sec(d*x+c),x, algorithm="giac")
```

output
```
a*x + 1/4*a*(log(abs(1/sin(d*x + c) + sin(d*x + c) + 2)) - log(abs(1/sin(d
*x + c) + sin(d*x + c) - 2)))/d
```

3.5.9 Mupad [B] (verification not implemented)

Time = 13.39 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.25

$$\int (a + a\sec(c + dx))\,dx = a\,x + \frac{2\,a\,\mathrm{atanh}\left(\tan\left(\frac{c}{2} + \frac{d\,x}{2}\right)\right)}{d}$$

input
```
int(a + a/cos(c + d*x),x)
```

output
```
a*x + (2*a*atanh(tan(c/2 + (d*x)/2)))/d
```

3.6 $\int \cos(c+dx)(a+a\sec(c+dx))\,dx$

3.6.1 Optimal result . . . 333
3.6.2 Mathematica [A] (verified) . . . 333
3.6.3 Rubi [A] (verified) . . . 334
3.6.4 Maple [A] (verified) . . . 335
3.6.5 Fricas [A] (verification not implemented) . . . 336
3.6.6 Sympy [A] (verification not implemented) . . . 336
3.6.7 Maxima [A] (verification not implemented) . . . 336
3.6.8 Giac [B] (verification not implemented) . . . 337
3.6.9 Mupad [B] (verification not implemented) . . . 337

3.6.1 Optimal result

Integrand size = 17, antiderivative size = 15

$$\int \cos(c+dx)(a+a\sec(c+dx))\,dx = ax+\frac{a\sin(c+dx)}{d}$$

output
```
a*x+a*sin(d*x+c)/d
```

3.6.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.73

$$\int \cos(c+dx)(a+a\sec(c+dx))\,dx = ax+\frac{a\cos(dx)\sin(c)}{d}+\frac{a\cos(c)\sin(dx)}{d}$$

input
```
Integrate[Cos[c + d*x]*(a + a*Sec[c + d*x]),x]
```

output
```
a*x + (a*Cos[d*x]*Sin[c])/d + (a*Cos[c]*Sin[d*x])/d
```

3.6.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {3042, 4274, 24, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(c+dx)(a\sec(c+dx)+a)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}{\csc\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{4274}$$

$$a\int \cos(c+dx)dx+a\int 1dx$$

$$\Big\downarrow \text{24}$$

$$a\int \cos(c+dx)dx+ax$$

$$\Big\downarrow \text{3042}$$

$$a\int \sin\left(c+dx+\frac{\pi}{2}\right)dx+ax$$

$$\Big\downarrow \text{3117}$$

$$\frac{a\sin(c+dx)}{d}+ax$$

input
```
Int[Cos[c + d*x]*(a + a*Sec[c + d*x]),x]
```

output
```
a*x + (a*Sin[c + d*x])/d
```

3.6.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

3.6.4 Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.07

method	result	size
risch	$ax+\frac{a\sin(dx+c)}{d}$	16
parallelrisch	$\frac{a(dx+\sin(dx+c))}{d}$	16
derivativedivides	$\frac{a\sin(dx+c)+a(dx+c)}{d}$	21
default	$\frac{a\sin(dx+c)+a(dx+c)}{d}$	21
norman	$\frac{ax+ax\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}}{1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}$	50

input
```
int(cos(d*x+c)*(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
a*x+a*sin(d*x+c)/d
```

3.6.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \cos(c+dx)(a+a\sec(c+dx))\,dx = \frac{adx+a\sin(dx+c)}{d}$$

input
```
integrate(cos(d*x+c)*(a+a*sec(d*x+c)),x, algorithm="fricas")
```

output
```
(a*d*x + a*sin(d*x + c))/d
```

3.6.6 Sympy [A] (verification not implemented)

Time = 0.78 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \cos(c+dx)(a+a\sec(c+dx))\,dx = ax+a\left(\begin{cases} x\cos(c) & \text{for } d=0 \\ \frac{\sin(c+dx)}{d} & \text{otherwise} \end{cases}\right)$$

input
```
integrate(cos(d*x+c)*(a+a*sec(d*x+c)),x)
```

output
```
a*x + a*Piecewise((x*cos(c), Eq(d, 0)), (sin(c + d*x)/d, True))
```

3.6.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.33

$$\int \cos(c+dx)(a+a\sec(c+dx))\,dx = \frac{(dx+c)a+a\sin(dx+c)}{d}$$

input
```
integrate(cos(d*x+c)*(a+a*sec(d*x+c)),x, algorithm="maxima")
```

output
```
((d*x + c)*a + a*sin(d*x + c))/d
```

3.6.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 39 vs. $2(15) = 30$.

Time = 0.28 (sec) , antiderivative size = 39, normalized size of antiderivative = 2.60

$$\int \cos(c+dx)(a+a\sec(c+dx))\,dx = \frac{(dx+c)a + \frac{2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1}}{d}$$

input
```
integrate(cos(d*x+c)*(a+a*sec(d*x+c)),x, algorithm="giac")
```

output
```
((d*x + c)*a + 2*a*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^2 + 1))/d
```

3.6.9 Mupad [B] (verification not implemented)

Time = 13.22 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \cos(c+dx)(a+a\sec(c+dx))\,dx = a\,x + \frac{a\,\sin\left(c+d\,x\right)}{d}$$

input
```
int(cos(c + d*x)*(a + a/cos(c + d*x)),x)
```

output
```
a*x + (a*sin(c + d*x))/d
```

3.7 $\int \cos^2(c+dx)(a+a\sec(c+dx))\,dx$

3.7.1 Optimal result . . . 338
3.7.2 Mathematica [A] (verified) . . . 338
3.7.3 Rubi [A] (verified) . . . 339
3.7.4 Maple [A] (verified) . . . 340
3.7.5 Fricas [A] (verification not implemented) . . . 341
3.7.6 Sympy [F] . . . 341
3.7.7 Maxima [A] (verification not implemented) . . . 341
3.7.8 Giac [A] (verification not implemented) . . . 342
3.7.9 Mupad [B] (verification not implemented) . . . 342

3.7.1 Optimal result

Integrand size = 19, antiderivative size = 38

$$\int \cos^2(c+dx)(a+a\sec(c+dx))\,dx=\frac{ax}{2}+\frac{a\sin(c+dx)}{d}+\frac{a\cos(c+dx)\sin(c+dx)}{2d}$$

output
```
1/2*a*x+a*sin(d*x+c)/d+1/2*a*cos(d*x+c)*sin(d*x+c)/d
```

3.7.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.84

$$\int \cos^2(c+dx)(a+a\sec(c+dx))\,dx=\frac{a(2(c+dx)+4\sin(c+dx)+\sin(2(c+dx)))}{4d}$$

input
```
Integrate[Cos[c + d*x]^2*(a + a*Sec[c + d*x]),x]
```

output
```
(a*(2*(c + d*x) + 4*Sin[c + d*x] + Sin[2*(c + d*x)]))/(4*d)
```

3.7.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {3042, 4274, 3042, 3115, 24, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(c+dx)(a\sec(c+dx)+a)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}{\csc\left(c+dx+\frac{\pi}{2}\right)^2}dx$$

$$\Big\downarrow \text{4274}$$

$$a\int \cos^2(c+dx)dx+a\int \cos(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$a\int \sin\left(c+dx+\frac{\pi}{2}\right)dx+a\int \sin\left(c+dx+\frac{\pi}{2}\right)^2dx$$

$$\Big\downarrow \text{3115}$$

$$a\int \sin\left(c+dx+\frac{\pi}{2}\right)dx+a\left(\frac{\int 1dx}{2}+\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)$$

$$\Big\downarrow \text{24}$$

$$a\int \sin\left(c+dx+\frac{\pi}{2}\right)dx+a\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)$$

$$\Big\downarrow \text{3117}$$

$$\frac{a\sin(c+dx)}{d}+a\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)$$

input
```
Int[Cos[c + d*x]^2*(a + a*Sec[c + d*x]),x]
```

output
```
(a*Sin[c + d*x])/d + a*(x/2 + (Cos[c + d*x]*Sin[c + d*x])/(2*d))
```

3.7.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

3.7.4 Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.76

method	result	size
parallelrisch	$\frac{a(2dx+4\sin(dx+c)+\sin(2dx+2c))}{4d}$	29
risch	$\frac{ax}{2}+\frac{a\sin(dx+c)}{d}+\frac{a\sin(2dx+2c)}{4d}$	32
derivativedivides	$\frac{a\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+a\sin(dx+c)}{d}$	38
default	$\frac{a\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+a\sin(dx+c)}{d}$	38
norman	$\frac{\frac{a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{d}+ax\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+\frac{ax}{2}+\frac{3a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{ax\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{2}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^2}$	82

input
```
int(cos(d*x+c)^2*(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/4*a*(2*d*x+4*sin(d*x+c)+sin(2*d*x+2*c))/d
```

3.7.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.76

$$\int \cos^2(c+dx)(a+a\sec(c+dx))\,dx = \frac{adx + (a\cos(dx+c)+2\,a)\sin(dx+c)}{2\,d}$$

input
```
integrate(cos(d*x+c)^2*(a+a*sec(d*x+c)),x, algorithm="fricas")
```

output
```
1/2*(a*d*x + (a*cos(d*x + c) + 2*a)*sin(d*x + c))/d
```

3.7.6 Sympy [F]

$$\int \cos^2(c+dx)(a+a\sec(c+dx))\,dx = a\left(\int \cos^2(c+dx)\sec(c+dx)\,dx + \int \cos^2(c+dx)\,dx\right)$$

input
```
integrate(cos(d*x+c)**2*(a+a*sec(d*x+c)),x)
```

output
```
a*(Integral(cos(c + d*x)**2*sec(c + d*x), x) + Integral(cos(c + d*x)**2, x
))
```

3.7.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.89

$$\int \cos^2(c+dx)(a+a\sec(c+dx))\,dx = \frac{(2\,dx+2\,c+\sin(2\,dx+2\,c))a + 4\,a\sin(dx+c)}{4\,d}$$

input
```
integrate(cos(d*x+c)^2*(a+a*sec(d*x+c)),x, algorithm="maxima")
```

output
```
1/4*((2*d*x + 2*c + sin(2*d*x + 2*c))*a + 4*a*sin(d*x + c))/d
```

3.7.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.47

$$\int \cos^2(c+dx)(a+a\sec(c+dx))\,dx = \frac{(dx+c)a + \frac{2\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+3\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^2}}{2\,d}$$

input
```
integrate(cos(d*x+c)^2*(a+a*sec(d*x+c)),x, algorithm="giac")
```

output
```
1/2*((d*x + c)*a + 2*(a*tan(1/2*d*x + 1/2*c)^3 + 3*a*tan(1/2*d*x + 1/2*c))
/(tan(1/2*d*x + 1/2*c)^2 + 1)^2)/d
```

3.7.9 Mupad [B] (verification not implemented)

Time = 13.76 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.32

$$\int \cos^2(c+dx)(a+a\sec(c+dx))\,dx = \frac{a\,x}{2} + \frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3 + 3\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)^2}$$

input
```
int(cos(c + d*x)^2*(a + a/cos(c + d*x)),x)
```

output
```
(a*x)/2 + (3*a*tan(c/2 + (d*x)/2) + a*tan(c/2 + (d*x)/2)^3)/(d*(tan(c/2 +
(d*x)/2)^2 + 1)^2)
```

3.8 $\int \cos^3(c+dx)(a+a\sec(c+dx))\,dx$

3.8.1 Optimal result . . . 343
3.8.2 Mathematica [A] (verified) . . . 343
3.8.3 Rubi [A] (verified) . . . 344
3.8.4 Maple [A] (verified) . . . 346
3.8.5 Fricas [A] (verification not implemented) . . . 346
3.8.6 Sympy [F] . . . 347
3.8.7 Maxima [A] (verification not implemented) . . . 347
3.8.8 Giac [A] (verification not implemented) . . . 347
3.8.9 Mupad [B] (verification not implemented) . . . 348

3.8.1 Optimal result

Integrand size = 19, antiderivative size = 54

$$\int \cos^3(c+dx)(a+a\sec(c+dx))\,dx = \frac{ax}{2}+\frac{a\sin(c+dx)}{d}+\frac{a\cos(c+dx)\sin(c+dx)}{2d}-\frac{a\sin^3(c+dx)}{3d}$$

output
```
1/2*a*x+a*sin(d*x+c)/d+1/2*a*cos(d*x+c)*sin(d*x+c)/d-1/3*a*sin(d*x+c)^3/d
```

3.8.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.06

$$\int \cos^3(c+dx)(a+a\sec(c+dx))\,dx = \frac{a(c+dx)}{2d}+\frac{a\sin(c+dx)}{d}-\frac{a\sin^3(c+dx)}{3d}+\frac{a\sin(2(c+dx))}{4d}$$

input
```
Integrate[Cos[c + d*x]^3*(a + a*Sec[c + d*x]),x]
```

output
```
(a*(c + d*x))/(2*d) + (a*Sin[c + d*x])/d - (a*Sin[c + d*x]^3)/(3*d) + (a*S
in[2*(c + d*x)])/(4*d)
```

3.8.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.02, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {3042, 4274, 3042, 3113, 2009, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^3(c+dx)(a\sec(c+dx)+a)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}{\csc\left(c+dx+\frac{\pi}{2}\right)^3}dx$$

$$\Big\downarrow \text{4274}$$

$$a\int \cos^3(c+dx)dx+a\int \cos^2(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$a\int \sin\left(c+dx+\frac{\pi}{2}\right)^2dx+a\int \sin\left(c+dx+\frac{\pi}{2}\right)^3dx$$

$$\Big\downarrow \text{3113}$$

$$a\int \sin\left(c+dx+\frac{\pi}{2}\right)^2dx-\frac{a\int \left(1-\sin^2(c+dx)\right)d(-\sin(c+dx))}{d}$$

$$\Big\downarrow \text{2009}$$

$$a\int \sin\left(c+dx+\frac{\pi}{2}\right)^2dx-\frac{a\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}$$

$$\Big\downarrow \text{3115}$$

$$a\left(\frac{\int 1dx}{2}+\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)-\frac{a\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}$$

$$\Big\downarrow \text{24}$$

$$a\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)-\frac{a\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}$$

input
```
Int[Cos[c + d*x]^3*(a + a*Sec[c + d*x]),x]
```

output
```
a*(x/2 + (Cos[c + d*x]*Sin[c + d*x])/(2*d)) - (a*(-Sin[c + d*x] + Sin[c +
d*x]^3/3))/d
```

3.8.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

3.8.4 Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.74

method	result	size
parallelrisch	$\frac{a(6dx+9\sin(dx+c)+\sin(3dx+3c)+3\sin(2dx+2c))}{12d}$	40
risch	$\frac{ax}{2}+\frac{3a\sin(dx+c)}{4d}+\frac{a\sin(3dx+3c)}{12d}+\frac{a\sin(2dx+2c)}{4d}$	48
derivativedivides	$\frac{\frac{a\left(2+\cos(dx+c)^2\right)\sin(dx+c)}{3}+a\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}$	49
default	$\frac{\frac{a\left(2+\cos(dx+c)^2\right)\sin(dx+c)}{3}+a\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}$	49
norman	$\frac{\frac{a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{d}+\frac{ax}{2}+\frac{3a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{4a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3d}+\frac{3ax\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{2}+\frac{3ax\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{2}+\frac{ax\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{2}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^3}$	115

input
```
int(cos(d*x+c)^3*(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/12*a*(6*d*x+9*sin(d*x+c)+sin(3*d*x+3*c)+3*sin(2*d*x+2*c))/d
```

3.8.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.78

$$\int \cos^3(c+dx)(a+a\sec(c+dx))\,dx = \frac{3\,adx+\left(2\,a\cos\left(dx+c\right)^2+3\,a\cos\left(dx+c\right)+4\,a\right)\sin\left(dx+c\right)}{6\,d}$$

input
```
integrate(cos(d*x+c)^3*(a+a*sec(d*x+c)),x, algorithm="fricas")
```

output
```
1/6*(3*a*d*x + (2*a*cos(d*x + c)^2 + 3*a*cos(d*x + c) + 4*a)*sin(d*x + c))
/d
```

3.8.6 Sympy [F]

$$\int \cos^3(c+dx)(a+a\sec(c+dx))\,dx = a\left(\int \cos^3(c+dx)\sec(c+dx)\,dx + \int \cos^3(c+dx)\,dx\right)$$

input
```
integrate(cos(d*x+c)**3*(a+a*sec(d*x+c)),x)
```

output
```
a*(Integral(cos(c + d*x)**3*sec(c + d*x), x) + Integral(cos(c + d*x)**3, x
))
```

3.8.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.85

$$\int \cos^3(c+dx)(a+a\sec(c+dx))\,dx = -\frac{4\left(\sin(dx+c)^3 - 3\sin(dx+c)\right)a - 3\left(2\,dx + 2\,c + \sin(2\,dx+2\,c)\right)a}{12\,d}$$

input
```
integrate(cos(d*x+c)^3*(a+a*sec(d*x+c)),x, algorithm="maxima")
```

output
```
-1/12*(4*(sin(d*x + c)^3 - 3*sin(d*x + c))*a - 3*(2*d*x + 2*c + sin(2*d*x
+ 2*c))*a)/d
```

3.8.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.33

$$\int \cos^3(c+dx)(a+a\sec(c+dx))\,dx = \frac{3\,(dx+c)a + \frac{2\left(3\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5 + 4\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 + 9\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^3}}{6\,d}$$

input
```
integrate(cos(d*x+c)^3*(a+a*sec(d*x+c)),x, algorithm="giac")
```

output
```
1/6*(3*(d*x + c)*a + 2*(3*a*tan(1/2*d*x + 1/2*c)^5 + 4*a*tan(1/2*d*x + 1/2
*c)^3 + 9*a*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^3)/d
```

3.8.9 Mupad [B] (verification not implemented)

Time = 13.25 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.02

$$\int \cos^3(c+dx)(a+a\sec(c+dx))\,dx = \frac{a\,x}{2}+\frac{2\,a\,\sin\left(c+d\,x\right)}{3\,d}+\frac{a\,\cos\left(c+d\,x\right)\,\sin\left(c+d\,x\right)}{2\,d}+\frac{a\cos\left(c+d\,x\right)^2\,\sin\left(c+d\,x\right)}{3\,d}$$

input
```
int(cos(c + d*x)^3*(a + a/cos(c + d*x)),x)
```

output
```
(a*x)/2 + (2*a*sin(c + d*x))/(3*d) + (a*cos(c + d*x)*sin(c + d*x))/(2*d) +
 (a*cos(c + d*x)^2*sin(c + d*x))/(3*d)
```

3.9 $\int \cos^4(c+dx)(a+a\sec(c+dx))\,dx$

3.9.1 Optimal result . . . 349
3.9.2 Mathematica [A] (verified) . . . 349
3.9.3 Rubi [A] (verified) . . . 350
3.9.4 Maple [A] (verified) . . . 352
3.9.5 Fricas [A] (verification not implemented) . . . 352
3.9.6 Sympy [F] . . . 353
3.9.7 Maxima [A] (verification not implemented) . . . 353
3.9.8 Giac [A] (verification not implemented) . . . 353
3.9.9 Mupad [B] (verification not implemented) . . . 354

3.9.1 Optimal result

Integrand size = 19, antiderivative size = 76

$$\int \cos^4(c+dx)(a+a\sec(c+dx))\,dx = \frac{3ax}{8}+\frac{a\sin(c+dx)}{d}+\frac{3a\cos(c+dx)\sin(c+dx)}{8d}+\frac{a\cos^3(c+dx)\sin(c+dx)}{4d}-\frac{a\sin^3(c+dx)}{3d}$$

output
```
3/8*a*x+a*sin(d*x+c)/d+3/8*a*cos(d*x+c)*sin(d*x+c)/d+1/4*a*cos(d*x+c)^3*si
n(d*x+c)/d-1/3*a*sin(d*x+c)^3/d
```

3.9.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.96

$$\int \cos^4(c+dx)(a+a\sec(c+dx))\,dx = \frac{3a(c+dx)}{8d}+\frac{a\sin(c+dx)}{d}-\frac{a\sin^3(c+dx)}{3d}+\frac{a\sin(2(c+dx))}{4d}+\frac{a\sin(4(c+dx))}{32d}$$

input
```
Integrate[Cos[c + d*x]^4*(a + a*Sec[c + d*x]),x]
```

output
```
(3*a*(c + d*x))/(8*d) + (a*Sin[c + d*x])/d - (a*Sin[c + d*x]^3)/(3*d) + (a
*Sin[2*(c + d*x)])/(4*d) + (a*Sin[4*(c + d*x)])/(32*d)
```

3.9.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.07, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {3042, 4274, 3042, 3113, 2009, 3115, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^4(c+dx)(a\sec(c+dx)+a)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}{\csc\left(c+dx+\frac{\pi}{2}\right)^4}dx$$

$$\Big\downarrow \text{4274}$$

$$a\int \cos^4(c+dx)dx+a\int \cos^3(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$a\int \sin\left(c+dx+\frac{\pi}{2}\right)^3dx+a\int \sin\left(c+dx+\frac{\pi}{2}\right)^4dx$$

$$\Big\downarrow \text{3113}$$

$$a\int \sin\left(c+dx+\frac{\pi}{2}\right)^4dx-\frac{a\int\left(1-\sin^2(c+dx)\right)d(-\sin(c+dx))}{d}$$

$$\Big\downarrow \text{2009}$$

$$a\int \sin\left(c+dx+\frac{\pi}{2}\right)^4dx-\frac{a\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}$$

$$\Big\downarrow \text{3115}$$

$$a\left(\frac{3}{4}\int \cos^2(c+dx)dx+\frac{\sin(c+dx)\cos^3(c+dx)}{4d}\right)-\frac{a\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{3}{4}\int \sin\left(c+dx+\frac{\pi}{2}\right)^2dx+\frac{\sin(c+dx)\cos^3(c+dx)}{4d}\right)-\frac{a\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}$$

$$\Big\downarrow \text{3115}$$

$$a\left(\frac{3}{4}\left(\frac{\int 1dx}{2}+\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)+\frac{\sin(c+dx)\cos^3(c+dx)}{4d}\right)-\frac{a\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}$$

$$\Big\downarrow \; 24$$

$$a\left(\frac{\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3}{4}\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)\right)-\frac{a\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}$$

input
```
Int[Cos[c + d*x]^4*(a + a*Sec[c + d*x]),x]
```

output
```
-((a*(-Sin[c + d*x] + Sin[c + d*x]^3/3))/d) + a*((Cos[c + d*x]^3*Sin[c + d
*x])/(4*d) + (3*(x/2 + (Cos[c + d*x]*Sin[c + d*x])/(2*d)))/4)
```

3.9.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

3.9.4 Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.70

method	result
parallelrisch	$\frac{a(36dx+72\sin(dx+c)+3\sin(4dx+4c)+8\sin(3dx+3c)+24\sin(2dx+2c))}{96d}$
derivativedivides	$\frac{a\left(\frac{\left(\cos(dx+c)^3+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+\frac{a\left(2+\cos(dx+c)^2\right)\sin(dx+c)}{3}}{d}$
default	$\frac{a\left(\frac{\left(\cos(dx+c)^3+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+\frac{a\left(2+\cos(dx+c)^2\right)\sin(dx+c)}{3}}{d}$
risch	$\frac{3ax}{8}+\frac{3a\sin(dx+c)}{4d}+\frac{a\sin(4dx+4c)}{32d}+\frac{a\sin(3dx+3c)}{12d}+\frac{a\sin(2dx+2c)}{4d}$
norman	$\frac{\frac{3ax}{8}+\frac{13a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d}+\frac{31a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{12d}+\frac{49a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{12d}+\frac{3a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{4d}+\frac{3ax\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{2}+\frac{9ax\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{4}+\frac{3ax}{}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^4}$

input
```
int(cos(d*x+c)^4*(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/96*a*(36*d*x+72*sin(d*x+c)+3*sin(4*d*x+4*c)+8*sin(3*d*x+3*c)+24*sin(2*d*
x+2*c))/d
```

3.9.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.70

$$\int \cos^4(c+dx)(a+a\sec(c+dx))\,dx = \frac{9\,adx+\left(6\,a\cos\left(dx+c\right)^3+8\,a\cos\left(dx+c\right)^2+9\,a\cos\left(dx+c\right)+16\,a\right)\sin\left(dx+c\right)}{24\,d}$$

input
```
integrate(cos(d*x+c)^4*(a+a*sec(d*x+c)),x, algorithm="fricas")
```

output
```
1/24*(9*a*d*x + (6*a*cos(d*x + c)^3 + 8*a*cos(d*x + c)^2 + 9*a*cos(d*x + c
) + 16*a)*sin(d*x + c))/d
```

3.9.6 Sympy [F]

$$\int \cos^4(c+dx)(a+a\sec(c+dx))\,dx = a\left(\int \cos^4(c+dx)\sec(c+dx)\,dx + \int \cos^4(c+dx)\,dx\right)$$

input
```
integrate(cos(d*x+c)**4*(a+a*sec(d*x+c)),x)
```

output
```
a*(Integral(cos(c + d*x)**4*sec(c + d*x), x) + Integral(cos(c + d*x)**4, x
))
```

3.9.7 Maxima [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.75

$$\int \cos^4(c+dx)(a+a\sec(c+dx))\,dx = -\frac{32\left(\sin(dx+c)^3 - 3\sin(dx+c)\right)a - 3\left(12\,dx + 12\,c + \sin(4\,dx+4\,c) + 8\sin(2\,dx+2\,c)\right)a}{96\,d}$$

input
```
integrate(cos(d*x+c)^4*(a+a*sec(d*x+c)),x, algorithm="maxima")
```

output
```
-1/96*(32*(sin(d*x + c)^3 - 3*sin(d*x + c))*a - 3*(12*d*x + 12*c + sin(4*d
*x + 4*c) + 8*sin(2*d*x + 2*c))*a)/d
```

3.9.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.13

$$\int \cos^4(c+dx)(a+a\sec(c+dx))\,dx = \frac{9\,(dx+c)a + \frac{2\left(9\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7 + 49\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5 + 31\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 + 39\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^4}}{24\,d}$$

input
```
integrate(cos(d*x+c)^4*(a+a*sec(d*x+c)),x, algorithm="giac")
```

output
```
1/24*(9*(d*x + c)*a + 2*(9*a*tan(1/2*d*x + 1/2*c)^7 + 49*a*tan(1/2*d*x + 1
/2*c)^5 + 31*a*tan(1/2*d*x + 1/2*c)^3 + 39*a*tan(1/2*d*x + 1/2*c))/(tan(1/
2*d*x + 1/2*c)^2 + 1)^4)/d
```

3.9.9 Mupad [B] (verification not implemented)

Time = 16.84 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.04

$$\int \cos^4(c+dx)(a+a\sec(c+dx))\,dx$$
$$= \frac{3\,a\,x}{8} + \frac{\frac{3\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{4} + \frac{49\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{12} + \frac{31\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{12} + \frac{13\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)^4}$$

input
```
int(cos(c + d*x)^4*(a + a/cos(c + d*x)),x)
```

output
```
(3*a*x)/8 + ((13*a*tan(c/2 + (d*x)/2))/4 + (31*a*tan(c/2 + (d*x)/2)^3)/12
+ (49*a*tan(c/2 + (d*x)/2)^5)/12 + (3*a*tan(c/2 + (d*x)/2)^7)/4)/(d*(tan(c
/2 + (d*x)/2)^2 + 1)^4)
```

3.10 $\int \sec^4(c+dx)(a+a\sec(c+dx))^2\,dx$

3.10.1 Optimal result . . . 355
3.10.2 Mathematica [A] (verified) . . . 355
3.10.3 Rubi [A] (verified) . . . 356
3.10.4 Maple [A] (verified) . . . 359
3.10.5 Fricas [A] (verification not implemented) . . . 359
3.10.6 Sympy [F] . . . 360
3.10.7 Maxima [A] (verification not implemented) . . . 360
3.10.8 Giac [A] (verification not implemented) . . . 361
3.10.9 Mupad [B] (verification not implemented) . . . 361

3.10.1 Optimal result

Integrand size = 21, antiderivative size = 122

$$\int \sec^4(c+dx)(a+a\sec(c+dx))^2\,dx = \frac{3a^2\text{arctanh}(\sin(c+dx))}{4d}+\frac{9a^2\tan(c+dx)}{5d}+\frac{3a^2\sec(c+dx)\tan(c+dx)}{4d}+\frac{a^2\sec^3(c+dx)\tan(c+dx)}{2d}+\frac{a^2\sec^4(c+dx)\tan(c+dx)}{5d}+\frac{3a^2\tan^3(c+dx)}{5d}$$

output
```
3/4*a^2*arctanh(sin(d*x+c))/d+9/5*a^2*tan(d*x+c)/d+3/4*a^2*sec(d*x+c)*tan(
d*x+c)/d+1/2*a^2*sec(d*x+c)^3*tan(d*x+c)/d+1/5*a^2*sec(d*x+c)^4*tan(d*x+c)
/d+3/5*a^2*tan(d*x+c)^3/d
```

3.10.2 Mathematica [A] (verified)

Time = 0.63 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.56

$$\int \sec^4(c+dx)(a+a\sec(c+dx))^2\,dx = \frac{a^2(15\text{arctanh}(\sin(c+dx))+\tan(c+dx)\,(15\sec(c+dx)+10\sec^3(c+dx)+4\sec^4(c+dx)+12(3+\tan}{20d}$$

input
```
Integrate[Sec[c + d*x]^4*(a + a*Sec[c + d*x])^2,x]
```

output
```
(a^2*(15*ArcTanh[Sin[c + d*x]] + Tan[c + d*x]*(15*Sec[c + d*x] + 10*Sec[c
+ d*x]^3 + 4*Sec[c + d*x]^4 + 12*(3 + Tan[c + d*x]^2))))/(20*d)
```

3.10.3 Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.99, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {3042, 4275, 3042, 4255, 3042, 4255, 3042, 4257, 4534, 3042, 4254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^4(c+dx)(a\sec(c+dx)+a)^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^4\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^2dx$$

$$\Big\downarrow \text{4275}$$

$$2a^2\int \sec^5(c+dx)dx+\int \sec^4(c+dx)\left(\sec^2(c+dx)a^2+a^2\right)dx$$

$$\Big\downarrow \text{3042}$$

$$2a^2\int \csc\left(c+dx+\frac{\pi}{2}\right)^5dx+\int \csc\left(c+dx+\frac{\pi}{2}\right)^4\left(\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2\right)dx$$

$$\Big\downarrow \text{4255}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^4\left(\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2\right)dx+2a^2\left(\frac{3}{4}\int \sec^3(c+dx)dx+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^4\left(\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2\right)dx+2a^2\left(\frac{3}{4}\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)$$

$$\Big\downarrow \text{4255}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^4\left(\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2\right)dx+$$
$$2a^2\left(\frac{3}{4}\left(\frac{1}{2}\int\sec(c+dx)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)$$

$\Big\downarrow$ 3042

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^4\left(\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2\right)dx+$$
$$2a^2\left(\frac{3}{4}\left(\frac{1}{2}\int\csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)$$

$\Big\downarrow$ 4257

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^4\left(\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2\right)dx+$$
$$2a^2\left(\frac{3}{4}\left(\frac{\text{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)$$

$\Big\downarrow$ 4534

$$\frac{9}{5}a^2\int\sec^4(c+dx)dx+$$
$$2a^2\left(\frac{3}{4}\left(\frac{\text{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)+$$
$$\frac{a^2\tan(c+dx)\sec^4(c+dx)}{5d}$$

$\Big\downarrow$ 3042

$$\frac{9}{5}a^2\int\csc\left(c+dx+\frac{\pi}{2}\right)^4dx+$$
$$2a^2\left(\frac{3}{4}\left(\frac{\text{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)+$$
$$\frac{a^2\tan(c+dx)\sec^4(c+dx)}{5d}$$

$\Big\downarrow$ 4254

$$-\frac{9a^2\int\left(\tan^2(c+dx)+1\right)d(-\tan(c+dx))}{5d}+$$
$$2a^2\left(\frac{3}{4}\left(\frac{\text{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)+$$
$$\frac{a^2\tan(c+dx)\sec^4(c+dx)}{5d}$$

$\Big\downarrow$ 2009

$$2a^2\left(\frac{3}{4}\left(\frac{\text{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)-$$
$$\frac{9a^2\left(-\frac{1}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{5d}+\frac{a^2\tan(c+dx)\sec^4(c+dx)}{5d}$$

3.10. $\int \sec^4(c+dx)(a+a\sec(c+dx))^2\,dx$

input
```
Int[Sec[c + d*x]^4*(a + a*Sec[c + d*x])^2,x]
```

output
```
(a^2*Sec[c + d*x]^4*Tan[c + d*x])/(5*d) - (9*a^2*(-Tan[c + d*x] - Tan[c +
d*x]^3/3))/(5*d) + 2*a^2*((Sec[c + d*x]^3*Tan[c + d*x])/(4*d) + (3*(ArcTan
h[Sin[c + d*x]]/(2*d) + (Sec[c + d*x]*Tan[c + d*x])/(2*d)))/4)
```

3.10.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4275
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Simp[2*a*(b/d)   Int[(d*Csc[e + f*x])^(n + 1), x], x]
 + Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
 e, f, n}, x]
```

rule 4534
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Simp[(C*m + A*(m + 1))/(m + 1)   Int[(b*Csc[e + f*x])^m, x], x] /;
 FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

3.10.4 Maple [A] (verified)

Time = 0.98 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.91

method	result
derivativedivides	$\frac{-a^2\left(-\frac{8}{15}-\frac{\sec(dx+c)^4}{5}-\frac{4\sec(dx+c)^2}{15}\right)\tan(dx+c)+2a^2\left(-\left(-\frac{\sec(dx+c)^3}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(}{8}\right.}{d}$
default	$\frac{-a^2\left(-\frac{8}{15}-\frac{\sec(dx+c)^4}{5}-\frac{4\sec(dx+c)^2}{15}\right)\tan(dx+c)+2a^2\left(-\left(-\frac{\sec(dx+c)^3}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(}{8}\right.}{d}$
parts	$-\frac{a^2\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right)\tan(dx+c)}{d}-\frac{a^2\left(-\frac{8}{15}-\frac{\sec(dx+c)^4}{5}-\frac{4\sec(dx+c)^2}{15}\right)\tan(dx+c)}{d}+\frac{2a^2\left(-\left(-\frac{\sec(dx+c)^3}{4}-\frac{3\,\mathrm{s}}{}\right.\right.}{}$
risch	$-\frac{ia^2\left(15\,\mathrm{e}^{9i(dx+c)}+70\,\mathrm{e}^{7i(dx+c)}-40\,\mathrm{e}^{6i(dx+c)}-200\,\mathrm{e}^{4i(dx+c)}-70\,\mathrm{e}^{3i(dx+c)}-120\,\mathrm{e}^{2i(dx+c)}-15\,\mathrm{e}^{i(dx+c)}-24\right)}{10d\left(\mathrm{e}^{2i(dx+c)}+1\right)^5}-\frac{3a^2\ln}{}$
norman	$\frac{-\frac{13a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2d}+\frac{9a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{d}-\frac{72a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{5d}+\frac{7a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{d}-\frac{3a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^9}{2d}}{\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^5}-\frac{3a^2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}{4d}$
parallelrisch	$\frac{6\left(\left(-\frac{5\cos(dx+c)}{4}-\frac{5\cos(3dx+3c)}{8}-\frac{\cos(5dx+5c)}{8}\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+\left(\frac{5\cos(dx+c)}{4}+\frac{5\cos(3dx+3c)}{8}+\frac{\cos(5dx+5c)}{8}\right)\ln\left(\tan\right.\right.}{d(\cos(5dx+5c)+5\cos(3dx+3c)+10\cos(}$

input
```
int(sec(d*x+c)^4*(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-a^2*(-8/15-1/5*sec(d*x+c)^4-4/15*sec(d*x+c)^2)*tan(d*x+c)+2*a^2*(-(-
1/4*sec(d*x+c)^3-3/8*sec(d*x+c))*tan(d*x+c)+3/8*ln(sec(d*x+c)+tan(d*x+c)))
-a^2*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c))
```

3.10.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.02

$$\int \sec^4(c+dx)(a+a\sec(c+dx))^2\,dx$$
$$=\frac{15\,a^2\cos\left(dx+c\right)^5\log\left(\sin\left(dx+c\right)+1\right)-15\,a^2\cos\left(dx+c\right)^5\log\left(-\sin\left(dx+c\right)+1\right)+2\left(24\,a^2\cos\left(dx+\right.\right.}{40\,d\cos\left(dx+c\right)^5}$$

input
```
integrate(sec(d*x+c)^4*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/40*(15*a^2*cos(d*x + c)^5*log(sin(d*x + c) + 1) - 15*a^2*cos(d*x + c)^5*
log(-sin(d*x + c) + 1) + 2*(24*a^2*cos(d*x + c)^4 + 15*a^2*cos(d*x + c)^3
+ 12*a^2*cos(d*x + c)^2 + 10*a^2*cos(d*x + c) + 4*a^2)*sin(d*x + c))/(d*co
s(d*x + c)^5)
```

3.10.6 Sympy [F]

$$\int \sec^4(c+dx)(a+a\sec(c+dx))^2\,dx = a^2\left(\int \sec^4(c+dx)\,dx + \int 2\sec^5(c+dx)\,dx + \int \sec^6(c+dx)\,dx\right)$$

input
```
integrate(sec(d*x+c)**4*(a+a*sec(d*x+c))**2,x)
```

output
```
a**2*(Integral(sec(c + d*x)**4, x) + Integral(2*sec(c + d*x)**5, x) + Inte
gral(sec(c + d*x)**6, x))
```

3.10.7 Maxima [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.09

$$\int \sec^4(c+dx)(a+a\sec(c+dx))^2\,dx$$
$$= \frac{8\left(3\tan(dx+c)^5 + 10\tan(dx+c)^3 + 15\tan(dx+c)\right)a^2 + 40\left(\tan(dx+c)^3 + 3\tan(dx+c)\right)a^2 - 15\,a}{120\,d}$$

input
```
integrate(sec(d*x+c)^4*(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
1/120*(8*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))*a^2 + 40
*(tan(d*x + c)^3 + 3*tan(d*x + c))*a^2 - 15*a^2*(2*(3*sin(d*x + c)^3 - 5*s
in(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c)
+ 1) + 3*log(sin(d*x + c) - 1)))/d
```

3.10.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.13

$$\int \sec^4(c+dx)(a+a\sec(c+dx))^2\,dx$$
$$= \frac{15\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-15\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(15\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9-70\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right.\right.}{20\,d}$$

input
```
integrate(sec(d*x+c)^4*(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
1/20*(15*a^2*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 15*a^2*log(abs(tan(1/2*d
*x + 1/2*c) - 1)) - 2*(15*a^2*tan(1/2*d*x + 1/2*c)^9 - 70*a^2*tan(1/2*d*x
+ 1/2*c)^7 + 144*a^2*tan(1/2*d*x + 1/2*c)^5 - 90*a^2*tan(1/2*d*x + 1/2*c)^
3 + 65*a^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^5)/d
```

3.10.9 Mupad [B] (verification not implemented)

Time = 17.89 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.39

$$\int \sec^4(c+dx)(a+a\sec(c+dx))^2\,dx = \frac{3\,a^2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{2\,d} - \frac{\frac{3\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9}{2}-7\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+\frac{72\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{5}-9\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+\frac{13\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2}}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}-5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+10\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-10\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)}$$

input
```
int((a + a/cos(c + d*x))^2/cos(c + d*x)^4,x)
```

output
```
(3*a^2*atanh(tan(c/2 + (d*x)/2)))/(2*d) - ((72*a^2*tan(c/2 + (d*x)/2)^5)/5
 - 9*a^2*tan(c/2 + (d*x)/2)^3 - 7*a^2*tan(c/2 + (d*x)/2)^7 + (3*a^2*tan(c/
2 + (d*x)/2)^9)/2 + (13*a^2*tan(c/2 + (d*x)/2))/2)/(d*(5*tan(c/2 + (d*x)/2
)^2 - 10*tan(c/2 + (d*x)/2)^4 + 10*tan(c/2 + (d*x)/2)^6 - 5*tan(c/2 + (d*x
)/2)^8 + tan(c/2 + (d*x)/2)^10 - 1))
```

3.11 $\int \sec^3(c+dx)(a+a\sec(c+dx))^2\,dx$

3.11.1 Optimal result . . . 362
3.11.2 Mathematica [A] (verified) . . . 362
3.11.3 Rubi [A] (verified) . . . 363
3.11.4 Maple [A] (verified) . . . 365
3.11.5 Fricas [A] (verification not implemented) . . . 366
3.11.6 Sympy [F] . . . 366
3.11.7 Maxima [A] (verification not implemented) . . . 366
3.11.8 Giac [A] (verification not implemented) . . . 367
3.11.9 Mupad [B] (verification not implemented) . . . 367

3.11.1 Optimal result

Integrand size = 21, antiderivative size = 96

$$\int \sec^3(c+dx)(a+a\sec(c+dx))^2\,dx = \frac{7a^2\text{arctanh}(\sin(c+dx))}{8d}+\frac{2a^2\tan(c+dx)}{d}+\frac{7a^2\sec(c+dx)\tan(c+dx)}{8d}+\frac{a^2\sec^3(c+dx)\tan(c+dx)}{4d}+\frac{2a^2\tan^3(c+dx)}{3d}$$

output
```
7/8*a^2*arctanh(sin(d*x+c))/d+2*a^2*tan(d*x+c)/d+7/8*a^2*sec(d*x+c)*tan(d*
x+c)/d+1/4*a^2*sec(d*x+c)^3*tan(d*x+c)/d+2/3*a^2*tan(d*x+c)^3/d
```

3.11.2 Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.60

$$\int \sec^3(c+dx)(a+a\sec(c+dx))^2\,dx = \frac{a^2(21\text{arctanh}(\sin(c+dx))+\tan(c+dx)\,(21\sec(c+dx)+6\sec^3(c+dx)+16(3+\tan^2(c+dx))))}{24d}$$

input
```
Integrate[Sec[c + d*x]^3*(a + a*Sec[c + d*x])^2,x]
```

output
```
(a^2*(21*ArcTanh[Sin[c + d*x]] + Tan[c + d*x]*(21*Sec[c + d*x] + 6*Sec[c +
 d*x]^3 + 16*(3 + Tan[c + d*x]^2))))/(24*d)
```

3.11.3 Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.99, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.476, Rules used = {3042, 4275, 3042, 4254, 2009, 4534, 3042, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^3(c+dx)(a\sec(c+dx)+a)^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^3\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^2dx$$

$$\Big\downarrow \text{4275}$$

$$2a^2\int \sec^4(c+dx)dx+\int \sec^3(c+dx)\left(\sec^2(c+dx)a^2+a^2\right)dx$$

$$\Big\downarrow \text{3042}$$

$$2a^2\int \csc\left(c+dx+\frac{\pi}{2}\right)^4dx+\int \csc\left(c+dx+\frac{\pi}{2}\right)^3\left(\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2\right)dx$$

$$\Big\downarrow \text{4254}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^3\left(\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2\right)dx-\frac{2a^2\int\left(\tan^2(c+dx)+1\right)d(-\tan(c+dx))}{d}$$

$$\Big\downarrow \text{2009}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^3\left(\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2\right)dx-\frac{2a^2\left(-\frac{1}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{d}$$

$$\Big\downarrow \text{4534}$$

$$\frac{7}{4}a^2\int \sec^3(c+dx)dx-\frac{2a^2\left(-\frac{1}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{d}+\frac{a^2\tan(c+dx)\sec^3(c+dx)}{4d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7}{4}a^2\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx-\frac{2a^2\left(-\frac{1}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{d}+\frac{a^2\tan(c+dx)\sec^3(c+dx)}{4d}$$

$$\Big\downarrow \text{4255}$$

$$\frac{7}{4}a^2\left(\frac{1}{2}\int \sec(c+dx)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)-\frac{2a^2\left(-\frac{1}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{d}+\frac{a^2\tan(c+dx)\sec^3(c+dx)}{4d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7}{4}a^2\left(\frac{1}{2}\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)-\frac{2a^2\left(-\frac{1}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{d}+\frac{a^2\tan(c+dx)\sec^3(c+dx)}{4d}$$

$$\Big\downarrow \text{4257}$$

$$\frac{7}{4}a^2\left(\frac{\text{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)-\frac{2a^2\left(-\frac{1}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{d}+\frac{a^2\tan(c+dx)\sec^3(c+dx)}{4d}$$

input
```
Int[Sec[c + d*x]^3*(a + a*Sec[c + d*x])^2,x]
```

output
```
(a^2*Sec[c + d*x]^3*Tan[c + d*x])/(4*d) + (7*a^2*(ArcTanh[Sin[c + d*x]]/(2
*d) + (Sec[c + d*x]*Tan[c + d*x])/(2*d)))/4 - (2*a^2*(-Tan[c + d*x] - Tan[
c + d*x]^3/3))/d
```

3.11.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4275
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Simp[2*a*(b/d)   Int[(d*Csc[e + f*x])^(n + 1), x], x]
 + Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
 e, f, n}, x]
```

rule 4534
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Simp[(C*m + A*(m + 1))/(m + 1)   Int[(b*Csc[e + f*x])^m, x], x] /;
 FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

3.11.4 Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.17

method	result
derivativedivides	$\frac{a^2\left(-\left(-\frac{\sec(dx+c)^3}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)-2a^2\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right)\tan(dx+c)+a^2\left(\sec\right.}{d}$
default	$\frac{a^2\left(-\left(-\frac{\sec(dx+c)^3}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)-2a^2\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right)\tan(dx+c)+a^2\left(\sec\right.}{d}$
parts	$\frac{a^2\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)}{d}+\frac{a^2\left(-\left(-\frac{\sec(dx+c)^3}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan}{8}\right.}{d}$
norman	$\frac{\frac{25a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d}-\frac{83a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{12d}+\frac{77a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{12d}-\frac{7a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{4d}}{\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^4}-\frac{7a^2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{8d}+\frac{7a^2\ln\left(\tan\right.}{}$
risch	$-\frac{ia^2\left(21\,\mathrm{e}^{7i(dx+c)}+45\,\mathrm{e}^{5i(dx+c)}-96\,\mathrm{e}^{4i(dx+c)}-45\,\mathrm{e}^{3i(dx+c)}-128\,\mathrm{e}^{2i(dx+c)}-21\,\mathrm{e}^{i(dx+c)}-32\right)}{12d\left(\mathrm{e}^{2i(dx+c)}+1\right)^4}-\frac{7a^2\ln\left(\mathrm{e}^{i(dx+c)}-i\right)}{8d}$
parallelrisch	$\frac{a^2\left(21(-\cos(4dx+4c)-4\cos(2dx+2c)-3)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+21(3+\cos(4dx+4c)+4\cos(2dx+2c))\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\right.\right.}{24d(3+\cos(4dx+4c)+4\cos(2dx+2c))}$

input
```
int(sec(d*x+c)^3*(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^2*(-(-1/4*sec(d*x+c)^3-3/8*sec(d*x+c))*tan(d*x+c)+3/8*ln(sec(d*x+c)
+tan(d*x+c)))-2*a^2*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c)+a^2*(1/2*sec(d*x+c)
*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c))))
```

3.11.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.16

$$\int \sec^3(c+dx)(a+a\sec(c+dx))^2\,dx = \frac{21\,a^2\cos(dx+c)^4\log(\sin(dx+c)+1) - 21\,a^2\cos(dx+c)^4\log(-\sin(dx+c)+1) + 2\,(32\,a^2\cos(dx+}{48\,d\cos(dx+c)^4}$$

input
```
integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/48*(21*a^2*cos(d*x + c)^4*log(sin(d*x + c) + 1) - 21*a^2*cos(d*x + c)^4*
log(-sin(d*x + c) + 1) + 2*(32*a^2*cos(d*x + c)^3 + 21*a^2*cos(d*x + c)^2
+ 16*a^2*cos(d*x + c) + 6*a^2)*sin(d*x + c))/(d*cos(d*x + c)^4)
```

3.11.6 Sympy [F]

$$\int \sec^3(c+dx)(a+a\sec(c+dx))^2\,dx = a^2\left(\int \sec^3(c+dx)\,dx + \int 2\sec^4(c+dx)\,dx + \int \sec^5(c+dx)\,dx\right)$$

input
```
integrate(sec(d*x+c)**3*(a+a*sec(d*x+c))**2,x)
```

output
```
a**2*(Integral(sec(c + d*x)**3, x) + Integral(2*sec(c + d*x)**4, x) + Inte
gral(sec(c + d*x)**5, x))
```

3.11.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.51

$$\int \sec^3(c+dx)(a+a\sec(c+dx))^2\,dx = \frac{32\left(\tan(dx+c)^3+3\tan(dx+c)\right)a^2 - 3\,a^2\left(\frac{2\left(3\sin(dx+c)^3-5\sin(dx+c)\right)}{\sin(dx+c)^4-2\sin(dx+c)^2+1} - 3\log(\sin(dx+c)+1) + 3\log(s}{48\,d}$$

input
```
integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
1/48*(32*(tan(d*x + c)^3 + 3*tan(d*x + c))*a^2 - 3*a^2*(2*(3*sin(d*x + c)^
3 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*
x + c) + 1) + 3*log(sin(d*x + c) - 1)) - 12*a^2*(2*sin(d*x + c)/(sin(d*x +
 c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)))/d
```

3.11.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.27

$$\int \sec^3(c+dx)(a+a\sec(c+dx))^2\,dx$$
$$= \frac{21\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-21\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(21\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-77\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right.\right.}{\left(\tan\left(\frac{1}{2}\,dx\right.\right.}}{24\,d}$$

input
```
integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
1/24*(21*a^2*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 21*a^2*log(abs(tan(1/2*d
*x + 1/2*c) - 1)) - 2*(21*a^2*tan(1/2*d*x + 1/2*c)^7 - 77*a^2*tan(1/2*d*x
+ 1/2*c)^5 + 83*a^2*tan(1/2*d*x + 1/2*c)^3 - 75*a^2*tan(1/2*d*x + 1/2*c))/
(tan(1/2*d*x + 1/2*c)^2 - 1)^4)/d
```

3.11.9 Mupad [B] (verification not implemented)

Time = 17.01 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.47

$$\int \sec^3(c+dx)(a+a\sec(c+dx))^2\,dx$$
$$= \frac{7\,a^2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{4\,d} - \frac{\frac{7\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{4}-\frac{77\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{12}+\frac{83\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{12}-\frac{25\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8-4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}$$

input
```
int((a + a/cos(c + d*x))^2/cos(c + d*x)^3,x)
```

output
```
(7*a^2*atanh(tan(c/2 + (d*x)/2)))/(4*d) - ((83*a^2*tan(c/2 + (d*x)/2)^3)/1
2 - (77*a^2*tan(c/2 + (d*x)/2)^5)/12 + (7*a^2*tan(c/2 + (d*x)/2)^7)/4 - (2
5*a^2*tan(c/2 + (d*x)/2))/4)/(d*(6*tan(c/2 + (d*x)/2)^4 - 4*tan(c/2 + (d*x
)/2)^2 - 4*tan(c/2 + (d*x)/2)^6 + tan(c/2 + (d*x)/2)^8 + 1))
```

3.11. $\int \sec^3(c+dx)(a+a\sec(c+dx))^2\,dx$

3.12 $\int \sec^2(c+dx)(a+a\sec(c+dx))^2\,dx$

3.12.1 Optimal result . . . 369
3.12.2 Mathematica [A] (verified) . . . 369
3.12.3 Rubi [A] (verified) . . . 370
3.12.4 Maple [A] (verified) . . . 373
3.12.5 Fricas [A] (verification not implemented) . . . 373
3.12.6 Sympy [F] . . . 374
3.12.7 Maxima [A] (verification not implemented) . . . 374
3.12.8 Giac [A] (verification not implemented) . . . 374
3.12.9 Mupad [B] (verification not implemented) . . . 375

3.12.1 Optimal result

Integrand size = 21, antiderivative size = 74

$$\int \sec^2(c+dx)(a+a\sec(c+dx))^2\,dx = \frac{a^2\text{arctanh}(\sin(c+dx))}{d}+\frac{5a^2\tan(c+dx)}{3d}+\frac{a^2\sec(c+dx)\tan(c+dx)}{d}+\frac{a^2\sec^2(c+dx)\tan(c+dx)}{3d}$$

output
```
a^2*arctanh(sin(d*x+c))/d+5/3*a^2*tan(d*x+c)/d+a^2*sec(d*x+c)*tan(d*x+c)/d
+1/3*a^2*sec(d*x+c)^2*tan(d*x+c)/d
```

3.12.2 Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00

$$\int \sec^2(c+dx)(a+a\sec(c+dx))^2\,dx = \frac{a^2\text{arctanh}(\sin(c+dx))}{d}+\frac{5a^2\tan(c+dx)}{3d}+\frac{a^2\sec(c+dx)\tan(c+dx)}{d}+\frac{a^2\sec^2(c+dx)\tan(c+dx)}{3d}$$

input
```
Integrate[Sec[c + d*x]^2*(a + a*Sec[c + d*x])^2,x]
```

output
```
(a^2*ArcTanh[Sin[c + d*x]])/d + (5*a^2*Tan[c + d*x])/(3*d) + (a^2*Sec[c +
d*x]*Tan[c + d*x])/d + (a^2*Sec[c + d*x]^2*Tan[c + d*x])/(3*d)
```

3.12.3 Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.08, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {3042, 4275, 3042, 4255, 3042, 4257, 4534, 3042, 4254, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(c+dx)(a\sec(c+dx)+a)^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^2\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^2dx$$

$$\Big\downarrow \text{4275}$$

$$2a^2\int \sec^3(c+dx)dx+\int \sec^2(c+dx)\left(\sec^2(c+dx)a^2+a^2\right)dx$$

$$\Big\downarrow \text{3042}$$

$$2a^2\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx+\int \csc\left(c+dx+\frac{\pi}{2}\right)^2\left(\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2\right)dx$$

$$\Big\downarrow \text{4255}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^2\left(\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2\right)dx+2a^2\left(\frac{1}{2}\int \sec(c+dx)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^2\left(\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2\right)dx+2a^2\left(\frac{1}{2}\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)$$

$$\Big\downarrow \text{4257}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^2\left(\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2\right)dx+2a^2\left(\frac{\text{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)$$

$$\Big\downarrow \text{4534}$$

$$\frac{5}{3}a^2\int \sec^2(c+dx)dx+2a^2\left(\frac{\text{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{a^2\tan(c+dx)\sec^2(c+dx)}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5}{3}a^2\int \csc\left(c+dx+\frac{\pi}{2}\right)^2dx+2a^2\left(\frac{\text{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{a^2\tan(c+dx)\sec^2(c+dx)}{3d}$$

$$\Big\downarrow \text{4254}$$

$$-\frac{5a^2\int 1d(-\tan(c+dx))}{3d}+2a^2\left(\frac{\text{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{a^2\tan(c+dx)\sec^2(c+dx)}{3d}$$

$$\Big\downarrow \text{24}$$

$$2a^2\left(\frac{\text{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{5a^2\tan(c+dx)}{3d}+\frac{a^2\tan(c+dx)\sec^2(c+dx)}{3d}$$

input
```
Int[Sec[c + d*x]^2*(a + a*Sec[c + d*x])^2,x]
```

output
```
(5*a^2*Tan[c + d*x])/(3*d) + (a^2*Sec[c + d*x]^2*Tan[c + d*x])/(3*d) + 2*a
^2*(ArcTanh[Sin[c + d*x]]/(2*d) + (Sec[c + d*x]*Tan[c + d*x])/(2*d))
```

3.12. $\int \sec^2(c+dx)(a+a\sec(c+dx))^2\,dx$

3.12.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4275
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Simp[2*a*(b/d)   Int[(d*Csc[e + f*x])^(n + 1), x], x]
 + Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
 e, f, n}, x]
```

rule 4534
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Simp[(C*m + A*(m + 1))/(m + 1)   Int[(b*Csc[e + f*x])^m, x], x] /;
 FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

3.12.4 Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.01

method	result
derivativedivides	$\frac{-a^2\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right)\tan(dx+c)+2a^2\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+a^2\tan(dx+c)}{d}$
default	$\frac{-a^2\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right)\tan(dx+c)+2a^2\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+a^2\tan(dx+c)}{d}$
parts	$\frac{a^2\tan(dx+c)}{d}-\frac{a^2\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right)\tan(dx+c)}{d}+\frac{a^2\sec(dx+c)\tan(dx+c)}{d}+\frac{a^2\ln(\sec(dx+c)+\tan(dx+c))}{d}$
risch	$-\frac{2ia^2\left(3\,\mathrm{e}^{5i(dx+c)}-3\,\mathrm{e}^{4i(dx+c)}-12\,\mathrm{e}^{2i(dx+c)}-3\,\mathrm{e}^{i(dx+c)}-5\right)}{3d\left(\mathrm{e}^{2i(dx+c)}+1\right)^3}+\frac{a^2\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{d}-\frac{a^2\ln\left(\mathrm{e}^{i(dx+c)}-i\right)}{d}$
norman	$\frac{-\frac{6a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{16a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3d}-\frac{2a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{d}}{\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^3}+\frac{a^2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{d}-\frac{a^2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{d}$
parallelrisch	$-\frac{3\left(\left(\cos(dx+c)+\frac{\cos(3dx+3c)}{3}\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+\left(-\cos(dx+c)-\frac{\cos(3dx+3c)}{3}\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-\sin(dx+c)-\cdots\right.}{d(\cos(3dx+3c)+3\cos(dx+c))}$

input
```
int(sec(d*x+c)^2*(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-a^2*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c)+2*a^2*(1/2*sec(d*x+c)*tan(d*x
+c)+1/2*ln(sec(d*x+c)+tan(d*x+c)))+a^2*tan(d*x+c))
```

3.12.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.30

$$\int \sec^2(c+dx)(a+a\sec(c+dx))^2\,dx$$
$$=\frac{3\,a^2\cos\left(dx+c\right)^3\log\left(\sin\left(dx+c\right)+1\right)-3\,a^2\cos\left(dx+c\right)^3\log\left(-\sin\left(dx+c\right)+1\right)+2\left(5\,a^2\cos\left(dx+c\right)^2\right.}{6\,d\cos\left(dx+c\right)^3}$$

input
```
integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/6*(3*a^2*cos(d*x + c)^3*log(sin(d*x + c) + 1) - 3*a^2*cos(d*x + c)^3*log
(-sin(d*x + c) + 1) + 2*(5*a^2*cos(d*x + c)^2 + 3*a^2*cos(d*x + c) + a^2)*
sin(d*x + c))/(d*cos(d*x + c)^3)
```

3.12.6 Sympy [F]

$$\int \sec^2(c+dx)(a+a\sec(c+dx))^2\,dx = a^2\left(\int \sec^2(c+dx)\,dx + \int 2\sec^3(c+dx)\,dx + \int \sec^4(c+dx)\,dx\right)$$

input
```
integrate(sec(d*x+c)**2*(a+a*sec(d*x+c))**2,x)
```

output
```
a**2*(Integral(sec(c + d*x)**2, x) + Integral(2*sec(c + d*x)**3, x) + Inte
gral(sec(c + d*x)**4, x))
```

3.12.7 Maxima [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.15

$$\int \sec^2(c+dx)(a+a\sec(c+dx))^2\,dx = \frac{2\left(\tan(dx+c)^3+3\tan(dx+c)\right)a^2 - 3a^2\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1} - \log(\sin(dx+c)+1) + \log(\sin(dx+c)-1)\right)}{6d}$$

input
```
integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
1/6*(2*(tan(d*x + c)^3 + 3*tan(d*x + c))*a^2 - 3*a^2*(2*sin(d*x + c)/(sin(
d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 6*a^2*t
an(d*x + c))/d
```

3.12.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.43

$$\int \sec^2(c+dx)(a+a\sec(c+dx))^2\,dx = \frac{3a^2\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right) - 3a^2\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right) - \frac{2\left(3a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5 - 8a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3 + 9\right.}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)^3}}{3d}$$

input
```
integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
1/3*(3*a^2*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*a^2*log(abs(tan(1/2*d*x
+ 1/2*c) - 1)) - 2*(3*a^2*tan(1/2*d*x + 1/2*c)^5 - 8*a^2*tan(1/2*d*x + 1/2
*c)^3 + 9*a^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^3)/d
```

3.12.9 Mupad [B] (verification not implemented)

Time = 15.22 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.51

$$\int \sec^2(c+dx)(a+a\sec(c+dx))^2\,dx$$
$$= \frac{2\,a^2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d} - \frac{2\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5 - \frac{16\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{3} + 6\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6 - 3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4 + 3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2 - 1\right)}$$

input
```
int((a + a/cos(c + d*x))^2/cos(c + d*x)^2,x)
```

output
```
(2*a^2*atanh(tan(c/2 + (d*x)/2)))/d - (2*a^2*tan(c/2 + (d*x)/2)^5 - (16*a^
2*tan(c/2 + (d*x)/2)^3)/3 + 6*a^2*tan(c/2 + (d*x)/2))/(d*(3*tan(c/2 + (d*x
)/2)^2 - 3*tan(c/2 + (d*x)/2)^4 + tan(c/2 + (d*x)/2)^6 - 1))
```

3.13 $\int \sec(c+dx)(a+a\sec(c+dx))^2\,dx$

3.13.1 Optimal result . . . 376
3.13.2 Mathematica [A] (verified) . . . 376
3.13.3 Rubi [A] (verified) . . . 377
3.13.4 Maple [A] (verified) . . . 379
3.13.5 Fricas [A] (verification not implemented) . . . 379
3.13.6 Sympy [F] . . . 380
3.13.7 Maxima [A] (verification not implemented) . . . 380
3.13.8 Giac [A] (verification not implemented) . . . 380
3.13.9 Mupad [B] (verification not implemented) . . . 381

3.13.1 Optimal result

Integrand size = 19, antiderivative size = 54

$$\int \sec(c+dx)(a+a\sec(c+dx))^2\,dx=\frac{3a^2\text{arctanh}(\sin(c+dx))}{2d}+\frac{2a^2\tan(c+dx)}{d}+\frac{a^2\sec(c+dx)\tan(c+dx)}{2d}$$

output
```
3/2*a^2*arctanh(sin(d*x+c))/d+2*a^2*tan(d*x+c)/d+1/2*a^2*sec(d*x+c)*tan(d*
x+c)/d
```

3.13.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00

$$\int \sec(c+dx)(a+a\sec(c+dx))^2\,dx=\frac{3a^2\text{arctanh}(\sin(c+dx))}{2d}+\frac{2a^2\tan(c+dx)}{d}+\frac{a^2\sec(c+dx)\tan(c+dx)}{2d}$$

input
```
Integrate[Sec[c + d*x]*(a + a*Sec[c + d*x])^2,x]
```

output
```
(3*a^2*ArcTanh[Sin[c + d*x]])/(2*d) + (2*a^2*Tan[c + d*x])/d + (a^2*Sec[c
+ d*x]*Tan[c + d*x])/(2*d)
```

3.13.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {3042, 4275, 3042, 4254, 24, 4534, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(c+dx)(a\sec(c+dx)+a)^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^2dx$$

$$\Big\downarrow \text{4275}$$

$$2a^2\int \sec^2(c+dx)dx+\int \sec(c+dx)\left(\sec^2(c+dx)a^2+a^2\right)dx$$

$$\Big\downarrow \text{3042}$$

$$2a^2\int \csc\left(c+dx+\frac{\pi}{2}\right)^2dx+\int \csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2\right)dx$$

$$\Big\downarrow \text{4254}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2\right)dx-\frac{2a^2\int 1d(-\tan(c+dx))}{d}$$

$$\Big\downarrow \text{24}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2\right)dx+\frac{2a^2\tan(c+dx)}{d}$$

$$\Big\downarrow \text{4534}$$

$$\frac{3}{2}a^2\int \sec(c+dx)dx+\frac{2a^2\tan(c+dx)}{d}+\frac{a^2\tan(c+dx)\sec(c+dx)}{2d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3}{2}a^2\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{2a^2\tan(c+dx)}{d}+\frac{a^2\tan(c+dx)\sec(c+dx)}{2d}$$

$$\Big\downarrow \text{4257}$$

$$\frac{3a^2\text{arctanh}(\sin(c+dx))}{2d}+\frac{2a^2\tan(c+dx)}{d}+\frac{a^2\tan(c+dx)\sec(c+dx)}{2d}$$

input
```
Int[Sec[c + d*x]*(a + a*Sec[c + d*x])^2,x]
```

output
```
(3*a^2*ArcTanh[Sin[c + d*x]])/(2*d) + (2*a^2*Tan[c + d*x])/d + (a^2*Sec[c
+ d*x]*Tan[c + d*x])/(2*d)
```

3.13.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4275
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Simp[2*a*(b/d)   Int[(d*Csc[e + f*x])^(n + 1), x], x]
 + Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
 e, f, n}, x]
```

rule 4534
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Simp[(C*m + A*(m + 1))/(m + 1)   Int[(b*Csc[e + f*x])^m, x], x] /;
 FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

3.13.4 Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.30

method	result
derivativedivides	$\frac{a^2\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+2a^2\tan(dx+c)+a^2\ln(\sec(dx+c)+\tan(dx+c))}{d}$
default	$\frac{a^2\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+2a^2\tan(dx+c)+a^2\ln(\sec(dx+c)+\tan(dx+c))}{d}$
parts	$\frac{a^2\ln(\sec(dx+c)+\tan(dx+c))}{d}+\frac{a^2\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)}{d}+\frac{2a^2\tan(dx+c)}{d}$
norman	$\frac{\frac{5a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}-\frac{3a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{d}}{\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^2}-\frac{3a^2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2d}+\frac{3a^2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{2d}$
risch	$-\frac{ia^2\left(e^{3i(dx+c)}-4e^{2i(dx+c)}-e^{i(dx+c)}-4\right)}{d\left(e^{2i(dx+c)}+1\right)^2}+\frac{3a^2\ln\left(e^{i(dx+c)}+i\right)}{2d}-\frac{3a^2\ln\left(e^{i(dx+c)}-i\right)}{2d}$
parallelrisch	$\frac{a^2\left(-3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\cos(2dx+2c)+3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)\cos(2dx+2c)+2\sin(dx+c)+4\sin(2dx+2c)-3\ln\left(\tan\left(\right.\right.\right.}{2d(1+\cos(2dx+2c))}$

input
```
int(sec(d*x+c)*(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^2*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c)))+2*a^2*t
an(d*x+c)+a^2*ln(sec(d*x+c)+tan(d*x+c)))
```

3.13.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.54

$$\int \sec(c+dx)(a+a\sec(c+dx))^2\,dx$$
$$=\frac{3\,a^2\cos\left(dx+c\right)^2\log\left(\sin\left(dx+c\right)+1\right)-3\,a^2\cos\left(dx+c\right)^2\log\left(-\sin\left(dx+c\right)+1\right)+2\left(4\,a^2\cos\left(dx+c\right)+\right.}{4\,d\cos\left(dx+c\right)^2}$$

input
```
integrate(sec(d*x+c)*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/4*(3*a^2*cos(d*x + c)^2*log(sin(d*x + c) + 1) - 3*a^2*cos(d*x + c)^2*log
(-sin(d*x + c) + 1) + 2*(4*a^2*cos(d*x + c) + a^2)*sin(d*x + c))/(d*cos(d*
x + c)^2)
```

3.13.6 Sympy [F]

$$\int \sec(c+dx)(a+a\sec(c+dx))^2\,dx = a^2\left(\int \sec(c+dx)\,dx + \int 2\sec^2(c+dx)\,dx + \int \sec^3(c+dx)\,dx\right)$$

input
```
integrate(sec(d*x+c)*(a+a*sec(d*x+c))**2,x)
```

output
```
a**2*(Integral(sec(c + d*x), x) + Integral(2*sec(c + d*x)**2, x) + Integra
l(sec(c + d*x)**3, x))
```

3.13.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.50

$$\int \sec(c+dx)(a+a\sec(c+dx))^2\,dx = -\frac{a^2\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1} - \log(\sin(dx+c)+1) + \log(\sin(dx+c)-1)\right) - 4a^2\log(\sec(dx+c)+\tan(dx+c)}{4d}$$

input
```
integrate(sec(d*x+c)*(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
-1/4*(a^2*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + l
og(sin(d*x + c) - 1)) - 4*a^2*log(sec(d*x + c) + tan(d*x + c)) - 8*a^2*tan
(d*x + c))/d
```

3.13.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.67

$$\int \sec(c+dx)(a+a\sec(c+dx))^2\,dx = \frac{3a^2\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right) - 3a^2\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right) - \frac{2\left(3a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3 - 5a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)^2}}{2d}$$

input
```
integrate(sec(d*x+c)*(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
1/2*(3*a^2*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*a^2*log(abs(tan(1/2*d*x
+ 1/2*c) - 1)) - 2*(3*a^2*tan(1/2*d*x + 1/2*c)^3 - 5*a^2*tan(1/2*d*x + 1/2
*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^2)/d
```

3.13.9 Mupad [B] (verification not implemented)

Time = 13.89 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.54

$$\int \sec(c+dx)(a+a\sec(c+dx))^2\,dx = \frac{3\,a^2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d} - \frac{3\,a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3 - 5\,a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4 - 2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2 + 1\right)}$$

input
```
int((a + a/cos(c + d*x))^2/cos(c + d*x),x)
```

output
```
(3*a^2*atanh(tan(c/2 + (d*x)/2)))/d - (3*a^2*tan(c/2 + (d*x)/2)^3 - 5*a^2*
tan(c/2 + (d*x)/2))/(d*(tan(c/2 + (d*x)/2)^4 - 2*tan(c/2 + (d*x)/2)^2 + 1)
)
```

3.14 $\int (a + a\sec(c + dx))^2 \, dx$

3.14.1 Optimal result . . . 382
3.14.2 Mathematica [A] (verified) . . . 382
3.14.3 Rubi [A] (verified) . . . 383
3.14.4 Maple [A] (verified) . . . 384
3.14.5 Fricas [B] (verification not implemented) . . . 385
3.14.6 Sympy [F] . . . 385
3.14.7 Maxima [A] (verification not implemented) . . . 386
3.14.8 Giac [B] (verification not implemented) . . . 386
3.14.9 Mupad [B] (verification not implemented) . . . 386

3.14.1 Optimal result

Integrand size = 12, antiderivative size = 34

$$\int (a + a\sec(c + dx))^2 \, dx = a^2 x + \frac{2a^2 \text{arctanh}(\sin(c + dx))}{d} + \frac{a^2 \tan(c + dx)}{d}$$

output
```
a^2*x+2*a^2*arctanh(sin(d*x+c))/d+a^2*tan(d*x+c)/d
```

3.14.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int (a + a\sec(c + dx))^2 \, dx = a^2 x + \frac{2a^2 \text{arctanh}(\sin(c + dx))}{d} + \frac{a^2 \tan(c + dx)}{d}$$

input
```
Integrate[(a + a*Sec[c + d*x])^2,x]
```

output
```
a^2*x + (2*a^2*ArcTanh[Sin[c + d*x]])/d + (a^2*Tan[c + d*x])/d
```

3.14.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3042, 4260, 3042, 4254, 24, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sec(c+dx)+a)^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^2dx$$

$$\Big\downarrow \text{4260}$$

$$a^2\int \sec^2(c+dx)dx+2a^2\int \sec(c+dx)dx+a^2x$$

$$\Big\downarrow \text{3042}$$

$$2a^2\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+a^2\int \csc\left(c+dx+\frac{\pi}{2}\right)^2dx+a^2x$$

$$\Big\downarrow \text{4254}$$

$$-\frac{a^2\int 1d(-\tan(c+dx))}{d}+2a^2\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+a^2x$$

$$\Big\downarrow \text{24}$$

$$2a^2\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{a^2\tan(c+dx)}{d}+a^2x$$

$$\Big\downarrow \text{4257}$$

$$\frac{2a^2\text{arctanh}(\sin(c+dx))}{d}+\frac{a^2\tan(c+dx)}{d}+a^2x$$

input
```
Int[(a + a*Sec[c + d*x])^2,x]
```

output
```
a^2*x + (2*a^2*ArcTanh[Sin[c + d*x]])/d + (a^2*Tan[c + d*x])/d
```

3.14.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4260
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^2, x_Symbol] :> Simp[a^2*x, x] +
 (Simp[2*a*b   Int[Csc[c + d*x], x], x] + Simp[b^2   Int[Csc[c + d*x]^2, x]
, x]) /; FreeQ[{a, b, c, d}, x]
```

3.14.4 Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.24

method	result	size
parts	$a^2x+\frac{a^2\tan(dx+c)}{d}+\frac{2a^2\ln(\sec(dx+c)+\tan(dx+c))}{d}$	42
derivativedivides	$\frac{a^2\tan(dx+c)+2a^2\ln(\sec(dx+c)+\tan(dx+c))+a^2(dx+c)}{d}$	44
default	$\frac{a^2\tan(dx+c)+2a^2\ln(\sec(dx+c)+\tan(dx+c))+a^2(dx+c)}{d}$	44
risch	$a^2x+\frac{2ia^2}{d(e^{2i(dx+c)}+1)}+\frac{2a^2\ln(e^{i(dx+c)}+i)}{d}-\frac{2a^2\ln(e^{i(dx+c)}-i)}{d}$	71
parallelrisch	$\frac{a^2\left(dx\cos(dx+c)-2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\cos(dx+c)+2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)\cos(dx+c)+\sin(dx+c)\right)}{d\cos(dx+c)}$	72
norman	$\frac{a^2x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2-a^2x-\frac{2a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}}{-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}-\frac{2a^2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{d}+\frac{2a^2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{d}$	98

input
```
int((a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
a^2*x+a^2*tan(d*x+c)/d+2*a^2/d*ln(sec(d*x+c)+tan(d*x+c))
```

3.14.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 76 vs. $2(34) = 68$.

Time = 0.28 (sec) , antiderivative size = 76, normalized size of antiderivative = 2.24

$$\int (a + a\sec(c + dx))^2 \, dx$$
$$= \frac{a^2 dx \cos(dx + c) + a^2 \cos(dx + c) \log(\sin(dx + c) + 1) - a^2 \cos(dx + c) \log(-\sin(dx + c) + 1) + a^2 \sin}{d \cos(dx + c)}$$

input
```
integrate((a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
(a^2*d*x*cos(d*x + c) + a^2*cos(d*x + c)*log(sin(d*x + c) + 1) - a^2*cos(d
*x + c)*log(-sin(d*x + c) + 1) + a^2*sin(d*x + c))/(d*cos(d*x + c))
```

3.14.6 Sympy [F]

$$\int (a + a\sec(c + dx))^2 \, dx = a^2 \left(\int 1 \, dx + \int 2\sec(c + dx) \, dx + \int \sec^2(c + dx) \, dx \right)$$

input
```
integrate((a+a*sec(d*x+c))**2,x)
```

output
```
a**2*(Integral(1, x) + Integral(2*sec(c + d*x), x) + Integral(sec(c + d*x)
**2, x))
```

3.14.7 Maxima [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.21

$$\int (a+a\sec(c+dx))^2\,dx = a^2x + \frac{2\,a^2\log\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)}{d} + \frac{a^2\tan\left(dx+c\right)}{d}$$

input
```
integrate((a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
a^2*x + 2*a^2*log(sec(d*x + c) + tan(d*x + c))/d + a^2*tan(d*x + c)/d
```

3.14.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 79 vs. 2(34) = 68.

Time = 0.29 (sec) , antiderivative size = 79, normalized size of antiderivative = 2.32

$$\int (a+a\sec(c+dx))^2\,dx = \frac{(dx+c)a^2 + 2\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right) - 2\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right) - \frac{2\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1}}{d}$$

input
```
integrate((a+a*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
((d*x + c)*a^2 + 2*a^2*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 2*a^2*log(abs(
tan(1/2*d*x + 1/2*c) - 1)) - 2*a^2*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2
*c)^2 - 1))/d
```

3.14.9 Mupad [B] (verification not implemented)

Time = 13.36 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.65

$$\int (a+a\sec(c+dx))^2\,dx = a^2\,x + \frac{4\,a^2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d} - \frac{2\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)}$$

input
```
int((a + a/cos(c + d*x))^2,x)
```

output
```
a^2*x + (4*a^2*atanh(tan(c/2 + (d*x)/2)))/d - (2*a^2*tan(c/2 + (d*x)/2))/(
d*(tan(c/2 + (d*x)/2)^2 - 1))
```

3.15 $\int \cos(c+dx)(a+a\sec(c+dx))^2\,dx$

3.15.1 Optimal result . . . 387
3.15.2 Mathematica [A] (verified) . . . 387
3.15.3 Rubi [A] (verified) . . . 388
3.15.4 Maple [A] (verified) . . . 389
3.15.5 Fricas [A] (verification not implemented) . . . 390
3.15.6 Sympy [F] . . . 390
3.15.7 Maxima [A] (verification not implemented) . . . 391
3.15.8 Giac [B] (verification not implemented) . . . 391
3.15.9 Mupad [B] (verification not implemented) . . . 392

3.15.1 Optimal result

Integrand size = 19, antiderivative size = 34

$$\int \cos(c+dx)(a+a\sec(c+dx))^2\,dx = 2a^2x+\frac{a^2\text{arctanh}(\sin(c+dx))}{d}+\frac{a^2\sin(c+dx)}{d}$$

output
```
2*a^2*x+a^2*arctanh(sin(d*x+c))/d+a^2*sin(d*x+c)/d
```

3.15.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.38

$$\int \cos(c+dx)(a+a\sec(c+dx))^2\,dx = 2a^2x+\frac{a^2\text{arctanh}(\sin(c+dx))}{d}+\frac{a^2\cos(dx)\sin(c)}{d}+\frac{a^2\cos(c)\sin(dx)}{d}$$

input
```
Integrate[Cos[c + d*x]*(a + a*Sec[c + d*x])^2,x]
```

output
```
2*a^2*x + (a^2*ArcTanh[Sin[c + d*x]])/d + (a^2*Cos[d*x]*Sin[c])/d + (a^2*C
os[c]*Sin[d*x])/d
```

3.15.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {3042, 4275, 24, 3042, 4533, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(c+dx)(a\sec(c+dx)+a)^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{4275}$$

$$\int \cos(c+dx)\left(\sec^2(c+dx)a^2+a^2\right)dx+2a^2\int 1dx$$

$$\Big\downarrow \text{24}$$

$$\int \cos(c+dx)\left(\sec^2(c+dx)a^2+a^2\right)dx+2a^2x$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2}{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+2a^2x$$

$$\Big\downarrow \text{4533}$$

$$a^2\int \sec(c+dx)dx+\frac{a^2\sin(c+dx)}{d}+2a^2x$$

$$\Big\downarrow \text{3042}$$

$$a^2\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{a^2\sin(c+dx)}{d}+2a^2x$$

$$\Big\downarrow \text{4257}$$

$$\frac{a^2\text{arctanh}(\sin(c+dx))}{d}+\frac{a^2\sin(c+dx)}{d}+2a^2x$$

input
```
Int[Cos[c + d*x]*(a + a*Sec[c + d*x])^2,x]
```

output
```
2*a^2*x + (a^2*ArcTanh[Sin[c + d*x]])/d + (a^2*Sin[c + d*x])/d
```

3.15.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4275
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Simp[2*a*(b/d)   Int[(d*Csc[e + f*x])^(n + 1), x], x]
 + Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
 e, f, n}, x]
```

rule 4533
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] +
Simp[(C*m + A*(m + 1))/(b^2*m)   Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fr
eeQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

3.15.4 Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.29

method	result
derivativedivides	$\frac{a^2 \ln(\sec(dx+c)+\tan(dx+c))+2a^2(dx+c)+a^2\sin(dx+c)}{d}$
default	$\frac{a^2 \ln(\sec(dx+c)+\tan(dx+c))+2a^2(dx+c)+a^2\sin(dx+c)}{d}$
parallelrisch	$\frac{a^2\left(2dx+\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+\sin(dx+c)\right)}{d}$
risch	$2a^2x-\frac{ia^2e^{i(dx+c)}}{2d}+\frac{ia^2e^{-i(dx+c)}}{2d}+\frac{a^2\ln\left(e^{i(dx+c)}+i\right)}{d}-\frac{a^2\ln\left(e^{i(dx+c)}-i\right)}{d}$
norman	$\frac{-2a^2x-\frac{2a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{2a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{d}+2a^2x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)}+\frac{a^2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{d}-\frac{a^2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{d}$

input
```
int(cos(d*x+c)*(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^2*ln(sec(d*x+c)+tan(d*x+c))+2*a^2*(d*x+c)+a^2*sin(d*x+c))
```

3.15.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.56

$$\int \cos(c+dx)(a+a\sec(c+dx))^2\,dx = \frac{4\,a^2dx + a^2\log\left(\sin\left(dx+c\right)+1\right) - a^2\log\left(-\sin\left(dx+c\right)+1\right) + 2\,a^2\sin\left(dx+c\right)}{2\,d}$$

input
```
integrate(cos(d*x+c)*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/2*(4*a^2*d*x + a^2*log(sin(d*x + c) + 1) - a^2*log(-sin(d*x + c) + 1) +
2*a^2*sin(d*x + c))/d
```

3.15.6 Sympy [F]

$$\int \cos(c+dx)(a+a\sec(c+dx))^2\,dx = a^2\left(\int 2\cos\left(c+dx\right)\sec\left(c+dx\right)dx + \int \cos\left(c+dx\right)\sec^2\left(c+dx\right)dx + \int \cos\left(c+dx\right)dx\right)$$

input
```
integrate(cos(d*x+c)*(a+a*sec(d*x+c))**2,x)
```

output
```
a**2*(Integral(2*cos(c + d*x)*sec(c + d*x), x) + Integral(cos(c + d*x)*sec
(c + d*x)**2, x) + Integral(cos(c + d*x), x))
```

3.15.7 Maxima [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.53

$$\int \cos(c+dx)(a+a\sec(c+dx))^2\,dx = \frac{4\,(dx+c)a^2+a^2(\log\left(\sin\left(dx+c\right)+1\right)-\log\left(\sin\left(dx+c\right)-1\right))+2\,a^2\sin\left(dx+c\right)}{2\,d}$$

input
```
integrate(cos(d*x+c)*(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
1/2*(4*(d*x + c)*a^2 + a^2*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1))
 + 2*a^2*sin(d*x + c))/d
```

3.15.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 79 vs. 2(34) = 68.

Time = 0.32 (sec) , antiderivative size = 79, normalized size of antiderivative = 2.32

$$\int \cos(c+dx)(a+a\sec(c+dx))^2\,dx = \frac{2\,(dx+c)a^2+a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{2\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1}}{d}$$

input
```
integrate(cos(d*x+c)*(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
(2*(d*x + c)*a^2 + a^2*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - a^2*log(abs(ta
n(1/2*d*x + 1/2*c) - 1)) + 2*a^2*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c
)^2 + 1))/d
```

3.15.9 Mupad [B] (verification not implemented)

Time = 13.43 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.97

$$\int \cos(c+dx)(a+a\sec(c+dx))^2\,dx = 2\,a^2\,x + \frac{a^2\left(2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)+\sin\left(c+d\,x\right)\right)}{d}$$

input
```
int(cos(c + d*x)*(a + a/cos(c + d*x))^2,x)
```

output
```
2*a^2*x + (a^2*(2*atanh(tan(c/2 + (d*x)/2)) + sin(c + d*x)))/d
```

3.16 $\int \cos^2(c+dx)(a+a\sec(c+dx))^2\,dx$

3.16.1 Optimal result . . . 393
3.16.2 Mathematica [A] (verified) . . . 393
3.16.3 Rubi [A] (verified) . . . 394
3.16.4 Maple [A] (verified) . . . 395
3.16.5 Fricas [A] (verification not implemented) . . . 396
3.16.6 Sympy [F] . . . 396
3.16.7 Maxima [A] (verification not implemented) . . . 396
3.16.8 Giac [A] (verification not implemented) . . . 397
3.16.9 Mupad [B] (verification not implemented) . . . 397

3.16.1 Optimal result

Integrand size = 21, antiderivative size = 45

$$\int \cos^2(c+dx)(a+a\sec(c+dx))^2\,dx=\frac{3a^2x}{2}+\frac{2a^2\sin(c+dx)}{d}+\frac{a^2\cos(c+dx)\sin(c+dx)}{2d}$$

output
```
3/2*a^2*x+2*a^2*sin(d*x+c)/d+1/2*a^2*cos(d*x+c)*sin(d*x+c)/d
```

3.16.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.76

$$\int \cos^2(c+dx)(a+a\sec(c+dx))^2\,dx=\frac{a^2(6(c+dx)+8\sin(c+dx)+\sin(2(c+dx)))}{4d}$$

input
```
Integrate[Cos[c + d*x]^2*(a + a*Sec[c + d*x])^2,x]
```

output
```
(a^2*(6*(c + d*x) + 8*Sin[c + d*x] + Sin[2*(c + d*x)]))/(4*d)
```

3.16.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3042, 4275, 3042, 3117, 4533, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(c+dx)(a\sec(c+dx)+a)^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^2}dx$$

$$\Big\downarrow \text{4275}$$

$$2a^2\int \cos(c+dx)dx+\int \cos^2(c+dx)\left(\sec^2(c+dx)a^2+a^2\right)dx$$

$$\Big\downarrow \text{3042}$$

$$2a^2\int \sin\left(c+dx+\frac{\pi}{2}\right)dx+\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^2}dx$$

$$\Big\downarrow \text{3117}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^2}dx+\frac{2a^2\sin(c+dx)}{d}$$

$$\Big\downarrow \text{4533}$$

$$\frac{3a^2\int 1dx}{2}+\frac{2a^2\sin(c+dx)}{d}+\frac{a^2\sin(c+dx)\cos(c+dx)}{2d}$$

$$\Big\downarrow \text{24}$$

$$\frac{2a^2\sin(c+dx)}{d}+\frac{a^2\sin(c+dx)\cos(c+dx)}{2d}+\frac{3a^2x}{2}$$

input
```
Int[Cos[c + d*x]^2*(a + a*Sec[c + d*x])^2,x]
```

output
```
(3*a^2*x)/2 + (2*a^2*Sin[c + d*x])/d + (a^2*Cos[c + d*x]*Sin[c + d*x])/(2*
d)
```

3.16.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 4275
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Simp[2*a*(b/d)   Int[(d*Csc[e + f*x])^(n + 1), x], x]
 + Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
 e, f, n}, x]
```

rule 4533
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] +
Simp[(C*m + A*(m + 1))/(b^2*m)   Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fr
eeQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

3.16.4 Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.69

method	result
parallelrisch	$\frac{a^2(6dx+8\sin(dx+c)+\sin(2dx+2c))}{4d}$
risch	$\frac{3a^2x}{2}+\frac{2a^2\sin(dx+c)}{d}+\frac{a^2\sin(2dx+2c)}{4d}$
derivativedivides	$\frac{a^2(dx+c)+2a^2\sin(dx+c)+a^2\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}$
default	$\frac{a^2(dx+c)+2a^2\sin(dx+c)+a^2\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}$
norman	$\frac{-\frac{3a^2x}{2}-\frac{5a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{2a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{d}+\frac{3a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{d}-\frac{3a^2x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{2}+\frac{3a^2x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{2}+\frac{3a^2x\tan\left(\frac{dx}{2}+\frac{c}{2}\right.}{2}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^2\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)}$

input
```
int(cos(d*x+c)^2*(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*a^2*(6*d*x+8*sin(d*x+c)+sin(2*d*x+2*c))/d
```

3.16.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.80

$$\int \cos^2(c+dx)(a+a\sec(c+dx))^2\,dx = \frac{3\,a^2dx + (a^2\cos(dx+c)+4\,a^2)\sin(dx+c)}{2\,d}$$

input
```
integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/2*(3*a^2*d*x + (a^2*cos(d*x + c) + 4*a^2)*sin(d*x + c))/d
```

3.16.6 Sympy [F]

$$\int \cos^2(c+dx)(a+a\sec(c+dx))^2\,dx = a^2\left(\int 2\cos^2(c+dx)\sec(c+dx)\,dx + \int \cos^2(c+dx)\sec^2(c+dx)\,dx + \int \cos^2(c+dx)\,dx\right)$$

input
```
integrate(cos(d*x+c)**2*(a+a*sec(d*x+c))**2,x)
```

output
```
a**2*(Integral(2*cos(c + d*x)**2*sec(c + d*x), x) + Integral(cos(c + d*x)*
*2*sec(c + d*x)**2, x) + Integral(cos(c + d*x)**2, x))
```

3.16.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.07

$$\int \cos^2(c+dx)(a+a\sec(c+dx))^2\,dx = \frac{(2\,dx+2\,c+\sin(2\,dx+2\,c))a^2+4\,(dx+c)a^2+8\,a^2\sin(dx+c)}{4\,d}$$

input
```
integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
1/4*((2*d*x + 2*c + sin(2*d*x + 2*c))*a^2 + 4*(d*x + c)*a^2 + 8*a^2*sin(d*
x + c))/d
```

3.16.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.42

$$\int \cos^2(c+dx)(a+a\sec(c+dx))^2\,dx = \frac{3\,(dx+c)a^2 + \frac{2\left(3\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+5\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^2}}{2\,d}$$

input
```
integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
1/2*(3*(d*x + c)*a^2 + 2*(3*a^2*tan(1/2*d*x + 1/2*c)^3 + 5*a^2*tan(1/2*d*x
 + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^2)/d
```

3.16.9 Mupad [B] (verification not implemented)

Time = 13.76 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.27

$$\int \cos^2(c+dx)(a+a\sec(c+dx))^2\,dx = \frac{3\,a^2\,x}{2} + \frac{3\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3 + 5\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)^2}$$

input
```
int(cos(c + d*x)^2*(a + a/cos(c + d*x))^2,x)
```

output
```
(3*a^2*x)/2 + (3*a^2*tan(c/2 + (d*x)/2)^3 + 5*a^2*tan(c/2 + (d*x)/2))/(d*(
tan(c/2 + (d*x)/2)^2 + 1)^2)
```

3.17 $\int \cos^3(c+dx)(a+a\sec(c+dx))^2\,dx$

3.17.1 Optimal result . . . 398
3.17.2 Mathematica [A] (verified) . . . 398
3.17.3 Rubi [A] (verified) . . . 399
3.17.4 Maple [A] (verified) . . . 401
3.17.5 Fricas [A] (verification not implemented) . . . 401
3.17.6 Sympy [F] . . . 402
3.17.7 Maxima [A] (verification not implemented) . . . 402
3.17.8 Giac [A] (verification not implemented) . . . 402
3.17.9 Mupad [B] (verification not implemented) . . . 403

3.17.1 Optimal result

Integrand size = 21, antiderivative size = 57

$$\int \cos^3(c+dx)(a+a\sec(c+dx))^2\,dx = a^2x+\frac{2a^2\sin(c+dx)}{d}+\frac{a^2\cos(c+dx)\sin(c+dx)}{d}-\frac{a^2\sin^3(c+dx)}{3d}$$

output
```
a^2*x+2*a^2*sin(d*x+c)/d+a^2*cos(d*x+c)*sin(d*x+c)/d-1/3*a^2*sin(d*x+c)^3/
d
```

3.17.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.72

$$\int \cos^3(c+dx)(a+a\sec(c+dx))^2\,dx = \frac{a^2(12dx+21\sin(c+dx)+6\sin(2(c+dx))+\sin(3(c+dx)))}{12d}$$

input
```
Integrate[Cos[c + d*x]^3*(a + a*Sec[c + d*x])^2,x]
```

output
```
(a^2*(12*d*x + 21*Sin[c + d*x] + 6*Sin[2*(c + d*x)] + Sin[3*(c + d*x)]))/(
12*d)
```

3.17.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.11, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 4275, 3042, 3115, 24, 4532, 3042, 3492, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^3(c+dx)(a\sec(c+dx)+a)^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^3}dx$$

$$\Big\downarrow \text{4275}$$

$$2a^2\int \cos^2(c+dx)dx+\int \cos^3(c+dx)\left(\sec^2(c+dx)a^2+a^2\right)dx$$

$$\Big\downarrow \text{3042}$$

$$2a^2\int \sin\left(c+dx+\frac{\pi}{2}\right)^2dx+\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^3}dx$$

$$\Big\downarrow \text{3115}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^3}dx+2a^2\left(\frac{\int 1dx}{2}+\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)$$

$$\Big\downarrow \text{24}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^3}dx+2a^2\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)$$

$$\Big\downarrow \text{4532}$$

$$\int \cos(c+dx)\left(\cos^2(c+dx)a^2+a^2\right)dx+2a^2\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)\left(\sin\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2\right)dx+2a^2\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)$$

$$\Big\downarrow \text{3492}$$

$$2a^2\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)-\frac{\int\left(2a^2-a^2\sin^2(c+dx)\right)d(-\sin(c+dx))}{d}$$

$$\Big\downarrow \text{2009}$$

$$2a^2\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)-\frac{\frac{1}{3}a^2\sin^3(c+dx)-2a^2\sin(c+dx)}{d}$$

input
```
Int[Cos[c + d*x]^3*(a + a*Sec[c + d*x])^2,x]
```

output
```
2*a^2*(x/2 + (Cos[c + d*x]*Sin[c + d*x])/(2*d)) - (-2*a^2*Sin[c + d*x] + (
a^2*Sin[c + d*x]^3)/3)/d
```

3.17.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3492
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2),
 x_Symbol] :> Simp[-f^(-1)   Subst[Int[(1 - x^2)^((m - 1)/2)*(A + C - C*x^2
), x], x, Cos[e + f*x]], x] /; FreeQ[{e, f, A, C}, x] && IGtQ[(m + 1)/2, 0]
```

rule 4275
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Simp[2*a*(b/d)   Int[(d*Csc[e + f*x])^(n + 1), x], x]
 + Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
 e, f, n}, x]
```

rule 4532
```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.) + (A_)),
 x_Symbol] :> Int[(C + A*Sin[e + f*x]^2)/Sin[e + f*x]^(m + 2), x] /; FreeQ[
{e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && ILtQ[(m + 1)/2, 0]
```

3.17.4 Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.74

method	result
parallelrisch	$\frac{a^2(12dx+21\sin(dx+c)+\sin(3dx+3c)+6\sin(2dx+2c))}{12d}$
risch	$a^2x+\frac{7a^2\sin(dx+c)}{4d}+\frac{a^2\sin(3dx+3c)}{12d}+\frac{a^2\sin(2dx+2c)}{2d}$
derivativedivides	$\frac{a^2\sin(dx+c)+2a^2\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+\frac{a^2\left(2+\cos(dx+c)^2\right)\sin(dx+c)}{3}}{d}$
default	$\frac{a^2\sin(dx+c)+2a^2\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+\frac{a^2\left(2+\cos(dx+c)^2\right)\sin(dx+c)}{3}}{d}$
norman	$\frac{a^2x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8-a^2x-\frac{6a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{2a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3d}+\frac{10a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{3d}+\frac{2a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{d}-2a^2x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^3\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)}$

input
```
int(cos(d*x+c)^3*(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/12*a^2*(12*d*x+21*sin(d*x+c)+sin(3*d*x+3*c)+6*sin(2*d*x+2*c))/d
```

3.17.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.86

$$\int \cos^3(c+dx)(a+a\sec(c+dx))^2\,dx = \frac{3\,a^2dx+\left(a^2\cos\left(dx+c\right)^2+3\,a^2\cos\left(dx+c\right)+5\,a^2\right)\sin\left(dx+c\right)}{3\,d}$$

input
```
integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/3*(3*a^2*d*x + (a^2*cos(d*x + c)^2 + 3*a^2*cos(d*x + c) + 5*a^2)*sin(d*x
 + c))/d
```

3.17.6 Sympy [F]

$$\int \cos^3(c+dx)(a+a\sec(c+dx))^2\,dx = a^2\left(\int 2\cos^3(c+dx)\sec(c+dx)\,dx + \int \cos^3(c+dx)\sec^2(c+dx)\,dx + \int \cos^3(c+dx)\,dx\right)$$

input
```
integrate(cos(d*x+c)**3*(a+a*sec(d*x+c))**2,x)
```

output
```
a**2*(Integral(2*cos(c + d*x)**3*sec(c + d*x), x) + Integral(cos(c + d*x)*
*3*sec(c + d*x)**2, x) + Integral(cos(c + d*x)**3, x))
```

3.17.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.07

$$\int \cos^3(c+dx)(a+a\sec(c+dx))^2\,dx = -\frac{2\left(\sin(dx+c)^3-3\sin(dx+c)\right)a^2-3\left(2\,dx+2\,c+\sin(2\,dx+2\,c)\right)a^2-6\,a^2\sin(dx+c)}{6\,d}$$

input
```
integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
-1/6*(2*(sin(d*x + c)^3 - 3*sin(d*x + c))*a^2 - 3*(2*d*x + 2*c + sin(2*d*x
 + 2*c))*a^2 - 6*a^2*sin(d*x + c))/d
```

3.17.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.40

$$\int \cos^3(c+dx)(a+a\sec(c+dx))^2\,dx = \frac{3\,(dx+c)a^2+\frac{2\left(3\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+8\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+9\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^3}}{3\,d}$$

input
```
integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
1/3*(3*(d*x + c)*a^2 + 2*(3*a^2*tan(1/2*d*x + 1/2*c)^5 + 8*a^2*tan(1/2*d*x
 + 1/2*c)^3 + 9*a^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^3)/
d
```

3.17.9 Mupad [B] (verification not implemented)

Time = 13.36 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.07

$$\int \cos^3(c+dx)(a+a\sec(c+dx))^2\,dx = a^2\,x+\frac{5\,a^2\,\sin\left(c+d\,x\right)}{3\,d}+\frac{a^2\cos\left(c+d\,x\right)^2\,\sin\left(c+d\,x\right)}{3\,d}+\frac{a^2\,\cos\left(c+d\,x\right)\,\sin\left(c+d\,x\right)}{d}$$

input
```
int(cos(c + d*x)^3*(a + a/cos(c + d*x))^2,x)
```

output
```
a^2*x + (5*a^2*sin(c + d*x))/(3*d) + (a^2*cos(c + d*x)^2*sin(c + d*x))/(3*
d) + (a^2*cos(c + d*x)*sin(c + d*x))/d
```

3.18 $\int \cos^4(c+dx)(a+a\sec(c+dx))^2\,dx$

3.18.1 Optimal result . . . 404
3.18.2 Mathematica [A] (verified) . . . 404
3.18.3 Rubi [A] (verified) . . . 405
3.18.4 Maple [A] (verified) . . . 407
3.18.5 Fricas [A] (verification not implemented) . . . 408
3.18.6 Sympy [F] . . . 408
3.18.7 Maxima [A] (verification not implemented) . . . 408
3.18.8 Giac [A] (verification not implemented) . . . 409
3.18.9 Mupad [B] (verification not implemented) . . . 409

3.18.1 Optimal result

Integrand size = 21, antiderivative size = 87

$$\int \cos^4(c+dx)(a+a\sec(c+dx))^2\,dx=\frac{7a^2x}{8}+\frac{2a^2\sin(c+dx)}{d}+\frac{7a^2\cos(c+dx)\sin(c+dx)}{8d}+\frac{a^2\cos^3(c+dx)\sin(c+dx)}{4d}-\frac{2a^2\sin^3(c+dx)}{3d}$$

output
```
7/8*a^2*x+2*a^2*sin(d*x+c)/d+7/8*a^2*cos(d*x+c)*sin(d*x+c)/d+1/4*a^2*cos(d
*x+c)^3*sin(d*x+c)/d-2/3*a^2*sin(d*x+c)^3/d
```

3.18.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.61

$$\int \cos^4(c+dx)(a+a\sec(c+dx))^2\,dx=\frac{a^2(84dx+144\sin(c+dx)+48\sin(2(c+dx))+16\sin(3(c+dx))+3\sin(4(c+dx)))}{96d}$$

input
```
Integrate[Cos[c + d*x]^4*(a + a*Sec[c + d*x])^2,x]
```

output
```
(a^2*(84*d*x + 144*Sin[c + d*x] + 48*Sin[2*(c + d*x)] + 16*Sin[3*(c + d*x)
] + 3*Sin[4*(c + d*x)]))/(96*d)
```

3.18.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.99, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 4275, 3042, 3113, 2009, 4533, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^4(c+dx)(a\sec(c+dx)+a)^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^4}dx$$

$$\Big\downarrow \text{4275}$$

$$2a^2\int \cos^3(c+dx)dx+\int \cos^4(c+dx)\left(\sec^2(c+dx)a^2+a^2\right)dx$$

$$\Big\downarrow \text{3042}$$

$$2a^2\int \sin\left(c+dx+\frac{\pi}{2}\right)^3dx+\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^4}dx$$

$$\Big\downarrow \text{3113}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^4}dx-\frac{2a^2\int\left(1-\sin^2(c+dx)\right)d(-\sin(c+dx))}{d}$$

$$\Big\downarrow \text{2009}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^4}dx-\frac{2a^2\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}$$

$$\Big\downarrow \text{4533}$$

$$\frac{7}{4}a^2\int \cos^2(c+dx)dx-\frac{2a^2\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}+\frac{a^2\sin(c+dx)\cos^3(c+dx)}{4d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7}{4}a^2\int \sin\left(c+dx+\frac{\pi}{2}\right)^2dx-\frac{2a^2\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}+\frac{a^2\sin(c+dx)\cos^3(c+dx)}{4d}$$

$$\Big\downarrow \text{3115}$$

$$\frac{7}{4}a^2\left(\frac{\int 1dx}{2}+\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)-\frac{2a^2\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}+\frac{a^2\sin(c+dx)\cos^3(c+dx)}{4d}$$

$$\Big\downarrow \; 24$$

$$-\frac{2a^2\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}+\frac{a^2\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{7}{4}a^2\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)$$

input
```
Int[Cos[c + d*x]^4*(a + a*Sec[c + d*x])^2,x]
```

output
```
(a^2*Cos[c + d*x]^3*Sin[c + d*x])/(4*d) + (7*a^2*(x/2 + (Cos[c + d*x]*Sin[
c + d*x])/(2*d)))/4 - (2*a^2*(-Sin[c + d*x] + Sin[c + d*x]^3/3))/d
```

3.18.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 4275
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Simp[2*a*(b/d)   Int[(d*Csc[e + f*x])^(n + 1), x], x]
 + Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
 e, f, n}, x]
```

rule 4533
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] +
Simp[(C*m + A*(m + 1))/(b^2*m)   Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fr
eeQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

3.18.4 Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.63

method	result
parallelrisch	$\frac{a^2(84dx+144\sin(dx+c)+3\sin(4dx+4c)+16\sin(3dx+3c)+48\sin(2dx+2c))}{96d}$
risch	$\frac{7a^2x}{8}+\frac{3a^2\sin(dx+c)}{2d}+\frac{a^2\sin(4dx+4c)}{32d}+\frac{a^2\sin(3dx+3c)}{6d}+\frac{a^2\sin(2dx+2c)}{2d}$
derivativedivides	$\frac{a^2\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+\frac{2a^2\left(2+\cos(dx+c)^2\right)\sin(dx+c)}{3}+a^2\left(\frac{\left(\cos(dx+c)^3+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)}{d}$
default	$\frac{a^2\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+\frac{2a^2\left(2+\cos(dx+c)^2\right)\sin(dx+c)}{3}+a^2\left(\frac{\left(\cos(dx+c)^3+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)}{d}$
norman	$\frac{-\frac{7a^2x}{8}-\frac{25a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d}-\frac{2a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3d}+\frac{a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{2d}+\frac{14a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{3d}+\frac{7a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^9}{4d}-\frac{21a^2x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^4\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}$

input
```
int(cos(d*x+c)^4*(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/96*a^2*(84*d*x+144*sin(d*x+c)+3*sin(4*d*x+4*c)+16*sin(3*d*x+3*c)+48*sin(
2*d*x+2*c))/d
```

3.18.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.72

$$\int \cos^4(c+dx)(a+a\sec(c+dx))^2\,dx = \frac{21\,a^2dx + \left(6\,a^2\cos\left(dx+c\right)^3 + 16\,a^2\cos\left(dx+c\right)^2 + 21\,a^2\cos\left(dx+c\right) + 32\,a^2\right)\sin\left(dx+c\right)}{24\,d}$$

input
```
integrate(cos(d*x+c)^4*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/24*(21*a^2*d*x + (6*a^2*cos(d*x + c)^3 + 16*a^2*cos(d*x + c)^2 + 21*a^2*
cos(d*x + c) + 32*a^2)*sin(d*x + c))/d
```

3.18.6 Sympy [F]

$$\int \cos^4(c+dx)(a+a\sec(c+dx))^2\,dx = a^2\left(\int 2\cos^4\left(c+dx\right)\sec\left(c+dx\right)dx + \int \cos^4\left(c+dx\right)\sec^2\left(c+dx\right)dx + \int \cos^4\left(c+dx\right)dx\right)$$

input
```
integrate(cos(d*x+c)**4*(a+a*sec(d*x+c))**2,x)
```

output
```
a**2*(Integral(2*cos(c + d*x)**4*sec(c + d*x), x) + Integral(cos(c + d*x)*
*4*sec(c + d*x)**2, x) + Integral(cos(c + d*x)**4, x))
```

3.18.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.95

$$\int \cos^4(c+dx)(a+a\sec(c+dx))^2\,dx = -\frac{64\left(\sin\left(dx+c\right)^3 - 3\,\sin\left(dx+c\right)\right)a^2 - 3\left(12\,dx + 12\,c + \sin\left(4\,dx+4\,c\right) + 8\,\sin\left(2\,dx+2\,c\right)\right)a^2 - 24\,(2}{96\,d}$$

input
```
integrate(cos(d*x+c)^4*(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
-1/96*(64*(sin(d*x + c)^3 - 3*sin(d*x + c))*a^2 - 3*(12*d*x + 12*c + sin(4
*d*x + 4*c) + 8*sin(2*d*x + 2*c))*a^2 - 24*(2*d*x + 2*c + sin(2*d*x + 2*c)
)*a^2)/d
```

3.18.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.10

$$\int \cos^4(c+dx)(a+a\sec(c+dx))^2\,dx = \frac{21\,(dx+c)a^2 + \frac{2\left(21\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+77\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+83\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+75\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^4}}{24\,d}$$

input
```
integrate(cos(d*x+c)^4*(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
1/24*(21*(d*x + c)*a^2 + 2*(21*a^2*tan(1/2*d*x + 1/2*c)^7 + 77*a^2*tan(1/2
*d*x + 1/2*c)^5 + 83*a^2*tan(1/2*d*x + 1/2*c)^3 + 75*a^2*tan(1/2*d*x + 1/2
*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^4)/d
```

3.18.9 Mupad [B] (verification not implemented)

Time = 16.49 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.02

$$\int \cos^4(c+dx)(a+a\sec(c+dx))^2\,dx = \frac{7\,a^2\,x}{8} + \frac{\frac{7\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{4} + \frac{77\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{12} + \frac{83\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{12} + \frac{25\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)^4}$$

input
```
int(cos(c + d*x)^4*(a + a/cos(c + d*x))^2,x)
```

output
```
(7*a^2*x)/8 + ((83*a^2*tan(c/2 + (d*x)/2)^3)/12 + (77*a^2*tan(c/2 + (d*x)/
2)^5)/12 + (7*a^2*tan(c/2 + (d*x)/2)^7)/4 + (25*a^2*tan(c/2 + (d*x)/2))/4)
/(d*(tan(c/2 + (d*x)/2)^2 + 1)^4)
```

3.19 $\int \cos^5(c+dx)(a+a\sec(c+dx))^2\,dx$

3.19.1 Optimal result . . . 410
3.19.2 Mathematica [A] (verified) . . . 410
3.19.3 Rubi [A] (verified) . . . 411
3.19.4 Maple [A] (verified) . . . 414
3.19.5 Fricas [A] (verification not implemented) . . . 415
3.19.6 Sympy [F] . . . 415
3.19.7 Maxima [A] (verification not implemented) . . . 415
3.19.8 Giac [A] (verification not implemented) . . . 416
3.19.9 Mupad [B] (verification not implemented) . . . 416

3.19.1 Optimal result

Integrand size = 21, antiderivative size = 103

$$\int \cos^5(c+dx)(a+a\sec(c+dx))^2\,dx = \frac{3a^2x}{4}+\frac{2a^2\sin(c+dx)}{d}+\frac{3a^2\cos(c+dx)\sin(c+dx)}{4d}+\frac{a^2\cos^3(c+dx)\sin(c+dx)}{2d}-\frac{a^2\sin^3(c+dx)}{d}+\frac{a^2\sin^5(c+dx)}{5d}$$

output
```
3/4*a^2*x+2*a^2*sin(d*x+c)/d+3/4*a^2*cos(d*x+c)*sin(d*x+c)/d+1/2*a^2*cos(d
*x+c)^3*sin(d*x+c)/d-a^2*sin(d*x+c)^3/d+1/5*a^2*sin(d*x+c)^5/d
```

3.19.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.59

$$\int \cos^5(c+dx)(a+a\sec(c+dx))^2\,dx = \frac{a^2(60dx+110\sin(c+dx)+40\sin(2(c+dx))+15\sin(3(c+dx))+5\sin(4(c+dx))+\sin(5(c+dx)))}{80d}$$

input
```
Integrate[Cos[c + d*x]^5*(a + a*Sec[c + d*x])^2,x]
```

output
```
(a^2*(60*d*x + 110*Sin[c + d*x] + 40*Sin[2*(c + d*x)] + 15*Sin[3*(c + d*x)
] + 5*Sin[4*(c + d*x)] + Sin[5*(c + d*x)]))/(80*d)
```

3.19.3 Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.91, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.619$, Rules used = {3042, 4275, 3042, 3115, 3042, 3115, 24, 4532, 3042, 3492, 27, 290, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^5(c+dx)(a\sec(c+dx)+a)^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^5}dx$$

$$\Big\downarrow \text{4275}$$

$$2a^2\int \cos^4(c+dx)dx+\int \cos^5(c+dx)\left(\sec^2(c+dx)a^2+a^2\right)dx$$

$$\Big\downarrow \text{3042}$$

$$2a^2\int \sin\left(c+dx+\frac{\pi}{2}\right)^4dx+\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^5}dx$$

$$\Big\downarrow \text{3115}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^5}dx+2a^2\left(\frac{3}{4}\int \cos^2(c+dx)dx+\frac{\sin(c+dx)\cos^3(c+dx)}{4d}\right)$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^5}dx+2a^2\left(\frac{3}{4}\int \sin\left(c+dx+\frac{\pi}{2}\right)^2dx+\frac{\sin(c+dx)\cos^3(c+dx)}{4d}\right)$$

$$\Big\downarrow \text{3115}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^5}dx+$$
$$2a^2\left(\frac{3}{4}\left(\frac{\int 1dx}{2}+\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)+\frac{\sin(c+dx)\cos^3(c+dx)}{4d}\right)$$

$$\Big\downarrow \; 24$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2 a^2+a^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^5}dx+$$
$$2a^2\left(\frac{\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3}{4}\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)\right)$$

$$\Big\downarrow \; 4532$$

$$\int \cos^3(c+dx)\left(\cos^2(c+dx)a^2+a^2\right)dx+$$
$$2a^2\left(\frac{\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3}{4}\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)\right)$$

$$\Big\downarrow \; 3042$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^3\left(\sin\left(c+dx+\frac{\pi}{2}\right)^2 a^2+a^2\right)dx+$$
$$2a^2\left(\frac{\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3}{4}\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)\right)$$

$$\Big\downarrow \; 3492$$

$$2a^2\left(\frac{\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3}{4}\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)\right)-\frac{\int a^2\left(1-\sin^2(c+dx)\right)\left(2-\sin^2(c+dx)\right)d(-\sin(c+dx))}{d}$$

$$\Big\downarrow \; 27$$

$$2a^2\left(\frac{\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3}{4}\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)\right)-\frac{a^2\int\left(1-\sin^2(c+dx)\right)\left(2-\sin^2(c+dx)\right)d(-\sin(c+dx))}{d}$$

$$\Big\downarrow \; 290$$

$$2a^2\left(\frac{\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3}{4}\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)\right)-\frac{a^2\int\left(\sin^4(c+dx)-3\sin^2(c+dx)+2\right)d(-\sin(c+dx))}{d}$$

$$\Big\downarrow \; 2009$$

$$2a^2\left(\frac{\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3}{4}\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)\right)-\frac{a^2\left(-\frac{1}{5}\sin^5(c+dx)+\sin^3(c+dx)-2\sin(c+dx)\right)}{d}$$

3.19. $\int \cos^5(c+dx)(a+a\sec(c+dx))^2\,dx$

input
```
Int[Cos[c + d*x]^5*(a + a*Sec[c + d*x])^2,x]
```

output
```
-((a^2*(-2*Sin[c + d*x] + Sin[c + d*x]^3 - Sin[c + d*x]^5/5))/d) + 2*a^2*(
(Cos[c + d*x]^3*Sin[c + d*x])/(4*d) + (3*(x/2 + (Cos[c + d*x]*Sin[c + d*x]
)/(2*d)))/4)
```

3.19.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 290
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol] :> I
nt[ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d
}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3492
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2),
 x_Symbol] :> Simp[-f^(-1)   Subst[Int[(1 - x^2)^((m - 1)/2)*(A + C - C*x^2
), x], x, Cos[e + f*x]], x] /; FreeQ[{e, f, A, C}, x] && IGtQ[(m + 1)/2, 0]
```

rule 4275
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Simp[2*a*(b/d)   Int[(d*Csc[e + f*x])^(n + 1), x], x]
 + Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
 e, f, n}, x]
```

rule 4532
```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.) + (A_)),
 x_Symbol] :> Int[(C + A*Sin[e + f*x]^2)/Sin[e + f*x]^(m + 2), x] /; FreeQ[
{e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && ILtQ[(m + 1)/2, 0]
```

3.19.4 Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.62

method	result
parallelrisch	$\frac{a^2(60dx+\sin(5dx+5c)+5\sin(4dx+4c)+15\sin(3dx+3c)+40\sin(2dx+2c)+110\sin(dx+c))}{80d}$
risch	$\frac{3a^2x}{4}+\frac{11a^2\sin(dx+c)}{8d}+\frac{a^2\sin(5dx+5c)}{80d}+\frac{a^2\sin(4dx+4c)}{16d}+\frac{3a^2\sin(3dx+3c)}{16d}+\frac{a^2\sin(2dx+2c)}{2d}$
derivativedivides	$\frac{\frac{a^2\left(2+\cos(dx+c)^2\right)\sin(dx+c)}{3}+2a^2\left(\frac{\left(\cos(dx+c)^3+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+\frac{a^2\left(\frac{8}{3}+\cos(dx+c)^4+\frac{4\cos(dx+c)^2}{3}\right)\mathrm{si}}{5}}{d}$
default	$\frac{\frac{a^2\left(2+\cos(dx+c)^2\right)\sin(dx+c)}{3}+2a^2\left(\frac{\left(\cos(dx+c)^3+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+\frac{a^2\left(\frac{8}{3}+\cos(dx+c)^4+\frac{4\cos(dx+c)^2}{3}\right)\mathrm{si}}{5}}{d}$
norman	$\frac{-\frac{3a^2x}{4}-\frac{13a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2d}-\frac{5a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{2d}-\frac{27a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{5d}+\frac{37a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{5d}+\frac{11a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^9}{2d}+\frac{3a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2d}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^5\left(-1\right.}$

input
```
int(cos(d*x+c)^5*(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/80*a^2*(60*d*x+sin(5*d*x+5*c)+5*sin(4*d*x+4*c)+15*sin(3*d*x+3*c)+40*sin(
2*d*x+2*c)+110*sin(d*x+c))/d
```

3.19.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.74

$$\int \cos^5(c+dx)(a+a\sec(c+dx))^2\,dx$$
$$= \frac{15\,a^2dx + \left(4\,a^2\cos\left(dx+c\right)^4 + 10\,a^2\cos\left(dx+c\right)^3 + 12\,a^2\cos\left(dx+c\right)^2 + 15\,a^2\cos\left(dx+c\right) + 24\,a^2\right)\sin\left(c\right.}{20\,d}$$

input
```
integrate(cos(d*x+c)^5*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/20*(15*a^2*d*x + (4*a^2*cos(d*x + c)^4 + 10*a^2*cos(d*x + c)^3 + 12*a^2*
cos(d*x + c)^2 + 15*a^2*cos(d*x + c) + 24*a^2)*sin(d*x + c))/d
```

3.19.6 Sympy [F]

$$\int \cos^5(c+dx)(a+a\sec(c+dx))^2\,dx = a^2\left(\int 2\cos^5\left(c+dx\right)\sec\left(c+dx\right)dx + \int \cos^5\left(c+dx\right)\sec^2\left(c+dx\right)dx + \int \cos^5\left(c+dx\right)dx\right)$$

input
```
integrate(cos(d*x+c)**5*(a+a*sec(d*x+c))**2,x)
```

output
```
a**2*(Integral(2*cos(c + d*x)**5*sec(c + d*x), x) + Integral(cos(c + d*x)*
*5*sec(c + d*x)**2, x) + Integral(cos(c + d*x)**5, x))
```

3.19.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.92

$$\int \cos^5(c+dx)(a+a\sec(c+dx))^2\,dx$$
$$= \frac{16\left(3\,\sin\left(dx+c\right)^5 - 10\,\sin\left(dx+c\right)^3 + 15\,\sin\left(dx+c\right)\right)a^2 - 80\left(\sin\left(dx+c\right)^3 - 3\,\sin\left(dx+c\right)\right)a^2 + 15\,(1}{240\,d}$$

input
```
integrate(cos(d*x+c)^5*(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
1/240*(16*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*a^2 - 8
0*(sin(d*x + c)^3 - 3*sin(d*x + c))*a^2 + 15*(12*d*x + 12*c + sin(4*d*x +
4*c) + 8*sin(2*d*x + 2*c))*a^2)/d
```

3.19.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.09

$$\int \cos^5(c+dx)(a+a\sec(c+dx))^2\,dx = \frac{15\,(dx+c)a^2 + \frac{2\left(15\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9+70\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+144\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+90\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+65\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^5}}{20\,d}$$

input
```
integrate(cos(d*x+c)^5*(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
1/20*(15*(d*x + c)*a^2 + 2*(15*a^2*tan(1/2*d*x + 1/2*c)^9 + 70*a^2*tan(1/2
*d*x + 1/2*c)^7 + 144*a^2*tan(1/2*d*x + 1/2*c)^5 + 90*a^2*tan(1/2*d*x + 1/
2*c)^3 + 65*a^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^5)/d
```

3.19.9 Mupad [B] (verification not implemented)

Time = 16.65 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.02

$$\int \cos^5(c+dx)(a+a\sec(c+dx))^2\,dx = \frac{3\,a^2\,x}{4} + \frac{\frac{3\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9}{2} + 7\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7 + \frac{72\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{5} + 9\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3 + \frac{13\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2}}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)^5}$$

input
```
int(cos(c + d*x)^5*(a + a/cos(c + d*x))^2,x)
```

output
```
(3*a^2*x)/4 + (9*a^2*tan(c/2 + (d*x)/2)^3 + (72*a^2*tan(c/2 + (d*x)/2)^5)/
5 + 7*a^2*tan(c/2 + (d*x)/2)^7 + (3*a^2*tan(c/2 + (d*x)/2)^9)/2 + (13*a^2*
tan(c/2 + (d*x)/2))/2)/(d*(tan(c/2 + (d*x)/2)^2 + 1)^5)
```

3.20 $\int \sec^3(c+dx)(a+a\sec(c+dx))^3\,dx$

3.20.1 Optimal result . . . 417
3.20.2 Mathematica [A] (verified) . . . 417
3.20.3 Rubi [A] (verified) . . . 418
3.20.4 Maple [C] (verified) . . . 419
3.20.5 Fricas [A] (verification not implemented) . . . 420
3.20.6 Sympy [F] . . . 420
3.20.7 Maxima [A] (verification not implemented) . . . 421
3.20.8 Giac [A] (verification not implemented) . . . 421
3.20.9 Mupad [B] (verification not implemented) . . . 422

3.20.1 Optimal result

Integrand size = 21, antiderivative size = 114

$$\int \sec^3(c+dx)(a+a\sec(c+dx))^3\,dx = \frac{13a^3\text{arctanh}(\sin(c+dx))}{8d}+\frac{4a^3\tan(c+dx)}{d}+\frac{13a^3\sec(c+dx)\tan(c+dx)}{8d}+\frac{3a^3\sec^3(c+dx)\tan(c+dx)}{4d}+\frac{5a^3\tan^3(c+dx)}{3d}+\frac{a^3\tan^5(c+dx)}{5d}$$

output
```
13/8*a^3*arctanh(sin(d*x+c))/d+4*a^3*tan(d*x+c)/d+13/8*a^3*sec(d*x+c)*tan(
d*x+c)/d+3/4*a^3*sec(d*x+c)^3*tan(d*x+c)/d+5/3*a^3*tan(d*x+c)^3/d+1/5*a^3*
tan(d*x+c)^5/d
```

3.20.2 Mathematica [A] (verified)

Time = 0.77 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.07

$$\int \sec^3(c+dx)(a+a\sec(c+dx))^3\,dx = \frac{13a^3\text{arctanh}(\sin(c+dx))}{8d}+\frac{19a^3\tan(c+dx)}{5d}+\frac{13a^3\sec(c+dx)\tan(c+dx)}{8d}+\frac{3a^3\sec^3(c+dx)\tan(c+dx)}{4d}+\frac{a^3\sec^4(c+dx)\tan(c+dx)}{5d}+\frac{19a^3\tan^3(c+dx)}{15d}$$

input
```
Integrate[Sec[c + d*x]^3*(a + a*Sec[c + d*x])^3,x]
```

output
```
(13*a^3*ArcTanh[Sin[c + d*x]])/(8*d) + (19*a^3*Tan[c + d*x])/(5*d) + (13*a
^3*Sec[c + d*x]*Tan[c + d*x])/(8*d) + (3*a^3*Sec[c + d*x]^3*Tan[c + d*x])/
(4*d) + (a^3*Sec[c + d*x]^4*Tan[c + d*x])/(5*d) + (19*a^3*Tan[c + d*x]^3)/
(15*d)
```

3.20.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3042, 4278, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^3(c+dx)(a\sec(c+dx)+a)^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^3\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^3dx$$

$$\Big\downarrow \text{4278}$$

$$\int \left(a^3\sec^6(c+dx)+3a^3\sec^5(c+dx)+3a^3\sec^4(c+dx)+a^3\sec^3(c+dx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{13a^3\text{arctanh}(\sin(c+dx))}{8d}+\frac{a^3\tan^5(c+dx)}{5d}+\frac{5a^3\tan^3(c+dx)}{3d}+\frac{4a^3\tan(c+dx)}{d}+\frac{3a^3\tan(c+dx)\sec^3(c+dx)}{4d}+\frac{13a^3\tan(c+dx)\sec(c+dx)}{8d}$$

input
```
Int[Sec[c + d*x]^3*(a + a*Sec[c + d*x])^3,x]
```

output
```
(13*a^3*ArcTanh[Sin[c + d*x]])/(8*d) + (4*a^3*Tan[c + d*x])/d + (13*a^3*Se
c[c + d*x]*Tan[c + d*x])/(8*d) + (3*a^3*Sec[c + d*x]^3*Tan[c + d*x])/(4*d)
 + (5*a^3*Tan[c + d*x]^3)/(3*d) + (a^3*Tan[c + d*x]^5)/(5*d)
```

3.20.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4278
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

3.20.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.10 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.27

method	result
risch	$-\frac{ia^3\left(195\,\mathrm{e}^{9i(dx+c)}+750\,\mathrm{e}^{7i(dx+c)}-720\,\mathrm{e}^{6i(dx+c)}-2320\,\mathrm{e}^{4i(dx+c)}-750\,\mathrm{e}^{3i(dx+c)}-1520\,\mathrm{e}^{2i(dx+c)}-195\,\mathrm{e}^{i(dx+c)}-304\right)}{60d\left(\mathrm{e}^{2i(dx+c)}+1\right)^5}$
derivativedivides	$\frac{-a^3\left(-\frac{8}{15}-\frac{\sec(dx+c)^4}{5}-\frac{4\sec(dx+c)^2}{15}\right)\tan(dx+c)+3a^3\left(-\left(-\frac{\sec(dx+c)^3}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(}{8}\right.}{d}$
default	$\frac{-a^3\left(-\frac{8}{15}-\frac{\sec(dx+c)^4}{5}-\frac{4\sec(dx+c)^2}{15}\right)\tan(dx+c)+3a^3\left(-\left(-\frac{\sec(dx+c)^3}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(}{8}\right.}{d}$
norman	$\frac{-\frac{51a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d}+\frac{133a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{6d}-\frac{416a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{15d}+\frac{91a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{6d}-\frac{13a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^9}{4d}}{\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^5}-\frac{13a^3\ln\left(\tan\left(\frac{d}{2}\right.\right.}{8d}$
parts	$\frac{a^3\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)}{d}-\frac{a^3\left(-\frac{8}{15}-\frac{\sec(dx+c)^4}{5}-\frac{4\sec(dx+c)^2}{15}\right)\tan(dx+c)}{d}-\frac{3a^3\left(-\frac{2}{3}-\right.}{}$
parallelrisch	$\frac{a^3\left(-1950\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\cos(dx+c)+1950\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)\cos(dx+c)-195\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\cos(5dx+5c)\right.}{}$

input
```
int(sec(d*x+c)^3*(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
-1/60*I*a^3*(195*exp(9*I*(d*x+c))+750*exp(7*I*(d*x+c))-720*exp(6*I*(d*x+c)
)-2320*exp(4*I*(d*x+c))-750*exp(3*I*(d*x+c))-1520*exp(2*I*(d*x+c))-195*exp
(I*(d*x+c))-304)/d/(exp(2*I*(d*x+c))+1)^5-13/8*a^3/d*ln(exp(I*(d*x+c))-I)+
13/8*a^3/d*ln(exp(I*(d*x+c))+I)
```

3.20.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.09

$$\int \sec^3(c+dx)(a+a\sec(c+dx))^3\,dx$$
$$= \frac{195\,a^3\cos(dx+c)^5\log(\sin(dx+c)+1) - 195\,a^3\cos(dx+c)^5\log(-\sin(dx+c)+1) + 2\,(304\,a^3\cos(d}{240\,d\cos(dx+}$$

input
```
integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/240*(195*a^3*cos(d*x + c)^5*log(sin(d*x + c) + 1) - 195*a^3*cos(d*x + c)
^5*log(-sin(d*x + c) + 1) + 2*(304*a^3*cos(d*x + c)^4 + 195*a^3*cos(d*x +
c)^3 + 152*a^3*cos(d*x + c)^2 + 90*a^3*cos(d*x + c) + 24*a^3)*sin(d*x + c)
)/(d*cos(d*x + c)^5)
```

3.20.6 Sympy [F]

$$\int \sec^3(c+dx)(a+a\sec(c+dx))^3\,dx = a^3\left(\int \sec^3(c+dx)\,dx + \int 3\sec^4(c+dx)\,dx + \int 3\sec^5(c+dx)\,dx + \int \sec^6(c+dx)\,dx\right)$$

input
```
integrate(sec(d*x+c)**3*(a+a*sec(d*x+c))**3,x)
```

output
```
a**3*(Integral(sec(c + d*x)**3, x) + Integral(3*sec(c + d*x)**4, x) + Inte
gral(3*sec(c + d*x)**5, x) + Integral(sec(c + d*x)**6, x))
```

3.20.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.57

$$\int \sec^3(c+dx)(a+a\sec(c+dx))^3\,dx$$
$$= \frac{16\left(3\tan\left(dx+c\right)^5+10\tan\left(dx+c\right)^3+15\tan\left(dx+c\right)\right)a^3+240\left(\tan\left(dx+c\right)^3+3\tan\left(dx+c\right)\right)a^3-4}{}$$

input
```
integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/240*(16*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))*a^3 + 2
40*(tan(d*x + c)^3 + 3*tan(d*x + c))*a^3 - 45*a^3*(2*(3*sin(d*x + c)^3 - 5
*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c
) + 1) + 3*log(sin(d*x + c) - 1)) - 60*a^3*(2*sin(d*x + c)/(sin(d*x + c)^2
 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)))/d
```

3.20.8 Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.21

$$\int \sec^3(c+dx)(a+a\sec(c+dx))^3\,dx$$
$$= \frac{195\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-195\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(195\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9-910\,a^3\tan\left(\frac{1}{2}\,dx\right.\right.}{}}{120\,d}$$

input
```
integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
1/120*(195*a^3*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 195*a^3*log(abs(tan(1/
2*d*x + 1/2*c) - 1)) - 2*(195*a^3*tan(1/2*d*x + 1/2*c)^9 - 910*a^3*tan(1/2
*d*x + 1/2*c)^7 + 1664*a^3*tan(1/2*d*x + 1/2*c)^5 - 1330*a^3*tan(1/2*d*x +
 1/2*c)^3 + 765*a^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^5)/
d
```

3.20.9 Mupad [B] (verification not implemented)

Time = 17.77 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.49

$$\int \sec^3(c+dx)(a+a\sec(c+dx))^3\,dx = \frac{13\,a^3\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{4\,d} - \frac{\frac{13\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9}{4} - \frac{91\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{6} + \frac{416\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{15} - \frac{133\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{6} + \frac{51\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10} - 5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8 + 10\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6 - 10\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4 + 5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2 - 1\right)}$$

input
```
int((a + a/cos(c + d*x))^3/cos(c + d*x)^3,x)
```

output
```
(13*a^3*atanh(tan(c/2 + (d*x)/2)))/(4*d) - ((416*a^3*tan(c/2 + (d*x)/2)^5)
/15 - (133*a^3*tan(c/2 + (d*x)/2)^3)/6 - (91*a^3*tan(c/2 + (d*x)/2)^7)/6 +
 (13*a^3*tan(c/2 + (d*x)/2)^9)/4 + (51*a^3*tan(c/2 + (d*x)/2))/4)/(d*(5*ta
n(c/2 + (d*x)/2)^2 - 10*tan(c/2 + (d*x)/2)^4 + 10*tan(c/2 + (d*x)/2)^6 - 5
*tan(c/2 + (d*x)/2)^8 + tan(c/2 + (d*x)/2)^10 - 1))
```

3.21 $\int \sec^2(c+dx)(a+a\sec(c+dx))^3\,dx$

3.21.1 Optimal result . . . 423
3.21.2 Mathematica [A] (verified) . . . 423
3.21.3 Rubi [A] (verified) . . . 424
3.21.4 Maple [A] (verified) . . . 425
3.21.5 Fricas [A] (verification not implemented) . . . 426
3.21.6 Sympy [F] . . . 426
3.21.7 Maxima [A] (verification not implemented) . . . 426
3.21.8 Giac [A] (verification not implemented) . . . 427
3.21.9 Mupad [B] (verification not implemented) . . . 427

3.21.1 Optimal result

Integrand size = 21, antiderivative size = 93

$$\int \sec^2(c+dx)(a+a\sec(c+dx))^3\,dx = \frac{15a^3\text{arctanh}(\sin(c+dx))}{8d}+\frac{4a^3\tan(c+dx)}{d}+\frac{15a^3\sec(c+dx)\tan(c+dx)}{8d}+\frac{a^3\sec^3(c+dx)\tan(c+dx)}{4d}+\frac{a^3\tan^3(c+dx)}{d}$$

output
```
15/8*a^3*arctanh(sin(d*x+c))/d+4*a^3*tan(d*x+c)/d+15/8*a^3*sec(d*x+c)*tan(
d*x+c)/d+1/4*a^3*sec(d*x+c)^3*tan(d*x+c)/d+a^3*tan(d*x+c)^3/d
```

3.21.2 Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.06

$$\int \sec^2(c+dx)(a+a\sec(c+dx))^3\,dx = \frac{15a^3\text{arctanh}(\sin(c+dx))}{8d}+\frac{3a^3\tan(c+dx)}{d}+\frac{15a^3\sec(c+dx)\tan(c+dx)}{8d}+\frac{a^3\sec^2(c+dx)\tan(c+dx)}{d}+\frac{a^3\sec^3(c+dx)\tan(c+dx)}{4d}$$

input
```
Integrate[Sec[c + d*x]^2*(a + a*Sec[c + d*x])^3,x]
```

output
```
(15*a^3*ArcTanh[Sin[c + d*x]])/(8*d) + (3*a^3*Tan[c + d*x])/d + (15*a^3*Se
c[c + d*x]*Tan[c + d*x])/(8*d) + (a^3*Sec[c + d*x]^2*Tan[c + d*x])/d + (a^
3*Sec[c + d*x]^3*Tan[c + d*x])/(4*d)
```

3.21.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3042, 4278, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(c+dx)(a\sec(c+dx)+a)^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^2\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^3dx$$

$$\Big\downarrow \text{4278}$$

$$\int \left(a^3\sec^5(c+dx)+3a^3\sec^4(c+dx)+3a^3\sec^3(c+dx)+a^3\sec^2(c+dx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{15a^3\text{arctanh}(\sin(c+dx))}{8d}+\frac{a^3\tan^3(c+dx)}{d}+\frac{4a^3\tan(c+dx)}{d}+\frac{a^3\tan(c+dx)\sec^3(c+dx)}{4d}+\frac{15a^3\tan(c+dx)\sec(c+dx)}{8d}$$

input
```
Int[Sec[c + d*x]^2*(a + a*Sec[c + d*x])^3,x]
```

output
```
(15*a^3*ArcTanh[Sin[c + d*x]])/(8*d) + (4*a^3*Tan[c + d*x])/d + (15*a^3*Se
c[c + d*x]*Tan[c + d*x])/(8*d) + (a^3*Sec[c + d*x]^3*Tan[c + d*x])/(4*d) +
 (a^3*Tan[c + d*x]^3)/d
```

3.21.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4278
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

3.21.4 Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.32

method	result
derivativedivides	$\frac{a^3\left(-\left(-\frac{\sec(dx+c)^3}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)-3a^3\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right)\tan(dx+c)+3a^3\left(\frac{\mathrm{se}}{}\right.}{d}$
default	$\frac{a^3\left(-\left(-\frac{\sec(dx+c)^3}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)-3a^3\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right)\tan(dx+c)+3a^3\left(\frac{\mathrm{se}}{}\right.}{d}$
parts	$\frac{a^3\tan(dx+c)}{d}+\frac{a^3\left(-\left(-\frac{\sec(dx+c)^3}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)}{d}+\frac{3a^3\left(\frac{\sec(dx+c)\tan(dx+}{2}\right.}{}$
norman	$\frac{\frac{49a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d}-\frac{73a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{4d}+\frac{55a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{4d}-\frac{15a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{4d}}{\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^4}-\frac{15a^3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{8d}+\frac{15a^3\ln\left(\right.}{}$
risch	$-\frac{ia^3\left(15\,\mathrm{e}^{7i(dx+c)}-8\,\mathrm{e}^{6i(dx+c)}+23\,\mathrm{e}^{5i(dx+c)}-72\,\mathrm{e}^{4i(dx+c)}-23\,\mathrm{e}^{3i(dx+c)}-88\,\mathrm{e}^{2i(dx+c)}-15\,\mathrm{e}^{i(dx+c)}-24\right)}{4d\left(\mathrm{e}^{2i(dx+c)}+1\right)^4}+\frac{15a^3\ln(\mathrm{e}}{}$
parallelrisch	$\frac{a^3\left(15(-\cos(4dx+4c)-4\cos(2dx+2c)-3)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+15(3+\cos(4dx+4c)+4\cos(2dx+2c))\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\right.\right.}{8d(3+\cos(4dx+4c)+4\cos(2dx+2c))}$

input
```
int(sec(d*x+c)^2*(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^3*(-(-1/4*sec(d*x+c)^3-3/8*sec(d*x+c))*tan(d*x+c)+3/8*ln(sec(d*x+c)
+tan(d*x+c)))-3*a^3*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c)+3*a^3*(1/2*sec(d*x+
c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c)))+a^3*tan(d*x+c))
```

3.21.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.19

$$\int \sec^2(c+dx)(a+a\sec(c+dx))^3\,dx = \frac{15\,a^3\cos(dx+c)^4\log(\sin(dx+c)+1) - 15\,a^3\cos(dx+c)^4\log(-\sin(dx+c)+1) + 2\,(24\,a^3\cos(dx+}{16\,d\cos(dx+c)^4}$$

input
```
integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/16*(15*a^3*cos(d*x + c)^4*log(sin(d*x + c) + 1) - 15*a^3*cos(d*x + c)^4*
log(-sin(d*x + c) + 1) + 2*(24*a^3*cos(d*x + c)^3 + 15*a^3*cos(d*x + c)^2
+ 8*a^3*cos(d*x + c) + 2*a^3)*sin(d*x + c))/(d*cos(d*x + c)^4)
```

3.21.6 Sympy [F]

$$\int \sec^2(c+dx)(a+a\sec(c+dx))^3\,dx = a^3\left(\int \sec^2(c+dx)\,dx + \int 3\sec^3(c+dx)\,dx + \int 3\sec^4(c+dx)\,dx + \int \sec^5(c+dx)\,dx\right)$$

input
```
integrate(sec(d*x+c)**2*(a+a*sec(d*x+c))**3,x)
```

output
```
a**3*(Integral(sec(c + d*x)**2, x) + Integral(3*sec(c + d*x)**3, x) + Inte
gral(3*sec(c + d*x)**4, x) + Integral(sec(c + d*x)**5, x))
```

3.21.7 Maxima [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.68

$$\int \sec^2(c+dx)(a+a\sec(c+dx))^3\,dx = \frac{16\left(\tan(dx+c)^3+3\tan(dx+c)\right)a^3 - a^3\left(\frac{2\left(3\sin(dx+c)^3-5\sin(dx+c)\right)}{\sin(dx+c)^4-2\sin(dx+c)^2+1} - 3\log(\sin(dx+c)+1) + 3\log(\sin}{1}$$

input
```
integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/16*(16*(tan(d*x + c)^3 + 3*tan(d*x + c))*a^3 - a^3*(2*(3*sin(d*x + c)^3
- 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x
+ c) + 1) + 3*log(sin(d*x + c) - 1)) - 12*a^3*(2*sin(d*x + c)/(sin(d*x + c
)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 16*a^3*tan(d*x
 + c))/d
```

3.21.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.31

$$\int \sec^2(c+dx)(a+a\sec(c+dx))^3\,dx$$
$$= \frac{15\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-15\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(15\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-55\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right.\right.}{\left(\tan\left(\frac{1}{2}\,dx\right.\right.}}{8\,d}$$

input
```
integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
1/8*(15*a^3*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 15*a^3*log(abs(tan(1/2*d*
x + 1/2*c) - 1)) - 2*(15*a^3*tan(1/2*d*x + 1/2*c)^7 - 55*a^3*tan(1/2*d*x +
 1/2*c)^5 + 73*a^3*tan(1/2*d*x + 1/2*c)^3 - 49*a^3*tan(1/2*d*x + 1/2*c))/(
tan(1/2*d*x + 1/2*c)^2 - 1)^4)/d
```

3.21.9 Mupad [B] (verification not implemented)

Time = 16.31 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.52

$$\int \sec^2(c+dx)(a+a\sec(c+dx))^3\,dx$$
$$= \frac{15\,a^3\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{4\,d} - \frac{\frac{15\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{4}-\frac{55\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{4}+\frac{73\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{4}-\frac{49\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8-4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}$$

input
```
int((a + a/cos(c + d*x))^3/cos(c + d*x)^2,x)
```

output
```
(15*a^3*atanh(tan(c/2 + (d*x)/2)))/(4*d) - ((73*a^3*tan(c/2 + (d*x)/2)^3)/
4 - (55*a^3*tan(c/2 + (d*x)/2)^5)/4 + (15*a^3*tan(c/2 + (d*x)/2)^7)/4 - (4
9*a^3*tan(c/2 + (d*x)/2))/4)/(d*(6*tan(c/2 + (d*x)/2)^4 - 4*tan(c/2 + (d*x
)/2)^2 - 4*tan(c/2 + (d*x)/2)^6 + tan(c/2 + (d*x)/2)^8 + 1))
```

3.21. $\int \sec^2(c+dx)(a+a\sec(c+dx))^3\,dx$

3.22 $\int \sec(c+dx)(a+a\sec(c+dx))^3\,dx$

3.22.1 Optimal result . . . 429
3.22.2 Mathematica [A] (verified) . . . 429
3.22.3 Rubi [A] (verified) . . . 430
3.22.4 Maple [A] (verified) . . . 431
3.22.5 Fricas [A] (verification not implemented) . . . 431
3.22.6 Sympy [F] . . . 432
3.22.7 Maxima [A] (verification not implemented) . . . 432
3.22.8 Giac [A] (verification not implemented) . . . 432
3.22.9 Mupad [B] (verification not implemented) . . . 433

3.22.1 Optimal result

Integrand size = 19, antiderivative size = 72

$$\int \sec(c+dx)(a+a\sec(c+dx))^3\,dx=\frac{5a^3\text{arctanh}(\sin(c+dx))}{2d}+\frac{4a^3\tan(c+dx)}{d}+\frac{3a^3\sec(c+dx)\tan(c+dx)}{2d}+\frac{a^3\tan^3(c+dx)}{3d}$$

output
```
5/2*a^3*arctanh(sin(d*x+c))/d+4*a^3*tan(d*x+c)/d+3/2*a^3*sec(d*x+c)*tan(d*
x+c)/d+1/3*a^3*tan(d*x+c)^3/d
```

3.22.2 Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00

$$\int \sec(c+dx)(a+a\sec(c+dx))^3\,dx=\frac{5a^3\text{arctanh}(\sin(c+dx))}{2d}+\frac{4a^3\tan(c+dx)}{d}+\frac{3a^3\sec(c+dx)\tan(c+dx)}{2d}+\frac{a^3\tan^3(c+dx)}{3d}$$

input
```
Integrate[Sec[c + d*x]*(a + a*Sec[c + d*x])^3,x]
```

output
```
(5*a^3*ArcTanh[Sin[c + d*x]])/(2*d) + (4*a^3*Tan[c + d*x])/d + (3*a^3*Sec[
c + d*x]*Tan[c + d*x])/(2*d) + (a^3*Tan[c + d*x]^3)/(3*d)
```

3.22.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {3042, 4278, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(c+dx)(a\sec(c+dx)+a)^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^3dx$$

$$\Big\downarrow \text{4278}$$

$$\int \left(a^3\sec^4(c+dx)+3a^3\sec^3(c+dx)+3a^3\sec^2(c+dx)+a^3\sec(c+dx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{5a^3\text{arctanh}(\sin(c+dx))}{2d}+\frac{a^3\tan^3(c+dx)}{3d}+\frac{4a^3\tan(c+dx)}{d}+\frac{3a^3\tan(c+dx)\sec(c+dx)}{2d}$$

input
```
Int[Sec[c + d*x]*(a + a*Sec[c + d*x])^3,x]
```

output
```
(5*a^3*ArcTanh[Sin[c + d*x]])/(2*d) + (4*a^3*Tan[c + d*x])/d + (3*a^3*Sec[
c + d*x]*Tan[c + d*x])/(2*d) + (a^3*Tan[c + d*x]^3)/(3*d)
```

3.22.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4278
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

3.22.4 Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.31

method	result
derivativedivides	$\frac{-a^3\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right)\tan(dx+c)+3a^3\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+3a^3\tan(dx+c)+a^3\ln(\sec(dx-}{d}$
default	$\frac{-a^3\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right)\tan(dx+c)+3a^3\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+3a^3\tan(dx+c)+a^3\ln(\sec(dx-}{d}$
parts	$\frac{a^3\ln(\sec(dx+c)+\tan(dx+c))}{d}-\frac{a^3\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right)\tan(dx+c)}{d}+\frac{3a^3\tan(dx+c)}{d}+\frac{3a^3\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\ln\right.}{d}$
risch	$-\frac{ia^3\left(9\,e^{5i(dx+c)}-18\,e^{4i(dx+c)}-48\,e^{2i(dx+c)}-9\,e^{i(dx+c)}-22\right)}{3d\left(e^{2i(dx+c)}+1\right)^3}+\frac{5a^3\ln\left(e^{i(dx+c)}+i\right)}{2d}-\frac{5a^3\ln\left(e^{i(dx+c)}-i\right)}{2d}$
norman	$\frac{-\frac{11a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{40a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3d}-\frac{5a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{d}}{\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^3}-\frac{5a^3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2d}+\frac{5a^3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{2d}$
parallelrisch	$-\frac{a^3\left(45\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\cos(dx+c)+15\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\cos(3dx+3c)-45\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)\cos(dx+c)-15\right.}{6d(\cos(3dx+3c)+3\cos(dx+c))}$

input
```
int(sec(d*x+c)*(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-a^3*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c)+3*a^3*(1/2*sec(d*x+c)*tan(d*x
+c)+1/2*ln(sec(d*x+c)+tan(d*x+c)))+3*a^3*tan(d*x+c)+a^3*ln(sec(d*x+c)+tan(
d*x+c)))
```

3.22.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.36

$$\int \sec(c+dx)(a+a\sec(c+dx))^3\,dx$$
$$=\frac{15\,a^3\cos(dx+c)^3\log(\sin(dx+c)+1)-15\,a^3\cos(dx+c)^3\log(-\sin(dx+c)+1)+2\,(22\,a^3\cos(dx+}{12\,d\cos(dx+c)^3}$$

input
```
integrate(sec(d*x+c)*(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/12*(15*a^3*cos(d*x + c)^3*log(sin(d*x + c) + 1) - 15*a^3*cos(d*x + c)^3*
log(-sin(d*x + c) + 1) + 2*(22*a^3*cos(d*x + c)^2 + 9*a^3*cos(d*x + c) + 2
*a^3)*sin(d*x + c))/(d*cos(d*x + c)^3)
```

3.22.6 Sympy [F]

$$\int \sec(c+dx)(a+a\sec(c+dx))^3\,dx = a^3\left(\int \sec(c+dx)\,dx + \int 3\sec^2(c+dx)\,dx + \int 3\sec^3(c+dx)\,dx + \int \sec^4(c+dx)\,dx\right)$$

input
```
integrate(sec(d*x+c)*(a+a*sec(d*x+c))**3,x)
```

output
```
a**3*(Integral(sec(c + d*x), x) + Integral(3*sec(c + d*x)**2, x) + Integra
l(3*sec(c + d*x)**3, x) + Integral(sec(c + d*x)**4, x))
```

3.22.7 Maxima [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.44

$$\int \sec(c+dx)(a+a\sec(c+dx))^3\,dx = \frac{4\left(\tan(dx+c)^3+3\tan(dx+c)\right)a^3 - 9a^3\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1} - \log(\sin(dx+c)+1) + \log(\sin(dx+c)-1)\right)}{12d}$$

input
```
integrate(sec(d*x+c)*(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/12*(4*(tan(d*x + c)^3 + 3*tan(d*x + c))*a^3 - 9*a^3*(2*sin(d*x + c)/(sin
(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 12*a^3
*log(sec(d*x + c) + tan(d*x + c)) + 36*a^3*tan(d*x + c))/d
```

3.22.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.47

$$\int \sec(c+dx)(a+a\sec(c+dx))^3\,dx = \frac{15a^3\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right) - 15a^3\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right) - \frac{2\left(15a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5 - 40a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right.\right.}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}}{6d}$$

input
```
integrate(sec(d*x+c)*(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
1/6*(15*a^3*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 15*a^3*log(abs(tan(1/2*d*
x + 1/2*c) - 1)) - 2*(15*a^3*tan(1/2*d*x + 1/2*c)^5 - 40*a^3*tan(1/2*d*x +
 1/2*c)^3 + 33*a^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^3)/d
```

3.22.9 Mupad [B] (verification not implemented)

Time = 14.48 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.56

$$\int \sec(c+dx)(a+a\sec(c+dx))^3\,dx$$
$$= \frac{5\,a^3\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d} - \frac{5\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5 - \frac{40\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{3} + 11\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6 - 3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4 + 3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2 - 1\right)}$$

input
```
int((a + a/cos(c + d*x))^3/cos(c + d*x),x)
```

output
```
(5*a^3*atanh(tan(c/2 + (d*x)/2)))/d - (5*a^3*tan(c/2 + (d*x)/2)^5 - (40*a^
3*tan(c/2 + (d*x)/2)^3)/3 + 11*a^3*tan(c/2 + (d*x)/2))/(d*(3*tan(c/2 + (d*
x)/2)^2 - 3*tan(c/2 + (d*x)/2)^4 + tan(c/2 + (d*x)/2)^6 - 1))
```

3.23 $\int (a + a\sec(c + dx))^3 \, dx$

3.23.1 Optimal result 434
3.23.2 Mathematica [A] (verified) 434
3.23.3 Rubi [A] (verified) 435
3.23.4 Maple [A] (verified) 437
3.23.5 Fricas [A] (verification not implemented) 437
3.23.6 Sympy [F] 438
3.23.7 Maxima [A] (verification not implemented) 438
3.23.8 Giac [A] (verification not implemented) 439
3.23.9 Mupad [B] (verification not implemented) 439

3.23.1 Optimal result

Integrand size = 12, antiderivative size = 66

$$\int (a + a\sec(c + dx))^3 \, dx = a^3x + \frac{7a^3\text{arctanh}(\sin(c + dx))}{2d} + \frac{5a^3\tan(c + dx)}{2d} + \frac{(a^3 + a^3\sec(c + dx))\tan(c + dx)}{2d}$$

output
```
a^3*x+7/2*a^3*arctanh(sin(d*x+c))/d+5/2*a^3*tan(d*x+c)/d+1/2*(a^3+a^3*sec(
d*x+c))*tan(d*x+c)/d
```

3.23.2 Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.89

$$\int (a + a\sec(c + dx))^3 \, dx = a^3x + \frac{7a^3\text{arctanh}(\sin(c + dx))}{2d} + \frac{3a^3\tan(c + dx)}{d} + \frac{a^3\sec(c + dx)\tan(c + dx)}{2d}$$

input
```
Integrate[(a + a*Sec[c + d*x])^3,x]
```

output
```
a^3*x + (7*a^3*ArcTanh[Sin[c + d*x]])/(2*d) + (3*a^3*Tan[c + d*x])/d + (a^
3*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

3.23.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.05, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {3042, 4262, 3042, 4402, 3042, 4254, 24, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sec(c+dx)+a)^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^3 dx$$

$$\Big\downarrow \text{4262}$$

$$\frac{1}{2}a\int (\sec(c+dx)a+a)(5\sec(c+dx)a+2a)dx+\frac{\tan(c+dx)\left(a^3\sec(c+dx)+a^3\right)}{2d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}a\int \left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)\left(5\csc\left(c+dx+\frac{\pi}{2}\right)a+2a\right)dx+\frac{\tan(c+dx)\left(a^3\sec(c+dx)+a^3\right)}{2d}$$

$$\Big\downarrow \text{4402}$$

$$\frac{1}{2}a\left(5a^2\int \sec^2(c+dx)dx+7a^2\int \sec(c+dx)dx+2a^2x\right)+\frac{\tan(c+dx)\left(a^3\sec(c+dx)+a^3\right)}{2d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}a\left(7a^2\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+5a^2\int \csc\left(c+dx+\frac{\pi}{2}\right)^2 dx+2a^2x\right)+\frac{\tan(c+dx)\left(a^3\sec(c+dx)+a^3\right)}{2d}$$

$$\Big\downarrow \text{4254}$$

$$\frac{1}{2}a\left(-\frac{5a^2\int 1d(-\tan(c+dx))}{d}+7a^2\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+2a^2x\right)+\frac{\tan(c+dx)\left(a^3\sec(c+dx)+a^3\right)}{2d}$$

$$\Big\downarrow \text{24}$$

$$\frac{1}{2}a\left(7a^2\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{5a^2\tan(c+dx)}{d}+2a^2x\right)+\frac{\tan(c+dx)\left(a^3\sec(c+dx)+a^3\right)}{2d}$$

$$\Big\downarrow \text{4257}$$

$$\frac{\tan(c+dx)\left(a^3\sec(c+dx)+a^3\right)}{2d}+\frac{1}{2}a\left(\frac{7a^2\text{arctanh}(\sin(c+dx))}{d}+\frac{5a^2\tan(c+dx)}{d}+2a^2x\right)$$

input
```
Int[(a + a*Sec[c + d*x])^3,x]
```

output
```
((a^3 + a^3*Sec[c + d*x])*Tan[c + d*x])/(2*d) + (a*(2*a^2*x + (7*a^2*ArcTa
nh[Sin[c + d*x]])/d + (5*a^2*Tan[c + d*x])/d))/2
```

3.23.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4262
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(-b^2)*C
ot[c + d*x]*((a + b*Csc[c + d*x])^(n - 2)/(d*(n - 1))), x] + Simp[a/(n - 1)
   Int[(a + b*Csc[c + d*x])^(n - 2)*(a*(n - 1) + b*(3*n - 4)*Csc[c + d*x]),
 x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && Inte
gerQ[2*n]
```

rule 4402
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))*(csc[(e_.) + (f_.)*(x_)]*(d_.) +
 (c_)), x_Symbol] :> Simp[a*c*x, x] + (Simp[b*d   Int[Csc[e + f*x]^2, x], x
] + Simp[(b*c + a*d)   Int[Csc[e + f*x], x], x]) /; FreeQ[{a, b, c, d, e, f
}, x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

3.23.4 Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.21

method	result
derivativedivides	$\frac{a^3\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+3a^3\tan(dx+c)+3a^3\ln(\sec(dx+c)+\tan(dx+c))+a^3(dx+c)}{d}$
default	$\frac{a^3\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+3a^3\tan(dx+c)+3a^3\ln(\sec(dx+c)+\tan(dx+c))+a^3(dx+c)}{d}$
parts	$a^3x+\frac{a^3\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)}{d}+\frac{3a^3\ln(\sec(dx+c)+\tan(dx+c))}{d}+\frac{3a^3\tan(dx+c)}{d}$
risch	$a^3x-\frac{ia^3\left(e^{3i(dx+c)}-6\,e^{2i(dx+c)}-e^{i(dx+c)}-6\right)}{d\left(e^{2i(dx+c)}+1\right)^2}+\frac{7a^3\ln\left(e^{i(dx+c)}+i\right)}{2d}-\frac{7a^3\ln\left(e^{i(dx+c)}-i\right)}{2d}$
norman	$\frac{a^3x+a^3x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\frac{7a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}-\frac{5a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{d}-2a^3x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^2}-\frac{7a^3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2d}+\frac{7a^3\,\mathrm{l}}{}$
parallelrisch	$\frac{a^3\left(2dx\cos(2dx+2c)-7\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\cos(2dx+2c)+7\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)\cos(2dx+2c)+2dx+2\sin(dx+c)+6\,\mathrm{si}\right.}{2d(1+\cos(2dx+2c))}$

input
```
int((a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^3*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c)))+3*a^3*t
an(d*x+c)+3*a^3*ln(sec(d*x+c)+tan(d*x+c))+a^3*(d*x+c))
```

3.23.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.48

$$\int (a+a\sec(c+dx))^3\,dx$$
$$=\frac{4\,a^3dx\cos\left(dx+c\right)^2+7\,a^3\cos\left(dx+c\right)^2\log\left(\sin\left(dx+c\right)+1\right)-7\,a^3\cos\left(dx+c\right)^2\log\left(-\sin\left(dx+c\right)+1\right)}{4\,d\cos\left(dx+c\right)^2}$$

input
```
integrate((a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/4*(4*a^3*d*x*cos(d*x + c)^2 + 7*a^3*cos(d*x + c)^2*log(sin(d*x + c) + 1)
 - 7*a^3*cos(d*x + c)^2*log(-sin(d*x + c) + 1) + 2*(6*a^3*cos(d*x + c) + a
^3)*sin(d*x + c))/(d*cos(d*x + c)^2)
```

3.23.6 Sympy [F]

$$\int (a+a\sec(c+dx))^3\,dx = a^3\left(\int 1\,dx+\int 3\sec(c+dx)\,dx+\int 3\sec^2(c+dx)\,dx+\int \sec^3(c+dx)\,dx\right)$$

input
```
integrate((a+a*sec(d*x+c))**3,x)
```

output
```
a**3*(Integral(1, x) + Integral(3*sec(c + d*x), x) + Integral(3*sec(c + d*
x)**2, x) + Integral(sec(c + d*x)**3, x))
```

3.23.7 Maxima [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.38

$$\int (a+a\sec(c+dx))^3\,dx = a^3x-\frac{a^3\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1}-\log(\sin(dx+c)+1)+\log(\sin(dx+c)-1)\right)}{4\,d}+\frac{3\,a^3\log(\sec(dx+c)+\tan(dx+c))}{d}+\frac{3\,a^3\tan(dx+c)}{d}$$

input
```
integrate((a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
a^3*x - 1/4*a^3*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) +
1) + log(sin(d*x + c) - 1))/d + 3*a^3*log(sec(d*x + c) + tan(d*x + c))/d +
 3*a^3*tan(d*x + c)/d
```

3.23.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.52

$$\int (a+a\sec(c+dx))^3\,dx$$
$$= \frac{2\,(dx+c)a^3 + 7\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right) - 7\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right) - \frac{2\left(5\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-7\,a^3\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2\right.}}{2\,d}$$

input
```
integrate((a+a*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
1/2*(2*(d*x + c)*a^3 + 7*a^3*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 7*a^3*lo
g(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(5*a^3*tan(1/2*d*x + 1/2*c)^3 - 7*a^3
*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^2)/d
```

3.23.9 Mupad [B] (verification not implemented)

Time = 13.47 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.33

$$\int (a+a\sec(c+dx))^3\,dx = a^3\,x + \frac{7\,a^3\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d} - \frac{5\,a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3 - 7\,a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4 - 2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)}$$

input
```
int((a + a/cos(c + d*x))^3,x)
```

output
```
a^3*x + (7*a^3*atanh(tan(c/2 + (d*x)/2)))/d - (5*a^3*tan(c/2 + (d*x)/2)^3
- 7*a^3*tan(c/2 + (d*x)/2))/(d*(tan(c/2 + (d*x)/2)^4 - 2*tan(c/2 + (d*x)/2
)^2 + 1))
```

3.24 $\int \cos(c+dx)(a+a\sec(c+dx))^3\,dx$

3.24.1 Optimal result . . . 440
3.24.2 Mathematica [A] (verified) . . . 440
3.24.3 Rubi [A] (verified) . . . 441
3.24.4 Maple [A] (verified) . . . 442
3.24.5 Fricas [A] (verification not implemented) . . . 442
3.24.6 Sympy [F] . . . 443
3.24.7 Maxima [A] (verification not implemented) . . . 443
3.24.8 Giac [A] (verification not implemented) . . . 444
3.24.9 Mupad [B] (verification not implemented) . . . 444

3.24.1 Optimal result

Integrand size = 19, antiderivative size = 48

$$\int \cos(c+dx)(a+a\sec(c+dx))^3\,dx = 3a^3x+\frac{3a^3\text{arctanh}(\sin(c+dx))}{d}+\frac{a^3\sin(c+dx)}{d}+\frac{a^3\tan(c+dx)}{d}$$

output
```
3*a^3*x+3*a^3*arctanh(sin(d*x+c))/d+a^3*sin(d*x+c)/d+a^3*tan(d*x+c)/d
```

3.24.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.69

$$\int \cos(c+dx)(a+a\sec(c+dx))^3\,dx = \frac{a^3(3dx+3\text{arctanh}(\sin(c+dx))+\sin(c+dx)+\tan(c+dx))}{d}$$

input
```
Integrate[Cos[c + d*x]*(a + a*Sec[c + d*x])^3,x]
```

output
```
(a^3*(3*d*x + 3*ArcTanh[Sin[c + d*x]] + Sin[c + d*x] + Tan[c + d*x]))/d
```

3.24.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {3042, 4278, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(c+dx)(a\sec(c+dx)+a)^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}{\csc\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{4278}$$

$$\int \left(a^3\cos(c+dx)+a^3\sec^2(c+dx)+3a^3\sec(c+dx)+3a^3\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{3a^3\text{arctanh}(\sin(c+dx))}{d}+\frac{a^3\sin(c+dx)}{d}+\frac{a^3\tan(c+dx)}{d}+3a^3x$$

input
```
Int[Cos[c + d*x]*(a + a*Sec[c + d*x])^3,x]
```

output
```
3*a^3*x + (3*a^3*ArcTanh[Sin[c + d*x]])/d + (a^3*Sin[c + d*x])/d + (a^3*Ta
n[c + d*x])/d
```

3.24.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4278
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

3.24.4 Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.15

method	result
derivativedivides	$\frac{a^3\tan(dx+c)+3a^3\ln(\sec(dx+c)+\tan(dx+c))+3a^3(dx+c)+a^3\sin(dx+c)}{d}$
default	$\frac{a^3\tan(dx+c)+3a^3\ln(\sec(dx+c)+\tan(dx+c))+3a^3(dx+c)+a^3\sin(dx+c)}{d}$
parallelrisch	$\frac{a^3\left(6dx\cos(dx+c)-6\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\cos(dx+c)+6\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)\cos(dx+c)+2\sin(dx+c)+\sin(2dx+2c)\right)}{2d\cos(dx+c)}$
risch	$3a^3x-\frac{ia^3e^{i(dx+c)}}{2d}+\frac{ia^3e^{-i(dx+c)}}{2d}+\frac{2ia^3}{d(e^{2i(dx+c)}+1)}-\frac{3a^3\ln(e^{i(dx+c)}-i)}{d}+\frac{3a^3\ln(e^{i(dx+c)}+i)}{d}$
norman	$\frac{3a^3x+\frac{4a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}-\frac{4a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{d}-3a^3x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2-3a^3x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4+3a^3x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^2}-\frac{3a^3\ln(\tan}{}$

input
```
int(cos(d*x+c)*(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^3*tan(d*x+c)+3*a^3*ln(sec(d*x+c)+tan(d*x+c))+3*a^3*(d*x+c)+a^3*sin(
d*x+c))
```

3.24.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.90

$$\int \cos(c+dx)(a+a\sec(c+dx))^3\,dx$$
$$=\frac{6\,a^3dx\cos(dx+c)+3\,a^3\cos(dx+c)\log(\sin(dx+c)+1)-3\,a^3\cos(dx+c)\log(-\sin(dx+c)+1)+2}{2\,d\cos(dx+c)}$$

input
```
integrate(cos(d*x+c)*(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/2*(6*a^3*d*x*cos(d*x + c) + 3*a^3*cos(d*x + c)*log(sin(d*x + c) + 1) - 3
*a^3*cos(d*x + c)*log(-sin(d*x + c) + 1) + 2*(a^3*cos(d*x + c) + a^3)*sin(
d*x + c))/(d*cos(d*x + c))
```

3.24.6 Sympy [F]

$$\int \cos(c+dx)(a+a\sec(c+dx))^3\,dx = a^3\left(\int 3\cos(c+dx)\sec(c+dx)\,dx + \int 3\cos(c+dx)\sec^2(c+dx)\,dx + \int \cos(c+dx)\sec^3(c+dx)\,dx + \int \cos(c+dx)\,dx\right)$$

input
```
integrate(cos(d*x+c)*(a+a*sec(d*x+c))**3,x)
```

output
```
a**3*(Integral(3*cos(c + d*x)*sec(c + d*x), x) + Integral(3*cos(c + d*x)*s
ec(c + d*x)**2, x) + Integral(cos(c + d*x)*sec(c + d*x)**3, x) + Integral(
cos(c + d*x), x))
```

3.24.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.33

$$\int \cos(c+dx)(a+a\sec(c+dx))^3\,dx = \frac{6\,(dx+c)a^3 + 3\,a^3(\log(\sin(dx+c)+1) - \log(\sin(dx+c)-1)) + 2\,a^3\sin(dx+c) + 2\,a^3\tan(dx+c)}{2\,d}$$

input
```
integrate(cos(d*x+c)*(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/2*(6*(d*x + c)*a^3 + 3*a^3*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1
)) + 2*a^3*sin(d*x + c) + 2*a^3*tan(d*x + c))/d
```

3.24.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.67

$$\int \cos(c+dx)(a+a\sec(c+dx))^3\,dx = \frac{3\,(dx+c)a^3+3\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{4\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-1}}{d}$$

input
```
integrate(cos(d*x+c)*(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
(3*(d*x + c)*a^3 + 3*a^3*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*a^3*log(ab
s(tan(1/2*d*x + 1/2*c) - 1)) - 4*a^3*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1
/2*c)^4 - 1))/d
```

3.24.9 Mupad [B] (verification not implemented)

Time = 13.23 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.19

$$\int \cos(c+dx)(a+a\sec(c+dx))^3\,dx = 3\,a^3\,x+\frac{6\,a^3\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{4\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-1\right)}$$

input
```
int(cos(c + d*x)*(a + a/cos(c + d*x))^3,x)
```

output
```
3*a^3*x + (6*a^3*atanh(tan(c/2 + (d*x)/2)))/d - (4*a^3*tan(c/2 + (d*x)/2))
/(d*(tan(c/2 + (d*x)/2)^4 - 1))
```

3.25 $\int \cos^2(c+dx)(a+a\sec(c+dx))^3\,dx$

3.25.1 Optimal result . . . 445
3.25.2 Mathematica [A] (verified) . . . 445
3.25.3 Rubi [A] (verified) . . . 446
3.25.4 Maple [A] (verified) . . . 447
3.25.5 Fricas [A] (verification not implemented) . . . 447
3.25.6 Sympy [F] . . . 448
3.25.7 Maxima [A] (verification not implemented) . . . 448
3.25.8 Giac [A] (verification not implemented) . . . 449
3.25.9 Mupad [B] (verification not implemented) . . . 449

3.25.1 Optimal result

Integrand size = 21, antiderivative size = 59

$$\int \cos^2(c+dx)(a+a\sec(c+dx))^3\,dx = \frac{7a^3x}{2}+\frac{a^3\text{arctanh}(\sin(c+dx))}{d}+\frac{3a^3\sin(c+dx)}{d}+\frac{a^3\cos(c+dx)\sin(c+dx)}{2d}$$

output
```
7/2*a^3*x+a^3*arctanh(sin(d*x+c))/d+3*a^3*sin(d*x+c)/d+1/2*a^3*cos(d*x+c)*
sin(d*x+c)/d
```

3.25.2 Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.37

$$\int \cos^2(c+dx)(a+a\sec(c+dx))^3\,dx = \frac{a^3\left(14dx-4\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+4\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)+12\sin(\right.}{4d}$$

input
```
Integrate[Cos[c + d*x]^2*(a + a*Sec[c + d*x])^3,x]
```

output
```
(a^3*(14*d*x - 4*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 4*Log[Cos[(c +
 d*x)/2] + Sin[(c + d*x)/2]] + 12*Sin[c + d*x] + Sin[2*(c + d*x)]))/(4*d)
```

3.25.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3042, 4278, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(c+dx)(a\sec(c+dx)+a)^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}{\csc\left(c+dx+\frac{\pi}{2}\right)^2}dx$$

$$\Big\downarrow \text{4278}$$

$$\int \left(a^3\cos^2(c+dx)+3a^3\cos(c+dx)+a^3\sec(c+dx)+3a^3\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^3\text{arctanh}(\sin(c+dx))}{d}+\frac{3a^3\sin(c+dx)}{d}+\frac{a^3\sin(c+dx)\cos(c+dx)}{2d}+\frac{7a^3x}{2}$$

input
```
Int[Cos[c + d*x]^2*(a + a*Sec[c + d*x])^3,x]
```

output
```
(7*a^3*x)/2 + (a^3*ArcTanh[Sin[c + d*x]])/d + (3*a^3*Sin[c + d*x])/d + (a^
3*Cos[c + d*x]*Sin[c + d*x])/(2*d)
```

3.25.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4278
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

3.25.4 Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00

method	result
parallelrisch	$\frac{a^3\left(14dx+12\sin(dx+c)+4\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-4\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+\sin(2dx+2c)\right)}{4d}$
derivativedivides	$\frac{a^3\ln(\sec(dx+c)+\tan(dx+c))+3a^3(dx+c)+3a^3\sin(dx+c)+a^3\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}$
default	$\frac{a^3\ln(\sec(dx+c)+\tan(dx+c))+3a^3(dx+c)+3a^3\sin(dx+c)+a^3\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}$
risch	$\frac{7a^3x}{2}-\frac{3ia^3e^{i(dx+c)}}{2d}+\frac{3ia^3e^{-i(dx+c)}}{2d}+\frac{a^3\ln\left(e^{i(dx+c)}+i\right)}{d}-\frac{a^3\ln\left(e^{i(dx+c)}-i\right)}{d}+\frac{a^3\sin(2dx+2c)}{4d}$
norman	$\frac{\frac{7a^3x}{2}+\frac{7a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}-\frac{9a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{d}-\frac{3a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{d}+\frac{5a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{d}-7a^3x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\frac{7a^3x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8}{2}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^2\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^2}$

input
```
int(cos(d*x+c)^2*(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/4*a^3*(14*d*x+12*sin(d*x+c)+4*ln(tan(1/2*d*x+1/2*c)+1)-4*ln(tan(1/2*d*x+
1/2*c)-1)+sin(2*d*x+2*c))/d
```

3.25.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.10

$$\int \cos^2(c+dx)(a+a\sec(c+dx))^3\,dx$$
$$=\frac{7\,a^3dx+a^3\log\left(\sin\left(dx+c\right)+1\right)-a^3\log\left(-\sin\left(dx+c\right)+1\right)+\left(a^3\cos\left(dx+c\right)+6\,a^3\right)\sin\left(dx+c\right)}{2\,d}$$

input
```
integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/2*(7*a^3*d*x + a^3*log(sin(d*x + c) + 1) - a^3*log(-sin(d*x + c) + 1) +
(a^3*cos(d*x + c) + 6*a^3)*sin(d*x + c))/d
```

3.25.6 Sympy [F]

$$\int \cos^2(c+dx)(a+a\sec(c+dx))^3\,dx = a^3\left(\int 3\cos^2(c+dx)\sec(c+dx)\,dx + \int 3\cos^2(c+dx)\sec^2(c+dx)\,dx + \int \cos^2(c+dx)\sec^3(c+dx)\,dx + \int \cos^2(c+dx)\,dx\right)$$

input
```
integrate(cos(d*x+c)**2*(a+a*sec(d*x+c))**3,x)
```

output
```
a**3*(Integral(3*cos(c + d*x)**2*sec(c + d*x), x) + Integral(3*cos(c + d*x
)**2*sec(c + d*x)**2, x) + Integral(cos(c + d*x)**2*sec(c + d*x)**3, x) +
Integral(cos(c + d*x)**2, x))
```

3.25.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.25

$$\int \cos^2(c+dx)(a+a\sec(c+dx))^3\,dx = \frac{(2\,dx+2\,c+\sin(2\,dx+2\,c))a^3+12\,(dx+c)a^3+2\,a^3(\log(\sin(dx+c)+1)-\log(\sin(dx+c)-1))+1}{4\,d}$$

input
```
integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/4*((2*d*x + 2*c + sin(2*d*x + 2*c))*a^3 + 12*(d*x + c)*a^3 + 2*a^3*(log(
sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 12*a^3*sin(d*x + c))/d
```

3.25.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.69

$$\int \cos^2(c+dx)(a+a\sec(c+dx))^3\,dx$$
$$= \frac{7\,(dx+c)a^3 + 2\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right) - 2\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right) + \frac{2\left(5\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+7\,a^3\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2\right.}}{2\,d}$$

input
```
integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
1/2*(7*(d*x + c)*a^3 + 2*a^3*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 2*a^3*lo
g(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(5*a^3*tan(1/2*d*x + 1/2*c)^3 + 7*a^3
*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^2)/d
```

3.25.9 Mupad [B] (verification not implemented)

Time = 13.28 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.49

$$\int \cos^2(c+dx)(a+a\sec(c+dx))^3\,dx = \frac{7\,a^3\,x}{2} + \frac{2\,a^3\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d} + \frac{5\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3 + 7\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4 + 2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2 + 1\right)}$$

input
```
int(cos(c + d*x)^2*(a + a/cos(c + d*x))^3,x)
```

output
```
(7*a^3*x)/2 + (2*a^3*atanh(tan(c/2 + (d*x)/2)))/d + (5*a^3*tan(c/2 + (d*x)
/2)^3 + 7*a^3*tan(c/2 + (d*x)/2))/(d*(2*tan(c/2 + (d*x)/2)^2 + tan(c/2 + (
d*x)/2)^4 + 1))
```

3.26 $\int \cos^3(c+dx)(a+a\sec(c+dx))^3\,dx$

3.26.1 Optimal result . . . 450
3.26.2 Mathematica [A] (verified) . . . 450
3.26.3 Rubi [A] (verified) . . . 451
3.26.4 Maple [A] (verified) . . . 452
3.26.5 Fricas [A] (verification not implemented) . . . 452
3.26.6 Sympy [F] . . . 453
3.26.7 Maxima [A] (verification not implemented) . . . 453
3.26.8 Giac [A] (verification not implemented) . . . 454
3.26.9 Mupad [B] (verification not implemented) . . . 454

3.26.1 Optimal result

Integrand size = 21, antiderivative size = 63

$$\int \cos^3(c+dx)(a+a\sec(c+dx))^3\,dx = \frac{5a^3x}{2}+\frac{4a^3\sin(c+dx)}{d}+\frac{3a^3\cos(c+dx)\sin(c+dx)}{2d}-\frac{a^3\sin^3(c+dx)}{3d}$$

output
```
5/2*a^3*x+4*a^3*sin(d*x+c)/d+3/2*a^3*cos(d*x+c)*sin(d*x+c)/d-1/3*a^3*sin(d
*x+c)^3/d
```

3.26.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.70

$$\int \cos^3(c+dx)(a+a\sec(c+dx))^3\,dx = \frac{a^3(30c+30dx+45\sin(c+dx)+9\sin(2(c+dx))+\sin(3(c+dx)))}{12d}$$

input
```
Integrate[Cos[c + d*x]^3*(a + a*Sec[c + d*x])^3,x]
```

output
```
(a^3*(30*c + 30*d*x + 45*Sin[c + d*x] + 9*Sin[2*(c + d*x)] + Sin[3*(c + d*
x)]))/(12*d)
```

3.26.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3042, 4278, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^3(c+dx)(a\sec(c+dx)+a)^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}{\csc\left(c+dx+\frac{\pi}{2}\right)^3}dx$$

$$\Big\downarrow \text{4278}$$

$$\int \left(a^3\cos^3(c+dx)+3a^3\cos^2(c+dx)+3a^3\cos(c+dx)+a^3\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^3\sin^3(c+dx)}{3d}+\frac{4a^3\sin(c+dx)}{d}+\frac{3a^3\sin(c+dx)\cos(c+dx)}{2d}+\frac{5a^3x}{2}$$

input
```
Int[Cos[c + d*x]^3*(a + a*Sec[c + d*x])^3,x]
```

output
```
(5*a^3*x)/2 + (4*a^3*Sin[c + d*x])/d + (3*a^3*Cos[c + d*x]*Sin[c + d*x])/(
2*d) - (a^3*Sin[c + d*x]^3)/(3*d)
```

3.26.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4278
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

3.26.4 Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.67

method	result
parallelrisch	$\frac{a^3(30dx+\sin(3dx+3c)+9\sin(2dx+2c)+45\sin(dx+c))}{12d}$
risch	$\frac{5a^3x}{2}+\frac{15a^3\sin(dx+c)}{4d}+\frac{a^3\sin(3dx+3c)}{12d}+\frac{3a^3\sin(2dx+2c)}{4d}$
derivativedivides	$\frac{a^3(dx+c)+3a^3\sin(dx+c)+3a^3\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+\frac{a^3\left(2+\cos(dx+c)^2\right)\sin(dx+c)}{3}}{d}$
default	$\frac{a^3(dx+c)+3a^3\sin(dx+c)+3a^3\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+\frac{a^3\left(2+\cos(dx+c)^2\right)\sin(dx+c)}{3}}{d}$
norman	$\frac{\frac{5a^3x}{2}+\frac{11a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}-\frac{26a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3d}-\frac{32a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{3d}+\frac{10a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{3d}+\frac{5a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^9}{d}+\frac{5a^3x\tan\left(\frac{dx}{2}+\frac{c}{2}\right.}{2}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^3\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}$

input
```
int(cos(d*x+c)^3*(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/12*a^3*(30*d*x+sin(3*d*x+3*c)+9*sin(2*d*x+2*c)+45*sin(d*x+c))/d
```

3.26.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.79

$$\int \cos^3(c+dx)(a+a\sec(c+dx))^3\,dx = \frac{15\,a^3dx+\left(2\,a^3\cos\left(dx+c\right)^2+9\,a^3\cos\left(dx+c\right)+22\,a^3\right)\sin\left(dx+c\right)}{6\,d}$$

input
```
integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/6*(15*a^3*d*x + (2*a^3*cos(d*x + c)^2 + 9*a^3*cos(d*x + c) + 22*a^3)*sin
(d*x + c))/d
```

3.26.6 Sympy [F]

$$\int \cos^3(c+dx)(a+a\sec(c+dx))^3\,dx = a^3\left(\int 3\cos^3(c+dx)\sec(c+dx)\,dx + \int 3\cos^3(c+dx)\sec^2(c+dx)\,dx + \int \cos^3(c+dx)\sec^3(c+dx)\,dx + \int \cos^3(c+dx)\,dx\right)$$

input
```
integrate(cos(d*x+c)**3*(a+a*sec(d*x+c))**3,x)
```

output
```
a**3*(Integral(3*cos(c + d*x)**3*sec(c + d*x), x) + Integral(3*cos(c + d*x
)**3*sec(c + d*x)**2, x) + Integral(cos(c + d*x)**3*sec(c + d*x)**3, x) +
Integral(cos(c + d*x)**3, x))
```

3.26.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.13

$$\int \cos^3(c+dx)(a+a\sec(c+dx))^3\,dx = -\frac{4\left(\sin(dx+c)^3-3\sin(dx+c)\right)a^3-9\left(2\,dx+2\,c+\sin(2\,dx+2\,c)\right)a^3-12\,(dx+c)a^3-36\,a^3\sin(dx}{12\,d}$$

input
```
integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
-1/12*(4*(sin(d*x + c)^3 - 3*sin(d*x + c))*a^3 - 9*(2*d*x + 2*c + sin(2*d*
x + 2*c))*a^3 - 12*(d*x + c)*a^3 - 36*a^3*sin(d*x + c))/d
```

3.26.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.27

$$\int \cos^3(c+dx)(a+a\sec(c+dx))^3\,dx = \frac{15\,(dx+c)a^3 + \frac{2\left(15\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+40\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+33\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^3}}{6\,d}$$

input
```
integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
1/6*(15*(d*x + c)*a^3 + 2*(15*a^3*tan(1/2*d*x + 1/2*c)^5 + 40*a^3*tan(1/2*
d*x + 1/2*c)^3 + 33*a^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)
^3)/d
```

3.26.9 Mupad [B] (verification not implemented)

Time = 13.39 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00

$$\int \cos^3(c+dx)(a+a\sec(c+dx))^3\,dx = \frac{5\,a^3\,x}{2} + \frac{11\,a^3\,\sin(c+d\,x)}{3\,d} + \frac{a^3\cos(c+d\,x)^2\,\sin(c+d\,x)}{3\,d} + \frac{3\,a^3\,\cos(c+d\,x)\,\sin(c+d\,x)}{2\,d}$$

input
```
int(cos(c + d*x)^3*(a + a/cos(c + d*x))^3,x)
```

output
```
(5*a^3*x)/2 + (11*a^3*sin(c + d*x))/(3*d) + (a^3*cos(c + d*x)^2*sin(c + d*
x))/(3*d) + (3*a^3*cos(c + d*x)*sin(c + d*x))/(2*d)
```

3.27 $\int \cos^4(c+dx)(a+a\sec(c+dx))^3\,dx$

3.27.1 Optimal result . . . 455
3.27.2 Mathematica [A] (verified) . . . 455
3.27.3 Rubi [A] (verified) . . . 456
3.27.4 Maple [A] (verified) . . . 457
3.27.5 Fricas [A] (verification not implemented) . . . 457
3.27.6 Sympy [F] . . . 458
3.27.7 Maxima [A] (verification not implemented) . . . 458
3.27.8 Giac [A] (verification not implemented) . . . 459
3.27.9 Mupad [B] (verification not implemented) . . . 459

3.27.1 Optimal result

Integrand size = 21, antiderivative size = 85

$$\int \cos^4(c+dx)(a+a\sec(c+dx))^3\,dx = \frac{15a^3x}{8}+\frac{4a^3\sin(c+dx)}{d}+\frac{15a^3\cos(c+dx)\sin(c+dx)}{8d}+\frac{a^3\cos^3(c+dx)\sin(c+dx)}{4d}-\frac{a^3\sin^3(c+dx)}{d}$$

output
```
15/8*a^3*x+4*a^3*sin(d*x+c)/d+15/8*a^3*cos(d*x+c)*sin(d*x+c)/d+1/4*a^3*cos
(d*x+c)^3*sin(d*x+c)/d-a^3*sin(d*x+c)^3/d
```

3.27.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.60

$$\int \cos^4(c+dx)(a+a\sec(c+dx))^3\,dx = \frac{a^3(60dx+104\sin(c+dx)+32\sin(2(c+dx))+8\sin(3(c+dx))+\sin(4(c+dx)))}{32d}$$

input
```
Integrate[Cos[c + d*x]^4*(a + a*Sec[c + d*x])^3,x]
```

output
```
(a^3*(60*d*x + 104*Sin[c + d*x] + 32*Sin[2*(c + d*x)] + 8*Sin[3*(c + d*x)]
 + Sin[4*(c + d*x)]))/(32*d)
```

3.27.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3042, 4278, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^4(c+dx)(a\sec(c+dx)+a)^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}{\csc\left(c+dx+\frac{\pi}{2}\right)^4}dx$$

$$\Big\downarrow \text{4278}$$

$$\int \left(a^3\cos^4(c+dx)+3a^3\cos^3(c+dx)+3a^3\cos^2(c+dx)+a^3\cos(c+dx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^3\sin^3(c+dx)}{d}+\frac{4a^3\sin(c+dx)}{d}+\frac{a^3\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{15a^3\sin(c+dx)\cos(c+dx)}{8d}+\frac{15a^3x}{8}$$

input
```
Int[Cos[c + d*x]^4*(a + a*Sec[c + d*x])^3,x]
```

output
```
(15*a^3*x)/8 + (4*a^3*Sin[c + d*x])/d + (15*a^3*Cos[c + d*x]*Sin[c + d*x])
/(8*d) + (a^3*Cos[c + d*x]^3*Sin[c + d*x])/(4*d) - (a^3*Sin[c + d*x]^3)/d
```

3.27.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4278
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

3.27.4 Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.62

method	result
parallelrisch	$\frac{a^3(60dx+104\sin(dx+c)+\sin(4dx+4c)+8\sin(3dx+3c)+32\sin(2dx+2c))}{32d}$
risch	$\frac{15a^3x}{8}+\frac{13a^3\sin(dx+c)}{4d}+\frac{a^3\sin(4dx+4c)}{32d}+\frac{a^3\sin(3dx+3c)}{4d}+\frac{a^3\sin(2dx+2c)}{d}$
derivativedivides	$\frac{a^3\sin(dx+c)+3a^3\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+a^3\left(2+\cos(dx+c)^2\right)\sin(dx+c)+a^3\left(\frac{\left(\cos(dx+c)^3+\frac{3\cos(dx+c)}{2}\right)\sin(dx\cdot}{4}\right.}{d}$
default	$\frac{a^3\sin(dx+c)+3a^3\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+a^3\left(2+\cos(dx+c)^2\right)\sin(dx+c)+a^3\left(\frac{\left(\cos(dx+c)^3+\frac{3\cos(dx+c)}{2}\right)\sin(dx\cdot}{4}\right.}{d}$
norman	$\frac{\frac{15a^3x}{8}+\frac{49a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d}-\frac{25a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{4d}-\frac{21a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{2d}-\frac{11a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{2d}+\frac{25a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^9}{4d}+\frac{15a^3\tan\left(\frac{dx}{2}+\right.}{4d}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}$

input
```
int(cos(d*x+c)^4*(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/32*a^3*(60*d*x+104*sin(d*x+c)+sin(4*d*x+4*c)+8*sin(3*d*x+3*c)+32*sin(2*d
*x+2*c))/d
```

3.27.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.74

$$\int \cos^4(c+dx)(a+a\sec(c+dx))^3\,dx$$
$$=\frac{15\,a^3dx+\left(2\,a^3\cos\left(dx+c\right)^3+8\,a^3\cos\left(dx+c\right)^2+15\,a^3\cos\left(dx+c\right)+24\,a^3\right)\sin\left(dx+c\right)}{8\,d}$$

input
```
integrate(cos(d*x+c)^4*(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/8*(15*a^3*d*x + (2*a^3*cos(d*x + c)^3 + 8*a^3*cos(d*x + c)^2 + 15*a^3*co
s(d*x + c) + 24*a^3)*sin(d*x + c))/d
```

3.27.6 Sympy [F]

$$\int \cos^4(c+dx)(a+a\sec(c+dx))^3\,dx = a^3\left(\int 3\cos^4(c+dx)\sec(c+dx)\,dx + \int 3\cos^4(c+dx)\sec^2(c+dx)\,dx + \int \cos^4(c+dx)\sec^3(c+dx)\,dx + \int \cos^4(c+dx)\,dx\right)$$

input
```
integrate(cos(d*x+c)**4*(a+a*sec(d*x+c))**3,x)
```

output
```
a**3*(Integral(3*cos(c + d*x)**4*sec(c + d*x), x) + Integral(3*cos(c + d*x
)**4*sec(c + d*x)**2, x) + Integral(cos(c + d*x)**4*sec(c + d*x)**3, x) +
Integral(cos(c + d*x)**4, x))
```

3.27.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.11

$$\int \cos^4(c+dx)(a+a\sec(c+dx))^3\,dx = -\frac{32\left(\sin(dx+c)^3-3\sin(dx+c)\right)a^3-(12\,dx+12\,c+\sin(4\,dx+4\,c)+8\sin(2\,dx+2\,c))a^3-24\,(2\,d}{32\,d}$$

input
```
integrate(cos(d*x+c)^4*(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
-1/32*(32*(sin(d*x + c)^3 - 3*sin(d*x + c))*a^3 - (12*d*x + 12*c + sin(4*d
*x + 4*c) + 8*sin(2*d*x + 2*c))*a^3 - 24*(2*d*x + 2*c + sin(2*d*x + 2*c))*
a^3 - 32*a^3*sin(d*x + c))/d
```

3.27.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.13

$$\int \cos^4(c+dx)(a+a\sec(c+dx))^3\,dx = \frac{15\,(dx+c)a^3 + \frac{2\left(15\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+55\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+73\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+49\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^4}}{8\,d}$$

input
```
integrate(cos(d*x+c)^4*(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
1/8*(15*(d*x + c)*a^3 + 2*(15*a^3*tan(1/2*d*x + 1/2*c)^7 + 55*a^3*tan(1/2*
d*x + 1/2*c)^5 + 73*a^3*tan(1/2*d*x + 1/2*c)^3 + 49*a^3*tan(1/2*d*x + 1/2*
c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^4)/d
```

3.27.9 Mupad [B] (verification not implemented)

Time = 16.72 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.05

$$\int \cos^4(c+dx)(a+a\sec(c+dx))^3\,dx = \frac{15\,a^3\,x}{8} + \frac{\frac{15\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{4} + \frac{55\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{4} + \frac{73\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{4} + \frac{49\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)^4}$$

input
```
int(cos(c + d*x)^4*(a + a/cos(c + d*x))^3,x)
```

output
```
(15*a^3*x)/8 + ((73*a^3*tan(c/2 + (d*x)/2)^3)/4 + (55*a^3*tan(c/2 + (d*x)/
2)^5)/4 + (15*a^3*tan(c/2 + (d*x)/2)^7)/4 + (49*a^3*tan(c/2 + (d*x)/2))/4)
/(d*(tan(c/2 + (d*x)/2)^2 + 1)^4)
```

3.28 $\int \cos^5(c+dx)(a+a\sec(c+dx))^3\,dx$

3.28.1 Optimal result . . . 460
3.28.2 Mathematica [A] (verified) . . . 460
3.28.3 Rubi [A] (verified) . . . 461
3.28.4 Maple [A] (verified) . . . 462
3.28.5 Fricas [A] (verification not implemented) . . . 463
3.28.6 Sympy [F(-1)] . . . 463
3.28.7 Maxima [A] (verification not implemented) . . . 463
3.28.8 Giac [A] (verification not implemented) . . . 464
3.28.9 Mupad [B] (verification not implemented) . . . 464

3.28.1 Optimal result

Integrand size = 21, antiderivative size = 105

$$\int \cos^5(c+dx)(a+a\sec(c+dx))^3\,dx = \frac{13a^3x}{8}+\frac{4a^3\sin(c+dx)}{d}+\frac{13a^3\cos(c+dx)\sin(c+dx)}{8d}+\frac{3a^3\cos^3(c+dx)\sin(c+dx)}{4d}-\frac{5a^3\sin^3(c+dx)}{3d}+\frac{a^3\sin^5(c+dx)}{5d}$$

output
```
13/8*a^3*x+4*a^3*sin(d*x+c)/d+13/8*a^3*cos(d*x+c)*sin(d*x+c)/d+3/4*a^3*cos
(d*x+c)^3*sin(d*x+c)/d-5/3*a^3*sin(d*x+c)^3/d+1/5*a^3*sin(d*x+c)^5/d
```

3.28.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.60

$$\int \cos^5(c+dx)(a+a\sec(c+dx))^3\,dx = \frac{a^3(780dx+1380\sin(c+dx)+480\sin(2(c+dx))+170\sin(3(c+dx))+45\sin(4(c+dx))+6\sin(5(c+}{480d}$$

input
```
Integrate[Cos[c + d*x]^5*(a + a*Sec[c + d*x])^3,x]
```

output
```
(a^3*(780*d*x + 1380*Sin[c + d*x] + 480*Sin[2*(c + d*x)] + 170*Sin[3*(c +
d*x)] + 45*Sin[4*(c + d*x)] + 6*Sin[5*(c + d*x)]))/(480*d)
```

3.28.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3042, 4278, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^5(c+dx)(a\sec(c+dx)+a)^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}{\csc\left(c+dx+\frac{\pi}{2}\right)^5}dx$$

$$\Big\downarrow \text{4278}$$

$$\int \left(a^3\cos^5(c+dx)+3a^3\cos^4(c+dx)+3a^3\cos^3(c+dx)+a^3\cos^2(c+dx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^3\sin^5(c+dx)}{5d}-\frac{5a^3\sin^3(c+dx)}{3d}+\frac{4a^3\sin(c+dx)}{d}+\frac{3a^3\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{13a^3\sin(c+dx)\cos(c+dx)}{8d}+\frac{13a^3x}{8}$$

input
```
Int[Cos[c + d*x]^5*(a + a*Sec[c + d*x])^3,x]
```

output
```
(13*a^3*x)/8 + (4*a^3*Sin[c + d*x])/d + (13*a^3*Cos[c + d*x]*Sin[c + d*x])
/(8*d) + (3*a^3*Cos[c + d*x]^3*Sin[c + d*x])/(4*d) - (5*a^3*Sin[c + d*x]^3
)/(3*d) + (a^3*Sin[c + d*x]^5)/(5*d)
```

3.28.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4278
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

3.28.4 Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.63

method	result
parallelrisch	$\frac{a^3(780dx+6\sin(5dx+5c)+45\sin(4dx+4c)+170\sin(3dx+3c)+480\sin(2dx+2c)+1380\sin(dx+c))}{480d}$
risch	$\frac{13a^3x}{8}+\frac{23a^3\sin(dx+c)}{8d}+\frac{a^3\sin(5dx+5c)}{80d}+\frac{3a^3\sin(4dx+4c)}{32d}+\frac{17a^3\sin(3dx+3c)}{48d}+\frac{a^3\sin(2dx+2c)}{d}$
derivativedivides	$\frac{a^3\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+a^3\left(2+\cos(dx+c)^2\right)\sin(dx+c)+3a^3\left(\frac{\left(\cos(dx+c)^3+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)}{d}$
default	$\frac{a^3\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+a^3\left(2+\cos(dx+c)^2\right)\sin(dx+c)+3a^3\left(\frac{\left(\cos(dx+c)^3+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)}{d}$
norman	$\frac{13a^3x}{8}+\frac{51a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d}-\frac{10a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3d}-\frac{77a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{20d}-\frac{272a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{15d}+\frac{13a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^9}{20d}+\frac{26a^3\tan\left(\frac{dx}{2}+\right.}{3d}$

input
```
int(cos(d*x+c)^5*(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/480*a^3*(780*d*x+6*sin(5*d*x+5*c)+45*sin(4*d*x+4*c)+170*sin(3*d*x+3*c)+4
80*sin(2*d*x+2*c)+1380*sin(d*x+c))/d
```

3.28.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.72

$$\int \cos^5(c+dx)(a+a\sec(c+dx))^3\,dx$$
$$= \frac{195\,a^3dx + \left(24\,a^3\cos\left(dx+c\right)^4 + 90\,a^3\cos\left(dx+c\right)^3 + 152\,a^3\cos\left(dx+c\right)^2 + 195\,a^3\cos\left(dx+c\right) + 304\,a^3\right.}{120\,d}$$

input
```
integrate(cos(d*x+c)^5*(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/120*(195*a^3*d*x + (24*a^3*cos(d*x + c)^4 + 90*a^3*cos(d*x + c)^3 + 152*
a^3*cos(d*x + c)^2 + 195*a^3*cos(d*x + c) + 304*a^3)*sin(d*x + c))/d
```

3.28.6 Sympy [F(-1)]

Timed out.

$$\int \cos^5(c+dx)(a+a\sec(c+dx))^3\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**5*(a+a*sec(d*x+c))**3,x)
```

output
```
Timed out
```

3.28.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.11

$$\int \cos^5(c+dx)(a+a\sec(c+dx))^3\,dx$$
$$= \frac{32\left(3\,\sin\left(dx+c\right)^5 - 10\,\sin\left(dx+c\right)^3 + 15\,\sin\left(dx+c\right)\right)a^3 - 480\left(\sin\left(dx+c\right)^3 - 3\,\sin\left(dx+c\right)\right)a^3 + 45\,(}{480\,d}$$

input
```
integrate(cos(d*x+c)^5*(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/480*(32*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*a^3 - 4
80*(sin(d*x + c)^3 - 3*sin(d*x + c))*a^3 + 45*(12*d*x + 12*c + sin(4*d*x +
 4*c) + 8*sin(2*d*x + 2*c))*a^3 + 120*(2*d*x + 2*c + sin(2*d*x + 2*c))*a^3
)/d
```

3.28.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.07

$$\int \cos^5(c+dx)(a+a\sec(c+dx))^3\,dx$$
$$= \frac{195\,(dx+c)a^3 + \frac{2\left(195\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9+910\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+1664\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+1330\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+765\,a^3\tan\left(\frac{1}{2}\,dx\right.\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^5}}{120\,d}$$

input
```
integrate(cos(d*x+c)^5*(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
1/120*(195*(d*x + c)*a^3 + 2*(195*a^3*tan(1/2*d*x + 1/2*c)^9 + 910*a^3*tan
(1/2*d*x + 1/2*c)^7 + 1664*a^3*tan(1/2*d*x + 1/2*c)^5 + 1330*a^3*tan(1/2*d
*x + 1/2*c)^3 + 765*a^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)
^5)/d
```

3.28.9 Mupad [B] (verification not implemented)

Time = 17.13 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00

$$\int \cos^5(c+dx)(a+a\sec(c+dx))^3\,dx$$
$$= \frac{13\,a^3\,x}{8} + \frac{\frac{13\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9}{4} + \frac{91\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{6} + \frac{416\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{15} + \frac{133\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{6} + \frac{51\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)^5}$$

input
```
int(cos(c + d*x)^5*(a + a/cos(c + d*x))^3,x)
```

output
```
(13*a^3*x)/8 + ((133*a^3*tan(c/2 + (d*x)/2)^3)/6 + (416*a^3*tan(c/2 + (d*x
)/2)^5)/15 + (91*a^3*tan(c/2 + (d*x)/2)^7)/6 + (13*a^3*tan(c/2 + (d*x)/2)^
9)/4 + (51*a^3*tan(c/2 + (d*x)/2))/4)/(d*(tan(c/2 + (d*x)/2)^2 + 1)^5)
```

3.29 $\int \cos^6(c+dx)(a+a\sec(c+dx))^3\,dx$

3.29.1 Optimal result . . . 465
3.29.2 Mathematica [A] (verified) . . . 466
3.29.3 Rubi [A] (verified) . . . 466
3.29.4 Maple [A] (verified) . . . 467
3.29.5 Fricas [A] (verification not implemented) . . . 468
3.29.6 Sympy [F(-1)] . . . 468
3.29.7 Maxima [A] (verification not implemented) . . . 469
3.29.8 Giac [A] (verification not implemented) . . . 469
3.29.9 Mupad [B] (verification not implemented) . . . 470

3.29.1 Optimal result

Integrand size = 21, antiderivative size = 129

$$\int \cos^6(c+dx)(a+a\sec(c+dx))^3\,dx = \frac{23a^3x}{16}+\frac{4a^3\sin(c+dx)}{d}+\frac{23a^3\cos(c+dx)\sin(c+dx)}{16d}+\frac{23a^3\cos^3(c+dx)\sin(c+dx)}{24d}+\frac{a^3\cos^5(c+dx)\sin(c+dx)}{6d}-\frac{7a^3\sin^3(c+dx)}{3d}+\frac{3a^3\sin^5(c+dx)}{5d}$$

output
```
23/16*a^3*x+4*a^3*sin(d*x+c)/d+23/16*a^3*cos(d*x+c)*sin(d*x+c)/d+23/24*a^3
*cos(d*x+c)^3*sin(d*x+c)/d+1/6*a^3*cos(d*x+c)^5*sin(d*x+c)/d-7/3*a^3*sin(d
*x+c)^3/d+3/5*a^3*sin(d*x+c)^5/d
```

3.29.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.57

$$\int \cos^6(c+dx)(a+a\sec(c+dx))^3\,dx$$
$$= \frac{a^3(1380dx+2520\sin(c+dx)+945\sin(2(c+dx))+380\sin(3(c+dx))+135\sin(4(c+dx))+36\sin(5(c$$

input
```
Integrate[Cos[c + d*x]^6*(a + a*Sec[c + d*x])^3,x]
```

output
```
(a^3*(1380*d*x + 2520*Sin[c + d*x] + 945*Sin[2*(c + d*x)] + 380*Sin[3*(c +
 d*x)] + 135*Sin[4*(c + d*x)] + 36*Sin[5*(c + d*x)] + 5*Sin[6*(c + d*x)]))
/(960*d)
```

3.29.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3042, 4278, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^6(c+dx)(a\sec(c+dx)+a)^3\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}{\csc\left(c+dx+\frac{\pi}{2}\right)^6}dx$$
$$\Big\downarrow \text{4278}$$
$$\int \left(a^3\cos^6(c+dx)+3a^3\cos^5(c+dx)+3a^3\cos^4(c+dx)+a^3\cos^3(c+dx)\right)dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{3a^3\sin^5(c+dx)}{5d}-\frac{7a^3\sin^3(c+dx)}{3d}+\frac{4a^3\sin(c+dx)}{d}+\frac{a^3\sin(c+dx)\cos^5(c+dx)}{6d}+\frac{23a^3\sin(c+dx)\cos^3(c+dx)}{24d}+\frac{23a^3\sin(c+dx)\cos(c+dx)}{16d}+\frac{23a^3x}{16}$$

input
```
Int[Cos[c + d*x]^6*(a + a*Sec[c + d*x])^3,x]
```

output
```
(23*a^3*x)/16 + (4*a^3*Sin[c + d*x])/d + (23*a^3*Cos[c + d*x]*Sin[c + d*x]
)/(16*d) + (23*a^3*Cos[c + d*x]^3*Sin[c + d*x])/(24*d) + (a^3*Cos[c + d*x]
^5*Sin[c + d*x])/(6*d) - (7*a^3*Sin[c + d*x]^3)/(3*d) + (3*a^3*Sin[c + d*x
]^5)/(5*d)
```

3.29.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4278
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

3.29.4 Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.58

method	result
parallelrisch	$\frac{\left(276dx+\sin(6dx+6c)+504\sin(dx+c)+189\sin(2dx+2c)+76\sin(3dx+3c)+27\sin(4dx+4c)+\frac{36\sin(5dx+5c)}{5}\right)a^3}{192d}$
risch	$\frac{23a^3x}{16}+\frac{21a^3\sin(dx+c)}{8d}+\frac{a^3\sin(6dx+6c)}{192d}+\frac{3a^3\sin(5dx+5c)}{80d}+\frac{9a^3\sin(4dx+4c)}{64d}+\frac{19a^3\sin(3dx+3c)}{48d}+\frac{63a}{}$
derivativedivides	$\frac{\frac{a^3\left(2+\cos(dx+c)^2\right)\sin(dx+c)}{3}+3a^3\left(\frac{\left(\cos(dx+c)^3+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+\frac{3a^3\left(\frac{8}{3}+\cos(dx+c)^4+\frac{4\cos(dx+c)^2}{3}\right)s}{5}}{d}$
default	$\frac{\frac{a^3\left(2+\cos(dx+c)^2\right)\sin(dx+c)}{3}+3a^3\left(\frac{\left(\cos(dx+c)^3+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+\frac{3a^3\left(\frac{8}{3}+\cos(dx+c)^4+\frac{4\cos(dx+c)^2}{3}\right)s}{5}}{d}$
norman	$\frac{23a^3x}{16}+\frac{105a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8d}+\frac{a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{8d}+\frac{353a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{40d}-\frac{1303a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{40d}-\frac{1339a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^9}{120d}+\frac{989a^3\tan(}{12}$

input
```
int(cos(d*x+c)^6*(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/192*(276*d*x+sin(6*d*x+6*c)+504*sin(d*x+c)+189*sin(2*d*x+2*c)+76*sin(3*d
*x+3*c)+27*sin(4*d*x+4*c)+36/5*sin(5*d*x+5*c))*a^3/d
```

3.29.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.69

$$\int \cos^6(c+dx)(a+a\sec(c+dx))^3\,dx$$
$$= \frac{345\,a^3dx + \left(40\,a^3\cos\left(dx+c\right)^5 + 144\,a^3\cos\left(dx+c\right)^4 + 230\,a^3\cos\left(dx+c\right)^3 + 272\,a^3\cos\left(dx+c\right)^2 + 345\,a^3\right.}{240\,d}$$

input
```
integrate(cos(d*x+c)^6*(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/240*(345*a^3*d*x + (40*a^3*cos(d*x + c)^5 + 144*a^3*cos(d*x + c)^4 + 230
*a^3*cos(d*x + c)^3 + 272*a^3*cos(d*x + c)^2 + 345*a^3*cos(d*x + c) + 544*
a^3)*sin(d*x + c))/d
```

3.29.6 Sympy [F(-1)]

Timed out.

$$\int \cos^6(c+dx)(a+a\sec(c+dx))^3\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**6*(a+a*sec(d*x+c))**3,x)
```

output
```
Timed out
```

3.29.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.11

$$\int \cos^6(c+dx)(a+a\sec(c+dx))^3\,dx$$
$$= \frac{192\left(3\sin(dx+c)^5-10\sin(dx+c)^3+15\sin(dx+c)\right)a^3-5\left(4\sin(2dx+2c)^3-60dx-60c-9\sin\right.}{}$$

input
```
integrate(cos(d*x+c)^6*(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/960*(192*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*a^3 -
5*(4*sin(2*d*x + 2*c)^3 - 60*d*x - 60*c - 9*sin(4*d*x + 4*c) - 48*sin(2*d*
x + 2*c))*a^3 - 320*(sin(d*x + c)^3 - 3*sin(d*x + c))*a^3 + 90*(12*d*x + 1
2*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*a^3)/d
```

3.29.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.99

$$\int \cos^6(c+dx)(a+a\sec(c+dx))^3\,dx$$
$$= \frac{345\,(dx+c)a^3+\frac{2\left(345\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}+1955\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9+4554\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+5814\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+3165\,a^3\tan\left(\frac{1}{2}\right.\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^6}}{240\,d}$$

input
```
integrate(cos(d*x+c)^6*(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
1/240*(345*(d*x + c)*a^3 + 2*(345*a^3*tan(1/2*d*x + 1/2*c)^11 + 1955*a^3*t
an(1/2*d*x + 1/2*c)^9 + 4554*a^3*tan(1/2*d*x + 1/2*c)^7 + 5814*a^3*tan(1/2
*d*x + 1/2*c)^5 + 3165*a^3*tan(1/2*d*x + 1/2*c)^3 + 1575*a^3*tan(1/2*d*x +
 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^6)/d
```

3.29.9 Mupad [B] (verification not implemented)

Time = 15.43 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.94

$$\int \cos^6(c+dx)(a+a\sec(c+dx))^3\,dx = \frac{23\,a^3\,x}{16} + \frac{\frac{23\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{8} + \frac{391\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}}{24} + \frac{759\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}}{20} + \frac{969\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{5}}{20} + \frac{211\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}}{8} + \frac{105\,a^3\tan\left(\frac{c}{2}\right.}{8}}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)^6}$$

input
```
int(cos(c + d*x)^6*(a + a/cos(c + d*x))^3,x)
```

output
```
(23*a^3*x)/16 + ((211*a^3*tan(c/2 + (d*x)/2)^3)/8 + (969*a^3*tan(c/2 + (d*
x)/2)^5)/20 + (759*a^3*tan(c/2 + (d*x)/2)^7)/20 + (391*a^3*tan(c/2 + (d*x)
/2)^9)/24 + (23*a^3*tan(c/2 + (d*x)/2)^11)/8 + (105*a^3*tan(c/2 + (d*x)/2)
)/8)/(d*(tan(c/2 + (d*x)/2)^2 + 1)^6)
```

3.30 $\int \sec^3(c+dx)(a+a\sec(c+dx))^4\,dx$

3.30.1 Optimal result . . . 471
3.30.2 Mathematica [A] (verified) . . . 472
3.30.3 Rubi [A] (verified) . . . 472
3.30.4 Maple [A] (verified) . . . 474
3.30.5 Fricas [A] (verification not implemented) . . . 474
3.30.6 Sympy [F] . . . 475
3.30.7 Maxima [B] (verification not implemented) . . . 475
3.30.8 Giac [A] (verification not implemented) . . . 476
3.30.9 Mupad [B] (verification not implemented) . . . 476

3.30.1 Optimal result

Integrand size = 21, antiderivative size = 136

$$\begin{aligned}\int \sec^3(c+dx)(a+a\sec(c+dx))^4\,dx = {} & \frac{49a^4\operatorname{arctanh}(\sin(c+dx))}{16d}+\frac{8a^4\tan(c+dx)}{d} \\ & +\frac{49a^4\sec(c+dx)\tan(c+dx)}{16d} \\ & +\frac{41a^4\sec^3(c+dx)\tan(c+dx)}{24d} \\ & +\frac{a^4\sec^5(c+dx)\tan(c+dx)}{6d} \\ & +\frac{4a^4\tan^3(c+dx)}{d}+\frac{4a^4\tan^5(c+dx)}{5d}\end{aligned}$$

output
```
49/16*a^4*arctanh(sin(d*x+c))/d+8*a^4*tan(d*x+c)/d+49/16*a^4*sec(d*x+c)*ta
n(d*x+c)/d+41/24*a^4*sec(d*x+c)^3*tan(d*x+c)/d+1/6*a^4*sec(d*x+c)^5*tan(d*
x+c)/d+4*a^4*tan(d*x+c)^3/d+4/5*a^4*tan(d*x+c)^5/d
```

3.30.2 Mathematica [A] (verified)

Time = 5.05 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.07

$$\int \sec^3(c+dx)(a+a\sec(c+dx))^4\,dx = \frac{49a^4\text{arctanh}(\sin(c+dx))}{16d} + \frac{36a^4\tan(c+dx)}{5d} + \frac{49a^4\sec(c+dx)\tan(c+dx)}{16d} + \frac{41a^4\sec^3(c+dx)\tan(c+dx)}{24d} + \frac{4a^4\sec^4(c+dx)\tan(c+dx)}{5d} + \frac{a^4\sec^5(c+dx)\tan(c+dx)}{6d} + \frac{12a^4\tan^3(c+dx)}{5d}$$

input
```
Integrate[Sec[c + d*x]^3*(a + a*Sec[c + d*x])^4,x]
```

output
```
(49*a^4*ArcTanh[Sin[c + d*x]])/(16*d) + (36*a^4*Tan[c + d*x])/(5*d) + (49*
a^4*Sec[c + d*x]*Tan[c + d*x])/(16*d) + (41*a^4*Sec[c + d*x]^3*Tan[c + d*x
])/(24*d) + (4*a^4*Sec[c + d*x]^4*Tan[c + d*x])/(5*d) + (a^4*Sec[c + d*x]^
5*Tan[c + d*x])/(6*d) + (12*a^4*Tan[c + d*x]^3)/(5*d)
```

3.30.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3042, 4278, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^3(c+dx)(a\sec(c+dx)+a)^4\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^3\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^4dx$$

$$\Big\downarrow \text{4278}$$

$$\int \left(a^4\sec^7(c+dx)+4a^4\sec^6(c+dx)+6a^4\sec^5(c+dx)+4a^4\sec^4(c+dx)+a^4\sec^3(c+dx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{49a^4\text{arctanh}(\sin(c+dx))}{16d}+\frac{4a^4\tan^5(c+dx)}{5d}+\frac{4a^4\tan^3(c+dx)}{d}+\frac{8a^4\tan(c+dx)}{d}+\frac{a^4\tan(c+dx)\sec^5(c+dx)}{6d}+\frac{41a^4\tan(c+dx)\sec^3(c+dx)}{24d}+\frac{49a^4\tan(c+dx)\sec(c+dx)}{16d}$$

input
```
Int[Sec[c + d*x]^3*(a + a*Sec[c + d*x])^4,x]
```

output
```
(49*a^4*ArcTanh[Sin[c + d*x]])/(16*d) + (8*a^4*Tan[c + d*x])/d + (49*a^4*S
ec[c + d*x]*Tan[c + d*x])/(16*d) + (41*a^4*Sec[c + d*x]^3*Tan[c + d*x])/(2
4*d) + (a^4*Sec[c + d*x]^5*Tan[c + d*x])/(6*d) + (4*a^4*Tan[c + d*x]^3)/d
+ (4*a^4*Tan[c + d*x]^5)/(5*d)
```

3.30.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4278
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

3.30.4 Maple [A] (verified)

Time = 1.48 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.26

method	result
norman	$\frac{\frac{207a^4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8d}-\frac{1471a^4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{24d}+\frac{1967a^4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{20d}-\frac{1617a^4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{20d}+\frac{833a^4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^9}{24d}-\frac{49a^4\tan\left(\frac{dx}{2}+\right.}{8d}}{\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^6}$
risch	$-\frac{ia^4\left(735\,\mathrm{e}^{11i(dx+c)}+3845\,\mathrm{e}^{9i(dx+c)}-1920\,\mathrm{e}^{8i(dx+c)}+3750\,\mathrm{e}^{7i(dx+c)}-11520\,\mathrm{e}^{6i(dx+c)}-3750\,\mathrm{e}^{5i(dx+c)}-15360\,\mathrm{e}^{4i(dx+}\right.}{120d\left(\mathrm{e}^{2i(dx+c)}+1\right)^6}$
derivativedivides	$\frac{a^4\left(-\left(-\frac{\sec(dx+c)^5}{6}-\frac{5\sec(dx+c)^3}{24}-\frac{5\sec(dx+c)}{16}\right)\tan(dx+c)+\frac{5\ln(\sec(dx+c)+\tan(dx+c))}{16}\right)-4a^4\left(-\frac{8}{15}-\frac{\sec(dx+c)^4}{5}-\frac{4\sec}{}\right.}{}$
default	$\frac{a^4\left(-\left(-\frac{\sec(dx+c)^5}{6}-\frac{5\sec(dx+c)^3}{24}-\frac{5\sec(dx+c)}{16}\right)\tan(dx+c)+\frac{5\ln(\sec(dx+c)+\tan(dx+c))}{16}\right)-4a^4\left(-\frac{8}{15}-\frac{\sec(dx+c)^4}{5}-\frac{4\sec}{}\right.}{}$
parallelrisch	$\frac{a^4\left(735(-10-\cos(6dx+6c)-6\cos(4dx+4c)-15\cos(2dx+2c))\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+735(10+\cos(6dx+6c)+6\cos(4dx+\right.}{240d(10-}$
parts	$\frac{a^4\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)}{d}+\frac{a^4\left(-\left(-\frac{\sec(dx+c)^5}{6}-\frac{5\sec(dx+c)^3}{24}-\frac{5\sec(dx+c)}{16}\right)\tan(dx+c)+\frac{5\ln}{}\right.}{d}$

input
```
int(sec(d*x+c)^3*(a+a*sec(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
(207/8*a^4/d*tan(1/2*d*x+1/2*c)-1471/24*a^4/d*tan(1/2*d*x+1/2*c)^3+1967/20
*a^4/d*tan(1/2*d*x+1/2*c)^5-1617/20*a^4/d*tan(1/2*d*x+1/2*c)^7+833/24*a^4/
d*tan(1/2*d*x+1/2*c)^9-49/8*a^4/d*tan(1/2*d*x+1/2*c)^11)/(-1+tan(1/2*d*x+1
/2*c)^2)^6-49/16*a^4/d*ln(tan(1/2*d*x+1/2*c)-1)+49/16*a^4/d*ln(tan(1/2*d*x
+1/2*c)+1)
```

3.30.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.01

$$\int \sec^3(c+dx)(a+a\sec(c+dx))^4\,dx$$
$$=\frac{735\,a^4\cos(dx+c)^6\log(\sin(dx+c)+1)-735\,a^4\cos(dx+c)^6\log(-\sin(dx+c)+1)+2\,(1152\,a^4\cos(}{480}$$

input
```
integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^4,x, algorithm="fricas")
```

output
```
1/480*(735*a^4*cos(d*x + c)^6*log(sin(d*x + c) + 1) - 735*a^4*cos(d*x + c)
^6*log(-sin(d*x + c) + 1) + 2*(1152*a^4*cos(d*x + c)^5 + 735*a^4*cos(d*x +
 c)^4 + 576*a^4*cos(d*x + c)^3 + 410*a^4*cos(d*x + c)^2 + 192*a^4*cos(d*x
+ c) + 40*a^4)*sin(d*x + c))/(d*cos(d*x + c)^6)
```

3.30.6 Sympy [F]

$$\int \sec^3(c+dx)(a+a\sec(c+dx))^4\,dx = a^4\left(\int \sec^3(c+dx)\,dx + \int 4\sec^4(c+dx)\,dx + \int 6\sec^5(c+dx)\,dx + \int 4\sec^6(c+dx)\,dx + \int \sec^7(c+dx)\,dx\right)$$

input
```
integrate(sec(d*x+c)**3*(a+a*sec(d*x+c))**4,x)
```

output
```
a**4*(Integral(sec(c + d*x)**3, x) + Integral(4*sec(c + d*x)**4, x) + Inte
gral(6*sec(c + d*x)**5, x) + Integral(4*sec(c + d*x)**6, x) + Integral(sec
(c + d*x)**7, x))
```

3.30.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 270 vs. $2(126) = 252$.

Time = 0.22 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.99

$$\int \sec^3(c+dx)(a+a\sec(c+dx))^4\,dx$$

$$= \frac{128\left(3\tan(dx+c)^5 + 10\tan(dx+c)^3 + 15\tan(dx+c)\right)a^4 + 640\left(\tan(dx+c)^3 + 3\tan(dx+c)\right)a^4 -}{}$$

input
```
integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^4,x, algorithm="maxima")
```

output
```
1/480*(128*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))*a^4 +
640*(tan(d*x + c)^3 + 3*tan(d*x + c))*a^4 - 5*a^4*(2*(15*sin(d*x + c)^5 -
40*sin(d*x + c)^3 + 33*sin(d*x + c))/(sin(d*x + c)^6 - 3*sin(d*x + c)^4 +
3*sin(d*x + c)^2 - 1) - 15*log(sin(d*x + c) + 1) + 15*log(sin(d*x + c) - 1
)) - 180*a^4*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*si
n(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1)) - 1
20*a^4*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(
sin(d*x + c) - 1)))/d
```

3.30.8 Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.13

$$\int \sec^3(c+dx)(a+a\sec(c+dx))^4\,dx$$
$$= \frac{735\,a^4\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-735\,a^4\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(735\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}-4165\,a^4\tan\left(\frac{1}{2}\right.\right.}{}}{240\,d}$$

input
```
integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^4,x, algorithm="giac")
```

output
```
1/240*(735*a^4*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 735*a^4*log(abs(tan(1/
2*d*x + 1/2*c) - 1)) - 2*(735*a^4*tan(1/2*d*x + 1/2*c)^11 - 4165*a^4*tan(1
/2*d*x + 1/2*c)^9 + 9702*a^4*tan(1/2*d*x + 1/2*c)^7 - 11802*a^4*tan(1/2*d*
x + 1/2*c)^5 + 7355*a^4*tan(1/2*d*x + 1/2*c)^3 - 3105*a^4*tan(1/2*d*x + 1/
2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^6)/d
```

3.30.9 Mupad [B] (verification not implemented)

Time = 17.06 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.46

$$\int \sec^3(c+dx)(a+a\sec(c+dx))^4\,dx = \frac{49\,a^4\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{8\,d}$$
$$-\frac{\frac{49\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{8}-\frac{833\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}}{24}+\frac{1617\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}}{20}-\frac{1967\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{5}}{20}+\frac{1471\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}}{24}-\frac{207}{}}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}-6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}+15\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8}-20\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6}+15\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}-6\,\mathrm{ta}\right.}$$

input
```
int((a + a/cos(c + d*x))^4/cos(c + d*x)^3,x)
```

output
```
(49*a^4*atanh(tan(c/2 + (d*x)/2)))/(8*d) - ((1471*a^4*tan(c/2 + (d*x)/2)^3
)/24 - (1967*a^4*tan(c/2 + (d*x)/2)^5)/20 + (1617*a^4*tan(c/2 + (d*x)/2)^7
)/20 - (833*a^4*tan(c/2 + (d*x)/2)^9)/24 + (49*a^4*tan(c/2 + (d*x)/2)^11)/
8 - (207*a^4*tan(c/2 + (d*x)/2))/8)/(d*(15*tan(c/2 + (d*x)/2)^4 - 6*tan(c/
2 + (d*x)/2)^2 - 20*tan(c/2 + (d*x)/2)^6 + 15*tan(c/2 + (d*x)/2)^8 - 6*tan
(c/2 + (d*x)/2)^10 + tan(c/2 + (d*x)/2)^12 + 1))
```

3.30. $\int \sec^3(c+dx)(a+a\sec(c+dx))^4\,dx$

3.31 $\int \sec^2(c+dx)(a+a\sec(c+dx))^4\,dx$

3.31.1 Optimal result . . . 478
3.31.2 Mathematica [A] (verified) . . . 479
3.31.3 Rubi [A] (verified) . . . 479
3.31.4 Maple [A] (verified) . . . 481
3.31.5 Fricas [A] (verification not implemented) . . . 481
3.31.6 Sympy [F] . . . 482
3.31.7 Maxima [A] (verification not implemented) . . . 482
3.31.8 Giac [A] (verification not implemented) . . . 483
3.31.9 Mupad [B] (verification not implemented) . . . 483

3.31.1 Optimal result

Integrand size = 21, antiderivative size = 111

$$\int \sec^2(c+dx)(a+a\sec(c+dx))^4\,dx = \frac{7a^4\text{arctanh}(\sin(c+dx))}{2d}+\frac{8a^4\tan(c+dx)}{d}+\frac{7a^4\sec(c+dx)\tan(c+dx)}{2d}+\frac{a^4\sec^3(c+dx)\tan(c+dx)}{d}+\frac{8a^4\tan^3(c+dx)}{3d}+\frac{a^4\tan^5(c+dx)}{5d}$$

output
```
7/2*a^4*arctanh(sin(d*x+c))/d+8*a^4*tan(d*x+c)/d+7/2*a^4*sec(d*x+c)*tan(d*
x+c)/d+a^4*sec(d*x+c)^3*tan(d*x+c)/d+8/3*a^4*tan(d*x+c)^3/d+1/5*a^4*tan(d*
x+c)^5/d
```

3.31.2 Mathematica [A] (verified)

Time = 2.55 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.13

$$\int \sec^2(c+dx)(a+a\sec(c+dx))^4\,dx = \frac{7a^4\text{arctanh}(\sin(c+dx))}{2d} + \frac{83a^4\tan(c+dx)}{15d} + \frac{7a^4\sec(c+dx)\tan(c+dx)}{2d} + \frac{34a^4\sec^2(c+dx)\tan(c+dx)}{15d} + \frac{a^4\sec^3(c+dx)\tan(c+dx)}{d} + \frac{a^4\sec^4(c+dx)\tan(c+dx)}{5d}$$

input
```
Integrate[Sec[c + d*x]^2*(a + a*Sec[c + d*x])^4,x]
```

output
```
(7*a^4*ArcTanh[Sin[c + d*x]])/(2*d) + (83*a^4*Tan[c + d*x])/(15*d) + (7*a^
4*Sec[c + d*x]*Tan[c + d*x])/(2*d) + (34*a^4*Sec[c + d*x]^2*Tan[c + d*x])/
(15*d) + (a^4*Sec[c + d*x]^3*Tan[c + d*x])/d + (a^4*Sec[c + d*x]^4*Tan[c +
 d*x])/(5*d)
```

3.31.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3042, 4278, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(c+dx)(a\sec(c+dx)+a)^4\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^2\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^4dx$$

$$\Big\downarrow \text{4278}$$

$$\int \left(a^4\sec^6(c+dx)+4a^4\sec^5(c+dx)+6a^4\sec^4(c+dx)+4a^4\sec^3(c+dx)+a^4\sec^2(c+dx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{7a^4\text{arctanh}(\sin(c+dx))}{2d}+\frac{a^4\tan^5(c+dx)}{5d}+\frac{8a^4\tan^3(c+dx)}{3d}+\frac{8a^4\tan(c+dx)}{d}+\frac{a^4\tan(c+dx)\sec^3(c+dx)}{d}+\frac{7a^4\tan(c+dx)\sec(c+dx)}{2d}$$

input
```
Int[Sec[c + d*x]^2*(a + a*Sec[c + d*x])^4,x]
```

output
```
(7*a^4*ArcTanh[Sin[c + d*x]])/(2*d) + (8*a^4*Tan[c + d*x])/d + (7*a^4*Sec[
c + d*x]*Tan[c + d*x])/(2*d) + (a^4*Sec[c + d*x]^3*Tan[c + d*x])/d + (8*a^
4*Tan[c + d*x]^3)/(3*d) + (a^4*Tan[c + d*x]^5)/(5*d)
```

3.31.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4278
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

3.31.4 Maple [A] (verified)

Time = 0.92 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.37

method	result
norman	$\frac{-\frac{25a^4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{158a^4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3d}-\frac{896a^4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{15d}+\frac{98a^4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{3d}-\frac{7a^4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^9}{d}}{\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^5}-\frac{7a^4\ln\left(\tan\left(\frac{dx}{2}+\right.\right.}{2d}$
risch	$-\frac{ia^4\left(105\,\mathrm{e}^{9i(dx+c)}-30\,\mathrm{e}^{8i(dx+c)}+330\,\mathrm{e}^{7i(dx+c)}-480\,\mathrm{e}^{6i(dx+c)}-1180\,\mathrm{e}^{4i(dx+c)}-330\,\mathrm{e}^{3i(dx+c)}-800\,\mathrm{e}^{2i(dx+c)}-105\,\mathrm{e}^{i(}\right.}{15d\left(\mathrm{e}^{2i(dx+c)}+1\right)^5}$
derivativedivides	$\frac{-a^4\left(-\frac{8}{15}-\frac{\sec(dx+c)^4}{5}-\frac{4\sec(dx+c)^2}{15}\right)\tan(dx+c)+4a^4\left(-\left(-\frac{\sec(dx+c)^3}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(}{8}\right.}{d}$
default	$\frac{-a^4\left(-\frac{8}{15}-\frac{\sec(dx+c)^4}{5}-\frac{4\sec(dx+c)^2}{15}\right)\tan(dx+c)+4a^4\left(-\left(-\frac{\sec(dx+c)^3}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(}{8}\right.}{d}$
parts	$\frac{a^4\tan(dx+c)}{d}-\frac{a^4\left(-\frac{8}{15}-\frac{\sec(dx+c)^4}{5}-\frac{4\sec(dx+c)^2}{15}\right)\tan(dx+c)}{d}+\frac{2a^4\sec(dx+c)\tan(dx+c)}{d}+\frac{2a^4\ln(\sec(dx+c)+}{d}$
parallelrisch	$\frac{77a^4\left(\left(-\frac{15\cos(dx+c)}{11}-\frac{15\cos(3dx+3c)}{22}-\frac{3\cos(5dx+5c)}{22}\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+\left(\frac{15\cos(dx+c)}{11}+\frac{15\cos(3dx+3c)}{22}+\frac{3\cos(5dx+5}{22}\right.\right.}{3d(\cos(5dx+5c)+5\cos(3dx+3c)+}$

input
```
int(sec(d*x+c)^2*(a+a*sec(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
(-25*a^4/d*tan(1/2*d*x+1/2*c)+158/3*a^4/d*tan(1/2*d*x+1/2*c)^3-896/15*a^4/
d*tan(1/2*d*x+1/2*c)^5+98/3*a^4/d*tan(1/2*d*x+1/2*c)^7-7*a^4/d*tan(1/2*d*x
+1/2*c)^9)/(-1+tan(1/2*d*x+1/2*c)^2)^5-7/2*a^4/d*ln(tan(1/2*d*x+1/2*c)-1)+
7/2*a^4/d*ln(tan(1/2*d*x+1/2*c)+1)
```

3.31.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.12

$$\int \sec^2(c+dx)(a+a\sec(c+dx))^4\,dx$$
$$=\frac{105\,a^4\cos\left(dx+c\right)^5\log\left(\sin\left(dx+c\right)+1\right)-105\,a^4\cos\left(dx+c\right)^5\log\left(-\sin\left(dx+c\right)+1\right)+2\left(166\,a^4\cos\left(d\right.\right.}{60\,d\cos\left(dx+c\right)}$$

input
```
integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^4,x, algorithm="fricas")
```

output
```
1/60*(105*a^4*cos(d*x + c)^5*log(sin(d*x + c) + 1) - 105*a^4*cos(d*x + c)^
5*log(-sin(d*x + c) + 1) + 2*(166*a^4*cos(d*x + c)^4 + 105*a^4*cos(d*x + c
)^3 + 68*a^4*cos(d*x + c)^2 + 30*a^4*cos(d*x + c) + 6*a^4)*sin(d*x + c))/(
d*cos(d*x + c)^5)
```

3.31.6 Sympy [F]

$$\int \sec^2(c+dx)(a+a\sec(c+dx))^4\,dx = a^4\left(\int \sec^2(c+dx)\,dx + \int 4\sec^3(c+dx)\,dx + \int 6\sec^4(c+dx)\,dx + \int 4\sec^5(c+dx)\,dx + \int \sec^6(c+dx)\,dx\right)$$

input
```
integrate(sec(d*x+c)**2*(a+a*sec(d*x+c))**4,x)
```

output
```
a**4*(Integral(sec(c + d*x)**2, x) + Integral(4*sec(c + d*x)**3, x) + Inte
gral(6*sec(c + d*x)**4, x) + Integral(4*sec(c + d*x)**5, x) + Integral(sec
(c + d*x)**6, x))
```

3.31.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.71

$$\int \sec^2(c+dx)(a+a\sec(c+dx))^4\,dx$$

$$= \frac{4\left(3\tan(dx+c)^5 + 10\tan(dx+c)^3 + 15\tan(dx+c)\right)a^4 + 120\left(\tan(dx+c)^3 + 3\tan(dx+c)\right)a^4 - 15}{}$$

input
```
integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^4,x, algorithm="maxima")
```

output
```
1/60*(4*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))*a^4 + 120
*(tan(d*x + c)^3 + 3*tan(d*x + c))*a^4 - 15*a^4*(2*(3*sin(d*x + c)^3 - 5*s
in(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c)
+ 1) + 3*log(sin(d*x + c) - 1)) - 60*a^4*(2*sin(d*x + c)/(sin(d*x + c)^2 -
 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 60*a^4*tan(d*x + c)
)/d
```

3.31.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.24

$$\int \sec^2(c+dx)(a+a\sec(c+dx))^4\,dx$$
$$= \frac{105\,a^4\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-105\,a^4\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(105\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9-490\,a^4\tan\left(\frac{1}{2}\,dx\right.\right.}{}}{30\,d}$$

input
```
integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^4,x, algorithm="giac")
```

output
```
1/30*(105*a^4*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 105*a^4*log(abs(tan(1/2
*d*x + 1/2*c) - 1)) - 2*(105*a^4*tan(1/2*d*x + 1/2*c)^9 - 490*a^4*tan(1/2*
d*x + 1/2*c)^7 + 896*a^4*tan(1/2*d*x + 1/2*c)^5 - 790*a^4*tan(1/2*d*x + 1/
2*c)^3 + 375*a^4*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^5)/d
```

3.31.9 Mupad [B] (verification not implemented)

Time = 17.80 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.53

$$\int \sec^2(c+dx)(a+a\sec(c+dx))^4\,dx = \frac{7\,a^4\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d} - \frac{7\,a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^9-\frac{98\,a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7}{3}+\frac{896\,a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5}{15}-\frac{158\,a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{3}+25\,a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}-5\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8+10\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6-10\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+5\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2-1\right)}$$

input
```
int((a + a/cos(c + d*x))^4/cos(c + d*x)^2,x)
```

output
```
(7*a^4*atanh(tan(c/2 + (d*x)/2)))/d - ((896*a^4*tan(c/2 + (d*x)/2)^5)/15 -
 (158*a^4*tan(c/2 + (d*x)/2)^3)/3 - (98*a^4*tan(c/2 + (d*x)/2)^7)/3 + 7*a^
4*tan(c/2 + (d*x)/2)^9 + 25*a^4*tan(c/2 + (d*x)/2))/(d*(5*tan(c/2 + (d*x)/
2)^2 - 10*tan(c/2 + (d*x)/2)^4 + 10*tan(c/2 + (d*x)/2)^6 - 5*tan(c/2 + (d*
x)/2)^8 + tan(c/2 + (d*x)/2)^10 - 1))
```

3.32 $\int \sec(c+dx)(a+a\sec(c+dx))^4\,dx$

3.32.1 Optimal result . . . 484
3.32.2 Mathematica [A] (verified) . . . 484
3.32.3 Rubi [A] (verified) . . . 485
3.32.4 Maple [A] (verified) . . . 486
3.32.5 Fricas [A] (verification not implemented) . . . 487
3.32.6 Sympy [F] . . . 487
3.32.7 Maxima [A] (verification not implemented) . . . 488
3.32.8 Giac [A] (verification not implemented) . . . 488
3.32.9 Mupad [B] (verification not implemented) . . . 489

3.32.1 Optimal result

Integrand size = 19, antiderivative size = 96

$$\int \sec(c+dx)(a+a\sec(c+dx))^4\,dx = \frac{35a^4\text{arctanh}(\sin(c+dx))}{8d}+\frac{8a^4\tan(c+dx)}{d}+\frac{27a^4\sec(c+dx)\tan(c+dx)}{8d}+\frac{a^4\sec^3(c+dx)\tan(c+dx)}{4d}+\frac{4a^4\tan^3(c+dx)}{3d}$$

output
```
35/8*a^4*arctanh(sin(d*x+c))/d+8*a^4*tan(d*x+c)/d+27/8*a^4*sec(d*x+c)*tan(
d*x+c)/d+1/4*a^4*sec(d*x+c)^3*tan(d*x+c)/d+4/3*a^4*tan(d*x+c)^3/d
```

3.32.2 Mathematica [A] (verified)

Time = 2.18 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00

$$\int \sec(c+dx)(a+a\sec(c+dx))^4\,dx = \frac{35a^4\text{arctanh}(\sin(c+dx))}{8d}+\frac{8a^4\tan(c+dx)}{d}+\frac{27a^4\sec(c+dx)\tan(c+dx)}{8d}+\frac{a^4\sec^3(c+dx)\tan(c+dx)}{4d}+\frac{4a^4\tan^3(c+dx)}{3d}$$

input
```
Integrate[Sec[c + d*x]*(a + a*Sec[c + d*x])^4,x]
```

output
```
(35*a^4*ArcTanh[Sin[c + d*x]])/(8*d) + (8*a^4*Tan[c + d*x])/d + (27*a^4*Se
c[c + d*x]*Tan[c + d*x])/(8*d) + (a^4*Sec[c + d*x]^3*Tan[c + d*x])/(4*d) +
 (4*a^4*Tan[c + d*x]^3)/(3*d)
```

3.32.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {3042, 4278, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(c+dx)(a\sec(c+dx)+a)^4\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^4dx$$

$$\Big\downarrow \text{4278}$$

$$\int \left(a^4\sec^5(c+dx)+4a^4\sec^4(c+dx)+6a^4\sec^3(c+dx)+4a^4\sec^2(c+dx)+a^4\sec(c+dx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{35a^4\text{arctanh}(\sin(c+dx))}{8d}+\frac{4a^4\tan^3(c+dx)}{3d}+\frac{8a^4\tan(c+dx)}{d}+\frac{a^4\tan(c+dx)\sec^3(c+dx)}{4d}+\frac{27a^4\tan(c+dx)\sec(c+dx)}{8d}$$

input
```
Int[Sec[c + d*x]*(a + a*Sec[c + d*x])^4,x]
```

output
```
(35*a^4*ArcTanh[Sin[c + d*x]])/(8*d) + (8*a^4*Tan[c + d*x])/d + (27*a^4*Se
c[c + d*x]*Tan[c + d*x])/(8*d) + (a^4*Sec[c + d*x]^3*Tan[c + d*x])/(4*d) +
 (4*a^4*Tan[c + d*x]^3)/(3*d)
```

3.32.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4278
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

3.32.4 Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.39

method	result
norman	$\frac{\frac{93a^4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d}-\frac{511a^4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{12d}+\frac{385a^4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{12d}-\frac{35a^4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{4d}}{\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^4}-\frac{35a^4\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{8d}+\frac{35a^4\ln}{}$
parts	$\frac{4a^4\ln(\sec(dx+c)+\tan(dx+c))}{d}+\frac{a^4\left(-\left(-\frac{\sec(dx+c)^3}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)}{d}+\frac{4a^4\,\mathrm{t}}{}$
derivativedivides	$\frac{a^4\left(-\left(-\frac{\sec(dx+c)^3}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)-4a^4\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right)\tan(dx+c)+6a^4\left(\mathrm{se}\right.}{d}$
default	$\frac{a^4\left(-\left(-\frac{\sec(dx+c)^3}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)-4a^4\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right)\tan(dx+c)+6a^4\left(\mathrm{se}\right.}{d}$
risch	$-\frac{ia^4\left(81\,\mathrm{e}^{7i(dx+c)}-96\,\mathrm{e}^{6i(dx+c)}+105\,\mathrm{e}^{5i(dx+c)}-480\,\mathrm{e}^{4i(dx+c)}-105\,\mathrm{e}^{3i(dx+c)}-544\,\mathrm{e}^{2i(dx+c)}-81\,\mathrm{e}^{i(dx+c)}-160\right)}{12d\left(\mathrm{e}^{2i(dx+c)}+1\right)^4}-\frac{35}{}$
parallelrisch	$\frac{a^4\left(105(-\cos(4dx+4c)-4\cos(2dx+2c)-3)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+105(3+\cos(4dx+4c)+4\cos(2dx+2c))\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right.\right.}{24d(3+\cos(4dx+4c)+4\cos(2dx+2c))}$

input
```
int(sec(d*x+c)*(a+a*sec(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
(93/4*a^4/d*tan(1/2*d*x+1/2*c)-511/12*a^4/d*tan(1/2*d*x+1/2*c)^3+385/12*a^
4/d*tan(1/2*d*x+1/2*c)^5-35/4*a^4/d*tan(1/2*d*x+1/2*c)^7)/(-1+tan(1/2*d*x+
1/2*c)^2)^4-35/8*a^4/d*ln(tan(1/2*d*x+1/2*c)-1)+35/8*a^4/d*ln(tan(1/2*d*x+
1/2*c)+1)
```

3.32.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.16

$$\int \sec(c+dx)(a+a\sec(c+dx))^4\,dx$$
$$= \frac{105\,a^4\cos(dx+c)^4\log(\sin(dx+c)+1) - 105\,a^4\cos(dx+c)^4\log(-\sin(dx+c)+1) + 2\left(160\,a^4\cos(d\right.}{48\,d\cos(dx+c)^4}$$

input
```
integrate(sec(d*x+c)*(a+a*sec(d*x+c))^4,x, algorithm="fricas")
```

output
```
1/48*(105*a^4*cos(d*x + c)^4*log(sin(d*x + c) + 1) - 105*a^4*cos(d*x + c)^
4*log(-sin(d*x + c) + 1) + 2*(160*a^4*cos(d*x + c)^3 + 81*a^4*cos(d*x + c)
^2 + 32*a^4*cos(d*x + c) + 6*a^4)*sin(d*x + c))/(d*cos(d*x + c)^4)
```

3.32.6 Sympy [F]

$$\int \sec(c+dx)(a+a\sec(c+dx))^4\,dx = a^4\left(\int \sec(c+dx)\,dx + \int 4\sec^2(c+dx)\,dx + \int 6\sec^3(c+dx)\,dx + \int 4\sec^4(c+dx)\,dx + \int \sec^5(c+dx)\,dx\right)$$

input
```
integrate(sec(d*x+c)*(a+a*sec(d*x+c))**4,x)
```

output
```
a**4*(Integral(sec(c + d*x), x) + Integral(4*sec(c + d*x)**2, x) + Integra
l(6*sec(c + d*x)**3, x) + Integral(4*sec(c + d*x)**4, x) + Integral(sec(c
+ d*x)**5, x))
```

3.32.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.82

$$\int \sec(c+dx)(a+a\sec(c+dx))^4\,dx$$
$$= \frac{64\left(\tan\left(dx+c\right)^3+3\,\tan\left(dx+c\right)\right)a^4-3\,a^4\left(\frac{2\left(3\,\sin(dx+c)^3-5\,\sin(dx+c)\right)}{\sin(dx+c)^4-2\,\sin(dx+c)^2+1}-3\,\log\left(\sin\left(dx+c\right)+1\right)+3\,\log\left(s\right.\right.}{}$$

input
```
integrate(sec(d*x+c)*(a+a*sec(d*x+c))^4,x, algorithm="maxima")
```

output
```
1/48*(64*(tan(d*x + c)^3 + 3*tan(d*x + c))*a^4 - 3*a^4*(2*(3*sin(d*x + c)^
3 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*
x + c) + 1) + 3*log(sin(d*x + c) - 1)) - 72*a^4*(2*sin(d*x + c)/(sin(d*x +
 c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 48*a^4*log(s
ec(d*x + c) + tan(d*x + c)) + 192*a^4*tan(d*x + c))/d
```

3.32.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.27

$$\int \sec(c+dx)(a+a\sec(c+dx))^4\,dx$$
$$= \frac{105\,a^4\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-105\,a^4\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(105\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-385\,a^4\tan\left(\frac{1}{2}\,dx\right.\right.}{\left(\tan\right.}}{24\,d}$$

input
```
integrate(sec(d*x+c)*(a+a*sec(d*x+c))^4,x, algorithm="giac")
```

output
```
1/24*(105*a^4*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 105*a^4*log(abs(tan(1/2
*d*x + 1/2*c) - 1)) - 2*(105*a^4*tan(1/2*d*x + 1/2*c)^7 - 385*a^4*tan(1/2*
d*x + 1/2*c)^5 + 511*a^4*tan(1/2*d*x + 1/2*c)^3 - 279*a^4*tan(1/2*d*x + 1/
2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^4)/d
```

3.32.9 Mupad [B] (verification not implemented)

Time = 16.53 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.47

$$\int \sec(c+dx)(a+a\sec(c+dx))^4\,dx = \frac{35\,a^4\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{4\,d} - \frac{\frac{35\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{4} - \frac{385\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{12} + \frac{511\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{12} - \frac{93\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8 - 4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6 + 6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4 - 4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2 + 1\right)}$$

input
```
int((a + a/cos(c + d*x))^4/cos(c + d*x),x)
```

output
```
(35*a^4*atanh(tan(c/2 + (d*x)/2)))/(4*d) - ((511*a^4*tan(c/2 + (d*x)/2)^3)
/12 - (385*a^4*tan(c/2 + (d*x)/2)^5)/12 + (35*a^4*tan(c/2 + (d*x)/2)^7)/4
- (93*a^4*tan(c/2 + (d*x)/2))/4)/(d*(6*tan(c/2 + (d*x)/2)^4 - 4*tan(c/2 +
(d*x)/2)^2 - 4*tan(c/2 + (d*x)/2)^6 + tan(c/2 + (d*x)/2)^8 + 1))
```

3.33 $\int (a + a\sec(c + dx))^4 \, dx$

3.33.1 Optimal result 490
3.33.2 Mathematica [A] (verified) 490
3.33.3 Rubi [A] (verified) 491
3.33.4 Maple [A] (verified) 494
3.33.5 Fricas [A] (verification not implemented) 494
3.33.6 Sympy [F] 495
3.33.7 Maxima [A] (verification not implemented) 495
3.33.8 Giac [A] (verification not implemented) 496
3.33.9 Mupad [B] (verification not implemented) 496

3.33.1 Optimal result

Integrand size = 12, antiderivative size = 91

$$\int (a + a\sec(c + dx))^4 \, dx = a^4 x + \frac{6a^4 \text{arctanh}(\sin(c + dx))}{d} + \frac{5a^4 \tan(c + dx)}{d} + \frac{(a^2 + a^2 \sec(c + dx))^2 \tan(c + dx)}{3d} + \frac{4(a^4 + a^4 \sec(c + dx)) \tan(c + dx)}{3d}$$

output
```
a^4*x+6*a^4*arctanh(sin(d*x+c))/d+5*a^4*tan(d*x+c)/d+1/3*(a^2+a^2*sec(d*x+
c))^2*tan(d*x+c)/d+4/3*(a^4+a^4*sec(d*x+c))*tan(d*x+c)/d
```

3.33.2 Mathematica [A] (verified)

Time = 2.12 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.80

$$\int (a + a\sec(c + dx))^4 \, dx = a^4 x + \frac{6a^4 \text{arctanh}(\sin(c + dx))}{d} + \frac{7a^4 \tan(c + dx)}{d} + \frac{2a^4 \sec(c + dx) \tan(c + dx)}{d} + \frac{a^4 \tan^3(c + dx)}{3d}$$

input
```
Integrate[(a + a*Sec[c + d*x])^4,x]
```

output
```
a^4*x + (6*a^4*ArcTanh[Sin[c + d*x]])/d + (7*a^4*Tan[c + d*x])/d + (2*a^4*
Sec[c + d*x]*Tan[c + d*x])/d + (a^4*Tan[c + d*x]^3)/(3*d)
```

3.33.3 Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.08, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.917, Rules used = {3042, 4262, 3042, 4405, 27, 3042, 4402, 3042, 4254, 24, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sec(c+dx)+a)^4\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^4 dx$$

$$\Big\downarrow \text{4262}$$

$$\frac{1}{3}a\int (\sec(c+dx)a+a)^2(8\sec(c+dx)a+3a)dx+\frac{\tan(c+dx)\left(a^2\sec(c+dx)+a^2\right)^2}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}a\int \left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2\left(8\csc\left(c+dx+\frac{\pi}{2}\right)a+3a\right)dx+\frac{\tan(c+dx)\left(a^2\sec(c+dx)+a^2\right)^2}{3d}$$

$$\Big\downarrow \text{4405}$$

$$\frac{1}{3}a\left(\frac{1}{2}\int 6(\sec(c+dx)a+a)\left(5\sec(c+dx)a^2+a^2\right)dx+\frac{4\tan(c+dx)\left(a^3\sec(c+dx)+a^3\right)}{d}\right)+\frac{\tan(c+dx)\left(a^2\sec(c+dx)+a^2\right)^2}{3d}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}a\left(3\int (\sec(c+dx)a+a)\left(5\sec(c+dx)a^2+a^2\right)dx+\frac{4\tan(c+dx)\left(a^3\sec(c+dx)+a^3\right)}{d}\right)+\frac{\tan(c+dx)\left(a^2\sec(c+dx)+a^2\right)^2}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}a\left(3\int \left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)\left(5\csc\left(c+dx+\frac{\pi}{2}\right)a^2+a^2\right)dx+\frac{4\tan(c+dx)\left(a^3\sec(c+dx)+a^3\right)}{d}\right)+\frac{\tan(c+dx)\left(a^2\sec(c+dx)+a^2\right)^2}{3d}$$

$$\Big\downarrow \text{4402}$$

$$\frac{1}{3}a\left(3\left(5a^3\int \sec^2(c+dx)dx+6a^3\int \sec(c+dx)dx+a^3x\right)+\frac{4\tan(c+dx)\left(a^3\sec(c+dx)+a^3\right)}{d}\right)+\frac{\tan(c+dx)\left(a^2\sec(c+dx)+a^2\right)^2}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}a\left(3\left(6a^3\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+5a^3\int \csc\left(c+dx+\frac{\pi}{2}\right)^2dx+a^3x\right)+\frac{4\tan(c+dx)\left(a^3\sec(c+dx)+a^3\right)}{d}\right)\frac{\tan(c+dx)\left(a^2\sec(c+dx)+a^2\right)^2}{3d}$$

$$\Big\downarrow \text{4254}$$

$$\frac{1}{3}a\left(3\left(-\frac{5a^3\int 1d(-\tan(c+dx))}{d}+6a^3\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+a^3x\right)+\frac{4\tan(c+dx)\left(a^3\sec(c+dx)+a^3\right)}{d}\right)+\frac{\tan(c+dx)\left(a^2\sec(c+dx)+a^2\right)^2}{3d}$$

$$\Big\downarrow \text{24}$$

$$\frac{1}{3}a\left(3\left(6a^3\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{5a^3\tan(c+dx)}{d}+a^3x\right)+\frac{4\tan(c+dx)\left(a^3\sec(c+dx)+a^3\right)}{d}\right)+\frac{\tan(c+dx)\left(a^2\sec(c+dx)+a^2\right)^2}{3d}$$

$$\Big\downarrow \text{4257}$$

$$\frac{1}{3}a\left(3\left(\frac{6a^3\text{arctanh}(\sin(c+dx))}{d}+\frac{5a^3\tan(c+dx)}{d}+a^3x\right)+\frac{4\tan(c+dx)\left(a^3\sec(c+dx)+a^3\right)}{d}\right)+\frac{\tan(c+dx)\left(a^2\sec(c+dx)+a^2\right)^2}{3d}$$

input
```
Int[(a + a*Sec[c + d*x])^4,x]
```

output
```
((a^2 + a^2*Sec[c + d*x])^2*Tan[c + d*x])/(3*d) + (a*((4*(a^3 + a^3*Sec[c
+ d*x])*Tan[c + d*x])/d + 3*(a^3*x + (6*a^3*ArcTanh[Sin[c + d*x]])/d + (5*
a^3*Tan[c + d*x])/d)))/3
```

3.33. $\int (a+a\sec(c+dx))^4\,dx$

3.33.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4262
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(-b^2)*C
ot[c + d*x]*((a + b*Csc[c + d*x])^(n - 2)/(d*(n - 1))), x] + Simp[a/(n - 1)
   Int[(a + b*Csc[c + d*x])^(n - 2)*(a*(n - 1) + b*(3*n - 4)*Csc[c + d*x]),
 x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && Inte
gerQ[2*n]
```

rule 4402
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))*(csc[(e_.) + (f_.)*(x_)]*(d_.) +
 (c_)), x_Symbol] :> Simp[a*c*x, x] + (Simp[b*d   Int[Csc[e + f*x]^2, x], x
] + Simp[(b*c + a*d)   Int[Csc[e + f*x], x], x]) /; FreeQ[{a, b, c, d, e, f
}, x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

rule 4405
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> Simp[(-b)*d*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m
 - 1)/(f*m)), x] + Simp[1/m   Int[(a + b*Csc[e + f*x])^(m - 1)*Simp[a*c*m +
 (b*c*m + a*d*(2*m - 1))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
f}, x] && NeQ[b*c - a*d, 0] && GtQ[m, 1] && EqQ[a^2 - b^2, 0] && IntegerQ[2
*m]
```

3.33.4 Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.98

method	result
parts	$a^4 x - \frac{a^4\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right)\tan(dx+c)}{d} + \frac{6a^4\ln(\sec(dx+c)+\tan(dx+c))}{d} + \frac{6a^4\tan(dx+c)}{d} + \frac{2a^4\sec(dx+c)\tan}{d}$
derivativedivides	$\frac{-a^4\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right)\tan(dx+c)+4a^4\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+6a^4\tan(dx+c)+4a^4\ln(\sec(dx}{d}$
default	$\frac{-a^4\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right)\tan(dx+c)+4a^4\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+6a^4\tan(dx+c)+4a^4\ln(\sec(dx}{d}$
risch	$a^4 x - \frac{4ia^4\left(3\,e^{5i(dx+c)}-9\,e^{4i(dx+c)}-21\,e^{2i(dx+c)}-3\,e^{i(dx+c)}-10\right)}{3d\left(e^{2i(dx+c)}+1\right)^3} + \frac{6a^4\ln\left(e^{i(dx+c)}+i\right)}{d} - \frac{6a^4\ln\left(e^{i(dx+c)}-i\right)}{d}$
parallelrisch	$-\frac{18a^4\left(\left(\cos(dx+c)+\frac{\cos(3dx+3c)}{3}\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+\left(-\cos(dx+c)-\frac{\cos(3dx+3c)}{3}\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-\frac{dx\cos(dx-}{6}\right.}{d(\cos(3dx+3c)+3\cos(dx+c))}$
norman	$\frac{a^4x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6-a^4x-\frac{18a^4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{76a^4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3d}-\frac{10a^4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{d}+3a^4x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2-3a^4x\tan\left(\frac{dx}{2}+\right.}{\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^3}$

input
```
int((a+a*sec(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
a^4*x-a^4/d*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c)+6*a^4/d*ln(sec(d*x+c)+tan(d
*x+c))+6*a^4*tan(d*x+c)/d+2*a^4*sec(d*x+c)*tan(d*x+c)/d
```

3.33.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.21

$$\int (a+a\sec(c+dx))^4\,dx = \frac{3\,a^4dx\cos\left(dx+c\right)^3+9\,a^4\cos\left(dx+c\right)^3\log\left(\sin\left(dx+c\right)+1\right)-9\,a^4\cos\left(dx+c\right)^3\log\left(-\sin\left(dx+c\right)+1\right)}{3\,d\cos\left(dx+c\right)^3}$$

input
```
integrate((a+a*sec(d*x+c))^4,x, algorithm="fricas")
```

output
```
1/3*(3*a^4*d*x*cos(d*x + c)^3 + 9*a^4*cos(d*x + c)^3*log(sin(d*x + c) + 1)
 - 9*a^4*cos(d*x + c)^3*log(-sin(d*x + c) + 1) + (20*a^4*cos(d*x + c)^2 +
6*a^4*cos(d*x + c) + a^4)*sin(d*x + c))/(d*cos(d*x + c)^3)
```

3.33.6 Sympy [F]

$$\int (a+a\sec(c+dx))^4\,dx = a^4\left(\int 1\,dx+\int 4\sec(c+dx)\,dx+\int 6\sec^2(c+dx)\,dx+\int 4\sec^3(c+dx)\,dx+\int \sec^4(c+dx)\,dx\right)$$

input
```
integrate((a+a*sec(d*x+c))**4,x)
```

output
```
a**4*(Integral(1, x) + Integral(4*sec(c + d*x), x) + Integral(6*sec(c + d*
x)**2, x) + Integral(4*sec(c + d*x)**3, x) + Integral(sec(c + d*x)**4, x))
```

3.33.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.27

$$\int (a+a\sec(c+dx))^4\,dx = a^4x+\frac{\left(\tan(dx+c)^3+3\tan(dx+c)\right)a^4}{3\,d}-\frac{a^4\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1}-\log(\sin(dx+c)+1)+\log(\sin(dx+c)-1)\right)}{d}+\frac{4\,a^4\log(\sec(dx+c)+\tan(dx+c))}{d}+\frac{6\,a^4\tan(dx+c)}{d}$$

input
```
integrate((a+a*sec(d*x+c))^4,x, algorithm="maxima")
```

output
```
a^4*x + 1/3*(tan(d*x + c)^3 + 3*tan(d*x + c))*a^4/d - a^4*(2*sin(d*x + c)/
(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1))/d +
4*a^4*log(sec(d*x + c) + tan(d*x + c))/d + 6*a^4*tan(d*x + c)/d
```

3.33.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.27

$$\int (a+a\sec(c+dx))^4\,dx$$
$$= \frac{3\,(dx+c)a^4 + 18\,a^4\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right) - 18\,a^4\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right) - \frac{2\left(15\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5 -\right.}{(}}{3\,d}$$

input
```
integrate((a+a*sec(d*x+c))^4,x, algorithm="giac")
```

output
```
1/3*(3*(d*x + c)*a^4 + 18*a^4*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 18*a^4*
log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(15*a^4*tan(1/2*d*x + 1/2*c)^5 - 38
*a^4*tan(1/2*d*x + 1/2*c)^3 + 27*a^4*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x +
1/2*c)^2 - 1)^3)/d
```

3.33.9 Mupad [B] (verification not implemented)

Time = 13.51 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.29

$$\int (a+a\sec(c+dx))^4\,dx = a^4\,x + \frac{12\,a^4\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d} - \frac{10\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5 - \frac{76\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{3} + 18\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6 - 3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4 + 3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2 - 1\right)}$$

input
```
int((a + a/cos(c + d*x))^4,x)
```

output
```
a^4*x + (12*a^4*atanh(tan(c/2 + (d*x)/2)))/d - (10*a^4*tan(c/2 + (d*x)/2)^
5 - (76*a^4*tan(c/2 + (d*x)/2)^3)/3 + 18*a^4*tan(c/2 + (d*x)/2))/(d*(3*tan
(c/2 + (d*x)/2)^2 - 3*tan(c/2 + (d*x)/2)^4 + tan(c/2 + (d*x)/2)^6 - 1))
```

3.34 $\int \cos(c+dx)(a+a\sec(c+dx))^4\,dx$

3.34.1 Optimal result . . . 497
3.34.2 Mathematica [B] (verified) . . . 498
3.34.3 Rubi [A] (verified) . . . 499
3.34.4 Maple [A] (verified) . . . 500
3.34.5 Fricas [A] (verification not implemented) . . . 500
3.34.6 Sympy [F] . . . 501
3.34.7 Maxima [A] (verification not implemented) . . . 501
3.34.8 Giac [A] (verification not implemented) . . . 502
3.34.9 Mupad [B] (verification not implemented) . . . 502

3.34.1 Optimal result

Integrand size = 19, antiderivative size = 73

$$\int \cos(c+dx)(a+a\sec(c+dx))^4\,dx = 4a^4x+\frac{13a^4\text{arctanh}(\sin(c+dx))}{2d}+\frac{a^4\sin(c+dx)}{d}+\frac{4a^4\tan(c+dx)}{d}+\frac{a^4\sec(c+dx)\tan(c+dx)}{2d}$$

output
```
4*a^4*x+13/2*a^4*arctanh(sin(d*x+c))/d+a^4*sin(d*x+c)/d+4*a^4*tan(d*x+c)/d
+1/2*a^4*sec(d*x+c)*tan(d*x+c)/d
```

3.34.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 272 vs. $2(73) = 146$.

Time = 5.28 (sec) , antiderivative size = 272, normalized size of antiderivative = 3.73

$$\int \cos(c+dx)(a+a\sec(c+dx))^4\,dx$$
$$= \frac{1}{64}a^4(1+\cos(c+dx))^4\sec^8\left(\frac{1}{2}(c+dx)\right)\left(16x - \frac{26\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}{d} + \frac{26\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}{d} + \frac{4\cos(dx)\sin(c)}{d} + \frac{4\cos(c)\sin(dx)}{d} + \frac{1}{d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^2} + \frac{16\sin\left(\frac{dx}{2}\right)}{d\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)} - \frac{1}{d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^2} + \frac{16\sin\left(\frac{dx}{2}\right)}{d\left(\cos\left(\frac{c}{2}\right)+\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}\right)$$

input
```
Integrate[Cos[c + d*x]*(a + a*Sec[c + d*x])^4,x]
```

output
```
(a^4*(1 + Cos[c + d*x])^4*Sec[(c + d*x)/2]^8*(16*x - (26*Log[Cos[(c + d*x)
/2] - Sin[(c + d*x)/2]])/d + (26*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]])
/d + (4*Cos[d*x]*Sin[c])/d + (4*Cos[c]*Sin[d*x])/d + 1/(d*(Cos[(c + d*x)/2
] - Sin[(c + d*x)/2])^2) + (16*Sin[(d*x)/2])/(d*(Cos[c/2] - Sin[c/2])*(Cos
[(c + d*x)/2] - Sin[(c + d*x)/2])) - 1/(d*(Cos[(c + d*x)/2] + Sin[(c + d*x
)/2])^2) + (16*Sin[(d*x)/2])/(d*(Cos[c/2] + Sin[c/2])*(Cos[(c + d*x)/2] +
Sin[(c + d*x)/2]))))/64
```

3.34.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {3042, 4278, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(c+dx)(a\sec(c+dx)+a)^4\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^4}{\csc\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{4278}$$

$$\int \left(a^4\cos(c+dx)+a^4\sec^3(c+dx)+4a^4\sec^2(c+dx)+6a^4\sec(c+dx)+4a^4\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{13a^4\text{arctanh}(\sin(c+dx))}{2d}+\frac{a^4\sin(c+dx)}{d}+\frac{4a^4\tan(c+dx)}{d}+\frac{a^4\tan(c+dx)\sec(c+dx)}{2d}+4a^4x$$

input
```
Int[Cos[c + d*x]*(a + a*Sec[c + d*x])^4,x]
```

output
```
4*a^4*x + (13*a^4*ArcTanh[Sin[c + d*x]])/(2*d) + (a^4*Sin[c + d*x])/d + (4
*a^4*Tan[c + d*x])/d + (a^4*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

3.34.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4278
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

3.34.4 Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.25

method	result
derivativedivides	$\frac{a^4\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+4a^4\tan(dx+c)+6a^4\ln(\sec(dx+c)+\tan(dx+c))+4a^4(dx+c)+a^4\sin}{d}$
default	$\frac{a^4\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+4a^4\tan(dx+c)+6a^4\ln(\sec(dx+c)+\tan(dx+c))+4a^4(dx+c)+a^4\sin}{d}$
parallelrisch	$\frac{a^4\left(-13(1+\cos(2dx+2c))\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+8dx\cos(2dx+2c)+13\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)\cos(2dx+2c)+8dx+13\ln\left(\tan\right.\right.}{2d(1+\cos(2dx+2c))}$
risch	$4a^4x-\frac{ia^4e^{i(dx+c)}}{2d}+\frac{ia^4e^{-i(dx+c)}}{2d}-\frac{ia^4\left(e^{3i(dx+c)}-8e^{2i(dx+c)}-e^{i(dx+c)}-8\right)}{d\left(e^{2i(dx+c)}+1\right)^2}-\frac{13a^4\ln\left(e^{i(dx+c)}-i\right)}{2d}+\frac{13a^4}{}$
norman	$\frac{-4a^4x-\frac{11a^4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{13a^4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{d}+\frac{3a^4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{d}-\frac{5a^4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{d}+8a^4x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2-8a^4x\tan\left(\frac{dx}{2}+\right.}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^3}$

input
```
int(cos(d*x+c)*(a+a*sec(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^4*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c)))+4*a^4*t
an(d*x+c)+6*a^4*ln(sec(d*x+c)+tan(d*x+c))+4*a^4*(d*x+c)+a^4*sin(d*x+c))
```

3.34.5 Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.52

$$\int \cos(c+dx)(a+a\sec(c+dx))^4\,dx$$
$$=\frac{16\,a^4dx\cos\left(dx+c\right)^2+13\,a^4\cos\left(dx+c\right)^2\log\left(\sin\left(dx+c\right)+1\right)-13\,a^4\cos\left(dx+c\right)^2\log\left(-\sin\left(dx+c\right)+\right.}{4\,d\cos\left(dx+c\right)^2}$$

input
```
integrate(cos(d*x+c)*(a+a*sec(d*x+c))^4,x, algorithm="fricas")
```

output
```
1/4*(16*a^4*d*x*cos(d*x + c)^2 + 13*a^4*cos(d*x + c)^2*log(sin(d*x + c) +
1) - 13*a^4*cos(d*x + c)^2*log(-sin(d*x + c) + 1) + 2*(2*a^4*cos(d*x + c)^
2 + 8*a^4*cos(d*x + c) + a^4)*sin(d*x + c))/(d*cos(d*x + c)^2)
```

3.34.6 Sympy [F]

$$\int \cos(c+dx)(a+a\sec(c+dx))^4\,dx = a^4\left(\int 4\cos(c+dx)\sec(c+dx)\,dx + \int 6\cos(c+dx)\sec^2(c+dx)\,dx + \int 4\cos(c+dx)\sec^3(c+dx)\,dx + \int \cos(c+dx)\sec^4(c+dx)\,dx + \int \cos(c+dx)\,dx\right)$$

input
```
integrate(cos(d*x+c)*(a+a*sec(d*x+c))**4,x)
```

output
```
a**4*(Integral(4*cos(c + d*x)*sec(c + d*x), x) + Integral(6*cos(c + d*x)*s
ec(c + d*x)**2, x) + Integral(4*cos(c + d*x)*sec(c + d*x)**3, x) + Integra
l(cos(c + d*x)*sec(c + d*x)**4, x) + Integral(cos(c + d*x), x))
```

3.34.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.51

$$\int \cos(c+dx)(a+a\sec(c+dx))^4\,dx = \frac{16\,(dx+c)a^4 - a^4\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1} - \log(\sin(dx+c)+1) + \log(\sin(dx+c)-1)\right) + 12\,a^4(\log(\sin(dx+c)}{4\,d}$$

input
```
integrate(cos(d*x+c)*(a+a*sec(d*x+c))^4,x, algorithm="maxima")
```

output
```
1/4*(16*(d*x + c)*a^4 - a^4*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin
(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 12*a^4*(log(sin(d*x + c) + 1) -
log(sin(d*x + c) - 1)) + 4*a^4*sin(d*x + c) + 16*a^4*tan(d*x + c))/d
```

3.34.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.77

$$\int \cos(c+dx)(a+a\sec(c+dx))^4\,dx$$
$$= \frac{8\,(dx+c)a^4 + 13\,a^4\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right) - 13\,a^4\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right) + \frac{4\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1} - \frac{2}{}}{2\,d}$$

input
```
integrate(cos(d*x+c)*(a+a*sec(d*x+c))^4,x, algorithm="giac")
```

output
```
1/2*(8*(d*x + c)*a^4 + 13*a^4*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 13*a^4*
log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 4*a^4*tan(1/2*d*x + 1/2*c)/(tan(1/2*d
*x + 1/2*c)^2 + 1) - 2*(7*a^4*tan(1/2*d*x + 1/2*c)^3 - 9*a^4*tan(1/2*d*x +
 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^2)/d
```

3.34.9 Mupad [B] (verification not implemented)

Time = 13.42 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.58

$$\int \cos(c+dx)(a+a\sec(c+dx))^4\,dx$$
$$= 4\,a^4\,x + \frac{13\,a^4\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d} + \frac{5\,a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5 + 2\,a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3 - 11\,a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\left(-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6 + \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4 + \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2 - 1\right)}$$

input
```
int(cos(c + d*x)*(a + a/cos(c + d*x))^4,x)
```

output
```
4*a^4*x + (13*a^4*atanh(tan(c/2 + (d*x)/2)))/d + (2*a^4*tan(c/2 + (d*x)/2)
^3 + 5*a^4*tan(c/2 + (d*x)/2)^5 - 11*a^4*tan(c/2 + (d*x)/2))/(d*(tan(c/2 +
 (d*x)/2)^2 + tan(c/2 + (d*x)/2)^4 - tan(c/2 + (d*x)/2)^6 - 1))
```

3.35 $\int \cos^2(c+dx)(a+a\sec(c+dx))^4\,dx$

3.35.1 Optimal result . . . 503
3.35.2 Mathematica [B] (verified) . . . 504
3.35.3 Rubi [A] (verified) . . . 504
3.35.4 Maple [A] (verified) . . . 506
3.35.5 Fricas [A] (verification not implemented) . . . 506
3.35.6 Sympy [F] . . . 507
3.35.7 Maxima [A] (verification not implemented) . . . 507
3.35.8 Giac [A] (verification not implemented) . . . 508
3.35.9 Mupad [B] (verification not implemented) . . . 508

3.35.1 Optimal result

Integrand size = 21, antiderivative size = 73

$$\int \cos^2(c+dx)(a+a\sec(c+dx))^4\,dx = \frac{13a^4x}{2}+\frac{4a^4\operatorname{arctanh}(\sin(c+dx))}{d}+\frac{4a^4\sin(c+dx)}{d}+\frac{a^4\cos(c+dx)\sin(c+dx)}{2d}+\frac{a^4\tan(c+dx)}{d}$$

output
```
13/2*a^4*x+4*a^4*arctanh(sin(d*x+c))/d+4*a^4*sin(d*x+c)/d+1/2*a^4*cos(d*x+
c)*sin(d*x+c)/d+a^4*tan(d*x+c)/d
```

3.35.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 241 vs. $2(73) = 146$.

Time = 4.82 (sec) , antiderivative size = 241, normalized size of antiderivative = 3.30

$$\int \cos^2(c+dx)(a+a\sec(c+dx))^4\,dx$$
$$= \frac{1}{64}a^4(1+\cos(c+dx))^4\sec^8\left(\frac{1}{2}(c+dx)\right)\left(26x - \frac{16\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}{d} + \frac{16\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}{d} + \frac{16\cos(dx)\sin(c)}{d} + \frac{\cos(2dx)\sin(2c)}{d} + \frac{16\cos(c)\sin(dx)}{d} + \frac{\cos(2c)\sin(2dx)}{d} + \frac{4\sin\left(\frac{dx}{2}\right)}{d\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)} + \frac{4\sin\left(\frac{dx}{2}\right)}{d\left(\cos\left(\frac{c}{2}\right)+\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}\right)$$

input
```
Integrate[Cos[c + d*x]^2*(a + a*Sec[c + d*x])^4,x]
```

output
```
(a^4*(1 + Cos[c + d*x])^4*Sec[(c + d*x)/2]^8*(26*x - (16*Log[Cos[(c + d*x)
/2] - Sin[(c + d*x)/2]])/d + (16*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]])
/d + (16*Cos[d*x]*Sin[c])/d + (Cos[2*d*x]*Sin[2*c])/d + (16*Cos[c]*Sin[d*x
])/d + (Cos[2*c]*Sin[2*d*x])/d + (4*Sin[(d*x)/2])/(d*(Cos[c/2] - Sin[c/2])
*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])) + (4*Sin[(d*x)/2])/(d*(Cos[c/2] +
Sin[c/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))))/64
```

3.35.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3042, 4278, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(c+dx)(a\sec(c+dx)+a)^4\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^4}{\csc\left(c+dx+\frac{\pi}{2}\right)^2}dx$$

$$\Big\downarrow \text{4278}$$

$$\int \left(a^4\cos^2(c+dx)+4a^4\cos(c+dx)+a^4\sec^2(c+dx)+4a^4\sec(c+dx)+6a^4\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{4a^4\text{arctanh}(\sin(c+dx))}{d}+\frac{4a^4\sin(c+dx)}{d}+\frac{a^4\tan(c+dx)}{d}+\frac{a^4\sin(c+dx)\cos(c+dx)}{2d}+\frac{13a^4x}{2}$$

input
```
Int[Cos[c + d*x]^2*(a + a*Sec[c + d*x])^4,x]
```

output
```
(13*a^4*x)/2 + (4*a^4*ArcTanh[Sin[c + d*x]])/d + (4*a^4*Sin[c + d*x])/d +
(a^4*Cos[c + d*x]*Sin[c + d*x])/(2*d) + (a^4*Tan[c + d*x])/d
```

3.35.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4278
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

3.35.4 Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.12

method	result
derivativedivides	$\frac{a^4 \tan(dx+c)+4a^4 \ln(\sec(dx+c)+\tan(dx+c))+6a^4(dx+c)+4a^4 \sin(dx+c)+a^4\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}$
default	$\frac{a^4 \tan(dx+c)+4a^4 \ln(\sec(dx+c)+\tan(dx+c))+6a^4(dx+c)+4a^4 \sin(dx+c)+a^4\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}$
parallelrisch	$\frac{a^4\left(52dx\cos(dx+c)-32\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\cos(dx+c)+32\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)\cos(dx+c)+\sin(3dx+3c)+16\sin(2dx+\right.}{8d\cos(dx+c)}$
risch	$\frac{13a^4x}{2}-\frac{ia^4e^{2i(dx+c)}}{8d}-\frac{2ia^4e^{i(dx+c)}}{d}+\frac{2ia^4e^{-i(dx+c)}}{d}+\frac{ia^4e^{-2i(dx+c)}}{8d}+\frac{2ia^4}{d(e^{2i(dx+c)}+1)}-\frac{4a^4\ln(e^{i(dx+c)}}{d}$
norman	$\frac{-\frac{13a^4x}{2}-\frac{11a^4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{20a^4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{d}-\frac{2a^4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{d}-\frac{12a^4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{d}+\frac{5a^4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^9}{d}+\frac{13a^4x\tan\left(\frac{dx}{2}+\right.}{2}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^2\left(-1+\tan\left(\frac{dx}{2}\right.\right.}$

input
```
int(cos(d*x+c)^2*(a+a*sec(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^4*tan(d*x+c)+4*a^4*ln(sec(d*x+c)+tan(d*x+c))+6*a^4*(d*x+c)+4*a^4*si
n(d*x+c)+a^4*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c))
```

3.35.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.44

$$\int \cos^2(c+dx)(a+a\sec(c+dx))^4\,dx$$
$$= \frac{13\,a^4dx\cos(dx+c)+4\,a^4\cos(dx+c)\log(\sin(dx+c)+1)-4\,a^4\cos(dx+c)\log(-\sin(dx+c)+1)+}{2\,d\cos(dx+c)}$$

input
```
integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))^4,x, algorithm="fricas")
```

output
```
1/2*(13*a^4*d*x*cos(d*x + c) + 4*a^4*cos(d*x + c)*log(sin(d*x + c) + 1) -
4*a^4*cos(d*x + c)*log(-sin(d*x + c) + 1) + (a^4*cos(d*x + c)^2 + 8*a^4*co
s(d*x + c) + 2*a^4)*sin(d*x + c))/(d*cos(d*x + c))
```

3.35.6 Sympy [F]

$$\int \cos^2(c+dx)(a+a\sec(c+dx))^4\,dx = a^4\left(\int 4\cos^2(c+dx)\sec(c+dx)\,dx + \int 6\cos^2(c+dx)\sec^2(c+dx)\,dx + \int 4\cos^2(c+dx)\sec^3(c+dx)\,dx + \int \cos^2(c+dx)\sec^4(c+dx)\,dx + \int \cos^2(c+dx)\,dx\right)$$

input
```
integrate(cos(d*x+c)**2*(a+a*sec(d*x+c))**4,x)
```

output
```
a**4*(Integral(4*cos(c + d*x)**2*sec(c + d*x), x) + Integral(6*cos(c + d*x
)**2*sec(c + d*x)**2, x) + Integral(4*cos(c + d*x)**2*sec(c + d*x)**3, x)
+ Integral(cos(c + d*x)**2*sec(c + d*x)**4, x) + Integral(cos(c + d*x)**2,
 x))
```

3.35.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.16

$$\int \cos^2(c+dx)(a+a\sec(c+dx))^4\,dx = \frac{(2\,dx+2\,c+\sin(2\,dx+2\,c))a^4+24\,(dx+c)a^4+8\,a^4(\log(\sin(dx+c)+1)-\log(\sin(dx+c)-1))+1}{4\,d}$$

input
```
integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))^4,x, algorithm="maxima")
```

output
```
1/4*((2*d*x + 2*c + sin(2*d*x + 2*c))*a^4 + 24*(d*x + c)*a^4 + 8*a^4*(log(
sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 16*a^4*sin(d*x + c) + 4*a^4*t
an(d*x + c))/d
```

3.35.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.77

$$\int \cos^2(c+dx)(a+a\sec(c+dx))^4\,dx$$
$$= \frac{13\,(dx+c)a^4 + 8\,a^4\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right) - 8\,a^4\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right) - \frac{4\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1} + \frac{2\left(\right.}{}}{2\,d}$$

input
```
integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))^4,x, algorithm="giac")
```

output
```
1/2*(13*(d*x + c)*a^4 + 8*a^4*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 8*a^4*l
og(abs(tan(1/2*d*x + 1/2*c) - 1)) - 4*a^4*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*
x + 1/2*c)^2 - 1) + 2*(7*a^4*tan(1/2*d*x + 1/2*c)^3 + 9*a^4*tan(1/2*d*x +
1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^2)/d
```

3.35.9 Mupad [B] (verification not implemented)

Time = 13.43 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.60

$$\int \cos^2(c+dx)(a+a\sec(c+dx))^4\,dx$$
$$= \frac{13\,a^4\,x}{2} + \frac{8\,a^4\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d} + \frac{-5\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5 + 2\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3 + 11\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\left(-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6 - \tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4 + \tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2 + 1\right)}$$

input
```
int(cos(c + d*x)^2*(a + a/cos(c + d*x))^4,x)
```

output
```
(13*a^4*x)/2 + (8*a^4*atanh(tan(c/2 + (d*x)/2)))/d + (2*a^4*tan(c/2 + (d*x
)/2)^3 - 5*a^4*tan(c/2 + (d*x)/2)^5 + 11*a^4*tan(c/2 + (d*x)/2))/(d*(tan(c
/2 + (d*x)/2)^2 - tan(c/2 + (d*x)/2)^4 - tan(c/2 + (d*x)/2)^6 + 1))
```

3.36 $\int \cos^3(c+dx)(a+a\sec(c+dx))^4\,dx$

3.36.1 Optimal result 509
3.36.2 Mathematica [A] (verified) 509
3.36.3 Rubi [A] (verified) 510
3.36.4 Maple [A] (verified) 511
3.36.5 Fricas [A] (verification not implemented) 511
3.36.6 Sympy [F] 512
3.36.7 Maxima [A] (verification not implemented) 512
3.36.8 Giac [A] (verification not implemented) 513
3.36.9 Mupad [B] (verification not implemented) 513

3.36.1 Optimal result

Integrand size = 21, antiderivative size = 73

$$\int \cos^3(c+dx)(a+a\sec(c+dx))^4\,dx = 6a^4x+\frac{a^4\text{arctanh}(\sin(c+dx))}{d}+\frac{7a^4\sin(c+dx)}{d}+\frac{2a^4\cos(c+dx)\sin(c+dx)}{d}-\frac{a^4\sin^3(c+dx)}{3d}$$

output
```
6*a^4*x+a^4*arctanh(sin(d*x+c))/d+7*a^4*sin(d*x+c)/d+2*a^4*cos(d*x+c)*sin(
d*x+c)/d-1/3*a^4*sin(d*x+c)^3/d
```

3.36.2 Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.25

$$\int \cos^3(c+dx)(a+a\sec(c+dx))^4\,dx = \frac{a^4\left(72dx-12\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+12\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)+81\,\text{si}}{12d}$$

input
```
Integrate[Cos[c + d*x]^3*(a + a*Sec[c + d*x])^4,x]
```

output
```
(a^4*(72*d*x - 12*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 12*Log[Cos[(c
 + d*x)/2] + Sin[(c + d*x)/2]] + 81*Sin[c + d*x] + 12*Sin[2*(c + d*x)] + S
in[3*(c + d*x)]))/(12*d)
```

3.36.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3042, 4278, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^3(c+dx)(a\sec(c+dx)+a)^4\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^4}{\csc\left(c+dx+\frac{\pi}{2}\right)^3}dx$$

$$\Big\downarrow \text{4278}$$

$$\int \left(a^4\cos^3(c+dx)+4a^4\cos^2(c+dx)+6a^4\cos(c+dx)+a^4\sec(c+dx)+4a^4\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^4\text{arctanh}(\sin(c+dx))}{d}-\frac{a^4\sin^3(c+dx)}{3d}+\frac{7a^4\sin(c+dx)}{d}+\frac{2a^4\sin(c+dx)\cos(c+dx)}{d}+6a^4x$$

input
```
Int[Cos[c + d*x]^3*(a + a*Sec[c + d*x])^4,x]
```

output
```
6*a^4*x + (a^4*ArcTanh[Sin[c + d*x]])/d + (7*a^4*Sin[c + d*x])/d + (2*a^4*
Cos[c + d*x]*Sin[c + d*x])/d - (a^4*Sin[c + d*x]^3)/(3*d)
```

3.36.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4278
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

3.36.4 Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.96

method	result
parallelrisch	$\frac{a^4\left(72dx+12\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-12\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+81\sin(dx+c)+\sin(3dx+3c)+12\sin(2dx+2c)\right)}{12d}$
derivativedivides	$\frac{a^4\ln(\sec(dx+c)+\tan(dx+c))+4a^4(dx+c)+6a^4\sin(dx+c)+4a^4\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+\frac{a^4\left(2+\cos(dx+c)^2\right)\sin(dx}{3}}{d}$
default	$\frac{a^4\ln(\sec(dx+c)+\tan(dx+c))+4a^4(dx+c)+6a^4\sin(dx+c)+4a^4\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+\frac{a^4\left(2+\cos(dx+c)^2\right)\sin(dx}{3}}{d}$
risch	$6a^4x-\frac{27ia^4e^{i(dx+c)}}{8d}+\frac{27ia^4e^{-i(dx+c)}}{8d}+\frac{a^4\ln\left(e^{i(dx+c)}+i\right)}{d}-\frac{a^4\ln\left(e^{i(dx+c)}-i\right)}{d}+\frac{a^4\sin(3dx+3c)}{12d}+\frac{a^4\sin}{}$
norman	$\frac{-6a^4x-\frac{18a^4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{86a^4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3d}+\frac{12a^4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{d}-\frac{28a^4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{d}-\frac{14a^4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^9}{3d}+\frac{10a^4\tan\left(\frac{dx}{2}+}{d}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^3\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^3}$

input
```
int(cos(d*x+c)^3*(a+a*sec(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
1/12*a^4*(72*d*x+12*ln(tan(1/2*d*x+1/2*c)+1)-12*ln(tan(1/2*d*x+1/2*c)-1)+8
1*sin(d*x+c)+sin(3*d*x+3*c)+12*sin(2*d*x+2*c))/d
```

3.36.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.10

$$\int \cos^3(c+dx)(a+a\sec(c+dx))^4\,dx$$
$$=\frac{36\,a^4dx+3\,a^4\log\left(\sin\left(dx+c\right)+1\right)-3\,a^4\log\left(-\sin\left(dx+c\right)+1\right)+2\left(a^4\cos\left(dx+c\right)^2+6\,a^4\cos\left(dx+c\right.\right.}{6\,d}$$

input
```
integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))^4,x, algorithm="fricas")
```

output
```
1/6*(36*a^4*d*x + 3*a^4*log(sin(d*x + c) + 1) - 3*a^4*log(-sin(d*x + c) +
1) + 2*(a^4*cos(d*x + c)^2 + 6*a^4*cos(d*x + c) + 20*a^4)*sin(d*x + c))/d
```

3.36.6 Sympy [F]

$$\int \cos^3(c+dx)(a+a\sec(c+dx))^4\,dx = a^4\left(\int 4\cos^3(c+dx)\sec(c+dx)\,dx + \int 6\cos^3(c+dx)\sec^2(c+dx)\,dx + \int 4\cos^3(c+dx)\sec^3(c+dx)\,dx + \int \cos^3(c+dx)\sec^4(c+dx)\,dx + \int \cos^3(c+dx)\,dx\right)$$

input
```
integrate(cos(d*x+c)**3*(a+a*sec(d*x+c))**4,x)
```

output
```
a**4*(Integral(4*cos(c + d*x)**3*sec(c + d*x), x) + Integral(6*cos(c + d*x
)**3*sec(c + d*x)**2, x) + Integral(4*cos(c + d*x)**3*sec(c + d*x)**3, x)
+ Integral(cos(c + d*x)**3*sec(c + d*x)**4, x) + Integral(cos(c + d*x)**3,
 x))
```

3.36.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.33

$$\int \cos^3(c+dx)(a+a\sec(c+dx))^4\,dx = -\frac{2\left(\sin\left(dx+c\right)^3-3\,\sin\left(dx+c\right)\right)a^4-6\left(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right)\right)a^4-24\left(dx+c\right)a^4-3\,a^4(\log\left(\sin\right.}{6\,d}$$

input
```
integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))^4,x, algorithm="maxima")
```

output
```
-1/6*(2*(sin(d*x + c)^3 - 3*sin(d*x + c))*a^4 - 6*(2*d*x + 2*c + sin(2*d*x
 + 2*c))*a^4 - 24*(d*x + c)*a^4 - 3*a^4*(log(sin(d*x + c) + 1) - log(sin(d
*x + c) - 1)) - 36*a^4*sin(d*x + c))/d
```

3.36.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.59

$$\int \cos^3(c+dx)(a+a\sec(c+dx))^4\,dx$$
$$= \frac{18\,(dx+c)a^4 + 3\,a^4\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right) - 3\,a^4\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right) + \frac{2\left(15\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+3\right.}{\left(t\right.}}{3\,d}$$

input
```
integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))^4,x, algorithm="giac")
```

output
```
1/3*(18*(d*x + c)*a^4 + 3*a^4*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*a^4*l
og(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(15*a^4*tan(1/2*d*x + 1/2*c)^5 + 38*
a^4*tan(1/2*d*x + 1/2*c)^3 + 27*a^4*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1
/2*c)^2 + 1)^3)/d
```

3.36.9 Mupad [B] (verification not implemented)

Time = 12.83 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.27

$$\int \cos^3(c+dx)(a+a\sec(c+dx))^4\,dx = 6\,a^4\,x + \frac{20\,a^4\,\sin(c+d\,x)}{3\,d} + \frac{2\,a^4\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d} + \frac{a^4\cos(c+d\,x)^2\,\sin(c+d\,x)}{3\,d} + \frac{2\,a^4\,\cos(c+d\,x)\,\sin(c+d\,x)}{d}$$

input
```
int(cos(c + d*x)^3*(a + a/cos(c + d*x))^4,x)
```

output
```
6*a^4*x + (20*a^4*sin(c + d*x))/(3*d) + (2*a^4*atanh(sin(c/2 + (d*x)/2)/co
s(c/2 + (d*x)/2)))/d + (a^4*cos(c + d*x)^2*sin(c + d*x))/(3*d) + (2*a^4*co
s(c + d*x)*sin(c + d*x))/d
```

3.37 $\int \cos^4(c+dx)(a+a\sec(c+dx))^4\,dx$

3.37.1 Optimal result . . . 514
3.37.2 Mathematica [A] (verified) . . . 514
3.37.3 Rubi [A] (verified) . . . 515
3.37.4 Maple [A] (verified) . . . 516
3.37.5 Fricas [A] (verification not implemented) . . . 516
3.37.6 Sympy [F] . . . 517
3.37.7 Maxima [A] (verification not implemented) . . . 517
3.37.8 Giac [A] (verification not implemented) . . . 518
3.37.9 Mupad [B] (verification not implemented) . . . 518

3.37.1 Optimal result

Integrand size = 21, antiderivative size = 87

$$\int \cos^4(c+dx)(a+a\sec(c+dx))^4\,dx = \frac{35a^4x}{8}+\frac{8a^4\sin(c+dx)}{d}+\frac{27a^4\cos(c+dx)\sin(c+dx)}{8d}+\frac{a^4\cos^3(c+dx)\sin(c+dx)}{4d}-\frac{4a^4\sin^3(c+dx)}{3d}$$

output
```
35/8*a^4*x+8*a^4*sin(d*x+c)/d+27/8*a^4*cos(d*x+c)*sin(d*x+c)/d+1/4*a^4*cos
(d*x+c)^3*sin(d*x+c)/d-4/3*a^4*sin(d*x+c)^3/d
```

3.37.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.64

$$\int \cos^4(c+dx)(a+a\sec(c+dx))^4\,dx = \frac{a^4(420c+420dx+672\sin(c+dx)+168\sin(2(c+dx))+32\sin(3(c+dx))+3\sin(4(c+dx)))}{96d}$$

input
```
Integrate[Cos[c + d*x]^4*(a + a*Sec[c + d*x])^4,x]
```

output
```
(a^4*(420*c + 420*d*x + 672*Sin[c + d*x] + 168*Sin[2*(c + d*x)] + 32*Sin[3
*(c + d*x)] + 3*Sin[4*(c + d*x)]))/(96*d)
```

3.37.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3042, 4278, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^4(c+dx)(a\sec(c+dx)+a)^4\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^4}{\csc\left(c+dx+\frac{\pi}{2}\right)^4}dx$$

$$\Big\downarrow \text{4278}$$

$$\int \left(a^4\cos^4(c+dx)+4a^4\cos^3(c+dx)+6a^4\cos^2(c+dx)+4a^4\cos(c+dx)+a^4\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{4a^4\sin^3(c+dx)}{3d}+\frac{8a^4\sin(c+dx)}{d}+\frac{a^4\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{27a^4\sin(c+dx)\cos(c+dx)}{8d}+\frac{35a^4x}{8}$$

input
```
Int[Cos[c + d*x]^4*(a + a*Sec[c + d*x])^4,x]
```

output
```
(35*a^4*x)/8 + (8*a^4*Sin[c + d*x])/d + (27*a^4*Cos[c + d*x]*Sin[c + d*x])
/(8*d) + (a^4*Cos[c + d*x]^3*Sin[c + d*x])/(4*d) - (4*a^4*Sin[c + d*x]^3)/
(3*d)
```

3.37.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4278
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

3.37.4 Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.63

method	result
parallelrisch	$\frac{a^4(420dx+672\sin(dx+c)+3\sin(4dx+4c)+32\sin(3dx+3c)+168\sin(2dx+2c))}{96d}$
risch	$\frac{35a^4x}{8}+\frac{7a^4\sin(dx+c)}{d}+\frac{a^4\sin(4dx+4c)}{32d}+\frac{a^4\sin(3dx+3c)}{3d}+\frac{7a^4\sin(2dx+2c)}{4d}$
derivativedivides	$\frac{a^4(dx+c)+4a^4\sin(dx+c)+6a^4\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+\frac{4a^4\left(2+\cos(dx+c)^2\right)\sin(dx+c)}{3}+a^4\left(\frac{\left(\cos(dx+c)^3+\frac{3\cos(dx+}{2}\right.}{4}\right.}{d}$
default	$\frac{a^4(dx+c)+4a^4\sin(dx+c)+6a^4\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+\frac{4a^4\left(2+\cos(dx+c)^2\right)\sin(dx+c)}{3}+a^4\left(\frac{\left(\cos(dx+c)^3+\frac{3\cos(dx+}{2}\right.}{4}\right.}{d}$
norman	$\frac{-\frac{35a^4x}{8}-\frac{93a^4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d}+\frac{163a^4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{6d}+\frac{311a^4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{12d}-\frac{17a^4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{d}-\frac{329a^4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^9}{12d}+\frac{35a^4\tan(}{}}{}$

input
```
int(cos(d*x+c)^4*(a+a*sec(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
1/96*a^4*(420*d*x+672*sin(d*x+c)+3*sin(4*d*x+4*c)+32*sin(3*d*x+3*c)+168*si
n(2*d*x+2*c))/d
```

3.37.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.72

$$\int \cos^4(c+dx)(a+a\sec(c+dx))^4\,dx$$
$$=\frac{105\,a^4dx+\left(6\,a^4\cos\left(dx+c\right)^3+32\,a^4\cos\left(dx+c\right)^2+81\,a^4\cos\left(dx+c\right)+160\,a^4\right)\sin\left(dx+c\right)}{24\,d}$$

input
```
integrate(cos(d*x+c)^4*(a+a*sec(d*x+c))^4,x, algorithm="fricas")
```

output
```
1/24*(105*a^4*d*x + (6*a^4*cos(d*x + c)^3 + 32*a^4*cos(d*x + c)^2 + 81*a^4
*cos(d*x + c) + 160*a^4)*sin(d*x + c))/d
```

3.37.6 Sympy [F]

$$\int \cos^4(c+dx)(a+a\sec(c+dx))^4\,dx = a^4\left(\int 4\cos^4(c+dx)\sec(c+dx)\,dx + \int 6\cos^4(c+dx)\sec^2(c+dx)\,dx + \int 4\cos^4(c+dx)\sec^3(c+dx)\,dx + \int \cos^4(c+dx)\sec^4(c+dx)\,dx + \int \cos^4(c+dx)\,dx\right)$$

input
```
integrate(cos(d*x+c)**4*(a+a*sec(d*x+c))**4,x)
```

output
```
a**4*(Integral(4*cos(c + d*x)**4*sec(c + d*x), x) + Integral(6*cos(c + d*x
)**4*sec(c + d*x)**2, x) + Integral(4*cos(c + d*x)**4*sec(c + d*x)**3, x)
+ Integral(cos(c + d*x)**4*sec(c + d*x)**4, x) + Integral(cos(c + d*x)**4,
 x))
```

3.37.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.20

$$\int \cos^4(c+dx)(a+a\sec(c+dx))^4\,dx = -\frac{128\left(\sin(dx+c)^3-3\sin(dx+c)\right)a^4-3\left(12\,dx+12\,c+\sin(4\,dx+4\,c)+8\sin(2\,dx+2\,c)\right)a^4-144}{96\,d}$$

input
```
integrate(cos(d*x+c)^4*(a+a*sec(d*x+c))^4,x, algorithm="maxima")
```

output
```
-1/96*(128*(sin(d*x + c)^3 - 3*sin(d*x + c))*a^4 - 3*(12*d*x + 12*c + sin(
4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*a^4 - 144*(2*d*x + 2*c + sin(2*d*x + 2*
c))*a^4 - 96*(d*x + c)*a^4 - 384*a^4*sin(d*x + c))/d
```

3.37.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.10

$$\int \cos^4(c+dx)(a+a\sec(c+dx))^4\,dx = \frac{105\,(dx+c)a^4 + \frac{2\left(105\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+385\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+511\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+279\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^4}}{24\,d}$$

input
```
integrate(cos(d*x+c)^4*(a+a*sec(d*x+c))^4,x, algorithm="giac")
```

output
```
1/24*(105*(d*x + c)*a^4 + 2*(105*a^4*tan(1/2*d*x + 1/2*c)^7 + 385*a^4*tan(
1/2*d*x + 1/2*c)^5 + 511*a^4*tan(1/2*d*x + 1/2*c)^3 + 279*a^4*tan(1/2*d*x
+ 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^4)/d
```

3.37.9 Mupad [B] (verification not implemented)

Time = 16.64 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.02

$$\int \cos^4(c+dx)(a+a\sec(c+dx))^4\,dx = \frac{35\,a^4\,x}{8} + \frac{\frac{35\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{4} + \frac{385\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{12} + \frac{511\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{12} + \frac{93\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)^4}$$

input
```
int(cos(c + d*x)^4*(a + a/cos(c + d*x))^4,x)
```

output
```
(35*a^4*x)/8 + ((511*a^4*tan(c/2 + (d*x)/2)^3)/12 + (385*a^4*tan(c/2 + (d*
x)/2)^5)/12 + (35*a^4*tan(c/2 + (d*x)/2)^7)/4 + (93*a^4*tan(c/2 + (d*x)/2)
)/4)/(d*(tan(c/2 + (d*x)/2)^2 + 1)^4)
```

3.38 $\int \cos^5(c+dx)(a+a\sec(c+dx))^4\,dx$

3.38.1 Optimal result . . . 519
3.38.2 Mathematica [A] (verified) . . . 519
3.38.3 Rubi [A] (verified) . . . 520
3.38.4 Maple [A] (verified) . . . 521
3.38.5 Fricas [A] (verification not implemented) . . . 522
3.38.6 Sympy [F(-1)] . . . 522
3.38.7 Maxima [A] (verification not implemented) . . . 522
3.38.8 Giac [A] (verification not implemented) . . . 523
3.38.9 Mupad [B] (verification not implemented) . . . 523

3.38.1 Optimal result

Integrand size = 21, antiderivative size = 102

$$\int \cos^5(c+dx)(a+a\sec(c+dx))^4\,dx = \frac{7a^4x}{2}+\frac{8a^4\sin(c+dx)}{d}+\frac{7a^4\cos(c+dx)\sin(c+dx)}{2d}+\frac{a^4\cos^3(c+dx)\sin(c+dx)}{d}-\frac{8a^4\sin^3(c+dx)}{3d}+\frac{a^4\sin^5(c+dx)}{5d}$$

output
```
7/2*a^4*x+8*a^4*sin(d*x+c)/d+7/2*a^4*cos(d*x+c)*sin(d*x+c)/d+a^4*cos(d*x+c
)^3*sin(d*x+c)/d-8/3*a^4*sin(d*x+c)^3/d+1/5*a^4*sin(d*x+c)^5/d
```

3.38.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.62

$$\int \cos^5(c+dx)(a+a\sec(c+dx))^4\,dx = \frac{a^4(840dx+1470\sin(c+dx)+480\sin(2(c+dx))+145\sin(3(c+dx))+30\sin(4(c+dx))+3\sin(5(c+}{240d}$$

input
```
Integrate[Cos[c + d*x]^5*(a + a*Sec[c + d*x])^4,x]
```

output
```
(a^4*(840*d*x + 1470*Sin[c + d*x] + 480*Sin[2*(c + d*x)] + 145*Sin[3*(c +
d*x)] + 30*Sin[4*(c + d*x)] + 3*Sin[5*(c + d*x)]))/(240*d)
```

3.38.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3042, 4278, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^5(c+dx)(a\sec(c+dx)+a)^4\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^4}{\csc\left(c+dx+\frac{\pi}{2}\right)^5}dx$$

$$\Big\downarrow \text{4278}$$

$$\int \left(a^4\cos^5(c+dx)+4a^4\cos^4(c+dx)+6a^4\cos^3(c+dx)+4a^4\cos^2(c+dx)+a^4\cos(c+dx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^4\sin^5(c+dx)}{5d}-\frac{8a^4\sin^3(c+dx)}{3d}+\frac{8a^4\sin(c+dx)}{d}+\frac{a^4\sin(c+dx)\cos^3(c+dx)}{d}+\frac{7a^4\sin(c+dx)\cos(c+dx)}{2d}+\frac{7a^4x}{2}$$

input
```
Int[Cos[c + d*x]^5*(a + a*Sec[c + d*x])^4,x]
```

output
```
(7*a^4*x)/2 + (8*a^4*Sin[c + d*x])/d + (7*a^4*Cos[c + d*x]*Sin[c + d*x])/(
2*d) + (a^4*Cos[c + d*x]^3*Sin[c + d*x])/d - (8*a^4*Sin[c + d*x]^3)/(3*d)
+ (a^4*Sin[c + d*x]^5)/(5*d)
```

3.38.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4278
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

3.38.4 Maple [A] (verified)

Time = 0.73 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.65

method	result
parallelrisch	$\frac{a^4(840dx+3\sin(5dx+5c)+30\sin(4dx+4c)+145\sin(3dx+3c)+480\sin(2dx+2c)+1470\sin(dx+c))}{240d}$
risch	$\frac{7a^4x}{2}+\frac{49a^4\sin(dx+c)}{8d}+\frac{a^4\sin(5dx+5c)}{80d}+\frac{a^4\sin(4dx+4c)}{8d}+\frac{29a^4\sin(3dx+3c)}{48d}+\frac{2a^4\sin(2dx+2c)}{d}$
derivativedivides	$\frac{a^4\sin(dx+c)+4a^4\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+2a^4\left(2+\cos(dx+c)^2\right)\sin(dx+c)+4a^4\left(\frac{\left(\cos(dx+c)^3+\frac{3\cos(dx+c)}{2}\right)\sin(}{4}\right.}{d}$
default	$\frac{a^4\sin(dx+c)+4a^4\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+2a^4\left(2+\cos(dx+c)^2\right)\sin(dx+c)+4a^4\left(\frac{\left(\cos(dx+c)^3+\frac{3\cos(dx+c)}{2}\right)\sin(}{4}\right.}{d}$
norman	$\frac{-\frac{7a^4x}{2}-\frac{25a^4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{67a^4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3d}+\frac{349a^4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{15d}+\frac{203a^4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{15d}-\frac{533a^4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^9}{15d}-\frac{259a^4\tan(}{15}}{}$

input
```
int(cos(d*x+c)^5*(a+a*sec(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
1/240*a^4*(840*d*x+3*sin(5*d*x+5*c)+30*sin(4*d*x+4*c)+145*sin(3*d*x+3*c)+4
80*sin(2*d*x+2*c)+1470*sin(d*x+c))/d
```

3.38.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.75

$$\int \cos^5(c+dx)(a+a\sec(c+dx))^4\,dx$$
$$= \frac{105\,a^4dx + \left(6\,a^4\cos\left(dx+c\right)^4 + 30\,a^4\cos\left(dx+c\right)^3 + 68\,a^4\cos\left(dx+c\right)^2 + 105\,a^4\cos\left(dx+c\right) + 166\,a^4\right)\mathrm{si}}{30\,d}$$

input
```
integrate(cos(d*x+c)^5*(a+a*sec(d*x+c))^4,x, algorithm="fricas")
```

output
```
1/30*(105*a^4*d*x + (6*a^4*cos(d*x + c)^4 + 30*a^4*cos(d*x + c)^3 + 68*a^4
*cos(d*x + c)^2 + 105*a^4*cos(d*x + c) + 166*a^4)*sin(d*x + c))/d
```

3.38.6 Sympy [F(-1)]

Timed out.

$$\int \cos^5(c+dx)(a+a\sec(c+dx))^4\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**5*(a+a*sec(d*x+c))**4,x)
```

output
```
Timed out
```

3.38.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.25

$$\int \cos^5(c+dx)(a+a\sec(c+dx))^4\,dx$$
$$= \frac{8\left(3\,\sin\left(dx+c\right)^5 - 10\,\sin\left(dx+c\right)^3 + 15\,\sin\left(dx+c\right)\right)a^4 - 240\left(\sin\left(dx+c\right)^3 - 3\,\sin\left(dx+c\right)\right)a^4 + 15\,(1}{}$$

input
```
integrate(cos(d*x+c)^5*(a+a*sec(d*x+c))^4,x, algorithm="maxima")
```

output
```
1/120*(8*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*a^4 - 24
0*(sin(d*x + c)^3 - 3*sin(d*x + c))*a^4 + 15*(12*d*x + 12*c + sin(4*d*x +
4*c) + 8*sin(2*d*x + 2*c))*a^4 + 120*(2*d*x + 2*c + sin(2*d*x + 2*c))*a^4
+ 120*a^4*sin(d*x + c))/d
```

3.38.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.10

$$\int \cos^5(c+dx)(a+a\sec(c+dx))^4\,dx$$
$$= \frac{105\,(dx+c)a^4 + \frac{2\left(105\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9+490\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+896\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+790\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+375\,a^4\tan\left(\frac{1}{2}\,dx+\right.\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^5}}{30\,d}$$

input
```
integrate(cos(d*x+c)^5*(a+a*sec(d*x+c))^4,x, algorithm="giac")
```

output
```
1/30*(105*(d*x + c)*a^4 + 2*(105*a^4*tan(1/2*d*x + 1/2*c)^9 + 490*a^4*tan(
1/2*d*x + 1/2*c)^7 + 896*a^4*tan(1/2*d*x + 1/2*c)^5 + 790*a^4*tan(1/2*d*x
+ 1/2*c)^3 + 375*a^4*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^5)
/d
```

3.38.9 Mupad [B] (verification not implemented)

Time = 16.86 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.03

$$\int \cos^5(c+dx)(a+a\sec(c+dx))^4\,dx = \frac{7\,a^4\,x}{2} + \frac{7\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9 + \frac{98\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{3} + \frac{896\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{15} + \frac{158\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{3} + 25\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)^5}$$

input
```
int(cos(c + d*x)^5*(a + a/cos(c + d*x))^4,x)
```

output
```
(7*a^4*x)/2 + ((158*a^4*tan(c/2 + (d*x)/2)^3)/3 + (896*a^4*tan(c/2 + (d*x)
/2)^5)/15 + (98*a^4*tan(c/2 + (d*x)/2)^7)/3 + 7*a^4*tan(c/2 + (d*x)/2)^9 +
 25*a^4*tan(c/2 + (d*x)/2))/(d*(tan(c/2 + (d*x)/2)^2 + 1)^5)
```

3.39 $\int \cos^6(c+dx)(a+a\sec(c+dx))^4\,dx$

3.39.1 Optimal result . . . 524
3.39.2 Mathematica [A] (verified) . . . 525
3.39.3 Rubi [A] (verified) . . . 525
3.39.4 Maple [A] (verified) . . . 526
3.39.5 Fricas [A] (verification not implemented) . . . 527
3.39.6 Sympy [F(-1)] . . . 527
3.39.7 Maxima [A] (verification not implemented) . . . 527
3.39.8 Giac [A] (verification not implemented) . . . 528
3.39.9 Mupad [B] (verification not implemented) . . . 528

3.39.1 Optimal result

Integrand size = 21, antiderivative size = 127

$$\int \cos^6(c+dx)(a+a\sec(c+dx))^4\,dx = \frac{49a^4x}{16}+\frac{8a^4\sin(c+dx)}{d}+\frac{49a^4\cos(c+dx)\sin(c+dx)}{16d}+\frac{41a^4\cos^3(c+dx)\sin(c+dx)}{24d}+\frac{a^4\cos^5(c+dx)\sin(c+dx)}{6d}-\frac{4a^4\sin^3(c+dx)}{d}+\frac{4a^4\sin^5(c+dx)}{5d}$$

output
```
49/16*a^4*x+8*a^4*sin(d*x+c)/d+49/16*a^4*cos(d*x+c)*sin(d*x+c)/d+41/24*a^4
*cos(d*x+c)^3*sin(d*x+c)/d+1/6*a^4*cos(d*x+c)^5*sin(d*x+c)/d-4*a^4*sin(d*x
+c)^3/d+4/5*a^4*sin(d*x+c)^5/d
```

3.39.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.57

$$\int \cos^6(c+dx)(a+a\sec(c+dx))^4\,dx$$
$$= \frac{a^4(2940dx+5280\sin(c+dx)+1905\sin(2(c+dx))+720\sin(3(c+dx))+225\sin(4(c+dx))+48\sin(5}{960d}$$

input
```
Integrate[Cos[c + d*x]^6*(a + a*Sec[c + d*x])^4,x]
```

output
```
(a^4*(2940*d*x + 5280*Sin[c + d*x] + 1905*Sin[2*(c + d*x)] + 720*Sin[3*(c
+ d*x)] + 225*Sin[4*(c + d*x)] + 48*Sin[5*(c + d*x)] + 5*Sin[6*(c + d*x)])
)/(960*d)
```

3.39.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3042, 4278, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^6(c+dx)(a\sec(c+dx)+a)^4\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^4}{\csc\left(c+dx+\frac{\pi}{2}\right)^6}dx$$
$$\Big\downarrow \text{4278}$$
$$\int \left(a^4\cos^6(c+dx)+4a^4\cos^5(c+dx)+6a^4\cos^4(c+dx)+4a^4\cos^3(c+dx)+a^4\cos^2(c+dx)\right)dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{4a^4\sin^5(c+dx)}{5d}-\frac{4a^4\sin^3(c+dx)}{d}+\frac{8a^4\sin(c+dx)}{d}+\frac{a^4\sin(c+dx)\cos^5(c+dx)}{6d}+\frac{41a^4\sin(c+dx)\cos^3(c+dx)}{24d}+\frac{49a^4\sin(c+dx)\cos(c+dx)}{16d}+\frac{49a^4x}{16}$$

input
```
Int[Cos[c + d*x]^6*(a + a*Sec[c + d*x])^4,x]
```

output
```
(49*a^4*x)/16 + (8*a^4*Sin[c + d*x])/d + (49*a^4*Cos[c + d*x]*Sin[c + d*x]
)/(16*d) + (41*a^4*Cos[c + d*x]^3*Sin[c + d*x])/(24*d) + (a^4*Cos[c + d*x]
^5*Sin[c + d*x])/(6*d) - (4*a^4*Sin[c + d*x]^3)/d + (4*a^4*Sin[c + d*x]^5)
/(5*d)
```

3.39.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4278
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

3.39.4 Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.59

method	result
parallelrisch	$\frac{\left(588dx+\sin(6dx+6c)+1056\sin(dx+c)+381\sin(2dx+2c)+144\sin(3dx+3c)+45\sin(4dx+4c)+\frac{48\sin(5dx+5c)}{5}\right)a^4}{192d}$
risch	$\frac{49a^4x}{16}+\frac{11a^4\sin(dx+c)}{2d}+\frac{a^4\sin(6dx+6c)}{192d}+\frac{a^4\sin(5dx+5c)}{20d}+\frac{15a^4\sin(4dx+4c)}{64d}+\frac{3a^4\sin(3dx+3c)}{4d}+\frac{127a}{}$
derivativedivides	$\frac{a^4\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+\frac{4a^4\left(2+\cos(dx+c)^2\right)\sin(dx+c)}{3}+6a^4\left(\frac{\left(\cos(dx+c)^3+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+}{d}$
default	$\frac{a^4\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+\frac{4a^4\left(2+\cos(dx+c)^2\right)\sin(dx+c)}{3}+6a^4\left(\frac{\left(\cos(dx+c)^3+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+}{d}$

input
```
int(cos(d*x+c)^6*(a+a*sec(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
1/192*(588*d*x+sin(6*d*x+6*c)+1056*sin(d*x+c)+381*sin(2*d*x+2*c)+144*sin(3
*d*x+3*c)+45*sin(4*d*x+4*c)+48/5*sin(5*d*x+5*c))*a^4/d
```

3.39.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.70

$$\int \cos^6(c+dx)(a+a\sec(c+dx))^4\,dx$$
$$= \frac{735\,a^4dx + \left(40\,a^4\cos\left(dx+c\right)^5 + 192\,a^4\cos\left(dx+c\right)^4 + 410\,a^4\cos\left(dx+c\right)^3 + 576\,a^4\cos\left(dx+c\right)^2 + 735\,a}{240\,d}$$

input
```
integrate(cos(d*x+c)^6*(a+a*sec(d*x+c))^4,x, algorithm="fricas")
```

output
```
1/240*(735*a^4*d*x + (40*a^4*cos(d*x + c)^5 + 192*a^4*cos(d*x + c)^4 + 410
*a^4*cos(d*x + c)^3 + 576*a^4*cos(d*x + c)^2 + 735*a^4*cos(d*x + c) + 1152
*a^4)*sin(d*x + c))/d
```

3.39.6 Sympy [F(-1)]

Timed out.

$$\int \cos^6(c+dx)(a+a\sec(c+dx))^4\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**6*(a+a*sec(d*x+c))**4,x)
```

output
```
Timed out
```

3.39.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.30

$$\int \cos^6(c+dx)(a+a\sec(c+dx))^4\,dx$$
$$= \frac{256\left(3\,\sin\left(dx+c\right)^5 - 10\,\sin\left(dx+c\right)^3 + 15\,\sin\left(dx+c\right)\right)a^4 - 5\left(4\,\sin\left(2\,dx+2\,c\right)^3 - 60\,dx - 60\,c - 9\,\sin}{}$$

input
```
integrate(cos(d*x+c)^6*(a+a*sec(d*x+c))^4,x, algorithm="maxima")
```

output
```
1/960*(256*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*a^4 -
5*(4*sin(2*d*x + 2*c)^3 - 60*d*x - 60*c - 9*sin(4*d*x + 4*c) - 48*sin(2*d*
x + 2*c))*a^4 - 1280*(sin(d*x + c)^3 - 3*sin(d*x + c))*a^4 + 180*(12*d*x +
 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*a^4 + 240*(2*d*x + 2*c + si
n(2*d*x + 2*c))*a^4)/d
```

3.39.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.01

$$\int \cos^6(c+dx)(a+a\sec(c+dx))^4\,dx$$
$$= \frac{735\,(dx+c)a^4 + \frac{2\left(735\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}+4165\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}+9702\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+11802\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+7355\,a^4\tan(\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{6}}}{240\,d}$$

input
```
integrate(cos(d*x+c)^6*(a+a*sec(d*x+c))^4,x, algorithm="giac")
```

output
```
1/240*(735*(d*x + c)*a^4 + 2*(735*a^4*tan(1/2*d*x + 1/2*c)^11 + 4165*a^4*t
an(1/2*d*x + 1/2*c)^9 + 9702*a^4*tan(1/2*d*x + 1/2*c)^7 + 11802*a^4*tan(1/
2*d*x + 1/2*c)^5 + 7355*a^4*tan(1/2*d*x + 1/2*c)^3 + 3105*a^4*tan(1/2*d*x
+ 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^6)/d
```

3.39.9 Mupad [B] (verification not implemented)

Time = 15.80 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.95

$$\int \cos^6(c+dx)(a+a\sec(c+dx))^4\,dx = \frac{49\,a^4\,x}{16} + \frac{\frac{49\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{8} + \frac{833\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}}{24} + \frac{1617\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}}{20} + \frac{1967\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{5}}{20} + \frac{1471\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}}{24} + \frac{207\,a^4\tan}{8}}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}+1\right)^{6}}$$

input
```
int(cos(c + d*x)^6*(a + a/cos(c + d*x))^4,x)
```

output
```
(49*a^4*x)/16 + ((1471*a^4*tan(c/2 + (d*x)/2)^3)/24 + (1967*a^4*tan(c/2 +
(d*x)/2)^5)/20 + (1617*a^4*tan(c/2 + (d*x)/2)^7)/20 + (833*a^4*tan(c/2 + (
d*x)/2)^9)/24 + (49*a^4*tan(c/2 + (d*x)/2)^11)/8 + (207*a^4*tan(c/2 + (d*x
)/2))/8)/(d*(tan(c/2 + (d*x)/2)^2 + 1)^6)
```

3.39. $\int \cos^6(c+dx)(a+a\sec(c+dx))^4\,dx$

3.40 $\int \cos^7(c+dx)(a+a\sec(c+dx))^4\,dx$

3.40.1 Optimal result . . . 530
3.40.2 Mathematica [A] (verified) . . . 531
3.40.3 Rubi [A] (verified) . . . 531
3.40.4 Maple [A] (verified) . . . 532
3.40.5 Fricas [A] (verification not implemented) . . . 533
3.40.6 Sympy [F(-1)] . . . 533
3.40.7 Maxima [A] (verification not implemented) . . . 534
3.40.8 Giac [A] (verification not implemented) . . . 534
3.40.9 Mupad [B] (verification not implemented) . . . 535

3.40.1 Optimal result

Integrand size = 21, antiderivative size = 147

$$\begin{aligned}\int \cos^7(c+dx)(a+a\sec(c+dx))^4\,dx = {} & \frac{11a^4x}{4}+\frac{8a^4\sin(c+dx)}{d} \\ & +\frac{11a^4\cos(c+dx)\sin(c+dx)}{4d} \\ & +\frac{11a^4\cos^3(c+dx)\sin(c+dx)}{6d} \\ & +\frac{2a^4\cos^5(c+dx)\sin(c+dx)}{3d}-\frac{16a^4\sin^3(c+dx)}{3d} \\ & +\frac{9a^4\sin^5(c+dx)}{5d}-\frac{a^4\sin^7(c+dx)}{7d}\end{aligned}$$

output
```
11/4*a^4*x+8*a^4*sin(d*x+c)/d+11/4*a^4*cos(d*x+c)*sin(d*x+c)/d+11/6*a^4*co
s(d*x+c)^3*sin(d*x+c)/d+2/3*a^4*cos(d*x+c)^5*sin(d*x+c)/d-16/3*a^4*sin(d*x
+c)^3/d+9/5*a^4*sin(d*x+c)^5/d-1/7*a^4*sin(d*x+c)^7/d
```

3.40.2 Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.56

$$\int \cos^7(c+dx)(a+a\sec(c+dx))^4\,dx$$
$$= \frac{a^4(18480dx+33915\sin(c+dx)+13020\sin(2(c+dx))+5495\sin(3(c+dx))+2100\sin(4(c+dx))+65}{6720d}$$

input
```
Integrate[Cos[c + d*x]^7*(a + a*Sec[c + d*x])^4,x]
```

output
```
(a^4*(18480*d*x + 33915*Sin[c + d*x] + 13020*Sin[2*(c + d*x)] + 5495*Sin[3
*(c + d*x)] + 2100*Sin[4*(c + d*x)] + 651*Sin[5*(c + d*x)] + 140*Sin[6*(c
+ d*x)] + 15*Sin[7*(c + d*x)]))/(6720*d)
```

3.40.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3042, 4278, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^7(c+dx)(a\sec(c+dx)+a)^4\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^4}{\csc\left(c+dx+\frac{\pi}{2}\right)^7}dx$$
$$\Big\downarrow \; 4278$$
$$\int \left(a^4\cos^7(c+dx)+4a^4\cos^6(c+dx)+6a^4\cos^5(c+dx)+4a^4\cos^4(c+dx)+a^4\cos^3(c+dx)\right)dx$$
$$\Big\downarrow \; 2009$$
$$-\frac{a^4\sin^7(c+dx)}{7d}+\frac{9a^4\sin^5(c+dx)}{5d}-\frac{16a^4\sin^3(c+dx)}{3d}+\frac{8a^4\sin(c+dx)}{d}+\frac{2a^4\sin(c+dx)\cos^5(c+dx)}{3d}+\frac{11a^4\sin(c+dx)\cos^3(c+dx)}{6d}+\frac{11a^4\sin(c+dx)\cos(c+dx)}{4d}+\frac{11a^4x}{4}$$

input
```
Int[Cos[c + d*x]^7*(a + a*Sec[c + d*x])^4,x]
```

output
```
(11*a^4*x)/4 + (8*a^4*Sin[c + d*x])/d + (11*a^4*Cos[c + d*x]*Sin[c + d*x])
/(4*d) + (11*a^4*Cos[c + d*x]^3*Sin[c + d*x])/(6*d) + (2*a^4*Cos[c + d*x]^
5*Sin[c + d*x])/(3*d) - (16*a^4*Sin[c + d*x]^3)/(3*d) + (9*a^4*Sin[c + d*x
]^5)/(5*d) - (a^4*Sin[c + d*x]^7)/(7*d)
```

3.40.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4278
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

3.40.4 Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.60

method	result
parallelrisch	$\frac{a^4(18480dx+33915\sin(dx+c)+15\sin(7dx+7c)+140\sin(6dx+6c)+651\sin(5dx+5c)+2100\sin(4dx+4c)+5495\sin(3dx+}{6720d}$
risch	$\frac{11a^4x}{4}+\frac{323a^4\sin(dx+c)}{64d}+\frac{a^4\sin(7dx+7c)}{448d}+\frac{a^4\sin(6dx+6c)}{48d}+\frac{31a^4\sin(5dx+5c)}{320d}+\frac{5a^4\sin(4dx+4c)}{16d}+\frac{157}{}$
derivativedivides	$\frac{\frac{a^4\left(2+\cos(dx+c)^2\right)\sin(dx+c)}{3}+4a^4\left(\frac{\left(\cos(dx+c)^3+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+\frac{6a^4\left(\frac{8}{3}+\cos(dx+c)^4+\frac{4\cos(dx+c)^2}{3}\right)}{5}}{}$
default	$\frac{\frac{a^4\left(2+\cos(dx+c)^2\right)\sin(dx+c)}{3}+4a^4\left(\frac{\left(\cos(dx+c)^3+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+\frac{6a^4\left(\frac{8}{3}+\cos(dx+c)^4+\frac{4\cos(dx+c)^2}{3}\right)}{5}}{}$

input
```
int(cos(d*x+c)^7*(a+a*sec(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
1/6720*a^4*(18480*d*x+33915*sin(d*x+c)+15*sin(7*d*x+7*c)+140*sin(6*d*x+6*c
)+651*sin(5*d*x+5*c)+2100*sin(4*d*x+4*c)+5495*sin(3*d*x+3*c)+13020*sin(2*d
*x+2*c))/d
```

3.40.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.69

$$\int \cos^7(c+dx)(a+a\sec(c+dx))^4\,dx$$
$$= \frac{1155\,a^4dx + \left(60\,a^4\cos\left(dx+c\right)^6 + 280\,a^4\cos\left(dx+c\right)^5 + 576\,a^4\cos\left(dx+c\right)^4 + 770\,a^4\cos\left(dx+c\right)^3 + 908}{420\,d}$$

input
```
integrate(cos(d*x+c)^7*(a+a*sec(d*x+c))^4,x, algorithm="fricas")
```

output
```
1/420*(1155*a^4*d*x + (60*a^4*cos(d*x + c)^6 + 280*a^4*cos(d*x + c)^5 + 57
6*a^4*cos(d*x + c)^4 + 770*a^4*cos(d*x + c)^3 + 908*a^4*cos(d*x + c)^2 + 1
155*a^4*cos(d*x + c) + 1816*a^4)*sin(d*x + c))/d
```

3.40.6 Sympy [F(-1)]

Timed out.

$$\int \cos^7(c+dx)(a+a\sec(c+dx))^4\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**7*(a+a*sec(d*x+c))**4,x)
```

output
```
Timed out
```

3.40.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.27

$$\int \cos^7(c+dx)(a+a\sec(c+dx))^4\,dx = -\frac{48\left(5\sin(dx+c)^7-21\sin(dx+c)^5+35\sin(dx+c)^3-35\sin(dx+c)\right)a^4-672\left(3\sin(dx+c)^5-\right.}{}$$

input
```
integrate(cos(d*x+c)^7*(a+a*sec(d*x+c))^4,x, algorithm="maxima")
```

output
```
-1/1680*(48*(5*sin(d*x + c)^7 - 21*sin(d*x + c)^5 + 35*sin(d*x + c)^3 - 35
*sin(d*x + c))*a^4 - 672*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*
x + c))*a^4 + 35*(4*sin(2*d*x + 2*c)^3 - 60*d*x - 60*c - 9*sin(4*d*x + 4*c
) - 48*sin(2*d*x + 2*c))*a^4 + 560*(sin(d*x + c)^3 - 3*sin(d*x + c))*a^4 -
 210*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*a^4)/d
```

3.40.8 Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.98

$$\int \cos^7(c+dx)(a+a\sec(c+dx))^4\,dx = \frac{1155\,(dx+c)a^4+\frac{2\left(1155\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{13}+7700\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}+21791\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}+33792\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+31521\,a\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^7}}{420\,d}$$

input
```
integrate(cos(d*x+c)^7*(a+a*sec(d*x+c))^4,x, algorithm="giac")
```

output
```
1/420*(1155*(d*x + c)*a^4 + 2*(1155*a^4*tan(1/2*d*x + 1/2*c)^13 + 7700*a^4
*tan(1/2*d*x + 1/2*c)^11 + 21791*a^4*tan(1/2*d*x + 1/2*c)^9 + 33792*a^4*ta
n(1/2*d*x + 1/2*c)^7 + 31521*a^4*tan(1/2*d*x + 1/2*c)^5 + 14700*a^4*tan(1/
2*d*x + 1/2*c)^3 + 5565*a^4*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2
+ 1)^7)/d
```

3.40.9 Mupad [B] (verification not implemented)

Time = 15.89 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.93

$$\int \cos^7(c+dx)(a+a\sec(c+dx))^4\,dx = \frac{11\,a^4\,x}{4} + \frac{\frac{11\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{13}}{2} + \frac{110\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{3} + \frac{3113\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}}{30} + \frac{5632\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}}{35} + \frac{1501\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{5}}{10} + 70\,a^4\,\mathrm{ta}}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)^7}$$

input
```
int(cos(c + d*x)^7*(a + a/cos(c + d*x))^4,x)
```

output
```
(11*a^4*x)/4 + (70*a^4*tan(c/2 + (d*x)/2)^3 + (1501*a^4*tan(c/2 + (d*x)/2)
^5)/10 + (5632*a^4*tan(c/2 + (d*x)/2)^7)/35 + (3113*a^4*tan(c/2 + (d*x)/2)
^9)/30 + (110*a^4*tan(c/2 + (d*x)/2)^11)/3 + (11*a^4*tan(c/2 + (d*x)/2)^13
)/2 + (53*a^4*tan(c/2 + (d*x)/2))/2)/(d*(tan(c/2 + (d*x)/2)^2 + 1)^7)
```

3.41 $\int \sec^3(c+dx)(a+a\sec(c+dx))^5\,dx$

3.41.1 Optimal result . . . 536
3.41.2 Mathematica [A] (verified) . . . 537
3.41.3 Rubi [A] (verified) . . . 537
3.41.4 Maple [C] (verified) . . . 539
3.41.5 Fricas [A] (verification not implemented) . . . 539
3.41.6 Sympy [F] . . . 540
3.41.7 Maxima [B] (verification not implemented) . . . 540
3.41.8 Giac [A] (verification not implemented) . . . 541
3.41.9 Mupad [B] (verification not implemented) . . . 541

3.41.1 Optimal result

Integrand size = 21, antiderivative size = 156

$$\begin{aligned}\int \sec^3(c+dx)(a+a\sec(c+dx))^5\,dx = {} & \frac{93a^5\text{arctanh}(\sin(c+dx))}{16d}+\frac{16a^5\tan(c+dx)}{d} \\ & +\frac{93a^5\sec(c+dx)\tan(c+dx)}{16d} \\ & +\frac{85a^5\sec^3(c+dx)\tan(c+dx)}{24d} \\ & +\frac{5a^5\sec^5(c+dx)\tan(c+dx)}{6d}+\frac{28a^5\tan^3(c+dx)}{3d} \\ & +\frac{13a^5\tan^5(c+dx)}{5d}+\frac{a^5\tan^7(c+dx)}{7d}\end{aligned}$$

output
```
93/16*a^5*arctanh(sin(d*x+c))/d+16*a^5*tan(d*x+c)/d+93/16*a^5*sec(d*x+c)*t
an(d*x+c)/d+85/24*a^5*sec(d*x+c)^3*tan(d*x+c)/d+5/6*a^5*sec(d*x+c)^5*tan(d
*x+c)/d+28/3*a^5*tan(d*x+c)^3/d+13/5*a^5*tan(d*x+c)^5/d+1/7*a^5*tan(d*x+c)
^7/d
```

3.41.2 Mathematica [A] (verified)

Time = 5.07 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.09

$$\int \sec^3(c+dx)(a+a\sec(c+dx))^5\,dx = \frac{93a^5\text{arctanh}(\sin(c+dx))}{16d} + \frac{479a^5\tan(c+dx)}{35d} + \frac{93a^5\sec(c+dx)\tan(c+dx)}{16d} + \frac{85a^5\sec^3(c+dx)\tan(c+dx)}{24d} + \frac{76a^5\sec^4(c+dx)\tan(c+dx)}{35d} + \frac{5a^5\sec^5(c+dx)\tan(c+dx)}{6d} + \frac{a^5\sec^6(c+dx)\tan(c+dx)}{7d} + \frac{479a^5\tan^3(c+dx)}{105d}$$

input
```
Integrate[Sec[c + d*x]^3*(a + a*Sec[c + d*x])^5,x]
```

output
```
(93*a^5*ArcTanh[Sin[c + d*x]])/(16*d) + (479*a^5*Tan[c + d*x])/(35*d) + (9
3*a^5*Sec[c + d*x]*Tan[c + d*x])/(16*d) + (85*a^5*Sec[c + d*x]^3*Tan[c + d
*x])/(24*d) + (76*a^5*Sec[c + d*x]^4*Tan[c + d*x])/(35*d) + (5*a^5*Sec[c +
 d*x]^5*Tan[c + d*x])/(6*d) + (a^5*Sec[c + d*x]^6*Tan[c + d*x])/(7*d) + (4
79*a^5*Tan[c + d*x]^3)/(105*d)
```

3.41.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3042, 4278, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^3(c+dx)(a\sec(c+dx)+a)^5\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^3\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^5dx$$

$$\Big\downarrow \; 4278$$

$$\int (a^5 \sec^8(c+dx)+5a^5 \sec^7(c+dx)+10a^5 \sec^6(c+dx)+10a^5 \sec^5(c+dx)+5a^5 \sec^4(c+dx)+a^5 \sec^3(c+dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{93a^5\text{arctanh}(\sin(c+dx))}{16d}+\frac{a^5 \tan^7(c+dx)}{7d}+\frac{13a^5 \tan^5(c+dx)}{5d}+\frac{28a^5 \tan^3(c+dx)}{3d}+\frac{16a^5 \tan(c+dx)}{d}+\frac{5a^5 \tan(c+dx)\sec^5(c+dx)}{6d}+\frac{85a^5 \tan(c+dx)\sec^3(c+dx)}{24d}+\frac{93a^5 \tan(c+dx)\sec(c+dx)}{16d}$$

input
```
Int[Sec[c + d*x]^3*(a + a*Sec[c + d*x])^5,x]
```

output
```
(93*a^5*ArcTanh[Sin[c + d*x]])/(16*d) + (16*a^5*Tan[c + d*x])/d + (93*a^5*
Sec[c + d*x]*Tan[c + d*x])/(16*d) + (85*a^5*Sec[c + d*x]^3*Tan[c + d*x])/(
24*d) + (5*a^5*Sec[c + d*x]^5*Tan[c + d*x])/(6*d) + (28*a^5*Tan[c + d*x]^3
)/(3*d) + (13*a^5*Tan[c + d*x]^5)/(5*d) + (a^5*Tan[c + d*x]^7)/(7*d)
```

3.41.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4278
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

3.41.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.46 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.21

method	result
risch	$-\frac{ia^5\left(9765\,\mathrm{e}^{13i(dx+c)}+62860\,\mathrm{e}^{11i(dx+c)}-16800\,\mathrm{e}^{10i(dx+c)}+118825\,\mathrm{e}^{9i(dx+c)}-162400\,\mathrm{e}^{8i(dx+c)}-374080\,\mathrm{e}^{6i(dx+c)}-11\right.}{840d\left(\mathrm{e}^{2i(dx+c)}+1\right)^7}$
norman	$\frac{-\frac{419a^5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8d}+\frac{943a^5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{6d}-\frac{37169a^5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{120d}+\frac{11904a^5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{35d}-\frac{8773a^5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^9}{40d}+\frac{155a^5\tan(}{2}}{\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^7}$
parallelrisch	$\frac{3395a^5\left(\frac{279\left(-\cos(dx+c)-\frac{3\cos(3dx+3c)}{5}-\frac{\cos(5dx+5c)}{5}-\frac{\cos(7dx+7c)}{35}\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{194}+\frac{279\left(\frac{\cos(7dx+7c)}{35}+\frac{\cos(5dx+5c)}{5}+\right.}{}\right.}{24d(\cos(7d}$
derivativedivides	$\frac{-a^5\left(-\frac{16}{35}-\frac{\sec(dx+c)^6}{7}-\frac{6\sec(dx+c)^4}{35}-\frac{8\sec(dx+c)^2}{35}\right)\tan(dx+c)+5a^5\left(-\left(-\frac{\sec(dx+c)^5}{6}-\frac{5\sec(dx+c)^3}{24}-\frac{5\sec(dx+c)}{16}\right)\tan\right.}{}$
default	$\frac{-a^5\left(-\frac{16}{35}-\frac{\sec(dx+c)^6}{7}-\frac{6\sec(dx+c)^4}{35}-\frac{8\sec(dx+c)^2}{35}\right)\tan(dx+c)+5a^5\left(-\left(-\frac{\sec(dx+c)^5}{6}-\frac{5\sec(dx+c)^3}{24}-\frac{5\sec(dx+c)}{16}\right)\tan\right.}{}$
parts	$\frac{a^5\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)}{d}-\frac{a^5\left(-\frac{16}{35}-\frac{\sec(dx+c)^6}{7}-\frac{6\sec(dx+c)^4}{35}-\frac{8\sec(dx+c)^2}{35}\right)\tan(dx+c)}{d}-$

input
```
int(sec(d*x+c)^3*(a+a*sec(d*x+c))^5,x,method=_RETURNVERBOSE)
```

output
```
-1/840*I*a^5*(9765*exp(13*I*(d*x+c))+62860*exp(11*I*(d*x+c))-16800*exp(10*
I*(d*x+c))+118825*exp(9*I*(d*x+c))-162400*exp(8*I*(d*x+c))-374080*exp(6*I*
(d*x+c))-118825*exp(5*I*(d*x+c))-305088*exp(4*I*(d*x+c))-62860*exp(3*I*(d*
x+c))-107296*exp(2*I*(d*x+c))-9765*exp(I*(d*x+c))-15328)/d/(exp(2*I*(d*x+c
))+1)^7+93/16*a^5/d*ln(exp(I*(d*x+c))+I)-93/16*a^5/d*ln(exp(I*(d*x+c))-I)
```

3.41.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.96

$$\int \sec^3(c+dx)(a+a\sec(c+dx))^5\,dx$$
$$=\frac{9765\,a^5\cos(dx+c)^7\log(\sin(dx+c)+1)-9765\,a^5\cos(dx+c)^7\log(-\sin(dx+c)+1)+2\left(15328\,a^5\,\mathrm{c}\right.}{}$$

input
```
integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^5,x, algorithm="fricas")
```

output
```
1/3360*(9765*a^5*cos(d*x + c)^7*log(sin(d*x + c) + 1) - 9765*a^5*cos(d*x +
 c)^7*log(-sin(d*x + c) + 1) + 2*(15328*a^5*cos(d*x + c)^6 + 9765*a^5*cos(
d*x + c)^5 + 7664*a^5*cos(d*x + c)^4 + 5950*a^5*cos(d*x + c)^3 + 3648*a^5*
cos(d*x + c)^2 + 1400*a^5*cos(d*x + c) + 240*a^5)*sin(d*x + c))/(d*cos(d*x
 + c)^7)
```

3.41.6 Sympy [F]

$$\int \sec^3(c+dx)(a+a\sec(c+dx))^5\,dx = a^5\left(\int \sec^3(c+dx)\,dx + \int 5\sec^4(c+dx)\,dx + \int 10\sec^5(c+dx)\,dx + \int 10\sec^6(c+dx)\,dx + \int 5\sec^7(c+dx)\,dx + \int \sec^8(c+dx)\,dx\right)$$

input
```
integrate(sec(d*x+c)**3*(a+a*sec(d*x+c))**5,x)
```

output
```
a**5*(Integral(sec(c + d*x)**3, x) + Integral(5*sec(c + d*x)**4, x) + Inte
gral(10*sec(c + d*x)**5, x) + Integral(10*sec(c + d*x)**6, x) + Integral(5
*sec(c + d*x)**7, x) + Integral(sec(c + d*x)**8, x))
```

3.41.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 314 vs. $2(142) = 284$.

Time = 0.32 (sec) , antiderivative size = 314, normalized size of antiderivative = 2.01

$$\int \sec^3(c+dx)(a+a\sec(c+dx))^5\,dx = \frac{96\left(5\tan(dx+c)^7 + 21\tan(dx+c)^5 + 35\tan(dx+c)^3 + 35\tan(dx+c)\right)a^5 + 2240\left(3\tan(dx+c)^5 + \right.}{}$$

input
```
integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^5,x, algorithm="maxima")
```

output
```
1/3360*(96*(5*tan(d*x + c)^7 + 21*tan(d*x + c)^5 + 35*tan(d*x + c)^3 + 35*
tan(d*x + c))*a^5 + 2240*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*
x + c))*a^5 + 5600*(tan(d*x + c)^3 + 3*tan(d*x + c))*a^5 - 175*a^5*(2*(15*
sin(d*x + c)^5 - 40*sin(d*x + c)^3 + 33*sin(d*x + c))/(sin(d*x + c)^6 - 3*
sin(d*x + c)^4 + 3*sin(d*x + c)^2 - 1) - 15*log(sin(d*x + c) + 1) + 15*log
(sin(d*x + c) - 1)) - 2100*a^5*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin
(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(
d*x + c) - 1)) - 840*a^5*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*
x + c) + 1) + log(sin(d*x + c) - 1)))/d
```

3.41.8 Giac [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.09

$$\int \sec^3(c+dx)(a+a\sec(c+dx))^5\,dx$$
$$= \frac{9765\,a^5\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-9765\,a^5\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(9765\,a^5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{13}-65100\,a^5\,t}{}}{}$$

input
```
integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^5,x, algorithm="giac")
```

output
```
1/1680*(9765*a^5*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 9765*a^5*log(abs(tan
(1/2*d*x + 1/2*c) - 1)) - 2*(9765*a^5*tan(1/2*d*x + 1/2*c)^13 - 65100*a^5*
tan(1/2*d*x + 1/2*c)^11 + 184233*a^5*tan(1/2*d*x + 1/2*c)^9 - 285696*a^5*t
an(1/2*d*x + 1/2*c)^7 + 260183*a^5*tan(1/2*d*x + 1/2*c)^5 - 132020*a^5*tan
(1/2*d*x + 1/2*c)^3 + 43995*a^5*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c
)^2 - 1)^7)/d
```

3.41.9 Mupad [B] (verification not implemented)

Time = 17.09 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.46

$$\int \sec^3(c+dx)(a+a\sec(c+dx))^5\,dx = \frac{93\,a^5\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{8\,d}$$
$$-\frac{\frac{93\,a^5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{13}}{8}-\frac{155\,a^5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{2}+\frac{8773\,a^5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}}{40}-\frac{11904\,a^5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}}{35}+\frac{37169\,a^5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{5}}{120}-}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{14}-7\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}+21\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}-35\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8}+35\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6}-21\right.}$$

input
```
int((a + a/cos(c + d*x))^5/cos(c + d*x)^3,x)
```

output
```
(93*a^5*atanh(tan(c/2 + (d*x)/2)))/(8*d) - ((37169*a^5*tan(c/2 + (d*x)/2)^
5)/120 - (943*a^5*tan(c/2 + (d*x)/2)^3)/6 - (11904*a^5*tan(c/2 + (d*x)/2)^
7)/35 + (8773*a^5*tan(c/2 + (d*x)/2)^9)/40 - (155*a^5*tan(c/2 + (d*x)/2)^1
1)/2 + (93*a^5*tan(c/2 + (d*x)/2)^13)/8 + (419*a^5*tan(c/2 + (d*x)/2))/8)/
(d*(7*tan(c/2 + (d*x)/2)^2 - 21*tan(c/2 + (d*x)/2)^4 + 35*tan(c/2 + (d*x)/
2)^6 - 35*tan(c/2 + (d*x)/2)^8 + 21*tan(c/2 + (d*x)/2)^10 - 7*tan(c/2 + (d
*x)/2)^12 + tan(c/2 + (d*x)/2)^14 - 1))
```

3.41. $\int \sec^3(c+dx)(a+a\sec(c+dx))^5\,dx$

3.42 $\int \frac{\sec^5(c+dx)}{a+a\sec(c+dx)}\,dx$

3.42.1 Optimal result . . . 543
3.42.2 Mathematica [A] (verified) . . . 543
3.42.3 Rubi [A] (verified) . . . 544
3.42.4 Maple [A] (verified) . . . 546
3.42.5 Fricas [A] (verification not implemented) . . . 547
3.42.6 Sympy [F] . . . 547
3.42.7 Maxima [B] (verification not implemented) . . . 548
3.42.8 Giac [A] (verification not implemented) . . . 548
3.42.9 Mupad [B] (verification not implemented) . . . 549

3.42.1 Optimal result

Integrand size = 21, antiderivative size = 103

$$\int \frac{\sec^5(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{3\text{arctanh}(\sin(c+dx))}{2ad}+\frac{4\tan(c+dx)}{ad} - \frac{3\sec(c+dx)\tan(c+dx)}{2ad} - \frac{\sec^3(c+dx)\tan(c+dx)}{d(a+a\sec(c+dx))}+\frac{4\tan^3(c+dx)}{3ad}$$

output
```
-3/2*arctanh(sin(d*x+c))/a/d+4*tan(d*x+c)/a/d-3/2*sec(d*x+c)*tan(d*x+c)/a/
d-sec(d*x+c)^3*tan(d*x+c)/d/(a+a*sec(d*x+c))+4/3*tan(d*x+c)^3/a/d
```

3.42.2 Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.81

$$\int \frac{\sec^5(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{-18\text{arctanh}(\sin(c+dx))(1+\sec(c+dx))+(11+22\cos(c+dx)+7\cos(2(c+dx))+8\cos(3(c+dx)))}{12ad(1+\sec(c+dx))}$$

input
```
Integrate[Sec[c + d*x]^5/(a + a*Sec[c + d*x]),x]
```

output
```
(-18*ArcTanh[Sin[c + d*x]]*(1 + Sec[c + d*x]) + (11 + 22*Cos[c + d*x] + 7*
Cos[2*(c + d*x)] + 8*Cos[3*(c + d*x)])*Sec[c + d*x]^3*Tan[c + d*x])/(12*a*
d*(1 + Sec[c + d*x]))
```

3.42.3 Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.99, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {3042, 4305, 3042, 4274, 3042, 4254, 2009, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^5(c+dx)}{a\sec(c+dx)+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^5}{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}dx$$

$$\Big\downarrow \text{4305}$$

$$-\frac{\int \sec^3(c+dx)(3a-4a\sec(c+dx))dx}{a^2}-\frac{\tan(c+dx)\sec^3(c+dx)}{d(a\sec(c+dx)+a)}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \csc\left(c+dx+\frac{\pi}{2}\right)^3\left(3a-4a\csc\left(c+dx+\frac{\pi}{2}\right)\right)dx}{a^2}-\frac{\tan(c+dx)\sec^3(c+dx)}{d(a\sec(c+dx)+a)}$$

$$\Big\downarrow \text{4274}$$

$$-\frac{3a\int \sec^3(c+dx)dx-4a\int \sec^4(c+dx)dx}{a^2}-\frac{\tan(c+dx)\sec^3(c+dx)}{d(a\sec(c+dx)+a)}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{3a\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx-4a\int \csc\left(c+dx+\frac{\pi}{2}\right)^4dx}{a^2}-\frac{\tan(c+dx)\sec^3(c+dx)}{d(a\sec(c+dx)+a)}$$

$$\Big\downarrow \text{4254}$$

$$-\frac{\frac{4a\int(\tan^2(c+dx)+1)d(-\tan(c+dx))}{d}+3a\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx}{a^2}-\frac{\tan(c+dx)\sec^3(c+dx)}{d(a\sec(c+dx)+a)}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{3a\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx+\frac{4a\left(-\frac{1}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{d}}{a^2}-\frac{\tan(c+dx)\sec^3(c+dx)}{d(a\sec(c+dx)+a)}$$

$$\Big\downarrow \text{4255}$$

$$-\frac{3a\left(\frac{1}{2}\int \sec(c+dx)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{4a\left(-\frac{1}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{d}}{a^2}-\frac{\tan(c+dx)\sec^3(c+dx)}{d(a\sec(c+dx)+a)}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{3a\left(\frac{1}{2}\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{4a\left(-\frac{1}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{d}}{a^2}-\frac{\tan(c+dx)\sec^3(c+dx)}{d(a\sec(c+dx)+a)}$$

$$\Big\downarrow \text{4257}$$

$$-\frac{3a\left(\frac{\text{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{4a\left(-\frac{1}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{d}}{a^2}-\frac{\tan(c+dx)\sec^3(c+dx)}{d(a\sec(c+dx)+a)}$$

input
```
Int[Sec[c + d*x]^5/(a + a*Sec[c + d*x]),x]
```

output
```
-((Sec[c + d*x]^3*Tan[c + d*x])/(d*(a + a*Sec[c + d*x]))) - (3*a*(ArcTanh[
Sin[c + d*x]]/(2*d) + (Sec[c + d*x]*Tan[c + d*x])/(2*d)) + (4*a*(-Tan[c +
d*x] - Tan[c + d*x]^3/3))/d)/a^2
```

3.42.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4305
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[d^2*Cot[e + f*x]*((d*Csc[e + f*x])^(n - 2)/(f*(a +
b*Csc[e + f*x]))), x] - Simp[d^2/(a*b)   Int[(d*Csc[e + f*x])^(n - 2)*(b*(n
 - 2) - a*(n - 1)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ
[a^2 - b^2, 0] && GtQ[n, 1]
```

3.42.4 Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.30

method	result
derivativedivides	$\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{3}}+\frac{1}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{2}}-\frac{5}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{2}-\frac{1}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{3}}-}{da}$
default	$\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{3}}+\frac{1}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{2}}-\frac{5}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{2}-\frac{1}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{3}}-}{da}$
parallelrisch	$\frac{(27\cos(dx+c)+9\cos(3dx+3c))\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+(-27\cos(dx+c)-9\cos(3dx+3c))\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+44\left(\cos(dx\right.}{6ad(\cos(3dx+3c)+3\cos(dx+c))}$
risch	$\frac{i\left(9\,\mathrm{e}^{6i(dx+c)}+9\,\mathrm{e}^{5i(dx+c)}+24\,\mathrm{e}^{4i(dx+c)}+24\,\mathrm{e}^{3i(dx+c)}+39\,\mathrm{e}^{2i(dx+c)}+7\,\mathrm{e}^{i(dx+c)}+16\right)}{3da\left(\mathrm{e}^{2i(dx+c)}+1\right)^{3}\left(\mathrm{e}^{i(dx+c)}+1\right)}+\frac{3\ln\left(\mathrm{e}^{i(dx+c)}-i\right)}{2ad}-\frac{3\ln\left(\mathrm{e}^{i(dx\right.}{2a}$
norman	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{9}}{ad}+\frac{4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}-\frac{37\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{3ad}+\frac{49\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{3ad}-\frac{9\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{ad}}{\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)^{4}}+\frac{3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2ad}-\frac{3\ln\left(\tan\right.}{}$

input
```
int(sec(d*x+c)^5/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d/a*(tan(1/2*d*x+1/2*c)-1/3/(tan(1/2*d*x+1/2*c)+1)^3+1/(tan(1/2*d*x+1/2*
c)+1)^2-5/2/(tan(1/2*d*x+1/2*c)+1)-3/2*ln(tan(1/2*d*x+1/2*c)+1)-1/3/(tan(1
/2*d*x+1/2*c)-1)^3-1/(tan(1/2*d*x+1/2*c)-1)^2-5/2/(tan(1/2*d*x+1/2*c)-1)+3
/2*ln(tan(1/2*d*x+1/2*c)-1))
```

3.42.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.20

$$\int \frac{\sec^5(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{9\left(\cos\left(dx+c\right)^4+\cos\left(dx+c\right)^3\right)\log\left(\sin\left(dx+c\right)+1\right)-9\left(\cos\left(dx+c\right)^4+\cos\left(dx+c\right)^3\right)\log\left(-\sin\left(dx\right.\right.}{12\left(ad\cos\left(dx+c\right)^4+ad\cos\left(\right.\right.}$$

input
```
integrate(sec(d*x+c)^5/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

output
```
-1/12*(9*(cos(d*x + c)^4 + cos(d*x + c)^3)*log(sin(d*x + c) + 1) - 9*(cos(
d*x + c)^4 + cos(d*x + c)^3)*log(-sin(d*x + c) + 1) - 2*(16*cos(d*x + c)^3
 + 7*cos(d*x + c)^2 - cos(d*x + c) + 2)*sin(d*x + c))/(a*d*cos(d*x + c)^4
+ a*d*cos(d*x + c)^3)
```

3.42.6 Sympy [F]

$$\int \frac{\sec^5(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\int \frac{\sec^5(c+dx)}{\sec(c+dx)+1}\,dx}{a}$$

input
```
integrate(sec(d*x+c)**5/(a+a*sec(d*x+c)),x)
```

output
```
Integral(sec(c + d*x)**5/(sec(c + d*x) + 1), x)/a
```

3.42.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 205 vs. $2(97) = 194$.

Time = 0.30 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.99

$$\int \frac{\sec^5(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\frac{2\left(\frac{9\sin(dx+c)}{\cos(dx+c)+1}-\frac{16\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{15\sin(dx+c)^5}{(\cos(dx+c)+1)^5}\right)}{a-\frac{3a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{3a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{a\sin(dx+c)^6}{(\cos(dx+c)+1)^6}}-\frac{9\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a}+\frac{9\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a}+\frac{6\sin(dx+c)}{a(\cos(dx+c)+1)}}{6d}$$

input
```
integrate(sec(d*x+c)^5/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

output
```
1/6*(2*(9*sin(d*x + c)/(cos(d*x + c) + 1) - 16*sin(d*x + c)^3/(cos(d*x + c
) + 1)^3 + 15*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/(a - 3*a*sin(d*x + c)^2
/(cos(d*x + c) + 1)^2 + 3*a*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - a*sin(d*
x + c)^6/(cos(d*x + c) + 1)^6) - 9*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1
)/a + 9*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a + 6*sin(d*x + c)/(a*(co
s(d*x + c) + 1)))/d
```

3.42.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.11

$$\int \frac{\sec^5(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{\frac{9\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a}-\frac{9\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{a}-\frac{6\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a}+\frac{2\left(15\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-16\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+9\tan\left(\frac{1}{2}dx\right.\right.}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)^3a}}{6d}$$

input
```
integrate(sec(d*x+c)^5/(a+a*sec(d*x+c)),x, algorithm="giac")
```

output
```
-1/6*(9*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a - 9*log(abs(tan(1/2*d*x + 1/2
*c) - 1))/a - 6*tan(1/2*d*x + 1/2*c)/a + 2*(15*tan(1/2*d*x + 1/2*c)^5 - 16
*tan(1/2*d*x + 1/2*c)^3 + 9*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2
 - 1)^3*a))/d
```

3.42.9 Mupad [B] (verification not implemented)

Time = 12.77 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.93

$$\int \frac{\sec^5(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{a\,d} - \frac{3\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{a\,d} - \frac{5\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5 - \frac{16\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{3} + 3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{a\,d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2-1\right)^3}$$

input
```
int(1/(cos(c + d*x)^5*(a + a/cos(c + d*x))),x)
```

output
```
tan(c/2 + (d*x)/2)/(a*d) - (3*atanh(tan(c/2 + (d*x)/2)))/(a*d) - (3*tan(c/
2 + (d*x)/2) - (16*tan(c/2 + (d*x)/2)^3)/3 + 5*tan(c/2 + (d*x)/2)^5)/(a*d*
(tan(c/2 + (d*x)/2)^2 - 1)^3)
```

3.43 $\int \frac{\sec^4(c+dx)}{a+a\sec(c+dx)}\,dx$

3.43.1 Optimal result . . . 550
3.43.2 Mathematica [A] (verified) . . . 550
3.43.3 Rubi [A] (verified) . . . 551
3.43.4 Maple [A] (verified) . . . 553
3.43.5 Fricas [A] (verification not implemented) . . . 554
3.43.6 Sympy [F] . . . 554
3.43.7 Maxima [A] (verification not implemented) . . . 554
3.43.8 Giac [A] (verification not implemented) . . . 555
3.43.9 Mupad [B] (verification not implemented) . . . 555

3.43.1 Optimal result

Integrand size = 21, antiderivative size = 85

$$\int \frac{\sec^4(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{3\text{arctanh}(\sin(c+dx))}{2ad} - \frac{2\tan(c+dx)}{ad} + \frac{3\sec(c+dx)\tan(c+dx)}{2ad} - \frac{\sec^2(c+dx)\tan(c+dx)}{d(a+a\sec(c+dx))}$$

output
```
3/2*arctanh(sin(d*x+c))/a/d-2*tan(d*x+c)/a/d+3/2*sec(d*x+c)*tan(d*x+c)/a/d
-sec(d*x+c)^2*tan(d*x+c)/d/(a+a*sec(d*x+c))
```

3.43.2 Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.85

$$\int \frac{\sec^4(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{3\text{arctanh}(\sin(c+dx))(1+\sec(c+dx)) - (1+\cos(c+dx)+2\cos(2(c+dx)))\sec^2(c+dx)\tan(c+dx)}{2ad(1+\sec(c+dx))}$$

input
```
Integrate[Sec[c + d*x]^4/(a + a*Sec[c + d*x]),x]
```

output
```
(3*ArcTanh[Sin[c + d*x]]*(1 + Sec[c + d*x]) - (1 + Cos[c + d*x] + 2*Cos[2*
(c + d*x)])*Sec[c + d*x]^2*Tan[c + d*x])/(2*a*d*(1 + Sec[c + d*x]))
```

3.43.3 Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.02, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.476, Rules used = {3042, 4305, 3042, 4274, 3042, 4254, 24, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^4(c+dx)}{a\sec(c+dx)+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^4}{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}dx$$

$$\Big\downarrow \text{4305}$$

$$-\frac{\int \sec^2(c+dx)(2a-3a\sec(c+dx))dx}{a^2}-\frac{\tan(c+dx)\sec^2(c+dx)}{d(a\sec(c+dx)+a)}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \csc\left(c+dx+\frac{\pi}{2}\right)^2\left(2a-3a\csc\left(c+dx+\frac{\pi}{2}\right)\right)dx}{a^2}-\frac{\tan(c+dx)\sec^2(c+dx)}{d(a\sec(c+dx)+a)}$$

$$\Big\downarrow \text{4274}$$

$$-\frac{2a\int \sec^2(c+dx)dx-3a\int \sec^3(c+dx)dx}{a^2}-\frac{\tan(c+dx)\sec^2(c+dx)}{d(a\sec(c+dx)+a)}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2a\int \csc\left(c+dx+\frac{\pi}{2}\right)^2dx-3a\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx}{a^2}-\frac{\tan(c+dx)\sec^2(c+dx)}{d(a\sec(c+dx)+a)}$$

$$\Big\downarrow \text{4254}$$

$$-\frac{-\frac{2a\int 1d(-\tan(c+dx))}{d}-3a\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx}{a^2}-\frac{\tan(c+dx)\sec^2(c+dx)}{d(a\sec(c+dx)+a)}$$

$$\Big\downarrow \text{24}$$

$$-\frac{\frac{2a\tan(c+dx)}{d}-3a\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx}{a^2}-\frac{\tan(c+dx)\sec^2(c+dx)}{d(a\sec(c+dx)+a)}$$

$$\Big\downarrow \text{4255}$$

$$-\frac{\frac{2a\tan(c+dx)}{d}-3a\left(\frac{1}{2}\int\sec(c+dx)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)}{a^2}-\frac{\tan(c+dx)\sec^2(c+dx)}{d(a\sec(c+dx)+a)}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\frac{2a\tan(c+dx)}{d}-3a\left(\frac{1}{2}\int\csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)}{a^2}-\frac{\tan(c+dx)\sec^2(c+dx)}{d(a\sec(c+dx)+a)}$$

$$\Big\downarrow \text{4257}$$

$$-\frac{\frac{2a\tan(c+dx)}{d}-3a\left(\frac{\text{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)}{a^2}-\frac{\tan(c+dx)\sec^2(c+dx)}{d(a\sec(c+dx)+a)}$$

input
```
Int[Sec[c + d*x]^4/(a + a*Sec[c + d*x]),x]
```

output
```
-((Sec[c + d*x]^2*Tan[c + d*x])/(d*(a + a*Sec[c + d*x]))) - ((2*a*Tan[c +
d*x])/d - 3*a*(ArcTanh[Sin[c + d*x]]/(2*d) + (Sec[c + d*x]*Tan[c + d*x])/(
2*d)))/a^2
```

3.43.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4305
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[d^2*Cot[e + f*x]*((d*Csc[e + f*x])^(n - 2)/(f*(a +
b*Csc[e + f*x]))), x] - Simp[d^2/(a*b)   Int[(d*Csc[e + f*x])^(n - 2)*(b*(n
 - 2) - a*(n - 1)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ
[a^2 - b^2, 0] && GtQ[n, 1]
```

3.43.4 Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.24

method	result
parallelrisch	$\frac{(-3\cos(2dx+2c)-3)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+(3\cos(2dx+2c)+3)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-2(1+2\cos(2dx+2c)+\cos(dx+c))\,t}{2ad(1+\cos(2dx+2c))}$
derivativedivides	$\frac{-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}+\frac{3}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}+\frac{3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{2}+\frac{1}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}+\frac{3}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}}{da}$
default	$\frac{-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}+\frac{3}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}+\frac{3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{2}+\frac{1}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}+\frac{3}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}}{da}$
risch	$-\frac{i\left(3\,\mathrm{e}^{4i(dx+c)}+3\,\mathrm{e}^{3i(dx+c)}+5\,\mathrm{e}^{2i(dx+c)}+\mathrm{e}^{i(dx+c)}+4\right)}{da\left(\mathrm{e}^{2i(dx+c)}+1\right)^2\left(\mathrm{e}^{i(dx+c)}+1\right)}+\frac{3\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{2ad}-\frac{3\ln\left(\mathrm{e}^{i(dx+c)}-i\right)}{2ad}$
norman	$\frac{\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}-\frac{7\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{ad}+\frac{6\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{ad}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{ad}}{\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^3}-\frac{3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2ad}+\frac{3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{2ad}$

input
```
int(sec(d*x+c)^4/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/2*((-3*cos(2*d*x+2*c)-3)*ln(tan(1/2*d*x+1/2*c)-1)+(3*cos(2*d*x+2*c)+3)*l
n(tan(1/2*d*x+1/2*c)+1)-2*(1+2*cos(2*d*x+2*c)+cos(d*x+c))*tan(1/2*d*x+1/2*
c))/a/d/(1+cos(2*d*x+2*c))
```

3.43.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.32

$$\int \frac{\sec^4(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{3\left(\cos\left(dx+c\right)^3+\cos\left(dx+c\right)^2\right)\log\left(\sin\left(dx+c\right)+1\right)-3\left(\cos\left(dx+c\right)^3+\cos\left(dx+c\right)^2\right)\log\left(-\sin\left(dx+\right.\right.}{4\left(ad\cos\left(dx+c\right)^3+ad\cos\left(dx+c\right)^2\right)}$$

input
```
integrate(sec(d*x+c)^4/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

output
```
1/4*(3*(cos(d*x + c)^3 + cos(d*x + c)^2)*log(sin(d*x + c) + 1) - 3*(cos(d*
x + c)^3 + cos(d*x + c)^2)*log(-sin(d*x + c) + 1) - 2*(4*cos(d*x + c)^2 +
cos(d*x + c) - 1)*sin(d*x + c))/(a*d*cos(d*x + c)^3 + a*d*cos(d*x + c)^2)
```

3.43.6 Sympy [F]

$$\int \frac{\sec^4(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\int \frac{\sec^4(c+dx)}{\sec(c+dx)+1}\,dx}{a}$$

input
```
integrate(sec(d*x+c)**4/(a+a*sec(d*x+c)),x)
```

output
```
Integral(sec(c + d*x)**4/(sec(c + d*x) + 1), x)/a
```

3.43.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.91

$$\int \frac{\sec^4(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{\frac{2\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-\frac{3\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a-\frac{2a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}-\frac{3\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a}+\frac{3\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a}+\frac{2\sin(dx+c)}{a(\cos(dx+c)+1)}}{2d}$$

input
```
integrate(sec(d*x+c)^4/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

output
```
-1/2*(2*(sin(d*x + c)/(cos(d*x + c) + 1) - 3*sin(d*x + c)^3/(cos(d*x + c)
+ 1)^3)/(a - 2*a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a*sin(d*x + c)^4/(c
os(d*x + c) + 1)^4) - 3*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a + 3*log
(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a + 2*sin(d*x + c)/(a*(cos(d*x + c)
+ 1)))/d
```

3.43.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.19

$$\int \frac{\sec^4(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\frac{3\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a} - \frac{3\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{a} - \frac{2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a} + \frac{2\left(3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)^2 a}}{2d}$$

input
```
integrate(sec(d*x+c)^4/(a+a*sec(d*x+c)),x, algorithm="giac")
```

output
```
1/2*(3*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a - 3*log(abs(tan(1/2*d*x + 1/2*
c) - 1))/a - 2*tan(1/2*d*x + 1/2*c)/a + 2*(3*tan(1/2*d*x + 1/2*c)^3 - tan(
1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 - 1)^2*a))/d
```

3.43.9 Mupad [B] (verification not implemented)

Time = 13.16 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.12

$$\int \frac{\sec^4(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{3\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{a\,d} - \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{a\,d} - \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{d\left(a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4-2\,a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+a\right)}$$

input
```
int(1/(cos(c + d*x)^4*(a + a/cos(c + d*x))),x)
```

output
```
(3*atanh(tan(c/2 + (d*x)/2)))/(a*d) - tan(c/2 + (d*x)/2)/(a*d) - (tan(c/2
+ (d*x)/2) - 3*tan(c/2 + (d*x)/2)^3)/(d*(a - 2*a*tan(c/2 + (d*x)/2)^2 + a*
tan(c/2 + (d*x)/2)^4))
```

3.43. $\int \frac{\sec^4(c+dx)}{a+a\sec(c+dx)}\,dx$

3.44 $\int \frac{\sec^3(c+dx)}{a+a\sec(c+dx)}\,dx$

3.44.1 Optimal result . . . 557
3.44.2 Mathematica [A] (verified) . . . 557
3.44.3 Rubi [A] (verified) . . . 558
3.44.4 Maple [A] (verified) . . . 560
3.44.5 Fricas [A] (verification not implemented) . . . 560
3.44.6 Sympy [F] . . . 561
3.44.7 Maxima [B] (verification not implemented) . . . 561
3.44.8 Giac [A] (verification not implemented) . . . 561
3.44.9 Mupad [B] (verification not implemented) . . . 562

3.44.1 Optimal result

Integrand size = 21, antiderivative size = 51

$$\int \frac{\sec^3(c+dx)}{a+a\sec(c+dx)}\,dx=-\frac{\operatorname{arctanh}(\sin(c+dx))}{ad}+\frac{\tan(c+dx)}{ad}+\frac{\tan(c+dx)}{d(a+a\sec(c+dx))}$$

output
```
-arctanh(sin(d*x+c))/a/d+tan(d*x+c)/a/d+tan(d*x+c)/d/(a+a*sec(d*x+c))
```

3.44.2 Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.82

$$\int \frac{\sec^3(c+dx)}{a+a\sec(c+dx)}\,dx=\frac{-\operatorname{arctanh}(\sin(c+dx))+\frac{(2+\sec(c+dx))\tan(c+dx)}{1+\sec(c+dx)}}{ad}$$

input
```
Integrate[Sec[c + d*x]^3/(a + a*Sec[c + d*x]),x]
```

output
```
(-ArcTanh[Sin[c + d*x]] + ((2 + Sec[c + d*x])*Tan[c + d*x])/(1 + Sec[c + d
*x]))/(a*d)
```

3.44.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 4277, 3042, 4276, 3042, 4257, 4281}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^3(c+dx)}{a\sec(c+dx)+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^3}{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}dx$$

$$\Big\downarrow \text{4277}$$

$$\frac{\tan(c+dx)}{ad}-\int \frac{\sec^2(c+dx)}{\sec(c+dx)a+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{\tan(c+dx)}{ad}-\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx$$

$$\Big\downarrow \text{4276}$$

$$-\frac{\int \sec(c+dx)dx}{a}+\int \frac{\sec(c+dx)}{\sec(c+dx)a+a}dx+\frac{\tan(c+dx)}{ad}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{a}+\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx+\frac{\tan(c+dx)}{ad}$$

$$\Big\downarrow \text{4257}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx-\frac{\text{arctanh}(\sin(c+dx))}{ad}+\frac{\tan(c+dx)}{ad}$$

$$\Big\downarrow \text{4281}$$

$$-\frac{\text{arctanh}(\sin(c+dx))}{ad}+\frac{\tan(c+dx)}{ad}+\frac{\tan(c+dx)}{d(a\sec(c+dx)+a)}$$

input
```
Int[Sec[c + d*x]^3/(a + a*Sec[c + d*x]),x]
```

output
```
-(ArcTanh[Sin[c + d*x]]/(a*d)) + Tan[c + d*x]/(a*d) + Tan[c + d*x]/(d*(a +
 a*Sec[c + d*x]))
```

3.44.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4276
```
Int[csc[(e_.) + (f_.)*(x_)]^2/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Sym
bol] :> Simp[1/b   Int[Csc[e + f*x], x], x] - Simp[a/b   Int[Csc[e + f*x]/(
a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x]
```

rule 4277
```
Int[csc[(e_.) + (f_.)*(x_)]^3/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Sym
bol] :> Simp[-Cot[e + f*x]/(b*f), x] - Simp[a/b   Int[Csc[e + f*x]^2/(a + b
*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x]
```

rule 4281
```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[-Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

3.44.4 Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.45

method	result	size
derivativedivides	$\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1}+\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)-\frac{1}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}-\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{da}$	74
default	$\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1}+\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)-\frac{1}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}-\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{da}$	74
parallelrisch	$\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\cos(dx+c)-\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)\cos(dx+c)+2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\cos(dx+c)+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad\cos(dx+c)}$	82
risch	$\frac{2i\left(e^{2i(dx+c)}+e^{i(dx+c)}+2\right)}{da\left(e^{i(dx+c)}+1\right)\left(e^{2i(dx+c)}+1\right)}-\frac{\ln\left(e^{i(dx+c)}+i\right)}{ad}+\frac{\ln\left(e^{i(dx+c)}-i\right)}{ad}$	98
norman	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{ad}+\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}-\frac{4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{ad}}{\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^2}+\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{ad}-\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{ad}$	112

input
```
int(sec(d*x+c)^3/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d/a*(tan(1/2*d*x+1/2*c)-1/(tan(1/2*d*x+1/2*c)-1)+ln(tan(1/2*d*x+1/2*c)-1
)-1/(tan(1/2*d*x+1/2*c)+1)-ln(tan(1/2*d*x+1/2*c)+1))
```

3.44.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.90

$$\int \frac{\sec^3(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{\left(\cos\left(dx+c\right)^2+\cos\left(dx+c\right)\right)\log\left(\sin\left(dx+c\right)+1\right)-\left(\cos\left(dx+c\right)^2+\cos\left(dx+c\right)\right)\log\left(-\sin\left(dx+c\right)\right.}{2\left(ad\cos\left(dx+c\right)^2+ad\cos\left(dx+c\right)\right)}$$

input
```
integrate(sec(d*x+c)^3/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

output
```
-1/2*((cos(d*x + c)^2 + cos(d*x + c))*log(sin(d*x + c) + 1) - (cos(d*x + c
)^2 + cos(d*x + c))*log(-sin(d*x + c) + 1) - 2*(2*cos(d*x + c) + 1)*sin(d*
x + c))/(a*d*cos(d*x + c)^2 + a*d*cos(d*x + c))
```

3.44.6 Sympy [F]

$$\int \frac{\sec^3(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\int \frac{\sec^3(c+dx)}{\sec(c+dx)+1}\,dx}{a}$$

input
```
integrate(sec(d*x+c)**3/(a+a*sec(d*x+c)),x)
```

output
```
Integral(sec(c + d*x)**3/(sec(c + d*x) + 1), x)/a
```

3.44.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 119 vs. $2(51) = 102$.

Time = 0.30 (sec) , antiderivative size = 119, normalized size of antiderivative = 2.33

$$\int \frac{\sec^3(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a} - \frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a} - \frac{2\sin(dx+c)}{\left(a-\frac{a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)} - \frac{\sin(dx+c)}{a(\cos(dx+c)+1)}}{d}$$

input
```
integrate(sec(d*x+c)^3/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

output
```
-(log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a - log(sin(d*x + c)/(cos(d*x +
 c) + 1) - 1)/a - 2*sin(d*x + c)/((a - a*sin(d*x + c)^2/(cos(d*x + c) + 1)
^2)*(cos(d*x + c) + 1)) - sin(d*x + c)/(a*(cos(d*x + c) + 1)))/d
```

3.44.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.65

$$\int \frac{\sec^3(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{\frac{\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a} - \frac{\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a} - \frac{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a} + \frac{2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)a}}{d}$$

input
```
integrate(sec(d*x+c)^3/(a+a*sec(d*x+c)),x, algorithm="giac")
```

output
```
-(log(abs(tan(1/2*d*x + 1/2*c) + 1))/a - log(abs(tan(1/2*d*x + 1/2*c) - 1)
)/a - tan(1/2*d*x + 1/2*c)/a + 2*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/2*
c)^2 - 1)*a))/d
```

3.44.9 Mupad [B] (verification not implemented)

Time = 13.28 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.31

$$\int \frac{\sec^3(c+dx)}{a+a\sec(c+dx)}\,dx=\frac{2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\left(a-a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\right)}-\frac{2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{a\,d}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{a\,d}$$

input
```
int(1/(cos(c + d*x)^3*(a + a/cos(c + d*x))),x)
```

output
```
(2*tan(c/2 + (d*x)/2))/(d*(a - a*tan(c/2 + (d*x)/2)^2)) - (2*atanh(tan(c/2
 + (d*x)/2)))/(a*d) + tan(c/2 + (d*x)/2)/(a*d)
```

3.45 $\int \frac{\sec^2(c+dx)}{a+a\sec(c+dx)}\,dx$

3.45.1 Optimal result 563
3.45.2 Mathematica [A] (verified) 563
3.45.3 Rubi [A] (verified) 564
3.45.4 Maple [A] (verified) 565
3.45.5 Fricas [A] (verification not implemented) 566
3.45.6 Sympy [F] 566
3.45.7 Maxima [A] (verification not implemented) 566
3.45.8 Giac [A] (verification not implemented) 567
3.45.9 Mupad [B] (verification not implemented) 567

3.45.1 Optimal result

Integrand size = 21, antiderivative size = 38

$$\int \frac{\sec^2(c+dx)}{a+a\sec(c+dx)}\,dx=\frac{\operatorname{arctanh}(\sin(c+dx))}{ad}-\frac{\tan(c+dx)}{d(a+a\sec(c+dx))}$$

output
```
arctanh(sin(d*x+c))/a/d-tan(d*x+c)/d/(a+a*sec(d*x+c))
```

3.45.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.71

$$\int \frac{\sec^2(c+dx)}{a+a\sec(c+dx)}\,dx=\frac{\operatorname{arctanh}(\sin(c+dx))-\tan\left(\frac{1}{2}(c+dx)\right)}{ad}$$

input
```
Integrate[Sec[c + d*x]^2/(a + a*Sec[c + d*x]),x]
```

output
```
(ArcTanh[Sin[c + d*x]] - Tan[(c + d*x)/2])/(a*d)
```

3.45.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3042, 4276, 3042, 4257, 4281}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(c+dx)}{a\sec(c+dx)+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2}{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}dx$$

$$\Big\downarrow \text{4276}$$

$$\frac{\int \sec(c+dx)dx}{a}-\int \frac{\sec(c+dx)}{\sec(c+dx)a+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{a}-\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx$$

$$\Big\downarrow \text{4257}$$

$$\frac{\text{arctanh}(\sin(c+dx))}{ad}-\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx$$

$$\Big\downarrow \text{4281}$$

$$\frac{\text{arctanh}(\sin(c+dx))}{ad}-\frac{\tan(c+dx)}{d(a\sec(c+dx)+a)}$$

input
```
Int[Sec[c + d*x]^2/(a + a*Sec[c + d*x]),x]
```

output
```
ArcTanh[Sin[c + d*x]]/(a*d) - Tan[c + d*x]/(d*(a + a*Sec[c + d*x]))
```

3.45.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4276
```
Int[csc[(e_.) + (f_.)*(x_)]^2/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Sym
bol] :> Simp[1/b   Int[Csc[e + f*x], x], x] - Simp[a/b   Int[Csc[e + f*x]/(
a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x]
```

rule 4281
```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[-Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

3.45.4 Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.21

method	result	size
derivativedivides	$\frac{-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{da}$	46
default	$\frac{-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{da}$	46
parallelrisch	$\frac{-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{da}$	46
risch	$-\frac{2i}{da\left(e^{i(dx+c)}+1\right)}-\frac{\ln\left(e^{i(dx+c)}-i\right)}{ad}+\frac{\ln\left(e^{i(dx+c)}+i\right)}{ad}$	65
norman	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{ad}}{-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}+\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{ad}-\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{ad}$	93

input
```
int(sec(d*x+c)^2/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d/a*(-tan(1/2*d*x+1/2*c)-ln(tan(1/2*d*x+1/2*c)-1)+ln(tan(1/2*d*x+1/2*c)+
1))
```

3.45.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.71

$$\int \frac{\sec^2(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{(\cos(dx+c)+1)\log(\sin(dx+c)+1)-(\cos(dx+c)+1)\log(-\sin(dx+c)+1)-2\sin(dx+c)}{2(ad\cos(dx+c)+ad)}$$

input
```
integrate(sec(d*x+c)^2/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

output
```
1/2*((cos(d*x + c) + 1)*log(sin(d*x + c) + 1) - (cos(d*x + c) + 1)*log(-si
n(d*x + c) + 1) - 2*sin(d*x + c))/(a*d*cos(d*x + c) + a*d)
```

3.45.6 Sympy [F]

$$\int \frac{\sec^2(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\int \frac{\sec^2(c+dx)}{\sec(c+dx)+1}\,dx}{a}$$

input
```
integrate(sec(d*x+c)**2/(a+a*sec(d*x+c)),x)
```

output
```
Integral(sec(c + d*x)**2/(sec(c + d*x) + 1), x)/a
```

3.45.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.97

$$\int \frac{\sec^2(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a}-\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a}-\frac{\sin(dx+c)}{a(\cos(dx+c)+1)}}{d}$$

input
```
integrate(sec(d*x+c)^2/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

output
```
(log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a - log(sin(d*x + c)/(cos(d*x +
c) + 1) - 1)/a - sin(d*x + c)/(a*(cos(d*x + c) + 1)))/d
```

3.45.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.42

$$\int \frac{\sec^2(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\frac{\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a} - \frac{\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a} - \frac{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a}}{d}$$

input
```
integrate(sec(d*x+c)^2/(a+a*sec(d*x+c)),x, algorithm="giac")
```

output
```
(log(abs(tan(1/2*d*x + 1/2*c) + 1))/a - log(abs(tan(1/2*d*x + 1/2*c) - 1))
/a - tan(1/2*d*x + 1/2*c)/a)/d
```

3.45.9 Mupad [B] (verification not implemented)

Time = 13.36 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.82

$$\int \frac{\sec^2(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{a\,d}$$

input
```
int(1/(cos(c + d*x)^2*(a + a/cos(c + d*x))),x)
```

output
```
(2*atanh(tan(c/2 + (d*x)/2)) - tan(c/2 + (d*x)/2))/(a*d)
```

3.46 $\int \frac{\sec(c+dx)}{a+a\sec(c+dx)}\,dx$

3.46.1 Optimal result 568
3.46.2 Mathematica [A] (verified) 568
3.46.3 Rubi [A] (verified) 569
3.46.4 Maple [A] (verified) 570
3.46.5 Fricas [A] (verification not implemented) 570
3.46.6 Sympy [F] 570
3.46.7 Maxima [A] (verification not implemented) 571
3.46.8 Giac [A] (verification not implemented) 571
3.46.9 Mupad [B] (verification not implemented) 571

3.46.1 Optimal result

Integrand size = 19, antiderivative size = 22

$$\int \frac{\sec(c+dx)}{a+a\sec(c+dx)}\,dx=\frac{\tan(c+dx)}{d(a+a\sec(c+dx))}$$

output
```
tan(d*x+c)/d/(a+a*sec(d*x+c))
```

3.46.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.77

$$\int \frac{\sec(c+dx)}{a+a\sec(c+dx)}\,dx=\frac{\tan\left(\frac{1}{2}(c+dx)\right)}{ad}$$

input
```
Integrate[Sec[c + d*x]/(a + a*Sec[c + d*x]),x]
```

output
```
Tan[(c + d*x)/2]/(a*d)
```

3.46.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {3042, 4281}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(c+dx)}{a\sec(c+dx)+a}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}dx$$

$$\Big\downarrow \; 4281$$

$$\frac{\tan(c+dx)}{d(a\sec(c+dx)+a)}$$

input
```
Int[Sec[c + d*x]/(a + a*Sec[c + d*x]),x]
```

output
```
Tan[c + d*x]/(d*(a + a*Sec[c + d*x]))
```

3.46.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4281
```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[-Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

3.46.4 Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.77

method	result	size
derivativedivides	$\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}$	17
default	$\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}$	17
norman	$\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}$	17
parallelrisch	$\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}$	17
risch	$\frac{2i}{da\left(e^{i(dx+c)}+1\right)}$	23

input
```
int(sec(d*x+c)/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/a/d*tan(1/2*d*x+1/2*c)
```

3.46.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\sec(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\sin(dx+c)}{ad\cos(dx+c)+ad}$$

input
```
integrate(sec(d*x+c)/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

output
```
sin(d*x + c)/(a*d*cos(d*x + c) + a*d)
```

3.46.6 Sympy [F]

$$\int \frac{\sec(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\int \frac{\sec(c+dx)}{\sec(c+dx)+1}\,dx}{a}$$

input
```
integrate(sec(d*x+c)/(a+a*sec(d*x+c)),x)
```

output
```
Integral(sec(c + d*x)/(sec(c + d*x) + 1), x)/a
```

3.46.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.05

$$\int \frac{\sec(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\sin(dx+c)}{ad(\cos(dx+c)+1)}$$

input
```
integrate(sec(d*x+c)/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

output
```
sin(d*x + c)/(a*d*(cos(d*x + c) + 1))
```

3.46.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

$$\int \frac{\sec(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{ad}$$

input
```
integrate(sec(d*x+c)/(a+a*sec(d*x+c)),x, algorithm="giac")
```

output
```
tan(1/2*d*x + 1/2*c)/(a*d)
```

3.46.9 Mupad [B] (verification not implemented)

Time = 13.36 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

$$\int \frac{\sec(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{a\,d}$$

input
```
int(1/(cos(c + d*x)*(a + a/cos(c + d*x))),x)
```

output
```
tan(c/2 + (d*x)/2)/(a*d)
```

3.47 $\int \frac{1}{a+a\sec(c+dx)}\,dx$

3.47.1 Optimal result . . . 572
3.47.2 Mathematica [A] (verified) . . . 572
3.47.3 Rubi [A] (verified) . . . 573
3.47.4 Maple [A] (verified) . . . 574
3.47.5 Fricas [A] (verification not implemented) . . . 574
3.47.6 Sympy [F] . . . 575
3.47.7 Maxima [A] (verification not implemented) . . . 575
3.47.8 Giac [A] (verification not implemented) . . . 575
3.47.9 Mupad [B] (verification not implemented) . . . 576

3.47.1 Optimal result

Integrand size = 12, antiderivative size = 29

$$\int \frac{1}{a+a\sec(c+dx)}\,dx=\frac{x}{a}-\frac{\tan(c+dx)}{d(a+a\sec(c+dx))}$$

output
```
x/a-tan(d*x+c)/d/(a+a*sec(d*x+c))
```

3.47.2 Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 58, normalized size of antiderivative = 2.00

$$\int \frac{1}{a+a\sec(c+dx)}\,dx=\frac{\sec\left(\frac{c}{2}\right)\sec\left(\frac{1}{2}(c+dx)\right)\left(dx\cos\left(\frac{dx}{2}\right)+dx\cos\left(c+\frac{dx}{2}\right)-2\sin\left(\frac{dx}{2}\right)\right)}{2ad}$$

input
```
Integrate[(a + a*Sec[c + d*x])^(-1),x]
```

output
```
(Sec[c/2]*Sec[(c + d*x)/2]*(d*x*Cos[(d*x)/2] + d*x*Cos[c + (d*x)/2] - 2*Si
n[(d*x)/2]))/(2*a*d)
```

3.47.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3042, 4264, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a\sec(c+dx)+a}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}dx$$

$$\Big\downarrow \text{ 4264}$$

$$-\frac{\int -adx}{a^2}-\frac{\tan(c+dx)}{d(a\sec(c+dx)+a)}$$

$$\Big\downarrow \text{ 24}$$

$$\frac{x}{a}-\frac{\tan(c+dx)}{d(a\sec(c+dx)+a)}$$

input
```
Int[(a + a*Sec[c + d*x])^(-1),x]
```

output
```
x/a - Tan[c + d*x]/(d*(a + a*Sec[c + d*x]))
```

3.47.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4264
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(-Cot[c
+ d*x])*((a + b*Csc[c + d*x])^n/(d*(2*n + 1))), x] + Simp[1/(a^2*(2*n + 1))
   Int[(a + b*Csc[c + d*x])^(n + 1)*(a*(2*n + 1) - b*(n + 1)*Csc[c + d*x]),
 x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LeQ[n, -1] && Int
egerQ[2*n]
```

3.47.4 Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.79

method	result	size
parallelrisch	$\frac{dx-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{da}$	23
norman	$\frac{x}{a}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}$	24
risch	$\frac{x}{a}-\frac{2i}{da\left(e^{i(dx+c)}+1\right)}$	29
derivativedivides	$\frac{-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}$	32
default	$\frac{-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}$	32

input
```
int(1/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
(d*x-tan(1/2*d*x+1/2*c))/d/a
```

3.47.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.28

$$\int \frac{1}{a+a\sec(c+dx)}\,dx=\frac{dx\cos\left(dx+c\right)+dx-\sin\left(dx+c\right)}{ad\cos\left(dx+c\right)+ad}$$

input
```
integrate(1/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

output
```
(d*x*cos(d*x + c) + d*x - sin(d*x + c))/(a*d*cos(d*x + c) + a*d)
```

3.47.6 Sympy [F]

$$\int \frac{1}{a+a\sec(c+dx)}\,dx=\frac{\int \frac{1}{\sec(c+dx)+1}\,dx}{a}$$

input
```
integrate(1/(a+a*sec(d*x+c)),x)
```

output
```
Integral(1/(sec(c + d*x) + 1), x)/a
```

3.47.7 Maxima [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.69

$$\int \frac{1}{a+a\sec(c+dx)}\,dx=\frac{\frac{2\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}-\frac{\sin(dx+c)}{a(\cos(dx+c)+1)}}{d}$$

input
```
integrate(1/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

output
```
(2*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a - sin(d*x + c)/(a*(cos(d*x +
c) + 1)))/d
```

3.47.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.97

$$\int \frac{1}{a+a\sec(c+dx)}\,dx=\frac{\frac{dx+c}{a}-\frac{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a}}{d}$$

input
```
integrate(1/(a+a*sec(d*x+c)),x, algorithm="giac")
```

output
```
((d*x + c)/a - tan(1/2*d*x + 1/2*c)/a)/d
```

3.47.9 Mupad [B] (verification not implemented)

Time = 13.17 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.79

$$\int \frac{1}{a+a\sec(c+dx)}\,dx=\frac{x}{a}-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{a\,d}$$

input
```
int(1/(a + a/cos(c + d*x)),x)
```

output
```
x/a - tan(c/2 + (d*x)/2)/(a*d)
```

3.48 $\int \frac{\cos(c+dx)}{a+a\sec(c+dx)}\,dx$

3.48.1 Optimal result . . . 577
3.48.2 Mathematica [B] (verified) . . . 577
3.48.3 Rubi [A] (verified) . . . 578
3.48.4 Maple [A] (verified) . . . 580
3.48.5 Fricas [A] (verification not implemented) . . . 580
3.48.6 Sympy [F] . . . 581
3.48.7 Maxima [B] (verification not implemented) . . . 581
3.48.8 Giac [A] (verification not implemented) . . . 581
3.48.9 Mupad [B] (verification not implemented) . . . 582

3.48.1 Optimal result

Integrand size = 19, antiderivative size = 44

$$\int \frac{\cos(c+dx)}{a+a\sec(c+dx)}\,dx=-\frac{x}{a}+\frac{2\sin(c+dx)}{ad}-\frac{\sin(c+dx)}{d(a+a\sec(c+dx))}$$

output
```
-x/a+2*sin(d*x+c)/a/d-sin(d*x+c)/d/(a+a*sec(d*x+c))
```

3.48.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 89 vs. 2(44) = 88.

Time = 0.82 (sec) , antiderivative size = 89, normalized size of antiderivative = 2.02

$$\int \frac{\cos(c+dx)}{a+a\sec(c+dx)}\,dx$$
$$=\frac{\sec\left(\frac{c}{2}\right)\sec\left(\frac{1}{2}(c+dx)\right)\left(-2dx\cos\left(\frac{dx}{2}\right)-2dx\cos\left(c+\frac{dx}{2}\right)+5\sin\left(\frac{dx}{2}\right)+\sin\left(c+\frac{dx}{2}\right)+\sin\left(c+\frac{3dx}{2}\right)+\sin\right.}{4ad}$$

input
```
Integrate[Cos[c + d*x]/(a + a*Sec[c + d*x]),x]
```

output
```
(Sec[c/2]*Sec[(c + d*x)/2]*(-2*d*x*Cos[(d*x)/2] - 2*d*x*Cos[c + (d*x)/2] +
 5*Sin[(d*x)/2] + Sin[c + (d*x)/2] + Sin[c + (3*d*x)/2] + Sin[2*c + (3*d*x
)/2]))/(4*a*d)
```

3.48.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.02, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {3042, 4306, 25, 3042, 4274, 24, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(c+dx)}{a\sec(c+dx)+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)}dx$$

$$\Big\downarrow \text{4306}$$

$$-\frac{\int -\cos(c+dx)(2a-a\sec(c+dx))dx}{a^2}-\frac{\sin(c+dx)}{d(a\sec(c+dx)+a)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \cos(c+dx)(2a-a\sec(c+dx))dx}{a^2}-\frac{\sin(c+dx)}{d(a\sec(c+dx)+a)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{2a-a\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}-\frac{\sin(c+dx)}{d(a\sec(c+dx)+a)}$$

$$\Big\downarrow \text{4274}$$

$$\frac{2a\int \cos(c+dx)dx-a\int 1dx}{a^2}-\frac{\sin(c+dx)}{d(a\sec(c+dx)+a)}$$

$$\Big\downarrow \text{24}$$

$$\frac{2a\int \cos(c+dx)dx-ax}{a^2}-\frac{\sin(c+dx)}{d(a\sec(c+dx)+a)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2a\int \sin\left(c+dx+\frac{\pi}{2}\right)dx-ax}{a^2}-\frac{\sin(c+dx)}{d(a\sec(c+dx)+a)}$$

$$\Big\downarrow \text{3117}$$

$$\frac{\frac{2a\sin(c+dx)}{d}-ax}{a^2}-\frac{\sin(c+dx)}{d(a\sec(c+dx)+a)}$$

input
```
Int[Cos[c + d*x]/(a + a*Sec[c + d*x]),x]
```

output
```
-(Sin[c + d*x]/(d*(a + a*Sec[c + d*x]))) + (-(a*x) + (2*a*Sin[c + d*x])/d)
/a^2
```

3.48.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4306
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*(a + b*Csc[e +
f*x]))), x] - Simp[1/a^2   Int[(d*Csc[e + f*x])^n*(a*(n - 1) - b*n*Csc[e +
f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, 0
]
```

3.48.4 Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.70

method	result	size
parallelrisch	$\frac{-dx+(2+\cos(dx+c))\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{da}$	31
derivativedivides	$\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}-2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}$	56
default	$\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}-2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}$	56
risch	$-\frac{x}{a}-\frac{i\mathrm{e}^{i(dx+c)}}{2da}+\frac{i\mathrm{e}^{-i(dx+c)}}{2da}+\frac{2i}{da\left(\mathrm{e}^{i(dx+c)}+1\right)}$	66
norman	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{ad}-\frac{x}{a}+\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}-\frac{x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{a}}{1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}$	76

input
```
int(cos(d*x+c)/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
(-d*x+(2+cos(d*x+c))*tan(1/2*d*x+1/2*c))/d/a
```

3.48.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.05

$$\int \frac{\cos(c+dx)}{a+a\sec(c+dx)}\,dx=-\frac{dx\cos\left(dx+c\right)+dx-\left(\cos\left(dx+c\right)+2\right)\sin\left(dx+c\right)}{ad\cos\left(dx+c\right)+ad}$$

input
```
integrate(cos(d*x+c)/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

output
```
-(d*x*cos(d*x + c) + d*x - (cos(d*x + c) + 2)*sin(d*x + c))/(a*d*cos(d*x +
 c) + a*d)
```

3.48.6 Sympy [F]

$$\int \frac{\cos(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\int \frac{\cos(c+dx)}{\sec(c+dx)+1}\,dx}{a}$$

input
```
integrate(cos(d*x+c)/(a+a*sec(d*x+c)),x)
```

output
```
Integral(cos(c + d*x)/(sec(c + d*x) + 1), x)/a
```

3.48.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 92 vs. $2(44) = 88$.

Time = 0.34 (sec) , antiderivative size = 92, normalized size of antiderivative = 2.09

$$\int \frac{\cos(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{\frac{2\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a} - \frac{2\sin(dx+c)}{\left(a+\frac{a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)} - \frac{\sin(dx+c)}{a(\cos(dx+c)+1)}}{d}$$

input
```
integrate(cos(d*x+c)/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

output
```
-(2*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a - 2*sin(d*x + c)/((a + a*sin
(d*x + c)^2/(cos(d*x + c) + 1)^2)*(cos(d*x + c) + 1)) - sin(d*x + c)/(a*(c
os(d*x + c) + 1)))/d
```

3.48.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.32

$$\int \frac{\cos(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{\frac{dx+c}{a} - \frac{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a} - \frac{2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)a}}{d}$$

input
```
integrate(cos(d*x+c)/(a+a*sec(d*x+c)),x, algorithm="giac")
```

output
```
-((d*x + c)/a - tan(1/2*d*x + 1/2*c)/a - 2*tan(1/2*d*x + 1/2*c)/((tan(1/2*
d*x + 1/2*c)^2 + 1)*a))/d
```

3.48.9 Mupad [B] (verification not implemented)

Time = 13.19 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.50

$$\int \frac{\cos(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{2\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+\left(-c-d\,x\right)\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{a\,d\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}$$

input
```
int(cos(c + d*x)/(a + a/cos(c + d*x)),x)
```

output
```
(sin(c/2 + (d*x)/2) - cos(c/2 + (d*x)/2)*(c + d*x) + 2*cos(c/2 + (d*x)/2)^
2*sin(c/2 + (d*x)/2))/(a*d*cos(c/2 + (d*x)/2))
```

3.49 $\int \frac{\cos^2(c+dx)}{a+a\sec(c+dx)}\,dx$

3.49.1 Optimal result . . . 583
3.49.2 Mathematica [A] (verified) . . . 583
3.49.3 Rubi [A] (verified) . . . 584
3.49.4 Maple [A] (verified) . . . 586
3.49.5 Fricas [A] (verification not implemented) . . . 586
3.49.6 Sympy [F] . . . 587
3.49.7 Maxima [A] (verification not implemented) . . . 587
3.49.8 Giac [A] (verification not implemented) . . . 588
3.49.9 Mupad [B] (verification not implemented) . . . 588

3.49.1 Optimal result

Integrand size = 21, antiderivative size = 74

$$\int \frac{\cos^2(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{3x}{2a} - \frac{2\sin(c+dx)}{ad} + \frac{3\cos(c+dx)\sin(c+dx)}{2ad} - \frac{\cos(c+dx)\sin(c+dx)}{d(a+a\sec(c+dx))}$$

output
```
3/2*x/a-2*sin(d*x+c)/a/d+3/2*cos(d*x+c)*sin(d*x+c)/a/d-cos(d*x+c)*sin(d*x+
c)/d/(a+a*sec(d*x+c))
```

3.49.2 Mathematica [A] (verified)

Time = 0.88 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.58

$$\int \frac{\cos^2(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\sec\left(\frac{c}{2}\right)\sec\left(\frac{1}{2}(c+dx)\right)\left(12dx\cos\left(\frac{dx}{2}\right)+12dx\cos\left(c+\frac{dx}{2}\right)-20\sin\left(\frac{dx}{2}\right)-4\sin\left(c+\frac{dx}{2}\right)-3\sin\left(c+\frac{3dx}{2}\right)\right.}{16ad}$$

input
```
Integrate[Cos[c + d*x]^2/(a + a*Sec[c + d*x]),x]
```

output
```
(Sec[c/2]*Sec[(c + d*x)/2]*(12*d*x*Cos[(d*x)/2] + 12*d*x*Cos[c + (d*x)/2]
- 20*Sin[(d*x)/2] - 4*Sin[c + (d*x)/2] - 3*Sin[c + (3*d*x)/2] - 3*Sin[2*c
+ (3*d*x)/2] + Sin[2*c + (5*d*x)/2] + Sin[3*c + (5*d*x)/2]))/(16*a*d)
```

3.49.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.01, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3042, 4306, 25, 3042, 4274, 3042, 3115, 24, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^2(c+dx)}{a\sec(c+dx)+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^2\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)}dx$$

$$\Big\downarrow \text{4306}$$

$$-\frac{\int -\cos^2(c+dx)(3a-2a\sec(c+dx))dx}{a^2}-\frac{\sin(c+dx)\cos(c+dx)}{d(a\sec(c+dx)+a)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \cos^2(c+dx)(3a-2a\sec(c+dx))dx}{a^2}-\frac{\sin(c+dx)\cos(c+dx)}{d(a\sec(c+dx)+a)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{3a-2a\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^2}dx}{a^2}-\frac{\sin(c+dx)\cos(c+dx)}{d(a\sec(c+dx)+a)}$$

$$\Big\downarrow \text{4274}$$

$$\frac{3a\int \cos^2(c+dx)dx-2a\int \cos(c+dx)dx}{a^2}-\frac{\sin(c+dx)\cos(c+dx)}{d(a\sec(c+dx)+a)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3a\int \sin\left(c+dx+\frac{\pi}{2}\right)^2dx-2a\int \sin\left(c+dx+\frac{\pi}{2}\right)dx}{a^2}-\frac{\sin(c+dx)\cos(c+dx)}{d(a\sec(c+dx)+a)}$$

$$\Big\downarrow \text{3115}$$

$$\frac{3a\left(\frac{\int 1dx}{2}+\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)-2a\int \sin\left(c+dx+\frac{\pi}{2}\right)dx}{a^2}-\frac{\sin(c+dx)\cos(c+dx)}{d(a\sec(c+dx)+a)}$$

$$\Big\downarrow \text{24}$$

$$\frac{3a\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)-2a\int\sin\left(c+dx+\frac{\pi}{2}\right)dx}{a^2}-\frac{\sin(c+dx)\cos(c+dx)}{d(a\sec(c+dx)+a)}$$

$$\Big\downarrow \; 3117$$

$$\frac{3a\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)-\frac{2a\sin(c+dx)}{d}}{a^2}-\frac{\sin(c+dx)\cos(c+dx)}{d(a\sec(c+dx)+a)}$$

input
```
Int[Cos[c + d*x]^2/(a + a*Sec[c + d*x]),x]
```

output
```
-((Cos[c + d*x]*Sin[c + d*x])/(d*(a + a*Sec[c + d*x]))) + ((-2*a*Sin[c + d
*x])/d + 3*a*(x/2 + (Cos[c + d*x]*Sin[c + d*x])/(2*d)))/a^2
```

3.49.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4306
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*(a + b*Csc[e +
f*x]))), x] - Simp[1/a^2   Int[(d*Csc[e + f*x])^n*(a*(n - 1) - b*n*Csc[e +
f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, 0
]
```

3.49.4 Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.58

method	result	size
parallelrisch	$\frac{6dx+(-7+\cos(2dx+2c)-2\cos(dx+c))\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4da}$	43
derivativedivides	$\frac{-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{-3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^2}+3\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}$	74
default	$\frac{-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{-3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^2}+3\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}$	74
risch	$\frac{3x}{2a}+\frac{ie^{i(dx+c)}}{2da}-\frac{ie^{-i(dx+c)}}{2da}-\frac{2i}{da\left(e^{i(dx+c)}+1\right)}+\frac{\sin(2dx+2c)}{4da}$	83
norman	$\frac{\frac{3x}{2a}-\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}-\frac{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{ad}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{ad}+\frac{3x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{a}+\frac{3x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{2a}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^2}$	113

input
```
int(cos(d*x+c)^2/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/4*(6*d*x+(-7+cos(2*d*x+2*c)-2*cos(d*x+c))*tan(1/2*d*x+1/2*c))/d/a
```

3.49.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.77

$$\int \frac{\cos^2(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{3\,dx\cos(dx+c)+3\,dx+\left(\cos(dx+c)^2-\cos(dx+c)-4\right)\sin(dx+c)}{2\left(ad\cos(dx+c)+ad\right)}$$

input
```
integrate(cos(d*x+c)^2/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

output
```
1/2*(3*d*x*cos(d*x + c) + 3*d*x + (cos(d*x + c)^2 - cos(d*x + c) - 4)*sin(
d*x + c))/(a*d*cos(d*x + c) + a*d)
```

3.49.6 Sympy [F]

$$\int \frac{\cos^2(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\int \frac{\cos^2(c+dx)}{\sec(c+dx)+1}\,dx}{a}$$

input
```
integrate(cos(d*x+c)**2/(a+a*sec(d*x+c)),x)
```

output
```
Integral(cos(c + d*x)**2/(sec(c + d*x) + 1), x)/a
```

3.49.7 Maxima [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.80

$$\int \frac{\cos^2(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{\frac{\frac{\sin(dx+c)}{\cos(dx+c)+1}+\frac{3\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a+\frac{2a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}-\frac{3\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}+\frac{\sin(dx+c)}{a(\cos(dx+c)+1)}}{d}$$

input
```
integrate(cos(d*x+c)^2/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

output
```
-((sin(d*x + c)/(cos(d*x + c) + 1) + 3*sin(d*x + c)^3/(cos(d*x + c) + 1)^3
)/(a + 2*a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a*sin(d*x + c)^4/(cos(d*x
 + c) + 1)^4) - 3*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a + sin(d*x + c)
/(a*(cos(d*x + c) + 1)))/d
```

3.49.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.99

$$\int \frac{\cos^2(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\frac{3\,(dx+c)}{a} - \frac{2\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a} - \frac{2\left(3\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^2 a}}{2\,d}$$

input
```
integrate(cos(d*x+c)^2/(a+a*sec(d*x+c)),x, algorithm="giac")
```

output
```
1/2*(3*(d*x + c)/a - 2*tan(1/2*d*x + 1/2*c)/a - 2*(3*tan(1/2*d*x + 1/2*c)^
3 + tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^2*a))/d
```

3.49.9 Mupad [B] (verification not implemented)

Time = 13.28 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.20

$$\int \frac{\cos^2(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right) - \frac{3\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,(c+d\,x)}{2} + 3\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right) - 2\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{a\,d\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}$$

input
```
int(cos(c + d*x)^2/(a + a/cos(c + d*x)),x)
```

output
```
-(sin(c/2 + (d*x)/2) - (3*cos(c/2 + (d*x)/2)*(c + d*x))/2 + 3*cos(c/2 + (d
*x)/2)^2*sin(c/2 + (d*x)/2) - 2*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2))/(
a*d*cos(c/2 + (d*x)/2))
```

3.50 $\int \frac{\cos^3(c+dx)}{a+a\sec(c+dx)}\,dx$

3.50.1 Optimal result . . . 589
3.50.2 Mathematica [A] (verified) . . . 589
3.50.3 Rubi [A] (verified) . . . 590
3.50.4 Maple [A] (verified) . . . 592
3.50.5 Fricas [A] (verification not implemented) . . . 593
3.50.6 Sympy [F(-1)] . . . 593
3.50.7 Maxima [A] (verification not implemented) . . . 593
3.50.8 Giac [A] (verification not implemented) . . . 594
3.50.9 Mupad [B] (verification not implemented) . . . 594

3.50.1 Optimal result

Integrand size = 21, antiderivative size = 94

$$\int \frac{\cos^3(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{3x}{2a}+\frac{4\sin(c+dx)}{ad}-\frac{3\cos(c+dx)\sin(c+dx)}{2ad}-\frac{\cos^2(c+dx)\sin(c+dx)}{d(a+a\sec(c+dx))}-\frac{4\sin^3(c+dx)}{3ad}$$

output
```
-3/2*x/a+4*sin(d*x+c)/a/d-3/2*cos(d*x+c)*sin(d*x+c)/a/d-cos(d*x+c)^2*sin(d
*x+c)/d/(a+a*sec(d*x+c))-4/3*sin(d*x+c)^3/a/d
```

3.50.2 Mathematica [A] (verified)

Time = 1.00 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.52

$$\int \frac{\cos^3(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\sec\left(\frac{c}{2}\right)\sec\left(\frac{1}{2}(c+dx)\right)\left(-36dx\cos\left(\frac{dx}{2}\right)-36dx\cos\left(c+\frac{dx}{2}\right)+69\sin\left(\frac{dx}{2}\right)+21\sin\left(c+\frac{dx}{2}\right)+18\sin\left(c+\right.\right.}{48ad}$$

input
```
Integrate[Cos[c + d*x]^3/(a + a*Sec[c + d*x]),x]
```

output
```
(Sec[c/2]*Sec[(c + d*x)/2]*(-36*d*x*Cos[(d*x)/2] - 36*d*x*Cos[c + (d*x)/2]
 + 69*Sin[(d*x)/2] + 21*Sin[c + (d*x)/2] + 18*Sin[c + (3*d*x)/2] + 18*Sin[
2*c + (3*d*x)/2] - 2*Sin[2*c + (5*d*x)/2] - 2*Sin[3*c + (5*d*x)/2] + Sin[3
*c + (7*d*x)/2] + Sin[4*c + (7*d*x)/2]))/(48*a*d)
```

3.50.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.98, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.476, Rules used = {3042, 4306, 25, 3042, 4274, 3042, 3113, 2009, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^3(c+dx)}{a\sec(c+dx)+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^3\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)}dx$$

$$\Big\downarrow \text{4306}$$

$$-\frac{\int -\cos^3(c+dx)(4a-3a\sec(c+dx))dx}{a^2}-\frac{\sin(c+dx)\cos^2(c+dx)}{d(a\sec(c+dx)+a)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \cos^3(c+dx)(4a-3a\sec(c+dx))dx}{a^2}-\frac{\sin(c+dx)\cos^2(c+dx)}{d(a\sec(c+dx)+a)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{4a-3a\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^3}dx}{a^2}-\frac{\sin(c+dx)\cos^2(c+dx)}{d(a\sec(c+dx)+a)}$$

$$\Big\downarrow \text{4274}$$

$$\frac{4a\int \cos^3(c+dx)dx-3a\int \cos^2(c+dx)dx}{a^2}-\frac{\sin(c+dx)\cos^2(c+dx)}{d(a\sec(c+dx)+a)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4a\int \sin\left(c+dx+\frac{\pi}{2}\right)^3dx-3a\int \sin\left(c+dx+\frac{\pi}{2}\right)^2dx}{a^2}-\frac{\sin(c+dx)\cos^2(c+dx)}{d(a\sec(c+dx)+a)}$$

$$\Big\downarrow \text{3113}$$

$$\frac{-\frac{4a\int \left(1-\sin^2(c+dx)\right)d(-\sin(c+dx))}{d}-3a\int \sin\left(c+dx+\frac{\pi}{2}\right)^2dx}{a^2}-\frac{\sin(c+dx)\cos^2(c+dx)}{d(a\sec(c+dx)+a)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-3a\int \sin\left(c+dx+\frac{\pi}{2}\right)^2dx-\frac{4a\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}}{a^2}-\frac{\sin(c+dx)\cos^2(c+dx)}{d(a\sec(c+dx)+a)}$$

$$\Big\downarrow \; 3115$$

$$\frac{-3a\left(\frac{\int 1dx}{2}+\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)-\frac{4a\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}}{a^2}-\frac{\sin(c+dx)\cos^2(c+dx)}{d(a\sec(c+dx)+a)}$$

$$\Big\downarrow \; 24$$

$$\frac{-\frac{4a\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}-3a\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)}{a^2}-\frac{\sin(c+dx)\cos^2(c+dx)}{d(a\sec(c+dx)+a)}$$

input
```
Int[Cos[c + d*x]^3/(a + a*Sec[c + d*x]),x]
```

output
```
-((Cos[c + d*x]^2*Sin[c + d*x])/(d*(a + a*Sec[c + d*x]))) + (-3*a*(x/2 + (
Cos[c + d*x]*Sin[c + d*x])/(2*d)) - (4*a*(-Sin[c + d*x] + Sin[c + d*x]^3/3
))/d)/a^2
```

3.50.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4306
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*(a + b*Csc[e +
f*x]))), x] - Simp[1/a^2   Int[(d*Csc[e + f*x])^n*(a*(n - 1) - b*n*Csc[e +
f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, 0
]
```

3.50.4 Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.57

method	result
parallelrisch	$\frac{(31+\cos(3dx+3c)-\cos(2dx+2c)+17\cos(dx+c))\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-18dx}{12da}$
derivativedivides	$\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{8\left(-\frac{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{8}-\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3}-\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8}\right)}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^3}-3\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}$
default	$\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{8\left(-\frac{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{8}-\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3}-\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8}\right)}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^3}-3\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}$
risch	$-\frac{3x}{2a}-\frac{7ie^{i(dx+c)}}{8da}+\frac{7ie^{-i(dx+c)}}{8da}+\frac{2i}{da\left(e^{i(dx+c)}+1\right)}+\frac{\sin(3dx+3c)}{12da}-\frac{\sin(2dx+2c)}{4da}$
norman	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{ad}-\frac{3x}{2a}+\frac{4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}+\frac{25\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3ad}+\frac{8\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{ad}-\frac{9x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{2a}-\frac{9x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{2a}-\frac{3x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{2a}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^3}$

input
```
int(cos(d*x+c)^3/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/12*((31+cos(3*d*x+3*c)-cos(2*d*x+2*c)+17*cos(d*x+c))*tan(1/2*d*x+1/2*c)-
18*d*x)/d/a
```

3.50.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.74

$$\int \frac{\cos^3(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{9\,dx\cos\left(dx+c\right)+9\,dx-\left(2\,\cos\left(dx+c\right)^3-\cos\left(dx+c\right)^2+7\,\cos\left(dx+c\right)+16\right)\sin\left(dx+c\right)}{6\left(ad\cos\left(dx+c\right)+ad\right)}$$

input
```
integrate(cos(d*x+c)^3/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

output
```
-1/6*(9*d*x*cos(d*x + c) + 9*d*x - (2*cos(d*x + c)^3 - cos(d*x + c)^2 + 7*
cos(d*x + c) + 16)*sin(d*x + c))/(a*d*cos(d*x + c) + a*d)
```

3.50.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^3(c+dx)}{a+a\sec(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**3/(a+a*sec(d*x+c)),x)
```

output
```
Timed out
```

3.50.7 Maxima [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.87

$$\int \frac{\cos^3(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\frac{\frac{9\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{16\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{15\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{a+\frac{3\,a\,\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{3\,a\,\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{a\,\sin(dx+c)^6}{(\cos(dx+c)+1)^6}}-\frac{9\,\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}+\frac{3\,\sin(dx+c)}{a(\cos(dx+c)+1)}}{3\,d}$$

input
```
integrate(cos(d*x+c)^3/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

output
```
1/3*((9*sin(d*x + c)/(cos(d*x + c) + 1) + 16*sin(d*x + c)^3/(cos(d*x + c)
+ 1)^3 + 15*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/(a + 3*a*sin(d*x + c)^2/(
cos(d*x + c) + 1)^2 + 3*a*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + a*sin(d*x
+ c)^6/(cos(d*x + c) + 1)^6) - 9*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a
 + 3*sin(d*x + c)/(a*(cos(d*x + c) + 1)))/d
```

3.50.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.94

$$\int \frac{\cos^3(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{\frac{9\,(dx+c)}{a} - \frac{6\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a} - \frac{2\left(15\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+16\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+9\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^3 a}}{6\,d}$$

input
```
integrate(cos(d*x+c)^3/(a+a*sec(d*x+c)),x, algorithm="giac")
```

output
```
-1/6*(9*(d*x + c)/a - 6*tan(1/2*d*x + 1/2*c)/a - 2*(15*tan(1/2*d*x + 1/2*c
)^5 + 16*tan(1/2*d*x + 1/2*c)^3 + 9*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x +
1/2*c)^2 + 1)^3*a))/d
```

3.50.9 Mupad [B] (verification not implemented)

Time = 12.87 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.74

$$\int \frac{\cos^3(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\frac{15\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8} + \frac{3\,\sin\left(\frac{3\,c}{2}+\frac{3\,d\,x}{2}\right)}{4} - \frac{\sin\left(\frac{5\,c}{2}+\frac{5\,d\,x}{2}\right)}{12} + \frac{\sin\left(\frac{7\,c}{2}+\frac{7\,d\,x}{2}\right)}{24}}{a\,d\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)} - \frac{3\,x}{2\,a}$$

input
```
int(cos(c + d*x)^3/(a + a/cos(c + d*x)),x)
```

output
```
((15*sin(c/2 + (d*x)/2))/8 + (3*sin((3*c)/2 + (3*d*x)/2))/4 - sin((5*c)/2
+ (5*d*x)/2)/12 + sin((7*c)/2 + (7*d*x)/2)/24)/(a*d*cos(c/2 + (d*x)/2)) -
(3*x)/(2*a)
```

3.51 $\int \frac{\cos^4(c+dx)}{a+a\sec(c+dx)}\,dx$

3.51.1 Optimal result 595
3.51.2 Mathematica [A] (verified) 595
3.51.3 Rubi [A] (verified) 596
3.51.4 Maple [A] (verified) 599
3.51.5 Fricas [A] (verification not implemented) 599
3.51.6 Sympy [F] 600
3.51.7 Maxima [A] (verification not implemented) 600
3.51.8 Giac [A] (verification not implemented) 600
3.51.9 Mupad [B] (verification not implemented) 601

3.51.1 Optimal result

Integrand size = 21, antiderivative size = 118

$$\int \frac{\cos^4(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{15x}{8a} - \frac{4\sin(c+dx)}{ad} + \frac{15\cos(c+dx)\sin(c+dx)}{8ad} + \frac{5\cos^3(c+dx)\sin(c+dx)}{4ad} - \frac{\cos^3(c+dx)\sin(c+dx)}{d(a+a\sec(c+dx))} + \frac{4\sin^3(c+dx)}{3ad}$$

output
```
15/8*x/a-4*sin(d*x+c)/a/d+15/8*cos(d*x+c)*sin(d*x+c)/a/d+5/4*cos(d*x+c)^3*
sin(d*x+c)/a/d-cos(d*x+c)^3*sin(d*x+c)/d/(a+a*sec(d*x+c))+4/3*sin(d*x+c)^3
/a/d
```

3.51.2 Mathematica [A] (verified)

Time = 1.03 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.47

$$\int \frac{\cos^4(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\sec\left(\frac{c}{2}\right)\sec\left(\frac{1}{2}(c+dx)\right)\left(360dx\cos\left(\frac{dx}{2}\right)+360dx\cos\left(c+\frac{dx}{2}\right)-552\sin\left(\frac{dx}{2}\right)-168\sin\left(c+\frac{dx}{2}\right)-120\sin\left(\right.\right.}{}$$

input
```
Integrate[Cos[c + d*x]^4/(a + a*Sec[c + d*x]),x]
```

output
```
(Sec[c/2]*Sec[(c + d*x)/2]*(360*d*x*Cos[(d*x)/2] + 360*d*x*Cos[c + (d*x)/2
] - 552*Sin[(d*x)/2] - 168*Sin[c + (d*x)/2] - 120*Sin[c + (3*d*x)/2] - 120
*Sin[2*c + (3*d*x)/2] + 40*Sin[2*c + (5*d*x)/2] + 40*Sin[3*c + (5*d*x)/2]
- 5*Sin[3*c + (7*d*x)/2] - 5*Sin[4*c + (7*d*x)/2] + 3*Sin[4*c + (9*d*x)/2]
 + 3*Sin[5*c + (9*d*x)/2]))/(384*a*d)
```

3.51.3 Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {3042, 4306, 25, 3042, 4274, 3042, 3113, 2009, 3115, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^4(c+dx)}{a\sec(c+dx)+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^4\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)}dx$$

$$\Big\downarrow \text{4306}$$

$$-\frac{\int -\cos^4(c+dx)(5a-4a\sec(c+dx))dx}{a^2}-\frac{\sin(c+dx)\cos^3(c+dx)}{d(a\sec(c+dx)+a)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \cos^4(c+dx)(5a-4a\sec(c+dx))dx}{a^2}-\frac{\sin(c+dx)\cos^3(c+dx)}{d(a\sec(c+dx)+a)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{5a-4a\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^4}dx}{a^2}-\frac{\sin(c+dx)\cos^3(c+dx)}{d(a\sec(c+dx)+a)}$$

$$\Big\downarrow \text{4274}$$

$$\frac{5a\int \cos^4(c+dx)dx-4a\int \cos^3(c+dx)dx}{a^2}-\frac{\sin(c+dx)\cos^3(c+dx)}{d(a\sec(c+dx)+a)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5a\int \sin\left(c+dx+\frac{\pi}{2}\right)^4dx-4a\int \sin\left(c+dx+\frac{\pi}{2}\right)^3dx}{a^2}-\frac{\sin(c+dx)\cos^3(c+dx)}{d(a\sec(c+dx)+a)}$$

$$\Big\downarrow \text{3113}$$

$$\frac{\frac{4a\int\left(1-\sin^2(c+dx)\right)d(-\sin(c+dx))}{d}+5a\int\sin\left(c+dx+\frac{\pi}{2}\right)^4dx}{a^2}-\frac{\sin(c+dx)\cos^3(c+dx)}{d(a\sec(c+dx)+a)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{5a\int\sin\left(c+dx+\frac{\pi}{2}\right)^4dx+\frac{4a\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}}{a^2}-\frac{\sin(c+dx)\cos^3(c+dx)}{d(a\sec(c+dx)+a)}$$

$$\Big\downarrow \text{3115}$$

$$\frac{5a\left(\frac{3}{4}\int\cos^2(c+dx)dx+\frac{\sin(c+dx)\cos^3(c+dx)}{4d}\right)+\frac{4a\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}}{a^2}-\frac{\sin(c+dx)\cos^3(c+dx)}{d(a\sec(c+dx)+a)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5a\left(\frac{3}{4}\int\sin\left(c+dx+\frac{\pi}{2}\right)^2dx+\frac{\sin(c+dx)\cos^3(c+dx)}{4d}\right)+\frac{4a\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}}{a^2}-\frac{\sin(c+dx)\cos^3(c+dx)}{d(a\sec(c+dx)+a)}$$

$$\Big\downarrow \text{3115}$$

$$\frac{5a\left(\frac{3}{4}\left(\frac{\int 1dx}{2}+\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)+\frac{\sin(c+dx)\cos^3(c+dx)}{4d}\right)+\frac{4a\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}}{a^2}-\frac{\sin(c+dx)\cos^3(c+dx)}{d(a\sec(c+dx)+a)}$$

$$\Big\downarrow \text{24}$$

$$\frac{\frac{4a\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}+5a\left(\frac{\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3}{4}\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)\right)}{a^2}-\frac{\sin(c+dx)\cos^3(c+dx)}{d(a\sec(c+dx)+a)}$$

input
```
Int[Cos[c + d*x]^4/(a + a*Sec[c + d*x]),x]
```

output
```
-((Cos[c + d*x]^3*Sin[c + d*x])/(d*(a + a*Sec[c + d*x]))) + ((4*a*(-Sin[c
+ d*x] + Sin[c + d*x]^3/3))/d + 5*a*((Cos[c + d*x]^3*Sin[c + d*x])/(4*d) +
 (3*(x/2 + (Cos[c + d*x]*Sin[c + d*x])/(2*d)))/4))/a^2
```

3.51. $\int \frac{\cos^4(c+dx)}{a+a\sec(c+dx)}\,dx$

3.51.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4306
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*(a + b*Csc[e +
f*x]))), x] - Simp[1/a^2   Int[(d*Csc[e + f*x])^n*(a*(n - 1) - b*n*Csc[e +
f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, 0
]
```

3.51.4 Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.57

method	result
parallelrisch	$\frac{180dx+(-221+3\cos(4dx+4c)-2\cos(3dx+3c)+38\cos(2dx+2c)-82\cos(dx+c))\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{96da}$
derivativedivides	$\frac{-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{-\frac{25\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{4}-\frac{115\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{12}-\frac{109\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{12}-\frac{7\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^4}+\frac{15\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4}}{da}$
default	$\frac{-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{-\frac{25\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{4}-\frac{115\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{12}-\frac{109\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{12}-\frac{7\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^4}+\frac{15\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4}}{da}$
risch	$\frac{15x}{8a}+\frac{7ie^{i(dx+c)}}{8da}-\frac{7ie^{-i(dx+c)}}{8da}-\frac{2i}{da\left(e^{i(dx+c)}+1\right)}+\frac{\sin(4dx+4c)}{32da}-\frac{\sin(3dx+3c)}{12da}+\frac{\sin(2dx+2c)}{2da}$
norman	$\frac{\frac{15x}{8a}-\frac{11\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4ad}-\frac{157\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{12ad}-\frac{187\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{12ad}-\frac{41\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{4ad}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^9}{ad}+\frac{15x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{2a}+\frac{45x\tan\left(\frac{d}{}\right.}{4a}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^4}$

input
```
int(cos(d*x+c)^4/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/96*(180*d*x+(-221+3*cos(4*d*x+4*c)-2*cos(3*d*x+3*c)+38*cos(2*d*x+2*c)-82
*cos(d*x+c))*tan(1/2*d*x+1/2*c))/d/a
```

3.51.5 Fricas [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.67

$$\int \frac{\cos^4(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{45\,dx\cos(dx+c)+45\,dx+\left(6\cos(dx+c)^4-2\cos(dx+c)^3+13\cos(dx+c)^2-19\cos(dx+c)-64\right.}{24\,(ad\cos(dx+c)+ad)}$$

input
```
integrate(cos(d*x+c)^4/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

output
```
1/24*(45*d*x*cos(d*x + c) + 45*d*x + (6*cos(d*x + c)^4 - 2*cos(d*x + c)^3
+ 13*cos(d*x + c)^2 - 19*cos(d*x + c) - 64)*sin(d*x + c))/(a*d*cos(d*x + c
) + a*d)
```

3.51.6 Sympy [F]

$$\int \frac{\cos^4(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\int \frac{\cos^4(c+dx)}{\sec(c+dx)+1}\,dx}{a}$$

input
```
integrate(cos(d*x+c)**4/(a+a*sec(d*x+c)),x)
```

output
```
Integral(cos(c + d*x)**4/(sec(c + d*x) + 1), x)/a
```

3.51.7 Maxima [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.84

$$\int \frac{\cos^4(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{\frac{\frac{21\sin(dx+c)}{\cos(dx+c)+1}+\frac{109\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{115\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{75\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{a+\frac{4a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{6a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{4a\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{a\sin(dx+c)^8}{(\cos(dx+c)+1)^8}} - \frac{45\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a} + \frac{12\sin(dx+c)}{a(\cos(dx+c)+1)}}{12\,d}$$

input
```
integrate(cos(d*x+c)^4/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

output
```
-1/12*((21*sin(d*x + c)/(cos(d*x + c) + 1) + 109*sin(d*x + c)^3/(cos(d*x +
 c) + 1)^3 + 115*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 75*sin(d*x + c)^7/(
cos(d*x + c) + 1)^7)/(a + 4*a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 6*a*si
n(d*x + c)^4/(cos(d*x + c) + 1)^4 + 4*a*sin(d*x + c)^6/(cos(d*x + c) + 1)^
6 + a*sin(d*x + c)^8/(cos(d*x + c) + 1)^8) - 45*arctan(sin(d*x + c)/(cos(d
*x + c) + 1))/a + 12*sin(d*x + c)/(a*(cos(d*x + c) + 1)))/d
```

3.51.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.86

$$\int \frac{\cos^4(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\frac{45\,(dx+c)}{a} - \frac{24\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a} - \frac{2\left(75\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+115\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+109\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+21\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^4 a}}{24\,d}$$

input
```
integrate(cos(d*x+c)^4/(a+a*sec(d*x+c)),x, algorithm="giac")
```

output
```
1/24*(45*(d*x + c)/a - 24*tan(1/2*d*x + 1/2*c)/a - 2*(75*tan(1/2*d*x + 1/2
*c)^7 + 115*tan(1/2*d*x + 1/2*c)^5 + 109*tan(1/2*d*x + 1/2*c)^3 + 21*tan(1
/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^4*a))/d
```

3.51.9 Mupad [B] (verification not implemented)

Time = 15.07 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.83

$$\int \frac{\cos^4(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{15\,x}{8\,a} - \frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{a\,d} - \frac{\frac{25\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{4} + \frac{115\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{12} + \frac{109\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{12} + \frac{7\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}}{a\,d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)^4}$$

input
```
int(cos(c + d*x)^4/(a + a/cos(c + d*x)),x)
```

output
```
(15*x)/(8*a) - tan(c/2 + (d*x)/2)/(a*d) - ((7*tan(c/2 + (d*x)/2))/4 + (109
*tan(c/2 + (d*x)/2)^3)/12 + (115*tan(c/2 + (d*x)/2)^5)/12 + (25*tan(c/2 +
(d*x)/2)^7)/4)/(a*d*(tan(c/2 + (d*x)/2)^2 + 1)^4)
```

3.52 $\int \frac{\sec^5(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

3.52.1 Optimal result 602
3.52.2 Mathematica [A] (verified) 602
3.52.3 Rubi [A] (verified) 603
3.52.4 Maple [A] (verified) 606
3.52.5 Fricas [A] (verification not implemented) 607
3.52.6 Sympy [F] 607
3.52.7 Maxima [A] (verification not implemented) 607
3.52.8 Giac [A] (verification not implemented) 608
3.52.9 Mupad [B] (verification not implemented) 608

3.52.1 Optimal result

Integrand size = 21, antiderivative size = 123

$$\int \frac{\sec^5(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{7\text{arctanh}(\sin(c+dx))}{2a^2d} - \frac{16\tan(c+dx)}{3a^2d} + \frac{7\sec(c+dx)\tan(c+dx)}{2a^2d} - \frac{8\sec^2(c+dx)\tan(c+dx)}{3a^2d(1+\sec(c+dx))} - \frac{\sec^3(c+dx)\tan(c+dx)}{3d(a+a\sec(c+dx))^2}$$

output
```
7/2*arctanh(sin(d*x+c))/a^2/d-16/3*tan(d*x+c)/a^2/d+7/2*sec(d*x+c)*tan(d*x
+c)/a^2/d-8/3*sec(d*x+c)^2*tan(d*x+c)/a^2/d/(1+sec(d*x+c))-1/3*sec(d*x+c)^
3*tan(d*x+c)/d/(a+a*sec(d*x+c))^2
```

3.52.2 Mathematica [A] (verified)

Time = 0.86 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.60

$$\int \frac{\sec^5(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{42\text{arctanh}(\sin(c+dx)) - \frac{(37+60\cos(c+dx)+43\cos(2(c+dx))+16\cos(3(c+dx)))\sec(c+dx)\tan(c+dx)}{(1+\cos(c+dx))^2}}{12a^2d}$$

input
```
Integrate[Sec[c + d*x]^5/(a + a*Sec[c + d*x])^2,x]
```

output
```
(42*ArcTanh[Sin[c + d*x]] - ((37 + 60*Cos[c + d*x] + 43*Cos[2*(c + d*x)] +
 16*Cos[3*(c + d*x)])*Sec[c + d*x]*Tan[c + d*x])/(1 + Cos[c + d*x])^2)/(12
*a^2*d)
```

3.52.3 Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.05, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {3042, 4303, 3042, 4507, 3042, 4274, 3042, 4254, 24, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^5(c+dx)}{(a\sec(c+dx)+a)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^5}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^2}dx$$

$$\Big\downarrow \text{4303}$$

$$-\frac{\int \frac{\sec^3(c+dx)(3a-5a\sec(c+dx))}{\sec(c+dx)a+a}dx}{3a^2}-\frac{\tan(c+dx)\sec^3(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^3\left(3a-5a\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{3a^2}-\frac{\tan(c+dx)\sec^3(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{4507}$$

$$-\frac{\frac{\int \sec^2(c+dx)\left(16a^2-21a^2\sec(c+dx)\right)dx}{a^2}+\frac{8\tan(c+dx)\sec^2(c+dx)}{d(\sec(c+dx)+1)}}{3a^2}-\frac{\tan(c+dx)\sec^3(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\frac{\int \csc\left(c+dx+\frac{\pi}{2}\right)^2\left(16a^2-21a^2\csc\left(c+dx+\frac{\pi}{2}\right)\right)dx}{a^2}+\frac{8\tan(c+dx)\sec^2(c+dx)}{d(\sec(c+dx)+1)}}{3a^2}-\frac{\tan(c+dx)\sec^3(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{4274}$$

$$-\frac{\frac{16a^2\int \sec^2(c+dx)dx-21a^2\int \sec^3(c+dx)dx}{a^2}+\frac{8\tan(c+dx)\sec^2(c+dx)}{d(\sec(c+dx)+1)}}{3a^2}-\frac{\tan(c+dx)\sec^3(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\frac{16a^2\int \csc\left(c+dx+\frac{\pi}{2}\right)^2dx-21a^2\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx}{a^2}+\frac{8\tan(c+dx)\sec^2(c+dx)}{d(\sec(c+dx)+1)}}{3a^2}-\frac{\tan(c+dx)\sec^3(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{4254}$$

$$-\frac{\frac{-\frac{16a^2\int 1d(-\tan(c+dx))}{d}-21a^2\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx}{a^2}+\frac{8\tan(c+dx)\sec^2(c+dx)}{d(\sec(c+dx)+1)}}{3a^2}-\frac{\tan(c+dx)\sec^3(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{24}$$

$$-\frac{\frac{\frac{16a^2\tan(c+dx)}{d}-21a^2\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx}{a^2}+\frac{8\tan(c+dx)\sec^2(c+dx)}{d(\sec(c+dx)+1)}}{3a^2}-\frac{\tan(c+dx)\sec^3(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{4255}$$

$$-\frac{\frac{\frac{16a^2\tan(c+dx)}{d}-21a^2\left(\frac{1}{2}\int \sec(c+dx)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)}{a^2}+\frac{8\tan(c+dx)\sec^2(c+dx)}{d(\sec(c+dx)+1)}}{3a^2}-\frac{\tan(c+dx)\sec^3(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\frac{\frac{16a^2\tan(c+dx)}{d}-21a^2\left(\frac{1}{2}\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)}{a^2}+\frac{8\tan(c+dx)\sec^2(c+dx)}{d(\sec(c+dx)+1)}}{3a^2}-\frac{\tan(c+dx)\sec^3(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{4257}$$

$$-\frac{\frac{\frac{16a^2\tan(c+dx)}{d}-21a^2\left(\frac{\text{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)}{a^2}+\frac{8\tan(c+dx)\sec^2(c+dx)}{d(\sec(c+dx)+1)}}{3a^2}-\frac{\tan(c+dx)\sec^3(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

input
```
Int[Sec[c + d*x]^5/(a + a*Sec[c + d*x])^2,x]
```

output
```
-1/3*(Sec[c + d*x]^3*Tan[c + d*x])/(d*(a + a*Sec[c + d*x])^2) - ((8*Sec[c
+ d*x]^2*Tan[c + d*x])/(d*(1 + Sec[c + d*x])) + ((16*a^2*Tan[c + d*x])/d -
 21*a^2*(ArcTanh[Sin[c + d*x]]/(2*d) + (Sec[c + d*x]*Tan[c + d*x])/(2*d)))
/a^2)/(3*a^2)
```

3.52. $\int \frac{\sec^5(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

3.52.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4303
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-d^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d
*Csc[e + f*x])^(n - 2)/(f*(2*m + 1))), x] + Simp[d^2/(a*b*(2*m + 1))   Int[
(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n
 + 2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2,
0] && LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

rule 4507
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[d*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(a*f*(
2*m + 1))), x] - Simp[1/(a*b*(2*m + 1))   Int[(a + b*Csc[e + f*x])^(m + 1)*
(d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
 - n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f,
A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && G
tQ[n, 0]
```

3.52.4 Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.98

method	result
derivativedivides	$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3}-7\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}+\frac{5}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}+7\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+\frac{1}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}+\frac{}{\tan\left(\frac{d}{2}\right.}}{2d\,a^2}$
default	$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3}-7\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}+\frac{5}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}+7\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+\frac{1}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}+\frac{}{\tan\left(\frac{d}{2}\right.}}{2d\,a^2}$
parallelrisch	$\frac{(-42\cos(2dx+2c)-42)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+(42\cos(2dx+2c)+42)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-60\sec\left(\frac{dx}{2}+\frac{c}{2}\right)^2\left(\cos(dx+c)+\right.}{12a^2d(1+\cos(2dx+2c))}$
risch	$-\frac{i\left(21\,\mathrm{e}^{6i(dx+c)}+63\,\mathrm{e}^{5i(dx+c)}+98\,\mathrm{e}^{4i(dx+c)}+126\,\mathrm{e}^{3i(dx+c)}+97\,\mathrm{e}^{2i(dx+c)}+75\,\mathrm{e}^{i(dx+c)}+32\right)}{3d\,a^2\left(\mathrm{e}^{2i(dx+c)}+1\right)^2\left(\mathrm{e}^{i(dx+c)}+1\right)^3}+\frac{7\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{2a^2d}-\frac{7\ln}{}$
norman	$\frac{-\frac{13\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2ad}+\frac{149\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{6ad}-\frac{100\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{3ad}+\frac{18\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{ad}-\frac{17\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^9}{6ad}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{11}}{6ad}}{\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^4a}-\frac{7\ln\left(\tan\left(\right.\right.}{2}$

input
```
int(sec(d*x+c)^5/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/2/d/a^2*(-1/3*tan(1/2*d*x+1/2*c)^3-7*tan(1/2*d*x+1/2*c)-1/(tan(1/2*d*x+1
/2*c)+1)^2+5/(tan(1/2*d*x+1/2*c)+1)+7*ln(tan(1/2*d*x+1/2*c)+1)+1/(tan(1/2*
d*x+1/2*c)-1)^2+5/(tan(1/2*d*x+1/2*c)-1)-7*ln(tan(1/2*d*x+1/2*c)-1))
```

3.52.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.32

$$\int \frac{\sec^5(c+dx)}{(a+a\sec(c+dx))^2}\,dx$$
$$= \frac{21\left(\cos\left(dx+c\right)^4+2\,\cos\left(dx+c\right)^3+\cos\left(dx+c\right)^2\right)\log\left(\sin\left(dx+c\right)+1\right)-21\left(\cos\left(dx+c\right)^4+2\,\cos\left(dx\right.\right.}{12\left(a^2d\cos\left(dx+c\right)^4+\right.}$$

input
```
integrate(sec(d*x+c)^5/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/12*(21*(cos(d*x + c)^4 + 2*cos(d*x + c)^3 + cos(d*x + c)^2)*log(sin(d*x
+ c) + 1) - 21*(cos(d*x + c)^4 + 2*cos(d*x + c)^3 + cos(d*x + c)^2)*log(-s
in(d*x + c) + 1) - 2*(32*cos(d*x + c)^3 + 43*cos(d*x + c)^2 + 6*cos(d*x +
c) - 3)*sin(d*x + c))/(a^2*d*cos(d*x + c)^4 + 2*a^2*d*cos(d*x + c)^3 + a^2
*d*cos(d*x + c)^2)
```

3.52.6 Sympy [F]

$$\int \frac{\sec^5(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{\int \frac{\sec^5(c+dx)}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx}{a^2}$$

input
```
integrate(sec(d*x+c)**5/(a+a*sec(d*x+c))**2,x)
```

output
```
Integral(sec(c + d*x)**5/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a**2
```

3.52.7 Maxima [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.54

$$\int \frac{\sec^5(c+dx)}{(a+a\sec(c+dx))^2}\,dx =$$
$$-\frac{\frac{6\left(\frac{3\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{5\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a^2-\frac{2\,a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}+\frac{\frac{21\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^2}-\frac{21\,\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^2}+\frac{21\,\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^2}}{6\,d}$$

input
```
integrate(sec(d*x+c)^5/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
-1/6*(6*(3*sin(d*x + c)/(cos(d*x + c) + 1) - 5*sin(d*x + c)^3/(cos(d*x + c
) + 1)^3)/(a^2 - 2*a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a^2*sin(d*x +
 c)^4/(cos(d*x + c) + 1)^4) + (21*sin(d*x + c)/(cos(d*x + c) + 1) + sin(d*
x + c)^3/(cos(d*x + c) + 1)^3)/a^2 - 21*log(sin(d*x + c)/(cos(d*x + c) + 1
) + 1)/a^2 + 21*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a^2)/d
```

3.52.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.99

$$\int \frac{\sec^5(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{\frac{21\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^2} - \frac{21\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a^2} + \frac{6\left(5\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-3\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^2 a^2} - \frac{a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+21\,a^4\tan\left(\frac{1}{2}\right.}{a^6}}{6\,d}$$

input
```
integrate(sec(d*x+c)^5/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
1/6*(21*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^2 - 21*log(abs(tan(1/2*d*x +
1/2*c) - 1))/a^2 + 6*(5*tan(1/2*d*x + 1/2*c)^3 - 3*tan(1/2*d*x + 1/2*c))/(
(tan(1/2*d*x + 1/2*c)^2 - 1)^2*a^2) - (a^4*tan(1/2*d*x + 1/2*c)^3 + 21*a^4
*tan(1/2*d*x + 1/2*c))/a^6)/d
```

3.52.9 Mupad [B] (verification not implemented)

Time = 13.36 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.99

$$\int \frac{\sec^5(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{7\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a^2\,d} - \frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{6\,a^2\,d} - \frac{3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{d\left(a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-2\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+a^2\right)} - \frac{7\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,a^2\,d}$$

input
```
int(1/(cos(c + d*x)^5*(a + a/cos(c + d*x))^2),x)
```

output
```
(7*atanh(tan(c/2 + (d*x)/2)))/(a^2*d) - tan(c/2 + (d*x)/2)^3/(6*a^2*d) - (
3*tan(c/2 + (d*x)/2) - 5*tan(c/2 + (d*x)/2)^3)/(d*(a^2*tan(c/2 + (d*x)/2)^
4 - 2*a^2*tan(c/2 + (d*x)/2)^2 + a^2)) - (7*tan(c/2 + (d*x)/2))/(2*a^2*d)
```

3.52. $\int \frac{\sec^5(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

3.53 $\int \frac{\sec^4(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

3.53.1 Optimal result . . . 610
3.53.2 Mathematica [A] (verified) . . . 610
3.53.3 Rubi [A] (verified) . . . 611
3.53.4 Maple [A] (verified) . . . 614
3.53.5 Fricas [A] (verification not implemented) . . . 614
3.53.6 Sympy [F] . . . 615
3.53.7 Maxima [A] (verification not implemented) . . . 615
3.53.8 Giac [A] (verification not implemented) . . . 615
3.53.9 Mupad [B] (verification not implemented) . . . 616

3.53.1 Optimal result

Integrand size = 21, antiderivative size = 89

$$\int \frac{\sec^4(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{2\text{arctanh}(\sin(c+dx))}{a^2d}+\frac{4\tan(c+dx)}{3a^2d}+\frac{2\tan(c+dx)}{a^2d(1+\sec(c+dx))}-\frac{\sec^2(c+dx)\tan(c+dx)}{3d(a+a\sec(c+dx))^2}$$

output
```
-2*arctanh(sin(d*x+c))/a^2/d+4/3*tan(d*x+c)/a^2/d+2*tan(d*x+c)/a^2/d/(1+se
c(d*x+c))-1/3*sec(d*x+c)^2*tan(d*x+c)/d/(a+a*sec(d*x+c))^2
```

3.53.2 Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.17

$$\int \frac{\sec^4(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{\cos\left(\frac{1}{2}(c+dx)\right)\sec^2(c+dx)\left(-24\text{arctanh}(\sin(c+dx))\cos^3\left(\frac{1}{2}(c+dx)\right)+\sec(c+dx)\left(2\sin\left(\frac{1}{2}(c+dx)\right)\right.\right.}{3a^2d(1+\sec(c+dx))^2}$$

input
```
Integrate[Sec[c + d*x]^4/(a + a*Sec[c + d*x])^2,x]
```

output
```
(Cos[(c + d*x)/2]*Sec[c + d*x]^2*(-24*ArcTanh[Sin[c + d*x]]*Cos[(c + d*x)/
2]^3 + Sec[c + d*x]*(2*Sin[(c + d*x)/2] + 9*Sin[(3*(c + d*x))/2] + 5*Sin[(
5*(c + d*x))/2])))/(3*a^2*d*(1 + Sec[c + d*x])^2)
```

3.53.3 Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.09, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {3042, 4303, 27, 3042, 4496, 25, 3042, 4274, 3042, 4254, 24, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^4(c+dx)}{(a\sec(c+dx)+a)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^4}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^2}dx$$

$$\Big\downarrow \text{4303}$$

$$-\frac{\int \frac{2\sec^2(c+dx)(a-2a\sec(c+dx))}{\sec(c+dx)a+a}dx}{3a^2}-\frac{\tan(c+dx)\sec^2(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{27}$$

$$-\frac{2\int \frac{\sec^2(c+dx)(a-2a\sec(c+dx))}{\sec(c+dx)a+a}dx}{3a^2}-\frac{\tan(c+dx)\sec^2(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2\left(a-2a\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{3a^2}-\frac{\tan(c+dx)\sec^2(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{4496}$$

$$-\frac{2\left(-\frac{\int -\sec(c+dx)\left(3a^2-2a^2\sec(c+dx)\right)dx}{a^2}-\frac{3\tan(c+dx)}{d(\sec(c+dx)+1)}\right)}{3a^2}-\frac{\tan(c+dx)\sec^2(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{25}$$

$$-\frac{2\left(\frac{\int \sec(c+dx)\left(3a^2-2a^2\sec(c+dx)\right)dx}{a^2}-\frac{3\tan(c+dx)}{d(\sec(c+dx)+1)}\right)}{3a^2}-\frac{\tan(c+dx)\sec^2(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2\left(\frac{\int \csc\left(c+dx+\frac{\pi}{2}\right)\left(3a^2-2a^2\csc\left(c+dx+\frac{\pi}{2}\right)\right)dx}{a^2}-\frac{3\tan(c+dx)}{d(\sec(c+dx)+1)}\right)}{3a^2}-\frac{\tan(c+dx)\sec^2(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{4274}$$

$$-\frac{2\left(\frac{3a^2\int \sec(c+dx)dx-2a^2\int \sec^2(c+dx)dx}{a^2}-\frac{3\tan(c+dx)}{d(\sec(c+dx)+1)}\right)}{3a^2}-\frac{\tan(c+dx)\sec^2(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$\Big\downarrow$ 3042

$$-\frac{2\left(\frac{3a^2\int \csc\left(c+dx+\frac{\pi}{2}\right)dx-2a^2\int \csc\left(c+dx+\frac{\pi}{2}\right)^2dx}{a^2}-\frac{3\tan(c+dx)}{d(\sec(c+dx)+1)}\right)}{3a^2}-\frac{\tan(c+dx)\sec^2(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$\Big\downarrow$ 4254

$$-\frac{2\left(\frac{\frac{2a^2\int 1d(-\tan(c+dx))}{d}+3a^2\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{a^2}-\frac{3\tan(c+dx)}{d(\sec(c+dx)+1)}\right)}{3a^2}-\frac{\tan(c+dx)\sec^2(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$\Big\downarrow$ 24

$$-\frac{2\left(\frac{3a^2\int \csc\left(c+dx+\frac{\pi}{2}\right)dx-\frac{2a^2\tan(c+dx)}{d}}{a^2}-\frac{3\tan(c+dx)}{d(\sec(c+dx)+1)}\right)}{3a^2}-\frac{\tan(c+dx)\sec^2(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$\Big\downarrow$ 4257

$$-\frac{2\left(\frac{\frac{3a^2\text{arctanh}(\sin(c+dx))}{d}-\frac{2a^2\tan(c+dx)}{d}}{a^2}-\frac{3\tan(c+dx)}{d(\sec(c+dx)+1)}\right)}{3a^2}-\frac{\tan(c+dx)\sec^2(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

input
```
Int[Sec[c + d*x]^4/(a + a*Sec[c + d*x])^2,x]
```

output
```
-1/3*(Sec[c + d*x]^2*Tan[c + d*x])/(d*(a + a*Sec[c + d*x])^2) - (2*((-3*Ta
n[c + d*x])/(d*(1 + Sec[c + d*x])) + ((3*a^2*ArcTanh[Sin[c + d*x]])/d - (2
*a^2*Tan[c + d*x])/d)/a^2))/(3*a^2)
```

3.53.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4303
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-d^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d
*Csc[e + f*x])^(n - 2)/(f*(2*m + 1))), x] + Simp[d^2/(a*b*(2*m + 1))   Int[
(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n
 + 2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2,
0] && LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

rule 4496
```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b - a*B))*Cot
[e + f*x]*((a + b*Csc[e + f*x])^m/(b*f*(2*m + 1))), x] + Simp[1/(b^2*(2*m +
 1))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[A*b*m - a*B*m + b
*B*(2*m + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && Ne
Q[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

3.53. $\int \frac{\sec^4(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

3.53.4 Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.03

method	result
derivativedivides	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3}+5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{2}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1}+4\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)-\frac{2}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}-4\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{2d\,a^2}$
default	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3}+5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{2}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1}+4\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)-\frac{2}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}-4\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{2d\,a^2}$
parallelrisch	$\frac{6\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\cos(dx+c)-6\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)\cos(dx+c)+7\left(\cos(dx+c)+\frac{5\cos(2dx+2c)}{14}+\frac{4}{7}\right)\sec\left(\frac{dx}{2}+\frac{c}{2}\right)^2\tan}{3a^2d\cos(dx+c)}$
risch	$\frac{4i\left(3\,\mathrm{e}^{4i(dx+c)}+9\,\mathrm{e}^{3i(dx+c)}+11\,\mathrm{e}^{2i(dx+c)}+12\,\mathrm{e}^{i(dx+c)}+5\right)}{3d\,a^2\left(\mathrm{e}^{i(dx+c)}+1\right)^3\left(\mathrm{e}^{2i(dx+c)}+1\right)}+\frac{2\ln\left(\mathrm{e}^{i(dx+c)}-i\right)}{a^2d}-\frac{2\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{a^2d}$
norman	$\frac{-\frac{9\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2ad}+\frac{34\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3ad}-\frac{9\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{ad}+\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{ad}+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^9}{6ad}}{\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^3a}+\frac{2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{a^2d}-\frac{2\ln\left(\tan\right.}{}$

input
```
int(sec(d*x+c)^4/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/2/d/a^2*(1/3*tan(1/2*d*x+1/2*c)^3+5*tan(1/2*d*x+1/2*c)-2/(tan(1/2*d*x+1/
2*c)-1)+4*ln(tan(1/2*d*x+1/2*c)-1)-2/(tan(1/2*d*x+1/2*c)+1)-4*ln(tan(1/2*d
*x+1/2*c)+1))
```

3.53.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.64

$$\int \frac{\sec^4(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{3\left(\cos\left(dx+c\right)^3+2\,\cos\left(dx+c\right)^2+\cos\left(dx+c\right)\right)\log\left(\sin\left(dx+c\right)+1\right)-3\left(\cos\left(dx+c\right)^3+2\,\cos\left(dx+\right.\right.}{3\left(a^2d\cos\left(dx+c\right)^3+2\,a^2d\cos\left(\right.\right.}$$

input
```
integrate(sec(d*x+c)^4/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
-1/3*(3*(cos(d*x + c)^3 + 2*cos(d*x + c)^2 + cos(d*x + c))*log(sin(d*x + c
) + 1) - 3*(cos(d*x + c)^3 + 2*cos(d*x + c)^2 + cos(d*x + c))*log(-sin(d*x
 + c) + 1) - (10*cos(d*x + c)^2 + 14*cos(d*x + c) + 3)*sin(d*x + c))/(a^2*
d*cos(d*x + c)^3 + 2*a^2*d*cos(d*x + c)^2 + a^2*d*cos(d*x + c))
```

3.53.6 Sympy [F]

$$\int \frac{\sec^4(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{\int \frac{\sec^4(c+dx)}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx}{a^2}$$

input
```
integrate(sec(d*x+c)**4/(a+a*sec(d*x+c))**2,x)
```

output
```
Integral(sec(c + d*x)**4/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a**2
```

3.53.7 Maxima [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.63

$$\int \frac{\sec^4(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{\frac{\frac{15\sin(dx+c)}{\cos(dx+c)+1}+\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^2} - \frac{12\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^2} + \frac{12\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^2} + \frac{12\sin(dx+c)}{\left(a^2-\frac{a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)}}{6\,d}$$

input
```
integrate(sec(d*x+c)^4/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
1/6*((15*sin(d*x + c)/(cos(d*x + c) + 1) + sin(d*x + c)^3/(cos(d*x + c) +
1)^3)/a^2 - 12*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^2 + 12*log(sin(d
*x + c)/(cos(d*x + c) + 1) - 1)/a^2 + 12*sin(d*x + c)/((a^2 - a^2*sin(d*x
+ c)^2/(cos(d*x + c) + 1)^2)*(cos(d*x + c) + 1)))/d
```

3.53.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.19

$$\int \frac{\sec^4(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{\frac{12\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a^2} - \frac{12\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{a^2} + \frac{12\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)a^2} - \frac{a^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+15a^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^6}}{6\,d}$$

input
```
integrate(sec(d*x+c)^4/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
-1/6*(12*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^2 - 12*log(abs(tan(1/2*d*x +
 1/2*c) - 1))/a^2 + 12*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/2*c)^2 - 1)*
a^2) - (a^4*tan(1/2*d*x + 1/2*c)^3 + 15*a^4*tan(1/2*d*x + 1/2*c))/a^6)/d
```

3.53.9 Mupad [B] (verification not implemented)

Time = 13.40 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.03

$$\int \frac{\sec^4(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{6\,a^2\,d} - \frac{4\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{a^2\,d} - \frac{2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\left(a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2-a^2\right)} + \frac{5\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{2\,a^2\,d}$$

input
```
int(1/(cos(c + d*x)^4*(a + a/cos(c + d*x))^2),x)
```

output
```
tan(c/2 + (d*x)/2)^3/(6*a^2*d) - (4*atanh(tan(c/2 + (d*x)/2)))/(a^2*d) - (
2*tan(c/2 + (d*x)/2))/(d*(a^2*tan(c/2 + (d*x)/2)^2 - a^2)) + (5*tan(c/2 +
(d*x)/2))/(2*a^2*d)
```

3.54 $\int \frac{\sec^3(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

3.54.1 Optimal result . . . 617
3.54.2 Mathematica [A] (verified) . . . 617
3.54.3 Rubi [A] (verified) . . . 618
3.54.4 Maple [A] (verified) . . . 620
3.54.5 Fricas [A] (verification not implemented) . . . 620
3.54.6 Sympy [F] . . . 621
3.54.7 Maxima [A] (verification not implemented) . . . 621
3.54.8 Giac [A] (verification not implemented) . . . 621
3.54.9 Mupad [B] (verification not implemented) . . . 622

3.54.1 Optimal result

Integrand size = 21, antiderivative size = 66

$$\int \frac{\sec^3(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{\operatorname{arctanh}(\sin(c+dx))}{a^2d} - \frac{5\tan(c+dx)}{3a^2d(1+\sec(c+dx))} + \frac{\tan(c+dx)}{3d(a+a\sec(c+dx))^2}$$

output
```
arctanh(sin(d*x+c))/a^2/d-5/3*tan(d*x+c)/a^2/d/(1+sec(d*x+c))+1/3*tan(d*x+
c)/d/(a+a*sec(d*x+c))^2
```

3.54.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.27

$$\int \frac{\sec^3(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{2\cos\left(\frac{1}{2}(c+dx)\right)\sec^2(c+dx)\left(6\operatorname{arctanh}(\sin(c+dx))\cos^3\left(\frac{1}{2}(c+dx)\right)-3\sin\left(\frac{1}{2}(c+dx)\right)-2\sin\left(\frac{3}{2}(c+\right.\right.}{3a^2d(1+\sec(c+dx))^2}$$

input
```
Integrate[Sec[c + d*x]^3/(a + a*Sec[c + d*x])^2,x]
```

output
```
(2*Cos[(c + d*x)/2]*Sec[c + d*x]^2*(6*ArcTanh[Sin[c + d*x]]*Cos[(c + d*x)/
2]^3 - 3*Sin[(c + d*x)/2] - 2*Sin[(3*(c + d*x))/2]))/(3*a^2*d*(1 + Sec[c +
 d*x])^2)
```

3.54.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.06, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {3042, 4286, 25, 3042, 4486, 3042, 4257, 4281}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^3(c+dx)}{(a\sec(c+dx)+a)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^3}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^2}dx$$

$$\Big\downarrow \text{4286}$$

$$\frac{\int -\frac{\sec(c+dx)(2a-3a\sec(c+dx))}{\sec(c+dx)a+a}dx}{3a^2}+\frac{\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}-\frac{\int \frac{\sec(c+dx)(2a-3a\sec(c+dx))}{\sec(c+dx)a+a}dx}{3a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}-\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(2a-3a\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{3a^2}$$

$$\Big\downarrow \text{4486}$$

$$\frac{\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}-\frac{5a\int \frac{\sec(c+dx)}{\sec(c+dx)a+a}dx-3\int \sec(c+dx)dx}{3a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}-\frac{5a\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx-3\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{3a^2}$$

$$\Big\downarrow \text{4257}$$

$$\frac{\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}-\frac{5a\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx-\frac{3\text{arctanh}(\sin(c+dx))}{d}}{3a^2}$$

$$\Big\downarrow \text{4281}$$

$$\frac{\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}-\frac{\frac{5a\tan(c+dx)}{d(a\sec(c+dx)+a)}-\frac{3\text{arctanh}(\sin(c+dx))}{d}}{3a^2}$$

input
```
Int[Sec[c + d*x]^3/(a + a*Sec[c + d*x])^2,x]
```

output
```
Tan[c + d*x]/(3*d*(a + a*Sec[c + d*x])^2) - ((-3*ArcTanh[Sin[c + d*x]])/d
+ (5*a*Tan[c + d*x])/(d*(a + a*Sec[c + d*x])))/(3*a^2)
```

3.54.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4281
```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[-Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

rule 4286
```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[b*Cot[e + f*x]*((a + b*Csc[e + f*x])^m/(a*f*(2*m + 1))),
x] - Simp[1/(a^2*(2*m + 1))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
*(a*m - b*(2*m + 1)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[
a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

rule 4486
```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/(csc[(
e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[B/b   Int[Csc[e + f*x],
 x], x] + Simp[(A*b - a*B)/b   Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x
] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0]
```

3.54.4 Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.94

method	result	s
derivativedivides	$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{3}-3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{2d\,a^{2}}$	6
default	$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{3}-3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{2d\,a^{2}}$	6
parallelrisch	$\frac{-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}-9\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-6\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+6\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{6a^{2}d}$	6
risch	$-\frac{2i\left(3\,\mathrm{e}^{2i(dx+c)}+9\,\mathrm{e}^{i(dx+c)}+4\right)}{3d\,a^{2}\left(\mathrm{e}^{i(dx+c)}+1\right)^{3}}+\frac{\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{a^{2}d}-\frac{\ln\left(\mathrm{e}^{i(dx+c)}-i\right)}{a^{2}d}$	8
norman	$\frac{-\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2ad}+\frac{17\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{6ad}-\frac{7\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{6ad}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{6ad}}{\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)^{2}a}+\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{a^{2}d}-\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{a^{2}d}$	1

input
```
int(sec(d*x+c)^3/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/2/d/a^2*(-1/3*tan(1/2*d*x+1/2*c)^3-3*tan(1/2*d*x+1/2*c)-2*ln(tan(1/2*d*x
+1/2*c)-1)+2*ln(tan(1/2*d*x+1/2*c)+1))
```

3.54.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.73

$$\int \frac{\sec^3(c+dx)}{(a+a\sec(c+dx))^2}\,dx$$
$$= \frac{3\left(\cos\left(dx+c\right)^2+2\,\cos\left(dx+c\right)+1\right)\log\left(\sin\left(dx+c\right)+1\right)-3\left(\cos\left(dx+c\right)^2+2\,\cos\left(dx+c\right)+1\right)\log\left(-\right.}{6\left(a^2d\cos\left(dx+c\right)^2+2\,a^2d\cos\left(dx+c\right)+a^2d\right)}$$

input
```
integrate(sec(d*x+c)^3/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/6*(3*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*log(sin(d*x + c) + 1) - 3*(co
s(d*x + c)^2 + 2*cos(d*x + c) + 1)*log(-sin(d*x + c) + 1) - 2*(4*cos(d*x +
 c) + 5)*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*
d)
```

3.54.6 Sympy [F]

$$\int \frac{\sec^3(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{\int \frac{\sec^3(c+dx)}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx}{a^2}$$

input
```
integrate(sec(d*x+c)**3/(a+a*sec(d*x+c))**2,x)
```

output
```
Integral(sec(c + d*x)**3/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a**2
```

3.54.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.48

$$\int \frac{\sec^3(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{\frac{\frac{9\sin(dx+c)}{\cos(dx+c)+1}+\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^2} - \frac{6\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^2} + \frac{6\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^2}}{6\,d}$$

input
```
integrate(sec(d*x+c)^3/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
-1/6*((9*sin(d*x + c)/(cos(d*x + c) + 1) + sin(d*x + c)^3/(cos(d*x + c) +
1)^3)/a^2 - 6*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^2 + 6*log(sin(d*x
 + c)/(cos(d*x + c) + 1) - 1)/a^2)/d
```

3.54.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.17

$$\int \frac{\sec^3(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{\frac{6\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a^2} - \frac{6\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{a^2} - \frac{a^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+9a^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^6}}{6\,d}$$

input
```
integrate(sec(d*x+c)^3/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
1/6*(6*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^2 - 6*log(abs(tan(1/2*d*x + 1/
2*c) - 1))/a^2 - (a^4*tan(1/2*d*x + 1/2*c)^3 + 9*a^4*tan(1/2*d*x + 1/2*c))
/a^6)/d
```

3.54.9 Mupad [B] (verification not implemented)

Time = 13.15 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.65

$$\int \frac{\sec^3(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{9\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-12\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{6\,a^2\,d}$$

input
```
int(1/(cos(c + d*x)^3*(a + a/cos(c + d*x))^2),x)
```

output
```
-(9*tan(c/2 + (d*x)/2) - 12*atanh(tan(c/2 + (d*x)/2)) + tan(c/2 + (d*x)/2)
^3)/(6*a^2*d)
```

3.55 $\int \frac{\sec^2(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

3.55.1 Optimal result 623
3.55.2 Mathematica [A] (verified) 623
3.55.3 Rubi [A] (verified) 624
3.55.4 Maple [A] (verified) 625
3.55.5 Fricas [A] (verification not implemented) 626
3.55.6 Sympy [F] 626
3.55.7 Maxima [A] (verification not implemented) 626
3.55.8 Giac [A] (verification not implemented) 627
3.55.9 Mupad [B] (verification not implemented) 627

3.55.1 Optimal result

Integrand size = 21, antiderivative size = 55

$$\int \frac{\sec^2(c+dx)}{(a+a\sec(c+dx))^2}\,dx=-\frac{\tan(c+dx)}{3d(a+a\sec(c+dx))^2}+\frac{2\tan(c+dx)}{3d\left(a^2+a^2\sec(c+dx)\right)}$$

output
```
-1/3*tan(d*x+c)/d/(a+a*sec(d*x+c))^2+2/3*tan(d*x+c)/d/(a^2+a^2*sec(d*x+c))
```

3.55.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.82

$$\int \frac{\sec^2(c+dx)}{(a+a\sec(c+dx))^2}\,dx=\frac{\sec^3\left(\frac{1}{2}(c+dx)\right)\left(3\sin\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{3}{2}(c+dx)\right)\right)}{12a^2d}$$

input
```
Integrate[Sec[c + d*x]^2/(a + a*Sec[c + d*x])^2,x]
```

output
```
(Sec[(c + d*x)/2]^3*(3*Sin[(c + d*x)/2] + Sin[(3*(c + d*x))/2]))/(12*a^2*d
)
```

3.55.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3042, 4284, 3042, 4281}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(c+dx)}{(a\sec(c+dx)+a)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^2}dx$$

$$\Big\downarrow \text{4284}$$

$$\frac{2\int \frac{\sec(c+dx)}{\sec(c+dx)a+a}dx}{3a}-\frac{\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{3a}-\frac{\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{4281}$$

$$\frac{2\tan(c+dx)}{3ad(a\sec(c+dx)+a)}-\frac{\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

input
```
Int[Sec[c + d*x]^2/(a + a*Sec[c + d*x])^2,x]
```

output
```
-1/3*Tan[c + d*x]/(d*(a + a*Sec[c + d*x])^2) + (2*Tan[c + d*x])/(3*a*d*(a
+ a*Sec[c + d*x]))
```

3.55.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4281
```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[-Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

rule 4284
```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(-Cot[e + f*x])*((a + b*Csc[e + f*x])^m/(f*(2*m + 1))), x
] + Simp[m/(b*(2*m + 1))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x
], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

3.55.4 Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.56

method	result	size
parallelrisch	$\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+3\right)}{6a^{2}d}$	31
derivativedivides	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{3}+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2d\,a^{2}}$	32
default	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{3}+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2d\,a^{2}}$	32
risch	$\frac{2i\left(1+3\,\mathrm{e}^{i(dx+c)}\right)}{3d\,a^{2}\left(\mathrm{e}^{i(dx+c)}+1\right)^{3}}$	36
norman	$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2ad}+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{3ad}+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{6ad}}{a\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)}$	76

input
```
int(sec(d*x+c)^2/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/6*tan(1/2*d*x+1/2*c)*(tan(1/2*d*x+1/2*c)^2+3)/a^2/d
```

3.55.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.89

$$\int \frac{\sec^2(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{(\cos(dx+c)+2)\sin(dx+c)}{3\left(a^2d\cos(dx+c)^2+2\,a^2d\cos(dx+c)+a^2d\right)}$$

input
```
integrate(sec(d*x+c)^2/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/3*(cos(d*x + c) + 2)*sin(d*x + c)/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*
x + c) + a^2*d)
```

3.55.6 Sympy [F]

$$\int \frac{\sec^2(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{\int \frac{\sec^2(c+dx)}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx}{a^2}$$

input
```
integrate(sec(d*x+c)**2/(a+a*sec(d*x+c))**2,x)
```

output
```
Integral(sec(c + d*x)**2/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a**2
```

3.55.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.84

$$\int \frac{\sec^2(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{\frac{3\sin(dx+c)}{\cos(dx+c)+1}+\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{6\,a^2d}$$

input
```
integrate(sec(d*x+c)^2/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
1/6*(3*sin(d*x + c)/(cos(d*x + c) + 1) + sin(d*x + c)^3/(cos(d*x + c) + 1)
^3)/(a^2*d)
```

3.55.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.56

$$\int \frac{\sec^2(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+3\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{6\,a^2d}$$

input
```
integrate(sec(d*x+c)^2/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
1/6*(tan(1/2*d*x + 1/2*c)^3 + 3*tan(1/2*d*x + 1/2*c))/(a^2*d)
```

3.55.9 Mupad [B] (verification not implemented)

Time = 13.48 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.55

$$\int \frac{\sec^2(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\,\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+3\right)}{6\,a^2\,d}$$

input
```
int(1/(cos(c + d*x)^2*(a + a/cos(c + d*x))^2),x)
```

output
```
(tan(c/2 + (d*x)/2)*(tan(c/2 + (d*x)/2)^2 + 3))/(6*a^2*d)
```

3.56 $\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

3.56.1 Optimal result . . . 628
3.56.2 Mathematica [A] (verified) . . . 628
3.56.3 Rubi [A] (verified) . . . 629
3.56.4 Maple [A] (verified) . . . 630
3.56.5 Fricas [A] (verification not implemented) . . . 631
3.56.6 Sympy [F] . . . 631
3.56.7 Maxima [A] (verification not implemented) . . . 631
3.56.8 Giac [A] (verification not implemented) . . . 632
3.56.9 Mupad [B] (verification not implemented) . . . 632

3.56.1 Optimal result

Integrand size = 19, antiderivative size = 55

$$\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^2}\,dx=\frac{\tan(c+dx)}{3d(a+a\sec(c+dx))^2}+\frac{\tan(c+dx)}{3d\left(a^2+a^2\sec(c+dx)\right)}$$

output
```
1/3*tan(d*x+c)/d/(a+a*sec(d*x+c))^2+1/3*tan(d*x+c)/d/(a^2+a^2*sec(d*x+c))
```

3.56.2 Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.09

$$\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^2}\,dx=\frac{\sec\left(\frac{c}{2}\right)\sec^3\left(\frac{1}{2}(c+dx)\right)\left(3\sin\left(\frac{dx}{2}\right)-3\sin\left(c+\frac{dx}{2}\right)+2\sin\left(c+\frac{3dx}{2}\right)\right)}{12a^2d}$$

input
```
Integrate[Sec[c + d*x]/(a + a*Sec[c + d*x])^2,x]
```

output
```
(Sec[c/2]*Sec[(c + d*x)/2]^3*(3*Sin[(d*x)/2] - 3*Sin[c + (d*x)/2] + 2*Sin[
c + (3*d*x)/2]))/(12*a^2*d)
```

3.56.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {3042, 4283, 3042, 4281}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(c+dx)}{(a\sec(c+dx)+a)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^2}dx$$

$$\Big\downarrow \text{4283}$$

$$\frac{\int \frac{\sec(c+dx)}{\sec(c+dx)a+a}dx}{3a}+\frac{\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{3a}+\frac{\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{4281}$$

$$\frac{\tan(c+dx)}{3ad(a\sec(c+dx)+a)}+\frac{\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

input
```
Int[Sec[c + d*x]/(a + a*Sec[c + d*x])^2,x]
```

output
```
Tan[c + d*x]/(3*d*(a + a*Sec[c + d*x])^2) + Tan[c + d*x]/(3*a*d*(a + a*Sec
[c + d*x]))
```

3.56.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4281
```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[-Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

rule 4283
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[b*Cot[e + f*x]*((a + b*Csc[e + f*x])^m/(a*f*(2*m + 1))), x]
 + Simp[(m + 1)/(a*(2*m + 1))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m +
1), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)
] && IntegerQ[2*m]
```

3.56.4 Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.56

method	result	size
parallelrisch	$-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-3\right)}{6a^{2}d}$	31
derivativedivides	$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{3}+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2d\,a^{2}}$	32
default	$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{3}+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2d\,a^{2}}$	32
norman	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2ad}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{6ad}}{a}$	42
risch	$\frac{2i\left(3\,\mathrm{e}^{2i(dx+c)}+3\,\mathrm{e}^{i(dx+c)}+2\right)}{3d\,a^{2}\left(\mathrm{e}^{i(dx+c)}+1\right)^{3}}$	47

input
```
int(sec(d*x+c)/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
-1/6*tan(1/2*d*x+1/2*c)*(tan(1/2*d*x+1/2*c)^2-3)/a^2/d
```

3.56.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.93

$$\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{(2\,\cos(dx+c)+1)\sin(dx+c)}{3\left(a^2d\cos(dx+c)^2+2\,a^2d\cos(dx+c)+a^2d\right)}$$

input
```
integrate(sec(d*x+c)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/3*(2*cos(d*x + c) + 1)*sin(d*x + c)/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(
d*x + c) + a^2*d)
```

3.56.6 Sympy [F]

$$\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{\int \frac{\sec(c+dx)}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx}{a^2}$$

input
```
integrate(sec(d*x+c)/(a+a*sec(d*x+c))**2,x)
```

output
```
Integral(sec(c + d*x)/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a**2
```

3.56.7 Maxima [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.85

$$\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{\frac{3\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{6\,a^2d}$$

input
```
integrate(sec(d*x+c)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
1/6*(3*sin(d*x + c)/(cos(d*x + c) + 1) - sin(d*x + c)^3/(cos(d*x + c) + 1)
^3)/(a^2*d)
```

3.56.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.56

$$\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-3\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{6\,a^2d}$$

input
```
integrate(sec(d*x+c)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
-1/6*(tan(1/2*d*x + 1/2*c)^3 - 3*tan(1/2*d*x + 1/2*c))/(a^2*d)
```

3.56.9 Mupad [B] (verification not implemented)

Time = 13.34 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.55

$$\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\,\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2-3\right)}{6\,a^2\,d}$$

input
```
int(1/(cos(c + d*x)*(a + a/cos(c + d*x))^2),x)
```

output
```
-(tan(c/2 + (d*x)/2)*(tan(c/2 + (d*x)/2)^2 - 3))/(6*a^2*d)
```

3.57 $\int \frac{1}{(a+a\sec(c+dx))^2}\,dx$

3.57.1 Optimal result 633
3.57.2 Mathematica [A] (verified) 633
3.57.3 Rubi [A] (verified) 634
3.57.4 Maple [A] (verified) 636
3.57.5 Fricas [A] (verification not implemented) 636
3.57.6 Sympy [F] 637
3.57.7 Maxima [A] (verification not implemented) 637
3.57.8 Giac [A] (verification not implemented) 637
3.57.9 Mupad [B] (verification not implemented) 638

3.57.1 Optimal result

Integrand size = 12, antiderivative size = 57

$$\int \frac{1}{(a+a\sec(c+dx))^2}\,dx=\frac{x}{a^2}-\frac{4\tan(c+dx)}{3a^2d(1+\sec(c+dx))}-\frac{\tan(c+dx)}{3d(a+a\sec(c+dx))^2}$$

output
```
x/a^2-4/3*tan(d*x+c)/a^2/d/(1+sec(d*x+c))-1/3*tan(d*x+c)/d/(a+a*sec(d*x+c)
)^2
```

3.57.2 Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.96

$$\int \frac{1}{(a+a\sec(c+dx))^2}\,dx$$
$$=\frac{\sec\left(\frac{c}{2}\right)\sec^3\left(\frac{1}{2}(c+dx)\right)\left(9dx\cos\left(\frac{dx}{2}\right)+9dx\cos\left(c+\frac{dx}{2}\right)+3dx\cos\left(c+\frac{3dx}{2}\right)+3dx\cos\left(2c+\frac{3dx}{2}\right)-18\,\mathrm{si}\right.}{24a^2d}$$

input
```
Integrate[(a + a*Sec[c + d*x])^(-2),x]
```

output
```
(Sec[c/2]*Sec[(c + d*x)/2]^3*(9*d*x*Cos[(d*x)/2] + 9*d*x*Cos[c + (d*x)/2]
+ 3*d*x*Cos[c + (3*d*x)/2] + 3*d*x*Cos[2*c + (3*d*x)/2] - 18*Sin[(d*x)/2]
+ 12*Sin[c + (d*x)/2] - 10*Sin[c + (3*d*x)/2]))/(24*a^2*d)
```

3.57.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.07, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {3042, 4264, 25, 3042, 4407, 3042, 4281}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\sec(c+dx)+a)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^2}dx$$

$$\Big\downarrow \text{4264}$$

$$-\frac{\int -\frac{3a-a\sec(c+dx)}{\sec(c+dx)a+a}dx}{3a^2}-\frac{\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{3a-a\sec(c+dx)}{\sec(c+dx)a+a}dx}{3a^2}-\frac{\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{3a-a\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{3a^2}-\frac{\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{4407}$$

$$\frac{3x-4a\int \frac{\sec(c+dx)}{\sec(c+dx)a+a}dx}{3a^2}-\frac{\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3x-4a\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{3a^2}-\frac{\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{4281}$$

$$\frac{3x-\frac{4a\tan(c+dx)}{d(a\sec(c+dx)+a)}}{3a^2}-\frac{\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

input
```
Int[(a + a*Sec[c + d*x])^(-2),x]
```

output
```
-1/3*Tan[c + d*x]/(d*(a + a*Sec[c + d*x])^2) + (3*x - (4*a*Tan[c + d*x])/(
d*(a + a*Sec[c + d*x])))/(3*a^2)
```

3.57.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4264
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(-Cot[c
+ d*x])*((a + b*Csc[c + d*x])^n/(d*(2*n + 1))), x] + Simp[1/(a^2*(2*n + 1))
   Int[(a + b*Csc[c + d*x])^(n + 1)*(a*(2*n + 1) - b*(n + 1)*Csc[c + d*x]),
 x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LeQ[n, -1] && Int
egerQ[2*n]
```

rule 4281
```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[-Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

rule 4407
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Simp[(b*c - a*d)/a   Int[Csc[e + f*
x]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
- a*d, 0]
```

3.57.4 Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.63

method	result	size
parallelrisch	$\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}+6dx-9\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{6a^{2}d}$	36
derivativedivides	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{3}-3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+4\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d\,a^{2}}$	46
default	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{3}-3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+4\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d\,a^{2}}$	46
norman	$\frac{\frac{x}{a}-\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2ad}+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{6ad}}{a}$	47
risch	$\frac{x}{a^{2}}-\frac{2i\left(6\,\mathrm{e}^{2i(dx+c)}+9\,\mathrm{e}^{i(dx+c)}+5\right)}{3d\,a^{2}\left(\mathrm{e}^{i(dx+c)}+1\right)^{3}}$	53

input
```
int(1/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/6*(tan(1/2*d*x+1/2*c)^3+6*d*x-9*tan(1/2*d*x+1/2*c))/a^2/d
```

3.57.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.40

$$\int \frac{1}{(a+a\sec(c+dx))^2}\,dx = \frac{3\,dx\cos\left(dx+c\right)^2+6\,dx\cos\left(dx+c\right)+3\,dx-\left(5\,\cos\left(dx+c\right)+4\right)\sin\left(dx+c\right)}{3\left(a^2d\cos\left(dx+c\right)^2+2\,a^2d\cos\left(dx+c\right)+a^2d\right)}$$

input
```
integrate(1/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/3*(3*d*x*cos(d*x + c)^2 + 6*d*x*cos(d*x + c) + 3*d*x - (5*cos(d*x + c) +
 4)*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)
```

3.57.6 Sympy [F]

$$\int \frac{1}{(a+a\sec(c+dx))^2}\,dx = \frac{\int \frac{1}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx}{a^2}$$

input
```
integrate(1/(a+a*sec(d*x+c))**2,x)
```

output
```
Integral(1/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a**2
```

3.57.7 Maxima [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.26

$$\int \frac{1}{(a+a\sec(c+dx))^2}\,dx = -\frac{\frac{\frac{9\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^2}-\frac{12\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}}{6\,d}$$

input
```
integrate(1/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
-1/6*((9*sin(d*x + c)/(cos(d*x + c) + 1) - sin(d*x + c)^3/(cos(d*x + c) +
1)^3)/a^2 - 12*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^2)/d
```

3.57.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.88

$$\int \frac{1}{(a+a\sec(c+dx))^2}\,dx = \frac{\frac{6\,(dx+c)}{a^2}+\frac{a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-9\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^6}}{6\,d}$$

input
```
integrate(1/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
1/6*(6*(d*x + c)/a^2 + (a^4*tan(1/2*d*x + 1/2*c)^3 - 9*a^4*tan(1/2*d*x + 1
/2*c))/a^6)/d
```

3.57.9 Mupad [B] (verification not implemented)

Time = 13.35 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.61

$$\int \frac{1}{(a+a\sec(c+dx))^2}\,dx=\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3-9\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+6\,d\,x}{6\,a^2\,d}$$

input
```
int(1/(a + a/cos(c + d*x))^2,x)
```

output
```
(tan(c/2 + (d*x)/2)^3 - 9*tan(c/2 + (d*x)/2) + 6*d*x)/(6*a^2*d)
```

3.58 $\int \frac{\cos(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

3.58.1 Optimal result 639
3.58.2 Mathematica [A] (verified) 639
3.58.3 Rubi [A] (verified) 640
3.58.4 Maple [A] (verified) 642
3.58.5 Fricas [A] (verification not implemented) 643
3.58.6 Sympy [F] 643
3.58.7 Maxima [A] (verification not implemented) 643
3.58.8 Giac [A] (verification not implemented) 644
3.58.9 Mupad [B] (verification not implemented) 644

3.58.1 Optimal result

Integrand size = 19, antiderivative size = 72

$$\int \frac{\cos(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{2x}{a^2}+\frac{10\sin(c+dx)}{3a^2d}-\frac{2\sin(c+dx)}{a^2d(1+\sec(c+dx))}-\frac{\sin(c+dx)}{3d(a+a\sec(c+dx))^2}$$

output
```
-2*x/a^2+10/3*sin(d*x+c)/a^2/d-2*sin(d*x+c)/a^2/d/(1+sec(d*x+c))-1/3*sin(d
*x+c)/d/(a+a*sec(d*x+c))^2
```

3.58.2 Mathematica [A] (verified)

Time = 0.91 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.35

$$\int \frac{\cos(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{\sin(c+dx)\left(24\arcsin(\cos(c+dx))\cos^4\left(\frac{1}{2}(c+dx)\right)+(10+14\cos(c+dx)+3\cos^2(c+dx))\sqrt{\sin^2(c+\right.}{3a^2d\sqrt{1-\cos(c+dx)}(1+\cos(c+dx))^{5/2}}$$

input
```
Integrate[Cos[c + d*x]/(a + a*Sec[c + d*x])^2,x]
```

output
```
(Sin[c + d*x]*(24*ArcSin[Cos[c + d*x]]*Cos[(c + d*x)/2]^4 + (10 + 14*Cos[c
 + d*x] + 3*Cos[c + d*x]^2)*Sqrt[Sin[c + d*x]^2]))/(3*a^2*d*Sqrt[1 - Cos[c
 + d*x]]*(1 + Cos[c + d*x])^(5/2))
```

3.58.3 Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.11, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.526, Rules used = {3042, 4304, 27, 3042, 4508, 3042, 4274, 24, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(c+dx)}{(a\sec(c+dx)+a)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^2}dx$$

$$\Big\downarrow \text{4304}$$

$$-\frac{\int -\frac{2\cos(c+dx)(2a-a\sec(c+dx))}{\sec(c+dx)a+a}dx}{3a^2}-\frac{\sin(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{2\int \frac{\cos(c+dx)(2a-a\sec(c+dx))}{\sec(c+dx)a+a}dx}{3a^2}-\frac{\sin(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\int \frac{2a-a\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)}dx}{3a^2}-\frac{\sin(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{4508}$$

$$\frac{2\left(\frac{\int \cos(c+dx)\left(5a^2-3a^2\sec(c+dx)\right)dx}{a^2}-\frac{3\sin(c+dx)}{d(\sec(c+dx)+1)}\right)}{3a^2}-\frac{\sin(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\left(\frac{\int \frac{5a^2-3a^2\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}-\frac{3\sin(c+dx)}{d(\sec(c+dx)+1)}\right)}{3a^2}-\frac{\sin(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{4274}$$

$$\frac{2\left(\frac{5a^2\int \cos(c+dx)dx-3a^2\int 1dx}{a^2}-\frac{3\sin(c+dx)}{d(\sec(c+dx)+1)}\right)}{3a^2}-\frac{\sin(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{24}$$

$$\frac{2\left(\frac{5a^2\int\cos(c+dx)dx-3a^2x}{a^2}-\frac{3\sin(c+dx)}{d(\sec(c+dx)+1)}\right)}{3a^2}-\frac{\sin(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\left(\frac{5a^2\int\sin\left(c+dx+\frac{\pi}{2}\right)dx-3a^2x}{a^2}-\frac{3\sin(c+dx)}{d(\sec(c+dx)+1)}\right)}{3a^2}-\frac{\sin(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{3117}$$

$$\frac{2\left(\frac{\frac{5a^2\sin(c+dx)}{d}-3a^2x}{a^2}-\frac{3\sin(c+dx)}{d(\sec(c+dx)+1)}\right)}{3a^2}-\frac{\sin(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

input
```
Int[Cos[c + d*x]/(a + a*Sec[c + d*x])^2,x]
```

output
```
-1/3*Sin[c + d*x]/(d*(a + a*Sec[c + d*x])^2) + (2*((-3*Sin[c + d*x])/(d*(1
 + Sec[c + d*x])) + (-3*a^2*x + (5*a^2*Sin[c + d*x])/d)/a^2))/(3*a^2)
```

3.58.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4304
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-Cot[e + f*x])*(a + b*Csc[e + f*x])^m*((d*Csc
[e + f*x])^n/(f*(2*m + 1))), x] + Simp[1/(a^2*(2*m + 1))   Int[(a + b*Csc[e
 + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e
+ f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ
[m, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

rule 4508
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
- a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
 1))), x] - Simp[1/(a^2*(2*m + 1))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Cs
c[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[
e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B
, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

3.58.4 Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.76

method	result	size
parallelrisch	$\frac{7\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\cos(dx+c)+\frac{3\cos(2dx+2c)}{28}+\frac{23}{28}\right)\sec\left(\frac{dx}{2}+\frac{c}{2}\right)^2-6dx}{3a^2d}$	55
derivativedivides	$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3}+5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}-8\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d\,a^2}$	72
default	$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3}+5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}-8\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d\,a^2}$	72
risch	$-\frac{2x}{a^2}-\frac{i\mathrm{e}^{i(dx+c)}}{2a^2d}+\frac{i\mathrm{e}^{-i(dx+c)}}{2a^2d}+\frac{2i\left(9\,\mathrm{e}^{2i(dx+c)}+15\,\mathrm{e}^{i(dx+c)}+8\right)}{3d\,a^2\left(\mathrm{e}^{i(dx+c)}+1\right)^3}$	90
norman	$\frac{-\frac{2x}{a}+\frac{9\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2ad}+\frac{7\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3ad}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{6ad}-\frac{2x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{a}}{a\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)}$	99

input
```
int(cos(d*x+c)/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/3*(7*tan(1/2*d*x+1/2*c)*(cos(d*x+c)+3/28*cos(2*d*x+2*c)+23/28)*sec(1/2*d
*x+1/2*c)^2-6*d*x)/a^2/d
```

3.58.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.25

$$\int \frac{\cos(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{6\,dx\cos\left(dx+c\right)^2+12\,dx\cos\left(dx+c\right)+6\,dx-\left(3\,\cos\left(dx+c\right)^2+14\,\cos\left(dx+c\right)+10\right)\sin\left(dx+c\right)}{3\left(a^2d\cos\left(dx+c\right)^2+2\,a^2d\cos\left(dx+c\right)+a^2d\right)}$$

input
```
integrate(cos(d*x+c)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
-1/3*(6*d*x*cos(d*x + c)^2 + 12*d*x*cos(d*x + c) + 6*d*x - (3*cos(d*x + c)
^2 + 14*cos(d*x + c) + 10)*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*c
os(d*x + c) + a^2*d)
```

3.58.6 Sympy [F]

$$\int \frac{\cos(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{\int \frac{\cos\left(c+dx\right)}{\sec^2\left(c+dx\right)+2\sec\left(c+dx\right)+1}\,dx}{a^2}$$

input
```
integrate(cos(d*x+c)/(a+a*sec(d*x+c))**2,x)
```

output
```
Integral(cos(c + d*x)/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a**2
```

3.58.7 Maxima [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.64

$$\int \frac{\cos(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{\frac{\frac{15\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^2}-\frac{24\,\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}+\frac{12\,\sin(dx+c)}{\left(a^2+\frac{a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)}}{6\,d}$$

input
```
integrate(cos(d*x+c)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
1/6*((15*sin(d*x + c)/(cos(d*x + c) + 1) - sin(d*x + c)^3/(cos(d*x + c) +
1)^3)/a^2 - 24*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^2 + 12*sin(d*x +
c)/((a^2 + a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)*(cos(d*x + c) + 1)))/d
```

3.58.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.10

$$\int \frac{\cos(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{\frac{12\,(dx+c)}{a^2} - \frac{12\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)a^2} + \frac{a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-15\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^6}}{6\,d}$$

input
```
integrate(cos(d*x+c)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
-1/6*(12*(d*x + c)/a^2 - 12*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/2*c)^2
+ 1)*a^2) + (a^4*tan(1/2*d*x + 1/2*c)^3 - 15*a^4*tan(1/2*d*x + 1/2*c))/a^6
)/d
```

3.58.9 Mupad [B] (verification not implemented)

Time = 13.50 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.26

$$\int \frac{\cos(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right) - 16\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^2\sin\left(\frac{c}{2}+\frac{dx}{2}\right) - 12\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^4\sin\left(\frac{c}{2}+\frac{dx}{2}\right) + 12\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^3(c+d}{6\,a^2\,d\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^3}$$

input
```
int(cos(c + d*x)/(a + a/cos(c + d*x))^2,x)
```

output
```
-(sin(c/2 + (d*x)/2) - 16*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2) - 12*cos
(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2) + 12*cos(c/2 + (d*x)/2)^3*(c + d*x))/
(6*a^2*d*cos(c/2 + (d*x)/2)^3)
```

3.59 $\int \frac{\cos^2(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

3.59.1 Optimal result 645
3.59.2 Mathematica [A] (verified) 645
3.59.3 Rubi [A] (verified) 646
3.59.4 Maple [A] (verified) 649
3.59.5 Fricas [A] (verification not implemented) 649
3.59.6 Sympy [F] 650
3.59.7 Maxima [A] (verification not implemented) 650
3.59.8 Giac [A] (verification not implemented) 650
3.59.9 Mupad [B] (verification not implemented) 651

3.59.1 Optimal result

Integrand size = 21, antiderivative size = 110

$$\int \frac{\cos^2(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{7x}{2a^2} - \frac{16\sin(c+dx)}{3a^2d} + \frac{7\cos(c+dx)\sin(c+dx)}{2a^2d} - \frac{8\cos(c+dx)\sin(c+dx)}{3a^2d(1+\sec(c+dx))} - \frac{\cos(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2}$$

output
```
7/2*x/a^2-16/3*sin(d*x+c)/a^2/d+7/2*cos(d*x+c)*sin(d*x+c)/a^2/d-8/3*cos(d*
x+c)*sin(d*x+c)/a^2/d/(1+sec(d*x+c))-1/3*cos(d*x+c)*sin(d*x+c)/d/(a+a*sec(
d*x+c))^2
```

3.59.2 Mathematica [A] (verified)

Time = 1.57 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.61

$$\int \frac{\cos^2(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{\sec\left(\frac{c}{2}\right)\sec^3\left(\frac{1}{2}(c+dx)\right)\left(252dx\cos\left(\frac{dx}{2}\right)+252dx\cos\left(c+\frac{dx}{2}\right)+84dx\cos\left(c+\frac{3dx}{2}\right)+84dx\cos\left(2c+\frac{3dx}{2}\right)\right.}{}$$

input
```
Integrate[Cos[c + d*x]^2/(a + a*Sec[c + d*x])^2,x]
```

output
```
(Sec[c/2]*Sec[(c + d*x)/2]^3*(252*d*x*Cos[(d*x)/2] + 252*d*x*Cos[c + (d*x)
/2] + 84*d*x*Cos[c + (3*d*x)/2] + 84*d*x*Cos[2*c + (3*d*x)/2] - 381*Sin[(d
*x)/2] + 147*Sin[c + (d*x)/2] - 239*Sin[c + (3*d*x)/2] - 63*Sin[2*c + (3*d
*x)/2] - 15*Sin[2*c + (5*d*x)/2] - 15*Sin[3*c + (5*d*x)/2] + 3*Sin[3*c + (
7*d*x)/2] + 3*Sin[4*c + (7*d*x)/2]))/(192*a^2*d)
```

3.59.3 Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.05, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.524$, Rules used = {3042, 4304, 25, 3042, 4508, 3042, 4274, 3042, 3115, 24, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^2(c+dx)}{(a\sec(c+dx)+a)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^2\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^2}dx$$

$$\Big\downarrow \text{4304}$$

$$-\frac{\int -\frac{\cos^2(c+dx)(5a-3a\sec(c+dx))}{\sec(c+dx)a+a}dx}{3a^2}-\frac{\sin(c+dx)\cos(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\cos^2(c+dx)(5a-3a\sec(c+dx))}{\sec(c+dx)a+a}dx}{3a^2}-\frac{\sin(c+dx)\cos(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{5a-3a\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^2\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)}dx}{3a^2}-\frac{\sin(c+dx)\cos(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{4508}$$

$$\frac{\frac{\int \cos^2(c+dx)\left(21a^2-16a^2\sec(c+dx)\right)dx}{a^2}-\frac{8\sin(c+dx)\cos(c+dx)}{d(\sec(c+dx)+1)}}{3a^2}-\frac{\sin(c+dx)\cos(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\int \frac{21a^2-16a^2\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^2}dx}{a^2}-\frac{8\sin(c+dx)\cos(c+dx)}{d(\sec(c+dx)+1)}}{3a^2}-\frac{\sin(c+dx)\cos(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{4274}$$

$$\frac{\frac{21a^2\int\cos^2(c+dx)dx-16a^2\int\cos(c+dx)dx}{a^2}-\frac{8\sin(c+dx)\cos(c+dx)}{d(\sec(c+dx)+1)}}{3a^2}-\frac{\sin(c+dx)\cos(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{21a^2\int\sin\left(c+dx+\frac{\pi}{2}\right)^2dx-16a^2\int\sin\left(c+dx+\frac{\pi}{2}\right)dx}{a^2}-\frac{8\sin(c+dx)\cos(c+dx)}{d(\sec(c+dx)+1)}}{3a^2}-\frac{\sin(c+dx)\cos(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{3115}$$

$$\frac{\frac{21a^2\left(\frac{\int 1dx}{2}+\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)-16a^2\int\sin\left(c+dx+\frac{\pi}{2}\right)dx}{a^2}-\frac{8\sin(c+dx)\cos(c+dx)}{d(\sec(c+dx)+1)}}{3a^2}-\frac{\sin(c+dx)\cos(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{24}$$

$$\frac{\frac{21a^2\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)-16a^2\int\sin\left(c+dx+\frac{\pi}{2}\right)dx}{a^2}-\frac{8\sin(c+dx)\cos(c+dx)}{d(\sec(c+dx)+1)}}{3a^2}-\frac{\sin(c+dx)\cos(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{3117}$$

$$\frac{\frac{21a^2\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)-\frac{16a^2\sin(c+dx)}{d}}{a^2}-\frac{8\sin(c+dx)\cos(c+dx)}{d(\sec(c+dx)+1)}}{3a^2}-\frac{\sin(c+dx)\cos(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

input
```
Int[Cos[c + d*x]^2/(a + a*Sec[c + d*x])^2,x]
```

output
```
-1/3*(Cos[c + d*x]*Sin[c + d*x])/(d*(a + a*Sec[c + d*x])^2) + ((-8*Cos[c +
 d*x]*Sin[c + d*x])/(d*(1 + Sec[c + d*x])) + ((-16*a^2*Sin[c + d*x])/d + 2
1*a^2*(x/2 + (Cos[c + d*x]*Sin[c + d*x])/(2*d)))/a^2)/(3*a^2)
```

3.59. $\int \frac{\cos^2(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

3.59.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4304
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-Cot[e + f*x])*(a + b*Csc[e + f*x])^m*((d*Csc
[e + f*x])^n/(f*(2*m + 1))), x] + Simp[1/(a^2*(2*m + 1))   Int[(a + b*Csc[e
 + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e
+ f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ
[m, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

rule 4508
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
- a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
 1))), x] - Simp[1/(a^2*(2*m + 1))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Cs
c[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[
e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B
, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

3.59. $\int \frac{\cos^2(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

3.59.4 Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.60

method	result	size
parallelrisch	$\frac{-163\left(\cos(dx+c)+\frac{12\cos(2dx+2c)}{163}-\frac{3\cos(3dx+3c)}{163}+\frac{140}{163}\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\sec\left(\frac{dx}{2}+\frac{c}{2}\right)^2+168dx}{48a^2d}$	66
derivativedivides	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3}-7\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{-10\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3-6\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^2}+14\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d\,a^2}$	88
default	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3}-7\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{-10\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3-6\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^2}+14\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d\,a^2}$	88
risch	$\frac{7x}{2a^2}-\frac{i\mathrm{e}^{2i(dx+c)}}{8a^2d}+\frac{i\mathrm{e}^{i(dx+c)}}{a^2d}-\frac{i\mathrm{e}^{-i(dx+c)}}{a^2d}+\frac{i\mathrm{e}^{-2i(dx+c)}}{8a^2d}-\frac{2i\left(12\,\mathrm{e}^{2i(dx+c)}+21\,\mathrm{e}^{i(dx+c)}+11\right)}{3d\,a^2\left(\mathrm{e}^{i(dx+c)}+1\right)^3}$	126
norman	$\frac{\frac{7x}{2a}-\frac{13\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2ad}-\frac{71\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{6ad}-\frac{19\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{6ad}+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{6ad}+\frac{7x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{a}+\frac{7x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{2a}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^2a}$	135

input
```
int(cos(d*x+c)^2/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/48*(-163*(cos(d*x+c)+12/163*cos(2*d*x+2*c)-3/163*cos(3*d*x+3*c)+140/163)
*tan(1/2*d*x+1/2*c)*sec(1/2*d*x+1/2*c)^2+168*d*x)/a^2/d
```

3.59.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.90

$$\int \frac{\cos^2(c+dx)}{(a+a\sec(c+dx))^2}\,dx$$
$$= \frac{21\,dx\cos\left(dx+c\right)^2+42\,dx\cos\left(dx+c\right)+21\,dx+\left(3\,\cos\left(dx+c\right)^3-6\,\cos\left(dx+c\right)^2-43\,\cos\left(dx+c\right)-\right.}{6\left(a^2d\cos\left(dx+c\right)^2+2\,a^2d\cos\left(dx+c\right)+a^2d\right)}$$

input
```
integrate(cos(d*x+c)^2/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/6*(21*d*x*cos(d*x + c)^2 + 42*d*x*cos(d*x + c) + 21*d*x + (3*cos(d*x + c
)^3 - 6*cos(d*x + c)^2 - 43*cos(d*x + c) - 32)*sin(d*x + c))/(a^2*d*cos(d*
x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)
```

3.59.6 Sympy [F]

$$\int \frac{\cos^2(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{\int \frac{\cos^2(c+dx)}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx}{a^2}$$

input
```
integrate(cos(d*x+c)**2/(a+a*sec(d*x+c))**2,x)
```

output
```
Integral(cos(c + d*x)**2/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a**2
```

3.59.7 Maxima [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.49

$$\int \frac{\cos^2(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{\frac{6\left(\frac{3\sin(dx+c)}{\cos(dx+c)+1}+\frac{5\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a^2+\frac{2a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}+\frac{\frac{21\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^2}-\frac{42\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}}{6d}$$

input
```
integrate(cos(d*x+c)^2/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
-1/6*(6*(3*sin(d*x + c)/(cos(d*x + c) + 1) + 5*sin(d*x + c)^3/(cos(d*x + c
) + 1)^3)/(a^2 + 2*a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a^2*sin(d*x +
 c)^4/(cos(d*x + c) + 1)^4) + (21*sin(d*x + c)/(cos(d*x + c) + 1) - sin(d*
x + c)^3/(cos(d*x + c) + 1)^3)/a^2 - 42*arctan(sin(d*x + c)/(cos(d*x + c)
+ 1))/a^2)/d
```

3.59.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.86

$$\int \frac{\cos^2(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{\frac{21\,(dx+c)}{a^2}-\frac{6\left(5\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)^2a^2}+\frac{a^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-21\,a^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^6}}{6d}$$

input
```
integrate(cos(d*x+c)^2/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
1/6*(21*(d*x + c)/a^2 - 6*(5*tan(1/2*d*x + 1/2*c)^3 + 3*tan(1/2*d*x + 1/2*
c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^2*a^2) + (a^4*tan(1/2*d*x + 1/2*c)^3 - 2
1*a^4*tan(1/2*d*x + 1/2*c))/a^6)/d
```

3.59.9 Mupad [B] (verification not implemented)

Time = 13.70 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.03

$$\int \frac{\cos^2(c+dx)}{(a+a\sec(c+dx))^2}\,dx$$
$$= \frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)-22\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^2\sin\left(\frac{c}{2}+\frac{dx}{2}\right)-30\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^4\sin\left(\frac{c}{2}+\frac{dx}{2}\right)+12\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^6\sin\left(\frac{c}{2}+\right.}{6\,a^2\,d\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^3}$$

input
```
int(cos(c + d*x)^2/(a + a/cos(c + d*x))^2,x)
```

output
```
(sin(c/2 + (d*x)/2) - 22*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2) - 30*cos(
c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2) + 12*cos(c/2 + (d*x)/2)^6*sin(c/2 + (d
*x)/2) + 21*cos(c/2 + (d*x)/2)^3*(c + d*x))/(6*a^2*d*cos(c/2 + (d*x)/2)^3)
```

3.60 $\int \frac{\cos^3(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

3.60.1 Optimal result 652
3.60.2 Mathematica [A] (verified) 652
3.60.3 Rubi [A] (verified) 653
3.60.4 Maple [A] (verified) 656
3.60.5 Fricas [A] (verification not implemented) 657
3.60.6 Sympy [F(-1)] 658
3.60.7 Maxima [A] (verification not implemented) 658
3.60.8 Giac [A] (verification not implemented) 658
3.60.9 Mupad [B] (verification not implemented) 659

3.60.1 Optimal result

Integrand size = 21, antiderivative size = 124

$$\int \frac{\cos^3(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{5x}{a^2}+\frac{12\sin(c+dx)}{a^2d}-\frac{5\cos(c+dx)\sin(c+dx)}{a^2d} - \frac{10\cos^2(c+dx)\sin(c+dx)}{3a^2d(1+\sec(c+dx))} - \frac{\cos^2(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2}-\frac{4\sin^3(c+dx)}{a^2d}$$

output
```
-5*x/a^2+12*sin(d*x+c)/a^2/d-5*cos(d*x+c)*sin(d*x+c)/a^2/d-10/3*cos(d*x+c)
^2*sin(d*x+c)/a^2/d/(1+sec(d*x+c))-1/3*cos(d*x+c)^2*sin(d*x+c)/d/(a+a*sec(
d*x+c))^2-4*sin(d*x+c)^3/a^2/d
```

3.60.2 Mathematica [A] (verified)

Time = 1.65 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.60

$$\int \frac{\cos^3(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{\sec\left(\frac{c}{2}\right)\sec^3\left(\frac{1}{2}(c+dx)\right)\left(-360dx\cos\left(\frac{dx}{2}\right)-360dx\cos\left(c+\frac{dx}{2}\right)-120dx\cos\left(c+\frac{3dx}{2}\right)-120dx\cos\left(2c+\right.\right.}{}$$

input
```
Integrate[Cos[c + d*x]^3/(a + a*Sec[c + d*x])^2,x]
```

output
```
(Sec[c/2]*Sec[(c + d*x)/2]^3*(-360*d*x*Cos[(d*x)/2] - 360*d*x*Cos[c + (d*x
)/2] - 120*d*x*Cos[c + (3*d*x)/2] - 120*d*x*Cos[2*c + (3*d*x)/2] + 516*Sin
[(d*x)/2] - 156*Sin[c + (d*x)/2] + 342*Sin[c + (3*d*x)/2] + 118*Sin[2*c +
(3*d*x)/2] + 30*Sin[2*c + (5*d*x)/2] + 30*Sin[3*c + (5*d*x)/2] - 3*Sin[3*c
 + (7*d*x)/2] - 3*Sin[4*c + (7*d*x)/2] + Sin[4*c + (9*d*x)/2] + Sin[5*c +
(9*d*x)/2]))/(192*a^2*d)
```

3.60.3 Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.10, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.619$, Rules used = {3042, 4304, 27, 3042, 4508, 27, 3042, 4274, 3042, 3113, 2009, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^3(c+dx)}{(a\sec(c+dx)+a)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^3\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^2}dx$$

$$\Big\downarrow \text{4304}$$

$$-\frac{\int -\frac{2\cos^3(c+dx)(3a-2a\sec(c+dx))}{\sec(c+dx)a+a}dx}{3a^2}-\frac{\sin(c+dx)\cos^2(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{2\int \frac{\cos^3(c+dx)(3a-2a\sec(c+dx))}{\sec(c+dx)a+a}dx}{3a^2}-\frac{\sin(c+dx)\cos^2(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\int \frac{3a-2a\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^3\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)}dx}{3a^2}-\frac{\sin(c+dx)\cos^2(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{4508}$$

$$\frac{2\left(\frac{\int 3\cos^3(c+dx)\left(6a^2-5a^2\sec(c+dx)\right)dx}{a^2}-\frac{5\sin(c+dx)\cos^2(c+dx)}{d(\sec(c+dx)+1)}\right)}{3a^2}-\frac{\sin(c+dx)\cos^2(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{2\left(\frac{3\int \cos^3(c+dx)\left(6a^2-5a^2\sec(c+dx)\right)dx}{a^2}-\frac{5\sin(c+dx)\cos^2(c+dx)}{d(\sec(c+dx)+1)}\right)}{3a^2}-\frac{\sin(c+dx)\cos^2(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$\Big\downarrow$ 3042

$$\frac{2\left(\frac{3\int \frac{6a^2-5a^2\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^3}dx}{a^2}-\frac{5\sin(c+dx)\cos^2(c+dx)}{d(\sec(c+dx)+1)}\right)}{3a^2}-\frac{\sin(c+dx)\cos^2(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$\Big\downarrow$ 4274

$$\frac{2\left(\frac{3\left(6a^2\int \cos^3(c+dx)dx-5a^2\int \cos^2(c+dx)dx\right)}{a^2}-\frac{5\sin(c+dx)\cos^2(c+dx)}{d(\sec(c+dx)+1)}\right)}{3a^2}-\frac{\sin(c+dx)\cos^2(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$\Big\downarrow$ 3042

$$\frac{2\left(\frac{3\left(6a^2\int \sin\left(c+dx+\frac{\pi}{2}\right)^3dx-5a^2\int \sin\left(c+dx+\frac{\pi}{2}\right)^2dx\right)}{a^2}-\frac{5\sin(c+dx)\cos^2(c+dx)}{d(\sec(c+dx)+1)}\right)}{3a^2}-\frac{\sin(c+dx)\cos^2(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$\Big\downarrow$ 3113

$$\frac{2\left(\frac{3\left(-\frac{6a^2\int\left(1-\sin^2(c+dx)\right)d(-\sin(c+dx))}{d}-5a^2\int \sin\left(c+dx+\frac{\pi}{2}\right)^2dx\right)}{a^2}-\frac{5\sin(c+dx)\cos^2(c+dx)}{d(\sec(c+dx)+1)}\right)}{3a^2}-\frac{\sin(c+dx)\cos^2(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$\Big\downarrow$ 2009

$$\frac{2\left(\frac{3\left(-5a^2\int \sin\left(c+dx+\frac{\pi}{2}\right)^2dx-\frac{6a^2\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}\right)}{a^2}-\frac{5\sin(c+dx)\cos^2(c+dx)}{d(\sec(c+dx)+1)}\right)}{3a^2}-\frac{\sin(c+dx)\cos^2(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$\Big\downarrow$ 3115

$$\frac{2\left(\frac{3\left(-5a^2\left(\frac{\int 1dx}{2}+\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)-\frac{6a^2\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}\right)}{a^2}-\frac{5\sin(c+dx)\cos^2(c+dx)}{d(\sec(c+dx)+1)}\right)}{3a^2}-\frac{\sin(c+dx)\cos^2(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

3.60. $\int \frac{\cos^3(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

$$\Big\downarrow \; 24$$

$$\frac{2\left(\frac{3\left(-\frac{6a^2\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}-5a^2\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)\right)}{a^2}-\frac{5\sin(c+dx)\cos^2(c+dx)}{d(\sec(c+dx)+1)}\right)}{3a^2}-\frac{\sin(c+dx)\cos^2(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

input
```
Int[Cos[c + d*x]^3/(a + a*Sec[c + d*x])^2,x]
```

output
```
-1/3*(Cos[c + d*x]^2*Sin[c + d*x])/(d*(a + a*Sec[c + d*x])^2) + (2*((-5*Co
s[c + d*x]^2*Sin[c + d*x])/(d*(1 + Sec[c + d*x])) + (3*(-5*a^2*(x/2 + (Cos
[c + d*x]*Sin[c + d*x])/(2*d)) - (6*a^2*(-Sin[c + d*x] + Sin[c + d*x]^3/3)
)/d))/a^2))/(3*a^2)
```

3.60.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4304
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-Cot[e + f*x])*(a + b*Csc[e + f*x])^m*((d*Csc
[e + f*x])^n/(f*(2*m + 1))), x] + Simp[1/(a^2*(2*m + 1))   Int[(a + b*Csc[e
 + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e
+ f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ
[m, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

rule 4508
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
- a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
 1))), x] - Simp[1/(a^2*(2*m + 1))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Cs
c[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[
e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B
, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

3.60.4 Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.62

method	result
parallelrisch	$\frac{43\left(\cos(dx+c)+\frac{14\cos(2dx+2c)}{129}-\frac{\cos(3dx+3c)}{129}+\frac{\cos(4dx+4c)}{258}+\frac{73}{86}\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\sec\left(\frac{dx}{2}+\frac{c}{2}\right)^2-40dx}{8a^2d}$
derivativedivides	$\frac{-\frac{8\left(-\frac{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{2}-\frac{10\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3}-\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}\right)}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^3}-20\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3}+9\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2d\,a^2}$
default	$\frac{-\frac{8\left(-\frac{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{2}-\frac{10\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3}-\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}\right)}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^3}-20\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3}+9\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2d\,a^2}$
risch	$-\frac{5x}{a^2}+\frac{ie^{2i(dx+c)}}{4a^2d}-\frac{15ie^{i(dx+c)}}{8a^2d}+\frac{15ie^{-i(dx+c)}}{8a^2d}-\frac{ie^{-2i(dx+c)}}{4a^2d}+\frac{2i\left(15\,e^{2i(dx+c)}+27\,e^{i(dx+c)}+14\right)}{3d\,a^2\left(e^{i(dx+c)}+1\right)^3}+\frac{\sin(3d}{12a}$
norman	$\frac{-\frac{5x}{a}+\frac{21\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2ad}+\frac{80\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3ad}+\frac{23\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{ad}+\frac{4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{ad}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^9}{6ad}-\frac{15x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{a}-\frac{15x\tan\left(\frac{dx}{2}\right.}{a}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^3a}$

input
```
int(cos(d*x+c)^3/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/8*(43*(cos(d*x+c)+14/129*cos(2*d*x+2*c)-1/129*cos(3*d*x+3*c)+1/258*cos(4
*d*x+4*c)+73/86)*tan(1/2*d*x+1/2*c)*sec(1/2*d*x+1/2*c)^2-40*d*x)/a^2/d
```

3.60.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.87

$$\int \frac{\cos^3(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{15\,dx\cos(dx+c)^2+30\,dx\cos(dx+c)+15\,dx-\left(\cos(dx+c)^4-\cos(dx+c)^3+6\cos(dx+c)^2+33\right.}{3\left(a^2d\cos(dx+c)^2+2\,a^2d\cos(dx+c)+a^2d\right)}$$

input
```
integrate(cos(d*x+c)^3/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
-1/3*(15*d*x*cos(d*x + c)^2 + 30*d*x*cos(d*x + c) + 15*d*x - (cos(d*x + c)
^4 - cos(d*x + c)^3 + 6*cos(d*x + c)^2 + 33*cos(d*x + c) + 24)*sin(d*x + c
))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)
```

3.60.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^3(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**3/(a+a*sec(d*x+c))**2,x)
```

output
```
Timed out
```

3.60.7 Maxima [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.67

$$\int \frac{\cos^3(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{\frac{4\left(\frac{9\sin(dx+c)}{\cos(dx+c)+1}+\frac{20\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{15\sin(dx+c)^5}{(\cos(dx+c)+1)^5}\right)}{a^2+\frac{3a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{3a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{a^2\sin(dx+c)^6}{(\cos(dx+c)+1)^6}}+\frac{\frac{27\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^2}-\frac{60\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}}{6d}$$

input
```
integrate(cos(d*x+c)^3/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
1/6*(4*(9*sin(d*x + c)/(cos(d*x + c) + 1) + 20*sin(d*x + c)^3/(cos(d*x + c
) + 1)^3 + 15*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/(a^2 + 3*a^2*sin(d*x +
c)^2/(cos(d*x + c) + 1)^2 + 3*a^2*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + a^
2*sin(d*x + c)^6/(cos(d*x + c) + 1)^6) + (27*sin(d*x + c)/(cos(d*x + c) +
1) - sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/a^2 - 60*arctan(sin(d*x + c)/(co
s(d*x + c) + 1))/a^2)/d
```

3.60.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.87

$$\int \frac{\cos^3(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{\frac{30(dx+c)}{a^2}-\frac{4\left(15\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+20\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+9\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)^3a^2}+\frac{a^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-27a^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^6}}{6d}$$

input
```
integrate(cos(d*x+c)^3/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
-1/6*(30*(d*x + c)/a^2 - 4*(15*tan(1/2*d*x + 1/2*c)^5 + 20*tan(1/2*d*x + 1
/2*c)^3 + 9*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^3*a^2) + (
a^4*tan(1/2*d*x + 1/2*c)^3 - 27*a^4*tan(1/2*d*x + 1/2*c))/a^6)/d
```

3.60.9 Mupad [B] (verification not implemented)

Time = 13.86 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.09

$$\int \frac{\cos^3(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)-28\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^2\sin\left(\frac{c}{2}+\frac{dx}{2}\right)-60\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^4\sin\left(\frac{c}{2}+\frac{dx}{2}\right)+40\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^6\sin\left(\frac{c}{2}\right.}{6\,a^2\,d\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^3}$$

input
```
int(cos(c + d*x)^3/(a + a/cos(c + d*x))^2,x)
```

output
```
-(sin(c/2 + (d*x)/2) - 28*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2) - 60*cos
(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2) + 40*cos(c/2 + (d*x)/2)^6*sin(c/2 + (
d*x)/2) - 16*cos(c/2 + (d*x)/2)^8*sin(c/2 + (d*x)/2) + 30*cos(c/2 + (d*x)/
2)^3*(c + d*x))/(6*a^2*d*cos(c/2 + (d*x)/2)^3)
```

3.61 $\int \frac{\sec^6(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

3.61.1 Optimal result 660
3.61.2 Mathematica [A] (verified) 660
3.61.3 Rubi [A] (verified) 661
3.61.4 Maple [A] (verified) 665
3.61.5 Fricas [A] (verification not implemented) 666
3.61.6 Sympy [F] 666
3.61.7 Maxima [A] (verification not implemented) 666
3.61.8 Giac [A] (verification not implemented) 667
3.61.9 Mupad [B] (verification not implemented) 667

3.61.1 Optimal result

Integrand size = 21, antiderivative size = 162

$$\int \frac{\sec^6(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{13\text{arctanh}(\sin(c+dx))}{2a^3d} - \frac{152\tan(c+dx)}{15a^3d} + \frac{13\sec(c+dx)\tan(c+dx)}{2a^3d} - \frac{\sec^4(c+dx)\tan(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{11\sec^3(c+dx)\tan(c+dx)}{15ad(a+a\sec(c+dx))^2} - \frac{76\sec^2(c+dx)\tan(c+dx)}{15d\,(a^3+a^3\sec(c+dx))}$$

output
```
13/2*arctanh(sin(d*x+c))/a^3/d-152/15*tan(d*x+c)/a^3/d+13/2*sec(d*x+c)*tan
(d*x+c)/a^3/d-1/5*sec(d*x+c)^4*tan(d*x+c)/d/(a+a*sec(d*x+c))^3-11/15*sec(d
*x+c)^3*tan(d*x+c)/a/d/(a+a*sec(d*x+c))^2-76/15*sec(d*x+c)^2*tan(d*x+c)/d/
(a^3+a^3*sec(d*x+c))
```

3.61.2 Mathematica [A] (verified)

Time = 1.10 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.69

$$\int \frac{\sec^6(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{1560\text{arctanh}(\sin(c+dx))\cos^6\left(\frac{1}{2}(c+dx)\right)\sec^3(c+dx) - \frac{1}{8}\sec^5(c+dx)(1142\sin(c+dx)+1614\sin(2(c}{30a^3d(1+\sec(c+dx))^3}$$

input
```
Integrate[Sec[c + d*x]^6/(a + a*Sec[c + d*x])^3,x]
```

output
```
(1560*ArcTanh[Sin[c + d*x]]*Cos[(c + d*x)/2]^6*Sec[c + d*x]^3 - (Sec[c + d
*x]^5*(1142*Sin[c + d*x] + 1614*Sin[2*(c + d*x)] + 1414*Sin[3*(c + d*x)] +
 717*Sin[4*(c + d*x)] + 152*Sin[5*(c + d*x)]))/8)/(30*a^3*d*(1 + Sec[c + d
*x])^3)
```

3.61.3 Rubi [A] (verified)

Time = 1.11 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.09, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {3042, 4303, 3042, 4507, 3042, 4507, 3042, 4274, 3042, 4254, 24, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^6(c+dx)}{(a\sec(c+dx)+a)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^6}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}dx$$

$$\Big\downarrow \text{4303}$$

$$-\frac{\int \frac{\sec^4(c+dx)(4a-7a\sec(c+dx))}{(\sec(c+dx)a+a)^2}dx}{5a^2}-\frac{\tan(c+dx)\sec^4(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^4\left(4a-7a\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}dx}{5a^2}-\frac{\tan(c+dx)\sec^4(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{4507}$$

$$-\frac{\frac{\int \frac{\sec^3(c+dx)\left(33a^2-43a^2\sec(c+dx)\right)}{\sec(c+dx)a+a}dx}{3a^2}+\frac{11a\tan(c+dx)\sec^3(c+dx)}{3d(a\sec(c+dx)+a)^2}}{5a^2}-\frac{\tan(c+dx)\sec^4(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^3\left(33a^2-43a^2\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{3a^2}+\frac{11a\tan(c+dx)\sec^3(c+dx)}{3d(a\sec(c+dx)+a)^2}}{5a^2}-\frac{\tan(c+dx)\sec^4(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$\Big\downarrow$ 4507

$$-\frac{\frac{\frac{\int \sec^2(c+dx)\left(152a^3-195a^3\sec(c+dx)\right)dx}{a^2}+\frac{76a^2\tan(c+dx)\sec^2(c+dx)}{d(a\sec(c+dx)+a)}}{3a^2}+\frac{11a\tan(c+dx)\sec^3(c+dx)}{3d(a\sec(c+dx)+a)^2}}{5a^2}-\frac{\tan(c+dx)\sec^4(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$\Big\downarrow$ 3042

$$-\frac{\frac{\frac{\int \csc\left(c+dx+\frac{\pi}{2}\right)^2\left(152a^3-195a^3\csc\left(c+dx+\frac{\pi}{2}\right)\right)dx}{a^2}+\frac{76a^2\tan(c+dx)\sec^2(c+dx)}{d(a\sec(c+dx)+a)}}{3a^2}+\frac{11a\tan(c+dx)\sec^3(c+dx)}{3d(a\sec(c+dx)+a)^2}}{5a^2}-\frac{\tan(c+dx)\sec^4(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$\Big\downarrow$ 4274

$$-\frac{\frac{\frac{152a^3\int \sec^2(c+dx)dx-195a^3\int \sec^3(c+dx)dx}{a^2}+\frac{76a^2\tan(c+dx)\sec^2(c+dx)}{d(a\sec(c+dx)+a)}}{3a^2}+\frac{11a\tan(c+dx)\sec^3(c+dx)}{3d(a\sec(c+dx)+a)^2}}{5a^2}-\frac{\tan(c+dx)\sec^4(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$\Big\downarrow$ 3042

$$-\frac{\frac{\frac{152a^3\int \csc\left(c+dx+\frac{\pi}{2}\right)^2dx-195a^3\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx}{a^2}+\frac{76a^2\tan(c+dx)\sec^2(c+dx)}{d(a\sec(c+dx)+a)}}{3a^2}+\frac{11a\tan(c+dx)\sec^3(c+dx)}{3d(a\sec(c+dx)+a)^2}}{5a^2}-\frac{\tan(c+dx)\sec^4(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$\Big\downarrow$ 4254

$$-\frac{\frac{\frac{-\frac{152a^3\int 1d(-\tan(c+dx))}{d}-195a^3\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx}{a^2}+\frac{76a^2\tan(c+dx)\sec^2(c+dx)}{d(a\sec(c+dx)+a)}}{3a^2}+\frac{11a\tan(c+dx)\sec^3(c+dx)}{3d(a\sec(c+dx)+a)^2}}{5a^2}-\frac{\tan(c+dx)\sec^4(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$\Big\downarrow$ 24

3.61. $\int \frac{\sec^6(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

$$-\frac{\frac{\frac{\frac{152a^3\tan(c+dx)}{d}-195a^3\int\csc\left(c+dx+\frac{\pi}{2}\right)^3dx}{a^2}+\frac{76a^2\tan(c+dx)\sec^2(c+dx)}{d(a\sec(c+dx)+a)}}{3a^2}+\frac{11a\tan(c+dx)\sec^3(c+dx)}{3d(a\sec(c+dx)+a)^2}}{5a^2}-\frac{\tan(c+dx)\sec^4(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \; 4255$$

$$-\frac{\frac{\frac{\frac{152a^3\tan(c+dx)}{d}-195a^3\left(\frac{1}{2}\int\sec(c+dx)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)}{a^2}+\frac{76a^2\tan(c+dx)\sec^2(c+dx)}{d(a\sec(c+dx)+a)}}{3a^2}+\frac{11a\tan(c+dx)\sec^3(c+dx)}{3d(a\sec(c+dx)+a)^2}}{5a^2}-\frac{\tan(c+dx)\sec^4(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\frac{\frac{\frac{152a^3\tan(c+dx)}{d}-195a^3\left(\frac{1}{2}\int\csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)}{a^2}+\frac{76a^2\tan(c+dx)\sec^2(c+dx)}{d(a\sec(c+dx)+a)}}{3a^2}+\frac{11a\tan(c+dx)\sec^3(c+dx)}{3d(a\sec(c+dx)+a)^2}}{5a^2}-\frac{\tan(c+dx)\sec^4(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \; 4257$$

$$-\frac{\frac{\frac{\frac{76a^2\tan(c+dx)\sec^2(c+dx)}{d(a\sec(c+dx)+a)}+\frac{\frac{152a^3\tan(c+dx)}{d}-195a^3\left(\frac{\text{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)}{a^2}}{3a^2}+\frac{11a\tan(c+dx)\sec^3(c+dx)}{3d(a\sec(c+dx)+a)^2}}{5a^2}-\frac{\tan(c+dx)\sec^4(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

input
```
Int[Sec[c + d*x]^6/(a + a*Sec[c + d*x])^3,x]
```

output
```
-1/5*(Sec[c + d*x]^4*Tan[c + d*x])/(d*(a + a*Sec[c + d*x])^3) - ((11*a*Sec
[c + d*x]^3*Tan[c + d*x])/(3*d*(a + a*Sec[c + d*x])^2) + ((76*a^2*Sec[c +
d*x]^2*Tan[c + d*x])/(d*(a + a*Sec[c + d*x])) + ((152*a^3*Tan[c + d*x])/d
- 195*a^3*(ArcTanh[Sin[c + d*x]]/(2*d) + (Sec[c + d*x]*Tan[c + d*x])/(2*d)
))/a^2)/(3*a^2))/(5*a^2)
```

3.61. $\int \frac{\sec^6(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

3.61.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4303
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-d^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d
*Csc[e + f*x])^(n - 2)/(f*(2*m + 1))), x] + Simp[d^2/(a*b*(2*m + 1))   Int[
(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n
 + 2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2,
0] && LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

rule 4507
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[d*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(a*f*(
2*m + 1))), x] - Simp[1/(a*b*(2*m + 1))   Int[(a + b*Csc[e + f*x])^(m + 1)*
(d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
 - n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f,
A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && G
tQ[n, 0]
```

3.61.4 Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.83

method	result
derivativedivides	$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{5}-\frac{8\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3}-31\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{2}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}+\frac{14}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1}-26\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)-\frac{}{\left(\tan\left(\frac{d}{2}\right.\right.}}{4d\,a^3}$
default	$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{5}-\frac{8\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3}-31\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{2}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}+\frac{14}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1}-26\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)-\frac{}{\left(\tan\left(\frac{d}{2}\right.\right.}}{4d\,a^3}$
parallelrisch	$\frac{(-1560\cos(2dx+2c)-1560)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+(1560\cos(2dx+2c)+1560)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-2331\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\mathrm{se}}{240a^3d(1+\cos(2dx+2c))}$
risch	$-\frac{i\left(195\,\mathrm{e}^{8i(dx+c)}+975\,\mathrm{e}^{7i(dx+c)}+2275\,\mathrm{e}^{6i(dx+c)}+3575\,\mathrm{e}^{5i(dx+c)}+4329\,\mathrm{e}^{4i(dx+c)}+3805\,\mathrm{e}^{3i(dx+c)}+2673\,\mathrm{e}^{2i(dx+c)}+132\right.}{15d\,a^3\left(\mathrm{e}^{2i(dx+c)}+1\right)^2\left(\mathrm{e}^{i(dx+c)}+1\right)^5}$
norman	$\frac{\frac{51\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4ad}-\frac{721\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{12ad}+\frac{6613\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{60ad}-\frac{1165\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{12ad}+\frac{475\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^9}{12ad}-\frac{59\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{11}}{12ad}-\frac{5\tan\left(\frac{dx}{2}\right.}{12a}}{\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^5a^2}$

input
```
int(sec(d*x+c)^6/(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/4/d/a^3*(-1/5*tan(1/2*d*x+1/2*c)^5-8/3*tan(1/2*d*x+1/2*c)^3-31*tan(1/2*d
*x+1/2*c)+2/(tan(1/2*d*x+1/2*c)-1)^2+14/(tan(1/2*d*x+1/2*c)-1)-26*ln(tan(1
/2*d*x+1/2*c)-1)-2/(tan(1/2*d*x+1/2*c)+1)^2+14/(tan(1/2*d*x+1/2*c)+1)+26*l
n(tan(1/2*d*x+1/2*c)+1))
```

3.61.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.27

$$\int \frac{\sec^6(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{195\left(\cos\left(dx+c\right)^5+3\cos\left(dx+c\right)^4+3\cos\left(dx+c\right)^3+\cos\left(dx+c\right)^2\right)\log\left(\sin\left(dx+c\right)+1\right)-195\left(\cos\left(d\right.\right.}{60}$$

input
```
integrate(sec(d*x+c)^6/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/60*(195*(cos(d*x + c)^5 + 3*cos(d*x + c)^4 + 3*cos(d*x + c)^3 + cos(d*x
+ c)^2)*log(sin(d*x + c) + 1) - 195*(cos(d*x + c)^5 + 3*cos(d*x + c)^4 + 3
*cos(d*x + c)^3 + cos(d*x + c)^2)*log(-sin(d*x + c) + 1) - 2*(304*cos(d*x
+ c)^4 + 717*cos(d*x + c)^3 + 479*cos(d*x + c)^2 + 45*cos(d*x + c) - 15)*s
in(d*x + c))/(a^3*d*cos(d*x + c)^5 + 3*a^3*d*cos(d*x + c)^4 + 3*a^3*d*cos(
d*x + c)^3 + a^3*d*cos(d*x + c)^2)
```

3.61.6 Sympy [F]

$$\int \frac{\sec^6(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\int \frac{\sec^6(c+dx)}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx}{a^3}$$

input
```
integrate(sec(d*x+c)**6/(a+a*sec(d*x+c))**3,x)
```

output
```
Integral(sec(c + d*x)**6/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c +
d*x) + 1), x)/a**3
```

3.61.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.30

$$\int \frac{\sec^6(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{\frac{60\left(\frac{5\sin(dx+c)}{\cos(dx+c)+1}-\frac{7\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a^3-\frac{2a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}+\frac{\frac{465\sin(dx+c)}{\cos(dx+c)+1}+\frac{40\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{a^3}-\frac{390\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^3}+\frac{390\log\left(\frac{\sin(dx+c}{\cos(dx+c)}\right.}{a^3}}{60\,d}$$

input
```
integrate(sec(d*x+c)^6/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
-1/60*(60*(5*sin(d*x + c)/(cos(d*x + c) + 1) - 7*sin(d*x + c)^3/(cos(d*x +
 c) + 1)^3)/(a^3 - 2*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a^3*sin(d*x
 + c)^4/(cos(d*x + c) + 1)^4) + (465*sin(d*x + c)/(cos(d*x + c) + 1) + 40*
sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 3*sin(d*x + c)^5/(cos(d*x + c) + 1)^
5)/a^3 - 390*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^3 + 390*log(sin(d*
x + c)/(cos(d*x + c) + 1) - 1)/a^3)/d
```

3.61.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.86

$$\int \frac{\sec^6(c+dx)}{(a+a\sec(c+dx))^3}\,dx$$
$$= \frac{\frac{390\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^3} - \frac{390\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a^3} + \frac{60\left(7\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-5\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^2 a^3} - \frac{3\,a^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+40\,a^{12}}{}}{60\,d}$$

input
```
integrate(sec(d*x+c)^6/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
1/60*(390*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^3 - 390*log(abs(tan(1/2*d*x
 + 1/2*c) - 1))/a^3 + 60*(7*tan(1/2*d*x + 1/2*c)^3 - 5*tan(1/2*d*x + 1/2*c
))/((tan(1/2*d*x + 1/2*c)^2 - 1)^2*a^3) - (3*a^12*tan(1/2*d*x + 1/2*c)^5 +
 40*a^12*tan(1/2*d*x + 1/2*c)^3 + 465*a^12*tan(1/2*d*x + 1/2*c))/a^15)/d
```

3.61.9 Mupad [B] (verification not implemented)

Time = 12.99 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.87

$$\int \frac{\sec^6(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{13\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{a^3\,d} - \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5}{20\,a^3\,d} - \frac{2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{3\,a^3\,d} - \frac{5\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-7\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{d\left(a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4-2\,a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+a^3\right)} - \frac{31\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{4\,a^3\,d}$$

input
```
int(1/(cos(c + d*x)^6*(a + a/cos(c + d*x))^3),x)
```

output
```
(13*atanh(tan(c/2 + (d*x)/2)))/(a^3*d) - tan(c/2 + (d*x)/2)^5/(20*a^3*d) -
 (2*tan(c/2 + (d*x)/2)^3)/(3*a^3*d) - (5*tan(c/2 + (d*x)/2) - 7*tan(c/2 +
(d*x)/2)^3)/(d*(a^3*tan(c/2 + (d*x)/2)^4 - 2*a^3*tan(c/2 + (d*x)/2)^2 + a^
3)) - (31*tan(c/2 + (d*x)/2))/(4*a^3*d)
```

3.61. $\int \frac{\sec^6(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

3.62 $\int \frac{\sec^5(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

3.62.1 Optimal result 669
3.62.2 Mathematica [A] (verified) 669
3.62.3 Rubi [A] (verified) 670
3.62.4 Maple [A] (verified) 674
3.62.5 Fricas [A] (verification not implemented) 675
3.62.6 Sympy [F] 675
3.62.7 Maxima [A] (verification not implemented) 676
3.62.8 Giac [A] (verification not implemented) 676
3.62.9 Mupad [B] (verification not implemented) 677

3.62.1 Optimal result

Integrand size = 21, antiderivative size = 128

$$\int \frac{\sec^5(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{3\text{arctanh}(\sin(c+dx))}{a^3d}+\frac{9\tan(c+dx)}{5a^3d} - \frac{\sec^3(c+dx)\tan(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{3\sec^2(c+dx)\tan(c+dx)}{5ad(a+a\sec(c+dx))^2}+\frac{3\tan(c+dx)}{d\left(a^3+a^3\sec(c+dx)\right)}$$

output
```
-3*arctanh(sin(d*x+c))/a^3/d+9/5*tan(d*x+c)/a^3/d-1/5*sec(d*x+c)^3*tan(d*x
+c)/d/(a+a*sec(d*x+c))^3-3/5*sec(d*x+c)^2*tan(d*x+c)/a/d/(a+a*sec(d*x+c))^
2+3*tan(d*x+c)/d/(a^3+a^3*sec(d*x+c))
```

3.62.2 Mathematica [A] (verified)

Time = 0.67 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.91

$$\int \frac{\sec^5(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\cos\left(\frac{1}{2}(c+dx)\right)\sec^3(c+dx)\left(-240\text{arctanh}(\sin(c+dx))\cos^5\left(\frac{1}{2}(c+dx)\right)+\sec(c+dx)\left(20\sin\left(\frac{1}{2}(c+dx)\right.\right.\right.}{10a^3d(1+\sec(c+dx))^3}$$

input
```
Integrate[Sec[c + d*x]^5/(a + a*Sec[c + d*x])^3,x]
```

output
```
(Cos[(c + d*x)/2]*Sec[c + d*x]^3*(-240*ArcTanh[Sin[c + d*x]]*Cos[(c + d*x)
/2]^5 + Sec[c + d*x]*(20*Sin[(c + d*x)/2] + 57*Sin[(3*(c + d*x))/2] + 45*S
in[(5*(c + d*x))/2] + 12*Sin[(7*(c + d*x))/2])))/(10*a^3*d*(1 + Sec[c + d*
x])^3)
```

3.62.3 Rubi [A] (verified)

Time = 0.94 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.08, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {3042, 4303, 27, 3042, 4507, 27, 3042, 4496, 25, 3042, 4274, 3042, 4254, 24, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^5(c+dx)}{(a\sec(c+dx)+a)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^5}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}dx$$

$$\Big\downarrow \text{4303}$$

$$-\frac{\int \frac{3\sec^3(c+dx)(a-2a\sec(c+dx))}{(\sec(c+dx)a+a)^2}dx}{5a^2}-\frac{\tan(c+dx)\sec^3(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{27}$$

$$-\frac{3\int \frac{\sec^3(c+dx)(a-2a\sec(c+dx))}{(\sec(c+dx)a+a)^2}dx}{5a^2}-\frac{\tan(c+dx)\sec^3(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{3\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^3\left(a-2a\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}dx}{5a^2}-\frac{\tan(c+dx)\sec^3(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{4507}$$

$$-\frac{3\left(\frac{\int \frac{3\sec^2(c+dx)\left(2a^2-3a^2\sec(c+dx)\right)}{\sec(c+dx)a+a}dx}{3a^2}+\frac{a\tan(c+dx)\sec^2(c+dx)}{d(a\sec(c+dx)+a)^2}\right)}{5a^2}-\frac{\tan(c+dx)\sec^3(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{27}$$

$$-\frac{3\left(\frac{\int \frac{\sec^2(c+dx)\left(2a^2-3a^2\sec(c+dx)\right)}{\sec(c+dx)a+a}dx}{a^2}+\frac{a\tan(c+dx)\sec^2(c+dx)}{d(a\sec(c+dx)+a)^2}\right)}{5a^2}-\frac{\tan(c+dx)\sec^3(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$\Big\downarrow$ 3042

$$-\frac{3\left(\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2\left(2a^2-3a^2\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{a^2}+\frac{a\tan(c+dx)\sec^2(c+dx)}{d(a\sec(c+dx)+a)^2}\right)}{5a^2}-\frac{\tan(c+dx)\sec^3(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$\Big\downarrow$ 4496

$$-\frac{3\left(\frac{-\frac{\int -\sec(c+dx)\left(5a^3-3a^3\sec(c+dx)\right)dx}{a^2}-\frac{5a^2\tan(c+dx)}{d(a\sec(c+dx)+a)}}{a^2}+\frac{a\tan(c+dx)\sec^2(c+dx)}{d(a\sec(c+dx)+a)^2}\right)}{5a^2}-\frac{\tan(c+dx)\sec^3(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$\Big\downarrow$ 25

$$-\frac{3\left(\frac{\frac{\int \sec(c+dx)\left(5a^3-3a^3\sec(c+dx)\right)dx}{a^2}-\frac{5a^2\tan(c+dx)}{d(a\sec(c+dx)+a)}}{a^2}+\frac{a\tan(c+dx)\sec^2(c+dx)}{d(a\sec(c+dx)+a)^2}\right)}{5a^2}-\frac{\tan(c+dx)\sec^3(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$\Big\downarrow$ 3042

$$-\frac{3\left(\frac{\frac{\int \csc\left(c+dx+\frac{\pi}{2}\right)\left(5a^3-3a^3\csc\left(c+dx+\frac{\pi}{2}\right)\right)dx}{a^2}-\frac{5a^2\tan(c+dx)}{d(a\sec(c+dx)+a)}}{a^2}+\frac{a\tan(c+dx)\sec^2(c+dx)}{d(a\sec(c+dx)+a)^2}\right)}{5a^2}-\frac{\tan(c+dx)\sec^3(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$\Big\downarrow$ 4274

$$-\frac{3\left(\frac{\frac{5a^3\int \sec(c+dx)dx-3a^3\int \sec^2(c+dx)dx}{a^2}-\frac{5a^2\tan(c+dx)}{d(a\sec(c+dx)+a)}}{a^2}+\frac{a\tan(c+dx)\sec^2(c+dx)}{d(a\sec(c+dx)+a)^2}\right)}{5a^2}-\frac{\tan(c+dx)\sec^3(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$\Big\downarrow$ 3042

3.62. $\int \frac{\sec^5(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

$$-\frac{3\left(\frac{\frac{5a^3\int \csc\left(c+dx+\frac{\pi}{2}\right)dx-3a^3\int \csc\left(c+dx+\frac{\pi}{2}\right)^2dx}{a^2}-\frac{5a^2\tan(c+dx)}{d(a\sec(c+dx)+a)}}{a^2}+\frac{a\tan(c+dx)\sec^2(c+dx)}{d(a\sec(c+dx)+a)^2}\right)}{5a^2}-\frac{\tan(c+dx)\sec^3(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$\Big\downarrow$ 4254

$$-\frac{3\left(\frac{\frac{\frac{3a^3\int 1d(-\tan(c+dx))}{d}+5a^3\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{a^2}-\frac{5a^2\tan(c+dx)}{d(a\sec(c+dx)+a)}}{a^2}+\frac{a\tan(c+dx)\sec^2(c+dx)}{d(a\sec(c+dx)+a)^2}\right)}{5a^2}-\frac{\tan(c+dx)\sec^3(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$\Big\downarrow$ 24

$$-\frac{3\left(\frac{\frac{5a^3\int \csc\left(c+dx+\frac{\pi}{2}\right)dx-\frac{3a^3\tan(c+dx)}{d}}{a^2}-\frac{5a^2\tan(c+dx)}{d(a\sec(c+dx)+a)}}{a^2}+\frac{a\tan(c+dx)\sec^2(c+dx)}{d(a\sec(c+dx)+a)^2}\right)}{5a^2}-\frac{\tan(c+dx)\sec^3(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$\Big\downarrow$ 4257

$$-\frac{3\left(\frac{\frac{\frac{5a^3\text{arctanh}(\sin(c+dx))}{d}-\frac{3a^3\tan(c+dx)}{d}}{a^2}-\frac{5a^2\tan(c+dx)}{d(a\sec(c+dx)+a)}}{a^2}+\frac{a\tan(c+dx)\sec^2(c+dx)}{d(a\sec(c+dx)+a)^2}\right)}{5a^2}-\frac{\tan(c+dx)\sec^3(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

input
```
Int[Sec[c + d*x]^5/(a + a*Sec[c + d*x])^3,x]
```

output
```
-1/5*(Sec[c + d*x]^3*Tan[c + d*x])/(d*(a + a*Sec[c + d*x])^3) - (3*((a*Sec
[c + d*x]^2*Tan[c + d*x])/(d*(a + a*Sec[c + d*x])^2) + ((-5*a^2*Tan[c + d*
x])/(d*(a + a*Sec[c + d*x])) + ((5*a^3*ArcTanh[Sin[c + d*x]])/d - (3*a^3*T
an[c + d*x])/d)/a^2)/a^2))/(5*a^2)
```

3.62. $\int \frac{\sec^5(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

3.62.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4303
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-d^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d
*Csc[e + f*x])^(n - 2)/(f*(2*m + 1))), x] + Simp[d^2/(a*b*(2*m + 1))   Int[
(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n
 + 2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2,
0] && LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

rule 4496
```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b - a*B))*Cot
[e + f*x]*((a + b*Csc[e + f*x])^m/(b*f*(2*m + 1))), x] + Simp[1/(b^2*(2*m +
 1))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[A*b*m - a*B*m + b
*B*(2*m + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && Ne
Q[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

rule 4507
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[d*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(a*f*(
2*m + 1))), x] - Simp[1/(a*b*(2*m + 1))   Int[(a + b*Csc[e + f*x])^(m + 1)*
(d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
 - n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f,
A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && G
tQ[n, 0]
```

3.62.4 Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.82

method	result
derivativedivides	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{5}+2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3+17\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{4}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}-12\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-\frac{4}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1}+12\ln\left(\tan\left(\right.\right.}{4d\,a^3}$
default	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{5}+2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3+17\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{4}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}-12\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-\frac{4}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1}+12\ln\left(\tan\left(\right.\right.}{4d\,a^3}$
parallelrisch	$\frac{3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\cos(dx+c)-3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)\cos(dx+c)+\frac{57\left(\cos(dx+c)+\frac{\cos(2dx+2c)}{2}+\frac{2\cos(3dx+3c)}{19}+\frac{67}{114}\right)\sec}{20}}{a^3d\cos(dx+c)}$
risch	$\frac{2i\left(15\,\mathrm{e}^{6i(dx+c)}+75\,\mathrm{e}^{5i(dx+c)}+160\,\mathrm{e}^{4i(dx+c)}+200\,\mathrm{e}^{3i(dx+c)}+189\,\mathrm{e}^{2i(dx+c)}+105\,\mathrm{e}^{i(dx+c)}+24\right)}{5d\,a^3\left(\mathrm{e}^{i(dx+c)}+1\right)^5\left(\mathrm{e}^{2i(dx+c)}+1\right)}+\frac{3\ln\left(\mathrm{e}^{i(dx+c)}-i\right)}{a^3d}-$
norman	$\frac{\frac{25\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4ad}-\frac{45\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{2ad}+\frac{591\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{20ad}-\frac{81\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{5ad}+\frac{51\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^9}{20ad}+\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{11}}{10ad}+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{13}}{20ad}}{\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^4a^2}$

input
```
int(sec(d*x+c)^5/(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/4/d/a^3*(1/5*tan(1/2*d*x+1/2*c)^5+2*tan(1/2*d*x+1/2*c)^3+17*tan(1/2*d*x+
1/2*c)-4/(tan(1/2*d*x+1/2*c)+1)-12*ln(tan(1/2*d*x+1/2*c)+1)-4/(tan(1/2*d*x
+1/2*c)-1)+12*ln(tan(1/2*d*x+1/2*c)-1))
```

3.62.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.48

$$\int \frac{\sec^5(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{15\left(\cos\left(dx+c\right)^4+3\,\cos\left(dx+c\right)^3+3\,\cos\left(dx+c\right)^2+\cos\left(dx+c\right)\right)\log\left(\sin\left(dx+c\right)+1\right)-15\left(\cos\left(d\right.\right.}{10\left(a^3d\cos\left(d\right.\right.}$$

input
```
integrate(sec(d*x+c)^5/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
-1/10*(15*(cos(d*x + c)^4 + 3*cos(d*x + c)^3 + 3*cos(d*x + c)^2 + cos(d*x
+ c))*log(sin(d*x + c) + 1) - 15*(cos(d*x + c)^4 + 3*cos(d*x + c)^3 + 3*co
s(d*x + c)^2 + cos(d*x + c))*log(-sin(d*x + c) + 1) - 2*(24*cos(d*x + c)^3
 + 57*cos(d*x + c)^2 + 39*cos(d*x + c) + 5)*sin(d*x + c))/(a^3*d*cos(d*x +
 c)^4 + 3*a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + a^3*d*cos(d*x +
c))
```

3.62.6 Sympy [F]

$$\int \frac{\sec^5(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\int \frac{\sec^5(c+dx)}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx}{a^3}$$

input
```
integrate(sec(d*x+c)**5/(a+a*sec(d*x+c))**3,x)
```

output
```
Integral(sec(c + d*x)**5/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c +
d*x) + 1), x)/a**3
```

3.62.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.29

$$\int \frac{\sec^5(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\frac{40\sin(dx+c)}{\left(a^3-\frac{a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)}+\frac{\frac{85\sin(dx+c)}{\cos(dx+c)+1}+\frac{10\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{a^3}-\frac{60\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^3}+\frac{60\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right.}{a^3}}{20\,d}$$

input
```
integrate(sec(d*x+c)^5/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/20*(40*sin(d*x + c)/((a^3 - a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)*(co
s(d*x + c) + 1)) + (85*sin(d*x + c)/(cos(d*x + c) + 1) + 10*sin(d*x + c)^3
/(cos(d*x + c) + 1)^3 + sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^3 - 60*log(
sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^3 + 60*log(sin(d*x + c)/(cos(d*x +
c) + 1) - 1)/a^3)/d
```

3.62.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.95

$$\int \frac{\sec^5(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{\frac{60\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a^3}-\frac{60\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{a^3}+\frac{40\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)a^3}-\frac{a^{12}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+10a^{12}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+}{a^{15}}}{20\,d}$$

input
```
integrate(sec(d*x+c)^5/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
-1/20*(60*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^3 - 60*log(abs(tan(1/2*d*x
+ 1/2*c) - 1))/a^3 + 40*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/2*c)^2 - 1)
*a^3) - (a^12*tan(1/2*d*x + 1/2*c)^5 + 10*a^12*tan(1/2*d*x + 1/2*c)^3 + 85
*a^12*tan(1/2*d*x + 1/2*c))/a^15)/d
```

3.62.9 Mupad [B] (verification not implemented)

Time = 12.66 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.87

$$\int \frac{\sec^5(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{2\,a^3\,d}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5}{20\,a^3\,d}-\frac{6\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{a^3\,d} - \frac{2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\left(a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2-a^3\right)}+\frac{17\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{4\,a^3\,d}$$

input
```
int(1/(cos(c + d*x)^5*(a + a/cos(c + d*x))^3),x)
```

output
```
tan(c/2 + (d*x)/2)^3/(2*a^3*d) + tan(c/2 + (d*x)/2)^5/(20*a^3*d) - (6*atan
h(tan(c/2 + (d*x)/2)))/(a^3*d) - (2*tan(c/2 + (d*x)/2))/(d*(a^3*tan(c/2 +
(d*x)/2)^2 - a^3)) + (17*tan(c/2 + (d*x)/2))/(4*a^3*d)
```

3.63 $\int \frac{\sec^4(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

3.63.1 Optimal result 678
3.63.2 Mathematica [A] (verified) 678
3.63.3 Rubi [A] (verified) 679
3.63.4 Maple [A] (verified) 682
3.63.5 Fricas [A] (verification not implemented) 682
3.63.6 Sympy [F] 683
3.63.7 Maxima [A] (verification not implemented) 683
3.63.8 Giac [A] (verification not implemented) 683
3.63.9 Mupad [B] (verification not implemented) 684

3.63.1 Optimal result

Integrand size = 21, antiderivative size = 105

$$\int \frac{\sec^4(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\operatorname{arctanh}(\sin(c+dx))}{a^3d} - \frac{\sec^2(c+dx)\tan(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{7\tan(c+dx)}{15ad(a+a\sec(c+dx))^2} - \frac{29\tan(c+dx)}{15d\left(a^3+a^3\sec(c+dx)\right)}$$

output
```
arctanh(sin(d*x+c))/a^3/d-1/5*sec(d*x+c)^2*tan(d*x+c)/d/(a+a*sec(d*x+c))^3
+7/15*tan(d*x+c)/a/d/(a+a*sec(d*x+c))^2-29/15*tan(d*x+c)/d/(a^3+a^3*sec(d*
x+c))
```

3.63.2 Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.91

$$\int \frac{\sec^4(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\cos\left(\frac{1}{2}(c+dx)\right)\sec^3(c+dx)\left(120\operatorname{arctanh}(\sin(c+dx))\cos^5\left(\frac{1}{2}(c+dx)\right) - 35\sin\left(\frac{1}{2}(c+dx)\right) - 40\sin\left(\frac{3}{2}(c\right.\right.}{15a^3d(1+\sec(c+dx))^3}$$

input
```
Integrate[Sec[c + d*x]^4/(a + a*Sec[c + d*x])^3,x]
```

output
```
(Cos[(c + d*x)/2]*Sec[c + d*x]^3*(120*ArcTanh[Sin[c + d*x]]*Cos[(c + d*x)/
2]^5 - 35*Sin[(c + d*x)/2] - 40*Sin[(3*(c + d*x))/2] - 11*Sin[(5*(c + d*x)
)/2]))/(15*a^3*d*(1 + Sec[c + d*x])^3)
```

3.63.3 Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.10, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {3042, 4303, 3042, 4496, 25, 3042, 4486, 3042, 4257, 4281}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^4(c+dx)}{(a\sec(c+dx)+a)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^4}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}dx$$

$$\Big\downarrow \text{4303}$$

$$-\frac{\int \frac{\sec^2(c+dx)(2a-5a\sec(c+dx))}{(\sec(c+dx)a+a)^2}dx}{5a^2}-\frac{\tan(c+dx)\sec^2(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2\left(2a-5a\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}dx}{5a^2}-\frac{\tan(c+dx)\sec^2(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{4496}$$

$$-\frac{-\frac{\int -\frac{\sec(c+dx)\left(14a^2-15a^2\sec(c+dx)\right)}{\sec(c+dx)a+a}dx}{3a^2}-\frac{7a\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}}{5a^2}-\frac{\tan(c+dx)\sec^2(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\frac{\int \frac{\sec(c+dx)\left(14a^2-15a^2\sec(c+dx)\right)}{\sec(c+dx)a+a}dx}{3a^2}-\frac{7a\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}}{5a^2}-\frac{\tan(c+dx)\sec^2(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(14a^2-15a^2\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{3a^2}-\frac{7a\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}}{5a^2}-\frac{\tan(c+dx)\sec^2(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$\Big\downarrow$ 4486

$$-\frac{\frac{29a^2\int \frac{\sec(c+dx)}{\sec(c+dx)a+a}dx-15a\int \sec(c+dx)dx}{3a^2}-\frac{7a\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}}{5a^2}-\frac{\tan(c+dx)\sec^2(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$\Big\downarrow$ 3042

$$-\frac{\frac{29a^2\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx-15a\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{3a^2}-\frac{7a\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}}{5a^2}-\frac{\tan(c+dx)\sec^2(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$\Big\downarrow$ 4257

$$-\frac{\frac{29a^2\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx-\frac{15a\text{arctanh}(\sin(c+dx))}{d}}{3a^2}-\frac{7a\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}}{5a^2}-\frac{\tan(c+dx)\sec^2(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$\Big\downarrow$ 4281

$$-\frac{\frac{\frac{29a^2\tan(c+dx)}{d(a\sec(c+dx)+a)}-\frac{15a\text{arctanh}(\sin(c+dx))}{d}}{3a^2}-\frac{7a\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}}{5a^2}-\frac{\tan(c+dx)\sec^2(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

input
```
Int[Sec[c + d*x]^4/(a + a*Sec[c + d*x])^3,x]
```

output
```
-1/5*(Sec[c + d*x]^2*Tan[c + d*x])/(d*(a + a*Sec[c + d*x])^3) - ((-7*a*Tan
[c + d*x])/(3*d*(a + a*Sec[c + d*x])^2) + ((-15*a*ArcTanh[Sin[c + d*x]])/d
 + (29*a^2*Tan[c + d*x])/(d*(a + a*Sec[c + d*x])))/(3*a^2))/(5*a^2)
```

3.63.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4281
```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[-Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

rule 4303
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-d^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d
*Csc[e + f*x])^(n - 2)/(f*(2*m + 1))), x] + Simp[d^2/(a*b*(2*m + 1))   Int[
(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n
 + 2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2,
0] && LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

rule 4486
```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/(csc[(
e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[B/b   Int[Csc[e + f*x],
 x], x] + Simp[(A*b - a*B)/b   Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x
] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0]
```

rule 4496
```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b - a*B))*Cot
[e + f*x]*((a + b*Csc[e + f*x])^m/(b*f*(2*m + 1))), x] + Simp[1/(b^2*(2*m +
 1))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[A*b*m - a*B*m + b
*B*(2*m + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && Ne
Q[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

3.63. $\int \frac{\sec^4(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

3.63.4 Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.71

method	result
derivativedivides	$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{5}-\frac{4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{3}-7\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-4\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+4\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{4d\,a^{3}}$
default	$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{5}-\frac{4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{3}-7\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-4\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+4\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{4d\,a^{3}}$
parallelrisch	$\frac{-3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}-20\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}-60\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+60\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-105\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{60a^{3}d}$
risch	$-\frac{2i\left(15\,\mathrm{e}^{4i(dx+c)}+75\,\mathrm{e}^{3i(dx+c)}+145\,\mathrm{e}^{2i(dx+c)}+95\,\mathrm{e}^{i(dx+c)}+22\right)}{15d\,a^{3}\left(\mathrm{e}^{i(dx+c)}+1\right)^{5}}+\frac{\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{a^{3}d}-\frac{\ln\left(\mathrm{e}^{i(dx+c)}-i\right)}{a^{3}d}$
norman	$\frac{\frac{7\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4ad}-\frac{59\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{12ad}+\frac{43\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{10ad}-\frac{9\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{10ad}-\frac{11\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{9}}{60ad}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{11}}{20ad}}{\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)^{3}a^{2}}+\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}{a^{3}d}$

input
```
int(sec(d*x+c)^4/(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/4/d/a^3*(-1/5*tan(1/2*d*x+1/2*c)^5-4/3*tan(1/2*d*x+1/2*c)^3-7*tan(1/2*d*
x+1/2*c)-4*ln(tan(1/2*d*x+1/2*c)-1)+4*ln(tan(1/2*d*x+1/2*c)+1))
```

3.63.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.50

$$\int \frac{\sec^4(c+dx)}{(a+a\sec(c+dx))^3}\,dx$$
$$= \frac{15\left(\cos\left(dx+c\right)^3+3\,\cos\left(dx+c\right)^2+3\,\cos\left(dx+c\right)+1\right)\log\left(\sin\left(dx+c\right)+1\right)-15\left(\cos\left(dx+c\right)^3+3\,\cos\right.}{30\left(a^3d\cos\left(dx+c\right)^3+3\,a^3d\,\mathrm{c}\right.}$$

input
```
integrate(sec(d*x+c)^4/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/30*(15*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*log(sin(
d*x + c) + 1) - 15*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1
)*log(-sin(d*x + c) + 1) - 2*(22*cos(d*x + c)^2 + 51*cos(d*x + c) + 32)*si
n(d*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d
*x + c) + a^3*d)
```

3.63.6 Sympy [F]

$$\int \frac{\sec^4(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\int \frac{\sec^4(c+dx)}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx}{a^3}$$

input
```
integrate(sec(d*x+c)**4/(a+a*sec(d*x+c))**3,x)
```

output
```
Integral(sec(c + d*x)**4/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c +
d*x) + 1), x)/a**3
```

3.63.7 Maxima [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.13

$$\int \frac{\sec^4(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{\frac{\frac{105\sin(dx+c)}{\cos(dx+c)+1}+\frac{20\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{a^3}-\frac{60\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^3}+\frac{60\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^3}}{60\,d}$$

input
```
integrate(sec(d*x+c)^4/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
-1/60*((105*sin(d*x + c)/(cos(d*x + c) + 1) + 20*sin(d*x + c)^3/(cos(d*x +
 c) + 1)^3 + 3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^3 - 60*log(sin(d*x +
 c)/(cos(d*x + c) + 1) + 1)/a^3 + 60*log(sin(d*x + c)/(cos(d*x + c) + 1) -
 1)/a^3)/d
```

3.63.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.90

$$\int \frac{\sec^4(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\frac{60\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a^3}-\frac{60\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{a^3}-\frac{3a^{12}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+20a^{12}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+105a^{12}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^{15}}}{60\,d}$$

input
```
integrate(sec(d*x+c)^4/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
1/60*(60*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^3 - 60*log(abs(tan(1/2*d*x +
 1/2*c) - 1))/a^3 - (3*a^12*tan(1/2*d*x + 1/2*c)^5 + 20*a^12*tan(1/2*d*x +
 1/2*c)^3 + 105*a^12*tan(1/2*d*x + 1/2*c))/a^15)/d
```

3.63.9 Mupad [B] (verification not implemented)

Time = 12.39 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.55

$$\int \frac{\sec^4(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{105\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-120\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)+20\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3+3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5}{60\,a^3\,d}$$

input
```
int(1/(cos(c + d*x)^4*(a + a/cos(c + d*x))^3),x)
```

output
```
-(105*tan(c/2 + (d*x)/2) - 120*atanh(tan(c/2 + (d*x)/2)) + 20*tan(c/2 + (d
*x)/2)^3 + 3*tan(c/2 + (d*x)/2)^5)/(60*a^3*d)
```

3.64 $\int \frac{\sec^3(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

3.64.1 Optimal result 685
3.64.2 Mathematica [A] (verified) 685
3.64.3 Rubi [A] (verified) 686
3.64.4 Maple [A] (verified) 688
3.64.5 Fricas [A] (verification not implemented) 688
3.64.6 Sympy [F] 689
3.64.7 Maxima [A] (verification not implemented) 689
3.64.8 Giac [A] (verification not implemented) 689
3.64.9 Mupad [B] (verification not implemented) 690

3.64.1 Optimal result

Integrand size = 21, antiderivative size = 83

$$\int \frac{\sec^3(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\tan(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{8\tan(c+dx)}{15ad(a+a\sec(c+dx))^2} + \frac{7\tan(c+dx)}{15d\left(a^3+a^3\sec(c+dx)\right)}$$

output
```
1/5*tan(d*x+c)/d/(a+a*sec(d*x+c))^3-8/15*tan(d*x+c)/a/d/(a+a*sec(d*x+c))^2
+7/15*tan(d*x+c)/d/(a^3+a^3*sec(d*x+c))
```

3.64.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.69

$$\int \frac{\sec^3(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\sec^5\left(\frac{1}{2}(c+dx)\right)\left(10\sin\left(\frac{1}{2}(c+dx)\right)+5\sin\left(\frac{3}{2}(c+dx)\right)+\sin\left(\frac{5}{2}(c+dx)\right)\right)}{120a^3d}$$

input
```
Integrate[Sec[c + d*x]^3/(a + a*Sec[c + d*x])^3,x]
```

output
```
(Sec[(c + d*x)/2]^5*(10*Sin[(c + d*x)/2] + 5*Sin[(3*(c + d*x))/2] + Sin[(5
*(c + d*x))/2]))/(120*a^3*d)
```

3.64.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.02, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 4286, 25, 3042, 4488, 3042, 4281}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^3(c+dx)}{(a\sec(c+dx)+a)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^3}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}dx$$

$$\Big\downarrow \text{4286}$$

$$\frac{\int -\frac{\sec(c+dx)(3a-5a\sec(c+dx))}{(\sec(c+dx)a+a)^2}dx}{5a^2}+\frac{\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{25}$$

$$\frac{\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}-\frac{\int \frac{\sec(c+dx)(3a-5a\sec(c+dx))}{(\sec(c+dx)a+a)^2}dx}{5a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}-\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(3a-5a\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}dx}{5a^2}$$

$$\Big\downarrow \text{4488}$$

$$\frac{\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}-\frac{\frac{8a\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}-\frac{7}{3}\int \frac{\sec(c+dx)}{\sec(c+dx)a+a}dx}{5a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}-\frac{\frac{8a\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}-\frac{7}{3}\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{5a^2}$$

$$\Big\downarrow \text{4281}$$

$$\frac{\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}-\frac{\frac{8a\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}-\frac{7\tan(c+dx)}{3d(a\sec(c+dx)+a)}}{5a^2}$$

input
```
Int[Sec[c + d*x]^3/(a + a*Sec[c + d*x])^3,x]
```

output
```
Tan[c + d*x]/(5*d*(a + a*Sec[c + d*x])^3) - ((8*a*Tan[c + d*x])/(3*d*(a +
a*Sec[c + d*x])^2) - (7*Tan[c + d*x])/(3*d*(a + a*Sec[c + d*x])))/(5*a^2)
```

3.64.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4281
```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[-Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

rule 4286
```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[b*Cot[e + f*x]*((a + b*Csc[e + f*x])^m/(a*f*(2*m + 1))),
x] - Simp[1/(a^2*(2*m + 1))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
*(a*m - b*(2*m + 1)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[
a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

rule 4488
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b - a*B)*Cot[e +
f*x]*((a + b*Csc[e + f*x])^m/(a*f*(2*m + 1))), x] + Simp[(a*B*m + A*b*(m +
1))/(a*b*(2*m + 1))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x], x]
 /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[a*B*m + A*b*(m + 1), 0] && LtQ[m, -2^(-1)]
```

3.64.4 Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.54

method	result	size
derivativedivides	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{5}+\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{3}+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d\,a^{3}}$	45
default	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{5}+\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{3}+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d\,a^{3}}$	45
risch	$\frac{4i\left(10\,\mathrm{e}^{2i(dx+c)}+5\,\mathrm{e}^{i(dx+c)}+1\right)}{15d\,a^{3}\left(\mathrm{e}^{i(dx+c)}+1\right)^{5}}$	47
parallelrisch	$\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}+10\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}+15\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{60a^{3}d}$	47
norman	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4ad}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{3ad}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{30ad}+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{15ad}+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{9}}{20ad}}{\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)^{2}a^{2}}$	114

input
```
int(sec(d*x+c)^3/(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/4/d/a^3*(1/5*tan(1/2*d*x+1/2*c)^5+2/3*tan(1/2*d*x+1/2*c)^3+tan(1/2*d*x+1
/2*c))
```

3.64.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.90

$$\int \frac{\sec^3(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\left(2\,\cos\left(dx+c\right)^2+6\,\cos\left(dx+c\right)+7\right)\sin\left(dx+c\right)}{15\left(a^3d\cos\left(dx+c\right)^3+3\,a^3d\cos\left(dx+c\right)^2+3\,a^3d\cos\left(dx+c\right)+a^3d\right)}$$

input
```
integrate(sec(d*x+c)^3/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/15*(2*cos(d*x + c)^2 + 6*cos(d*x + c) + 7)*sin(d*x + c)/(a^3*d*cos(d*x +
 c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

3.64.6 Sympy [F]

$$\int \frac{\sec^3(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\int \frac{\sec^3(c+dx)}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx}{a^3}$$

input
```
integrate(sec(d*x+c)**3/(a+a*sec(d*x+c))**3,x)
```

output
```
Integral(sec(c + d*x)**3/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c +
d*x) + 1), x)/a**3
```

3.64.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.81

$$\int \frac{\sec^3(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\frac{15\,\sin(dx+c)}{\cos(dx+c)+1} + \frac{10\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{3\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{60\,a^3d}$$

input
```
integrate(sec(d*x+c)^3/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/60*(15*sin(d*x + c)/(cos(d*x + c) + 1) + 10*sin(d*x + c)^3/(cos(d*x + c)
 + 1)^3 + 3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/(a^3*d)
```

3.64.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.55

$$\int \frac{\sec^3(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{3\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5 + 10\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 + 15\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{60\,a^3d}$$

input
```
integrate(sec(d*x+c)^3/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
1/60*(3*tan(1/2*d*x + 1/2*c)^5 + 10*tan(1/2*d*x + 1/2*c)^3 + 15*tan(1/2*d*
x + 1/2*c))/(a^3*d)
```

3.64.9 Mupad [B] (verification not implemented)

Time = 12.09 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.54

$$\int \frac{\sec^3(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\,\left(3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+10\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+15\right)}{60\,a^3\,d}$$

input
```
int(1/(cos(c + d*x)^3*(a + a/cos(c + d*x))^3),x)
```

output
```
(tan(c/2 + (d*x)/2)*(10*tan(c/2 + (d*x)/2)^2 + 3*tan(c/2 + (d*x)/2)^4 + 15
))/(60*a^3*d)
```

3.65 $\int \frac{\sec^2(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

3.65.1 Optimal result . . . 691
3.65.2 Mathematica [A] (verified) . . . 691
3.65.3 Rubi [A] (verified) . . . 692
3.65.4 Maple [A] (verified) . . . 693
3.65.5 Fricas [A] (verification not implemented) . . . 694
3.65.6 Sympy [F] . . . 695
3.65.7 Maxima [A] (verification not implemented) . . . 695
3.65.8 Giac [A] (verification not implemented) . . . 695
3.65.9 Mupad [B] (verification not implemented) . . . 696

3.65.1 Optimal result

Integrand size = 21, antiderivative size = 83

$$\int \frac{\sec^2(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{\tan(c+dx)}{5d(a+a\sec(c+dx))^3}+\frac{\tan(c+dx)}{5ad(a+a\sec(c+dx))^2}+\frac{\tan(c+dx)}{5d\left(a^3+a^3\sec(c+dx)\right)}$$

output
```
-1/5*tan(d*x+c)/d/(a+a*sec(d*x+c))^3+1/5*tan(d*x+c)/a/d/(a+a*sec(d*x+c))^2
+1/5*tan(d*x+c)/d/(a^3+a^3*sec(d*x+c))
```

3.65.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.53

$$\int \frac{\sec^2(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{(1+3\sec(c+dx)+\sec^2(c+dx))\tan(c+dx)}{5a^3d(1+\sec(c+dx))^3}$$

input
```
Integrate[Sec[c + d*x]^2/(a + a*Sec[c + d*x])^3,x]
```

output
```
((1 + 3*Sec[c + d*x] + Sec[c + d*x]^2)*Tan[c + d*x])/(5*a^3*d*(1 + Sec[c +
 d*x])^3)
```

3.65.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3042, 4284, 3042, 4283, 3042, 4281}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(c+dx)}{(a\sec(c+dx)+a)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}dx$$

$$\Big\downarrow \text{4284}$$

$$\frac{3\int \frac{\sec(c+dx)}{(\sec(c+dx)a+a)^2}dx}{5a}-\frac{\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}dx}{5a}-\frac{\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{4283}$$

$$\frac{3\left(\frac{\int \frac{\sec(c+dx)}{\sec(c+dx)a+a}dx}{3a}+\frac{\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}\right)}{5a}-\frac{\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\left(\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{3a}+\frac{\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}\right)}{5a}-\frac{\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{4281}$$

$$\frac{3\left(\frac{\tan(c+dx)}{3ad(a\sec(c+dx)+a)}+\frac{\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}\right)}{5a}-\frac{\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

input
```
Int[Sec[c + d*x]^2/(a + a*Sec[c + d*x])^3,x]
```

output
```
-1/5*Tan[c + d*x]/(d*(a + a*Sec[c + d*x])^3) + (3*(Tan[c + d*x]/(3*d*(a +
a*Sec[c + d*x])^2) + Tan[c + d*x]/(3*a*d*(a + a*Sec[c + d*x]))))/(5*a)
```

3.65.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4281
```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[-Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

rule 4283
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[b*Cot[e + f*x]*((a + b*Csc[e + f*x])^m/(a*f*(2*m + 1))), x]
 + Simp[(m + 1)/(a*(2*m + 1))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m +
1), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)
] && IntegerQ[2*m]
```

rule 4284
```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(-Cot[e + f*x])*((a + b*Csc[e + f*x])^m/(f*(2*m + 1))), x
] + Simp[m/(b*(2*m + 1))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x
], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

3.65.4 Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.37

method	result	size
parallelrisch	$-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}-5\right)}{20a^{3}d}$	31
derivativedivides	$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{5}+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d\,a^{3}}$	32
default	$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{5}+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d\,a^{3}}$	32
risch	$\frac{2i\left(5\,\mathrm{e}^{3i(dx+c)}+5\,\mathrm{e}^{2i(dx+c)}+5\,\mathrm{e}^{i(dx+c)}+1\right)}{5d\,a^{3}\left(\mathrm{e}^{i(dx+c)}+1\right)^{5}}$	58
norman	$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4ad}+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{4ad}+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{20ad}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{20ad}}{\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)a^{2}}$	95

input
```
int(sec(d*x+c)^2/(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
-1/20*tan(1/2*d*x+1/2*c)*(tan(1/2*d*x+1/2*c)^4-5)/a^3/d
```

3.65.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.88

$$\int \frac{\sec^2(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\left(\cos\left(dx+c\right)^2+3\,\cos\left(dx+c\right)+1\right)\sin\left(dx+c\right)}{5\left(a^3d\cos\left(dx+c\right)^3+3\,a^3d\cos\left(dx+c\right)^2+3\,a^3d\cos\left(dx+c\right)+a^3d\right)}$$

input
```
integrate(sec(d*x+c)^2/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/5*(cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*sin(d*x + c)/(a^3*d*cos(d*x + c)
^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

3.65.6 Sympy [F]

$$\int \frac{\sec^2(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\int \frac{\sec^2(c+dx)}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx}{a^3}$$

input
```
integrate(sec(d*x+c)**2/(a+a*sec(d*x+c))**3,x)
```

output
```
Integral(sec(c + d*x)**2/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c +
d*x) + 1), x)/a**3
```

3.65.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.57

$$\int \frac{\sec^2(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\frac{5\,\sin(dx+c)}{\cos(dx+c)+1} - \frac{\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{20\,a^3 d}$$

input
```
integrate(sec(d*x+c)^2/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/20*(5*sin(d*x + c)/(cos(d*x + c) + 1) - sin(d*x + c)^5/(cos(d*x + c) + 1
)^5)/(a^3*d)
```

3.65.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.37

$$\int \frac{\sec^2(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5 - 5\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{20\,a^3 d}$$

input
```
integrate(sec(d*x+c)^2/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
-1/20*(tan(1/2*d*x + 1/2*c)^5 - 5*tan(1/2*d*x + 1/2*c))/(a^3*d)
```

3.65.9 Mupad [B] (verification not implemented)

Time = 13.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.36

$$\int \frac{\sec^2(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\,\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4-5\right)}{20\,a^3\,d}$$

input
```
int(1/(cos(c + d*x)^2*(a + a/cos(c + d*x))^3),x)
```

output
```
-(tan(c/2 + (d*x)/2)*(tan(c/2 + (d*x)/2)^4 - 5))/(20*a^3*d)
```

3.66 $\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

3.66.1 Optimal result . . . 697
3.66.2 Mathematica [A] (verified) . . . 697
3.66.3 Rubi [A] (verified) . . . 698
3.66.4 Maple [A] (verified) . . . 699
3.66.5 Fricas [A] (verification not implemented) . . . 700
3.66.6 Sympy [F] . . . 700
3.66.7 Maxima [A] (verification not implemented) . . . 701
3.66.8 Giac [A] (verification not implemented) . . . 701
3.66.9 Mupad [B] (verification not implemented) . . . 701

3.66.1 Optimal result

Integrand size = 19, antiderivative size = 83

$$\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\tan(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{2\tan(c+dx)}{15ad(a+a\sec(c+dx))^2} + \frac{2\tan(c+dx)}{15d\left(a^3+a^3\sec(c+dx)\right)}$$

output
```
1/5*tan(d*x+c)/d/(a+a*sec(d*x+c))^3+2/15*tan(d*x+c)/a/d/(a+a*sec(d*x+c))^2
+2/15*tan(d*x+c)/d/(a^3+a^3*sec(d*x+c))
```

3.66.2 Mathematica [A] (verified)

Time = 0.58 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.04

$$\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\sec\left(\frac{c}{2}\right)\sec^5\left(\frac{1}{2}(c+dx)\right)\left(40\sin\left(\frac{dx}{2}\right)-30\sin\left(c+\frac{dx}{2}\right)+20\sin\left(c+\frac{3dx}{2}\right)-15\sin\left(2c+\frac{3dx}{2}\right)+7\sin\left(2c+\right.\right.}{240a^3d}$$

input
```
Integrate[Sec[c + d*x]/(a + a*Sec[c + d*x])^3,x]
```

output
```
(Sec[c/2]*Sec[(c + d*x)/2]^5*(40*Sin[(d*x)/2] - 30*Sin[c + (d*x)/2] + 20*S
in[c + (3*d*x)/2] - 15*Sin[2*c + (3*d*x)/2] + 7*Sin[2*c + (5*d*x)/2]))/(24
0*a^3*d)
```

3.66.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {3042, 4283, 3042, 4283, 3042, 4281}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(c+dx)}{(a\sec(c+dx)+a)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}dx$$

$$\Big\downarrow \text{4283}$$

$$\frac{2\int \frac{\sec(c+dx)}{(\sec(c+dx)a+a)^2}dx}{5a}+\frac{\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}dx}{5a}+\frac{\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{4283}$$

$$\frac{2\left(\frac{\int \frac{\sec(c+dx)}{\sec(c+dx)a+a}dx}{3a}+\frac{\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}\right)}{5a}+\frac{\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\left(\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{3a}+\frac{\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}\right)}{5a}+\frac{\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{4281}$$

$$\frac{\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}+\frac{2\left(\frac{\tan(c+dx)}{3ad(a\sec(c+dx)+a)}+\frac{\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}\right)}{5a}$$

input
```
Int[Sec[c + d*x]/(a + a*Sec[c + d*x])^3,x]
```

output
```
Tan[c + d*x]/(5*d*(a + a*Sec[c + d*x])^3) + (2*(Tan[c + d*x]/(3*d*(a + a*S
ec[c + d*x])^2) + Tan[c + d*x]/(3*a*d*(a + a*Sec[c + d*x]))))/(5*a)
```

3.66.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4281
```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[-Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

rule 4283
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[b*Cot[e + f*x]*((a + b*Csc[e + f*x])^m/(a*f*(2*m + 1))), x]
 + Simp[(m + 1)/(a*(2*m + 1))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m +
1), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)
] && IntegerQ[2*m]
```

3.66.4 Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.54

method	result	size
derivativedivides	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{5}-\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{3}+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d\,a^{3}}$	45
default	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{5}-\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{3}+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d\,a^{3}}$	45
parallelrisch	$\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}-10\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}+15\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{60a^{3}d}$	47
norman	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4ad}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{6ad}+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{20ad}}{a^{2}}$	61
risch	$\frac{2i\left(15\,\mathrm{e}^{4i(dx+c)}+30\,\mathrm{e}^{3i(dx+c)}+40\,\mathrm{e}^{2i(dx+c)}+20\,\mathrm{e}^{i(dx+c)}+7\right)}{15d\,a^{3}\left(\mathrm{e}^{i(dx+c)}+1\right)^{5}}$	69

input
```
int(sec(d*x+c)/(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/4/d/a^3*(1/5*tan(1/2*d*x+1/2*c)^5-2/3*tan(1/2*d*x+1/2*c)^3+tan(1/2*d*x+1
/2*c))
```

3.66.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.90

$$\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\left(7\cos\left(dx+c\right)^2+6\cos\left(dx+c\right)+2\right)\sin\left(dx+c\right)}{15\left(a^3d\cos\left(dx+c\right)^3+3\,a^3d\cos\left(dx+c\right)^2+3\,a^3d\cos\left(dx+c\right)+a^3d\right)}$$

input
```
integrate(sec(d*x+c)/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/15*(7*cos(d*x + c)^2 + 6*cos(d*x + c) + 2)*sin(d*x + c)/(a^3*d*cos(d*x +
 c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

3.66.6 Sympy [F]

$$\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\int \frac{\sec\left(c+dx\right)}{\sec^3\left(c+dx\right)+3\sec^2\left(c+dx\right)+3\sec\left(c+dx\right)+1}\,dx}{a^3}$$

input
```
integrate(sec(d*x+c)/(a+a*sec(d*x+c))**3,x)
```

output
```
Integral(sec(c + d*x)/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c + d*x
) + 1), x)/a**3
```

3.66.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.81

$$\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\frac{15\,\sin(dx+c)}{\cos(dx+c)+1} - \frac{10\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{3\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{60\,a^3d}$$

input
```
integrate(sec(d*x+c)/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/60*(15*sin(d*x + c)/(cos(d*x + c) + 1) - 10*sin(d*x + c)^3/(cos(d*x + c)
 + 1)^3 + 3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/(a^3*d)
```

3.66.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.55

$$\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{3\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5 - 10\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 + 15\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{60\,a^3d}$$

input
```
integrate(sec(d*x+c)/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
1/60*(3*tan(1/2*d*x + 1/2*c)^5 - 10*tan(1/2*d*x + 1/2*c)^3 + 15*tan(1/2*d*
x + 1/2*c))/(a^3*d)
```

3.66.9 Mupad [B] (verification not implemented)

Time = 13.16 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.54

$$\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4 - 10\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2 + 15\right)}{60\,a^3\,d}$$

input
```
int(1/(cos(c + d*x)*(a + a/cos(c + d*x))^3),x)
```

output
```
(tan(c/2 + (d*x)/2)*(3*tan(c/2 + (d*x)/2)^4 - 10*tan(c/2 + (d*x)/2)^2 + 15
))/(60*a^3*d)
```

3.67 $\int \frac{1}{(a+a\sec(c+dx))^3}\,dx$

3.67.1 Optimal result . . . 702
3.67.2 Mathematica [A] (verified) . . . 702
3.67.3 Rubi [A] (verified) . . . 703
3.67.4 Maple [A] (verified) . . . 705
3.67.5 Fricas [A] (verification not implemented) . . . 706
3.67.6 Sympy [F] . . . 706
3.67.7 Maxima [A] (verification not implemented) . . . 706
3.67.8 Giac [A] (verification not implemented) . . . 707
3.67.9 Mupad [B] (verification not implemented) . . . 707

3.67.1 Optimal result

Integrand size = 12, antiderivative size = 88

$$\int \frac{1}{(a+a\sec(c+dx))^3}\,dx = \frac{x}{a^3} - \frac{\tan(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{7\tan(c+dx)}{15ad(a+a\sec(c+dx))^2} - \frac{22\tan(c+dx)}{15d\left(a^3+a^3\sec(c+dx)\right)}$$

output
```
x/a^3-1/5*tan(d*x+c)/d/(a+a*sec(d*x+c))^3-7/15*tan(d*x+c)/a/d/(a+a*sec(d*x
+c))^2-22/15*tan(d*x+c)/d/(a^3+a^3*sec(d*x+c))
```

3.67.2 Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.84

$$\int \frac{1}{(a+a\sec(c+dx))^3}\,dx = \frac{2\cos\left(\frac{1}{2}(c+dx)\right)\sec^3(c+dx)\left(60dx\cos^5\left(\frac{1}{2}(c+dx)\right)-3\sec\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)+26\cos^2\left(\frac{1}{2}(c+dx)\right)\sec\left(\frac{c}{2}\right)\sin\right.}{15a^3d(1+\sec}$$

input
```
Integrate[(a + a*Sec[c + d*x])^(-3),x]
```

output
```
(2*Cos[(c + d*x)/2]*Sec[c + d*x]^3*(60*d*x*Cos[(c + d*x)/2]^5 - 3*Sec[c/2]
*Sin[(d*x)/2] + 26*Cos[(c + d*x)/2]^2*Sec[c/2]*Sin[(d*x)/2] - 128*Cos[(c +
 d*x)/2]^4*Sec[c/2]*Sin[(d*x)/2] - 3*Cos[(c + d*x)/2]*Tan[c/2] + 26*Cos[(c
 + d*x)/2]^3*Tan[c/2]))/(15*a^3*d*(1 + Sec[c + d*x])^3)
```

3.67.3 Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.11, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.833, Rules used = {3042, 4264, 25, 3042, 4410, 25, 3042, 4407, 3042, 4281}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\sec(c+dx)+a)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}dx$$

$$\Big\downarrow \text{4264}$$

$$-\frac{\int -\frac{5a-2a\sec(c+dx)}{(\sec(c+dx)a+a)^2}dx}{5a^2}-\frac{\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{5a-2a\sec(c+dx)}{(\sec(c+dx)a+a)^2}dx}{5a^2}-\frac{\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{5a-2a\csc\left(c+dx+\frac{\pi}{2}\right)}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}dx}{5a^2}-\frac{\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{4410}$$

$$\frac{-\frac{\int -\frac{15a^2-7a^2\sec(c+dx)}{\sec(c+dx)a+a}dx}{3a^2}-\frac{7a\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}}{5a^2}-\frac{\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{15a^2-7a^2\sec(c+dx)}{\sec(c+dx)a+a}dx}{3a^2}-\frac{7a\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}}{5a^2}-\frac{\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\int \frac{15a^2-7a^2\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{3a^2}-\frac{7a\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}}{5a^2}-\frac{\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{4407}$$

$$\frac{\frac{15ax-22a^2\int \frac{\sec(c+dx)}{\sec(c+dx)a+a}dx}{3a^2}-\frac{7a\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}}{5a^2}-\frac{\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{15ax-22a^2\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{3a^2}-\frac{7a\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}}{5a^2}-\frac{\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{4281}$$

$$\frac{\frac{15ax-\frac{22a^2\tan(c+dx)}{d(a\sec(c+dx)+a)}}{3a^2}-\frac{7a\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}}{5a^2}-\frac{\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

input
```
Int[(a + a*Sec[c + d*x])^(-3),x]
```

output
```
-1/5*Tan[c + d*x]/(d*(a + a*Sec[c + d*x])^3) + ((-7*a*Tan[c + d*x])/(3*d*(
a + a*Sec[c + d*x])^2) + (15*a*x - (22*a^2*Tan[c + d*x])/(d*(a + a*Sec[c +
 d*x])))/(3*a^2))/(5*a^2)
```

3.67.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4264
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(-Cot[c
+ d*x])*((a + b*Csc[c + d*x])^n/(d*(2*n + 1))), x] + Simp[1/(a^2*(2*n + 1))
   Int[(a + b*Csc[c + d*x])^(n + 1)*(a*(2*n + 1) - b*(n + 1)*Csc[c + d*x]),
 x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LeQ[n, -1] && Int
egerQ[2*n]
```

rule 4281
```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[-Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

rule 4407
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Simp[(b*c - a*d)/a   Int[Csc[e + f*
x]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
- a*d, 0]
```

rule 4410
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> Simp[(-(b*c - a*d))*Cot[e + f*x]*((a + b*Csc[e +
f*x])^m/(b*f*(2*m + 1))), x] + Simp[1/(a^2*(2*m + 1))   Int[(a + b*Csc[e +
f*x])^(m + 1)*Simp[a*c*(2*m + 1) - (b*c - a*d)*(m + 1)*Csc[e + f*x], x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&
EqQ[a^2 - b^2, 0] && IntegerQ[2*m]
```

3.67.4 Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.58

method	result	size
parallelrisch	$\frac{-3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}+20\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}+60dx-105\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{60a^{3}d}$	51
derivativedivides	$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{5}+\frac{4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{3}-7\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+8\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d\,a^{3}}$	59
default	$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{5}+\frac{4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{3}-7\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+8\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d\,a^{3}}$	59
norman	$\frac{\frac{x}{a}-\frac{7\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4ad}+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{3ad}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{20ad}}{a^{2}}$	66
risch	$\frac{x}{a^{3}}-\frac{2i\left(45\,\mathrm{e}^{4i(dx+c)}+135\,\mathrm{e}^{3i(dx+c)}+185\,\mathrm{e}^{2i(dx+c)}+115\,\mathrm{e}^{i(dx+c)}+32\right)}{15d\,a^{3}\left(\mathrm{e}^{i(dx+c)}+1\right)^{5}}$	75

input
```
int(1/(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/60*(-3*tan(1/2*d*x+1/2*c)^5+20*tan(1/2*d*x+1/2*c)^3+60*d*x-105*tan(1/2*d
*x+1/2*c))/a^3/d
```

3.67.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.32

$$\int \frac{1}{(a+a\sec(c+dx))^3}\,dx = \frac{15\,dx\cos(dx+c)^3+45\,dx\cos(dx+c)^2+45\,dx\cos(dx+c)+15\,dx-\left(32\,\cos(dx+c)^2+51\,\cos(dx+\right.}{15\left(a^3d\cos(dx+c)^3+3\,a^3d\cos(dx+c)^2+3\,a^3d\cos(dx+c)+a^3d\right)}$$

input
```
integrate(1/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/15*(15*d*x*cos(d*x + c)^3 + 45*d*x*cos(d*x + c)^2 + 45*d*x*cos(d*x + c)
+ 15*d*x - (32*cos(d*x + c)^2 + 51*cos(d*x + c) + 22)*sin(d*x + c))/(a^3*d
*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

3.67.6 Sympy [F]

$$\int \frac{1}{(a+a\sec(c+dx))^3}\,dx = \frac{\int \frac{1}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx}{a^3}$$

input
```
integrate(1/(a+a*sec(d*x+c))**3,x)
```

output
```
Integral(1/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c + d*x) + 1), x)/
a**3
```

3.67.7 Maxima [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.05

$$\int \frac{1}{(a+a\sec(c+dx))^3}\,dx = -\frac{\frac{\frac{105\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{20\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{a^3}-\frac{120\,\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}}{60\,d}$$

input
```
integrate(1/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
-1/60*((105*sin(d*x + c)/(cos(d*x + c) + 1) - 20*sin(d*x + c)^3/(cos(d*x +
 c) + 1)^3 + 3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^3 - 120*arctan(sin(d
*x + c)/(cos(d*x + c) + 1))/a^3)/d
```

3.67.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.77

$$\int \frac{1}{(a+a\sec(c+dx))^3}\,dx = \frac{\frac{60\,(dx+c)}{a^3} - \frac{3\,a^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5 - 20\,a^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 + 105\,a^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^{15}}}{60\,d}$$

input
```
integrate(1/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
1/60*(60*(d*x + c)/a^3 - (3*a^12*tan(1/2*d*x + 1/2*c)^5 - 20*a^12*tan(1/2*
d*x + 1/2*c)^3 + 105*a^12*tan(1/2*d*x + 1/2*c))/a^15)/d
```

3.67.9 Mupad [B] (verification not implemented)

Time = 13.26 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.92

$$\int \frac{1}{(a+a\sec(c+dx))^3}\,dx = \frac{x}{a^3} - \frac{\frac{32\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{15} - \frac{13\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{30} + \frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{20}}{a^3\,d\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}$$

input
```
int(1/(a + a/cos(c + d*x))^3,x)
```

output
```
x/a^3 - (sin(c/2 + (d*x)/2)/20 - (13*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/
2))/30 + (32*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2))/15)/(a^3*d*cos(c/2 +
 (d*x)/2)^5)
```

3.68 $\int \frac{\cos(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

3.68.1 Optimal result 708
3.68.2 Mathematica [A] (verified) 708
3.68.3 Rubi [A] (verified) 709
3.68.4 Maple [A] (verified) 712
3.68.5 Fricas [A] (verification not implemented) 712
3.68.6 Sympy [F] 713
3.68.7 Maxima [A] (verification not implemented) 713
3.68.8 Giac [A] (verification not implemented) 713
3.68.9 Mupad [B] (verification not implemented) 714

3.68.1 Optimal result

Integrand size = 19, antiderivative size = 103

$$\int \frac{\cos(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{3x}{a^3}+\frac{24\sin(c+dx)}{5a^3d}-\frac{\sin(c+dx)}{5d(a+a\sec(c+dx))^3}-\frac{3\sin(c+dx)}{5ad(a+a\sec(c+dx))^2}-\frac{3\sin(c+dx)}{d\left(a^3+a^3\sec(c+dx)\right)}$$

output
```
-3*x/a^3+24/5*sin(d*x+c)/a^3/d-1/5*sin(d*x+c)/d/(a+a*sec(d*x+c))^3-3/5*sin
(d*x+c)/a/d/(a+a*sec(d*x+c))^2-3*sin(d*x+c)/d/(a^3+a^3*sec(d*x+c))
```

3.68.2 Mathematica [A] (verified)

Time = 1.05 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.04

$$\int \frac{\cos(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\sin(c+dx)\left(120\arcsin(\cos(c+dx))\cos^6\left(\frac{1}{2}(c+dx)\right)+(24+57\cos(c+dx)+39\cos^2(c+dx)+5\cos^3(c\right.}{5a^3d\sqrt{1-\cos(c+dx)}(1+\cos(c+dx))^{7/2}}$$

input
```
Integrate[Cos[c + d*x]/(a + a*Sec[c + d*x])^3,x]
```

output
```
(Sin[c + d*x]*(120*ArcSin[Cos[c + d*x]]*Cos[(c + d*x)/2]^6 + (24 + 57*Cos[
c + d*x] + 39*Cos[c + d*x]^2 + 5*Cos[c + d*x]^3)*Sqrt[Sin[c + d*x]^2]))/(5
*a^3*d*Sqrt[1 - Cos[c + d*x]]*(1 + Cos[c + d*x])^(7/2))
```

3.68.3 Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.11, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.684, Rules used = {3042, 4304, 27, 3042, 4508, 27, 3042, 4508, 3042, 4274, 24, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(c+dx)}{(a\sec(c+dx)+a)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}dx$$

$$\Big\downarrow \text{4304}$$

$$-\frac{\int -\frac{3\cos(c+dx)(2a-a\sec(c+dx))}{(\sec(c+dx)a+a)^2}dx}{5a^2}-\frac{\sin(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{3\int \frac{\cos(c+dx)(2a-a\sec(c+dx))}{(\sec(c+dx)a+a)^2}dx}{5a^2}-\frac{\sin(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\int \frac{2a-a\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}dx}{5a^2}-\frac{\sin(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{4508}$$

$$\frac{3\left(\frac{\int \frac{3\cos(c+dx)\left(3a^2-2a^2\sec(c+dx)\right)}{\sec(c+dx)a+a}dx}{3a^2}-\frac{a\sin(c+dx)}{d(a\sec(c+dx)+a)^2}\right)}{5a^2}-\frac{\sin(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{3\left(\frac{\int \frac{\cos(c+dx)\left(3a^2-2a^2\sec(c+dx)\right)}{\sec(c+dx)a+a}dx}{a^2}-\frac{a\sin(c+dx)}{d(a\sec(c+dx)+a)^2}\right)}{5a^2}-\frac{\sin(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\left(\frac{\int \frac{3a^2-2a^2\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)}dx}{a^2}-\frac{a\sin(c+dx)}{d(a\sec(c+dx)+a)^2}\right)}{5a^2}-\frac{\sin(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$\downarrow$ 4508

$$\frac{3\left(\frac{\frac{\int \cos(c+dx)\left(8a^3-5a^3\sec(c+dx)\right)dx}{a^2}-\frac{5a^2\sin(c+dx)}{d(a\sec(c+dx)+a)}}{a^2}-\frac{a\sin(c+dx)}{d(a\sec(c+dx)+a)^2}\right)}{5a^2}-\frac{\sin(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$\downarrow$ 3042

$$\frac{3\left(\frac{\frac{\int \frac{8a^3-5a^3\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}-\frac{5a^2\sin(c+dx)}{d(a\sec(c+dx)+a)}}{a^2}-\frac{a\sin(c+dx)}{d(a\sec(c+dx)+a)^2}\right)}{5a^2}-\frac{\sin(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$\downarrow$ 4274

$$\frac{3\left(\frac{\frac{8a^3\int \cos(c+dx)dx-5a^3\int 1dx}{a^2}-\frac{5a^2\sin(c+dx)}{d(a\sec(c+dx)+a)}}{a^2}-\frac{a\sin(c+dx)}{d(a\sec(c+dx)+a)^2}\right)}{5a^2}-\frac{\sin(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$\downarrow$ 24

$$\frac{3\left(\frac{\frac{8a^3\int \cos(c+dx)dx-5a^3x}{a^2}-\frac{5a^2\sin(c+dx)}{d(a\sec(c+dx)+a)}}{a^2}-\frac{a\sin(c+dx)}{d(a\sec(c+dx)+a)^2}\right)}{5a^2}-\frac{\sin(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$\downarrow$ 3042

$$\frac{3\left(\frac{\frac{8a^3\int \sin\left(c+dx+\frac{\pi}{2}\right)dx-5a^3x}{a^2}-\frac{5a^2\sin(c+dx)}{d(a\sec(c+dx)+a)}}{a^2}-\frac{a\sin(c+dx)}{d(a\sec(c+dx)+a)^2}\right)}{5a^2}-\frac{\sin(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$\downarrow$ 3117

$$\frac{3\left(\frac{\frac{8a^3\frac{\sin(c+dx)}{d}-5a^3x}{a^2}-\frac{5a^2\sin(c+dx)}{d(a\sec(c+dx)+a)}}{a^2}-\frac{a\sin(c+dx)}{d(a\sec(c+dx)+a)^2}\right)}{5a^2}-\frac{\sin(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

input
```
Int[Cos[c + d*x]/(a + a*Sec[c + d*x])^3,x]
```

3.68. $\int \frac{\cos(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

output
```
-1/5*Sin[c + d*x]/(d*(a + a*Sec[c + d*x])^3) + (3*(-((a*Sin[c + d*x])/(d*(
a + a*Sec[c + d*x])^2)) + ((-5*a^2*Sin[c + d*x])/(d*(a + a*Sec[c + d*x]))
+ (-5*a^3*x + (8*a^3*Sin[c + d*x])/d)/a^2)/a^2))/(5*a^2)
```

3.68.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4304
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-Cot[e + f*x])*(a + b*Csc[e + f*x])^m*((d*Csc
[e + f*x])^n/(f*(2*m + 1))), x] + Simp[1/(a^2*(2*m + 1))   Int[(a + b*Csc[e
 + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e
+ f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ
[m, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

rule 4508
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
- a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
 1))), x] - Simp[1/(a^2*(2*m + 1))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Cs
c[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[
e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B
, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

3.68.4 Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.64

method	result	size
parallelrisch	$\frac{\frac{243\left(\cos(dx+c)+\frac{26\cos(2dx+2c)}{81}+\frac{5\cos(3dx+3c)}{243}+\frac{58}{81}\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\sec\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{80}-3dx}{a^3d}$	66
derivativedivides	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{5}-2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3+17\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{8\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}-24\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4da^3}$	85
default	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{5}-2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3+17\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{8\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}-24\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4da^3}$	85
risch	$-\frac{3x}{a^3}-\frac{ie^{i(dx+c)}}{2a^3d}+\frac{ie^{-i(dx+c)}}{2a^3d}+\frac{4i\left(15e^{4i(dx+c)}+50e^{3i(dx+c)}+70e^{2i(dx+c)}+45e^{i(dx+c)}+12\right)}{5da^3\left(e^{i(dx+c)}+1\right)^5}$	112
norman	$\frac{-\frac{3x}{a}+\frac{25\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4ad}+\frac{15\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{4ad}-\frac{9\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{20ad}+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{20ad}-\frac{3x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{a}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)a^2}$	118

input
```
int(cos(d*x+c)/(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
3/80*(81*(cos(d*x+c)+26/81*cos(2*d*x+2*c)+5/243*cos(3*d*x+3*c)+58/81)*tan(
1/2*d*x+1/2*c)*sec(1/2*d*x+1/2*c)^4-80*d*x)/a^3/d
```

3.68.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.22

$$\int \frac{\cos(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{15\,dx\cos\left(dx+c\right)^3+45\,dx\cos\left(dx+c\right)^2+45\,dx\cos\left(dx+c\right)+15\,dx-\left(5\,\cos\left(dx+c\right)^3+39\,\cos\left(dx+\right.\right.}{5\left(a^3d\cos\left(dx+c\right)^3+3\,a^3d\cos\left(dx+c\right)^2+3\,a^3d\cos\left(dx+c\right)+a^{3}\right.}$$

input
```
integrate(cos(d*x+c)/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
-1/5*(15*d*x*cos(d*x + c)^3 + 45*d*x*cos(d*x + c)^2 + 45*d*x*cos(d*x + c)
+ 15*d*x - (5*cos(d*x + c)^3 + 39*cos(d*x + c)^2 + 57*cos(d*x + c) + 24)*s
in(d*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(
d*x + c) + a^3*d)
```

3.68.6 Sympy [F]

$$\int \frac{\cos(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\int \frac{\cos(c+dx)}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx}{a^3}$$

input
```
integrate(cos(d*x+c)/(a+a*sec(d*x+c))**3,x)
```

output
```
Integral(cos(c + d*x)/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c + d*x
) + 1), x)/a**3
```

3.68.7 Maxima [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.33

$$\int \frac{\cos(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\frac{40\sin(dx+c)}{\left(a^3+\frac{a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)} + \frac{\frac{85\sin(dx+c)}{\cos(dx+c)+1} - \frac{10\sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{a^3} - \frac{120\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}}{20\,d}$$

input
```
integrate(cos(d*x+c)/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/20*(40*sin(d*x + c)/((a^3 + a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)*(co
s(d*x + c) + 1)) + (85*sin(d*x + c)/(cos(d*x + c) + 1) - 10*sin(d*x + c)^3
/(cos(d*x + c) + 1)^3 + sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^3 - 120*arc
tan(sin(d*x + c)/(cos(d*x + c) + 1))/a^3)/d
```

3.68.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.93

$$\int \frac{\cos(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{\frac{60\,(dx+c)}{a^3} - \frac{40\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)a^3} - \frac{a^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5 - 10\,a^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 + 85\,a^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^{15}}}{20\,d}$$

input
```
integrate(cos(d*x+c)/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
-1/20*(60*(d*x + c)/a^3 - 40*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/2*c)^2
 + 1)*a^3) - (a^12*tan(1/2*d*x + 1/2*c)^5 - 10*a^12*tan(1/2*d*x + 1/2*c)^3
 + 85*a^12*tan(1/2*d*x + 1/2*c))/a^15)/d
```

3.68.9 Mupad [B] (verification not implemented)

Time = 13.22 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.10

$$\int \frac{\cos(c+dx)}{(a+a\sec(c+dx))^3}\,dx$$
$$= \frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)-12\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^2\sin\left(\frac{c}{2}+\frac{dx}{2}\right)+96\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^4\sin\left(\frac{c}{2}+\frac{dx}{2}\right)+40\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^6\sin\left(\frac{c}{2}+\right.}{20\,a^3\,d\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^5}$$

input
```
int(cos(c + d*x)/(a + a/cos(c + d*x))^3,x)
```

output
```
(sin(c/2 + (d*x)/2) - 12*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2) + 96*cos(
c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2) + 40*cos(c/2 + (d*x)/2)^6*sin(c/2 + (d
*x)/2) - 60*cos(c/2 + (d*x)/2)^5*(c + d*x))/(20*a^3*d*cos(c/2 + (d*x)/2)^5
)
```

3.69 $\int \frac{\cos^2(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

3.69.1 Optimal result 715
3.69.2 Mathematica [A] (verified) 715
3.69.3 Rubi [A] (verified) 716
3.69.4 Maple [A] (verified) 719
3.69.5 Fricas [A] (verification not implemented) 720
3.69.6 Sympy [F] 720
3.69.7 Maxima [A] (verification not implemented) 720
3.69.8 Giac [A] (verification not implemented) 721
3.69.9 Mupad [B] (verification not implemented) 721

3.69.1 Optimal result

Integrand size = 21, antiderivative size = 147

$$\int \frac{\cos^2(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{13x}{2a^3} - \frac{152\sin(c+dx)}{15a^3d} + \frac{13\cos(c+dx)\sin(c+dx)}{2a^3d} - \frac{\cos(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{11\cos(c+dx)\sin(c+dx)}{15ad(a+a\sec(c+dx))^2} - \frac{76\cos(c+dx)\sin(c+dx)}{15d\left(a^3+a^3\sec(c+dx)\right)}$$

output
```
13/2*x/a^3-152/15*sin(d*x+c)/a^3/d+13/2*cos(d*x+c)*sin(d*x+c)/a^3/d-1/5*co
s(d*x+c)*sin(d*x+c)/d/(a+a*sec(d*x+c))^3-11/15*cos(d*x+c)*sin(d*x+c)/a/d/(
a+a*sec(d*x+c))^2-76/15*cos(d*x+c)*sin(d*x+c)/d/(a^3+a^3*sec(d*x+c))
```

3.69.2 Mathematica [A] (verified)

Time = 2.08 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.23

$$\int \frac{\cos^2(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{2\cos\left(\frac{1}{2}(c+dx)\right)\sec^3(c+dx)\left(-3\sec\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)+46\cos^2\left(\frac{1}{2}(c+dx)\right)\sec\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)-508\cos^4\left(\frac{1}{2}(c+\right.\right.}{}$$

input
```
Integrate[Cos[c + d*x]^2/(a + a*Sec[c + d*x])^3,x]
```

output
```
(2*Cos[(c + d*x)/2]*Sec[c + d*x]^3*(-3*Sec[c/2]*Sin[(d*x)/2] + 46*Cos[(c +
 d*x)/2]^2*Sec[c/2]*Sin[(d*x)/2] - 508*Cos[(c + d*x)/2]^4*Sec[c/2]*Sin[(d*
x)/2] + 15*Cos[(c + d*x)/2]^5*(26*d*x - 12*Sin[c + d*x] + Sin[2*(c + d*x)]
) - 3*Cos[(c + d*x)/2]*Tan[c/2] + 46*Cos[(c + d*x)/2]^3*Tan[c/2]))/(15*a^3
*d*(1 + Sec[c + d*x])^3)
```

3.69.3 Rubi [A] (verified)

Time = 0.93 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.10, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.619$, Rules used = {3042, 4304, 25, 3042, 4508, 3042, 4508, 3042, 4274, 3042, 3115, 24, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^2(c+dx)}{(a\sec(c+dx)+a)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^2\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}dx$$

$$\Big\downarrow \text{4304}$$

$$-\frac{\int -\frac{\cos^2(c+dx)(7a-4a\sec(c+dx))}{(\sec(c+dx)a+a)^2}dx}{5a^2}-\frac{\sin(c+dx)\cos(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\cos^2(c+dx)(7a-4a\sec(c+dx))}{(\sec(c+dx)a+a)^2}dx}{5a^2}-\frac{\sin(c+dx)\cos(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{7a-4a\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^2\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}dx}{5a^2}-\frac{\sin(c+dx)\cos(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{4508}$$

$$\frac{\frac{\int \frac{\cos^2(c+dx)\left(43a^2-33a^2\sec(c+dx)\right)}{\sec(c+dx)a+a}dx}{3a^2}-\frac{11a\sin(c+dx)\cos(c+dx)}{3d(a\sec(c+dx)+a)^2}}{5a^2}-\frac{\sin(c+dx)\cos(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\int \frac{43a^2-33a^2\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^2\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)}dx}{3a^2}-\frac{11a\sin(c+dx)\cos(c+dx)}{3d(a\sec(c+dx)+a)^2}}{5a^2}-\frac{\sin(c+dx)\cos(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$\downarrow$ 4508

$$\frac{\frac{\frac{\int\cos^2(c+dx)\left(195a^3-152a^3\sec(c+dx)\right)dx}{a^2}-\frac{76a^2\sin(c+dx)\cos(c+dx)}{d(a\sec(c+dx)+a)}}{3a^2}-\frac{11a\sin(c+dx)\cos(c+dx)}{3d(a\sec(c+dx)+a)^2}}{5a^2}-\frac{\sin(c+dx)\cos(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$\downarrow$ 3042

$$\frac{\frac{\frac{\int\frac{195a^3-152a^3\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^2}dx}{a^2}-\frac{76a^2\sin(c+dx)\cos(c+dx)}{d(a\sec(c+dx)+a)}}{3a^2}-\frac{11a\sin(c+dx)\cos(c+dx)}{3d(a\sec(c+dx)+a)^2}}{5a^2}-\frac{\sin(c+dx)\cos(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$\downarrow$ 4274

$$\frac{\frac{\frac{195a^3\int\cos^2(c+dx)dx-152a^3\int\cos(c+dx)dx}{a^2}-\frac{76a^2\sin(c+dx)\cos(c+dx)}{d(a\sec(c+dx)+a)}}{3a^2}-\frac{11a\sin(c+dx)\cos(c+dx)}{3d(a\sec(c+dx)+a)^2}}{5a^2}-\frac{\sin(c+dx)\cos(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$\downarrow$ 3042

$$\frac{\frac{\frac{195a^3\int\sin\left(c+dx+\frac{\pi}{2}\right)^2dx-152a^3\int\sin\left(c+dx+\frac{\pi}{2}\right)dx}{a^2}-\frac{76a^2\sin(c+dx)\cos(c+dx)}{d(a\sec(c+dx)+a)}}{3a^2}-\frac{11a\sin(c+dx)\cos(c+dx)}{3d(a\sec(c+dx)+a)^2}}{5a^2}-\frac{\sin(c+dx)\cos(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$\downarrow$ 3115

$$\frac{\frac{\frac{195a^3\left(\frac{\int 1dx}{2}+\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)-152a^3\int\sin\left(c+dx+\frac{\pi}{2}\right)dx}{a^2}-\frac{76a^2\sin(c+dx)\cos(c+dx)}{d(a\sec(c+dx)+a)}}{3a^2}-\frac{11a\sin(c+dx)\cos(c+dx)}{3d(a\sec(c+dx)+a)^2}}{5a^2}-\frac{\sin(c+dx)\cos(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$\downarrow$ 24

$$\frac{\frac{\frac{195a^3\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)-152a^3\int\sin\left(c+dx+\frac{\pi}{2}\right)dx}{a^2}-\frac{76a^2\sin(c+dx)\cos(c+dx)}{d(a\sec(c+dx)+a)}}{3a^2}-\frac{11a\sin(c+dx)\cos(c+dx)}{3d(a\sec(c+dx)+a)^2}}{5a^2}-\frac{\sin(c+dx)\cos(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$\downarrow$ 3117

3.69. $\int \frac{\cos^2(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

$$\frac{\frac{\frac{195a^3\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)-\frac{152a^3\sin(c+dx)}{d}}{a^2}-\frac{76a^2\sin(c+dx)\cos(c+dx)}{d(a\sec(c+dx)+a)}}{3a^2}-\frac{11a\sin(c+dx)\cos(c+dx)}{3d(a\sec(c+dx)+a)^2}}{5a^2}-\frac{\sin(c+dx)\cos(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

input
```
Int[Cos[c + d*x]^2/(a + a*Sec[c + d*x])^3,x]
```

output
```
-1/5*(Cos[c + d*x]*Sin[c + d*x])/(d*(a + a*Sec[c + d*x])^3) + ((-11*a*Cos[
c + d*x]*Sin[c + d*x])/(3*d*(a + a*Sec[c + d*x])^2) + ((-76*a^2*Cos[c + d*
x]*Sin[c + d*x])/(d*(a + a*Sec[c + d*x])) + ((-152*a^3*Sin[c + d*x])/d + 1
95*a^3*(x/2 + (Cos[c + d*x]*Sin[c + d*x])/(2*d)))/a^2)/(3*a^2))/(5*a^2)
```

3.69.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4304
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-Cot[e + f*x])*(a + b*Csc[e + f*x])^m*((d*Csc
[e + f*x])^n/(f*(2*m + 1))), x] + Simp[1/(a^2*(2*m + 1))   Int[(a + b*Csc[e
 + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e
+ f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ
[m, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

rule 4508
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
- a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
 1))), x] - Simp[1/(a^2*(2*m + 1))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Cs
c[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[
e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B
, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

3.69.4 Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.52

method	result
parallelrisch	$\frac{-\frac{1001\left(\cos(dx+c)+\frac{928\cos(2dx+2c)}{3003}+\frac{15\cos(3dx+3c)}{1001}-\frac{5\cos(4dx+4c)}{2002}+\frac{331}{462}\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\sec\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{160}+\frac{13dx}{2}}{a^3d}$
derivativedivides	$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{5}+\frac{8\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3}-31\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{-28\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3-20\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^2}+52\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d\,a^3}$
default	$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{5}+\frac{8\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3}-31\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{-28\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3-20\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^2}+52\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d\,a^3}$
risch	$\frac{13x}{2a^3}-\frac{ie^{2i(dx+c)}}{8a^3d}+\frac{3ie^{i(dx+c)}}{2a^3d}-\frac{3ie^{-i(dx+c)}}{2a^3d}+\frac{ie^{-2i(dx+c)}}{8a^3d}-\frac{2i\left(150\,e^{4i(dx+c)}+525\,e^{3i(dx+c)}+745\,e^{2i(dx+c)}+\right.}{15d\,a^3\left(e^{i(dx+c)}+1\right)^5}$
norman	$\frac{\frac{13x}{2a}-\frac{51\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4ad}-\frac{131\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{6ad}-\frac{97\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{15ad}+\frac{17\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{30ad}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^9}{20ad}+\frac{13x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{a}+\frac{13x\tan\left(\frac{dx}{2}\right.}{2a}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^2a^2}$

input
```
int(cos(d*x+c)^2/(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
13/160*(-77*(cos(d*x+c)+928/3003*cos(2*d*x+2*c)+15/1001*cos(3*d*x+3*c)-5/2
002*cos(4*d*x+4*c)+331/462)*tan(1/2*d*x+1/2*c)*sec(1/2*d*x+1/2*c)^4+80*d*x
)/a^3/d
```

3.69.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.92

$$\int \frac{\cos^2(c+dx)}{(a+a\sec(c+dx))^3}\,dx$$
$$= \frac{195\,dx\cos(dx+c)^3 + 585\,dx\cos(dx+c)^2 + 585\,dx\cos(dx+c) + 195\,dx + \left(15\,\cos(dx+c)^4 - 45\,\cos(d\right.}{30\left(a^3d\cos(dx+c)^3 + 3\,a^3d\cos(dx+c)^2 + 3\,a^3d\cos(\right.}$$

input
```
integrate(cos(d*x+c)^2/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/30*(195*d*x*cos(d*x + c)^3 + 585*d*x*cos(d*x + c)^2 + 585*d*x*cos(d*x +
c) + 195*d*x + (15*cos(d*x + c)^4 - 45*cos(d*x + c)^3 - 479*cos(d*x + c)^2
 - 717*cos(d*x + c) - 304)*sin(d*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*c
os(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

3.69.6 Sympy [F]

$$\int \frac{\cos^2(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\int \frac{\cos^2(c+dx)}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx}{a^3}$$

input
```
integrate(cos(d*x+c)**2/(a+a*sec(d*x+c))**3,x)
```

output
```
Integral(cos(c + d*x)**2/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c +
d*x) + 1), x)/a**3
```

3.69.7 Maxima [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.25

$$\int \frac{\cos^2(c+dx)}{(a+a\sec(c+dx))^3}\,dx$$
$$= -\frac{\frac{60\left(\frac{5\sin(dx+c)}{\cos(dx+c)+1}+\frac{7\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a^3+\frac{2\,a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}} + \frac{\frac{465\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{40\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{a^3} - \frac{780\,\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}}{60\,d}$$

input
```
integrate(cos(d*x+c)^2/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
-1/60*(60*(5*sin(d*x + c)/(cos(d*x + c) + 1) + 7*sin(d*x + c)^3/(cos(d*x +
 c) + 1)^3)/(a^3 + 2*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a^3*sin(d*x
 + c)^4/(cos(d*x + c) + 1)^4) + (465*sin(d*x + c)/(cos(d*x + c) + 1) - 40*
sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 3*sin(d*x + c)^5/(cos(d*x + c) + 1)^
5)/a^3 - 780*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^3)/d
```

3.69.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.77

$$\int \frac{\cos^2(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\frac{390\,(dx+c)}{a^3} - \frac{60\left(7\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+5\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^2 a^3} - \frac{3\,a^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-40\,a^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+465\,a^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^{15}}}{60\,d}$$

input
```
integrate(cos(d*x+c)^2/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
1/60*(390*(d*x + c)/a^3 - 60*(7*tan(1/2*d*x + 1/2*c)^3 + 5*tan(1/2*d*x + 1
/2*c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^2*a^3) - (3*a^12*tan(1/2*d*x + 1/2*c)
^5 - 40*a^12*tan(1/2*d*x + 1/2*c)^3 + 465*a^12*tan(1/2*d*x + 1/2*c))/a^15)
/d
```

3.69.9 Mupad [B] (verification not implemented)

Time = 13.16 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.93

$$\int \frac{\cos^2(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{3\,\sin\left(\frac{c}{2}+\frac{dx}{2}\right)-46\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^2\sin\left(\frac{c}{2}+\frac{dx}{2}\right)+508\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^4\sin\left(\frac{c}{2}+\frac{dx}{2}\right)+420\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^6 \mathrm{s}}{60\,a^3\,d\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^5}$$

input
```
int(cos(c + d*x)^2/(a + a/cos(c + d*x))^3,x)
```

output
```
-(3*sin(c/2 + (d*x)/2) - 46*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2) + 508*
cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2) + 420*cos(c/2 + (d*x)/2)^6*sin(c/2
 + (d*x)/2) - 120*cos(c/2 + (d*x)/2)^8*sin(c/2 + (d*x)/2) - 390*cos(c/2 +
(d*x)/2)^5*(c + d*x))/(60*a^3*d*cos(c/2 + (d*x)/2)^5)
```

3.69. $\int \frac{\cos^2(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

3.70 $\int \frac{\sec^7(c+dx)}{(a+a\sec(c+dx))^4}\,dx$

3.70.1 Optimal result 723
3.70.2 Mathematica [A] (verified) 724
3.70.3 Rubi [A] (verified) 724
3.70.4 Maple [A] (verified) 729
3.70.5 Fricas [A] (verification not implemented) 730
3.70.6 Sympy [F] 730
3.70.7 Maxima [A] (verification not implemented) 731
3.70.8 Giac [A] (verification not implemented) 731
3.70.9 Mupad [B] (verification not implemented) 732

3.70.1 Optimal result

Integrand size = 21, antiderivative size = 193

$$\begin{aligned}\int \frac{\sec^7(c+dx)}{(a+a\sec(c+dx))^4}\,dx = {} & \frac{21\operatorname{arctanh}(\sin(c+dx))}{2a^4d} - \frac{576\tan(c+dx)}{35a^4d} \\ & + \frac{21\sec(c+dx)\tan(c+dx)}{2a^4d} - \frac{43\sec^3(c+dx)\tan(c+dx)}{35a^4d(1+\sec(c+dx))^2} \\ & - \frac{288\sec^2(c+dx)\tan(c+dx)}{35a^4d(1+\sec(c+dx))} \\ & - \frac{\sec^5(c+dx)\tan(c+dx)}{7d(a+a\sec(c+dx))^4} - \frac{2\sec^4(c+dx)\tan(c+dx)}{5ad(a+a\sec(c+dx))^3}\end{aligned}$$

output
```
21/2*arctanh(sin(d*x+c))/a^4/d-576/35*tan(d*x+c)/a^4/d+21/2*sec(d*x+c)*tan
(d*x+c)/a^4/d-43/35*sec(d*x+c)^3*tan(d*x+c)/a^4/d/(1+sec(d*x+c))^2-288/35*
sec(d*x+c)^2*tan(d*x+c)/a^4/d/(1+sec(d*x+c))-1/7*sec(d*x+c)^5*tan(d*x+c)/d
/(a+a*sec(d*x+c))^4-2/5*sec(d*x+c)^4*tan(d*x+c)/a/d/(a+a*sec(d*x+c))^3
```

3.70.2 Mathematica [A] (verified)

Time = 1.72 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.63

$$\int \frac{\sec^7(c+dx)}{(a+a\sec(c+dx))^4}\,dx$$
$$= \frac{11760\text{arctanh}(\sin(c+dx))\cos^8\left(\frac{1}{2}(c+dx)\right)\sec^4(c+dx)-\frac{1}{16}\sec^6(c+dx)(15234\sin(c+dx)+22192\sin}{70a^4d(1+\text{sec}}$$

input
```
Integrate[Sec[c + d*x]^7/(a + a*Sec[c + d*x])^4,x]
```

output
```
(11760*ArcTanh[Sin[c + d*x]]*Cos[(c + d*x)/2]^8*Sec[c + d*x]^4 - (Sec[c +
d*x]^6*(15234*Sin[c + d*x] + 22192*Sin[2*(c + d*x)] + 19667*Sin[3*(c + d*x
)] + 11400*Sin[4*(c + d*x)] + 3873*Sin[5*(c + d*x)] + 576*Sin[6*(c + d*x)]
))/16)/(70*a^4*d*(1 + Sec[c + d*x])^4)
```

3.70.3 Rubi [A] (verified)

Time = 1.42 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.09, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.810$, Rules used = {3042, 4303, 3042, 4507, 3042, 4507, 27, 3042, 4507, 3042, 4274, 3042, 4254, 24, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^7(c+dx)}{(a\sec(c+dx)+a)^4}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^7}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^4}dx$$
$$\Big\downarrow \; 4303$$
$$-\frac{\int \frac{\sec^5(c+dx)(5a-9a\sec(c+dx))}{(\sec(c+dx)a+a)^3}dx}{7a^2}-\frac{\tan(c+dx)\sec^5(c+dx)}{7d(a\sec(c+dx)+a)^4}$$
$$\Big\downarrow \; 3042$$

$$-\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^5\left(5a-9a\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^3}dx}{7a^2}-\frac{\tan(c+dx)\sec^5(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

$\Big\downarrow$ 4507

$$-\frac{\frac{\int \frac{\sec^4(c+dx)\left(56a^2-73a^2\sec(c+dx)\right)}{(\sec(c+dx)a+a)^2}dx}{5a^2}+\frac{14a\tan(c+dx)\sec^4(c+dx)}{5d(a\sec(c+dx)+a)^3}}{7a^2}-\frac{\tan(c+dx)\sec^5(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

$\Big\downarrow$ 3042

$$-\frac{\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^4\left(56a^2-73a^2\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}dx}{5a^2}+\frac{14a\tan(c+dx)\sec^4(c+dx)}{5d(a\sec(c+dx)+a)^3}}{7a^2}-\frac{\tan(c+dx)\sec^5(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

$\Big\downarrow$ 4507

$$-\frac{\frac{\frac{\int \frac{9\sec^3(c+dx)\left(43a^3-53a^3\sec(c+dx)\right)}{\sec(c+dx)a+a}dx}{3a^2}+\frac{43\tan(c+dx)\sec^3(c+dx)}{d(\sec(c+dx)+1)^2}}{5a^2}+\frac{14a\tan(c+dx)\sec^4(c+dx)}{5d(a\sec(c+dx)+a)^3}}{7a^2}-\frac{\tan(c+dx)\sec^5(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

$\Big\downarrow$ 27

$$-\frac{\frac{\frac{3\int \frac{\sec^3(c+dx)\left(43a^3-53a^3\sec(c+dx)\right)}{\sec(c+dx)a+a}dx}{a^2}+\frac{43\tan(c+dx)\sec^3(c+dx)}{d(\sec(c+dx)+1)^2}}{5a^2}+\frac{14a\tan(c+dx)\sec^4(c+dx)}{5d(a\sec(c+dx)+a)^3}}{7a^2}-\frac{\tan(c+dx)\sec^5(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

$\Big\downarrow$ 3042

$$-\frac{\frac{\frac{3\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^3\left(43a^3-53a^3\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{a^2}+\frac{43\tan(c+dx)\sec^3(c+dx)}{d(\sec(c+dx)+1)^2}}{5a^2}+\frac{14a\tan(c+dx)\sec^4(c+dx)}{5d(a\sec(c+dx)+a)^3}}{7a^2}-\frac{\tan(c+dx)\sec^5(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

$\Big\downarrow$ 4507

$$-\frac{\frac{\frac{3\left(\frac{\int \sec^2(c+dx)\left(192a^4-245a^4\sec(c+dx)\right)dx}{a^2}+\frac{96a^3\tan(c+dx)\sec^2(c+dx)}{d(a\sec(c+dx)+a)}\right)}{a^2}+\frac{43\tan(c+dx)\sec^3(c+dx)}{d(\sec(c+dx)+1)^2}}{5a^2}+\frac{14a\tan(c+dx)\sec^4(c+dx)}{5d(a\sec(c+dx)+a)^3}}{7a^2}-\frac{\tan(c+dx)\sec^5(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

3.70. $\int \frac{\sec^7(c+dx)}{(a+a\sec(c+dx))^4}\,dx$

$$\Big\downarrow \; 3042$$

$$-\frac{\frac{\frac{3\left(\frac{\int \csc\left(c+dx+\frac{\pi}{2}\right)^2\left(192a^4-245a^4\csc\left(c+dx+\frac{\pi}{2}\right)\right)dx}{a^2}+\frac{96a^3\tan(c+dx)\sec^2(c+dx)}{d(a\sec(c+dx)+a)}\right)}{a^2}+\frac{43\tan(c+dx)\sec^3(c+dx)}{d(\sec(c+dx)+1)^2}}{5a^2}+\frac{14a\tan(c+dx)\sec^4(c+dx)}{5d(a\sec(c+dx)+a)^3}}{7a^2}-\frac{\tan(c+dx)\sec^5(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

$$\Big\downarrow \; 4274$$

$$-\frac{\frac{\frac{3\left(\frac{192a^4\int \sec^2(c+dx)dx-245a^4\int \sec^3(c+dx)dx}{a^2}+\frac{96a^3\tan(c+dx)\sec^2(c+dx)}{d(a\sec(c+dx)+a)}\right)}{a^2}+\frac{43\tan(c+dx)\sec^3(c+dx)}{d(\sec(c+dx)+1)^2}}{5a^2}+\frac{14a\tan(c+dx)\sec^4(c+dx)}{5d(a\sec(c+dx)+a)^3}}{7a^2}-\frac{\tan(c+dx)\sec^5(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\frac{\frac{3\left(\frac{192a^4\int \csc\left(c+dx+\frac{\pi}{2}\right)^2dx-245a^4\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx}{a^2}+\frac{96a^3\tan(c+dx)\sec^2(c+dx)}{d(a\sec(c+dx)+a)}\right)}{a^2}+\frac{43\tan(c+dx)\sec^3(c+dx)}{d(\sec(c+dx)+1)^2}}{5a^2}+\frac{14a\tan(c+dx)\sec^4(c+dx)}{5d(a\sec(c+dx)+a)^3}}{7a^2}-\frac{\tan(c+dx)\sec^5(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

$$\Big\downarrow \; 4254$$

$$-\frac{\frac{\frac{3\left(\frac{-\frac{192a^4\int 1d(-\tan(c+dx))}{d}-245a^4\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx}{a^2}+\frac{96a^3\tan(c+dx)\sec^2(c+dx)}{d(a\sec(c+dx)+a)}\right)}{a^2}+\frac{43\tan(c+dx)\sec^3(c+dx)}{d(\sec(c+dx)+1)^2}}{5a^2}+\frac{14a\tan(c+dx)\sec^4(c+dx)}{5d(a\sec(c+dx)+a)^3}}{7a^2}-\frac{\tan(c+dx)\sec^5(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

$$\Big\downarrow \; 24$$

$$-\frac{\frac{\frac{3\left(\frac{192a^4\tan(c+dx)}{d}-245a^4\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx}{a^2}+\frac{96a^3\tan(c+dx)\sec^2(c+dx)}{d(a\sec(c+dx)+a)}\right)}{a^2}+\frac{43\tan(c+dx)\sec^3(c+dx)}{d(\sec(c+dx)+1)^2}}{5a^2}+\frac{14a\tan(c+dx)\sec^4(c+dx)}{5d(a\sec(c+dx)+a)^3}}{7a^2}-\frac{\tan(c+dx)\sec^5(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

3.70. $\int \frac{\sec^7(c+dx)}{(a+a\sec(c+dx))^4}\,dx$

$$\Big\downarrow \; 4255$$

$$-\frac{\frac{\frac{3\left(\frac{\frac{192a^4\tan(c+dx)}{d}-245a^4\left(\frac{1}{2}\int\sec(c+dx)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)}{a^2}+\frac{96a^3\tan(c+dx)\sec^2(c+dx)}{d(a\sec(c+dx)+a)}\right)}{a^2}+\frac{43\tan(c+dx)\sec^3(c+dx)}{d(\sec(c+dx)+1)^2}}{5a^2}+\frac{14a\tan(c+dx)\sec^4(c}{5d(a\sec(c+dx)+a}}{7a^2}$$
$$\frac{\tan(c+dx)\sec^5(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\frac{\frac{3\left(\frac{\frac{192a^4\tan(c+dx)}{d}-245a^4\left(\frac{1}{2}\int\csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)}{a^2}+\frac{96a^3\tan(c+dx)\sec^2(c+dx)}{d(a\sec(c+dx)+a)}\right)}{a^2}+\frac{43\tan(c+dx)\sec^3(c+dx)}{d(\sec(c+dx)+1)^2}}{5a^2}+\frac{14a\tan(c+dx)\sec}{5d(a\sec(c+dx}}{7a^2}$$
$$\frac{\tan(c+dx)\sec^5(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

$$\Big\downarrow \; 4257$$

$$-\frac{\frac{\frac{3\left(\frac{96a^3\tan(c+dx)\sec^2(c+dx)}{d(a\sec(c+dx)+a)}+\frac{\frac{192a^4\tan(c+dx)}{d}-245a^4\left(\frac{\text{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)}{a^2}\right)}{a^2}+\frac{43\tan(c+dx)\sec^3(c+dx)}{d(\sec(c+dx)+1)^2}}{5a^2}+\frac{14a\tan(c+dx)}{5d(a\sec(c+}}{7a^2}$$
$$\frac{\tan(c+dx)\sec^5(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

input
```
Int[Sec[c + d*x]^7/(a + a*Sec[c + d*x])^4,x]
```

output
```
-1/7*(Sec[c + d*x]^5*Tan[c + d*x])/(d*(a + a*Sec[c + d*x])^4) - ((14*a*Sec
[c + d*x]^4*Tan[c + d*x])/(5*d*(a + a*Sec[c + d*x])^3) + ((43*Sec[c + d*x]
^3*Tan[c + d*x])/(d*(1 + Sec[c + d*x])^2) + (3*((96*a^3*Sec[c + d*x]^2*Tan
[c + d*x])/(d*(a + a*Sec[c + d*x])) + ((192*a^4*Tan[c + d*x])/d - 245*a^4*
(ArcTanh[Sin[c + d*x]]/(2*d) + (Sec[c + d*x]*Tan[c + d*x])/(2*d)))/a^2))/a
^2)/(5*a^2))/(7*a^2)
```

3.70. $\int \frac{\sec^7(c+dx)}{(a+a\sec(c+dx))^4}\,dx$

3.70.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4303
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-d^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d
*Csc[e + f*x])^(n - 2)/(f*(2*m + 1))), x] + Simp[d^2/(a*b*(2*m + 1))   Int[
(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n
 + 2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2,
0] && LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

rule 4507
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[d*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(a*f*(
2*m + 1))), x] - Simp[1/(a*b*(2*m + 1))   Int[(a + b*Csc[e + f*x])^(m + 1)*
(d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
 - n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f,
A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && G
tQ[n, 0]
```

3.70.4 Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.77

method	result
derivativedivides	$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{7}-\frac{9\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{5}-13\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}-111\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{4}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}+\frac{36}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1}-84\ln\left(\tan\left(\frac{d}{2}\right.\right.}{8d\,a^{4}}$
default	$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{7}-\frac{9\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{5}-13\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}-111\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{4}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}+\frac{36}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1}-84\ln\left(\tan\left(\frac{d}{2}\right.\right.}{8d\,a^{4}}$
parallelrisch	$\frac{(-23520\cos(2dx+2c)-23520)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+(23520\cos(2dx+2c)+23520)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-34168\left(\cos(dx+\right.}{2240a^{4}d(1+\cos(2d}$
risch	$-\frac{i\left(735\,\mathrm{e}^{10i(dx+c)}+5145\,\mathrm{e}^{9i(dx+c)}+16660\,\mathrm{e}^{8i(dx+c)}+34300\,\mathrm{e}^{7i(dx+c)}+51842\,\mathrm{e}^{6i(dx+c)}+61054\,\mathrm{e}^{5i(dx+c)}+55556\,\mathrm{e}^{4i(dx+}\right.}{35d\,a^{4}\left(\mathrm{e}^{2i(dx+c)}+1\right)^{2}\left(\mathrm{e}^{i(dx+c)}+1\right)^{7}}$

input
```
int(sec(d*x+c)^7/(a+a*sec(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
1/8/d/a^4*(-1/7*tan(1/2*d*x+1/2*c)^7-9/5*tan(1/2*d*x+1/2*c)^5-13*tan(1/2*d
*x+1/2*c)^3-111*tan(1/2*d*x+1/2*c)+4/(tan(1/2*d*x+1/2*c)-1)^2+36/(tan(1/2*
d*x+1/2*c)-1)-84*ln(tan(1/2*d*x+1/2*c)-1)-4/(tan(1/2*d*x+1/2*c)+1)^2+36/(t
an(1/2*d*x+1/2*c)+1)+84*ln(tan(1/2*d*x+1/2*c)+1))
```

3.70.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.30

$$\int \frac{\sec^7(c+dx)}{(a+a\sec(c+dx))^4}\,dx$$
$$= \frac{735\left(\cos\left(dx+c\right)^6+4\,\cos\left(dx+c\right)^5+6\,\cos\left(dx+c\right)^4+4\,\cos\left(dx+c\right)^3+\cos\left(dx+c\right)^2\right)\log\left(\sin\left(dx+c\right)\right.}{}$$

input
```
integrate(sec(d*x+c)^7/(a+a*sec(d*x+c))^4,x, algorithm="fricas")
```

output
```
1/140*(735*(cos(d*x + c)^6 + 4*cos(d*x + c)^5 + 6*cos(d*x + c)^4 + 4*cos(d
*x + c)^3 + cos(d*x + c)^2)*log(sin(d*x + c) + 1) - 735*(cos(d*x + c)^6 +
4*cos(d*x + c)^5 + 6*cos(d*x + c)^4 + 4*cos(d*x + c)^3 + cos(d*x + c)^2)*l
og(-sin(d*x + c) + 1) - 2*(1152*cos(d*x + c)^5 + 3873*cos(d*x + c)^4 + 454
8*cos(d*x + c)^3 + 2012*cos(d*x + c)^2 + 140*cos(d*x + c) - 35)*sin(d*x +
c))/(a^4*d*cos(d*x + c)^6 + 4*a^4*d*cos(d*x + c)^5 + 6*a^4*d*cos(d*x + c)^
4 + 4*a^4*d*cos(d*x + c)^3 + a^4*d*cos(d*x + c)^2)
```

3.70.6 Sympy [F]

$$\int \frac{\sec^7(c+dx)}{(a+a\sec(c+dx))^4}\,dx = \frac{\int \frac{\sec^7(c+dx)}{\sec^4(c+dx)+4\sec^3(c+dx)+6\sec^2(c+dx)+4\sec(c+dx)+1}\,dx}{a^4}$$

input
```
integrate(sec(d*x+c)**7/(a+a*sec(d*x+c))**4,x)
```

output
```
Integral(sec(c + d*x)**7/(sec(c + d*x)**4 + 4*sec(c + d*x)**3 + 6*sec(c +
d*x)**2 + 4*sec(c + d*x) + 1), x)/a**4
```

3.70.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.20

$$\int \frac{\sec^7(c+dx)}{(a+a\sec(c+dx))^4}\,dx = -\frac{\frac{280\left(\frac{7\sin(dx+c)}{\cos(dx+c)+1}-\frac{9\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a^4-\frac{2a^4\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a^4\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}+\frac{\frac{3885\sin(dx+c)}{\cos(dx+c)+1}+\frac{455\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{63\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{5\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{a^4}-\frac{2940\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^4}+}{280\,d}$$

input
```
integrate(sec(d*x+c)^7/(a+a*sec(d*x+c))^4,x, algorithm="maxima")
```

output
```
-1/280*(280*(7*sin(d*x + c)/(cos(d*x + c) + 1) - 9*sin(d*x + c)^3/(cos(d*x
 + c) + 1)^3)/(a^4 - 2*a^4*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a^4*sin(d
*x + c)^4/(cos(d*x + c) + 1)^4) + (3885*sin(d*x + c)/(cos(d*x + c) + 1) +
455*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 63*sin(d*x + c)^5/(cos(d*x + c)
+ 1)^5 + 5*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)/a^4 - 2940*log(sin(d*x + c
)/(cos(d*x + c) + 1) + 1)/a^4 + 2940*log(sin(d*x + c)/(cos(d*x + c) + 1) -
 1)/a^4)/d
```

3.70.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.80

$$\int \frac{\sec^7(c+dx)}{(a+a\sec(c+dx))^4}\,dx = \frac{\frac{2940\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a^4}-\frac{2940\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{a^4}+\frac{280\left(9\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-7\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)^2a^4}-\frac{5a^{24}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7+63}{}}{280\,d}$$

input
```
integrate(sec(d*x+c)^7/(a+a*sec(d*x+c))^4,x, algorithm="giac")
```

output
```
1/280*(2940*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^4 - 2940*log(abs(tan(1/2*
d*x + 1/2*c) - 1))/a^4 + 280*(9*tan(1/2*d*x + 1/2*c)^3 - 7*tan(1/2*d*x + 1
/2*c))/((tan(1/2*d*x + 1/2*c)^2 - 1)^2*a^4) - (5*a^24*tan(1/2*d*x + 1/2*c)
^7 + 63*a^24*tan(1/2*d*x + 1/2*c)^5 + 455*a^24*tan(1/2*d*x + 1/2*c)^3 + 38
85*a^24*tan(1/2*d*x + 1/2*c))/a^28)/d
```

3.70.9 Mupad [B] (verification not implemented)

Time = 13.46 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.83

$$\int \frac{\sec^7(c+dx)}{(a+a\sec(c+dx))^4}\,dx = \frac{21\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{a^4\,d} - \frac{9\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5}{40\,a^4\,d} - \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7}{56\,a^4\,d} - \frac{13\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{8\,a^4\,d} - \frac{7\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-9\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{d\left(a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4-2\,a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+a^4\right)} - \frac{111\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{8\,a^4\,d}$$

input
```
int(1/(cos(c + d*x)^7*(a + a/cos(c + d*x))^4),x)
```

output
```
(21*atanh(tan(c/2 + (d*x)/2)))/(a^4*d) - (9*tan(c/2 + (d*x)/2)^5)/(40*a^4*
d) - tan(c/2 + (d*x)/2)^7/(56*a^4*d) - (13*tan(c/2 + (d*x)/2)^3)/(8*a^4*d)
 - (7*tan(c/2 + (d*x)/2) - 9*tan(c/2 + (d*x)/2)^3)/(d*(a^4*tan(c/2 + (d*x)
/2)^4 - 2*a^4*tan(c/2 + (d*x)/2)^2 + a^4)) - (111*tan(c/2 + (d*x)/2))/(8*a
^4*d)
```

3.71 $\int \frac{\sec^6(c+dx)}{(a+a\sec(c+dx))^4}\,dx$

3.71.1 Optimal result 733
3.71.2 Mathematica [A] (verified) 733
3.71.3 Rubi [A] (verified) 734
3.71.4 Maple [A] (verified) 739
3.71.5 Fricas [A] (verification not implemented) 739
3.71.6 Sympy [F] 740
3.71.7 Maxima [A] (verification not implemented) 740
3.71.8 Giac [A] (verification not implemented) 740
3.71.9 Mupad [B] (verification not implemented) 741

3.71.1 Optimal result

Integrand size = 21, antiderivative size = 159

$$\int \frac{\sec^6(c+dx)}{(a+a\sec(c+dx))^4}\,dx = -\frac{4\text{arctanh}(\sin(c+dx))}{a^4d}+\frac{244\tan(c+dx)}{105a^4d} - \frac{88\sec^2(c+dx)\tan(c+dx)}{105a^4d(1+\sec(c+dx))^2}+\frac{4\tan(c+dx)}{a^4d(1+\sec(c+dx))} - \frac{\sec^4(c+dx)\tan(c+dx)}{7d(a+a\sec(c+dx))^4}-\frac{12\sec^3(c+dx)\tan(c+dx)}{35ad(a+a\sec(c+dx))^3}$$

output
```
-4*arctanh(sin(d*x+c))/a^4/d+244/105*tan(d*x+c)/a^4/d-88/105*sec(d*x+c)^2*
tan(d*x+c)/a^4/d/(1+sec(d*x+c))^2+4*tan(d*x+c)/a^4/d/(1+sec(d*x+c))-1/7*se
c(d*x+c)^4*tan(d*x+c)/d/(a+a*sec(d*x+c))^4-12/35*sec(d*x+c)^3*tan(d*x+c)/a
/d/(a+a*sec(d*x+c))^3
```

3.71.2 Mathematica [A] (verified)

Time = 1.19 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.81

$$\int \frac{\sec^6(c+dx)}{(a+a\sec(c+dx))^4}\,dx = \frac{\cos\left(\frac{1}{2}(c+dx)\right)\sec^4(c+dx)\left(-6720\text{arctanh}(\sin(c+dx))\cos^7\left(\frac{1}{2}(c+dx)\right)+\sec(c+dx)\left(483\sin\left(\frac{1}{2}(c+d\right.\right.\right.}{105a^4d(1+\sec(c}$$

input
```
Integrate[Sec[c + d*x]^6/(a + a*Sec[c + d*x])^4,x]
```

output
```
(Cos[(c + d*x)/2]*Sec[c + d*x]^4*(-6720*ArcTanh[Sin[c + d*x]]*Cos[(c + d*x
)/2]^7 + Sec[c + d*x]*(483*Sin[(c + d*x)/2] + 1211*Sin[(3*(c + d*x))/2] +
1091*Sin[(5*(c + d*x))/2] + 476*Sin[(7*(c + d*x))/2] + 83*Sin[(9*(c + d*x)
)/2])))/(105*a^4*d*(1 + Sec[c + d*x])^4)
```

3.71.3 Rubi [A] (verified)

Time = 1.25 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.15, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.762$, Rules used = {3042, 4303, 27, 3042, 4507, 3042, 4507, 3042, 4496, 25, 3042, 4274, 3042, 4254, 24, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^6(c+dx)}{(a\sec(c+dx)+a)^4}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^6}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^4}dx$$

$$\Big\downarrow \text{4303}$$

$$-\frac{\int \frac{4\sec^4(c+dx)(a-2a\sec(c+dx))}{(\sec(c+dx)a+a)^3}dx}{7a^2}-\frac{\tan(c+dx)\sec^4(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

$$\Big\downarrow \text{27}$$

$$-\frac{4\int \frac{\sec^4(c+dx)(a-2a\sec(c+dx))}{(\sec(c+dx)a+a)^3}dx}{7a^2}-\frac{\tan(c+dx)\sec^4(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{4\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^4\left(a-2a\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^3}dx}{7a^2}-\frac{\tan(c+dx)\sec^4(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

$$\Big\downarrow \text{4507}$$

$$-\frac{4\left(\frac{\int \frac{\sec^3(c+dx)\left(9a^2-13a^2\sec(c+dx)\right)}{(\sec(c+dx)a+a)^2}dx}{5a^2}+\frac{3a\tan(c+dx)\sec^3(c+dx)}{5d(a\sec(c+dx)+a)^3}\right)}{7a^2}-\frac{\tan(c+dx)\sec^4(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

$\Big\downarrow$ 3042

$$-\frac{4\left(\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^3\left(9a^2-13a^2\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}dx}{5a^2}+\frac{3a\tan(c+dx)\sec^3(c+dx)}{5d(a\sec(c+dx)+a)^3}\right)}{7a^2}-\frac{\tan(c+dx)\sec^4(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

$\Big\downarrow$ 4507

$$-\frac{4\left(\frac{\frac{\int \frac{\sec^2(c+dx)\left(44a^3-61a^3\sec(c+dx)\right)}{\sec(c+dx)a+a}dx}{3a^2}+\frac{22\tan(c+dx)\sec^2(c+dx)}{3d(\sec(c+dx)+1)^2}}{5a^2}+\frac{3a\tan(c+dx)\sec^3(c+dx)}{5d(a\sec(c+dx)+a)^3}\right)}{7a^2}-\frac{\tan(c+dx)\sec^4(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

$\Big\downarrow$ 3042

$$-\frac{4\left(\frac{\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2\left(44a^3-61a^3\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{3a^2}+\frac{22\tan(c+dx)\sec^2(c+dx)}{3d(\sec(c+dx)+1)^2}}{5a^2}+\frac{3a\tan(c+dx)\sec^3(c+dx)}{5d(a\sec(c+dx)+a)^3}\right)}{7a^2}-\frac{\tan(c+dx)\sec^4(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

$\Big\downarrow$ 4496

$$-\frac{4\left(\frac{\frac{-\frac{\int -\sec(c+dx)\left(105a^4-61a^4\sec(c+dx)\right)dx}{a^2}-\frac{105a^3\tan(c+dx)}{d(a\sec(c+dx)+a)}}{3a^2}+\frac{22\tan(c+dx)\sec^2(c+dx)}{3d(\sec(c+dx)+1)^2}}{5a^2}+\frac{3a\tan(c+dx)\sec^3(c+dx)}{5d(a\sec(c+dx)+a)^3}\right)}{7a^2}-\frac{\tan(c+dx)\sec^4(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

$\Big\downarrow$ 25

3.71. $\int \frac{\sec^6(c+dx)}{(a+a\sec(c+dx))^4}\,dx$

$$-\frac{4\left(\frac{\frac{\frac{\int \sec(c+dx)\left(105a^4-61a^4\sec(c+dx)\right)dx}{a^2}-\frac{105a^3\tan(c+dx)}{d(a\sec(c+dx)+a)}}{3a^2}+\frac{22\tan(c+dx)\sec^2(c+dx)}{3d(\sec(c+dx)+1)^2}}{5a^2}+\frac{3a\tan(c+dx)\sec^3(c+dx)}{5d(a\sec(c+dx)+a)^3}\right)}{7a^2}-\frac{\tan(c+dx)\sec^4(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

$\downarrow$ 3042

$$-\frac{4\left(\frac{\frac{\frac{\int \csc\left(c+dx+\frac{\pi}{2}\right)\left(105a^4-61a^4\csc\left(c+dx+\frac{\pi}{2}\right)\right)dx}{a^2}-\frac{105a^3\tan(c+dx)}{d(a\sec(c+dx)+a)}}{3a^2}+\frac{22\tan(c+dx)\sec^2(c+dx)}{3d(\sec(c+dx)+1)^2}}{5a^2}+\frac{3a\tan(c+dx)\sec^3(c+dx)}{5d(a\sec(c+dx)+a)^3}\right)}{7a^2}-\frac{\tan(c+dx)\sec^4(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

$\downarrow$ 4274

$$-\frac{4\left(\frac{\frac{\frac{105a^4\int \sec(c+dx)dx-61a^4\int \sec^2(c+dx)dx}{a^2}-\frac{105a^3\tan(c+dx)}{d(a\sec(c+dx)+a)}}{3a^2}+\frac{22\tan(c+dx)\sec^2(c+dx)}{3d(\sec(c+dx)+1)^2}}{5a^2}+\frac{3a\tan(c+dx)\sec^3(c+dx)}{5d(a\sec(c+dx)+a)^3}\right)}{7a^2}-\frac{\tan(c+dx)\sec^4(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

$\downarrow$ 3042

$$-\frac{4\left(\frac{\frac{\frac{105a^4\int \csc\left(c+dx+\frac{\pi}{2}\right)dx-61a^4\int \csc\left(c+dx+\frac{\pi}{2}\right)^2dx}{a^2}-\frac{105a^3\tan(c+dx)}{d(a\sec(c+dx)+a)}}{3a^2}+\frac{22\tan(c+dx)\sec^2(c+dx)}{3d(\sec(c+dx)+1)^2}}{5a^2}+\frac{3a\tan(c+dx)\sec^3(c+dx)}{5d(a\sec(c+dx)+a)^3}\right)}{7a^2}-\frac{\tan(c+dx)\sec^4(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

$\downarrow$ 4254

$$-\frac{4\left(\frac{\frac{\frac{61a^4\int 1d(-\tan(c+dx))}{d}+105a^4\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{a^2}-\frac{105a^3\tan(c+dx)}{d(a\sec(c+dx)+a)}}{3a^2}+\frac{22\tan(c+dx)\sec^2(c+dx)}{3d(\sec(c+dx)+1)^2}}{5a^2}+\frac{3a\tan(c+dx)\sec^3(c+dx)}{5d(a\sec(c+dx)+a)^3}\right)}{7a^2}-\frac{\tan(c+dx)\sec^4(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

3.71. $\int \frac{\sec^6(c+dx)}{(a+a\sec(c+dx))^4}\,dx$

$$\Big\downarrow \; 24$$

$$-\frac{4\left(\frac{\frac{\frac{105a^4\int \csc\left(c+dx+\frac{\pi}{2}\right)dx-\frac{61a^4\tan(c+dx)}{d}}{a^2}-\frac{105a^3\tan(c+dx)}{d(a\sec(c+dx)+a)}}{3a^2}+\frac{22\tan(c+dx)\sec^2(c+dx)}{3d(\sec(c+dx)+1)^2}}{5a^2}+\frac{3a\tan(c+dx)\sec^3(c+dx)}{5d(a\sec(c+dx)+a)^3}\right)}{7a^2}-\frac{\tan(c+dx)\sec^4(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

$$\Big\downarrow \; 4257$$

$$-\frac{4\left(\frac{\frac{\frac{105a^4\text{arctanh}(\sin(c+dx))}{d}-\frac{61a^4\tan(c+dx)}{d}}{a^2}-\frac{105a^3\tan(c+dx)}{d(a\sec(c+dx)+a)}}{3a^2}+\frac{22\tan(c+dx)\sec^2(c+dx)}{3d(\sec(c+dx)+1)^2}}{5a^2}+\frac{3a\tan(c+dx)\sec^3(c+dx)}{5d(a\sec(c+dx)+a)^3}\right)}{7a^2}-\frac{\tan(c+dx)\sec^4(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

input
```
Int[Sec[c + d*x]^6/(a + a*Sec[c + d*x])^4,x]
```

output
```
-1/7*(Sec[c + d*x]^4*Tan[c + d*x])/(d*(a + a*Sec[c + d*x])^4) - (4*((3*a*S
ec[c + d*x]^3*Tan[c + d*x])/(5*d*(a + a*Sec[c + d*x])^3) + ((22*Sec[c + d*
x]^2*Tan[c + d*x])/(3*d*(1 + Sec[c + d*x])^2) + ((-105*a^3*Tan[c + d*x])/(
d*(a + a*Sec[c + d*x])) + ((105*a^4*ArcTanh[Sin[c + d*x]])/d - (61*a^4*Tan
[c + d*x])/d)/a^2)/(3*a^2))/(5*a^2)))/(7*a^2)
```

3.71.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4303
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-d^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d
*Csc[e + f*x])^(n - 2)/(f*(2*m + 1))), x] + Simp[d^2/(a*b*(2*m + 1))   Int[
(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n
 + 2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2,
0] && LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

rule 4496
```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b - a*B))*Cot
[e + f*x]*((a + b*Csc[e + f*x])^m/(b*f*(2*m + 1))), x] + Simp[1/(b^2*(2*m +
 1))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[A*b*m - a*B*m + b
*B*(2*m + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && Ne
Q[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

rule 4507
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[d*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(a*f*(
2*m + 1))), x] - Simp[1/(a*b*(2*m + 1))   Int[(a + b*Csc[e + f*x])^(m + 1)*
(d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
 - n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f,
A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && G
tQ[n, 0]
```

3.71. $\int \frac{\sec^6(c+dx)}{(a+a\sec(c+dx))^4}\,dx$

3.71.4 Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.74

method	result
derivativedivides	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{7}+\frac{7\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{5}+\frac{23\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{3}+49\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{8}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}-32\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-\frac{8}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}}{8d\,a^{4}}$
default	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{7}+\frac{7\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{5}+\frac{23\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{3}+49\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{8}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}-32\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-\frac{8}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}}{8d\,a^{4}}$
parallelrisch	$\frac{3360\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\cos(dx+c)-3360\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)\cos(dx+c)+2861\left(\cos(dx+c)+\frac{1650\cos(2dx+2c)}{2861}+\frac{559\cos}{2}\right.}{840a^{4}d\cos(dx+c)}$
risch	$\frac{8i\left(105\,\mathrm{e}^{8i(dx+c)}+735\,\mathrm{e}^{7i(dx+c)}+2275\,\mathrm{e}^{6i(dx+c)}+4165\,\mathrm{e}^{5i(dx+c)}+5131\,\mathrm{e}^{4i(dx+c)}+4697\,\mathrm{e}^{3i(dx+c)}+2917\,\mathrm{e}^{2i(dx+c)}+1057\right.}{105d\,a^{4}\left(\mathrm{e}^{i(dx+c)}+1\right)^{7}\left(\mathrm{e}^{2i(dx+c)}+1\right)}$

input
```
int(sec(d*x+c)^6/(a+a*sec(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
1/8/d/a^4*(1/7*tan(1/2*d*x+1/2*c)^7+7/5*tan(1/2*d*x+1/2*c)^5+23/3*tan(1/2*
d*x+1/2*c)^3+49*tan(1/2*d*x+1/2*c)-8/(tan(1/2*d*x+1/2*c)+1)-32*ln(tan(1/2*
d*x+1/2*c)+1)-8/(tan(1/2*d*x+1/2*c)-1)+32*ln(tan(1/2*d*x+1/2*c)-1))
```

3.71.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.47

$$\int \frac{\sec^6(c+dx)}{(a+a\sec(c+dx))^4}\,dx = -\frac{210\left(\cos\left(dx+c\right)^5+4\,\cos\left(dx+c\right)^4+6\,\cos\left(dx+c\right)^3+4\,\cos\left(dx+c\right)^2+\cos\left(dx+c\right)\right)\log\left(\sin\left(dx+c\right.\right.}{}$$

input
```
integrate(sec(d*x+c)^6/(a+a*sec(d*x+c))^4,x, algorithm="fricas")
```

output
```
-1/105*(210*(cos(d*x + c)^5 + 4*cos(d*x + c)^4 + 6*cos(d*x + c)^3 + 4*cos(
d*x + c)^2 + cos(d*x + c))*log(sin(d*x + c) + 1) - 210*(cos(d*x + c)^5 + 4
*cos(d*x + c)^4 + 6*cos(d*x + c)^3 + 4*cos(d*x + c)^2 + cos(d*x + c))*log(
-sin(d*x + c) + 1) - (664*cos(d*x + c)^4 + 2236*cos(d*x + c)^3 + 2636*cos(
d*x + c)^2 + 1184*cos(d*x + c) + 105)*sin(d*x + c))/(a^4*d*cos(d*x + c)^5
+ 4*a^4*d*cos(d*x + c)^4 + 6*a^4*d*cos(d*x + c)^3 + 4*a^4*d*cos(d*x + c)^2
 + a^4*d*cos(d*x + c))
```

3.71.6 Sympy [F]

$$\int \frac{\sec^6(c+dx)}{(a+a\sec(c+dx))^4}\,dx = \frac{\int \frac{\sec^6(c+dx)}{\sec^4(c+dx)+4\sec^3(c+dx)+6\sec^2(c+dx)+4\sec(c+dx)+1}\,dx}{a^4}$$

input
```
integrate(sec(d*x+c)**6/(a+a*sec(d*x+c))**4,x)
```

output
```
Integral(sec(c + d*x)**6/(sec(c + d*x)**4 + 4*sec(c + d*x)**3 + 6*sec(c +
d*x)**2 + 4*sec(c + d*x) + 1), x)/a**4
```

3.71.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.17

$$\int \frac{\sec^6(c+dx)}{(a+a\sec(c+dx))^4}\,dx = \frac{\frac{1680\sin(dx+c)}{\left(a^4-\frac{a^4\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)} + \frac{\frac{5145\sin(dx+c)}{\cos(dx+c)+1}+\frac{805\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{147\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{15\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{a^4} - \frac{3360\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^4} + \frac{3}{}}{840\,d}$$

input
```
integrate(sec(d*x+c)^6/(a+a*sec(d*x+c))^4,x, algorithm="maxima")
```

output
```
1/840*(1680*sin(d*x + c)/((a^4 - a^4*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)*
(cos(d*x + c) + 1)) + (5145*sin(d*x + c)/(cos(d*x + c) + 1) + 805*sin(d*x
+ c)^3/(cos(d*x + c) + 1)^3 + 147*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 15
*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)/a^4 - 3360*log(sin(d*x + c)/(cos(d*x
 + c) + 1) + 1)/a^4 + 3360*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a^4)/d
```

3.71.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.87

$$\int \frac{\sec^6(c+dx)}{(a+a\sec(c+dx))^4}\,dx = -\frac{\frac{3360\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a^4} - \frac{3360\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{a^4} + \frac{1680\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)a^4} - \frac{15a^{24}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7+147a^{24}\tan\left(\frac{1}{2}dx\right.}{}}{840\,d}$$

input
```
integrate(sec(d*x+c)^6/(a+a*sec(d*x+c))^4,x, algorithm="giac")
```

output
```
-1/840*(3360*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^4 - 3360*log(abs(tan(1/2
*d*x + 1/2*c) - 1))/a^4 + 1680*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/2*c)
^2 - 1)*a^4) - (15*a^24*tan(1/2*d*x + 1/2*c)^7 + 147*a^24*tan(1/2*d*x + 1/
2*c)^5 + 805*a^24*tan(1/2*d*x + 1/2*c)^3 + 5145*a^24*tan(1/2*d*x + 1/2*c))
/a^28)/d
```

3.71.9 Mupad [B] (verification not implemented)

Time = 13.25 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.82

$$\int \frac{\sec^6(c+dx)}{(a+a\sec(c+dx))^4}\,dx = \frac{23\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{24\,a^4\,d} + \frac{7\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5}{40\,a^4\,d} + \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7}{56\,a^4\,d} - \frac{8\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{a^4\,d} - \frac{2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\left(a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2-a^4\right)} + \frac{49\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{8\,a^4\,d}$$

input
```
int(1/(cos(c + d*x)^6*(a + a/cos(c + d*x))^4),x)
```

output
```
(23*tan(c/2 + (d*x)/2)^3)/(24*a^4*d) + (7*tan(c/2 + (d*x)/2)^5)/(40*a^4*d)
 + tan(c/2 + (d*x)/2)^7/(56*a^4*d) - (8*atanh(tan(c/2 + (d*x)/2)))/(a^4*d)
 - (2*tan(c/2 + (d*x)/2))/(d*(a^4*tan(c/2 + (d*x)/2)^2 - a^4)) + (49*tan(c
/2 + (d*x)/2))/(8*a^4*d)
```

3.72 $\int \frac{\sec^5(c+dx)}{(a+a\sec(c+dx))^4}\,dx$

3.72.1 Optimal result 742
3.72.2 Mathematica [A] (verified) 742
3.72.3 Rubi [A] (verified) 743
3.72.4 Maple [A] (verified) 747
3.72.5 Fricas [A] (verification not implemented) 747
3.72.6 Sympy [F] 748
3.72.7 Maxima [A] (verification not implemented) 748
3.72.8 Giac [A] (verification not implemented) 749
3.72.9 Mupad [B] (verification not implemented) 749

3.72.1 Optimal result

Integrand size = 21, antiderivative size = 136

$$\int \frac{\sec^5(c+dx)}{(a+a\sec(c+dx))^4}\,dx = \frac{\operatorname{arctanh}(\sin(c+dx))}{a^4d} + \frac{11\tan(c+dx)}{21a^4d(1+\sec(c+dx))^2} - \frac{43\tan(c+dx)}{21a^4d(1+\sec(c+dx))} - \frac{\sec^3(c+dx)\tan(c+dx)}{7d(a+a\sec(c+dx))^4} - \frac{2\sec^2(c+dx)\tan(c+dx)}{7ad(a+a\sec(c+dx))^3}$$

output
```
arctanh(sin(d*x+c))/a^4/d+11/21*tan(d*x+c)/a^4/d/(1+sec(d*x+c))^2-43/21*ta
n(d*x+c)/a^4/d/(1+sec(d*x+c))-1/7*sec(d*x+c)^3*tan(d*x+c)/d/(a+a*sec(d*x+c
))^4-2/7*sec(d*x+c)^2*tan(d*x+c)/a/d/(a+a*sec(d*x+c))^3
```

3.72.2 Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.79

$$\int \frac{\sec^5(c+dx)}{(a+a\sec(c+dx))^4}\,dx = \frac{\cos\left(\frac{1}{2}(c+dx)\right)\sec^4(c+dx)\left(672\operatorname{arctanh}(\sin(c+dx))\cos^7\left(\frac{1}{2}(c+dx)\right) - 126\sin\left(\frac{1}{2}(c+dx)\right) - 189\sin\left(\frac{3}{2}\right.\right.}{42a^4d(1+\sec(c+dx))^4}$$

input
```
Integrate[Sec[c + d*x]^5/(a + a*Sec[c + d*x])^4,x]
```

output
```
(Cos[(c + d*x)/2]*Sec[c + d*x]^4*(672*ArcTanh[Sin[c + d*x]]*Cos[(c + d*x)/
2]^7 - 126*Sin[(c + d*x)/2] - 189*Sin[(3*(c + d*x))/2] - 91*Sin[(5*(c + d*
x))/2] - 16*Sin[(7*(c + d*x))/2]))/(42*a^4*d*(1 + Sec[c + d*x])^4)
```

3.72.3 Rubi [A] (verified)

Time = 1.00 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.11, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.619$, Rules used = {3042, 4303, 3042, 4507, 27, 3042, 4496, 25, 3042, 4486, 3042, 4257, 4281}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^5(c+dx)}{(a\sec(c+dx)+a)^4}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^5}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^4}dx$$

$$\Big\downarrow \text{4303}$$

$$-\frac{\int \frac{\sec^3(c+dx)(3a-7a\sec(c+dx))}{(\sec(c+dx)a+a)^3}dx}{7a^2}-\frac{\tan(c+dx)\sec^3(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^3\left(3a-7a\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^3}dx}{7a^2}-\frac{\tan(c+dx)\sec^3(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

$$\Big\downarrow \text{4507}$$

$$-\frac{\frac{\int \frac{5\sec^2(c+dx)\left(4a^2-7a^2\sec(c+dx)\right)}{(\sec(c+dx)a+a)^2}dx}{5a^2}+\frac{2a\tan(c+dx)\sec^2(c+dx)}{d(a\sec(c+dx)+a)^3}}{7a^2}-\frac{\tan(c+dx)\sec^3(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\frac{\int \frac{\sec^2(c+dx)\left(4a^2-7a^2\sec(c+dx)\right)}{(\sec(c+dx)a+a)^2}dx}{a^2}+\frac{2a\tan(c+dx)\sec^2(c+dx)}{d(a\sec(c+dx)+a)^3}}{7a^2}-\frac{\tan(c+dx)\sec^3(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2\left(4a^2-7a^2\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}dx}{a^2}+\frac{2a\tan(c+dx)\sec^2(c+dx)}{d(a\sec(c+dx)+a)^3}}{7a^2}-\frac{\tan(c+dx)\sec^3(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

$\downarrow$ 4496

$$-\frac{\frac{-\int -\frac{\sec(c+dx)\left(22a^3-21a^3\sec(c+dx)\right)}{\sec(c+dx)a+a}dx}{3a^2}-\frac{11\tan(c+dx)}{3d(\sec(c+dx)+1)^2}}{a^2}+\frac{2a\tan(c+dx)\sec^2(c+dx)}{d(a\sec(c+dx)+a)^3}}{7a^2}-\frac{\tan(c+dx)\sec^3(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

$\downarrow$ 25

$$-\frac{\frac{\frac{\int \frac{\sec(c+dx)\left(22a^3-21a^3\sec(c+dx)\right)}{\sec(c+dx)a+a}dx}{3a^2}-\frac{11\tan(c+dx)}{3d(\sec(c+dx)+1)^2}}{a^2}+\frac{2a\tan(c+dx)\sec^2(c+dx)}{d(a\sec(c+dx)+a)^3}}{7a^2}-\frac{\tan(c+dx)\sec^3(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

$\downarrow$ 3042

$$-\frac{\frac{\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(22a^3-21a^3\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{3a^2}-\frac{11\tan(c+dx)}{3d(\sec(c+dx)+1)^2}}{a^2}+\frac{2a\tan(c+dx)\sec^2(c+dx)}{d(a\sec(c+dx)+a)^3}}{7a^2}-\frac{\tan(c+dx)\sec^3(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

$\downarrow$ 4486

$$-\frac{\frac{\frac{43a^3\int \frac{\sec(c+dx)}{\sec(c+dx)a+a}dx-21a^2\int \sec(c+dx)dx}{3a^2}-\frac{11\tan(c+dx)}{3d(\sec(c+dx)+1)^2}}{a^2}+\frac{2a\tan(c+dx)\sec^2(c+dx)}{d(a\sec(c+dx)+a)^3}}{7a^2}-\frac{\tan(c+dx)\sec^3(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

$\downarrow$ 3042

$$-\frac{\frac{\frac{43a^3\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx-21a^2\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{3a^2}-\frac{11\tan(c+dx)}{3d(\sec(c+dx)+1)^2}}{a^2}+\frac{2a\tan(c+dx)\sec^2(c+dx)}{d(a\sec(c+dx)+a)^3}}{7a^2}-\frac{\tan(c+dx)\sec^3(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

$\downarrow$ 4257

3.72. $\int \frac{\sec^5(c+dx)}{(a+a\sec(c+dx))^4}\,dx$

$$-\frac{\frac{\frac{43a^3\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx-\frac{21a^2\text{arctanh}(\sin(c+dx))}{d}}{3a^2}-\frac{11\tan(c+dx)}{3d(\sec(c+dx)+1)^2}}{a^2}+\frac{2a\tan(c+dx)\sec^2(c+dx)}{d(a\sec(c+dx)+a)^3}}{7a^2}-\frac{\tan(c+dx)\sec^3(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

$$\Big\downarrow \; 4281$$

$$-\frac{\frac{\frac{\frac{43a^3\tan(c+dx)}{d(a\sec(c+dx)+a)}-\frac{21a^2\text{arctanh}(\sin(c+dx))}{d}}{3a^2}-\frac{11\tan(c+dx)}{3d(\sec(c+dx)+1)^2}}{a^2}+\frac{2a\tan(c+dx)\sec^2(c+dx)}{d(a\sec(c+dx)+a)^3}}{7a^2}-\frac{\tan(c+dx)\sec^3(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

input
```
Int[Sec[c + d*x]^5/(a + a*Sec[c + d*x])^4,x]
```

output
```
-1/7*(Sec[c + d*x]^3*Tan[c + d*x])/(d*(a + a*Sec[c + d*x])^4) - ((2*a*Sec[
c + d*x]^2*Tan[c + d*x])/(d*(a + a*Sec[c + d*x])^3) + ((-11*Tan[c + d*x])/
(3*d*(1 + Sec[c + d*x])^2) + ((-21*a^2*ArcTanh[Sin[c + d*x]])/d + (43*a^3*
Tan[c + d*x])/(d*(a + a*Sec[c + d*x])))/(3*a^2))/a^2)/(7*a^2)
```

3.72.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4281
```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[-Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

rule 4303
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-d^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d
*Csc[e + f*x])^(n - 2)/(f*(2*m + 1))), x] + Simp[d^2/(a*b*(2*m + 1))   Int[
(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n
 + 2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2,
0] && LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

rule 4486
```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/(csc[(
e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[B/b   Int[Csc[e + f*x],
 x], x] + Simp[(A*b - a*B)/b   Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x
] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0]
```

rule 4496
```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b - a*B))*Cot
[e + f*x]*((a + b*Csc[e + f*x])^m/(b*f*(2*m + 1))), x] + Simp[1/(b^2*(2*m +
 1))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[A*b*m - a*B*m + b
*B*(2*m + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && Ne
Q[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

rule 4507
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[d*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(a*f*(
2*m + 1))), x] - Simp[1/(a*b*(2*m + 1))   Int[(a + b*Csc[e + f*x])^(m + 1)*
(d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
 - n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f,
A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && G
tQ[n, 0]
```

3.72. $\int \frac{\sec^5(c+dx)}{(a+a\sec(c+dx))^4}\,dx$

3.72.4 Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.65

method	result
derivativedivides	$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{7}-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}-\frac{11\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{3}-15\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-8\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+8\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{8d\,a^{4}}$
default	$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{7}-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}-\frac{11\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{3}-15\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-8\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+8\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{8d\,a^{4}}$
parallelrisch	$\frac{-3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}-21\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}-77\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}-315\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-168\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+168\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}{168a^{4}d}$
risch	$-\frac{2i\left(21\,\mathrm{e}^{6i(dx+c)}+147\,\mathrm{e}^{5i(dx+c)}+434\,\mathrm{e}^{4i(dx+c)}+686\,\mathrm{e}^{3i(dx+c)}+525\,\mathrm{e}^{2i(dx+c)}+203\,\mathrm{e}^{i(dx+c)}+32\right)}{21d\,a^{4}\left(\mathrm{e}^{i(dx+c)}+1\right)^{7}}+\frac{\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{a^{4}d}-$
norman	$\frac{-\frac{15\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8ad}+\frac{169\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{24ad}-\frac{229\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{24ad}+\frac{293\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{56ad}-\frac{121\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{9}}{168ad}-\frac{11\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{11}}{168ad}-\frac{3\tan\left(\frac{dx}{2}\right.}{56a}}{\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)^{4}a^{3}}$

input
```
int(sec(d*x+c)^5/(a+a*sec(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
1/8/d/a^4*(-1/7*tan(1/2*d*x+1/2*c)^7-tan(1/2*d*x+1/2*c)^5-11/3*tan(1/2*d*x
+1/2*c)^3-15*tan(1/2*d*x+1/2*c)-8*ln(tan(1/2*d*x+1/2*c)-1)+8*ln(tan(1/2*d*
x+1/2*c)+1))
```

3.72.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.49

$$\int \frac{\sec^5(c+dx)}{(a+a\sec(c+dx))^4}\,dx$$
$$= \frac{21\left(\cos\left(dx+c\right)^4+4\,\cos\left(dx+c\right)^3+6\,\cos\left(dx+c\right)^2+4\,\cos\left(dx+c\right)+1\right)\log\left(\sin\left(dx+c\right)+1\right)-21\left(\cos\right.}{42\left(a^4d\cos\left(\right.\right.}$$

input
```
integrate(sec(d*x+c)^5/(a+a*sec(d*x+c))^4,x, algorithm="fricas")
```

output
```
1/42*(21*(cos(d*x + c)^4 + 4*cos(d*x + c)^3 + 6*cos(d*x + c)^2 + 4*cos(d*x
 + c) + 1)*log(sin(d*x + c) + 1) - 21*(cos(d*x + c)^4 + 4*cos(d*x + c)^3 +
 6*cos(d*x + c)^2 + 4*cos(d*x + c) + 1)*log(-sin(d*x + c) + 1) - 2*(32*cos
(d*x + c)^3 + 107*cos(d*x + c)^2 + 124*cos(d*x + c) + 52)*sin(d*x + c))/(a
^4*d*cos(d*x + c)^4 + 4*a^4*d*cos(d*x + c)^3 + 6*a^4*d*cos(d*x + c)^2 + 4*
a^4*d*cos(d*x + c) + a^4*d)
```

3.72.6 Sympy [F]

$$\int \frac{\sec^5(c+dx)}{(a+a\sec(c+dx))^4}\,dx = \frac{\int \frac{\sec^5(c+dx)}{\sec^4(c+dx)+4\sec^3(c+dx)+6\sec^2(c+dx)+4\sec(c+dx)+1}\,dx}{a^4}$$

input
```
integrate(sec(d*x+c)**5/(a+a*sec(d*x+c))**4,x)
```

output
```
Integral(sec(c + d*x)**5/(sec(c + d*x)**4 + 4*sec(c + d*x)**3 + 6*sec(c +
d*x)**2 + 4*sec(c + d*x) + 1), x)/a**4
```

3.72.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.02

$$\int \frac{\sec^5(c+dx)}{(a+a\sec(c+dx))^4}\,dx = -\frac{\frac{\frac{315\sin(dx+c)}{\cos(dx+c)+1}+\frac{77\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{21\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{3\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{a^4}-\frac{168\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^4}+\frac{168\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^4}}{168\,d}$$

input
```
integrate(sec(d*x+c)^5/(a+a*sec(d*x+c))^4,x, algorithm="maxima")
```

output
```
-1/168*((315*sin(d*x + c)/(cos(d*x + c) + 1) + 77*sin(d*x + c)^3/(cos(d*x
+ c) + 1)^3 + 21*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 3*sin(d*x + c)^7/(c
os(d*x + c) + 1)^7)/a^4 - 168*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^4
 + 168*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a^4)/d
```

3.72.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.81

$$\int \frac{\sec^5(c+dx)}{(a+a\sec(c+dx))^4}\,dx = \frac{\frac{168\log(|\tan(\frac{1}{2}dx+\frac{1}{2}c)+1|)}{a^4} - \frac{168\log(|\tan(\frac{1}{2}dx+\frac{1}{2}c)-1|)}{a^4} - \frac{3a^{24}\tan(\frac{1}{2}dx+\frac{1}{2}c)^7+21a^{24}\tan(\frac{1}{2}dx+\frac{1}{2}c)^5+77a^{24}\tan(\frac{1}{2}dx+\frac{1}{2}c)^3+315}{a^{28}}}{168d}$$

input
```
integrate(sec(d*x+c)^5/(a+a*sec(d*x+c))^4,x, algorithm="giac")
```

output
```
1/168*(168*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^4 - 168*log(abs(tan(1/2*d*
x + 1/2*c) - 1))/a^4 - (3*a^24*tan(1/2*d*x + 1/2*c)^7 + 21*a^24*tan(1/2*d*
x + 1/2*c)^5 + 77*a^24*tan(1/2*d*x + 1/2*c)^3 + 315*a^24*tan(1/2*d*x + 1/2
*c))/a^28)/d
```

3.72.9 Mupad [B] (verification not implemented)

Time = 13.36 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.61

$$\int \frac{\sec^5(c+dx)}{(a+a\sec(c+dx))^4}\,dx = -\frac{\frac{11\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{24a^4} + \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5}{8a^4} + \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7}{56a^4} - \frac{2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{a^4} + \frac{15\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{8a^4}}{d}$$

input
```
int(1/(cos(c + d*x)^5*(a + a/cos(c + d*x))^4),x)
```

output
```
-((11*tan(c/2 + (d*x)/2)^3)/(24*a^4) + tan(c/2 + (d*x)/2)^5/(8*a^4) + tan(
c/2 + (d*x)/2)^7/(56*a^4) - (2*atanh(tan(c/2 + (d*x)/2)))/a^4 + (15*tan(c/
2 + (d*x)/2))/(8*a^4))/d
```

3.73 $\int \frac{\sec^4(c+dx)}{(a+a\sec(c+dx))^4}\,dx$

3.73.1 Optimal result . . . 750
3.73.2 Mathematica [A] (verified) . . . 750
3.73.3 Rubi [A] (verified) . . . 751
3.73.4 Maple [A] (verified) . . . 753
3.73.5 Fricas [A] (verification not implemented) . . . 754
3.73.6 Sympy [F] . . . 754
3.73.7 Maxima [A] (verification not implemented) . . . 755
3.73.8 Giac [A] (verification not implemented) . . . 755
3.73.9 Mupad [B] (verification not implemented) . . . 755

3.73.1 Optimal result

Integrand size = 21, antiderivative size = 120

$$\int \frac{\sec^4(c+dx)}{(a+a\sec(c+dx))^4}\,dx = \frac{\sec^3(c+dx)\tan(c+dx)}{7d(a+a\sec(c+dx))^4} + \frac{3\tan(c+dx)}{35ad(a+a\sec(c+dx))^3} - \frac{8\tan(c+dx)}{35d\left(a^2+a^2\sec(c+dx)\right)^2} + \frac{\tan(c+dx)}{5d\left(a^4+a^4\sec(c+dx)\right)}$$

output
```
1/7*sec(d*x+c)^3*tan(d*x+c)/d/(a+a*sec(d*x+c))^4+3/35*tan(d*x+c)/a/d/(a+a*
sec(d*x+c))^3-8/35*tan(d*x+c)/d/(a^2+a^2*sec(d*x+c))^2+1/5*tan(d*x+c)/d/(a
^4+a^4*sec(d*x+c))
```

3.73.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.58

$$\int \frac{\sec^4(c+dx)}{(a+a\sec(c+dx))^4}\,dx = \frac{\sec^7\left(\frac{1}{2}(c+dx)\right)\left(35\sin\left(\frac{1}{2}(c+dx)\right)+21\sin\left(\frac{3}{2}(c+dx)\right)+7\sin\left(\frac{5}{2}(c+dx)\right)+\sin\left(\frac{7}{2}(c+dx)\right)\right)}{1120a^4d}$$

input
```
Integrate[Sec[c + d*x]^4/(a + a*Sec[c + d*x])^4,x]
```

output
```
(Sec[(c + d*x)/2]^7*(35*Sin[(c + d*x)/2] + 21*Sin[(3*(c + d*x))/2] + 7*Sin
[(5*(c + d*x))/2] + Sin[(7*(c + d*x))/2]))/(1120*a^4*d)
```

3.73.3 Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.05, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3042, 4297, 3042, 4286, 25, 3042, 4488, 3042, 4281}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^4(c+dx)}{(a\sec(c+dx)+a)^4}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^4}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^4}dx$$

$$\Big\downarrow \text{4297}$$

$$\frac{3\int \frac{\sec^3(c+dx)}{(\sec(c+dx)a+a)^3}dx}{7a}+\frac{\tan(c+dx)\sec^3(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^3}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^3}dx}{7a}+\frac{\tan(c+dx)\sec^3(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

$$\Big\downarrow \text{4286}$$

$$\frac{3\left(\frac{\int -\frac{\sec(c+dx)(3a-5a\sec(c+dx))}{(\sec(c+dx)a+a)^2}dx}{5a^2}+\frac{\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}\right)}{7a}+\frac{\tan(c+dx)\sec^3(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

$$\Big\downarrow \text{25}$$

$$\frac{3\left(\frac{\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}-\frac{\int \frac{\sec(c+dx)(3a-5a\sec(c+dx))}{(\sec(c+dx)a+a)^2}dx}{5a^2}\right)}{7a}+\frac{\tan(c+dx)\sec^3(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\left(\frac{\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}-\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(3a-5a\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}dx}{5a^2}\right)}{7a}+\frac{\tan(c+dx)\sec^3(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

$$\Big\downarrow \text{4488}$$

$$\frac{3\left(\frac{\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}-\frac{\frac{8a\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}-\frac{7}{3}\int\frac{\sec(c+dx)}{\sec(c+dx)a+a}dx}{5a^2}\right)}{7a}+\frac{\tan(c+dx)\sec^3(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

$$\Big\downarrow \; 3042$$

$$\frac{3\left(\frac{\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}-\frac{\frac{8a\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}-\frac{7}{3}\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{5a^2}\right)}{7a}+\frac{\tan(c+dx)\sec^3(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

$$\Big\downarrow \; 4281$$

$$\frac{3\left(\frac{\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}-\frac{\frac{8a\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}-\frac{7\tan(c+dx)}{3d(a\sec(c+dx)+a)}}{5a^2}\right)}{7a}+\frac{\tan(c+dx)\sec^3(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

input
```
Int[Sec[c + d*x]^4/(a + a*Sec[c + d*x])^4,x]
```

output
```
(Sec[c + d*x]^3*Tan[c + d*x])/(7*d*(a + a*Sec[c + d*x])^4) + (3*(Tan[c + d
*x]/(5*d*(a + a*Sec[c + d*x])^3) - ((8*a*Tan[c + d*x])/(3*d*(a + a*Sec[c +
 d*x])^2) - (7*Tan[c + d*x])/(3*d*(a + a*Sec[c + d*x])))/(5*a^2)))/(7*a)
```

3.73.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4281
```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[-Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

rule 4286
```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[b*Cot[e + f*x]*((a + b*Csc[e + f*x])^m/(a*f*(2*m + 1))),
x] - Simp[1/(a^2*(2*m + 1))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
*(a*m - b*(2*m + 1)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[
a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

rule 4297
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[b*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Cs
c[e + f*x])^(n - 1)/(a*f*(2*m + 1))), x] + Simp[d*((m + 1)/(b*(2*m + 1)))
 Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ
[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && EqQ[m + n, 0] && LtQ[m,
-2^(-1)] && IntegerQ[2*m]
```

rule 4488
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b - a*B)*Cot[e +
f*x]*((a + b*Csc[e + f*x])^m/(a*f*(2*m + 1))), x] + Simp[(a*B*m + A*b*(m +
1))/(a*b*(2*m + 1))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x], x]
 /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[a*B*m + A*b*(m + 1), 0] && LtQ[m, -2^(-1)]
```

3.73.4 Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.47

method	result	siz
derivativedivides	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{7}+\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{5}+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8d\,a^{4}}$	56
default	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{7}+\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{5}+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8d\,a^{4}}$	56
risch	$\frac{4i\left(35\,\mathrm{e}^{3i(dx+c)}+21\,\mathrm{e}^{2i(dx+c)}+7\,\mathrm{e}^{i(dx+c)}+1\right)}{35d\,a^{4}\left(\mathrm{e}^{i(dx+c)}+1\right)^{7}}$	58
parallelrisch	$\frac{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}+21\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}+35\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}+35\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{280a^{4}d}$	60
norman	$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8ad}+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{4ad}-\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{40ad}-\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{70ad}-\frac{13\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{9}}{280ad}+\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{11}}{140ad}+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{13}}{56ad}}{\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)^{3}a^{3}}$	15

input
```
int(sec(d*x+c)^4/(a+a*sec(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
1/8/d/a^4*(1/7*tan(1/2*d*x+1/2*c)^7+3/5*tan(1/2*d*x+1/2*c)^5+tan(1/2*d*x+1
/2*c)^3+tan(1/2*d*x+1/2*c))
```

3.73.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.82

$$\int \frac{\sec^4(c+dx)}{(a+a\sec(c+dx))^4}\,dx = \frac{\left(2\cos(dx+c)^3+8\cos(dx+c)^2+13\cos(dx+c)+12\right)\sin(dx+c)}{35\left(a^4d\cos(dx+c)^4+4a^4d\cos(dx+c)^3+6a^4d\cos(dx+c)^2+4a^4d\cos(dx+c)+a^4d\right)}$$

input
```
integrate(sec(d*x+c)^4/(a+a*sec(d*x+c))^4,x, algorithm="fricas")
```

output
```
1/35*(2*cos(d*x + c)^3 + 8*cos(d*x + c)^2 + 13*cos(d*x + c) + 12)*sin(d*x
+ c)/(a^4*d*cos(d*x + c)^4 + 4*a^4*d*cos(d*x + c)^3 + 6*a^4*d*cos(d*x + c)
^2 + 4*a^4*d*cos(d*x + c) + a^4*d)
```

3.73.6 Sympy [F]

$$\int \frac{\sec^4(c+dx)}{(a+a\sec(c+dx))^4}\,dx = \frac{\int \frac{\sec^4(c+dx)}{\sec^4(c+dx)+4\sec^3(c+dx)+6\sec^2(c+dx)+4\sec(c+dx)+1}\,dx}{a^4}$$

input
```
integrate(sec(d*x+c)**4/(a+a*sec(d*x+c))**4,x)
```

output
```
Integral(sec(c + d*x)**4/(sec(c + d*x)**4 + 4*sec(c + d*x)**3 + 6*sec(c +
d*x)**2 + 4*sec(c + d*x) + 1), x)/a**4
```

3.73.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.72

$$\int \frac{\sec^4(c+dx)}{(a+a\sec(c+dx))^4}\,dx = \frac{\frac{35\,\sin(dx+c)}{\cos(dx+c)+1} + \frac{35\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{21\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5} + \frac{5\,\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{280\,a^4 d}$$

input
```
integrate(sec(d*x+c)^4/(a+a*sec(d*x+c))^4,x, algorithm="maxima")
```

output
```
1/280*(35*sin(d*x + c)/(cos(d*x + c) + 1) + 35*sin(d*x + c)^3/(cos(d*x + c
) + 1)^3 + 21*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 5*sin(d*x + c)^7/(cos(
d*x + c) + 1)^7)/(a^4*d)
```

3.73.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.49

$$\int \frac{\sec^4(c+dx)}{(a+a\sec(c+dx))^4}\,dx = \frac{5\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7 + 21\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5 + 35\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 + 35\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{280\,a^4 d}$$

input
```
integrate(sec(d*x+c)^4/(a+a*sec(d*x+c))^4,x, algorithm="giac")
```

output
```
1/280*(5*tan(1/2*d*x + 1/2*c)^7 + 21*tan(1/2*d*x + 1/2*c)^5 + 35*tan(1/2*d
*x + 1/2*c)^3 + 35*tan(1/2*d*x + 1/2*c))/(a^4*d)
```

3.73.9 Mupad [B] (verification not implemented)

Time = 13.15 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.48

$$\int \frac{\sec^4(c+dx)}{(a+a\sec(c+dx))^4}\,dx = \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\,\left(5\,\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6 + 21\,\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4 + 35\,\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2 + 35\right)}{280\,a^4\,d}$$

input
```
int(1/(cos(c + d*x)^4*(a + a/cos(c + d*x))^4),x)
```

output
```
(tan(c/2 + (d*x)/2)*(35*tan(c/2 + (d*x)/2)^2 + 21*tan(c/2 + (d*x)/2)^4 + 5
*tan(c/2 + (d*x)/2)^6 + 35))/(280*a^4*d)
```

3.73. $\int \frac{\sec^4(c+dx)}{(a+a\sec(c+dx))^4}\,dx$

3.74 $\int \frac{\sec^3(c+dx)}{(a+a\sec(c+dx))^4}\,dx$

3.74.1 Optimal result . . . 757
3.74.2 Mathematica [A] (verified) . . . 757
3.74.3 Rubi [A] (verified) . . . 758
3.74.4 Maple [A] (verified) . . . 760
3.74.5 Fricas [A] (verification not implemented) . . . 761
3.74.6 Sympy [F] . . . 761
3.74.7 Maxima [A] (verification not implemented) . . . 761
3.74.8 Giac [A] (verification not implemented) . . . 762
3.74.9 Mupad [B] (verification not implemented) . . . 762

3.74.1 Optimal result

Integrand size = 21, antiderivative size = 112

$$\int \frac{\sec^3(c+dx)}{(a+a\sec(c+dx))^4}\,dx = \frac{\tan(c+dx)}{7d(a+a\sec(c+dx))^4} - \frac{11\tan(c+dx)}{35ad(a+a\sec(c+dx))^3} + \frac{13\tan(c+dx)}{105d\left(a^2+a^2\sec(c+dx)\right)^2} + \frac{13\tan(c+dx)}{105d\left(a^4+a^4\sec(c+dx)\right)}$$

output
```
1/7*tan(d*x+c)/d/(a+a*sec(d*x+c))^4-11/35*tan(d*x+c)/a/d/(a+a*sec(d*x+c))^
3+13/105*tan(d*x+c)/d/(a^2+a^2*sec(d*x+c))^2+13/105*tan(d*x+c)/d/(a^4+a^4*
sec(d*x+c))
```

3.74.2 Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.50

$$\int \frac{\sec^3(c+dx)}{(a+a\sec(c+dx))^4}\,dx = \frac{(29+58\cos(c+dx)+16\cos(2(c+dx))+2\cos(3(c+dx)))\sin(c+dx)}{105a^4d(1+\cos(c+dx))^4}$$

input
```
Integrate[Sec[c + d*x]^3/(a + a*Sec[c + d*x])^4,x]
```

output
```
((29 + 58*Cos[c + d*x] + 16*Cos[2*(c + d*x)] + 2*Cos[3*(c + d*x)])*Sin[c +
 d*x])/(105*a^4*d*(1 + Cos[c + d*x])^4)
```

3.74.3 Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.05, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3042, 4286, 25, 3042, 4488, 3042, 4283, 3042, 4281}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^3(c+dx)}{(a\sec(c+dx)+a)^4}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^3}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^4}dx$$

$$\Big\downarrow \text{4286}$$

$$\frac{\int -\frac{\sec(c+dx)(4a-7a\sec(c+dx))}{(\sec(c+dx)a+a)^3}dx}{7a^2}+\frac{\tan(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

$$\Big\downarrow \text{25}$$

$$\frac{\tan(c+dx)}{7d(a\sec(c+dx)+a)^4}-\frac{\int \frac{\sec(c+dx)(4a-7a\sec(c+dx))}{(\sec(c+dx)a+a)^3}dx}{7a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\tan(c+dx)}{7d(a\sec(c+dx)+a)^4}-\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(4a-7a\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^3}dx}{7a^2}$$

$$\Big\downarrow \text{4488}$$

$$\frac{\tan(c+dx)}{7d(a\sec(c+dx)+a)^4}-\frac{\frac{11a\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}-\frac{13}{5}\int \frac{\sec(c+dx)}{(\sec(c+dx)a+a)^2}dx}{7a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\tan(c+dx)}{7d(a\sec(c+dx)+a)^4}-\frac{\frac{11a\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}-\frac{13}{5}\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}dx}{7a^2}$$

$$\Big\downarrow \text{4283}$$

$$\frac{\tan(c+dx)}{7d(a\sec(c+dx)+a)^4}-\frac{\frac{11a\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}-\frac{13}{5}\left(\frac{\int \frac{\sec(c+dx)}{\sec(c+dx)a+a}dx}{3a}+\frac{\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}\right)}{7a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\tan(c+dx)}{7d(a\sec(c+dx)+a)^4}-\frac{\frac{11a\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}-\frac{13}{5}\left(\frac{\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{3a}+\frac{\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}\right)}{7a^2}$$

$$\Big\downarrow \text{4281}$$

$$\frac{\tan(c+dx)}{7d(a\sec(c+dx)+a)^4}-\frac{\frac{11a\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}-\frac{13}{5}\left(\frac{\tan(c+dx)}{3ad(a\sec(c+dx)+a)}+\frac{\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}\right)}{7a^2}$$

input
```
Int[Sec[c + d*x]^3/(a + a*Sec[c + d*x])^4,x]
```

output
```
Tan[c + d*x]/(7*d*(a + a*Sec[c + d*x])^4) - ((11*a*Tan[c + d*x])/(5*d*(a +
 a*Sec[c + d*x])^3) - (13*(Tan[c + d*x]/(3*d*(a + a*Sec[c + d*x])^2) + Tan
[c + d*x]/(3*a*d*(a + a*Sec[c + d*x]))))/5)/(7*a^2)
```

3.74.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4281
```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[-Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

rule 4283
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[b*Cot[e + f*x]*((a + b*Csc[e + f*x])^m/(a*f*(2*m + 1))), x]
 + Simp[(m + 1)/(a*(2*m + 1))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m +
1), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)
] && IntegerQ[2*m]
```

rule 4286
```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[b*Cot[e + f*x]*((a + b*Csc[e + f*x])^m/(a*f*(2*m + 1))),
x] - Simp[1/(a^2*(2*m + 1))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
*(a*m - b*(2*m + 1)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[
a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

rule 4488
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b - a*B)*Cot[e +
f*x]*((a + b*Csc[e + f*x])^m/(a*f*(2*m + 1))), x] + Simp[(a*B*m + A*b*(m +
1))/(a*b*(2*m + 1))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x], x]
 /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[a*B*m + A*b*(m + 1), 0] && LtQ[m, -2^(-1)]
```

3.74.4 Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.51

method	result	size
parallelrisch	$-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}+\frac{7\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}}{5}-\frac{7\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}{3}-7\right)}{56a^{4}d}$	57
derivativedivides	$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{7}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{5}+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{3}+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8d\,a^{4}}$	58
default	$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{7}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{5}+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{3}+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8d\,a^{4}}$	58
risch	$\frac{8i\left(35\,\mathrm{e}^{4i(dx+c)}+35\,\mathrm{e}^{3i(dx+c)}+42\,\mathrm{e}^{2i(dx+c)}+14\,\mathrm{e}^{i(dx+c)}+2\right)}{105d\,a^{4}\left(\mathrm{e}^{i(dx+c)}+1\right)^{7}}$	69
norman	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8ad}-\frac{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{24ad}+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{60ad}+\frac{31\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{420ad}+\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{9}}{280ad}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{11}}{56ad}}{\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)^{2}a^{3}}$	133

input
```
int(sec(d*x+c)^3/(a+a*sec(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
-1/56*tan(1/2*d*x+1/2*c)*(tan(1/2*d*x+1/2*c)^6+7/5*tan(1/2*d*x+1/2*c)^4-7/
3*tan(1/2*d*x+1/2*c)^2-7)/a^4/d
```

3.74.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.88

$$\int \frac{\sec^3(c+dx)}{(a+a\sec(c+dx))^4}\,dx = \frac{\left(8\cos\left(dx+c\right)^3+32\cos\left(dx+c\right)^2+52\cos\left(dx+c\right)+13\right)\sin\left(dx+c\right)}{105\left(a^4d\cos\left(dx+c\right)^4+4\,a^4d\cos\left(dx+c\right)^3+6\,a^4d\cos\left(dx+c\right)^2+4\,a^4d\cos\left(dx+c\right)+a^4d\right)}$$

input
```
integrate(sec(d*x+c)^3/(a+a*sec(d*x+c))^4,x, algorithm="fricas")
```

output
```
1/105*(8*cos(d*x + c)^3 + 32*cos(d*x + c)^2 + 52*cos(d*x + c) + 13)*sin(d*
x + c)/(a^4*d*cos(d*x + c)^4 + 4*a^4*d*cos(d*x + c)^3 + 6*a^4*d*cos(d*x +
c)^2 + 4*a^4*d*cos(d*x + c) + a^4*d)
```

3.74.6 Sympy [F]

$$\int \frac{\sec^3(c+dx)}{(a+a\sec(c+dx))^4}\,dx = \frac{\int \frac{\sec^3(c+dx)}{\sec^4(c+dx)+4\sec^3(c+dx)+6\sec^2(c+dx)+4\sec(c+dx)+1}\,dx}{a^4}$$

input
```
integrate(sec(d*x+c)**3/(a+a*sec(d*x+c))**4,x)
```

output
```
Integral(sec(c + d*x)**3/(sec(c + d*x)**4 + 4*sec(c + d*x)**3 + 6*sec(c +
d*x)**2 + 4*sec(c + d*x) + 1), x)/a**4
```

3.74.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.78

$$\int \frac{\sec^3(c+dx)}{(a+a\sec(c+dx))^4}\,dx = \frac{\frac{105\sin(dx+c)}{\cos(dx+c)+1}+\frac{35\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{21\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{15\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{840\,a^4d}$$

input
```
integrate(sec(d*x+c)^3/(a+a*sec(d*x+c))^4,x, algorithm="maxima")
```

output
```
1/840*(105*sin(d*x + c)/(cos(d*x + c) + 1) + 35*sin(d*x + c)^3/(cos(d*x +
c) + 1)^3 - 21*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 15*sin(d*x + c)^7/(co
s(d*x + c) + 1)^7)/(a^4*d)
```

3.74.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.53

$$\int \frac{\sec^3(c+dx)}{(a+a\sec(c+dx))^4}\,dx = -\frac{15\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+21\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-35\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-105\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{840\,a^4d}$$

input
```
integrate(sec(d*x+c)^3/(a+a*sec(d*x+c))^4,x, algorithm="giac")
```

output
```
-1/840*(15*tan(1/2*d*x + 1/2*c)^7 + 21*tan(1/2*d*x + 1/2*c)^5 - 35*tan(1/2
*d*x + 1/2*c)^3 - 105*tan(1/2*d*x + 1/2*c))/(a^4*d)
```

3.74.9 Mupad [B] (verification not implemented)

Time = 13.87 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.52

$$\int \frac{\sec^3(c+dx)}{(a+a\sec(c+dx))^4}\,dx = \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\,\left(-15\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6-21\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+35\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+105\right)}{840\,a^4\,d}$$

input
```
int(1/(cos(c + d*x)^3*(a + a/cos(c + d*x))^4),x)
```

output
```
(tan(c/2 + (d*x)/2)*(35*tan(c/2 + (d*x)/2)^2 - 21*tan(c/2 + (d*x)/2)^4 - 1
5*tan(c/2 + (d*x)/2)^6 + 105))/(840*a^4*d)
```

3.75 $\int \frac{\sec^2(c+dx)}{(a+a\sec(c+dx))^4}\,dx$

3.75.1 Optimal result . . . 763
3.75.2 Mathematica [A] (verified) . . . 763
3.75.3 Rubi [A] (verified) . . . 764
3.75.4 Maple [A] (verified) . . . 766
3.75.5 Fricas [A] (verification not implemented) . . . 766
3.75.6 Sympy [F] . . . 767
3.75.7 Maxima [A] (verification not implemented) . . . 767
3.75.8 Giac [A] (verification not implemented) . . . 768
3.75.9 Mupad [B] (verification not implemented) . . . 768

3.75.1 Optimal result

Integrand size = 21, antiderivative size = 112

$$\int \frac{\sec^2(c+dx)}{(a+a\sec(c+dx))^4}\,dx = -\frac{\tan(c+dx)}{7d(a+a\sec(c+dx))^4}+\frac{4\tan(c+dx)}{35ad(a+a\sec(c+dx))^3}+\frac{8\tan(c+dx)}{105d\left(a^2+a^2\sec(c+dx)\right)^2}+\frac{8\tan(c+dx)}{105d\left(a^4+a^4\sec(c+dx)\right)}$$

output
```
-1/7*tan(d*x+c)/d/(a+a*sec(d*x+c))^4+4/35*tan(d*x+c)/a/d/(a+a*sec(d*x+c))^
3+8/105*tan(d*x+c)/d/(a^2+a^2*sec(d*x+c))^2+8/105*tan(d*x+c)/d/(a^4+a^4*se
c(d*x+c))
```

3.75.2 Mathematica [A] (verified)

Time = 1.55 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.50

$$\int \frac{\sec^2(c+dx)}{(a+a\sec(c+dx))^4}\,dx = \frac{(13+52\sec(c+dx)+32\sec^2(c+dx)+8\sec^3(c+dx))\tan(c+dx)}{105a^4d(1+\sec(c+dx))^4}$$

input
```
Integrate[Sec[c + d*x]^2/(a + a*Sec[c + d*x])^4,x]
```

output
```
((13 + 52*Sec[c + d*x] + 32*Sec[c + d*x]^2 + 8*Sec[c + d*x]^3)*Tan[c + d*x
])/(105*a^4*d*(1 + Sec[c + d*x])^4)
```

3.75.3 Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.07, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {3042, 4284, 3042, 4283, 3042, 4283, 3042, 4281}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(c+dx)}{(a\sec(c+dx)+a)^4}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^4}dx$$

$$\Big\downarrow \text{4284}$$

$$\frac{4\int \frac{\sec(c+dx)}{(\sec(c+dx)a+a)^3}dx}{7a}-\frac{\tan(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^3}dx}{7a}-\frac{\tan(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

$$\Big\downarrow \text{4283}$$

$$\frac{4\left(\frac{2\int \frac{\sec(c+dx)}{(\sec(c+dx)a+a)^2}dx}{5a}+\frac{\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}\right)}{7a}-\frac{\tan(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4\left(\frac{2\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}dx}{5a}+\frac{\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}\right)}{7a}-\frac{\tan(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

$$\Big\downarrow \text{4283}$$

$$\frac{4\left(\frac{2\left(\frac{\int \frac{\sec(c+dx)}{\sec(c+dx)a+a}dx}{3a}+\frac{\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}\right)}{5a}+\frac{\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}\right)}{7a}-\frac{\tan(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4\left(\frac{2\left(\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{3a}+\frac{\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}\right)}{5a}+\frac{\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}\right)}{7a}-\frac{\tan(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

$$\Big\downarrow \text{4281}$$

$$\frac{4\left(\frac{\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}+\frac{2\left(\frac{\tan(c+dx)}{3ad(a\sec(c+dx)+a)}+\frac{\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}\right)}{5a}\right)}{7a}-\frac{\tan(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

input
```
Int[Sec[c + d*x]^2/(a + a*Sec[c + d*x])^4,x]
```

output
```
-1/7*Tan[c + d*x]/(d*(a + a*Sec[c + d*x])^4) + (4*(Tan[c + d*x]/(5*d*(a +
a*Sec[c + d*x])^3) + (2*(Tan[c + d*x]/(3*d*(a + a*Sec[c + d*x])^2) + Tan[c
 + d*x]/(3*a*d*(a + a*Sec[c + d*x]))))/(5*a)))/(7*a)
```

3.75.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4281
```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[-Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

rule 4283
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[b*Cot[e + f*x]*((a + b*Csc[e + f*x])^m/(a*f*(2*m + 1))), x]
 + Simp[(m + 1)/(a*(2*m + 1))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m +
1), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)
] && IntegerQ[2*m]
```

rule 4284
```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(-Cot[e + f*x])*((a + b*Csc[e + f*x])^m/(f*(2*m + 1))), x
] + Simp[m/(b*(2*m + 1))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x
], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

3.75.4 Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.52

method	result	size
derivativedivides	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{7}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{5}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{3}+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8d\,a^{4}}$	58
default	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{7}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{5}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{3}+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8d\,a^{4}}$	58
parallelrisch	$\frac{15\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}-21\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}-35\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}+105\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{840a^{4}d}$	60
risch	$\frac{2i\left(105\,\mathrm{e}^{5i(dx+c)}+175\,\mathrm{e}^{4i(dx+c)}+280\,\mathrm{e}^{3i(dx+c)}+168\,\mathrm{e}^{2i(dx+c)}+91\,\mathrm{e}^{i(dx+c)}+13\right)}{105d\,a^{4}\left(\mathrm{e}^{i(dx+c)}+1\right)^{7}}$	80
norman	$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8ad}+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{6ad}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{60ad}-\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{70ad}+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{9}}{56ad}}{\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)a^{3}}$	114

input
```
int(sec(d*x+c)^2/(a+a*sec(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
1/8/d/a^4*(1/7*tan(1/2*d*x+1/2*c)^7-1/5*tan(1/2*d*x+1/2*c)^5-1/3*tan(1/2*d
*x+1/2*c)^3+tan(1/2*d*x+1/2*c))
```

3.75.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.88

$$\int \frac{\sec^2(c+dx)}{(a+a\sec(c+dx))^4}\,dx = \frac{\left(13\cos\left(dx+c\right)^3+52\cos\left(dx+c\right)^2+32\cos\left(dx+c\right)+8\right)\sin\left(dx+c\right)}{105\left(a^4d\cos\left(dx+c\right)^4+4\,a^4d\cos\left(dx+c\right)^3+6\,a^4d\cos\left(dx+c\right)^2+4\,a^4d\cos\left(dx+c\right)+a^4d\right)}$$

input
```
integrate(sec(d*x+c)^2/(a+a*sec(d*x+c))^4,x, algorithm="fricas")
```

output
```
1/105*(13*cos(d*x + c)^3 + 52*cos(d*x + c)^2 + 32*cos(d*x + c) + 8)*sin(d*
x + c)/(a^4*d*cos(d*x + c)^4 + 4*a^4*d*cos(d*x + c)^3 + 6*a^4*d*cos(d*x +
c)^2 + 4*a^4*d*cos(d*x + c) + a^4*d)
```

3.75.6 Sympy [F]

$$\int \frac{\sec^2(c+dx)}{(a+a\sec(c+dx))^4}\,dx = \frac{\int \frac{\sec^2(c+dx)}{\sec^4(c+dx)+4\sec^3(c+dx)+6\sec^2(c+dx)+4\sec(c+dx)+1}\,dx}{a^4}$$

input
```
integrate(sec(d*x+c)**2/(a+a*sec(d*x+c))**4,x)
```

output
```
Integral(sec(c + d*x)**2/(sec(c + d*x)**4 + 4*sec(c + d*x)**3 + 6*sec(c +
d*x)**2 + 4*sec(c + d*x) + 1), x)/a**4
```

3.75.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.78

$$\int \frac{\sec^2(c+dx)}{(a+a\sec(c+dx))^4}\,dx = \frac{\frac{105\sin(dx+c)}{\cos(dx+c)+1} - \frac{35\sin(dx+c)^3}{(\cos(dx+c)+1)^3} - \frac{21\sin(dx+c)^5}{(\cos(dx+c)+1)^5} + \frac{15\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{840\,a^4 d}$$

input
```
integrate(sec(d*x+c)^2/(a+a*sec(d*x+c))^4,x, algorithm="maxima")
```

output
```
1/840*(105*sin(d*x + c)/(cos(d*x + c) + 1) - 35*sin(d*x + c)^3/(cos(d*x +
c) + 1)^3 - 21*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 15*sin(d*x + c)^7/(co
s(d*x + c) + 1)^7)/(a^4*d)
```

3.75.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.53

$$\int \frac{\sec^2(c+dx)}{(a+a\sec(c+dx))^4}\,dx = \frac{15\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-21\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-35\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+105\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{840\,a^4d}$$

input
```
integrate(sec(d*x+c)^2/(a+a*sec(d*x+c))^4,x, algorithm="giac")
```

output
```
1/840*(15*tan(1/2*d*x + 1/2*c)^7 - 21*tan(1/2*d*x + 1/2*c)^5 - 35*tan(1/2*
d*x + 1/2*c)^3 + 105*tan(1/2*d*x + 1/2*c))/(a^4*d)
```

3.75.9 Mupad [B] (verification not implemented)

Time = 13.98 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.52

$$\int \frac{\sec^2(c+dx)}{(a+a\sec(c+dx))^4}\,dx = -\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\,\left(-15\,\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6+21\,\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+35\,\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2-105\right)}{840\,a^4\,d}$$

input
```
int(1/(cos(c + d*x)^2*(a + a/cos(c + d*x))^4),x)
```

output
```
-(tan(c/2 + (d*x)/2)*(35*tan(c/2 + (d*x)/2)^2 + 21*tan(c/2 + (d*x)/2)^4 -
15*tan(c/2 + (d*x)/2)^6 - 105))/(840*a^4*d)
```

3.76 $\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^4}\,dx$

3.76.1 Optimal result . . . 769
3.76.2 Mathematica [A] (verified) . . . 769
3.76.3 Rubi [A] (verified) . . . 770
3.76.4 Maple [A] (verified) . . . 772
3.76.5 Fricas [A] (verification not implemented) . . . 772
3.76.6 Sympy [F] . . . 773
3.76.7 Maxima [A] (verification not implemented) . . . 773
3.76.8 Giac [A] (verification not implemented) . . . 773
3.76.9 Mupad [B] (verification not implemented) . . . 774

3.76.1 Optimal result

Integrand size = 19, antiderivative size = 112

$$\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^4}\,dx = \frac{\tan(c+dx)}{7d(a+a\sec(c+dx))^4} + \frac{3\tan(c+dx)}{35ad(a+a\sec(c+dx))^3} + \frac{2\tan(c+dx)}{35d\left(a^2+a^2\sec(c+dx)\right)^2} + \frac{2\tan(c+dx)}{35d\left(a^4+a^4\sec(c+dx)\right)}$$

output
```
1/7*tan(d*x+c)/d/(a+a*sec(d*x+c))^4+3/35*tan(d*x+c)/a/d/(a+a*sec(d*x+c))^3
+2/35*tan(d*x+c)/d/(a^2+a^2*sec(d*x+c))^2+2/35*tan(d*x+c)/d/(a^4+a^4*sec(d
*x+c))
```

3.76.2 Mathematica [A] (verified)

Time = 2.52 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.00

$$\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^4}\,dx = \frac{\sec\left(\frac{c}{2}\right)\sec^7\left(\frac{1}{2}(c+dx)\right)\left(210\sin\left(\frac{dx}{2}\right)-210\sin\left(c+\frac{dx}{2}\right)+147\sin\left(c+\frac{3dx}{2}\right)-105\sin\left(2c+\frac{3dx}{2}\right)+49\sin\right.}{2240a^4d}$$

input
```
Integrate[Sec[c + d*x]/(a + a*Sec[c + d*x])^4,x]
```

output
```
(Sec[c/2]*Sec[(c + d*x)/2]^7*(210*Sin[(d*x)/2] - 210*Sin[c + (d*x)/2] + 14
7*Sin[c + (3*d*x)/2] - 105*Sin[2*c + (3*d*x)/2] + 49*Sin[2*c + (5*d*x)/2]
- 35*Sin[3*c + (5*d*x)/2] + 12*Sin[3*c + (7*d*x)/2]))/(2240*a^4*d)
```

3.76.3 Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.07, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {3042, 4283, 3042, 4283, 3042, 4283, 3042, 4281}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(c+dx)}{(a\sec(c+dx)+a)^4}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^4}dx$$

$$\Big\downarrow \text{4283}$$

$$\frac{3\int \frac{\sec(c+dx)}{(\sec(c+dx)a+a)^3}dx}{7a}+\frac{\tan(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^3}dx}{7a}+\frac{\tan(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

$$\Big\downarrow \text{4283}$$

$$\frac{3\left(\frac{2\int \frac{\sec(c+dx)}{(\sec(c+dx)a+a)^2}dx}{5a}+\frac{\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}\right)}{7a}+\frac{\tan(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\left(\frac{2\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}dx}{5a}+\frac{\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}\right)}{7a}+\frac{\tan(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

$$\Big\downarrow \text{4283}$$

$$\frac{3\left(\frac{2\left(\frac{\int \frac{\sec(c+dx)}{\sec(c+dx)a+a}dx}{3a}+\frac{\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}\right)}{5a}+\frac{\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}\right)}{7a}+\frac{\tan(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

$$\Big\downarrow \; 3042$$

$$\frac{3\left(\frac{2\left(\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{3a}+\frac{\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}\right)}{5a}+\frac{\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}\right)}{7a}+\frac{\tan(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

$$\Big\downarrow \; 4281$$

$$\frac{\tan(c+dx)}{7d(a\sec(c+dx)+a)^4}+\frac{3\left(\frac{\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}+\frac{2\left(\frac{\tan(c+dx)}{3ad(a\sec(c+dx)+a)}+\frac{\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}\right)}{5a}\right)}{7a}$$

input
```
Int[Sec[c + d*x]/(a + a*Sec[c + d*x])^4,x]
```

output
```
Tan[c + d*x]/(7*d*(a + a*Sec[c + d*x])^4) + (3*(Tan[c + d*x]/(5*d*(a + a*S
ec[c + d*x])^3) + (2*(Tan[c + d*x]/(3*d*(a + a*Sec[c + d*x])^2) + Tan[c +
d*x]/(3*a*d*(a + a*Sec[c + d*x]))))/(5*a)))/(7*a)
```

3.76.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4281
```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[-Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

rule 4283
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[b*Cot[e + f*x]*((a + b*Csc[e + f*x])^m/(a*f*(2*m + 1))), x]
 + Simp[(m + 1)/(a*(2*m + 1))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m +
1), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)
] && IntegerQ[2*m]
```

3.76.4 Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.51

method	result	size
parallelrisch	$-\frac{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}-\frac{21\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}}{5}+7\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-7\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{56a^{4}d}$	57
derivativedivides	$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{7}+\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{5}-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8d\,a^{4}}$	58
default	$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{7}+\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{5}-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8d\,a^{4}}$	58
norman	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8ad}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{8ad}+\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{40ad}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{56ad}}{a^{3}}$	80
risch	$\frac{2i\left(35\,\mathrm{e}^{6i(dx+c)}+105\,\mathrm{e}^{5i(dx+c)}+210\,\mathrm{e}^{4i(dx+c)}+210\,\mathrm{e}^{3i(dx+c)}+147\,\mathrm{e}^{2i(dx+c)}+49\,\mathrm{e}^{i(dx+c)}+12\right)}{35d\,a^{4}\left(\mathrm{e}^{i(dx+c)}+1\right)^{7}}$	91

input
```
int(sec(d*x+c)/(a+a*sec(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
-1/56*(tan(1/2*d*x+1/2*c)^6-21/5*tan(1/2*d*x+1/2*c)^4+7*tan(1/2*d*x+1/2*c)
^2-7)*tan(1/2*d*x+1/2*c)/a^4/d
```

3.76.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.88

$$\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^4}\,dx = \frac{\left(12\,\cos\left(dx+c\right)^{3}+13\,\cos\left(dx+c\right)^{2}+8\,\cos\left(dx+c\right)+2\right)\sin\left(dx+c\right)}{35\left(a^{4}d\cos\left(dx+c\right)^{4}+4\,a^{4}d\cos\left(dx+c\right)^{3}+6\,a^{4}d\cos\left(dx+c\right)^{2}+4\,a^{4}d\cos\left(dx+c\right)+a^{4}d\right)}$$

input
```
integrate(sec(d*x+c)/(a+a*sec(d*x+c))^4,x, algorithm="fricas")
```

output
```
1/35*(12*cos(d*x + c)^3 + 13*cos(d*x + c)^2 + 8*cos(d*x + c) + 2)*sin(d*x
+ c)/(a^4*d*cos(d*x + c)^4 + 4*a^4*d*cos(d*x + c)^3 + 6*a^4*d*cos(d*x + c)
^2 + 4*a^4*d*cos(d*x + c) + a^4*d)
```

3.76.6 Sympy [F]

$$\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^4}\,dx = \frac{\int \frac{\sec(c+dx)}{\sec^4(c+dx)+4\sec^3(c+dx)+6\sec^2(c+dx)+4\sec(c+dx)+1}\,dx}{a^4}$$

input
```
integrate(sec(d*x+c)/(a+a*sec(d*x+c))**4,x)
```

output
```
Integral(sec(c + d*x)/(sec(c + d*x)**4 + 4*sec(c + d*x)**3 + 6*sec(c + d*x
)**2 + 4*sec(c + d*x) + 1), x)/a**4
```

3.76.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.78

$$\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^4}\,dx = \frac{\frac{35\sin(dx+c)}{\cos(dx+c)+1} - \frac{35\sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{21\sin(dx+c)^5}{(\cos(dx+c)+1)^5} - \frac{5\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{280\,a^4 d}$$

input
```
integrate(sec(d*x+c)/(a+a*sec(d*x+c))^4,x, algorithm="maxima")
```

output
```
1/280*(35*sin(d*x + c)/(cos(d*x + c) + 1) - 35*sin(d*x + c)^3/(cos(d*x + c
) + 1)^3 + 21*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 5*sin(d*x + c)^7/(cos(
d*x + c) + 1)^7)/(a^4*d)
```

3.76.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.53

$$\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^4}\,dx = -\frac{5\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7 - 21\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5 + 35\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3 - 35\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{280\,a^4 d}$$

input
```
integrate(sec(d*x+c)/(a+a*sec(d*x+c))^4,x, algorithm="giac")
```

output
```
-1/280*(5*tan(1/2*d*x + 1/2*c)^7 - 21*tan(1/2*d*x + 1/2*c)^5 + 35*tan(1/2*
d*x + 1/2*c)^3 - 35*tan(1/2*d*x + 1/2*c))/(a^4*d)
```

3.76.9 Mupad [B] (verification not implemented)

Time = 13.46 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.52

$$\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^4}\,dx = -\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\left(5\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6-21\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+35\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2-35\right)}{280\,a^4\,d}$$

input
```
int(1/(cos(c + d*x)*(a + a/cos(c + d*x))^4),x)
```

output
```
-(tan(c/2 + (d*x)/2)*(35*tan(c/2 + (d*x)/2)^2 - 21*tan(c/2 + (d*x)/2)^4 +
5*tan(c/2 + (d*x)/2)^6 - 35))/(280*a^4*d)
```

3.77 $\int \frac{1}{(a+a\sec(c+dx))^4}\,dx$

3.77.1 Optimal result . . . 775
3.77.2 Mathematica [B] (verified) . . . 775
3.77.3 Rubi [A] (verified) . . . 776
3.77.4 Maple [A] (verified) . . . 779
3.77.5 Fricas [A] (verification not implemented) . . . 779
3.77.6 Sympy [F] . . . 780
3.77.7 Maxima [A] (verification not implemented) . . . 780
3.77.8 Giac [A] (verification not implemented) . . . 781
3.77.9 Mupad [B] (verification not implemented) . . . 781

3.77.1 Optimal result

Integrand size = 12, antiderivative size = 111

$$\int \frac{1}{(a+a\sec(c+dx))^4}\,dx = \frac{x}{a^4}-\frac{11\tan(c+dx)}{21a^4d(1+\sec(c+dx))^2}-\frac{32\tan(c+dx)}{21a^4d(1+\sec(c+dx))}-\frac{\tan(c+dx)}{7d(a+a\sec(c+dx))^4}-\frac{2\tan(c+dx)}{7ad(a+a\sec(c+dx))^3}$$

output
```
x/a^4-11/21*tan(d*x+c)/a^4/d/(1+sec(d*x+c))^2-32/21*tan(d*x+c)/a^4/d/(1+se
c(d*x+c))-1/7*tan(d*x+c)/d/(a+a*sec(d*x+c))^4-2/7*tan(d*x+c)/a/d/(a+a*sec(
d*x+c))^3
```

3.77.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 224 vs. 2(111) = 222.

Time = 2.53 (sec) , antiderivative size = 224, normalized size of antiderivative = 2.02

$$\int \frac{1}{(a+a\sec(c+dx))^4}\,dx = \frac{\sec\left(\frac{c}{2}\right)\sec^7\left(\frac{1}{2}(c+dx)\right)\left(735dx\cos\left(\frac{dx}{2}\right)+735dx\cos\left(c+\frac{dx}{2}\right)+441dx\cos\left(c+\frac{3dx}{2}\right)+441dx\cos\left(2c+\frac{3dx}{2}\right.\right.}{}$$

input
```
Integrate[(a + a*Sec[c + d*x])^(-4),x]
```

output
```
(Sec[c/2]*Sec[(c + d*x)/2]^7*(735*d*x*Cos[(d*x)/2] + 735*d*x*Cos[c + (d*x)
/2] + 441*d*x*Cos[c + (3*d*x)/2] + 441*d*x*Cos[2*c + (3*d*x)/2] + 147*d*x*
Cos[2*c + (5*d*x)/2] + 147*d*x*Cos[3*c + (5*d*x)/2] + 21*d*x*Cos[3*c + (7*
d*x)/2] + 21*d*x*Cos[4*c + (7*d*x)/2] - 1988*Sin[(d*x)/2] + 1652*Sin[c + (
d*x)/2] - 1428*Sin[c + (3*d*x)/2] + 756*Sin[2*c + (3*d*x)/2] - 560*Sin[2*c
 + (5*d*x)/2] + 168*Sin[3*c + (5*d*x)/2] - 104*Sin[3*c + (7*d*x)/2]))/(268
8*a^4*d)
```

3.77.3 Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.14, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 1.083$, Rules used = {3042, 4264, 25, 3042, 4410, 27, 3042, 4410, 25, 3042, 4407, 3042, 4281}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\sec(c+dx)+a)^4}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^4}dx$$

$$\Big\downarrow \text{4264}$$

$$-\frac{\int -\frac{7a-3a\sec(c+dx)}{(\sec(c+dx)a+a)^3}dx}{7a^2}-\frac{\tan(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{7a-3a\sec(c+dx)}{(\sec(c+dx)a+a)^3}dx}{7a^2}-\frac{\tan(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{7a-3a\csc\left(c+dx+\frac{\pi}{2}\right)}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^3}dx}{7a^2}-\frac{\tan(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

$$\Big\downarrow \text{4410}$$

$$\frac{-\frac{\int -\frac{5\left(7a^2-4a^2\sec(c+dx)\right)}{(\sec(c+dx)a+a)^2}dx}{5a^2}-\frac{2a\tan(c+dx)}{d(a\sec(c+dx)+a)^3}}{7a^2}-\frac{\tan(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\int \frac{7a^2-4a^2\sec(c+dx)}{(\sec(c+dx)a+a)^2}dx}{a^2}-\frac{2a\tan(c+dx)}{d(a\sec(c+dx)+a)^3}}{7a^2}-\frac{\tan(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{\int \frac{7a^2-4a^2\csc\left(c+dx+\frac{\pi}{2}\right)}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}dx}{a^2}-\frac{2a\tan(c+dx)}{d(a\sec(c+dx)+a)^3}}{7a^2}-\frac{\tan(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

$$\Big\downarrow \; 4410$$

$$\frac{\frac{-\frac{\int -\frac{21a^3-11a^3\sec(c+dx)}{\sec(c+dx)a+a}dx}{3a^2}-\frac{11\tan(c+dx)}{3d(\sec(c+dx)+1)^2}}{a^2}-\frac{2a\tan(c+dx)}{d(a\sec(c+dx)+a)^3}}{7a^2}-\frac{\tan(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\frac{\int \frac{21a^3-11a^3\sec(c+dx)}{\sec(c+dx)a+a}dx}{3a^2}-\frac{11\tan(c+dx)}{3d(\sec(c+dx)+1)^2}}{a^2}-\frac{2a\tan(c+dx)}{d(a\sec(c+dx)+a)^3}}{7a^2}-\frac{\tan(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{\frac{\int \frac{21a^3-11a^3\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{3a^2}-\frac{11\tan(c+dx)}{3d(\sec(c+dx)+1)^2}}{a^2}-\frac{2a\tan(c+dx)}{d(a\sec(c+dx)+a)^3}}{7a^2}-\frac{\tan(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

$$\Big\downarrow \; 4407$$

$$\frac{\frac{\frac{21a^2x-32a^3\int \frac{\sec(c+dx)}{\sec(c+dx)a+a}dx}{3a^2}-\frac{11\tan(c+dx)}{3d(\sec(c+dx)+1)^2}}{a^2}-\frac{2a\tan(c+dx)}{d(a\sec(c+dx)+a)^3}}{7a^2}-\frac{\tan(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{\frac{21a^2x-32a^3\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{3a^2}-\frac{11\tan(c+dx)}{3d(\sec(c+dx)+1)^2}}{a^2}-\frac{2a\tan(c+dx)}{d(a\sec(c+dx)+a)^3}}{7a^2}-\frac{\tan(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

$$\Big\downarrow \; 4281$$

$$\frac{\frac{\frac{21a^2x-\frac{32a^3\tan(c+dx)}{d(a\sec(c+dx)+a)}}{3a^2}-\frac{11\tan(c+dx)}{3d(\sec(c+dx)+1)^2}}{a^2}-\frac{2a\tan(c+dx)}{d(a\sec(c+dx)+a)^3}}{7a^2}-\frac{\tan(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

3.77. $\int \frac{1}{(a+a\sec(c+dx))^4}\,dx$

input
```
Int[(a + a*Sec[c + d*x])^(-4),x]
```

output
```
-1/7*Tan[c + d*x]/(d*(a + a*Sec[c + d*x])^4) + ((-2*a*Tan[c + d*x])/(d*(a
+ a*Sec[c + d*x])^3) + ((-11*Tan[c + d*x])/(3*d*(1 + Sec[c + d*x])^2) + (2
1*a^2*x - (32*a^3*Tan[c + d*x])/(d*(a + a*Sec[c + d*x])))/(3*a^2))/a^2)/(7
*a^2)
```

3.77.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4264
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(-Cot[c
+ d*x])*((a + b*Csc[c + d*x])^n/(d*(2*n + 1))), x] + Simp[1/(a^2*(2*n + 1))
   Int[(a + b*Csc[c + d*x])^(n + 1)*(a*(2*n + 1) - b*(n + 1)*Csc[c + d*x]),
 x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LeQ[n, -1] && Int
egerQ[2*n]
```

rule 4281
```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[-Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

rule 4407
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Simp[(b*c - a*d)/a   Int[Csc[e + f*
x]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
- a*d, 0]
```

rule 4410
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> Simp[(-(b*c - a*d))*Cot[e + f*x]*((a + b*Csc[e +
f*x])^m/(b*f*(2*m + 1))), x] + Simp[1/(a^2*(2*m + 1))   Int[(a + b*Csc[e +
f*x])^(m + 1)*Simp[a*c*(2*m + 1) - (b*c - a*d)*(m + 1)*Csc[e + f*x], x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&
EqQ[a^2 - b^2, 0] && IntegerQ[2*m]
```

3.77.4 Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.58

method	result	size
parallelrisch	$\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}-21\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}+77\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}+168dx-315\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{168a^{4}d}$	64
derivativedivides	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{7}-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}+\frac{11\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{3}-15\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+16\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8d\,a^{4}}$	72
default	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{7}-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}+\frac{11\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{3}-15\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+16\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8d\,a^{4}}$	72
norman	$\frac{\frac{x}{a}-\frac{15\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8ad}+\frac{11\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{24ad}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{8ad}+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{56ad}}{a^{3}}$	85
risch	$\frac{x}{a^{4}}-\frac{4i\left(42\,\mathrm{e}^{6i(dx+c)}+189\,\mathrm{e}^{5i(dx+c)}+413\,\mathrm{e}^{4i(dx+c)}+497\,\mathrm{e}^{3i(dx+c)}+357\,\mathrm{e}^{2i(dx+c)}+140\,\mathrm{e}^{i(dx+c)}+26\right)}{21d\,a^{4}\left(\mathrm{e}^{i(dx+c)}+1\right)^{7}}$	97

input
```
int(1/(a+a*sec(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
1/168*(3*tan(1/2*d*x+1/2*c)^7-21*tan(1/2*d*x+1/2*c)^5+77*tan(1/2*d*x+1/2*c
)^3+168*d*x-315*tan(1/2*d*x+1/2*c))/a^4/d
```

3.77.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.37

$$\int \frac{1}{(a+a\sec(c+dx))^4}\,dx$$
$$= \frac{21\,dx\cos(dx+c)^4+84\,dx\cos(dx+c)^3+126\,dx\cos(dx+c)^2+84\,dx\cos(dx+c)+21\,dx-(52\,\cos(dx}{21\left(a^4d\cos(dx+c)^4+4\,a^4d\cos(dx+c)^3+6\,a^4d\cos(dx+c)^2+\right.}$$

input
```
integrate(1/(a+a*sec(d*x+c))^4,x, algorithm="fricas")
```

output
```
1/21*(21*d*x*cos(d*x + c)^4 + 84*d*x*cos(d*x + c)^3 + 126*d*x*cos(d*x + c)
^2 + 84*d*x*cos(d*x + c) + 21*d*x - (52*cos(d*x + c)^3 + 124*cos(d*x + c)^
2 + 107*cos(d*x + c) + 32)*sin(d*x + c))/(a^4*d*cos(d*x + c)^4 + 4*a^4*d*c
os(d*x + c)^3 + 6*a^4*d*cos(d*x + c)^2 + 4*a^4*d*cos(d*x + c) + a^4*d)
```

3.77.6 Sympy [F]

$$\int \frac{1}{(a+a\sec(c+dx))^4}\,dx = \frac{\int \frac{1}{\sec^4(c+dx)+4\sec^3(c+dx)+6\sec^2(c+dx)+4\sec(c+dx)+1}\,dx}{a^4}$$

input
```
integrate(1/(a+a*sec(d*x+c))**4,x)
```

output
```
Integral(1/(sec(c + d*x)**4 + 4*sec(c + d*x)**3 + 6*sec(c + d*x)**2 + 4*se
c(c + d*x) + 1), x)/a**4
```

3.77.7 Maxima [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.01

$$\int \frac{1}{(a+a\sec(c+dx))^4}\,dx = -\frac{\frac{\frac{315\sin(dx+c)}{\cos(dx+c)+1}-\frac{77\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{21\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{3\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{a^4}-\frac{336\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^4}}{168\,d}$$

input
```
integrate(1/(a+a*sec(d*x+c))^4,x, algorithm="maxima")
```

output
```
-1/168*((315*sin(d*x + c)/(cos(d*x + c) + 1) - 77*sin(d*x + c)^3/(cos(d*x
+ c) + 1)^3 + 21*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 3*sin(d*x + c)^7/(c
os(d*x + c) + 1)^7)/a^4 - 336*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^4)
/d
```

3.77.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.75

$$\int \frac{1}{(a+a\sec(c+dx))^4}\,dx = \frac{\frac{168\,(dx+c)}{a^4}+\frac{3\,a^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-21\,a^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+77\,a^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-315\,a^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^{28}}}{168\,d}$$

input
```
integrate(1/(a+a*sec(d*x+c))^4,x, algorithm="giac")
```

output
```
1/168*(168*(d*x + c)/a^4 + (3*a^24*tan(1/2*d*x + 1/2*c)^7 - 21*a^24*tan(1/
2*d*x + 1/2*c)^5 + 77*a^24*tan(1/2*d*x + 1/2*c)^3 - 315*a^24*tan(1/2*d*x +
 1/2*c))/a^28)/d
```

3.77.9 Mupad [B] (verification not implemented)

Time = 13.58 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.92

$$\int \frac{1}{(a+a\sec(c+dx))^4}\,dx = \frac{x}{a^4} + \frac{-\frac{52\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6}{21}+\frac{16\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{21}-\frac{5\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{28}+\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{56}}{a^4\,d\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}$$

input
```
int(1/(a + a/cos(c + d*x))^4,x)
```

output
```
x/a^4 + (sin(c/2 + (d*x)/2)/56 - (5*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2
))/28 + (16*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2))/21 - (52*cos(c/2 + (d
*x)/2)^6*sin(c/2 + (d*x)/2))/21)/(a^4*d*cos(c/2 + (d*x)/2)^7)
```

3.78 $\int \frac{\cos(c+dx)}{(a+a\sec(c+dx))^4}\,dx$

3.78.1 Optimal result . . . 782
3.78.2 Mathematica [A] (verified) . . . 782
3.78.3 Rubi [A] (verified) . . . 783
3.78.4 Maple [A] (verified) . . . 787
3.78.5 Fricas [A] (verification not implemented) . . . 787
3.78.6 Sympy [F] . . . 788
3.78.7 Maxima [A] (verification not implemented) . . . 788
3.78.8 Giac [A] (verification not implemented) . . . 788
3.78.9 Mupad [B] (verification not implemented) . . . 789

3.78.1 Optimal result

Integrand size = 19, antiderivative size = 126

$$\int \frac{\cos(c+dx)}{(a+a\sec(c+dx))^4}\,dx = -\frac{4x}{a^4}+\frac{664\sin(c+dx)}{105a^4d}-\frac{88\sin(c+dx)}{105a^4d(1+\sec(c+dx))^2}-\frac{4\sin(c+dx)}{a^4d(1+\sec(c+dx))}-\frac{\sin(c+dx)}{7d(a+a\sec(c+dx))^4}-\frac{12\sin(c+dx)}{35ad(a+a\sec(c+dx))^3}$$

output
```
-4*x/a^4+664/105*sin(d*x+c)/a^4/d-88/105*sin(d*x+c)/a^4/d/(1+sec(d*x+c))^2
-4*sin(d*x+c)/a^4/d/(1+sec(d*x+c))-1/7*sin(d*x+c)/d/(a+a*sec(d*x+c))^4-12/
35*sin(d*x+c)/a/d/(a+a*sec(d*x+c))^3
```

3.78.2 Mathematica [A] (verified)

Time = 2.77 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.93

$$\int \frac{\cos(c+dx)}{(a+a\sec(c+dx))^4}\,dx = \frac{\sin(c+dx)\left(6720\arcsin(\cos(c+dx))\cos^8\left(\frac{1}{2}(c+dx)\right)+(664+2236\cos(c+dx)+2636\cos^2(c+dx)+\right.}{105a^4d\sqrt{1-\cos(c+dx)}(1+\cos(c+dx))^{9/2}}$$

input
```
Integrate[Cos[c + d*x]/(a + a*Sec[c + d*x])^4,x]
```

output
```
(Sin[c + d*x]*(6720*ArcSin[Cos[c + d*x]]*Cos[(c + d*x)/2]^8 + (664 + 2236*
Cos[c + d*x] + 2636*Cos[c + d*x]^2 + 1184*Cos[c + d*x]^3 + 105*Cos[c + d*x
]^4)*Sqrt[Sin[c + d*x]^2]))/(105*a^4*d*Sqrt[1 - Cos[c + d*x]]*(1 + Cos[c +
 d*x])^(9/2))
```

3.78.3 Rubi [A] (verified)

Time = 1.00 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.19, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.737$, Rules used = {3042, 4304, 27, 3042, 4508, 3042, 4508, 3042, 4508, 3042, 4274, 24, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(c+dx)}{(a\sec(c+dx)+a)^4}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^4}dx$$

$$\Big\downarrow \text{4304}$$

$$-\frac{\int -\frac{4\cos(c+dx)(2a-a\sec(c+dx))}{(\sec(c+dx)a+a)^3}dx}{7a^2}-\frac{\sin(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

$$\Big\downarrow \text{27}$$

$$\frac{4\int \frac{\cos(c+dx)(2a-a\sec(c+dx))}{(\sec(c+dx)a+a)^3}dx}{7a^2}-\frac{\sin(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4\int \frac{2a-a\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^3}dx}{7a^2}-\frac{\sin(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

$$\Big\downarrow \text{4508}$$

$$\frac{4\left(\frac{\int \frac{\cos(c+dx)\left(13a^2-9a^2\sec(c+dx)\right)}{(\sec(c+dx)a+a)^2}dx}{5a^2}-\frac{3a\sin(c+dx)}{5d(a\sec(c+dx)+a)^3}\right)}{7a^2}-\frac{\sin(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

$$\Big\downarrow \; 3042$$

$$\frac{4\left(\frac{\int \frac{13a^2-9a^2\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}dx}{5a^2}-\frac{3a\sin(c+dx)}{5d(a\sec(c+dx)+a)^3}\right)}{7a^2}-\frac{\sin(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

$$\Big\downarrow \; 4508$$

$$\frac{4\left(\frac{\frac{\int \frac{\cos(c+dx)\left(61a^3-44a^3\sec(c+dx)\right)}{\sec(c+dx)a+a}dx}{3a^2}-\frac{22\sin(c+dx)}{3d(\sec(c+dx)+1)^2}}{5a^2}-\frac{3a\sin(c+dx)}{5d(a\sec(c+dx)+a)^3}\right)}{7a^2}-\frac{\sin(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

$$\Big\downarrow \; 3042$$

$$\frac{4\left(\frac{\frac{\int \frac{61a^3-44a^3\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)}dx}{3a^2}-\frac{22\sin(c+dx)}{3d(\sec(c+dx)+1)^2}}{5a^2}-\frac{3a\sin(c+dx)}{5d(a\sec(c+dx)+a)^3}\right)}{7a^2}-\frac{\sin(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

$$\Big\downarrow \; 4508$$

$$\frac{4\left(\frac{\frac{\frac{\int \cos(c+dx)\left(166a^4-105a^4\sec(c+dx)\right)dx}{a^2}-\frac{105a^3\sin(c+dx)}{d(a\sec(c+dx)+a)}}{3a^2}-\frac{22\sin(c+dx)}{3d(\sec(c+dx)+1)^2}}{5a^2}-\frac{3a\sin(c+dx)}{5d(a\sec(c+dx)+a)^3}\right)}{7a^2}-\frac{\sin(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

$$\Big\downarrow \; 3042$$

$$\frac{4\left(\frac{\frac{\frac{\int \frac{166a^4-105a^4\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}-\frac{105a^3\sin(c+dx)}{d(a\sec(c+dx)+a)}}{3a^2}-\frac{22\sin(c+dx)}{3d(\sec(c+dx)+1)^2}}{5a^2}-\frac{3a\sin(c+dx)}{5d(a\sec(c+dx)+a)^3}\right)}{7a^2}-\frac{\sin(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

$$\Big\downarrow \; 4274$$

3.78. $\int \frac{\cos(c+dx)}{(a+a\sec(c+dx))^4}\,dx$

$$\frac{4\left(\frac{\frac{\frac{166a^4\int\cos(c+dx)dx-105a^4\int 1dx}{a^2}-\frac{105a^3\sin(c+dx)}{d(a\sec(c+dx)+a)}}{3a^2}-\frac{22\sin(c+dx)}{3d(\sec(c+dx)+1)^2}}{5a^2}-\frac{3a\sin(c+dx)}{5d(a\sec(c+dx)+a)^3}\right)}{7a^2}-\frac{\sin(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

$\downarrow$ 24

$$\frac{4\left(\frac{\frac{\frac{166a^4\int\cos(c+dx)dx-105a^4x}{a^2}-\frac{105a^3\sin(c+dx)}{d(a\sec(c+dx)+a)}}{3a^2}-\frac{22\sin(c+dx)}{3d(\sec(c+dx)+1)^2}}{5a^2}-\frac{3a\sin(c+dx)}{5d(a\sec(c+dx)+a)^3}\right)}{7a^2}-\frac{\sin(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

$\downarrow$ 3042

$$\frac{4\left(\frac{\frac{\frac{166a^4\int\sin\left(c+dx+\frac{\pi}{2}\right)dx-105a^4x}{a^2}-\frac{105a^3\sin(c+dx)}{d(a\sec(c+dx)+a)}}{3a^2}-\frac{22\sin(c+dx)}{3d(\sec(c+dx)+1)^2}}{5a^2}-\frac{3a\sin(c+dx)}{5d(a\sec(c+dx)+a)^3}\right)}{7a^2}-\frac{\sin(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

$\downarrow$ 3117

$$\frac{4\left(\frac{\frac{\frac{166a^4\frac{\sin(c+dx)}{d}-105a^4x}{a^2}-\frac{105a^3\sin(c+dx)}{d(a\sec(c+dx)+a)}}{3a^2}-\frac{22\sin(c+dx)}{3d(\sec(c+dx)+1)^2}}{5a^2}-\frac{3a\sin(c+dx)}{5d(a\sec(c+dx)+a)^3}\right)}{7a^2}-\frac{\sin(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

input
```
Int[Cos[c + d*x]/(a + a*Sec[c + d*x])^4,x]
```

output
```
-1/7*Sin[c + d*x]/(d*(a + a*Sec[c + d*x])^4) + (4*((-3*a*Sin[c + d*x])/(5*
d*(a + a*Sec[c + d*x])^3) + ((-22*Sin[c + d*x])/(3*d*(1 + Sec[c + d*x])^2)
 + ((-105*a^3*Sin[c + d*x])/(d*(a + a*Sec[c + d*x])) + (-105*a^4*x + (166*
a^4*Sin[c + d*x])/d)/a^2)/(3*a^2))/(5*a^2)))/(7*a^2)
```

3.78. $\int \frac{\cos(c+dx)}{(a+a\sec(c+dx))^4}\,dx$

3.78.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4304
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-Cot[e + f*x])*(a + b*Csc[e + f*x])^m*((d*Csc
[e + f*x])^n/(f*(2*m + 1))), x] + Simp[1/(a^2*(2*m + 1))   Int[(a + b*Csc[e
 + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e
+ f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ
[m, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

rule 4508
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
- a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
 1))), x] - Simp[1/(a^2*(2*m + 1))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Cs
c[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[
e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B
, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

3.78.4 Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.61

method	result
parallelrisch	$\frac{781\left(\cos(dx+c)+\frac{2741\cos(2dx+2c)}{6248}+\frac{74\cos(3dx+3c)}{781}+\frac{105\cos(4dx+4c)}{24992}+\frac{16171}{24992}\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\sec\left(\frac{dx}{2}+\frac{c}{2}\right)^6-840dx}{210a^4d}$
derivativedivides	$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{7}+\frac{7\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{5}-\frac{23\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3}+49\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{16\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}-64\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8d\,a^4}$
default	$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{7}+\frac{7\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{5}-\frac{23\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3}+49\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{16\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}-64\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8d\,a^4}$
risch	$-\frac{4x}{a^4}-\frac{ie^{i(dx+c)}}{2a^4d}+\frac{ie^{-i(dx+c)}}{2a^4d}+\frac{4i\left(525\,e^{6i(dx+c)}+2625\,e^{5i(dx+c)}+5950\,e^{4i(dx+c)}+7420\,e^{3i(dx+c)}+5397\,e^{2i(dx+c}\right.}{105d\,a^4\left(e^{i(dx+c)}+1\right)^7}$
norman	$\frac{-\frac{4x}{a}+\frac{65\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8ad}+\frac{31\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{6ad}-\frac{47\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{60ad}+\frac{11\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{70ad}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^9}{56ad}-\frac{4x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{a}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)a^3}$

input
```
int(cos(d*x+c)/(a+a*sec(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
1/210*(781*(cos(d*x+c)+2741/6248*cos(2*d*x+2*c)+74/781*cos(3*d*x+3*c)+105/
24992*cos(4*d*x+4*c)+16171/24992)*tan(1/2*d*x+1/2*c)*sec(1/2*d*x+1/2*c)^6-
840*d*x)/a^4/d
```

3.78.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.29

$$\int \frac{\cos(c+dx)}{(a+a\sec(c+dx))^4}\,dx = -\frac{420\,dx\cos(dx+c)^4+1680\,dx\cos(dx+c)^3+2520\,dx\cos(dx+c)^2+1680\,dx\cos(dx+c)+420\,dx-}{105\left(a^4d\cos(dx+c)^4+4\,a^4d\cos(dx+c)^3+6\right.}$$

input
```
integrate(cos(d*x+c)/(a+a*sec(d*x+c))^4,x, algorithm="fricas")
```

output
```
-1/105*(420*d*x*cos(d*x + c)^4 + 1680*d*x*cos(d*x + c)^3 + 2520*d*x*cos(d*
x + c)^2 + 1680*d*x*cos(d*x + c) + 420*d*x - (105*cos(d*x + c)^4 + 1184*co
s(d*x + c)^3 + 2636*cos(d*x + c)^2 + 2236*cos(d*x + c) + 664)*sin(d*x + c)
)/(a^4*d*cos(d*x + c)^4 + 4*a^4*d*cos(d*x + c)^3 + 6*a^4*d*cos(d*x + c)^2
+ 4*a^4*d*cos(d*x + c) + a^4*d)
```

3.78.6 Sympy [F]

$$\int \frac{\cos(c+dx)}{(a+a\sec(c+dx))^4}\,dx = \frac{\int \frac{\cos(c+dx)}{\sec^4(c+dx)+4\sec^3(c+dx)+6\sec^2(c+dx)+4\sec(c+dx)+1}\,dx}{a^4}$$

input
```
integrate(cos(d*x+c)/(a+a*sec(d*x+c))**4,x)
```

output
```
Integral(cos(c + d*x)/(sec(c + d*x)**4 + 4*sec(c + d*x)**3 + 6*sec(c + d*x
)**2 + 4*sec(c + d*x) + 1), x)/a**4
```

3.78.7 Maxima [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.25

$$\int \frac{\cos(c+dx)}{(a+a\sec(c+dx))^4}\,dx = \frac{\frac{1680\sin(dx+c)}{\left(a^4+\frac{a^4\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)} + \frac{\frac{5145\sin(dx+c)}{\cos(dx+c)+1} - \frac{805\sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{147\sin(dx+c)^5}{(\cos(dx+c)+1)^5} - \frac{15\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{a^4} - \frac{6720\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^4}}{840\,d}$$

input
```
integrate(cos(d*x+c)/(a+a*sec(d*x+c))^4,x, algorithm="maxima")
```

output
```
1/840*(1680*sin(d*x + c)/((a^4 + a^4*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)*
(cos(d*x + c) + 1)) + (5145*sin(d*x + c)/(cos(d*x + c) + 1) - 805*sin(d*x
+ c)^3/(cos(d*x + c) + 1)^3 + 147*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 15
*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)/a^4 - 6720*arctan(sin(d*x + c)/(cos(
d*x + c) + 1))/a^4)/d
```

3.78.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.89

$$\int \frac{\cos(c+dx)}{(a+a\sec(c+dx))^4}\,dx = -\frac{\frac{3360\,(dx+c)}{a^4} - \frac{1680\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)a^4} + \frac{15\,a^{24}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7 - 147\,a^{24}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5 + 805\,a^{24}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3 - 5145\,a^{24}\tan\left(\frac{1}{2}dx\right.}{a^{28}}}{840\,d}$$

input
```
integrate(cos(d*x+c)/(a+a*sec(d*x+c))^4,x, algorithm="giac")
```

output
```
-1/840*(3360*(d*x + c)/a^4 - 1680*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/2
*c)^2 + 1)*a^4) + (15*a^24*tan(1/2*d*x + 1/2*c)^7 - 147*a^24*tan(1/2*d*x +
 1/2*c)^5 + 805*a^24*tan(1/2*d*x + 1/2*c)^3 - 5145*a^24*tan(1/2*d*x + 1/2*
c))/a^28)/d
```

3.78.9 Mupad [B] (verification not implemented)

Time = 13.71 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.09

$$\int \frac{\cos(c+dx)}{(a+a\sec(c+dx))^4}\,dx = -\frac{15\,\sin\left(\frac{c}{2}+\frac{dx}{2}\right)-192\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^2\sin\left(\frac{c}{2}+\frac{dx}{2}\right)+1144\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^4\sin\left(\frac{c}{2}+\frac{dx}{2}\right)-6112\cos\left(\frac{c}{2}+\frac{d}{2}\right.}{840\,a^4\,d\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{7}}$$

input
```
int(cos(c + d*x)/(a + a/cos(c + d*x))^4,x)
```

output
```
-(15*sin(c/2 + (d*x)/2) - 192*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2) + 11
44*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2) - 6112*cos(c/2 + (d*x)/2)^6*sin
(c/2 + (d*x)/2) - 1680*cos(c/2 + (d*x)/2)^8*sin(c/2 + (d*x)/2) + 3360*cos(
c/2 + (d*x)/2)^7*(c + d*x))/(840*a^4*d*cos(c/2 + (d*x)/2)^7)
```

3.79 $\int \frac{\cos^2(c+dx)}{(a+a\sec(c+dx))^4}\,dx$

3.79.1 Optimal result . . . 790
3.79.2 Mathematica [A] (verified) . . . 790
3.79.3 Rubi [A] (verified) . . . 791
3.79.4 Maple [A] (verified) . . . 795
3.79.5 Fricas [A] (verification not implemented) . . . 796
3.79.6 Sympy [F] . . . 796
3.79.7 Maxima [A] (verification not implemented) . . . 797
3.79.8 Giac [A] (verification not implemented) . . . 797
3.79.9 Mupad [B] (verification not implemented) . . . 798

3.79.1 Optimal result

Integrand size = 21, antiderivative size = 176

$$\int \frac{\cos^2(c+dx)}{(a+a\sec(c+dx))^4}\,dx = \frac{21x}{2a^4} - \frac{576\sin(c+dx)}{35a^4d} + \frac{21\cos(c+dx)\sin(c+dx)}{2a^4d} - \frac{43\cos(c+dx)\sin(c+dx)}{35a^4d(1+\sec(c+dx))^2} - \frac{288\cos(c+dx)\sin(c+dx)}{35a^4d(1+\sec(c+dx))} - \frac{\cos(c+dx)\sin(c+dx)}{7d(a+a\sec(c+dx))^4} - \frac{2\cos(c+dx)\sin(c+dx)}{5ad(a+a\sec(c+dx))^3}$$

output
```
21/2*x/a^4-576/35*sin(d*x+c)/a^4/d+21/2*cos(d*x+c)*sin(d*x+c)/a^4/d-43/35*
cos(d*x+c)*sin(d*x+c)/a^4/d/(1+sec(d*x+c))^2-288/35*cos(d*x+c)*sin(d*x+c)/
a^4/d/(1+sec(d*x+c))-1/7*cos(d*x+c)*sin(d*x+c)/d/(a+a*sec(d*x+c))^4-2/5*co
s(d*x+c)*sin(d*x+c)/a/d/(a+a*sec(d*x+c))^3
```

3.79.2 Mathematica [A] (verified)

Time = 3.64 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.64

$$\int \frac{\cos^2(c+dx)}{(a+a\sec(c+dx))^4}\,dx = \frac{\sec\left(\frac{c}{2}\right)\sec^7\left(\frac{1}{2}(c+dx)\right)\left(102900dx\cos\left(\frac{dx}{2}\right)+102900dx\cos\left(c+\frac{dx}{2}\right)+61740dx\cos\left(c+\frac{3dx}{2}\right)+61740dx\right.}{}$$

input
```
Integrate[Cos[c + d*x]^2/(a + a*Sec[c + d*x])^4,x]
```

output
```
(Sec[c/2]*Sec[(c + d*x)/2]^7*(102900*d*x*Cos[(d*x)/2] + 102900*d*x*Cos[c +
 (d*x)/2] + 61740*d*x*Cos[c + (3*d*x)/2] + 61740*d*x*Cos[2*c + (3*d*x)/2]
+ 20580*d*x*Cos[2*c + (5*d*x)/2] + 20580*d*x*Cos[3*c + (5*d*x)/2] + 2940*d
*x*Cos[3*c + (7*d*x)/2] + 2940*d*x*Cos[4*c + (7*d*x)/2] - 179830*Sin[(d*x)
/2] + 128730*Sin[c + (d*x)/2] - 140826*Sin[c + (3*d*x)/2] + 44310*Sin[2*c
+ (3*d*x)/2] - 60487*Sin[2*c + (5*d*x)/2] + 1225*Sin[3*c + (5*d*x)/2] - 12
001*Sin[3*c + (7*d*x)/2] - 3185*Sin[4*c + (7*d*x)/2] - 315*Sin[4*c + (9*d*
x)/2] - 315*Sin[5*c + (9*d*x)/2] + 35*Sin[5*c + (11*d*x)/2] + 35*Sin[6*c +
 (11*d*x)/2]))/(35840*a^4*d)
```

3.79.3 Rubi [A] (verified)

Time = 1.22 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.10, number of steps used = 16, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.762$, Rules used = {3042, 4304, 25, 3042, 4508, 3042, 4508, 27, 3042, 4508, 3042, 4274, 3042, 3115, 24, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^2(c+dx)}{(a\sec(c+dx)+a)^4}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^2\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^4}dx$$

$$\Big\downarrow \text{4304}$$

$$-\frac{\int -\frac{\cos^2(c+dx)(9a-5a\sec(c+dx))}{(\sec(c+dx)a+a)^3}dx}{7a^2}-\frac{\sin(c+dx)\cos(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\cos^2(c+dx)(9a-5a\sec(c+dx))}{(\sec(c+dx)a+a)^3}dx}{7a^2}-\frac{\sin(c+dx)\cos(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{9a-5a\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^2\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^3}dx}{7a^2}-\frac{\sin(c+dx)\cos(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

$$\Big\downarrow \; 4508$$

$$\frac{\frac{\int \frac{\cos^2(c+dx)\left(73a^2-56a^2\sec(c+dx)\right)}{(\sec(c+dx)a+a)^2}dx}{5a^2}-\frac{14a\sin(c+dx)\cos(c+dx)}{5d(a\sec(c+dx)+a)^3}}{7a^2}-\frac{\sin(c+dx)\cos(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{\int \frac{73a^2-56a^2\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^2\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}dx}{5a^2}-\frac{14a\sin(c+dx)\cos(c+dx)}{5d(a\sec(c+dx)+a)^3}}{7a^2}-\frac{\sin(c+dx)\cos(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

$$\Big\downarrow \; 4508$$

$$\frac{\frac{\frac{\int \frac{9\cos^2(c+dx)\left(53a^3-43a^3\sec(c+dx)\right)}{\sec(c+dx)a+a}dx}{3a^2}-\frac{43\sin(c+dx)\cos(c+dx)}{d(\sec(c+dx)+1)^2}}{5a^2}-\frac{14a\sin(c+dx)\cos(c+dx)}{5d(a\sec(c+dx)+a)^3}}{7a^2}-\frac{\sin(c+dx)\cos(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\frac{3\int \frac{\cos^2(c+dx)\left(53a^3-43a^3\sec(c+dx)\right)}{\sec(c+dx)a+a}dx}{a^2}-\frac{43\sin(c+dx)\cos(c+dx)}{d(\sec(c+dx)+1)^2}}{5a^2}-\frac{14a\sin(c+dx)\cos(c+dx)}{5d(a\sec(c+dx)+a)^3}}{7a^2}-\frac{\sin(c+dx)\cos(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{\frac{3\int \frac{53a^3-43a^3\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^2\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)}dx}{a^2}-\frac{43\sin(c+dx)\cos(c+dx)}{d(\sec(c+dx)+1)^2}}{5a^2}-\frac{14a\sin(c+dx)\cos(c+dx)}{5d(a\sec(c+dx)+a)^3}}{7a^2}-\frac{\sin(c+dx)\cos(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

$$\Big\downarrow \; 4508$$

$$\frac{\frac{\frac{3\left(\frac{\int \cos^2(c+dx)\left(245a^4-192a^4\sec(c+dx)\right)dx}{a^2}-\frac{96a^3\sin(c+dx)\cos(c+dx)}{d(a\sec(c+dx)+a)}\right)}{a^2}-\frac{43\sin(c+dx)\cos(c+dx)}{d(\sec(c+dx)+1)^2}}{5a^2}-\frac{14a\sin(c+dx)\cos(c+dx)}{5d(a\sec(c+dx)+a)^3}}{7a^2}-\frac{\sin(c+dx)\cos(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

$$\Big\downarrow \; 3042$$

3.79. $\int \frac{\cos^2(c+dx)}{(a+a\sec(c+dx))^4}\,dx$

$$\frac{\frac{\frac{3\left(\frac{\int \frac{245a^4-192a^4\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^2}dx}{a^2}-\frac{96a^3\sin(c+dx)\cos(c+dx)}{d(a\sec(c+dx)+a)}\right)}{a^2}-\frac{43\sin(c+dx)\cos(c+dx)}{d(\sec(c+dx)+1)^2}}{5a^2}-\frac{14a\sin(c+dx)\cos(c+dx)}{5d(a\sec(c+dx)+a)^3}}{7a^2}-\frac{\sin(c+dx)\cos(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

$\downarrow$ 4274

$$\frac{\frac{\frac{3\left(\frac{245a^4\int\cos^2(c+dx)dx-192a^4\int\cos(c+dx)dx}{a^2}-\frac{96a^3\sin(c+dx)\cos(c+dx)}{d(a\sec(c+dx)+a)}\right)}{a^2}-\frac{43\sin(c+dx)\cos(c+dx)}{d(\sec(c+dx)+1)^2}}{5a^2}-\frac{14a\sin(c+dx)\cos(c+dx)}{5d(a\sec(c+dx)+a)^3}}{7a^2}-\frac{\sin(c+dx)\cos(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

$\downarrow$ 3042

$$\frac{\frac{\frac{3\left(\frac{245a^4\int\sin\left(c+dx+\frac{\pi}{2}\right)^2dx-192a^4\int\sin\left(c+dx+\frac{\pi}{2}\right)dx}{a^2}-\frac{96a^3\sin(c+dx)\cos(c+dx)}{d(a\sec(c+dx)+a)}\right)}{a^2}-\frac{43\sin(c+dx)\cos(c+dx)}{d(\sec(c+dx)+1)^2}}{5a^2}-\frac{14a\sin(c+dx)\cos(c+dx)}{5d(a\sec(c+dx)+a)^3}}{7a^2}-\frac{\sin(c+dx)\cos(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

$\downarrow$ 3115

$$\frac{\frac{\frac{3\left(\frac{245a^4\left(\frac{\int 1dx}{2}+\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)-192a^4\int\sin\left(c+dx+\frac{\pi}{2}\right)dx}{a^2}-\frac{96a^3\sin(c+dx)\cos(c+dx)}{d(a\sec(c+dx)+a)}\right)}{a^2}-\frac{43\sin(c+dx)\cos(c+dx)}{d(\sec(c+dx)+1)^2}}{5a^2}-\frac{14a\sin(c+dx)\cos(c+dx)}{5d(a\sec(c+dx)+a)^3}}{7a^2}-\frac{\sin(c+dx)\cos(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

$\downarrow$ 24

$$\frac{\frac{\frac{3\left(\frac{245a^4\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)-192a^4\int\sin\left(c+dx+\frac{\pi}{2}\right)dx}{a^2}-\frac{96a^3\sin(c+dx)\cos(c+dx)}{d(a\sec(c+dx)+a)}\right)}{a^2}-\frac{43\sin(c+dx)\cos(c+dx)}{d(\sec(c+dx)+1)^2}}{5a^2}-\frac{14a\sin(c+dx)\cos(c+dx)}{5d(a\sec(c+dx)+a)^3}}{7a^2}-\frac{\sin(c+dx)\cos(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

$\downarrow$ 3117

3.79. $\int \frac{\cos^2(c+dx)}{(a+a\sec(c+dx))^4}\,dx$

$$\frac{\frac{\frac{3\left(\frac{245a^4\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)-\frac{192a^4\sin(c+dx)}{d}}{a^2}-\frac{96a^3\sin(c+dx)\cos(c+dx)}{d(a\sec(c+dx)+a)}\right)}{a^2}-\frac{43\sin(c+dx)\cos(c+dx)}{d(\sec(c+dx)+1)^2}}{5a^2}-\frac{14a\sin(c+dx)\cos(c+dx)}{5d(a\sec(c+dx)+a)^3}}{7a^2}-\frac{\sin(c+dx)\cos(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

input
```
Int[Cos[c + d*x]^2/(a + a*Sec[c + d*x])^4,x]
```

output
```
-1/7*(Cos[c + d*x]*Sin[c + d*x])/(d*(a + a*Sec[c + d*x])^4) + ((-14*a*Cos[
c + d*x]*Sin[c + d*x])/(5*d*(a + a*Sec[c + d*x])^3) + ((-43*Cos[c + d*x]*S
in[c + d*x])/(d*(1 + Sec[c + d*x])^2) + (3*((-96*a^3*Cos[c + d*x]*Sin[c +
d*x])/(d*(a + a*Sec[c + d*x])) + ((-192*a^4*Sin[c + d*x])/d + 245*a^4*(x/2
 + (Cos[c + d*x]*Sin[c + d*x])/(2*d)))/a^2))/a^2)/(5*a^2))/(7*a^2)
```

3.79.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4304
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-Cot[e + f*x])*(a + b*Csc[e + f*x])^m*((d*Csc
[e + f*x])^n/(f*(2*m + 1))), x] + Simp[1/(a^2*(2*m + 1))   Int[(a + b*Csc[e
 + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e
+ f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ
[m, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

rule 4508
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
- a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
 1))), x] - Simp[1/(a^2*(2*m + 1))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Cs
c[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[
e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B
, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

3.79.4 Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.50

method	result
parallelrisch	$\frac{-85762\left(\cos(dx+c)+\frac{18752\cos(2dx+2c)}{42881}+\frac{7873\cos(3dx+3c)}{85762}+\frac{140\cos(4dx+4c)}{42881}-\frac{35\cos(5dx+5c)}{85762}+\frac{27828}{42881}\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\sec\left(\frac{dx}{2}\right.}{8960a^4d}$
derivativedivides	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{7}-\frac{9\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{5}+13\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3-111\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{-72\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3-56\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^2}+168\arctan\left(\tan\left(\right.\right.}{8d\,a^4}$
default	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{7}-\frac{9\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{5}+13\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3-111\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{-72\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3-56\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^2}+168\arctan\left(\tan\left(\right.\right.}{8d\,a^4}$
risch	$\frac{21x}{2a^4}-\frac{ie^{2i(dx+c)}}{8a^4d}+\frac{2ie^{i(dx+c)}}{a^4d}-\frac{2ie^{-i(dx+c)}}{a^4d}+\frac{ie^{-2i(dx+c)}}{8a^4d}-\frac{2i\left(700\,e^{6i(dx+c)}+3675\,e^{5i(dx+c)}+8505\,e^{4i(dx+c)}\right.}{35d\,a^4}$
norman	$\frac{\frac{21x}{2a}-\frac{167\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8ad}-\frac{281\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{8ad}-\frac{217\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{20ad}+\frac{167\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{140ad}-\frac{53\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^9}{280ad}+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{11}}{56ad}+\frac{21x\tan\left(\right.}{}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^2a^3}$

input
```
int(cos(d*x+c)^2/(a+a*sec(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
1/8960*(-85762*(cos(d*x+c)+18752/42881*cos(2*d*x+2*c)+7873/85762*cos(3*d*x
+3*c)+140/42881*cos(4*d*x+4*c)-35/85762*cos(5*d*x+5*c)+27828/42881)*tan(1/
2*d*x+1/2*c)*sec(1/2*d*x+1/2*c)^6+94080*d*x)/a^4/d
```

3.79.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.97

$$\int \frac{\cos^2(c+dx)}{(a+a\sec(c+dx))^4}\,dx$$
$$= \frac{735\,dx\cos\left(dx+c\right)^4 + 2940\,dx\cos\left(dx+c\right)^3 + 4410\,dx\cos\left(dx+c\right)^2 + 2940\,dx\cos\left(dx+c\right) + 735\,dx + \left(3\right.}{70\left(a^4d\cos\left(dx+c\right)^4 + 4\,a^4d\cos\left(dx+\right.\right.}$$

input
```
integrate(cos(d*x+c)^2/(a+a*sec(d*x+c))^4,x, algorithm="fricas")
```

output
```
1/70*(735*d*x*cos(d*x + c)^4 + 2940*d*x*cos(d*x + c)^3 + 4410*d*x*cos(d*x
+ c)^2 + 2940*d*x*cos(d*x + c) + 735*d*x + (35*cos(d*x + c)^5 - 140*cos(d*
x + c)^4 - 2012*cos(d*x + c)^3 - 4548*cos(d*x + c)^2 - 3873*cos(d*x + c) -
 1152)*sin(d*x + c))/(a^4*d*cos(d*x + c)^4 + 4*a^4*d*cos(d*x + c)^3 + 6*a^
4*d*cos(d*x + c)^2 + 4*a^4*d*cos(d*x + c) + a^4*d)
```

3.79.6 Sympy [F]

$$\int \frac{\cos^2(c+dx)}{(a+a\sec(c+dx))^4}\,dx = \frac{\int \frac{\cos^2(c+dx)}{\sec^4(c+dx)+4\sec^3(c+dx)+6\sec^2(c+dx)+4\sec(c+dx)+1}\,dx}{a^4}$$

input
```
integrate(cos(d*x+c)**2/(a+a*sec(d*x+c))**4,x)
```

output
```
Integral(cos(c + d*x)**2/(sec(c + d*x)**4 + 4*sec(c + d*x)**3 + 6*sec(c +
d*x)**2 + 4*sec(c + d*x) + 1), x)/a**4
```

3.79.7 Maxima [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.16

$$\int \frac{\cos^2(c+dx)}{(a+a\sec(c+dx))^4}\,dx = -\frac{\frac{280\left(\frac{7\sin(dx+c)}{\cos(dx+c)+1}+\frac{9\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a^4+\frac{2a^4\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a^4\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}+\frac{\frac{3885\sin(dx+c)}{\cos(dx+c)+1}-\frac{455\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{63\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{5\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{a^4}-\frac{5880\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^4}}{280\,d}$$

input
```
integrate(cos(d*x+c)^2/(a+a*sec(d*x+c))^4,x, algorithm="maxima")
```

output
```
-1/280*(280*(7*sin(d*x + c)/(cos(d*x + c) + 1) + 9*sin(d*x + c)^3/(cos(d*x
 + c) + 1)^3)/(a^4 + 2*a^4*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a^4*sin(d
*x + c)^4/(cos(d*x + c) + 1)^4) + (3885*sin(d*x + c)/(cos(d*x + c) + 1) -
455*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 63*sin(d*x + c)^5/(cos(d*x + c)
+ 1)^5 - 5*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)/a^4 - 5880*arctan(sin(d*x
+ c)/(cos(d*x + c) + 1))/a^4)/d
```

3.79.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.73

$$\int \frac{\cos^2(c+dx)}{(a+a\sec(c+dx))^4}\,dx = \frac{\frac{2940\,(dx+c)}{a^4}-\frac{280\left(9\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+7\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)^2a^4}+\frac{5a^{24}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7-63a^{24}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+455a^{24}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-3885}{a^{28}}}{280\,d}$$

input
```
integrate(cos(d*x+c)^2/(a+a*sec(d*x+c))^4,x, algorithm="giac")
```

output
```
1/280*(2940*(d*x + c)/a^4 - 280*(9*tan(1/2*d*x + 1/2*c)^3 + 7*tan(1/2*d*x
+ 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^2*a^4) + (5*a^24*tan(1/2*d*x + 1/2
*c)^7 - 63*a^24*tan(1/2*d*x + 1/2*c)^5 + 455*a^24*tan(1/2*d*x + 1/2*c)^3 -
 3885*a^24*tan(1/2*d*x + 1/2*c))/a^28)/d
```

3.79.9 Mupad [B] (verification not implemented)

Time = 13.61 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.90

$$\int \frac{\cos^2(c+dx)}{(a+a\sec(c+dx))^4}\,dx$$
$$= \frac{5\,\sin\left(\frac{c}{2}+\frac{dx}{2}\right)-78\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^2\sin\left(\frac{c}{2}+\frac{dx}{2}\right)+596\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^4\sin\left(\frac{c}{2}+\frac{dx}{2}\right)-4408\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^6\sin}{280\,a^4\,d}$$

input
```
int(cos(c + d*x)^2/(a + a/cos(c + d*x))^4,x)
```

output
```
(5*sin(c/2 + (d*x)/2) - 78*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2) + 596*c
os(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2) - 4408*cos(c/2 + (d*x)/2)^6*sin(c/2
 + (d*x)/2) - 2520*cos(c/2 + (d*x)/2)^8*sin(c/2 + (d*x)/2) + 560*cos(c/2 +
 (d*x)/2)^10*sin(c/2 + (d*x)/2) + 2940*cos(c/2 + (d*x)/2)^7*(c + d*x))/(28
0*a^4*d*cos(c/2 + (d*x)/2)^7)
```

3.80 $\int \frac{\sec^7(c+dx)}{(a+a\sec(c+dx))^5}\,dx$

3.80.1 Optimal result . . . 799
3.80.2 Mathematica [A] (verified) . . . 800
3.80.3 Rubi [A] (verified) . . . 800
3.80.4 Maple [A] (verified) . . . 807
3.80.5 Fricas [A] (verification not implemented) . . . 807
3.80.6 Sympy [F] . . . 808
3.80.7 Maxima [A] (verification not implemented) . . . 808
3.80.8 Giac [A] (verification not implemented) . . . 809
3.80.9 Mupad [B] (verification not implemented) . . . 809

3.80.1 Optimal result

Integrand size = 21, antiderivative size = 200

$$\begin{aligned}\int \frac{\sec^7(c+dx)}{(a+a\sec(c+dx))^5}\,dx = &-\frac{5\operatorname{arctanh}(\sin(c+dx))}{a^5d}+\frac{181\tan(c+dx)}{63a^5d}\\ &-\frac{\sec^5(c+dx)\tan(c+dx)}{9d(a+a\sec(c+dx))^5}-\frac{5\sec^4(c+dx)\tan(c+dx)}{21ad(a+a\sec(c+dx))^4}\\ &-\frac{29\sec^3(c+dx)\tan(c+dx)}{63a^2d(a+a\sec(c+dx))^3}\\ &-\frac{67\sec^2(c+dx)\tan(c+dx)}{63a^3d(a+a\sec(c+dx))^2}+\frac{5\tan(c+dx)}{d\left(a^5+a^5\sec(c+dx)\right)}\end{aligned}$$

output
```
-5*arctanh(sin(d*x+c))/a^5/d+181/63*tan(d*x+c)/a^5/d-1/9*sec(d*x+c)^5*tan(
d*x+c)/d/(a+a*sec(d*x+c))^5-5/21*sec(d*x+c)^4*tan(d*x+c)/a/d/(a+a*sec(d*x+
c))^4-29/63*sec(d*x+c)^3*tan(d*x+c)/a^2/d/(a+a*sec(d*x+c))^3-67/63*sec(d*x
+c)^2*tan(d*x+c)/a^3/d/(a+a*sec(d*x+c))^2+5*tan(d*x+c)/d/(a^5+a^5*sec(d*x+
c))
```

3.80.2 Mathematica [A] (verified)

Time = 1.82 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.70

$$\int \frac{\sec^7(c+dx)}{(a+a\sec(c+dx))^5}\,dx$$
$$= \frac{\cos\left(\frac{1}{2}(c+dx)\right)\sec^5(c+dx)\left(-80640\text{arctanh}(\sin(c+dx))\cos^9\left(\frac{1}{2}(c+dx)\right)+\sec(c+dx)\left(4872\sin\left(\frac{1}{2}(c\right.\right.}{50}$$

input
```
Integrate[Sec[c + d*x]^7/(a + a*Sec[c + d*x])^5,x]
```

output
```
(Cos[(c + d*x)/2]*Sec[c + d*x]^5*(-80640*ArcTanh[Sin[c + d*x]]*Cos[(c + d*
x)/2]^9 + Sec[c + d*x]*(4872*Sin[(c + d*x)/2] + 11826*Sin[(3*(c + d*x))/2]
 + 11514*Sin[(5*(c + d*x))/2] + 6341*Sin[(7*(c + d*x))/2] + 1917*Sin[(9*(c
 + d*x))/2] + 248*Sin[(11*(c + d*x))/2])))/(504*a^5*d*(1 + Sec[c + d*x])^5
)
```

3.80.3 Rubi [A] (verified)

Time = 1.55 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.16, number of steps used = 20, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.905$, Rules used = {3042, 4303, 27, 3042, 4507, 3042, 4507, 27, 3042, 4507, 3042, 4496, 25, 3042, 4274, 3042, 4254, 24, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^7(c+dx)}{(a\sec(c+dx)+a)^5}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^7}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^5}dx$$
$$\Big\downarrow \; 4303$$
$$-\frac{\int \frac{5\sec^5(c+dx)(a-2a\sec(c+dx))}{(\sec(c+dx)a+a)^4}dx}{9a^2}-\frac{\tan(c+dx)\sec^5(c+dx)}{9d(a\sec(c+dx)+a)^5}$$
$$\Big\downarrow \; 27$$

$$-\frac{5\int \frac{\sec^5(c+dx)(a-2a\sec(c+dx))}{(\sec(c+dx)a+a)^4}dx}{9a^2}-\frac{\tan(c+dx)\sec^5(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$$\Big\downarrow \; 3042$$

$$-\frac{5\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^5\left(a-2a\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^4}dx}{9a^2}-\frac{\tan(c+dx)\sec^5(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$$\Big\downarrow \; 4507$$

$$-\frac{5\left(\frac{\int \frac{\sec^4(c+dx)\left(12a^2-17a^2\sec(c+dx)\right)}{(\sec(c+dx)a+a)^3}dx}{7a^2}+\frac{3a\tan(c+dx)\sec^4(c+dx)}{7d(a\sec(c+dx)+a)^4}\right)}{9a^2}-\frac{\tan(c+dx)\sec^5(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$$\Big\downarrow \; 3042$$

$$-\frac{5\left(\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^4\left(12a^2-17a^2\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^3}dx}{7a^2}+\frac{3a\tan(c+dx)\sec^4(c+dx)}{7d(a\sec(c+dx)+a)^4}\right)}{9a^2}-\frac{\tan(c+dx)\sec^5(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$$\Big\downarrow \; 4507$$

$$-\frac{5\left(\frac{\frac{\int \frac{3\sec^3(c+dx)\left(29a^3-38a^3\sec(c+dx)\right)}{(\sec(c+dx)a+a)^2}dx}{5a^2}+\frac{29a^2\tan(c+dx)\sec^3(c+dx)}{5d(a\sec(c+dx)+a)^3}}{7a^2}+\frac{3a\tan(c+dx)\sec^4(c+dx)}{7d(a\sec(c+dx)+a)^4}\right)}{9a^2}-\frac{\tan(c+dx)\sec^5(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$$\Big\downarrow \; 27$$

$$-\frac{5\left(\frac{\frac{3\int \frac{\sec^3(c+dx)\left(29a^3-38a^3\sec(c+dx)\right)}{(\sec(c+dx)a+a)^2}dx}{5a^2}+\frac{29a^2\tan(c+dx)\sec^3(c+dx)}{5d(a\sec(c+dx)+a)^3}}{7a^2}+\frac{3a\tan(c+dx)\sec^4(c+dx)}{7d(a\sec(c+dx)+a)^4}\right)}{9a^2}-\frac{\tan(c+dx)\sec^5(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$$\Big\downarrow \; 3042$$

3.80. $\int \frac{\sec^7(c+dx)}{(a+a\sec(c+dx))^5}\,dx$

$$-\frac{5\left(\frac{\frac{3\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^3\left(29a^3-38a^3\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}dx}{5a^2}+\frac{29a^2\tan(c+dx)\sec^3(c+dx)}{5d(a\sec(c+dx)+a)^3}}{7a^2}+\frac{3a\tan(c+dx)\sec^4(c+dx)}{7d(a\sec(c+dx)+a)^4}\right)}{9a^2}-\frac{\tan(c+dx)\sec^5(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$\downarrow$ 4507

$$-\frac{5\left(\frac{\frac{3\left(\frac{\int \frac{\sec^2(c+dx)\left(134a^4-181a^4\sec(c+dx)\right)}{\sec(c+dx)a+a}dx}{3a^2}+\frac{67a^3\tan(c+dx)\sec^2(c+dx)}{3d(a\sec(c+dx)+a)^2}\right)}{5a^2}+\frac{29a^2\tan(c+dx)\sec^3(c+dx)}{5d(a\sec(c+dx)+a)^3}}{7a^2}+\frac{3a\tan(c+dx)\sec^4(c+dx)}{7d(a\sec(c+dx)+a)^4}\right)}{9a^2}-\frac{\tan(c+dx)\sec^5(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$\downarrow$ 3042

$$-\frac{5\left(\frac{\frac{3\left(\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2\left(134a^4-181a^4\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{3a^2}+\frac{67a^3\tan(c+dx)\sec^2(c+dx)}{3d(a\sec(c+dx)+a)^2}\right)}{5a^2}+\frac{29a^2\tan(c+dx)\sec^3(c+dx)}{5d(a\sec(c+dx)+a)^3}}{7a^2}+\frac{3a\tan(c+dx)\sec^4(c+dx)}{7d(a\sec(c+dx)+a)^4}\right)}{9a^2}-\frac{\tan(c+dx)\sec^5(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$\downarrow$ 4496

$$-\frac{5\left(\frac{\frac{3\left(\frac{-\frac{\int -\sec(c+dx)\left(315a^5-181a^5\sec(c+dx)\right)dx}{a^2}-\frac{315a^4\tan(c+dx)}{d(a\sec(c+dx)+a)}}{3a^2}+\frac{67a^3\tan(c+dx)\sec^2(c+dx)}{3d(a\sec(c+dx)+a)^2}\right)}{5a^2}+\frac{29a^2\tan(c+dx)\sec^3(c+dx)}{5d(a\sec(c+dx)+a)^3}}{7a^2}+\frac{3a\tan(c+dx)\sec}{7d(a\sec(c+dx}\right.}{9a^2}-\frac{\tan(c+dx)\sec^5(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

3.80. $\int \frac{\sec^7(c+dx)}{(a+a\sec(c+dx))^5}\,dx$

$$\Big\downarrow \; 25$$

$$-\frac{5\left(\frac{\frac{3\left(\frac{\frac{\int \sec(c+dx)\left(315a^5-181a^5\sec(c+dx)\right)dx}{a^2}-\frac{315a^4\tan(c+dx)}{d(a\sec(c+dx)+a)}}{3a^2}+\frac{67a^3\tan(c+dx)\sec^2(c+dx)}{3d(a\sec(c+dx)+a)^2}\right)}{5a^2}+\frac{29a^2\tan(c+dx)\sec^3(c+dx)}{5d(a\sec(c+dx)+a)^3}}{7a^2}+\frac{3a\tan(c+dx)\sec^4(c+\cdots}{7d(a\sec(c+dx)+a\cdots}\right)}{\frac{\tan(c+dx)\sec^5(c+dx)}{9d(a\sec(c+dx)+a)^5}}\,9a^2$$

$$\Big\downarrow \; 3042$$

$$-\frac{5\left(\frac{\frac{3\left(\frac{\frac{\int \csc\left(c+dx+\frac{\pi}{2}\right)\left(315a^5-181a^5\csc\left(c+dx+\frac{\pi}{2}\right)\right)dx}{a^2}-\frac{315a^4\tan(c+dx)}{d(a\sec(c+dx)+a)}}{3a^2}+\frac{67a^3\tan(c+dx)\sec^2(c+dx)}{3d(a\sec(c+dx)+a)^2}\right)}{5a^2}+\frac{29a^2\tan(c+dx)\sec^3(c+dx)}{5d(a\sec(c+dx)+a)^3}}{7a^2}+\frac{3a\tan(c+dx\cdots}{7d(a\sec(c\cdots}\right)}{\frac{\tan(c+dx)\sec^5(c+dx)}{9d(a\sec(c+dx)+a)^5}}\,9a^2$$

$$\Big\downarrow \; 4274$$

$$-\frac{5\left(\frac{\frac{3\left(\frac{\frac{315a^5\int \sec(c+dx)dx-181a^5\int \sec^2(c+dx)dx}{a^2}-\frac{315a^4\tan(c+dx)}{d(a\sec(c+dx)+a)}}{3a^2}+\frac{67a^3\tan(c+dx)\sec^2(c+dx)}{3d(a\sec(c+dx)+a)^2}\right)}{5a^2}+\frac{29a^2\tan(c+dx)\sec^3(c+dx)}{5d(a\sec(c+dx)+a)^3}}{7a^2}+\frac{3a\tan(c+dx)\sec\cdots}{7d(a\sec(c+dx)\cdots}\right)}{\frac{\tan(c+dx)\sec^5(c+dx)}{9d(a\sec(c+dx)+a)^5}}\,9a^2$$

$$\Big\downarrow \; 3042$$

3.80. $\int \frac{\sec^7(c+dx)}{(a+a\sec(c+dx))^5}\,dx$

$$-\frac{5\left(\frac{3\left(\frac{\frac{315a^5\int \csc\left(c+dx+\frac{\pi}{2}\right)dx-181a^5\int \csc\left(c+dx+\frac{\pi}{2}\right)^2dx}{a^2}}{3a^2}-\frac{315a^4\tan(c+dx)}{d(a\sec(c+dx)+a)}+\frac{67a^3\tan(c+dx)\sec^2(c+dx)}{3d(a\sec(c+dx)+a)^2}\right)}{5a^2}+\frac{29a^2\tan(c+dx)\sec^3(c+dx)}{5d(a\sec(c+dx)+a)^3}\right)}{7a^2}+\frac{3a\tan(c+}{7d(a\,\mathrm{se}}}{9a^2}$$

$$\frac{\tan(c+dx)\sec^5(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$\downarrow$ 4254

$$-\frac{5\left(\frac{3\left(\frac{\frac{\frac{181a^5\int 1d(-\tan(c+dx))}{d}+315a^5\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{a^2}}{3a^2}-\frac{315a^4\tan(c+dx)}{d(a\sec(c+dx)+a)}+\frac{67a^3\tan(c+dx)\sec^2(c+dx)}{3d(a\sec(c+dx)+a)^2}\right)}{5a^2}+\frac{29a^2\tan(c+dx)\sec^3(c+dx)}{5d(a\sec(c+dx)+a)^3}\right)}{7a^2}+\frac{3a\tan(c+}{7d(a\,\mathrm{sec}}}{9a^2}$$

$$\frac{\tan(c+dx)\sec^5(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$\downarrow$ 24

$$-\frac{5\left(\frac{3\left(\frac{\frac{315a^5\int \csc\left(c+dx+\frac{\pi}{2}\right)dx-\frac{181a^5\tan(c+dx)}{d}}{a^2}}{3a^2}-\frac{315a^4\tan(c+dx)}{d(a\sec(c+dx)+a)}+\frac{67a^3\tan(c+dx)\sec^2(c+dx)}{3d(a\sec(c+dx)+a)^2}\right)}{5a^2}+\frac{29a^2\tan(c+dx)\sec^3(c+dx)}{5d(a\sec(c+dx)+a)^3}\right)}{7a^2}+\frac{3a\tan(c+dx)\sec^{\prime}}{7d(a\sec(c+dx)}}{9a^2}$$

$$\frac{\tan(c+dx)\sec^5(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$\downarrow$ 4257

3.80. $\int \frac{\sec^7(c+dx)}{(a+a\sec(c+dx))^5}\,dx$

$$-\frac{5\left(\frac{\frac{29a^2\tan(c+dx)\sec^3(c+dx)}{5d(a\sec(c+dx)+a)^3}+\frac{3\left(\frac{67a^3\tan(c+dx)\sec^2(c+dx)}{3d(a\sec(c+dx)+a)^2}+\frac{\frac{315a^5\text{arctanh}(\sin(c+dx))}{d}-\frac{181a^5\tan(c+dx)}{d}}{a^2}-\frac{315a^4\tan(c+dx)}{d(a\sec(c+dx)+a)}}{3a^2}\right)}{5a^2}}{7a^2}+\frac{3a\tan(c+dx)}{7d(a\sec(c+}\right)}{9a^2}}{\frac{\tan(c+dx)\sec^5(c+dx)}{9d(a\sec(c+dx)+a)^5}}$$

input
```
Int[Sec[c + d*x]^7/(a + a*Sec[c + d*x])^5,x]
```

output
```
-1/9*(Sec[c + d*x]^5*Tan[c + d*x])/(d*(a + a*Sec[c + d*x])^5) - (5*((3*a*S
ec[c + d*x]^4*Tan[c + d*x])/(7*d*(a + a*Sec[c + d*x])^4) + ((29*a^2*Sec[c
+ d*x]^3*Tan[c + d*x])/(5*d*(a + a*Sec[c + d*x])^3) + (3*((67*a^3*Sec[c +
d*x]^2*Tan[c + d*x])/(3*d*(a + a*Sec[c + d*x])^2) + ((-315*a^4*Tan[c + d*x
])/(d*(a + a*Sec[c + d*x])) + ((315*a^5*ArcTanh[Sin[c + d*x]])/d - (181*a^
5*Tan[c + d*x])/d)/a^2)/(3*a^2)))/(5*a^2))/(7*a^2)))/(9*a^2)
```

3.80.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4303
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-d^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d
*Csc[e + f*x])^(n - 2)/(f*(2*m + 1))), x] + Simp[d^2/(a*b*(2*m + 1))   Int[
(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n
 + 2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2,
0] && LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

rule 4496
```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b - a*B))*Cot
[e + f*x]*((a + b*Csc[e + f*x])^m/(b*f*(2*m + 1))), x] + Simp[1/(b^2*(2*m +
 1))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[A*b*m - a*B*m + b
*B*(2*m + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && Ne
Q[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

rule 4507
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[d*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(a*f*(
2*m + 1))), x] - Simp[1/(a*b*(2*m + 1))   Int[(a + b*Csc[e + f*x])^(m + 1)*
(d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
 - n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f,
A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && G
tQ[n, 0]
```

3.80. $\int \frac{\sec^7(c+dx)}{(a+a\sec(c+dx))^5}\,dx$

3.80.4 Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.66

method	result
derivativedivides	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{9}}{9}+\frac{8\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{7}+6\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}+24\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}+129\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{16}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1}+80\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}{16d\,a^{5}}$
default	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{9}}{9}+\frac{8\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{7}+6\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}+24\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}+129\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{16}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1}+80\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}{16d\,a^{5}}$
parallelrisch	$\frac{40320\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\cos(dx+c)-40320\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)\cos(dx+c)+31846\sec\left(\frac{dx}{2}+\frac{c}{2}\right)^{8}\left(\cos(dx+c)+\frac{10010\,c}{1}\right.}{8064d\,a^{5}\cos(dx+c)}$
risch	$\frac{2i\left(315\,\mathrm{e}^{10i(dx+c)}+2835\,\mathrm{e}^{9i(dx+c)}+11550\,\mathrm{e}^{8i(dx+c)}+28350\,\mathrm{e}^{7i(dx+c)}+47208\,\mathrm{e}^{6i(dx+c)}+56952\,\mathrm{e}^{5i(dx+c)}+52002\,\mathrm{e}^{4i(dx+c}\right.}{63d\,a^{5}\left(\mathrm{e}^{i(dx+c)}+1\right)^{9}\left(\mathrm{e}^{2i(dx+c)}+1\right)}$

input
```
int(sec(d*x+c)^7/(a+a*sec(d*x+c))^5,x,method=_RETURNVERBOSE)
```

output
```
1/16/d/a^5*(1/9*tan(1/2*d*x+1/2*c)^9+8/7*tan(1/2*d*x+1/2*c)^7+6*tan(1/2*d*
x+1/2*c)^5+24*tan(1/2*d*x+1/2*c)^3+129*tan(1/2*d*x+1/2*c)-16/(tan(1/2*d*x+
1/2*c)-1)+80*ln(tan(1/2*d*x+1/2*c)-1)-16/(tan(1/2*d*x+1/2*c)+1)-80*ln(tan(
1/2*d*x+1/2*c)+1))
```

3.80.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.39

$$\int \frac{\sec^7(c+dx)}{(a+a\sec(c+dx))^5}\,dx = -\frac{315\left(\cos\left(dx+c\right)^6+5\,\cos\left(dx+c\right)^5+10\,\cos\left(dx+c\right)^4+10\,\cos\left(dx+c\right)^3+5\,\cos\left(dx+c\right)^2+\cos\left(dx+\right.\right.}{}$$

input
```
integrate(sec(d*x+c)^7/(a+a*sec(d*x+c))^5,x, algorithm="fricas")
```

output
```
-1/126*(315*(cos(d*x + c)^6 + 5*cos(d*x + c)^5 + 10*cos(d*x + c)^4 + 10*co
s(d*x + c)^3 + 5*cos(d*x + c)^2 + cos(d*x + c))*log(sin(d*x + c) + 1) - 31
5*(cos(d*x + c)^6 + 5*cos(d*x + c)^5 + 10*cos(d*x + c)^4 + 10*cos(d*x + c)
^3 + 5*cos(d*x + c)^2 + cos(d*x + c))*log(-sin(d*x + c) + 1) - 2*(496*cos(
d*x + c)^5 + 2165*cos(d*x + c)^4 + 3633*cos(d*x + c)^3 + 2840*cos(d*x + c)
^2 + 946*cos(d*x + c) + 63)*sin(d*x + c))/(a^5*d*cos(d*x + c)^6 + 5*a^5*d*
cos(d*x + c)^5 + 10*a^5*d*cos(d*x + c)^4 + 10*a^5*d*cos(d*x + c)^3 + 5*a^5
*d*cos(d*x + c)^2 + a^5*d*cos(d*x + c))
```

3.80.6 Sympy [F]

$$\int \frac{\sec^7(c+dx)}{(a+a\sec(c+dx))^5}\,dx = \frac{\int \frac{\sec^7(c+dx)}{\sec^5(c+dx)+5\sec^4(c+dx)+10\sec^3(c+dx)+10\sec^2(c+dx)+5\sec(c+dx)+1}\,dx}{a^5}$$

input
```
integrate(sec(d*x+c)**7/(a+a*sec(d*x+c))**5,x)
```

output
```
Integral(sec(c + d*x)**7/(sec(c + d*x)**5 + 5*sec(c + d*x)**4 + 10*sec(c +
 d*x)**3 + 10*sec(c + d*x)**2 + 5*sec(c + d*x) + 1), x)/a**5
```

3.80.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.03

$$\int \frac{\sec^7(c+dx)}{(a+a\sec(c+dx))^5}\,dx$$
$$= \frac{\frac{2016\sin(dx+c)}{\left(a^5-\frac{a^5\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)} + \frac{\frac{8127\sin(dx+c)}{\cos(dx+c)+1}+\frac{1512\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{378\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{72\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{7\sin(dx+c)^9}{(\cos(dx+c)+1)^9}}{a^5} - \frac{5040\log\left(\frac{\sin(c}{\cos(dx}\right.}{a^5}}{1008\,d}$$

input
```
integrate(sec(d*x+c)^7/(a+a*sec(d*x+c))^5,x, algorithm="maxima")
```

output
```
1/1008*(2016*sin(d*x + c)/((a^5 - a^5*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)
*(cos(d*x + c) + 1)) + (8127*sin(d*x + c)/(cos(d*x + c) + 1) + 1512*sin(d*
x + c)^3/(cos(d*x + c) + 1)^3 + 378*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 +
72*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 7*sin(d*x + c)^9/(cos(d*x + c) +
1)^9)/a^5 - 5040*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^5 + 5040*log(s
in(d*x + c)/(cos(d*x + c) + 1) - 1)/a^5)/d
```

3.80.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.78

$$\int \frac{\sec^7(c+dx)}{(a+a\sec(c+dx))^5}\,dx = -\frac{\frac{5040\log(|\tan(\frac{1}{2}dx+\frac{1}{2}c)+1|)}{a^5} - \frac{5040\log(|\tan(\frac{1}{2}dx+\frac{1}{2}c)-1|)}{a^5} + \frac{2016\tan(\frac{1}{2}dx+\frac{1}{2}c)}{(\tan(\frac{1}{2}dx+\frac{1}{2}c)^2-1)a^5} - \frac{7a^{40}\tan(\frac{1}{2}dx+\frac{1}{2}c)^9+72a^{40}\tan(\frac{1}{2}dx+\frac{1}{2}}{}}{1008\,d}$$

input
```
integrate(sec(d*x+c)^7/(a+a*sec(d*x+c))^5,x, algorithm="giac")
```

output
```
-1/1008*(5040*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^5 - 5040*log(abs(tan(1/
2*d*x + 1/2*c) - 1))/a^5 + 2016*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/2*c
)^2 - 1)*a^5) - (7*a^40*tan(1/2*d*x + 1/2*c)^9 + 72*a^40*tan(1/2*d*x + 1/2
*c)^7 + 378*a^40*tan(1/2*d*x + 1/2*c)^5 + 1512*a^40*tan(1/2*d*x + 1/2*c)^3
 + 8127*a^40*tan(1/2*d*x + 1/2*c))/a^45)/d
```

3.80.9 Mupad [B] (verification not implemented)

Time = 13.76 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.74

$$\int \frac{\sec^7(c+dx)}{(a+a\sec(c+dx))^5}\,dx = \frac{3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{2a^5d} + \frac{3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5}{8a^5d} + \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7}{14a^5d} + \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^9}{144a^5d} - \frac{10\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{a^5d} - \frac{2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\left(a^5\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2-a^5\right)} + \frac{129\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{16a^5d}$$

input
```
int(1/(cos(c + d*x)^7*(a + a/cos(c + d*x))^5),x)
```

output
```
(3*tan(c/2 + (d*x)/2)^3)/(2*a^5*d) + (3*tan(c/2 + (d*x)/2)^5)/(8*a^5*d) +
tan(c/2 + (d*x)/2)^7/(14*a^5*d) + tan(c/2 + (d*x)/2)^9/(144*a^5*d) - (10*a
tanh(tan(c/2 + (d*x)/2)))/(a^5*d) - (2*tan(c/2 + (d*x)/2))/(d*(a^5*tan(c/2
 + (d*x)/2)^2 - a^5)) + (129*tan(c/2 + (d*x)/2))/(16*a^5*d)
```

3.81 $\int \frac{\sec^6(c+dx)}{(a+a\sec(c+dx))^5}\,dx$

3.81.1 Optimal result 810
3.81.2 Mathematica [A] (verified) 810
3.81.3 Rubi [A] (verified) 811
3.81.4 Maple [A] (verified) 816
3.81.5 Fricas [A] (verification not implemented) 816
3.81.6 Sympy [F] 817
3.81.7 Maxima [A] (verification not implemented) 817
3.81.8 Giac [A] (verification not implemented) 817
3.81.9 Mupad [B] (verification not implemented) 818

3.81.1 Optimal result

Integrand size = 21, antiderivative size = 177

$$\begin{aligned}\int \frac{\sec^6(c+dx)}{(a+a\sec(c+dx))^5}\,dx &= \frac{\operatorname{arctanh}(\sin(c+dx))}{a^5d} - \frac{\sec^4(c+dx)\tan(c+dx)}{9d(a+a\sec(c+dx))^5} \\ &\quad - \frac{13\sec^3(c+dx)\tan(c+dx)}{63ad(a+a\sec(c+dx))^4} - \frac{34\sec^2(c+dx)\tan(c+dx)}{105a^2d(a+a\sec(c+dx))^3} \\ &\quad + \frac{173\tan(c+dx)}{315a^3d(a+a\sec(c+dx))^2} - \frac{661\tan(c+dx)}{315d\left(a^5+a^5\sec(c+dx)\right)}\end{aligned}$$

output
```
arctanh(sin(d*x+c))/a^5/d-1/9*sec(d*x+c)^4*tan(d*x+c)/d/(a+a*sec(d*x+c))^5
-13/63*sec(d*x+c)^3*tan(d*x+c)/a/d/(a+a*sec(d*x+c))^4-34/105*sec(d*x+c)^2*
tan(d*x+c)/a^2/d/(a+a*sec(d*x+c))^3+173/315*tan(d*x+c)/a^3/d/(a+a*sec(d*x+
c))^2-661/315*tan(d*x+c)/d/(a^5+a^5*sec(d*x+c))
```

3.81.2 Mathematica [A] (verified)

Time = 0.74 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.69

$$\int \frac{\sec^6(c+dx)}{(a+a\sec(c+dx))^5}\,dx$$
$$= \frac{\cos\left(\frac{1}{2}(c+dx)\right)\sec^5(c+dx)\left(80640\operatorname{arctanh}(\sin(c+dx))\cos^9\left(\frac{1}{2}(c+dx)\right) - 2\left(5229\sin\left(\frac{1}{2}(c+dx)\right) + 926\right.\right.}{2520a^5d(1+\sec(c+d}$$

input
```
Integrate[Sec[c + d*x]^6/(a + a*Sec[c + d*x])^5,x]
```

output
```
(Cos[(c + d*x)/2]*Sec[c + d*x]^5*(80640*ArcTanh[Sin[c + d*x]]*Cos[(c + d*x
)/2]^9 - 2*(5229*Sin[(c + d*x)/2] + 9261*Sin[(3*(c + d*x))/2] + 5949*Sin[(
5*(c + d*x))/2] + 1881*Sin[(7*(c + d*x))/2] + 244*Sin[(9*(c + d*x))/2])))/
(2520*a^5*d*(1 + Sec[c + d*x])^5)
```

3.81.3 Rubi [A] (verified)

Time = 1.33 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.16, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {3042, 4303, 3042, 4507, 27, 3042, 4507, 3042, 4496, 25, 3042, 4486, 3042, 4257, 4281}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^6(c+dx)}{(a\sec(c+dx)+a)^5}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^6}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^5}dx$$

$$\Big\downarrow \text{4303}$$

$$-\frac{\int \frac{\sec^4(c+dx)(4a-9a\sec(c+dx))}{(\sec(c+dx)a+a)^4}dx}{9a^2}-\frac{\tan(c+dx)\sec^4(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^4\left(4a-9a\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^4}dx}{9a^2}-\frac{\tan(c+dx)\sec^4(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$$\Big\downarrow \text{4507}$$

$$-\frac{\frac{\int \frac{3\sec^3(c+dx)\left(13a^2-21a^2\sec(c+dx)\right)}{(\sec(c+dx)a+a)^3}dx}{7a^2}+\frac{13a\tan(c+dx)\sec^3(c+dx)}{7d(a\sec(c+dx)+a)^4}}{9a^2}-\frac{\tan(c+dx)\sec^4(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\frac{3\int \frac{\sec^3(c+dx)\left(13a^2-21a^2\sec(c+dx)\right)}{(\sec(c+dx)a+a)^3}dx}{7a^2}+\frac{13a\tan(c+dx)\sec^3(c+dx)}{7d(a\sec(c+dx)+a)^4}}{9a^2}-\frac{\tan(c+dx)\sec^4(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\frac{3\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^3\left(13a^2-21a^2\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^3}dx}{7a^2}+\frac{13a\tan(c+dx)\sec^3(c+dx)}{7d(a\sec(c+dx)+a)^4}}{9a^2}-\frac{\tan(c+dx)\sec^4(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$$\Big\downarrow \; 4507$$

$$-\frac{\frac{3\left(\frac{\int \frac{\sec^2(c+dx)\left(68a^3-105a^3\sec(c+dx)\right)}{(\sec(c+dx)a+a)^2}dx}{5a^2}+\frac{34a^2\tan(c+dx)\sec^2(c+dx)}{5d(a\sec(c+dx)+a)^3}\right)}{7a^2}+\frac{13a\tan(c+dx)\sec^3(c+dx)}{7d(a\sec(c+dx)+a)^4}}{9a^2}-\frac{\tan(c+dx)\sec^4(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\frac{3\left(\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2\left(68a^3-105a^3\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}dx}{5a^2}+\frac{34a^2\tan(c+dx)\sec^2(c+dx)}{5d(a\sec(c+dx)+a)^3}\right)}{7a^2}+\frac{13a\tan(c+dx)\sec^3(c+dx)}{7d(a\sec(c+dx)+a)^4}}{9a^2}-\frac{\tan(c+dx)\sec^4(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$$\Big\downarrow \; 4496$$

$$-\frac{\frac{3\left(\frac{-\frac{\int -\frac{\sec(c+dx)\left(346a^4-315a^4\sec(c+dx)\right)}{\sec(c+dx)a+a}dx}{3a^2}-\frac{173a^3\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}}{5a^2}+\frac{34a^2\tan(c+dx)\sec^2(c+dx)}{5d(a\sec(c+dx)+a)^3}\right)}{7a^2}+\frac{13a\tan(c+dx)\sec^3(c+dx)}{7d(a\sec(c+dx)+a)^4}}{9a^2}-\frac{\tan(c+dx)\sec^4(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$$\Big\downarrow \; 25$$

$$-\frac{\frac{3\left(\frac{\frac{\int \frac{\sec(c+dx)\left(346a^4-315a^4\sec(c+dx)\right)}{\sec(c+dx)a+a}dx}{3a^2}-\frac{173a^3\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}}{5a^2}+\frac{34a^2\tan(c+dx)\sec^2(c+dx)}{5d(a\sec(c+dx)+a)^3}\right)}{7a^2}+\frac{13a\tan(c+dx)\sec^3(c+dx)}{7d(a\sec(c+dx)+a)^4}}{9a^2}-\frac{\tan(c+dx)\sec^4(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$$\Big\downarrow \; 3042$$

3.81. $\int \frac{\sec^6(c+dx)}{(a+a\sec(c+dx))^5}\,dx$

$$-\frac{\frac{3\left(\frac{\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(346a^4-315a^4\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{3a^2}-\frac{173a^3\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}}{5a^2}+\frac{34a^2\tan(c+dx)\sec^2(c+dx)}{5d(a\sec(c+dx)+a)^3}\right)}{7a^2}+\frac{13a\tan(c+dx)\sec^3(c+dx)}{7d(a\sec(c+dx)+a)^4}}{9a^2}-\frac{\tan(c+dx)\sec^4(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$\downarrow$ 4486

$$-\frac{\frac{3\left(\frac{\frac{661a^4\int\frac{\sec(c+dx)}{\sec(c+dx)a+a}dx-315a^3\int\sec(c+dx)dx}{3a^2}-\frac{173a^3\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}}{5a^2}+\frac{34a^2\tan(c+dx)\sec^2(c+dx)}{5d(a\sec(c+dx)+a)^3}\right)}{7a^2}+\frac{13a\tan(c+dx)\sec^3(c+dx)}{7d(a\sec(c+dx)+a)^4}}{9a^2}-\frac{\tan(c+dx)\sec^4(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$\downarrow$ 3042

$$-\frac{\frac{3\left(\frac{\frac{661a^4\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx-315a^3\int\csc\left(c+dx+\frac{\pi}{2}\right)dx}{3a^2}-\frac{173a^3\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}}{5a^2}+\frac{34a^2\tan(c+dx)\sec^2(c+dx)}{5d(a\sec(c+dx)+a)^3}\right)}{7a^2}+\frac{13a\tan(c+dx)\sec^3(c+dx)}{7d(a\sec(c+dx)+a)^4}}{9a^2}-\frac{\tan(c+dx)\sec^4(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$\downarrow$ 4257

$$-\frac{\frac{3\left(\frac{\frac{661a^4\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx-\frac{315a^3\text{arctanh}(\sin(c+dx))}{d}}{3a^2}-\frac{173a^3\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}}{5a^2}+\frac{34a^2\tan(c+dx)\sec^2(c+dx)}{5d(a\sec(c+dx)+a)^3}\right)}{7a^2}+\frac{13a\tan(c+dx)\sec^3(c+dx)}{7d(a\sec(c+dx)+a)^4}}{9a^2}-\frac{\tan(c+dx)\sec^4(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$\downarrow$ 4281

3.81. $\int \frac{\sec^6(c+dx)}{(a+a\sec(c+dx))^5}\,dx$

$$-\frac{\frac{3\left(\frac{34a^2\tan(c+dx)\sec^2(c+dx)}{5d(a\sec(c+dx)+a)^3}+\frac{\frac{661a^4\tan(c+dx)}{d(a\sec(c+dx)+a)}-\frac{315a^3\text{arctanh}(\sin(c+dx))}{d}}{3a^2}-\frac{173a^3\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}}{5a^2}\right)}{7a^2}+\frac{13a\tan(c+dx)\sec^3(c+dx)}{7d(a\sec(c+dx)+a)^4}}{9a^2}-\frac{\tan(c+dx)\sec^4(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

input
```
Int[Sec[c + d*x]^6/(a + a*Sec[c + d*x])^5,x]
```

output
```
-1/9*(Sec[c + d*x]^4*Tan[c + d*x])/(d*(a + a*Sec[c + d*x])^5) - ((13*a*Sec
[c + d*x]^3*Tan[c + d*x])/(7*d*(a + a*Sec[c + d*x])^4) + (3*((34*a^2*Sec[c
 + d*x]^2*Tan[c + d*x])/(5*d*(a + a*Sec[c + d*x])^3) + ((-173*a^3*Tan[c +
d*x])/(3*d*(a + a*Sec[c + d*x])^2) + ((-315*a^3*ArcTanh[Sin[c + d*x]])/d +
 (661*a^4*Tan[c + d*x])/(d*(a + a*Sec[c + d*x])))/(3*a^2))/(5*a^2)))/(7*a^
2))/(9*a^2)
```

3.81.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4281
```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[-Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

rule 4303
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-d^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d
*Csc[e + f*x])^(n - 2)/(f*(2*m + 1))), x] + Simp[d^2/(a*b*(2*m + 1))   Int[
(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n
 + 2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2,
0] && LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

rule 4486
```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/(csc[(
e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[B/b   Int[Csc[e + f*x],
 x], x] + Simp[(A*b - a*B)/b   Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x
] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0]
```

rule 4496
```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b - a*B))*Cot
[e + f*x]*((a + b*Csc[e + f*x])^m/(b*f*(2*m + 1))), x] + Simp[1/(b^2*(2*m +
 1))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[A*b*m - a*B*m + b
*B*(2*m + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && Ne
Q[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

rule 4507
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[d*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(a*f*(
2*m + 1))), x] - Simp[1/(a*b*(2*m + 1))   Int[(a + b*Csc[e + f*x])^(m + 1)*
(d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
 - n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f,
A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && G
tQ[n, 0]
```

3.81. $\int \frac{\sec^6(c+dx)}{(a+a\sec(c+dx))^5}\,dx$

3.81.4 Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.57

method	result
derivativedivides	$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{9}}{9}-\frac{6\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{7}-\frac{16\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{5}-\frac{26\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{3}-31\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-16\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+16\ln\left(\tan\right.}{16d\,a^{5}}$
default	$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{9}}{9}-\frac{6\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{7}-\frac{16\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{5}-\frac{26\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{3}-31\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-16\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+16\ln\left(\tan\right.}{16d\,a^{5}}$
parallelrisch	$\frac{-35\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{9}-270\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}-1008\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}-2730\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}-5040\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+5040\ln\left(\tan\right.}{5040d\,a^{5}}$
risch	$-\frac{2i\left(315\,\mathrm{e}^{8i(dx+c)}+2835\,\mathrm{e}^{7i(dx+c)}+11235\,\mathrm{e}^{6i(dx+c)}+25515\,\mathrm{e}^{5i(dx+c)}+35973\,\mathrm{e}^{4i(dx+c)}+29757\,\mathrm{e}^{3i(dx+c)}+14733\,\mathrm{e}^{2i(dx+}\right.}{315d\,a^{5}\left(\mathrm{e}^{i(dx+c)}+1\right)^{9}}$

input
```
int(sec(d*x+c)^6/(a+a*sec(d*x+c))^5,x,method=_RETURNVERBOSE)
```

output
```
1/16/d/a^5*(-1/9*tan(1/2*d*x+1/2*c)^9-6/7*tan(1/2*d*x+1/2*c)^7-16/5*tan(1/
2*d*x+1/2*c)^5-26/3*tan(1/2*d*x+1/2*c)^3-31*tan(1/2*d*x+1/2*c)-16*ln(tan(1
/2*d*x+1/2*c)-1)+16*ln(tan(1/2*d*x+1/2*c)+1))
```

3.81.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.39

$$\int \frac{\sec^6(c+dx)}{(a+a\sec(c+dx))^5}\,dx$$
$$= \frac{315\left(\cos\left(dx+c\right)^5+5\,\cos\left(dx+c\right)^4+10\,\cos\left(dx+c\right)^3+10\,\cos\left(dx+c\right)^2+5\,\cos\left(dx+c\right)+1\right)\log\left(\sin\left(\right.\right.}{}$$

input
```
integrate(sec(d*x+c)^6/(a+a*sec(d*x+c))^5,x, algorithm="fricas")
```

output
```
1/630*(315*(cos(d*x + c)^5 + 5*cos(d*x + c)^4 + 10*cos(d*x + c)^3 + 10*cos
(d*x + c)^2 + 5*cos(d*x + c) + 1)*log(sin(d*x + c) + 1) - 315*(cos(d*x + c
)^5 + 5*cos(d*x + c)^4 + 10*cos(d*x + c)^3 + 10*cos(d*x + c)^2 + 5*cos(d*x
 + c) + 1)*log(-sin(d*x + c) + 1) - 2*(488*cos(d*x + c)^4 + 2125*cos(d*x +
 c)^3 + 3549*cos(d*x + c)^2 + 2740*cos(d*x + c) + 863)*sin(d*x + c))/(a^5*
d*cos(d*x + c)^5 + 5*a^5*d*cos(d*x + c)^4 + 10*a^5*d*cos(d*x + c)^3 + 10*a
^5*d*cos(d*x + c)^2 + 5*a^5*d*cos(d*x + c) + a^5*d)
```

3.81.6 Sympy [F]

$$\int \frac{\sec^6(c+dx)}{(a+a\sec(c+dx))^5}\,dx = \frac{\int \frac{\sec^6(c+dx)}{\sec^5(c+dx)+5\sec^4(c+dx)+10\sec^3(c+dx)+10\sec^2(c+dx)+5\sec(c+dx)+1}\,dx}{a^5}$$

input
```
integrate(sec(d*x+c)**6/(a+a*sec(d*x+c))**5,x)
```

output
```
Integral(sec(c + d*x)**6/(sec(c + d*x)**5 + 5*sec(c + d*x)**4 + 10*sec(c +
 d*x)**3 + 10*sec(c + d*x)**2 + 5*sec(c + d*x) + 1), x)/a**5
```

3.81.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.90

$$\int \frac{\sec^6(c+dx)}{(a+a\sec(c+dx))^5}\,dx = -\frac{\frac{\frac{9765\sin(dx+c)}{\cos(dx+c)+1}+\frac{2730\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{1008\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{270\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{35\sin(dx+c)^9}{(\cos(dx+c)+1)^9}}{a^5}-\frac{5040\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^5}+\frac{5040\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right.}{a^5}}{5040\,d}$$

input
```
integrate(sec(d*x+c)^6/(a+a*sec(d*x+c))^5,x, algorithm="maxima")
```

output
```
-1/5040*((9765*sin(d*x + c)/(cos(d*x + c) + 1) + 2730*sin(d*x + c)^3/(cos(
d*x + c) + 1)^3 + 1008*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 270*sin(d*x +
 c)^7/(cos(d*x + c) + 1)^7 + 35*sin(d*x + c)^9/(cos(d*x + c) + 1)^9)/a^5 -
 5040*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^5 + 5040*log(sin(d*x + c)
/(cos(d*x + c) + 1) - 1)/a^5)/d
```

3.81.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.71

$$\int \frac{\sec^6(c+dx)}{(a+a\sec(c+dx))^5}\,dx = \frac{\frac{5040\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a^5}-\frac{5040\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{a^5}-\frac{35a^{40}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^9+270a^{40}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7+1008a^{40}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^{45}}}{5040\,d}$$

input
```
integrate(sec(d*x+c)^6/(a+a*sec(d*x+c))^5,x, algorithm="giac")
```

output
```
1/5040*(5040*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^5 - 5040*log(abs(tan(1/2
*d*x + 1/2*c) - 1))/a^5 - (35*a^40*tan(1/2*d*x + 1/2*c)^9 + 270*a^40*tan(1
/2*d*x + 1/2*c)^7 + 1008*a^40*tan(1/2*d*x + 1/2*c)^5 + 2730*a^40*tan(1/2*d
*x + 1/2*c)^3 + 9765*a^40*tan(1/2*d*x + 1/2*c))/a^45)/d
```

3.81.9 Mupad [B] (verification not implemented)

Time = 13.56 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.56

$$\int \frac{\sec^6(c+dx)}{(a+a\sec(c+dx))^5}\,dx = -\frac{\frac{13\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{24\,a^5}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5}{5\,a^5}+\frac{3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7}{56\,a^5}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^9}{144\,a^5}-\frac{2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{a^5}+\frac{31\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{16\,a^5}}{d}$$

input
```
int(1/(cos(c + d*x)^6*(a + a/cos(c + d*x))^5),x)
```

output
```
-((13*tan(c/2 + (d*x)/2)^3)/(24*a^5) + tan(c/2 + (d*x)/2)^5/(5*a^5) + (3*t
an(c/2 + (d*x)/2)^7)/(56*a^5) + tan(c/2 + (d*x)/2)^9/(144*a^5) - (2*atanh(
tan(c/2 + (d*x)/2)))/a^5 + (31*tan(c/2 + (d*x)/2))/(16*a^5))/d
```

3.82 $\int \frac{\sec^5(c+dx)}{(a+a\sec(c+dx))^5}\,dx$

3.82.1 Optimal result 819
3.82.2 Mathematica [A] (verified) 819
3.82.3 Rubi [A] (verified) 820
3.82.4 Maple [C] (verified) 823
3.82.5 Fricas [A] (verification not implemented) 824
3.82.6 Sympy [F] 824
3.82.7 Maxima [A] (verification not implemented) 825
3.82.8 Giac [A] (verification not implemented) 825
3.82.9 Mupad [B] (verification not implemented) 826

3.82.1 Optimal result

Integrand size = 21, antiderivative size = 159

$$\int \frac{\sec^5(c+dx)}{(a+a\sec(c+dx))^5}\,dx = \frac{\sec^4(c+dx)\tan(c+dx)}{9d(a+a\sec(c+dx))^5} + \frac{4\sec^3(c+dx)\tan(c+dx)}{63ad(a+a\sec(c+dx))^4} + \frac{4\tan(c+dx)}{105a^2d(a+a\sec(c+dx))^3} - \frac{32\tan(c+dx)}{315ad\left(a^2+a^2\sec(c+dx)\right)^2} + \frac{4\tan(c+dx)}{45d\left(a^5+a^5\sec(c+dx)\right)}$$

output
```
1/9*sec(d*x+c)^4*tan(d*x+c)/d/(a+a*sec(d*x+c))^5+4/63*sec(d*x+c)^3*tan(d*x
+c)/a/d/(a+a*sec(d*x+c))^4+4/105*tan(d*x+c)/a^2/d/(a+a*sec(d*x+c))^3-32/31
5*tan(d*x+c)/a/d/(a^2+a^2*sec(d*x+c))^2+4/45*tan(d*x+c)/d/(a^5+a^5*sec(d*x
+c))
```

3.82.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.61

$$\int \frac{\sec^5(c+dx)}{(a+a\sec(c+dx))^5}\,dx = \frac{\cos\left(\frac{1}{2}(c+dx)\right)\sec^5(c+dx)\left(126\sin\left(\frac{1}{2}(c+dx)\right)+84\sin\left(\frac{3}{2}(c+dx)\right)+36\sin\left(\frac{5}{2}(c+dx)\right)+9\sin\left(\frac{7}{2}(c+\right.\right.}{315a^5d(1+\sec(c+dx))^5}$$

input
```
Integrate[Sec[c + d*x]^5/(a + a*Sec[c + d*x])^5,x]
```

output
```
(Cos[(c + d*x)/2]*Sec[c + d*x]^5*(126*Sin[(c + d*x)/2] + 84*Sin[(3*(c + d*
x))/2] + 36*Sin[(5*(c + d*x))/2] + 9*Sin[(7*(c + d*x))/2] + Sin[(9*(c + d*
x))/2]))/(315*a^5*d*(1 + Sec[c + d*x])^5)
```

3.82.3 Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.05, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.524$, Rules used = {3042, 4297, 3042, 4297, 3042, 4286, 25, 3042, 4488, 3042, 4281}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^5(c+dx)}{(a\sec(c+dx)+a)^5}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^5}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^5}dx$$

$$\Big\downarrow \text{4297}$$

$$\frac{4\int \frac{\sec^4(c+dx)}{(\sec(c+dx)a+a)^4}dx}{9a}+\frac{\tan(c+dx)\sec^4(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^4}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^4}dx}{9a}+\frac{\tan(c+dx)\sec^4(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$$\Big\downarrow \text{4297}$$

$$\frac{4\left(\frac{3\int \frac{\sec^3(c+dx)}{(\sec(c+dx)a+a)^3}dx}{7a}+\frac{\tan(c+dx)\sec^3(c+dx)}{7d(a\sec(c+dx)+a)^4}\right)}{9a}+\frac{\tan(c+dx)\sec^4(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4\left(\frac{3\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^3}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^3}dx}{7a}+\frac{\tan(c+dx)\sec^3(c+dx)}{7d(a\sec(c+dx)+a)^4}\right)}{9a}+\frac{\tan(c+dx)\sec^4(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$\downarrow$ 4286

$$\frac{4\left(\frac{3\left(\frac{\int -\frac{\sec(c+dx)(3a-5a\sec(c+dx))}{(\sec(c+dx)a+a)^2}dx}{5a^2}+\frac{\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}\right)}{7a}+\frac{\tan(c+dx)\sec^3(c+dx)}{7d(a\sec(c+dx)+a)^4}\right)}{9a}+\frac{\tan(c+dx)\sec^4(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$\downarrow$ 25

$$\frac{4\left(\frac{3\left(\frac{\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}-\frac{\int \frac{\sec(c+dx)(3a-5a\sec(c+dx))}{(\sec(c+dx)a+a)^2}dx}{5a^2}\right)}{7a}+\frac{\tan(c+dx)\sec^3(c+dx)}{7d(a\sec(c+dx)+a)^4}\right)}{9a}+\frac{\tan(c+dx)\sec^4(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$\downarrow$ 3042

$$\frac{4\left(\frac{3\left(\frac{\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}-\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(3a-5a\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}dx}{5a^2}\right)}{7a}+\frac{\tan(c+dx)\sec^3(c+dx)}{7d(a\sec(c+dx)+a)^4}\right)}{9a}+\frac{\tan(c+dx)\sec^4(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$\downarrow$ 4488

$$\frac{4\left(\frac{3\left(\frac{\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}-\frac{\frac{8a\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}-\frac{7}{3}\int \frac{\sec(c+dx)}{\sec(c+dx)a+a}dx}{5a^2}\right)}{7a}+\frac{\tan(c+dx)\sec^3(c+dx)}{7d(a\sec(c+dx)+a)^4}\right)}{9a}+\frac{\tan(c+dx)\sec^4(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$\downarrow$ 3042

3.82. $\int \frac{\sec^5(c+dx)}{(a+a\sec(c+dx))^5}\,dx$

$$\frac{4\left(\frac{3\left(\frac{\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}-\frac{\frac{8a\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}-\frac{7}{3}\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{5a^2}\right)}{7a}+\frac{\tan(c+dx)\sec^3(c+dx)}{7d(a\sec(c+dx)+a)^4}\right)}{9a}+\frac{\tan(c+dx)\sec^4(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$\Big\downarrow$ 4281

$$\frac{4\left(\frac{3\left(\frac{\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}-\frac{\frac{8a\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}-\frac{7\tan(c+dx)}{3d(a\sec(c+dx)+a)}}{5a^2}\right)}{7a}+\frac{\tan(c+dx)\sec^3(c+dx)}{7d(a\sec(c+dx)+a)^4}\right)}{9a}+\frac{\tan(c+dx)\sec^4(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

input
```
Int[Sec[c + d*x]^5/(a + a*Sec[c + d*x])^5,x]
```

output
```
(Sec[c + d*x]^4*Tan[c + d*x])/(9*d*(a + a*Sec[c + d*x])^5) + (4*((Sec[c +
d*x]^3*Tan[c + d*x])/(7*d*(a + a*Sec[c + d*x])^4) + (3*(Tan[c + d*x]/(5*d*
(a + a*Sec[c + d*x])^3) - ((8*a*Tan[c + d*x])/(3*d*(a + a*Sec[c + d*x])^2)
 - (7*Tan[c + d*x])/(3*d*(a + a*Sec[c + d*x])))/(5*a^2)))/(7*a)))/(9*a)
```

3.82.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4281
```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[-Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

rule 4286
```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[b*Cot[e + f*x]*((a + b*Csc[e + f*x])^m/(a*f*(2*m + 1))),
x] - Simp[1/(a^2*(2*m + 1))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
*(a*m - b*(2*m + 1)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[
a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

rule 4297
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[b*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Cs
c[e + f*x])^(n - 1)/(a*f*(2*m + 1))), x] + Simp[d*((m + 1)/(b*(2*m + 1)))
 Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ
[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && EqQ[m + n, 0] && LtQ[m,
-2^(-1)] && IntegerQ[2*m]
```

rule 4488
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b - a*B)*Cot[e +
f*x]*((a + b*Csc[e + f*x])^m/(a*f*(2*m + 1))), x] + Simp[(a*B*m + A*b*(m +
1))/(a*b*(2*m + 1))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x], x]
 /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[a*B*m + A*b*(m + 1), 0] && LtQ[m, -2^(-1)]
```

3.82.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.43

method	result	size
risch	$\frac{16i\left(126\,\mathrm{e}^{4i(dx+c)}+84\,\mathrm{e}^{3i(dx+c)}+36\,\mathrm{e}^{2i(dx+c)}+9\,\mathrm{e}^{i(dx+c)}+1\right)}{315d\,a^5\left(\mathrm{e}^{i(dx+c)}+1\right)^9}$	69
derivativedivides	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^9}{9}+\frac{4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{7}+\frac{6\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{5}+\frac{4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3}+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{16d\,a^5}$	71
default	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^9}{9}+\frac{4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{7}+\frac{6\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{5}+\frac{4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3}+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{16d\,a^5}$	71
parallelrisch	$\frac{35\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^9+180\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7+378\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5+420\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3+315\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{5040d\,a^5}$	73

input
```
int(sec(d*x+c)^5/(a+a*sec(d*x+c))^5,x,method=_RETURNVERBOSE)
```

output
```
16/315*I*(126*exp(4*I*(d*x+c))+84*exp(3*I*(d*x+c))+36*exp(2*I*(d*x+c))+9*e
xp(I*(d*x+c))+1)/d/a^5/(exp(I*(d*x+c))+1)^9
```

3.82.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.77

$$\int \frac{\sec^5(c+dx)}{(a+a\sec(c+dx))^5}\,dx = \frac{\left(8\cos(dx+c)^4+40\cos(dx+c)^3+84\cos(dx+c)^2+100\cos(dx+c)+83\right)\sin(dx+c)}{315\left(a^5d\cos(dx+c)^5+5a^5d\cos(dx+c)^4+10a^5d\cos(dx+c)^3+10a^5d\cos(dx+c)^2+5a^5d\cos(dx+\right.}$$

input
```
integrate(sec(d*x+c)^5/(a+a*sec(d*x+c))^5,x, algorithm="fricas")
```

output
```
1/315*(8*cos(d*x + c)^4 + 40*cos(d*x + c)^3 + 84*cos(d*x + c)^2 + 100*cos(
d*x + c) + 83)*sin(d*x + c)/(a^5*d*cos(d*x + c)^5 + 5*a^5*d*cos(d*x + c)^4
 + 10*a^5*d*cos(d*x + c)^3 + 10*a^5*d*cos(d*x + c)^2 + 5*a^5*d*cos(d*x + c
) + a^5*d)
```

3.82.6 Sympy [F]

$$\int \frac{\sec^5(c+dx)}{(a+a\sec(c+dx))^5}\,dx = \frac{\int \frac{\sec^5(c+dx)}{\sec^5(c+dx)+5\sec^4(c+dx)+10\sec^3(c+dx)+10\sec^2(c+dx)+5\sec(c+dx)+1}\,dx}{a^5}$$

input
```
integrate(sec(d*x+c)**5/(a+a*sec(d*x+c))**5,x)
```

output
```
Integral(sec(c + d*x)**5/(sec(c + d*x)**5 + 5*sec(c + d*x)**4 + 10*sec(c +
 d*x)**3 + 10*sec(c + d*x)**2 + 5*sec(c + d*x) + 1), x)/a**5
```

3.82.7 Maxima [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.67

$$\int \frac{\sec^5(c+dx)}{(a+a\sec(c+dx))^5}\,dx = \frac{\frac{315\sin(dx+c)}{\cos(dx+c)+1}+\frac{420\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{378\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{180\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{35\sin(dx+c)^9}{(\cos(dx+c)+1)^9}}{5040\,a^5d}$$

input
```
integrate(sec(d*x+c)^5/(a+a*sec(d*x+c))^5,x, algorithm="maxima")
```

output
```
1/5040*(315*sin(d*x + c)/(cos(d*x + c) + 1) + 420*sin(d*x + c)^3/(cos(d*x
+ c) + 1)^3 + 378*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 180*sin(d*x + c)^7
/(cos(d*x + c) + 1)^7 + 35*sin(d*x + c)^9/(cos(d*x + c) + 1)^9)/(a^5*d)
```

3.82.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.45

$$\int \frac{\sec^5(c+dx)}{(a+a\sec(c+dx))^5}\,dx = \frac{35\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^9+180\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7+378\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+420\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+315\tan\left(\right.}{5040\,a^5d}$$

input
```
integrate(sec(d*x+c)^5/(a+a*sec(d*x+c))^5,x, algorithm="giac")
```

output
```
1/5040*(35*tan(1/2*d*x + 1/2*c)^9 + 180*tan(1/2*d*x + 1/2*c)^7 + 378*tan(1
/2*d*x + 1/2*c)^5 + 420*tan(1/2*d*x + 1/2*c)^3 + 315*tan(1/2*d*x + 1/2*c))
/(a^5*d)
```

3.82.9 Mupad [B] (verification not implemented)

Time = 13.59 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.80

$$\int \frac{\sec^5(c+dx)}{(a+a\sec(c+dx))^5}\,dx$$
$$= \frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\left(315\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^8+420\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^6\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^2+378\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^4\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^4+18\right.}{5040\,a^5\,d\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^9}$$

input
```
int(1/(cos(c + d*x)^5*(a + a/cos(c + d*x))^5),x)
```

output
```
(sin(c/2 + (d*x)/2)*(315*cos(c/2 + (d*x)/2)^8 + 35*sin(c/2 + (d*x)/2)^8 +
180*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^6 + 378*cos(c/2 + (d*x)/2)^4*s
in(c/2 + (d*x)/2)^4 + 420*cos(c/2 + (d*x)/2)^6*sin(c/2 + (d*x)/2)^2))/(504
0*a^5*d*cos(c/2 + (d*x)/2)^9)
```

3.83 $\int \frac{\sec^4(c+dx)}{(a+a\sec(c+dx))^5}\,dx$

3.83.1 Optimal result . . . 827
3.83.2 Mathematica [A] (verified) . . . 827
3.83.3 Rubi [A] (verified) . . . 828
3.83.4 Maple [A] (verified) . . . 831
3.83.5 Fricas [A] (verification not implemented) . . . 832
3.83.6 Sympy [F] . . . 833
3.83.7 Maxima [A] (verification not implemented) . . . 833
3.83.8 Giac [A] (verification not implemented) . . . 833
3.83.9 Mupad [B] (verification not implemented) . . . 834

3.83.1 Optimal result

Integrand size = 21, antiderivative size = 159

$$\int \frac{\sec^4(c+dx)}{(a+a\sec(c+dx))^5}\,dx = -\frac{\sec^4(c+dx)\tan(c+dx)}{9d(a+a\sec(c+dx))^5}+\frac{5\sec^3(c+dx)\tan(c+dx)}{63ad(a+a\sec(c+dx))^4}+\frac{\tan(c+dx)}{21a^2d(a+a\sec(c+dx))^3}-\frac{8\tan(c+dx)}{63ad\left(a^2+a^2\sec(c+dx)\right)^2}+\frac{\tan(c+dx)}{9d\left(a^5+a^5\sec(c+dx)\right)}$$

output
```
-1/9*sec(d*x+c)^4*tan(d*x+c)/d/(a+a*sec(d*x+c))^5+5/63*sec(d*x+c)^3*tan(d*
x+c)/a/d/(a+a*sec(d*x+c))^4+1/21*tan(d*x+c)/a^2/d/(a+a*sec(d*x+c))^3-8/63*
tan(d*x+c)/a/d/(a^2+a^2*sec(d*x+c))^2+1/9*tan(d*x+c)/d/(a^5+a^5*sec(d*x+c)
)
```

3.83.2 Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.40

$$\int \frac{\sec^4(c+dx)}{(a+a\sec(c+dx))^5}\,dx = \frac{(65+130\cos(c+dx)+46\cos(2(c+dx))+10\cos(3(c+dx))+\cos(4(c+dx)))\sin(c+dx)}{252a^5d(1+\cos(c+dx))^5}$$

input
```
Integrate[Sec[c + d*x]^4/(a + a*Sec[c + d*x])^5,x]
```

output
```
((65 + 130*Cos[c + d*x] + 46*Cos[2*(c + d*x)] + 10*Cos[3*(c + d*x)] + Cos[
4*(c + d*x)])*Sin[c + d*x])/(252*a^5*d*(1 + Cos[c + d*x])^5)
```

3.83.3 Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.05, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.524$, Rules used = {3042, 4298, 3042, 4297, 3042, 4286, 25, 3042, 4488, 3042, 4281}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^4(c+dx)}{(a\sec(c+dx)+a)^5}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^4}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^5}dx$$

$$\Big\downarrow \text{4298}$$

$$\frac{5\int \frac{\sec^4(c+dx)}{(\sec(c+dx)a+a)^4}dx}{9a}-\frac{\tan(c+dx)\sec^4(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^4}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^4}dx}{9a}-\frac{\tan(c+dx)\sec^4(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$$\Big\downarrow \text{4297}$$

$$\frac{5\left(\frac{3\int \frac{\sec^3(c+dx)}{(\sec(c+dx)a+a)^3}dx}{7a}+\frac{\tan(c+dx)\sec^3(c+dx)}{7d(a\sec(c+dx)+a)^4}\right)}{9a}-\frac{\tan(c+dx)\sec^4(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5\left(\frac{3\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^3}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^3}dx}{7a}+\frac{\tan(c+dx)\sec^3(c+dx)}{7d(a\sec(c+dx)+a)^4}\right)}{9a}-\frac{\tan(c+dx)\sec^4(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$$\Big\downarrow \text{4286}$$

$$\frac{5\left(\frac{3\left(\frac{\int -\frac{\sec(c+dx)(3a-5a\sec(c+dx))}{(\sec(c+dx)a+a)^2}dx}{5a^2}+\frac{\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}\right)}{7a}+\frac{\tan(c+dx)\sec^3(c+dx)}{7d(a\sec(c+dx)+a)^4}\right)}{9a}-\frac{\tan(c+dx)\sec^4(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$$\Big\downarrow \text{25}$$

$$\frac{5\left(\frac{3\left(\frac{\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}-\frac{\int \frac{\sec(c+dx)(3a-5a\sec(c+dx))}{(\sec(c+dx)a+a)^2}dx}{5a^2}\right)}{7a}+\frac{\tan(c+dx)\sec^3(c+dx)}{7d(a\sec(c+dx)+a)^4}\right)}{9a}-\frac{\tan(c+dx)\sec^4(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5\left(\frac{3\left(\frac{\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}-\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(3a-5a\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}dx}{5a^2}\right)}{7a}+\frac{\tan(c+dx)\sec^3(c+dx)}{7d(a\sec(c+dx)+a)^4}\right)}{9a}-\frac{\tan(c+dx)\sec^4(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$$\Big\downarrow \text{4488}$$

$$\frac{5\left(\frac{3\left(\frac{\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}-\frac{\frac{8a\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}-\frac{7}{3}\int \frac{\sec(c+dx)}{\sec(c+dx)a+a}dx}{5a^2}\right)}{7a}+\frac{\tan(c+dx)\sec^3(c+dx)}{7d(a\sec(c+dx)+a)^4}\right)}{9a}-\frac{\tan(c+dx)\sec^4(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$$\Big\downarrow \text{3042}$$

3.83. $\int \frac{\sec^4(c+dx)}{(a+a\sec(c+dx))^5}\,dx$

$$\frac{5\left(\frac{3\left(\frac{\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}-\frac{\frac{8a\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}-\frac{7}{3}\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{5a^2}\right)}{7a}+\frac{\tan(c+dx)\sec^3(c+dx)}{7d(a\sec(c+dx)+a)^4}\right)}{9a}-\frac{\tan(c+dx)\sec^4(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$$\Big\downarrow \; 4281$$

$$\frac{5\left(\frac{3\left(\frac{\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}-\frac{\frac{8a\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}-\frac{7\tan(c+dx)}{3d(a\sec(c+dx)+a)}}{5a^2}\right)}{7a}+\frac{\tan(c+dx)\sec^3(c+dx)}{7d(a\sec(c+dx)+a)^4}\right)}{9a}-\frac{\tan(c+dx)\sec^4(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

input
```
Int[Sec[c + d*x]^4/(a + a*Sec[c + d*x])^5,x]
```

output
```
-1/9*(Sec[c + d*x]^4*Tan[c + d*x])/(d*(a + a*Sec[c + d*x])^5) + (5*((Sec[c
 + d*x]^3*Tan[c + d*x])/(7*d*(a + a*Sec[c + d*x])^4) + (3*(Tan[c + d*x]/(5
*d*(a + a*Sec[c + d*x])^3) - ((8*a*Tan[c + d*x])/(3*d*(a + a*Sec[c + d*x])
^2) - (7*Tan[c + d*x])/(3*d*(a + a*Sec[c + d*x])))/(5*a^2)))/(7*a)))/(9*a)
```

3.83.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4281
```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[-Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

rule 4286
```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[b*Cot[e + f*x]*((a + b*Csc[e + f*x])^m/(a*f*(2*m + 1))),
x] - Simp[1/(a^2*(2*m + 1))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
*(a*m - b*(2*m + 1)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[
a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

rule 4297
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[b*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Cs
c[e + f*x])^(n - 1)/(a*f*(2*m + 1))), x] + Simp[d*((m + 1)/(b*(2*m + 1)))
 Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ
[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && EqQ[m + n, 0] && LtQ[m,
-2^(-1)] && IntegerQ[2*m]
```

rule 4298
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-Cot[e + f*x])*(a + b*Csc[e + f*x])^m*((d*Csc
[e + f*x])^n/(f*(2*m + 1))), x] + Simp[m/(a*(2*m + 1))   Int[(a + b*Csc[e +
 f*x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f}, x] && E
qQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && LtQ[m, -2^(-1)]
```

rule 4488
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b - a*B)*Cot[e +
f*x]*((a + b*Csc[e + f*x])^m/(a*f*(2*m + 1))), x] + Simp[(a*B*m + A*b*(m +
1))/(a*b*(2*m + 1))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x], x]
 /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[a*B*m + A*b*(m + 1), 0] && LtQ[m, -2^(-1)]
```

3.83.4 Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.36

method	result
parallelrisch	$-\frac{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{8}+\frac{18\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}}{7}-6\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-9\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{144d\,a^{5}}$
derivativedivides	$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{9}}{9}-\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{7}+\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{3}+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{16d\,a^{5}}$
default	$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{9}}{9}-\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{7}+\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{3}+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{16d\,a^{5}}$
risch	$\frac{4i\left(63\,\mathrm{e}^{5i(dx+c)}+63\,\mathrm{e}^{4i(dx+c)}+84\,\mathrm{e}^{3i(dx+c)}+36\,\mathrm{e}^{2i(dx+c)}+9\,\mathrm{e}^{i(dx+c)}+1\right)}{63d\,a^{5}\left(\mathrm{e}^{i(dx+c)}+1\right)^{9}}$
norman	$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{16ad}+\frac{7\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{48ad}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{16ad}-\frac{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{112ad}-\frac{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{9}}{1008ad}+\frac{11\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{11}}{336ad}+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{13}}{336ad}-\frac{\tan(}{}}{\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)^{3}a^{4}}$

input
```
int(sec(d*x+c)^4/(a+a*sec(d*x+c))^5,x,method=_RETURNVERBOSE)
```

output
```
-1/144*(tan(1/2*d*x+1/2*c)^8+18/7*tan(1/2*d*x+1/2*c)^6-6*tan(1/2*d*x+1/2*c
)^2-9)*tan(1/2*d*x+1/2*c)/d/a^5
```

3.83.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.77

$$\int \frac{\sec^4(c+dx)}{(a+a\sec(c+dx))^5}\,dx = \frac{\left(2\cos(dx+c)^4+10\cos(dx+c)^3+21\cos(dx+c)^2+25\cos(dx+c)+5\right)\sin(dx+c)}{63\left(a^5d\cos(dx+c)^5+5a^5d\cos(dx+c)^4+10a^5d\cos(dx+c)^3+10a^5d\cos(dx+c)^2+5a^5d\cos(dx+c\right.}$$

input
```
integrate(sec(d*x+c)^4/(a+a*sec(d*x+c))^5,x, algorithm="fricas")
```

output
```
1/63*(2*cos(d*x + c)^4 + 10*cos(d*x + c)^3 + 21*cos(d*x + c)^2 + 25*cos(d*
x + c) + 5)*sin(d*x + c)/(a^5*d*cos(d*x + c)^5 + 5*a^5*d*cos(d*x + c)^4 +
10*a^5*d*cos(d*x + c)^3 + 10*a^5*d*cos(d*x + c)^2 + 5*a^5*d*cos(d*x + c) +
 a^5*d)
```

3.83.6 Sympy [F]

$$\int \frac{\sec^4(c+dx)}{(a+a\sec(c+dx))^5}\,dx = \frac{\int \frac{\sec^4(c+dx)}{\sec^5(c+dx)+5\sec^4(c+dx)+10\sec^3(c+dx)+10\sec^2(c+dx)+5\sec(c+dx)+1}\,dx}{a^5}$$

input
```
integrate(sec(d*x+c)**4/(a+a*sec(d*x+c))**5,x)
```

output
```
Integral(sec(c + d*x)**4/(sec(c + d*x)**5 + 5*sec(c + d*x)**4 + 10*sec(c +
 d*x)**3 + 10*sec(c + d*x)**2 + 5*sec(c + d*x) + 1), x)/a**5
```

3.83.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.55

$$\int \frac{\sec^4(c+dx)}{(a+a\sec(c+dx))^5}\,dx = \frac{\frac{63\sin(dx+c)}{\cos(dx+c)+1} + \frac{42\sin(dx+c)^3}{(\cos(dx+c)+1)^3} - \frac{18\sin(dx+c)^7}{(\cos(dx+c)+1)^7} - \frac{7\sin(dx+c)^9}{(\cos(dx+c)+1)^9}}{1008\,a^5 d}$$

input
```
integrate(sec(d*x+c)^4/(a+a*sec(d*x+c))^5,x, algorithm="maxima")
```

output
```
1/1008*(63*sin(d*x + c)/(cos(d*x + c) + 1) + 42*sin(d*x + c)^3/(cos(d*x +
c) + 1)^3 - 18*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 - 7*sin(d*x + c)^9/(cos
(d*x + c) + 1)^9)/(a^5*d)
```

3.83.8 Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.37

$$\int \frac{\sec^4(c+dx)}{(a+a\sec(c+dx))^5}\,dx = -\frac{7\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^9 + 18\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7 - 42\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3 - 63\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{1008\,a^5 d}$$

input
```
integrate(sec(d*x+c)^4/(a+a*sec(d*x+c))^5,x, algorithm="giac")
```

output
```
-1/1008*(7*tan(1/2*d*x + 1/2*c)^9 + 18*tan(1/2*d*x + 1/2*c)^7 - 42*tan(1/2
*d*x + 1/2*c)^3 - 63*tan(1/2*d*x + 1/2*c))/(a^5*d)
```

3.83.9 Mupad [B] (verification not implemented)

Time = 13.62 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.36

$$\int \frac{\sec^4(c+dx)}{(a+a\sec(c+dx))^5}\,dx = \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\left(-7\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8-18\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6+42\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+63\right)}{1008\,a^5\,d}$$

input
```
int(1/(cos(c + d*x)^4*(a + a/cos(c + d*x))^5),x)
```

output
```
(tan(c/2 + (d*x)/2)*(42*tan(c/2 + (d*x)/2)^2 - 18*tan(c/2 + (d*x)/2)^6 - 7
*tan(c/2 + (d*x)/2)^8 + 63))/(1008*a^5*d)
```

3.84 $\int \frac{\sec^3(c+dx)}{(a+a\sec(c+dx))^5}\,dx$

3.84.1 Optimal result 835
3.84.2 Mathematica [A] (verified) 835
3.84.3 Rubi [A] (verified) 836
3.84.4 Maple [A] (verified) 839
3.84.5 Fricas [A] (verification not implemented) 839
3.84.6 Sympy [F] 840
3.84.7 Maxima [A] (verification not implemented) 840
3.84.8 Giac [A] (verification not implemented) 840
3.84.9 Mupad [B] (verification not implemented) 841

3.84.1 Optimal result

Integrand size = 21, antiderivative size = 139

$$\int \frac{\sec^3(c+dx)}{(a+a\sec(c+dx))^5}\,dx = \frac{\tan(c+dx)}{9d(a+a\sec(c+dx))^5} - \frac{2\tan(c+dx)}{9ad(a+a\sec(c+dx))^4} + \frac{\tan(c+dx)}{15a^2d(a+a\sec(c+dx))^3} + \frac{2\tan(c+dx)}{45a^3d(a+a\sec(c+dx))^2} + \frac{2\tan(c+dx)}{45d\left(a^5+a^5\sec(c+dx)\right)}$$

output
```
1/9*tan(d*x+c)/d/(a+a*sec(d*x+c))^5-2/9*tan(d*x+c)/a/d/(a+a*sec(d*x+c))^4+
1/15*tan(d*x+c)/a^2/d/(a+a*sec(d*x+c))^3+2/45*tan(d*x+c)/a^3/d/(a+a*sec(d*
x+c))^2+2/45*tan(d*x+c)/d/(a^5+a^5*sec(d*x+c))
```

3.84.2 Mathematica [A] (verified)

Time = 4.11 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.46

$$\int \frac{\sec^3(c+dx)}{(a+a\sec(c+dx))^5}\,dx = \frac{(53+70\cos(c+dx)+46\cos(2(c+dx))+10\cos(3(c+dx))+\cos(4(c+dx)))\sin(c+dx)}{180a^5d(1+\cos(c+dx))^5}$$

input
```
Integrate[Sec[c + d*x]^3/(a + a*Sec[c + d*x])^5,x]
```

output
```
((53 + 70*Cos[c + d*x] + 46*Cos[2*(c + d*x)] + 10*Cos[3*(c + d*x)] + Cos[4
*(c + d*x)])*Sin[c + d*x])/(180*a^5*d*(1 + Cos[c + d*x])^5)
```

3.84.3 Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.06, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.524$, Rules used = {3042, 4286, 25, 3042, 4488, 3042, 4283, 3042, 4283, 3042, 4281}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^3(c+dx)}{(a\sec(c+dx)+a)^5}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^3}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^5}dx$$

$$\Big\downarrow \text{4286}$$

$$\frac{\int -\frac{\sec(c+dx)(5a-9a\sec(c+dx))}{(\sec(c+dx)a+a)^4}dx}{9a^2}+\frac{\tan(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$$\Big\downarrow \text{25}$$

$$\frac{\tan(c+dx)}{9d(a\sec(c+dx)+a)^5}-\frac{\int \frac{\sec(c+dx)(5a-9a\sec(c+dx))}{(\sec(c+dx)a+a)^4}dx}{9a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\tan(c+dx)}{9d(a\sec(c+dx)+a)^5}-\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(5a-9a\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^4}dx}{9a^2}$$

$$\Big\downarrow \text{4488}$$

$$\frac{\tan(c+dx)}{9d(a\sec(c+dx)+a)^5}-\frac{\frac{2a\tan(c+dx)}{d(a\sec(c+dx)+a)^4}-3\int \frac{\sec(c+dx)}{(\sec(c+dx)a+a)^3}dx}{9a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\tan(c+dx)}{9d(a\sec(c+dx)+a)^5}-\frac{\frac{2a\tan(c+dx)}{d(a\sec(c+dx)+a)^4}-3\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^3}dx}{9a^2}$$

$$\Big\downarrow \text{4283}$$

$$\frac{\tan(c+dx)}{9d(a\sec(c+dx)+a)^5}-\frac{\frac{2a\tan(c+dx)}{d(a\sec(c+dx)+a)^4}-3\left(\frac{2\int\frac{\sec(c+dx)}{(\sec(c+dx)a+a)^2}dx}{5a}+\frac{\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}\right)}{9a^2}$$

$\Big\downarrow$ 3042

$$\frac{\tan(c+dx)}{9d(a\sec(c+dx)+a)^5}-\frac{\frac{2a\tan(c+dx)}{d(a\sec(c+dx)+a)^4}-3\left(\frac{2\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}dx}{5a}+\frac{\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}\right)}{9a^2}$$

$\Big\downarrow$ 4283

$$\frac{\tan(c+dx)}{9d(a\sec(c+dx)+a)^5}-\frac{\frac{2a\tan(c+dx)}{d(a\sec(c+dx)+a)^4}-3\left(\frac{2\left(\frac{\int\frac{\sec(c+dx)}{\sec(c+dx)a+a}dx}{3a}+\frac{\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}\right)}{5a}+\frac{\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}\right)}{9a^2}$$

$\Big\downarrow$ 3042

$$\frac{\tan(c+dx)}{9d(a\sec(c+dx)+a)^5}-\frac{\frac{2a\tan(c+dx)}{d(a\sec(c+dx)+a)^4}-3\left(\frac{2\left(\frac{\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{3a}+\frac{\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}\right)}{5a}+\frac{\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}\right)}{9a^2}$$

$\Big\downarrow$ 4281

$$\frac{\tan(c+dx)}{9d(a\sec(c+dx)+a)^5}-\frac{\frac{2a\tan(c+dx)}{d(a\sec(c+dx)+a)^4}-3\left(\frac{\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}+\frac{2\left(\frac{\tan(c+dx)}{3ad(a\sec(c+dx)+a)}+\frac{\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}\right)}{5a}\right)}{9a^2}$$

input
```
Int[Sec[c + d*x]^3/(a + a*Sec[c + d*x])^5,x]
```

3.84. $\int\frac{\sec^3(c+dx)}{(a+a\sec(c+dx))^5}\,dx$

output
```
Tan[c + d*x]/(9*d*(a + a*Sec[c + d*x])^5) - ((2*a*Tan[c + d*x])/(d*(a + a*
Sec[c + d*x])^4) - 3*(Tan[c + d*x]/(5*d*(a + a*Sec[c + d*x])^3) + (2*(Tan[
c + d*x]/(3*d*(a + a*Sec[c + d*x])^2) + Tan[c + d*x]/(3*a*d*(a + a*Sec[c +
 d*x]))))/(5*a)))/(9*a^2)
```

3.84.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4281
```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[-Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

rule 4283
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[b*Cot[e + f*x]*((a + b*Csc[e + f*x])^m/(a*f*(2*m + 1))), x]
 + Simp[(m + 1)/(a*(2*m + 1))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m +
1), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)
] && IntegerQ[2*m]
```

rule 4286
```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[b*Cot[e + f*x]*((a + b*Csc[e + f*x])^m/(a*f*(2*m + 1))),
x] - Simp[1/(a^2*(2*m + 1))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
*(a*m - b*(2*m + 1)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[
a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

rule 4488
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b - a*B)*Cot[e +
f*x]*((a + b*Csc[e + f*x])^m/(a*f*(2*m + 1))), x] + Simp[(a*B*m + A*b*(m +
1))/(a*b*(2*m + 1))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x], x]
 /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[a*B*m + A*b*(m + 1), 0] && LtQ[m, -2^(-1)]
```

3.84.4 Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.32

method	result	size
derivativedivides	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{9}}{9}-\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{5}+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{16d\,a^{5}}$	45
default	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{9}}{9}-\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{5}+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{16d\,a^{5}}$	45
parallelrisch	$\frac{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{9}-18\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}+45\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{720d\,a^{5}}$	47
risch	$\frac{4i\left(30\,\mathrm{e}^{6i(dx+c)}+45\,\mathrm{e}^{5i(dx+c)}+81\,\mathrm{e}^{4i(dx+c)}+54\,\mathrm{e}^{3i(dx+c)}+36\,\mathrm{e}^{2i(dx+c)}+9\,\mathrm{e}^{i(dx+c)}+1\right)}{45d\,a^{5}\left(\mathrm{e}^{i(dx+c)}+1\right)^{9}}$	91
norman	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{16ad}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{8ad}+\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{80ad}+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{20ad}-\frac{13\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{9}}{720ad}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{11}}{72ad}+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{13}}{144ad}}{\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)^{2}a^{4}}$	152

input
```
int(sec(d*x+c)^3/(a+a*sec(d*x+c))^5,x,method=_RETURNVERBOSE)
```

output
```
1/16/d/a^5*(1/9*tan(1/2*d*x+1/2*c)^9-2/5*tan(1/2*d*x+1/2*c)^5+tan(1/2*d*x+
1/2*c))
```

3.84.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.88

$$\int \frac{\sec^3(c+dx)}{(a+a\sec(c+dx))^5}\,dx$$
$$= \frac{\left(2\,\cos\left(dx+c\right)^4+10\,\cos\left(dx+c\right)^3+21\,\cos\left(dx+c\right)^2+10\,\cos\left(dx+c\right)+2\right)\sin\left(dx+c\right)}{45\left(a^5d\cos\left(dx+c\right)^5+5\,a^5d\cos\left(dx+c\right)^4+10\,a^5d\cos\left(dx+c\right)^3+10\,a^5d\cos\left(dx+c\right)^2+5\,a^5d\cos\left(dx+c\right.\right.}$$

input
```
integrate(sec(d*x+c)^3/(a+a*sec(d*x+c))^5,x, algorithm="fricas")
```

output
```
1/45*(2*cos(d*x + c)^4 + 10*cos(d*x + c)^3 + 21*cos(d*x + c)^2 + 10*cos(d*
x + c) + 2)*sin(d*x + c)/(a^5*d*cos(d*x + c)^5 + 5*a^5*d*cos(d*x + c)^4 +
10*a^5*d*cos(d*x + c)^3 + 10*a^5*d*cos(d*x + c)^2 + 5*a^5*d*cos(d*x + c) +
 a^5*d)
```

3.84.6 Sympy [F]

$$\int \frac{\sec^3(c+dx)}{(a+a\sec(c+dx))^5}\,dx = \frac{\int \frac{\sec^3(c+dx)}{\sec^5(c+dx)+5\sec^4(c+dx)+10\sec^3(c+dx)+10\sec^2(c+dx)+5\sec(c+dx)+1}\,dx}{a^5}$$

input
```
integrate(sec(d*x+c)**3/(a+a*sec(d*x+c))**5,x)
```

output
```
Integral(sec(c + d*x)**3/(sec(c + d*x)**5 + 5*sec(c + d*x)**4 + 10*sec(c +
 d*x)**3 + 10*sec(c + d*x)**2 + 5*sec(c + d*x) + 1), x)/a**5
```

3.84.7 Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.48

$$\int \frac{\sec^3(c+dx)}{(a+a\sec(c+dx))^5}\,dx = \frac{\frac{45\,\sin(dx+c)}{\cos(dx+c)+1} - \frac{18\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5} + \frac{5\,\sin(dx+c)^9}{(\cos(dx+c)+1)^9}}{720\,a^5 d}$$

input
```
integrate(sec(d*x+c)^3/(a+a*sec(d*x+c))^5,x, algorithm="maxima")
```

output
```
1/720*(45*sin(d*x + c)/(cos(d*x + c) + 1) - 18*sin(d*x + c)^5/(cos(d*x + c
) + 1)^5 + 5*sin(d*x + c)^9/(cos(d*x + c) + 1)^9)/(a^5*d)
```

3.84.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.33

$$\int \frac{\sec^3(c+dx)}{(a+a\sec(c+dx))^5}\,dx = \frac{5\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9 - 18\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5 + 45\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{720\,a^5 d}$$

input
```
integrate(sec(d*x+c)^3/(a+a*sec(d*x+c))^5,x, algorithm="giac")
```

output
```
1/720*(5*tan(1/2*d*x + 1/2*c)^9 - 18*tan(1/2*d*x + 1/2*c)^5 + 45*tan(1/2*d
*x + 1/2*c))/(a^5*d)
```

3.84.9 Mupad [B] (verification not implemented)

Time = 13.38 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.32

$$\int \frac{\sec^3(c+dx)}{(a+a\sec(c+dx))^5}\,dx = \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\,\left(5\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8-18\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+45\right)}{720\,a^5\,d}$$

input
```
int(1/(cos(c + d*x)^3*(a + a/cos(c + d*x))^5),x)
```

output
```
(tan(c/2 + (d*x)/2)*(5*tan(c/2 + (d*x)/2)^8 - 18*tan(c/2 + (d*x)/2)^4 + 45
))/(720*a^5*d)
```

3.85 $\int \frac{\sec^2(c+dx)}{(a+a\sec(c+dx))^5}\,dx$

3.85.1 Optimal result 842
3.85.2 Mathematica [A] (verified) 842
3.85.3 Rubi [A] (verified) 843
3.85.4 Maple [A] (verified) 846
3.85.5 Fricas [A] (verification not implemented) 846
3.85.6 Sympy [F] 847
3.85.7 Maxima [A] (verification not implemented) 847
3.85.8 Giac [A] (verification not implemented) 847
3.85.9 Mupad [B] (verification not implemented) 848

3.85.1 Optimal result

Integrand size = 21, antiderivative size = 143

$$\int \frac{\sec^2(c+dx)}{(a+a\sec(c+dx))^5}\,dx = -\frac{\tan(c+dx)}{9d(a+a\sec(c+dx))^5}+\frac{5\tan(c+dx)}{63ad(a+a\sec(c+dx))^4}+\frac{\tan(c+dx)}{21a^2d(a+a\sec(c+dx))^3}+\frac{2\tan(c+dx)}{63ad\left(a^2+a^2\sec(c+dx)\right)^2}+\frac{2\tan(c+dx)}{63d\left(a^5+a^5\sec(c+dx)\right)}$$

output
```
-1/9*tan(d*x+c)/d/(a+a*sec(d*x+c))^5+5/63*tan(d*x+c)/a/d/(a+a*sec(d*x+c))^
4+1/21*tan(d*x+c)/a^2/d/(a+a*sec(d*x+c))^3+2/63*tan(d*x+c)/a/d/(a^2+a^2*se
c(d*x+c))^2+2/63*tan(d*x+c)/d/(a^5+a^5*sec(d*x+c))
```

3.85.2 Mathematica [A] (verified)

Time = 4.93 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.46

$$\int \frac{\sec^2(c+dx)}{(a+a\sec(c+dx))^5}\,dx = \frac{(5+25\sec(c+dx)+21\sec^2(c+dx)+10\sec^3(c+dx)+2\sec^4(c+dx))\tan(c+dx)}{63a^5d(1+\sec(c+dx))^5}$$

input
```
Integrate[Sec[c + d*x]^2/(a + a*Sec[c + d*x])^5,x]
```

output
```
((5 + 25*Sec[c + d*x] + 21*Sec[c + d*x]^2 + 10*Sec[c + d*x]^3 + 2*Sec[c +
d*x]^4)*Tan[c + d*x])/(63*a^5*d*(1 + Sec[c + d*x])^5)
```

3.85.3 Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.07, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {3042, 4284, 3042, 4283, 3042, 4283, 3042, 4283, 3042, 4281}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(c+dx)}{(a\sec(c+dx)+a)^5}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^5}dx$$

$$\Big\downarrow \text{4284}$$

$$\frac{5\int \frac{\sec(c+dx)}{(\sec(c+dx)a+a)^4}dx}{9a}-\frac{\tan(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^4}dx}{9a}-\frac{\tan(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$$\Big\downarrow \text{4283}$$

$$\frac{5\left(\frac{3\int \frac{\sec(c+dx)}{(\sec(c+dx)a+a)^3}dx}{7a}+\frac{\tan(c+dx)}{7d(a\sec(c+dx)+a)^4}\right)}{9a}-\frac{\tan(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5\left(\frac{3\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^3}dx}{7a}+\frac{\tan(c+dx)}{7d(a\sec(c+dx)+a)^4}\right)}{9a}-\frac{\tan(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$$\Big\downarrow \text{4283}$$

$$\frac{5\left(\frac{3\left(\frac{2\int \frac{\sec(c+dx)}{(\sec(c+dx)a+a)^2}dx}{5a}+\frac{\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}\right)}{7a}+\frac{\tan(c+dx)}{7d(a\sec(c+dx)+a)^4}\right)}{9a}-\frac{\tan(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$\downarrow$ 3042

$$\frac{5\left(\frac{3\left(\frac{2\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}dx}{5a}+\frac{\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}\right)}{7a}+\frac{\tan(c+dx)}{7d(a\sec(c+dx)+a)^4}\right)}{9a}-\frac{\tan(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$\downarrow$ 4283

$$\frac{5\left(\frac{3\left(\frac{2\left(\frac{\int \frac{\sec(c+dx)}{\sec(c+dx)a+a}dx}{3a}+\frac{\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}\right)}{5a}+\frac{\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}\right)}{7a}+\frac{\tan(c+dx)}{7d(a\sec(c+dx)+a)^4}\right)}{9a}-\frac{\tan(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$\downarrow$ 3042

$$\frac{5\left(\frac{3\left(\frac{2\left(\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{3a}+\frac{\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}\right)}{5a}+\frac{\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}\right)}{7a}+\frac{\tan(c+dx)}{7d(a\sec(c+dx)+a)^4}\right)}{9a}-\frac{\tan(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

3.85. $\int \frac{\sec^2(c+dx)}{(a+a\sec(c+dx))^5}\,dx$

$$\Big\downarrow \text{4281}$$

$$\frac{5\left(\frac{\tan(c+dx)}{7d(a\sec(c+dx)+a)^4}+\frac{3\left(\frac{\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}+\frac{2\left(\frac{\tan(c+dx)}{3ad(a\sec(c+dx)+a)}+\frac{\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}\right)}{5a}\right)}{7a}\right)}{9a}-\frac{\tan(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

input
```
Int[Sec[c + d*x]^2/(a + a*Sec[c + d*x])^5,x]
```

output
```
-1/9*Tan[c + d*x]/(d*(a + a*Sec[c + d*x])^5) + (5*(Tan[c + d*x]/(7*d*(a +
a*Sec[c + d*x])^4) + (3*(Tan[c + d*x]/(5*d*(a + a*Sec[c + d*x])^3) + (2*(T
an[c + d*x]/(3*d*(a + a*Sec[c + d*x])^2) + Tan[c + d*x]/(3*a*d*(a + a*Sec[
c + d*x])))))/(5*a)))/(7*a)))/(9*a)
```

3.85.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4281
```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[-Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

rule 4283
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[b*Cot[e + f*x]*((a + b*Csc[e + f*x])^m/(a*f*(2*m + 1))), x]
 + Simp[(m + 1)/(a*(2*m + 1))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m +
1), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)
] && IntegerQ[2*m]
```

rule 4284
```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(-Cot[e + f*x])*((a + b*Csc[e + f*x])^m/(f*(2*m + 1))), x
] + Simp[m/(b*(2*m + 1))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x
], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

3.85.4 Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.40

method	result	size
parallelrisch	$-\frac{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{8}-\frac{18\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}}{7}+6\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-9\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{144d\,a^{5}}$	57
derivativedivides	$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{9}}{9}+\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{7}-\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{3}+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{16d\,a^{5}}$	58
default	$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{9}}{9}+\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{7}-\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{3}+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{16d\,a^{5}}$	58
risch	$\frac{2i\left(63\,\mathrm{e}^{7i(dx+c)}+147\,\mathrm{e}^{6i(dx+c)}+315\,\mathrm{e}^{5i(dx+c)}+315\,\mathrm{e}^{4i(dx+c)}+273\,\mathrm{e}^{3i(dx+c)}+117\,\mathrm{e}^{2i(dx+c)}+45\,\mathrm{e}^{i(dx+c)}+5\right)}{63d\,a^{5}\left(\mathrm{e}^{i(dx+c)}+1\right)^{9}}$	102
norman	$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{16ad}+\frac{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{48ad}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{24ad}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{56ad}+\frac{25\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{9}}{1008ad}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{11}}{144ad}}{\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)a^{4}}$	133

input
```
int(sec(d*x+c)^2/(a+a*sec(d*x+c))^5,x,method=_RETURNVERBOSE)
```

output
```
-1/144*(tan(1/2*d*x+1/2*c)^8-18/7*tan(1/2*d*x+1/2*c)^6+6*tan(1/2*d*x+1/2*c
)^2-9)*tan(1/2*d*x+1/2*c)/d/a^5
```

3.85.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.86

$$\int \frac{\sec^2(c+dx)}{(a+a\sec(c+dx))^5}\,dx = \frac{\left(5\cos(dx+c)^4+25\cos(dx+c)^3+21\cos(dx+c)^2+10\cos(dx+c)+2\right)\sin(dx+c)}{63\left(a^5d\cos(dx+c)^5+5a^5d\cos(dx+c)^4+10a^5d\cos(dx+c)^3+10a^5d\cos(dx+c)^2+5a^5d\cos(dx+c\right.}$$

input
```
integrate(sec(d*x+c)^2/(a+a*sec(d*x+c))^5,x, algorithm="fricas")
```

output
```
1/63*(5*cos(d*x + c)^4 + 25*cos(d*x + c)^3 + 21*cos(d*x + c)^2 + 10*cos(d*
x + c) + 2)*sin(d*x + c)/(a^5*d*cos(d*x + c)^5 + 5*a^5*d*cos(d*x + c)^4 +
10*a^5*d*cos(d*x + c)^3 + 10*a^5*d*cos(d*x + c)^2 + 5*a^5*d*cos(d*x + c) +
 a^5*d)
```

3.85.6 Sympy [F]

$$\int \frac{\sec^2(c+dx)}{(a+a\sec(c+dx))^5}\,dx = \frac{\int \frac{\sec^2(c+dx)}{\sec^5(c+dx)+5\sec^4(c+dx)+10\sec^3(c+dx)+10\sec^2(c+dx)+5\sec(c+dx)+1}\,dx}{a^5}$$

input
```
integrate(sec(d*x+c)**2/(a+a*sec(d*x+c))**5,x)
```

output
```
Integral(sec(c + d*x)**2/(sec(c + d*x)**5 + 5*sec(c + d*x)**4 + 10*sec(c +
 d*x)**3 + 10*sec(c + d*x)**2 + 5*sec(c + d*x) + 1), x)/a**5
```

3.85.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.61

$$\int \frac{\sec^2(c+dx)}{(a+a\sec(c+dx))^5}\,dx = \frac{\frac{63\sin(dx+c)}{\cos(dx+c)+1} - \frac{42\sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{18\sin(dx+c)^7}{(\cos(dx+c)+1)^7} - \frac{7\sin(dx+c)^9}{(\cos(dx+c)+1)^9}}{1008\,a^5 d}$$

input
```
integrate(sec(d*x+c)^2/(a+a*sec(d*x+c))^5,x, algorithm="maxima")
```

output
```
1/1008*(63*sin(d*x + c)/(cos(d*x + c) + 1) - 42*sin(d*x + c)^3/(cos(d*x +
c) + 1)^3 + 18*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 - 7*sin(d*x + c)^9/(cos
(d*x + c) + 1)^9)/(a^5*d)
```

3.85.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.41

$$\int \frac{\sec^2(c+dx)}{(a+a\sec(c+dx))^5}\,dx = -\frac{7\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^9 - 18\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7 + 42\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3 - 63\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{1008\,a^5 d}$$

input
```
integrate(sec(d*x+c)^2/(a+a*sec(d*x+c))^5,x, algorithm="giac")
```

output
```
-1/1008*(7*tan(1/2*d*x + 1/2*c)^9 - 18*tan(1/2*d*x + 1/2*c)^7 + 42*tan(1/2
*d*x + 1/2*c)^3 - 63*tan(1/2*d*x + 1/2*c))/(a^5*d)
```

3.85.9 Mupad [B] (verification not implemented)

Time = 13.26 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.41

$$\int \frac{\sec^2(c+dx)}{(a+a\sec(c+dx))^5}\,dx = -\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\left(7\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8-18\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6+42\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2-63\right)}{1008\,a^5\,d}$$

input
```
int(1/(cos(c + d*x)^2*(a + a/cos(c + d*x))^5),x)
```

output
```
-(tan(c/2 + (d*x)/2)*(42*tan(c/2 + (d*x)/2)^2 - 18*tan(c/2 + (d*x)/2)^6 +
7*tan(c/2 + (d*x)/2)^8 - 63))/(1008*a^5*d)
```

3.86 $\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^5}\,dx$

3.86.1 Optimal result . . . 849
3.86.2 Mathematica [A] (verified) . . . 849
3.86.3 Rubi [A] (verified) . . . 850
3.86.4 Maple [A] (verified) . . . 853
3.86.5 Fricas [A] (verification not implemented) . . . 854
3.86.6 Sympy [F] . . . 854
3.86.7 Maxima [A] (verification not implemented) . . . 854
3.86.8 Giac [A] (verification not implemented) . . . 855
3.86.9 Mupad [B] (verification not implemented) . . . 855

3.86.1 Optimal result

Integrand size = 19, antiderivative size = 143

$$\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^5}\,dx = \frac{\tan(c+dx)}{9d(a+a\sec(c+dx))^5} + \frac{4\tan(c+dx)}{63ad(a+a\sec(c+dx))^4} + \frac{4\tan(c+dx)}{105a^2d(a+a\sec(c+dx))^3} + \frac{8\tan(c+dx)}{315ad\left(a^2+a^2\sec(c+dx)\right)^2} + \frac{8\tan(c+dx)}{315d\left(a^5+a^5\sec(c+dx)\right)}$$

output
```
1/9*tan(d*x+c)/d/(a+a*sec(d*x+c))^5+4/63*tan(d*x+c)/a/d/(a+a*sec(d*x+c))^4
+4/105*tan(d*x+c)/a^2/d/(a+a*sec(d*x+c))^3+8/315*tan(d*x+c)/a/d/(a^2+a^2*s
ec(d*x+c))^2+8/315*tan(d*x+c)/d/(a^5+a^5*sec(d*x+c))
```

3.86.2 Mathematica [A] (verified)

Time = 5.42 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.97

$$\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^5}\,dx = \frac{\sec\left(\frac{c}{2}\right)\sec^9\left(\frac{1}{2}(c+dx)\right)\left(5418\sin\left(\frac{dx}{2}\right)-5040\sin\left(c+\frac{dx}{2}\right)+3612\sin\left(c+\frac{3dx}{2}\right)-3360\sin\left(2c+\frac{3dx}{2}\right)+1\right.}{80640d}$$

input
```
Integrate[Sec[c + d*x]/(a + a*Sec[c + d*x])^5,x]
```

output
```
(Sec[c/2]*Sec[(c + d*x)/2]^9*(5418*Sin[(d*x)/2] - 5040*Sin[c + (d*x)/2] +
3612*Sin[c + (3*d*x)/2] - 3360*Sin[2*c + (3*d*x)/2] + 1728*Sin[2*c + (5*d*
x)/2] - 1260*Sin[3*c + (5*d*x)/2] + 432*Sin[3*c + (7*d*x)/2] - 315*Sin[4*c
 + (7*d*x)/2] + 83*Sin[4*c + (9*d*x)/2]))/(80640*a^5*d)
```

3.86.3 Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.07, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.526$, Rules used = {3042, 4283, 3042, 4283, 3042, 4283, 3042, 4283, 3042, 4281}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(c+dx)}{(a\sec(c+dx)+a)^5}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^5}dx$$

$$\Big\downarrow \text{4283}$$

$$\frac{4\int \frac{\sec(c+dx)}{(\sec(c+dx)a+a)^4}dx}{9a}+\frac{\tan(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^4}dx}{9a}+\frac{\tan(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$$\Big\downarrow \text{4283}$$

$$\frac{4\left(\frac{3\int \frac{\sec(c+dx)}{(\sec(c+dx)a+a)^3}dx}{7a}+\frac{\tan(c+dx)}{7d(a\sec(c+dx)+a)^4}\right)}{9a}+\frac{\tan(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4\left(\frac{3\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^3}dx}{7a}+\frac{\tan(c+dx)}{7d(a\sec(c+dx)+a)^4}\right)}{9a}+\frac{\tan(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$$\Big\downarrow \text{4283}$$

$$\frac{4\left(\frac{3\left(\frac{2\int \frac{\sec(c+dx)}{(\sec(c+dx)a+a)^2}dx}{5a}+\frac{\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}\right)}{7a}+\frac{\tan(c+dx)}{7d(a\sec(c+dx)+a)^4}\right)}{9a}+\frac{\tan(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4\left(\frac{3\left(\frac{2\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}dx}{5a}+\frac{\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}\right)}{7a}+\frac{\tan(c+dx)}{7d(a\sec(c+dx)+a)^4}\right)}{9a}+\frac{\tan(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$$\Big\downarrow \text{4283}$$

$$\frac{4\left(\frac{3\left(\frac{2\left(\frac{\int \frac{\sec(c+dx)}{\sec(c+dx)a+a}dx}{3a}+\frac{\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}\right)}{5a}+\frac{\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}\right)}{7a}+\frac{\tan(c+dx)}{7d(a\sec(c+dx)+a)^4}\right)}{9a}+\frac{\tan(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$$\Big\downarrow \text{3042}$$

3.86. $\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^5}\,dx$

$$\frac{4\left(\frac{3\left(\frac{2\left(\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{3a}+\frac{\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}\right)}{5a}+\frac{\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}\right)}{7a}+\frac{\tan(c+dx)}{7d(a\sec(c+dx)+a)^4}\right)}{9a}+\frac{\tan(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$$\Big\downarrow \; 4281$$

$$\frac{\tan(c+dx)}{9d(a\sec(c+dx)+a)^5}+\frac{4\left(\frac{\tan(c+dx)}{7d(a\sec(c+dx)+a)^4}+\frac{3\left(\frac{\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}+\frac{2\left(\frac{\tan(c+dx)}{3ad(a\sec(c+dx)+a)}+\frac{\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}\right)}{5a}\right)}{7a}\right)}{9a}$$

input
```
Int[Sec[c + d*x]/(a + a*Sec[c + d*x])^5,x]
```

output
```
Tan[c + d*x]/(9*d*(a + a*Sec[c + d*x])^5) + (4*(Tan[c + d*x]/(7*d*(a + a*S
ec[c + d*x])^4) + (3*(Tan[c + d*x]/(5*d*(a + a*Sec[c + d*x])^3) + (2*(Tan[
c + d*x]/(3*d*(a + a*Sec[c + d*x])^2) + Tan[c + d*x]/(3*a*d*(a + a*Sec[c +
 d*x])))))/(5*a)))/(7*a)))/(9*a)
```

3.86. $\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^5}\,dx$

3.86.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4281
```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[-Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

rule 4283
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[b*Cot[e + f*x]*((a + b*Csc[e + f*x])^m/(a*f*(2*m + 1))), x]
 + Simp[(m + 1)/(a*(2*m + 1))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m +
1), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)
] && IntegerQ[2*m]
```

3.86.4 Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.50

method	result
derivativedivides	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{9}}{9}-\frac{4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{7}+\frac{6\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{5}-\frac{4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{3}+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{16d\,a^{5}}$
default	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{9}}{9}-\frac{4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{7}+\frac{6\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{5}-\frac{4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{3}+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{16d\,a^{5}}$
parallelrisch	$\frac{35\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{9}-180\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}+378\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}-420\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}+315\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{5040d\,a^{5}}$
norman	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{16ad}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{12ad}+\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{40ad}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{28ad}+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{9}}{144ad}}{a^{4}}$
risch	$\frac{2i\left(315\,\mathrm{e}^{8i(dx+c)}+1260\,\mathrm{e}^{7i(dx+c)}+3360\,\mathrm{e}^{6i(dx+c)}+5040\,\mathrm{e}^{5i(dx+c)}+5418\,\mathrm{e}^{4i(dx+c)}+3612\,\mathrm{e}^{3i(dx+c)}+1728\,\mathrm{e}^{2i(dx+c)}+432\right.}{315d\,a^{5}\left(\mathrm{e}^{i(dx+c)}+1\right)^{9}}$

input
```
int(sec(d*x+c)/(a+a*sec(d*x+c))^5,x,method=_RETURNVERBOSE)
```

output
```
1/16/d/a^5*(1/9*tan(1/2*d*x+1/2*c)^9-4/7*tan(1/2*d*x+1/2*c)^7+6/5*tan(1/2*
d*x+1/2*c)^5-4/3*tan(1/2*d*x+1/2*c)^3+tan(1/2*d*x+1/2*c))
```

3.86.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.86

$$\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^5}\,dx = \frac{\left(83\cos(dx+c)^4+100\cos(dx+c)^3+84\cos(dx+c)^2+40\cos(dx+c)+8\right)\sin(dx+c)}{315\left(a^5d\cos(dx+c)^5+5a^5d\cos(dx+c)^4+10a^5d\cos(dx+c)^3+10a^5d\cos(dx+c)^2+5a^5d\cos(dx+\right.}$$

input
```
integrate(sec(d*x+c)/(a+a*sec(d*x+c))^5,x, algorithm="fricas")
```

output
```
1/315*(83*cos(d*x + c)^4 + 100*cos(d*x + c)^3 + 84*cos(d*x + c)^2 + 40*cos
(d*x + c) + 8)*sin(d*x + c)/(a^5*d*cos(d*x + c)^5 + 5*a^5*d*cos(d*x + c)^4
 + 10*a^5*d*cos(d*x + c)^3 + 10*a^5*d*cos(d*x + c)^2 + 5*a^5*d*cos(d*x + c
) + a^5*d)
```

3.86.6 Sympy [F]

$$\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^5}\,dx = \frac{\int \frac{\sec(c+dx)}{\sec^5(c+dx)+5\sec^4(c+dx)+10\sec^3(c+dx)+10\sec^2(c+dx)+5\sec(c+dx)+1}\,dx}{a^5}$$

input
```
integrate(sec(d*x+c)/(a+a*sec(d*x+c))**5,x)
```

output
```
Integral(sec(c + d*x)/(sec(c + d*x)**5 + 5*sec(c + d*x)**4 + 10*sec(c + d*
x)**3 + 10*sec(c + d*x)**2 + 5*sec(c + d*x) + 1), x)/a**5
```

3.86.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.75

$$\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^5}\,dx = \frac{\frac{315\sin(dx+c)}{\cos(dx+c)+1}-\frac{420\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{378\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{180\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{35\sin(dx+c)^9}{(\cos(dx+c)+1)^9}}{5040\,a^5d}$$

input
```
integrate(sec(d*x+c)/(a+a*sec(d*x+c))^5,x, algorithm="maxima")
```

output
```
1/5040*(315*sin(d*x + c)/(cos(d*x + c) + 1) - 420*sin(d*x + c)^3/(cos(d*x
+ c) + 1)^3 + 378*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 180*sin(d*x + c)^7
/(cos(d*x + c) + 1)^7 + 35*sin(d*x + c)^9/(cos(d*x + c) + 1)^9)/(a^5*d)
```

3.86.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.50

$$\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^5}\,dx$$
$$= \frac{35\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9 - 180\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7 + 378\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5 - 420\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 + 315\,\tan\left(\right.}{5040\,a^5 d}$$

input
```
integrate(sec(d*x+c)/(a+a*sec(d*x+c))^5,x, algorithm="giac")
```

output
```
1/5040*(35*tan(1/2*d*x + 1/2*c)^9 - 180*tan(1/2*d*x + 1/2*c)^7 + 378*tan(1
/2*d*x + 1/2*c)^5 - 420*tan(1/2*d*x + 1/2*c)^3 + 315*tan(1/2*d*x + 1/2*c))
/(a^5*d)
```

3.86.9 Mupad [B] (verification not implemented)

Time = 13.03 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.89

$$\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^5}\,dx$$
$$= \frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\left(315\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^8 - 420\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^6\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^2 + 378\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^4\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^4 - 18\right.}{5040\,a^5\,d\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^9}$$

input
```
int(1/(cos(c + d*x)*(a + a/cos(c + d*x))^5),x)
```

output
```
(sin(c/2 + (d*x)/2)*(315*cos(c/2 + (d*x)/2)^8 + 35*sin(c/2 + (d*x)/2)^8 -
180*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^6 + 378*cos(c/2 + (d*x)/2)^4*s
in(c/2 + (d*x)/2)^4 - 420*cos(c/2 + (d*x)/2)^6*sin(c/2 + (d*x)/2)^2))/(504
0*a^5*d*cos(c/2 + (d*x)/2)^9)
```

3.87 $\int \frac{1}{(a+a\sec(c+dx))^5}\,dx$

3.87.1 Optimal result 856
3.87.2 Mathematica [A] (verified) 856
3.87.3 Rubi [A] (verified) 857
3.87.4 Maple [A] (verified) 861
3.87.5 Fricas [A] (verification not implemented) 861
3.87.6 Sympy [F] 862
3.87.7 Maxima [A] (verification not implemented) 862
3.87.8 Giac [A] (verification not implemented) 863
3.87.9 Mupad [B] (verification not implemented) 863

3.87.1 Optimal result

Integrand size = 12, antiderivative size = 144

$$\int \frac{1}{(a+a\sec(c+dx))^5}\,dx = \frac{x}{a^5}-\frac{\tan(c+dx)}{9d(a+a\sec(c+dx))^5}-\frac{13\tan(c+dx)}{63ad(a+a\sec(c+dx))^4}-\frac{34\tan(c+dx)}{105a^2d(a+a\sec(c+dx))^3}-\frac{173\tan(c+dx)}{315a^3d(a+a\sec(c+dx))^2}-\frac{488\tan(c+dx)}{315d\left(a^5+a^5\sec(c+dx)\right)}$$

output
```
x/a^5-1/9*tan(d*x+c)/d/(a+a*sec(d*x+c))^5-13/63*tan(d*x+c)/a/d/(a+a*sec(d*
x+c))^4-34/105*tan(d*x+c)/a^2/d/(a+a*sec(d*x+c))^3-173/315*tan(d*x+c)/a^3/
d/(a+a*sec(d*x+c))^2-488/315*tan(d*x+c)/d/(a^5+a^5*sec(d*x+c))
```

3.87.2 Mathematica [A] (verified)

Time = 4.87 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.94

$$\int \frac{1}{(a+a\sec(c+dx))^5}\,dx = \frac{\sec\left(\frac{c}{2}\right)\sec^9\left(\frac{1}{2}(c+dx)\right)\left(39690dx\cos\left(\frac{dx}{2}\right)+39690dx\cos\left(c+\frac{dx}{2}\right)+26460dx\cos\left(c+\frac{3dx}{2}\right)+26460dx\cos\right.}{}$$

input
```
Integrate[(a + a*Sec[c + d*x])^(-5),x]
```

output
```
(Sec[c/2]*Sec[(c + d*x)/2]^9*(39690*d*x*Cos[(d*x)/2] + 39690*d*x*Cos[c + (
d*x)/2] + 26460*d*x*Cos[c + (3*d*x)/2] + 26460*d*x*Cos[2*c + (3*d*x)/2] +
11340*d*x*Cos[2*c + (5*d*x)/2] + 11340*d*x*Cos[3*c + (5*d*x)/2] + 2835*d*x
*Cos[3*c + (7*d*x)/2] + 2835*d*x*Cos[4*c + (7*d*x)/2] + 315*d*x*Cos[4*c +
(9*d*x)/2] + 315*d*x*Cos[5*c + (9*d*x)/2] - 116676*Sin[(d*x)/2] + 100800*S
in[c + (d*x)/2] - 88284*Sin[c + (3*d*x)/2] + 56700*Sin[2*c + (3*d*x)/2] -
43236*Sin[2*c + (5*d*x)/2] + 18900*Sin[3*c + (5*d*x)/2] - 12384*Sin[3*c +
(7*d*x)/2] + 3150*Sin[4*c + (7*d*x)/2] - 1726*Sin[4*c + (9*d*x)/2]))/(1612
80*a^5*d)
```

3.87.3 Rubi [A] (verified)

Time = 0.96 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.19, number of steps used = 16, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 1.333$, Rules used = {3042, 4264, 25, 3042, 4410, 27, 3042, 4410, 25, 3042, 4410, 25, 3042, 4407, 3042, 4281}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\sec(c+dx)+a)^5}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^5}dx$$

$$\Big\downarrow \text{4264}$$

$$-\frac{\int -\frac{9a-4a\sec(c+dx)}{(\sec(c+dx)a+a)^4}dx}{9a^2}-\frac{\tan(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{9a-4a\sec(c+dx)}{(\sec(c+dx)a+a)^4}dx}{9a^2}-\frac{\tan(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{9a-4a\csc\left(c+dx+\frac{\pi}{2}\right)}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^4}dx}{9a^2}-\frac{\tan(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$$\Big\downarrow \text{4410}$$

$$\frac{-\frac{\int -\frac{3\left(21a^2-13a^2\sec(c+dx)\right)}{(\sec(c+dx)a+a)^3}dx}{7a^2}-\frac{13a\tan(c+dx)}{7d(a\sec(c+dx)+a)^4}}{9a^2}-\frac{\tan(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$\Big\downarrow$ 27

$$\frac{\frac{3\int \frac{21a^2-13a^2\sec(c+dx)}{(\sec(c+dx)a+a)^3}dx}{7a^2}-\frac{13a\tan(c+dx)}{7d(a\sec(c+dx)+a)^4}}{9a^2}-\frac{\tan(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$\Big\downarrow$ 3042

$$\frac{\frac{3\int \frac{21a^2-13a^2\csc\left(c+dx+\frac{\pi}{2}\right)}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^3}dx}{7a^2}-\frac{13a\tan(c+dx)}{7d(a\sec(c+dx)+a)^4}}{9a^2}-\frac{\tan(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$\Big\downarrow$ 4410

$$\frac{\frac{3\left(-\frac{\int -\frac{105a^3-68a^3\sec(c+dx)}{(\sec(c+dx)a+a)^2}dx}{5a^2}-\frac{34a^2\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}\right)}{7a^2}-\frac{13a\tan(c+dx)}{7d(a\sec(c+dx)+a)^4}}{9a^2}-\frac{\tan(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$\Big\downarrow$ 25

$$\frac{\frac{3\left(\frac{\int \frac{105a^3-68a^3\sec(c+dx)}{(\sec(c+dx)a+a)^2}dx}{5a^2}-\frac{34a^2\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}\right)}{7a^2}-\frac{13a\tan(c+dx)}{7d(a\sec(c+dx)+a)^4}}{9a^2}-\frac{\tan(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$\Big\downarrow$ 3042

$$\frac{\frac{3\left(\frac{\int \frac{105a^3-68a^3\csc\left(c+dx+\frac{\pi}{2}\right)}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}dx}{5a^2}-\frac{34a^2\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}\right)}{7a^2}-\frac{13a\tan(c+dx)}{7d(a\sec(c+dx)+a)^4}}{9a^2}-\frac{\tan(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$\Big\downarrow$ 4410

$$\frac{\frac{3\left(\frac{-\frac{\int -\frac{315a^4-173a^4\sec(c+dx)}{\sec(c+dx)a+a}dx}{3a^2}-\frac{173a^3\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}}{5a^2}-\frac{34a^2\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}\right)}{7a^2}-\frac{13a\tan(c+dx)}{7d(a\sec(c+dx)+a)^4}}{9a^2}-\frac{\tan(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$\Big\downarrow$ 25

3.87. $\int \frac{1}{(a+a\sec(c+dx))^5}\,dx$

$$\frac{\frac{3\left(\frac{\frac{\int \frac{315a^4-173a^4\sec(c+dx)}{\sec(c+dx)a+a}dx}{3a^2}-\frac{173a^3\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}}{5a^2}-\frac{34a^2\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}\right)}{7a^2}-\frac{13a\tan(c+dx)}{7d(a\sec(c+dx)+a)^4}}{9a^2}-\frac{\tan(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$\downarrow$ 3042

$$\frac{\frac{3\left(\frac{\frac{\int \frac{315a^4-173a^4\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{3a^2}-\frac{173a^3\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}}{5a^2}-\frac{34a^2\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}\right)}{7a^2}-\frac{13a\tan(c+dx)}{7d(a\sec(c+dx)+a)^4}}{9a^2}-\frac{\tan(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$\downarrow$ 4407

$$\frac{\frac{3\left(\frac{\frac{315a^3x-488a^4\int \frac{\sec(c+dx)}{\sec(c+dx)a+a}dx}{3a^2}-\frac{173a^3\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}}{5a^2}-\frac{34a^2\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}\right)}{7a^2}-\frac{13a\tan(c+dx)}{7d(a\sec(c+dx)+a)^4}}{9a^2}-\frac{\tan(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$\downarrow$ 3042

$$\frac{\frac{3\left(\frac{\frac{315a^3x-488a^4\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{3a^2}-\frac{173a^3\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}}{5a^2}-\frac{34a^2\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}\right)}{7a^2}-\frac{13a\tan(c+dx)}{7d(a\sec(c+dx)+a)^4}}{9a^2}-\frac{\tan(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$\downarrow$ 4281

$$\frac{\frac{3\left(\frac{\frac{315a^3x-\frac{488a^4\tan(c+dx)}{d(a\sec(c+dx)+a)}}{3a^2}-\frac{173a^3\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}}{5a^2}-\frac{34a^2\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}\right)}{7a^2}-\frac{13a\tan(c+dx)}{7d(a\sec(c+dx)+a)^4}}{9a^2}-\frac{\tan(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

3.87. $\int \frac{1}{(a+a\sec(c+dx))^5}\,dx$

input
```
Int[(a + a*Sec[c + d*x])^(-5),x]
```

output
```
-1/9*Tan[c + d*x]/(d*(a + a*Sec[c + d*x])^5) + ((-13*a*Tan[c + d*x])/(7*d*
(a + a*Sec[c + d*x])^4) + (3*((-34*a^2*Tan[c + d*x])/(5*d*(a + a*Sec[c + d
*x])^3) + ((-173*a^3*Tan[c + d*x])/(3*d*(a + a*Sec[c + d*x])^2) + (315*a^3
*x - (488*a^4*Tan[c + d*x])/(d*(a + a*Sec[c + d*x])))/(3*a^2))/(5*a^2)))/(
7*a^2))/(9*a^2)
```

3.87.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4264
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(-Cot[c
+ d*x])*((a + b*Csc[c + d*x])^n/(d*(2*n + 1))), x] + Simp[1/(a^2*(2*n + 1))
   Int[(a + b*Csc[c + d*x])^(n + 1)*(a*(2*n + 1) - b*(n + 1)*Csc[c + d*x]),
 x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LeQ[n, -1] && Int
egerQ[2*n]
```

rule 4281
```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[-Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

rule 4407
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Simp[(b*c - a*d)/a   Int[Csc[e + f*
x]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
- a*d, 0]
```

rule 4410
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> Simp[(-(b*c - a*d))*Cot[e + f*x]*((a + b*Csc[e +
f*x])^m/(b*f*(2*m + 1))), x] + Simp[1/(a^2*(2*m + 1))   Int[(a + b*Csc[e +
f*x])^(m + 1)*Simp[a*c*(2*m + 1) - (b*c - a*d)*(m + 1)*Csc[e + f*x], x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&
EqQ[a^2 - b^2, 0] && IntegerQ[2*m]
```

3.87.4 Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.53

method	result
parallelrisch	$\frac{-35\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{9}+270\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}-1008\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}+2730\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}+5040dx-9765\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{5040d\,a^{5}}$
derivativedivides	$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{9}}{9}+\frac{6\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{7}-\frac{16\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{5}+\frac{26\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{3}-31\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+32\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{16d\,a^{5}}$
default	$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{9}}{9}+\frac{6\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{7}-\frac{16\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{5}+\frac{26\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{3}-31\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+32\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{16d\,a^{5}}$
norman	$\frac{\frac{x}{a}-\frac{31\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{16ad}+\frac{13\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{24ad}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{5ad}+\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{56ad}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{9}}{144ad}}{a^{4}}$
risch	$\frac{x}{a^{5}}-\frac{2i\left(1575\,\mathrm{e}^{8i(dx+c)}+9450\,\mathrm{e}^{7i(dx+c)}+28350\,\mathrm{e}^{6i(dx+c)}+50400\,\mathrm{e}^{5i(dx+c)}+58338\,\mathrm{e}^{4i(dx+c)}+44142\,\mathrm{e}^{3i(dx+c)}+21618\,\mathrm{e}\right.}{315d\,a^{5}\left(\mathrm{e}^{i(dx+c)}+1\right)^{9}}$

input
```
int(1/(a+a*sec(d*x+c))^5,x,method=_RETURNVERBOSE)
```

output
```
1/5040*(-35*tan(1/2*d*x+1/2*c)^9+270*tan(1/2*d*x+1/2*c)^7-1008*tan(1/2*d*x
+1/2*c)^5+2730*tan(1/2*d*x+1/2*c)^3+5040*d*x-9765*tan(1/2*d*x+1/2*c))/d/a^
5
```

3.87.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.31

$$\int \frac{1}{(a+a\sec(c+dx))^5}\,dx$$
$$= \frac{315\,dx\cos(dx+c)^5+1575\,dx\cos(dx+c)^4+3150\,dx\cos(dx+c)^3+3150\,dx\cos(dx+c)^2+1575\,dx\cos}{315\left(a^5d\cos(dx+c)^5+5\,a^5d\cos(dx+c)^4+10\right.}$$

input
```
integrate(1/(a+a*sec(d*x+c))^5,x, algorithm="fricas")
```

output
```
1/315*(315*d*x*cos(d*x + c)^5 + 1575*d*x*cos(d*x + c)^4 + 3150*d*x*cos(d*x
 + c)^3 + 3150*d*x*cos(d*x + c)^2 + 1575*d*x*cos(d*x + c) + 315*d*x - (863
*cos(d*x + c)^4 + 2740*cos(d*x + c)^3 + 3549*cos(d*x + c)^2 + 2125*cos(d*x
 + c) + 488)*sin(d*x + c))/(a^5*d*cos(d*x + c)^5 + 5*a^5*d*cos(d*x + c)^4
+ 10*a^5*d*cos(d*x + c)^3 + 10*a^5*d*cos(d*x + c)^2 + 5*a^5*d*cos(d*x + c)
 + a^5*d)
```

3.87.6 Sympy [F]

$$\int \frac{1}{(a+a\sec(c+dx))^5}\,dx = \frac{\int \frac{1}{\sec^5(c+dx)+5\sec^4(c+dx)+10\sec^3(c+dx)+10\sec^2(c+dx)+5\sec(c+dx)+1}\,dx}{a^5}$$

input
```
integrate(1/(a+a*sec(d*x+c))**5,x)
```

output
```
Integral(1/(sec(c + d*x)**5 + 5*sec(c + d*x)**4 + 10*sec(c + d*x)**3 + 10*
sec(c + d*x)**2 + 5*sec(c + d*x) + 1), x)/a**5
```

3.87.7 Maxima [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.92

$$\int \frac{1}{(a+a\sec(c+dx))^5}\,dx = -\frac{\frac{\frac{9765\sin(dx+c)}{\cos(dx+c)+1}-\frac{2730\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{1008\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{270\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{35\sin(dx+c)^9}{(\cos(dx+c)+1)^9}}{a^5}-\frac{10080\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^5}}{5040\,d}$$

input
```
integrate(1/(a+a*sec(d*x+c))^5,x, algorithm="maxima")
```

output
```
-1/5040*((9765*sin(d*x + c)/(cos(d*x + c) + 1) - 2730*sin(d*x + c)^3/(cos(
d*x + c) + 1)^3 + 1008*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 270*sin(d*x +
 c)^7/(cos(d*x + c) + 1)^7 + 35*sin(d*x + c)^9/(cos(d*x + c) + 1)^9)/a^5 -
 10080*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^5)/d
```

3.87.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.69

$$\int \frac{1}{(a+a\sec(c+dx))^5}\,dx = \frac{\frac{5040\,(dx+c)}{a^5} - \frac{35\,a^{40}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9 - 270\,a^{40}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7 + 1008\,a^{40}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5 - 2730\,a^{40}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 + 9765\,a^{40}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right.}{a^{45}}}{5040\,d}$$

input
```
integrate(1/(a+a*sec(d*x+c))^5,x, algorithm="giac")
```

output
```
1/5040*(5040*(d*x + c)/a^5 - (35*a^40*tan(1/2*d*x + 1/2*c)^9 - 270*a^40*ta
n(1/2*d*x + 1/2*c)^7 + 1008*a^40*tan(1/2*d*x + 1/2*c)^5 - 2730*a^40*tan(1/
2*d*x + 1/2*c)^3 + 9765*a^40*tan(1/2*d*x + 1/2*c))/a^45)/d
```

3.87.9 Mupad [B] (verification not implemented)

Time = 13.44 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.87

$$\int \frac{1}{(a+a\sec(c+dx))^5}\,dx = \frac{x}{a^5} - \frac{\frac{863\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8}{315} - \frac{356\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6}{315} + \frac{169\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{420} - \frac{41\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{504} +}{a^5\,d\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9}$$

input
```
int(1/(a + a/cos(c + d*x))^5,x)
```

output
```
x/a^5 - (sin(c/2 + (d*x)/2)/144 - (41*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)
/2))/504 + (169*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2))/420 - (356*cos(c/
2 + (d*x)/2)^6*sin(c/2 + (d*x)/2))/315 + (863*cos(c/2 + (d*x)/2)^8*sin(c/2
 + (d*x)/2))/315)/(a^5*d*cos(c/2 + (d*x)/2)^9)
```

3.88 $\int \frac{\cos(c+dx)}{(a+a\sec(c+dx))^5}\,dx$

3.88.1 Optimal result . . . 864
3.88.2 Mathematica [A] (verified) . . . 864
3.88.3 Rubi [A] (verified) . . . 865
3.88.4 Maple [A] (verified) . . . 870
3.88.5 Fricas [A] (verification not implemented) . . . 871
3.88.6 Sympy [F] . . . 871
3.88.7 Maxima [A] (verification not implemented) . . . 871
3.88.8 Giac [A] (verification not implemented) . . . 872
3.88.9 Mupad [B] (verification not implemented) . . . 872

3.88.1 Optimal result

Integrand size = 19, antiderivative size = 159

$$\int \frac{\cos(c+dx)}{(a+a\sec(c+dx))^5}\,dx = -\frac{5x}{a^5}+\frac{496\sin(c+dx)}{63a^5d}-\frac{\sin(c+dx)}{9d(a+a\sec(c+dx))^5} - \frac{5\sin(c+dx)}{21ad(a+a\sec(c+dx))^4}-\frac{29\sin(c+dx)}{63a^2d(a+a\sec(c+dx))^3} - \frac{67\sin(c+dx)}{63a^3d(a+a\sec(c+dx))^2}-\frac{5\sin(c+dx)}{d\left(a^5+a^5\sec(c+dx)\right)}$$

output
```
-5*x/a^5+496/63*sin(d*x+c)/a^5/d-1/9*sin(d*x+c)/d/(a+a*sec(d*x+c))^5-5/21*
sin(d*x+c)/a/d/(a+a*sec(d*x+c))^4-29/63*sin(d*x+c)/a^2/d/(a+a*sec(d*x+c))^
3-67/63*sin(d*x+c)/a^3/d/(a+a*sec(d*x+c))^2-5*sin(d*x+c)/d/(a^5+a^5*sec(d*
x+c))
```

3.88.2 Mathematica [A] (verified)

Time = 6.16 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.80

$$\int \frac{\cos(c+dx)}{(a+a\sec(c+dx))^5}\,dx = \frac{\sin(c+dx)\left(10080\arcsin(\cos(c+dx))\cos^{10}\left(\frac{1}{2}(c+dx)\right)+(496+2165\cos(c+dx)+3633\cos^2(c+dx)\right.}{63a^5d\sqrt{1-\cos(c+dx)}(1+\cos(c+d}$$

input
```
Integrate[Cos[c + d*x]/(a + a*Sec[c + d*x])^5,x]
```

output
```
(Sin[c + d*x]*(10080*ArcSin[Cos[c + d*x]]*Cos[(c + d*x)/2]^10 + (496 + 216
5*Cos[c + d*x] + 3633*Cos[c + d*x]^2 + 2840*Cos[c + d*x]^3 + 946*Cos[c + d
*x]^4 + 63*Cos[c + d*x]^5)*Sqrt[Sin[c + d*x]^2]))/(63*a^5*d*Sqrt[1 - Cos[c
 + d*x]]*(1 + Cos[c + d*x])^(11/2))
```

3.88.3 Rubi [A] (verified)

Time = 1.26 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.20, number of steps used = 17, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.895$, Rules used = {3042, 4304, 27, 3042, 4508, 3042, 4508, 27, 3042, 4508, 3042, 4508, 3042, 4274, 24, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(c+dx)}{(a\sec(c+dx)+a)^5}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^5}dx$$

$$\Big\downarrow \text{4304}$$

$$-\frac{\int -\frac{5\cos(c+dx)(2a-a\sec(c+dx))}{(\sec(c+dx)a+a)^4}dx}{9a^2}-\frac{\sin(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$$\Big\downarrow \text{27}$$

$$\frac{5\int \frac{\cos(c+dx)(2a-a\sec(c+dx))}{(\sec(c+dx)a+a)^4}dx}{9a^2}-\frac{\sin(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5\int \frac{2a-a\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^4}dx}{9a^2}-\frac{\sin(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$$\Big\downarrow \text{4508}$$

$$\frac{5\left(\frac{\int \frac{\cos(c+dx)\left(17a^2-12a^2\sec(c+dx)\right)}{(\sec(c+dx)a+a)^3}dx}{7a^2}-\frac{3a\sin(c+dx)}{7d(a\sec(c+dx)+a)^4}\right)}{9a^2}-\frac{\sin(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$\Big\downarrow$ 3042

$$\frac{5\left(\frac{\int \frac{17a^2-12a^2\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^3}dx}{7a^2}-\frac{3a\sin(c+dx)}{7d(a\sec(c+dx)+a)^4}\right)}{9a^2}-\frac{\sin(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$\Big\downarrow$ 4508

$$\frac{5\left(\frac{\frac{\int \frac{3\cos(c+dx)\left(38a^3-29a^3\sec(c+dx)\right)}{(\sec(c+dx)a+a)^2}dx}{5a^2}-\frac{29a^2\sin(c+dx)}{5d(a\sec(c+dx)+a)^3}}{7a^2}-\frac{3a\sin(c+dx)}{7d(a\sec(c+dx)+a)^4}\right)}{9a^2}-\frac{\sin(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$\Big\downarrow$ 27

$$\frac{5\left(\frac{\frac{3\int \frac{\cos(c+dx)\left(38a^3-29a^3\sec(c+dx)\right)}{(\sec(c+dx)a+a)^2}dx}{5a^2}-\frac{29a^2\sin(c+dx)}{5d(a\sec(c+dx)+a)^3}}{7a^2}-\frac{3a\sin(c+dx)}{7d(a\sec(c+dx)+a)^4}\right)}{9a^2}-\frac{\sin(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$\Big\downarrow$ 3042

$$\frac{5\left(\frac{\frac{3\int \frac{38a^3-29a^3\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}dx}{5a^2}-\frac{29a^2\sin(c+dx)}{5d(a\sec(c+dx)+a)^3}}{7a^2}-\frac{3a\sin(c+dx)}{7d(a\sec(c+dx)+a)^4}\right)}{9a^2}-\frac{\sin(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$\Big\downarrow$ 4508

$$\frac{5\left(\frac{\frac{3\left(\frac{\int \frac{\cos(c+dx)\left(181a^4-134a^4\sec(c+dx)\right)}{\sec(c+dx)a+a}dx}{3a^2}-\frac{67a^3\sin(c+dx)}{3d(a\sec(c+dx)+a)^2}\right)}{5a^2}-\frac{29a^2\sin(c+dx)}{5d(a\sec(c+dx)+a)^3}}{7a^2}-\frac{3a\sin(c+dx)}{7d(a\sec(c+dx)+a)^4}\right)}{9a^2}-\frac{\sin(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

3.88. $\int \frac{\cos(c+dx)}{(a+a\sec(c+dx))^5}\,dx$

$$\Big\downarrow \; 3042$$

$$\frac{5\left(\frac{3\left(\frac{\int \frac{181a^4-134a^4\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)}dx}{3a^2}-\frac{67a^3\sin(c+dx)}{3d(a\sec(c+dx)+a)^2}\right)}{5a^2}-\frac{29a^2\sin(c+dx)}{5d(a\sec(c+dx)+a)^3}}{7a^2}-\frac{3a\sin(c+dx)}{7d(a\sec(c+dx)+a)^4}\right)}{9a^2}-\frac{\sin(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$$\Big\downarrow \; 4508$$

$$\frac{5\left(\frac{3\left(\frac{\frac{\int \cos(c+dx)\left(496a^5-315a^5\sec(c+dx)\right)dx}{a^2}}{3a^2}-\frac{315a^4\sin(c+dx)}{d(a\sec(c+dx)+a)}\right)}{5a^2}-\frac{29a^2\sin(c+dx)}{5d(a\sec(c+dx)+a)^3}}{7a^2}-\frac{3a\sin(c+dx)}{7d(a\sec(c+dx)+a)^4}\right)}{9a^2}-\frac{\sin(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$$\Big\downarrow \; 3042$$

$$\frac{5\left(\frac{3\left(\frac{\frac{\int \frac{496a^5-315a^5\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}}{3a^2}-\frac{315a^4\sin(c+dx)}{d(a\sec(c+dx)+a)}\right)}{5a^2}-\frac{29a^2\sin(c+dx)}{5d(a\sec(c+dx)+a)^3}}{7a^2}-\frac{3a\sin(c+dx)}{7d(a\sec(c+dx)+a)^4}\right)}{9a^2}-\frac{\sin(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$$\Big\downarrow \; 4274$$

3.88. $\int \frac{\cos(c+dx)}{(a+a\sec(c+dx))^5}\,dx$

$$\frac{5\left(\frac{\frac{3\left(\frac{\frac{496a^5\int\cos(c+dx)dx-315a^5\int 1dx}{a^2}-\frac{315a^4\sin(c+dx)}{d(a\sec(c+dx)+a)}}{3a^2}-\frac{67a^3\sin(c+dx)}{3d(a\sec(c+dx)+a)^2}\right)}{5a^2}-\frac{29a^2\sin(c+dx)}{5d(a\sec(c+dx)+a)^3}}{7a^2}-\frac{3a\sin(c+dx)}{7d(a\sec(c+dx)+a)^4}\right)}{9a^2}-\frac{\sin(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$\downarrow$ 24

$$\frac{5\left(\frac{\frac{3\left(\frac{\frac{496a^5\int\cos(c+dx)dx-315a^5x}{a^2}-\frac{315a^4\sin(c+dx)}{d(a\sec(c+dx)+a)}}{3a^2}-\frac{67a^3\sin(c+dx)}{3d(a\sec(c+dx)+a)^2}\right)}{5a^2}-\frac{29a^2\sin(c+dx)}{5d(a\sec(c+dx)+a)^3}}{7a^2}-\frac{3a\sin(c+dx)}{7d(a\sec(c+dx)+a)^4}\right)}{9a^2}-\frac{\sin(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$\downarrow$ 3042

$$\frac{5\left(\frac{\frac{3\left(\frac{\frac{496a^5\int\sin\left(c+dx+\frac{\pi}{2}\right)dx-315a^5x}{a^2}-\frac{315a^4\sin(c+dx)}{d(a\sec(c+dx)+a)}}{3a^2}-\frac{67a^3\sin(c+dx)}{3d(a\sec(c+dx)+a)^2}\right)}{5a^2}-\frac{29a^2\sin(c+dx)}{5d(a\sec(c+dx)+a)^3}}{7a^2}-\frac{3a\sin(c+dx)}{7d(a\sec(c+dx)+a)^4}\right)}{9a^2}-\frac{\sin(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$\downarrow$ 3117

$$\frac{5\left(\frac{\frac{3\left(\frac{\frac{\frac{496a^5\sin(c+dx)}{d}-315a^5x}{a^2}-\frac{315a^4\sin(c+dx)}{d(a\sec(c+dx)+a)}}{3a^2}-\frac{67a^3\sin(c+dx)}{3d(a\sec(c+dx)+a)^2}\right)}{5a^2}-\frac{29a^2\sin(c+dx)}{5d(a\sec(c+dx)+a)^3}}{7a^2}-\frac{3a\sin(c+dx)}{7d(a\sec(c+dx)+a)^4}\right)}{9a^2}-\frac{\sin(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

3.88. $\int \frac{\cos(c+dx)}{(a+a\sec(c+dx))^5}\,dx$

input
```
Int[Cos[c + d*x]/(a + a*Sec[c + d*x])^5,x]
```

output
```
-1/9*Sin[c + d*x]/(d*(a + a*Sec[c + d*x])^5) + (5*((-3*a*Sin[c + d*x])/(7*
d*(a + a*Sec[c + d*x])^4) + ((-29*a^2*Sin[c + d*x])/(5*d*(a + a*Sec[c + d*
x])^3) + (3*((-67*a^3*Sin[c + d*x])/(3*d*(a + a*Sec[c + d*x])^2) + ((-315*
a^4*Sin[c + d*x])/(d*(a + a*Sec[c + d*x])) + (-315*a^5*x + (496*a^5*Sin[c
+ d*x])/d)/a^2)/(3*a^2)))/(5*a^2))/(7*a^2)))/(9*a^2)
```

3.88.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4304
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-Cot[e + f*x])*(a + b*Csc[e + f*x])^m*((d*Csc
[e + f*x])^n/(f*(2*m + 1))), x] + Simp[1/(a^2*(2*m + 1))   Int[(a + b*Csc[e
 + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e
+ f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ
[m, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

rule 4508
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
- a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
 1))), x] - Simp[1/(a^2*(2*m + 1))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Cs
c[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[
e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B
, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

3.88.4 Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.55

method	result
parallelrisch	$\frac{\frac{34675\left(\cos(dx+c)+\frac{964\cos(2dx+2c)}{1825}+\frac{467\cos(3dx+3c)}{2774}+\frac{946\cos(4dx+4c)}{34675}+\frac{63\cos(5dx+5c)}{69350}+\frac{21338}{34675}\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\sec\left(\frac{dx}{2}+\frac{c}{2}\right)^8}{8064}-5d}{d\,a^5}$
derivativedivides	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^9}{9}-\frac{8\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{7}+6\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5-24\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3+129\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{32\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}-160\arctan\left(\tan\right.}{16d\,a^5}$
default	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^9}{9}-\frac{8\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{7}+6\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5-24\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3+129\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{32\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}-160\arctan\left(\tan\right.}{16d\,a^5}$
norman	$\frac{-\frac{5x}{a}+\frac{161\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{16ad}+\frac{105\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{16ad}-\frac{9\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{8ad}+\frac{17\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{56ad}-\frac{65\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^9}{1008ad}+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{11}}{144ad}-\frac{5x\tan\left(\frac{dx}{2}\right.}{a}}{a^4\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)}$
risch	$-\frac{5x}{a^5}-\frac{ie^{i(dx+c)}}{2d\,a^5}+\frac{ie^{-i(dx+c)}}{2d\,a^5}+\frac{2i\left(945\,e^{8i(dx+c)}+6300\,e^{7i(dx+c)}+19740\,e^{6i(dx+c)}+36414\,e^{5i(dx+c)}+43092\,e^{4i(d}\right.}{63d\,a^5\left(e^{i(dx+c)}+1\right)^9}$

input
```
int(cos(d*x+c)/(a+a*sec(d*x+c))^5,x,method=_RETURNVERBOSE)
```

output
```
5/8064*(6935*(cos(d*x+c)+964/1825*cos(2*d*x+2*c)+467/2774*cos(3*d*x+3*c)+9
46/34675*cos(4*d*x+4*c)+63/69350*cos(5*d*x+5*c)+21338/34675)*tan(1/2*d*x+1
/2*c)*sec(1/2*d*x+1/2*c)^8-8064*d*x)/d/a^5
```

3.88.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.25

$$\int \frac{\cos(c+dx)}{(a+a\sec(c+dx))^5}\,dx = -\frac{315\,dx\cos(dx+c)^5+1575\,dx\cos(dx+c)^4+3150\,dx\cos(dx+c)^3+3150\,dx\cos(dx+c)^2+1575\,dx}{63\left(a^5d\cos(dx+c)^5+5\,a^5d\cos(dx}$$

input
```
integrate(cos(d*x+c)/(a+a*sec(d*x+c))^5,x, algorithm="fricas")
```

output
```
-1/63*(315*d*x*cos(d*x + c)^5 + 1575*d*x*cos(d*x + c)^4 + 3150*d*x*cos(d*x
 + c)^3 + 3150*d*x*cos(d*x + c)^2 + 1575*d*x*cos(d*x + c) + 315*d*x - (63*
cos(d*x + c)^5 + 946*cos(d*x + c)^4 + 2840*cos(d*x + c)^3 + 3633*cos(d*x +
 c)^2 + 2165*cos(d*x + c) + 496)*sin(d*x + c))/(a^5*d*cos(d*x + c)^5 + 5*a
^5*d*cos(d*x + c)^4 + 10*a^5*d*cos(d*x + c)^3 + 10*a^5*d*cos(d*x + c)^2 +
5*a^5*d*cos(d*x + c) + a^5*d)
```

3.88.6 Sympy [F]

$$\int \frac{\cos(c+dx)}{(a+a\sec(c+dx))^5}\,dx = \frac{\int \frac{\cos(c+dx)}{\sec^5(c+dx)+5\sec^4(c+dx)+10\sec^3(c+dx)+10\sec^2(c+dx)+5\sec(c+dx)+1}\,dx}{a^5}$$

input
```
integrate(cos(d*x+c)/(a+a*sec(d*x+c))**5,x)
```

output
```
Integral(cos(c + d*x)/(sec(c + d*x)**5 + 5*sec(c + d*x)**4 + 10*sec(c + d*
x)**3 + 10*sec(c + d*x)**2 + 5*sec(c + d*x) + 1), x)/a**5
```

3.88.7 Maxima [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.12

$$\int \frac{\cos(c+dx)}{(a+a\sec(c+dx))^5}\,dx = \frac{\frac{2016\sin(dx+c)}{\left(a^5+\frac{a^5\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)} + \frac{\frac{8127\sin(dx+c)}{\cos(dx+c)+1}-\frac{1512\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{378\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{72\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{7\sin(dx+c)^9}{(\cos(dx+c)+1)^9}}{a^5} - \frac{10080\arctan\left(\frac{}{}\right.}{a^5}}{1008\,d}$$

input
```
integrate(cos(d*x+c)/(a+a*sec(d*x+c))^5,x, algorithm="maxima")
```

output
```
1/1008*(2016*sin(d*x + c)/((a^5 + a^5*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)
*(cos(d*x + c) + 1)) + (8127*sin(d*x + c)/(cos(d*x + c) + 1) - 1512*sin(d*
x + c)^3/(cos(d*x + c) + 1)^3 + 378*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 -
72*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 7*sin(d*x + c)^9/(cos(d*x + c) +
1)^9)/a^5 - 10080*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^5)/d
```

3.88.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.81

$$\int \frac{\cos(c+dx)}{(a+a\sec(c+dx))^5}\,dx = -\frac{\frac{5040\,(dx+c)}{a^5} - \frac{2016\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)a^5} - \frac{7\,a^{40}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9-72\,a^{40}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+378\,a^{40}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-1512\,a^{40}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\right.}{a^{45}}}{1008\,d}$$

input
```
integrate(cos(d*x+c)/(a+a*sec(d*x+c))^5,x, algorithm="giac")
```

output
```
-1/1008*(5040*(d*x + c)/a^5 - 2016*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/
2*c)^2 + 1)*a^5) - (7*a^40*tan(1/2*d*x + 1/2*c)^9 - 72*a^40*tan(1/2*d*x +
1/2*c)^7 + 378*a^40*tan(1/2*d*x + 1/2*c)^5 - 1512*a^40*tan(1/2*d*x + 1/2*c
)^3 + 8127*a^40*tan(1/2*d*x + 1/2*c))/a^45)/d
```

3.88.9 Mupad [B] (verification not implemented)

Time = 13.61 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.00

$$\int \frac{\cos(c+dx)}{(a+a\sec(c+dx))^5}\,dx = \frac{7\,\sin\left(\frac{c}{2}+\frac{dx}{2}\right) - 100\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^2\sin\left(\frac{c}{2}+\frac{dx}{2}\right) + 636\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^4\sin\left(\frac{c}{2}+\frac{dx}{2}\right) - 2512\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^6\,\mathrm{s}}{1008\,a^5}$$

input
```
int(cos(c + d*x)/(a + a/cos(c + d*x))^5,x)
```

output
```
(7*sin(c/2 + (d*x)/2) - 100*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2) + 636*
cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2) - 2512*cos(c/2 + (d*x)/2)^6*sin(c/
2 + (d*x)/2) + 10096*cos(c/2 + (d*x)/2)^8*sin(c/2 + (d*x)/2) + 2016*cos(c/
2 + (d*x)/2)^10*sin(c/2 + (d*x)/2) - 5040*cos(c/2 + (d*x)/2)^9*(c + d*x))/
(1008*a^5*d*cos(c/2 + (d*x)/2)^9)
```

3.88. $\int \frac{\cos(c+dx)}{(a+a\sec(c+dx))^5}\,dx$

3.89 $\int \frac{\cos^2(c+dx)}{(a+a\sec(c+dx))^5}\,dx$

3.89.1 Optimal result . . . 874
3.89.2 Mathematica [A] (verified) . . . 875
3.89.3 Rubi [A] (verified) . . . 875
3.89.4 Maple [A] (verified) . . . 880
3.89.5 Fricas [A] (verification not implemented) . . . 881
3.89.6 Sympy [F] . . . 881
3.89.7 Maxima [A] (verification not implemented) . . . 881
3.89.8 Giac [A] (verification not implemented) . . . 882
3.89.9 Mupad [B] (verification not implemented) . . . 882

3.89.1 Optimal result

Integrand size = 21, antiderivative size = 215

$$\begin{aligned}\int \frac{\cos^2(c+dx)}{(a+a\sec(c+dx))^5}\,dx = {} & \frac{31x}{2a^5} - \frac{7664\sin(c+dx)}{315a^5d} + \frac{31\cos(c+dx)\sin(c+dx)}{2a^5d} \\ & - \frac{\cos(c+dx)\sin(c+dx)}{9d(a+a\sec(c+dx))^5} - \frac{17\cos(c+dx)\sin(c+dx)}{63ad(a+a\sec(c+dx))^4} \\ & - \frac{28\cos(c+dx)\sin(c+dx)}{45a^2d(a+a\sec(c+dx))^3} - \frac{577\cos(c+dx)\sin(c+dx)}{315a^3d(a+a\sec(c+dx))^2} \\ & - \frac{3832\cos(c+dx)\sin(c+dx)}{315d\left(a^5+a^5\sec(c+dx)\right)}\end{aligned}$$

output
```
31/2*x/a^5-7664/315*sin(d*x+c)/a^5/d+31/2*cos(d*x+c)*sin(d*x+c)/a^5/d-1/9*
cos(d*x+c)*sin(d*x+c)/d/(a+a*sec(d*x+c))^5-17/63*cos(d*x+c)*sin(d*x+c)/a/d
/(a+a*sec(d*x+c))^4-28/45*cos(d*x+c)*sin(d*x+c)/a^2/d/(a+a*sec(d*x+c))^3-5
77/315*cos(d*x+c)*sin(d*x+c)/a^3/d/(a+a*sec(d*x+c))^2-3832/315*cos(d*x+c)*
sin(d*x+c)/d/(a^5+a^5*sec(d*x+c))
```

3.89.2 Mathematica [A] (verified)

Time = 6.82 (sec) , antiderivative size = 345, normalized size of antiderivative = 1.60

$$\int \frac{\cos^2(c+dx)}{(a+a\sec(c+dx))^5}\,dx$$
$$= \frac{\sec\left(\frac{c}{2}\right)\sec^9\left(\frac{1}{2}(c+dx)\right)\left(4921560dx\cos\left(\frac{dx}{2}\right)+4921560dx\cos\left(c+\frac{dx}{2}\right)+3281040dx\cos\left(c+\frac{3dx}{2}\right)+3281\right.}{}$$

input
```
Integrate[Cos[c + d*x]^2/(a + a*Sec[c + d*x])^5,x]
```

output
```
(Sec[c/2]*Sec[(c + d*x)/2]^9*(4921560*d*x*Cos[(d*x)/2] + 4921560*d*x*Cos[c
 + (d*x)/2] + 3281040*d*x*Cos[c + (3*d*x)/2] + 3281040*d*x*Cos[2*c + (3*d*
x)/2] + 1406160*d*x*Cos[2*c + (5*d*x)/2] + 1406160*d*x*Cos[3*c + (5*d*x)/2
] + 351540*d*x*Cos[3*c + (7*d*x)/2] + 351540*d*x*Cos[4*c + (7*d*x)/2] + 39
060*d*x*Cos[4*c + (9*d*x)/2] + 39060*d*x*Cos[5*c + (9*d*x)/2] - 9163224*Si
n[(d*x)/2] + 7194600*Sin[c + (d*x)/2] - 7472241*Sin[c + (3*d*x)/2] + 34329
75*Sin[2*c + (3*d*x)/2] - 3871989*Sin[2*c + (5*d*x)/2] + 801675*Sin[3*c +
(5*d*x)/2] - 1186056*Sin[3*c + (7*d*x)/2] - 17640*Sin[4*c + (7*d*x)/2] - 1
75184*Sin[4*c + (9*d*x)/2] - 45360*Sin[5*c + (9*d*x)/2] - 3465*Sin[5*c + (
11*d*x)/2] - 3465*Sin[6*c + (11*d*x)/2] + 315*Sin[6*c + (13*d*x)/2] + 315*
Sin[7*c + (13*d*x)/2]))/(1290240*a^5*d)
```

3.89.3 Rubi [A] (verified)

Time = 1.52 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.12, number of steps used = 18, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.857$, Rules used = {3042, 4304, 25, 3042, 4508, 3042, 4508, 3042, 4508, 27, 3042, 4508, 3042, 4274, 3042, 3115, 24, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^2(c+dx)}{(a\sec(c+dx)+a)^5}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^2\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^5}dx$$
$$\Big\downarrow \; 4304$$

$$-\frac{\int -\frac{\cos^2(c+dx)(11a-6a\sec(c+dx))}{(\sec(c+dx)a+a)^4}dx}{9a^2}-\frac{\sin(c+dx)\cos(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$\Big\downarrow$ 25

$$\frac{\int \frac{\cos^2(c+dx)(11a-6a\sec(c+dx))}{(\sec(c+dx)a+a)^4}dx}{9a^2}-\frac{\sin(c+dx)\cos(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$\Big\downarrow$ 3042

$$\frac{\int \frac{11a-6a\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^2\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^4}dx}{9a^2}-\frac{\sin(c+dx)\cos(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$\Big\downarrow$ 4508

$$\frac{\frac{\int \frac{\cos^2(c+dx)\left(111a^2-85a^2\sec(c+dx)\right)}{(\sec(c+dx)a+a)^3}dx}{7a^2}-\frac{17a\sin(c+dx)\cos(c+dx)}{7d(a\sec(c+dx)+a)^4}}{9a^2}-\frac{\sin(c+dx)\cos(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\int \frac{111a^2-85a^2\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^2\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^3}dx}{7a^2}-\frac{17a\sin(c+dx)\cos(c+dx)}{7d(a\sec(c+dx)+a)^4}}{9a^2}-\frac{\sin(c+dx)\cos(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$\Big\downarrow$ 4508

$$\frac{\frac{\frac{\int \frac{\cos^2(c+dx)\left(947a^3-784a^3\sec(c+dx)\right)}{(\sec(c+dx)a+a)^2}dx}{5a^2}-\frac{196a^2\sin(c+dx)\cos(c+dx)}{5d(a\sec(c+dx)+a)^3}}{7a^2}-\frac{17a\sin(c+dx)\cos(c+dx)}{7d(a\sec(c+dx)+a)^4}}{9a^2}-\frac{\sin(c+dx)\cos(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\frac{\int \frac{947a^3-784a^3\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^2\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}dx}{5a^2}-\frac{196a^2\sin(c+dx)\cos(c+dx)}{5d(a\sec(c+dx)+a)^3}}{7a^2}-\frac{17a\sin(c+dx)\cos(c+dx)}{7d(a\sec(c+dx)+a)^4}}{9a^2}-\frac{\sin(c+dx)\cos(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$\Big\downarrow$ 4508

3.89. $\int \frac{\cos^2(c+dx)}{(a+a\sec(c+dx))^5}\,dx$

$$\frac{\frac{\frac{\int \frac{3\cos^2(c+dx)\left(2101a^4-1731a^4\sec(c+dx)\right)}{\sec(c+dx)a+a}dx}{3a^2}-\frac{577a^3\sin(c+dx)\cos(c+dx)}{d(a\sec(c+dx)+a)^2}}{5a^2}-\frac{196a^2\sin(c+dx)\cos(c+dx)}{5d(a\sec(c+dx)+a)^3}}{7a^2}-\frac{17a\sin(c+dx)\cos(c+dx)}{7d(a\sec(c+dx)+a)^4}}{9a^2}-\frac{\sin(c+dx)\cos(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$\downarrow$ 27

$$\frac{\frac{\frac{\int \frac{\cos^2(c+dx)\left(2101a^4-1731a^4\sec(c+dx)\right)}{\sec(c+dx)a+a}dx}{a^2}-\frac{577a^3\sin(c+dx)\cos(c+dx)}{d(a\sec(c+dx)+a)^2}}{5a^2}-\frac{196a^2\sin(c+dx)\cos(c+dx)}{5d(a\sec(c+dx)+a)^3}}{7a^2}-\frac{17a\sin(c+dx)\cos(c+dx)}{7d(a\sec(c+dx)+a)^4}}{9a^2}-\frac{\sin(c+dx)\cos(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$\downarrow$ 3042

$$\frac{\frac{\frac{\int \frac{2101a^4-1731a^4\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^2\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)}dx}{a^2}-\frac{577a^3\sin(c+dx)\cos(c+dx)}{d(a\sec(c+dx)+a)^2}}{5a^2}-\frac{196a^2\sin(c+dx)\cos(c+dx)}{5d(a\sec(c+dx)+a)^3}}{7a^2}-\frac{17a\sin(c+dx)\cos(c+dx)}{7d(a\sec(c+dx)+a)^4}}{9a^2}-\frac{\sin(c+dx)\cos(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$\downarrow$ 4508

$$\frac{\frac{\frac{\frac{\int \cos^2(c+dx)\left(9765a^5-7664a^5\sec(c+dx)\right)dx}{a^2}-\frac{3832a^4\sin(c+dx)\cos(c+dx)}{d(a\sec(c+dx)+a)}}{a^2}-\frac{577a^3\sin(c+dx)\cos(c+dx)}{d(a\sec(c+dx)+a)^2}}{5a^2}-\frac{196a^2\sin(c+dx)\cos(c+dx)}{5d(a\sec(c+dx)+a)^3}}{7a^2}-\frac{17a\sin(c+dx)\cos(c}{7d(a\sec(c+dx)+}}{9a^2}-\frac{\sin(c+dx)\cos(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$\downarrow$ 3042

$$\frac{\frac{\frac{\frac{\frac{\int \frac{9765a^5-7664a^5\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^2}dx}{a^2}-\frac{3832a^4\sin(c+dx)\cos(c+dx)}{d(a\sec(c+dx)+a)}}{a^2}-\frac{577a^3\sin(c+dx)\cos(c+dx)}{d(a\sec(c+dx)+a)^2}}{5a^2}-\frac{196a^2\sin(c+dx)\cos(c+dx)}{5d(a\sec(c+dx)+a)^3}}{7a^2}-\frac{17a\sin(c+dx)\cos(c+dx)}{7d(a\sec(c+dx)+a)^4}}{9a^2}-\frac{\sin(c+dx)\cos(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$\downarrow$ 4274

3.89. $\int \frac{\cos^2(c+dx)}{(a+a\sec(c+dx))^5}\,dx$

$$\frac{\frac{\frac{\frac{9765a^5\int\cos^2(c+dx)dx-7664a^5\int\cos(c+dx)dx}{a^2}-\frac{3832a^4\sin(c+dx)\cos(c+dx)}{d(a\sec(c+dx)+a)}}{a^2}-\frac{577a^3\sin(c+dx)\cos(c+dx)}{d(a\sec(c+dx)+a)^2}}{5a^2}-\frac{196a^2\sin(c+dx)\cos(c+dx)}{5d(a\sec(c+dx)+a)^3}}{7a^2}-\frac{17a\sin(c+dx)\cos}{7d(a\sec(c+dx)}}{9a^2}$$

$$\frac{\sin(c+dx)\cos(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$\downarrow$ 3042

$$\frac{\frac{\frac{\frac{9765a^5\int\sin\left(c+dx+\frac{\pi}{2}\right)^2dx-7664a^5\int\sin\left(c+dx+\frac{\pi}{2}\right)dx}{a^2}-\frac{3832a^4\sin(c+dx)\cos(c+dx)}{d(a\sec(c+dx)+a)}}{a^2}-\frac{577a^3\sin(c+dx)\cos(c+dx)}{d(a\sec(c+dx)+a)^2}}{5a^2}-\frac{196a^2\sin(c+dx)\cos(c+dx)}{5d(a\sec(c+dx)+a)^3}}{7a^2}-\frac{17a\sin(c+}{7d(a\sec}}{9a^2}$$

$$\frac{\sin(c+dx)\cos(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$\downarrow$ 3115

$$\frac{\frac{\frac{\frac{9765a^5\left(\frac{\int 1dx}{2}+\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)-7664a^5\int\sin\left(c+dx+\frac{\pi}{2}\right)dx}{a^2}-\frac{3832a^4\sin(c+dx)\cos(c+dx)}{d(a\sec(c+dx)+a)}}{a^2}-\frac{577a^3\sin(c+dx)\cos(c+dx)}{d(a\sec(c+dx)+a)^2}}{5a^2}-\frac{196a^2\sin(c+dx)\cos(c+dx)}{5d(a\sec(c+dx)+a)^3}}{7a^2}-}{9a^2}$$

$$\frac{\sin(c+dx)\cos(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$\downarrow$ 24

$$\frac{\frac{\frac{\frac{9765a^5\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)-7664a^5\int\sin\left(c+dx+\frac{\pi}{2}\right)dx}{a^2}-\frac{3832a^4\sin(c+dx)\cos(c+dx)}{d(a\sec(c+dx)+a)}}{a^2}-\frac{577a^3\sin(c+dx)\cos(c+dx)}{d(a\sec(c+dx)+a)^2}}{5a^2}-\frac{196a^2\sin(c+dx)\cos(c+dx)}{5d(a\sec(c+dx)+a)^3}}{7a^2}-\frac{17}{7}}{9a^2}$$

$$\frac{\sin(c+dx)\cos(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

$\downarrow$ 3117

$$\frac{\frac{\frac{\frac{9765a^5\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)-\frac{7664a^5\sin(c+dx)}{d}}{a^2}-\frac{3832a^4\sin(c+dx)\cos(c+dx)}{d(a\sec(c+dx)+a)}}{a^2}-\frac{577a^3\sin(c+dx)\cos(c+dx)}{d(a\sec(c+dx)+a)^2}}{5a^2}-\frac{196a^2\sin(c+dx)\cos(c+dx)}{5d(a\sec(c+dx)+a)^3}}{7a^2}-\frac{17a\sin(c+}{7d(a\sec}}{9a^2}$$

$$\frac{\sin(c+dx)\cos(c+dx)}{9d(a\sec(c+dx)+a)^5}$$

3.89. $\int \frac{\cos^2(c+dx)}{(a+a\sec(c+dx))^5}\,dx$

input
```
Int[Cos[c + d*x]^2/(a + a*Sec[c + d*x])^5,x]
```

output
```
-1/9*(Cos[c + d*x]*Sin[c + d*x])/(d*(a + a*Sec[c + d*x])^5) + ((-17*a*Cos[
c + d*x]*Sin[c + d*x])/(7*d*(a + a*Sec[c + d*x])^4) + ((-196*a^2*Cos[c + d
*x]*Sin[c + d*x])/(5*d*(a + a*Sec[c + d*x])^3) + ((-577*a^3*Cos[c + d*x]*S
in[c + d*x])/(d*(a + a*Sec[c + d*x])^2) + ((-3832*a^4*Cos[c + d*x]*Sin[c +
 d*x])/(d*(a + a*Sec[c + d*x])) + ((-7664*a^5*Sin[c + d*x])/d + 9765*a^5*(
x/2 + (Cos[c + d*x]*Sin[c + d*x])/(2*d)))/a^2)/a^2)/(5*a^2))/(7*a^2))/(9*a
^2)
```

3.89.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4304
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-Cot[e + f*x])*(a + b*Csc[e + f*x])^m*((d*Csc
[e + f*x])^n/(f*(2*m + 1))), x] + Simp[1/(a^2*(2*m + 1))   Int[(a + b*Csc[e
 + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e
+ f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ
[m, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

rule 4508
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
- a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
 1))), x] - Simp[1/(a^2*(2*m + 1))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Cs
c[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[
e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B
, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

3.89.4 Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.46

method	result
parallelrisch	$\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\cos(6dx+6c)-\frac{854012\cos(dx+c)}{63}-\frac{2250427\cos(2dx+2c)}{315}-\frac{143054\cos(3dx+3c)}{63}-\frac{113422\cos(4dx+4c)}{315}-10\cos(5dx-}{1024d\,a^5}$
derivativedivides	$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^9}{9}+\frac{10\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{7}-\frac{48\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{5}+50\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3-351\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{-176\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3-144\tan\left(\frac{dx}{2}-}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^2}}{16d\,a^5}$
default	$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^9}{9}+\frac{10\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{7}-\frac{48\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{5}+50\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3-351\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{-176\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3-144\tan\left(\frac{dx}{2}-}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^2}}{16d\,a^5}$
norman	$\frac{\frac{31x}{2a}-\frac{495\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{16ad}-\frac{207\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{4ad}-\frac{1303\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{80ad}+\frac{141\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{70ad}-\frac{2159\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^9}{5040ad}+\frac{19\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{11}}{252ad}-\mathrm{ta}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^2a^4}$
risch	$\frac{31x}{2a^5}-\frac{i\mathrm{e}^{2i(dx+c)}}{8d\,a^5}+\frac{5i\mathrm{e}^{i(dx+c)}}{2d\,a^5}-\frac{5i\mathrm{e}^{-i(dx+c)}}{2d\,a^5}+\frac{i\mathrm{e}^{-2i(dx+c)}}{8d\,a^5}-\frac{2i\left(11025\,\mathrm{e}^{8i(dx+c)}+77175\,\mathrm{e}^{7i(dx+c)}+247695\,\mathrm{e}^{6i}\right.}{}$

input
```
int(cos(d*x+c)^2/(a+a*sec(d*x+c))^5,x,method=_RETURNVERBOSE)
```

output
```
1/1024*(tan(1/2*d*x+1/2*c)*(cos(6*d*x+6*c)-854012/63*cos(d*x+c)-2250427/31
5*cos(2*d*x+2*c)-143054/63*cos(3*d*x+3*c)-113422/315*cos(4*d*x+4*c)-10*cos
(5*d*x+5*c)-2627186/315)*sec(1/2*d*x+1/2*c)^8+15872*d*x)/d/a^5
```

3.89.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.96

$$\int \frac{\cos^2(c+dx)}{(a+a\sec(c+dx))^5}\,dx$$
$$= \frac{9765\,dx\cos(dx+c)^5 + 48825\,dx\cos(dx+c)^4 + 97650\,dx\cos(dx+c)^3 + 97650\,dx\cos(dx+c)^2 + 48825}{630\,(a^5 d\cos(dx+c}$$

input
```
integrate(cos(d*x+c)^2/(a+a*sec(d*x+c))^5,x, algorithm="fricas")
```

output
```
1/630*(9765*d*x*cos(d*x + c)^5 + 48825*d*x*cos(d*x + c)^4 + 97650*d*x*cos(
d*x + c)^3 + 97650*d*x*cos(d*x + c)^2 + 48825*d*x*cos(d*x + c) + 9765*d*x
+ (315*cos(d*x + c)^6 - 1575*cos(d*x + c)^5 - 28828*cos(d*x + c)^4 - 87440
*cos(d*x + c)^3 - 112119*cos(d*x + c)^2 - 66875*cos(d*x + c) - 15328)*sin(
d*x + c))/(a^5*d*cos(d*x + c)^5 + 5*a^5*d*cos(d*x + c)^4 + 10*a^5*d*cos(d*
x + c)^3 + 10*a^5*d*cos(d*x + c)^2 + 5*a^5*d*cos(d*x + c) + a^5*d)
```

3.89.6 Sympy [F]

$$\int \frac{\cos^2(c+dx)}{(a+a\sec(c+dx))^5}\,dx = \frac{\int \frac{\cos^2(c+dx)}{\sec^5(c+dx)+5\sec^4(c+dx)+10\sec^3(c+dx)+10\sec^2(c+dx)+5\sec(c+dx)+1}\,dx}{a^5}$$

input
```
integrate(cos(d*x+c)**2/(a+a*sec(d*x+c))**5,x)
```

output
```
Integral(cos(c + d*x)**2/(sec(c + d*x)**5 + 5*sec(c + d*x)**4 + 10*sec(c +
 d*x)**3 + 10*sec(c + d*x)**2 + 5*sec(c + d*x) + 1), x)/a**5
```

3.89.7 Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.04

$$\int \frac{\cos^2(c+dx)}{(a+a\sec(c+dx))^5}\,dx =$$
$$-\frac{\frac{5040\left(\frac{9\sin(dx+c)}{\cos(dx+c)+1}+\frac{11\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a^5+\frac{2a^5\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a^5\sin(dx+c)^4}{(\cos(dx+c)+1)^4}} + \frac{\frac{110565\sin(dx+c)}{\cos(dx+c)+1}-\frac{15750\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3024\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{450\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{35\sin(dx+c)^9}{(\cos(dx+c)+1)^9}}{a^5} - \frac{156240\,a}{}}{5040\,d}$$

input
```
integrate(cos(d*x+c)^2/(a+a*sec(d*x+c))^5,x, algorithm="maxima")
```

output
```
-1/5040*(5040*(9*sin(d*x + c)/(cos(d*x + c) + 1) + 11*sin(d*x + c)^3/(cos(
d*x + c) + 1)^3)/(a^5 + 2*a^5*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a^5*si
n(d*x + c)^4/(cos(d*x + c) + 1)^4) + (110565*sin(d*x + c)/(cos(d*x + c) +
1) - 15750*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 3024*sin(d*x + c)^5/(cos(
d*x + c) + 1)^5 - 450*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 35*sin(d*x + c
)^9/(cos(d*x + c) + 1)^9)/a^5 - 156240*arctan(sin(d*x + c)/(cos(d*x + c) +
 1))/a^5)/d
```

3.89.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.67

$$\int \frac{\cos^2(c+dx)}{(a+a\sec(c+dx))^5}\,dx = \frac{\frac{78120\,(dx+c)}{a^5} - \frac{5040\left(11\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+9\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^2 a^5} - \frac{35\,a^{40}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9-450\,a^{40}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+3024\,a^{40}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5}{a^{45}}}{5040\,d}$$

input
```
integrate(cos(d*x+c)^2/(a+a*sec(d*x+c))^5,x, algorithm="giac")
```

output
```
1/5040*(78120*(d*x + c)/a^5 - 5040*(11*tan(1/2*d*x + 1/2*c)^3 + 9*tan(1/2*
d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^2*a^5) - (35*a^40*tan(1/2*d*x
+ 1/2*c)^9 - 450*a^40*tan(1/2*d*x + 1/2*c)^7 + 3024*a^40*tan(1/2*d*x + 1/2
*c)^5 - 15750*a^40*tan(1/2*d*x + 1/2*c)^3 + 110565*a^40*tan(1/2*d*x + 1/2*
c))/a^45)/d
```

3.89.9 Mupad [B] (verification not implemented)

Time = 13.25 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.84

$$\int \frac{\cos^2(c+dx)}{(a+a\sec(c+dx))^5}\,dx = -\frac{35\,\sin\left(\frac{c}{2}+\frac{dx}{2}\right)-590\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^2\sin\left(\frac{c}{2}+\frac{dx}{2}\right)+4584\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^4\sin\left(\frac{c}{2}+\frac{dx}{2}\right)-23288\cos\left(\frac{c}{2}+\right.}{}$$

input
```
int(cos(c + d*x)^2/(a + a/cos(c + d*x))^5,x)
```

output
```
-(35*sin(c/2 + (d*x)/2) - 590*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2) + 45
84*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2) - 23288*cos(c/2 + (d*x)/2)^6*si
n(c/2 + (d*x)/2) + 129824*cos(c/2 + (d*x)/2)^8*sin(c/2 + (d*x)/2) + 55440*
cos(c/2 + (d*x)/2)^10*sin(c/2 + (d*x)/2) - 10080*cos(c/2 + (d*x)/2)^12*sin
(c/2 + (d*x)/2) - 78120*cos(c/2 + (d*x)/2)^9*(c + d*x))/(5040*a^5*d*cos(c/
2 + (d*x)/2)^9)
```

3.89. $\int \frac{\cos^2(c+dx)}{(a+a\sec(c+dx))^5}\,dx$

3.90 $\int \sec^4(c+dx)\sqrt{a+a\sec(c+dx)}\,dx$

3.90.1 Optimal result . . . 884
3.90.2 Mathematica [A] (verified) . . . 884
3.90.3 Rubi [A] (verified) . . . 885
3.90.4 Maple [A] (verified) . . . 888
3.90.5 Fricas [A] (verification not implemented) . . . 888
3.90.6 Sympy [F] . . . 888
3.90.7 Maxima [F] . . . 889
3.90.8 Giac [F] . . . 889
3.90.9 Mupad [B] (verification not implemented) . . . 890

3.90.1 Optimal result

Integrand size = 23, antiderivative size = 122

$$\int \sec^4(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \frac{4a\tan(c+dx)}{5d\sqrt{a+a\sec(c+dx)}}+\frac{2a\sec^3(c+dx)\tan(c+dx)}{7d\sqrt{a+a\sec(c+dx)}} - \frac{8\sqrt{a+a\sec(c+dx)}\tan(c+dx)}{35d} + \frac{12(a+a\sec(c+dx))^{3/2}\tan(c+dx)}{35ad}$$

output
```
12/35*(a+a*sec(d*x+c))^(3/2)*tan(d*x+c)/a/d+4/5*a*tan(d*x+c)/d/(a+a*sec(d*
x+c))^(1/2)+2/7*a*sec(d*x+c)^3*tan(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)-8/35*(a
+a*sec(d*x+c))^(1/2)*tan(d*x+c)/d
```

3.90.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.48

$$\int \sec^4(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \frac{2a(16+8\sec(c+dx)+6\sec^2(c+dx)+5\sec^3(c+dx))\tan(c+dx)}{35d\sqrt{a(1+\sec(c+dx))}}$$

input
```
Integrate[Sec[c + d*x]^4*Sqrt[a + a*Sec[c + d*x]],x]
```

output
```
(2*a*(16 + 8*Sec[c + d*x] + 6*Sec[c + d*x]^2 + 5*Sec[c + d*x]^3)*Tan[c + d
*x])/(35*d*Sqrt[a*(1 + Sec[c + d*x])])
```

3.90.3 Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.13, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {3042, 4290, 3042, 4287, 27, 3042, 4489, 3042, 4279}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^4(c+dx)\sqrt{a\sec(c+dx)+a}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^4\sqrt{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}dx$$

$$\Big\downarrow \text{4290}$$

$$\frac{6}{7}\int \sec^3(c+dx)\sqrt{\sec(c+dx)a+a}dx+\frac{2a\tan(c+dx)\sec^3(c+dx)}{7d\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{6}{7}\int \csc\left(c+dx+\frac{\pi}{2}\right)^3\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx+\frac{2a\tan(c+dx)\sec^3(c+dx)}{7d\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{4287}$$

$$\frac{6}{7}\left(\frac{2\int \frac{1}{2}\sec(c+dx)(3a-2a\sec(c+dx))\sqrt{\sec(c+dx)a+a}dx}{5a}+\frac{2\tan(c+dx)(a\sec(c+dx)+a)^{3/2}}{5ad}\right)+\frac{2a\tan(c+dx)\sec^3(c+dx)}{7d\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{27}$$

$$\frac{6}{7}\left(\frac{\int \sec(c+dx)(3a-2a\sec(c+dx))\sqrt{\sec(c+dx)a+a}dx}{5a}+\frac{2\tan(c+dx)(a\sec(c+dx)+a)^{3/2}}{5ad}\right)+\frac{2a\tan(c+dx)\sec^3(c+dx)}{7d\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{6}{7}\left(\frac{\int \csc\left(c+dx+\frac{\pi}{2}\right)\left(3a-2a\csc\left(c+dx+\frac{\pi}{2}\right)\right)\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{5a}+\frac{2\tan(c+dx)(a\sec(c+dx)+a)^3}{5ad}\right.$$

$$\frac{2a\tan(c+dx)\sec^3(c+dx)}{7d\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \; 4489$$

$$\frac{6}{7}\left(\frac{\frac{7}{3}a\int \sec(c+dx)\sqrt{\sec(c+dx)a+a}dx-\frac{4a\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}}{5a}+\frac{2\tan(c+dx)(a\sec(c+dx)+a)^{3/2}}{5ad}\right)+$$

$$\frac{2a\tan(c+dx)\sec^3(c+dx)}{7d\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \; 3042$$

$$\frac{6}{7}\left(\frac{\frac{7}{3}a\int \csc\left(c+dx+\frac{\pi}{2}\right)\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx-\frac{4a\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}}{5a}+\frac{2\tan(c+dx)(a\sec(c+dx)+}{5ad}\right.$$

$$\frac{2a\tan(c+dx)\sec^3(c+dx)}{7d\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \; 4279$$

$$\frac{6}{7}\left(\frac{\frac{14a^2\tan(c+dx)}{3d\sqrt{a\sec(c+dx)+a}}-\frac{4a\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}}{5a}+\frac{2\tan(c+dx)(a\sec(c+dx)+a)^{3/2}}{5ad}\right)+$$

$$\frac{2a\tan(c+dx)\sec^3(c+dx)}{7d\sqrt{a\sec(c+dx)+a}}$$

input
```
Int[Sec[c + d*x]^4*Sqrt[a + a*Sec[c + d*x]],x]
```

output
```
(2*a*Sec[c + d*x]^3*Tan[c + d*x])/(7*d*Sqrt[a + a*Sec[c + d*x]]) + (6*((2*
(a + a*Sec[c + d*x])^(3/2)*Tan[c + d*x])/(5*a*d) + ((14*a^2*Tan[c + d*x])/
(3*d*Sqrt[a + a*Sec[c + d*x]]) - (4*a*Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x
])/(3*d))/(5*a)))/7
```

3.90. $\int \sec^4(c+dx)\sqrt{a+a\sec(c+dx)}\,dx$

3.90.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4279
```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*b*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]])), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4287
```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(-Cot[e + f*x])*((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2
))), x] + Simp[1/(b*(m + 2))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(b*(
m + 1) - a*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 -
b^2, 0] &&  !LtQ[m, -2^(-1)]
```

rule 4290
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[-2*b*d*Cot[e + f*x]*((d*Csc[e + f*x])^(n - 1)/(
f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]])), x] + Simp[2*a*d*((n - 1)/(b*(2*n -
1)))   Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e + f*x])^(n - 1), x], x] /; Fre
eQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

rule 4489
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*((
a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Simp[(a*B*m + A*b*(m + 1))/(b*(m +
 1))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B
, e, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b
*(m + 1), 0] &&  !LtQ[m, -2^(-1)]
```

3.90.4 Maple [A] (verified)

Time = 0.98 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.59

method	result	size
default	$\frac{2\left(16\cos(dx+c)^{3}+8\cos(dx+c)^{2}+6\cos(dx+c)+5\right)\sqrt{a(1+\sec(dx+c))}\,\tan(dx+c)\sec(dx+c)^{2}}{35d(\cos(dx+c)+1)}$	72

input
```
int(sec(d*x+c)^4*(a+a*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/35/d*(16*cos(d*x+c)^3+8*cos(d*x+c)^2+6*cos(d*x+c)+5)*(a*(1+sec(d*x+c)))^
(1/2)/(cos(d*x+c)+1)*tan(d*x+c)*sec(d*x+c)^2
```

3.90.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.67

$$\int \sec^4(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \frac{2\left(16\cos\left(dx+c\right)^3+8\cos\left(dx+c\right)^2+6\cos\left(dx+c\right)+5\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin\left(dx+c\right)}{35\left(d\cos\left(dx+c\right)^4+d\cos\left(dx+c\right)^3\right)}$$

input
```
integrate(sec(d*x+c)^4*(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
2/35*(16*cos(d*x + c)^3 + 8*cos(d*x + c)^2 + 6*cos(d*x + c) + 5)*sqrt((a*c
os(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c)^4 + d*cos(d*x
+ c)^3)
```

3.90.6 Sympy [F]

$$\int \sec^4(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \int \sqrt{a\left(\sec\left(c+dx\right)+1\right)}\sec^4\left(c+dx\right)dx$$

input
```
integrate(sec(d*x+c)**4*(a+a*sec(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(a*(sec(c + d*x) + 1))*sec(c + d*x)**4, x)
```

3.90.7 Maxima [F]

$$\int \sec^4(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \int \sqrt{a\sec(dx+c)+a}\sec(dx+c)^4\,dx$$

input
```
integrate(sec(d*x+c)^4*(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
16/35*(35*(d*cos(2*d*x + 2*c)^2 + d*sin(2*d*x + 2*c)^2 + 2*d*cos(2*d*x + 2
*c) + d)*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1
)^(3/4)*sqrt(a)*integrate((((cos(10*d*x + 10*c)*cos(2*d*x + 2*c) + 4*cos(8
*d*x + 8*c)*cos(2*d*x + 2*c) + 6*cos(6*d*x + 6*c)*cos(2*d*x + 2*c) + 4*cos
(4*d*x + 4*c)*cos(2*d*x + 2*c) + cos(2*d*x + 2*c)^2 + sin(10*d*x + 10*c)*s
in(2*d*x + 2*c) + 4*sin(8*d*x + 8*c)*sin(2*d*x + 2*c) + 6*sin(6*d*x + 6*c)
*sin(2*d*x + 2*c) + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + sin(2*d*x + 2*c)
^2)*cos(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + (cos(2*d*x + 2*
c)*sin(10*d*x + 10*c) + 4*cos(2*d*x + 2*c)*sin(8*d*x + 8*c) + 6*cos(2*d*x
+ 2*c)*sin(6*d*x + 6*c) + 4*cos(2*d*x + 2*c)*sin(4*d*x + 4*c) - cos(10*d*x
 + 10*c)*sin(2*d*x + 2*c) - 4*cos(8*d*x + 8*c)*sin(2*d*x + 2*c) - 6*cos(6*
d*x + 6*c)*sin(2*d*x + 2*c) - 4*cos(4*d*x + 4*c)*sin(2*d*x + 2*c))*sin(5/2
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(1/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c) + 1)) - ((cos(2*d*x + 2*c)*sin(10*d*x + 10*c) + 4*
cos(2*d*x + 2*c)*sin(8*d*x + 8*c) + 6*cos(2*d*x + 2*c)*sin(6*d*x + 6*c) +
4*cos(2*d*x + 2*c)*sin(4*d*x + 4*c) - cos(10*d*x + 10*c)*sin(2*d*x + 2*c)
- 4*cos(8*d*x + 8*c)*sin(2*d*x + 2*c) - 6*cos(6*d*x + 6*c)*sin(2*d*x + 2*c
) - 4*cos(4*d*x + 4*c)*sin(2*d*x + 2*c))*cos(5/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c))) - (cos(10*d*x + 10*c)*cos(2*d*x + 2*c) + 4*cos(8*d*x +
 8*c)*cos(2*d*x + 2*c) + 6*cos(6*d*x + 6*c)*cos(2*d*x + 2*c) + 4*cos(4*...
```

3.90.8 Giac [F]

$$\int \sec^4(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \int \sqrt{a\sec(dx+c)+a}\sec(dx+c)^4\,dx$$

input
```
integrate(sec(d*x+c)^4*(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.90.9 Mupad [B] (verification not implemented)

Time = 17.95 (sec) , antiderivative size = 331, normalized size of antiderivative = 2.71

$$\int \sec^4(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = -\frac{e^{c\,1i+d\,x\,1i}\sqrt{a+\frac{a}{\frac{e^{-c\,1i-d\,x\,1i}}{2}+\frac{e^{c\,1i+d\,x\,1i}}{2}}}\,32i}{35\,d\,\left(e^{c\,1i+d\,x\,1i}+1\right)} - \frac{\left(\frac{16i}{7\,d}+\frac{e^{c\,1i+d\,x\,1i}\,16i}{7\,d}\right)\sqrt{a+\frac{a}{\frac{e^{-c\,1i-d\,x\,1i}}{2}+\frac{e^{c\,1i+d\,x\,1i}}{2}}}}{\left(e^{c\,1i+d\,x\,1i}+1\right)\left(e^{c\,2i+d\,x\,2i}+1\right)^3} + \frac{\left(\frac{16i}{5\,d}+\frac{e^{c\,1i+d\,x\,1i}\,128i}{35\,d}\right)\sqrt{a+\frac{a}{\frac{e^{-c\,1i-d\,x\,1i}}{2}+\frac{e^{c\,1i+d\,x\,1i}}{2}}}}{\left(e^{c\,1i+d\,x\,1i}+1\right)\left(e^{c\,2i+d\,x\,2i}+1\right)^2} - \frac{e^{c\,1i+d\,x\,1i}\sqrt{a+\frac{a}{\frac{e^{-c\,1i-d\,x\,1i}}{2}+\frac{e^{c\,1i+d\,x\,1i}}{2}}}\,16i}{35\,d\,\left(e^{c\,1i+d\,x\,1i}+1\right)\left(e^{c\,2i+d\,x\,2i}+1\right)}$$

input
```
int((a + a/cos(c + d*x))^(1/2)/cos(c + d*x)^4,x)
```

output
```
((16i/(5*d) + (exp(c*1i + d*x*1i)*128i)/(35*d))*(a + a/(exp(- c*1i - d*x*1
i)/2 + exp(c*1i + d*x*1i)/2))^(1/2))/((exp(c*1i + d*x*1i) + 1)*(exp(c*2i +
 d*x*2i) + 1)^2) - ((16i/(7*d) + (exp(c*1i + d*x*1i)*16i)/(7*d))*(a + a/(e
xp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2))/((exp(c*1i + d*x*1i)
 + 1)*(exp(c*2i + d*x*2i) + 1)^3) - (exp(c*1i + d*x*1i)*(a + a/(exp(- c*1i
 - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*32i)/(35*d*(exp(c*1i + d*x*1i)
 + 1)) - (exp(c*1i + d*x*1i)*(a + a/(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d
*x*1i)/2))^(1/2)*16i)/(35*d*(exp(c*1i + d*x*1i) + 1)*(exp(c*2i + d*x*2i) +
 1))
```

3.91 $\int \sec^3(c+dx)\sqrt{a+a\sec(c+dx)}\,dx$

3.91.1 Optimal result . . . 891
3.91.2 Mathematica [A] (verified) . . . 891
3.91.3 Rubi [A] (verified) . . . 892
3.91.4 Maple [A] (verified) . . . 894
3.91.5 Fricas [A] (verification not implemented) . . . 894
3.91.6 Sympy [F] . . . 894
3.91.7 Maxima [F] . . . 895
3.91.8 Giac [F] . . . 895
3.91.9 Mupad [B] (verification not implemented) . . . 896

3.91.1 Optimal result

Integrand size = 23, antiderivative size = 86

$$\int \sec^3(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \frac{14a\tan(c+dx)}{15d\sqrt{a+a\sec(c+dx)}} - \frac{4\sqrt{a+a\sec(c+dx)}\tan(c+dx)}{15d} + \frac{2(a+a\sec(c+dx))^{3/2}\tan(c+dx)}{5ad}$$

output
```
2/5*(a+a*sec(d*x+c))^(3/2)*tan(d*x+c)/a/d+14/15*a*tan(d*x+c)/d/(a+a*sec(d*
x+c))^(1/2)-4/15*(a+a*sec(d*x+c))^(1/2)*tan(d*x+c)/d
```

3.91.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.56

$$\int \sec^3(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \frac{2a(8+4\sec(c+dx)+3\sec^2(c+dx))\tan(c+dx)}{15d\sqrt{a(1+\sec(c+dx))}}$$

input
```
Integrate[Sec[c + d*x]^3*Sqrt[a + a*Sec[c + d*x]],x]
```

output
```
(2*a*(8 + 4*Sec[c + d*x] + 3*Sec[c + d*x]^2)*Tan[c + d*x])/(15*d*Sqrt[a*(1
 + Sec[c + d*x])])
```

3.91.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.13, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3042, 4287, 27, 3042, 4489, 3042, 4279}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^3(c+dx)\sqrt{a\sec(c+dx)+a}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^3\sqrt{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}dx$$

$$\Big\downarrow \text{4287}$$

$$\frac{2\int \frac{1}{2}\sec(c+dx)(3a-2a\sec(c+dx))\sqrt{\sec(c+dx)a+a}dx}{5a}+\frac{2\tan(c+dx)(a\sec(c+dx)+a)^{3/2}}{5ad}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \sec(c+dx)(3a-2a\sec(c+dx))\sqrt{\sec(c+dx)a+a}dx}{5a}+\frac{2\tan(c+dx)(a\sec(c+dx)+a)^{3/2}}{5ad}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \csc\left(c+dx+\frac{\pi}{2}\right)\left(3a-2a\csc\left(c+dx+\frac{\pi}{2}\right)\right)\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{5a}+\frac{2\tan(c+dx)(a\sec(c+dx)+a)^{3/2}}{5ad}$$

$$\Big\downarrow \text{4489}$$

$$\frac{\frac{7}{3}a\int \sec(c+dx)\sqrt{\sec(c+dx)a+a}dx-\frac{4a\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}}{5a}+\frac{2\tan(c+dx)(a\sec(c+dx)+a)^{3/2}}{5ad}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{7}{3}a\int \csc\left(c+dx+\frac{\pi}{2}\right)\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx-\frac{4a\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}}{5a}+\frac{2\tan(c+dx)(a\sec(c+dx)+a)^{3/2}}{5ad}$$

$$\Big\downarrow \text{4279}$$

$$\frac{\frac{14a^2\tan(c+dx)}{3d\sqrt{a\sec(c+dx)+a}}-\frac{4a\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}}{5a}+\frac{2\tan(c+dx)(a\sec(c+dx)+a)^{3/2}}{5ad}$$

input
```
Int[Sec[c + d*x]^3*Sqrt[a + a*Sec[c + d*x]],x]
```

output
```
(2*(a + a*Sec[c + d*x])^(3/2)*Tan[c + d*x])/(5*a*d) + ((14*a^2*Tan[c + d*x
])/(3*d*Sqrt[a + a*Sec[c + d*x]]) - (4*a*Sqrt[a + a*Sec[c + d*x]]*Tan[c +
d*x])/(3*d))/(5*a)
```

3.91.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4279
```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*b*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]])), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4287
```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(-Cot[e + f*x])*((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2
))), x] + Simp[1/(b*(m + 2))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(b*(
m + 1) - a*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 -
b^2, 0] &&  !LtQ[m, -2^(-1)]
```

rule 4489
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*((
a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Simp[(a*B*m + A*b*(m + 1))/(b*(m +
 1))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B
, e, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b
*(m + 1), 0] &&  !LtQ[m, -2^(-1)]
```

3.91.4 Maple [A] (verified)

Time = 0.87 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.69

method	result	size
default	$\frac{2\sqrt{a(1+\sec(dx+c))}\,(8\sin(dx+c)+4\tan(dx+c)+3\sec(dx+c)\tan(dx+c))}{15d(\cos(dx+c)+1)}$	59

input
```
int(sec(d*x+c)^3*(a+a*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/15/d*(a*(1+sec(d*x+c)))^(1/2)/(cos(d*x+c)+1)*(8*sin(d*x+c)+4*tan(d*x+c)+
3*sec(d*x+c)*tan(d*x+c))
```

3.91.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.84

$$\int \sec^3(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \frac{2\left(8\cos\left(dx+c\right)^2+4\cos\left(dx+c\right)+3\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin\left(dx+c\right)}{15\left(d\cos\left(dx+c\right)^3+d\cos\left(dx+c\right)^2\right)}$$

input
```
integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
2/15*(8*cos(d*x + c)^2 + 4*cos(d*x + c) + 3)*sqrt((a*cos(d*x + c) + a)/cos
(d*x + c))*sin(d*x + c)/(d*cos(d*x + c)^3 + d*cos(d*x + c)^2)
```

3.91.6 Sympy [F]

$$\int \sec^3(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \int \sqrt{a\left(\sec\left(c+dx\right)+1\right)}\sec^3\left(c+dx\right)dx$$

input
```
integrate(sec(d*x+c)**3*(a+a*sec(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(a*(sec(c + d*x) + 1))*sec(c + d*x)**3, x)
```

3.91.7 Maxima [F]

$$\int \sec^3(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \int \sqrt{a\sec(dx+c)+a}\sec(dx+c)^3\,dx$$

input
```
integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
8/15*(15*(d*cos(2*d*x + 2*c)^2 + d*sin(2*d*x + 2*c)^2 + 2*d*cos(2*d*x + 2*
c) + d)*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)
^(1/4)*sqrt(a)*integrate((((cos(8*d*x + 8*c)*cos(2*d*x + 2*c) + 3*cos(6*d*
x + 6*c)*cos(2*d*x + 2*c) + 3*cos(4*d*x + 4*c)*cos(2*d*x + 2*c) + cos(2*d*
x + 2*c)^2 + sin(8*d*x + 8*c)*sin(2*d*x + 2*c) + 3*sin(6*d*x + 6*c)*sin(2*
d*x + 2*c) + 3*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + sin(2*d*x + 2*c)^2)*cos
(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + (cos(2*d*x + 2*c)*sin(
8*d*x + 8*c) + 3*cos(2*d*x + 2*c)*sin(6*d*x + 6*c) + 3*cos(2*d*x + 2*c)*si
n(4*d*x + 4*c) - cos(8*d*x + 8*c)*sin(2*d*x + 2*c) - 3*cos(6*d*x + 6*c)*si
n(2*d*x + 2*c) - 3*cos(4*d*x + 4*c)*sin(2*d*x + 2*c))*sin(3/2*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c) + 1)) - ((cos(2*d*x + 2*c)*sin(8*d*x + 8*c) + 3*cos(2*d*x + 2*c
)*sin(6*d*x + 6*c) + 3*cos(2*d*x + 2*c)*sin(4*d*x + 4*c) - cos(8*d*x + 8*c
)*sin(2*d*x + 2*c) - 3*cos(6*d*x + 6*c)*sin(2*d*x + 2*c) - 3*cos(4*d*x + 4
*c)*sin(2*d*x + 2*c))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
 - (cos(8*d*x + 8*c)*cos(2*d*x + 2*c) + 3*cos(6*d*x + 6*c)*cos(2*d*x + 2*c
) + 3*cos(4*d*x + 4*c)*cos(2*d*x + 2*c) + cos(2*d*x + 2*c)^2 + sin(8*d*x +
 8*c)*sin(2*d*x + 2*c) + 3*sin(6*d*x + 6*c)*sin(2*d*x + 2*c) + 3*sin(4*d*x
 + 4*c)*sin(2*d*x + 2*c) + sin(2*d*x + 2*c)^2)*sin(3/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c))))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x ...
```

3.91.8 Giac [F]

$$\int \sec^3(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \int \sqrt{a\sec(dx+c)+a}\sec(dx+c)^3\,dx$$

input
```
integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.91.9 Mupad [B] (verification not implemented)

Time = 16.66 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.34

$$\int \sec^3(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \frac{8\sqrt{a+\frac{a}{\frac{e^{-c\,1i-d\,x\,1i}}{2}+\frac{e^{c\,1i+d\,x\,1i}}{2}}}\left(e^{c\,2i+d\,x\,2i}\,5i-e^{c\,3i+d\,x\,3i}\,5i-e^{c\,5i+d\,x\,5i}\,2i+2i\right)}{15\,d\,\left(e^{c\,1i+d\,x\,1i}+1\right)\left(e^{c\,2i+d\,x\,2i}+1\right)^2}$$

input
```
int((a + a/cos(c + d*x))^(1/2)/cos(c + d*x)^3,x)
```

output
```
(8*(a + a/(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*(exp(c*2i
 + d*x*2i)*5i - exp(c*3i + d*x*3i)*5i - exp(c*5i + d*x*5i)*2i + 2i))/(15*d
*(exp(c*1i + d*x*1i) + 1)*(exp(c*2i + d*x*2i) + 1)^2)
```

3.92 $\int \sec^2(c+dx)\sqrt{a+a\sec(c+dx)}\,dx$

3.92.1 Optimal result . . . 897
3.92.2 Mathematica [A] (verified) . . . 897
3.92.3 Rubi [A] (verified) . . . 898
3.92.4 Maple [A] (verified) . . . 899
3.92.5 Fricas [A] (verification not implemented) . . . 899
3.92.6 Sympy [F] . . . 900
3.92.7 Maxima [F] . . . 900
3.92.8 Giac [F] . . . 901
3.92.9 Mupad [B] (verification not implemented) . . . 902

3.92.1 Optimal result

Integrand size = 23, antiderivative size = 56

$$\int \sec^2(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \frac{2a\tan(c+dx)}{3d\sqrt{a+a\sec(c+dx)}} + \frac{2\sqrt{a+a\sec(c+dx)}\tan(c+dx)}{3d}$$

output
```
2/3*a*tan(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+2/3*(a+a*sec(d*x+c))^(1/2)*tan(d
*x+c)/d
```

3.92.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.64

$$\int \sec^2(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \frac{2a(2+\sec(c+dx))\tan(c+dx)}{3d\sqrt{a(1+\sec(c+dx))}}$$

input
```
Integrate[Sec[c + d*x]^2*Sqrt[a + a*Sec[c + d*x]],x]
```

output
```
(2*a*(2 + Sec[c + d*x])*Tan[c + d*x])/(3*d*Sqrt[a*(1 + Sec[c + d*x])])
```

3.92.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 4285, 3042, 4279}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(c+dx)\sqrt{a\sec(c+dx)+a}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^2\sqrt{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}dx$$

$$\Big\downarrow \text{4285}$$

$$\frac{1}{3}\int \sec(c+dx)\sqrt{\sec(c+dx)a+a}dx+\frac{2\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\int \csc\left(c+dx+\frac{\pi}{2}\right)\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx+\frac{2\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}$$

$$\Big\downarrow \text{4279}$$

$$\frac{2\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}+\frac{2a\tan(c+dx)}{3d\sqrt{a\sec(c+dx)+a}}$$

input
```
Int[Sec[c + d*x]^2*Sqrt[a + a*Sec[c + d*x]],x]
```

output
```
(2*a*Tan[c + d*x])/(3*d*Sqrt[a + a*Sec[c + d*x]]) + (2*Sqrt[a + a*Sec[c +
d*x]]*Tan[c + d*x])/(3*d)
```

3.92.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4279
```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*b*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]])), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4285
```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(-Cot[e + f*x])*((a + b*Csc[e + f*x])^m/(f*(m + 1))), x]
+ Simp[a*(m/(b*(m + 1)))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x]
/; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

3.92.4 Maple [A] (verified)

Time = 0.86 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.77

method	result	size
default	$\frac{2\sqrt{a(1+\sec(dx+c))}\,(2\sin(dx+c)+\tan(dx+c))}{3d(\cos(dx+c)+1)}$	43

input
```
int(sec(d*x+c)^2*(a+a*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3/d*(a*(1+sec(d*x+c)))^(1/2)/(cos(d*x+c)+1)*(2*sin(d*x+c)+tan(d*x+c))
```

3.92.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.07

$$\int \sec^2(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \frac{2\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}(2\cos(dx+c)+1)\sin(dx+c)}{3\left(d\cos(dx+c)^2+d\cos(dx+c)\right)}$$

input
```
integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
2/3*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(2*cos(d*x + c) + 1)*sin(d*x +
 c)/(d*cos(d*x + c)^2 + d*cos(d*x + c))
```

3.92.6 Sympy [F]

$$\int \sec^2(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \int \sqrt{a\left(\sec\left(c+dx\right)+1\right)}\sec^2\left(c+dx\right)dx$$

input
```
integrate(sec(d*x+c)**2*(a+a*sec(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(a*(sec(c + d*x) + 1))*sec(c + d*x)**2, x)
```

3.92.7 Maxima [F]

$$\int \sec^2(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \int \sqrt{a\sec\left(dx+c\right)+a}\sec\left(dx+c\right)^2\,dx$$

input
```
integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
4/3*(3*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^
(3/4)*sqrt(a)*d*integrate((((cos(6*d*x + 6*c)*cos(2*d*x + 2*c) + 2*cos(4*d
*x + 4*c)*cos(2*d*x + 2*c) + cos(2*d*x + 2*c)^2 + sin(6*d*x + 6*c)*sin(2*d
*x + 2*c) + 2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + sin(2*d*x + 2*c)^2)*cos(
3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + (cos(2*d*x + 2*c)*sin(6
*d*x + 6*c) + 2*cos(2*d*x + 2*c)*sin(4*d*x + 4*c) - cos(6*d*x + 6*c)*sin(2
*d*x + 2*c) - 2*cos(4*d*x + 4*c)*sin(2*d*x + 2*c))*sin(3/2*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c))))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c) + 1)) - ((cos(2*d*x + 2*c)*sin(6*d*x + 6*c) + 2*cos(2*d*x + 2*c)*s
in(4*d*x + 4*c) - cos(6*d*x + 6*c)*sin(2*d*x + 2*c) - 2*cos(4*d*x + 4*c)*s
in(2*d*x + 2*c))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - (c
os(6*d*x + 6*c)*cos(2*d*x + 2*c) + 2*cos(4*d*x + 4*c)*cos(2*d*x + 2*c) + c
os(2*d*x + 2*c)^2 + sin(6*d*x + 6*c)*sin(2*d*x + 2*c) + 2*sin(4*d*x + 4*c)
*sin(2*d*x + 2*c) + sin(2*d*x + 2*c)^2)*sin(3/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c))))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1
)))/(((2*(2*cos(4*d*x + 4*c) + cos(2*d*x + 2*c))*cos(6*d*x + 6*c) + cos(6*
d*x + 6*c)^2 + 4*cos(4*d*x + 4*c)^2 + 4*cos(4*d*x + 4*c)*cos(2*d*x + 2*c)
+ cos(2*d*x + 2*c)^2 + 2*(2*sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(6*d*x
 + 6*c) + sin(6*d*x + 6*c)^2 + 4*sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*s
in(2*d*x + 2*c) + sin(2*d*x + 2*c)^2)*cos(1/2*arctan2(sin(2*d*x + 2*c),...
```

3.92.8 Giac [F]

$$\int \sec^2(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \int \sqrt{a\sec(dx+c)+a}\sec(dx+c)^2\,dx$$

input
```
integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.92.9 Mupad [B] (verification not implemented)

Time = 1.38 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.93

$$\int \sec^2(c+dx)\sqrt{a+a\sec(c+dx)}\,dx$$
$$=\frac{4\sqrt{\frac{a\,(\cos(c+d\,x)+1)}{\cos(c+d\,x)}}\,(3\,\sin(c+d\,x)+4\,\sin(2\,c+2\,d\,x)+3\,\sin(3\,c+3\,d\,x)+\sin(4\,c+4\,d\,x))}{3\,d\,(12\,\cos(c+d\,x)+8\,\cos(2\,c+2\,d\,x)+4\,\cos(3\,c+3\,d\,x)+\cos(4\,c+4\,d\,x)+7)}$$

input
```
int((a + a/cos(c + d*x))^(1/2)/cos(c + d*x)^2,x)
```

output
```
(4*((a*(cos(c + d*x) + 1))/cos(c + d*x))^(1/2)*(3*sin(c + d*x) + 4*sin(2*c
 + 2*d*x) + 3*sin(3*c + 3*d*x) + sin(4*c + 4*d*x)))/(3*d*(12*cos(c + d*x)
+ 8*cos(2*c + 2*d*x) + 4*cos(3*c + 3*d*x) + cos(4*c + 4*d*x) + 7))
```

3.93 $\int \sec(c+dx)\sqrt{a+a\sec(c+dx)}\,dx$

3.93.1 Optimal result . . . 903
3.93.2 Mathematica [A] (verified) . . . 903
3.93.3 Rubi [A] (verified) . . . 904
3.93.4 Maple [A] (verified) . . . 905
3.93.5 Fricas [A] (verification not implemented) . . . 905
3.93.6 Sympy [F] . . . 905
3.93.7 Maxima [F] . . . 906
3.93.8 Giac [F] . . . 906
3.93.9 Mupad [B] (verification not implemented) . . . 906

3.93.1 Optimal result

Integrand size = 21, antiderivative size = 26

$$\int \sec(c+dx)\sqrt{a+a\sec(c+dx)}\,dx=\frac{2a\tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}}$$

output
```
2*a*tan(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)
```

3.93.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.12

$$\int \sec(c+dx)\sqrt{a+a\sec(c+dx)}\,dx=\frac{2\sqrt{a(1+\sec(c+dx))}\tan\left(\frac{1}{2}(c+dx)\right)}{d}$$

input
```
Integrate[Sec[c + d*x]*Sqrt[a + a*Sec[c + d*x]],x]
```

output
```
(2*Sqrt[a*(1 + Sec[c + d*x])]*Tan[(c + d*x)/2])/d
```

3.93.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3042, 4279}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(c+dx)\sqrt{a\sec(c+dx)+a}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)\sqrt{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}dx$$

$$\Big\downarrow \text{4279}$$

$$\frac{2a\tan(c+dx)}{d\sqrt{a\sec(c+dx)+a}}$$

input
```
Int[Sec[c + d*x]*Sqrt[a + a*Sec[c + d*x]],x]
```

output
```
(2*a*Tan[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]])
```

3.93.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4279
```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*b*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]])), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

3.93.4 Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.27

method	result	size
default	$-\frac{2\sqrt{a(1+\sec(dx+c))}\,(\cot(dx+c)-\csc(dx+c))}{d}$	33

input
```
int(sec(d*x+c)*(a+a*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/d*(a*(1+sec(d*x+c)))^(1/2)*(cot(d*x+c)-csc(d*x+c))
```

3.93.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.58

$$\int \sec(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \frac{2\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{d\cos(dx+c)+d}$$

input
```
integrate(sec(d*x+c)*(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
2*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c) + d
)
```

3.93.6 Sympy [F]

$$\int \sec(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \int \sqrt{a\left(\sec(c+dx)+1\right)}\sec(c+dx)\,dx$$

input
```
integrate(sec(d*x+c)*(a+a*sec(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(a*(sec(c + d*x) + 1))*sec(c + d*x), x)
```

3.93.7 Maxima [F]

$$\int \sec(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \int \sqrt{a\sec(dx+c)+a}\sec(dx+c)\,dx$$

input
```
integrate(sec(d*x+c)*(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(a*sec(d*x + c) + a)*sec(d*x + c), x)
```

3.93.8 Giac [F]

$$\int \sec(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \int \sqrt{a\sec(dx+c)+a}\sec(dx+c)\,dx$$

input
```
integrate(sec(d*x+c)*(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.93.9 Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.58

$$\int \sec(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \frac{2\sin(c+d\,x)\,\sqrt{\frac{a\,(\cos(c+d\,x)+1)}{\cos(c+d\,x)}}}{d\,(\cos(c+d\,x)+1)}$$

input
```
int((a + a/cos(c + d*x))^(1/2)/cos(c + d*x),x)
```

output
```
(2*sin(c + d*x)*((a*(cos(c + d*x) + 1))/cos(c + d*x))^(1/2))/(d*(cos(c + d
*x) + 1))
```

3.94 $\int \sqrt{a+a\sec(c+dx)}\,dx$

3.94.1 Optimal result . . . 907
3.94.2 Mathematica [A] (verified) . . . 907
3.94.3 Rubi [A] (verified) . . . 908
3.94.4 Maple [B] (verified) . . . 909
3.94.5 Fricas [A] (verification not implemented) . . . 909
3.94.6 Sympy [F] . . . 910
3.94.7 Maxima [B] (verification not implemented) . . . 910
3.94.8 Giac [F] . . . 911
3.94.9 Mupad [F(-1)] . . . 911

3.94.1 Optimal result

Integrand size = 14, antiderivative size = 37

$$\int \sqrt{a+a\sec(c+dx)}\,dx=\frac{2\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}$$

output
```
2*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))*a^(1/2)/d
```

3.94.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.62

$$\int \sqrt{a+a\sec(c+dx)}\,dx=\frac{\sqrt{2}\arcsin\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)\sqrt{\cos(c+dx)}\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(1+\sec(c+dx))}}{d}$$

input
```
Integrate[Sqrt[a + a*Sec[c + d*x]],x]
```

output
```
(Sqrt[2]*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]]*Sqrt[Cos[c + d*x]]*Sec[(c + d*x)
/2]*Sqrt[a*(1 + Sec[c + d*x])])/d
```

3.94.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3042, 4261, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a\sec(c+dx)+a}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}dx$$

$$\Big\downarrow \text{4261}$$

$$-\frac{2a\int \frac{1}{\frac{a^2\tan^2(c+dx)}{\sec(c+dx)a+a}+a}d\left(-\frac{a\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}$$

$$\Big\downarrow \text{216}$$

$$\frac{2\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}$$

input
```
Int[Sqrt[a + a*Sec[c + d*x]],x]
```

output
```
(2*Sqrt[a]*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d
```

3.94.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4261
```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(b/d)
  Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

3.94.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 75 vs. $2(31) = 62$.

Time = 0.83 (sec) , antiderivative size = 76, normalized size of antiderivative = 2.05

method	result	size
default	$\frac{2\sqrt{a(1+\sec(dx+c))}\,\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\operatorname{arctanh}\left(\frac{\sin(dx+c)}{(\cos(dx+c)+1)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}}\right)}{d}$	76

input
```
int((a+a*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/d*(a*(1+sec(d*x+c)))^(1/2)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(si
n(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))
```

3.94.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 133, normalized size of antiderivative = 3.59

$$\int \sqrt{a+a\sec(c+dx)}\,dx = \left[\frac{\sqrt{-a}\log\left(\frac{2\,a\cos(dx+c)^2-2\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+a\cos(dx+c)-a}{\cos(dx+c)+1}\right)}{d},\ -\frac{2\sqrt{a}\arctan\left(\frac{\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{\sqrt{a}\sin(dx+c)}\right)}{d}\right]$$

input
```
integrate((a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
[sqrt(-a)*log((2*a*cos(d*x + c)^2 - 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/c
os(d*x + c))*cos(d*x + c)*sin(d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c)
 + 1))/d, -2*sqrt(a)*arctan(sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*
x + c)/(sqrt(a)*sin(d*x + c)))/d]
```

3.94.6 Sympy [F]

$$\int \sqrt{a + a\sec(c + dx)}\,dx = \int \sqrt{a\sec(c + dx) + a}\,dx$$

input
```
integrate((a+a*sec(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(a*sec(c + d*x) + a), x)
```

3.94.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 146 vs. $2(31) = 62$.

Time = 0.38 (sec) , antiderivative size = 146, normalized size of antiderivative = 3.95

$$\int \sqrt{a + a\sec(c + dx)}\,dx$$
$$= \frac{\sqrt{a}\arctan\left(\left(\cos\left(2\,dx + 2\,c\right)^2 + \sin\left(2\,dx + 2\,c\right)^2 + 2\,\cos\left(2\,dx + 2\,c\right) + 1\right)^{\frac{1}{4}}\sin\left(\frac{1}{2}\,\arctan\left(\sin\left(2\,dx + 2\,c\right),\right.\right.\right.}{}$$

input
```
integrate((a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
sqrt(a)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2
*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) +
sin(d*x + c), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c
) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + co
s(d*x + c))/d
```

3.94.8 Giac [F]

$$\int \sqrt{a+a\sec(c+dx)}\,dx=\int \sqrt{a\sec{(dx+c)}+a}\,dx$$

input
```
integrate((a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.94.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+a\sec(c+dx)}\,dx=\int \sqrt{a+\frac{a}{\cos{(c+d\,x)}}}\,dx$$

input
```
int((a + a/cos(c + d*x))^(1/2),x)
```

output
```
int((a + a/cos(c + d*x))^(1/2), x)
```

3.95 $\int \cos(c+dx)\sqrt{a+a\sec(c+dx)}\,dx$

3.95.1 Optimal result . . . 912
3.95.2 Mathematica [A] (verified) . . . 912
3.95.3 Rubi [A] (verified) . . . 913
3.95.4 Maple [B] (verified) . . . 914
3.95.5 Fricas [A] (verification not implemented) . . . 915
3.95.6 Sympy [F] . . . 915
3.95.7 Maxima [B] (verification not implemented) . . . 916
3.95.8 Giac [F] . . . 916
3.95.9 Mupad [F(-1)] . . . 917

3.95.1 Optimal result

Integrand size = 21, antiderivative size = 62

$$\int \cos(c+dx)\sqrt{a+a\sec(c+dx)}\,dx=\frac{\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}+\frac{a\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}}$$

output
```
arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))*a^(1/2)/d+a*sin(d*x+c)/d
/(a+a*sec(d*x+c))^(1/2)
```

3.95.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.27

$$\int \cos(c+dx)\sqrt{a+a\sec(c+dx)}\,dx$$
$$=\frac{\left(\operatorname{arctanh}\left(\sqrt{1-\sec(c+dx)}\right)+\cos(c+dx)\sqrt{1-\sec(c+dx)}\right)\sqrt{a(1+\sec(c+dx))}\tan\left(\frac{1}{2}(c+dx)\right)}{d\sqrt{1-\sec(c+dx)}}$$

input
```
Integrate[Cos[c + d*x]*Sqrt[a + a*Sec[c + d*x]],x]
```

output
```
((ArcTanh[Sqrt[1 - Sec[c + d*x]]] + Cos[c + d*x]*Sqrt[1 - Sec[c + d*x]])*S
qrt[a*(1 + Sec[c + d*x])]*Tan[(c + d*x)/2])/(d*Sqrt[1 - Sec[c + d*x]])
```

3.95.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3042, 4292, 3042, 4261, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(c+dx)\sqrt{a\sec(c+dx)+a}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}}{\csc\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{4292}$$

$$\frac{1}{2}\int \sqrt{\sec(c+dx)a+a}dx+\frac{a\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx+\frac{a\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{4261}$$

$$\frac{a\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}-\frac{a\int \frac{1}{\frac{a^2\tan^2(c+dx)}{\sec(c+dx)a+a}+a}d\left(-\frac{a\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}$$

$$\Big\downarrow \text{216}$$

$$\frac{\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{a\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}$$

input
```
Int[Cos[c + d*x]*Sqrt[a + a*Sec[c + d*x]],x]
```

output
```
(Sqrt[a]*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d + (a*S
in[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]])
```

3.95.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4261
```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(b/d)
  Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 4292
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[a*Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*n*Sqrt[a
+ b*Csc[e + f*x]])), x] + Simp[a*((2*n + 1)/(2*b*d*n))   Int[Sqrt[a + b*Csc
[e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
 EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

3.95.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 164 vs. $2(54) = 108$.

Time = 24.22 (sec) , antiderivative size = 165, normalized size of antiderivative = 2.66

method	result
default	$\frac{\left(\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\operatorname{arctanh}\left(\frac{\sin(dx+c)}{(\cos(dx+c)+1)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}}\right)\cos(dx+c)+\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\operatorname{arctanh}\left(\frac{\sin(dx+c)}{(\cos(dx+c)+1)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}}\right)\right.}{d(\cos(dx+c)+1)}$

input
```
int(cos(d*x+c)*(a+a*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/d*((-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(sin(d*x+c)/(cos(d*x+c)+1)/
(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)+(-cos(d*x+c)/(cos(d*x+c)+1)
)^(1/2)*arctanh(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/
2))+cos(d*x+c)*sin(d*x+c))*(a*(1+sec(d*x+c)))^(1/2)/(cos(d*x+c)+1)
```

3.95.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 242, normalized size of antiderivative = 3.90

$$\int \cos(c+dx)\sqrt{a+a\sec(c+dx)}\,dx$$
$$= \left[\frac{\sqrt{-a}(\cos(dx+c)+1)\log\left(\frac{2\,a\cos(dx+c)^2-2\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+a\cos(dx+c)-a}{\cos(dx+c)+1}\right)+2\sqrt{\frac{a\cos(dx+c)+}{\cos(dx+c)}}}{2\,(d\cos(dx+c)+d)}\right.$$
$$\left.-\frac{\sqrt{a}(\cos(dx+c)+1)\arctan\left(\frac{\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{\sqrt{a}\sin(dx+c)}\right)-\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)}{d\cos(dx+c)+d}\right]$$

input
```
integrate(cos(d*x+c)*(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
[1/2*(sqrt(-a)*(cos(d*x + c) + 1)*log((2*a*cos(d*x + c)^2 - 2*sqrt(-a)*sqr
t((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) + a*cos(d*x
 + c) - a)/(cos(d*x + c) + 1)) + 2*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))
*cos(d*x + c)*sin(d*x + c))/(d*cos(d*x + c) + d), -(sqrt(a)*(cos(d*x + c)
+ 1)*arctan(sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*
sin(d*x + c))) - sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(
d*x + c))/(d*cos(d*x + c) + d)]
```

3.95.6 Sympy [F]

$$\int \cos(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \int \sqrt{a\,(\sec(c+dx)+1)}\cos(c+dx)\,dx$$

input
```
integrate(cos(d*x+c)*(a+a*sec(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(a*(sec(c + d*x) + 1))*cos(c + d*x), x)
```

3.95.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 791 vs. $2(54) = 108$.

Time = 0.42 (sec) , antiderivative size = 791, normalized size of antiderivative = 12.76

$$\int \cos(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)*(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
1/4*(2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^
(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(d*x +
c) - (cos(d*x + c) - 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
 + 1)))*sqrt(a) + sqrt(a)*(arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)
^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 +
 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c) + 1))) + 1) - arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d*x
 + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2
*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c) + 1))) - 1) - arctan2((cos(2*d*x + 2*c)^2 + sin
(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 +
 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c) + 1)) + 1) + arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 +...
```

3.95.8 Giac [F]

$$\int \cos(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \int \sqrt{a\sec(dx+c)+a}\cos(dx+c)\,dx$$

input
```
integrate(cos(d*x+c)*(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.95.9 Mupad [F(-1)]

Timed out.

$$\int \cos(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \int \cos\left(c+d\,x\right)\,\sqrt{a+\frac{a}{\cos\left(c+d\,x\right)}}\,dx$$

input
```
int(cos(c + d*x)*(a + a/cos(c + d*x))^(1/2),x)
```

output
```
int(cos(c + d*x)*(a + a/cos(c + d*x))^(1/2), x)
```

3.96 $\int \cos^2(c+dx)\sqrt{a+a\sec(c+dx)}\,dx$

3.96.1 Optimal result . . . 918
3.96.2 Mathematica [C] (verified) . . . 918
3.96.3 Rubi [A] (verified) . . . 919
3.96.4 Maple [B] (verified) . . . 921
3.96.5 Fricas [A] (verification not implemented) . . . 921
3.96.6 Sympy [F] . . . 922
3.96.7 Maxima [B] (verification not implemented) . . . 922
3.96.8 Giac [F] . . . 923
3.96.9 Mupad [F(-1)] . . . 924

3.96.1 Optimal result

Integrand size = 23, antiderivative size = 102

$$\int \cos^2(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \frac{3\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{4d} + \frac{3a\sin(c+dx)}{4d\sqrt{a+a\sec(c+dx)}} + \frac{a\cos(c+dx)\sin(c+dx)}{2d\sqrt{a+a\sec(c+dx)}}$$

output
```
3/4*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))*a^(1/2)/d+3/4*a*sin(
d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+1/2*a*cos(d*x+c)*sin(d*x+c)/d/(a+a*sec(d*x
+c))^(1/2)
```

3.96.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.11 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.46

$$\int \cos^2(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \frac{2\,\text{Hypergeometric2F1}\left(\frac{1}{2},3,\frac{3}{2},1-\sec(c+dx)\right)\sqrt{a(1+\sec(c+dx))}\tan\left(\frac{1}{2}(c+dx)\right)}{d}$$

input
```
Integrate[Cos[c + d*x]^2*Sqrt[a + a*Sec[c + d*x]],x]
```

output
```
(2*Hypergeometric2F1[1/2, 3, 3/2, 1 - Sec[c + d*x]]*Sqrt[a*(1 + Sec[c + d*
x])]*Tan[(c + d*x)/2])/d
```

3.96.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.99, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3042, 4292, 3042, 4292, 3042, 4261, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(c+dx)\sqrt{a\sec(c+dx)+a}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}}{\csc\left(c+dx+\frac{\pi}{2}\right)^2}dx$$

$$\Big\downarrow \text{4292}$$

$$\frac{3}{4}\int \cos(c+dx)\sqrt{\sec(c+dx)a+a}dx+\frac{a\sin(c+dx)\cos(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3}{4}\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{a\sin(c+dx)\cos(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{4292}$$

$$\frac{3}{4}\left(\frac{1}{2}\int \sqrt{\sec(c+dx)a+a}dx+\frac{a\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}\right)+\frac{a\sin(c+dx)\cos(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3}{4}\left(\frac{1}{2}\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx+\frac{a\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}\right)+\frac{a\sin(c+dx)\cos(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{4261}$$

$$\frac{3}{4}\left(\frac{a\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}-\frac{a\int \frac{1}{\frac{a^2\tan^2(c+dx)}{\sec(c+dx)a+a}+a}d\left(-\frac{a\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}\right)+\frac{a\sin(c+dx)\cos(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{216}$$

$$\frac{3}{4}\left(\frac{\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{a\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}\right)+\frac{a\sin(c+dx)\cos(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}$$

input
```
Int[Cos[c + d*x]^2*Sqrt[a + a*Sec[c + d*x]],x]
```

output
```
(a*Cos[c + d*x]*Sin[c + d*x])/(2*d*Sqrt[a + a*Sec[c + d*x]]) + (3*((Sqrt[a
]*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d + (a*Sin[c +
d*x])/(d*Sqrt[a + a*Sec[c + d*x]])))/4
```

3.96.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4261
```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(b/d)
  Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 4292
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[a*Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*n*Sqrt[a
+ b*Csc[e + f*x]])), x] + Simp[a*((2*n + 1)/(2*b*d*n))   Int[Sqrt[a + b*Csc
[e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
 EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

3.96.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 184 vs. $2(86) = 172$.

Time = 22.84 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.81

method	result
default	$\frac{\left(3\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\operatorname{arctanh}\left(\frac{\sin(dx+c)}{(\cos(dx+c)+1)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}}\right)\cos(dx+c)+2\cos(dx+c)^2\sin(dx+c)+3\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\operatorname{arctanh}\left(\frac{}{(\cos}\right.\right.}{4d(\cos(dx+c)+1)}$

input
```
int(cos(d*x+c)^2*(a+a*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4/d*(3*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(sin(d*x+c)/(cos(d*x+c)
+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)+2*cos(d*x+c)^2*sin(d*x+
c)+3*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(sin(d*x+c)/(cos(d*x+c)+1)/
(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+3*cos(d*x+c)*sin(d*x+c))*(a*(1+sec(d*x
+c)))^(1/2)/(cos(d*x+c)+1)
```

3.96.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 270, normalized size of antiderivative = 2.65

$$\int \cos^2(c+dx)\sqrt{a+a\sec(c+dx)}\,dx$$
$$= \left[\frac{3\sqrt{-a}(\cos(dx+c)+1)\log\left(\frac{2\,a\cos(dx+c)^2-2\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+a\cos(dx+c)-a}{\cos(dx+c)+1}\right)+2\left(2\cos(dx\right.}{8\left(d\cos(dx+c)+d\right)}\right.$$
$$-\frac{3\sqrt{a}(\cos(dx+c)+1)\arctan\left(\frac{\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{\sqrt{a}\sin(dx+c)}\right)-\left(2\cos(dx+c)^2+3\cos(dx+c)\right)\sqrt{\frac{a\cos(dx+c)-}{\cos(dx+c)}}}{4\left(d\cos(dx+c)+d\right)}$$

input
```
integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
[1/8*(3*sqrt(-a)*(cos(d*x + c) + 1)*log((2*a*cos(d*x + c)^2 - 2*sqrt(-a)*s
qrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) + a*cos(d
*x + c) - a)/(cos(d*x + c) + 1)) + 2*(2*cos(d*x + c)^2 + 3*cos(d*x + c))*s
qrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c) + d),
 -1/4*(3*sqrt(a)*(cos(d*x + c) + 1)*arctan(sqrt((a*cos(d*x + c) + a)/cos(d
*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) - (2*cos(d*x + c)^2 + 3*cos(
d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x
 + c) + d)]
```

3.96.6 Sympy [F]

$$\int \cos^2(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \int \sqrt{a\left(\sec\left(c+dx\right)+1\right)}\cos^2\left(c+dx\right)dx$$

input
```
integrate(cos(d*x+c)**2*(a+a*sec(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(a*(sec(c + d*x) + 1))*cos(c + d*x)**2, x)
```

3.96.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1059 vs. $2(86) = 172$.

Time = 0.47 (sec) , antiderivative size = 1059, normalized size of antiderivative = 10.38

$$\int \cos^2(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
1/16*(2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)
^(1/4)*((cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(2*d*x +
2*c) - (cos(2*d*x + 2*c) - 2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c))) + sin(2*d*x + 2*c))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c) + 1)) + ((cos(2*d*x + 2*c) - 2)*cos(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c))) + sin(2*d*x + 2*c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c))) - cos(2*d*x + 2*c) + 2)*sin(1/2*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c) + 1)))*sqrt(a) + 3*sqrt(a)*(arctan2((cos(2*d*x + 2*c)^2 +
sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c) + 1)) - cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*s
in(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2
+ sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))
*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) + 1) - arctan2((cos
(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(
1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))),...
```

3.96.8 Giac [F]

$$\int \cos^2(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \int \sqrt{a\sec(dx+c)+a}\cos(dx+c)^2\,dx$$

input
```
integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.96.9 Mupad [F(-1)]

Timed out.

$$\int \cos^2(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \int \cos\left(c+d\,x\right)^2\,\sqrt{a+\frac{a}{\cos\left(c+d\,x\right)}}\,dx$$

input
```
int(cos(c + d*x)^2*(a + a/cos(c + d*x))^(1/2),x)
```

output
```
int(cos(c + d*x)^2*(a + a/cos(c + d*x))^(1/2), x)
```

3.97 $\int \cos^3(c+dx)\sqrt{a+a\sec(c+dx)}\,dx$

3.97.1 Optimal result . . . 925
3.97.2 Mathematica [C] (verified) . . . 925
3.97.3 Rubi [A] (verified) . . . 926
3.97.4 Maple [A] (verified) . . . 928
3.97.5 Fricas [A] (verification not implemented) . . . 929
3.97.6 Sympy [F(-1)] . . . 929
3.97.7 Maxima [B] (verification not implemented) . . . 930
3.97.8 Giac [F] . . . 930
3.97.9 Mupad [F(-1)] . . . 931

3.97.1 Optimal result

Integrand size = 23, antiderivative size = 138

$$\int \cos^3(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \frac{5\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{8d} + \frac{5a\sin(c+dx)}{8d\sqrt{a+a\sec(c+dx)}} + \frac{5a\cos(c+dx)\sin(c+dx)}{12d\sqrt{a+a\sec(c+dx)}} + \frac{a\cos^2(c+dx)\sin(c+dx)}{3d\sqrt{a+a\sec(c+dx)}}$$

output
```
5/8*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))*a^(1/2)/d+5/8*a*sin(
d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+5/12*a*cos(d*x+c)*sin(d*x+c)/d/(a+a*sec(d*
x+c))^(1/2)+1/3*a*cos(d*x+c)^2*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)
```

3.97.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.10 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.34

$$\int \cos^3(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \frac{2\,\text{Hypergeometric2F1}\left(\frac{1}{2},4,\frac{3}{2},1-\sec(c+dx)\right)\sqrt{a(1+\sec(c+dx))}\tan\left(\frac{1}{2}(c+dx)\right)}{d}$$

input
```
Integrate[Cos[c + d*x]^3*Sqrt[a + a*Sec[c + d*x]],x]
```

output
```
(2*Hypergeometric2F1[1/2, 4, 3/2, 1 - Sec[c + d*x]]*Sqrt[a*(1 + Sec[c + d*
x])]*Tan[(c + d*x)/2])/d
```

3.97.3 Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.03, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {3042, 4292, 3042, 4292, 3042, 4292, 3042, 4261, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^3(c+dx)\sqrt{a\sec(c+dx)+a}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}}{\csc\left(c+dx+\frac{\pi}{2}\right)^3}dx$$

$$\Big\downarrow \text{4292}$$

$$\frac{5}{6}\int \cos^2(c+dx)\sqrt{\sec(c+dx)a+a}dx+\frac{a\sin(c+dx)\cos^2(c+dx)}{3d\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5}{6}\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}{\csc\left(c+dx+\frac{\pi}{2}\right)^2}dx+\frac{a\sin(c+dx)\cos^2(c+dx)}{3d\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{4292}$$

$$\frac{5}{6}\left(\frac{3}{4}\int \cos(c+dx)\sqrt{\sec(c+dx)a+a}dx+\frac{a\sin(c+dx)\cos(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}\right)+\frac{a\sin(c+dx)\cos^2(c+dx)}{3d\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5}{6}\left(\frac{3}{4}\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{a\sin(c+dx)\cos(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}\right)+\frac{a\sin(c+dx)\cos^2(c+dx)}{3d\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{4292}$$

$$\frac{5}{6}\left(\frac{3}{4}\left(\frac{1}{2}\int\sqrt{\sec(c+dx)a+a}dx+\frac{a\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}\right)+\frac{a\sin(c+dx)\cos(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}\right)+\frac{a\sin(c+dx)\cos^2(c+dx)}{3d\sqrt{a\sec(c+dx)+a}}$$

$\Big\downarrow$ 3042

$$\frac{5}{6}\left(\frac{3}{4}\left(\frac{1}{2}\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx+\frac{a\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}\right)+\frac{a\sin(c+dx)\cos(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}\right)+\frac{a\sin(c+dx)\cos^2(c+dx)}{3d\sqrt{a\sec(c+dx)+a}}$$

$\Big\downarrow$ 4261

$$\frac{5}{6}\left(\frac{3}{4}\left(\frac{a\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}-\frac{a\int\frac{1}{\frac{a^2\tan^2(c+dx)}{\sec(c+dx)a+a}+a}d\left(-\frac{a\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}\right)+\frac{a\sin(c+dx)\cos(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}\right)+\frac{a\sin(c+dx)\cos^2(c+dx)}{3d\sqrt{a\sec(c+dx)+a}}$$

$\Big\downarrow$ 216

$$\frac{5}{6}\left(\frac{3}{4}\left(\frac{\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{a\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}\right)+\frac{a\sin(c+dx)\cos(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}\right)+\frac{a\sin(c+dx)\cos^2(c+dx)}{3d\sqrt{a\sec(c+dx)+a}}$$

input
```
Int[Cos[c + d*x]^3*Sqrt[a + a*Sec[c + d*x]],x]
```

output
```
(a*Cos[c + d*x]^2*Sin[c + d*x])/(3*d*Sqrt[a + a*Sec[c + d*x]]) + (5*((a*Co
s[c + d*x]*Sin[c + d*x])/(2*d*Sqrt[a + a*Sec[c + d*x]]) + (3*((Sqrt[a]*Arc
Tan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d + (a*Sin[c + d*x])
/(d*Sqrt[a + a*Sec[c + d*x]])))/4))/6
```

3.97. $\int\cos^3(c+dx)\sqrt{a+a\sec(c+dx)}\,dx$

3.97.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4261
```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(b/d)
  Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 4292
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[a*Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*n*Sqrt[a
+ b*Csc[e + f*x]])), x] + Simp[a*((2*n + 1)/(2*b*d*n))   Int[Sqrt[a + b*Csc
[e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
 EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

3.97.4 Maple [A] (verified)

Time = 22.69 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.46

method	result
default	$\frac{\left(8\cos(dx+c)^3\sin(dx+c)+15\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\operatorname{arctanh}\left(\frac{\sin(dx+c)}{(\cos(dx+c)+1)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}}\right)\cos(dx+c)+10\cos(dx+c)^2\sin(dx+c)+15\sqrt{}\right.}{24d(\cos(dx+c)+1)}$

input
```
int(cos(d*x+c)^3*(a+a*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/24/d*(8*cos(d*x+c)^3*sin(d*x+c)+15*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*ar
ctanh(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*
x+c)+10*cos(d*x+c)^2*sin(d*x+c)+15*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arct
anh(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+15*cos(d
*x+c)*sin(d*x+c))*(a*(1+sec(d*x+c)))^(1/2)/(cos(d*x+c)+1)
```

3.97.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 290, normalized size of antiderivative = 2.10

$$\int \cos^3(c+dx)\sqrt{a+a\sec(c+dx)}\,dx$$
$$= \left[\frac{15\sqrt{-a}(\cos(dx+c)+1)\log\left(\frac{2\,a\cos(dx+c)^2-2\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+a\cos(dx+c)-a}{\cos(dx+c)+1}\right)+2\left(8\cos(d\right.}{48\,(d\cos(dx+c)+d)}\right.$$
$$-\frac{15\sqrt{a}(\cos(dx+c)+1)\arctan\left(\frac{\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{\sqrt{a}\sin(dx+c)}\right)-\left(8\cos(dx+c)^3+10\cos(dx+c)^2+15\cos\right.}{24\,(d\cos(dx+c)+d)}$$

input
```
integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
[1/48*(15*sqrt(-a)*(cos(d*x + c) + 1)*log((2*a*cos(d*x + c)^2 - 2*sqrt(-a)
*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) + a*cos
(d*x + c) - a)/(cos(d*x + c) + 1)) + 2*(8*cos(d*x + c)^3 + 10*cos(d*x + c)
^2 + 15*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)
)/(d*cos(d*x + c) + d), -1/24*(15*sqrt(a)*(cos(d*x + c) + 1)*arctan(sqrt((
a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) - (
8*cos(d*x + c)^3 + 10*cos(d*x + c)^2 + 15*cos(d*x + c))*sqrt((a*cos(d*x +
c) + a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c) + d)]
```

3.97.6 Sympy [F(-1)]

Timed out.

$$\int \cos^3(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**3*(a+a*sec(d*x+c))**(1/2),x)
```

output
```
Timed out
```

3.97.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1921 vs. $2(118) = 236$.

Time = 0.54 (sec) , antiderivative size = 1921, normalized size of antiderivative = 13.92

$$\int \cos^3(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
1/96*(4*(cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*
arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d
*x + 3*c), cos(3*d*x + 3*c))) + 1)^(3/4)*(cos(3/2*arctan2(sin(2/3*arctan2(
sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), co
s(3*d*x + 3*c))) + 1))*sin(3*d*x + 3*c) - (cos(3*d*x + 3*c) - 1)*sin(3/2*a
rctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arcta
n2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)))*sqrt(a) + 6*(cos(2/3*arctan
2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c
), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x +
3*c))) + 1)^(1/4)*((sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) +
 5*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))))*cos(1/2*arctan2(s
in(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3
*d*x + 3*c), cos(3*d*x + 3*c))) + 1)) - (cos(2/3*arctan2(sin(3*d*x + 3*c),
 cos(3*d*x + 3*c))) + 3*cos(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)
)) - 4)*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)
)), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)))*sqrt(a) +
15*sqrt(a)*(arctan2(-(cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))
^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*ar
ctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*(cos(1/2*arctan2(sin
(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(...
```

3.97.8 Giac [F]

$$\int \cos^3(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \int \sqrt{a\sec(dx+c)+a}\cos(dx+c)^3\,dx$$

input
```
integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.97.9 Mupad [F(-1)]

Timed out.

$$\int \cos^3(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \int \cos\left(c+d\,x\right)^3\,\sqrt{a+\frac{a}{\cos\left(c+d\,x\right)}}\,dx$$

input
```
int(cos(c + d*x)^3*(a + a/cos(c + d*x))^(1/2),x)
```

output
```
int(cos(c + d*x)^3*(a + a/cos(c + d*x))^(1/2), x)
```

3.98 $\int \cos^4(c+dx)\sqrt{a+a\sec(c+dx)}\,dx$

3.98.1 Optimal result . . . 932
3.98.2 Mathematica [C] (verified) . . . 933
3.98.3 Rubi [A] (verified) . . . 933
3.98.4 Maple [A] (verified) . . . 936
3.98.5 Fricas [A] (verification not implemented) . . . 936
3.98.6 Sympy [F] . . . 937
3.98.7 Maxima [B] (verification not implemented) . . . 937
3.98.8 Giac [F] . . . 938
3.98.9 Mupad [F(-1)] . . . 939

3.98.1 Optimal result

Integrand size = 23, antiderivative size = 174

$$\int \cos^4(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \frac{35\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{64d} + \frac{35a\sin(c+dx)}{64d\sqrt{a+a\sec(c+dx)}} + \frac{35a\cos(c+dx)\sin(c+dx)}{96d\sqrt{a+a\sec(c+dx)}} + \frac{7a\cos^2(c+dx)\sin(c+dx)}{24d\sqrt{a+a\sec(c+dx)}} + \frac{a\cos^3(c+dx)\sin(c+dx)}{4d\sqrt{a+a\sec(c+dx)}}$$

output
```
35/64*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))*a^(1/2)/d+35/64*a*
sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+35/96*a*cos(d*x+c)*sin(d*x+c)/d/(a+a*s
ec(d*x+c))^(1/2)+7/24*a*cos(d*x+c)^2*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+1
/4*a*cos(d*x+c)^3*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)
```

3.98.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.10 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.27

$$\int \cos^4(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \frac{2\,\text{Hypergeometric2F1}\left(\frac{1}{2},5,\frac{3}{2},1-\sec(c+dx)\right)\sqrt{a(1+\sec(c+dx))}\tan\left(\frac{1}{2}(c+dx)\right)}{d}$$

input
```
Integrate[Cos[c + d*x]^4*Sqrt[a + a*Sec[c + d*x]],x]
```

output
```
(2*Hypergeometric2F1[1/2, 5, 3/2, 1 - Sec[c + d*x]]*Sqrt[a*(1 + Sec[c + d*
x])]*Tan[(c + d*x)/2])/d
```

3.98.3 Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.05, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules used = {3042, 4292, 3042, 4292, 3042, 4292, 3042, 4292, 3042, 4261, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^4(c+dx)\sqrt{a\sec(c+dx)+a}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}}{\csc\left(c+dx+\frac{\pi}{2}\right)^4}dx$$

$$\Big\downarrow \text{4292}$$

$$\frac{7}{8}\int \cos^3(c+dx)\sqrt{\sec(c+dx)a+a}dx+\frac{a\sin(c+dx)\cos^3(c+dx)}{4d\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7}{8}\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}{\csc\left(c+dx+\frac{\pi}{2}\right)^3}dx+\frac{a\sin(c+dx)\cos^3(c+dx)}{4d\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{4292}$$

$$\frac{7}{8}\left(\frac{5}{6}\int \cos^2(c+dx)\sqrt{\sec(c+dx)a+a}dx+\frac{a\sin(c+dx)\cos^2(c+dx)}{3d\sqrt{a\sec(c+dx)+a}}\right)+\frac{a\sin(c+dx)\cos^3(c+dx)}{4d\sqrt{a\sec(c+dx)+a}}$$

$\downarrow$ 3042

$$\frac{7}{8}\left(\frac{5}{6}\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}{\csc\left(c+dx+\frac{\pi}{2}\right)^2}dx+\frac{a\sin(c+dx)\cos^2(c+dx)}{3d\sqrt{a\sec(c+dx)+a}}\right)+\frac{a\sin(c+dx)\cos^3(c+dx)}{4d\sqrt{a\sec(c+dx)+a}}$$

$\downarrow$ 4292

$$\frac{7}{8}\left(\frac{5}{6}\left(\frac{3}{4}\int \cos(c+dx)\sqrt{\sec(c+dx)a+a}dx+\frac{a\sin(c+dx)\cos(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}\right)+\frac{a\sin(c+dx)\cos^2(c+dx)}{3d\sqrt{a\sec(c+dx)+a}}\right)+\frac{a\sin(c+dx)\cos^3(c+dx)}{4d\sqrt{a\sec(c+dx)+a}}$$

$\downarrow$ 3042

$$\frac{7}{8}\left(\frac{5}{6}\left(\frac{3}{4}\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{a\sin(c+dx)\cos(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}\right)+\frac{a\sin(c+dx)\cos^2(c+dx)}{3d\sqrt{a\sec(c+dx)+a}}\right)+\frac{a\sin(c+dx)\cos^3(c+dx)}{4d\sqrt{a\sec(c+dx)+a}}$$

$\downarrow$ 4292

$$\frac{7}{8}\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{1}{2}\int \sqrt{\sec(c+dx)a+a}dx+\frac{a\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}\right)+\frac{a\sin(c+dx)\cos(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}\right)+\frac{a\sin(c+dx)\cos^2}{3d\sqrt{a\sec(c+d}}\right.$$
$$\frac{a\sin(c+dx)\cos^3(c+dx)}{4d\sqrt{a\sec(c+dx)+a}}$$

$\downarrow$ 3042

$$\frac{7}{8}\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{1}{2}\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx+\frac{a\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}\right)+\frac{a\sin(c+dx)\cos(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}\right)+\frac{a\sin(c+d}{3d\sqrt{a\sec}}\right.$$
$$\frac{a\sin(c+dx)\cos^3(c+dx)}{4d\sqrt{a\sec(c+dx)+a}}$$

$\downarrow$ 4261

3.98. $\int \cos^4(c+dx)\sqrt{a+a\sec(c+dx)}\,dx$

$$\frac{7}{8}\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{a\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}-\frac{a\int\frac{1}{\frac{a^2\tan^2(c+dx)}{\sec(c+dx)a+a}+a}d\left(-\frac{a\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}\right)+\frac{a\sin(c+dx)\cos(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}\right)+\frac{a\sin(}{3d\sqrt{}}\right.$$

$$\frac{a\sin(c+dx)\cos^3(c+dx)}{4d\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \; 216$$

$$\frac{7}{8}\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{a\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}\right)+\frac{a\sin(c+dx)\cos(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}\right)+\frac{a\sin(c+dx)\,\mathrm{co}}{3d\sqrt{a\sec(c+}}\right.$$

$$\frac{a\sin(c+dx)\cos^3(c+dx)}{4d\sqrt{a\sec(c+dx)+a}}$$

input
```
Int[Cos[c + d*x]^4*Sqrt[a + a*Sec[c + d*x]],x]
```

output
```
(a*Cos[c + d*x]^3*Sin[c + d*x])/(4*d*Sqrt[a + a*Sec[c + d*x]]) + (7*((a*Co
s[c + d*x]^2*Sin[c + d*x])/(3*d*Sqrt[a + a*Sec[c + d*x]]) + (5*((a*Cos[c +
 d*x]*Sin[c + d*x])/(2*d*Sqrt[a + a*Sec[c + d*x]]) + (3*((Sqrt[a]*ArcTan[(
Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d + (a*Sin[c + d*x])/(d*S
qrt[a + a*Sec[c + d*x]])))/4))/6))/8
```

3.98.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4261
```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(b/d)
  Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 4292
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[a*Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*n*Sqrt[a
+ b*Csc[e + f*x]])), x] + Simp[a*((2*n + 1)/(2*b*d*n))   Int[Sqrt[a + b*Csc
[e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
 EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

3.98.4 Maple [A] (verified)

Time = 26.78 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.25

method	result
default	$\frac{\left(48\cos(dx+c)^4\sin(dx+c)+56\cos(dx+c)^3\sin(dx+c)+105\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\operatorname{arctanh}\left(\frac{\sin(dx+c)}{(\cos(dx+c)+1)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}}\right)\cos(dx+c)+70\right.}{192d(\cos}$

input
```
int(cos(d*x+c)^4*(a+a*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/192/d*(48*cos(d*x+c)^4*sin(d*x+c)+56*cos(d*x+c)^3*sin(d*x+c)+105*(-cos(d
*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)
/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)+70*cos(d*x+c)^2*sin(d*x+c)+105*(-cos(d*
x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/
(cos(d*x+c)+1))^(1/2))+105*cos(d*x+c)*sin(d*x+c))*(a*(1+sec(d*x+c)))^(1/2)
/(cos(d*x+c)+1)
```

3.98.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 310, normalized size of antiderivative = 1.78

$$\int \cos^4(c+dx)\sqrt{a+a\sec(c+dx)}\,dx$$
$$= \left[\frac{105\sqrt{-a}(\cos(dx+c)+1)\log\left(\frac{2\,a\cos(dx+c)^2-2\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+a\cos(dx+c)-a}{\cos(dx+c)+1}\right)+2\left(48\cos\right.}{384\left(d\cos(dx+\right.}\right.$$
$$-\frac{105\sqrt{a}(\cos(dx+c)+1)\arctan\left(\frac{\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{\sqrt{a}\sin(dx+c)}\right)-\left(48\cos(dx+c)^4+56\cos(dx+c)^3+70\,c\right.}{192\left(d\cos(dx+c)+d\right)}$$

input
```
integrate(cos(d*x+c)^4*(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
[1/384*(105*sqrt(-a)*(cos(d*x + c) + 1)*log((2*a*cos(d*x + c)^2 - 2*sqrt(-
a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) + a*c
os(d*x + c) - a)/(cos(d*x + c) + 1)) + 2*(48*cos(d*x + c)^4 + 56*cos(d*x +
 c)^3 + 70*cos(d*x + c)^2 + 105*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/co
s(d*x + c))*sin(d*x + c))/(d*cos(d*x + c) + d), -1/192*(105*sqrt(a)*(cos(d
*x + c) + 1)*arctan(sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(
sqrt(a)*sin(d*x + c))) - (48*cos(d*x + c)^4 + 56*cos(d*x + c)^3 + 70*cos(d
*x + c)^2 + 105*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(
d*x + c))/(d*cos(d*x + c) + d)]
```

3.98.6 Sympy [F]

$$\int \cos^4(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \int \sqrt{a\left(\sec\left(c+dx\right)+1\right)}\cos^4\left(c+dx\right)dx$$

input
```
integrate(cos(d*x+c)**4*(a+a*sec(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(a*(sec(c + d*x) + 1))*cos(c + d*x)**4, x)
```

3.98.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6638 vs. $2(150) = 300$.

Time = 0.71 (sec) , antiderivative size = 6638, normalized size of antiderivative = 38.15

$$\int \cos^4(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^4*(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
-1/768*(2*(cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/
2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4
*d*x + 4*c), cos(4*d*x + 4*c))) + 1)^(3/4)*((36*(sin(4*d*x + 4*c)^3 + (cos
(4*d*x + 4*c)^2 - 2*cos(4*d*x + 4*c) + 1)*sin(4*d*x + 4*c))*cos(1/2*arctan
2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 9*cos(4*d*x + 4*c)^2*sin(4*d*x
+ 4*c) + 9*sin(4*d*x + 4*c)^3 + 36*(sin(4*d*x + 4*c)^3 + (cos(4*d*x + 4*c)
^2 + 2*cos(4*d*x + 4*c) + 1)*sin(4*d*x + 4*c))*sin(1/2*arctan2(sin(4*d*x +
 4*c), cos(4*d*x + 4*c)))^2 + 9*(2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4
*d*x + 4*c)))*sin(4*d*x + 4*c) - 2*(cos(4*d*x + 4*c) + 1)*sin(1/2*arctan2(
sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + sin(4*d*x + 4*c))*cos(3/4*arctan2(s
in(4*d*x + 4*c), cos(4*d*x + 4*c))) + 36*(sin(4*d*x + 4*c)^3 + (cos(4*d*x
+ 4*c)^2 - cos(4*d*x + 4*c))*sin(4*d*x + 4*c))*cos(1/2*arctan2(sin(4*d*x +
 4*c), cos(4*d*x + 4*c))) - (32*(cos(4*d*x + 4*c)^2 + sin(4*d*x + 4*c)^2 -
 2*cos(4*d*x + 4*c) + 1)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c
)))^2 + 32*(cos(4*d*x + 4*c)^2 + sin(4*d*x + 4*c)^2 + 2*cos(4*d*x + 4*c) +
 1)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 8*cos(4*d*x +
 4*c)^2 + 2*(16*cos(4*d*x + 4*c)^2 + 16*sin(4*d*x + 4*c)^2 - 7*cos(4*d*x +
 4*c) - 9)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 8*sin(4*
d*x + 4*c)^2 - 2*(64*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*
sin(4*d*x + 4*c) + 7*sin(4*d*x + 4*c))*sin(1/2*arctan2(sin(4*d*x + 4*c)...
```

3.98.8 Giac [F]

$$\int \cos^4(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \int \sqrt{a\sec(dx+c)+a}\cos(dx+c)^4\,dx$$

input
```
integrate(cos(d*x+c)^4*(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.98.9 Mupad [F(-1)]

Timed out.

$$\int \cos^4(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \int \cos{(c+d\,x)}^4\,\sqrt{a+\frac{a}{\cos{(c+d\,x)}}}\,dx$$

input
```
int(cos(c + d*x)^4*(a + a/cos(c + d*x))^(1/2),x)
```

output
```
int(cos(c + d*x)^4*(a + a/cos(c + d*x))^(1/2), x)
```

3.99 $\int \sec^4(c+dx)(a+a\sec(c+dx))^{3/2}\,dx$

3.99.1 Optimal result . . . 940
3.99.2 Mathematica [A] (verified) . . . 940
3.99.3 Rubi [A] (verified) . . . 941
3.99.4 Maple [A] (verified) . . . 944
3.99.5 Fricas [A] (verification not implemented) . . . 945
3.99.6 Sympy [F] . . . 945
3.99.7 Maxima [F] . . . 945
3.99.8 Giac [F] . . . 946
3.99.9 Mupad [B] (verification not implemented) . . . 947

3.99.1 Optimal result

Integrand size = 23, antiderivative size = 162

$$\int \sec^4(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \frac{68a^2\tan(c+dx)}{45d\sqrt{a+a\sec(c+dx)}} + \frac{34a^2\sec^3(c+dx)\tan(c+dx)}{63d\sqrt{a+a\sec(c+dx)}} + \frac{2a^2\sec^4(c+dx)\tan(c+dx)}{9d\sqrt{a+a\sec(c+dx)}} - \frac{136a\sqrt{a+a\sec(c+dx)}\tan(c+dx)}{315d} + \frac{68(a+a\sec(c+dx))^{3/2}\tan(c+dx)}{105d}$$

output
```
68/105*(a+a*sec(d*x+c))^(3/2)*tan(d*x+c)/d+68/45*a^2*tan(d*x+c)/d/(a+a*sec
(d*x+c))^(1/2)+34/63*a^2*sec(d*x+c)^3*tan(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+
2/9*a^2*sec(d*x+c)^4*tan(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)-136/315*a*(a+a*se
c(d*x+c))^(1/2)*tan(d*x+c)/d
```

3.99.2 Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.43

$$\int \sec^4(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \frac{2a^2(272+136\sec(c+dx)+102\sec^2(c+dx)+85\sec^3(c+dx)+35\sec^4(c+dx))\tan(c+dx)}{315d\sqrt{a(1+\sec(c+dx))}}$$

input
```
Integrate[Sec[c + d*x]^4*(a + a*Sec[c + d*x])^(3/2),x]
```

output
```
(2*a^2*(272 + 136*Sec[c + d*x] + 102*Sec[c + d*x]^2 + 85*Sec[c + d*x]^3 +
35*Sec[c + d*x]^4)*Tan[c + d*x])/(315*d*Sqrt[a*(1 + Sec[c + d*x])])
```

3.99.3 Rubi [A] (verified)

Time = 0.88 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.12, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.522$, Rules used = {3042, 4301, 27, 3042, 4290, 3042, 4287, 27, 3042, 4489, 3042, 4279}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^4(c+dx)(a\sec(c+dx)+a)^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^4\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}dx$$

$$\Big\downarrow \text{4301}$$

$$\frac{2}{9}a\int \frac{17}{2}\sec^4(c+dx)\sqrt{\sec(c+dx)a+a}dx+\frac{2a^2\tan(c+dx)\sec^4(c+dx)}{9d\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{27}$$

$$\frac{17}{9}a\int \sec^4(c+dx)\sqrt{\sec(c+dx)a+a}dx+\frac{2a^2\tan(c+dx)\sec^4(c+dx)}{9d\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{17}{9}a\int \csc\left(c+dx+\frac{\pi}{2}\right)^4\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx+\frac{2a^2\tan(c+dx)\sec^4(c+dx)}{9d\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{4290}$$

$$\frac{17}{9}a\left(\frac{6}{7}\int \sec^3(c+dx)\sqrt{\sec(c+dx)a+a}dx+\frac{2a\tan(c+dx)\sec^3(c+dx)}{7d\sqrt{a\sec(c+dx)+a}}\right)+\frac{2a^2\tan(c+dx)\sec^4(c+dx)}{9d\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{17}{9}a\left(\frac{6}{7}\int \csc\left(c+dx+\frac{\pi}{2}\right)^3\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx+\frac{2a\tan(c+dx)\sec^3(c+dx)}{7d\sqrt{a\sec(c+dx)+a}}\right)+\frac{2a^2\tan(c+dx)\sec^4(c+dx)}{9d\sqrt{a\sec(c+dx)+a}}$$

$\Big\downarrow$ 4287

$$\frac{17}{9}a\left(\frac{6}{7}\left(\frac{2\int\frac{1}{2}\sec(c+dx)(3a-2a\sec(c+dx))\sqrt{\sec(c+dx)a+a}dx}{5a}+\frac{2\tan(c+dx)(a\sec(c+dx)+a)^{3/2}}{5ad}\right)+\frac{2a^2\tan(c+dx)\sec^4(c+dx)}{9d\sqrt{a\sec(c+dx)+a}}\right.$$

$\Big\downarrow$ 27

$$\frac{17}{9}a\left(\frac{6}{7}\left(\frac{\int\sec(c+dx)(3a-2a\sec(c+dx))\sqrt{\sec(c+dx)a+a}dx}{5a}+\frac{2\tan(c+dx)(a\sec(c+dx)+a)^{3/2}}{5ad}\right)+\frac{2a^2\tan(c+dx)\sec^4(c+dx)}{9d\sqrt{a\sec(c+dx)+a}}\right.$$

$\Big\downarrow$ 3042

$$\frac{17}{9}a\left(\frac{6}{7}\left(\frac{\int\csc\left(c+dx+\frac{\pi}{2}\right)\left(3a-2a\csc\left(c+dx+\frac{\pi}{2}\right)\right)\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{5a}+\frac{2\tan(c+dx)(a\sec(c+dx)}{5ad}\right.\right.$$
$$\frac{2a^2\tan(c+dx)\sec^4(c+dx)}{9d\sqrt{a\sec(c+dx)+a}}$$

$\Big\downarrow$ 4489

$$\frac{17}{9}a\left(\frac{6}{7}\left(\frac{\frac{7}{3}a\int\sec(c+dx)\sqrt{\sec(c+dx)a+a}dx-\frac{4a\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}}{5a}+\frac{2\tan(c+dx)(a\sec(c+dx)+a)^{3/2}}{5ad}\right.\right.$$
$$\frac{2a^2\tan(c+dx)\sec^4(c+dx)}{9d\sqrt{a\sec(c+dx)+a}}$$

$\Big\downarrow$ 3042

$$\frac{17}{9}a\left(\frac{6}{7}\left(\frac{\frac{7}{3}a\int\csc\left(c+dx+\frac{\pi}{2}\right)\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx-\frac{4a\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}}{5a}+\frac{2\tan(c+dx)(a\sec(c+}{5ad}\right.\right.$$
$$\frac{2a^2\tan(c+dx)\sec^4(c+dx)}{9d\sqrt{a\sec(c+dx)+a}}$$

$\Big\downarrow$ 4279

3.99. $\int\sec^4(c+dx)(a+a\sec(c+dx))^{3/2}\,dx$

$$\frac{2a^2\tan(c+dx)\sec^4(c+dx)}{9d\sqrt{a\sec(c+dx)+a}}+\frac{17}{9}a\left(\frac{6}{7}\left(\frac{\frac{14a^2\tan(c+dx)}{3d\sqrt{a\sec(c+dx)+a}}-\frac{4a\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}}{5a}+\frac{2\tan(c+dx)(a\sec(c+dx)+a)^{3/2}}{5ad}\right)+\frac{2a\tan(c+dx)\,\mathrm{se}}{7d\sqrt{a\sec(c+}}\right.$$

input
```
Int[Sec[c + d*x]^4*(a + a*Sec[c + d*x])^(3/2),x]
```

output
```
(2*a^2*Sec[c + d*x]^4*Tan[c + d*x])/(9*d*Sqrt[a + a*Sec[c + d*x]]) + (17*a
*((2*a*Sec[c + d*x]^3*Tan[c + d*x])/(7*d*Sqrt[a + a*Sec[c + d*x]]) + (6*((
2*(a + a*Sec[c + d*x])^(3/2)*Tan[c + d*x])/(5*a*d) + ((14*a^2*Tan[c + d*x]
)/(3*d*Sqrt[a + a*Sec[c + d*x]]) - (4*a*Sqrt[a + a*Sec[c + d*x]]*Tan[c + d
*x])/(3*d))/(5*a)))/7))/9
```

3.99.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4279
```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*b*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]])), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4287
```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(-Cot[e + f*x])*((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2
))), x] + Simp[1/(b*(m + 2))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(b*(
m + 1) - a*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 -
b^2, 0] &&  !LtQ[m, -2^(-1)]
```

rule 4290
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[-2*b*d*Cot[e + f*x]*((d*Csc[e + f*x])^(n - 1)/(
f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]])), x] + Simp[2*a*d*((n - 1)/(b*(2*n -
1)))   Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e + f*x])^(n - 1), x], x] /; Fre
eQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

rule 4301
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
2)*((d*Csc[e + f*x])^n/(f*(m + n - 1))), x] + Simp[b/(m + n - 1)   Int[(a +
 b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*(b*(m + 2*n - 1) + a*(3*m + 2*n
 - 4)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^
2, 0] && GtQ[m, 1] && NeQ[m + n - 1, 0] && IntegerQ[2*m]
```

rule 4489
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*((
a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Simp[(a*B*m + A*b*(m + 1))/(b*(m +
 1))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B
, e, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b
*(m + 1), 0] &&  !LtQ[m, -2^(-1)]
```

3.99.4 Maple [A] (verified)

Time = 0.87 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.51

method	result	size
default	$\frac{2a\left(272\cos(dx+c)^4+136\cos(dx+c)^3+102\cos(dx+c)^2+85\cos(dx+c)+35\right)\sqrt{a(1+\sec(dx+c))}\,\tan(dx+c)\sec(dx+c)^3}{315d(\cos(dx+c)+1)}$	83

input
```
int(sec(d*x+c)^4*(a+a*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/315/d*a*(272*cos(d*x+c)^4+136*cos(d*x+c)^3+102*cos(d*x+c)^2+85*cos(d*x+c
)+35)*(a*(1+sec(d*x+c)))^(1/2)/(cos(d*x+c)+1)*tan(d*x+c)*sec(d*x+c)^3
```

3.99.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.60

$$\int \sec^4(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \frac{2\left(272\,a\cos\left(dx+c\right)^4+136\,a\cos\left(dx+c\right)^3+102\,a\cos\left(dx+c\right)^2+85\,a\cos\left(dx+c\right)+35\,a\right)\sqrt{}}{315\left(d\cos\left(dx+c\right)^5+d\cos\left(dx+c\right)^4\right)}$$

input
```
integrate(sec(d*x+c)^4*(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
2/315*(272*a*cos(d*x + c)^4 + 136*a*cos(d*x + c)^3 + 102*a*cos(d*x + c)^2
+ 85*a*cos(d*x + c) + 35*a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*
x + c)/(d*cos(d*x + c)^5 + d*cos(d*x + c)^4)
```

3.99.6 Sympy [F]

$$\int \sec^4(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \int (a(\sec(c+dx)+1))^{\frac{3}{2}}\sec^4(c+dx)\,dx$$

input
```
integrate(sec(d*x+c)**4*(a+a*sec(d*x+c))**(3/2),x)
```

output
```
Integral((a*(sec(c + d*x) + 1))**(3/2)*sec(c + d*x)**4, x)
```

3.99.7 Maxima [F]

$$\int \sec^4(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \int (a\sec(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^4\,dx$$

input
```
integrate(sec(d*x+c)^4*(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
16/315*(315*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)
+ 1)^(1/4)*((a*d*cos(2*d*x + 2*c)^4 + a*d*sin(2*d*x + 2*c)^4 + 4*a*d*cos(2
*d*x + 2*c)^3 + 6*a*d*cos(2*d*x + 2*c)^2 + 4*a*d*cos(2*d*x + 2*c) + 2*(a*d
*cos(2*d*x + 2*c)^2 + 2*a*d*cos(2*d*x + 2*c) + a*d)*sin(2*d*x + 2*c)^2 + a
*d)*integrate((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c
) + 1)^(1/4)*(((cos(10*d*x + 10*c)*cos(2*d*x + 2*c) + 4*cos(8*d*x + 8*c)*c
os(2*d*x + 2*c) + 6*cos(6*d*x + 6*c)*cos(2*d*x + 2*c) + 4*cos(4*d*x + 4*c)
*cos(2*d*x + 2*c) + cos(2*d*x + 2*c)^2 + sin(10*d*x + 10*c)*sin(2*d*x + 2*
c) + 4*sin(8*d*x + 8*c)*sin(2*d*x + 2*c) + 6*sin(6*d*x + 6*c)*sin(2*d*x +
2*c) + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + sin(2*d*x + 2*c)^2)*cos(7/2*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + (cos(2*d*x + 2*c)*sin(10*d*x
 + 10*c) + 4*cos(2*d*x + 2*c)*sin(8*d*x + 8*c) + 6*cos(2*d*x + 2*c)*sin(6*
d*x + 6*c) + 4*cos(2*d*x + 2*c)*sin(4*d*x + 4*c) - cos(10*d*x + 10*c)*sin(
2*d*x + 2*c) - 4*cos(8*d*x + 8*c)*sin(2*d*x + 2*c) - 6*cos(6*d*x + 6*c)*si
n(2*d*x + 2*c) - 4*cos(4*d*x + 4*c)*sin(2*d*x + 2*c))*sin(7/2*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c) + 1)) - ((cos(2*d*x + 2*c)*sin(10*d*x + 10*c) + 4*cos(2*d*x + 2
*c)*sin(8*d*x + 8*c) + 6*cos(2*d*x + 2*c)*sin(6*d*x + 6*c) + 4*cos(2*d*x +
 2*c)*sin(4*d*x + 4*c) - cos(10*d*x + 10*c)*sin(2*d*x + 2*c) - 4*cos(8*d*x
 + 8*c)*sin(2*d*x + 2*c) - 6*cos(6*d*x + 6*c)*sin(2*d*x + 2*c) - 4*cos(...
```

3.99.8 Giac [F]

$$\int \sec^4(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \int (a\sec(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^4\,dx$$

input
```
integrate(sec(d*x+c)^4*(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.99.9 Mupad [B] (verification not implemented)

Time = 21.19 (sec) , antiderivative size = 429, normalized size of antiderivative = 2.65

$$\int \sec^4(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \frac{\left(\frac{a\,32\mathrm{i}}{9\,d}-\frac{a\,\mathrm{e}^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}\,32\mathrm{i}}{9\,d}\right)\sqrt{a+\frac{a}{\frac{\mathrm{e}^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}}{2}}}}{(\mathrm{e}^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}+1)\,(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1)^4} - \frac{\left(\frac{a\,80\mathrm{i}}{7\,d}-\frac{a\,\mathrm{e}^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}\,176\mathrm{i}}{63\,d}\right)\sqrt{a+\frac{a}{\frac{\mathrm{e}^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}}{2}}}}{(\mathrm{e}^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}+1)\,(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1)^3} + \frac{\left(\frac{a\,48\mathrm{i}}{5\,d}+\frac{a\,\mathrm{e}^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}\,352\mathrm{i}}{105\,d}\right)\sqrt{a+\frac{a}{\frac{\mathrm{e}^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}}{2}}}}{(\mathrm{e}^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}+1)\,(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1)^2} - \frac{a\,\mathrm{e}^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}\sqrt{a+\frac{a}{\frac{\mathrm{e}^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}}{2}}}\,544\mathrm{i}}{315\,d\,(\mathrm{e}^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}+1)} - \frac{a\,\mathrm{e}^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}\sqrt{a+\frac{a}{\frac{\mathrm{e}^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}}{2}}}\,272\mathrm{i}}{315\,d\,(\mathrm{e}^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}+1)\,(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1)}$$

input
```
int((a + a/cos(c + d*x))^(3/2)/cos(c + d*x)^4,x)
```

output
```
(((a*32i)/(9*d) - (a*exp(c*1i + d*x*1i)*32i)/(9*d))*(a + a/(exp(- c*1i - d
*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2))/((exp(c*1i + d*x*1i) + 1)*(exp(c*
2i + d*x*2i) + 1)^4) - (((a*80i)/(7*d) - (a*exp(c*1i + d*x*1i)*176i)/(63*d
))*(a + a/(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2))/((exp(c*
1i + d*x*1i) + 1)*(exp(c*2i + d*x*2i) + 1)^3) + (((a*48i)/(5*d) + (a*exp(c
*1i + d*x*1i)*352i)/(105*d))*(a + a/(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d
*x*1i)/2))^(1/2))/((exp(c*1i + d*x*1i) + 1)*(exp(c*2i + d*x*2i) + 1)^2) -
(a*exp(c*1i + d*x*1i)*(a + a/(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/
2))^(1/2)*544i)/(315*d*(exp(c*1i + d*x*1i) + 1)) - (a*exp(c*1i + d*x*1i)*(
a + a/(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*272i)/(315*d*
(exp(c*1i + d*x*1i) + 1)*(exp(c*2i + d*x*2i) + 1))
```

3.100 $\int \sec^3(c+dx)(a+a\sec(c+dx))^{3/2}\,dx$

3.100.1 Optimal result . . . 948
3.100.2 Mathematica [A] (verified) . . . 948
3.100.3 Rubi [A] (verified) . . . 949
3.100.4 Maple [A] (verified) . . . 951
3.100.5 Fricas [A] (verification not implemented) . . . 952
3.100.6 Sympy [F] . . . 952
3.100.7 Maxima [F] . . . 952
3.100.8 Giac [F] . . . 953
3.100.9 Mupad [B] (verification not implemented) . . . 954

3.100.1 Optimal result

Integrand size = 23, antiderivative size = 116

$$\int \sec^3(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \frac{152a^2\tan(c+dx)}{105d\sqrt{a+a\sec(c+dx)}} + \frac{38a\sqrt{a+a\sec(c+dx)}\tan(c+dx)}{105d} - \frac{4(a+a\sec(c+dx))^{3/2}\tan(c+dx)}{35d} + \frac{2(a+a\sec(c+dx))^{5/2}\tan(c+dx)}{7ad}$$

output
```
-4/35*(a+a*sec(d*x+c))^(3/2)*tan(d*x+c)/d+2/7*(a+a*sec(d*x+c))^(5/2)*tan(d
*x+c)/a/d+152/105*a^2*tan(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+38/105*a*(a+a*se
c(d*x+c))^(1/2)*tan(d*x+c)/d
```

3.100.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.52

$$\int \sec^3(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \frac{2a^2(104+52\sec(c+dx)+39\sec^2(c+dx)+15\sec^3(c+dx))\tan(c+dx)}{105d\sqrt{a(1+\sec(c+dx))}}$$

input
```
Integrate[Sec[c + d*x]^3*(a + a*Sec[c + d*x])^(3/2),x]
```

output
```
(2*a^2*(104 + 52*Sec[c + d*x] + 39*Sec[c + d*x]^2 + 15*Sec[c + d*x]^3)*Tan
[c + d*x])/(105*d*Sqrt[a*(1 + Sec[c + d*x])])
```

3.100.3 Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.13, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {3042, 4287, 27, 3042, 4489, 3042, 4280, 3042, 4279}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^3(c+dx)(a\sec(c+dx)+a)^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^3\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}dx$$

$$\Big\downarrow \text{4287}$$

$$\frac{2\int \frac{1}{2}\sec(c+dx)(5a-2a\sec(c+dx))(\sec(c+dx)a+a)^{3/2}dx}{7a}+\frac{2\tan(c+dx)(a\sec(c+dx)+a)^{5/2}}{7ad}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \sec(c+dx)(5a-2a\sec(c+dx))(\sec(c+dx)a+a)^{3/2}dx}{7a}+\frac{2\tan(c+dx)(a\sec(c+dx)+a)^{5/2}}{7ad}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \csc\left(c+dx+\frac{\pi}{2}\right)\left(5a-2a\csc\left(c+dx+\frac{\pi}{2}\right)\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}dx}{7a}+\frac{2\tan(c+dx)(a\sec(c+dx)+a)^{5/2}}{7ad}$$

$$\Big\downarrow \text{4489}$$

$$\frac{\frac{19}{5}a\int \sec(c+dx)(\sec(c+dx)a+a)^{3/2}dx-\frac{4a\tan(c+dx)(a\sec(c+dx)+a)^{3/2}}{5d}}{7a}+\frac{2\tan(c+dx)(a\sec(c+dx)+a)^{5/2}}{7ad}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{19}{5}a\int \csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}dx-\frac{4a\tan(c+dx)(a\sec(c+dx)+a)^{3/2}}{5d}}{7a}+\frac{2\tan(c+dx)(a\sec(c+dx)+a)^{5/2}}{7ad}$$

$$\Big\downarrow \; 4280$$

$$\frac{\frac{19}{5}a\left(\frac{4}{3}a\int \sec(c+dx)\sqrt{\sec(c+dx)a+a}dx+\frac{2a\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}\right)-\frac{4a\tan(c+dx)(a\sec(c+dx)+a)^{3/2}}{5d}}{7a}+\frac{2\tan(c+dx)(a\sec(c+dx)+a)^{5/2}}{7ad}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{19}{5}a\left(\frac{4}{3}a\int \csc\left(c+dx+\frac{\pi}{2}\right)\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx+\frac{2a\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}\right)-\frac{4a\tan(c+dx)(a\sec(c+dx)+a)^{3/2}}{5d}}{7a}+\frac{2\tan(c+dx)(a\sec(c+dx)+a)^{5/2}}{7ad}$$

$$\Big\downarrow \; 4279$$

$$\frac{\frac{19}{5}a\left(\frac{8a^2\tan(c+dx)}{3d\sqrt{a\sec(c+dx)+a}}+\frac{2a\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}\right)-\frac{4a\tan(c+dx)(a\sec(c+dx)+a)^{3/2}}{5d}}{7a}+\frac{2\tan(c+dx)(a\sec(c+dx)+a)^{5/2}}{7ad}$$

input
```
Int[Sec[c + d*x]^3*(a + a*Sec[c + d*x])^(3/2),x]
```

output
```
(2*(a + a*Sec[c + d*x])^(5/2)*Tan[c + d*x])/(7*a*d) + ((-4*a*(a + a*Sec[c
+ d*x])^(3/2)*Tan[c + d*x])/(5*d) + (19*a*((8*a^2*Tan[c + d*x])/(3*d*Sqrt[
a + a*Sec[c + d*x]]) + (2*a*Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x])/(3*d)))
/5)/(7*a)
```

3.100.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4279
```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*b*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]])), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4280
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[(-b)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m - 1)/(f*m)), x]
+ Simp[a*((2*m - 1)/m)   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m - 1), x],
 x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] && Intege
rQ[2*m]
```

rule 4287
```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(-Cot[e + f*x])*((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2
))), x] + Simp[1/(b*(m + 2))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(b*(
m + 1) - a*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 -
b^2, 0] &&  !LtQ[m, -2^(-1)]
```

rule 4489
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*((
a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Simp[(a*B*m + A*b*(m + 1))/(b*(m +
 1))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B
, e, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b
*(m + 1), 0] &&  !LtQ[m, -2^(-1)]
```

3.100.4 Maple [A] (verified)

Time = 0.83 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.63

method	result	size
default	$\frac{2a\left(104\cos(dx+c)^3+52\cos(dx+c)^2+39\cos(dx+c)+15\right)\sqrt{a(1+\sec(dx+c))}\,\tan(dx+c)\sec(dx+c)^2}{105d(\cos(dx+c)+1)}$	73

input
```
int(sec(d*x+c)^3*(a+a*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/105/d*a*(104*cos(d*x+c)^3+52*cos(d*x+c)^2+39*cos(d*x+c)+15)*(a*(1+sec(d*
x+c)))^(1/2)/(cos(d*x+c)+1)*tan(d*x+c)*sec(d*x+c)^2
```

3.100.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.75

$$\int \sec^3(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \frac{2\left(104\,a\cos\left(dx+c\right)^3+52\,a\cos\left(dx+c\right)^2+39\,a\cos\left(dx+c\right)+15\,a\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin\left(dx+c\right.}{105\left(d\cos\left(dx+c\right)^4+d\cos\left(dx+c\right)^3\right)}$$

input
```
integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
2/105*(104*a*cos(d*x + c)^3 + 52*a*cos(d*x + c)^2 + 39*a*cos(d*x + c) + 15
*a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c)^4
 + d*cos(d*x + c)^3)
```

3.100.6 Sympy [F]

$$\int \sec^3(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \int (a(\sec(c+dx)+1))^{\frac{3}{2}}\sec^3(c+dx)\,dx$$

input
```
integrate(sec(d*x+c)**3*(a+a*sec(d*x+c))**(3/2),x)
```

output
```
Integral((a*(sec(c + d*x) + 1))**(3/2)*sec(c + d*x)**3, x)
```

3.100.7 Maxima [F]

$$\int \sec^3(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \int (a\sec(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^3\,dx$$

input
```
integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
8/105*(105*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) +
 1)^(3/4)*(3*(a*d*cos(2*d*x + 2*c)^2 + a*d*sin(2*d*x + 2*c)^2 + 2*a*d*cos(
2*d*x + 2*c) + a*d)*integrate((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2
*cos(2*d*x + 2*c) + 1)^(1/4)*(((cos(8*d*x + 8*c)*cos(2*d*x + 2*c) + 3*cos(
6*d*x + 6*c)*cos(2*d*x + 2*c) + 3*cos(4*d*x + 4*c)*cos(2*d*x + 2*c) + cos(
2*d*x + 2*c)^2 + sin(8*d*x + 8*c)*sin(2*d*x + 2*c) + 3*sin(6*d*x + 6*c)*si
n(2*d*x + 2*c) + 3*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + sin(2*d*x + 2*c)^2)
*cos(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + (cos(2*d*x + 2*c)*
sin(8*d*x + 8*c) + 3*cos(2*d*x + 2*c)*sin(6*d*x + 6*c) + 3*cos(2*d*x + 2*c
)*sin(4*d*x + 4*c) - cos(8*d*x + 8*c)*sin(2*d*x + 2*c) - 3*cos(6*d*x + 6*c
)*sin(2*d*x + 2*c) - 3*cos(4*d*x + 4*c)*sin(2*d*x + 2*c))*sin(5/2*arctan2(
sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(3/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c) + 1)) - ((cos(2*d*x + 2*c)*sin(8*d*x + 8*c) + 3*cos(2*d*x +
 2*c)*sin(6*d*x + 6*c) + 3*cos(2*d*x + 2*c)*sin(4*d*x + 4*c) - cos(8*d*x +
 8*c)*sin(2*d*x + 2*c) - 3*cos(6*d*x + 6*c)*sin(2*d*x + 2*c) - 3*cos(4*d*x
 + 4*c)*sin(2*d*x + 2*c))*cos(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c))) - (cos(8*d*x + 8*c)*cos(2*d*x + 2*c) + 3*cos(6*d*x + 6*c)*cos(2*d*x +
 2*c) + 3*cos(4*d*x + 4*c)*cos(2*d*x + 2*c) + cos(2*d*x + 2*c)^2 + sin(8*d
*x + 8*c)*sin(2*d*x + 2*c) + 3*sin(6*d*x + 6*c)*sin(2*d*x + 2*c) + 3*sin(4
*d*x + 4*c)*sin(2*d*x + 2*c) + sin(2*d*x + 2*c)^2)*sin(5/2*arctan2(sin(...
```

3.100.8 Giac [F]

$$\int \sec^3(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \int (a\sec(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^3\,dx$$

input
```
integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.100.9 Mupad [B] (verification not implemented)

Time = 17.84 (sec) , antiderivative size = 346, normalized size of antiderivative = 2.98

$$\int \sec^3(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = -\frac{\left(\frac{a\,16\mathrm{i}}{7\,d}+\frac{a\,\mathrm{e}^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}\,16\mathrm{i}}{7\,d}\right)\sqrt{a+\frac{a}{\frac{\mathrm{e}^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}}{2}}}}{\left(\mathrm{e}^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}+1\right)\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)^3} + \frac{\left(\frac{a\,8\mathrm{i}}{3\,d}-\frac{a\,\mathrm{e}^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}\,104\mathrm{i}}{105\,d}\right)\sqrt{a+\frac{a}{\frac{\mathrm{e}^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}}{2}}}}{\left(\mathrm{e}^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}+1\right)\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)} + \frac{\left(\frac{a\,8\mathrm{i}}{5\,d}+\frac{a\,\mathrm{e}^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}\,184\mathrm{i}}{35\,d}\right)\sqrt{a+\frac{a}{\frac{\mathrm{e}^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}}{2}}}}{\left(\mathrm{e}^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}+1\right)\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)^2} - \frac{a\,\mathrm{e}^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}\sqrt{a+\frac{a}{\frac{\mathrm{e}^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}}{2}}}\,208\mathrm{i}}{105\,d\left(\mathrm{e}^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}+1\right)}$$

input
```
int((a + a/cos(c + d*x))^(3/2)/cos(c + d*x)^3,x)
```

output
```
(((a*8i)/(3*d) - (a*exp(c*1i + d*x*1i)*104i)/(105*d))*(a + a/(exp(- c*1i -
 d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2))/((exp(c*1i + d*x*1i) + 1)*(exp(
c*2i + d*x*2i) + 1)) - (((a*16i)/(7*d) + (a*exp(c*1i + d*x*1i)*16i)/(7*d))
*(a + a/(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2))/((exp(c*1i
 + d*x*1i) + 1)*(exp(c*2i + d*x*2i) + 1)^3) + (((a*8i)/(5*d) + (a*exp(c*1i
 + d*x*1i)*184i)/(35*d))*(a + a/(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1
i)/2))^(1/2))/((exp(c*1i + d*x*1i) + 1)*(exp(c*2i + d*x*2i) + 1)^2) - (a*e
xp(c*1i + d*x*1i)*(a + a/(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^
(1/2)*208i)/(105*d*(exp(c*1i + d*x*1i) + 1))
```

3.101 $\int \sec^2(c+dx)(a+a\sec(c+dx))^{3/2}\,dx$

3.101.1 Optimal result 955
3.101.2 Mathematica [A] (verified) 955
3.101.3 Rubi [A] (verified) 956
3.101.4 Maple [A] (verified) 957
3.101.5 Fricas [A] (verification not implemented) 958
3.101.6 Sympy [F] 958
3.101.7 Maxima [F] 958
3.101.8 Giac [F] 959
3.101.9 Mupad [B] (verification not implemented) 960

3.101.1 Optimal result

Integrand size = 23, antiderivative size = 86

$$\int \sec^2(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \frac{8a^2\tan(c+dx)}{5d\sqrt{a+a\sec(c+dx)}} + \frac{2a\sqrt{a+a\sec(c+dx)}\tan(c+dx)}{5d} + \frac{2(a+a\sec(c+dx))^{3/2}\tan(c+dx)}{5d}$$

output
```
2/5*(a+a*sec(d*x+c))^(3/2)*tan(d*x+c)/d+8/5*a^2*tan(d*x+c)/d/(a+a*sec(d*x+
c))^(1/2)+2/5*a*(a+a*sec(d*x+c))^(1/2)*tan(d*x+c)/d
```

3.101.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.56

$$\int \sec^2(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \frac{2a^2(6+3\sec(c+dx)+\sec^2(c+dx))\tan(c+dx)}{5d\sqrt{a(1+\sec(c+dx))}}$$

input
```
Integrate[Sec[c + d*x]^2*(a + a*Sec[c + d*x])^(3/2),x]
```

output
```
(2*a^2*(6 + 3*Sec[c + d*x] + Sec[c + d*x]^2)*Tan[c + d*x])/(5*d*Sqrt[a*(1
+ Sec[c + d*x])])
```

3.101.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.06, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3042, 4285, 3042, 4280, 3042, 4279}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(c+dx)(a\sec(c+dx)+a)^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^2\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}dx$$

$$\Big\downarrow \text{4285}$$

$$\frac{3}{5}\int \sec(c+dx)(\sec(c+dx)a+a)^{3/2}dx+\frac{2\tan(c+dx)(a\sec(c+dx)+a)^{3/2}}{5d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3}{5}\int \csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}dx+\frac{2\tan(c+dx)(a\sec(c+dx)+a)^{3/2}}{5d}$$

$$\Big\downarrow \text{4280}$$

$$\frac{3}{5}\left(\frac{4}{3}a\int \sec(c+dx)\sqrt{\sec(c+dx)a+a}dx+\frac{2a\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}\right)+\frac{2\tan(c+dx)(a\sec(c+dx)+a)^{3/2}}{5d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3}{5}\left(\frac{4}{3}a\int \csc\left(c+dx+\frac{\pi}{2}\right)\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx+\frac{2a\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}\right)+\frac{2\tan(c+dx)(a\sec(c+dx)+a)^{3/2}}{5d}$$

$$\Big\downarrow \text{4279}$$

$$\frac{3}{5}\left(\frac{8a^2\tan(c+dx)}{3d\sqrt{a\sec(c+dx)+a}}+\frac{2a\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}\right)+\frac{2\tan(c+dx)(a\sec(c+dx)+a)^{3/2}}{5d}$$

input
```
Int[Sec[c + d*x]^2*(a + a*Sec[c + d*x])^(3/2),x]
```

output
```
(2*(a + a*Sec[c + d*x])^(3/2)*Tan[c + d*x])/(5*d) + (3*((8*a^2*Tan[c + d*x
])/(3*d*Sqrt[a + a*Sec[c + d*x]]) + (2*a*Sqrt[a + a*Sec[c + d*x]]*Tan[c +
d*x])/(3*d)))/5
```

3.101.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4279
```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*b*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]])), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4280
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[(-b)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m - 1)/(f*m)), x]
+ Simp[a*((2*m - 1)/m)   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m - 1), x],
 x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] && Intege
rQ[2*m]
```

rule 4285
```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(-Cot[e + f*x])*((a + b*Csc[e + f*x])^m/(f*(m + 1))), x]
+ Simp[a*(m/(b*(m + 1)))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x]
/; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

3.101.4 Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.69

method	result	size
default	$\frac{2a\sqrt{a(1+\sec(dx+c))}\,(6\sin(dx+c)+3\tan(dx+c)+\sec(dx+c)\tan(dx+c))}{5d(\cos(dx+c)+1)}$	59

input
```
int(sec(d*x+c)^2*(a+a*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/5/d*a*(a*(1+sec(d*x+c)))^(1/2)/(cos(d*x+c)+1)*(6*sin(d*x+c)+3*tan(d*x+c)
+sec(d*x+c)*tan(d*x+c))
```

3.101.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.86

$$\int \sec^2(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \frac{2\left(6\,a\cos\left(dx+c\right)^2+3\,a\cos\left(dx+c\right)+a\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin\left(dx+c\right)}{5\left(d\cos\left(dx+c\right)^3+d\cos\left(dx+c\right)^2\right)}$$

input
```
integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
2/5*(6*a*cos(d*x + c)^2 + 3*a*cos(d*x + c) + a)*sqrt((a*cos(d*x + c) + a)/
cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c)^3 + d*cos(d*x + c)^2)
```

3.101.6 Sympy [F]

$$\int \sec^2(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \int \left(a(\sec\left(c+dx\right)+1)\right)^{\frac{3}{2}}\sec^2\left(c+dx\right)dx$$

input
```
integrate(sec(d*x+c)**2*(a+a*sec(d*x+c))**(3/2),x)
```

output
```
Integral((a*(sec(c + d*x) + 1))**(3/2)*sec(c + d*x)**2, x)
```

3.101.7 Maxima [F]

$$\int \sec^2(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \int \left(a\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}\sec\left(dx+c\right)^2\,dx$$

input
```
integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
4/5*(5*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^
(1/4)*((a*d*cos(2*d*x + 2*c)^2 + a*d*sin(2*d*x + 2*c)^2 + 2*a*d*cos(2*d*x
+ 2*c) + a*d)*integrate((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2
*d*x + 2*c) + 1)^(1/4)*(((cos(6*d*x + 6*c)*cos(2*d*x + 2*c) + 2*cos(4*d*x
+ 4*c)*cos(2*d*x + 2*c) + cos(2*d*x + 2*c)^2 + sin(6*d*x + 6*c)*sin(2*d*x
+ 2*c) + 2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + sin(2*d*x + 2*c)^2)*cos(5/2
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + (cos(2*d*x + 2*c)*sin(6*d*
x + 6*c) + 2*cos(2*d*x + 2*c)*sin(4*d*x + 4*c) - cos(6*d*x + 6*c)*sin(2*d*
x + 2*c) - 2*cos(4*d*x + 4*c)*sin(2*d*x + 2*c))*sin(5/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c))))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c) + 1)) - ((cos(2*d*x + 2*c)*sin(6*d*x + 6*c) + 2*cos(2*d*x + 2*c)*sin(
4*d*x + 4*c) - cos(6*d*x + 6*c)*sin(2*d*x + 2*c) - 2*cos(4*d*x + 4*c)*sin(
2*d*x + 2*c))*cos(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - (cos(
6*d*x + 6*c)*cos(2*d*x + 2*c) + 2*cos(4*d*x + 4*c)*cos(2*d*x + 2*c) + cos(
2*d*x + 2*c)^2 + sin(6*d*x + 6*c)*sin(2*d*x + 2*c) + 2*sin(4*d*x + 4*c)*si
n(2*d*x + 2*c) + sin(2*d*x + 2*c)^2)*sin(5/2*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c))))*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))
/((cos(2*d*x + 2*c)^4 + sin(2*d*x + 2*c)^4 + (cos(2*d*x + 2*c)^2 + sin(2*d
*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c)^2 + 4*(cos(2*d*x +
2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c)^...
```

3.101.8 Giac [F]

$$\int \sec^2(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \int (a\sec(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^2\,dx$$

input
```
integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.101.9 Mupad [B] (verification not implemented)

Time = 17.19 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.35

$$\int \sec^2(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \frac{4\,a\,\sqrt{a+\frac{a}{\frac{e^{-c\,1i-d\,x\,1i}}{2}+\frac{e^{c\,1i+d\,x\,1i}}{2}}}\,\left(e^{c\,2i+d\,x\,2i}\,5i-e^{c\,3i+d\,x\,3i}\,5i-e^{c\,5i+d\,x\,5i}\,3i+3i\right)}{5\,d\,\left(e^{c\,1i+d\,x\,1i}+1\right)\,\left(e^{c\,2i+d\,x\,2i}+1\right)^2}$$

input
```
int((a + a/cos(c + d*x))^(3/2)/cos(c + d*x)^2,x)
```

output
```
(4*a*(a + a/(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*(exp(c*
2i + d*x*2i)*5i - exp(c*3i + d*x*3i)*5i - exp(c*5i + d*x*5i)*3i + 3i))/(5*
d*(exp(c*1i + d*x*1i) + 1)*(exp(c*2i + d*x*2i) + 1)^2)
```

3.102 $\int \sec(c+dx)(a+a\sec(c+dx))^{3/2}\,dx$

3.102.1 Optimal result . . . 961
3.102.2 Mathematica [A] (verified) . . . 961
3.102.3 Rubi [A] (verified) . . . 962
3.102.4 Maple [A] (verified) . . . 963
3.102.5 Fricas [A] (verification not implemented) . . . 963
3.102.6 Sympy [F] . . . 964
3.102.7 Maxima [F] . . . 964
3.102.8 Giac [F] . . . 964
3.102.9 Mupad [B] (verification not implemented) . . . 965

3.102.1 Optimal result

Integrand size = 21, antiderivative size = 59

$$\int \sec(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \frac{8a^2\tan(c+dx)}{3d\sqrt{a+a\sec(c+dx)}} + \frac{2a\sqrt{a+a\sec(c+dx)}\tan(c+dx)}{3d}$$

output
```
8/3*a^2*tan(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+2/3*a*(a+a*sec(d*x+c))^(1/2)*t
an(d*x+c)/d
```

3.102.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.64

$$\int \sec(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \frac{2a^2(5+\sec(c+dx))\tan(c+dx)}{3d\sqrt{a(1+\sec(c+dx))}}$$

input
```
Integrate[Sec[c + d*x]*(a + a*Sec[c + d*x])^(3/2),x]
```

output
```
(2*a^2*(5 + Sec[c + d*x])*Tan[c + d*x])/(3*d*Sqrt[a*(1 + Sec[c + d*x])])
```

3.102.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3042, 4280, 3042, 4279}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(c+dx)(a\sec(c+dx)+a)^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}dx$$

$$\Big\downarrow \text{4280}$$

$$\frac{4}{3}a\int \sec(c+dx)\sqrt{\sec(c+dx)a+a}dx+\frac{2a\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4}{3}a\int \csc\left(c+dx+\frac{\pi}{2}\right)\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx+\frac{2a\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}$$

$$\Big\downarrow \text{4279}$$

$$\frac{8a^2\tan(c+dx)}{3d\sqrt{a\sec(c+dx)+a}}+\frac{2a\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}$$

input
```
Int[Sec[c + d*x]*(a + a*Sec[c + d*x])^(3/2),x]
```

output
```
(8*a^2*Tan[c + d*x])/(3*d*Sqrt[a + a*Sec[c + d*x]]) + (2*a*Sqrt[a + a*Sec[
c + d*x]]*Tan[c + d*x])/(3*d)
```

3.102.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4279
```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*b*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]])), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4280
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[(-b)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m - 1)/(f*m)), x]
+ Simp[a*((2*m - 1)/m)   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m - 1), x],
 x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] && Intege
rQ[2*m]
```

3.102.4 Maple [A] (verified)

Time = 0.89 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.75

method	result	size
default	$\frac{2a\sqrt{a(1+\sec(dx+c))}\,(5\sin(dx+c)+\tan(dx+c))}{3d(\cos(dx+c)+1)}$	44

input
```
int(sec(d*x+c)*(a+a*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/3/d*a*(a*(1+sec(d*x+c)))^(1/2)/(cos(d*x+c)+1)*(5*sin(d*x+c)+tan(d*x+c))
```

3.102.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.03

$$\int \sec(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \frac{2\left(5\,a\cos\left(dx+c\right)+a\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin\left(dx+c\right)}{3\left(d\cos\left(dx+c\right)^2+d\cos\left(dx+c\right)\right)}$$

input
```
integrate(sec(d*x+c)*(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
2/3*(5*a*cos(d*x + c) + a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x
 + c)/(d*cos(d*x + c)^2 + d*cos(d*x + c))
```

3.102.6 Sympy [F]

$$\int \sec(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \int (a(\sec(c+dx)+1))^{\frac{3}{2}}\sec(c+dx)\,dx$$

input
```
integrate(sec(d*x+c)*(a+a*sec(d*x+c))**(3/2),x)
```

output
```
Integral((a*(sec(c + d*x) + 1))**(3/2)*sec(c + d*x), x)
```

3.102.7 Maxima [F]

$$\int \sec(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \int (a\sec(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)\,dx$$

input
```
integrate(sec(d*x+c)*(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)^(3/2)*sec(d*x + c), x)
```

3.102.8 Giac [F]

$$\int \sec(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \int (a\sec(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)\,dx$$

input
```
integrate(sec(d*x+c)*(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.102.9 Mupad [B] (verification not implemented)

Time = 1.25 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.88

$$\int \sec(c+dx)(a+a\sec(c+dx))^{3/2}\,dx=\frac{2\,a\,\sqrt{\frac{a\,(\cos(c+d\,x)+1)}{\cos(c+d\,x)}}\,(12\,\sin(c+d\,x)+14\,\sin(2\,c+2\,d\,x)+12\,\sin(3\,c+3\,d\,x)+5\,\sin(4\,c+4}{3\,d\,(12\,\cos(c+d\,x)+8\,\cos(2\,c+2\,d\,x)+4\,\cos(3\,c+3\,d\,x)+\cos(4\,c+4\,d\,x)+7)}$$

input
```
int((a + a/cos(c + d*x))^(3/2)/cos(c + d*x),x)
```

output
```
(2*a*((a*(cos(c + d*x) + 1))/cos(c + d*x))^(1/2)*(12*sin(c + d*x) + 14*sin
(2*c + 2*d*x) + 12*sin(3*c + 3*d*x) + 5*sin(4*c + 4*d*x)))/(3*d*(12*cos(c
+ d*x) + 8*cos(2*c + 2*d*x) + 4*cos(3*c + 3*d*x) + cos(4*c + 4*d*x) + 7))
```

3.103 $\int (a+a\sec(c+dx))^{3/2}\,dx$

3.103.1 Optimal result 966
3.103.2 Mathematica [A] (verified) 966
3.103.3 Rubi [A] (verified) 967
3.103.4 Maple [B] (verified) 968
3.103.5 Fricas [A] (verification not implemented) 969
3.103.6 Sympy [F] 970
3.103.7 Maxima [B] (verification not implemented) 970
3.103.8 Giac [F] 971
3.103.9 Mupad [F(-1)] 971

3.103.1 Optimal result

Integrand size = 14, antiderivative size = 66

$$\int (a+a\sec(c+dx))^{3/2}\,dx=\frac{2a^{3/2}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}+\frac{2a^2\tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}}$$

output
```
2*a^(3/2)*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/d+2*a^2*tan(d*
x+c)/d/(a+a*sec(d*x+c))^(1/2)
```

3.103.2 Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.14

$$\int (a+a\sec(c+dx))^{3/2}\,dx=\frac{a\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(1+\sec(c+dx))}\left(\sqrt{2}\arcsin\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)\sqrt{\cos(c+dx)}+2\sin\right.}{d}$$

input
```
Integrate[(a + a*Sec[c + d*x])^(3/2),x]
```

output
```
(a*Sec[(c + d*x)/2]*Sqrt[a*(1 + Sec[c + d*x])]*(Sqrt[2]*ArcSin[Sqrt[2]*Sin
[(c + d*x)/2]]*Sqrt[Cos[c + d*x]] + 2*Sin[(c + d*x)/2]))/d
```

3.103.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 4262, 27, 3042, 4261, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sec(c+dx)+a)^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}dx$$

$$\Big\downarrow \text{4262}$$

$$2a\int \frac{1}{2}\sqrt{\sec(c+dx)a+a}dx+\frac{2a^2\tan(c+dx)}{d\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{27}$$

$$a\int \sqrt{\sec(c+dx)a+a}dx+\frac{2a^2\tan(c+dx)}{d\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$a\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx+\frac{2a^2\tan(c+dx)}{d\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{4261}$$

$$\frac{2a^2\tan(c+dx)}{d\sqrt{a\sec(c+dx)+a}}-\frac{2a^2\int \frac{1}{\frac{a^2\tan^2(c+dx)}{\sec(c+dx)a+a}+a}d\left(-\frac{a\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}$$

$$\Big\downarrow \text{216}$$

$$\frac{2a^{3/2}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{2a^2\tan(c+dx)}{d\sqrt{a\sec(c+dx)+a}}$$

input
```
Int[(a + a*Sec[c + d*x])^(3/2),x]
```

output
```
(2*a^(3/2)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d + (2
*a^2*Tan[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]])
```

3.103.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4261
```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(b/d)
  Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 4262
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(-b^2)*C
ot[c + d*x]*((a + b*Csc[c + d*x])^(n - 2)/(d*(n - 1))), x] + Simp[a/(n - 1)
   Int[(a + b*Csc[c + d*x])^(n - 2)*(a*(n - 1) + b*(3*n - 4)*Csc[c + d*x]),
 x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && Inte
gerQ[2*n]
```

3.103.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 159 vs. $2(58) = 116$.

Time = 1.29 (sec) , antiderivative size = 160, normalized size of antiderivative = 2.42

method	result
default	$\frac{2a\sqrt{a(1+\sec(dx+c))}\left(\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\operatorname{arctanh}\left(\frac{\sin(dx+c)}{(\cos(dx+c)+1)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}}\right)\cos(dx+c)+\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\operatorname{arctanh}\left(\frac{}{(\cos(dx+c}\right.\right.}{d(\cos(dx+c)+1)}$

input
```
int((a+a*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/d*a*(a*(1+sec(d*x+c)))^(1/2)*((-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh
(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)+
(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(sin(d*x+c)/(cos(d*x+c)+1)/(-cos
(d*x+c)/(cos(d*x+c)+1))^(1/2))+sin(d*x+c))/(cos(d*x+c)+1)
```

3.103.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 235, normalized size of antiderivative = 3.56

$$\int (a+a\sec(c+dx))^{3/2}\,dx = \left[\frac{(a\cos(dx+c)+a)\sqrt{-a}\log\left(\frac{2\,a\cos(dx+c)^2-2\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+a\cos(dx+c)-a}{\cos(dx+c)+1}\right)+2\left((a\cos(dx+c)+a)\sqrt{a}\arctan\left(\frac{\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{\sqrt{a}\sin(dx+c)}\right)-a\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)\right)}{d\cos(dx+c)+d}\right]$$

input
```
integrate((a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
[((a*cos(d*x + c) + a)*sqrt(-a)*log((2*a*cos(d*x + c)^2 - 2*sqrt(-a)*sqrt(
(a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) + a*cos(d*x +
 c) - a)/(cos(d*x + c) + 1)) + 2*a*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))
*sin(d*x + c))/(d*cos(d*x + c) + d), -2*((a*cos(d*x + c) + a)*sqrt(a)*arct
an(sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x +
 c))) - a*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x
 + c) + d)]
```

3.103.6 Sympy [F]

$$\int (a + a\sec(c + dx))^{3/2}\,dx = \int (a\sec(c + dx) + a)^{\frac{3}{2}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))**(3/2),x)
```

output
```
Integral((a*sec(c + d*x) + a)**(3/2), x)
```

3.103.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 997 vs. $2(58) = 116$.

Time = 0.42 (sec) , antiderivative size = 997, normalized size of antiderivative = 15.11

$$\int (a + a\sec(c + dx))^{3/2}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
1/2*((a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2
*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1
/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x +
 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))
*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c)))) + 1) - a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)
^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) -
cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*
c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c) + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
+ sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) - 1) - a*arctan2((cos(2*d*x + 2*c)^
2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin
(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2
*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c),...
```

3.103.8 Giac [F]

$$\int (a + a\sec(c + dx))^{3/2}\,dx = \int (a\sec(dx + c) + a)^{\frac{3}{2}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.103.9 Mupad [F(-1)]

Timed out.

$$\int (a + a\sec(c + dx))^{3/2}\,dx = \int \left(a + \frac{a}{\cos(c + d\,x)}\right)^{3/2}\,dx$$

input
```
int((a + a/cos(c + d*x))^(3/2),x)
```

output
```
int((a + a/cos(c + d*x))^(3/2), x)
```

3.104 $\int \cos(c+dx)(a+a\sec(c+dx))^{3/2}\,dx$

3.104.1 Optimal result . . . 972
3.104.2 Mathematica [A] (verified) . . . 972
3.104.3 Rubi [A] (verified) . . . 973
3.104.4 Maple [B] (verified) . . . 975
3.104.5 Fricas [A] (verification not implemented) . . . 975
3.104.6 Sympy [F] . . . 976
3.104.7 Maxima [B] (verification not implemented) . . . 976
3.104.8 Giac [F] . . . 977
3.104.9 Mupad [F(-1)] . . . 978

3.104.1 Optimal result

Integrand size = 21, antiderivative size = 65

$$\int \cos(c+dx)(a+a\sec(c+dx))^{3/2}\,dx=\frac{3a^{3/2}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}+\frac{a^2\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}}$$

output
```
3*a^(3/2)*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/d+a^2*sin(d*x+
c)/d/(a+a*sec(d*x+c))^(1/2)
```

3.104.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.26

$$\int \cos(c+dx)(a+a\sec(c+dx))^{3/2}\,dx=\frac{a\left(3\text{arctanh}\left(\sqrt{1-\sec(c+dx)}\right)+\cos(c+dx)\sqrt{1-\sec(c+dx)}\right)\sqrt{a(1+\sec(c+dx))}\tan\left(\right.}{d\sqrt{1-\sec(c+dx)}}$$

input
```
Integrate[Cos[c + d*x]*(a + a*Sec[c + d*x])^(3/2),x]
```

output
```
(a*(3*ArcTanh[Sqrt[1 - Sec[c + d*x]]] + Cos[c + d*x]*Sqrt[1 - Sec[c + d*x]
])*Sqrt[a*(1 + Sec[c + d*x])]*Tan[(c + d*x)/2])/(d*Sqrt[1 - Sec[c + d*x]])
```

3.104.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.45, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {3042, 4301, 27, 3042, 4292, 3042, 4261, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(c+dx)(a\sec(c+dx)+a)^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}}{\csc\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{4301}$$

$$-2a\int -\frac{3}{2}\cos(c+dx)\sqrt{\sec(c+dx)a+a}dx-\frac{2a^2\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{27}$$

$$3a\int \cos(c+dx)\sqrt{\sec(c+dx)a+a}dx-\frac{2a^2\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$3a\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}{\csc\left(c+dx+\frac{\pi}{2}\right)}dx-\frac{2a^2\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{4292}$$

$$3a\left(\frac{1}{2}\int \sqrt{\sec(c+dx)a+a}dx+\frac{a\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}\right)-\frac{2a^2\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$3a\left(\frac{1}{2}\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx+\frac{a\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}\right)-\frac{2a^2\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{4261}$$

$$3a\left(\frac{a\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}-\frac{a\int \frac{1}{\frac{a^2\tan^2(c+dx)}{\sec(c+dx)a+a}+a}d\left(-\frac{a\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}\right)-\frac{2a^2\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{216}$$

$$3a\left(\frac{\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{a\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}\right)-\frac{2a^2\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}$$

input
```
Int[Cos[c + d*x]*(a + a*Sec[c + d*x])^(3/2),x]
```

output
```
(-2*a^2*Sin[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]]) + 3*a*((Sqrt[a]*ArcTan[
(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d + (a*Sin[c + d*x])/(d*
Sqrt[a + a*Sec[c + d*x]]))
```

3.104.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4261
```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(b/d)
  Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 4292
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[a*Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*n*Sqrt[a
+ b*Csc[e + f*x]])), x] + Simp[a*((2*n + 1)/(2*b*d*n))   Int[Sqrt[a + b*Csc
[e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
 EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

rule 4301
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
2)*((d*Csc[e + f*x])^n/(f*(m + n - 1))), x] + Simp[b/(m + n - 1)   Int[(a +
 b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*(b*(m + 2*n - 1) + a*(3*m + 2*n
 - 4)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^
2, 0] && GtQ[m, 1] && NeQ[m + n - 1, 0] && IntegerQ[2*m]
```

3.104.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 167 vs. $2(57) = 114$.

Time = 23.60 (sec) , antiderivative size = 168, normalized size of antiderivative = 2.58

method	result
default	$\frac{a\left(3\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\operatorname{arctanh}\left(\frac{\sin(dx+c)}{(\cos(dx+c)+1)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}}\right)\cos(dx+c)+3\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\operatorname{arctanh}\left(\frac{\sin(dx+c)}{(\cos(dx+c)+1)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+}}}\right.\right.}{d(\cos(dx+c)+1)}$

input
```
int(cos(d*x+c)*(a+a*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/d*a*(3*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(sin(d*x+c)/(cos(d*x+c)
+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)+3*(-cos(d*x+c)/(cos(d*x
+c)+1))^(1/2)*arctanh(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1
))^(1/2))+cos(d*x+c)*sin(d*x+c))*(a*(1+sec(d*x+c)))^(1/2)/(cos(d*x+c)+1)
```

3.104.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 248, normalized size of antiderivative = 3.82

$$\int \cos(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \left[\frac{2\,a\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+3\,(a\cos(dx+c)+a)\sqrt{-a}\log\left(\frac{2\,a\cos(dx+c)^2-2\sqrt{-}}{}\right.}{2\,(d\cos(dx+c)+d)}\right.$$

input
```
integrate(cos(d*x+c)*(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
[1/2*(2*a*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c
) + 3*(a*cos(d*x + c) + a)*sqrt(-a)*log((2*a*cos(d*x + c)^2 - 2*sqrt(-a)*s
qrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) + a*cos(d
*x + c) - a)/(cos(d*x + c) + 1)))/(d*cos(d*x + c) + d), (a*sqrt((a*cos(d*x
 + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) - 3*(a*cos(d*x + c) + a
)*sqrt(a)*arctan(sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqr
t(a)*sin(d*x + c))))/(d*cos(d*x + c) + d)]
```

3.104.6 Sympy [F]

$$\int \cos(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \int (a(\sec(c+dx)+1))^{\frac{3}{2}}\cos(c+dx)\,dx$$

input
```
integrate(cos(d*x+c)*(a+a*sec(d*x+c))**(3/2),x)
```

output
```
Integral((a*(sec(c + d*x) + 1))**(3/2)*cos(c + d*x), x)
```

3.104.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 803 vs. $2(57) = 114$.

Time = 0.44 (sec) , antiderivative size = 803, normalized size of antiderivative = 12.35

$$\int \cos(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)*(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
1/4*(2*(a*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(d*x
 + c) - (a*cos(d*x + c) - a)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c) + 1)))*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)
 + 1)^(1/4)*sqrt(a) + 3*(a*arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)
^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 +
 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c) + 1))) + 1) - a*arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d
*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x +
 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c) + 1))) - 1) - a*arctan2((cos(2*d*x + 2*c)^2 +
 sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)
^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c) + 1)) + 1) + a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*...
```

3.104.8 Giac [F]

$$\int \cos(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \int (a\sec(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)\,dx$$

input
```
integrate(cos(d*x+c)*(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.104.9 Mupad [F(-1)]

Timed out.

$$\int \cos(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \int \cos\left(c+d\,x\right)\,\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)^{3/2}\,dx$$

input
```
int(cos(c + d*x)*(a + a/cos(c + d*x))^(3/2),x)
```

output
```
int(cos(c + d*x)*(a + a/cos(c + d*x))^(3/2), x)
```

3.105 $\int \cos^2(c+dx)(a+a\sec(c+dx))^{3/2}\,dx$

3.105.1 Optimal result 979
3.105.2 Mathematica [A] (verified) 979
3.105.3 Rubi [A] (verified) 980
3.105.4 Maple [B] (verified) 982
3.105.5 Fricas [A] (verification not implemented) 983
3.105.6 Sympy [F(-1)] 983
3.105.7 Maxima [F(-1)] 984
3.105.8 Giac [F] 984
3.105.9 Mupad [F(-1)] 984

3.105.1 Optimal result

Integrand size = 23, antiderivative size = 106

$$\int \cos^2(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \frac{7a^{3/2}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{4d} + \frac{7a^2\sin(c+dx)}{4d\sqrt{a+a\sec(c+dx)}} + \frac{a^2\cos(c+dx)\sin(c+dx)}{2d\sqrt{a+a\sec(c+dx)}}$$

output
```
7/4*a^(3/2)*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/d+7/4*a^2*si
n(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+1/2*a^2*cos(d*x+c)*sin(d*x+c)/d/(a+a*sec
(d*x+c))^(1/2)
```

3.105.2 Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.02

$$\int \cos^2(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \frac{a\cos(c+dx)\sqrt{a(1+\sec(c+dx))}\left(\sqrt{1-\sec(c+dx)}(7\sin(c+dx)+\sin(2(c+dx)))+7\text{arct}\right.}{4d(1+\cos(c+dx))\sqrt{1-\sec(c+dx)}}$$

input
```
Integrate[Cos[c + d*x]^2*(a + a*Sec[c + d*x])^(3/2),x]
```

output
```
(a*Cos[c + d*x]*Sqrt[a*(1 + Sec[c + d*x])]*(Sqrt[1 - Sec[c + d*x]]*(7*Sin[
c + d*x] + Sin[2*(c + d*x)]) + 7*ArcTanh[Sqrt[1 - Sec[c + d*x]]]*Tan[c + d
*x]))/(4*d*(1 + Cos[c + d*x])*Sqrt[1 - Sec[c + d*x]])
```

3.105.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.98, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {3042, 4300, 27, 3042, 4292, 3042, 4261, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(c+dx)(a\sec(c+dx)+a)^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}}{\csc\left(c+dx+\frac{\pi}{2}\right)^2}dx$$

$$\Big\downarrow \text{4300}$$

$$\frac{1}{2}a\int \frac{7}{2}\cos(c+dx)\sqrt{\sec(c+dx)a+a}dx+\frac{a^2\sin(c+dx)\cos(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{27}$$

$$\frac{7}{4}a\int \cos(c+dx)\sqrt{\sec(c+dx)a+a}dx+\frac{a^2\sin(c+dx)\cos(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7}{4}a\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{a^2\sin(c+dx)\cos(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{4292}$$

$$\frac{7}{4}a\left(\frac{1}{2}\int \sqrt{\sec(c+dx)a+a}dx+\frac{a\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}\right)+\frac{a^2\sin(c+dx)\cos(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7}{4}a\left(\frac{1}{2}\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx+\frac{a\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}\right)+\frac{a^2\sin(c+dx)\cos(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{4261}$$

$$\frac{7}{4}a\left(\frac{a\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}-\frac{a\int\frac{1}{\frac{a^2\tan^2(c+dx)}{\sec(c+dx)a+a}+a}d\left(-\frac{a\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}\right)+\frac{a^2\sin(c+dx)\cos(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{216}$$

$$\frac{a^2\sin(c+dx)\cos(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}+\frac{7}{4}a\left(\frac{\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{a\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}\right)$$

input
```
Int[Cos[c + d*x]^2*(a + a*Sec[c + d*x])^(3/2),x]
```

output
```
(a^2*Cos[c + d*x]*Sin[c + d*x])/(2*d*Sqrt[a + a*Sec[c + d*x]]) + (7*a*((Sq
rt[a]*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d + (a*Sin[
c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]])))/4
```

3.105.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4261
```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(b/d)
  Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 4292
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[a*Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*n*Sqrt[a
+ b*Csc[e + f*x]])), x] + Simp[a*((2*n + 1)/(2*b*d*n))   Int[Sqrt[a + b*Csc
[e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
 EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

rule 4300
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)*
((d*Csc[e + f*x])^n/(f*n)), x] - Simp[a/(d*n)   Int[(a + b*Csc[e + f*x])^(m
 - 2)*(d*Csc[e + f*x])^(n + 1)*(b*(m - 2*n - 2) - a*(m + 2*n - 1)*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1]
 && (LtQ[n, -1] || (EqQ[m, 3/2] && EqQ[n, -2^(-1)])) && IntegerQ[2*m]
```

3.105.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 185 vs. $2(90) = 180$.

Time = 23.84 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.75

method	result
default	$\frac{a\left(7\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\operatorname{arctanh}\left(\frac{\sin(dx+c)}{(\cos(dx+c)+1)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}}\right)\cos(dx+c)+2\cos(dx+c)^2\sin(dx+c)+7\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\operatorname{arctanh}\left(-\right.\right.}{4d(\cos(dx+c)+1)}$

input
```
int(cos(d*x+c)^2*(a+a*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/4/d*a*(7*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(sin(d*x+c)/(cos(d*x+
c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)+2*cos(d*x+c)^2*sin(d*
x+c)+7*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(sin(d*x+c)/(cos(d*x+c)+1
)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+7*cos(d*x+c)*sin(d*x+c))*(a*(1+sec(d
*x+c)))^(1/2)/(cos(d*x+c)+1)
```

3.105.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 278, normalized size of antiderivative = 2.62

$$\int \cos^2(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \left[\frac{7\,(a\cos(dx+c)+a)\sqrt{-a}\log\left(\frac{2\,a\cos(dx+c)^2-2\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+a\cos(dx+c)-a}{\cos(dx+c)+1}\right)}{8\,(d\cos(dx+c)+d)} - \frac{7\,(a\cos(dx+c)+a)\sqrt{a}\arctan\left(\frac{\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{\sqrt{a}\sin(dx+c)}\right)-\left(2\,a\cos(dx+c)^2+7\,a\cos(dx+c)\right)\sqrt{\frac{a\cos(dx+c}{\cos(dx+c}}}{4\,(d\cos(dx+c)+d)}\right.$$

input
```
integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
[1/8*(7*(a*cos(d*x + c) + a)*sqrt(-a)*log((2*a*cos(d*x + c)^2 - 2*sqrt(-a)
*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) + a*cos
(d*x + c) - a)/(cos(d*x + c) + 1)) + 2*(2*a*cos(d*x + c)^2 + 7*a*cos(d*x +
 c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)
 + d), -1/4*(7*(a*cos(d*x + c) + a)*sqrt(a)*arctan(sqrt((a*cos(d*x + c) +
a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) - (2*a*cos(d*x + c)^
2 + 7*a*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)
)/(d*cos(d*x + c) + d)]
```

3.105.6 Sympy [F(-1)]

Timed out.

$$\int \cos^2(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**2*(a+a*sec(d*x+c))**(3/2),x)
```

output
```
Timed out
```

3.105.7 Maxima [F(-1)]

Timed out.

$$\int \cos^2(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
Timed out
```

3.105.8 Giac [F]

$$\int \cos^2(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \int (a\sec(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^2\,dx$$

input
```
integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.105.9 Mupad [F(-1)]

Timed out.

$$\int \cos^2(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \int \cos(c+d\,x)^2\left(a+\frac{a}{\cos(c+d\,x)}\right)^{3/2}dx$$

input
```
int(cos(c + d*x)^2*(a + a/cos(c + d*x))^(3/2),x)
```

output
```
int(cos(c + d*x)^2*(a + a/cos(c + d*x))^(3/2), x)
```

3.106 $\int \cos^3(c+dx)(a+a\sec(c+dx))^{3/2}\,dx$

3.106.1 Optimal result 985
3.106.2 Mathematica [A] (verified) 985
3.106.3 Rubi [A] (verified) 986
3.106.4 Maple [A] (verified) 989
3.106.5 Fricas [A] (verification not implemented) 989
3.106.6 Sympy [F(-1)] 990
3.106.7 Maxima [F(-1)] 990
3.106.8 Giac [F] 991
3.106.9 Mupad [F(-1)] 991

3.106.1 Optimal result

Integrand size = 23, antiderivative size = 144

$$\int \cos^3(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \frac{11a^{3/2}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{8d} + \frac{11a^2\sin(c+dx)}{8d\sqrt{a+a\sec(c+dx)}} + \frac{11a^2\cos(c+dx)\sin(c+dx)}{12d\sqrt{a+a\sec(c+dx)}} + \frac{a^2\cos^2(c+dx)\sin(c+dx)}{3d\sqrt{a+a\sec(c+dx)}}$$

output
```
11/8*a^(3/2)*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/d+11/8*a^2*
sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+11/12*a^2*cos(d*x+c)*sin(d*x+c)/d/(a+a
*sec(d*x+c))^(1/2)+1/3*a^2*cos(d*x+c)^2*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2
)
```

3.106.2 Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.83

$$\int \cos^3(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \frac{a\cos(c+dx)\sqrt{a(1+\sec(c+dx))}\left(\sqrt{1-\sec(c+dx)}(35\sin(c+dx)+11\sin(2(c+dx))+2\,\text{s}\right.}{24d(1+\cos(c+dx))\sqrt{1-\sec(c+}}$$

input
```
Integrate[Cos[c + d*x]^3*(a + a*Sec[c + d*x])^(3/2),x]
```

output
```
(a*Cos[c + d*x]*Sqrt[a*(1 + Sec[c + d*x])]*(Sqrt[1 - Sec[c + d*x]]*(35*Sin
[c + d*x] + 11*Sin[2*(c + d*x)] + 2*Sin[3*(c + d*x)]) + 33*ArcTanh[Sqrt[1
- Sec[c + d*x]]]*Tan[c + d*x]))/(24*d*(1 + Cos[c + d*x])*Sqrt[1 - Sec[c +
d*x]])
```

3.106.3 Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.01, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {3042, 4300, 27, 3042, 4292, 3042, 4292, 3042, 4261, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^3(c+dx)(a\sec(c+dx)+a)^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}}{\csc\left(c+dx+\frac{\pi}{2}\right)^3}dx$$

$$\Big\downarrow \text{4300}$$

$$\frac{1}{3}a\int \frac{11}{2}\cos^2(c+dx)\sqrt{\sec(c+dx)a+a}dx+\frac{a^2\sin(c+dx)\cos^2(c+dx)}{3d\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{27}$$

$$\frac{11}{6}a\int \cos^2(c+dx)\sqrt{\sec(c+dx)a+a}dx+\frac{a^2\sin(c+dx)\cos^2(c+dx)}{3d\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{11}{6}a\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}{\csc\left(c+dx+\frac{\pi}{2}\right)^2}dx+\frac{a^2\sin(c+dx)\cos^2(c+dx)}{3d\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{4292}$$

$$\frac{11}{6}a\left(\frac{3}{4}\int \cos(c+dx)\sqrt{\sec(c+dx)a+a}dx+\frac{a\sin(c+dx)\cos(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}\right)+\frac{a^2\sin(c+dx)\cos^2(c+dx)}{3d\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{11}{6}a\left(\frac{3}{4}\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{a\sin(c+dx)\cos(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}\right)+\frac{a^2\sin(c+dx)\cos^2(c+dx)}{3d\sqrt{a\sec(c+dx)+a}}$$

$\Big\downarrow$ 4292

$$\frac{11}{6}a\left(\frac{3}{4}\left(\frac{1}{2}\int\sqrt{\sec(c+dx)a+a}dx+\frac{a\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}\right)+\frac{a\sin(c+dx)\cos(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}\right)+\frac{a^2\sin(c+dx)\cos^2(c+dx)}{3d\sqrt{a\sec(c+dx)+a}}$$

$\Big\downarrow$ 3042

$$\frac{11}{6}a\left(\frac{3}{4}\left(\frac{1}{2}\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx+\frac{a\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}\right)+\frac{a\sin(c+dx)\cos(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}\right)+\frac{a^2\sin(c+dx)\cos^2(c+dx)}{3d\sqrt{a\sec(c+dx)+a}}$$

$\Big\downarrow$ 4261

$$\frac{11}{6}a\left(\frac{3}{4}\left(\frac{a\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}-\frac{a\int\frac{1}{\frac{a^2\tan^2(c+dx)}{\sec(c+dx)a+a}+a}d\left(-\frac{a\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}\right)+\frac{a\sin(c+dx)\cos(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}\right)+\frac{a^2\sin(c+dx)\cos^2(c+dx)}{3d\sqrt{a\sec(c+dx)+a}}$$

$\Big\downarrow$ 216

$$\frac{a^2\sin(c+dx)\cos^2(c+dx)}{3d\sqrt{a\sec(c+dx)+a}}+\frac{11}{6}a\left(\frac{3}{4}\left(\frac{\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{a\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}\right)+\frac{a\sin(c+dx)\cos(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}\right)$$

input
```
Int[Cos[c + d*x]^3*(a + a*Sec[c + d*x])^(3/2),x]
```

output
```
(a^2*Cos[c + d*x]^2*Sin[c + d*x])/(3*d*Sqrt[a + a*Sec[c + d*x]]) + (11*a*(
(a*Cos[c + d*x]*Sin[c + d*x])/(2*d*Sqrt[a + a*Sec[c + d*x]]) + (3*((Sqrt[a
]*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d + (a*Sin[c +
d*x])/(d*Sqrt[a + a*Sec[c + d*x]])))/4))/6
```

3.106. $\int\cos^3(c+dx)(a+a\sec(c+dx))^{3/2}\,dx$

3.106.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4261
```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(b/d)
  Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 4292
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[a*Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*n*Sqrt[a
+ b*Csc[e + f*x]])), x] + Simp[a*((2*n + 1)/(2*b*d*n))   Int[Sqrt[a + b*Csc
[e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
 EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

rule 4300
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)*
((d*Csc[e + f*x])^n/(f*n)), x] - Simp[a/(d*n)   Int[(a + b*Csc[e + f*x])^(m
 - 2)*(d*Csc[e + f*x])^(n + 1)*(b*(m - 2*n - 2) - a*(m + 2*n - 1)*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1]
 && (LtQ[n, -1] || (EqQ[m, 3/2] && EqQ[n, -2^(-1)])) && IntegerQ[2*m]
```

3.106. $\int \cos^3(c+dx)(a+a\sec(c+dx))^{3/2}\,dx$

3.106.4 Maple [A] (verified)

Time = 26.06 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.40

method	result
default	$\frac{a\left(8\cos(dx+c)^3\sin(dx+c)+33\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\operatorname{arctanh}\left(\frac{\sin(dx+c)}{(\cos(dx+c)+1)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}}\right)\cos(dx+c)+22\cos(dx+c)^2\sin(dx+c)+33\right.}{24d(\cos(dx+c)+1)}$

input
```
int(cos(d*x+c)^3*(a+a*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/24/d*a*(8*cos(d*x+c)^3*sin(d*x+c)+33*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*
arctanh(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(
d*x+c)+22*cos(d*x+c)^2*sin(d*x+c)+33*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*ar
ctanh(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+33*cos
(d*x+c)*sin(d*x+c))*(a*(1+sec(d*x+c)))^(1/2)/(cos(d*x+c)+1)
```

3.106.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 300, normalized size of antiderivative = 2.08

$$\int \cos^3(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \left[\frac{33\,(a\cos(dx+c)+a)\sqrt{-a}\log\left(\frac{2\,a\cos(dx+c)^2-2\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+a\cos(dx+c)-a}{\cos(dx+c)+1}\right)}{48\,(d\cos(dx}\right.$$

$$-\frac{33\,(a\cos(dx+c)+a)\sqrt{a}\arctan\left(\frac{\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{\sqrt{a}\sin(dx+c)}\right)-(8\,a\cos(dx+c)^3+22\,a\cos(dx+c)^2+33\,a\,c}{24\,(d\cos(dx+c)+d)}$$

input
```
integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
[1/48*(33*(a*cos(d*x + c) + a)*sqrt(-a)*log((2*a*cos(d*x + c)^2 - 2*sqrt(-
a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) + a*c
os(d*x + c) - a)/(cos(d*x + c) + 1)) + 2*(8*a*cos(d*x + c)^3 + 22*a*cos(d*
x + c)^2 + 33*a*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(
d*x + c))/(d*cos(d*x + c) + d), -1/24*(33*(a*cos(d*x + c) + a)*sqrt(a)*arc
tan(sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x
+ c))) - (8*a*cos(d*x + c)^3 + 22*a*cos(d*x + c)^2 + 33*a*cos(d*x + c))*sq
rt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c) + d)]
```

3.106.6 Sympy [F(-1)]

Timed out.

$$\int \cos^3(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**3*(a+a*sec(d*x+c))**(3/2),x)
```

output
```
Timed out
```

3.106.7 Maxima [F(-1)]

Timed out.

$$\int \cos^3(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
Timed out
```

3.106.8 Giac [F]

$$\int \cos^3(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \int (a\sec(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^3\,dx$$

input
```
integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.106.9 Mupad [F(-1)]

Timed out.

$$\int \cos^3(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \int \cos(c+d\,x)^3\left(a+\frac{a}{\cos(c+d\,x)}\right)^{3/2}dx$$

input
```
int(cos(c + d*x)^3*(a + a/cos(c + d*x))^(3/2),x)
```

output
```
int(cos(c + d*x)^3*(a + a/cos(c + d*x))^(3/2), x)
```

3.107 $\int \sec^4(c+dx)(a+a\sec(c+dx))^{5/2}\,dx$

3.107.1 Optimal result . . . 992
3.107.2 Mathematica [A] (verified) . . . 993
3.107.3 Rubi [A] (verified) . . . 993
3.107.4 Maple [A] (verified) . . . 997
3.107.5 Fricas [A] (verification not implemented) . . . 998
3.107.6 Sympy [F(-1)] . . . 998
3.107.7 Maxima [F(-1)] . . . 998
3.107.8 Giac [F] . . . 999
3.107.9 Mupad [B] (verification not implemented) . . . 999

3.107.1 Optimal result

Integrand size = 23, antiderivative size = 203

$$\begin{aligned}\int \sec^4(c+dx)(a+a\sec(c+dx))^{5/2}\,dx &= \frac{284a^3\tan(c+dx)}{99d\sqrt{a+a\sec(c+dx)}}\\ &+\frac{710a^3\sec^3(c+dx)\tan(c+dx)}{693d\sqrt{a+a\sec(c+dx)}}+\frac{46a^3\sec^4(c+dx)\tan(c+dx)}{99d\sqrt{a+a\sec(c+dx)}}\\ &-\frac{568a^2\sqrt{a+a\sec(c+dx)}\tan(c+dx)}{693d}\\ &+\frac{2a^2\sec^4(c+dx)\sqrt{a+a\sec(c+dx)}\tan(c+dx)}{11d}\\ &+\frac{284a(a+a\sec(c+dx))^{3/2}\tan(c+dx)}{231d}\end{aligned}$$

output
```
284/231*a*(a+a*sec(d*x+c))^(3/2)*tan(d*x+c)/d+284/99*a^3*tan(d*x+c)/d/(a+a
*sec(d*x+c))^(1/2)+710/693*a^3*sec(d*x+c)^3*tan(d*x+c)/d/(a+a*sec(d*x+c))^
(1/2)+46/99*a^3*sec(d*x+c)^4*tan(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)-568/693*a
^2*(a+a*sec(d*x+c))^(1/2)*tan(d*x+c)/d+2/11*a^2*sec(d*x+c)^4*(a+a*sec(d*x+
c))^(1/2)*tan(d*x+c)/d
```

3.107.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.39

$$\int \sec^4(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \frac{2a^3(1136+568\sec(c+dx)+426\sec^2(c+dx)+355\sec^3(c+dx)+224\sec^4(c+dx)+63\sec}{693d\sqrt{a(1+\sec(c+dx))}}$$

input
```
Integrate[Sec[c + d*x]^4*(a + a*Sec[c + d*x])^(5/2),x]
```

output
```
(2*a^3*(1136 + 568*Sec[c + d*x] + 426*Sec[c + d*x]^2 + 355*Sec[c + d*x]^3
+ 224*Sec[c + d*x]^4 + 63*Sec[c + d*x]^5)*Tan[c + d*x])/(693*d*Sqrt[a*(1 +
 Sec[c + d*x])])
```

3.107.3 Rubi [A] (verified)

Time = 1.16 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.11, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.609$, Rules used = {3042, 4301, 27, 3042, 4504, 3042, 4290, 3042, 4287, 27, 3042, 4489, 3042, 4279}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^4(c+dx)(a\sec(c+dx)+a)^{5/2}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^4\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}dx$$

$$\Big\downarrow \; 4301$$

$$\frac{2}{11}a\int \frac{1}{2}\sec^4(c+dx)\sqrt{\sec(c+dx)a+a}(23\sec(c+dx)a+19a)dx+\frac{2a^2\tan(c+dx)\sec^4(c+dx)\sqrt{a\sec(c+dx)+a}}{11d}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{11}a\int \sec^4(c+dx)\sqrt{\sec(c+dx)a+a}(23\sec(c+dx)a+19a)dx+\frac{2a^2\tan(c+dx)\sec^4(c+dx)\sqrt{a\sec(c+dx)+a}}{11d}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{11}a\int \csc\left(c+dx+\frac{\pi}{2}\right)^4\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}\left(23\csc\left(c+dx+\frac{\pi}{2}\right)a+19a\right)dx+\frac{2a^2\tan(c+dx)\sec^4(c+dx)\sqrt{a\sec(c+dx)+a}}{11d}$$

$$\Big\downarrow \; 4504$$

$$\frac{1}{11}a\left(\frac{355}{9}a\int \sec^4(c+dx)\sqrt{\sec(c+dx)a+a}dx+\frac{46a^2\tan(c+dx)\sec^4(c+dx)}{9d\sqrt{a\sec(c+dx)+a}}\right)+\frac{2a^2\tan(c+dx)\sec^4(c+dx)\sqrt{a\sec(c+dx)+a}}{11d}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{11}a\left(\frac{355}{9}a\int \csc\left(c+dx+\frac{\pi}{2}\right)^4\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx+\frac{46a^2\tan(c+dx)\sec^4(c+dx)}{9d\sqrt{a\sec(c+dx)+a}}\right)+\frac{2a^2\tan(c+dx)\sec^4(c+dx)\sqrt{a\sec(c+dx)+a}}{11d}$$

$$\Big\downarrow \; 4290$$

$$\frac{1}{11}a\left(\frac{355}{9}a\left(\frac{6}{7}\int \sec^3(c+dx)\sqrt{\sec(c+dx)a+a}dx+\frac{2a\tan(c+dx)\sec^3(c+dx)}{7d\sqrt{a\sec(c+dx)+a}}\right)+\frac{46a^2\tan(c+dx)\sec^4(c+\right.$$
$$\frac{2a^2\tan(c+dx)\sec^4(c+dx)\sqrt{a\sec(c+dx)+a}}{11d}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{11}a\left(\frac{355}{9}a\left(\frac{6}{7}\int \csc\left(c+dx+\frac{\pi}{2}\right)^3\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx+\frac{2a\tan(c+dx)\sec^3(c+dx)}{7d\sqrt{a\sec(c+dx)+a}}\right)+\frac{46a^2\tan(c+\right.$$
$$\frac{2a^2\tan(c+dx)\sec^4(c+dx)\sqrt{a\sec(c+dx)+a}}{11d}$$

$$\Big\downarrow \; 4287$$

$$\frac{1}{11}a\left(\frac{355}{9}a\left(\frac{6}{7}\left(\frac{2\int \frac{1}{2}\sec(c+dx)(3a-2a\sec(c+dx))\sqrt{\sec(c+dx)a+a}dx}{5a}+\frac{2\tan(c+dx)(a\sec(c+dx)+a)^{3/}}{5ad}\right.\right.\right.$$
$$\frac{2a^2\tan(c+dx)\sec^4(c+dx)\sqrt{a\sec(c+dx)+a}}{11d}$$

$$\Big\downarrow \; 27$$

3.107. $\int \sec^4(c+dx)(a+a\sec(c+dx))^{5/2}\,dx$

$$\frac{1}{11}a\left(\frac{355}{9}a\left(\frac{6}{7}\left(\frac{\int \sec(c+dx)(3a-2a\sec(c+dx))\sqrt{\sec(c+dx)a+a}dx}{5a}+\frac{2\tan(c+dx)(a\sec(c+dx)+a)^{3/2}}{5ad}\right)\right.\right.$$
$$\frac{2a^2\tan(c+dx)\sec^4(c+dx)\sqrt{a\sec(c+dx)+a}}{11d}$$

$\Big\downarrow$ 3042

$$\frac{1}{11}a\left(\frac{355}{9}a\left(\frac{6}{7}\left(\frac{\int \csc\left(c+dx+\frac{\pi}{2}\right)\left(3a-2a\csc\left(c+dx+\frac{\pi}{2}\right)\right)\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{5a}+\frac{2\tan(c+dx)(a\sec(}{5ad}\right.\right.\right.$$
$$\frac{2a^2\tan(c+dx)\sec^4(c+dx)\sqrt{a\sec(c+dx)+a}}{11d}$$

$\Big\downarrow$ 4489

$$\frac{1}{11}a\left(\frac{355}{9}a\left(\frac{6}{7}\left(\frac{\frac{7}{3}a\int \sec(c+dx)\sqrt{\sec(c+dx)a+a}dx-\frac{4a\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}}{5a}+\frac{2\tan(c+dx)(a\sec(c+dx)}{5ad}\right.\right.\right.$$
$$\frac{2a^2\tan(c+dx)\sec^4(c+dx)\sqrt{a\sec(c+dx)+a}}{11d}$$

$\Big\downarrow$ 3042

$$\frac{1}{11}a\left(\frac{355}{9}a\left(\frac{6}{7}\left(\frac{\frac{7}{3}a\int \csc\left(c+dx+\frac{\pi}{2}\right)\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx-\frac{4a\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}}{5a}+\frac{2\tan(c+dx)(a}{5}\right.\right.\right.$$
$$\frac{2a^2\tan(c+dx)\sec^4(c+dx)\sqrt{a\sec(c+dx)+a}}{11d}$$

$\Big\downarrow$ 4279

$$\frac{2a^2\tan(c+dx)\sec^4(c+dx)\sqrt{a\sec(c+dx)+a}}{11d}+$$
$$\frac{1}{11}a\left(\frac{46a^2\tan(c+dx)\sec^4(c+dx)}{9d\sqrt{a\sec(c+dx)+a}}+\frac{355}{9}a\left(\frac{6}{7}\left(\frac{\frac{14a^2\tan(c+dx)}{3d\sqrt{a\sec(c+dx)+a}}-\frac{4a\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}}{5a}+\frac{2\tan(c+dx)(a}{5}\right.\right.\right.$$

input
```
Int[Sec[c + d*x]^4*(a + a*Sec[c + d*x])^(5/2),x]
```

3.107. $\int \sec^4(c+dx)(a+a\sec(c+dx))^{5/2}\,dx$

output
```
(2*a^2*Sec[c + d*x]^4*Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x])/(11*d) + (a*(
(46*a^2*Sec[c + d*x]^4*Tan[c + d*x])/(9*d*Sqrt[a + a*Sec[c + d*x]]) + (355
*a*((2*a*Sec[c + d*x]^3*Tan[c + d*x])/(7*d*Sqrt[a + a*Sec[c + d*x]]) + (6*
((2*(a + a*Sec[c + d*x])^(3/2)*Tan[c + d*x])/(5*a*d) + ((14*a^2*Tan[c + d*
x])/(3*d*Sqrt[a + a*Sec[c + d*x]]) - (4*a*Sqrt[a + a*Sec[c + d*x]]*Tan[c +
 d*x])/(3*d))/(5*a)))/7))/9))/11
```

3.107.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4279
```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*b*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]])), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4287
```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(-Cot[e + f*x])*((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2
))), x] + Simp[1/(b*(m + 2))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(b*(
m + 1) - a*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 -
b^2, 0] &&  !LtQ[m, -2^(-1)]
```

rule 4290
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[-2*b*d*Cot[e + f*x]*((d*Csc[e + f*x])^(n - 1)/(
f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]])), x] + Simp[2*a*d*((n - 1)/(b*(2*n -
1)))   Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e + f*x])^(n - 1), x], x] /; Fre
eQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

rule 4301
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
2)*((d*Csc[e + f*x])^n/(f*(m + n - 1))), x] + Simp[b/(m + n - 1)   Int[(a +
 b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*(b*(m + 2*n - 1) + a*(3*m + 2*n
 - 4)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^
2, 0] && GtQ[m, 1] && NeQ[m + n - 1, 0] && IntegerQ[2*m]
```

rule 4489
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*((
a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Simp[(a*B*m + A*b*(m + 1))/(b*(m +
 1))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B
, e, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b
*(m + 1), 0] &&  !LtQ[m, -2^(-1)]
```

rule 4504
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[-2*b*B*C
ot[e + f*x]*((d*Csc[e + f*x])^n/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]])), x]
 + Simp[(A*b*(2*n + 1) + 2*a*B*n)/(b*(2*n + 1))   Int[Sqrt[a + b*Csc[e + f*
x]]*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ
[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] &&
!LtQ[n, 0]
```

3.107.4 Maple [A] (verified)

Time = 145.82 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.47

method	result
default	$\frac{2a^2\left(1136\cos(dx+c)^5+568\cos(dx+c)^4+426\cos(dx+c)^3+355\cos(dx+c)^2+224\cos(dx+c)+63\right)\sqrt{a(1+\sec(dx+c))}\,\tan(dx+c)\sec(dx}{693d(\cos(dx+c)+1)}$

input
```
int(sec(d*x+c)^4*(a+a*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/693/d*a^2*(1136*cos(d*x+c)^5+568*cos(d*x+c)^4+426*cos(d*x+c)^3+355*cos(d
*x+c)^2+224*cos(d*x+c)+63)*(a*(1+sec(d*x+c)))^(1/2)/(cos(d*x+c)+1)*tan(d*x
+c)*sec(d*x+c)^4
```

3.107.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.60

$$\int \sec^4(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \frac{2\left(1136\,a^2\cos\left(dx+c\right)^5+568\,a^2\cos\left(dx+c\right)^4+426\,a^2\cos\left(dx+c\right)^3+355\,a^2\cos\left(dx+c\right)^2+\right.}{693\left(d\cos\left(dx+c\right)^6+d\cos\left(dx+c\right)^5\right)}$$

input
```
integrate(sec(d*x+c)^4*(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
2/693*(1136*a^2*cos(d*x + c)^5 + 568*a^2*cos(d*x + c)^4 + 426*a^2*cos(d*x
+ c)^3 + 355*a^2*cos(d*x + c)^2 + 224*a^2*cos(d*x + c) + 63*a^2)*sqrt((a*c
os(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c)^6 + d*cos(d*x
+ c)^5)
```

3.107.6 Sympy [F(-1)]

Timed out.

$$\int \sec^4(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**4*(a+a*sec(d*x+c))**(5/2),x)
```

output
```
Timed out
```

3.107.7 Maxima [F(-1)]

Timed out.

$$\int \sec^4(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)^4*(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
Timed out
```

3.107.8 Giac [F]

$$\int \sec^4(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \int (a\sec(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^4\,dx$$

input
```
integrate(sec(d*x+c)^4*(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.107.9 Mupad [B] (verification not implemented)

Time = 24.76 (sec) , antiderivative size = 542, normalized size of antiderivative = 2.67

$$\int \sec^4(c+dx)(a + a\sec(c+dx))^{5/2}\,dx = \frac{\sqrt{a+\frac{a}{\frac{e^{-c\,1i-d\,x\,1i}}{2}+\frac{e^{c\,1i+d\,x\,1i}}{2}}}\left(\frac{a^2\,64i}{11\,d}+\frac{a^2\,e^{c\,1i+d\,x\,1i}\,64i}{11\,d}\right)}{(e^{c\,1i+d\,x\,1i}+1)\,(e^{c\,2i+d\,x\,2i}+1)^5} + \frac{\sqrt{a+\frac{a}{\frac{e^{-c\,1i-d\,x\,1i}}{2}+\frac{e^{c\,1i+d\,x\,1i}}{2}}}\left(\frac{a^2\,16i}{d}+\frac{a^2\,e^{c\,1i+d\,x\,1i}\,640i}{231\,d}\right)}{(e^{c\,1i+d\,x\,1i}+1)\,(e^{c\,2i+d\,x\,2i}+1)^2} - \frac{\sqrt{a+\frac{a}{\frac{e^{-c\,1i-d\,x\,1i}}{2}+\frac{e^{c\,1i+d\,x\,1i}}{2}}}\left(\frac{a^2\,64i}{9\,d}+\frac{a^2\,e^{c\,1i+d\,x\,1i}\,2176i}{99\,d}\right)}{(e^{c\,1i+d\,x\,1i}+1)\,(e^{c\,2i+d\,x\,2i}+1)^4} - \frac{\sqrt{a+\frac{a}{\frac{e^{-c\,1i-d\,x\,1i}}{2}+\frac{e^{c\,1i+d\,x\,1i}}{2}}}\left(\frac{a^2\,80i}{7\,d}-\frac{a^2\,e^{c\,1i+d\,x\,1i}\,12688i}{693\,d}\right)}{(e^{c\,1i+d\,x\,1i}+1)\,(e^{c\,2i+d\,x\,2i}+1)^3} - \frac{a^2\,e^{c\,1i+d\,x\,1i}\,\sqrt{a+\frac{a}{\frac{e^{-c\,1i-d\,x\,1i}}{2}+\frac{e^{c\,1i+d\,x\,1i}}{2}}}\,2272i}{693\,d\,(e^{c\,1i+d\,x\,1i}+1)} - \frac{a^2\,e^{c\,1i+d\,x\,1i}\,\sqrt{a+\frac{a}{\frac{e^{-c\,1i-d\,x\,1i}}{2}+\frac{e^{c\,1i+d\,x\,1i}}{2}}}\,1136i}{693\,d\,(e^{c\,1i+d\,x\,1i}+1)\,(e^{c\,2i+d\,x\,2i}+1)}$$

input
```
int((a + a/cos(c + d*x))^(5/2)/cos(c + d*x)^4,x)
```

output
```
((a + a/(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*((a^2*64i)/
(11*d) + (a^2*exp(c*1i + d*x*1i)*64i)/(11*d)))/((exp(c*1i + d*x*1i) + 1)*(
exp(c*2i + d*x*2i) + 1)^5) + ((a + a/(exp(- c*1i - d*x*1i)/2 + exp(c*1i +
d*x*1i)/2))^(1/2)*((a^2*16i)/d + (a^2*exp(c*1i + d*x*1i)*640i)/(231*d)))/(
(exp(c*1i + d*x*1i) + 1)*(exp(c*2i + d*x*2i) + 1)^2) - ((a + a/(exp(- c*1i
 - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*((a^2*64i)/(9*d) + (a^2*exp(c*
1i + d*x*1i)*2176i)/(99*d)))/((exp(c*1i + d*x*1i) + 1)*(exp(c*2i + d*x*2i)
 + 1)^4) - ((a + a/(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*
((a^2*80i)/(7*d) - (a^2*exp(c*1i + d*x*1i)*12688i)/(693*d)))/((exp(c*1i +
d*x*1i) + 1)*(exp(c*2i + d*x*2i) + 1)^3) - (a^2*exp(c*1i + d*x*1i)*(a + a/
(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*2272i)/(693*d*(exp(
c*1i + d*x*1i) + 1)) - (a^2*exp(c*1i + d*x*1i)*(a + a/(exp(- c*1i - d*x*1i
)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*1136i)/(693*d*(exp(c*1i + d*x*1i) + 1)*
(exp(c*2i + d*x*2i) + 1))
```

3.107. $\int \sec^4(c+dx)(a+a\sec(c+dx))^{5/2}\,dx$

3.108 $\int \sec^3(c+dx)(a+a\sec(c+dx))^{5/2}\,dx$

3.108.1 Optimal result 1001
3.108.2 Mathematica [A] (verified) 1001
3.108.3 Rubi [A] (verified) 1002
3.108.4 Maple [A] (verified) 1005
3.108.5 Fricas [A] (verification not implemented) 1005
3.108.6 Sympy [F] 1005
3.108.7 Maxima [F] 1006
3.108.8 Giac [F] 1006
3.108.9 Mupad [B] (verification not implemented) 1007

3.108.1 Optimal result

Integrand size = 23, antiderivative size = 146

$$\int \sec^3(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \frac{832a^3\tan(c+dx)}{315d\sqrt{a+a\sec(c+dx)}} + \frac{208a^2\sqrt{a+a\sec(c+dx)}\tan(c+dx)}{315d} + \frac{26a(a+a\sec(c+dx))^{3/2}\tan(c+dx)}{105d} - \frac{4(a+a\sec(c+dx))^{5/2}\tan(c+dx)}{63d} + \frac{2(a+a\sec(c+dx))^{7/2}\tan(c+dx)}{9ad}$$

output
```
26/105*a*(a+a*sec(d*x+c))^(3/2)*tan(d*x+c)/d-4/63*(a+a*sec(d*x+c))^(5/2)*t
an(d*x+c)/d+2/9*(a+a*sec(d*x+c))^(7/2)*tan(d*x+c)/a/d+832/315*a^3*tan(d*x+
c)/d/(a+a*sec(d*x+c))^(1/2)+208/315*a^2*(a+a*sec(d*x+c))^(1/2)*tan(d*x+c)/
d
```

3.108.2 Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.48

$$\int \sec^3(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \frac{2a^3(584+292\sec(c+dx)+219\sec^2(c+dx)+130\sec^3(c+dx)+35\sec^4(c+dx))\tan(c+d}{315d\sqrt{a(1+\sec(c+dx))}}$$

input
```
Integrate[Sec[c + d*x]^3*(a + a*Sec[c + d*x])^(5/2),x]
```

output
```
(2*a^3*(584 + 292*Sec[c + d*x] + 219*Sec[c + d*x]^2 + 130*Sec[c + d*x]^3 +
 35*Sec[c + d*x]^4)*Tan[c + d*x])/(315*d*Sqrt[a*(1 + Sec[c + d*x])])
```

3.108.3 Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.13, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules used = {3042, 4287, 27, 3042, 4489, 3042, 4280, 3042, 4280, 3042, 4279}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^3(c+dx)(a\sec(c+dx)+a)^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^3\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}dx$$

$$\Big\downarrow \text{4287}$$

$$\frac{2\int \frac{1}{2}\sec(c+dx)(7a-2a\sec(c+dx))(\sec(c+dx)a+a)^{5/2}dx}{9a}+\frac{2\tan(c+dx)(a\sec(c+dx)+a)^{7/2}}{9ad}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \sec(c+dx)(7a-2a\sec(c+dx))(\sec(c+dx)a+a)^{5/2}dx}{9a}+\frac{2\tan(c+dx)(a\sec(c+dx)+a)^{7/2}}{9ad}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \csc\left(c+dx+\frac{\pi}{2}\right)\left(7a-2a\csc\left(c+dx+\frac{\pi}{2}\right)\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{5/2}dx}{9a}+\frac{2\tan(c+dx)(a\sec(c+dx)+a)^{7/2}}{9ad}$$

$$\Big\downarrow \text{4489}$$

$$\frac{\frac{39}{7}a\int \sec(c+dx)(\sec(c+dx)a+a)^{5/2}dx-\frac{4a\tan(c+dx)(a\sec(c+dx)+a)^{5/2}}{7d}}{9a}+\frac{2\tan(c+dx)(a\sec(c+dx)+a)^{7/2}}{9ad}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{39}{7}a\int \csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{5/2}dx-\frac{4a\tan(c+dx)(a\sec(c+dx)+a)^{5/2}}{7d}}{9a}+\frac{2\tan(c+dx)(a\sec(c+dx)+a)^{7/2}}{9ad}$$

$$\Big\downarrow \; 4280$$

$$\frac{\frac{39}{7}a\left(\frac{8}{5}a\int \sec(c+dx)(\sec(c+dx)a+a)^{3/2}dx+\frac{2a\tan(c+dx)(a\sec(c+dx)+a)^{3/2}}{5d}\right)-\frac{4a\tan(c+dx)(a\sec(c+dx)+a)^{5/2}}{7d}}{9a}+\frac{2\tan(c+dx)(a\sec(c+dx)+a)^{7/2}}{9ad}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{39}{7}a\left(\frac{8}{5}a\int \csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}dx+\frac{2a\tan(c+dx)(a\sec(c+dx)+a)^{3/2}}{5d}\right)-\frac{4a\tan(c+dx)(a\sec(c+dx)+a}{7d}}{9a}$$
$$\frac{2\tan(c+dx)(a\sec(c+dx)+a)^{7/2}}{9ad}$$

$$\Big\downarrow \; 4280$$

$$\frac{\frac{39}{7}a\left(\frac{8}{5}a\left(\frac{4}{3}a\int \sec(c+dx)\sqrt{\sec(c+dx)a+a}dx+\frac{2a\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}\right)+\frac{2a\tan(c+dx)(a\sec(c+dx)+a)^{3/2}}{5d}\right)-\frac{4a\tan}{}}{9a}$$
$$\frac{2\tan(c+dx)(a\sec(c+dx)+a)^{7/2}}{9ad}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{39}{7}a\left(\frac{8}{5}a\left(\frac{4}{3}a\int \csc\left(c+dx+\frac{\pi}{2}\right)\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx+\frac{2a\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}\right)+\frac{2a\tan(c+dx)(a\sec(c+dx)+a)}{5d}\right.}{9a}$$
$$\frac{2\tan(c+dx)(a\sec(c+dx)+a)^{7/2}}{9ad}$$

$$\Big\downarrow \; 4279$$

$$\frac{\frac{39}{7}a\left(\frac{8}{5}a\left(\frac{8a^2\tan(c+dx)}{3d\sqrt{a\sec(c+dx)+a}}+\frac{2a\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}\right)+\frac{2a\tan(c+dx)(a\sec(c+dx)+a)^{3/2}}{5d}\right)-\frac{4a\tan(c+dx)(a\sec(c+dx)+a)^{5/2}}{7d}}{9a}$$
$$\frac{2\tan(c+dx)(a\sec(c+dx)+a)^{7/2}}{9ad}$$

input
```
Int[Sec[c + d*x]^3*(a + a*Sec[c + d*x])^(5/2),x]
```

3.108. $\int \sec^3(c+dx)(a+a\sec(c+dx))^{5/2}\,dx$

output
```
(2*(a + a*Sec[c + d*x])^(7/2)*Tan[c + d*x])/(9*a*d) + ((-4*a*(a + a*Sec[c
+ d*x])^(5/2)*Tan[c + d*x])/(7*d) + (39*a*((2*a*(a + a*Sec[c + d*x])^(3/2)
*Tan[c + d*x])/(5*d) + (8*a*((8*a^2*Tan[c + d*x])/(3*d*Sqrt[a + a*Sec[c +
d*x]]) + (2*a*Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x])/(3*d)))/5))/7)/(9*a)
```

3.108.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4279
```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*b*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]])), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4280
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[(-b)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m - 1)/(f*m)), x]
+ Simp[a*((2*m - 1)/m)   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m - 1), x],
 x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] && Intege
rQ[2*m]
```

rule 4287
```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(-Cot[e + f*x])*((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2
))), x] + Simp[1/(b*(m + 2))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(b*(
m + 1) - a*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 -
b^2, 0] &&  !LtQ[m, -2^(-1)]
```

rule 4489
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*((
a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Simp[(a*B*m + A*b*(m + 1))/(b*(m +
 1))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B
, e, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b
*(m + 1), 0] &&  !LtQ[m, -2^(-1)]
```

3.108.4 Maple [A] (verified)

Time = 47.74 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.58

method	result	size
default	$\frac{2a^2\left(584\cos(dx+c)^4+292\cos(dx+c)^3+219\cos(dx+c)^2+130\cos(dx+c)+35\right)\sqrt{a(1+\sec(dx+c))}\,\tan(dx+c)\sec(dx+c)^3}{315d(\cos(dx+c)+1)}$	85

input
```
int(sec(d*x+c)^3*(a+a*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/315/d*a^2*(584*cos(d*x+c)^4+292*cos(d*x+c)^3+219*cos(d*x+c)^2+130*cos(d*
x+c)+35)*(a*(1+sec(d*x+c)))^(1/2)/(cos(d*x+c)+1)*tan(d*x+c)*sec(d*x+c)^3
```

3.108.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.74

$$\int \sec^3(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \frac{2\left(584\,a^2\cos\left(dx+c\right)^4+292\,a^2\cos\left(dx+c\right)^3+219\,a^2\cos\left(dx+c\right)^2+130\,a^2\cos\left(dx+c\right)+35\right.}{315\left(d\cos\left(dx+c\right)^5+d\cos\left(dx+c\right)^4\right)}$$

input
```
integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
2/315*(584*a^2*cos(d*x + c)^4 + 292*a^2*cos(d*x + c)^3 + 219*a^2*cos(d*x +
 c)^2 + 130*a^2*cos(d*x + c) + 35*a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x +
 c))*sin(d*x + c)/(d*cos(d*x + c)^5 + d*cos(d*x + c)^4)
```

3.108.6 Sympy [F]

$$\int \sec^3(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \int (a(\sec(c+dx)+1))^{\frac{5}{2}}\sec^3(c+dx)\,dx$$

input
```
integrate(sec(d*x+c)**3*(a+a*sec(d*x+c))**(5/2),x)
```

output
```
Integral((a*(sec(c + d*x) + 1))**(5/2)*sec(c + d*x)**3, x)
```

3.108.7 Maxima [F]

$$\int \sec^3(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \int (a\sec(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^3\,dx$$

input
```
integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
8/315*(315*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) +
 1)^(1/4)*(5*(a^2*d*cos(2*d*x + 2*c)^4 + a^2*d*sin(2*d*x + 2*c)^4 + 4*a^2*
d*cos(2*d*x + 2*c)^3 + 6*a^2*d*cos(2*d*x + 2*c)^2 + 4*a^2*d*cos(2*d*x + 2*
c) + a^2*d + 2*(a^2*d*cos(2*d*x + 2*c)^2 + 2*a^2*d*cos(2*d*x + 2*c) + a^2*
d)*sin(2*d*x + 2*c)^2)*integrate((((cos(8*d*x + 8*c)*cos(2*d*x + 2*c) + 3*
cos(6*d*x + 6*c)*cos(2*d*x + 2*c) + 3*cos(4*d*x + 4*c)*cos(2*d*x + 2*c) +
cos(2*d*x + 2*c)^2 + sin(8*d*x + 8*c)*sin(2*d*x + 2*c) + 3*sin(6*d*x + 6*c
)*sin(2*d*x + 2*c) + 3*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + sin(2*d*x + 2*c
)^2)*cos(7/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + (cos(2*d*x + 2
*c)*sin(8*d*x + 8*c) + 3*cos(2*d*x + 2*c)*sin(6*d*x + 6*c) + 3*cos(2*d*x +
 2*c)*sin(4*d*x + 4*c) - cos(8*d*x + 8*c)*sin(2*d*x + 2*c) - 3*cos(6*d*x +
 6*c)*sin(2*d*x + 2*c) - 3*cos(4*d*x + 4*c)*sin(2*d*x + 2*c))*sin(7/2*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(5/2*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c) + 1)) - ((cos(2*d*x + 2*c)*sin(8*d*x + 8*c) + 3*cos(2*d
*x + 2*c)*sin(6*d*x + 6*c) + 3*cos(2*d*x + 2*c)*sin(4*d*x + 4*c) - cos(8*d
*x + 8*c)*sin(2*d*x + 2*c) - 3*cos(6*d*x + 6*c)*sin(2*d*x + 2*c) - 3*cos(4
*d*x + 4*c)*sin(2*d*x + 2*c))*cos(7/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c))) - (cos(8*d*x + 8*c)*cos(2*d*x + 2*c) + 3*cos(6*d*x + 6*c)*cos(2*d
*x + 2*c) + 3*cos(4*d*x + 4*c)*cos(2*d*x + 2*c) + cos(2*d*x + 2*c)^2 + sin
(8*d*x + 8*c)*sin(2*d*x + 2*c) + 3*sin(6*d*x + 6*c)*sin(2*d*x + 2*c) + ...
```

3.108.8 Giac [F]

$$\int \sec^3(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \int (a\sec(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^3\,dx$$

input
```
integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.108.9 Mupad [B] (verification not implemented)

Time = 21.98 (sec) , antiderivative size = 456, normalized size of antiderivative = 3.12

$$\int \sec^3(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \frac{\sqrt{a+\frac{a}{\frac{e^{-c\,1i-d\,x\,1i}}{2}+\frac{e^{c\,1i+d\,x\,1i}}{2}}}\left(\frac{a^2\,32i}{9\,d}-\frac{a^2\,e^{c\,1i+d\,x\,1i}\,32i}{9\,d}\right)}{(e^{c\,1i+d\,x\,1i}+1)\,(e^{c\,2i+d\,x\,2i}+1)^4} - \frac{\sqrt{a+\frac{a}{\frac{e^{-c\,1i-d\,x\,1i}}{2}+\frac{e^{c\,1i+d\,x\,1i}}{2}}}\left(\frac{a^2\,96i}{7\,d}-\frac{a^2\,e^{c\,1i+d\,x\,1i}\,32i}{63\,d}\right)}{(e^{c\,1i+d\,x\,1i}+1)\,(e^{c\,2i+d\,x\,2i}+1)^3} + \frac{\sqrt{a+\frac{a}{\frac{e^{-c\,1i-d\,x\,1i}}{2}+\frac{e^{c\,1i+d\,x\,1i}}{2}}}\left(\frac{a^2\,8i}{3\,d}-\frac{a^2\,e^{c\,1i+d\,x\,1i}\,584i}{315\,d}\right)}{(e^{c\,1i+d\,x\,1i}+1)\,(e^{c\,2i+d\,x\,2i}+1)} + \frac{\sqrt{a+\frac{a}{\frac{e^{-c\,1i-d\,x\,1i}}{2}+\frac{e^{c\,1i+d\,x\,1i}}{2}}}\left(\frac{a^2\,56i}{5\,d}+\frac{a^2\,e^{c\,1i+d\,x\,1i}\,904i}{105\,d}\right)}{(e^{c\,1i+d\,x\,1i}+1)\,(e^{c\,2i+d\,x\,2i}+1)^2} - \frac{a^2\,e^{c\,1i+d\,x\,1i}\,\sqrt{a+\frac{a}{\frac{e^{-c\,1i-d\,x\,1i}}{2}+\frac{e^{c\,1i+d\,x\,1i}}{2}}}\,1168i}{315\,d\,(e^{c\,1i+d\,x\,1i}+1)}$$

input
```
int((a + a/cos(c + d*x))^(5/2)/cos(c + d*x)^3,x)
```

output
```
((a + a/(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*((a^2*32i)/
(9*d) - (a^2*exp(c*1i + d*x*1i)*32i)/(9*d)))/((exp(c*1i + d*x*1i) + 1)*(ex
p(c*2i + d*x*2i) + 1)^4) - ((a + a/(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*
x*1i)/2))^(1/2)*((a^2*96i)/(7*d) - (a^2*exp(c*1i + d*x*1i)*32i)/(63*d)))/(
(exp(c*1i + d*x*1i) + 1)*(exp(c*2i + d*x*2i) + 1)^3) + ((a + a/(exp(- c*1i
 - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*((a^2*8i)/(3*d) - (a^2*exp(c*1
i + d*x*1i)*584i)/(315*d)))/((exp(c*1i + d*x*1i) + 1)*(exp(c*2i + d*x*2i)
+ 1)) + ((a + a/(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*((a
^2*56i)/(5*d) + (a^2*exp(c*1i + d*x*1i)*904i)/(105*d)))/((exp(c*1i + d*x*1
i) + 1)*(exp(c*2i + d*x*2i) + 1)^2) - (a^2*exp(c*1i + d*x*1i)*(a + a/(exp(
- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*1168i)/(315*d*(exp(c*1i
+ d*x*1i) + 1))
```

3.109 $\int \sec^2(c+dx)(a+a\sec(c+dx))^{5/2}\,dx$

3.109.1 Optimal result 1008
3.109.2 Mathematica [A] (verified) 1008
3.109.3 Rubi [A] (verified) 1009
3.109.4 Maple [A] (verified) 1011
3.109.5 Fricas [A] (verification not implemented) 1011
3.109.6 Sympy [F] 1012
3.109.7 Maxima [F] 1012
3.109.8 Giac [F] 1013
3.109.9 Mupad [B] (verification not implemented) 1014

3.109.1 Optimal result

Integrand size = 23, antiderivative size = 116

$$\int \sec^2(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \frac{64a^3\tan(c+dx)}{21d\sqrt{a+a\sec(c+dx)}} + \frac{16a^2\sqrt{a+a\sec(c+dx)}\tan(c+dx)}{21d} + \frac{2a(a+a\sec(c+dx))^{3/2}\tan(c+dx)}{7d} + \frac{2(a+a\sec(c+dx))^{5/2}\tan(c+dx)}{7d}$$

output
```
2/7*a*(a+a*sec(d*x+c))^(3/2)*tan(d*x+c)/d+2/7*(a+a*sec(d*x+c))^(5/2)*tan(d
*x+c)/d+64/21*a^3*tan(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+16/21*a^2*(a+a*sec(d
*x+c))^(1/2)*tan(d*x+c)/d
```

3.109.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.52

$$\int \sec^2(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \frac{2a^3(46+23\sec(c+dx)+12\sec^2(c+dx)+3\sec^3(c+dx))\tan(c+dx)}{21d\sqrt{a(1+\sec(c+dx))}}$$

input
```
Integrate[Sec[c + d*x]^2*(a + a*Sec[c + d*x])^(5/2),x]
```

output
```
(2*a^3*(46 + 23*Sec[c + d*x] + 12*Sec[c + d*x]^2 + 3*Sec[c + d*x]^3)*Tan[c
 + d*x])/(21*d*Sqrt[a*(1 + Sec[c + d*x])])
```

3.109.3 Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.08, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {3042, 4285, 3042, 4280, 3042, 4280, 3042, 4279}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(c+dx)(a\sec(c+dx)+a)^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^2\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}dx$$

$$\Big\downarrow \text{4285}$$

$$\frac{5}{7}\int \sec(c+dx)(\sec(c+dx)a+a)^{5/2}dx+\frac{2\tan(c+dx)(a\sec(c+dx)+a)^{5/2}}{7d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5}{7}\int \csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{5/2}dx+\frac{2\tan(c+dx)(a\sec(c+dx)+a)^{5/2}}{7d}$$

$$\Big\downarrow \text{4280}$$

$$\frac{5}{7}\left(\frac{8}{5}a\int \sec(c+dx)(\sec(c+dx)a+a)^{3/2}dx+\frac{2a\tan(c+dx)(a\sec(c+dx)+a)^{3/2}}{5d}\right)+\frac{2\tan(c+dx)(a\sec(c+dx)+a)^{5/2}}{7d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5}{7}\left(\frac{8}{5}a\int \csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}dx+\frac{2a\tan(c+dx)(a\sec(c+dx)+a)^{3/2}}{5d}\right)+\frac{2\tan(c+dx)(a\sec(c+dx)+a)^{5/2}}{7d}$$

$$\Big\downarrow \text{4280}$$

$$\frac{5}{7}\left(\frac{8}{5}a\left(\frac{4}{3}a\int \sec(c+dx)\sqrt{\sec(c+dx)a+a}dx+\frac{2a\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}\right)+\frac{2a\tan(c+dx)(a\sec(c+}{5d}\right.$$
$$\frac{2\tan(c+dx)(a\sec(c+dx)+a)^{5/2}}{7d}$$
$$\Big\downarrow \; 3042$$
$$\frac{5}{7}\left(\frac{8}{5}a\left(\frac{4}{3}a\int \csc\left(c+dx+\frac{\pi}{2}\right)\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx+\frac{2a\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}\right)+\frac{2a\tan(c+}{}\right.$$
$$\frac{2\tan(c+dx)(a\sec(c+dx)+a)^{5/2}}{7d}$$
$$\Big\downarrow \; 4279$$
$$\frac{5}{7}\left(\frac{8}{5}a\left(\frac{8a^2\tan(c+dx)}{3d\sqrt{a\sec(c+dx)+a}}+\frac{2a\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}\right)+\frac{2a\tan(c+dx)(a\sec(c+dx)+a)^{3/2}}{5d}\right)+$$
$$\frac{2\tan(c+dx)(a\sec(c+dx)+a)^{5/2}}{7d}$$

input
```
Int[Sec[c + d*x]^2*(a + a*Sec[c + d*x])^(5/2),x]
```

output
```
(2*(a + a*Sec[c + d*x])^(5/2)*Tan[c + d*x])/(7*d) + (5*((2*a*(a + a*Sec[c
+ d*x])^(3/2)*Tan[c + d*x])/(5*d) + (8*a*((8*a^2*Tan[c + d*x])/(3*d*Sqrt[a
 + a*Sec[c + d*x]]) + (2*a*Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x])/(3*d)))/
5))/7
```

3.109.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4279
```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*b*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]])), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4280
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[(-b)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m - 1)/(f*m)), x]
+ Simp[a*((2*m - 1)/m)   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m - 1), x],
 x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] && Intege
rQ[2*m]
```

rule 4285
```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(-Cot[e + f*x])*((a + b*Csc[e + f*x])^m/(f*(m + 1))), x]
+ Simp[a*(m/(b*(m + 1)))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x]
/; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

3.109.4 Maple [A] (verified)

Time = 5.28 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.65

method	result	size
default	$\frac{2a^2\left(46\cos(dx+c)^3+23\cos(dx+c)^2+12\cos(dx+c)+3\right)\sqrt{a(1+\sec(dx+c))}\,\tan(dx+c)\sec(dx+c)^2}{21d(\cos(dx+c)+1)}$	75

input
```
int(sec(d*x+c)^2*(a+a*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/21/d*a^2*(46*cos(d*x+c)^3+23*cos(d*x+c)^2+12*cos(d*x+c)+3)*(a*(1+sec(d*x
+c)))^(1/2)/(cos(d*x+c)+1)*tan(d*x+c)*sec(d*x+c)^2
```

3.109.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.82

$$\int \sec^2(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \frac{2\left(46\,a^2\cos\left(dx+c\right)^3+23\,a^2\cos\left(dx+c\right)^2+12\,a^2\cos\left(dx+c\right)+3\,a^2\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin\left(dx+\right.}{21\left(d\cos\left(dx+c\right)^4+d\cos\left(dx+c\right)^3\right)}$$

input
```
integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
2/21*(46*a^2*cos(d*x + c)^3 + 23*a^2*cos(d*x + c)^2 + 12*a^2*cos(d*x + c)
+ 3*a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/(d*cos(d*x +
 c)^4 + d*cos(d*x + c)^3)
```

3.109.6 Sympy [F]

$$\int \sec^2(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \int (a(\sec(c+dx)+1))^{\frac{5}{2}}\sec^2(c+dx)\,dx$$

input
```
integrate(sec(d*x+c)**2*(a+a*sec(d*x+c))**(5/2),x)
```

output
```
Integral((a*(sec(c + d*x) + 1))**(5/2)*sec(c + d*x)**2, x)
```

3.109.7 Maxima [F]

$$\int \sec^2(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \int (a\sec(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^2\,dx$$

input
```
integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
4/21*(21*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1
)^(3/4)*((a^2*d*cos(2*d*x + 2*c)^2 + a^2*d*sin(2*d*x + 2*c)^2 + 2*a^2*d*co
s(2*d*x + 2*c) + a^2*d)*integrate((((cos(6*d*x + 6*c)*cos(2*d*x + 2*c) + 2
*cos(4*d*x + 4*c)*cos(2*d*x + 2*c) + cos(2*d*x + 2*c)^2 + sin(6*d*x + 6*c)
*sin(2*d*x + 2*c) + 2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + sin(2*d*x + 2*c)
^2)*cos(7/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + (cos(2*d*x + 2*
c)*sin(6*d*x + 6*c) + 2*cos(2*d*x + 2*c)*sin(4*d*x + 4*c) - cos(6*d*x + 6*
c)*sin(2*d*x + 2*c) - 2*cos(4*d*x + 4*c)*sin(2*d*x + 2*c))*sin(7/2*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(5/2*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c) + 1)) - ((cos(2*d*x + 2*c)*sin(6*d*x + 6*c) + 2*cos(2*d*x
+ 2*c)*sin(4*d*x + 4*c) - cos(6*d*x + 6*c)*sin(2*d*x + 2*c) - 2*cos(4*d*x
+ 4*c)*sin(2*d*x + 2*c))*cos(7/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
))) - (cos(6*d*x + 6*c)*cos(2*d*x + 2*c) + 2*cos(4*d*x + 4*c)*cos(2*d*x +
2*c) + cos(2*d*x + 2*c)^2 + sin(6*d*x + 6*c)*sin(2*d*x + 2*c) + 2*sin(4*d*
x + 4*c)*sin(2*d*x + 2*c) + sin(2*d*x + 2*c)^2)*sin(7/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c))))*sin(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c) + 1)))/(((cos(2*d*x + 2*c)^4 + sin(2*d*x + 2*c)^4 + (cos(2*d*x + 2*c)
^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c)^2 + 4*(
cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*cos(4*d*
x + 4*c)^2 + 2*cos(2*d*x + 2*c)^3 + (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2...
```

3.109.8 Giac [F]

$$\int \sec^2(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \int (a\sec(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^2\,dx$$

input
```
integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.109.9 Mupad [B] (verification not implemented)

Time = 17.26 (sec) , antiderivative size = 349, normalized size of antiderivative = 3.01

$$\int \sec^2(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \frac{\sqrt{a+\frac{a}{\frac{e^{-c\,1i-d\,x\,1i}}{2}+\frac{e^{c\,1i+d\,x\,1i}}{2}}}\left(\frac{a^2\,20i}{3\,d}-\frac{a^2\,e^{c\,1i+d\,x\,1i}\,4i}{21\,d}\right)}{(e^{c\,1i+d\,x\,1i}+1)\,(e^{c\,2i+d\,x\,2i}+1)} - \frac{\sqrt{a+\frac{a}{\frac{e^{-c\,1i-d\,x\,1i}}{2}+\frac{e^{c\,1i+d\,x\,1i}}{2}}}\left(\frac{a^2\,16i}{7\,d}+\frac{a^2\,e^{c\,1i+d\,x\,1i}\,16i}{7\,d}\right)}{(e^{c\,1i+d\,x\,1i}+1)\,(e^{c\,2i+d\,x\,2i}+1)^3} - \frac{a^2\,e^{c\,1i+d\,x\,1i}\,\sqrt{a+\frac{a}{\frac{e^{-c\,1i-d\,x\,1i}}{2}+\frac{e^{c\,1i+d\,x\,1i}}{2}}}\,92i}{21\,d\,(e^{c\,1i+d\,x\,1i}+1)} + \frac{a^2\,e^{c\,1i+d\,x\,1i}\,\sqrt{a+\frac{a}{\frac{e^{-c\,1i-d\,x\,1i}}{2}+\frac{e^{c\,1i+d\,x\,1i}}{2}}}\,48i}{7\,d\,(e^{c\,1i+d\,x\,1i}+1)\,(e^{c\,2i+d\,x\,2i}+1)^2}$$

input
```
int((a + a/cos(c + d*x))^(5/2)/cos(c + d*x)^2,x)
```

output
```
((a + a/(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*((a^2*20i)/
(3*d) - (a^2*exp(c*1i + d*x*1i)*4i)/(21*d)))/((exp(c*1i + d*x*1i) + 1)*(ex
p(c*2i + d*x*2i) + 1)) - ((a + a/(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*
1i)/2))^(1/2)*((a^2*16i)/(7*d) + (a^2*exp(c*1i + d*x*1i)*16i)/(7*d)))/((ex
p(c*1i + d*x*1i) + 1)*(exp(c*2i + d*x*2i) + 1)^3) - (a^2*exp(c*1i + d*x*1i
)*(a + a/(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*92i)/(21*d
*(exp(c*1i + d*x*1i) + 1)) + (a^2*exp(c*1i + d*x*1i)*(a + a/(exp(- c*1i -
d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*48i)/(7*d*(exp(c*1i + d*x*1i) + 1
)*(exp(c*2i + d*x*2i) + 1)^2)
```

3.110 $\int \sec(c+dx)(a+a\sec(c+dx))^{5/2}\,dx$

3.110.1 Optimal result . . . 1015
3.110.2 Mathematica [A] (verified) . . . 1015
3.110.3 Rubi [A] (verified) . . . 1016
3.110.4 Maple [A] (verified) . . . 1017
3.110.5 Fricas [A] (verification not implemented) . . . 1018
3.110.6 Sympy [F] . . . 1018
3.110.7 Maxima [F] . . . 1018
3.110.8 Giac [F] . . . 1019
3.110.9 Mupad [B] (verification not implemented) . . . 1019

3.110.1 Optimal result

Integrand size = 21, antiderivative size = 89

$$\int \sec(c+dx)(a+a\sec(c+dx))^{5/2}\,dx=\frac{64a^3\tan(c+dx)}{15d\sqrt{a+a\sec(c+dx)}}+\frac{16a^2\sqrt{a+a\sec(c+dx)}\tan(c+dx)}{15d}+\frac{2a(a+a\sec(c+dx))^{3/2}\tan(c+dx)}{5d}$$

output
```
2/5*a*(a+a*sec(d*x+c))^(3/2)*tan(d*x+c)/d+64/15*a^3*tan(d*x+c)/d/(a+a*sec(
d*x+c))^(1/2)+16/15*a^2*(a+a*sec(d*x+c))^(1/2)*tan(d*x+c)/d
```

3.110.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.56

$$\int \sec(c+dx)(a+a\sec(c+dx))^{5/2}\,dx=\frac{2a^3(43+14\sec(c+dx)+3\sec^2(c+dx))\tan(c+dx)}{15d\sqrt{a(1+\sec(c+dx))}}$$

input
```
Integrate[Sec[c + d*x]*(a + a*Sec[c + d*x])^(5/2),x]
```

output
```
(2*a^3*(43 + 14*Sec[c + d*x] + 3*Sec[c + d*x]^2)*Tan[c + d*x])/(15*d*Sqrt[
a*(1 + Sec[c + d*x])])
```

3.110.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3042, 4280, 3042, 4280, 3042, 4279}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(c+dx)(a\sec(c+dx)+a)^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}dx$$

$$\Big\downarrow \text{4280}$$

$$\frac{8}{5}a\int \sec(c+dx)(\sec(c+dx)a+a)^{3/2}dx+\frac{2a\tan(c+dx)(a\sec(c+dx)+a)^{3/2}}{5d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{8}{5}a\int \csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}dx+\frac{2a\tan(c+dx)(a\sec(c+dx)+a)^{3/2}}{5d}$$

$$\Big\downarrow \text{4280}$$

$$\frac{8}{5}a\left(\frac{4}{3}a\int \sec(c+dx)\sqrt{\sec(c+dx)a+a}dx+\frac{2a\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}\right)+\frac{2a\tan(c+dx)(a\sec(c+dx)+a)^{3/2}}{5d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{8}{5}a\left(\frac{4}{3}a\int \csc\left(c+dx+\frac{\pi}{2}\right)\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx+\frac{2a\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}\right)+\frac{2a\tan(c+dx)(a\sec(c+dx)+a)^{3/2}}{5d}$$

$$\Big\downarrow \text{4279}$$

$$\frac{8}{5}a\left(\frac{8a^2\tan(c+dx)}{3d\sqrt{a\sec(c+dx)+a}}+\frac{2a\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}\right)+\frac{2a\tan(c+dx)(a\sec(c+dx)+a)^{3/2}}{5d}$$

input
```
Int[Sec[c + d*x]*(a + a*Sec[c + d*x])^(5/2),x]
```

output
```
(2*a*(a + a*Sec[c + d*x])^(3/2)*Tan[c + d*x])/(5*d) + (8*a*((8*a^2*Tan[c +
 d*x])/(3*d*Sqrt[a + a*Sec[c + d*x]]) + (2*a*Sqrt[a + a*Sec[c + d*x]]*Tan[
c + d*x])/(3*d)))/5
```

3.110.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4279
```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*b*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]])), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4280
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[(-b)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m - 1)/(f*m)), x]
+ Simp[a*((2*m - 1)/m)   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m - 1), x],
 x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] && Intege
rQ[2*m]
```

3.110.4 Maple [A] (verified)

Time = 1.91 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.70

method	result	size
default	$\frac{2a^2\sqrt{a(1+\sec(dx+c))}\,(43\sin(dx+c)+14\tan(dx+c)+3\sec(dx+c)\tan(dx+c))}{15d(\cos(dx+c)+1)}$	62

input
```
int(sec(d*x+c)*(a+a*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/15/d*a^2*(a*(1+sec(d*x+c)))^(1/2)/(cos(d*x+c)+1)*(43*sin(d*x+c)+14*tan(d
*x+c)+3*sec(d*x+c)*tan(d*x+c))
```

3.110.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.92

$$\int \sec(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \frac{2\left(43\,a^2\cos\left(dx+c\right)^2+14\,a^2\cos\left(dx+c\right)+3\,a^2\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin\left(dx+c\right)}{15\left(d\cos\left(dx+c\right)^3+d\cos\left(dx+c\right)^2\right)}$$

input
```
integrate(sec(d*x+c)*(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
2/15*(43*a^2*cos(d*x + c)^2 + 14*a^2*cos(d*x + c) + 3*a^2)*sqrt((a*cos(d*x
 + c) + a)/cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c)^3 + d*cos(d*x + c)^2
)
```

3.110.6 Sympy [F]

$$\int \sec(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \int \left(a(\sec\left(c+dx\right)+1)\right)^{\frac{5}{2}}\sec\left(c+dx\right)dx$$

input
```
integrate(sec(d*x+c)*(a+a*sec(d*x+c))**(5/2),x)
```

output
```
Integral((a*(sec(c + d*x) + 1))**(5/2)*sec(c + d*x), x)
```

3.110.7 Maxima [F]

$$\int \sec(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \int \left(a\sec\left(dx+c\right)+a\right)^{\frac{5}{2}}\sec\left(dx+c\right)\,dx$$

input
```
integrate(sec(d*x+c)*(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)^(5/2)*sec(d*x + c), x)
```

3.110.8 Giac [F]

$$\int \sec(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \int (a\sec(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)\,dx$$

input
```
integrate(sec(d*x+c)*(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.110.9 Mupad [B] (verification not implemented)

Time = 17.42 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.64

$$\int \sec(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = -\frac{2\,a^2\,\sqrt{a+\frac{a}{\frac{e^{-c\,1i-d\,x\,1i}}{2}+\frac{e^{c\,1i+d\,x\,1i}}{2}}}\,\left(e^{c\,1i+d\,x\,1i}\,15i-e^{c\,2i+d\,x\,2i}\,70i+e^{c\,3i+d\,x\,3i}\,70i-e^{c\,4i+d\,x\,4i}\,15i+e^{c\,5i+d\,x\,5i}\,43i-\right.}{15\,d\,\left(e^{c\,1i+d\,x\,1i}+1\right)\,\left(e^{c\,2i+d\,x\,2i}+1\right)^2}$$

input
```
int((a + a/cos(c + d*x))^(5/2)/cos(c + d*x),x)
```

output
```
-(2*a^2*(a + a/(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*(exp
(c*1i + d*x*1i)*15i - exp(c*2i + d*x*2i)*70i + exp(c*3i + d*x*3i)*70i - ex
p(c*4i + d*x*4i)*15i + exp(c*5i + d*x*5i)*43i - 43i))/(15*d*(exp(c*1i + d*
x*1i) + 1)*(exp(c*2i + d*x*2i) + 1)^2)
```

3.111 $\int (a+a\sec(c+dx))^{5/2}\,dx$

3.111.1 Optimal result 1020
3.111.2 Mathematica [C] (warning: unable to verify) 1020
3.111.3 Rubi [A] (verified) 1021
3.111.4 Maple [B] (verified) 1024
3.111.5 Fricas [A] (verification not implemented) 1024
3.111.6 Sympy [F] 1025
3.111.7 Maxima [B] (verification not implemented) 1025
3.111.8 Giac [F] 1026
3.111.9 Mupad [F(-1)] 1027

3.111.1 Optimal result

Integrand size = 14, antiderivative size = 98

$$\int (a+a\sec(c+dx))^{5/2}\,dx=\frac{2a^{5/2}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}+\frac{14a^3\tan(c+dx)}{3d\sqrt{a+a\sec(c+dx)}}+\frac{2a^2\sqrt{a+a\sec(c+dx)}\tan(c+dx)}{3d}$$

output
```
2*a^(5/2)*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/d+14/3*a^3*tan
(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+2/3*a^2*(a+a*sec(d*x+c))^(1/2)*tan(d*x+c)
/d
```

3.111.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 6.45 (sec) , antiderivative size = 360, normalized size of antiderivative = 3.67

$$\int (a+a\sec(c+dx))^{5/2}\,dx=\frac{\csc^3\left(\frac{1}{2}(c+dx)\right)\sec^5\left(\frac{1}{2}(c+dx)\right)(a(1+\sec(c+dx)))^{5/2}\sqrt{\frac{1}{1-2\sin^2\left(\frac{1}{2}(c+dx)\right)}}\sqrt{1-2\sin^2\left(\frac{1}{2}(c+\right.}}{}$$

input
```
Integrate[(a + a*Sec[c + d*x])^(5/2),x]
```

output
```
(Csc[(c + d*x)/2]^3*Sec[(c + d*x)/2]^5*(a*(1 + Sec[c + d*x]))^(5/2)*Sqrt[(
1 - 2*Sin[(c + d*x)/2]^2)^(-1)]*Sqrt[1 - 2*Sin[(c + d*x)/2]^2]*(256*Cos[(c
 + d*x)/2]^4*HypergeometricPFQ[{3/2, 2, 7/2}, {1, 9/2}, 2*Sin[(c + d*x)/2]
^2]*Sin[(c + d*x)/2]^6 + 512*Hypergeometric2F1[3/2, 7/2, 9/2, 2*Sin[(c + d
*x)/2]^2]*Sin[(c + d*x)/2]^6*(2 - 3*Sin[(c + d*x)/2]^2 + Sin[(c + d*x)/2]^
4) + (21*Sqrt[2]*ArcSin[Sqrt[2]*Sqrt[Sin[(c + d*x)/2]^2]]*(15 - 10*Sin[(c
+ d*x)/2]^2 + 3*Sin[(c + d*x)/2]^4))/Sqrt[Sin[(c + d*x)/2]^2] - 14*Sqrt[1
- 2*Sin[(c + d*x)/2]^2]*(45 + 30*Sin[(c + d*x)/2]^2 - 31*Sin[(c + d*x)/2]^
4 + 12*Sin[(c + d*x)/2]^6)))/(672*d*Sec[c + d*x]^(5/2))
```

3.111.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.04, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.643$, Rules used = {3042, 4262, 27, 3042, 4403, 3042, 4261, 216, 4279}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sec(c+dx)+a)^{5/2}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}dx$$

$$\Big\downarrow \; 4262$$

$$\frac{2}{3}a\int \frac{1}{2}\sqrt{\sec(c+dx)a+a}(7\sec(c+dx)a+3a)dx+\frac{2a^2\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{3}a\int \sqrt{\sec(c+dx)a+a}(7\sec(c+dx)a+3a)dx+\frac{2a^2\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{3}a\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}\left(7\csc\left(c+dx+\frac{\pi}{2}\right)a+3a\right)dx+\frac{2a^2\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}$$

$$\Big\downarrow \text{4403}$$

$$\frac{1}{3}a\left(3a\int \sqrt{\sec(c+dx)a+a}dx+7a\int \sec(c+dx)\sqrt{\sec(c+dx)a+a}dx\right)+\frac{2a^2\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}a\left(3a\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx+7a\int \csc\left(c+dx+\frac{\pi}{2}\right)\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx\right)+\frac{2a^2\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}$$

$$\Big\downarrow \text{4261}$$

$$\frac{1}{3}a\left(7a\int \csc\left(c+dx+\frac{\pi}{2}\right)\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx-\frac{6a^2\int \frac{1}{\frac{a^2\tan^2(c+dx)}{\sec(c+dx)a+a}+a}d\left(-\frac{a\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}\right)+\frac{2a^2\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{3}a\left(7a\int \csc\left(c+dx+\frac{\pi}{2}\right)\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx+\frac{6a^{3/2}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}\right)+\frac{2a^2\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}$$

$$\Big\downarrow \text{4279}$$

$$\frac{2a^2\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}+\frac{1}{3}a\left(\frac{6a^{3/2}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{14a^2\tan(c+dx)}{d\sqrt{a\sec(c+dx)+a}}\right)$$

input
```
Int[(a + a*Sec[c + d*x])^(5/2),x]
```

output
```
(2*a^2*Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x])/(3*d) + (a*((6*a^(3/2)*ArcTa
n[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d + (14*a^2*Tan[c + d*
x])/(d*Sqrt[a + a*Sec[c + d*x]])))/3
```

3.111. $\int (a+a\sec(c+dx))^{5/2}\,dx$

3.111.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4261
```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(b/d)
  Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 4262
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(-b^2)*C
ot[c + d*x]*((a + b*Csc[c + d*x])^(n - 2)/(d*(n - 1))), x] + Simp[a/(n - 1)
   Int[(a + b*Csc[c + d*x])^(n - 2)*(a*(n - 1) + b*(3*n - 4)*Csc[c + d*x]),
 x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && Inte
gerQ[2*n]
```

rule 4279
```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*b*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]])), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4403
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_
.) + (c_)), x_Symbol] :> Simp[c   Int[Sqrt[a + b*Csc[e + f*x]], x], x] + Si
mp[d   Int[Sqrt[a + b*Csc[e + f*x]]*Csc[e + f*x], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

3.111.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 171 vs. $2(84) = 168$.

Time = 1.53 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.76

method	result
default	$\frac{2a^2\sqrt{a(1+\sec(dx+c))}\left(3\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\operatorname{arctanh}\left(\frac{\sin(dx+c)}{(\cos(dx+c)+1)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}}\right)\cos(dx+c)+3\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\operatorname{arctanh}\left(\frac{}{(\cos(d}\right.\right.}{3d(\cos(dx+c)+1)}$

input
```
int((a+a*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/3/d*a^2*(a*(1+sec(d*x+c)))^(1/2)/(cos(d*x+c)+1)*(3*(-cos(d*x+c)/(cos(d*x
+c)+1))^(1/2)*arctanh(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1
))^(1/2))*cos(d*x+c)+3*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(sin(d*x+
c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+8*sin(d*x+c)+tan(d*x
+c))
```

3.111.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 310, normalized size of antiderivative = 3.16

$$\int (a + a\sec(c + dx))^{5/2}\,dx = \left[\frac{3\left(a^2\cos\left(dx+c\right)^2 + a^2\cos\left(dx+c\right)\right)\sqrt{-a}\log\left(\frac{2\,a\cos(dx+c)^2 - 2\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)}{\cos(dx+c)+1}\right.}{3\left(d\cos\left(dx+c\right)^2 + d\cos\left(dx\right.\right.}\right.$$

$$-\frac{2\left(3\left(a^2\cos\left(dx+c\right)^2 + a^2\cos\left(dx+c\right)\right)\sqrt{a}\arctan\left(\frac{\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{\sqrt{a}\sin(dx+c)}\right) - \left(8\,a^2\cos\left(dx+c\right) + a^2\right)\sqrt{\frac{a\,\mathrm{co}}{\mathrm{c}}}\right.}{3\left(d\cos\left(dx+c\right)^2 + d\cos\left(dx+c\right)\right)}$$

input
```
integrate((a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
[1/3*(3*(a^2*cos(d*x + c)^2 + a^2*cos(d*x + c))*sqrt(-a)*log((2*a*cos(d*x
+ c)^2 - 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*s
in(d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) + 2*(8*a^2*cos(d*x +
 c) + a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d*
x + c)^2 + d*cos(d*x + c)), -2/3*(3*(a^2*cos(d*x + c)^2 + a^2*cos(d*x + c)
)*sqrt(a)*arctan(sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqr
t(a)*sin(d*x + c))) - (8*a^2*cos(d*x + c) + a^2)*sqrt((a*cos(d*x + c) + a)
/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^2 + d*cos(d*x + c))]
```

3.111.6 Sympy [F]

$$\int (a + a\sec(c + dx))^{5/2}\,dx = \int (a\sec(c + dx) + a)^{\frac{5}{2}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))**(5/2),x)
```

output
```
Integral((a*sec(c + d*x) + a)**(5/2), x)
```

3.111.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1395 vs. $2(84) = 168$.

Time = 0.54 (sec) , antiderivative size = 1395, normalized size of antiderivative = 14.23

$$\int (a + a\sec(c + dx))^{5/2}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
1/6*(30*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)
^(3/4)*a^(5/2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) -
2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)
*((12*a^2*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(2*d*x +
 2*c) - 3*a^2*sin(2*d*x + 2*c) - 4*(3*a^2*cos(2*d*x + 2*c) + 4*a^2)*sin(3/
2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(3/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c) + 1)) + (12*a^2*sin(2*d*x + 2*c)*sin(3/2*arctan2(
sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 3*a^2*cos(2*d*x + 2*c) - a^2 + 4*(3
*a^2*cos(2*d*x + 2*c) + 4*a^2)*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c))))*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))*sqrt(
a) + 3*((a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x + 2*c)^2 + 2*a^2*cos(2*d*x
 + 2*c) + a^2)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*
d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))
)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(
2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
) + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c)))) + 1) - (a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*...
```

3.111.8 Giac [F]

$$\int (a + a\sec(c + dx))^{5/2}\,dx = \int (a\sec(dx + c) + a)^{\frac{5}{2}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.111.9 Mupad [F(-1)]

Timed out.

$$\int (a + a\sec(c + dx))^{5/2}\,dx = \int \left(a + \frac{a}{\cos(c + d\,x)}\right)^{5/2}\,dx$$

input
```
int((a + a/cos(c + d*x))^(5/2),x)
```

output
```
int((a + a/cos(c + d*x))^(5/2), x)
```

3.112 $\int \cos(c+dx)(a+a\sec(c+dx))^{5/2}\,dx$

3.112.1 Optimal result 1028
3.112.2 Mathematica [A] (verified) 1028
3.112.3 Rubi [A] (verified) 1029
3.112.4 Maple [B] (verified) 1031
3.112.5 Fricas [A] (verification not implemented) 1032
3.112.6 Sympy [F(-1)] 1032
3.112.7 Maxima [B] (verification not implemented) 1033
3.112.8 Giac [F] 1033
3.112.9 Mupad [F(-1)] 1034

3.112.1 Optimal result

Integrand size = 21, antiderivative size = 94

$$\int \cos(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \frac{5a^{5/2}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} - \frac{a^3\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}} + \frac{2a^2\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{d}$$

output
```
5*a^(5/2)*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/d-a^3*sin(d*x+
c)/d/(a+a*sec(d*x+c))^(1/2)+2*a^2*sin(d*x+c)*(a+a*sec(d*x+c))^(1/2)/d
```

3.112.2 Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.87

$$\int \cos(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \frac{a^3\left(5\text{arctanh}\left(\sqrt{1-\sec(c+dx)}\right)+(2+\cos(c+dx))\sqrt{1-\sec(c+dx)}\right)\tan(c+dx)}{d\sqrt{1-\sec(c+dx)}\sqrt{a(1+\sec(c+dx))}}$$

input
```
Integrate[Cos[c + d*x]*(a + a*Sec[c + d*x])^(5/2),x]
```

output
```
(a^3*(5*ArcTanh[Sqrt[1 - Sec[c + d*x]]] + (2 + Cos[c + d*x])*Sqrt[1 - Sec[
c + d*x]])*Tan[c + d*x])/(d*Sqrt[1 - Sec[c + d*x]]*Sqrt[a*(1 + Sec[c + d*x
])])
```

3.112.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.03, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {3042, 4301, 27, 3042, 4503, 3042, 4261, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(c+dx)(a\sec(c+dx)+a)^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}}{\csc\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{4301}$$

$$2a\int -\frac{1}{2}\cos(c+dx)(a-3a\sec(c+dx))\sqrt{\sec(c+dx)a+a}dx+\frac{2a^2\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{2a^2\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{d}-a\int \cos(c+dx)(a-3a\sec(c+dx))\sqrt{\sec(c+dx)a+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{2a^2\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{d}-a\int \frac{\left(a-3a\csc\left(c+dx+\frac{\pi}{2}\right)\right)\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}{\csc\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{4503}$$

$$\frac{2a^2\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{d}-a\left(\frac{a^2\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}-\frac{5}{2}a\int \sqrt{\sec(c+dx)a+a}dx\right)$$

$$\Big\downarrow \text{3042}$$

$$\frac{2a^2\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{d}-a\left(\frac{a^2\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}-\frac{5}{2}a\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx\right)$$

$$\Big\downarrow \text{4261}$$

$$\frac{2a^2\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{d}-a\left(\frac{5a^2\int\frac{1}{\frac{a^2\tan^2(c+dx)}{\sec(c+dx)a+a}+a}d\left(-\frac{a\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}+\frac{a^2\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}\right)$$

$$\Big\downarrow \text{216}$$

$$\frac{2a^2\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{d}-a\left(\frac{a^2\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}-\frac{5a^{3/2}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}\right)$$

input
```
Int[Cos[c + d*x]*(a + a*Sec[c + d*x])^(5/2),x]
```

output
```
(2*a^2*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/d - a*((-5*a^(3/2)*ArcTan[(S
qrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d + (a^2*Sin[c + d*x])/(d*
Sqrt[a + a*Sec[c + d*x]]))
```

3.112.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4261
```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(b/d)
  Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 4301
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
2)*((d*Csc[e + f*x])^n/(f*(m + n - 1))), x] + Simp[b/(m + n - 1)   Int[(a +
 b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*(b*(m + 2*n - 1) + a*(3*m + 2*n
 - 4)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^
2, 0] && GtQ[m, 1] && NeQ[m + n - 1, 0] && IntegerQ[2*m]
```

rule 4503
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*b^2*Co
t[e + f*x]*((d*Csc[e + f*x])^n/(a*f*n*Sqrt[a + b*Csc[e + f*x]])), x] + Simp
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n)   Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[
e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a
*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

3.112.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 177 vs. $2(84) = 168$.

Time = 32.65 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.89

method	result
default	$\frac{a^2\left(5\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\operatorname{arctanh}\left(\frac{\sin(dx+c)}{(\cos(dx+c)+1)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}}\right)\cos(dx+c)+5\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\operatorname{arctanh}\left(\frac{\sin(dx+c)}{(\cos(dx+c)+1)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)}}}\right.\right.}{d(\cos(dx+c)+1)}$

input
```
int(cos(d*x+c)*(a+a*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/d*a^2*(5*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(sin(d*x+c)/(cos(d*x+
c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)+5*(-cos(d*x+c)/(cos(d
*x+c)+1))^(1/2)*arctanh(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)
+1))^(1/2))+cos(d*x+c)*sin(d*x+c)+2*sin(d*x+c))*(a*(1+sec(d*x+c)))^(1/2)/(
cos(d*x+c)+1)
```

3.112.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 276, normalized size of antiderivative = 2.94

$$\int \cos(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \left[\frac{5\,(a^2\cos(dx+c)+a^2)\sqrt{-a}\log\left(\frac{2\,a\cos(dx+c)^2-2\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+a\cos(dx+c)-a}{\cos(dx+c)+1}\right)}{2\,(d\cos(dx+c)+d)}\right.$$

$$-\frac{5\,(a^2\cos(dx+c)+a^2)\sqrt{a}\arctan\left(\frac{\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{\sqrt{a}\sin(dx+c)}\right)-(a^2\cos(dx+c)+2\,a^2)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+}{d\cos(dx+c)+d}$$

input
```
integrate(cos(d*x+c)*(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
[1/2*(5*(a^2*cos(d*x + c) + a^2)*sqrt(-a)*log((2*a*cos(d*x + c)^2 - 2*sqrt
(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) + a
*cos(d*x + c) - a)/(cos(d*x + c) + 1)) + 2*(a^2*cos(d*x + c) + 2*a^2)*sqrt
((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c) + d), -(
5*(a^2*cos(d*x + c) + a^2)*sqrt(a)*arctan(sqrt((a*cos(d*x + c) + a)/cos(d*
x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) - (a^2*cos(d*x + c) + 2*a^2)*
sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c) + d)
]
```

3.112.6 Sympy [F(-1)]

Timed out.

$$\int \cos(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)*(a+a*sec(d*x+c))**(5/2),x)
```

output
```
Timed out
```

3.112.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1383 vs. $2(84) = 168$.

Time = 0.54 (sec) , antiderivative size = 1383, normalized size of antiderivative = 14.71

$$\int \cos(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)*(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
1/4*(18*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)
^(3/4)*a^(5/2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) +
2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)
*((4*a^2*sin(3*d*x + 3*c) + 5*a^2*sin(2*d*x + 2*c) + 4*a^2*sin(d*x + c))*c
os(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + (a^2*cos(2*d*x +
 2*c)^2*sin(d*x + c) + a^2*sin(2*d*x + 2*c)^2*sin(d*x + c) + 2*a^2*cos(2*d
*x + 2*c)*sin(d*x + c) + a^2*sin(d*x + c))*cos(1/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c) + 1)) - (4*a^2*cos(3*d*x + 3*c) + 5*a^2*cos(2*d*x + 2*
c) + 4*a^2*cos(d*x + c) + 5*a^2)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c) + 1)) - ((a^2*cos(d*x + c) - a^2)*cos(2*d*x + 2*c)^2 + a^2*cos(d
*x + c) + (a^2*cos(d*x + c) - a^2)*sin(2*d*x + 2*c)^2 - a^2 + 2*(a^2*cos(d
*x + c) - a^2)*cos(2*d*x + 2*c))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c) + 1)))*sqrt(a) + 5*((a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x + 2*
c)^2 + 2*a^2*cos(2*d*x + 2*c) + a^2)*arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*
d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x
+ 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin
(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c) + 1))) + 1) - (a^2*cos(2*d*x + 2*c)^2 + a...
```

3.112.8 Giac [F]

$$\int \cos(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \int (a\sec(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)\,dx$$

input
```
integrate(cos(d*x+c)*(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.112.9 Mupad [F(-1)]

Timed out.

$$\int \cos(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \int \cos\left(c+d\,x\right)\,\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)^{5/2}\,dx$$

input
```
int(cos(c + d*x)*(a + a/cos(c + d*x))^(5/2),x)
```

output
```
int(cos(c + d*x)*(a + a/cos(c + d*x))^(5/2), x)
```

3.113 $\int \cos^2(c+dx)(a+a\sec(c+dx))^{5/2}\,dx$

3.113.1 Optimal result 1035
3.113.2 Mathematica [C] (verified) 1035
3.113.3 Rubi [A] (verified) 1036
3.113.4 Maple [B] (verified) 1039
3.113.5 Fricas [A] (verification not implemented) 1039
3.113.6 Sympy [F(-1)] 1040
3.113.7 Maxima [F(-1)] 1040
3.113.8 Giac [F] 1041
3.113.9 Mupad [F(-1)] 1041

3.113.1 Optimal result

Integrand size = 23, antiderivative size = 106

$$\int \cos^2(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \frac{19a^{5/2}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{4d} + \frac{9a^3\sin(c+dx)}{4d\sqrt{a+a\sec(c+dx)}} + \frac{a^2\cos(c+dx)\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{2d}$$

output
```
19/4*a^(5/2)*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/d+9/4*a^3*s
in(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+1/2*a^2*cos(d*x+c)*sin(d*x+c)*(a+a*sec(
d*x+c))^(1/2)/d
```

3.113.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.57 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.42

$$\int \cos^2(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = -\frac{a^2\cos(c+dx)\sqrt{a(1+\sec(c+dx))}\left(\sqrt{1-\sec(c+dx)}(\sin(c+dx)+3\sin(2(c+dx)))-7\text{arctanh}\left(\sqrt{1-\right.\right.}{4d(1+\cos(c+d}$$

input
```
Integrate[Cos[c + d*x]^2*(a + a*Sec[c + d*x])^(5/2),x]
```

output
```
-1/4*(a^2*Cos[c + d*x]*Sqrt[a*(1 + Sec[c + d*x])]*(Sqrt[1 - Sec[c + d*x]]*
(Sin[c + d*x] + 3*Sin[2*(c + d*x)]) - 7*ArcTanh[Sqrt[1 - Sec[c + d*x]]]*Ta
n[c + d*x] - 32*Hypergeometric2F1[1/2, 3, 3/2, 1 - Sec[c + d*x]]*Sqrt[1 -
Sec[c + d*x]]*Tan[c + d*x]))/(d*(1 + Cos[c + d*x])*Sqrt[1 - Sec[c + d*x]])
```

3.113.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.02, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {3042, 4300, 27, 3042, 4503, 3042, 4261, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(c+dx)(a\sec(c+dx)+a)^{5/2}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}}{\csc\left(c+dx+\frac{\pi}{2}\right)^2}dx$$

$$\Big\downarrow \; 4300$$

$$\frac{1}{2}a\int \frac{1}{2}\cos(c+dx)\sqrt{\sec(c+dx)a+a}(5\sec(c+dx)a+9a)dx+\frac{a^2\sin(c+dx)\cos(c+dx)\sqrt{a\sec(c+dx)+a}}{2d}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{4}a\int \cos(c+dx)\sqrt{\sec(c+dx)a+a}(5\sec(c+dx)a+9a)dx+\frac{a^2\sin(c+dx)\cos(c+dx)\sqrt{a\sec(c+dx)+a}}{2d}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{4}a\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}\left(5\csc\left(c+dx+\frac{\pi}{2}\right)a+9a\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{a^2\sin(c+dx)\cos(c+dx)\sqrt{a\sec(c+dx)+a}}{2d}$$

$$\Big\downarrow \; 4503$$

$$\frac{1}{4}a\left(\frac{19}{2}a\int\sqrt{\sec(c+dx)a+a}dx+\frac{9a^2\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}\right)+\frac{a^2\sin(c+dx)\cos(c+dx)\sqrt{a\sec(c+dx)+a}}{2d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}a\left(\frac{19}{2}a\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx+\frac{9a^2\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}\right)+\frac{a^2\sin(c+dx)\cos(c+dx)\sqrt{a\sec(c+dx)+a}}{2d}$$

$$\Big\downarrow \text{4261}$$

$$\frac{1}{4}a\left(\frac{9a^2\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}-\frac{19a^2\int\frac{1}{\frac{a^2\tan^2(c+dx)}{\sec(c+dx)a+a}+a}d\left(-\frac{a\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}\right)+\frac{a^2\sin(c+dx)\cos(c+dx)\sqrt{a\sec(c+dx)+a}}{2d}$$

$$\Big\downarrow \text{216}$$

$$\frac{a^2\sin(c+dx)\cos(c+dx)\sqrt{a\sec(c+dx)+a}}{2d}+\frac{1}{4}a\left(\frac{19a^{3/2}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{9a^2\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}\right)$$

input
```
Int[Cos[c + d*x]^2*(a + a*Sec[c + d*x])^(5/2),x]
```

output
```
(a^2*Cos[c + d*x]*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(2*d) + (a*((19*a
^(3/2)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d + (9*a^2
*Sin[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]])))/4
```

3.113. $\int \cos^2(c+dx)(a+a\sec(c+dx))^{5/2}\,dx$

3.113.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4261
```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(b/d)
  Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 4300
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)*
((d*Csc[e + f*x])^n/(f*n)), x] - Simp[a/(d*n)   Int[(a + b*Csc[e + f*x])^(m
 - 2)*(d*Csc[e + f*x])^(n + 1)*(b*(m - 2*n - 2) - a*(m + 2*n - 1)*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1]
 && (LtQ[n, -1] || (EqQ[m, 3/2] && EqQ[n, -2^(-1)])) && IntegerQ[2*m]
```

rule 4503
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*b^2*Co
t[e + f*x]*((d*Csc[e + f*x])^n/(a*f*n*Sqrt[a + b*Csc[e + f*x]])), x] + Simp
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n)   Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[
e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a
*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

3.113. $\int \cos^2(c+dx)(a+a\sec(c+dx))^{5/2}\,dx$

3.113.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 187 vs. $2(90) = 180$.

Time = 58.66 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.77

method	result
default	$\frac{a^2\left(19\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\operatorname{arctanh}\left(\frac{\sin(dx+c)}{(\cos(dx+c)+1)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}}\right)\cos(dx+c)+2\cos(dx+c)^2\sin(dx+c)+19\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\operatorname{arctanh}\right.}{4d(\cos(dx+c)+1)}$

input
```
int(cos(d*x+c)^2*(a+a*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/4/d*a^2*(19*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(sin(d*x+c)/(cos(d
*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)+2*cos(d*x+c)^2*sin
(d*x+c)+19*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(sin(d*x+c)/(cos(d*x+
c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+11*cos(d*x+c)*sin(d*x+c))*(a*(1+
sec(d*x+c)))^(1/2)/(cos(d*x+c)+1)
```

3.113.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 294, normalized size of antiderivative = 2.77

$$\int \cos^2(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \left[\frac{19\,(a^2\cos(dx+c)+a^2)\sqrt{-a}\log\left(\frac{2\,a\cos(dx+c)^2-2\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+a\cos(dx+c)-a}{\cos(dx+c)+1}\right.}{8\,(d\cos(dx+c)+d)}\right.$$

$$-\frac{19\,(a^2\cos(dx+c)+a^2)\sqrt{a}\arctan\left(\frac{\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{\sqrt{a}\sin(dx+c)}\right)-\left(2\,a^2\cos(dx+c)^2+11\,a^2\cos(dx+c)\right)\sqrt{\frac{a\,c}{}}}{4\,(d\cos(dx+c)+d)}$$

input
```
integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
[1/8*(19*(a^2*cos(d*x + c) + a^2)*sqrt(-a)*log((2*a*cos(d*x + c)^2 - 2*sqr
t(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) +
a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) + 2*(2*a^2*cos(d*x + c)^2 + 11*a^2
*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(d*co
s(d*x + c) + d), -1/4*(19*(a^2*cos(d*x + c) + a^2)*sqrt(a)*arctan(sqrt((a*
cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) - (2*
a^2*cos(d*x + c)^2 + 11*a^2*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*
x + c))*sin(d*x + c))/(d*cos(d*x + c) + d)]
```

3.113.6 Sympy [F(-1)]

Timed out.

$$\int \cos^2(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**2*(a+a*sec(d*x+c))**(5/2),x)
```

output
```
Timed out
```

3.113.7 Maxima [F(-1)]

Timed out.

$$\int \cos^2(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
Timed out
```

3.113.8 Giac [F]

$$\int \cos^2(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \int (a\sec(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^2\,dx$$

input
```
integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.113.9 Mupad [F(-1)]

Timed out.

$$\int \cos^2(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \int \cos(c+d\,x)^2\left(a+\frac{a}{\cos(c+d\,x)}\right)^{5/2}dx$$

input
```
int(cos(c + d*x)^2*(a + a/cos(c + d*x))^(5/2),x)
```

output
```
int(cos(c + d*x)^2*(a + a/cos(c + d*x))^(5/2), x)
```

3.114 $\int \cos^3(c+dx)(a+a\sec(c+dx))^{5/2}\,dx$

3.114.1 Optimal result . . . 1042
3.114.2 Mathematica [C] (verified) . . . 1042
3.114.3 Rubi [A] (verified) . . . 1043
3.114.4 Maple [A] (verified) . . . 1046
3.114.5 Fricas [A] (verification not implemented) . . . 1046
3.114.6 Sympy [F(-1)] . . . 1047
3.114.7 Maxima [F(-1)] . . . 1047
3.114.8 Giac [F] . . . 1048
3.114.9 Mupad [F(-1)] . . . 1048

3.114.1 Optimal result

Integrand size = 23, antiderivative size = 144

$$\int \cos^3(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \frac{25a^{5/2}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{8d} + \frac{25a^3\sin(c+dx)}{8d\sqrt{a+a\sec(c+dx)}} + \frac{13a^3\cos(c+dx)\sin(c+dx)}{12d\sqrt{a+a\sec(c+dx)}} + \frac{a^2\cos^2(c+dx)\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{3d}$$

output
```
25/8*a^(5/2)*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/d+25/8*a^3*
sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+13/12*a^3*cos(d*x+c)*sin(d*x+c)/d/(a+a
*sec(d*x+c))^(1/2)+1/3*a^2*cos(d*x+c)^2*sin(d*x+c)*(a+a*sec(d*x+c))^(1/2)/
d
```

3.114.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.84 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.05

$$\int \cos^3(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \frac{a^2\left(165\text{arctanh}\left(\sqrt{1-\sec(c+dx)}\right)+(31+159\cos(c+dx)+31\cos(2(c+dx))-2\cos(3(c+\right.\right.}{7}$$

input
```
Integrate[Cos[c + d*x]^3*(a + a*Sec[c + d*x])^(5/2),x]
```

output
```
(a^2*(165*ArcTanh[Sqrt[1 - Sec[c + d*x]]] + (31 + 159*Cos[c + d*x] + 31*Co
s[2*(c + d*x)] - 2*Cos[3*(c + d*x)])*Sqrt[1 - Sec[c + d*x]] + 192*Hypergeo
metric2F1[1/2, 4, 3/2, 1 - Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]])*Sqrt[a*(1
 + Sec[c + d*x])]*Sin[c + d*x])/(72*d*(1 + Cos[c + d*x])*Sqrt[1 - Sec[c +
d*x]])
```

3.114.3 Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.03, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {3042, 4300, 27, 3042, 4503, 3042, 4292, 3042, 4261, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^3(c+dx)(a\sec(c+dx)+a)^{5/2}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}}{\csc\left(c+dx+\frac{\pi}{2}\right)^3}dx$$

$$\Big\downarrow \; 4300$$

$$\frac{1}{3}a\int \frac{1}{2}\cos^2(c+dx)\sqrt{\sec(c+dx)a+a}(9\sec(c+dx)a+13a)dx+\frac{a^2\sin(c+dx)\cos^2(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{6}a\int \cos^2(c+dx)\sqrt{\sec(c+dx)a+a}(9\sec(c+dx)a+13a)dx+\frac{a^2\sin(c+dx)\cos^2(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{6}a\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}\left(9\csc\left(c+dx+\frac{\pi}{2}\right)a+13a\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^2}dx+\frac{a^2\sin(c+dx)\cos^2(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}$$

$$\Big\downarrow \text{4503}$$

$$\frac{1}{6}a\left(\frac{75}{4}a\int \cos(c+dx)\sqrt{\sec(c+dx)a+a}dx+\frac{13a^2\sin(c+dx)\cos(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}\right)+\frac{a^2\sin(c+dx)\cos^2(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{6}a\left(\frac{75}{4}a\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{13a^2\sin(c+dx)\cos(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}\right)+\frac{a^2\sin(c+dx)\cos^2(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}$$

$$\Big\downarrow \text{4292}$$

$$\frac{1}{6}a\left(\frac{75}{4}a\left(\frac{1}{2}\int \sqrt{\sec(c+dx)a+a}dx+\frac{a\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}\right)+\frac{13a^2\sin(c+dx)\cos(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}\right)+\frac{a^2\sin(c+dx)\cos^2(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{6}a\left(\frac{75}{4}a\left(\frac{1}{2}\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx+\frac{a\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}\right)+\frac{13a^2\sin(c+dx)\cos(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}\right)+\frac{a^2\sin(c+dx)\cos^2(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}$$

$$\Big\downarrow \text{4261}$$

$$\frac{1}{6}a\left(\frac{75}{4}a\left(\frac{a\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}-\frac{a\int \frac{1}{\frac{a^2\tan^2(c+dx)}{\sec(c+dx)a+a}+a}d\left(-\frac{a\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}\right)+\frac{13a^2\sin(c+dx)\cos(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}\right)+\frac{a^2\sin(c+dx)\cos^2(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{6}a\left(\frac{13a^2\sin(c+dx)\cos(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}+\frac{75}{4}a\left(\frac{\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{a\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}\right)\right)+\frac{a^2\sin(c+dx)\cos^2(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}$$

3.114. $\int \cos^3(c+dx)(a+a\sec(c+dx))^{5/2}\,dx$

input
```
Int[Cos[c + d*x]^3*(a + a*Sec[c + d*x])^(5/2),x]
```

output
```
(a^2*Cos[c + d*x]^2*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(3*d) + (a*((13
*a^2*Cos[c + d*x]*Sin[c + d*x])/(2*d*Sqrt[a + a*Sec[c + d*x]]) + (75*a*((S
qrt[a]*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d + (a*Sin
[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]])))/4))/6
```

3.114.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4261
```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(b/d)
  Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 4292
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[a*Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*n*Sqrt[a
+ b*Csc[e + f*x]])), x] + Simp[a*((2*n + 1)/(2*b*d*n))   Int[Sqrt[a + b*Csc
[e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
 EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

rule 4300
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)*
((d*Csc[e + f*x])^n/(f*n)), x] - Simp[a/(d*n)   Int[(a + b*Csc[e + f*x])^(m
 - 2)*(d*Csc[e + f*x])^(n + 1)*(b*(m - 2*n - 2) - a*(m + 2*n - 1)*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1]
 && (LtQ[n, -1] || (EqQ[m, 3/2] && EqQ[n, -2^(-1)])) && IntegerQ[2*m]
```

rule 4503
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*b^2*Co
t[e + f*x]*((d*Csc[e + f*x])^n/(a*f*n*Sqrt[a + b*Csc[e + f*x]])), x] + Simp
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n)   Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[
e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a
*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

3.114.4 Maple [A] (verified)

Time = 157.07 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.42

method	result
default	$\frac{a^2\left(8\cos(dx+c)^3\sin(dx+c)+75\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\operatorname{arctanh}\left(\frac{\sin(dx+c)}{(\cos(dx+c)+1)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}}\right)\cos(dx+c)+34\cos(dx+c)^2\sin(dx+c)+75\right.}{24d(\cos(dx+c)+1)}$

input
```
int(cos(d*x+c)^3*(a+a*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/24/d*a^2*(8*cos(d*x+c)^3*sin(d*x+c)+75*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2
)*arctanh(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*co
s(d*x+c)+34*cos(d*x+c)^2*sin(d*x+c)+75*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*
arctanh(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+75*c
os(d*x+c)*sin(d*x+c))*(a*(1+sec(d*x+c)))^(1/2)/(cos(d*x+c)+1)
```

3.114.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 320, normalized size of antiderivative = 2.22

$$\int \cos^3(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \left[\frac{75\,(a^2\cos(dx+c)+a^2)\sqrt{-a}\log\left(\frac{2\,a\cos(dx+c)^2-2\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+a\cos(dx+c)-a}{\cos(dx+c)+1}\right.}{48\,(d\cos(d}\right.$$

$$-\frac{75\,(a^2\cos(dx+c)+a^2)\sqrt{a}\arctan\left(\frac{\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{\sqrt{a}\sin(dx+c)}\right)-\left(8\,a^2\cos(dx+c)^3+34\,a^2\cos(dx+c)^2+75\right.}{24\,(d\cos(dx+c)+d)}$$

input
```
integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
[1/48*(75*(a^2*cos(d*x + c) + a^2)*sqrt(-a)*log((2*a*cos(d*x + c)^2 - 2*sq
rt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) +
 a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) + 2*(8*a^2*cos(d*x + c)^3 + 34*a^
2*cos(d*x + c)^2 + 75*a^2*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x
+ c))*sin(d*x + c))/(d*cos(d*x + c) + d), -1/24*(75*(a^2*cos(d*x + c) + a^
2)*sqrt(a)*arctan(sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sq
rt(a)*sin(d*x + c))) - (8*a^2*cos(d*x + c)^3 + 34*a^2*cos(d*x + c)^2 + 75*
a^2*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(d
*cos(d*x + c) + d)]
```

3.114.6 Sympy [F(-1)]

Timed out.

$$\int \cos^3(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**3*(a+a*sec(d*x+c))**(5/2),x)
```

output
```
Timed out
```

3.114.7 Maxima [F(-1)]

Timed out.

$$\int \cos^3(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
Timed out
```

3.114.8 Giac [F]

$$\int \cos^3(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \int (a\sec(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^3\,dx$$

input
```
integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.114.9 Mupad [F(-1)]

Timed out.

$$\int \cos^3(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \int \cos(c+d\,x)^3\left(a+\frac{a}{\cos(c+d\,x)}\right)^{5/2}dx$$

input
```
int(cos(c + d*x)^3*(a + a/cos(c + d*x))^(5/2),x)
```

output
```
int(cos(c + d*x)^3*(a + a/cos(c + d*x))^(5/2), x)
```

3.115 $\int \cos^4(c+dx)(a+a\sec(c+dx))^{5/2}\,dx$

3.115.1 Optimal result . . . 1049
3.115.2 Mathematica [C] (verified) . . . 1049
3.115.3 Rubi [A] (verified) . . . 1050
3.115.4 Maple [A] (verified) . . . 1053
3.115.5 Fricas [A] (verification not implemented) . . . 1054
3.115.6 Sympy [F(-1)] . . . 1055
3.115.7 Maxima [F(-1)] . . . 1055
3.115.8 Giac [F] . . . 1055
3.115.9 Mupad [F(-1)] . . . 1056

3.115.1 Optimal result

Integrand size = 23, antiderivative size = 182

$$\int \cos^4(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \frac{163a^{5/2}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{64d} + \frac{163a^3\sin(c+dx)}{64d\sqrt{a+a\sec(c+dx)}} + \frac{163a^3\cos(c+dx)\sin(c+dx)}{96d\sqrt{a+a\sec(c+dx)}} + \frac{17a^3\cos^2(c+dx)\sin(c+dx)}{24d\sqrt{a+a\sec(c+dx)}} + \frac{a^2\cos^3(c+dx)\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{4d}$$

output
```
163/64*a^(5/2)*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/d+163/64*
a^3*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+163/96*a^3*cos(d*x+c)*sin(d*x+c)/d
/(a+a*sec(d*x+c))^(1/2)+17/24*a^3*cos(d*x+c)^2*sin(d*x+c)/d/(a+a*sec(d*x+c
))^(1/2)+1/4*a^2*cos(d*x+c)^3*sin(d*x+c)*(a+a*sec(d*x+c))^(1/2)/d
```

3.115.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.82 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.88

$$\int \cos^4(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \frac{a^2\left(675\text{arctanh}\left(\sqrt{1-\sec(c+dx)}\right)+(231+849\cos(c+dx)+233\cos(2(c+dx))+58\cos(3\right.}{}$$

input
```
Integrate[Cos[c + d*x]^4*(a + a*Sec[c + d*x])^(5/2),x]
```

output
```
(a^2*(675*ArcTanh[Sqrt[1 - Sec[c + d*x]]] + (231 + 849*Cos[c + d*x] + 233*
Cos[2*(c + d*x)] + 58*Cos[3*(c + d*x)] + 2*Cos[4*(c + d*x)])*Sqrt[1 - Sec[
c + d*x]] + 512*Hypergeometric2F1[1/2, 5, 3/2, 1 - Sec[c + d*x]]*Sqrt[1 -
Sec[c + d*x]])*Sqrt[a*(1 + Sec[c + d*x])]*Sin[c + d*x])/(320*d*(1 + Cos[c
+ d*x])*Sqrt[1 - Sec[c + d*x]])
```

3.115.3 Rubi [A] (verified)

Time = 0.91 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.04, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.522$, Rules used = {3042, 4300, 27, 3042, 4503, 3042, 4292, 3042, 4292, 3042, 4261, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^4(c+dx)(a\sec(c+dx)+a)^{5/2}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}}{\csc\left(c+dx+\frac{\pi}{2}\right)^4}dx$$

$$\Big\downarrow \; 4300$$

$$\frac{1}{4}a\int \frac{1}{2}\cos^3(c+dx)\sqrt{\sec(c+dx)a+a}(13\sec(c+dx)a+17a)dx+\frac{a^2\sin(c+dx)\cos^3(c+dx)\sqrt{a\sec(c+dx)+a}}{4d}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{8}a\int \cos^3(c+dx)\sqrt{\sec(c+dx)a+a}(13\sec(c+dx)a+17a)dx+\frac{a^2\sin(c+dx)\cos^3(c+dx)\sqrt{a\sec(c+dx)+a}}{4d}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{8}a\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}\left(13\csc\left(c+dx+\frac{\pi}{2}\right)a+17a\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^3}dx+\frac{a^2\sin(c+dx)\cos^3(c+dx)\sqrt{a\sec(c+dx)+a}}{4d}$$

$$\Big\downarrow \text{4503}$$

$$\frac{1}{8}a\left(\frac{163}{6}a\int \cos^2(c+dx)\sqrt{\sec(c+dx)a+a}dx+\frac{17a^2\sin(c+dx)\cos^2(c+dx)}{3d\sqrt{a\sec(c+dx)+a}}\right)+\frac{a^2\sin(c+dx)\cos^3(c+dx)\sqrt{a\sec(c+dx)+a}}{4d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{8}a\left(\frac{163}{6}a\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}{\csc\left(c+dx+\frac{\pi}{2}\right)^2}dx+\frac{17a^2\sin(c+dx)\cos^2(c+dx)}{3d\sqrt{a\sec(c+dx)+a}}\right)+\frac{a^2\sin(c+dx)\cos^3(c+dx)\sqrt{a\sec(c+dx)+a}}{4d}$$

$$\Big\downarrow \text{4292}$$

$$\frac{1}{8}a\left(\frac{163}{6}a\left(\frac{3}{4}\int \cos(c+dx)\sqrt{\sec(c+dx)a+a}dx+\frac{a\sin(c+dx)\cos(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}\right)+\frac{17a^2\sin(c+dx)\cos^2(c+dx)}{3d\sqrt{a\sec(c+dx)+a}}\right)$$
$$\frac{a^2\sin(c+dx)\cos^3(c+dx)\sqrt{a\sec(c+dx)+a}}{4d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{8}a\left(\frac{163}{6}a\left(\frac{3}{4}\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{a\sin(c+dx)\cos(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}\right)+\frac{17a^2\sin(c+dx)\cos^2(c+dx)}{3d\sqrt{a\sec(c+dx)+a}}\right)+$$
$$\frac{a^2\sin(c+dx)\cos^3(c+dx)\sqrt{a\sec(c+dx)+a}}{4d}$$

$$\Big\downarrow \text{4292}$$

$$\frac{1}{8}a\left(\frac{163}{6}a\left(\frac{3}{4}\left(\frac{1}{2}\int \sqrt{\sec(c+dx)a+a}dx+\frac{a\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}\right)+\frac{a\sin(c+dx)\cos(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}\right)+\frac{17a^2\sin(c+}{3d\sqrt{a\sec}}\right.$$
$$\frac{a^2\sin(c+dx)\cos^3(c+dx)\sqrt{a\sec(c+dx)+a}}{4d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{8}a\left(\frac{163}{6}a\left(\frac{3}{4}\left(\frac{1}{2}\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx+\frac{a\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}\right)+\frac{a\sin(c+dx)\cos(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}\right)+\frac{17a^2\,\mathrm{si}}{3d}\right.$$
$$\frac{a^2\sin(c+dx)\cos^3(c+dx)\sqrt{a\sec(c+dx)+a}}{4d}$$

$$\Big\downarrow \text{4261}$$

3.115. $\int \cos^4(c+dx)(a+a\sec(c+dx))^{5/2}\,dx$

$$\frac{1}{8}a\left(\frac{163}{6}a\left(\frac{3}{4}\left(\frac{a\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}-\frac{a\int\frac{1}{\frac{a^2\tan^2(c+dx)}{\sec(c+dx)a+a}+a}d\left(-\frac{a\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}\right)+\frac{a\sin(c+dx)\cos(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}\right)+\frac{1}{}$$

$$\frac{a^2\sin(c+dx)\cos^3(c+dx)\sqrt{a\sec(c+dx)+a}}{4d}$$

$$\Big\downarrow \; 216$$

$$\frac{1}{8}a\left(\frac{17a^2\sin(c+dx)\cos^2(c+dx)}{3d\sqrt{a\sec(c+dx)+a}}+\frac{163}{6}a\left(\frac{3}{4}\left(\frac{\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{a\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}\right)+\frac{a\sin(c+}{2d\sqrt{a}}\right.\right.$$

$$\frac{a^2\sin(c+dx)\cos^3(c+dx)\sqrt{a\sec(c+dx)+a}}{4d}$$

input
```
Int[Cos[c + d*x]^4*(a + a*Sec[c + d*x])^(5/2),x]
```

output
```
(a^2*Cos[c + d*x]^3*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(4*d) + (a*((17
*a^2*Cos[c + d*x]^2*Sin[c + d*x])/(3*d*Sqrt[a + a*Sec[c + d*x]]) + (163*a*
((a*Cos[c + d*x]*Sin[c + d*x])/(2*d*Sqrt[a + a*Sec[c + d*x]]) + (3*((Sqrt[
a]*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d + (a*Sin[c +
 d*x])/(d*Sqrt[a + a*Sec[c + d*x]])))/4))/6))/8
```

3.115.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4261
```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(b/d)
  Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 4292
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[a*Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*n*Sqrt[a
+ b*Csc[e + f*x]])), x] + Simp[a*((2*n + 1)/(2*b*d*n))   Int[Sqrt[a + b*Csc
[e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
 EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

rule 4300
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)*
((d*Csc[e + f*x])^n/(f*n)), x] - Simp[a/(d*n)   Int[(a + b*Csc[e + f*x])^(m
 - 2)*(d*Csc[e + f*x])^(n + 1)*(b*(m - 2*n - 2) - a*(m + 2*n - 1)*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1]
 && (LtQ[n, -1] || (EqQ[m, 3/2] && EqQ[n, -2^(-1)])) && IntegerQ[2*m]
```

rule 4503
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*b^2*Co
t[e + f*x]*((d*Csc[e + f*x])^n/(a*f*n*Sqrt[a + b*Csc[e + f*x]])), x] + Simp
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n)   Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[
e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a
*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

3.115.4 Maple [A] (verified)

Time = 0.94 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.21

$$a^2\left(48\cos(dx+c)^4\sin(dx+c)+184\cos(dx+c)^3\sin(dx+c)+489\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\operatorname{arctanh}\left(\frac{\sin(d}{(\cos(dx+c)+1)\sqrt{}}\right.\right.$$

input
```
int(cos(d*x+c)^4*(a+a*sec(d*x+c))^(5/2),x)
```

output
```
1/192/d*a^2*(48*cos(d*x+c)^4*sin(d*x+c)+184*cos(d*x+c)^3*sin(d*x+c)+489*(-
cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d
*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)+326*cos(d*x+c)^2*sin(d*x+c)+489*(-
cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d
*x+c)/(cos(d*x+c)+1))^(1/2))+489*cos(d*x+c)*sin(d*x+c))*(a*(1+sec(d*x+c)))
^(1/2)/(cos(d*x+c)+1)
```

3.115.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 346, normalized size of antiderivative = 1.90

$$\int \cos^4(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \left[\frac{489\,(a^2\cos(dx+c)+a^2)\sqrt{-a}\log\left(\frac{2\,a\cos(dx+c)^2-2\,\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+a\cos(dx+c)-}{\cos(dx+c)+1}\right.}{}\right.$$

$$-\frac{489\,(a^2\cos(dx+c)+a^2)\sqrt{a}\arctan\left(\frac{\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{\sqrt{a}\sin(dx+c)}\right)-\left(48\,a^2\cos(dx+c)^4+184\,a^2\cos(dx+c)^3-\right.}{192\,(d\cos(dx+c)+d)}$$

input
```
integrate(cos(d*x+c)^4*(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
[1/384*(489*(a^2*cos(d*x + c) + a^2)*sqrt(-a)*log((2*a*cos(d*x + c)^2 - 2*
sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c)
 + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) + 2*(48*a^2*cos(d*x + c)^4 + 18
4*a^2*cos(d*x + c)^3 + 326*a^2*cos(d*x + c)^2 + 489*a^2*cos(d*x + c))*sqrt
((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c) + d), -1
/192*(489*(a^2*cos(d*x + c) + a^2)*sqrt(a)*arctan(sqrt((a*cos(d*x + c) + a
)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) - (48*a^2*cos(d*x + c
)^4 + 184*a^2*cos(d*x + c)^3 + 326*a^2*cos(d*x + c)^2 + 489*a^2*cos(d*x +
c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)
+ d)]
```

3.115.6 Sympy [F(-1)]

Timed out.

$$\int \cos^4(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**4*(a+a*sec(d*x+c))**(5/2),x)
```

output
```
Timed out
```

3.115.7 Maxima [F(-1)]

Timed out.

$$\int \cos^4(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)^4*(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
Timed out
```

3.115.8 Giac [F]

$$\int \cos^4(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \int (a\sec(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^4\,dx$$

input
```
integrate(cos(d*x+c)^4*(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.115.9 Mupad [F(-1)]

Timed out.

$$\int \cos^4(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \int \cos{(c+d\,x)}^4\,{\left(a+\frac{a}{\cos{(c+d\,x)}}\right)}^{5/2}\,dx$$

input
```
int(cos(c + d*x)^4*(a + a/cos(c + d*x))^(5/2),x)
```

output
```
int(cos(c + d*x)^4*(a + a/cos(c + d*x))^(5/2), x)
```

3.116 $\int \sec(c+dx)\sqrt{a-a\sec(c+dx)}\,dx$

3.116.1 Optimal result . . . 1057
3.116.2 Mathematica [A] (verified) . . . 1057
3.116.3 Rubi [A] (verified) . . . 1058
3.116.4 Maple [A] (verified) . . . 1059
3.116.5 Fricas [A] (verification not implemented) . . . 1059
3.116.6 Sympy [F] . . . 1059
3.116.7 Maxima [F] . . . 1060
3.116.8 Giac [B] (verification not implemented) . . . 1060
3.116.9 Mupad [B] (verification not implemented) . . . 1060

3.116.1 Optimal result

Integrand size = 22, antiderivative size = 27

$$\int \sec(c+dx)\sqrt{a-a\sec(c+dx)}\,dx=-\frac{2a\tan(c+dx)}{d\sqrt{a-a\sec(c+dx)}}$$

output
```
-2*a*tan(d*x+c)/d/(a-a*sec(d*x+c))^(1/2)
```

3.116.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.11

$$\int \sec(c+dx)\sqrt{a-a\sec(c+dx)}\,dx=\frac{2\cot\left(\frac{1}{2}(c+dx)\right)\sqrt{a-a\sec(c+dx)}}{d}$$

input
```
Integrate[Sec[c + d*x]*Sqrt[a - a*Sec[c + d*x]],x]
```

output
```
(2*Cot[(c + d*x)/2]*Sqrt[a - a*Sec[c + d*x]])/d
```

3.116.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {3042, 4279}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(c+dx)\sqrt{a-a\sec(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)\sqrt{a-a\csc\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{4279}$$

$$-\frac{2a\tan(c+dx)}{d\sqrt{a-a\sec(c+dx)}}$$

input
```
Int[Sec[c + d*x]*Sqrt[a - a*Sec[c + d*x]],x]
```

output
```
(-2*a*Tan[c + d*x])/(d*Sqrt[a - a*Sec[c + d*x]])
```

3.116.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4279
```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*b*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]])), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

3.116.4 Maple [A] (verified)

Time = 0.86 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.30

method	result	size
default	$-\frac{2\sqrt{-a(\sec(dx+c)-1)}\,\sin(dx+c)}{d(\cos(dx+c)-1)}$	35

input
```
int(sec(d*x+c)*(a-a*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/d*(-a*(sec(d*x+c)-1))^(1/2)*sin(d*x+c)/(cos(d*x+c)-1)
```

3.116.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.63

$$\int \sec(c+dx)\sqrt{a-a\sec(c+dx)}\,dx = \frac{2\sqrt{\frac{a\cos(dx+c)-a}{\cos(dx+c)}}(\cos(dx+c)+1)}{d\sin(dx+c)}$$

input
```
integrate(sec(d*x+c)*(a-a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
2*sqrt((a*cos(d*x + c) - a)/cos(d*x + c))*(cos(d*x + c) + 1)/(d*sin(d*x +
c))
```

3.116.6 Sympy [F]

$$\int \sec(c+dx)\sqrt{a-a\sec(c+dx)}\,dx = \int \sqrt{-a(\sec(c+dx)-1)}\sec(c+dx)\,dx$$

input
```
integrate(sec(d*x+c)*(a-a*sec(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(-a*(sec(c + d*x) - 1))*sec(c + d*x), x)
```

3.116.7 Maxima [F]

$$\int \sec(c+dx)\sqrt{a-a\sec(c+dx)}\,dx = \int \sqrt{-a\sec(dx+c)+a}\sec(dx+c)\,dx$$

input
```
integrate(sec(d*x+c)*(a-a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(-a*sec(d*x + c) + a)*sec(d*x + c), x)
```

3.116.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 57 vs. $2(25) = 50$.

Time = 0.56 (sec) , antiderivative size = 57, normalized size of antiderivative = 2.11

$$\int \sec(c+dx)\sqrt{a-a\sec(c+dx)}\,dx = -\frac{2\sqrt{2}a\operatorname{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)\operatorname{sgn}(\cos(dx+c))}{\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a}d}$$

input
```
integrate(sec(d*x+c)*(a-a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
-2*sqrt(2)*a*sgn(tan(1/2*d*x + 1/2*c)^3 + tan(1/2*d*x + 1/2*c))*sgn(cos(d*
x + c))/(sqrt(a*tan(1/2*d*x + 1/2*c)^2 - a)*d)
```

3.116.9 Mupad [B] (verification not implemented)

Time = 13.59 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.33

$$\int \sec(c+dx)\sqrt{a-a\sec(c+dx)}\,dx = \frac{\sin(c+d\,x)\,\sqrt{a-\frac{a}{\cos(c+d\,x)}}}{d\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}$$

input
```
int((a - a/cos(c + d*x))^(1/2)/cos(c + d*x),x)
```

output
```
(sin(c + d*x)*(a - a/cos(c + d*x))^(1/2))/(d*sin(c/2 + (d*x)/2)^2)
```

3.117 $\int \sqrt{a - a\sec(c + dx)}\,dx$

3.117.1 Optimal result 1061
3.117.2 Mathematica [C] (verified) 1061
3.117.3 Rubi [A] (verified) 1062
3.117.4 Maple [B] (verified) 1063
3.117.5 Fricas [B] (verification not implemented) 1063
3.117.6 Sympy [F] 1064
3.117.7 Maxima [B] (verification not implemented) 1064
3.117.8 Giac [B] (verification not implemented) 1065
3.117.9 Mupad [F(-1)] 1065

3.117.1 Optimal result

Integrand size = 15, antiderivative size = 38

$$\int \sqrt{a - a\sec(c + dx)}\,dx = \frac{2\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a-a\sec(c+dx)}}\right)}{d}$$

output
```
2*arctan(a^(1/2)*tan(d*x+c)/(a-a*sec(d*x+c))^(1/2))*a^(1/2)/d
```

3.117.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.62 (sec) , antiderivative size = 85, normalized size of antiderivative = 2.24

$$\int \sqrt{a - a\sec(c + dx)}\,dx = -\frac{i\sqrt{1+e^{2i(c+dx)}}\left(\operatorname{arcsinh}\left(e^{i(c+dx)}\right) + \operatorname{arctanh}\left(\sqrt{1+e^{2i(c+dx)}}\right)\right)\sqrt{a - a\sec(c+dx)}}{d\left(-1+e^{i(c+dx)}\right)}$$

input
```
Integrate[Sqrt[a - a*Sec[c + d*x]],x]
```

output
```
((-I)*Sqrt[1 + E^((2*I)*(c + d*x))]*(ArcSinh[E^(I*(c + d*x))] + ArcTanh[Sq
rt[1 + E^((2*I)*(c + d*x))]])*Sqrt[a - a*Sec[c + d*x]])/(d*(-1 + E^(I*(c +
 d*x))))
```

3.117.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3042, 4261, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a-a\sec(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{a-a\csc\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{4261}$$

$$\frac{2a\int \frac{1}{\frac{a^2\tan^2(c+dx)}{a-a\sec(c+dx)}+a}d\frac{a\tan(c+dx)}{\sqrt{a-a\sec(c+dx)}}}{d}$$

$$\Big\downarrow \text{216}$$

$$\frac{2\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a-a\sec(c+dx)}}\right)}{d}$$

input
```
Int[Sqrt[a - a*Sec[c + d*x]],x]
```

output
```
(2*Sqrt[a]*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a - a*Sec[c + d*x]]])/d
```

3.117.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4261
```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(b/d)
  Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

3.117.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 72 vs. $2(32) = 64$.

Time = 0.90 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.92

method	result	size
default	$\frac{2\sqrt{-a(\sec(dx+c)-1)}\,\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\arctan\left(\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\right)(\cot(dx+c)+\csc(dx+c))}{d}$	73

input
```
int((a-a*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/d*(-a*(sec(d*x+c)-1))^(1/2)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan((-
cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*(cot(d*x+c)+csc(d*x+c))
```

3.117.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 73 vs. $2(32) = 64$.

Time = 0.30 (sec) , antiderivative size = 182, normalized size of antiderivative = 4.79

$$\int \sqrt{a - a\sec(c+dx)}\,dx = \left[\frac{\sqrt{-a}\log\left(-\frac{4\left(2\cos(dx+c)^3+3\cos(dx+c)^2+\cos(dx+c)\right)\sqrt{-a}\sqrt{\frac{a\cos(dx+c)-a}{\cos(dx+c)}}+\left(8a\cos(dx+c)^2+8a\cos(dx+c)+a\right)\sin(dx+c)}{\sin(dx+c)}\right)}{2d}, -\frac{\sqrt{a}\arctan\left(\frac{2\left(\cos(dx+c)^2+\cos(dx+c)\right)\sqrt{a}\sqrt{\frac{a\cos(dx+c)-a}{\cos(dx+c)}}}{(2a\cos(dx+c)+a)\sin(dx+c)}\right)}{d}\right]$$

input
```
integrate((a-a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
[1/2*sqrt(-a)*log(-(4*(2*cos(d*x + c)^3 + 3*cos(d*x + c)^2 + cos(d*x + c))
*sqrt(-a)*sqrt((a*cos(d*x + c) - a)/cos(d*x + c)) + (8*a*cos(d*x + c)^2 +
8*a*cos(d*x + c) + a)*sin(d*x + c))/sin(d*x + c))/d, -sqrt(a)*arctan(2*(co
s(d*x + c)^2 + cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) - a)/cos(d*x + c
))/((2*a*cos(d*x + c) + a)*sin(d*x + c)))/d]
```

3.117.6 Sympy [F]

$$\int \sqrt{a - a\sec(c + dx)}\,dx = \int \sqrt{-a\sec(c + dx) + a}\,dx$$

input
```
integrate((a-a*sec(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(-a*sec(c + d*x) + a), x)
```

3.117.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 146 vs. $2(32) = 64$.

Time = 0.42 (sec) , antiderivative size = 146, normalized size of antiderivative = 3.84

$$\int \sqrt{a - a\sec(c + dx)}\,dx$$
$$= \frac{\sqrt{a}\arctan\left(\left(\cos(2\,dx + 2\,c)^2 + \sin(2\,dx + 2\,c)^2 + 2\,\cos(2\,dx + 2\,c) + 1\right)^{\frac{1}{4}}\sin\left(\frac{1}{2}\,\arctan(\sin(2\,dx + 2\,c),\right.}{}$$

input
```
integrate((a-a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
sqrt(a)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2
*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) +
sin(d*x + c), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c
) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + co
s(d*x + c))/d
```

3.117.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 65 vs. $2(32) = 64$.

Time = 0.56 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.71

$$\int \sqrt{a - a\sec(c + dx)}\,dx = -\frac{2\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a}}{2\sqrt{a}}\right)\operatorname{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)\operatorname{sgn}(\cos(dx+c))}{d}$$

input
```
integrate((a-a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
-2*sqrt(a)*arctan(1/2*sqrt(2)*sqrt(a*tan(1/2*d*x + 1/2*c)^2 - a)/sqrt(a))*
sgn(tan(1/2*d*x + 1/2*c)^3 + tan(1/2*d*x + 1/2*c))*sgn(cos(d*x + c))/d
```

3.117.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{a - a\sec(c + dx)}\,dx = \int \sqrt{a - \frac{a}{\cos(c + d\,x)}}\,dx$$

input
```
int((a - a/cos(c + d*x))^(1/2),x)
```

output
```
int((a - a/cos(c + d*x))^(1/2), x)
```

3.118 $\int \cos(c+dx)\sqrt{a-a\sec(c+dx)}\,dx$

3.118.1 Optimal result 1066
3.118.2 Mathematica [A] (verified) 1066
3.118.3 Rubi [A] (verified) 1067
3.118.4 Maple [A] (verified) 1068
3.118.5 Fricas [B] (verification not implemented) 1069
3.118.6 Sympy [F] 1069
3.118.7 Maxima [B] (verification not implemented) 1070
3.118.8 Giac [B] (verification not implemented) 1070
3.118.9 Mupad [F(-1)] 1071

3.118.1 Optimal result

Integrand size = 22, antiderivative size = 65

$$\int \cos(c+dx)\sqrt{a-a\sec(c+dx)}\,dx=-\frac{\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a-a\sec(c+dx)}}\right)}{d}+\frac{a\sin(c+dx)}{d\sqrt{a-a\sec(c+dx)}}$$

output
```
-arctan(a^(1/2)*tan(d*x+c)/(a-a*sec(d*x+c))^(1/2))*a^(1/2)/d+a*sin(d*x+c)/
d/(a-a*sec(d*x+c))^(1/2)
```

3.118.2 Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.18

$$\int \cos(c+dx)\sqrt{a-a\sec(c+dx)}\,dx=-\frac{\cot\left(\frac{1}{2}(c+dx)\right)\sqrt{a-a\sec(c+dx)}\left(-\operatorname{arctanh}\left(\sqrt{1+\sec(c+dx)}\right)+\cos(c+dx)\sqrt{1+\sec(c+dx)}\right)}{d\sqrt{1+\sec(c+dx)}}$$

input
```
Integrate[Cos[c + d*x]*Sqrt[a - a*Sec[c + d*x]],x]
```

output
```
-((Cot[(c + d*x)/2]*Sqrt[a - a*Sec[c + d*x]]*(-ArcTanh[Sqrt[1 + Sec[c + d*
x]]] + Cos[c + d*x]*Sqrt[1 + Sec[c + d*x]]))/(d*Sqrt[1 + Sec[c + d*x]]))
```

3.118.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {3042, 4292, 3042, 4261, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(c+dx)\sqrt{a-a\sec(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a-a\csc\left(c+dx+\frac{\pi}{2}\right)}}{\csc\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{4292}$$

$$\frac{a\sin(c+dx)}{d\sqrt{a-a\sec(c+dx)}}-\frac{1}{2}\int \sqrt{a-a\sec(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{a\sin(c+dx)}{d\sqrt{a-a\sec(c+dx)}}-\frac{1}{2}\int \sqrt{a-a\csc\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{4261}$$

$$\frac{a\sin(c+dx)}{d\sqrt{a-a\sec(c+dx)}}-\frac{a\int \frac{1}{\frac{a^2\tan^2(c+dx)}{a-a\sec(c+dx)}+a}d\frac{a\tan(c+dx)}{\sqrt{a-a\sec(c+dx)}}}{d}$$

$$\Big\downarrow \text{216}$$

$$\frac{a\sin(c+dx)}{d\sqrt{a-a\sec(c+dx)}}-\frac{\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a-a\sec(c+dx)}}\right)}{d}$$

input
```
Int[Cos[c + d*x]*Sqrt[a - a*Sec[c + d*x]],x]
```

output
```
-((Sqrt[a]*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a - a*Sec[c + d*x]]])/d) + (
a*Sin[c + d*x])/(d*Sqrt[a - a*Sec[c + d*x]])
```

3.118.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4261
```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(b/d)
  Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 4292
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[a*Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*n*Sqrt[a
+ b*Csc[e + f*x]])), x] + Simp[a*((2*n + 1)/(2*b*d*n))   Int[Sqrt[a + b*Csc
[e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
 EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

3.118.4 Maple [A] (verified)

Time = 22.38 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.26

method	result	size
default	$-\frac{\left(\arctan\left(\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\right)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}+\cos(dx+c)\right)\sqrt{-a(\sec(dx+c)-1)}\,(\cos(dx+c)+1)\csc(dx+c)}{d}$	82

input
```
int(cos(d*x+c)*(a-a*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/d*(arctan((-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*(-cos(d*x+c)/(cos(d*x+c)+
1))^(1/2)+cos(d*x+c))*(-a*(sec(d*x+c)-1))^(1/2)*(cos(d*x+c)+1)*csc(d*x+c)
```

3.118.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 129 vs. $2(57) = 114$.

Time = 0.32 (sec) , antiderivative size = 294, normalized size of antiderivative = 4.52

$$\int \cos(c+dx)\sqrt{a-a\sec(c+dx)}\,dx$$
$$= \left[\frac{\sqrt{-a}\log\left(\frac{4\left(2\cos(dx+c)^3+3\cos(dx+c)^2+\cos(dx+c)\right)\sqrt{-a}\sqrt{\frac{a\cos(dx+c)-a}{\cos(dx+c)}}-\left(8\,a\cos(dx+c)^2+8\,a\cos(dx+c)+a\right)\sin(dx+c)}{\sin(dx+c)}\right)\sin(dx}{4\,d\sin(dx+c)}\right.$$

input
```
integrate(cos(d*x+c)*(a-a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
[1/4*(sqrt(-a)*log((4*(2*cos(d*x + c)^3 + 3*cos(d*x + c)^2 + cos(d*x + c))
*sqrt(-a)*sqrt((a*cos(d*x + c) - a)/cos(d*x + c)) - (8*a*cos(d*x + c)^2 +
8*a*cos(d*x + c) + a)*sin(d*x + c))/sin(d*x + c))*sin(d*x + c) - 4*(cos(d*
x + c)^2 + cos(d*x + c))*sqrt((a*cos(d*x + c) - a)/cos(d*x + c)))/(d*sin(d
*x + c)), 1/2*(sqrt(a)*arctan(2*(cos(d*x + c)^2 + cos(d*x + c))*sqrt(a)*sq
rt((a*cos(d*x + c) - a)/cos(d*x + c))/((2*a*cos(d*x + c) + a)*sin(d*x + c)
))*sin(d*x + c) - 2*(cos(d*x + c)^2 + cos(d*x + c))*sqrt((a*cos(d*x + c) -
 a)/cos(d*x + c)))/(d*sin(d*x + c))]
```

3.118.6 Sympy [F]

$$\int \cos(c+dx)\sqrt{a-a\sec(c+dx)}\,dx = \int \sqrt{-a\left(\sec\left(c+dx\right)-1\right)}\cos\left(c+dx\right)dx$$

input
```
integrate(cos(d*x+c)*(a-a*sec(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(-a*(sec(c + d*x) - 1))*cos(c + d*x), x)
```

3.118.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 791 vs. $2(57) = 114$.

Time = 0.48 (sec) , antiderivative size = 791, normalized size of antiderivative = 12.17

$$\int \cos(c+dx)\sqrt{a-a\sec(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)*(a-a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
-1/4*(2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)
^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(d*x +
 c) - (cos(d*x + c) + 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
) + 1)))*sqrt(a) + sqrt(a)*(arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c
)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2
+ 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c) + 1))) + 1) - arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d*
x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(sin
(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x +
2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c) + 1))) - 1) + arctan2((cos(2*d*x + 2*c)^2 + si
n(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2
+ 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c) + 1)) + 1) - arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 ...
```

3.118.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(57) = 114$.

Time = 0.42 (sec) , antiderivative size = 134, normalized size of antiderivative = 2.06

$$\int \cos(c+dx)\sqrt{a-a\sec(c+dx)}\,dx$$
$$= \frac{\sqrt{2}\left(\sqrt{2}\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a}}{2\sqrt{a}}\right)\operatorname{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)-\frac{2\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-}}{a}\right.}{2d}$$

input
```
integrate(cos(d*x+c)*(a-a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
1/2*sqrt(2)*(sqrt(2)*sqrt(a)*arctan(1/2*sqrt(2)*sqrt(a*tan(1/2*d*x + 1/2*c
)^2 - a)/sqrt(a))*sgn(tan(1/2*d*x + 1/2*c)^3 + tan(1/2*d*x + 1/2*c)) - 2*s
qrt(a*tan(1/2*d*x + 1/2*c)^2 - a)*a*sgn(tan(1/2*d*x + 1/2*c)^3 + tan(1/2*d
*x + 1/2*c))/(a*tan(1/2*d*x + 1/2*c)^2 + a))*sgn(cos(d*x + c))/d
```

3.118.9 Mupad [F(-1)]

Timed out.

$$\int \cos(c+dx)\sqrt{a-a\sec(c+dx)}\,dx=\int \cos\left(c+d\,x\right)\,\sqrt{a-\frac{a}{\cos\left(c+d\,x\right)}}\,dx$$

input
```
int(cos(c + d*x)*(a - a/cos(c + d*x))^(1/2),x)
```

output
```
int(cos(c + d*x)*(a - a/cos(c + d*x))^(1/2), x)
```

3.119 $\int \frac{\sec^4(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$

3.119.1 Optimal result 1072
3.119.2 Mathematica [A] (verified) 1072
3.119.3 Rubi [A] (verified) 1073
3.119.4 Maple [A] (verified) 1076
3.119.5 Fricas [A] (verification not implemented) 1076
3.119.6 Sympy [F] 1077
3.119.7 Maxima [F] 1077
3.119.8 Giac [A] (verification not implemented) 1078
3.119.9 Mupad [F(-1)] 1078

3.119.1 Optimal result

Integrand size = 23, antiderivative size = 140

$$\int \frac{\sec^4(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = -\frac{\sqrt{2}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}d}+\frac{28\tan(c+dx)}{15d\sqrt{a+a\sec(c+dx)}}+\frac{2\sec^2(c+dx)\tan(c+dx)}{5d\sqrt{a+a\sec(c+dx)}}-\frac{2\sqrt{a+a\sec(c+dx)}\tan(c+dx)}{15ad}$$

output
```
-arctan(1/2*a^(1/2)*tan(d*x+c)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))*2^(1/2)/d/a
^(1/2)+28/15*tan(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+2/5*sec(d*x+c)^2*tan(d*x+
c)/d/(a+a*sec(d*x+c))^(1/2)-2/15*(a+a*sec(d*x+c))^(1/2)*tan(d*x+c)/a/d
```

3.119.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.76

$$\int \frac{\sec^4(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \frac{\left(-15\sqrt{2}\text{arctanh}\left(\frac{\sqrt{1-\sec(c+dx)}}{\sqrt{2}}\right)+2\sqrt{1-\sec(c+dx)}(13-\sec(c+dx)+3\sec^2(c+dx))\right)\tan(c+dx)}{15d\sqrt{1-\sec(c+dx)}\sqrt{a(1+\sec(c+dx))}}$$

input
```
Integrate[Sec[c + d*x]^4/Sqrt[a + a*Sec[c + d*x]],x]
```

output
```
((-15*Sqrt[2]*ArcTanh[Sqrt[1 - Sec[c + d*x]]/Sqrt[2]] + 2*Sqrt[1 - Sec[c +
 d*x]]*(13 - Sec[c + d*x] + 3*Sec[c + d*x]^2))*Tan[c + d*x])/(15*d*Sqrt[1
- Sec[c + d*x]]*Sqrt[a*(1 + Sec[c + d*x])])
```

3.119.3 Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.10, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {3042, 4309, 3042, 4498, 27, 3042, 4489, 3042, 4282, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^4(c+dx)}{\sqrt{a\sec(c+dx)+a}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^4}{\sqrt{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}}dx$$

$$\Big\downarrow \text{4309}$$

$$\frac{\int \frac{\sec^2(c+dx)(4a-a\sec(c+dx))}{\sqrt{\sec(c+dx)a+a}}dx}{5a}+\frac{2\tan(c+dx)\sec^2(c+dx)}{5d\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2\left(4a-a\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{5a}+\frac{2\tan(c+dx)\sec^2(c+dx)}{5d\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{4498}$$

$$\frac{\frac{2\int -\frac{\sec(c+dx)\left(a^2-14a^2\sec(c+dx)\right)}{2\sqrt{\sec(c+dx)a+a}}dx}{3a}-\frac{2\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}}{5a}+\frac{2\tan(c+dx)\sec^2(c+dx)}{5d\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{27}$$

$$\frac{-\frac{\int \frac{\sec(c+dx)\left(a^2-14a^2\sec(c+dx)\right)}{\sqrt{\sec(c+dx)a+a}}dx}{3a}-\frac{2\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}}{5a}+\frac{2\tan(c+dx)\sec^2(c+dx)}{5d\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{-\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(a^2-14a^2\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{3a}-\frac{2\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}}{5a}+\frac{2\tan(c+dx)\sec^2(c+dx)}{5d\sqrt{a\sec(c+dx)+a}}$$

$\Big\downarrow$ 4489

$$\frac{-\frac{15a^2\int \frac{\sec(c+dx)}{\sqrt{\sec(c+dx)a+a}}dx-\frac{28a^2\tan(c+dx)}{d\sqrt{a\sec(c+dx)+a}}}{3a}-\frac{2\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}}{5a}+\frac{2\tan(c+dx)\sec^2(c+dx)}{5d\sqrt{a\sec(c+dx)+a}}$$

$\Big\downarrow$ 3042

$$\frac{-\frac{15a^2\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx-\frac{28a^2\tan(c+dx)}{d\sqrt{a\sec(c+dx)+a}}}{3a}-\frac{2\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}}{5a}+\frac{2\tan(c+dx)\sec^2(c+dx)}{5d\sqrt{a\sec(c+dx)+a}}$$

$\Big\downarrow$ 4282

$$\frac{-\frac{-\frac{30a^2\int \frac{1}{\frac{a^2\tan^2(c+dx)}{\sec(c+dx)a+a}+2a}d\left(-\frac{a\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}-\frac{28a^2\tan(c+dx)}{d\sqrt{a\sec(c+dx)+a}}}{3a}-\frac{2\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}}{5a}+\frac{2\tan(c+dx)\sec^2(c+dx)}{5d\sqrt{a\sec(c+dx)+a}}$$

$\Big\downarrow$ 216

$$\frac{-\frac{-\frac{15\sqrt{2}a^{3/2}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{d}-\frac{28a^2\tan(c+dx)}{d\sqrt{a\sec(c+dx)+a}}}{3a}-\frac{2\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}}{5a}+\frac{2\tan(c+dx)\sec^2(c+dx)}{5d\sqrt{a\sec(c+dx)+a}}$$

input
```
Int[Sec[c + d*x]^4/Sqrt[a + a*Sec[c + d*x]],x]
```

output
```
(2*Sec[c + d*x]^2*Tan[c + d*x])/(5*d*Sqrt[a + a*Sec[c + d*x]]) + ((-2*Sqrt
[a + a*Sec[c + d*x]]*Tan[c + d*x])/(3*d) - ((15*Sqrt[2]*a^(3/2)*ArcTan[(Sq
rt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/d - (28*a^2*Tan[c
 + d*x])/(d*Sqrt[a + a*Sec[c + d*x]]))/(3*a))/(5*a)
```

3.119. $\int \frac{\sec^4(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$

3.119.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4282
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2/f   Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[
a + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4309
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[-2*d^2*Cot[e + f*x]*((d*Csc[e + f*x])^(n - 2)/(
f*(2*n - 3)*Sqrt[a + b*Csc[e + f*x]])), x] + Simp[d^2/(b*(2*n - 3))   Int[(
d*Csc[e + f*x])^(n - 2)*((2*b*(n - 2) - a*Csc[e + f*x])/Sqrt[a + b*Csc[e +
f*x]]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[n,
2] && IntegerQ[2*n]
```

rule 4489
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*((
a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Simp[(a*B*m + A*b*(m + 1))/(b*(m +
 1))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B
, e, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b
*(m + 1), 0] &&  !LtQ[m, -2^(-1)]
```

rule 4498
```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*
((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m + 2))   Int
[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*B*(m + 1) + (A*b*(m + 2) - a*B)
*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, m}, x] && NeQ[A*b - a
*B, 0] &&  !LtQ[m, -1]
```

3.119. $\int \frac{\sec^4(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$

3.119.4 Maple [A] (verified)

Time = 1.22 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.38

method	result
default	$-\frac{\sqrt{-\frac{2a}{(1-\cos(dx+c))^2\csc(dx+c)^2-1}}\left(15\ln\left(\csc(dx+c)-\cot(dx+c)+\sqrt{(1-\cos(dx+c))^2\csc(dx+c)^2-1}\right)\left((1-\cos(dx+c))^2\csc(dx+c)^2\right.\right.}{15da\left((1-\cos(dx+c))^2\csc(dx+c)\right.}$

input
```
int(sec(d*x+c)^4/(a+a*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/15/d/a*(-2*a/((1-cos(d*x+c))^2*csc(d*x+c)^2-1))^(1/2)*(15*ln(csc(d*x+c)
-cot(d*x+c)+((1-cos(d*x+c))^2*csc(d*x+c)^2-1)^(1/2))*((1-cos(d*x+c))^2*csc
(d*x+c)^2-1)^(5/2)-34*(1-cos(d*x+c))^5*csc(d*x+c)^5+40*(1-cos(d*x+c))^3*cs
c(d*x+c)^3-30*csc(d*x+c)+30*cot(d*x+c))/((1-cos(d*x+c))^2*csc(d*x+c)^2-1)^
2
```

3.119.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 347, normalized size of antiderivative = 2.48

$$\int \frac{\sec^4(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$$
$$= \left[\frac{15\sqrt{2}\left(a\cos(dx+c)^3+a\cos(dx+c)^2\right)\sqrt{-\frac{1}{a}}\log\left(\frac{2\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{-\frac{1}{a}}\cos(dx+c)\sin(dx+c)+3\cos(dx+c)^2+2\cos(}{\cos(dx+c)^2+2\cos(dx+c)+1}\right.}{30\left(ad\cos(dx+c)^3+ad\cos(dx\right.}\right.$$

input
```
integrate(sec(d*x+c)^4/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
[1/30*(15*sqrt(2)*(a*cos(d*x + c)^3 + a*cos(d*x + c)^2)*sqrt(-1/a)*log((2*
sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(-1/a)*cos(d*x + c)*si
n(d*x + c) + 3*cos(d*x + c)^2 + 2*cos(d*x + c) - 1)/(cos(d*x + c)^2 + 2*co
s(d*x + c) + 1)) + 4*(13*cos(d*x + c)^2 - cos(d*x + c) + 3)*sqrt((a*cos(d*
x + c) + a)/cos(d*x + c))*sin(d*x + c))/(a*d*cos(d*x + c)^3 + a*d*cos(d*x
+ c)^2), 1/15*(2*(13*cos(d*x + c)^2 - cos(d*x + c) + 3)*sqrt((a*cos(d*x +
c) + a)/cos(d*x + c))*sin(d*x + c) + 15*sqrt(2)*(a*cos(d*x + c)^3 + a*cos(
d*x + c)^2)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x
 + c)/(sqrt(a)*sin(d*x + c)))/sqrt(a))/(a*d*cos(d*x + c)^3 + a*d*cos(d*x +
 c)^2)]
```

3.119.6 Sympy [F]

$$\int \frac{\sec^4(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \int \frac{\sec^4(c+dx)}{\sqrt{a\left(\sec(c+dx)+1\right)}}\,dx$$

input
```
integrate(sec(d*x+c)**4/(a+a*sec(d*x+c))**(1/2),x)
```

output
```
Integral(sec(c + d*x)**4/sqrt(a*(sec(c + d*x) + 1)), x)
```

3.119.7 Maxima [F]

$$\int \frac{\sec^4(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \int \frac{\sec(dx+c)^4}{\sqrt{a\sec(dx+c)+a}}\,dx$$

input
```
integrate(sec(d*x+c)^4/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)^4/sqrt(a*sec(d*x + c) + a), x)
```

3.119.8 Giac [A] (verification not implemented)

Time = 1.04 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.07

$$\int \frac{\sec^4(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \frac{\sqrt{2}\left(\frac{15\log\left(\left|-\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right|\right)}{\sqrt{-a}}+\frac{2\left(\left(17a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-20a^2\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+15a^2\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}\right.}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a\right)^2\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}\right.}{15\,d\,\mathrm{sgn}\left(\cos\left(dx+c\right)\right)}$$

input
```
integrate(sec(d*x+c)^4/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
1/15*sqrt(2)*(15*log(abs(-sqrt(-a)*tan(1/2*d*x + 1/2*c) + sqrt(-a*tan(1/2*
d*x + 1/2*c)^2 + a)))/sqrt(-a) + 2*((17*a^2*tan(1/2*d*x + 1/2*c)^2 - 20*a^
2)*tan(1/2*d*x + 1/2*c)^2 + 15*a^2)*tan(1/2*d*x + 1/2*c)/((a*tan(1/2*d*x +
 1/2*c)^2 - a)^2*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)))/(d*sgn(cos(d*x + c)
))
```

3.119.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^4(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \int \frac{1}{\cos\left(c+d\,x\right)^4\,\sqrt{a+\frac{a}{\cos\left(c+d\,x\right)}}}\,dx$$

input
```
int(1/(cos(c + d*x)^4*(a + a/cos(c + d*x))^(1/2)),x)
```

output
```
int(1/(cos(c + d*x)^4*(a + a/cos(c + d*x))^(1/2)), x)
```

3.120 $\int \frac{\sec^3(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$

3.120.1 Optimal result . . . 1079
3.120.2 Mathematica [A] (verified) . . . 1079
3.120.3 Rubi [A] (verified) . . . 1080
3.120.4 Maple [A] (verified) . . . 1082
3.120.5 Fricas [A] (verification not implemented) . . . 1083
3.120.6 Sympy [F] . . . 1083
3.120.7 Maxima [F] . . . 1084
3.120.8 Giac [A] (verification not implemented) . . . 1084
3.120.9 Mupad [F(-1)] . . . 1084

3.120.1 Optimal result

Integrand size = 23, antiderivative size = 104

$$\int \frac{\sec^3(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \frac{\sqrt{2}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}d} - \frac{4\tan(c+dx)}{3d\sqrt{a+a\sec(c+dx)}} + \frac{2\sqrt{a+a\sec(c+dx)}\tan(c+dx)}{3ad}$$

output
```
arctan(1/2*a^(1/2)*tan(d*x+c)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))*2^(1/2)/d/a^
(1/2)-4/3*tan(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+2/3*(a+a*sec(d*x+c))^(1/2)*t
an(d*x+c)/a/d
```

3.120.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.83

$$\int \frac{\sec^3(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = -\frac{\left(-\sqrt{2}\text{arctanh}\left(\frac{\sqrt{1-\sec(c+dx)}}{\sqrt{2}}\right)+\frac{2}{3}(1-\sec(c+dx))^{3/2}\right)\tan(c+dx)}{d\sqrt{1-\sec(c+dx)}\sqrt{a(1+\sec(c+dx))}}$$

input
```
Integrate[Sec[c + d*x]^3/Sqrt[a + a*Sec[c + d*x]],x]
```

output
```
-(((-(Sqrt[2]*ArcTanh[Sqrt[1 - Sec[c + d*x]]/Sqrt[2]]) + (2*(1 - Sec[c + d
*x])^(3/2))/3)*Tan[c + d*x])/(d*Sqrt[1 - Sec[c + d*x]]*Sqrt[a*(1 + Sec[c +
 d*x])]))
```

3.120.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.08, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {3042, 4287, 27, 3042, 4489, 3042, 4282, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^3(c+dx)}{\sqrt{a\sec(c+dx)+a}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^3}{\sqrt{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}}dx$$

$$\Big\downarrow \text{4287}$$

$$\frac{2\int \frac{\sec(c+dx)(a-2a\sec(c+dx))}{2\sqrt{\sec(c+dx)a+a}}dx}{3a}+\frac{2\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3ad}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\sec(c+dx)(a-2a\sec(c+dx))}{\sqrt{\sec(c+dx)a+a}}dx}{3a}+\frac{2\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3ad}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(a-2a\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{3a}+\frac{2\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3ad}$$

$$\Big\downarrow \text{4489}$$

$$\frac{3a\int \frac{\sec(c+dx)}{\sqrt{\sec(c+dx)a+a}}dx-\frac{4a\tan(c+dx)}{d\sqrt{a\sec(c+dx)+a}}}{3a}+\frac{2\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3ad}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3a\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx-\frac{4a\tan(c+dx)}{d\sqrt{a\sec(c+dx)+a}}}{3a}+\frac{2\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3ad}$$

$$\Big\downarrow \; 4282$$

$$\frac{-\frac{6a\int \frac{1}{\frac{a^2\tan^2(c+dx)}{\sec(c+dx)a+a}+2a}d\left(-\frac{a\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}-\frac{4a\tan(c+dx)}{d\sqrt{a\sec(c+dx)+a}}}{3a}+\frac{2\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3ad}$$

$$\Big\downarrow \; 216$$

$$\frac{\frac{3\sqrt{2}\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{d}-\frac{4a\tan(c+dx)}{d\sqrt{a\sec(c+dx)+a}}}{3a}+\frac{2\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3ad}$$

input
```
Int[Sec[c + d*x]^3/Sqrt[a + a*Sec[c + d*x]],x]
```

output
```
(2*Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x])/(3*a*d) + ((3*Sqrt[2]*Sqrt[a]*Ar
cTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/d - (4*a*
Tan[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]]))/(3*a)
```

3.120.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4282
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2/f   Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[
a + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4287
```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(-Cot[e + f*x])*((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2
))), x] + Simp[1/(b*(m + 2))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(b*(
m + 1) - a*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 -
b^2, 0] &&  !LtQ[m, -2^(-1)]
```

rule 4489
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*((
a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Simp[(a*B*m + A*b*(m + 1))/(b*(m +
 1))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B
, e, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b
*(m + 1), 0] &&  !LtQ[m, -2^(-1)]
```

3.120.4 Maple [A] (verified)

Time = 1.05 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.49

method	result
default	$\frac{\sqrt{-\frac{2a}{(1-\cos(dx+c))^2\csc(dx+c)^2-1}}\left(3\ln\left(\csc(dx+c)-\cot(dx+c)+\sqrt{(1-\cos(dx+c))^2\csc(dx+c)^2-1}\right)\left((1-\cos(dx+c))^2\csc(dx+c)^2-1\right.\right.}{3da\left((1-\cos(dx+c))^2\csc(dx+c)^2-1\right)}$

input
```
int(sec(d*x+c)^3/(a+a*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3/d/a*(-2*a/((1-cos(d*x+c))^2*csc(d*x+c)^2-1))^(1/2)*(3*ln(csc(d*x+c)-co
t(d*x+c)+((1-cos(d*x+c))^2*csc(d*x+c)^2-1)^(1/2))*((1-cos(d*x+c))^2*csc(d*
x+c)^2-1)^(3/2)-4*(1-cos(d*x+c))^3*csc(d*x+c)^3)/((1-cos(d*x+c))^2*csc(d*x
+c)^2-1)
```

3.120.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 316, normalized size of antiderivative = 3.04

$$\int \frac{\sec^3(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$$
$$= \left[\frac{3\sqrt{2}\left(a\cos\left(dx+c\right)^2+a\cos\left(dx+c\right)\right)\sqrt{-\frac{1}{a}}\log\left(-\frac{2\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{-\frac{1}{a}}\cos(dx+c)\sin(dx+c)-3\cos(dx+c)^2-2\cos(\right.}{\cos(dx+c)^2+2\cos(dx+c)+1}\right.}{6\left(ad\cos\left(dx+c\right)^2+ad\cos\left(dx+c\right)\right)}\right.$$
$$-\frac{2\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\left(\cos\left(dx+c\right)-1\right)\sin\left(dx+c\right)+\frac{3\sqrt{2}\left(a\cos(dx+c)^2+a\cos(dx+c)\right)\arctan\left(\frac{\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{\sqrt{a}\sin(dx+c)}\right)}{\sqrt{a}}}{3\left(ad\cos\left(dx+c\right)^2+ad\cos\left(dx+c\right)\right)}$$

input
```
integrate(sec(d*x+c)^3/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
[1/6*(3*sqrt(2)*(a*cos(d*x + c)^2 + a*cos(d*x + c))*sqrt(-1/a)*log(-(2*sqr
t(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(-1/a)*cos(d*x + c)*sin(d
*x + c) - 3*cos(d*x + c)^2 - 2*cos(d*x + c) + 1)/(cos(d*x + c)^2 + 2*cos(d
*x + c) + 1)) - 4*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(cos(d*x + c) -
1)*sin(d*x + c))/(a*d*cos(d*x + c)^2 + a*d*cos(d*x + c)), -1/3*(2*sqrt((a*
cos(d*x + c) + a)/cos(d*x + c))*(cos(d*x + c) - 1)*sin(d*x + c) + 3*sqrt(2
)*(a*cos(d*x + c)^2 + a*cos(d*x + c))*arctan(sqrt(2)*sqrt((a*cos(d*x + c)
+ a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c)))/sqrt(a))/(a*d*cos(
d*x + c)^2 + a*d*cos(d*x + c))]
```

3.120.6 Sympy [F]

$$\int \frac{\sec^3(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \int \frac{\sec^3\left(c+dx\right)}{\sqrt{a\left(\sec\left(c+dx\right)+1\right)}}\,dx$$

input
```
integrate(sec(d*x+c)**3/(a+a*sec(d*x+c))**(1/2),x)
```

output
```
Integral(sec(c + d*x)**3/sqrt(a*(sec(c + d*x) + 1)), x)
```

3.120.7 Maxima [F]

$$\int \frac{\sec^3(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \int \frac{\sec(dx+c)^3}{\sqrt{a\sec(dx+c)+a}}\,dx$$

input
```
integrate(sec(d*x+c)^3/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)^3/sqrt(a*sec(d*x + c) + a), x)
```

3.120.8 Giac [A] (verification not implemented)

Time = 1.00 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.09

$$\int \frac{\sec^3(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = -\frac{\sqrt{2}\left(\frac{4\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3}{\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a\right)\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}}+\frac{3\,\log\left(\left|-\sqrt{-a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\right|\right)}{\sqrt{-a}}\right)}{3\,d\,\mathrm{sgn}\left(\cos\left(dx+c\right)\right)}$$

input
```
integrate(sec(d*x+c)^3/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
-1/3*sqrt(2)*(4*a*tan(1/2*d*x + 1/2*c)^3/((a*tan(1/2*d*x + 1/2*c)^2 - a)*s
qrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)) + 3*log(abs(-sqrt(-a)*tan(1/2*d*x + 1/
2*c) + sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)))/sqrt(-a))/(d*sgn(cos(d*x + c)
))
```

3.120.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^3(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \int \frac{1}{\cos\left(c+d\,x\right)^3\,\sqrt{a+\frac{a}{\cos(c+d\,x)}}}\,dx$$

input
```
int(1/(cos(c + d*x)^3*(a + a/cos(c + d*x))^(1/2)),x)
```

output
```
int(1/(cos(c + d*x)^3*(a + a/cos(c + d*x))^(1/2)), x)
```

3.121 $\int \frac{\sec^2(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$

3.121.1 Optimal result 1085
3.121.2 Mathematica [A] (verified) 1085
3.121.3 Rubi [A] (verified) 1086
3.121.4 Maple [A] (verified) 1087
3.121.5 Fricas [A] (verification not implemented) 1088
3.121.6 Sympy [F] 1088
3.121.7 Maxima [F] 1089
3.121.8 Giac [A] (verification not implemented) 1089
3.121.9 Mupad [F(-1)] 1089

3.121.1 Optimal result

Integrand size = 23, antiderivative size = 73

$$\int \frac{\sec^2(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx=-\frac{\sqrt{2}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}d}+\frac{2\tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}}$$

output
```
-arctan(1/2*a^(1/2)*tan(d*x+c)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))*2^(1/2)/d/a
^(1/2)+2*tan(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)
```

3.121.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.14

$$\int \frac{\sec^2(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx=-\frac{\left(\sqrt{2}\text{arctanh}\left(\frac{\sqrt{1-\sec(c+dx)}}{\sqrt{2}}\right)-2\sqrt{1-\sec(c+dx)}\right)\tan(c+dx)}{d\sqrt{1-\sec(c+dx)}\sqrt{a(1+\sec(c+dx))}}$$

input
```
Integrate[Sec[c + d*x]^2/Sqrt[a + a*Sec[c + d*x]],x]
```

output
```
-(((Sqrt[2]*ArcTanh[Sqrt[1 - Sec[c + d*x]]/Sqrt[2]] - 2*Sqrt[1 - Sec[c + d
*x]])*Tan[c + d*x])/(d*Sqrt[1 - Sec[c + d*x]]*Sqrt[a*(1 + Sec[c + d*x])]))
```

3.121.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3042, 4285, 3042, 4282, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(c+dx)}{\sqrt{a\sec(c+dx)+a}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}}dx$$

$$\Big\downarrow \text{4285}$$

$$\frac{2\tan(c+dx)}{d\sqrt{a\sec(c+dx)+a}}-\int \frac{\sec(c+dx)}{\sqrt{\sec(c+dx)a+a}}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\tan(c+dx)}{d\sqrt{a\sec(c+dx)+a}}-\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx$$

$$\Big\downarrow \text{4282}$$

$$\frac{2\int \frac{1}{\frac{a^2\tan^2(c+dx)}{\sec(c+dx)a+a}+2a}d\left(-\frac{a\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}+\frac{2\tan(c+dx)}{d\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{216}$$

$$\frac{2\tan(c+dx)}{d\sqrt{a\sec(c+dx)+a}}-\frac{\sqrt{2}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}$$

input
```
Int[Sec[c + d*x]^2/Sqrt[a + a*Sec[c + d*x]],x]
```

output
```
-((Sqrt[2]*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]]
)])/(Sqrt[a]*d)) + (2*Tan[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]])
```

3.121.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4282
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2/f   Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[
a + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4285
```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(-Cot[e + f*x])*((a + b*Csc[e + f*x])^m/(f*(m + 1))), x]
+ Simp[a*(m/(b*(m + 1)))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x]
/; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

3.121.4 Maple [A] (verified)

Time = 0.97 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.53

method	result
default	$-\frac{\sqrt{a(1+\sec(dx+c))}\left(\ln\left(\csc(dx+c)-\cot(dx+c)+\sqrt{\cot(dx+c)^2-2\cot(dx+c)\csc(dx+c)+\csc(dx+c)^2-1}\right)\sqrt{2}\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}+2\,\mathrm{c}\right.}{da}$

input
```
int(sec(d*x+c)^2/(a+a*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/d/a*(a*(1+sec(d*x+c)))^(1/2)*(ln(csc(d*x+c)-cot(d*x+c)+(cot(d*x+c)^2-2*
cot(d*x+c)*csc(d*x+c)+csc(d*x+c)^2-1)^(1/2))*2^(1/2)*(-cos(d*x+c)/(cos(d*x
+c)+1))^(1/2)+2*cot(d*x+c)-2*csc(d*x+c))
```

3.121.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 262, normalized size of antiderivative = 3.59

$$\int \frac{\sec^2(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$$
$$= \left[\frac{\sqrt{2}(a\cos(dx+c)+a)\sqrt{-\frac{1}{a}}\log\left(\frac{2\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{-\frac{1}{a}}\cos(dx+c)\sin(dx+c)+3\cos(dx+c)^2+2\cos(dx+c)-1}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)+4\sqrt{\frac{a\cdots}{}}}{2(ad\cos(dx+c)+ad)}\right.$$

input
```
integrate(sec(d*x+c)^2/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
[1/2*(sqrt(2)*(a*cos(d*x + c) + a)*sqrt(-1/a)*log((2*sqrt(2)*sqrt((a*cos(d
*x + c) + a)/cos(d*x + c))*sqrt(-1/a)*cos(d*x + c)*sin(d*x + c) + 3*cos(d*
x + c)^2 + 2*cos(d*x + c) - 1)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + 4*
sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(a*d*cos(d*x + c) +
a*d), (sqrt(2)*(a*cos(d*x + c) + a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) +
a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c)))/sqrt(a) + 2*sqrt((a*
cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(a*d*cos(d*x + c) + a*d)]
```

3.121.6 Sympy [F]

$$\int \frac{\sec^2(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \int \frac{\sec^2(c+dx)}{\sqrt{a(\sec(c+dx)+1)}}\,dx$$

input
```
integrate(sec(d*x+c)**2/(a+a*sec(d*x+c))**(1/2),x)
```

output
```
Integral(sec(c + d*x)**2/sqrt(a*(sec(c + d*x) + 1)), x)
```

3.121.7 Maxima [F]

$$\int \frac{\sec^2(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \int \frac{\sec(dx+c)^2}{\sqrt{a\sec(dx+c)+a}}\,dx$$

input
```
integrate(sec(d*x+c)^2/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)^2/sqrt(a*sec(d*x + c) + a), x)
```

3.121.8 Giac [A] (verification not implemented)

Time = 0.99 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.48

$$\int \frac{\sec^2(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \frac{\sqrt{2}\left(\frac{\log\left(\left|-\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right|\right)}{\sqrt{-a}} - \frac{2\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a}\right)}{d\,\mathrm{sgn}\left(\cos\left(dx+c\right)\right)}$$

input
```
integrate(sec(d*x+c)^2/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
sqrt(2)*(log(abs(-sqrt(-a)*tan(1/2*d*x + 1/2*c) + sqrt(-a*tan(1/2*d*x + 1/
2*c)^2 + a)))/sqrt(-a) - 2*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*tan(1/2*d*x
 + 1/2*c)/(a*tan(1/2*d*x + 1/2*c)^2 - a))/(d*sgn(cos(d*x + c)))
```

3.121.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^2(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \int \frac{1}{\cos\left(c+d\,x\right)^2\,\sqrt{a+\frac{a}{\cos(c+d\,x)}}}\,dx$$

input
```
int(1/(cos(c + d*x)^2*(a + a/cos(c + d*x))^(1/2)),x)
```

output
```
int(1/(cos(c + d*x)^2*(a + a/cos(c + d*x))^(1/2)), x)
```

3.122 $\int \frac{\sec(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$

3.122.1 Optimal result 1090
3.122.2 Mathematica [A] (verified) 1090
3.122.3 Rubi [A] (verified) 1091
3.122.4 Maple [B] (verified) 1092
3.122.5 Fricas [A] (verification not implemented) 1092
3.122.6 Sympy [F] 1093
3.122.7 Maxima [F] 1093
3.122.8 Giac [A] (verification not implemented) 1093
3.122.9 Mupad [F(-1)] 1094

3.122.1 Optimal result

Integrand size = 21, antiderivative size = 46

$$\int \frac{\sec(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx=\frac{\sqrt{2}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}d}$$

output
```
arctan(1/2*a^(1/2)*tan(d*x+c)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))*2^(1/2)/d/a^
(1/2)
```

3.122.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.39

$$\int \frac{\sec(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx=\frac{\sqrt{2}\text{arctanh}\left(\frac{\sqrt{1-\sec(c+dx)}}{\sqrt{2}}\right)\tan(c+dx)}{d\sqrt{1-\sec(c+dx)}\sqrt{a(1+\sec(c+dx))}}$$

input
```
Integrate[Sec[c + d*x]/Sqrt[a + a*Sec[c + d*x]],x]
```

output
```
(Sqrt[2]*ArcTanh[Sqrt[1 - Sec[c + d*x]]/Sqrt[2]]*Tan[c + d*x])/(d*Sqrt[1 -
 Sec[c + d*x]]*Sqrt[a*(1 + Sec[c + d*x])])
```

3.122.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3042, 4282, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(c+dx)}{\sqrt{a\sec(c+dx)+a}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}}dx$$

$$\Big\downarrow \text{ 4282}$$

$$-\frac{2\int \frac{1}{\frac{a^2\tan^2(c+dx)}{\sec(c+dx)a+a}+2a}d\left(-\frac{a\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}$$

$$\Big\downarrow \text{ 216}$$

$$\frac{\sqrt{2}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}$$

input
```
Int[Sec[c + d*x]/Sqrt[a + a*Sec[c + d*x]],x]
```

output
```
(Sqrt[2]*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])]
)/(Sqrt[a]*d)
```

3.122.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4282
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2/f   Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[
a + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

3.122.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 92 vs. $2(37) = 74$.

Time = 0.88 (sec) , antiderivative size = 93, normalized size of antiderivative = 2.02

method	result	size
default	$\frac{\sqrt{a(1+\sec(dx+c))}\,\sqrt{2}\,\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\ln\left(\csc(dx+c)-\cot(dx+c)+\sqrt{\cot(dx+c)^2-2\cot(dx+c)\csc(dx+c)+\csc(dx+c)^2-1}\right)}{da}$	93

input
```
int(sec(d*x+c)/(a+a*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/d/a*(a*(1+sec(d*x+c)))^(1/2)*2^(1/2)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*
ln(csc(d*x+c)-cot(d*x+c)+(cot(d*x+c)^2-2*cot(d*x+c)*csc(d*x+c)+csc(d*x+c)^
2-1)^(1/2))
```

3.122.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 158, normalized size of antiderivative = 3.43

$$\int \frac{\sec(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \left[\frac{\sqrt{2}\sqrt{-\frac{1}{a}}\log\left(-\frac{2\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{-\frac{1}{a}}\cos(dx+c)\sin(dx+c)-3\cos(dx+c)^2-2\cos(dx+c)+1}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)}{2d},\ -\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{\sqrt{a}\sin(dx+c)}\right)}{\sqrt{a}d}\right]$$

input
```
integrate(sec(d*x+c)/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
[1/2*sqrt(2)*sqrt(-1/a)*log(-(2*sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x
+ c))*sqrt(-1/a)*cos(d*x + c)*sin(d*x + c) - 3*cos(d*x + c)^2 - 2*cos(d*x
+ c) + 1)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1))/d, -sqrt(2)*arctan(sqrt(2
)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x +
c)))/(sqrt(a)*d)]
```

3.122.6 Sympy [F]

$$\int \frac{\sec(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \int \frac{\sec(c+dx)}{\sqrt{a(\sec(c+dx)+1)}}\,dx$$

input
```
integrate(sec(d*x+c)/(a+a*sec(d*x+c))**(1/2),x)
```

output
```
Integral(sec(c + d*x)/sqrt(a*(sec(c + d*x) + 1)), x)
```

3.122.7 Maxima [F]

$$\int \frac{\sec(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \int \frac{\sec(dx+c)}{\sqrt{a\sec(dx+c)+a}}\,dx$$

input
```
integrate(sec(d*x+c)/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)/sqrt(a*sec(d*x + c) + a), x)
```

3.122.8 Giac [A] (verification not implemented)

Time = 0.65 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.28

$$\int \frac{\sec(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = -\frac{\sqrt{2}\log\left(\left|-\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right|\right)}{\sqrt{-a}d\operatorname{sgn}(\cos(dx+c))}$$

input
```
integrate(sec(d*x+c)/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
-sqrt(2)*log(abs(-sqrt(-a)*tan(1/2*d*x + 1/2*c) + sqrt(-a*tan(1/2*d*x + 1/
2*c)^2 + a)))/(sqrt(-a)*d*sgn(cos(d*x + c)))
```

3.122.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx=\int \frac{1}{\cos\left(c+d\,x\right)\,\sqrt{a+\frac{a}{\cos\left(c+d\,x\right)}}}\,dx$$

input
```
int(1/(cos(c + d*x)*(a + a/cos(c + d*x))^(1/2)),x)
```

output
```
int(1/(cos(c + d*x)*(a + a/cos(c + d*x))^(1/2)), x)
```

3.123 $\int \frac{1}{\sqrt{a+a\sec(c+dx)}}\,dx$

3.123.1 Optimal result 1095
3.123.2 Mathematica [A] (verified) 1095
3.123.3 Rubi [A] (verified) 1096
3.123.4 Maple [A] (verified) 1098
3.123.5 Fricas [A] (verification not implemented) 1098
3.123.6 Sympy [F] 1099
3.123.7 Maxima [C] (verification not implemented) 1099
3.123.8 Giac [F] 1100
3.123.9 Mupad [F(-1)] 1101

3.123.1 Optimal result

Integrand size = 14, antiderivative size = 85

$$\int \frac{1}{\sqrt{a+a\sec(c+dx)}}\,dx=\frac{2\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}d}-\frac{\sqrt{2}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}d}$$

output
```
2*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/d/a^(1/2)-arctan(1/2*a
^(1/2)*tan(d*x+c)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))*2^(1/2)/d/a^(1/2)
```

3.123.2 Mathematica [A] (verified)

Time = 0.80 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.53

$$\int \frac{1}{\sqrt{a+a\sec(c+dx)}}\,dx=-\frac{2\left(\arcsin\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)-\sqrt{2}\arctan\left(\frac{\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}}\right)\right)\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\sqrt{\sec(c+dx)}\sqrt{1+\sec(c+dx)}}{d\sqrt{\sec^2\left(\frac{1}{2}(c+dx)\right)}\sqrt{a(1+\sec(c+dx))}}$$

input
```
Integrate[1/Sqrt[a + a*Sec[c + d*x]],x]
```

output
```
(-2*(ArcSin[Tan[(c + d*x)/2]] - Sqrt[2]*ArcTan[Tan[(c + d*x)/2]/Sqrt[Cos[c
 + d*x]/(1 + Cos[c + d*x])]])*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[S
ec[c + d*x]]*Sqrt[1 + Sec[c + d*x]])/(d*Sqrt[Sec[(c + d*x)/2]^2]*Sqrt[a*(1
 + Sec[c + d*x])])
```

3.123.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 4263, 3042, 4261, 216, 4282, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a\sec(c+dx)+a}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}}dx$$

$$\Big\downarrow \text{4263}$$

$$\frac{\int \sqrt{\sec(c+dx)a+a}dx}{a}-\int \frac{\sec(c+dx)}{\sqrt{\sec(c+dx)a+a}}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{a}-\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx$$

$$\Big\downarrow \text{4261}$$

$$-\frac{2\int \frac{1}{\frac{a^2\tan^2(c+dx)}{\sec(c+dx)a+a}+a}d\left(-\frac{a\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}-\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx$$

$$\Big\downarrow \text{216}$$

$$\frac{2\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}-\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx$$

$$\Big\downarrow \text{4282}$$

$$\frac{2\int \frac{1}{\frac{a^2\tan^2(c+dx)}{\sec(c+dx)a+a}+2a}d\left(-\frac{a\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}+\frac{2\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}$$

$$\Big\downarrow \; 216$$

$$\frac{2\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}-\frac{\sqrt{2}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}$$

input
```
Int[1/Sqrt[a + a*Sec[c + d*x]],x]
```

output
```
(2*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(Sqrt[a]*d) -
(Sqrt[2]*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])]
)/(Sqrt[a]*d)
```

3.123.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4261
```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(b/d)
  Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 4263
```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[1/a   I
nt[Sqrt[a + b*Csc[c + d*x]], x], x] - Simp[b/a   Int[Csc[c + d*x]/Sqrt[a +
b*Csc[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 4282
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2/f   Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[
a + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

3.123.4 Maple [A] (verified)

Time = 0.86 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.60

method	result
default	$\frac{\sqrt{a(1+\sec(dx+c))}\,\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\left(2\,\operatorname{arctanh}\left(\frac{\sin(dx+c)}{(\cos(dx+c)+1)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}}\right)-\ln\left(\csc(dx+c)-\cot(dx+c)+\sqrt{\cot(dx+c)^2-2\cot}\right.\right.}{da}$

input
```
int(1/(a+a*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/d/a*(a*(1+sec(d*x+c)))^(1/2)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(2*arcta
nh(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))-ln(csc(d*
x+c)-cot(d*x+c)+(cot(d*x+c)^2-2*cot(d*x+c)*csc(d*x+c)+csc(d*x+c)^2-1)^(1/2
))*2^(1/2))
```

3.123.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 294, normalized size of antiderivative = 3.46

$$\int \frac{1}{\sqrt{a+a\sec(c+dx)}}\,dx = \left[\frac{\sqrt{2}a\sqrt{-\frac{1}{a}}\log\left(\frac{2\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{-\frac{1}{a}}\cos(dx+c)\sin(dx+c)+3\cos(dx+c)^2+2\cos(dx+c)-1}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)-2\sqrt{-a}\log\left(\frac{2a\cos(dx+c)^2+}{}\right.}{2ad}\right.$$

input
```
integrate(1/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
[1/2*(sqrt(2)*a*sqrt(-1/a)*log((2*sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*
x + c))*sqrt(-1/a)*cos(d*x + c)*sin(d*x + c) + 3*cos(d*x + c)^2 + 2*cos(d*
x + c) - 1)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) - 2*sqrt(-a)*log((2*a*c
os(d*x + c)^2 + 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x
 + c)*sin(d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)))/(a*d), (sqrt
(2)*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x
 + c)/(sqrt(a)*sin(d*x + c))) - 2*sqrt(a)*arctan(sqrt((a*cos(d*x + c) + a)
/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))))/(a*d)]
```

3.123.6 Sympy [F]

$$\int \frac{1}{\sqrt{a + a\sec(c + dx)}}\,dx = \int \frac{1}{\sqrt{a\sec(c + dx) + a}}\,dx$$

input
```
integrate(1/(a+a*sec(d*x+c))**(1/2),x)
```

output
```
Integral(1/sqrt(a*sec(c + d*x) + a), x)
```

3.123.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.68 (sec) , antiderivative size = 698, normalized size of antiderivative = 8.21

$$\int \frac{1}{\sqrt{a + a\sec(c + dx)}}\,dx =$$

$$-\frac{\sqrt{2}\sqrt{a}\arctan\left(\frac{\left(\left|2\,e^{(i\,dx+i\,c)}+2\right|^4+16\,\cos(dx+c)^4+16\,\sin(dx+c)^4+8\left(\cos(dx+c)^2-\sin(dx+c)^2-2\,\cos(dx+c)+1\right)\left|2\,e^{(i\,dx+i\,c)}+2\right|^2-6\right.}{}\right.}{}$$

input
```
integrate(1/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
-(sqrt(2)*sqrt(a)*arctan2(((abs(2*e^(I*d*x + I*c) + 2)^4 + 16*cos(d*x + c)
^4 + 16*sin(d*x + c)^4 + 8*(cos(d*x + c)^2 - sin(d*x + c)^2 - 2*cos(d*x +
c) + 1)*abs(2*e^(I*d*x + I*c) + 2)^2 - 64*cos(d*x + c)^3 + 32*(cos(d*x + c
)^2 - 2*cos(d*x + c) + 1)*sin(d*x + c)^2 + 96*cos(d*x + c)^2 - 64*cos(d*x
+ c) + 16)^(1/4)*sin(1/2*arctan2(8*(cos(d*x + c) - 1)*sin(d*x + c)/abs(2*e
^(I*d*x + I*c) + 2)^2, (abs(2*e^(I*d*x + I*c) + 2)^2 + 4*cos(d*x + c)^2 -
4*sin(d*x + c)^2 - 8*cos(d*x + c) + 4)/abs(2*e^(I*d*x + I*c) + 2)^2)) + 2*
sin(d*x + c))/abs(2*e^(I*d*x + I*c) + 2), ((abs(2*e^(I*d*x + I*c) + 2)^4 +
 16*cos(d*x + c)^4 + 16*sin(d*x + c)^4 + 8*(cos(d*x + c)^2 - sin(d*x + c)^
2 - 2*cos(d*x + c) + 1)*abs(2*e^(I*d*x + I*c) + 2)^2 - 64*cos(d*x + c)^3 +
 32*(cos(d*x + c)^2 - 2*cos(d*x + c) + 1)*sin(d*x + c)^2 + 96*cos(d*x + c)
^2 - 64*cos(d*x + c) + 16)^(1/4)*cos(1/2*arctan2(8*(cos(d*x + c) - 1)*sin(
d*x + c)/abs(2*e^(I*d*x + I*c) + 2)^2, (abs(2*e^(I*d*x + I*c) + 2)^2 + 4*c
os(d*x + c)^2 - 4*sin(d*x + c)^2 - 8*cos(d*x + c) + 4)/abs(2*e^(I*d*x + I*
c) + 2)^2)) + 2*cos(d*x + c) - 2)/abs(2*e^(I*d*x + I*c) + 2)) - sqrt(a)*ar
ctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(
1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + sin(d*x +
c), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/
4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + cos(d*x + c)
))/(a*d)
```

3.123.8 Giac [F]

$$\int \frac{1}{\sqrt{a+a\sec(c+dx)}}\,dx=\int \frac{1}{\sqrt{a\sec\left(dx+c\right)+a}}\,dx$$

input
```
integrate(1/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.123.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+a\sec(c+dx)}}\,dx=\int \frac{1}{\sqrt{a+\frac{a}{\cos(c+d\,x)}}}\,dx$$

input
```
int(1/(a + a/cos(c + d*x))^(1/2),x)
```

output
```
int(1/(a + a/cos(c + d*x))^(1/2), x)
```

3.124 $\int \frac{\cos(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$

3.124.1 Optimal result 1102
3.124.2 Mathematica [A] (verified) 1102
3.124.3 Rubi [A] (verified) 1103
3.124.4 Maple [A] (verified) 1105
3.124.5 Fricas [A] (verification not implemented) 1106
3.124.6 Sympy [F] 1106
3.124.7 Maxima [F] 1107
3.124.8 Giac [B] (verification not implemented) 1107
3.124.9 Mupad [F(-1)] 1108

3.124.1 Optimal result

Integrand size = 21, antiderivative size = 108

$$\int \frac{\cos(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = -\frac{\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}d} + \frac{\sqrt{2}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}d} + \frac{\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}}$$

output
```
-arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/d/a^(1/2)+arctan(1/2*a^
(1/2)*tan(d*x+c)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))*2^(1/2)/d/a^(1/2)+sin(d*x
+c)/d/(a+a*sec(d*x+c))^(1/2)
```

3.124.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.97

$$\int \frac{\cos(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = -\frac{\left(\operatorname{arctanh}\left(\sqrt{1-\sec(c+dx)}\right)-\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{1-\sec(c+dx)}}{\sqrt{2}}\right)-\cos(c+dx)\sqrt{1-\sec(c+dx)}\right)\tan(c+d}{d\sqrt{1-\sec(c+dx)}\sqrt{a(1+\sec(c+dx))}}$$

input
```
Integrate[Cos[c + d*x]/Sqrt[a + a*Sec[c + d*x]],x]
```

output
```
-(((ArcTanh[Sqrt[1 - Sec[c + d*x]]] - Sqrt[2]*ArcTanh[Sqrt[1 - Sec[c + d*x
]]/Sqrt[2]] - Cos[c + d*x]*Sqrt[1 - Sec[c + d*x]])*Tan[c + d*x])/(d*Sqrt[1
 - Sec[c + d*x]]*Sqrt[a*(1 + Sec[c + d*x])]))
```

3.124.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.01, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {3042, 4310, 3042, 4392, 3042, 4375, 383, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(c+dx)}{\sqrt{a\sec(c+dx)+a}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)\sqrt{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}}dx$$

$$\Big\downarrow \text{4310}$$

$$\frac{\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}-\frac{\int \frac{a-a\sec(c+dx)}{\sqrt{\sec(c+dx)a+a}}dx}{2a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}-\frac{\int \frac{a-a\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{2a}$$

$$\Big\downarrow \text{4392}$$

$$\frac{1}{2}a\int \frac{\tan^2(c+dx)}{(\sec(c+dx)a+a)^{3/2}}dx+\frac{\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}a\int \frac{\cot\left(c+dx+\frac{\pi}{2}\right)^2}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}}dx+\frac{\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{4375}$$

$$\frac{\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}-\frac{a\int\frac{\tan^2(c+dx)}{(\sec(c+dx)a+a)\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1\right)\left(\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+2\right)}d\left(-\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}$$

$$\Big\downarrow \; 383$$

$$\frac{\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}-\frac{a\left(\frac{2\int\frac{1}{\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+2}d\left(-\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{a}-\frac{\int\frac{1}{\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1}d\left(-\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{a}\right)}{d}$$

$$\Big\downarrow \; 216$$

$$\frac{\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}-\frac{a\left(\frac{\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{3/2}}-\frac{\sqrt{2}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{a^{3/2}}\right)}{d}$$

input
```
Int[Cos[c + d*x]/Sqrt[a + a*Sec[c + d*x]],x]
```

output
```
-((a*(ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]/a^(3/2) - (S
qrt[2]*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/
a^(3/2)))/d) + Sin[c + d*x]/(d*Sqrt[a + a*Sec[c + d*x]])
```

3.124.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 383
```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_Sym
bol] :> Simp[(-a)*(e^2/(b*c - a*d))   Int[(e*x)^(m - 2)/(a + b*x^2), x], x]
 + Simp[c*(e^2/(b*c - a*d))   Int[(e*x)^(m - 2)/(c + d*x^2), x], x] /; Free
Q[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && LeQ[2, m, 3]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4310
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*n*Sqrt[a +
b*Csc[e + f*x]])), x] + Simp[1/(2*b*d*n)   Int[(d*Csc[e + f*x])^(n + 1)*((a
 + b*(2*n + 1)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, 0] && IntegerQ[2*n]
```

rule 4375
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Simp[-2*(a^(m/2 + n + 1/2)/d)   Subst[Int[x^m*((2 + a*x^2
)^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]
]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && I
ntegerQ[n - 1/2]
```

rule 4392
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[((-a)*c)^m   Int[Cot[e + f*x]^(2*m)*(
c + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && E
qQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(
IntegerQ[n] && GtQ[m - n, 0])
```

3.124.4 Maple [A] (verified)

Time = 23.94 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.62

method	result
default	$\frac{\sqrt{a(1+\sec(dx+c))}\left(\ln\left(\csc(dx+c)-\cot(dx+c)+\sqrt{\cot(dx+c)^2-2\cot(dx+c)\csc(dx+c)+\csc(dx+c)^2-1}\right)\sqrt{2}\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}-\sqrt{-}\right.}{da}$

input
```
int(cos(d*x+c)/(a+a*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/d/a*(a*(1+sec(d*x+c)))^(1/2)*(ln(csc(d*x+c)-cot(d*x+c)+(cot(d*x+c)^2-2*c
ot(d*x+c)*csc(d*x+c)+csc(d*x+c)^2-1)^(1/2))*2^(1/2)*(-cos(d*x+c)/(cos(d*x+
c)+1))^(1/2)-(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(sin(d*x+c)/(cos(d*
x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))-cot(d*x+c)*cos(d*x+c)+cot(d*x+
c))
```

3.124.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 417, normalized size of antiderivative = 3.86

$$\int \frac{\cos(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$$
$$= \left[\frac{\sqrt{2}(a\cos(dx+c)+a)\sqrt{-\frac{1}{a}}\log\left(-\frac{2\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{-\frac{1}{a}}\cos(dx+c)\sin(dx+c)-3\cos(dx+c)^2-2\cos(dx+c)+1}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)-\sqrt{-}}{2}\right.$$

input
```
integrate(cos(d*x+c)/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
[1/2*(sqrt(2)*(a*cos(d*x + c) + a)*sqrt(-1/a)*log(-(2*sqrt(2)*sqrt((a*cos(
d*x + c) + a)/cos(d*x + c))*sqrt(-1/a)*cos(d*x + c)*sin(d*x + c) - 3*cos(d
*x + c)^2 - 2*cos(d*x + c) + 1)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) - s
qrt(-a)*(cos(d*x + c) + 1)*log((2*a*cos(d*x + c)^2 - 2*sqrt(-a)*sqrt((a*co
s(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) + a*cos(d*x + c) -
 a)/(cos(d*x + c) + 1)) + 2*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*
x + c)*sin(d*x + c))/(a*d*cos(d*x + c) + a*d), (sqrt(a)*(cos(d*x + c) + 1)
*arctan(sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(
d*x + c))) + sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x
+ c) - sqrt(2)*(a*cos(d*x + c) + a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) +
a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c)))/sqrt(a))/(a*d*cos(d*
x + c) + a*d)]
```

3.124.6 Sympy [F]

$$\int \frac{\cos(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \int \frac{\cos(c+dx)}{\sqrt{a(\sec(c+dx)+1)}}\,dx$$

input
```
integrate(cos(d*x+c)/(a+a*sec(d*x+c))**(1/2),x)
```

output
```
Integral(cos(c + d*x)/sqrt(a*(sec(c + d*x) + 1)), x)
```

3.124.7 Maxima [F]

$$\int \frac{\cos(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \int \frac{\cos(dx+c)}{\sqrt{a\sec(dx+c)+a}}\,dx$$

input
```
integrate(cos(d*x+c)/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(cos(d*x + c)/sqrt(a*sec(d*x + c) + a), x)
```

3.124.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 326 vs. $2(91) = 182$.

Time = 1.01 (sec) , antiderivative size = 326, normalized size of antiderivative = 3.02

$$\int \frac{\cos(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$$
$$= \frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt{-a}\log\left(\frac{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2-4\sqrt{2}|a|-6a\right|}{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2+4\sqrt{2}|a|-6a\right|}\right)}{|a|} - \frac{2\log\left(\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+}\right.\right.}{\sqrt{-a}}\right.}{4\,d\operatorname{sgn}(\cos(dx+}$$

input
```
integrate(cos(d*x+c)/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
1/4*sqrt(2)*(sqrt(2)*sqrt(-a)*log(abs(2*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - s
qrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/abs(2*(sqr
t(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + 4*sq
rt(2)*abs(a) - 6*a))/abs(a) - 2*log((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(
-a*tan(1/2*d*x + 1/2*c)^2 + a))^2)/sqrt(-a) - 8*(3*(sqrt(-a)*tan(1/2*d*x +
 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*sqrt(-a) - sqrt(-a)*a)/((
sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4 - 6
*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a
 + a^2))/(d*sgn(cos(d*x + c)))
```

3.124.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx=\int \frac{\cos\left(c+d\,x\right)}{\sqrt{a+\frac{a}{\cos\left(c+d\,x\right)}}}\,dx$$

input
```
int(cos(c + d*x)/(a + a/cos(c + d*x))^(1/2),x)
```

output
```
int(cos(c + d*x)/(a + a/cos(c + d*x))^(1/2), x)
```

3.125 $\int \frac{\cos^2(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$

3.125.1 Optimal result . . . 1109
3.125.2 Mathematica [A] (verified) . . . 1109
3.125.3 Rubi [A] (verified) . . . 1110
3.125.4 Maple [B] (verified) . . . 1113
3.125.5 Fricas [A] (verification not implemented) . . . 1114
3.125.6 Sympy [F] . . . 1115
3.125.7 Maxima [F] . . . 1115
3.125.8 Giac [B] (verification not implemented) . . . 1116
3.125.9 Mupad [F(-1)] . . . 1116

3.125.1 Optimal result

Integrand size = 23, antiderivative size = 147

$$\int \frac{\cos^2(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \frac{7\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{4\sqrt{a}d} - \frac{\sqrt{2}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}d} - \frac{\sin(c+dx)}{4d\sqrt{a+a\sec(c+dx)}} + \frac{\cos(c+dx)\sin(c+dx)}{2d\sqrt{a+a\sec(c+dx)}}$$

output
```
7/4*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/d/a^(1/2)-arctan(1/2
*a^(1/2)*tan(d*x+c)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))*2^(1/2)/d/a^(1/2)-1/4*
sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+1/2*cos(d*x+c)*sin(d*x+c)/d/(a+a*sec(d
*x+c))^(1/2)
```

3.125.2 Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.80

$$\int \frac{\cos^2(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \frac{\left(7\text{arctanh}\left(\sqrt{1-\sec(c+dx)}\right) - 4\sqrt{2}\text{arctanh}\left(\frac{\sqrt{1-\sec(c+dx)}}{\sqrt{2}}\right) + \cos(c+dx)(-1+2\cos(c+dx))\sqrt{1-\text{se}}\right.}{4d\sqrt{1-\sec(c+dx)}\sqrt{a(1+\sec(c+dx))}}$$

input
```
Integrate[Cos[c + d*x]^2/Sqrt[a + a*Sec[c + d*x]],x]
```

output
```
((7*ArcTanh[Sqrt[1 - Sec[c + d*x]]] - 4*Sqrt[2]*ArcTanh[Sqrt[1 - Sec[c + d
*x]]/Sqrt[2]] + Cos[c + d*x]*(-1 + 2*Cos[c + d*x])*Sqrt[1 - Sec[c + d*x]])
*Tan[c + d*x])/(4*d*Sqrt[1 - Sec[c + d*x]]*Sqrt[a*(1 + Sec[c + d*x])])
```

3.125.3 Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.08, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.522$, Rules used = {3042, 4310, 3042, 4510, 27, 3042, 4408, 3042, 4261, 216, 4282, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^2(c+dx)}{\sqrt{a\sec(c+dx)+a}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^2\sqrt{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}}dx$$

$$\Big\downarrow \text{4310}$$

$$\frac{\sin(c+dx)\cos(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}-\frac{\int \frac{\cos(c+dx)(a-3a\sec(c+dx))}{\sqrt{\sec(c+dx)a+a}}dx}{4a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sin(c+dx)\cos(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}-\frac{\int \frac{a-3a\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a}$$

$$\Big\downarrow \text{4510}$$

$$\frac{\sin(c+dx)\cos(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}-\frac{\frac{\int -\frac{7a^2-a^2\sec(c+dx)}{2\sqrt{\sec(c+dx)a+a}}dx}{a}+\frac{a\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}}{4a}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sin(c+dx)\cos(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}-\frac{\frac{a\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}-\frac{\int \frac{7a^2-a^2\sec(c+dx)}{\sqrt{\sec(c+dx)a+a}}dx}{2a}}{4a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sin(c+dx)\cos(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}-\frac{\frac{a\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}-\frac{\int\frac{7a^2-a^2\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{2a}}{4a}$$

$$\Big\downarrow \text{4408}$$

$$\frac{\sin(c+dx)\cos(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}-\frac{\frac{a\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}-\frac{7a\int\sqrt{\sec(c+dx)a+a}dx-8a^2\int\frac{\sec(c+dx)}{\sqrt{\sec(c+dx)a+a}}dx}{2a}}{4a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sin(c+dx)\cos(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}-\frac{\frac{a\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}-\frac{7a\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx-8a^2\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{2a}}{4a}$$

$$\Big\downarrow \text{4261}$$

$$\frac{\sin(c+dx)\cos(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}-\frac{\frac{a\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}-\frac{-8a^2\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx-\frac{14a^2\int\frac{1}{\frac{a^2\tan^2(c+dx)}{\sec(c+dx)a+a}+a}d\left(-\frac{a\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}}{2a}}{4a}$$

$$\Big\downarrow \text{216}$$

$$\frac{\sin(c+dx)\cos(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}-\frac{\frac{a\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}-\frac{\frac{14a^{3/2}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}-8a^2\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{2a}}{4a}$$

$$\Big\downarrow \text{4282}$$

$$\frac{\sin(c+dx)\cos(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}-\frac{\frac{a\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}-\frac{\frac{16a^2\int\frac{1}{\frac{a^2\tan^2(c+dx)}{\sec(c+dx)a+a}+2a}d\left(-\frac{a\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}+\frac{14a^{3/2}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}}{2a}}{4a}$$

$$\Big\downarrow \text{216}$$

3.125. $\int\frac{\cos^2(c+dx)}{\sqrt{a+a\sec(c+dx)}}dx$

$$\frac{\sin(c+dx)\cos(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}-\frac{\frac{a\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}-\frac{\frac{14a^{3/2}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}-\frac{8\sqrt{2}a^{3/2}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{d}}{2a}}{4a}$$

input
```
Int[Cos[c + d*x]^2/Sqrt[a + a*Sec[c + d*x]],x]
```

output
```
(Cos[c + d*x]*Sin[c + d*x])/(2*d*Sqrt[a + a*Sec[c + d*x]]) - (-1/2*((14*a^
(3/2)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d - (8*Sqrt
[2]*a^(3/2)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]
])])/d)/a + (a*Sin[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]]))/(4*a)
```

3.125.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4261
```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(b/d)
  Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 4282
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2/f   Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[
a + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

3.125. $\int \frac{\cos^2(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$

rule 4310
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*n*Sqrt[a +
b*Csc[e + f*x]])), x] + Simp[1/(2*b*d*n)   Int[(d*Csc[e + f*x])^(n + 1)*((a
 + b*(2*n + 1)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, 0] && IntegerQ[2*n]
```

rule 4408
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Simp[c/a   Int[Sqrt[a + b*Csc[e + f*x]], x], x] -
Simp[(b*c - a*d)/a   Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

rule 4510
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*Cot[e
 + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(f*n)), x] - Simp[1/(b*d
*n)   Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*
n - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

3.125.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 342 vs. $2(122) = 244$.

Time = 25.80 (sec) , antiderivative size = 343, normalized size of antiderivative = 2.33

method	result
default	$-\frac{\left(4\sqrt{2}\,\ln\left(\csc(dx+c)-\cot(dx+c)+\sqrt{\cot(dx+c)^2-2\cot(dx+c)\csc(dx+c)+\csc(dx+c)^2-1}\right)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\cos(dx+c)+4\ln\left(\csc\right.\right.}{}$

input
```
int(cos(d*x+c)^2/(a+a*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4/d/a*(4*2^(1/2)*ln(csc(d*x+c)-cot(d*x+c)+(cot(d*x+c)^2-2*cot(d*x+c)*cs
c(d*x+c)+csc(d*x+c)^2-1)^(1/2))*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*cos(d*x
+c)+4*ln(csc(d*x+c)-cot(d*x+c)+(cot(d*x+c)^2-2*cot(d*x+c)*csc(d*x+c)+csc(d
*x+c)^2-1)^(1/2))*2^(1/2)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-7*(-cos(d*x+c
)/(cos(d*x+c)+1))^(1/2)*arctanh(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(co
s(d*x+c)+1))^(1/2))*cos(d*x+c)-2*cos(d*x+c)^2*sin(d*x+c)-7*(-cos(d*x+c)/(c
os(d*x+c)+1))^(1/2)*arctanh(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*
x+c)+1))^(1/2))+cos(d*x+c)*sin(d*x+c))*(a*(1+sec(d*x+c)))^(1/2)/(cos(d*x+c
)+1)
```

3.125.5 Fricas [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 446, normalized size of antiderivative = 3.03

$$\int \frac{\cos^2(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$$

$$= \left[\frac{4\sqrt{2}(a\cos(dx+c)+a)\sqrt{-\frac{1}{a}}\log\left(\frac{2\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{-\frac{1}{a}}\cos(dx+c)\sin(dx+c)+3\cos(dx+c)^2+2\cos(dx+c)-1}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)-7\sqrt{}}{}\right.$$

$$-\frac{7\sqrt{a}(\cos(dx+c)+1)\arctan\left(\frac{\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{\sqrt{a}\sin(dx+c)}\right)-\left(2\cos(dx+c)^2-\cos(dx+c)\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}}{4\left(ad\cos(dx+c)+ad\right)}$$

input
```
integrate(cos(d*x+c)^2/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
[1/8*(4*sqrt(2)*(a*cos(d*x + c) + a)*sqrt(-1/a)*log((2*sqrt(2)*sqrt((a*cos
(d*x + c) + a)/cos(d*x + c))*sqrt(-1/a)*cos(d*x + c)*sin(d*x + c) + 3*cos(
d*x + c)^2 + 2*cos(d*x + c) - 1)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) -
7*sqrt(-a)*(cos(d*x + c) + 1)*log((2*a*cos(d*x + c)^2 + 2*sqrt(-a)*sqrt((a
*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) + a*cos(d*x + c
) - a)/(cos(d*x + c) + 1)) + 2*(2*cos(d*x + c)^2 - cos(d*x + c))*sqrt((a*c
os(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(a*d*cos(d*x + c) + a*d), -1/
4*(7*sqrt(a)*(cos(d*x + c) + 1)*arctan(sqrt((a*cos(d*x + c) + a)/cos(d*x +
 c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) - (2*cos(d*x + c)^2 - cos(d*x +
c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c) - 4*sqrt(2)*(a*co
s(d*x + c) + a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos
(d*x + c)/(sqrt(a)*sin(d*x + c)))/sqrt(a))/(a*d*cos(d*x + c) + a*d)]
```

3.125.6 Sympy [F]

$$\int \frac{\cos^2(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \int \frac{\cos^2(c+dx)}{\sqrt{a(\sec(c+dx)+1)}}\,dx$$

input
```
integrate(cos(d*x+c)**2/(a+a*sec(d*x+c))**(1/2),x)
```

output
```
Integral(cos(c + d*x)**2/sqrt(a*(sec(c + d*x) + 1)), x)
```

3.125.7 Maxima [F]

$$\int \frac{\cos^2(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \int \frac{\cos(dx+c)^2}{\sqrt{a\sec(dx+c)+a}}\,dx$$

input
```
integrate(cos(d*x+c)^2/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(cos(d*x + c)^2/sqrt(a*sec(d*x + c) + a), x)
```

3.125.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 423 vs. $2(122) = 244$.

Time = 1.01 (sec) , antiderivative size = 423, normalized size of antiderivative = 2.88

$$\int \frac{\cos^2(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = -\frac{\sqrt{2}\left(\frac{7\sqrt{2}\sqrt{-a}\log\left(\frac{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2-4\sqrt{2}|a|-6a\right|}{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2+4\sqrt{2}|a|-6a\right|}\right)}{|a|} - \frac{8\log\left(\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}\right.}\right.}{\sqrt{-a}}\right.}{}$$

input
```
integrate(cos(d*x+c)^2/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
-1/16*sqrt(2)*(7*sqrt(2)*sqrt(-a)*log(abs(2*(sqrt(-a)*tan(1/2*d*x + 1/2*c)
 - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/abs(2*
(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 +
4*sqrt(2)*abs(a) - 6*a))/abs(a) - 8*log((sqrt(-a)*tan(1/2*d*x + 1/2*c) - s
qrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2)/sqrt(-a) - 8*(17*(sqrt(-a)*tan(1/2*
d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^6*sqrt(-a) - 57*(sqrt(
-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4*sqrt(-a)
*a + 19*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 +
a))^2*sqrt(-a)*a^2 - 3*sqrt(-a)*a^3)/((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqr
t(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4 - 6*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - s
qrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a + a^2)^2)/(d*sgn(cos(d*x + c)))
```

3.125.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^2(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \int \frac{\cos(c+d\,x)^2}{\sqrt{a+\frac{a}{\cos(c+d\,x)}}}\,dx$$

input
```
int(cos(c + d*x)^2/(a + a/cos(c + d*x))^(1/2),x)
```

output
```
int(cos(c + d*x)^2/(a + a/cos(c + d*x))^(1/2), x)
```

3.126 $\int \frac{\sec^5(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx$

3.126.1 Optimal result 1117
3.126.2 Mathematica [A] (verified) 1117
3.126.3 Rubi [A] (verified) 1118
3.126.4 Maple [A] (warning: unable to verify) 1122
3.126.5 Fricas [A] (verification not implemented) 1123
3.126.6 Sympy [F] 1123
3.126.7 Maxima [F] 1124
3.126.8 Giac [A] (verification not implemented) 1124
3.126.9 Mupad [F(-1)] 1124

3.126.1 Optimal result

Integrand size = 23, antiderivative size = 183

$$\int \frac{\sec^5(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = -\frac{15\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} - \frac{\sec^3(c+dx)\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{31\tan(c+dx)}{5ad\sqrt{a+a\sec(c+dx)}} + \frac{9\sec^2(c+dx)\tan(c+dx)}{10ad\sqrt{a+a\sec(c+dx)}} - \frac{13\sqrt{a+a\sec(c+dx)}\tan(c+dx)}{10a^2d}$$

output
```
-15/4*arctan(1/2*a^(1/2)*tan(d*x+c)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))/a^(3/2
)/d*2^(1/2)-1/2*sec(d*x+c)^3*tan(d*x+c)/d/(a+a*sec(d*x+c))^(3/2)+31/5*tan(
d*x+c)/a/d/(a+a*sec(d*x+c))^(1/2)+9/10*sec(d*x+c)^2*tan(d*x+c)/a/d/(a+a*se
c(d*x+c))^(1/2)-13/10*(a+a*sec(d*x+c))^(1/2)*tan(d*x+c)/a^2/d
```

3.126.2 Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.68

$$\int \frac{\sec^5(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \frac{\left(-75\sqrt{2}\text{arctanh}\left(\frac{\sqrt{1-\sec(c+dx)}}{\sqrt{2}}\right)(1+\sec(c+dx)) + 2\sqrt{1-\sec(c+dx)}(49+\right.}{20d\sqrt{1-\sec(c+dx)}(a(1+s}$$

input
```
Integrate[Sec[c + d*x]^5/(a + a*Sec[c + d*x])^(3/2),x]
```

output
```
((-75*Sqrt[2]*ArcTanh[Sqrt[1 - Sec[c + d*x]]/Sqrt[2]]*(1 + Sec[c + d*x]) +
 2*Sqrt[1 - Sec[c + d*x]]*(49 + 36*Sec[c + d*x] - 4*Sec[c + d*x]^2 + 4*Sec
[c + d*x]^3))*Tan[c + d*x])/(20*d*Sqrt[1 - Sec[c + d*x]]*(a*(1 + Sec[c + d
*x]))^(3/2))
```

3.126.3 Rubi [A] (verified)

Time = 1.07 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.09, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.609$, Rules used = {3042, 4303, 27, 3042, 4509, 27, 3042, 4498, 27, 3042, 4489, 3042, 4282, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^5(c+dx)}{(a\sec(c+dx)+a)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^5}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}}dx$$

$$\Big\downarrow \text{4303}$$

$$-\frac{\int \frac{3\sec^3(c+dx)(2a-3a\sec(c+dx))}{2\sqrt{\sec(c+dx)a+a}}dx}{2a^2}-\frac{\tan(c+dx)\sec^3(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{3\int \frac{\sec^3(c+dx)(2a-3a\sec(c+dx))}{\sqrt{\sec(c+dx)a+a}}dx}{4a^2}-\frac{\tan(c+dx)\sec^3(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{3\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^3\left(2a-3a\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a^2}-\frac{\tan(c+dx)\sec^3(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{4509}$$

$$-\frac{3\left(\frac{2\int -\frac{\sec^2(c+dx)\left(12a^2-13a^2\sec(c+dx)\right)}{2\sqrt{\sec(c+dx)a+a}}dx}{5a}-\frac{6a\tan(c+dx)\sec^2(c+dx)}{5d\sqrt{a\sec(c+dx)+a}}\right)}{4a^2}-\frac{\tan(c+dx)\sec^3(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

$\downarrow$ 27

$$-\frac{3\left(-\frac{\int \frac{\sec^2(c+dx)\left(12a^2-13a^2\sec(c+dx)\right)}{\sqrt{\sec(c+dx)a+a}}dx}{5a}-\frac{6a\tan(c+dx)\sec^2(c+dx)}{5d\sqrt{a\sec(c+dx)+a}}\right)}{4a^2}-\frac{\tan(c+dx)\sec^3(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

$\downarrow$ 3042

$$-\frac{3\left(-\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2\left(12a^2-13a^2\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{5a}-\frac{6a\tan(c+dx)\sec^2(c+dx)}{5d\sqrt{a\sec(c+dx)+a}}\right)}{4a^2}-\frac{\tan(c+dx)\sec^3(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

$\downarrow$ 4498

$$-\frac{3\left(-\frac{\frac{2\int -\frac{\sec(c+dx)\left(13a^3-62a^3\sec(c+dx)\right)}{2\sqrt{\sec(c+dx)a+a}}dx}{3a}-\frac{26a\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}}{5a}-\frac{6a\tan(c+dx)\sec^2(c+dx)}{5d\sqrt{a\sec(c+dx)+a}}\right)}{4a^2}-\frac{\tan(c+dx)\sec^3(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

$\downarrow$ 27

$$-\frac{3\left(-\frac{-\frac{\int \frac{\sec(c+dx)\left(13a^3-62a^3\sec(c+dx)\right)}{\sqrt{\sec(c+dx)a+a}}dx}{3a}-\frac{26a\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}}{5a}-\frac{6a\tan(c+dx)\sec^2(c+dx)}{5d\sqrt{a\sec(c+dx)+a}}\right)}{4a^2}-\frac{\tan(c+dx)\sec^3(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

$\downarrow$ 3042

$$-\frac{3\left(-\frac{-\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(13a^3-62a^3\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{3a}-\frac{26a\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}}{5a}-\frac{6a\tan(c+dx)\sec^2(c+dx)}{5d\sqrt{a\sec(c+dx)+a}}\right)}{4a^2}-\frac{\tan(c+dx)\sec^3(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

3.126. $\int \frac{\sec^5(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx$

$$\Big\downarrow \; 4489$$

$$-\frac{3\left(-\frac{-\frac{75a^3\int \frac{\sec(c+dx)}{\sqrt{\sec(c+dx)a+a}}dx-\frac{124a^3\tan(c+dx)}{d\sqrt{a\sec(c+dx)+a}}}{3a}-\frac{26a\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}}{5a}-\frac{6a\tan(c+dx)\sec^2(c+dx)}{5d\sqrt{a\sec(c+dx)+a}}\right)}{4a^2}-\frac{\tan(c+dx)\sec^3(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \; 3042$$

$$-\frac{3\left(-\frac{-\frac{75a^3\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx-\frac{124a^3\tan(c+dx)}{d\sqrt{a\sec(c+dx)+a}}}{3a}-\frac{26a\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}}{5a}-\frac{6a\tan(c+dx)\sec^2(c+dx)}{5d\sqrt{a\sec(c+dx)+a}}\right)}{4a^2}-\frac{\tan(c+dx)\sec^3(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \; 4282$$

$$-\frac{3\left(-\frac{-\frac{150a^3\int \frac{1}{\frac{a^2\tan^2(c+dx)}{\sec(c+dx)a+a}+2a}d\left(-\frac{a\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}-\frac{124a^3\tan(c+dx)}{d\sqrt{a\sec(c+dx)+a}}}{3a}-\frac{26a\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}}{5a}-\frac{6a\tan(c+dx)\sec^2(c+dx)}{5d\sqrt{a\sec(c+dx)+a}}\right)}{4a^2}-\frac{\tan(c+dx)\sec^3(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \; 216$$

$$-\frac{3\left(-\frac{-\frac{\frac{75\sqrt{2}a^{5/2}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{d}-\frac{124a^3\tan(c+dx)}{d\sqrt{a\sec(c+dx)+a}}}{3a}-\frac{26a\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}}{5a}-\frac{6a\tan(c+dx)\sec^2(c+dx)}{5d\sqrt{a\sec(c+dx)+a}}\right)}{4a^2}-\frac{\tan(c+dx)\sec^3(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

input
```
Int[Sec[c + d*x]^5/(a + a*Sec[c + d*x])^(3/2),x]
```

3.126. $\int \frac{\sec^5(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx$

output
```
-1/2*(Sec[c + d*x]^3*Tan[c + d*x])/(d*(a + a*Sec[c + d*x])^(3/2)) - (3*((-
6*a*Sec[c + d*x]^2*Tan[c + d*x])/(5*d*Sqrt[a + a*Sec[c + d*x]]) - ((-26*a*
Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x])/(3*d) - ((75*Sqrt[2]*a^(5/2)*ArcTan
[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/d - (124*a^3*
Tan[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]]))/(3*a))/(5*a)))/(4*a^2)
```

3.126.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4282
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2/f   Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[
a + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4303
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-d^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d
*Csc[e + f*x])^(n - 2)/(f*(2*m + 1))), x] + Simp[d^2/(a*b*(2*m + 1))   Int[
(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n
 + 2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2,
0] && LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

rule 4489
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*((
a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Simp[(a*B*m + A*b*(m + 1))/(b*(m +
 1))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B
, e, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b
*(m + 1), 0] &&  !LtQ[m, -2^(-1)]
```

3.126. $\int \frac{\sec^5(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx$

rule 4498
```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*
((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m + 2))   Int
[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*B*(m + 1) + (A*b*(m + 2) - a*B)
*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, m}, x] && NeQ[A*b - a
*B, 0] &&  !LtQ[m, -1]
```

rule 4509
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*d*
Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(f*(m + n))),
 x] + Simp[d/(b*(m + n))   Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n -
 1)*Simp[b*B*(n - 1) + (A*b*(m + n) + a*B*m)*Csc[e + f*x], x], x], x] /; Fr
eeQ[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
&& GtQ[n, 1]
```

3.126.4 Maple [A] (warning: unable to verify)

Time = 1.19 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.17

method	result
default	$-\frac{\sqrt{-\frac{2a}{(1-\cos(dx+c))^2\csc(dx+c)^2-1}}\left(5(1-\cos(dx+c))^7\csc(dx+c)^7+75\ln\left(\csc(dx+c)-\cot(dx+c)+\sqrt{(1-\cos(dx+c))^2\csc(dx+c)^2-1}\right.\right.}{20d\,a^2\left((1-\cos(d\right.}$

input
```
int(sec(d*x+c)^5/(a+a*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/20/d/a^2*(-2*a/((1-cos(d*x+c))^2*csc(d*x+c)^2-1))^(1/2)*(5*(1-cos(d*x+c
))^7*csc(d*x+c)^7+75*ln(csc(d*x+c)-cot(d*x+c)+((1-cos(d*x+c))^2*csc(d*x+c)
^2-1)^(1/2))*((1-cos(d*x+c))^2*csc(d*x+c)^2-1)^(5/2)-127*(1-cos(d*x+c))^5*
csc(d*x+c)^5+175*(1-cos(d*x+c))^3*csc(d*x+c)^3-85*csc(d*x+c)+85*cot(d*x+c)
)/((1-cos(d*x+c))^2*csc(d*x+c)^2-1)^2
```

3.126.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 414, normalized size of antiderivative = 2.26

$$\int \frac{\sec^5(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \left[-\frac{75\sqrt{2}\left(\cos\left(dx+c\right)^4+2\cos\left(dx+c\right)^3+\cos\left(dx+c\right)^2\right)\sqrt{-a}\log\left(-\frac{2\sqrt{2}\sqrt{-}}{}\right.}{}\right.$$

input
```
integrate(sec(d*x+c)^5/(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
[-1/40*(75*sqrt(2)*(cos(d*x + c)^4 + 2*cos(d*x + c)^3 + cos(d*x + c)^2)*sq
rt(-a)*log(-(2*sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*co
s(d*x + c)*sin(d*x + c) - 3*a*cos(d*x + c)^2 - 2*a*cos(d*x + c) + a)/(cos(
d*x + c)^2 + 2*cos(d*x + c) + 1)) - 4*(49*cos(d*x + c)^3 + 36*cos(d*x + c)
^2 - 4*cos(d*x + c) + 4)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x +
 c))/(a^2*d*cos(d*x + c)^4 + 2*a^2*d*cos(d*x + c)^3 + a^2*d*cos(d*x + c)^2
), 1/20*(75*sqrt(2)*(cos(d*x + c)^4 + 2*cos(d*x + c)^3 + cos(d*x + c)^2)*s
qrt(a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)
/(sqrt(a)*sin(d*x + c))) + 2*(49*cos(d*x + c)^3 + 36*cos(d*x + c)^2 - 4*co
s(d*x + c) + 4)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(a^2
*d*cos(d*x + c)^4 + 2*a^2*d*cos(d*x + c)^3 + a^2*d*cos(d*x + c)^2)]
```

3.126.6 Sympy [F]

$$\int \frac{\sec^5(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \int \frac{\sec^5\left(c+dx\right)}{\left(a\left(\sec\left(c+dx\right)+1\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)**5/(a+a*sec(d*x+c))**(3/2),x)
```

output
```
Integral(sec(c + d*x)**5/(a*(sec(c + d*x) + 1))**(3/2), x)
```

3.126.7 Maxima [F]

$$\int \frac{\sec^5(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \int \frac{\sec(dx+c)^5}{(a\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^5/(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)^5/(a*sec(d*x + c) + a)^(3/2), x)
```

3.126.8 Giac [A] (verification not implemented)

Time = 1.00 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.15

$$\int \frac{\sec^5(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = -\frac{\frac{\left(\left(\left(\frac{5\sqrt{2}a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2}{\operatorname{sgn}(\cos(dx+c))}-\frac{127\sqrt{2}a}{\operatorname{sgn}(\cos(dx+c))}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+\frac{175\sqrt{2}a}{\operatorname{sgn}(\cos(dx+c))}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-\frac{85\sqrt{2}a}{\operatorname{sgn}(\cos(dx+c))}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a\right)^2\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}} - \frac{75\sqrt{2}\log\left(\right.}{}}{20\,d}$$

input
```
integrate(sec(d*x+c)^5/(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
-1/20*((((5*sqrt(2)*a*tan(1/2*d*x + 1/2*c)^2/sgn(cos(d*x + c)) - 127*sqrt(
2)*a/sgn(cos(d*x + c)))*tan(1/2*d*x + 1/2*c)^2 + 175*sqrt(2)*a/sgn(cos(d*x
 + c)))*tan(1/2*d*x + 1/2*c)^2 - 85*sqrt(2)*a/sgn(cos(d*x + c)))*tan(1/2*d
*x + 1/2*c)/((a*tan(1/2*d*x + 1/2*c)^2 - a)^2*sqrt(-a*tan(1/2*d*x + 1/2*c)
^2 + a)) - 75*sqrt(2)*log(abs(-sqrt(-a)*tan(1/2*d*x + 1/2*c) + sqrt(-a*tan
(1/2*d*x + 1/2*c)^2 + a)))/(sqrt(-a)*a*sgn(cos(d*x + c))))/d
```

3.126.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^5(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \int \frac{1}{\cos(c+d\,x)^5\left(a+\frac{a}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

input
```
int(1/(cos(c + d*x)^5*(a + a/cos(c + d*x))^(3/2)),x)
```

output
```
int(1/(cos(c + d*x)^5*(a + a/cos(c + d*x))^(3/2)), x)
```

3.126. $\int \frac{\sec^5(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx$

3.127 $\int \frac{\sec^4(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx$

3.127.1 Optimal result . . . 1126
3.127.2 Mathematica [A] (verified) . . . 1126
3.127.3 Rubi [A] (verified) . . . 1127
3.127.4 Maple [A] (verified) . . . 1130
3.127.5 Fricas [A] (verification not implemented) . . . 1130
3.127.6 Sympy [F] . . . 1131
3.127.7 Maxima [F] . . . 1131
3.127.8 Giac [A] (verification not implemented) . . . 1132
3.127.9 Mupad [F(-1)] . . . 1132

3.127.1 Optimal result

Integrand size = 23, antiderivative size = 145

$$\int \frac{\sec^4(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \frac{11\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} - \frac{\sec^2(c+dx)\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} - \frac{13\tan(c+dx)}{3ad\sqrt{a+a\sec(c+dx)}} + \frac{7\sqrt{a+a\sec(c+dx)}\tan(c+dx)}{6a^2d}$$

output
```
11/4*arctan(1/2*a^(1/2)*tan(d*x+c)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))/a^(3/2)
/d*2^(1/2)-1/2*sec(d*x+c)^2*tan(d*x+c)/d/(a+a*sec(d*x+c))^(3/2)-13/3*tan(d
*x+c)/a/d/(a+a*sec(d*x+c))^(1/2)+7/6*(a+a*sec(d*x+c))^(1/2)*tan(d*x+c)/a^2
/d
```

3.127.2 Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.79

$$\int \frac{\sec^4(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \frac{\left(33\sqrt{2}\text{arctanh}\left(\frac{\sqrt{1-\sec(c+dx)}}{\sqrt{2}}\right)(1+\sec(c+dx)) + 2\sqrt{1-\sec(c+dx)}(-19 -}{12d\sqrt{1-\sec(c+dx)}(a(1+\sec(c+d}$$

input
```
Integrate[Sec[c + d*x]^4/(a + a*Sec[c + d*x])^(3/2),x]
```

output
```
((33*Sqrt[2]*ArcTanh[Sqrt[1 - Sec[c + d*x]]/Sqrt[2]]*(1 + Sec[c + d*x]) +
2*Sqrt[1 - Sec[c + d*x]]*(-19 - 12*Sec[c + d*x] + 4*Sec[c + d*x]^2))*Tan[c
 + d*x])/(12*d*Sqrt[1 - Sec[c + d*x]]*(a*(1 + Sec[c + d*x]))^(3/2))
```

3.127.3 Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.06, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules used = {3042, 4303, 27, 3042, 4498, 27, 3042, 4489, 3042, 4282, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^4(c+dx)}{(a\sec(c+dx)+a)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^4}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}}dx$$

$$\Big\downarrow \text{4303}$$

$$-\frac{\int \frac{\sec^2(c+dx)(4a-7a\sec(c+dx))}{2\sqrt{\sec(c+dx)a+a}}dx}{2a^2}-\frac{\tan(c+dx)\sec^2(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{\sec^2(c+dx)(4a-7a\sec(c+dx))}{\sqrt{\sec(c+dx)a+a}}dx}{4a^2}-\frac{\tan(c+dx)\sec^2(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2\left(4a-7a\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a^2}-\frac{\tan(c+dx)\sec^2(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{4498}$$

$$-\frac{\frac{2\int -\frac{\sec(c+dx)\left(7a^2-26a^2\sec(c+dx)\right)}{2\sqrt{\sec(c+dx)a+a}}dx}{3a}-\frac{14\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}}{4a^2}-\frac{\tan(c+dx)\sec^2(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{-\frac{\int \frac{\sec(c+dx)\left(7a^2-26a^2\sec(c+dx)\right)}{\sqrt{\sec(c+dx)a+a}}dx}{3a}-\frac{14\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}}{4a^2}-\frac{\tan(c+dx)\sec^2(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

$\downarrow$ 3042

$$-\frac{-\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(7a^2-26a^2\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{3a}-\frac{14\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}}{4a^2}-\frac{\tan(c+dx)\sec^2(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

$\downarrow$ 4489

$$-\frac{-\frac{33a^2\int \frac{\sec(c+dx)}{\sqrt{\sec(c+dx)a+a}}dx-\frac{52a^2\tan(c+dx)}{d\sqrt{a\sec(c+dx)+a}}}{3a}-\frac{14\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}}{4a^2}-\frac{\tan(c+dx)\sec^2(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

$\downarrow$ 3042

$$-\frac{-\frac{33a^2\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx-\frac{52a^2\tan(c+dx)}{d\sqrt{a\sec(c+dx)+a}}}{3a}-\frac{14\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}}{4a^2}-\frac{\tan(c+dx)\sec^2(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

$\downarrow$ 4282

$$-\frac{-\frac{-\frac{66a^2\int \frac{1}{\frac{a^2\tan^2(c+dx)}{\sec(c+dx)a+a}+2a}d\left(-\frac{a\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}-\frac{52a^2\tan(c+dx)}{d\sqrt{a\sec(c+dx)+a}}}{3a}-\frac{14\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}}{4a^2}-\frac{\tan(c+dx)\sec^2(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

$\downarrow$ 216

$$-\frac{-\frac{-\frac{33\sqrt{2}a^{3/2}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{d}-\frac{52a^2\tan(c+dx)}{d\sqrt{a\sec(c+dx)+a}}}{3a}-\frac{14\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}}{4a^2}-\frac{\tan(c+dx)\sec^2(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

input
```
Int[Sec[c + d*x]^4/(a + a*Sec[c + d*x])^(3/2),x]
```

output
```
-1/2*(Sec[c + d*x]^2*Tan[c + d*x])/(d*(a + a*Sec[c + d*x])^(3/2)) - ((-14*
Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x])/(3*d) - ((33*Sqrt[2]*a^(3/2)*ArcTan
[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/d - (52*a^2*T
an[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]]))/(3*a))/(4*a^2)
```

3.127. $\int \frac{\sec^4(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx$

3.127.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4282
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2/f   Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[
a + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4303
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-d^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d
*Csc[e + f*x])^(n - 2)/(f*(2*m + 1))), x] + Simp[d^2/(a*b*(2*m + 1))   Int[
(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n
 + 2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2,
0] && LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

rule 4489
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*((
a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Simp[(a*B*m + A*b*(m + 1))/(b*(m +
 1))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B
, e, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b
*(m + 1), 0] &&  !LtQ[m, -2^(-1)]
```

rule 4498
```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*
((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m + 2))   Int
[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*B*(m + 1) + (A*b*(m + 2) - a*B)
*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, m}, x] && NeQ[A*b - a
*B, 0] &&  !LtQ[m, -1]
```

3.127. $\int \frac{\sec^4(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx$

3.127.4 Maple [A] (verified)

Time = 0.98 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.33

method	result
default	$\frac{\sqrt{-\frac{2a}{(1-\cos(dx+c))^2\csc(dx+c)^2-1}}\left(3(1-\cos(dx+c))^5\csc(dx+c)^5+33\ln\left(\csc(dx+c)-\cot(dx+c)+\sqrt{(1-\cos(dx+c))^2\csc(dx+c)^2-1}\right)\right.}{12d\,a^2\left((1-\cos(dx+c))^2\csc(dx+c)^2\right.}$

input
```
int(sec(d*x+c)^4/(a+a*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/12/d/a^2*(-2*a/((1-cos(d*x+c))^2*csc(d*x+c)^2-1))^(1/2)*(3*(1-cos(d*x+c)
)^5*csc(d*x+c)^5+33*ln(csc(d*x+c)-cot(d*x+c)+((1-cos(d*x+c))^2*csc(d*x+c)^
2-1)^(1/2))*((1-cos(d*x+c))^2*csc(d*x+c)^2-1)^(3/2)-46*(1-cos(d*x+c))^3*cs
c(d*x+c)^3+27*csc(d*x+c)-27*cot(d*x+c))/((1-cos(d*x+c))^2*csc(d*x+c)^2-1)
```

3.127.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 387, normalized size of antiderivative = 2.67

$$\int \frac{\sec^4(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \left[-\frac{33\sqrt{2}\left(\cos(dx+c)^3+2\cos(dx+c)^2+\cos(dx+c)\right)\sqrt{-a}\log\left(\frac{2\sqrt{2}\sqrt{-a}\sqrt{}}{}\right.}{24\,(a^2}\right.$$

$$-\frac{33\sqrt{2}\left(\cos(dx+c)^3+2\cos(dx+c)^2+\cos(dx+c)\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{\sqrt{a}\sin(dx+c)}\right)+2\left(19\cos(dx\right.}{12\left(a^2d\cos(dx+c)^3+2\,a^2d\cos(dx+c)^2+a^2d\cos(dx\right.}$$

input
```
integrate(sec(d*x+c)^4/(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
[-1/24*(33*sqrt(2)*(cos(d*x + c)^3 + 2*cos(d*x + c)^2 + cos(d*x + c))*sqrt
(-a)*log((2*sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d
*x + c)*sin(d*x + c) + 3*a*cos(d*x + c)^2 + 2*a*cos(d*x + c) - a)/(cos(d*x
 + c)^2 + 2*cos(d*x + c) + 1)) + 4*(19*cos(d*x + c)^2 + 12*cos(d*x + c) -
4)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(a^2*d*cos(d*x +
c)^3 + 2*a^2*d*cos(d*x + c)^2 + a^2*d*cos(d*x + c)), -1/12*(33*sqrt(2)*(co
s(d*x + c)^3 + 2*cos(d*x + c)^2 + cos(d*x + c))*sqrt(a)*arctan(sqrt(2)*sqr
t((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c)))
+ 2*(19*cos(d*x + c)^2 + 12*cos(d*x + c) - 4)*sqrt((a*cos(d*x + c) + a)/co
s(d*x + c))*sin(d*x + c))/(a^2*d*cos(d*x + c)^3 + 2*a^2*d*cos(d*x + c)^2 +
 a^2*d*cos(d*x + c))]
```

3.127.6 Sympy [F]

$$\int \frac{\sec^4(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \int \frac{\sec^4(c+dx)}{(a(\sec(c+dx)+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)**4/(a+a*sec(d*x+c))**(3/2),x)
```

output
```
Integral(sec(c + d*x)**4/(a*(sec(c + d*x) + 1))**(3/2), x)
```

3.127.7 Maxima [F]

$$\int \frac{\sec^4(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \int \frac{\sec(dx+c)^4}{(a\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^4/(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)^4/(a*sec(d*x + c) + a)^(3/2), x)
```

3.127.8 Giac [A] (verification not implemented)

Time = 1.19 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.23

$$\int \frac{\sec^4(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \frac{\frac{\left(\left(\frac{3\sqrt{2}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2}{\operatorname{sgn}(\cos(dx+c))}-\frac{46\sqrt{2}}{\operatorname{sgn}(\cos(dx+c))}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+\frac{27\sqrt{2}}{\operatorname{sgn}(\cos(dx+c))}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a\right)\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}} - \frac{33\sqrt{2}\log}{}}{12\,d}$$

input
```
integrate(sec(d*x+c)^4/(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
1/12*(((3*sqrt(2)*tan(1/2*d*x + 1/2*c)^2/sgn(cos(d*x + c)) - 46*sqrt(2)/sg
n(cos(d*x + c)))*tan(1/2*d*x + 1/2*c)^2 + 27*sqrt(2)/sgn(cos(d*x + c)))*ta
n(1/2*d*x + 1/2*c)/((a*tan(1/2*d*x + 1/2*c)^2 - a)*sqrt(-a*tan(1/2*d*x + 1
/2*c)^2 + a)) - 33*sqrt(2)*log(abs(-sqrt(-a)*tan(1/2*d*x + 1/2*c) + sqrt(-
a*tan(1/2*d*x + 1/2*c)^2 + a)))/(sqrt(-a)*a*sgn(cos(d*x + c))))/d
```

3.127.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^4(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \int \frac{1}{\cos(c+d\,x)^4\left(a+\frac{a}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

input
```
int(1/(cos(c + d*x)^4*(a + a/cos(c + d*x))^(3/2)),x)
```

output
```
int(1/(cos(c + d*x)^4*(a + a/cos(c + d*x))^(3/2)), x)
```

3.128 $\int \frac{\sec^3(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx$

3.128.1 Optimal result 1133
3.128.2 Mathematica [A] (verified) 1133
3.128.3 Rubi [A] (verified) 1134
3.128.4 Maple [A] (verified) 1136
3.128.5 Fricas [A] (verification not implemented) 1136
3.128.6 Sympy [F] 1137
3.128.7 Maxima [F] 1137
3.128.8 Giac [A] (verification not implemented) 1138
3.128.9 Mupad [F(-1)] 1138

3.128.1 Optimal result

Integrand size = 23, antiderivative size = 105

$$\int \frac{\sec^3(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = -\frac{7\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} + \frac{\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{2\tan(c+dx)}{ad\sqrt{a+a\sec(c+dx)}}$$

output
```
-7/4*arctan(1/2*a^(1/2)*tan(d*x+c)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))/a^(3/2)
/d*2^(1/2)+1/2*tan(d*x+c)/d/(a+a*sec(d*x+c))^(3/2)+2*tan(d*x+c)/a/d/(a+a*s
ec(d*x+c))^(1/2)
```

3.128.2 Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.99

$$\int \frac{\sec^3(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \frac{\left(-7\sqrt{2}\text{arctanh}\left(\frac{\sqrt{1-\sec(c+dx)}}{\sqrt{2}}\right)(1+\sec(c+dx)) + 2\sqrt{1-\sec(c+dx)}(5+4\right.}{4d\sqrt{1-\sec(c+dx)}(a(1+\sec(c+dx)))^{3/2}}$$

input
```
Integrate[Sec[c + d*x]^3/(a + a*Sec[c + d*x])^(3/2),x]
```

output
```
((-7*Sqrt[2]*ArcTanh[Sqrt[1 - Sec[c + d*x]]/Sqrt[2]]*(1 + Sec[c + d*x]) +
2*Sqrt[1 - Sec[c + d*x]]*(5 + 4*Sec[c + d*x]))*Tan[c + d*x])/(4*d*Sqrt[1 -
 Sec[c + d*x]]*(a*(1 + Sec[c + d*x]))^(3/2))
```

3.128.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.04, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {3042, 4286, 27, 3042, 4489, 3042, 4282, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^3(c+dx)}{(a\sec(c+dx)+a)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^3}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}}dx$$

$$\Big\downarrow \text{4286}$$

$$\frac{\int -\frac{\sec(c+dx)(3a-4a\sec(c+dx))}{2\sqrt{\sec(c+dx)a+a}}dx}{2a^2}+\frac{\tan(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\tan(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}-\frac{\int \frac{\sec(c+dx)(3a-4a\sec(c+dx))}{\sqrt{\sec(c+dx)a+a}}dx}{4a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\tan(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}-\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(3a-4a\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a^2}$$

$$\Big\downarrow \text{4489}$$

$$\frac{\tan(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}-\frac{7a\int \frac{\sec(c+dx)}{\sqrt{\sec(c+dx)a+a}}dx-\frac{8a\tan(c+dx)}{d\sqrt{a\sec(c+dx)+a}}}{4a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\tan(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}-\frac{7a\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx-\frac{8a\tan(c+dx)}{d\sqrt{a\sec(c+dx)+a}}}{4a^2}$$

$$\Big\downarrow \text{4282}$$

$$\frac{\tan(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}-\frac{-\frac{14a\int \frac{1}{\frac{a^2\tan^2(c+dx)}{\sec(c+dx)a+a}+2a}d\left(-\frac{a\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}-\frac{8a\tan(c+dx)}{d\sqrt{a\sec(c+dx)+a}}}{4a^2}$$

$$\Big\downarrow \; 216$$

$$\frac{\tan(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}-\frac{\frac{7\sqrt{2}\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{d}-\frac{8a\tan(c+dx)}{d\sqrt{a\sec(c+dx)+a}}}{4a^2}$$

input
```
Int[Sec[c + d*x]^3/(a + a*Sec[c + d*x])^(3/2),x]
```

output
```
Tan[c + d*x]/(2*d*(a + a*Sec[c + d*x])^(3/2)) - ((7*Sqrt[2]*Sqrt[a]*ArcTan
[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/d - (8*a*Tan[
c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]]))/(4*a^2)
```

3.128.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4282
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2/f   Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[
a + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4286
```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[b*Cot[e + f*x]*((a + b*Csc[e + f*x])^m/(a*f*(2*m + 1))),
x] - Simp[1/(a^2*(2*m + 1))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
*(a*m - b*(2*m + 1)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[
a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

rule 4489
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*((
a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Simp[(a*B*m + A*b*(m + 1))/(b*(m +
 1))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B
, e, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b
*(m + 1), 0] &&  !LtQ[m, -2^(-1)]
```

3.128.4 Maple [A] (verified)

Time = 0.96 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.38

method	result
default	$-\frac{\sqrt{-\frac{2a}{(1-\cos(dx+c))^2\csc(dx+c)^2-1}}\left((1-\cos(dx+c))^3\csc(dx+c)^3+7\ln\left(\csc(dx+c)-\cot(dx+c)+\sqrt{(1-\cos(dx+c))^2\csc(dx+c)^2-1}\right)\right.}{4d\,a^2}$

input
```
int(sec(d*x+c)^3/(a+a*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4/d/a^2*(-2*a/((1-cos(d*x+c))^2*csc(d*x+c)^2-1))^(1/2)*((1-cos(d*x+c))^
3*csc(d*x+c)^3+7*ln(csc(d*x+c)-cot(d*x+c)+((1-cos(d*x+c))^2*csc(d*x+c)^2-1
)^(1/2))*((1-cos(d*x+c))^2*csc(d*x+c)^2-1)^(1/2)-9*csc(d*x+c)+9*cot(d*x+c)
)
```

3.128.5 Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 336, normalized size of antiderivative = 3.20

$$\int \frac{\sec^3(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \left[-\frac{7\sqrt{2}\left(\cos(dx+c)^2+2\cos(dx+c)+1\right)\sqrt{-a}\log\left(-\frac{2\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}}{}\right.}{8\left(a^2d\cos(dx\right.}\right.$$

input
```
integrate(sec(d*x+c)^3/(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
[-1/8*(7*sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(-a)*log(-(2*sq
rt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*
x + c) - 3*a*cos(d*x + c)^2 - 2*a*cos(d*x + c) + a)/(cos(d*x + c)^2 + 2*co
s(d*x + c) + 1)) - 4*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(5*cos(d*x +
c) + 4)*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d
), 1/4*(7*sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*arctan(sqr
t(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x
 + c))) + 2*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(5*cos(d*x + c) + 4)*s
in(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)]
```

3.128.6 Sympy [F]

$$\int \frac{\sec^3(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \int \frac{\sec^3(c+dx)}{(a(\sec(c+dx)+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)**3/(a+a*sec(d*x+c))**(3/2),x)
```

output
```
Integral(sec(c + d*x)**3/(a*(sec(c + d*x) + 1))**(3/2), x)
```

3.128.7 Maxima [F]

$$\int \frac{\sec^3(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \int \frac{\sec(dx+c)^3}{(a\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^3/(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)^3/(a*sec(d*x + c) + a)^(3/2), x)
```

3.128.8 Giac [A] (verification not implemented)

Time = 1.15 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.50

$$\int \frac{\sec^3(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \frac{\frac{\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\left(\frac{\sqrt{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2}{a\operatorname{sgn}(\cos(dx+c))}-\frac{9\sqrt{2}}{a\operatorname{sgn}(\cos(dx+c))}\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a}+\frac{7\sqrt{2}\log\left(\left|-\sqrt{-a}\tan\right.\right.}{}}{4\,d}$$

input
```
integrate(sec(d*x+c)^3/(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
1/4*(sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*(sqrt(2)*tan(1/2*d*x + 1/2*c)^2/(
a*sgn(cos(d*x + c))) - 9*sqrt(2)/(a*sgn(cos(d*x + c))))*tan(1/2*d*x + 1/2*
c)/(a*tan(1/2*d*x + 1/2*c)^2 - a) + 7*sqrt(2)*log(abs(-sqrt(-a)*tan(1/2*d*
x + 1/2*c) + sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)))/(sqrt(-a)*a*sgn(cos(d*x
 + c))))/d
```

3.128.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^3(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \int \frac{1}{\cos\left(c+d\,x\right)^3\,\left(a+\frac{a}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

input
```
int(1/(cos(c + d*x)^3*(a + a/cos(c + d*x))^(3/2)),x)
```

output
```
int(1/(cos(c + d*x)^3*(a + a/cos(c + d*x))^(3/2)), x)
```

3.129 $\int \frac{\sec^2(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx$

3.129.1 Optimal result 1139
3.129.2 Mathematica [A] (verified) 1139
3.129.3 Rubi [A] (verified) 1140
3.129.4 Maple [B] (verified) 1141
3.129.5 Fricas [B] (verification not implemented) 1142
3.129.6 Sympy [F] 1142
3.129.7 Maxima [F] 1143
3.129.8 Giac [A] (verification not implemented) 1143
3.129.9 Mupad [F(-1)] 1143

3.129.1 Optimal result

Integrand size = 23, antiderivative size = 77

$$\int \frac{\sec^2(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx=\frac{3\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}}$$

output
```
3/4*arctan(1/2*a^(1/2)*tan(d*x+c)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))/a^(3/2)/
d*2^(1/2)-1/2*tan(d*x+c)/d/(a+a*sec(d*x+c))^(3/2)
```

3.129.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.22

$$\int \frac{\sec^2(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx=\frac{\left(-2\sqrt{1-\sec(c+dx)}+3\sqrt{2}\text{arctanh}\left(\frac{\sqrt{1-\sec(c+dx)}}{\sqrt{2}}\right)(1+\sec(c+dx))\right)\tan(c}{4d\sqrt{1-\sec(c+dx)}(a(1+\sec(c+dx)))^{3/2}}$$

input
```
Integrate[Sec[c + d*x]^2/(a + a*Sec[c + d*x])^(3/2),x]
```

output
```
((-2*Sqrt[1 - Sec[c + d*x]] + 3*Sqrt[2]*ArcTanh[Sqrt[1 - Sec[c + d*x]]/Sqr
t[2]]*(1 + Sec[c + d*x]))*Tan[c + d*x])/(4*d*Sqrt[1 - Sec[c + d*x]]*(a*(1
+ Sec[c + d*x]))^(3/2))
```

3.129.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3042, 4284, 3042, 4282, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(c+dx)}{(a\sec(c+dx)+a)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}}dx$$

$$\Big\downarrow \text{4284}$$

$$\frac{3\int \frac{\sec(c+dx)}{\sqrt{\sec(c+dx)a+a}}dx}{4a}-\frac{\tan(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a}-\frac{\tan(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{4282}$$

$$-\frac{3\int \frac{1}{\frac{a^2\tan^2(c+dx)}{\sec(c+dx)a+a}+2a}d\left(-\frac{a\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{2ad}-\frac{\tan(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{216}$$

$$\frac{3\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{\tan(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

input
```
Int[Sec[c + d*x]^2/(a + a*Sec[c + d*x])^(3/2),x]
```

output
```
(3*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/(2*S
qrt[2]*a^(3/2)*d) - Tan[c + d*x]/(2*d*(a + a*Sec[c + d*x])^(3/2))
```

3.129.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4282
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2/f   Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[
a + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4284
```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(-Cot[e + f*x])*((a + b*Csc[e + f*x])^m/(f*(2*m + 1))), x
] + Simp[m/(b*(2*m + 1))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x
], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

3.129.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 148 vs. $2(62) = 124$.

Time = 0.85 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.94

method	result
default	$\frac{\sqrt{-\frac{2a}{(1-\cos(dx+c))^2\csc(dx+c)^2-1}}\sqrt{(1-\cos(dx+c))^2\csc(dx+c)^2-1}\left(\sqrt{(1-\cos(dx+c))^2\csc(dx+c)^2-1}(-\cot(dx+c)+\csc(dx+c))+3\right.}{4d\,a^2}$

input
```
int(sec(d*x+c)^2/(a+a*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/4/d/a^2*(-2*a/((1-cos(d*x+c))^2*csc(d*x+c)^2-1))^(1/2)*((1-cos(d*x+c))^2
*csc(d*x+c)^2-1)^(1/2)*(((1-cos(d*x+c))^2*csc(d*x+c)^2-1)^(1/2)*(-cot(d*x+
c)+csc(d*x+c))+3*ln(csc(d*x+c)-cot(d*x+c)+((1-cos(d*x+c))^2*csc(d*x+c)^2-1
)^(1/2)))
```

3.129.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 140 vs. $2(62) = 124$.

Time = 0.31 (sec) , antiderivative size = 329, normalized size of antiderivative = 4.27

$$\int \frac{\sec^2(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \left[-\frac{3\sqrt{2}\left(\cos\left(dx+c\right)^2+2\cos\left(dx+c\right)+1\right)\sqrt{-a}\log\left(\frac{2\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos}{\cos}\right.}{8\left(a^2d\cos\left(dx+c\right)^2\right.}\right.$$

$$-\frac{3\sqrt{2}\left(\cos\left(dx+c\right)^2+2\cos\left(dx+c\right)+1\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{\sqrt{a}\sin(dx+c)}\right)+2\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos\left(dx+\right.}{4\left(a^2d\cos\left(dx+c\right)^2+2a^2d\cos\left(dx+c\right)+a^2d\right)}$$

input
```
integrate(sec(d*x+c)^2/(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
[-1/8*(3*sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(-a)*log((2*sqr
t(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x
 + c) + 3*a*cos(d*x + c)^2 + 2*a*cos(d*x + c) - a)/(cos(d*x + c)^2 + 2*cos
(d*x + c) + 1)) + 4*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*s
in(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d), -1/4*(
3*sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*arctan(sqrt(2)*sqr
t((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c)))
+ 2*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c))/(a^
2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)]
```

3.129.6 Sympy [F]

$$\int \frac{\sec^2(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \int \frac{\sec^2\left(c+dx\right)}{\left(a\left(\sec\left(c+dx\right)+1\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)**2/(a+a*sec(d*x+c))**(3/2),x)
```

output
```
Integral(sec(c + d*x)**2/(a*(sec(c + d*x) + 1))**(3/2), x)
```

3.129.7 Maxima [F]

$$\int \frac{\sec^2(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \int \frac{\sec{(dx+c)}^2}{(a\sec{(dx+c)}+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^2/(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)^2/(a*sec(d*x + c) + a)^(3/2), x)
```

3.129.8 Giac [A] (verification not implemented)

Time = 1.04 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.40

$$\int \frac{\sec^2(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = -\frac{\frac{3\sqrt{2}\log\left(\left|-\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right|\right)}{\sqrt{-a}a\operatorname{sgn}(\cos(dx+c))}+\frac{\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^2\operatorname{sgn}(\cos(dx+c))}}{4d}$$

input
```
integrate(sec(d*x+c)^2/(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
-1/4*(3*sqrt(2)*log(abs(-sqrt(-a)*tan(1/2*d*x + 1/2*c) + sqrt(-a*tan(1/2*d
*x + 1/2*c)^2 + a)))/(sqrt(-a)*a*sgn(cos(d*x + c))) + sqrt(2)*sqrt(-a*tan(
1/2*d*x + 1/2*c)^2 + a)*tan(1/2*d*x + 1/2*c)/(a^2*sgn(cos(d*x + c))))/d
```

3.129.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^2(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \int \frac{1}{\cos{(c+d\,x)}^2\,\left(a+\frac{a}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

input
```
int(1/(cos(c + d*x)^2*(a + a/cos(c + d*x))^(3/2)),x)
```

output
```
int(1/(cos(c + d*x)^2*(a + a/cos(c + d*x))^(3/2)), x)
```

3.130 $\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx$

3.130.1 Optimal result 1144
3.130.2 Mathematica [A] (verified) 1144
3.130.3 Rubi [A] (verified) 1145
3.130.4 Maple [B] (verified) 1146
3.130.5 Fricas [B] (verification not implemented) 1147
3.130.6 Sympy [F] 1147
3.130.7 Maxima [F] 1148
3.130.8 Giac [A] (verification not implemented) 1148
3.130.9 Mupad [F(-1)] 1148

3.130.1 Optimal result

Integrand size = 21, antiderivative size = 77

$$\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx=\frac{\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}}$$

output
```
1/4*arctan(1/2*a^(1/2)*tan(d*x+c)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))/a^(3/2)/
d*2^(1/2)+1/2*tan(d*x+c)/d/(a+a*sec(d*x+c))^(3/2)
```

3.130.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.21

$$\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx=\frac{\left(2\sqrt{1-\sec(c+dx)}+\sqrt{2}\text{arctanh}\left(\frac{\sqrt{1-\sec(c+dx)}}{\sqrt{2}}\right)(1+\sec(c+dx))\right)\tan(c+}{4d\sqrt{1-\sec(c+dx)}(a(1+\sec(c+dx)))^{3/2}}$$

input
```
Integrate[Sec[c + d*x]/(a + a*Sec[c + d*x])^(3/2),x]
```

output
```
((2*Sqrt[1 - Sec[c + d*x]] + Sqrt[2]*ArcTanh[Sqrt[1 - Sec[c + d*x]]/Sqrt[2
]]*(1 + Sec[c + d*x]))*Tan[c + d*x])/(4*d*Sqrt[1 - Sec[c + d*x]]*(a*(1 + S
ec[c + d*x]))^(3/2))
```

3.130.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3042, 4283, 3042, 4282, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(c+dx)}{(a\sec(c+dx)+a)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}}dx$$

$$\Big\downarrow \text{4283}$$

$$\frac{\int \frac{\sec(c+dx)}{\sqrt{\sec(c+dx)a+a}}dx}{4a}+\frac{\tan(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a}+\frac{\tan(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{4282}$$

$$\frac{\tan(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}-\frac{\int \frac{1}{\frac{a^2\tan^2(c+dx)}{\sec(c+dx)a+a}+2a}d\left(-\frac{a\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{2ad}$$

$$\Big\downarrow \text{216}$$

$$\frac{\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{\tan(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

input
```
Int[Sec[c + d*x]/(a + a*Sec[c + d*x])^(3/2),x]
```

output
```
ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])]/(2*Sqrt[
2]*a^(3/2)*d) + Tan[c + d*x]/(2*d*(a + a*Sec[c + d*x])^(3/2))
```

3.130.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4282
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2/f   Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[
a + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4283
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[b*Cot[e + f*x]*((a + b*Csc[e + f*x])^m/(a*f*(2*m + 1))), x]
 + Simp[(m + 1)/(a*(2*m + 1))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m +
1), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)
] && IntegerQ[2*m]
```

3.130.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 148 vs. $2(62) = 124$.

Time = 0.86 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.94

method	result
default	$-\frac{\sqrt{-\frac{2a}{(1-\cos(dx+c))^2\csc(dx+c)^2-1}}\sqrt{(1-\cos(dx+c))^2\csc(dx+c)^2-1}\left(\sqrt{(1-\cos(dx+c))^2\csc(dx+c)^2-1}\,(-\cot(dx+c)+\csc(dx+c))\right.}{4d\,a^2}$

input
```
int(sec(d*x+c)/(a+a*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4/d/a^2*(-2*a/((1-cos(d*x+c))^2*csc(d*x+c)^2-1))^(1/2)*((1-cos(d*x+c))^
2*csc(d*x+c)^2-1)^(1/2)*(((1-cos(d*x+c))^2*csc(d*x+c)^2-1)^(1/2)*(-cot(d*x
+c)+csc(d*x+c))-ln(csc(d*x+c)-cot(d*x+c)+((1-cos(d*x+c))^2*csc(d*x+c)^2-1)
^(1/2)))
```

3.130.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 139 vs. $2(62) = 124$.

Time = 0.33 (sec) , antiderivative size = 327, normalized size of antiderivative = 4.25

$$\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \left[-\frac{\sqrt{2}\left(\cos\left(dx+c\right)^2+2\,\cos\left(dx+c\right)+1\right)\sqrt{-a}\log\left(\frac{2\,\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(d}{\cos(d}\right.}{8\left(a^2d\cos\left(dx+c\right)^2\right.}\right.$$

$$-\frac{\sqrt{2}\left(\cos\left(dx+c\right)^2+2\,\cos\left(dx+c\right)+1\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{\sqrt{a}\sin(dx+c)}\right)-2\,\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos\left(dx+c\right.}{4\left(a^2d\cos\left(dx+c\right)^2+2\,a^2d\cos\left(dx+c\right)+a^2d\right)}$$

input
```
integrate(sec(d*x+c)/(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
[-1/8*(sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(-a)*log((2*sqrt(
2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x +
 c) + 3*a*cos(d*x + c)^2 + 2*a*cos(d*x + c) - a)/(cos(d*x + c)^2 + 2*cos(d
*x + c) + 1)) - 4*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin
(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d), -1/4*(sq
rt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*arctan(sqrt(2)*sqrt((a
*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) - 2*
sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c))/(a^2*d*
cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)]
```

3.130.6 Sympy [F]

$$\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \int \frac{\sec\left(c+dx\right)}{\left(a\left(\sec\left(c+dx\right)+1\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)/(a+a*sec(d*x+c))**(3/2),x)
```

output
```
Integral(sec(c + d*x)/(a*(sec(c + d*x) + 1))**(3/2), x)
```

3.130.7 Maxima [F]

$$\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \int \frac{\sec(dx+c)}{(a\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)/(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)/(a*sec(d*x + c) + a)^(3/2), x)
```

3.130.8 Giac [A] (verification not implemented)

Time = 0.74 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.40

$$\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = -\frac{\frac{\sqrt{2}\log\left(\left|-\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right|\right)}{\sqrt{-a}a\operatorname{sgn}(\cos(dx+c))} - \frac{\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^2\operatorname{sgn}(\cos(dx+c))}}{4d}$$

input
```
integrate(sec(d*x+c)/(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
-1/4*(sqrt(2)*log(abs(-sqrt(-a)*tan(1/2*d*x + 1/2*c) + sqrt(-a*tan(1/2*d*x
 + 1/2*c)^2 + a)))/(sqrt(-a)*a*sgn(cos(d*x + c))) - sqrt(2)*sqrt(-a*tan(1/
2*d*x + 1/2*c)^2 + a)*tan(1/2*d*x + 1/2*c)/(a^2*sgn(cos(d*x + c))))/d
```

3.130.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \int \frac{1}{\cos(c+d\,x)\,\left(a+\frac{a}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

input
```
int(1/(cos(c + d*x)*(a + a/cos(c + d*x))^(3/2)),x)
```

output
```
int(1/(cos(c + d*x)*(a + a/cos(c + d*x))^(3/2)), x)
```

3.131 $\int \frac{1}{(a+a\sec(c+dx))^{3/2}}\,dx$

3.131.1 Optimal result . . . 1149
3.131.2 Mathematica [A] (verified) . . . 1149
3.131.3 Rubi [A] (verified) . . . 1150
3.131.4 Maple [B] (verified) . . . 1153
3.131.5 Fricas [B] (verification not implemented) . . . 1153
3.131.6 Sympy [F] . . . 1154
3.131.7 Maxima [F] . . . 1154
3.131.8 Giac [A] (verification not implemented) . . . 1155
3.131.9 Mupad [F(-1)] . . . 1155

3.131.1 Optimal result

Integrand size = 14, antiderivative size = 114

$$\int \frac{1}{(a+a\sec(c+dx))^{3/2}}\,dx = \frac{2\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^{3/2}d} - \frac{5\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} - \frac{\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}}$$

output
```
2*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/a^(3/2)/d-5/4*arctan(1
/2*a^(1/2)*tan(d*x+c)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))/a^(3/2)/d*2^(1/2)-1/
2*tan(d*x+c)/d/(a+a*sec(d*x+c))^(3/2)
```

3.131.2 Mathematica [A] (verified)

Time = 2.61 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.59

$$\int \frac{1}{(a+a\sec(c+dx))^{3/2}}\,dx = -\frac{\sec^{\frac{3}{2}}(c+dx)\left(5\arcsin\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)\sqrt{\frac{1}{1+\sec(c+dx)}}\sqrt{1+\sec(c+dx)}+\sqrt{2}\left(-4\arctan\left(\frac{\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{\frac{1}{1+\sec(c+dx)}}}\right)\right.\right.}{d\sec^2\left(\frac{1}{2}(c+dx)\right)^{3/2}(a(1+\sec(c+dx)))^{3/2}}$$

input
```
Integrate[(a + a*Sec[c + d*x])^(-3/2),x]
```

output
```
-((Sec[c + d*x]^(3/2)*(5*ArcSin[Tan[(c + d*x)/2]]*Sqrt[(1 + Sec[c + d*x])^
(-1)]*Sqrt[1 + Sec[c + d*x]] + Sqrt[2]*(-4*ArcTan[Tan[(c + d*x)/2]/Sqrt[(1
 + Sec[c + d*x])^(-1)]]*Sqrt[(1 + Sec[c + d*x])^(-1)]*Sqrt[1 + Sec[c + d*x
]] + (Sqrt[(1 + Cos[c + d*x])^(-1)]*Tan[(c + d*x)/2])/Sqrt[Sec[c + d*x]]))
)/(d*(Sec[(c + d*x)/2]^2)^(3/2)*(a*(1 + Sec[c + d*x]))^(3/2)))
```

3.131.3 Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.05, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {3042, 4264, 27, 3042, 4408, 3042, 4261, 216, 4282, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\sec(c+dx)+a)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}}dx$$

$$\Big\downarrow \text{4264}$$

$$-\frac{\int -\frac{4a-a\sec(c+dx)}{2\sqrt{\sec(c+dx)a+a}}dx}{2a^2}-\frac{\tan(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{4a-a\sec(c+dx)}{\sqrt{\sec(c+dx)a+a}}dx}{4a^2}-\frac{\tan(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{4a-a\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a^2}-\frac{\tan(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{4408}$$

$$\frac{4\int \sqrt{\sec(c+dx)a+a}dx-5a\int \frac{\sec(c+dx)}{\sqrt{\sec(c+dx)a+a}}dx}{4a^2}-\frac{\tan(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx-5a\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a^2}-\frac{\tan(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

$\Big\downarrow$ 4261

$$\frac{-\frac{8a\int \frac{1}{\frac{a^2\tan^2(c+dx)}{\sec(c+dx)a+a}+a}d\left(-\frac{a\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}-5a\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a^2}-\frac{\tan(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

$\Big\downarrow$ 216

$$\frac{\frac{8\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}-5a\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a^2}-\frac{\tan(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

$\Big\downarrow$ 4282

$$\frac{\frac{10a\int \frac{1}{\frac{a^2\tan^2(c+dx)}{\sec(c+dx)a+a}+2a}d\left(-\frac{a\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}+\frac{8\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}}{4a^2}-\frac{\tan(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

$\Big\downarrow$ 216

$$\frac{\frac{8\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}-\frac{5\sqrt{2}\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{d}}{4a^2}-\frac{\tan(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

input
```
Int[(a + a*Sec[c + d*x])^(-3/2),x]
```

output
```
((8*Sqrt[a]*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d - (
5*Sqrt[2]*Sqrt[a]*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c
+ d*x]])])/d)/(4*a^2) - Tan[c + d*x]/(2*d*(a + a*Sec[c + d*x])^(3/2))
```

3.131. $\int \frac{1}{(a+a\sec(c+dx))^{3/2}}\,dx$

3.131.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4261
```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(b/d)
  Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 4264
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(-Cot[c
+ d*x])*((a + b*Csc[c + d*x])^n/(d*(2*n + 1))), x] + Simp[1/(a^2*(2*n + 1))
   Int[(a + b*Csc[c + d*x])^(n + 1)*(a*(2*n + 1) - b*(n + 1)*Csc[c + d*x]),
 x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LeQ[n, -1] && Int
egerQ[2*n]
```

rule 4282
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2/f   Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[
a + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4408
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Simp[c/a   Int[Sqrt[a + b*Csc[e + f*x]], x], x] -
Simp[(b*c - a*d)/a   Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

3.131. $\int \frac{1}{(a+a\sec(c+dx))^{3/2}}\,dx$

3.131.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 198 vs. $2(93) = 186$.

Time = 0.91 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.75

method	result
default	$\frac{\sqrt{-\frac{2a}{(1-\cos(dx+c))^2\csc(dx+c)^2-1}}\sqrt{(1-\cos(dx+c))^2\csc(dx+c)^2-1}\left(4\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{2}\,(-\cot(dx+c)+\csc(dx+c))}{\sqrt{(1-\cos(dx+c))^2\csc(dx+c)^2-1}}\right)+\sqrt{(1-\cos(dx+}\right.}{4d\,a^2}$

input
```
int(1/(a+a*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/4/d/a^2*(-2*a/((1-cos(d*x+c))^2*csc(d*x+c)^2-1))^(1/2)*((1-cos(d*x+c))^2
*csc(d*x+c)^2-1)^(1/2)*(4*2^(1/2)*arctanh(2^(1/2)/((1-cos(d*x+c))^2*csc(d*
x+c)^2-1)^(1/2)*(-cot(d*x+c)+csc(d*x+c)))+((1-cos(d*x+c))^2*csc(d*x+c)^2-1
)^(1/2)*(-cot(d*x+c)+csc(d*x+c))-5*ln(csc(d*x+c)-cot(d*x+c)+((1-cos(d*x+c)
)^2*csc(d*x+c)^2-1)^(1/2)))
```

3.131.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 203 vs. $2(93) = 186$.

Time = 0.34 (sec) , antiderivative size = 491, normalized size of antiderivative = 4.31

$$\int \frac{1}{(a+a\sec(c+dx))^{3/2}}\,dx = \left[-\frac{5\sqrt{2}\left(\cos(dx+c)^2+2\cos(dx+c)+1\right)\sqrt{-a}\log\left(-\frac{2\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}}{}\right.}{}\right.$$

input
```
integrate(1/(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
[-1/8*(5*sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(-a)*log(-(2*sq
rt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*
x + c) - 3*a*cos(d*x + c)^2 - 2*a*cos(d*x + c) + a)/(cos(d*x + c)^2 + 2*co
s(d*x + c) + 1)) + 8*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(-a)*log((2
*a*cos(d*x + c)^2 + 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos
(d*x + c)*sin(d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) + 4*sqrt(
(a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c))/(a^2*d*cos(d
*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d), 1/4*(5*sqrt(2)*(cos(d*x + c)^2
+ 2*cos(d*x + c) + 1)*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos
(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) - 8*(cos(d*x + c)^2 + 2*co
s(d*x + c) + 1)*sqrt(a)*arctan(sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos
(d*x + c)/(sqrt(a)*sin(d*x + c))) - 2*sqrt((a*cos(d*x + c) + a)/cos(d*x +
c))*cos(d*x + c)*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c
) + a^2*d)]
```

3.131.6 Sympy [F]

$$\int \frac{1}{(a + a\sec(c + dx))^{3/2}}\,dx = \int \frac{1}{(a\sec{(c + dx)} + a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+a*sec(d*x+c))**(3/2),x)
```

output
```
Integral((a*sec(c + d*x) + a)**(-3/2), x)
```

3.131.7 Maxima [F]

$$\int \frac{1}{(a + a\sec(c + dx))^{3/2}}\,dx = \int \frac{1}{(a\sec{(dx + c)} + a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)^(-3/2), x)
```

3.131.8 Giac [A] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.41

$$\int \frac{1}{(a+a\sec(c+dx))^{3/2}}\,dx = -\frac{\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{4\,a^2d\mathrm{sgn}\left(\cos\left(dx+c\right)\right)}$$

input
```
integrate(1/(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
-1/4*sqrt(2)*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*tan(1/2*d*x + 1/2*c)/(a^2
*d*sgn(cos(d*x + c)))
```

3.131.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\sec(c+dx))^{3/2}}\,dx = \int \frac{1}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

input
```
int(1/(a + a/cos(c + d*x))^(3/2),x)
```

output
```
int(1/(a + a/cos(c + d*x))^(3/2), x)
```

3.132 $\int \frac{\cos(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx$

3.132.1 Optimal result 1156
3.132.2 Mathematica [A] (verified) 1156
3.132.3 Rubi [A] (verified) 1157
3.132.4 Maple [B] (warning: unable to verify) 1160
3.132.5 Fricas [A] (verification not implemented) 1161
3.132.6 Sympy [F] 1162
3.132.7 Maxima [F] 1162
3.132.8 Giac [B] (verification not implemented) 1163
3.132.9 Mupad [F(-1)] 1163

3.132.1 Optimal result

Integrand size = 21, antiderivative size = 144

$$\int \frac{\cos(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = -\frac{3\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^{3/2}d} + \frac{9\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} - \frac{\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{3\sin(c+dx)}{2ad\sqrt{a+a\sec(c+dx)}}$$

output
```
-3*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/a^(3/2)/d-1/2*sin(d*x
+c)/d/(a+a*sec(d*x+c))^(3/2)+9/4*arctan(1/2*a^(1/2)*tan(d*x+c)*2^(1/2)/(a+
a*sec(d*x+c))^(1/2))/a^(3/2)/d*2^(1/2)+3/2*sin(d*x+c)/a/d/(a+a*sec(d*x+c))
^(1/2)
```

3.132.2 Mathematica [A] (verified)

Time = 0.85 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.90

$$\int \frac{\cos(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \frac{\left(2(3+2\cos(c+dx))\sqrt{1-\sec(c+dx)} - 12\text{arctanh}\left(\sqrt{1-\sec(c+dx)}\right)(1\right.}{4d\sqrt{1-\sec(c+dx)}(a}$$

input
```
Integrate[Cos[c + d*x]/(a + a*Sec[c + d*x])^(3/2),x]
```

output
```
((2*(3 + 2*Cos[c + d*x])*Sqrt[1 - Sec[c + d*x]] - 12*ArcTanh[Sqrt[1 - Sec[
c + d*x]]]*(1 + Sec[c + d*x]) + 9*Sqrt[2]*ArcTanh[Sqrt[1 - Sec[c + d*x]]/S
qrt[2]]*(1 + Sec[c + d*x]))*Tan[c + d*x])/(4*d*Sqrt[1 - Sec[c + d*x]]*(a*(
1 + Sec[c + d*x]))^(3/2))
```

3.132.3 Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.06, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.619$, Rules used = {3042, 4304, 27, 3042, 4510, 25, 3042, 4408, 3042, 4261, 216, 4282, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(c+dx)}{(a\sec(c+dx)+a)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}}dx$$

$$\Big\downarrow \text{4304}$$

$$-\frac{\int -\frac{3\cos(c+dx)(2a-a\sec(c+dx))}{2\sqrt{\sec(c+dx)a+a}}dx}{2a^2}-\frac{\sin(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{3\int \frac{\cos(c+dx)(2a-a\sec(c+dx))}{\sqrt{\sec(c+dx)a+a}}dx}{4a^2}-\frac{\sin(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\int \frac{2a-a\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a^2}-\frac{\sin(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{4510}$$

$$\frac{3\left(\frac{\int -\frac{2a^2-a^2\sec(c+dx)}{\sqrt{\sec(c+dx)a+a}}dx}{a}+\frac{2a\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}\right)}{4a^2}-\frac{\sin(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{3\left(\frac{2a\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}-\frac{\int\frac{2a^2-a^2\sec(c+dx)}{\sqrt{\sec(c+dx)a+a}}dx}{a}\right)}{4a^2}-\frac{\sin(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

$\Big\downarrow$ 3042

$$\frac{3\left(\frac{2a\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}-\frac{\int\frac{2a^2-a^2\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{a}\right)}{4a^2}-\frac{\sin(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

$\Big\downarrow$ 4408

$$\frac{3\left(\frac{2a\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}-\frac{2a\int\sqrt{\sec(c+dx)a+a}dx-3a^2\int\frac{\sec(c+dx)}{\sqrt{\sec(c+dx)a+a}}dx}{a}\right)}{4a^2}-\frac{\sin(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

$\Big\downarrow$ 3042

$$\frac{3\left(\frac{2a\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}-\frac{2a\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx-3a^2\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{a}\right)}{4a^2}-\frac{\sin(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

$\Big\downarrow$ 4261

$$\frac{3\left(\frac{2a\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}-\frac{-3a^2\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx-\frac{4a^2\int\frac{1}{\frac{a^2\tan^2(c+dx)}{\sec(c+dx)a+a}+a}d\left(-\frac{a\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}}{a}\right)}{4a^2}-\frac{\sin(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

$\Big\downarrow$ 216

$$\frac{3\left(\frac{2a\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}-\frac{\frac{4a^{3/2}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}-3a^2\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{a}\right)}{4a^2}-\frac{\sin(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

$\Big\downarrow$ 4282

3.132. $\int\frac{\cos(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx$

$$\frac{3\left(\frac{2a\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}-\frac{\frac{6a^2\int\frac{1}{\frac{a^2\tan^2(c+dx)}{\sec(c+dx)a+a}+2a}d\left(-\frac{a\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}+\frac{4a^{3/2}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}}{a}\right)}{4a^2}-\frac{\sin(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \; 216$$

$$\frac{3\left(\frac{2a\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}-\frac{\frac{4a^{3/2}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}-\frac{3\sqrt{2}a^{3/2}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{d}}{a}\right)}{4a^2}-\frac{\sin(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

input
```
Int[Cos[c + d*x]/(a + a*Sec[c + d*x])^(3/2),x]
```

output
```
-1/2*Sin[c + d*x]/(d*(a + a*Sec[c + d*x])^(3/2)) + (3*(-(((4*a^(3/2)*ArcTa
n[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d - (3*Sqrt[2]*a^(3/2)
*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/d)/a)
+ (2*a*Sin[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]])))/(4*a^2)
```

3.132.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4261
```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(b/d)
  Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 4282
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2/f   Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[
a + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4304
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-Cot[e + f*x])*(a + b*Csc[e + f*x])^m*((d*Csc
[e + f*x])^n/(f*(2*m + 1))), x] + Simp[1/(a^2*(2*m + 1))   Int[(a + b*Csc[e
 + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e
+ f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ
[m, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

rule 4408
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Simp[c/a   Int[Sqrt[a + b*Csc[e + f*x]], x], x] -
Simp[(b*c - a*d)/a   Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

rule 4510
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*Cot[e
 + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(f*n)), x] - Simp[1/(b*d
*n)   Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*
n - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

3.132.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 449 vs. $2(119) = 238$.

Time = 26.79 (sec) , antiderivative size = 450, normalized size of antiderivative = 3.12

method	result
default	$-\frac{\sqrt{-\frac{2a}{(1-\cos(dx+c))^2\csc(dx+c)^2-1}}\left(6\sqrt{(1-\cos(dx+c))^2\csc(dx+c)^2-1}\,\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{2}\,(-\cot(dx+c)+\csc(dx+c))}{\sqrt{(1-\cos(dx+c))^2\csc(dx+c)^2-1}}\right)(1-\cos(dx+c)\right.}{}$

input
```
int(cos(d*x+c)/(a+a*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4/d/a^2*(-2*a/((1-cos(d*x+c))^2*csc(d*x+c)^2-1))^(1/2)*(6*((1-cos(d*x+c
))^2*csc(d*x+c)^2-1)^(1/2)*2^(1/2)*arctanh(2^(1/2)/((1-cos(d*x+c))^2*csc(d
*x+c)^2-1)^(1/2)*(-cot(d*x+c)+csc(d*x+c)))*(1-cos(d*x+c))^2*csc(d*x+c)^2+(
1-cos(d*x+c))^5*csc(d*x+c)^5-9*((1-cos(d*x+c))^2*csc(d*x+c)^2-1)^(1/2)*ln(
csc(d*x+c)-cot(d*x+c)+((1-cos(d*x+c))^2*csc(d*x+c)^2-1)^(1/2))*(1-cos(d*x+
c))^2*csc(d*x+c)^2+6*2^(1/2)*arctanh(2^(1/2)/((1-cos(d*x+c))^2*csc(d*x+c)^
2-1)^(1/2)*(-cot(d*x+c)+csc(d*x+c)))*((1-cos(d*x+c))^2*csc(d*x+c)^2-1)^(1/
2)+4*(1-cos(d*x+c))^3*csc(d*x+c)^3-9*ln(csc(d*x+c)-cot(d*x+c)+((1-cos(d*x+
c))^2*csc(d*x+c)^2-1)^(1/2))*((1-cos(d*x+c))^2*csc(d*x+c)^2-1)^(1/2)-5*csc
(d*x+c)+5*cot(d*x+c))/((1-cos(d*x+c))^2*csc(d*x+c)^2+1)
```

3.132.5 Fricas [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 518, normalized size of antiderivative = 3.60

$$\int \frac{\cos(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \left[-\frac{9\sqrt{2}\left(\cos\left(dx+c\right)^2+2\,\cos\left(dx+c\right)+1\right)\sqrt{-a}\log\left(\frac{2\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos}{\cos}\right.}{} \right.$$

$$-\frac{9\sqrt{2}\left(\cos\left(dx+c\right)^2+2\,\cos\left(dx+c\right)+1\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{\sqrt{a}\sin(dx+c)}\right)-12\left(\cos\left(dx+c\right)^2+2\,\cos\right.}{4\left(a^2d\cos\left(dx+c\right)^2\right.}$$

input
```
integrate(cos(d*x+c)/(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
[-1/8*(9*sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(-a)*log((2*sqr
t(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x
 + c) + 3*a*cos(d*x + c)^2 + 2*a*cos(d*x + c) - a)/(cos(d*x + c)^2 + 2*cos
(d*x + c) + 1)) + 12*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(-a)*log((2
*a*cos(d*x + c)^2 - 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos
(d*x + c)*sin(d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) - 4*(2*co
s(d*x + c)^2 + 3*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin
(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d), -1/4*(9*
sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*arctan(sqrt(2)*sqrt(
(a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) -
12*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*arctan(sqrt((a*cos(d*x +
c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) - 2*(2*cos(d*x
+ c)^2 + 3*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x +
 c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)]
```

3.132.6 Sympy [F]

$$\int \frac{\cos(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \int \frac{\cos{(c+dx)}}{(a(\sec{(c+dx)}+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)/(a+a*sec(d*x+c))**(3/2),x)
```

output
```
Integral(cos(c + d*x)/(a*(sec(c + d*x) + 1))**(3/2), x)
```

3.132.7 Maxima [F]

$$\int \frac{\cos(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \int \frac{\cos{(dx+c)}}{(a\sec{(dx+c)}+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)/(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate(cos(d*x + c)/(a*sec(d*x + c) + a)^(3/2), x)
```

3.132.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 387 vs. $2(119) = 238$.

Time = 1.17 (sec) , antiderivative size = 387, normalized size of antiderivative = 2.69

$$\int \frac{\cos(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \frac{16\sqrt{2}\left(3\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2-a\right)}{\left(\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^4-6\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+}\right.\right.}$$

input
```
integrate(cos(d*x+c)/(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
1/8*(16*sqrt(2)*(3*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x +
1/2*c)^2 + a))^2 - a)/(((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d
*x + 1/2*c)^2 + a))^4 - 6*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2
*d*x + 1/2*c)^2 + a))^2*a + a^2)*sqrt(-a)*sgn(cos(d*x + c))) - 9*sqrt(2)*l
og((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2
)/(sqrt(-a)*a*sgn(cos(d*x + c))) + 2*sqrt(2)*sqrt(-a*tan(1/2*d*x + 1/2*c)^
2 + a)*tan(1/2*d*x + 1/2*c)/(a^2*sgn(cos(d*x + c))) + 12*log(abs(-2*(sqrt(
-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - 4*sqrt
(2)*abs(a) + 6*a)/abs(-2*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*
d*x + 1/2*c)^2 + a))^2 + 4*sqrt(2)*abs(a) + 6*a))/(sqrt(-a)*abs(a)*sgn(cos
(d*x + c))))/d
```

3.132.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \int \frac{\cos(c+d\,x)}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

input
```
int(cos(c + d*x)/(a + a/cos(c + d*x))^(3/2),x)
```

output
```
int(cos(c + d*x)/(a + a/cos(c + d*x))^(3/2), x)
```

3.133 $\int \frac{\cos^2(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx$

3.133.1 Optimal result 1164
3.133.2 Mathematica [C] (verified) 1164
3.133.3 Rubi [A] (verified) 1165
3.133.4 Maple [B] (verified) 1169
3.133.5 Fricas [A] (verification not implemented) 1170
3.133.6 Sympy [F] 1170
3.133.7 Maxima [F] 1171
3.133.8 Giac [B] (verification not implemented) 1171
3.133.9 Mupad [F(-1)] 1172

3.133.1 Optimal result

Integrand size = 23, antiderivative size = 185

$$\int \frac{\cos^2(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \frac{19\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{4a^{3/2}d} - \frac{13\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} - \frac{\cos(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} - \frac{7\sin(c+dx)}{4ad\sqrt{a+a\sec(c+dx)}} + \frac{\cos(c+dx)\sin(c+dx)}{ad\sqrt{a+a\sec(c+dx)}}$$

output
```
19/4*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/a^(3/2)/d-1/2*cos(d
*x+c)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(3/2)-13/4*arctan(1/2*a^(1/2)*tan(d*x+
c)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))/a^(3/2)/d*2^(1/2)-7/4*sin(d*x+c)/a/d/(a
+a*sec(d*x+c))^(1/2)+cos(d*x+c)*sin(d*x+c)/a/d/(a+a*sec(d*x+c))^(1/2)
```

3.133.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 1.74 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.96

$$\int \frac{\cos^2(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \frac{-\sin(2(c+dx)) + \frac{\left(\frac{13}{4}\left(7\operatorname{arctanh}\left(\sqrt{1-\sec(c+dx)}\right)-4\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{1-\sec(c+dx)}}{\sqrt{2}}\right)+\cos(c-\right.\right.}{}}{4d(}$$

input
```
Integrate[Cos[c + d*x]^2/(a + a*Sec[c + d*x])^(3/2),x]
```

output
```
(-Sin[2*(c + d*x)] + (((13*(7*ArcTanh[Sqrt[1 - Sec[c + d*x]]] - 4*Sqrt[2]*
ArcTanh[Sqrt[1 - Sec[c + d*x]]/Sqrt[2]] + Cos[c + d*x]*(-1 + 2*Cos[c + d*x
])*Sqrt[1 - Sec[c + d*x]]))/4 - 10*Hypergeometric2F1[1/2, 3, 3/2, 1 - Sec[
c + d*x]]*Sqrt[1 - Sec[c + d*x]])*(1 + Sec[c + d*x])*Tan[c + d*x])/Sqrt[1
- Sec[c + d*x]])/(4*d*(a*(1 + Sec[c + d*x]))^(3/2))
```

3.133.3 Rubi [A] (verified)

Time = 1.13 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.08, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.696$, Rules used = {3042, 4304, 27, 3042, 4510, 27, 3042, 4510, 27, 3042, 4408, 3042, 4261, 216, 4282, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^2(c+dx)}{(a\sec(c+dx)+a)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^2\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}}dx$$

$$\Big\downarrow \text{4304}$$

$$-\frac{\int -\frac{\cos^2(c+dx)(8a-5a\sec(c+dx))}{2\sqrt{\sec(c+dx)a+a}}dx}{2a^2}-\frac{\sin(c+dx)\cos(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\cos^2(c+dx)(8a-5a\sec(c+dx))}{\sqrt{\sec(c+dx)a+a}}dx}{4a^2}-\frac{\sin(c+dx)\cos(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{8a-5a\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^2\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a^2}-\frac{\sin(c+dx)\cos(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{4510}$$

$$\frac{\frac{\int -\frac{2\cos(c+dx)\left(7a^2-6a^2\sec(c+dx)\right)}{\sqrt{\sec(c+dx)a+a}}dx}{2a}+\frac{4a\sin(c+dx)\cos(c+dx)}{d\sqrt{a\sec(c+dx)+a}}}{4a^2}-\frac{\sin(c+dx)\cos(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{4a\sin(c+dx)\cos(c+dx)}{d\sqrt{a\sec(c+dx)+a}}-\frac{\int \frac{\cos(c+dx)\left(7a^2-6a^2\sec(c+dx)\right)}{\sqrt{\sec(c+dx)a+a}}dx}{a}}{4a^2}-\frac{\sin(c+dx)\cos(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{4a\sin(c+dx)\cos(c+dx)}{d\sqrt{a\sec(c+dx)+a}}-\frac{\int \frac{7a^2-6a^2\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{a}}{4a^2}-\frac{\sin(c+dx)\cos(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \; 4510$$

$$\frac{\frac{4a\sin(c+dx)\cos(c+dx)}{d\sqrt{a\sec(c+dx)+a}}-\frac{\frac{\int -\frac{19a^3-7a^3\sec(c+dx)}{2\sqrt{\sec(c+dx)a+a}}dx}{a}+\frac{7a^2\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}}{a}}{4a^2}-\frac{\sin(c+dx)\cos(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{4a\sin(c+dx)\cos(c+dx)}{d\sqrt{a\sec(c+dx)+a}}-\frac{\frac{7a^2\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}-\frac{\int \frac{19a^3-7a^3\sec(c+dx)}{\sqrt{\sec(c+dx)a+a}}dx}{2a}}{a}}{4a^2}-\frac{\sin(c+dx)\cos(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{4a\sin(c+dx)\cos(c+dx)}{d\sqrt{a\sec(c+dx)+a}}-\frac{\frac{7a^2\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}-\frac{\int \frac{19a^3-7a^3\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{2a}}{a}}{4a^2}-\frac{\sin(c+dx)\cos(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \; 4408$$

$$\frac{\frac{4a\sin(c+dx)\cos(c+dx)}{d\sqrt{a\sec(c+dx)+a}}-\frac{\frac{7a^2\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}-\frac{19a^2\int \sqrt{\sec(c+dx)a+a}dx-26a^3\int \frac{\sec(c+dx)}{\sqrt{\sec(c+dx)a+a}}dx}{2a}}{a}}{4a^2}-\frac{\sin(c+dx)\cos(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{4a\sin(c+dx)\cos(c+dx)}{d\sqrt{a\sec(c+dx)+a}}-\frac{\frac{7a^2\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}-\frac{19a^2\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx-26a^3\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{2a}}{a}}{4a^2}-\frac{\sin(c+dx)\cos(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

3.133. $\int \frac{\cos^2(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx$

$$\Big\downarrow \; 4261$$

$$\frac{\frac{4a\sin(c+dx)\cos(c+dx)}{d\sqrt{a\sec(c+dx)+a}}-\frac{\frac{7a^2\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}-\frac{-26a^3\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx-\frac{38a^3\int\frac{1}{\frac{a^2\tan^2(c+dx)}{\sec(c+dx)a+a}+a}d\left(-\frac{a\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}}{2a}}{a}}{\frac{4a^2\sin(c+dx)\cos(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}}-$$

$$\Big\downarrow \; 216$$

$$\frac{\frac{4a\sin(c+dx)\cos(c+dx)}{d\sqrt{a\sec(c+dx)+a}}-\frac{\frac{7a^2\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}-\frac{\frac{38a^{5/2}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}-26a^3\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{2a}}{a}}{\frac{4a^2\sin(c+dx)\cos(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}}-$$

$$\Big\downarrow \; 4282$$

$$\frac{\frac{4a\sin(c+dx)\cos(c+dx)}{d\sqrt{a\sec(c+dx)+a}}-\frac{\frac{7a^2\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}-\frac{\frac{52a^3\int\frac{1}{\frac{a^2\tan^2(c+dx)}{\sec(c+dx)a+a}+2a}d\left(-\frac{a\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}+\frac{38a^{5/2}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}}{2a}}{a}}{\frac{4a^2\sin(c+dx)\cos(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}}-$$

$$\Big\downarrow \; 216$$

$$\frac{\frac{4a\sin(c+dx)\cos(c+dx)}{d\sqrt{a\sec(c+dx)+a}}-\frac{\frac{7a^2\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}-\frac{\frac{38a^{5/2}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}-\frac{26\sqrt{2}a^{5/2}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{d}}{2a}}{a}}{\frac{4a^2\sin(c+dx)\cos(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}}-$$

input
```
Int[Cos[c + d*x]^2/(a + a*Sec[c + d*x])^(3/2),x]
```

output
```
-1/2*(Cos[c + d*x]*Sin[c + d*x])/(d*(a + a*Sec[c + d*x])^(3/2)) + ((4*a*Co
s[c + d*x]*Sin[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]]) - (-1/2*((38*a^(5/2)
*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d - (26*Sqrt[2]*
a^(5/2)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])
/d)/a + (7*a^2*Sin[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]]))/a)/(4*a^2)
```

3.133. $\int \frac{\cos^2(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx$

3.133.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4261
```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(b/d)
  Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 4282
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2/f   Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[
a + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4304
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-Cot[e + f*x])*(a + b*Csc[e + f*x])^m*((d*Csc
[e + f*x])^n/(f*(2*m + 1))), x] + Simp[1/(a^2*(2*m + 1))   Int[(a + b*Csc[e
 + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e
+ f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ
[m, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

rule 4408
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Simp[c/a   Int[Sqrt[a + b*Csc[e + f*x]], x], x] -
Simp[(b*c - a*d)/a   Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

3.133. $\int \frac{\cos^2(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx$

rule 4510
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*Cot[e
 + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(f*n)), x] - Simp[1/(b*d
*n)   Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*
n - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

3.133.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 510 vs. $2(156) = 312$.

Time = 22.39 (sec) , antiderivative size = 511, normalized size of antiderivative = 2.76

method	result
default	$-\frac{\left(13\sqrt{2}\,\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\ln\left(\csc(dx+c)-\cot(dx+c)+\sqrt{\cot(dx+c)^2-2\cot(dx+c)\csc(dx+c)+\csc(dx+c)^2-1}\right)\cos(dx+c)^2+26\sqrt{2}\right.}{}$

input
```
int(cos(d*x+c)^2/(a+a*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4/d/a^2*(13*2^(1/2)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*ln(csc(d*x+c)-co
t(d*x+c)+(cot(d*x+c)^2-2*cot(d*x+c)*csc(d*x+c)+csc(d*x+c)^2-1)^(1/2))*cos(
d*x+c)^2+26*2^(1/2)*ln(csc(d*x+c)-cot(d*x+c)+(cot(d*x+c)^2-2*cot(d*x+c)*cs
c(d*x+c)+csc(d*x+c)^2-1)^(1/2))*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*cos(d*x
+c)-19*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(sin(d*x+c)/(cos(d*x+c)+1
)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)^2-2*cos(d*x+c)^3*sin(d*x+
c)+13*ln(csc(d*x+c)-cot(d*x+c)+(cot(d*x+c)^2-2*cot(d*x+c)*csc(d*x+c)+csc(d
*x+c)^2-1)^(1/2))*2^(1/2)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-38*(-cos(d*x+
c)/(cos(d*x+c)+1))^(1/2)*arctanh(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(c
os(d*x+c)+1))^(1/2))*cos(d*x+c)+3*cos(d*x+c)^2*sin(d*x+c)-19*(-cos(d*x+c)/
(cos(d*x+c)+1))^(1/2)*arctanh(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(
d*x+c)+1))^(1/2))+7*cos(d*x+c)*sin(d*x+c))*(a*(1+sec(d*x+c)))^(1/2)/(cos(d
*x+c)+1)^2
```

3.133.5 Fricas [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 536, normalized size of antiderivative = 2.90

$$\int \frac{\cos^2(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \left[-\frac{13\sqrt{2}\left(\cos\left(dx+c\right)^2+2\cos\left(dx+c\right)+1\right)\sqrt{-a}\log\left(-\frac{2\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}}{}\right.}{}\right.$$

input
```
integrate(cos(d*x+c)^2/(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
[-1/8*(13*sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(-a)*log(-(2*s
qrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d
*x + c) - 3*a*cos(d*x + c)^2 - 2*a*cos(d*x + c) + a)/(cos(d*x + c)^2 + 2*c
os(d*x + c) + 1)) + 19*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(-a)*log(
(2*a*cos(d*x + c)^2 + 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*c
os(d*x + c)*sin(d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) - 2*(2*
cos(d*x + c)^3 - 3*cos(d*x + c)^2 - 7*cos(d*x + c))*sqrt((a*cos(d*x + c) +
 a)/cos(d*x + c))*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x +
c) + a^2*d), 1/4*(13*sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)
*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt
(a)*sin(d*x + c))) - 19*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*arct
an(sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x +
 c))) + (2*cos(d*x + c)^3 - 3*cos(d*x + c)^2 - 7*cos(d*x + c))*sqrt((a*cos
(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d
*cos(d*x + c) + a^2*d)]
```

3.133.6 Sympy [F]

$$\int \frac{\cos^2(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \int \frac{\cos^2\left(c+dx\right)}{\left(a\left(\sec\left(c+dx\right)+1\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)**2/(a+a*sec(d*x+c))**(3/2),x)
```

output
```
Integral(cos(c + d*x)**2/(a*(sec(c + d*x) + 1))**(3/2), x)
```

3.133.7 Maxima [F]

$$\int \frac{\cos^2(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \int \frac{\cos\left(dx+c\right)^2}{\left(a\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^2/(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate(cos(d*x + c)^2/(a*sec(d*x + c) + a)^(3/2), x)
```

3.133.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 473 vs. $2(156) = 312$.

Time = 1.76 (sec) , antiderivative size = 473, normalized size of antiderivative = 2.56

$$\int \frac{\cos^2(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \frac{\frac{13\sqrt{2}\log\left(\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2\right)}{\sqrt{-a}a\operatorname{sgn}(\cos(dx+c))} - \frac{2\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\tan(}{a^2\operatorname{sgn}(\cos(dx+c))}}{}$$

input
```
integrate(cos(d*x+c)^2/(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
1/8*(13*sqrt(2)*log((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x +
 1/2*c)^2 + a))^2)/(sqrt(-a)*a*sgn(cos(d*x + c))) - 2*sqrt(2)*sqrt(-a*tan(
1/2*d*x + 1/2*c)^2 + a)*tan(1/2*d*x + 1/2*c)/(a^2*sgn(cos(d*x + c))) + 19*
log(abs(8*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2
+ a))^2 - 16*sqrt(2)*abs(a) - 24*a)/abs(8*(sqrt(-a)*tan(1/2*d*x + 1/2*c) -
 sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + 16*sqrt(2)*abs(a) - 24*a))/(sqrt
(-a)*abs(a)*sgn(cos(d*x + c))) - 4*sqrt(2)*(29*(sqrt(-a)*tan(1/2*d*x + 1/2
*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^6 - 133*(sqrt(-a)*tan(1/2*d*x +
 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4*a + 55*(sqrt(-a)*tan(1/2*
d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a^2 - 7*a^3)/(((sqrt
(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4 - 6*(sq
rt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a + a
^2)^2*sqrt(-a)*sgn(cos(d*x + c))))/d
```

3.133.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^2(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \int \frac{\cos\left(c+d\,x\right)^2}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

input
```
int(cos(c + d*x)^2/(a + a/cos(c + d*x))^(3/2),x)
```

output
```
int(cos(c + d*x)^2/(a + a/cos(c + d*x))^(3/2), x)
```

3.134 $\int \frac{\sec^5(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx$

3.134.1 Optimal result . . . 1173
3.134.2 Mathematica [A] (verified) . . . 1173
3.134.3 Rubi [A] (verified) . . . 1174
3.134.4 Maple [A] (warning: unable to verify) . . . 1178
3.134.5 Fricas [A] (verification not implemented) . . . 1178
3.134.6 Sympy [F] . . . 1179
3.134.7 Maxima [F] . . . 1179
3.134.8 Giac [A] (verification not implemented) . . . 1180
3.134.9 Mupad [F(-1)] . . . 1180

3.134.1 Optimal result

Integrand size = 23, antiderivative size = 183

$$\int \frac{\sec^5(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \frac{163\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} - \frac{\sec^3(c+dx)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{17\sec^2(c+dx)\tan(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} - \frac{197\tan(c+dx)}{24a^2d\sqrt{a+a\sec(c+dx)}} + \frac{95\sqrt{a+a\sec(c+dx)}\tan(c+dx)}{48a^3d}$$

output
```
163/32*arctan(1/2*a^(1/2)*tan(d*x+c)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))/a^(5/
2)/d*2^(1/2)-1/4*sec(d*x+c)^3*tan(d*x+c)/d/(a+a*sec(d*x+c))^(5/2)-17/16*se
c(d*x+c)^2*tan(d*x+c)/a/d/(a+a*sec(d*x+c))^(3/2)-197/24*tan(d*x+c)/a^2/d/(
a+a*sec(d*x+c))^(1/2)+95/48*(a+a*sec(d*x+c))^(1/2)*tan(d*x+c)/a^3/d
```

3.134.2 Mathematica [A] (verified)

Time = 1.35 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.74

$$\int \frac{\sec^5(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \frac{\left(978\sqrt{2}\text{arctanh}\left(\frac{\sqrt{1-\sec(c+dx)}}{\sqrt{2}}\right)\cos^4\left(\frac{1}{2}(c+dx)\right)\sec^2(c+dx)+\sqrt{1-\sec(c+}\right.}{48d\sqrt{1-\sec(c+dx}}$$

input
```
Integrate[Sec[c + d*x]^5/(a + a*Sec[c + d*x])^(5/2),x]
```

output
```
((978*Sqrt[2]*ArcTanh[Sqrt[1 - Sec[c + d*x]]/Sqrt[2]]*Cos[(c + d*x)/2]^4*S
ec[c + d*x]^2 + Sqrt[1 - Sec[c + d*x]]*(-299 - 503*Sec[c + d*x] - 160*Sec[
c + d*x]^2 + 32*Sec[c + d*x]^3))*Tan[c + d*x])/(48*d*Sqrt[1 - Sec[c + d*x]
]*(a*(1 + Sec[c + d*x]))^(5/2))
```

3.134.3 Rubi [A] (verified)

Time = 1.09 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.09, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.609$, Rules used = {3042, 4303, 27, 3042, 4507, 27, 3042, 4498, 27, 3042, 4489, 3042, 4282, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^5(c+dx)}{(a\sec(c+dx)+a)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^5}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}}dx$$

$$\Big\downarrow \text{4303}$$

$$-\frac{\int \frac{\sec^3(c+dx)(6a-11a\sec(c+dx))}{2(\sec(c+dx)a+a)^{3/2}}dx}{4a^2}-\frac{\tan(c+dx)\sec^3(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{\sec^3(c+dx)(6a-11a\sec(c+dx))}{(\sec(c+dx)a+a)^{3/2}}dx}{8a^2}-\frac{\tan(c+dx)\sec^3(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^3\left(6a-11a\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}}dx}{8a^2}-\frac{\tan(c+dx)\sec^3(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{4507}$$

$$-\frac{\frac{\int \frac{\sec^2(c+dx)\left(68a^2-95a^2\sec(c+dx)\right)}{2\sqrt{\sec(c+dx)a+a}}dx}{2a^2}+\frac{17a\tan(c+dx)\sec^2(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}}{8a^2}-\frac{\tan(c+dx)\sec^3(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \; 27$$

$$-\frac{\frac{\int \frac{\sec^2(c+dx)\left(68a^2-95a^2\sec(c+dx)\right)}{\sqrt{\sec(c+dx)a+a}}dx}{4a^2}+\frac{17a\tan(c+dx)\sec^2(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}}{8a^2}-\frac{\tan(c+dx)\sec^3(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2\left(68a^2-95a^2\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a^2}+\frac{17a\tan(c+dx)\sec^2(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}}{8a^2}-\frac{\tan(c+dx)\sec^3(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \; 4498$$

$$-\frac{\frac{\frac{2\int -\frac{\sec(c+dx)\left(95a^3-394a^3\sec(c+dx)\right)}{2\sqrt{\sec(c+dx)a+a}}dx}{3a}-\frac{190a\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}}{4a^2}+\frac{17a\tan(c+dx)\sec^2(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}}{8a^2}-\frac{\tan(c+dx)\sec^3(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \; 27$$

$$-\frac{\frac{\frac{-\int \frac{\sec(c+dx)\left(95a^3-394a^3\sec(c+dx)\right)}{\sqrt{\sec(c+dx)a+a}}dx}{3a}-\frac{190a\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}}{4a^2}+\frac{17a\tan(c+dx)\sec^2(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}}{8a^2}-\frac{\tan(c+dx)\sec^3(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\frac{\frac{-\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(95a^3-394a^3\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{3a}-\frac{190a\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}}{4a^2}+\frac{17a\tan(c+dx)\sec^2(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}}{8a^2}-\frac{\tan(c+dx)\sec^3(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \; 4489$$

$$-\frac{\frac{\frac{-\frac{489a^3\int \frac{\sec(c+dx)}{\sqrt{\sec(c+dx)a+a}}dx-\frac{788a^3\tan(c+dx)}{d\sqrt{a\sec(c+dx)+a}}}{3a}-\frac{190a\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}}{4a^2}+\frac{17a\tan(c+dx)\sec^2(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}}{8a^2}-\frac{\tan(c+dx)\sec^3(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \; 3042$$

3.134. $\int \frac{\sec^5(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx$

$$-\frac{-\frac{-\frac{489a^3\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx-\frac{788a^3\tan(c+dx)}{d\sqrt{a\sec(c+dx)+a}}}{3a}-\frac{190a\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}}{4a^2}+\frac{17a\tan(c+dx)\sec^2(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}}{8a^2}-\frac{\tan(c+dx)\sec^3(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \; 4282$$

$$-\frac{-\frac{-\frac{-\frac{978a^3\int\frac{1}{\frac{a^2\tan^2(c+dx)}{\sec(c+dx)a+a}+2a}d\left(-\frac{a\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}-\frac{788a^3\tan(c+dx)}{d\sqrt{a\sec(c+dx)+a}}}{3a}-\frac{190a\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}}{4a^2}+\frac{17a\tan(c+dx)\sec^2(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}}{8a^2}-\frac{\tan(c+dx)\sec^3(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \; 216$$

$$-\frac{-\frac{-\frac{\frac{489\sqrt{2}a^{5/2}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{d}-\frac{788a^3\tan(c+dx)}{d\sqrt{a\sec(c+dx)+a}}}{3a}-\frac{190a\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}}{4a^2}+\frac{17a\tan(c+dx)\sec^2(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}}{8a^2}-\frac{\tan(c+dx)\sec^3(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

input
```
Int[Sec[c + d*x]^5/(a + a*Sec[c + d*x])^(5/2),x]
```

output
```
-1/4*(Sec[c + d*x]^3*Tan[c + d*x])/(d*(a + a*Sec[c + d*x])^(5/2)) - ((17*a
*Sec[c + d*x]^2*Tan[c + d*x])/(2*d*(a + a*Sec[c + d*x])^(3/2)) + ((-190*a*
Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x])/(3*d) - ((489*Sqrt[2]*a^(5/2)*ArcTa
n[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/d - (788*a^3
*Tan[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]]))/(3*a))/(4*a^2))/(8*a^2)
```

3.134.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4282
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2/f   Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[
a + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4303
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-d^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d
*Csc[e + f*x])^(n - 2)/(f*(2*m + 1))), x] + Simp[d^2/(a*b*(2*m + 1))   Int[
(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n
 + 2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2,
0] && LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

rule 4489
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*((
a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Simp[(a*B*m + A*b*(m + 1))/(b*(m +
 1))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B
, e, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b
*(m + 1), 0] &&  !LtQ[m, -2^(-1)]
```

rule 4498
```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*
((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m + 2))   Int
[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*B*(m + 1) + (A*b*(m + 2) - a*B)
*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, m}, x] && NeQ[A*b - a
*B, 0] &&  !LtQ[m, -1]
```

rule 4507
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[d*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(a*f*(
2*m + 1))), x] - Simp[1/(a*b*(2*m + 1))   Int[(a + b*Csc[e + f*x])^(m + 1)*
(d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
 - n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f,
A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && G
tQ[n, 0]
```

3.134. $\int \frac{\sec^5(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx$

3.134.4 Maple [A] (warning: unable to verify)

Time = 1.12 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.17

method	result
default	$\frac{\sqrt{-\frac{2a}{(1-\cos(dx+c))^2\csc(dx+c)^2-1}}\left(6(1-\cos(dx+c))^7\csc(dx+c)^7+69(1-\cos(dx+c))^5\csc(dx+c)^5+489\ln\left(\csc(dx+c)-\cot(dx+c)+\sqrt{\right.\right.}{96d\,a^3\left((1-\cos(dx\right.}$

input
```
int(sec(d*x+c)^5/(a+a*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/96/d/a^3*(-2*a/((1-cos(d*x+c))^2*csc(d*x+c)^2-1))^(1/2)*(6*(1-cos(d*x+c)
)^7*csc(d*x+c)^7+69*(1-cos(d*x+c))^5*csc(d*x+c)^5+489*ln(csc(d*x+c)-cot(d*
x+c)+((1-cos(d*x+c))^2*csc(d*x+c)^2-1)^(1/2))*((1-cos(d*x+c))^2*csc(d*x+c)
^2-1)^(3/2)-668*(1-cos(d*x+c))^3*csc(d*x+c)^3+465*csc(d*x+c)-465*cot(d*x+c
))/((1-cos(d*x+c))^2*csc(d*x+c)^2-1)
```

3.134.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 455, normalized size of antiderivative = 2.49

$$\int \frac{\sec^5(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \left[-\frac{489\sqrt{2}\left(\cos(dx+c)^4+3\cos(dx+c)^3+3\cos(dx+c)^2+\cos(dx+c)\right)\sqrt{}}{}\right.$$

$$-\frac{489\sqrt{2}\left(\cos(dx+c)^4+3\cos(dx+c)^3+3\cos(dx+c)^2+\cos(dx+c)\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+}{\sqrt{a}\sin(dx+c)}\right.}{96\left(a^3d\cos(dx+c)^4+3a^3d\cos(dx+c)^3+3\right.}$$

input
```
integrate(sec(d*x+c)^5/(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
[-1/192*(489*sqrt(2)*(cos(d*x + c)^4 + 3*cos(d*x + c)^3 + 3*cos(d*x + c)^2
 + cos(d*x + c))*sqrt(-a)*log((2*sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a
)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) + 3*a*cos(d*x + c)^2 + 2*a*cos(d
*x + c) - a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + 4*(299*cos(d*x + c)^
3 + 503*cos(d*x + c)^2 + 160*cos(d*x + c) - 32)*sqrt((a*cos(d*x + c) + a)/
cos(d*x + c))*sin(d*x + c))/(a^3*d*cos(d*x + c)^4 + 3*a^3*d*cos(d*x + c)^3
 + 3*a^3*d*cos(d*x + c)^2 + a^3*d*cos(d*x + c)), -1/96*(489*sqrt(2)*(cos(d
*x + c)^4 + 3*cos(d*x + c)^3 + 3*cos(d*x + c)^2 + cos(d*x + c))*sqrt(a)*ar
ctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)
*sin(d*x + c))) + 2*(299*cos(d*x + c)^3 + 503*cos(d*x + c)^2 + 160*cos(d*x
 + c) - 32)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(a^3*d*c
os(d*x + c)^4 + 3*a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + a^3*d*co
s(d*x + c))]
```

3.134.6 Sympy [F]

$$\int \frac{\sec^5(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \int \frac{\sec^5(c+dx)}{(a(\sec(c+dx)+1))^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)**5/(a+a*sec(d*x+c))**(5/2),x)
```

output
```
Integral(sec(c + d*x)**5/(a*(sec(c + d*x) + 1))**(5/2), x)
```

3.134.7 Maxima [F]

$$\int \frac{\sec^5(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \int \frac{\sec(dx+c)^5}{(a\sec(dx+c)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^5/(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)^5/(a*sec(d*x + c) + a)^(5/2), x)
```

3.134.8 Giac [A] (verification not implemented)

Time = 1.10 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.20

$$\int \frac{\sec^5(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \frac{\frac{\left(\left(3\left(\frac{2\sqrt{2}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2}{a\operatorname{sgn}(\cos(dx+c))}+\frac{23\sqrt{2}}{a\operatorname{sgn}(\cos(dx+c))}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-\frac{668\sqrt{2}}{a\operatorname{sgn}(\cos(dx+c))}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+\frac{}{a\operatorname{sgn}(}\right.}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a\right)\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}}{96\,d}$$

input
```
integrate(sec(d*x+c)^5/(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
1/96*(((3*(2*sqrt(2)*tan(1/2*d*x + 1/2*c)^2/(a*sgn(cos(d*x + c))) + 23*sqr
t(2)/(a*sgn(cos(d*x + c))))*tan(1/2*d*x + 1/2*c)^2 - 668*sqrt(2)/(a*sgn(co
s(d*x + c))))*tan(1/2*d*x + 1/2*c)^2 + 465*sqrt(2)/(a*sgn(cos(d*x + c))))*
tan(1/2*d*x + 1/2*c)/((a*tan(1/2*d*x + 1/2*c)^2 - a)*sqrt(-a*tan(1/2*d*x +
 1/2*c)^2 + a)) - 489*sqrt(2)*log(abs(-sqrt(-a)*tan(1/2*d*x + 1/2*c) + sqr
t(-a*tan(1/2*d*x + 1/2*c)^2 + a)))/(sqrt(-a)*a^2*sgn(cos(d*x + c))))/d
```

3.134.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^5(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \int \frac{1}{\cos(c+d\,x)^5\left(a+\frac{a}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

input
```
int(1/(cos(c + d*x)^5*(a + a/cos(c + d*x))^(5/2)),x)
```

output
```
int(1/(cos(c + d*x)^5*(a + a/cos(c + d*x))^(5/2)), x)
```

3.135 $\int \frac{\sec^4(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx$

3.135.1 Optimal result 1181
3.135.2 Mathematica [A] (verified) 1181
3.135.3 Rubi [A] (verified) 1182
3.135.4 Maple [A] (verified) 1185
3.135.5 Fricas [A] (verification not implemented) 1185
3.135.6 Sympy [F] 1186
3.135.7 Maxima [F] 1186
3.135.8 Giac [A] (verification not implemented) 1186
3.135.9 Mupad [F(-1)] 1187

3.135.1 Optimal result

Integrand size = 23, antiderivative size = 145

$$\int \frac{\sec^4(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = -\frac{75\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} - \frac{\sec^2(c+dx)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \frac{13\tan(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} + \frac{9\tan(c+dx)}{4a^2d\sqrt{a+a\sec(c+dx)}}$$

output
```
-75/32*arctan(1/2*a^(1/2)*tan(d*x+c)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))/a^(5/
2)/d*2^(1/2)-1/4*sec(d*x+c)^2*tan(d*x+c)/d/(a+a*sec(d*x+c))^(5/2)+13/16*ta
n(d*x+c)/a/d/(a+a*sec(d*x+c))^(3/2)+9/4*tan(d*x+c)/a^2/d/(a+a*sec(d*x+c))^
(1/2)
```

3.135.2 Mathematica [A] (verified)

Time = 0.87 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.86

$$\int \frac{\sec^4(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \frac{\left(-150\sqrt{2}\text{arctanh}\left(\frac{\sqrt{1-\sec(c+dx)}}{\sqrt{2}}\right)\cos^4\left(\frac{1}{2}(c+dx)\right)\sec^2(c+dx)+\sqrt{1-\sec(c}\right.}{16d\sqrt{1-\sec(c+dx)}(a(1+\sec}$$

input
```
Integrate[Sec[c + d*x]^4/(a + a*Sec[c + d*x])^(5/2),x]
```

output
```
((-150*Sqrt[2]*ArcTanh[Sqrt[1 - Sec[c + d*x]]/Sqrt[2]]*Cos[(c + d*x)/2]^4*
Sec[c + d*x]^2 + Sqrt[1 - Sec[c + d*x]]*(49 + 85*Sec[c + d*x] + 32*Sec[c +
 d*x]^2))*Tan[c + d*x])/(16*d*Sqrt[1 - Sec[c + d*x]]*(a*(1 + Sec[c + d*x])
)^(5/2))
```

3.135.3 Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.07, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules used = {3042, 4303, 27, 3042, 4496, 27, 3042, 4489, 3042, 4282, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^4(c+dx)}{(a\sec(c+dx)+a)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^4}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}}dx$$

$$\Big\downarrow \text{4303}$$

$$-\frac{\int \frac{\sec^2(c+dx)(4a-9a\sec(c+dx))}{2(\sec(c+dx)a+a)^{3/2}}dx}{4a^2}-\frac{\tan(c+dx)\sec^2(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{\sec^2(c+dx)(4a-9a\sec(c+dx))}{(\sec(c+dx)a+a)^{3/2}}dx}{8a^2}-\frac{\tan(c+dx)\sec^2(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2\left(4a-9a\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}}dx}{8a^2}-\frac{\tan(c+dx)\sec^2(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{4496}$$

$$-\frac{-\frac{\int -\frac{3\sec(c+dx)\left(13a^2-12a^2\sec(c+dx)\right)}{2\sqrt{\sec(c+dx)a+a}}dx}{2a^2}-\frac{13a\tan(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}}{8a^2}-\frac{\tan(c+dx)\sec^2(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\frac{3\int \frac{\sec(c+dx)\left(13a^2-12a^2\sec(c+dx)\right)}{\sqrt{\sec(c+dx)a+a}}dx}{4a^2}-\frac{13a\tan(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}}{8a^2}-\frac{\tan(c+dx)\sec^2(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

$\downarrow$ 3042

$$-\frac{\frac{3\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(13a^2-12a^2\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a^2}-\frac{13a\tan(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}}{8a^2}-\frac{\tan(c+dx)\sec^2(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

$\downarrow$ 4489

$$-\frac{\frac{3\left(25a^2\int \frac{\sec(c+dx)}{\sqrt{\sec(c+dx)a+a}}dx-\frac{24a^2\tan(c+dx)}{d\sqrt{a\sec(c+dx)+a}}\right)}{4a^2}-\frac{13a\tan(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}}{8a^2}-\frac{\tan(c+dx)\sec^2(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

$\downarrow$ 3042

$$-\frac{\frac{3\left(25a^2\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx-\frac{24a^2\tan(c+dx)}{d\sqrt{a\sec(c+dx)+a}}\right)}{4a^2}-\frac{13a\tan(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}}{8a^2}-\frac{\tan(c+dx)\sec^2(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

$\downarrow$ 4282

$$-\frac{\frac{3\left(-\frac{50a^2\int \frac{1}{\frac{a^2\tan^2(c+dx)}{\sec(c+dx)a+a}+2a}d\left(-\frac{a\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}-\frac{24a^2\tan(c+dx)}{d\sqrt{a\sec(c+dx)+a}}\right)}{4a^2}-\frac{13a\tan(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}}{8a^2}-\frac{\tan(c+dx)\sec^2(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

$\downarrow$ 216

$$-\frac{\frac{3\left(\frac{25\sqrt{2}a^{3/2}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{d}-\frac{24a^2\tan(c+dx)}{d\sqrt{a\sec(c+dx)+a}}\right)}{4a^2}-\frac{13a\tan(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}}{8a^2}-\frac{\tan(c+dx)\sec^2(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

input
```
Int[Sec[c + d*x]^4/(a + a*Sec[c + d*x])^(5/2),x]
```

output
```
-1/4*(Sec[c + d*x]^2*Tan[c + d*x])/(d*(a + a*Sec[c + d*x])^(5/2)) - ((-13*
a*Tan[c + d*x])/(2*d*(a + a*Sec[c + d*x])^(3/2)) + (3*((25*Sqrt[2]*a^(3/2)
*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/d - (2
4*a^2*Tan[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]])))/(4*a^2))/(8*a^2)
```

3.135. $\int \frac{\sec^4(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx$

3.135.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4282
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2/f   Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[
a + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4303
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-d^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d
*Csc[e + f*x])^(n - 2)/(f*(2*m + 1))), x] + Simp[d^2/(a*b*(2*m + 1))   Int[
(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n
 + 2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2,
0] && LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

rule 4489
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*((
a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Simp[(a*B*m + A*b*(m + 1))/(b*(m +
 1))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B
, e, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b
*(m + 1), 0] &&  !LtQ[m, -2^(-1)]
```

rule 4496
```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b - a*B))*Cot
[e + f*x]*((a + b*Csc[e + f*x])^m/(b*f*(2*m + 1))), x] + Simp[1/(b^2*(2*m +
 1))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[A*b*m - a*B*m + b
*B*(2*m + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && Ne
Q[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

3.135. $\int \frac{\sec^4(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx$

3.135.4 Maple [A] (verified)

Time = 1.11 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.16

method	result
default	$-\frac{\sqrt{-\frac{2a}{(1-\cos(dx+c))^2\csc(dx+c)^2-1}}\left(2(1-\cos(dx+c))^5\csc(dx+c)^5+17(1-\cos(dx+c))^3\csc(dx+c)^3+75\ln\left(\csc(dx+c)-\cot(dx+c)+\right.\right.}{32d\,a^3}$

input
```
int(sec(d*x+c)^4/(a+a*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/32/d/a^3*(-2*a/((1-cos(d*x+c))^2*csc(d*x+c)^2-1))^(1/2)*(2*(1-cos(d*x+c
))^5*csc(d*x+c)^5+17*(1-cos(d*x+c))^3*csc(d*x+c)^3+75*ln(csc(d*x+c)-cot(d*
x+c)+((1-cos(d*x+c))^2*csc(d*x+c)^2-1)^(1/2))*((1-cos(d*x+c))^2*csc(d*x+c)
^2-1)^(1/2)-83*csc(d*x+c)+83*cot(d*x+c))
```

3.135.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 404, normalized size of antiderivative = 2.79

$$\int \frac{\sec^4(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \left[-\frac{75\sqrt{2}\left(\cos(dx+c)^3+3\cos(dx+c)^2+3\cos(dx+c)+1\right)\sqrt{-a}\log\left(-\frac{2}{}\right.}{64\,(a}\right.$$

input
```
integrate(sec(d*x+c)^4/(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
[-1/64*(75*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1
)*sqrt(-a)*log(-(2*sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)
)*cos(d*x + c)*sin(d*x + c) - 3*a*cos(d*x + c)^2 - 2*a*cos(d*x + c) + a)/(
cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) - 4*(49*cos(d*x + c)^2 + 85*cos(d*x
+ c) + 32)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(a^3*d*co
s(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d), 1/3
2*(75*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*sqr
t(a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(
sqrt(a)*sin(d*x + c))) + 2*(49*cos(d*x + c)^2 + 85*cos(d*x + c) + 32)*sqrt
((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(a^3*d*cos(d*x + c)^3 +
3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)]
```

3.135.6 Sympy [F]

$$\int \frac{\sec^4(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \int \frac{\sec^4(c+dx)}{(a(\sec(c+dx)+1))^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)**4/(a+a*sec(d*x+c))**(5/2),x)
```

output
```
Integral(sec(c + d*x)**4/(a*(sec(c + d*x) + 1))**(5/2), x)
```

3.135.7 Maxima [F]

$$\int \frac{\sec^4(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \int \frac{\sec(dx+c)^4}{(a\sec(dx+c)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^4/(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)^4/(a*sec(d*x + c) + a)^(5/2), x)
```

3.135.8 Giac [A] (verification not implemented)

Time = 1.05 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.30

$$\int \frac{\sec^4(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \frac{\frac{\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\left(\left(\frac{2\sqrt{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2}{a^2\mathrm{sgn}(\cos(dx+c))}+\frac{17\sqrt{2}}{a^2\mathrm{sgn}(\cos(dx+c))}\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-\frac{83\sqrt{2}}{a^2\mathrm{sgn}(\cos(dx+c))}\right)}{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a}}{32\,d}$$

input
```
integrate(sec(d*x+c)^4/(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
1/32*(sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*((2*sqrt(2)*tan(1/2*d*x + 1/2*c)
^2/(a^2*sgn(cos(d*x + c))) + 17*sqrt(2)/(a^2*sgn(cos(d*x + c))))*tan(1/2*d
*x + 1/2*c)^2 - 83*sqrt(2)/(a^2*sgn(cos(d*x + c))))*tan(1/2*d*x + 1/2*c)/(
a*tan(1/2*d*x + 1/2*c)^2 - a) + 75*sqrt(2)*log(abs(-sqrt(-a)*tan(1/2*d*x +
 1/2*c) + sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)))/(sqrt(-a)*a^2*sgn(cos(d*x
+ c))))/d
```

3.135.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^4(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \int \frac{1}{\cos\left(c+d\,x\right)^4\left(a+\frac{a}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

input
```
int(1/(cos(c + d*x)^4*(a + a/cos(c + d*x))^(5/2)),x)
```

output
```
int(1/(cos(c + d*x)^4*(a + a/cos(c + d*x))^(5/2)), x)
```

3.136 $\int \frac{\sec^3(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx$

3.136.1 Optimal result 1188
3.136.2 Mathematica [A] (verified) 1188
3.136.3 Rubi [A] (verified) 1189
3.136.4 Maple [B] (verified) 1191
3.136.5 Fricas [B] (verification not implemented) 1191
3.136.6 Sympy [F] 1192
3.136.7 Maxima [F] 1192
3.136.8 Giac [A] (verification not implemented) 1193
3.136.9 Mupad [F(-1)] 1193

3.136.1 Optimal result

Integrand size = 23, antiderivative size = 107

$$\int \frac{\sec^3(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \frac{19\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} + \frac{\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{13\tan(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}}$$

output
```
19/32*arctan(1/2*a^(1/2)*tan(d*x+c)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))/a^(5/2
)/d*2^(1/2)+1/4*tan(d*x+c)/d/(a+a*sec(d*x+c))^(5/2)-13/16*tan(d*x+c)/a/d/(
a+a*sec(d*x+c))^(3/2)
```

3.136.2 Mathematica [A] (verified)

Time = 0.68 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.08

$$\int \frac{\sec^3(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \frac{\left(76\sqrt{2}\text{arctanh}\left(\frac{\sqrt{1-\sec(c+dx)}}{\sqrt{2}}\right)\cos^4\left(\frac{1}{2}(c+dx)\right)\sec^2(c+dx) - 2\sqrt{1-\sec(c+}\right.}{32d\sqrt{1-\sec(c+dx)}(a(1+\sec(c+dx)))}$$

input
```
Integrate[Sec[c + d*x]^3/(a + a*Sec[c + d*x])^(5/2),x]
```

output
```
((76*Sqrt[2]*ArcTanh[Sqrt[1 - Sec[c + d*x]]/Sqrt[2]]*Cos[(c + d*x)/2]^4*Se
c[c + d*x]^2 - 2*Sqrt[1 - Sec[c + d*x]]*(9 + 13*Sec[c + d*x]))*Tan[c + d*x
])/(32*d*Sqrt[1 - Sec[c + d*x]]*(a*(1 + Sec[c + d*x]))^(5/2))
```

3.136.3 Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.06, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {3042, 4286, 27, 3042, 4488, 3042, 4282, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^3(c+dx)}{(a\sec(c+dx)+a)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^3}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}}dx$$

$$\Big\downarrow \text{4286}$$

$$\frac{\int -\frac{\sec(c+dx)(5a-8a\sec(c+dx))}{2(\sec(c+dx)a+a)^{3/2}}dx}{4a^2}+\frac{\tan(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\tan(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}-\frac{\int \frac{\sec(c+dx)(5a-8a\sec(c+dx))}{(\sec(c+dx)a+a)^{3/2}}dx}{8a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\tan(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}-\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(5a-8a\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}}dx}{8a^2}$$

$$\Big\downarrow \text{4488}$$

$$\frac{\tan(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}-\frac{\frac{13a\tan(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}-\frac{19}{4}\int \frac{\sec(c+dx)}{\sqrt{\sec(c+dx)a+a}}dx}{8a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\tan(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}-\frac{\frac{13a\tan(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}-\frac{19}{4}\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{8a^2}$$

$$\Big\downarrow \text{4282}$$

$$\frac{\tan(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}-\frac{\frac{19\int \frac{1}{\frac{a^2\tan^2(c+dx)}{\sec(c+dx)a+a}+2a}d\left(-\frac{a\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{2d}+\frac{13a\tan(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}}{8a^2}$$

$$\Big\downarrow \text{216}$$

$$\frac{\tan(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}-\frac{\frac{13a\tan(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}-\frac{19\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}\sqrt{a}d}}{8a^2}$$

input
```
Int[Sec[c + d*x]^3/(a + a*Sec[c + d*x])^(5/2),x]
```

output
```
Tan[c + d*x]/(4*d*(a + a*Sec[c + d*x])^(5/2)) - ((-19*ArcTan[(Sqrt[a]*Tan[
c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/(2*Sqrt[2]*Sqrt[a]*d) + (13
*a*Tan[c + d*x])/(2*d*(a + a*Sec[c + d*x])^(3/2)))/(8*a^2)
```

3.136.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4282
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2/f   Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[
a + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4286
```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[b*Cot[e + f*x]*((a + b*Csc[e + f*x])^m/(a*f*(2*m + 1))),
x] - Simp[1/(a^2*(2*m + 1))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
*(a*m - b*(2*m + 1)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[
a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

rule 4488
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b - a*B)*Cot[e +
f*x]*((a + b*Csc[e + f*x])^m/(a*f*(2*m + 1))), x] + Simp[(a*B*m + A*b*(m +
1))/(a*b*(2*m + 1))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x], x]
 /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[a*B*m + A*b*(m + 1), 0] && LtQ[m, -2^(-1)]
```

3.136.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 196 vs. $2(88) = 176$.

Time = 1.09 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.84

method	result
default	$\frac{\sqrt{-\frac{2a}{(1-\cos(dx+c))^2\csc(dx+c)^2-1}}\sqrt{(1-\cos(dx+c))^2\csc(dx+c)^2-1}\left(2(1-\cos(dx+c))^3\sqrt{(1-\cos(dx+c))^2\csc(dx+c)^2-1}\csc(dx+c)^3\right.}{32}$

input
```
int(sec(d*x+c)^3/(a+a*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/32/d/a^3*(-2*a/((1-cos(d*x+c))^2*csc(d*x+c)^2-1))^(1/2)*((1-cos(d*x+c))^
2*csc(d*x+c)^2-1)^(1/2)*(2*(1-cos(d*x+c))^3*((1-cos(d*x+c))^2*csc(d*x+c)^2
-1)^(1/2)*csc(d*x+c)^3+11*((1-cos(d*x+c))^2*csc(d*x+c)^2-1)^(1/2)*(-cot(d*
x+c)+csc(d*x+c))+19*ln(csc(d*x+c)-cot(d*x+c)+((1-cos(d*x+c))^2*csc(d*x+c)^
2-1)^(1/2)))
```

3.136.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 177 vs. $2(88) = 176$.

Time = 0.31 (sec) , antiderivative size = 403, normalized size of antiderivative = 3.77

$$\int \frac{\sec^3(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \left[-\frac{19\sqrt{2}\left(\cos(dx+c)^3+3\cos(dx+c)^2+3\cos(dx+c)+1\right)\sqrt{-a}\log\left(\frac{2\sqrt{}}{}\right.}{64\left(a^3d\,c\right.}\right.$$

$$-\frac{19\sqrt{2}\left(\cos(dx+c)^3+3\cos(dx+c)^2+3\cos(dx+c)+1\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{\sqrt{a}\sin(dx+c)}\right)+2\left(9\,c\right.}{32\left(a^3d\cos(dx+c)^3+3\,a^3d\cos(dx+c)^2+3\,a^3d\cos(dx+\right.}$$

input
```
integrate(sec(d*x+c)^3/(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
[-1/64*(19*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1
)*sqrt(-a)*log((2*sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))
*cos(d*x + c)*sin(d*x + c) + 3*a*cos(d*x + c)^2 + 2*a*cos(d*x + c) - a)/(c
os(d*x + c)^2 + 2*cos(d*x + c) + 1)) + 4*(9*cos(d*x + c)^2 + 13*cos(d*x +
c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(a^3*d*cos(d*x +
 c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d), -1/32*(19*
sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*sqrt(a)*a
rctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a
)*sin(d*x + c))) + 2*(9*cos(d*x + c)^2 + 13*cos(d*x + c))*sqrt((a*cos(d*x
+ c) + a)/cos(d*x + c))*sin(d*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(
d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)]
```

3.136.6 Sympy [F]

$$\int \frac{\sec^3(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \int \frac{\sec^3(c+dx)}{(a(\sec(c+dx)+1))^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)**3/(a+a*sec(d*x+c))**(5/2),x)
```

output
```
Integral(sec(c + d*x)**3/(a*(sec(c + d*x) + 1))**(5/2), x)
```

3.136.7 Maxima [F]

$$\int \frac{\sec^3(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \int \frac{\sec(dx+c)^3}{(a\sec(dx+c)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^3/(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)^3/(a*sec(d*x + c) + a)^(5/2), x)
```

3.136.8 Giac [A] (verification not implemented)

Time = 1.15 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.30

$$\int \frac{\sec^3(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx =$$
$$-\frac{\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\left(\frac{2\sqrt{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2}{a^3\mathrm{sgn}(\cos(dx+c))}+\frac{11\sqrt{2}}{a^3\mathrm{sgn}(\cos(dx+c))}\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{19\sqrt{2}\log\left(\left|-\sqrt{-a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\right.\right.\right.}{\sqrt{-a}a^2\mathrm{sgn}}}{32\,d}$$

input
```
integrate(sec(d*x+c)^3/(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
-1/32*(sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*(2*sqrt(2)*tan(1/2*d*x + 1/2*c)
^2/(a^3*sgn(cos(d*x + c))) + 11*sqrt(2)/(a^3*sgn(cos(d*x + c))))*tan(1/2*d
*x + 1/2*c) + 19*sqrt(2)*log(abs(-sqrt(-a)*tan(1/2*d*x + 1/2*c) + sqrt(-a*
tan(1/2*d*x + 1/2*c)^2 + a)))/(sqrt(-a)*a^2*sgn(cos(d*x + c))))/d
```

3.136.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^3(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \int \frac{1}{\cos\left(c+d\,x\right)^3\,\left(a+\frac{a}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

input
```
int(1/(cos(c + d*x)^3*(a + a/cos(c + d*x))^(5/2)),x)
```

output
```
int(1/(cos(c + d*x)^3*(a + a/cos(c + d*x))^(5/2)), x)
```

3.137 $\int \frac{\sec^2(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx$

3.137.1 Optimal result 1194
3.137.2 Mathematica [A] (verified) 1194
3.137.3 Rubi [A] (verified) 1195
3.137.4 Maple [B] (verified) 1197
3.137.5 Fricas [A] (verification not implemented) 1197
3.137.6 Sympy [F] 1198
3.137.7 Maxima [F] 1198
3.137.8 Giac [A] (verification not implemented) 1199
3.137.9 Mupad [F(-1)] 1199

3.137.1 Optimal result

Integrand size = 23, antiderivative size = 107

$$\int \frac{\sec^2(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \frac{5\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} - \frac{\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \frac{5\tan(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}}$$

output
```
5/32*arctan(1/2*a^(1/2)*tan(d*x+c)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))/a^(5/2)
/d*2^(1/2)-1/4*tan(d*x+c)/d/(a+a*sec(d*x+c))^(5/2)+5/16*tan(d*x+c)/a/d/(a+
a*sec(d*x+c))^(3/2)
```

3.137.2 Mathematica [A] (verified)

Time = 0.66 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.07

$$\int \frac{\sec^2(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \frac{\left(10\sqrt{2}\text{arctanh}\left(\frac{\sqrt{1-\sec(c+dx)}}{\sqrt{2}}\right)\cos^4\left(\frac{1}{2}(c+dx)\right)\sec^2(c+dx)+\sqrt{1-\sec(c+\right.}{16d\sqrt{1-\sec(c+dx)}(a(1+\sec(c+dx)))^5}$$

input
```
Integrate[Sec[c + d*x]^2/(a + a*Sec[c + d*x])^(5/2),x]
```

output
```
((10*Sqrt[2]*ArcTanh[Sqrt[1 - Sec[c + d*x]]/Sqrt[2]]*Cos[(c + d*x)/2]^4*Se
c[c + d*x]^2 + Sqrt[1 - Sec[c + d*x]]*(1 + 5*Sec[c + d*x]))*Tan[c + d*x])/
(16*d*Sqrt[1 - Sec[c + d*x]]*(a*(1 + Sec[c + d*x]))^(5/2))
```

3.137.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3042, 4284, 3042, 4283, 3042, 4282, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(c+dx)}{(a\sec(c+dx)+a)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}}dx$$

$$\Big\downarrow \text{4284}$$

$$\frac{5\int \frac{\sec(c+dx)}{(\sec(c+dx)a+a)^{3/2}}dx}{8a}-\frac{\tan(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}}dx}{8a}-\frac{\tan(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{4283}$$

$$\frac{5\left(\frac{\int \frac{\sec(c+dx)}{\sqrt{\sec(c+dx)a+a}}dx}{4a}+\frac{\tan(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}\right)}{8a}-\frac{\tan(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5\left(\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a}+\frac{\tan(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}\right)}{8a}-\frac{\tan(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{4282}$$

$$\frac{5\left(\frac{\tan(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}-\frac{\int \frac{1}{\frac{a^2\tan^2(c+dx)}{\sec(c+dx)a+a}+2a}d\left(-\frac{a\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{2ad}\right)}{8a}-\frac{\tan(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{216}$$

$$\frac{5\left(\frac{\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{\tan(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}\right)}{8a}-\frac{\tan(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

input
```
Int[Sec[c + d*x]^2/(a + a*Sec[c + d*x])^(5/2),x]
```

output
```
-1/4*Tan[c + d*x]/(d*(a + a*Sec[c + d*x])^(5/2)) + (5*(ArcTan[(Sqrt[a]*Tan
[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])]/(2*Sqrt[2]*a^(3/2)*d) + Tan
[c + d*x]/(2*d*(a + a*Sec[c + d*x])^(3/2))))/(8*a)
```

3.137.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4282
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2/f   Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[
a + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4283
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[b*Cot[e + f*x]*((a + b*Csc[e + f*x])^m/(a*f*(2*m + 1))), x]
 + Simp[(m + 1)/(a*(2*m + 1))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m +
1), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)
] && IntegerQ[2*m]
```

rule 4284
```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(-Cot[e + f*x])*((a + b*Csc[e + f*x])^m/(f*(2*m + 1))), x
] + Simp[m/(b*(2*m + 1))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x
], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

3.137.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 191 vs. $2(88) = 176$.

Time = 0.85 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.79

method	result
default	$-\frac{\sqrt{-\frac{2a}{(1-\cos(dx+c))^2\csc(dx+c)^2-1}}\sqrt{(1-\cos(dx+c))^2\csc(dx+c)^2-1}\left(2\left((1-\cos(dx+c))^2\csc(dx+c)^2-1\right)^{\frac{3}{2}}(-\cot(dx+c)+\csc(dx+c\right.}{3}$

input
```
int(sec(d*x+c)^2/(a+a*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/32/d/a^3*(-2*a/((1-cos(d*x+c))^2*csc(d*x+c)^2-1))^(1/2)*((1-cos(d*x+c))
^2*csc(d*x+c)^2-1)^(1/2)*(2*((1-cos(d*x+c))^2*csc(d*x+c)^2-1)^(3/2)*(-cot(
d*x+c)+csc(d*x+c))+5*((1-cos(d*x+c))^2*csc(d*x+c)^2-1)^(1/2)*(-cot(d*x+c)+
csc(d*x+c))-5*ln(csc(d*x+c)-cot(d*x+c)+((1-cos(d*x+c))^2*csc(d*x+c)^2-1)^(
1/2)))
```

3.137.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 399, normalized size of antiderivative = 3.73

$$\int \frac{\sec^2(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \left[-\frac{5\sqrt{2}\left(\cos(dx+c)^3+3\cos(dx+c)^2+3\cos(dx+c)+1\right)\sqrt{-a}\log\left(\frac{2\sqrt{2}\cdots}{}\right.}{64\left(a^3d\cos\cdots\right.}\right.$$

$$-\frac{5\sqrt{2}\left(\cos(dx+c)^3+3\cos(dx+c)^2+3\cos(dx+c)+1\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{\sqrt{a}\sin(dx+c)}\right)-2\left(\cos(\cdots\right.}{32\left(a^3d\cos(dx+c)^3+3a^3d\cos(dx+c)^2+3a^3d\cos(dx+c\cdots\right.}$$

input
```
integrate(sec(d*x+c)^2/(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
[-1/64*(5*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)
*sqrt(-a)*log((2*sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*
cos(d*x + c)*sin(d*x + c) + 3*a*cos(d*x + c)^2 + 2*a*cos(d*x + c) - a)/(co
s(d*x + c)^2 + 2*cos(d*x + c) + 1)) - 4*(cos(d*x + c)^2 + 5*cos(d*x + c))*
sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(a^3*d*cos(d*x + c)^
3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d), -1/32*(5*sqrt(
2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*sqrt(a)*arctan
(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin
(d*x + c))) - 2*(cos(d*x + c)^2 + 5*cos(d*x + c))*sqrt((a*cos(d*x + c) + a
)/cos(d*x + c))*sin(d*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)
^2 + 3*a^3*d*cos(d*x + c) + a^3*d)]
```

3.137.6 Sympy [F]

$$\int \frac{\sec^2(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \int \frac{\sec^2(c+dx)}{(a(\sec(c+dx)+1))^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)**2/(a+a*sec(d*x+c))**(5/2),x)
```

output
```
Integral(sec(c + d*x)**2/(a*(sec(c + d*x) + 1))**(5/2), x)
```

3.137.7 Maxima [F]

$$\int \frac{\sec^2(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \int \frac{\sec(dx+c)^2}{(a\sec(dx+c)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^2/(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)^2/(a*sec(d*x + c) + a)^(5/2), x)
```

3.137.8 Giac [A] (verification not implemented)

Time = 1.20 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.30

$$\int \frac{\sec^2(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \frac{\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\left(\frac{2\sqrt{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2}{a^3\mathrm{sgn}(\cos(dx+c))}+\frac{3\sqrt{2}}{a^3\mathrm{sgn}(\cos(dx+c))}\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\right.}{32\,d}$$

input
```
integrate(sec(d*x+c)^2/(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
1/32*(sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*(2*sqrt(2)*tan(1/2*d*x + 1/2*c)^
2/(a^3*sgn(cos(d*x + c))) + 3*sqrt(2)/(a^3*sgn(cos(d*x + c))))*tan(1/2*d*x
 + 1/2*c) - 5*sqrt(2)*log(abs(-sqrt(-a)*tan(1/2*d*x + 1/2*c) + sqrt(-a*tan
(1/2*d*x + 1/2*c)^2 + a)))/(sqrt(-a)*a^2*sgn(cos(d*x + c))))/d
```

3.137.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^2(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \int \frac{1}{\cos\left(c+d\,x\right)^2\,\left(a+\frac{a}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

input
```
int(1/(cos(c + d*x)^2*(a + a/cos(c + d*x))^(5/2)),x)
```

output
```
int(1/(cos(c + d*x)^2*(a + a/cos(c + d*x))^(5/2)), x)
```

3.138 $\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx$

3.138.1 Optimal result 1200
3.138.2 Mathematica [C] (verified) 1200
3.138.3 Rubi [A] (verified) 1201
3.138.4 Maple [B] (verified) 1203
3.138.5 Fricas [B] (verification not implemented) 1203
3.138.6 Sympy [F] 1204
3.138.7 Maxima [F] 1204
3.138.8 Giac [A] (verification not implemented) 1205
3.138.9 Mupad [F(-1)] 1205

3.138.1 Optimal result

Integrand size = 21, antiderivative size = 107

$$\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \frac{3\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} + \frac{\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \frac{3\tan(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}}$$

output
```
3/32*arctan(1/2*a^(1/2)*tan(d*x+c)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))/a^(5/2)
/d*2^(1/2)+1/4*tan(d*x+c)/d/(a+a*sec(d*x+c))^(5/2)+3/16*tan(d*x+c)/a/d/(a+
a*sec(d*x+c))^(3/2)
```

3.138.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.07 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.49

$$\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \frac{\text{Hypergeometric2F1}\left(\frac{1}{2},3,\frac{3}{2},\frac{1}{2}(1-\sec(c+dx))\right)\tan(c+dx)}{4a^2d\sqrt{a(1+\sec(c+dx))}}$$

input
```
Integrate[Sec[c + d*x]/(a + a*Sec[c + d*x])^(5/2),x]
```

output
```
(Hypergeometric2F1[1/2, 3, 3/2, (1 - Sec[c + d*x])/2]*Tan[c + d*x])/(4*a^2
*d*Sqrt[a*(1 + Sec[c + d*x])])
```

3.138.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 4283, 3042, 4283, 3042, 4282, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(c+dx)}{(a\sec(c+dx)+a)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}}dx$$

$$\Big\downarrow \text{4283}$$

$$\frac{3\int \frac{\sec(c+dx)}{(\sec(c+dx)a+a)^{3/2}}dx}{8a}+\frac{\tan(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}}dx}{8a}+\frac{\tan(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{4283}$$

$$\frac{3\left(\frac{\int \frac{\sec(c+dx)}{\sqrt{\sec(c+dx)a+a}}dx}{4a}+\frac{\tan(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}\right)}{8a}+\frac{\tan(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\left(\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a}+\frac{\tan(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}\right)}{8a}+\frac{\tan(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{4282}$$

$$\frac{3\left(\frac{\tan(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}-\frac{\int \frac{1}{\frac{a^2\tan^2(c+dx)}{\sec(c+dx)a+a}+2a}d\left(-\frac{a\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{2ad}\right)}{8a}+\frac{\tan(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{216}$$

$$\frac{3\left(\frac{\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{\tan(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}\right)}{8a}+\frac{\tan(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

input
```
Int[Sec[c + d*x]/(a + a*Sec[c + d*x])^(5/2),x]
```

output
```
Tan[c + d*x]/(4*d*(a + a*Sec[c + d*x])^(5/2)) + (3*(ArcTan[(Sqrt[a]*Tan[c
+ d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])]/(2*Sqrt[2]*a^(3/2)*d) + Tan[c
+ d*x]/(2*d*(a + a*Sec[c + d*x])^(3/2))))/(8*a)
```

3.138.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4282
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2/f   Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[
a + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4283
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[b*Cot[e + f*x]*((a + b*Csc[e + f*x])^m/(a*f*(2*m + 1))), x]
 + Simp[(m + 1)/(a*(2*m + 1))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m +
1), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)
] && IntegerQ[2*m]
```

3.138.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 191 vs. $2(88) = 176$.

Time = 0.96 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.79

method	result
default	$\frac{\sqrt{-\frac{2a}{(1-\cos(dx+c))^2\csc(dx+c)^2-1}}\sqrt{(1-\cos(dx+c))^2\csc(dx+c)^2-1}\left(2\left((1-\cos(dx+c))^2\csc(dx+c)^2-1\right)^{\frac{3}{2}}(-\cot(dx+c)+\csc(dx+c))\right.}{32a}$

input
```
int(sec(d*x+c)/(a+a*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/32/d/a^3*(-2*a/((1-cos(d*x+c))^2*csc(d*x+c)^2-1))^(1/2)*((1-cos(d*x+c))^
2*csc(d*x+c)^2-1)^(1/2)*(2*((1-cos(d*x+c))^2*csc(d*x+c)^2-1)^(3/2)*(-cot(d
*x+c)+csc(d*x+c))-3*((1-cos(d*x+c))^2*csc(d*x+c)^2-1)^(1/2)*(-cot(d*x+c)+c
sc(d*x+c))+3*ln(csc(d*x+c)-cot(d*x+c)+((1-cos(d*x+c))^2*csc(d*x+c)^2-1)^(1
/2)))
```

3.138.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 177 vs. $2(88) = 176$.

Time = 0.33 (sec) , antiderivative size = 403, normalized size of antiderivative = 3.77

$$\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \left[-\frac{3\sqrt{2}\left(\cos(dx+c)^3+3\cos(dx+c)^2+3\cos(dx+c)+1\right)\sqrt{-a}\log\left(\frac{2\sqrt{2}}{}\right.}{64\left(a^3d\cos\right.}\right.$$

$$-\frac{3\sqrt{2}\left(\cos(dx+c)^3+3\cos(dx+c)^2+3\cos(dx+c)+1\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{\sqrt{a}\sin(dx+c)}\right)-2\left(7\cos\right.}{32\left(a^3d\cos(dx+c)^3+3a^3d\cos(dx+c)^2+3a^3d\cos(dx+c\right.}$$

input
```
integrate(sec(d*x+c)/(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
[-1/64*(3*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)
*sqrt(-a)*log((2*sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*
cos(d*x + c)*sin(d*x + c) + 3*a*cos(d*x + c)^2 + 2*a*cos(d*x + c) - a)/(co
s(d*x + c)^2 + 2*cos(d*x + c) + 1)) - 4*(7*cos(d*x + c)^2 + 3*cos(d*x + c)
)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(a^3*d*cos(d*x + c
)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d), -1/32*(3*sqr
t(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*sqrt(a)*arct
an(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*s
in(d*x + c))) - 2*(7*cos(d*x + c)^2 + 3*cos(d*x + c))*sqrt((a*cos(d*x + c)
 + a)/cos(d*x + c))*sin(d*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x
+ c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)]
```

3.138.6 Sympy [F]

$$\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \int \frac{\sec{(c+dx)}}{(a(\sec{(c+dx)}+1))^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)/(a+a*sec(d*x+c))**(5/2),x)
```

output
```
Integral(sec(c + d*x)/(a*(sec(c + d*x) + 1))**(5/2), x)
```

3.138.7 Maxima [F]

$$\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \int \frac{\sec{(dx+c)}}{(a\sec{(dx+c)}+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)/(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)/(a*sec(d*x + c) + a)^(5/2), x)
```

3.138.8 Giac [A] (verification not implemented)

Time = 0.86 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.30

$$\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = -\frac{\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\left(\frac{2\sqrt{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2}{a^3\mathrm{sgn}(\cos(dx+c))}-\frac{5\sqrt{2}}{a^3\mathrm{sgn}(\cos(dx+c))}\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{3\sqrt{2}\log\left(\left|-\sqrt{-a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right.\right.\right.}{\sqrt{-a}a^2\mathrm{sgn}(}}{32\,d}$$

input
```
integrate(sec(d*x+c)/(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
-1/32*(sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*(2*sqrt(2)*tan(1/2*d*x + 1/2*c)
^2/(a^3*sgn(cos(d*x + c))) - 5*sqrt(2)/(a^3*sgn(cos(d*x + c))))*tan(1/2*d*
x + 1/2*c) + 3*sqrt(2)*log(abs(-sqrt(-a)*tan(1/2*d*x + 1/2*c) + sqrt(-a*ta
n(1/2*d*x + 1/2*c)^2 + a)))/(sqrt(-a)*a^2*sgn(cos(d*x + c))))/d
```

3.138.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \int \frac{1}{\cos\left(c+d\,x\right)\,\left(a+\frac{a}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

input
```
int(1/(cos(c + d*x)*(a + a/cos(c + d*x))^(5/2)),x)
```

output
```
int(1/(cos(c + d*x)*(a + a/cos(c + d*x))^(5/2)), x)
```

3.139 $\int \frac{1}{(a+a\sec(c+dx))^{5/2}}\,dx$

3.139.1 Optimal result . . . 1206
3.139.2 Mathematica [A] (warning: unable to verify) . . . 1206
3.139.3 Rubi [A] (verified) . . . 1207
3.139.4 Maple [B] (warning: unable to verify) . . . 1210
3.139.5 Fricas [B] (verification not implemented) . . . 1211
3.139.6 Sympy [F] . . . 1211
3.139.7 Maxima [F] . . . 1212
3.139.8 Giac [A] (verification not implemented) . . . 1212
3.139.9 Mupad [F(-1)] . . . 1212

3.139.1 Optimal result

Integrand size = 14, antiderivative size = 144

$$\int \frac{1}{(a+a\sec(c+dx))^{5/2}}\,dx = \frac{2\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^{5/2}d} - \frac{43\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} - \frac{\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{11\tan(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}}$$

output
```
2*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/a^(5/2)/d-43/32*arctan
(1/2*a^(1/2)*tan(d*x+c)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))/a^(5/2)/d*2^(1/2)-
1/4*tan(d*x+c)/d/(a+a*sec(d*x+c))^(5/2)-11/16*tan(d*x+c)/a/d/(a+a*sec(d*x+
c))^(3/2)
```

3.139.2 Mathematica [A] (warning: unable to verify)

Time = 6.45 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.47

$$\int \frac{1}{(a+a\sec(c+dx))^{5/2}}\,dx = -\frac{\cos^6\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{5}{2}}(c+dx)\left(-64\arctan\left(\frac{\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{\frac{1}{1+\sec(c+dx)}}}\right)\sqrt{\frac{\cos(c+dx)}{(1+\cos(c+dx))^2}}\sqrt{1+\sec(c+dx)}+43\arcsin\left(\text{ta}\right.\right.}{}$$

input
```
Integrate[(a + a*Sec[c + d*x])^(-5/2),x]
```

output
```
-1/4*(Cos[(c + d*x)/2]^6*Sec[c + d*x]^(5/2)*(-64*ArcTan[Tan[(c + d*x)/2]/S
qrt[(1 + Sec[c + d*x])^(-1)]]*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])^2]*Sqrt
[1 + Sec[c + d*x]] + 43*ArcSin[Tan[(c + d*x)/2]]*Sqrt[Sec[(c + d*x)/2]^2]*
Sqrt[(1 + Sec[c + d*x])^(-1)]*Sqrt[1 + Sec[c + d*x]] + (30 - 19*Sec[(c + d
*x)/2]^2 + 2*Sec[(c + d*x)/2]^4)*Sqrt[Sec[c + d*x]]*Tan[(c + d*x)/2]))/(d*
(a*(1 + Sec[c + d*x]))^(5/2))
```

3.139.3 Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.08, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.929$, Rules used = {3042, 4264, 27, 3042, 4410, 27, 3042, 4408, 3042, 4261, 216, 4282, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\sec(c+dx)+a)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}}dx$$

$$\Big\downarrow \text{4264}$$

$$-\frac{\int -\frac{8a-3a\sec(c+dx)}{2(\sec(c+dx)a+a)^{3/2}}dx}{4a^2}-\frac{\tan(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{8a-3a\sec(c+dx)}{(\sec(c+dx)a+a)^{3/2}}dx}{8a^2}-\frac{\tan(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{8a-3a\csc\left(c+dx+\frac{\pi}{2}\right)}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}}dx}{8a^2}-\frac{\tan(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{4410}$$

$$\frac{-\frac{\int -\frac{32a^2-11a^2\sec(c+dx)}{2\sqrt{\sec(c+dx)a+a}}dx}{2a^2}-\frac{11a\tan(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}}{8a^2}-\frac{\tan(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int \frac{32a^2-11a^2\sec(c+dx)}{\sqrt{\sec(c+dx)a+a}}dx}{4a^2}-\frac{11a\tan(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}}{8a^2}-\frac{\tan(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\int \frac{32a^2-11a^2\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a^2}-\frac{11a\tan(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}}{8a^2}-\frac{\tan(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

$\Big\downarrow$ 4408

$$\frac{\frac{32a\int \sqrt{\sec(c+dx)a+a}dx-43a^2\int \frac{\sec(c+dx)}{\sqrt{\sec(c+dx)a+a}}dx}{4a^2}-\frac{11a\tan(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}}{8a^2}-\frac{\tan(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

$\Big\downarrow$ 3042

$$\frac{\frac{32a\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx-43a^2\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a^2}-\frac{11a\tan(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}}{8a^2}-\frac{\tan(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

$\Big\downarrow$ 4261

$$\frac{\frac{-43a^2\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx-\frac{64a^2\int \frac{1}{\frac{a^2\tan^2(c+dx)}{\sec(c+dx)a+a}+a}d\left(-\frac{a\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}}{4a^2}-\frac{11a\tan(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}}{8a^2}-\frac{\tan(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

$\Big\downarrow$ 216

$$\frac{\frac{\frac{64a^{3/2}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}-43a^2\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a^2}-\frac{11a\tan(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}}{8a^2}-\frac{\tan(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

$\Big\downarrow$ 4282

$$\frac{\frac{\frac{86a^2\int \frac{1}{\frac{a^2\tan^2(c+dx)}{\sec(c+dx)a+a}+2a}d\left(-\frac{a\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}+\frac{64a^{3/2}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}}{4a^2}-\frac{11a\tan(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}}{8a^2}-\frac{\tan(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

$\Big\downarrow$ 216

3.139. $\int \frac{1}{(a+a\sec(c+dx))^{5/2}}\,dx$

$$\frac{\frac{\frac{64a^{3/2}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}-\frac{43\sqrt{2}a^{3/2}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{d}}{4a^2}-\frac{11a\tan(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}}{8a^2}-\frac{\tan(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

input
```
Int[(a + a*Sec[c + d*x])^(-5/2),x]
```

output
```
-1/4*Tan[c + d*x]/(d*(a + a*Sec[c + d*x])^(5/2)) + (((64*a^(3/2)*ArcTan[(S
qrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d - (43*Sqrt[2]*a^(3/2)*Ar
cTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/d)/(4*a^2
) - (11*a*Tan[c + d*x])/(2*d*(a + a*Sec[c + d*x])^(3/2)))/(8*a^2)
```

3.139.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4261
```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(b/d)
  Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 4264
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(-Cot[c
+ d*x])*((a + b*Csc[c + d*x])^n/(d*(2*n + 1))), x] + Simp[1/(a^2*(2*n + 1))
   Int[(a + b*Csc[c + d*x])^(n + 1)*(a*(2*n + 1) - b*(n + 1)*Csc[c + d*x]),
 x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LeQ[n, -1] && Int
egerQ[2*n]
```

rule 4282
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2/f   Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[
a + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4408
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Simp[c/a   Int[Sqrt[a + b*Csc[e + f*x]], x], x] -
Simp[(b*c - a*d)/a   Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

rule 4410
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> Simp[(-(b*c - a*d))*Cot[e + f*x]*((a + b*Csc[e +
f*x])^m/(b*f*(2*m + 1))), x] + Simp[1/(a^2*(2*m + 1))   Int[(a + b*Csc[e +
f*x])^(m + 1)*Simp[a*c*(2*m + 1) - (b*c - a*d)*(m + 1)*Csc[e + f*x], x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&
EqQ[a^2 - b^2, 0] && IntegerQ[2*m]
```

3.139.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 246 vs. $2(119) = 238$.

Time = 1.01 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.72

method	result
default	$-\frac{\sqrt{-\frac{2a}{(1-\cos(dx+c))^2\csc(dx+c)^2-1}}\sqrt{(1-\cos(dx+c))^2\csc(dx+c)^2-1}\left(2(1-\cos(dx+c))^3\sqrt{(1-\cos(dx+c))^2\csc(dx+c)^2-1}\csc(dx+c\right.}{}$

input
```
int(1/(a+a*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/32/d/a^3*(-2*a/((1-cos(d*x+c))^2*csc(d*x+c)^2-1))^(1/2)*((1-cos(d*x+c))
^2*csc(d*x+c)^2-1)^(1/2)*(2*(1-cos(d*x+c))^3*((1-cos(d*x+c))^2*csc(d*x+c)^
2-1)^(1/2)*csc(d*x+c)^3-13*((1-cos(d*x+c))^2*csc(d*x+c)^2-1)^(1/2)*(-cot(d
*x+c)+csc(d*x+c))-32*2^(1/2)*arctanh(2^(1/2)/((1-cos(d*x+c))^2*csc(d*x+c)^
2-1)^(1/2)*(-cot(d*x+c)+csc(d*x+c)))+43*ln(csc(d*x+c)-cot(d*x+c)+((1-cos(d
*x+c))^2*csc(d*x+c)^2-1)^(1/2)))
```

3.139.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 250 vs. $2(119) = 238$.

Time = 0.39 (sec) , antiderivative size = 585, normalized size of antiderivative = 4.06

$$\int \frac{1}{(a+a\sec(c+dx))^{5/2}}\,dx = \left[-\frac{43\sqrt{2}\left(\cos\left(dx+c\right)^{3}+3\cos\left(dx+c\right)^{2}+3\cos\left(dx+c\right)+1\right)\sqrt{-a}\log\left(-\frac{2}{}\right.}{}\right.$$

input
```
integrate(1/(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
[-1/64*(43*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1
)*sqrt(-a)*log(-(2*sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)
)*cos(d*x + c)*sin(d*x + c) - 3*a*cos(d*x + c)^2 - 2*a*cos(d*x + c) + a)/(
cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + 64*(cos(d*x + c)^3 + 3*cos(d*x + c
)^2 + 3*cos(d*x + c) + 1)*sqrt(-a)*log((2*a*cos(d*x + c)^2 + 2*sqrt(-a)*sq
rt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) + a*cos(d*
x + c) - a)/(cos(d*x + c) + 1)) + 4*(15*cos(d*x + c)^2 + 11*cos(d*x + c))*
sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(a^3*d*cos(d*x + c)^
3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d), 1/32*(43*sqrt(
2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*sqrt(a)*arctan
(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin
(d*x + c))) - 64*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*
sqrt(a)*arctan(sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(
a)*sin(d*x + c))) - 2*(15*cos(d*x + c)^2 + 11*cos(d*x + c))*sqrt((a*cos(d*
x + c) + a)/cos(d*x + c))*sin(d*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*co
s(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)]
```

3.139.6 Sympy [F]

$$\int \frac{1}{(a+a\sec(c+dx))^{5/2}}\,dx = \int \frac{1}{(a\sec\left(c+dx\right)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a+a*sec(d*x+c))**(5/2),x)
```

output
```
Integral((a*sec(c + d*x) + a)**(-5/2), x)
```

3.139.7 Maxima [F]

$$\int \frac{1}{(a+a\sec(c+dx))^{5/2}}\,dx = \int \frac{1}{(a\sec(dx+c)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)^(-5/2), x)
```

3.139.8 Giac [A] (verification not implemented)

Time = 0.66 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.54

$$\int \frac{1}{(a+a\sec(c+dx))^{5/2}}\,dx = \frac{\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\left(\frac{2\sqrt{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2}{a^3\mathrm{sgn}(\cos(dx+c))}-\frac{13\sqrt{2}}{a^3\mathrm{sgn}(\cos(dx+c))}\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\right.}{32\,d}$$

input
```
integrate(1/(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
1/32*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*(2*sqrt(2)*tan(1/2*d*x + 1/2*c)^2
/(a^3*sgn(cos(d*x + c))) - 13*sqrt(2)/(a^3*sgn(cos(d*x + c))))*tan(1/2*d*x
 + 1/2*c)/d
```

3.139.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\sec(c+dx))^{5/2}}\,dx = \int \frac{1}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

input
```
int(1/(a + a/cos(c + d*x))^(5/2),x)
```

output
```
int(1/(a + a/cos(c + d*x))^(5/2), x)
```

3.140 $\int \frac{\cos(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx$

3.140.1 Optimal result 1213
3.140.2 Mathematica [A] (verified) 1213
3.140.3 Rubi [A] (verified) 1214
3.140.4 Maple [B] (warning: unable to verify) 1219
3.140.5 Fricas [A] (verification not implemented) 1219
3.140.6 Sympy [F] 1220
3.140.7 Maxima [F] 1220
3.140.8 Giac [B] (verification not implemented) 1221
3.140.9 Mupad [F(-1)] 1221

3.140.1 Optimal result

Integrand size = 21, antiderivative size = 174

$$\int \frac{\cos(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = -\frac{5\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^{5/2}d} + \frac{115\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} - \frac{\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{15\sin(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} + \frac{35\sin(c+dx)}{16a^2d\sqrt{a+a\sec(c+dx)}}$$

output
```
-5*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/a^(5/2)/d-1/4*sin(d*x
+c)/d/(a+a*sec(d*x+c))^(5/2)-15/16*sin(d*x+c)/a/d/(a+a*sec(d*x+c))^(3/2)+1
15/32*arctan(1/2*a^(1/2)*tan(d*x+c)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))/a^(5/2
)/d*2^(1/2)+35/16*sin(d*x+c)/a^2/d/(a+a*sec(d*x+c))^(1/2)
```

3.140.2 Mathematica [A] (verified)

Time = 1.96 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.97

$$\int \frac{\cos(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \frac{460\sqrt{2}\text{arctanh}\left(\frac{\sqrt{1-\sec(c+dx)}}{\sqrt{2}}\right)\cos^5\left(\frac{1}{2}(c+dx)\right)\sec^3(c+dx)\sin\left(\frac{1}{2}(c+dx)\right) -}{}$$

input
```
Integrate[Cos[c + d*x]/(a + a*Sec[c + d*x])^(5/2),x]
```

output
```
(460*Sqrt[2]*ArcTanh[Sqrt[1 - Sec[c + d*x]]/Sqrt[2]]*Cos[(c + d*x)/2]^5*Se
c[c + d*x]^3*Sin[(c + d*x)/2] - 80*ArcTanh[Sqrt[1 - Sec[c + d*x]]]*(1 + Se
c[c + d*x])^2*Tan[c + d*x] + Sqrt[1 - Sec[c + d*x]]*(16*Sin[c + d*x] + 5*(
11 + 7*Sec[c + d*x])*Tan[c + d*x]))/(16*d*Sqrt[1 - Sec[c + d*x]]*(a*(1 + S
ec[c + d*x]))^(5/2))
```

3.140.3 Rubi [A] (verified)

Time = 1.09 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.09, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.762$, Rules used = {3042, 4304, 27, 3042, 4508, 27, 3042, 4510, 25, 3042, 4408, 3042, 4261, 216, 4282, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(c+dx)}{(a\sec(c+dx)+a)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}}dx$$

$$\Big\downarrow \text{4304}$$

$$-\frac{\int -\frac{5\cos(c+dx)(2a-a\sec(c+dx))}{2(\sec(c+dx)a+a)^{3/2}}dx}{4a^2}-\frac{\sin(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{5\int \frac{\cos(c+dx)(2a-a\sec(c+dx))}{(\sec(c+dx)a+a)^{3/2}}dx}{8a^2}-\frac{\sin(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5\int \frac{2a-a\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}}dx}{8a^2}-\frac{\sin(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{4508}$$

$$\frac{5\left(\frac{\int \frac{\cos(c+dx)\left(14a^2-9a^2\sec(c+dx)\right)}{2\sqrt{\sec(c+dx)a+a}}dx}{2a^2}-\frac{3a\sin(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}\right)}{8a^2}-\frac{\sin(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{5\left(\frac{\int \frac{\cos(c+dx)\left(14a^2-9a^2\sec(c+dx)\right)}{\sqrt{\sec(c+dx)a+a}}dx}{4a^2}-\frac{3a\sin(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}\right)}{8a^2}-\frac{\sin(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \; 3042$$

$$\frac{5\left(\frac{\int \frac{14a^2-9a^2\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a^2}-\frac{3a\sin(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}\right)}{8a^2}-\frac{\sin(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \; 4510$$

$$\frac{5\left(\frac{\frac{\int -\frac{16a^3-7a^3\sec(c+dx)}{\sqrt{\sec(c+dx)a+a}}dx}{a}+\frac{14a^2\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}}{4a^2}-\frac{3a\sin(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}\right)}{8a^2}-\frac{\sin(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \; 25$$

$$\frac{5\left(\frac{\frac{14a^2\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}-\frac{\int \frac{16a^3-7a^3\sec(c+dx)}{\sqrt{\sec(c+dx)a+a}}dx}{a}}{4a^2}-\frac{3a\sin(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}\right)}{8a^2}-\frac{\sin(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \; 3042$$

$$\frac{5\left(\frac{\frac{14a^2\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}-\frac{\int \frac{16a^3-7a^3\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{a}}{4a^2}-\frac{3a\sin(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}\right)}{8a^2}-\frac{\sin(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \; 4408$$

$$\frac{5\left(\frac{\frac{14a^2\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}-\frac{16a^2\int\sqrt{\sec(c+dx)a+a}dx-23a^3\int\frac{\sec(c+dx)}{\sqrt{\sec(c+dx)a+a}}dx}{a}}{4a^2}-\frac{3a\sin(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}\right)}{8a^2}-\frac{\sin(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \; 3042$$

3.140. $\int \frac{\cos(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx$

$$\frac{5\left(\frac{\frac{14a^2\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}-\frac{16a^2\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx-23a^3\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{a}}{4a^2}-\frac{3a\sin(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}\right)}{8a^2}-\frac{\sin(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

$\downarrow$ 4261

$$\frac{5\left(\frac{\frac{14a^2\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}-\frac{-23a^3\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx-\frac{32a^3\int\frac{1}{\frac{a^2\tan^2(c+dx)}{\sec(c+dx)a+a}+a}d\left(-\frac{a\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}}{a}}{4a^2}-\frac{3a\sin(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}\right)}{8a^2}-\frac{\sin(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

$\downarrow$ 216

$$\frac{5\left(\frac{\frac{14a^2\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}-\frac{\frac{32a^{5/2}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}-23a^3\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{a}}{4a^2}-\frac{3a\sin(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}\right)}{8a^2}-\frac{\sin(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

$\downarrow$ 4282

$$\frac{5\left(\frac{\frac{14a^2\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}-\frac{\frac{46a^3\int\frac{1}{\frac{a^2\tan^2(c+dx)}{\sec(c+dx)a+a}+2a}d\left(-\frac{a\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}+\frac{32a^{5/2}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}}{a}}{4a^2}-\frac{3a\sin(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}\right)}{8a^2}-\frac{\sin(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

$\downarrow$ 216

3.140. $\int\frac{\cos(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx$

$$\frac{5\left(\frac{\frac{14a^2\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}-\frac{\frac{32a^{5/2}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}-\frac{23\sqrt{2}a^{5/2}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{d}}{a}}{4a^2}-\frac{3a\sin(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}\right)}{8a^2}-\frac{\sin(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

input
```
Int[Cos[c + d*x]/(a + a*Sec[c + d*x])^(5/2),x]
```

output
```
-1/4*Sin[c + d*x]/(d*(a + a*Sec[c + d*x])^(5/2)) + (5*((-3*a*Sin[c + d*x])
/(2*d*(a + a*Sec[c + d*x])^(3/2)) + (-(((32*a^(5/2)*ArcTan[(Sqrt[a]*Tan[c
+ d*x])/Sqrt[a + a*Sec[c + d*x]]])/d - (23*Sqrt[2]*a^(5/2)*ArcTan[(Sqrt[a]
*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/d)/a) + (14*a^2*Sin[c
+ d*x])/(d*Sqrt[a + a*Sec[c + d*x]]))/(4*a^2)))/(8*a^2)
```

3.140.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4261
```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(b/d)
  Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 4282
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2/f   Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[
a + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4304
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-Cot[e + f*x])*(a + b*Csc[e + f*x])^m*((d*Csc
[e + f*x])^n/(f*(2*m + 1))), x] + Simp[1/(a^2*(2*m + 1))   Int[(a + b*Csc[e
 + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e
+ f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ
[m, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

rule 4408
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Simp[c/a   Int[Sqrt[a + b*Csc[e + f*x]], x], x] -
Simp[(b*c - a*d)/a   Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

rule 4508
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
- a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
 1))), x] - Simp[1/(a^2*(2*m + 1))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Cs
c[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[
e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B
, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

rule 4510
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*Cot[e
 + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(f*n)), x] - Simp[1/(b*d
*n)   Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*
n - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

3.140. $\int \frac{\cos(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx$

3.140.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 472 vs. $2(145) = 290$.

Time = 22.85 (sec) , antiderivative size = 473, normalized size of antiderivative = 2.72

method	result
default	$-\frac{\sqrt{-\frac{2a}{(1-\cos(dx+c))^2\csc(dx+c)^2-1}}\left(-2(1-\cos(dx+c))^7\csc(dx+c)^7+80\sqrt{(1-\cos(dx+c))^2\csc(dx+c)^2-1}\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{2}(-\cot(\cdots}{\sqrt{(1-\cos(dx+\cdots}}\right.\right.}{\cdots}$

input
```
int(cos(d*x+c)/(a+a*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/32/d/a^3*(-2*a/((1-cos(d*x+c))^2*csc(d*x+c)^2-1))^(1/2)*(-2*(1-cos(d*x+
c))^7*csc(d*x+c)^7+80*((1-cos(d*x+c))^2*csc(d*x+c)^2-1)^(1/2)*2^(1/2)*arct
anh(2^(1/2)/((1-cos(d*x+c))^2*csc(d*x+c)^2-1)^(1/2)*(-cot(d*x+c)+csc(d*x+c
)))*(1-cos(d*x+c))^2*csc(d*x+c)^2+21*(1-cos(d*x+c))^5*csc(d*x+c)^5-115*((1
-cos(d*x+c))^2*csc(d*x+c)^2-1)^(1/2)*ln(csc(d*x+c)-cot(d*x+c)+((1-cos(d*x+
c))^2*csc(d*x+c)^2-1)^(1/2))*(1-cos(d*x+c))^2*csc(d*x+c)^2+80*2^(1/2)*arct
anh(2^(1/2)/((1-cos(d*x+c))^2*csc(d*x+c)^2-1)^(1/2)*(-cot(d*x+c)+csc(d*x+c
)))*((1-cos(d*x+c))^2*csc(d*x+c)^2-1)^(1/2)+34*(1-cos(d*x+c))^3*csc(d*x+c)
^3-115*ln(csc(d*x+c)-cot(d*x+c)+((1-cos(d*x+c))^2*csc(d*x+c)^2-1)^(1/2))*(
(1-cos(d*x+c))^2*csc(d*x+c)^2-1)^(1/2)-53*csc(d*x+c)+53*cot(d*x+c))/((1-co
s(d*x+c))^2*csc(d*x+c)^2+1)
```

3.140.5 Fricas [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 606, normalized size of antiderivative = 3.48

$$\int \frac{\cos(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \left[-\frac{115\sqrt{2}\left(\cos(dx+c)^3+3\cos(dx+c)^2+3\cos(dx+c)+1\right)\sqrt{-a}\log\left(\frac{2\cdots}{\cdots}\right.}{\cdots}\right.$$

$$-\frac{115\sqrt{2}\left(\cos(dx+c)^3+3\cos(dx+c)^2+3\cos(dx+c)+1\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{\sqrt{a}\sin(dx+c)}\right)-160\,(\cdots}{3\cdots}$$

input
```
integrate(cos(d*x+c)/(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
[-1/64*(115*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) +
1)*sqrt(-a)*log((2*sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)
)*cos(d*x + c)*sin(d*x + c) + 3*a*cos(d*x + c)^2 + 2*a*cos(d*x + c) - a)/(
cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + 160*(cos(d*x + c)^3 + 3*cos(d*x +
c)^2 + 3*cos(d*x + c) + 1)*sqrt(-a)*log((2*a*cos(d*x + c)^2 - 2*sqrt(-a)*s
qrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) + a*cos(d
*x + c) - a)/(cos(d*x + c) + 1)) - 4*(16*cos(d*x + c)^3 + 55*cos(d*x + c)^
2 + 35*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))
/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a
^3*d), -1/32*(115*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x +
 c) + 1)*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*co
s(d*x + c)/(sqrt(a)*sin(d*x + c))) - 160*(cos(d*x + c)^3 + 3*cos(d*x + c)^
2 + 3*cos(d*x + c) + 1)*sqrt(a)*arctan(sqrt((a*cos(d*x + c) + a)/cos(d*x +
 c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) - 2*(16*cos(d*x + c)^3 + 55*cos(
d*x + c)^2 + 35*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(
d*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x
 + c) + a^3*d)]
```

3.140.6 Sympy [F]

$$\int \frac{\cos(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \int \frac{\cos{(c+dx)}}{(a\,(\sec{(c+dx)}+1))^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)/(a+a*sec(d*x+c))**(5/2),x)
```

output
```
Integral(cos(c + d*x)/(a*(sec(c + d*x) + 1))**(5/2), x)
```

3.140.7 Maxima [F]

$$\int \frac{\cos(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \int \frac{\cos{(dx+c)}}{(a\sec{(dx+c)}+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)/(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate(cos(d*x + c)/(a*sec(d*x + c) + a)^(5/2), x)
```

3.140.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 424 vs. $2(145) = 290$.

Time = 1.26 (sec) , antiderivative size = 424, normalized size of antiderivative = 2.44

$$\int \frac{\cos(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx =$$

$$-\frac{2\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\left(\frac{2\sqrt{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2}{a^3\mathrm{sgn}(\cos(dx+c))}-\frac{21\sqrt{2}}{a^3\mathrm{sgn}(\cos(dx+c))}\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\frac{}{\left(\left(\sqrt{-a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{-}\right.\right.}}{}$$

input
```
integrate(cos(d*x+c)/(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
-1/64*(2*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*(2*sqrt(2)*tan(1/2*d*x + 1/2*
c)^2/(a^3*sgn(cos(d*x + c))) - 21*sqrt(2)/(a^3*sgn(cos(d*x + c))))*tan(1/2
*d*x + 1/2*c) - 128*sqrt(2)*(3*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*ta
n(1/2*d*x + 1/2*c)^2 + a))^2 - a)/(((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(
-a*tan(1/2*d*x + 1/2*c)^2 + a))^4 - 6*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqr
t(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a + a^2)*sqrt(-a)*a*sgn(cos(d*x + c)))
 + 115*sqrt(2)*log((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x +
1/2*c)^2 + a))^2)/(sqrt(-a)*a^2*sgn(cos(d*x + c))) - 160*log(abs(-2*(sqrt(
-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - 4*sqrt
(2)*abs(a) + 6*a)/abs(-2*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*
d*x + 1/2*c)^2 + a))^2 + 4*sqrt(2)*abs(a) + 6*a))/(sqrt(-a)*a*abs(a)*sgn(c
os(d*x + c))))/d
```

3.140.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \int \frac{\cos\left(c+d\,x\right)}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

input
```
int(cos(c + d*x)/(a + a/cos(c + d*x))^(5/2),x)
```

output
```
int(cos(c + d*x)/(a + a/cos(c + d*x))^(5/2), x)
```

3.141 $\int \frac{\sec(c+dx)}{\sqrt{a-a\sec(c+dx)}}\,dx$

3.141.1 Optimal result 1222
3.141.2 Mathematica [A] (verified) 1222
3.141.3 Rubi [A] (verified) 1223
3.141.4 Maple [B] (verified) 1224
3.141.5 Fricas [A] (verification not implemented) 1224
3.141.6 Sympy [F] 1225
3.141.7 Maxima [F] 1225
3.141.8 Giac [A] (verification not implemented) 1225
3.141.9 Mupad [F(-1)] 1226

3.141.1 Optimal result

Integrand size = 22, antiderivative size = 48

$$\int \frac{\sec(c+dx)}{\sqrt{a-a\sec(c+dx)}}\,dx=-\frac{\sqrt{2}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a-a\sec(c+dx)}}\right)}{\sqrt{a}d}$$

output
```
-arctan(1/2*a^(1/2)*tan(d*x+c)*2^(1/2)/(a-a*sec(d*x+c))^(1/2))*2^(1/2)/d/a
^(1/2)
```

3.141.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.29

$$\int \frac{\sec(c+dx)}{\sqrt{a-a\sec(c+dx)}}\,dx=-\frac{\sqrt{2}\text{arctanh}\left(\frac{\sqrt{1+\sec(c+dx)}}{\sqrt{2}}\right)\tan(c+dx)}{d\sqrt{1+\sec(c+dx)}\sqrt{a-a\sec(c+dx)}}$$

input
```
Integrate[Sec[c + d*x]/Sqrt[a - a*Sec[c + d*x]],x]
```

output
```
-((Sqrt[2]*ArcTanh[Sqrt[1 + Sec[c + d*x]]/Sqrt[2]]*Tan[c + d*x])/(d*Sqrt[1
 + Sec[c + d*x]]*Sqrt[a - a*Sec[c + d*x]]))
```

3.141.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {3042, 4282, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(c+dx)}{\sqrt{a-a\sec(c+dx)}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a-a\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{ 4282}$$

$$-\frac{2\int \frac{1}{\frac{a^2\tan^2(c+dx)}{a-a\sec(c+dx)}+2a}d\frac{a\tan(c+dx)}{\sqrt{a-a\sec(c+dx)}}}{d}$$

$$\Big\downarrow \text{ 216}$$

$$-\frac{\sqrt{2}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a-a\sec(c+dx)}}\right)}{\sqrt{a}d}$$

input
```
Int[Sec[c + d*x]/Sqrt[a - a*Sec[c + d*x]],x]
```

output
```
-((Sqrt[2]*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a - a*Sec[c + d*x]]
)])/(Sqrt[a]*d))
```

3.141.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4282
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2/f   Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[
a + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

3.141.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 82 vs. $2(39) = 78$.

Time = 0.94 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.73

method	result	size
default	$\frac{\sqrt{2}\,\sin(dx+c)\arctan\left(\frac{\sqrt{2}}{2\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}}\right)}{d(\cos(dx+c)+1)\sqrt{-a(\sec(dx+c)-1)}\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}}$	83

input
```
int(sec(d*x+c)/(a-a*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/d*2^(1/2)*sin(d*x+c)*arctan(1/2*2^(1/2)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/
2))/(cos(d*x+c)+1)/(-a*(sec(d*x+c)-1))^(1/2)/(-cos(d*x+c)/(cos(d*x+c)+1))^
(1/2)
```

3.141.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 161, normalized size of antiderivative = 3.35

$$\int \frac{\sec(c+dx)}{\sqrt{a-a\sec(c+dx)}}\,dx$$
$$= \left[\frac{\sqrt{2}\sqrt{-\frac{1}{a}}\log\left(-\frac{2\sqrt{2}\left(\cos(dx+c)^2+\cos(dx+c)\right)\sqrt{\frac{a\cos(dx+c)-a}{\cos(dx+c)}}\sqrt{-\frac{1}{a}}-(3\cos(dx+c)+1)\sin(dx+c)}{(\cos(dx+c)-1)\sin(dx+c)}\right)}{2\,d}, \frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{a\cos(dx+}{\cos(dx+}}}{\sqrt{a}\sin}\right.}{\sqrt{a}d}\right.$$

input
```
integrate(sec(d*x+c)/(a-a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
[1/2*sqrt(2)*sqrt(-1/a)*log(-(2*sqrt(2)*(cos(d*x + c)^2 + cos(d*x + c))*sq
rt((a*cos(d*x + c) - a)/cos(d*x + c))*sqrt(-1/a) - (3*cos(d*x + c) + 1)*si
n(d*x + c))/((cos(d*x + c) - 1)*sin(d*x + c)))/d, sqrt(2)*arctan(sqrt(2)*s
qrt((a*cos(d*x + c) - a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))
)/(sqrt(a)*d)]
```

3.141.6 Sympy [F]

$$\int \frac{\sec(c+dx)}{\sqrt{a-a\sec(c+dx)}}\,dx=\int \frac{\sec{(c+dx)}}{\sqrt{-a\left(\sec{(c+dx)}-1\right)}}\,dx$$

input
```
integrate(sec(d*x+c)/(a-a*sec(d*x+c))**(1/2),x)
```

output
```
Integral(sec(c + d*x)/sqrt(-a*(sec(c + d*x) - 1)), x)
```

3.141.7 Maxima [F]

$$\int \frac{\sec(c+dx)}{\sqrt{a-a\sec(c+dx)}}\,dx=\int \frac{\sec{(dx+c)}}{\sqrt{-a\sec{(dx+c)}+a}}\,dx$$

input
```
integrate(sec(d*x+c)/(a-a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)/sqrt(-a*sec(d*x + c) + a), x)
```

3.141.8 Giac [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.40

$$\int \frac{\sec(c+dx)}{\sqrt{a-a\sec(c+dx)}}\,dx$$
$$=\frac{\sqrt{2}\arctan\left(\frac{\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a}}{\sqrt{a}}\right)}{\sqrt{a}d\operatorname{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\operatorname{sgn}\left(\cos\left(dx+c\right)\right)}$$

input
```
integrate(sec(d*x+c)/(a-a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
sqrt(2)*arctan(sqrt(a*tan(1/2*d*x + 1/2*c)^2 - a)/sqrt(a))/(sqrt(a)*d*sgn(
tan(1/2*d*x + 1/2*c)^3 + tan(1/2*d*x + 1/2*c))*sgn(cos(d*x + c)))
```

3.141.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(c+dx)}{\sqrt{a-a\sec(c+dx)}}\,dx=\int \frac{1}{\cos\left(c+d\,x\right)\,\sqrt{a-\frac{a}{\cos\left(c+d\,x\right)}}}\,dx$$

input
```
int(1/(cos(c + d*x)*(a - a/cos(c + d*x))^(1/2)),x)
```

output
```
int(1/(cos(c + d*x)*(a - a/cos(c + d*x))^(1/2)), x)
```

3.142 $\int \frac{1}{\sqrt{a-a\sec(c+dx)}}\,dx$

3.142.1 Optimal result 1227
3.142.2 Mathematica [C] (verified) 1227
3.142.3 Rubi [A] (verified) 1228
3.142.4 Maple [A] (verified) 1230
3.142.5 Fricas [A] (verification not implemented) 1230
3.142.6 Sympy [F] 1231
3.142.7 Maxima [C] (verification not implemented) 1231
3.142.8 Giac [A] (verification not implemented) 1232
3.142.9 Mupad [F(-1)] 1233

3.142.1 Optimal result

Integrand size = 15, antiderivative size = 87

$$\int \frac{1}{\sqrt{a-a\sec(c+dx)}}\,dx=\frac{2\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a-a\sec(c+dx)}}\right)}{\sqrt{a}d}-\frac{\sqrt{2}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a-a\sec(c+dx)}}\right)}{\sqrt{a}d}$$

output
```
2*arctan(a^(1/2)*tan(d*x+c)/(a-a*sec(d*x+c))^(1/2))/d/a^(1/2)-arctan(1/2*a
^(1/2)*tan(d*x+c)*2^(1/2)/(a-a*sec(d*x+c))^(1/2))*2^(1/2)/d/a^(1/2)
```

3.142.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.74 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.46

$$\int \frac{1}{\sqrt{a-a\sec(c+dx)}}\,dx=-\frac{i\left(-1+e^{i(c+dx)}\right)\left(\operatorname{arcsinh}\left(e^{i(c+dx)}\right)-\sqrt{2}\operatorname{arctanh}\left(\frac{1+e^{i(c+dx)}}{\sqrt{2}\sqrt{1+e^{2i(c+dx)}}}\right)+\operatorname{arctanh}\left(\sqrt{1+e^{2i(c+dx)}}\right)\right)}{d\sqrt{1+e^{2i(c+dx)}}\sqrt{a-a\sec(c+dx)}}$$

input
```
Integrate[1/Sqrt[a - a*Sec[c + d*x]],x]
```

output
```
((-I)*(-1 + E^(I*(c + d*x)))*(ArcSinh[E^(I*(c + d*x))] - Sqrt[2]*ArcTanh[(
1 + E^(I*(c + d*x)))/(Sqrt[2]*Sqrt[1 + E^((2*I)*(c + d*x))])] + ArcTanh[Sq
rt[1 + E^((2*I)*(c + d*x))]]))/(d*Sqrt[1 + E^((2*I)*(c + d*x))]*Sqrt[a - a
*Sec[c + d*x]])
```

3.142.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {3042, 4263, 3042, 4261, 216, 4282, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a-a\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{a-a\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4263}$$

$$\int \frac{\sec(c+dx)}{\sqrt{a-a\sec(c+dx)}}dx+\frac{\int \sqrt{a-a\sec(c+dx)}dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a-a\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{\int \sqrt{a-a\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a}$$

$$\Big\downarrow \text{4261}$$

$$\frac{2\int \frac{1}{\frac{a^2\tan^2(c+dx)}{a-a\sec(c+dx)}+a}d\frac{a\tan(c+dx)}{\sqrt{a-a\sec(c+dx)}}}{d}+\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a-a\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{216}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a-a\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a-a\sec(c+dx)}}\right)}{\sqrt{a}d}$$

$$\Big\downarrow \text{4282}$$

$$\frac{2\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a-a\sec(c+dx)}}\right)}{\sqrt{a}d}-\frac{2\int\frac{1}{\frac{a^2\tan^2(c+dx)}{a-a\sec(c+dx)}+2a}d\frac{a\tan(c+dx)}{\sqrt{a-a\sec(c+dx)}}}{d}$$

$$\Big\downarrow \; 216$$

$$\frac{2\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a-a\sec(c+dx)}}\right)}{\sqrt{a}d}-\frac{\sqrt{2}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a-a\sec(c+dx)}}\right)}{\sqrt{a}d}$$

input
```
Int[1/Sqrt[a - a*Sec[c + d*x]],x]
```

output
```
(2*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a - a*Sec[c + d*x]]])/(Sqrt[a]*d) -
(Sqrt[2]*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a - a*Sec[c + d*x]])]
)/(Sqrt[a]*d)
```

3.142.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4261
```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(b/d)
  Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 4263
```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[1/a   I
nt[Sqrt[a + b*Csc[c + d*x]], x], x] - Simp[b/a   Int[Csc[c + d*x]/Sqrt[a +
b*Csc[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 4282
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2/f   Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[
a + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

3.142.4 Maple [A] (verified)

Time = 0.89 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.25

method	result	size
default	$\frac{\sqrt{2}\,\sin(dx+c)\left(\arctan\left(\frac{\sqrt{2}}{2\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}}\right)+\sqrt{2}\,\arctan\left(\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\right)\right)}{d(\cos(dx+c)+1)\sqrt{-a(\sec(dx+c)-1)}\,\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}}$	109

input
```
int(1/(a-a*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/d*2^(1/2)*sin(d*x+c)*(arctan(1/2*2^(1/2)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1
/2))+2^(1/2)*arctan((-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)))/(cos(d*x+c)+1)/(-
a*(sec(d*x+c)-1))^(1/2)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)
```

3.142.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 301, normalized size of antiderivative = 3.46

$$\int \frac{1}{\sqrt{a-a\sec(c+dx)}}\,dx$$
$$= \left[\frac{\sqrt{2}a\sqrt{-\frac{1}{a}}\log\left(-\frac{2\sqrt{2}\left(\cos(dx+c)^2+\cos(dx+c)\right)\sqrt{\frac{a\cos(dx+c)-a}{\cos(dx+c)}}\sqrt{-\frac{1}{a}}-(3\cos(dx+c)+1)\sin(dx+c)}{(\cos(dx+c)-1)\sin(dx+c)}\right)-2\sqrt{-a}\log\left(\frac{2\left(\cos(dx+\right.}{}\right.}{2\,ad}\right.$$

input
```
integrate(1/(a-a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
[1/2*(sqrt(2)*a*sqrt(-1/a)*log(-(2*sqrt(2)*(cos(d*x + c)^2 + cos(d*x + c))
*sqrt((a*cos(d*x + c) - a)/cos(d*x + c))*sqrt(-1/a) - (3*cos(d*x + c) + 1)
*sin(d*x + c))/((cos(d*x + c) - 1)*sin(d*x + c))) - 2*sqrt(-a)*log((2*(cos
(d*x + c)^2 + cos(d*x + c))*sqrt(-a)*sqrt((a*cos(d*x + c) - a)/cos(d*x + c
)) - (2*a*cos(d*x + c) + a)*sin(d*x + c))/sin(d*x + c)))/(a*d), (sqrt(2)*s
qrt(a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) - a)/cos(d*x + c))*cos(d*x + c)
/(sqrt(a)*sin(d*x + c))) - 2*sqrt(a)*arctan(sqrt((a*cos(d*x + c) - a)/cos(
d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))))/(a*d)]
```

3.142.6 Sympy [F]

$$\int \frac{1}{\sqrt{a - a\sec(c + dx)}}\,dx = \int \frac{1}{\sqrt{-a\sec(c + dx) + a}}\,dx$$

input
```
integrate(1/(a-a*sec(d*x+c))**(1/2),x)
```

output
```
Integral(1/sqrt(-a*sec(c + d*x) + a), x)
```

3.142.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.93 (sec) , antiderivative size = 698, normalized size of antiderivative = 8.02

$$\int \frac{1}{\sqrt{a - a\sec(c + dx)}}\,dx =$$

$$-\frac{\sqrt{2}\sqrt{a}\arctan\left(\frac{\left(\left|2\,e^{(i\,dx+i\,c)}-2\right|^{4}+16\,\cos(dx+c)^{4}+16\,\sin(dx+c)^{4}+8\left(\cos(dx+c)^{2}-\sin(dx+c)^{2}+2\,\cos(dx+c)+1\right)\left|2\,e^{(i\,dx+i\,c)}-2\right|^{2}+6\right.}{}\right.}{}$$

input
```
integrate(1/(a-a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
-(sqrt(2)*sqrt(a)*arctan2(((abs(2*e^(I*d*x + I*c) - 2)^4 + 16*cos(d*x + c)
^4 + 16*sin(d*x + c)^4 + 8*(cos(d*x + c)^2 - sin(d*x + c)^2 + 2*cos(d*x +
c) + 1)*abs(2*e^(I*d*x + I*c) - 2)^2 + 64*cos(d*x + c)^3 + 32*(cos(d*x + c
)^2 + 2*cos(d*x + c) + 1)*sin(d*x + c)^2 + 96*cos(d*x + c)^2 + 64*cos(d*x
+ c) + 16)^(1/4)*sin(1/2*arctan2(8*(cos(d*x + c) + 1)*sin(d*x + c)/abs(2*e
^(I*d*x + I*c) - 2)^2, (abs(2*e^(I*d*x + I*c) - 2)^2 + 4*cos(d*x + c)^2 -
4*sin(d*x + c)^2 + 8*cos(d*x + c) + 4)/abs(2*e^(I*d*x + I*c) - 2)^2)) + 2*
sin(d*x + c))/abs(2*e^(I*d*x + I*c) - 2), ((abs(2*e^(I*d*x + I*c) - 2)^4 +
 16*cos(d*x + c)^4 + 16*sin(d*x + c)^4 + 8*(cos(d*x + c)^2 - sin(d*x + c)^
2 + 2*cos(d*x + c) + 1)*abs(2*e^(I*d*x + I*c) - 2)^2 + 64*cos(d*x + c)^3 +
 32*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sin(d*x + c)^2 + 96*cos(d*x + c)
^2 + 64*cos(d*x + c) + 16)^(1/4)*cos(1/2*arctan2(8*(cos(d*x + c) + 1)*sin(
d*x + c)/abs(2*e^(I*d*x + I*c) - 2)^2, (abs(2*e^(I*d*x + I*c) - 2)^2 + 4*c
os(d*x + c)^2 - 4*sin(d*x + c)^2 + 8*cos(d*x + c) + 4)/abs(2*e^(I*d*x + I*
c) - 2)^2)) + 2*cos(d*x + c) + 2)/abs(2*e^(I*d*x + I*c) - 2)) - sqrt(a)*ar
ctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(
1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + sin(d*x +
c), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/
4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + cos(d*x + c)
))/(a*d)
```

3.142.8 Giac [A] (verification not implemented)

Time = 0.57 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.79

$$\int \frac{1}{\sqrt{a-a\sec(c+dx)}}\,dx = \frac{\frac{\sqrt{2}\arctan\left(\frac{\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a}}{\sqrt{a}}\right)}{\sqrt{a}} - \frac{2\arctan\left(\frac{\sqrt{2}\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a}}{2\sqrt{a}}\right)}{\sqrt{a}}}{d}$$

input
```
integrate(1/(a-a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
(sqrt(2)*arctan(sqrt(a*tan(1/2*d*x + 1/2*c)^2 - a)/sqrt(a))/sqrt(a) - 2*ar
ctan(1/2*sqrt(2)*sqrt(a*tan(1/2*d*x + 1/2*c)^2 - a)/sqrt(a))/sqrt(a))/d
```

3.142.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a-a\sec(c+dx)}}\,dx=\int \frac{1}{\sqrt{a-\frac{a}{\cos(c+d\,x)}}}\,dx$$

input
```
int(1/(a - a/cos(c + d*x))^(1/2),x)
```

output
```
int(1/(a - a/cos(c + d*x))^(1/2), x)
```

3.143 $\int \sec^3(c+dx)(a+a\sec(c+dx))^{2/3}\,dx$

3.143.1 Optimal result 1234
3.143.2 Mathematica [C] (verified) 1235
3.143.3 Rubi [A] (verified) 1235
3.143.4 Maple [F] 1239
3.143.5 Fricas [F] 1239
3.143.6 Sympy [F] 1240
3.143.7 Maxima [F] 1240
3.143.8 Giac [F] 1240
3.143.9 Mupad [F(-1)] 1241

3.143.1 Optimal result

Integrand size = 23, antiderivative size = 383

$$\int \sec^3(c+dx)(a+a\sec(c+dx))^{2/3}\,dx = -\frac{9(a+a\sec(c+dx))^{2/3}\tan(c+dx)}{40d} + \frac{57(a+a\sec(c+dx))^{2/3}\tan(c+dx)}{80d(1+\sec(c+dx))} + \frac{3(a+a\sec(c+dx))^{5/3}\tan(c+dx)}{8ad} - \frac{19\,3^{3/4}\,\text{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{2}-\left(1-\sqrt{3}\right)\sqrt[3]{1+\sec(c+dx)}}{\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{1+\sec(c+dx)}}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)(a+a\sec(c+dx))^{2/3}\left(\sqrt[3]{2}-\sqrt[3]{\cdots}\right.}{80\sqrt[3]{2}d(1-\sec(c+dx))(1+\sec(c+dx))\sqrt{-\frac{\sqrt[3]{1+\sec(c+\cdots}}{\left(\sqrt[3]{2}-(1+\cdots\right.}}}$$

output
```
-9/40*(a+a*sec(d*x+c))^(2/3)*tan(d*x+c)/d+57/80*(a+a*sec(d*x+c))^(2/3)*tan
(d*x+c)/d/(1+sec(d*x+c))+3/8*(a+a*sec(d*x+c))^(5/3)*tan(d*x+c)/a/d-19/160*
3^(3/4)*((2^(1/3)-(1+sec(d*x+c))^(1/3)*(1-3^(1/2)))^2/(2^(1/3)-(1+sec(d*x+
c))^(1/3)*(1+3^(1/2)))^2)^(1/2)/(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1-3^(1/2)))
*(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1+3^(1/2)))*EllipticF((1-(2^(1/3)-(1+sec(d
*x+c))^(1/3)*(1-3^(1/2)))^2/(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1+3^(1/2)))^2)^
(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(a+a*sec(d*x+c))^(2/3)*(2^(1/3)-(1+sec(d*x+
c))^(1/3))*((2^(2/3)+2^(1/3)*(1+sec(d*x+c))^(1/3)+(1+sec(d*x+c))^(2/3))/(2
^(1/3)-(1+sec(d*x+c))^(1/3)*(1+3^(1/2)))^2)^(1/2)*tan(d*x+c)*2^(2/3)/d/(1-
sec(d*x+c))/(1+sec(d*x+c))/(-(1+sec(d*x+c))^(1/3)*(2^(1/3)-(1+sec(d*x+c))^
(1/3))/(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

3.143.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.28 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.27

$$\int \sec^3(c+dx)(a+a\sec(c+dx))^{2/3}\,dx = \frac{(a(1+\sec(c+dx)))^{2/3}\left(38\sqrt[6]{2}\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{2},\frac{3}{2},\frac{1}{2}(1-\sec(c+dx))\right)+3\sqrt[6]{1+\text{s}}\right.}{40d(1+\sec(c+dx))^{7/6}}$$

input
```
Integrate[Sec[c + d*x]^3*(a + a*Sec[c + d*x])^(2/3),x]
```

output
```
((a*(1 + Sec[c + d*x]))^(2/3)*(38*2^(1/6)*Hypergeometric2F1[-1/6, 1/2, 3/2
, (1 - Sec[c + d*x])/2] + 3*(1 + Sec[c + d*x])^(1/6)*(2 + 7*Sec[c + d*x] +
 5*Sec[c + d*x]^2))*Tan[c + d*x])/(40*d*(1 + Sec[c + d*x])^(7/6))
```

3.143.3 Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 421, normalized size of antiderivative = 1.10, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.522$, Rules used = {3042, 4287, 27, 3042, 4489, 3042, 4315, 3042, 4314, 60, 73, 766}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^3(c+dx)(a\sec(c+dx)+a)^{2/3}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^3\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{2/3}dx$$

$$\Big\downarrow \; 4287$$

$$\frac{3\int \frac{1}{3}\sec(c+dx)(5a-3a\sec(c+dx))(\sec(c+dx)a+a)^{2/3}dx}{8a}+\frac{3\tan(c+dx)(a\sec(c+dx)+a)^{5/3}}{8ad}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \sec(c+dx)(5a-3a\sec(c+dx))(\sec(c+dx)a+a)^{2/3}dx}{8a}+\frac{3\tan(c+dx)(a\sec(c+dx)+a)^{5/3}}{8ad}$$

$$\Big\downarrow \; 3042$$

$$\frac{\int \csc\left(c+dx+\frac{\pi}{2}\right)\left(5a-3a\csc\left(c+dx+\frac{\pi}{2}\right)\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{2/3}dx}{8a}+\frac{3\tan(c+dx)(a\sec(c+dx)+a)^{5/3}}{8ad}$$

$$\Big\downarrow \; 4489$$

$$\frac{\frac{19}{5}a\int \sec(c+dx)(\sec(c+dx)a+a)^{2/3}dx-\frac{9a\tan(c+dx)(a\sec(c+dx)+a)^{2/3}}{5d}}{8a}+\frac{3\tan(c+dx)(a\sec(c+dx)+a)^{5/3}}{8ad}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{19}{5}a\int \csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{2/3}dx-\frac{9a\tan(c+dx)(a\sec(c+dx)+a)^{2/3}}{5d}}{8a}+\frac{3\tan(c+dx)(a\sec(c+dx)+a)^{5/3}}{8ad}$$

$$\Big\downarrow \; 4315$$

$$\frac{\frac{19a(a\sec(c+dx)+a)^{2/3}\int \sec(c+dx)(\sec(c+dx)+1)^{2/3}dx}{5(\sec(c+dx)+1)^{2/3}}-\frac{9a\tan(c+dx)(a\sec(c+dx)+a)^{2/3}}{5d}}{8a}+\frac{3\tan(c+dx)(a\sec(c+dx)+a)^{5/3}}{8ad}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{19a(a\sec(c+dx)+a)^{2/3}\int \csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)^{2/3}dx}{5(\sec(c+dx)+1)^{2/3}}-\frac{9a\tan(c+dx)(a\sec(c+dx)+a)^{2/3}}{5d}}{8a}+\frac{3\tan(c+dx)(a\sec(c+dx)+a)^{5/3}}{8ad}$$

$$\Big\downarrow \; 4314$$

$$\frac{-\frac{19a\tan(c+dx)(a\sec(c+dx)+a)^{2/3}\int \frac{\sqrt[6]{\sec(c+dx)+1}}{\sqrt{1-\sec(c+dx)}}d\sec(c+dx)}{5d\sqrt{1-\sec(c+dx)}(\sec(c+dx)+1)^{7/6}}-\frac{9a\tan(c+dx)(a\sec(c+dx)+a)^{2/3}}{5d}}{8a}+\frac{3\tan(c+dx)(a\sec(c+dx)+a)^{5/3}}{8ad}$$

$$\Big\downarrow \; 60$$

3.143. $\int \sec^3(c+dx)(a+a\sec(c+dx))^{2/3}\,dx$

$$\frac{-\frac{19a\tan(c+dx)(a\sec(c+dx)+a)^{2/3}\left(\frac{1}{2}\int\frac{1}{\sqrt{1-\sec(c+dx)}(\sec(c+dx)+1)^{5/6}}d\sec(c+dx)-\frac{3}{2}\sqrt{1-\sec(c+dx)}\sqrt[6]{\sec(c+dx)+1}\right)}{5d\sqrt{1-\sec(c+dx)}(\sec(c+dx)+1)^{7/6}}-\frac{9a\tan(c+dx}{}}{\frac{3\tan(c+dx)(a\sec(c+dx)+a)^{5/3}}{8ad}}$$

$$\Big\downarrow \; 73$$

$$\frac{-\frac{19a\tan(c+dx)(a\sec(c+dx)+a)^{2/3}\left(3\int\frac{1}{\sqrt{1-\sec(c+dx)}}d\sqrt[6]{\sec(c+dx)+1}-\frac{3}{2}\sqrt{1-\sec(c+dx)}\sqrt[6]{\sec(c+dx)+1}\right)}{5d\sqrt{1-\sec(c+dx)}(\sec(c+dx)+1)^{7/6}}-\frac{9a\tan(c+dx)(a\,s}{5}}{\frac{3\tan(c+dx)(a\sec(c+dx)+a)^{5/3}}{8ad}}$$

$$\Big\downarrow \; 766$$

$$-\frac{19a\tan(c+dx)(a\sec(c+dx)+a)^{2/3}\left(\frac{3^{3/4}\sqrt[6]{\sec(c+dx)+1}\left(\sqrt[3]{2}-\sqrt[3]{\sec(c+dx)+1}\right)\sqrt{\frac{(\sec(c+dx)+1)^{2/3}+\sqrt[3]{2}\sqrt[3]{\sec(c+dx)+}}{\left(\sqrt[3]{2}-(1+\sqrt{3})\sqrt[3]{\sec(c+dx)+1}\right)}}}{2\sqrt[3]{2}\sqrt{1-\sec(c+dx)}\sqrt{-\frac{\sqrt[3]{\sec(c+dx)+1}\left(\sqrt[3]{2}-\sqrt[3]{}\right.}{\left(\sqrt[3]{2}-(1+\sqrt{3})\sqrt[3]{\sec(c}\right.}}}\right.}{5d\sqrt{1-\sec(c+dx)}(\sec(c+dx)-}$$

$$\frac{3\tan(c+dx)(a\sec(c+dx)+a)^{5/3}}{8ad}$$

input
```
Int[Sec[c + d*x]^3*(a + a*Sec[c + d*x])^(2/3),x]
```

output
```
(3*(a + a*Sec[c + d*x])^(5/3)*Tan[c + d*x])/(8*a*d) + ((-9*a*(a + a*Sec[c
+ d*x])^(2/3)*Tan[c + d*x])/(5*d) - (19*a*(a + a*Sec[c + d*x])^(2/3)*((-3*
Sqrt[1 - Sec[c + d*x]]*(1 + Sec[c + d*x])^(1/6))/2 + (3^(3/4)*EllipticF[Ar
cCos[(2^(1/3) - (1 - Sqrt[3])*(1 + Sec[c + d*x])^(1/3))/(2^(1/3) - (1 + Sq
rt[3])*(1 + Sec[c + d*x])^(1/3))], (2 + Sqrt[3])/4]*(1 + Sec[c + d*x])^(1/
6)*(2^(1/3) - (1 + Sec[c + d*x])^(1/3))*Sqrt[(2^(2/3) + 2^(1/3)*(1 + Sec[c
 + d*x])^(1/3) + (1 + Sec[c + d*x])^(2/3))/(2^(1/3) - (1 + Sqrt[3])*(1 + S
ec[c + d*x])^(1/3))^2])/(2*2^(1/3)*Sqrt[1 - Sec[c + d*x]]*Sqrt[-(((1 + Sec
[c + d*x])^(1/3)*(2^(1/3) - (1 + Sec[c + d*x])^(1/3)))/(2^(1/3) - (1 + Sqr
t[3])*(1 + Sec[c + d*x])^(1/3))^2)]))*Tan[c + d*x])/(5*d*Sqrt[1 - Sec[c +
d*x]]*(1 + Sec[c + d*x])^(7/6)))/(8*a)
```

3.143. $\int \sec^3(c+dx)(a+a\sec(c+dx))^{2/3}\,dx$

3.143.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 766
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4287
```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(-Cot[e + f*x])*((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2
))), x] + Simp[1/(b*(m + 2))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(b*(
m + 1) - a*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 -
b^2, 0] &&  !LtQ[m, -2^(-1)]
```

3.143. $\int \sec^3(c+dx)(a+a\sec(c+dx))^{2/3}\,dx$

rule 4314
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Simp[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x
]]*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(d*x)^(n - 1)*((a + b*x)^(m - 1/2
)/Sqrt[a - b*x]), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n},
x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

rule 4315
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Simp[a^IntPart[m]*((a + b*Csc[e + f*x])^FracPart[m
]/(1 + (b/a)*Csc[e + f*x])^FracPart[m])   Int[(1 + (b/a)*Csc[e + f*x])^m*(d
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^
2, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

rule 4489
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*((
a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Simp[(a*B*m + A*b*(m + 1))/(b*(m +
 1))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B
, e, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b
*(m + 1), 0] &&  !LtQ[m, -2^(-1)]
```

3.143.4 Maple [F]

$$\int \sec (dx+c)^3 (a+a\sec (dx+c))^{\frac{2}{3}} dx$$

input
```
int(sec(d*x+c)^3*(a+a*sec(d*x+c))^(2/3),x)
```

output
```
int(sec(d*x+c)^3*(a+a*sec(d*x+c))^(2/3),x)
```

3.143.5 Fricas [F]

$$\int \sec^3(c+dx)(a+a\sec(c+dx))^{2/3}\,dx = \int (a\sec (dx+c)+a)^{\frac{2}{3}} \sec (dx+c)^3\,dx$$

input
```
integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^(2/3),x, algorithm="fricas")
```

output
```
integral((a*sec(d*x + c) + a)^(2/3)*sec(d*x + c)^3, x)
```

3.143.6 Sympy [F]

$$\int \sec^3(c+dx)(a+a\sec(c+dx))^{2/3}\,dx = \int (a(\sec(c+dx)+1))^{\frac{2}{3}}\sec^3(c+dx)\,dx$$

input
```
integrate(sec(d*x+c)**3*(a+a*sec(d*x+c))**(2/3),x)
```

output
```
Integral((a*(sec(c + d*x) + 1))**(2/3)*sec(c + d*x)**3, x)
```

3.143.7 Maxima [F]

$$\int \sec^3(c+dx)(a+a\sec(c+dx))^{2/3}\,dx = \int (a\sec(dx+c)+a)^{\frac{2}{3}}\sec(dx+c)^3\,dx$$

input
```
integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^(2/3),x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)^(2/3)*sec(d*x + c)^3, x)
```

3.143.8 Giac [F]

$$\int \sec^3(c+dx)(a+a\sec(c+dx))^{2/3}\,dx = \int (a\sec(dx+c)+a)^{\frac{2}{3}}\sec(dx+c)^3\,dx$$

input
```
integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^(2/3),x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)^(2/3)*sec(d*x + c)^3, x)
```

3.143.9 Mupad [F(-1)]

Timed out.

$$\int \sec^3(c+dx)(a+a\sec(c+dx))^{2/3}\,dx = \int \frac{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{2/3}}{\cos\left(c+d\,x\right)^3}\,dx$$

input
```
int((a + a/cos(c + d*x))^(2/3)/cos(c + d*x)^3,x)
```

output
```
int((a + a/cos(c + d*x))^(2/3)/cos(c + d*x)^3, x)
```

3.144 $\int \sec^2(c+dx)(a+a\sec(c+dx))^{2/3}\,dx$

3.144.1 Optimal result 1242
3.144.2 Mathematica [C] (verified) 1243
3.144.3 Rubi [A] (verified) 1243
3.144.4 Maple [F] 1246
3.144.5 Fricas [F] 1246
3.144.6 Sympy [F] 1247
3.144.7 Maxima [F] 1247
3.144.8 Giac [F] 1247
3.144.9 Mupad [F(-1)] 1248

3.144.1 Optimal result

Integrand size = 23, antiderivative size = 353

$$\int \sec^2(c+dx)(a+a\sec(c+dx))^{2/3}\,dx = \frac{3(a+a\sec(c+dx))^{2/3}\tan(c+dx)}{5d} + \frac{3(a+a\sec(c+dx))^{2/3}\tan(c+dx)}{5d(1+\sec(c+dx))} - \frac{3^{3/4}\,\text{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{2}-\left(1-\sqrt{3}\right)\sqrt[3]{1+\sec(c+dx)}}{\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{1+\sec(c+dx)}}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)(a+a\sec(c+dx))^{2/3}\left(\sqrt[3]{2}-\sqrt[3]{1-}\right.}{5\sqrt[3]{2}d(1-\sec(c+dx))(1+\sec(c+dx))\sqrt{-\frac{\sqrt[3]{1+\sec(c+dx}}{\left(\sqrt[3]{2}-\left(1+\sqrt{3}\right.\right.}}}$$

output
```
3/5*(a+a*sec(d*x+c))^(2/3)*tan(d*x+c)/d+3/5*(a+a*sec(d*x+c))^(2/3)*tan(d*x
+c)/d/(1+sec(d*x+c))-1/10*3^(3/4)*((2^(1/3)-(1+sec(d*x+c))^(1/3)*(1-3^(1/2
)))^2/(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1+3^(1/2)))^2)^(1/2)/(2^(1/3)-(1+sec(
d*x+c))^(1/3)*(1-3^(1/2)))*(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1+3^(1/2)))*Elli
pticF((1-(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1-3^(1/2)))^2/(2^(1/3)-(1+sec(d*x+
c))^(1/3)*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(a+a*sec(d*x+c))^
(2/3)*(2^(1/3)-(1+sec(d*x+c))^(1/3))*((2^(2/3)+2^(1/3)*(1+sec(d*x+c))^(1/3
)+(1+sec(d*x+c))^(2/3))/(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1+3^(1/2)))^2)^(1/2
)*tan(d*x+c)*2^(2/3)/d/(1-sec(d*x+c))/(1+sec(d*x+c))/(-(1+sec(d*x+c))^(1/3
)*(2^(1/3)-(1+sec(d*x+c))^(1/3))/(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1+3^(1/2))
)^2)^(1/2)
```

3.144.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.14 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.24

$$\int \sec^2(c+dx)(a+a\sec(c+dx))^{2/3}\,dx = \frac{(a(1+\sec(c+dx)))^{2/3}\left(4\sqrt[6]{2}\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{2},\frac{3}{2},\frac{1}{2}(1-\sec(c+dx))\right)+3(1+\sec(\right.}{5d(1+\sec(c+dx))^{7/6}}$$

input
```
Integrate[Sec[c + d*x]^2*(a + a*Sec[c + d*x])^(2/3),x]
```

output
```
((a*(1 + Sec[c + d*x]))^(2/3)*(4*2^(1/6)*Hypergeometric2F1[-1/6, 1/2, 3/2,
 (1 - Sec[c + d*x])/2] + 3*(1 + Sec[c + d*x])^(7/6))*Tan[c + d*x])/(5*d*(1
 + Sec[c + d*x])^(7/6))
```

3.144.3 Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 381, normalized size of antiderivative = 1.08, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {3042, 4285, 3042, 4315, 3042, 4314, 60, 73, 766}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(c+dx)(a\sec(c+dx)+a)^{2/3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^2\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{2/3}dx$$

$$\Big\downarrow \text{4285}$$

$$\frac{2}{5}\int \sec(c+dx)(\sec(c+dx)a+a)^{2/3}dx+\frac{3\tan(c+dx)(a\sec(c+dx)+a)^{2/3}}{5d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{5}\int \csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{2/3}dx+\frac{3\tan(c+dx)(a\sec(c+dx)+a)^{2/3}}{5d}$$

$$\Big\downarrow \text{4315}$$

$$\frac{2(a\sec(c+dx)+a)^{2/3}\int \sec(c+dx)(\sec(c+dx)+1)^{2/3}dx}{5(\sec(c+dx)+1)^{2/3}}+\frac{3\tan(c+dx)(a\sec(c+dx)+a)^{2/3}}{5d}$$

$$\Big\downarrow \; 3042$$

$$\frac{2(a\sec(c+dx)+a)^{2/3}\int \csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)^{2/3}dx}{5(\sec(c+dx)+1)^{2/3}}+\frac{3\tan(c+dx)(a\sec(c+dx)+a)^{2/3}}{5d}$$

$$\Big\downarrow \; 4314$$

$$\frac{3\tan(c+dx)(a\sec(c+dx)+a)^{2/3}}{5d}-\frac{2\tan(c+dx)(a\sec(c+dx)+a)^{2/3}\int \frac{\sqrt[6]{\sec(c+dx)+1}}{\sqrt{1-\sec(c+dx)}}d\sec(c+dx)}{5d\sqrt{1-\sec(c+dx)}(\sec(c+dx)+1)^{7/6}}$$

$$\Big\downarrow \; 60$$

$$\frac{3\tan(c+dx)(a\sec(c+dx)+a)^{2/3}}{5d}-\frac{2\tan(c+dx)(a\sec(c+dx)+a)^{2/3}\left(\frac{1}{2}\int \frac{1}{\sqrt{1-\sec(c+dx)}(\sec(c+dx)+1)^{5/6}}d\sec(c+dx)-\frac{3}{2}\sqrt{1-\sec(c+dx)}\sqrt[6]{\sec(c+d}\right.}{5d\sqrt{1-\sec(c+dx)}(\sec(c+dx)+1)^{7/6}}$$

$$\Big\downarrow \; 73$$

$$\frac{3\tan(c+dx)(a\sec(c+dx)+a)^{2/3}}{5d}-\frac{2\tan(c+dx)(a\sec(c+dx)+a)^{2/3}\left(3\int \frac{1}{\sqrt{1-\sec(c+dx)}}d\sqrt[6]{\sec(c+dx)+1}-\frac{3}{2}\sqrt{1-\sec(c+dx)}\sqrt[6]{\sec(c+dx)+1}\right)}{5d\sqrt{1-\sec(c+dx)}(\sec(c+dx)+1)^{7/6}}$$

$$\Big\downarrow \; 766$$

$$\frac{3\tan(c+dx)(a\sec(c+dx)+a)^{2/3}}{5d}-\frac{2\tan(c+dx)(a\sec(c+dx)+a)^{2/3}\left(\frac{3^{3/4}\sqrt[6]{\sec(c+dx)+1}\left(\sqrt[3]{2}-\sqrt[3]{\sec(c+dx)+1}\right)\sqrt{\frac{(\sec(c+dx)+1)^{2/3}+\sqrt[3]{2}\sqrt[3]{\sec(c+}}{\left(\sqrt[3]{2}-(1+\sqrt{3})\sqrt[3]{\sec(c+}\right.}}}{2\sqrt[3]{2}\sqrt{1-\sec(c+dx)}\sqrt{-\frac{\sqrt[3]{\sec(c+dx)}}{\left(\sqrt[3]{2}-(1+\right.}}}\right.}{5d\sqrt{1-\sec(c+dx)}(}$$

input
```
Int[Sec[c + d*x]^2*(a + a*Sec[c + d*x])^(2/3),x]
```

3.144. $\int \sec^2(c+dx)(a+a\sec(c+dx))^{2/3}\,dx$

output
```
(3*(a + a*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(5*d) - (2*(a + a*Sec[c + d*x]
)^(2/3)*((-3*Sqrt[1 - Sec[c + d*x]]*(1 + Sec[c + d*x])^(1/6))/2 + (3^(3/4)
*EllipticF[ArcCos[(2^(1/3) - (1 - Sqrt[3])*(1 + Sec[c + d*x])^(1/3))/(2^(1
/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))], (2 + Sqrt[3])/4]*(1 + Sec[
c + d*x])^(1/6)*(2^(1/3) - (1 + Sec[c + d*x])^(1/3))*Sqrt[(2^(2/3) + 2^(1/
3)*(1 + Sec[c + d*x])^(1/3) + (1 + Sec[c + d*x])^(2/3))/(2^(1/3) - (1 + Sq
rt[3])*(1 + Sec[c + d*x])^(1/3))^2])/(2*2^(1/3)*Sqrt[1 - Sec[c + d*x]]*Sqr
t[-(((1 + Sec[c + d*x])^(1/3)*(2^(1/3) - (1 + Sec[c + d*x])^(1/3)))/(2^(1/
3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2)]))*Tan[c + d*x])/(5*d*Sqrt
[1 - Sec[c + d*x]]*(1 + Sec[c + d*x])^(7/6))
```

3.144.3.1 Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 766
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4285
```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(-Cot[e + f*x])*((a + b*Csc[e + f*x])^m/(f*(m + 1))), x]
+ Simp[a*(m/(b*(m + 1)))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x]
/; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

rule 4314
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Simp[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x
]]*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(d*x)^(n - 1)*((a + b*x)^(m - 1/2
)/Sqrt[a - b*x]), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n},
x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

rule 4315
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Simp[a^IntPart[m]*((a + b*Csc[e + f*x])^FracPart[m
]/(1 + (b/a)*Csc[e + f*x])^FracPart[m])   Int[(1 + (b/a)*Csc[e + f*x])^m*(d
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^
2, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

3.144.4 Maple [F]

$$\int \sec (dx+c)^2 (a+a\sec (dx+c))^{\frac{2}{3}} dx$$

input
```
int(sec(d*x+c)^2*(a+a*sec(d*x+c))^(2/3),x)
```

output
```
int(sec(d*x+c)^2*(a+a*sec(d*x+c))^(2/3),x)
```

3.144.5 Fricas [F]

$$\int \sec^2(c+dx)(a+a\sec(c+dx))^{2/3}\,dx = \int (a\sec (dx+c)+a)^{\frac{2}{3}} \sec (dx+c)^2 \, dx$$

input
```
integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^(2/3),x, algorithm="fricas")
```

output
```
integral((a*sec(d*x + c) + a)^(2/3)*sec(d*x + c)^2, x)
```

3.144.6 Sympy [F]

$$\int \sec^2(c+dx)(a+a\sec(c+dx))^{2/3}\,dx = \int (a(\sec(c+dx)+1))^{\frac{2}{3}}\sec^2(c+dx)\,dx$$

input
```
integrate(sec(d*x+c)**2*(a+a*sec(d*x+c))**(2/3),x)
```

output
```
Integral((a*(sec(c + d*x) + 1))**(2/3)*sec(c + d*x)**2, x)
```

3.144.7 Maxima [F]

$$\int \sec^2(c+dx)(a+a\sec(c+dx))^{2/3}\,dx = \int (a\sec(dx+c)+a)^{\frac{2}{3}}\sec(dx+c)^2\,dx$$

input
```
integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^(2/3),x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)^(2/3)*sec(d*x + c)^2, x)
```

3.144.8 Giac [F]

$$\int \sec^2(c+dx)(a+a\sec(c+dx))^{2/3}\,dx = \int (a\sec(dx+c)+a)^{\frac{2}{3}}\sec(dx+c)^2\,dx$$

input
```
integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^(2/3),x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)^(2/3)*sec(d*x + c)^2, x)
```

3.144.9 Mupad [F(-1)]

Timed out.

$$\int \sec^2(c+dx)(a+a\sec(c+dx))^{2/3}\,dx = \int \frac{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{2/3}}{\cos\left(c+d\,x\right)^2}\,dx$$

input
```
int((a + a/cos(c + d*x))^(2/3)/cos(c + d*x)^2,x)
```

output
```
int((a + a/cos(c + d*x))^(2/3)/cos(c + d*x)^2, x)
```

3.145 $\int \sec(c+dx)(a+a\sec(c+dx))^{2/3}\,dx$

3.145.1 Optimal result 1249
3.145.2 Mathematica [C] (verified) 1250
3.145.3 Rubi [A] (verified) 1250
3.145.4 Maple [F] 1253
3.145.5 Fricas [F] 1253
3.145.6 Sympy [F] 1253
3.145.7 Maxima [F] 1254
3.145.8 Giac [F] 1254
3.145.9 Mupad [F(-1)] 1254

3.145.1 Optimal result

Integrand size = 21, antiderivative size = 326

$$\int \sec(c+dx)(a+a\sec(c+dx))^{2/3}\,dx = \frac{3(a+a\sec(c+dx))^{2/3}\tan(c+dx)}{2d(1+\sec(c+dx))} - \frac{3^{3/4}\,\text{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{2}-\left(1-\sqrt{3}\right)\sqrt[3]{1+\sec(c+dx)}}{\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{1+\sec(c+dx)}}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)(a+a\sec(c+dx))^{2/3}\left(\sqrt[3]{2}-\sqrt[3]{1-}\right.}{2\sqrt[3]{2}d(1-\sec(c+dx))(1+\sec(c+dx))\sqrt{-\frac{\sqrt[3]{1+\sec(c+dx}}{\left(\sqrt[3]{2}-\left(1+\sqrt{3}\right.\right.}}}$$

output
```
3/2*(a+a*sec(d*x+c))^(2/3)*tan(d*x+c)/d/(1+sec(d*x+c))-1/4*3^(3/4)*((2^(1/
3)-(1+sec(d*x+c))^(1/3)*(1-3^(1/2)))^2/(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1+3^
(1/2)))^2)^(1/2)/(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1-3^(1/2)))*(2^(1/3)-(1+se
c(d*x+c))^(1/3)*(1+3^(1/2)))*EllipticF((1-(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1
-3^(1/2)))^2/(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/
2)+1/4*2^(1/2))*(a+a*sec(d*x+c))^(2/3)*(2^(1/3)-(1+sec(d*x+c))^(1/3))*((2^
(2/3)+2^(1/3)*(1+sec(d*x+c))^(1/3)+(1+sec(d*x+c))^(2/3))/(2^(1/3)-(1+sec(d
*x+c))^(1/3)*(1+3^(1/2)))^2)^(1/2)*tan(d*x+c)*2^(2/3)/d/(1-sec(d*x+c))/(1+
sec(d*x+c))/(-(1+sec(d*x+c))^(1/3)*(2^(1/3)-(1+sec(d*x+c))^(1/3))/(2^(1/3)
-(1+sec(d*x+c))^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

3.145.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.06 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.20

$$\int \sec(c+dx)(a+a\sec(c+dx))^{2/3}\,dx = \frac{2\sqrt[6]{2}\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{2},\frac{3}{2},\frac{1}{2}(1-\sec(c+dx))\right)(a(1+\sec(c+dx)))^{2/3}\tan(c+dx)}{d(1+\sec(c+dx))^{7/6}}$$

input
```
Integrate[Sec[c + d*x]*(a + a*Sec[c + d*x])^(2/3),x]
```

output
```
(2*2^(1/6)*Hypergeometric2F1[-1/6, 1/2, 3/2, (1 - Sec[c + d*x])/2]*(a*(1 +
 Sec[c + d*x]))^(2/3)*Tan[c + d*x])/(d*(1 + Sec[c + d*x])^(7/6))
```

3.145.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 351, normalized size of antiderivative = 1.08, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 4315, 3042, 4314, 60, 73, 766}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(c+dx)(a\sec(c+dx)+a)^{2/3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{2/3}dx$$

$$\Big\downarrow \text{4315}$$

$$\frac{(a\sec(c+dx)+a)^{2/3}\int \sec(c+dx)(\sec(c+dx)+1)^{2/3}dx}{(\sec(c+dx)+1)^{2/3}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(a\sec(c+dx)+a)^{2/3}\int \csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)^{2/3}dx}{(\sec(c+dx)+1)^{2/3}}$$

$$\Big\downarrow \text{4314}$$

$$-\frac{\tan(c+dx)(a\sec(c+dx)+a)^{2/3}\int \frac{\sqrt[6]{\sec(c+dx)+1}}{\sqrt{1-\sec(c+dx)}}d\sec(c+dx)}{d\sqrt{1-\sec(c+dx)}(\sec(c+dx)+1)^{7/6}}$$

$\Big\downarrow$ 60

$$-\frac{\tan(c+dx)(a\sec(c+dx)+a)^{2/3}\left(\frac{1}{2}\int \frac{1}{\sqrt{1-\sec(c+dx)}(\sec(c+dx)+1)^{5/6}}d\sec(c+dx)-\frac{3}{2}\sqrt{1-\sec(c+dx)}\sqrt[6]{\sec(c+\right.}{d\sqrt{1-\sec(c+dx)}(\sec(c+dx)+1)^{7/6}}$$

$\Big\downarrow$ 73

$$-\frac{\tan(c+dx)(a\sec(c+dx)+a)^{2/3}\left(3\int \frac{1}{\sqrt{1-\sec(c+dx)}}d\sqrt[6]{\sec(c+dx)+1}-\frac{3}{2}\sqrt{1-\sec(c+dx)}\sqrt[6]{\sec(c+dx)+1}\right)}{d\sqrt{1-\sec(c+dx)}(\sec(c+dx)+1)^{7/6}}$$

$\Big\downarrow$ 766

$$-\frac{\tan(c+dx)(a\sec(c+dx)+a)^{2/3}\left(\frac{3^{3/4}\sqrt[6]{\sec(c+dx)+1}\left(\sqrt[3]{2}-\sqrt[3]{\sec(c+dx)+1}\right)\sqrt{\frac{(\sec(c+dx)+1)^{2/3}+\sqrt[3]{2}\sqrt[3]{\sec(c-}}{\left(\sqrt[3]{2}-(1+\sqrt{3})\sqrt[3]{\sec(c+}\right.}}}{2\sqrt[3]{2}\sqrt{1-\sec(c+dx)}\sqrt{-\frac{\sqrt[3]{\sec(c+dx)}}{\left(\sqrt[3]{2}-(1+\right.}}}\right.}{d\sqrt{1-\sec(c+dx)}}$$

input
```
Int[Sec[c + d*x]*(a + a*Sec[c + d*x])^(2/3),x]
```

output
```
-(((a + a*Sec[c + d*x])^(2/3)*((-3*Sqrt[1 - Sec[c + d*x]]*(1 + Sec[c + d*x
])^(1/6))/2 + (3^(3/4)*EllipticF[ArcCos[(2^(1/3) - (1 - Sqrt[3])*(1 + Sec[
c + d*x])^(1/3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))], (2 +
 Sqrt[3])/4]*(1 + Sec[c + d*x])^(1/6)*(2^(1/3) - (1 + Sec[c + d*x])^(1/3))
*Sqrt[(2^(2/3) + 2^(1/3)*(1 + Sec[c + d*x])^(1/3) + (1 + Sec[c + d*x])^(2/
3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2])/(2*2^(1/3)*Sqrt
[1 - Sec[c + d*x]]*Sqrt[-(((1 + Sec[c + d*x])^(1/3)*(2^(1/3) - (1 + Sec[c
+ d*x])^(1/3)))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2)]))*T
an[c + d*x])/(d*Sqrt[1 - Sec[c + d*x]]*(1 + Sec[c + d*x])^(7/6)))
```

3.145. $\int \sec(c+dx)(a+a\sec(c+dx))^{2/3}\,dx$

3.145.3.1 Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 766
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4314
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Simp[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x
]]*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(d*x)^(n - 1)*((a + b*x)^(m - 1/2
)/Sqrt[a - b*x]), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n},
x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

rule 4315
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Simp[a^IntPart[m]*((a + b*Csc[e + f*x])^FracPart[m
]/(1 + (b/a)*Csc[e + f*x])^FracPart[m])   Int[(1 + (b/a)*Csc[e + f*x])^m*(d
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^
2, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

3.145. $\int \sec(c+dx)(a+a\sec(c+dx))^{2/3}\,dx$

3.145.4 Maple [F]

$$\int \sec(dx+c)\,(a+a\sec(dx+c))^{\frac{2}{3}}\,dx$$

input
```
int(sec(d*x+c)*(a+a*sec(d*x+c))^(2/3),x)
```

output
```
int(sec(d*x+c)*(a+a*sec(d*x+c))^(2/3),x)
```

3.145.5 Fricas [F]

$$\int \sec(c+dx)(a+a\sec(c+dx))^{2/3}\,dx = \int (a\sec(dx+c)+a)^{\frac{2}{3}}\sec(dx+c)\,dx$$

input
```
integrate(sec(d*x+c)*(a+a*sec(d*x+c))^(2/3),x, algorithm="fricas")
```

output
```
integral((a*sec(d*x + c) + a)^(2/3)*sec(d*x + c), x)
```

3.145.6 Sympy [F]

$$\int \sec(c+dx)(a+a\sec(c+dx))^{2/3}\,dx = \int (a(\sec(c+dx)+1))^{\frac{2}{3}}\sec(c+dx)\,dx$$

input
```
integrate(sec(d*x+c)*(a+a*sec(d*x+c))**(2/3),x)
```

output
```
Integral((a*(sec(c + d*x) + 1))**(2/3)*sec(c + d*x), x)
```

3.145.7 Maxima [F]

$$\int \sec(c+dx)(a+a\sec(c+dx))^{2/3}\,dx = \int (a\sec(dx+c)+a)^{\frac{2}{3}}\sec(dx+c)\,dx$$

input
```
integrate(sec(d*x+c)*(a+a*sec(d*x+c))^(2/3),x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)^(2/3)*sec(d*x + c), x)
```

3.145.8 Giac [F]

$$\int \sec(c+dx)(a+a\sec(c+dx))^{2/3}\,dx = \int (a\sec(dx+c)+a)^{\frac{2}{3}}\sec(dx+c)\,dx$$

input
```
integrate(sec(d*x+c)*(a+a*sec(d*x+c))^(2/3),x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)^(2/3)*sec(d*x + c), x)
```

3.145.9 Mupad [F(-1)]

Timed out.

$$\int \sec(c+dx)(a+a\sec(c+dx))^{2/3}\,dx = \int \frac{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{2/3}}{\cos(c+d\,x)}\,dx$$

input
```
int((a + a/cos(c + d*x))^(2/3)/cos(c + d*x),x)
```

output
```
int((a + a/cos(c + d*x))^(2/3)/cos(c + d*x), x)
```

3.146 $\int (a+a\sec(c+dx))^{2/3}\,dx$

3.146.1 Optimal result 1255
3.146.2 Mathematica [B] (warning: unable to verify) 1255
3.146.3 Rubi [A] (verified) 1256
3.146.4 Maple [F] 1258
3.146.5 Fricas [F(-1)] 1259
3.146.6 Sympy [F] 1259
3.146.7 Maxima [F] 1259
3.146.8 Giac [F] 1260
3.146.9 Mupad [F(-1)] 1260

3.146.1 Optimal result

Integrand size = 14, antiderivative size = 77

$$\int (a+a\sec(c+dx))^{2/3}\,dx=\frac{3\sqrt{2}\,\text{AppellF1}\left(\frac{7}{6},\frac{1}{2},1,\frac{13}{6},\frac{1}{2}(1+\sec(c+dx)),1+\sec(c+dx)\right)(a+a\sec(c+dx))^{2/3}\tan(c+$$

output
```
3/7*AppellF1(7/6,1,1/2,13/6,1+sec(d*x+c),1/2+1/2*sec(d*x+c))*(a+a*sec(d*x+
c))^(2/3)*2^(1/2)*tan(d*x+c)/d/(1-sec(d*x+c))^(1/2)
```

3.146.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 691 vs. 2(77) = 154.

Time = 5.21 (sec) , antiderivative size = 691, normalized size of antiderivative = 8.97

$$\int (a+a\sec(c+dx))^{2/3}\,dx=\frac{}{ad\left(40\left(3\,\text{AppellF1}\left(\frac{3}{2},\frac{2}{3},2,\frac{5}{2},\tan^2\left(\frac{1}{2}(c+dx)\right),-\tan^2\left(\frac{1}{2}(c+dx)\right)\right)-2\,\text{AppellF1}\left(\frac{3}{2},\frac{5}{3},1,\frac{5}{2},\right.\right.\right.}$$

input
```
Integrate[(a + a*Sec[c + d*x])^(2/3),x]
```

output
```
(45*AppellF1[1/2, 2/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(a
*(1 + Sec[c + d*x]))^(5/3)*Sin[c + d*x]*(9*AppellF1[1/2, 2/3, 1, 3/2, Tan[
(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + 2*(-3*AppellF1[3/2, 2/3, 2, 5/2, Ta
n[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + 2*AppellF1[3/2, 5/3, 1, 5/2, Tan[
(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Tan[(c + d*x)/2]^2))/(a*d*(40*(3*App
ellF1[3/2, 2/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - 2*Appel
lF1[3/2, 5/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])^2*Sec[c +
d*x]*Sin[(c + d*x)/2]^2*Tan[(c + d*x)/2]^2 + 6*AppellF1[1/2, 2/3, 1, 3/2,
Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[c + d*x]^2*Sin[(c + d*x)/2]^2
*(15*AppellF1[3/2, 2/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(
-7 + 16*Cos[c + d*x] - 3*Cos[2*(c + d*x)]) + 10*AppellF1[3/2, 5/3, 1, 5/2,
 Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(7 - 16*Cos[c + d*x] + 3*Cos[2*(
c + d*x)]) - 24*(9*AppellF1[5/2, 2/3, 3, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c
+ d*x)/2]^2] - 6*AppellF1[5/2, 5/3, 2, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c +
d*x)/2]^2] + 5*AppellF1[5/2, 8/3, 1, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*
x)/2]^2])*Cos[c + d*x]*Tan[(c + d*x)/2]^2) + 135*AppellF1[1/2, 2/3, 1, 3/2
, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]^2*(3 + 3*Cos[c + d*x] + 2*Tan[c
 + d*x]^2)))
```

3.146.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 4266, 3042, 4265, 149, 25, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sec(c+dx)+a)^{2/3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{2/3}dx$$

$$\Big\downarrow \text{4266}$$

$$\frac{(a\sec(c+dx)+a)^{2/3}\int(\sec(c+dx)+1)^{2/3}dx}{(\sec(c+dx)+1)^{2/3}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(a\sec(c+dx)+a)^{2/3}\int\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)^{2/3}dx}{(\sec(c+dx)+1)^{2/3}}$$

$$\Big\downarrow \text{4265}$$

$$-\frac{\tan(c+dx)(a\sec(c+dx)+a)^{2/3}\int \frac{\cos(c+dx)\sqrt[6]{\sec(c+dx)+1}}{\sqrt{1-\sec(c+dx)}}d\sec(c+dx)}{d\sqrt{1-\sec(c+dx)}(\sec(c+dx)+1)^{7/6}}$$

$$\Big\downarrow \text{149}$$

$$-\frac{6\tan(c+dx)(a\sec(c+dx)+a)^{2/3}\int \frac{\cos(c+dx)(\sec(c+dx)+1)}{\sqrt{1-\sec(c+dx)}}d\sqrt[6]{\sec(c+dx)+1}}{d\sqrt{1-\sec(c+dx)}(\sec(c+dx)+1)^{7/6}}$$

$$\Big\downarrow \text{25}$$

$$\frac{6\tan(c+dx)(a\sec(c+dx)+a)^{2/3}\int -\frac{\cos(c+dx)(\sec(c+dx)+1)}{\sqrt{1-\sec(c+dx)}}d\sqrt[6]{\sec(c+dx)+1}}{d\sqrt{1-\sec(c+dx)}(\sec(c+dx)+1)^{7/6}}$$

$$\Big\downarrow \text{1012}$$

$$\frac{3\sqrt{2}\tan(c+dx)(a\sec(c+dx)+a)^{2/3}\,\text{AppellF1}\left(\frac{7}{6},1,\frac{1}{2},\frac{13}{6},\sec(c+dx)+1,\frac{1}{2}(\sec(c+dx)+1)\right)}{7d\sqrt{1-\sec(c+dx)}}$$

input
```
Int[(a + a*Sec[c + d*x])^(2/3),x]
```

output
```
(3*Sqrt[2]*AppellF1[7/6, 1, 1/2, 13/6, 1 + Sec[c + d*x], (1 + Sec[c + d*x]
)/2]*(a + a*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(7*d*Sqrt[1 - Sec[c + d*x]])
```

3.146.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 149
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_)
)^(p_.), x_] :> With[{k = Denominator[m]}, Simp[k/b   Subst[Int[x^(k*(m + 1
) - 1)*(c - a*(d/b) + d*(x^k/b))^n*(e - a*(f/b) + f*(x^k/b))^p, x], x, (a +
 b*x)^(1/k)], x]] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && FractionQ[m] &&
IntegerQ[2*n] && IntegerQ[p]
```

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4265
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[a^n*(Cot
[c + d*x]/(d*Sqrt[1 + Csc[c + d*x]]*Sqrt[1 - Csc[c + d*x]]))   Subst[Int[(1
 + b*(x/a))^(n - 1/2)/(x*Sqrt[1 - b*(x/a)]), x], x, Csc[c + d*x]], x] /; Fr
eeQ[{a, b, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0
]
```

rule 4266
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[a^IntPar
t[n]*((a + b*Csc[c + d*x])^FracPart[n]/(1 + (b/a)*Csc[c + d*x])^FracPart[n]
)   Int[(1 + (b/a)*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] &&
EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

3.146.4 Maple [F]

$$\int (a + a\sec(dx + c))^{\frac{2}{3}}\,dx$$

input
```
int((a+a*sec(d*x+c))^(2/3),x)
```

output
```
int((a+a*sec(d*x+c))^(2/3),x)
```

3.146.5 Fricas [F(-1)]

Timed out.

$$\int (a + a\sec(c + dx))^{2/3}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sec(d*x+c))^(2/3),x, algorithm="fricas")
```

output
```
Timed out
```

3.146.6 Sympy [F]

$$\int (a + a\sec(c + dx))^{2/3}\,dx = \int (a\sec(c + dx) + a)^{\frac{2}{3}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))**(2/3),x)
```

output
```
Integral((a*sec(c + d*x) + a)**(2/3), x)
```

3.146.7 Maxima [F]

$$\int (a + a\sec(c + dx))^{2/3}\,dx = \int (a\sec(dx + c) + a)^{\frac{2}{3}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))^(2/3),x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)^(2/3), x)
```

3.146.8 Giac [F]

$$\int (a + a\sec(c + dx))^{2/3}\,dx = \int (a\sec(dx + c) + a)^{\frac{2}{3}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))^(2/3),x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)^(2/3), x)
```

3.146.9 Mupad [F(-1)]

Timed out.

$$\int (a + a\sec(c + dx))^{2/3}\,dx = \int \left(a + \frac{a}{\cos(c + d\,x)}\right)^{2/3}\,dx$$

input
```
int((a + a/cos(c + d*x))^(2/3),x)
```

output
```
int((a + a/cos(c + d*x))^(2/3), x)
```

3.147 $\int \cos(c+dx)(a+a\sec(c+dx))^{2/3}\,dx$

3.147.1 Optimal result 1261
3.147.2 Mathematica [B] (warning: unable to verify) 1261
3.147.3 Rubi [A] (verified) 1262
3.147.4 Maple [F] 1264
3.147.5 Fricas [F(-1)] 1264
3.147.6 Sympy [F] 1265
3.147.7 Maxima [F] 1265
3.147.8 Giac [F] 1265
3.147.9 Mupad [F(-1)] 1266

3.147.1 Optimal result

Integrand size = 21, antiderivative size = 77

$$\int \cos(c+dx)(a+a\sec(c+dx))^{2/3}\,dx = -\frac{3\sqrt{2}\,\text{AppellF1}\left(\frac{7}{6},\frac{1}{2},2,\frac{13}{6},\frac{1}{2}(1+\sec(c+dx)),1+\sec(c+dx)\right)(a+a\sec(c+dx))^{2/3}\tan(c+dx)}{7d\sqrt{1-\sec(c+dx)}}$$

output
```
-3/7*AppellF1(7/6,2,1/2,13/6,1+sec(d*x+c),1/2+1/2*sec(d*x+c))*(a+a*sec(d*x
+c))^(2/3)*2^(1/2)*tan(d*x+c)/d/(1-sec(d*x+c))^(1/2)
```

3.147.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 2700 vs. 2(77) = 154.

Time = 16.05 (sec) , antiderivative size = 2700, normalized size of antiderivative = 35.06

$$\int \cos(c+dx)(a+a\sec(c+dx))^{2/3}\,dx = \text{Result too large to show}$$

input
```
Integrate[Cos[c + d*x]*(a + a*Sec[c + d*x])^(2/3),x]
```

output
```
(((1 + Cos[c + d*x])*Sec[c + d*x])^(2/3)*(a*(1 + Sec[c + d*x]))^(2/3)*(Sin
[c + d*x] - Tan[(c + d*x)/2]))/(d*(1 + Sec[c + d*x])^(2/3)) - (2^(2/3)*(Co
s[(c + d*x)/2]^2*Sec[c + d*x])^(2/3)*(a*(1 + Sec[c + d*x]))^(2/3)*((Sec[(c
 + d*x)/2]^2*(1 + Sec[c + d*x])^(2/3))/6 + (Cos[c + d*x]*Sec[(c + d*x)/2]^
2*(1 + Sec[c + d*x])^(2/3))/3)*Tan[(c + d*x)/2]*(AppellF1[3/2, 2/3, 1, 5/2
, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(Cos[c + d*x]*Sec[(c + d*x)/2]^
2)^(2/3)*Tan[(c + d*x)/2]^2 + (81*AppellF1[1/2, 2/3, 1, 3/2, Tan[(c + d*x)
/2]^2, -Tan[(c + d*x)/2]^2]*Cos[(c + d*x)/2]^2)/(-9*AppellF1[1/2, 2/3, 1,
3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + 2*(3*AppellF1[3/2, 2/3, 2,
 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - 2*AppellF1[3/2, 5/3, 1, 5
/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Tan[(c + d*x)/2]^2)))/(9*d*(
1 + Sec[c + d*x])^(2/3)*(-1/9*(Sec[(c + d*x)/2]^2*(Cos[(c + d*x)/2]^2*Sec[
c + d*x])^(2/3)*(AppellF1[3/2, 2/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c +
d*x)/2]^2]*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(2/3)*Tan[(c + d*x)/2]^2 + (8
1*AppellF1[1/2, 2/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Cos[
(c + d*x)/2]^2)/(-9*AppellF1[1/2, 2/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c
 + d*x)/2]^2] + 2*(3*AppellF1[3/2, 2/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(
c + d*x)/2]^2] - 2*AppellF1[3/2, 5/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c
+ d*x)/2]^2])*Tan[(c + d*x)/2]^2)))/2^(1/3) - (2^(2/3)*(Cos[(c + d*x)/2]^2
*Sec[c + d*x])^(2/3)*Tan[(c + d*x)/2]*(AppellF1[3/2, 2/3, 1, 5/2, Tan[(...
```

3.147.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 4315, 3042, 4314, 149, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(c+dx)(a\sec(c+dx)+a)^{2/3}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{2/3}}{\csc\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \; 4315$$

$$\frac{(a\sec(c+dx)+a)^{2/3}\int \cos(c+dx)(\sec(c+dx)+1)^{2/3}dx}{(\sec(c+dx)+1)^{2/3}}$$

$$\Big\downarrow \; 3042$$

$$\frac{(a\sec(c+dx)+a)^{2/3}\int \frac{\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)^{2/3}}{\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{(\sec(c+dx)+1)^{2/3}}$$

$$\Big\downarrow \; 4314$$

$$-\frac{\tan(c+dx)(a\sec(c+dx)+a)^{2/3}\int \frac{\cos^2(c+dx)\sqrt[6]{\sec(c+dx)+1}}{\sqrt{1-\sec(c+dx)}}d\sec(c+dx)}{d\sqrt{1-\sec(c+dx)}(\sec(c+dx)+1)^{7/6}}$$

$$\Big\downarrow \; 149$$

$$-\frac{6\tan(c+dx)(a\sec(c+dx)+a)^{2/3}\int \frac{\cos^2(c+dx)(\sec(c+dx)+1)}{\sqrt{1-\sec(c+dx)}}d\sqrt[6]{\sec(c+dx)+1}}{d\sqrt{1-\sec(c+dx)}(\sec(c+dx)+1)^{7/6}}$$

$$\Big\downarrow \; 1012$$

$$-\frac{3\sqrt{2}\tan(c+dx)(a\sec(c+dx)+a)^{2/3}\operatorname{AppellF1}\left(\frac{7}{6},2,\frac{1}{2},\frac{13}{6},\sec(c+dx)+1,\frac{1}{2}(\sec(c+dx)+1)\right)}{7d\sqrt{1-\sec(c+dx)}}$$

input
```
Int[Cos[c + d*x]*(a + a*Sec[c + d*x])^(2/3),x]
```

output
```
(-3*Sqrt[2]*AppellF1[7/6, 2, 1/2, 13/6, 1 + Sec[c + d*x], (1 + Sec[c + d*x
])/2]*(a + a*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(7*d*Sqrt[1 - Sec[c + d*x]]
)
```

3.147.3.1 Defintions of rubi rules used

rule 149
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_)
)^(p_.), x_] :> With[{k = Denominator[m]}, Simp[k/b   Subst[Int[x^(k*(m + 1
) - 1)*(c - a*(d/b) + d*(x^k/b))^n*(e - a*(f/b) + f*(x^k/b))^p, x], x, (a +
 b*x)^(1/k)], x]] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && FractionQ[m] &&
IntegerQ[2*n] && IntegerQ[p]
```

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4314
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Simp[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x
]]*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(d*x)^(n - 1)*((a + b*x)^(m - 1/2
)/Sqrt[a - b*x]), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n},
x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

rule 4315
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Simp[a^IntPart[m]*((a + b*Csc[e + f*x])^FracPart[m
]/(1 + (b/a)*Csc[e + f*x])^FracPart[m])   Int[(1 + (b/a)*Csc[e + f*x])^m*(d
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^
2, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

3.147.4 Maple [F]

$$\int \cos\left(dx+c\right)\left(a+a\sec\left(dx+c\right)\right)^{\frac{2}{3}}dx$$

input
```
int(cos(d*x+c)*(a+a*sec(d*x+c))^(2/3),x)
```

output
```
int(cos(d*x+c)*(a+a*sec(d*x+c))^(2/3),x)
```

3.147.5 Fricas [F(-1)]

Timed out.

$$\int \cos(c+dx)(a+a\sec(c+dx))^{2/3}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)*(a+a*sec(d*x+c))^(2/3),x, algorithm="fricas")
```

output
```
Timed out
```

3.147.6 Sympy [F]

$$\int \cos(c+dx)(a+a\sec(c+dx))^{2/3}\,dx = \int (a(\sec(c+dx)+1))^{\frac{2}{3}}\cos(c+dx)\,dx$$

input
```
integrate(cos(d*x+c)*(a+a*sec(d*x+c))**(2/3),x)
```

output
```
Integral((a*(sec(c + d*x) + 1))**(2/3)*cos(c + d*x), x)
```

3.147.7 Maxima [F]

$$\int \cos(c+dx)(a+a\sec(c+dx))^{2/3}\,dx = \int (a\sec(dx+c)+a)^{\frac{2}{3}}\cos(dx+c)\,dx$$

input
```
integrate(cos(d*x+c)*(a+a*sec(d*x+c))^(2/3),x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)^(2/3)*cos(d*x + c), x)
```

3.147.8 Giac [F]

$$\int \cos(c+dx)(a+a\sec(c+dx))^{2/3}\,dx = \int (a\sec(dx+c)+a)^{\frac{2}{3}}\cos(dx+c)\,dx$$

input
```
integrate(cos(d*x+c)*(a+a*sec(d*x+c))^(2/3),x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)^(2/3)*cos(d*x + c), x)
```

3.147.9 Mupad [F(-1)]

Timed out.

$$\int \cos(c+dx)(a+a\sec(c+dx))^{2/3}\,dx = \int \cos\left(c+d\,x\right)\,\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)^{2/3}\,dx$$

input
```
int(cos(c + d*x)*(a + a/cos(c + d*x))^(2/3),x)
```

output
```
int(cos(c + d*x)*(a + a/cos(c + d*x))^(2/3), x)
```

3.148 $\int \sec^3(c+dx)(a+a\sec(c+dx))^{5/3}\,dx$

3.148.1 Optimal result 1267
3.148.2 Mathematica [C] (verified) 1268
3.148.3 Rubi [A] (verified) 1268
3.148.4 Maple [F] 1273
3.148.5 Fricas [F] 1273
3.148.6 Sympy [F(-1)] 1273
3.148.7 Maxima [F] 1274
3.148.8 Giac [F] 1274
3.148.9 Mupad [F(-1)] 1274

3.148.1 Optimal result

Integrand size = 23, antiderivative size = 413

$$\int \sec^3(c+dx)(a+a\sec(c+dx))^{5/3}\,dx = \frac{147a(a+a\sec(c+dx))^{2/3}\tan(c+dx)}{440d} + \frac{1029a(a+a\sec(c+dx))^{2/3}\tan(c+dx)}{880d(1+\sec(c+dx))} - \frac{9(a+a\sec(c+dx))^{5/3}\tan(c+dx)}{88d} + \frac{3(a+a\sec(c+dx))^{8/3}\tan(c+dx)}{11ad} - \frac{343\ 3^{3/4}a\,\text{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{2}-(1-\sqrt{3})\sqrt[3]{1+\sec(c+dx)}}{\sqrt[3]{2}-(1+\sqrt{3})\sqrt[3]{1+\sec(c+dx)}}\right),\frac{1}{4}(2+\sqrt{3})\right)(a+a\sec(c+dx))^{2/3}\left(\sqrt[3]{2}-\right.}{880\sqrt[3]{2}d(1-\sec(c+dx))(1+\sec(c+dx))\sqrt{-\frac{\sqrt[3]{1+\sec(c}}{\left(\sqrt[3]{2}-(\right.}}}$$

output
```
147/440*a*(a+a*sec(d*x+c))^(2/3)*tan(d*x+c)/d+1029/880*a*(a+a*sec(d*x+c))^
(2/3)*tan(d*x+c)/d/(1+sec(d*x+c))-9/88*(a+a*sec(d*x+c))^(5/3)*tan(d*x+c)/d
+3/11*(a+a*sec(d*x+c))^(8/3)*tan(d*x+c)/a/d-343/1760*3^(3/4)*a*((2^(1/3)-(
1+sec(d*x+c))^(1/3)*(1-3^(1/2)))^2/(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1+3^(1/2
)))^2)^(1/2)/(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1-3^(1/2)))*(2^(1/3)-(1+sec(d*
x+c))^(1/3)*(1+3^(1/2)))*EllipticF((1-(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1-3^(
1/2)))^2/(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1
/4*2^(1/2))*(a+a*sec(d*x+c))^(2/3)*(2^(1/3)-(1+sec(d*x+c))^(1/3))*((2^(2/3
)+2^(1/3)*(1+sec(d*x+c))^(1/3)+(1+sec(d*x+c))^(2/3))/(2^(1/3)-(1+sec(d*x+c
))^(1/3)*(1+3^(1/2)))^2)^(1/2)*tan(d*x+c)*2^(2/3)/d/(1-sec(d*x+c))/(1+sec(
d*x+c))/(-(1+sec(d*x+c))^(1/3)*(2^(1/3)-(1+sec(d*x+c))^(1/3))/(2^(1/3)-(1+
sec(d*x+c))^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

3.148.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.37 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.23

$$\int \sec^3(c+dx)(a+a\sec(c+dx))^{5/3}\,dx = \frac{a(a(1+\sec(c+dx)))^{2/3}\left(196\sqrt[6]{2}\,\text{Hypergeometric2F1}\left(-\frac{7}{6},\frac{1}{2},\frac{3}{2},\frac{1}{2}(1-\sec(c+dx))\right)+3(1+\right.}{88d(1+\sec(c+dx))^{7/6}}$$

input
```
Integrate[Sec[c + d*x]^3*(a + a*Sec[c + d*x])^(5/3),x]
```

output
```
(a*(a*(1 + Sec[c + d*x]))^(2/3)*(196*2^(1/6)*Hypergeometric2F1[-7/6, 1/2,
3/2, (1 - Sec[c + d*x])/2] + 3*(1 + Sec[c + d*x])^(13/6)*(5 + 8*Sec[c + d*
x]))*Tan[c + d*x])/(88*d*(1 + Sec[c + d*x])^(7/6))
```

3.148.3 Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 458, normalized size of antiderivative = 1.11, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.565$, Rules used = {3042, 4287, 27, 3042, 4489, 3042, 4315, 3042, 4314, 60, 60, 73, 766}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^3(c+dx)(a\sec(c+dx)+a)^{5/3}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^3\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/3}dx$$

$$\Big\downarrow \; 4287$$

$$\frac{3\int \frac{1}{3}\sec(c+dx)(8a-3a\sec(c+dx))(\sec(c+dx)a+a)^{5/3}dx}{11a}+\frac{3\tan(c+dx)(a\sec(c+dx)+a)^{8/3}}{11ad}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \sec(c+dx)(8a-3a\sec(c+dx))(\sec(c+dx)a+a)^{5/3}dx}{11a}+\frac{3\tan(c+dx)(a\sec(c+dx)+a)^{8/3}}{11ad}$$

$$\Big\downarrow \; 3042$$

$$\frac{\int \csc\left(c+dx+\frac{\pi}{2}\right)\left(8a-3a\csc\left(c+dx+\frac{\pi}{2}\right)\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{5/3}dx}{11a}+\frac{3\tan(c+dx)(a\sec(c+dx)+a)^{8/3}}{11ad}$$

$$\Big\downarrow \; 4489$$

$$\frac{\frac{49}{8}a\int \sec(c+dx)(\sec(c+dx)a+a)^{5/3}dx-\frac{9a\tan(c+dx)(a\sec(c+dx)+a)^{5/3}}{8d}}{11a}+\frac{3\tan(c+dx)(a\sec(c+dx)+a)^{8/3}}{11ad}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{49}{8}a\int \csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{5/3}dx-\frac{9a\tan(c+dx)(a\sec(c+dx)+a)^{5/3}}{8d}}{11a}+\frac{3\tan(c+dx)(a\sec(c+dx)+a)^{8/3}}{11ad}$$

$$\Big\downarrow \; 4315$$

$$\frac{\frac{49a^2(a\sec(c+dx)+a)^{2/3}\int \sec(c+dx)(\sec(c+dx)+1)^{5/3}dx}{8(\sec(c+dx)+1)^{2/3}}-\frac{9a\tan(c+dx)(a\sec(c+dx)+a)^{5/3}}{8d}}{11a}+\frac{3\tan(c+dx)(a\sec(c+dx)+a)^{8/3}}{11ad}$$

$$\Big\downarrow \; 3042$$

3.148. $\int \sec^3(c+dx)(a+a\sec(c+dx))^{5/3}\,dx$

$$\frac{\frac{49a^2(a\sec(c+dx)+a)^{2/3}\int \csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)^{5/3}dx}{8(\sec(c+dx)+1)^{2/3}}-\frac{9a\tan(c+dx)(a\sec(c+dx)+a)^{5/3}}{8d}}{11a}+\frac{3\tan(c+dx)(a\sec(c+dx)+a)^{8/3}}{11ad}$$

$\downarrow$ 4314

$$\frac{-\frac{49a^2\tan(c+dx)(a\sec(c+dx)+a)^{2/3}\int \frac{(\sec(c+dx)+1)^{7/6}}{\sqrt{1-\sec(c+dx)}}d\sec(c+dx)}{8d\sqrt{1-\sec(c+dx)}(\sec(c+dx)+1)^{7/6}}-\frac{9a\tan(c+dx)(a\sec(c+dx)+a)^{5/3}}{8d}}{11a}+\frac{3\tan(c+dx)(a\sec(c+dx)+a)^{8/3}}{11ad}$$

$\downarrow$ 60

$$\frac{-\frac{49a^2\tan(c+dx)(a\sec(c+dx)+a)^{2/3}\left(\frac{7}{5}\int \frac{\sqrt[6]{\sec(c+dx)+1}}{\sqrt{1-\sec(c+dx)}}d\sec(c+dx)-\frac{3}{5}\sqrt{1-\sec(c+dx)}(\sec(c+dx)+1)^{7/6}\right)}{8d\sqrt{1-\sec(c+dx)}(\sec(c+dx)+1)^{7/6}}-\frac{9a\tan(c+dx)(a\sec(c+}{8d}}{11a}+\frac{3\tan(c+dx)(a\sec(c+dx)+a)^{8/3}}{11ad}$$

$\downarrow$ 60

$$\frac{-\frac{49a^2\tan(c+dx)(a\sec(c+dx)+a)^{2/3}\left(\frac{7}{5}\left(\frac{1}{2}\int \frac{1}{\sqrt{1-\sec(c+dx)}(\sec(c+dx)+1)^{5/6}}d\sec(c+dx)-\frac{3}{2}\sqrt{1-\sec(c+dx)}\sqrt[6]{\sec(c+dx)+1}\right)-\frac{3}{5}\sqrt{1-\sec}\right.}{8d\sqrt{1-\sec(c+dx)}(\sec(c+dx)+1)^{7/6}}}{11a}+\frac{3\tan(c+dx)(a\sec(c+dx)+a)^{8/3}}{11ad}$$

$\downarrow$ 73

$$\frac{-\frac{49a^2\tan(c+dx)(a\sec(c+dx)+a)^{2/3}\left(\frac{7}{5}\left(3\int \frac{1}{\sqrt{1-\sec(c+dx)}}d\sqrt[6]{\sec(c+dx)+1}-\frac{3}{2}\sqrt{1-\sec(c+dx)}\sqrt[6]{\sec(c+dx)+1}\right)-\frac{3}{5}\sqrt{1-\sec(c+}\right.}{8d\sqrt{1-\sec(c+dx)}(\sec(c+dx)+1)^{7/6}}}{11a}+\frac{3\tan(c+dx)(a\sec(c+dx)+a)^{8/3}}{11ad}$$

$\downarrow$ 766

3.148. $\int \sec^3(c+dx)(a+a\sec(c+dx))^{5/3}\,dx$

$$-\frac{49a^2\tan(c+dx)(a\sec(c+dx)+a)^{2/3}\left(\frac{7}{5}\left(\frac{3^{3/4}\sqrt[6]{\sec(c+dx)+1}\left(\sqrt[3]{2}-\sqrt[3]{\sec(c+dx)+1}\right)\sqrt{\frac{(\sec(c+dx)+1)^{2/3}+\sqrt[3]{2}\sqrt[3]{\sec(c+dx}}{\left(\sqrt[3]{2}-(1+\sqrt{3})\sqrt[3]{\sec(c+dx)}\right.}}}{2\sqrt[3]{2}\sqrt{1-\sec(c+dx)}}\sqrt{-\frac{\sqrt[3]{\sec(c+dx)+1}\left(\sqrt[3]{\right.}}{\left(\sqrt[3]{2}-(1+\sqrt{3})\sqrt[3]{\mathrm{s}}\right.}}\right.\right.}{8d\sqrt{1-}}$$

$$\frac{3\tan(c+dx)(a\sec(c+dx)+a)^{8/3}}{11ad}$$

input
```
Int[Sec[c + d*x]^3*(a + a*Sec[c + d*x])^(5/3),x]
```

output
```
(3*(a + a*Sec[c + d*x])^(8/3)*Tan[c + d*x])/(11*a*d) + ((-9*a*(a + a*Sec[c
 + d*x])^(5/3)*Tan[c + d*x])/(8*d) - (49*a^2*(a + a*Sec[c + d*x])^(2/3)*((
-3*Sqrt[1 - Sec[c + d*x]]*(1 + Sec[c + d*x])^(7/6))/5 + (7*((-3*Sqrt[1 - S
ec[c + d*x]]*(1 + Sec[c + d*x])^(1/6))/2 + (3^(3/4)*EllipticF[ArcCos[(2^(1
/3) - (1 - Sqrt[3])*(1 + Sec[c + d*x])^(1/3))/(2^(1/3) - (1 + Sqrt[3])*(1
+ Sec[c + d*x])^(1/3))], (2 + Sqrt[3])/4]*(1 + Sec[c + d*x])^(1/6)*(2^(1/3
) - (1 + Sec[c + d*x])^(1/3))*Sqrt[(2^(2/3) + 2^(1/3)*(1 + Sec[c + d*x])^(
1/3) + (1 + Sec[c + d*x])^(2/3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x
])^(1/3))^2])/(2*2^(1/3)*Sqrt[1 - Sec[c + d*x]]*Sqrt[-(((1 + Sec[c + d*x])
^(1/3)*(2^(1/3) - (1 + Sec[c + d*x])^(1/3)))/(2^(1/3) - (1 + Sqrt[3])*(1 +
 Sec[c + d*x])^(1/3))^2)])))/5)*Tan[c + d*x])/(8*d*Sqrt[1 - Sec[c + d*x]]*
(1 + Sec[c + d*x])^(7/6)))/(11*a)
```

3.148.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 766
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4287
```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(-Cot[e + f*x])*((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2
))), x] + Simp[1/(b*(m + 2))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(b*(
m + 1) - a*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 -
b^2, 0] &&  !LtQ[m, -2^(-1)]
```

rule 4314
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Simp[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x
]]*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(d*x)^(n - 1)*((a + b*x)^(m - 1/2
)/Sqrt[a - b*x]), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n},
x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

rule 4315
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Simp[a^IntPart[m]*((a + b*Csc[e + f*x])^FracPart[m
]/(1 + (b/a)*Csc[e + f*x])^FracPart[m])   Int[(1 + (b/a)*Csc[e + f*x])^m*(d
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^
2, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

3.148. $\int \sec^3(c+dx)(a+a\sec(c+dx))^{5/3}\,dx$

rule 4489
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*((
a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Simp[(a*B*m + A*b*(m + 1))/(b*(m +
 1))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B
, e, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b
*(m + 1), 0] &&  !LtQ[m, -2^(-1)]
```

3.148.4 Maple [F]

$$\int \sec(dx+c)^3 (a+a\sec(dx+c))^{\frac{5}{3}}\,dx$$

input
```
int(sec(d*x+c)^3*(a+a*sec(d*x+c))^(5/3),x)
```

output
```
int(sec(d*x+c)^3*(a+a*sec(d*x+c))^(5/3),x)
```

3.148.5 Fricas [F]

$$\int \sec^3(c+dx)(a+a\sec(c+dx))^{5/3}\,dx = \int (a\sec(dx+c)+a)^{\frac{5}{3}}\sec(dx+c)^3\,dx$$

input
```
integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^(5/3),x, algorithm="fricas")
```

output
```
integral((a*sec(d*x + c)^4 + a*sec(d*x + c)^3)*(a*sec(d*x + c) + a)^(2/3),
 x)
```

3.148.6 Sympy [F(-1)]

Timed out.

$$\int \sec^3(c+dx)(a+a\sec(c+dx))^{5/3}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**3*(a+a*sec(d*x+c))**(5/3),x)
```

output
```
Timed out
```

3.148.7 Maxima [F]

$$\int \sec^3(c+dx)(a+a\sec(c+dx))^{5/3}\,dx = \int (a\sec(dx+c)+a)^{\frac{5}{3}}\sec(dx+c)^3\,dx$$

input
```
integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^(5/3),x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)^(5/3)*sec(d*x + c)^3, x)
```

3.148.8 Giac [F]

$$\int \sec^3(c+dx)(a+a\sec(c+dx))^{5/3}\,dx = \int (a\sec(dx+c)+a)^{\frac{5}{3}}\sec(dx+c)^3\,dx$$

input
```
integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^(5/3),x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)^(5/3)*sec(d*x + c)^3, x)
```

3.148.9 Mupad [F(-1)]

Timed out.

$$\int \sec^3(c+dx)(a+a\sec(c+dx))^{5/3}\,dx = \int \frac{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{5/3}}{\cos(c+d\,x)^3}\,dx$$

input
```
int((a + a/cos(c + d*x))^(5/3)/cos(c + d*x)^3,x)
```

output
```
int((a + a/cos(c + d*x))^(5/3)/cos(c + d*x)^3, x)
```

3.149 $\int \sec^2(c+dx)(a+a\sec(c+dx))^{5/3}\,dx$

3.149.1 Optimal result . . . 1275
3.149.2 Mathematica [C] (verified) . . . 1276
3.149.3 Rubi [A] (verified) . . . 1276
3.149.4 Maple [F] . . . 1279
3.149.5 Fricas [F] . . . 1280
3.149.6 Sympy [F] . . . 1280
3.149.7 Maxima [F] . . . 1280
3.149.8 Giac [F] . . . 1281
3.149.9 Mupad [F(-1)] . . . 1281

3.149.1 Optimal result

Integrand size = 23, antiderivative size = 383

$$\int \sec^2(c+dx)(a+a\sec(c+dx))^{5/3}\,dx = \frac{3a(a+a\sec(c+dx))^{2/3}\tan(c+dx)}{8d} + \frac{21a(a+a\sec(c+dx))^{2/3}\tan(c+dx)}{16d(1+\sec(c+dx))} + \frac{3(a+a\sec(c+dx))^{5/3}\tan(c+dx)}{8d} - \frac{7\,3^{3/4}a\,\text{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{2}-\left(1-\sqrt{3}\right)\sqrt[3]{1+\sec(c+dx)}}{\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{1+\sec(c+dx)}}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)(a+a\sec(c+dx))^{2/3}\left(\sqrt[3]{2}-\sqrt[3]{\cdots}\right.}{16\sqrt[3]{2}d(1-\sec(c+dx))(1+\sec(c+dx))\sqrt{-\frac{\sqrt[3]{1+\sec(c+\cdots}}{\left(\sqrt[3]{2}-(1+\cdots\right.}}}$$

output
```
3/8*a*(a+a*sec(d*x+c))^(2/3)*tan(d*x+c)/d+21/16*a*(a+a*sec(d*x+c))^(2/3)*t
an(d*x+c)/d/(1+sec(d*x+c))+3/8*(a+a*sec(d*x+c))^(5/3)*tan(d*x+c)/d-7/32*3^
(3/4)*a*((2^(1/3)-(1+sec(d*x+c))^(1/3)*(1-3^(1/2)))^2/(2^(1/3)-(1+sec(d*x+
c))^(1/3)*(1+3^(1/2)))^2)^(1/2)/(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1-3^(1/2)))
*(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1+3^(1/2)))*EllipticF((1-(2^(1/3)-(1+sec(d
*x+c))^(1/3)*(1-3^(1/2)))^2/(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1+3^(1/2)))^2)^
(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(a+a*sec(d*x+c))^(2/3)*(2^(1/3)-(1+sec(d*x+
c))^(1/3))*((2^(2/3)+2^(1/3)*(1+sec(d*x+c))^(1/3)+(1+sec(d*x+c))^(2/3))/(2
^(1/3)-(1+sec(d*x+c))^(1/3)*(1+3^(1/2)))^2)^(1/2)*tan(d*x+c)*2^(2/3)/d/(1-
sec(d*x+c))/(1+sec(d*x+c))/(-(1+sec(d*x+c))^(1/3)*(2^(1/3)-(1+sec(d*x+c))^
(1/3))/(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

3.149.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.48 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.28

$$\int \sec^2(c+dx)(a+a\sec(c+dx))^{5/3}\,dx = \frac{a(a(1+\sec(c+dx)))^{2/3}\left(5\sqrt[6]{2}\,\text{Hypergeometric2F1}\left(-\frac{7}{6},\frac{1}{2},\frac{3}{2},\frac{1}{2}(1-\sec(c+dx))\right)+3\cos^4\left(\frac{1}{2}\right.\right.}{2d(1+\sec(c+dx))^{7/6}}$$

input
```
Integrate[Sec[c + d*x]^2*(a + a*Sec[c + d*x])^(5/3),x]
```

output
```
(a*(a*(1 + Sec[c + d*x]))^(2/3)*(5*2^(1/6)*Hypergeometric2F1[-7/6, 1/2, 3/
2, (1 - Sec[c + d*x])/2] + 3*Cos[(c + d*x)/2]^4*Sec[c + d*x]^2*(1 + Sec[c
+ d*x])^(1/6))*Tan[c + d*x])/(2*d*(1 + Sec[c + d*x])^(7/6))
```

3.149.3 Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 417, normalized size of antiderivative = 1.09, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {3042, 4285, 3042, 4315, 3042, 4314, 60, 60, 73, 766}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(c+dx)(a\sec(c+dx)+a)^{5/3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^2\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/3}dx$$

$$\Big\downarrow \text{4285}$$

$$\frac{5}{8}\int \sec(c+dx)(\sec(c+dx)a+a)^{5/3}dx+\frac{3\tan(c+dx)(a\sec(c+dx)+a)^{5/3}}{8d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5}{8}\int \csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{5/3}dx+\frac{3\tan(c+dx)(a\sec(c+dx)+a)^{5/3}}{8d}$$

$$\Big\downarrow \text{4315}$$

$$\frac{5a(a\sec(c+dx)+a)^{2/3}\int \sec(c+dx)(\sec(c+dx)+1)^{5/3}dx}{8(\sec(c+dx)+1)^{2/3}}+\frac{3\tan(c+dx)(a\sec(c+dx)+a)^{5/3}}{8d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5a(a\sec(c+dx)+a)^{2/3}\int \csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)^{5/3}dx}{8(\sec(c+dx)+1)^{2/3}}+\frac{3\tan(c+dx)(a\sec(c+dx)+a)^{5/3}}{8d}$$

$$\Big\downarrow \text{4314}$$

$$\frac{3\tan(c+dx)(a\sec(c+dx)+a)^{5/3}}{8d}-\frac{5a\tan(c+dx)(a\sec(c+dx)+a)^{2/3}\int \frac{(\sec(c+dx)+1)^{7/6}}{\sqrt{1-\sec(c+dx)}}d\sec(c+dx)}{8d\sqrt{1-\sec(c+dx)}(\sec(c+dx)+1)^{7/6}}$$

$$\Big\downarrow \text{60}$$

$$\frac{3\tan(c+dx)(a\sec(c+dx)+a)^{5/3}}{8d}-\frac{5a\tan(c+dx)(a\sec(c+dx)+a)^{2/3}\left(\frac{7}{5}\int \frac{\sqrt[6]{\sec(c+dx)+1}}{\sqrt{1-\sec(c+dx)}}d\sec(c+dx)-\frac{3}{5}\sqrt{1-\sec(c+dx)}(\sec(c+dx)+1)^{7}\right.}{8d\sqrt{1-\sec(c+dx)}(\sec(c+dx)+1)^{7/6}}$$

$$\Big\downarrow \text{60}$$

$$\frac{3\tan(c+dx)(a\sec(c+dx)+a)^{5/3}}{8d}-\frac{5a\tan(c+dx)(a\sec(c+dx)+a)^{2/3}\left(\frac{7}{5}\left(\frac{1}{2}\int \frac{1}{\sqrt{1-\sec(c+dx)}(\sec(c+dx)+1)^{5/6}}d\sec(c+dx)-\frac{3}{2}\sqrt{1-\sec(c+dx)}\sqrt[6]{\sec(c\right.\right.}{8d\sqrt{1-\sec(c+dx)}(\sec(c+dx)+1)^{7/6}}$$

$$\Big\downarrow \text{73}$$

$$\frac{3\tan(c+dx)(a\sec(c+dx)+a)^{5/3}}{8d}-\frac{5a\tan(c+dx)(a\sec(c+dx)+a)^{2/3}\left(\frac{7}{5}\left(3\int \frac{1}{\sqrt{1-\sec(c+dx)}}d\sqrt[6]{\sec(c+dx)+1}-\frac{3}{2}\sqrt{1-\sec(c+dx)}\sqrt[6]{\sec(c+dx)+}\right.\right.}{8d\sqrt{1-\sec(c+dx)}(\sec(c+dx)+1)^{7/6}}$$

$$\Big\downarrow \text{766}$$

3.149. $\int \sec^2(c+dx)(a+a\sec(c+dx))^{5/3}\,dx$

$$\frac{3\tan(c+dx)(a\sec(c+dx)+a)^{5/3}}{8d} - 5a\tan(c+dx)(a\sec(c+dx)+a)^{2/3}\left(\frac{7}{5}\left(\frac{3^{3/4}\sqrt[6]{\sec(c+dx)+1}\left(\sqrt[3]{2}-\sqrt[3]{\sec(c+dx)+1}\right)\sqrt{\frac{(\sec(c+dx)+1)^{2/3}+\sqrt[3]{2}\sqrt[3]{\sec}}{\left(\sqrt[3]{2}-(1+\sqrt{3})\sqrt[3]{\sec}\right.}}}{2\sqrt[3]{2}\sqrt{1-\sec(c+dx)}\sqrt{-\frac{\sqrt[3]{\sec(c+}}{\left(\sqrt[3]{2}\right.}}}\right.\right.$$

input
```
Int[Sec[c + d*x]^2*(a + a*Sec[c + d*x])^(5/3),x]
```

output
```
(3*(a + a*Sec[c + d*x])^(5/3)*Tan[c + d*x])/(8*d) - (5*a*(a + a*Sec[c + d*
x])^(2/3)*((-3*Sqrt[1 - Sec[c + d*x]]*(1 + Sec[c + d*x])^(7/6))/5 + (7*((-
3*Sqrt[1 - Sec[c + d*x]]*(1 + Sec[c + d*x])^(1/6))/2 + (3^(3/4)*EllipticF[
ArcCos[(2^(1/3) - (1 - Sqrt[3])*(1 + Sec[c + d*x])^(1/3))/(2^(1/3) - (1 +
Sqrt[3])*(1 + Sec[c + d*x])^(1/3))], (2 + Sqrt[3])/4]*(1 + Sec[c + d*x])^(
1/6)*(2^(1/3) - (1 + Sec[c + d*x])^(1/3))*Sqrt[(2^(2/3) + 2^(1/3)*(1 + Sec
[c + d*x])^(1/3) + (1 + Sec[c + d*x])^(2/3))/(2^(1/3) - (1 + Sqrt[3])*(1 +
 Sec[c + d*x])^(1/3))^2])/(2*2^(1/3)*Sqrt[1 - Sec[c + d*x]]*Sqrt[-(((1 + S
ec[c + d*x])^(1/3)*(2^(1/3) - (1 + Sec[c + d*x])^(1/3)))/(2^(1/3) - (1 + S
qrt[3])*(1 + Sec[c + d*x])^(1/3))^2)])))/5)*Tan[c + d*x])/(8*d*Sqrt[1 - Se
c[c + d*x]]*(1 + Sec[c + d*x])^(7/6))
```

3.149.3.1 Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 766
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4285
```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(-Cot[e + f*x])*((a + b*Csc[e + f*x])^m/(f*(m + 1))), x]
+ Simp[a*(m/(b*(m + 1)))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x]
/; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

rule 4314
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Simp[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x
]]*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(d*x)^(n - 1)*((a + b*x)^(m - 1/2
)/Sqrt[a - b*x]), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n},
x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

rule 4315
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Simp[a^IntPart[m]*((a + b*Csc[e + f*x])^FracPart[m
]/(1 + (b/a)*Csc[e + f*x])^FracPart[m])   Int[(1 + (b/a)*Csc[e + f*x])^m*(d
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^
2, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

3.149.4 Maple [F]

$$\int \sec (dx+c)^2 (a+a\sec (dx+c))^{\frac{5}{3}} dx$$

input
```
int(sec(d*x+c)^2*(a+a*sec(d*x+c))^(5/3),x)
```

output
```
int(sec(d*x+c)^2*(a+a*sec(d*x+c))^(5/3),x)
```

3.149.5 Fricas [F]

$$\int \sec^2(c+dx)(a+a\sec(c+dx))^{5/3}\,dx = \int (a\sec(dx+c)+a)^{\frac{5}{3}}\sec(dx+c)^2\,dx$$

input
```
integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^(5/3),x, algorithm="fricas")
```

output
```
integral((a*sec(d*x + c)^3 + a*sec(d*x + c)^2)*(a*sec(d*x + c) + a)^(2/3),
 x)
```

3.149.6 Sympy [F]

$$\int \sec^2(c+dx)(a+a\sec(c+dx))^{5/3}\,dx = \int (a(\sec(c+dx)+1))^{\frac{5}{3}}\sec^2(c+dx)\,dx$$

input
```
integrate(sec(d*x+c)**2*(a+a*sec(d*x+c))**(5/3),x)
```

output
```
Integral((a*(sec(c + d*x) + 1))**(5/3)*sec(c + d*x)**2, x)
```

3.149.7 Maxima [F]

$$\int \sec^2(c+dx)(a+a\sec(c+dx))^{5/3}\,dx = \int (a\sec(dx+c)+a)^{\frac{5}{3}}\sec(dx+c)^2\,dx$$

input
```
integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^(5/3),x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)^(5/3)*sec(d*x + c)^2, x)
```

3.149.8 Giac [F]

$$\int \sec^2(c+dx)(a+a\sec(c+dx))^{5/3}\,dx = \int (a\sec(dx+c)+a)^{\frac{5}{3}}\sec(dx+c)^2\,dx$$

input
```
integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^(5/3),x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)^(5/3)*sec(d*x + c)^2, x)
```

3.149.9 Mupad [F(-1)]

Timed out.

$$\int \sec^2(c+dx)(a+a\sec(c+dx))^{5/3}\,dx = \int \frac{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{5/3}}{\cos(c+d\,x)^2}\,dx$$

input
```
int((a + a/cos(c + d*x))^(5/3)/cos(c + d*x)^2,x)
```

output
```
int((a + a/cos(c + d*x))^(5/3)/cos(c + d*x)^2, x)
```

3.150 $\int \sec(c+dx)(a+a\sec(c+dx))^{5/3}\,dx$

3.150.1 Optimal result 1282
3.150.2 Mathematica [C] (verified) 1283
3.150.3 Rubi [A] (verified) 1283
3.150.4 Maple [F] 1286
3.150.5 Fricas [F] 1286
3.150.6 Sympy [F] 1287
3.150.7 Maxima [F] 1287
3.150.8 Giac [F] 1287
3.150.9 Mupad [F(-1)] 1288

3.150.1 Optimal result

Integrand size = 21, antiderivative size = 356

$$\int \sec(c+dx)(a+a\sec(c+dx))^{5/3}\,dx = \frac{3a(a+a\sec(c+dx))^{2/3}\tan(c+dx)}{5d} + \frac{21a(a+a\sec(c+dx))^{2/3}\tan(c+dx)}{10d(1+\sec(c+dx))} - \frac{7\,3^{3/4}a\,\text{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{2}-\left(1-\sqrt{3}\right)\sqrt[3]{1+\sec(c+dx)}}{\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{1+\sec(c+dx)}}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)(a+a\sec(c+dx))^{2/3}\left(\sqrt[3]{2}-\sqrt[3]{\cdots}\right.}{10\sqrt[3]{2}d(1-\sec(c+dx))(1+\sec(c+dx))\sqrt{-\frac{\sqrt[3]{1+\sec(c+\cdots}}{\left(\sqrt[3]{2}-(1+\cdots\right.}}}$$

output
```
3/5*a*(a+a*sec(d*x+c))^(2/3)*tan(d*x+c)/d+21/10*a*(a+a*sec(d*x+c))^(2/3)*t
an(d*x+c)/d/(1+sec(d*x+c))-7/20*3^(3/4)*a*((2^(1/3)-(1+sec(d*x+c))^(1/3)*(
1-3^(1/2)))^2/(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1+3^(1/2)))^2)^(1/2)/(2^(1/3)
-(1+sec(d*x+c))^(1/3)*(1-3^(1/2)))*(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1+3^(1/2
)))*EllipticF((1-(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1-3^(1/2)))^2/(2^(1/3)-(1+
sec(d*x+c))^(1/3)*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(a+a*sec(
d*x+c))^(2/3)*(2^(1/3)-(1+sec(d*x+c))^(1/3))*((2^(2/3)+2^(1/3)*(1+sec(d*x+
c))^(1/3)+(1+sec(d*x+c))^(2/3))/(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1+3^(1/2)))
^2)^(1/2)*tan(d*x+c)*2^(2/3)/d/(1-sec(d*x+c))/(1+sec(d*x+c))/(-(1+sec(d*x+
c))^(1/3)*(2^(1/3)-(1+sec(d*x+c))^(1/3))/(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1+
3^(1/2)))^2)^(1/2)
```

3.150.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.09 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.19

$$\int \sec(c+dx)(a+a\sec(c+dx))^{5/3}\,dx = \frac{4\sqrt[6]{2}\,\text{Hypergeometric2F1}\left(-\frac{7}{6},\frac{1}{2},\frac{3}{2},\frac{1}{2}(1-\sec(c+dx))\right)(a(1+\sec(c+dx)))^{5/3}\tan(c+dx)}{d(1+\sec(c+dx))^{13/6}}$$

input
```
Integrate[Sec[c + d*x]*(a + a*Sec[c + d*x])^(5/3),x]
```

output
```
(4*2^(1/6)*Hypergeometric2F1[-7/6, 1/2, 3/2, (1 - Sec[c + d*x])/2]*(a*(1 +
 Sec[c + d*x]))^(5/3)*Tan[c + d*x])/(d*(1 + Sec[c + d*x])^(13/6))
```

3.150.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 387, normalized size of antiderivative = 1.09, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {3042, 4315, 3042, 4314, 60, 60, 73, 766}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(c+dx)(a\sec(c+dx)+a)^{5/3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/3}dx$$

$$\Big\downarrow \text{4315}$$

$$\frac{a(a\sec(c+dx)+a)^{2/3}\int \sec(c+dx)(\sec(c+dx)+1)^{5/3}dx}{(\sec(c+dx)+1)^{2/3}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a(a\sec(c+dx)+a)^{2/3}\int \csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)^{5/3}dx}{(\sec(c+dx)+1)^{2/3}}$$

$$\Big\downarrow \text{4314}$$

$$-\frac{a\tan(c+dx)(a\sec(c+dx)+a)^{2/3}\int\frac{(\sec(c+dx)+1)^{7/6}}{\sqrt{1-\sec(c+dx)}}d\sec(c+dx)}{d\sqrt{1-\sec(c+dx)}(\sec(c+dx)+1)^{7/6}}$$

$\Big\downarrow$ 60

$$-\frac{a\tan(c+dx)(a\sec(c+dx)+a)^{2/3}\left(\frac{7}{5}\int\frac{\sqrt[6]{\sec(c+dx)+1}}{\sqrt{1-\sec(c+dx)}}d\sec(c+dx)-\frac{3}{5}\sqrt{1-\sec(c+dx)}(\sec(c+dx)+1)\right.}{d\sqrt{1-\sec(c+dx)}(\sec(c+dx)+1)^{7/6}}$$

$\Big\downarrow$ 60

$$-\frac{a\tan(c+dx)(a\sec(c+dx)+a)^{2/3}\left(\frac{7}{5}\left(\frac{1}{2}\int\frac{1}{\sqrt{1-\sec(c+dx)}(\sec(c+dx)+1)^{5/6}}d\sec(c+dx)-\frac{3}{2}\sqrt{1-\sec(c+dx)}\sqrt[6]{\sec(\right.\right.}{d\sqrt{1-\sec(c+dx)}(\sec(c+dx)+1)^{7/6}}$$

$\Big\downarrow$ 73

$$-\frac{a\tan(c+dx)(a\sec(c+dx)+a)^{2/3}\left(\frac{7}{5}\left(3\int\frac{1}{\sqrt{1-\sec(c+dx)}}d\sqrt[6]{\sec(c+dx)+1}-\frac{3}{2}\sqrt{1-\sec(c+dx)}\sqrt[6]{\sec(c+dx)\right.\right.}{d\sqrt{1-\sec(c+dx)}(\sec(c+dx)+1)^{7/6}}$$

$\Big\downarrow$ 766

$$-\frac{a\tan(c+dx)(a\sec(c+dx)+a)^{2/3}\left(\frac{7}{5}\left(\frac{3^{3/4}\sqrt[6]{\sec(c+dx)+1}\left(\sqrt[3]{2}-\sqrt[3]{\sec(c+dx)+1}\right)\sqrt{\frac{(\sec(c+dx)+1)^{2/3}+\sqrt[3]{2}\sqrt[3]{\mathrm{s}}}{\left(\sqrt[3]{2}-(1+\sqrt{3})\sqrt[3]{\mathrm{sec}}\right.}}}{2\sqrt[3]{2}\sqrt{1-\sec(c+dx)}\sqrt{-\frac{\sqrt[3]{\sec(c+}}{(\sqrt[3]{}}}}\right.\right.}{}$$

input
```
Int[Sec[c + d*x]*(a + a*Sec[c + d*x])^(5/3),x]
```

3.150. $\int \sec(c+dx)(a+a\sec(c+dx))^{5/3}\,dx$

output
```
-((a*(a + a*Sec[c + d*x])^(2/3)*((-3*Sqrt[1 - Sec[c + d*x]]*(1 + Sec[c + d
*x])^(7/6))/5 + (7*((-3*Sqrt[1 - Sec[c + d*x]]*(1 + Sec[c + d*x])^(1/6))/2
 + (3^(3/4)*EllipticF[ArcCos[(2^(1/3) - (1 - Sqrt[3])*(1 + Sec[c + d*x])^(
1/3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))], (2 + Sqrt[3])/4
]*(1 + Sec[c + d*x])^(1/6)*(2^(1/3) - (1 + Sec[c + d*x])^(1/3))*Sqrt[(2^(2
/3) + 2^(1/3)*(1 + Sec[c + d*x])^(1/3) + (1 + Sec[c + d*x])^(2/3))/(2^(1/3
) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2])/(2*2^(1/3)*Sqrt[1 - Sec[c
+ d*x]]*Sqrt[-(((1 + Sec[c + d*x])^(1/3)*(2^(1/3) - (1 + Sec[c + d*x])^(1/
3)))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2)])))/5)*Tan[c +
d*x])/(d*Sqrt[1 - Sec[c + d*x]]*(1 + Sec[c + d*x])^(7/6)))
```

3.150.3.1 Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 766
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.150. $\int \sec(c+dx)(a+a\sec(c+dx))^{5/3}\,dx$

rule 4314
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Simp[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x
]]*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(d*x)^(n - 1)*((a + b*x)^(m - 1/2
)/Sqrt[a - b*x]), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n},
x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

rule 4315
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Simp[a^IntPart[m]*((a + b*Csc[e + f*x])^FracPart[m
]/(1 + (b/a)*Csc[e + f*x])^FracPart[m])   Int[(1 + (b/a)*Csc[e + f*x])^m*(d
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^
2, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

3.150.4 Maple [F]

$$\int \sec\left(dx+c\right)\left(a+a\sec\left(dx+c\right)\right)^{\frac{5}{3}}dx$$

input
```
int(sec(d*x+c)*(a+a*sec(d*x+c))^(5/3),x)
```

output
```
int(sec(d*x+c)*(a+a*sec(d*x+c))^(5/3),x)
```

3.150.5 Fricas [F]

$$\int \sec(c+dx)(a+a\sec(c+dx))^{5/3}\,dx=\int \left(a\sec\left(dx+c\right)+a\right)^{\frac{5}{3}}\sec\left(dx+c\right)\,dx$$

input
```
integrate(sec(d*x+c)*(a+a*sec(d*x+c))^(5/3),x, algorithm="fricas")
```

output
```
integral((a*sec(d*x + c)^2 + a*sec(d*x + c))*(a*sec(d*x + c) + a)^(2/3), x
)
```

3.150.6 Sympy [F]

$$\int \sec(c+dx)(a+a\sec(c+dx))^{5/3}\,dx = \int (a(\sec(c+dx)+1))^{\frac{5}{3}}\sec(c+dx)\,dx$$

input
```
integrate(sec(d*x+c)*(a+a*sec(d*x+c))**(5/3),x)
```

output
```
Integral((a*(sec(c + d*x) + 1))**(5/3)*sec(c + d*x), x)
```

3.150.7 Maxima [F]

$$\int \sec(c+dx)(a+a\sec(c+dx))^{5/3}\,dx = \int (a\sec(dx+c)+a)^{\frac{5}{3}}\sec(dx+c)\,dx$$

input
```
integrate(sec(d*x+c)*(a+a*sec(d*x+c))^(5/3),x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)^(5/3)*sec(d*x + c), x)
```

3.150.8 Giac [F]

$$\int \sec(c+dx)(a+a\sec(c+dx))^{5/3}\,dx = \int (a\sec(dx+c)+a)^{\frac{5}{3}}\sec(dx+c)\,dx$$

input
```
integrate(sec(d*x+c)*(a+a*sec(d*x+c))^(5/3),x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)^(5/3)*sec(d*x + c), x)
```

3.150.9 Mupad [F(-1)]

Timed out.

$$\int \sec(c+dx)(a+a\sec(c+dx))^{5/3}\,dx = \int \frac{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{5/3}}{\cos\left(c+d\,x\right)}\,dx$$

input
```
int((a + a/cos(c + d*x))^(5/3)/cos(c + d*x),x)
```

output
```
int((a + a/cos(c + d*x))^(5/3)/cos(c + d*x), x)
```

3.151 $\int (a+a\sec(c+dx))^{5/3}\,dx$

3.151.1 Optimal result 1289
3.151.2 Mathematica [B] (warning: unable to verify) 1289
3.151.3 Rubi [A] (verified) 1290
3.151.4 Maple [F] 1292
3.151.5 Fricas [F(-1)] 1293
3.151.6 Sympy [F] 1293
3.151.7 Maxima [F] 1293
3.151.8 Giac [F] 1294
3.151.9 Mupad [F(-1)] 1294

3.151.1 Optimal result

Integrand size = 14, antiderivative size = 86

$$\int (a+a\sec(c+dx))^{5/3}\,dx=\frac{3\sqrt{2}a\,\text{AppellF1}\left(\frac{13}{6},\frac{1}{2},1,\frac{19}{6},\frac{1}{2}(1+\sec(c+dx)),1+\sec(c+dx)\right)(1+\sec(c+dx))(a+a\sec(c$$

$$13d\sqrt{1-\sec(c+dx)}$$

output
```
3/13*a*AppellF1(13/6,1,1/2,19/6,1+sec(d*x+c),1/2+1/2*sec(d*x+c))*(1+sec(d*
x+c))*(a+a*sec(d*x+c))^(2/3)*2^(1/2)*tan(d*x+c)/d/(1-sec(d*x+c))^(1/2)
```

3.151.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 2694 vs. 2(86) = 172.

Time = 15.93 (sec) , antiderivative size = 2694, normalized size of antiderivative = 31.33

$$\int (a+a\sec(c+dx))^{5/3}\,dx=\text{Result too large to show}$$

input
```
Integrate[(a + a*Sec[c + d*x])^(5/3),x]
```

output
```
(3*((1 + Cos[c + d*x])*Sec[c + d*x])^(2/3)*(a*(1 + Sec[c + d*x]))^(5/3)*Ta
n[(c + d*x)/2])/(2*d*(1 + Sec[c + d*x])^(5/3)) + ((Cos[(c + d*x)/2]^2*Sec[
c + d*x])^(2/3)*(a*(1 + Sec[c + d*x]))^(5/3)*((3*Sec[(c + d*x)/2]^2*(1 + S
ec[c + d*x])^(2/3))/4 + (Cos[c + d*x]*Sec[(c + d*x)/2]^2*(1 + Sec[c + d*x]
)^(2/3))/2)*Tan[(c + d*x)/2]*(AppellF1[3/2, 2/3, 1, 5/2, Tan[(c + d*x)/2]^
2, -Tan[(c + d*x)/2]^2]*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(2/3)*Tan[(c + d
*x)/2]^2 + (135*AppellF1[1/2, 2/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d
*x)/2]^2]*Cos[(c + d*x)/2]^2)/(9*AppellF1[1/2, 2/3, 1, 3/2, Tan[(c + d*x)/
2]^2, -Tan[(c + d*x)/2]^2] + 2*(-3*AppellF1[3/2, 2/3, 2, 5/2, Tan[(c + d*x
)/2]^2, -Tan[(c + d*x)/2]^2] + 2*AppellF1[3/2, 5/3, 1, 5/2, Tan[(c + d*x)/
2]^2, -Tan[(c + d*x)/2]^2])*Tan[(c + d*x)/2]^2)))/(3*2^(1/3)*d*(1 + Sec[c
+ d*x])^(5/3)*((Sec[(c + d*x)/2]^2*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(2/3)
*(AppellF1[3/2, 2/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(Cos
[c + d*x]*Sec[(c + d*x)/2]^2)^(2/3)*Tan[(c + d*x)/2]^2 + (135*AppellF1[1/2
, 2/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Cos[(c + d*x)/2]^2
)/(9*AppellF1[1/2, 2/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] +
 2*(-3*AppellF1[3/2, 2/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]
 + 2*AppellF1[3/2, 5/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*
Tan[(c + d*x)/2]^2)))/(6*2^(1/3)) + ((Cos[(c + d*x)/2]^2*Sec[c + d*x])^(2/
3)*Tan[(c + d*x)/2]*(AppellF1[3/2, 2/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Ta...
```

3.151.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 4266, 3042, 4265, 149, 25, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sec(c+dx)+a)^{5/3}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/3}dx$$

$$\Big\downarrow \; 4266$$

$$\frac{a(a\sec(c+dx)+a)^{2/3}\int(\sec(c+dx)+1)^{5/3}dx}{(\sec(c+dx)+1)^{2/3}}$$

$$\Big\downarrow \; 3042$$

$$\frac{a(a\sec(c+dx)+a)^{2/3}\int\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)^{5/3}dx}{(\sec(c+dx)+1)^{2/3}}$$

$$\Big\downarrow \text{4265}$$

$$-\frac{a\tan(c+dx)(a\sec(c+dx)+a)^{2/3}\int\frac{\cos(c+dx)(\sec(c+dx)+1)^{7/6}}{\sqrt{1-\sec(c+dx)}}d\sec(c+dx)}{d\sqrt{1-\sec(c+dx)}(\sec(c+dx)+1)^{7/6}}$$

$$\Big\downarrow \text{149}$$

$$-\frac{6a\tan(c+dx)(a\sec(c+dx)+a)^{2/3}\int\frac{\cos(c+dx)(\sec(c+dx)+1)^{2}}{\sqrt{1-\sec(c+dx)}}d\sqrt[6]{\sec(c+dx)+1}}{d\sqrt{1-\sec(c+dx)}(\sec(c+dx)+1)^{7/6}}$$

$$\Big\downarrow \text{25}$$

$$\frac{6a\tan(c+dx)(a\sec(c+dx)+a)^{2/3}\int-\frac{\cos(c+dx)(\sec(c+dx)+1)^{2}}{\sqrt{1-\sec(c+dx)}}d\sqrt[6]{\sec(c+dx)+1}}{d\sqrt{1-\sec(c+dx)}(\sec(c+dx)+1)^{7/6}}$$

$$\Big\downarrow \text{1012}$$

$$\frac{3\sqrt{2}a\tan(c+dx)(\sec(c+dx)+1)(a\sec(c+dx)+a)^{2/3}\operatorname{AppellF1}\left(\frac{13}{6},1,\frac{1}{2},\frac{19}{6},\sec(c+dx)+1,\frac{1}{2}(\sec(c+dx)+1\right.}{13d\sqrt{1-\sec(c+dx)}}$$

input
```
Int[(a + a*Sec[c + d*x])^(5/3),x]
```

output
```
(3*Sqrt[2]*a*AppellF1[13/6, 1, 1/2, 19/6, 1 + Sec[c + d*x], (1 + Sec[c + d
*x])/2]*(1 + Sec[c + d*x])*(a + a*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(13*d*
Sqrt[1 - Sec[c + d*x]])
```

3.151.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 149
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_)
)^(p_.), x_] :> With[{k = Denominator[m]}, Simp[k/b   Subst[Int[x^(k*(m + 1
) - 1)*(c - a*(d/b) + d*(x^k/b))^n*(e - a*(f/b) + f*(x^k/b))^p, x], x, (a +
 b*x)^(1/k)], x]] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && FractionQ[m] &&
IntegerQ[2*n] && IntegerQ[p]
```

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4265
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[a^n*(Cot
[c + d*x]/(d*Sqrt[1 + Csc[c + d*x]]*Sqrt[1 - Csc[c + d*x]]))   Subst[Int[(1
 + b*(x/a))^(n - 1/2)/(x*Sqrt[1 - b*(x/a)]), x], x, Csc[c + d*x]], x] /; Fr
eeQ[{a, b, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0
]
```

rule 4266
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[a^IntPar
t[n]*((a + b*Csc[c + d*x])^FracPart[n]/(1 + (b/a)*Csc[c + d*x])^FracPart[n]
)   Int[(1 + (b/a)*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] &&
EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

3.151.4 Maple [F]

$$\int (a + a\sec(dx + c))^{\frac{5}{3}}\,dx$$

input
```
int((a+a*sec(d*x+c))^(5/3),x)
```

output
```
int((a+a*sec(d*x+c))^(5/3),x)
```

3.151.5 Fricas [F(-1)]

Timed out.

$$\int (a + a\sec(c + dx))^{5/3}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sec(d*x+c))^(5/3),x, algorithm="fricas")
```

output
```
Timed out
```

3.151.6 Sympy [F]

$$\int (a + a\sec(c + dx))^{5/3}\,dx = \int (a\sec(c + dx) + a)^{\frac{5}{3}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))**(5/3),x)
```

output
```
Integral((a*sec(c + d*x) + a)**(5/3), x)
```

3.151.7 Maxima [F]

$$\int (a + a\sec(c + dx))^{5/3}\,dx = \int (a\sec(dx + c) + a)^{\frac{5}{3}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))^(5/3),x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)^(5/3), x)
```

3.151.8 Giac [F]

$$\int (a + a\sec(c + dx))^{5/3}\,dx = \int (a\sec(dx + c) + a)^{\frac{5}{3}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))^(5/3),x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)^(5/3), x)
```

3.151.9 Mupad [F(-1)]

Timed out.

$$\int (a + a\sec(c + dx))^{5/3}\,dx = \int \left(a + \frac{a}{\cos(c + d\,x)}\right)^{5/3}\,dx$$

input
```
int((a + a/cos(c + d*x))^(5/3),x)
```

output
```
int((a + a/cos(c + d*x))^(5/3), x)
```

3.152 $\int \cos(c+dx)(a+a\sec(c+dx))^{5/3}\,dx$

3.152.1 Optimal result 1295
3.152.2 Mathematica [B] (warning: unable to verify) 1295
3.152.3 Rubi [A] (verified) 1296
3.152.4 Maple [F] 1298
3.152.5 Fricas [F(-1)] 1298
3.152.6 Sympy [F(-1)] 1299
3.152.7 Maxima [F] 1299
3.152.8 Giac [F] 1299
3.152.9 Mupad [F(-1)] 1300

3.152.1 Optimal result

Integrand size = 21, antiderivative size = 86

$$\int \cos(c+dx)(a+a\sec(c+dx))^{5/3}\,dx = -\frac{3\sqrt{2}a\,\text{AppellF1}\left(\frac{13}{6},\frac{1}{2},2,\frac{19}{6},\frac{1}{2}(1+\sec(c+dx)),1+\sec(c+dx)\right)(1+\sec(c+dx))(a+a\sec(c+dx))^{2/3}\,\text{ta}}{13d\sqrt{1-\sec(c+dx)}}$$

output
```
-3/13*a*AppellF1(13/6,2,1/2,19/6,1+sec(d*x+c),1/2+1/2*sec(d*x+c))*(1+sec(d
*x+c))*(a+a*sec(d*x+c))^(2/3)*2^(1/2)*tan(d*x+c)/d/(1-sec(d*x+c))^(1/2)
```

3.152.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 2700 vs. 2(86) = 172.

Time = 16.01 (sec) , antiderivative size = 2700, normalized size of antiderivative = 31.40

$$\int \cos(c+dx)(a+a\sec(c+dx))^{5/3}\,dx = \text{Result too large to show}$$

input
```
Integrate[Cos[c + d*x]*(a + a*Sec[c + d*x])^(5/3),x]
```

output
```
(((1 + Cos[c + d*x])*Sec[c + d*x])^(2/3)*(a*(1 + Sec[c + d*x]))^(5/3)*(Sin
[c + d*x] - Tan[(c + d*x)/2]))/(d*(1 + Sec[c + d*x])^(5/3)) - (2^(2/3)*(Co
s[(c + d*x)/2]^2*Sec[c + d*x])^(2/3)*(a*(1 + Sec[c + d*x]))^(5/3)*((2*Sec[
(c + d*x)/2]^2*(1 + Sec[c + d*x])^(2/3))/3 + (5*Cos[c + d*x]*Sec[(c + d*x)
/2]^2*(1 + Sec[c + d*x])^(2/3))/6)*Tan[(c + d*x)/2]*(AppellF1[3/2, 2/3, 1,
 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(Cos[c + d*x]*Sec[(c + d*x)
/2]^2)^(2/3)*Tan[(c + d*x)/2]^2 + (243*AppellF1[1/2, 2/3, 1, 3/2, Tan[(c +
 d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Cos[(c + d*x)/2]^2)/(-9*AppellF1[1/2, 2/3
, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + 2*(3*AppellF1[3/2, 2/
3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - 2*AppellF1[3/2, 5/3,
 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Tan[(c + d*x)/2]^2)))/(
9*d*(1 + Sec[c + d*x])^(5/3)*(-1/9*(Sec[(c + d*x)/2]^2*(Cos[(c + d*x)/2]^2
*Sec[c + d*x])^(2/3)*(AppellF1[3/2, 2/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[
(c + d*x)/2]^2]*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(2/3)*Tan[(c + d*x)/2]^2
 + (243*AppellF1[1/2, 2/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2
]*Cos[(c + d*x)/2]^2)/(-9*AppellF1[1/2, 2/3, 1, 3/2, Tan[(c + d*x)/2]^2, -
Tan[(c + d*x)/2]^2] + 2*(3*AppellF1[3/2, 2/3, 2, 5/2, Tan[(c + d*x)/2]^2,
-Tan[(c + d*x)/2]^2] - 2*AppellF1[3/2, 5/3, 1, 5/2, Tan[(c + d*x)/2]^2, -T
an[(c + d*x)/2]^2])*Tan[(c + d*x)/2]^2)))/2^(1/3) - (2^(2/3)*(Cos[(c + d*x
)/2]^2*Sec[c + d*x])^(2/3)*Tan[(c + d*x)/2]*(AppellF1[3/2, 2/3, 1, 5/2,...
```

3.152.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 4315, 3042, 4314, 149, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(c+dx)(a\sec(c+dx)+a)^{5/3}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/3}}{\csc\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \; 4315$$

$$\frac{a(a\sec(c+dx)+a)^{2/3}\int \cos(c+dx)(\sec(c+dx)+1)^{5/3}dx}{(\sec(c+dx)+1)^{2/3}}$$

$$\Big\downarrow \; 3042$$

$$\frac{a(a\sec(c+dx)+a)^{2/3}\int \frac{\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)^{5/3}}{\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{(\sec(c+dx)+1)^{2/3}}$$

$$\Big\downarrow \text{4314}$$

$$-\frac{a\tan(c+dx)(a\sec(c+dx)+a)^{2/3}\int \frac{\cos^2(c+dx)(\sec(c+dx)+1)^{7/6}}{\sqrt{1-\sec(c+dx)}}d\sec(c+dx)}{d\sqrt{1-\sec(c+dx)}(\sec(c+dx)+1)^{7/6}}$$

$$\Big\downarrow \text{149}$$

$$-\frac{6a\tan(c+dx)(a\sec(c+dx)+a)^{2/3}\int \frac{\cos^2(c+dx)(\sec(c+dx)+1)^2}{\sqrt{1-\sec(c+dx)}}d\sqrt[6]{\sec(c+dx)+1}}{d\sqrt{1-\sec(c+dx)}(\sec(c+dx)+1)^{7/6}}$$

$$\Big\downarrow \text{1012}$$

$$-\frac{3\sqrt{2}a\tan(c+dx)(\sec(c+dx)+1)(a\sec(c+dx)+a)^{2/3}\operatorname{AppellF1}\left(\frac{13}{6},2,\frac{1}{2},\frac{19}{6},\sec(c+dx)+1,\frac{1}{2}(\sec(c+dx)+\right.}{13d\sqrt{1-\sec(c+dx)}}$$

input
```
Int[Cos[c + d*x]*(a + a*Sec[c + d*x])^(5/3),x]
```

output
```
(-3*Sqrt[2]*a*AppellF1[13/6, 2, 1/2, 19/6, 1 + Sec[c + d*x], (1 + Sec[c +
d*x])/2]*(1 + Sec[c + d*x])*(a + a*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(13*d
*Sqrt[1 - Sec[c + d*x]])
```

3.152.3.1 Defintions of rubi rules used

rule 149
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_)
)^(p_.), x_] :> With[{k = Denominator[m]}, Simp[k/b   Subst[Int[x^(k*(m + 1
) - 1)*(c - a*(d/b) + d*(x^k/b))^n*(e - a*(f/b) + f*(x^k/b))^p, x], x, (a +
 b*x)^(1/k)], x]] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && FractionQ[m] &&
IntegerQ[2*n] && IntegerQ[p]
```

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4314
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Simp[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x
]]*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(d*x)^(n - 1)*((a + b*x)^(m - 1/2
)/Sqrt[a - b*x]), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n},
x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

rule 4315
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Simp[a^IntPart[m]*((a + b*Csc[e + f*x])^FracPart[m
]/(1 + (b/a)*Csc[e + f*x])^FracPart[m])   Int[(1 + (b/a)*Csc[e + f*x])^m*(d
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^
2, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

3.152.4 Maple [F]

$$\int \cos\left(dx+c\right)\left(a+a\sec\left(dx+c\right)\right)^{\frac{5}{3}}dx$$

input
```
int(cos(d*x+c)*(a+a*sec(d*x+c))^(5/3),x)
```

output
```
int(cos(d*x+c)*(a+a*sec(d*x+c))^(5/3),x)
```

3.152.5 Fricas [F(-1)]

Timed out.

$$\int \cos(c+dx)(a+a\sec(c+dx))^{5/3}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)*(a+a*sec(d*x+c))^(5/3),x, algorithm="fricas")
```

output
```
Timed out
```

3.152.6 Sympy [F(-1)]

Timed out.

$$\int \cos(c+dx)(a+a\sec(c+dx))^{5/3}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)*(a+a*sec(d*x+c))**(5/3),x)
```

output
```
Timed out
```

3.152.7 Maxima [F]

$$\int \cos(c+dx)(a+a\sec(c+dx))^{5/3}\,dx = \int (a\sec(dx+c)+a)^{\frac{5}{3}}\cos(dx+c)\,dx$$

input
```
integrate(cos(d*x+c)*(a+a*sec(d*x+c))^(5/3),x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)^(5/3)*cos(d*x + c), x)
```

3.152.8 Giac [F]

$$\int \cos(c+dx)(a+a\sec(c+dx))^{5/3}\,dx = \int (a\sec(dx+c)+a)^{\frac{5}{3}}\cos(dx+c)\,dx$$

input
```
integrate(cos(d*x+c)*(a+a*sec(d*x+c))^(5/3),x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)^(5/3)*cos(d*x + c), x)
```

3.152.9 Mupad [F(-1)]

Timed out.

$$\int \cos(c+dx)(a+a\sec(c+dx))^{5/3}\,dx = \int \cos\left(c+d\,x\right)\,\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)^{5/3}\,dx$$

input
```
int(cos(c + d*x)*(a + a/cos(c + d*x))^(5/3),x)
```

output
```
int(cos(c + d*x)*(a + a/cos(c + d*x))^(5/3), x)
```

3.153 $\int \frac{\sec^4(c+dx)}{\sqrt[3]{a+a\sec(c+dx)}}\,dx$

3.153.1 Optimal result 1301
3.153.2 Mathematica [C] (verified) 1302
3.153.3 Rubi [A] (verified) 1302
3.153.4 Maple [F] 1307
3.153.5 Fricas [F] 1307
3.153.6 Sympy [F] 1307
3.153.7 Maxima [F] 1308
3.153.8 Giac [F] 1308
3.153.9 Mupad [F(-1)] 1308

3.153.1 Optimal result

Integrand size = 23, antiderivative size = 371

$$\int \frac{\sec^4(c+dx)}{\sqrt[3]{a+a\sec(c+dx)}}\,dx$$
$$= \frac{99\tan(c+dx)}{80d\sqrt[3]{a+a\sec(c+dx)}} + \frac{3\sec^2(c+dx)\tan(c+dx)}{8d\sqrt[3]{a+a\sec(c+dx)}} - \frac{3(a+a\sec(c+dx))^{2/3}\tan(c+dx)}{40ad}$$
$$+ \frac{37\ 3^{3/4}\,\text{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{2}-\left(1-\sqrt{3}\right)\sqrt[3]{1+\sec(c+dx)}}{\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{1+\sec(c+dx)}}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)\left(\sqrt[3]{2}-\sqrt[3]{1+\sec(c+dx)}\right)\sqrt{}}{80\sqrt[3]{2}d(1-\sec(c+dx))\sqrt[3]{a+a\sec(c+dx)}\sqrt{-\frac{\sqrt[3]{1+\sec(c+dx)}\left(\sqrt[3]{}\right.}{\left(\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{1}\right.}}}$$

output
```
99/80*tan(d*x+c)/d/(a+a*sec(d*x+c))^(1/3)+3/8*sec(d*x+c)^2*tan(d*x+c)/d/(a
+a*sec(d*x+c))^(1/3)-3/40*(a+a*sec(d*x+c))^(2/3)*tan(d*x+c)/a/d+37/160*3^(
3/4)*((2^(1/3)-(1+sec(d*x+c))^(1/3)*(1-3^(1/2)))^2/(2^(1/3)-(1+sec(d*x+c))
^(1/3)*(1+3^(1/2)))^2)^(1/2)/(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1-3^(1/2)))*(2
^(1/3)-(1+sec(d*x+c))^(1/3)*(1+3^(1/2)))*EllipticF((1-(2^(1/3)-(1+sec(d*x+
c))^(1/3)*(1-3^(1/2)))^2/(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1+3^(1/2)))^2)^(1/
2),1/4*6^(1/2)+1/4*2^(1/2))*(2^(1/3)-(1+sec(d*x+c))^(1/3))*((2^(2/3)+2^(1/
3)*(1+sec(d*x+c))^(1/3)+(1+sec(d*x+c))^(2/3))/(2^(1/3)-(1+sec(d*x+c))^(1/3
)*(1+3^(1/2)))^2)^(1/2)*tan(d*x+c)*2^(2/3)/d/(1-sec(d*x+c))/(a+a*sec(d*x+c
))^(1/3)/(-(1+sec(d*x+c))^(1/3)*(2^(1/3)-(1+sec(d*x+c))^(1/3))/(2^(1/3)-(1
+sec(d*x+c))^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

3.153.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.40 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.42

$$\int \frac{\sec^4(c+dx)}{\sqrt[3]{a+a\sec(c+dx)}}\,dx$$
$$= \frac{\left(-4\sqrt[6]{2}\,\text{Hypergeometric2F1}\left(-\frac{7}{6},\frac{1}{2},\frac{3}{2},\frac{1}{2}(1-\sec(c+dx))\right)+16\sqrt[6]{2}\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{2},\frac{3}{2},\frac{1}{2}(1-\right.\right.}{8d\sqrt[6]{1+\sec(c}}$$

input
```
Integrate[Sec[c + d*x]^4/(a + a*Sec[c + d*x])^(1/3),x]
```

output
```
((-4*2^(1/6)*Hypergeometric2F1[-7/6, 1/2, 3/2, (1 - Sec[c + d*x])/2] + 16*
2^(1/6)*Hypergeometric2F1[-1/6, 1/2, 3/2, (1 - Sec[c + d*x])/2] - 7*2^(1/6
)*Hypergeometric2F1[1/2, 5/6, 3/2, (1 - Sec[c + d*x])/2] + 3*Sec[c + d*x]^
2*(1 + Sec[c + d*x])^(1/6))*Tan[c + d*x])/(8*d*(1 + Sec[c + d*x])^(1/6)*(a
*(1 + Sec[c + d*x]))^(1/3))
```

3.153.3 Rubi [A] (verified)

Time = 1.05 (sec) , antiderivative size = 390, normalized size of antiderivative = 1.05, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.609$, Rules used = {3042, 4311, 27, 3042, 4498, 27, 3042, 4489, 3042, 4315, 3042, 4314, 73, 766}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^4(c+dx)}{\sqrt[3]{a\sec(c+dx)+a}}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^4}{\sqrt[3]{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}}dx$$
$$\Big\downarrow \; 4311$$
$$\frac{3\int \frac{\sec^2(c+dx)(6a-a\sec(c+dx))}{3\sqrt[3]{\sec(c+dx)a+a}}dx}{8a}+\frac{3\tan(c+dx)\sec^2(c+dx)}{8d\sqrt[3]{a\sec(c+dx)+a}}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{\sec^2(c+dx)(6a-a\sec(c+dx))}{\sqrt[3]{\sec(c+dx)a+a}}dx}{8a}+\frac{3\tan(c+dx)\sec^2(c+dx)}{8d\sqrt[3]{a\sec(c+dx)+a}}$$

$$\Big\downarrow \; 3042$$

$$\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2\left(6a-a\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt[3]{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{8a}+\frac{3\tan(c+dx)\sec^2(c+dx)}{8d\sqrt[3]{a\sec(c+dx)+a}}$$

$$\Big\downarrow \; 4498$$

$$\frac{\frac{3\int -\frac{\sec(c+dx)\left(2a^2-33a^2\sec(c+dx)\right)}{3\sqrt[3]{\sec(c+dx)a+a}}dx}{5a}-\frac{3\tan(c+dx)(a\sec(c+dx)+a)^{2/3}}{5d}}{8a}+\frac{3\tan(c+dx)\sec^2(c+dx)}{8d\sqrt[3]{a\sec(c+dx)+a}}$$

$$\Big\downarrow \; 27$$

$$\frac{-\frac{\int \frac{\sec(c+dx)\left(2a^2-33a^2\sec(c+dx)\right)}{\sqrt[3]{\sec(c+dx)a+a}}dx}{5a}-\frac{3\tan(c+dx)(a\sec(c+dx)+a)^{2/3}}{5d}}{8a}+\frac{3\tan(c+dx)\sec^2(c+dx)}{8d\sqrt[3]{a\sec(c+dx)+a}}$$

$$\Big\downarrow \; 3042$$

$$\frac{-\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(2a^2-33a^2\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt[3]{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{5a}-\frac{3\tan(c+dx)(a\sec(c+dx)+a)^{2/3}}{5d}}{8a}+\frac{3\tan(c+dx)\sec^2(c+dx)}{8d\sqrt[3]{a\sec(c+dx)+a}}$$

$$\Big\downarrow \; 4489$$

$$\frac{-\frac{\frac{37}{2}a^2\int \frac{\sec(c+dx)}{\sqrt[3]{\sec(c+dx)a+a}}dx-\frac{99a^2\tan(c+dx)}{2d\sqrt[3]{a\sec(c+dx)+a}}}{5a}-\frac{3\tan(c+dx)(a\sec(c+dx)+a)^{2/3}}{5d}}{8a}+\frac{3\tan(c+dx)\sec^2(c+dx)}{8d\sqrt[3]{a\sec(c+dx)+a}}$$

$$\Big\downarrow \; 3042$$

$$\frac{-\frac{\frac{37}{2}a^2\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt[3]{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx-\frac{99a^2\tan(c+dx)}{2d\sqrt[3]{a\sec(c+dx)+a}}}{5a}-\frac{3\tan(c+dx)(a\sec(c+dx)+a)^{2/3}}{5d}}{8a}+\frac{3\tan(c+dx)\sec^2(c+dx)}{8d\sqrt[3]{a\sec(c+dx)+a}}$$

3.153. $\int \frac{\sec^4(c+dx)}{\sqrt[3]{a+a\sec(c+dx)}}dx$

$$\Big\downarrow \; 4315$$

$$\frac{-\frac{\frac{37a^2\sqrt[3]{\sec(c+dx)+1}\int \frac{\sec(c+dx)}{\sqrt[3]{\sec(c+dx)+1}}dx}{2\sqrt[3]{a\sec(c+dx)+a}}-\frac{99a^2\tan(c+dx)}{2d\sqrt[3]{a\sec(c+dx)+a}}}{5a}-\frac{3\tan(c+dx)(a\sec(c+dx)+a)^{2/3}}{5d}}{8a}+\frac{3\tan(c+dx)\sec^2(c+dx)}{8d\sqrt[3]{a\sec(c+dx)+a}}$$

$$\Big\downarrow \; 3042$$

$$\frac{-\frac{\frac{37a^2\sqrt[3]{\sec(c+dx)+1}\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt[3]{\csc\left(c+dx+\frac{\pi}{2}\right)+1}}dx}{2\sqrt[3]{a\sec(c+dx)+a}}-\frac{99a^2\tan(c+dx)}{2d\sqrt[3]{a\sec(c+dx)+a}}}{5a}-\frac{3\tan(c+dx)(a\sec(c+dx)+a)^{2/3}}{5d}}{8a}+\frac{3\tan(c+dx)\sec^2(c+dx)}{8d\sqrt[3]{a\sec(c+dx)+a}}$$

$$\Big\downarrow \; 4314$$

$$\frac{-\frac{-\frac{37a^2\tan(c+dx)\int \frac{1}{\sqrt{1-\sec(c+dx)}(\sec(c+dx)+1)^{5/6}}d\sec(c+dx)}{2d\sqrt{1-\sec(c+dx)}\sqrt[6]{\sec(c+dx)+1}\sqrt[3]{a\sec(c+dx)+a}}-\frac{99a^2\tan(c+dx)}{2d\sqrt[3]{a\sec(c+dx)+a}}}{5a}-\frac{3\tan(c+dx)(a\sec(c+dx)+a)^{2/3}}{5d}}{8a}+\frac{3\tan(c+dx)\sec^2(c+dx)}{8d\sqrt[3]{a\sec(c+dx)+a}}$$

$$\Big\downarrow \; 73$$

$$\frac{-\frac{-\frac{111a^2\tan(c+dx)\int \frac{1}{\sqrt{1-\sec(c+dx)}}d\sqrt[6]{\sec(c+dx)+1}}{d\sqrt{1-\sec(c+dx)}\sqrt[6]{\sec(c+dx)+1}\sqrt[3]{a\sec(c+dx)+a}}-\frac{99a^2\tan(c+dx)}{2d\sqrt[3]{a\sec(c+dx)+a}}}{5a}-\frac{3\tan(c+dx)(a\sec(c+dx)+a)^{2/3}}{5d}}{8a}+\frac{3\tan(c+dx)\sec^2(c+dx)}{8d\sqrt[3]{a\sec(c+dx)+a}}$$

$$\Big\downarrow \; 766$$

3.153. $\int \frac{\sec^4(c+dx)}{\sqrt[3]{a+a\sec(c+dx)}}dx$

$$-\frac{-\frac{37\ 3^{3/4}a^2\tan(c+dx)\left(\sqrt[3]{2}-\sqrt[3]{\sec(c+dx)+1}\right)\sqrt{\frac{(\sec(c+dx)+1)^{2/3}+\sqrt[3]{2}\sqrt[3]{\sec(c+dx)+1}+2^{2/3}}{\left(\sqrt[3]{2}-(1+\sqrt{3})\sqrt[3]{\sec(c+dx)+1}\right)^2}}\,\text{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{2}-(1-\sqrt{3})\sqrt[3]{\text{s}}}{\sqrt[3]{2}-(1+\sqrt{3})\sqrt[3]{\text{s}}}\right.\right.}{2\sqrt[3]{2}d(1-\sec(c+dx))\sqrt{-\frac{\sqrt[3]{\sec(c+dx)+1}\left(\sqrt[3]{2}-\sqrt[3]{\sec(c+dx)+1}\right)}{\left(\sqrt[3]{2}-(1+\sqrt{3})\sqrt[3]{\sec(c+dx)+1}\right)^2}}}\sqrt[3]{a\sec(c+dx)+a}}{5a}}{8a}$$

$$\frac{3\tan(c+dx)\sec^2(c+dx)}{8d\sqrt[3]{a\sec(c+dx)+a}}$$

input
```
Int[Sec[c + d*x]^4/(a + a*Sec[c + d*x])^(1/3),x]
```

output
```
(3*Sec[c + d*x]^2*Tan[c + d*x])/(8*d*(a + a*Sec[c + d*x])^(1/3)) + ((-3*(a
 + a*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(5*d) - ((-99*a^2*Tan[c + d*x])/(2*
d*(a + a*Sec[c + d*x])^(1/3)) - (37*3^(3/4)*a^2*EllipticF[ArcCos[(2^(1/3)
- (1 - Sqrt[3])*(1 + Sec[c + d*x])^(1/3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Se
c[c + d*x])^(1/3))], (2 + Sqrt[3])/4]*(2^(1/3) - (1 + Sec[c + d*x])^(1/3))
*Sqrt[(2^(2/3) + 2^(1/3)*(1 + Sec[c + d*x])^(1/3) + (1 + Sec[c + d*x])^(2/
3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2]*Tan[c + d*x])/(2
*2^(1/3)*d*(1 - Sec[c + d*x])*(a + a*Sec[c + d*x])^(1/3)*Sqrt[-(((1 + Sec[
c + d*x])^(1/3)*(2^(1/3) - (1 + Sec[c + d*x])^(1/3)))/(2^(1/3) - (1 + Sqrt
[3])*(1 + Sec[c + d*x])^(1/3))^2)]))/(5*a))/(8*a)
```

3.153.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

3.153. $\int \frac{\sec^4(c+dx)}{\sqrt[3]{a+a\sec(c+dx)}}\,dx$

rule 766
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4311
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-d^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d
*Csc[e + f*x])^(n - 2)/(f*(m + n - 1))), x] + Simp[d^2/(b*(m + n - 1))   In
t[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*m*Csc[e +
f*x]), x], x] /; FreeQ[{a, b, d, e, f, m}, x] && EqQ[a^2 - b^2, 0] && GtQ[n
, 2] && NeQ[m + n - 1, 0] && IntegerQ[n]
```

rule 4314
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Simp[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x
]]*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(d*x)^(n - 1)*((a + b*x)^(m - 1/2
)/Sqrt[a - b*x]), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n},
x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

rule 4315
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Simp[a^IntPart[m]*((a + b*Csc[e + f*x])^FracPart[m
]/(1 + (b/a)*Csc[e + f*x])^FracPart[m])   Int[(1 + (b/a)*Csc[e + f*x])^m*(d
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^
2, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

rule 4489
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*((
a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Simp[(a*B*m + A*b*(m + 1))/(b*(m +
 1))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B
, e, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b
*(m + 1), 0] &&  !LtQ[m, -2^(-1)]
```

3.153. $\int \frac{\sec^4(c+dx)}{\sqrt[3]{a+a\sec(c+dx)}}\,dx$

rule 4498
```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*
((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m + 2))   Int
[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*B*(m + 1) + (A*b*(m + 2) - a*B)
*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, m}, x] && NeQ[A*b - a
*B, 0] &&  !LtQ[m, -1]
```

3.153.4 Maple [F]

$$\int \frac{\sec(dx+c)^4}{(a+a\sec(dx+c))^{\frac{1}{3}}}dx$$

input
```
int(sec(d*x+c)^4/(a+a*sec(d*x+c))^(1/3),x)
```

output
```
int(sec(d*x+c)^4/(a+a*sec(d*x+c))^(1/3),x)
```

3.153.5 Fricas [F]

$$\int \frac{\sec^4(c+dx)}{\sqrt[3]{a+a\sec(c+dx)}}\,dx = \int \frac{\sec(dx+c)^4}{(a\sec(dx+c)+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate(sec(d*x+c)^4/(a+a*sec(d*x+c))^(1/3),x, algorithm="fricas")
```

output
```
integral(sec(d*x + c)^4/(a*sec(d*x + c) + a)^(1/3), x)
```

3.153.6 Sympy [F]

$$\int \frac{\sec^4(c+dx)}{\sqrt[3]{a+a\sec(c+dx)}}\,dx = \int \frac{\sec^4(c+dx)}{\sqrt[3]{a(\sec(c+dx)+1)}}\,dx$$

input
```
integrate(sec(d*x+c)**4/(a+a*sec(d*x+c))**(1/3),x)
```

output
```
Integral(sec(c + d*x)**4/(a*(sec(c + d*x) + 1))**(1/3), x)
```

3.153.7 Maxima [F]

$$\int \frac{\sec^4(c+dx)}{\sqrt[3]{a+a\sec(c+dx)}}\,dx = \int \frac{\sec(dx+c)^4}{(a\sec(dx+c)+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate(sec(d*x+c)^4/(a+a*sec(d*x+c))^(1/3),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)^4/(a*sec(d*x + c) + a)^(1/3), x)
```

3.153.8 Giac [F]

$$\int \frac{\sec^4(c+dx)}{\sqrt[3]{a+a\sec(c+dx)}}\,dx = \int \frac{\sec(dx+c)^4}{(a\sec(dx+c)+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate(sec(d*x+c)^4/(a+a*sec(d*x+c))^(1/3),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^4/(a*sec(d*x + c) + a)^(1/3), x)
```

3.153.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^4(c+dx)}{\sqrt[3]{a+a\sec(c+dx)}}\,dx = \int \frac{1}{\cos(c+d\,x)^4\left(a+\frac{a}{\cos(c+d\,x)}\right)^{1/3}}\,dx$$

input
```
int(1/(cos(c + d*x)^4*(a + a/cos(c + d*x))^(1/3)),x)
```

output
```
int(1/(cos(c + d*x)^4*(a + a/cos(c + d*x))^(1/3)), x)
```

3.154 $\int \frac{\sec^3(c+dx)}{\sqrt[3]{a+a\sec(c+dx)}}\,dx$

3.154.1 Optimal result . . . 1309
3.154.2 Mathematica [C] (verified) . . . 1310
3.154.3 Rubi [A] (verified) . . . 1310
3.154.4 Maple [F] . . . 1314
3.154.5 Fricas [F] . . . 1314
3.154.6 Sympy [F] . . . 1314
3.154.7 Maxima [F] . . . 1315
3.154.8 Giac [F] . . . 1315
3.154.9 Mupad [F(-1)] . . . 1315

3.154.1 Optimal result

Integrand size = 23, antiderivative size = 336

$$\int \frac{\sec^3(c+dx)}{\sqrt[3]{a+a\sec(c+dx)}}\,dx = -\frac{9\tan(c+dx)}{10d\sqrt[3]{a+a\sec(c+dx)}} + \frac{3(a+a\sec(c+dx))^{2/3}\tan(c+dx)}{5ad} - \frac{7\,3^{3/4}\,\text{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{2}-\left(1-\sqrt{3}\right)\sqrt[3]{1+\sec(c+dx)}}{\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{1+\sec(c+dx)}}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)\left(\sqrt[3]{2}-\sqrt[3]{1+\sec(c+dx)}\right)\sqrt{\cdots}}{10\sqrt[3]{2}d(1-\sec(c+dx))\sqrt[3]{a+a\sec(c+dx)}\sqrt{-\frac{\sqrt[3]{1+\sec(c+dx)}\left(\sqrt[3]{2}\cdots\right)}{\left(\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{1\cdots}\right.}}}$$

output
```
-9/10*tan(d*x+c)/d/(a+a*sec(d*x+c))^(1/3)+3/5*(a+a*sec(d*x+c))^(2/3)*tan(d
*x+c)/a/d-7/20*3^(3/4)*((2^(1/3)-(1+sec(d*x+c))^(1/3)*(1-3^(1/2)))^2/(2^(1
/3)-(1+sec(d*x+c))^(1/3)*(1+3^(1/2)))^2)^(1/2)/(2^(1/3)-(1+sec(d*x+c))^(1/
3)*(1-3^(1/2)))*(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1+3^(1/2)))*EllipticF((1-(2
^(1/3)-(1+sec(d*x+c))^(1/3)*(1-3^(1/2)))^2/(2^(1/3)-(1+sec(d*x+c))^(1/3)*(
1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(2^(1/3)-(1+sec(d*x+c))^(1/3
))*((2^(2/3)+2^(1/3)*(1+sec(d*x+c))^(1/3)+(1+sec(d*x+c))^(2/3))/(2^(1/3)-(
1+sec(d*x+c))^(1/3)*(1+3^(1/2)))^2)^(1/2)*tan(d*x+c)*2^(2/3)/d/(1-sec(d*x+
c))/(a+a*sec(d*x+c))^(1/3)/(-(1+sec(d*x+c))^(1/3)*(2^(1/3)-(1+sec(d*x+c))^
(1/3))/(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

3.154.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.20 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.28

$$\int \frac{\sec^3(c+dx)}{\sqrt[3]{a+a\sec(c+dx)}}\,dx = \frac{\left(7\sqrt[6]{2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{3}{2},\frac{1}{2}(1-\sec(c+dx))\right)+3\sqrt[6]{1+\sec(c+dx)}(-1+2\sec(c+dx))\right)\tan(c}{10d\sqrt[6]{1+\sec(c+dx)}\sqrt[3]{a(1+\sec(c+dx))}}$$

input
```
Integrate[Sec[c + d*x]^3/(a + a*Sec[c + d*x])^(1/3),x]
```

output
```
((7*2^(1/6)*Hypergeometric2F1[1/2, 5/6, 3/2, (1 - Sec[c + d*x])/2] + 3*(1
+ Sec[c + d*x])^(1/6)*(-1 + 2*Sec[c + d*x]))*Tan[c + d*x])/(10*d*(1 + Sec[
c + d*x])^(1/6)*(a*(1 + Sec[c + d*x]))^(1/3))
```

3.154.3 Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 346, normalized size of antiderivative = 1.03, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules used = {3042, 4287, 27, 3042, 4489, 3042, 4315, 3042, 4314, 73, 766}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^3(c+dx)}{\sqrt[3]{a\sec(c+dx)+a}}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^3}{\sqrt[3]{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}}dx$$

$$\Big\downarrow \; 4287$$

$$\frac{3\int \frac{\sec(c+dx)(2a-3a\sec(c+dx))}{3\sqrt[3]{\sec(c+dx)a+a}}dx}{5a}+\frac{3\tan(c+dx)(a\sec(c+dx)+a)^{2/3}}{5ad}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{\sec(c+dx)(2a-3a\sec(c+dx))}{\sqrt[3]{\sec(c+dx)a+a}}dx}{5a}+\frac{3\tan(c+dx)(a\sec(c+dx)+a)^{2/3}}{5ad}$$

$\Big\downarrow$ 3042

$$\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(2a-3a\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt[3]{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{5a}+\frac{3\tan(c+dx)(a\sec(c+dx)+a)^{2/3}}{5ad}$$

$\Big\downarrow$ 4489

$$\frac{\frac{7}{2}a\int \frac{\sec(c+dx)}{\sqrt[3]{\sec(c+dx)a+a}}dx-\frac{9a\tan(c+dx)}{2d\sqrt[3]{a\sec(c+dx)+a}}}{5a}+\frac{3\tan(c+dx)(a\sec(c+dx)+a)^{2/3}}{5ad}$$

$\Big\downarrow$ 3042

$$\frac{\frac{7}{2}a\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt[3]{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx-\frac{9a\tan(c+dx)}{2d\sqrt[3]{a\sec(c+dx)+a}}}{5a}+\frac{3\tan(c+dx)(a\sec(c+dx)+a)^{2/3}}{5ad}$$

$\Big\downarrow$ 4315

$$\frac{\frac{7a\sqrt[3]{\sec(c+dx)+1}\int \frac{\sec(c+dx)}{\sqrt[3]{\sec(c+dx)+1}}dx}{2\sqrt[3]{a\sec(c+dx)+a}}-\frac{9a\tan(c+dx)}{2d\sqrt[3]{a\sec(c+dx)+a}}}{5a}+\frac{3\tan(c+dx)(a\sec(c+dx)+a)^{2/3}}{5ad}$$

$\Big\downarrow$ 3042

$$\frac{\frac{7a\sqrt[3]{\sec(c+dx)+1}\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt[3]{\csc\left(c+dx+\frac{\pi}{2}\right)+1}}dx}{2\sqrt[3]{a\sec(c+dx)+a}}-\frac{9a\tan(c+dx)}{2d\sqrt[3]{a\sec(c+dx)+a}}}{5a}+\frac{3\tan(c+dx)(a\sec(c+dx)+a)^{2/3}}{5ad}$$

$\Big\downarrow$ 4314

$$\frac{-\frac{7a\tan(c+dx)\int \frac{1}{\sqrt{1-\sec(c+dx)}(\sec(c+dx)+1)^{5/6}}d\sec(c+dx)}{2d\sqrt{1-\sec(c+dx)}\sqrt[6]{\sec(c+dx)+1}\sqrt[3]{a\sec(c+dx)+a}}-\frac{9a\tan(c+dx)}{2d\sqrt[3]{a\sec(c+dx)+a}}}{5a}+\frac{3\tan(c+dx)(a\sec(c+dx)+a)^{2/3}}{5ad}$$

$\Big\downarrow$ 73

3.154. $\int \frac{\sec^3(c+dx)}{\sqrt[3]{a+a\sec(c+dx)}}dx$

$$\frac{-\frac{21a\tan(c+dx)\int\frac{1}{\sqrt{1-\sec(c+dx)}}d\sqrt[6]{\sec(c+dx)+1}}{d\sqrt{1-\sec(c+dx)}\sqrt[6]{\sec(c+dx)+1}\sqrt[3]{a\sec(c+dx)+a}}-\frac{9a\tan(c+dx)}{2d\sqrt[3]{a\sec(c+dx)+a}}}{5a}+\frac{3\tan(c+dx)(a\sec(c+dx)+a)^{2/3}}{5ad}$$

$\Big\downarrow$ 766

$$-\frac{\frac{7\,3^{3/4}a\tan(c+dx)\left(\sqrt[3]{2}-\sqrt[3]{\sec(c+dx)+1}\right)\sqrt{\frac{(\sec(c+dx)+1)^{2/3}+\sqrt[3]{2}\sqrt[3]{\sec(c+dx)+1}+2^{2/3}}{\left(\sqrt[3]{2}-(1+\sqrt{3})\sqrt[3]{\sec(c+dx)+1}\right)^2}}\,\text{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{2}-(1-\sqrt{3})\sqrt[3]{\cdots}}{\sqrt[3]{2}-(1+\sqrt{3})\sqrt[3]{\cdots}}\right.\right.}{2\sqrt[3]{2}d(1-\sec(c+dx))\sqrt{-\frac{\sqrt[3]{\sec(c+dx)+1}\left(\sqrt[3]{2}-\sqrt[3]{\sec(c+dx)+1}\right)}{\left(\sqrt[3]{2}-(1+\sqrt{3})\sqrt[3]{\sec(c+dx)+1}\right)^2}}\sqrt[3]{a\sec(c+dx)+a\cdots}}}{5a}}{\frac{3\tan(c+dx)(a\sec(c+dx)+a)^{2/3}}{5ad}}$$

input
```
Int[Sec[c + d*x]^3/(a + a*Sec[c + d*x])^(1/3),x]
```

output
```
(3*(a + a*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(5*a*d) + ((-9*a*Tan[c + d*x])
/(2*d*(a + a*Sec[c + d*x])^(1/3)) - (7*3^(3/4)*a*EllipticF[ArcCos[(2^(1/3)
 - (1 - Sqrt[3])*(1 + Sec[c + d*x])^(1/3))/(2^(1/3) - (1 + Sqrt[3])*(1 + S
ec[c + d*x])^(1/3))], (2 + Sqrt[3])/4]*(2^(1/3) - (1 + Sec[c + d*x])^(1/3)
)*Sqrt[(2^(2/3) + 2^(1/3)*(1 + Sec[c + d*x])^(1/3) + (1 + Sec[c + d*x])^(2
/3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2]*Tan[c + d*x])/(
2*2^(1/3)*d*(1 - Sec[c + d*x])*(a + a*Sec[c + d*x])^(1/3)*Sqrt[-(((1 + Sec
[c + d*x])^(1/3)*(2^(1/3) - (1 + Sec[c + d*x])^(1/3)))/(2^(1/3) - (1 + Sqr
t[3])*(1 + Sec[c + d*x])^(1/3))^2)]))/(5*a)
```

3.154.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

3.154. $\int \frac{\sec^3(c+dx)}{\sqrt[3]{a+a\sec(c+dx)}}\,dx$

rule 766
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4287
```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(-Cot[e + f*x])*((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2
))), x] + Simp[1/(b*(m + 2))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(b*(
m + 1) - a*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 -
b^2, 0] &&  !LtQ[m, -2^(-1)]
```

rule 4314
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Simp[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x
]]*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(d*x)^(n - 1)*((a + b*x)^(m - 1/2
)/Sqrt[a - b*x]), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n},
x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

rule 4315
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Simp[a^IntPart[m]*((a + b*Csc[e + f*x])^FracPart[m
]/(1 + (b/a)*Csc[e + f*x])^FracPart[m])   Int[(1 + (b/a)*Csc[e + f*x])^m*(d
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^
2, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

rule 4489
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*((
a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Simp[(a*B*m + A*b*(m + 1))/(b*(m +
 1))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B
, e, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b
*(m + 1), 0] &&  !LtQ[m, -2^(-1)]
```

3.154. $\int \frac{\sec^3(c+dx)}{\sqrt[3]{a+a\sec(c+dx)}}\,dx$

3.154.4 Maple [F]

$$\int \frac{\sec (dx+c)^{3}}{(a+a\sec (dx+c))^{\frac{1}{3}}}dx$$

input
```
int(sec(d*x+c)^3/(a+a*sec(d*x+c))^(1/3),x)
```

output
```
int(sec(d*x+c)^3/(a+a*sec(d*x+c))^(1/3),x)
```

3.154.5 Fricas [F]

$$\int \frac{\sec^3(c+dx)}{\sqrt[3]{a+a\sec(c+dx)}}\,dx=\int \frac{\sec (dx+c)^{3}}{(a\sec (dx+c)+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate(sec(d*x+c)^3/(a+a*sec(d*x+c))^(1/3),x, algorithm="fricas")
```

output
```
integral(sec(d*x + c)^3/(a*sec(d*x + c) + a)^(1/3), x)
```

3.154.6 Sympy [F]

$$\int \frac{\sec^3(c+dx)}{\sqrt[3]{a+a\sec(c+dx)}}\,dx=\int \frac{\sec^{3}(c+dx)}{\sqrt[3]{a\left(\sec (c+dx)+1\right)}}\,dx$$

input
```
integrate(sec(d*x+c)**3/(a+a*sec(d*x+c))**(1/3),x)
```

output
```
Integral(sec(c + d*x)**3/(a*(sec(c + d*x) + 1))**(1/3), x)
```

3.154.7 Maxima [F]

$$\int \frac{\sec^3(c+dx)}{\sqrt[3]{a+a\sec(c+dx)}}\,dx = \int \frac{\sec(dx+c)^3}{(a\sec(dx+c)+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate(sec(d*x+c)^3/(a+a*sec(d*x+c))^(1/3),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)^3/(a*sec(d*x + c) + a)^(1/3), x)
```

3.154.8 Giac [F]

$$\int \frac{\sec^3(c+dx)}{\sqrt[3]{a+a\sec(c+dx)}}\,dx = \int \frac{\sec(dx+c)^3}{(a\sec(dx+c)+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate(sec(d*x+c)^3/(a+a*sec(d*x+c))^(1/3),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^3/(a*sec(d*x + c) + a)^(1/3), x)
```

3.154.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^3(c+dx)}{\sqrt[3]{a+a\sec(c+dx)}}\,dx = \int \frac{1}{\cos(c+d\,x)^3\left(a+\frac{a}{\cos(c+d\,x)}\right)^{1/3}}\,dx$$

input
```
int(1/(cos(c + d*x)^3*(a + a/cos(c + d*x))^(1/3)),x)
```

output
```
int(1/(cos(c + d*x)^3*(a + a/cos(c + d*x))^(1/3)), x)
```

3.155 $\int \frac{\sec^2(c+dx)}{\sqrt[3]{a+a\sec(c+dx)}}\,dx$

3.155.1 Optimal result 1316
3.155.2 Mathematica [C] (verified) 1317
3.155.3 Rubi [A] (verified) 1317
3.155.4 Maple [F] 1320
3.155.5 Fricas [F] 1320
3.155.6 Sympy [F] 1320
3.155.7 Maxima [F] 1321
3.155.8 Giac [F] 1321
3.155.9 Mupad [F(-1)] 1321

3.155.1 Optimal result

Integrand size = 23, antiderivative size = 306

$$\int \frac{\sec^2(c+dx)}{\sqrt[3]{a+a\sec(c+dx)}}\,dx = \frac{3\tan(c+dx)}{2d\sqrt[3]{a+a\sec(c+dx)}} + \frac{3^{3/4}\,\text{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{2}-\left(1-\sqrt{3}\right)\sqrt[3]{1+\sec(c+dx)}}{\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{1+\sec(c+dx)}}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)\left(\sqrt[3]{2}-\sqrt[3]{1+\sec(c+dx)}\right)\sqrt{\frac{2^{2/}}{}}}{2\sqrt[3]{2}d(1-\sec(c+dx))\sqrt[3]{a+a\sec(c+dx)}\sqrt{-\frac{\sqrt[3]{1+\sec(c+dx)}\left(\sqrt[3]{2}-\right.}{\left(\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{1+}\right.}}}$$

output
```
3/2*tan(d*x+c)/d/(a+a*sec(d*x+c))^(1/3)+1/4*3^(3/4)*((2^(1/3)-(1+sec(d*x+c
))^(1/3)*(1-3^(1/2)))^2/(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1+3^(1/2)))^2)^(1/2
)/(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1-3^(1/2)))*(2^(1/3)-(1+sec(d*x+c))^(1/3)
*(1+3^(1/2)))*EllipticF((1-(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1-3^(1/2)))^2/(2
^(1/3)-(1+sec(d*x+c))^(1/3)*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))
*(2^(1/3)-(1+sec(d*x+c))^(1/3))*((2^(2/3)+2^(1/3)*(1+sec(d*x+c))^(1/3)+(1+
sec(d*x+c))^(2/3))/(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1+3^(1/2)))^2)^(1/2)*tan
(d*x+c)*2^(2/3)/d/(1-sec(d*x+c))/(a+a*sec(d*x+c))^(1/3)/(-(1+sec(d*x+c))^(
1/3)*(2^(1/3)-(1+sec(d*x+c))^(1/3))/(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1+3^(1/
2)))^2)^(1/2)
```

3.155.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.12 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.28

$$\int \frac{\sec^2(c+dx)}{\sqrt[3]{a+a\sec(c+dx)}}\,dx = \frac{\left(-\sqrt[6]{2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{3}{2},\frac{1}{2}(1-\sec(c+dx))\right)+3\sqrt[6]{1+\sec(c+dx)}\right)\tan(c+dx)}{2d\sqrt[6]{1+\sec(c+dx)}\sqrt[3]{a(1+\sec(c+dx))}}$$

input
```
Integrate[Sec[c + d*x]^2/(a + a*Sec[c + d*x])^(1/3),x]
```

output
```
((-(2^(1/6)*Hypergeometric2F1[1/2, 5/6, 3/2, (1 - Sec[c + d*x])/2]) + 3*(1
 + Sec[c + d*x])^(1/6))*Tan[c + d*x])/(2*d*(1 + Sec[c + d*x])^(1/6)*(a*(1
+ Sec[c + d*x]))^(1/3))
```

3.155.3 Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 306, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {3042, 4285, 3042, 4315, 3042, 4314, 73, 766}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(c+dx)}{\sqrt[3]{a\sec(c+dx)+a}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt[3]{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}}dx$$

$$\Big\downarrow \text{4285}$$

$$\frac{3\tan(c+dx)}{2d\sqrt[3]{a\sec(c+dx)+a}}-\frac{1}{2}\int \frac{\sec(c+dx)}{\sqrt[3]{\sec(c+dx)a+a}}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\tan(c+dx)}{2d\sqrt[3]{a\sec(c+dx)+a}}-\frac{1}{2}\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt[3]{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx$$

$$\Big\downarrow \text{4315}$$

$$\frac{3\tan(c+dx)}{2d\sqrt[3]{a\sec(c+dx)+a}}-\frac{\sqrt[3]{\sec(c+dx)+1}\int\frac{\sec(c+dx)}{\sqrt[3]{\sec(c+dx)+1}}dx}{2\sqrt[3]{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\tan(c+dx)}{2d\sqrt[3]{a\sec(c+dx)+a}}-\frac{\sqrt[3]{\sec(c+dx)+1}\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt[3]{\csc\left(c+dx+\frac{\pi}{2}\right)+1}}dx}{2\sqrt[3]{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{4314}$$

$$\frac{\tan(c+dx)\int\frac{1}{\sqrt{1-\sec(c+dx)}(\sec(c+dx)+1)^{5/6}}d\sec(c+dx)}{2d\sqrt{1-\sec(c+dx)}\sqrt[6]{\sec(c+dx)+1}\sqrt[3]{a\sec(c+dx)+a}}+\frac{3\tan(c+dx)}{2d\sqrt[3]{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{73}$$

$$\frac{3\tan(c+dx)\int\frac{1}{\sqrt{1-\sec(c+dx)}}d\sqrt[6]{\sec(c+dx)+1}}{d\sqrt{1-\sec(c+dx)}\sqrt[6]{\sec(c+dx)+1}\sqrt[3]{a\sec(c+dx)+a}}+\frac{3\tan(c+dx)}{2d\sqrt[3]{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{766}$$

$$\frac{3^{3/4}\tan(c+dx)\left(\sqrt[3]{2}-\sqrt[3]{\sec(c+dx)+1}\right)\sqrt{\frac{(\sec(c+dx)+1)^{2/3}+\sqrt[3]{2}\sqrt[3]{\sec(c+dx)+1}+2^{2/3}}{\left(\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}\right)^2}}\text{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{2}-}{\sqrt[3]{2}-}\right.\right.}{2\sqrt[3]{2}d(1-\sec(c+dx))\sqrt{-\frac{\sqrt[3]{\sec(c+dx)+1}\left(\sqrt[3]{2}-\sqrt[3]{\sec(c+dx)+1}\right)}{\left(\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}\right)^2}}\sqrt[3]{a\sec(c}}$$

$$\frac{3\tan(c+dx)}{2d\sqrt[3]{a\sec(c+dx)+a}}$$

input
```
Int[Sec[c + d*x]^2/(a + a*Sec[c + d*x])^(1/3),x]
```

3.155. $\int\frac{\sec^2(c+dx)}{\sqrt[3]{a+a\sec(c+dx)}}dx$

output
```
(3*Tan[c + d*x])/(2*d*(a + a*Sec[c + d*x])^(1/3)) + (3^(3/4)*EllipticF[Arc
Cos[(2^(1/3) - (1 - Sqrt[3])*(1 + Sec[c + d*x])^(1/3))/(2^(1/3) - (1 + Sqr
t[3])*(1 + Sec[c + d*x])^(1/3))], (2 + Sqrt[3])/4]*(2^(1/3) - (1 + Sec[c +
 d*x])^(1/3))*Sqrt[(2^(2/3) + 2^(1/3)*(1 + Sec[c + d*x])^(1/3) + (1 + Sec[
c + d*x])^(2/3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2]*Tan
[c + d*x])/(2*2^(1/3)*d*(1 - Sec[c + d*x])*(a + a*Sec[c + d*x])^(1/3)*Sqrt
[-(((1 + Sec[c + d*x])^(1/3)*(2^(1/3) - (1 + Sec[c + d*x])^(1/3)))/(2^(1/3
) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2)])
```

3.155.3.1 Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 766
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4285
```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(-Cot[e + f*x])*((a + b*Csc[e + f*x])^m/(f*(m + 1))), x]
+ Simp[a*(m/(b*(m + 1)))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x]
/; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

rule 4314
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Simp[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x
]]*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(d*x)^(n - 1)*((a + b*x)^(m - 1/2
)/Sqrt[a - b*x]), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n},
x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

3.155. $\int \frac{\sec^2(c+dx)}{\sqrt[3]{a+a\sec(c+dx)}}\,dx$

rule 4315
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Simp[a^IntPart[m]*((a + b*Csc[e + f*x])^FracPart[m
]/(1 + (b/a)*Csc[e + f*x])^FracPart[m])   Int[(1 + (b/a)*Csc[e + f*x])^m*(d
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^
2, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

3.155.4 Maple [F]

$$\int \frac{\sec (dx+c)^2}{(a+a\sec (dx+c))^{\frac{1}{3}}}dx$$

input
```
int(sec(d*x+c)^2/(a+a*sec(d*x+c))^(1/3),x)
```

output
```
int(sec(d*x+c)^2/(a+a*sec(d*x+c))^(1/3),x)
```

3.155.5 Fricas [F]

$$\int \frac{\sec^2(c+dx)}{\sqrt[3]{a+a\sec(c+dx)}}\,dx=\int \frac{\sec (dx+c)^2}{(a\sec (dx+c)+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate(sec(d*x+c)^2/(a+a*sec(d*x+c))^(1/3),x, algorithm="fricas")
```

output
```
integral(sec(d*x + c)^2/(a*sec(d*x + c) + a)^(1/3), x)
```

3.155.6 Sympy [F]

$$\int \frac{\sec^2(c+dx)}{\sqrt[3]{a+a\sec(c+dx)}}\,dx=\int \frac{\sec^2 (c+dx)}{\sqrt[3]{a (\sec (c+dx)+1)}}\,dx$$

input
```
integrate(sec(d*x+c)**2/(a+a*sec(d*x+c))**(1/3),x)
```

output
```
Integral(sec(c + d*x)**2/(a*(sec(c + d*x) + 1))**(1/3), x)
```

3.155.7 Maxima [F]

$$\int \frac{\sec^2(c+dx)}{\sqrt[3]{a+a\sec(c+dx)}}\,dx = \int \frac{\sec(dx+c)^2}{(a\sec(dx+c)+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate(sec(d*x+c)^2/(a+a*sec(d*x+c))^(1/3),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)^2/(a*sec(d*x + c) + a)^(1/3), x)
```

3.155.8 Giac [F]

$$\int \frac{\sec^2(c+dx)}{\sqrt[3]{a+a\sec(c+dx)}}\,dx = \int \frac{\sec(dx+c)^2}{(a\sec(dx+c)+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate(sec(d*x+c)^2/(a+a*sec(d*x+c))^(1/3),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^2/(a*sec(d*x + c) + a)^(1/3), x)
```

3.155.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^2(c+dx)}{\sqrt[3]{a+a\sec(c+dx)}}\,dx = \int \frac{1}{\cos(c+d\,x)^2\left(a+\frac{a}{\cos(c+d\,x)}\right)^{1/3}}\,dx$$

input
```
int(1/(cos(c + d*x)^2*(a + a/cos(c + d*x))^(1/3)),x)
```

output
```
int(1/(cos(c + d*x)^2*(a + a/cos(c + d*x))^(1/3)), x)
```

3.156 $\int \frac{\sec(c+dx)}{\sqrt[3]{a+a\sec(c+dx)}}\,dx$

3.156.1 Optimal result 1322
3.156.2 Mathematica [C] (verified) 1323
3.156.3 Rubi [A] (verified) 1323
3.156.4 Maple [F] 1325
3.156.5 Fricas [F] 1326
3.156.6 Sympy [F] 1326
3.156.7 Maxima [F] 1326
3.156.8 Giac [F] 1327
3.156.9 Mupad [F(-1)] 1327

3.156.1 Optimal result

Integrand size = 21, antiderivative size = 276

$$\int \frac{\sec(c+dx)}{\sqrt[3]{a+a\sec(c+dx)}}\,dx = -\frac{3^{3/4}\,\text{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{2}-\left(1-\sqrt{3}\right)\sqrt[3]{1+\sec(c+dx)}}{\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{1+\sec(c+dx)}}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)\left(\sqrt[3]{2}-\sqrt[3]{1+\sec(c+dx)}\right)\sqrt{\frac{2^{2}}{}}}{\sqrt[3]{2}d(1-\sec(c+dx))\sqrt[3]{a+a\sec(c+dx)}\sqrt{-\frac{\sqrt[3]{1+\sec(c+dx)}\left(\sqrt[3]{2}-\right.}{\left(\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{1+}\right.}}}$$

output
```
-1/2*3^(3/4)*((2^(1/3)-(1+sec(d*x+c))^(1/3)*(1-3^(1/2)))^2/(2^(1/3)-(1+sec
(d*x+c))^(1/3)*(1+3^(1/2)))^2)^(1/2)/(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1-3^(1
/2)))*(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1+3^(1/2)))*EllipticF((1-(2^(1/3)-(1+
sec(d*x+c))^(1/3)*(1-3^(1/2)))^2/(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1+3^(1/2))
)^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(2^(1/3)-(1+sec(d*x+c))^(1/3))*((2^(2/
3)+2^(1/3)*(1+sec(d*x+c))^(1/3)+(1+sec(d*x+c))^(2/3))/(2^(1/3)-(1+sec(d*x+
c))^(1/3)*(1+3^(1/2)))^2)^(1/2)*tan(d*x+c)*2^(2/3)/d/(1-sec(d*x+c))/(a+a*s
ec(d*x+c))^(1/3)/(-(1+sec(d*x+c))^(1/3)*(2^(1/3)-(1+sec(d*x+c))^(1/3))/(2^
(1/3)-(1+sec(d*x+c))^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

3.156.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.07 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.24

$$\int \frac{\sec(c+dx)}{\sqrt[3]{a+a\sec(c+dx)}}\,dx = \frac{\sqrt[6]{2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{3}{2},\frac{1}{2}(1-\sec(c+dx))\right)\tan(c+dx)}{d\sqrt[6]{1+\sec(c+dx)}\sqrt[3]{a(1+\sec(c+dx))}}$$

input
```
Integrate[Sec[c + d*x]/(a + a*Sec[c + d*x])^(1/3),x]
```

output
```
(2^(1/6)*Hypergeometric2F1[1/2, 5/6, 3/2, (1 - Sec[c + d*x])/2]*Tan[c + d*
x])/(d*(1 + Sec[c + d*x])^(1/6)*(a*(1 + Sec[c + d*x]))^(1/3))
```

3.156.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 276, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 4315, 3042, 4314, 73, 766}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(c+dx)}{\sqrt[3]{a\sec(c+dx)+a}}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt[3]{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}}dx$$

$$\Big\downarrow \; 4315$$

$$\frac{\sqrt[3]{\sec(c+dx)+1}\int \frac{\sec(c+dx)}{\sqrt[3]{\sec(c+dx)+1}}dx}{\sqrt[3]{a\sec(c+dx)+a}}$$

$$\Big\downarrow \; 3042$$

$$\frac{\sqrt[3]{\sec(c+dx)+1}\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt[3]{\csc\left(c+dx+\frac{\pi}{2}\right)+1}}dx}{\sqrt[3]{a\sec(c+dx)+a}}$$

$$\Big\downarrow \; 4314$$

$$-\frac{\tan(c+dx)\int \frac{1}{\sqrt{1-\sec(c+dx)}(\sec(c+dx)+1)^{5/6}}d\sec(c+dx)}{d\sqrt{1-\sec(c+dx)}\sqrt[6]{\sec(c+dx)+1}\sqrt[3]{a\sec(c+dx)+a}}$$

$$\Big\downarrow \; 73$$

$$-\frac{6\tan(c+dx)\int \frac{1}{\sqrt{1-\sec(c+dx)}}d\sqrt[6]{\sec(c+dx)+1}}{d\sqrt{1-\sec(c+dx)}\sqrt[6]{\sec(c+dx)+1}\sqrt[3]{a\sec(c+dx)+a}}$$

$$\Big\downarrow \; 766$$

$$-\frac{3^{3/4}\tan(c+dx)\left(\sqrt[3]{2}-\sqrt[3]{\sec(c+dx)+1}\right)\sqrt{\frac{(\sec(c+dx)+1)^{2/3}+\sqrt[3]{2}\sqrt[3]{\sec(c+dx)+1}+2^{2/3}}{\left(\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}\right)^2}}\,\text{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{2}}{\sqrt[3]{2}}\right.\right.}{\sqrt[3]{2}d(1-\sec(c+dx))\sqrt{-\frac{\sqrt[3]{\sec(c+dx)+1}\left(\sqrt[3]{2}-\sqrt[3]{\sec(c+dx)+1}\right)}{\left(\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}\right)^2}}\sqrt[3]{a\sec(c}}$$

input
```
Int[Sec[c + d*x]/(a + a*Sec[c + d*x])^(1/3),x]
```

output
```
-((3^(3/4)*EllipticF[ArcCos[(2^(1/3) - (1 - Sqrt[3])*(1 + Sec[c + d*x])^(1
/3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))], (2 + Sqrt[3])/4]
*(2^(1/3) - (1 + Sec[c + d*x])^(1/3))*Sqrt[(2^(2/3) + 2^(1/3)*(1 + Sec[c +
 d*x])^(1/3) + (1 + Sec[c + d*x])^(2/3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec
[c + d*x])^(1/3))^2]*Tan[c + d*x])/(2^(1/3)*d*(1 - Sec[c + d*x])*(a + a*Se
c[c + d*x])^(1/3)*Sqrt[-(((1 + Sec[c + d*x])^(1/3)*(2^(1/3) - (1 + Sec[c +
 d*x])^(1/3)))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2)]))
```

3.156.3.1 Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 766
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4314
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Simp[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x
]]*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(d*x)^(n - 1)*((a + b*x)^(m - 1/2
)/Sqrt[a - b*x]), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n},
x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

rule 4315
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Simp[a^IntPart[m]*((a + b*Csc[e + f*x])^FracPart[m
]/(1 + (b/a)*Csc[e + f*x])^FracPart[m])   Int[(1 + (b/a)*Csc[e + f*x])^m*(d
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^
2, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

3.156.4 Maple [F]

$$\int \frac{\sec(dx+c)}{(a+a\sec(dx+c))^{\frac{1}{3}}}dx$$

input
```
int(sec(d*x+c)/(a+a*sec(d*x+c))^(1/3),x)
```

output
```
int(sec(d*x+c)/(a+a*sec(d*x+c))^(1/3),x)
```

3.156.5 Fricas [F]

$$\int \frac{\sec(c+dx)}{\sqrt[3]{a+a\sec(c+dx)}}\,dx = \int \frac{\sec(dx+c)}{(a\sec(dx+c)+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate(sec(d*x+c)/(a+a*sec(d*x+c))^(1/3),x, algorithm="fricas")
```

output
```
integral(sec(d*x + c)/(a*sec(d*x + c) + a)^(1/3), x)
```

3.156.6 Sympy [F]

$$\int \frac{\sec(c+dx)}{\sqrt[3]{a+a\sec(c+dx)}}\,dx = \int \frac{\sec(c+dx)}{\sqrt[3]{a(\sec(c+dx)+1)}}\,dx$$

input
```
integrate(sec(d*x+c)/(a+a*sec(d*x+c))**(1/3),x)
```

output
```
Integral(sec(c + d*x)/(a*(sec(c + d*x) + 1))**(1/3), x)
```

3.156.7 Maxima [F]

$$\int \frac{\sec(c+dx)}{\sqrt[3]{a+a\sec(c+dx)}}\,dx = \int \frac{\sec(dx+c)}{(a\sec(dx+c)+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate(sec(d*x+c)/(a+a*sec(d*x+c))^(1/3),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)/(a*sec(d*x + c) + a)^(1/3), x)
```

3.156.8 Giac [F]

$$\int \frac{\sec(c+dx)}{\sqrt[3]{a+a\sec(c+dx)}}\,dx = \int \frac{\sec(dx+c)}{(a\sec(dx+c)+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate(sec(d*x+c)/(a+a*sec(d*x+c))^(1/3),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)/(a*sec(d*x + c) + a)^(1/3), x)
```

3.156.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(c+dx)}{\sqrt[3]{a+a\sec(c+dx)}}\,dx = \int \frac{1}{\cos(c+d\,x)\,\left(a+\frac{a}{\cos(c+d\,x)}\right)^{1/3}}\,dx$$

input
```
int(1/(cos(c + d*x)*(a + a/cos(c + d*x))^(1/3)),x)
```

output
```
int(1/(cos(c + d*x)*(a + a/cos(c + d*x))^(1/3)), x)
```

3.157 $\int \frac{1}{\sqrt[3]{a+a\sec(c+dx)}}\,dx$

3.157.1 Optimal result 1328
3.157.2 Mathematica [B] (warning: unable to verify) 1328
3.157.3 Rubi [A] (verified) 1329
3.157.4 Maple [F] 1331
3.157.5 Fricas [F(-1)] 1331
3.157.6 Sympy [F] 1332
3.157.7 Maxima [F] 1332
3.157.8 Giac [F] 1332
3.157.9 Mupad [F(-1)] 1333

3.157.1 Optimal result

Integrand size = 14, antiderivative size = 75

$$\int \frac{1}{\sqrt[3]{a+a\sec(c+dx)}}\,dx = \frac{3\sqrt{2}\,\text{AppellF1}\left(\frac{1}{6},\frac{1}{2},1,\frac{7}{6},\frac{1}{2}(1+\sec(c+dx)),1+\sec(c+dx)\right)\tan(c+dx)}{d\sqrt{1-\sec(c+dx)}\,\sqrt[3]{a+a\sec(c+dx)}}$$

output
```
3*AppellF1(1/6,1,1/2,7/6,1+sec(d*x+c),1/2+1/2*sec(d*x+c))*2^(1/2)*tan(d*x+
c)/d/(a+a*sec(d*x+c))^(1/3)/(1-sec(d*x+c))^(1/2)
```

3.157.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 718 vs. 2(75) = 150.

Time = 4.78 (sec) , antiderivative size = 718, normalized size of antiderivative = 9.57

$$\int \frac{1}{\sqrt[3]{a+a\sec(c+dx)}}\,dx = \frac{}{d\sqrt[3]{a(1+\sec(c+dx))}\left(40\left(3\,\text{AppellF1}\left(\frac{3}{2},-\frac{1}{3},2,\frac{5}{2},\tan^2\left(\frac{1}{2}(c+dx)\right),-\tan^2\left(\frac{1}{2}(c+dx)\right)\right)+\text{AppellF1}\left(\frac{3}{2}\right.\right.\right.}$$

input
```
Integrate[(a + a*Sec[c + d*x])^(-1/3),x]
```

output
```
(45*AppellF1[1/2, -1/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*C
os[c + d*x]*(1 + Sec[c + d*x])^2*Tan[(c + d*x)/2]*(9*AppellF1[1/2, -1/3, 1
, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - 2*(3*AppellF1[3/2, -1/3,
 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + AppellF1[3/2, 2/3, 1,
5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Tan[(c + d*x)/2]^2))/(d*(a*
(1 + Sec[c + d*x]))^(1/3)*(40*(3*AppellF1[3/2, -1/3, 2, 5/2, Tan[(c + d*x)
/2]^2, -Tan[(c + d*x)/2]^2] + AppellF1[3/2, 2/3, 1, 5/2, Tan[(c + d*x)/2]^
2, -Tan[(c + d*x)/2]^2])^2*Sec[c + d*x]*Sin[(c + d*x)/2]^2*Tan[(c + d*x)/2
]^2 + 6*AppellF1[1/2, -1/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^
2]*Sec[c + d*x]^2*Sin[(c + d*x)/2]^2*(-15*AppellF1[3/2, -1/3, 2, 5/2, Tan[
(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(1 - 10*Cos[c + d*x] + 3*Cos[2*(c + d
*x)]) - 5*AppellF1[3/2, 2/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]
^2]*(1 - 10*Cos[c + d*x] + 3*Cos[2*(c + d*x)]) - 24*(9*AppellF1[5/2, -1/3,
 3, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + 3*AppellF1[5/2, 2/3, 2
, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - AppellF1[5/2, 5/3, 1, 7/
2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Cos[c + d*x]*Tan[(c + d*x)/2]
^2) + 135*AppellF1[1/2, -1/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2
]^2]^2*(3 + 3*Sec[c + d*x] - 3*Sin[c + d*x]*Tan[c + d*x] - Tan[c + d*x]^2)
))
```

3.157.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 4266, 3042, 4265, 149, 25, 936}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[3]{a\sec(c+dx)+a}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt[3]{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}}dx$$

$$\Big\downarrow \text{4266}$$

$$\frac{\sqrt[3]{\sec(c+dx)+1}\int \frac{1}{\sqrt[3]{\sec(c+dx)+1}}dx}{\sqrt[3]{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt[3]{\sec(c+dx)+1}\int \frac{1}{\sqrt[3]{\csc\left(c+dx+\frac{\pi}{2}\right)+1}}dx}{\sqrt[3]{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{4265}$$

$$-\frac{\tan(c+dx)\int \frac{\cos(c+dx)}{\sqrt{1-\sec(c+dx)}(\sec(c+dx)+1)^{5/6}}d\sec(c+dx)}{d\sqrt{1-\sec(c+dx)}\sqrt[6]{\sec(c+dx)+1}\sqrt[3]{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{149}$$

$$-\frac{6\tan(c+dx)\int \frac{\cos(c+dx)}{\sqrt{1-\sec(c+dx)}}d\sqrt[6]{\sec(c+dx)+1}}{d\sqrt{1-\sec(c+dx)}\sqrt[6]{\sec(c+dx)+1}\sqrt[3]{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{25}$$

$$\frac{6\tan(c+dx)\int -\frac{\cos(c+dx)}{\sqrt{1-\sec(c+dx)}}d\sqrt[6]{\sec(c+dx)+1}}{d\sqrt{1-\sec(c+dx)}\sqrt[6]{\sec(c+dx)+1}\sqrt[3]{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{936}$$

$$\frac{3\sqrt{2}\tan(c+dx)\,\text{AppellF1}\left(\frac{1}{6},1,\frac{1}{2},\frac{7}{6},\sec(c+dx)+1,\frac{1}{2}(\sec(c+dx)+1)\right)}{d\sqrt{1-\sec(c+dx)}\sqrt[3]{a\sec(c+dx)+a}}$$

input
```
Int[(a + a*Sec[c + d*x])^(-1/3),x]
```

output
```
(3*Sqrt[2]*AppellF1[1/6, 1, 1/2, 7/6, 1 + Sec[c + d*x], (1 + Sec[c + d*x])
/2]*Tan[c + d*x])/(d*Sqrt[1 - Sec[c + d*x]]*(a + a*Sec[c + d*x])^(1/3))
```

3.157.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 149
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_)
)^(p_.), x_] :> With[{k = Denominator[m]}, Simp[k/b   Subst[Int[x^(k*(m + 1
) - 1)*(c - a*(d/b) + d*(x^k/b))^n*(e - a*(f/b) + f*(x^k/b))^p, x], x, (a +
 b*x)^(1/k)], x]] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && FractionQ[m] &&
IntegerQ[2*n] && IntegerQ[p]
```

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4265
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[a^n*(Cot
[c + d*x]/(d*Sqrt[1 + Csc[c + d*x]]*Sqrt[1 - Csc[c + d*x]]))   Subst[Int[(1
 + b*(x/a))^(n - 1/2)/(x*Sqrt[1 - b*(x/a)]), x], x, Csc[c + d*x]], x] /; Fr
eeQ[{a, b, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0
]
```

rule 4266
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[a^IntPar
t[n]*((a + b*Csc[c + d*x])^FracPart[n]/(1 + (b/a)*Csc[c + d*x])^FracPart[n]
)   Int[(1 + (b/a)*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] &&
EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

3.157.4 Maple [F]

$$\int \frac{1}{(a+a\sec(dx+c))^{\frac{1}{3}}}dx$$

input
```
int(1/(a+a*sec(d*x+c))^(1/3),x)
```

output
```
int(1/(a+a*sec(d*x+c))^(1/3),x)
```

3.157.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{a+a\sec(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+a*sec(d*x+c))^(1/3),x, algorithm="fricas")
```

output
```
Timed out
```

3.157.6 Sympy [F]

$$\int \frac{1}{\sqrt[3]{a+a\sec(c+dx)}}\,dx=\int \frac{1}{\sqrt[3]{a\sec{(c+dx)}+a}}\,dx$$

input
```
integrate(1/(a+a*sec(d*x+c))**(1/3),x)
```

output
```
Integral((a*sec(c + d*x) + a)**(-1/3), x)
```

3.157.7 Maxima [F]

$$\int \frac{1}{\sqrt[3]{a+a\sec(c+dx)}}\,dx=\int \frac{1}{(a\sec{(dx+c)}+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/(a+a*sec(d*x+c))^(1/3),x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)^(-1/3), x)
```

3.157.8 Giac [F]

$$\int \frac{1}{\sqrt[3]{a+a\sec(c+dx)}}\,dx=\int \frac{1}{(a\sec{(dx+c)}+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/(a+a*sec(d*x+c))^(1/3),x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)^(-1/3), x)
```

3.157.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{a+a\sec(c+dx)}}\,dx=\int \frac{1}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{1/3}}\,dx$$

input
```
int(1/(a + a/cos(c + d*x))^(1/3),x)
```

output
```
int(1/(a + a/cos(c + d*x))^(1/3), x)
```

3.158 $\int \frac{\cos(c+dx)}{\sqrt[3]{a+a\sec(c+dx)}}\,dx$

3.158.1 Optimal result . . . 1334
3.158.2 Mathematica [B] (warning: unable to verify) . . . 1334
3.158.3 Rubi [A] (verified) . . . 1335
3.158.4 Maple [F] . . . 1337
3.158.5 Fricas [F(-1)] . . . 1337
3.158.6 Sympy [F] . . . 1337
3.158.7 Maxima [F] . . . 1338
3.158.8 Giac [F] . . . 1338
3.158.9 Mupad [F(-1)] . . . 1338

3.158.1 Optimal result

Integrand size = 21, antiderivative size = 75

$$\int \frac{\cos(c+dx)}{\sqrt[3]{a+a\sec(c+dx)}}\,dx = -\frac{3\sqrt{2}\,\text{AppellF1}\left(\frac{1}{6},\frac{1}{2},2,\frac{7}{6},\frac{1}{2}(1+\sec(c+dx)),1+\sec(c+dx)\right)\tan(c+dx)}{d\sqrt{1-\sec(c+dx)}\,\sqrt[3]{a+a\sec(c+dx)}}$$

output
```
-3*AppellF1(1/6,2,1/2,7/6,1+sec(d*x+c),1/2+1/2*sec(d*x+c))*2^(1/2)*tan(d*x
+c)/d/(a+a*sec(d*x+c))^(1/3)/(1-sec(d*x+c))^(1/2)
```

3.158.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 240 vs. 2(75) = 150.

Time = 1.90 (sec) , antiderivative size = 240, normalized size of antiderivative = 3.20

$$\int \frac{\cos(c+dx)}{\sqrt[3]{a+a\sec(c+dx)}}\,dx = \frac{(a(1+\sec(c+dx)))^{2/3}\left(\frac{20\,\text{AppellF1}\left(\frac{3}{2},\frac{2}{3},1,\frac{5}{2},\tan^2\left(\frac{1}{2}(c+dx)\right),-\tan^2\left(\frac{1}{2}(c\right.\right.}{6\left(3\,\text{AppellF1}\left(\frac{5}{2},\frac{2}{3},2,\frac{7}{2},\tan^2\left(\frac{1}{2}(c+dx)\right),-\tan^2\left(\frac{1}{2}(c+dx)\right)\right)-2\,\text{AppellF1}\left(\frac{5}{2},\frac{5}{3},1,\frac{7}{2},\tan^2\left(\frac{1}{2}(c+dx)\right),-\tan^2\left(\frac{1}{2}(c+\right.\right.}\right)}{a}$$

input
```
Integrate[Cos[c + d*x]/(a + a*Sec[c + d*x])^(1/3),x]
```

output
```
((a*(1 + Sec[c + d*x]))^(2/3)*((20*AppellF1[3/2, 2/3, 1, 5/2, Tan[(c + d*x
)/2]^2, -Tan[(c + d*x)/2]^2]*Cos[(c + d*x)/2]*Sin[(c + d*x)/2]^3)/(6*(3*Ap
pellF1[5/2, 2/3, 2, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - 2*Appe
llF1[5/2, 5/3, 1, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*(-1 + Cos
[c + d*x]) + 45*AppellF1[3/2, 2/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d
*x)/2]^2]*(1 + Cos[c + d*x])) + Sin[c + d*x] - Tan[(c + d*x)/2]))/(a*d)
```

3.158.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 4315, 3042, 4314, 149, 936}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(c+dx)}{\sqrt[3]{a\sec(c+dx)+a}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)\sqrt[3]{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}}dx$$

$$\Big\downarrow \text{4315}$$

$$\frac{\sqrt[3]{\sec(c+dx)+1}\int \frac{\cos(c+dx)}{\sqrt[3]{\sec(c+dx)+1}}dx}{\sqrt[3]{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt[3]{\sec(c+dx)+1}\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)\sqrt[3]{\csc\left(c+dx+\frac{\pi}{2}\right)+1}}dx}{\sqrt[3]{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{4314}$$

$$-\frac{\tan(c+dx)\int \frac{\cos^2(c+dx)}{\sqrt{1-\sec(c+dx)}(\sec(c+dx)+1)^{5/6}}d\sec(c+dx)}{d\sqrt{1-\sec(c+dx)}\sqrt[6]{\sec(c+dx)+1}\sqrt[3]{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{149}$$

$$-\frac{6\tan(c+dx)\int \frac{\cos^2(c+dx)}{\sqrt{1-\sec(c+dx)}}d\sqrt[6]{\sec(c+dx)+1}}{d\sqrt{1-\sec(c+dx)}\sqrt[6]{\sec(c+dx)+1}\sqrt[3]{a\sec(c+dx)+a}}$$

$$\Big\downarrow \; 936$$

$$-\frac{3\sqrt{2}\tan(c+dx)\,\text{AppellF1}\left(\frac{1}{6},2,\frac{1}{2},\frac{7}{6},\sec(c+dx)+1,\frac{1}{2}(\sec(c+dx)+1)\right)}{d\sqrt{1-\sec(c+dx)}\sqrt[3]{a\sec(c+dx)+a}}$$

input
```
Int[Cos[c + d*x]/(a + a*Sec[c + d*x])^(1/3),x]
```

output
```
(-3*Sqrt[2]*AppellF1[1/6, 2, 1/2, 7/6, 1 + Sec[c + d*x], (1 + Sec[c + d*x]
)/2]*Tan[c + d*x])/(d*Sqrt[1 - Sec[c + d*x]]*(a + a*Sec[c + d*x])^(1/3))
```

3.158.3.1 Defintions of rubi rules used

rule 149
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_)
)^(p_.), x_] :> With[{k = Denominator[m]}, Simp[k/b   Subst[Int[x^(k*(m + 1
) - 1)*(c - a*(d/b) + d*(x^k/b))^n*(e - a*(f/b) + f*(x^k/b))^p, x], x, (a +
 b*x)^(1/k)], x]] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && FractionQ[m] &&
IntegerQ[2*n] && IntegerQ[p]
```

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4314
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Simp[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x
]]*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(d*x)^(n - 1)*((a + b*x)^(m - 1/2
)/Sqrt[a - b*x]), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n},
x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

3.158. $\int \frac{\cos(c+dx)}{\sqrt[3]{a+a\sec(c+dx)}}\,dx$

rule 4315
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Simp[a^IntPart[m]*((a + b*Csc[e + f*x])^FracPart[m
]/(1 + (b/a)*Csc[e + f*x])^FracPart[m])   Int[(1 + (b/a)*Csc[e + f*x])^m*(d
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^
2, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

3.158.4 Maple [F]

$$\int \frac{\cos(dx+c)}{(a+a\sec(dx+c))^{\frac{1}{3}}}dx$$

input
```
int(cos(d*x+c)/(a+a*sec(d*x+c))^(1/3),x)
```

output
```
int(cos(d*x+c)/(a+a*sec(d*x+c))^(1/3),x)
```

3.158.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\cos(c+dx)}{\sqrt[3]{a+a\sec(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)/(a+a*sec(d*x+c))^(1/3),x, algorithm="fricas")
```

output
```
Timed out
```

3.158.6 Sympy [F]

$$\int \frac{\cos(c+dx)}{\sqrt[3]{a+a\sec(c+dx)}}\,dx = \int \frac{\cos(c+dx)}{\sqrt[3]{a(\sec(c+dx)+1)}}\,dx$$

input
```
integrate(cos(d*x+c)/(a+a*sec(d*x+c))**(1/3),x)
```

output
```
Integral(cos(c + d*x)/(a*(sec(c + d*x) + 1))**(1/3), x)
```

3.158.7 Maxima [F]

$$\int \frac{\cos(c+dx)}{\sqrt[3]{a+a\sec(c+dx)}}\,dx = \int \frac{\cos(dx+c)}{(a\sec(dx+c)+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate(cos(d*x+c)/(a+a*sec(d*x+c))^(1/3),x, algorithm="maxima")
```

output
```
integrate(cos(d*x + c)/(a*sec(d*x + c) + a)^(1/3), x)
```

3.158.8 Giac [F]

$$\int \frac{\cos(c+dx)}{\sqrt[3]{a+a\sec(c+dx)}}\,dx = \int \frac{\cos(dx+c)}{(a\sec(dx+c)+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate(cos(d*x+c)/(a+a*sec(d*x+c))^(1/3),x, algorithm="giac")
```

output
```
integrate(cos(d*x + c)/(a*sec(d*x + c) + a)^(1/3), x)
```

3.158.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos(c+dx)}{\sqrt[3]{a+a\sec(c+dx)}}\,dx = \int \frac{\cos(c+d\,x)}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{1/3}}\,dx$$

input
```
int(cos(c + d*x)/(a + a/cos(c + d*x))^(1/3),x)
```

output
```
int(cos(c + d*x)/(a + a/cos(c + d*x))^(1/3), x)
```

3.159 $\int \frac{\sec^4(c+dx)}{(a+a\sec(c+dx))^{5/3}}\,dx$

3.159.1 Optimal result . . . 1339
3.159.2 Mathematica [C] (verified) . . . 1340
3.159.3 Rubi [A] (warning: unable to verify) . . . 1341
3.159.4 Maple [F] . . . 1348
3.159.5 Fricas [F] . . . 1349
3.159.6 Sympy [F] . . . 1349
3.159.7 Maxima [F] . . . 1349
3.159.8 Giac [F] . . . 1350
3.159.9 Mupad [F(-1)] . . . 1350

3.159.1 Optimal result

Integrand size = 23, antiderivative size = 766

$$\int \frac{\sec^4(c+dx)}{(a+a\sec(c+dx))^{5/3}}\,dx = -\frac{33\tan(c+dx)}{28d(a+a\sec(c+dx))^{5/3}} + \frac{3\sec^2(c+dx)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/3}} + \frac{135\tan(c+dx)}{14ad(a+a\sec(c+dx))^{2/3}} + \frac{375\left(1+\sqrt{3}\right)\sqrt[3]{a+a\sec(c+dx)}\tan(c+dx)}{28a^2d(1+\sec(c+dx))^{2/3}\left(\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{1+\sec(c+dx)}\right)}$$

$$-\frac{375\sqrt[4]{3}E\left(\arccos\left(\frac{\sqrt[3]{2}-\left(1-\sqrt{3}\right)\sqrt[3]{1+\sec(c+dx)}}{\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{1+\sec(c+dx)}}\right)|\tfrac{1}{4}\left(2+\sqrt{3}\right)\right)\sqrt[3]{a+a\sec(c+dx)}\left(\sqrt[3]{2}-\sqrt[3]{1+\sec(c+\cdots}\right.}{14\,2^{2/3}a^2d(1-\sec(c+dx))(1+\sec(c+dx))^{2/3}\sqrt{-\frac{\sqrt[3]{1+\sec(c+dx\cdots}}{\left(\sqrt[3]{2}-\left(1+\sqrt{3}\cdots\right.\right.}}}$$

$$-\frac{125\,3^{3/4}\left(1-\sqrt{3}\right)\text{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{2}-\left(1-\sqrt{3}\right)\sqrt[3]{1+\sec(c+dx)}}{\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{1+\sec(c+dx)}}\right),\tfrac{1}{4}\left(2+\sqrt{3}\right)\right)\sqrt[3]{a+a\sec(c+dx)}\left(\cdots\right.}{28\,2^{2/3}a^2d(1-\sec(c+dx))(1+\sec(c+dx))^{2/3}\sqrt{-\frac{\sqrt[3]{1+s\cdots}}{\left(\cdots\right.}}}$$

output
```
-33/28*tan(d*x+c)/d/(a+a*sec(d*x+c))^(5/3)+3/4*sec(d*x+c)^2*tan(d*x+c)/d/(
a+a*sec(d*x+c))^(5/3)+135/14*tan(d*x+c)/a/d/(a+a*sec(d*x+c))^(2/3)+375/28*
(a+a*sec(d*x+c))^(1/3)*(1+3^(1/2))*tan(d*x+c)/a^2/d/(1+sec(d*x+c))^(2/3)/(
2^(1/3)-(1+sec(d*x+c))^(1/3)*(1+3^(1/2)))-375/28*3^(1/4)*((2^(1/3)-(1+sec(
d*x+c))^(1/3)*(1-3^(1/2)))^2/(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1+3^(1/2)))^2)
^(1/2)/(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1-3^(1/2)))*(2^(1/3)-(1+sec(d*x+c))^
(1/3)*(1+3^(1/2)))*EllipticE((1-(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1-3^(1/2)))
^2/(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(
1/2))*(a+a*sec(d*x+c))^(1/3)*(2^(1/3)-(1+sec(d*x+c))^(1/3))*((2^(2/3)+2^(1
/3)*(1+sec(d*x+c))^(1/3)+(1+sec(d*x+c))^(2/3))/(2^(1/3)-(1+sec(d*x+c))^(1/
3)*(1+3^(1/2)))^2)^(1/2)*tan(d*x+c)*2^(1/3)/a^2/d/(1-sec(d*x+c))/(1+sec(d*
x+c))^(2/3)/(-(1+sec(d*x+c))^(1/3)*(2^(1/3)-(1+sec(d*x+c))^(1/3))/(2^(1/3)
-(1+sec(d*x+c))^(1/3)*(1+3^(1/2)))^2)^(1/2)-125/56*3^(3/4)*((2^(1/3)-(1+se
c(d*x+c))^(1/3)*(1-3^(1/2)))^2/(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1+3^(1/2)))^
2)^(1/2)/(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1-3^(1/2)))*(2^(1/3)-(1+sec(d*x+c)
)^(1/3)*(1+3^(1/2)))*EllipticF((1-(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1-3^(1/2)
))^2/(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2
^(1/2))*(a+a*sec(d*x+c))^(1/3)*(2^(1/3)-(1+sec(d*x+c))^(1/3))*(1-3^(1/2))*
((2^(2/3)+2^(1/3)*(1+sec(d*x+c))^(1/3)+(1+sec(d*x+c))^(2/3))/(2^(1/3)-(1+s
ec(d*x+c))^(1/3)*(1+3^(1/2)))^2)^(1/2)*tan(d*x+c)*2^(1/3)/a^2/d/(1-sec(...
```

3.159.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.59 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.14

$$\int \frac{\sec^4(c+dx)}{(a+a\sec(c+dx))^{5/3}}\,dx = \frac{\left(-2502^{5/6}\cos^2\left(\frac{1}{2}(c+dx)\right)\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{3}{2},\frac{1}{2}(1-\sec(c+dx))\right)\right.}{28d(a(1-}$$

input
```
Integrate[Sec[c + d*x]^4/(a + a*Sec[c + d*x])^(5/3),x]
```

output
```
((-250*2^(5/6)*Cos[(c + d*x)/2]^2*Hypergeometric2F1[1/6, 1/2, 3/2, (1 - Se
c[c + d*x])/2]*Sec[c + d*x]*(1 + Sec[c + d*x])^(1/6) + 3*(79 + 90*Sec[c +
d*x] + 7*Sec[c + d*x]^2))*Tan[c + d*x])/(28*d*(a*(1 + Sec[c + d*x]))^(5/3)
)
```

3.159.3 Rubi [A] (warning: unable to verify)

Time = 1.37 (sec) , antiderivative size = 781, normalized size of antiderivative = 1.02, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.783, Rules used = {3042, 4311, 27, 3042, 4496, 27, 3042, 4488, 3042, 4315, 3042, 4314, 73, 837, 25, 27, 766, 2420}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^4(c+dx)}{(a\sec(c+dx)+a)^{5/3}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^4}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/3}}dx$$

$$\Big\downarrow \text{4311}$$

$$\frac{3\int \frac{\sec^2(c+dx)(6a-5a\sec(c+dx))}{3(\sec(c+dx)a+a)^{5/3}}dx}{4a}+\frac{3\tan(c+dx)\sec^2(c+dx)}{4d(a\sec(c+dx)+a)^{5/3}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\sec^2(c+dx)(6a-5a\sec(c+dx))}{(\sec(c+dx)a+a)^{5/3}}dx}{4a}+\frac{3\tan(c+dx)\sec^2(c+dx)}{4d(a\sec(c+dx)+a)^{5/3}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2\left(6a-5a\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{5/3}}dx}{4a}+\frac{3\tan(c+dx)\sec^2(c+dx)}{4d(a\sec(c+dx)+a)^{5/3}}$$

$$\Big\downarrow \text{4496}$$

$$\frac{-\frac{3\int -\frac{5\sec(c+dx)\left(11a^2-7a^2\sec(c+dx)\right)}{3(\sec(c+dx)a+a)^{2/3}}dx}{7a^2}-\frac{33a\tan(c+dx)}{7d(a\sec(c+dx)+a)^{5/3}}}{4a}+\frac{3\tan(c+dx)\sec^2(c+dx)}{4d(a\sec(c+dx)+a)^{5/3}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{5\int \frac{\sec(c+dx)\left(11a^2-7a^2\sec(c+dx)\right)}{(\sec(c+dx)a+a)^{2/3}}dx}{7a^2}-\frac{33a\tan(c+dx)}{7d(a\sec(c+dx)+a)^{5/3}}}{4a}+\frac{3\tan(c+dx)\sec^2(c+dx)}{4d(a\sec(c+dx)+a)^{5/3}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{5\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(11a^2-7a^2\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{2/3}}dx}{7a^2}-\frac{33a\tan(c+dx)}{7d(a\sec(c+dx)+a)^{5/3}}}{4a}+\frac{3\tan(c+dx)\sec^2(c+dx)}{4d(a\sec(c+dx)+a)^{5/3}}$$

$\Big\downarrow$ 4488

$$\frac{\frac{5\left(\frac{54a^2\tan(c+dx)}{d(a\sec(c+dx)+a)^{2/3}}-25a\int\sec(c+dx)\sqrt[3]{\sec(c+dx)a+a}dx\right)}{7a^2}-\frac{33a\tan(c+dx)}{7d(a\sec(c+dx)+a)^{5/3}}}{4a}+\frac{3\tan(c+dx)\sec^2(c+dx)}{4d(a\sec(c+dx)+a)^{5/3}}$$

$\Big\downarrow$ 3042

$$\frac{\frac{5\left(\frac{54a^2\tan(c+dx)}{d(a\sec(c+dx)+a)^{2/3}}-25a\int\csc\left(c+dx+\frac{\pi}{2}\right)\sqrt[3]{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx\right)}{7a^2}-\frac{33a\tan(c+dx)}{7d(a\sec(c+dx)+a)^{5/3}}}{4a}+\frac{3\tan(c+dx)\sec^2(c+dx)}{4d(a\sec(c+dx)+a)^{5/3}}$$

$\Big\downarrow$ 4315

$$\frac{\frac{5\left(\frac{54a^2\tan(c+dx)}{d(a\sec(c+dx)+a)^{2/3}}-\frac{25a\sqrt[3]{a\sec(c+dx)+a}\int\sec(c+dx)\sqrt[3]{\sec(c+dx)+1}dx}{\sqrt[3]{\sec(c+dx)+1}}\right)}{7a^2}-\frac{33a\tan(c+dx)}{7d(a\sec(c+dx)+a)^{5/3}}}{4a}+\frac{3\tan(c+dx)\sec^2(c+dx)}{4d(a\sec(c+dx)+a)^{5/3}}$$

$\Big\downarrow$ 3042

$$\frac{\frac{5\left(\frac{54a^2\tan(c+dx)}{d(a\sec(c+dx)+a)^{2/3}}-\frac{25a\sqrt[3]{a\sec(c+dx)+a}\int\csc\left(c+dx+\frac{\pi}{2}\right)\sqrt[3]{\csc\left(c+dx+\frac{\pi}{2}\right)+1}dx}{\sqrt[3]{\sec(c+dx)+1}}\right)}{7a^2}-\frac{33a\tan(c+dx)}{7d(a\sec(c+dx)+a)^{5/3}}}{4a}+\frac{3\tan(c+dx)\sec^2(c+dx)}{4d(a\sec(c+dx)+a)^{5/3}}$$

$\Big\downarrow$ 4314

3.159. $\int \frac{\sec^4(c+dx)}{(a+a\sec(c+dx))^{5/3}}\,dx$

$$\frac{\frac{5\left(\frac{25a\tan(c+dx)\sqrt[3]{a\sec(c+dx)+a}\int\frac{1}{\sqrt{1-\sec(c+dx)}\sqrt[6]{\sec(c+dx)+1}}d\sec(c+dx)}{d\sqrt{1-\sec(c+dx)}(\sec(c+dx)+1)^{5/6}}+\frac{54a^2\tan(c+dx)}{d(a\sec(c+dx)+a)^{2/3}}\right)}{7a^2}-\frac{33a\tan(c+dx)}{7d(a\sec(c+dx)+a)^{5/3}}}{4a}+\frac{3\tan(c+dx)\sec^2(c+dx)}{4d(a\sec(c+dx)+a)^{5/3}}$$

$\downarrow$ 73

$$\frac{\frac{5\left(\frac{150a\tan(c+dx)\sqrt[3]{a\sec(c+dx)+a}\int\frac{(\sec(c+dx)+1)^{2/3}}{\sqrt{1-\sec(c+dx)}}d\sqrt[6]{\sec(c+dx)+1}}{d\sqrt{1-\sec(c+dx)}(\sec(c+dx)+1)^{5/6}}+\frac{54a^2\tan(c+dx)}{d(a\sec(c+dx)+a)^{2/3}}\right)}{7a^2}-\frac{33a\tan(c+dx)}{7d(a\sec(c+dx)+a)^{5/3}}}{4a}+\frac{3\tan(c+dx)\sec^2(c+dx)}{4d(a\sec(c+dx)+a)^{5/3}}$$

$\downarrow$ 837

$$\frac{\frac{5\left(\frac{150a\tan(c+dx)\sqrt[3]{a\sec(c+dx)+a}\left(-\frac{\left(1-\sqrt{3}\right)\int\frac{1}{\sqrt{1-\sec(c+dx)}}d\sqrt[6]{\sec(c+dx)+1}}{\sqrt[3]{2}}-\frac{1}{2}\int-\frac{2(\sec(c+dx)+1)^{2/3}+2^{2/3}\left(1-\sqrt{3}\right)}{\sqrt{1-\sec(c+dx)}}d\sqrt[6]{\sec(c+\right.}{d\sqrt{1-\sec(c+dx)}(\sec(c+dx)+1)^{5/6}}\right.}{7a^2}}{4a}$$

$$\frac{3\tan(c+dx)\sec^2(c+dx)}{4d(a\sec(c+dx)+a)^{5/3}}$$

$\downarrow$ 25

$$\frac{\frac{5\left(\frac{150a\tan(c+dx)\sqrt[3]{a\sec(c+dx)+a}\left(\frac{1}{2}\int\frac{2^{2/3}\left(\sqrt[3]{2}(\sec(c+dx)+1)^{2/3}-\sqrt{3}+1\right)}{\sqrt{1-\sec(c+dx)}}d\sqrt[6]{\sec(c+dx)+1}-\frac{\left(1-\sqrt{3}\right)\int\frac{1}{\sqrt{1-\sec(c+dx)}}d\sqrt[6]{\sec(c-}}{\sqrt[3]{2}}\right.}{d\sqrt{1-\sec(c+dx)}(\sec(c+dx)+1)^{5/6}}\right.}{7a^2}}{4a}$$

$$\frac{3\tan(c+dx)\sec^2(c+dx)}{4d(a\sec(c+dx)+a)^{5/3}}$$

$\downarrow$ 27

3.159. $\int \frac{\sec^4(c+dx)}{(a+a\sec(c+dx))^{5/3}}\,dx$

$$\frac{5\left(\frac{150a\tan(c+dx)\sqrt[3]{a\sec(c+dx)+a}\left(\frac{\int\frac{\sqrt[3]{2}(\sec(c+dx)+1)^{2/3}-\sqrt{3}+1}{\sqrt{1-\sec(c+dx)}}d\sqrt[6]{\sec(c+dx)+1}}{\sqrt[3]{2}}-\frac{\left(1-\sqrt{3}\right)\int\frac{1}{\sqrt{1-\sec(c+dx)}}d\sqrt[6]{\sec(c+dx)+}}{\sqrt[3]{2}}\right.}{d\sqrt{1-\sec(c+dx)}(\sec(c+dx)+1)^{5/6}}\right.}{7a^2}}{4a}$$

$$\frac{3\tan(c+dx)\sec^2(c+dx)}{4d(a\sec(c+dx)+a)^{5/3}}$$

$$\Big\downarrow \; 766$$

$$\frac{5\left(\frac{150a\tan(c+dx)\sqrt[3]{a\sec(c+dx)+a}\left(\frac{\int\frac{\sqrt[3]{2}(\sec(c+dx)+1)^{2/3}-\sqrt{3}+1}{\sqrt{1-\sec(c+dx)}}d\sqrt[6]{\sec(c+dx)+1}}{\sqrt[3]{2}}-\frac{\left(1-\sqrt{3}\right)\sqrt[6]{\sec(c+dx)+1}\left(\sqrt[3]{2}-\sqrt[3]{se}\right.}{}\right.}{d\sqrt{1-\sec(c+dx)}(}\right.}{}$$

$$\frac{3\tan(c+dx)\sec^2(c+dx)}{4d(a\sec(c+dx)+a)^{5/3}}$$

$$\Big\downarrow \; 2420$$

3.159. $\int \frac{\sec^4(c+dx)}{(a+a\sec(c+dx))^{5/3}}\,dx$

$$\frac{5\left(\frac{54a^2\tan(c+dx)}{d(a\sec(c+dx)+a)^{2/3}}+\frac{150a\tan(c+dx)\sqrt[3]{a\sec(c+dx)+a}\left(\frac{\left(1+\sqrt{3}\right)\sqrt{1-\sec(c+dx)}\sqrt[6]{\sec(c+dx)+1}}{2^{2/3}\left(\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}\right)}-\frac{\sqrt[4]{3}\sqrt[6]{\sec(c+dx)+1}\left(\sqrt[3]{2}\right.}{}\right.}{}\right.}{}$$

$$\frac{3\tan(c+dx)\sec^2(c+dx)}{4d(a\sec(c+dx)+a)^{5/3}}$$

input
```
Int[Sec[c + d*x]^4/(a + a*Sec[c + d*x])^(5/3),x]
```

3.159. $\int \frac{\sec^4(c+dx)}{(a+a\sec(c+dx))^{5/3}}\,dx$

output
```
(3*Sec[c + d*x]^2*Tan[c + d*x])/(4*d*(a + a*Sec[c + d*x])^(5/3)) + ((-33*a
*Tan[c + d*x])/(7*d*(a + a*Sec[c + d*x])^(5/3)) + (5*((54*a^2*Tan[c + d*x]
)/(d*(a + a*Sec[c + d*x])^(2/3)) + (150*a*(a + a*Sec[c + d*x])^(1/3)*(-1/2
*((1 - Sqrt[3])*EllipticF[ArcCos[(2^(1/3) - (1 - Sqrt[3])*(1 + Sec[c + d*x
])^(1/3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))], (2 + Sqrt[3
])/4]*(1 + Sec[c + d*x])^(1/6)*(2^(1/3) - (1 + Sec[c + d*x])^(1/3))*Sqrt[(
2^(2/3) + 2^(1/3)*(1 + Sec[c + d*x])^(1/3) + (1 + Sec[c + d*x])^(2/3))/(2^
(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2])/(2^(2/3)*3^(1/4)*Sqrt[
1 - Sec[c + d*x]]*Sqrt[-(((1 + Sec[c + d*x])^(1/3)*(2^(1/3) - (1 + Sec[c +
 d*x])^(1/3)))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2)]) + (
((1 + Sqrt[3])*Sqrt[1 - Sec[c + d*x]]*(1 + Sec[c + d*x])^(1/6))/(2^(2/3)*(
2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))) - (3^(1/4)*EllipticE[Ar
cCos[(2^(1/3) - (1 - Sqrt[3])*(1 + Sec[c + d*x])^(1/3))/(2^(1/3) - (1 + Sq
rt[3])*(1 + Sec[c + d*x])^(1/3))], (2 + Sqrt[3])/4]*(1 + Sec[c + d*x])^(1/
6)*(2^(1/3) - (1 + Sec[c + d*x])^(1/3))*Sqrt[(2^(2/3) + 2^(1/3)*(1 + Sec[c
 + d*x])^(1/3) + (1 + Sec[c + d*x])^(2/3))/(2^(1/3) - (1 + Sqrt[3])*(1 + S
ec[c + d*x])^(1/3))^2])/(2^(1/3)*Sqrt[1 - Sec[c + d*x]]*Sqrt[-(((1 + Sec[c
 + d*x])^(1/3)*(2^(1/3) - (1 + Sec[c + d*x])^(1/3)))/(2^(1/3) - (1 + Sqrt[
3])*(1 + Sec[c + d*x])^(1/3))^2)]))/2^(1/3))*Tan[c + d*x])/(d*Sqrt[1 - Sec
[c + d*x]]*(1 + Sec[c + d*x])^(5/6))))/(7*a^2))/(4*a)
```

3.159.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 766
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

rule 837
```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Simp[(Sqrt[3] - 1)*(s^2/(2*r^2))   Int[1/Sqrt[
a + b*x^6], x], x] - Simp[1/(2*r^2)   Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/S
qrt[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

rule 2420
```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(1 + Sqrt[3])*d*s^3*x*(Sqr
t[a + b*x^6]/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2))), x] - Simp[3^(1/4)*d*s*x*
(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]/(2
*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sqrt[a + b*x^6])
)*EllipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2
 + Sqrt[3])/4], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4311
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-d^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d
*Csc[e + f*x])^(n - 2)/(f*(m + n - 1))), x] + Simp[d^2/(b*(m + n - 1))   In
t[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*m*Csc[e +
f*x]), x], x] /; FreeQ[{a, b, d, e, f, m}, x] && EqQ[a^2 - b^2, 0] && GtQ[n
, 2] && NeQ[m + n - 1, 0] && IntegerQ[n]
```

rule 4314
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Simp[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x
]]*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(d*x)^(n - 1)*((a + b*x)^(m - 1/2
)/Sqrt[a - b*x]), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n},
x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

3.159. $\int \frac{\sec^4(c+dx)}{(a+a\sec(c+dx))^{5/3}}\,dx$

rule 4315
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Simp[a^IntPart[m]*((a + b*Csc[e + f*x])^FracPart[m
]/(1 + (b/a)*Csc[e + f*x])^FracPart[m])   Int[(1 + (b/a)*Csc[e + f*x])^m*(d
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^
2, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

rule 4488
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b - a*B)*Cot[e +
f*x]*((a + b*Csc[e + f*x])^m/(a*f*(2*m + 1))), x] + Simp[(a*B*m + A*b*(m +
1))/(a*b*(2*m + 1))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x], x]
 /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[a*B*m + A*b*(m + 1), 0] && LtQ[m, -2^(-1)]
```

rule 4496
```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b - a*B))*Cot
[e + f*x]*((a + b*Csc[e + f*x])^m/(b*f*(2*m + 1))), x] + Simp[1/(b^2*(2*m +
 1))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[A*b*m - a*B*m + b
*B*(2*m + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && Ne
Q[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

3.159.4 Maple [F]

$$\int \frac{\sec(dx+c)^4}{(a+a\sec(dx+c))^{\frac{5}{3}}}dx$$

input
```
int(sec(d*x+c)^4/(a+a*sec(d*x+c))^(5/3),x)
```

output
```
int(sec(d*x+c)^4/(a+a*sec(d*x+c))^(5/3),x)
```

3.159.5 Fricas [F]

$$\int \frac{\sec^4(c+dx)}{(a+a\sec(c+dx))^{5/3}}\,dx = \int \frac{\sec{(dx+c)}^4}{(a\sec{(dx+c)}+a)^{\frac{5}{3}}}\,dx$$

input
```
integrate(sec(d*x+c)^4/(a+a*sec(d*x+c))^(5/3),x, algorithm="fricas")
```

output
```
integral((a*sec(d*x + c) + a)^(1/3)*sec(d*x + c)^4/(a^2*sec(d*x + c)^2 + 2
*a^2*sec(d*x + c) + a^2), x)
```

3.159.6 Sympy [F]

$$\int \frac{\sec^4(c+dx)}{(a+a\sec(c+dx))^{5/3}}\,dx = \int \frac{\sec^4{(c+dx)}}{(a(\sec{(c+dx)}+1))^{\frac{5}{3}}}\,dx$$

input
```
integrate(sec(d*x+c)**4/(a+a*sec(d*x+c))**(5/3),x)
```

output
```
Integral(sec(c + d*x)**4/(a*(sec(c + d*x) + 1))**(5/3), x)
```

3.159.7 Maxima [F]

$$\int \frac{\sec^4(c+dx)}{(a+a\sec(c+dx))^{5/3}}\,dx = \int \frac{\sec{(dx+c)}^4}{(a\sec{(dx+c)}+a)^{\frac{5}{3}}}\,dx$$

input
```
integrate(sec(d*x+c)^4/(a+a*sec(d*x+c))^(5/3),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)^4/(a*sec(d*x + c) + a)^(5/3), x)
```

3.159.8 Giac [F]

$$\int \frac{\sec^4(c+dx)}{(a+a\sec(c+dx))^{5/3}}\,dx = \int \frac{\sec{(dx+c)}^4}{(a\sec{(dx+c)}+a)^{\frac{5}{3}}}\,dx$$

input
```
integrate(sec(d*x+c)^4/(a+a*sec(d*x+c))^(5/3),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^4/(a*sec(d*x + c) + a)^(5/3), x)
```

3.159.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^4(c+dx)}{(a+a\sec(c+dx))^{5/3}}\,dx = \int \frac{1}{\cos{(c+d\,x)}^4\,\left(a+\frac{a}{\cos(c+d\,x)}\right)^{5/3}}\,dx$$

input
```
int(1/(cos(c + d*x)^4*(a + a/cos(c + d*x))^(5/3)),x)
```

output
```
int(1/(cos(c + d*x)^4*(a + a/cos(c + d*x))^(5/3)), x)
```

3.160 $\int \frac{\sec^3(c+dx)}{(a+a\sec(c+dx))^{5/3}}\,dx$

3.160.1 Optimal result 1351
3.160.2 Mathematica [C] (verified) 1352
3.160.3 Rubi [A] (warning: unable to verify) 1353
3.160.4 Maple [F] 1358
3.160.5 Fricas [F] 1358
3.160.6 Sympy [F] 1359
3.160.7 Maxima [F] 1359
3.160.8 Giac [F] 1359
3.160.9 Mupad [F(-1)] 1360

3.160.1 Optimal result

Integrand size = 23, antiderivative size = 731

$$\int \frac{\sec^3(c+dx)}{(a+a\sec(c+dx))^{5/3}}\,dx = \frac{3\tan(c+dx)}{7d(a+a\sec(c+dx))^{5/3}} - \frac{36\tan(c+dx)}{7ad(a+a\sec(c+dx))^{2/3}} - \frac{57\left(1+\sqrt{3}\right)\sqrt[3]{a+a\sec(c+dx)}\tan(c+dx)}{7a^2d(1+\sec(c+dx))^{2/3}\left(\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{1+\sec(c+dx)}\right)}$$

$$+\frac{57\sqrt[3]{2}\sqrt[4]{3}E\left(\arccos\left(\frac{\sqrt[3]{2}-\left(1-\sqrt{3}\right)\sqrt[3]{1+\sec(c+dx)}}{\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{1+\sec(c+dx)}}\right)|\tfrac{1}{4}\left(2+\sqrt{3}\right)\right)\sqrt[3]{a+a\sec(c+dx)}\left(\sqrt[3]{2}-\sqrt[3]{1+\sec(c\right.}{7a^2d(1-\sec(c+dx))(1+\sec(c+dx))^{2/3}\sqrt{-\frac{\sqrt[3]{1+\sec(c+dx)}\left(\right.}{\left(\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{\right.}}}}$$

$$+\frac{19\ 3^{3/4}\left(1-\sqrt{3}\right)\text{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{2}-\left(1-\sqrt{3}\right)\sqrt[3]{1+\sec(c+dx)}}{\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{1+\sec(c+dx)}}\right),\tfrac{1}{4}\left(2+\sqrt{3}\right)\right)\sqrt[3]{a+a\sec(c+dx)}\left(\sqrt[3]{\right.}{7\ 2^{2/3}a^2d(1-\sec(c+dx))(1+\sec(c+dx))^{2/3}\sqrt{-\frac{\sqrt[3]{1+\sec}}{\left(\sqrt[3]{\right.}}}}$$

output
```
3/7*tan(d*x+c)/d/(a+a*sec(d*x+c))^(5/3)-36/7*tan(d*x+c)/a/d/(a+a*sec(d*x+c
))^(2/3)-57/7*(a+a*sec(d*x+c))^(1/3)*(1+3^(1/2))*tan(d*x+c)/a^2/d/(1+sec(d
*x+c))^(2/3)/(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1+3^(1/2)))+57/7*3^(1/4)*((2^(
1/3)-(1+sec(d*x+c))^(1/3)*(1-3^(1/2)))^2/(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1+
3^(1/2)))^2)^(1/2)/(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1-3^(1/2)))*(2^(1/3)-(1+
sec(d*x+c))^(1/3)*(1+3^(1/2)))*EllipticE((1-(2^(1/3)-(1+sec(d*x+c))^(1/3)*
(1-3^(1/2)))^2/(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1+3^(1/2)))^2)^(1/2),1/4*6^(
1/2)+1/4*2^(1/2))*(a+a*sec(d*x+c))^(1/3)*(2^(1/3)-(1+sec(d*x+c))^(1/3))*((
2^(2/3)+2^(1/3)*(1+sec(d*x+c))^(1/3)+(1+sec(d*x+c))^(2/3))/(2^(1/3)-(1+sec
(d*x+c))^(1/3)*(1+3^(1/2)))^2)^(1/2)*tan(d*x+c)*2^(1/3)/a^2/d/(1-sec(d*x+c
))/(1+sec(d*x+c))^(2/3)/(-(1+sec(d*x+c))^(1/3)*(2^(1/3)-(1+sec(d*x+c))^(1/
3))/(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1+3^(1/2)))^2)^(1/2)+19/14*3^(3/4)*((2^
(1/3)-(1+sec(d*x+c))^(1/3)*(1-3^(1/2)))^2/(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1
+3^(1/2)))^2)^(1/2)/(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1-3^(1/2)))*(2^(1/3)-(1
+sec(d*x+c))^(1/3)*(1+3^(1/2)))*EllipticF((1-(2^(1/3)-(1+sec(d*x+c))^(1/3)
*(1-3^(1/2)))^2/(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1+3^(1/2)))^2)^(1/2),1/4*6^
(1/2)+1/4*2^(1/2))*(a+a*sec(d*x+c))^(1/3)*(2^(1/3)-(1+sec(d*x+c))^(1/3))*(
1-3^(1/2))*((2^(2/3)+2^(1/3)*(1+sec(d*x+c))^(1/3)+(1+sec(d*x+c))^(2/3))/(2
^(1/3)-(1+sec(d*x+c))^(1/3)*(1+3^(1/2)))^2)^(1/2)*tan(d*x+c)*2^(1/3)/a^2/d
/(1-sec(d*x+c))/(1+sec(d*x+c))^(2/3)/(-(1+sec(d*x+c))^(1/3)*(2^(1/3)-(1...
```

3.160.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.29 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.13

$$\int \frac{\sec^3(c+dx)}{(a+a\sec(c+dx))^{5/3}}\,dx = \frac{\left(-33-36\sec(c+dx)+38\ 2^{5/6}\cos^2\left(\frac{1}{2}(c+dx)\right)\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\right.\right.}{7d(a(1+\sec(c+d}$$

input
```
Integrate[Sec[c + d*x]^3/(a + a*Sec[c + d*x])^(5/3),x]
```

output
```
((-33 - 36*Sec[c + d*x] + 38*2^(5/6)*Cos[(c + d*x)/2]^2*Hypergeometric2F1[
1/6, 1/2, 3/2, (1 - Sec[c + d*x])/2]*Sec[c + d*x]*(1 + Sec[c + d*x])^(1/6)
)*Tan[c + d*x])/(7*d*(a*(1 + Sec[c + d*x]))^(5/3))
```

3.160.3 Rubi [A] (warning: unable to verify)

Time = 0.99 (sec) , antiderivative size = 734, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.652, Rules used = {3042, 4286, 27, 3042, 4488, 3042, 4315, 3042, 4314, 73, 837, 25, 27, 766, 2420}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^3(c+dx)}{(a\sec(c+dx)+a)^{5/3}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^3}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/3}}dx$$

$$\Big\downarrow \text{4286}$$

$$\frac{3\int -\frac{\sec(c+dx)(5a-7a\sec(c+dx))}{3(\sec(c+dx)a+a)^{2/3}}dx}{7a^2}+\frac{3\tan(c+dx)}{7d(a\sec(c+dx)+a)^{5/3}}$$

$$\Big\downarrow \text{27}$$

$$\frac{3\tan(c+dx)}{7d(a\sec(c+dx)+a)^{5/3}}-\frac{\int \frac{\sec(c+dx)(5a-7a\sec(c+dx))}{(\sec(c+dx)a+a)^{2/3}}dx}{7a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\tan(c+dx)}{7d(a\sec(c+dx)+a)^{5/3}}-\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(5a-7a\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{2/3}}dx}{7a^2}$$

$$\Big\downarrow \text{4488}$$

$$\frac{3\tan(c+dx)}{7d(a\sec(c+dx)+a)^{5/3}}-\frac{\frac{36a\tan(c+dx)}{d(a\sec(c+dx)+a)^{2/3}}-19\int \sec(c+dx)\sqrt[3]{\sec(c+dx)a+a}dx}{7a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\tan(c+dx)}{7d(a\sec(c+dx)+a)^{5/3}}-\frac{\frac{36a\tan(c+dx)}{d(a\sec(c+dx)+a)^{2/3}}-19\int \csc\left(c+dx+\frac{\pi}{2}\right)\sqrt[3]{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{7a^2}$$

$$\Big\downarrow \text{4315}$$

$$\frac{3\tan(c+dx)}{7d(a\sec(c+dx)+a)^{5/3}}-\frac{\frac{36a\tan(c+dx)}{d(a\sec(c+dx)+a)^{2/3}}-\frac{19\sqrt[3]{a\sec(c+dx)+a}\int \sec(c+dx)\sqrt[3]{\sec(c+dx)+1}dx}{\sqrt[3]{\sec(c+dx)+1}}}{7a^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{3\tan(c+dx)}{7d(a\sec(c+dx)+a)^{5/3}}-\frac{\frac{36a\tan(c+dx)}{d(a\sec(c+dx)+a)^{2/3}}-\frac{19\sqrt[3]{a\sec(c+dx)+a}\int\csc\left(c+dx+\frac{\pi}{2}\right)\sqrt[3]{\csc\left(c+dx+\frac{\pi}{2}\right)+1}dx}{\sqrt[3]{\sec(c+dx)+1}}}{7a^2}$$

$$\Big\downarrow \; 4314$$

$$\frac{3\tan(c+dx)}{7d(a\sec(c+dx)+a)^{5/3}}-\frac{\frac{19\tan(c+dx)\sqrt[3]{a\sec(c+dx)+a}\int\frac{1}{\sqrt{1-\sec(c+dx)}\sqrt[6]{\sec(c+dx)+1}}d\sec(c+dx)}{d\sqrt{1-\sec(c+dx)}(\sec(c+dx)+1)^{5/6}}+\frac{36a\tan(c+dx)}{d(a\sec(c+dx)+a)^{2/3}}}{7a^2}$$

$$\Big\downarrow \; 73$$

$$\frac{3\tan(c+dx)}{7d(a\sec(c+dx)+a)^{5/3}}-\frac{\frac{114\tan(c+dx)\sqrt[3]{a\sec(c+dx)+a}\int\frac{(\sec(c+dx)+1)^{2/3}}{\sqrt{1-\sec(c+dx)}}d\sqrt[6]{\sec(c+dx)+1}}{d\sqrt{1-\sec(c+dx)}(\sec(c+dx)+1)^{5/6}}+\frac{36a\tan(c+dx)}{d(a\sec(c+dx)+a)^{2/3}}}{7a^2}$$

$$\Big\downarrow \; 837$$

$$\frac{3\tan(c+dx)}{7d(a\sec(c+dx)+a)^{5/3}}-\frac{\frac{114\tan(c+dx)\sqrt[3]{a\sec(c+dx)+a}\left(-\frac{\left(1-\sqrt{3}\right)\int\frac{1}{\sqrt{1-\sec(c+dx)}}d\sqrt[6]{\sec(c+dx)+1}}{\sqrt[3]{2}}-\frac{1}{2}\int-\frac{2(\sec(c+dx)+1)^{2/3}+2^{2/3}\left(1-\sqrt{3}\right)}{\sqrt{1-\sec(c+dx)}}d\sqrt[6]{\sec(c+}\right.}{d\sqrt{1-\sec(c+dx)}(\sec(c+dx)+1)^{5/6}}}{7a^2}$$

$$\Big\downarrow \; 25$$

$$\frac{3\tan(c+dx)}{7d(a\sec(c+dx)+a)^{5/3}}-\frac{\frac{114\tan(c+dx)\sqrt[3]{a\sec(c+dx)+a}\left(\frac{1}{2}\int\frac{2^{2/3}\left(\sqrt[3]{2}(\sec(c+dx)+1)^{2/3}-\sqrt{3}+1\right)}{\sqrt{1-\sec(c+dx)}}d\sqrt[6]{\sec(c+dx)+1}-\frac{\left(1-\sqrt{3}\right)\int\frac{1}{\sqrt{1-\sec(c+dx)}}d\sqrt[6]{\sec(c+}}{\sqrt[3]{2}}\right.}{d\sqrt{1-\sec(c+dx)}(\sec(c+dx)+1)^{5/6}}}{7a^2}$$

$$\Big\downarrow \; 27$$

3.160. $\int\frac{\sec^3(c+dx)}{(a+a\sec(c+dx))^{5/3}}\,dx$

$$\frac{\frac{3\tan(c+dx)}{7d(a\sec(c+dx)+a)^{5/3}}-\frac{114\tan(c+dx)\sqrt[3]{a\sec(c+dx)+a}\left(\frac{\int\frac{\sqrt[3]{2}(\sec(c+dx)+1)^{2/3}-\sqrt{3}+1}{\sqrt{1-\sec(c+dx)}}d\sqrt[6]{\sec(c+dx)+1}}{\sqrt[3]{2}}-\frac{\left(1-\sqrt{3}\right)\int\frac{1}{\sqrt{1-\sec(c+dx)}}d\sqrt[6]{\sec(c+dx)+}}{\sqrt[3]{2}}\right.}{d\sqrt{1-\sec(c+dx)}(\sec(c+dx)+1)^{5/6}}}{7a^2}$$

$\Big\downarrow$ 766

$$\frac{\frac{3\tan(c+dx)}{7d(a\sec(c+dx)+a)^{5/3}}-\frac{114\tan(c+dx)\sqrt[3]{a\sec(c+dx)+a}\left(\frac{\int\frac{\sqrt[3]{2}(\sec(c+dx)+1)^{2/3}-\sqrt{3}+1}{\sqrt{1-\sec(c+dx)}}d\sqrt[6]{\sec(c+dx)+1}}{\sqrt[3]{2}}-\frac{\left(1-\sqrt{3}\right)\sqrt[6]{\sec(c+dx)+1}\left(\sqrt[3]{2}-\sqrt[3]{\sec(\right.}}{}\right.}{d\sqrt{1-\sec(c+dx)}(\sec}}{}$$

$\Big\downarrow$ 2420

$$\frac{\frac{3\tan(c+dx)}{7d(a\sec(c+dx)+a)^{5/3}}-\frac{114\tan(c+dx)\sqrt[3]{a\sec(c+dx)+a}\left(\frac{\left(1+\sqrt{3}\right)\sqrt{1-\sec(c+dx)}\sqrt[6]{\sec(c+dx)+1}}{2^{2/3}\left(\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}\right)}-\frac{\sqrt[4]{3}\sqrt[6]{\sec(c+dx)+1}\left(\sqrt[3]{2}-\sqrt[3]{\sec(c+dx)+}\right.}{\sqrt[3]{2}\sqrt{1-}}\right.}{}}{}$$

input
```
Int[Sec[c + d*x]^3/(a + a*Sec[c + d*x])^(5/3),x]
```

3.160. $\int \frac{\sec^3(c+dx)}{(a+a\sec(c+dx))^{5/3}}\,dx$

output
```
(3*Tan[c + d*x])/(7*d*(a + a*Sec[c + d*x])^(5/3)) - ((36*a*Tan[c + d*x])/(
d*(a + a*Sec[c + d*x])^(2/3)) + (114*(a + a*Sec[c + d*x])^(1/3)*(-1/2*((1
- Sqrt[3])*EllipticF[ArcCos[(2^(1/3) - (1 - Sqrt[3])*(1 + Sec[c + d*x])^(1
/3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))], (2 + Sqrt[3])/4]
*(1 + Sec[c + d*x])^(1/6)*(2^(1/3) - (1 + Sec[c + d*x])^(1/3))*Sqrt[(2^(2/
3) + 2^(1/3)*(1 + Sec[c + d*x])^(1/3) + (1 + Sec[c + d*x])^(2/3))/(2^(1/3)
 - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2])/(2^(2/3)*3^(1/4)*Sqrt[1 - S
ec[c + d*x]]*Sqrt[-(((1 + Sec[c + d*x])^(1/3)*(2^(1/3) - (1 + Sec[c + d*x]
)^(1/3)))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2)]) + (((1 +
 Sqrt[3])*Sqrt[1 - Sec[c + d*x]]*(1 + Sec[c + d*x])^(1/6))/(2^(2/3)*(2^(1/
3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))) - (3^(1/4)*EllipticE[ArcCos[
(2^(1/3) - (1 - Sqrt[3])*(1 + Sec[c + d*x])^(1/3))/(2^(1/3) - (1 + Sqrt[3]
)*(1 + Sec[c + d*x])^(1/3))], (2 + Sqrt[3])/4]*(1 + Sec[c + d*x])^(1/6)*(2
^(1/3) - (1 + Sec[c + d*x])^(1/3))*Sqrt[(2^(2/3) + 2^(1/3)*(1 + Sec[c + d*
x])^(1/3) + (1 + Sec[c + d*x])^(2/3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c
+ d*x])^(1/3))^2])/(2^(1/3)*Sqrt[1 - Sec[c + d*x]]*Sqrt[-(((1 + Sec[c + d*
x])^(1/3)*(2^(1/3) - (1 + Sec[c + d*x])^(1/3)))/(2^(1/3) - (1 + Sqrt[3])*(
1 + Sec[c + d*x])^(1/3))^2)]))/2^(1/3))*Tan[c + d*x])/(d*Sqrt[1 - Sec[c +
d*x]]*(1 + Sec[c + d*x])^(5/6)))/(7*a^2)
```

3.160.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 766
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

rule 837
```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Simp[(Sqrt[3] - 1)*(s^2/(2*r^2))   Int[1/Sqrt[
a + b*x^6], x], x] - Simp[1/(2*r^2)   Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/S
qrt[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

rule 2420
```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(1 + Sqrt[3])*d*s^3*x*(Sqr
t[a + b*x^6]/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2))), x] - Simp[3^(1/4)*d*s*x*
(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]/(2
*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sqrt[a + b*x^6])
)*EllipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2
 + Sqrt[3])/4], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4286
```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[b*Cot[e + f*x]*((a + b*Csc[e + f*x])^m/(a*f*(2*m + 1))),
x] - Simp[1/(a^2*(2*m + 1))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
*(a*m - b*(2*m + 1)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[
a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

rule 4314
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Simp[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x
]]*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(d*x)^(n - 1)*((a + b*x)^(m - 1/2
)/Sqrt[a - b*x]), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n},
x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

3.160. $\int \frac{\sec^3(c+dx)}{(a+a\sec(c+dx))^{5/3}}\,dx$

rule 4315
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Simp[a^IntPart[m]*((a + b*Csc[e + f*x])^FracPart[m
]/(1 + (b/a)*Csc[e + f*x])^FracPart[m])   Int[(1 + (b/a)*Csc[e + f*x])^m*(d
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^
2, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

rule 4488
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b - a*B)*Cot[e +
f*x]*((a + b*Csc[e + f*x])^m/(a*f*(2*m + 1))), x] + Simp[(a*B*m + A*b*(m +
1))/(a*b*(2*m + 1))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x], x]
 /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[a*B*m + A*b*(m + 1), 0] && LtQ[m, -2^(-1)]
```

3.160.4 Maple [F]

$$\int \frac{\sec (dx+c)^{3}}{(a+a\sec (dx+c))^{\frac{5}{3}}}dx$$

input
```
int(sec(d*x+c)^3/(a+a*sec(d*x+c))^(5/3),x)
```

output
```
int(sec(d*x+c)^3/(a+a*sec(d*x+c))^(5/3),x)
```

3.160.5 Fricas [F]

$$\int \frac{\sec^3(c+dx)}{(a+a\sec(c+dx))^{5/3}}\,dx=\int \frac{\sec (dx+c)^{3}}{(a\sec (dx+c)+a)^{\frac{5}{3}}}\,dx$$

input
```
integrate(sec(d*x+c)^3/(a+a*sec(d*x+c))^(5/3),x, algorithm="fricas")
```

output
```
integral((a*sec(d*x + c) + a)^(1/3)*sec(d*x + c)^3/(a^2*sec(d*x + c)^2 + 2
*a^2*sec(d*x + c) + a^2), x)
```

3.160.6 Sympy [F]

$$\int \frac{\sec^3(c+dx)}{(a+a\sec(c+dx))^{5/3}}\,dx = \int \frac{\sec^3(c+dx)}{(a(\sec(c+dx)+1))^{\frac{5}{3}}}\,dx$$

input
```
integrate(sec(d*x+c)**3/(a+a*sec(d*x+c))**(5/3),x)
```

output
```
Integral(sec(c + d*x)**3/(a*(sec(c + d*x) + 1))**(5/3), x)
```

3.160.7 Maxima [F]

$$\int \frac{\sec^3(c+dx)}{(a+a\sec(c+dx))^{5/3}}\,dx = \int \frac{\sec(dx+c)^3}{(a\sec(dx+c)+a)^{\frac{5}{3}}}\,dx$$

input
```
integrate(sec(d*x+c)^3/(a+a*sec(d*x+c))^(5/3),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)^3/(a*sec(d*x + c) + a)^(5/3), x)
```

3.160.8 Giac [F]

$$\int \frac{\sec^3(c+dx)}{(a+a\sec(c+dx))^{5/3}}\,dx = \int \frac{\sec(dx+c)^3}{(a\sec(dx+c)+a)^{\frac{5}{3}}}\,dx$$

input
```
integrate(sec(d*x+c)^3/(a+a*sec(d*x+c))^(5/3),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^3/(a*sec(d*x + c) + a)^(5/3), x)
```

3.160.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^3(c+dx)}{(a+a\sec(c+dx))^{5/3}}\,dx = \int \frac{1}{\cos\left(c+d\,x\right)^3\left(a+\frac{a}{\cos(c+d\,x)}\right)^{5/3}}\,dx$$

input
```
int(1/(cos(c + d*x)^3*(a + a/cos(c + d*x))^(5/3)),x)
```

output
```
int(1/(cos(c + d*x)^3*(a + a/cos(c + d*x))^(5/3)), x)
```

3.161 $\int \frac{\sec^2(c+dx)}{(a+a\sec(c+dx))^{5/3}}\,dx$

3.161.1 Optimal result 1361
3.161.2 Mathematica [C] (verified) 1362
3.161.3 Rubi [A] (warning: unable to verify) 1363
3.161.4 Maple [F] 1368
3.161.5 Fricas [F] 1368
3.161.6 Sympy [F] 1369
3.161.7 Maxima [F] 1369
3.161.8 Giac [F] 1369
3.161.9 Mupad [F(-1)] 1370

3.161.1 Optimal result

Integrand size = 23, antiderivative size = 731

$$\int \frac{\sec^2(c+dx)}{(a+a\sec(c+dx))^{5/3}}\,dx = -\frac{3\tan(c+dx)}{7d(a+a\sec(c+dx))^{5/3}} + \frac{15\tan(c+dx)}{7ad(a+a\sec(c+dx))^{2/3}} + \frac{15\left(1+\sqrt{3}\right)\sqrt[3]{1+\sec(c+dx)}\tan(c+dx)}{7ad(a+a\sec(c+dx))^{2/3}\left(\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{1+\sec(c+dx)}\right)}$$

$$-\frac{15\sqrt[3]{2}\sqrt[4]{3}E\left(\arccos\left(\frac{\sqrt[3]{2}-\left(1-\sqrt{3}\right)\sqrt[3]{1+\sec(c+dx)}}{\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{1+\sec(c+dx)}}\right)|\tfrac{1}{4}\left(2+\sqrt{3}\right)\right)\sqrt[3]{1+\sec(c+dx)}\left(\sqrt[3]{2}-\sqrt[3]{1+\sec(c\right.}{7ad(1-\sec(c+dx))(a+a\sec(c+dx))^{2/3}\sqrt{-\frac{\sqrt[3]{1+\sec(c+dx)}\left(\right.}{\left(\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{\right.}}}}$$

$$-\frac{5\,3^{3/4}\left(1-\sqrt{3}\right)\text{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{2}-\left(1-\sqrt{3}\right)\sqrt[3]{1+\sec(c+dx)}}{\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{1+\sec(c+dx)}}\right),\tfrac{1}{4}\left(2+\sqrt{3}\right)\right)\sqrt[3]{1+\sec(c+dx)}\left(\sqrt[3]{2}\right.}{7\,2^{2/3}ad(1-\sec(c+dx))(a+a\sec(c+dx))^{2/3}\sqrt{-\frac{\sqrt[3]{1+\sec}}{\left(\sqrt[3]{2}\right.}}}$$

output
```
-3/7*tan(d*x+c)/d/(a+a*sec(d*x+c))^(5/3)+15/7*tan(d*x+c)/a/d/(a+a*sec(d*x+
c))^(2/3)+15/7*(1+sec(d*x+c))^(1/3)*(1+3^(1/2))*tan(d*x+c)/a/d/(a+a*sec(d*
x+c))^(2/3)/(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1+3^(1/2)))-15/7*2^(1/3)*3^(1/4
)*((2^(1/3)-(1+sec(d*x+c))^(1/3)*(1-3^(1/2)))^2/(2^(1/3)-(1+sec(d*x+c))^(1
/3)*(1+3^(1/2)))^2)^(1/2)/(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1-3^(1/2)))*(2^(1
/3)-(1+sec(d*x+c))^(1/3)*(1+3^(1/2)))*EllipticE((1-(2^(1/3)-(1+sec(d*x+c))
^(1/3)*(1-3^(1/2)))^2/(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1+3^(1/2)))^2)^(1/2),
1/4*6^(1/2)+1/4*2^(1/2))*(1+sec(d*x+c))^(1/3)*(2^(1/3)-(1+sec(d*x+c))^(1/3
))*((2^(2/3)+2^(1/3)*(1+sec(d*x+c))^(1/3)+(1+sec(d*x+c))^(2/3))/(2^(1/3)-(
1+sec(d*x+c))^(1/3)*(1+3^(1/2)))^2)^(1/2)*tan(d*x+c)/a/d/(1-sec(d*x+c))/(a
+a*sec(d*x+c))^(2/3)/(-(1+sec(d*x+c))^(1/3)*(2^(1/3)-(1+sec(d*x+c))^(1/3))
/(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1+3^(1/2)))^2)^(1/2)-5/14*3^(3/4)*((2^(1/3
)-(1+sec(d*x+c))^(1/3)*(1-3^(1/2)))^2/(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1+3^(
1/2)))^2)^(1/2)/(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1-3^(1/2)))*(2^(1/3)-(1+sec
(d*x+c))^(1/3)*(1+3^(1/2)))*EllipticF((1-(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1-
3^(1/2)))^2/(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2
)+1/4*2^(1/2))*(1+sec(d*x+c))^(1/3)*(2^(1/3)-(1+sec(d*x+c))^(1/3))*(1-3^(1
/2))*((2^(2/3)+2^(1/3)*(1+sec(d*x+c))^(1/3)+(1+sec(d*x+c))^(2/3))/(2^(1/3)
-(1+sec(d*x+c))^(1/3)*(1+3^(1/2)))^2)^(1/2)*tan(d*x+c)*2^(1/3)/a/d/(1-sec(
d*x+c))/(a+a*sec(d*x+c))^(2/3)/(-(1+sec(d*x+c))^(1/3)*(2^(1/3)-(1+sec(d...
```

3.161.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.34 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.12

$$\int \frac{\sec^2(c+dx)}{(a+a\sec(c+dx))^{5/3}}\,dx = \frac{\left(-3+5\ 2^{5/6}\cos^2\left(\frac{1}{2}(c+dx)\right)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{7}{6},\frac{3}{2},\frac{1}{2}(1-\sec(c+dx)\right.\right.}{7d(a(1+\sec(c+dx)))^{5/3}}$$

input
```
Integrate[Sec[c + d*x]^2/(a + a*Sec[c + d*x])^(5/3),x]
```

output
```
((-3 + 5*2^(5/6)*Cos[(c + d*x)/2]^2*Hypergeometric2F1[1/2, 7/6, 3/2, (1 -
Sec[c + d*x])/2]*Sec[c + d*x]*(1 + Sec[c + d*x])^(1/6))*Tan[c + d*x])/(7*d
*(a*(1 + Sec[c + d*x]))^(5/3))
```

3.161.3 Rubi [A] (warning: unable to verify)

Time = 0.81 (sec) , antiderivative size = 736, normalized size of antiderivative = 1.01, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.565, Rules used = {3042, 4284, 3042, 4315, 3042, 4314, 61, 73, 837, 25, 27, 766, 2420}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(c+dx)}{(a\sec(c+dx)+a)^{5/3}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/3}}dx$$

$$\Big\downarrow \text{4284}$$

$$\frac{5\int \frac{\sec(c+dx)}{(\sec(c+dx)a+a)^{2/3}}dx}{7a}-\frac{3\tan(c+dx)}{7d(a\sec(c+dx)+a)^{5/3}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{2/3}}dx}{7a}-\frac{3\tan(c+dx)}{7d(a\sec(c+dx)+a)^{5/3}}$$

$$\Big\downarrow \text{4315}$$

$$\frac{5(\sec(c+dx)+1)^{2/3}\int \frac{\sec(c+dx)}{(\sec(c+dx)+1)^{2/3}}dx}{7a(a\sec(c+dx)+a)^{2/3}}-\frac{3\tan(c+dx)}{7d(a\sec(c+dx)+a)^{5/3}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5(\sec(c+dx)+1)^{2/3}\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)^{2/3}}dx}{7a(a\sec(c+dx)+a)^{2/3}}-\frac{3\tan(c+dx)}{7d(a\sec(c+dx)+a)^{5/3}}$$

$$\Big\downarrow \text{4314}$$

$$-\frac{5\tan(c+dx)\sqrt[6]{\sec(c+dx)+1}\int \frac{1}{\sqrt{1-\sec(c+dx)}(\sec(c+dx)+1)^{7/6}}d\sec(c+dx)}{7ad\sqrt{1-\sec(c+dx)}(a\sec(c+dx)+a)^{2/3}}-\frac{3\tan(c+dx)}{7d(a\sec(c+dx)+a)^{5/3}}$$

$$\Big\downarrow \text{61}$$

$$-\frac{5\tan(c+dx)\sqrt[6]{\sec(c+dx)+1}\left(-\int\frac{1}{\sqrt{1-\sec(c+dx)}\sqrt[6]{\sec(c+dx)+1}}d\sec(c+dx)-\frac{3\sqrt{1-\sec(c+dx)}}{\sqrt[6]{\sec(c+dx)+1}}\right)}{7ad\sqrt{1-\sec(c+dx)}(a\sec(c+dx)+a)^{2/3}}-\frac{3\tan(c+dx)}{7d(a\sec(c+dx)+a)^{5/3}}$$

$\Big\downarrow$ 73

$$-\frac{5\tan(c+dx)\sqrt[6]{\sec(c+dx)+1}\left(-6\int\frac{(\sec(c+dx)+1)^{2/3}}{\sqrt{1-\sec(c+dx)}}d\sqrt[6]{\sec(c+dx)+1}-\frac{3\sqrt{1-\sec(c+dx)}}{\sqrt[6]{\sec(c+dx)+1}}\right)}{7ad\sqrt{1-\sec(c+dx)}(a\sec(c+dx)+a)^{2/3}}-\frac{3\tan(c+dx)}{7d(a\sec(c+dx)+a)^{5/3}}$$

$\Big\downarrow$ 837

$$-\frac{5\tan(c+dx)\sqrt[6]{\sec(c+dx)+1}\left(-6\left(-\frac{\left(1-\sqrt{3}\right)\int\frac{1}{\sqrt{1-\sec(c+dx)}}d\sqrt[6]{\sec(c+dx)+1}}{\sqrt[3]{2}}-\frac{1}{2}\int-\frac{2(\sec(c+dx)+1)^{2/3}+2^{2/3}\left(1-\sqrt{\right.}}{\sqrt{1-\sec(c+dx)}}\right.\right.}{7ad\sqrt{1-\sec(c+dx)}(a\sec(c+dx)+a)^{2/3}}-\frac{3\tan(c+dx)}{7d(a\sec(c+dx)+a)^{5/3}}$$

$\Big\downarrow$ 25

$$-\frac{5\tan(c+dx)\sqrt[6]{\sec(c+dx)+1}\left(-6\left(\frac{1}{2}\int\frac{2^{2/3}\left(\sqrt[3]{2}(\sec(c+dx)+1)^{2/3}-\sqrt{3}+1\right)}{\sqrt{1-\sec(c+dx)}}d\sqrt[6]{\sec(c+dx)+1}-\frac{\left(1-\sqrt{3}\right)\int\frac{1}{\sqrt{1-\sec(c+}}}{}\right.\right.}{7ad\sqrt{1-\sec(c+dx)}(a\sec(c+dx)+a)^{2/3}}-\frac{3\tan(c+dx)}{7d(a\sec(c+dx)+a)^{5/3}}$$

$\Big\downarrow$ 27

$$-\frac{5\tan(c+dx)\sqrt[6]{\sec(c+dx)+1}\left(-6\left(\frac{\int\frac{\sqrt[3]{2}(\sec(c+dx)+1)^{2/3}-\sqrt{3}+1}{\sqrt{1-\sec(c+dx)}}d\sqrt[6]{\sec(c+dx)+1}}{\sqrt[3]{2}}-\frac{\left(1-\sqrt{3}\right)\int\frac{1}{\sqrt{1-\sec(c+dx)}}d\sqrt[6]{\sec(}}{\sqrt[3]{2}}\right.\right.}{7ad\sqrt{1-\sec(c+dx)}(a\sec(c+dx)+a)^{2/3}}-\frac{3\tan(c+dx)}{7d(a\sec(c+dx)+a)^{5/3}}$$

$\Big\downarrow$ 766

3.161. $\int\frac{\sec^2(c+dx)}{(a+a\sec(c+dx))^{5/3}}\,dx$

$$-\frac{5\tan(c+dx)\sqrt[6]{\sec(c+dx)+1}\left(-6\left(\frac{\int \frac{\sqrt[3]{2}(\sec(c+dx)+1)^{2/3}-\sqrt{3}+1}{\sqrt{1-\sec(c+dx)}}d\sqrt[6]{\sec(c+dx)+1}}{\sqrt[3]{2}}-\frac{\left(1-\sqrt{3}\right)\sqrt[6]{\sec(c+dx)+1}\left(\sqrt[3]{\right.}{7ad}\right.\right.}{}$$

$$\frac{3\tan(c+dx)}{7d(a\sec(c+dx)+a)^{5/3}}$$

$$\Big\downarrow \; 2420$$

$$-\frac{5\tan(c+dx)\sqrt[6]{\sec(c+dx)+1}\left(-6\left(\frac{\frac{\left(1+\sqrt{3}\right)\sqrt{1-\sec(c+dx)}\sqrt[6]{\sec(c+dx)+1}}{2^{2/3}\left(\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}\right)}-\frac{\sqrt[4]{3}\sqrt[6]{\sec(c+dx)+1}\left(\sqrt[3]{2}-\sqrt[3]{\sec(c+\right.}{}}{}\right.\right.}{}$$

$$\frac{3\tan(c+dx)}{7d(a\sec(c+dx)+a)^{5/3}}$$

input
```
Int[Sec[c + d*x]^2/(a + a*Sec[c + d*x])^(5/3),x]
```

3.161. $\int \frac{\sec^2(c+dx)}{(a+a\sec(c+dx))^{5/3}}\,dx$

output
```
(-3*Tan[c + d*x])/(7*d*(a + a*Sec[c + d*x])^(5/3)) - (5*(1 + Sec[c + d*x])
^(1/6)*((-3*Sqrt[1 - Sec[c + d*x]])/(1 + Sec[c + d*x])^(1/6) - 6*(-1/2*((1
 - Sqrt[3])*EllipticF[ArcCos[(2^(1/3) - (1 - Sqrt[3])*(1 + Sec[c + d*x])^(
1/3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))], (2 + Sqrt[3])/4
]*(1 + Sec[c + d*x])^(1/6)*(2^(1/3) - (1 + Sec[c + d*x])^(1/3))*Sqrt[(2^(2
/3) + 2^(1/3)*(1 + Sec[c + d*x])^(1/3) + (1 + Sec[c + d*x])^(2/3))/(2^(1/3
) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2])/(2^(2/3)*3^(1/4)*Sqrt[1 -
Sec[c + d*x]]*Sqrt[-(((1 + Sec[c + d*x])^(1/3)*(2^(1/3) - (1 + Sec[c + d*x
])^(1/3)))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2)]) + (((1
+ Sqrt[3])*Sqrt[1 - Sec[c + d*x]]*(1 + Sec[c + d*x])^(1/6))/(2^(2/3)*(2^(1
/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))) - (3^(1/4)*EllipticE[ArcCos
[(2^(1/3) - (1 - Sqrt[3])*(1 + Sec[c + d*x])^(1/3))/(2^(1/3) - (1 + Sqrt[3
])*(1 + Sec[c + d*x])^(1/3))], (2 + Sqrt[3])/4]*(1 + Sec[c + d*x])^(1/6)*(
2^(1/3) - (1 + Sec[c + d*x])^(1/3))*Sqrt[(2^(2/3) + 2^(1/3)*(1 + Sec[c + d
*x])^(1/3) + (1 + Sec[c + d*x])^(2/3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c
 + d*x])^(1/3))^2])/(2^(1/3)*Sqrt[1 - Sec[c + d*x]]*Sqrt[-(((1 + Sec[c + d
*x])^(1/3)*(2^(1/3) - (1 + Sec[c + d*x])^(1/3)))/(2^(1/3) - (1 + Sqrt[3])*
(1 + Sec[c + d*x])^(1/3))^2)]))/2^(1/3)))*Tan[c + d*x])/(7*a*d*Sqrt[1 - Se
c[c + d*x]]*(a + a*Sec[c + d*x])^(2/3))
```

3.161.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 766
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

rule 837
```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Simp[(Sqrt[3] - 1)*(s^2/(2*r^2))   Int[1/Sqrt[
a + b*x^6], x], x] - Simp[1/(2*r^2)   Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/S
qrt[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

rule 2420
```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(1 + Sqrt[3])*d*s^3*x*(Sqr
t[a + b*x^6]/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2))), x] - Simp[3^(1/4)*d*s*x*
(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]/(2
*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sqrt[a + b*x^6])
)*EllipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2
 + Sqrt[3])/4], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4284
```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(-Cot[e + f*x])*((a + b*Csc[e + f*x])^m/(f*(2*m + 1))), x
] + Simp[m/(b*(2*m + 1))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x
], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

3.161. $\int \frac{\sec^2(c+dx)}{(a+a\sec(c+dx))^{5/3}}\,dx$

rule 4314
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Simp[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x
]]*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(d*x)^(n - 1)*((a + b*x)^(m - 1/2
)/Sqrt[a - b*x]), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n},
x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

rule 4315
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Simp[a^IntPart[m]*((a + b*Csc[e + f*x])^FracPart[m
]/(1 + (b/a)*Csc[e + f*x])^FracPart[m])   Int[(1 + (b/a)*Csc[e + f*x])^m*(d
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^
2, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

3.161.4 Maple [F]

$$\int \frac{\sec (dx+c)^2}{(a+a\sec (dx+c))^{\frac{5}{3}}}dx$$

input
```
int(sec(d*x+c)^2/(a+a*sec(d*x+c))^(5/3),x)
```

output
```
int(sec(d*x+c)^2/(a+a*sec(d*x+c))^(5/3),x)
```

3.161.5 Fricas [F]

$$\int \frac{\sec^2(c+dx)}{(a+a\sec(c+dx))^{5/3}}\,dx=\int \frac{\sec (dx+c)^2}{(a\sec (dx+c)+a)^{\frac{5}{3}}}\,dx$$

input
```
integrate(sec(d*x+c)^2/(a+a*sec(d*x+c))^(5/3),x, algorithm="fricas")
```

output
```
integral((a*sec(d*x + c) + a)^(1/3)*sec(d*x + c)^2/(a^2*sec(d*x + c)^2 + 2
*a^2*sec(d*x + c) + a^2), x)
```

3.161.6 Sympy [F]

$$\int \frac{\sec^2(c+dx)}{(a+a\sec(c+dx))^{5/3}}\,dx = \int \frac{\sec^2(c+dx)}{(a(\sec(c+dx)+1))^{\frac{5}{3}}}\,dx$$

input
```
integrate(sec(d*x+c)**2/(a+a*sec(d*x+c))**(5/3),x)
```

output
```
Integral(sec(c + d*x)**2/(a*(sec(c + d*x) + 1))**(5/3), x)
```

3.161.7 Maxima [F]

$$\int \frac{\sec^2(c+dx)}{(a+a\sec(c+dx))^{5/3}}\,dx = \int \frac{\sec(dx+c)^2}{(a\sec(dx+c)+a)^{\frac{5}{3}}}\,dx$$

input
```
integrate(sec(d*x+c)^2/(a+a*sec(d*x+c))^(5/3),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)^2/(a*sec(d*x + c) + a)^(5/3), x)
```

3.161.8 Giac [F]

$$\int \frac{\sec^2(c+dx)}{(a+a\sec(c+dx))^{5/3}}\,dx = \int \frac{\sec(dx+c)^2}{(a\sec(dx+c)+a)^{\frac{5}{3}}}\,dx$$

input
```
integrate(sec(d*x+c)^2/(a+a*sec(d*x+c))^(5/3),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^2/(a*sec(d*x + c) + a)^(5/3), x)
```

3.161.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^2(c+dx)}{(a+a\sec(c+dx))^{5/3}}\,dx=\int \frac{1}{\cos\left(c+d\,x\right)^2\,\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)^{5/3}}\,dx$$

input
```
int(1/(cos(c + d*x)^2*(a + a/cos(c + d*x))^(5/3)),x)
```

output
```
int(1/(cos(c + d*x)^2*(a + a/cos(c + d*x))^(5/3)), x)
```

3.162 $\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^{5/3}}\,dx$

3.162.1 Optimal result 1371
3.162.2 Mathematica [C] (verified) 1372
3.162.3 Rubi [A] (warning: unable to verify) 1373
3.162.4 Maple [F] 1377
3.162.5 Fricas [F] 1378
3.162.6 Sympy [F] 1378
3.162.7 Maxima [F] 1378
3.162.8 Giac [F] 1379
3.162.9 Mupad [F(-1)] 1379

3.162.1 Optimal result

Integrand size = 21, antiderivative size = 744

$$\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^{5/3}}\,dx = \frac{6\tan(c+dx)}{7ad(a+a\sec(c+dx))^{2/3}} + \frac{3\tan(c+dx)}{7ad(1+\sec(c+dx))(a+a\sec(c+dx))^{2/3}} + \frac{6(1+\sqrt{3})\sqrt[3]{1+\sec(c+dx)}\tan(c+dx)}{7ad(a+a\sec(c+dx))^{2/3}\left(\sqrt[3]{2}-(1+\sqrt{3})\sqrt[3]{1+\sec(c+dx)}\right)}$$

$$-\frac{6\sqrt[3]{2}\sqrt[4]{3}E\left(\arccos\left(\frac{\sqrt[3]{2}-(1-\sqrt{3})\sqrt[3]{1+\sec(c+dx)}}{\sqrt[3]{2}-(1+\sqrt{3})\sqrt[3]{1+\sec(c+dx)}}\right)|\frac{1}{4}(2+\sqrt{3})\right)\sqrt[3]{1+\sec(c+dx)}\left(\sqrt[3]{2}-\sqrt[3]{1+\sec(c+}\right.}{7ad(1-\sec(c+dx))(a+a\sec(c+dx))^{2/3}\sqrt{-\frac{\sqrt[3]{1+\sec(c+dx)}\left(\right.}{\left(\sqrt[3]{2}-(1+\sqrt{3})\sqrt[3]{}\right.}}}$$

$$-\frac{\sqrt[3]{2}3^{3/4}(1-\sqrt{3})\,\text{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{2}-(1-\sqrt{3})\sqrt[3]{1+\sec(c+dx)}}{\sqrt[3]{2}-(1+\sqrt{3})\sqrt[3]{1+\sec(c+dx)}}\right),\frac{1}{4}(2+\sqrt{3})\right)\sqrt[3]{1+\sec(c+dx)}\left(\sqrt[3]{2}\right.}{7ad(1-\sec(c+dx))(a+a\sec(c+dx))^{2/3}\sqrt{-\frac{\sqrt[3]{1+\sec(}}{\left(\sqrt[3]{2}-\right.}}}$$

output
```
6/7*tan(d*x+c)/a/d/(a+a*sec(d*x+c))^(2/3)+3/7*tan(d*x+c)/a/d/(1+sec(d*x+c)
)/(a+a*sec(d*x+c))^(2/3)+6/7*(1+sec(d*x+c))^(1/3)*(1+3^(1/2))*tan(d*x+c)/a
/d/(a+a*sec(d*x+c))^(2/3)/(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1+3^(1/2)))-6/7*2
^(1/3)*3^(1/4)*((2^(1/3)-(1+sec(d*x+c))^(1/3)*(1-3^(1/2)))^2/(2^(1/3)-(1+s
ec(d*x+c))^(1/3)*(1+3^(1/2)))^2)^(1/2)/(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1-3^
(1/2)))*(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1+3^(1/2)))*EllipticE((1-(2^(1/3)-(
1+sec(d*x+c))^(1/3)*(1-3^(1/2)))^2/(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1+3^(1/2
)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(1+sec(d*x+c))^(1/3)*(2^(1/3)-(1+sec
(d*x+c))^(1/3))*((2^(2/3)+2^(1/3)*(1+sec(d*x+c))^(1/3)+(1+sec(d*x+c))^(2/3
))/(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1+3^(1/2)))^2)^(1/2)*tan(d*x+c)/a/d/(1-s
ec(d*x+c))/(a+a*sec(d*x+c))^(2/3)/(-(1+sec(d*x+c))^(1/3)*(2^(1/3)-(1+sec(d
*x+c))^(1/3))/(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1+3^(1/2)))^2)^(1/2)-1/7*3^(3
/4)*((2^(1/3)-(1+sec(d*x+c))^(1/3)*(1-3^(1/2)))^2/(2^(1/3)-(1+sec(d*x+c))^
(1/3)*(1+3^(1/2)))^2)^(1/2)/(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1-3^(1/2)))*(2^
(1/3)-(1+sec(d*x+c))^(1/3)*(1+3^(1/2)))*EllipticF((1-(2^(1/3)-(1+sec(d*x+c
))^(1/3)*(1-3^(1/2)))^2/(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1+3^(1/2)))^2)^(1/2
),1/4*6^(1/2)+1/4*2^(1/2))*(1+sec(d*x+c))^(1/3)*(2^(1/3)-(1+sec(d*x+c))^(1
/3))*(1-3^(1/2))*((2^(2/3)+2^(1/3)*(1+sec(d*x+c))^(1/3)+(1+sec(d*x+c))^(2/
3))/(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1+3^(1/2)))^2)^(1/2)*tan(d*x+c)*2^(1/3)
/a/d/(1-sec(d*x+c))/(a+a*sec(d*x+c))^(2/3)/(-(1+sec(d*x+c))^(1/3)*(2^(1...
```

3.162.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.07 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.09

$$\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^{5/3}}\,dx = \frac{\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{13}{6},\frac{3}{2},\frac{1}{2}(1-\sec(c+dx))\right)(1+\sec(c+dx))^{7/6}\tan(c+}{2\sqrt[6]{2}d(a(1+\sec(c+dx)))^{5/3}}$$

input
```
Integrate[Sec[c + d*x]/(a + a*Sec[c + d*x])^(5/3),x]
```

output
```
(Hypergeometric2F1[1/2, 13/6, 3/2, (1 - Sec[c + d*x])/2]*(1 + Sec[c + d*x]
)^(7/6)*Tan[c + d*x])/(2*2^(1/6)*d*(a*(1 + Sec[c + d*x]))^(5/3))
```

3.162.3 Rubi [A] (warning: unable to verify)

Time = 0.69 (sec) , antiderivative size = 741, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {3042, 4315, 3042, 4314, 61, 61, 73, 837, 25, 27, 766, 2420}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(c+dx)}{(a\sec(c+dx)+a)^{5/3}}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/3}}dx$$

$$\Big\downarrow \; 4315$$

$$\frac{(\sec(c+dx)+1)^{2/3}\int \frac{\sec(c+dx)}{(\sec(c+dx)+1)^{5/3}}dx}{a(a\sec(c+dx)+a)^{2/3}}$$

$$\Big\downarrow \; 3042$$

$$\frac{(\sec(c+dx)+1)^{2/3}\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)^{5/3}}dx}{a(a\sec(c+dx)+a)^{2/3}}$$

$$\Big\downarrow \; 4314$$

$$-\frac{\tan(c+dx)\sqrt[6]{\sec(c+dx)+1}\int \frac{1}{\sqrt{1-\sec(c+dx)}(\sec(c+dx)+1)^{13/6}}d\sec(c+dx)}{ad\sqrt{1-\sec(c+dx)}(a\sec(c+dx)+a)^{2/3}}$$

$$\Big\downarrow \; 61$$

$$-\frac{\tan(c+dx)\sqrt[6]{\sec(c+dx)+1}\left(\frac{2}{7}\int \frac{1}{\sqrt{1-\sec(c+dx)}(\sec(c+dx)+1)^{7/6}}d\sec(c+dx)-\frac{3\sqrt{1-\sec(c+dx)}}{7(\sec(c+dx)+1)^{7/6}}\right)}{ad\sqrt{1-\sec(c+dx)}(a\sec(c+dx)+a)^{2/3}}$$

$$\Big\downarrow \; 61$$

$$-\frac{\tan(c+dx)\sqrt[6]{\sec(c+dx)+1}\left(\frac{2}{7}\left(-\int \frac{1}{\sqrt{1-\sec(c+dx)}\sqrt[6]{\sec(c+dx)+1}}d\sec(c+dx)-\frac{3\sqrt{1-\sec(c+dx)}}{\sqrt[6]{\sec(c+dx)+1}}\right)-\frac{3\sqrt{}}{7(\text{se}}\right.}{ad\sqrt{1-\sec(c+dx)}(a\sec(c+dx)+a)^{2/3}}$$

$$\Big\downarrow \; 73$$

$$-\frac{\tan(c+dx)\sqrt[6]{\sec(c+dx)+1}\left(\frac{2}{7}\left(-6\int\frac{(\sec(c+dx)+1)^{2/3}}{\sqrt{1-\sec(c+dx)}}d\sqrt[6]{\sec(c+dx)+1}-\frac{3\sqrt{1-\sec(c+dx)}}{\sqrt[6]{\sec(c+dx)+1}}\right)-\frac{3\sqrt{1-\sec(c+\cdots}}{7(\sec(c+dx)+1\cdots}\right.}{ad\sqrt{1-\sec(c+dx)}(a\sec(c+dx)+a)^{2/3}}$$

$\downarrow$ 837

$$-\frac{\tan(c+dx)\sqrt[6]{\sec(c+dx)+1}\left(\frac{2}{7}\left(-6\left(-\frac{\left(1-\sqrt{3}\right)\int\frac{1}{\sqrt{1-\sec(c+dx)}}d\sqrt[6]{\sec(c+dx)+1}}{\sqrt[3]{2}}-\frac{1}{2}\int-\frac{2(\sec(c+dx)+1)^{2/3}+2^{2/3}\left(1\cdots\right.}{\sqrt{1-\sec(c+dx)}}\cdots\right.\right.\right.}{ad\sqrt{1-\sec(c+dx)}(a\sec(c+dx)+a)^{2/3}}$$

$\downarrow$ 25

$$-\frac{\tan(c+dx)\sqrt[6]{\sec(c+dx)+1}\left(\frac{2}{7}\left(-6\left(\frac{1}{2}\int\frac{2^{2/3}\left(\sqrt[3]{2}(\sec(c+dx)+1)^{2/3}-\sqrt{3}+1\right)}{\sqrt{1-\sec(c+dx)}}d\sqrt[6]{\sec(c+dx)+1}-\frac{\left(1-\sqrt{3}\right)\int\frac{1}{\sqrt{1-\sec(\cdots}}\cdots}{\cdots}\right.\right.\right.}{ad\sqrt{1-\sec(c+dx)}(a\sec(c+dx)+a)^{2/3}}$$

$\downarrow$ 27

$$-\frac{\tan(c+dx)\sqrt[6]{\sec(c+dx)+1}\left(\frac{2}{7}\left(-6\left(\frac{\int\frac{\sqrt[3]{2}(\sec(c+dx)+1)^{2/3}-\sqrt{3}+1}{\sqrt{1-\sec(c+dx)}}d\sqrt[6]{\sec(c+dx)+1}}{\sqrt[3]{2}}-\frac{\left(1-\sqrt{3}\right)\int\frac{1}{\sqrt{1-\sec(c+dx)}}d\sqrt[6]{\mathrm{se}\cdots}}{\sqrt[3]{2}}\right.\right.\right.}{ad\sqrt{1-\sec(c+dx)}(a\sec(c+dx)+a)^{2/3}}$$

$\downarrow$ 766

$$-\frac{\tan(c+dx)\sqrt[6]{\sec(c+dx)+1}\left(\frac{2}{7}\left(-6\left(\frac{\int\frac{\sqrt[3]{2}(\sec(c+dx)+1)^{2/3}-\sqrt{3}+1}{\sqrt{1-\sec(c+dx)}}d\sqrt[6]{\sec(c+dx)+1}}{\sqrt[3]{2}}-\frac{\left(1-\sqrt{3}\right)\sqrt[6]{\sec(c+dx)+1}\cdots}{\cdots}\right.\right.\right.}{\cdots}$$

$\downarrow$ 2420

3.162. $\int\frac{\sec(c+dx)}{(a+a\sec(c+dx))^{5/3}}\,dx$

$$-\frac{\tan(c+dx)\sqrt[6]{\sec(c+dx)+1}\left(\frac{2}{7}\left(-6\left(\frac{\frac{\left(1+\sqrt{3}\right)\sqrt{1-\sec(c+dx)}\sqrt[6]{\sec(c+dx)+1}}{2^{2/3}\left(\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}\right)}-\frac{\sqrt[4]{3}\sqrt[6]{\sec(c+dx)+1}\left(\sqrt[3]{2}-\sqrt[3]{\sec(c\right.}{}}{}\right.\right.\right.}{}$$

input
```
Int[Sec[c + d*x]/(a + a*Sec[c + d*x])^(5/3),x]
```

output
```
-(((1 + Sec[c + d*x])^(1/6)*((-3*Sqrt[1 - Sec[c + d*x]])/(7*(1 + Sec[c + d
*x])^(7/6)) + (2*((-3*Sqrt[1 - Sec[c + d*x]])/(1 + Sec[c + d*x])^(1/6) - 6
*(-1/2*((1 - Sqrt[3])*EllipticF[ArcCos[(2^(1/3) - (1 - Sqrt[3])*(1 + Sec[c
 + d*x])^(1/3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))], (2 +
Sqrt[3])/4]*(1 + Sec[c + d*x])^(1/6)*(2^(1/3) - (1 + Sec[c + d*x])^(1/3))*
Sqrt[(2^(2/3) + 2^(1/3)*(1 + Sec[c + d*x])^(1/3) + (1 + Sec[c + d*x])^(2/3
))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2])/(2^(2/3)*3^(1/4)
*Sqrt[1 - Sec[c + d*x]]*Sqrt[-(((1 + Sec[c + d*x])^(1/3)*(2^(1/3) - (1 + S
ec[c + d*x])^(1/3)))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2)
]) + (((1 + Sqrt[3])*Sqrt[1 - Sec[c + d*x]]*(1 + Sec[c + d*x])^(1/6))/(2^(
2/3)*(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))) - (3^(1/4)*Ellipt
icE[ArcCos[(2^(1/3) - (1 - Sqrt[3])*(1 + Sec[c + d*x])^(1/3))/(2^(1/3) - (
1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))], (2 + Sqrt[3])/4]*(1 + Sec[c + d*x
])^(1/6)*(2^(1/3) - (1 + Sec[c + d*x])^(1/3))*Sqrt[(2^(2/3) + 2^(1/3)*(1 +
 Sec[c + d*x])^(1/3) + (1 + Sec[c + d*x])^(2/3))/(2^(1/3) - (1 + Sqrt[3])*
(1 + Sec[c + d*x])^(1/3))^2])/(2^(1/3)*Sqrt[1 - Sec[c + d*x]]*Sqrt[-(((1 +
 Sec[c + d*x])^(1/3)*(2^(1/3) - (1 + Sec[c + d*x])^(1/3)))/(2^(1/3) - (1 +
 Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2)]))/2^(1/3))))/7)*Tan[c + d*x])/(a*d
*Sqrt[1 - Sec[c + d*x]]*(a + a*Sec[c + d*x])^(2/3)))
```

3.162. $\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^{5/3}}\,dx$

3.162.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 766
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

rule 837
```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Simp[(Sqrt[3] - 1)*(s^2/(2*r^2))   Int[1/Sqrt[
a + b*x^6], x], x] - Simp[1/(2*r^2)   Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/S
qrt[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

rule 2420
```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(1 + Sqrt[3])*d*s^3*x*(Sqr
t[a + b*x^6]/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2))), x] - Simp[3^(1/4)*d*s*x*
(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]/(2
*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sqrt[a + b*x^6])
)*EllipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2
 + Sqrt[3])/4], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4314
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Simp[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x
]]*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(d*x)^(n - 1)*((a + b*x)^(m - 1/2
)/Sqrt[a - b*x]), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n},
x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

rule 4315
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Simp[a^IntPart[m]*((a + b*Csc[e + f*x])^FracPart[m
]/(1 + (b/a)*Csc[e + f*x])^FracPart[m])   Int[(1 + (b/a)*Csc[e + f*x])^m*(d
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^
2, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

3.162.4 Maple [F]

$$\int \frac{\sec(dx+c)}{(a+a\sec(dx+c))^{\frac{5}{3}}}dx$$

input
```
int(sec(d*x+c)/(a+a*sec(d*x+c))^(5/3),x)
```

output
```
int(sec(d*x+c)/(a+a*sec(d*x+c))^(5/3),x)
```

3.162.5 Fricas [F]

$$\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^{5/3}}\,dx = \int \frac{\sec(dx+c)}{(a\sec(dx+c)+a)^{\frac{5}{3}}}\,dx$$

input
```
integrate(sec(d*x+c)/(a+a*sec(d*x+c))^(5/3),x, algorithm="fricas")
```

output
```
integral((a*sec(d*x + c) + a)^(1/3)*sec(d*x + c)/(a^2*sec(d*x + c)^2 + 2*a
^2*sec(d*x + c) + a^2), x)
```

3.162.6 Sympy [F]

$$\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^{5/3}}\,dx = \int \frac{\sec(c+dx)}{(a(\sec(c+dx)+1))^{\frac{5}{3}}}\,dx$$

input
```
integrate(sec(d*x+c)/(a+a*sec(d*x+c))**(5/3),x)
```

output
```
Integral(sec(c + d*x)/(a*(sec(c + d*x) + 1))**(5/3), x)
```

3.162.7 Maxima [F]

$$\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^{5/3}}\,dx = \int \frac{\sec(dx+c)}{(a\sec(dx+c)+a)^{\frac{5}{3}}}\,dx$$

input
```
integrate(sec(d*x+c)/(a+a*sec(d*x+c))^(5/3),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)/(a*sec(d*x + c) + a)^(5/3), x)
```

3.162.8 Giac [F]

$$\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^{5/3}}\,dx = \int \frac{\sec(dx+c)}{(a\sec(dx+c)+a)^{\frac{5}{3}}}\,dx$$

input
```
integrate(sec(d*x+c)/(a+a*sec(d*x+c))^(5/3),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)/(a*sec(d*x + c) + a)^(5/3), x)
```

3.162.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^{5/3}}\,dx = \int \frac{1}{\cos(c+d\,x)\,\left(a+\frac{a}{\cos(c+d\,x)}\right)^{5/3}}\,dx$$

input
```
int(1/(cos(c + d*x)*(a + a/cos(c + d*x))^(5/3)),x)
```

output
```
int(1/(cos(c + d*x)*(a + a/cos(c + d*x))^(5/3)), x)
```

3.163 $\int \frac{1}{(a+a\sec(c+dx))^{5/3}}\,dx$

3.163.1 Optimal result . . . 1380
3.163.2 Mathematica [B] (warning: unable to verify) . . . 1380
3.163.3 Rubi [A] (warning: unable to verify) . . . 1381
3.163.4 Maple [F] . . . 1383
3.163.5 Fricas [F(-1)] . . . 1384
3.163.6 Sympy [F] . . . 1384
3.163.7 Maxima [F] . . . 1384
3.163.8 Giac [F] . . . 1385
3.163.9 Mupad [F(-1)] . . . 1385

3.163.1 Optimal result

Integrand size = 14, antiderivative size = 90

$$\int \frac{1}{(a+a\sec(c+dx))^{5/3}}\,dx = -\frac{3\sqrt{2}\,\text{AppellF1}\left(-\frac{7}{6},\frac{1}{2},1,-\frac{1}{6},\frac{1}{2}(1+\sec(c+dx)),1+\sec(c+dx)\right)\tan(c+dx)}{7ad\sqrt{1-\sec(c+dx)}(1+\sec(c+dx))(a+a\sec(c+dx))^{2/3}}$$

output
```
-3/7*AppellF1(-7/6,1,1/2,-1/6,1+sec(d*x+c),1/2+1/2*sec(d*x+c))*2^(1/2)*tan
(d*x+c)/a/d/(1+sec(d*x+c))/(a+a*sec(d*x+c))^(2/3)/(1-sec(d*x+c))^(1/2)
```

3.163.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 3007 vs. 2(90) = 180.

Time = 17.58 (sec) , antiderivative size = 3007, normalized size of antiderivative = 33.41

$$\int \frac{1}{(a+a\sec(c+dx))^{5/3}}\,dx = \text{Result too large to show}$$

input
```
Integrate[(a + a*Sec[c + d*x])^(-5/3),x]
```

output
```
(((1 + Cos[c + d*x])*Sec[c + d*x])^(1/3)*(1 + Sec[c + d*x])^(5/3)*((27*Sin
[c + d*x])/7 - (30*Tan[(c + d*x)/2])/7 + (3*Sec[(c + d*x)/2]^2*Tan[(c + d*
x)/2])/14))/(d*(a*(1 + Sec[c + d*x]))^(5/3)) + (2^(1/3)*(1 + Sec[c + d*x])
^(5/3)*((16*(1 + Sec[c + d*x])^(1/3))/7 - (27*Cos[c + d*x]*(1 + Sec[c + d*
x])^(1/3))/7)*Tan[(c + d*x)/2]*((-3*AppellF1[3/2, 1/3, 1, 5/2, Tan[(c + d*
x)/2]^2, -Tan[(c + d*x)/2]^2]*Tan[(c + d*x)/2]^2)/(Cos[c + d*x]*Sec[(c + d
*x)/2]^2)^(2/3) + Cos[(c + d*x)/2]^2*(-27 - (5*AppellF1[1/2, 1/3, 1, 3/2,
Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])/((-1 + Tan[(c + d*x)/2]^2)*(Appe
llF1[1/2, 1/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + (2*(-3*A
ppellF1[3/2, 1/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + Appel
lF1[3/2, 4/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Tan[(c + d
*x)/2]^2)/9)))))/(7*d*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(2/3)*(a*(1 + Sec[
c + d*x]))^(5/3)*((Sec[(c + d*x)/2]^2*((-3*AppellF1[3/2, 1/3, 1, 5/2, Tan[
(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Tan[(c + d*x)/2]^2)/(Cos[c + d*x]*Sec
[(c + d*x)/2]^2)^(2/3) + Cos[(c + d*x)/2]^2*(-27 - (5*AppellF1[1/2, 1/3, 1
, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])/((-1 + Tan[(c + d*x)/2]^2
)*(AppellF1[1/2, 1/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + (
2*(-3*AppellF1[3/2, 1/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]
+ AppellF1[3/2, 4/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Tan
[(c + d*x)/2]^2)/9)))))/(7*2^(2/3)*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(2...
```

3.163.3 Rubi [A] (warning: unable to verify)

Time = 0.35 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.11, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 4266, 3042, 4265, 149, 25, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\sec(c+dx)+a)^{5/3}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/3}}dx$$

$$\Big\downarrow \text{4266}$$

$$\frac{(\sec(c+dx)+1)^{2/3}\int \frac{1}{(\sec(c+dx)+1)^{5/3}}dx}{a(a\sec(c+dx)+a)^{2/3}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(\sec(c+dx)+1)^{2/3}\int \frac{1}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)^{5/3}}dx}{a(a\sec(c+dx)+a)^{2/3}}$$

$$\Big\downarrow \text{4265}$$

$$-\frac{\tan(c+dx)\sqrt[6]{\sec(c+dx)+1}\int \frac{\cos(c+dx)}{\sqrt{1-\sec(c+dx)}(\sec(c+dx)+1)^{13/6}}d\sec(c+dx)}{ad\sqrt{1-\sec(c+dx)}(a\sec(c+dx)+a)^{2/3}}$$

$$\Big\downarrow \text{149}$$

$$-\frac{6\tan(c+dx)\sqrt[6]{\sec(c+dx)+1}\int \frac{\cos^9(c+dx)}{\sqrt{1-\sec(c+dx)}}d\sqrt[6]{\sec(c+dx)+1}}{ad\sqrt{1-\sec(c+dx)}(a\sec(c+dx)+a)^{2/3}}$$

$$\Big\downarrow \text{25}$$

$$\frac{6\tan(c+dx)\sqrt[6]{\sec(c+dx)+1}\int -\frac{\cos^9(c+dx)}{\sqrt{1-\sec(c+dx)}}d\sqrt[6]{\sec(c+dx)+1}}{ad\sqrt{1-\sec(c+dx)}(a\sec(c+dx)+a)^{2/3}}$$

$$\Big\downarrow \text{1012}$$

$$-\frac{3\sqrt{2}\sin(c+dx)\cos^6(c+dx)\sqrt[6]{\sec(c+dx)+1}\,\text{AppellF1}\left(-\frac{7}{6},1,\frac{1}{2},-\frac{1}{6},\sec(c+dx)+1,\frac{1}{2}(\sec(c+dx)+1)\right)}{7ad\sqrt{1-\sec(c+dx)}(a\sec(c+dx)+a)^{2/3}}$$

input
```
Int[(a + a*Sec[c + d*x])^(-5/3),x]
```

output
```
(-3*Sqrt[2]*AppellF1[-7/6, 1, 1/2, -1/6, 1 + Sec[c + d*x], (1 + Sec[c + d*
x])/2]*Cos[c + d*x]^6*(1 + Sec[c + d*x])^(1/6)*Sin[c + d*x])/(7*a*d*Sqrt[1
 - Sec[c + d*x]]*(a + a*Sec[c + d*x])^(2/3))
```

3.163.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 149
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_)
)^(p_.), x_] :> With[{k = Denominator[m]}, Simp[k/b   Subst[Int[x^(k*(m + 1
) - 1)*(c - a*(d/b) + d*(x^k/b))^n*(e - a*(f/b) + f*(x^k/b))^p, x], x, (a +
 b*x)^(1/k)], x]] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && FractionQ[m] &&
IntegerQ[2*n] && IntegerQ[p]
```

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4265
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[a^n*(Cot
[c + d*x]/(d*Sqrt[1 + Csc[c + d*x]]*Sqrt[1 - Csc[c + d*x]]))   Subst[Int[(1
 + b*(x/a))^(n - 1/2)/(x*Sqrt[1 - b*(x/a)]), x], x, Csc[c + d*x]], x] /; Fr
eeQ[{a, b, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0
]
```

rule 4266
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[a^IntPar
t[n]*((a + b*Csc[c + d*x])^FracPart[n]/(1 + (b/a)*Csc[c + d*x])^FracPart[n]
)   Int[(1 + (b/a)*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] &&
EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

3.163.4 Maple [F]

$$\int \frac{1}{(a+a\sec(dx+c))^{\frac{5}{3}}}dx$$

input
```
int(1/(a+a*sec(d*x+c))^(5/3),x)
```

output
```
int(1/(a+a*sec(d*x+c))^(5/3),x)
```

3.163.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\sec(c+dx))^{5/3}}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+a*sec(d*x+c))^(5/3),x, algorithm="fricas")
```

output
```
Timed out
```

3.163.6 Sympy [F]

$$\int \frac{1}{(a+a\sec(c+dx))^{5/3}}\,dx = \int \frac{1}{(a\sec(c+dx)+a)^{\frac{5}{3}}}\,dx$$

input
```
integrate(1/(a+a*sec(d*x+c))**(5/3),x)
```

output
```
Integral((a*sec(c + d*x) + a)**(-5/3), x)
```

3.163.7 Maxima [F]

$$\int \frac{1}{(a+a\sec(c+dx))^{5/3}}\,dx = \int \frac{1}{(a\sec(dx+c)+a)^{\frac{5}{3}}}\,dx$$

input
```
integrate(1/(a+a*sec(d*x+c))^(5/3),x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)^(-5/3), x)
```

3.163.8 Giac [F]

$$\int \frac{1}{(a+a\sec(c+dx))^{5/3}}\,dx = \int \frac{1}{(a\sec(dx+c)+a)^{\frac{5}{3}}}\,dx$$

input
```
integrate(1/(a+a*sec(d*x+c))^(5/3),x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)^(-5/3), x)
```

3.163.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\sec(c+dx))^{5/3}}\,dx = \int \frac{1}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{5/3}}\,dx$$

input
```
int(1/(a + a/cos(c + d*x))^(5/3),x)
```

output
```
int(1/(a + a/cos(c + d*x))^(5/3), x)
```

3.164 $\int \frac{\cos(c+dx)}{(a+a\sec(c+dx))^{5/3}}\,dx$

3.164.1 Optimal result 1386
3.164.2 Mathematica [B] (warning: unable to verify) 1386
3.164.3 Rubi [A] (warning: unable to verify) 1387
3.164.4 Maple [F] 1389
3.164.5 Fricas [F(-1)] 1389
3.164.6 Sympy [F] 1390
3.164.7 Maxima [F] 1390
3.164.8 Giac [F] 1390
3.164.9 Mupad [F(-1)] 1391

3.164.1 Optimal result

Integrand size = 21, antiderivative size = 90

$$\int \frac{\cos(c+dx)}{(a+a\sec(c+dx))^{5/3}}\,dx=\frac{3\sqrt{2}\,\text{AppellF1}\left(-\frac{7}{6},\frac{1}{2},2,-\frac{1}{6},\frac{1}{2}(1+\sec(c+dx)),1+\sec(c+dx)\right)\tan(c+dx)}{7ad\sqrt{1-\sec(c+dx)}(1+\sec(c+dx))(a+a\sec(c+dx))^{2/3}}$$

output
```
3/7*AppellF1(-7/6,2,1/2,-1/6,1+sec(d*x+c),1/2+1/2*sec(d*x+c))*2^(1/2)*tan(
d*x+c)/a/d/(1+sec(d*x+c))/(a+a*sec(d*x+c))^(2/3)/(1-sec(d*x+c))^(1/2)
```

3.164.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 3011 vs. 2(90) = 180.

Time = 16.20 (sec) , antiderivative size = 3011, normalized size of antiderivative = 33.46

$$\int \frac{\cos(c+dx)}{(a+a\sec(c+dx))^{5/3}}\,dx=\text{Result too large to show}$$

input
```
Integrate[Cos[c + d*x]/(a + a*Sec[c + d*x])^(5/3),x]
```

output
```
(((1 + Cos[c + d*x])*Sec[c + d*x])^(1/3)*(1 + Sec[c + d*x])^(5/3)*((-48*Si
n[c + d*x])/7 + (51*Tan[(c + d*x)/2])/7 - (3*Sec[(c + d*x)/2]^2*Tan[(c + d
*x)/2])/14))/(d*(a*(1 + Sec[c + d*x]))^(5/3)) - (5*2^(1/3)*(1 + Sec[c + d*
x])^(5/3)*((-30*(1 + Sec[c + d*x])^(1/3))/7 + (55*Cos[c + d*x]*(1 + Sec[c
+ d*x])^(1/3))/7)*Tan[(c + d*x)/2]*((-11*AppellF1[3/2, 1/3, 1, 5/2, Tan[(c
 + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Tan[(c + d*x)/2]^2)/(Cos[c + d*x]*Sec[(
c + d*x)/2]^2)^(2/3) + 9*Cos[(c + d*x)/2]^2*(-11 - AppellF1[1/2, 1/3, 1, 3
/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]/((-1 + Tan[(c + d*x)/2]^2)*(A
ppellF1[1/2, 1/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + (2*(-
3*AppellF1[3/2, 1/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + Ap
pellF1[3/2, 4/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Tan[(c
+ d*x)/2]^2)/9)))))/(63*d*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(2/3)*(a*(1 +
Sec[c + d*x]))^(5/3)*((-5*Sec[(c + d*x)/2]^2*((-11*AppellF1[3/2, 1/3, 1, 5
/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Tan[(c + d*x)/2]^2)/(Cos[c +
d*x]*Sec[(c + d*x)/2]^2)^(2/3) + 9*Cos[(c + d*x)/2]^2*(-11 - AppellF1[1/2,
 1/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]/((-1 + Tan[(c + d*x
)/2]^2)*(AppellF1[1/2, 1/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^
2] + (2*(-3*AppellF1[3/2, 1/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/
2]^2] + AppellF1[3/2, 4/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2
])*Tan[(c + d*x)/2]^2)/9)))))/(63*2^(2/3)*(Cos[(c + d*x)/2]^2*Sec[c + d...
```

3.164.3 Rubi [A] (warning: unable to verify)

Time = 0.42 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.11, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 4315, 3042, 4314, 149, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(c+dx)}{(a\sec(c+dx)+a)^{5/3}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/3}}dx$$

$$\Big\downarrow \text{4315}$$

$$\frac{(\sec(c+dx)+1)^{2/3}\int \frac{\cos(c+dx)}{(\sec(c+dx)+1)^{5/3}}dx}{a(a\sec(c+dx)+a)^{2/3}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(\sec(c+dx)+1)^{2/3}\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)^{5/3}}dx}{a(a\sec(c+dx)+a)^{2/3}}$$

$$\Big\downarrow \; 4314$$

$$-\frac{\tan(c+dx)\sqrt[6]{\sec(c+dx)+1}\int \frac{\cos^2(c+dx)}{\sqrt{1-\sec(c+dx)}(\sec(c+dx)+1)^{13/6}}d\sec(c+dx)}{ad\sqrt{1-\sec(c+dx)}(a\sec(c+dx)+a)^{2/3}}$$

$$\Big\downarrow \; 149$$

$$-\frac{6\tan(c+dx)\sqrt[6]{\sec(c+dx)+1}\int \frac{\cos^{10}(c+dx)}{\sqrt{1-\sec(c+dx)}}d\sqrt[6]{\sec(c+dx)+1}}{ad\sqrt{1-\sec(c+dx)}(a\sec(c+dx)+a)^{2/3}}$$

$$\Big\downarrow \; 1012$$

$$\frac{3\sqrt{2}\sin(c+dx)\cos^6(c+dx)\sqrt[6]{\sec(c+dx)+1}\,\text{AppellF1}\left(-\frac{7}{6},2,\frac{1}{2},-\frac{1}{6},\sec(c+dx)+1,\frac{1}{2}(\sec(c+dx)+1)\right)}{7ad\sqrt{1-\sec(c+dx)}(a\sec(c+dx)+a)^{2/3}}$$

input
```
Int[Cos[c + d*x]/(a + a*Sec[c + d*x])^(5/3),x]
```

output
```
(3*Sqrt[2]*AppellF1[-7/6, 2, 1/2, -1/6, 1 + Sec[c + d*x], (1 + Sec[c + d*x
])/2]*Cos[c + d*x]^6*(1 + Sec[c + d*x])^(1/6)*Sin[c + d*x])/(7*a*d*Sqrt[1
- Sec[c + d*x]]*(a + a*Sec[c + d*x])^(2/3))
```

3.164.3.1 Defintions of rubi rules used

rule 149
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_)
)^(p_.), x_] :> With[{k = Denominator[m]}, Simp[k/b   Subst[Int[x^(k*(m + 1
) - 1)*(c - a*(d/b) + d*(x^k/b))^n*(e - a*(f/b) + f*(x^k/b))^p, x], x, (a +
 b*x)^(1/k)], x]] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && FractionQ[m] &&
IntegerQ[2*n] && IntegerQ[p]
```

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4314
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Simp[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x
]]*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(d*x)^(n - 1)*((a + b*x)^(m - 1/2
)/Sqrt[a - b*x]), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n},
x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

rule 4315
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Simp[a^IntPart[m]*((a + b*Csc[e + f*x])^FracPart[m
]/(1 + (b/a)*Csc[e + f*x])^FracPart[m])   Int[(1 + (b/a)*Csc[e + f*x])^m*(d
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^
2, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

3.164.4 Maple [F]

$$\int \frac{\cos(dx+c)}{(a+a\sec(dx+c))^{\frac{5}{3}}}\,dx$$

input
```
int(cos(d*x+c)/(a+a*sec(d*x+c))^(5/3),x)
```

output
```
int(cos(d*x+c)/(a+a*sec(d*x+c))^(5/3),x)
```

3.164.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\cos(c+dx)}{(a+a\sec(c+dx))^{5/3}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)/(a+a*sec(d*x+c))^(5/3),x, algorithm="fricas")
```

output
```
Timed out
```

3.164.6 Sympy [F]

$$\int \frac{\cos(c+dx)}{(a+a\sec(c+dx))^{5/3}}\,dx = \int \frac{\cos(c+dx)}{(a(\sec(c+dx)+1))^{\frac{5}{3}}}\,dx$$

input
```
integrate(cos(d*x+c)/(a+a*sec(d*x+c))**(5/3),x)
```

output
```
Integral(cos(c + d*x)/(a*(sec(c + d*x) + 1))**(5/3), x)
```

3.164.7 Maxima [F]

$$\int \frac{\cos(c+dx)}{(a+a\sec(c+dx))^{5/3}}\,dx = \int \frac{\cos(dx+c)}{(a\sec(dx+c)+a)^{\frac{5}{3}}}\,dx$$

input
```
integrate(cos(d*x+c)/(a+a*sec(d*x+c))^(5/3),x, algorithm="maxima")
```

output
```
integrate(cos(d*x + c)/(a*sec(d*x + c) + a)^(5/3), x)
```

3.164.8 Giac [F]

$$\int \frac{\cos(c+dx)}{(a+a\sec(c+dx))^{5/3}}\,dx = \int \frac{\cos(dx+c)}{(a\sec(dx+c)+a)^{\frac{5}{3}}}\,dx$$

input
```
integrate(cos(d*x+c)/(a+a*sec(d*x+c))^(5/3),x, algorithm="giac")
```

output
```
integrate(cos(d*x + c)/(a*sec(d*x + c) + a)^(5/3), x)
```

3.164.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos(c+dx)}{(a+a\sec(c+dx))^{5/3}}\,dx=\int \frac{\cos\left(c+d\,x\right)}{\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)^{5/3}}\,dx$$

input
```
int(cos(c + d*x)/(a + a/cos(c + d*x))^(5/3),x)
```

output
```
int(cos(c + d*x)/(a + a/cos(c + d*x))^(5/3), x)
```

3.165 $\int \sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))\,dx$

3.165.1 Optimal result . . . 1392
3.165.2 Mathematica [A] (verified) . . . 1393
3.165.3 Rubi [A] (verified) . . . 1393
3.165.4 Maple [B] (verified) . . . 1396
3.165.5 Fricas [C] (verification not implemented) . . . 1397
3.165.6 Sympy [F(-1)] . . . 1397
3.165.7 Maxima [F] . . . 1398
3.165.8 Giac [F] . . . 1398
3.165.9 Mupad [F(-1)] . . . 1398

3.165.1 Optimal result

Integrand size = 21, antiderivative size = 151

$$\begin{aligned}\int \sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))\,dx &= -\frac{6a\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} \\ &\quad+\frac{2a\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{3d}+\frac{6a\sqrt{\sec(c+dx)}\sin(c+dx)}{5d} \\ &\quad+\frac{2a\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d}+\frac{2a\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d}\end{aligned}$$

output
```
2/3*a*sec(d*x+c)^(3/2)*sin(d*x+c)/d+2/5*a*sec(d*x+c)^(5/2)*sin(d*x+c)/d+6/
5*a*sin(d*x+c)*sec(d*x+c)^(1/2)/d-6/5*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1
/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d
*x+c)^(1/2)/d+2/3*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellipt
icF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

3.165.2 Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.76

$$\int \sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))\,dx$$
$$= \frac{a\sec^2\left(\frac{1}{2}(c+dx)\right)(1+\sec(c+dx))\left(-9\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)+5\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+\right.\right.}{15d\sqrt{\sec(c+dx)}}$$

input
```
Integrate[Sec[c + d*x]^(5/2)*(a + a*Sec[c + d*x]),x]
```

output
```
(a*Sec[(c + d*x)/2]^2*(1 + Sec[c + d*x])*(-9*Sqrt[Cos[c + d*x]]*EllipticE[
(c + d*x)/2, 2] + 5*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + 9*Sin[c
 + d*x] + 5*Tan[c + d*x] + 3*Sec[c + d*x]*Tan[c + d*x]))/(15*d*Sqrt[Sec[c
+ d*x]])
```

3.165.3 Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.01, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.524$, Rules used = {3042, 4274, 3042, 4255, 3042, 4255, 3042, 4258, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{\frac{5}{2}}(c+dx)(a\sec(c+dx)+a)\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)dx$$
$$\Big\downarrow \text{4274}$$
$$a\int \sec^{\frac{5}{2}}(c+dx)dx+a\int \sec^{\frac{7}{2}}(c+dx)dx$$
$$\Big\downarrow \text{3042}$$
$$a\int \csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}dx+a\int \csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}dx$$
$$\Big\downarrow \text{4255}$$

$$a\left(\frac{1}{3}\int \sqrt{\sec(c+dx)}dx+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}\right)+a\left(\frac{3}{5}\int \sec^{\frac{3}{2}}(c+dx)dx+\frac{2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}\right)$$

$\Big\downarrow$ 3042

$$a\left(\frac{1}{3}\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}\right)+a\left(\frac{3}{5}\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}dx+\frac{2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}\right)$$

$\Big\downarrow$ 4255

$$a\left(\frac{3}{5}\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\int \frac{1}{\sqrt{\sec(c+dx)}}dx\right)+\frac{2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}\right)+a\left(\frac{1}{3}\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}\right)$$

$\Big\downarrow$ 3042

$$a\left(\frac{3}{5}\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)+\frac{2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}\right)+a\left(\frac{1}{3}\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}\right)$$

$\Big\downarrow$ 4258

$$a\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}\right)+a\left(\frac{3}{5}\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\cos(c+dx)}dx\right)+\frac{2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}\right.$$

$\Big\downarrow$ 3042

$$a\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}\right)+a\left(\frac{3}{5}\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx\right)+\frac{2\sin(c+dx)\sec^{\frac{5}{2}}(c}{5d}\right.$$

$\Big\downarrow$ 3119

3.165. $\int \sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))\,dx$

$$a\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}\right)+$$
$$a\left(\frac{2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}+\frac{3}{5}\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)\right)$$

$$\Big\downarrow \text{3120}$$

$$a\left(\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}\right)+$$
$$a\left(\frac{2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}+\frac{3}{5}\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)\right)$$

input
```
Int[Sec[c + d*x]^(5/2)*(a + a*Sec[c + d*x]),x]
```

output
```
a*((2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*
d) + (2*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d)) + a*((2*Sec[c + d*x]^(5/2)
*Sin[c + d*x])/(5*d) + (3*((-2*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2
]*Sqrt[Sec[c + d*x]])/d + (2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d))/5)
```

3.165.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

3.165.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 383 vs. $2(179) = 358$.

Time = 24.59 (sec) , antiderivative size = 384, normalized size of antiderivative = 2.54

method	result
default	$-\frac{a\sqrt{-\left(-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(-\frac{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}}{3\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-\frac{1}{2}\right)^2}+\frac{28\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1}}{15\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\sin}}\right.}{}$
parts	$-\frac{2a\left(-2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1}\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2+\sqrt{\frac{1}{2}-\frac{\cos(dx}{2}}\right.}{3\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right.}$

input
```
int(sec(d*x+c)^(5/2)*(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-a*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-1/3*cos(1/2
*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*
d*x+1/2*c)^2-1/2)^2+28/15*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2
*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipt
icF(cos(1/2*d*x+1/2*c),2^(1/2))-1/10*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^3-12/5*sin(1
/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)/(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d
*x+1/2*c)^2)^(1/2)-6/5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)
^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(Elliptic
F(cos(1/2*d*x+1/2*c),2^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))))/sin(
1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

3.165.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.25

$$\int \sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))\,dx$$
$$= \frac{-5i\sqrt{2}a\cos(dx+c)^2\,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+5i\sqrt{2}a\cos(dx+c)^2\,\text{wei}}{}$$

input
```
integrate(sec(d*x+c)^(5/2)*(a+a*sec(d*x+c)),x, algorithm="fricas")
```

output
```
1/15*(-5*I*sqrt(2)*a*cos(d*x + c)^2*weierstrassPInverse(-4, 0, cos(d*x + c
) + I*sin(d*x + c)) + 5*I*sqrt(2)*a*cos(d*x + c)^2*weierstrassPInverse(-4,
 0, cos(d*x + c) - I*sin(d*x + c)) - 9*I*sqrt(2)*a*cos(d*x + c)^2*weierstr
assZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)))
+ 9*I*sqrt(2)*a*cos(d*x + c)^2*weierstrassZeta(-4, 0, weierstrassPInverse(
-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*(9*a*cos(d*x + c)^2 + 5*a*cos(d
*x + c) + 3*a)*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c)^2)
```

3.165.6 Sympy [F(-1)]

Timed out.

$$\int \sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**(5/2)*(a+a*sec(d*x+c)),x)
```

output
```
Timed out
```

3.165.7 Maxima [F]

$$\int \sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))\,dx = \int (a\sec(dx+c)+a)\sec(dx+c)^{\frac{5}{2}}\,dx$$

input
```
integrate(sec(d*x+c)^(5/2)*(a+a*sec(d*x+c)),x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)*sec(d*x + c)^(5/2), x)
```

3.165.8 Giac [F]

$$\int \sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))\,dx = \int (a\sec(dx+c)+a)\sec(dx+c)^{\frac{5}{2}}\,dx$$

input
```
integrate(sec(d*x+c)^(5/2)*(a+a*sec(d*x+c)),x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)*sec(d*x + c)^(5/2), x)
```

3.165.9 Mupad [F(-1)]

Timed out.

$$\int \sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))\,dx = \int \left(a+\frac{a}{\cos(c+d\,x)}\right)\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}dx$$

input
```
int((a + a/cos(c + d*x))*(1/cos(c + d*x))^(5/2),x)
```

output
```
int((a + a/cos(c + d*x))*(1/cos(c + d*x))^(5/2), x)
```

3.166 $\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))\,dx$

3.166.1 Optimal result . . . 1399
3.166.2 Mathematica [A] (verified) . . . 1400
3.166.3 Rubi [A] (verified) . . . 1400
3.166.4 Maple [B] (verified) . . . 1403
3.166.5 Fricas [C] (verification not implemented) . . . 1403
3.166.6 Sympy [F] . . . 1404
3.166.7 Maxima [F] . . . 1404
3.166.8 Giac [F] . . . 1404
3.166.9 Mupad [F(-1)] . . . 1405

3.166.1 Optimal result

Integrand size = 21, antiderivative size = 123

$$\begin{aligned}\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))\,dx &= -\frac{2a\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} \\ &\quad+\frac{2a\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{3d} \\ &\quad+\frac{2a\sqrt{\sec(c+dx)}\sin(c+dx)}{d}+\frac{2a\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d}\end{aligned}$$

output
```
2/3*a*sec(d*x+c)^(3/2)*sin(d*x+c)/d+2*a*sin(d*x+c)*sec(d*x+c)^(1/2)/d-2*a*
(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*
c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+2/3*a*(cos(1/2*d*x+1/2*c)^
2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+
c)^(1/2)*sec(d*x+c)^(1/2)/d
```

3.166.2 Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.67

$$\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))\,dx$$
$$=\frac{a\sec^{\frac{3}{2}}(c+dx)\left(-6\cos^{\frac{3}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)+2\cos^{\frac{3}{2}}(c+dx)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+2\sin(c+dx\right.}{3d}$$

input
```
Integrate[Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x]),x]
```

output
```
(a*Sec[c + d*x]^(3/2)*(-6*Cos[c + d*x]^(3/2)*EllipticE[(c + d*x)/2, 2] + 2
*Cos[c + d*x]^(3/2)*EllipticF[(c + d*x)/2, 2] + 2*Sin[c + d*x] + 3*Sin[2*(
c + d*x)]))/(3*d)
```

3.166.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.02, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3042, 4274, 3042, 4255, 3042, 4258, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)dx$$
$$\Big\downarrow \text{4274}$$
$$a\int \sec^{\frac{3}{2}}(c+dx)dx+a\int \sec^{\frac{5}{2}}(c+dx)dx$$
$$\Big\downarrow \text{3042}$$
$$a\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}dx+a\int \csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}dx$$
$$\Big\downarrow \text{4255}$$

$$a\left(\frac{1}{3}\int \sqrt{\sec(c+dx)}dx+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}\right)+a\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\int \frac{1}{\sqrt{\sec(c+dx)}}dx\right)$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{1}{3}\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}\right)+a\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)$$

$$\Big\downarrow \text{4258}$$

$$a\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}\right)+a\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\cos(c+dx)}dx\right)$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}\right)+a\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx\right)$$

$$\Big\downarrow \text{3119}$$

$$a\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}\right)+a\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)$$

$$\Big\downarrow \text{3120}$$

$$a\left(\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}\right)+a\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)$$

input
```
Int[Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x]),x]
```

3.166. $\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))\,dx$

output
```
a*((-2*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d
+ (2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d) + a*((2*Sqrt[Cos[c + d*x]]*Ellipt
icF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (2*Sec[c + d*x]^(3/2)*Sin[
c + d*x])/(3*d))
```

3.166.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

3.166.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 367 vs. $2(159) = 318$.

Time = 19.54 (sec) , antiderivative size = 368, normalized size of antiderivative = 2.99

method	result
default	$-\frac{2a\sqrt{-\left(-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\left(12\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1}\,\mathrm{EllipticF}\left(\cos\left(\right.\right.}{}$
parts	$-\frac{2a\left(-2\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1}\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\mathrm{si}}\right.}{\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1}\,d}$

input
```
int(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-2/3*a*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(4*sin(1/
2*d*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)/sin(1/2*d*x+1/2*c)^3*(12*sin(1/2*
d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/
2*c)^2-6*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)
)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2-8*cos(1/2*d*x+1/2*
c)*sin(1/2*d*x+1/2*c)^2+(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)
^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1
/2))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1
/2*c)^2-1)^(1/2)/d
```

3.166.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.36

$$\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))\,dx$$
$$= \frac{-i\sqrt{2}a\cos(dx+c)\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+i\sqrt{2}a\cos(dx+c)\,\mathrm{weierst}}{}$$

input
```
integrate(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c)),x, algorithm="fricas")
```

output
```
1/3*(-I*sqrt(2)*a*cos(d*x + c)*weierstrassPInverse(-4, 0, cos(d*x + c) + I
*sin(d*x + c)) + I*sqrt(2)*a*cos(d*x + c)*weierstrassPInverse(-4, 0, cos(d
*x + c) - I*sin(d*x + c)) - 3*I*sqrt(2)*a*cos(d*x + c)*weierstrassZeta(-4,
 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 3*I*sqrt(
2)*a*cos(d*x + c)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*
x + c) - I*sin(d*x + c))) + 2*(3*a*cos(d*x + c) + a)*sin(d*x + c)/sqrt(cos
(d*x + c)))/(d*cos(d*x + c))
```

3.166.6 Sympy [F]

$$\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))\,dx = a\left(\int \sec^{\frac{3}{2}}(c+dx)\,dx + \int \sec^{\frac{5}{2}}(c+dx)\,dx\right)$$

input
```
integrate(sec(d*x+c)**(3/2)*(a+a*sec(d*x+c)),x)
```

output
```
a*(Integral(sec(c + d*x)**(3/2), x) + Integral(sec(c + d*x)**(5/2), x))
```

3.166.7 Maxima [F]

$$\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))\,dx = \int (a\sec(dx+c)+a)\sec(dx+c)^{\frac{3}{2}}\,dx$$

input
```
integrate(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c)),x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)*sec(d*x + c)^(3/2), x)
```

3.166.8 Giac [F]

$$\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))\,dx = \int (a\sec(dx+c)+a)\sec(dx+c)^{\frac{3}{2}}\,dx$$

input
```
integrate(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c)),x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)*sec(d*x + c)^(3/2), x)
```

3.166.9 Mupad [F(-1)]

Timed out.

$$\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))\,dx=\int \left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)\,{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{3/2}\,dx$$

input
```
int((a + a/cos(c + d*x))*(1/cos(c + d*x))^(3/2),x)
```

output
```
int((a + a/cos(c + d*x))*(1/cos(c + d*x))^(3/2), x)
```

3.167 $\int \sqrt{\sec(c+dx)}(a+a\sec(c+dx))\,dx$

3.167.1 Optimal result 1406
3.167.2 Mathematica [A] (verified) 1406
3.167.3 Rubi [A] (verified) 1407
3.167.4 Maple [A] (verified) 1409
3.167.5 Fricas [C] (verification not implemented) 1410
3.167.6 Sympy [F] 1410
3.167.7 Maxima [F] 1410
3.167.8 Giac [F] 1411
3.167.9 Mupad [F(-1)] 1411

3.167.1 Optimal result

Integrand size = 21, antiderivative size = 97

$$\int \sqrt{\sec(c+dx)}(a+a\sec(c+dx))\,dx = -\frac{2a\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{2a\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{d} + \frac{2a\sqrt{\sec(c+dx)}\sin(c+dx)}{d}$$

output
```
2*a*sin(d*x+c)*sec(d*x+c)^(1/2)/d-2*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2
*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x
+c)^(1/2)/d+2*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(
sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

3.167.2 Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.70

$$\int \sqrt{\sec(c+dx)}(a+a\sec(c+dx))\,dx = \frac{2a\sqrt{\sec(c+dx)}\left(-\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)+\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+\sin(c+dx)\right)}{d}$$

input
```
Integrate[Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x]),x]
```

output
```
(2*a*Sqrt[Sec[c + d*x]]*(-(Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]) +
 Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + Sin[c + d*x]))/d
```

3.167.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.01, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 4274, 3042, 4255, 3042, 4258, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\sec(c+dx)}(a\sec(c+dx)+a)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)dx$$

$$\Big\downarrow \text{4274}$$

$$a\int \sec^{\frac{3}{2}}(c+dx)dx+a\int \sqrt{\sec(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$a\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+a\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}dx$$

$$\Big\downarrow \text{4255}$$

$$a\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+a\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\int \frac{1}{\sqrt{\sec(c+dx)}}dx\right)$$

$$\Big\downarrow \text{3042}$$

$$a\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+a\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)$$

$$\Big\downarrow \text{4258}$$

$$a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx+$$
$$a\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\cos(c+dx)}dx\right)$$

$\Big\downarrow$ 3042

$$a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+$$
$$a\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx\right)$$

$\Big\downarrow$ 3119

$$a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+$$
$$a\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)$$

$\Big\downarrow$ 3120

$$\frac{2a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+$$
$$a\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)$$

input
```
Int[Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x]),x]
```

output
```
(2*a*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d +
a*((-2*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d
+ (2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d)
```

3.167.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

3.167.4 Maple [A] (verified)

Time = 15.67 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.53

method	result
default	$\frac{2a\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2-\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1}\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)-\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\mathrm{EllipticE}\left(\cos\left(\right.\right.\right.}{\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1}\,d}$
parts	$-\frac{2a\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1}\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1}\,d}-\frac{2a\left(-2\sqrt{-2\sin\left(\right.}\right.}{}$

input
```
int(sec(d*x+c)^(1/2)*(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
2*a*(2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-(
sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1
/2*d*x+1/2*c)^2-1)^(1/2))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1
/2)/d
```

3.167.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.28

$$\int \sqrt{\sec(c+dx)}(a+a\sec(c+dx))\,dx$$
$$= \frac{-i\sqrt{2}a\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+i\sqrt{2}a\text{weierstrassPInverse}(-4,0,\cos(d}{}$$

input
```
integrate(sec(d*x+c)^(1/2)*(a+a*sec(d*x+c)),x, algorithm="fricas")
```

output
```
(-I*sqrt(2)*a*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) +
I*sqrt(2)*a*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - I*
sqrt(2)*a*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) +
 I*sin(d*x + c))) + I*sqrt(2)*a*weierstrassZeta(-4, 0, weierstrassPInverse
(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*a*sin(d*x + c)/sqrt(cos(d*x +
c)))/d
```

3.167.6 Sympy [F]

$$\int \sqrt{\sec(c+dx)}(a+a\sec(c+dx))\,dx = a\left(\int \sqrt{\sec(c+dx)}\,dx + \int \sec^{\frac{3}{2}}(c+dx)\,dx\right)$$

input
```
integrate(sec(d*x+c)**(1/2)*(a+a*sec(d*x+c)),x)
```

output
```
a*(Integral(sqrt(sec(c + d*x)), x) + Integral(sec(c + d*x)**(3/2), x))
```

3.167.7 Maxima [F]

$$\int \sqrt{\sec(c+dx)}(a+a\sec(c+dx))\,dx = \int (a\sec(dx+c)+a)\sqrt{\sec(dx+c)}\,dx$$

input
```
integrate(sec(d*x+c)^(1/2)*(a+a*sec(d*x+c)),x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)*sqrt(sec(d*x + c)), x)
```

3.167.8 Giac [F]

$$\int \sqrt{\sec(c+dx)}(a+a\sec(c+dx))\,dx = \int (a\sec(dx+c)+a)\sqrt{\sec(dx+c)}\,dx$$

input
```
integrate(sec(d*x+c)^(1/2)*(a+a*sec(d*x+c)),x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)*sqrt(sec(d*x + c)), x)
```

3.167.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{\sec(c+dx)}(a+a\sec(c+dx))\,dx = \int \left(a+\frac{a}{\cos(c+d\,x)}\right)\sqrt{\frac{1}{\cos(c+d\,x)}}\,dx$$

input
```
int((a + a/cos(c + d*x))*(1/cos(c + d*x))^(1/2),x)
```

output
```
int((a + a/cos(c + d*x))*(1/cos(c + d*x))^(1/2), x)
```

3.168 $\int \frac{a+a\sec(c+dx)}{\sqrt{\sec(c+dx)}}\,dx$

3.168.1 Optimal result 1412
3.168.2 Mathematica [A] (verified) 1412
3.168.3 Rubi [A] (verified) 1413
3.168.4 Maple [A] (verified) 1415
3.168.5 Fricas [C] (verification not implemented) 1415
3.168.6 Sympy [F] 1416
3.168.7 Maxima [F] 1416
3.168.8 Giac [F] 1416
3.168.9 Mupad [F(-1)] 1417

3.168.1 Optimal result

Integrand size = 21, antiderivative size = 75

$$\int \frac{a+a\sec(c+dx)}{\sqrt{\sec(c+dx)}}\,dx = \frac{2a\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{2a\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{d}$$

output
```
2*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+
1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+2*a*(cos(1/2*d*x+1/2*c
)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*
x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

3.168.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.65

$$\int \frac{a+a\sec(c+dx)}{\sqrt{\sec(c+dx)}}\,dx = \frac{2a\sqrt{\cos(c+dx)}\left(E\left(\frac{1}{2}(c+dx)\middle|2\right)+\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\right)\sqrt{\sec(c+dx)}}{d}$$

input
```
Integrate[(a + a*Sec[c + d*x])/Sqrt[Sec[c + d*x]],x]
```

output
```
(2*a*Sqrt[Cos[c + d*x]]*(EllipticE[(c + d*x)/2, 2] + EllipticF[(c + d*x)/2
, 2])*Sqrt[Sec[c + d*x]])/d
```

3.168.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 4274, 3042, 4258, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a\sec(c+dx)+a}{\sqrt{\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4274}$$

$$a\int \frac{1}{\sqrt{\sec(c+dx)}}dx+a\int \sqrt{\sec(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$a\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+a\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{4258}$$

$$a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx+a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\cos(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+$$
$$a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3119}$$

$$a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

$$\Big\downarrow \; 3120$$

$$\frac{2a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+\frac{2a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

input
```
Int[(a + a*Sec[c + d*x])/Sqrt[Sec[c + d*x]],x]
```

output
```
(2*a*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d +
(2*a*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d
```

3.168.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

3.168.4 Maple [A] (verified)

Time = 12.25 (sec) , antiderivative size = 150, normalized size of antiderivative = 2.00

method	result
default	$-\frac{2\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,a\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+1}\,\left(\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)-\mathrm{EllipticE}\left(\cos\left(\frac{dx}{2}+\right.\right.\right.}{\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1}\,d}$
parts	$\frac{2a\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+1}\,\mathrm{EllipticE}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1}\,d}-\frac{2a\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}}{}$
risch	$-\frac{ia\sqrt{2}}{d\sqrt{\frac{e^{i(dx+c)}}{e^{2i(dx+c)}+1}}}-\frac{i\left(\frac{i\sqrt{-i\left(e^{i(dx+c)}+i\right)}\,\sqrt{2}\,\sqrt{i\left(e^{i(dx+c)}-i\right)}\,\sqrt{ie^{i(dx+c)}}\,\mathrm{EllipticF}\left(\sqrt{-i\left(e^{i(dx+c)}+i\right)},\frac{\sqrt{2}}{2}\right)}{\sqrt{e^{3i(dx+c)}+e^{i(dx+c)}}}-\frac{2\left(e^{2i(dx+c)}+1\right)}{\sqrt{e^{i(dx+c)}\left(e^{2i(dx+c)}+1\right.}}\right.}{}$

input
```
int((a+a*sec(d*x+c))/sec(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-Ellipt
icE(cos(1/2*d*x+1/2*c),2^(1/2)))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^
2-1)^(1/2)/d
```

3.168.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.43

$$\int \frac{a+a\sec(c+dx)}{\sqrt{\sec(c+dx)}}\,dx$$
$$=\frac{-i\sqrt{2}a\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+i\sqrt{2}a\mathrm{weierstrassPInverse}(-4,0,\cos(d}{}$$

input
```
integrate((a+a*sec(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
(-I*sqrt(2)*a*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) +
I*sqrt(2)*a*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + I*
sqrt(2)*a*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) +
 I*sin(d*x + c))) - I*sqrt(2)*a*weierstrassZeta(-4, 0, weierstrassPInverse
(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/d
```

3.168.6 Sympy [F]

$$\int \frac{a + a\sec(c + dx)}{\sqrt{\sec(c + dx)}}\,dx = a\left(\int \frac{1}{\sqrt{\sec(c + dx)}}\,dx + \int \sqrt{\sec(c + dx)}\,dx\right)$$

input
```
integrate((a+a*sec(d*x+c))/sec(d*x+c)**(1/2),x)
```

output
```
a*(Integral(1/sqrt(sec(c + d*x)), x) + Integral(sqrt(sec(c + d*x)), x))
```

3.168.7 Maxima [F]

$$\int \frac{a + a\sec(c + dx)}{\sqrt{\sec(c + dx)}}\,dx = \int \frac{a\sec(dx + c) + a}{\sqrt{\sec(dx + c)}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)/sqrt(sec(d*x + c)), x)
```

3.168.8 Giac [F]

$$\int \frac{a + a\sec(c + dx)}{\sqrt{\sec(c + dx)}}\,dx = \int \frac{a\sec(dx + c) + a}{\sqrt{\sec(dx + c)}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)/sqrt(sec(d*x + c)), x)
```

3.168.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+a\sec(c+dx)}{\sqrt{\sec(c+dx)}}\,dx=\int \frac{a+\frac{a}{\cos(c+d\,x)}}{\sqrt{\frac{1}{\cos(c+d\,x)}}}\,dx$$

input
```
int((a + a/cos(c + d*x))/(1/cos(c + d*x))^(1/2),x)
```

output
```
int((a + a/cos(c + d*x))/(1/cos(c + d*x))^(1/2), x)
```

3.169 $\int \frac{a+a\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

3.169.1 Optimal result 1418
3.169.2 Mathematica [A] (verified) 1418
3.169.3 Rubi [A] (verified) 1419
3.169.4 Maple [A] (verified) 1421
3.169.5 Fricas [C] (verification not implemented) 1422
3.169.6 Sympy [F] 1422
3.169.7 Maxima [F] 1423
3.169.8 Giac [F] 1423
3.169.9 Mupad [F(-1)] 1423

3.169.1 Optimal result

Integrand size = 21, antiderivative size = 101

$$\int \frac{a+a\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \frac{2a\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{2a\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{3d} + \frac{2a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}$$

output
```
2/3*a*sin(d*x+c)/d/sec(d*x+c)^(1/2)+2*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1
/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d
*x+c)^(1/2)/d+2/3*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellipt
icF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

3.169.2 Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.72

$$\int \frac{a+a\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \frac{a\sqrt{\sec(c+dx)}\left(6\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)+2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+\sin(2(c+dx\right.}{3d}$$

input
```
Integrate[(a + a*Sec[c + d*x])/Sec[c + d*x]^(3/2),x]
```

output
```
(a*Sqrt[Sec[c + d*x]]*(6*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 2*
Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + Sin[2*(c + d*x)]))/(3*d)
```

3.169.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.01, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 4274, 3042, 4256, 3042, 4258, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a\sec(c+dx)+a}{\sec^{\frac{3}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{4274}$$

$$a\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}dx+a\int \frac{1}{\sqrt{\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$a\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+a\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4256}$$

$$a\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+a\left(\frac{1}{3}\int \sqrt{\sec(c+dx)}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)$$

$$\Big\downarrow \text{3042}$$

$$a\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+a\left(\frac{1}{3}\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)$$

$$\Big\downarrow \text{4258}$$

$$a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\cos(c+dx)}dx+$$
$$a\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)$$

$\Big\downarrow$ 3042

$$a\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)+$$
$$a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx$$

$\Big\downarrow$ 3119

$$a\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)+$$
$$\frac{2a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

$\Big\downarrow$ 3120

$$\frac{2a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+$$
$$a\left(\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}\right)$$

input
```
Int[(a + a*Sec[c + d*x])/Sec[c + d*x]^(3/2),x]
```

output
```
(2*a*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d +
a*((2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*
d) + (2*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]]))
```

3.169.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

3.169.4 Maple [A] (verified)

Time = 18.04 (sec) , antiderivative size = 225, normalized size of antiderivative = 2.23

method	result
default	$-\frac{2\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\,a\left(4\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\sin\left(\frac{dx}{2}+\right.}\right.}{3\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right.}$
parts	$-\frac{2a\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(4\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\sin\left(\frac{dx}{2}+\right.}\right.}{3\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1}\,d}$

input
```
int((a+a*sec(d*x+c))/sec(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a*(4*sin(1/2*
d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(co
s(1/2*d*x+1/2*c),2^(1/2))-3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2
*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2))/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)
^2-1)^(1/2)/d
```

3.169.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.24

$$\int \frac{a+a\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)}\,dx$$
$$= \frac{2\,a\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)-i\,\sqrt{2}a\text{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right))+i\,\sqrt{2}a\text{we}}{}$$

input
```
integrate((a+a*sec(d*x+c))/sec(d*x+c)^(3/2),x, algorithm="fricas")
```

output
```
1/3*(2*a*sqrt(cos(d*x + c))*sin(d*x + c) - I*sqrt(2)*a*weierstrassPInverse
(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + I*sqrt(2)*a*weierstrassPInverse(-
4, 0, cos(d*x + c) - I*sin(d*x + c)) + 3*I*sqrt(2)*a*weierstrassZeta(-4, 0
, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 3*I*sqrt(2)
*a*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(
d*x + c))))/d
```

3.169.6 Sympy [F]

$$\int \frac{a+a\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)}\,dx = a\left(\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}\,dx + \int \frac{1}{\sqrt{\sec(c+dx)}}\,dx\right)$$

input
```
integrate((a+a*sec(d*x+c))/sec(d*x+c)**(3/2),x)
```

output
```
a*(Integral(sec(c + d*x)**(-3/2), x) + Integral(1/sqrt(sec(c + d*x)), x))
```

3.169.7 Maxima [F]

$$\int \frac{a+a\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)}\,dx=\int \frac{a\sec(dx+c)+a}{\sec(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))/sec(d*x+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)/sec(d*x + c)^(3/2), x)
```

3.169.8 Giac [F]

$$\int \frac{a+a\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)}\,dx=\int \frac{a\sec(dx+c)+a}{\sec(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))/sec(d*x+c)^(3/2),x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)/sec(d*x + c)^(3/2), x)
```

3.169.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+a\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)}\,dx=\int \frac{a+\frac{a}{\cos(c+d\,x)}}{\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

input
```
int((a + a/cos(c + d*x))/(1/cos(c + d*x))^(3/2),x)
```

output
```
int((a + a/cos(c + d*x))/(1/cos(c + d*x))^(3/2), x)
```

3.170 $\int \frac{a+a\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)}\,dx$

3.170.1 Optimal result 1424
3.170.2 Mathematica [A] (verified) 1424
3.170.3 Rubi [A] (verified) 1425
3.170.4 Maple [A] (verified) 1427
3.170.5 Fricas [C] (verification not implemented) 1428
3.170.6 Sympy [F] 1429
3.170.7 Maxima [F] 1429
3.170.8 Giac [F] 1429
3.170.9 Mupad [F(-1)] 1430

3.170.1 Optimal result

Integrand size = 21, antiderivative size = 127

$$\int \frac{a+a\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \frac{6a\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{2a\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{3d} + \frac{2a\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{2a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}$$

output
```
2/5*a*sin(d*x+c)/d/sec(d*x+c)^(3/2)+2/3*a*sin(d*x+c)/d/sec(d*x+c)^(1/2)+6/
5*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+
1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+2/3*a*(cos(1/2*d*x+1/2
*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(
d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

3.170.2 Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.73

$$\int \frac{a+a\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \frac{a\sqrt{\sec(c+dx)}\left(36\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)+20\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+3\sin(c+d\right.}{30d}$$

input
```
Integrate[(a + a*Sec[c + d*x])/Sec[c + d*x]^(5/2),x]
```

output
```
(a*Sqrt[Sec[c + d*x]]*(36*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 2
0*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + 3*Sin[c + d*x] + 10*Sin[2
*(c + d*x)] + 3*Sin[3*(c + d*x)]))/(30*d)
```

3.170.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.02, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 4274, 3042, 4256, 3042, 4258, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a\sec(c+dx)+a}{\sec^{\frac{5}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx$$

$$\Big\downarrow \text{4274}$$

$$a\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)}dx+a\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$a\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+a\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{4256}$$

$$a\left(\frac{3}{5}\int \frac{1}{\sqrt{\sec(c+dx)}}dx+\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}\right)+a\left(\frac{1}{3}\int \sqrt{\sec(c+dx)}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{3}{5}\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}\right)+ a\left(\frac{1}{3}\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)$$

$$\Big\downarrow \text{4258}$$

$$a\left(\frac{3}{5}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\cos(c+dx)}dx+\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}\right)+a\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{3}{5}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}\right)+a\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)$$

$$\Big\downarrow \text{3119}$$

$$a\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)+a\left(\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{6\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}\right)$$

$$\Big\downarrow \text{3120}$$

$$a\left(\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}\right)+a\left(\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{6\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}\right)$$

input
```
Int[(a + a*Sec[c + d*x])/Sec[c + d*x]^(5/2),x]
```

output
```
a*((6*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*
d) + (2*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2))) + a*((2*Sqrt[Cos[c + d*x]]
*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (2*Sin[c + d*x])/(3
*d*Sqrt[Sec[c + d*x]]))
```

3.170. $\int \frac{a+a\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)}\,dx$

3.170.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

3.170.4 Maple [A] (verified)

Time = 16.01 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.72

method	result
default	$-\frac{2\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\,a\left(24\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^7-28\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^5+5\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1}\,\mathrm{EllipticF}\left(\right.\right.}{15\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right.}$
parts	$-\frac{2a\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(-8\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^6\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+8\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}{5\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)}}$

input
```
int((a+a*sec(d*x+c))/sec(d*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/15*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a*(24*cos(1/
2*d*x+1/2*c)^7-28*cos(1/2*d*x+1/2*c)^5+5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*
cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-9*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1
/2*d*x+1/2*c),2^(1/2))+4*cos(1/2*d*x+1/2*c))/(-2*sin(1/2*d*x+1/2*c)^4+sin(
1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2
)/d
```

3.170.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.14

$$\int \frac{a+a\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)}\,dx$$

$$= \frac{-5i\sqrt{2}a\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+5i\sqrt{2}a\text{weierstrassPInverse}(-4,0,\cos}{}$$

input
```
integrate((a+a*sec(d*x+c))/sec(d*x+c)^(5/2),x, algorithm="fricas")
```

output
```
1/15*(-5*I*sqrt(2)*a*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x +
 c)) + 5*I*sqrt(2)*a*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x +
 c)) + 9*I*sqrt(2)*a*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos
(d*x + c) + I*sin(d*x + c))) - 9*I*sqrt(2)*a*weierstrassZeta(-4, 0, weiers
trassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*(3*a*cos(d*x + c)
^2 + 5*a*cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/d
```

3.170.6 Sympy [F]

$$\int \frac{a + a\sec(c + dx)}{\sec^{\frac{5}{2}}(c + dx)}\,dx = a\left(\int \frac{1}{\sec^{\frac{5}{2}}(c + dx)}\,dx + \int \frac{1}{\sec^{\frac{3}{2}}(c + dx)}\,dx\right)$$

input
```
integrate((a+a*sec(d*x+c))/sec(d*x+c)**(5/2),x)
```

output
```
a*(Integral(sec(c + d*x)**(-5/2), x) + Integral(sec(c + d*x)**(-3/2), x))
```

3.170.7 Maxima [F]

$$\int \frac{a + a\sec(c + dx)}{\sec^{\frac{5}{2}}(c + dx)}\,dx = \int \frac{a\sec(dx + c) + a}{\sec(dx + c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))/sec(d*x+c)^(5/2),x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)/sec(d*x + c)^(5/2), x)
```

3.170.8 Giac [F]

$$\int \frac{a + a\sec(c + dx)}{\sec^{\frac{5}{2}}(c + dx)}\,dx = \int \frac{a\sec(dx + c) + a}{\sec(dx + c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))/sec(d*x+c)^(5/2),x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)/sec(d*x + c)^(5/2), x)
```

3.170.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+a\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)}\,dx=\int \frac{a+\frac{a}{\cos(c+d\,x)}}{\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

input
```
int((a + a/cos(c + d*x))/(1/cos(c + d*x))^(5/2),x)
```

output
```
int((a + a/cos(c + d*x))/(1/cos(c + d*x))^(5/2), x)
```

3.171 $\int \frac{a+a\sec(c+dx)}{\sec^{\frac{7}{2}}(c+dx)}\,dx$

3.171.1 Optimal result 1431
3.171.2 Mathematica [A] (verified) 1431
3.171.3 Rubi [A] (verified) 1432
3.171.4 Maple [A] (verified) 1435
3.171.5 Fricas [C] (verification not implemented) 1435
3.171.6 Sympy [F] 1436
3.171.7 Maxima [F] 1436
3.171.8 Giac [F] 1437
3.171.9 Mupad [F(-1)] 1437

3.171.1 Optimal result

Integrand size = 21, antiderivative size = 151

$$\int \frac{a+a\sec(c+dx)}{\sec^{\frac{7}{2}}(c+dx)}\,dx = \frac{6a\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{10a\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{21d} + \frac{2a\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{2a\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{10a\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}$$

output
```
2/7*a*sin(d*x+c)/d/sec(d*x+c)^(5/2)+2/5*a*sin(d*x+c)/d/sec(d*x+c)^(3/2)+10
/21*a*sin(d*x+c)/d/sec(d*x+c)^(1/2)+6/5*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos
(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec
(d*x+c)^(1/2)/d+10/21*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*El
lipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

3.171.2 Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.68

$$\int \frac{a+a\sec(c+dx)}{\sec^{\frac{7}{2}}(c+dx)}\,dx = \frac{a\sqrt{\sec(c+dx)}\left(504\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)+200\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+42\sin(c\right.}{420d}$$

input
```
Integrate[(a + a*Sec[c + d*x])/Sec[c + d*x]^(7/2),x]
```

output
```
(a*Sqrt[Sec[c + d*x]]*(504*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] +
200*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + 42*Sin[c + d*x] + 130*S
in[2*(c + d*x)] + 42*Sin[3*(c + d*x)] + 15*Sin[4*(c + d*x)]))/(420*d)
```

3.171.3 Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.04, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.524$, Rules used = {3042, 4274, 3042, 4256, 3042, 4256, 3042, 4258, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a\sec(c+dx)+a}{\sec^{\frac{7}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx$$

$$\Big\downarrow \text{4274}$$

$$a\int \frac{1}{\sec^{\frac{7}{2}}(c+dx)}dx+a\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$a\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx+a\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx$$

$$\Big\downarrow \text{4256}$$

$$a\left(\frac{5}{7}\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}dx+\frac{2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}\right)+a\left(\frac{3}{5}\int \frac{1}{\sqrt{\sec(c+dx)}}dx+\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}\right)$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{5}{7}\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+\frac{2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}\right)+$$
$$a\left(\frac{3}{5}\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}\right)$$

$$\Big\downarrow \text{4256}$$

$$a\left(\frac{5}{7}\left(\frac{1}{3}\int\sqrt{\sec(c+dx)}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)+\frac{2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}\right)+a\left(\frac{3}{5}\int\frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}\right)$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{3}{5}\int\frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}\right)+a\left(\frac{5}{7}\left(\frac{1}{3}\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)+\frac{2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}\right)$$

$$\Big\downarrow \text{4258}$$

$$a\left(\frac{5}{7}\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)+\frac{2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}\right)+a\left(\frac{3}{5}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\cos(c+dx)}dx+\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}\right)$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{5}{7}\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)+\frac{2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}\right)+a\left(\frac{3}{5}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}\right)$$

$$\Big\downarrow \text{3119}$$

$$a\left(\frac{5}{7}\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)+\frac{2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}\right)+a\left(\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{6\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}\right)$$

$$\Big\downarrow \text{3120}$$

3.171. $\int\frac{a+a\sec(c+dx)}{\sec^{\frac{7}{2}}(c+dx)}\,dx$

$$a\left(\frac{2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+\frac{5}{7}\left(\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}\right)\right)+a\left(\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{6\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}\right)$$

input
```
Int[(a + a*Sec[c + d*x])/Sec[c + d*x]^(7/2),x]
```

output
```
a*((6*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*
d) + (2*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2))) + a*((2*Sin[c + d*x])/(7*d
*Sec[c + d*x]^(5/2)) + (5*((2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]
*Sqrt[Sec[c + d*x]])/(3*d) + (2*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])))/7
)
```

3.171.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

3.171.4 Maple [A] (verified)

Time = 19.47 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.79

method	result
default	$-\frac{2\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,a\left(240\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{8}-528\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+448\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}\cos\left(\frac{d}{2}\right.\right.}{105\sqrt{-2\sin}}$
parts	$-\frac{2a\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\left(48\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{9}-120\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}+128\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}-72\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}+5\sqrt{\frac{1}{2}-\frac{\cos(d}{2}}\right.}{21\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\cos\left(\frac{dx}{2}+\right.}}$

input
```
int((a+a*sec(d*x+c))/sec(d*x+c)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-2/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a*(240*cos(
1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8-528*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1
/2*c)+448*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-122*cos(1/2*d*x+1/2*c)*s
in(1/2*d*x+1/2*c)^2+25*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^
2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-63*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1
/2))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*
c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

3.171.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.03

$$\int \frac{a+a\sec(c+dx)}{\sec^{\frac{7}{2}}(c+dx)}\,dx$$

$$= \frac{-25i\sqrt{2}a\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+25i\sqrt{2}a\text{weierstrassPInverse}(-4,0,c}{}$$

input
```
integrate((a+a*sec(d*x+c))/sec(d*x+c)^(7/2),x, algorithm="fricas")
```

output
```
1/105*(-25*I*sqrt(2)*a*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x
 + c)) + 25*I*sqrt(2)*a*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*
x + c)) + 63*I*sqrt(2)*a*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0,
 cos(d*x + c) + I*sin(d*x + c))) - 63*I*sqrt(2)*a*weierstrassZeta(-4, 0, w
eierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*(15*a*cos(d*
x + c)^3 + 21*a*cos(d*x + c)^2 + 25*a*cos(d*x + c))*sin(d*x + c)/sqrt(cos(
d*x + c)))/d
```

3.171.6 Sympy [F]

$$\int \frac{a + a\sec(c + dx)}{\sec^{\frac{7}{2}}(c + dx)}\,dx = a\left(\int \frac{1}{\sec^{\frac{7}{2}}(c + dx)}\,dx + \int \frac{1}{\sec^{\frac{5}{2}}(c + dx)}\,dx\right)$$

input
```
integrate((a+a*sec(d*x+c))/sec(d*x+c)**(7/2),x)
```

output
```
a*(Integral(sec(c + d*x)**(-7/2), x) + Integral(sec(c + d*x)**(-5/2), x))
```

3.171.7 Maxima [F]

$$\int \frac{a + a\sec(c + dx)}{\sec^{\frac{7}{2}}(c + dx)}\,dx = \int \frac{a\sec(dx + c) + a}{\sec(dx + c)^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))/sec(d*x+c)^(7/2),x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)/sec(d*x + c)^(7/2), x)
```

3.171.8 Giac [F]

$$\int \frac{a+a\sec(c+dx)}{\sec^{\frac{7}{2}}(c+dx)}\,dx=\int \frac{a\sec(dx+c)+a}{\sec(dx+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))/sec(d*x+c)^(7/2),x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)/sec(d*x + c)^(7/2), x)
```

3.171.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+a\sec(c+dx)}{\sec^{\frac{7}{2}}(c+dx)}\,dx=\int \frac{a+\frac{a}{\cos(c+d\,x)}}{\left(\frac{1}{\cos(c+d\,x)}\right)^{7/2}}\,dx$$

input
```
int((a + a/cos(c + d*x))/(1/cos(c + d*x))^(7/2),x)
```

output
```
int((a + a/cos(c + d*x))/(1/cos(c + d*x))^(7/2), x)
```

3.172 $\int \sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^2\,dx$

3.172.1 Optimal result 1438
3.172.2 Mathematica [C] (verified) 1439
3.172.3 Rubi [A] (verified) 1439
3.172.4 Maple [B] (verified) 1443
3.172.5 Fricas [C] (verification not implemented) 1444
3.172.6 Sympy [F(-1)] 1445
3.172.7 Maxima [F] 1445
3.172.8 Giac [F] 1445
3.172.9 Mupad [F(-1)] 1446

3.172.1 Optimal result

Integrand size = 23, antiderivative size = 187

$$\begin{aligned}\int \sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^2\,dx &= -\frac{12a^2\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} \\ &\quad+\frac{8a^2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{7d} \\ &\quad+\frac{12a^2\sqrt{\sec(c+dx)}\sin(c+dx)}{5d}+\frac{8a^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{7d} \\ &\quad+\frac{4a^2\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d}+\frac{2a^2\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{7d}\end{aligned}$$

output
```
8/7*a^2*sec(d*x+c)^(3/2)*sin(d*x+c)/d+4/5*a^2*sec(d*x+c)^(5/2)*sin(d*x+c)/
d+2/7*a^2*sec(d*x+c)^(7/2)*sin(d*x+c)/d+12/5*a^2*sin(d*x+c)*sec(d*x+c)^(1/
2)/d-12/5*a^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(si
n(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+8/7*a^2*(cos
(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2
^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

3.172.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 3.22 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.53

$$\int \sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^2\,dx$$
$$= \frac{a^2\sec^4\left(\frac{1}{2}(c+dx)\right)(1+\sec(c+dx))^2\left(-\frac{2i\sqrt{2}e^{-i(c+dx)}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\cos^2(c+dx)\left(21\left(1+e^{2i(c+dx)}\right)+21\left(-1+e^{2ic}\right)\sqrt{1+e^{2i(c+}}\right.}{}\right.}{}$$

input
```
Integrate[Sec[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^2,x]
```

output
```
(a^2*Sec[(c + d*x)/2]^4*(1 + Sec[c + d*x])^2*(((-2*I)*Sqrt[2]*Sqrt[E^(I*(c
 + d*x))/(1 + E^((2*I)*(c + d*x)))]*Cos[c + d*x]^2*(21*(1 + E^((2*I)*(c +
d*x))) + 21*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometri
c2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))] + 10*E^(I*(c + d*x))*(-1 + E^((
2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E
^((2*I)*(c + d*x))]))/(E^(I*(c + d*x))*(-1 + E^((2*I)*c))) + (42*Cos[d*x]*
Csc[c] + (15 + 14*Cos[c + d*x] + 10*Cos[2*(c + d*x)])*Sec[c + d*x]^2*Tan[c
 + d*x])/Sec[c + d*x]^(3/2)))/(70*d)
```

3.172.3 Rubi [A] (verified)

Time = 1.02 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.739$, Rules used = {3042, 4275, 3042, 4255, 3042, 4255, 3042, 4258, 3042, 3119, 4534, 3042, 4255, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{\frac{5}{2}}(c+dx)(a\sec(c+dx)+a)^2\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^2dx$$
$$\Big\downarrow \; 4275$$

$$2a^2\int \sec^{\frac{7}{2}}(c+dx)dx+\int \sec^{\frac{5}{2}}(c+dx)\left(\sec^2(c+dx)a^2+a^2\right)dx$$

$$\Big\downarrow \text{3042}$$

$$2a^2\int \csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}dx+\int \csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2\right)dx$$

$$\Big\downarrow \text{4255}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2\right)dx+$$
$$2a^2\left(\frac{3}{5}\int \sec^{\frac{3}{2}}(c+dx)dx+\frac{2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}\right)$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2\right)dx+$$
$$2a^2\left(\frac{3}{5}\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}dx+\frac{2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}\right)$$

$$\Big\downarrow \text{4255}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2\right)dx+$$
$$2a^2\left(\frac{3}{5}\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\int \frac{1}{\sqrt{\sec(c+dx)}}dx\right)+\frac{2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}\right)$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2\right)dx+$$
$$2a^2\left(\frac{3}{5}\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)+\frac{2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}\right)$$

$$\Big\downarrow \text{4258}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2\right)dx+$$
$$2a^2\left(\frac{3}{5}\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\cos(c+dx)}dx\right)+\frac{2\sin(c+dx)\sec^{\frac{5}{2}}(c+d}{5d}\right.$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2\right)dx+$$
$$2a^2\left(\frac{3}{5}\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx\right)+\frac{2\sin(c+dx)\sec^{\frac{5}{2}}}{5d}\right.$$

3.172. $\int \sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^2\,dx$

$$\Big\downarrow \text{3119}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(\csc\left(c+dx+\frac{\pi}{2}\right)^2 a^2+a^2\right)dx+$$
$$2a^2\left(\frac{2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}+\frac{3}{5}\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right.\right.$$

$$\Big\downarrow \text{4534}$$

$$\frac{12}{7}a^2\int \sec^{\frac{5}{2}}(c+dx)dx+\frac{2a^2\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{7d}+$$
$$2a^2\left(\frac{2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}+\frac{3}{5}\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right.\right.$$

$$\Big\downarrow \text{3042}$$

$$\frac{12}{7}a^2\int \csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}dx+\frac{2a^2\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{7d}+$$
$$2a^2\left(\frac{2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}+\frac{3}{5}\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right.\right.$$

$$\Big\downarrow \text{4255}$$

$$\frac{12}{7}a^2\left(\frac{1}{3}\int\sqrt{\sec(c+dx)}dx+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}\right)+\frac{2a^2\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{7d}+$$
$$2a^2\left(\frac{2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}+\frac{3}{5}\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right.\right.$$

$$\Big\downarrow \text{3042}$$

$$\frac{12}{7}a^2\left(\frac{1}{3}\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}\right)+\frac{2a^2\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{7d}+$$
$$2a^2\left(\frac{2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}+\frac{3}{5}\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right.\right.$$

$$\Big\downarrow \text{4258}$$

$$\frac{12}{7}a^2\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}\right)+$$
$$\frac{2a^2\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{7d}+$$
$$2a^2\left(\frac{2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}+\frac{3}{5}\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right.\right.$$

$$\Big\downarrow \text{3042}$$

3.172. $\int \sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^2\,dx$

$$\frac{12}{7}a^2\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}\right)+$$
$$\frac{2a^2\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{7d}+$$
$$2a^2\left(\frac{2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}+\frac{3}{5}\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right.\right.$$

$\Big\downarrow$ 3120

$$\frac{2a^2\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{7d}+$$
$$\frac{12}{7}a^2\left(\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}\right)+$$
$$2a^2\left(\frac{2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}+\frac{3}{5}\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right.\right.$$

input
```
Int[Sec[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^2,x]
```

output
```
(2*a^2*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(7*d) + (12*a^2*((2*Sqrt[Cos[c + d
*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (2*Sec[c + d*x]
^(3/2)*Sin[c + d*x])/(3*d)))/7 + 2*a^2*((2*Sec[c + d*x]^(5/2)*Sin[c + d*x]
)/(5*d) + (3*((-2*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
+ d*x]])/d + (2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d))/5)
```

3.172.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4275
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Simp[2*a*(b/d)   Int[(d*Csc[e + f*x])^(n + 1), x], x]
 + Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
 e, f, n}, x]
```

rule 4534
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Simp[(C*m + A*(m + 1))/(m + 1)   Int[(b*Csc[e + f*x])^m, x], x] /;
 FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

3.172.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 438 vs. $2(211) = 422$.

Time = 36.25 (sec) , antiderivative size = 439, normalized size of antiderivative = 2.35

method	result
default	$-\frac{a^2\sqrt{-\left(-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(-\frac{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}}{28\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-\frac{1}{2}\right)^4}-\frac{4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\sin\left(\frac{dx}{2}\right.}}{7\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-\frac{1}{2}\right)^2}\right.}{}$
parts	Expression too large to display

input
```
int(sec(d*x+c)^(5/2)*(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
-a^2*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-1/28*cos(
1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1
/2*d*x+1/2*c)^2-1/2)^4-4/7*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin
(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+124/35*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/5*cos
(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(
1/2*d*x+1/2*c)^2-1/2)^3-24/5*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)/(-(-2
*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)-12/5*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+si
n(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-EllipticE
(cos(1/2*d*x+1/2*c),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-
1)^(1/2)/d
```

3.172.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.15

$$\int \sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^2\,dx = -\frac{2\left(10i\sqrt{2}a^2\cos(dx+c)^3\,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))-10i\sqrt{2}a^2\cos(dx+\right.}{}$$

input
```
integrate(sec(d*x+c)^(5/2)*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
-2/35*(10*I*sqrt(2)*a^2*cos(d*x + c)^3*weierstrassPInverse(-4, 0, cos(d*x
+ c) + I*sin(d*x + c)) - 10*I*sqrt(2)*a^2*cos(d*x + c)^3*weierstrassPInver
se(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 21*I*sqrt(2)*a^2*cos(d*x + c)^3
*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*
x + c))) - 21*I*sqrt(2)*a^2*cos(d*x + c)^3*weierstrassZeta(-4, 0, weierstr
assPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - (42*a^2*cos(d*x + c)^
3 + 20*a^2*cos(d*x + c)^2 + 14*a^2*cos(d*x + c) + 5*a^2)*sin(d*x + c)/sqrt
(cos(d*x + c)))/(d*cos(d*x + c)^3)
```

3.172.6 Sympy [F(-1)]

Timed out.

$$\int \sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^2\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**(5/2)*(a+a*sec(d*x+c))**2,x)
```

output
```
Timed out
```

3.172.7 Maxima [F]

$$\int \sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^2\,dx = \int (a\sec(dx+c)+a)^2\sec(dx+c)^{\frac{5}{2}}\,dx$$

input
```
integrate(sec(d*x+c)^(5/2)*(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)^2*sec(d*x + c)^(5/2), x)
```

3.172.8 Giac [F]

$$\int \sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^2\,dx = \int (a\sec(dx+c)+a)^2\sec(dx+c)^{\frac{5}{2}}\,dx$$

input
```
integrate(sec(d*x+c)^(5/2)*(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)^2*sec(d*x + c)^(5/2), x)
```

3.172.9 Mupad [F(-1)]

Timed out.

$$\int \sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^2\,dx=\int \left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)^2\left(\frac{1}{\cos\left(c+d\,x\right)}\right)^{5/2}dx$$

input
```
int((a + a/cos(c + d*x))^2*(1/cos(c + d*x))^(5/2),x)
```

output
```
int((a + a/cos(c + d*x))^2*(1/cos(c + d*x))^(5/2), x)
```

3.173 $\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2\,dx$

3.173.1 Optimal result 1447
3.173.2 Mathematica [C] (verified) 1448
3.173.3 Rubi [A] (verified) 1448
3.173.4 Maple [B] (verified) 1452
3.173.5 Fricas [C] (verification not implemented) 1453
3.173.6 Sympy [F(-1)] 1453
3.173.7 Maxima [F] 1454
3.173.8 Giac [F] 1454
3.173.9 Mupad [F(-1)] 1454

3.173.1 Optimal result

Integrand size = 23, antiderivative size = 161

$$\begin{aligned}\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2\,dx &= -\frac{16a^2\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} \\ &\quad+\frac{4a^2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{3d} \\ &\quad+\frac{16a^2\sqrt{\sec(c+dx)}\sin(c+dx)}{5d} \\ &\quad+\frac{4a^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d}+\frac{2a^2\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d}\end{aligned}$$

output
```
4/3*a^2*sec(d*x+c)^(3/2)*sin(d*x+c)/d+2/5*a^2*sec(d*x+c)^(5/2)*sin(d*x+c)/
d+16/5*a^2*sin(d*x+c)*sec(d*x+c)^(1/2)/d-16/5*a^2*(cos(1/2*d*x+1/2*c)^2)^(
1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(
1/2)*sec(d*x+c)^(1/2)/d+4/3*a^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1
/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1
/2)/d
```

3.173.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 2.06 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.67

$$\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2\,dx$$
$$= \frac{a^2\sec^4\left(\frac{1}{2}(c+dx)\right)(1+\sec(c+dx))^2\left(-\frac{2i\sqrt{2}e^{-i(c+dx)}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\cos^2(c+dx)\left(12\left(1+e^{2i(c+dx)}\right)+12\left(-1+e^{2ic}\right)\sqrt{1+e^{2i(c+}}\right.}{}\right.}{}$$

input
```
Integrate[Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^2,x]
```

output
```
(a^2*Sec[(c + d*x)/2]^4*(1 + Sec[c + d*x])^2*(((-2*I)*Sqrt[2]*Sqrt[E^(I*(c
 + d*x))/(1 + E^((2*I)*(c + d*x)))]*Cos[c + d*x]^2*(12*(1 + E^((2*I)*(c +
d*x))) + 12*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometri
c2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))] + 5*E^(I*(c + d*x))*(-1 + E^((2
*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^
((2*I)*(c + d*x))]))/(E^(I*(c + d*x))*(-1 + E^((2*I)*c))) + (24*Cos[d*x]*C
sc[c] + (10 + 3*Sec[c + d*x])*Tan[c + d*x])/Sec[c + d*x]^(3/2)))/(30*d)
```

3.173.3 Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.99, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.652$, Rules used = {3042, 4275, 3042, 4255, 3042, 4258, 3042, 3120, 4534, 3042, 4255, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^2\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^2dx$$
$$\Big\downarrow \text{4275}$$

$$2a^2\int \sec^{\frac{5}{2}}(c+dx)dx+\int \sec^{\frac{3}{2}}(c+dx)\left(\sec^2(c+dx)a^2+a^2\right)dx$$

$$\Big\downarrow \text{3042}$$

$$2a^2\int \csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}dx+\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2\right)dx$$

$$\Big\downarrow \text{4255}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2\right)dx+$$
$$2a^2\left(\frac{1}{3}\int \sqrt{\sec(c+dx)}dx+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}\right)$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2\right)dx+$$
$$2a^2\left(\frac{1}{3}\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}\right)$$

$$\Big\downarrow \text{4258}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2\right)dx+$$
$$2a^2\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}\right)$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2\right)dx+$$
$$2a^2\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}\right)$$

$$\Big\downarrow \text{3120}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2\right)dx+$$
$$2a^2\left(\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}\right)$$

$$\Big\downarrow \text{4534}$$

3.173. $\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2\,dx$

$$\frac{8}{5}a^2\int \sec^{\frac{3}{2}}(c+dx)dx+\frac{2a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}+2a^2\left(\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}\right)$$

$\Big\downarrow$ 3042

$$\frac{8}{5}a^2\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}dx+\frac{2a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}+2a^2\left(\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}\right)$$

$\Big\downarrow$ 4255

$$\frac{8}{5}a^2\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\int\frac{1}{\sqrt{\sec(c+dx)}}dx\right)+\frac{2a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}+2a^2\left(\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}\right)$$

$\Big\downarrow$ 3042

$$\frac{8}{5}a^2\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\int\frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)+\frac{2a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}+2a^2\left(\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}\right)$$

$\Big\downarrow$ 4258

$$\frac{8}{5}a^2\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\cos(c+dx)}dx\right)+\frac{2a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}+2a^2\left(\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}\right)$$

$\Big\downarrow$ 3042

$$\frac{8}{5}a^2\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx\right)+\frac{2a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}+2a^2\left(\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}\right)$$

$\Big\downarrow$ 3119

3.173. $\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2\,dx$

$$\frac{2a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}+2a^2\left(\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}\right)+\frac{8}{5}a^2\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)$$

input
```
Int[Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^2,x]
```

output
```
(2*a^2*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5*d) + (8*a^2*((-2*Sqrt[Cos[c + d
*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (2*Sqrt[Sec[c + d*x
]]*Sin[c + d*x])/d))/5 + 2*a^2*((2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/
2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (2*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d
))
```

3.173.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4275
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Simp[2*a*(b/d)   Int[(d*Csc[e + f*x])^(n + 1), x], x]
 + Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
 e, f, n}, x]
```

rule 4534
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Simp[(C*m + A*(m + 1))/(m + 1)   Int[(b*Csc[e + f*x])^m, x], x] /;
 FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

3.173.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 385 vs. $2(189) = 378$.

Time = 26.28 (sec) , antiderivative size = 386, normalized size of antiderivative = 2.40

method	result
default	$-\frac{a^2\sqrt{-\left(-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(-\frac{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}}{3\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-\frac{1}{2}\right)^2}+\frac{68\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2+}}{15\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+}}\right.}{}$
parts	Expression too large to display

input
```
int(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
-a^2*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2/3*cos(1
/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/
2*d*x+1/2*c)^2-1/2)^2+68/15*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1
/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Elli
pticF(cos(1/2*d*x+1/2*c),2^(1/2))-32/5*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/
2*c)/(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)-16/5*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))
-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))-1/10*cos(1/2*d*x+1/2*c)*(-2*sin(1/
2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^3)/s
in(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

3.173.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.25

$$\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2\,dx = -\frac{2\left(5i\sqrt{2}a^2\cos(dx+c)^2\,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))-5i\sqrt{2}a^2\cos(dx+c\right.}{}$$

input
```
integrate(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
-2/15*(5*I*sqrt(2)*a^2*cos(d*x + c)^2*weierstrassPInverse(-4, 0, cos(d*x +
 c) + I*sin(d*x + c)) - 5*I*sqrt(2)*a^2*cos(d*x + c)^2*weierstrassPInverse
(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 12*I*sqrt(2)*a^2*cos(d*x + c)^2*w
eierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x
+ c))) - 12*I*sqrt(2)*a^2*cos(d*x + c)^2*weierstrassZeta(-4, 0, weierstras
sPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - (24*a^2*cos(d*x + c)^2
+ 10*a^2*cos(d*x + c) + 3*a^2)*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x
 + c)^2)
```

3.173.6 Sympy [F(-1)]

Timed out.

$$\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**(3/2)*(a+a*sec(d*x+c))**2,x)
```

output
```
Timed out
```

3.173.7 Maxima [F]

$$\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2\,dx = \int (a\sec(dx+c)+a)^2\sec(dx+c)^{\frac{3}{2}}\,dx$$

input
```
integrate(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)^2*sec(d*x + c)^(3/2), x)
```

3.173.8 Giac [F]

$$\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2\,dx = \int (a\sec(dx+c)+a)^2\sec(dx+c)^{\frac{3}{2}}\,dx$$

input
```
integrate(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)^2*sec(d*x + c)^(3/2), x)
```

3.173.9 Mupad [F(-1)]

Timed out.

$$\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2\,dx = \int \left(a+\frac{a}{\cos(c+d\,x)}\right)^2\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}dx$$

input
```
int((a + a/cos(c + d*x))^2*(1/cos(c + d*x))^(3/2),x)
```

output
```
int((a + a/cos(c + d*x))^2*(1/cos(c + d*x))^(3/2), x)
```

3.174 $\int \sqrt{\sec(c+dx)}(a+a\sec(c+dx))^2\,dx$

3.174.1 Optimal result 1455
3.174.2 Mathematica [C] (verified) 1456
3.174.3 Rubi [A] (verified) 1456
3.174.4 Maple [B] (verified) 1460
3.174.5 Fricas [C] (verification not implemented) 1460
3.174.6 Sympy [F] 1461
3.174.7 Maxima [F] 1461
3.174.8 Giac [F] 1462
3.174.9 Mupad [F(-1)] 1462

3.174.1 Optimal result

Integrand size = 23, antiderivative size = 131

$$\begin{aligned}\int \sqrt{\sec(c+dx)}(a+a\sec(c+dx))^2\,dx &= -\frac{4a^2\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} \\ &\quad+\frac{8a^2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{3d} \\ &\quad+\frac{4a^2\sqrt{\sec(c+dx)}\sin(c+dx)}{d}+\frac{2a^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d}\end{aligned}$$

output
```
2/3*a^2*sec(d*x+c)^(3/2)*sin(d*x+c)/d+4*a^2*sin(d*x+c)*sec(d*x+c)^(1/2)/d-
4*a^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*
x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+8/3*a^2*(cos(1/2*d*x
+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*
cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

3.174.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.71 (sec) , antiderivative size = 264, normalized size of antiderivative = 2.02

$$\int \sqrt{\sec(c+dx)}(a+a\sec(c+dx))^2\,dx = \frac{1}{3}a^2\sec^4\left(\frac{1}{2}(c+dx)\right)(1+\sec(c+dx))^2\left(-\frac{i\sqrt{2}e^{-i(c+dx)}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\cos^2(c+dx)\left(3\left(1+e^{2i(c+dx)}\right)+3\left(-1+e^{2ic}\right)\sqrt{1+e^{2i(c+dx)}}\,\text{Hyperge}\right.}{}\right.$$
$$\left.+\frac{6\cos(dx)\csc(c)+\tan(c+dx)}{2d\sec^{\frac{3}{2}}(c+dx)}\right)$$

input
```
Integrate[Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^2,x]
```

output
```
(a^2*Sec[(c + d*x)/2]^4*(1 + Sec[c + d*x])^2*(((-I)*Sqrt[2]*Sqrt[E^(I*(c +
 d*x))/(1 + E^((2*I)*(c + d*x)))]*Cos[c + d*x]^2*(3*(1 + E^((2*I)*(c + d*x
))) + 3*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1
[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))] + 2*E^(I*(c + d*x))*(-1 + E^((2*I)*
c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*
I)*(c + d*x))]))/(d*E^(I*(c + d*x))*(-1 + E^((2*I)*c))) + (6*Cos[d*x]*Csc[
c] + Tan[c + d*x])/(2*d*Sec[c + d*x]^(3/2))))/3
```

3.174.3 Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.565$, Rules used = {3042, 4275, 3042, 4255, 3042, 4258, 3042, 3119, 4534, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^2\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^2dx$$

$$\Big\downarrow \text{4275}$$

$$2a^2\int \sec^{\frac{3}{2}}(c+dx)dx+\int \sqrt{\sec(c+dx)}\left(\sec^2(c+dx)a^2+a^2\right)dx$$

$$\Big\downarrow \text{3042}$$

$$2a^2\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}dx+\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2\right)dx$$

$$\Big\downarrow \text{4255}$$

$$\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2\right)dx+2a^2\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\int \frac{1}{\sqrt{\sec(c+dx)}}dx\right)$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2\right)dx+2a^2\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)$$

$$\Big\downarrow \text{4258}$$

$$\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2\right)dx+2a^2\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\cos(c+dx)}dx\right)$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2\right)dx+2a^2\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx\right)$$

$$\Big\downarrow \text{3119}$$

$$\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2\right)dx+2a^2\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)$$

$$\Big\downarrow \text{4534}$$

3.174. $\int \sqrt{\sec(c+dx)}(a+a\sec(c+dx))^2\,dx$

$$\frac{4}{3}a^2\int\sqrt{\sec(c+dx)}dx+\frac{2a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+2a^2\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)$$

$\Big\downarrow$ 3042

$$\frac{4}{3}a^2\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+2a^2\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)$$

$\Big\downarrow$ 4258

$$\frac{4}{3}a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+2a^2\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)$$

$\Big\downarrow$ 3042

$$\frac{4}{3}a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+2a^2\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)$$

$\Big\downarrow$ 3120

$$\frac{2a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{8a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+2a^2\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)$$

input
```
Int[Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^2,x]
```

output
```
(8*a^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3
*d) + (2*a^2*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d) + 2*a^2*((-2*Sqrt[Cos[
c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (2*Sqrt[Sec[c
+ d*x]]*Sin[c + d*x])/d)
```

3.174. $\int\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^2\,dx$

3.174.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4275
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Simp[2*a*(b/d)   Int[(d*Csc[e + f*x])^(n + 1), x], x]
 + Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
 e, f, n}, x]
```

rule 4534
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Simp[(C*m + A*(m + 1))/(m + 1)   Int[(b*Csc[e + f*x])^m, x], x] /;
 FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

3.174. $\int \sqrt{\sec(c+dx)}(a+a\sec(c+dx))^2\,dx$

3.174.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 370 vs. $2(167) = 334$.

Time = 18.74 (sec) , antiderivative size = 371, normalized size of antiderivative = 2.83

method	result
default	$-\frac{4a^2\sqrt{-\left(-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(12\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-4\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1}\,\mathrm{EllipticF}\left(\cos\right.\right.}{}$
parts	$-\frac{2a^2\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1}\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1}\,d}-\frac{2a^2\left(-2\sqrt{\frac{1}{2}-\mathrm{co}}\right.}{}$

input
```
int(sec(d*x+c)^(1/2)*(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
-4/3*a^2*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(4*sin(
1/2*d*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)/sin(1/2*d*x+1/2*c)^3*(12*sin(1/
2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/
2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+
1/2*c)^2-6*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/
2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2-7*cos(1/2*d*x+1/
2*c)*sin(1/2*d*x+1/2*c)^2+2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/
2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1
)^(1/2))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d
*x+1/2*c)^2-1)^(1/2)/d
```

3.174.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.37

$$\int \sqrt{\sec(c+dx)}(a+a\sec(c+dx))^2\,dx =$$
$$-\frac{2\left(2i\sqrt{2}a^2\cos(dx+c)\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))-2i\sqrt{2}a^2\cos(dx+c)\right.}{}$$

input
```
integrate(sec(d*x+c)^(1/2)*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
-2/3*(2*I*sqrt(2)*a^2*cos(d*x + c)*weierstrassPInverse(-4, 0, cos(d*x + c)
 + I*sin(d*x + c)) - 2*I*sqrt(2)*a^2*cos(d*x + c)*weierstrassPInverse(-4,
0, cos(d*x + c) - I*sin(d*x + c)) + 3*I*sqrt(2)*a^2*cos(d*x + c)*weierstra
ssZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) -
 3*I*sqrt(2)*a^2*cos(d*x + c)*weierstrassZeta(-4, 0, weierstrassPInverse(-
4, 0, cos(d*x + c) - I*sin(d*x + c))) - (6*a^2*cos(d*x + c) + a^2)*sin(d*x
 + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c))
```

3.174.6 Sympy [F]

$$\int \sqrt{\sec(c+dx)}(a+a\sec(c+dx))^2\,dx = a^2\left(\int \sqrt{\sec(c+dx)}\,dx + \int 2\sec^{\frac{3}{2}}(c+dx)\,dx + \int \sec^{\frac{5}{2}}(c+dx)\,dx\right)$$

input
```
integrate(sec(d*x+c)**(1/2)*(a+a*sec(d*x+c))**2,x)
```

output
```
a**2*(Integral(sqrt(sec(c + d*x)), x) + Integral(2*sec(c + d*x)**(3/2), x)
 + Integral(sec(c + d*x)**(5/2), x))
```

3.174.7 Maxima [F]

$$\int \sqrt{\sec(c+dx)}(a+a\sec(c+dx))^2\,dx = \int (a\sec(dx+c)+a)^2\sqrt{\sec(dx+c)}\,dx$$

input
```
integrate(sec(d*x+c)^(1/2)*(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)^2*sqrt(sec(d*x + c)), x)
```

3.174.8 Giac [F]

$$\int \sqrt{\sec(c+dx)}(a+a\sec(c+dx))^2\,dx = \int (a\sec(dx+c)+a)^2\sqrt{\sec(dx+c)}\,dx$$

input
```
integrate(sec(d*x+c)^(1/2)*(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)^2*sqrt(sec(d*x + c)), x)
```

3.174.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{\sec(c+dx)}(a+a\sec(c+dx))^2\,dx = \int \left(a+\frac{a}{\cos(c+d\,x)}\right)^2\sqrt{\frac{1}{\cos(c+d\,x)}}\,dx$$

input
```
int((a + a/cos(c + d*x))^2*(1/cos(c + d*x))^(1/2),x)
```

output
```
int((a + a/cos(c + d*x))^2*(1/cos(c + d*x))^(1/2), x)
```

3.175 $\int \frac{(a+a\sec(c+dx))^2}{\sqrt{\sec(c+dx)}}\,dx$

3.175.1 Optimal result . . . 1463
3.175.2 Mathematica [A] (verified) . . . 1463
3.175.3 Rubi [A] (verified) . . . 1464
3.175.4 Maple [B] (verified) . . . 1466
3.175.5 Fricas [C] (verification not implemented) . . . 1466
3.175.6 Sympy [F] . . . 1467
3.175.7 Maxima [F] . . . 1467
3.175.8 Giac [F] . . . 1467
3.175.9 Mupad [F(-1)] . . . 1468

3.175.1 Optimal result

Integrand size = 23, antiderivative size = 64

$$\int \frac{(a+a\sec(c+dx))^2}{\sqrt{\sec(c+dx)}}\,dx = \frac{4a^2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{d} + \frac{2a^2\sqrt{\sec(c+dx)}\sin(c+dx)}{d}$$

output
```
2*a^2*sin(d*x+c)*sec(d*x+c)^(1/2)/d+4*a^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos
(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec
(d*x+c)^(1/2)/d
```

3.175.2 Mathematica [A] (verified)

Time = 1.03 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.75

$$\int \frac{(a+a\sec(c+dx))^2}{\sqrt{\sec(c+dx)}}\,dx = \frac{2a^2\sqrt{\sec(c+dx)}\left(2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+\sin(c+dx)\right)}{d}$$

input
```
Integrate[(a + a*Sec[c + d*x])^2/Sqrt[Sec[c + d*x]],x]
```

output
```
(2*a^2*Sqrt[Sec[c + d*x]]*(2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]
+ Sin[c + d*x]))/d
```

3.175.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3042, 4275, 3042, 4258, 3042, 3120, 4531}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sec(c+dx)+a)^2}{\sqrt{\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4275}$$

$$\int \frac{\sec^2(c+dx)a^2+a^2}{\sqrt{\sec(c+dx)}}dx+2a^2\int \sqrt{\sec(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$2a^2\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4258}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+2a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+2a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{3120}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{4a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}$$

$$\Big\downarrow \text{4531}$$

$$\frac{2a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}+\frac{4a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}$$

input
```
Int[(a + a*Sec[c + d*x])^2/Sqrt[Sec[c + d*x]],x]
```

output
```
(4*a^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d
+ (2*a^2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d
```

3.175.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4275
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Simp[2*a*(b/d)   Int[(d*Csc[e + f*x])^(n + 1), x], x]
 + Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
 e, f, n}, x]
```

rule 4531
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] /;
 FreeQ[{b, e, f, A, C, m}, x] && EqQ[C*m + A*(m + 1), 0]
```

3.175.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 184 vs. $2(84) = 168$.

Time = 14.96 (sec) , antiderivative size = 185, normalized size of antiderivative = 2.89

method	result
default	$-\frac{4a^2\left(-\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1}\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\right.\right.}{\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1}\,d}$
parts	$\frac{2a^2\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1}\,\mathrm{EllipticE}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1}\,d}-\frac{2a^2\left(-2\sqrt{-2\sin\left(\right.}\right.}{}$

input
```
int((a+a*sec(d*x+c))^2/sec(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-4*a^2*(-(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+
1/2*c)*sin(1/2*d*x+1/2*c)^2+(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/
2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(-2*sin(1/2*d*x+1/2*
c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2
*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

3.175.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.20

$$\int \frac{(a+a\sec(c+dx))^2}{\sqrt{\sec(c+dx)}}\,dx = -\frac{2\left(i\sqrt{2}a^2\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))-i\sqrt{2}a^2\mathrm{weierstrassPInverse}(-4,0,\right.}{d}$$

input
```
integrate((a+a*sec(d*x+c))^2/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
-2*(I*sqrt(2)*a^2*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)
) - I*sqrt(2)*a^2*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)
) - a^2*sin(d*x + c)/sqrt(cos(d*x + c)))/d
```

3.175.6 Sympy [F]

$$\int \frac{(a+a\sec(c+dx))^2}{\sqrt{\sec(c+dx)}}\,dx = a^2\left(\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx + \int 2\sqrt{\sec(c+dx)}\,dx + \int \sec^{\frac{3}{2}}(c+dx)\,dx\right)$$

input
```
integrate((a+a*sec(d*x+c))**2/sec(d*x+c)**(1/2),x)
```

output
```
a**2*(Integral(1/sqrt(sec(c + d*x)), x) + Integral(2*sqrt(sec(c + d*x)), x
) + Integral(sec(c + d*x)**(3/2), x))
```

3.175.7 Maxima [F]

$$\int \frac{(a+a\sec(c+dx))^2}{\sqrt{\sec(c+dx)}}\,dx = \int \frac{(a\sec(dx+c)+a)^2}{\sqrt{\sec(dx+c)}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))^2/sec(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)^2/sqrt(sec(d*x + c)), x)
```

3.175.8 Giac [F]

$$\int \frac{(a+a\sec(c+dx))^2}{\sqrt{\sec(c+dx)}}\,dx = \int \frac{(a\sec(dx+c)+a)^2}{\sqrt{\sec(dx+c)}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))^2/sec(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)^2/sqrt(sec(d*x + c)), x)
```

3.175.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(c+dx))^2}{\sqrt{\sec(c+dx)}}\,dx=\int \frac{\left(a+\frac{a}{\cos(c+d\,x)}\right)^2}{\sqrt{\frac{1}{\cos(c+d\,x)}}}\,dx$$

input
```
int((a + a/cos(c + d*x))^2/(1/cos(c + d*x))^(1/2),x)
```

output
```
int((a + a/cos(c + d*x))^2/(1/cos(c + d*x))^(1/2), x)
```

3.176 $\int \frac{(a+a\sec(c+dx))^2}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

3.176.1 Optimal result . . . 1469
3.176.2 Mathematica [C] (verified) . . . 1469
3.176.3 Rubi [A] (verified) . . . 1470
3.176.4 Maple [A] (verified) . . . 1472
3.176.5 Fricas [C] (verification not implemented) . . . 1473
3.176.6 Sympy [F] . . . 1473
3.176.7 Maxima [F] . . . 1474
3.176.8 Giac [F] . . . 1474
3.176.9 Mupad [F(-1)] . . . 1474

3.176.1 Optimal result

Integrand size = 23, antiderivative size = 107

$$\int \frac{(a+a\sec(c+dx))^2}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \frac{4a^2\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{8a^2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{3d} + \frac{2a^2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}$$

output
```
2/3*a^2*sin(d*x+c)/d/sec(d*x+c)^(1/2)+4*a^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/c
os(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*s
ec(d*x+c)^(1/2)/d+8/3*a^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*
EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

3.176.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.61 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.46

$$\int \frac{(a+a\sec(c+dx))^2}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \frac{a^2\left(\cos\left(\frac{c}{2}\right)-i\sin\left(\frac{c}{2}\right)\right)\left(-i\cos\left(\frac{c}{2}\right)+\sin\left(\frac{c}{2}\right)\right)\left(12-\frac{24\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},-e^{2i(c+dx)}\right)}{\sqrt{1+e^{2i(c+dx)}}}+8\sqrt{1+e^{2i(c+dx)}}\,\text{H}\right.}{3d\sqrt{\sec(c+dx)}}$$

input
```
Integrate[(a + a*Sec[c + d*x])^2/Sec[c + d*x]^(3/2),x]
```

output
```
(a^2*(Cos[c/2] - I*Sin[c/2])*((-I)*Cos[c/2] + Sin[c/2])*(12 - (24*Hypergeo
metric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))])/Sqrt[1 + E^((2*I)*(c + d*
x))] + 8*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E
^((2*I)*(c + d*x))]*Sec[c + d*x] + (2*I)*Sin[c + d*x]))/(3*d*Sqrt[Sec[c +
d*x]])
```

3.176.3 Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules used = {3042, 4275, 3042, 4258, 3042, 3119, 4533, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sec(c+dx)+a)^2}{\sec^{\frac{3}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{4275}$$

$$\int \frac{\sec^2(c+dx)a^2+a^2}{\sec^{\frac{3}{2}}(c+dx)}dx+2a^2\int \frac{1}{\sqrt{\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$2a^2\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{4258}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+2a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\cos(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+2a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3119}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2 a^2+a^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+\frac{4a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

$$\Big\downarrow \text{4533}$$

$$\frac{4}{3}a^2\int\sqrt{\sec(c+dx)}dx+\frac{2a^2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{4a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4}{3}a^2\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2a^2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{4a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

$$\Big\downarrow \text{4258}$$

$$\frac{4}{3}a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2a^2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{4a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4}{3}a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a^2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{4a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

$$\Big\downarrow \text{3120}$$

$$\frac{2a^2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{8a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{4a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

input
```
Int[(a + a*Sec[c + d*x])^2/Sec[c + d*x]^(3/2),x]
```

output
```
(4*a^2*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d
+ (8*a^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/
(3*d) + (2*a^2*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])
```

3.176. $\int \frac{(a+a\sec(c+dx))^2}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

3.176.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4275
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Simp[2*a*(b/d)   Int[(d*Csc[e + f*x])^(n + 1), x], x]
 + Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
 e, f, n}, x]
```

rule 4533
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] +
Simp[(C*m + A*(m + 1))/(b^2*m)   Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fr
eeQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

3.176.4 Maple [A] (verified)

Time = 15.16 (sec) , antiderivative size = 228, normalized size of antiderivative = 2.13

method	result
default	$-\frac{4\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\,a^2\left(2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2+2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\sin\left(\frac{dx}{2}\right.}\right.}{3\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right.}$
parts	$-\frac{2a^2\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(4\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\sin\left(\frac{dx}{2}\right.}\right.}{3\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1}\,d}$

input
```
int((a+a*sec(d*x+c))^2/sec(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-4/3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*(2*sin(1/
2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+
2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(
cos(1/2*d*x+1/2*c),2^(1/2))-3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1
/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2))/(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*
c)^2-1)^(1/2)/d
```

3.176.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.25

$$\int \frac{(a+a\sec(c+dx))^2}{\sec^{\frac{3}{2}}(c+dx)}\,dx$$
$$= \frac{2\left(a^2\sqrt{\cos(dx+c)}\sin(dx+c) - 2i\sqrt{2}a^2\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)) + 2i\sqrt{}\right.}{}$$

input
```
integrate((a+a*sec(d*x+c))^2/sec(d*x+c)^(3/2),x, algorithm="fricas")
```

output
```
2/3*(a^2*sqrt(cos(d*x + c))*sin(d*x + c) - 2*I*sqrt(2)*a^2*weierstrassPInv
erse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 2*I*sqrt(2)*a^2*weierstrassPI
nverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 3*I*sqrt(2)*a^2*weierstrass
Zeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 3
*I*sqrt(2)*a^2*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x +
 c) - I*sin(d*x + c))))/d
```

3.176.6 Sympy [F]

$$\int \frac{(a+a\sec(c+dx))^2}{\sec^{\frac{3}{2}}(c+dx)}\,dx = a^2\left(\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}\,dx + \int \frac{2}{\sqrt{\sec(c+dx)}}\,dx + \int \sqrt{\sec(c+dx)}\,dx\right)$$

input
```
integrate((a+a*sec(d*x+c))**2/sec(d*x+c)**(3/2),x)
```

output
```
a**2*(Integral(sec(c + d*x)**(-3/2), x) + Integral(2/sqrt(sec(c + d*x)), x
) + Integral(sqrt(sec(c + d*x)), x))
```

3.176.7 Maxima [F]

$$\int \frac{(a+a\sec(c+dx))^2}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(a\sec(dx+c)+a)^2}{\sec(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))^2/sec(d*x+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)^2/sec(d*x + c)^(3/2), x)
```

3.176.8 Giac [F]

$$\int \frac{(a+a\sec(c+dx))^2}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(a\sec(dx+c)+a)^2}{\sec(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))^2/sec(d*x+c)^(3/2),x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)^2/sec(d*x + c)^(3/2), x)
```

3.176.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(c+dx))^2}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{\left(a+\frac{a}{\cos(c+d\,x)}\right)^2}{\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

input
```
int((a + a/cos(c + d*x))^2/(1/cos(c + d*x))^(3/2),x)
```

output
```
int((a + a/cos(c + d*x))^2/(1/cos(c + d*x))^(3/2), x)
```

3.177 $\int \frac{(a+a\sec(c+dx))^2}{\sec^{\frac{5}{2}}(c+dx)}\,dx$

3.177.1 Optimal result 1475
3.177.2 Mathematica [C] (verified) 1476
3.177.3 Rubi [A] (verified) 1476
3.177.4 Maple [A] (verified) 1480
3.177.5 Fricas [C] (verification not implemented) 1480
3.177.6 Sympy [F] 1481
3.177.7 Maxima [F] 1481
3.177.8 Giac [F] 1482
3.177.9 Mupad [F(-1)] 1482

3.177.1 Optimal result

Integrand size = 23, antiderivative size = 135

$$\int \frac{(a+a\sec(c+dx))^2}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \frac{16a^2\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{4a^2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{3d} + \frac{2a^2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{4a^2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}$$

output
```
2/5*a^2*sin(d*x+c)/d/sec(d*x+c)^(3/2)+4/3*a^2*sin(d*x+c)/d/sec(d*x+c)^(1/2
)+16/5*a^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1
/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+4/3*a^2*(cos(1/
2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1
/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

3.177.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 2.00 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.01

$$\int \frac{(a+a\sec(c+dx))^2}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \frac{a^2\left(-96i+\frac{192i\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},-e^{2i(c+dx)}\right)}{\sqrt{1+e^{2i(c+dx)}}}-40i\sqrt{1+e^{2i(c+dx)}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-e^{2i(c+dx)}\right.\right.}{30d\sqrt{\sec(c+dx)}}$$

input
```
Integrate[(a + a*Sec[c + d*x])^2/Sec[c + d*x]^(5/2),x]
```

output
```
(a^2*(-96*I + ((192*I)*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*
x))])/Sqrt[1 + E^((2*I)*(c + d*x))] - (40*I)*Sqrt[1 + E^((2*I)*(c + d*x))]
*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]*Sec[c + d*x] + 40*
Sin[c + d*x] + 6*Sin[2*(c + d*x)]))/(30*d*Sqrt[Sec[c + d*x]])
```

3.177.3 Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.565$, Rules used = {3042, 4275, 3042, 4256, 3042, 4258, 3042, 3120, 4533, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sec(c+dx)+a)^2}{\sec^{\frac{5}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx$$

$$\Big\downarrow \text{4275}$$

$$2a^2\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}dx+\int \frac{\sec^2(c+dx)a^2+a^2}{\sec^{\frac{5}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$2a^2\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx$$

$\Big\downarrow$ 4256

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+2a^2\left(\frac{1}{3}\int \sqrt{\sec(c+dx)}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)$$

$\Big\downarrow$ 3042

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+2a^2\left(\frac{1}{3}\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)$$

$\Big\downarrow$ 4258

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+$$
$$2a^2\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)$$

$\Big\downarrow$ 3042

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+$$
$$2a^2\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)$$

$\Big\downarrow$ 3120

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+$$
$$2a^2\left(\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}\right)$$

$\Big\downarrow$ 4533

$$\frac{8}{5}a^2\int \frac{1}{\sqrt{\sec(c+dx)}}dx+\frac{2a^2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+$$
$$2a^2\left(\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}\right)$$

$\Big\downarrow$ 3042

3.177. $\int \frac{(a+a\sec(c+dx))^2}{\sec^{\frac{5}{2}}(c+dx)}\,dx$

$$\frac{8}{5}a^2\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a^2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+2a^2\left(\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}\right)$$

$\Big\downarrow$ 4258

$$\frac{8}{5}a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\cos(c+dx)}dx+\frac{2a^2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+2a^2\left(\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}\right)$$

$\Big\downarrow$ 3042

$$\frac{8}{5}a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2a^2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+2a^2\left(\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}\right)$$

$\Big\downarrow$ 3119

$$\frac{2a^2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{16a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+2a^2\left(\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}\right)$$

input
```
Int[(a + a*Sec[c + d*x])^2/Sec[c + d*x]^(5/2),x]
```

output
```
(16*a^2*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(
5*d) + (2*a^2*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)) + 2*a^2*((2*Sqrt[Cos[
c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (2*Sin[c +
 d*x])/(3*d*Sqrt[Sec[c + d*x]]))
```

3.177. $\int \frac{(a+a\sec(c+dx))^2}{\sec^{\frac{5}{2}}(c+dx)}\,dx$

3.177.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4275
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Simp[2*a*(b/d)   Int[(d*Csc[e + f*x])^(n + 1), x], x]
 + Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
 e, f, n}, x]
```

rule 4533
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] +
Simp[(C*m + A*(m + 1))/(b^2*m)   Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fr
eeQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

3.177. $\int \frac{(a+a\sec(c+dx))^2}{\sec^{\frac{5}{2}}(c+dx)}\,dx$

3.177.4 Maple [A] (verified)

Time = 19.77 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.85

method	result
default	$-\frac{4\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,a^{2}\left(-12\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+32\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-13\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right.\right.}{15\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\sin\left(\right.}}$
parts	$-\frac{2a^{2}\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\left(-8\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+8\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}{5\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right.}}$

input
```
int((a+a*sec(d*x+c))^2/sec(d*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-4/15*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*(-12*sin
(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+32*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1
/2*c)-13*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+5*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2
))-12*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(
2*sin(1/2*d*x+1/2*c)^2-1)^(1/2))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*
c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

3.177.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.16

$$\int \frac{(a+a\sec(c+dx))^2}{\sec^{\frac{5}{2}}(c+dx)}\,dx = -\frac{2\left(5i\sqrt{2}a^2\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))-5i\sqrt{2}a^2\text{weierstrassPInverse}(-4,\right.}{}$$

input
```
integrate((a+a*sec(d*x+c))^2/sec(d*x+c)^(5/2),x, algorithm="fricas")
```

output
```
-2/15*(5*I*sqrt(2)*a^2*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x
 + c)) - 5*I*sqrt(2)*a^2*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d
*x + c)) - 12*I*sqrt(2)*a^2*weierstrassZeta(-4, 0, weierstrassPInverse(-4,
 0, cos(d*x + c) + I*sin(d*x + c))) + 12*I*sqrt(2)*a^2*weierstrassZeta(-4,
 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - (3*a^2*co
s(d*x + c)^2 + 10*a^2*cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/d
```

3.177.6 Sympy [F]

$$\int \frac{(a+a\sec(c+dx))^2}{\sec^{\frac{5}{2}}(c+dx)}\,dx = a^2\left(\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)}\,dx + \int \frac{2}{\sec^{\frac{3}{2}}(c+dx)}\,dx + \int \frac{1}{\sqrt{\sec(c+dx)}}\,dx\right)$$

input
```
integrate((a+a*sec(d*x+c))**2/sec(d*x+c)**(5/2),x)
```

output
```
a**2*(Integral(sec(c + d*x)**(-5/2), x) + Integral(2/sec(c + d*x)**(3/2),
x) + Integral(1/sqrt(sec(c + d*x)), x))
```

3.177.7 Maxima [F]

$$\int \frac{(a+a\sec(c+dx))^2}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{(a\sec(dx+c)+a)^2}{\sec(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))^2/sec(d*x+c)^(5/2),x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)^2/sec(d*x + c)^(5/2), x)
```

3.177.8 Giac [F]

$$\int \frac{(a+a\sec(c+dx))^2}{\sec^{\frac{5}{2}}(c+dx)}\,dx=\int \frac{(a\sec(dx+c)+a)^2}{\sec(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))^2/sec(d*x+c)^(5/2),x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)^2/sec(d*x + c)^(5/2), x)
```

3.177.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(c+dx))^2}{\sec^{\frac{5}{2}}(c+dx)}\,dx=\int \frac{\left(a+\frac{a}{\cos(c+d\,x)}\right)^2}{\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

input
```
int((a + a/cos(c + d*x))^2/(1/cos(c + d*x))^(5/2),x)
```

output
```
int((a + a/cos(c + d*x))^2/(1/cos(c + d*x))^(5/2), x)
```

3.178 $\int \frac{(a+a\sec(c+dx))^2}{\sec^{\frac{7}{2}}(c+dx)}\,dx$

3.178.1 Optimal result 1483
3.178.2 Mathematica [C] (verified) 1484
3.178.3 Rubi [A] (verified) 1484
3.178.4 Maple [A] (verified) 1488
3.178.5 Fricas [C] (verification not implemented) 1488
3.178.6 Sympy [F] 1489
3.178.7 Maxima [F] 1489
3.178.8 Giac [F] 1490
3.178.9 Mupad [F(-1)] 1490

3.178.1 Optimal result

Integrand size = 23, antiderivative size = 161

$$\int \frac{(a+a\sec(c+dx))^2}{\sec^{\frac{7}{2}}(c+dx)}\,dx = \frac{12a^2\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{8a^2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{7d} + \frac{2a^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{4a^2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{8a^2\sin(c+dx)}{7d\sqrt{\sec(c+dx)}}$$

output
```
2/7*a^2*sin(d*x+c)/d/sec(d*x+c)^(5/2)+4/5*a^2*sin(d*x+c)/d/sec(d*x+c)^(3/2
)+8/7*a^2*sin(d*x+c)/d/sec(d*x+c)^(1/2)+12/5*a^2*(cos(1/2*d*x+1/2*c)^2)^(1
/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1
/2)*sec(d*x+c)^(1/2)/d+8/7*a^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/
2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/
2)/d
```

3.178.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 2.33 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.93

$$\int \frac{(a+a\sec(c+dx))^2}{\sec^{\frac{7}{2}}(c+dx)}\,dx = \frac{a^2\left(\frac{672i\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},-e^{2i(c+dx)}\right)}{\sqrt{1+e^{2i(c+dx)}}}+2\left(-168i-80i\sqrt{1+e^{2i(c+dx)}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-e^{2i(c}\right.\right.\right.}{140d\sqrt{\sec(c+dx)}}$$

input
```
Integrate[(a + a*Sec[c + d*x])^2/Sec[c + d*x]^(7/2),x]
```

output
```
(a^2*(((672*I)*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))])/Sq
rt[1 + E^((2*I)*(c + d*x))] + 2*(-168*I - (80*I)*Sqrt[1 + E^((2*I)*(c + d*
x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]*Sec[c + d*x] +
 85*Sin[c + d*x] + 28*Sin[2*(c + d*x)] + 5*Sin[3*(c + d*x)])))/(140*d*Sqrt
[Sec[c + d*x]])
```

3.178.3 Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.01, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.652$, Rules used = {3042, 4275, 3042, 4256, 3042, 4258, 3042, 3119, 4533, 3042, 4256, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sec(c+dx)+a)^2}{\sec^{\frac{7}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx$$

$$\Big\downarrow \text{4275}$$

$$2a^2\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)}dx+\int \frac{\sec^2(c+dx)a^2+a^2}{\sec^{\frac{7}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$2a^2\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx$$

$$\Big\downarrow \text{4256}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx+2a^2\left(\frac{3}{5}\int \frac{1}{\sqrt{\sec(c+dx)}}dx+\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}\right)$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx+2a^2\left(\frac{3}{5}\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}\right)$$

$$\Big\downarrow \text{4258}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx+$$
$$2a^2\left(\frac{3}{5}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\cos(c+dx)}dx+\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}\right)$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx+$$
$$2a^2\left(\frac{3}{5}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}\right)$$

$$\Big\downarrow \text{3119}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx+$$
$$2a^2\left(\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{6\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}\right)$$

$$\Big\downarrow \text{4533}$$

$$\frac{12}{7}a^2\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}dx+\frac{2a^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+$$
$$2a^2\left(\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{6\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}\right)$$

$$\Big\downarrow \text{3042}$$

3.178. $\int \frac{(a+a\sec(c+dx))^2}{\sec^{\frac{7}{2}}(c+dx)}\,dx$

$$\frac{12}{7}a^2\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+\frac{2a^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+2a^2\left(\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{6\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}\right)$$

$\downarrow$ 4256

$$\frac{12}{7}a^2\left(\frac{1}{3}\int \sqrt{\sec(c+dx)}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)+\frac{2a^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+2a^2\left(\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{6\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}\right)$$

$\downarrow$ 3042

$$\frac{12}{7}a^2\left(\frac{1}{3}\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)+\frac{2a^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+2a^2\left(\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{6\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}\right)$$

$\downarrow$ 4258

$$\frac{12}{7}a^2\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)+\frac{2a^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+2a^2\left(\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{6\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}\right)$$

$\downarrow$ 3042

$$\frac{12}{7}a^2\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)+\frac{2a^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+2a^2\left(\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{6\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}\right)$$

$\downarrow$ 3120

$$\frac{2a^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+\frac{12}{7}a^2\left(\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}\right)+2a^2\left(\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{6\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}\right)$$

3.178. $\int \frac{(a+a\sec(c+dx))^2}{\sec^{\frac{7}{2}}(c+dx)}\,dx$

input
```
Int[(a + a*Sec[c + d*x])^2/Sec[c + d*x]^(7/2),x]
```

output
```
(2*a^2*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/2)) + 2*a^2*((6*Sqrt[Cos[c + d*x
]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d) + (2*Sin[c + d*x])/
(5*d*Sec[c + d*x]^(3/2))) + (12*a^2*((2*Sqrt[Cos[c + d*x]]*EllipticF[(c +
d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (2*Sin[c + d*x])/(3*d*Sqrt[Sec[c +
d*x]])))/7
```

3.178.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4275
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Simp[2*a*(b/d)   Int[(d*Csc[e + f*x])^(n + 1), x], x]
 + Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
 e, f, n}, x]
```

3.178. $\int \frac{(a+a\sec(c+dx))^2}{\sec^{\frac{7}{2}}(c+dx)}\,dx$

rule 4533
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] +
Simp[(C*m + A*(m + 1))/(b^2*m)   Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fr
eeQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

3.178.4 Maple [A] (verified)

Time = 24.69 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.69

method	result
default	$-\frac{4\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\,a^2\left(40\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^8-116\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^6\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+126\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4\cos\left(\frac{d}{2}\right.\right.}{35\sqrt{-2\sin\left(\right.}}$
parts	$-\frac{2a^2\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(48\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^9-120\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^7+128\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^5-72\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^3+5\sqrt{\frac{1}{2}-\frac{\cos(}{}}\right.}{21\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\cos\left(\frac{dx}{2}+\right.}}$

input
```
int((a+a*sec(d*x+c))^2/sec(d*x+c)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-4/35*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*(40*cos(
1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8-116*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1
/2*c)+126*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-39*cos(1/2*d*x+1/2*c)*si
n(1/2*d*x+1/2*c)^2+10*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2
-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-21*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/
2))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c
)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

3.178.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.06

$$\int \frac{(a+a\sec(c+dx))^2}{\sec^{\frac{7}{2}}(c+dx)}\,dx =$$
$$-\frac{2\left(10i\sqrt{2}a^2\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))-10i\sqrt{2}a^2\text{weierstrassPInverse}(-\right.}{}$$

input
```
integrate((a+a*sec(d*x+c))^2/sec(d*x+c)^(7/2),x, algorithm="fricas")
```

output
```
-2/35*(10*I*sqrt(2)*a^2*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*
x + c)) - 10*I*sqrt(2)*a^2*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin
(d*x + c)) - 21*I*sqrt(2)*a^2*weierstrassZeta(-4, 0, weierstrassPInverse(-
4, 0, cos(d*x + c) + I*sin(d*x + c))) + 21*I*sqrt(2)*a^2*weierstrassZeta(-
4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - (5*a^2*
cos(d*x + c)^3 + 14*a^2*cos(d*x + c)^2 + 20*a^2*cos(d*x + c))*sin(d*x + c)
/sqrt(cos(d*x + c)))/d
```

3.178.6 Sympy [F]

$$\int \frac{(a+a\sec(c+dx))^2}{\sec^{\frac{7}{2}}(c+dx)}\,dx = a^2\left(\int \frac{1}{\sec^{\frac{7}{2}}(c+dx)}\,dx + \int \frac{2}{\sec^{\frac{5}{2}}(c+dx)}\,dx + \int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}\,dx\right)$$

input
```
integrate((a+a*sec(d*x+c))**2/sec(d*x+c)**(7/2),x)
```

output
```
a**2*(Integral(sec(c + d*x)**(-7/2), x) + Integral(2/sec(c + d*x)**(5/2),
x) + Integral(sec(c + d*x)**(-3/2), x))
```

3.178.7 Maxima [F]

$$\int \frac{(a+a\sec(c+dx))^2}{\sec^{\frac{7}{2}}(c+dx)}\,dx = \int \frac{(a\sec(dx+c)+a)^2}{\sec(dx+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))^2/sec(d*x+c)^(7/2),x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)^2/sec(d*x + c)^(7/2), x)
```

3.178.8 Giac [F]

$$\int \frac{(a+a\sec(c+dx))^2}{\sec^{\frac{7}{2}}(c+dx)}\,dx=\int \frac{(a\sec(dx+c)+a)^2}{\sec(dx+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))^2/sec(d*x+c)^(7/2),x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)^2/sec(d*x + c)^(7/2), x)
```

3.178.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(c+dx))^2}{\sec^{\frac{7}{2}}(c+dx)}\,dx=\int \frac{\left(a+\frac{a}{\cos(c+d\,x)}\right)^2}{\left(\frac{1}{\cos(c+d\,x)}\right)^{7/2}}\,dx$$

input
```
int((a + a/cos(c + d*x))^2/(1/cos(c + d*x))^(7/2),x)
```

output
```
int((a + a/cos(c + d*x))^2/(1/cos(c + d*x))^(7/2), x)
```

3.179 $\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3\,dx$

3.179.1 Optimal result . . . 1491
3.179.2 Mathematica [C] (verified) . . . 1492
3.179.3 Rubi [A] (verified) . . . 1492
3.179.4 Maple [B] (verified) . . . 1494
3.179.5 Fricas [C] (verification not implemented) . . . 1494
3.179.6 Sympy [F(-1)] . . . 1495
3.179.7 Maxima [F] . . . 1495
3.179.8 Giac [F] . . . 1496
3.179.9 Mupad [F(-1)] . . . 1496

3.179.1 Optimal result

Integrand size = 23, antiderivative size = 187

$$\begin{aligned}\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3\,dx &= -\frac{28a^3\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} \\ &\quad+\frac{52a^3\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{21d} \\ &\quad+\frac{28a^3\sqrt{\sec(c+dx)}\sin(c+dx)}{5d}+\frac{52a^3\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{21d} \\ &\quad+\frac{6a^3\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d}+\frac{2a^3\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{7d}\end{aligned}$$

output
```
52/21*a^3*sec(d*x+c)^(3/2)*sin(d*x+c)/d+6/5*a^3*sec(d*x+c)^(5/2)*sin(d*x+c
)/d+2/7*a^3*sec(d*x+c)^(7/2)*sin(d*x+c)/d+28/5*a^3*sin(d*x+c)*sec(d*x+c)^(
1/2)/d-28/5*a^3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(
sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+52/21*a^3*
(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*
c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

3.179.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 3.45 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.53

$$\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3\,dx$$
$$= \frac{a^3\sec^6\left(\frac{1}{2}(c+dx)\right)(1+\sec(c+dx))^3\left(-\frac{2i\sqrt{2}e^{-i(c+dx)}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\cos^3(c+dx)\left(147(1+e^{2i(c+dx)})+147(-1+e^{2ic})\sqrt{1+e^{2i(}}\right.}{}\right.}{}$$

input
```
Integrate[Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^3,x]
```

output
```
(a^3*Sec[(c + d*x)/2]^6*(1 + Sec[c + d*x])^3*(((-2*I)*Sqrt[2]*Sqrt[E^(I*(c
 + d*x))/(1 + E^((2*I)*(c + d*x)))]*Cos[c + d*x]^3*(147*(1 + E^((2*I)*(c +
 d*x))) + 147*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeomet
ric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))] + 65*E^(I*(c + d*x))*(-1 + E^
((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4,
-E^((2*I)*(c + d*x))]))/(E^(I*(c + d*x))*(-1 + E^((2*I)*c))) + (294*Cos[d*
x]*Csc[c] + (80 + 63*Cos[c + d*x] + 65*Cos[2*(c + d*x)])*Sec[c + d*x]^2*Ta
n[c + d*x])/Sec[c + d*x]^(5/2)))/(420*d)
```

3.179.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3042, 4278, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^3\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^3dx$$
$$\Big\downarrow \; 4278$$

$$\int \left(a^3 \sec^{\frac{9}{2}}(c+dx)+3a^3 \sec^{\frac{7}{2}}(c+dx)+3a^3 \sec^{\frac{5}{2}}(c+dx)+a^3 \sec^{\frac{3}{2}}(c+dx)\right) dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2a^3 \sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{7d}+\frac{6a^3 \sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}+\frac{52a^3 \sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{21d}+\frac{28a^3 \sin(c+dx)\sqrt{\sec(c+dx)}}{5d}+\frac{52a^3 \sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d}-\frac{28a^3 \sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}$$

input
```
Int[Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^3,x]
```

output
```
(-28*a^3*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/
(5*d) + (52*a^3*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c +
d*x]])/(21*d) + (28*a^3*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(5*d) + (52*a^3*S
ec[c + d*x]^(3/2)*Sin[c + d*x])/(21*d) + (6*a^3*Sec[c + d*x]^(5/2)*Sin[c +
 d*x])/(5*d) + (2*a^3*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(7*d)
```

3.179.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4278
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

3.179.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 438 vs. $2(211) = 422$.

Time = 38.06 (sec) , antiderivative size = 439, normalized size of antiderivative = 2.35

method	result
default	$-\frac{a^3\sqrt{-\left(-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(-\frac{26\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}}{21\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-\frac{1}{2}\right)^2}+\frac{848\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)}}{105\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4}}\right.}{}$
parts	Expression too large to display

input
```
int(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
-a^3*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-26/21*cos
(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(
1/2*d*x+1/2*c)^2-1/2)^2+848/105*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d
*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*
EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-56/5*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*
x+1/2*c)/(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)-28/5*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d
*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1
/2))-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))-1/28*cos(1/2*d*x+1/2*c)*(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^
4-3/10*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(
1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^3)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c
)^2-1)^(1/2)/d
```

3.179.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.15

$$\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3\,dx =$$

$$-\frac{2\left(65i\sqrt{2}a^3\cos(dx+c)^3\,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))-65i\sqrt{2}a^3\cos(dx+\right.}{}$$

input
```
integrate(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
-2/105*(65*I*sqrt(2)*a^3*cos(d*x + c)^3*weierstrassPInverse(-4, 0, cos(d*x
 + c) + I*sin(d*x + c)) - 65*I*sqrt(2)*a^3*cos(d*x + c)^3*weierstrassPInve
rse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 147*I*sqrt(2)*a^3*cos(d*x + c)
^3*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(
d*x + c))) - 147*I*sqrt(2)*a^3*cos(d*x + c)^3*weierstrassZeta(-4, 0, weier
strassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - (294*a^3*cos(d*x +
 c)^3 + 130*a^3*cos(d*x + c)^2 + 63*a^3*cos(d*x + c) + 15*a^3)*sin(d*x + c
)/sqrt(cos(d*x + c)))/(d*cos(d*x + c)^3)
```

3.179.6 Sympy [F(-1)]

Timed out.

$$\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**(3/2)*(a+a*sec(d*x+c))**3,x)
```

output
```
Timed out
```

3.179.7 Maxima [F]

$$\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3\,dx = \int (a\sec(dx+c)+a)^3\sec(dx+c)^{\frac{3}{2}}\,dx$$

input
```
integrate(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)^3*sec(d*x + c)^(3/2), x)
```

3.179.8 Giac [F]

$$\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3\,dx = \int (a\sec(dx+c)+a)^3\sec(dx+c)^{\frac{3}{2}}\,dx$$

input
```
integrate(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)^3*sec(d*x + c)^(3/2), x)
```

3.179.9 Mupad [F(-1)]

Timed out.

$$\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3\,dx = \int \left(a+\frac{a}{\cos(c+d\,x)}\right)^3\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}dx$$

input
```
int((a + a/cos(c + d*x))^3*(1/cos(c + d*x))^(3/2),x)
```

output
```
int((a + a/cos(c + d*x))^3*(1/cos(c + d*x))^(3/2), x)
```

3.180 $\int \sqrt{\sec(c+dx)}(a+a\sec(c+dx))^3\,dx$

3.180.1 Optimal result . . . 1497
3.180.2 Mathematica [C] (verified) . . . 1498
3.180.3 Rubi [A] (verified) . . . 1498
3.180.4 Maple [B] (verified) . . . 1499
3.180.5 Fricas [C] (verification not implemented) . . . 1500
3.180.6 Sympy [F(-1)] . . . 1501
3.180.7 Maxima [F] . . . 1501
3.180.8 Giac [F] . . . 1502
3.180.9 Mupad [F(-1)] . . . 1502

3.180.1 Optimal result

Integrand size = 23, antiderivative size = 157

$$\begin{aligned}\int \sqrt{\sec(c+dx)}(a+a\sec(c+dx))^3\,dx &= -\frac{36a^3\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} \\ &\quad+\frac{4a^3\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{d} \\ &\quad+\frac{36a^3\sqrt{\sec(c+dx)}\sin(c+dx)}{5d} \\ &\quad+\frac{2a^3\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d}+\frac{2a^3\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d}\end{aligned}$$

output
```
2*a^3*sec(d*x+c)^(3/2)*sin(d*x+c)/d+2/5*a^3*sec(d*x+c)^(5/2)*sin(d*x+c)/d+
36/5*a^3*sin(d*x+c)*sec(d*x+c)^(1/2)/d-36/5*a^3*(cos(1/2*d*x+1/2*c)^2)^(1/
2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/
2)*sec(d*x+c)^(1/2)/d+4*a^3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c
)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/
d
```

3.180.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 3.06 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.70

$$\int \sqrt{\sec(c+dx)}(a+a\sec(c+dx))^3\,dx$$
$$= \frac{a^3\sec^6\left(\frac{1}{2}(c+dx)\right)(1+\sec(c+dx))^3\left(-\frac{2i\sqrt{2}e^{-i(c+dx)}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\cos^3(c+dx)\left(9\left(1+e^{2i(c+dx)}\right)+9\left(-1+e^{2ic}\right)\sqrt{1+e^{2i(c+dx)}}\right.}{}\right.}{}$$

input
```
Integrate[Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^3,x]
```

output
```
(a^3*Sec[(c + d*x)/2]^6*(1 + Sec[c + d*x])^3*(((-2*I)*Sqrt[2]*Sqrt[E^(I*(c
 + d*x))/(1 + E^((2*I)*(c + d*x)))]*Cos[c + d*x]^3*(9*(1 + E^((2*I)*(c + d
*x))) + 9*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2
F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))] + 5*E^(I*(c + d*x))*(-1 + E^((2*I
)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((
2*I)*(c + d*x))]))/(E^(I*(c + d*x))*(-1 + E^((2*I)*c))) + (18*Cos[d*x]*Csc
[c] + (5 + Sec[c + d*x])*Tan[c + d*x])/Sec[c + d*x]^(5/2)))/(20*d)
```

3.180.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3042, 4278, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^3\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^3dx$$
$$\Big\downarrow \; 4278$$
$$\int \left(a^3\sec^{\frac{7}{2}}(c+dx)+3a^3\sec^{\frac{5}{2}}(c+dx)+3a^3\sec^{\frac{3}{2}}(c+dx)+a^3\sqrt{\sec(c+dx)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2a^3\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}+\frac{2a^3\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d}+\frac{36a^3\sin(c+dx)\sqrt{\sec(c+dx)}}{5d}+\frac{4a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}-\frac{36a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}$$

input
```
Int[Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^3,x]
```

output
```
(-36*a^3*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/
(5*d) + (4*a^3*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d
*x]])/d + (36*a^3*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(5*d) + (2*a^3*Sec[c +
d*x]^(3/2)*Sin[c + d*x])/d + (2*a^3*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5*d)
```

3.180.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4278
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

3.180.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 385 vs. $2(189) = 378$.

Time = 29.17 (sec) , antiderivative size = 386, normalized size of antiderivative = 2.46

method	result
default	$-\frac{a^3\sqrt{-\left(-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(\frac{56\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1}\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{5\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}}-\frac{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\sin}}{\left(\cos\left(\frac{d}{2}\right.\right.}\right.}{}$
parts	Expression too large to display

input
```
int(sec(d*x+c)^(1/2)*(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
-a^3*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(56/5*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+
1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))
-cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(
cos(1/2*d*x+1/2*c)^2-1/2)^2-72/5*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)/(
-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)-36/5*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-Ellip
ticE(cos(1/2*d*x+1/2*c),2^(1/2)))-1/10*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+
1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^3)/sin(1/2
*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

3.180.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.27

$$\int \sqrt{\sec(c+dx)}(a+a\sec(c+dx))^3\,dx =$$
$$-\frac{2\left(5i\sqrt{2}a^3\cos(dx+c)^2\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))-5i\sqrt{2}a^3\cos(dx+c\right.}{}$$

input
```
integrate(sec(d*x+c)^(1/2)*(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
-2/5*(5*I*sqrt(2)*a^3*cos(d*x + c)^2*weierstrassPInverse(-4, 0, cos(d*x +
c) + I*sin(d*x + c)) - 5*I*sqrt(2)*a^3*cos(d*x + c)^2*weierstrassPInverse(
-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 9*I*sqrt(2)*a^3*cos(d*x + c)^2*wei
erstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x +
c))) - 9*I*sqrt(2)*a^3*cos(d*x + c)^2*weierstrassZeta(-4, 0, weierstrassPI
nverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - (18*a^3*cos(d*x + c)^2 + 5
*a^3*cos(d*x + c) + a^3)*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c)^
2)
```

3.180.6 Sympy [F(-1)]

Timed out.

$$\int \sqrt{\sec(c+dx)}(a+a\sec(c+dx))^3\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**(1/2)*(a+a*sec(d*x+c))**3,x)
```

output
```
Timed out
```

3.180.7 Maxima [F]

$$\int \sqrt{\sec(c+dx)}(a+a\sec(c+dx))^3\,dx = \int (a\sec(dx+c)+a)^3\sqrt{\sec(dx+c)}\,dx$$

input
```
integrate(sec(d*x+c)^(1/2)*(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)^3*sqrt(sec(d*x + c)), x)
```

3.180.8 Giac [F]

$$\int \sqrt{\sec(c+dx)}(a+a\sec(c+dx))^3\,dx = \int (a\sec(dx+c)+a)^3\sqrt{\sec(dx+c)}\,dx$$

input
```
integrate(sec(d*x+c)^(1/2)*(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)^3*sqrt(sec(d*x + c)), x)
```

3.180.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{\sec(c+dx)}(a+a\sec(c+dx))^3\,dx = \int \left(a+\frac{a}{\cos(c+d\,x)}\right)^3\sqrt{\frac{1}{\cos(c+d\,x)}}\,dx$$

input
```
int((a + a/cos(c + d*x))^3*(1/cos(c + d*x))^(1/2),x)
```

output
```
int((a + a/cos(c + d*x))^3*(1/cos(c + d*x))^(1/2), x)
```

3.181 $\int \frac{(a+a\sec(c+dx))^3}{\sqrt{\sec(c+dx)}}\,dx$

3.181.1 Optimal result . . . 1503
3.181.2 Mathematica [C] (verified) . . . 1503
3.181.3 Rubi [A] (verified) . . . 1504
3.181.4 Maple [B] (verified) . . . 1505
3.181.5 Fricas [C] (verification not implemented) . . . 1506
3.181.6 Sympy [F] . . . 1507
3.181.7 Maxima [F] . . . 1507
3.181.8 Giac [F] . . . 1507
3.181.9 Mupad [F(-1)] . . . 1508

3.181.1 Optimal result

Integrand size = 23, antiderivative size = 131

$$\int \frac{(a+a\sec(c+dx))^3}{\sqrt{\sec(c+dx)}}\,dx = -\frac{4a^3\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{20a^3\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{3d} + \frac{6a^3\sqrt{\sec(c+dx)}\sin(c+dx)}{d} + \frac{2a^3\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d}$$

output
```
2/3*a^3*sec(d*x+c)^(3/2)*sin(d*x+c)/d+6*a^3*sin(d*x+c)*sec(d*x+c)^(1/2)/d-
4*a^3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*
x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+20/3*a^3*(cos(1/2*d*
x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))
*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

3.181.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 2.53 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.43

$$\int \frac{(a+a\sec(c+dx))^3}{\sqrt{\sec(c+dx)}}\,dx = \frac{a^3e^{-2i(c+dx)}\sec^{\frac{3}{2}}(c+dx)\left(-6-6\cos(2(c+dx))+6e^{-2i(c+dx)}\left(1+e^{2i(c+dx)}\right)^{3/2}\text{Hypergeometric2F1}\left(-\frac{1}{4},\right.\right.}{}$$

input
```
Integrate[(a + a*Sec[c + d*x])^3/Sqrt[Sec[c + d*x]],x]
```

output
```
(a^3*Sec[c + d*x]^(3/2)*(-6 - 6*Cos[2*(c + d*x)] + (6*(1 + E^((2*I)*(c + d
*x)))^(3/2)*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))])/E^((2
*I)*(c + d*x)) + 20*Sqrt[1 + E^((2*I)*(c + d*x))]*Cos[c + d*x]*Hypergeomet
ric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))] + (2*I)*Sin[c + d*x] + (9*I)*S
in[2*(c + d*x)])*((-I)*Cos[2*(c + d*x)] + Sin[2*(c + d*x)]))/(3*d*E^((2*I)
*(c + d*x)))
```

3.181.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3042, 4278, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sec(c+dx)+a)^3}{\sqrt{\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4278}$$

$$\int \left(a^3\sec^{\frac{5}{2}}(c+dx)+3a^3\sec^{\frac{3}{2}}(c+dx)+3a^3\sqrt{\sec(c+dx)}+\frac{a^3}{\sqrt{\sec(c+dx)}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2a^3\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{6a^3\sin(c+dx)\sqrt{\sec(c+dx)}}{d}+\frac{20a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}-\frac{4a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

input
```
Int[(a + a*Sec[c + d*x])^3/Sqrt[Sec[c + d*x]],x]
```

output
```
(-4*a^3*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d
 + (20*a^3*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]]
)/(3*d) + (6*a^3*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d + (2*a^3*Sec[c + d*x]^
(3/2)*Sin[c + d*x])/(3*d)
```

3.181.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4278
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

3.181.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 370 vs. $2(167) = 334$.

Time = 22.08 (sec) , antiderivative size = 371, normalized size of antiderivative = 2.83

method	result
default	$-\frac{4a^3\sqrt{-\left(-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(18\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-10\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1}\,\mathrm{EllipticF}\left(\mathrm{co}\right.\right.}{}$
parts	$\frac{2a^3\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1}\,\mathrm{EllipticE}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1}\,d}-\frac{2a^3\left(-2\sqrt{\frac{1}{2}-\frac{\cos(}{}}\right.}{}$

input
```
int((a+a*sec(d*x+c))^3/sec(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-4/3*a^3*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(4*sin(
1/2*d*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)/sin(1/2*d*x+1/2*c)^3*(18*sin(1/
2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-10*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1
/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x
+1/2*c)^2-6*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1
/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2-10*cos(1/2*d*x+
1/2*c)*sin(1/2*d*x+1/2*c)^2+5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+
1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2
-1)^(1/2))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2
*d*x+1/2*c)^2-1)^(1/2)/d
```

3.181.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.37

$$\int \frac{(a+a\sec(c+dx))^3}{\sqrt{\sec(c+dx)}}\,dx = -\frac{2\left(5i\sqrt{2}a^3\cos(dx+c)\,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))-5i\sqrt{2}a^3\cos(dx+c)\right.}{}$$

input
```
integrate((a+a*sec(d*x+c))^3/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
-2/3*(5*I*sqrt(2)*a^3*cos(d*x + c)*weierstrassPInverse(-4, 0, cos(d*x + c)
 + I*sin(d*x + c)) - 5*I*sqrt(2)*a^3*cos(d*x + c)*weierstrassPInverse(-4,
0, cos(d*x + c) - I*sin(d*x + c)) + 3*I*sqrt(2)*a^3*cos(d*x + c)*weierstra
ssZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) -
 3*I*sqrt(2)*a^3*cos(d*x + c)*weierstrassZeta(-4, 0, weierstrassPInverse(-
4, 0, cos(d*x + c) - I*sin(d*x + c))) - (9*a^3*cos(d*x + c) + a^3)*sin(d*x
 + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c))
```

3.181.6 Sympy [F]

$$\int \frac{(a+a\sec(c+dx))^3}{\sqrt{\sec(c+dx)}}\,dx = a^3\left(\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx + \int 3\sqrt{\sec(c+dx)}\,dx + \int 3\sec^{\frac{3}{2}}(c+dx)\,dx + \int \sec^{\frac{5}{2}}(c+dx)\,dx\right)$$

input
```
integrate((a+a*sec(d*x+c))**3/sec(d*x+c)**(1/2),x)
```

output
```
a**3*(Integral(1/sqrt(sec(c + d*x)), x) + Integral(3*sqrt(sec(c + d*x)), x
) + Integral(3*sec(c + d*x)**(3/2), x) + Integral(sec(c + d*x)**(5/2), x))
```

3.181.7 Maxima [F]

$$\int \frac{(a+a\sec(c+dx))^3}{\sqrt{\sec(c+dx)}}\,dx = \int \frac{(a\sec(dx+c)+a)^3}{\sqrt{\sec(dx+c)}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))^3/sec(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)^3/sqrt(sec(d*x + c)), x)
```

3.181.8 Giac [F]

$$\int \frac{(a+a\sec(c+dx))^3}{\sqrt{\sec(c+dx)}}\,dx = \int \frac{(a\sec(dx+c)+a)^3}{\sqrt{\sec(dx+c)}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))^3/sec(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)^3/sqrt(sec(d*x + c)), x)
```

3.181.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(c+dx))^3}{\sqrt{\sec(c+dx)}}\,dx=\int \frac{\left(a+\frac{a}{\cos(c+d\,x)}\right)^3}{\sqrt{\frac{1}{\cos(c+d\,x)}}}\,dx$$

input
```
int((a + a/cos(c + d*x))^3/(1/cos(c + d*x))^(1/2),x)
```

output
```
int((a + a/cos(c + d*x))^3/(1/cos(c + d*x))^(1/2), x)
```

3.182 $\int \frac{(a+a\sec(c+dx))^3}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

3.182.1 Optimal result 1509
3.182.2 Mathematica [C] (verified) 1510
3.182.3 Rubi [A] (verified) 1510
3.182.4 Maple [A] (verified) 1512
3.182.5 Fricas [C] (verification not implemented) 1512
3.182.6 Sympy [F] 1513
3.182.7 Maxima [F] 1513
3.182.8 Giac [F] 1513
3.182.9 Mupad [F(-1)] 1514

3.182.1 Optimal result

Integrand size = 23, antiderivative size = 131

$$\int \frac{(a+a\sec(c+dx))^3}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \frac{4a^3\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{20a^3\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{3d} + \frac{2a^3\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{2a^3\sqrt{\sec(c+dx)}\sin(c+dx)}{d}$$

output
```
2/3*a^3*sin(d*x+c)/d/sec(d*x+c)^(1/2)+2*a^3*sin(d*x+c)*sec(d*x+c)^(1/2)/d+
4*a^3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*
x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+20/3*a^3*(cos(1/2*d*
x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))
*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

3.182.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 2.31 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.29

$$\int \frac{(a+a\sec(c+dx))^3}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \frac{a^3\left(\cos\left(\frac{c}{2}\right)-i\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{c}{2}\right)+i\sin\left(\frac{c}{2}\right)\right)\left(\frac{24i\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},-e^{2i(c+dx)}\right)}{\sqrt{1+e^{2i(c+dx)}}}+2\left(-6i-10i\sqrt{1+e^{2i(c+}}\right.\right.}{3d\sqrt{\sec(c+dx)}}$$

input
```
Integrate[(a + a*Sec[c + d*x])^3/Sec[c + d*x]^(3/2),x]
```

output
```
(a^3*(Cos[c/2] - I*Sin[c/2])*(Cos[c/2] + I*Sin[c/2])*(((24*I)*Hypergeometr
ic2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))])/Sqrt[1 + E^((2*I)*(c + d*x))]
 + 2*(-6*I - (10*I)*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1
/2, 5/4, -E^((2*I)*(c + d*x))]*Sec[c + d*x] + Sin[c + d*x] + 3*Tan[c + d*x
])))/(3*d*Sqrt[Sec[c + d*x]])
```

3.182.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3042, 4278, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sec(c+dx)+a)^3}{\sec^{\frac{3}{2}}(c+dx)}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx$$

$$\Big\downarrow \; 4278$$

$$\int \left(a^3\sec^{\frac{3}{2}}(c+dx)+\frac{a^3}{\sec^{\frac{3}{2}}(c+dx)}+3a^3\sqrt{\sec(c+dx)}+\frac{3a^3}{\sqrt{\sec(c+dx)}}\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{2a^3\sin(c+dx)\sqrt{\sec(c+dx)}}{d}+\frac{2a^3\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{20a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{4a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

input
```
Int[(a + a*Sec[c + d*x])^3/Sec[c + d*x]^(3/2),x]
```

output
```
(4*a^3*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d
+ (20*a^3*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])
/(3*d) + (2*a^3*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]]) + (2*a^3*Sqrt[Sec[c
 + d*x]]*Sin[c + d*x])/d
```

3.182.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4278
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

3.182.4 Maple [A] (verified)

Time = 20.02 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.31

method	result
default	$-\frac{4a^3\left(2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2+5\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1}\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right.}{3\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1}\,d}$
parts	$-\frac{2a^3\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(4\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\sin\left(\frac{dx}{2}\right.}\right.}{3\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1}\,d}$

input
```
int((a+a*sec(d*x+c))^3/sec(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-4/3*a^3*(2*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-4*cos(1/2*d*x+1/2*c)*s
in(1/2*d*x+1/2*c)^2+5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2
-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2
))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

3.182.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.13

$$\int \frac{(a+a\sec(c+dx))^3}{\sec^{\frac{3}{2}}(c+dx)}\,dx = -\frac{2\left(5i\sqrt{2}a^3\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))-5i\sqrt{2}a^3\mathrm{weierstrassPInverse}(-4,\right.}{}$$

input
```
integrate((a+a*sec(d*x+c))^3/sec(d*x+c)^(3/2),x, algorithm="fricas")
```

output
```
-2/3*(5*I*sqrt(2)*a^3*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x
+ c)) - 5*I*sqrt(2)*a^3*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*
x + c)) - 3*I*sqrt(2)*a^3*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0
, cos(d*x + c) + I*sin(d*x + c))) + 3*I*sqrt(2)*a^3*weierstrassZeta(-4, 0,
 weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - (a^3*cos(d*x
 + c) + 3*a^3)*sin(d*x + c)/sqrt(cos(d*x + c)))/d
```

3.182.6 Sympy [F]

$$\int \frac{(a+a\sec(c+dx))^3}{\sec^{\frac{3}{2}}(c+dx)}\,dx = a^3\left(\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}\,dx + \int \frac{3}{\sqrt{\sec(c+dx)}}\,dx + \int 3\sqrt{\sec(c+dx)}\,dx + \int \sec^{\frac{3}{2}}(c+dx)\,dx\right)$$

input
```
integrate((a+a*sec(d*x+c))**3/sec(d*x+c)**(3/2),x)
```

output
```
a**3*(Integral(sec(c + d*x)**(-3/2), x) + Integral(3/sqrt(sec(c + d*x)), x
) + Integral(3*sqrt(sec(c + d*x)), x) + Integral(sec(c + d*x)**(3/2), x))
```

3.182.7 Maxima [F]

$$\int \frac{(a+a\sec(c+dx))^3}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(a\sec(dx+c)+a)^3}{\sec(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))^3/sec(d*x+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)^3/sec(d*x + c)^(3/2), x)
```

3.182.8 Giac [F]

$$\int \frac{(a+a\sec(c+dx))^3}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(a\sec(dx+c)+a)^3}{\sec(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))^3/sec(d*x+c)^(3/2),x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)^3/sec(d*x + c)^(3/2), x)
```

3.182.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(c+dx))^3}{\sec^{\frac{3}{2}}(c+dx)}\,dx=\int \frac{\left(a+\frac{a}{\cos(c+d\,x)}\right)^3}{\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

input
```
int((a + a/cos(c + d*x))^3/(1/cos(c + d*x))^(3/2),x)
```

output
```
int((a + a/cos(c + d*x))^3/(1/cos(c + d*x))^(3/2), x)
```

3.183 $\int \frac{(a+a\sec(c+dx))^3}{\sec^{\frac{5}{2}}(c+dx)}\,dx$

3.183.1 Optimal result 1515
3.183.2 Mathematica [C] (verified) 1516
3.183.3 Rubi [A] (verified) 1516
3.183.4 Maple [A] (verified) 1517
3.183.5 Fricas [C] (verification not implemented) 1518
3.183.6 Sympy [F] 1519
3.183.7 Maxima [F] 1519
3.183.8 Giac [F] 1519
3.183.9 Mupad [F(-1)] 1520

3.183.1 Optimal result

Integrand size = 23, antiderivative size = 131

$$\int \frac{(a+a\sec(c+dx))^3}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \frac{36a^3\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{4a^3\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{d} + \frac{2a^3\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{2a^3\sin(c+dx)}{d\sqrt{\sec(c+dx)}}$$

output
```
2/5*a^3*sin(d*x+c)/d/sec(d*x+c)^(3/2)+2*a^3*sin(d*x+c)/d/sec(d*x+c)^(1/2)+
36/5*a^3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2
*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+4*a^3*(cos(1/2*d*
x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))
*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

3.183.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 2.13 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.31

$$\int \frac{(a+a\sec(c+dx))^3}{\sec^{\frac{5}{2}}(c+dx)}\,dx$$
$$= \frac{a^3\left(\cos\left(\frac{c}{2}\right)-i\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{c}{2}\right)+i\sin\left(\frac{c}{2}\right)\right)\left(\frac{144i\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},-e^{2i(c+dx)}\right)}{\sqrt{1+e^{2i(c+dx)}}}+2\left(-36i-20i\sqrt{1+e^{2i}}\right.\right.}{10d\sqrt{\sec(c+}}$$

input
```
Integrate[(a + a*Sec[c + d*x])^3/Sec[c + d*x]^(5/2),x]
```

output
```
(a^3*(Cos[c/2] - I*Sin[c/2])*(Cos[c/2] + I*Sin[c/2])*(((144*I)*Hypergeomet
ric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))])/Sqrt[1 + E^((2*I)*(c + d*x))
] + 2*(-36*I - (20*I)*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/4,
 1/2, 5/4, -E^((2*I)*(c + d*x))]*Sec[c + d*x] + 10*Sin[c + d*x] + Sin[2*(c
 + d*x)])))/(10*d*Sqrt[Sec[c + d*x]])
```

3.183.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3042, 4278, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sec(c+dx)+a)^3}{\sec^{\frac{5}{2}}(c+dx)}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx$$
$$\Big\downarrow \; 4278$$
$$\int \left(\frac{3a^3}{\sec^{\frac{3}{2}}(c+dx)}+\frac{a^3}{\sec^{\frac{5}{2}}(c+dx)}+a^3\sqrt{\sec(c+dx)}+\frac{3a^3}{\sqrt{\sec(c+dx)}}\right)dx$$
$$\Big\downarrow \; 2009$$

$$\frac{2a^3\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{2a^3\sin(c+dx)}{d\sqrt{\sec(c+dx)}}+\frac{4a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+\frac{36a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}$$

input
```
Int[(a + a*Sec[c + d*x])^3/Sec[c + d*x]^(5/2),x]
```

output
```
(36*a^3*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(
5*d) + (4*a^3*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*
x]])/d + (2*a^3*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)) + (2*a^3*Sin[c + d*
x])/(d*Sqrt[Sec[c + d*x]])
```

3.183.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4278
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

3.183.4 Maple [A] (verified)

Time = 18.33 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.91

method	result
default	$-\frac{4\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\,a^3\left(-4\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^6\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+14\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-6\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\right.\right.}{5\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\sin\left(\frac{dx}{2}\right.}}$
parts	$-\frac{2a^3\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(-8\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^6\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+8\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}{5\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right.}}$

input
```
int((a+a*sec(d*x+c))^3/sec(d*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-4/5*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3*(-4*sin(1
/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+14*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2
*c)-6*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+5*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-
9*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*si
n(1/2*d*x+1/2*c)^2-1)^(1/2))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2
)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

3.183.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.19

$$\int \frac{(a+a\sec(c+dx))^3}{\sec^{\frac{5}{2}}(c+dx)}\,dx = -\frac{2\left(5i\sqrt{2}a^3\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))-5i\sqrt{2}a^3\text{weierstrassPInverse}(-4,\right.}{}$$

input
```
integrate((a+a*sec(d*x+c))^3/sec(d*x+c)^(5/2),x, algorithm="fricas")
```

output
```
-2/5*(5*I*sqrt(2)*a^3*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x
+ c)) - 5*I*sqrt(2)*a^3*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*
x + c)) - 9*I*sqrt(2)*a^3*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0
, cos(d*x + c) + I*sin(d*x + c))) + 9*I*sqrt(2)*a^3*weierstrassZeta(-4, 0,
 weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - (a^3*cos(d*x
 + c)^2 + 5*a^3*cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/d
```

3.183.6 Sympy [F]

$$\int \frac{(a+a\sec(c+dx))^3}{\sec^{\frac{5}{2}}(c+dx)}\,dx = a^3\left(\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)}\,dx + \int \frac{3}{\sec^{\frac{3}{2}}(c+dx)}\,dx + \int \frac{3}{\sqrt{\sec(c+dx)}}\,dx + \int \sqrt{\sec(c+dx)}\,dx\right)$$

input
```
integrate((a+a*sec(d*x+c))**3/sec(d*x+c)**(5/2),x)
```

output
```
a**3*(Integral(sec(c + d*x)**(-5/2), x) + Integral(3/sec(c + d*x)**(3/2),
x) + Integral(3/sqrt(sec(c + d*x)), x) + Integral(sqrt(sec(c + d*x)), x))
```

3.183.7 Maxima [F]

$$\int \frac{(a+a\sec(c+dx))^3}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{(a\sec(dx+c)+a)^3}{\sec(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))^3/sec(d*x+c)^(5/2),x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)^3/sec(d*x + c)^(5/2), x)
```

3.183.8 Giac [F]

$$\int \frac{(a+a\sec(c+dx))^3}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{(a\sec(dx+c)+a)^3}{\sec(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))^3/sec(d*x+c)^(5/2),x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)^3/sec(d*x + c)^(5/2), x)
```

3.183.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(c+dx))^3}{\sec^{\frac{5}{2}}(c+dx)}\,dx=\int \frac{\left(a+\frac{a}{\cos(c+d\,x)}\right)^3}{\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

input
```
int((a + a/cos(c + d*x))^3/(1/cos(c + d*x))^(5/2),x)
```

output
```
int((a + a/cos(c + d*x))^3/(1/cos(c + d*x))^(5/2), x)
```

3.184 $\int \frac{(a+a\sec(c+dx))^3}{\sec^{\frac{7}{2}}(c+dx)}\,dx$

3.184.1 Optimal result 1521
3.184.2 Mathematica [C] (verified) 1522
3.184.3 Rubi [A] (verified) 1522
3.184.4 Maple [A] (verified) 1524
3.184.5 Fricas [C] (verification not implemented) 1524
3.184.6 Sympy [F] 1525
3.184.7 Maxima [F] 1525
3.184.8 Giac [F] 1526
3.184.9 Mupad [F(-1)] 1526

3.184.1 Optimal result

Integrand size = 23, antiderivative size = 161

$$\int \frac{(a+a\sec(c+dx))^3}{\sec^{\frac{7}{2}}(c+dx)}\,dx = \frac{28a^3\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{52a^3\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{21d} + \frac{2a^3\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{6a^3\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{52a^3\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}$$

output
```
2/7*a^3*sin(d*x+c)/d/sec(d*x+c)^(5/2)+6/5*a^3*sin(d*x+c)/d/sec(d*x+c)^(3/2
)+52/21*a^3*sin(d*x+c)/d/sec(d*x+c)^(1/2)+28/5*a^3*(cos(1/2*d*x+1/2*c)^2)^
(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^
(1/2)*sec(d*x+c)^(1/2)/d+52/21*a^3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*
x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)
^(1/2)/d
```

3.184.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 2.45 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.91

$$\int \frac{(a+a\sec(c+dx))^3}{\sec^{\frac{7}{2}}(c+dx)}\,dx = \frac{a^3\left(-2352i+\frac{4704i\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},-e^{2i(c+dx)}\right)}{\sqrt{1+e^{2i(c+dx)}}}-1040i\sqrt{1+e^{2i(c+dx)}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-e^{2}\right.\right.}{420d\sqrt{\sec(c+dx)}}$$

input
```
Integrate[(a + a*Sec[c + d*x])^3/Sec[c + d*x]^(7/2),x]
```

output
```
(a^3*(-2352*I + ((4704*I)*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c +
 d*x))])/Sqrt[1 + E^((2*I)*(c + d*x))] - (1040*I)*Sqrt[1 + E^((2*I)*(c + d
*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]*Sec[c + d*x]
+ 1070*Sin[c + d*x] + 252*Sin[2*(c + d*x)] + 30*Sin[3*(c + d*x)]))/(420*d*
Sqrt[Sec[c + d*x]])
```

3.184.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3042, 4278, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sec(c+dx)+a)^3}{\sec^{\frac{7}{2}}(c+dx)}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx$$

$$\Big\downarrow \; 4278$$

$$\int \left(\frac{3a^3}{\sec^{\frac{3}{2}}(c+dx)}+\frac{3a^3}{\sec^{\frac{5}{2}}(c+dx)}+\frac{a^3}{\sec^{\frac{7}{2}}(c+dx)}+\frac{a^3}{\sqrt{\sec(c+dx)}}\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{6a^3\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{2a^3\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+\frac{52a^3\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}+\frac{52a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d}+\frac{28a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}$$

input
```
Int[(a + a*Sec[c + d*x])^3/Sec[c + d*x]^(7/2),x]
```

output
```
(28*a^3*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(
5*d) + (52*a^3*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d
*x]])/(21*d) + (2*a^3*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/2)) + (6*a^3*Sin[
c + d*x])/(5*d*Sec[c + d*x]^(3/2)) + (52*a^3*Sin[c + d*x])/(21*d*Sqrt[Sec[
c + d*x]])
```

3.184.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4278
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

3.184.4 Maple [A] (verified)

Time = 25.94 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.69

method	result
default	$-\frac{4\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,a^{3}\left(120\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{8}-432\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+602\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}\cos\left(\right.\right.}{105\sqrt{-2\,\mathrm{si}}}$
parts	$-\frac{2a^{3}\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\left(48\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{9}-120\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}+128\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}-72\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}+5\sqrt{\frac{1}{2}-\frac{\cos(}{}}\right.}{21\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\cos\left(\frac{dx}{2}+\right.}}$

input
```
int((a+a*sec(d*x+c))^3/sec(d*x+c)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-4/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3*(120*co
s(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8-432*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x
+1/2*c)+602*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-208*cos(1/2*d*x+1/2*c)
*sin(1/2*d*x+1/2*c)^2+65*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c
)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-147*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)
^(1/2))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1
/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

3.184.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.06

$$\int \frac{(a+a\sec(c+dx))^3}{\sec^{\frac{7}{2}}(c+dx)}\,dx = -\frac{2\left(65i\sqrt{2}a^3\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))-65i\sqrt{2}a^3\mathrm{weierstrassPInverse}(-\right.}{}$$

input
```
integrate((a+a*sec(d*x+c))^3/sec(d*x+c)^(7/2),x, algorithm="fricas")
```

output
```
-2/105*(65*I*sqrt(2)*a^3*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d
*x + c)) - 65*I*sqrt(2)*a^3*weierstrassPInverse(-4, 0, cos(d*x + c) - I*si
n(d*x + c)) - 147*I*sqrt(2)*a^3*weierstrassZeta(-4, 0, weierstrassPInverse
(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 147*I*sqrt(2)*a^3*weierstrassZet
a(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - (15*
a^3*cos(d*x + c)^3 + 63*a^3*cos(d*x + c)^2 + 130*a^3*cos(d*x + c))*sin(d*x
 + c)/sqrt(cos(d*x + c)))/d
```

3.184.6 Sympy [F]

$$\int \frac{(a+a\sec(c+dx))^3}{\sec^{\frac{7}{2}}(c+dx)}\,dx = a^3\left(\int \frac{1}{\sec^{\frac{7}{2}}(c+dx)}\,dx + \int \frac{3}{\sec^{\frac{5}{2}}(c+dx)}\,dx + \int \frac{3}{\sec^{\frac{3}{2}}(c+dx)}\,dx + \int \frac{1}{\sqrt{\sec(c+dx)}}\,dx\right)$$

input
```
integrate((a+a*sec(d*x+c))**3/sec(d*x+c)**(7/2),x)
```

output
```
a**3*(Integral(sec(c + d*x)**(-7/2), x) + Integral(3/sec(c + d*x)**(5/2),
x) + Integral(3/sec(c + d*x)**(3/2), x) + Integral(1/sqrt(sec(c + d*x)), x
))
```

3.184.7 Maxima [F]

$$\int \frac{(a+a\sec(c+dx))^3}{\sec^{\frac{7}{2}}(c+dx)}\,dx = \int \frac{(a\sec(dx+c)+a)^3}{\sec(dx+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))^3/sec(d*x+c)^(7/2),x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)^3/sec(d*x + c)^(7/2), x)
```

3.184.8 Giac [F]

$$\int \frac{(a+a\sec(c+dx))^3}{\sec^{\frac{7}{2}}(c+dx)}\,dx=\int \frac{(a\sec(dx+c)+a)^3}{\sec(dx+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))^3/sec(d*x+c)^(7/2),x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)^3/sec(d*x + c)^(7/2), x)
```

3.184.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(c+dx))^3}{\sec^{\frac{7}{2}}(c+dx)}\,dx=\int \frac{\left(a+\frac{a}{\cos(c+d\,x)}\right)^3}{\left(\frac{1}{\cos(c+d\,x)}\right)^{7/2}}\,dx$$

input
```
int((a + a/cos(c + d*x))^3/(1/cos(c + d*x))^(7/2),x)
```

output
```
int((a + a/cos(c + d*x))^3/(1/cos(c + d*x))^(7/2), x)
```

3.185 $\int \frac{(a+a\sec(c+dx))^3}{\sec^{\frac{9}{2}}(c+dx)}\,dx$

3.185.1 Optimal result 1527
3.185.2 Mathematica [C] (verified) 1528
3.185.3 Rubi [A] (verified) 1528
3.185.4 Maple [A] (verified) 1529
3.185.5 Fricas [C] (verification not implemented) 1530
3.185.6 Sympy [F(-1)] 1531
3.185.7 Maxima [F] 1531
3.185.8 Giac [F] 1531
3.185.9 Mupad [F(-1)] 1532

3.185.1 Optimal result

Integrand size = 23, antiderivative size = 187

$$\begin{aligned}\int \frac{(a+a\sec(c+dx))^3}{\sec^{\frac{9}{2}}(c+dx)}\,dx &= \frac{68a^3\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{15d} \\ &\quad+\frac{44a^3\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{21d} \\ &\quad+\frac{2a^3\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)}+\frac{6a^3\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} \\ &\quad+\frac{68a^3\sin(c+dx)}{45d\sec^{\frac{3}{2}}(c+dx)}+\frac{44a^3\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}\end{aligned}$$

output
```
2/9*a^3*sin(d*x+c)/d/sec(d*x+c)^(7/2)+6/7*a^3*sin(d*x+c)/d/sec(d*x+c)^(5/2
)+68/45*a^3*sin(d*x+c)/d/sec(d*x+c)^(3/2)+44/21*a^3*sin(d*x+c)/d/sec(d*x+c
)^(1/2)+68/15*a^3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elliptic
E(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+44/21*a^
3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/
2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

3.185.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 3.07 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.83

$$\int \frac{(a+a\sec(c+dx))^3}{\sec^{\frac{9}{2}}(c+dx)}\,dx$$
$$= \frac{a^3\left(-11424i + \frac{22848i\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},-e^{2i(c+dx)}\right)}{\sqrt{1+e^{2i(c+dx)}}} - 5280i\sqrt{1+e^{2i(c+dx)}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\right.\right.}{2520d\sqrt{\text{se}}}$$

input
```
Integrate[(a + a*Sec[c + d*x])^3/Sec[c + d*x]^(9/2),x]
```

output
```
(a^3*(-11424*I + ((22848*I)*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c
 + d*x))])/Sqrt[1 + E^((2*I)*(c + d*x))] - (5280*I)*Sqrt[1 + E^((2*I)*(c +
 d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]*Sec[c + d*x
] + 5820*Sin[c + d*x] + 2044*Sin[2*(c + d*x)] + 540*Sin[3*(c + d*x)] + 70*
Sin[4*(c + d*x)]))/(2520*d*Sqrt[Sec[c + d*x]])
```

3.185.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3042, 4278, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sec(c+dx)+a)^3}{\sec^{\frac{9}{2}}(c+dx)}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}{\csc\left(c+dx+\frac{\pi}{2}\right)^{9/2}}dx$$
$$\Big\downarrow \; 4278$$
$$\int \left(\frac{a^3}{\sec^{\frac{3}{2}}(c+dx)}+\frac{3a^3}{\sec^{\frac{5}{2}}(c+dx)}+\frac{3a^3}{\sec^{\frac{7}{2}}(c+dx)}+\frac{a^3}{\sec^{\frac{9}{2}}(c+dx)}\right)dx$$
$$\Big\downarrow \; 2009$$

$$\frac{68a^3\sin(c+dx)}{45d\sec^{\frac{3}{2}}(c+dx)}+\frac{6a^3\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+\frac{2a^3\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)}+\frac{44a^3\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}+\frac{44a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d}+\frac{68a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}$$

input
```
Int[(a + a*Sec[c + d*x])^3/Sec[c + d*x]^(9/2),x]
```

output
```
(68*a^3*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(
15*d) + (44*a^3*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c +
d*x]])/(21*d) + (2*a^3*Sin[c + d*x])/(9*d*Sec[c + d*x]^(7/2)) + (6*a^3*Sin
[c + d*x])/(7*d*Sec[c + d*x]^(5/2)) + (68*a^3*Sin[c + d*x])/(45*d*Sec[c +
d*x]^(3/2)) + (44*a^3*Sin[c + d*x])/(21*d*Sqrt[Sec[c + d*x]])
```

3.185.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4278
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

3.185.4 Maple [A] (verified)

Time = 34.23 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.39

method	result
default	$-\frac{4\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\,a^3\left(560\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{11}-600\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^9+212\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^7+66\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^5-430\cos\left(\right.\right.}{315\sqrt{-2\sin\left(\right.}}$
parts	Expression too large to display

input
```
int((a+a*sec(d*x+c))^3/sec(d*x+c)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
-4/315*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3*(560*co
s(1/2*d*x+1/2*c)^11-600*cos(1/2*d*x+1/2*c)^9+212*cos(1/2*d*x+1/2*c)^7+66*c
os(1/2*d*x+1/2*c)^5-430*cos(1/2*d*x+1/2*c)^3+165*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)
)-357*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*Ellip
ticE(cos(1/2*d*x+1/2*c),2^(1/2))+192*cos(1/2*d*x+1/2*c))/(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*
c)^2-1)^(1/2)/d
```

3.185.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.98

$$\int \frac{(a+a\sec(c+dx))^3}{\sec^{\frac{9}{2}}(c+dx)}\,dx = -\frac{2\left(165i\sqrt{2}a^3\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))-165i\sqrt{2}a^3\text{weierstrassPInverse}\right.}{}$$

input
```
integrate((a+a*sec(d*x+c))^3/sec(d*x+c)^(9/2),x, algorithm="fricas")
```

output
```
-2/315*(165*I*sqrt(2)*a^3*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(
d*x + c)) - 165*I*sqrt(2)*a^3*weierstrassPInverse(-4, 0, cos(d*x + c) - I*
sin(d*x + c)) - 357*I*sqrt(2)*a^3*weierstrassZeta(-4, 0, weierstrassPInver
se(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 357*I*sqrt(2)*a^3*weierstrassZ
eta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - (3
5*a^3*cos(d*x + c)^4 + 135*a^3*cos(d*x + c)^3 + 238*a^3*cos(d*x + c)^2 + 3
30*a^3*cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/d
```

3.185.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(c+dx))^3}{\sec^{\frac{9}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sec(d*x+c))**3/sec(d*x+c)**(9/2),x)
```

output
```
Timed out
```

3.185.7 Maxima [F]

$$\int \frac{(a+a\sec(c+dx))^3}{\sec^{\frac{9}{2}}(c+dx)}\,dx = \int \frac{(a\sec(dx+c)+a)^3}{\sec(dx+c)^{\frac{9}{2}}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))^3/sec(d*x+c)^(9/2),x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)^3/sec(d*x + c)^(9/2), x)
```

3.185.8 Giac [F]

$$\int \frac{(a+a\sec(c+dx))^3}{\sec^{\frac{9}{2}}(c+dx)}\,dx = \int \frac{(a\sec(dx+c)+a)^3}{\sec(dx+c)^{\frac{9}{2}}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))^3/sec(d*x+c)^(9/2),x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)^3/sec(d*x + c)^(9/2), x)
```

3.185.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(c+dx))^3}{\sec^{\frac{9}{2}}(c+dx)}\,dx=\int \frac{\left(a+\frac{a}{\cos(c+d\,x)}\right)^3}{\left(\frac{1}{\cos(c+d\,x)}\right)^{9/2}}\,dx$$

input
```
int((a + a/cos(c + d*x))^3/(1/cos(c + d*x))^(9/2),x)
```

output
```
int((a + a/cos(c + d*x))^3/(1/cos(c + d*x))^(9/2), x)
```

3.186 $\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^4\,dx$

3.186.1 Optimal result . . . 1533
3.186.2 Mathematica [C] (verified) . . . 1534
3.186.3 Rubi [A] (verified) . . . 1534
3.186.4 Maple [B] (verified) . . . 1536
3.186.5 Fricas [C] (verification not implemented) . . . 1536
3.186.6 Sympy [F(-1)] . . . 1537
3.186.7 Maxima [F(-1)] . . . 1537
3.186.8 Giac [F] . . . 1538
3.186.9 Mupad [F(-1)] . . . 1538

3.186.1 Optimal result

Integrand size = 23, antiderivative size = 213

$$\begin{aligned}\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^4\,dx &= -\frac{152a^4\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{15d} \\ &\quad+\frac{32a^4\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{7d} \\ &\quad+\frac{152a^4\sqrt{\sec(c+dx)}\sin(c+dx)}{15d} \\ &\quad+\frac{32a^4\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{7d}+\frac{122a^4\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{45d} \\ &\quad+\frac{8a^4\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{7d}+\frac{2a^4\sec^{\frac{9}{2}}(c+dx)\sin(c+dx)}{9d}\end{aligned}$$

output
```
32/7*a^4*sec(d*x+c)^(3/2)*sin(d*x+c)/d+122/45*a^4*sec(d*x+c)^(5/2)*sin(d*x
+c)/d+8/7*a^4*sec(d*x+c)^(7/2)*sin(d*x+c)/d+2/9*a^4*sec(d*x+c)^(9/2)*sin(d
*x+c)/d+152/15*a^4*sin(d*x+c)*sec(d*x+c)^(1/2)/d-152/15*a^4*(cos(1/2*d*x+1
/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*co
s(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+32/7*a^4*(cos(1/2*d*x+1/2*c)^2)^(1/2)/co
s(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*se
c(d*x+c)^(1/2)/d
```

3.186.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 7.61 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.36

$$\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^4\,dx$$
$$= \frac{a^4\sec^8\left(\frac{1}{2}(c+dx)\right)(1+\sec(c+dx))^4\left(-\frac{12i\sqrt{2}e^{-i(c+dx)}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\cos^4(c+dx)\left(133\left(1+e^{2i(c+dx)}\right)+133\left(-1+e^{2ic}\right)\sqrt{1+e^{2i}}\right.}{}\right.}{}$$

input
```
Integrate[Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^4,x]
```

output
```
(a^4*Sec[(c + d*x)/2]^8*(1 + Sec[c + d*x])^4*(((-12*I)*Sqrt[2]*Sqrt[E^(I*(
c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Cos[c + d*x]^4*(133*(1 + E^((2*I)*(c
+ d*x))) + 133*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeome
tric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))] + 60*E^(I*(c + d*x))*(-1 + E
^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4,
 -E^((2*I)*(c + d*x))]))/(E^(I*(c + d*x))*(-1 + E^((2*I)*c))) + (1596*Cos[
d*x]*Csc[c] + (720 + 427*Sec[c + d*x] + 180*Sec[c + d*x]^2 + 35*Sec[c + d*
x]^3)*Tan[c + d*x])/Sec[c + d*x]^(7/2)))/(2520*d)
```

3.186.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3042, 4278, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^4\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^4dx$$
$$\Big\downarrow \; 4278$$

$$\int \left(a^4 \sec^{\frac{11}{2}}(c+dx)+4a^4 \sec^{\frac{9}{2}}(c+dx)+6a^4 \sec^{\frac{7}{2}}(c+dx)+4a^4 \sec^{\frac{5}{2}}(c+dx)+a^4 \sec^{\frac{3}{2}}(c+dx)\right) dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2a^4 \sin(c+dx)\sec^{\frac{9}{2}}(c+dx)}{9d}+\frac{8a^4 \sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{7d}+\frac{122a^4 \sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{45d}+\frac{32a^4 \sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{7d}+\frac{152a^4 \sin(c+dx)\sqrt{\sec(c+dx)}}{15d}+\frac{32a^4 \sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{7d}-\frac{152a^4 \sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}$$

input
```
Int[Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^4,x]
```

output
```
(-152*a^4*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])
/(15*d) + (32*a^4*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c
+ d*x]])/(7*d) + (152*a^4*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(15*d) + (32*a^
4*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(7*d) + (122*a^4*Sec[c + d*x]^(5/2)*Sin
[c + d*x])/(45*d) + (8*a^4*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(7*d) + (2*a^4
*Sec[c + d*x]^(9/2)*Sin[c + d*x])/(9*d)
```

3.186.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4278
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

3.186.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 491 vs. $2(233) = 466$.

Time = 48.24 (sec) , antiderivative size = 492, normalized size of antiderivative = 2.31

method	result
default	$-\frac{a^4\sqrt{-\left(-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(-\frac{16\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}}{7\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-\frac{1}{2}\right)^2}+\frac{1544\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right.}}{105\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)}}\right.}{}$
parts	Expression too large to display

input
```
int(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
-a^4*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-16/7*cos(
1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1
/2*d*x+1/2*c)^2-1/2)^2+1544/105*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d
*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*
EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/7*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d
*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^4-61/90
*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(
cos(1/2*d*x+1/2*c)^2-1/2)^3-304/15*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)
/(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)-152/15*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-E
llipticE(cos(1/2*d*x+1/2*c),2^(1/2)))-1/72*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*
d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^5)/sin
(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

3.186.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.07

$$\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^4\,dx =$$

$$-\frac{2\left(360i\sqrt{2}a^4\cos(dx+c)^4\,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))-360i\sqrt{2}a^4\cos(d\right.}{}$$

input
```
integrate(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^4,x, algorithm="fricas")
```

output
```
-2/315*(360*I*sqrt(2)*a^4*cos(d*x + c)^4*weierstrassPInverse(-4, 0, cos(d*
x + c) + I*sin(d*x + c)) - 360*I*sqrt(2)*a^4*cos(d*x + c)^4*weierstrassPIn
verse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 798*I*sqrt(2)*a^4*cos(d*x +
c)^4*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*si
n(d*x + c))) - 798*I*sqrt(2)*a^4*cos(d*x + c)^4*weierstrassZeta(-4, 0, wei
erstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - (1596*a^4*cos(d*
x + c)^4 + 720*a^4*cos(d*x + c)^3 + 427*a^4*cos(d*x + c)^2 + 180*a^4*cos(d
*x + c) + 35*a^4)*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c)^4)
```

3.186.6 Sympy [F(-1)]

Timed out.

$$\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^4\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**(3/2)*(a+a*sec(d*x+c))**4,x)
```

output
```
Timed out
```

3.186.7 Maxima [F(-1)]

Timed out.

$$\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^4\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^4,x, algorithm="maxima")
```

output
```
Timed out
```

3.186.8 Giac [F]

$$\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^4\,dx = \int (a\sec(dx+c)+a)^4\sec(dx+c)^{\frac{3}{2}}\,dx$$

input
```
integrate(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^4,x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)^4*sec(d*x + c)^(3/2), x)
```

3.186.9 Mupad [F(-1)]

Timed out.

$$\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^4\,dx = \int \left(a+\frac{a}{\cos(c+d\,x)}\right)^4\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}dx$$

input
```
int((a + a/cos(c + d*x))^4*(1/cos(c + d*x))^(3/2),x)
```

output
```
int((a + a/cos(c + d*x))^4*(1/cos(c + d*x))^(3/2), x)
```

3.187 $\int \sqrt{\sec(c+dx)}(a+a\sec(c+dx))^4\,dx$

3.187.1 Optimal result . . . 1539
3.187.2 Mathematica [C] (verified) . . . 1540
3.187.3 Rubi [A] (verified) . . . 1540
3.187.4 Maple [B] (verified) . . . 1542
3.187.5 Fricas [C] (verification not implemented) . . . 1542
3.187.6 Sympy [F(-1)] . . . 1543
3.187.7 Maxima [F] . . . 1543
3.187.8 Giac [F] . . . 1544
3.187.9 Mupad [F(-1)] . . . 1544

3.187.1 Optimal result

Integrand size = 23, antiderivative size = 187

$$\begin{aligned}\int \sqrt{\sec(c+dx)}(a+a\sec(c+dx))^4\,dx &= -\frac{64a^4\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} \\ &\quad+\frac{136a^4\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{21d} \\ &\quad+\frac{64a^4\sqrt{\sec(c+dx)}\sin(c+dx)}{5d}+\frac{94a^4\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{21d} \\ &\quad+\frac{8a^4\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d}+\frac{2a^4\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{7d}\end{aligned}$$

output
```
94/21*a^4*sec(d*x+c)^(3/2)*sin(d*x+c)/d+8/5*a^4*sec(d*x+c)^(5/2)*sin(d*x+c
)/d+2/7*a^4*sec(d*x+c)^(7/2)*sin(d*x+c)/d+64/5*a^4*sin(d*x+c)*sec(d*x+c)^(
1/2)/d-64/5*a^4*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(
sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+136/21*a^4
*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2
*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

3.187.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 5.84 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.49

$$\int \sqrt{\sec(c+dx)}(a+a\sec(c+dx))^4\,dx$$
$$= \frac{a^4\sec^8\left(\frac{1}{2}(c+dx)\right)(1+\sec(c+dx))^4\left(-\frac{4i\sqrt{2}e^{-i(c+dx)}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\cos^4(c+dx)\left(168\left(1+e^{2i(c+dx)}\right)+168\left(-1+e^{2ic}\right)\sqrt{1+e^{2i(}}\right.}{}\right.}{}$$

input
```
Integrate[Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^4,x]
```

output
```
(a^4*Sec[(c + d*x)/2]^8*(1 + Sec[c + d*x])^4*(((-4*I)*Sqrt[2]*Sqrt[E^(I*(c
 + d*x))/(1 + E^((2*I)*(c + d*x)))]*Cos[c + d*x]^4*(168*(1 + E^((2*I)*(c +
 d*x))) + 168*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeomet
ric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))] + 85*E^(I*(c + d*x))*(-1 + E^
((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4,
-E^((2*I)*(c + d*x))]))/(E^(I*(c + d*x))*(-1 + E^((2*I)*c))) + (672*Cos[d*
x]*Csc[c] + (235 + 84*Sec[c + d*x] + 15*Sec[c + d*x]^2)*Tan[c + d*x])/Sec[
c + d*x]^(7/2)))/(840*d)
```

3.187.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3042, 4278, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^4\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^4dx$$
$$\Big\downarrow \; 4278$$

$$\int \left(a^4 \sec^{\frac{9}{2}}(c+dx)+4a^4 \sec^{\frac{7}{2}}(c+dx)+6a^4 \sec^{\frac{5}{2}}(c+dx)+4a^4 \sec^{\frac{3}{2}}(c+dx)+a^4\sqrt{\sec(c+dx)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2a^4 \sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{7d}+\frac{8a^4 \sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}+\frac{94a^4 \sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{21d}+\frac{64a^4 \sin(c+dx)\sqrt{\sec(c+dx)}}{5d}+\frac{136a^4\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d}-\frac{64a^4\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}$$

input
```
Int[Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^4,x]
```

output
```
(-64*a^4*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/
(5*d) + (136*a^4*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c +
 d*x]])/(21*d) + (64*a^4*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(5*d) + (94*a^4*
Sec[c + d*x]^(3/2)*Sin[c + d*x])/(21*d) + (8*a^4*Sec[c + d*x]^(5/2)*Sin[c
+ d*x])/(5*d) + (2*a^4*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(7*d)
```

3.187.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4278
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

3.187.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 438 vs. $2(211) = 422$.

Time = 38.79 (sec) , antiderivative size = 439, normalized size of antiderivative = 2.35

method	result
default	$-\frac{a^4\sqrt{-\left(-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(\frac{2024\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1}\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{105\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}}-\frac{47\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-}}{21\left(\mathrm{co}\right.}\right.}{}$
parts	Expression too large to display

input
```
int(sec(d*x+c)^(1/2)*(a+a*sec(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
-a^4*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2024/105*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*
d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1
/2))-47/21*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^
2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2-128/5*sin(1/2*d*x+1/2*c)^2*cos(1/2*d
*x+1/2*c)/(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)-64/5*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*
d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(
1/2))-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))-2/5*cos(1/2*d*x+1/2*c)*(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^
3-1/28*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(
1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^4)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c
)^2-1)^(1/2)/d
```

3.187.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.15

$$\int \sqrt{\sec(c+dx)}(a+a\sec(c+dx))^4\,dx =$$

$$-\frac{2\left(170i\sqrt{2}a^4\cos(dx+c)^3\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))-170i\sqrt{2}a^4\cos(d\right.}{}$$

input
```
integrate(sec(d*x+c)^(1/2)*(a+a*sec(d*x+c))^4,x, algorithm="fricas")
```

output
```
-2/105*(170*I*sqrt(2)*a^4*cos(d*x + c)^3*weierstrassPInverse(-4, 0, cos(d*
x + c) + I*sin(d*x + c)) - 170*I*sqrt(2)*a^4*cos(d*x + c)^3*weierstrassPIn
verse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 336*I*sqrt(2)*a^4*cos(d*x +
c)^3*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*si
n(d*x + c))) - 336*I*sqrt(2)*a^4*cos(d*x + c)^3*weierstrassZeta(-4, 0, wei
erstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - (672*a^4*cos(d*x
 + c)^3 + 235*a^4*cos(d*x + c)^2 + 84*a^4*cos(d*x + c) + 15*a^4)*sin(d*x +
 c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c)^3)
```

3.187.6 Sympy [F(-1)]

Timed out.

$$\int \sqrt{\sec(c+dx)}(a+a\sec(c+dx))^4\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**(1/2)*(a+a*sec(d*x+c))**4,x)
```

output
```
Timed out
```

3.187.7 Maxima [F]

$$\int \sqrt{\sec(c+dx)}(a+a\sec(c+dx))^4\,dx = \int (a\sec(dx+c)+a)^4\sqrt{\sec(dx+c)}\,dx$$

input
```
integrate(sec(d*x+c)^(1/2)*(a+a*sec(d*x+c))^4,x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)^4*sqrt(sec(d*x + c)), x)
```

3.187.8 Giac [F]

$$\int \sqrt{\sec(c+dx)}(a+a\sec(c+dx))^4\,dx = \int (a\sec(dx+c)+a)^4\sqrt{\sec(dx+c)}\,dx$$

input
```
integrate(sec(d*x+c)^(1/2)*(a+a*sec(d*x+c))^4,x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)^4*sqrt(sec(d*x + c)), x)
```

3.187.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{\sec(c+dx)}(a+a\sec(c+dx))^4\,dx = \int \left(a+\frac{a}{\cos(c+d\,x)}\right)^4\sqrt{\frac{1}{\cos(c+d\,x)}}\,dx$$

input
```
int((a + a/cos(c + d*x))^4*(1/cos(c + d*x))^(1/2),x)
```

output
```
int((a + a/cos(c + d*x))^4*(1/cos(c + d*x))^(1/2), x)
```

3.188 $\int \frac{(a+a\sec(c+dx))^4}{\sqrt{\sec(c+dx)}}\,dx$

3.188.1 Optimal result . . . 1545
3.188.2 Mathematica [C] (verified) . . . 1546
3.188.3 Rubi [A] (verified) . . . 1546
3.188.4 Maple [B] (verified) . . . 1548
3.188.5 Fricas [C] (verification not implemented) . . . 1548
3.188.6 Sympy [F(-1)] . . . 1549
3.188.7 Maxima [F] . . . 1549
3.188.8 Giac [F] . . . 1550
3.188.9 Mupad [F(-1)] . . . 1550

3.188.1 Optimal result

Integrand size = 23, antiderivative size = 161

$$\begin{aligned}\int \frac{(a+a\sec(c+dx))^4}{\sqrt{\sec(c+dx)}}\,dx = &-\frac{56a^4\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d}\\ &+\frac{32a^4\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{3d}\\ &+\frac{66a^4\sqrt{\sec(c+dx)}\sin(c+dx)}{5d}\\ &+\frac{8a^4\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d}+\frac{2a^4\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d}\end{aligned}$$

output
```
8/3*a^4*sec(d*x+c)^(3/2)*sin(d*x+c)/d+2/5*a^4*sec(d*x+c)^(5/2)*sin(d*x+c)/
d+66/5*a^4*sin(d*x+c)*sec(d*x+c)^(1/2)/d-56/5*a^4*(cos(1/2*d*x+1/2*c)^2)^(
1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(
1/2)*sec(d*x+c)^(1/2)/d+32/3*a^4*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+
1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(
1/2)/d
```

3.188.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 6.15 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.78

$$\int \frac{(a+a\sec(c+dx))^4}{\sqrt{\sec(c+dx)}}\,dx$$
$$= \frac{a^4\sec^8\left(\frac{1}{2}(c+dx)\right)(1+\sec(c+dx))^4\left(-\frac{8i\sqrt{2}e^{-i(c+dx)}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\cos^4(c+dx)\left(21\left(1+e^{2i(c+dx)}\right)+21\left(-1+e^{2ic}\right)\sqrt{1+e^{2i(c+}}\right.}{}\right.}{}$$

input
```
Integrate[(a + a*Sec[c + d*x])^4/Sqrt[Sec[c + d*x]],x]
```

output
```
(a^4*Sec[(c + d*x)/2]^8*(1 + Sec[c + d*x])^4*(((-8*I)*Sqrt[2]*Sqrt[E^(I*(c
 + d*x))/(1 + E^((2*I)*(c + d*x)))]*Cos[c + d*x]^4*(21*(1 + E^((2*I)*(c +
d*x))) + 21*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometri
c2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))] + 20*E^(I*(c + d*x))*(-1 + E^((
2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E
^((2*I)*(c + d*x))]))/(E^(I*(c + d*x))*(-1 + E^((2*I)*c))) + (-3*(-61 + 5*
Cos[2*c])*Cos[d*x]*Csc[c] + 30*Cos[c]*Sin[d*x] + 2*(20 + 3*Sec[c + d*x])*T
an[c + d*x])/Sec[c + d*x]^(7/2)))/(240*d)
```

3.188.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3042, 4278, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sec(c+dx)+a)^4}{\sqrt{\sec(c+dx)}}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^4}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$
$$\Big\downarrow \; 4278$$

$$\int \left(a^4 \sec^{\frac{7}{2}}(c+dx)+4a^4 \sec^{\frac{5}{2}}(c+dx)+6a^4 \sec^{\frac{3}{2}}(c+dx)+4a^4\sqrt{\sec(c+dx)}+\frac{a^4}{\sqrt{\sec(c+dx)}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2a^4 \sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}+\frac{8a^4 \sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{66a^4 \sin(c+dx)\sqrt{\sec(c+dx)}}{5d}+\frac{32a^4\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}-\frac{56a^4\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}$$

input
```
Int[(a + a*Sec[c + d*x])^4/Sqrt[Sec[c + d*x]],x]
```

output
```
(-56*a^4*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/
(5*d) + (32*a^4*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c +
d*x]])/(3*d) + (66*a^4*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(5*d) + (8*a^4*Sec
[c + d*x]^(3/2)*Sin[c + d*x])/(3*d) + (2*a^4*Sec[c + d*x]^(5/2)*Sin[c + d*
x])/(5*d)
```

3.188.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4278
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

3.188.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 385 vs. $2(189) = 378$.

Time = 32.19 (sec) , antiderivative size = 386, normalized size of antiderivative = 2.40

method	result
default	$-\frac{a^4\sqrt{-\left(-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(\frac{328\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1}\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{15\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}}-\frac{56\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}}{}\right.}{}$
parts	Expression too large to display

input
```
int((a+a*sec(d*x+c))^4/sec(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-a^4*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(328/15*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*
x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2
))-56/5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2
*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1
/2*c),2^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))-132/5*sin(1/2*d*x+1/
2*c)^2*cos(1/2*d*x+1/2*c)/(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)
^2)^(1/2)-4/3*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*
c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2-1/10*cos(1/2*d*x+1/2*c)*(-2*sin(1
/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^3)/
sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

3.188.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.25

$$\int \frac{(a+a\sec(c+dx))^4}{\sqrt{\sec(c+dx)}}\,dx =$$
$$-\frac{2\left(40i\sqrt{2}a^4\cos(dx+c)^2\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))-40i\sqrt{2}a^4\cos(dx+\right.}{}$$

input
```
integrate((a+a*sec(d*x+c))^4/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
-2/15*(40*I*sqrt(2)*a^4*cos(d*x + c)^2*weierstrassPInverse(-4, 0, cos(d*x
+ c) + I*sin(d*x + c)) - 40*I*sqrt(2)*a^4*cos(d*x + c)^2*weierstrassPInver
se(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 42*I*sqrt(2)*a^4*cos(d*x + c)^2
*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*
x + c))) - 42*I*sqrt(2)*a^4*cos(d*x + c)^2*weierstrassZeta(-4, 0, weierstr
assPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - (99*a^4*cos(d*x + c)^
2 + 20*a^4*cos(d*x + c) + 3*a^4)*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d
*x + c)^2)
```

3.188.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(c+dx))^4}{\sqrt{\sec(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sec(d*x+c))**4/sec(d*x+c)**(1/2),x)
```

output
```
Timed out
```

3.188.7 Maxima [F]

$$\int \frac{(a+a\sec(c+dx))^4}{\sqrt{\sec(c+dx)}}\,dx = \int \frac{(a\sec(dx+c)+a)^4}{\sqrt{\sec(dx+c)}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))^4/sec(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)^4/sqrt(sec(d*x + c)), x)
```

3.188.8 Giac [F]

$$\int \frac{(a+a\sec(c+dx))^4}{\sqrt{\sec(c+dx)}}\,dx=\int \frac{(a\sec(dx+c)+a)^4}{\sqrt{\sec(dx+c)}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))^4/sec(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)^4/sqrt(sec(d*x + c)), x)
```

3.188.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(c+dx))^4}{\sqrt{\sec(c+dx)}}\,dx=\int \frac{\left(a+\frac{a}{\cos(c+d\,x)}\right)^4}{\sqrt{\frac{1}{\cos(c+d\,x)}}}\,dx$$

input
```
int((a + a/cos(c + d*x))^4/(1/cos(c + d*x))^(1/2),x)
```

output
```
int((a + a/cos(c + d*x))^4/(1/cos(c + d*x))^(1/2), x)
```

3.189 $\int \frac{(a+a\sec(c+dx))^4}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

3.189.1 Optimal result 1551
3.189.2 Mathematica [A] (verified) 1551
3.189.3 Rubi [A] (verified) 1552
3.189.4 Maple [B] (verified) 1553
3.189.5 Fricas [C] (verification not implemented) 1554
3.189.6 Sympy [F] 1554
3.189.7 Maxima [F] 1555
3.189.8 Giac [F] 1555
3.189.9 Mupad [F(-1)] 1555

3.189.1 Optimal result

Integrand size = 23, antiderivative size = 118

$$\int \frac{(a+a\sec(c+dx))^4}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \frac{40a^4\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{3d} + \frac{2a^4\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{8a^4\sqrt{\sec(c+dx)}\sin(c+dx)}{d} + \frac{2a^4\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d}$$

output
```
2/3*a^4*sec(d*x+c)^(3/2)*sin(d*x+c)/d+2/3*a^4*sin(d*x+c)/d/sec(d*x+c)^(1/2
)+8*a^4*sin(d*x+c)*sec(d*x+c)^(1/2)/d+40/3*a^4*(cos(1/2*d*x+1/2*c)^2)^(1/2
)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2
)*sec(d*x+c)^(1/2)/d
```

3.189.2 Mathematica [A] (verified)

Time = 2.97 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.59

$$\int \frac{(a+a\sec(c+dx))^4}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \frac{a^4\sec^{\frac{3}{2}}(c+dx)\left(80\cos^{\frac{3}{2}}(c+dx)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+5\sin(c+dx)+24\sin(2(c+dx))+\sin(3(c+\right.}{6d}$$

input
```
Integrate[(a + a*Sec[c + d*x])^4/Sec[c + d*x]^(3/2),x]
```

output
```
(a^4*Sec[c + d*x]^(3/2)*(80*Cos[c + d*x]^(3/2)*EllipticF[(c + d*x)/2, 2] +
 5*Sin[c + d*x] + 24*Sin[2*(c + d*x)] + Sin[3*(c + d*x)]))/(6*d)
```

3.189.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3042, 4278, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sec(c+dx)+a)^4}{\sec^{\frac{3}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^4}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{4278}$$

$$\int \left(a^4\sec^{\frac{5}{2}}(c+dx)+4a^4\sec^{\frac{3}{2}}(c+dx)+\frac{a^4}{\sec^{\frac{3}{2}}(c+dx)}+6a^4\sqrt{\sec(c+dx)}+\frac{4a^4}{\sqrt{\sec(c+dx)}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2a^4\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{8a^4\sin(c+dx)\sqrt{\sec(c+dx)}}{d}+\frac{2a^4\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{40a^4\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}$$

input
```
Int[(a + a*Sec[c + d*x])^4/Sec[c + d*x]^(3/2),x]
```

output
```
(40*a^4*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(
3*d) + (2*a^4*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]]) + (8*a^4*Sqrt[Sec[c +
 d*x]]*Sin[c + d*x])/d + (2*a^4*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d)
```

3.189.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4278
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

3.189.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 291 vs. $2(128) = 256$.

Time = 24.69 (sec) , antiderivative size = 292, normalized size of antiderivative = 2.47

method	result
default	$\frac{8a^4\sqrt{-\left(-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^6\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-14\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+10\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\sin}\right.}{3\left(4\sin\left(\right.\right.}$
parts	Expression too large to display

input
```
int((a+a*sec(d*x+c))^4/sec(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
8/3*a^4*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(4*sin(1
/2*d*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)/sin(1/2*d*x+1/2*c)^3*(2*sin(1/2*
d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-14*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)
+10*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elliptic
F(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2+7*cos(1/2*d*x+1/2*c)*si
n(1/2*d*x+1/2*c)^2-5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-
1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))*(-2*sin(1/2*d*x+1/2*c)^4+s
in(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

3.189.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.03

$$\int \frac{(a+a\sec(c+dx))^4}{\sec^{\frac{3}{2}}(c+dx)}\,dx = -\frac{2\left(10i\sqrt{2}a^4\cos(dx+c)\,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))-10i\sqrt{2}a^4\cos(dx+\right.}{3\,d\cos(d}$$

input
```
integrate((a+a*sec(d*x+c))^4/sec(d*x+c)^(3/2),x, algorithm="fricas")
```

output
```
-2/3*(10*I*sqrt(2)*a^4*cos(d*x + c)*weierstrassPInverse(-4, 0, cos(d*x + c
) + I*sin(d*x + c)) - 10*I*sqrt(2)*a^4*cos(d*x + c)*weierstrassPInverse(-4
, 0, cos(d*x + c) - I*sin(d*x + c)) - (a^4*cos(d*x + c)^2 + 12*a^4*cos(d*x
 + c) + a^4)*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c))
```

3.189.6 Sympy [F]

$$\int \frac{(a+a\sec(c+dx))^4}{\sec^{\frac{3}{2}}(c+dx)}\,dx = a^4\left(\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}\,dx + \int \frac{4}{\sqrt{\sec(c+dx)}}\,dx + \int 6\sqrt{\sec(c+dx)}\,dx + \int 4\sec^{\frac{3}{2}}(c+dx)\,dx + \int \sec^{\frac{5}{2}}(c+dx)\,dx\right)$$

input
```
integrate((a+a*sec(d*x+c))**4/sec(d*x+c)**(3/2),x)
```

output
```
a**4*(Integral(sec(c + d*x)**(-3/2), x) + Integral(4/sqrt(sec(c + d*x)), x
) + Integral(6*sqrt(sec(c + d*x)), x) + Integral(4*sec(c + d*x)**(3/2), x)
 + Integral(sec(c + d*x)**(5/2), x))
```

3.189.7 Maxima [F]

$$\int \frac{(a+a\sec(c+dx))^4}{\sec^{\frac{3}{2}}(c+dx)}\,dx=\int \frac{(a\sec(dx+c)+a)^4}{\sec(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))^4/sec(d*x+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)^4/sec(d*x + c)^(3/2), x)
```

3.189.8 Giac [F]

$$\int \frac{(a+a\sec(c+dx))^4}{\sec^{\frac{3}{2}}(c+dx)}\,dx=\int \frac{(a\sec(dx+c)+a)^4}{\sec(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))^4/sec(d*x+c)^(3/2),x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)^4/sec(d*x + c)^(3/2), x)
```

3.189.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(c+dx))^4}{\sec^{\frac{3}{2}}(c+dx)}\,dx=\int \frac{\left(a+\frac{a}{\cos(c+d\,x)}\right)^4}{\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

input
```
int((a + a/cos(c + d*x))^4/(1/cos(c + d*x))^(3/2),x)
```

output
```
int((a + a/cos(c + d*x))^4/(1/cos(c + d*x))^(3/2), x)
```

3.190 $\int \frac{(a+a\sec(c+dx))^4}{\sec^{\frac{5}{2}}(c+dx)}\,dx$

3.190.1 Optimal result 1556
3.190.2 Mathematica [C] (verified) 1557
3.190.3 Rubi [A] (verified) 1557
3.190.4 Maple [A] (verified) 1558
3.190.5 Fricas [C] (verification not implemented) 1559
3.190.6 Sympy [F] 1560
3.190.7 Maxima [F] 1560
3.190.8 Giac [F] 1560
3.190.9 Mupad [F(-1)] 1561

3.190.1 Optimal result

Integrand size = 23, antiderivative size = 159

$$\begin{aligned}\int \frac{(a+a\sec(c+dx))^4}{\sec^{\frac{5}{2}}(c+dx)}\,dx = {} & \frac{56a^4\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} \\ & + \frac{32a^4\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{3d} \\ & + \frac{2a^4\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{8a^4\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} \\ & + \frac{2a^4\sqrt{\sec(c+dx)}\sin(c+dx)}{d}\end{aligned}$$

output
```
2/5*a^4*sin(d*x+c)/d/sec(d*x+c)^(3/2)+8/3*a^4*sin(d*x+c)/d/sec(d*x+c)^(1/2
)+2*a^4*sin(d*x+c)*sec(d*x+c)^(1/2)/d+56/5*a^4*(cos(1/2*d*x+1/2*c)^2)^(1/2
)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2
)*sec(d*x+c)^(1/2)/d+32/3*a^4*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2
*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2
)/d
```

3.190.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 4.03 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.16

$$\int \frac{(a+a\sec(c+dx))^4}{\sec^{\frac{5}{2}}(c+dx)}\,dx$$
$$= \frac{a^4\left(\cos\left(\frac{c}{2}\right)-i\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{c}{2}\right)+i\sin\left(\frac{c}{2}\right)\right)\left(-336i+\frac{672i\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},-e^{2i(c+dx)}\right)}{\sqrt{1+e^{2i(c+dx)}}}-320i\sqrt{1+e^{2i(}}\right.}{}$$

input
```
Integrate[(a + a*Sec[c + d*x])^4/Sec[c + d*x]^(5/2),x]
```

output
```
(a^4*(Cos[c/2] - I*Sin[c/2])*(Cos[c/2] + I*Sin[c/2])*(-336*I + ((672*I)*Hy
pergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))])/Sqrt[1 + E^((2*I)*(
c + d*x))] - (320*I)*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/4,
1/2, 5/4, -E^((2*I)*(c + d*x))]*Sec[c + d*x] + 80*Sin[c + d*x] + 3*Sec[c +
 d*x]*Sin[3*(c + d*x)] + 63*Tan[c + d*x]))/(30*d*Sqrt[Sec[c + d*x]])
```

3.190.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3042, 4278, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sec(c+dx)+a)^4}{\sec^{\frac{5}{2}}(c+dx)}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^4}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx$$
$$\Big\downarrow \; 4278$$
$$\int \left(a^4\sec^{\frac{3}{2}}(c+dx)+\frac{4a^4}{\sec^{\frac{3}{2}}(c+dx)}+\frac{a^4}{\sec^{\frac{5}{2}}(c+dx)}+4a^4\sqrt{\sec(c+dx)}+\frac{6a^4}{\sqrt{\sec(c+dx)}}\right)dx$$
$$\Big\downarrow \; 2009$$

$$\frac{2a^4\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{2a^4\sin(c+dx)\sqrt{\sec(c+dx)}}{d}+\frac{8a^4\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{32a^4\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{56a^4\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}$$

input
```
Int[(a + a*Sec[c + d*x])^4/Sec[c + d*x]^(5/2),x]
```

output
```
(56*a^4*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(
5*d) + (32*a^4*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d
*x]])/(3*d) + (2*a^4*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)) + (8*a^4*Sin[c
 + d*x])/(3*d*Sqrt[Sec[c + d*x]]) + (2*a^4*Sqrt[Sec[c + d*x]]*Sin[c + d*x]
)/d
```

3.190.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4278
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

3.190.4 Maple [A] (verified)

Time = 27.48 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.22

method	result
default	$\frac{8a^4\left(6\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^6\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-26\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+19\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2-20\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\right.}{15\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-}}$
parts	Expression too large to display

input
```
int((a+a*sec(d*x+c))^4/sec(d*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
8/15*a^4*(6*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-26*sin(1/2*d*x+1/2*c)^
4*cos(1/2*d*x+1/2*c)+19*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-20*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d
*x+1/2*c),2^(1/2))+21*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1
/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2))/sin(1/2*d*x+1/2*c)/(2*cos
(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

3.190.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.02

$$\int \frac{(a+a\sec(c+dx))^4}{\sec^{\frac{5}{2}}(c+dx)}\,dx = -\frac{2\left(40i\sqrt{2}a^4\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))-40i\sqrt{2}a^4\text{weierstrassPInverse}(-\right.}{}$$

input
```
integrate((a+a*sec(d*x+c))^4/sec(d*x+c)^(5/2),x, algorithm="fricas")
```

output
```
-2/15*(40*I*sqrt(2)*a^4*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*
x + c)) - 40*I*sqrt(2)*a^4*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin
(d*x + c)) - 42*I*sqrt(2)*a^4*weierstrassZeta(-4, 0, weierstrassPInverse(-
4, 0, cos(d*x + c) + I*sin(d*x + c))) + 42*I*sqrt(2)*a^4*weierstrassZeta(-
4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - (3*a^4*
cos(d*x + c)^2 + 20*a^4*cos(d*x + c) + 15*a^4)*sin(d*x + c)/sqrt(cos(d*x +
 c)))/d
```

3.190.6 Sympy [F]

$$\int \frac{(a+a\sec(c+dx))^4}{\sec^{\frac{5}{2}}(c+dx)}\,dx = a^4\left(\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)}\,dx + \int \frac{4}{\sec^{\frac{3}{2}}(c+dx)}\,dx + \int \frac{6}{\sqrt{\sec(c+dx)}}\,dx + \int 4\sqrt{\sec(c+dx)}\,dx + \int \sec^{\frac{3}{2}}(c+dx)\,dx\right)$$

input
```
integrate((a+a*sec(d*x+c))**4/sec(d*x+c)**(5/2),x)
```

output
```
a**4*(Integral(sec(c + d*x)**(-5/2), x) + Integral(4/sec(c + d*x)**(3/2),
x) + Integral(6/sqrt(sec(c + d*x)), x) + Integral(4*sqrt(sec(c + d*x)), x)
 + Integral(sec(c + d*x)**(3/2), x))
```

3.190.7 Maxima [F]

$$\int \frac{(a+a\sec(c+dx))^4}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{(a\sec(dx+c)+a)^4}{\sec(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))^4/sec(d*x+c)^(5/2),x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)^4/sec(d*x + c)^(5/2), x)
```

3.190.8 Giac [F]

$$\int \frac{(a+a\sec(c+dx))^4}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{(a\sec(dx+c)+a)^4}{\sec(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))^4/sec(d*x+c)^(5/2),x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)^4/sec(d*x + c)^(5/2), x)
```

3.190.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(c+dx))^4}{\sec^{\frac{5}{2}}(c+dx)}\,dx=\int \frac{\left(a+\frac{a}{\cos(c+d\,x)}\right)^4}{\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

input
```
int((a + a/cos(c + d*x))^4/(1/cos(c + d*x))^(5/2),x)
```

output
```
int((a + a/cos(c + d*x))^4/(1/cos(c + d*x))^(5/2), x)
```

3.191 $\int \frac{(a+a\sec(c+dx))^4}{\sec^{\frac{7}{2}}(c+dx)}\,dx$

3.191.1 Optimal result . . . 1562
3.191.2 Mathematica [C] (verified) . . . 1563
3.191.3 Rubi [A] (verified) . . . 1563
3.191.4 Maple [A] (verified) . . . 1564
3.191.5 Fricas [C] (verification not implemented) . . . 1565
3.191.6 Sympy [F] . . . 1566
3.191.7 Maxima [F] . . . 1566
3.191.8 Giac [F] . . . 1566
3.191.9 Mupad [F(-1)] . . . 1567

3.191.1 Optimal result

Integrand size = 23, antiderivative size = 161

$$\int \frac{(a+a\sec(c+dx))^4}{\sec^{\frac{7}{2}}(c+dx)}\,dx = \frac{64a^4\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{136a^4\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{21d} + \frac{2a^4\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{8a^4\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{94a^4\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}$$

output
```
2/7*a^4*sin(d*x+c)/d/sec(d*x+c)^(5/2)+8/5*a^4*sin(d*x+c)/d/sec(d*x+c)^(3/2
)+94/21*a^4*sin(d*x+c)/d/sec(d*x+c)^(1/2)+64/5*a^4*(cos(1/2*d*x+1/2*c)^2)^
(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^
(1/2)*sec(d*x+c)^(1/2)/d+136/21*a^4*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d
*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c
)^(1/2)/d
```

3.191.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 3.93 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.12

$$\int \frac{(a+a\sec(c+dx))^4}{\sec^{\frac{7}{2}}(c+dx)}\,dx$$
$$= \frac{a^4\left(\cos\left(\frac{c}{2}\right)-i\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{c}{2}\right)+i\sin\left(\frac{c}{2}\right)\right)\left(-5376i+\frac{10752i\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},-e^{2i(c+dx)}\right)}{\sqrt{1+e^{2i(c+dx)}}}-2720i\sqrt{1+}\right.}{}$$

input
```
Integrate[(a + a*Sec[c + d*x])^4/Sec[c + d*x]^(7/2),x]
```

output
```
(a^4*(Cos[c/2] - I*Sin[c/2])*(Cos[c/2] + I*Sin[c/2])*(-5376*I + ((10752*I)
*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))])/Sqrt[1 + E^((2*I
)*(c + d*x))] - (2720*I)*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1
/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]*Sec[c + d*x] + 1910*Sin[c + d*x] + 336
*Sin[2*(c + d*x)] + 30*Sin[3*(c + d*x)]))/(420*d*Sqrt[Sec[c + d*x]])
```

3.191.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3042, 4278, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sec(c+dx)+a)^4}{\sec^{\frac{7}{2}}(c+dx)}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^4}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx$$
$$\Big\downarrow \; 4278$$
$$\int \left(\frac{6a^4}{\sec^{\frac{3}{2}}(c+dx)}+\frac{4a^4}{\sec^{\frac{5}{2}}(c+dx)}+\frac{a^4}{\sec^{\frac{7}{2}}(c+dx)}+a^4\sqrt{\sec(c+dx)}+\frac{4a^4}{\sqrt{\sec(c+dx)}}\right)dx$$
$$\Big\downarrow \; 2009$$

$$\frac{8a^4\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{2a^4\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+\frac{94a^4\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}+\frac{136a^4\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d}+\frac{64a^4\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}$$

input
```
Int[(a + a*Sec[c + d*x])^4/Sec[c + d*x]^(7/2),x]
```

output
```
(64*a^4*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(
5*d) + (136*a^4*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c +
d*x]])/(21*d) + (2*a^4*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/2)) + (8*a^4*Sin
[c + d*x])/(5*d*Sec[c + d*x]^(3/2)) + (94*a^4*Sin[c + d*x])/(21*d*Sqrt[Sec
[c + d*x]])
```

3.191.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4278
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

3.191.4 Maple [A] (verified)

Time = 25.63 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.69

method	result
default	$-\frac{8\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\,a^4\left(60\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^8-258\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^6\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+448\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4\cos\left(\frac{d}{2}\right.\right.}{105\sqrt{-2\sin}}$
parts	Expression too large to display

input
```
int((a+a*sec(d*x+c))^4/sec(d*x+c)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-8/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^4*(60*cos
(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8-258*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+
1/2*c)+448*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-167*cos(1/2*d*x+1/2*c)*
sin(1/2*d*x+1/2*c)^2+85*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)
^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-168*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^
(1/2))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/
2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

3.191.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.06

$$\int \frac{(a+a\sec(c+dx))^4}{\sec^{\frac{7}{2}}(c+dx)}\,dx = -\frac{2\left(170i\sqrt{2}a^4\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))-170i\sqrt{2}a^4\text{weierstrassPInverse}\right.}{}$$

input
```
integrate((a+a*sec(d*x+c))^4/sec(d*x+c)^(7/2),x, algorithm="fricas")
```

output
```
-2/105*(170*I*sqrt(2)*a^4*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(
d*x + c)) - 170*I*sqrt(2)*a^4*weierstrassPInverse(-4, 0, cos(d*x + c) - I*
sin(d*x + c)) - 336*I*sqrt(2)*a^4*weierstrassZeta(-4, 0, weierstrassPInver
se(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 336*I*sqrt(2)*a^4*weierstrassZ
eta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - (1
5*a^4*cos(d*x + c)^3 + 84*a^4*cos(d*x + c)^2 + 235*a^4*cos(d*x + c))*sin(d
*x + c)/sqrt(cos(d*x + c)))/d
```

3.191.6 Sympy [F]

$$\int \frac{(a+a\sec(c+dx))^4}{\sec^{\frac{7}{2}}(c+dx)}\,dx = a^4\left(\int \frac{1}{\sec^{\frac{7}{2}}(c+dx)}\,dx + \int \frac{4}{\sec^{\frac{5}{2}}(c+dx)}\,dx + \int \frac{6}{\sec^{\frac{3}{2}}(c+dx)}\,dx + \int \frac{4}{\sqrt{\sec(c+dx)}}\,dx + \int \sqrt{\sec(c+dx)}\,dx\right)$$

input
```
integrate((a+a*sec(d*x+c))**4/sec(d*x+c)**(7/2),x)
```

output
```
a**4*(Integral(sec(c + d*x)**(-7/2), x) + Integral(4/sec(c + d*x)**(5/2),
x) + Integral(6/sec(c + d*x)**(3/2), x) + Integral(4/sqrt(sec(c + d*x)), x
) + Integral(sqrt(sec(c + d*x)), x))
```

3.191.7 Maxima [F]

$$\int \frac{(a+a\sec(c+dx))^4}{\sec^{\frac{7}{2}}(c+dx)}\,dx = \int \frac{(a\sec(dx+c)+a)^4}{\sec(dx+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))^4/sec(d*x+c)^(7/2),x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)^4/sec(d*x + c)^(7/2), x)
```

3.191.8 Giac [F]

$$\int \frac{(a+a\sec(c+dx))^4}{\sec^{\frac{7}{2}}(c+dx)}\,dx = \int \frac{(a\sec(dx+c)+a)^4}{\sec(dx+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))^4/sec(d*x+c)^(7/2),x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)^4/sec(d*x + c)^(7/2), x)
```

3.191.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(c+dx))^4}{\sec^{\frac{7}{2}}(c+dx)}\,dx=\int \frac{\left(a+\frac{a}{\cos(c+d\,x)}\right)^4}{\left(\frac{1}{\cos(c+d\,x)}\right)^{7/2}}\,dx$$

input
```
int((a + a/cos(c + d*x))^4/(1/cos(c + d*x))^(7/2),x)
```

output
```
int((a + a/cos(c + d*x))^4/(1/cos(c + d*x))^(7/2), x)
```

3.192 $\int \frac{(a+a\sec(c+dx))^4}{\sec^{\frac{9}{2}}(c+dx)}\,dx$

3.192.1 Optimal result 1568
3.192.2 Mathematica [C] (verified) 1569
3.192.3 Rubi [A] (verified) 1569
3.192.4 Maple [A] (verified) 1570
3.192.5 Fricas [C] (verification not implemented) 1571
3.192.6 Sympy [F(-1)] 1572
3.192.7 Maxima [F] 1572
3.192.8 Giac [F] 1572
3.192.9 Mupad [F(-1)] 1573

3.192.1 Optimal result

Integrand size = 23, antiderivative size = 187

$$\begin{aligned}\int \frac{(a+a\sec(c+dx))^4}{\sec^{\frac{9}{2}}(c+dx)}\,dx &= \frac{152a^4\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{15d} \\ &\quad+\frac{32a^4\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{7d} \\ &\quad+\frac{2a^4\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)}+\frac{8a^4\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} \\ &\quad+\frac{122a^4\sin(c+dx)}{45d\sec^{\frac{3}{2}}(c+dx)}+\frac{32a^4\sin(c+dx)}{7d\sqrt{\sec(c+dx)}}\end{aligned}$$

output
```
2/9*a^4*sin(d*x+c)/d/sec(d*x+c)^(7/2)+8/7*a^4*sin(d*x+c)/d/sec(d*x+c)^(5/2
)+122/45*a^4*sin(d*x+c)/d/sec(d*x+c)^(3/2)+32/7*a^4*sin(d*x+c)/d/sec(d*x+c
)^(1/2)+152/15*a^4*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellipti
cE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+32/7*a^
4*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/
2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

3.192.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 4.22 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.83

$$\int \frac{(a+a\sec(c+dx))^4}{\sec^{\frac{9}{2}}(c+dx)}\,dx$$
$$= \frac{a^4\left(-25536i + \frac{51072i\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},-e^{2i(c+dx)}\right)}{\sqrt{1+e^{2i(c+dx)}}} - 11520i\sqrt{1+e^{2i(c+dx)}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},\right.\right.}{2520d\sqrt{}}$$

input
```
Integrate[(a + a*Sec[c + d*x])^4/Sec[c + d*x]^(9/2),x]
```

output
```
(a^4*(-25536*I + ((51072*I)*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c
 + d*x))])/Sqrt[1 + E^((2*I)*(c + d*x))] - (11520*I)*Sqrt[1 + E^((2*I)*(c
+ d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]*Sec[c + d*
x] + 12240*Sin[c + d*x] + 3556*Sin[2*(c + d*x)] + 720*Sin[3*(c + d*x)] + 7
0*Sin[4*(c + d*x)]))/(2520*d*Sqrt[Sec[c + d*x]])
```

3.192.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3042, 4278, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sec(c+dx)+a)^4}{\sec^{\frac{9}{2}}(c+dx)}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^4}{\csc\left(c+dx+\frac{\pi}{2}\right)^{9/2}}dx$$
$$\Big\downarrow \; 4278$$
$$\int \left(\frac{4a^4}{\sec^{\frac{3}{2}}(c+dx)}+\frac{6a^4}{\sec^{\frac{5}{2}}(c+dx)}+\frac{4a^4}{\sec^{\frac{7}{2}}(c+dx)}+\frac{a^4}{\sec^{\frac{9}{2}}(c+dx)}+\frac{a^4}{\sqrt{\sec(c+dx)}}\right)dx$$
$$\Big\downarrow \; 2009$$

$$\frac{122a^4\sin(c+dx)}{45d\sec^{\frac{3}{2}}(c+dx)}+\frac{8a^4\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+\frac{2a^4\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)}+\frac{32a^4\sin(c+dx)}{7d\sqrt{\sec(c+dx)}}+\frac{32a^4\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{7d}+\frac{152a^4\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}$$

input
```
Int[(a + a*Sec[c + d*x])^4/Sec[c + d*x]^(9/2),x]
```

output
```
(152*a^4*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/
(15*d) + (32*a^4*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c +
 d*x]])/(7*d) + (2*a^4*Sin[c + d*x])/(9*d*Sec[c + d*x]^(7/2)) + (8*a^4*Sin
[c + d*x])/(7*d*Sec[c + d*x]^(5/2)) + (122*a^4*Sin[c + d*x])/(45*d*Sec[c +
 d*x]^(3/2)) + (32*a^4*Sin[c + d*x])/(7*d*Sqrt[Sec[c + d*x]])
```

3.192.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4278
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

3.192.4 Maple [A] (verified)

Time = 37.33 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.39

method	result
default	$-\frac{8\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\,a^4\left(280\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{11}-120\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^9+34\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^7+72\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^5-485\cos\left(\frac{d}{2}\right.\right.}{315\sqrt{-2\sin\left(\frac{d}{2}\right.}}$
parts	Expression too large to display

input
```
int((a+a*sec(d*x+c))^4/sec(d*x+c)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
-8/315*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^4*(280*co
s(1/2*d*x+1/2*c)^11-120*cos(1/2*d*x+1/2*c)^9+34*cos(1/2*d*x+1/2*c)^7+72*co
s(1/2*d*x+1/2*c)^5-485*cos(1/2*d*x+1/2*c)^3+180*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))
-399*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*Ellipt
icE(cos(1/2*d*x+1/2*c),2^(1/2))+219*cos(1/2*d*x+1/2*c))/(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c
)^2-1)^(1/2)/d
```

3.192.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.98

$$\int \frac{(a+a\sec(c+dx))^4}{\sec^{\frac{9}{2}}(c+dx)}\,dx = -\frac{2\left(360i\sqrt{2}a^4\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))-360i\sqrt{2}a^4\text{weierstrassPInverse}\right.}{}$$

input
```
integrate((a+a*sec(d*x+c))^4/sec(d*x+c)^(9/2),x, algorithm="fricas")
```

output
```
-2/315*(360*I*sqrt(2)*a^4*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(
d*x + c)) - 360*I*sqrt(2)*a^4*weierstrassPInverse(-4, 0, cos(d*x + c) - I*
sin(d*x + c)) - 798*I*sqrt(2)*a^4*weierstrassZeta(-4, 0, weierstrassPInver
se(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 798*I*sqrt(2)*a^4*weierstrassZ
eta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - (3
5*a^4*cos(d*x + c)^4 + 180*a^4*cos(d*x + c)^3 + 427*a^4*cos(d*x + c)^2 + 7
20*a^4*cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/d
```

3.192.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(c+dx))^4}{\sec^{\frac{9}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sec(d*x+c))**4/sec(d*x+c)**(9/2),x)
```

output
```
Timed out
```

3.192.7 Maxima [F]

$$\int \frac{(a+a\sec(c+dx))^4}{\sec^{\frac{9}{2}}(c+dx)}\,dx = \int \frac{(a\sec(dx+c)+a)^4}{\sec(dx+c)^{\frac{9}{2}}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))^4/sec(d*x+c)^(9/2),x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)^4/sec(d*x + c)^(9/2), x)
```

3.192.8 Giac [F]

$$\int \frac{(a+a\sec(c+dx))^4}{\sec^{\frac{9}{2}}(c+dx)}\,dx = \int \frac{(a\sec(dx+c)+a)^4}{\sec(dx+c)^{\frac{9}{2}}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))^4/sec(d*x+c)^(9/2),x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)^4/sec(d*x + c)^(9/2), x)
```

3.192.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(c+dx))^4}{\sec^{\frac{9}{2}}(c+dx)}\,dx=\int \frac{\left(a+\frac{a}{\cos(c+d\,x)}\right)^4}{\left(\frac{1}{\cos(c+d\,x)}\right)^{9/2}}\,dx$$

input
```
int((a + a/cos(c + d*x))^4/(1/cos(c + d*x))^(9/2),x)
```

output
```
int((a + a/cos(c + d*x))^4/(1/cos(c + d*x))^(9/2), x)
```

3.193 $\int \frac{(a+a\sec(c+dx))^4}{\sec^{\frac{11}{2}}(c+dx)}\,dx$

3.193.1 Optimal result 1574
3.193.2 Mathematica [C] (verified) 1575
3.193.3 Rubi [A] (verified) 1575
3.193.4 Maple [A] (verified) 1577
3.193.5 Fricas [C] (verification not implemented) 1577
3.193.6 Sympy [F(-1)] 1578
3.193.7 Maxima [F] 1578
3.193.8 Giac [F] 1579
3.193.9 Mupad [F(-1)] 1579

3.193.1 Optimal result

Integrand size = 23, antiderivative size = 213

$$\begin{aligned}\int \frac{(a+a\sec(c+dx))^4}{\sec^{\frac{11}{2}}(c+dx)}\,dx &= \frac{128a^4\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{15d} \\ &\quad+\frac{904a^4\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{231d} \\ &\quad+\frac{2a^4\sin(c+dx)}{11d\sec^{\frac{9}{2}}(c+dx)}+\frac{8a^4\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)}+\frac{150a^4\sin(c+dx)}{77d\sec^{\frac{5}{2}}(c+dx)} \\ &\quad+\frac{128a^4\sin(c+dx)}{45d\sec^{\frac{3}{2}}(c+dx)}+\frac{904a^4\sin(c+dx)}{231d\sqrt{\sec(c+dx)}}\end{aligned}$$

output
```
2/11*a^4*sin(d*x+c)/d/sec(d*x+c)^(9/2)+8/9*a^4*sin(d*x+c)/d/sec(d*x+c)^(7/
2)+150/77*a^4*sin(d*x+c)/d/sec(d*x+c)^(5/2)+128/45*a^4*sin(d*x+c)/d/sec(d*
x+c)^(3/2)+904/231*a^4*sin(d*x+c)/d/sec(d*x+c)^(1/2)+128/15*a^4*(cos(1/2*d
*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)
)*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+904/231*a^4*(cos(1/2*d*x+1/2*c)^2)^(
1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(
1/2)*sec(d*x+c)^(1/2)/d
```

3.193.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 6.02 (sec) , antiderivative size = 306, normalized size of antiderivative = 1.44

$$\int \frac{(a+a\sec(c+dx))^4}{\sec^{\frac{11}{2}}(c+dx)}\,dx = -\frac{ia^4e^{-6i(c+dx)}\left(-315-3080e^{i(c+dx)}-14760e^{2i(c+dx)}-48664e^{3i(c+dx)}-137055e^{4i(c+dx)}+427504e^{5i(c+dx)}\right.}{}$$

input
```
Integrate[(a + a*Sec[c + d*x])^4/Sec[c + d*x]^(11/2),x]
```

output
```
((-1/1774080*I)*a^4*(-315 - 3080*E^(I*(c + d*x)) - 14760*E^((2*I)*(c + d*x
)) - 48664*E^((3*I)*(c + d*x)) - 137055*E^((4*I)*(c + d*x)) + 427504*E^((5
*I)*(c + d*x)) + 518672*E^((7*I)*(c + d*x)) + 137055*E^((8*I)*(c + d*x)) +
 48664*E^((9*I)*(c + d*x)) + 14760*E^((10*I)*(c + d*x)) + 3080*E^((11*I)*(
c + d*x)) + 315*E^((12*I)*(c + d*x)) - 946176*E^((5*I)*(c + d*x))*Sqrt[1 +
 E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x
))] + 433920*E^((6*I)*(c + d*x))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeomet
ric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))])*Sec[(c + d*x)/2]^8*(1 + Sec[c
 + d*x])^4)/(d*E^((6*I)*(c + d*x))*Sec[c + d*x]^(7/2))
```

3.193.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3042, 4278, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sec(c+dx)+a)^4}{\sec^{\frac{11}{2}}(c+dx)}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^4}{\csc\left(c+dx+\frac{\pi}{2}\right)^{11/2}}dx$$

$$\Big\downarrow \; 4278$$

$$\int \left(\frac{a^4}{\sec^{\frac{3}{2}}(c+dx)}+\frac{4a^4}{\sec^{\frac{5}{2}}(c+dx)}+\frac{6a^4}{\sec^{\frac{7}{2}}(c+dx)}+\frac{4a^4}{\sec^{\frac{9}{2}}(c+dx)}+\frac{a^4}{\sec^{\frac{11}{2}}(c+dx)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{128a^4\sin(c+dx)}{45d\sec^{\frac{3}{2}}(c+dx)}+\frac{150a^4\sin(c+dx)}{77d\sec^{\frac{5}{2}}(c+dx)}+\frac{8a^4\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)}+\frac{2a^4\sin(c+dx)}{11d\sec^{\frac{9}{2}}(c+dx)}+\frac{904a^4\sin(c+dx)}{231d\sqrt{\sec(c+dx)}}+\frac{904a^4\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{231d}+\frac{128a^4\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}$$

input
```
Int[(a + a*Sec[c + d*x])^4/Sec[c + d*x]^(11/2),x]
```

output
```
(128*a^4*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/
(15*d) + (904*a^4*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c
+ d*x]])/(231*d) + (2*a^4*Sin[c + d*x])/(11*d*Sec[c + d*x]^(9/2)) + (8*a^4
*Sin[c + d*x])/(9*d*Sec[c + d*x]^(7/2)) + (150*a^4*Sin[c + d*x])/(77*d*Sec
[c + d*x]^(5/2)) + (128*a^4*Sin[c + d*x])/(45*d*Sec[c + d*x]^(3/2)) + (904
*a^4*Sin[c + d*x])/(231*d*Sqrt[Sec[c + d*x]])
```

3.193.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4278
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

3.193.4 Maple [A] (verified)

Time = 45.27 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.28

method	result
default	$-\frac{8\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,a^{4}\left(5040\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{13}-5320\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{11}+1740\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{9}+326\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}+678\right.}{}$
parts	Expression too large to display

input
```
int((a+a*sec(d*x+c))^4/sec(d*x+c)^(11/2),x,method=_RETURNVERBOSE)
```

output
```
-8/3465*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^4*(5040*
cos(1/2*d*x+1/2*c)^13-5320*cos(1/2*d*x+1/2*c)^11+1740*cos(1/2*d*x+1/2*c)^9
+326*cos(1/2*d*x+1/2*c)^7+678*cos(1/2*d*x+1/2*c)^5-4465*cos(1/2*d*x+1/2*c)
^3+1695*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*Ell
ipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3696*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*c
os(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2001*co
s(1/2*d*x+1/2*c))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin
(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

3.193.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.92

$$\int \frac{(a+a\sec(c+dx))^4}{\sec^{\frac{11}{2}}(c+dx)}\,dx = -\frac{2\left(3390i\sqrt{2}a^4\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))-3390i\sqrt{2}a^4\text{weierstrassPInver}\right.}{}$$

input
```
integrate((a+a*sec(d*x+c))^4/sec(d*x+c)^(11/2),x, algorithm="fricas")
```

output
```
-2/3465*(3390*I*sqrt(2)*a^4*weierstrassPInverse(-4, 0, cos(d*x + c) + I*si
n(d*x + c)) - 3390*I*sqrt(2)*a^4*weierstrassPInverse(-4, 0, cos(d*x + c) -
 I*sin(d*x + c)) - 7392*I*sqrt(2)*a^4*weierstrassZeta(-4, 0, weierstrassPI
nverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 7392*I*sqrt(2)*a^4*weierst
rassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)))
 - (315*a^4*cos(d*x + c)^5 + 1540*a^4*cos(d*x + c)^4 + 3375*a^4*cos(d*x +
c)^3 + 4928*a^4*cos(d*x + c)^2 + 6780*a^4*cos(d*x + c))*sin(d*x + c)/sqrt(
cos(d*x + c)))/d
```

3.193.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(c+dx))^4}{\sec^{\frac{11}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sec(d*x+c))**4/sec(d*x+c)**(11/2),x)
```

output
```
Timed out
```

3.193.7 Maxima [F]

$$\int \frac{(a+a\sec(c+dx))^4}{\sec^{\frac{11}{2}}(c+dx)}\,dx = \int \frac{(a\sec(dx+c)+a)^4}{\sec(dx+c)^{\frac{11}{2}}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))^4/sec(d*x+c)^(11/2),x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)^4/sec(d*x + c)^(11/2), x)
```

3.193.8 Giac [F]

$$\int \frac{(a+a\sec(c+dx))^4}{\sec^{\frac{11}{2}}(c+dx)}\,dx=\int \frac{(a\sec(dx+c)+a)^4}{\sec(dx+c)^{\frac{11}{2}}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))^4/sec(d*x+c)^(11/2),x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)^4/sec(d*x + c)^(11/2), x)
```

3.193.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(c+dx))^4}{\sec^{\frac{11}{2}}(c+dx)}\,dx=\int \frac{\left(a+\frac{a}{\cos(c+d\,x)}\right)^4}{\left(\frac{1}{\cos(c+d\,x)}\right)^{11/2}}\,dx$$

input
```
int((a + a/cos(c + d*x))^4/(1/cos(c + d*x))^(11/2),x)
```

output
```
int((a + a/cos(c + d*x))^4/(1/cos(c + d*x))^(11/2), x)
```

3.194 $\int \frac{\sec^{\frac{7}{2}}(c+dx)}{a+a\sec(c+dx)}\,dx$

3.194.1 Optimal result 1580
3.194.2 Mathematica [C] (verified) 1581
3.194.3 Rubi [A] (verified) 1581
3.194.4 Maple [B] (verified) 1584
3.194.5 Fricas [C] (verification not implemented) 1585
3.194.6 Sympy [F(-1)] 1586
3.194.7 Maxima [F] 1586
3.194.8 Giac [F] 1586
3.194.9 Mupad [F(-1)] 1587

3.194.1 Optimal result

Integrand size = 23, antiderivative size = 164

$$\begin{aligned}\int \frac{\sec^{\frac{7}{2}}(c+dx)}{a+a\sec(c+dx)}\,dx &= \frac{3\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{ad} \\ &\quad+\frac{5\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{3ad} \\ &\quad-\frac{3\sqrt{\sec(c+dx)}\sin(c+dx)}{ad} \\ &\quad+\frac{5\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3ad}-\frac{\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{d(a+a\sec(c+dx))}\end{aligned}$$

output
```
5/3*sec(d*x+c)^(3/2)*sin(d*x+c)/a/d-sec(d*x+c)^(5/2)*sin(d*x+c)/d/(a+a*sec
(d*x+c))-3*sin(d*x+c)*sec(d*x+c)^(1/2)/a/d+3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/
cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*
sec(d*x+c)^(1/2)/a/d+5/3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*E
llipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a/d
```

3.194.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 4.23 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.77

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\left(\frac{2i\sqrt{2}e^{-i(c+dx)}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\left(9\left(1+e^{2i(c+dx)}\right)+9\left(-1+e^{2ic}\right)\sqrt{1+e^{2i(c+dx)}}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2}\right.\right.}{-1+e^{2i}}\right.}{}$$

input
```
Integrate[Sec[c + d*x]^(7/2)/(a + a*Sec[c + d*x]),x]
```

output
```
(Cos[(c + d*x)/2]^2*Sec[c + d*x]*(((2*I)*Sqrt[2]*Sqrt[E^(I*(c + d*x))/(1 +
 E^((2*I)*(c + d*x)))]*(9*(1 + E^((2*I)*(c + d*x))) + 9*(-1 + E^((2*I)*c))
*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)
*(c + d*x))] - 5*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c +
 d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]))/(E^(I*(c
+ d*x))*(-1 + E^((2*I)*c))) - Sqrt[Sec[c + d*x]]*(18*Cos[d*x]*Csc[c] + Sec
[c + d*x]*(-5*Sec[(c + d*x)/2]*Sin[(3*(c + d*x))/2] + Tan[(c + d*x)/2]))))
/(3*a*d*(1 + Sec[c + d*x]))
```

3.194.3 Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.02, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.522$, Rules used = {3042, 4305, 27, 3042, 4274, 3042, 4255, 3042, 4258, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{a\sec(c+dx)+a}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}dx$$

$$\Big\downarrow \; 4305$$

$$-\frac{\int \frac{1}{2}\sec^{\frac{3}{2}}(c+dx)(3a-5a\sec(c+dx))dx}{a^2}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{d(a\sec(c+dx)+a)}$$

$\Big\downarrow$ 27

$$-\frac{\int \sec^{\frac{3}{2}}(c+dx)(3a-5a\sec(c+dx))dx}{2a^2}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{d(a\sec(c+dx)+a)}$$

$\Big\downarrow$ 3042

$$-\frac{\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(3a-5a\csc\left(c+dx+\frac{\pi}{2}\right)\right)dx}{2a^2}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{d(a\sec(c+dx)+a)}$$

$\Big\downarrow$ 4274

$$-\frac{3a\int \sec^{\frac{3}{2}}(c+dx)dx-5a\int \sec^{\frac{5}{2}}(c+dx)dx}{2a^2}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{d(a\sec(c+dx)+a)}$$

$\Big\downarrow$ 3042

$$-\frac{3a\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}dx-5a\int \csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}dx}{2a^2}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{d(a\sec(c+dx)+a)}$$

$\Big\downarrow$ 4255

$$-\frac{3a\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\int \frac{1}{\sqrt{\sec(c+dx)}}dx\right)-5a\left(\frac{1}{3}\int \sqrt{\sec(c+dx)}dx+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}\right)}{2a^2}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{d(a\sec(c+dx)+a)}$$

$\Big\downarrow$ 3042

$$-\frac{3a\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)-5a\left(\frac{1}{3}\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}\right)}{2a^2}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{d(a\sec(c+dx)+a)}$$

$\Big\downarrow$ 4258

$$-\frac{3a\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\cos(c+dx)}dx\right)-5a\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\right.}{2a^2}$$

$$\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{d(a\sec(c+dx)+a)}$$

$\Big\downarrow$ 3042

3.194. $\int \frac{\sec^{\frac{7}{2}}(c+dx)}{a+a\sec(c+dx)}\,dx$

$$-\frac{3a\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx\right)-5a\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+\right.}{2a^2}}{\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{d(a\sec(c+dx)+a)}}$$

$\Big\downarrow$ 3119

$$-\frac{3a\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|2\right)}{d}\right)-5a\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx+\frac{\pi}{2}}}\right.}{2a^2}}{\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{d(a\sec(c+dx)+a)}}$$

$\Big\downarrow$ 3120

$$-\frac{3a\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|2\right)}{d}\right)-5a\left(\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{3d}\right.}{2a^2}}{\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{d(a\sec(c+dx)+a)}}$$

input
```
Int[Sec[c + d*x]^(7/2)/(a + a*Sec[c + d*x]),x]
```

output
```
-((Sec[c + d*x]^(5/2)*Sin[c + d*x])/(d*(a + a*Sec[c + d*x]))) - (3*a*((-2*
Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (2*Sq
rt[Sec[c + d*x]]*Sin[c + d*x])/d) - 5*a*((2*Sqrt[Cos[c + d*x]]*EllipticF[(
c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (2*Sec[c + d*x]^(3/2)*Sin[c + d
*x])/(3*d)))/(2*a^2)
```

3.194.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.194. $\int \frac{\sec^{\frac{7}{2}}(c+dx)}{a+a\sec(c+dx)}\,dx$

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4305
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[d^2*Cot[e + f*x]*((d*Csc[e + f*x])^(n - 2)/(f*(a +
b*Csc[e + f*x]))), x] - Simp[d^2/(a*b)   Int[(d*Csc[e + f*x])^(n - 2)*(b*(n
 - 2) - a*(n - 1)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ
[a^2 - b^2, 0] && GtQ[n, 1]
```

3.194.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 412 vs. $2(198) = 396$.

Time = 10.85 (sec) , antiderivative size = 413, normalized size of antiderivative = 2.52

method	result
default	$\frac{\sqrt{-\left(-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(10\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\operatorname{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sqrt{2\sin\left(\frac{dx}{2}\right.}\right.}{}$

input
```
int(sec(d*x+c)^(7/2)/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/3*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/a/sin(1/2*d*
x+1/2*c)^3/cos(1/2*d*x+1/2*c)/(4*sin(1/2*d*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)
^2+1)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(10*cos(1/2*d*x
+1/2*c)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(
1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2-18*cos(1/2*d*x+1/2*
c)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2-36*sin(1/2*d*x+1/2*c)^6-
5*cos(1/2*d*x+1/2*c)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+
1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)+9*cos(1/2*d*x+1/2*c)*Ellipt
icE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*
d*x+1/2*c)^2)^(1/2)+44*sin(1/2*d*x+1/2*c)^4-11*sin(1/2*d*x+1/2*c)^2)/(2*co
s(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

3.194.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.51

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{a+a\sec(c+dx)}\,dx =$$

$$-\frac{5\left(i\sqrt{2}\cos\left(dx+c\right)^2+i\sqrt{2}\cos\left(dx+c\right)\right)\text{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right))+5\,(}{}$$

input
```
integrate(sec(d*x+c)^(7/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

output
```
-1/6*(5*(I*sqrt(2)*cos(d*x + c)^2 + I*sqrt(2)*cos(d*x + c))*weierstrassPIn
verse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 5*(-I*sqrt(2)*cos(d*x + c)^2
 - I*sqrt(2)*cos(d*x + c))*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin
(d*x + c)) + 9*(-I*sqrt(2)*cos(d*x + c)^2 - I*sqrt(2)*cos(d*x + c))*weiers
trassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))
) + 9*(I*sqrt(2)*cos(d*x + c)^2 + I*sqrt(2)*cos(d*x + c))*weierstrassZeta(
-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*(9*c
os(d*x + c)^2 + 4*cos(d*x + c) - 2)*sin(d*x + c)/sqrt(cos(d*x + c)))/(a*d*
cos(d*x + c)^2 + a*d*cos(d*x + c))
```

3.194.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{a+a\sec(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**(7/2)/(a+a*sec(d*x+c)),x)
```

output
```
Timed out
```

3.194.7 Maxima [F]

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{a+a\sec(c+dx)}\,dx = \int \frac{\sec(dx+c)^{\frac{7}{2}}}{a\sec(dx+c)+a}\,dx$$

input
```
integrate(sec(d*x+c)^(7/2)/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)^(7/2)/(a*sec(d*x + c) + a), x)
```

3.194.8 Giac [F]

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{a+a\sec(c+dx)}\,dx = \int \frac{\sec(dx+c)^{\frac{7}{2}}}{a\sec(dx+c)+a}\,dx$$

input
```
integrate(sec(d*x+c)^(7/2)/(a+a*sec(d*x+c)),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^(7/2)/(a*sec(d*x + c) + a), x)
```

3.194.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{a+a\sec(c+dx)}\,dx=\int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{7/2}}{a+\frac{a}{\cos(c+d\,x)}}\,dx$$

input
```
int((1/cos(c + d*x))^(7/2)/(a + a/cos(c + d*x)),x)
```

output
```
int((1/cos(c + d*x))^(7/2)/(a + a/cos(c + d*x)), x)
```

3.195 $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{a+a\sec(c+dx)}\,dx$

3.195.1 Optimal result 1588
3.195.2 Mathematica [C] (verified) 1588
3.195.3 Rubi [A] (verified) 1589
3.195.4 Maple [A] (verified) 1592
3.195.5 Fricas [C] (verification not implemented) 1593
3.195.6 Sympy [F] 1593
3.195.7 Maxima [F] 1594
3.195.8 Giac [F] 1594
3.195.9 Mupad [F(-1)] 1594

3.195.1 Optimal result

Integrand size = 23, antiderivative size = 136

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{3\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{ad} - \frac{\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{ad} + \frac{3\sqrt{\sec(c+dx)}\sin(c+dx)}{ad} - \frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d(a+a\sec(c+dx))}$$

output
```
-sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))+3*sin(d*x+c)*sec(d*x+c)^(1
/2)/a/d-3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/
2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a/d-(cos(1/2*d*x+1
/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*co
s(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a/d
```

3.195.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 2.60 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.93

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{a+a\sec(c+dx)}\,dx$$
$$= \frac{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\left(-\frac{2i\sqrt{2}e^{-i(c+dx)}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\left(3\left(1+e^{2i(c+dx)}\right)+3\left(-1+e^{2ic}\right)\sqrt{1+e^{2i(c+dx)}}\,\text{Hypergeometric2F1}(-\right.}{d(-1+}\right.}{a(1+}$$

input
```
Integrate[Sec[c + d*x]^(5/2)/(a + a*Sec[c + d*x]),x]
```

output
```
(Cos[(c + d*x)/2]^2*Sec[c + d*x]*(((-2*I)*Sqrt[2]*Sqrt[E^(I*(c + d*x))/(1
+ E^((2*I)*(c + d*x)))]*(3*(1 + E^((2*I)*(c + d*x))) + 3*(-1 + E^((2*I)*c)
)*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I
)*(c + d*x))] - E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c +
d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]))/(d*E^(I*(c
 + d*x))*(-1 + E^((2*I)*c))) + (Sqrt[Sec[c + d*x]]*(6*Cos[d*x]*Csc[c] - 2*
Tan[(c + d*x)/2]))/d))/(a*(1 + Sec[c + d*x]))
```

3.195.3 Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.03, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.522$, Rules used = {3042, 4305, 27, 3042, 4274, 3042, 4255, 3042, 4258, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{a\sec(c+dx)+a}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}dx$$
$$\Big\downarrow \; 4305$$
$$-\frac{\int \frac{1}{2}\sqrt{\sec(c+dx)}(a-3a\sec(c+dx))dx}{a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d(a\sec(c+dx)+a)}$$
$$\Big\downarrow \; 27$$

$$-\frac{\int \sqrt{\sec(c+dx)}(a-3a\sec(c+dx))dx}{2a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d(a\sec(c+dx)+a)}$$

$\Big\downarrow$ 3042

$$-\frac{\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a-3a\csc\left(c+dx+\frac{\pi}{2}\right)\right)dx}{2a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d(a\sec(c+dx)+a)}$$

$\Big\downarrow$ 4274

$$-\frac{a\int \sqrt{\sec(c+dx)}dx-3a\int \sec^{\frac{3}{2}}(c+dx)dx}{2a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d(a\sec(c+dx)+a)}$$

$\Big\downarrow$ 3042

$$-\frac{a\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx-3a\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}dx}{2a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d(a\sec(c+dx)+a)}$$

$\Big\downarrow$ 4255

$$-\frac{a\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx-3a\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\int \frac{1}{\sqrt{\sec(c+dx)}}dx\right)}{2a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d(a\sec(c+dx)+a)}$$

$\Big\downarrow$ 3042

$$-\frac{a\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx-3a\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)}{2a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d(a\sec(c+dx)+a)}$$

$\Big\downarrow$ 4258

$$-\frac{a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx-3a\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\cos(c-}\right.}{2a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d(a\sec(c+dx)+a)}$$

$\Big\downarrow$ 3042

$$-\frac{a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-3a\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\sin}\right.}{2a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d(a\sec(c+dx)+a)}$$

$\Big\downarrow$ 3119

3.195. $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{a+a\sec(c+dx)}\,dx$

$$-\frac{a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-3a\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|2\right)}{d}\right)}{2a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d(a\sec(c+dx)+a)}$$

$$\Big\downarrow \; 3120$$

$$-\frac{\frac{2a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}-3a\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|2\right)}{d}\right)}{2a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d(a\sec(c+dx)+a)}$$

input
```
Int[Sec[c + d*x]^(5/2)/(a + a*Sec[c + d*x]),x]
```

output
```
-((Sec[c + d*x]^(3/2)*Sin[c + d*x])/(d*(a + a*Sec[c + d*x]))) - ((2*a*Sqrt
[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d - 3*a*((-2*
Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (2*Sq
rt[Sec[c + d*x]]*Sin[c + d*x])/d))/(2*a^2)
```

3.195.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4305
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[d^2*Cot[e + f*x]*((d*Csc[e + f*x])^(n - 2)/(f*(a +
b*Csc[e + f*x]))), x] - Simp[d^2/(a*b)   Int[(d*Csc[e + f*x])^(n - 2)*(b*(n
 - 2) - a*(n - 1)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ
[a^2 - b^2, 0] && GtQ[n, 1]
```

3.195.4 Maple [A] (verified)

Time = 7.78 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.86

method	result
default	$-\frac{-\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1}\left(\operatorname{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)-3\operatorname{EllipticE}\left(\right.\right.}{a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\sin\left(\frac{dx}{2}+\frac{c}{2}\right.}}$

input
```
int(sec(d*x+c)^(5/2)/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-(-cos(1/2*d*x+1/2*c)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(EllipticF(
cos(1/2*d*x+1/2*c),2^(1/2))-3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))+6*(-2
*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-5*(
-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2)/
a/cos(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/
sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

3.195.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.44

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\left(i\sqrt{2}\cos\left(dx+c\right)+i\sqrt{2}\right)\text{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right))+\left(-i\sqrt{2}\cos\left(dx+c\right)\right.}{}$$

input
```
integrate(sec(d*x+c)^(5/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

output
```
1/2*((I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierstrassPInverse(-4, 0, cos(d
*x + c) + I*sin(d*x + c)) + (-I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstr
assPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 3*(I*sqrt(2)*cos(d*x +
 c) + I*sqrt(2))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x
 + c) + I*sin(d*x + c))) - 3*(-I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierst
rassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)))
 + 2*(3*cos(d*x + c) + 2)*sin(d*x + c)/sqrt(cos(d*x + c)))/(a*d*cos(d*x +
c) + a*d)
```

3.195.6 Sympy [F]

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\int \frac{\sec^{\frac{5}{2}}(c+dx)}{\sec(c+dx)+1}\,dx}{a}$$

input
```
integrate(sec(d*x+c)**(5/2)/(a+a*sec(d*x+c)),x)
```

output
```
Integral(sec(c + d*x)**(5/2)/(sec(c + d*x) + 1), x)/a
```

3.195.7 Maxima [F]

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{a+a\sec(c+dx)}\,dx = \int \frac{\sec(dx+c)^{\frac{5}{2}}}{a\sec(dx+c)+a}\,dx$$

input
```
integrate(sec(d*x+c)^(5/2)/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)^(5/2)/(a*sec(d*x + c) + a), x)
```

3.195.8 Giac [F]

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{a+a\sec(c+dx)}\,dx = \int \frac{\sec(dx+c)^{\frac{5}{2}}}{a\sec(dx+c)+a}\,dx$$

input
```
integrate(sec(d*x+c)^(5/2)/(a+a*sec(d*x+c)),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^(5/2)/(a*sec(d*x + c) + a), x)
```

3.195.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{a+a\sec(c+dx)}\,dx = \int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}}{a+\frac{a}{\cos(c+d\,x)}}\,dx$$

input
```
int((1/cos(c + d*x))^(5/2)/(a + a/cos(c + d*x)),x)
```

output
```
int((1/cos(c + d*x))^(5/2)/(a + a/cos(c + d*x)), x)
```

3.196 $\int \frac{\sec^{\frac{3}{2}}(c+dx)}{a+a\sec(c+dx)}\,dx$

3.196.1 Optimal result 1595
3.196.2 Mathematica [C] (verified) 1595
3.196.3 Rubi [A] (verified) 1596
3.196.4 Maple [A] (verified) 1599
3.196.5 Fricas [C] (verification not implemented) 1599
3.196.6 Sympy [F] 1600
3.196.7 Maxima [F] 1600
3.196.8 Giac [F] 1600
3.196.9 Mupad [F(-1)] 1601

3.196.1 Optimal result

Integrand size = 23, antiderivative size = 110

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{ad} + \frac{\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{ad} - \frac{\sqrt{\sec(c+dx)}\sin(c+dx)}{d(a+a\sec(c+dx))}$$

output
```
-sin(d*x+c)*sec(d*x+c)^(1/2)/d/(a+a*sec(d*x+c))+(cos(1/2*d*x+1/2*c)^2)^(1/
2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/
2)*sec(d*x+c)^(1/2)/a/d+(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*El
lipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a/d
```

3.196.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.42 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.83

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{2ie^{-i(c+dx)}\cos^2\left(\frac{1}{2}(c+dx)\right)\left(1+e^{2i(c+dx)}-\left(1+e^{i(c+dx)}\right)\sqrt{1+e^{2i(c+dx)}}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},\right.\right.}{ad\left(1+e^{i(c+dx)}\right)}$$

input
```
Integrate[Sec[c + d*x]^(3/2)/(a + a*Sec[c + d*x]),x]
```

output
```
((-2*I)*Cos[(c + d*x)/2]^2*(1 + E^((2*I)*(c + d*x)) - (1 + E^(I*(c + d*x))
)*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I
)*(c + d*x))] + E^(I*(c + d*x))*(1 + E^(I*(c + d*x)))*Sqrt[1 + E^((2*I)*(c
 + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))])*Sec[c +
d*x]^(3/2))/(a*d*E^(I*(c + d*x))*(1 + E^(I*(c + d*x)))*(1 + Sec[c + d*x]))
```

3.196.3 Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.05, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {3042, 4305, 27, 3042, 4274, 3042, 4258, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{a\sec(c+dx)+a}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}dx$$

$$\Big\downarrow \; 4305$$

$$-\frac{\int -\frac{\sec(c+dx)a+a}{2\sqrt{\sec(c+dx)}}dx}{a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{\sec(c+dx)a+a}{\sqrt{\sec(c+dx)}}dx}{2a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}$$

$$\Big\downarrow \; 3042$$

$$\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{2a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}$$

$$\Big\downarrow \; 4274$$

$$\frac{a\int \frac{1}{\sqrt{\sec(c+dx)}}dx+a\int \sqrt{\sec(c+dx)}dx}{2a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+a\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{2a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}$$

$$\Big\downarrow \text{4258}$$

$$\frac{a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx+a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\cos(c+dx)}dx}{2a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{2a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}$$

$$\Big\downarrow \text{3119}$$

$$\frac{a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|2\right)}{d}}{2a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}$$

$$\Big\downarrow \text{3120}$$

$$\frac{\frac{2a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+\frac{2a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|2\right)}{d}}{2a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}$$

input
```
Int[Sec[c + d*x]^(3/2)/(a + a*Sec[c + d*x]),x]
```

output
```
((2*a*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d +
 (2*a*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d)/
(2*a^2) - (Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(d*(a + a*Sec[c + d*x]))
```

3.196. $\int \frac{\sec^{\frac{3}{2}}(c+dx)}{a+a\sec(c+dx)}\,dx$

3.196.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4305
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[d^2*Cot[e + f*x]*((d*Csc[e + f*x])^(n - 2)/(f*(a +
b*Csc[e + f*x]))), x] - Simp[d^2/(a*b)   Int[(d*Csc[e + f*x])^(n - 2)*(b*(n
 - 2) - a*(n - 1)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ
[a^2 - b^2, 0] && GtQ[n, 1]
```

3.196. $\int \frac{\sec^{\frac{3}{2}}(c+dx)}{a+a\sec(c+dx)}\,dx$

3.196.4 Maple [A] (verified)

Time = 3.94 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.82

method	result
default	$\frac{\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(-\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1}\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\left(\text{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)-\text{EllipticE}\right.\right.}{a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1}}$

input
```
int(sec(d*x+c)^(3/2)/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-cos(1/2*d*x+1/2*
c)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(Elliptic
F(cos(1/2*d*x+1/2*c),2^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))+2*sin
(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2)/a/cos(1/2*d*x+1/2*c)/(-2*sin(1/2*d
*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+
1/2*c)^2-1)^(1/2)/d
```

3.196.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.67

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{a+a\sec(c+dx)}\,dx$$
$$= \frac{\left(-i\sqrt{2}\cos\left(dx+c\right)-i\sqrt{2}\right)\text{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right))+\left(i\sqrt{2}\cos\left(dx+c\right)\right.}{}$$

input
```
integrate(sec(d*x+c)^(3/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

output
```
1/2*((-I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassPInverse(-4, 0, cos(
d*x + c) + I*sin(d*x + c)) + (I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierstr
assPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + (I*sqrt(2)*cos(d*x + c
) + I*sqrt(2))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x +
 c) + I*sin(d*x + c))) + (-I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrass
Zeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - 2
*sqrt(cos(d*x + c))*sin(d*x + c))/(a*d*cos(d*x + c) + a*d)
```

3.196.6 Sympy [F]

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sec(c+dx)+1}\,dx}{a}$$

input
```
integrate(sec(d*x+c)**(3/2)/(a+a*sec(d*x+c)),x)
```

output
```
Integral(sec(c + d*x)**(3/2)/(sec(c + d*x) + 1), x)/a
```

3.196.7 Maxima [F]

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{a+a\sec(c+dx)}\,dx = \int \frac{\sec(dx+c)^{\frac{3}{2}}}{a\sec(dx+c)+a}\,dx$$

input
```
integrate(sec(d*x+c)^(3/2)/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)^(3/2)/(a*sec(d*x + c) + a), x)
```

3.196.8 Giac [F]

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{a+a\sec(c+dx)}\,dx = \int \frac{\sec(dx+c)^{\frac{3}{2}}}{a\sec(dx+c)+a}\,dx$$

input
```
integrate(sec(d*x+c)^(3/2)/(a+a*sec(d*x+c)),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^(3/2)/(a*sec(d*x + c) + a), x)
```

3.196.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{a+a\sec(c+dx)}\,dx=\int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}{a+\frac{a}{\cos(c+d\,x)}}\,dx$$

input
```
int((1/cos(c + d*x))^(3/2)/(a + a/cos(c + d*x)),x)
```

output
```
int((1/cos(c + d*x))^(3/2)/(a + a/cos(c + d*x)), x)
```

3.197 $\int \frac{\sqrt{\sec(c+dx)}}{a+a\sec(c+dx)}\,dx$

3.197.1 Optimal result 1602
3.197.2 Mathematica [C] (verified) 1602
3.197.3 Rubi [A] (verified) 1603
3.197.4 Maple [A] (verified) 1605
3.197.5 Fricas [C] (verification not implemented) 1606
3.197.6 Sympy [F] 1606
3.197.7 Maxima [F] 1607
3.197.8 Giac [F] 1607
3.197.9 Mupad [F(-1)] 1607

3.197.1 Optimal result

Integrand size = 23, antiderivative size = 110

$$\int \frac{\sqrt{\sec(c+dx)}}{a+a\sec(c+dx)}\,dx = -\frac{\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{ad} + \frac{\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{ad} + \frac{\sqrt{\sec(c+dx)}\sin(c+dx)}{d(a+a\sec(c+dx))}$$

output
```
sin(d*x+c)*sec(d*x+c)^(1/2)/d/(a+a*sec(d*x+c))-(cos(1/2*d*x+1/2*c)^2)^(1/2
)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2
)*sec(d*x+c)^(1/2)/a/d+(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ell
ipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a/d
```

3.197.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.05 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.84

$$\int \frac{\sqrt{\sec(c+dx)}}{a+a\sec(c+dx)}\,dx = -\frac{2ie^{-i(c+dx)}\cos^2\left(\frac{1}{2}(c+dx)\right)\left(-1-e^{2i(c+dx)}+\left(1+e^{i(c+dx)}\right)\sqrt{1+e^{2i(c+dx)}}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\right.\right.}{ad\left(1+e^{i(c+dx)}\right)}$$

input
```
Integrate[Sqrt[Sec[c + d*x]]/(a + a*Sec[c + d*x]),x]
```

output
```
((-2*I)*Cos[(c + d*x)/2]^2*(-1 - E^((2*I)*(c + d*x)) + (1 + E^(I*(c + d*x)
))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*
I)*(c + d*x))] + E^(I*(c + d*x))*(1 + E^(I*(c + d*x)))*Sqrt[1 + E^((2*I)*(
c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))])*Sec[c +
 d*x]^(3/2))/(a*d*E^(I*(c + d*x))*(1 + E^(I*(c + d*x)))*(1 + Sec[c + d*x])
)
```

3.197.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.05, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {3042, 4307, 3042, 4274, 3042, 4258, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\sec(c+dx)}}{a\sec(c+dx)+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}dx$$

$$\Big\downarrow \text{4307}$$

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}-\frac{\int \frac{a-a\sec(c+dx)}{\sqrt{\sec(c+dx)}}dx}{2a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}-\frac{\int \frac{a-a\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{2a^2}$$

$$\Big\downarrow \text{4274}$$

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}-\frac{a\int \frac{1}{\sqrt{\sec(c+dx)}}dx-a\int \sqrt{\sec(c+dx)}dx}{2a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}-\frac{a\int\frac{1}{\sqrt{\csc(c+dx+\frac{\pi}{2})}}dx-a\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{2a^2}$$

$\Big\downarrow$ 4258

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}-\frac{a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\cos(c+dx)}dx-a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx}{2a^2}$$

$\Big\downarrow$ 3042

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}-\frac{a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx-a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx+\frac{\pi}{2})}}dx}{2a^2}$$

$\Big\downarrow$ 3119

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}-\frac{\frac{2a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|2\right)}{d}-a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx+\frac{\pi}{2})}}dx}{2a^2}$$

$\Big\downarrow$ 3120

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}-\frac{\frac{2a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|2\right)}{d}-\frac{2a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}}{2a^2}$$

input
```
Int[Sqrt[Sec[c + d*x]]/(a + a*Sec[c + d*x]),x]
```

output
```
-1/2*((2*a*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]]
)/d - (2*a*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]]
)/d)/a^2 + (Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(d*(a + a*Sec[c + d*x]))
```

3.197. $\int\frac{\sqrt{\sec(c+dx)}}{a+a\sec(c+dx)}\,dx$

3.197.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4307
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-b)*d*Cot[e + f*x]*((d*Csc[e + f*x])^(n - 1)/(a*f*
(a + b*Csc[e + f*x]))), x] + Simp[d*((n - 1)/(a*b))   Int[(d*Csc[e + f*x])^
(n - 1)*(a - b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ
[a^2 - b^2, 0]
```

3.197.4 Maple [A] (verified)

Time = 5.03 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.80

method	result
default	$-\frac{\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1}\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\left(\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)+\mathrm{EllipticE}\right.\right.}{a\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-}}$

input
```
int(sec(d*x+c)^(1/2)/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(cos(1/2*d*x+1/2*
c)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(Elliptic
F(cos(1/2*d*x+1/2*c),2^(1/2))+EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))+2*sin
(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2)/a/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2
*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+
1/2*c)^2-1)^(1/2)/d
```

3.197.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.67

$$\int \frac{\sqrt{\sec(c+dx)}}{a+a\sec(c+dx)}\,dx$$
$$= \frac{\left(-i\sqrt{2}\cos\left(dx+c\right)-i\sqrt{2}\right)\text{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right))+\left(i\sqrt{2}\cos\left(dx+c\right)\right.}{}$$

input
```
integrate(sec(d*x+c)^(1/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

output
```
1/2*((-I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassPInverse(-4, 0, cos(
d*x + c) + I*sin(d*x + c)) + (I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierstr
assPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + (-I*sqrt(2)*cos(d*x +
c) - I*sqrt(2))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x
+ c) + I*sin(d*x + c))) + (I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierstrass
Zeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2
*sqrt(cos(d*x + c))*sin(d*x + c))/(a*d*cos(d*x + c) + a*d)
```

3.197.6 Sympy [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{a+a\sec(c+dx)}\,dx = \frac{\int \frac{\sqrt{\sec(c+dx)}}{\sec(c+dx)+1}\,dx}{a}$$

input
```
integrate(sec(d*x+c)**(1/2)/(a+a*sec(d*x+c)),x)
```

output
```
Integral(sqrt(sec(c + d*x))/(sec(c + d*x) + 1), x)/a
```

3.197.7 Maxima [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{a+a\sec(c+dx)}\,dx = \int \frac{\sqrt{\sec(dx+c)}}{a\sec(dx+c)+a}\,dx$$

input
```
integrate(sec(d*x+c)^(1/2)/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

output
```
integrate(sqrt(sec(d*x + c))/(a*sec(d*x + c) + a), x)
```

3.197.8 Giac [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{a+a\sec(c+dx)}\,dx = \int \frac{\sqrt{\sec(dx+c)}}{a\sec(dx+c)+a}\,dx$$

input
```
integrate(sec(d*x+c)^(1/2)/(a+a*sec(d*x+c)),x, algorithm="giac")
```

output
```
integrate(sqrt(sec(d*x + c))/(a*sec(d*x + c) + a), x)
```

3.197.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\sec(c+dx)}}{a+a\sec(c+dx)}\,dx = \int \frac{\sqrt{\frac{1}{\cos(c+d\,x)}}}{a+\frac{a}{\cos(c+d\,x)}}\,dx$$

input
```
int((1/cos(c + d*x))^(1/2)/(a + a/cos(c + d*x)),x)
```

output
```
int((1/cos(c + d*x))^(1/2)/(a + a/cos(c + d*x)), x)
```

3.198 $\int \frac{1}{\sqrt{\sec(c+dx)}(a+a\sec(c+dx))}\,dx$

3.198.1 Optimal result 1608
3.198.2 Mathematica [C] (verified) 1608
3.198.3 Rubi [A] (verified) 1609
3.198.4 Maple [A] (verified) 1612
3.198.5 Fricas [C] (verification not implemented) 1612
3.198.6 Sympy [F] 1613
3.198.7 Maxima [F] 1613
3.198.8 Giac [F] 1614
3.198.9 Mupad [F(-1)] 1614

3.198.1 Optimal result

Integrand size = 23, antiderivative size = 112

$$\int \frac{1}{\sqrt{\sec(c+dx)}(a+a\sec(c+dx))}\,dx = \frac{3\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{ad} - \frac{\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{ad} - \frac{\sqrt{\sec(c+dx)}\sin(c+dx)}{d(a+a\sec(c+dx))}$$

output
```
-sin(d*x+c)*sec(d*x+c)^(1/2)/d/(a+a*sec(d*x+c))+3*(cos(1/2*d*x+1/2*c)^2)^(
1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(
1/2)*sec(d*x+c)^(1/2)/a/d-(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*
EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a/
d
```

3.198.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 2.28 (sec) , antiderivative size = 317, normalized size of antiderivative = 2.83

$$\int \frac{1}{\sqrt{\sec(c+dx)}(a+a\sec(c+dx))}\,dx$$
$$= \frac{\cos^2\left(\frac{1}{2}(c+dx)\right)\left(\frac{2i\sqrt{2}e^{-i(c+dx)}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\left(3(1+e^{2i(c+dx)})+3(-1+e^{2ic})\sqrt{1+e^{2i(c+dx)}}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},-e^{2i(c+dx)}\right)\right.}{d(-1+e^{2ic})}\right.}{}$$

input
```
Integrate[1/(Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])),x]
```

output
```
(Cos[(c + d*x)/2]^2*(((2*I)*Sqrt[2]*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c
+ d*x)))]*(3*(1 + E^((2*I)*(c + d*x))) + 3*(-1 + E^((2*I)*c))*Sqrt[1 + E^(
(2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))]
+ E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hyperge
ometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]))/(d*E^(I*(c + d*x))*(-1 +
 E^((2*I)*c))) - ((Cos[(c - d*x)/2] + 2*Cos[(3*c + d*x)/2] + 2*Cos[(c + 3*
d*x)/2] + Cos[(5*c + 3*d*x)/2])*Csc[c/2]*Sec[c/2]*Sec[(c + d*x)/2]*Sqrt[Se
c[c + d*x]])/(2*d))*Sec[c + d*x])/(a*(1 + Sec[c + d*x]))
```

3.198.3 Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.04, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {3042, 4306, 27, 3042, 4274, 3042, 4258, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)}dx$$
$$\Big\downarrow \; 4306$$
$$-\frac{\int -\frac{3a-a\sec(c+dx)}{2\sqrt{\sec(c+dx)}}dx}{a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{3a-a\sec(c+dx)}{\sqrt{\sec(c+dx)}}dx}{2a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{3a-a\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{2a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}$$

$$\Big\downarrow \text{4274}$$

$$\frac{3a\int \frac{1}{\sqrt{\sec(c+dx)}}dx-a\int \sqrt{\sec(c+dx)}dx}{2a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3a\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-a\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{2a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}$$

$$\Big\downarrow \text{4258}$$

$$\frac{3a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\cos(c+dx)}dx-a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{2a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx-a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{2a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}$$

$$\Big\downarrow \text{3119}$$

$$\frac{\frac{6a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|2\right)}{d}-a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{2a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}$$

$$\Big\downarrow \text{3120}$$

3.198. $\int \frac{1}{\sqrt{\sec(c+dx)}(a+a\sec(c+dx))}\,dx$

$$\frac{\frac{6a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|2\right)}{d}-\frac{2a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}}{2a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}$$

input
```
Int[1/(Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])),x]
```

output
```
((6*a*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d -
 (2*a*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d)/
(2*a^2) - (Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(d*(a + a*Sec[c + d*x]))
```

3.198.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4306
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*(a + b*Csc[e +
f*x]))), x] - Simp[1/a^2   Int[(d*Csc[e + f*x])^n*(a*(n - 1) - b*n*Csc[e +
f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, 0
]
```

3.198.4 Maple [A] (verified)

Time = 5.84 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.78

method	result
default	$\frac{\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1}\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\left(\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)+3\,\mathrm{EllipticE}\left(\right.\right.\right.}{a\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1}}$

input
```
int(1/sec(d*x+c)^(1/2)/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/a*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(cos(1/2*d*x+1
/2*c)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(Ellip
ticF(cos(1/2*d*x+1/2*c),2^(1/2))+3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))+
2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(
1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d
*x+1/2*c)^2-1)^(1/2)/d
```

3.198.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.66

$$\int \frac{1}{\sqrt{\sec(c+dx)}(a+a\sec(c+dx))}\,dx$$
$$= \frac{\left(i\sqrt{2}\cos\left(dx+c\right)+i\sqrt{2}\right)\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right))+\left(-i\sqrt{2}\cos\left(dx+c\right)\right.}{}$$

input
```
integrate(1/sec(d*x+c)^(1/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

output
```
1/2*((I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierstrassPInverse(-4, 0, cos(d
*x + c) + I*sin(d*x + c)) + (-I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstr
assPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 3*(-I*sqrt(2)*cos(d*x
+ c) - I*sqrt(2))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*
x + c) + I*sin(d*x + c))) - 3*(I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierst
rassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)))
 - 2*sqrt(cos(d*x + c))*sin(d*x + c))/(a*d*cos(d*x + c) + a*d)
```

3.198.6 Sympy [F]

$$\int \frac{1}{\sqrt{\sec(c+dx)}(a+a\sec(c+dx))}\,dx = \frac{\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)+\sqrt{\sec(c+dx)}}\,dx}{a}$$

input
```
integrate(1/sec(d*x+c)**(1/2)/(a+a*sec(d*x+c)),x)
```

output
```
Integral(1/(sec(c + d*x)**(3/2) + sqrt(sec(c + d*x))), x)/a
```

3.198.7 Maxima [F]

$$\int \frac{1}{\sqrt{\sec(c+dx)}(a+a\sec(c+dx))}\,dx = \int \frac{1}{(a\sec(dx+c)+a)\sqrt{\sec(dx+c)}}\,dx$$

input
```
integrate(1/sec(d*x+c)^(1/2)/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

output
```
integrate(1/((a*sec(d*x + c) + a)*sqrt(sec(d*x + c))), x)
```

3.198.8 Giac [F]

$$\int \frac{1}{\sqrt{\sec(c+dx)}(a+a\sec(c+dx))}\,dx = \int \frac{1}{(a\sec(dx+c)+a)\sqrt{\sec(dx+c)}}\,dx$$

input
```
integrate(1/sec(d*x+c)^(1/2)/(a+a*sec(d*x+c)),x, algorithm="giac")
```

output
```
integrate(1/((a*sec(d*x + c) + a)*sqrt(sec(d*x + c))), x)
```

3.198.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\sec(c+dx)}(a+a\sec(c+dx))}\,dx = \int \frac{1}{\left(a+\frac{a}{\cos(c+d\,x)}\right)\,\sqrt{\frac{1}{\cos(c+d\,x)}}}\,dx$$

input
```
int(1/((a + a/cos(c + d*x))*(1/cos(c + d*x))^(1/2)),x)
```

output
```
int(1/((a + a/cos(c + d*x))*(1/cos(c + d*x))^(1/2)), x)
```

3.199 $\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))}\,dx$

3.199.1 Optimal result 1615
3.199.2 Mathematica [C] (verified) 1616
3.199.3 Rubi [A] (verified) 1616
3.199.4 Maple [A] (verified) 1620
3.199.5 Fricas [C] (verification not implemented) 1620
3.199.6 Sympy [F] 1621
3.199.7 Maxima [F] 1621
3.199.8 Giac [F] 1621
3.199.9 Mupad [F(-1)] 1622

3.199.1 Optimal result

Integrand size = 23, antiderivative size = 140

$$\begin{aligned}\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))}\,dx &= -\frac{3\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{ad} \\ &\quad+\frac{5\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{3ad} \\ &\quad+\frac{5\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{\sin(c+dx)}{d\sqrt{\sec(c+dx)}(a+a\sec(c+dx))}\end{aligned}$$

output
```
5/3*sin(d*x+c)/a/d/sec(d*x+c)^(1/2)-sin(d*x+c)/d/(a+a*sec(d*x+c))/sec(d*x+
c)^(1/2)-3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1
/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a/d+5/3*(cos(1/2*
d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2
))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a/d
```

3.199.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 4.96 (sec) , antiderivative size = 318, normalized size of antiderivative = 2.27

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))}\,dx$$
$$= \frac{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\left(-\frac{2i\sqrt{2}e^{-i(c+dx)}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\left(9\left(1+e^{2i(c+dx)}\right)+9\left(-1+e^{2ic}\right)\sqrt{1+e^{2i(c+dx)}}\,\text{Hypergeometric2F1}\left(-\right.\right.}{-1+e}\right.}{}$$

input
```
Integrate[1/(Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])),x]
```

output
```
(Cos[(c + d*x)/2]^2*Sec[c + d*x]*(((-2*I)*Sqrt[2]*Sqrt[E^(I*(c + d*x))/(1
+ E^((2*I)*(c + d*x)))]*(9*(1 + E^((2*I)*(c + d*x))) + 9*(-1 + E^((2*I)*c)
)*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I
)*(c + d*x))] + 5*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c
+ d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]))/(E^(I*(c
 + d*x))*(-1 + E^((2*I)*c))) + 2*Sqrt[Sec[c + d*x]]*(3*(2 + Cos[2*c])*Cos[
d*x]*Csc[c] + Cos[2*d*x]*Sin[2*c] - 3*Sec[c/2]*Sec[(c + d*x)/2]*Sin[(d*x)/
2] - 6*Cos[c]*Sin[d*x] + Cos[2*c]*Sin[2*d*x] - 3*Tan[c/2])))/(3*a*d*(1 + S
ec[c + d*x]))
```

3.199.3 Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.03, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.522$, Rules used = {3042, 4306, 27, 3042, 4274, 3042, 4256, 3042, 4258, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)}dx$$

$$\Big\downarrow \text{4306}$$

$$-\frac{\int -\frac{5a-3a\sec(c+dx)}{2\sec^{\frac{3}{2}}(c+dx)}dx}{a^2}-\frac{\sin(c+dx)}{d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{5a-3a\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)}dx}{2a^2}-\frac{\sin(c+dx)}{d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{5a-3a\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx}{2a^2}-\frac{\sin(c+dx)}{d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)}$$

$$\Big\downarrow \text{4274}$$

$$\frac{5a\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}dx-3a\int \frac{1}{\sqrt{\sec(c+dx)}}dx}{2a^2}-\frac{\sin(c+dx)}{d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5a\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx-3a\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{2a^2}-\frac{\sin(c+dx)}{d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)}$$

$$\Big\downarrow \text{4256}$$

$$\frac{5a\left(\frac{1}{3}\int \sqrt{\sec(c+dx)}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)-3a\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{2a^2}-\frac{\sin(c+dx)}{d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5a\left(\frac{1}{3}\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)-3a\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{2a^2}-\frac{\sin(c+dx)}{d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)}$$

$$\Big\downarrow \text{4258}$$

3.199. $\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))}dx$

$$\frac{5a\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)-3a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\cos(c+dx)}}{2a^2}$$
$$\frac{\sin(c+dx)}{d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)}$$

$\Big\downarrow$ 3042

$$\frac{5a\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)-3a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+\right.}}{2a^2}$$
$$\frac{\sin(c+dx)}{d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)}$$

$\Big\downarrow$ 3119

$$\frac{5a\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)-\frac{6a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|2\right)}{d}}{2a^2}-$$
$$\frac{\sin(c+dx)}{d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)}$$

$\Big\downarrow$ 3120

$$\frac{5a\left(\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}\right)-\frac{6a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|2\right)}{d}}{2a^2}-$$
$$\frac{\sin(c+dx)}{d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)}$$

input
```
Int[1/(Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])),x]
```

output
```
-(Sin[c + d*x]/(d*Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x]))) + ((-6*a*Sqrt[
Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + 5*a*((2*Sq
rt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (2*
Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])))/(2*a^2)
```

3.199. $\int\frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))}dx$

3.199.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4306
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*(a + b*Csc[e +
f*x]))), x] - Simp[1/a^2   Int[(d*Csc[e + f*x])^n*(a*(n - 1) - b*n*Csc[e +
f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, 0
]
```

3.199. $\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))}\,dx$

3.199.4 Maple [A] (verified)

Time = 6.58 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.54

method	result
default	$-\frac{\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1}\left(5\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)+9\,\mathrm{Ellipti}\right.\right.}{3a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\cos}}$

input
```
int(1/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-1/3/a*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(cos(1/2*d*
x+1/2*c)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(5*
EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+9*EllipticE(cos(1/2*d*x+1/2*c),2^(1/
2)))-8*sin(1/2*d*x+1/2*c)^6+18*sin(1/2*d*x+1/2*c)^4-7*sin(1/2*d*x+1/2*c)^2
)/cos(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/
sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

3.199.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.48

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))}\,dx =$$

$$-\frac{5\left(i\sqrt{2}\cos\left(dx+c\right)+i\sqrt{2}\right)\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right))+5\left(-i\sqrt{2}\cos\left(dx\right.\right.}{}$$

input
```
integrate(1/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

output
```
-1/6*(5*(I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierstrassPInverse(-4, 0, co
s(d*x + c) + I*sin(d*x + c)) + 5*(-I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*wei
erstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 9*(I*sqrt(2)*cos(
d*x + c) + I*sqrt(2))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, co
s(d*x + c) + I*sin(d*x + c))) + 9*(-I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*we
ierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x +
 c))) - 2*(2*cos(d*x + c)^2 + 5*cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x +
c)))/(a*d*cos(d*x + c) + a*d)
```

3.199.6 Sympy [F]

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))}\,dx = \frac{\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)+\sec^{\frac{3}{2}}(c+dx)}\,dx}{a}$$

input
```
integrate(1/sec(d*x+c)**(3/2)/(a+a*sec(d*x+c)),x)
```

output
```
Integral(1/(sec(c + d*x)**(5/2) + sec(c + d*x)**(3/2)), x)/a
```

3.199.7 Maxima [F]

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))}\,dx = \int \frac{1}{(a\sec(dx+c)+a)\sec(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

output
```
integrate(1/((a*sec(d*x + c) + a)*sec(d*x + c)^(3/2)), x)
```

3.199.8 Giac [F]

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))}\,dx = \int \frac{1}{(a\sec(dx+c)+a)\sec(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c)),x, algorithm="giac")
```

output
```
integrate(1/((a*sec(d*x + c) + a)*sec(d*x + c)^(3/2)), x)
```

3.199.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))}\,dx=\int \frac{1}{\left(a+\frac{a}{\cos(c+d\,x)}\right)\,{\left(\frac{1}{\cos(c+d\,x)}\right)}^{3/2}}\,dx$$

input
```
int(1/((a + a/cos(c + d*x))*(1/cos(c + d*x))^(3/2)),x)
```

output
```
int(1/((a + a/cos(c + d*x))*(1/cos(c + d*x))^(3/2)), x)
```

3.200 $\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))}\,dx$

3.200.1 Optimal result . . . 1623
3.200.2 Mathematica [C] (verified) . . . 1624
3.200.3 Rubi [A] (verified) . . . 1624
3.200.4 Maple [A] (verified) . . . 1628
3.200.5 Fricas [C] (verification not implemented) . . . 1628
3.200.6 Sympy [F] . . . 1629
3.200.7 Maxima [F] . . . 1629
3.200.8 Giac [F] . . . 1630
3.200.9 Mupad [F(-1)] . . . 1630

3.200.1 Optimal result

Integrand size = 23, antiderivative size = 168

$$\begin{aligned}\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))}\,dx &= \frac{21\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5ad} \\ &\quad - \frac{5\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{3ad} + \frac{7\sin(c+dx)}{5ad\sec^{\frac{3}{2}}(c+dx)} \\ &\quad - \frac{5\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}} - \frac{\sin(c+dx)}{d\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))}\end{aligned}$$

output
```
7/5*sin(d*x+c)/a/d/sec(d*x+c)^(3/2)-sin(d*x+c)/d/sec(d*x+c)^(3/2)/(a+a*sec
(d*x+c))-5/3*sin(d*x+c)/a/d/sec(d*x+c)^(1/2)+21/5*(cos(1/2*d*x+1/2*c)^2)^(
1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(
1/2)*sec(d*x+c)^(1/2)/a/d-5/3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2
*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2
)/a/d
```

3.200.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 3.34 (sec) , antiderivative size = 347, normalized size of antiderivative = 2.07

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))}\,dx = \frac{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\left(\frac{8i\sqrt{2}e^{-i(c+dx)}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\left(63\left(1+e^{2i(c+dx)}\right)+63\left(-1+e^{2ic}\right)\sqrt{1+e^{2i(c+dx)}}\,\text{Hypergeometric2F1}\left(-\frac{1}{4}\right.\right.}{-1+e}\right.}{}$$

input
```
Integrate[1/(Sec[c + d*x]^(5/2)*(a + a*Sec[c + d*x])),x]
```

output
```
(Cos[(c + d*x)/2]^2*Sec[c + d*x]*(((8*I)*Sqrt[2]*Sqrt[E^(I*(c + d*x))/(1 +
 E^((2*I)*(c + d*x)))]*(63*(1 + E^((2*I)*(c + d*x))) + 63*(-1 + E^((2*I)*c
))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*
I)*(c + d*x))] + 25*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(
c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]))/(E^(I*
(c + d*x))*(-1 + E^((2*I)*c))) - Sqrt[Sec[c + d*x]]*(18*(17 + 11*Cos[2*c])
*Cos[d*x]*Csc[c] + 4*(10*Cos[2*d*x]*Sin[2*c] - 3*Cos[3*d*x]*Sin[3*c] - 30*
Sec[c/2]*Sec[(c + d*x)/2]*Sin[(d*x)/2] - 99*Cos[c]*Sin[d*x] + 10*Cos[2*c]*
Sin[2*d*x] - 3*Cos[3*c]*Sin[3*d*x] - 30*Tan[c/2]))))/(60*a*d*(1 + Sec[c +
d*x]))
```

3.200.3 Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.02, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.522$, Rules used = {3042, 4306, 27, 3042, 4274, 3042, 4256, 3042, 4258, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a\sec(c+dx)+a)}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)}dx$$

$$\Big\downarrow \text{4306}$$

$$-\frac{\int -\frac{7a-5a\sec(c+dx)}{2\sec^{\frac{5}{2}}(c+dx)}dx}{a^2}-\frac{\sin(c+dx)}{d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{7a-5a\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)}dx}{2a^2}-\frac{\sin(c+dx)}{d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{7a-5a\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx}{2a^2}-\frac{\sin(c+dx)}{d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)}$$

$$\Big\downarrow \text{4274}$$

$$\frac{7a\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)}dx-5a\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}dx}{2a^2}-\frac{\sin(c+dx)}{d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7a\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx-5a\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx}{2a^2}-\frac{\sin(c+dx)}{d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)}$$

$$\Big\downarrow \text{4256}$$

$$\frac{7a\left(\frac{3}{5}\int \frac{1}{\sqrt{\sec(c+dx)}}dx+\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}\right)-5a\left(\frac{1}{3}\int \sqrt{\sec(c+dx)}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)}{2a^2}-\frac{\sin(c+dx)}{d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7a\left(\frac{3}{5}\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}\right)-5a\left(\frac{1}{3}\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)}{2a^2}-\frac{\sin(c+dx)}{d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)}$$

$$\Big\downarrow \text{4258}$$

3.200. $\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))}dx$

$$\frac{7a\left(\frac{3}{5}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\cos(c+dx)}dx+\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}\right)-5a\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+}}\right.}{2a^2}$$

$$\frac{\sin(c+dx)}{d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)}$$

$$\Big\downarrow \; 3042$$

$$\frac{7a\left(\frac{3}{5}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}\right)-5a\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{}{\sqrt{}}\right.}{2a^2}$$

$$\frac{\sin(c+dx)}{d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)}$$

$$\Big\downarrow \; 3119$$

$$\frac{7a\left(\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{6\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|2\right)}{5d}\right)-5a\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2}{3d}\right.}{2a^2}$$

$$\frac{\sin(c+dx)}{d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)}$$

$$\Big\downarrow \; 3120$$

$$\frac{7a\left(\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{6\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|2\right)}{5d}\right)-5a\left(\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}\right.}{2a^2}$$

$$\frac{\sin(c+dx)}{d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)}$$

input
```
Int[1/(Sec[c + d*x]^(5/2)*(a + a*Sec[c + d*x])),x]
```

output
```
-(Sin[c + d*x]/(d*Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x]))) + (7*a*((6*Sqr
t[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d) + (2*S
in[c + d*x])/(5*d*Sec[c + d*x]^(3/2))) - 5*a*((2*Sqrt[Cos[c + d*x]]*Ellipt
icF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (2*Sin[c + d*x])/(3*d*Sqrt
[Sec[c + d*x]])))/(2*a^2)
```

3.200. $\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))}\,dx$

3.200.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4306
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*(a + b*Csc[e +
f*x]))), x] - Simp[1/a^2   Int[(d*Csc[e + f*x])^n*(a*(n - 1) - b*n*Csc[e +
f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, 0
]
```

3.200. $\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))}\,dx$

3.200.4 Maple [A] (verified)

Time = 6.96 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.36

method	result
default	$-\frac{\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(-\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1}\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\left(25\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)+63\,\mathrm{E}\right.\right.}{15a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\,\sin\left(\frac{d}{2}\right.}$

input
```
int(1/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-1/15/a*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-cos(1/2*
d*x+1/2*c)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
25*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+63*EllipticE(cos(1/2*d*x+1/2*c),2
^(1/2)))+48*sin(1/2*d*x+1/2*c)^8-56*sin(1/2*d*x+1/2*c)^6-30*sin(1/2*d*x+1/
2*c)^4+23*sin(1/2*d*x+1/2*c)^2)/cos(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1
)^(1/2)/d
```

3.200.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.29

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))}\,dx =$$

$$-\frac{25\left(-i\sqrt{2}\cos\left(dx+c\right)-i\sqrt{2}\right)\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right))+25\left(i\sqrt{2}\cos\right.}{}$$

input
```
integrate(1/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

output
```
-1/30*(25*(-I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassPInverse(-4, 0,
 cos(d*x + c) + I*sin(d*x + c)) + 25*(I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*
weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 63*(-I*sqrt(2)
*cos(d*x + c) - I*sqrt(2))*weierstrassZeta(-4, 0, weierstrassPInverse(-4,
0, cos(d*x + c) + I*sin(d*x + c))) + 63*(I*sqrt(2)*cos(d*x + c) + I*sqrt(2
))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(
d*x + c))) - 2*(6*cos(d*x + c)^3 - 4*cos(d*x + c)^2 - 25*cos(d*x + c))*sin
(d*x + c)/sqrt(cos(d*x + c)))/(a*d*cos(d*x + c) + a*d)
```

3.200.6 Sympy [F]

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))}\,dx = \frac{\int \frac{1}{\sec^{\frac{7}{2}}(c+dx)+\sec^{\frac{5}{2}}(c+dx)}\,dx}{a}$$

input
```
integrate(1/sec(d*x+c)**(5/2)/(a+a*sec(d*x+c)),x)
```

output
```
Integral(1/(sec(c + d*x)**(7/2) + sec(c + d*x)**(5/2)), x)/a
```

3.200.7 Maxima [F]

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))}\,dx = \int \frac{1}{(a\sec(dx+c)+a)\sec(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

output
```
integrate(1/((a*sec(d*x + c) + a)*sec(d*x + c)^(5/2)), x)
```

3.200.8 Giac [F]

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))}\,dx = \int \frac{1}{(a\sec(dx+c)+a)\sec(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c)),x, algorithm="giac")
```

output
```
integrate(1/((a*sec(d*x + c) + a)*sec(d*x + c)^(5/2)), x)
```

3.200.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))}\,dx = \int \frac{1}{\left(a+\frac{a}{\cos(c+d\,x)}\right)\,\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

input
```
int(1/((a + a/cos(c + d*x))*(1/cos(c + d*x))^(5/2)),x)
```

output
```
int(1/((a + a/cos(c + d*x))*(1/cos(c + d*x))^(5/2)), x)
```

3.201 $\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

3.201.1 Optimal result . . . 1631
3.201.2 Mathematica [C] (verified) . . . 1632
3.201.3 Rubi [A] (verified) . . . 1632
3.201.4 Maple [A] (verified) . . . 1636
3.201.5 Fricas [C] (verification not implemented) . . . 1637
3.201.6 Sympy [F(-1)] . . . 1638
3.201.7 Maxima [F(-1)] . . . 1638
3.201.8 Giac [F] . . . 1638
3.201.9 Mupad [F(-1)] . . . 1639

3.201.1 Optimal result

Integrand size = 23, antiderivative size = 202

$$\begin{aligned}\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a+a\sec(c+dx))^2}\,dx = {} & \frac{7\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^2d} \\ & + \frac{10\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{3a^2d} \\ & - \frac{7\sqrt{\sec(c+dx)}\sin(c+dx)}{a^2d} + \frac{10\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3a^2d} \\ & - \frac{7\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{3a^2d(1+\sec(c+dx))} - \frac{\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2}\end{aligned}$$

output
```
10/3*sec(d*x+c)^(3/2)*sin(d*x+c)/a^2/d-7/3*sec(d*x+c)^(5/2)*sin(d*x+c)/a^2
/d/(1+sec(d*x+c))-1/3*sec(d*x+c)^(7/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^2-7*s
in(d*x+c)*sec(d*x+c)^(1/2)/a^2/d+7*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*
x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)
^(1/2)/a^2/d+10/3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elliptic
F(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^2/d
```

3.201.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 4.72 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.42

$$\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{e^{-\frac{1}{2}i(4c+3dx)}(-1+e^{ic})\cos\left(\frac{1}{2}(c+dx)\right)\csc\left(\frac{c}{2}\right)\left(-10-37e^{i(c+dx)}-65e^{2i(c+dx)}-82e^{3i(c+dx)}-68e^{4i(c+dx)}\right.}{}$$

input
```
Integrate[Sec[c + d*x]^(9/2)/(a + a*Sec[c + d*x])^2,x]
```

output
```
-1/12*((-1 + E^(I*c))*Cos[(c + d*x)/2]*Csc[c/2]*(-10 - 37*E^(I*(c + d*x))
- 65*E^((2*I)*(c + d*x)) - 82*E^((3*I)*(c + d*x)) - 68*E^((4*I)*(c + d*x))
 - 53*E^((5*I)*(c + d*x)) - 21*E^((6*I)*(c + d*x)) + (10*I)*(1 + E^(I*(c +
 d*x)))^3*(1 + E^((2*I)*(c + d*x)))*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)
/2, 2] + 7*E^(I*(c + d*x))*(1 + E^(I*(c + d*x)))^3*(1 + E^((2*I)*(c + d*x)
))^(3/2)*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))])*Sec[c + d
*x]^(5/2))/(a^2*d*E^((I/2)*(4*c + 3*d*x))*(1 + E^((2*I)*(c + d*x)))*(1 + S
ec[c + d*x])^2)
```

3.201.3 Rubi [A] (verified)

Time = 1.04 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.04, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.652$, Rules used = {3042, 4303, 27, 3042, 4507, 27, 3042, 4274, 3042, 4255, 3042, 4258, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a\sec(c+dx)+a)^2}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{9/2}}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^2}dx$$

$$\Big\downarrow \; 4303$$

$$-\frac{\int \frac{\sec^{\frac{5}{2}}(c+dx)(5a-9a\sec(c+dx))}{2(\sec(c+dx)a+a)}dx}{3a^2}-\frac{\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$\Big\downarrow$ 27

$$-\frac{\int \frac{\sec^{\frac{5}{2}}(c+dx)(5a-9a\sec(c+dx))}{\sec(c+dx)a+a}dx}{6a^2}-\frac{\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$\Big\downarrow$ 3042

$$-\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(5a-9a\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{6a^2}-\frac{\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$\Big\downarrow$ 4507

$$-\frac{\frac{\int 3\sec^{\frac{3}{2}}(c+dx)\left(7a^2-10a^2\sec(c+dx)\right)dx}{a^2}+\frac{14\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{d(\sec(c+dx)+1)}}{6a^2}-\frac{\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$\Big\downarrow$ 27

$$-\frac{\frac{3\int \sec^{\frac{3}{2}}(c+dx)\left(7a^2-10a^2\sec(c+dx)\right)dx}{a^2}+\frac{14\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{d(\sec(c+dx)+1)}}{6a^2}-\frac{\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$\Big\downarrow$ 3042

$$-\frac{\frac{3\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(7a^2-10a^2\csc\left(c+dx+\frac{\pi}{2}\right)\right)dx}{a^2}+\frac{14\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{d(\sec(c+dx)+1)}}{6a^2}-\frac{\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$\Big\downarrow$ 4274

$$-\frac{\frac{3\left(7a^2\int \sec^{\frac{3}{2}}(c+dx)dx-10a^2\int \sec^{\frac{5}{2}}(c+dx)dx\right)}{a^2}+\frac{14\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{d(\sec(c+dx)+1)}}{6a^2}-\frac{\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$\Big\downarrow$ 3042

$$-\frac{\frac{3\left(7a^2\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}dx-10a^2\int \csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}dx\right)}{a^2}+\frac{14\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{d(\sec(c+dx)+1)}}{6a^2}-\frac{\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$\Big\downarrow$ 4255

$$-\frac{\frac{3\left(7a^2\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\int \frac{1}{\sqrt{\sec(c+dx)}}dx\right)-10a^2\left(\frac{1}{3}\int \sqrt{\sec(c+dx)}dx+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}\right)\right)}{a^2}+\frac{14\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{d(\sec(c+dx)+1)}}{6a^2}-\frac{\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

3.201. $\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

$$\Big\downarrow \; 3042$$

$$-\frac{\frac{3\left(7a^2\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\int\frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)-10a^2\left(\frac{1}{3}\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}\right)\right)}{a^2}+\frac{14\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{d(\sec(c+dx)+1)}}{6a^2}$$
$$\frac{\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \; 4258$$

$$-\frac{\frac{3\left(7a^2\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\cos(c+dx)}dx\right)-10a^2\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2\sin(c+dx)\sec}{3d}\right.\right.}{a^2}}{6a^2}$$
$$\frac{\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\frac{3\left(7a^2\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx\right)-10a^2\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(}{}\right.\right.}{a^2}}{6a^2}$$
$$\frac{\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \; 3119$$

$$-\frac{\frac{3\left(7a^2\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)-10a^2\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)\sec^{\frac{3}{}}}{3d}\right.\right.}{a^2}}{6a^2}$$
$$\frac{\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \; 3120$$

$$-\frac{\frac{3\left(7a^2\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)-10a^2\left(\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx)\right.}{3d}\right.\right.}{a^2}}{6a^2}$$
$$\frac{\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

3.201. $\int\frac{\sec^{\frac{9}{2}}(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

input
```
Int[Sec[c + d*x]^(9/2)/(a + a*Sec[c + d*x])^2,x]
```

output
```
-1/3*(Sec[c + d*x]^(7/2)*Sin[c + d*x])/(d*(a + a*Sec[c + d*x])^2) - ((14*S
ec[c + d*x]^(5/2)*Sin[c + d*x])/(d*(1 + Sec[c + d*x])) + (3*(7*a^2*((-2*Sq
rt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (2*Sqrt
[Sec[c + d*x]]*Sin[c + d*x])/d) - 10*a^2*((2*Sqrt[Cos[c + d*x]]*EllipticF[
(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (2*Sec[c + d*x]^(3/2)*Sin[c +
d*x])/(3*d))))/a^2)/(6*a^2)
```

3.201.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4303
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-d^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d
*Csc[e + f*x])^(n - 2)/(f*(2*m + 1))), x] + Simp[d^2/(a*b*(2*m + 1))   Int[
(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n
 + 2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2,
0] && LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

rule 4507
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[d*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(a*f*(
2*m + 1))), x] - Simp[1/(a*b*(2*m + 1))   Int[(a + b*Csc[e + f*x])^(m + 1)*
(d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
 - n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f,
A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && G
tQ[n, 0]
```

3.201.4 Maple [A] (verified)

Time = 16.44 (sec) , antiderivative size = 413, normalized size of antiderivative = 2.04

method	result
default	$-\frac{\sqrt{-\left(-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\left(\frac{\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}}{3\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}+\frac{6\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}}{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{22\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}}{3\sqrt{}}\right.}{}$

input
```
int(sec(d*x+c)^(9/2)/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/a^2*(1/3*(-
2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)^3+6*
(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)-22
/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(
1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),
2^(1/2))+14*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)
/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d
*x+1/2*c),2^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))+16*sin(1/2*d*x+1
/2*c)^2*cos(1/2*d*x+1/2*c)/(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c
)^2)^(1/2)-2/3*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2
*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d
*x+1/2*c)^2-1)^(1/2)/d
```

3.201.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 328, normalized size of antiderivative = 1.62

$$\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a+a\sec(c+dx))^2}\,dx =$$

$$-\frac{10\left(i\sqrt{2}\cos\left(dx+c\right)^3+2i\sqrt{2}\cos\left(dx+c\right)^2+i\sqrt{2}\cos\left(dx+c\right)\right)\text{weierstrassPInverse}(-4,0,\cos\left(dx+c\right.}{}$$

input
```
integrate(sec(d*x+c)^(9/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
-1/6*(10*(I*sqrt(2)*cos(d*x + c)^3 + 2*I*sqrt(2)*cos(d*x + c)^2 + I*sqrt(2
)*cos(d*x + c))*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))
+ 10*(-I*sqrt(2)*cos(d*x + c)^3 - 2*I*sqrt(2)*cos(d*x + c)^2 - I*sqrt(2)*c
os(d*x + c))*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 2
1*(-I*sqrt(2)*cos(d*x + c)^3 - 2*I*sqrt(2)*cos(d*x + c)^2 - I*sqrt(2)*cos(
d*x + c))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) +
 I*sin(d*x + c))) + 21*(I*sqrt(2)*cos(d*x + c)^3 + 2*I*sqrt(2)*cos(d*x + c
)^2 + I*sqrt(2)*cos(d*x + c))*weierstrassZeta(-4, 0, weierstrassPInverse(-
4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*(21*cos(d*x + c)^3 + 32*cos(d*x
+ c)^2 + 8*cos(d*x + c) - 2)*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^2*d*cos(d
*x + c)^3 + 2*a^2*d*cos(d*x + c)^2 + a^2*d*cos(d*x + c))
```

3.201.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**(9/2)/(a+a*sec(d*x+c))**2,x)
```

output
```
Timed out
```

3.201.7 Maxima [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)^(9/2)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
Timed out
```

3.201.8 Giac [F]

$$\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \int \frac{\sec(dx+c)^{\frac{9}{2}}}{(a\sec(dx+c)+a)^2}\,dx$$

input
```
integrate(sec(d*x+c)^(9/2)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^(9/2)/(a*sec(d*x + c) + a)^2, x)
```

3.201.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{9/2}}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^2}\,dx$$

input
```
int((1/cos(c + d*x))^(9/2)/(a + a/cos(c + d*x))^2,x)
```

output
```
int((1/cos(c + d*x))^(9/2)/(a + a/cos(c + d*x))^2, x)
```

3.202 $\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

3.202.1 Optimal result 1640
3.202.2 Mathematica [C] (verified) 1641
3.202.3 Rubi [A] (verified) 1641
3.202.4 Maple [A] (verified) 1645
3.202.5 Fricas [C] (verification not implemented) 1646
3.202.6 Sympy [F(-1)] 1646
3.202.7 Maxima [F(-1)] 1647
3.202.8 Giac [F] 1647
3.202.9 Mupad [F(-1)] 1647

3.202.1 Optimal result

Integrand size = 23, antiderivative size = 176

$$\begin{aligned}\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+a\sec(c+dx))^2}\,dx = &-\frac{4\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^2d}\\ &-\frac{5\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{3a^2d}\\ &+\frac{4\sqrt{\sec(c+dx)}\sin(c+dx)}{a^2d}\\ &-\frac{5\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3a^2d(1+\sec(c+dx))}-\frac{\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2}\end{aligned}$$

output
```
-5/3*sec(d*x+c)^(3/2)*sin(d*x+c)/a^2/d/(1+sec(d*x+c))-1/3*sec(d*x+c)^(5/2)
*sin(d*x+c)/d/(a+a*sec(d*x+c))^2+4*sin(d*x+c)*sec(d*x+c)^(1/2)/a^2/d-4*(co
s(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),
2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^2/d-5/3*(cos(1/2*d*x+1/2*c)^2
)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c
)^(1/2)*sec(d*x+c)^(1/2)/a^2/d
```

3.202.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 2.49 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.43

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{e^{-idx}\cos\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{5}{2}}(c+dx)\left(-4ie^{-i(c+dx)}\left(1+e^{i(c+dx)}\right)^3\sqrt{1+e^{2i(c+dx)}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4}\right.\right.}{}$$

input
```
Integrate[Sec[c + d*x]^(7/2)/(a + a*Sec[c + d*x])^2,x]
```

output
```
-1/6*(Cos[(c + d*x)/2]*Sec[c + d*x]^(5/2)*(((-4*I)*(1 + E^(I*(c + d*x)))^3
*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*
(c + d*x))])/E^(I*(c + d*x)) + 40*Cos[(c + d*x)/2]^3*Sqrt[Cos[c + d*x]]*El
lipticF[(c + d*x)/2, 2]*(Cos[(c + d*x)/2] - I*Sin[(c + d*x)/2]) + I*(29 +
50*Cos[c + d*x] + 17*Cos[2*(c + d*x)] + (12*I)*Sin[c + d*x] + (7*I)*Sin[2*
(c + d*x)]))*(Cos[(c + 3*d*x)/2] + I*Sin[(c + 3*d*x)/2]))/(a^2*d*E^(I*d*x)
*(1 + Sec[c + d*x])^2)
```

3.202.3 Rubi [A] (verified)

Time = 0.98 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.03, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.609$, Rules used = {3042, 4303, 27, 3042, 4507, 3042, 4274, 3042, 4255, 3042, 4258, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a\sec(c+dx)+a)^2}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^2}dx$$

$$\Big\downarrow \; 4303$$

$$-\frac{\int \frac{\sec^{\frac{3}{2}}(c+dx)(3a-7a\sec(c+dx))}{2(\sec(c+dx)a+a)}dx}{3a^2}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \; 27$$

$$-\frac{\int \frac{\sec^{\frac{3}{2}}(c+dx)(3a-7a\sec(c+dx))}{\sec(c+dx)a+a}dx}{6a^2}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(3a-7a\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{6a^2}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \; 4507$$

$$-\frac{\frac{\int \sqrt{\sec(c+dx)}\left(5a^2-12a^2\sec(c+dx)\right)dx}{a^2}+\frac{10\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d(\sec(c+dx)+1)}}{6a^2}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\frac{\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(5a^2-12a^2\csc\left(c+dx+\frac{\pi}{2}\right)\right)dx}{a^2}+\frac{10\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d(\sec(c+dx)+1)}}{6a^2}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \; 4274$$

$$-\frac{\frac{5a^2\int \sqrt{\sec(c+dx)}dx-12a^2\int \sec^{\frac{3}{2}}(c+dx)dx}{a^2}+\frac{10\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d(\sec(c+dx)+1)}}{6a^2}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\frac{5a^2\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx-12a^2\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}dx}{a^2}+\frac{10\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d(\sec(c+dx)+1)}}{6a^2}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \; 4255$$

$$-\frac{\frac{5a^2\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx-12a^2\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\int \frac{1}{\sqrt{\sec(c+dx)}}dx\right)}{a^2}+\frac{10\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d(\sec(c+dx)+1)}}{6a^2}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\frac{5a^2\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx-12a^2\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)}{a^2}+\frac{10\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d(\sec(c+dx)+1)}}{6a^2}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

3.202. $\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

$$\Big\downarrow \; 4258$$

$$-\frac{\frac{5a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx-12a^2\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\cos(c+dx)}dx\right)}{a^2}+\frac{10\sin(c+dx)\sec^{\frac{3}{2}}}{d(\sec(c+dx)}}{6a^2}$$
$$\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\frac{5a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-12a^2\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx\right)}{a^2}+\frac{10\sin(c+}{d(\sec}}{6a^2}$$
$$\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \; 3119$$

$$-\frac{\frac{5a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-12a^2\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)}{a^2}+\frac{10\sin(c+dx)\sec^{\frac{3}{2}}(}{d(\sec(c+dx)+}}{6a^2}$$
$$\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \; 3120$$

$$-\frac{\frac{\frac{10a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}-12a^2\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)}{a^2}+\frac{10\sin(c+dx)\sec^{\frac{3}{2}}(c+}{d(\sec(c+dx)+1)}}{6a^2}$$
$$\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

input
```
Int[Sec[c + d*x]^(7/2)/(a + a*Sec[c + d*x])^2,x]
```

output
```
-1/3*(Sec[c + d*x]^(5/2)*Sin[c + d*x])/(d*(a + a*Sec[c + d*x])^2) - ((10*S
ec[c + d*x]^(3/2)*Sin[c + d*x])/(d*(1 + Sec[c + d*x])) + ((10*a^2*Sqrt[Cos
[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d - 12*a^2*((-2*S
qrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (2*Sqr
t[Sec[c + d*x]]*Sin[c + d*x])/d))/a^2)/(6*a^2)
```

3.202. $\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

3.202.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4303
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-d^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d
*Csc[e + f*x])^(n - 2)/(f*(2*m + 1))), x] + Simp[d^2/(a*b*(2*m + 1))   Int[
(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n
 + 2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2,
0] && LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

3.202. $\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

rule 4507
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[d*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(a*f*(
2*m + 1))), x] - Simp[1/(a*b*(2*m + 1))   Int[(a + b*Csc[e + f*x])^(m + 1)*
(d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
 - n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f,
A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && G
tQ[n, 0]
```

3.202.4 Maple [A] (verified)

Time = 13.24 (sec) , antiderivative size = 405, normalized size of antiderivative = 2.30

method	result
default	$-\frac{2\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\left(5\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)-12\,\mathrm{EllipticE}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right.\right.\right.}{}$

input
```
int(sec(d*x+c)^(7/2)/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
-1/6*(2*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*
sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(5*EllipticF(cos(1/2*d*x+
1/2*c),2^(1/2))-12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*cos(1/2*d*x+1/2*
c)*sin(1/2*d*x+1/2*c)^2-2*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(5*Elli
pticF(cos(1/2*d*x+1/2*c),2^(1/2))-12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))
)*cos(1/2*d*x+1/2*c)-48*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/
2)*sin(1/2*d*x+1/2*c)^6+86*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^
(1/2)*sin(1/2*d*x+1/2*c)^4-37*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^
2)^(1/2)*sin(1/2*d*x+1/2*c)^2)/a^2/cos(1/2*d*x+1/2*c)^3/(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c
)^2-1)^(1/2)/d
```

3.202.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.58

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{5\left(-i\sqrt{2}\cos\left(dx+c\right)^2-2i\sqrt{2}\cos\left(dx+c\right)-i\sqrt{2}\right)\text{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\sin\left(dx\right.}{}$$

input
```
integrate(sec(d*x+c)^(7/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
-1/6*(5*(-I*sqrt(2)*cos(d*x + c)^2 - 2*I*sqrt(2)*cos(d*x + c) - I*sqrt(2))
*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 5*(I*sqrt(2)*
cos(d*x + c)^2 + 2*I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierstrassPInverse
(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 12*(I*sqrt(2)*cos(d*x + c)^2 + 2*
I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierstrassZeta(-4, 0, weierstrassPInv
erse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 12*(-I*sqrt(2)*cos(d*x + c)^
2 - 2*I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassZeta(-4, 0, weierstra
ssPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - 2*(12*cos(d*x + c)^2 +
 19*cos(d*x + c) + 6)*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^2*d*cos(d*x + c)
^2 + 2*a^2*d*cos(d*x + c) + a^2*d)
```

3.202.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**(7/2)/(a+a*sec(d*x+c))**2,x)
```

output
```
Timed out
```

3.202.7 Maxima [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)^(7/2)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
Timed out
```

3.202.8 Giac [F]

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \int \frac{\sec(dx+c)^{\frac{7}{2}}}{(a\sec(dx+c)+a)^2}\,dx$$

input
```
integrate(sec(d*x+c)^(7/2)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^(7/2)/(a*sec(d*x + c) + a)^2, x)
```

3.202.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{7/2}}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^2}\,dx$$

input
```
int((1/cos(c + d*x))^(7/2)/(a + a/cos(c + d*x))^2,x)
```

output
```
int((1/cos(c + d*x))^(7/2)/(a + a/cos(c + d*x))^2, x)
```

3.203 $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

3.203.1 Optimal result 1648
3.203.2 Mathematica [C] (verified) 1648
3.203.3 Rubi [A] (verified) 1649
3.203.4 Maple [A] (verified) 1652
3.203.5 Fricas [C] (verification not implemented) 1653
3.203.6 Sympy [F] 1654
3.203.7 Maxima [F] 1654
3.203.8 Giac [F] 1654
3.203.9 Mupad [F(-1)] 1655

3.203.1 Optimal result

Integrand size = 23, antiderivative size = 149

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^2d} + \frac{2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{3a^2d} - \frac{\sqrt{\sec(c+dx)}\sin(c+dx)}{a^2d(1+\sec(c+dx))} - \frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2}$$

output
```
-1/3*sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^2-sin(d*x+c)*sec(d*x+c
)^(1/2)/a^2/d/(1+sec(d*x+c))+(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*
c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)
/a^2/d+2/3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1
/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^2/d
```

3.203.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 2.36 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.62

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{e^{-idx}\cos\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{5}{2}}(c+dx)\left(-ie^{-i(c+dx)}\left(1+e^{i(c+dx)}\right)^3\sqrt{1+e^{2i(c+dx)}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},\right.\right.}{}$$

input
```
Integrate[Sec[c + d*x]^(5/2)/(a + a*Sec[c + d*x])^2,x]
```

output
```
(Cos[(c + d*x)/2]*Sec[c + d*x]^(5/2)*(((-I)*(1 + E^(I*(c + d*x)))^3*Sqrt[1
 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*
x))])/E^(I*(c + d*x)) + 16*Cos[(c + d*x)/2]^3*Sqrt[Cos[c + d*x]]*EllipticF
[(c + d*x)/2, 2]*(Cos[(c + d*x)/2] - I*Sin[(c + d*x)/2]) + I*(5 + 14*Cos[c
 + d*x] + 5*Cos[2*(c + d*x)] + I*Sin[2*(c + d*x)]))*(Cos[(c + 3*d*x)/2] +
I*Sin[(c + 3*d*x)/2]))/(6*a^2*d*E^(I*d*x)*(1 + Sec[c + d*x])^2)
```

3.203.3 Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.07, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.565$, Rules used = {3042, 4303, 27, 3042, 4507, 25, 3042, 4274, 3042, 4258, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a\sec(c+dx)+a)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^2}dx$$

$$\Big\downarrow \text{4303}$$

$$-\frac{\int \frac{\sqrt{\sec(c+dx)}(a-5a\sec(c+dx))}{2(\sec(c+dx)a+a)}dx}{3a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{\sqrt{\sec(c+dx)}(a-5a\sec(c+dx))}{\sec(c+dx)a+a}dx}{6a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a-5a\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{6a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{4507}$$

$$-\frac{\frac{\int -\frac{2\sec(c+dx)a^2+3a^2}{\sqrt{\sec(c+dx)}}dx}{a^2}+\frac{6\sin(c+dx)\sqrt{\sec(c+dx)}}{d(\sec(c+dx)+1)}}{6a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$\downarrow$ 25

$$-\frac{\frac{6\sin(c+dx)\sqrt{\sec(c+dx)}}{d(\sec(c+dx)+1)}-\frac{\int \frac{2\sec(c+dx)a^2+3a^2}{\sqrt{\sec(c+dx)}}dx}{a^2}}{6a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$\downarrow$ 3042

$$-\frac{\frac{6\sin(c+dx)\sqrt{\sec(c+dx)}}{d(\sec(c+dx)+1)}-\frac{\int \frac{2\csc\left(c+dx+\frac{\pi}{2}\right)a^2+3a^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2}}{6a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$\downarrow$ 4274

$$-\frac{\frac{6\sin(c+dx)\sqrt{\sec(c+dx)}}{d(\sec(c+dx)+1)}-\frac{3a^2\int \frac{1}{\sqrt{\sec(c+dx)}}dx+2a^2\int \sqrt{\sec(c+dx)}dx}{a^2}}{6a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$\downarrow$ 3042

$$-\frac{\frac{6\sin(c+dx)\sqrt{\sec(c+dx)}}{d(\sec(c+dx)+1)}-\frac{3a^2\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+2a^2\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}}{6a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$\downarrow$ 4258

$$-\frac{\frac{6\sin(c+dx)\sqrt{\sec(c+dx)}}{d(\sec(c+dx)+1)}-\frac{2a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx+3a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\cos(c+dx)}dx}{a^2}}{6a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$\downarrow$ 3042

$$-\frac{\frac{6\sin(c+dx)\sqrt{\sec(c+dx)}}{d(\sec(c+dx)+1)}-\frac{2a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+3a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}}{6a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$\downarrow$ 3119

3.203. $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

$$-\frac{\frac{6\sin(c+dx)\sqrt{\sec(c+dx)}}{d(\sec(c+dx)+1)}-\frac{2a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{6a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}}{a^2}}{6a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \; 3120$$

$$-\frac{\frac{6\sin(c+dx)\sqrt{\sec(c+dx)}}{d(\sec(c+dx)+1)}-\frac{\frac{4a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+\frac{6a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}}{a^2}}{6a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

input
```
Int[Sec[c + d*x]^(5/2)/(a + a*Sec[c + d*x])^2,x]
```

output
```
-1/3*(Sec[c + d*x]^(3/2)*Sin[c + d*x])/(d*(a + a*Sec[c + d*x])^2) - (-(((6
*a^2*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d +
(4*a^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d)
/a^2) + (6*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(d*(1 + Sec[c + d*x])))/(6*a^2
)
```

3.203.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

3.203. $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4303
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-d^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d
*Csc[e + f*x])^(n - 2)/(f*(2*m + 1))), x] + Simp[d^2/(a*b*(2*m + 1))   Int[
(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n
 + 2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2,
0] && LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

rule 4507
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[d*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(a*f*(
2*m + 1))), x] - Simp[1/(a*b*(2*m + 1))   Int[(a + b*Csc[e + f*x])^(m + 1)*
(d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
 - n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f,
A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && G
tQ[n, 0]
```

3.203.4 Maple [A] (verified)

Time = 10.57 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.72

method	result
default	$\frac{\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(12\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^6-4\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1}\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\cos\left(\right.\right.}{6a^2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^3\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\sin\left(\right.}}$

input
```
int(sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/6*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(12*cos(1/2*d*
x+1/2*c)^6-4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2
)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^3+6*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*cos(1/2*d*x+1/2*c)^3*E
llipticE(cos(1/2*d*x+1/2*c),2^(1/2))-16*cos(1/2*d*x+1/2*c)^4+3*cos(1/2*d*x
+1/2*c)^2+1)/a^2/cos(1/2*d*x+1/2*c)^3/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x
+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

3.203.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.86

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^2}\,dx =$$

$$-\frac{2\left(i\sqrt{2}\cos\left(dx+c\right)^2+2i\sqrt{2}\cos\left(dx+c\right)+i\sqrt{2}\right)\text{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+\right.}{}$$

input
```
integrate(sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
-1/6*(2*(I*sqrt(2)*cos(d*x + c)^2 + 2*I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*
weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 2*(-I*sqrt(2)*
cos(d*x + c)^2 - 2*I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassPInverse
(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 3*(-I*sqrt(2)*cos(d*x + c)^2 - 2*
I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassZeta(-4, 0, weierstrassPInv
erse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 3*(I*sqrt(2)*cos(d*x + c)^2
+ 2*I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierstrassZeta(-4, 0, weierstrass
PInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*(3*cos(d*x + c)^2 + 4*
cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^2*d*cos(d*x + c)^2 + 2*a
^2*d*cos(d*x + c) + a^2*d)
```

3.203.6 Sympy [F]

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{\int \frac{\sec^{\frac{5}{2}}(c+dx)}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx}{a^2}$$

input
```
integrate(sec(d*x+c)**(5/2)/(a+a*sec(d*x+c))**2,x)
```

output
```
Integral(sec(c + d*x)**(5/2)/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a*
*2
```

3.203.7 Maxima [F]

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \int \frac{\sec(dx+c)^{\frac{5}{2}}}{(a\sec(dx+c)+a)^2}\,dx$$

input
```
integrate(sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)^(5/2)/(a*sec(d*x + c) + a)^2, x)
```

3.203.8 Giac [F]

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \int \frac{\sec(dx+c)^{\frac{5}{2}}}{(a\sec(dx+c)+a)^2}\,dx$$

input
```
integrate(sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^(5/2)/(a*sec(d*x + c) + a)^2, x)
```

3.203.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^2}\,dx$$

input
```
int((1/cos(c + d*x))^(5/2)/(a + a/cos(c + d*x))^2,x)
```

output
```
int((1/cos(c + d*x))^(5/2)/(a + a/cos(c + d*x))^2, x)
```

3.204 $\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

3.204.1 Optimal result 1656
3.204.2 Mathematica [A] (verified) 1656
3.204.3 Rubi [A] (verified) 1657
3.204.4 Maple [B] (verified) 1659
3.204.5 Fricas [C] (verification not implemented) 1659
3.204.6 Sympy [F] 1660
3.204.7 Maxima [F] 1660
3.204.8 Giac [F] 1660
3.204.9 Mupad [F(-1)] 1661

3.204.1 Optimal result

Integrand size = 23, antiderivative size = 77

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{3a^2d} + \frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2}$$

output
```
1/3*sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^2+1/3*(cos(1/2*d*x+1/2*
c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d
*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^2/d
```

3.204.2 Mathematica [A] (verified)

Time = 1.47 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.27

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{\cos\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{5}{2}}(c+dx)\left(4\cos^3\left(\frac{1}{2}(c+dx)\right)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}{3a^2d(1+\sec(c+dx))^2}$$

input
```
Integrate[Sec[c + d*x]^(3/2)/(a + a*Sec[c + d*x])^2,x]
```

output
```
(Cos[(c + d*x)/2]*Sec[c + d*x]^(5/2)*(4*Cos[(c + d*x)/2]^3*Sqrt[Cos[c + d*
x]]*EllipticF[(c + d*x)/2, 2] - Sin[(c + d*x)/2] + Sin[(3*(c + d*x))/2]))/
(3*a^2*d*(1 + Sec[c + d*x])^2)
```

3.204.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3042, 4302, 27, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a\sec(c+dx)+a)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^2}dx$$

$$\Big\downarrow \text{4302}$$

$$\frac{\int \frac{1}{2}\sqrt{\sec(c+dx)}dx}{3a^2}+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \sqrt{\sec(c+dx)}dx}{6a^2}+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{6a^2}+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{4258}$$

$$\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{6a^2}+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{6a^2}+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \; 3120$$

$$\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3a^2d}+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

input
```
Int[Sec[c + d*x]^(3/2)/(a + a*Sec[c + d*x])^2,x]
```

output
```
(Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*a^2*d
) + (Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d*(a + a*Sec[c + d*x])^2)
```

3.204.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4302
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[b*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Cs
c[e + f*x])^(n - 1)/(a*f*(2*m + 1))), x] - Simp[d/(a*b*(2*m + 1))   Int[(a
+ b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1)*(a*(n - 1) - b*(m + n)*C
sc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && L
tQ[m, -1] && LtQ[1, n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

3.204.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 187 vs. $2(93) = 186$.

Time = 4.95 (sec) , antiderivative size = 188, normalized size of antiderivative = 2.44

method	result
default	$-\frac{\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1}\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^3+2\cos\left(\right.\right.}{6a^2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^3\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1}\,d}$

input
```
int(sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
-1/6*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x
+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^3+2*cos(1/2*d*x+1/2*c)^4-3*cos(1/2*d*x
+1/2*c)^2+1)/a^2/cos(1/2*d*x+1/2*c)^3/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x
+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

3.204.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.95

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^2}\,dx$$
$$= \frac{\left(-i\sqrt{2}\cos(dx+c)^2-2i\sqrt{2}\cos(dx+c)-i\sqrt{2}\right)\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)}{6\left(a^2d\right.}$$

input
```
integrate(sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/6*((-I*sqrt(2)*cos(d*x + c)^2 - 2*I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*we
ierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + (I*sqrt(2)*cos(d
*x + c)^2 + 2*I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierstrassPInverse(-4,
0, cos(d*x + c) - I*sin(d*x + c)) + 2*sqrt(cos(d*x + c))*sin(d*x + c))/(a^
2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)
```

3.204.6 Sympy [F]

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx}{a^2}$$

input
```
integrate(sec(d*x+c)**(3/2)/(a+a*sec(d*x+c))**2,x)
```

output
```
Integral(sec(c + d*x)**(3/2)/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a*
*2
```

3.204.7 Maxima [F]

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \int \frac{\sec(dx+c)^{\frac{3}{2}}}{(a\sec(dx+c)+a)^2}\,dx$$

input
```
integrate(sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)^(3/2)/(a*sec(d*x + c) + a)^2, x)
```

3.204.8 Giac [F]

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \int \frac{\sec(dx+c)^{\frac{3}{2}}}{(a\sec(dx+c)+a)^2}\,dx$$

input
```
integrate(sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^(3/2)/(a*sec(d*x + c) + a)^2, x)
```

3.204.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^2}\,dx$$

input
```
int((1/cos(c + d*x))^(3/2)/(a + a/cos(c + d*x))^2,x)
```

output
```
int((1/cos(c + d*x))^(3/2)/(a + a/cos(c + d*x))^2, x)
```

3.205 $\int \frac{\sqrt{\sec(c+dx)}}{(a+a\sec(c+dx))^2}\,dx$

3.205.1 Optimal result 1662
3.205.2 Mathematica [C] (verified) 1662
3.205.3 Rubi [A] (verified) 1663
3.205.4 Maple [A] (verified) 1666
3.205.5 Fricas [C] (verification not implemented) 1667
3.205.6 Sympy [F] 1668
3.205.7 Maxima [F] 1668
3.205.8 Giac [F] 1668
3.205.9 Mupad [F(-1)] 1669

3.205.1 Optimal result

Integrand size = 23, antiderivative size = 149

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+a\sec(c+dx))^2}\,dx = -\frac{\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^2d} + \frac{2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{3a^2d} + \frac{\sqrt{\sec(c+dx)}\sin(c+dx)}{a^2d(1+\sec(c+dx))} - \frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2}$$

output
```
-1/3*sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^2+sin(d*x+c)*sec(d*x+c
)^(1/2)/a^2/d/(1+sec(d*x+c))-(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*
c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)
/a^2/d+2/3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1
/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^2/d
```

3.205.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 2.56 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.60

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+a\sec(c+dx))^2}\,dx = \frac{e^{-idx}\cos\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{5}{2}}(c+dx)\left(16\cos^3\left(\frac{1}{2}(c+dx)\right)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\left(\cos\left(\frac{1}{2}(c+\right.\right.\right.}{}$$

input
```
Integrate[Sqrt[Sec[c + d*x]]/(a + a*Sec[c + d*x])^2,x]
```

output
```
(Cos[(c + d*x)/2]*Sec[c + d*x]^(5/2)*(16*Cos[(c + d*x)/2]^3*Sqrt[Cos[c + d
*x]]*EllipticF[(c + d*x)/2, 2]*(Cos[(c + d*x)/2] - I*Sin[(c + d*x)/2]) + I
*(-7 - 10*Cos[c + d*x] - 7*Cos[2*(c + d*x)] + ((1 + E^(I*(c + d*x)))^3*Sqr
t[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c +
 d*x))])/E^(I*(c + d*x)) + I*Sin[2*(c + d*x)]))*(Cos[(c + 3*d*x)/2] + I*Si
n[(c + 3*d*x)/2]))/(6*a^2*d*E^(I*d*x)*(1 + Sec[c + d*x])^2)
```

3.205.3 Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.07, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.565$, Rules used = {3042, 4304, 27, 3042, 4507, 25, 3042, 4274, 3042, 4258, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\sec(c+dx)}}{(a\sec(c+dx)+a)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^2}dx$$

$$\Big\downarrow \text{4304}$$

$$-\frac{\int -\frac{\sqrt{\sec(c+dx)}(5a-a\sec(c+dx))}{2(\sec(c+dx)a+a)}dx}{3a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\sqrt{\sec(c+dx)}(5a-a\sec(c+dx))}{\sec(c+dx)a+a}dx}{6a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(5a-a\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{6a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{4507}$$

$$\frac{\frac{\int -\frac{3a^2-2a^2\sec(c+dx)}{\sqrt{\sec(c+dx)}}dx}{a^2}+\frac{6\sin(c+dx)\sqrt{\sec(c+dx)}}{d(\sec(c+dx)+1)}}{6a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$\Big\downarrow$ 25

$$\frac{\frac{6\sin(c+dx)\sqrt{\sec(c+dx)}}{d(\sec(c+dx)+1)}-\frac{\int\frac{3a^2-2a^2\sec(c+dx)}{\sqrt{\sec(c+dx)}}dx}{a^2}}{6a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$\Big\downarrow$ 3042

$$\frac{\frac{6\sin(c+dx)\sqrt{\sec(c+dx)}}{d(\sec(c+dx)+1)}-\frac{\int\frac{3a^2-2a^2\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2}}{6a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$\Big\downarrow$ 4274

$$\frac{\frac{6\sin(c+dx)\sqrt{\sec(c+dx)}}{d(\sec(c+dx)+1)}-\frac{3a^2\int\frac{1}{\sqrt{\sec(c+dx)}}dx-2a^2\int\sqrt{\sec(c+dx)}dx}{a^2}}{6a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$\Big\downarrow$ 3042

$$\frac{\frac{6\sin(c+dx)\sqrt{\sec(c+dx)}}{d(\sec(c+dx)+1)}-\frac{3a^2\int\frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-2a^2\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}}{6a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$\Big\downarrow$ 4258

$$\frac{\frac{6\sin(c+dx)\sqrt{\sec(c+dx)}}{d(\sec(c+dx)+1)}-\frac{3a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\cos(c+dx)}dx-2a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx}{a^2}}{6a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$\Big\downarrow$ 3042

$$\frac{\frac{6\sin(c+dx)\sqrt{\sec(c+dx)}}{d(\sec(c+dx)+1)}-\frac{3a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx-2a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2}}{6a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$\Big\downarrow$ 3119

3.205. $\int\frac{\sqrt{\sec(c+dx)}}{(a+a\sec(c+dx))^2}\,dx$

$$\frac{\frac{6\sin(c+dx)\sqrt{\sec(c+dx)}}{d(\sec(c+dx)+1)}-\frac{\frac{6a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-2a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2}}{6a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \; 3120$$

$$\frac{\frac{6\sin(c+dx)\sqrt{\sec(c+dx)}}{d(\sec(c+dx)+1)}-\frac{\frac{6a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-\frac{4a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}}{a^2}}{6a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

input
```
Int[Sqrt[Sec[c + d*x]]/(a + a*Sec[c + d*x])^2,x]
```

output
```
-1/3*(Sec[c + d*x]^(3/2)*Sin[c + d*x])/(d*(a + a*Sec[c + d*x])^2) + (-(((6
*a^2*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d -
(4*a^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d)
/a^2) + (6*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(d*(1 + Sec[c + d*x])))/(6*a^2
)
```

3.205.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4304
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-Cot[e + f*x])*(a + b*Csc[e + f*x])^m*((d*Csc
[e + f*x])^n/(f*(2*m + 1))), x] + Simp[1/(a^2*(2*m + 1))   Int[(a + b*Csc[e
 + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e
+ f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ
[m, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

rule 4507
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[d*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(a*f*(
2*m + 1))), x] - Simp[1/(a*b*(2*m + 1))   Int[(a + b*Csc[e + f*x])^(m + 1)*
(d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
 - n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f,
A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && G
tQ[n, 0]
```

3.205.4 Maple [A] (verified)

Time = 7.06 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.72

method	result
default	$-\frac{\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(12\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^6+4\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1}\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\cos}{6a^2\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\,\mathrm{co}}$

input
```
int(sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
-1/6/a^2*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(12*cos(1
/2*d*x+1/2*c)^6+4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^3+6*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*cos(1/2*d*x+1/2*c
)^3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-20*cos(1/2*d*x+1/2*c)^4+9*cos(1/
2*d*x+1/2*c)^2-1)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/cos
(1/2*d*x+1/2*c)^3/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

3.205.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.86

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+a\sec(c+dx))^2}\,dx =$$

$$-\frac{2\left(i\sqrt{2}\cos\left(dx+c\right)^2+2i\sqrt{2}\cos\left(dx+c\right)+i\sqrt{2}\right)\text{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+\right.}{}$$

input
```
integrate(sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
-1/6*(2*(I*sqrt(2)*cos(d*x + c)^2 + 2*I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*
weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 2*(-I*sqrt(2)*
cos(d*x + c)^2 - 2*I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassPInverse
(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 3*(I*sqrt(2)*cos(d*x + c)^2 + 2*I
*sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierstrassZeta(-4, 0, weierstrassPInve
rse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 3*(-I*sqrt(2)*cos(d*x + c)^2
- 2*I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassZeta(-4, 0, weierstrass
PInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - 2*(3*cos(d*x + c)^2 + 2*
cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^2*d*cos(d*x + c)^2 + 2*a
^2*d*cos(d*x + c) + a^2*d)
```

3.205.6 Sympy [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+a\sec(c+dx))^2}\,dx = \frac{\int \frac{\sqrt{\sec(c+dx)}}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx}{a^2}$$

input
```
integrate(sec(d*x+c)**(1/2)/(a+a*sec(d*x+c))**2,x)
```

output
```
Integral(sqrt(sec(c + d*x))/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a**
2
```

3.205.7 Maxima [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+a\sec(c+dx))^2}\,dx = \int \frac{\sqrt{\sec(dx+c)}}{(a\sec(dx+c)+a)^2}\,dx$$

input
```
integrate(sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(sec(d*x + c))/(a*sec(d*x + c) + a)^2, x)
```

3.205.8 Giac [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+a\sec(c+dx))^2}\,dx = \int \frac{\sqrt{\sec(dx+c)}}{(a\sec(dx+c)+a)^2}\,dx$$

input
```
integrate(sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate(sqrt(sec(d*x + c))/(a*sec(d*x + c) + a)^2, x)
```

3.205.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+a\sec(c+dx))^2}\,dx = \int \frac{\sqrt{\frac{1}{\cos(c+d\,x)}}}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^2}\,dx$$

input
```
int((1/cos(c + d*x))^(1/2)/(a + a/cos(c + d*x))^2,x)
```

output
```
int((1/cos(c + d*x))^(1/2)/(a + a/cos(c + d*x))^2, x)
```

3.206 $\int \frac{1}{\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^2}\,dx$

3.206.1 Optimal result 1670
3.206.2 Mathematica [C] (verified) 1671
3.206.3 Rubi [A] (verified) 1671
3.206.4 Maple [A] (verified) 1675
3.206.5 Fricas [C] (verification not implemented) 1675
3.206.6 Sympy [F] 1676
3.206.7 Maxima [F] 1676
3.206.8 Giac [F] 1677
3.206.9 Mupad [F(-1)] 1677

3.206.1 Optimal result

Integrand size = 23, antiderivative size = 152

$$\begin{aligned}\int \frac{1}{\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^2}\,dx &= \frac{4\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^2d} \\ &\quad - \frac{5\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{3a^2d} \\ &\quad - \frac{5\sqrt{\sec(c+dx)}\sin(c+dx)}{3a^2d(1+\sec(c+dx))} - \frac{\sqrt{\sec(c+dx)}\sin(c+dx)}{3d(a+a\sec(c+dx))^2}\end{aligned}$$

output
```
-5/3*sin(d*x+c)*sec(d*x+c)^(1/2)/a^2/d/(1+sec(d*x+c))-1/3*sin(d*x+c)*sec(d
*x+c)^(1/2)/d/(a+a*sec(d*x+c))^2+4*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*
x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)
^(1/2)/a^2/d-5/3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF
(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^2/d
```

3.206.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 2.28 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.71

$$\int \frac{1}{\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^2}\,dx = -\frac{i\sqrt{2}e^{-i(c+dx)}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\left(-3-16e^{i(c+dx)}-23e^{2i(c+dx)}-25e^{3i(c+dx)}-20e^{4i(c+dx)}-9e^{5i(c+dx)}-5ie^{i(c+d}\right.}{}$$

input
```
Integrate[1/(Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^2),x]
```

output
```
((-1/3*I)*Sqrt[2]*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*(-3 - 16
*E^(I*(c + d*x)) - 23*E^((2*I)*(c + d*x)) - 25*E^((3*I)*(c + d*x)) - 20*E^
((4*I)*(c + d*x)) - 9*E^((5*I)*(c + d*x)) - (5*I)*E^(I*(c + d*x))*(1 + E^(
I*(c + d*x)))^3*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + 4*E^((2*I)*
(c + d*x))*(1 + E^(I*(c + d*x)))^3*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeom
etric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))]))/(a^2*d*E^(I*(c + d*x))*(1
+ E^(I*(c + d*x)))^3)
```

3.206.3 Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.04, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.522$, Rules used = {3042, 4304, 27, 3042, 4508, 3042, 4274, 3042, 4258, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^2}dx$$

$$\Big\downarrow \text{4304}$$

$$-\frac{\int -\frac{7a-3a\sec(c+dx)}{2\sqrt{\sec(c+dx)}(\sec(c+dx)a+a)}dx}{3a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{7a-3a\sec(c+dx)}{\sqrt{\sec(c+dx)}(\sec(c+dx)a+a)}dx}{6a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{\int \frac{7a-3a\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)}dx}{6a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \; 4508$$

$$\frac{\frac{\int \frac{12a^2-5a^2\sec(c+dx)}{\sqrt{\sec(c+dx)}}dx}{a^2}-\frac{10\sin(c+dx)\sqrt{\sec(c+dx)}}{d(\sec(c+dx)+1)}}{6a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{\int \frac{12a^2-5a^2\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2}-\frac{10\sin(c+dx)\sqrt{\sec(c+dx)}}{d(\sec(c+dx)+1)}}{6a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \; 4274$$

$$\frac{\frac{12a^2\int \frac{1}{\sqrt{\sec(c+dx)}}dx-5a^2\int \sqrt{\sec(c+dx)}dx}{a^2}-\frac{10\sin(c+dx)\sqrt{\sec(c+dx)}}{d(\sec(c+dx)+1)}}{6a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{12a^2\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-5a^2\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}-\frac{10\sin(c+dx)\sqrt{\sec(c+dx)}}{d(\sec(c+dx)+1)}}{6a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \; 4258$$

$$\frac{\frac{12a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\cos(c+dx)}dx-5a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{a^2}-\frac{10\sin(c+dx)\sqrt{\sec(c+dx)}}{d(\sec(c+dx)+1)}}{6a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \; 3042$$

3.206. $\int \frac{1}{\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^2}\,dx$

$$\frac{\frac{\frac{12a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx-5a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2}-\frac{10\sin(c+dx)\sqrt{\sec(c+dx)}}{d(\sec(c+dx)+1)}}{6a^2}}{3d(a\sec(c+dx)+a)^2}\sin(c+dx)\sqrt{\sec(c+dx)}-$$

$$\Big\downarrow \; 3119$$

$$\frac{\frac{\frac{24a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-5a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2}-\frac{10\sin(c+dx)\sqrt{\sec(c+dx)}}{d(\sec(c+dx)+1)}}{6a^2}}{3d(a\sec(c+dx)+a)^2}\sin(c+dx)\sqrt{\sec(c+dx)}-$$

$$\Big\downarrow \; 3120$$

$$\frac{\frac{\frac{24a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-\frac{10a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}}{a^2}-\frac{10\sin(c+dx)\sqrt{\sec(c+dx)}}{d(\sec(c+dx)+1)}}{6a^2}}{3d(a\sec(c+dx)+a)^2}\sin(c+dx)\sqrt{\sec(c+dx)}-$$

input
```
Int[1/(Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^2),x]
```

output
```
-1/3*(Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(d*(a + a*Sec[c + d*x])^2) + (((24*
a^2*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d - (
10*a^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d)
/a^2 - (10*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(d*(1 + Sec[c + d*x])))/(6*a^2
)
```

3.206.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4304
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-Cot[e + f*x])*(a + b*Csc[e + f*x])^m*((d*Csc
[e + f*x])^n/(f*(2*m + 1))), x] + Simp[1/(a^2*(2*m + 1))   Int[(a + b*Csc[e
 + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e
+ f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ
[m, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

rule 4508
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
- a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
 1))), x] - Simp[1/(a^2*(2*m + 1))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Cs
c[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[
e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B
, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

3.206. $\int \frac{1}{\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^2}\,dx$

3.206.4 Maple [A] (verified)

Time = 7.19 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.69

method	result
default	$\frac{\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\left(24\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}+10\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+1}\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\cos\left(\right.\right.}{6a^{2}\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\cos}$

input
```
int(1/sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/6/a^2*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(24*cos(1/
2*d*x+1/2*c)^6+10*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^3+24*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*cos(1/2*d*x+1/2*
c)^3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-38*cos(1/2*d*x+1/2*c)^4+15*cos(
1/2*d*x+1/2*c)^2-1)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/c
os(1/2*d*x+1/2*c)^3/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

3.206.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.82

$$\int \frac{1}{\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^2}\,dx =$$
$$-\frac{5\left(-i\sqrt{2}\cos\left(dx+c\right)^{2}-2i\sqrt{2}\cos\left(dx+c\right)-i\sqrt{2}\right)\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\sin\left(dx\right.}{}$$

input
```
integrate(1/sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
-1/6*(5*(-I*sqrt(2)*cos(d*x + c)^2 - 2*I*sqrt(2)*cos(d*x + c) - I*sqrt(2))
*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 5*(I*sqrt(2)*
cos(d*x + c)^2 + 2*I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierstrassPInverse
(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 12*(-I*sqrt(2)*cos(d*x + c)^2 - 2
*I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassZeta(-4, 0, weierstrassPIn
verse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 12*(I*sqrt(2)*cos(d*x + c)^
2 + 2*I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierstrassZeta(-4, 0, weierstra
ssPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*(6*cos(d*x + c)^2 +
5*cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^2*d*cos(d*x + c)^2 + 2
*a^2*d*cos(d*x + c) + a^2*d)
```

3.206.6 Sympy [F]

$$\int \frac{1}{\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^2}\,dx = \frac{\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)+2\sec^{\frac{3}{2}}(c+dx)+\sqrt{\sec(c+dx)}}\,dx}{a^2}$$

input
```
integrate(1/sec(d*x+c)**(1/2)/(a+a*sec(d*x+c))**2,x)
```

output
```
Integral(1/(sec(c + d*x)**(5/2) + 2*sec(c + d*x)**(3/2) + sqrt(sec(c + d*x
))), x)/a**2
```

3.206.7 Maxima [F]

$$\int \frac{1}{\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^2}\,dx = \int \frac{1}{(a\sec(dx+c)+a)^2\sqrt{\sec(dx+c)}}\,dx$$

input
```
integrate(1/sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
integrate(1/((a*sec(d*x + c) + a)^2*sqrt(sec(d*x + c))), x)
```

3.206.8 Giac [F]

$$\int \frac{1}{\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^2}\,dx = \int \frac{1}{(a\sec(dx+c)+a)^2\sqrt{\sec(dx+c)}}\,dx$$

input
```
integrate(1/sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate(1/((a*sec(d*x + c) + a)^2*sqrt(sec(d*x + c))), x)
```

3.206.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^2}\,dx = \int \frac{1}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^2\sqrt{\frac{1}{\cos(c+d\,x)}}}\,dx$$

input
```
int(1/((a + a/cos(c + d*x))^2*(1/cos(c + d*x))^(1/2)),x)
```

output
```
int(1/((a + a/cos(c + d*x))^2*(1/cos(c + d*x))^(1/2)), x)
```

3.207 $\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx$

3.207.1 Optimal result . . . 1678
3.207.2 Mathematica [C] (verified) . . . 1679
3.207.3 Rubi [A] (verified) . . . 1679
3.207.4 Maple [A] (verified) . . . 1683
3.207.5 Fricas [C] (verification not implemented) . . . 1684
3.207.6 Sympy [F] . . . 1685
3.207.7 Maxima [F] . . . 1685
3.207.8 Giac [F] . . . 1685
3.207.9 Mupad [F(-1)] . . . 1686

3.207.1 Optimal result

Integrand size = 23, antiderivative size = 178

$$\begin{aligned}\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx &= -\frac{7\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^2d} \\ &\quad + \frac{10\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{3a^2d} + \frac{10\sin(c+dx)}{3a^2d\sqrt{\sec(c+dx)}} \\ &\quad - \frac{7\sin(c+dx)}{3a^2d\sqrt{\sec(c+dx)}(1+\sec(c+dx))} - \frac{\sin(c+dx)}{3d\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^2}\end{aligned}$$

output
```
10/3*sin(d*x+c)/a^2/d/sec(d*x+c)^(1/2)-7/3*sin(d*x+c)/a^2/d/(1+sec(d*x+c))
/sec(d*x+c)^(1/2)-1/3*sin(d*x+c)/d/(a+a*sec(d*x+c))^2/sec(d*x+c)^(1/2)-7*(
cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c
),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^2/d+10/3*(cos(1/2*d*x+1/2*c
)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*
x+c)^(1/2)*sec(d*x+c)^(1/2)/a^2/d
```

3.207.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 2.92 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.44

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx$$
$$= \frac{e^{-idx}\cos\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{5}{2}}(c+dx)(\cos(dx)+i\sin(dx))\left(-84i\cos\left(\frac{1}{2}(c+dx)\right)-63i\cos\left(\frac{3}{2}(c+dx)\right)-21i\right.}{}$$

input
```
Integrate[1/(Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^2),x]
```

output
```
(Cos[(c + d*x)/2]*Sec[c + d*x]^(5/2)*(Cos[d*x] + I*Sin[d*x])*((-84*I)*Cos[
(c + d*x)/2] - (63*I)*Cos[(3*(c + d*x))/2] - (21*I)*Cos[(5*(c + d*x))/2] +
 80*Cos[(c + d*x)/2]^3*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + ((7*
I)*(1 + E^(I*(c + d*x)))^3*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1
[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))])/E^((I/2)*(c + d*x)) + 3*Sin[(c + d*
x)/2] + 10*Sin[(3*(c + d*x))/2] + 12*Sin[(5*(c + d*x))/2] + Sin[(7*(c + d*
x))/2]))/(6*a^2*d*E^(I*d*x)*(1 + Sec[c + d*x])^2)
```

3.207.3 Rubi [A] (verified)

Time = 0.96 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.05, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.652$, Rules used = {3042, 4304, 27, 3042, 4508, 27, 3042, 4274, 3042, 4256, 3042, 4258, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^2}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^2}dx$$
$$\Big\downarrow \text{4304}$$
$$-\frac{\int -\frac{9a-5a\sec(c+dx)}{2\sec^{\frac{3}{2}}(c+dx)(\sec(c+dx)a+a)}dx}{3a^2}-\frac{\sin(c+dx)}{3d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{9a-5a\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(\sec(c+dx)a+a)}dx}{6a^2}-\frac{\sin(c+dx)}{3d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{\int \frac{9a-5a\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)}dx}{6a^2}-\frac{\sin(c+dx)}{3d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \; 4508$$

$$\frac{\frac{\int \frac{3\left(10a^2-7a^2\sec(c+dx)\right)}{\sec^{\frac{3}{2}}(c+dx)}dx}{a^2}-\frac{14\sin(c+dx)}{d\sqrt{\sec(c+dx)}(\sec(c+dx)+1)}}{6a^2}-\frac{\sin(c+dx)}{3d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{3\int \frac{10a^2-7a^2\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)}dx}{a^2}-\frac{14\sin(c+dx)}{d\sqrt{\sec(c+dx)}(\sec(c+dx)+1)}}{6a^2}-\frac{\sin(c+dx)}{3d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{3\int \frac{10a^2-7a^2\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx}{a^2}-\frac{14\sin(c+dx)}{d\sqrt{\sec(c+dx)}(\sec(c+dx)+1)}}{6a^2}-\frac{\sin(c+dx)}{3d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \; 4274$$

$$\frac{\frac{3\left(10a^2\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}dx-7a^2\int \frac{1}{\sqrt{\sec(c+dx)}}dx\right)}{a^2}-\frac{14\sin(c+dx)}{d\sqrt{\sec(c+dx)}(\sec(c+dx)+1)}}{6a^2}-\frac{\sin(c+dx)}{3d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{3\left(10a^2\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx-7a^2\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)}{a^2}-\frac{14\sin(c+dx)}{d\sqrt{\sec(c+dx)}(\sec(c+dx)+1)}}{6a^2}-\frac{\sin(c+dx)}{3d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \; 4256$$

3.207. $\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2}dx$

$$\frac{\frac{3\left(10a^2\left(\frac{1}{3}\int\sqrt{\sec(c+dx)}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)-7a^2\int\frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)}{a^2}-\frac{14\sin(c+dx)}{d\sqrt{\sec(c+dx)}(\sec(c+dx)+1)}}{6a^2}-\frac{\sin(c+dx)}{3d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^2}$$

$\downarrow$ 3042

$$\frac{\frac{3\left(10a^2\left(\frac{1}{3}\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)-7a^2\int\frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)}{a^2}-\frac{14\sin(c+dx)}{d\sqrt{\sec(c+dx)}(\sec(c+dx)+1)}}{6a^2}-\frac{\sin(c+dx)}{3d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^2}$$

$\downarrow$ 4258

$$\frac{\frac{3\left(10a^2\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)-7a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\cos(c+dx)}dx\right)}{a^2}-\frac{14\sin(c+dx)}{d\sqrt{\sec(c+dx)}(\sec(c+}}{6a^2}$$
$$\frac{\sin(c+dx)}{3d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^2}$$

$\downarrow$ 3042

$$\frac{\frac{3\left(10a^2\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)-7a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx\right)}{a^2}-\frac{14\,\mathrm{si}}{d\sqrt{\sec(c+d}}}{6a^2}$$
$$\frac{\sin(c+dx)}{3d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^2}$$

$\downarrow$ 3119

$$\frac{\frac{3\left(10a^2\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)-\frac{14a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)}{a^2}-\frac{14\sin(c+dx)}{d\sqrt{\sec(c+dx)}(\sec(c+}}{6a^2}$$
$$\frac{\sin(c+dx)}{3d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^2}$$

$\downarrow$ 3120

3.207. $\int\frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx$

$$\frac{\frac{3\left(10a^2\left(\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}\right)-\frac{14a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)}{a^2}}{6a^2}-\frac{14\sin(c+dx)}{d\sqrt{\sec(c+dx)}(\sec(c+dx)+}$$

$$\frac{\sin(c+dx)}{3d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^2}$$

input
```
Int[1/(Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^2),x]
```

output
```
-1/3*Sin[c + d*x]/(d*Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^2) + ((-14*Si
n[c + d*x])/(d*Sqrt[Sec[c + d*x]]*(1 + Sec[c + d*x])) + (3*((-14*a^2*Sqrt[
Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + 10*a^2*((2
*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) +
(2*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]]))))/a^2)/(6*a^2)
```

3.207.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4304
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-Cot[e + f*x])*(a + b*Csc[e + f*x])^m*((d*Csc
[e + f*x])^n/(f*(2*m + 1))), x] + Simp[1/(a^2*(2*m + 1))   Int[(a + b*Csc[e
 + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e
+ f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ
[m, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

rule 4508
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
- a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
 1))), x] - Simp[1/(a^2*(2*m + 1))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Cs
c[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[
e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B
, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

3.207.4 Maple [A] (verified)

Time = 8.14 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.52

method	result
default	$-\frac{\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(16\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^8+12\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^6+20\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1}\,\mathrm{EllipticF}\left(c\right.\right.}{6a^2\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\sin\left(\right.}}$

input
```
int(1/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
-1/6/a^2*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(16*cos(1
/2*d*x+1/2*c)^8+12*cos(1/2*d*x+1/2*c)^6+20*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-
2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(
1/2*d*x+1/2*c)^3+42*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+
1)^(1/2)*cos(1/2*d*x+1/2*c)^3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-48*cos
(1/2*d*x+1/2*c)^4+21*cos(1/2*d*x+1/2*c)^2-1)/(-2*sin(1/2*d*x+1/2*c)^4+sin(
1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)^3/sin(1/2*d*x+1/2*c)/(2*cos(1/2
*d*x+1/2*c)^2-1)^(1/2)/d
```

3.207.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.13 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.61

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx =$$

$$-\frac{10\left(i\sqrt{2}\cos\left(dx+c\right)^2+2i\sqrt{2}\cos\left(dx+c\right)+i\sqrt{2}\right)\text{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+\right.}{}$$

input
```
integrate(1/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
-1/6*(10*(I*sqrt(2)*cos(d*x + c)^2 + 2*I*sqrt(2)*cos(d*x + c) + I*sqrt(2))
*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 10*(-I*sqrt(2
)*cos(d*x + c)^2 - 2*I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassPInver
se(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 21*(I*sqrt(2)*cos(d*x + c)^2 +
2*I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierstrassZeta(-4, 0, weierstrassPI
nverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 21*(-I*sqrt(2)*cos(d*x + c
)^2 - 2*I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassZeta(-4, 0, weierst
rassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - 2*(2*cos(d*x + c)^3
+ 13*cos(d*x + c)^2 + 10*cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a
^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)
```

3.207.6 Sympy [F]

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx = \frac{\int \frac{1}{\sec^{\frac{7}{2}}(c+dx)+2\sec^{\frac{5}{2}}(c+dx)+\sec^{\frac{3}{2}}(c+dx)}\,dx}{a^2}$$

input
```
integrate(1/sec(d*x+c)**(3/2)/(a+a*sec(d*x+c))**2,x)
```

output
```
Integral(1/(sec(c + d*x)**(7/2) + 2*sec(c + d*x)**(5/2) + sec(c + d*x)**(3
/2)), x)/a**2
```

3.207.7 Maxima [F]

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx = \int \frac{1}{(a\sec(dx+c)+a)^2\sec(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
integrate(1/((a*sec(d*x + c) + a)^2*sec(d*x + c)^(3/2)), x)
```

3.207.8 Giac [F]

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx = \int \frac{1}{(a\sec(dx+c)+a)^2\sec(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate(1/((a*sec(d*x + c) + a)^2*sec(d*x + c)^(3/2)), x)
```

3.207.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx=\int \frac{1}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^2\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

input
```
int(1/((a + a/cos(c + d*x))^2*(1/cos(c + d*x))^(3/2)),x)
```

output
```
int(1/((a + a/cos(c + d*x))^2*(1/cos(c + d*x))^(3/2)), x)
```

3.208 $\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx$

3.208.1 Optimal result . . . 1687
3.208.2 Mathematica [C] (verified) . . . 1688
3.208.3 Rubi [A] (verified) . . . 1688
3.208.4 Maple [A] (verified) . . . 1692
3.208.5 Fricas [C] (verification not implemented) . . . 1693
3.208.6 Sympy [F] . . . 1694
3.208.7 Maxima [F] . . . 1694
3.208.8 Giac [F] . . . 1694
3.208.9 Mupad [F(-1)] . . . 1695

3.208.1 Optimal result

Integrand size = 23, antiderivative size = 200

$$\begin{aligned}\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx &= \frac{56\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5a^2d} \\ &\quad - \frac{5\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{a^2d} \\ &\quad + \frac{56\sin(c+dx)}{15a^2d\sec^{\frac{3}{2}}(c+dx)} - \frac{5\sin(c+dx)}{a^2d\sqrt{\sec(c+dx)}} \\ &\quad - \frac{3\sin(c+dx)}{a^2d\sec^{\frac{3}{2}}(c+dx)(1+\sec(c+dx))} - \frac{\sin(c+dx)}{3d\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2}\end{aligned}$$

output
```
56/15*sin(d*x+c)/a^2/d/sec(d*x+c)^(3/2)-3*sin(d*x+c)/a^2/d/sec(d*x+c)^(3/2
)/(1+sec(d*x+c))-1/3*sin(d*x+c)/d/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^2-5*si
n(d*x+c)/a^2/d/sec(d*x+c)^(1/2)+56/5*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*
d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+
c)^(1/2)/a^2/d-5*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF
(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^2/d
```

3.208.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 3.15 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.36

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx$$
$$= \frac{e^{-idx}\cos\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{5}{2}}(c+dx)(\cos(dx)+i\sin(dx))\left(1344i\cos\left(\frac{1}{2}(c+dx)\right)+1008i\cos\left(\frac{3}{2}(c+dx)\right)+\right.}{}$$

input
```
Integrate[1/(Sec[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^2),x]
```

output
```
(Cos[(c + d*x)/2]*Sec[c + d*x]^(5/2)*(Cos[d*x] + I*Sin[d*x])*((1344*I)*Cos
[(c + d*x)/2] + (1008*I)*Cos[(3*(c + d*x))/2] + (336*I)*Cos[(5*(c + d*x))/
2] - 1200*Cos[(c + d*x)/2]^3*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]
- ((112*I)*(1 + E^(I*(c + d*x)))^3*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeom
etric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))])/E^((I/2)*(c + d*x)) - 34*Si
n[(c + d*x)/2] - 148*Sin[(3*(c + d*x))/2] - 168*Sin[(5*(c + d*x))/2] - 11*
Sin[(7*(c + d*x))/2] + 3*Sin[(9*(c + d*x))/2]))/(60*a^2*d*E^(I*d*x)*(1 + S
ec[c + d*x])^2)
```

3.208.3 Rubi [A] (verified)

Time = 1.00 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.07, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.609$, Rules used = {3042, 4304, 27, 3042, 4508, 3042, 4274, 3042, 4256, 3042, 4258, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a\sec(c+dx)+a)^2}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^2}dx$$
$$\Big\downarrow \; 4304$$

$$-\frac{\int -\frac{11a-7a\sec(c+dx)}{2\sec^{\frac{5}{2}}(c+dx)(\sec(c+dx)a+a)}dx}{3a^2}-\frac{\sin(c+dx)}{3d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^2}$$

$\Big\downarrow$ 27

$$\frac{\int \frac{11a-7a\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)(\sec(c+dx)a+a)}dx}{6a^2}-\frac{\sin(c+dx)}{3d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^2}$$

$\Big\downarrow$ 3042

$$\frac{\int \frac{11a-7a\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)}dx}{6a^2}-\frac{\sin(c+dx)}{3d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^2}$$

$\Big\downarrow$ 4508

$$\frac{\frac{\int \frac{56a^2-45a^2\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)}dx}{a^2}-\frac{18\sin(c+dx)}{d\sec^{\frac{3}{2}}(c+dx)(\sec(c+dx)+1)}}{6a^2}-\frac{\sin(c+dx)}{3d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^2}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\int \frac{56a^2-45a^2\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx}{a^2}-\frac{18\sin(c+dx)}{d\sec^{\frac{3}{2}}(c+dx)(\sec(c+dx)+1)}}{6a^2}-\frac{\sin(c+dx)}{3d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^2}$$

$\Big\downarrow$ 4274

$$\frac{\frac{56a^2\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)}dx-45a^2\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}dx}{a^2}-\frac{18\sin(c+dx)}{d\sec^{\frac{3}{2}}(c+dx)(\sec(c+dx)+1)}}{6a^2}-\frac{\sin(c+dx)}{3d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^2}$$

$\Big\downarrow$ 3042

$$\frac{\frac{56a^2\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx-45a^2\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx}{a^2}-\frac{18\sin(c+dx)}{d\sec^{\frac{3}{2}}(c+dx)(\sec(c+dx)+1)}}{6a^2}-\frac{\sin(c+dx)}{3d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^2}$$

$\Big\downarrow$ 4256

3.208. $\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^2}dx$

$$\frac{\frac{56a^2\left(\frac{3}{5}\int \frac{1}{\sqrt{\sec(c+dx)}}dx+\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}\right)-45a^2\left(\frac{1}{3}\int \sqrt{\sec(c+dx)}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)}{a^2}-\frac{18\sin(c+dx)}{d\sec^{\frac{3}{2}}(c+dx)(\sec(c+dx)+1)}}{6a^2}-\frac{\sin(c+dx)}{3d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^2}$$

$\downarrow$ 3042

$$\frac{\frac{56a^2\left(\frac{3}{5}\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}\right)-45a^2\left(\frac{1}{3}\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)}{a^2}-\frac{18\sin(c+dx)}{d\sec^{\frac{3}{2}}(c+dx)(\sec(c+dx)+1)}}{6a^2}-\frac{\sin(c+dx)}{3d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^2}$$

$\downarrow$ 4258

$$\frac{\frac{56a^2\left(\frac{3}{5}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\cos(c+dx)}dx+\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}\right)-45a^2\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)}{a^2}-\frac{}{d\sec}}{6a^2}-\frac{\sin(c+dx)}{3d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^2}$$

$\downarrow$ 3042

$$\frac{\frac{56a^2\left(\frac{3}{5}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}\right)-45a^2\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)}{a^2}}{6a^2}-\frac{\sin(c+dx)}{3d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^2}$$

$\downarrow$ 3119

$$\frac{\frac{56a^2\left(\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{6\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}\right)-45a^2\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)}{a^2}-\frac{}{d\sec^{\frac{3}{2}}}}{6a^2}-\frac{\sin(c+dx)}{3d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^2}$$

$\downarrow$ 3120

3.208. $\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx$

$$\frac{\frac{\frac{56a^2\left(\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{6\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}\right)-45a^2\left(\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}\right)}{a^2}-\frac{18}{d\sec^{\frac{3}{2}}(c+}}{6a^2}}{\frac{\sin(c+dx)}{3d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^2}}$$

input
```
Int[1/(Sec[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^2),x]
```

output
```
-1/3*Sin[c + d*x]/(d*Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^2) + ((-18*Si
n[c + d*x])/(d*Sec[c + d*x]^(3/2)*(1 + Sec[c + d*x])) + (56*a^2*((6*Sqrt[C
os[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d) + (2*Sin[
c + d*x])/(5*d*Sec[c + d*x]^(3/2))) - 45*a^2*((2*Sqrt[Cos[c + d*x]]*Ellipt
icF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (2*Sin[c + d*x])/(3*d*Sqrt
[Sec[c + d*x]])))/a^2)/(6*a^2)
```

3.208.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4304
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-Cot[e + f*x])*(a + b*Csc[e + f*x])^m*((d*Csc
[e + f*x])^n/(f*(2*m + 1))), x] + Simp[1/(a^2*(2*m + 1))   Int[(a + b*Csc[e
 + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e
+ f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ
[m, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

rule 4508
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
- a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
 1))), x] - Simp[1/(a^2*(2*m + 1))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Cs
c[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[
e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B
, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

3.208.4 Maple [A] (verified)

Time = 7.92 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.42

method	result
default	$-\frac{\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(96\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}-352\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^8+120\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^6-150\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\cos\left(\frac{dx}{2}\right.}\right.}{30a^2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)}$

input
```
int(1/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
-1/30/a^2*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(96*cos(
1/2*d*x+1/2*c)^10-352*cos(1/2*d*x+1/2*c)^8+120*cos(1/2*d*x+1/2*c)^6-150*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos
(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^3-336*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*cos(1/2*d*x+1/2*c)^3*EllipticE(cos(
1/2*d*x+1/2*c),2^(1/2))+266*cos(1/2*d*x+1/2*c)^4-135*cos(1/2*d*x+1/2*c)^2+
5)/cos(1/2*d*x+1/2*c)^3/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/
2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

3.208.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.48

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx =$$

$$-\frac{75\left(-i\sqrt{2}\cos\left(dx+c\right)^2-2i\sqrt{2}\cos\left(dx+c\right)-i\sqrt{2}\right)\text{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\sin\left(dx\right.}{}$$

input
```
integrate(1/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
-1/30*(75*(-I*sqrt(2)*cos(d*x + c)^2 - 2*I*sqrt(2)*cos(d*x + c) - I*sqrt(2
))*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 75*(I*sqrt(
2)*cos(d*x + c)^2 + 2*I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierstrassPInve
rse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 168*(-I*sqrt(2)*cos(d*x + c)^2
 - 2*I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassZeta(-4, 0, weierstras
sPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 168*(I*sqrt(2)*cos(d*x
+ c)^2 + 2*I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierstrassZeta(-4, 0, weie
rstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - 2*(6*cos(d*x + c)
^4 - 8*cos(d*x + c)^3 - 94*cos(d*x + c)^2 - 75*cos(d*x + c))*sin(d*x + c)/
sqrt(cos(d*x + c)))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)
```

3.208.6 Sympy [F]

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx = \frac{\int \frac{1}{\sec^{\frac{9}{2}}(c+dx)+2\sec^{\frac{7}{2}}(c+dx)+\sec^{\frac{5}{2}}(c+dx)}\,dx}{a^2}$$

input
```
integrate(1/sec(d*x+c)**(5/2)/(a+a*sec(d*x+c))**2,x)
```

output
```
Integral(1/(sec(c + d*x)**(9/2) + 2*sec(c + d*x)**(7/2) + sec(c + d*x)**(5
/2)), x)/a**2
```

3.208.7 Maxima [F]

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx = \int \frac{1}{(a\sec(dx+c)+a)^2\sec(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
integrate(1/((a*sec(d*x + c) + a)^2*sec(d*x + c)^(5/2)), x)
```

3.208.8 Giac [F]

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx = \int \frac{1}{(a\sec(dx+c)+a)^2\sec(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate(1/((a*sec(d*x + c) + a)^2*sec(d*x + c)^(5/2)), x)
```

3.208.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx = \int \frac{1}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^2\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

input
```
int(1/((a + a/cos(c + d*x))^2*(1/cos(c + d*x))^(5/2)),x)
```

output
```
int(1/((a + a/cos(c + d*x))^2*(1/cos(c + d*x))^(5/2)), x)
```

3.209 $\int \frac{\sec^{\frac{11}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

3.209.1 Optimal result 1696
3.209.2 Mathematica [C] (verified) 1697
3.209.3 Rubi [A] (verified) 1697
3.209.4 Maple [A] (verified) 1702
3.209.5 Fricas [C] (verification not implemented) 1703
3.209.6 Sympy [F(-1)] 1704
3.209.7 Maxima [F(-1)] 1704
3.209.8 Giac [F] 1705
3.209.9 Mupad [F(-1)] 1705

3.209.1 Optimal result

Integrand size = 23, antiderivative size = 247

$$\begin{aligned}\int \frac{\sec^{\frac{11}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx &= \frac{119\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{10a^3d} \\ &\quad + \frac{11\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{2a^3d} \\ &\quad - \frac{119\sqrt{\sec(c+dx)}\sin(c+dx)}{10a^3d} \\ &\quad + \frac{11\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2a^3d} - \frac{\sec^{\frac{9}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} \\ &\quad - \frac{2\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{3ad(a+a\sec(c+dx))^2} - \frac{119\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{30d\left(a^3+a^3\sec(c+dx)\right)}\end{aligned}$$

output
```
11/2*sec(d*x+c)^(3/2)*sin(d*x+c)/a^3/d-1/5*sec(d*x+c)^(9/2)*sin(d*x+c)/d/(
a+a*sec(d*x+c))^3-2/3*sec(d*x+c)^(7/2)*sin(d*x+c)/a/d/(a+a*sec(d*x+c))^2-1
19/30*sec(d*x+c)^(5/2)*sin(d*x+c)/d/(a^3+a^3*sec(d*x+c))-119/10*sin(d*x+c)
*sec(d*x+c)^(1/2)/a^3/d+119/10*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/
2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/
2)/a^3/d+11/2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(si
n(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^3/d
```

3.209.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 6.22 (sec) , antiderivative size = 378, normalized size of antiderivative = 1.53

$$\int \frac{\sec^{\frac{11}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx$$
$$= \frac{e^{-idx}\csc\left(\frac{c}{2}\right)\left(-119\sqrt{2}e^{2idx}(-1+e^{2ic})\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\sqrt{1+e^{2i(c+dx)}}\cos^6\left(\frac{1}{2}(c+dx)\right)\text{Hypergeometric2F1}\left(\frac{1}{2}\right.\right.}{}$$

input
```
Integrate[Sec[c + d*x]^(11/2)/(a + a*Sec[c + d*x])^3,x]
```

output
```
(Csc[c/2]*(-119*Sqrt[2]*E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Sqrt[E^(I*(c + d*
x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*Cos[(c + d*x)
/2]^6*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))]*Sec[c/2]*Sec[
c + d*x]^3 + ((-1 + E^(I*c))*Cos[(c + d*x)/2]*(165 + 944*E^(I*(c + d*x)) +
 2476*E^((2*I)*(c + d*x)) + 4148*E^((3*I)*(c + d*x)) + 5134*E^((4*I)*(c +
d*x)) + 4664*E^((5*I)*(c + d*x)) + 3340*E^((6*I)*(c + d*x)) + 1620*E^((7*I
)*(c + d*x)) + 357*E^((8*I)*(c + d*x)) - (165*I)*(1 + E^(I*(c + d*x)))^5*(
1 + E^((2*I)*(c + d*x)))*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])*Sec
[c + d*x]^(7/2))/(16*E^(((3*I)/2)*(2*c + d*x))*(1 + E^((2*I)*(c + d*x)))))
)/(15*a^3*d*E^(I*d*x)*(1 + Sec[c + d*x])^3)
```

3.209.3 Rubi [A] (verified)

Time = 1.33 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.06, number of steps used = 17, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.739$, Rules used = {3042, 4303, 27, 3042, 4507, 3042, 4507, 27, 3042, 4274, 3042, 4255, 3042, 4258, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{\frac{11}{2}}(c+dx)}{(a\sec(c+dx)+a)^3}\,dx$$
$$\Big\downarrow \; 3042$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{11/2}}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}dx$$

$$\Big\downarrow \; 4303$$

$$-\frac{\int \frac{\sec^{\frac{7}{2}}(c+dx)(7a-13a\sec(c+dx))}{2(\sec(c+dx)a+a)^2}dx}{5a^2}-\frac{\sin(c+dx)\sec^{\frac{9}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \; 27$$

$$-\frac{\int \frac{\sec^{\frac{7}{2}}(c+dx)(7a-13a\sec(c+dx))}{(\sec(c+dx)a+a)^2}dx}{10a^2}-\frac{\sin(c+dx)\sec^{\frac{9}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}\left(7a-13a\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}dx}{10a^2}-\frac{\sin(c+dx)\sec^{\frac{9}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \; 4507$$

$$-\frac{\frac{\int \frac{\sec^{\frac{5}{2}}(c+dx)\left(50a^2-69a^2\sec(c+dx)\right)}{\sec(c+dx)a+a}dx}{3a^2}+\frac{20a\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}}{10a^2}-\frac{\sin(c+dx)\sec^{\frac{9}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(50a^2-69a^2\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{3a^2}+\frac{20a\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}}{10a^2}-\frac{\sin(c+dx)\sec^{\frac{9}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \; 4507$$

$$-\frac{\frac{\frac{\int \frac{3}{2}\sec^{\frac{3}{2}}(c+dx)\left(119a^3-165a^3\sec(c+dx)\right)dx}{a^2}+\frac{119a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{d(a\sec(c+dx)+a)}}{3a^2}+\frac{20a\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}}{10a^2}-\frac{\sin(c+dx)\sec^{\frac{9}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \; 27$$

$$-\frac{\frac{\frac{3\int \sec^{\frac{3}{2}}(c+dx)\left(119a^3-165a^3\sec(c+dx)\right)dx}{2a^2}+\frac{119a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{d(a\sec(c+dx)+a)}}{3a^2}+\frac{20a\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}}{10a^2}-\frac{\sin(c+dx)\sec^{\frac{9}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \; 3042$$

3.209. $\int \frac{\sec^{\frac{11}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

$$-\frac{\frac{\frac{3\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(119a^3-165a^3\csc\left(c+dx+\frac{\pi}{2}\right)\right)dx}{2a^2}+\frac{119a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{d(a\sec(c+dx)+a)}}{3a^2}+\frac{20a\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}}{10a^2}-\frac{\sin(c+dx)\sec^{\frac{9}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$\downarrow$ 4274

$$-\frac{\frac{\frac{3\left(119a^3\int \sec^{\frac{3}{2}}(c+dx)dx-165a^3\int \sec^{\frac{5}{2}}(c+dx)dx\right)}{2a^2}+\frac{119a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{d(a\sec(c+dx)+a)}}{3a^2}+\frac{20a\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}}{10a^2}-\frac{\sin(c+dx)\sec^{\frac{9}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$\downarrow$ 3042

$$-\frac{\frac{\frac{3\left(119a^3\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}dx-165a^3\int \csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}dx\right)}{2a^2}+\frac{119a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{d(a\sec(c+dx)+a)}}{3a^2}+\frac{20a\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}}{10a^2}-\frac{\sin(c+dx)\sec^{\frac{9}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$\downarrow$ 4255

$$-\frac{\frac{\frac{3\left(119a^3\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\int \frac{1}{\sqrt{\sec(c+dx)}}dx\right)-165a^3\left(\frac{1}{3}\int \sqrt{\sec(c+dx)}dx+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}\right)\right)}{2a^2}+\frac{119a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{d(a\sec(c+dx)+a)}}{3a^2}+\frac{20a\sin(c}{3d(a\,s}}{10a^2}-\frac{\sin(c+dx)\sec^{\frac{9}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$\downarrow$ 3042

$$-\frac{\frac{\frac{3\left(119a^3\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)-165a^3\left(\frac{1}{3}\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}\right)\right)}{2a^2}+\frac{119a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{d(a\sec(c+dx)+a)}}{3a^2}+}{10a^2}-\frac{\sin(c+dx)\sec^{\frac{9}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$\downarrow$ 4258

3.209. $\int \frac{\sec^{\frac{11}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

$$-\frac{\frac{\frac{3\left(119a^3\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\cos(c+dx)}dx\right)-165a^3\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}\right)}{2a^2}}{3a^2}}{10a^2}-\frac{\sin(c+dx)\sec^{\frac{9}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$\downarrow$ 3042

$$-\frac{\frac{\frac{3\left(119a^3\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx\right)-165a^3\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}}{3d}\right.}{2a^2}}{3a^2}}{10a^2}-\frac{\sin(c+dx)\sec^{\frac{9}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$\downarrow$ 3119

$$-\frac{\frac{\frac{3\left(119a^3\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)-165a^3\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+}{3d}\right.}{2a^2}}{3a^2}}{10a^2}-\frac{\sin(c+dx)\sec^{\frac{9}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$\downarrow$ 3120

$$-\frac{\frac{\frac{119a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{d(a\sec(c+dx)+a)}+\frac{3\left(119a^3\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)-165a^3\left(\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{2\sqrt{\cos(c+dx}}{}\right.}{2a^2}}{3a^2}}{10a^2}-\frac{\sin(c+dx)\sec^{\frac{9}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

input
```
Int[Sec[c + d*x]^(11/2)/(a + a*Sec[c + d*x])^3,x]
```

3.209. $\int \frac{\sec^{\frac{11}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

output
```
-1/5*(Sec[c + d*x]^(9/2)*Sin[c + d*x])/(d*(a + a*Sec[c + d*x])^3) - ((20*a
*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(3*d*(a + a*Sec[c + d*x])^2) + ((119*a^2
*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(d*(a + a*Sec[c + d*x])) + (3*(119*a^3*(
(-2*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (
2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d) - 165*a^3*((2*Sqrt[Cos[c + d*x]]*Ell
ipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (2*Sec[c + d*x]^(3/2)*S
in[c + d*x])/(3*d))))/(2*a^2))/(3*a^2))/(10*a^2)
```

3.209.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

3.209. $\int \frac{\sec^{\frac{11}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

rule 4303
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-d^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d
*Csc[e + f*x])^(n - 2)/(f*(2*m + 1))), x] + Simp[d^2/(a*b*(2*m + 1))   Int[
(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n
 + 2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2,
0] && LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

rule 4507
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[d*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(a*f*(
2*m + 1))), x] - Simp[1/(a*b*(2*m + 1))   Int[(a + b*Csc[e + f*x])^(m + 1)*
(d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
 - n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f,
A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && G
tQ[n, 0]
```

3.209.4 Maple [A] (verified)

Time = 61.12 (sec) , antiderivative size = 453, normalized size of antiderivative = 1.83

method	result
default	$-\frac{\sqrt{-\left(-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\left(\frac{\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}}{5\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}+\frac{32\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}}{15\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}+\frac{118\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}}{5\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}\right.}{}$

input
```
int(sec(d*x+c)^(11/2)/(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
-1/4*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/a^3*(1/5*(-
2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)^5+32
/15*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c
)^3+118/5*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x
+1/2*c)-128/5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/
2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*
d*x+1/2*c),2^(1/2))+238/5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2
*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(Ellip
ticF(cos(1/2*d*x+1/2*c),2^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))-4/
3*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/
(cos(1/2*d*x+1/2*c)^2-1/2)^2+48*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)/(-
(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2))/sin(1/2*d*x+1/2*c
)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

3.209.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 404, normalized size of antiderivative = 1.64

$$\int \frac{\sec^{\frac{11}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx =$$

$$-\frac{165\left(i\sqrt{2}\cos\left(dx+c\right)^4+3i\sqrt{2}\cos\left(dx+c\right)^3+3i\sqrt{2}\cos\left(dx+c\right)^2+i\sqrt{2}\cos\left(dx+c\right)\right)\text{weierstrassPInv}}{}$$

input
```
integrate(sec(d*x+c)^(11/2)/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
-1/60*(165*(I*sqrt(2)*cos(d*x + c)^4 + 3*I*sqrt(2)*cos(d*x + c)^3 + 3*I*sq
rt(2)*cos(d*x + c)^2 + I*sqrt(2)*cos(d*x + c))*weierstrassPInverse(-4, 0,
cos(d*x + c) + I*sin(d*x + c)) + 165*(-I*sqrt(2)*cos(d*x + c)^4 - 3*I*sqrt
(2)*cos(d*x + c)^3 - 3*I*sqrt(2)*cos(d*x + c)^2 - I*sqrt(2)*cos(d*x + c))*
weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 357*(-I*sqrt(2
)*cos(d*x + c)^4 - 3*I*sqrt(2)*cos(d*x + c)^3 - 3*I*sqrt(2)*cos(d*x + c)^2
 - I*sqrt(2)*cos(d*x + c))*weierstrassZeta(-4, 0, weierstrassPInverse(-4,
0, cos(d*x + c) + I*sin(d*x + c))) + 357*(I*sqrt(2)*cos(d*x + c)^4 + 3*I*s
qrt(2)*cos(d*x + c)^3 + 3*I*sqrt(2)*cos(d*x + c)^2 + I*sqrt(2)*cos(d*x + c
))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(
d*x + c))) + 2*(357*cos(d*x + c)^4 + 906*cos(d*x + c)^3 + 695*cos(d*x + c)
^2 + 120*cos(d*x + c) - 20)*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^3*d*cos(d*
x + c)^4 + 3*a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + a^3*d*cos(d*x
 + c))
```

3.209.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{11}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**(11/2)/(a+a*sec(d*x+c))**3,x)
```

output
```
Timed out
```

3.209.7 Maxima [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{11}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)^(11/2)/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
Timed out
```

3.209.8 Giac [F]

$$\int \frac{\sec^{\frac{11}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \int \frac{\sec(dx+c)^{\frac{11}{2}}}{(a\sec(dx+c)+a)^3}\,dx$$

input
```
integrate(sec(d*x+c)^(11/2)/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^(11/2)/(a*sec(d*x + c) + a)^3, x)
```

3.209.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{11}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{11/2}}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^3}\,dx$$

input
```
int((1/cos(c + d*x))^(11/2)/(a + a/cos(c + d*x))^3,x)
```

output
```
int((1/cos(c + d*x))^(11/2)/(a + a/cos(c + d*x))^3, x)
```

3.210 $\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

3.210.1 Optimal result . . . 1706
3.210.2 Mathematica [C] (verified) . . . 1707
3.210.3 Rubi [A] (verified) . . . 1707
3.210.4 Maple [B] (verified) . . . 1712
3.210.5 Fricas [C] (verification not implemented) . . . 1713
3.210.6 Sympy [F(-1)] . . . 1713
3.210.7 Maxima [F(-1)] . . . 1714
3.210.8 Giac [F] . . . 1714
3.210.9 Mupad [F(-1)] . . . 1714

3.210.1 Optimal result

Integrand size = 23, antiderivative size = 221

$$\begin{aligned}\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx = &-\frac{49\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{10a^3d}\\ &-\frac{13\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{6a^3d}\\ &+\frac{49\sqrt{\sec(c+dx)}\sin(c+dx)}{10a^3d}-\frac{\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3}\\ &-\frac{8\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{15ad(a+a\sec(c+dx))^2}-\frac{13\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{6d\left(a^3+a^3\sec(c+dx)\right)}\end{aligned}$$

output
```
-1/5*sec(d*x+c)^(7/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^3-8/15*sec(d*x+c)^(5/2
)*sin(d*x+c)/a/d/(a+a*sec(d*x+c))^2-13/6*sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a^
3+a^3*sec(d*x+c))+49/10*sin(d*x+c)*sec(d*x+c)^(1/2)/a^3/d-49/10*(cos(1/2*d
*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)
)*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^3/d-13/6*(cos(1/2*d*x+1/2*c)^2)^(1/2
)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2
)*sec(d*x+c)^(1/2)/a^3/d
```

3.210.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 3.12 (sec) , antiderivative size = 371, normalized size of antiderivative = 1.68

$$\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{2\cos^6\left(\frac{1}{2}(c+dx)\right)\left(-\frac{2i\sqrt{2}e^{-i(c+dx)}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\left(147\left(1+e^{2i(c+dx)}\right)+147\left(-1+e^{2ic}\right)\sqrt{1+e^{2i(c+dx)}}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},-e\right.\right.}{-1+e^{2ic}}\right.}{}$$

input
```
Integrate[Sec[c + d*x]^(9/2)/(a + a*Sec[c + d*x])^3,x]
```

output
```
(2*Cos[(c + d*x)/2]^6*(((-2*I)*Sqrt[2]*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*
(c + d*x)))]*(147*(1 + E^((2*I)*(c + d*x))) + 147*(-1 + E^((2*I)*c))*Sqrt[
1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c +
d*x))] - 65*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x)
)]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]))/(E^(I*(c + d*x
))*(-1 + E^((2*I)*c))) + ((1284*Cos[(c - d*x)/2] + 921*Cos[(3*c + d*x)/2]
+ 1243*Cos[(c + 3*d*x)/2] + 374*Cos[(5*c + 3*d*x)/2] + 670*Cos[(3*c + 5*d*
x)/2] + 65*Cos[(7*c + 5*d*x)/2] + 147*Cos[(5*c + 7*d*x)/2])*Csc[c/2]*Sec[c
/2]*Sec[(c + d*x)/2]^5*Sqrt[Sec[c + d*x]])/32)*Sec[c + d*x]^3)/(15*a^3*d*(
1 + Sec[c + d*x])^3)
```

3.210.3 Rubi [A] (verified)

Time = 1.30 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.06, number of steps used = 17, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.739$, Rules used = {3042, 4303, 27, 3042, 4507, 3042, 4507, 27, 3042, 4274, 3042, 4255, 3042, 4258, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a\sec(c+dx)+a)^3}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{9/2}}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}dx$$

$$\Big\downarrow \; 4303$$

$$-\frac{\int \frac{\sec^{\frac{5}{2}}(c+dx)(5a-11a\sec(c+dx))}{2(\sec(c+dx)a+a)^2}dx}{5a^2}-\frac{\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \; 27$$

$$-\frac{\int \frac{\sec^{\frac{5}{2}}(c+dx)(5a-11a\sec(c+dx))}{(\sec(c+dx)a+a)^2}dx}{10a^2}-\frac{\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(5a-11a\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}dx}{10a^2}-\frac{\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \; 4507$$

$$-\frac{\frac{\int \frac{\sec^{\frac{3}{2}}(c+dx)\left(24a^2-41a^2\sec(c+dx)\right)}{\sec(c+dx)a+a}dx}{3a^2}+\frac{16a\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}}{10a^2}-\frac{\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(24a^2-41a^2\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{3a^2}+\frac{16a\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}}{10a^2}-\frac{\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \; 4507$$

$$-\frac{\frac{\frac{\int \frac{1}{2}\sqrt{\sec(c+dx)}\left(65a^3-147a^3\sec(c+dx)\right)dx}{a^2}+\frac{65a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d(a\sec(c+dx)+a)}}{3a^2}+\frac{16a\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}}{10a^2}-\frac{\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \; 27$$

$$-\frac{\frac{\frac{\int \sqrt{\sec(c+dx)}\left(65a^3-147a^3\sec(c+dx)\right)dx}{2a^2}+\frac{65a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d(a\sec(c+dx)+a)}}{3a^2}+\frac{16a\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}}{10a^2}-\frac{\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \; 3042$$

3.210. $\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

$$-\frac{\frac{\frac{\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(65a^3-147a^3\csc\left(c+dx+\frac{\pi}{2}\right)\right)dx}{2a^2}+\frac{65a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d(a\sec(c+dx)+a)}}{3a^2}+\frac{16a\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}}{10a^2}-\frac{\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$\downarrow$ 4274

$$-\frac{\frac{\frac{65a^3\int\sqrt{\sec(c+dx)}dx-147a^3\int\sec^{\frac{3}{2}}(c+dx)dx}{2a^2}+\frac{65a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d(a\sec(c+dx)+a)}}{3a^2}+\frac{16a\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}}{10a^2}-\frac{\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$\downarrow$ 3042

$$-\frac{\frac{\frac{65a^3\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx-147a^3\int\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}dx}{2a^2}+\frac{65a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d(a\sec(c+dx)+a)}}{3a^2}+\frac{16a\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}}{10a^2}-\frac{\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$\downarrow$ 4255

$$-\frac{\frac{\frac{65a^3\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx-147a^3\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\int\frac{1}{\sqrt{\sec(c+dx)}}dx\right)}{2a^2}+\frac{65a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d(a\sec(c+dx)+a)}}{3a^2}+\frac{16a\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}}{10a^2}-\frac{\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$\downarrow$ 3042

$$-\frac{\frac{\frac{65a^3\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx-147a^3\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\int\frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)}{2a^2}+\frac{65a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d(a\sec(c+dx)+a)}}{3a^2}+\frac{16a\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}}{10a^2}-\frac{\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$\downarrow$ 4258

3.210. $\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

$$-\frac{\frac{\frac{65a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx-147a^3\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\cos(c+dx)}dx\right)}{2a^2}+\frac{65a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d(a\sec(c+dx)+a)}}{3a^2}+\frac{\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}}{10a^2}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\frac{\frac{65a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-147a^3\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx\right)}{2a^2}+\frac{65a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+}{d(a\sec(c+dx)+a)}}{3a^2}+\frac{\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}}{10a^2}$$

$$\Big\downarrow \; 3119$$

$$-\frac{\frac{\frac{65a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-147a^3\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)}{2a^2}+\frac{65a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx}{d(a\sec(c+dx)+a)}}{3a^2}+\frac{\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}}{10a^2}$$

$$\Big\downarrow \; 3120$$

$$-\frac{\frac{\frac{65a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d(a\sec(c+dx)+a)}+\frac{\frac{130a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}-147a^3\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)}{2a^2}}{3a^2}}{10a^2}+\frac{\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

input
```
Int[Sec[c + d*x]^(9/2)/(a + a*Sec[c + d*x])^3,x]
```

output
```
-1/5*(Sec[c + d*x]^(7/2)*Sin[c + d*x])/(d*(a + a*Sec[c + d*x])^3) - ((16*a
*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(3*d*(a + a*Sec[c + d*x])^2) + ((65*a^2*
Sec[c + d*x]^(3/2)*Sin[c + d*x])/(d*(a + a*Sec[c + d*x])) + ((130*a^3*Sqrt
[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d - 147*a^3*(
(-2*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (
2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d))/(2*a^2))/(3*a^2))/(10*a^2)
```

3.210. $\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

3.210.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4303
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-d^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d
*Csc[e + f*x])^(n - 2)/(f*(2*m + 1))), x] + Simp[d^2/(a*b*(2*m + 1))   Int[
(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n
 + 2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2,
0] && LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

3.210. $\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

rule 4507
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[d*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(a*f*(
2*m + 1))), x] - Simp[1/(a*b*(2*m + 1))   Int[(a + b*Csc[e + f*x])^(m + 1)*
(d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
 - n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f,
A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && G
tQ[n, 0]
```

3.210.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 554 vs. $2(245) = 490$.

Time = 57.49 (sec) , antiderivative size = 555, normalized size of antiderivative = 2.51

method	result
default	$-\frac{-2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1}\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(65\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)-147\,\mathrm{EllipticE}\left(\cos\left(\frac{dx}{2}\right.\right.\right.}{}$

input
```
int(sec(d*x+c)^(9/2)/(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
-1/60*(-2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-
2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(65*EllipticF(cos(1/2*d
*x+1/2*c),2^(1/2))-147*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*cos(1/2*d*x+
1/2*c)*sin(1/2*d*x+1/2*c)^4+4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+
1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(65
*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-147*EllipticE(cos(1/2*d*x+1/2*c),2^
(1/2)))*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-2*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x
+1/2*c)^2)^(1/2)*(65*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-147*EllipticE(c
os(1/2*d*x+1/2*c),2^(1/2)))*cos(1/2*d*x+1/2*c)+588*(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^8-1634*(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^6+1488*(-2*sin(1/2*d
*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-439*(-2*sin(1
/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2)/a^3/cos(
1/2*d*x+1/2*c)^5/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(
1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

3.210.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 354, normalized size of antiderivative = 1.60

$$\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{65\left(-i\sqrt{2}\cos(dx+c)^3-3i\sqrt{2}\cos(dx+c)^2-3i\sqrt{2}\cos(dx+c)-i\sqrt{2}\right)\text{weierstrassPInverse}(-4,0,c}{}$$

input
```
integrate(sec(d*x+c)^(9/2)/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
-1/60*(65*(-I*sqrt(2)*cos(d*x + c)^3 - 3*I*sqrt(2)*cos(d*x + c)^2 - 3*I*sq
rt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassPInverse(-4, 0, cos(d*x + c) +
I*sin(d*x + c)) + 65*(I*sqrt(2)*cos(d*x + c)^3 + 3*I*sqrt(2)*cos(d*x + c)^
2 + 3*I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierstrassPInverse(-4, 0, cos(d
*x + c) - I*sin(d*x + c)) + 147*(I*sqrt(2)*cos(d*x + c)^3 + 3*I*sqrt(2)*co
s(d*x + c)^2 + 3*I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierstrassZeta(-4, 0
, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 147*(-I*sqr
t(2)*cos(d*x + c)^3 - 3*I*sqrt(2)*cos(d*x + c)^2 - 3*I*sqrt(2)*cos(d*x + c
) - I*sqrt(2))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x +
 c) - I*sin(d*x + c))) - 2*(147*cos(d*x + c)^3 + 376*cos(d*x + c)^2 + 295*
cos(d*x + c) + 60)*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^3*d*cos(d*x + c)^3
+ 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

3.210.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**(9/2)/(a+a*sec(d*x+c))**3,x)
```

output
```
Timed out
```

3.210.7 Maxima [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)^(9/2)/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
Timed out
```

3.210.8 Giac [F]

$$\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \int \frac{\sec(dx+c)^{\frac{9}{2}}}{(a\sec(dx+c)+a)^3}\,dx$$

input
```
integrate(sec(d*x+c)^(9/2)/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^(9/2)/(a*sec(d*x + c) + a)^3, x)
```

3.210.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{9/2}}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^3}\,dx$$

input
```
int((1/cos(c + d*x))^(9/2)/(a + a/cos(c + d*x))^3,x)
```

output
```
int((1/cos(c + d*x))^(9/2)/(a + a/cos(c + d*x))^3, x)
```

3.211 $\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

3.211.1 Optimal result 1715
3.211.2 Mathematica [C] (verified) 1716
3.211.3 Rubi [A] (verified) 1716
3.211.4 Maple [A] (verified) 1720
3.211.5 Fricas [C] (verification not implemented) 1721
3.211.6 Sympy [F(-1)] 1722
3.211.7 Maxima [F(-1)] 1722
3.211.8 Giac [F] 1722
3.211.9 Mupad [F(-1)] 1723

3.211.1 Optimal result

Integrand size = 23, antiderivative size = 195

$$\begin{aligned}\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx &= \frac{9\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{10a^3d} \\ &\quad+\frac{\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{2a^3d} \\ &\quad-\frac{\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3}-\frac{2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5ad(a+a\sec(c+dx))^2} \\ &\quad-\frac{9\sqrt{\sec(c+dx)}\sin(c+dx)}{10d\left(a^3+a^3\sec(c+dx)\right)}\end{aligned}$$

output
```
-1/5*sec(d*x+c)^(5/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^3-2/5*sec(d*x+c)^(3/2)
*sin(d*x+c)/a/d/(a+a*sec(d*x+c))^2-9/10*sin(d*x+c)*sec(d*x+c)^(1/2)/d/(a^3
+a^3*sec(d*x+c))+9/10*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elli
pticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^3/d+
1/2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+
1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^3/d
```

3.211.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 5.97 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.41

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx$$
$$= \frac{e^{-idx}\cos\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{7}{2}}(c+dx)\left(-3ie^{-2i(c+dx)}\left(1+e^{i(c+dx)}\right)^5\sqrt{1+e^{2i(c+dx)}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4}\right.\right.}{}$$

input
```
Integrate[Sec[c + d*x]^(7/2)/(a + a*Sec[c + d*x])^3,x]
```

output
```
(Cos[(c + d*x)/2]*Sec[c + d*x]^(7/2)*(((-3*I)*(1 + E^(I*(c + d*x)))^5*Sqrt
[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c +
d*x))])/E^((2*I)*(c + d*x)) + 160*Cos[(c + d*x)/2]^5*Sqrt[Cos[c + d*x]]*El
lipticF[(c + d*x)/2, 2]*(Cos[(c + d*x)/2] - I*Sin[(c + d*x)/2]) + (2*I)*(3
4 + 69*Cos[c + d*x] + 34*Cos[2*(c + d*x)] + 7*Cos[3*(c + d*x)] + (2*I)*Sin
[c + d*x] + (6*I)*Sin[2*(c + d*x)] + (2*I)*Sin[3*(c + d*x)]))*(Cos[(c + 3*
d*x)/2] + I*Sin[(c + 3*d*x)/2]))/(40*a^3*d*E^(I*d*x)*(1 + Sec[c + d*x])^3)
```

3.211.3 Rubi [A] (verified)

Time = 1.11 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.08, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.652$, Rules used = {3042, 4303, 27, 3042, 4507, 3042, 4507, 27, 3042, 4274, 3042, 4258, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a\sec(c+dx)+a)^3}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}dx$$
$$\Big\downarrow \; 4303$$
$$-\frac{\int \frac{3\sec^{\frac{3}{2}}(c+dx)(a-3a\sec(c+dx))}{2(\sec(c+dx)a+a)^2}dx}{5a^2}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \; 27$$

$$-\frac{3\int \frac{\sec^{\frac{3}{2}}(c+dx)(a-3a\sec(c+dx))}{(\sec(c+dx)a+a)^2}dx}{10a^2}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \; 3042$$

$$-\frac{3\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a-3a\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}dx}{10a^2}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \; 4507$$

$$-\frac{3\left(\frac{\int \frac{\sqrt{\sec(c+dx)}\left(2a^2-7a^2\sec(c+dx)\right)}{\sec(c+dx)a+a}dx}{3a^2}+\frac{4a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}\right)}{10a^2}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \; 3042$$

$$-\frac{3\left(\frac{\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(2a^2-7a^2\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{3a^2}+\frac{4a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}\right)}{10a^2}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \; 4507$$

$$-\frac{3\left(\frac{\frac{\int -\frac{5\sec(c+dx)a^3+9a^3}{2\sqrt{\sec(c+dx)}}dx}{a^2}+\frac{9a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}}{3a^2}+\frac{4a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}\right)}{10a^2}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \; 27$$

$$-\frac{3\left(\frac{\frac{9a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}-\frac{\int \frac{5\sec(c+dx)a^3+9a^3}{\sqrt{\sec(c+dx)}}dx}{2a^2}}{3a^2}+\frac{4a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}\right)}{10a^2}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \; 3042$$

$$-\frac{3\left(\frac{\frac{9a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}-\frac{\int \frac{5\csc\left(c+dx+\frac{\pi}{2}\right)a^3+9a^3}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{2a^2}}{3a^2}+\frac{4a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}\right)}{10a^2}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

3.211. $\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

$$\Big\downarrow \text{4274}$$

$$-\frac{3\left(\frac{\frac{9a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}-\frac{9a^3\int\frac{1}{\sqrt{\sec(c+dx)}}dx+5a^3\int\sqrt{\sec(c+dx)}dx}{2a^2}}{3a^2}+\frac{4a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}\right)}{10a^2}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{3\left(\frac{\frac{9a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}-\frac{9a^3\int\frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+5a^3\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{2a^2}}{3a^2}+\frac{4a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}\right)}{10a^2}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{4258}$$

$$-\frac{3\left(\frac{\frac{9a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}-\frac{5a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx+9a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\cos(c+dx)}dx}{2a^2}}{3a^2}+\frac{4a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}\right)}{10a^2}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{3\left(\frac{\frac{9a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}-\frac{5a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+9a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{2a^2}}{3a^2}+\frac{4a\sin(c+dx)\sec^{\frac{3}{2}}}{3d(a\sec(c+dx)}\right.}{10a^2}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{3119}$$

3.211. $\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

$$-\frac{3\left(\frac{\frac{9a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}-\frac{5a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{18a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}}{2a^2}}{3a^2}+\frac{4a\sin(c+dx)\sec^{\frac{3}{2}}(c+}{3d(a\sec(c+dx)+a)}\right)}{10a^2}$$

$$\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \; 3120$$

$$-\frac{3\left(\frac{\frac{9a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}-\frac{\frac{10a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+\frac{18a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}}{2a^2}}{3a^2}+\frac{4a\sin(c+dx)\sec^{\frac{3}{2}}(c}{3d(a\sec(c+dx)+}\right)}{10a^2}$$

$$\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

input
```
Int[Sec[c + d*x]^(7/2)/(a + a*Sec[c + d*x])^3,x]
```

output
```
-1/5*(Sec[c + d*x]^(5/2)*Sin[c + d*x])/(d*(a + a*Sec[c + d*x])^3) - (3*((4
*a*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d*(a + a*Sec[c + d*x])^2) + (-1/2*(
(18*a^3*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d
 + (10*a^3*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]]
)/d)/a^2 + (9*a^2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(d*(a + a*Sec[c + d*x])
))/(3*a^2)))/(10*a^2)
```

3.211.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4303
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-d^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d
*Csc[e + f*x])^(n - 2)/(f*(2*m + 1))), x] + Simp[d^2/(a*b*(2*m + 1))   Int[
(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n
 + 2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2,
0] && LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

rule 4507
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[d*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(a*f*(
2*m + 1))), x] - Simp[1/(a*b*(2*m + 1))   Int[(a + b*Csc[e + f*x])^(m + 1)*
(d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
 - n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f,
A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && G
tQ[n, 0]
```

3.211.4 Maple [A] (verified)

Time = 55.45 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.37

method	result
default	$\frac{\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(36\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^8-10\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1}\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\cos\left(\right.\right.}{20a^3\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^5\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right.}}$

input
```
int(sec(d*x+c)^(7/2)/(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/20*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(36*cos(1/2*d
*x+1/2*c)^8-10*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1
/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5+18*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*cos(1/2*d*x+1/2*c)^
5*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-46*cos(1/2*d*x+1/2*c)^6+8*cos(1/2*
d*x+1/2*c)^4+cos(1/2*d*x+1/2*c)^2+1)/a^3/cos(1/2*d*x+1/2*c)^5/(-2*sin(1/2*
d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x
+1/2*c)^2-1)^(1/2)/d
```

3.211.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 353, normalized size of antiderivative = 1.81

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx =$$

$$-\frac{5\left(i\sqrt{2}\cos\left(dx+c\right)^3+3i\sqrt{2}\cos\left(dx+c\right)^2+3i\sqrt{2}\cos\left(dx+c\right)+i\sqrt{2}\right)\text{weierstrassPInverse}(-4,0,\cos($$

input
```
integrate(sec(d*x+c)^(7/2)/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
-1/20*(5*(I*sqrt(2)*cos(d*x + c)^3 + 3*I*sqrt(2)*cos(d*x + c)^2 + 3*I*sqrt
(2)*cos(d*x + c) + I*sqrt(2))*weierstrassPInverse(-4, 0, cos(d*x + c) + I*
sin(d*x + c)) + 5*(-I*sqrt(2)*cos(d*x + c)^3 - 3*I*sqrt(2)*cos(d*x + c)^2
- 3*I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassPInverse(-4, 0, cos(d*x
 + c) - I*sin(d*x + c)) + 9*(-I*sqrt(2)*cos(d*x + c)^3 - 3*I*sqrt(2)*cos(d
*x + c)^2 - 3*I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassZeta(-4, 0, w
eierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 9*(I*sqrt(2)*c
os(d*x + c)^3 + 3*I*sqrt(2)*cos(d*x + c)^2 + 3*I*sqrt(2)*cos(d*x + c) + I*
sqrt(2))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) -
I*sin(d*x + c))) + 2*(9*cos(d*x + c)^3 + 22*cos(d*x + c)^2 + 15*cos(d*x +
c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d
*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

3.211.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**(7/2)/(a+a*sec(d*x+c))**3,x)
```

output
```
Timed out
```

3.211.7 Maxima [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)^(7/2)/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
Timed out
```

3.211.8 Giac [F]

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \int \frac{\sec(dx+c)^{\frac{7}{2}}}{(a\sec(dx+c)+a)^3}\,dx$$

input
```
integrate(sec(d*x+c)^(7/2)/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^(7/2)/(a*sec(d*x + c) + a)^3, x)
```

3.211.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{7/2}}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^3}\,dx$$

input
```
int((1/cos(c + d*x))^(7/2)/(a + a/cos(c + d*x))^3,x)
```

output
```
int((1/cos(c + d*x))^(7/2)/(a + a/cos(c + d*x))^3, x)
```

3.212 $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

3.212.1 Optimal result . . . 1724
3.212.2 Mathematica [C] (verified) . . . 1725
3.212.3 Rubi [A] (verified) . . . 1725
3.212.4 Maple [A] (verified) . . . 1730
3.212.5 Fricas [C] (verification not implemented) . . . 1730
3.212.6 Sympy [F] . . . 1731
3.212.7 Maxima [F(-1)] . . . 1731
3.212.8 Giac [F] . . . 1732
3.212.9 Mupad [F(-1)] . . . 1732

3.212.1 Optimal result

Integrand size = 23, antiderivative size = 195

$$\begin{aligned}\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx = &\frac{\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{10a^3d}\\ &+\frac{\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{6a^3d}\\ &-\frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3}-\frac{4\sqrt{\sec(c+dx)}\sin(c+dx)}{15ad(a+a\sec(c+dx))^2}\\ &+\frac{\sqrt{\sec(c+dx)}\sin(c+dx)}{6d\left(a^3+a^3\sec(c+dx)\right)}\end{aligned}$$

output
```
-1/5*sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^3-4/15*sin(d*x+c)*sec(
d*x+c)^(1/2)/a/d/(a+a*sec(d*x+c))^2+1/6*sin(d*x+c)*sec(d*x+c)^(1/2)/d/(a^3
+a^3*sec(d*x+c))+1/10*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elli
pticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^3/d+
1/6*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+
1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^3/d
```

3.212.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 2.90 (sec) , antiderivative size = 371, normalized size of antiderivative = 1.90

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{2\cos^6\left(\frac{1}{2}(c+dx)\right)\left(\frac{2i\sqrt{2}e^{-i(c+dx)}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\left(3\left(1+e^{2i(c+dx)}\right)+3\left(-1+e^{2ic}\right)\sqrt{1+e^{2i(c+dx)}}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},-e^{2i(c+dx}\right.\right.}{-1+e^{2ic}}\right.}{}$$

input
```
Integrate[Sec[c + d*x]^(5/2)/(a + a*Sec[c + d*x])^3,x]
```

output
```
(2*Cos[(c + d*x)/2]^6*(((2*I)*Sqrt[2]*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(
c + d*x)))]*(3*(1 + E^((2*I)*(c + d*x))) + 3*(-1 + E^((2*I)*c))*Sqrt[1 + E
^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))
] - 5*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hyp
ergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]))/(E^(I*(c + d*x))*(-1
 + E^((2*I)*c))) - ((36*Cos[(c - d*x)/2] + 9*Cos[(3*c + d*x)/2] + 7*Cos[(c
 + 3*d*x)/2] + 26*Cos[(5*c + 3*d*x)/2] + 10*Cos[(3*c + 5*d*x)/2] + 5*Cos[(
7*c + 5*d*x)/2] + 3*Cos[(5*c + 7*d*x)/2])*Csc[c/2]*Sec[c/2]*Sec[(c + d*x)/
2]^5*Sqrt[Sec[c + d*x]])/32)*Sec[c + d*x]^3)/(15*a^3*d*(1 + Sec[c + d*x])^
3)
```

3.212.3 Rubi [A] (verified)

Time = 1.11 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.08, number of steps used = 16, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.696$, Rules used = {3042, 4303, 27, 3042, 4507, 25, 3042, 4508, 27, 3042, 4274, 3042, 4258, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a\sec(c+dx)+a)^3}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}dx$$

$$\Big\downarrow \; 4303$$

$$-\frac{\int \frac{\sqrt{\sec(c+dx)}(a-7a\sec(c+dx))}{2(\sec(c+dx)a+a)^2}dx}{5a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \; 27$$

$$-\frac{\int \frac{\sqrt{\sec(c+dx)}(a-7a\sec(c+dx))}{(\sec(c+dx)a+a)^2}dx}{10a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a-7a\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}dx}{10a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \; 4507$$

$$-\frac{\frac{\int -\frac{9\sec(c+dx)a^2+4a^2}{\sqrt{\sec(c+dx)}(\sec(c+dx)a+a)}dx}{3a^2}+\frac{8a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}}{10a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \; 25$$

$$-\frac{\frac{8a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}-\frac{\int \frac{9\sec(c+dx)a^2+4a^2}{\sqrt{\sec(c+dx)}(\sec(c+dx)a+a)}dx}{3a^2}}{10a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\frac{8a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}-\frac{\int \frac{9\csc\left(c+dx+\frac{\pi}{2}\right)a^2+4a^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)}dx}{3a^2}}{10a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \; 4508$$

$$-\frac{\frac{8a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}-\frac{\frac{\int \frac{5\sec(c+dx)a^3+3a^3}{2\sqrt{\sec(c+dx)}}dx}{a^2}+\frac{5a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}}{3a^2}}{10a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \; 27$$

$$-\frac{\frac{8a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}-\frac{\frac{\int \frac{5\sec(c+dx)a^3+3a^3}{\sqrt{\sec(c+dx)}}dx}{2a^2}+\frac{5a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}}{3a^2}}{10a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

3.212. $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

$$\Big\downarrow \; 3042$$

$$-\frac{\frac{8a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}-\frac{\frac{\int \frac{5\csc\left(c+dx+\frac{\pi}{2}\right)a^3+3a^3}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{2a^2}+\frac{5a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}}{3a^2}}{10a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \; 4274$$

$$-\frac{\frac{8a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}-\frac{\frac{3a^3\int \frac{1}{\sqrt{\sec(c+dx)}}dx+5a^3\int \sqrt{\sec(c+dx)}dx}{2a^2}+\frac{5a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}}{3a^2}}{10a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\frac{8a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}-\frac{\frac{3a^3\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+5a^3\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{2a^2}+\frac{5a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}}{3a^2}}{10a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \; 4258$$

$$-\frac{\frac{8a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}-\frac{\frac{5a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx+3a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\cos(c+dx)}dx}{2a^2}+\frac{5a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}}{3a^2}}{10a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\frac{8a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}-\frac{\frac{5a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+3a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{2a^2}+\frac{5a^2\sin(c+dx)\sqrt{\sec(c+}}{d(a\sec(c+dx)+a)}}{3a^2}}{10a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \; 3119$$

3.212. $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

$$-\frac{\frac{8a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}-\frac{\frac{5a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{6a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}}{2a^2}+\frac{5a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}}{3a^2}}{10a^2}$$

$$\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \; 3120$$

$$-\frac{\frac{8a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}-\frac{\frac{5a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}+\frac{\frac{10a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+\frac{6a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}}{2a^2}}{3a^2}}{10a^2}$$

$$\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

input
```
Int[Sec[c + d*x]^(5/2)/(a + a*Sec[c + d*x])^3,x]
```

output
```
-1/5*(Sec[c + d*x]^(3/2)*Sin[c + d*x])/(d*(a + a*Sec[c + d*x])^3) - ((8*a*
Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*d*(a + a*Sec[c + d*x])^2) - (((6*a^3*S
qrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (10*a^
3*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d)/(2*a
^2) + (5*a^2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(d*(a + a*Sec[c + d*x])))/(3
*a^2))/(10*a^2)
```

3.212.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4303
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-d^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d
*Csc[e + f*x])^(n - 2)/(f*(2*m + 1))), x] + Simp[d^2/(a*b*(2*m + 1))   Int[
(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n
 + 2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2,
0] && LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

rule 4507
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[d*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(a*f*(
2*m + 1))), x] - Simp[1/(a*b*(2*m + 1))   Int[(a + b*Csc[e + f*x])^(m + 1)*
(d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
 - n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f,
A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && G
tQ[n, 0]
```

rule 4508
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
- a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
 1))), x] - Simp[1/(a^2*(2*m + 1))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Cs
c[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[
e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B
, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

3.212. $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

3.212.4 Maple [A] (verified)

Time = 57.17 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.38

method	result
default	$\frac{\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\left(12\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{8}-10\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+1}\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\cos\left(\right.\right.}{60a^{3}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right.}}$

input
```
int(sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/60*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(12*cos(1/2*d
*x+1/2*c)^8-10*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1
/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5+6*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*cos(1/2*d*x+1/2*c)^5
*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-22*cos(1/2*d*x+1/2*c)^6+6*cos(1/2*d
*x+1/2*c)^4+7*cos(1/2*d*x+1/2*c)^2-3)/a^3/cos(1/2*d*x+1/2*c)^5/(-2*sin(1/2
*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*
x+1/2*c)^2-1)^(1/2)/d
```

3.212.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 353, normalized size of antiderivative = 1.81

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^{3}}\,dx =$$

$$-\frac{5\left(i\sqrt{2}\cos\left(dx+c\right)^{3}+3i\sqrt{2}\cos\left(dx+c\right)^{2}+3i\sqrt{2}\cos\left(dx+c\right)+i\sqrt{2}\right)\mathrm{weierstrassPInverse}(-4,0,\cos($$

input
```
integrate(sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
-1/60*(5*(I*sqrt(2)*cos(d*x + c)^3 + 3*I*sqrt(2)*cos(d*x + c)^2 + 3*I*sqrt
(2)*cos(d*x + c) + I*sqrt(2))*weierstrassPInverse(-4, 0, cos(d*x + c) + I*
sin(d*x + c)) + 5*(-I*sqrt(2)*cos(d*x + c)^3 - 3*I*sqrt(2)*cos(d*x + c)^2
- 3*I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassPInverse(-4, 0, cos(d*x
 + c) - I*sin(d*x + c)) + 3*(-I*sqrt(2)*cos(d*x + c)^3 - 3*I*sqrt(2)*cos(d
*x + c)^2 - 3*I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassZeta(-4, 0, w
eierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 3*(I*sqrt(2)*c
os(d*x + c)^3 + 3*I*sqrt(2)*cos(d*x + c)^2 + 3*I*sqrt(2)*cos(d*x + c) + I*
sqrt(2))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) -
I*sin(d*x + c))) + 2*(3*cos(d*x + c)^3 + 4*cos(d*x + c)^2 - 5*cos(d*x + c)
)*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x
 + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

3.212.6 Sympy [F]

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\int \frac{\sec^{\frac{5}{2}}(c+dx)}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx}{a^3}$$

input
```
integrate(sec(d*x+c)**(5/2)/(a+a*sec(d*x+c))**3,x)
```

output
```
Integral(sec(c + d*x)**(5/2)/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(
c + d*x) + 1), x)/a**3
```

3.212.7 Maxima [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
Timed out
```

3.212.8 Giac [F]

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \int \frac{\sec{(dx+c)}^{\frac{5}{2}}}{(a\sec{(dx+c)}+a)^3}\,dx$$

input
```
integrate(sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^(5/2)/(a*sec(d*x + c) + a)^3, x)
```

3.212.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^3}\,dx$$

input
```
int((1/cos(c + d*x))^(5/2)/(a + a/cos(c + d*x))^3,x)
```

output
```
int((1/cos(c + d*x))^(5/2)/(a + a/cos(c + d*x))^3, x)
```

3.213 $\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

3.213.1 Optimal result . . . 1733
3.213.2 Mathematica [C] (verified) . . . 1734
3.213.3 Rubi [A] (verified) . . . 1734
3.213.4 Maple [A] (verified) . . . 1738
3.213.5 Fricas [C] (verification not implemented) . . . 1739
3.213.6 Sympy [F] . . . 1740
3.213.7 Maxima [F(-1)] . . . 1740
3.213.8 Giac [F] . . . 1740
3.213.9 Mupad [F(-1)] . . . 1741

3.213.1 Optimal result

Integrand size = 23, antiderivative size = 195

$$\begin{aligned}\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx &= -\frac{\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{10a^3d} \\ &\quad+\frac{\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{6a^3d} \\ &\quad+\frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3}-\frac{\sqrt{\sec(c+dx)}\sin(c+dx)}{15ad(a+a\sec(c+dx))^2} \\ &\quad+\frac{\sqrt{\sec(c+dx)}\sin(c+dx)}{6d\left(a^3+a^3\sec(c+dx)\right)}\end{aligned}$$

output
```
1/5*sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^3-1/15*sin(d*x+c)*sec(d
*x+c)^(1/2)/a/d/(a+a*sec(d*x+c))^2+1/6*sin(d*x+c)*sec(d*x+c)^(1/2)/d/(a^3+
a^3*sec(d*x+c))-1/10*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellip
ticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^3/d+1
/6*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1
/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^3/d
```

3.213.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 2.92 (sec) , antiderivative size = 371, normalized size of antiderivative = 1.90

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{2\cos^6\left(\frac{1}{2}(c+dx)\right)\left(-\frac{2i\sqrt{2}e^{-i(c+dx)}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\left(3\left(1+e^{2i(c+dx)}\right)+3\left(-1+e^{2ic}\right)\sqrt{1+e^{2i(c+dx)}}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},-e^{2i(c+}\right.\right.}{-1+e^{2ic}}\right.}{}$$

input
```
Integrate[Sec[c + d*x]^(3/2)/(a + a*Sec[c + d*x])^3,x]
```

output
```
(2*Cos[(c + d*x)/2]^6*(((-2*I)*Sqrt[2]*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*
(c + d*x)))]*(3*(1 + E^((2*I)*(c + d*x))) + 3*(-1 + E^((2*I)*c))*Sqrt[1 +
E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x)
)] + 5*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hy
pergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]))/(E^(I*(c + d*x))*(-
1 + E^((2*I)*c))) + ((36*Cos[(c - d*x)/2] + 9*Cos[(3*c + d*x)/2] + 17*Cos[
(c + 3*d*x)/2] + 16*Cos[(5*c + 3*d*x)/2] + 20*Cos[(3*c + 5*d*x)/2] - 5*Cos
[(7*c + 5*d*x)/2] + 3*Cos[(5*c + 7*d*x)/2])*Csc[c/2]*Sec[c/2]*Sec[(c + d*x
)/2]^5*Sqrt[Sec[c + d*x]])/32)*Sec[c + d*x]^3)/(15*a^3*d*(1 + Sec[c + d*x]
)^3)
```

3.213.3 Rubi [A] (verified)

Time = 1.10 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.08, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.652$, Rules used = {3042, 4302, 27, 3042, 4507, 3042, 4508, 27, 3042, 4274, 3042, 4258, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a\sec(c+dx)+a)^3}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}dx$$

$$\Big\downarrow \text{4302}$$

$$\frac{\int \frac{\sqrt{\sec(c+dx)}(3\sec(c+dx)a+a)}{2(\sec(c+dx)a+a)^2}dx}{5a^2}+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\sqrt{\sec(c+dx)}(3\sec(c+dx)a+a)}{(\sec(c+dx)a+a)^2}dx}{10a^2}+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(3\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}dx}{10a^2}+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{4507}$$

$$\frac{\frac{\int \frac{6\sec(c+dx)a^2+a^2}{\sqrt{\sec(c+dx)}(\sec(c+dx)a+a)}dx}{3a^2}-\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}}{10a^2}+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\int \frac{6\csc\left(c+dx+\frac{\pi}{2}\right)a^2+a^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)}dx}{3a^2}-\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}}{10a^2}+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{4508}$$

$$\frac{\frac{\frac{\int -\frac{3a^3-5a^3\sec(c+dx)}{2\sqrt{\sec(c+dx)}}dx}{a^2}+\frac{5a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}}{3a^2}-\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}}{10a^2}+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\frac{5a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}-\frac{\int \frac{3a^3-5a^3\sec(c+dx)}{\sqrt{\sec(c+dx)}}dx}{2a^2}}{3a^2}-\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}}{10a^2}+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\frac{5a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}-\frac{\int \frac{3a^3-5a^3\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{2a^2}}{3a^2}-\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}}{10a^2}+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \text{4274}$$

3.213. $\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

$$\frac{\frac{\frac{5a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}-\frac{3a^3\int\frac{1}{\sqrt{\sec(c+dx)}}dx-5a^3\int\sqrt{\sec(c+dx)}dx}{2a^2}}{3a^2}-\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}}{10a^2}+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$\downarrow$ 3042

$$\frac{\frac{\frac{5a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}-\frac{3a^3\int\frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-5a^3\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{2a^2}}{3a^2}-\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}}{10a^2}+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$\downarrow$ 4258

$$\frac{\frac{\frac{5a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}-\frac{3a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\cos(c+dx)}dx-5a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx}{2a^2}}{3a^2}-\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}}{10a^2}+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$\downarrow$ 3042

$$\frac{\frac{\frac{5a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}-\frac{3a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx-5a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{2a^2}}{3a^2}-\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}}{10a^2}+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$\downarrow$ 3119

$$\frac{\frac{\frac{5a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}-\frac{\frac{6a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-5a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{2a^2}}{3a^2}-\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}}{10a^2}+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$\downarrow$ 3120

3.213. $\int\frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

$$\frac{\frac{\frac{5a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}-\frac{\frac{6a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-\frac{10a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}}{2a^2}}{3a^2}-\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}}{10a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

input
```
Int[Sec[c + d*x]^(3/2)/(a + a*Sec[c + d*x])^3,x]
```

output
```
(Sec[c + d*x]^(3/2)*Sin[c + d*x])/(5*d*(a + a*Sec[c + d*x])^3) + ((-2*a*Sq
rt[Sec[c + d*x]]*Sin[c + d*x])/(3*d*(a + a*Sec[c + d*x])^2) + (-1/2*((6*a^
3*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d - (10
*a^3*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d)/a
^2 + (5*a^2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(d*(a + a*Sec[c + d*x])))/(3*
a^2))/(10*a^2)
```

3.213.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4302
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[b*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Cs
c[e + f*x])^(n - 1)/(a*f*(2*m + 1))), x] - Simp[d/(a*b*(2*m + 1))   Int[(a
+ b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1)*(a*(n - 1) - b*(m + n)*C
sc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && L
tQ[m, -1] && LtQ[1, n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

rule 4507
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[d*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(a*f*(
2*m + 1))), x] - Simp[1/(a*b*(2*m + 1))   Int[(a + b*Csc[e + f*x])^(m + 1)*
(d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
 - n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f,
A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && G
tQ[n, 0]
```

rule 4508
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
- a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
 1))), x] - Simp[1/(a^2*(2*m + 1))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Cs
c[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[
e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B
, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

3.213.4 Maple [A] (verified)

Time = 7.54 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.38

method	result
default	$-\frac{\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(12\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^8+10\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1}\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\mathrm{co}\right.}{60a^3\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^5\sqrt{-2\sin\left(\frac{dx}{2}\right.}}$

input
```
int(sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
-1/60*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(12*cos(1/2*
d*x+1/2*c)^8+10*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(
1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5+6*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*cos(1/2*d*x+1/2*c)^
5*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-2*cos(1/2*d*x+1/2*c)^6-24*cos(1/2*
d*x+1/2*c)^4+17*cos(1/2*d*x+1/2*c)^2-3)/a^3/cos(1/2*d*x+1/2*c)^5/(-2*sin(1
/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*
d*x+1/2*c)^2-1)^(1/2)/d
```

3.213.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 353, normalized size of antiderivative = 1.81

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx =$$

$$-\frac{5\left(i\sqrt{2}\cos\left(dx+c\right)^3+3i\sqrt{2}\cos\left(dx+c\right)^2+3i\sqrt{2}\cos\left(dx+c\right)+i\sqrt{2}\right)\text{weierstrassPInverse}(-4,0,\cos($$

input
```
integrate(sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
-1/60*(5*(I*sqrt(2)*cos(d*x + c)^3 + 3*I*sqrt(2)*cos(d*x + c)^2 + 3*I*sqrt
(2)*cos(d*x + c) + I*sqrt(2))*weierstrassPInverse(-4, 0, cos(d*x + c) + I*
sin(d*x + c)) + 5*(-I*sqrt(2)*cos(d*x + c)^3 - 3*I*sqrt(2)*cos(d*x + c)^2
- 3*I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassPInverse(-4, 0, cos(d*x
 + c) - I*sin(d*x + c)) + 3*(I*sqrt(2)*cos(d*x + c)^3 + 3*I*sqrt(2)*cos(d*
x + c)^2 + 3*I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierstrassZeta(-4, 0, we
ierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 3*(-I*sqrt(2)*c
os(d*x + c)^3 - 3*I*sqrt(2)*cos(d*x + c)^2 - 3*I*sqrt(2)*cos(d*x + c) - I*
sqrt(2))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) -
I*sin(d*x + c))) - 2*(3*cos(d*x + c)^3 + 14*cos(d*x + c)^2 + 5*cos(d*x + c
))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*
x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

3.213.6 Sympy [F]

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx}{a^3}$$

input
```
integrate(sec(d*x+c)**(3/2)/(a+a*sec(d*x+c))**3,x)
```

output
```
Integral(sec(c + d*x)**(3/2)/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(
c + d*x) + 1), x)/a**3
```

3.213.7 Maxima [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
Timed out
```

3.213.8 Giac [F]

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \int \frac{\sec(dx+c)^{\frac{3}{2}}}{(a\sec(dx+c)+a)^3}\,dx$$

input
```
integrate(sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^(3/2)/(a*sec(d*x + c) + a)^3, x)
```

3.213.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^3}\,dx$$

input
```
int((1/cos(c + d*x))^(3/2)/(a + a/cos(c + d*x))^3,x)
```

output
```
int((1/cos(c + d*x))^(3/2)/(a + a/cos(c + d*x))^3, x)
```

3.214 $\int \frac{\sqrt{\sec(c+dx)}}{(a+a\sec(c+dx))^3}\,dx$

3.214.1 Optimal result . . . 1742
3.214.2 Mathematica [C] (verified) . . . 1743
3.214.3 Rubi [A] (verified) . . . 1743
3.214.4 Maple [A] (verified) . . . 1748
3.214.5 Fricas [C] (verification not implemented) . . . 1748
3.214.6 Sympy [F] . . . 1749
3.214.7 Maxima [F] . . . 1749
3.214.8 Giac [F] . . . 1750
3.214.9 Mupad [F(-1)] . . . 1750

3.214.1 Optimal result

Integrand size = 23, antiderivative size = 195

$$\begin{aligned}\int \frac{\sqrt{\sec(c+dx)}}{(a+a\sec(c+dx))^3}\,dx = &-\frac{9\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{10a^3d}\\ &+\frac{\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{2a^3d}\\ &-\frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3}+\frac{2\sqrt{\sec(c+dx)}\sin(c+dx)}{5ad(a+a\sec(c+dx))^2}\\ &+\frac{\sqrt{\sec(c+dx)}\sin(c+dx)}{2d\left(a^3+a^3\sec(c+dx)\right)}\end{aligned}$$

output
```
-1/5*sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^3+2/5*sin(d*x+c)*sec(d
*x+c)^(1/2)/a/d/(a+a*sec(d*x+c))^2+1/2*sin(d*x+c)*sec(d*x+c)^(1/2)/d/(a^3+
a^3*sec(d*x+c))-9/10*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellip
ticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^3/d+1
/2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1
/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^3/d
```

3.214.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 6.51 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.39

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+a\sec(c+dx))^3}\,dx$$
$$= \frac{e^{-idx}\cos\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{7}{2}}(c+dx)\left(160\cos^5\left(\frac{1}{2}(c+dx)\right)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\left(\cos\left(\frac{1}{2}(c-\right.\right.}{}$$

input
```
Integrate[Sqrt[Sec[c + d*x]]/(a + a*Sec[c + d*x])^3,x]
```

output
```
(Cos[(c + d*x)/2]*Sec[c + d*x]^(7/2)*(160*Cos[(c + d*x)/2]^5*Sqrt[Cos[c +
d*x]]*EllipticF[(c + d*x)/2, 2]*(Cos[(c + d*x)/2] - I*Sin[(c + d*x)/2]) +
I*(-68 - 128*Cos[c + d*x] - 68*Cos[2*(c + d*x)] - 24*Cos[3*(c + d*x)] + (3
*(1 + E^(I*(c + d*x)))^5*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1
/2, 3/4, 7/4, -E^((2*I)*(c + d*x))])/E^((2*I)*(c + d*x)) + (6*I)*Sin[c + d
*x] + (8*I)*Sin[2*(c + d*x)] + (6*I)*Sin[3*(c + d*x)]))*(Cos[(c + 3*d*x)/2
] + I*Sin[(c + 3*d*x)/2]))/(40*a^3*d*E^(I*d*x)*(1 + Sec[c + d*x])^3)
```

3.214.3 Rubi [A] (verified)

Time = 1.10 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.08, number of steps used = 16, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.696$, Rules used = {3042, 4304, 27, 3042, 4507, 25, 3042, 4508, 27, 3042, 4274, 3042, 4258, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\sec(c+dx)}}{(a\sec(c+dx)+a)^3}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}dx$$
$$\Big\downarrow \; 4304$$

$$-\frac{\int -\frac{3\sqrt{\sec(c+dx)}(3a-a\sec(c+dx))}{2(\sec(c+dx)a+a)^2}dx}{5a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$\Big\downarrow$ 27

$$\frac{3\int \frac{\sqrt{\sec(c+dx)}(3a-a\sec(c+dx))}{(\sec(c+dx)a+a)^2}dx}{10a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$\Big\downarrow$ 3042

$$\frac{3\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(3a-a\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}dx}{10a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$\Big\downarrow$ 4507

$$\frac{3\left(\frac{\int -\frac{2a^2-3a^2\sec(c+dx)}{\sqrt{\sec(c+dx)}(\sec(c+dx)a+a)}dx}{3a^2}+\frac{4a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}\right)}{10a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$\Big\downarrow$ 25

$$\frac{3\left(\frac{4a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}-\frac{\int \frac{2a^2-3a^2\sec(c+dx)}{\sqrt{\sec(c+dx)}(\sec(c+dx)a+a)}dx}{3a^2}\right)}{10a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$\Big\downarrow$ 3042

$$\frac{3\left(\frac{4a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}-\frac{\int \frac{2a^2-3a^2\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)}dx}{3a^2}\right)}{10a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$\Big\downarrow$ 4508

$$\frac{3\left(\frac{4a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}-\frac{\frac{\int \frac{9a^3-5a^3\sec(c+dx)}{2\sqrt{\sec(c+dx)}}dx}{a^2}-\frac{5a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}}{3a^2}\right)}{10a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$\Big\downarrow$ 27

$$\frac{3\left(\frac{4a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}-\frac{\frac{\int \frac{9a^3-5a^3\sec(c+dx)}{\sqrt{\sec(c+dx)}}dx}{2a^2}-\frac{5a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}}{3a^2}\right)}{10a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

3.214. $\int \frac{\sqrt{\sec(c+dx)}}{(a+a\sec(c+dx))^3}\,dx$

$$\Big\downarrow \; 3042$$

$$\frac{3\left(\frac{4a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}-\frac{\frac{\int\frac{9a^3-5a^3\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{2a^2}-\frac{5a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}}{3a^2}\right)}{10a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \; 4274$$

$$\frac{3\left(\frac{4a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}-\frac{\frac{9a^3\int\frac{1}{\sqrt{\sec(c+dx)}}dx-5a^3\int\sqrt{\sec(c+dx)}dx}{2a^2}-\frac{5a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}}{3a^2}\right)}{10a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \; 3042$$

$$\frac{3\left(\frac{4a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}-\frac{\frac{9a^3\int\frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-5a^3\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{2a^2}-\frac{5a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}}{3a^2}\right)}{10a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \; 4258$$

$$\frac{3\left(\frac{4a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}-\frac{\frac{9a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\cos(c+dx)}dx-5a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx}{2a^2}-\frac{5a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}}{3a^2}\right)}{10a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \; 3042$$

$$\frac{3\left(\frac{4a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}-\frac{\frac{9a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx-5a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{2a^2}-\frac{5a^2\sin(c+dx)\sqrt{\sec(c}}{d(a\sec(c+dx)+a}}{3a^2}\right)}{10a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

3.214. $\int \frac{\sqrt{\sec(c+dx)}}{(a+a\sec(c+dx))^3}\,dx$

$$\Big\downarrow \; 3119$$

$$\frac{3\left(\frac{4a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}-\frac{\frac{\frac{18a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-5a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{2a^2}-\frac{5a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}}{3a^2}\right)}{10a^2}}{\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}}$$

$$\Big\downarrow \; 3120$$

$$\frac{3\left(\frac{4a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}-\frac{\frac{\frac{18a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-\frac{10a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}}{2a^2}-\frac{5a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}}{3a^2}\right)}{10a^2}}{\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}}$$

input
```
Int[Sqrt[Sec[c + d*x]]/(a + a*Sec[c + d*x])^3,x]
```

output
```
-1/5*(Sec[c + d*x]^(3/2)*Sin[c + d*x])/(d*(a + a*Sec[c + d*x])^3) + (3*((4
*a*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*d*(a + a*Sec[c + d*x])^2) - (((18*a
^3*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d - (1
0*a^3*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d)/
(2*a^2) - (5*a^2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(d*(a + a*Sec[c + d*x]))
)/(3*a^2)))/(10*a^2)
```

3.214.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4304
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-Cot[e + f*x])*(a + b*Csc[e + f*x])^m*((d*Csc
[e + f*x])^n/(f*(2*m + 1))), x] + Simp[1/(a^2*(2*m + 1))   Int[(a + b*Csc[e
 + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e
+ f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ
[m, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

rule 4507
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[d*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(a*f*(
2*m + 1))), x] - Simp[1/(a*b*(2*m + 1))   Int[(a + b*Csc[e + f*x])^(m + 1)*
(d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
 - n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f,
A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && G
tQ[n, 0]
```

3.214. $\int \frac{\sqrt{\sec(c+dx)}}{(a+a\sec(c+dx))^3}\,dx$

rule 4508
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
- a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
 1))), x] - Simp[1/(a^2*(2*m + 1))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Cs
c[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[
e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B
, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

3.214.4 Maple [A] (verified)

Time = 7.91 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.38

method	result
default	$-\frac{\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(36\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^8+10\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1}\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\mathrm{co}\right.}{20a^3\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\sin\left(\right.}}$

input
```
int(sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
-1/20/a^3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(36*cos(
1/2*d*x+1/2*c)^8+10*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+
1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5+18*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*cos(1/2*d*x+1/
2*c)^5*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-66*cos(1/2*d*x+1/2*c)^6+38*co
s(1/2*d*x+1/2*c)^4-9*cos(1/2*d*x+1/2*c)^2+1)/(-2*sin(1/2*d*x+1/2*c)^4+sin(
1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)^5/sin(1/2*d*x+1/2*c)/(2*cos(1/2
*d*x+1/2*c)^2-1)^(1/2)/d
```

3.214.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 353, normalized size of antiderivative = 1.81

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+a\sec(c+dx))^3}\,dx =$$

$$-\frac{5\left(i\sqrt{2}\cos(dx+c)^3+3i\sqrt{2}\cos(dx+c)^2+3i\sqrt{2}\cos(dx+c)+i\sqrt{2}\right)\mathrm{weierstrassPInverse}(-4,0,\cos(}{}$$

input
```
integrate(sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
-1/20*(5*(I*sqrt(2)*cos(d*x + c)^3 + 3*I*sqrt(2)*cos(d*x + c)^2 + 3*I*sqrt
(2)*cos(d*x + c) + I*sqrt(2))*weierstrassPInverse(-4, 0, cos(d*x + c) + I*
sin(d*x + c)) + 5*(-I*sqrt(2)*cos(d*x + c)^3 - 3*I*sqrt(2)*cos(d*x + c)^2
- 3*I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassPInverse(-4, 0, cos(d*x
 + c) - I*sin(d*x + c)) + 9*(I*sqrt(2)*cos(d*x + c)^3 + 3*I*sqrt(2)*cos(d*
x + c)^2 + 3*I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierstrassZeta(-4, 0, we
ierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 9*(-I*sqrt(2)*c
os(d*x + c)^3 - 3*I*sqrt(2)*cos(d*x + c)^2 - 3*I*sqrt(2)*cos(d*x + c) - I*
sqrt(2))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) -
I*sin(d*x + c))) - 2*(9*cos(d*x + c)^3 + 12*cos(d*x + c)^2 + 5*cos(d*x + c
))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*
x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

3.214.6 Sympy [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+a\sec(c+dx))^3}\,dx = \frac{\int \frac{\sqrt{\sec(c+dx)}}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx}{a^3}$$

input
```
integrate(sec(d*x+c)**(1/2)/(a+a*sec(d*x+c))**3,x)
```

output
```
Integral(sqrt(sec(c + d*x))/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c
 + d*x) + 1), x)/a**3
```

3.214.7 Maxima [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+a\sec(c+dx))^3}\,dx = \int \frac{\sqrt{\sec(dx+c)}}{(a\sec(dx+c)+a)^3}\,dx$$

input
```
integrate(sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
integrate(sqrt(sec(d*x + c))/(a*sec(d*x + c) + a)^3, x)
```

3.214.8 Giac [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+a\sec(c+dx))^3}\,dx = \int \frac{\sqrt{\sec(dx+c)}}{(a\sec(dx+c)+a)^3}\,dx$$

input
```
integrate(sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate(sqrt(sec(d*x + c))/(a*sec(d*x + c) + a)^3, x)
```

3.214.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+a\sec(c+dx))^3}\,dx = \int \frac{\sqrt{\frac{1}{\cos(c+d\,x)}}}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^3}\,dx$$

input
```
int((1/cos(c + d*x))^(1/2)/(a + a/cos(c + d*x))^3,x)
```

output
```
int((1/cos(c + d*x))^(1/2)/(a + a/cos(c + d*x))^3, x)
```

3.215 $\int \frac{1}{\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^3}\,dx$

3.215.1 Optimal result 1751
3.215.2 Mathematica [C] (verified) 1752
3.215.3 Rubi [A] (verified) 1752
3.215.4 Maple [A] (verified) 1756
3.215.5 Fricas [C] (verification not implemented) 1757
3.215.6 Sympy [F] 1758
3.215.7 Maxima [F] 1758
3.215.8 Giac [F] 1758
3.215.9 Mupad [F(-1)] 1759

3.215.1 Optimal result

Integrand size = 23, antiderivative size = 195

$$\begin{aligned}\int \frac{1}{\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^3}\,dx &= \frac{49\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{10a^3d} \\ &\quad - \frac{13\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{6a^3d} - \frac{\sqrt{\sec(c+dx)}\sin(c+dx)}{5d(a+a\sec(c+dx))^3} \\ &\quad - \frac{8\sqrt{\sec(c+dx)}\sin(c+dx)}{15ad(a+a\sec(c+dx))^2} - \frac{13\sqrt{\sec(c+dx)}\sin(c+dx)}{6d\left(a^3+a^3\sec(c+dx)\right)}\end{aligned}$$

output
```
-1/5*sin(d*x+c)*sec(d*x+c)^(1/2)/d/(a+a*sec(d*x+c))^3-8/15*sin(d*x+c)*sec(
d*x+c)^(1/2)/a/d/(a+a*sec(d*x+c))^2-13/6*sin(d*x+c)*sec(d*x+c)^(1/2)/d/(a^
3+a^3*sec(d*x+c))+49/10*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*El
lipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^3/
d-13/6*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d
*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^3/d
```

3.215.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 3.30 (sec) , antiderivative size = 386, normalized size of antiderivative = 1.98

$$\int \frac{1}{\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^3}\,dx = \frac{2\cos^6\left(\frac{1}{2}(c+dx)\right)\left(\frac{2i\sqrt{2}e^{-i(c+dx)}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\left(147\left(1+e^{2i(c+dx)}\right)+147\left(-1+e^{2ic}\right)\sqrt{1+e^{2i(c+dx)}}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},-e^{2i(}\right.\right.}{-1+e^{2ic}}\right.}{}$$

input
```
Integrate[1/(Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^3),x]
```

output
```
(2*Cos[(c + d*x)/2]^6*(((2*I)*Sqrt[2]*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(
c + d*x)))]*(147*(1 + E^((2*I)*(c + d*x))) + 147*(-1 + E^((2*I)*c))*Sqrt[1
 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d
*x))] + 65*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))
]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]))/(E^(I*(c + d*x)
)*(-1 + E^((2*I)*c))) - ((1134*Cos[(c - d*x)/2] + 1071*Cos[(3*c + d*x)/2]
+ 923*Cos[(c + 3*d*x)/2] + 694*Cos[(5*c + 3*d*x)/2] + 470*Cos[(3*c + 5*d*x
)/2] + 265*Cos[(7*c + 5*d*x)/2] + 117*Cos[(5*c + 7*d*x)/2] + 30*Cos[(9*c +
 7*d*x)/2])*Csc[c/2]*Sec[c/2]*Sec[(c + d*x)/2]^5*Sqrt[Sec[c + d*x]])/32)*S
ec[c + d*x]^3)/(15*a^3*d*(1 + Sec[c + d*x])^3)
```

3.215.3 Rubi [A] (verified)

Time = 1.13 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.08, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.652$, Rules used = {3042, 4304, 27, 3042, 4508, 3042, 4508, 27, 3042, 4274, 3042, 4258, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^3}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}dx$$

$$\Big\downarrow \; 4304$$

$$-\frac{\int -\frac{11a-5a\sec(c+dx)}{2\sqrt{\sec(c+dx)}(\sec(c+dx)a+a)^2}dx}{5a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{11a-5a\sec(c+dx)}{\sqrt{\sec(c+dx)}(\sec(c+dx)a+a)^2}dx}{10a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \; 3042$$

$$\frac{\int \frac{11a-5a\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}dx}{10a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \; 4508$$

$$\frac{\frac{\int \frac{41a^2-24a^2\sec(c+dx)}{\sqrt{\sec(c+dx)}(\sec(c+dx)a+a)}dx}{3a^2}-\frac{16a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}}{10a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{\int \frac{41a^2-24a^2\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)}dx}{3a^2}-\frac{16a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}}{10a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \; 4508$$

$$\frac{\frac{\frac{\int \frac{147a^3-65a^3\sec(c+dx)}{2\sqrt{\sec(c+dx)}}dx}{a^2}-\frac{65a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}}{3a^2}-\frac{16a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}}{10a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\frac{\int \frac{147a^3-65a^3\sec(c+dx)}{\sqrt{\sec(c+dx)}}dx}{2a^2}-\frac{65a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}}{3a^2}-\frac{16a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}}{10a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{\frac{\int \frac{147a^3-65a^3\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{2a^2}-\frac{65a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}}{3a^2}-\frac{16a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}}{10a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \; 4274$$

3.215. $\int \frac{1}{\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^3}\,dx$

$$\frac{\frac{\frac{147a^3\int\frac{1}{\sqrt{\sec(c+dx)}}dx-65a^3\int\sqrt{\sec(c+dx)}dx}{2a^2}-\frac{65a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}}{3a^2}-\frac{16a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}}{10a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{\frac{147a^3\int\frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-65a^3\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{2a^2}-\frac{65a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}}{3a^2}-\frac{16a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}}{10a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \; 4258$$

$$\frac{\frac{\frac{147a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\cos(c+dx)}dx-65a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx}{2a^2}-\frac{65a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}}{3a^2}-\frac{16a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}}{10a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{\frac{147a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx-65a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{2a^2}-\frac{65a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}}{3a^2}-\frac{16a\sin(c+dx)\sqrt{\sec(}}{3d(a\sec(c+dx)+}}{10a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \; 3119$$

$$\frac{\frac{\frac{294a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-65a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{2a^2}-\frac{65a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}}{3a^2}-\frac{16a\sin(c+dx)\sqrt{\sec(c+d}}{3d(a\sec(c+dx)+a)^2}}{10a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{5d(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \; 3120$$

3.215. $\int \frac{1}{\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^3}\,dx$

$$\frac{\frac{\frac{294a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-\frac{130a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}}{2a^2}-\frac{65a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}}{3a^2}-\frac{16a\sin(c+dx)\sqrt{\sec(c+}}{3d(a\sec(c+dx)+a)^2}$$

$$\frac{10a^2}{\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{5d(a\sec(c+dx)+a)^3}}$$

input
```
Int[1/(Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^3),x]
```

output
```
-1/5*(Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(d*(a + a*Sec[c + d*x])^3) + ((-16*
a*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*d*(a + a*Sec[c + d*x])^2) + (((294*a
^3*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d - (1
30*a^3*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d)
/(2*a^2) - (65*a^2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(d*(a + a*Sec[c + d*x]
)))/(3*a^2))/(10*a^2)
```

3.215.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4304
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-Cot[e + f*x])*(a + b*Csc[e + f*x])^m*((d*Csc
[e + f*x])^n/(f*(2*m + 1))), x] + Simp[1/(a^2*(2*m + 1))   Int[(a + b*Csc[e
 + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e
+ f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ
[m, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

rule 4508
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
- a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
 1))), x] - Simp[1/(a^2*(2*m + 1))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Cs
c[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[
e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B
, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

3.215.4 Maple [A] (verified)

Time = 8.13 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.38

method	result
default	$\frac{\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(348\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^8+130\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1}\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\mathrm{co}\right.}{60a^3\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\sin\left(\right.}}$

input
```
int(1/sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/60/a^3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(348*cos(
1/2*d*x+1/2*c)^8+130*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2
+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5+294*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*cos(1/2*d*x+
1/2*c)^5*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-578*cos(1/2*d*x+1/2*c)^6+26
4*cos(1/2*d*x+1/2*c)^4-37*cos(1/2*d*x+1/2*c)^2+3)/(-2*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)^5/sin(1/2*d*x+1/2*c)/(2*co
s(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

3.215.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 353, normalized size of antiderivative = 1.81

$$\int \frac{1}{\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^3}\,dx =$$

$$-\frac{65\left(-i\sqrt{2}\cos\left(dx+c\right)^3-3i\sqrt{2}\cos\left(dx+c\right)^2-3i\sqrt{2}\cos\left(dx+c\right)-i\sqrt{2}\right)\text{weierstrassPInverse}(-4,0,c}{}$$

input
```
integrate(1/sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
-1/60*(65*(-I*sqrt(2)*cos(d*x + c)^3 - 3*I*sqrt(2)*cos(d*x + c)^2 - 3*I*sq
rt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassPInverse(-4, 0, cos(d*x + c) +
I*sin(d*x + c)) + 65*(I*sqrt(2)*cos(d*x + c)^3 + 3*I*sqrt(2)*cos(d*x + c)^
2 + 3*I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierstrassPInverse(-4, 0, cos(d
*x + c) - I*sin(d*x + c)) + 147*(-I*sqrt(2)*cos(d*x + c)^3 - 3*I*sqrt(2)*c
os(d*x + c)^2 - 3*I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassZeta(-4,
0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 147*(I*sqr
t(2)*cos(d*x + c)^3 + 3*I*sqrt(2)*cos(d*x + c)^2 + 3*I*sqrt(2)*cos(d*x + c
) + I*sqrt(2))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x +
 c) - I*sin(d*x + c))) + 2*(87*cos(d*x + c)^3 + 146*cos(d*x + c)^2 + 65*co
s(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^3*d*cos(d*x + c)^3 + 3*a^3
*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

3.215.6 Sympy [F]

$$\int \frac{1}{\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^3}\,dx = \frac{\int \frac{1}{\sec^{\frac{7}{2}}(c+dx)+3\sec^{\frac{5}{2}}(c+dx)+3\sec^{\frac{3}{2}}(c+dx)+\sqrt{\sec(c+dx)}}\,dx}{a^3}$$

input
```
integrate(1/sec(d*x+c)**(1/2)/(a+a*sec(d*x+c))**3,x)
```

output
```
Integral(1/(sec(c + d*x)**(7/2) + 3*sec(c + d*x)**(5/2) + 3*sec(c + d*x)**
(3/2) + sqrt(sec(c + d*x))), x)/a**3
```

3.215.7 Maxima [F]

$$\int \frac{1}{\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^3}\,dx = \int \frac{1}{(a\sec(dx+c)+a)^3\sqrt{\sec(dx+c)}}\,dx$$

input
```
integrate(1/sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
integrate(1/((a*sec(d*x + c) + a)^3*sqrt(sec(d*x + c))), x)
```

3.215.8 Giac [F]

$$\int \frac{1}{\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^3}\,dx = \int \frac{1}{(a\sec(dx+c)+a)^3\sqrt{\sec(dx+c)}}\,dx$$

input
```
integrate(1/sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate(1/((a*sec(d*x + c) + a)^3*sqrt(sec(d*x + c))), x)
```

3.215.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^3}\,dx = \int \frac{1}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^3\sqrt{\frac{1}{\cos(c+d\,x)}}}\,dx$$

input
```
int(1/((a + a/cos(c + d*x))^3*(1/cos(c + d*x))^(1/2)),x)
```

output
```
int(1/((a + a/cos(c + d*x))^3*(1/cos(c + d*x))^(1/2)), x)
```

3.216 $\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx$

3.216.1 Optimal result . . . 1760
3.216.2 Mathematica [C] (verified) . . . 1761
3.216.3 Rubi [A] (verified) . . . 1761
3.216.4 Maple [A] (verified) . . . 1766
3.216.5 Fricas [C] (verification not implemented) . . . 1767
3.216.6 Sympy [F] . . . 1767
3.216.7 Maxima [F(-2)] . . . 1768
3.216.8 Giac [F] . . . 1768
3.216.9 Mupad [F(-1)] . . . 1768

3.216.1 Optimal result

Integrand size = 23, antiderivative size = 221

$$\begin{aligned}\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx &= -\frac{119\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{10a^3d} \\ &\quad + \frac{11\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{2a^3d} \\ &\quad + \frac{11\sin(c+dx)}{2a^3d\sqrt{\sec(c+dx)}} - \frac{\sin(c+dx)}{5d\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^3} \\ &\quad - \frac{2\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^2} - \frac{119\sin(c+dx)}{30d\sqrt{\sec(c+dx)}\,(a^3+a^3\sec(c+dx))}\end{aligned}$$

output
```
11/2*sin(d*x+c)/a^3/d/sec(d*x+c)^(1/2)-1/5*sin(d*x+c)/d/(a+a*sec(d*x+c))^3
/sec(d*x+c)^(1/2)-2/3*sin(d*x+c)/a/d/(a+a*sec(d*x+c))^2/sec(d*x+c)^(1/2)-1
19/30*sin(d*x+c)/d/(a^3+a^3*sec(d*x+c))/sec(d*x+c)^(1/2)-119/10*(cos(1/2*d
*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)
)*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^3/d+11/2*(cos(1/2*d*x+1/2*c)^2)^(1/2
)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2
)*sec(d*x+c)^(1/2)/a^3/d
```

3.216.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 3.87 (sec) , antiderivative size = 285, normalized size of antiderivative = 1.29

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx$$
$$= \frac{e^{-idx}\cos\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{7}{2}}(c+dx)(\cos(dx)+i\sin(dx))\left(-5355i\cos\left(\frac{1}{2}(c+dx)\right)-3927i\cos\left(\frac{3}{2}(c+dx)\right)\right.}{}$$

input
```
Integrate[1/(Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^3),x]
```

output
```
(Cos[(c + d*x)/2]*Sec[c + d*x]^(7/2)*(Cos[d*x] + I*Sin[d*x])*((-5355*I)*Co
s[(c + d*x)/2] - (3927*I)*Cos[(3*(c + d*x))/2] - (1785*I)*Cos[(5*(c + d*x)
)/2] - (357*I)*Cos[(7*(c + d*x))/2] + 5280*Cos[(c + d*x)/2]^5*Sqrt[Cos[c +
 d*x]]*EllipticF[(c + d*x)/2, 2] + ((119*I)*(1 + E^(I*(c + d*x)))^5*Sqrt[1
 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*
x))])/E^(((3*I)/2)*(c + d*x)) + 193*Sin[(c + d*x)/2] + 579*Sin[(3*(c + d*x
))/2] + 555*Sin[(5*(c + d*x))/2] + 227*Sin[(7*(c + d*x))/2] + 10*Sin[(9*(c
 + d*x))/2]))/(120*a^3*d*E^(I*d*x)*(1 + Sec[c + d*x])^3)
```

3.216.3 Rubi [A] (verified)

Time = 1.32 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.08, number of steps used = 17, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.739$, Rules used = {3042, 4304, 27, 3042, 4508, 3042, 4508, 27, 3042, 4274, 3042, 4256, 3042, 4258, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^3}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}dx$$
$$\Big\downarrow \; 4304$$

$$-\frac{\int -\frac{13a-7a\sec(c+dx)}{2\sec^{\frac{3}{2}}(c+dx)(\sec(c+dx)a+a)^2}dx}{5a^2}-\frac{\sin(c+dx)}{5d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^3}$$

$\Big\downarrow$ 27

$$\frac{\int \frac{13a-7a\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(\sec(c+dx)a+a)^2}dx}{10a^2}-\frac{\sin(c+dx)}{5d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^3}$$

$\Big\downarrow$ 3042

$$\frac{\int \frac{13a-7a\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}dx}{10a^2}-\frac{\sin(c+dx)}{5d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^3}$$

$\Big\downarrow$ 4508

$$\frac{\frac{\int \frac{69a^2-50a^2\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(\sec(c+dx)a+a)}dx}{3a^2}-\frac{20a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^2}}{10a^2}-\frac{\sin(c+dx)}{5d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^3}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\int \frac{69a^2-50a^2\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)}dx}{3a^2}-\frac{20a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^2}}{10a^2}-\frac{\sin(c+dx)}{5d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^3}$$

$\Big\downarrow$ 4508

$$\frac{\frac{\frac{\int \frac{3\left(165a^3-119a^3\sec(c+dx)\right)}{2\sec^{\frac{3}{2}}(c+dx)}dx}{a^2}-\frac{119a^2\sin(c+dx)}{d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)}}{3a^2}-\frac{20a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^2}}{10a^2}-\frac{\sin(c+dx)}{5d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^3}$$

$\Big\downarrow$ 27

$$\frac{\frac{\frac{3\int \frac{165a^3-119a^3\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)}dx}{2a^2}-\frac{119a^2\sin(c+dx)}{d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)}}{3a^2}-\frac{20a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^2}}{10a^2}-\frac{\sin(c+dx)}{5d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^3}$$

$\Big\downarrow$ 3042

3.216. $\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3}dx$

$$\frac{\frac{\frac{3\int \frac{165a^3-119a^3\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx}{2a^2}-\frac{119a^2\sin(c+dx)}{d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)}}{3a^2}-\frac{20a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^2}}{10a^2}-\frac{\sin(c+dx)}{5d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^3}$$

$\downarrow$ 4274

$$\frac{\frac{\frac{3\left(165a^3\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}dx-119a^3\int \frac{1}{\sqrt{\sec(c+dx)}}dx\right)}{2a^2}-\frac{119a^2\sin(c+dx)}{d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)}}{3a^2}-\frac{20a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^2}}{10a^2}-\frac{\sin(c+dx)}{5d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^3}$$

$\downarrow$ 3042

$$\frac{\frac{\frac{3\left(165a^3\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx-119a^3\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)}{2a^2}-\frac{119a^2\sin(c+dx)}{d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)}}{3a^2}-\frac{20a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^2}}{10a^2}-\frac{\sin(c+dx)}{5d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^3}$$

$\downarrow$ 4256

$$\frac{\frac{\frac{3\left(165a^3\left(\frac{1}{3}\int \sqrt{\sec(c+dx)}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)-119a^3\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)}{2a^2}-\frac{119a^2\sin(c+dx)}{d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)}}{3a^2}-\frac{20a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^2}}{10a^2}-\frac{\sin(c+dx)}{5d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^3}$$

$\downarrow$ 3042

$$\frac{\frac{\frac{3\left(165a^3\left(\frac{1}{3}\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)-119a^3\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)}{2a^2}-\frac{119a^2\sin(c+dx)}{d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)}}{3a^2}-\frac{20a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^2}}{10a^2}-\frac{\sin(c+dx)}{5d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^3}$$

$\downarrow$ 4258

3.216. $\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3}dx$

$$\frac{\frac{\frac{3\left(165a^3\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)-119a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\cos(c+dx)}dx\right)}{2a^2}-\frac{119a^2\sin(c+dx)}{d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)}}{3a^2}-\frac{}{3a}}{10a^2}$$

$$\frac{\sin(c+dx)}{5d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^3}$$

$\downarrow$ 3042

$$\frac{\frac{\frac{3\left(165a^3\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)-119a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx\right)}{2a^2}-\frac{119a^2\sin(c+dx)}{d\sqrt{\sec(c+dx)}(a\sec(c+dx)+}}{3a^2}}{10a^2}$$

$$\frac{\sin(c+dx)}{5d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^3}$$

$\downarrow$ 3119

$$\frac{\frac{\frac{3\left(165a^3\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)-\frac{238a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)}{2a^2}-\frac{119a^2\sin(c+dx)}{d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)}}{3a^2}}{10a^2}$$

$$\frac{\sin(c+dx)}{5d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^3}$$

$\downarrow$ 3120

$$\frac{\frac{\frac{3\left(165a^3\left(\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}\right)-\frac{238a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)}{2a^2}-\frac{119a^2\sin(c+dx)}{d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)}}{3a^2}}{10a^2}$$

$$\frac{\sin(c+dx)}{5d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^3}$$

input
```
Int[1/(Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^3),x]
```

3.216. $\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx$

output
```
-1/5*Sin[c + d*x]/(d*Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^3) + ((-20*a*
Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^2) + ((-119*a^2
*Sin[c + d*x])/(d*Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])) + (3*((-238*a^3
*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + 165*
a^3*((2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(
3*d) + (2*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]]))))/(2*a^2))/(3*a^2))/(10*
a^2)
```

3.216.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

3.216. $\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx$

rule 4304
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-Cot[e + f*x])*(a + b*Csc[e + f*x])^m*((d*Csc
[e + f*x])^n/(f*(2*m + 1))), x] + Simp[1/(a^2*(2*m + 1))   Int[(a + b*Csc[e
 + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e
+ f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ
[m, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

rule 4508
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
- a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
 1))), x] - Simp[1/(a^2*(2*m + 1))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Cs
c[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[
e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B
, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

3.216.4 Maple [A] (verified)

Time = 9.15 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.28

method	result
default	$-\frac{\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(160\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}+468\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^8+330\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1}\ \text{Elliptic}}{60a^3\sqrt{-2\sin\left(\frac{d}{2}\right.}}$

input
```
int(1/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
-1/60/a^3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(160*cos
(1/2*d*x+1/2*c)^10+468*cos(1/2*d*x+1/2*c)^8+330*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))
*cos(1/2*d*x+1/2*c)^5+714*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2
*c)^2+1)^(1/2)*cos(1/2*d*x+1/2*c)^5*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-
1058*cos(1/2*d*x+1/2*c)^6+474*cos(1/2*d*x+1/2*c)^4-47*cos(1/2*d*x+1/2*c)^2
+3)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c
)^5/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

3.216.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 363, normalized size of antiderivative = 1.64

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx =$$

$$-\frac{165\left(i\sqrt{2}\cos\left(dx+c\right)^3+3i\sqrt{2}\cos\left(dx+c\right)^2+3i\sqrt{2}\cos\left(dx+c\right)+i\sqrt{2}\right)\text{weierstrassPInverse}(-4,0,\text{co}}{}$$

input
```
integrate(1/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
-1/60*(165*(I*sqrt(2)*cos(d*x + c)^3 + 3*I*sqrt(2)*cos(d*x + c)^2 + 3*I*sq
rt(2)*cos(d*x + c) + I*sqrt(2))*weierstrassPInverse(-4, 0, cos(d*x + c) +
I*sin(d*x + c)) + 165*(-I*sqrt(2)*cos(d*x + c)^3 - 3*I*sqrt(2)*cos(d*x + c
)^2 - 3*I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassPInverse(-4, 0, cos
(d*x + c) - I*sin(d*x + c)) + 357*(I*sqrt(2)*cos(d*x + c)^3 + 3*I*sqrt(2)*
cos(d*x + c)^2 + 3*I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierstrassZeta(-4,
 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 357*(-I*s
qrt(2)*cos(d*x + c)^3 - 3*I*sqrt(2)*cos(d*x + c)^2 - 3*I*sqrt(2)*cos(d*x +
 c) - I*sqrt(2))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x
 + c) - I*sin(d*x + c))) - 2*(20*cos(d*x + c)^4 + 237*cos(d*x + c)^3 + 376
*cos(d*x + c)^2 + 165*cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^3*
d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

3.216.6 Sympy [F]

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx = \frac{\int \frac{1}{\sec^{\frac{9}{2}}(c+dx)+3\sec^{\frac{7}{2}}(c+dx)+3\sec^{\frac{5}{2}}(c+dx)+\sec^{\frac{3}{2}}(c+dx)}\,dx}{a^3}$$

input
```
integrate(1/sec(d*x+c)**(3/2)/(a+a*sec(d*x+c))**3,x)
```

output
```
Integral(1/(sec(c + d*x)**(9/2) + 3*sec(c + d*x)**(7/2) + 3*sec(c + d*x)**
(5/2) + sec(c + d*x)**(3/2)), x)/a**3
```

3.216.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

3.216.8 Giac [F]

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx = \int \frac{1}{(a\sec(dx+c)+a)^3\sec(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate(1/((a*sec(d*x + c) + a)^3*sec(d*x + c)^(3/2)), x)
```

3.216.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx = \int \frac{1}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^3\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

input
```
int(1/((a + a/cos(c + d*x))^3*(1/cos(c + d*x))^(3/2)),x)
```

output
```
int(1/((a + a/cos(c + d*x))^3*(1/cos(c + d*x))^(3/2)), x)
```

3.217 $\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx$

3.217.1 Optimal result . . . 1769
3.217.2 Mathematica [C] (verified) . . . 1770
3.217.3 Rubi [A] (verified) . . . 1770
3.217.4 Maple [A] (verified) . . . 1776
3.217.5 Fricas [C] (verification not implemented) . . . 1777
3.217.6 Sympy [F] . . . 1778
3.217.7 Maxima [F] . . . 1778
3.217.8 Giac [F] . . . 1778
3.217.9 Mupad [F(-1)] . . . 1779

3.217.1 Optimal result

Integrand size = 23, antiderivative size = 247

$$\begin{aligned}\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx &= \frac{231\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{10a^3d} \\ &\quad - \frac{21\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{2a^3d} + \frac{77\sin(c+dx)}{10a^3d\sec^{\frac{3}{2}}(c+dx)} \\ &\quad - \frac{21\sin(c+dx)}{2a^3d\sqrt{\sec(c+dx)}} - \frac{\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3} \\ &\quad - \frac{4\sin(c+dx)}{5ad\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2} - \frac{63\sin(c+dx)}{10d\sec^{\frac{3}{2}}(c+dx)\left(a^3+a^3\sec(c+dx)\right)}\end{aligned}$$

output
```
77/10*sin(d*x+c)/a^3/d/sec(d*x+c)^(3/2)-1/5*sin(d*x+c)/d/sec(d*x+c)^(3/2)/
(a+a*sec(d*x+c))^3-4/5*sin(d*x+c)/a/d/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^2-
63/10*sin(d*x+c)/d/sec(d*x+c)^(3/2)/(a^3+a^3*sec(d*x+c))-21/2*sin(d*x+c)/a
^3/d/sec(d*x+c)^(1/2)+231/10*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*
c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)
/a^3/d-21/2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(
1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^3/d
```

3.217.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 4.51 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.20

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx = -\frac{e^{-idx}\cos\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{7}{2}}(c+dx)(\cos(dx)+i\sin(dx))\left(-3465i\cos\left(\frac{1}{2}(c+dx)\right)-2541i\cos\left(\frac{3}{2}(c+dx\right.\right.}{}$$

input
```
Integrate[1/(Sec[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^3),x]
```

output
```
-1/40*(Cos[(c + d*x)/2]*Sec[c + d*x]^(7/2)*(Cos[d*x] + I*Sin[d*x])*((-3465
*I)*Cos[(c + d*x)/2] - (2541*I)*Cos[(3*(c + d*x))/2] - (1155*I)*Cos[(5*(c
+ d*x))/2] - (231*I)*Cos[(7*(c + d*x))/2] + 3360*Cos[(c + d*x)/2]^5*Sqrt[C
os[c + d*x]]*EllipticF[(c + d*x)/2, 2] + ((77*I)*(1 + E^(I*(c + d*x)))^5*S
qrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c
 + d*x))])/E^(((3*I)/2)*(c + d*x)) + 125*Sin[(c + d*x)/2] + 359*Sin[(3*(c
+ d*x))/2] + 350*Sin[(5*(c + d*x))/2] + 138*Sin[(7*(c + d*x))/2] + 5*Sin[(
9*(c + d*x))/2] - Sin[(11*(c + d*x))/2]))/(a^3*d*E^(I*d*x)*(1 + Sec[c + d*
x])^3)
```

3.217.3 Rubi [A] (verified)

Time = 1.33 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.08, number of steps used = 18, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.783$, Rules used = {3042, 4304, 27, 3042, 4508, 27, 3042, 4508, 27, 3042, 4274, 3042, 4256, 3042, 4258, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a\sec(c+dx)+a)^3}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}dx$$

$$\Big\downarrow \; 4304$$

$$-\frac{\int -\frac{3(5a-3a\sec(c+dx))}{2\sec^{\frac{5}{2}}(c+dx)(\sec(c+dx)a+a)^2}dx}{5a^2}-\frac{\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^3}$$

$\Big\downarrow$ 27

$$\frac{3\int \frac{5a-3a\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)(\sec(c+dx)a+a)^2}dx}{10a^2}-\frac{\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^3}$$

$\Big\downarrow$ 3042

$$\frac{3\int \frac{5a-3a\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}dx}{10a^2}-\frac{\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^3}$$

$\Big\downarrow$ 4508

$$\frac{3\left(\frac{\int \frac{7\left(5a^2-4a^2\sec(c+dx)\right)}{\sec^{\frac{5}{2}}(c+dx)(\sec(c+dx)a+a)}dx}{3a^2}-\frac{8a\sin(c+dx)}{3d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^2}\right)}{10a^2}-\frac{\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^3}$$

$\Big\downarrow$ 27

$$\frac{3\left(\frac{7\int \frac{5a^2-4a^2\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)(\sec(c+dx)a+a)}dx}{3a^2}-\frac{8a\sin(c+dx)}{3d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^2}\right)}{10a^2}-\frac{\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^3}$$

$\Big\downarrow$ 3042

$$\frac{3\left(\frac{7\int \frac{5a^2-4a^2\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)}dx}{3a^2}-\frac{8a\sin(c+dx)}{3d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^2}\right)}{10a^2}-\frac{\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^3}$$

$\Big\downarrow$ 4508

3.217. $\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^3}dx$

$$\frac{3\left(\frac{7\left(\frac{\int \frac{5\left(11a^3-9a^3\sec(c+dx)\right)}{\sec^{\frac{5}{2}}(c+dx)}dx}{2a^2}-\frac{9a^2\sin(c+dx)}{d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)}\right)}{3a^2}-\frac{8a\sin(c+dx)}{3d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^2}\right)}{10a^2}-\frac{\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^3}$$

$\Big\downarrow$ 27

$$\frac{3\left(\frac{7\left(\frac{5\int \frac{11a^3-9a^3\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)}dx}{2a^2}-\frac{9a^2\sin(c+dx)}{d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)}\right)}{3a^2}-\frac{8a\sin(c+dx)}{3d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^2}\right)}{10a^2}-\frac{\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^3}$$

$\Big\downarrow$ 3042

$$\frac{3\left(\frac{7\left(\frac{5\int \frac{11a^3-9a^3\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx}{2a^2}-\frac{9a^2\sin(c+dx)}{d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)}\right)}{3a^2}-\frac{8a\sin(c+dx)}{3d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^2}\right)}{10a^2}-\frac{\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^3}$$

$\Big\downarrow$ 4274

$$\frac{3\left(\frac{7\left(\frac{5\left(11a^3\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)}dx-9a^3\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}dx\right)}{2a^2}-\frac{9a^2\sin(c+dx)}{d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)}\right)}{3a^2}-\frac{8a\sin(c+dx)}{3d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^2}\right)}{10a^2}-\frac{\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^3}$$

3.217. $\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx$

$$\Big\downarrow \; 3042$$

$$\frac{3\left(\frac{7\left(\frac{5\left(11a^3\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx-9a^3\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx\right)}{2a^2}-\frac{9a^2\sin(c+dx)}{d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)}\right)}{3a^2}-\frac{8a\sin(c+dx)}{3d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^2}\right)}{10a^2}-\frac{\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \; 4256$$

$$\frac{3\left(\frac{7\left(\frac{5\left(11a^3\left(\frac{3}{5}\int \frac{1}{\sqrt{\sec(c+dx)}}dx+\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}\right)-9a^3\left(\frac{1}{3}\int \sqrt{\sec(c+dx)}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)\right)}{2a^2}-\frac{9a^2\sin(c+dx)}{d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)}\right)}{3a^2}-\frac{8a\sin(c}{3d\sec^{\frac{3}{2}}(c+dx)(a}}{10a^2}-\frac{\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \; 3042$$

$$\frac{3\left(\frac{7\left(\frac{5\left(11a^3\left(\frac{3}{5}\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}\right)-9a^3\left(\frac{1}{3}\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)\right)}{2a^2}-\frac{9a^2\sin(c+dx)}{d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)}\right)}{3a^2}-\frac{}{3d\sec^{\frac{3}{2}}(c}}{10a^2}-\frac{\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \; 4258$$

3.217. $\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx$

$$\frac{3\left(\frac{7\left(\frac{5\left(11a^3\left(\frac{3}{5}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\cos(c+dx)}dx+\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}\right)-9a^3\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)\right)}{2a^2}-\frac{9}{d\sec^{\frac{3}{2}}(c}\right.}{3a^2}\right.}{10a^2}$$

$$\frac{\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \; 3042$$

$$\frac{3\left(\frac{7\left(\frac{5\left(11a^3\left(\frac{3}{5}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}\right)-9a^3\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)\right)}{2a^2}-\right.}{3a^2}\right.}{10a^2}$$

$$\frac{\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \; 3119$$

$$\frac{3\left(\frac{7\left(\frac{5\left(11a^3\left(\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{6\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}\right)-9a^3\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)\right)}{2a^2}-\right.}{3a^2}\right.}{10a^2}$$

$$\frac{\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^3}$$

$$\Big\downarrow \; 3120$$

3.217. $\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx$

$$\frac{3\left(\frac{7\left(\frac{5\left(11a^3\left(\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{6\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}\right)-9a^3\left(\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}\right)\right)}{2a^2}-\frac{}{d\sec}}{3a^2}\right)}{10a^2}}{\frac{\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^3}}$$

input
```
Int[1/(Sec[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^3),x]
```

output
```
-1/5*Sin[c + d*x]/(d*Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^3) + (3*((-8*
a*Sin[c + d*x])/(3*d*Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^2) + (7*((-9*
a^2*Sin[c + d*x])/(d*Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])) + (5*(11*a^3
*((6*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d
) + (2*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2))) - 9*a^3*((2*Sqrt[Cos[c + d*
x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (2*Sin[c + d*x])
/(3*d*Sqrt[Sec[c + d*x]])))/(2*a^2)))/(3*a^2)))/(10*a^2)
```

3.217.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4304
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-Cot[e + f*x])*(a + b*Csc[e + f*x])^m*((d*Csc
[e + f*x])^n/(f*(2*m + 1))), x] + Simp[1/(a^2*(2*m + 1))   Int[(a + b*Csc[e
 + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e
+ f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ
[m, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

rule 4508
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
- a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
 1))), x] - Simp[1/(a^2*(2*m + 1))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Cs
c[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[
e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B
, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

3.217.4 Maple [A] (verified)

Time = 9.51 (sec) , antiderivative size = 296, normalized size of antiderivative = 1.20

method	result
default	$-\frac{\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(64\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{12}-288\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}-76\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^8-210\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\cos\left(\frac{dx}{2}\right.}\right.}{20a^3\sqrt{-}}$

input
```
int(1/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
-1/20/a^3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(64*cos(
1/2*d*x+1/2*c)^12-288*cos(1/2*d*x+1/2*c)^10-76*cos(1/2*d*x+1/2*c)^8-210*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos
(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5-462*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*cos(1/2*d*x+1/2*c)^5*EllipticE(cos(
1/2*d*x+1/2*c),2^(1/2))+530*cos(1/2*d*x+1/2*c)^6-248*cos(1/2*d*x+1/2*c)^4+
19*cos(1/2*d*x+1/2*c)^2-1)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^
(1/2)/cos(1/2*d*x+1/2*c)^5/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(
1/2)/d
```

3.217.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.13 (sec) , antiderivative size = 373, normalized size of antiderivative = 1.51

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx =$$
$$-\frac{105\left(-i\sqrt{2}\cos\left(dx+c\right)^3-3i\sqrt{2}\cos\left(dx+c\right)^2-3i\sqrt{2}\cos\left(dx+c\right)-i\sqrt{2}\right)\text{weierstrassPInverse}(-4,0,}{}$$

input
```
integrate(1/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
-1/20*(105*(-I*sqrt(2)*cos(d*x + c)^3 - 3*I*sqrt(2)*cos(d*x + c)^2 - 3*I*s
qrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassPInverse(-4, 0, cos(d*x + c) +
 I*sin(d*x + c)) + 105*(I*sqrt(2)*cos(d*x + c)^3 + 3*I*sqrt(2)*cos(d*x + c
)^2 + 3*I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierstrassPInverse(-4, 0, cos
(d*x + c) - I*sin(d*x + c)) + 231*(-I*sqrt(2)*cos(d*x + c)^3 - 3*I*sqrt(2)
*cos(d*x + c)^2 - 3*I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassZeta(-4
, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 231*(I*s
qrt(2)*cos(d*x + c)^3 + 3*I*sqrt(2)*cos(d*x + c)^2 + 3*I*sqrt(2)*cos(d*x +
 c) + I*sqrt(2))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x
 + c) - I*sin(d*x + c))) - 2*(4*cos(d*x + c)^5 - 8*cos(d*x + c)^4 - 147*co
s(d*x + c)^3 - 238*cos(d*x + c)^2 - 105*cos(d*x + c))*sin(d*x + c)/sqrt(co
s(d*x + c)))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(
d*x + c) + a^3*d)
```

3.217.6 Sympy [F]

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx = \frac{\int \frac{1}{\sec^{\frac{11}{2}}(c+dx)+3\sec^{\frac{9}{2}}(c+dx)+3\sec^{\frac{7}{2}}(c+dx)+\sec^{\frac{5}{2}}(c+dx)}\,dx}{a^3}$$

input
```
integrate(1/sec(d*x+c)**(5/2)/(a+a*sec(d*x+c))**3,x)
```

output
```
Integral(1/(sec(c + d*x)**(11/2) + 3*sec(c + d*x)**(9/2) + 3*sec(c + d*x)*
*(7/2) + sec(c + d*x)**(5/2)), x)/a**3
```

3.217.7 Maxima [F]

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx = \int \frac{1}{(a\sec(dx+c)+a)^3\sec(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
integrate(1/((a*sec(d*x + c) + a)^3*sec(d*x + c)^(5/2)), x)
```

3.217.8 Giac [F]

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx = \int \frac{1}{(a\sec(dx+c)+a)^3\sec(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate(1/((a*sec(d*x + c) + a)^3*sec(d*x + c)^(5/2)), x)
```

3.217.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx = \int \frac{1}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^3\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

input
```
int(1/((a + a/cos(c + d*x))^3*(1/cos(c + d*x))^(5/2)),x)
```

output
```
int(1/((a + a/cos(c + d*x))^3*(1/cos(c + d*x))^(5/2)), x)
```

3.218 $\int \sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\,dx$

3.218.1 Optimal result 1780
3.218.2 Mathematica [A] (verified) 1780
3.218.3 Rubi [A] (verified) 1781
3.218.4 Maple [B] (verified) 1783
3.218.5 Fricas [A] (verification not implemented) 1783
3.218.6 Sympy [F(-1)] 1784
3.218.7 Maxima [B] (verification not implemented) 1784
3.218.8 Giac [F] 1785
3.218.9 Mupad [F(-1)] 1786

3.218.1 Optimal result

Integrand size = 25, antiderivative size = 116

$$\int \sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \frac{3\sqrt{a}\operatorname{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{4d} + \frac{3a\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4d\sqrt{a+a\sec(c+dx)}} + \frac{a\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2d\sqrt{a+a\sec(c+dx)}}$$

output
```
3/4*arcsinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))*a^(1/2)/d+3/4*a*sec
(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+1/2*a*sec(d*x+c)^(5/2)*s
in(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)
```

3.218.2 Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.93

$$\int \sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \frac{a\left(3\arcsin\left(\sqrt{1-\sec(c+dx)}\right)+2\sqrt{1-\sec(c+dx)}\sec^{\frac{3}{2}}(c+dx)+3\sqrt{-((-1+\sec(c+dx))\sec(c+d}\right.}{4d\sqrt{1-\sec(c+dx)}\sqrt{a(1+\sec(c+dx))}}$$

input
```
Integrate[Sec[c + d*x]^(5/2)*Sqrt[a + a*Sec[c + d*x]],x]
```

output
```
(a*(3*ArcSin[Sqrt[1 - Sec[c + d*x]]] + 2*Sqrt[1 - Sec[c + d*x]]*Sec[c + d*
x]^(3/2) + 3*Sqrt[-((-1 + Sec[c + d*x])*Sec[c + d*x])])*Tan[c + d*x])/(4*d
*Sqrt[1 - Sec[c + d*x]]*Sqrt[a*(1 + Sec[c + d*x])])
```

3.218.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.99, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3042, 4290, 3042, 4290, 3042, 4288, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\sqrt{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}dx$$

$$\Big\downarrow \text{4290}$$

$$\frac{3}{4}\int \sec^{\frac{3}{2}}(c+dx)\sqrt{\sec(c+dx)a+a}dx+\frac{a\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3}{4}\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx+\frac{a\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{4290}$$

$$\frac{3}{4}\left(\frac{1}{2}\int \sqrt{\sec(c+dx)}\sqrt{\sec(c+dx)a+a}dx+\frac{a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}\right)+\frac{a\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3}{4}\left(\frac{1}{2}\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx+\frac{a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}\right)+\frac{a\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}$$

$\Big\downarrow$ 4288

$$\frac{3}{4}\left(\frac{a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}-\frac{\int\frac{1}{\sqrt{\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1}}d\left(-\frac{a\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}\right)+\frac{a\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}$$

$\Big\downarrow$ 222

$$\frac{3}{4}\left(\frac{\sqrt{a}\text{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}\right)+\frac{a\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}$$

input
```
Int[Sec[c + d*x]^(5/2)*Sqrt[a + a*Sec[c + d*x]],x]
```

output
```
(a*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(2*d*Sqrt[a + a*Sec[c + d*x]]) + (3*((
Sqrt[a]*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d + (a*S
ec[c + d*x]^(3/2)*Sin[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]])))/4
```

3.218.3.1 Defintions of rubi rules used

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4288
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[-2*(a/(b*f))*Sqrt[a*(d/b)]   Subst[Int[1/Sqrt[1
+ x^2/a], x], x, b*(Cot[e + f*x]/Sqrt[a + b*Csc[e + f*x]])], x] /; FreeQ[{a
, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[a*(d/b), 0]
```

rule 4290
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[-2*b*d*Cot[e + f*x]*((d*Csc[e + f*x])^(n - 1)/(
f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]])), x] + Simp[2*a*d*((n - 1)/(b*(2*n -
1)))   Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e + f*x])^(n - 1), x], x] /; Fre
eQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

3.218.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 212 vs. $2(96) = 192$.

Time = 1.64 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.84

method	result
default	$\frac{\sec(dx+c)^{\frac{5}{2}}\sqrt{a(1+\sec(dx+c))}\left(6\sqrt{-\frac{1}{\cos(dx+c)+1}}\,\sin(dx+c)\cos(dx+c)^2-3\cos(dx+c)^3\arctan\left(\frac{-\cos(dx+c)+\sin(dx+c)-1}{2(\cos(dx+c)+1)\sqrt{-\frac{1}{\cos(dx+c)+1}}}\right)-\right.}{8d(\cos(dx+c)+1)\sqrt{-\frac{1}{\cos(dx+c)+1}}}$

input
```
int(sec(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/8/d*sec(d*x+c)^(5/2)*(a*(1+sec(d*x+c)))^(1/2)/(cos(d*x+c)+1)/(-1/(cos(d*
x+c)+1))^(1/2)*(6*(-1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*cos(d*x+c)^2-3*cos(
d*x+c)^3*arctan(1/2*(-cos(d*x+c)+sin(d*x+c)-1)/(cos(d*x+c)+1)/(-1/(cos(d*x
+c)+1))^(1/2))-3*cos(d*x+c)^3*arctan(1/2*(cos(d*x+c)+sin(d*x+c)+1)/(cos(d*
x+c)+1)/(-1/(cos(d*x+c)+1))^(1/2))+4*(-1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*
cos(d*x+c))
```

3.218.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 356, normalized size of antiderivative = 3.07

$$\int \sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\,dx$$
$$= \left[\frac{3\left(\cos\left(dx+c\right)^2+\cos\left(dx+c\right)\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7\,a\cos(dx+c)^2-\frac{4\left(\cos(dx+c)^2-2\,\cos(dx+c)\right)\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{\cos(dx+c)^3+\cos(dx+c)^2}\right.}{16\left(d\cos\left(dx+c\right)^2+d\cos\left(dx+c\right)\right)}\right.$$

input
```
integrate(sec(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
[1/16*(3*(cos(d*x + c)^2 + cos(d*x + c))*sqrt(a)*log((a*cos(d*x + c)^3 - 7
*a*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 - 2*cos(d*x + c))*sqrt(a)*sqrt((a*co
s(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)) + 8*a)/(cos(
d*x + c)^3 + cos(d*x + c)^2)) + 4*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*
(3*cos(d*x + c) + 2)*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c)^2 +
d*cos(d*x + c)), 1/8*(3*(cos(d*x + c)^2 + cos(d*x + c))*sqrt(-a)*arctan(2*
sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*
x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)) + 2*sqrt((a*cos(d*x + c)
 + a)/cos(d*x + c))*(3*cos(d*x + c) + 2)*sin(d*x + c)/sqrt(cos(d*x + c)))/
(d*cos(d*x + c)^2 + d*cos(d*x + c))]
```

3.218.6 Sympy [F(-1)]

Timed out.

$$\int \sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**(5/2)*(a+a*sec(d*x+c))**(1/2),x)
```

output
```
Timed out
```

3.218.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1264 vs. $2(96) = 192$.

Time = 0.44 (sec) , antiderivative size = 1264, normalized size of antiderivative = 10.90

$$\int \sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(sec(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
-1/16*(12*(sqrt(2)*sin(4*d*x + 4*c) + 2*sqrt(2)*sin(2*d*x + 2*c))*cos(7/2*
arctan2(sin(d*x + c), cos(d*x + c))) + 4*(sqrt(2)*sin(4*d*x + 4*c) + 2*sqr
t(2)*sin(2*d*x + 2*c))*cos(5/2*arctan2(sin(d*x + c), cos(d*x + c))) - 4*(s
qrt(2)*sin(4*d*x + 4*c) + 2*sqrt(2)*sin(2*d*x + 2*c))*cos(3/2*arctan2(sin(
d*x + c), cos(d*x + c))) - 12*(sqrt(2)*sin(4*d*x + 4*c) + 2*sqrt(2)*sin(2*
d*x + 2*c))*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) - 3*(2*(2*cos(2*d
*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^
2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x
 + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*x + c), co
s(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sqrt
(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2*sqrt(2)*sin(1/2*arcta
n2(sin(d*x + c), cos(d*x + c))) + 2) + 3*(2*(2*cos(2*d*x + 2*c) + 1)*cos(4
*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)
^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*
d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*s
in(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(
sin(d*x + c), cos(d*x + c))) - 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos
(d*x + c))) + 2) - 3*(2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*
d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4
*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*l...
```

3.218.8 Giac [F]

$$\int \sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \int \sqrt{a\sec(dx+c)+a}\sec(dx+c)^{\frac{5}{2}}\,dx$$

input
```
integrate(sec(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.218.9 Mupad [F(-1)]

Timed out.

$$\int \sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\,dx=\int \sqrt{a+\frac{a}{\cos\left(c+d\,x\right)}}\,\left(\frac{1}{\cos\left(c+d\,x\right)}\right)^{5/2}\,dx$$

input
```
int((a + a/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(5/2),x)
```

output
```
int((a + a/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(5/2), x)
```

3.219 $\int \sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\,dx$

3.219.1 Optimal result 1787
3.219.2 Mathematica [A] (verified) 1787
3.219.3 Rubi [A] (verified) 1788
3.219.4 Maple [B] (verified) 1789
3.219.5 Fricas [B] (verification not implemented) 1790
3.219.6 Sympy [F] 1790
3.219.7 Maxima [B] (verification not implemented) 1791
3.219.8 Giac [F] 1791
3.219.9 Mupad [F(-1)] 1792

3.219.1 Optimal result

Integrand size = 25, antiderivative size = 72

$$\int \sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\,dx=\frac{\sqrt{a}\operatorname{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}+\frac{a\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}}$$

output
```
arcsinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))*a^(1/2)/d+a*sec(d*x+c)^
(3/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)
```

3.219.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.04

$$\int \sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\,dx$$
$$=\frac{a\left(\arcsin\left(\sqrt{1-\sec(c+dx)}\right)+\sqrt{-((-1+\sec(c+dx))\sec(c+dx))}\right)\tan(c+dx)}{d\sqrt{1-\sec(c+dx)}\sqrt{a(1+\sec(c+dx))}}$$

input
```
Integrate[Sec[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]],x]
```

output
```
(a*(ArcSin[Sqrt[1 - Sec[c + d*x]]] + Sqrt[-((-1 + Sec[c + d*x])*Sec[c + d*
x])])*Tan[c + d*x])/(d*Sqrt[1 - Sec[c + d*x]]*Sqrt[a*(1 + Sec[c + d*x])])
```

3.219.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3042, 4290, 3042, 4288, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}dx$$

$$\Big\downarrow \text{4290}$$

$$\frac{1}{2}\int \sqrt{\sec(c+dx)}\sqrt{\sec(c+dx)a+a}dx+\frac{a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx+\frac{a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{4288}$$

$$\frac{a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}-\frac{\int \frac{1}{\sqrt{\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}}+1}d\left(-\frac{a\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}$$

$$\Big\downarrow \text{222}$$

$$\frac{\sqrt{a}\text{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}$$

input
```
Int[Sec[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]],x]
```

output
```
(Sqrt[a]*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d + (a*
Sec[c + d*x]^(3/2)*Sin[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]])
```

3.219.3.1 Defintions of rubi rules used

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4288
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[-2*(a/(b*f))*Sqrt[a*(d/b)]   Subst[Int[1/Sqrt[1
+ x^2/a], x], x, b*(Cot[e + f*x]/Sqrt[a + b*Csc[e + f*x]])], x] /; FreeQ[{a
, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[a*(d/b), 0]
```

rule 4290
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[-2*b*d*Cot[e + f*x]*((d*Csc[e + f*x])^(n - 1)/(
f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]])), x] + Simp[2*a*d*((n - 1)/(b*(2*n -
1)))   Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e + f*x])^(n - 1), x], x] /; Fre
eQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

3.219.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 182 vs. $2(62) = 124$.

Time = 1.71 (sec) , antiderivative size = 183, normalized size of antiderivative = 2.54

method	result
default	$\frac{\sec(dx+c)^{\frac{3}{2}}\sqrt{a(1+\sec(dx+c))}\left(2\sqrt{-\frac{1}{\cos(dx+c)+1}}\,\sin(dx+c)\cos(dx+c)-\cos(dx+c)^{2}\arctan\left(\frac{-\cos(dx+c)+\sin(dx+c)-1}{2(\cos(dx+c)+1)\sqrt{-\frac{1}{\cos(dx+c)+1}}}\right)-\mathrm{arc}\right.}{2d(\cos(dx+c)+1)\sqrt{-\frac{1}{\cos(dx+c)+1}}}$

input
```
int(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/d*sec(d*x+c)^(3/2)*(a*(1+sec(d*x+c)))^(1/2)/(cos(d*x+c)+1)/(-1/(cos(d*
x+c)+1))^(1/2)*(2*(-1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*cos(d*x+c)-cos(d*x+
c)^2*arctan(1/2*(-cos(d*x+c)+sin(d*x+c)-1)/(cos(d*x+c)+1)/(-1/(cos(d*x+c)+
1))^(1/2))-arctan(1/2*(cos(d*x+c)+sin(d*x+c)+1)/(cos(d*x+c)+1)/(-1/(cos(d*
x+c)+1))^(1/2))*cos(d*x+c)^2)
```

3.219.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(62) = 124$.

Time = 0.31 (sec) , antiderivative size = 302, normalized size of antiderivative = 4.19

$$\int \sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\,dx$$
$$= \left[\frac{\sqrt{a}(\cos(dx+c)+1)\log\left(\frac{a\cos(dx+c)^3-7\,a\cos(dx+c)^2-\frac{4\left(\cos(dx+c)^2-2\,\cos(dx+c)\right)\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{\sqrt{\cos(dx+c)}}+8\,a}{\cos(dx+c)^3+\cos(dx+c)^2}\right)+\frac{4\sqrt{\frac{a\,c}{}}}{}}{4\,(d\cos(dx+c)+d)}\right.$$

input
```
integrate(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
[1/4*(sqrt(a)*(cos(d*x + c) + 1)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^
2 - 4*(cos(d*x + c)^2 - 2*cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/
cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)) + 8*a)/(cos(d*x + c)^3 + cos
(d*x + c)^2)) + 4*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqr
t(cos(d*x + c)))/(d*cos(d*x + c) + d), 1/2*(sqrt(-a)*(cos(d*x + c) + 1)*ar
ctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))
*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)) + 2*sqrt((a*cos(d
*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c
) + d)]
```

3.219.6 Sympy [F]

$$\int \sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \int \sqrt{a\left(\sec\left(c+dx\right)+1\right)}\sec^{\frac{3}{2}}\left(c+dx\right)dx$$

input
```
integrate(sec(d*x+c)**(3/2)*(a+a*sec(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(a*(sec(c + d*x) + 1))*sec(c + d*x)**(3/2), x)
```

3.219.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 662 vs. $2(62) = 124$.

Time = 0.39 (sec) , antiderivative size = 662, normalized size of antiderivative = 9.19

$$\int \sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
-1/4*(4*sqrt(2)*cos(3/2*arctan2(sin(d*x + c), cos(d*x + c)))*sin(2*d*x + 2
*c) - 4*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))*sin(2*d*x + 2
*c) - (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*l
og(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(si
n(d*x + c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos
(d*x + c))) + 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2)
+ (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2
*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*
x + c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x
 + c))) - 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) - (c
os(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2*cos
(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x +
c), cos(d*x + c)))^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c
))) + 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) + (cos(2
*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2
*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c),
cos(d*x + c)))^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))
- 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) - 4*(sqrt(2)
*cos(2*d*x + 2*c) + sqrt(2))*sin(3/2*arctan2(sin(d*x + c), cos(d*x + c)))
+ 4*(sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*sin(1/2*arctan2(sin(d*x + c), ...
```

3.219.8 Giac [F]

$$\int \sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \int \sqrt{a\sec(dx+c)+a}\sec(dx+c)^{\frac{3}{2}}\,dx$$

input
```
integrate(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.219.9 Mupad [F(-1)]

Timed out.

$$\int \sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\,dx=\int \sqrt{a+\frac{a}{\cos\left(c+d\,x\right)}}\,\left(\frac{1}{\cos\left(c+d\,x\right)}\right)^{3/2}\,dx$$

input
```
int((a + a/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(3/2),x)
```

output
```
int((a + a/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(3/2), x)
```

3.220 $\int \sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+dx)}\,dx$

3.220.1 Optimal result 1793
3.220.2 Mathematica [A] (verified) 1793
3.220.3 Rubi [A] (verified) 1794
3.220.4 Maple [B] (verified) 1795
3.220.5 Fricas [B] (verification not implemented) 1795
3.220.6 Sympy [F] 1796
3.220.7 Maxima [B] (verification not implemented) 1796
3.220.8 Giac [F] 1797
3.220.9 Mupad [F(-1)] 1797

3.220.1 Optimal result

Integrand size = 25, antiderivative size = 37

$$\int \sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+dx)}\,dx=\frac{2\sqrt{a}\operatorname{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}$$

output
```
2*arcsinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))*a^(1/2)/d
```

3.220.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.46

$$\int \sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+dx)}\,dx=-\frac{2\arcsin\left(\sqrt{\sec(c+dx)}\right)\sqrt{a(1+\sec(c+dx))}\tan\left(\frac{1}{2}(c+dx)\right)}{d\sqrt{1-\sec(c+dx)}}$$

input
```
Integrate[Sqrt[Sec[c + d*x]]*Sqrt[a + a*Sec[c + d*x]],x]
```

output
```
(-2*ArcSin[Sqrt[Sec[c + d*x]]]*Sqrt[a*(1 + Sec[c + d*x])]*Tan[(c + d*x)/2]
)/(d*Sqrt[1 - Sec[c + d*x]])
```

3.220.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {3042, 4288, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}dx$$

$$\Big\downarrow \text{4288}$$

$$-\frac{2\int \frac{1}{\sqrt{\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1}}d\left(-\frac{a\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}$$

$$\Big\downarrow \text{222}$$

$$\frac{2\sqrt{a}\text{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}$$

input
```
Int[Sqrt[Sec[c + d*x]]*Sqrt[a + a*Sec[c + d*x]],x]
```

output
```
(2*Sqrt[a]*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d
```

3.220.3.1 Defintions of rubi rules used

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4288
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[-2*(a/(b*f))*Sqrt[a*(d/b)]   Subst[Int[1/Sqrt[1
+ x^2/a], x], x, b*(Cot[e + f*x]/Sqrt[a + b*Csc[e + f*x]])], x] /; FreeQ[{a
, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[a*(d/b), 0]
```

3.220.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 142 vs. $2(31) = 62$.

Time = 1.85 (sec) , antiderivative size = 143, normalized size of antiderivative = 3.86

method	result
default	$-\frac{\sqrt{a(1+\sec(dx+c))}\,\sqrt{\sec(dx+c)}\left(\arctan\left(\frac{\cos(dx+c)+\sin(dx+c)+1}{2(\cos(dx+c)+1)\sqrt{-\frac{1}{\cos(dx+c)+1}}}\right)-\arctan\left(\frac{\cos(dx+c)-\sin(dx+c)+1}{2(\cos(dx+c)+1)\sqrt{-\frac{1}{\cos(dx+c)+1}}}\right)\right)\cos(dx+}{d(\cos(dx+c)+1)\sqrt{-\frac{1}{\cos(dx+c)+1}}}$

input
```
int(sec(d*x+c)^(1/2)*(a+a*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/d*(a*(1+sec(d*x+c)))^(1/2)*sec(d*x+c)^(1/2)*(arctan(1/2*(cos(d*x+c)+sin
(d*x+c)+1)/(cos(d*x+c)+1)/(-1/(cos(d*x+c)+1))^(1/2))-arctan(1/2*(cos(d*x+c
)-sin(d*x+c)+1)/(cos(d*x+c)+1)/(-1/(cos(d*x+c)+1))^(1/2)))*cos(d*x+c)/(cos
(d*x+c)+1)/(-1/(cos(d*x+c)+1))^(1/2)
```

3.220.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 77 vs. $2(31) = 62$.

Time = 0.30 (sec) , antiderivative size = 189, normalized size of antiderivative = 5.11

$$\int \sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+dx)}\,dx$$
$$= \left[\frac{\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7\,a\cos(dx+c)^2-\frac{4\left(\cos(dx+c)^2-2\,\cos(dx+c)\right)\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{\sqrt{\cos(dx+c)}}+8\,a}{\cos(dx+c)^3+\cos(dx+c)^2}\right)}{2\,d}, \frac{\sqrt{-a}\arctan\left(\frac{2\sqrt{-a}\sqrt{\frac{a\cos}{co}}}{a\cos(}\right.}{}\right.$$

input
```
integrate(sec(d*x+c)^(1/2)*(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
[1/2*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*(cos(d*x + c)^
2 - 2*cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*
x + c)/sqrt(cos(d*x + c)) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2))/d, sqr
t(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d
*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a))/d]
```

3.220.6 Sympy [F]

$$\int \sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+dx)}\,dx = \int \sqrt{a\left(\sec\left(c+dx\right)+1\right)}\sqrt{\sec\left(c+dx\right)}\,dx$$

input
```
integrate(sec(d*x+c)**(1/2)*(a+a*sec(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(a*(sec(c + d*x) + 1))*sqrt(sec(c + d*x)), x)
```

3.220.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 241 vs. $2(31) = 62$.

Time = 0.38 (sec) , antiderivative size = 241, normalized size of antiderivative = 6.51

$$\int \sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+dx)}\,dx$$
$$= \frac{\sqrt{a}\left(\log\left(2\,\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+2\,\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+2\,\sqrt{2}\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+2\,\sqrt{2}\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+2\right)-}{}$$

input
```
integrate(sec(d*x+c)^(1/2)*(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
1/2*sqrt(a)*(log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*s
qrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - log(2*
cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x
+ 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + log(2*cos(1/2*d*x + 1/2*c
)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2
)*sin(1/2*d*x + 1/2*c) + 2) - log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x
 + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2
*c) + 2))/d
```

3.220.8 Giac [F]

$$\int \sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+dx)}\,dx = \int \sqrt{a\sec(dx+c)+a}\sqrt{\sec(dx+c)}\,dx$$

input
```
integrate(sec(d*x+c)^(1/2)*(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.220.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+dx)}\,dx = \int \sqrt{a+\frac{a}{\cos(c+d\,x)}}\,\sqrt{\frac{1}{\cos(c+d\,x)}}\,dx$$

input
```
int((a + a/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(1/2),x)
```

output
```
int((a + a/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(1/2), x)
```

3.221 $\int \frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{\sec(c+dx)}}\,dx$

3.221.1 Optimal result 1798
3.221.2 Mathematica [A] (verified) 1798
3.221.3 Rubi [A] (verified) 1799
3.221.4 Maple [A] (verified) 1800
3.221.5 Fricas [A] (verification not implemented) 1800
3.221.6 Sympy [F] 1800
3.221.7 Maxima [A] (verification not implemented) 1801
3.221.8 Giac [F] 1801
3.221.9 Mupad [B] (verification not implemented) 1801

3.221.1 Optimal result

Integrand size = 25, antiderivative size = 36

$$\int \frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{\sec(c+dx)}}\,dx=\frac{2a\sqrt{\sec(c+dx)}\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}}$$

output
```
2*a*sin(d*x+c)*sec(d*x+c)^(1/2)/d/(a+a*sec(d*x+c))^(1/2)
```

3.221.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.08

$$\int \frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{\sec(c+dx)}}\,dx=\frac{2\sqrt{a(1+\sec(c+dx))}\tan\left(\frac{1}{2}(c+dx)\right)}{d\sqrt{\sec(c+dx)}}$$

input
```
Integrate[Sqrt[a + a*Sec[c + d*x]]/Sqrt[Sec[c + d*x]],x]
```

output
```
(2*Sqrt[a*(1 + Sec[c + d*x])]*Tan[(c + d*x)/2])/(d*Sqrt[Sec[c + d*x]])
```

3.221.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {3042, 4291}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4291}$$

$$\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\sec(c+dx)+a}}$$

input
```
Int[Sqrt[a + a*Sec[c + d*x]]/Sqrt[Sec[c + d*x]],x]
```

output
```
(2*a*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]])
```

3.221.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4291
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[-2*a*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*S
qrt[d*Csc[e + f*x]])), x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

3.221.4 Maple [A] (verified)

Time = 1.21 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.14

method	result	size
default	$-\frac{2\sqrt{a(1+\sec(dx+c))}\,(\cot(dx+c)-\csc(dx+c))}{d\sqrt{\sec(dx+c)}}$	41
risch	$-\frac{i\sqrt{2}\sqrt{\frac{a\left(e^{i(dx+c)}+1\right)^2}{e^{2i(dx+c)}+1}}\,(e^{i(dx+c)}-1)}{\sqrt{\frac{e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,(e^{i(dx+c)}+1)d}$	89

input
```
int((a+a*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/d*(a*(1+sec(d*x+c)))^(1/2)/sec(d*x+c)^(1/2)*(cot(d*x+c)-csc(d*x+c))
```

3.221.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.36

$$\int \frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{\sec(c+dx)}}\,dx = \frac{2\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)}{d\cos(dx+c)+d}$$

input
```
integrate((a+a*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
2*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/
(d*cos(d*x + c) + d)
```

3.221.6 Sympy [F]

$$\int \frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{\sec(c+dx)}}\,dx = \int \frac{\sqrt{a\left(\sec(c+dx)+1\right)}}{\sqrt{\sec(c+dx)}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))**(1/2)/sec(d*x+c)**(1/2),x)
```

output
```
Integral(sqrt(a*(sec(c + d*x) + 1))/sqrt(sec(c + d*x)), x)
```

3.221.7 Maxima [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.56

$$\int \frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{\sec(c+dx)}}\,dx = \frac{2\sqrt{2}\sqrt{a}\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{d}$$

input
```
integrate((a+a*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
2*sqrt(2)*sqrt(a)*sin(1/2*d*x + 1/2*c)/d
```

3.221.8 Giac [F]

$$\int \frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{\sec(c+dx)}}\,dx = \int \frac{\sqrt{a\sec(dx+c)+a}}{\sqrt{\sec(dx+c)}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.221.9 Mupad [B] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.47

$$\int \frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{\sec(c+dx)}}\,dx = \frac{\sin(2c+2dx)\sqrt{\frac{1}{\cos(c+dx)}}\sqrt{\frac{a(\cos(c+dx)+1)}{\cos(c+dx)}}}{d(\cos(c+dx)+1)}$$

input
```
int((a + a/cos(c + d*x))^(1/2)/(1/cos(c + d*x))^(1/2),x)
```

output
```
(sin(2*c + 2*d*x)*(1/cos(c + d*x))^(1/2)*((a*(cos(c + d*x) + 1))/cos(c + d
*x))^(1/2))/(d*(cos(c + d*x) + 1))
```

3.222 $\int \frac{\sqrt{a+a\sec(c+dx)}}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

3.222.1 Optimal result 1802
3.222.2 Mathematica [A] (verified) 1802
3.222.3 Rubi [A] (verified) 1803
3.222.4 Maple [A] (verified) 1804
3.222.5 Fricas [A] (verification not implemented) 1804
3.222.6 Sympy [F] 1805
3.222.7 Maxima [A] (verification not implemented) 1805
3.222.8 Giac [F] 1806
3.222.9 Mupad [B] (verification not implemented) 1806

3.222.1 Optimal result

Integrand size = 25, antiderivative size = 77

$$\int \frac{\sqrt{a+a\sec(c+dx)}}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \frac{2a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+dx)}}+\frac{4a\sqrt{\sec(c+dx)}\sin(c+dx)}{3d\sqrt{a+a\sec(c+dx)}}$$

output
```
2/3*a*sin(d*x+c)/d/sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2)+4/3*a*sin(d*x+c
)*sec(d*x+c)^(1/2)/d/(a+a*sec(d*x+c))^(1/2)
```

3.222.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.64

$$\int \frac{\sqrt{a+a\sec(c+dx)}}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \frac{2(2+\cos(c+dx))\sqrt{a(1+\sec(c+dx))}\tan\left(\frac{1}{2}(c+dx)\right)}{3d\sqrt{\sec(c+dx)}}$$

input
```
Integrate[Sqrt[a + a*Sec[c + d*x]]/Sec[c + d*x]^(3/2),x]
```

output
```
(2*(2 + Cos[c + d*x])*Sqrt[a*(1 + Sec[c + d*x])]*Tan[(c + d*x)/2])/(3*d*Sq
rt[Sec[c + d*x]])
```

3.222.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {3042, 4292, 3042, 4291}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a\sec(c+dx)+a}}{\sec^{\frac{3}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{4292}$$

$$\frac{2}{3}\int \frac{\sqrt{\sec(c+dx)a+a}}{\sqrt{\sec(c+dx)}}dx+\frac{2a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{3}\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{4291}$$

$$\frac{4a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\sqrt{a\sec(c+dx)+a}}+\frac{2a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}$$

input
```
Int[Sqrt[a + a*Sec[c + d*x]]/Sec[c + d*x]^(3/2),x]
```

output
```
(2*a*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) + (4*
a*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*d*Sqrt[a + a*Sec[c + d*x]])
```

3.222.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4291
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[-2*a*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*S
qrt[d*Csc[e + f*x]])), x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4292
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[a*Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*n*Sqrt[a
+ b*Csc[e + f*x]])), x] + Simp[a*((2*n + 1)/(2*b*d*n))   Int[Sqrt[a + b*Csc
[e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
 EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

3.222.4 Maple [A] (verified)

Time = 1.21 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.66

method	result	size
default	$\frac{2\sqrt{a(1+\sec(dx+c))}\,(\sin(dx+c)+2\tan(dx+c))}{3d(\cos(dx+c)+1)\sec(dx+c)^{\frac{3}{2}}}$	51

input
```
int((a+a*sec(d*x+c))^(1/2)/sec(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/3/d*(a*(1+sec(d*x+c)))^(1/2)/(cos(d*x+c)+1)/sec(d*x+c)^(3/2)*(sin(d*x+c)
+2*tan(d*x+c))
```

3.222.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.86

$$\int \frac{\sqrt{a+a\sec(c+dx)}}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \frac{2\left(\cos\left(dx+c\right)^2+2\,\cos\left(dx+c\right)\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin\left(dx+c\right)}{3\left(d\cos\left(dx+c\right)+d\right)\sqrt{\cos\left(dx+c\right)}}$$

input
```
integrate((a+a*sec(d*x+c))^(1/2)/sec(d*x+c)^(3/2),x, algorithm="fricas")
```

output
```
2/3*(cos(d*x + c)^2 + 2*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x +
c))*sin(d*x + c)/((d*cos(d*x + c) + d)*sqrt(cos(d*x + c)))
```

3.222.6 Sympy [F]

$$\int \frac{\sqrt{a+a\sec(c+dx)}}{\sec^{\frac{3}{2}}(c+dx)}\,dx=\int \frac{\sqrt{a\left(\sec\left(c+dx\right)+1\right)}}{\sec^{\frac{3}{2}}\left(c+dx\right)}\,dx$$

input
```
integrate((a+a*sec(d*x+c))**(1/2)/sec(d*x+c)**(3/2),x)
```

output
```
Integral(sqrt(a*(sec(c + d*x) + 1))/sec(c + d*x)**(3/2), x)
```

3.222.7 Maxima [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.47

$$\int \frac{\sqrt{a+a\sec(c+dx)}}{\sec^{\frac{3}{2}}(c+dx)}\,dx$$
$$=\frac{\sqrt{2}\left(3\cos\left(\frac{2}{3}\arctan\left(\sin\left(\frac{3}{2}dx+\frac{3}{2}c\right),\cos\left(\frac{3}{2}dx+\frac{3}{2}c\right)\right)\right)\sin\left(\frac{3}{2}dx+\frac{3}{2}c\right)-3\cos\left(\frac{3}{2}dx+\frac{3}{2}c\right)\sin\left(\frac{2}{3}\arctan\right.\right.}{}$$

input
```
integrate((a+a*sec(d*x+c))^(1/2)/sec(d*x+c)^(3/2),x, algorithm="maxima")
```

output
```
1/6*sqrt(2)*(3*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))
)*sin(3/2*d*x + 3/2*c) - 3*cos(3/2*d*x + 3/2*c)*sin(2/3*arctan2(sin(3/2*d*
x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sin(3/2*d*x + 3/2*c) + 3*sin(1/3*ar
ctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sqrt(a)/d
```

3.222.8 Giac [F]

$$\int \frac{\sqrt{a+a\sec(c+dx)}}{\sec^{\frac{3}{2}}(c+dx)}\,dx=\int \frac{\sqrt{a\sec(dx+c)+a}}{\sec(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))^(1/2)/sec(d*x+c)^(3/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.222.9 Mupad [B] (verification not implemented)

Time = 13.92 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.90

$$\int \frac{\sqrt{a+a\sec(c+dx)}}{\sec^{\frac{3}{2}}(c+dx)}\,dx$$
$$=\frac{\cos(c+d\,x)\,\left(4\,\sin(c+d\,x)+\sin(2\,c+2\,d\,x)\right)\,\sqrt{\frac{1}{\cos(c+d\,x)}}\,\sqrt{\frac{a\,(\cos(c+d\,x)+1)}{\cos(c+d\,x)}}}{3\,d\,\left(\cos(c+d\,x)+1\right)}$$

input
```
int((a + a/cos(c + d*x))^(1/2)/(1/cos(c + d*x))^(3/2),x)
```

output
```
(cos(c + d*x)*(4*sin(c + d*x) + sin(2*c + 2*d*x))*(1/cos(c + d*x))^(1/2)*(
(a*(cos(c + d*x) + 1))/cos(c + d*x))^(1/2))/(3*d*(cos(c + d*x) + 1))
```

3.223 $\int \frac{\sqrt{a+a\sec(c+dx)}}{\sec^{\frac{5}{2}}(c+dx)}\,dx$

3.223.1 Optimal result . . . 1807
3.223.2 Mathematica [A] (verified) . . . 1807
3.223.3 Rubi [A] (verified) . . . 1808
3.223.4 Maple [A] (verified) . . . 1810
3.223.5 Fricas [A] (verification not implemented) . . . 1810
3.223.6 Sympy [F] . . . 1810
3.223.7 Maxima [B] (verification not implemented) . . . 1811
3.223.8 Giac [F] . . . 1811
3.223.9 Mupad [B] (verification not implemented) . . . 1812

3.223.1 Optimal result

Integrand size = 25, antiderivative size = 115

$$\int \frac{\sqrt{a+a\sec(c+dx)}}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \frac{2a\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{8a\sin(c+dx)}{15d\sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+dx)}} + \frac{16a\sqrt{\sec(c+dx)}\sin(c+dx)}{15d\sqrt{a+a\sec(c+dx)}}$$

output
```
2/5*a*sin(d*x+c)/d/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2)+8/15*a*sin(d*x+
c)/d/sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2)+16/15*a*sin(d*x+c)*sec(d*x+c)
^(1/2)/d/(a+a*sec(d*x+c))^(1/2)
```

3.223.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.53

$$\int \frac{\sqrt{a+a\sec(c+dx)}}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \frac{(19+8\cos(c+dx)+3\cos(2(c+dx)))\sqrt{a(1+\sec(c+dx))}\tan\left(\frac{1}{2}(c+dx)\right)}{15d\sqrt{\sec(c+dx)}}$$

input
```
Integrate[Sqrt[a + a*Sec[c + d*x]]/Sec[c + d*x]^(5/2),x]
```

output
```
((19 + 8*Cos[c + d*x] + 3*Cos[2*(c + d*x)])*Sqrt[a*(1 + Sec[c + d*x])]*Tan
[(c + d*x)/2])/(15*d*Sqrt[Sec[c + d*x]])
```

3.223.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3042, 4292, 3042, 4292, 3042, 4291}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a\sec(c+dx)+a}}{\sec^{\frac{5}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx$$

$$\Big\downarrow \text{4292}$$

$$\frac{4}{5}\int \frac{\sqrt{\sec(c+dx)a+a}}{\sec^{\frac{3}{2}}(c+dx)}dx+\frac{2a\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4}{5}\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+\frac{2a\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{4292}$$

$$\frac{4}{5}\left(\frac{2}{3}\int \frac{\sqrt{\sec(c+dx)a+a}}{\sqrt{\sec(c+dx)}}dx+\frac{2a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}\right)+\frac{2a\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4}{5}\left(\frac{2}{3}\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}\right)+\frac{2a\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{4291}$$

$$\frac{2a\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{4}{5}\left(\frac{4a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\sqrt{a\sec(c+dx)+a}}+\frac{2a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}\right)$$

input
```
Int[Sqrt[a + a*Sec[c + d*x]]/Sec[c + d*x]^(5/2),x]
```

output
```
(2*a*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]]) + (4*
((2*a*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) + (4
*a*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*d*Sqrt[a + a*Sec[c + d*x]])))/5
```

3.223.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4291
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[-2*a*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*S
qrt[d*Csc[e + f*x]])), x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4292
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[a*Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*n*Sqrt[a
+ b*Csc[e + f*x]])), x] + Simp[a*((2*n + 1)/(2*b*d*n))   Int[Sqrt[a + b*Csc
[e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
 EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

3.223.4 Maple [A] (verified)

Time = 1.22 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.58

method	result	size
default	$\frac{2\sqrt{a(1+\sec(dx+c))}\,(3\sin(dx+c)+4\tan(dx+c)+8\sec(dx+c)\tan(dx+c))}{15d(\cos(dx+c)+1)\sec(dx+c)^{\frac{5}{2}}}$	67

input
```
int((a+a*sec(d*x+c))^(1/2)/sec(d*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/15/d*(a*(1+sec(d*x+c)))^(1/2)/(cos(d*x+c)+1)/sec(d*x+c)^(5/2)*(3*sin(d*x
+c)+4*tan(d*x+c)+8*sec(d*x+c)*tan(d*x+c))
```

3.223.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.68

$$\int \frac{\sqrt{a+a\sec(c+dx)}}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \frac{2\left(3\cos\left(dx+c\right)^3+4\cos\left(dx+c\right)^2+8\cos\left(dx+c\right)\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin\left(dx+c\right)}{15\left(d\cos\left(dx+c\right)+d\right)\sqrt{\cos\left(dx+c\right)}}$$

input
```
integrate((a+a*sec(d*x+c))^(1/2)/sec(d*x+c)^(5/2),x, algorithm="fricas")
```

output
```
2/15*(3*cos(d*x + c)^3 + 4*cos(d*x + c)^2 + 8*cos(d*x + c))*sqrt((a*cos(d*
x + c) + a)/cos(d*x + c))*sin(d*x + c)/((d*cos(d*x + c) + d)*sqrt(cos(d*x
+ c)))
```

3.223.6 Sympy [F]

$$\int \frac{\sqrt{a+a\sec(c+dx)}}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{\sqrt{a\left(\sec\left(c+dx\right)+1\right)}}{\sec^{\frac{5}{2}}\left(c+dx\right)}\,dx$$

input
```
integrate((a+a*sec(d*x+c))**(1/2)/sec(d*x+c)**(5/2),x)
```

output
```
Integral(sqrt(a*(sec(c + d*x) + 1))/sec(c + d*x)**(5/2), x)
```

3.223.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 203 vs. $2(97) = 194$.

Time = 0.39 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.77

$$\int \frac{\sqrt{a+a\sec(c+dx)}}{\sec^{\frac{5}{2}}(c+dx)}\,dx$$
$$= \frac{\sqrt{2}\left(30\cos\left(\frac{4}{5}\arctan\left(\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right),\cos\left(\frac{5}{2}dx+\frac{5}{2}c\right)\right)\right)\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right)+5\cos\left(\frac{2}{5}\arctan\left(\sin\left(\frac{5}{2}dx+\right.\right.\right.\right.}{}$$

input
```
integrate((a+a*sec(d*x+c))^(1/2)/sec(d*x+c)^(5/2),x, algorithm="maxima")
```

output
```
1/60*sqrt(2)*(30*cos(4/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c
)))*sin(5/2*d*x + 5/2*c) + 5*cos(2/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2
*d*x + 5/2*c)))*sin(5/2*d*x + 5/2*c) - 30*cos(5/2*d*x + 5/2*c)*sin(4/5*arc
tan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) - 5*cos(5/2*d*x + 5/2*c)
*sin(2/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 6*sin(5/2*
d*x + 5/2*c) + 5*sin(3/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c
))) + 30*sin(1/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))))*sqr
t(a)/d
```

3.223.8 Giac [F]

$$\int \frac{\sqrt{a+a\sec(c+dx)}}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{\sqrt{a\sec(dx+c)+a}}{\sec(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))^(1/2)/sec(d*x+c)^(5/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.223.9 Mupad [B] (verification not implemented)

Time = 14.26 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.71

$$\int \frac{\sqrt{a+a\sec(c+dx)}}{\sec^{\frac{5}{2}}(c+dx)}\,dx$$
$$=\frac{\cos\left(c+d\,x\right)\,\sqrt{\frac{1}{\cos\left(c+d\,x\right)}}\,\sqrt{\frac{a\,\left(\cos\left(c+d\,x\right)+1\right)}{\cos\left(c+d\,x\right)}}\,\left(35\,\sin\left(c+d\,x\right)+8\,\sin\left(2\,c+2\,d\,x\right)+3\,\sin\left(3\,c+3\,d\,x\right)\right)}{30\,d\,\left(\cos\left(c+d\,x\right)+1\right)}$$

input
```
int((a + a/cos(c + d*x))^(1/2)/(1/cos(c + d*x))^(5/2),x)
```

output
```
(cos(c + d*x)*(1/cos(c + d*x))^(1/2)*((a*(cos(c + d*x) + 1))/cos(c + d*x))
^(1/2)*(35*sin(c + d*x) + 8*sin(2*c + 2*d*x) + 3*sin(3*c + 3*d*x)))/(30*d*
(cos(c + d*x) + 1))
```

3.224 $\int \frac{\sqrt{a+a\sec(c+dx)}}{\sec^{\frac{7}{2}}(c+dx)}\,dx$

3.224.1 Optimal result . . . 1813
3.224.2 Mathematica [A] (verified) . . . 1814
3.224.3 Rubi [A] (verified) . . . 1814
3.224.4 Maple [A] (verified) . . . 1816
3.224.5 Fricas [A] (verification not implemented) . . . 1816
3.224.6 Sympy [F(-1)] . . . 1817
3.224.7 Maxima [B] (verification not implemented) . . . 1817
3.224.8 Giac [F] . . . 1818
3.224.9 Mupad [B] (verification not implemented) . . . 1818

3.224.1 Optimal result

Integrand size = 25, antiderivative size = 153

$$\begin{aligned}\int \frac{\sqrt{a+a\sec(c+dx)}}{\sec^{\frac{7}{2}}(c+dx)}\,dx &= \frac{2a\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} \\ &+ \frac{12a\sin(c+dx)}{35d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} \\ &+ \frac{16a\sin(c+dx)}{35d\sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+dx)}} \\ &+ \frac{32a\sqrt{\sec(c+dx)}\sin(c+dx)}{35d\sqrt{a+a\sec(c+dx)}}\end{aligned}$$

output
```
2/7*a*sin(d*x+c)/d/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(1/2)+12/35*a*sin(d*x
+c)/d/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2)+16/35*a*sin(d*x+c)/d/sec(d*x
+c)^(1/2)/(a+a*sec(d*x+c))^(1/2)+32/35*a*sin(d*x+c)*sec(d*x+c)^(1/2)/d/(a+
a*sec(d*x+c))^(1/2)
```

3.224.2 Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.46

$$\int \frac{\sqrt{a+a\sec(c+dx)}}{\sec^{\frac{7}{2}}(c+dx)}\,dx = \frac{(76+47\cos(c+dx)+12\cos(2(c+dx))+5\cos(3(c+dx)))\sqrt{a(1+\sec(c+dx))}\tan\left(\frac{1}{2}(c+dx)\right)}{70d\sqrt{\sec(c+dx)}}$$

input
```
Integrate[Sqrt[a + a*Sec[c + d*x]]/Sec[c + d*x]^(7/2),x]
```

output
```
((76 + 47*Cos[c + d*x] + 12*Cos[2*(c + d*x)] + 5*Cos[3*(c + d*x)])*Sqrt[a*
(1 + Sec[c + d*x])]*Tan[(c + d*x)/2])/(70*d*Sqrt[Sec[c + d*x]])
```

3.224.3 Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.07, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3042, 4292, 3042, 4292, 3042, 4292, 3042, 4291}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a\sec(c+dx)+a}}{\sec^{\frac{7}{2}}(c+dx)}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\sqrt{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx$$

$$\Big\downarrow \; 4292$$

$$\frac{6}{7}\int \frac{\sqrt{\sec(c+dx)a+a}}{\sec^{\frac{5}{2}}(c+dx)}dx+\frac{2a\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \; 3042$$

$$\frac{6}{7}\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+\frac{2a\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \; 4292$$

$$\frac{6}{7}\left(\frac{4}{5}\int \frac{\sqrt{\sec(c+dx)a+a}}{\sec^{\frac{3}{2}}(c+dx)}dx+\frac{2a\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}\right)+\frac{2a\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \; 3042$$

$$\frac{6}{7}\left(\frac{4}{5}\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+\frac{2a\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}\right)+\frac{2a\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \; 4292$$

$$\frac{6}{7}\left(\frac{4}{5}\left(\frac{2}{3}\int \frac{\sqrt{\sec(c+dx)a+a}}{\sqrt{\sec(c+dx)}}dx+\frac{2a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}\right)+\frac{2a\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}\right)+\frac{2a\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \; 3042$$

$$\frac{6}{7}\left(\frac{4}{5}\left(\frac{2}{3}\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}\right)+\frac{2a\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx}}\right.$$

$$\frac{2a\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \; 4291$$

$$\frac{2a\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{6}{7}\left(\frac{2a\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{4}{5}\left(\frac{4a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\sqrt{a\sec(c+dx)+a}}+\frac{2a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}\right)\right.$$

input
```
Int[Sqrt[a + a*Sec[c + d*x]]/Sec[c + d*x]^(7/2),x]
```

output
```
(2*a*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/2)*Sqrt[a + a*Sec[c + d*x]]) + (6*
((2*a*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]]) + (4
*((2*a*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) + (
4*a*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*d*Sqrt[a + a*Sec[c + d*x]])))/5))/
7
```

3.224. $\int \frac{\sqrt{a+a\sec(c+dx)}}{\sec^{\frac{7}{2}}(c+dx)}\,dx$

3.224.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4291
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[-2*a*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*S
qrt[d*Csc[e + f*x]])), x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4292
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[a*Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*n*Sqrt[a
+ b*Csc[e + f*x]])), x] + Simp[a*((2*n + 1)/(2*b*d*n))   Int[Sqrt[a + b*Csc
[e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
 EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

3.224.4 Maple [A] (verified)

Time = 1.33 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.47

method	result	size
default	$\frac{2\left(5\cos(dx+c)^3+6\cos(dx+c)^2+8\cos(dx+c)+16\right)\sqrt{a(1+\sec(dx+c))}\,\tan(dx+c)}{35d(\cos(dx+c)+1)\sec(dx+c)^{\frac{3}{2}}}$	72

input
```
int((a+a*sec(d*x+c))^(1/2)/sec(d*x+c)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
2/35/d*(5*cos(d*x+c)^3+6*cos(d*x+c)^2+8*cos(d*x+c)+16)*(a*(1+sec(d*x+c)))^
(1/2)/(cos(d*x+c)+1)/sec(d*x+c)^(3/2)*tan(d*x+c)
```

3.224.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.58

$$\int \frac{\sqrt{a+a\sec(c+dx)}}{\sec^{\frac{7}{2}}(c+dx)}\,dx = \frac{2\left(5\cos(dx+c)^4+6\cos(dx+c)^3+8\cos(dx+c)^2+16\cos(dx+c)\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{35\left(d\cos(dx+c)+d\right)\sqrt{\cos(dx+c)}}$$

input
```
integrate((a+a*sec(d*x+c))^(1/2)/sec(d*x+c)^(7/2),x, algorithm="fricas")
```

output
```
2/35*(5*cos(d*x + c)^4 + 6*cos(d*x + c)^3 + 8*cos(d*x + c)^2 + 16*cos(d*x
+ c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/((d*cos(d*x + c
) + d)*sqrt(cos(d*x + c)))
```

3.224.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+a\sec(c+dx)}}{\sec^{\frac{7}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sec(d*x+c))**(1/2)/sec(d*x+c)**(7/2),x)
```

output
```
Timed out
```

3.224.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 293 vs. $2(129) = 258$.

Time = 0.38 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.92

$$\int \frac{\sqrt{a+a\sec(c+dx)}}{\sec^{\frac{7}{2}}(c+dx)}\,dx$$
$$= \frac{\sqrt{2}\left(105\cos\left(\frac{6}{7}\arctan\left(\sin\left(\frac{7}{2}dx+\frac{7}{2}c\right),\cos\left(\frac{7}{2}dx+\frac{7}{2}c\right)\right)\right)\sin\left(\frac{7}{2}dx+\frac{7}{2}c\right)+35\cos\left(\frac{4}{7}\arctan\left(\sin\left(\frac{7}{2}dx\right.\right.\right.\right.}{}$$

input
```
integrate((a+a*sec(d*x+c))^(1/2)/sec(d*x+c)^(7/2),x, algorithm="maxima")
```

output
```
1/280*sqrt(2)*(105*cos(6/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2
*c)))*sin(7/2*d*x + 7/2*c) + 35*cos(4/7*arctan2(sin(7/2*d*x + 7/2*c), cos(
7/2*d*x + 7/2*c)))*sin(7/2*d*x + 7/2*c) + 7*cos(2/7*arctan2(sin(7/2*d*x +
7/2*c), cos(7/2*d*x + 7/2*c)))*sin(7/2*d*x + 7/2*c) - 105*cos(7/2*d*x + 7/
2*c)*sin(6/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) - 35*cos
(7/2*d*x + 7/2*c)*sin(4/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*
c))) - 7*cos(7/2*d*x + 7/2*c)*sin(2/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/
2*d*x + 7/2*c))) + 10*sin(7/2*d*x + 7/2*c) + 7*sin(5/7*arctan2(sin(7/2*d*x
 + 7/2*c), cos(7/2*d*x + 7/2*c))) + 35*sin(3/7*arctan2(sin(7/2*d*x + 7/2*c
), cos(7/2*d*x + 7/2*c))) + 105*sin(1/7*arctan2(sin(7/2*d*x + 7/2*c), cos(
7/2*d*x + 7/2*c))))*sqrt(a)/d
```

3.224.8 Giac [F]

$$\int \frac{\sqrt{a+a\sec(c+dx)}}{\sec^{\frac{7}{2}}(c+dx)}\,dx=\int \frac{\sqrt{a\sec{(dx+c)}+a}}{\sec{(dx+c)}^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))^(1/2)/sec(d*x+c)^(7/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.224.9 Mupad [B] (verification not implemented)

Time = 14.86 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.61

$$\int \frac{\sqrt{a+a\sec(c+dx)}}{\sec^{\frac{7}{2}}(c+dx)}\,dx$$
$$=\frac{\cos{(c+d\,x)}\,\sqrt{\frac{1}{\cos(c+d\,x)}}\,\sqrt{\frac{a\,(\cos(c+d\,x)+1)}{\cos(c+d\,x)}}\,(140\,\sin{(c+d\,x)}+42\,\sin{(2\,c+2\,d\,x)}+12\,\sin{(3\,c+3\,d\,x)}+5\,s}{140\,d\,(\cos{(c+d\,x)}+1)}$$

input
```
int((a + a/cos(c + d*x))^(1/2)/(1/cos(c + d*x))^(7/2),x)
```

output
```
(cos(c + d*x)*(1/cos(c + d*x))^(1/2)*((a*(cos(c + d*x) + 1))/cos(c + d*x))
^(1/2)*(140*sin(c + d*x) + 42*sin(2*c + 2*d*x) + 12*sin(3*c + 3*d*x) + 5*s
in(4*c + 4*d*x)))/(140*d*(cos(c + d*x) + 1))
```

3.225 $\int \sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\,dx$

3.225.1 Optimal result . . . 1819
3.225.2 Mathematica [A] (verified) . . . 1819
3.225.3 Rubi [A] (verified) . . . 1820
3.225.4 Maple [A] (verified) . . . 1823
3.225.5 Fricas [A] (verification not implemented) . . . 1823
3.225.6 Sympy [F(-1)] . . . 1824
3.225.7 Maxima [B] (verification not implemented) . . . 1824
3.225.8 Giac [F] . . . 1825
3.225.9 Mupad [F(-1)] . . . 1826

3.225.1 Optimal result

Integrand size = 25, antiderivative size = 160

$$\int \sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \frac{11a^{3/2}\operatorname{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{8d} + \frac{11a^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{8d\sqrt{a+a\sec(c+dx)}} + \frac{11a^2\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{12d\sqrt{a+a\sec(c+dx)}} + \frac{a^2\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{3d\sqrt{a+a\sec(c+dx)}}$$

output
```
11/8*a^(3/2)*arcsinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/d+11/8*a^2
*sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+11/12*a^2*sec(d*x+c)
^(5/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+1/3*a^2*sec(d*x+c)^(7/2)*sin(d*
x+c)/d/(a+a*sec(d*x+c))^(1/2)
```

3.225.2 Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.85

$$\int \sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \frac{a^2\left(33\arcsin\left(\sqrt{1-\sec(c+dx)}\right)+22\sqrt{1-\sec(c+dx)}\sec^{\frac{3}{2}}(c+dx)+8\sqrt{1-\sec(c+dx)}\,\mathrm{s}\right.}{24d\sqrt{1-\sec(c+dx)}\sqrt{a(1+\sec(}}$$

input
```
Integrate[Sec[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^(3/2),x]
```

output
```
(a^2*(33*ArcSin[Sqrt[1 - Sec[c + d*x]]] + 22*Sqrt[1 - Sec[c + d*x]]*Sec[c
+ d*x]^(3/2) + 8*Sqrt[1 - Sec[c + d*x]]*Sec[c + d*x]^(5/2) + 33*Sqrt[-((-1
 + Sec[c + d*x])*Sec[c + d*x])])*Tan[c + d*x])/(24*d*Sqrt[1 - Sec[c + d*x]
]*Sqrt[a*(1 + Sec[c + d*x])])
```

3.225.3 Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.01, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 4301, 27, 3042, 4290, 3042, 4290, 3042, 4288, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{\frac{5}{2}}(c+dx)(a\sec(c+dx)+a)^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}dx$$

$$\Big\downarrow \text{4301}$$

$$\frac{1}{3}a\int \frac{11}{2}\sec^{\frac{5}{2}}(c+dx)\sqrt{\sec(c+dx)a+a}dx+\frac{a^2\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{3d\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{27}$$

$$\frac{11}{6}a\int \sec^{\frac{5}{2}}(c+dx)\sqrt{\sec(c+dx)a+a}dx+\frac{a^2\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{3d\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{11}{6}a\int \csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx+\frac{a^2\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{3d\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{4290}$$

$$\frac{11}{6}a\left(\frac{3}{4}\int \sec^{\frac{3}{2}}(c+dx)\sqrt{\sec(c+dx)a+a}dx+\frac{a\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}\right)+\frac{a^2\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{3d\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \; 3042$$

$$\frac{11}{6}a\left(\frac{3}{4}\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx+\frac{a\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}\right)+\frac{a^2\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{3d\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \; 4290$$

$$\frac{11}{6}a\left(\frac{3}{4}\left(\frac{1}{2}\int\sqrt{\sec(c+dx)}\sqrt{\sec(c+dx)a+a}dx+\frac{a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}\right)+\frac{a\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}\right)+\frac{a^2\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{3d\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \; 3042$$

$$\frac{11}{6}a\left(\frac{3}{4}\left(\frac{1}{2}\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx+\frac{a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}\right)+\frac{a\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}\right)+\frac{a^2\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{3d\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \; 4288$$

$$\frac{11}{6}a\left(\frac{3}{4}\left(\frac{a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}-\frac{\int\frac{1}{\sqrt{\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1}}d\left(-\frac{a\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}\right)+\frac{a\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}\right)+\frac{a^2\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{3d\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \; 222$$

$$\frac{a^2\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{3d\sqrt{a\sec(c+dx)+a}}+\frac{11}{6}a\left(\frac{3}{4}\left(\frac{\sqrt{a}\text{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}\right)+\frac{a\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}\right)$$

input
```
Int[Sec[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^(3/2),x]
```

3.225. $\int \sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\,dx$

output
```
(a^2*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(3*d*Sqrt[a + a*Sec[c + d*x]]) + (11
*a*((a*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(2*d*Sqrt[a + a*Sec[c + d*x]]) + (
3*((Sqrt[a]*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d +
(a*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]])))/4))/6
```

3.225.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4288
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[-2*(a/(b*f))*Sqrt[a*(d/b)]   Subst[Int[1/Sqrt[1
+ x^2/a], x], x, b*(Cot[e + f*x]/Sqrt[a + b*Csc[e + f*x]])], x] /; FreeQ[{a
, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[a*(d/b), 0]
```

rule 4290
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[-2*b*d*Cot[e + f*x]*((d*Csc[e + f*x])^(n - 1)/(
f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]])), x] + Simp[2*a*d*((n - 1)/(b*(2*n -
1)))   Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e + f*x])^(n - 1), x], x] /; Fre
eQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

rule 4301
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
2)*((d*Csc[e + f*x])^n/(f*(m + n - 1))), x] + Simp[b/(m + n - 1)   Int[(a +
 b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*(b*(m + 2*n - 1) + a*(3*m + 2*n
 - 4)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^
2, 0] && GtQ[m, 1] && NeQ[m + n - 1, 0] && IntegerQ[2*m]
```

3.225.4 Maple [A] (verified)

Time = 1.80 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.48

method	result
default	$\frac{a\left(66\sqrt{-\frac{1}{\cos(dx+c)+1}}\,\sin(dx+c)\cos(dx+c)^2-33\cos(dx+c)^3\arctan\left(\frac{-\cos(dx+c)+\sin(dx+c)-1}{2(\cos(dx+c)+1)\sqrt{-\frac{1}{\cos(dx+c)+1}}}\right)-33\cos(dx+c)^3\arctan\left(\frac{}{2(\mathrm{co}}\right.\right.}{48d(\cos(dx+c)-}$

input
```
int(sec(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/48/d*a*(66*(-1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*cos(d*x+c)^2-33*cos(d*x+
c)^3*arctan(1/2*(-cos(d*x+c)+sin(d*x+c)-1)/(cos(d*x+c)+1)/(-1/(cos(d*x+c)+
1))^(1/2))-33*cos(d*x+c)^3*arctan(1/2*(cos(d*x+c)+sin(d*x+c)+1)/(cos(d*x+c
)+1)/(-1/(cos(d*x+c)+1))^(1/2))+44*(-1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*co
s(d*x+c)+16*sin(d*x+c)*(-1/(cos(d*x+c)+1))^(1/2))*(a*(1+sec(d*x+c)))^(1/2)
*sec(d*x+c)^(5/2)/(cos(d*x+c)+1)/(-1/(cos(d*x+c)+1))^(1/2)
```

3.225.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 400, normalized size of antiderivative = 2.50

$$\int \sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \left[\frac{33\left(a\cos\left(dx+c\right)^3+a\cos\left(dx+c\right)^2\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7\,a\cos(dx+c)^2-\frac{4\left(\cos(dx+c)^2-2\,\cos(dx+c)\right)\sqrt{a}\sqrt{}}{\sqrt{\cos(dx+}}}{\cos(dx+c)^3+\cos(dx+c)^2}\right.}{96\left(d\cos\left(dx+c\right)^3+d\cos\right.}\right.$$

input
```
integrate(sec(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
[1/96*(33*(a*cos(d*x + c)^3 + a*cos(d*x + c)^2)*sqrt(a)*log((a*cos(d*x + c
)^3 - 7*a*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 - 2*cos(d*x + c))*sqrt(a)*sqr
t((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)) + 8*a
)/(cos(d*x + c)^3 + cos(d*x + c)^2)) + 4*(33*a*cos(d*x + c)^2 + 22*a*cos(d
*x + c) + 8*a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(c
os(d*x + c)))/(d*cos(d*x + c)^3 + d*cos(d*x + c)^2), 1/48*(33*(a*cos(d*x +
 c)^3 + a*cos(d*x + c)^2)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c)
+ a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*c
os(d*x + c) - 2*a)) + 2*(33*a*cos(d*x + c)^2 + 22*a*cos(d*x + c) + 8*a)*sq
rt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*
cos(d*x + c)^3 + d*cos(d*x + c)^2)]
```

3.225.6 Sympy [F(-1)]

Timed out.

$$\int \sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**(5/2)*(a+a*sec(d*x+c))**(3/2),x)
```

output
```
Timed out
```

3.225.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2361 vs. $2(134) = 268$.

Time = 0.49 (sec) , antiderivative size = 2361, normalized size of antiderivative = 14.76

$$\int \sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \text{Too large to display}$$

input
```
integrate(sec(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
-1/96*(132*(sqrt(2)*a*sin(6*d*x + 6*c) + 3*sqrt(2)*a*sin(4*d*x + 4*c) + 3*
sqrt(2)*a*sin(2*d*x + 2*c))*cos(11/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c))) + 44*(sqrt(2)*a*sin(6*d*x + 6*c) + 3*sqrt(2)*a*sin(4*d*x + 4*c) +
3*sqrt(2)*a*sin(2*d*x + 2*c))*cos(9/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c))) + 216*(sqrt(2)*a*sin(6*d*x + 6*c) + 3*sqrt(2)*a*sin(4*d*x + 4*c)
+ 3*sqrt(2)*a*sin(2*d*x + 2*c))*cos(7/4*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c))) - 216*(sqrt(2)*a*sin(6*d*x + 6*c) + 3*sqrt(2)*a*sin(4*d*x + 4*c
) + 3*sqrt(2)*a*sin(2*d*x + 2*c))*cos(5/4*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c))) - 44*(sqrt(2)*a*sin(6*d*x + 6*c) + 3*sqrt(2)*a*sin(4*d*x + 4*
c) + 3*sqrt(2)*a*sin(2*d*x + 2*c))*cos(3/4*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c))) - 132*(sqrt(2)*a*sin(6*d*x + 6*c) + 3*sqrt(2)*a*sin(4*d*x +
4*c) + 3*sqrt(2)*a*sin(2*d*x + 2*c))*cos(1/4*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c))) - 33*(a*cos(6*d*x + 6*c)^2 + 9*a*cos(4*d*x + 4*c)^2 + 9*a*
cos(2*d*x + 2*c)^2 + a*sin(6*d*x + 6*c)^2 + 9*a*sin(4*d*x + 4*c)^2 + 18*a*
sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*a*sin(2*d*x + 2*c)^2 + 2*(3*a*cos(4*
d*x + 4*c) + 3*a*cos(2*d*x + 2*c) + a)*cos(6*d*x + 6*c) + 6*(3*a*cos(2*d*x
 + 2*c) + a)*cos(4*d*x + 4*c) + 6*a*cos(2*d*x + 2*c) + 6*(a*sin(4*d*x + 4*
c) + a*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + a)*log(2*cos(1/4*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c)))^2 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*...
```

3.225.8 Giac [F]

$$\int \sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \int (a\sec(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{5}{2}}\,dx$$

input
```
integrate(sec(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.225.9 Mupad [F(-1)]

Timed out.

$$\int \sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\,dx=\int \left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)^{3/2}\left(\frac{1}{\cos\left(c+d\,x\right)}\right)^{5/2}dx$$

input
```
int((a + a/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(5/2),x)
```

output
```
int((a + a/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(5/2), x)
```

3.226 $\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\,dx$

3.226.1 Optimal result 1827
3.226.2 Mathematica [A] (verified) 1827
3.226.3 Rubi [A] (verified) 1828
3.226.4 Maple [B] (verified) 1830
3.226.5 Fricas [A] (verification not implemented) 1831
3.226.6 Sympy [F(-1)] 1831
3.226.7 Maxima [B] (verification not implemented) 1832
3.226.8 Giac [F] 1832
3.226.9 Mupad [F(-1)] 1833

3.226.1 Optimal result

Integrand size = 25, antiderivative size = 120

$$\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \frac{7a^{3/2}\text{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{4d} + \frac{7a^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4d\sqrt{a+a\sec(c+dx)}} + \frac{a^2\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2d\sqrt{a+a\sec(c+dx)}}$$

output
```
7/4*a^(3/2)*arcsinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/d+7/4*a^2*s
ec(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+1/2*a^2*sec(d*x+c)^(5/
2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)
```

3.226.2 Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.92

$$\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \frac{a^2\left(7\arcsin\left(\sqrt{1-\sec(c+dx)}\right)+2\sqrt{1-\sec(c+dx)}\sec^{\frac{3}{2}}(c+dx)+7\sqrt{-((-1+\sec(c+d}\right.}{4d\sqrt{1-\sec(c+dx)}\sqrt{a(1+\sec(c+dx))}}$$

input
```
Integrate[Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(3/2),x]
```

output
```
(a^2*(7*ArcSin[Sqrt[1 - Sec[c + d*x]]] + 2*Sqrt[1 - Sec[c + d*x]]*Sec[c +
d*x]^(3/2) + 7*Sqrt[-((-1 + Sec[c + d*x])*Sec[c + d*x])])*Tan[c + d*x])/(4
*d*Sqrt[1 - Sec[c + d*x]]*Sqrt[a*(1 + Sec[c + d*x])])
```

3.226.3 Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.98, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3042, 4301, 27, 3042, 4290, 3042, 4288, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}dx$$

$$\Big\downarrow \text{4301}$$

$$\frac{1}{2}a\int \frac{7}{2}\sec^{\frac{3}{2}}(c+dx)\sqrt{\sec(c+dx)a+a}dx+\frac{a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{27}$$

$$\frac{7}{4}a\int \sec^{\frac{3}{2}}(c+dx)\sqrt{\sec(c+dx)a+a}dx+\frac{a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7}{4}a\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx+\frac{a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{4290}$$

$$\frac{7}{4}a\left(\frac{1}{2}\int \sqrt{\sec(c+dx)}\sqrt{\sec(c+dx)a+a}dx+\frac{a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}\right)+\frac{a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7}{4}a\left(\frac{1}{2}\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx+\frac{a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}\right)+\frac{a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}$$

$\Big\downarrow$ 4288

$$\frac{7}{4}a\left(\frac{a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}-\frac{\int\frac{1}{\sqrt{\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1}}d\left(-\frac{a\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}\right)+\frac{a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}$$

$\Big\downarrow$ 222

$$\frac{a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}+\frac{7}{4}a\left(\frac{\sqrt{a}\text{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}\right)$$

input
```
Int[Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(3/2),x]
```

output
```
(a^2*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(2*d*Sqrt[a + a*Sec[c + d*x]]) + (7*
a*((Sqrt[a]*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d +
(a*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]])))/4
```

3.226.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4288
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[-2*(a/(b*f))*Sqrt[a*(d/b)]   Subst[Int[1/Sqrt[1
+ x^2/a], x], x, b*(Cot[e + f*x]/Sqrt[a + b*Csc[e + f*x]])], x] /; FreeQ[{a
, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[a*(d/b), 0]
```

rule 4290
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[-2*b*d*Cot[e + f*x]*((d*Csc[e + f*x])^(n - 1)/(
f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]])), x] + Simp[2*a*d*((n - 1)/(b*(2*n -
1)))   Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e + f*x])^(n - 1), x], x] /; Fre
eQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

rule 4301
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
2)*((d*Csc[e + f*x])^n/(f*(m + n - 1))), x] + Simp[b/(m + n - 1)   Int[(a +
 b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*(b*(m + 2*n - 1) + a*(3*m + 2*n
 - 4)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^
2, 0] && GtQ[m, 1] && NeQ[m + n - 1, 0] && IntegerQ[2*m]
```

3.226.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 205 vs. $2(100) = 200$.

Time = 1.57 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.72

method	result
default	$\frac{a\left(14\sqrt{-\frac{1}{\cos(dx+c)+1}}\,\sin(dx+c)\cos(dx+c)-7\cos(dx+c)^2\arctan\left(\frac{-\cos(dx+c)+\sin(dx+c)-1}{2(\cos(dx+c)+1)\sqrt{-\frac{1}{\cos(dx+c)+1}}}\right)-7\arctan\left(\frac{\cos(dx+c)+\sin(d}{2(\cos(dx+c)+1)\sqrt{-\frac{}{c}}}\right.\right.}{8d(\cos(dx+c)+1)\sqrt{-\frac{1}{\cos(dx+c)+1}}}$

input
```
int(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/8/d*a*(14*(-1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*cos(d*x+c)-7*cos(d*x+c)^2
*arctan(1/2*(-cos(d*x+c)+sin(d*x+c)-1)/(cos(d*x+c)+1)/(-1/(cos(d*x+c)+1))^
(1/2))-7*arctan(1/2*(cos(d*x+c)+sin(d*x+c)+1)/(cos(d*x+c)+1)/(-1/(cos(d*x+
c)+1))^(1/2))*cos(d*x+c)^2+4*sin(d*x+c)*(-1/(cos(d*x+c)+1))^(1/2))*(a*(1+s
ec(d*x+c)))^(1/2)*sec(d*x+c)^(3/2)/(cos(d*x+c)+1)/(-1/(cos(d*x+c)+1))^(1/2
)
```

3.226.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 370, normalized size of antiderivative = 3.08

$$\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \left[\frac{7\left(a\cos\left(dx+c\right)^{2}+a\cos\left(dx+c\right)\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^{3}-7\,a\cos(dx+c)^{2}-\frac{4\left(\cos(dx+c)^{2}-2\,\cos(dx+c)\right)\sqrt{a}\sqrt{\frac{a\,c}{\sqrt{\cos(dx+c)}}}}{\cos(dx+c)^{3}+\cos(dx+c)^{2}}}{16\left(d\cos\left(dx+c\right)^{2}+d\cos\left(dx+c\right)\right.}\right.}{}\right.$$

input
```
integrate(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
[1/16*(7*(a*cos(d*x + c)^2 + a*cos(d*x + c))*sqrt(a)*log((a*cos(d*x + c)^3
 - 7*a*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 - 2*cos(d*x + c))*sqrt(a)*sqrt((
a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)) + 8*a)/(
cos(d*x + c)^3 + cos(d*x + c)^2)) + 4*(7*a*cos(d*x + c) + 2*a)*sqrt((a*cos
(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x +
 c)^2 + d*cos(d*x + c)), 1/8*(7*(a*cos(d*x + c)^2 + a*cos(d*x + c))*sqrt(-
a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x
+ c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)) + 2*(7*a*cos
(d*x + c) + 2*a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt
(cos(d*x + c)))/(d*cos(d*x + c)^2 + d*cos(d*x + c))]
```

3.226.6 Sympy [F(-1)]

Timed out.

$$\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**(3/2)*(a+a*sec(d*x+c))**(3/2),x)
```

output
```
Timed out
```

3.226.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2244 vs. $2(100) = 200$.

Time = 0.45 (sec) , antiderivative size = 2244, normalized size of antiderivative = 18.70

$$\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \text{Too large to display}$$

input
```
integrate(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
-1/16*(56*sqrt(2)*a*cos(7/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/
2*c)))*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 24*s
qrt(2)*a*cos(5/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))*sin(
4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 12*sqrt(2)*a*si
n(3/2*d*x + 3/2*c) + 28*sqrt(2)*a*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), co
s(3/2*d*x + 3/2*c))) - 4*(3*sqrt(2)*a*sin(3/2*d*x + 3/2*c) + 7*sqrt(2)*a*s
in(7/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 3*sqrt(2)*a*
sin(5/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 7*sqrt(2)*a
*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*cos(8/3*arc
tan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 8*(3*sqrt(2)*a*sin(3/2
*d*x + 3/2*c) - 7*sqrt(2)*a*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*
d*x + 3/2*c))))*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)
)) - 7*(a*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 +
 4*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + a*si
n(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*a*sin(8/3
*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))*sin(4/3*arctan2(sin(
3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 4*a*sin(4/3*arctan2(sin(3/2*d*x
 + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*(2*a*cos(4/3*arctan2(sin(3/2*d*x +
 3/2*c), cos(3/2*d*x + 3/2*c))) + a)*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c),
 cos(3/2*d*x + 3/2*c))) + 4*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos...
```

3.226.8 Giac [F]

$$\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \int (a\sec(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{3}{2}}\,dx$$

input
```
integrate(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.226.9 Mupad [F(-1)]

Timed out.

$$\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\,dx=\int \left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)^{3/2}\left(\frac{1}{\cos\left(c+d\,x\right)}\right)^{3/2}dx$$

input
```
int((a + a/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(3/2),x)
```

output
```
int((a + a/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(3/2), x)
```

3.227 $\int \sqrt{\sec(c+dx)}(a+a\sec(c+dx))^{3/2}\,dx$

3.227.1 Optimal result . . . 1834
3.227.2 Mathematica [A] (verified) . . . 1834
3.227.3 Rubi [A] (verified) . . . 1835
3.227.4 Maple [B] (verified) . . . 1836
3.227.5 Fricas [B] (verification not implemented) . . . 1837
3.227.6 Sympy [F] . . . 1838
3.227.7 Maxima [B] (verification not implemented) . . . 1838
3.227.8 Giac [F] . . . 1839
3.227.9 Mupad [F(-1)] . . . 1839

3.227.1 Optimal result

Integrand size = 25, antiderivative size = 75

$$\int \sqrt{\sec(c+dx)}(a+a\sec(c+dx))^{3/2}\,dx = \frac{3a^{3/2}\operatorname{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} + \frac{a^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}}$$

output
```
3*a^(3/2)*arcsinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/d+a^2*sec(d*x
+c)^(3/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)
```

3.227.2 Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00

$$\int \sqrt{\sec(c+dx)}(a+a\sec(c+dx))^{3/2}\,dx = \frac{a^2\left(-3\arcsin\left(\sqrt{\sec(c+dx)}\right)+\sqrt{-((-1+\sec(c+dx))\sec(c+dx))}\right)\tan(c+dx)}{d\sqrt{1-\sec(c+dx)}\sqrt{a(1+\sec(c+dx))}}$$

input
```
Integrate[Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^(3/2),x]
```

output
```
(a^2*(-3*ArcSin[Sqrt[Sec[c + d*x]]] + Sqrt[-((-1 + Sec[c + d*x])*Sec[c + d
*x])])*Tan[c + d*x])/(d*Sqrt[1 - Sec[c + d*x]]*Sqrt[a*(1 + Sec[c + d*x])])
```

3.227.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {3042, 4301, 27, 3042, 4288, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}dx$$

$$\Big\downarrow \text{4301}$$

$$a\int \frac{3}{2}\sqrt{\sec(c+dx)}\sqrt{\sec(c+dx)a+a}dx+\frac{a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{27}$$

$$\frac{3}{2}a\int \sqrt{\sec(c+dx)}\sqrt{\sec(c+dx)a+a}dx+\frac{a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3}{2}a\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx+\frac{a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{4288}$$

$$\frac{a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}-\frac{3a\int \frac{1}{\sqrt{\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1}}d\left(-\frac{a\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}$$

$$\Big\downarrow \text{222}$$

$$\frac{3a^{3/2}\text{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}$$

input
```
Int[Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^(3/2),x]
```

output
```
(3*a^(3/2)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d + (
a^2*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]])
```

3.227.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4288
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[-2*(a/(b*f))*Sqrt[a*(d/b)]   Subst[Int[1/Sqrt[1
+ x^2/a], x], x, b*(Cot[e + f*x]/Sqrt[a + b*Csc[e + f*x]])], x] /; FreeQ[{a
, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[a*(d/b), 0]
```

rule 4301
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
2)*((d*Csc[e + f*x])^n/(f*(m + n - 1))), x] + Simp[b/(m + n - 1)   Int[(a +
 b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*(b*(m + 2*n - 1) + a*(3*m + 2*n
 - 4)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^
2, 0] && GtQ[m, 1] && NeQ[m + n - 1, 0] && IntegerQ[2*m]
```

3.227.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 173 vs. $2(65) = 130$.

Time = 1.64 (sec) , antiderivative size = 174, normalized size of antiderivative = 2.32

method	result
default	$\frac{a\sqrt{a(1+\sec(dx+c))}\,\sqrt{\sec(dx+c)}\left(2\sin(dx+c)\sqrt{-\frac{1}{\cos(dx+c)+1}}-3\cos(dx+c)\arctan\left(\frac{-\cos(dx+c)+\sin(dx+c)-1}{2(\cos(dx+c)+1)\sqrt{-\frac{1}{\cos(dx+c)+1}}}\right)-3\cos(dx+c\right.}{2d(\cos(dx+c)+1)\sqrt{-\frac{1}{\cos(dx+c)+1}}}$

input
```
int(sec(d*x+c)^(1/2)*(a+a*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/d*a*(a*(1+sec(d*x+c)))^(1/2)*sec(d*x+c)^(1/2)*(2*sin(d*x+c)*(-1/(cos(d
*x+c)+1))^(1/2)-3*cos(d*x+c)*arctan(1/2*(-cos(d*x+c)+sin(d*x+c)-1)/(cos(d*
x+c)+1)/(-1/(cos(d*x+c)+1))^(1/2))-3*cos(d*x+c)*arctan(1/2*(cos(d*x+c)+sin
(d*x+c)+1)/(cos(d*x+c)+1)/(-1/(cos(d*x+c)+1))^(1/2)))/(cos(d*x+c)+1)/(-1/(
cos(d*x+c)+1))^(1/2)
```

3.227.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 138 vs. $2(65) = 130$.

Time = 0.30 (sec) , antiderivative size = 310, normalized size of antiderivative = 4.13

$$\int \sqrt{\sec(c+dx)}(a+a\sec(c+dx))^{3/2}\,dx = \left[\frac{3\,(a\cos(dx+c)+a)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7\,a\cos(dx+c)^2-\frac{4\left(\cos(dx+c)^2-2\,\cos(dx+c)\right)\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+}{\sqrt{\cos(dx+c)}}}{\cos(dx+c)^3+\cos(dx+c)^2}\right.}{4\,(d\cos(dx+c)+d)}\right.$$

input
```
integrate(sec(d*x+c)^(1/2)*(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
[1/4*(3*(a*cos(d*x + c) + a)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x +
 c)^2 - 4*(cos(d*x + c)^2 - 2*cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) +
 a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)) + 8*a)/(cos(d*x + c)^3 +
 cos(d*x + c)^2)) + 4*a*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x +
c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c) + d), 1/2*(3*(a*cos(d*x + c) + a)*s
qrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos
(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)) + 2*a*s
qrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(d
*cos(d*x + c) + d)]
```

3.227.6 Sympy [F]

$$\int \sqrt{\sec(c+dx)}(a+a\sec(c+dx))^{3/2}\,dx = \int (a(\sec(c+dx)+1))^{\frac{3}{2}}\sqrt{\sec(c+dx)}\,dx$$

input
```
integrate(sec(d*x+c)**(1/2)*(a+a*sec(d*x+c))**(3/2),x)
```

output
```
Integral((a*(sec(c + d*x) + 1))**(3/2)*sqrt(sec(c + d*x)), x)
```

3.227.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1143 vs. $2(65) = 130$.

Time = 0.39 (sec) , antiderivative size = 1143, normalized size of antiderivative = 15.24

$$\int \sqrt{\sec(c+dx)}(a+a\sec(c+dx))^{3/2}\,dx = \text{Too large to display}$$

input
```
integrate(sec(d*x+c)^(1/2)*(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
1/4*(3*(a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt
(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - a*log(2*c
os(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x +
 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + a*log(2*cos(1/2*d*x + 1/2*
c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(
2)*sin(1/2*d*x + 1/2*c) + 2) - a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*
d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x +
1/2*c) + 2))*cos(2*d*x + 2*c)^2 + 3*(a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*si
n(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*
d*x + 1/2*c) + 2) - a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c
)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2)
 + a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*c
os(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - a*log(2*cos(1/
2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*
c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*sin(2*d*x + 2*c)^2 + 4*sqrt(2)*a
*sin(3/2*d*x + 3/2*c) - 4*sqrt(2)*a*sin(1/2*d*x + 1/2*c) + 2*(2*sqrt(2)*a*
sin(3/2*d*x + 3/2*c) - 2*sqrt(2)*a*sin(1/2*d*x + 1/2*c) + 3*a*log(2*cos(1/
2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*
c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 3*a*log(2*cos(1/2*d*x + 1/2*c)^
2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(...
```

3.227.8 Giac [F]

$$\int \sqrt{\sec(c+dx)}(a+a\sec(c+dx))^{3/2}\,dx = \int (a\sec(dx+c)+a)^{\frac{3}{2}}\sqrt{\sec(dx+c)}\,dx$$

input
```
integrate(sec(d*x+c)^(1/2)*(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.227.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{\sec(c+dx)}(a+a\sec(c+dx))^{3/2}\,dx = \int \left(a+\frac{a}{\cos(c+d\,x)}\right)^{3/2}\sqrt{\frac{1}{\cos(c+d\,x)}}\,dx$$

input
```
int((a + a/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(1/2),x)
```

output
```
int((a + a/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(1/2), x)
```

3.228 $\int \frac{(a+a\sec(c+dx))^{3/2}}{\sqrt{\sec(c+dx)}}\,dx$

3.228.1 Optimal result 1840
3.228.2 Mathematica [A] (verified) 1840
3.228.3 Rubi [A] (verified) 1841
3.228.4 Maple [B] (verified) 1842
3.228.5 Fricas [B] (verification not implemented) 1843
3.228.6 Sympy [F] 1844
3.228.7 Maxima [B] (verification not implemented) 1844
3.228.8 Giac [F] 1845
3.228.9 Mupad [F(-1)] 1845

3.228.1 Optimal result

Integrand size = 25, antiderivative size = 76

$$\int \frac{(a+a\sec(c+dx))^{3/2}}{\sqrt{\sec(c+dx)}}\,dx=\frac{2a^{3/2}\operatorname{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}+\frac{2a^2\sqrt{\sec(c+dx)}\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}}$$

output
```
2*a^(3/2)*arcsinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/d+2*a^2*sin(d
*x+c)*sec(d*x+c)^(1/2)/d/(a+a*sec(d*x+c))^(1/2)
```

3.228.2 Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.13

$$\int \frac{(a+a\sec(c+dx))^{3/2}}{\sqrt{\sec(c+dx)}}\,dx=\frac{2a^2\left(\sqrt{-((-1+\sec(c+dx))\sec(c+dx))}\sin(c+dx)+\arcsin\left(\sqrt{1-\sec(c\right.\right.}{d\sqrt{1-\sec(c+dx)}\sqrt{a(1+\sec(c+dx))}}$$

input
```
Integrate[(a + a*Sec[c + d*x])^(3/2)/Sqrt[Sec[c + d*x]],x]
```

output
```
(2*a^2*(Sqrt[-((-1 + Sec[c + d*x])*Sec[c + d*x])]*Sin[c + d*x] + ArcSin[Sq
rt[1 - Sec[c + d*x]]]*Tan[c + d*x]))/(d*Sqrt[1 - Sec[c + d*x]]*Sqrt[a*(1 +
 Sec[c + d*x])])
```

3.228.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {3042, 4300, 27, 3042, 4288, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sec(c+dx)+a)^{3/2}}{\sqrt{\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4300}$$

$$2a\int \frac{1}{2}\sqrt{\sec(c+dx)}\sqrt{\sec(c+dx)a+a}dx+\frac{2a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{27}$$

$$a\int \sqrt{\sec(c+dx)}\sqrt{\sec(c+dx)a+a}dx+\frac{2a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$a\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx+\frac{2a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{4288}$$

$$\frac{2a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\sec(c+dx)+a}}-\frac{2a\int \frac{1}{\sqrt{\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1}}d\left(-\frac{a\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}$$

$$\Big\downarrow \text{222}$$

$$\frac{2a^{3/2}\text{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{2a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\sec(c+dx)+a}}$$

input
```
Int[(a + a*Sec[c + d*x])^(3/2)/Sqrt[Sec[c + d*x]],x]
```

output
```
(2*a^(3/2)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d + (
2*a^2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]])
```

3.228.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4288
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[-2*(a/(b*f))*Sqrt[a*(d/b)]   Subst[Int[1/Sqrt[1
+ x^2/a], x], x, b*(Cot[e + f*x]/Sqrt[a + b*Csc[e + f*x]])], x] /; FreeQ[{a
, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[a*(d/b), 0]
```

rule 4300
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)*
((d*Csc[e + f*x])^n/(f*n)), x] - Simp[a/(d*n)   Int[(a + b*Csc[e + f*x])^(m
 - 2)*(d*Csc[e + f*x])^(n + 1)*(b*(m - 2*n - 2) - a*(m + 2*n - 1)*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1]
 && (LtQ[n, -1] || (EqQ[m, 3/2] && EqQ[n, -2^(-1)])) && IntegerQ[2*m]
```

3.228.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 156 vs. $2(66) = 132$.

Time = 1.57 (sec) , antiderivative size = 157, normalized size of antiderivative = 2.07

method	result
default	$\frac{a\sqrt{a(1+\sec(dx+c))}\left(\arctan\left(\frac{-\cos(dx+c)+\sin(dx+c)-1}{2(\cos(dx+c)+1)\sqrt{-\frac{1}{\cos(dx+c)+1}}}\right)\sqrt{-\frac{1}{\cos(dx+c)+1}}+\sqrt{-\frac{1}{\cos(dx+c)+1}}\arctan\left(\frac{\cos(dx+c)+\sin(dx+c)}{2(\cos(dx+c)+1)\sqrt{-\frac{}{\cos(dx}}}\right.\right.}{d\sqrt{\sec(dx+c)}}$

input
```
int((a+a*sec(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/d*a*(a*(1+sec(d*x+c)))^(1/2)/sec(d*x+c)^(1/2)*(arctan(1/2*(-cos(d*x+c)+s
in(d*x+c)-1)/(cos(d*x+c)+1)/(-1/(cos(d*x+c)+1))^(1/2))*(-1/(cos(d*x+c)+1))
^(1/2)+(-1/(cos(d*x+c)+1))^(1/2)*arctan(1/2*(cos(d*x+c)+sin(d*x+c)+1)/(cos
(d*x+c)+1)/(-1/(cos(d*x+c)+1))^(1/2))-2*cot(d*x+c)+2*csc(d*x+c))
```

3.228.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 136 vs. $2(66) = 132$.

Time = 0.31 (sec) , antiderivative size = 307, normalized size of antiderivative = 4.04

$$\int \frac{(a+a\sec(c+dx))^{3/2}}{\sqrt{\sec(c+dx)}}\,dx = \left[\frac{4\,a\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)+\left(a\cos\left(dx+c\right)+a\right)\sqrt{a}\log\left(\right.}{2\left(d\cos\left(dx+\right.\right.}\right.$$

input
```
integrate((a+a*sec(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
[1/2*(4*a*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d
*x + c) + (a*cos(d*x + c) + a)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x
 + c)^2 - 4*(cos(d*x + c)^2 - 2*cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c)
 + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)) + 8*a)/(cos(d*x + c)^3
 + cos(d*x + c)^2)))/(d*cos(d*x + c) + d), (2*a*sqrt((a*cos(d*x + c) + a)/
cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) + (a*cos(d*x + c) + a)*sqrt(
-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x
 + c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)))/(d*cos(d*x
 + c) + d)]
```

3.228.6 Sympy [F]

$$\int \frac{(a+a\sec(c+dx))^{3/2}}{\sqrt{\sec(c+dx)}}\,dx=\int \frac{(a(\sec(c+dx)+1))^{\frac{3}{2}}}{\sqrt{\sec(c+dx)}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))**(3/2)/sec(d*x+c)**(1/2),x)
```

output
```
Integral((a*(sec(c + d*x) + 1))**(3/2)/sqrt(sec(c + d*x)), x)
```

3.228.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 274 vs. $2(66) = 132$.

Time = 0.38 (sec) , antiderivative size = 274, normalized size of antiderivative = 3.61

$$\int \frac{(a+a\sec(c+dx))^{3/2}}{\sqrt{\sec(c+dx)}}\,dx=\frac{\sqrt{2}\left(\sqrt{2}a\log\left(2\,\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+2\,\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+2\,\sqrt{2}\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right.\right.\right.}{}$$

input
```
integrate((a+a*sec(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
1/4*sqrt(2)*(sqrt(2)*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*
c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2
) - sqrt(2)*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*
sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + sqrt(
2)*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*c
os(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - sqrt(2)*a*log(
2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*
x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 8*a*sin(1/2*d*x + 1/2*c
))*sqrt(a)/d
```

3.228.8 Giac [F]

$$\int \frac{(a+a\sec(c+dx))^{3/2}}{\sqrt{\sec(c+dx)}}\,dx=\int \frac{(a\sec(dx+c)+a)^{\frac{3}{2}}}{\sqrt{\sec(dx+c)}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.228.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(c+dx))^{3/2}}{\sqrt{\sec(c+dx)}}\,dx=\int \frac{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{3/2}}{\sqrt{\frac{1}{\cos(c+d\,x)}}}\,dx$$

input
```
int((a + a/cos(c + d*x))^(3/2)/(1/cos(c + d*x))^(1/2),x)
```

output
```
int((a + a/cos(c + d*x))^(3/2)/(1/cos(c + d*x))^(1/2), x)
```

3.229 $\int \frac{(a+a\sec(c+dx))^{3/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

3.229.1 Optimal result 1846
3.229.2 Mathematica [A] (verified) 1846
3.229.3 Rubi [A] (verified) 1847
3.229.4 Maple [A] (verified) 1848
3.229.5 Fricas [A] (verification not implemented) 1849
3.229.6 Sympy [F] 1849
3.229.7 Maxima [A] (verification not implemented) 1849
3.229.8 Giac [F] 1850
3.229.9 Mupad [B] (verification not implemented) 1850

3.229.1 Optimal result

Integrand size = 25, antiderivative size = 79

$$\int \frac{(a+a\sec(c+dx))^{3/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx=\frac{8a^2\sqrt{\sec(c+dx)}\sin(c+dx)}{3d\sqrt{a+a\sec(c+dx)}}+\frac{2a\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}$$

output
```
8/3*a^2*sin(d*x+c)*sec(d*x+c)^(1/2)/d/(a+a*sec(d*x+c))^(1/2)+2/3*a*sin(d*x
+c)*(a+a*sec(d*x+c))^(1/2)/d/sec(d*x+c)^(1/2)
```

3.229.2 Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.63

$$\int \frac{(a+a\sec(c+dx))^{3/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx=\frac{2a(5+\cos(c+dx))\sqrt{a(1+\sec(c+dx))}\tan\left(\frac{1}{2}(c+dx)\right)}{3d\sqrt{\sec(c+dx)}}$$

input
```
Integrate[(a + a*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(3/2),x]
```

output
```
(2*a*(5 + Cos[c + d*x])*Sqrt[a*(1 + Sec[c + d*x])]*Tan[(c + d*x)/2])/(3*d*
Sqrt[Sec[c + d*x]])
```

3.229.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {3042, 4296, 3042, 4291}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sec(c+dx)+a)^{3/2}}{\sec^{\frac{3}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 4296}$$

$$\frac{4}{3}a\int \frac{\sqrt{\sec(c+dx)a+a}}{\sqrt{\sec(c+dx)}}dx+\frac{2a\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{3d\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{4}{3}a\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{3d\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{ 4291}$$

$$\frac{8a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\sqrt{a\sec(c+dx)+a}}+\frac{2a\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{3d\sqrt{\sec(c+dx)}}$$

input
```
Int[(a + a*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(3/2),x]
```

output
```
(8*a^2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*d*Sqrt[a + a*Sec[c + d*x]]) + (
2*a*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])
```

3.229.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4291
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[-2*a*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*S
qrt[d*Csc[e + f*x]])), x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4296
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-a)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)
*((d*Csc[e + f*x])^n/(f*m)), x] + Simp[b*((2*m - 1)/(d*m))   Int[(a + b*Csc
[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f
, m, n}, x] && EqQ[a^2 - b^2, 0] && EqQ[m + n, 0] && GtQ[m, 1/2] && Integer
Q[2*m]
```

3.229.4 Maple [A] (verified)

Time = 1.20 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.66

method	result	size
default	$\frac{2a\sqrt{a(1+\sec(dx+c))}\,(\sin(dx+c)+5\tan(dx+c))}{3d(\cos(dx+c)+1)\sec(dx+c)^{\frac{3}{2}}}$	52

input
```
int((a+a*sec(d*x+c))^(3/2)/sec(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/3/d*a*(a*(1+sec(d*x+c)))^(1/2)/(cos(d*x+c)+1)/sec(d*x+c)^(3/2)*(sin(d*x+
c)+5*tan(d*x+c))
```

3.229.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.87

$$\int \frac{(a+a\sec(c+dx))^{3/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx=\frac{2\left(a\cos\left(dx+c\right)^{2}+5\,a\cos\left(dx+c\right)\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin\left(dx+c\right)}{3\left(d\cos\left(dx+c\right)+d\right)\sqrt{\cos\left(dx+c\right)}}$$

input
```
integrate((a+a*sec(d*x+c))^(3/2)/sec(d*x+c)^(3/2),x, algorithm="fricas")
```

output
```
2/3*(a*cos(d*x + c)^2 + 5*a*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*
x + c))*sin(d*x + c)/((d*cos(d*x + c) + d)*sqrt(cos(d*x + c)))
```

3.229.6 Sympy [F]

$$\int \frac{(a+a\sec(c+dx))^{3/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx=\int \frac{\left(a(\sec\left(c+dx\right)+1)\right)^{\frac{3}{2}}}{\sec^{\frac{3}{2}}\left(c+dx\right)}\,dx$$

input
```
integrate((a+a*sec(d*x+c))**(3/2)/sec(d*x+c)**(3/2),x)
```

output
```
Integral((a*(sec(c + d*x) + 1))**(3/2)/sec(c + d*x)**(3/2), x)
```

3.229.7 Maxima [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.48

$$\int \frac{(a+a\sec(c+dx))^{3/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx=\frac{\left(\sqrt{2}a\sin\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)+9\,\sqrt{2}a\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sqrt{a}}{3\,d}$$

input
```
integrate((a+a*sec(d*x+c))^(3/2)/sec(d*x+c)^(3/2),x, algorithm="maxima")
```

output
```
1/3*(sqrt(2)*a*sin(3/2*d*x + 3/2*c) + 9*sqrt(2)*a*sin(1/2*d*x + 1/2*c))*sq
rt(a)/d
```

3.229.8 Giac [F]

$$\int \frac{(a+a\sec(c+dx))^{3/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx=\int \frac{(a\sec(dx+c)+a)^{\frac{3}{2}}}{\sec(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))^(3/2)/sec(d*x+c)^(3/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.229.9 Mupad [B] (verification not implemented)

Time = 14.37 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.89

$$\int \frac{(a+a\sec(c+dx))^{3/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx=\frac{a\,\cos(c+d\,x)\,\left(10\,\sin(c+d\,x)+\sin(2\,c+2\,d\,x)\right)\,\sqrt{\frac{1}{\cos(c+d\,x)}}\,\sqrt{\frac{a\,(\cos(c+d\,x)+1}{\cos(c+d\,x)}}}{3\,d\,\left(\cos(c+d\,x)+1\right)}$$

input
```
int((a + a/cos(c + d*x))^(3/2)/(1/cos(c + d*x))^(3/2),x)
```

output
```
(a*cos(c + d*x)*(10*sin(c + d*x) + sin(2*c + 2*d*x))*(1/cos(c + d*x))^(1/2
)*((a*(cos(c + d*x) + 1))/cos(c + d*x))^(1/2))/(3*d*(cos(c + d*x) + 1))
```

3.230 $\int \frac{(a+a\sec(c+dx))^{3/2}}{\sec^{\frac{5}{2}}(c+dx)}\,dx$

3.230.1 Optimal result 1851
3.230.2 Mathematica [A] (verified) 1851
3.230.3 Rubi [A] (verified) 1852
3.230.4 Maple [A] (verified) 1854
3.230.5 Fricas [A] (verification not implemented) 1854
3.230.6 Sympy [F] 1854
3.230.7 Maxima [B] (verification not implemented) 1855
3.230.8 Giac [F] 1855
3.230.9 Mupad [B] (verification not implemented) 1855

3.230.1 Optimal result

Integrand size = 25, antiderivative size = 116

$$\int \frac{(a+a\sec(c+dx))^{3/2}}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \frac{8a^2\sqrt{\sec(c+dx)}\sin(c+dx)}{5d\sqrt{a+a\sec(c+dx)}} + \frac{2a\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{5d\sqrt{\sec(c+dx)}} + \frac{2(a+a\sec(c+dx))^{3/2}\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}$$

output
```
2/5*(a+a*sec(d*x+c))^(3/2)*sin(d*x+c)/d/sec(d*x+c)^(3/2)+8/5*a^2*sin(d*x+c
)*sec(d*x+c)^(1/2)/d/(a+a*sec(d*x+c))^(1/2)+2/5*a*sin(d*x+c)*(a+a*sec(d*x+
c))^(1/2)/d/sec(d*x+c)^(1/2)
```

3.230.2 Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.52

$$\int \frac{(a+a\sec(c+dx))^{3/2}}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \frac{a(13+6\cos(c+dx)+\cos(2(c+dx)))\sqrt{a(1+\sec(c+dx))}\tan\left(\frac{1}{2}(c+dx)\right)}{5d\sqrt{\sec(c+dx)}}$$

input
```
Integrate[(a + a*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(5/2),x]
```

output
```
(a*(13 + 6*Cos[c + d*x] + Cos[2*(c + d*x)])*Sqrt[a*(1 + Sec[c + d*x])]*Tan
[(c + d*x)/2])/(5*d*Sqrt[Sec[c + d*x]])
```

3.230.3 Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3042, 4299, 3042, 4296, 3042, 4291}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sec(c+dx)+a)^{3/2}}{\sec^{\frac{5}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx$$

$$\Big\downarrow \text{4299}$$

$$\frac{3}{5}\int \frac{(\sec(c+dx)a+a)^{3/2}}{\sec^{\frac{3}{2}}(c+dx)}dx+\frac{2\sin(c+dx)(a\sec(c+dx)+a)^{3/2}}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3}{5}\int \frac{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+\frac{2\sin(c+dx)(a\sec(c+dx)+a)^{3/2}}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{4296}$$

$$\frac{3}{5}\left(\frac{4}{3}a\int \frac{\sqrt{\sec(c+dx)a+a}}{\sqrt{\sec(c+dx)}}dx+\frac{2a\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{3d\sqrt{\sec(c+dx)}}\right)+\frac{2\sin(c+dx)(a\sec(c+dx)+a)^{3/2}}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3}{5}\left(\frac{4}{3}a\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{3d\sqrt{\sec(c+dx)}}\right)+\frac{2\sin(c+dx)(a\sec(c+dx)+a)^{3/2}}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{4291}$$

$$\frac{3}{5}\left(\frac{8a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\sqrt{a\sec(c+dx)+a}}+\frac{2a\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{3d\sqrt{\sec(c+dx)}}\right)+\frac{2\sin(c+dx)(a\sec(c+dx)+a)^{3/2}}{5d\sec^{\frac{3}{2}}(c+dx)}$$

input
```
Int[(a + a*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(5/2),x]
```

output
```
(2*(a + a*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)) + (3*
((8*a^2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*d*Sqrt[a + a*Sec[c + d*x]]) +
(2*a*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])))/5
```

3.230.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4291
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[-2*a*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*S
qrt[d*Csc[e + f*x]])), x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4296
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-a)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)
*((d*Csc[e + f*x])^n/(f*m)), x] + Simp[b*((2*m - 1)/(d*m))   Int[(a + b*Csc
[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f
, m, n}, x] && EqQ[a^2 - b^2, 0] && EqQ[m + n, 0] && GtQ[m, 1/2] && Integer
Q[2*m]
```

rule 4299
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-Cot[e + f*x])*(a + b*Csc[e + f*x])^m*((d*Csc
[e + f*x])^n/(f*(m + 1))), x] + Simp[a*(m/(b*d*(m + 1)))   Int[(a + b*Csc[e
 + f*x])^m*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m, n},
 x] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LtQ[m, -2^(-1)]
```

3.230. $\int \frac{(a+a\sec(c+dx))^{3/2}}{\sec^{\frac{5}{2}}(c+dx)}\,dx$

3.230.4 Maple [A] (verified)

Time = 1.28 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.57

method	result	size
default	$\frac{2a\sqrt{a(1+\sec(dx+c))}\,(\sin(dx+c)+3\tan(dx+c)+6\sec(dx+c)\tan(dx+c))}{5d(\cos(dx+c)+1)\sec(dx+c)^{\frac{5}{2}}}$	66

input
```
int((a+a*sec(d*x+c))^(3/2)/sec(d*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/5/d*a*(a*(1+sec(d*x+c)))^(1/2)/(cos(d*x+c)+1)/sec(d*x+c)^(5/2)*(sin(d*x+
c)+3*tan(d*x+c)+6*sec(d*x+c)*tan(d*x+c))
```

3.230.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.69

$$\int \frac{(a+a\sec(c+dx))^{3/2}}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \frac{2\left(a\cos\left(dx+c\right)^{3}+3\,a\cos\left(dx+c\right)^{2}+6\,a\cos\left(dx+c\right)\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin\left(dx\right.}{5\left(d\cos\left(dx+c\right)+d\right)\sqrt{\cos\left(dx+c\right)}}$$

input
```
integrate((a+a*sec(d*x+c))^(3/2)/sec(d*x+c)^(5/2),x, algorithm="fricas")
```

output
```
2/5*(a*cos(d*x + c)^3 + 3*a*cos(d*x + c)^2 + 6*a*cos(d*x + c))*sqrt((a*cos
(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/((d*cos(d*x + c) + d)*sqrt(cos(d
*x + c)))
```

3.230.6 Sympy [F]

$$\int \frac{(a+a\sec(c+dx))^{3/2}}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{(a(\sec(c+dx)+1))^{\frac{3}{2}}}{\sec^{\frac{5}{2}}(c+dx)}\,dx$$

input
```
integrate((a+a*sec(d*x+c))**(3/2)/sec(d*x+c)**(5/2),x)
```

output
```
Integral((a*(sec(c + d*x) + 1))**(3/2)/sec(c + d*x)**(5/2), x)
```

3.230.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 210 vs. $2(98) = 196$.

Time = 0.37 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.81

$$\int \frac{(a+a\sec(c+dx))^{3/2}}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \frac{\sqrt{2}\left(20\,a\cos\left(\frac{4}{5}\arctan\left(\sin\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right),\cos\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right)\right)\right)\sin\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right)+\right.}{}$$

input
```
integrate((a+a*sec(d*x+c))^(3/2)/sec(d*x+c)^(5/2),x, algorithm="maxima")
```

output
```
1/20*sqrt(2)*(20*a*cos(4/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2
*c)))*sin(5/2*d*x + 5/2*c) + 5*a*cos(2/5*arctan2(sin(5/2*d*x + 5/2*c), cos
(5/2*d*x + 5/2*c)))*sin(5/2*d*x + 5/2*c) - 20*a*cos(5/2*d*x + 5/2*c)*sin(4
/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) - 5*a*cos(5/2*d*x
+ 5/2*c)*sin(2/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 2*
a*sin(5/2*d*x + 5/2*c) + 5*a*sin(3/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2
*d*x + 5/2*c))) + 20*a*sin(1/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x +
 5/2*c))))*sqrt(a)/d
```

3.230.8 Giac [F]

$$\int \frac{(a+a\sec(c+dx))^{3/2}}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{(a\sec(dx+c)+a)^{\frac{3}{2}}}{\sec(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))^(3/2)/sec(d*x+c)^(5/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.230.9 Mupad [B] (verification not implemented)

Time = 14.55 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.70

$$\int \frac{(a+a\sec(c+dx))^{3/2}}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \frac{a\,\cos(c+d\,x)\,\sqrt{\frac{1}{\cos(c+d\,x)}}\,(25\,\sin(c+d\,x)+6\,\sin(2\,c+2\,d\,x)+\sin(3\,c+3}{10\,d\,(\cos(c+d\,x)+1)}$$

input
```
int((a + a/cos(c + d*x))^(3/2)/(1/cos(c + d*x))^(5/2),x)
```

output
```
(a*cos(c + d*x)*(1/cos(c + d*x))^(1/2)*(25*sin(c + d*x) + 6*sin(2*c + 2*d*
x) + sin(3*c + 3*d*x))*((a*(cos(c + d*x) + 1))/cos(c + d*x))^(1/2))/(10*d*
(cos(c + d*x) + 1))
```

3.230. $\int \frac{(a+a\sec(c+dx))^{3/2}}{\sec^{\frac{5}{2}}(c+dx)}\,dx$

3.231 $\int \frac{(a+a\sec(c+dx))^{3/2}}{\sec^{\frac{7}{2}}(c+dx)}\,dx$

3.231.1 Optimal result 1857
3.231.2 Mathematica [A] (verified) 1857
3.231.3 Rubi [A] (verified) 1858
3.231.4 Maple [A] (verified) 1860
3.231.5 Fricas [A] (verification not implemented) 1861
3.231.6 Sympy [F(-1)] 1861
3.231.7 Maxima [B] (verification not implemented) 1861
3.231.8 Giac [F] 1862
3.231.9 Mupad [B] (verification not implemented) 1862

3.231.1 Optimal result

Integrand size = 25, antiderivative size = 161

$$\int \frac{(a+a\sec(c+dx))^{3/2}}{\sec^{\frac{7}{2}}(c+dx)}\,dx = \frac{2a^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{26a^2\sin(c+dx)}{35d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{104a^2\sin(c+dx)}{105d\sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+dx)}} + \frac{208a^2\sqrt{\sec(c+dx)}\sin(c+dx)}{105d\sqrt{a+a\sec(c+dx)}}$$

output
```
2/7*a^2*sin(d*x+c)/d/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(1/2)+26/35*a^2*sin
(d*x+c)/d/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2)+104/105*a^2*sin(d*x+c)/d
/sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2)+208/105*a^2*sin(d*x+c)*sec(d*x+c)
^(1/2)/d/(a+a*sec(d*x+c))^(1/2)
```

3.231.2 Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.45

$$\int \frac{(a+a\sec(c+dx))^{3/2}}{\sec^{\frac{7}{2}}(c+dx)}\,dx = \frac{a(494+253\cos(c+dx)+78\cos(2(c+dx))+15\cos(3(c+dx)))\sqrt{a(1+\sec}}{210d\sqrt{\sec(c+dx)}}$$

input
```
Integrate[(a + a*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(7/2),x]
```

output
```
(a*(494 + 253*Cos[c + d*x] + 78*Cos[2*(c + d*x)] + 15*Cos[3*(c + d*x)])*Sq
rt[a*(1 + Sec[c + d*x])]*Tan[(c + d*x)/2])/(210*d*Sqrt[Sec[c + d*x]])
```

3.231.3 Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.03, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {3042, 4300, 27, 3042, 4292, 3042, 4292, 3042, 4291}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sec(c+dx)+a)^{3/2}}{\sec^{\frac{7}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx$$

$$\Big\downarrow \text{4300}$$

$$\frac{2}{7}a\int \frac{13\sqrt{\sec(c+dx)a+a}}{2\sec^{\frac{5}{2}}(c+dx)}dx+\frac{2a^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{27}$$

$$\frac{13}{7}a\int \frac{\sqrt{\sec(c+dx)a+a}}{\sec^{\frac{5}{2}}(c+dx)}dx+\frac{2a^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{13}{7}a\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+\frac{2a^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{4292}$$

$$\frac{13}{7}a\left(\frac{4}{5}\int \frac{\sqrt{\sec(c+dx)a+a}}{\sec^{\frac{3}{2}}(c+dx)}dx+\frac{2a\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}\right)+\frac{2a^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{13}{7}a\left(\frac{4}{5}\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+\frac{2a\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}\right)+\frac{2a^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \; 4292$$

$$\frac{13}{7}a\left(\frac{4}{5}\left(\frac{2}{3}\int\frac{\sqrt{\sec(c+dx)a+a}}{\sqrt{\sec(c+dx)}}dx+\frac{2a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}\right)+\frac{2a\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+}}\right.$$
$$\frac{2a^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \; 3042$$

$$\frac{13}{7}a\left(\frac{4}{5}\left(\frac{2}{3}\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}\right)+\frac{2a\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+}}\right.$$
$$\frac{2a^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \; 4291$$

$$\frac{2a^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+$$
$$\frac{13}{7}a\left(\frac{2a\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{4}{5}\left(\frac{4a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\sqrt{a\sec(c+dx)+a}}+\frac{2a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+}}\right.\right.$$

input
```
Int[(a + a*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(7/2),x]
```

output
```
(2*a^2*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/2)*Sqrt[a + a*Sec[c + d*x]]) + (
13*a*((2*a*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]])
 + (4*((2*a*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]
) + (4*a*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*d*Sqrt[a + a*Sec[c + d*x]])))
/5))/7
```

3.231. $\int \frac{(a+a\sec(c+dx))^{3/2}}{\sec^{\frac{7}{2}}(c+dx)}\,dx$

3.231.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4291
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[-2*a*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*S
qrt[d*Csc[e + f*x]])), x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4292
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[a*Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*n*Sqrt[a
+ b*Csc[e + f*x]])), x] + Simp[a*((2*n + 1)/(2*b*d*n))   Int[Sqrt[a + b*Csc
[e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
 EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

rule 4300
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)*
((d*Csc[e + f*x])^n/(f*n)), x] - Simp[a/(d*n)   Int[(a + b*Csc[e + f*x])^(m
 - 2)*(d*Csc[e + f*x])^(n + 1)*(b*(m - 2*n - 2) - a*(m + 2*n - 1)*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1]
 && (LtQ[n, -1] || (EqQ[m, 3/2] && EqQ[n, -2^(-1)])) && IntegerQ[2*m]
```

3.231.4 Maple [A] (verified)

Time = 1.25 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.45

method	result	size
default	$\frac{2a\left(15\cos(dx+c)^3+39\cos(dx+c)^2+52\cos(dx+c)+104\right)\sqrt{a(1+\sec(dx+c))}\,\tan(dx+c)}{105d(\cos(dx+c)+1)\sec(dx+c)^{\frac{3}{2}}}$	73

input
```
int((a+a*sec(d*x+c))^(3/2)/sec(d*x+c)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
2/105/d*a*(15*cos(d*x+c)^3+39*cos(d*x+c)^2+52*cos(d*x+c)+104)*(a*(1+sec(d*
x+c)))^(1/2)/(cos(d*x+c)+1)/sec(d*x+c)^(3/2)*tan(d*x+c)
```

3.231.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.57

$$\int \frac{(a+a\sec(c+dx))^{3/2}}{\sec^{\frac{7}{2}}(c+dx)}\,dx = \frac{2\left(15\,a\cos\left(dx+c\right)^4+39\,a\cos\left(dx+c\right)^3+52\,a\cos\left(dx+c\right)^2+104\,a\cos\left(dx\right.\right.}{105\left(d\cos\left(dx+c\right)+d\right)\sqrt{\cos\left(dx+c\right)}}$$

input
```
integrate((a+a*sec(d*x+c))^(3/2)/sec(d*x+c)^(7/2),x, algorithm="fricas")
```

output
```
2/105*(15*a*cos(d*x + c)^4 + 39*a*cos(d*x + c)^3 + 52*a*cos(d*x + c)^2 + 1
04*a*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/((
d*cos(d*x + c) + d)*sqrt(cos(d*x + c)))
```

3.231.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(c+dx))^{3/2}}{\sec^{\frac{7}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sec(d*x+c))**(3/2)/sec(d*x+c)**(7/2),x)
```

output
```
Timed out
```

3.231.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 303 vs. $2(137) = 274$.

Time = 0.40 (sec) , antiderivative size = 303, normalized size of antiderivative = 1.88

$$\int \frac{(a+a\sec(c+dx))^{3/2}}{\sec^{\frac{7}{2}}(c+dx)}\,dx = \frac{\sqrt{2}\left(735\,a\cos\left(\frac{6}{7}\arctan\left(\sin\left(\frac{7}{2}\,dx+\frac{7}{2}\,c\right),\cos\left(\frac{7}{2}\,dx+\frac{7}{2}\,c\right)\right)\right)\sin\left(\frac{7}{2}\,dx+\frac{7}{2}\,c\right)\right.}{}$$

input
```
integrate((a+a*sec(d*x+c))^(3/2)/sec(d*x+c)^(7/2),x, algorithm="maxima")
```

output
```
1/840*sqrt(2)*(735*a*cos(6/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7
/2*c)))*sin(7/2*d*x + 7/2*c) + 175*a*cos(4/7*arctan2(sin(7/2*d*x + 7/2*c),
 cos(7/2*d*x + 7/2*c)))*sin(7/2*d*x + 7/2*c) + 63*a*cos(2/7*arctan2(sin(7/
2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c)))*sin(7/2*d*x + 7/2*c) - 735*a*cos(7/
2*d*x + 7/2*c)*sin(6/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))
) - 175*a*cos(7/2*d*x + 7/2*c)*sin(4/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7
/2*d*x + 7/2*c))) - 63*a*cos(7/2*d*x + 7/2*c)*sin(2/7*arctan2(sin(7/2*d*x
+ 7/2*c), cos(7/2*d*x + 7/2*c))) + 30*a*sin(7/2*d*x + 7/2*c) + 63*a*sin(5/
7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) + 175*a*sin(3/7*arc
tan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) + 735*a*sin(1/7*arctan2(
sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))))*sqrt(a)/d
```

3.231.8 Giac [F]

$$\int \frac{(a+a\sec(c+dx))^{3/2}}{\sec^{\frac{7}{2}}(c+dx)}\,dx=\int \frac{(a\sec(dx+c)+a)^{\frac{3}{2}}}{\sec(dx+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))^(3/2)/sec(d*x+c)^(7/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.231.9 Mupad [B] (verification not implemented)

Time = 14.82 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.58

$$\int \frac{(a+a\sec(c+dx))^{3/2}}{\sec^{\frac{7}{2}}(c+dx)}\,dx=\frac{a\,\cos(c+d\,x)\,\sqrt{\frac{1}{\cos(c+d\,x)}}\,\sqrt{\frac{a\,(\cos(c+d\,x)+1)}{\cos(c+d\,x)}}\,(910\,\sin(c+d\,x)+238\,\sin(2\,c+2}{420\,d\,(\cos(c+d\,x)+1)}$$

input
```
int((a + a/cos(c + d*x))^(3/2)/(1/cos(c + d*x))^(7/2),x)
```

output
```
(a*cos(c + d*x)*(1/cos(c + d*x))^(1/2)*((a*(cos(c + d*x) + 1))/cos(c + d*x
))^(1/2)*(910*sin(c + d*x) + 238*sin(2*c + 2*d*x) + 78*sin(3*c + 3*d*x) +
15*sin(4*c + 4*d*x)))/(420*d*(cos(c + d*x) + 1))
```

3.232 $\int \frac{(a+a\sec(c+dx))^{3/2}}{\sec^{\frac{9}{2}}(c+dx)}\,dx$

3.232.1 Optimal result 1863
3.232.2 Mathematica [A] (verified) 1864
3.232.3 Rubi [A] (verified) 1864
3.232.4 Maple [A] (verified) 1867
3.232.5 Fricas [A] (verification not implemented) 1867
3.232.6 Sympy [F(-1)] 1868
3.232.7 Maxima [B] (verification not implemented) 1868
3.232.8 Giac [F] 1869
3.232.9 Mupad [B] (verification not implemented) 1869

3.232.1 Optimal result

Integrand size = 25, antiderivative size = 201

$$\int \frac{(a+a\sec(c+dx))^{3/2}}{\sec^{\frac{9}{2}}(c+dx)}\,dx = \frac{2a^2\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{34a^2\sin(c+dx)}{63d\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{68a^2\sin(c+dx)}{105d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{272a^2\sin(c+dx)}{315d\sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+dx)}} + \frac{544a^2\sqrt{\sec(c+dx)}\sin(c+dx)}{315d\sqrt{a+a\sec(c+dx)}}$$

output
```
2/9*a^2*sin(d*x+c)/d/sec(d*x+c)^(7/2)/(a+a*sec(d*x+c))^(1/2)+34/63*a^2*sin
(d*x+c)/d/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(1/2)+68/105*a^2*sin(d*x+c)/d/
sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2)+272/315*a^2*sin(d*x+c)/d/sec(d*x+c
)^(1/2)/(a+a*sec(d*x+c))^(1/2)+544/315*a^2*sin(d*x+c)*sec(d*x+c)^(1/2)/d/(
a+a*sec(d*x+c))^(1/2)
```

3.232.2 Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.40

$$\int \frac{(a+a\sec(c+dx))^{3/2}}{\sec^{\frac{9}{2}}(c+dx)}\,dx = \frac{2a^2(35+85\sec(c+dx)+102\sec^2(c+dx)+136\sec^3(c+dx)+272\sec^4(c+}{315d\sec^{\frac{7}{2}}(c+dx)\sqrt{a(1+\sec(c+dx))}}$$

input
```
Integrate[(a + a*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(9/2),x]
```

output
```
(2*a^2*(35 + 85*Sec[c + d*x] + 102*Sec[c + d*x]^2 + 136*Sec[c + d*x]^3 + 2
72*Sec[c + d*x]^4)*Sin[c + d*x])/(315*d*Sec[c + d*x]^(7/2)*Sqrt[a*(1 + Sec
[c + d*x])])
```

3.232.3 Rubi [A] (verified)

Time = 0.98 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.04, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {3042, 4300, 27, 3042, 4292, 3042, 4292, 3042, 4292, 3042, 4291}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sec(c+dx)+a)^{3/2}}{\sec^{\frac{9}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}}{\csc\left(c+dx+\frac{\pi}{2}\right)^{9/2}}dx$$

$$\Big\downarrow \text{4300}$$

$$\frac{2}{9}a\int \frac{17\sqrt{\sec(c+dx)a+a}}{2\sec^{\frac{7}{2}}(c+dx)}dx+\frac{2a^2\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{27}$$

$$\frac{17}{9}a\int \frac{\sqrt{\sec(c+dx)a+a}}{\sec^{\frac{7}{2}}(c+dx)}dx+\frac{2a^2\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{17}{9}a\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx+\frac{2a^2\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}$$

$\Big\downarrow$ 4292

$$\frac{17}{9}a\left(\frac{6}{7}\int \frac{\sqrt{\sec(c+dx)a+a}}{\sec^{\frac{5}{2}}(c+dx)}dx+\frac{2a\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}\right)+\frac{2a^2\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}$$

$\Big\downarrow$ 3042

$$\frac{17}{9}a\left(\frac{6}{7}\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+\frac{2a\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}\right)+\frac{2a^2\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}$$

$\Big\downarrow$ 4292

$$\frac{17}{9}a\left(\frac{6}{7}\left(\frac{4}{5}\int \frac{\sqrt{\sec(c+dx)a+a}}{\sec^{\frac{3}{2}}(c+dx)}dx+\frac{2a\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}\right)+\frac{2a\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+}}\right.$$
$$\frac{2a^2\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}$$

$\Big\downarrow$ 3042

$$\frac{17}{9}a\left(\frac{6}{7}\left(\frac{4}{5}\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+\frac{2a\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}\right)+\frac{2a\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+}}\right.$$
$$\frac{2a^2\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}$$

$\Big\downarrow$ 4292

$$\frac{17}{9}a\left(\frac{6}{7}\left(\frac{4}{5}\left(\frac{2}{3}\int \frac{\sqrt{\sec(c+dx)a+a}}{\sqrt{\sec(c+dx)}}dx+\frac{2a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}\right)+\frac{2a\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx}}\right.\right.$$
$$\frac{2a^2\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}$$

$\Big\downarrow$ 3042

3.232. $\int \frac{(a+a\sec(c+dx))^{3/2}}{\sec^{\frac{9}{2}}(c+dx)}\,dx$

$$\frac{17}{9}a\left(\frac{6}{7}\left(\frac{4}{5}\left(\frac{2}{3}\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}\right)+\frac{2a\sin(c+dx}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(}}\right.\right.$$

$$\frac{2a^2\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \; 4291$$

$$\frac{2a^2\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+$$

$$\frac{17}{9}a\left(\frac{2a\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{6}{7}\left(\frac{2a\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{4}{5}\left(\frac{4a\sin(c+dx)\sqrt{\sec(c+}}{3d\sqrt{a\sec(c+dx)+}}\right.\right.\right.$$

input
```
Int[(a + a*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(9/2),x]
```

output
```
(2*a^2*Sin[c + d*x])/(9*d*Sec[c + d*x]^(7/2)*Sqrt[a + a*Sec[c + d*x]]) + (
17*a*((2*a*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/2)*Sqrt[a + a*Sec[c + d*x]])
 + (6*((2*a*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]]
) + (4*((2*a*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]]*Sqrt[a + a*Sec[c + d*x]
]) + (4*a*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*d*Sqrt[a + a*Sec[c + d*x]]))
)/5))/7))/9
```

3.232.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4291
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[-2*a*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*S
qrt[d*Csc[e + f*x]])), x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4292
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[a*Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*n*Sqrt[a
+ b*Csc[e + f*x]])), x] + Simp[a*((2*n + 1)/(2*b*d*n))   Int[Sqrt[a + b*Csc
[e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
 EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

rule 4300
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)*
((d*Csc[e + f*x])^n/(f*n)), x] - Simp[a/(d*n)   Int[(a + b*Csc[e + f*x])^(m
 - 2)*(d*Csc[e + f*x])^(n + 1)*(b*(m - 2*n - 2) - a*(m + 2*n - 1)*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1]
 && (LtQ[n, -1] || (EqQ[m, 3/2] && EqQ[n, -2^(-1)])) && IntegerQ[2*m]
```

3.232.4 Maple [A] (verified)

Time = 1.21 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.41

method	result	size
default	$\frac{2a\left(35\cos(dx+c)^4+85\cos(dx+c)^3+102\cos(dx+c)^2+136\cos(dx+c)+272\right)\sqrt{a(1+\sec(dx+c))}\,\tan(dx+c)}{315d(\cos(dx+c)+1)\sec(dx+c)^{\frac{3}{2}}}$	83

input
```
int((a+a*sec(d*x+c))^(3/2)/sec(d*x+c)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
2/315/d*a*(35*cos(d*x+c)^4+85*cos(d*x+c)^3+102*cos(d*x+c)^2+136*cos(d*x+c)
+272)*(a*(1+sec(d*x+c)))^(1/2)/(cos(d*x+c)+1)/sec(d*x+c)^(3/2)*tan(d*x+c)
```

3.232.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.51

$$\int \frac{(a+a\sec(c+dx))^{3/2}}{\sec^{\frac{9}{2}}(c+dx)}\,dx = \frac{2\left(35\,a\cos\left(dx+c\right)^5+85\,a\cos\left(dx+c\right)^4+102\,a\cos\left(dx+c\right)^3+136\,a\cos\left(dx\right.\right.}{315\left(d\cos\left(dx+c\right)+d\right)\sqrt{\cos\left(\right.}}$$

input
```
integrate((a+a*sec(d*x+c))^(3/2)/sec(d*x+c)^(9/2),x, algorithm="fricas")
```

output
```
2/315*(35*a*cos(d*x + c)^5 + 85*a*cos(d*x + c)^4 + 102*a*cos(d*x + c)^3 +
136*a*cos(d*x + c)^2 + 272*a*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d
*x + c))*sin(d*x + c)/((d*cos(d*x + c) + d)*sqrt(cos(d*x + c)))
```

3.232.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(c+dx))^{3/2}}{\sec^{\frac{9}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sec(d*x+c))**(3/2)/sec(d*x+c)**(9/2),x)
```

output
```
Timed out
```

3.232.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 396 vs. $2(171) = 342$.

Time = 0.38 (sec) , antiderivative size = 396, normalized size of antiderivative = 1.97

$$\int \frac{(a+a\sec(c+dx))^{3/2}}{\sec^{\frac{9}{2}}(c+dx)}\,dx = \frac{\sqrt{2}\left(3780\,a\cos\left(\frac{8}{9}\arctan\left(\sin\left(\frac{9}{2}\,dx+\frac{9}{2}\,c\right),\cos\left(\frac{9}{2}\,dx+\frac{9}{2}\,c\right)\right)\right)\sin\left(\frac{9}{2}\,dx+\frac{9}{2}\,c\right)\right.}{}$$

input
```
integrate((a+a*sec(d*x+c))^(3/2)/sec(d*x+c)^(9/2),x, algorithm="maxima")
```

output
```
1/5040*sqrt(2)*(3780*a*cos(8/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x +
 9/2*c)))*sin(9/2*d*x + 9/2*c) + 1050*a*cos(2/3*arctan2(sin(9/2*d*x + 9/2*
c), cos(9/2*d*x + 9/2*c)))*sin(9/2*d*x + 9/2*c) + 378*a*cos(4/9*arctan2(si
n(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c)))*sin(9/2*d*x + 9/2*c) + 135*a*co
s(2/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c)))*sin(9/2*d*x + 9
/2*c) - 3780*a*cos(9/2*d*x + 9/2*c)*sin(8/9*arctan2(sin(9/2*d*x + 9/2*c),
cos(9/2*d*x + 9/2*c))) - 1050*a*cos(9/2*d*x + 9/2*c)*sin(2/3*arctan2(sin(9
/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c))) - 378*a*cos(9/2*d*x + 9/2*c)*sin(4
/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c))) - 135*a*cos(9/2*d*
x + 9/2*c)*sin(2/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c))) +
70*a*sin(9/2*d*x + 9/2*c) + 135*a*sin(7/9*arctan2(sin(9/2*d*x + 9/2*c), co
s(9/2*d*x + 9/2*c))) + 378*a*sin(5/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2
*d*x + 9/2*c))) + 1050*a*sin(1/3*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x
 + 9/2*c))) + 3780*a*sin(1/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9
/2*c))))*sqrt(a)/d
```

3.232.8 Giac [F]

$$\int \frac{(a+a\sec(c+dx))^{3/2}}{\sec^{\frac{9}{2}}(c+dx)}\,dx=\int \frac{(a\sec(dx+c)+a)^{\frac{3}{2}}}{\sec(dx+c)^{\frac{9}{2}}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))^(3/2)/sec(d*x+c)^(9/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.232.9 Mupad [B] (verification not implemented)

Time = 15.70 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.52

$$\int \frac{(a+a\sec(c+dx))^{3/2}}{\sec^{\frac{9}{2}}(c+dx)}\,dx=\frac{a\,\cos(c+d\,x)\,\sqrt{\frac{1}{\cos(c+d\,x)}}\,\sqrt{\frac{a\,(\cos(c+d\,x)+1)}{\cos(c+d\,x)}}\,(4830\,\sin(c+d\,x)+1428\,\sin(2\,c+}{2520\,d\,(\cos(c}$$

input
```
int((a + a/cos(c + d*x))^(3/2)/(1/cos(c + d*x))^(9/2),x)
```

output
```
(a*cos(c + d*x)*(1/cos(c + d*x))^(1/2)*((a*(cos(c + d*x) + 1))/cos(c + d*x
))^(1/2)*(4830*sin(c + d*x) + 1428*sin(2*c + 2*d*x) + 513*sin(3*c + 3*d*x)
 + 170*sin(4*c + 4*d*x) + 35*sin(5*c + 5*d*x)))/(2520*d*(cos(c + d*x) + 1)
)
```

3.232. $\int \frac{(a+a\sec(c+dx))^{3/2}}{\sec^{\frac{9}{2}}(c+dx)}\,dx$

3.233 $\int \sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\,dx$

3.233.1 Optimal result 1871
3.233.2 Mathematica [A] (verified) 1871
3.233.3 Rubi [A] (verified) 1872
3.233.4 Maple [A] (verified) 1875
3.233.5 Fricas [A] (verification not implemented) 1876
3.233.6 Sympy [F(-1)] 1877
3.233.7 Maxima [B] (verification not implemented) 1877
3.233.8 Giac [F] 1878
3.233.9 Mupad [F(-1)] 1878

3.233.1 Optimal result

Integrand size = 25, antiderivative size = 200

$$\int \sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \frac{163a^{5/2}\operatorname{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{64d} + \frac{163a^3\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{64d\sqrt{a+a\sec(c+dx)}} + \frac{163a^3\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{96d\sqrt{a+a\sec(c+dx)}} + \frac{17a^3\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{24d\sqrt{a+a\sec(c+dx)}} + \frac{a^2\sec^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{4d}$$

output
```
163/64*a^(5/2)*arcsinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/d+163/64
*a^3*sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+163/96*a^3*sec(d
*x+c)^(5/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+17/24*a^3*sec(d*x+c)^(7/2)
*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+1/4*a^2*sec(d*x+c)^(7/2)*sin(d*x+c)*(
a+a*sec(d*x+c))^(1/2)/d
```

3.233.2 Mathematica [A] (verified)

Time = 0.76 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.81

$$\int \sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \frac{a^3\left(489\arcsin\left(\sqrt{1-\sec(c+dx)}\right)+326\sqrt{1-\sec(c+dx)}\sec^{\frac{3}{2}}(c+dx)+184\sqrt{1-\sec(c+\cdots}\right.}{192d\sqrt{1-\sec\cdots}}$$

input
```
Integrate[Sec[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^(5/2),x]
```

output
```
(a^3*(489*ArcSin[Sqrt[1 - Sec[c + d*x]]] + 326*Sqrt[1 - Sec[c + d*x]]*Sec[
c + d*x]^(3/2) + 184*Sqrt[1 - Sec[c + d*x]]*Sec[c + d*x]^(5/2) + 48*Sqrt[1
 - Sec[c + d*x]]*Sec[c + d*x]^(7/2) + 489*Sqrt[-((-1 + Sec[c + d*x])*Sec[c
 + d*x])])*Tan[c + d*x])/(192*d*Sqrt[1 - Sec[c + d*x]]*Sqrt[a*(1 + Sec[c +
 d*x])])
```

3.233.3 Rubi [A] (verified)

Time = 0.97 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.04, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {3042, 4301, 27, 3042, 4504, 3042, 4290, 3042, 4290, 3042, 4288, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{\frac{5}{2}}(c+dx)(a\sec(c+dx)+a)^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}dx$$

$$\Big\downarrow \text{4301}$$

$$\frac{1}{4}a\int \frac{1}{2}\sec^{\frac{5}{2}}(c+dx)\sqrt{\sec(c+dx)a+a}(17\sec(c+dx)a+13a)dx+\frac{a^2\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}{4d}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{8}a\int \sec^{\frac{5}{2}}(c+dx)\sqrt{\sec(c+dx)a+a}(17\sec(c+dx)a+13a)dx+\frac{a^2\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}{4d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{8}a\int \csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}\left(17\csc\left(c+dx+\frac{\pi}{2}\right)a+13a\right)dx+\frac{a^2\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}{4d}$$

$$\Big\downarrow \text{4504}$$

$$\frac{1}{8}a\left(\frac{163}{6}a\int \sec^{\frac{5}{2}}(c+dx)\sqrt{\sec(c+dx)a+a}dx+\frac{17a^2\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{3d\sqrt{a\sec(c+dx)+a}}\right)+\frac{a^2\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}{4d}$$

$\Big\downarrow$ 3042

$$\frac{1}{8}a\left(\frac{163}{6}a\int \csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx+\frac{17a^2\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{3d\sqrt{a\sec(c+dx)+a}}\right)+\frac{a^2\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}{4d}$$

$\Big\downarrow$ 4290

$$\frac{1}{8}a\left(\frac{163}{6}a\left(\frac{3}{4}\int \sec^{\frac{3}{2}}(c+dx)\sqrt{\sec(c+dx)a+a}dx+\frac{a\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}\right)+\frac{17a^2\sin(c+dx)\sec^{\frac{7}{2}}(c+dx}{3d\sqrt{a\sec(c+dx)+a}}\right.$$
$$\frac{a^2\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}{4d}$$

$\Big\downarrow$ 3042

$$\frac{1}{8}a\left(\frac{163}{6}a\left(\frac{3}{4}\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx+\frac{a\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}\right)+\frac{17a^2\sin(c+a}{3d\sqrt{a\sec(}}\right.$$
$$\frac{a^2\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}{4d}$$

$\Big\downarrow$ 4290

$$\frac{1}{8}a\left(\frac{163}{6}a\left(\frac{3}{4}\left(\frac{1}{2}\int \sqrt{\sec(c+dx)}\sqrt{\sec(c+dx)a+a}dx+\frac{a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}\right)+\frac{a\sin(c+dx)\sec^{\frac{5}{2}}(c+d}{2d\sqrt{a\sec(c+dx)+a}}\right.$$
$$\frac{a^2\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}{4d}$$

$\Big\downarrow$ 3042

$$\frac{1}{8}a\left(\frac{163}{6}a\left(\frac{3}{4}\left(\frac{1}{2}\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx+\frac{a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}\right)+\frac{a\sin(c+d}{2d\sqrt{a\sec}}\right.$$
$$\frac{a^2\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}{4d}$$

$\Big\downarrow$ 4288

3.233. $\int \sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\,dx$

$$\frac{1}{8}a\left(\frac{163}{6}a\left(\frac{3}{4}\left(\frac{a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}-\frac{\int\frac{1}{\sqrt{\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1}}d\left(-\frac{a\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}\right)+\frac{a\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}\right.\right.$$

$$\frac{a^2\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}{4d}$$

$$\Big\downarrow \; 222$$

$$\frac{1}{8}a\left(\frac{17a^2\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{3d\sqrt{a\sec(c+dx)+a}}+\frac{163}{6}a\left(\frac{3}{4}\left(\frac{\sqrt{a}\text{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}\right)+\frac{a\,s}{2}\right.\right.$$

$$\frac{a^2\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}{4d}$$

input
```
Int[Sec[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^(5/2),x]
```

output
```
(a^2*Sec[c + d*x]^(7/2)*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(4*d) + (a*
((17*a^2*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(3*d*Sqrt[a + a*Sec[c + d*x]]) +
 (163*a*((a*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(2*d*Sqrt[a + a*Sec[c + d*x]]
) + (3*((Sqrt[a]*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])
/d + (a*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]])))/4)
)/6))/8
```

3.233.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4288
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[-2*(a/(b*f))*Sqrt[a*(d/b)]   Subst[Int[1/Sqrt[1
+ x^2/a], x], x, b*(Cot[e + f*x]/Sqrt[a + b*Csc[e + f*x]])], x] /; FreeQ[{a
, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[a*(d/b), 0]
```

rule 4290
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[-2*b*d*Cot[e + f*x]*((d*Csc[e + f*x])^(n - 1)/(
f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]])), x] + Simp[2*a*d*((n - 1)/(b*(2*n -
1)))   Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e + f*x])^(n - 1), x], x] /; Fre
eQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

rule 4301
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
2)*((d*Csc[e + f*x])^n/(f*(m + n - 1))), x] + Simp[b/(m + n - 1)   Int[(a +
 b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*(b*(m + 2*n - 1) + a*(3*m + 2*n
 - 4)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^
2, 0] && GtQ[m, 1] && NeQ[m + n - 1, 0] && IntegerQ[2*m]
```

rule 4504
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[-2*b*B*C
ot[e + f*x]*((d*Csc[e + f*x])^n/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]])), x]
 + Simp[(A*b*(2*n + 1) + 2*a*B*n)/(b*(2*n + 1))   Int[Sqrt[a + b*Csc[e + f*
x]]*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ
[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] &&
!LtQ[n, 0]
```

3.233.4 Maple [A] (verified)

Time = 63.25 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.30

method	result
default	$\frac{a^2\sqrt{a(1+\sec(dx+c))}\,\sec(dx+c)^{\frac{5}{2}}\left(489\cos(dx+c)^3\arctan\left(\frac{\cos(dx+c)-\sin(dx+c)+1}{2(\cos(dx+c)+1)\sqrt{-\frac{1}{\cos(dx+c)+1}}}\right)-489\cos(dx+c)^3\arctan\left(\frac{\cos(dx+c}{2(\cos(dx+c)+}\right.\right.}{}$

input
```
int(sec(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/384/d*a^2*(a*(1+sec(d*x+c)))^(1/2)*sec(d*x+c)^(5/2)/(cos(d*x+c)+1)/(-1/(
cos(d*x+c)+1))^(1/2)*(489*cos(d*x+c)^3*arctan(1/2*(cos(d*x+c)-sin(d*x+c)+1
)/(cos(d*x+c)+1)/(-1/(cos(d*x+c)+1))^(1/2))-489*cos(d*x+c)^3*arctan(1/2*(c
os(d*x+c)+sin(d*x+c)+1)/(cos(d*x+c)+1)/(-1/(cos(d*x+c)+1))^(1/2))+978*(-1/
(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*cos(d*x+c)^2+652*(-1/(cos(d*x+c)+1))^(1/2
)*sin(d*x+c)*cos(d*x+c)+368*sin(d*x+c)*(-1/(cos(d*x+c)+1))^(1/2)+96*(-1/(c
os(d*x+c)+1))^(1/2)*tan(d*x+c))
```

3.233.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 446, normalized size of antiderivative = 2.23

$$\int \sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \left[\frac{489\left(a^2\cos\left(dx+c\right)^4+a^2\cos\left(dx+c\right)^3\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7\,a\cos(dx+c)^2-\frac{4\left(\cos(dx+c)^2-2\,\cos(dx+c)\right)}{\sqrt{\cos}}}{\cos(dx+c)^3+\cos(dx+c)^2}\right.}{768\left(d\cos\left(dx+\right.\right.}\right.$$

input
```
integrate(sec(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
[1/768*(489*(a^2*cos(d*x + c)^4 + a^2*cos(d*x + c)^3)*sqrt(a)*log((a*cos(d
*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 - 2*cos(d*x + c))*sqrt(
a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c))
 + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) + 4*(489*a^2*cos(d*x + c)^3 + 3
26*a^2*cos(d*x + c)^2 + 184*a^2*cos(d*x + c) + 48*a^2)*sqrt((a*cos(d*x + c
) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c)^4 +
d*cos(d*x + c)^3), 1/384*(489*(a^2*cos(d*x + c)^4 + a^2*cos(d*x + c)^3)*sq
rt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(
d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)) + 2*(489
*a^2*cos(d*x + c)^3 + 326*a^2*cos(d*x + c)^2 + 184*a^2*cos(d*x + c) + 48*a
^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)
))/(d*cos(d*x + c)^4 + d*cos(d*x + c)^3)]
```

3.233.6 Sympy [F(-1)]

Timed out.

$$\int \sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**(5/2)*(a+a*sec(d*x+c))**(5/2),x)
```

output
```
Timed out
```

3.233.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3860 vs. $2(168) = 336$.

Time = 0.64 (sec) , antiderivative size = 3860, normalized size of antiderivative = 19.30

$$\int \sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \text{Too large to display}$$

input
```
integrate(sec(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
-1/768*(1956*(sqrt(2)*a^2*sin(8*d*x + 8*c) + 4*sqrt(2)*a^2*sin(6*d*x + 6*c
) + 6*sqrt(2)*a^2*sin(4*d*x + 4*c) + 4*sqrt(2)*a^2*sin(2*d*x + 2*c))*cos(1
5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 652*(sqrt(2)*a^2*sin(8*
d*x + 8*c) + 4*sqrt(2)*a^2*sin(6*d*x + 6*c) + 6*sqrt(2)*a^2*sin(4*d*x + 4*
c) + 4*sqrt(2)*a^2*sin(2*d*x + 2*c))*cos(13/4*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c))) + 6204*(sqrt(2)*a^2*sin(8*d*x + 8*c) + 4*sqrt(2)*a^2*sin(
6*d*x + 6*c) + 6*sqrt(2)*a^2*sin(4*d*x + 4*c) + 4*sqrt(2)*a^2*sin(2*d*x +
2*c))*cos(11/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2060*(sqrt(2
)*a^2*sin(8*d*x + 8*c) + 4*sqrt(2)*a^2*sin(6*d*x + 6*c) + 6*sqrt(2)*a^2*si
n(4*d*x + 4*c) + 4*sqrt(2)*a^2*sin(2*d*x + 2*c))*cos(9/4*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c))) + 2060*(sqrt(2)*a^2*sin(8*d*x + 8*c) + 4*sqrt(
2)*a^2*sin(6*d*x + 6*c) + 6*sqrt(2)*a^2*sin(4*d*x + 4*c) + 4*sqrt(2)*a^2*s
in(2*d*x + 2*c))*cos(7/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 62
04*(sqrt(2)*a^2*sin(8*d*x + 8*c) + 4*sqrt(2)*a^2*sin(6*d*x + 6*c) + 6*sqrt
(2)*a^2*sin(4*d*x + 4*c) + 4*sqrt(2)*a^2*sin(2*d*x + 2*c))*cos(5/4*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 652*(sqrt(2)*a^2*sin(8*d*x + 8*c)
+ 4*sqrt(2)*a^2*sin(6*d*x + 6*c) + 6*sqrt(2)*a^2*sin(4*d*x + 4*c) + 4*sqrt
(2)*a^2*sin(2*d*x + 2*c))*cos(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c))) - 1956*(sqrt(2)*a^2*sin(8*d*x + 8*c) + 4*sqrt(2)*a^2*sin(6*d*x + 6*c)
 + 6*sqrt(2)*a^2*sin(4*d*x + 4*c) + 4*sqrt(2)*a^2*sin(2*d*x + 2*c))*cos...
```

3.233.8 Giac [F]

$$\int \sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \int (a\sec(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^{\frac{5}{2}}\,dx$$

input
```
integrate(sec(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.233.9 Mupad [F(-1)]

Timed out.

$$\int \sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \int \left(a+\frac{a}{\cos(c+d\,x)}\right)^{5/2}\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}dx$$

input
```
int((a + a/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(5/2),x)
```

output
```
int((a + a/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(5/2), x)
```

3.234 $\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\,dx$

3.234.1 Optimal result . . . 1879
3.234.2 Mathematica [A] (verified) . . . 1879
3.234.3 Rubi [A] (verified) . . . 1880
3.234.4 Maple [A] (verified) . . . 1883
3.234.5 Fricas [A] (verification not implemented) . . . 1884
3.234.6 Sympy [F(-1)] . . . 1884
3.234.7 Maxima [B] (verification not implemented) . . . 1885
3.234.8 Giac [F] . . . 1885
3.234.9 Mupad [F(-1)] . . . 1886

3.234.1 Optimal result

Integrand size = 25, antiderivative size = 160

$$\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \frac{25a^{5/2}\operatorname{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{8d} + \frac{25a^3\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{8d\sqrt{a+a\sec(c+dx)}} + \frac{13a^3\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{12d\sqrt{a+a\sec(c+dx)}} + \frac{a^2\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{3d}$$

output
```
25/8*a^(5/2)*arcsinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/d+25/8*a^3
*sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+13/12*a^3*sec(d*x+c)
^(5/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+1/3*a^2*sec(d*x+c)^(5/2)*sin(d*
x+c)*(a+a*sec(d*x+c))^(1/2)/d
```

3.234.2 Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.85

$$\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \frac{a^3\left(75\arcsin\left(\sqrt{1-\sec(c+dx)}\right)+34\sqrt{1-\sec(c+dx)}\sec^{\frac{3}{2}}(c+dx)+8\sqrt{1-\sec(c+dx)}\,\mathrm{s}\right.}{24d\sqrt{1-\sec(c+dx)}\sqrt{a(1+\sec(}}$$

input
```
Integrate[Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(5/2),x]
```

output
```
(a^3*(75*ArcSin[Sqrt[1 - Sec[c + d*x]]] + 34*Sqrt[1 - Sec[c + d*x]]*Sec[c
+ d*x]^(3/2) + 8*Sqrt[1 - Sec[c + d*x]]*Sec[c + d*x]^(5/2) + 75*Sqrt[-((-1
 + Sec[c + d*x])*Sec[c + d*x])])*Tan[c + d*x])/(24*d*Sqrt[1 - Sec[c + d*x]
]*Sqrt[a*(1 + Sec[c + d*x])])
```

3.234.3 Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.02, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 4301, 27, 3042, 4504, 3042, 4290, 3042, 4288, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^{5/2}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}dx$$

$$\Big\downarrow \; 4301$$

$$\frac{1}{3}a\int \frac{1}{2}\sec^{\frac{3}{2}}(c+dx)\sqrt{\sec(c+dx)a+a}(13\sec(c+dx)a+9a)dx+\frac{a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{6}a\int \sec^{\frac{3}{2}}(c+dx)\sqrt{\sec(c+dx)a+a}(13\sec(c+dx)a+9a)dx+\frac{a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{6}a\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}\left(13\csc\left(c+dx+\frac{\pi}{2}\right)a+9a\right)dx+\frac{a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}$$

$$\Big\downarrow \; 4504$$

$$\frac{1}{6}a\left(\frac{75}{4}a\int \sec^{\frac{3}{2}}(c+dx)\sqrt{\sec(c+dx)a+a}dx+\frac{13a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}\right)+\frac{a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}$$

$\Big\downarrow$ 3042

$$\frac{1}{6}a\left(\frac{75}{4}a\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx+\frac{13a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}\right)+\frac{a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}$$

$\Big\downarrow$ 4290

$$\frac{1}{6}a\left(\frac{75}{4}a\left(\frac{1}{2}\int \sqrt{\sec(c+dx)}\sqrt{\sec(c+dx)a+a}dx+\frac{a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}\right)+\frac{13a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx}{2d\sqrt{a\sec(c+dx)+a}}\right.$$
$$\frac{a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}$$

$\Big\downarrow$ 3042

$$\frac{1}{6}a\left(\frac{75}{4}a\left(\frac{1}{2}\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx+\frac{a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}\right)+\frac{13a^2\sin(c+dx)}{2d\sqrt{a\sec(c}}\right.$$
$$\frac{a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}$$

$\Big\downarrow$ 4288

$$\frac{1}{6}a\left(\frac{75}{4}a\left(\frac{a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}-\frac{\int \frac{1}{\sqrt{\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1}}d\left(-\frac{a\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}\right)+\frac{13a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}\right)$$
$$\frac{a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}$$

$\Big\downarrow$ 222

$$\frac{1}{6}a\left(\frac{13a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}+\frac{75}{4}a\left(\frac{\sqrt{a}\text{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}\right)\right)+\frac{a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}$$

3.234. $\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\,dx$

input
```
Int[Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(5/2),x]
```

output
```
(a^2*Sec[c + d*x]^(5/2)*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(3*d) + (a*
((13*a^2*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(2*d*Sqrt[a + a*Sec[c + d*x]]) +
 (75*a*((Sqrt[a]*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])
/d + (a*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]])))/4)
)/6
```

3.234.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4288
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[-2*(a/(b*f))*Sqrt[a*(d/b)]   Subst[Int[1/Sqrt[1
+ x^2/a], x], x, b*(Cot[e + f*x]/Sqrt[a + b*Csc[e + f*x]])], x] /; FreeQ[{a
, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[a*(d/b), 0]
```

rule 4290
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[-2*b*d*Cot[e + f*x]*((d*Csc[e + f*x])^(n - 1)/(
f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]])), x] + Simp[2*a*d*((n - 1)/(b*(2*n -
1)))   Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e + f*x])^(n - 1), x], x] /; Fre
eQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

rule 4301
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
2)*((d*Csc[e + f*x])^n/(f*(m + n - 1))), x] + Simp[b/(m + n - 1)   Int[(a +
 b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*(b*(m + 2*n - 1) + a*(3*m + 2*n
 - 4)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^
2, 0] && GtQ[m, 1] && NeQ[m + n - 1, 0] && IntegerQ[2*m]
```

rule 4504
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[-2*b*B*C
ot[e + f*x]*((d*Csc[e + f*x])^n/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]])), x]
 + Simp[(A*b*(2*n + 1) + 2*a*B*n)/(b*(2*n + 1))   Int[Sqrt[a + b*Csc[e + f*
x]]*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ
[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] &&
!LtQ[n, 0]
```

3.234.4 Maple [A] (verified)

Time = 1.88 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.44

method	result
default	$\frac{a^2\sqrt{a(1+\sec(dx+c))}\,\sec(dx+c)^{\frac{3}{2}}\left(75\cos(dx+c)^2\arctan\left(\frac{\cos(dx+c)-\sin(dx+c)+1}{2(\cos(dx+c)+1)\sqrt{-\frac{1}{\cos(dx+c)+1}}}\right)-75\arctan\left(\frac{\cos(dx+c)+\sin(dx+c)+1}{2(\cos(dx+c)+1)\sqrt{-\frac{1}{\cos(dx+c}}}\right.\right.}{48d(\cos(dx+c)+1)\sqrt{}}$

input
```
int(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/48/d*a^2*(a*(1+sec(d*x+c)))^(1/2)*sec(d*x+c)^(3/2)/(cos(d*x+c)+1)/(-1/(c
os(d*x+c)+1))^(1/2)*(75*cos(d*x+c)^2*arctan(1/2*(cos(d*x+c)-sin(d*x+c)+1)/
(cos(d*x+c)+1)/(-1/(cos(d*x+c)+1))^(1/2))-75*arctan(1/2*(cos(d*x+c)+sin(d*
x+c)+1)/(cos(d*x+c)+1)/(-1/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)^2+150*(-1/(co
s(d*x+c)+1))^(1/2)*sin(d*x+c)*cos(d*x+c)+68*sin(d*x+c)*(-1/(cos(d*x+c)+1))
^(1/2)+16*(-1/(cos(d*x+c)+1))^(1/2)*tan(d*x+c))
```

3.234.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 420, normalized size of antiderivative = 2.62

$$\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \left[\frac{75\left(a^2\cos\left(dx+c\right)^3+a^2\cos\left(dx+c\right)^2\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7\,a\cos(dx+c)^2-\frac{4\left(\cos(dx+c)^2-2\,\cos(dx+c)\right)\sqrt{\cdots}}{\sqrt{\cos(d\cdots}}}{\cos(dx+c)^3+\cos(dx+c)^2}\right.}{96\left(d\cos\left(dx+c\right)^3+d\,c\cdots\right.}\right.$$

input
```
integrate(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
[1/96*(75*(a^2*cos(d*x + c)^3 + a^2*cos(d*x + c)^2)*sqrt(a)*log((a*cos(d*x
 + c)^3 - 7*a*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 - 2*cos(d*x + c))*sqrt(a)
*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)) +
 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) + 4*(75*a^2*cos(d*x + c)^2 + 34*a
^2*cos(d*x + c) + 8*a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x +
 c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c)^3 + d*cos(d*x + c)^2), 1/48*(75*(a
^2*cos(d*x + c)^3 + a^2*cos(d*x + c)^2)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a
*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*
x + c)^2 - a*cos(d*x + c) - 2*a)) + 2*(75*a^2*cos(d*x + c)^2 + 34*a^2*cos(
d*x + c) + 8*a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqr
t(cos(d*x + c)))/(d*cos(d*x + c)^3 + d*cos(d*x + c)^2)]
```

3.234.6 Sympy [F(-1)]

Timed out.

$$\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**(3/2)*(a+a*sec(d*x+c))**(5/2),x)
```

output
```
Timed out
```

3.234.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3469 vs. $2(134) = 268$.

Time = 0.55 (sec) , antiderivative size = 3469, normalized size of antiderivative = 21.68

$$\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \text{Too large to display}$$

input
```
integrate(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
1/96*(300*sqrt(2)*a^2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x +
3/2*c)))*sin(6*d*x + 6*c) - 28*sqrt(2)*a^2*sin(9/2*d*x + 9/2*c) + 28*sqrt(
2)*a^2*sin(3/2*d*x + 3/2*c) - 28*(sqrt(2)*a^2*sin(9/2*d*x + 9/2*c) - sqrt(
2)*a^2*sin(3/2*d*x + 3/2*c))*cos(6*d*x + 6*c) - 300*(sqrt(2)*a^2*sin(6*d*x
 + 6*c) + 3*sqrt(2)*a^2*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x
+ 3/2*c))) + 3*sqrt(2)*a^2*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d
*x + 3/2*c))))*cos(11/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)
)) - 12*(7*sqrt(2)*a^2*sin(9/2*d*x + 9/2*c) - 7*sqrt(2)*a^2*sin(3/2*d*x +
3/2*c) - 114*sqrt(2)*a^2*sin(7/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x
 + 3/2*c))) + 114*sqrt(2)*a^2*sin(5/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/
2*d*x + 3/2*c))) + 75*sqrt(2)*a^2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), co
s(3/2*d*x + 3/2*c))))*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x +
3/2*c))) - 456*(sqrt(2)*a^2*sin(6*d*x + 6*c) + 3*sqrt(2)*a^2*sin(4/3*arcta
n2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*cos(7/3*arctan2(sin(3/2*d
*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 456*(sqrt(2)*a^2*sin(6*d*x + 6*c) +
3*sqrt(2)*a^2*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))
)*cos(5/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 12*(7*sqr
t(2)*a^2*sin(9/2*d*x + 9/2*c) - 7*sqrt(2)*a^2*sin(3/2*d*x + 3/2*c) + 75*sq
rt(2)*a^2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*co
s(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 75*(a^2*co...
```

3.234.8 Giac [F]

$$\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \int (a\sec(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^{\frac{3}{2}}\,dx$$

input
```
integrate(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.234.9 Mupad [F(-1)]

Timed out.

$$\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\,dx=\int \left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)^{5/2}\left(\frac{1}{\cos\left(c+d\,x\right)}\right)^{3/2}dx$$

input
```
int((a + a/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(3/2),x)
```

output
```
int((a + a/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(3/2), x)
```

3.235 $\int \sqrt{\sec(c+dx)}(a+a\sec(c+dx))^{5/2}\,dx$

3.235.1 Optimal result 1887
3.235.2 Mathematica [A] (verified) 1887
3.235.3 Rubi [A] (verified) 1888
3.235.4 Maple [A] (verified) 1890
3.235.5 Fricas [A] (verification not implemented) 1891
3.235.6 Sympy [F(-1)] 1891
3.235.7 Maxima [B] (verification not implemented) 1892
3.235.8 Giac [F] 1892
3.235.9 Mupad [F(-1)] 1893

3.235.1 Optimal result

Integrand size = 25, antiderivative size = 120

$$\int \sqrt{\sec(c+dx)}(a+a\sec(c+dx))^{5/2}\,dx = \frac{19a^{5/2}\operatorname{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{4d} + \frac{9a^3\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4d\sqrt{a+a\sec(c+dx)}} + \frac{a^2\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{2d}$$

output
```
19/4*a^(5/2)*arcsinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/d+9/4*a^3*
sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+1/2*a^2*sec(d*x+c)^(3
/2)*sin(d*x+c)*(a+a*sec(d*x+c))^(1/2)/d
```

3.235.2 Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.88

$$\int \sqrt{\sec(c+dx)}(a+a\sec(c+dx))^{5/2}\,dx = \frac{a^3\left(-19\arcsin\left(\sqrt{\sec(c+dx)}\right)+2\sqrt{1-\sec(c+dx)}\sec^{\frac{3}{2}}(c+dx)+11\sqrt{-((-1+\sec(c+d}\right.}{4d\sqrt{1-\sec(c+dx)}\sqrt{a(1+\sec(c+dx))}}$$

input
```
Integrate[Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^(5/2),x]
```

output
```
(a^3*(-19*ArcSin[Sqrt[Sec[c + d*x]]] + 2*Sqrt[1 - Sec[c + d*x]]*Sec[c + d*
x]^(3/2) + 11*Sqrt[-((-1 + Sec[c + d*x])*Sec[c + d*x])])*Tan[c + d*x])/(4*
d*Sqrt[1 - Sec[c + d*x]]*Sqrt[a*(1 + Sec[c + d*x])])
```

3.235.3 Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.02, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3042, 4301, 27, 3042, 4504, 3042, 4288, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^{5/2}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}dx$$

$$\Big\downarrow \; 4301$$

$$\frac{1}{2}a\int \frac{1}{2}\sqrt{\sec(c+dx)}\sqrt{\sec(c+dx)a+a}(9\sec(c+dx)a+5a)dx+\frac{a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}{2d}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{4}a\int \sqrt{\sec(c+dx)}\sqrt{\sec(c+dx)a+a}(9\sec(c+dx)a+5a)dx+\frac{a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}{2d}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{4}a\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}\left(9\csc\left(c+dx+\frac{\pi}{2}\right)a+5a\right)dx+\frac{a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}{2d}$$

$$\Big\downarrow \; 4504$$

$$\frac{1}{4}a\left(\frac{19}{2}a\int \sqrt{\sec(c+dx)}\sqrt{\sec(c+dx)a+a}dx+\frac{9a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}\right)+\frac{a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}{2d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}a\left(\frac{19}{2}a\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx+\frac{9a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}\right)+\frac{a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}{2d}$$

$$\Big\downarrow \text{4288}$$

$$\frac{1}{4}a\left(\frac{9a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}-\frac{19a\int\frac{1}{\sqrt{\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1}}d\left(-\frac{a\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}\right)+\frac{a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}{2d}$$

$$\Big\downarrow \text{222}$$

$$\frac{a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}{2d}+\frac{1}{4}a\left(\frac{19a^{3/2}\text{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{9a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}\right)$$

input
```
Int[Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^(5/2),x]
```

output
```
(a^2*Sec[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(2*d) + (a*
((19*a^(3/2)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d +
 (9*a^2*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]])))/4
```

3.235.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4288
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[-2*(a/(b*f))*Sqrt[a*(d/b)]   Subst[Int[1/Sqrt[1
+ x^2/a], x], x, b*(Cot[e + f*x]/Sqrt[a + b*Csc[e + f*x]])], x] /; FreeQ[{a
, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[a*(d/b), 0]
```

rule 4301
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
2)*((d*Csc[e + f*x])^n/(f*(m + n - 1))), x] + Simp[b/(m + n - 1)   Int[(a +
 b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*(b*(m + 2*n - 1) + a*(3*m + 2*n
 - 4)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^
2, 0] && GtQ[m, 1] && NeQ[m + n - 1, 0] && IntegerQ[2*m]
```

rule 4504
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[-2*b*B*C
ot[e + f*x]*((d*Csc[e + f*x])^n/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]])), x]
 + Simp[(A*b*(2*n + 1) + 2*a*B*n)/(b*(2*n + 1))   Int[Sqrt[a + b*Csc[e + f*
x]]*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ
[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] &&
!LtQ[n, 0]
```

3.235.4 Maple [A] (verified)

Time = 1.81 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.65

method	result
default	$-\frac{a^2\sqrt{a(1+\sec(dx+c))}\,\sqrt{\sec(dx+c)}\left(19\cos(dx+c)\arctan\left(\frac{-\cos(dx+c)+\sin(dx+c)-1}{2(\cos(dx+c)+1)\sqrt{-\frac{1}{\cos(dx+c)+1}}}\right)+19\cos(dx+c)\arctan\left(\frac{\cos(dx+c)-}{2(\cos(dx+c)+1}\right.\right.}{8d(\cos(dx+c)+1)\sqrt{-\frac{1}{\cos(dx+c)+1}}}$

input
```
int(sec(d*x+c)^(1/2)*(a+a*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/8/d*a^2*(a*(1+sec(d*x+c)))^(1/2)*sec(d*x+c)^(1/2)/(cos(d*x+c)+1)/(-1/(c
os(d*x+c)+1))^(1/2)*(19*cos(d*x+c)*arctan(1/2*(-cos(d*x+c)+sin(d*x+c)-1)/(
cos(d*x+c)+1)/(-1/(cos(d*x+c)+1))^(1/2))+19*cos(d*x+c)*arctan(1/2*(cos(d*x
+c)+sin(d*x+c)+1)/(cos(d*x+c)+1)/(-1/(cos(d*x+c)+1))^(1/2))-22*sin(d*x+c)*
(-1/(cos(d*x+c)+1))^(1/2)-4*(-1/(cos(d*x+c)+1))^(1/2)*tan(d*x+c))
```

3.235.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 386, normalized size of antiderivative = 3.22

$$\int \sqrt{\sec(c+dx)}(a+a\sec(c+dx))^{5/2}\,dx = \left[\frac{19\left(a^2\cos\left(dx+c\right)^2+a^2\cos\left(dx+c\right)\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7\,a\cos(dx+c)^2-\frac{4\left(\cos(dx+c)^2-2\,\cos(dx+c)\right)\sqrt{a}}{\sqrt{\cos(dx}}}{\cos(dx+c)^3+\cos(dx+c)^2}\right.}{16\left(d\cos\left(dx+c\right)^2+d\cos\left(dx\right.\right.}\right.$$

input
```
integrate(sec(d*x+c)^(1/2)*(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
[1/16*(19*(a^2*cos(d*x + c)^2 + a^2*cos(d*x + c))*sqrt(a)*log((a*cos(d*x +
 c)^3 - 7*a*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 - 2*cos(d*x + c))*sqrt(a)*s
qrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)) + 8
*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) + 4*(11*a^2*cos(d*x + c) + 2*a^2)*s
qrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(d
*cos(d*x + c)^2 + d*cos(d*x + c)), 1/8*(19*(a^2*cos(d*x + c)^2 + a^2*cos(d
*x + c))*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)
)*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a
)) + 2*(11*a^2*cos(d*x + c) + 2*a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c
))*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c)^2 + d*cos(d*x + c))]
```

3.235.6 Sympy [F(-1)]

Timed out.

$$\int \sqrt{\sec(c+dx)}(a+a\sec(c+dx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**(1/2)*(a+a*sec(d*x+c))**(5/2),x)
```

output
```
Timed out
```

3.235.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2826 vs. $2(100) = 200$.

Time = 3.18 (sec) , antiderivative size = 2826, normalized size of antiderivative = 23.55

$$\int \sqrt{\sec(c+dx)}(a+a\sec(c+dx))^{5/2}\,dx = \text{Too large to display}$$

input
```
integrate(sec(d*x+c)^(1/2)*(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
-1/16*(88*sqrt(2)*a^2*cos(7/2*d*x + 7/2*c)*sin(2*d*x + 2*c) - 56*sqrt(2)*a
^2*cos(5/2*d*x + 5/2*c)*sin(2*d*x + 2*c) - 28*sqrt(2)*a^2*sin(3/2*d*x + 3/
2*c) + 44*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c) - 19*(a^2*log(2*cos(1/2*d*x + 1
/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sq
rt(2)*sin(1/2*d*x + 1/2*c) + 2) - a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin
(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d
*x + 1/2*c) + 2) + a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*
c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2
) - a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2
)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*cos(4*d*x +
4*c)^2 - 76*(a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 +
 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - a^
2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(
1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + a^2*log(2*cos(1/2
*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c
) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - a^2*log(2*cos(1/2*d*x + 1/2*c)^2
 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*s
in(1/2*d*x + 1/2*c) + 2))*cos(2*d*x + 2*c)^2 - 19*a^2*log(2*cos(1/2*d*x +
1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*s
qrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 ...
```

3.235.8 Giac [F]

$$\int \sqrt{\sec(c+dx)}(a+a\sec(c+dx))^{5/2}\,dx = \int (a\sec(dx+c)+a)^{\frac{5}{2}}\sqrt{\sec(dx+c)}\,dx$$

input
```
integrate(sec(d*x+c)^(1/2)*(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.235.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{\sec(c+dx)}(a+a\sec(c+dx))^{5/2}\,dx=\int \left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)^{5/2}\sqrt{\frac{1}{\cos\left(c+d\,x\right)}}\,dx$$

input
```
int((a + a/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(1/2),x)
```

output
```
int((a + a/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(1/2), x)
```

3.236 $\int \frac{(a+a\sec(c+dx))^{5/2}}{\sqrt{\sec(c+dx)}}\,dx$

3.236.1 Optimal result 1894
3.236.2 Mathematica [A] (verified) 1894
3.236.3 Rubi [A] (verified) 1895
3.236.4 Maple [B] (verified) 1897
3.236.5 Fricas [A] (verification not implemented) 1898
3.236.6 Sympy [F(-1)] 1899
3.236.7 Maxima [B] (verification not implemented) 1899
3.236.8 Giac [F] 1900
3.236.9 Mupad [F(-1)] 1900

3.236.1 Optimal result

Integrand size = 25, antiderivative size = 112

$$\int \frac{(a+a\sec(c+dx))^{5/2}}{\sqrt{\sec(c+dx)}}\,dx = \frac{5a^{5/2}\operatorname{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} + \frac{a^3\sqrt{\sec(c+dx)}\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}} + \frac{a^2\sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{d}$$

output
```
5*a^(5/2)*arcsinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/d+a^3*sin(d*x
+c)*sec(d*x+c)^(1/2)/d/(a+a*sec(d*x+c))^(1/2)+a^2*sin(d*x+c)*sec(d*x+c)^(1
/2)*(a+a*sec(d*x+c))^(1/2)/d
```

3.236.2 Mathematica [A] (verified)

Time = 0.69 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.81

$$\int \frac{(a+a\sec(c+dx))^{5/2}}{\sqrt{\sec(c+dx)}}\,dx = \frac{a^3\left(5\arcsin\left(\sqrt{1-\sec(c+dx)}\right)+(1+2\cos(c+dx))\sqrt{(-1+\cos(c+dx))\,s}\right.}{d\sqrt{1-\sec(c+dx)}\sqrt{a(1+\sec(c+dx))}}$$

input
```
Integrate[(a + a*Sec[c + d*x])^(5/2)/Sqrt[Sec[c + d*x]],x]
```

output
```
(a^3*(5*ArcSin[Sqrt[1 - Sec[c + d*x]]] + (1 + 2*Cos[c + d*x])*Sqrt[(-1 + C
os[c + d*x])*Sec[c + d*x]^2])*Tan[c + d*x])/(d*Sqrt[1 - Sec[c + d*x]]*Sqrt
[a*(1 + Sec[c + d*x])])
```

3.236.3 Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.06, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3042, 4301, 27, 3042, 4503, 3042, 4288, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sec(c+dx)+a)^{5/2}}{\sqrt{\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4301}$$

$$a\int \frac{\sqrt{\sec(c+dx)a+a}(5\sec(c+dx)a+a)}{2\sqrt{\sec(c+dx)}}dx+\frac{a^2\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}a\int \frac{\sqrt{\sec(c+dx)a+a}(5\sec(c+dx)a+a)}{\sqrt{\sec(c+dx)}}dx+\frac{a^2\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}{d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}a\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}\left(5\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{a^2\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}{d}$$

$$\Big\downarrow \text{4503}$$

$$\frac{1}{2}a\left(5a\int \sqrt{\sec(c+dx)}\sqrt{\sec(c+dx)a+a}dx+\frac{2a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\sec(c+dx)+a}}\right)+\frac{a^2\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}{d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}a\left(5a\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx+\frac{2a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\sec(c+dx)+a}}\right)+\frac{a^2\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}{d}$$

$$\Big\downarrow \text{4288}$$

$$\frac{1}{2}a\left(\frac{2a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\sec(c+dx)+a}}-\frac{10a\int\frac{1}{\sqrt{\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1}}d\left(-\frac{a\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}\right)+\frac{a^2\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}{d}$$

$$\Big\downarrow \text{222}$$

$$\frac{a^2\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}{d}+\frac{1}{2}a\left(\frac{10a^{3/2}\text{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{2a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\sec(c+dx)+a}}\right)$$

input
```
Int[(a + a*Sec[c + d*x])^(5/2)/Sqrt[Sec[c + d*x]],x]
```

output
```
(a^2*Sqrt[Sec[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/d + (a*((10
*a^(3/2)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d + (2*
a^2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]])))/2
```

3.236.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4288
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[-2*(a/(b*f))*Sqrt[a*(d/b)]   Subst[Int[1/Sqrt[1
+ x^2/a], x], x, b*(Cot[e + f*x]/Sqrt[a + b*Csc[e + f*x]])], x] /; FreeQ[{a
, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[a*(d/b), 0]
```

rule 4301
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
2)*((d*Csc[e + f*x])^n/(f*(m + n - 1))), x] + Simp[b/(m + n - 1)   Int[(a +
 b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*(b*(m + 2*n - 1) + a*(3*m + 2*n
 - 4)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^
2, 0] && GtQ[m, 1] && NeQ[m + n - 1, 0] && IntegerQ[2*m]
```

rule 4503
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*b^2*Co
t[e + f*x]*((d*Csc[e + f*x])^n/(a*f*n*Sqrt[a + b*Csc[e + f*x]])), x] + Simp
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n)   Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[
e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a
*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

3.236.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 299 vs. $2(98) = 196$.

Time = 1.66 (sec) , antiderivative size = 300, normalized size of antiderivative = 2.68

method	result
default	$-\frac{a^2\sqrt{a(1+\sec(dx+c))}\left(5\sqrt{-\frac{1}{\cos(dx+c)+1}}\arctan\left(\frac{\cos(dx+c)-\sin(dx+c)+1}{2(\cos(dx+c)+1)\sqrt{-\frac{1}{\cos(dx+c)+1}}}\right)\cos(dx+c)-5\sqrt{-\frac{1}{\cos(dx+c)+1}}\arctan\left(\frac{\cos}{2(\cos(}\right.\right.}{}$

input
```
int((a+a*sec(d*x+c))^(5/2)/sec(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2/d*a^2*(a*(1+sec(d*x+c)))^(1/2)/(cos(d*x+c)+1)/sec(d*x+c)^(1/2)*(5*(-1
/(cos(d*x+c)+1))^(1/2)*arctan(1/2*(cos(d*x+c)-sin(d*x+c)+1)/(cos(d*x+c)+1)
/(-1/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)-5*(-1/(cos(d*x+c)+1))^(1/2)*arctan(
1/2*(cos(d*x+c)+sin(d*x+c)+1)/(cos(d*x+c)+1)/(-1/(cos(d*x+c)+1))^(1/2))*co
s(d*x+c)+5*(-1/(cos(d*x+c)+1))^(1/2)*arctan(1/2*(cos(d*x+c)-sin(d*x+c)+1)/
(cos(d*x+c)+1)/(-1/(cos(d*x+c)+1))^(1/2))-5*(-1/(cos(d*x+c)+1))^(1/2)*arct
an(1/2*(cos(d*x+c)+sin(d*x+c)+1)/(cos(d*x+c)+1)/(-1/(cos(d*x+c)+1))^(1/2))
-4*sin(d*x+c)-2*tan(d*x+c))
```

3.236.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 346, normalized size of antiderivative = 3.09

$$\int \frac{(a+a\sec(c+dx))^{5/2}}{\sqrt{\sec(c+dx)}}\,dx = \left[\frac{5\left(a^2\cos\left(dx+c\right)+a^2\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7\,a\cos(dx+c)^2-\frac{4\left(\cos(dx+c)^2-2\,\cos(dx+c)\right)\cdots}{\sqrt{\cos(\cdots}}}{\cos(dx+c)^3+\cos(dx+c)^2}\cdots\right)}{4\left(d\cos\left(dx+c\right)\cdots\right.}\right.$$

input
```
integrate((a+a*sec(d*x+c))^(5/2)/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
[1/4*(5*(a^2*cos(d*x + c) + a^2)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d
*x + c)^2 - 4*(cos(d*x + c)^2 - 2*cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x +
c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)) + 8*a)/(cos(d*x + c)
^3 + cos(d*x + c)^2)) + 4*(2*a^2*cos(d*x + c) + a^2)*sqrt((a*cos(d*x + c)
+ a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c) + d),
1/2*(5*(a^2*cos(d*x + c) + a^2)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x
 + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2
 - a*cos(d*x + c) - 2*a)) + 2*(2*a^2*cos(d*x + c) + a^2)*sqrt((a*cos(d*x +
 c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c) +
d)]
```

3.236.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(c+dx))^{5/2}}{\sqrt{\sec(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sec(d*x+c))**(5/2)/sec(d*x+c)**(1/2),x)
```

output
```
Timed out
```

3.236.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 11494 vs. $2(98) = 196$.

Time = 0.49 (sec) , antiderivative size = 11494, normalized size of antiderivative = 102.62

$$\int \frac{(a+a\sec(c+dx))^{5/2}}{\sqrt{\sec(c+dx)}}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sec(d*x+c))^(5/2)/sec(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
1/4*(8*a^2*cos(1/2*d*x + 1/2*c)^4*sin(1/2*d*x + 1/2*c) + 16*a^2*cos(1/2*d*
x + 1/2*c)^2*sin(1/2*d*x + 1/2*c)^3 + 8*a^2*sin(1/2*d*x + 1/2*c)^5 + 5*(sq
rt(2)*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt
(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - sqrt(2)*a
^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos
(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + sqrt(2)*a^2*log(
2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*
x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - sqrt(2)*a^2*log(2*cos(1
/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2
*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*cos(1/2*d*x + 1/2*c)^4 + 10*(sq
rt(2)*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt
(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - sqrt(2)*a
^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos
(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + sqrt(2)*a^2*log(
2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*
x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - sqrt(2)*a^2*log(2*cos(1
/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2
*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*cos(1/2*d*x + 1/2*c)^2*sin(1/2*
d*x + 1/2*c)^2 + 5*(sqrt(2)*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d
*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x ...
```

3.236.8 Giac [F]

$$\int \frac{(a+a\sec(c+dx))^{5/2}}{\sqrt{\sec(c+dx)}}\,dx=\int \frac{(a\sec(dx+c)+a)^{\frac{5}{2}}}{\sqrt{\sec(dx+c)}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))^(5/2)/sec(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.236.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(c+dx))^{5/2}}{\sqrt{\sec(c+dx)}}\,dx=\int \frac{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{5/2}}{\sqrt{\frac{1}{\cos(c+d\,x)}}}\,dx$$

input
```
int((a + a/cos(c + d*x))^(5/2)/(1/cos(c + d*x))^(1/2),x)
```

output
```
int((a + a/cos(c + d*x))^(5/2)/(1/cos(c + d*x))^(1/2), x)
```

3.237 $\int \frac{(a+a\sec(c+dx))^{5/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

3.237.1 Optimal result 1901
3.237.2 Mathematica [A] (verified) 1901
3.237.3 Rubi [A] (verified) 1902
3.237.4 Maple [B] (verified) 1904
3.237.5 Fricas [A] (verification not implemented) 1905
3.237.6 Sympy [F(-1)] 1906
3.237.7 Maxima [B] (verification not implemented) 1906
3.237.8 Giac [F] 1907
3.237.9 Mupad [F(-1)] 1907

3.237.1 Optimal result

Integrand size = 25, antiderivative size = 118

$$\int \frac{(a+a\sec(c+dx))^{5/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \frac{2a^{5/2}\operatorname{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} + \frac{14a^3\sqrt{\sec(c+dx)}\sin(c+dx)}{3d\sqrt{a+a\sec(c+dx)}} + \frac{2a^2\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}$$

output
```
2*a^(5/2)*arcsinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/d+14/3*a^3*si
n(d*x+c)*sec(d*x+c)^(1/2)/d/(a+a*sec(d*x+c))^(1/2)+2/3*a^2*sin(d*x+c)*(a+a
*sec(d*x+c))^(1/2)/d/sec(d*x+c)^(1/2)
```

3.237.2 Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.87

$$\int \frac{(a+a\sec(c+dx))^{5/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \frac{2a^3\left(3\arcsin\left(\sqrt{1-\sec(c+dx)}\right)\sec^{\frac{3}{2}}(c+dx)+\sqrt{1-\sec(c+dx)}(1+8\sec\right.}{3d\sqrt{-((-1+\sec(c+dx))\sec(c+dx))}\sqrt{a(1+\sec(c+d}}$$

input
```
Integrate[(a + a*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(3/2),x]
```

output
```
(2*a^3*(3*ArcSin[Sqrt[1 - Sec[c + d*x]]]*Sec[c + d*x]^(3/2) + Sqrt[1 - Sec
[c + d*x]]*(1 + 8*Sec[c + d*x]))*Sin[c + d*x])/(3*d*Sqrt[-((-1 + Sec[c + d
*x])*Sec[c + d*x])]*Sqrt[a*(1 + Sec[c + d*x])])
```

3.237.3 Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.03, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3042, 4300, 27, 3042, 4503, 3042, 4288, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sec(c+dx)+a)^{5/2}}{\sec^{\frac{3}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{4300}$$

$$\frac{2}{3}a\int \frac{\sqrt{\sec(c+dx)a+a}(3\sec(c+dx)a+7a)}{2\sqrt{\sec(c+dx)}}dx+\frac{2a^2\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{3d\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}a\int \frac{\sqrt{\sec(c+dx)a+a}(3\sec(c+dx)a+7a)}{\sqrt{\sec(c+dx)}}dx+\frac{2a^2\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{3d\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}a\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}\left(3\csc\left(c+dx+\frac{\pi}{2}\right)a+7a\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a^2\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{3d\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{4503}$$

$$\frac{1}{3}a\left(3a\int \sqrt{\sec(c+dx)}\sqrt{\sec(c+dx)a+a}dx+\frac{14a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\sec(c+dx)+a}}\right)+\frac{2a^2\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{3d\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}a\left(3a\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx+\frac{14a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\sec(c+dx)+a}}\right)+\frac{2a^2\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{3d\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \; 4288$$

$$\frac{1}{3}a\left(\frac{14a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\sec(c+dx)+a}}-\frac{6a\int\frac{1}{\sqrt{\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1}}d\left(-\frac{a\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}\right)+\frac{2a^2\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{3d\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \; 222$$

$$\frac{2a^2\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{3d\sqrt{\sec(c+dx)}}+\frac{1}{3}a\left(\frac{6a^{3/2}\text{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{14a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\sec(c+dx)+a}}\right)$$

input
```
Int[(a + a*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(3/2),x]
```

output
```
(2*a^2*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]]) + (
a*((6*a^(3/2)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d
+ (14*a^2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]])))/
3
```

3.237.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.237. $\int \frac{(a+a\sec(c+dx))^{5/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

rule 4288
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[-2*(a/(b*f))*Sqrt[a*(d/b)]   Subst[Int[1/Sqrt[1
+ x^2/a], x], x, b*(Cot[e + f*x]/Sqrt[a + b*Csc[e + f*x]])], x] /; FreeQ[{a
, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[a*(d/b), 0]
```

rule 4300
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)*
((d*Csc[e + f*x])^n/(f*n)), x] - Simp[a/(d*n)   Int[(a + b*Csc[e + f*x])^(m
 - 2)*(d*Csc[e + f*x])^(n + 1)*(b*(m - 2*n - 2) - a*(m + 2*n - 1)*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1]
 && (LtQ[n, -1] || (EqQ[m, 3/2] && EqQ[n, -2^(-1)])) && IntegerQ[2*m]
```

rule 4503
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*b^2*Co
t[e + f*x]*((d*Csc[e + f*x])^n/(a*f*n*Sqrt[a + b*Csc[e + f*x]])), x] + Simp
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n)   Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[
e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a
*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

3.237.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 299 vs. $2(100) = 200$.

Time = 1.82 (sec) , antiderivative size = 300, normalized size of antiderivative = 2.54

method	result
default	$\frac{a^2\sqrt{a(1+\sec(dx+c))}\left(-3\sqrt{-\frac{1}{\cos(dx+c)+1}}\arctan\left(\frac{\cos(dx+c)-\sin(dx+c)+1}{2(\cos(dx+c)+1)\sqrt{-\frac{1}{\cos(dx+c)+1}}}\right)+3\sqrt{-\frac{1}{\cos(dx+c)+1}}\arctan\left(\frac{\cos(dx+c)+\sin(}{2(\cos(dx+c)+1)\sqrt{-}}\right.\right.}{}$

input
```
int((a+a*sec(d*x+c))^(5/2)/sec(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/3/d*a^2*(a*(1+sec(d*x+c)))^(1/2)/(cos(d*x+c)+1)/sec(d*x+c)^(3/2)*(-3*(-1
/(cos(d*x+c)+1))^(1/2)*arctan(1/2*(cos(d*x+c)-sin(d*x+c)+1)/(cos(d*x+c)+1)
/(-1/(cos(d*x+c)+1))^(1/2))+3*(-1/(cos(d*x+c)+1))^(1/2)*arctan(1/2*(cos(d*
x+c)+sin(d*x+c)+1)/(cos(d*x+c)+1)/(-1/(cos(d*x+c)+1))^(1/2))+2*sin(d*x+c)-
3*(-1/(cos(d*x+c)+1))^(1/2)*arctan(1/2*(cos(d*x+c)-sin(d*x+c)+1)/(cos(d*x+
c)+1)/(-1/(cos(d*x+c)+1))^(1/2))*sec(d*x+c)+3*(-1/(cos(d*x+c)+1))^(1/2)*ar
ctan(1/2*(cos(d*x+c)+sin(d*x+c)+1)/(cos(d*x+c)+1)/(-1/(cos(d*x+c)+1))^(1/2
))*sec(d*x+c)+16*tan(d*x+c))
```

3.237.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 364, normalized size of antiderivative = 3.08

$$\int \frac{(a+a\sec(c+dx))^{5/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \left[\frac{3\left(a^2\cos\left(dx+c\right)+a^2\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7\,a\cos(dx+c)^2-\frac{4\left(\cos(dx+c)^2-2\,\cos(dx+c)\right)\cdots}{\sqrt{\cos(\cdots}}}{\cos(dx+c)^3+\cos(dx+c)^2}\cdots\right)}{6\left(d\cos\left(dx\cdots\right.\right.}\right.$$

input
```
integrate((a+a*sec(d*x+c))^(5/2)/sec(d*x+c)^(3/2),x, algorithm="fricas")
```

output
```
[1/6*(3*(a^2*cos(d*x + c) + a^2)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d
*x + c)^2 - 4*(cos(d*x + c)^2 - 2*cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x +
c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)) + 8*a)/(cos(d*x + c)
^3 + cos(d*x + c)^2)) + 4*(a^2*cos(d*x + c)^2 + 8*a^2*cos(d*x + c))*sqrt((
a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(
d*x + c) + d), 1/3*(3*(a^2*cos(d*x + c) + a^2)*sqrt(-a)*arctan(2*sqrt(-a)*
sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a
*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)) + 2*(a^2*cos(d*x + c)^2 + 8*a^2*c
os(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos
(d*x + c)))/(d*cos(d*x + c) + d)]
```

3.237.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(c+dx))^{5/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sec(d*x+c))**(5/2)/sec(d*x+c)**(3/2),x)
```

output
```
Timed out
```

3.237.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 593 vs. $2(100) = 200$.

Time = 0.41 (sec) , antiderivative size = 593, normalized size of antiderivative = 5.03

$$\int \frac{(a+a\sec(c+dx))^{5/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sec(d*x+c))^(5/2)/sec(d*x+c)^(3/2),x, algorithm="maxima")
```

output
```
1/12*sqrt(2)*(30*a^2*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3
/2*c)))*sin(3/2*d*x + 3/2*c) - 30*a^2*cos(3/2*d*x + 3/2*c)*sin(2/3*arctan2
(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 3*sqrt(2)*a^2*log(2*cos(1/
3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arcta
n2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sqrt(2)*cos(1/3*arct
an2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sqrt(2)*sin(1/3*arcta
n2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) - 3*sqrt(2)*a^2*log(2
*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/
3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sqrt(2)*cos(1
/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 2*sqrt(2)*sin(1/
3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) + 3*sqrt(2)*a^
2*log(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2
*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 - 2*sqrt(2
)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sqrt(2)
*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) - 3*sqr
t(2)*a^2*log(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))
)^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 - 2
*sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 2*
sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2)
+ 4*a^2*sin(3/2*d*x + 3/2*c) + 30*a^2*sin(1/3*arctan2(sin(3/2*d*x + 3/2...
```

3.237.8 Giac [F]

$$\int \frac{(a+a\sec(c+dx))^{5/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx=\int \frac{(a\sec(dx+c)+a)^{\frac{5}{2}}}{\sec(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))^(5/2)/sec(d*x+c)^(3/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.237.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(c+dx))^{5/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx=\int \frac{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{5/2}}{\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

input
```
int((a + a/cos(c + d*x))^(5/2)/(1/cos(c + d*x))^(3/2),x)
```

output
```
int((a + a/cos(c + d*x))^(5/2)/(1/cos(c + d*x))^(3/2), x)
```

3.238 $\int \frac{(a+a\sec(c+dx))^{5/2}}{\sec^{\frac{5}{2}}(c+dx)}\,dx$

3.238.1 Optimal result 1908
3.238.2 Mathematica [A] (verified) 1908
3.238.3 Rubi [A] (verified) 1909
3.238.4 Maple [A] (verified) 1911
3.238.5 Fricas [A] (verification not implemented) 1911
3.238.6 Sympy [F(-1)] 1911
3.238.7 Maxima [A] (verification not implemented) 1912
3.238.8 Giac [F] 1912
3.238.9 Mupad [B] (verification not implemented) 1912

3.238.1 Optimal result

Integrand size = 25, antiderivative size = 119

$$\int \frac{(a+a\sec(c+dx))^{5/2}}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \frac{64a^3\sqrt{\sec(c+dx)}\sin(c+dx)}{15d\sqrt{a+a\sec(c+dx)}} + \frac{16a^2\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{15d\sqrt{\sec(c+dx)}} + \frac{2a(a+a\sec(c+dx))^{3/2}\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}$$

output
```
2/5*a*(a+a*sec(d*x+c))^(3/2)*sin(d*x+c)/d/sec(d*x+c)^(3/2)+64/15*a^3*sin(d
*x+c)*sec(d*x+c)^(1/2)/d/(a+a*sec(d*x+c))^(1/2)+16/15*a^2*sin(d*x+c)*(a+a*
sec(d*x+c))^(1/2)/d/sec(d*x+c)^(1/2)
```

3.238.2 Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.54

$$\int \frac{(a+a\sec(c+dx))^{5/2}}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \frac{a^2(89+28\cos(c+dx)+3\cos(2(c+dx)))\sqrt{a(1+\sec(c+dx))}\tan\left(\frac{1}{2}(c+d\right.}{15d\sqrt{\sec(c+dx)}}$$

input
```
Integrate[(a + a*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(5/2),x]
```

output
```
(a^2*(89 + 28*Cos[c + d*x] + 3*Cos[2*(c + d*x)])*Sqrt[a*(1 + Sec[c + d*x])
]*Tan[(c + d*x)/2])/(15*d*Sqrt[Sec[c + d*x]])
```

3.238.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {3042, 4296, 3042, 4296, 3042, 4291}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sec(c+dx)+a)^{5/2}}{\sec^{\frac{5}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx$$

$$\Big\downarrow \text{4296}$$

$$\frac{8}{5}a\int \frac{(\sec(c+dx)a+a)^{3/2}}{\sec^{\frac{3}{2}}(c+dx)}dx+\frac{2a\sin(c+dx)(a\sec(c+dx)+a)^{3/2}}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{8}{5}a\int \frac{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+\frac{2a\sin(c+dx)(a\sec(c+dx)+a)^{3/2}}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{4296}$$

$$\frac{8}{5}a\left(\frac{4}{3}a\int \frac{\sqrt{\sec(c+dx)a+a}}{\sqrt{\sec(c+dx)}}dx+\frac{2a\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{3d\sqrt{\sec(c+dx)}}\right)+\frac{2a\sin(c+dx)(a\sec(c+dx)+a)^{3/2}}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{8}{5}a\left(\frac{4}{3}a\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{3d\sqrt{\sec(c+dx)}}\right)+\frac{2a\sin(c+dx)(a\sec(c+dx)+a)^{3/2}}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{4291}$$

$$\frac{8}{5}a\left(\frac{8a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\sqrt{a\sec(c+dx)+a}}+\frac{2a\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{3d\sqrt{\sec(c+dx)}}\right)+\frac{2a\sin(c+dx)(a\sec(c+dx)+a)^{3/2}}{5d\sec^{\frac{3}{2}}(c+dx)}$$

input
```
Int[(a + a*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(5/2),x]
```

output
```
(2*a*(a + a*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)) + (
8*a*((8*a^2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*d*Sqrt[a + a*Sec[c + d*x]]
) + (2*a*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])))
/5
```

3.238.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4291
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[-2*a*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*S
qrt[d*Csc[e + f*x]])), x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4296
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-a)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)
*((d*Csc[e + f*x])^n/(f*m)), x] + Simp[b*((2*m - 1)/(d*m))   Int[(a + b*Csc
[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f
, m, n}, x] && EqQ[a^2 - b^2, 0] && EqQ[m + n, 0] && GtQ[m, 1/2] && Integer
Q[2*m]
```

3.238.4 Maple [A] (verified)

Time = 1.41 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.59

method	result	size
default	$\frac{2a^2\sqrt{a(1+\sec(dx+c))}\,(3\sin(dx+c)+14\tan(dx+c)+43\sec(dx+c)\tan(dx+c))}{15d(\cos(dx+c)+1)\sec(dx+c)^{\frac{5}{2}}}$	70

input
```
int((a+a*sec(d*x+c))^(5/2)/sec(d*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/15/d*a^2*(a*(1+sec(d*x+c)))^(1/2)/(cos(d*x+c)+1)/sec(d*x+c)^(5/2)*(3*sin
(d*x+c)+14*tan(d*x+c)+43*sec(d*x+c)*tan(d*x+c))
```

3.238.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.73

$$\int \frac{(a+a\sec(c+dx))^{5/2}}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \frac{2\left(3\,a^2\cos\left(dx+c\right)^3+14\,a^2\cos\left(dx+c\right)^2+43\,a^2\cos\left(dx+c\right)\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\,s}{15\left(d\cos\left(dx+c\right)+d\right)\sqrt{\cos\left(dx+c\right)}}$$

input
```
integrate((a+a*sec(d*x+c))^(5/2)/sec(d*x+c)^(5/2),x, algorithm="fricas")
```

output
```
2/15*(3*a^2*cos(d*x + c)^3 + 14*a^2*cos(d*x + c)^2 + 43*a^2*cos(d*x + c))*
sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/((d*cos(d*x + c) + d)
*sqrt(cos(d*x + c)))
```

3.238.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(c+dx))^{5/2}}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sec(d*x+c))**(5/2)/sec(d*x+c)**(5/2),x)
```

output
```
Timed out
```

3.238.7 Maxima [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.50

$$\int \frac{(a+a\sec(c+dx))^{5/2}}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \frac{\left(3\sqrt{2}a^2\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right)+25\sqrt{2}a^2\sin\left(\frac{3}{2}dx+\frac{3}{2}c\right)+150\sqrt{2}a^2\sin\left(\frac{1}{2}dx+\frac{1}{2}\right.\right.}{30\,d}$$

input
```
integrate((a+a*sec(d*x+c))^(5/2)/sec(d*x+c)^(5/2),x, algorithm="maxima")
```

output
```
1/30*(3*sqrt(2)*a^2*sin(5/2*d*x + 5/2*c) + 25*sqrt(2)*a^2*sin(3/2*d*x + 3/
2*c) + 150*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c))*sqrt(a)/d
```

3.238.8 Giac [F]

$$\int \frac{(a+a\sec(c+dx))^{5/2}}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{(a\sec(dx+c)+a)^{\frac{5}{2}}}{\sec(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))^(5/2)/sec(d*x+c)^(5/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.238.9 Mupad [B] (verification not implemented)

Time = 14.73 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.71

$$\int \frac{(a+a\sec(c+dx))^{5/2}}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \frac{a^2\,\cos(c+d\,x)\,\sqrt{\frac{1}{\cos(c+d\,x)}}\,\sqrt{\frac{a\,(\cos(c+d\,x)+1)}{\cos(c+d\,x)}}\,(175\,\sin(c+d\,x)+28\,\sin(2\,c+2}{30\,d\,(\cos(c+d\,x)+1)}$$

input
```
int((a + a/cos(c + d*x))^(5/2)/(1/cos(c + d*x))^(5/2),x)
```

output
```
(a^2*cos(c + d*x)*(1/cos(c + d*x))^(1/2)*((a*(cos(c + d*x) + 1))/cos(c + d
*x))^(1/2)*(175*sin(c + d*x) + 28*sin(2*c + 2*d*x) + 3*sin(3*c + 3*d*x)))/
(30*d*(cos(c + d*x) + 1))
```

3.239 $\int \frac{(a+a\sec(c+dx))^{5/2}}{\sec^{\frac{7}{2}}(c+dx)}\,dx$

3.239.1 Optimal result 1913
3.239.2 Mathematica [A] (verified) 1913
3.239.3 Rubi [A] (verified) 1914
3.239.4 Maple [A] (verified) 1916
3.239.5 Fricas [A] (verification not implemented) 1916
3.239.6 Sympy [F(-1)] 1917
3.239.7 Maxima [B] (verification not implemented) 1917
3.239.8 Giac [F] 1918
3.239.9 Mupad [B] (verification not implemented) 1918

3.239.1 Optimal result

Integrand size = 25, antiderivative size = 156

$$\int \frac{(a+a\sec(c+dx))^{5/2}}{\sec^{\frac{7}{2}}(c+dx)}\,dx = \frac{64a^3\sqrt{\sec(c+dx)}\sin(c+dx)}{21d\sqrt{a+a\sec(c+dx)}} + \frac{16a^2\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{21d\sqrt{\sec(c+dx)}} + \frac{2a(a+a\sec(c+dx))^{3/2}\sin(c+dx)}{7d\sec^{\frac{3}{2}}(c+dx)} + \frac{2(a+a\sec(c+dx))^{5/2}\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}$$

output
```
2/7*a*(a+a*sec(d*x+c))^(3/2)*sin(d*x+c)/d/sec(d*x+c)^(3/2)+2/7*(a+a*sec(d*
x+c))^(5/2)*sin(d*x+c)/d/sec(d*x+c)^(5/2)+64/21*a^3*sin(d*x+c)*sec(d*x+c)^
(1/2)/d/(a+a*sec(d*x+c))^(1/2)+16/21*a^2*sin(d*x+c)*(a+a*sec(d*x+c))^(1/2)
/d/sec(d*x+c)^(1/2)
```

3.239.2 Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.47

$$\int \frac{(a+a\sec(c+dx))^{5/2}}{\sec^{\frac{7}{2}}(c+dx)}\,dx = \frac{a^2(208+101\cos(c+dx)+24\cos(2(c+dx))+3\cos(3(c+dx)))\sqrt{a(1+\sec}}{42d\sqrt{\sec(c+dx)}}$$

input
```
Integrate[(a + a*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(7/2),x]
```

output
```
(a^2*(208 + 101*Cos[c + d*x] + 24*Cos[2*(c + d*x)] + 3*Cos[3*(c + d*x)])*S
qrt[a*(1 + Sec[c + d*x])]*Tan[(c + d*x)/2])/(42*d*Sqrt[Sec[c + d*x]])
```

3.239.3 Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.06, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3042, 4299, 3042, 4296, 3042, 4296, 3042, 4291}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sec(c+dx)+a)^{5/2}}{\sec^{\frac{7}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx$$

$$\Big\downarrow \text{4299}$$

$$\frac{5}{7}\int \frac{(\sec(c+dx)a+a)^{5/2}}{\sec^{\frac{5}{2}}(c+dx)}dx+\frac{2\sin(c+dx)(a\sec(c+dx)+a)^{5/2}}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5}{7}\int \frac{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{5/2}}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+\frac{2\sin(c+dx)(a\sec(c+dx)+a)^{5/2}}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{4296}$$

$$\frac{5}{7}\left(\frac{8}{5}a\int \frac{(\sec(c+dx)a+a)^{3/2}}{\sec^{\frac{3}{2}}(c+dx)}dx+\frac{2a\sin(c+dx)(a\sec(c+dx)+a)^{3/2}}{5d\sec^{\frac{3}{2}}(c+dx)}\right)+\frac{2\sin(c+dx)(a\sec(c+dx)+a)^{5/2}}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5}{7}\left(\frac{8}{5}a\int \frac{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+\frac{2a\sin(c+dx)(a\sec(c+dx)+a)^{3/2}}{5d\sec^{\frac{3}{2}}(c+dx)}\right)+\frac{2\sin(c+dx)(a\sec(c+dx)+a)^{5/2}}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{4296}$$

$$\frac{5}{7}\left(\frac{8}{5}a\left(\frac{4}{3}a\int\frac{\sqrt{\sec(c+dx)a+a}}{\sqrt{\sec(c+dx)}}dx+\frac{2a\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{3d\sqrt{\sec(c+dx)}}\right)+\frac{2a\sin(c+dx)(a\sec(c+dx)+a)^{3/2}}{5d\sec^{\frac{3}{2}}(c+dx)}\right.$$
$$\frac{2\sin(c+dx)(a\sec(c+dx)+a)^{5/2}}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$\Big\downarrow$ 3042

$$\frac{5}{7}\left(\frac{8}{5}a\left(\frac{4}{3}a\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{3d\sqrt{\sec(c+dx)}}\right)+\frac{2a\sin(c+dx)(a\sec(c+dx)+}{5d\sec^{\frac{3}{2}}(c+dx)}\right.$$
$$\frac{2\sin(c+dx)(a\sec(c+dx)+a)^{5/2}}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$\Big\downarrow$ 4291

$$\frac{5}{7}\left(\frac{8}{5}a\left(\frac{8a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\sqrt{a\sec(c+dx)+a}}+\frac{2a\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{3d\sqrt{\sec(c+dx)}}\right)+\frac{2a\sin(c+dx)(a\sec(c+dx)+a)^{3/}}{5d\sec^{\frac{3}{2}}(c+dx)}\right.$$
$$\frac{2\sin(c+dx)(a\sec(c+dx)+a)^{5/2}}{7d\sec^{\frac{5}{2}}(c+dx)}$$

input
```
Int[(a + a*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(7/2),x]
```

output
```
(2*(a + a*Sec[c + d*x])^(5/2)*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/2)) + (5*
((2*a*(a + a*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)) +
(8*a*((8*a^2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*d*Sqrt[a + a*Sec[c + d*x]
]) + (2*a*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]]))
)/5))/7
```

3.239.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4291
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[-2*a*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*S
qrt[d*Csc[e + f*x]])), x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

3.239. $\int \frac{(a+a\sec(c+dx))^{5/2}}{\sec^{\frac{7}{2}}(c+dx)}\,dx$

rule 4296
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-a)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)
*((d*Csc[e + f*x])^n/(f*m)), x] + Simp[b*((2*m - 1)/(d*m))   Int[(a + b*Csc
[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f
, m, n}, x] && EqQ[a^2 - b^2, 0] && EqQ[m + n, 0] && GtQ[m, 1/2] && Integer
Q[2*m]
```

rule 4299
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-Cot[e + f*x])*(a + b*Csc[e + f*x])^m*((d*Csc
[e + f*x])^n/(f*(m + 1))), x] + Simp[a*(m/(b*d*(m + 1)))   Int[(a + b*Csc[e
 + f*x])^m*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m, n},
 x] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LtQ[m, -2^(-1)]
```

3.239.4 Maple [A] (verified)

Time = 1.49 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.48

method	result	size
default	$\frac{2a^2\left(3\cos(dx+c)^3+12\cos(dx+c)^2+23\cos(dx+c)+46\right)\sqrt{a(1+\sec(dx+c))}\,\tan(dx+c)}{21d(\cos(dx+c)+1)\sec(dx+c)^{\frac{3}{2}}}$	75

input
```
int((a+a*sec(d*x+c))^(5/2)/sec(d*x+c)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
2/21/d*a^2*(3*cos(d*x+c)^3+12*cos(d*x+c)^2+23*cos(d*x+c)+46)*(a*(1+sec(d*x
+c)))^(1/2)/(cos(d*x+c)+1)/sec(d*x+c)^(3/2)*tan(d*x+c)
```

3.239.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.64

$$\int \frac{(a+a\sec(c+dx))^{5/2}}{\sec^{\frac{7}{2}}(c+dx)}\,dx = \frac{2\left(3\,a^2\cos\left(dx+c\right)^4+12\,a^2\cos\left(dx+c\right)^3+23\,a^2\cos\left(dx+c\right)^2+46\,a^2\cos\left(dx\right.\right.}{21\left(d\cos\left(dx+c\right)+d\right)\sqrt{\cos\left(dx+c\right)}}$$

input
```
integrate((a+a*sec(d*x+c))^(5/2)/sec(d*x+c)^(7/2),x, algorithm="fricas")
```

output
```
2/21*(3*a^2*cos(d*x + c)^4 + 12*a^2*cos(d*x + c)^3 + 23*a^2*cos(d*x + c)^2
 + 46*a^2*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x +
c)/((d*cos(d*x + c) + d)*sqrt(cos(d*x + c)))
```

3.239.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(c+dx))^{5/2}}{\sec^{\frac{7}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sec(d*x+c))**(5/2)/sec(d*x+c)**(7/2),x)
```

output
```
Timed out
```

3.239.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 323 vs. $2(132) = 264$.

Time = 0.38 (sec) , antiderivative size = 323, normalized size of antiderivative = 2.07

$$\int \frac{(a+a\sec(c+dx))^{5/2}}{\sec^{\frac{7}{2}}(c+dx)}\,dx = \frac{\sqrt{2}\left(315\,a^2\cos\left(\frac{6}{7}\arctan\left(\sin\left(\frac{7}{2}\,dx+\frac{7}{2}\,c\right),\cos\left(\frac{7}{2}\,dx+\frac{7}{2}\,c\right)\right)\right)\sin\left(\frac{7}{2}\,dx+\frac{7}{2}\,c\right)\right.}{}$$

input
```
integrate((a+a*sec(d*x+c))^(5/2)/sec(d*x+c)^(7/2),x, algorithm="maxima")
```

output
```
1/168*sqrt(2)*(315*a^2*cos(6/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x +
 7/2*c)))*sin(7/2*d*x + 7/2*c) + 77*a^2*cos(4/7*arctan2(sin(7/2*d*x + 7/2*
c), cos(7/2*d*x + 7/2*c)))*sin(7/2*d*x + 7/2*c) + 21*a^2*cos(2/7*arctan2(s
in(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c)))*sin(7/2*d*x + 7/2*c) - 315*a^2
*cos(7/2*d*x + 7/2*c)*sin(6/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x +
7/2*c))) - 77*a^2*cos(7/2*d*x + 7/2*c)*sin(4/7*arctan2(sin(7/2*d*x + 7/2*c
), cos(7/2*d*x + 7/2*c))) - 21*a^2*cos(7/2*d*x + 7/2*c)*sin(2/7*arctan2(si
n(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) + 6*a^2*sin(7/2*d*x + 7/2*c) +
21*a^2*sin(5/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) + 77*a
^2*sin(3/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) + 315*a^2*
sin(1/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))))*sqrt(a)/d
```

3.239.8 Giac [F]

$$\int \frac{(a+a\sec(c+dx))^{5/2}}{\sec^{\frac{7}{2}}(c+dx)}\,dx=\int \frac{(a\sec(dx+c)+a)^{\frac{5}{2}}}{\sec(dx+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))^(5/2)/sec(d*x+c)^(7/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.239.9 Mupad [B] (verification not implemented)

Time = 14.77 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.62

$$\int \frac{(a+a\sec(c+dx))^{5/2}}{\sec^{\frac{7}{2}}(c+dx)}\,dx=\frac{a^2\,\cos(c+d\,x)\,\sqrt{\frac{1}{\cos(c+d\,x)}}\,\sqrt{\frac{a\,(\cos(c+d\,x)+1)}{\cos(c+d\,x)}}\,(392\,\sin(c+d\,x)+98\,\sin(2\,c+2}{84\,d\,(\cos(c+d\,x)+1)}$$

input
```
int((a + a/cos(c + d*x))^(5/2)/(1/cos(c + d*x))^(7/2),x)
```

output
```
(a^2*cos(c + d*x)*(1/cos(c + d*x))^(1/2)*((a*(cos(c + d*x) + 1))/cos(c + d
*x))^(1/2)*(392*sin(c + d*x) + 98*sin(2*c + 2*d*x) + 24*sin(3*c + 3*d*x) +
 3*sin(4*c + 4*d*x)))/(84*d*(cos(c + d*x) + 1))
```

3.240 $\int \frac{(a+a\sec(c+dx))^{5/2}}{\sec^{\frac{9}{2}}(c+dx)}\,dx$

3.240.1 Optimal result 1919
3.240.2 Mathematica [A] (verified) 1920
3.240.3 Rubi [A] (verified) 1920
3.240.4 Maple [A] (verified) 1923
3.240.5 Fricas [A] (verification not implemented) 1924
3.240.6 Sympy [F(-1)] 1924
3.240.7 Maxima [B] (verification not implemented) 1924
3.240.8 Giac [F] 1925
3.240.9 Mupad [B] (verification not implemented) 1925

3.240.1 Optimal result

Integrand size = 25, antiderivative size = 201

$$\int \frac{(a+a\sec(c+dx))^{5/2}}{\sec^{\frac{9}{2}}(c+dx)}\,dx = \frac{38a^3\sin(c+dx)}{63d\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{146a^3\sin(c+dx)}{105d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{584a^3\sin(c+dx)}{315d\sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+dx)}} + \frac{1168a^3\sqrt{\sec(c+dx)}\sin(c+dx)}{315d\sqrt{a+a\sec(c+dx)}} + \frac{2a^2\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)}$$

output
```
38/63*a^3*sin(d*x+c)/d/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(1/2)+146/105*a^3
*sin(d*x+c)/d/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2)+584/315*a^3*sin(d*x+
c)/d/sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2)+1168/315*a^3*sin(d*x+c)*sec(d
*x+c)^(1/2)/d/(a+a*sec(d*x+c))^(1/2)+2/9*a^2*sin(d*x+c)*(a+a*sec(d*x+c))^(
1/2)/d/sec(d*x+c)^(7/2)
```

3.240.2 Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.40

$$\int \frac{(a+a\sec(c+dx))^{5/2}}{\sec^{\frac{9}{2}}(c+dx)}\,dx = \frac{2a^3(35+130\sec(c+dx)+219\sec^2(c+dx)+292\sec^3(c+dx)+584\sec^4(c}{315d\sec^{\frac{7}{2}}(c+dx)\sqrt{a(1+\sec(c+dx))}}$$

input
```
Integrate[(a + a*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(9/2),x]
```

output
```
(2*a^3*(35 + 130*Sec[c + d*x] + 219*Sec[c + d*x]^2 + 292*Sec[c + d*x]^3 +
584*Sec[c + d*x]^4)*Sin[c + d*x])/(315*d*Sec[c + d*x]^(7/2)*Sqrt[a*(1 + Se
c[c + d*x])])
```

3.240.3 Rubi [A] (verified)

Time = 0.98 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.05, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {3042, 4300, 27, 3042, 4503, 3042, 4292, 3042, 4292, 3042, 4291}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sec(c+dx)+a)^{5/2}}{\sec^{\frac{9}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}}{\csc\left(c+dx+\frac{\pi}{2}\right)^{9/2}}dx$$

$$\Big\downarrow \text{4300}$$

$$\frac{2}{9}a\int \frac{\sqrt{\sec(c+dx)a+a}(15\sec(c+dx)a+19a)}{2\sec^{\frac{7}{2}}(c+dx)}dx+\frac{2a^2\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{9d\sec^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{9}a\int \frac{\sqrt{\sec(c+dx)a+a}(15\sec(c+dx)a+19a)}{\sec^{\frac{7}{2}}(c+dx)}dx+\frac{2a^2\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{9d\sec^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{9}a\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}\left(15\csc\left(c+dx+\frac{\pi}{2}\right)a+19a\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx+\frac{2a^2\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{9d\sec^{\frac{7}{2}}(c+dx)}$$

$\Big\downarrow$ 4503

$$\frac{1}{9}a\left(\frac{219}{7}a\int \frac{\sqrt{\sec(c+dx)a+a}}{\sec^{\frac{5}{2}}(c+dx)}dx+\frac{38a^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}\right)+\frac{2a^2\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{9d\sec^{\frac{7}{2}}(c+dx)}$$

$\Big\downarrow$ 3042

$$\frac{1}{9}a\left(\frac{219}{7}a\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+\frac{38a^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}\right)+\frac{2a^2\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{9d\sec^{\frac{7}{2}}(c+dx)}$$

$\Big\downarrow$ 4292

$$\frac{1}{9}a\left(\frac{219}{7}a\left(\frac{4}{5}\int \frac{\sqrt{\sec(c+dx)a+a}}{\sec^{\frac{3}{2}}(c+dx)}dx+\frac{2a\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}\right)+\frac{38a^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)}}\right.$$
$$\frac{2a^2\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{9d\sec^{\frac{7}{2}}(c+dx)}$$

$\Big\downarrow$ 3042

$$\frac{1}{9}a\left(\frac{219}{7}a\left(\frac{4}{5}\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+\frac{2a\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}\right)+\frac{38a^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c}}\right.$$
$$\frac{2a^2\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{9d\sec^{\frac{7}{2}}(c+dx)}$$

$\Big\downarrow$ 4292

$$\frac{1}{9}a\left(\frac{219}{7}a\left(\frac{4}{5}\left(\frac{2}{3}\int \frac{\sqrt{\sec(c+dx)a+a}}{\sqrt{\sec(c+dx)}}dx+\frac{2a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}\right)+\frac{2a\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+}}\right.$$
$$\frac{2a^2\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{9d\sec^{\frac{7}{2}}(c+dx)}$$

$\Big\downarrow$ 3042

3.240. $\int \frac{(a+a\sec(c+dx))^{5/2}}{\sec^{\frac{9}{2}}(c+dx)}\,dx$

$$\frac{1}{9}a\left(\frac{219}{7}a\left(\frac{4}{5}\left(\frac{2}{3}\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}\right)+\frac{2a\sin(c+}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\,\mathrm{se}}}\right.$$

$$\frac{2a^2\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{9d\sec^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \; 4291$$

$$\frac{2a^2\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{9d\sec^{\frac{7}{2}}(c+dx)}+$$

$$\frac{1}{9}a\left(\frac{38a^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{219}{7}a\left(\frac{2a\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{4}{5}\left(\frac{4a\sin(c+dx)\sqrt{\sec(c}}{3d\sqrt{a\sec(c+dx)}}\right.\right.\right.$$

input
```
Int[(a + a*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(9/2),x]
```

output
```
(2*a^2*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(9*d*Sec[c + d*x]^(7/2)) + (
a*((38*a^2*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/2)*Sqrt[a + a*Sec[c + d*x]])
 + (219*a*((2*a*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d
*x]]) + (4*((2*a*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]]*Sqrt[a + a*Sec[c +
d*x]]) + (4*a*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*d*Sqrt[a + a*Sec[c + d*x
]]))))/5))/7))/9
```

3.240.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4291
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[-2*a*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*S
qrt[d*Csc[e + f*x]])), x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4292
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[a*Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*n*Sqrt[a
+ b*Csc[e + f*x]])), x] + Simp[a*((2*n + 1)/(2*b*d*n))   Int[Sqrt[a + b*Csc
[e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
 EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

rule 4300
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)*
((d*Csc[e + f*x])^n/(f*n)), x] - Simp[a/(d*n)   Int[(a + b*Csc[e + f*x])^(m
 - 2)*(d*Csc[e + f*x])^(n + 1)*(b*(m - 2*n - 2) - a*(m + 2*n - 1)*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1]
 && (LtQ[n, -1] || (EqQ[m, 3/2] && EqQ[n, -2^(-1)])) && IntegerQ[2*m]
```

rule 4503
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*b^2*Co
t[e + f*x]*((d*Csc[e + f*x])^n/(a*f*n*Sqrt[a + b*Csc[e + f*x]])), x] + Simp
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n)   Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[
e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a
*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

3.240.4 Maple [A] (verified)

Time = 1.25 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.42

method	result	size
default	$\frac{2a^2\left(35\cos(dx+c)^4+130\cos(dx+c)^3+219\cos(dx+c)^2+292\cos(dx+c)+584\right)\sqrt{a(1+\sec(dx+c))}\,\tan(dx+c)}{315d(\cos(dx+c)+1)\sec(dx+c)^{\frac{3}{2}}}$	85

input
```
int((a+a*sec(d*x+c))^(5/2)/sec(d*x+c)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
2/315/d*a^2*(35*cos(d*x+c)^4+130*cos(d*x+c)^3+219*cos(d*x+c)^2+292*cos(d*x
+c)+584)*(a*(1+sec(d*x+c)))^(1/2)/(cos(d*x+c)+1)/sec(d*x+c)^(3/2)*tan(d*x+
c)
```

3.240.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.56

$$\int \frac{(a+a\sec(c+dx))^{5/2}}{\sec^{\frac{9}{2}}(c+dx)}\,dx = \frac{2\left(35\,a^2\cos\left(dx+c\right)^5+130\,a^2\cos\left(dx+c\right)^4+219\,a^2\cos\left(dx+c\right)^3+292\,a^2\,\mathrm{co}\right.}{315\left(d\cos\left(dx+c\right)+d\right)\sqrt{\mathrm{co}}}$$

input
```
integrate((a+a*sec(d*x+c))^(5/2)/sec(d*x+c)^(9/2),x, algorithm="fricas")
```

output
```
2/315*(35*a^2*cos(d*x + c)^5 + 130*a^2*cos(d*x + c)^4 + 219*a^2*cos(d*x +
c)^3 + 292*a^2*cos(d*x + c)^2 + 584*a^2*cos(d*x + c))*sqrt((a*cos(d*x + c)
 + a)/cos(d*x + c))*sin(d*x + c)/((d*cos(d*x + c) + d)*sqrt(cos(d*x + c)))
```

3.240.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(c+dx))^{5/2}}{\sec^{\frac{9}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sec(d*x+c))**(5/2)/sec(d*x+c)**(9/2),x)
```

output
```
Timed out
```

3.240.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 422 vs. $2(171) = 342$.

Time = 0.39 (sec) , antiderivative size = 422, normalized size of antiderivative = 2.10

$$\int \frac{(a+a\sec(c+dx))^{5/2}}{\sec^{\frac{9}{2}}(c+dx)}\,dx = \frac{\sqrt{2}\left(8190\,a^2\cos\left(\frac{8}{9}\arctan\left(\sin\left(\frac{9}{2}\,dx+\frac{9}{2}\,c\right),\cos\left(\frac{9}{2}\,dx+\frac{9}{2}\,c\right)\right)\right)\sin\left(\frac{9}{2}\,dx+\frac{9}{2}\,c\right.\right.}{}$$

input
```
integrate((a+a*sec(d*x+c))^(5/2)/sec(d*x+c)^(9/2),x, algorithm="maxima")
```

output
```
1/5040*sqrt(2)*(8190*a^2*cos(8/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x
 + 9/2*c)))*sin(9/2*d*x + 9/2*c) + 2100*a^2*cos(2/3*arctan2(sin(9/2*d*x +
9/2*c), cos(9/2*d*x + 9/2*c)))*sin(9/2*d*x + 9/2*c) + 756*a^2*cos(4/9*arct
an2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c)))*sin(9/2*d*x + 9/2*c) + 22
5*a^2*cos(2/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c)))*sin(9/2
*d*x + 9/2*c) - 8190*a^2*cos(9/2*d*x + 9/2*c)*sin(8/9*arctan2(sin(9/2*d*x
+ 9/2*c), cos(9/2*d*x + 9/2*c))) - 2100*a^2*cos(9/2*d*x + 9/2*c)*sin(2/3*a
rctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c))) - 756*a^2*cos(9/2*d*x
+ 9/2*c)*sin(4/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c))) - 22
5*a^2*cos(9/2*d*x + 9/2*c)*sin(2/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d
*x + 9/2*c))) + 70*a^2*sin(9/2*d*x + 9/2*c) + 225*a^2*sin(7/9*arctan2(sin(
9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c))) + 756*a^2*sin(5/9*arctan2(sin(9/2
*d*x + 9/2*c), cos(9/2*d*x + 9/2*c))) + 2100*a^2*sin(1/3*arctan2(sin(9/2*d
*x + 9/2*c), cos(9/2*d*x + 9/2*c))) + 8190*a^2*sin(1/9*arctan2(sin(9/2*d*x
 + 9/2*c), cos(9/2*d*x + 9/2*c))))*sqrt(a)/d
```

3.240.8 Giac [F]

$$\int \frac{(a+a\sec(c+dx))^{5/2}}{\sec^{\frac{9}{2}}(c+dx)}\,dx=\int \frac{(a\sec{(dx+c)}+a)^{\frac{5}{2}}}{\sec{(dx+c)}^{\frac{9}{2}}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))^(5/2)/sec(d*x+c)^(9/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.240.9 Mupad [B] (verification not implemented)

Time = 15.74 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.53

$$\int \frac{(a+a\sec(c+dx))^{5/2}}{\sec^{\frac{9}{2}}(c+dx)}\,dx=\frac{a^2\,\cos{(c+d\,x)}\,\sqrt{\frac{1}{\cos(c+d\,x)}}\,\sqrt{\frac{a\,(\cos(c+d\,x)+1)}{\cos(c+d\,x)}}\,(10290\,\sin{(c+d\,x)}+2856\,\sin{(2}}{2520\,d\,(\cos{(}}$$

input
```
int((a + a/cos(c + d*x))^(5/2)/(1/cos(c + d*x))^(9/2),x)
```

output
```
(a^2*cos(c + d*x)*(1/cos(c + d*x))^(1/2)*((a*(cos(c + d*x) + 1))/cos(c + d
*x))^(1/2)*(10290*sin(c + d*x) + 2856*sin(2*c + 2*d*x) + 981*sin(3*c + 3*d
*x) + 260*sin(4*c + 4*d*x) + 35*sin(5*c + 5*d*x)))/(2520*d*(cos(c + d*x) +
 1))
```

3.240. $\int \frac{(a+a\sec(c+dx))^{5/2}}{\sec^{\frac{9}{2}}(c+dx)}\,dx$

3.241 $\int \frac{(a+a\sec(c+dx))^{5/2}}{\sec^{\frac{11}{2}}(c+dx)}\,dx$

3.241.1 Optimal result 1927
3.241.2 Mathematica [A] (verified) 1928
3.241.3 Rubi [A] (verified) 1928
3.241.4 Maple [A] (verified) 1932
3.241.5 Fricas [A] (verification not implemented) 1932
3.241.6 Sympy [F(-1)] 1932
3.241.7 Maxima [B] (verification not implemented) 1933
3.241.8 Giac [F] 1933
3.241.9 Mupad [B] (verification not implemented) 1934

3.241.1 Optimal result

Integrand size = 25, antiderivative size = 241

$$\begin{aligned}\int \frac{(a+a\sec(c+dx))^{5/2}}{\sec^{\frac{11}{2}}(c+dx)}\,dx &= \frac{46a^3\sin(c+dx)}{99d\sec^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} \\ &+ \frac{710a^3\sin(c+dx)}{693d\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{284a^3\sin(c+dx)}{231d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} \\ &+ \frac{1136a^3\sin(c+dx)}{693d\sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+dx)}} \\ &+ \frac{2272a^3\sqrt{\sec(c+dx)}\sin(c+dx)}{693d\sqrt{a+a\sec(c+dx)}} + \frac{2a^2\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{11d\sec^{\frac{9}{2}}(c+dx)}\end{aligned}$$

output
```
46/99*a^3*sin(d*x+c)/d/sec(d*x+c)^(7/2)/(a+a*sec(d*x+c))^(1/2)+710/693*a^3
*sin(d*x+c)/d/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(1/2)+284/231*a^3*sin(d*x+
c)/d/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2)+1136/693*a^3*sin(d*x+c)/d/sec
(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2)+2272/693*a^3*sin(d*x+c)*sec(d*x+c)^(1
/2)/d/(a+a*sec(d*x+c))^(1/2)+2/11*a^2*sin(d*x+c)*(a+a*sec(d*x+c))^(1/2)/d/
sec(d*x+c)^(9/2)
```

3.241.2 Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.37

$$\int \frac{(a+a\sec(c+dx))^{5/2}}{\sec^{\frac{11}{2}}(c+dx)}\,dx = \frac{2a^3(63+224\sec(c+dx)+355\sec^2(c+dx)+426\sec^3(c+dx)+568\sec^4(c}{693d\sec^{\frac{9}{2}}(c+dx)\sqrt{a(1+\sec(c+dx))}}$$

input
```
Integrate[(a + a*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(11/2),x]
```

output
```
(2*a^3*(63 + 224*Sec[c + d*x] + 355*Sec[c + d*x]^2 + 426*Sec[c + d*x]^3 +
568*Sec[c + d*x]^4 + 1136*Sec[c + d*x]^5)*Sin[c + d*x])/(693*d*Sec[c + d*x
]^(9/2)*Sqrt[a*(1 + Sec[c + d*x])])
```

3.241.3 Rubi [A] (verified)

Time = 1.22 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.06, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.520$, Rules used = {3042, 4300, 27, 3042, 4503, 3042, 4292, 3042, 4292, 3042, 4292, 3042, 4291}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sec(c+dx)+a)^{5/2}}{\sec^{\frac{11}{2}}(c+dx)}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}}{\csc\left(c+dx+\frac{\pi}{2}\right)^{11/2}}dx$$

$$\Big\downarrow \; 4300$$

$$\frac{2}{11}a\int \frac{\sqrt{\sec(c+dx)a+a}(19\sec(c+dx)a+23a)}{2\sec^{\frac{9}{2}}(c+dx)}dx+\frac{2a^2\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{11d\sec^{\frac{9}{2}}(c+dx)}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{11}a\int \frac{\sqrt{\sec(c+dx)a+a}(19\sec(c+dx)a+23a)}{\sec^{\frac{9}{2}}(c+dx)}dx+\frac{2a^2\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{11d\sec^{\frac{9}{2}}(c+dx)}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{11}a\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}\left(19\csc\left(c+dx+\frac{\pi}{2}\right)a+23a\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{9/2}}dx+\frac{2a^2\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{11d\sec^{\frac{9}{2}}(c+dx)}$$

$\Big\downarrow$ 4503

$$\frac{1}{11}a\left(\frac{355}{9}a\int \frac{\sqrt{\sec(c+dx)a+a}}{\sec^{\frac{7}{2}}(c+dx)}dx+\frac{46a^2\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}\right)+\frac{2a^2\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{11d\sec^{\frac{9}{2}}(c+dx)}$$

$\Big\downarrow$ 3042

$$\frac{1}{11}a\left(\frac{355}{9}a\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx+\frac{46a^2\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}\right)+\frac{2a^2\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{11d\sec^{\frac{9}{2}}(c+dx)}$$

$\Big\downarrow$ 4292

$$\frac{1}{11}a\left(\frac{355}{9}a\left(\frac{6}{7}\int \frac{\sqrt{\sec(c+dx)a+a}}{\sec^{\frac{5}{2}}(c+dx)}dx+\frac{2a\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}\right)+\frac{46a^2\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)\sqrt{a\sec(c+dx)}}\right.$$
$$\frac{2a^2\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{11d\sec^{\frac{9}{2}}(c+dx)}$$

$\Big\downarrow$ 3042

$$\frac{1}{11}a\left(\frac{355}{9}a\left(\frac{6}{7}\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+\frac{2a\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}\right)+\frac{46a^2\sin(c+dx}{9d\sec^{\frac{7}{2}}(c+dx)\sqrt{a\sec(c}}\right.$$
$$\frac{2a^2\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{11d\sec^{\frac{9}{2}}(c+dx)}$$

$\Big\downarrow$ 4292

$$\frac{1}{11}a\left(\frac{355}{9}a\left(\frac{6}{7}\left(\frac{4}{5}\int \frac{\sqrt{\sec(c+dx)a+a}}{\sec^{\frac{3}{2}}(c+dx)}dx+\frac{2a\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}\right)+\frac{2a\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+}}\right.$$
$$\frac{2a^2\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{11d\sec^{\frac{9}{2}}(c+dx)}$$

$\Big\downarrow$ 3042

3.241. $\int \frac{(a+a\sec(c+dx))^{5/2}}{\sec^{\frac{11}{2}}(c+dx)}\,dx$

$$\frac{1}{11}a\left(\frac{355}{9}a\left(\frac{6}{7}\left(\frac{4}{5}\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+\frac{2a\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}\right)+\frac{2a\sin(c+}{7d\sec^{\frac{5}{2}}(c+dx)\sqrt{a\,\mathrm{s}}}\right.\right.$$

$$\frac{2a^2\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{11d\sec^{\frac{9}{2}}(c+dx)}$$

$$\Big\downarrow \; 4292$$

$$\frac{1}{11}a\left(\frac{355}{9}a\left(\frac{6}{7}\left(\frac{4}{5}\left(\frac{2}{3}\int\frac{\sqrt{\sec(c+dx)a+a}}{\sqrt{\sec(c+dx)}}dx+\frac{2a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}\right)+\frac{2a\sin(c+d}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec}}\right.\right.\right.$$

$$\frac{2a^2\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{11d\sec^{\frac{9}{2}}(c+dx)}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{11}a\left(\frac{355}{9}a\left(\frac{6}{7}\left(\frac{4}{5}\left(\frac{2}{3}\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}\right)+\frac{2a\sin}{5d\sec^{\frac{3}{2}}(c+dx)}\right.\right.\right.$$

$$\frac{2a^2\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{11d\sec^{\frac{9}{2}}(c+dx)}$$

$$\Big\downarrow \; 4291$$

$$\frac{2a^2\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{11d\sec^{\frac{9}{2}}(c+dx)}+$$

$$\frac{1}{11}a\left(\frac{46a^2\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{355}{9}a\left(\frac{2a\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{6}{7}\left(\frac{2a\sin(c+}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\,\mathrm{s}}}\right.\right.\right.$$

input
```
Int[(a + a*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(11/2),x]
```

output
```
(2*a^2*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(11*d*Sec[c + d*x]^(9/2)) +
(a*((46*a^2*Sin[c + d*x])/(9*d*Sec[c + d*x]^(7/2)*Sqrt[a + a*Sec[c + d*x]]
) + (355*a*((2*a*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/2)*Sqrt[a + a*Sec[c +
d*x]]) + (6*((2*a*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)*Sqrt[a + a*Sec[c +
 d*x]]) + (4*((2*a*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]]*Sqrt[a + a*Sec[c
+ d*x]]) + (4*a*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*d*Sqrt[a + a*Sec[c + d
*x]])))/5))/7))/9))/11
```

3.241. $\int \frac{(a+a\sec(c+dx))^{5/2}}{\sec^{\frac{11}{2}}(c+dx)}\,dx$

3.241.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4291
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[-2*a*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*S
qrt[d*Csc[e + f*x]])), x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4292
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[a*Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*n*Sqrt[a
+ b*Csc[e + f*x]])), x] + Simp[a*((2*n + 1)/(2*b*d*n))   Int[Sqrt[a + b*Csc
[e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
 EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

rule 4300
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)*
((d*Csc[e + f*x])^n/(f*n)), x] - Simp[a/(d*n)   Int[(a + b*Csc[e + f*x])^(m
 - 2)*(d*Csc[e + f*x])^(n + 1)*(b*(m - 2*n - 2) - a*(m + 2*n - 1)*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1]
 && (LtQ[n, -1] || (EqQ[m, 3/2] && EqQ[n, -2^(-1)])) && IntegerQ[2*m]
```

rule 4503
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*b^2*Co
t[e + f*x]*((d*Csc[e + f*x])^n/(a*f*n*Sqrt[a + b*Csc[e + f*x]])), x] + Simp
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n)   Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[
e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a
*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

3.241. $\int \frac{(a+a\sec(c+dx))^{5/2}}{\sec^{\frac{11}{2}}(c+dx)}\,dx$

3.241.4 Maple [A] (verified)

Time = 1.19 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.39

method	result	siz
default	$\frac{2a^2\left(63\cos(dx+c)^5+224\cos(dx+c)^4+355\cos(dx+c)^3+426\cos(dx+c)^2+568\cos(dx+c)+1136\right)\sqrt{a(1+\sec(dx+c))}\,\tan(dx+c)}{693d(\cos(dx+c)+1)\sec(dx+c)^{\frac{3}{2}}}$	95

input
```
int((a+a*sec(d*x+c))^(5/2)/sec(d*x+c)^(11/2),x,method=_RETURNVERBOSE)
```

output
```
2/693/d*a^2*(63*cos(d*x+c)^5+224*cos(d*x+c)^4+355*cos(d*x+c)^3+426*cos(d*x
+c)^2+568*cos(d*x+c)+1136)*(a*(1+sec(d*x+c)))^(1/2)/(cos(d*x+c)+1)/sec(d*x
+c)^(3/2)*tan(d*x+c)
```

3.241.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.52

$$\int \frac{(a+a\sec(c+dx))^{5/2}}{\sec^{\frac{11}{2}}(c+dx)}\,dx = \frac{2\left(63\,a^2\cos\left(dx+c\right)^6+224\,a^2\cos\left(dx+c\right)^5+355\,a^2\cos\left(dx+c\right)^4+426\,a^2\,\mathrm{co}\right.}{693\left(d\cos\left(dx+\right.\right.}$$

input
```
integrate((a+a*sec(d*x+c))^(5/2)/sec(d*x+c)^(11/2),x, algorithm="fricas")
```

output
```
2/693*(63*a^2*cos(d*x + c)^6 + 224*a^2*cos(d*x + c)^5 + 355*a^2*cos(d*x +
c)^4 + 426*a^2*cos(d*x + c)^3 + 568*a^2*cos(d*x + c)^2 + 1136*a^2*cos(d*x
+ c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/((d*cos(d*x + c
) + d)*sqrt(cos(d*x + c)))
```

3.241.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(c+dx))^{5/2}}{\sec^{\frac{11}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sec(d*x+c))**(5/2)/sec(d*x+c)**(11/2),x)
```

output
```
Timed out
```

3.241.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 521 vs. $2(205) = 410$.

Time = 0.40 (sec) , antiderivative size = 521, normalized size of antiderivative = 2.16

$$\int \frac{(a + a\sec(c + dx))^{5/2}}{\sec^{\frac{11}{2}}(c + dx)}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sec(d*x+c))^(5/2)/sec(d*x+c)^(11/2),x, algorithm="maxima")
```

output
```
1/22176*sqrt(2)*(31878*a^2*cos(10/11*arctan2(sin(11/2*d*x + 11/2*c), cos(1
1/2*d*x + 11/2*c)))*sin(11/2*d*x + 11/2*c) + 8778*a^2*cos(8/11*arctan2(sin
(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c)))*sin(11/2*d*x + 11/2*c) + 346
5*a^2*cos(6/11*arctan2(sin(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c)))*si
n(11/2*d*x + 11/2*c) + 1287*a^2*cos(4/11*arctan2(sin(11/2*d*x + 11/2*c), c
os(11/2*d*x + 11/2*c)))*sin(11/2*d*x + 11/2*c) + 385*a^2*cos(2/11*arctan2(
sin(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c)))*sin(11/2*d*x + 11/2*c) -
31878*a^2*cos(11/2*d*x + 11/2*c)*sin(10/11*arctan2(sin(11/2*d*x + 11/2*c),
 cos(11/2*d*x + 11/2*c))) - 8778*a^2*cos(11/2*d*x + 11/2*c)*sin(8/11*arcta
n2(sin(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c))) - 3465*a^2*cos(11/2*d*
x + 11/2*c)*sin(6/11*arctan2(sin(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c
))) - 1287*a^2*cos(11/2*d*x + 11/2*c)*sin(4/11*arctan2(sin(11/2*d*x + 11/2
*c), cos(11/2*d*x + 11/2*c))) - 385*a^2*cos(11/2*d*x + 11/2*c)*sin(2/11*ar
ctan2(sin(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c))) + 126*a^2*sin(11/2*
d*x + 11/2*c) + 385*a^2*sin(9/11*arctan2(sin(11/2*d*x + 11/2*c), cos(11/2*
d*x + 11/2*c))) + 1287*a^2*sin(7/11*arctan2(sin(11/2*d*x + 11/2*c), cos(11
/2*d*x + 11/2*c))) + 3465*a^2*sin(5/11*arctan2(sin(11/2*d*x + 11/2*c), cos
(11/2*d*x + 11/2*c))) + 8778*a^2*sin(3/11*arctan2(sin(11/2*d*x + 11/2*c),
cos(11/2*d*x + 11/2*c))) + 31878*a^2*sin(1/11*arctan2(sin(11/2*d*x + 11/2*
c), cos(11/2*d*x + 11/2*c))))*sqrt(a)/d
```

3.241.8 Giac [F]

$$\int \frac{(a + a\sec(c + dx))^{5/2}}{\sec^{\frac{11}{2}}(c + dx)}\,dx = \int \frac{(a\sec(dx + c) + a)^{\frac{5}{2}}}{\sec(dx + c)^{\frac{11}{2}}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))^(5/2)/sec(d*x+c)^(11/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.241.9 Mupad [B] (verification not implemented)

Time = 19.51 (sec) , antiderivative size = 356, normalized size of antiderivative = 1.48

$$\int \frac{(a+a\sec(c+dx))^{5/2}}{\sec^{\frac{11}{2}}(c+dx)}\,dx = \frac{\sqrt{a-\frac{a}{2\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1}}\left(2\sin\left(\frac{11\,c}{4}+\frac{11\,d\,x}{4}\right)^2+\sin\left(\frac{11\,c}{2}+\frac{11\,d\,x}{2}\right)\,1\mathrm{i}-1\right)\left(\frac{23\,a^2\,\mathrm{si}}{}\right.}{}$$

input
```
int((a + a/cos(c + d*x))^(5/2)/(1/cos(c + d*x))^(11/2),x)
```

output
```
((a - a/(2*sin(c/2 + (d*x)/2)^2 - 1))^(1/2)*(sin((11*c)/2 + (11*d*x)/2)*1i
 + 2*sin((11*c)/4 + (11*d*x)/4)^2 - 1)*((23*a^2*sin(c/2 + (d*x)/2)*(sin((1
1*c)/2 + (11*d*x)/2)*1i - 2*sin((11*c)/4 + (11*d*x)/4)^2 + 1))/(4*d) + (19
*a^2*sin((3*c)/2 + (3*d*x)/2)*(sin((11*c)/2 + (11*d*x)/2)*1i - 2*sin((11*c
)/4 + (11*d*x)/4)^2 + 1))/(12*d) + (5*a^2*sin((5*c)/2 + (5*d*x)/2)*(sin((1
1*c)/2 + (11*d*x)/2)*1i - 2*sin((11*c)/4 + (11*d*x)/4)^2 + 1))/(8*d) + (13
*a^2*sin((7*c)/2 + (7*d*x)/2)*(sin((11*c)/2 + (11*d*x)/2)*1i - 2*sin((11*c
)/4 + (11*d*x)/4)^2 + 1))/(56*d) + (5*a^2*sin((9*c)/2 + (9*d*x)/2)*(sin((1
1*c)/2 + (11*d*x)/2)*1i - 2*sin((11*c)/4 + (11*d*x)/4)^2 + 1))/(72*d) + (a
^2*sin((11*c)/2 + (11*d*x)/2)*(sin((11*c)/2 + (11*d*x)/2)*1i - 2*sin((11*c
)/4 + (11*d*x)/4)^2 + 1))/(88*d)))/(2*(-1/(2*sin(c/2 + (d*x)/2)^2 - 1))^(1
/2)*(2*sin(c/4 + (d*x)/4)^2 - 1))
```

3.242 $\int \frac{(a+a\sec(c+dx))^{3/2}}{\sqrt[4]{\sec(c+dx)}}\,dx$

3.242.1 Optimal result . . . 1935
3.242.2 Mathematica [A] (verified) . . . 1935
3.242.3 Rubi [A] (verified) . . . 1936
3.242.4 Maple [F] . . . 1937
3.242.5 Fricas [A] (verification not implemented) . . . 1937
3.242.6 Sympy [F] . . . 1938
3.242.7 Maxima [B] (verification not implemented) . . . 1938
3.242.8 Giac [F] . . . 1938
3.242.9 Mupad [B] (verification not implemented) . . . 1939

3.242.1 Optimal result

Integrand size = 25, antiderivative size = 38

$$\int \frac{(a+a\sec(c+dx))^{3/2}}{\sqrt[4]{\sec(c+dx)}}\,dx=\frac{4a^2\sec^{\frac{3}{4}}(c+dx)\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}}$$

output
```
4*a^2*sec(d*x+c)^(3/4)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)
```

3.242.2 Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.34

$$\int \frac{(a+a\sec(c+dx))^{3/2}}{\sqrt[4]{\sec(c+dx)}}\,dx=\frac{2\sec^2\left(\frac{1}{2}(c+dx)\right)(a(1+\sec(c+dx)))^{3/2}\tan\left(\frac{1}{2}(c+dx)\right)}{d\sec^{\frac{5}{4}}(c+dx)}$$

input
```
Integrate[(a + a*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(1/4),x]
```

output
```
(2*Sec[(c + d*x)/2]^2*(a*(1 + Sec[c + d*x]))^(3/2)*Tan[(c + d*x)/2])/(d*Se
c[c + d*x]^(5/4))
```

3.242.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {3042, 4301, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sec(c+dx)+a)^{3/2}}{\sqrt[4]{\sec(c+dx)}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}}{\sqrt[4]{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{ 4301}$$

$$4a\int 0dx+\frac{4a^2\sin(c+dx)\sec^{\frac{3}{4}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{ 24}$$

$$\frac{4a^2\sin(c+dx)\sec^{\frac{3}{4}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}$$

input
```
Int[(a + a*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(1/4),x]
```

output
```
(4*a^2*Sec[c + d*x]^(3/4)*Sin[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]])
```

3.242.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4301
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
2)*((d*Csc[e + f*x])^n/(f*(m + n - 1))), x] + Simp[b/(m + n - 1)   Int[(a +
 b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*(b*(m + 2*n - 1) + a*(3*m + 2*n
 - 4)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^
2, 0] && GtQ[m, 1] && NeQ[m + n - 1, 0] && IntegerQ[2*m]
```

3.242.4 Maple [F]

$$\int \frac{(a+a\sec(dx+c))^{\frac{3}{2}}}{\sec(dx+c)^{\frac{1}{4}}}dx$$

input
```
int((a+a*sec(d*x+c))^(3/2)/sec(d*x+c)^(1/4),x)
```

output
```
int((a+a*sec(d*x+c))^(3/2)/sec(d*x+c)^(1/4),x)
```

3.242.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.32

$$\int \frac{(a+a\sec(c+dx))^{3/2}}{\sqrt[4]{\sec(c+dx)}}\,dx = \frac{4\,a\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)^{\frac{1}{4}}\sin(dx+c)}{d\cos(dx+c)+d}$$

input
```
integrate((a+a*sec(d*x+c))^(3/2)/sec(d*x+c)^(1/4),x, algorithm="fricas")
```

output
```
4*a*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)^(1/4)*sin(d*x + c
)/(d*cos(d*x + c) + d)
```

3.242.6 Sympy [F]

$$\int \frac{(a+a\sec(c+dx))^{3/2}}{\sqrt[4]{\sec(c+dx)}}\,dx=\int \frac{(a(\sec(c+dx)+1))^{\frac{3}{2}}}{\sqrt[4]{\sec(c+dx)}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))**(3/2)/sec(d*x+c)**(1/4),x)
```

output
```
Integral((a*(sec(c + d*x) + 1))**(3/2)/sec(c + d*x)**(1/4), x)
```

3.242.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 121 vs. $2(34) = 68$.

Time = 0.30 (sec) , antiderivative size = 121, normalized size of antiderivative = 3.18

$$\int \frac{(a+a\sec(c+dx))^{3/2}}{\sqrt[4]{\sec(c+dx)}}\,dx=\frac{4\left(\frac{\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{d\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{5}{4}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{5}{4}}\left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+1\right)^{\frac{1}{4}}}$$

input
```
integrate((a+a*sec(d*x+c))^(3/2)/sec(d*x+c)^(1/4),x, algorithm="maxima")
```

output
```
4*(sqrt(2)*a^(3/2)*sin(d*x + c)/(cos(d*x + c) + 1) - sqrt(2)*a^(3/2)*sin(d
*x + c)^3/(cos(d*x + c) + 1)^3)/(d*(sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(
5/4)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(5/4)*(sin(d*x + c)^2/(cos(d*x
 + c) + 1)^2 + 1)^(1/4))
```

3.242.8 Giac [F]

$$\int \frac{(a+a\sec(c+dx))^{3/2}}{\sqrt[4]{\sec(c+dx)}}\,dx=\int \frac{(a\sec(dx+c)+a)^{\frac{3}{2}}}{\sec(dx+c)^{\frac{1}{4}}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))^(3/2)/sec(d*x+c)^(1/4),x, algorithm="giac")
```

output
```
sage0*x
```

3.242.9 Mupad [B] (verification not implemented)

Time = 0.76 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.45

$$\int \frac{(a+a\sec(c+dx))^{3/2}}{\sqrt[4]{\sec(c+dx)}}\,dx=\frac{2\,a\,\sin\left(2\,c+2\,d\,x\right)\,\left(\frac{1}{\cos\left(c+d\,x\right)}\right)^{3/4}\,\sqrt{\frac{a\,\left(\cos\left(c+d\,x\right)+1\right)}{\cos\left(c+d\,x\right)}}}{d\,\left(\cos\left(c+d\,x\right)+1\right)}$$

input
```
int((a + a/cos(c + d*x))^(3/2)/(1/cos(c + d*x))^(1/4),x)
```

output
```
(2*a*sin(2*c + 2*d*x)*(1/cos(c + d*x))^(3/4)*((a*(cos(c + d*x) + 1))/cos(c
 + d*x))^(1/2))/(d*(cos(c + d*x) + 1))
```

3.243 $\int \sqrt{\sec(e+fx)}\sqrt{a+a\sec(e+fx)}\,dx$

3.243.1 Optimal result 1940
3.243.2 Mathematica [A] (verified) 1940
3.243.3 Rubi [A] (verified) 1941
3.243.4 Maple [B] (verified) 1942
3.243.5 Fricas [B] (verification not implemented) 1942
3.243.6 Sympy [F] 1943
3.243.7 Maxima [B] (verification not implemented) 1943
3.243.8 Giac [F] 1944
3.243.9 Mupad [F(-1)] 1944

3.243.1 Optimal result

Integrand size = 25, antiderivative size = 37

$$\int \sqrt{\sec(e+fx)}\sqrt{a+a\sec(e+fx)}\,dx=\frac{2\sqrt{a}\operatorname{arcsinh}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f}$$

output
```
2*arcsinh(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))*a^(1/2)/f
```

3.243.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.46

$$\int \sqrt{\sec(e+fx)}\sqrt{a+a\sec(e+fx)}\,dx=-\frac{2\arcsin\left(\sqrt{\sec(e+fx)}\right)\sqrt{a(1+\sec(e+fx))}\tan\left(\frac{1}{2}(e+fx)\right)}{f\sqrt{1-\sec(e+fx)}}$$

input
```
Integrate[Sqrt[Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]],x]
```

output
```
(-2*ArcSin[Sqrt[Sec[e + f*x]]]*Sqrt[a*(1 + Sec[e + f*x])]*Tan[(e + f*x)/2]
)/(f*Sqrt[1 - Sec[e + f*x]])
```

3.243.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {3042, 4288, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\sec(e+fx)}\sqrt{a\sec(e+fx)+a}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)}\sqrt{a\csc\left(e+fx+\frac{\pi}{2}\right)+a}dx$$

$$\Big\downarrow \text{4288}$$

$$-\frac{2\int \frac{1}{\sqrt{\frac{a\tan^2(e+fx)}{\sec(e+fx)a+a}+1}}d\left(-\frac{a\tan(e+fx)}{\sqrt{\sec(e+fx)a+a}}\right)}{f}$$

$$\Big\downarrow \text{222}$$

$$\frac{2\sqrt{a}\text{arcsinh}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{f}$$

input
```
Int[Sqrt[Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]],x]
```

output
```
(2*Sqrt[a]*ArcSinh[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/f
```

3.243.3.1 Defintions of rubi rules used

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4288
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[-2*(a/(b*f))*Sqrt[a*(d/b)]   Subst[Int[1/Sqrt[1
+ x^2/a], x], x, b*(Cot[e + f*x]/Sqrt[a + b*Csc[e + f*x]])], x] /; FreeQ[{a
, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[a*(d/b), 0]
```

3.243.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 140 vs. $2(31) = 62$.

Time = 1.82 (sec) , antiderivative size = 141, normalized size of antiderivative = 3.81

method	result
default	$-\frac{\sqrt{\sec(fx+e)}\,\sqrt{a(\sec(fx+e)+1)}\left(\arctan\left(\frac{-\cos(fx+e)+\sin(fx+e)-1}{2(\cos(fx+e)+1)\sqrt{-\frac{1}{\cos(fx+e)+1}}}\right)+\arctan\left(\frac{\cos(fx+e)+\sin(fx+e)+1}{2(\cos(fx+e)+1)\sqrt{-\frac{1}{\cos(fx+e)+1}}}\right)\right)\cos(fx}{f(\cos(fx+e)+1)\sqrt{-\frac{1}{\cos(fx+e)+1}}}$

input
```
int(sec(f*x+e)^(1/2)*(a+a*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/f*sec(f*x+e)^(1/2)*(a*(sec(f*x+e)+1))^(1/2)*(arctan(1/2*(-cos(f*x+e)+si
n(f*x+e)-1)/(cos(f*x+e)+1)/(-1/(cos(f*x+e)+1))^(1/2))+arctan(1/2*(cos(f*x+
e)+sin(f*x+e)+1)/(cos(f*x+e)+1)/(-1/(cos(f*x+e)+1))^(1/2)))*cos(f*x+e)/(co
s(f*x+e)+1)/(-1/(cos(f*x+e)+1))^(1/2)
```

3.243.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 77 vs. $2(31) = 62$.

Time = 0.28 (sec) , antiderivative size = 189, normalized size of antiderivative = 5.11

$$\int \sqrt{\sec(e+fx)}\sqrt{a+a\sec(e+fx)}\,dx$$
$$= \left[\frac{\sqrt{a}\log\left(\frac{a\cos(fx+e)^3-7\,a\cos(fx+e)^2-\frac{4\left(\cos(fx+e)^2-2\,\cos(fx+e)\right)\sqrt{a}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sin(fx+e)}{\sqrt{\cos(fx+e)}}+8\,a}{\cos(fx+e)^3+\cos(fx+e)^2}\right)}{2\,f}, \frac{\sqrt{-a}\arctan\left(\frac{2\sqrt{-a}\sqrt{\frac{a\,\mathrm{co}}{\mathrm{c}}}}{a\cos}\right.}{}\right.$$

input
```
integrate(sec(f*x+e)^(1/2)*(a+a*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
[1/2*sqrt(a)*log((a*cos(f*x + e)^3 - 7*a*cos(f*x + e)^2 - 4*(cos(f*x + e)^
2 - 2*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*
x + e)/sqrt(cos(f*x + e)) + 8*a)/(cos(f*x + e)^3 + cos(f*x + e)^2))/f, sqr
t(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt(cos(f
*x + e))*sin(f*x + e)/(a*cos(f*x + e)^2 - a*cos(f*x + e) - 2*a))/f]
```

3.243.6 Sympy [F]

$$\int \sqrt{\sec(e+fx)}\sqrt{a+a\sec(e+fx)}\,dx = \int \sqrt{a\left(\sec\left(e+fx\right)+1\right)}\sqrt{\sec\left(e+fx\right)}\,dx$$

input
```
integrate(sec(f*x+e)**(1/2)*(a+a*sec(f*x+e))**(1/2),x)
```

output
```
Integral(sqrt(a*(sec(e + f*x) + 1))*sqrt(sec(e + f*x)), x)
```

3.243.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 241 vs. $2(31) = 62$.

Time = 0.37 (sec) , antiderivative size = 241, normalized size of antiderivative = 6.51

$$\int \sqrt{\sec(e+fx)}\sqrt{a+a\sec(e+fx)}\,dx$$
$$= \frac{\sqrt{a}\left(\log\left(2\cos\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+2\sin\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+2\sqrt{2}\cos\left(\frac{1}{2}fx+\frac{1}{2}e\right)+2\sqrt{2}\sin\left(\frac{1}{2}fx+\frac{1}{2}e\right)+2\right)-}{}$$

input
```
integrate(sec(f*x+e)^(1/2)*(a+a*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
1/2*sqrt(a)*(log(2*cos(1/2*f*x + 1/2*e)^2 + 2*sin(1/2*f*x + 1/2*e)^2 + 2*s
qrt(2)*cos(1/2*f*x + 1/2*e) + 2*sqrt(2)*sin(1/2*f*x + 1/2*e) + 2) - log(2*
cos(1/2*f*x + 1/2*e)^2 + 2*sin(1/2*f*x + 1/2*e)^2 + 2*sqrt(2)*cos(1/2*f*x
+ 1/2*e) - 2*sqrt(2)*sin(1/2*f*x + 1/2*e) + 2) + log(2*cos(1/2*f*x + 1/2*e
)^2 + 2*sin(1/2*f*x + 1/2*e)^2 - 2*sqrt(2)*cos(1/2*f*x + 1/2*e) + 2*sqrt(2
)*sin(1/2*f*x + 1/2*e) + 2) - log(2*cos(1/2*f*x + 1/2*e)^2 + 2*sin(1/2*f*x
 + 1/2*e)^2 - 2*sqrt(2)*cos(1/2*f*x + 1/2*e) - 2*sqrt(2)*sin(1/2*f*x + 1/2
*e) + 2))/f
```

3.243.8 Giac [F]

$$\int \sqrt{\sec(e+fx)}\sqrt{a+a\sec(e+fx)}\,dx = \int \sqrt{a\sec(fx+e)+a}\sqrt{\sec(fx+e)}\,dx$$

input
```
integrate(sec(f*x+e)^(1/2)*(a+a*sec(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.243.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{\sec(e+fx)}\sqrt{a+a\sec(e+fx)}\,dx = \int \sqrt{a+\frac{a}{\cos(e+f\,x)}}\,\sqrt{\frac{1}{\cos(e+f\,x)}}\,dx$$

input
```
int((a + a/cos(e + f*x))^(1/2)*(1/cos(e + f*x))^(1/2),x)
```

output
```
int((a + a/cos(e + f*x))^(1/2)*(1/cos(e + f*x))^(1/2), x)
```

3.244 $\int \sqrt{-\sec(e+fx)}\sqrt{a-a\sec(e+fx)}\,dx$

3.244.1 Optimal result 1945
3.244.2 Mathematica [A] (verified) 1945
3.244.3 Rubi [A] (verified) 1946
3.244.4 Maple [B] (verified) 1947
3.244.5 Fricas [B] (verification not implemented) 1947
3.244.6 Sympy [F] 1948
3.244.7 Maxima [B] (verification not implemented) 1948
3.244.8 Giac [B] (verification not implemented) 1949
3.244.9 Mupad [F(-1)] 1950

3.244.1 Optimal result

Integrand size = 28, antiderivative size = 38

$$\int \sqrt{-\sec(e+fx)}\sqrt{a-a\sec(e+fx)}\,dx=\frac{2\sqrt{a}\operatorname{arcsinh}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a-a\sec(e+fx)}}\right)}{f}$$

output
```
2*arcsinh(a^(1/2)*tan(f*x+e)/(a-a*sec(f*x+e))^(1/2))*a^(1/2)/f
```

3.244.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.45

$$\int \sqrt{-\sec(e+fx)}\sqrt{a-a\sec(e+fx)}\,dx=\frac{2\arcsin\left(\sqrt{-\sec(e+fx)}\right)\cot\left(\frac{1}{2}(e+fx)\right)\sqrt{a-a\sec(e+fx)}}{f\sqrt{1+\sec(e+fx)}}$$

input
```
Integrate[Sqrt[-Sec[e + f*x]]*Sqrt[a - a*Sec[e + f*x]],x]
```

output
```
(2*ArcSin[Sqrt[-Sec[e + f*x]]]*Cot[(e + f*x)/2]*Sqrt[a - a*Sec[e + f*x]])/
(f*Sqrt[1 + Sec[e + f*x]])
```

3.244.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {3042, 4288, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{-\sec(e+fx)}\sqrt{a-a\sec(e+fx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{-\csc\left(e+fx+\frac{\pi}{2}\right)}\sqrt{a-a\csc\left(e+fx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{4288}$$

$$\frac{2\int \frac{1}{\sqrt{\frac{a\tan^2(e+fx)}{a-a\sec(e+fx)}+1}}d\frac{a\tan(e+fx)}{\sqrt{a-a\sec(e+fx)}}}{f}$$

$$\Big\downarrow \text{222}$$

$$\frac{2\sqrt{a}\text{arcsinh}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a-a\sec(e+fx)}}\right)}{f}$$

input
```
Int[Sqrt[-Sec[e + f*x]]*Sqrt[a - a*Sec[e + f*x]],x]
```

output
```
(2*Sqrt[a]*ArcSinh[(Sqrt[a]*Tan[e + f*x])/Sqrt[a - a*Sec[e + f*x]]])/f
```

3.244.3.1 Defintions of rubi rules used

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4288
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[-2*(a/(b*f))*Sqrt[a*(d/b)]   Subst[Int[1/Sqrt[1
+ x^2/a], x], x, b*(Cot[e + f*x]/Sqrt[a + b*Csc[e + f*x]])], x] /; FreeQ[{a
, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[a*(d/b), 0]
```

3.244.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 129 vs. $2(32) = 64$.

Time = 1.72 (sec) , antiderivative size = 130, normalized size of antiderivative = 3.42

method	result
default	$-\frac{\sqrt{-a(\sec(fx+e)-1)}\,\sqrt{-\sec(fx+e)}\left(\operatorname{arctanh}\left(\frac{-\cos(fx+e)+\sin(fx+e)-1}{2(\cos(fx+e)+1)\sqrt{\frac{1}{\cos(fx+e)+1}}}\right)-\operatorname{arctanh}\left(\frac{\cos(fx+e)+\sin(fx+e)+1}{2(\cos(fx+e)+1)\sqrt{\frac{1}{\cos(fx+e)+1}}}\right)\right)\cot(}{f\sqrt{\frac{1}{\cos(fx+e)+1}}}$

input
```
int((-sec(f*x+e))^(1/2)*(a-a*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/f*(-a*(sec(f*x+e)-1))^(1/2)*(-sec(f*x+e))^(1/2)*(arctanh(1/2*(-cos(f*x+
e)+sin(f*x+e)-1)/(cos(f*x+e)+1)/(1/(cos(f*x+e)+1))^(1/2))-arctanh(1/2*(cos
(f*x+e)+sin(f*x+e)+1)/(cos(f*x+e)+1)/(1/(cos(f*x+e)+1))^(1/2)))/(1/(cos(f*
x+e)+1))^(1/2)*cot(f*x+e)
```

3.244.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 90 vs. $2(32) = 64$.

Time = 0.30 (sec) , antiderivative size = 215, normalized size of antiderivative = 5.66

$$\int \sqrt{-\sec(e+fx)}\sqrt{a-a\sec(e+fx)}\,dx$$
$$= \left[\frac{\sqrt{a}\log\left(\frac{4\left(\cos(fx+e)^3+3\cos(fx+e)^2+2\cos(fx+e)\right)\sqrt{a}\sqrt{\frac{a\cos(fx+e)-a}{\cos(fx+e)}}\sqrt{-\frac{1}{\cos(fx+e)}}+\left(a\cos(fx+e)^2+8a\cos(fx+e)+8a\right)\sin(fx+e)}{\cos(fx+e)^2\sin(fx+e)}\right)}{2f}\right.$$
$$\left.-\frac{\sqrt{-a}\arctan\left(\frac{2\left(\cos(fx+e)^2+\cos(fx+e)\right)\sqrt{-a}\sqrt{\frac{a\cos(fx+e)-a}{\cos(fx+e)}}\sqrt{-\frac{1}{\cos(fx+e)}}}{(a\cos(fx+e)+2a)\sin(fx+e)}\right)}{f}\right]$$

input
```
integrate((-sec(f*x+e))^(1/2)*(a-a*sec(f*x+e))^(1/2),x, algorithm="fricas"
)
```

output
```
[1/2*sqrt(a)*log((4*(cos(f*x + e)^3 + 3*cos(f*x + e)^2 + 2*cos(f*x + e))*s
qrt(a)*sqrt((a*cos(f*x + e) - a)/cos(f*x + e))*sqrt(-1/cos(f*x + e)) + (a*
cos(f*x + e)^2 + 8*a*cos(f*x + e) + 8*a)*sin(f*x + e))/(cos(f*x + e)^2*sin
(f*x + e)))/f, -sqrt(-a)*arctan(2*(cos(f*x + e)^2 + cos(f*x + e))*sqrt(-a)
*sqrt((a*cos(f*x + e) - a)/cos(f*x + e))*sqrt(-1/cos(f*x + e))/((a*cos(f*x
 + e) + 2*a)*sin(f*x + e)))/f]
```

3.244.6 Sympy [F]

$$\int \sqrt{-\sec(e+fx)}\sqrt{a-a\sec(e+fx)}\,dx = \int \sqrt{-\sec(e+fx)}\sqrt{-a(\sec(e+fx)-1)}\,dx$$

input
```
integrate((-sec(f*x+e))**(1/2)*(a-a*sec(f*x+e))**(1/2),x)
```

output
```
Integral(sqrt(-sec(e + f*x))*sqrt(-a*(sec(e + f*x) - 1)), x)
```

3.244.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 353 vs. $2(32) = 64$.

Time = 0.41 (sec) , antiderivative size = 353, normalized size of antiderivative = 9.29

$$\int \sqrt{-\sec(e+fx)}\sqrt{a-a\sec(e+fx)}\,dx = -\frac{\sqrt{a}\left(\log\left(2\cos\left(\frac{1}{2}\arctan\left(\sin(fx+e),\cos(fx+e)\right)\right)^2+2\sin\left(\frac{1}{2}\arctan\left(\sin(fx+e),\cos(fx+e)\right)\right)^2\right.\right.}{}$$

input
```
integrate((-sec(f*x+e))^(1/2)*(a-a*sec(f*x+e))^(1/2),x, algorithm="maxima"
)
```

output
```
-1/2*sqrt(a)*(log(2*cos(1/2*arctan2(sin(f*x + e), cos(f*x + e)))^2 + 2*sin
(1/2*arctan2(sin(f*x + e), cos(f*x + e)))^2 + 2*sqrt(2)*cos(1/2*arctan2(si
n(f*x + e), cos(f*x + e))) + 2*sqrt(2)*sin(1/2*arctan2(sin(f*x + e), cos(f
*x + e))) + 2) + log(2*cos(1/2*arctan2(sin(f*x + e), cos(f*x + e)))^2 + 2*
sin(1/2*arctan2(sin(f*x + e), cos(f*x + e)))^2 + 2*sqrt(2)*cos(1/2*arctan2
(sin(f*x + e), cos(f*x + e))) - 2*sqrt(2)*sin(1/2*arctan2(sin(f*x + e), co
s(f*x + e))) + 2) - log(2*cos(1/2*arctan2(sin(f*x + e), cos(f*x + e)))^2 +
 2*sin(1/2*arctan2(sin(f*x + e), cos(f*x + e)))^2 - 2*sqrt(2)*cos(1/2*arct
an2(sin(f*x + e), cos(f*x + e))) + 2*sqrt(2)*sin(1/2*arctan2(sin(f*x + e),
 cos(f*x + e))) + 2) - log(2*cos(1/2*arctan2(sin(f*x + e), cos(f*x + e)))^
2 + 2*sin(1/2*arctan2(sin(f*x + e), cos(f*x + e)))^2 - 2*sqrt(2)*cos(1/2*a
rctan2(sin(f*x + e), cos(f*x + e))) - 2*sqrt(2)*sin(1/2*arctan2(sin(f*x +
e), cos(f*x + e))) + 2))/f
```

3.244.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 98 vs. $2(32) = 64$.

Time = 0.66 (sec) , antiderivative size = 98, normalized size of antiderivative = 2.58

$$\int \sqrt{-\sec(e+fx)}\sqrt{a-a\sec(e+fx)}\,dx = \frac{\sqrt{2}\left(\frac{\sqrt{2}a^2\arctan\left(\frac{\sqrt{2}\sqrt{a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a}}{2\sqrt{-a}}\right)}{\sqrt{-a}}-\frac{\sqrt{2}a^2\arctan\left(\frac{\sqrt{a}}{\sqrt{-a}}\right)}{\sqrt{-a}}\right)|a|\operatorname{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{a^2f}$$

input
```
integrate((-sec(f*x+e))^(1/2)*(a-a*sec(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
sqrt(2)*(sqrt(2)*a^2*arctan(1/2*sqrt(2)*sqrt(a*tan(1/2*f*x + 1/2*e)^2 + a)
/sqrt(-a))/sqrt(-a) - sqrt(2)*a^2*arctan(sqrt(a)/sqrt(-a))/sqrt(-a))*abs(a
)*sgn(tan(1/2*f*x + 1/2*e)^3 + tan(1/2*f*x + 1/2*e))/(a^2*f)
```

3.244.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{-\sec(e+fx)}\sqrt{a-a\sec(e+fx)}\,dx=\int \sqrt{a-\frac{a}{\cos\left(e+f\,x\right)}}\,\sqrt{-\frac{1}{\cos\left(e+f\,x\right)}}\,dx$$

input
```
int((a - a/cos(e + f*x))^(1/2)*(-1/cos(e + f*x))^(1/2),x)
```

output
```
int((a - a/cos(e + f*x))^(1/2)*(-1/cos(e + f*x))^(1/2), x)
```

3.245 $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$

3.245.1 Optimal result 1951
3.245.2 Mathematica [A] (verified) 1951
3.245.3 Rubi [A] (verified) 1952
3.245.4 Maple [B] (verified) 1955
3.245.5 Fricas [A] (verification not implemented) 1956
3.245.6 Sympy [F(-1)] 1957
3.245.7 Maxima [B] (verification not implemented) 1957
3.245.8 Giac [F] 1958
3.245.9 Mupad [F(-1)] 1959

3.245.1 Optimal result

Integrand size = 25, antiderivative size = 128

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = -\frac{\operatorname{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}d} + \frac{\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}d} + \frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}}$$

output
```
-arcsinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/d/a^(1/2)+arctanh(1/2*
sin(d*x+c)*a^(1/2)*sec(d*x+c)^(1/2)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))*2^(1/2
)/d/a^(1/2)+sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)
```

3.245.2 Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.98

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \frac{\left(\arcsin\left(\sqrt{1-\sec(c+dx)}\right)+2\arcsin\left(\sqrt{\sec(c+dx)}\right)-\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)+\sqrt{-((-1+\sec(c}\right.}{d\sqrt{1-\sec(c+dx)}\sqrt{a(1+\sec(c+dx))}}$$

input
```
Integrate[Sec[c + d*x]^(5/2)/Sqrt[a + a*Sec[c + d*x]],x]
```

output
```
((ArcSin[Sqrt[1 - Sec[c + d*x]]] + 2*ArcSin[Sqrt[Sec[c + d*x]]] - Sqrt[2]*
ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]] + Sqrt[-((-1 +
 Sec[c + d*x])*Sec[c + d*x])])*Tan[c + d*x])/(d*Sqrt[1 - Sec[c + d*x]]*Sqr
t[a*(1 + Sec[c + d*x])])
```

3.245.3 Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.07, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {3042, 4309, 3042, 4511, 3042, 4288, 222, 4295, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{\sqrt{a\sec(c+dx)+a}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}{\sqrt{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}}dx$$

$$\Big\downarrow \text{4309}$$

$$\frac{\int \frac{\sqrt{\sec(c+dx)}(a-a\sec(c+dx))}{\sqrt{\sec(c+dx)a+a}}dx}{2a}+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a-a\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{2a}+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{4511}$$

$$\frac{2a\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{\sec(c+dx)a+a}}dx-\int \sqrt{\sec(c+dx)}\sqrt{\sec(c+dx)a+a}dx}{2a}+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2a\int \frac{\sqrt{\csc(c+dx+\frac{\pi}{2})}}{\sqrt{\csc(c+dx+\frac{\pi}{2})a+a}}dx-\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{2a}+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}$$

$\Big\downarrow$ 4288

$$\frac{2a\int \frac{\sqrt{\csc(c+dx+\frac{\pi}{2})}}{\sqrt{\csc(c+dx+\frac{\pi}{2})a+a}}dx+\frac{2\int \frac{1}{\sqrt{\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1}}d\left(-\frac{a\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}}{2a}+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}$$

$\Big\downarrow$ 222

$$\frac{2a\int \frac{\sqrt{\csc(c+dx+\frac{\pi}{2})}}{\sqrt{\csc(c+dx+\frac{\pi}{2})a+a}}dx-\frac{2\sqrt{a}\text{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}}{2a}+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}$$

$\Big\downarrow$ 4295

$$\frac{-\frac{4a\int \frac{1}{2a-\frac{a^2\sin(c+dx)\tan(c+dx)}{\sec(c+dx)a+a}}d\left(-\frac{a\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}-\frac{2\sqrt{a}\text{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}}{2a}+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}$$

$\Big\downarrow$ 219

$$\frac{\frac{2\sqrt{2}\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{d}-\frac{2\sqrt{a}\text{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}}{2a}+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}$$

input
```
Int[Sec[c + d*x]^(5/2)/Sqrt[a + a*Sec[c + d*x]],x]
```

output
```
((-2*Sqrt[a]*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d +
 (2*Sqrt[2]*Sqrt[a]*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqr
t[2]*Sqrt[a + a*Sec[c + d*x]])])/d)/(2*a) + (Sec[c + d*x]^(3/2)*Sin[c + d*
x])/(d*Sqrt[a + a*Sec[c + d*x]])
```

3.245. $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$

3.245.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4288
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[-2*(a/(b*f))*Sqrt[a*(d/b)]   Subst[Int[1/Sqrt[1
+ x^2/a], x], x, b*(Cot[e + f*x]/Sqrt[a + b*Csc[e + f*x]])], x] /; FreeQ[{a
, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[a*(d/b), 0]
```

rule 4295
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[-2*b*(d/(a*f))   Subst[Int[1/(2*b - d*x^2), x],
x, b*(Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]]))], x] /;
 FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4309
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[-2*d^2*Cot[e + f*x]*((d*Csc[e + f*x])^(n - 2)/(
f*(2*n - 3)*Sqrt[a + b*Csc[e + f*x]])), x] + Simp[d^2/(b*(2*n - 3))   Int[(
d*Csc[e + f*x])^(n - 2)*((2*b*(n - 2) - a*Csc[e + f*x])/Sqrt[a + b*Csc[e +
f*x]]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[n,
2] && IntegerQ[2*n]
```

rule 4511
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b -
a*B)/b   Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n, x], x] + Simp[B/b
 Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b
, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
```

3.245. $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$

3.245.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 234 vs. $2(107) = 214$.

Time = 1.72 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.84

method	result
default	$\frac{\sec(dx+c)^{\frac{5}{2}}\sqrt{a(1+\sec(dx+c))}\left(-2\arctan\left(\frac{\sin(dx+c)\sqrt{2}}{2(\cos(dx+c)+1)\sqrt{-\frac{1}{\cos(dx+c)+1}}}\right)\cos(dx+c)^{3}\sqrt{2}+2\sqrt{-\frac{1}{\cos(dx+c)+1}}\,\sin(dx+c)\cos(dx+\right.}{2da(\cos(dx+c)+1)\sqrt{-\frac{}{\cos}}}$

input
```
int(sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/d/a*sec(d*x+c)^(5/2)*(a*(1+sec(d*x+c)))^(1/2)/(cos(d*x+c)+1)/(-1/(cos(
d*x+c)+1))^(1/2)*(-2*arctan(1/2*sin(d*x+c)*2^(1/2)/(cos(d*x+c)+1)/(-1/(cos
(d*x+c)+1))^(1/2))*cos(d*x+c)^3*2^(1/2)+2*(-1/(cos(d*x+c)+1))^(1/2)*sin(d*
x+c)*cos(d*x+c)^2+cos(d*x+c)^3*arctan(1/2*(-cos(d*x+c)+sin(d*x+c)-1)/(cos(
d*x+c)+1)/(-1/(cos(d*x+c)+1))^(1/2))+cos(d*x+c)^3*arctan(1/2*(cos(d*x+c)+s
in(d*x+c)+1)/(cos(d*x+c)+1)/(-1/(cos(d*x+c)+1))^(1/2)))
```

3.245.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 481, normalized size of antiderivative = 3.76

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$$

$$= \left[\frac{\sqrt{a}(\cos(dx+c)+1)\log\left(\frac{a\cos(dx+c)^3-7\,a\cos(dx+c)^2+\frac{4\left(\cos(dx+c)^2-2\,\cos(dx+c)\right)\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{\sqrt{\cos(dx+c)}}+8\,a}{\cos(dx+c)^3+\cos(dx+c)^2}\right)+\frac{2\sqrt{2}(a}{}}{4\,(ad\cos(dx+c)} \right.$$

$$-\frac{2\sqrt{2}(a\cos(dx+c)+a)\sqrt{-\frac{1}{a}}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{-\frac{1}{a}}\sqrt{\cos(dx+c)}}{\sin(dx+c)}\right)+\sqrt{-a}(\cos(dx+c)+1)\arctan}{2\,(ad\cos(dx+c)+ad)}$$

input
```
integrate(sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
[1/4*(sqrt(a)*(cos(d*x + c) + 1)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^
2 + 4*(cos(d*x + c)^2 - 2*cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/
cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)) + 8*a)/(cos(d*x + c)^3 + cos
(d*x + c)^2)) + 2*sqrt(2)*(a*cos(d*x + c) + a)*log(-(cos(d*x + c)^2 - 2*sq
rt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x +
 c)/sqrt(a) - 2*cos(d*x + c) - 3)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1))/s
qrt(a) + 4*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d
*x + c)))/(a*d*cos(d*x + c) + a*d), -1/2*(2*sqrt(2)*(a*cos(d*x + c) + a)*s
qrt(-1/a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(-1/a
)*sqrt(cos(d*x + c))/sin(d*x + c)) + sqrt(-a)*(cos(d*x + c) + 1)*arctan(2*
sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*
x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)) - 2*sqrt((a*cos(d*x + c)
 + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a*d*cos(d*x + c) + a
*d)]
```

3.245.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**(5/2)/(a+a*sec(d*x+c))**(1/2),x)
```

output
```
Timed out
```

3.245.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 876 vs. $2(107) = 214$.

Time = 0.41 (sec) , antiderivative size = 876, normalized size of antiderivative = 6.84

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \text{Too large to display}$$

input
```
integrate(sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
-1/4*(4*sqrt(2)*cos(3/2*arctan2(sin(d*x + c), cos(d*x + c)))*sin(2*d*x + 2
*c) - 4*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))*sin(2*d*x + 2
*c) + (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*l
og(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(si
n(d*x + c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos
(d*x + c))) + 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2)
- (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2
*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*
x + c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x
 + c))) - 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) + (c
os(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2*cos
(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x +
c), cos(d*x + c)))^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c
))) + 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) - (cos(2
*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2
*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c),
cos(d*x + c)))^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))
- 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) - 2*(sqrt(2)
*cos(2*d*x + 2*c)^2 + sqrt(2)*sin(2*d*x + 2*c)^2 + 2*sqrt(2)*cos(2*d*x + 2
*c) + sqrt(2))*log(cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + sin...
```

3.245.8 Giac [F]

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \int \frac{\sec\left(dx+c\right)^{\frac{5}{2}}}{\sqrt{a\sec\left(dx+c\right)+a}}\,dx$$

input
```
integrate(sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^(5/2)/sqrt(a*sec(d*x + c) + a), x)
```

3.245.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx=\int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}}{\sqrt{a+\frac{a}{\cos(c+d\,x)}}}\,dx$$

input
```
int((1/cos(c + d*x))^(5/2)/(a + a/cos(c + d*x))^(1/2),x)
```

output
```
int((1/cos(c + d*x))^(5/2)/(a + a/cos(c + d*x))^(1/2), x)
```

3.246 $\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$

3.246.1 Optimal result 1960
3.246.2 Mathematica [A] (verified) 1960
3.246.3 Rubi [A] (verified) 1961
3.246.4 Maple [B] (verified) 1963
3.246.5 Fricas [A] (verification not implemented) 1963
3.246.6 Sympy [F] 1964
3.246.7 Maxima [B] (verification not implemented) 1964
3.246.8 Giac [F] 1965
3.246.9 Mupad [F(-1)] 1965

3.246.1 Optimal result

Integrand size = 25, antiderivative size = 95

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx=\frac{2\text{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}d}-\frac{\sqrt{2}\text{arctanh}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}d}$$

output
```
2*arcsinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/d/a^(1/2)-arctanh(1/2
*sin(d*x+c)*a^(1/2)*sec(d*x+c)^(1/2)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))*2^(1/
2)/d/a^(1/2)
```

3.246.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.94

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx=\frac{\left(-2\arcsin\left(\sqrt{\sec(c+dx)}\right)+\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)\right)\tan(c+dx)}{d\sqrt{1-\sec(c+dx)}\sqrt{a(1+\sec(c+dx))}}$$

input
```
Integrate[Sec[c + d*x]^(3/2)/Sqrt[a + a*Sec[c + d*x]],x]
```

output
```
((-2*ArcSin[Sqrt[Sec[c + d*x]]] + Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x
]])/Sqrt[1 - Sec[c + d*x]]])*Tan[c + d*x])/(d*Sqrt[1 - Sec[c + d*x]]*Sqrt[
a*(1 + Sec[c + d*x])])
```

3.246.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3042, 4308, 3042, 4288, 222, 4295, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a\sec(c+dx)+a}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\sqrt{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}}dx$$

$$\Big\downarrow \text{4308}$$

$$\frac{\int \sqrt{\sec(c+dx)}\sqrt{\sec(c+dx)a+a}dx}{a}-\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{\sec(c+dx)a+a}}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{a}-\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx$$

$$\Big\downarrow \text{4288}$$

$$-\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx-\frac{2\int \frac{1}{\sqrt{\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1}}d\left(-\frac{a\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{ad}$$

$$\Big\downarrow \text{222}$$

$$\frac{2\text{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}-\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx$$

$$\Big\downarrow \text{4295}$$

$$\frac{2\int \frac{1}{2a-\frac{a^2\sin(c+dx)\tan(c+dx)}{\sec(c+dx)a+a}}d\left(-\frac{a\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}+\frac{2\text{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}$$

$$\Big\downarrow \; 219$$

$$\frac{2\text{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}-\frac{\sqrt{2}\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}$$

input
```
Int[Sec[c + d*x]^(3/2)/Sqrt[a + a*Sec[c + d*x]],x]
```

output
```
(2*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(Sqrt[a]*d) -
 (Sqrt[2]*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[
a + a*Sec[c + d*x]])])/(Sqrt[a]*d)
```

3.246.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4288
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[-2*(a/(b*f))*Sqrt[a*(d/b)]   Subst[Int[1/Sqrt[1
+ x^2/a], x], x, b*(Cot[e + f*x]/Sqrt[a + b*Csc[e + f*x]])], x] /; FreeQ[{a
, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[a*(d/b), 0]
```

rule 4295
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[-2*b*(d/(a*f))   Subst[Int[1/(2*b - d*x^2), x],
x, b*(Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]]))], x] /;
 FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

3.246. $\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$

rule 4308
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Simp[d/b   Int[Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[
e + f*x]], x], x] - Simp[a*(d/b)   Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[
e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

3.246.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 186 vs. $2(78) = 156$.

Time = 1.86 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.97

method	result
default	$\frac{\left(\arctan\left(\frac{\sin(dx+c)\sqrt{2}}{2(\cos(dx+c)+1)\sqrt{-\frac{1}{\cos(dx+c)+1}}}\right)\sqrt{2}+\arctan\left(\frac{\cos(dx+c)-\sin(dx+c)+1}{2(\cos(dx+c)+1)\sqrt{-\frac{1}{\cos(dx+c)+1}}}\right)-\arctan\left(\frac{\cos(dx+c)+\sin(dx+c)+1}{2(\cos(dx+c)+1)\sqrt{-\frac{1}{\cos(dx+c)+1}}}\right)\right.}{da(\cos(dx+c)+1)\sqrt{-\frac{1}{\cos(dx+c)+1}}}$

input
```
int(sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/d/a*(arctan(1/2*sin(d*x+c)*2^(1/2)/(cos(d*x+c)+1)/(-1/(cos(d*x+c)+1))^(1
/2))*2^(1/2)+arctan(1/2*(cos(d*x+c)-sin(d*x+c)+1)/(cos(d*x+c)+1)/(-1/(cos(
d*x+c)+1))^(1/2))-arctan(1/2*(cos(d*x+c)+sin(d*x+c)+1)/(cos(d*x+c)+1)/(-1/
(cos(d*x+c)+1))^(1/2)))*sec(d*x+c)^(3/2)*(a*(1+sec(d*x+c)))^(1/2)*cos(d*x+
c)^2/(cos(d*x+c)+1)/(-1/(cos(d*x+c)+1))^(1/2)
```

3.246.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 351, normalized size of antiderivative = 3.69

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$$
$$= \left[\frac{\sqrt{2}\sqrt{a}\log\left(-\frac{\cos(dx+c)^2+\frac{2\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)}{\sqrt{a}}-2\cos(dx+c)-3}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)+\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7a\cos(dx+c)^2-}{}\right.}{2ad}\right.$$

input
```
integrate(sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
[1/2*(sqrt(2)*sqrt(a)*log(-(cos(d*x + c)^2 + 2*sqrt(2)*sqrt((a*cos(d*x + c
) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/sqrt(a) - 2*cos(d*x +
 c) - 3)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + sqrt(a)*log((a*cos(d*x +
 c)^3 - 7*a*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 - 2*cos(d*x + c))*sqrt(a)*s
qrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)) + 8
*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)))/(a*d), (sqrt(2)*a*sqrt(-1/a)*arcta
n(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(-1/a)*sqrt(cos(d*x
+ c))/sin(d*x + c)) + sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)
/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d
*x + c) - 2*a)))/(a*d)]
```

3.246.6 Sympy [F]

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a\left(\sec(c+dx)+1\right)}}\,dx$$

input
```
integrate(sec(d*x+c)**(3/2)/(a+a*sec(d*x+c))**(1/2),x)
```

output
```
Integral(sec(c + d*x)**(3/2)/sqrt(a*(sec(c + d*x) + 1)), x)
```

3.246.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 476 vs. $2(78) = 156$.

Time = 0.41 (sec) , antiderivative size = 476, normalized size of antiderivative = 5.01

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = -\frac{\sqrt{2}\log\left(\cos\left(\frac{1}{2}\arctan\left(\sin\left(dx+c\right),\cos\left(dx+c\right)\right)\right)^2+\sin\left(\frac{1}{2}\arctan\left(\sin\left(dx+c\right),\cos\left(dx+c\right)\right)\right)^2+2\,\mathrm{si}\right.}{}$$

input
```
integrate(sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
-1/2*(sqrt(2)*log(cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + sin(1/2
*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c),
cos(d*x + c))) + 1) - sqrt(2)*log(cos(1/2*arctan2(sin(d*x + c), cos(d*x +
c)))^2 + sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 - 2*sin(1/2*arctan
2(sin(d*x + c), cos(d*x + c))) + 1) - log(2*cos(1/2*arctan2(sin(d*x + c),
cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sq
rt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2*sqrt(2)*sin(1/2*arc
tan2(sin(d*x + c), cos(d*x + c))) + 2) + log(2*cos(1/2*arctan2(sin(d*x + c
), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2
*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) - 2*sqrt(2)*sin(1/2*
arctan2(sin(d*x + c), cos(d*x + c))) + 2) - log(2*cos(1/2*arctan2(sin(d*x
+ c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2
- 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2*sqrt(2)*sin(1
/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) + log(2*cos(1/2*arctan2(sin(d
*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))
^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) - 2*sqrt(2)*si
n(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2))/(sqrt(a)*d)
```

3.246.8 Giac [F]

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \int \frac{\sec\left(dx+c\right)^{\frac{3}{2}}}{\sqrt{a\sec\left(dx+c\right)+a}}\,dx$$

input
```
integrate(sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^(3/2)/sqrt(a*sec(d*x + c) + a), x)
```

3.246.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}{\sqrt{a+\frac{a}{\cos(c+d\,x)}}}\,dx$$

input
```
int((1/cos(c + d*x))^(3/2)/(a + a/cos(c + d*x))^(1/2),x)
```

output
```
int((1/cos(c + d*x))^(3/2)/(a + a/cos(c + d*x))^(1/2), x)
```

3.247 $\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+a\sec(c+dx)}}\,dx$

3.247.1 Optimal result 1966
3.247.2 Mathematica [A] (verified) 1966
3.247.3 Rubi [A] (verified) 1967
3.247.4 Maple [B] (verified) 1968
3.247.5 Fricas [A] (verification not implemented) 1969
3.247.6 Sympy [F] 1969
3.247.7 Maxima [A] (verification not implemented) 1970
3.247.8 Giac [F] 1970
3.247.9 Mupad [F(-1)] 1970

3.247.1 Optimal result

Integrand size = 25, antiderivative size = 56

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+a\sec(c+dx)}}\,dx=\frac{\sqrt{2}\text{arctanh}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}d}$$

output
```
arctanh(1/2*sin(d*x+c)*a^(1/2)*sec(d*x+c)^(1/2)*2^(1/2)/(a+a*sec(d*x+c))^(
1/2))*2^(1/2)/d/a^(1/2)
```

3.247.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.34

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+a\sec(c+dx)}}\,dx=-\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)\tan(c+dx)}{d\sqrt{1-\sec(c+dx)}\sqrt{a(1+\sec(c+dx))}}$$

input
```
Integrate[Sqrt[Sec[c + d*x]]/Sqrt[a + a*Sec[c + d*x]],x]
```

output
```
-((Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*Tan
[c + d*x])/(d*Sqrt[1 - Sec[c + d*x]]*Sqrt[a*(1 + Sec[c + d*x])]))
```

3.247.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {3042, 4295, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a\sec(c+dx)+a}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}}dx$$

$$\Big\downarrow \text{4295}$$

$$-\frac{2\int \frac{1}{2a-\frac{a^2\sin(c+dx)\tan(c+dx)}{\sec(c+dx)a+a}}d\left(-\frac{a\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}$$

$$\Big\downarrow \text{219}$$

$$\frac{\sqrt{2}\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}$$

input
```
Int[Sqrt[Sec[c + d*x]]/Sqrt[a + a*Sec[c + d*x]],x]
```

output
```
(Sqrt[2]*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a
 + a*Sec[c + d*x]])])/(Sqrt[a]*d)
```

3.247.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4295
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[-2*b*(d/(a*f))   Subst[Int[1/(2*b - d*x^2), x],
x, b*(Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]]))], x] /;
 FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

3.247.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 97 vs. $2(45) = 90$.

Time = 1.26 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.75

method	result	size
default	$-\frac{\sqrt{2}\,\sqrt{\sec(dx+c)}\,\sqrt{a(1+\sec(dx+c))}\,\arctan\left(\frac{\sin(dx+c)\sqrt{2}}{2(\cos(dx+c)+1)\sqrt{-\frac{1}{\cos(dx+c)+1}}}\right)\cos(dx+c)}{da(\cos(dx+c)+1)\sqrt{-\frac{1}{\cos(dx+c)+1}}}$	98

input
```
int(sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/d/a*2^(1/2)*sec(d*x+c)^(1/2)*(a*(1+sec(d*x+c)))^(1/2)*arctan(1/2*sin(d*
x+c)*2^(1/2)/(cos(d*x+c)+1)/(-1/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)/(cos(d*x
+c)+1)/(-1/(cos(d*x+c)+1))^(1/2)
```

3.247.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 160, normalized size of antiderivative = 2.86

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+a\sec(c+dx)}}\,dx = \left[\frac{\sqrt{2}\log\left(-\frac{\cos(dx+c)^2-\frac{2\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)}{\sqrt{a}}-2\cos(dx+c)-3}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)}{2\sqrt{a}d}, -\frac{\sqrt{2}\sqrt{-\frac{1}{a}}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{-\frac{1}{a}}\sqrt{\cos(dx+c)}}{\sin(dx+c)}\right)}{d}\right]$$

input
```
integrate(sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
[1/2*sqrt(2)*log(-(cos(d*x + c)^2 - 2*sqrt(2)*sqrt((a*cos(d*x + c) + a)/co
s(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/sqrt(a) - 2*cos(d*x + c) - 3)/
(cos(d*x + c)^2 + 2*cos(d*x + c) + 1))/(sqrt(a)*d), -sqrt(2)*sqrt(-1/a)*ar
ctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(-1/a)*sqrt(cos(d
*x + c))/sin(d*x + c))/d]
```

3.247.6 Sympy [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+a\sec(c+dx)}}\,dx = \int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a(\sec(c+dx)+1)}}\,dx$$

input
```
integrate(sec(d*x+c)**(1/2)/(a+a*sec(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(sec(c + d*x))/sqrt(a*(sec(c + d*x) + 1)), x)
```

3.247.7 Maxima [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.61

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+a\sec(c+dx)}}\,dx = \frac{\sqrt{2}\log\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+2\,\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)-\sqrt{2}\log\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+\sin\right.}{2\sqrt{a}d}$$

input
```
integrate(sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
1/2*(sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1
/2*d*x + 1/2*c) + 1) - sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x +
1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))/(sqrt(a)*d)
```

3.247.8 Giac [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+a\sec(c+dx)}}\,dx = \int \frac{\sqrt{\sec\left(dx+c\right)}}{\sqrt{a\sec\left(dx+c\right)+a}}\,dx$$

input
```
integrate(sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(sec(d*x + c))/sqrt(a*sec(d*x + c) + a), x)
```

3.247.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+a\sec(c+dx)}}\,dx = \int \frac{\sqrt{\frac{1}{\cos(c+d\,x)}}}{\sqrt{a+\frac{a}{\cos(c+d\,x)}}}\,dx$$

input
```
int((1/cos(c + d*x))^(1/2)/(a + a/cos(c + d*x))^(1/2),x)
```

output
```
int((1/cos(c + d*x))^(1/2)/(a + a/cos(c + d*x))^(1/2), x)
```

3.248 $\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+dx)}}\,dx$

3.248.1 Optimal result 1971
3.248.2 Mathematica [A] (verified) 1971
3.248.3 Rubi [A] (verified) 1972
3.248.4 Maple [A] (verified) 1973
3.248.5 Fricas [A] (verification not implemented) 1974
3.248.6 Sympy [F] 1974
3.248.7 Maxima [A] (verification not implemented) 1975
3.248.8 Giac [F] 1975
3.248.9 Mupad [F(-1)] 1975

3.248.1 Optimal result

Integrand size = 25, antiderivative size = 93

$$\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+dx)}}\,dx = -\frac{\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}d} + \frac{2\sqrt{\sec(c+dx)}\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}}$$

output
```
-arctanh(1/2*sin(d*x+c)*a^(1/2)*sec(d*x+c)^(1/2)*2^(1/2)/(a+a*sec(d*x+c))^
(1/2))*2^(1/2)/d/a^(1/2)+2*sin(d*x+c)*sec(d*x+c)^(1/2)/d/(a+a*sec(d*x+c))^
(1/2)
```

3.248.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.10

$$\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+dx)}}\,dx = \frac{\left(\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)+\frac{2\sqrt{1-\sec(c+dx)}}{\sqrt{\sec(c+dx)}}\right)\tan(c+dx)}{d\sqrt{1-\sec(c+dx)}\sqrt{a(1+\sec(c+dx))}}$$

input
```
Integrate[1/(Sqrt[Sec[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]),x]
```

output
```
((Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]] + (2
*Sqrt[1 - Sec[c + d*x]])/Sqrt[Sec[c + d*x]])*Tan[c + d*x])/(d*Sqrt[1 - Sec
[c + d*x]]*Sqrt[a*(1 + Sec[c + d*x])])
```

3.248.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 4299, 3042, 4295, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}}dx$$

$$\Big\downarrow \text{4299}$$

$$\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\sec(c+dx)+a}}-\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{\sec(c+dx)a+a}}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\sec(c+dx)+a}}-\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx$$

$$\Big\downarrow \text{4295}$$

$$\frac{2\int \frac{1}{2a-\frac{a^2\sin(c+dx)\tan(c+dx)}{\sec(c+dx)a+a}}d\left(-\frac{a\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}+\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{219}$$

$$\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\sec(c+dx)+a}}-\frac{\sqrt{2}\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}$$

input
```
Int[1/(Sqrt[Sec[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]),x]
```

output
```
-((Sqrt[2]*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt
[a + a*Sec[c + d*x]])])/(Sqrt[a]*d)) + (2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])
/(d*Sqrt[a + a*Sec[c + d*x]])
```

3.248.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4295
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[-2*b*(d/(a*f))   Subst[Int[1/(2*b - d*x^2), x],
x, b*(Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]]))], x] /;
 FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4299
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-Cot[e + f*x])*(a + b*Csc[e + f*x])^m*((d*Csc
[e + f*x])^n/(f*(m + 1))), x] + Simp[a*(m/(b*d*(m + 1)))   Int[(a + b*Csc[e
 + f*x])^m*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m, n},
 x] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LtQ[m, -2^(-1)]
```

3.248.4 Maple [A] (verified)

Time = 1.08 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.08

method	result	size
default	$-\frac{\sqrt{a(1+\sec(dx+c))}\left(\sqrt{2}\,\arctan\left(\frac{\sin(dx+c)\sqrt{2}}{2(\cos(dx+c)+1)\sqrt{-\frac{1}{\cos(dx+c)+1}}}\right)\sqrt{-\frac{1}{\cos(dx+c)+1}}+2\cot(dx+c)-2\csc(dx+c)\right)}{da\sqrt{\sec(dx+c)}}$	100

input
```
int(1/sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/d/a*(a*(1+sec(d*x+c)))^(1/2)/sec(d*x+c)^(1/2)*(2^(1/2)*arctan(1/2*sin(d
*x+c)*2^(1/2)/(cos(d*x+c)+1)/(-1/(cos(d*x+c)+1))^(1/2))*(-1/(cos(d*x+c)+1)
)^(1/2)+2*cot(d*x+c)-2*csc(d*x+c))
```

3.248.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 281, normalized size of antiderivative = 3.02

$$\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+dx)}}\,dx = \left[\frac{\frac{\sqrt{2}(a\cos(dx+c)+a)\log\left(-\frac{\cos(dx+c)^2+\frac{2\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)}{\sqrt{a}}-2\cos(dx+c)-3}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)}{\sqrt{a}}+4\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}}{2(ad\cos(dx+c)+ad)}\right.$$

input
```
integrate(1/sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
[1/2*(sqrt(2)*(a*cos(d*x + c) + a)*log(-(cos(d*x + c)^2 + 2*sqrt(2)*sqrt((
a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/sqrt(a)
- 2*cos(d*x + c) - 3)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1))/sqrt(a) + 4*s
qrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(a
*d*cos(d*x + c) + a*d), (sqrt(2)*(a*cos(d*x + c) + a)*sqrt(-1/a)*arctan(sq
rt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(-1/a)*sqrt(cos(d*x + c)
)/sin(d*x + c)) + 2*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x +
 c))*sin(d*x + c))/(a*d*cos(d*x + c) + a*d)]
```

3.248.6 Sympy [F]

$$\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{a(\sec(c+dx)+1)}\sqrt{\sec(c+dx)}}\,dx$$

input
```
integrate(1/sec(d*x+c)**(1/2)/(a+a*sec(d*x+c))**(1/2),x)
```

output
```
Integral(1/(sqrt(a*(sec(c + d*x) + 1))*sqrt(sec(c + d*x))), x)
```

3.248.7 Maxima [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.12

$$\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+dx)}}\,dx = -\frac{\sqrt{2}\log\left(\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+2\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)-\sqrt{2}\log\left(\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+\mathrm{s}\right.}{2\sqrt{a}d}$$

input
```
integrate(1/sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
-1/2*(sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(
1/2*d*x + 1/2*c) + 1) - sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x +
 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) - 4*sqrt(2)*sin(1/2*d*x + 1/2*c))/
(sqrt(a)*d)
```

3.248.8 Giac [F]

$$\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{a\sec(dx+c)+a}\sqrt{\sec(dx+c)}}\,dx$$

input
```
integrate(1/sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(a*sec(d*x + c) + a)*sqrt(sec(d*x + c))), x)
```

3.248.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{a+\frac{a}{\cos(c+d\,x)}}\,\sqrt{\frac{1}{\cos(c+d\,x)}}}\,dx$$

input
```
int(1/((a + a/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(1/2)),x)
```

output
```
int(1/((a + a/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(1/2)), x)
```

3.249 $\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\,dx$

3.249.1 Optimal result 1976
3.249.2 Mathematica [A] (verified) 1976
3.249.3 Rubi [A] (verified) 1977
3.249.4 Maple [A] (verified) 1979
3.249.5 Fricas [A] (verification not implemented) 1980
3.249.6 Sympy [F] 1981
3.249.7 Maxima [B] (verification not implemented) 1981
3.249.8 Giac [F] 1982
3.249.9 Mupad [F(-1)] 1982

3.249.1 Optimal result

Integrand size = 25, antiderivative size = 131

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\,dx = \frac{\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}d} + \frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+dx)}} - \frac{2\sqrt{\sec(c+dx)}\sin(c+dx)}{3d\sqrt{a+a\sec(c+dx)}}$$

output
```
arctanh(1/2*sin(d*x+c)*a^(1/2)*sec(d*x+c)^(1/2)*2^(1/2)/(a+a*sec(d*x+c))^(
1/2))*2^(1/2)/d/a^(1/2)+2/3*sin(d*x+c)/d/sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))
^(1/2)-2/3*sin(d*x+c)*sec(d*x+c)^(1/2)/d/(a+a*sec(d*x+c))^(1/2)
```

3.249.2 Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.92

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\,dx = \frac{\left(2(-1+\cos(c+dx))\sqrt{1-\sec(c+dx)} - 3\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)\sqrt{\sec(c+dx)}\right)\tan(c+dx)}{3d\sqrt{-((-1+\sec(c+dx))\sec(c+dx))}\sqrt{a(1+\sec(c+dx))}}$$

input
```
Integrate[1/(Sec[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]]),x]
```

output
```
((2*(-1 + Cos[c + d*x])*Sqrt[1 - Sec[c + d*x]] - 3*Sqrt[2]*ArcTan[(Sqrt[2]
*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*Sqrt[Sec[c + d*x]])*Tan[c + d
*x])/(3*d*Sqrt[-((-1 + Sec[c + d*x])*Sec[c + d*x])]*Sqrt[a*(1 + Sec[c + d*
x])])
```

3.249.3 Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.06, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3042, 4310, 3042, 4501, 3042, 4295, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}}dx$$

$$\Big\downarrow \text{4310}$$

$$\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}-\frac{\int \frac{a-2a\sec(c+dx)}{\sqrt{\sec(c+dx)}\sqrt{\sec(c+dx)a+a}}dx}{3a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}-\frac{\int \frac{a-2a\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{3a}$$

$$\Big\downarrow \text{4501}$$

$$\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}-\frac{\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\sec(c+dx)+a}}-3a\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{\sec(c+dx)a+a}}dx}{3a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}-\frac{\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\sec(c+dx)+a}}-3a\int\frac{\sqrt{\csc(c+dx+\frac{\pi}{2})}}{\sqrt{\csc(c+dx+\frac{\pi}{2})a+a}}dx}{3a}$$

$\Big\downarrow$ 4295

$$\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}-\frac{\frac{6a\int\frac{1}{2a-\frac{a^2\sin(c+dx)\tan(c+dx)}{\sec(c+dx)a+a}}d\left(-\frac{a\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}+\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\sec(c+dx)+a}}}{3a}$$

$\Big\downarrow$ 219

$$\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}-\frac{\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\sec(c+dx)+a}}-\frac{3\sqrt{2}\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{d}}{3a}$$

input
```
Int[1/(Sec[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]]),x]
```

output
```
(2*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) - ((-3*
Sqrt[2]*Sqrt[a]*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]
*Sqrt[a + a*Sec[c + d*x]])])/d + (2*a*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(d*
Sqrt[a + a*Sec[c + d*x]]))/(3*a)
```

3.249.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4295
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[-2*b*(d/(a*f))   Subst[Int[1/(2*b - d*x^2), x],
x, b*(Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]]))], x] /;
 FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

3.249. $\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\,dx$

rule 4310
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*n*Sqrt[a +
b*Csc[e + f*x]])), x] + Simp[1/(2*b*d*n)   Int[(d*Csc[e + f*x])^(n + 1)*((a
 + b*(2*n + 1)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, 0] && IntegerQ[2*n]
```

rule 4501
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*Cot[e
 + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(f*n)), x] - Simp[(a*A*m
 - b*B*n)/(b*d*n)   Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x]
, x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a
^2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LeQ[m, -1]
```

3.249.4 Maple [A] (verified)

Time = 1.34 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.31

method	result
default	$\frac{\sqrt{a(1+\sec(dx+c))}\left(3\sqrt{2}\,\arctan\left(\frac{\sin(dx+c)\sqrt{2}}{2(\cos(dx+c)+1)\sqrt{-\frac{1}{\cos(dx+c)+1}}}\right)\sqrt{-\frac{1}{\cos(dx+c)+1}}+3\sqrt{2}\,\arctan\left(\frac{\sin(dx+c)\sqrt{2}}{2(\cos(dx+c)+1)\sqrt{-\frac{1}{\cos(dx+c)+1}}}\right)\right.}{3da(\cos(dx+c)+1)\sec(dx+c)^{\frac{3}{2}}}$

input
```
int(1/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3/d/a*(a*(1+sec(d*x+c)))^(1/2)/(cos(d*x+c)+1)/sec(d*x+c)^(3/2)*(3*2^(1/2
)*arctan(1/2*sin(d*x+c)*2^(1/2)/(cos(d*x+c)+1)/(-1/(cos(d*x+c)+1))^(1/2))*
(-1/(cos(d*x+c)+1))^(1/2)+3*2^(1/2)*arctan(1/2*sin(d*x+c)*2^(1/2)/(cos(d*x
+c)+1)/(-1/(cos(d*x+c)+1))^(1/2))*(-1/(cos(d*x+c)+1))^(1/2)*sec(d*x+c)+2*s
in(d*x+c)-2*tan(d*x+c))
```

3.249.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 318, normalized size of antiderivative = 2.43

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\,dx$$
$$= \left[\frac{\frac{3\sqrt{2}(a\cos(dx+c)+a)\log\left(-\frac{\cos(dx+c)^2-\frac{2\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)}{\sqrt{a}}-2\cos(dx+c)-3}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)}{\sqrt{a}}+\frac{4\left(\cos(dx+c)^2-\cos(dx+c)\right)\sqrt{\frac{a\cos(dx+c)}{\cos(dx}}}{\sqrt{\cos(dx+c)}}}{6\left(ad\cos\left(dx+c\right)+ad\right)}\right.$$

$$-\frac{3\sqrt{2}(a\cos\left(dx+c\right)+a)\sqrt{-\frac{1}{a}}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{-\frac{1}{a}}\sqrt{\cos(dx+c)}}{\sin(dx+c)}\right)-\frac{2\left(\cos(dx+c)^2-\cos(dx+c)\right)\sqrt{\frac{a\cos(dx+c)}{\cos(dx+c)}}}{\sqrt{\cos(dx+c)}}}{3\left(ad\cos\left(dx+c\right)+ad\right)}$$

input
```
integrate(1/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
[1/6*(3*sqrt(2)*(a*cos(d*x + c) + a)*log(-(cos(d*x + c)^2 - 2*sqrt(2)*sqrt
((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/sqrt(a
) - 2*cos(d*x + c) - 3)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1))/sqrt(a) + 4
*(cos(d*x + c)^2 - cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*s
in(d*x + c)/sqrt(cos(d*x + c)))/(a*d*cos(d*x + c) + a*d), -1/3*(3*sqrt(2)*
(a*cos(d*x + c) + a)*sqrt(-1/a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/c
os(d*x + c))*sqrt(-1/a)*sqrt(cos(d*x + c))/sin(d*x + c)) - 2*(cos(d*x + c)
^2 - cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sq
rt(cos(d*x + c)))/(a*d*cos(d*x + c) + a*d)]
```

3.249.6 Sympy [F]

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{a\left(\sec\left(c+dx\right)+1\right)}\sec^{\frac{3}{2}}\left(c+dx\right)}\,dx$$

input
```
integrate(1/sec(d*x+c)**(3/2)/(a+a*sec(d*x+c))**(1/2),x)
```

output
```
Integral(1/(sqrt(a*(sec(c + d*x) + 1))*sec(c + d*x)**(3/2)), x)
```

3.249.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 282 vs. $2(108) = 216$.

Time = 0.39 (sec) , antiderivative size = 282, normalized size of antiderivative = 2.15

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\,dx = -\frac{3\sqrt{2}\cos\left(\frac{2}{3}\arctan\left(\sin\left(\frac{3}{2}dx+\frac{3}{2}c\right),\cos\left(\frac{3}{2}dx+\frac{3}{2}c\right)\right)\right)\sin\left(\frac{3}{2}dx+\frac{3}{2}c\right)-3\sqrt{2}\cos\left(\frac{3}{2}dx+\frac{3}{2}c\right)\sin\left(\frac{2}{3}\,a\right.}{}$$

input
```
integrate(1/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
-1/6*(3*sqrt(2)*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)
))*sin(3/2*d*x + 3/2*c) - 3*sqrt(2)*cos(3/2*d*x + 3/2*c)*sin(2/3*arctan2(s
in(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 3*sqrt(2)*log(cos(1/3*arctan
2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + sin(1/3*arctan2(sin(3/2
*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x +
3/2*c), cos(3/2*d*x + 3/2*c))) + 1) + 3*sqrt(2)*log(cos(1/3*arctan2(sin(3/
2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + sin(1/3*arctan2(sin(3/2*d*x + 3
/2*c), cos(3/2*d*x + 3/2*c)))^2 - 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c),
cos(3/2*d*x + 3/2*c))) + 1) - 2*sqrt(2)*sin(3/2*d*x + 3/2*c) + 3*sqrt(2)*s
in(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))/(sqrt(a)*d)
```

3.249.8 Giac [F]

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{a\sec(dx+c)+a}\sec(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(a*sec(d*x + c) + a)*sec(d*x + c)^(3/2)), x)
```

3.249.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{a+\frac{a}{\cos(c+d\,x)}}\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

input
```
int(1/((a + a/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(3/2)),x)
```

output
```
int(1/((a + a/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(3/2)), x)
```

3.250 $\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\,dx$

3.250.1 Optimal result 1983
3.250.2 Mathematica [A] (verified) 1984
3.250.3 Rubi [A] (verified) 1984
3.250.4 Maple [A] (verified) 1987
3.250.5 Fricas [A] (verification not implemented) 1988
3.250.6 Sympy [F] 1988
3.250.7 Maxima [B] (verification not implemented) 1989
3.250.8 Giac [F] 1989
3.250.9 Mupad [F(-1)] 1990

3.250.1 Optimal result

Integrand size = 25, antiderivative size = 169

$$\begin{aligned}\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\,dx = &-\frac{\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}d}\\ &+\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\\ &-\frac{2\sin(c+dx)}{15d\sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+dx)}}\\ &+\frac{26\sqrt{\sec(c+dx)}\sin(c+dx)}{15d\sqrt{a+a\sec(c+dx)}}\end{aligned}$$

output
```
-arctanh(1/2*sin(d*x+c)*a^(1/2)*sec(d*x+c)^(1/2)*2^(1/2)/(a+a*sec(d*x+c))^
(1/2))*2^(1/2)/d/a^(1/2)+2/5*sin(d*x+c)/d/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c)
)^(1/2)-2/15*sin(d*x+c)/d/sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2)+26/15*si
n(d*x+c)*sec(d*x+c)^(1/2)/d/(a+a*sec(d*x+c))^(1/2)
```

3.250.2 Mathematica [A] (verified)

Time = 1.16 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.69

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\,dx$$
$$= \frac{(29-2\cos(c+dx)+3\cos(2(c+dx)))\sqrt{\sec(c+dx)}\sin(c+dx)+\frac{15\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)\tan(c+dx)}{\sqrt{1-\sec(c+dx)}}}{15d\sqrt{a(1+\sec(c+dx))}}$$

input
```
Integrate[1/(Sec[c + d*x]^(5/2)*Sqrt[a + a*Sec[c + d*x]]),x]
```

output
```
((29 - 2*Cos[c + d*x] + 3*Cos[2*(c + d*x)])*Sqrt[Sec[c + d*x]]*Sin[c + d*x
] + (15*Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]
]*Tan[c + d*x])/Sqrt[1 - Sec[c + d*x]])/(15*d*Sqrt[a*(1 + Sec[c + d*x])])
```

3.250.3 Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.11, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 4310, 3042, 4510, 27, 3042, 4501, 3042, 4295, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\sqrt{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}}dx$$
$$\Big\downarrow \text{4310}$$
$$\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}-\frac{\int \frac{a-4a\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)\sqrt{\sec(c+dx)a+a}}dx}{5a}$$
$$\Big\downarrow \text{3042}$$

$$\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}-\frac{\int\frac{a-4a\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{5a}$$

$\Big\downarrow$ 4510

$$\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}-\frac{\frac{2\int-\frac{13a^2-2a^2\sec(c+dx)}{2\sqrt{\sec(c+dx)}\sqrt{\sec(c+dx)a+a}}dx}{3a}+\frac{2a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}}{5a}$$

$\Big\downarrow$ 27

$$\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}-\frac{\frac{2a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}-\frac{\int\frac{13a^2-2a^2\sec(c+dx)}{\sqrt{\sec(c+dx)}\sqrt{\sec(c+dx)a+a}}dx}{3a}}{5a}$$

$\Big\downarrow$ 3042

$$\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}-\frac{\frac{2a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}-\frac{\int\frac{13a^2-2a^2\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{3a}}{5a}$$

$\Big\downarrow$ 4501

$$\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}-\frac{\frac{2a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}-\frac{\frac{26a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\sec(c+dx)+a}}-15a^2\int\frac{\sqrt{\sec(c+dx)}}{\sqrt{\sec(c+dx)a+a}}dx}{3a}}{5a}$$

$\Big\downarrow$ 3042

$$\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}-\frac{\frac{2a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}-\frac{\frac{26a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\sec(c+dx)+a}}-15a^2\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{3a}}{5a}$$

$\Big\downarrow$ 4295

$$\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}-\frac{\frac{2a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}-\frac{\frac{30a^2\int\frac{1}{2a-\frac{a^2\sin(c+dx)\tan(c+dx)}{\sec(c+dx)a+a}}d\left(-\frac{a\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}+\frac{26a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\sec(c+dx)+a}}}{3a}}{5a}$$

3.250. $\int\frac{1}{\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}dx$

$$\Big\downarrow \text{219}$$

$$\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}-\frac{\frac{2a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}-\frac{\frac{26a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\sec(c+dx)+a}}-\frac{15\sqrt{2}a^{3/2}\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{d}}{3a}}{5a}$$

input
```
Int[1/(Sec[c + d*x]^(5/2)*Sqrt[a + a*Sec[c + d*x]]),x]
```

output
```
(2*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]]) - ((2*a
*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) - ((-15*S
qrt[2]*a^(3/2)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*
Sqrt[a + a*Sec[c + d*x]])])/d + (26*a^2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(
d*Sqrt[a + a*Sec[c + d*x]]))/(3*a))/(5*a)
```

3.250.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4295
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[-2*b*(d/(a*f))   Subst[Int[1/(2*b - d*x^2), x],
x, b*(Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]]))], x] /;
 FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4310
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*n*Sqrt[a +
b*Csc[e + f*x]])), x] + Simp[1/(2*b*d*n)   Int[(d*Csc[e + f*x])^(n + 1)*((a
 + b*(2*n + 1)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, 0] && IntegerQ[2*n]
```

rule 4501
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*Cot[e
 + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(f*n)), x] - Simp[(a*A*m
 - b*B*n)/(b*d*n)   Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x]
, x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a
^2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LeQ[m, -1]
```

rule 4510
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*Cot[e
 + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(f*n)), x] - Simp[1/(b*d
*n)   Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*
n - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

3.250.4 Maple [A] (verified)

Time = 1.42 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.15

method	result
default	$-\frac{\sqrt{a(1+\sec(dx+c))}\left(15\sqrt{2}\arctan\left(\frac{\sin(dx+c)\sqrt{2}}{2(\cos(dx+c)+1)\sqrt{-\frac{1}{\cos(dx+c)+1}}}\right)\sqrt{-\frac{1}{\cos(dx+c)+1}}\sec(dx+c)+15\sqrt{2}\arctan\left(\frac{\sin(dx+}{2(\cos(dx+c)+1)\sqrt{}}\right.\right.}{15da(\cos(dx+c)+1)\sec(dx+c)}$

input
```
int(1/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/15/d/a*(a*(1+sec(d*x+c)))^(1/2)/(cos(d*x+c)+1)/sec(d*x+c)^(5/2)*(15*2^(
1/2)*arctan(1/2*sin(d*x+c)*2^(1/2)/(cos(d*x+c)+1)/(-1/(cos(d*x+c)+1))^(1/2
))*(-1/(cos(d*x+c)+1))^(1/2)*sec(d*x+c)+15*2^(1/2)*arctan(1/2*sin(d*x+c)*2
^(1/2)/(cos(d*x+c)+1)/(-1/(cos(d*x+c)+1))^(1/2))*(-1/(cos(d*x+c)+1))^(1/2)
*sec(d*x+c)^2-6*sin(d*x+c)+2*tan(d*x+c)-26*sec(d*x+c)*tan(d*x+c))
```

3.250.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 342, normalized size of antiderivative = 2.02

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\,dx$$
$$= \left[\frac{\frac{15\sqrt{2}(a\cos(dx+c)+a)\log\left(-\frac{\cos(dx+c)^2+\frac{2\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)}{\sqrt{a}}-2\cos(dx+c)-3}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)}{\sqrt{a}}+\frac{4\left(3\cos(dx+c)^3-\cos(dx+c)^2+13\cos\right.}{\sqrt{\cos(}}}{30\,(ad\cos(dx+c)+ad)}\right.$$

input
```
integrate(1/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
[1/30*(15*sqrt(2)*(a*cos(d*x + c) + a)*log(-(cos(d*x + c)^2 + 2*sqrt(2)*sq
rt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/sqrt
(a) - 2*cos(d*x + c) - 3)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1))/sqrt(a) +
 4*(3*cos(d*x + c)^3 - cos(d*x + c)^2 + 13*cos(d*x + c))*sqrt((a*cos(d*x +
 c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a*d*cos(d*x + c)
+ a*d), 1/15*(15*sqrt(2)*(a*cos(d*x + c) + a)*sqrt(-1/a)*arctan(sqrt(2)*sq
rt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(-1/a)*sqrt(cos(d*x + c))/sin(d*
x + c)) + 2*(3*cos(d*x + c)^3 - cos(d*x + c)^2 + 13*cos(d*x + c))*sqrt((a*
cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a*d*cos(
d*x + c) + a*d)]
```

3.250.6 Sympy [F]

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{a\left(\sec\left(c+dx\right)+1\right)}\sec^{\frac{5}{2}}\left(c+dx\right)}\,dx$$

input
```
integrate(1/sec(d*x+c)**(5/2)/(a+a*sec(d*x+c))**(1/2),x)
```

output
```
Integral(1/(sqrt(a*(sec(c + d*x) + 1))*sec(c + d*x)**(5/2)), x)
```

3.250.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 357 vs. $2(140) = 280$.

Time = 0.44 (sec) , antiderivative size = 357, normalized size of antiderivative = 2.11

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\,dx$$
$$= \frac{\sqrt{2}\left(60\cos\left(\frac{4}{5}\arctan\left(\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right),\cos\left(\frac{5}{2}dx+\frac{5}{2}c\right)\right)\right)\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right)-5\cos\left(\frac{2}{5}\arctan\left(\sin\left(\frac{5}{2}dx+\right.\right.\right.\right.}{}$$

input
```
integrate(1/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
1/60*sqrt(2)*(60*cos(4/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c
)))*sin(5/2*d*x + 5/2*c) - 5*cos(2/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2
*d*x + 5/2*c)))*sin(5/2*d*x + 5/2*c) - 60*cos(5/2*d*x + 5/2*c)*sin(4/5*arc
tan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 5*cos(5/2*d*x + 5/2*c)
*sin(2/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) - 30*log(cos
(1/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c)))^2 + sin(1/5*arct
an2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c)))^2 + 2*sin(1/5*arctan2(sin
(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 1) + 30*log(cos(1/5*arctan2(si
n(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c)))^2 + sin(1/5*arctan2(sin(5/2*d*x
 + 5/2*c), cos(5/2*d*x + 5/2*c)))^2 - 2*sin(1/5*arctan2(sin(5/2*d*x + 5/2*
c), cos(5/2*d*x + 5/2*c))) + 1) + 6*sin(5/2*d*x + 5/2*c) - 5*sin(3/5*arcta
n2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 60*sin(1/5*arctan2(sin(5
/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))))/(sqrt(a)*d)
```

3.250.8 Giac [F]

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{a\sec(dx+c)+a}\sec(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(a*sec(d*x + c) + a)*sec(d*x + c)^(5/2)), x)
```

3.250.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\,dx=\int \frac{1}{\sqrt{a+\frac{a}{\cos(c+d\,x)}}\,\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

input
```
int(1/((a + a/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(5/2)),x)
```

output
```
int(1/((a + a/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(5/2)), x)
```

3.251 $\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx$

3.251.1 Optimal result 1991
3.251.2 Mathematica [A] (verified) 1991
3.251.3 Rubi [A] (verified) 1992
3.251.4 Maple [B] (verified) 1996
3.251.5 Fricas [A] (verification not implemented) 1997
3.251.6 Sympy [F(-1)] 1998
3.251.7 Maxima [B] (verification not implemented) 1998
3.251.8 Giac [F] 1999
3.251.9 Mupad [F(-1)] 2000

3.251.1 Optimal result

Integrand size = 25, antiderivative size = 174

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = -\frac{3\operatorname{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^{3/2}d} + \frac{9\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} - \frac{\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{3\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2ad\sqrt{a+a\sec(c+dx)}}$$

output
```
-3*arcsinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/a^(3/2)/d-1/2*sec(d*
x+c)^(5/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(3/2)+9/4*arctanh(1/2*sin(d*x+c)*
a^(1/2)*sec(d*x+c)^(1/2)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))/a^(3/2)/d*2^(1/2)
+3/2*sec(d*x+c)^(3/2)*sin(d*x+c)/a/d/(a+a*sec(d*x+c))^(1/2)
```

3.251.2 Mathematica [A] (verified)

Time = 0.63 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.45

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \frac{6\sqrt{1-\sec(c+dx)}\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)+4\sqrt{1-\sec(c+dx)}\sec^{\frac{5}{2}}(c+d}{}$$

input
```
Integrate[Sec[c + d*x]^(7/2)/(a + a*Sec[c + d*x])^(3/2),x]
```

output
```
(6*Sqrt[1 - Sec[c + d*x]]*Sec[c + d*x]^(3/2)*Sin[c + d*x] + 4*Sqrt[1 - Sec
[c + d*x]]*Sec[c + d*x]^(5/2)*Sin[c + d*x] - 9*Sqrt[2]*ArcTan[(Sqrt[2]*Sqr
t[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*Tan[c + d*x] - 9*Sqrt[2]*ArcTan[(
Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*Sec[c + d*x]*Tan[c + d
*x] + 6*ArcSin[Sqrt[1 - Sec[c + d*x]]]*(1 + Sec[c + d*x])*Tan[c + d*x] + 1
8*ArcSin[Sqrt[Sec[c + d*x]]]*(1 + Sec[c + d*x])*Tan[c + d*x])/(4*d*Sqrt[1
- Sec[c + d*x]]*(a*(1 + Sec[c + d*x]))^(3/2))
```

3.251.3 Rubi [A] (verified)

Time = 1.02 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.05, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.520$, Rules used = {3042, 4303, 27, 3042, 4509, 25, 3042, 4511, 3042, 4288, 222, 4295, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a\sec(c+dx)+a)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}}dx$$

$$\Big\downarrow \text{4303}$$

$$-\frac{\int \frac{3\sec^{\frac{3}{2}}(c+dx)(a-2a\sec(c+dx))}{2\sqrt{\sec(c+dx)a+a}}dx}{2a^2}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{3\int \frac{\sec^{\frac{3}{2}}(c+dx)(a-2a\sec(c+dx))}{\sqrt{\sec(c+dx)a+a}}dx}{4a^2}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{3\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a-2a\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a^2}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{4509}$$

$$-\frac{3\left(\frac{\int -\frac{\sqrt{\sec(c+dx)}\left(a^2-2a^2\sec(c+dx)\right)}{\sqrt{\sec(c+dx)a+a}}dx}{a}-\frac{2a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}\right)}{4a^2}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

$\Big\downarrow$ 25

$$-\frac{3\left(-\frac{\int \frac{\sqrt{\sec(c+dx)}\left(a^2-2a^2\sec(c+dx)\right)}{\sqrt{\sec(c+dx)a+a}}dx}{a}-\frac{2a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}\right)}{4a^2}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

$\Big\downarrow$ 3042

$$-\frac{3\left(-\frac{\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a^2-2a^2\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{a}-\frac{2a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}\right)}{4a^2}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

$\Big\downarrow$ 4511

$$-\frac{3\left(-\frac{3a^2\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{\sec(c+dx)a+a}}dx-2a\int \sqrt{\sec(c+dx)}\sqrt{\sec(c+dx)a+a}dx}{a}-\frac{2a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}\right)}{4a^2}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

$\Big\downarrow$ 3042

$$-\frac{3\left(-\frac{3a^2\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx-2a\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{a}-\frac{2a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}\right)}{4a^2}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

$\Big\downarrow$ 4288

3.251. $\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx$

$$-\frac{3\left(-\frac{3a^2\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx+\frac{4a\int \frac{1}{\sqrt{\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1}}d\left(-\frac{a\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}}{a}-\frac{2a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}\right)}{4a^2}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

$\downarrow$ 222

$$-\frac{3\left(-\frac{3a^2\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx-\frac{4a^{3/2}\text{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}}{a}-\frac{2a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}\right)}{4a^2}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

$\downarrow$ 4295

$$-\frac{3\left(-\frac{-\frac{6a^2\int \frac{1}{2a-\frac{a^2\sin(c+dx)\tan(c+dx)}{\sec(c+dx)a+a}}d\left(-\frac{a\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}-\frac{4a^{3/2}\text{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}}{a}-\frac{2a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}\right)}{4a^2}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

$\downarrow$ 219

$$-\frac{3\left(-\frac{\frac{3\sqrt{2}a^{3/2}\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{d}-\frac{4a^{3/2}\text{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}}{a}-\frac{2a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}\right)}{4a^2}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

input
```
Int[Sec[c + d*x]^(7/2)/(a + a*Sec[c + d*x])^(3/2),x]
```

3.251. $\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx$

output
```
-1/2*(Sec[c + d*x]^(5/2)*Sin[c + d*x])/(d*(a + a*Sec[c + d*x])^(3/2)) - (3
*(-(((-4*a^(3/2)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])
/d + (3*Sqrt[2]*a^(3/2)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/
(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/d)/a) - (2*a*Sec[c + d*x]^(3/2)*Sin[c
 + d*x])/(d*Sqrt[a + a*Sec[c + d*x]])))/(4*a^2)
```

3.251.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4288
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[-2*(a/(b*f))*Sqrt[a*(d/b)]   Subst[Int[1/Sqrt[1
+ x^2/a], x], x, b*(Cot[e + f*x]/Sqrt[a + b*Csc[e + f*x]])], x] /; FreeQ[{a
, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[a*(d/b), 0]
```

rule 4295
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[-2*b*(d/(a*f))   Subst[Int[1/(2*b - d*x^2), x],
x, b*(Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]]))], x] /;
 FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

3.251. $\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx$

rule 4303
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-d^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d
*Csc[e + f*x])^(n - 2)/(f*(2*m + 1))), x] + Simp[d^2/(a*b*(2*m + 1))   Int[
(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n
 + 2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2,
0] && LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

rule 4509
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*d*
Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(f*(m + n))),
 x] + Simp[d/(b*(m + n))   Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n -
 1)*Simp[b*B*(n - 1) + (A*b*(m + n) + a*B*m)*Csc[e + f*x], x], x], x] /; Fr
eeQ[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
&& GtQ[n, 1]
```

rule 4511
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b -
a*B)/b   Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n, x], x] + Simp[B/b
 Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b
, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
```

3.251.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 419 vs. $2(143) = 286$.

Time = 1.89 (sec) , antiderivative size = 420, normalized size of antiderivative = 2.41

method	result
default	$-\frac{\sec(dx+c)^{\frac{7}{2}}\sqrt{a(1+\sec(dx+c))}\left(9\arctan\left(\frac{\sin(dx+c)\sqrt{2}}{2(\cos(dx+c)+1)\sqrt{-\frac{1}{\cos(dx+c)+1}}}\right)\sqrt{2}\cos(dx+c)^5+6\arctan\left(\frac{\cos(dx+c)-\sin(dx+c)+1}{2(\cos(dx+c)+1)\sqrt{-\frac{1}{\cos(dx+}}}\right.\right.}{}$

input
```
int(sec(d*x+c)^(7/2)/(a+a*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4/d/a^2*sec(d*x+c)^(7/2)*(a*(1+sec(d*x+c)))^(1/2)/(cos(d*x+c)+1)^2/(-1/
(cos(d*x+c)+1))^(1/2)*(9*arctan(1/2*sin(d*x+c)*2^(1/2)/(cos(d*x+c)+1)/(-1/
(cos(d*x+c)+1))^(1/2))*2^(1/2)*cos(d*x+c)^5+6*arctan(1/2*(cos(d*x+c)-sin(d
*x+c)+1)/(cos(d*x+c)+1)/(-1/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)^5-6*arctan(1
/2*(cos(d*x+c)+sin(d*x+c)+1)/(cos(d*x+c)+1)/(-1/(cos(d*x+c)+1))^(1/2))*cos
(d*x+c)^5+9*arctan(1/2*sin(d*x+c)*2^(1/2)/(cos(d*x+c)+1)/(-1/(cos(d*x+c)+1
))^(1/2))*2^(1/2)*cos(d*x+c)^4-6*(-1/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^4*si
n(d*x+c)+6*arctan(1/2*(cos(d*x+c)-sin(d*x+c)+1)/(cos(d*x+c)+1)/(-1/(cos(d*
x+c)+1))^(1/2))*cos(d*x+c)^4-6*arctan(1/2*(cos(d*x+c)+sin(d*x+c)+1)/(cos(d
*x+c)+1)/(-1/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)^4-4*sin(d*x+c)*(-1/(cos(d*x
+c)+1))^(1/2)*cos(d*x+c)^3)
```

3.251.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 579, normalized size of antiderivative = 3.33

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \left[\frac{9\sqrt{2}\left(\cos\left(dx+c\right)^2+2\,\cos\left(dx+c\right)+1\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^2-2\sqrt{2}\sqrt{a}\sqrt{\frac{a\cos(dx}{\cos(d}}}{\cos(d}\right.}{}\right.$$

$$-\frac{9\sqrt{2}\left(\cos\left(dx+c\right)^2+2\,\cos\left(dx+c\right)+1\right)\sqrt{-a}\arctan\left(\frac{\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}}{a\sin(dx+c)}\right)+6\left(\cos\left(dx+c\right)^2+\right.}{4\left(a^2d\cos\left(dx+c\right)^2+2\right.}$$

input
```
integrate(sec(d*x+c)^(7/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
[1/8*(9*sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*log(-(a*cos(
d*x + c)^2 - 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqr
t(cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2
*cos(d*x + c) + 1)) + 6*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*log(
(a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 + 4*(cos(d*x + c)^2 - 2*cos(d*x + c
))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d
*x + c)) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) + 4*sqrt((a*cos(d*x + c
) + a)/cos(d*x + c))*(3*cos(d*x + c) + 2)*sin(d*x + c)/sqrt(cos(d*x + c)))
/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d), -1/4*(9*sqrt(2)*(c
os(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt(
(a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))/(a*sin(d*x + c))) +
6*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a
*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*
x + c)^2 - a*cos(d*x + c) - 2*a)) - 2*sqrt((a*cos(d*x + c) + a)/cos(d*x +
c))*(3*cos(d*x + c) + 2)*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^2*d*cos(d*x +
 c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)]
```

3.251.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**(7/2)/(a+a*sec(d*x+c))**(3/2),x)
```

output
```
Timed out
```

3.251.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4934 vs. $2(143) = 286$.

Time = 0.67 (sec) , antiderivative size = 4934, normalized size of antiderivative = 28.36

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(sec(d*x+c)^(7/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
-1/4*(12*(sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c) + 2*sin(3/2*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c))) + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c))))*cos(7/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 8*(s
in(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - sin(3/4*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c))) - 3*sin(1/4*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c))))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*
(sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c) + 2*sin(1/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c))))*cos(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))
) - 4*(sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c) + 2*sin(1/2*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c))))*cos(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c))) - 12*(sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*cos(1/4*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c))) + 3*(sqrt(2)*cos(4*d*x + 4*c)^2 + 4*sqrt(2
)*cos(2*d*x + 2*c)^2 + 4*sqrt(2)*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c)))^2 + 4*sqrt(2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c)))^2 + sqrt(2)*sin(4*d*x + 4*c)^2 + 4*sqrt(2)*sin(4*d*x + 4*c)*sin(2*d*x
 + 2*c) + 4*sqrt(2)*sin(2*d*x + 2*c)^2 + 4*sqrt(2)*sin(3/2*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c)))^2 + 4*sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c)))^2 + 2*(2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(4*
d*x + 4*c) + 4*(sqrt(2)*cos(4*d*x + 4*c) + 2*sqrt(2)*cos(2*d*x + 2*c) + 2*
sqrt(2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sqrt(2))...
```

3.251.8 Giac [F]

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \int \frac{\sec\left(dx+c\right)^{\frac{7}{2}}}{\left(a\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^(7/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^(7/2)/(a*sec(d*x + c) + a)^(3/2), x)
```

3.251.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx=\int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{7/2}}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

input
```
int((1/cos(c + d*x))^(7/2)/(a + a/cos(c + d*x))^(3/2),x)
```

output
```
int((1/cos(c + d*x))^(7/2)/(a + a/cos(c + d*x))^(3/2), x)
```

3.252 $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx$

3.252.1 Optimal result 2001
3.252.2 Mathematica [A] (verified) 2001
3.252.3 Rubi [A] (verified) 2002
3.252.4 Maple [B] (warning: unable to verify) 2005
3.252.5 Fricas [B] (verification not implemented) 2005
3.252.6 Sympy [F(-1)] 2006
3.252.7 Maxima [B] (verification not implemented) 2006
3.252.8 Giac [F] 2007
3.252.9 Mupad [F(-1)] 2008

3.252.1 Optimal result

Integrand size = 25, antiderivative size = 134

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \frac{2\operatorname{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^{3/2}d} - \frac{5\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} - \frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}}$$

output
```
2*arcsinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/a^(3/2)/d-1/2*sec(d*x
+c)^(3/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(3/2)-5/4*arctanh(1/2*sin(d*x+c)*a
^(1/2)*sec(d*x+c)^(1/2)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))/a^(3/2)/d*2^(1/2)
```

3.252.2 Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.64

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \frac{-2\sqrt{1-\sec(c+dx)}\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)+5\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)\text{ta}}{}$$

input
```
Integrate[Sec[c + d*x]^(5/2)/(a + a*Sec[c + d*x])^(3/2),x]
```

output
```
(-2*Sqrt[1 - Sec[c + d*x]]*Sec[c + d*x]^(3/2)*Sin[c + d*x] + 5*Sqrt[2]*Arc
Tan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*Tan[c + d*x] + 5*
Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*Sec[c
+ d*x]*Tan[c + d*x] - 2*ArcSin[Sqrt[1 - Sec[c + d*x]]]*(1 + Sec[c + d*x])*
Tan[c + d*x] - 10*ArcSin[Sqrt[Sec[c + d*x]]]*(1 + Sec[c + d*x])*Tan[c + d*
x])/(4*d*Sqrt[1 - Sec[c + d*x]]*(a*(1 + Sec[c + d*x]))^(3/2))
```

3.252.3 Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.04, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 4303, 27, 3042, 4511, 3042, 4288, 222, 4295, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a\sec(c+dx)+a)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}}dx$$

$$\Big\downarrow \text{4303}$$

$$-\frac{\int \frac{\sqrt{\sec(c+dx)}(a-4a\sec(c+dx))}{2\sqrt{\sec(c+dx)a+a}}dx}{2a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{\sqrt{\sec(c+dx)}(a-4a\sec(c+dx))}{\sqrt{\sec(c+dx)a+a}}dx}{4a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a-4a\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{4511}$$

$$-\frac{5a\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{\sec(c+dx)a+a}}dx-4\int \sqrt{\sec(c+dx)}\sqrt{\sec(c+dx)a+a}dx}{4a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\frac{5a\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx-4\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{4a^2}}{} - \frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \; 4288$$

$$-\frac{\frac{5a\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx+\frac{8\int \frac{1}{\sqrt{\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1}}d\left(-\frac{a\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}}{4a^2}}{} - \frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \; 222$$

$$-\frac{\frac{5a\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx-\frac{8\sqrt{a}\text{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}}{4a^2}}{} - \frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \; 4295$$

$$-\frac{-\frac{\frac{10a\int \frac{1}{2a-\frac{a^2\sin(c+dx)\tan(c+dx)}{\sec(c+dx)a+a}}d\left(-\frac{a\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}-\frac{8\sqrt{a}\text{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}}{4a^2}}{} - \frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \; 219$$

$$-\frac{\frac{\frac{5\sqrt{2}\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{d}-\frac{8\sqrt{a}\text{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}}{4a^2}}{} - \frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

input
```
Int[Sec[c + d*x]^(5/2)/(a + a*Sec[c + d*x])^(3/2),x]
```

output
```
-1/4*((-8*Sqrt[a]*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]
)/d + (5*Sqrt[2]*Sqrt[a]*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])
/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/d)/a^2 - (Sec[c + d*x]^(3/2)*Sin[c +
 d*x])/(2*d*(a + a*Sec[c + d*x])^(3/2))
```

3.252. $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx$

3.252.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4288
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[-2*(a/(b*f))*Sqrt[a*(d/b)]   Subst[Int[1/Sqrt[1
+ x^2/a], x], x, b*(Cot[e + f*x]/Sqrt[a + b*Csc[e + f*x]])], x] /; FreeQ[{a
, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[a*(d/b), 0]
```

rule 4295
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[-2*b*(d/(a*f))   Subst[Int[1/(2*b - d*x^2), x],
x, b*(Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]]))], x] /;
 FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4303
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-d^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d
*Csc[e + f*x])^(n - 2)/(f*(2*m + 1))), x] + Simp[d^2/(a*b*(2*m + 1))   Int[
(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n
 + 2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2,
0] && LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

3.252. $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx$

rule 4511
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b -
a*B)/b   Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n, x], x] + Simp[B/b
 Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b
, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
```

3.252.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 360 vs. $2(109) = 218$.

Time = 1.76 (sec) , antiderivative size = 361, normalized size of antiderivative = 2.69

method	result
default	$\frac{\left(-\frac{(1-\cos(dx+c))^2\csc(dx+c)^2+1}{(1-\cos(dx+c))^2\csc(dx+c)^2-1}\right)^{\frac{5}{2}}\left((1-\cos(dx+c))^2\csc(dx+c)^2-1\right)^3\sqrt{-\frac{2a}{(1-\cos(dx+c))^2\csc(dx+c)^2-1}}\left(2\sqrt{2}\arctan\left(\frac{(\csc(dx+c)\cdots}{2\sqrt{-(1-\cos(d\cdots}}\right.\right.}{4d\,a^2\left((1-\mathrm{co}\cdots\right.}$

input
```
int(sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/4/d/a^2*(-((1-cos(d*x+c))^2*csc(d*x+c)^2+1)/((1-cos(d*x+c))^2*csc(d*x+c)
^2-1))^(5/2)*((1-cos(d*x+c))^2*csc(d*x+c)^2-1)^3*(-2*a/((1-cos(d*x+c))^2*c
sc(d*x+c)^2-1))^(1/2)*(2*2^(1/2)*arctan(1/2*(csc(d*x+c)-cot(d*x+c)+1)*2^(1
/2)/(-(1-cos(d*x+c))^2*csc(d*x+c)^2-1)^(1/2))+2*2^(1/2)*arctan(1/2*(-cot(d
*x+c)+csc(d*x+c)-1)*2^(1/2)/(-(1-cos(d*x+c))^2*csc(d*x+c)^2-1)^(1/2))+(-(1
-cos(d*x+c))^2*csc(d*x+c)^2-1)^(1/2)*(-cot(d*x+c)+csc(d*x+c))-5*arctan(1/(
-(1-cos(d*x+c))^2*csc(d*x+c)^2-1)^(1/2)*(-cot(d*x+c)+csc(d*x+c))))/((1-cos
(d*x+c))^2*csc(d*x+c)^2+1)^2/(-(1-cos(d*x+c))^2*csc(d*x+c)^2-1)^(1/2)
```

3.252.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 244 vs. $2(109) = 218$.

Time = 0.31 (sec) , antiderivative size = 559, normalized size of antiderivative = 4.17

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \left[\frac{5\sqrt{2}\left(\cos(dx+c)^2+2\cos(dx+c)+1\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^2+2\sqrt{2}\sqrt{a}\sqrt{\frac{a\cos(d\cdots}{\cos(d\cdots}}}{\cos(d\cdots}\right.}{\cdots}\right.$$

input
```
integrate(sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
[1/8*(5*sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*log(-(a*cos(
d*x + c)^2 + 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqr
t(cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2
*cos(d*x + c) + 1)) + 4*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*log(
(a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 - 2*cos(d*x + c
))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d
*x + c)) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) - 4*sqrt((a*cos(d*x + c
) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(a^2*d*cos(d*x + c)^
2 + 2*a^2*d*cos(d*x + c) + a^2*d), 1/4*(5*sqrt(2)*(cos(d*x + c)^2 + 2*cos(
d*x + c) + 1)*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/c
os(d*x + c))*sqrt(cos(d*x + c))/(a*sin(d*x + c))) + 4*(cos(d*x + c)^2 + 2*
cos(d*x + c) + 1)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos
(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x +
 c) - 2*a)) - 2*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))
*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)]
```

3.252.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**(5/2)/(a+a*sec(d*x+c))**(3/2),x)
```

output
```
Timed out
```

3.252.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2122 vs. $2(109) = 218$.

Time = 0.44 (sec) , antiderivative size = 2122, normalized size of antiderivative = 15.84

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
1/4*(4*(sin(2*d*x + 2*c) + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c))))*cos(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*(sqrt(2)
*cos(2*d*x + 2*c)^2 + 4*sqrt(2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c)))^2 + sqrt(2)*sin(2*d*x + 2*c)^2 + 4*sqrt(2)*sin(2*d*x + 2*c)*sin
(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*sqrt(2)*sin(1/2*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 4*(sqrt(2)*cos(2*d*x + 2*c) +
 sqrt(2))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)
*cos(2*d*x + 2*c) + sqrt(2))*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2
 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt
(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) - 2*(sqrt(2)
*cos(2*d*x + 2*c)^2 + 4*sqrt(2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c)))^2 + sqrt(2)*sin(2*d*x + 2*c)^2 + 4*sqrt(2)*sin(2*d*x + 2*c)*sin
(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*sqrt(2)*sin(1/2*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 4*(sqrt(2)*cos(2*d*x + 2*c) +
 sqrt(2))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)
*cos(2*d*x + 2*c) + sqrt(2))*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2
 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2*sqrt
(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) + 2*(sqrt...
```

3.252.8 Giac [F]

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \int \frac{\sec\left(dx+c\right)^{\frac{5}{2}}}{\left(a\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^(5/2)/(a*sec(d*x + c) + a)^(3/2), x)
```

3.252.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx=\int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

input
```
int((1/cos(c + d*x))^(5/2)/(a + a/cos(c + d*x))^(3/2),x)
```

output
```
int((1/cos(c + d*x))^(5/2)/(a + a/cos(c + d*x))^(3/2), x)
```

3.253 $\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx$

3.253.1 Optimal result 2009
3.253.2 Mathematica [B] (verified) 2009
3.253.3 Rubi [A] (verified) 2010
3.253.4 Maple [B] (verified) 2011
3.253.5 Fricas [A] (verification not implemented) 2012
3.253.6 Sympy [F] 2013
3.253.7 Maxima [B] (verification not implemented) 2013
3.253.8 Giac [F] 2014
3.253.9 Mupad [F(-1)] 2014

3.253.1 Optimal result

Integrand size = 25, antiderivative size = 97

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx=\frac{\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}}$$

output
```
1/2*sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(3/2)+1/4*arctanh(1/2*s
in(d*x+c)*a^(1/2)*sec(d*x+c)^(1/2)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))/a^(3/2)
/d*2^(1/2)
```

3.253.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 220 vs. 2(97) = 194.

Time = 0.52 (sec) , antiderivative size = 220, normalized size of antiderivative = 2.27

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx=\frac{2\sqrt{1-\sec(c+dx)}\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)-\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)\tan(}{}$$

input
```
Integrate[Sec[c + d*x]^(3/2)/(a + a*Sec[c + d*x])^(3/2),x]
```

output
```
(2*Sqrt[1 - Sec[c + d*x]]*Sec[c + d*x]^(3/2)*Sin[c + d*x] - Sqrt[2]*ArcTan
[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*Tan[c + d*x] - Sqrt[
2]*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*Sec[c + d*x
]*Tan[c + d*x] + 2*ArcSin[Sqrt[1 - Sec[c + d*x]]]*(1 + Sec[c + d*x])*Tan[c
 + d*x] + 2*ArcSin[Sqrt[Sec[c + d*x]]]*(1 + Sec[c + d*x])*Tan[c + d*x])/(4
*d*Sqrt[1 - Sec[c + d*x]]*(a*(1 + Sec[c + d*x]))^(3/2))
```

3.253.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 4297, 3042, 4295, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a\sec(c+dx)+a)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}}dx$$

$$\Big\downarrow \text{4297}$$

$$\frac{\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{\sec(c+dx)a+a}}dx}{4a}+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a}+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{4295}$$

$$\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}-\frac{\int \frac{1}{2a-\frac{a^2\sin(c+dx)\tan(c+dx)}{\sec(c+dx)a+a}}d\left(-\frac{a\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{2ad}$$

$$\Big\downarrow \text{219}$$

$$\frac{\operatorname{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

input
```
Int[Sec[c + d*x]^(3/2)/(a + a*Sec[c + d*x])^(3/2),x]
```

output
```
ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[
c + d*x]])]/(2*Sqrt[2]*a^(3/2)*d) + (Sec[c + d*x]^(3/2)*Sin[c + d*x])/(2*d
*(a + a*Sec[c + d*x])^(3/2))
```

3.253.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4295
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[-2*b*(d/(a*f))   Subst[Int[1/(2*b - d*x^2), x],
x, b*(Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]]))], x] /;
 FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4297
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[b*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Cs
c[e + f*x])^(n - 1)/(a*f*(2*m + 1))), x] + Simp[d*((m + 1)/(b*(2*m + 1)))
 Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ
[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && EqQ[m + n, 0] && LtQ[m,
-2^(-1)] && IntegerQ[2*m]
```

3.253.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 169 vs. $2(78) = 156$.

Time = 1.28 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.75

method	result
default	$-\frac{\left(\arctan\left(\frac{\sin(dx+c)\sqrt{2}}{2(\cos(dx+c)+1)\sqrt{-\frac{1}{\cos(dx+c)+1}}}\right)\sqrt{2}\,\cos(dx+c)+\arctan\left(\frac{\sin(dx+c)\sqrt{2}}{2(\cos(dx+c)+1)\sqrt{-\frac{1}{\cos(dx+c)+1}}}\right)\sqrt{2}-2\sin(dx+c)\sqrt{-\frac{1}{\cos(dx+}}\right.}{4d\,a^2(\cos(dx+c)+1)^2\sqrt{-\frac{1}{\cos(dx+c)+1}}}$

input
```
int(sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4/d/a^2*(arctan(1/2*sin(d*x+c)*2^(1/2)/(cos(d*x+c)+1)/(-1/(cos(d*x+c)+1
))^(1/2))*2^(1/2)*cos(d*x+c)+arctan(1/2*sin(d*x+c)*2^(1/2)/(cos(d*x+c)+1)/
(-1/(cos(d*x+c)+1))^(1/2))*2^(1/2)-2*sin(d*x+c)*(-1/(cos(d*x+c)+1))^(1/2))
*sec(d*x+c)^(3/2)*(a*(1+sec(d*x+c)))^(1/2)*cos(d*x+c)^2/(cos(d*x+c)+1)^2/(
-1/(cos(d*x+c)+1))^(1/2)
```

3.253.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 338, normalized size of antiderivative = 3.48

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \left[\frac{\sqrt{2}\left(\cos\left(dx+c\right)^2+2\,\cos\left(dx+c\right)+1\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^2-2\sqrt{2}\sqrt{a}\sqrt{\frac{a\cos(dx+}{\cos(dx+}}}{\cos(dx+}\right.}{8\left(a^2d\cos\left(dx+c\right)^2\right.}\right.$$

$$-\frac{\sqrt{2}\left(\cos\left(dx+c\right)^2+2\,\cos\left(dx+c\right)+1\right)\sqrt{-a}\arctan\left(\frac{\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}}{a\sin(dx+c)}\right)-2\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\mathrm{co}}}{4\left(a^2d\cos\left(dx+c\right)^2+2\,a^2d\cos\left(dx+c\right)+a^2d\right)}$$

input
```
integrate(sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
[1/8*(sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*log(-(a*cos(d*
x + c)^2 - 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(
cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*c
os(d*x + c) + 1)) + 4*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x
 + c))*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)
, -1/4*(sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(-a)*arctan(sqrt
(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))/(a
*sin(d*x + c))) - 2*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x +
 c))*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)]
```

3.253.6 Sympy [F]

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a(\sec(c+dx)+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)**(3/2)/(a+a*sec(d*x+c))**(3/2),x)
```

output
```
Integral(sec(c + d*x)**(3/2)/(a*(sec(c + d*x) + 1))**(3/2), x)
```

3.253.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 15721 vs. $2(78) = 156$.

Time = 0.89 (sec) , antiderivative size = 15721, normalized size of antiderivative = 162.07

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
1/4*(32*(cos(3/2*d*x + 3/2*c)*sin(2*d*x + 2*c) + cos(2*d*x + 2*c)*sin(3/2*
d*x + 3/2*c) + cos(d*x + c)*sin(3/2*d*x + 3/2*c) + cos(3/2*d*x + 3/2*c)*si
n(d*x + c))*cos(3*d*x + 3*c)^2 + 96*(cos(3/2*d*x + 3/2*c)*sin(3*d*x + 3*c)
 + 3*cos(3/2*d*x + 3/2*c)*sin(2*d*x + 2*c) - (3*cos(d*x + c) + 1)*sin(3/2*
d*x + 3/2*c) - cos(3*d*x + 3*c)*sin(3/2*d*x + 3/2*c) - 3*cos(2*d*x + 2*c)*
sin(3/2*d*x + 3/2*c) + 3*cos(3/2*d*x + 3/2*c)*sin(d*x + c))*cos(4/3*arctan
2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 96*(cos(3/2*d*x + 3/2*c
)*sin(3*d*x + 3*c) + 3*cos(3/2*d*x + 3/2*c)*sin(2*d*x + 2*c) - (3*cos(d*x
+ c) + 1)*sin(3/2*d*x + 3/2*c) - cos(3*d*x + 3*c)*sin(3/2*d*x + 3/2*c) - 3
*cos(2*d*x + 2*c)*sin(3/2*d*x + 3/2*c) + 3*cos(3/2*d*x + 3/2*c)*sin(d*x +
c))*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 - 32*(c
os(3/2*d*x + 3/2*c)*sin(2*d*x + 2*c) + cos(2*d*x + 2*c)*sin(3/2*d*x + 3/2*
c) + cos(d*x + c)*sin(3/2*d*x + 3/2*c) + cos(3/2*d*x + 3/2*c)*sin(d*x + c)
)*sin(3*d*x + 3*c)^2 + 32*(6*cos(d*x + c) + 1)*cos(2*d*x + 2*c)*sin(3/2*d*
x + 3/2*c) + 96*cos(2*d*x + 2*c)^2*sin(3/2*d*x + 3/2*c) + 96*sin(2*d*x + 2
*c)^2*sin(3/2*d*x + 3/2*c) + 96*(cos(3/2*d*x + 3/2*c)*sin(3*d*x + 3*c) + 3
*cos(3/2*d*x + 3/2*c)*sin(2*d*x + 2*c) - (3*cos(d*x + c) + 1)*sin(3/2*d*x
+ 3/2*c) - cos(3*d*x + 3*c)*sin(3/2*d*x + 3/2*c) - 3*cos(2*d*x + 2*c)*sin(
3/2*d*x + 3/2*c) + 3*cos(3/2*d*x + 3/2*c)*sin(d*x + c))*sin(4/3*arctan2(si
n(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 96*(cos(3/2*d*x + 3/2*c)...
```

3.253.8 Giac [F]

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \int \frac{\sec{(dx+c)}^{\frac{3}{2}}}{(a\sec{(dx+c)}+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^(3/2)/(a*sec(d*x + c) + a)^(3/2), x)
```

3.253.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

input
```
int((1/cos(c + d*x))^(3/2)/(a + a/cos(c + d*x))^(3/2),x)
```

output
```
int((1/cos(c + d*x))^(3/2)/(a + a/cos(c + d*x))^(3/2), x)
```

3.254 $\int \frac{\sqrt{\sec(c+dx)}}{(a+a\sec(c+dx))^{3/2}}\,dx$

3.254.1 Optimal result 2015
3.254.2 Mathematica [A] (verified) 2015
3.254.3 Rubi [A] (verified) 2016
3.254.4 Maple [B] (verified) 2017
3.254.5 Fricas [A] (verification not implemented) 2018
3.254.6 Sympy [F] 2019
3.254.7 Maxima [B] (verification not implemented) 2019
3.254.8 Giac [F] 2020
3.254.9 Mupad [F(-1)] 2020

3.254.1 Optimal result

Integrand size = 25, antiderivative size = 97

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+a\sec(c+dx))^{3/2}}\,dx=\frac{3\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}}$$

output
```
-1/2*sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(3/2)+3/4*arctanh(1/2*
sin(d*x+c)*a^(1/2)*sec(d*x+c)^(1/2)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))/a^(3/2
)/d*2^(1/2)
```

3.254.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.24

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+a\sec(c+dx))^{3/2}}\,dx=\frac{-2\sqrt{1-\sec(c+dx)}\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)-3\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)(}{4d\sqrt{1-\sec(c+dx)}(a(1+\sec(c+dx)))^{3/2}}$$

input
```
Integrate[Sqrt[Sec[c + d*x]]/(a + a*Sec[c + d*x])^(3/2),x]
```

output
```
(-2*Sqrt[1 - Sec[c + d*x]]*Sec[c + d*x]^(3/2)*Sin[c + d*x] - 3*Sqrt[2]*Arc
Tan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*(1 + Sec[c + d*x]
)*Tan[c + d*x])/(4*d*Sqrt[1 - Sec[c + d*x]]*(a*(1 + Sec[c + d*x]))^(3/2))
```

3.254.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3042, 4298, 3042, 4295, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\sec(c+dx)}}{(a\sec(c+dx)+a)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}}dx$$

$$\Big\downarrow \text{4298}$$

$$\frac{3\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{\sec(c+dx)a+a}}dx}{4a}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{4295}$$

$$-\frac{3\int \frac{1}{2a-\frac{a^2\sin(c+dx)\tan(c+dx)}{\sec(c+dx)a+a}}d\left(-\frac{a\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{2ad}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{3\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

input
```
Int[Sqrt[Sec[c + d*x]]/(a + a*Sec[c + d*x])^(3/2),x]
```

output
```
(3*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*S
ec[c + d*x]])])/(2*Sqrt[2]*a^(3/2)*d) - (Sec[c + d*x]^(3/2)*Sin[c + d*x])/
(2*d*(a + a*Sec[c + d*x])^(3/2))
```

3.254.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4295
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[-2*b*(d/(a*f))   Subst[Int[1/(2*b - d*x^2), x],
x, b*(Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]]))], x] /;
 FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4298
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-Cot[e + f*x])*(a + b*Csc[e + f*x])^m*((d*Csc
[e + f*x])^n/(f*(2*m + 1))), x] + Simp[m/(a*(2*m + 1))   Int[(a + b*Csc[e +
 f*x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f}, x] && E
qQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && LtQ[m, -2^(-1)]
```

3.254.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 169 vs. $2(78) = 156$.

Time = 1.39 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.75

method	result
default	$-\frac{\left(3\arctan\left(\frac{\sin(dx+c)\sqrt{2}}{2(\cos(dx+c)+1)\sqrt{-\frac{1}{\cos(dx+c)+1}}}\right)\sqrt{2}\cos(dx+c)+3\arctan\left(\frac{\sin(dx+c)\sqrt{2}}{2(\cos(dx+c)+1)\sqrt{-\frac{1}{\cos(dx+c)+1}}}\right)\sqrt{2}+2\sin(dx+c)\sqrt{-\frac{}{\cos(}}\right.}{4d\,a^2(\cos(dx+c)+1)^2\sqrt{-\frac{1}{\cos(dx+c)+1}}}$

input
```
int(sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4/d/a^2*(3*arctan(1/2*sin(d*x+c)*2^(1/2)/(cos(d*x+c)+1)/(-1/(cos(d*x+c)
+1))^(1/2))*2^(1/2)*cos(d*x+c)+3*arctan(1/2*sin(d*x+c)*2^(1/2)/(cos(d*x+c)
+1)/(-1/(cos(d*x+c)+1))^(1/2))*2^(1/2)+2*sin(d*x+c)*(-1/(cos(d*x+c)+1))^(1
/2))*sec(d*x+c)^(1/2)*(a*(1+sec(d*x+c)))^(1/2)*cos(d*x+c)/(cos(d*x+c)+1)^2
/(-1/(cos(d*x+c)+1))^(1/2)
```

3.254.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 340, normalized size of antiderivative = 3.51

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+a\sec(c+dx))^{3/2}}\,dx = \left[\frac{3\sqrt{2}\left(\cos\left(dx+c\right)^2+2\cos\left(dx+c\right)+1\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^2-2\sqrt{2}\sqrt{a}\sqrt{\frac{a\cos(dx}{\cos(d}}}{\cos(d}\right.}{8\left(a^2d\cos\left(dx+c\right)^2\right.}\right.$$

$$-\frac{3\sqrt{2}\left(\cos\left(dx+c\right)^2+2\cos\left(dx+c\right)+1\right)\sqrt{-a}\arctan\left(\frac{\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}}{a\sin(dx+c)}\right)+2\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{c}}{4\left(a^2d\cos\left(dx+c\right)^2+2\,a^2d\cos\left(dx+c\right)+a^2d\right)}$$

input
```
integrate(sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
[1/8*(3*sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*log(-(a*cos(
d*x + c)^2 - 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqr
t(cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2
*cos(d*x + c) + 1)) - 4*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d
*x + c))*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*
d), -1/4*(3*sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(-a)*arctan(
sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c)
)/(a*sin(d*x + c))) + 2*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d
*x + c))*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*
d)]
```

3.254.6 Sympy [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+a\sec(c+dx))^{3/2}}\,dx = \int \frac{\sqrt{\sec(c+dx)}}{(a(\sec(c+dx)+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)**(1/2)/(a+a*sec(d*x+c))**(3/2),x)
```

output
```
Integral(sqrt(sec(c + d*x))/(a*(sec(c + d*x) + 1))**(3/2), x)
```

3.254.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1031 vs. $2(78) = 156$.

Time = 0.52 (sec) , antiderivative size = 1031, normalized size of antiderivative = 10.63

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+a\sec(c+dx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
1/4*(3*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*
x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*
sin(1/2*d*x + 1/2*c) + 1))*cos(2*d*x + 2*c)^2 + 12*(log(cos(1/2*d*x + 1/2*
c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*
d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos
(d*x + c)^2 + 3*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*s
in(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*
c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(2*d*x + 2*c)^2 + 12*(log(cos(1/2*d
*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log
(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c)
+ 1))*sin(d*x + c)^2 + 2*(6*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/
2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/
2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(d*x + c) + 3*log(cos(1
/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) -
 3*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1
/2*c) + 1) - 2*sin(3/2*d*x + 3/2*c) + 2*sin(1/2*d*x + 1/2*c))*cos(2*d*x +
2*c) + 4*(3*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/
2*d*x + 1/2*c) + 1) - 3*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^
2 - 2*sin(1/2*d*x + 1/2*c) + 1) + 2*sin(1/2*d*x + 1/2*c))*cos(d*x + c) + 4
*(3*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*...
```

3.254.8 Giac [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+a\sec(c+dx))^{3/2}}\,dx = \int \frac{\sqrt{\sec(dx+c)}}{(a\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate(sqrt(sec(d*x + c))/(a*sec(d*x + c) + a)^(3/2), x)
```

3.254.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+a\sec(c+dx))^{3/2}}\,dx = \int \frac{\sqrt{\frac{1}{\cos(c+d\,x)}}}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

input
```
int((1/cos(c + d*x))^(1/2)/(a + a/cos(c + d*x))^(3/2),x)
```

output
```
int((1/cos(c + d*x))^(1/2)/(a + a/cos(c + d*x))^(3/2), x)
```

3.255 $\int \frac{1}{\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^{3/2}}\,dx$

3.255.1 Optimal result 2021
3.255.2 Mathematica [A] (verified) 2021
3.255.3 Rubi [A] (verified) 2022
3.255.4 Maple [A] (verified) 2024
3.255.5 Fricas [A] (verification not implemented) 2025
3.255.6 Sympy [F] 2025
3.255.7 Maxima [B] (verification not implemented) 2026
3.255.8 Giac [F] 2026
3.255.9 Mupad [F(-1)] 2027

3.255.1 Optimal result

Integrand size = 25, antiderivative size = 137

$$\int \frac{1}{\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^{3/2}}\,dx = -\frac{7\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} - \frac{\sqrt{\sec(c+dx)}\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{5\sqrt{\sec(c+dx)}\sin(c+dx)}{2ad\sqrt{a+a\sec(c+dx)}}$$

output
```
-7/4*arctanh(1/2*sin(d*x+c)*a^(1/2)*sec(d*x+c)^(1/2)*2^(1/2)/(a+a*sec(d*x+
c))^(1/2))/a^(3/2)/d*2^(1/2)-1/2*sin(d*x+c)*sec(d*x+c)^(1/2)/d/(a+a*sec(d*
x+c))^(3/2)+5/2*sin(d*x+c)*sec(d*x+c)^(1/2)/a/d/(a+a*sec(d*x+c))^(1/2)
```

3.255.2 Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.06

$$\int \frac{1}{\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^{3/2}}\,dx = \frac{2\left(5\sqrt{1-\sec(c+dx)}\sec^{\frac{3}{2}}(c+dx)+4\sqrt{-((-1+\sec(c+dx))}\right.}{4d\sqrt{1-\mathrm{se}}}$$

input
```
Integrate[1/(Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^(3/2)),x]
```

output
```
(2*(5*Sqrt[1 - Sec[c + d*x]]*Sec[c + d*x]^(3/2) + 4*Sqrt[-((-1 + Sec[c + d
*x])*Sec[c + d*x])])*Sin[c + d*x] + 7*Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[Sec[c +
 d*x]])/Sqrt[1 - Sec[c + d*x]]]*(1 + Sec[c + d*x])*Tan[c + d*x])/(4*d*Sqrt
[1 - Sec[c + d*x]]*(a*(1 + Sec[c + d*x]))^(3/2))
```

3.255.3 Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.01, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3042, 4304, 27, 3042, 4501, 3042, 4295, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}}dx$$

$$\Big\downarrow \text{4304}$$

$$-\frac{\int -\frac{5a-2a\sec(c+dx)}{2\sqrt{\sec(c+dx)}\sqrt{\sec(c+dx)a+a}}dx}{2a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{2d(a\sec(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{5a-2a\sec(c+dx)}{\sqrt{\sec(c+dx)}\sqrt{\sec(c+dx)a+a}}dx}{4a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{2d(a\sec(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{5a-2a\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{2d(a\sec(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{4501}$$

$$\frac{\frac{10a\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\sec(c+dx)+a}}-7a\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{\sec(c+dx)a+a}}dx}{4a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{2d(a\sec(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{10a\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\sec(c+dx)+a}}-7a\int\frac{\sqrt{\csc(c+dx+\frac{\pi}{2})}}{\sqrt{\csc(c+dx+\frac{\pi}{2})a+a}}dx}{4a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{2d(a\sec(c+dx)+a)^{3/2}}$$

$\Big\downarrow$ 4295

$$\frac{\frac{14a\int\frac{1}{2a-\frac{a^2\sin(c+dx)\tan(c+dx)}{\sec(c+dx)a+a}}d\left(-\frac{a\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}+\frac{10a\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\sec(c+dx)+a}}}{4a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{2d(a\sec(c+dx)+a)^{3/2}}$$

$\Big\downarrow$ 219

$$\frac{\frac{10a\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\sec(c+dx)+a}}-\frac{7\sqrt{2}\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{d}}{4a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{2d(a\sec(c+dx)+a)^{3/2}}$$

input
```
Int[1/(Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^(3/2)),x]
```

output
```
-1/2*(Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(d*(a + a*Sec[c + d*x])^(3/2)) + ((
-7*Sqrt[2]*Sqrt[a]*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt
[2]*Sqrt[a + a*Sec[c + d*x]])])/d + (10*a*Sqrt[Sec[c + d*x]]*Sin[c + d*x])
/(d*Sqrt[a + a*Sec[c + d*x]]))/(4*a^2)
```

3.255.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4295
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[-2*b*(d/(a*f))   Subst[Int[1/(2*b - d*x^2), x],
x, b*(Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]]))], x] /;
 FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4304
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-Cot[e + f*x])*(a + b*Csc[e + f*x])^m*((d*Csc
[e + f*x])^n/(f*(2*m + 1))), x] + Simp[1/(a^2*(2*m + 1))   Int[(a + b*Csc[e
 + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e
+ f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ
[m, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

rule 4501
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*Cot[e
 + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(f*n)), x] - Simp[(a*A*m
 - b*B*n)/(b*d*n)   Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x]
, x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a
^2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LeQ[m, -1]
```

3.255.4 Maple [A] (verified)

Time = 1.43 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.47

method	result
default	$-\frac{\left(-(1-\cos(dx+c))^3\csc(dx+c)^3+7\arctan\left(\frac{-\cot(dx+c)+\csc(dx+c)}{\sqrt{-(1-\cos(dx+c))^2\csc(dx+c)^2-1}}\right)\sqrt{-(1-\cos(dx+c))^2\csc(dx+c)^2-1}-9\csc(dx+c)+9}{4d\,a^2\sqrt{-\frac{(1-\cos(dx+c))^2\csc(dx+c)^2+1}{(1-\cos(dx+c))^2\csc(dx+c)^2-1}}}$

input
```
int(1/(a+a*sec(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4/d/a^2*(-(1-cos(d*x+c))^3*csc(d*x+c)^3+7*arctan(1/(-(1-cos(d*x+c))^2*c
sc(d*x+c)^2-1)^(1/2)*(-cot(d*x+c)+csc(d*x+c)))*(-(1-cos(d*x+c))^2*csc(d*x+
c)^2-1)^(1/2)-9*csc(d*x+c)+9*cot(d*x+c))*(-2*a/((1-cos(d*x+c))^2*csc(d*x+c
)^2-1))^(1/2)/(-((1-cos(d*x+c))^2*csc(d*x+c)^2+1)/((1-cos(d*x+c))^2*csc(d*
x+c)^2-1))^(1/2)
```

3.255.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 378, normalized size of antiderivative = 2.76

$$\int \frac{1}{\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^{3/2}}\,dx = \left[\frac{7\sqrt{2}\left(\cos\left(dx+c\right)^2+2\cos\left(dx+c\right)+1\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^2}{}\right.}{8\left(a^2\right.}\right.$$

input
```
integrate(1/(a+a*sec(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
[1/8*(7*sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*log(-(a*cos(
d*x + c)^2 + 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqr
t(cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2
*cos(d*x + c) + 1)) + 4*(4*cos(d*x + c)^2 + 5*cos(d*x + c))*sqrt((a*cos(d*
x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^2*d*cos(d*x
+ c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d), 1/4*(7*sqrt(2)*(cos(d*x + c)^2 + 2
*cos(d*x + c) + 1)*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) +
 a)/cos(d*x + c))*sqrt(cos(d*x + c))/(a*sin(d*x + c))) + 2*(4*cos(d*x + c)
^2 + 5*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/
sqrt(cos(d*x + c)))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)]
```

3.255.6 Sympy [F]

$$\int \frac{1}{\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^{3/2}}\,dx = \int \frac{1}{\left(a\left(\sec\left(c+dx\right)+1\right)\right)^{\frac{3}{2}}\sqrt{\sec\left(c+dx\right)}}\,dx$$

input
```
integrate(1/(a+a*sec(d*x+c))**(3/2)/sec(d*x+c)**(1/2),x)
```

output
```
Integral(1/((a*(sec(c + d*x) + 1))**(3/2)*sqrt(sec(c + d*x))), x)
```

3.255.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7176 vs. $2(112) = 224$.

Time = 0.49 (sec) , antiderivative size = 7176, normalized size of antiderivative = 52.38

$$\int \frac{1}{\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+a*sec(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
-1/4*(4*(7*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2
*d*x + 1/2*c) + 1) - 7*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2
 - 2*sin(1/2*d*x + 1/2*c) + 1) - 8*sin(1/2*d*x + 1/2*c))*cos(3/2*d*x + 3/2
*c)^4 + 63*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/
2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2
- 2*sin(1/2*d*x + 1/2*c) + 1))*cos(1/2*d*x + 1/2*c)^4 + 4*(7*log(cos(1/2*d
*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - 7*l
og(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c
) + 1) - 8*sin(1/2*d*x + 1/2*c))*sin(3/2*d*x + 3/2*c)^4 + 70*(log(cos(1/2*
d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - lo
g(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c)
 + 1))*cos(1/2*d*x + 1/2*c)^2*sin(1/2*d*x + 1/2*c)^2 + 7*(log(cos(1/2*d*x
+ 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(co
s(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1
))*sin(1/2*d*x + 1/2*c)^4 - 8*sin(1/2*d*x + 1/2*c)^5 + 28*(7*(log(cos(1/2*
d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - lo
g(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c)
 + 1))*cos(1/2*d*x + 1/2*c) - 8*cos(1/2*d*x + 1/2*c)*sin(1/2*d*x + 1/2*c))
*cos(3/2*d*x + 3/2*c)^3 + 4*(21*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x
+ 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 +...
```

3.255.8 Giac [F]

$$\int \frac{1}{\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^{3/2}}\,dx = \int \frac{1}{(a\sec(dx+c)+a)^{\frac{3}{2}}\sqrt{\sec(dx+c)}}\,dx$$

input
```
integrate(1/(a+a*sec(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/((a*sec(d*x + c) + a)^(3/2)*sqrt(sec(d*x + c))), x)
```

3.255.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^{3/2}}\,dx=\int \frac{1}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{3/2}\,\sqrt{\frac{1}{\cos(c+d\,x)}}}\,dx$$

input
```
int(1/((a + a/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(1/2)),x)
```

output
```
int(1/((a + a/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(1/2)), x)
```

3.256 $\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}\,dx$

3.256.1 Optimal result 2028
3.256.2 Mathematica [A] (verified) 2028
3.256.3 Rubi [A] (verified) 2029
3.256.4 Maple [A] (verified) 2032
3.256.5 Fricas [A] (verification not implemented) 2033
3.256.6 Sympy [F] 2034
3.256.7 Maxima [B] (verification not implemented) 2034
3.256.8 Giac [F] 2035
3.256.9 Mupad [F(-1)] 2035

3.256.1 Optimal result

Integrand size = 25, antiderivative size = 177

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}\,dx = \frac{11\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} - \frac{\sin(c+dx)}{2d\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^{3/2}} + \frac{7\sin(c+dx)}{6ad\sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+dx)}} - \frac{19\sqrt{\sec(c+dx)}\sin(c+dx)}{6ad\sqrt{a+a\sec(c+dx)}}$$

output
```
11/4*arctanh(1/2*sin(d*x+c)*a^(1/2)*sec(d*x+c)^(1/2)*2^(1/2)/(a+a*sec(d*x+
c))^(1/2))/a^(3/2)/d*2^(1/2)-1/2*sin(d*x+c)/d/(a+a*sec(d*x+c))^(3/2)/sec(d
*x+c)^(1/2)+7/6*sin(d*x+c)/a/d/sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2)-19/
6*sin(d*x+c)*sec(d*x+c)^(1/2)/a/d/(a+a*sec(d*x+c))^(1/2)
```

3.256.2 Mathematica [A] (verified)

Time = 0.98 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.85

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}\,dx = \frac{-33\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)\cos^2\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{5}{2}}(c+dx)\sin(}{6d\sqrt{-((-1+\sec(c+d}}$$

input
```
Integrate[1/(Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(3/2)),x]
```

output
```
(-33*Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*C
os[(c + d*x)/2]^2*Sec[c + d*x]^(5/2)*Sin[c + d*x] + Sqrt[1 - Sec[c + d*x]]
*(4*Sin[c + d*x] - (12 + 19*Sec[c + d*x])*Tan[c + d*x]))/(6*d*Sqrt[-((-1 +
 Sec[c + d*x])*Sec[c + d*x])]*(a*(1 + Sec[c + d*x]))^(3/2))
```

3.256.3 Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.06, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {3042, 4304, 27, 3042, 4510, 27, 3042, 4501, 3042, 4295, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}}dx$$

$$\Big\downarrow \text{4304}$$

$$-\frac{\int -\frac{7a-4a\sec(c+dx)}{2\sec^{\frac{3}{2}}(c+dx)\sqrt{\sec(c+dx)a+a}}dx}{2a^2}-\frac{\sin(c+dx)}{2d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{7a-4a\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)\sqrt{\sec(c+dx)a+a}}dx}{4a^2}-\frac{\sin(c+dx)}{2d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{7a-4a\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a^2}-\frac{\sin(c+dx)}{2d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{4510}$$

$$\frac{\frac{2\int -\frac{19a^2-14a^2\sec(c+dx)}{2\sqrt{\sec(c+dx)}\sqrt{\sec(c+dx)a+a}}dx}{3a}+\frac{14a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}}{4a^2}-\frac{\sin(c+dx)}{2d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^{3/2}}$$

$\Big\downarrow$ 27

$$\frac{\frac{14a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}-\frac{\int\frac{19a^2-14a^2\sec(c+dx)}{\sqrt{\sec(c+dx)}\sqrt{\sec(c+dx)a+a}}dx}{3a}}{4a^2}-\frac{\sin(c+dx)}{2d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^{3/2}}$$

$\Big\downarrow$ 3042

$$\frac{\frac{14a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}-\frac{\int\frac{19a^2-14a^2\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{3a}}{4a^2}-\frac{\sin(c+dx)}{2d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^{3/2}}$$

$\Big\downarrow$ 4501

$$\frac{\frac{14a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}-\frac{\frac{38a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\sec(c+dx)+a}}-33a^2\int\frac{\sqrt{\sec(c+dx)}}{\sqrt{\sec(c+dx)a+a}}dx}{3a}}{4a^2}-\frac{\sin(c+dx)}{2d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^{3/2}}$$

$\Big\downarrow$ 3042

$$\frac{\frac{14a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}-\frac{\frac{38a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\sec(c+dx)+a}}-33a^2\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{3a}}{4a^2}-\frac{\sin(c+dx)}{2d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^{3/2}}$$

$\Big\downarrow$ 4295

$$\frac{\frac{14a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}-\frac{\frac{66a^2\int\frac{1}{2a-\frac{a^2\sin(c+dx)\tan(c+dx)}{\sec(c+dx)a+a}}d\left(-\frac{a\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}+\frac{38a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\sec(c+dx)+a}}}{3a}}{4a^2}-\frac{\sin(c+dx)}{2d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^{3/2}}$$

$\Big\downarrow$ 219

3.256. $\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}\,dx$

$$\frac{\frac{14a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}-\frac{\frac{38a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\sec(c+dx)+a}}-\frac{33\sqrt{2}a^{3/2}\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{d}}{3a}}{4a^2}-\frac{\sin(c+dx)}{2d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^{3/2}}$$

input
```
Int[1/(Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(3/2)),x]
```

output
```
-1/2*Sin[c + d*x]/(d*Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^(3/2)) + ((14
*a*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) - ((-33
*Sqrt[2]*a^(3/2)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2
]*Sqrt[a + a*Sec[c + d*x]])])/d + (38*a^2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])
/(d*Sqrt[a + a*Sec[c + d*x]]))/(3*a))/(4*a^2)
```

3.256.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4295
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[-2*b*(d/(a*f))   Subst[Int[1/(2*b - d*x^2), x],
x, b*(Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]]))], x] /;
 FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4304
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-Cot[e + f*x])*(a + b*Csc[e + f*x])^m*((d*Csc
[e + f*x])^n/(f*(2*m + 1))), x] + Simp[1/(a^2*(2*m + 1))   Int[(a + b*Csc[e
 + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e
+ f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ
[m, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

rule 4501
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*Cot[e
 + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(f*n)), x] - Simp[(a*A*m
 - b*B*n)/(b*d*n)   Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x]
, x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a
^2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LeQ[m, -1]
```

rule 4510
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*Cot[e
 + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(f*n)), x] - Simp[1/(b*d
*n)   Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*
n - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

3.256.4 Maple [A] (verified)

Time = 1.29 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.40

method	result
default	$\frac{\sqrt{a(1+\sec(dx+c))}\left(33\sqrt{2}\arctan\left(\frac{\sin(dx+c)\sqrt{2}}{2(\cos(dx+c)+1)\sqrt{-\frac{1}{\cos(dx+c)+1}}}\right)\sqrt{-\frac{1}{\cos(dx+c)+1}}\cos(dx+c)+66\sqrt{2}\arctan\left(\frac{\sin(dx+c)}{2(\cos(dx+c)+1)\sqrt{-}}\right.\right.}{12}$

input
```
int(1/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/12/d/a^2*(a*(1+sec(d*x+c)))^(1/2)/(cos(d*x+c)+1)^2/sec(d*x+c)^(3/2)*(33*
2^(1/2)*arctan(1/2*sin(d*x+c)*2^(1/2)/(cos(d*x+c)+1)/(-1/(cos(d*x+c)+1))^(
1/2))*(-1/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)+66*2^(1/2)*arctan(1/2*sin(d*x+c
)*2^(1/2)/(cos(d*x+c)+1)/(-1/(cos(d*x+c)+1))^(1/2))*(-1/(cos(d*x+c)+1))^(1
/2)+33*2^(1/2)*arctan(1/2*sin(d*x+c)*2^(1/2)/(cos(d*x+c)+1)/(-1/(cos(d*x+c
)+1))^(1/2))*(-1/(cos(d*x+c)+1))^(1/2)*sec(d*x+c)+8*cos(d*x+c)*sin(d*x+c)-
24*sin(d*x+c)-38*tan(d*x+c))
```

3.256.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 398, normalized size of antiderivative = 2.25

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}\,dx = \left[\frac{33\sqrt{2}\left(\cos\left(dx+c\right)^2+2\cos\left(dx+c\right)+1\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^2}{}\right.}{}\right.$$

$$-\frac{33\sqrt{2}\left(\cos\left(dx+c\right)^2+2\cos\left(dx+c\right)+1\right)\sqrt{-a}\arctan\left(\frac{\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}}{a\sin(dx+c)}\right)-\frac{2\left(4\cos(dx+c)^3-12\,\mathrm{co}\right.}{}}{12\left(a^2d\cos\left(dx+c\right)^2+2\,a^2d\cos\left(dx+c\right)+a^2d\right)}$$

input
```
integrate(1/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
[1/24*(33*sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*log(-(a*co
s(d*x + c)^2 - 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*s
qrt(cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 +
 2*cos(d*x + c) + 1)) + 4*(4*cos(d*x + c)^3 - 12*cos(d*x + c)^2 - 19*cos(d
*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x
 + c)))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d), -1/12*(33*s
qrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(-a)*arctan(sqrt(2)*sqrt(
-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))/(a*sin(d*x
+ c))) - 2*(4*cos(d*x + c)^3 - 12*cos(d*x + c)^2 - 19*cos(d*x + c))*sqrt((
a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^2*d*
cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)]
```

3.256.6 Sympy [F]

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}\,dx = \int \frac{1}{(a(\sec(c+dx)+1))^{\frac{3}{2}}\sec^{\frac{3}{2}}(c+dx)}\,dx$$

input
```
integrate(1/sec(d*x+c)**(3/2)/(a+a*sec(d*x+c))**(3/2),x)
```

output
```
Integral(1/((a*(sec(c + d*x) + 1))**(3/2)*sec(c + d*x)**(3/2)), x)
```

3.256.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 33960 vs. $2(146) = 292$.

Time = 0.93 (sec) , antiderivative size = 33960, normalized size of antiderivative = 191.86

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
1/12*(4*(cos(3*d*x + 3*c)^2*sin(3/2*d*x + 3/2*c) + sin(3*d*x + 3*c)^2*sin(
3/2*d*x + 3/2*c) - 9*(cos(3*d*x + 3*c)^2 + sin(3*d*x + 3*c)^2)*sin(1/3*arc
tan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*cos(7/3*arctan2(sin(3/2
*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^4 + 64*(cos(3*d*x + 3*c)^2*sin(3/2*d
*x + 3/2*c) + sin(3*d*x + 3*c)^2*sin(3/2*d*x + 3/2*c) - 9*(cos(3*d*x + 3*c
)^2 + sin(3*d*x + 3*c)^2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*
x + 3/2*c))))*cos(5/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))
^4 + 4*sin(3/2*d*x + 3/2*c)^5 + 4*(cos(3*d*x + 3*c)^2*sin(3/2*d*x + 3/2*c)
 + sin(3*d*x + 3*c)^2*sin(3/2*d*x + 3/2*c) - 9*(cos(3*d*x + 3*c)^2 + sin(3
*d*x + 3*c)^2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))
))*sin(7/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^4 + 64*(co
s(3*d*x + 3*c)^2*sin(3/2*d*x + 3/2*c) + sin(3*d*x + 3*c)^2*sin(3/2*d*x + 3
/2*c) - 9*(cos(3*d*x + 3*c)^2 + sin(3*d*x + 3*c)^2)*sin(1/3*arctan2(sin(3/
2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sin(5/3*arctan2(sin(3/2*d*x + 3/2*
c), cos(3/2*d*x + 3/2*c)))^4 + 4*(2*cos(3*d*x + 3*c)^2*cos(3/2*d*x + 3/2*c
)*sin(3/2*d*x + 3/2*c) + 2*cos(3/2*d*x + 3/2*c)*sin(3*d*x + 3*c)^2*sin(3/2
*d*x + 3/2*c) + 2*cos(3*d*x + 3*c)*cos(3/2*d*x + 3/2*c)*sin(3/2*d*x + 3/2*
c) + 8*(cos(3*d*x + 3*c)^2*sin(3/2*d*x + 3/2*c) + sin(3*d*x + 3*c)^2*sin(3
/2*d*x + 3/2*c) - 9*(cos(3*d*x + 3*c)^2 + sin(3*d*x + 3*c)^2)*sin(1/3*arct
an2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*cos(5/3*arctan2(sin(3...
```

3.256.8 Giac [F]

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}\,dx = \int \frac{1}{(a\sec(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((a*sec(d*x + c) + a)^(3/2)*sec(d*x + c)^(3/2)), x)
```

3.256.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}\,dx = \int \frac{1}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{3/2}\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

input
```
int(1/((a + a/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(3/2)),x)
```

output
```
int(1/((a + a/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(3/2)), x)
```

3.257 $\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}\,dx$

3.257.1 Optimal result 2036
3.257.2 Mathematica [A] (verified) 2037
3.257.3 Rubi [A] (verified) 2037
3.257.4 Maple [A] (verified) 2041
3.257.5 Fricas [A] (verification not implemented) 2042
3.257.6 Sympy [F(-1)] 2043
3.257.7 Maxima [F(-2)] 2043
3.257.8 Giac [F] 2043
3.257.9 Mupad [F(-1)] 2044

3.257.1 Optimal result

Integrand size = 25, antiderivative size = 217

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}\,dx = -\frac{15\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} - \frac{\sin(c+dx)}{2d\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}} + \frac{9\sin(c+dx)}{10ad\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} - \frac{13\sin(c+dx)}{10ad\sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+dx)}} + \frac{49\sqrt{\sec(c+dx)}\sin(c+dx)}{10ad\sqrt{a+a\sec(c+dx)}}$$

output
```
-1/2*sin(d*x+c)/d/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(3/2)-15/4*arctanh(1/2
*sin(d*x+c)*a^(1/2)*sec(d*x+c)^(1/2)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))/a^(3/
2)/d*2^(1/2)+9/10*sin(d*x+c)/a/d/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2)-1
3/10*sin(d*x+c)/a/d/sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2)+49/10*sin(d*x+
c)*sec(d*x+c)^(1/2)/a/d/(a+a*sec(d*x+c))^(1/2)
```

3.257.2 Mathematica [A] (verified)

Time = 1.35 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.75

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}\,dx = \frac{75\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)\cos^2\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{5}{2}}(c+dx)\sin(c+}{10d\sqrt{-((}}$$

input
```
Integrate[1/(Sec[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^(3/2)),x]
```

output
```
(75*Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*Co
s[(c + d*x)/2]^2*Sec[c + d*x]^(5/2)*Sin[c + d*x] + (47 + 39*Cos[c + d*x] -
 2*Cos[2*(c + d*x)] + Cos[3*(c + d*x)])*Sqrt[1 - Sec[c + d*x]]*Sec[c + d*x
]*Tan[c + d*x])/(10*d*Sqrt[-((-1 + Sec[c + d*x])*Sec[c + d*x])]*(a*(1 + Se
c[c + d*x]))^(3/2))
```

3.257.3 Rubi [A] (verified)

Time = 1.16 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.08, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.560$, Rules used = {3042, 4304, 27, 3042, 4510, 27, 3042, 4510, 27, 3042, 4501, 3042, 4295, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a\sec(c+dx)+a)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}}dx$$

$$\Big\downarrow \text{4304}$$

$$-\frac{\int -\frac{3(3a-2a\sec(c+dx))}{2\sec^{\frac{5}{2}}(c+dx)\sqrt{\sec(c+dx)a+a}}dx}{2a^2}-\frac{\sin(c+dx)}{2d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{3\int \frac{3a-2a\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)\sqrt{\sec(c+dx)a+a}}dx}{4a^2}-\frac{\sin(c+dx)}{2d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\int \frac{3a-2a\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a^2}-\frac{\sin(c+dx)}{2d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{4510}$$

$$\frac{3\left(\frac{2\int -\frac{13a^2-12a^2\sec(c+dx)}{2\sec^{\frac{3}{2}}(c+dx)\sqrt{\sec(c+dx)a+a}}dx}{5a}+\frac{6a\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}\right)}{4a^2}-\frac{\sin(c+dx)}{2d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{3\left(\frac{6a\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}-\frac{\int \frac{13a^2-12a^2\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)\sqrt{\sec(c+dx)a+a}}dx}{5a}\right)}{4a^2}-\frac{\sin(c+dx)}{2d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\left(\frac{6a\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}-\frac{\int \frac{13a^2-12a^2\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{5a}\right)}{4a^2}-\frac{\sin(c+dx)}{2d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{4510}$$

$$\frac{3\left(\frac{6a\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}-\frac{\frac{2\int -\frac{49a^3-26a^3\sec(c+dx)}{2\sqrt{\sec(c+dx)}\sqrt{\sec(c+dx)a+a}}dx}{3a}+\frac{26a^2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}}{5a}\right)}{4a^2}-\frac{\sin(c+dx)}{2d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{3\left(\frac{6a\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}-\frac{\frac{26a^2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}-\frac{\int \frac{49a^3-26a^3\sec(c+dx)}{\sqrt{\sec(c+dx)}\sqrt{\sec(c+dx)a+a}}dx}{3a}}{5a}\right)}{4a^2}-\frac{\sin(c+dx)}{2d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^{3/2}}$$

3.257. $\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}\,dx$

$$\Big\downarrow \; 3042$$

$$\frac{3\left(\frac{6a\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}-\frac{\frac{26a^2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}-\frac{\int\frac{49a^3-26a^3\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{3a}}{5a}\right)}{4a^2}-\frac{\sin(c+dx)}{2d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \; 4501$$

$$\frac{3\left(\frac{6a\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}-\frac{\frac{26a^2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}-\frac{\frac{98a^3\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\sec(c+dx)+a}}-75a^3\int\frac{\sqrt{\sec(c+dx)}}{\sqrt{\sec(c+dx)a+a}}dx}{3a}}{5a}\right)}{4a^2}-\frac{\sin(c+dx)}{2d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \; 3042$$

$$\frac{3\left(\frac{6a\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}-\frac{\frac{26a^2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}-\frac{\frac{98a^3\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\sec(c+dx)+a}}-75a^3\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{3a}}{5a}\right)}{4a^2}-\frac{\sin(c+dx)}{2d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \; 4295$$

$$\frac{3\left(\frac{6a\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}-\frac{\frac{26a^2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}-\frac{\frac{150a^3\int\frac{1}{2a-\frac{a^2\sin(c+dx)\tan(c+dx)}{\sec(c+dx)a+a}}d\left(-\frac{a\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}+\frac{98a^3\sin(c+d}{d\sqrt{a\sec}}}{3a}}{5a}\right)}{4a^2}-\frac{\sin(c+dx)}{2d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^{3/2}}$$

$$\Big\downarrow \; 219$$

3.257. $\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}\,dx$

$$\frac{3\left(\frac{6a\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}-\frac{\frac{26a^2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}-\frac{\frac{98a^3\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\sec(c+dx)+a}}-\frac{75\sqrt{2}a^{5/2}\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{d}}{3a}}{5a}\right)}{4a^2}}{\frac{\sin(c+dx)}{2d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^{3/2}}}$$

input
```
Int[1/(Sec[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^(3/2)),x]
```

output
```
-1/2*Sin[c + d*x]/(d*Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(3/2)) + (3*(
(6*a*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]]) - ((2
6*a^2*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) - ((
-75*Sqrt[2]*a^(5/2)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqr
t[2]*Sqrt[a + a*Sec[c + d*x]])])/d + (98*a^3*Sqrt[Sec[c + d*x]]*Sin[c + d*
x])/(d*Sqrt[a + a*Sec[c + d*x]]))/(3*a))/(5*a)))/(4*a^2)
```

3.257.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4295
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[-2*b*(d/(a*f))   Subst[Int[1/(2*b - d*x^2), x],
x, b*(Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]]))], x] /;
 FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4304
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-Cot[e + f*x])*(a + b*Csc[e + f*x])^m*((d*Csc
[e + f*x])^n/(f*(2*m + 1))), x] + Simp[1/(a^2*(2*m + 1))   Int[(a + b*Csc[e
 + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e
+ f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ
[m, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

rule 4501
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*Cot[e
 + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(f*n)), x] - Simp[(a*A*m
 - b*B*n)/(b*d*n)   Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x]
, x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a
^2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LeQ[m, -1]
```

rule 4510
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*Cot[e
 + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(f*n)), x] - Simp[1/(b*d
*n)   Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*
n - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

3.257.4 Maple [A] (verified)

Time = 1.47 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.21

method	result
default	$\frac{\sqrt{a(1+\sec(dx+c))}\left(-75\sqrt{2}\arctan\left(\frac{\sin(dx+c)\sqrt{2}}{2(\cos(dx+c)+1)\sqrt{-\frac{1}{\cos(dx+c)+1}}}\right)\sqrt{-\frac{1}{\cos(dx+c)+1}}+8\cos(dx+c)\sin(dx+c)-150\sqrt{2}\arctan\left(\frac{}{2(c}\right.\right.}{}$

input
```
int(1/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/20/d/a^2*(a*(1+sec(d*x+c)))^(1/2)/(cos(d*x+c)+1)^2/sec(d*x+c)^(5/2)*(-75
*2^(1/2)*arctan(1/2*sin(d*x+c)*2^(1/2)/(cos(d*x+c)+1)/(-1/(cos(d*x+c)+1))^
(1/2))*(-1/(cos(d*x+c)+1))^(1/2)+8*cos(d*x+c)*sin(d*x+c)-150*2^(1/2)*arcta
n(1/2*sin(d*x+c)*2^(1/2)/(cos(d*x+c)+1)/(-1/(cos(d*x+c)+1))^(1/2))*(-1/(co
s(d*x+c)+1))^(1/2)*sec(d*x+c)-8*sin(d*x+c)-75*2^(1/2)*arctan(1/2*sin(d*x+c
)*2^(1/2)/(cos(d*x+c)+1)/(-1/(cos(d*x+c)+1))^(1/2))*(-1/(cos(d*x+c)+1))^(1
/2)*sec(d*x+c)^2+72*tan(d*x+c)+98*sec(d*x+c)*tan(d*x+c))
```

3.257.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 418, normalized size of antiderivative = 1.93

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}\,dx = \left[\frac{75\sqrt{2}\left(\cos\left(dx+c\right)^2+2\,\cos\left(dx+c\right)+1\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^2}{}\right.}{}\right.$$

input
```
integrate(1/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
[1/40*(75*sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*log(-(a*co
s(d*x + c)^2 + 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*s
qrt(cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 +
 2*cos(d*x + c) + 1)) + 4*(4*cos(d*x + c)^4 - 4*cos(d*x + c)^3 + 36*cos(d*
x + c)^2 + 49*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*
x + c)/sqrt(cos(d*x + c)))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) +
a^2*d), 1/20*(75*sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(-a)*ar
ctan(sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x
 + c))/(a*sin(d*x + c))) + 2*(4*cos(d*x + c)^4 - 4*cos(d*x + c)^3 + 36*cos
(d*x + c)^2 + 49*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin
(d*x + c)/sqrt(cos(d*x + c)))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c)
 + a^2*d)]
```

3.257.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/sec(d*x+c)**(5/2)/(a+a*sec(d*x+c))**(3/2),x)
```

output
```
Timed out
```

3.257.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

3.257.8 Giac [F]

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}\,dx = \int \frac{1}{(a\sec(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((a*sec(d*x + c) + a)^(3/2)*sec(d*x + c)^(5/2)), x)
```

3.257.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}\,dx=\int \frac{1}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{3/2}\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

input
```
int(1/((a + a/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(5/2)),x)
```

output
```
int(1/((a + a/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(5/2)), x)
```

3.258 $\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx$

3.258.1 Optimal result 2045
3.258.2 Mathematica [A] (verified) 2046
3.258.3 Rubi [A] (verified) 2046
3.258.4 Maple [B] (warning: unable to verify) 2052
3.258.5 Fricas [A] (verification not implemented) 2053
3.258.6 Sympy [F(-1)] 2054
3.258.7 Maxima [B] (verification not implemented) 2054
3.258.8 Giac [F] 2055
3.258.9 Mupad [F(-1)] 2055

3.258.1 Optimal result

Integrand size = 25, antiderivative size = 214

$$\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = -\frac{5\operatorname{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^{5/2}d} + \frac{115\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} - \frac{\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{15\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} + \frac{35\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{16a^2d\sqrt{a+a\sec(c+dx)}}$$

output
```
-5*arcsinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/a^(5/2)/d-1/4*sec(d*
x+c)^(7/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(5/2)-15/16*sec(d*x+c)^(5/2)*sin(
d*x+c)/a/d/(a+a*sec(d*x+c))^(3/2)+115/32*arctanh(1/2*sin(d*x+c)*a^(1/2)*se
c(d*x+c)^(1/2)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))/a^(5/2)/d*2^(1/2)+35/16*sec
(d*x+c)^(3/2)*sin(d*x+c)/a^2/d/(a+a*sec(d*x+c))^(1/2)
```

3.258.2 Mathematica [A] (verified)

Time = 1.30 (sec) , antiderivative size = 340, normalized size of antiderivative = 1.59

$$\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \frac{70\sqrt{1-\sec(c+dx)}\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)+110\sqrt{1-\sec(c+dx)}\sec^{\frac{5}{2}}(c}{}$$

input
```
Integrate[Sec[c + d*x]^(9/2)/(a + a*Sec[c + d*x])^(5/2),x]
```

output
```
(70*Sqrt[1 - Sec[c + d*x]]*Sec[c + d*x]^(3/2)*Sin[c + d*x] + 110*Sqrt[1 -
Sec[c + d*x]]*Sec[c + d*x]^(5/2)*Sin[c + d*x] + 32*Sqrt[1 - Sec[c + d*x]]*
Sec[c + d*x]^(7/2)*Sin[c + d*x] - 115*Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[Sec[c +
 d*x]])/Sqrt[1 - Sec[c + d*x]]]*Tan[c + d*x] - 230*Sqrt[2]*ArcTan[(Sqrt[2]
*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*Sec[c + d*x]*Tan[c + d*x] - 1
15*Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*Sec
[c + d*x]^2*Tan[c + d*x] + 70*ArcSin[Sqrt[1 - Sec[c + d*x]]]*(1 + Sec[c +
d*x])^2*Tan[c + d*x] + 230*ArcSin[Sqrt[Sec[c + d*x]]]*(1 + Sec[c + d*x])^2
*Tan[c + d*x])/(32*d*Sqrt[1 - Sec[c + d*x]]*(a*(1 + Sec[c + d*x]))^(5/2))
```

3.258.3 Rubi [A] (verified)

Time = 1.37 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.07, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.640$, Rules used = {3042, 4303, 27, 3042, 4507, 27, 3042, 4509, 25, 3042, 4511, 3042, 4288, 222, 4295, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a\sec(c+dx)+a)^{5/2}}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{9/2}}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}}dx$$

$$\Big\downarrow \; 4303$$

$$-\frac{\int \frac{5\sec^{\frac{5}{2}}(c+dx)(a-2a\sec(c+dx))}{2(\sec(c+dx)a+a)^{3/2}}dx}{4a^2}-\frac{\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \; 27$$

$$-\frac{5\int \frac{\sec^{\frac{5}{2}}(c+dx)(a-2a\sec(c+dx))}{(\sec(c+dx)a+a)^{3/2}}dx}{8a^2}-\frac{\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \; 3042$$

$$-\frac{5\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a-2a\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}}dx}{8a^2}-\frac{\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \; 4507$$

$$-\frac{5\left(\frac{\int \frac{\sec^{\frac{3}{2}}(c+dx)\left(9a^2-14a^2\sec(c+dx)\right)}{2\sqrt{\sec(c+dx)a+a}}dx}{2a^2}+\frac{3a\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}\right)}{8a^2}-\frac{\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \; 27$$

$$-\frac{5\left(\frac{\int \frac{\sec^{\frac{3}{2}}(c+dx)\left(9a^2-14a^2\sec(c+dx)\right)}{\sqrt{\sec(c+dx)a+a}}dx}{4a^2}+\frac{3a\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}\right)}{8a^2}-\frac{\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \; 3042$$

$$-\frac{5\left(\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(9a^2-14a^2\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a^2}+\frac{3a\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}\right)}{8a^2}-\frac{\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \; 4509$$

$$-\frac{5\left(\frac{\frac{\int -\frac{\sqrt{\sec(c+dx)}\left(7a^3-16a^3\sec(c+dx)\right)}{\sqrt{\sec(c+dx)a+a}}dx}{a}-\frac{14a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}}{4a^2}+\frac{3a\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}\right)}{8a^2}-\frac{\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \; 25$$

3.258. $\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx$

$$-\frac{5\left(\frac{-\frac{\int \frac{\sqrt{\sec(c+dx)}\left(7a^3-16a^3\sec(c+dx)\right)}{\sqrt{\sec(c+dx)a+a}}dx}{a}-\frac{14a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}}{4a^2}+\frac{3a\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}\right)}{8a^2}-\frac{\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

$\downarrow$ 3042

$$-\frac{5\left(\frac{-\frac{\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(7a^3-16a^3\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{a}-\frac{14a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}}{4a^2}+\frac{3a\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}\right)}{8a^2}-\frac{\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

$\downarrow$ 4511

$$-\frac{5\left(\frac{-\frac{23a^3\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{\sec(c+dx)a+a}}dx-16a^2\int \sqrt{\sec(c+dx)}\sqrt{\sec(c+dx)a+a}dx}{a}-\frac{14a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}}{4a^2}+\frac{3a\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}\right)}{8a^2}-\frac{\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

$\downarrow$ 3042

$$-\frac{5\left(\frac{-\frac{23a^3\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx-16a^2\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{a}-\frac{14a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}}{4a^2}+\frac{3a\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}\right)}{8a^2}-\frac{\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

$\downarrow$ 4288

3.258. $\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx$

$$-\frac{5\left(\frac{-\frac{23a^3\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx+\frac{32a^2\int \frac{1}{\sqrt{\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1}}d\left(-\frac{a\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}}{a}-\frac{14a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}}{4a^2}+\frac{3a\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}\right)}{8a^2}-\frac{\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

$\downarrow$ 222

$$-\frac{5\left(\frac{-\frac{23a^3\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx-\frac{32a^{5/2}\text{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}}{a}-\frac{14a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}}{4a^2}+\frac{3a\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}\right)}{8a^2}-\frac{\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

$\downarrow$ 4295

$$-\frac{5\left(\frac{-\frac{-\frac{46a^3\int \frac{1}{2a-\frac{a^2\sin(c+dx)\tan(c+dx)}{\sec(c+dx)a+a}}d\left(-\frac{a\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}-\frac{32a^{5/2}\text{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}}{a}-\frac{14a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}}{4a^2}+\frac{3a\sin(}{2d(a\,}\right.}{8a^2}-\frac{\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

$\downarrow$ 219

$$-\frac{5\left(\frac{-\frac{\frac{23\sqrt{2}a^{5/2}\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{d}-\frac{32a^{5/2}\text{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}}{a}-\frac{14a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}}{4a^2}+\frac{3a\sin(c+dx)\sec^{\frac{5}{2}}(c+}{2d(a\sec(c+dx)+a)^{3/}}\right.}{8a^2}-\frac{\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

3.258. $\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx$

input
```
Int[Sec[c + d*x]^(9/2)/(a + a*Sec[c + d*x])^(5/2),x]
```

output
```
-1/4*(Sec[c + d*x]^(7/2)*Sin[c + d*x])/(d*(a + a*Sec[c + d*x])^(5/2)) - (5
*((3*a*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(2*d*(a + a*Sec[c + d*x])^(3/2)) +
 (-(((-32*a^(5/2)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]
)/d + (23*Sqrt[2]*a^(5/2)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x]
)/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/d)/a) - (14*a^2*Sec[c + d*x]^(3/2)*
Sin[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]]))/(4*a^2)))/(8*a^2)
```

3.258.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4288
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[-2*(a/(b*f))*Sqrt[a*(d/b)]   Subst[Int[1/Sqrt[1
+ x^2/a], x], x, b*(Cot[e + f*x]/Sqrt[a + b*Csc[e + f*x]])], x] /; FreeQ[{a
, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[a*(d/b), 0]
```

3.258. $\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx$

rule 4295
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[-2*b*(d/(a*f))   Subst[Int[1/(2*b - d*x^2), x],
x, b*(Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]]))], x] /;
 FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4303
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-d^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d
*Csc[e + f*x])^(n - 2)/(f*(2*m + 1))), x] + Simp[d^2/(a*b*(2*m + 1))   Int[
(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n
 + 2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2,
0] && LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

rule 4507
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[d*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(a*f*(
2*m + 1))), x] - Simp[1/(a*b*(2*m + 1))   Int[(a + b*Csc[e + f*x])^(m + 1)*
(d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
 - n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f,
A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && G
tQ[n, 0]
```

rule 4509
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*d*
Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(f*(m + n))),
 x] + Simp[d/(b*(m + n))   Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n -
 1)*Simp[b*B*(n - 1) + (A*b*(m + n) + a*B*m)*Csc[e + f*x], x], x], x] /; Fr
eeQ[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
&& GtQ[n, 1]
```

rule 4511
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b -
a*B)/b   Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n, x], x] + Simp[B/b
 Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b
, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
```

3.258. $\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx$

3.258.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 668 vs. $2(177) = 354$.

Time = 1.84 (sec) , antiderivative size = 669, normalized size of antiderivative = 3.13

method	result
default	$-\frac{\left(-\frac{(1-\cos(dx+c))^2\csc(dx+c)^2+1}{(1-\cos(dx+c))^2\csc(dx+c)^2-1}\right)^{\frac{9}{2}}\left((1-\cos(dx+c))^2\csc(dx+c)^2-1\right)^4\sqrt{-\frac{2a}{(1-\cos(dx+c))^2\csc(dx+c)^2-1}}\left(2\sqrt{-(1-\cos(dx+c))^2\csc}\right.}{}$

input
```
int(sec(d*x+c)^(9/2)/(a+a*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/32/d/a^3*(-((1-cos(d*x+c))^2*csc(d*x+c)^2+1)/((1-cos(d*x+c))^2*csc(d*x+
c)^2-1))^(9/2)*((1-cos(d*x+c))^2*csc(d*x+c)^2-1)^4*(-2*a/((1-cos(d*x+c))^2
*csc(d*x+c)^2-1))^(1/2)*(2*(-(1-cos(d*x+c))^2*csc(d*x+c)^2-1)^(1/2)*(1-cos
(d*x+c))^5*csc(d*x+c)^5+40*2^(1/2)*arctan(1/2*(-cot(d*x+c)+csc(d*x+c)-1)*2
^(1/2)/(-(1-cos(d*x+c))^2*csc(d*x+c)^2-1)^(1/2))*(1-cos(d*x+c))^2*csc(d*x+
c)^2+40*2^(1/2)*arctan(1/2*(csc(d*x+c)-cot(d*x+c)+1)*2^(1/2)/(-(1-cos(d*x+
c))^2*csc(d*x+c)^2-1)^(1/2))*(1-cos(d*x+c))^2*csc(d*x+c)^2+19*(-(1-cos(d*x
+c))^2*csc(d*x+c)^2-1)^(1/2)*(1-cos(d*x+c))^3*csc(d*x+c)^3-115*arctan(1/(-
(1-cos(d*x+c))^2*csc(d*x+c)^2-1)^(1/2)*(-cot(d*x+c)+csc(d*x+c)))*(1-cos(d*
x+c))^2*csc(d*x+c)^2-40*2^(1/2)*arctan(1/2*(-cot(d*x+c)+csc(d*x+c)-1)*2^(1
/2)/(-(1-cos(d*x+c))^2*csc(d*x+c)^2-1)^(1/2))-40*2^(1/2)*arctan(1/2*(csc(d
*x+c)-cot(d*x+c)+1)*2^(1/2)/(-(1-cos(d*x+c))^2*csc(d*x+c)^2-1)^(1/2))-53*(
-(1-cos(d*x+c))^2*csc(d*x+c)^2-1)^(1/2)*(-cot(d*x+c)+csc(d*x+c))+115*arcta
n(1/(-(1-cos(d*x+c))^2*csc(d*x+c)^2-1)^(1/2)*(-cot(d*x+c)+csc(d*x+c))))/(-
(1-cos(d*x+c))^2*csc(d*x+c)^2-1)^(1/2)/((1-cos(d*x+c))^2*csc(d*x+c)^2+1)^4
```

3.258.5 Fricas [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 667, normalized size of antiderivative = 3.12

$$\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \left[\frac{115\sqrt{2}\left(\cos\left(dx+c\right)^{3}+3\,\cos\left(dx+c\right)^{2}+3\,\cos\left(dx+c\right)+1\right)\sqrt{a}\log\left(-\frac{a\cos}{}\right.}{}\right.$$

$$-\frac{115\sqrt{2}\left(\cos\left(dx+c\right)^{3}+3\,\cos\left(dx+c\right)^{2}+3\,\cos\left(dx+c\right)+1\right)\sqrt{-a}\arctan\left(\frac{\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}}{a\sin(dx+c)}\right)}{32\left(a^{3}d\right.}$$

input
```
integrate(sec(d*x+c)^(9/2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
[1/64*(115*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1
)*sqrt(a)*log(-(a*cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c)
+ a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*
a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + 80*(cos(d*x + c)^3 + 3*cos(d*x
 + c)^2 + 3*cos(d*x + c) + 1)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x
+ c)^2 + 4*(cos(d*x + c)^2 - 2*cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c)
+ a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)) + 8*a)/(cos(d*x + c)^3
+ cos(d*x + c)^2)) + 4*(35*cos(d*x + c)^2 + 55*cos(d*x + c) + 16)*sqrt((a*
cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^3*d*co
s(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d), -1/
32*(115*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*s
qrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sq
rt(cos(d*x + c))/(a*sin(d*x + c))) + 80*(cos(d*x + c)^3 + 3*cos(d*x + c)^2
 + 3*cos(d*x + c) + 1)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a
)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(
d*x + c) - 2*a)) - 2*(35*cos(d*x + c)^2 + 55*cos(d*x + c) + 16)*sqrt((a*co
s(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^3*d*cos(
d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)]
```

3.258.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**(9/2)/(a+a*sec(d*x+c))**(5/2),x)
```

output
```
Timed out
```

3.258.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 9048 vs. $2(177) = 354$.

Time = 2.72 (sec) , antiderivative size = 9048, normalized size of antiderivative = 42.28

$$\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(sec(d*x+c)^(9/2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
-1/32*(140*(sin(6*d*x + 6*c) + 7*sin(4*d*x + 4*c) + 7*sin(2*d*x + 2*c) + 4
*sin(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 8*sin(3/2*arctan2(
sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*sin(1/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c))))*cos(11/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
 - 16*(75*sin(9/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 24*sin(7/
4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 24*sin(5/4*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c))) - 75*sin(3/4*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c))) - 35*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*
cos(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 300*(sin(6*d*x + 6*
c) + 7*sin(4*d*x + 4*c) + 7*sin(2*d*x + 2*c) + 8*sin(3/2*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c))) + 4*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c))))*cos(9/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 96*(sin
(6*d*x + 6*c) + 7*sin(4*d*x + 4*c) + 7*sin(2*d*x + 2*c) + 8*sin(3/2*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*sin(1/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c))))*cos(7/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))
) + 32*(24*sin(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 75*sin(3
/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 35*sin(1/4*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c))))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c))) - 96*(sin(6*d*x + 6*c) + 7*sin(4*d*x + 4*c) + 7*sin(2*d*x + 2*
c) + 4*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(5/4*ar...
```

3.258.8 Giac [F]

$$\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \int \frac{\sec{(dx+c)}^{\frac{9}{2}}}{(a\sec{(dx+c)}+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^(9/2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^(9/2)/(a*sec(d*x + c) + a)^(5/2), x)
```

3.258.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{9/2}}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

input
```
int((1/cos(c + d*x))^(9/2)/(a + a/cos(c + d*x))^(5/2),x)
```

output
```
int((1/cos(c + d*x))^(9/2)/(a + a/cos(c + d*x))^(5/2), x)
```

3.259 $\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx$

3.259.1 Optimal result 2056
3.259.2 Mathematica [A] (verified) 2057
3.259.3 Rubi [A] (verified) 2057
3.259.4 Maple [B] (warning: unable to verify) 2061
3.259.5 Fricas [B] (verification not implemented) 2062
3.259.6 Sympy [F(-1)] 2063
3.259.7 Maxima [B] (verification not implemented) 2063
3.259.8 Giac [F] 2064
3.259.9 Mupad [F(-1)] 2064

3.259.1 Optimal result

Integrand size = 25, antiderivative size = 174

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \frac{2\operatorname{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^{5/2}d} - \frac{43\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} - \frac{\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{11\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}}$$

output
```
2*arcsinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/a^(5/2)/d-1/4*sec(d*x
+c)^(5/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(5/2)-11/16*sec(d*x+c)^(3/2)*sin(d
*x+c)/a/d/(a+a*sec(d*x+c))^(3/2)-43/32*arctanh(1/2*sin(d*x+c)*a^(1/2)*sec(
d*x+c)^(1/2)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))/a^(5/2)/d*2^(1/2)
```

3.259.2 Mathematica [A] (verified)

Time = 0.71 (sec) , antiderivative size = 308, normalized size of antiderivative = 1.77

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \frac{-22\sqrt{1-\sec(c+dx)}\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)-30\sqrt{1-\sec(c+dx)}\sec^{\frac{5}{2}}(c}{}$$

input
```
Integrate[Sec[c + d*x]^(7/2)/(a + a*Sec[c + d*x])^(5/2),x]
```

output
```
(-22*Sqrt[1 - Sec[c + d*x]]*Sec[c + d*x]^(3/2)*Sin[c + d*x] - 30*Sqrt[1 -
Sec[c + d*x]]*Sec[c + d*x]^(5/2)*Sin[c + d*x] + 43*Sqrt[2]*ArcTan[(Sqrt[2]
*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*Tan[c + d*x] + 86*Sqrt[2]*Arc
Tan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*Sec[c + d*x]*Tan[
c + d*x] + 43*Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c +
 d*x]]]*Sec[c + d*x]^2*Tan[c + d*x] - 22*ArcSin[Sqrt[1 - Sec[c + d*x]]]*(1
 + Sec[c + d*x])^2*Tan[c + d*x] - 86*ArcSin[Sqrt[Sec[c + d*x]]]*(1 + Sec[c
 + d*x])^2*Tan[c + d*x])/(32*d*Sqrt[1 - Sec[c + d*x]]*(a*(1 + Sec[c + d*x]
))^(5/2))
```

3.259.3 Rubi [A] (verified)

Time = 1.02 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.07, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.520$, Rules used = {3042, 4303, 27, 3042, 4507, 27, 3042, 4511, 3042, 4288, 222, 4295, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a\sec(c+dx)+a)^{5/2}}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}}dx$$

$$\Big\downarrow \; 4303$$

$$-\frac{\int \frac{\sec^{\frac{3}{2}}(c+dx)(3a-8a\sec(c+dx))}{2(\sec(c+dx)a+a)^{3/2}}dx}{4a^2}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \; 27$$

$$-\frac{\int \frac{\sec^{\frac{3}{2}}(c+dx)(3a-8a\sec(c+dx))}{(\sec(c+dx)a+a)^{3/2}}dx}{8a^2}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(3a-8a\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}}dx}{8a^2}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \; 4507$$

$$-\frac{\frac{\int \frac{\sqrt{\sec(c+dx)}\left(11a^2-32a^2\sec(c+dx)\right)}{2\sqrt{\sec(c+dx)a+a}}dx}{2a^2}+\frac{11a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}}{8a^2}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \; 27$$

$$-\frac{\frac{\int \frac{\sqrt{\sec(c+dx)}\left(11a^2-32a^2\sec(c+dx)\right)}{\sqrt{\sec(c+dx)a+a}}dx}{4a^2}+\frac{11a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}}{8a^2}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\frac{\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(11a^2-32a^2\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a^2}+\frac{11a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}}{8a^2}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \; 4511$$

$$-\frac{\frac{43a^2\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{\sec(c+dx)a+a}}dx-32a\int \sqrt{\sec(c+dx)}\sqrt{\sec(c+dx)a+a}dx}{4a^2}+\frac{11a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}}{8a^2}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\frac{43a^2\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx-32a\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{4a^2}+\frac{11a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}}{8a^2}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \; 4288$$

3.259. $\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx$

$$-\frac{\frac{43a^2\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx+\frac{64a\int \frac{1}{\sqrt{\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1}}d\left(-\frac{a\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}}{4a^2}+\frac{11a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}}{8a^2}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

$\downarrow$ 222

$$-\frac{\frac{43a^2\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx-\frac{64a^{3/2}\text{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}}{4a^2}+\frac{11a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}}{8a^2}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

$\downarrow$ 4295

$$-\frac{\frac{-\frac{86a^2\int \frac{1}{2a-\frac{a^2\sin(c+dx)\tan(c+dx)}{\sec(c+dx)a+a}}d\left(-\frac{a\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}-\frac{64a^{3/2}\text{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}}{4a^2}+\frac{11a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}}{8a^2}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

$\downarrow$ 219

$$-\frac{\frac{\frac{43\sqrt{2}a^{3/2}\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{d}-\frac{64a^{3/2}\text{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}}{4a^2}+\frac{11a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}}{8a^2}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

input
```
Int[Sec[c + d*x]^(7/2)/(a + a*Sec[c + d*x])^(5/2),x]
```

output
```
-1/4*(Sec[c + d*x]^(5/2)*Sin[c + d*x])/(d*(a + a*Sec[c + d*x])^(5/2)) - ((
(-64*a^(3/2)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d +
 (43*Sqrt[2]*a^(3/2)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sq
rt[2]*Sqrt[a + a*Sec[c + d*x]])])/d)/(4*a^2) + (11*a*Sec[c + d*x]^(3/2)*Si
n[c + d*x])/(2*d*(a + a*Sec[c + d*x])^(3/2)))/(8*a^2)
```

3.259. $\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx$

3.259.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4288
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[-2*(a/(b*f))*Sqrt[a*(d/b)]   Subst[Int[1/Sqrt[1
+ x^2/a], x], x, b*(Cot[e + f*x]/Sqrt[a + b*Csc[e + f*x]])], x] /; FreeQ[{a
, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[a*(d/b), 0]
```

rule 4295
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[-2*b*(d/(a*f))   Subst[Int[1/(2*b - d*x^2), x],
x, b*(Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]]))], x] /;
 FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4303
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-d^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d
*Csc[e + f*x])^(n - 2)/(f*(2*m + 1))), x] + Simp[d^2/(a*b*(2*m + 1))   Int[
(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n
 + 2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2,
0] && LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

3.259. $\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx$

rule 4507
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[d*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(a*f*(
2*m + 1))), x] - Simp[1/(a*b*(2*m + 1))   Int[(a + b*Csc[e + f*x])^(m + 1)*
(d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
 - n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f,
A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && G
tQ[n, 0]
```

rule 4511
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b -
a*B)/b   Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n, x], x] + Simp[B/b
 Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b
, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
```

3.259.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 409 vs. $2(143) = 286$.

Time = 1.63 (sec) , antiderivative size = 410, normalized size of antiderivative = 2.36

method	result
default	$-\frac{\left(-\frac{(1-\cos(dx+c))^2\csc(dx+c)^2+1}{(1-\cos(dx+c))^2\csc(dx+c)^2-1}\right)^{\frac{7}{2}}\left((1-\cos(dx+c))^2\csc(dx+c)^2-1\right)^4\sqrt{-\frac{2a}{(1-\cos(dx+c))^2\csc(dx+c)^2-1}}\left(2\sqrt{-(1-\cos(dx+c))^2\csc}\right.}{}$

input
```
int(sec(d*x+c)^(7/2)/(a+a*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/32/d/a^3*(-((1-cos(d*x+c))^2*csc(d*x+c)^2+1)/((1-cos(d*x+c))^2*csc(d*x+
c)^2-1))^(7/2)*((1-cos(d*x+c))^2*csc(d*x+c)^2-1)^4*(-2*a/((1-cos(d*x+c))^2
*csc(d*x+c)^2-1))^(1/2)*(2*(-(1-cos(d*x+c))^2*csc(d*x+c)^2-1)^(1/2)*(1-cos
(d*x+c))^3*csc(d*x+c)^3+16*2^(1/2)*arctan(1/2*(csc(d*x+c)-cot(d*x+c)+1)*2^
(1/2)/(-(1-cos(d*x+c))^2*csc(d*x+c)^2-1)^(1/2))+16*2^(1/2)*arctan(1/2*(-co
t(d*x+c)+csc(d*x+c)-1)*2^(1/2)/(-(1-cos(d*x+c))^2*csc(d*x+c)^2-1)^(1/2))+1
3*(-(1-cos(d*x+c))^2*csc(d*x+c)^2-1)^(1/2)*(-cot(d*x+c)+csc(d*x+c))-43*arc
tan(1/(-(1-cos(d*x+c))^2*csc(d*x+c)^2-1)^(1/2)*(-cot(d*x+c)+csc(d*x+c))))/
((1-cos(d*x+c))^2*csc(d*x+c)^2+1)^3/(-(1-cos(d*x+c))^2*csc(d*x+c)^2-1)^(1/
2)
```

3.259.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 297 vs. $2(143) = 286$.

Time = 0.34 (sec) , antiderivative size = 665, normalized size of antiderivative = 3.82

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \left[\frac{43\sqrt{2}\left(\cos\left(dx+c\right)^{3}+3\,\cos\left(dx+c\right)^{2}+3\,\cos\left(dx+c\right)+1\right)\sqrt{a}\log\left(-\frac{a\cos(}{}\right.}{}\right.$$

input
```
integrate(sec(d*x+c)^(7/2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
[1/64*(43*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)
*sqrt(a)*log(-(a*cos(d*x + c)^2 + 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) +
 a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a
)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + 32*(cos(d*x + c)^3 + 3*cos(d*x
+ c)^2 + 3*cos(d*x + c) + 1)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x +
 c)^2 - 4*(cos(d*x + c)^2 - 2*cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) +
 a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)) + 8*a)/(cos(d*x + c)^3 +
 cos(d*x + c)^2)) - 4*(11*cos(d*x + c)^2 + 15*cos(d*x + c))*sqrt((a*cos(d*
x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^3*d*cos(d*x
+ c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d), 1/32*(43*
sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*sqrt(-a)*
arctan(sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d
*x + c))/(a*sin(d*x + c))) + 32*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos
(d*x + c) + 1)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*
x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c)
 - 2*a)) - 2*(11*cos(d*x + c)^2 + 15*cos(d*x + c))*sqrt((a*cos(d*x + c) +
a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^3*d*cos(d*x + c)^3 +
3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)]
```

3.259.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**(7/2)/(a+a*sec(d*x+c))**(5/2),x)
```

output
```
Timed out
```

3.259.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4988 vs. $2(143) = 286$.

Time = 0.60 (sec) , antiderivative size = 4988, normalized size of antiderivative = 28.67

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(sec(d*x+c)^(7/2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
1/32*(44*(sin(4*d*x + 4*c) + 6*sin(2*d*x + 2*c) + 4*sin(3/2*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c))) + 4*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c))))*cos(7/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 16*(
19*sin(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 19*sin(3/4*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 11*sin(1/4*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c))))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
))) + 76*(sin(4*d*x + 4*c) + 6*sin(2*d*x + 2*c) + 4*sin(1/2*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c))))*cos(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c))) - 76*(sin(4*d*x + 4*c) + 6*sin(2*d*x + 2*c) + 4*sin(1/2*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(3/4*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c))) - 44*(sin(4*d*x + 4*c) + 6*sin(2*d*x + 2*c))*cos(1/4*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 16*(sqrt(2)*cos(4*d*x + 4*c)^2
 + 36*sqrt(2)*cos(2*d*x + 2*c)^2 + 16*sqrt(2)*cos(3/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c)))^2 + 16*sqrt(2)*cos(1/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c)))^2 + sqrt(2)*sin(4*d*x + 4*c)^2 + 12*sqrt(2)*sin(4*d*x +
 4*c)*sin(2*d*x + 2*c) + 36*sqrt(2)*sin(2*d*x + 2*c)^2 + 16*sqrt(2)*sin(3/
2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 16*sqrt(2)*sin(1/2*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*(6*sqrt(2)*cos(2*d*x + 2*c)
 + sqrt(2))*cos(4*d*x + 4*c) + 8*(sqrt(2)*cos(4*d*x + 4*c) + 6*sqrt(2)*cos
(2*d*x + 2*c) + 4*sqrt(2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +...
```

3.259.8 Giac [F]

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \int \frac{\sec{(dx+c)}^{\frac{7}{2}}}{(a\sec{(dx+c)}+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^(7/2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^(7/2)/(a*sec(d*x + c) + a)^(5/2), x)
```

3.259.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{7/2}}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

input
```
int((1/cos(c + d*x))^(7/2)/(a + a/cos(c + d*x))^(5/2),x)
```

output
```
int((1/cos(c + d*x))^(7/2)/(a + a/cos(c + d*x))^(5/2), x)
```

3.260 $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx$

3.260.1 Optimal result 2065
3.260.2 Mathematica [B] (verified) 2065
3.260.3 Rubi [A] (verified) 2066
3.260.4 Maple [B] (verified) 2068
3.260.5 Fricas [A] (verification not implemented) 2069
3.260.6 Sympy [F(-1)] 2069
3.260.7 Maxima [B] (verification not implemented) 2070
3.260.8 Giac [F] 2070
3.260.9 Mupad [F(-1)] 2071

3.260.1 Optimal result

Integrand size = 25, antiderivative size = 137

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \frac{3\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} + \frac{\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \frac{3\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}}$$

output
```
1/4*sec(d*x+c)^(5/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(5/2)+3/16*sec(d*x+c)^(
3/2)*sin(d*x+c)/a/d/(a+a*sec(d*x+c))^(3/2)+3/32*arctanh(1/2*sin(d*x+c)*a^(
1/2)*sec(d*x+c)^(1/2)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))/a^(5/2)/d*2^(1/2)
```

3.260.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 308 vs. 2(137) = 274.

Time = 0.71 (sec) , antiderivative size = 308, normalized size of antiderivative = 2.25

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \frac{6\sqrt{1-\sec(c+dx)}\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)+14\sqrt{1-\sec(c+dx)}\sec^{\frac{5}{2}}(c+}{}$$

input
```
Integrate[Sec[c + d*x]^(5/2)/(a + a*Sec[c + d*x])^(5/2),x]
```

output
```
(6*Sqrt[1 - Sec[c + d*x]]*Sec[c + d*x]^(3/2)*Sin[c + d*x] + 14*Sqrt[1 - Se
c[c + d*x]]*Sec[c + d*x]^(5/2)*Sin[c + d*x] - 3*Sqrt[2]*ArcTan[(Sqrt[2]*Sq
rt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*Tan[c + d*x] - 6*Sqrt[2]*ArcTan[
(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*Sec[c + d*x]*Tan[c +
d*x] - 3*Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]
]]*Sec[c + d*x]^2*Tan[c + d*x] + 6*ArcSin[Sqrt[1 - Sec[c + d*x]]]*(1 + Sec
[c + d*x])^2*Tan[c + d*x] + 6*ArcSin[Sqrt[Sec[c + d*x]]]*(1 + Sec[c + d*x]
)^2*Tan[c + d*x])/(32*d*Sqrt[1 - Sec[c + d*x]]*(a*(1 + Sec[c + d*x]))^(5/2
))
```

3.260.3 Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.04, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3042, 4297, 3042, 4297, 3042, 4295, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a\sec(c+dx)+a)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}}dx$$

$$\Big\downarrow \text{4297}$$

$$\frac{3\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(\sec(c+dx)a+a)^{3/2}}dx}{8a}+\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}}dx}{8a}+\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{4297}$$

$$\frac{3\left(\frac{\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{\sec(c+dx)a+a}}dx}{4a}+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}\right)}{8a}+\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\left(\frac{\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a}+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}\right)}{8a}+\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

$\Big\downarrow$ 4295

$$\frac{3\left(\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}-\frac{\int \frac{1}{2a-\frac{a^2\sin(c+dx)\tan(c+dx)}{\sec(c+dx)a+a}}d\left(-\frac{a\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{2ad}\right)}{8a}+\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

$\Big\downarrow$ 219

$$\frac{3\left(\frac{\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}\right)}{8a}+\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

input
```
Int[Sec[c + d*x]^(5/2)/(a + a*Sec[c + d*x])^(5/2),x]
```

output
```
(Sec[c + d*x]^(5/2)*Sin[c + d*x])/(4*d*(a + a*Sec[c + d*x])^(5/2)) + (3*(A
rcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c
 + d*x]])]/(2*Sqrt[2]*a^(3/2)*d) + (Sec[c + d*x]^(3/2)*Sin[c + d*x])/(2*d*
(a + a*Sec[c + d*x])^(3/2))))/(8*a)
```

3.260.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4295
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[-2*b*(d/(a*f))   Subst[Int[1/(2*b - d*x^2), x],
x, b*(Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]]))], x] /;
 FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4297
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[b*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Cs
c[e + f*x])^(n - 1)/(a*f*(2*m + 1))), x] + Simp[d*((m + 1)/(b*(2*m + 1)))
 Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ
[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && EqQ[m + n, 0] && LtQ[m,
-2^(-1)] && IntegerQ[2*m]
```

3.260.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 303 vs. $2(112) = 224$.

Time = 1.26 (sec) , antiderivative size = 304, normalized size of antiderivative = 2.22

method	result
default	$\frac{\left(-\frac{(1-\cos(dx+c))^2\csc(dx+c)^2+1}{(1-\cos(dx+c))^2\csc(dx+c)^2-1}\right)^{\frac{5}{2}}\left((1-\cos(dx+c))^2\csc(dx+c)^2-1\right)^3\sqrt{-\frac{2a}{(1-\cos(dx+c))^2\csc(dx+c)^2-1}}\left(-2\sqrt{-(1-\cos(dx+c))^2\csc}\right.}{32d\,a^3\left((1-\cos(dx+c))^2\csc(\right.}$

input
```
int(sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/32/d/a^3*(-((1-cos(d*x+c))^2*csc(d*x+c)^2+1)/((1-cos(d*x+c))^2*csc(d*x+c
)^2-1))^(5/2)*((1-cos(d*x+c))^2*csc(d*x+c)^2-1)^3*(-2*a/((1-cos(d*x+c))^2*
csc(d*x+c)^2-1))^(1/2)*(-2*(-(1-cos(d*x+c))^2*csc(d*x+c)^2-1)^(1/2)*(1-cos
(d*x+c))^3*csc(d*x+c)^3-5*(-(1-cos(d*x+c))^2*csc(d*x+c)^2-1)^(1/2)*(-cot(d
*x+c)+csc(d*x+c))+3*arctan(1/(-(1-cos(d*x+c))^2*csc(d*x+c)^2-1)^(1/2)*(-co
t(d*x+c)+csc(d*x+c))))/((1-cos(d*x+c))^2*csc(d*x+c)^2+1)^2/(-(1-cos(d*x+c)
)^2*csc(d*x+c)^2-1)^(1/2)
```

3.260.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 426, normalized size of antiderivative = 3.11

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \left[\frac{3\sqrt{2}\left(\cos\left(dx+c\right)^{3}+3\,\cos\left(dx+c\right)^{2}+3\,\cos\left(dx+c\right)+1\right)\sqrt{a}\log\left(-\frac{a\cos(dx}{}\right.}{64\left(a^{3}d\cos\left(dx+c\right)^{3}\right.}\right.$$

$$-\frac{3\sqrt{2}\left(\cos\left(dx+c\right)^{3}+3\,\cos\left(dx+c\right)^{2}+3\,\cos\left(dx+c\right)+1\right)\sqrt{-a}\arctan\left(\frac{\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}}{a\sin(dx+c)}\right)-}{32\left(a^{3}d\cos\left(dx+c\right)^{3}+3\,a^{3}d\cos\left(dx+c\right)^{2}+3\,a^{3}d\cos\left(dx+c\right)+a\right.}$$

input
```
integrate(sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
[1/64*(3*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*
sqrt(a)*log(-(a*cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) +
a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)
/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + 4*(3*cos(d*x + c)^2 + 7*cos(d*x
+ c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x +
c)))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c)
 + a^3*d), -1/32*(3*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x
 + c) + 1)*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(
d*x + c))*sqrt(cos(d*x + c))/(a*sin(d*x + c))) - 2*(3*cos(d*x + c)^2 + 7*c
os(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos
(d*x + c)))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d
*x + c) + a^3*d)]
```

3.260.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**(5/2)/(a+a*sec(d*x+c))**(5/2),x)
```

output
```
Timed out
```

3.260.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 84332 vs. $2(112) = 224$.

Time = 14.47 (sec) , antiderivative size = 84332, normalized size of antiderivative = 615.56

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
1/32*(512*((2*sin(2*d*x + 2*c) + sin(d*x + c))*cos(5/2*d*x + 5/2*c) + cos(
5/2*d*x + 5/2*c)*sin(4*d*x + 4*c) + 2*cos(5/2*d*x + 5/2*c)*sin(3*d*x + 3*c
) + (2*cos(2*d*x + 2*c) + cos(d*x + c))*sin(5/2*d*x + 5/2*c) + cos(4*d*x +
 4*c)*sin(5/2*d*x + 5/2*c) + 2*cos(3*d*x + 3*c)*sin(5/2*d*x + 5/2*c))*cos(
5*d*x + 5*c)^2 + 2560*(5*(2*sin(2*d*x + 2*c) + sin(d*x + c))*cos(5/2*d*x +
 5/2*c) + cos(5/2*d*x + 5/2*c)*sin(5*d*x + 5*c) + 5*cos(5/2*d*x + 5/2*c)*s
in(4*d*x + 4*c) + 10*cos(5/2*d*x + 5/2*c)*sin(3*d*x + 3*c) - (10*cos(2*d*x
 + 2*c) + 5*cos(d*x + c) + 1)*sin(5/2*d*x + 5/2*c) - cos(5*d*x + 5*c)*sin(
5/2*d*x + 5/2*c) - 5*cos(4*d*x + 4*c)*sin(5/2*d*x + 5/2*c) - 10*cos(3*d*x
+ 3*c)*sin(5/2*d*x + 5/2*c))*cos(8/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2
*d*x + 5/2*c)))^2 + 10240*(5*(2*sin(2*d*x + 2*c) + sin(d*x + c))*cos(5/2*d
*x + 5/2*c) + cos(5/2*d*x + 5/2*c)*sin(5*d*x + 5*c) + 5*cos(5/2*d*x + 5/2*
c)*sin(4*d*x + 4*c) + 10*cos(5/2*d*x + 5/2*c)*sin(3*d*x + 3*c) - (10*cos(2
*d*x + 2*c) + 5*cos(d*x + c) + 1)*sin(5/2*d*x + 5/2*c) - cos(5*d*x + 5*c)*
sin(5/2*d*x + 5/2*c) - 5*cos(4*d*x + 4*c)*sin(5/2*d*x + 5/2*c) - 10*cos(3*
d*x + 3*c)*sin(5/2*d*x + 5/2*c))*cos(6/5*arctan2(sin(5/2*d*x + 5/2*c), cos
(5/2*d*x + 5/2*c)))^2 + 10240*(5*(2*sin(2*d*x + 2*c) + sin(d*x + c))*cos(5
/2*d*x + 5/2*c) + cos(5/2*d*x + 5/2*c)*sin(5*d*x + 5*c) + 5*cos(5/2*d*x +
5/2*c)*sin(4*d*x + 4*c) + 10*cos(5/2*d*x + 5/2*c)*sin(3*d*x + 3*c) - (10*c
os(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*sin(5/2*d*x + 5/2*c) - cos(5*d*x ...
```

3.260.8 Giac [F]

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \int \frac{\sec\left(dx+c\right)^{\frac{5}{2}}}{\left(a\sec\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^(5/2)/(a*sec(d*x + c) + a)^(5/2), x)
```

3.260.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx=\int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

input
```
int((1/cos(c + d*x))^(5/2)/(a + a/cos(c + d*x))^(5/2),x)
```

output
```
int((1/cos(c + d*x))^(5/2)/(a + a/cos(c + d*x))^(5/2), x)
```

3.261 $\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx$

3.261.1 Optimal result 2072
3.261.2 Mathematica [A] (verified) 2072
3.261.3 Rubi [A] (verified) 2073
3.261.4 Maple [B] (verified) 2075
3.261.5 Fricas [A] (verification not implemented) 2076
3.261.6 Sympy [F] 2076
3.261.7 Maxima [B] (verification not implemented) 2077
3.261.8 Giac [F] 2077
3.261.9 Mupad [F(-1)] 2078

3.261.1 Optimal result

Integrand size = 25, antiderivative size = 137

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \frac{5\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} - \frac{\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \frac{5\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}}$$

output
```
-1/4*sec(d*x+c)^(5/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(5/2)+5/16*sec(d*x+c)^
(3/2)*sin(d*x+c)/a/d/(a+a*sec(d*x+c))^(3/2)+5/32*arctanh(1/2*sin(d*x+c)*a^
(1/2)*sec(d*x+c)^(1/2)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))/a^(5/2)/d*2^(1/2)
```

3.261.2 Mathematica [A] (verified)

Time = 3.41 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.49

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = -\frac{8\sec^{\frac{5}{2}}(c+dx)\sin(c+dx) + \frac{5(1+\sec(c+dx))\left(2\sqrt{1-\sec(c+dx)}\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)-(1+\sec(c+dx))\left(2\arcsin\left(\sqrt{1-\sec(c+dx)}\right)+2\right.\right.}{\sqrt{1-\sec(}}}{32d(a(1+\sec(c+dx)))^{5/2}}$$

input
```
Integrate[Sec[c + d*x]^(3/2)/(a + a*Sec[c + d*x])^(5/2),x]
```

output
```
-1/32*(8*Sec[c + d*x]^(5/2)*Sin[c + d*x] + (5*(1 + Sec[c + d*x])*(2*Sqrt[1
 - Sec[c + d*x]]*Sec[c + d*x]^(5/2)*Sin[c + d*x] - (1 + Sec[c + d*x])*(2*A
rcSin[Sqrt[1 - Sec[c + d*x]]] + 2*ArcSin[Sqrt[Sec[c + d*x]]] - Sqrt[2]*Arc
Tan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]] + 2*Sqrt[-((-1 +
Sec[c + d*x])*Sec[c + d*x])])*Tan[c + d*x]))/Sqrt[1 - Sec[c + d*x]])/(d*(a
*(1 + Sec[c + d*x]))^(5/2))
```

3.261.3 Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.04, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3042, 4298, 3042, 4297, 3042, 4295, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a\sec(c+dx)+a)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}}dx$$

$$\Big\downarrow \text{4298}$$

$$\frac{5\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(\sec(c+dx)a+a)^{3/2}}dx}{8a}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}}dx}{8a}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{4297}$$

$$\frac{5\left(\frac{\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{\sec(c+dx)a+a}}dx}{4a}+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}\right)}{8a}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5\left(\frac{\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a}+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}\right)}{8a}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

$\Big\downarrow$ 4295

$$\frac{5\left(\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}-\frac{\int \frac{1}{2a-\frac{a^2\sin(c+dx)\tan(c+dx)}{\sec(c+dx)a+a}}d\left(-\frac{a\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{2ad}\right)}{8a}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

$\Big\downarrow$ 219

$$\frac{5\left(\frac{\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}\right)}{8a}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

input
```
Int[Sec[c + d*x]^(3/2)/(a + a*Sec[c + d*x])^(5/2),x]
```

output
```
-1/4*(Sec[c + d*x]^(5/2)*Sin[c + d*x])/(d*(a + a*Sec[c + d*x])^(5/2)) + (5
*(ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Se
c[c + d*x]])]/(2*Sqrt[2]*a^(3/2)*d) + (Sec[c + d*x]^(3/2)*Sin[c + d*x])/(2
*d*(a + a*Sec[c + d*x])^(3/2))))/(8*a)
```

3.261.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4295
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[-2*b*(d/(a*f))   Subst[Int[1/(2*b - d*x^2), x],
x, b*(Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]]))], x] /;
 FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4297
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[b*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Cs
c[e + f*x])^(n - 1)/(a*f*(2*m + 1))), x] + Simp[d*((m + 1)/(b*(2*m + 1)))
 Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ
[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && EqQ[m + n, 0] && LtQ[m,
-2^(-1)] && IntegerQ[2*m]
```

rule 4298
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-Cot[e + f*x])*(a + b*Csc[e + f*x])^m*((d*Csc
[e + f*x])^n/(f*(2*m + 1))), x] + Simp[m/(a*(2*m + 1))   Int[(a + b*Csc[e +
 f*x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f}, x] && E
qQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && LtQ[m, -2^(-1)]
```

3.261.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 303 vs. $2(112) = 224$.

Time = 1.34 (sec) , antiderivative size = 304, normalized size of antiderivative = 2.22

method	result
default	$-\frac{\left(-\frac{(1-\cos(dx+c))^2\csc(dx+c)^2+1}{(1-\cos(dx+c))^2\csc(dx+c)^2-1}\right)^{\frac{3}{2}}\left((1-\cos(dx+c))^2\csc(dx+c)^2-1\right)^2\sqrt{-\frac{2a}{(1-\cos(dx+c))^2\csc(dx+c)^2-1}}\left(2\sqrt{-(1-\cos(dx+c))^2\cos}\right.}{32da^3\left((1-\cos(dx+c))^2\cos\right.}$

input
```
int(sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/32/d/a^3*(-((1-cos(d*x+c))^2*csc(d*x+c)^2+1)/((1-cos(d*x+c))^2*csc(d*x+
c)^2-1))^(3/2)*((1-cos(d*x+c))^2*csc(d*x+c)^2-1)^2*(-2*a/((1-cos(d*x+c))^2
*csc(d*x+c)^2-1))^(1/2)*(2*(-(1-cos(d*x+c))^2*csc(d*x+c)^2-1)^(1/2)*(1-cos
(d*x+c))^3*csc(d*x+c)^3-3*(-(1-cos(d*x+c))^2*csc(d*x+c)^2-1)^(1/2)*(-cot(d
*x+c)+csc(d*x+c))+5*arctan(1/(-(1-cos(d*x+c))^2*csc(d*x+c)^2-1)^(1/2)*(-co
t(d*x+c)+csc(d*x+c))))/((1-cos(d*x+c))^2*csc(d*x+c)^2+1)/(-(1-cos(d*x+c))^
2*csc(d*x+c)^2-1)^(1/2)
```

3.261.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 422, normalized size of antiderivative = 3.08

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \left[\frac{5\sqrt{2}\left(\cos\left(dx+c\right)^{3}+3\,\cos\left(dx+c\right)^{2}+3\,\cos\left(dx+c\right)+1\right)\sqrt{a}\log\left(-\frac{a\cos(dx}{}\right.}{64\left(a^{3}d\cos\left(dx+c\right)^{3}-\right.}\right.$$

$$-\frac{5\sqrt{2}\left(\cos\left(dx+c\right)^{3}+3\,\cos\left(dx+c\right)^{2}+3\,\cos\left(dx+c\right)+1\right)\sqrt{-a}\arctan\left(\frac{\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}}{a\sin(dx+c)}\right)-}{32\left(a^{3}d\cos\left(dx+c\right)^{3}+3\,a^{3}d\cos\left(dx+c\right)^{2}+3\,a^{3}d\cos\left(dx+c\right)+a^{3}\right.}$$

input
```
integrate(sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
[1/64*(5*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*
sqrt(a)*log(-(a*cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) +
a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)
/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + 4*(5*cos(d*x + c)^2 + cos(d*x +
c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)
))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) +
 a^3*d), -1/32*(5*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x +
 c) + 1)*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*
x + c))*sqrt(cos(d*x + c))/(a*sin(d*x + c))) - 2*(5*cos(d*x + c)^2 + cos(d
*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x
 + c)))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x +
 c) + a^3*d)]
```

3.261.6 Sympy [F]

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a(\sec(c+dx)+1))^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)**(3/2)/(a+a*sec(d*x+c))**(5/2),x)
```

output
```
Integral(sec(c + d*x)**(3/2)/(a*(sec(c + d*x) + 1))**(5/2), x)
```

3.261.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2875 vs. $2(112) = 224$.

Time = 0.57 (sec) , antiderivative size = 2875, normalized size of antiderivative = 20.99

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
1/32*(4*(3*sin(3/2*d*x + 3/2*c) + 5*sin(7/3*arctan2(sin(3/2*d*x + 3/2*c),
cos(3/2*d*x + 3/2*c))) - 3*sin(5/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d
*x + 3/2*c))) - 5*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*
c))))*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 40*(2
*sin(3*d*x + 3*c) + 3*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x +
3/2*c))) + 2*sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))
*cos(7/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 24*(2*sin(
3*d*x + 3*c) + 3*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c
))) + 2*sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*cos(
5/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 24*(3*sin(3/2*d
*x + 3/2*c) - 5*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)
)))*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 16*(3*s
in(3/2*d*x + 3/2*c) - 5*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x
+ 3/2*c))))*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) +
 5*(16*cos(3*d*x + 3*c)^2 + 2*(4*cos(3*d*x + 3*c) + 6*cos(4/3*arctan2(sin(
3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 4*cos(2/3*arctan2(sin(3/2*d*x +
 3/2*c), cos(3/2*d*x + 3/2*c))) + 1)*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c),
 cos(3/2*d*x + 3/2*c))) + cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*
x + 3/2*c)))^2 + 12*(4*cos(3*d*x + 3*c) + 4*cos(2/3*arctan2(sin(3/2*d*x +
3/2*c), cos(3/2*d*x + 3/2*c))) + 1)*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c...
```

3.261.8 Giac [F]

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \int \frac{\sec\left(dx+c\right)^{\frac{3}{2}}}{\left(a\sec\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^(3/2)/(a*sec(d*x + c) + a)^(5/2), x)
```

3.261.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx=\int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

input
```
int((1/cos(c + d*x))^(3/2)/(a + a/cos(c + d*x))^(5/2),x)
```

output
```
int((1/cos(c + d*x))^(3/2)/(a + a/cos(c + d*x))^(5/2), x)
```

3.262 $\int \frac{\sqrt{\sec(c+dx)}}{(a+a\sec(c+dx))^{5/2}}\,dx$

3.262.1 Optimal result . . . 2079
3.262.2 Mathematica [A] (verified) . . . 2079
3.262.3 Rubi [A] (verified) . . . 2080
3.262.4 Maple [B] (verified) . . . 2082
3.262.5 Fricas [A] (verification not implemented) . . . 2083
3.262.6 Sympy [F] . . . 2084
3.262.7 Maxima [B] (verification not implemented) . . . 2084
3.262.8 Giac [F] . . . 2085
3.262.9 Mupad [F(-1)] . . . 2085

3.262.1 Optimal result

Integrand size = 25, antiderivative size = 137

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+a\sec(c+dx))^{5/2}}\,dx = \frac{19\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} - \frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{9\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}}$$

output
```
-1/4*sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(5/2)-9/16*sec(d*x+c)^
(3/2)*sin(d*x+c)/a/d/(a+a*sec(d*x+c))^(3/2)+19/32*arctanh(1/2*sin(d*x+c)*a
^(1/2)*sec(d*x+c)^(1/2)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))/a^(5/2)/d*2^(1/2)
```

3.262.2 Mathematica [A] (verified)

Time = 1.00 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.07

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+a\sec(c+dx))^{5/2}}\,dx = \frac{-76\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)\cos^5\left(\frac{1}{2}(c+dx)\right)\sec^3(c+dx)\sin\left(\frac{1}{2}(c+dx)\right) -}{16d\sqrt{1-\sec(c+dx)}(a(1+}$$

input
```
Integrate[Sqrt[Sec[c + d*x]]/(a + a*Sec[c + d*x])^(5/2),x]
```

output
```
(-76*Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*C
os[(c + d*x)/2]^5*Sec[c + d*x]^3*Sin[(c + d*x)/2] - (9 + 13*Cos[c + d*x])*
Sqrt[1 - Sec[c + d*x]]*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(16*d*Sqrt[1 - Sec
[c + d*x]]*(a*(1 + Sec[c + d*x]))^(5/2))
```

3.262.3 Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.04, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3042, 4304, 27, 3042, 4500, 3042, 4295, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\sec(c+dx)}}{(a\sec(c+dx)+a)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}}dx$$

$$\Big\downarrow \text{4304}$$

$$-\frac{\int -\frac{\sqrt{\sec(c+dx)}(7a-2a\sec(c+dx))}{2(\sec(c+dx)a+a)^{3/2}}dx}{4a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\sqrt{\sec(c+dx)}(7a-2a\sec(c+dx))}{(\sec(c+dx)a+a)^{3/2}}dx}{8a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(7a-2a\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}}dx}{8a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{4500}$$

$$\frac{\frac{19}{4}\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{\sec(c+dx)}a+a}dx-\frac{9a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}}{8a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{19}{4}\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx-\frac{9a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}}{8a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \; 4295$$

$$\frac{-\frac{19\int \frac{1}{2a-\frac{a^2\sin(c+dx)\tan(c+dx)}{\sec(c+dx)a+a}}d\left(-\frac{a\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{2d}-\frac{9a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}}{8a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{19\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}\sqrt{a}d}-\frac{9a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}}{8a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

input
```
Int[Sqrt[Sec[c + d*x]]/(a + a*Sec[c + d*x])^(5/2),x]
```

output
```
-1/4*(Sec[c + d*x]^(3/2)*Sin[c + d*x])/(d*(a + a*Sec[c + d*x])^(5/2)) + ((
19*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*S
ec[c + d*x]])])/(2*Sqrt[2]*Sqrt[a]*d) - (9*a*Sec[c + d*x]^(3/2)*Sin[c + d*
x])/(2*d*(a + a*Sec[c + d*x])^(3/2)))/(8*a^2)
```

3.262.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4295
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[-2*b*(d/(a*f))   Subst[Int[1/(2*b - d*x^2), x],
x, b*(Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]]))], x] /;
 FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4304
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-Cot[e + f*x])*(a + b*Csc[e + f*x])^m*((d*Csc
[e + f*x])^n/(f*(2*m + 1))), x] + Simp[1/(a^2*(2*m + 1))   Int[(a + b*Csc[e
 + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e
+ f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ
[m, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

rule 4500
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
- a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
 1))), x] + Simp[(a*A*m + b*B*(m + 1))/(a^2*(2*m + 1))   Int[(a + b*Csc[e +
 f*x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, n}
, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && LeQ[
m, -1]
```

3.262.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 276 vs. $2(112) = 224$.

Time = 1.47 (sec) , antiderivative size = 277, normalized size of antiderivative = 2.02

method	result
default	$\frac{\sqrt{-\frac{(1-\cos(dx+c))^2\csc(dx+c)^2+1}{(1-\cos(dx+c))^2\csc(dx+c)^2-1}}\left((1-\cos(dx+c))^2\csc(dx+c)^2-1\right)\sqrt{-\frac{2a}{(1-\cos(dx+c))^2\csc(dx+c)^2-1}}\left(-2\sqrt{-(1-\cos(dx+c))^2\csc(dx}\right.}{32d\,a^3\sqrt{-(}}$

input
```
int(sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/32/d/a^3*(-((1-cos(d*x+c))^2*csc(d*x+c)^2+1)/((1-cos(d*x+c))^2*csc(d*x+c
)^2-1))^(1/2)*((1-cos(d*x+c))^2*csc(d*x+c)^2-1)*(-2*a/((1-cos(d*x+c))^2*cs
c(d*x+c)^2-1))^(1/2)*(-2*(-(1-cos(d*x+c))^2*csc(d*x+c)^2-1)^(1/2)*(1-cos(d
*x+c))^3*csc(d*x+c)^3+11*(-(1-cos(d*x+c))^2*csc(d*x+c)^2-1)^(1/2)*(-cot(d*
x+c)+csc(d*x+c))+19*arctan(1/(-(1-cos(d*x+c))^2*csc(d*x+c)^2-1)^(1/2)*(-co
t(d*x+c)+csc(d*x+c))))/(-(1-cos(d*x+c))^2*csc(d*x+c)^2-1)^(1/2)
```

3.262.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 426, normalized size of antiderivative = 3.11

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+a\sec(c+dx))^{5/2}}\,dx = \left[\frac{19\sqrt{2}\left(\cos\left(dx+c\right)^3+3\cos\left(dx+c\right)^2+3\cos\left(dx+c\right)+1\right)\sqrt{a}\log\left(-\frac{a\cos(}{}\right.}{64\left(a^3d\cos\left(dx+c\right)^{3}\right.}\right.$$

$$-\frac{19\sqrt{2}\left(\cos\left(dx+c\right)^3+3\cos\left(dx+c\right)^2+3\cos\left(dx+c\right)+1\right)\sqrt{-a}\arctan\left(\frac{\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}}{a\sin(dx+c)}\right)+}{32\left(a^3d\cos\left(dx+c\right)^3+3\,a^3d\cos\left(dx+c\right)^2+3\,a^3d\cos\left(dx+c\right)+\right.}$$

input
```
integrate(sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
[1/64*(19*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)
*sqrt(a)*log(-(a*cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) +
 a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a
)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) - 4*(13*cos(d*x + c)^2 + 9*cos(d*
x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x
+ c)))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x +
c) + a^3*d), -1/32*(19*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(
d*x + c) + 1)*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/c
os(d*x + c))*sqrt(cos(d*x + c))/(a*sin(d*x + c))) + 2*(13*cos(d*x + c)^2 +
 9*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt
(cos(d*x + c)))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*c
os(d*x + c) + a^3*d)]
```

3.262.6 Sympy [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+a\sec(c+dx))^{5/2}}\,dx = \int \frac{\sqrt{\sec(c+dx)}}{(a(\sec(c+dx)+1))^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)**(1/2)/(a+a*sec(d*x+c))**(5/2),x)
```

output
```
Integral(sqrt(sec(c + d*x))/(a*(sec(c + d*x) + 1))**(5/2), x)
```

3.262.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3049 vs. $2(112) = 224$.

Time = 0.64 (sec) , antiderivative size = 3049, normalized size of antiderivative = 22.26

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+a\sec(c+dx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
1/32*(19*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*
d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 -
2*sin(1/2*d*x + 1/2*c) + 1))*cos(4*d*x + 4*c)^2 + 304*(log(cos(1/2*d*x + 1
/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1
/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*
cos(3*d*x + 3*c)^2 + 684*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c
)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d
*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(2*d*x + 2*c)^2 + 304*(log
(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)
+ 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x
 + 1/2*c) + 1))*cos(d*x + c)^2 + 19*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*
d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2
+ sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(4*d*x + 4*c)^2
 + 304*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*
x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*
sin(1/2*d*x + 1/2*c) + 1))*sin(3*d*x + 3*c)^2 + 684*(log(cos(1/2*d*x + 1/2
*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2
*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*si
n(2*d*x + 2*c)^2 + 304*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^
2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*...
```

3.262.8 Giac [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+a\sec(c+dx))^{5/2}}\,dx = \int \frac{\sqrt{\sec(dx+c)}}{(a\sec(dx+c)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate(sqrt(sec(d*x + c))/(a*sec(d*x + c) + a)^(5/2), x)
```

3.262.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+a\sec(c+dx))^{5/2}}\,dx = \int \frac{\sqrt{\frac{1}{\cos(c+d\,x)}}}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

input
```
int((1/cos(c + d*x))^(1/2)/(a + a/cos(c + d*x))^(5/2),x)
```

output
```
int((1/cos(c + d*x))^(1/2)/(a + a/cos(c + d*x))^(5/2), x)
```

3.263 $\int \frac{1}{\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^{5/2}}\,dx$

3.263.1 Optimal result 2086
3.263.2 Mathematica [A] (verified) 2086
3.263.3 Rubi [A] (verified) 2087
3.263.4 Maple [A] (warning: unable to verify) 2090
3.263.5 Fricas [A] (verification not implemented) 2090
3.263.6 Sympy [F] 2091
3.263.7 Maxima [B] (verification not implemented) 2091
3.263.8 Giac [F] 2092
3.263.9 Mupad [F(-1)] 2093

3.263.1 Optimal result

Integrand size = 25, antiderivative size = 177

$$\int \frac{1}{\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^{5/2}}\,dx = -\frac{75\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} - \frac{\sqrt{\sec(c+dx)}\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{13\sqrt{\sec(c+dx)}\sin(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} + \frac{49\sqrt{\sec(c+dx)}\sin(c+dx)}{16a^2d\sqrt{a+a\sec(c+dx)}}$$

output
```
-75/32*arctanh(1/2*sin(d*x+c)*a^(1/2)*sec(d*x+c)^(1/2)*2^(1/2)/(a+a*sec(d*
x+c))^(1/2))/a^(5/2)/d*2^(1/2)-1/4*sin(d*x+c)*sec(d*x+c)^(1/2)/d/(a+a*sec(
d*x+c))^(5/2)-13/16*sin(d*x+c)*sec(d*x+c)^(1/2)/a/d/(a+a*sec(d*x+c))^(3/2)
+49/16*sin(d*x+c)*sec(d*x+c)^(1/2)/a^2/d/(a+a*sec(d*x+c))^(1/2)
```

3.263.2 Mathematica [A] (verified)

Time = 1.39 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.05

$$\int \frac{1}{\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^{5/2}}\,dx = \frac{300\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)\cos^5\left(\frac{1}{2}(c+dx)\right)\sec^3(c+dx)\sin(}{}$$

input
```
Integrate[1/(Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^(5/2)),x]
```

output
```
(300*Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*C
os[(c + d*x)/2]^5*Sec[c + d*x]^3*Sin[(c + d*x)/2] + (85*Sqrt[1 - Sec[c + d
*x]]*Sec[c + d*x]^(3/2) + 49*Sqrt[1 - Sec[c + d*x]]*Sec[c + d*x]^(5/2) + 3
2*Sqrt[-((-1 + Sec[c + d*x])*Sec[c + d*x])])*Sin[c + d*x])/(16*d*Sqrt[1 -
Sec[c + d*x]]*(a*(1 + Sec[c + d*x]))^(5/2))
```

3.263.3 Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.06, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {3042, 4304, 27, 3042, 4508, 27, 3042, 4501, 3042, 4295, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}}dx$$

$$\Big\downarrow \text{4304}$$

$$-\frac{\int -\frac{9a-4a\sec(c+dx)}{2\sqrt{\sec(c+dx)}(\sec(c+dx)a+a)^{3/2}}dx}{4a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{4d(a\sec(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{9a-4a\sec(c+dx)}{\sqrt{\sec(c+dx)}(\sec(c+dx)a+a)^{3/2}}dx}{8a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{4d(a\sec(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{9a-4a\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}}dx}{8a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{4d(a\sec(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{4508}$$

$$\frac{\frac{\int \frac{49a^2-26a^2\sec(c+dx)}{2\sqrt{\sec(c+dx)}\sqrt{\sec(c+dx)a+a}}dx}{2a^2}-\frac{13a\sin(c+dx)\sqrt{\sec(c+dx)}}{2d(a\sec(c+dx)+a)^{3/2}}}{8a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{4d(a\sec(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int \frac{49a^2-26a^2\sec(c+dx)}{\sqrt{\sec(c+dx)}\sqrt{\sec(c+dx)a+a}}dx}{4a^2}-\frac{13a\sin(c+dx)\sqrt{\sec(c+dx)}}{2d(a\sec(c+dx)+a)^{3/2}}}{8a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{4d(a\sec(c+dx)+a)^{5/2}}$$

$\downarrow$ 3042

$$\frac{\frac{\int \frac{49a^2-26a^2\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a^2}-\frac{13a\sin(c+dx)\sqrt{\sec(c+dx)}}{2d(a\sec(c+dx)+a)^{3/2}}}{8a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{4d(a\sec(c+dx)+a)^{5/2}}$$

$\downarrow$ 4501

$$\frac{\frac{\frac{98a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\sec(c+dx)+a}}-75a^2\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{\sec(c+dx)a+a}}dx}{4a^2}-\frac{13a\sin(c+dx)\sqrt{\sec(c+dx)}}{2d(a\sec(c+dx)+a)^{3/2}}}{8a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{4d(a\sec(c+dx)+a)^{5/2}}$$

$\downarrow$ 3042

$$\frac{\frac{\frac{98a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\sec(c+dx)+a}}-75a^2\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a^2}-\frac{13a\sin(c+dx)\sqrt{\sec(c+dx)}}{2d(a\sec(c+dx)+a)^{3/2}}}{8a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{4d(a\sec(c+dx)+a)^{5/2}}$$

$\downarrow$ 4295

$$\frac{\frac{\frac{150a^2\int \frac{1}{2a-\frac{a^2\sin(c+dx)\tan(c+dx)}{\sec(c+dx)a+a}}d\left(-\frac{a\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}+\frac{98a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\sec(c+dx)+a}}}{4a^2}-\frac{13a\sin(c+dx)\sqrt{\sec(c+dx)}}{2d(a\sec(c+dx)+a)^{3/2}}}{8a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{4d(a\sec(c+dx)+a)^{5/2}}$$

$\downarrow$ 219

$$\frac{\frac{\frac{98a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\sec(c+dx)+a}}-\frac{75\sqrt{2}a^{3/2}\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{d}}{4a^2}-\frac{13a\sin(c+dx)\sqrt{\sec(c+dx)}}{2d(a\sec(c+dx)+a)^{3/2}}}{8a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{4d(a\sec(c+dx)+a)^{5/2}}$$

input
```
Int[1/(Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^(5/2)),x]
```

3.263. $\int \frac{1}{\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^{5/2}}\,dx$

output
```
-1/4*(Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(d*(a + a*Sec[c + d*x])^(5/2)) + ((
-13*a*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(2*d*(a + a*Sec[c + d*x])^(3/2)) +
((-75*Sqrt[2]*a^(3/2)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(S
qrt[2]*Sqrt[a + a*Sec[c + d*x]])])/d + (98*a^2*Sqrt[Sec[c + d*x]]*Sin[c +
d*x])/(d*Sqrt[a + a*Sec[c + d*x]]))/(4*a^2))/(8*a^2)
```

3.263.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4295
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[-2*b*(d/(a*f))   Subst[Int[1/(2*b - d*x^2), x],
x, b*(Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]]))], x] /;
 FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4304
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-Cot[e + f*x])*(a + b*Csc[e + f*x])^m*((d*Csc
[e + f*x])^n/(f*(2*m + 1))), x] + Simp[1/(a^2*(2*m + 1))   Int[(a + b*Csc[e
 + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e
+ f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ
[m, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

rule 4501
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*Cot[e
 + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(f*n)), x] - Simp[(a*A*m
 - b*B*n)/(b*d*n)   Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x]
, x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a
^2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LeQ[m, -1]
```

3.263. $\int \frac{1}{\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^{5/2}}\,dx$

rule 4508
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
- a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
 1))), x] - Simp[1/(a^2*(2*m + 1))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Cs
c[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[
e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B
, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

3.263.4 Maple [A] (warning: unable to verify)

Time = 1.44 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.26

method	result
default	$-\frac{\left(2(1-\cos(dx+c))^5\csc(dx+c)^5-17(1-\cos(dx+c))^3\csc(dx+c)^3+75\arctan\left(\frac{-\cot(dx+c)+\csc(dx+c)}{\sqrt{-(1-\cos(dx+c))^2\csc(dx+c)^2-1}}\right)\sqrt{-(1-\cos(dx+c))}\right.}{32d\,a^3\sqrt{-\frac{(1-\cos(dx+c))^2\csc(dx+c)^2+1}{(1-\cos(dx+c))^2\csc(dx+c)^2-1}}}$

input
```
int(1/(a+a*sec(d*x+c))^(5/2)/sec(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/32/d/a^3*(2*(1-cos(d*x+c))^5*csc(d*x+c)^5-17*(1-cos(d*x+c))^3*csc(d*x+c
)^3+75*arctan(1/(-(1-cos(d*x+c))^2*csc(d*x+c)^2-1)^(1/2)*(-cot(d*x+c)+csc(
d*x+c)))*(-(1-cos(d*x+c))^2*csc(d*x+c)^2-1)^(1/2)-83*csc(d*x+c)+83*cot(d*x
+c))*(-2*a/((1-cos(d*x+c))^2*csc(d*x+c)^2-1))^(1/2)/(-((1-cos(d*x+c))^2*cs
c(d*x+c)^2+1)/((1-cos(d*x+c))^2*csc(d*x+c)^2-1))^(1/2)
```

3.263.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 446, normalized size of antiderivative = 2.52

$$\int \frac{1}{\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^{5/2}}\,dx = \left[\frac{75\sqrt{2}\left(\cos\left(dx+c\right)^3+3\,\cos\left(dx+c\right)^2+3\,\cos\left(dx+c\right)+1\right)\sqrt{}}{}\right.$$

input
```
integrate(1/(a+a*sec(d*x+c))^(5/2)/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
[1/64*(75*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)
*sqrt(a)*log(-(a*cos(d*x + c)^2 + 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) +
 a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a
)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + 4*(32*cos(d*x + c)^3 + 85*cos(d
*x + c)^2 + 49*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d
*x + c)/sqrt(cos(d*x + c)))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2
 + 3*a^3*d*cos(d*x + c) + a^3*d), 1/32*(75*sqrt(2)*(cos(d*x + c)^3 + 3*cos
(d*x + c)^2 + 3*cos(d*x + c) + 1)*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt((a
*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))/(a*sin(d*x + c))) + 2*
(32*cos(d*x + c)^3 + 85*cos(d*x + c)^2 + 49*cos(d*x + c))*sqrt((a*cos(d*x
+ c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^3*d*cos(d*x +
c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)]
```

3.263.6 Sympy [F]

$$\int \frac{1}{\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^{5/2}}\,dx = \int \frac{1}{(a(\sec(c+dx)+1))^{\frac{5}{2}}\sqrt{\sec(c+dx)}}\,dx$$

input
```
integrate(1/(a+a*sec(d*x+c))**(5/2)/sec(d*x+c)**(1/2),x)
```

output
```
Integral(1/((a*(sec(c + d*x) + 1))**(5/2)*sqrt(sec(c + d*x))), x)
```

3.263.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 258456 vs. $2(146) = 292$.

Time = 2.87 (sec) , antiderivative size = 258456, normalized size of antiderivative = 1460.20

$$\int \frac{1}{\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+a*sec(d*x+c))^(5/2)/sec(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
-1/32*(576*(75*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin
(1/2*d*x + 1/2*c) + 1) - 75*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2
*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) - 64*sin(1/2*d*x + 1/2*c))*cos(5/2*d*x
 + 5/2*c)^6 + 14400*(75*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^
2 + 2*sin(1/2*d*x + 1/2*c) + 1) - 75*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*
d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) - 64*sin(1/2*d*x + 1/2*c))*co
s(3/2*d*x + 3/2*c)^6 + 187500*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x +
1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(
1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(1/2*d*x + 1/2*c)^6 +
 576*(75*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d
*x + 1/2*c) + 1) - 75*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2
- 2*sin(1/2*d*x + 1/2*c) + 1) - 64*sin(1/2*d*x + 1/2*c))*sin(5/2*d*x + 5/2
*c)^6 + 5184*(75*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*s
in(1/2*d*x + 1/2*c) + 1) - 75*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1
/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) - 64*sin(1/2*d*x + 1/2*c))*sin(3/2*d
*x + 3/2*c)^6 + 262500*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^
2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x
 + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(1/2*d*x + 1/2*c)^4*sin(1/2*
d*x + 1/2*c)^2 + 77700*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^
2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*...
```

3.263.8 Giac [F]

$$\int \frac{1}{\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^{5/2}}\,dx = \int \frac{1}{(a\sec(dx+c)+a)^{\frac{5}{2}}\sqrt{\sec(dx+c)}}\,dx$$

input
```
integrate(1/(a+a*sec(d*x+c))^(5/2)/sec(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/((a*sec(d*x + c) + a)^(5/2)*sqrt(sec(d*x + c))), x)
```

3.263.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^{5/2}}\,dx=\int \frac{1}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{5/2}\,\sqrt{\frac{1}{\cos(c+d\,x)}}}\,dx$$

input
```
int(1/((a + a/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(1/2)),x)
```

output
```
int(1/((a + a/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(1/2)), x)
```

3.264 $\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}}\,dx$

3.264.1 Optimal result . . . 2094
3.264.2 Mathematica [A] (verified) . . . 2095
3.264.3 Rubi [A] (verified) . . . 2095
3.264.4 Maple [A] (warning: unable to verify) . . . 2099
3.264.5 Fricas [A] (verification not implemented) . . . 2100
3.264.6 Sympy [F(-1)] . . . 2101
3.264.7 Maxima [B] (verification not implemented) . . . 2101
3.264.8 Giac [F] . . . 2102
3.264.9 Mupad [F(-1)] . . . 2102

3.264.1 Optimal result

Integrand size = 25, antiderivative size = 217

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}}\,dx = \frac{163\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} - \frac{\sin(c+dx)}{4d\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^{5/2}} - \frac{17\sin(c+dx)}{16ad\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^{3/2}} + \frac{95\sin(c+dx)}{48a^2d\sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+dx)}} - \frac{299\sqrt{\sec(c+dx)}\sin(c+dx)}{48a^2d\sqrt{a+a\sec(c+dx)}}$$

output
```
163/32*arctanh(1/2*sin(d*x+c)*a^(1/2)*sec(d*x+c)^(1/2)*2^(1/2)/(a+a*sec(d*
x+c))^(1/2))/a^(5/2)/d*2^(1/2)-1/4*sin(d*x+c)/d/(a+a*sec(d*x+c))^(5/2)/sec
(d*x+c)^(1/2)-17/16*sin(d*x+c)/a/d/(a+a*sec(d*x+c))^(3/2)/sec(d*x+c)^(1/2)
+95/48*sin(d*x+c)/a^2/d/sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2)-299/48*sin
(d*x+c)*sec(d*x+c)^(1/2)/a^2/d/(a+a*sec(d*x+c))^(1/2)
```

3.264.2 Mathematica [A] (verified)

Time = 2.65 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.76

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}}\,dx = -\frac{\sec(c+dx)\left(978\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)\cos^4\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)+\sqrt{1-\sec(c+dx)}(\right.}{48d\sqrt{-((-1+\sec(c+dx))\sec(c+dx))}(a(1-}$$

input
```
Integrate[1/(Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(5/2)),x]
```

output
```
-1/48*(Sec[c + d*x]*(978*Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[
1 - Sec[c + d*x]]]*Cos[(c + d*x)/2]^4*Sec[c + d*x]^(5/2)*Sin[c + d*x] + Sq
rt[1 - Sec[c + d*x]]*(487 + 379*Sec[c + d*x] + 16*Cos[2*(c + d*x)]*(-1 + 5
*Sec[c + d*x]))*Tan[c + d*x]))/(d*Sqrt[-((-1 + Sec[c + d*x])*Sec[c + d*x])
]*(a*(1 + Sec[c + d*x]))^(5/2))
```

3.264.3 Rubi [A] (verified)

Time = 1.18 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.08, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.560$, Rules used = {3042, 4304, 27, 3042, 4508, 27, 3042, 4510, 27, 3042, 4501, 3042, 4295, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}}dx$$

$$\Big\downarrow \text{4304}$$

$$-\frac{\int -\frac{11a-6a\sec(c+dx)}{2\sec^{\frac{3}{2}}(c+dx)(\sec(c+dx)a+a)^{3/2}}dx}{4a^2}-\frac{\sin(c+dx)}{4d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{11a-6a\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(\sec(c+dx)a+a)^{3/2}}dx}{8a^2}-\frac{\sin(c+dx)}{4d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^{5/2}}$$

$\Big\downarrow$ 3042

$$\frac{\int \frac{11a-6a\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}}dx}{8a^2}-\frac{\sin(c+dx)}{4d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^{5/2}}$$

$\Big\downarrow$ 4508

$$\frac{\frac{\int \frac{95a^2-68a^2\sec(c+dx)}{2\sec^{\frac{3}{2}}(c+dx)\sqrt{\sec(c+dx)a+a}}dx}{2a^2}-\frac{17a\sin(c+dx)}{2d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^{3/2}}}{8a^2}-\frac{\sin(c+dx)}{4d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^{5/2}}$$

$\Big\downarrow$ 27

$$\frac{\frac{\int \frac{95a^2-68a^2\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)\sqrt{\sec(c+dx)a+a}}dx}{4a^2}-\frac{17a\sin(c+dx)}{2d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^{3/2}}}{8a^2}-\frac{\sin(c+dx)}{4d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^{5/2}}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\int \frac{95a^2-68a^2\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a^2}-\frac{17a\sin(c+dx)}{2d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^{3/2}}}{8a^2}-\frac{\sin(c+dx)}{4d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^{5/2}}$$

$\Big\downarrow$ 4510

$$\frac{\frac{2\int -\frac{299a^3-190a^3\sec(c+dx)}{2\sqrt{\sec(c+dx)}\sqrt{\sec(c+dx)a+a}}dx}{3a}+\frac{190a^2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}}{4a^2}-\frac{17a\sin(c+dx)}{2d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^{3/2}}}{8a^2}-\frac{\sin(c+dx)}{4d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^{5/2}}$$

$\Big\downarrow$ 27

$$\frac{\frac{\frac{190a^2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}-\frac{\int \frac{299a^3-190a^3\sec(c+dx)}{\sqrt{\sec(c+dx)}\sqrt{\sec(c+dx)a+a}}dx}{3a}}{4a^2}-\frac{17a\sin(c+dx)}{2d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^{3/2}}}{8a^2}-\frac{\sin(c+dx)}{4d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^{5/2}}$$

$\Big\downarrow$ 3042

3.264. $\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}}dx$

$$\frac{\frac{\frac{190a^2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}-\frac{\int\frac{299a^3-190a^3\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{3a}}{4a^2}-\frac{17a\sin(c+dx)}{2d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^{3/2}}}{8a^2}-\frac{\sin(c+dx)}{4d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^{5/2}}$$

$\downarrow$ 4501

$$\frac{\frac{\frac{190a^2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}-\frac{\frac{598a^3\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\sec(c+dx)+a}}-489a^3\int\frac{\sqrt{\sec(c+dx)}}{\sqrt{\sec(c+dx)a+a}}dx}{3a}}{4a^2}-\frac{17a\sin(c+dx)}{2d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^{3/2}}}{8a^2}-\frac{\sin(c+dx)}{4d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^{5/2}}$$

$\downarrow$ 3042

$$\frac{\frac{\frac{190a^2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}-\frac{\frac{598a^3\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\sec(c+dx)+a}}-489a^3\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{3a}}{4a^2}-\frac{17a\sin(c+dx)}{2d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^{3/2}}}{8a^2}-\frac{\sin(c+dx)}{4d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^{5/2}}$$

$\downarrow$ 4295

$$\frac{\frac{\frac{190a^2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}-\frac{\frac{978a^3\int\frac{1}{2a-\frac{a^2\sin(c+dx)\tan(c+dx)}{\sec(c+dx)a+a}}d\left(-\frac{a\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}+\frac{598a^3\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\sec(c+dx)+a}}}{3a}}{4a^2}-\frac{17a\sin(c+d}{2d\sqrt{\sec(c+dx)}(a\sec(}}{8a^2}-\frac{\sin(c+dx)}{4d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^{5/2}}$$

$\downarrow$ 219

$$\frac{\frac{\frac{190a^2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}-\frac{\frac{598a^3\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\sec(c+dx)+a}}-\frac{489\sqrt{2}a^{5/2}\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{d}}{3a}}{4a^2}-\frac{17a\sin(c+dx)}{2d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^{3/2}}}{8a^2}-\frac{\sin(c+dx)}{4d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^{5/2}}$$

input
```
Int[1/(Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(5/2)),x]
```

3.264. $\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}}\,dx$

output
```
-1/4*Sin[c + d*x]/(d*Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^(5/2)) + ((-1
7*a*Sin[c + d*x])/(2*d*Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^(3/2)) + ((
190*a^2*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) -
((-489*Sqrt[2]*a^(5/2)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(
Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/d + (598*a^3*Sqrt[Sec[c + d*x]]*Sin[c
+ d*x])/(d*Sqrt[a + a*Sec[c + d*x]]))/(3*a))/(4*a^2))/(8*a^2)
```

3.264.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4295
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[-2*b*(d/(a*f))   Subst[Int[1/(2*b - d*x^2), x],
x, b*(Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]]))], x] /;
 FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4304
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-Cot[e + f*x])*(a + b*Csc[e + f*x])^m*((d*Csc
[e + f*x])^n/(f*(2*m + 1))), x] + Simp[1/(a^2*(2*m + 1))   Int[(a + b*Csc[e
 + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e
+ f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ
[m, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

3.264. $\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}}\,dx$

rule 4501
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*Cot[e
 + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(f*n)), x] - Simp[(a*A*m
 - b*B*n)/(b*d*n)   Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x]
, x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a
^2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LeQ[m, -1]
```

rule 4508
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
- a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
 1))), x] - Simp[1/(a^2*(2*m + 1))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Cs
c[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[
e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B
, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

rule 4510
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*Cot[e
 + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(f*n)), x] - Simp[1/(b*d
*n)   Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*
n - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

3.264.4 Maple [A] (warning: unable to verify)

Time = 1.40 (sec) , antiderivative size = 361, normalized size of antiderivative = 1.66

method	result
default	$-\frac{\left(6(1-\cos(dx+c))^7\csc(dx+c)^7-69(1-\cos(dx+c))^5\csc(dx+c)^5+489\arctan\left(\frac{-\cot(dx+c)+\csc(dx+c)}{\sqrt{-(1-\cos(dx+c))^2\csc(dx+c)^2-1}}\right)\sqrt{-(1-\cos(dx+c)}\right.}{}$

input
```
int(1/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/96/d/a^3*(6*(1-cos(d*x+c))^7*csc(d*x+c)^7-69*(1-cos(d*x+c))^5*csc(d*x+c
)^5+489*arctan(1/(-(1-cos(d*x+c))^2*csc(d*x+c)^2-1)^(1/2)*(-cot(d*x+c)+csc
(d*x+c)))*(-(1-cos(d*x+c))^2*csc(d*x+c)^2-1)^(1/2)*(1-cos(d*x+c))^2*csc(d*
x+c)^2-668*(1-cos(d*x+c))^3*csc(d*x+c)^3+489*arctan(1/(-(1-cos(d*x+c))^2*c
sc(d*x+c)^2-1)^(1/2)*(-cot(d*x+c)+csc(d*x+c)))*(-(1-cos(d*x+c))^2*csc(d*x+
c)^2-1)^(1/2)-465*csc(d*x+c)+465*cot(d*x+c))*(-2*a/((1-cos(d*x+c))^2*csc(d
*x+c)^2-1))^(1/2)/((1-cos(d*x+c))^2*csc(d*x+c)^2-1)/(-((1-cos(d*x+c))^2*cs
c(d*x+c)^2+1)/((1-cos(d*x+c))^2*csc(d*x+c)^2-1))^(3/2)
```

3.264.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 466, normalized size of antiderivative = 2.15

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}}\,dx = \left[\frac{489\sqrt{2}\left(\cos\left(dx+c\right)^3+3\cos\left(dx+c\right)^2+3\cos\left(dx+c\right)+1\right)\sqrt{}}{}\right.$$

$$-\frac{489\sqrt{2}\left(\cos\left(dx+c\right)^3+3\cos\left(dx+c\right)^2+3\cos\left(dx+c\right)+1\right)\sqrt{-a}\arctan\left(\frac{\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}}{a\sin(dx+c)}\right)}{96\left(a^3d\cos\left(dx+c\right)^3+3\,a^3d\cos\left(dx+c\right)^2+3\,a^3d\,\mathrm{c}\right.}$$

input
```
integrate(1/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
[1/192*(489*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) +
1)*sqrt(a)*log(-(a*cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c)
 + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3
*a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + 4*(32*cos(d*x + c)^4 - 160*co
s(d*x + c)^3 - 503*cos(d*x + c)^2 - 299*cos(d*x + c))*sqrt((a*cos(d*x + c)
 + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^3*d*cos(d*x + c)^3
 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d), -1/96*(489*sqrt
(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*sqrt(-a)*arct
an(sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x +
 c))/(a*sin(d*x + c))) - 2*(32*cos(d*x + c)^4 - 160*cos(d*x + c)^3 - 503*c
os(d*x + c)^2 - 299*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*
sin(d*x + c)/sqrt(cos(d*x + c)))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x +
 c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)]
```

3.264.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/sec(d*x+c)**(3/2)/(a+a*sec(d*x+c))**(5/2),x)
```

output
```
Timed out
```

3.264.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 148823 vs. $2(180) = 360$.

Time = 2.75 (sec) , antiderivative size = 148823, normalized size of antiderivative = 685.82

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
1/96*(32*(cos(3*d*x + 3*c)^2*sin(3/2*d*x + 3/2*c) + sin(3*d*x + 3*c)^2*sin
(3/2*d*x + 3/2*c) - 15*(cos(3*d*x + 3*c)^2 + sin(3*d*x + 3*c)^2)*sin(1/3*a
rctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*cos(11/3*arctan2(sin(
3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^4 + 41472*(cos(3*d*x + 3*c)^2*sin
(3/2*d*x + 3/2*c) + sin(3*d*x + 3*c)^2*sin(3/2*d*x + 3/2*c) - 15*(cos(3*d*
x + 3*c)^2 + sin(3*d*x + 3*c)^2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos
(3/2*d*x + 3/2*c))))*cos(7/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3
/2*c)))^4 + 8192*(cos(3*d*x + 3*c)^2*sin(3/2*d*x + 3/2*c) + sin(3*d*x + 3*
c)^2*sin(3/2*d*x + 3/2*c) - 15*(cos(3*d*x + 3*c)^2 + sin(3*d*x + 3*c)^2)*s
in(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*cos(5/3*arcta
n2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^4 + 288*sin(3/2*d*x + 3/2*
c)^5 + 32*(cos(3*d*x + 3*c)^2*sin(3/2*d*x + 3/2*c) + sin(3*d*x + 3*c)^2*si
n(3/2*d*x + 3/2*c) - 15*(cos(3*d*x + 3*c)^2 + sin(3*d*x + 3*c)^2)*sin(1/3*
arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sin(11/3*arctan2(sin
(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^4 + 41472*(cos(3*d*x + 3*c)^2*si
n(3/2*d*x + 3/2*c) + sin(3*d*x + 3*c)^2*sin(3/2*d*x + 3/2*c) - 15*(cos(3*d
*x + 3*c)^2 + sin(3*d*x + 3*c)^2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), co
s(3/2*d*x + 3/2*c))))*sin(7/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x +
3/2*c)))^4 + 8192*(cos(3*d*x + 3*c)^2*sin(3/2*d*x + 3/2*c) + sin(3*d*x + 3
*c)^2*sin(3/2*d*x + 3/2*c) - 15*(cos(3*d*x + 3*c)^2 + sin(3*d*x + 3*c)^...
```

3.264.8 Giac [F]

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}}\,dx = \int \frac{1}{(a\sec(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate(1/((a*sec(d*x + c) + a)^(5/2)*sec(d*x + c)^(3/2)), x)
```

3.264.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}}\,dx = \int \frac{1}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{5/2}\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

input
```
int(1/((a + a/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(3/2)),x)
```

output
```
int(1/((a + a/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(3/2)), x)
```

3.265 $\int \frac{\sec^{\frac{7}{2}}(c+dx)}{\sqrt{1+\sec(c+dx)}}\,dx$

3.265.1 Optimal result 2103
3.265.2 Mathematica [A] (verified) 2103
3.265.3 Rubi [A] (verified) 2104
3.265.4 Maple [B] (verified) 2108
3.265.5 Fricas [B] (verification not implemented) 2108
3.265.6 Sympy [F(-1)] 2109
3.265.7 Maxima [B] (verification not implemented) 2109
3.265.8 Giac [F] 2110
3.265.9 Mupad [F(-1)] 2111

3.265.1 Optimal result

Integrand size = 23, antiderivative size = 126

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{\sqrt{1+\sec(c+dx)}}\,dx = -\frac{\sqrt{2}\operatorname{arcsinh}\left(\frac{\tan(c+dx)}{1+\sec(c+dx)}\right)}{d}+\frac{7\operatorname{arcsinh}\left(\frac{\tan(c+dx)}{\sqrt{1+\sec(c+dx)}}\right)}{4d} - \frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4d\sqrt{1+\sec(c+dx)}}+\frac{\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2d\sqrt{1+\sec(c+dx)}}$$

output
```
7/4*arcsinh(tan(d*x+c)/(1+sec(d*x+c))^(1/2))/d-arcsinh(tan(d*x+c)/(1+sec(d
*x+c)))*2^(1/2)/d-1/4*sec(d*x+c)^(3/2)*sin(d*x+c)/d/(1+sec(d*x+c))^(1/2)+1
/2*sec(d*x+c)^(5/2)*sin(d*x+c)/d/(1+sec(d*x+c))^(1/2)
```

3.265.2 Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.11

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{\sqrt{1+\sec(c+dx)}}\,dx = \frac{\cot(c+dx)\left(\arcsin\left(\sqrt{1-\sec(c+dx)}\right)+8\arcsin\left(\sqrt{\sec(c+dx)}\right)-4\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)-2\sqrt{1}\right.}{4d}$$

input
```
Integrate[Sec[c + d*x]^(7/2)/Sqrt[1 + Sec[c + d*x]],x]
```

output
```
(Cot[c + d*x]*(ArcSin[Sqrt[1 - Sec[c + d*x]]] + 8*ArcSin[Sqrt[Sec[c + d*x]
]] - 4*Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]
 - 2*Sqrt[1 - Sec[c + d*x]]*Sec[c + d*x]^(3/2) + Sqrt[-((-1 + Sec[c + d*x]
)*Sec[c + d*x])])*Sqrt[-Tan[c + d*x]^2])/(4*d)
```

3.265.3 Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.05, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.522$, Rules used = {3042, 4309, 3042, 4509, 27, 3042, 4511, 3042, 4288, 222, 4294, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{\sqrt{\sec(c+dx)+1}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)+1}}dx$$

$$\Big\downarrow \text{4309}$$

$$\frac{1}{4}\int \frac{(3-\sec(c+dx))\sec^{\frac{3}{2}}(c+dx)}{\sqrt{\sec(c+dx)+1}}dx+\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2d\sqrt{\sec(c+dx)+1}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}\int \frac{\left(3-\csc\left(c+dx+\frac{\pi}{2}\right)\right)\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)+1}}dx+\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2d\sqrt{\sec(c+dx)+1}}$$

$$\Big\downarrow \text{4509}$$

$$\frac{1}{4}\left(\int -\frac{(1-7\sec(c+dx))\sqrt{\sec(c+dx)}}{2\sqrt{\sec(c+dx)+1}}dx-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{\sec(c+dx)+1}}\right)+\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2d\sqrt{\sec(c+dx)+1}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{4}\left(-\frac{1}{2}\int \frac{(1-7\sec(c+dx))\sqrt{\sec(c+dx)}}{\sqrt{\sec(c+dx)+1}}dx-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{\sec(c+dx)+1}}\right)+\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2d\sqrt{\sec(c+dx)+1}}$$

$\Big\downarrow$ 3042

$$\frac{1}{4}\left(-\frac{1}{2}\int \frac{\left(1-7\csc\left(c+dx+\frac{\pi}{2}\right)\right)\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)+1}}dx-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{\sec(c+dx)+1}}\right)+\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2d\sqrt{\sec(c+dx)+1}}$$

$\Big\downarrow$ 4511

$$\frac{1}{4}\left(\frac{1}{2}\left(7\int \sqrt{\sec(c+dx)}\sqrt{\sec(c+dx)+1}dx-8\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{\sec(c+dx)+1}}dx\right)-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{\sec(c+dx)+1}}\right)+\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2d\sqrt{\sec(c+dx)+1}}$$

$\Big\downarrow$ 3042

$$\frac{1}{4}\left(\frac{1}{2}\left(7\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)+1}dx-8\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)+1}}dx\right)-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c}{d\sqrt{\sec(c+dx)}}\right.$$
$$\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2d\sqrt{\sec(c+dx)+1}}$$

$\Big\downarrow$ 4288

$$\frac{1}{4}\left(\frac{1}{2}\left(-8\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)+1}}dx-\frac{14\int \frac{1}{\sqrt{\frac{\tan^2(c+dx)}{\sec(c+dx)+1}+1}}d\left(-\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)+1}}\right)}{d}\right)-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{\sec(c+dx)+1}}\right)+$$
$$\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2d\sqrt{\sec(c+dx)+1}}$$

$\Big\downarrow$ 222

3.265. $\int \frac{\sec^{\frac{7}{2}}(c+dx)}{\sqrt{1+\sec(c+dx)}}\,dx$

$$\frac{1}{4}\left(\frac{1}{2}\left(\frac{14\text{arcsinh}\left(\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)+1}}\right)}{d}-8\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)+1}}dx\right)-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{\sec(c+dx)+1}}\right)+\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2d\sqrt{\sec(c+dx)+1}}$$

$\Big\downarrow$ 4294

$$\frac{1}{4}\left(\frac{1}{2}\left(\frac{8\sqrt{2}\int\frac{1}{\sqrt{\frac{\tan^2(c+dx)}{(\sec(c+dx)+1)^2}+1}}d\left(-\frac{\tan(c+dx)}{\sec(c+dx)+1}\right)}{d}+\frac{14\text{arcsinh}\left(\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)+1}}\right)}{d}\right)-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{\sec(c+dx)+1}}\right)+\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2d\sqrt{\sec(c+dx)+1}}$$

$\Big\downarrow$ 222

$$\frac{1}{4}\left(\frac{1}{2}\left(\frac{14\text{arcsinh}\left(\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)+1}}\right)}{d}-\frac{8\sqrt{2}\text{arcsinh}\left(\frac{\tan(c+dx)}{\sec(c+dx)+1}\right)}{d}\right)-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{\sec(c+dx)+1}}\right)+\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2d\sqrt{\sec(c+dx)+1}}$$

input
```
Int[Sec[c + d*x]^(7/2)/Sqrt[1 + Sec[c + d*x]],x]
```

output
```
(Sec[c + d*x]^(5/2)*Sin[c + d*x])/(2*d*Sqrt[1 + Sec[c + d*x]]) + (((-8*Sqr
t[2]*ArcSinh[Tan[c + d*x]/(1 + Sec[c + d*x])])/d + (14*ArcSinh[Tan[c + d*x
]/Sqrt[1 + Sec[c + d*x]]])/d)/2 - (Sec[c + d*x]^(3/2)*Sin[c + d*x])/(d*Sqr
t[1 + Sec[c + d*x]]))/4
```

3.265.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

3.265. $\int \frac{\sec^{\frac{7}{2}}(c+dx)}{\sqrt{1+\sec(c+dx)}}\,dx$

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4288
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[-2*(a/(b*f))*Sqrt[a*(d/b)]   Subst[Int[1/Sqrt[1
+ x^2/a], x], x, b*(Cot[e + f*x]/Sqrt[a + b*Csc[e + f*x]])], x] /; FreeQ[{a
, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[a*(d/b), 0]
```

rule 4294
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[(-Sqrt[2])*(Sqrt[a]/(b*f))   Subst[Int[1/Sqrt[1
+ x^2], x], x, b*(Cot[e + f*x]/(a + b*Csc[e + f*x]))], x] /; FreeQ[{a, b, d
, e, f}, x] && EqQ[a^2 - b^2, 0] && EqQ[d - a/b, 0] && GtQ[a, 0]
```

rule 4309
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[-2*d^2*Cot[e + f*x]*((d*Csc[e + f*x])^(n - 2)/(
f*(2*n - 3)*Sqrt[a + b*Csc[e + f*x]])), x] + Simp[d^2/(b*(2*n - 3))   Int[(
d*Csc[e + f*x])^(n - 2)*((2*b*(n - 2) - a*Csc[e + f*x])/Sqrt[a + b*Csc[e +
f*x]]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[n,
2] && IntegerQ[2*n]
```

rule 4509
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*d*
Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(f*(m + n))),
 x] + Simp[d/(b*(m + n))   Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n -
 1)*Simp[b*B*(n - 1) + (A*b*(m + n) + a*B*m)*Csc[e + f*x], x], x], x] /; Fr
eeQ[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
&& GtQ[n, 1]
```

rule 4511
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b -
a*B)/b   Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n, x], x] + Simp[B/b
 Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b
, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
```

3.265. $\int \frac{\sec^{\frac{7}{2}}(c+dx)}{\sqrt{1+\sec(c+dx)}}\,dx$

3.265.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 261 vs. $2(108) = 216$.

Time = 1.63 (sec) , antiderivative size = 262, normalized size of antiderivative = 2.08

method	result
default	$\frac{\sqrt{1+\sec(dx+c)}\,\sec(dx+c)^{\frac{7}{2}}\left(8\arctan\left(\frac{\sin(dx+c)\sqrt{2}}{2(\cos(dx+c)+1)\sqrt{-\frac{1}{\cos(dx+c)+1}}}\right)\sqrt{2}\,\cos(dx+c)^4+7\arctan\left(\frac{\cos(dx+c)-\sin(dx+c)+1}{2(\cos(dx+c)+1)\sqrt{-\frac{1}{\cos(dx+c)+1}}}\right.\right.}{8d(c}$

input
```
int(sec(d*x+c)^(7/2)/(1+sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/8/d*(1+sec(d*x+c))^(1/2)*sec(d*x+c)^(7/2)/(cos(d*x+c)+1)/(-1/(cos(d*x+c)
+1))^(1/2)*(8*arctan(1/2*sin(d*x+c)*2^(1/2)/(cos(d*x+c)+1)/(-1/(cos(d*x+c)
+1))^(1/2))*2^(1/2)*cos(d*x+c)^4+7*arctan(1/2*(cos(d*x+c)-sin(d*x+c)+1)/(c
os(d*x+c)+1)/(-1/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)^4-7*arctan(1/2*(cos(d*x
+c)+sin(d*x+c)+1)/(cos(d*x+c)+1)/(-1/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)^4-2
*sin(d*x+c)*(-1/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^3+4*(-1/(cos(d*x+c)+1))^(
1/2)*sin(d*x+c)*cos(d*x+c)^2)
```

3.265.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 337 vs. $2(108) = 216$.

Time = 0.30 (sec) , antiderivative size = 337, normalized size of antiderivative = 2.67

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{\sqrt{1+\sec(c+dx)}}\,dx$$
$$= \frac{8\left(\sqrt{2}\cos\left(dx+c\right)^2+\sqrt{2}\cos\left(dx+c\right)\right)\log\left(-\frac{2\sqrt{2}\sqrt{\frac{\cos(dx+c)+1}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)+\cos(dx+c)^2-2\cos(dx+c)-3}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)-7}{}$$

input
```
integrate(sec(d*x+c)^(7/2)/(1+sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
1/16*(8*(sqrt(2)*cos(d*x + c)^2 + sqrt(2)*cos(d*x + c))*log(-(2*sqrt(2)*sq
rt((cos(d*x + c) + 1)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) + cos(
d*x + c)^2 - 2*cos(d*x + c) - 3)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) -
7*(cos(d*x + c)^2 + cos(d*x + c))*log(-(cos(d*x + c)^2 + 2*sqrt((cos(d*x +
 c) + 1)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - cos(d*x + c) - 2)
/(cos(d*x + c) + 1)) + 7*(cos(d*x + c)^2 + cos(d*x + c))*log(-(cos(d*x + c
)^2 - 2*sqrt((cos(d*x + c) + 1)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x +
 c) - cos(d*x + c) - 2)/(cos(d*x + c) + 1)) - 4*sqrt((cos(d*x + c) + 1)/co
s(d*x + c))*(cos(d*x + c) - 2)*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x
 + c)^2 + d*cos(d*x + c))
```

3.265.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{\sqrt{1+\sec(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**(7/2)/(1+sec(d*x+c))**(1/2),x)
```

output
```
Timed out
```

3.265.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1643 vs. $2(108) = 216$.

Time = 0.44 (sec) , antiderivative size = 1643, normalized size of antiderivative = 13.04

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{\sqrt{1+\sec(c+dx)}}\,dx = \text{Too large to display}$$

input
```
integrate(sec(d*x+c)^(7/2)/(1+sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
1/16*(4*(sqrt(2)*sin(4*d*x + 4*c) + 2*sqrt(2)*sin(2*d*x + 2*c))*cos(7/2*ar
ctan2(sin(d*x + c), cos(d*x + c))) - 20*(sqrt(2)*sin(4*d*x + 4*c) + 2*sqrt
(2)*sin(2*d*x + 2*c))*cos(5/2*arctan2(sin(d*x + c), cos(d*x + c))) + 20*(s
qrt(2)*sin(4*d*x + 4*c) + 2*sqrt(2)*sin(2*d*x + 2*c))*cos(3/2*arctan2(sin(
d*x + c), cos(d*x + c))) - 4*(sqrt(2)*sin(4*d*x + 4*c) + 2*sqrt(2)*sin(2*d
*x + 2*c))*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 7*(2*(2*cos(2*d*
x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2
 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x
+ 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*x + c), cos
(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sqrt(
2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2*sqrt(2)*sin(1/2*arctan
2(sin(d*x + c), cos(d*x + c))) + 2) - 7*(2*(2*cos(2*d*x + 2*c) + 1)*cos(4*
d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^
2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d
*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*si
n(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(s
in(d*x + c), cos(d*x + c))) - 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(
d*x + c))) + 2) + 7*(2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d
*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*
c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*lo...
```

3.265.8 Giac [F]

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{\sqrt{1+\sec(c+dx)}}\,dx = \int \frac{\sec{(dx+c)}^{\frac{7}{2}}}{\sqrt{\sec{(dx+c)}+1}}\,dx$$

input
```
integrate(sec(d*x+c)^(7/2)/(1+sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^(7/2)/sqrt(sec(d*x + c) + 1), x)
```

3.265.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{\sqrt{1+\sec(c+dx)}}\,dx = \int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{7/2}}{\sqrt{\frac{1}{\cos(c+d\,x)}+1}}\,dx$$

input
```
int((1/cos(c + d*x))^(7/2)/(1/cos(c + d*x) + 1)^(1/2),x)
```

output
```
int((1/cos(c + d*x))^(7/2)/(1/cos(c + d*x) + 1)^(1/2), x)
```

3.266 $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{\sqrt{1+\sec(c+dx)}}\,dx$

3.266.1 Optimal result 2112
3.266.2 Mathematica [A] (verified) 2112
3.266.3 Rubi [A] (verified) 2113
3.266.4 Maple [B] (verified) 2116
3.266.5 Fricas [B] (verification not implemented) 2116
3.266.6 Sympy [F(-1)] 2117
3.266.7 Maxima [B] (verification not implemented) 2117
3.266.8 Giac [F] 2118
3.266.9 Mupad [F(-1)] 2119

3.266.1 Optimal result

Integrand size = 23, antiderivative size = 85

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{\sqrt{1+\sec(c+dx)}}\,dx = \frac{\sqrt{2}\operatorname{arcsinh}\left(\frac{\tan(c+dx)}{1+\sec(c+dx)}\right)}{d} - \frac{\operatorname{arcsinh}\left(\frac{\tan(c+dx)}{\sqrt{1+\sec(c+dx)}}\right)}{d} + \frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d\sqrt{1+\sec(c+dx)}}$$

output
```
-arcsinh(tan(d*x+c)/(1+sec(d*x+c))^(1/2))/d+arcsinh(tan(d*x+c)/(1+sec(d*x+
c)))*2^(1/2)/d+sec(d*x+c)^(3/2)*sin(d*x+c)/d/(1+sec(d*x+c))^(1/2)
```

3.266.2 Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.31

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{\sqrt{1+\sec(c+dx)}}\,dx = \frac{\left(\arcsin\left(\sqrt{1-\sec(c+dx)}\right)+2\arcsin\left(\sqrt{\sec(c+dx)}\right)-\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)+\sqrt{-((-1+\sec(c}\right.}{d\sqrt{-\tan^2(c+dx)}}$$

input
```
Integrate[Sec[c + d*x]^(5/2)/Sqrt[1 + Sec[c + d*x]],x]
```

output
```
((ArcSin[Sqrt[1 - Sec[c + d*x]]] + 2*ArcSin[Sqrt[Sec[c + d*x]]] - Sqrt[2]*
ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]] + Sqrt[-((-1 +
 Sec[c + d*x])*Sec[c + d*x])])*Tan[c + d*x])/(d*Sqrt[-Tan[c + d*x]^2])
```

3.266.3 Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.07, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {3042, 4309, 3042, 4511, 3042, 4288, 222, 4294, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{\sqrt{\sec(c+dx)+1}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)+1}}dx$$

$$\Big\downarrow \text{4309}$$

$$\frac{1}{2}\int \frac{(1-\sec(c+dx))\sqrt{\sec(c+dx)}}{\sqrt{\sec(c+dx)+1}}dx+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{\sec(c+dx)+1}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\int \frac{\left(1-\csc\left(c+dx+\frac{\pi}{2}\right)\right)\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)+1}}dx+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{\sec(c+dx)+1}}$$

$$\Big\downarrow \text{4511}$$

$$\frac{1}{2}\left(2\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{\sec(c+dx)+1}}dx-\int \sqrt{\sec(c+dx)}\sqrt{\sec(c+dx)+1}dx\right)+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{\sec(c+dx)+1}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\left(2\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)+1}}dx-\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)+1}dx\right)+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{\sec(c+dx)+1}}$$

$$\Big\downarrow \text{4288}$$

$$\frac{1}{2}\left(2\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)+1}}dx+\frac{2\int \frac{1}{\sqrt{\frac{\tan^2(c+dx)}{\sec(c+dx)+1}+1}}d\left(-\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)+1}}\right)}{d}\right)+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{\sec(c+dx)+1}}$$

$$\Big\downarrow \text{222}$$

$$\frac{1}{2}\left(2\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)+1}}dx-\frac{2\text{arcsinh}\left(\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)+1}}\right)}{d}\right)+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{\sec(c+dx)+1}}$$

$$\Big\downarrow \text{4294}$$

$$\frac{1}{2}\left(-\frac{2\sqrt{2}\int \frac{1}{\sqrt{\frac{\tan^2(c+dx)}{(\sec(c+dx)+1)^2}+1}}d\left(-\frac{\tan(c+dx)}{\sec(c+dx)+1}\right)}{d}-\frac{2\text{arcsinh}\left(\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)+1}}\right)}{d}\right)+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{\sec(c+dx)+1}}$$

$$\Big\downarrow \text{222}$$

$$\frac{1}{2}\left(\frac{2\sqrt{2}\text{arcsinh}\left(\frac{\tan(c+dx)}{\sec(c+dx)+1}\right)}{d}-\frac{2\text{arcsinh}\left(\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)+1}}\right)}{d}\right)+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{\sec(c+dx)+1}}$$

input
```
Int[Sec[c + d*x]^(5/2)/Sqrt[1 + Sec[c + d*x]],x]
```

output
```
((2*Sqrt[2]*ArcSinh[Tan[c + d*x]/(1 + Sec[c + d*x])])/d - (2*ArcSinh[Tan[c
 + d*x]/Sqrt[1 + Sec[c + d*x]]])/d)/2 + (Sec[c + d*x]^(3/2)*Sin[c + d*x])/
(d*Sqrt[1 + Sec[c + d*x]])
```

3.266. $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{\sqrt{1+\sec(c+dx)}}\,dx$

3.266.3.1 Defintions of rubi rules used

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4288
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[-2*(a/(b*f))*Sqrt[a*(d/b)]   Subst[Int[1/Sqrt[1
+ x^2/a], x], x, b*(Cot[e + f*x]/Sqrt[a + b*Csc[e + f*x]])], x] /; FreeQ[{a
, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[a*(d/b), 0]
```

rule 4294
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[(-Sqrt[2])*(Sqrt[a]/(b*f))   Subst[Int[1/Sqrt[1
+ x^2], x], x, b*(Cot[e + f*x]/(a + b*Csc[e + f*x]))], x] /; FreeQ[{a, b, d
, e, f}, x] && EqQ[a^2 - b^2, 0] && EqQ[d - a/b, 0] && GtQ[a, 0]
```

rule 4309
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[-2*d^2*Cot[e + f*x]*((d*Csc[e + f*x])^(n - 2)/(
f*(2*n - 3)*Sqrt[a + b*Csc[e + f*x]])), x] + Simp[d^2/(b*(2*n - 3))   Int[(
d*Csc[e + f*x])^(n - 2)*((2*b*(n - 2) - a*Csc[e + f*x])/Sqrt[a + b*Csc[e +
f*x]]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[n,
2] && IntegerQ[2*n]
```

rule 4511
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b -
a*B)/b   Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n, x], x] + Simp[B/b
 Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b
, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
```

3.266. $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{\sqrt{1+\sec(c+dx)}}\,dx$

3.266.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 230 vs. $2(77) = 154$.

Time = 1.51 (sec) , antiderivative size = 231, normalized size of antiderivative = 2.72

method	result
default	$\frac{\sec(dx+c)^{\frac{5}{2}}\sqrt{1+\sec(dx+c)}\left(-2\arctan\left(\frac{\sin(dx+c)\sqrt{2}}{2(\cos(dx+c)+1)\sqrt{-\frac{1}{\cos(dx+c)+1}}}\right)\cos(dx+c)^3\sqrt{2}+2\sqrt{-\frac{1}{\cos(dx+c)+1}}\sin(dx+c)\cos(dx+c)^2\right.}{2d(\cos(dx+c)+1)\sqrt{-\frac{}{\cos(d}}}$

input
```
int(sec(d*x+c)^(5/2)/(1+sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/d*sec(d*x+c)^(5/2)*(1+sec(d*x+c))^(1/2)/(cos(d*x+c)+1)/(-1/(cos(d*x+c)
+1))^(1/2)*(-2*arctan(1/2*sin(d*x+c)*2^(1/2)/(cos(d*x+c)+1)/(-1/(cos(d*x+c
)+1))^(1/2))*cos(d*x+c)^3*2^(1/2)+2*(-1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*c
os(d*x+c)^2-cos(d*x+c)^3*arctan(1/2*(cos(d*x+c)-sin(d*x+c)+1)/(cos(d*x+c)+
1)/(-1/(cos(d*x+c)+1))^(1/2))+cos(d*x+c)^3*arctan(1/2*(cos(d*x+c)+sin(d*x+
c)+1)/(cos(d*x+c)+1)/(-1/(cos(d*x+c)+1))^(1/2)))
```

3.266.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 297 vs. $2(77) = 154$.

Time = 0.29 (sec) , antiderivative size = 297, normalized size of antiderivative = 3.49

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{\sqrt{1+\sec(c+dx)}}\,dx$$
$$= \frac{2\left(\sqrt{2}\cos\left(dx+c\right)+\sqrt{2}\right)\log\left(\frac{2\sqrt{2}\sqrt{\frac{\cos(dx+c)+1}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)-\cos(dx+c)^2+2\cos(dx+c)+3}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)+\left(\cos\left(dx+c\right)+\right.}{}$$

input
```
integrate(sec(d*x+c)^(5/2)/(1+sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
1/4*(2*(sqrt(2)*cos(d*x + c) + sqrt(2))*log((2*sqrt(2)*sqrt((cos(d*x + c)
+ 1)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - cos(d*x + c)^2 + 2*co
s(d*x + c) + 3)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + (cos(d*x + c) + 1
)*log(-(cos(d*x + c)^2 + 2*sqrt((cos(d*x + c) + 1)/cos(d*x + c))*sqrt(cos(
d*x + c))*sin(d*x + c) - cos(d*x + c) - 2)/(cos(d*x + c) + 1)) - (cos(d*x
+ c) + 1)*log(-(cos(d*x + c)^2 - 2*sqrt((cos(d*x + c) + 1)/cos(d*x + c))*s
qrt(cos(d*x + c))*sin(d*x + c) - cos(d*x + c) - 2)/(cos(d*x + c) + 1)) + 4
*sqrt((cos(d*x + c) + 1)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(d
*cos(d*x + c) + d)
```

3.266.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{\sqrt{1+\sec(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**(5/2)/(1+sec(d*x+c))**(1/2),x)
```

output
```
Timed out
```

3.266.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 873 vs. $2(77) = 154$.

Time = 0.46 (sec) , antiderivative size = 873, normalized size of antiderivative = 10.27

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{\sqrt{1+\sec(c+dx)}}\,dx = \text{Too large to display}$$

input
```
integrate(sec(d*x+c)^(5/2)/(1+sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
-1/4*(4*sqrt(2)*cos(3/2*arctan2(sin(d*x + c), cos(d*x + c)))*sin(2*d*x + 2
*c) - 4*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))*sin(2*d*x + 2
*c) + (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*l
og(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(si
n(d*x + c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos
(d*x + c))) + 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2)
- (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2
*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*
x + c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x
 + c))) - 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) + (c
os(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2*cos
(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x +
c), cos(d*x + c)))^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c
))) + 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) - (cos(2
*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2
*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c),
cos(d*x + c)))^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))
- 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) - 2*(sqrt(2)
*cos(2*d*x + 2*c)^2 + sqrt(2)*sin(2*d*x + 2*c)^2 + 2*sqrt(2)*cos(2*d*x + 2
*c) + sqrt(2))*log(cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + sin...
```

3.266.8 Giac [F]

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{\sqrt{1+\sec(c+dx)}}\,dx = \int \frac{\sec{(dx+c)}^{\frac{5}{2}}}{\sqrt{\sec{(dx+c)}+1}}\,dx$$

input
```
integrate(sec(d*x+c)^(5/2)/(1+sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^(5/2)/sqrt(sec(d*x + c) + 1), x)
```

3.266.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{\sqrt{1+\sec(c+dx)}}\,dx = \int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}}{\sqrt{\frac{1}{\cos(c+d\,x)}+1}}\,dx$$

input
```
int((1/cos(c + d*x))^(5/2)/(1/cos(c + d*x) + 1)^(1/2),x)
```

output
```
int((1/cos(c + d*x))^(5/2)/(1/cos(c + d*x) + 1)^(1/2), x)
```

3.267 $\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{1+\sec(c+dx)}}\,dx$

3.267.1 Optimal result 2120
3.267.2 Mathematica [A] (verified) 2120
3.267.3 Rubi [A] (verified) 2121
3.267.4 Maple [B] (verified) 2123
3.267.5 Fricas [B] (verification not implemented) 2123
3.267.6 Sympy [F] 2124
3.267.7 Maxima [B] (verification not implemented) 2124
3.267.8 Giac [F] 2125
3.267.9 Mupad [F(-1)] 2125

3.267.1 Optimal result

Integrand size = 23, antiderivative size = 54

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{1+\sec(c+dx)}}\,dx=-\frac{\sqrt{2}\operatorname{arcsinh}\left(\frac{\tan(c+dx)}{1+\sec(c+dx)}\right)}{d}+\frac{2\operatorname{arcsinh}\left(\frac{\tan(c+dx)}{\sqrt{1+\sec(c+dx)}}\right)}{d}$$

output
```
2*arcsinh(tan(d*x+c)/(1+sec(d*x+c))^(1/2))/d-arcsinh(tan(d*x+c)/(1+sec(d*x
+c)))*2^(1/2)/d
```

3.267.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.41

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{1+\sec(c+dx)}}\,dx=\frac{\left(2\arcsin\left(\sqrt{\sec(c+dx)}\right)-\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)\right)\cot(c+dx)\sqrt{-\tan^2(c+dx)}}{d}$$

input
```
Integrate[Sec[c + d*x]^(3/2)/Sqrt[1 + Sec[c + d*x]],x]
```

output
```
((2*ArcSin[Sqrt[Sec[c + d*x]]] - Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]
])/Sqrt[1 - Sec[c + d*x]]])*Cot[c + d*x]*Sqrt[-Tan[c + d*x]^2])/d
```

3.267.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3042, 4308, 3042, 4288, 222, 4294, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{\sec(c+dx)+1}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)+1}}dx$$

$$\Big\downarrow \text{4308}$$

$$\int \sqrt{\sec(c+dx)}\sqrt{\sec(c+dx)+1}dx-\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{\sec(c+dx)+1}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)+1}dx-\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)+1}}dx$$

$$\Big\downarrow \text{4288}$$

$$-\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)+1}}dx-\frac{2\int \frac{1}{\sqrt{\frac{\tan^2(c+dx)}{\sec(c+dx)+1}+1}}d\left(-\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)+1}}\right)}{d}$$

$$\Big\downarrow \text{222}$$

$$\frac{2\text{arcsinh}\left(\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)+1}}\right)}{d}-\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)+1}}dx$$

$$\Big\downarrow \text{4294}$$

$$\frac{\sqrt{2}\int \frac{1}{\sqrt{\frac{\tan^2(c+dx)}{(\sec(c+dx)+1)^2}+1}}d\left(-\frac{\tan(c+dx)}{\sec(c+dx)+1}\right)}{d}+\frac{2\text{arcsinh}\left(\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)+1}}\right)}{d}$$

$$\Big\downarrow \text{222}$$

$$\frac{2\text{arcsinh}\left(\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)+1}}\right)}{d}-\frac{\sqrt{2}\text{arcsinh}\left(\frac{\tan(c+dx)}{\sec(c+dx)+1}\right)}{d}$$

input
```
Int[Sec[c + d*x]^(3/2)/Sqrt[1 + Sec[c + d*x]],x]
```

output
```
-((Sqrt[2]*ArcSinh[Tan[c + d*x]/(1 + Sec[c + d*x])])/d) + (2*ArcSinh[Tan[c
 + d*x]/Sqrt[1 + Sec[c + d*x]]])/d
```

3.267.3.1 Defintions of rubi rules used

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4288
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[-2*(a/(b*f))*Sqrt[a*(d/b)]   Subst[Int[1/Sqrt[1
+ x^2/a], x], x, b*(Cot[e + f*x]/Sqrt[a + b*Csc[e + f*x]])], x] /; FreeQ[{a
, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[a*(d/b), 0]
```

rule 4294
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[(-Sqrt[2])*(Sqrt[a]/(b*f))   Subst[Int[1/Sqrt[1
+ x^2], x], x, b*(Cot[e + f*x]/(a + b*Csc[e + f*x]))], x] /; FreeQ[{a, b, d
, e, f}, x] && EqQ[a^2 - b^2, 0] && EqQ[d - a/b, 0] && GtQ[a, 0]
```

rule 4308
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Simp[d/b   Int[Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[
e + f*x]], x], x] - Simp[a*(d/b)   Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[
e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

3.267.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 181 vs. $2(50) = 100$.

Time = 1.72 (sec) , antiderivative size = 182, normalized size of antiderivative = 3.37

method	result
default	$\frac{\left(\arctan\left(\frac{\sin(dx+c)\sqrt{2}}{2(\cos(dx+c)+1)\sqrt{-\frac{1}{\cos(dx+c)+1}}}\right)\sqrt{2}+\arctan\left(\frac{\cos(dx+c)-\sin(dx+c)+1}{2(\cos(dx+c)+1)\sqrt{-\frac{1}{\cos(dx+c)+1}}}\right)-\arctan\left(\frac{\cos(dx+c)+\sin(dx+c)+1}{2(\cos(dx+c)+1)\sqrt{-\frac{1}{\cos(dx+c)+1}}}\right)\right.}{d(\cos(dx+c)+1)\sqrt{-\frac{1}{\cos(dx+c)+1}}}$

input
```
int(sec(d*x+c)^(3/2)/(1+sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(arctan(1/2*sin(d*x+c)*2^(1/2)/(cos(d*x+c)+1)/(-1/(cos(d*x+c)+1))^(1/2
))*2^(1/2)+arctan(1/2*(cos(d*x+c)-sin(d*x+c)+1)/(cos(d*x+c)+1)/(-1/(cos(d*
x+c)+1))^(1/2))-arctan(1/2*(cos(d*x+c)+sin(d*x+c)+1)/(cos(d*x+c)+1)/(-1/(c
os(d*x+c)+1))^(1/2)))*(1+sec(d*x+c))^(1/2)*sec(d*x+c)^(3/2)*cos(d*x+c)^2/(
cos(d*x+c)+1)/(-1/(cos(d*x+c)+1))^(1/2)
```

3.267.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 223 vs. $2(50) = 100$.

Time = 0.29 (sec) , antiderivative size = 223, normalized size of antiderivative = 4.13

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{1+\sec(c+dx)}}\,dx = \frac{\sqrt{2}\log\left(-\frac{2\sqrt{2}\sqrt{\frac{\cos(dx+c)+1}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)+\cos(dx+c)^2-2\cos(dx+c)-3}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)-\log\left(-\frac{\cos(dx+c)^2+2\sqrt{\frac{\cos(dx+c)+1}{\cos(dx+c)}}\sqrt{\cos(dx+}}{\cos(dx+c)+}\right.}{2\,d}$$

input
```
integrate(sec(d*x+c)^(3/2)/(1+sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
1/2*(sqrt(2)*log(-(2*sqrt(2)*sqrt((cos(d*x + c) + 1)/cos(d*x + c))*sqrt(co
s(d*x + c))*sin(d*x + c) + cos(d*x + c)^2 - 2*cos(d*x + c) - 3)/(cos(d*x +
 c)^2 + 2*cos(d*x + c) + 1)) - log(-(cos(d*x + c)^2 + 2*sqrt((cos(d*x + c)
 + 1)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - cos(d*x + c) - 2)/(c
os(d*x + c) + 1)) + log(-(cos(d*x + c)^2 - 2*sqrt((cos(d*x + c) + 1)/cos(d
*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - cos(d*x + c) - 2)/(cos(d*x + c)
 + 1)))/d
```

3.267.6 Sympy [F]

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{1+\sec(c+dx)}}\,dx = \int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{\sec(c+dx)+1}}\,dx$$

input
```
integrate(sec(d*x+c)**(3/2)/(1+sec(d*x+c))**(1/2),x)
```

output
```
Integral(sec(c + d*x)**(3/2)/sqrt(sec(c + d*x) + 1), x)
```

3.267.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 473 vs. $2(50) = 100$.

Time = 0.43 (sec) , antiderivative size = 473, normalized size of antiderivative = 8.76

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{1+\sec(c+dx)}}\,dx = -\frac{\sqrt{2}\log\left(\cos\left(\frac{1}{2}\arctan\left(\sin\left(dx+c\right),\cos\left(dx+c\right)\right)\right)^2+\sin\left(\frac{1}{2}\arctan\left(\sin\left(dx+c\right),\cos\left(dx+c\right)\right)\right)^2+2\,\mathrm{si}\right.}{}$$

input
```
integrate(sec(d*x+c)^(3/2)/(1+sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
-1/2*(sqrt(2)*log(cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + sin(1/2
*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c),
cos(d*x + c))) + 1) - sqrt(2)*log(cos(1/2*arctan2(sin(d*x + c), cos(d*x +
c)))^2 + sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 - 2*sin(1/2*arctan
2(sin(d*x + c), cos(d*x + c))) + 1) - log(2*cos(1/2*arctan2(sin(d*x + c),
cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sq
rt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2*sqrt(2)*sin(1/2*arc
tan2(sin(d*x + c), cos(d*x + c))) + 2) + log(2*cos(1/2*arctan2(sin(d*x + c
), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2
*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) - 2*sqrt(2)*sin(1/2*
arctan2(sin(d*x + c), cos(d*x + c))) + 2) - log(2*cos(1/2*arctan2(sin(d*x
+ c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2
- 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2*sqrt(2)*sin(1
/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) + log(2*cos(1/2*arctan2(sin(d
*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))
^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) - 2*sqrt(2)*si
n(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2))/d
```

3.267.8 Giac [F]

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{1+\sec(c+dx)}}\,dx = \int \frac{\sec{(dx+c)}^{\frac{3}{2}}}{\sqrt{\sec{(dx+c)}+1}}\,dx$$

input
```
integrate(sec(d*x+c)^(3/2)/(1+sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^(3/2)/sqrt(sec(d*x + c) + 1), x)
```

3.267.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{1+\sec(c+dx)}}\,dx = \int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}{\sqrt{\frac{1}{\cos(c+d\,x)}+1}}\,dx$$

input
```
int((1/cos(c + d*x))^(3/2)/(1/cos(c + d*x) + 1)^(1/2),x)
```

output
```
int((1/cos(c + d*x))^(3/2)/(1/cos(c + d*x) + 1)^(1/2), x)
```

3.268 $\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{1+\sec(c+dx)}}\,dx$

3.268.1 Optimal result . . . 2126
3.268.2 Mathematica [A] (verified) . . . 2126
3.268.3 Rubi [A] (verified) . . . 2127
3.268.4 Maple [B] (verified) . . . 2128
3.268.5 Fricas [B] (verification not implemented) . . . 2128
3.268.6 Sympy [F] . . . 2129
3.268.7 Maxima [B] (verification not implemented) . . . 2129
3.268.8 Giac [F] . . . 2129
3.268.9 Mupad [F(-1)] . . . 2130

3.268.1 Optimal result

Integrand size = 23, antiderivative size = 27

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{1+\sec(c+dx)}}\,dx=\frac{\sqrt{2}\operatorname{arcsinh}\left(\frac{\tan(c+dx)}{1+\sec(c+dx)}\right)}{d}$$

output
```
arcsinh(tan(d*x+c)/(1+sec(d*x+c)))*2^(1/2)/d
```

3.268.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.48

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{1+\sec(c+dx)}}\,dx=\frac{2\operatorname{arctanh}\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\cos\left(\frac{1}{2}(c+dx)\right)\sqrt{\frac{1}{1+\cos(c+dx)}}}{d}$$

input
```
Integrate[Sqrt[Sec[c + d*x]]/Sqrt[1 + Sec[c + d*x]],x]
```

output
```
(2*ArcTanh[Sin[(c + d*x)/2]]*Cos[(c + d*x)/2]*Sqrt[(1 + Cos[c + d*x])^(-1)
])/d
```

3.268.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3042, 4294, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{\sec(c+dx)+1}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)+1}}dx$$

$$\Big\downarrow \text{4294}$$

$$-\frac{\sqrt{2}\int \frac{1}{\sqrt{\frac{\tan^2(c+dx)}{(\sec(c+dx)+1)^2}+1}}d\left(-\frac{\tan(c+dx)}{\sec(c+dx)+1}\right)}{d}$$

$$\Big\downarrow \text{222}$$

$$\frac{\sqrt{2}\text{arcsinh}\left(\frac{\tan(c+dx)}{\sec(c+dx)+1}\right)}{d}$$

input
```
Int[Sqrt[Sec[c + d*x]]/Sqrt[1 + Sec[c + d*x]],x]
```

output
```
(Sqrt[2]*ArcSinh[Tan[c + d*x]/(1 + Sec[c + d*x])])/d
```

3.268.3.1 Defintions of rubi rules used

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4294
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[(-Sqrt[2])*(Sqrt[a]/(b*f))   Subst[Int[1/Sqrt[1
+ x^2], x], x, b*(Cot[e + f*x]/(a + b*Csc[e + f*x]))], x] /; FreeQ[{a, b, d
, e, f}, x] && EqQ[a^2 - b^2, 0] && EqQ[d - a/b, 0] && GtQ[a, 0]
```

3.268.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 92 vs. $2(25) = 50$.

Time = 1.29 (sec) , antiderivative size = 93, normalized size of antiderivative = 3.44

method	result	size
default	$-\frac{\sqrt{2}\,\sqrt{\sec(dx+c)}\,\sqrt{1+\sec(dx+c)}\,\arctan\left(\frac{\sin(dx+c)\sqrt{2}}{2(\cos(dx+c)+1)\sqrt{-\frac{1}{\cos(dx+c)+1}}}\right)\cos(dx+c)}{d(\cos(dx+c)+1)\sqrt{-\frac{1}{\cos(dx+c)+1}}}$	93

input
```
int(sec(d*x+c)^(1/2)/(1+sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/d*2^(1/2)*sec(d*x+c)^(1/2)*(1+sec(d*x+c))^(1/2)*arctan(1/2*sin(d*x+c)*2
^(1/2)/(cos(d*x+c)+1)/(-1/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)/(cos(d*x+c)+1)
/(-1/(cos(d*x+c)+1))^(1/2)
```

3.268.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 88 vs. $2(25) = 50$.

Time = 0.28 (sec) , antiderivative size = 88, normalized size of antiderivative = 3.26

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{1+\sec(c+dx)}}\,dx = \frac{\sqrt{2}\log\left(\frac{2\sqrt{2}\sqrt{\frac{\cos(dx+c)+1}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)-\cos(dx+c)^2+2\cos(dx+c)+3}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)}{2\,d}$$

input
```
integrate(sec(d*x+c)^(1/2)/(1+sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
1/2*sqrt(2)*log((2*sqrt(2)*sqrt((cos(d*x + c) + 1)/cos(d*x + c))*sqrt(cos(
d*x + c))*sin(d*x + c) - cos(d*x + c)^2 + 2*cos(d*x + c) + 3)/(cos(d*x + c
)^2 + 2*cos(d*x + c) + 1))/d
```

3.268.6 Sympy [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{1+\sec(c+dx)}}\,dx = \int \frac{\sqrt{\sec{(c+dx)}}}{\sqrt{\sec{(c+dx)}+1}}\,dx$$

input
```
integrate(sec(d*x+c)**(1/2)/(1+sec(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(sec(c + d*x))/sqrt(sec(c + d*x) + 1), x)
```

3.268.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 87 vs. $2(25) = 50$.

Time = 0.39 (sec) , antiderivative size = 87, normalized size of antiderivative = 3.22

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{1+\sec(c+dx)}}\,dx = \frac{\sqrt{2}\log\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+2\,\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)-\sqrt{2}\log\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+\sin\right.}{2\,d}$$

input
```
integrate(sec(d*x+c)^(1/2)/(1+sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
1/2*(sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1
/2*d*x + 1/2*c) + 1) - sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x +
1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))/d
```

3.268.8 Giac [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{1+\sec(c+dx)}}\,dx = \int \frac{\sqrt{\sec{(dx+c)}}}{\sqrt{\sec{(dx+c)}+1}}\,dx$$

input
```
integrate(sec(d*x+c)^(1/2)/(1+sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(sec(d*x + c))/sqrt(sec(d*x + c) + 1), x)
```

3.268.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{1+\sec(c+dx)}}\,dx=\int \frac{\sqrt{\frac{1}{\cos(c+d\,x)}}}{\sqrt{\frac{1}{\cos(c+d\,x)}+1}}\,dx$$

input
```
int((1/cos(c + d*x))^(1/2)/(1/cos(c + d*x) + 1)^(1/2),x)
```

output
```
int((1/cos(c + d*x))^(1/2)/(1/cos(c + d*x) + 1)^(1/2), x)
```

3.269 $\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{1+\sec(c+dx)}}\,dx$

3.269.1 Optimal result 2131
3.269.2 Mathematica [A] (verified) 2131
3.269.3 Rubi [A] (verified) 2132
3.269.4 Maple [A] (verified) 2133
3.269.5 Fricas [B] (verification not implemented) 2134
3.269.6 Sympy [F] 2134
3.269.7 Maxima [A] (verification not implemented) 2135
3.269.8 Giac [F] 2135
3.269.9 Mupad [F(-1)] 2135

3.269.1 Optimal result

Integrand size = 23, antiderivative size = 62

$$\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{1+\sec(c+dx)}}\,dx = -\frac{\sqrt{2}\operatorname{arcsinh}\left(\frac{\tan(c+dx)}{1+\sec(c+dx)}\right)}{d}+\frac{2\sqrt{\sec(c+dx)}\sin(c+dx)}{d\sqrt{1+\sec(c+dx)}}$$

output
```
-arcsinh(tan(d*x+c)/(1+sec(d*x+c)))*2^(1/2)/d+2*sin(d*x+c)*sec(d*x+c)^(1/2
)/d/(1+sec(d*x+c))^(1/2)
```

3.269.2 Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.45

$$\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{1+\sec(c+dx)}}\,dx = \frac{2\sqrt{-((-1+\sec(c+dx))\sec(c+dx))}\sin(c+dx)+\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)\tan(c+dx)}{d\sqrt{-\tan^2(c+dx)}}$$

input
```
Integrate[1/(Sqrt[Sec[c + d*x]]*Sqrt[1 + Sec[c + d*x]]),x]
```

output
```
(2*Sqrt[-((-1 + Sec[c + d*x])*Sec[c + d*x])]*Sin[c + d*x] + Sqrt[2]*ArcTan
[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*Tan[c + d*x])/(d*Sqr
t[-Tan[c + d*x]^2])
```

3.269.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 4299, 3042, 4294, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{\sec(c+dx)+1}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)+1}}dx$$

$$\Big\downarrow \text{4299}$$

$$\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{\sec(c+dx)+1}}-\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{\sec(c+dx)+1}}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{\sec(c+dx)+1}}-\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)+1}}dx$$

$$\Big\downarrow \text{4294}$$

$$\frac{\sqrt{2}\int \frac{1}{\sqrt{\frac{\tan^2(c+dx)}{(\sec(c+dx)+1)^2}+1}}d\left(-\frac{\tan(c+dx)}{\sec(c+dx)+1}\right)}{d}+\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{\sec(c+dx)+1}}$$

$$\Big\downarrow \text{222}$$

$$\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{\sec(c+dx)+1}}-\frac{\sqrt{2}\text{arcsinh}\left(\frac{\tan(c+dx)}{\sec(c+dx)+1}\right)}{d}$$

input
```
Int[1/(Sqrt[Sec[c + d*x]]*Sqrt[1 + Sec[c + d*x]]),x]
```

output
```
-((Sqrt[2]*ArcSinh[Tan[c + d*x]/(1 + Sec[c + d*x])])/d) + (2*Sqrt[Sec[c +
d*x]]*Sin[c + d*x])/(d*Sqrt[1 + Sec[c + d*x]])
```

3.269.3.1 Defintions of rubi rules used

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4294
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[(-Sqrt[2])*(Sqrt[a]/(b*f))   Subst[Int[1/Sqrt[1
+ x^2], x], x, b*(Cot[e + f*x]/(a + b*Csc[e + f*x]))], x] /; FreeQ[{a, b, d
, e, f}, x] && EqQ[a^2 - b^2, 0] && EqQ[d - a/b, 0] && GtQ[a, 0]
```

rule 4299
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-Cot[e + f*x])*(a + b*Csc[e + f*x])^m*((d*Csc
[e + f*x])^n/(f*(m + 1))), x] + Simp[a*(m/(b*d*(m + 1)))   Int[(a + b*Csc[e
 + f*x])^m*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m, n},
 x] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LtQ[m, -2^(-1)]
```

3.269.4 Maple [A] (verified)

Time = 0.97 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.53

method	result	size
default	$-\frac{\sqrt{1+\sec(dx+c)}\left(\sqrt{2}\,\arctan\left(\frac{\sin(dx+c)\sqrt{2}}{2(\cos(dx+c)+1)\sqrt{-\frac{1}{\cos(dx+c)+1}}}\right)\sqrt{-\frac{1}{\cos(dx+c)+1}}+2\cot(dx+c)-2\csc(dx+c)\right)}{d\sqrt{\sec(dx+c)}}$	95

input
```
int(1/sec(d*x+c)^(1/2)/(1+sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/d*(1+sec(d*x+c))^(1/2)/sec(d*x+c)^(1/2)*(2^(1/2)*arctan(1/2*sin(d*x+c)*
2^(1/2)/(cos(d*x+c)+1)/(-1/(cos(d*x+c)+1))^(1/2))*(-1/(cos(d*x+c)+1))^(1/2
)+2*cot(d*x+c)-2*csc(d*x+c))
```

3.269.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 144 vs. $2(56) = 112$.

Time = 0.30 (sec) , antiderivative size = 144, normalized size of antiderivative = 2.32

$$\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{1+\sec(c+dx)}}\,dx = \frac{\left(\sqrt{2}\cos\left(dx+c\right)+\sqrt{2}\right)\log\left(-\frac{2\sqrt{2}\sqrt{\frac{\cos(dx+c)+1}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)+\cos(dx+c)^2-2\cos(dx+c)-3}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)+4\sqrt{\frac{\cos(dx+c)+1}{\cos(dx+c)}}\sqrt{}}{2\left(d\cos\left(dx+c\right)+d\right)}$$

input
```
integrate(1/sec(d*x+c)^(1/2)/(1+sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
1/2*((sqrt(2)*cos(d*x + c) + sqrt(2))*log(-(2*sqrt(2)*sqrt((cos(d*x + c) +
 1)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) + cos(d*x + c)^2 - 2*cos
(d*x + c) - 3)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + 4*sqrt((cos(d*x +
c) + 1)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c) + d
)
```

3.269.6 Sympy [F]

$$\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{1+\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{\sec\left(c+dx\right)+1}\sqrt{\sec\left(c+dx\right)}}\,dx$$

input
```
integrate(1/sec(d*x+c)**(1/2)/(1+sec(d*x+c))**(1/2),x)
```

output
```
Integral(1/(sqrt(sec(c + d*x) + 1)*sqrt(sec(c + d*x))), x)
```

3.269.7 Maxima [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.63

$$\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{1+\sec(c+dx)}}\,dx = -\frac{\sqrt{2}\log\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+2\,\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)-\sqrt{2}\log\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+\mathrm{s}\right.}{2\,d}$$

input
```
integrate(1/sec(d*x+c)^(1/2)/(1+sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
-1/2*(sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(
1/2*d*x + 1/2*c) + 1) - sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x +
 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) - 4*sqrt(2)*sin(1/2*d*x + 1/2*c))/
d
```

3.269.8 Giac [F]

$$\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{1+\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{\sec(dx+c)+1}\sqrt{\sec(dx+c)}}\,dx$$

input
```
integrate(1/sec(d*x+c)^(1/2)/(1+sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(sec(d*x + c) + 1)*sqrt(sec(d*x + c))), x)
```

3.269.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{1+\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{\frac{1}{\cos(c+d\,x)}+1}\,\sqrt{\frac{1}{\cos(c+d\,x)}}}\,dx$$

input
```
int(1/((1/cos(c + d*x) + 1)^(1/2)*(1/cos(c + d*x))^(1/2)),x)
```

output
```
int(1/((1/cos(c + d*x) + 1)^(1/2)*(1/cos(c + d*x))^(1/2)), x)
```

3.270 $\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)\sqrt{1+\sec(c+dx)}}\,dx$

3.270.1 Optimal result 2136
3.270.2 Mathematica [A] (verified) 2136
3.270.3 Rubi [A] (verified) 2137
3.270.4 Maple [A] (verified) 2139
3.270.5 Fricas [A] (verification not implemented) 2140
3.270.6 Sympy [F] 2140
3.270.7 Maxima [B] (verification not implemented) 2140
3.270.8 Giac [F] 2141
3.270.9 Mupad [F(-1)] 2141

3.270.1 Optimal result

Integrand size = 23, antiderivative size = 98

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)\sqrt{1+\sec(c+dx)}}\,dx = \frac{\sqrt{2}\operatorname{arcsinh}\left(\frac{\tan(c+dx)}{1+\sec(c+dx)}\right)}{d} + \frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{1+\sec(c+dx)}} - \frac{2\sqrt{\sec(c+dx)}\sin(c+dx)}{3d\sqrt{1+\sec(c+dx)}}$$

output
```
arcsinh(tan(d*x+c)/(1+sec(d*x+c)))*2^(1/2)/d+2/3*sin(d*x+c)/d/sec(d*x+c)^(
1/2)/(1+sec(d*x+c))^(1/2)-2/3*sin(d*x+c)*sec(d*x+c)^(1/2)/d/(1+sec(d*x+c))
^(1/2)
```

3.270.2 Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.20

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)\sqrt{1+\sec(c+dx)}}\,dx = \frac{\left(2(-1+\cos(c+dx))\sqrt{1-\sec(c+dx)} - 3\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)\sqrt{\sec(c+dx)}\right)\tan(c+dx)}{3d\sqrt{-((-1+\sec(c+dx))\sec(c+dx))}\sqrt{1+\sec(c+dx)}}$$

input
```
Integrate[1/(Sec[c + d*x]^(3/2)*Sqrt[1 + Sec[c + d*x]]),x]
```

output
```
((2*(-1 + Cos[c + d*x])*Sqrt[1 - Sec[c + d*x]] - 3*Sqrt[2]*ArcTan[(Sqrt[2]
*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*Sqrt[Sec[c + d*x]])*Tan[c + d
*x])/(3*d*Sqrt[-((-1 + Sec[c + d*x])*Sec[c + d*x])]*Sqrt[1 + Sec[c + d*x]]
)
```

3.270.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.04, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3042, 4310, 3042, 4501, 3042, 4294, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)\sqrt{\sec(c+dx)+1}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)+1}}dx$$

$$\Big\downarrow \text{4310}$$

$$\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{\sec(c+dx)+1}}-\frac{1}{3}\int \frac{1-2\sec(c+dx)}{\sqrt{\sec(c+dx)}\sqrt{\sec(c+dx)+1}}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{\sec(c+dx)+1}}-\frac{1}{3}\int \frac{1-2\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)+1}}dx$$

$$\Big\downarrow \text{4501}$$

$$\frac{1}{3}\left(3\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{\sec(c+dx)+1}}dx-\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{\sec(c+dx)+1}}\right)+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{\sec(c+dx)+1}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\left(3\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)+1}}dx-\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{\sec(c+dx)+1}}\right)+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{\sec(c+dx)+1}}$$

$\Big\downarrow$ 4294

$$\frac{1}{3}\left(-\frac{3\sqrt{2}\int \frac{1}{\sqrt{\frac{\tan^2(c+dx)}{(\sec(c+dx)+1)^2}+1}}d\left(-\frac{\tan(c+dx)}{\sec(c+dx)+1}\right)}{d}-\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{\sec(c+dx)+1}}\right)+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{\sec(c+dx)+1}}$$

$\Big\downarrow$ 222

$$\frac{1}{3}\left(\frac{3\sqrt{2}\text{arcsinh}\left(\frac{\tan(c+dx)}{\sec(c+dx)+1}\right)}{d}-\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{\sec(c+dx)+1}}\right)+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{\sec(c+dx)+1}}$$

input
```
Int[1/(Sec[c + d*x]^(3/2)*Sqrt[1 + Sec[c + d*x]]),x]
```

output
```
(2*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]]*Sqrt[1 + Sec[c + d*x]]) + ((3*Sqr
t[2]*ArcSinh[Tan[c + d*x]/(1 + Sec[c + d*x])])/d - (2*Sqrt[Sec[c + d*x]]*S
in[c + d*x])/(d*Sqrt[1 + Sec[c + d*x]]))/3
```

3.270.3.1 Defintions of rubi rules used

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4294
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[(-Sqrt[2])*(Sqrt[a]/(b*f))   Subst[Int[1/Sqrt[1
+ x^2], x], x, b*(Cot[e + f*x]/(a + b*Csc[e + f*x]))], x] /; FreeQ[{a, b, d
, e, f}, x] && EqQ[a^2 - b^2, 0] && EqQ[d - a/b, 0] && GtQ[a, 0]
```

rule 4310
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*n*Sqrt[a +
b*Csc[e + f*x]])), x] + Simp[1/(2*b*d*n)   Int[(d*Csc[e + f*x])^(n + 1)*((a
 + b*(2*n + 1)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, 0] && IntegerQ[2*n]
```

rule 4501
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*Cot[e
 + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(f*n)), x] - Simp[(a*A*m
 - b*B*n)/(b*d*n)   Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x]
, x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a
^2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LeQ[m, -1]
```

3.270.4 Maple [A] (verified)

Time = 1.14 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.70

method	result
default	$\frac{\sqrt{1+\sec(dx+c)}\left(3\sqrt{2}\,\arctan\left(\frac{\sin(dx+c)\sqrt{2}}{2(\cos(dx+c)+1)\sqrt{-\frac{1}{\cos(dx+c)+1}}}\right)\sqrt{-\frac{1}{\cos(dx+c)+1}}+3\sqrt{2}\,\arctan\left(\frac{\sin(dx+c)\sqrt{2}}{2(\cos(dx+c)+1)\sqrt{-\frac{1}{\cos(dx+c)+1}}}\right)\sqrt{}}{3d(\cos(dx+c)+1)\sec(dx+c)^{\frac{3}{2}}}$

input
```
int(1/sec(d*x+c)^(3/2)/(1+sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3/d*(1+sec(d*x+c))^(1/2)/(cos(d*x+c)+1)/sec(d*x+c)^(3/2)*(3*2^(1/2)*arct
an(1/2*sin(d*x+c)*2^(1/2)/(cos(d*x+c)+1)/(-1/(cos(d*x+c)+1))^(1/2))*(-1/(c
os(d*x+c)+1))^(1/2)+3*2^(1/2)*arctan(1/2*sin(d*x+c)*2^(1/2)/(cos(d*x+c)+1)
/(-1/(cos(d*x+c)+1))^(1/2))*(-1/(cos(d*x+c)+1))^(1/2)*sec(d*x+c)+2*sin(d*x
+c)-2*tan(d*x+c))
```

3.270.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.66

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)\sqrt{1+\sec(c+dx)}}\,dx$$
$$= \frac{3\left(\sqrt{2}\cos\left(dx+c\right)+\sqrt{2}\right)\log\left(\frac{2\sqrt{2}\sqrt{\frac{\cos(dx+c)+1}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)-\cos(dx+c)^2+2\cos(dx+c)+3}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)+\frac{4\left(\cos(dx+c)^2-\cos(d\right.}{\sqrt{}}}{6\left(d\cos\left(dx+c\right)+d\right)}$$

input
```
integrate(1/sec(d*x+c)^(3/2)/(1+sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
1/6*(3*(sqrt(2)*cos(d*x + c) + sqrt(2))*log((2*sqrt(2)*sqrt((cos(d*x + c)
+ 1)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - cos(d*x + c)^2 + 2*co
s(d*x + c) + 3)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + 4*(cos(d*x + c)^2
 - cos(d*x + c))*sqrt((cos(d*x + c) + 1)/cos(d*x + c))*sin(d*x + c)/sqrt(c
os(d*x + c)))/(d*cos(d*x + c) + d)
```

3.270.6 Sympy [F]

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)\sqrt{1+\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{\sec\left(c+dx\right)+1}\sec^{\frac{3}{2}}\left(c+dx\right)}\,dx$$

input
```
integrate(1/sec(d*x+c)**(3/2)/(1+sec(d*x+c))**(1/2),x)
```

output
```
Integral(1/(sqrt(sec(c + d*x) + 1)*sec(c + d*x)**(3/2)), x)
```

3.270.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 279 vs. 2(84) = 168.

Time = 0.41 (sec) , antiderivative size = 279, normalized size of antiderivative = 2.85

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)\sqrt{1+\sec(c+dx)}}\,dx =$$
$$-\frac{3\sqrt{2}\cos\left(\frac{2}{3}\arctan\left(\sin\left(\frac{3}{2}dx+\frac{3}{2}c\right),\cos\left(\frac{3}{2}dx+\frac{3}{2}c\right)\right)\right)\sin\left(\frac{3}{2}dx+\frac{3}{2}c\right)-3\sqrt{2}\cos\left(\frac{3}{2}dx+\frac{3}{2}c\right)\sin\left(\frac{2}{3}a\right.}{}$$

input
```
integrate(1/sec(d*x+c)^(3/2)/(1+sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
-1/6*(3*sqrt(2)*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)
))*sin(3/2*d*x + 3/2*c) - 3*sqrt(2)*cos(3/2*d*x + 3/2*c)*sin(2/3*arctan2(s
in(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 3*sqrt(2)*log(cos(1/3*arctan
2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + sin(1/3*arctan2(sin(3/2
*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x +
3/2*c), cos(3/2*d*x + 3/2*c))) + 1) + 3*sqrt(2)*log(cos(1/3*arctan2(sin(3/
2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + sin(1/3*arctan2(sin(3/2*d*x + 3
/2*c), cos(3/2*d*x + 3/2*c)))^2 - 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c),
cos(3/2*d*x + 3/2*c))) + 1) - 2*sqrt(2)*sin(3/2*d*x + 3/2*c) + 3*sqrt(2)*s
in(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))/d
```

3.270.8 Giac [F]

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)\sqrt{1+\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{\sec(dx+c)+1}\sec(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/sec(d*x+c)^(3/2)/(1+sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(sec(d*x + c) + 1)*sec(d*x + c)^(3/2)), x)
```

3.270.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)\sqrt{1+\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{\frac{1}{\cos(c+d\,x)}+1}\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

input
```
int(1/((1/cos(c + d*x) + 1)^(1/2)*(1/cos(c + d*x))^(3/2)),x)
```

output
```
int(1/((1/cos(c + d*x) + 1)^(1/2)*(1/cos(c + d*x))^(3/2)), x)
```

3.271 $\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)\sqrt{1+\sec(c+dx)}}\,dx$

3.271.1 Optimal result 2142
3.271.2 Mathematica [A] (verified) 2143
3.271.3 Rubi [A] (verified) 2143
3.271.4 Maple [A] (verified) 2146
3.271.5 Fricas [A] (verification not implemented) 2147
3.271.6 Sympy [F] 2147
3.271.7 Maxima [B] (verification not implemented) 2148
3.271.8 Giac [F] 2148
3.271.9 Mupad [F(-1)] 2149

3.271.1 Optimal result

Integrand size = 23, antiderivative size = 134

$$\begin{aligned}\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)\sqrt{1+\sec(c+dx)}}\,dx = &-\frac{\sqrt{2}\operatorname{arcsinh}\left(\frac{\tan(c+dx)}{1+\sec(c+dx)}\right)}{d}\\ &+\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{1+\sec(c+dx)}}\\ &-\frac{2\sin(c+dx)}{15d\sqrt{\sec(c+dx)}\sqrt{1+\sec(c+dx)}}\\ &+\frac{26\sqrt{\sec(c+dx)}\sin(c+dx)}{15d\sqrt{1+\sec(c+dx)}}\end{aligned}$$

output
```
-arcsinh(tan(d*x+c)/(1+sec(d*x+c)))*2^(1/2)/d+2/5*sin(d*x+c)/d/sec(d*x+c)^
(3/2)/(1+sec(d*x+c))^(1/2)-2/15*sin(d*x+c)/d/sec(d*x+c)^(1/2)/(1+sec(d*x+c
))^(1/2)+26/15*sin(d*x+c)*sec(d*x+c)^(1/2)/d/(1+sec(d*x+c))^(1/2)
```

3.271.2 Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.91

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)\sqrt{1+\sec(c+dx)}}\,dx = \frac{\left(15\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)\sec^{\frac{5}{2}}(c+dx)+2\sqrt{1-\sec(c+dx)}(3-\sec(c+dx)+13\sec^2(c+dx))\right)\sin(}{15d\sec^{\frac{3}{2}}(c+dx)\sqrt{-\tan^2(c+dx)}}$$

input
```
Integrate[1/(Sec[c + d*x]^(5/2)*Sqrt[1 + Sec[c + d*x]]),x]
```

output
```
((15*Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*S
ec[c + d*x]^(5/2) + 2*Sqrt[1 - Sec[c + d*x]]*(3 - Sec[c + d*x] + 13*Sec[c
+ d*x]^2))*Sin[c + d*x])/(15*d*Sec[c + d*x]^(3/2)*Sqrt[-Tan[c + d*x]^2])
```

3.271.3 Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.06, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {3042, 4310, 3042, 4510, 27, 3042, 4501, 3042, 4294, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)\sqrt{\sec(c+dx)+1}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)+1}}dx$$

$$\Big\downarrow \text{4310}$$

$$\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{\sec(c+dx)+1}}-\frac{1}{5}\int \frac{1-4\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)\sqrt{\sec(c+dx)+1}}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{\sec(c+dx)+1}}-\frac{1}{5}\int \frac{1-4\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)+1}}dx$$

$$\Big\downarrow \text{4510}$$

$$\frac{1}{5}\left(-\frac{2}{3}\int -\frac{13-2\sec(c+dx)}{2\sqrt{\sec(c+dx)}\sqrt{\sec(c+dx)+1}}dx-\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{\sec(c+dx)+1}}\right)+\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{\sec(c+dx)+1}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{5}\left(\frac{1}{3}\int \frac{13-2\sec(c+dx)}{\sqrt{\sec(c+dx)}\sqrt{\sec(c+dx)+1}}dx-\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{\sec(c+dx)+1}}\right)+\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{\sec(c+dx)+1}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}\left(\frac{1}{3}\int \frac{13-2\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)+1}}dx-\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{\sec(c+dx)+1}}\right)+\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{\sec(c+dx)+1}}$$

$$\Big\downarrow \text{4501}$$

$$\frac{1}{5}\left(\frac{1}{3}\left(\frac{26\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{\sec(c+dx)+1}}-15\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{\sec(c+dx)+1}}dx\right)-\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{\sec(c+dx)+1}}\right)+\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{\sec(c+dx)+1}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}\left(\frac{1}{3}\left(\frac{26\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{\sec(c+dx)+1}}-15\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)+1}}dx\right)-\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{\sec(c+dx)+1}}\right)+\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{\sec(c+dx)+1}}$$

$$\Big\downarrow \text{4294}$$

3.271. $\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)\sqrt{1+\sec(c+dx)}}\,dx$

$$\frac{1}{5}\left(\frac{1}{3}\left(\frac{15\sqrt{2}\int \frac{1}{\sqrt{\frac{\tan^2(c+dx)}{(\sec(c+dx)+1)^2}+1}}d\left(-\frac{\tan(c+dx)}{\sec(c+dx)+1}\right)}{d}+\frac{26\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{\sec(c+dx)+1}}\right)-\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{\sec(c+dx}}\right.$$

$$\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{\sec(c+dx)+1}}$$

$$\Big\downarrow \; 222$$

$$\frac{1}{5}\left(\frac{1}{3}\left(\frac{26\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{\sec(c+dx)+1}}-\frac{15\sqrt{2}\text{arcsinh}\left(\frac{\tan(c+dx)}{\sec(c+dx)+1}\right)}{d}\right)-\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{\sec(c+dx)+1}}\right)+$$

$$\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{\sec(c+dx)+1}}$$

input
```
Int[1/(Sec[c + d*x]^(5/2)*Sqrt[1 + Sec[c + d*x]]),x]
```

output
```
(2*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)*Sqrt[1 + Sec[c + d*x]]) + ((-2*Si
n[c + d*x])/(3*d*Sqrt[Sec[c + d*x]]*Sqrt[1 + Sec[c + d*x]]) + ((-15*Sqrt[2
]*ArcSinh[Tan[c + d*x]/(1 + Sec[c + d*x])])/d + (26*Sqrt[Sec[c + d*x]]*Sin
[c + d*x])/(d*Sqrt[1 + Sec[c + d*x]]))/3)/5
```

3.271.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4294
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[(-Sqrt[2])*(Sqrt[a]/(b*f))   Subst[Int[1/Sqrt[1
+ x^2], x], x, b*(Cot[e + f*x]/(a + b*Csc[e + f*x]))], x] /; FreeQ[{a, b, d
, e, f}, x] && EqQ[a^2 - b^2, 0] && EqQ[d - a/b, 0] && GtQ[a, 0]
```

rule 4310
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*n*Sqrt[a +
b*Csc[e + f*x]])), x] + Simp[1/(2*b*d*n)   Int[(d*Csc[e + f*x])^(n + 1)*((a
 + b*(2*n + 1)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, 0] && IntegerQ[2*n]
```

rule 4501
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*Cot[e
 + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(f*n)), x] - Simp[(a*A*m
 - b*B*n)/(b*d*n)   Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x]
, x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a
^2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LeQ[m, -1]
```

rule 4510
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*Cot[e
 + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(f*n)), x] - Simp[1/(b*d
*n)   Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*
n - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

3.271.4 Maple [A] (verified)

Time = 1.27 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.41

method	result
default	$-\frac{\sqrt{1+\sec(dx+c)}\left(15\sqrt{2}\arctan\left(\frac{\sin(dx+c)\sqrt{2}}{2(\cos(dx+c)+1)\sqrt{-\frac{1}{\cos(dx+c)+1}}}\right)\sqrt{-\frac{1}{\cos(dx+c)+1}}\sec(dx+c)+15\sqrt{2}\arctan\left(\frac{\sin(dx+c)\sqrt{}}{2(\cos(dx+c)+1)\sqrt{-}}\right.\right.}{15d(\cos(dx+c)+1)\sec(dx+c)^{\frac{5}{2}}}$

input
```
int(1/sec(d*x+c)^(5/2)/(1+sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/15/d*(1+sec(d*x+c))^(1/2)/(cos(d*x+c)+1)/sec(d*x+c)^(5/2)*(15*2^(1/2)*a
rctan(1/2*sin(d*x+c)*2^(1/2)/(cos(d*x+c)+1)/(-1/(cos(d*x+c)+1))^(1/2))*(-1
/(cos(d*x+c)+1))^(1/2)*sec(d*x+c)+15*2^(1/2)*arctan(1/2*sin(d*x+c)*2^(1/2)
/(cos(d*x+c)+1)/(-1/(cos(d*x+c)+1))^(1/2))*(-1/(cos(d*x+c)+1))^(1/2)*sec(d
*x+c)^2-6*sin(d*x+c)+2*tan(d*x+c)-26*sec(d*x+c)*tan(d*x+c))
```

3.271.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.30

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)\sqrt{1+\sec(c+dx)}}\,dx$$
$$= \frac{15\left(\sqrt{2}\cos\left(dx+c\right)+\sqrt{2}\right)\log\left(-\frac{2\sqrt{2}\sqrt{\frac{\cos(dx+c)+1}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)+\cos(dx+c)^2-2\cos(dx+c)-3}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)+\frac{4\left(3\cos(dx+c)^3-\right.}{}}{30\left(d\cos\left(dx+c\right)+d\right)}$$

input
```
integrate(1/sec(d*x+c)^(5/2)/(1+sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
1/30*(15*(sqrt(2)*cos(d*x + c) + sqrt(2))*log(-(2*sqrt(2)*sqrt((cos(d*x +
c) + 1)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) + cos(d*x + c)^2 - 2
*cos(d*x + c) - 3)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + 4*(3*cos(d*x +
 c)^3 - cos(d*x + c)^2 + 13*cos(d*x + c))*sqrt((cos(d*x + c) + 1)/cos(d*x
+ c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c) + d)
```

3.271.6 Sympy [F]

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)\sqrt{1+\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{\sec\left(c+dx\right)+1}\sec^{\frac{5}{2}}\left(c+dx\right)}\,dx$$

input
```
integrate(1/sec(d*x+c)**(5/2)/(1+sec(d*x+c))**(1/2),x)
```

output
```
Integral(1/(sqrt(sec(c + d*x) + 1)*sec(c + d*x)**(5/2)), x)
```

3.271.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 354 vs. $2(114) = 228$.

Time = 0.52 (sec) , antiderivative size = 354, normalized size of antiderivative = 2.64

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)\sqrt{1+\sec(c+dx)}}\,dx$$
$$= \frac{\sqrt{2}\left(60\cos\left(\frac{4}{5}\arctan\left(\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right),\cos\left(\frac{5}{2}dx+\frac{5}{2}c\right)\right)\right)\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right)-5\cos\left(\frac{2}{5}\arctan\left(\sin\left(\frac{5}{2}dx+\right.\right.\right.\right.}{}$$

input
```
integrate(1/sec(d*x+c)^(5/2)/(1+sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
1/60*sqrt(2)*(60*cos(4/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c
)))*sin(5/2*d*x + 5/2*c) - 5*cos(2/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2
*d*x + 5/2*c)))*sin(5/2*d*x + 5/2*c) - 60*cos(5/2*d*x + 5/2*c)*sin(4/5*arc
tan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 5*cos(5/2*d*x + 5/2*c)
*sin(2/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) - 30*log(cos
(1/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c)))^2 + sin(1/5*arct
an2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c)))^2 + 2*sin(1/5*arctan2(sin
(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 1) + 30*log(cos(1/5*arctan2(si
n(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c)))^2 + sin(1/5*arctan2(sin(5/2*d*x
 + 5/2*c), cos(5/2*d*x + 5/2*c)))^2 - 2*sin(1/5*arctan2(sin(5/2*d*x + 5/2*
c), cos(5/2*d*x + 5/2*c))) + 1) + 6*sin(5/2*d*x + 5/2*c) - 5*sin(3/5*arcta
n2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 60*sin(1/5*arctan2(sin(5
/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))))/d
```

3.271.8 Giac [F]

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)\sqrt{1+\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{\sec(dx+c)+1}\sec(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/sec(d*x+c)^(5/2)/(1+sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(sec(d*x + c) + 1)*sec(d*x + c)^(5/2)), x)
```

3.271.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)\sqrt{1+\sec(c+dx)}}\,dx=\int \frac{1}{\sqrt{\frac{1}{\cos(c+d\,x)}+1}\,{\left(\frac{1}{\cos(c+d\,x)}\right)}^{5/2}}\,dx$$

input
```
int(1/((1/cos(c + d*x) + 1)^(1/2)*(1/cos(c + d*x))^(5/2)),x)
```

output
```
int(1/((1/cos(c + d*x) + 1)^(1/2)*(1/cos(c + d*x))^(5/2)), x)
```

3.272 $\int (e\sec(c+dx))^{4/3}\sqrt{a+a\sec(c+dx)}\,dx$

3.272.1 Optimal result 2150
3.272.2 Mathematica [C] (verified) 2151
3.272.3 Rubi [A] (verified) 2151
3.272.4 Maple [F] 2153
3.272.5 Fricas [F] 2154
3.272.6 Sympy [F(-1)] 2154
3.272.7 Maxima [F] 2154
3.272.8 Giac [F] 2155
3.272.9 Mupad [F(-1)] 2155

3.272.1 Optimal result

Integrand size = 27, antiderivative size = 325

$$\int (e\sec(c+dx))^{4/3}\sqrt{a+a\sec(c+dx)}\,dx = \frac{6ae\sqrt[3]{e\sec(c+dx)}\tan(c+dx)}{5d\sqrt{a+a\sec(c+dx)}} + \frac{4\,3^{3/4}\sqrt{2+\sqrt{3}}a^2e\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}}{\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}}\right),-7-4\sqrt{3}\right)\left(\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)}{5d(a-a\sec(c+dx))\sqrt{a+a\sec(c+dx)}\sqrt{\frac{\sqrt[3]{e}\left(\sqrt[3]{e}-\sqrt[3]{e\sec(\cdots}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec\cdots}\right.}}}$$

output
```
6/5*a*e*(e*sec(d*x+c))^(1/3)*tan(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+4/5*3^(3/
4)*a^2*e*EllipticF((-(e*sec(d*x+c))^(1/3)+e^(1/3)*(1-3^(1/2)))/(-(e*sec(d*
x+c))^(1/3)+e^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(e^(1/3)-(e*sec(d*x+c))^(1
/3))*(1/2*6^(1/2)+1/2*2^(1/2))*((e^(2/3)+e^(1/3)*(e*sec(d*x+c))^(1/3)+(e*s
ec(d*x+c))^(2/3))/(-(e*sec(d*x+c))^(1/3)+e^(1/3)*(1+3^(1/2)))^2)^(1/2)*tan
(d*x+c)/d/(a-a*sec(d*x+c))/(a+a*sec(d*x+c))^(1/2)/(e^(1/3)*(e^(1/3)-(e*sec
(d*x+c))^(1/3))/(-(e*sec(d*x+c))^(1/3)+e^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

3.272.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.25 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.22

$$\int (e\sec(c+dx))^{4/3}\sqrt{a+a\sec(c+dx)}\,dx = \frac{2\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{2},\frac{3}{2},1-\sec(c+dx)\right)(e\sec(c+dx))^{4/3}\sqrt{a(1-}}{d\sec^{\frac{4}{3}}(c+dx)}$$

input
```
Integrate[(e*Sec[c + d*x])^(4/3)*Sqrt[a + a*Sec[c + d*x]],x]
```

output
```
(2*Hypergeometric2F1[-1/3, 1/2, 3/2, 1 - Sec[c + d*x]]*(e*Sec[c + d*x])^(4
/3)*Sqrt[a*(1 + Sec[c + d*x])]*Tan[(c + d*x)/2])/(d*Sec[c + d*x]^(4/3))
```

3.272.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 337, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {3042, 4293, 60, 73, 759}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a\sec(c+dx)+a}(e\sec(c+dx))^{4/3}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \sqrt{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}\left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{4/3}dx$$

$$\Big\downarrow \; 4293$$

$$-\frac{a^2e\tan(c+dx)\int \frac{\sqrt[3]{e\sec(c+dx)}}{\sqrt{a-a\sec(c+dx)}}d\sec(c+dx)}{d\sqrt{a-a\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \; 60$$

$$-\frac{a^2e\tan(c+dx)\left(\frac{2}{5}e\int \frac{1}{(e\sec(c+dx))^{2/3}\sqrt{a-a\sec(c+dx)}}d\sec(c+dx)-\frac{6\sqrt{a-a\sec(c+dx)}\sqrt[3]{e\sec(c+dx)}}{5a}\right)}{d\sqrt{a-a\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{73}$$

$$-\frac{a^2e\tan(c+dx)\left(\frac{6}{5}\int\frac{1}{\sqrt{a-a\sec(c+dx)}}d\sqrt[3]{e\sec(c+dx)}-\frac{6\sqrt{a-a\sec(c+dx)}\sqrt[3]{e\sec(c+dx)}}{5a}\right)}{d\sqrt{a-a\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{759}$$

$$-\frac{a^2e\tan(c+dx)\left(-\frac{4\ 3^{3/4}\sqrt{2+\sqrt{3}}\sqrt{\frac{\sqrt[3]{e}\sqrt[3]{e\sec(c+dx)}+(e\sec(c+dx))^{2/3}+e^{2/3}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)^2}}\left(\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)\text{EllipticF}\left(\arcsin\left(\frac{(1-}{(1+}\right.\right.}{5\sqrt{a-a\sec(c+dx)}\sqrt{\frac{\sqrt[3]{e}\left(\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)^2}}}\right)}{d\sqrt{a-a\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}$$

input
```
Int[(e*Sec[c + d*x])^(4/3)*Sqrt[a + a*Sec[c + d*x]],x]
```

output
```
-((a^2*e*((-6*(e*Sec[c + d*x])^(1/3)*Sqrt[a - a*Sec[c + d*x]])/(5*a) - (4*
3^(3/4)*Sqrt[2 + Sqrt[3]]*EllipticF[ArcSin[((1 - Sqrt[3])*e^(1/3) - (e*Sec
[c + d*x])^(1/3))/((1 + Sqrt[3])*e^(1/3) - (e*Sec[c + d*x])^(1/3))], -7 -
4*Sqrt[3]]*(e^(1/3) - (e*Sec[c + d*x])^(1/3))*Sqrt[(e^(2/3) + e^(1/3)*(e*S
ec[c + d*x])^(1/3) + (e*Sec[c + d*x])^(2/3))/((1 + Sqrt[3])*e^(1/3) - (e*S
ec[c + d*x])^(1/3))^2])/(5*Sqrt[a - a*Sec[c + d*x]]*Sqrt[(e^(1/3)*(e^(1/3)
 - (e*Sec[c + d*x])^(1/3)))/((1 + Sqrt[3])*e^(1/3) - (e*Sec[c + d*x])^(1/3
))^2]))*Tan[c + d*x])/(d*Sqrt[a - a*Sec[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]
))
```

3.272.3.1 Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4293
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]
*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(d*x)^(n - 1)/Sqrt[a - b*x], x], x,
 Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0]
```

3.272.4 Maple [F]

$$\int (e\sec(dx+c))^{\frac{4}{3}}\sqrt{a+a\sec(dx+c)}dx$$

input
```
int((e*sec(d*x+c))^(4/3)*(a+a*sec(d*x+c))^(1/2),x)
```

output
```
int((e*sec(d*x+c))^(4/3)*(a+a*sec(d*x+c))^(1/2),x)
```

3.272.5 Fricas [F]

$$\int (e\sec(c+dx))^{4/3}\sqrt{a+a\sec(c+dx)}\,dx = \int \sqrt{a\sec(dx+c)+a}(e\sec(dx+c))^{\frac{4}{3}}\,dx$$

input
```
integrate((e*sec(d*x+c))^(4/3)*(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas
")
```

output
```
integral(sqrt(a*sec(d*x + c) + a)*(e*sec(d*x + c))^(1/3)*e*sec(d*x + c), x
)
```

3.272.6 Sympy [F(-1)]

Timed out.

$$\int (e\sec(c+dx))^{4/3}\sqrt{a+a\sec(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((e*sec(d*x+c))**(4/3)*(a+a*sec(d*x+c))**(1/2),x)
```

output
```
Timed out
```

3.272.7 Maxima [F]

$$\int (e\sec(c+dx))^{4/3}\sqrt{a+a\sec(c+dx)}\,dx = \int \sqrt{a\sec(dx+c)+a}(e\sec(dx+c))^{\frac{4}{3}}\,dx$$

input
```
integrate((e*sec(d*x+c))^(4/3)*(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima
")
```

output
```
integrate(sqrt(a*sec(d*x + c) + a)*(e*sec(d*x + c))^(4/3), x)
```

3.272.8 Giac [F]

$$\int (e\sec(c+dx))^{4/3}\sqrt{a+a\sec(c+dx)}\,dx = \int \sqrt{a\sec(dx+c)+a}(e\sec(dx+c))^{\frac{4}{3}}\,dx$$

input
```
integrate((e*sec(d*x+c))^(4/3)*(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.272.9 Mupad [F(-1)]

Timed out.

$$\int (e\sec(c+dx))^{4/3}\sqrt{a+a\sec(c+dx)}\,dx = \int \sqrt{a+\frac{a}{\cos(c+d\,x)}}\left(\frac{e}{\cos(c+d\,x)}\right)^{4/3}dx$$

input
```
int((a + a/cos(c + d*x))^(1/2)*(e/cos(c + d*x))^(4/3),x)
```

output
```
int((a + a/cos(c + d*x))^(1/2)*(e/cos(c + d*x))^(4/3), x)
```

3.273 $\int \sqrt[3]{e\sec(c+dx)}\sqrt{a+a\sec(c+dx)}\,dx$

3.273.1 Optimal result 2156
3.273.2 Mathematica [C] (verified) 2157
3.273.3 Rubi [A] (verified) 2157
3.273.4 Maple [F] 2159
3.273.5 Fricas [F] 2159
3.273.6 Sympy [F] 2159
3.273.7 Maxima [F] 2160
3.273.8 Giac [F] 2160
3.273.9 Mupad [F(-1)] 2160

3.273.1 Optimal result

Integrand size = 27, antiderivative size = 280

$$\int \sqrt[3]{e\sec(c+dx)}\sqrt{a+a\sec(c+dx)}\,dx = \frac{2\,3^{3/4}\sqrt{2+\sqrt{3}}a^2\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}}{\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}}\right),-7-4\sqrt{3}\right)\left(\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)}{d(a-a\sec(c+dx))\sqrt{a+a\sec(c+dx)}\sqrt{\frac{\sqrt[3]{e}\left(\sqrt[3]{e}-\sqrt[3]{e\sec(c}\right.}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec}\right.}}}$$

output
```
2*3^(3/4)*a^2*EllipticF((-(e*sec(d*x+c))^(1/3)+e^(1/3)*(1-3^(1/2)))/(-(e*s
ec(d*x+c))^(1/3)+e^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(e^(1/3)-(e*sec(d*x+c
))^(1/3))*(1/2*6^(1/2)+1/2*2^(1/2))*((e^(2/3)+e^(1/3)*(e*sec(d*x+c))^(1/3)
+(e*sec(d*x+c))^(2/3))/(-(e*sec(d*x+c))^(1/3)+e^(1/3)*(1+3^(1/2)))^2)^(1/2
)*tan(d*x+c)/d/(a-a*sec(d*x+c))/(a+a*sec(d*x+c))^(1/2)/(e^(1/3)*(e^(1/3)-(
e*sec(d*x+c))^(1/3))/(-(e*sec(d*x+c))^(1/3)+e^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

3.273.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.14 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.25

$$\int \sqrt[3]{e\sec(c+dx)}\sqrt{a+a\sec(c+dx)}\,dx = \frac{2\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{3}{2},1-\sec(c+dx)\right)\sqrt[3]{e\sec(c+dx)}\sqrt{a(1+\sec(c+dx))}\tan\left(\frac{1}{2}(c+dx)\right)}{d\sqrt[3]{\sec(c+dx)}}$$

input
```
Integrate[(e*Sec[c + d*x])^(1/3)*Sqrt[a + a*Sec[c + d*x]],x]
```

output
```
(2*Hypergeometric2F1[1/2, 2/3, 3/2, 1 - Sec[c + d*x]]*(e*Sec[c + d*x])^(1/
3)*Sqrt[a*(1 + Sec[c + d*x])]*Tan[(c + d*x)/2])/(d*Sec[c + d*x]^(1/3))
```

3.273.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {3042, 4293, 73, 759}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a\sec(c+dx)+a}\sqrt[3]{e\sec(c+dx)}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \sqrt{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}\sqrt[3]{e\csc\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \; 4293$$

$$-\frac{a^2e\tan(c+dx)\int \frac{1}{(e\sec(c+dx))^{2/3}\sqrt{a-a\sec(c+dx)}}d\sec(c+dx)}{d\sqrt{a-a\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \; 73$$

$$-\frac{3a^2\tan(c+dx)\int \frac{1}{\sqrt{a-a\sec(c+dx)}}d\sqrt[3]{e\sec(c+dx)}}{d\sqrt{a-a\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \; 759$$

$$\frac{2\,3^{3/4}\sqrt{2+\sqrt{3}}a^2\tan(c+dx)\left(\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)\sqrt{\frac{\sqrt[3]{e}\sqrt[3]{e\sec(c+dx)}+(e\sec(c+dx))^{2/3}+e^{2/3}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)^2}}\,\text{EllipticF}\left(\arcsin\right.}{d(a-a\sec(c+dx))\sqrt{a\sec(c+dx)+a}\sqrt{\frac{\sqrt[3]{e}\left(\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)}}}$$

input
```
Int[(e*Sec[c + d*x])^(1/3)*Sqrt[a + a*Sec[c + d*x]],x]
```

output
```
(2*3^(3/4)*Sqrt[2 + Sqrt[3]]*a^2*EllipticF[ArcSin[((1 - Sqrt[3])*e^(1/3) -
 (e*Sec[c + d*x])^(1/3))/((1 + Sqrt[3])*e^(1/3) - (e*Sec[c + d*x])^(1/3))]
, -7 - 4*Sqrt[3]]*(e^(1/3) - (e*Sec[c + d*x])^(1/3))*Sqrt[(e^(2/3) + e^(1/
3)*(e*Sec[c + d*x])^(1/3) + (e*Sec[c + d*x])^(2/3))/((1 + Sqrt[3])*e^(1/3)
 - (e*Sec[c + d*x])^(1/3))^2]*Tan[c + d*x])/(d*(a - a*Sec[c + d*x])*Sqrt[a
 + a*Sec[c + d*x]]*Sqrt[(e^(1/3)*(e^(1/3) - (e*Sec[c + d*x])^(1/3)))/((1 +
 Sqrt[3])*e^(1/3) - (e*Sec[c + d*x])^(1/3))^2])
```

3.273.3.1 Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4293
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]
*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(d*x)^(n - 1)/Sqrt[a - b*x], x], x,
 Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0]
```

3.273.4 Maple [F]

$$\int (e\sec(dx+c))^{\frac{1}{3}}\sqrt{a+a\sec(dx+c)}dx$$

input
```
int((e*sec(d*x+c))^(1/3)*(a+a*sec(d*x+c))^(1/2),x)
```

output
```
int((e*sec(d*x+c))^(1/3)*(a+a*sec(d*x+c))^(1/2),x)
```

3.273.5 Fricas [F]

$$\int \sqrt[3]{e\sec(c+dx)}\sqrt{a+a\sec(c+dx)}\,dx=\int \sqrt{a\sec(dx+c)+a}(e\sec(dx+c))^{\frac{1}{3}}\,dx$$

input
```
integrate((e*sec(d*x+c))^(1/3)*(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas
")
```

output
```
integral(sqrt(a*sec(d*x + c) + a)*(e*sec(d*x + c))^(1/3), x)
```

3.273.6 Sympy [F]

$$\int \sqrt[3]{e\sec(c+dx)}\sqrt{a+a\sec(c+dx)}\,dx=\int \sqrt{a(\sec(c+dx)+1)}\sqrt[3]{e\sec(c+dx)}\,dx$$

input
```
integrate((e*sec(d*x+c))**(1/3)*(a+a*sec(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(a*(sec(c + d*x) + 1))*(e*sec(c + d*x))**(1/3), x)
```

3.273.7 Maxima [F]

$$\int \sqrt[3]{e\sec(c+dx)}\sqrt{a+a\sec(c+dx)}\,dx = \int \sqrt{a\sec(dx+c)+a}(e\sec(dx+c))^{\frac{1}{3}}\,dx$$

input
```
integrate((e*sec(d*x+c))^(1/3)*(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima
")
```

output
```
integrate(sqrt(a*sec(d*x + c) + a)*(e*sec(d*x + c))^(1/3), x)
```

3.273.8 Giac [F]

$$\int \sqrt[3]{e\sec(c+dx)}\sqrt{a+a\sec(c+dx)}\,dx = \int \sqrt{a\sec(dx+c)+a}(e\sec(dx+c))^{\frac{1}{3}}\,dx$$

input
```
integrate((e*sec(d*x+c))^(1/3)*(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.273.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt[3]{e\sec(c+dx)}\sqrt{a+a\sec(c+dx)}\,dx = \int \sqrt{a+\frac{a}{\cos(c+d\,x)}}\left(\frac{e}{\cos(c+d\,x)}\right)^{1/3}dx$$

input
```
int((a + a/cos(c + d*x))^(1/2)*(e/cos(c + d*x))^(1/3),x)
```

output
```
int((a + a/cos(c + d*x))^(1/2)*(e/cos(c + d*x))^(1/3), x)
```

3.274 $\int \frac{\sqrt{a+a\sec(c+dx)}}{(e\sec(c+dx))^{2/3}}\,dx$

3.274.1 Optimal result 2161
3.274.2 Mathematica [C] (verified) 2162
3.274.3 Rubi [A] (verified) 2162
3.274.4 Maple [F] 2164
3.274.5 Fricas [F] 2165
3.274.6 Sympy [F] 2165
3.274.7 Maxima [F] 2165
3.274.8 Giac [F] 2166
3.274.9 Mupad [F(-1)] 2166

3.274.1 Optimal result

Integrand size = 27, antiderivative size = 326

$$\int \frac{\sqrt{a+a\sec(c+dx)}}{(e\sec(c+dx))^{2/3}}\,dx = \frac{3a\tan(c+dx)}{2d(e\sec(c+dx))^{2/3}\sqrt{a+a\sec(c+dx)}} + \frac{3^{3/4}\sqrt{2+\sqrt{3}}a^2\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}}{\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}}\right),-7-4\sqrt{3}\right)\left(\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)\sqrt{\cdots}}{2de(a-a\sec(c+dx))\sqrt{a+a\sec(c+dx)}\sqrt{\frac{\sqrt[3]{e}\left(\sqrt[3]{e}-\sqrt[3]{e\sec(c\cdots}\right.}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(\cdots}\right.}}}$$

output
```
3/2*a*tan(d*x+c)/d/(e*sec(d*x+c))^(2/3)/(a+a*sec(d*x+c))^(1/2)+1/2*3^(3/4)
*a^2*EllipticF((-(e*sec(d*x+c))^(1/3)+e^(1/3)*(1-3^(1/2)))/(-(e*sec(d*x+c)
)^(1/3)+e^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(e^(1/3)-(e*sec(d*x+c))^(1/3))
*(1/2*6^(1/2)+1/2*2^(1/2))*((e^(2/3)+e^(1/3)*(e*sec(d*x+c))^(1/3)+(e*sec(d
*x+c))^(2/3))/(-(e*sec(d*x+c))^(1/3)+e^(1/3)*(1+3^(1/2)))^2)^(1/2)*tan(d*x
+c)/d/e/(a-a*sec(d*x+c))/(a+a*sec(d*x+c))^(1/2)/(e^(1/3)*(e^(1/3)-(e*sec(d
*x+c))^(1/3))/(-(e*sec(d*x+c))^(1/3)+e^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

3.274.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.16 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.22

$$\int \frac{\sqrt{a+a\sec(c+dx)}}{(e\sec(c+dx))^{2/3}}\,dx = \frac{2\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{3},\frac{3}{2},1-\sec(c+dx)\right)\sec^{\frac{2}{3}}(c+dx)\sqrt{a(1+\sec(c+dx}}{d(e\sec(c+dx))^{2/3}}$$

input
```
Integrate[Sqrt[a + a*Sec[c + d*x]]/(e*Sec[c + d*x])^(2/3),x]
```

output
```
(2*Hypergeometric2F1[1/2, 5/3, 3/2, 1 - Sec[c + d*x]]*Sec[c + d*x]^(2/3)*S
qrt[a*(1 + Sec[c + d*x])]*Tan[(c + d*x)/2])/(d*(e*Sec[c + d*x])^(2/3))
```

3.274.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 343, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {3042, 4293, 61, 73, 759}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a\sec(c+dx)+a}}{(e\sec(c+dx))^{2/3}}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\sqrt{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}}{\left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{2/3}}dx$$

$$\Big\downarrow \; 4293$$

$$-\frac{a^2e\tan(c+dx)\int \frac{1}{(e\sec(c+dx))^{5/3}\sqrt{a-a\sec(c+dx)}}d\sec(c+dx)}{d\sqrt{a-a\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \; 61$$

$$-\frac{a^2e\tan(c+dx)\left(\frac{\int \frac{1}{(e\sec(c+dx))^{2/3}\sqrt{a-a\sec(c+dx)}}d\sec(c+dx)}{4e}-\frac{3\sqrt{a-a\sec(c+dx)}}{2ae(e\sec(c+dx))^{2/3}}\right)}{d\sqrt{a-a\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \; 73$$

$$-\frac{a^2 e\tan(c+dx)\left(\frac{3\int \frac{1}{\sqrt{a-a\sec(c+dx)}}d\sqrt[3]{e\sec(c+dx)}}{4e^2}-\frac{3\sqrt{a-a\sec(c+dx)}}{2ae(e\sec(c+dx))^{2/3}}\right)}{d\sqrt{a-a\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \; 759$$

$$-\frac{a^2 e\tan(c+dx)\left(-\frac{3^{3/4}\sqrt{2+\sqrt{3}}\sqrt{\frac{\sqrt[3]{e}\sqrt[3]{e\sec(c+dx)}+(e\sec(c+dx))^{2/3}+e^{2/3}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)^2}}\left(\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right.}{\left(1+\sqrt{3}\right.}\right.\right.}{2e^2\sqrt{a-a\sec(c+dx)}\sqrt{\frac{\sqrt[3]{e}\left(\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)^2}}}\right.}{d\sqrt{a-a\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}$$

input
```
Int[Sqrt[a + a*Sec[c + d*x]]/(e*Sec[c + d*x])^(2/3),x]
```

output
```
-((a^2*e*((-3*Sqrt[a - a*Sec[c + d*x]])/(2*a*e*(e*Sec[c + d*x])^(2/3)) - (
3^(3/4)*Sqrt[2 + Sqrt[3]]*EllipticF[ArcSin[((1 - Sqrt[3])*e^(1/3) - (e*Sec
[c + d*x])^(1/3))/((1 + Sqrt[3])*e^(1/3) - (e*Sec[c + d*x])^(1/3))], -7 -
4*Sqrt[3]]*(e^(1/3) - (e*Sec[c + d*x])^(1/3))*Sqrt[(e^(2/3) + e^(1/3)*(e*S
ec[c + d*x])^(1/3) + (e*Sec[c + d*x])^(2/3))/((1 + Sqrt[3])*e^(1/3) - (e*S
ec[c + d*x])^(1/3))^2])/(2*e^2*Sqrt[a - a*Sec[c + d*x]]*Sqrt[(e^(1/3)*(e^(
1/3) - (e*Sec[c + d*x])^(1/3)))/((1 + Sqrt[3])*e^(1/3) - (e*Sec[c + d*x])^
(1/3))^2]))*Tan[c + d*x])/(d*Sqrt[a - a*Sec[c + d*x]]*Sqrt[a + a*Sec[c + d
*x]]))
```

3.274.3.1 Defintions of rubi rules used

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4293
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]
*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(d*x)^(n - 1)/Sqrt[a - b*x], x], x,
 Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0]
```

3.274.4 Maple [F]

$$\int \frac{\sqrt{a+a\sec\left(dx+c\right)}}{\left(e\sec\left(dx+c\right)\right)^{\frac{2}{3}}}dx$$

input
```
int((a+a*sec(d*x+c))^(1/2)/(e*sec(d*x+c))^(2/3),x)
```

output
```
int((a+a*sec(d*x+c))^(1/2)/(e*sec(d*x+c))^(2/3),x)
```

3.274.5 Fricas [F]

$$\int \frac{\sqrt{a+a\sec(c+dx)}}{(e\sec(c+dx))^{2/3}}\,dx=\int \frac{\sqrt{a\sec(dx+c)+a}}{(e\sec(dx+c))^{\frac{2}{3}}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))^(1/2)/(e*sec(d*x+c))^(2/3),x, algorithm="fricas
")
```

output
```
integral(sqrt(a*sec(d*x + c) + a)*(e*sec(d*x + c))^(1/3)/(e*sec(d*x + c)),
 x)
```

3.274.6 Sympy [F]

$$\int \frac{\sqrt{a+a\sec(c+dx)}}{(e\sec(c+dx))^{2/3}}\,dx=\int \frac{\sqrt{a(\sec(c+dx)+1)}}{(e\sec(c+dx))^{\frac{2}{3}}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))**(1/2)/(e*sec(d*x+c))**(2/3),x)
```

output
```
Integral(sqrt(a*(sec(c + d*x) + 1))/(e*sec(c + d*x))**(2/3), x)
```

3.274.7 Maxima [F]

$$\int \frac{\sqrt{a+a\sec(c+dx)}}{(e\sec(c+dx))^{2/3}}\,dx=\int \frac{\sqrt{a\sec(dx+c)+a}}{(e\sec(dx+c))^{\frac{2}{3}}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))^(1/2)/(e*sec(d*x+c))^(2/3),x, algorithm="maxima
")
```

output
```
integrate(sqrt(a*sec(d*x + c) + a)/(e*sec(d*x + c))^(2/3), x)
```

3.274.8 Giac [F]

$$\int \frac{\sqrt{a+a\sec(c+dx)}}{(e\sec(c+dx))^{2/3}}\,dx=\int \frac{\sqrt{a\sec(dx+c)+a}}{(e\sec(dx+c))^{\frac{2}{3}}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))^(1/2)/(e*sec(d*x+c))^(2/3),x, algorithm="giac")
```

output
```
sage0*x
```

3.274.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+a\sec(c+dx)}}{(e\sec(c+dx))^{2/3}}\,dx=\int \frac{\sqrt{a+\frac{a}{\cos(c+d\,x)}}}{\left(\frac{e}{\cos(c+d\,x)}\right)^{2/3}}\,dx$$

input
```
int((a + a/cos(c + d*x))^(1/2)/(e/cos(c + d*x))^(2/3),x)
```

output
```
int((a + a/cos(c + d*x))^(1/2)/(e/cos(c + d*x))^(2/3), x)
```

3.275 $\int (e\sec(c+dx))^{8/3}\sqrt{a+a\sec(c+dx)}\,dx$

3.275.1 Optimal result 2167
3.275.2 Mathematica [C] (verified) 2168
3.275.3 Rubi [A] (warning: unable to verify) 2169
3.275.4 Maple [F] 2172
3.275.5 Fricas [F] 2172
3.275.6 Sympy [F(-1)] 2173
3.275.7 Maxima [F] 2173
3.275.8 Giac [F] 2173
3.275.9 Mupad [F(-1)] 2174

3.275.1 Optimal result

Integrand size = 27, antiderivative size = 716

$$\int (e\sec(c+dx))^{8/3}\sqrt{a+a\sec(c+dx)}\,dx = \frac{60ae^2(e\sec(c+dx))^{2/3}\tan(c+dx)}{91d\sqrt{a+a\sec(c+dx)}}$$
$$+\frac{6ae(e\sec(c+dx))^{5/3}\tan(c+dx)}{13d\sqrt{a+a\sec(c+dx)}}$$
$$-\frac{240ae^3\tan(c+dx)}{91d\sqrt{a+a\sec(c+dx)}\left(\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)}$$
$$+\frac{120\sqrt[4]{3}\sqrt{2-\sqrt{3}}a^2e^{7/3}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}}{\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}}\right)|-7-4\sqrt{3}\right)\left(\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)\sqrt{\frac{e}{}}}{91d(a-a\sec(c+dx))\sqrt{a+a\sec(c+dx)}\sqrt{\frac{\sqrt[3]{e}\left(\sqrt[3]{e}-\sqrt[3]{e\sec(c}\right.}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c}\right.}}}$$
$$-\frac{80\sqrt{2}3^{3/4}a^2e^{7/3}\,\mathrm{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}}{\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}}\right),-7-4\sqrt{3}\right)\left(\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)\sqrt{}}{91d(a-a\sec(c+dx))\sqrt{a+a\sec(c+dx)}\sqrt{\frac{\sqrt[3]{e}\left(\sqrt[3]{e}-\sqrt[3]{e\sec(c}\right.}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(}\right.}}}$$

output
```
60/91*a*e^2*(e*sec(d*x+c))^(2/3)*tan(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+6/13*
a*e*(e*sec(d*x+c))^(5/3)*tan(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)-240/91*a*e^3*
tan(d*x+c)/d/(-(e*sec(d*x+c))^(1/3)+e^(1/3)*(1+3^(1/2)))/(a+a*sec(d*x+c))^
(1/2)-80/91*3^(3/4)*a^2*e^(7/3)*EllipticF((-(e*sec(d*x+c))^(1/3)+e^(1/3)*(
1-3^(1/2)))/(-(e*sec(d*x+c))^(1/3)+e^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(e^
(1/3)-(e*sec(d*x+c))^(1/3))*2^(1/2)*((e^(2/3)+e^(1/3)*(e*sec(d*x+c))^(1/3)
+(e*sec(d*x+c))^(2/3))/(-(e*sec(d*x+c))^(1/3)+e^(1/3)*(1+3^(1/2)))^2)^(1/2
)*tan(d*x+c)/d/(a-a*sec(d*x+c))/(a+a*sec(d*x+c))^(1/2)/(e^(1/3)*(e^(1/3)-(
e*sec(d*x+c))^(1/3))/(-(e*sec(d*x+c))^(1/3)+e^(1/3)*(1+3^(1/2)))^2)^(1/2)+
120/91*3^(1/4)*a^2*e^(7/3)*EllipticE((-(e*sec(d*x+c))^(1/3)+e^(1/3)*(1-3^(
1/2)))/(-(e*sec(d*x+c))^(1/3)+e^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(e^(1/3)
-(e*sec(d*x+c))^(1/3))*(1/2*6^(1/2)-1/2*2^(1/2))*((e^(2/3)+e^(1/3)*(e*sec(
d*x+c))^(1/3)+(e*sec(d*x+c))^(2/3))/(-(e*sec(d*x+c))^(1/3)+e^(1/3)*(1+3^(1
/2)))^2)^(1/2)*tan(d*x+c)/d/(a-a*sec(d*x+c))/(a+a*sec(d*x+c))^(1/2)/(e^(1/
3)*(e^(1/3)-(e*sec(d*x+c))^(1/3))/(-(e*sec(d*x+c))^(1/3)+e^(1/3)*(1+3^(1/2
)))^2)^(1/2)
```

3.275.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.23 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.10

$$\int (e\sec(c+dx))^{8/3}\sqrt{a+a\sec(c+dx)}\,dx = \frac{2\,\text{Hypergeometric2F1}\left(-\frac{5}{3},\frac{1}{2},\frac{3}{2},1-\sec(c+dx)\right)(e\sec(c+dx))^{8/3}\sqrt{a(1-}}{d\sec^{\frac{8}{3}}(c+dx)}$$

input
```
Integrate[(e*Sec[c + d*x])^(8/3)*Sqrt[a + a*Sec[c + d*x]],x]
```

output
```
(2*Hypergeometric2F1[-5/3, 1/2, 3/2, 1 - Sec[c + d*x]]*(e*Sec[c + d*x])^(8
/3)*Sqrt[a*(1 + Sec[c + d*x])]*Tan[(c + d*x)/2])/(d*Sec[c + d*x]^(8/3))
```

3.275.3 Rubi [A] (warning: unable to verify)

Time = 0.65 (sec) , antiderivative size = 708, normalized size of antiderivative = 0.99, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$, Rules used = {3042, 4293, 60, 60, 73, 832, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a\sec(c+dx)+a}(e\sec(c+dx))^{8/3}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \sqrt{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}\left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{8/3}dx$$

$$\Big\downarrow \; 4293$$

$$-\frac{a^2e\tan(c+dx)\int \frac{(e\sec(c+dx))^{5/3}}{\sqrt{a-a\sec(c+dx)}}d\sec(c+dx)}{d\sqrt{a-a\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \; 60$$

$$-\frac{a^2e\tan(c+dx)\left(\frac{10}{13}e\int \frac{(e\sec(c+dx))^{2/3}}{\sqrt{a-a\sec(c+dx)}}d\sec(c+dx)-\frac{6\sqrt{a-a\sec(c+dx)}(e\sec(c+dx))^{5/3}}{13a}\right)}{d\sqrt{a-a\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \; 60$$

$$-\frac{a^2e\tan(c+dx)\left(\frac{10}{13}e\left(\frac{4}{7}e\int \frac{1}{\sqrt[3]{e\sec(c+dx)}\sqrt{a-a\sec(c+dx)}}d\sec(c+dx)-\frac{6\sqrt{a-a\sec(c+dx)}(e\sec(c+dx))^{2/3}}{7a}\right)-\frac{6\sqrt{a-a}}{}\right.}{d\sqrt{a-a\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \; 73$$

$$-\frac{a^2e\tan(c+dx)\left(\frac{10}{13}e\left(\frac{12}{7}\int \frac{\sqrt[3]{e\sec(c+dx)}}{\sqrt{a-a\sec(c+dx)}}d\sqrt[3]{e\sec(c+dx)}-\frac{6\sqrt{a-a\sec(c+dx)}(e\sec(c+dx))^{2/3}}{7a}\right)-\frac{6\sqrt{a-a\sec(c+dx)}(e}{13a}\right.}{d\sqrt{a-a\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \; 832$$

$$-\frac{a^2e\tan(c+dx)\left(\frac{10}{13}e\left(\frac{12}{7}\left(\left(1-\sqrt{3}\right)\sqrt[3]{e}\int \frac{1}{\sqrt{a-a\sec(c+dx)}}d\sqrt[3]{e\sec(c+dx)}-\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}}{\sqrt{a-a\sec(c+dx)}}d\sqrt[3]{e\,\mathrm{s}}\right.\right.}{d\sqrt{a-a\sec(c+dx)}\sqrt{a\sec(c+dx)}}$$

$$\Big\downarrow \; 759$$

$$-\frac{a^2 e\tan(c+dx)\left(\frac{10}{13}e\left(\frac{12}{7}\left(-\int\frac{\left(1-\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}}{\sqrt{a-a\sec(c+dx)}}d\sqrt[3]{e\sec(c+dx)}-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}\sqrt[3]{e}\sqrt{\frac{\sqrt[3]{e}\sqrt[3]{e\sec(c}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{}\right.}}}{}\right.\right.\right.}{}$$

$$\Big\downarrow \; 2416$$

$$-\frac{a^2 e\tan(c+dx)\left(\frac{10}{13}e\left(\frac{12}{7}\left(-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}\sqrt[3]{e}\left(\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)\sqrt{\frac{\sqrt[3]{e}\sqrt[3]{e\sec(c+dx)}+(e\sec(c+dx))^{2/3}+e^{2/3}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)^2}}\,\mathrm{El}}{\sqrt[4]{3}\sqrt{a-a\sec(c+dx)}\sqrt{\frac{\sqrt[3]{e}\left(\sqrt[3]{e}-\sqrt[3]{e\sec(c}\right.}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c}\right.}}}\right.\right.\right.}{}$$

input
```
Int[(e*Sec[c + d*x])^(8/3)*Sqrt[a + a*Sec[c + d*x]],x]
```

output
```
-((a^2*e*((-6*(e*Sec[c + d*x])^(5/3)*Sqrt[a - a*Sec[c + d*x]])/(13*a) + (1
0*e*((-6*(e*Sec[c + d*x])^(2/3)*Sqrt[a - a*Sec[c + d*x]])/(7*a) + (12*((2*
e*Sqrt[a - a*Sec[c + d*x]])/(a*((1 + Sqrt[3])*e^(1/3) - (e*Sec[c + d*x])^(
1/3))) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*e^(1/3)*EllipticE[ArcSin[((1 - Sqrt[3]
)*e^(1/3) - (e*Sec[c + d*x])^(1/3))/((1 + Sqrt[3])*e^(1/3) - (e*Sec[c + d*
x])^(1/3))], -7 - 4*Sqrt[3]]*(e^(1/3) - (e*Sec[c + d*x])^(1/3))*Sqrt[(e^(2
/3) + e^(1/3)*(e*Sec[c + d*x])^(1/3) + (e*Sec[c + d*x])^(2/3))/((1 + Sqrt[
3])*e^(1/3) - (e*Sec[c + d*x])^(1/3))^2])/(Sqrt[a - a*Sec[c + d*x]]*Sqrt[(
e^(1/3)*(e^(1/3) - (e*Sec[c + d*x])^(1/3)))/((1 + Sqrt[3])*e^(1/3) - (e*Se
c[c + d*x])^(1/3))^2]) - (2*(1 - Sqrt[3])*Sqrt[2 + Sqrt[3]]*e^(1/3)*Ellipt
icF[ArcSin[((1 - Sqrt[3])*e^(1/3) - (e*Sec[c + d*x])^(1/3))/((1 + Sqrt[3])
*e^(1/3) - (e*Sec[c + d*x])^(1/3))], -7 - 4*Sqrt[3]]*(e^(1/3) - (e*Sec[c +
 d*x])^(1/3))*Sqrt[(e^(2/3) + e^(1/3)*(e*Sec[c + d*x])^(1/3) + (e*Sec[c +
d*x])^(2/3))/((1 + Sqrt[3])*e^(1/3) - (e*Sec[c + d*x])^(1/3))^2])/(3^(1/4)
*Sqrt[a - a*Sec[c + d*x]]*Sqrt[(e^(1/3)*(e^(1/3) - (e*Sec[c + d*x])^(1/3))
)/((1 + Sqrt[3])*e^(1/3) - (e*Sec[c + d*x])^(1/3))^2])))/7))/13)*Tan[c + d
*x])/(d*Sqrt[a - a*Sec[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]))
```

3.275. $\int (e\sec(c+dx))^{8/3}\sqrt{a+a\sec(c+dx)}\,dx$

3.275.3.1 Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 832
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 - Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && PosQ[a]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

3.275. $\int (e\sec(c+dx))^{8/3}\sqrt{a+a\sec(c+dx)}\,dx$

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4293
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]
*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(d*x)^(n - 1)/Sqrt[a - b*x], x], x,
 Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0]
```

3.275.4 Maple [F]

$$\int (e\sec(dx+c))^{\frac{8}{3}}\sqrt{a+a\sec(dx+c)}dx$$

input
```
int((e*sec(d*x+c))^(8/3)*(a+a*sec(d*x+c))^(1/2),x)
```

output
```
int((e*sec(d*x+c))^(8/3)*(a+a*sec(d*x+c))^(1/2),x)
```

3.275.5 Fricas [F]

$$\int (e\sec(c+dx))^{8/3}\sqrt{a+a\sec(c+dx)}\,dx=\int \sqrt{a\sec(dx+c)+a}(e\sec(dx+c))^{\frac{8}{3}}\,dx$$

input
```
integrate((e*sec(d*x+c))^(8/3)*(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas
")
```

output
```
integral(sqrt(a*sec(d*x + c) + a)*(e*sec(d*x + c))^(2/3)*e^2*sec(d*x + c)^
2, x)
```

3.275.6 Sympy [F(-1)]

Timed out.

$$\int (e\sec(c+dx))^{8/3}\sqrt{a+a\sec(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((e*sec(d*x+c))**(8/3)*(a+a*sec(d*x+c))**(1/2),x)
```

output
```
Timed out
```

3.275.7 Maxima [F]

$$\int (e\sec(c+dx))^{8/3}\sqrt{a+a\sec(c+dx)}\,dx = \int \sqrt{a\sec(dx+c)+a}(e\sec(dx+c))^{\frac{8}{3}}\,dx$$

input
```
integrate((e*sec(d*x+c))^(8/3)*(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima
")
```

output
```
integrate(sqrt(a*sec(d*x + c) + a)*(e*sec(d*x + c))^(8/3), x)
```

3.275.8 Giac [F]

$$\int (e\sec(c+dx))^{8/3}\sqrt{a+a\sec(c+dx)}\,dx = \int \sqrt{a\sec(dx+c)+a}(e\sec(dx+c))^{\frac{8}{3}}\,dx$$

input
```
integrate((e*sec(d*x+c))^(8/3)*(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.275.9 Mupad [F(-1)]

Timed out.

$$\int (e\sec(c+dx))^{8/3}\sqrt{a+a\sec(c+dx)}\,dx=\int \sqrt{a+\frac{a}{\cos(c+d\,x)}}\,\left(\frac{e}{\cos(c+d\,x)}\right)^{8/3}\,dx$$

input
```
int((a + a/cos(c + d*x))^(1/2)*(e/cos(c + d*x))^(8/3),x)
```

output
```
int((a + a/cos(c + d*x))^(1/2)*(e/cos(c + d*x))^(8/3), x)
```

3.276 $\int (e\sec(c+dx))^{5/3}\sqrt{a+a\sec(c+dx)}\,dx$

3.276.1 Optimal result . . . 2175
3.276.2 Mathematica [C] (verified) . . . 2176
3.276.3 Rubi [A] (warning: unable to verify) . . . 2177
3.276.4 Maple [F] . . . 2180
3.276.5 Fricas [F] . . . 2180
3.276.6 Sympy [F(-1)] . . . 2181
3.276.7 Maxima [F] . . . 2181
3.276.8 Giac [F] . . . 2181
3.276.9 Mupad [F(-1)] . . . 2182

3.276.1 Optimal result

Integrand size = 27, antiderivative size = 673

$$\int (e\sec(c+dx))^{5/3}\sqrt{a+a\sec(c+dx)}\,dx = \frac{6ae(e\sec(c+dx))^{2/3}\tan(c+dx)}{7d\sqrt{a+a\sec(c+dx)}} - \frac{24ae^2\tan(c+dx)}{7d\sqrt{a+a\sec(c+dx)}\left((1+\sqrt{3})\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)}$$

$$+\frac{12\sqrt[4]{3}\sqrt{2-\sqrt{3}}a^2e^{4/3}E\left(\arcsin\left(\frac{(1-\sqrt{3})\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}}{(1+\sqrt{3})\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}}\right)|-7-4\sqrt{3}\right)\left(\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)\sqrt{\frac{e^{2/}}{}}}{7d(a-a\sec(c+dx))\sqrt{a+a\sec(c+dx)}\sqrt{\frac{\sqrt[3]{e}\left(\sqrt[3]{e}-\sqrt[3]{e\sec(c+}\right.}{\left((1+\sqrt{3})\sqrt[3]{e}-\sqrt[3]{e\sec(c}\right.}}}$$

$$-\frac{8\sqrt{2}3^{3/4}a^2e^{4/3}\,\text{EllipticF}\left(\arcsin\left(\frac{(1-\sqrt{3})\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}}{(1+\sqrt{3})\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}}\right),-7-4\sqrt{3}\right)\left(\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)\sqrt{}}{7d(a-a\sec(c+dx))\sqrt{a+a\sec(c+dx)}\sqrt{\frac{\sqrt[3]{e}\left(\sqrt[3]{e}-\sqrt[3]{e\sec(c}\right.}{\left((1+\sqrt{3})\sqrt[3]{e}-\sqrt[3]{e\sec(c}\right.}}}$$

output
```
6/7*a*e*(e*sec(d*x+c))^(2/3)*tan(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)-24/7*a*e^
2*tan(d*x+c)/d/(-(e*sec(d*x+c))^(1/3)+e^(1/3)*(1+3^(1/2)))/(a+a*sec(d*x+c)
)^(1/2)-8/7*3^(3/4)*a^2*e^(4/3)*EllipticF((-(e*sec(d*x+c))^(1/3)+e^(1/3)*(
1-3^(1/2)))/(-(e*sec(d*x+c))^(1/3)+e^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(e^
(1/3)-(e*sec(d*x+c))^(1/3))*2^(1/2)*((e^(2/3)+e^(1/3)*(e*sec(d*x+c))^(1/3)
+(e*sec(d*x+c))^(2/3))/(-(e*sec(d*x+c))^(1/3)+e^(1/3)*(1+3^(1/2)))^2)^(1/2
)*tan(d*x+c)/d/(a-a*sec(d*x+c))/(a+a*sec(d*x+c))^(1/2)/(e^(1/3)*(e^(1/3)-(
e*sec(d*x+c))^(1/3))/(-(e*sec(d*x+c))^(1/3)+e^(1/3)*(1+3^(1/2)))^2)^(1/2)+
12/7*3^(1/4)*a^2*e^(4/3)*EllipticE((-(e*sec(d*x+c))^(1/3)+e^(1/3)*(1-3^(1/
2)))/(-(e*sec(d*x+c))^(1/3)+e^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(e^(1/3)-(
e*sec(d*x+c))^(1/3))*(1/2*6^(1/2)-1/2*2^(1/2))*((e^(2/3)+e^(1/3)*(e*sec(d*
x+c))^(1/3)+(e*sec(d*x+c))^(2/3))/(-(e*sec(d*x+c))^(1/3)+e^(1/3)*(1+3^(1/2
)))^2)^(1/2)*tan(d*x+c)/d/(a-a*sec(d*x+c))/(a+a*sec(d*x+c))^(1/2)/(e^(1/3)
*(e^(1/3)-(e*sec(d*x+c))^(1/3))/(-(e*sec(d*x+c))^(1/3)+e^(1/3)*(1+3^(1/2))
)^2)^(1/2)
```

3.276.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.21 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.11

$$\int (e\sec(c+dx))^{5/3}\sqrt{a+a\sec(c+dx)}\,dx = \frac{2\,\text{Hypergeometric2F1}\left(-\frac{2}{3},\frac{1}{2},\frac{3}{2},1-\sec(c+dx)\right)(e\sec(c+dx))^{5/3}\sqrt{a(1-}}{d\sec^{\frac{5}{3}}(c+dx)}$$

input
```
Integrate[(e*Sec[c + d*x])^(5/3)*Sqrt[a + a*Sec[c + d*x]],x]
```

output
```
(2*Hypergeometric2F1[-2/3, 1/2, 3/2, 1 - Sec[c + d*x]]*(e*Sec[c + d*x])^(5
/3)*Sqrt[a*(1 + Sec[c + d*x])]*Tan[(c + d*x)/2])/(d*Sec[c + d*x]^(5/3))
```

3.276.3 Rubi [A] (warning: unable to verify)

Time = 0.61 (sec) , antiderivative size = 668, normalized size of antiderivative = 0.99, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {3042, 4293, 60, 73, 832, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a\sec(c+dx)+a}(e\sec(c+dx))^{5/3}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \sqrt{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}\left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/3}dx$$

$$\Big\downarrow \; 4293$$

$$-\frac{a^2e\tan(c+dx)\int \frac{(e\sec(c+dx))^{2/3}}{\sqrt{a-a\sec(c+dx)}}d\sec(c+dx)}{d\sqrt{a-a\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \; 60$$

$$-\frac{a^2e\tan(c+dx)\left(\frac{4}{7}e\int \frac{1}{\sqrt[3]{e\sec(c+dx)}\sqrt{a-a\sec(c+dx)}}d\sec(c+dx)-\frac{6\sqrt{a-a\sec(c+dx)}(e\sec(c+dx))^{2/3}}{7a}\right)}{d\sqrt{a-a\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \; 73$$

$$-\frac{a^2e\tan(c+dx)\left(\frac{12}{7}\int \frac{\sqrt[3]{e\sec(c+dx)}}{\sqrt{a-a\sec(c+dx)}}d\sqrt[3]{e\sec(c+dx)}-\frac{6\sqrt{a-a\sec(c+dx)}(e\sec(c+dx))^{2/3}}{7a}\right)}{d\sqrt{a-a\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \; 832$$

$$-\frac{a^2e\tan(c+dx)\left(\frac{12}{7}\left(\left(1-\sqrt{3}\right)\sqrt[3]{e}\int \frac{1}{\sqrt{a-a\sec(c+dx)}}d\sqrt[3]{e\sec(c+dx)}-\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}}{\sqrt{a-a\sec(c+dx)}}d\sqrt[3]{e\sec(c-}\right.\right.}{d\sqrt{a-a\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \; 759$$

$$-\frac{a^2 e\tan(c+dx)\left(\frac{12}{7}\left(-\int\frac{\left(1-\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}}{\sqrt{a-a\sec(c+dx)}}d\sqrt[3]{e\sec(c+dx)}-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}\sqrt[3]{e}\sqrt{\frac{\sqrt[3]{e}\sqrt[3]{e\sec(c+dx}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{}\right.}}}{}\right.\right.}{d\sqrt{a-a\,\mathrm{se}}}$$

$$\Big\downarrow \; 2416$$

$$-\frac{a^2 e\tan(c+dx)\left(\frac{12}{7}\left(-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}\sqrt[3]{e}\left(\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)\sqrt{\frac{\sqrt[3]{e}\sqrt[3]{e\sec(c+dx)}+(e\sec(c+dx))^{2/3}+e^{2/3}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)^2}}\,\mathrm{EllipticF}}{\sqrt[4]{3}\sqrt{a-a\sec(c+dx)}\sqrt{\frac{\sqrt[3]{e}\left(\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx}\right.}}}\right.\right.}{}$$

input
```
Int[(e*Sec[c + d*x])^(5/3)*Sqrt[a + a*Sec[c + d*x]],x]
```

output
```
-((a^2*e*((-6*(e*Sec[c + d*x])^(2/3)*Sqrt[a - a*Sec[c + d*x]])/(7*a) + (12
*((2*e*Sqrt[a - a*Sec[c + d*x]])/(a*((1 + Sqrt[3])*e^(1/3) - (e*Sec[c + d*
x])^(1/3))) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*e^(1/3)*EllipticE[ArcSin[((1 - Sq
rt[3])*e^(1/3) - (e*Sec[c + d*x])^(1/3))/((1 + Sqrt[3])*e^(1/3) - (e*Sec[c
 + d*x])^(1/3))], -7 - 4*Sqrt[3]]*(e^(1/3) - (e*Sec[c + d*x])^(1/3))*Sqrt[
(e^(2/3) + e^(1/3)*(e*Sec[c + d*x])^(1/3) + (e*Sec[c + d*x])^(2/3))/((1 +
Sqrt[3])*e^(1/3) - (e*Sec[c + d*x])^(1/3))^2])/(Sqrt[a - a*Sec[c + d*x]]*S
qrt[(e^(1/3)*(e^(1/3) - (e*Sec[c + d*x])^(1/3)))/((1 + Sqrt[3])*e^(1/3) -
(e*Sec[c + d*x])^(1/3))^2]) - (2*(1 - Sqrt[3])*Sqrt[2 + Sqrt[3]]*e^(1/3)*E
llipticF[ArcSin[((1 - Sqrt[3])*e^(1/3) - (e*Sec[c + d*x])^(1/3))/((1 + Sqr
t[3])*e^(1/3) - (e*Sec[c + d*x])^(1/3))], -7 - 4*Sqrt[3]]*(e^(1/3) - (e*Se
c[c + d*x])^(1/3))*Sqrt[(e^(2/3) + e^(1/3)*(e*Sec[c + d*x])^(1/3) + (e*Sec
[c + d*x])^(2/3))/((1 + Sqrt[3])*e^(1/3) - (e*Sec[c + d*x])^(1/3))^2])/(3^
(1/4)*Sqrt[a - a*Sec[c + d*x]]*Sqrt[(e^(1/3)*(e^(1/3) - (e*Sec[c + d*x])^(
1/3)))/((1 + Sqrt[3])*e^(1/3) - (e*Sec[c + d*x])^(1/3))^2])))/7)*Tan[c + d
*x])/(d*Sqrt[a - a*Sec[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]))
```

3.276. $\int (e\sec(c+dx))^{5/3}\sqrt{a+a\sec(c+dx)}\,dx$

3.276.3.1 Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 832
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 - Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && PosQ[a]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

3.276. $\int (e\sec(c+dx))^{5/3}\sqrt{a+a\sec(c+dx)}\,dx$

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4293
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]
*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(d*x)^(n - 1)/Sqrt[a - b*x], x], x,
 Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0]
```

3.276.4 Maple [F]

$$\int (e\sec(dx+c))^{\frac{5}{3}}\sqrt{a+a\sec(dx+c)}dx$$

input
```
int((e*sec(d*x+c))^(5/3)*(a+a*sec(d*x+c))^(1/2),x)
```

output
```
int((e*sec(d*x+c))^(5/3)*(a+a*sec(d*x+c))^(1/2),x)
```

3.276.5 Fricas [F]

$$\int (e\sec(c+dx))^{5/3}\sqrt{a+a\sec(c+dx)}\,dx=\int \sqrt{a\sec(dx+c)+a}(e\sec(dx+c))^{\frac{5}{3}}\,dx$$

input
```
integrate((e*sec(d*x+c))^(5/3)*(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas
")
```

output
```
integral(sqrt(a*sec(d*x + c) + a)*(e*sec(d*x + c))^(2/3)*e*sec(d*x + c), x
)
```

3.276.6 Sympy [F(-1)]

Timed out.

$$\int (e\sec(c+dx))^{5/3}\sqrt{a+a\sec(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((e*sec(d*x+c))**(5/3)*(a+a*sec(d*x+c))**(1/2),x)
```

output
```
Timed out
```

3.276.7 Maxima [F]

$$\int (e\sec(c+dx))^{5/3}\sqrt{a+a\sec(c+dx)}\,dx = \int \sqrt{a\sec(dx+c)+a}(e\sec(dx+c))^{\frac{5}{3}}\,dx$$

input
```
integrate((e*sec(d*x+c))^(5/3)*(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima
")
```

output
```
integrate(sqrt(a*sec(d*x + c) + a)*(e*sec(d*x + c))^(5/3), x)
```

3.276.8 Giac [F]

$$\int (e\sec(c+dx))^{5/3}\sqrt{a+a\sec(c+dx)}\,dx = \int \sqrt{a\sec(dx+c)+a}(e\sec(dx+c))^{\frac{5}{3}}\,dx$$

input
```
integrate((e*sec(d*x+c))^(5/3)*(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.276.9 Mupad [F(-1)]

Timed out.

$$\int (e\sec(c+dx))^{5/3}\sqrt{a+a\sec(c+dx)}\,dx=\int \sqrt{a+\frac{a}{\cos(c+d\,x)}}\,\left(\frac{e}{\cos(c+d\,x)}\right)^{5/3}dx$$

input
```
int((a + a/cos(c + d*x))^(1/2)*(e/cos(c + d*x))^(5/3),x)
```

output
```
int((a + a/cos(c + d*x))^(1/2)*(e/cos(c + d*x))^(5/3), x)
```

3.277 $\int (e\sec(c+dx))^{2/3}\sqrt{a+a\sec(c+dx)}\,dx$

3.277.1 Optimal result . . . 2183
3.277.2 Mathematica [C] (verified) . . . 2184
3.277.3 Rubi [A] (warning: unable to verify) . . . 2184
3.277.4 Maple [F] . . . 2187
3.277.5 Fricas [F] . . . 2187
3.277.6 Sympy [F] . . . 2188
3.277.7 Maxima [F] . . . 2188
3.277.8 Giac [F] . . . 2188
3.277.9 Mupad [F(-1)] . . . 2189

3.277.1 Optimal result

Integrand size = 27, antiderivative size = 624

$$\int (e\sec(c+dx))^{2/3}\sqrt{a+a\sec(c+dx)}\,dx =$$

$$-\frac{6ae\tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}\left(\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)}$$

$$+\frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}a^2\sqrt[3]{e}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}}{\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}}\right)|-7-4\sqrt{3}\right)\left(\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)\sqrt{\frac{e^{2/3}-}{}}}{d(a-a\sec(c+dx))\sqrt{a+a\sec(c+dx)}\sqrt{\frac{\sqrt[3]{e}\left(\sqrt[3]{e}-\sqrt[3]{e\sec(c+}\right.}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+}\right.}}}$$

$$-\frac{2\sqrt{2}3^{3/4}a^2\sqrt[3]{e}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}}{\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}}\right),-7-4\sqrt{3}\right)\left(\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)\sqrt{\frac{e}{}}}{d(a-a\sec(c+dx))\sqrt{a+a\sec(c+dx)}\sqrt{\frac{\sqrt[3]{e}\left(\sqrt[3]{e}-\sqrt[3]{e\sec(c+}\right.}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c}\right.}}}$$

output
```
-6*a*e*tan(d*x+c)/d/(-(e*sec(d*x+c))^(1/3)+e^(1/3)*(1+3^(1/2)))/(a+a*sec(d
*x+c))^(1/2)-2*3^(3/4)*a^2*e^(1/3)*EllipticF((-(e*sec(d*x+c))^(1/3)+e^(1/3
)*(1-3^(1/2)))/(-(e*sec(d*x+c))^(1/3)+e^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*
(e^(1/3)-(e*sec(d*x+c))^(1/3))*2^(1/2)*((e^(2/3)+e^(1/3)*(e*sec(d*x+c))^(1
/3)+(e*sec(d*x+c))^(2/3))/(-(e*sec(d*x+c))^(1/3)+e^(1/3)*(1+3^(1/2)))^2)^(
1/2)*tan(d*x+c)/d/(a-a*sec(d*x+c))/(a+a*sec(d*x+c))^(1/2)/(e^(1/3)*(e^(1/3
)-(e*sec(d*x+c))^(1/3))/(-(e*sec(d*x+c))^(1/3)+e^(1/3)*(1+3^(1/2)))^2)^(1/
2)+3*3^(1/4)*a^2*e^(1/3)*EllipticE((-(e*sec(d*x+c))^(1/3)+e^(1/3)*(1-3^(1/
2)))/(-(e*sec(d*x+c))^(1/3)+e^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(e^(1/3)-(
e*sec(d*x+c))^(1/3))*(1/2*6^(1/2)-1/2*2^(1/2))*((e^(2/3)+e^(1/3)*(e*sec(d*
x+c))^(1/3)+(e*sec(d*x+c))^(2/3))/(-(e*sec(d*x+c))^(1/3)+e^(1/3)*(1+3^(1/2
)))^2)^(1/2)*tan(d*x+c)/d/(a-a*sec(d*x+c))/(a+a*sec(d*x+c))^(1/2)/(e^(1/3)
*(e^(1/3)-(e*sec(d*x+c))^(1/3))/(-(e*sec(d*x+c))^(1/3)+e^(1/3)*(1+3^(1/2))
)^2)^(1/2)
```

3.277.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.15 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.11

$$\int (e\sec(c+dx))^{2/3}\sqrt{a+a\sec(c+dx)}\,dx = \frac{2\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{3}{2},1-\sec(c+dx)\right)(e\sec(c+dx))^{2/3}\sqrt{a(1+}}{d\sec^{\frac{2}{3}}(c+dx)}$$

input
```
Integrate[(e*Sec[c + d*x])^(2/3)*Sqrt[a + a*Sec[c + d*x]],x]
```

output
```
(2*Hypergeometric2F1[1/3, 1/2, 3/2, 1 - Sec[c + d*x]]*(e*Sec[c + d*x])^(2/
3)*Sqrt[a*(1 + Sec[c + d*x])]*Tan[(c + d*x)/2])/(d*Sec[c + d*x]^(2/3))
```

3.277.3 Rubi [A] (warning: unable to verify)

Time = 0.60 (sec) , antiderivative size = 628, normalized size of antiderivative = 1.01, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {3042, 4293, 73, 832, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a\sec(c+dx)+a}(e\sec(c+dx))^{2/3}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \sqrt{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}\left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{2/3}dx$$

$$\Big\downarrow \; 4293$$

$$-\frac{a^2e\tan(c+dx)\int \frac{1}{\sqrt[3]{e\sec(c+dx)}\sqrt{a-a\sec(c+dx)}}d\sec(c+dx)}{d\sqrt{a-a\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \; 73$$

$$-\frac{3a^2\tan(c+dx)\int \frac{\sqrt[3]{e\sec(c+dx)}}{\sqrt{a-a\sec(c+dx)}}d\sqrt[3]{e\sec(c+dx)}}{d\sqrt{a-a\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \; 832$$

$$-\frac{3a^2\tan(c+dx)\left(\left(1-\sqrt{3}\right)\sqrt[3]{e}\int \frac{1}{\sqrt{a-a\sec(c+dx)}}d\sqrt[3]{e\sec(c+dx)}-\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}}{\sqrt{a-a\sec(c+dx)}}d\sqrt[3]{e\sec(c+dx}\right.}{d\sqrt{a-a\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \; 759$$

$$-\frac{3a^2\tan(c+dx)\left(-\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}}{\sqrt{a-a\sec(c+dx)}}d\sqrt[3]{e\sec(c+dx)}-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}\sqrt[3]{e}\sqrt{\frac{\sqrt[3]{e}\sqrt[3]{e\sec(c+dx)}+(e\,s}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec}\right.}}}{}\right.}{d\sqrt{a-a\sec(c+dx)}\sqrt{a\sec(c}}$$

$$\Big\downarrow \; 2416$$

$$-\frac{3a^2\tan(c+dx)\left(-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}\sqrt[3]{e}\left(\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)\sqrt{\frac{\sqrt[3]{e}\sqrt[3]{e\sec(c+dx)}+(e\sec(c+dx))^{2/3}+e^{2/3}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)^2}}\,\text{EllipticF}\left(\text{arcs}\right.}{\sqrt[4]{3}\sqrt{a-a\sec(c+dx)}\sqrt{\frac{\sqrt[3]{e}\left(\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)^2}}}\right.}{}$$

3.277. $\int (e\sec(c+dx))^{2/3}\sqrt{a+a\sec(c+dx)}\,dx$

input
```
Int[(e*Sec[c + d*x])^(2/3)*Sqrt[a + a*Sec[c + d*x]],x]
```

output
```
(-3*a^2*((2*e*Sqrt[a - a*Sec[c + d*x]])/(a*((1 + Sqrt[3])*e^(1/3) - (e*Sec
[c + d*x])^(1/3))) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*e^(1/3)*EllipticE[ArcSin[(
(1 - Sqrt[3])*e^(1/3) - (e*Sec[c + d*x])^(1/3))/((1 + Sqrt[3])*e^(1/3) - (
e*Sec[c + d*x])^(1/3))], -7 - 4*Sqrt[3]]*(e^(1/3) - (e*Sec[c + d*x])^(1/3)
)*Sqrt[(e^(2/3) + e^(1/3)*(e*Sec[c + d*x])^(1/3) + (e*Sec[c + d*x])^(2/3))
/((1 + Sqrt[3])*e^(1/3) - (e*Sec[c + d*x])^(1/3))^2])/(Sqrt[a - a*Sec[c +
d*x]]*Sqrt[(e^(1/3)*(e^(1/3) - (e*Sec[c + d*x])^(1/3)))/((1 + Sqrt[3])*e^(
1/3) - (e*Sec[c + d*x])^(1/3))^2]) - (2*(1 - Sqrt[3])*Sqrt[2 + Sqrt[3]]*e^
(1/3)*EllipticF[ArcSin[((1 - Sqrt[3])*e^(1/3) - (e*Sec[c + d*x])^(1/3))/((
1 + Sqrt[3])*e^(1/3) - (e*Sec[c + d*x])^(1/3))], -7 - 4*Sqrt[3]]*(e^(1/3)
- (e*Sec[c + d*x])^(1/3))*Sqrt[(e^(2/3) + e^(1/3)*(e*Sec[c + d*x])^(1/3) +
 (e*Sec[c + d*x])^(2/3))/((1 + Sqrt[3])*e^(1/3) - (e*Sec[c + d*x])^(1/3))^
2])/(3^(1/4)*Sqrt[a - a*Sec[c + d*x]]*Sqrt[(e^(1/3)*(e^(1/3) - (e*Sec[c +
d*x])^(1/3)))/((1 + Sqrt[3])*e^(1/3) - (e*Sec[c + d*x])^(1/3))^2]))*Tan[c
+ d*x])/(d*Sqrt[a - a*Sec[c + d*x]]*Sqrt[a + a*Sec[c + d*x]])
```

3.277.3.1 Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 832
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 - Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && PosQ[a]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4293
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]
*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(d*x)^(n - 1)/Sqrt[a - b*x], x], x,
 Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0]
```

3.277.4 Maple [F]

$$\int (e\sec(dx+c))^{\frac{2}{3}}\sqrt{a+a\sec(dx+c)}dx$$

input
```
int((e*sec(d*x+c))^(2/3)*(a+a*sec(d*x+c))^(1/2),x)
```

output
```
int((e*sec(d*x+c))^(2/3)*(a+a*sec(d*x+c))^(1/2),x)
```

3.277.5 Fricas [F]

$$\int (e\sec(c+dx))^{2/3}\sqrt{a+a\sec(c+dx)}\,dx=\int \sqrt{a\sec(dx+c)+a}(e\sec(dx+c))^{\frac{2}{3}}\,dx$$

input
```
integrate((e*sec(d*x+c))^(2/3)*(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas
")
```

output
```
integral(sqrt(a*sec(d*x + c) + a)*(e*sec(d*x + c))^(2/3), x)
```

3.277.6 Sympy [F]

$$\int (e\sec(c+dx))^{2/3}\sqrt{a+a\sec(c+dx)}\,dx=\int \sqrt{a\left(\sec\left(c+dx\right)+1\right)}\left(e\sec\left(c+dx\right)\right)^{\frac{2}{3}}\,dx$$

input
```
integrate((e*sec(d*x+c))**(2/3)*(a+a*sec(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(a*(sec(c + d*x) + 1))*(e*sec(c + d*x))**(2/3), x)
```

3.277.7 Maxima [F]

$$\int (e\sec(c+dx))^{2/3}\sqrt{a+a\sec(c+dx)}\,dx=\int \sqrt{a\sec\left(dx+c\right)+a}\left(e\sec\left(dx+c\right)\right)^{\frac{2}{3}}\,dx$$

input
```
integrate((e*sec(d*x+c))^(2/3)*(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima
")
```

output
```
integrate(sqrt(a*sec(d*x + c) + a)*(e*sec(d*x + c))^(2/3), x)
```

3.277.8 Giac [F]

$$\int (e\sec(c+dx))^{2/3}\sqrt{a+a\sec(c+dx)}\,dx=\int \sqrt{a\sec\left(dx+c\right)+a}\left(e\sec\left(dx+c\right)\right)^{\frac{2}{3}}\,dx$$

input
```
integrate((e*sec(d*x+c))^(2/3)*(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.277.9 Mupad [F(-1)]

Timed out.

$$\int (e\sec(c+dx))^{2/3}\sqrt{a+a\sec(c+dx)}\,dx=\int \sqrt{a+\frac{a}{\cos(c+d\,x)}}\,\left(\frac{e}{\cos(c+d\,x)}\right)^{2/3}dx$$

input
```
int((a + a/cos(c + d*x))^(1/2)*(e/cos(c + d*x))^(2/3),x)
```

output
```
int((a + a/cos(c + d*x))^(1/2)*(e/cos(c + d*x))^(2/3), x)
```

3.278 $\int \frac{\sqrt{a+a\sec(c+dx)}}{\sqrt[3]{e\sec(c+dx)}}\,dx$

3.278.1 Optimal result 2190
3.278.2 Mathematica [C] (verified) 2191
3.278.3 Rubi [A] (warning: unable to verify) 2192
3.278.4 Maple [F] 2195
3.278.5 Fricas [F] 2196
3.278.6 Sympy [F] 2196
3.278.7 Maxima [F] 2196
3.278.8 Giac [F] 2197
3.278.9 Mupad [F(-1)] 2197

3.278.1 Optimal result

Integrand size = 27, antiderivative size = 662

$$\int \frac{\sqrt{a+a\sec(c+dx)}}{\sqrt[3]{e\sec(c+dx)}}\,dx = \frac{3a\tan(c+dx)}{d\sqrt[3]{e\sec(c+dx)}\sqrt{a+a\sec(c+dx)}} + \frac{3a\tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}\left(\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)}$$

$$-\frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}a^2E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}}{\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}}\right)|-7-4\sqrt{3}\right)\left(\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)\sqrt{\frac{e^{2/3}\cdots}{\cdots}}}{2de^{2/3}(a-a\sec(c+dx))\sqrt{a+a\sec(c+dx)}\sqrt{\frac{\sqrt[3]{e}\left(\sqrt[3]{e}-\sqrt[3]{e\sec(\cdots}\right.}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(\cdots}\right.}}}$$

$$+\frac{\sqrt{2}3^{3/4}a^2\,\mathrm{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}}{\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}}\right),-7-4\sqrt{3}\right)\left(\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)\sqrt{\frac{e^{2/3}\cdots}{\cdots}}}{de^{2/3}(a-a\sec(c+dx))\sqrt{a+a\sec(c+dx)}\sqrt{\frac{\sqrt[3]{e}\left(\sqrt[3]{e}-\sqrt[3]{e\sec(\cdots}\right.}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(\cdots}\right.}}}$$

output
```
3*a*tan(d*x+c)/d/(e*sec(d*x+c))^(1/3)/(a+a*sec(d*x+c))^(1/2)+3*a*tan(d*x+c
)/d/(-(e*sec(d*x+c))^(1/3)+e^(1/3)*(1+3^(1/2)))/(a+a*sec(d*x+c))^(1/2)+3^(
3/4)*a^2*EllipticF((-(e*sec(d*x+c))^(1/3)+e^(1/3)*(1-3^(1/2)))/(-(e*sec(d*
x+c))^(1/3)+e^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(e^(1/3)-(e*sec(d*x+c))^(1
/3))*2^(1/2)*((e^(2/3)+e^(1/3)*(e*sec(d*x+c))^(1/3)+(e*sec(d*x+c))^(2/3))/
(-(e*sec(d*x+c))^(1/3)+e^(1/3)*(1+3^(1/2)))^2)^(1/2)*tan(d*x+c)/d/e^(2/3)/
(a-a*sec(d*x+c))/(a+a*sec(d*x+c))^(1/2)/(e^(1/3)*(e^(1/3)-(e*sec(d*x+c))^(
1/3))/(-(e*sec(d*x+c))^(1/3)+e^(1/3)*(1+3^(1/2)))^2)^(1/2)-3/2*3^(1/4)*a^2
*EllipticE((-(e*sec(d*x+c))^(1/3)+e^(1/3)*(1-3^(1/2)))/(-(e*sec(d*x+c))^(1
/3)+e^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(e^(1/3)-(e*sec(d*x+c))^(1/3))*(1/
2*6^(1/2)-1/2*2^(1/2))*((e^(2/3)+e^(1/3)*(e*sec(d*x+c))^(1/3)+(e*sec(d*x+c
))^(2/3))/(-(e*sec(d*x+c))^(1/3)+e^(1/3)*(1+3^(1/2)))^2)^(1/2)*tan(d*x+c)/
d/e^(2/3)/(a-a*sec(d*x+c))/(a+a*sec(d*x+c))^(1/2)/(e^(1/3)*(e^(1/3)-(e*sec
(d*x+c))^(1/3))/(-(e*sec(d*x+c))^(1/3)+e^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

3.278.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.16 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.11

$$\int \frac{\sqrt{a+a\sec(c+dx)}}{\sqrt[3]{e\sec(c+dx)}}\,dx = \frac{2\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{4}{3},\frac{3}{2},1-\sec(c+dx)\right)\sqrt[3]{\sec(c+dx)}\sqrt{a(1+\sec(c+dx))}\tan\left(\frac{1}{2}(c+dx)\right)}{d\sqrt[3]{e\sec(c+dx)}}$$

input
```
Integrate[Sqrt[a + a*Sec[c + d*x]]/(e*Sec[c + d*x])^(1/3),x]
```

output
```
(2*Hypergeometric2F1[1/2, 4/3, 3/2, 1 - Sec[c + d*x]]*Sec[c + d*x]^(1/3)*S
qrt[a*(1 + Sec[c + d*x])]*Tan[(c + d*x)/2])/(d*(e*Sec[c + d*x])^(1/3))
```

3.278.3 Rubi [A] (warning: unable to verify)

Time = 0.63 (sec) , antiderivative size = 672, normalized size of antiderivative = 1.02, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {3042, 4293, 61, 73, 832, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a\sec(c+dx)+a}}{\sqrt[3]{e\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}}{\sqrt[3]{e\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4293}$$

$$-\frac{a^2e\tan(c+dx)\int \frac{1}{(e\sec(c+dx))^{4/3}\sqrt{a-a\sec(c+dx)}}d\sec(c+dx)}{d\sqrt{a-a\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{61}$$

$$-\frac{a^2e\tan(c+dx)\left(-\frac{\int \frac{1}{\sqrt[3]{e\sec(c+dx)}\sqrt{a-a\sec(c+dx)}}d\sec(c+dx)}{2e}-\frac{3\sqrt{a-a\sec(c+dx)}}{ae\sqrt[3]{e\sec(c+dx)}}\right)}{d\sqrt{a-a\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{73}$$

$$-\frac{a^2e\tan(c+dx)\left(-\frac{3\int \frac{\sqrt[3]{e\sec(c+dx)}}{\sqrt{a-a\sec(c+dx)}}d\sqrt[3]{e\sec(c+dx)}}{2e^2}-\frac{3\sqrt{a-a\sec(c+dx)}}{ae\sqrt[3]{e\sec(c+dx)}}\right)}{d\sqrt{a-a\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{832}$$

$$-\frac{a^2e\tan(c+dx)\left(-\frac{3\left(\left(1-\sqrt{3}\right)\sqrt[3]{e}\int \frac{1}{\sqrt{a-a\sec(c+dx)}}d\sqrt[3]{e\sec(c+dx)}-\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}}{\sqrt{a-a\sec(c+dx)}}d\sqrt[3]{e\sec(c+dx)}\right)}{2e^2}\right)}{d\sqrt{a-a\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{759}$$

$$-\frac{a^2 e\tan(c+dx)\left(-\frac{3\left(-\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}}{\sqrt{a-a\sec(c+dx)}}d\sqrt[3]{e\sec(c+dx)}-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}\sqrt[3]{e}\sqrt{\frac{\sqrt[3]{e}\sqrt[3]{e\sec(c+dx)}+(e\sec}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(}\right.}}}{\sqrt[4]{}}\right.}{2e^2}\right.}{d\sqrt{a-a\sec(c+dx)}\sqrt{}}$$

$\downarrow$ 2416

$$-\frac{a^2 e\tan(c+dx)\left(-\frac{3\left(-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}\sqrt[3]{e}\left(\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)\sqrt{\frac{\sqrt[3]{e}\sqrt[3]{e\sec(c+dx)}+(e\sec(c+dx))^{2/3}+e^{2/3}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)^2}}\,\text{EllipticF}\left(\arcsin\right.}{\sqrt[4]{3}\sqrt{a-a\sec(c+dx)}\sqrt{\frac{\sqrt[3]{e}\left(\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)^2}}}}\right.}{}$$

input
```
Int[Sqrt[a + a*Sec[c + d*x]]/(e*Sec[c + d*x])^(1/3),x]
```

3.278. $\int \frac{\sqrt{a+a\sec(c+dx)}}{\sqrt[3]{e\sec(c+dx)}}\,dx$

output
```
-((a^2*e*((-3*Sqrt[a - a*Sec[c + d*x]])/(a*e*(e*Sec[c + d*x])^(1/3)) - (3*
((2*e*Sqrt[a - a*Sec[c + d*x]])/(a*((1 + Sqrt[3])*e^(1/3) - (e*Sec[c + d*x
])^(1/3))) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*e^(1/3)*EllipticE[ArcSin[((1 - Sqr
t[3])*e^(1/3) - (e*Sec[c + d*x])^(1/3))/((1 + Sqrt[3])*e^(1/3) - (e*Sec[c
+ d*x])^(1/3))], -7 - 4*Sqrt[3]]*(e^(1/3) - (e*Sec[c + d*x])^(1/3))*Sqrt[(
e^(2/3) + e^(1/3)*(e*Sec[c + d*x])^(1/3) + (e*Sec[c + d*x])^(2/3))/((1 + S
qrt[3])*e^(1/3) - (e*Sec[c + d*x])^(1/3))^2])/(Sqrt[a - a*Sec[c + d*x]]*Sq
rt[(e^(1/3)*(e^(1/3) - (e*Sec[c + d*x])^(1/3)))/((1 + Sqrt[3])*e^(1/3) - (
e*Sec[c + d*x])^(1/3))^2]) - (2*(1 - Sqrt[3])*Sqrt[2 + Sqrt[3]]*e^(1/3)*El
lipticF[ArcSin[((1 - Sqrt[3])*e^(1/3) - (e*Sec[c + d*x])^(1/3))/((1 + Sqrt
[3])*e^(1/3) - (e*Sec[c + d*x])^(1/3))], -7 - 4*Sqrt[3]]*(e^(1/3) - (e*Sec
[c + d*x])^(1/3))*Sqrt[(e^(2/3) + e^(1/3)*(e*Sec[c + d*x])^(1/3) + (e*Sec[
c + d*x])^(2/3))/((1 + Sqrt[3])*e^(1/3) - (e*Sec[c + d*x])^(1/3))^2])/(3^(
1/4)*Sqrt[a - a*Sec[c + d*x]]*Sqrt[(e^(1/3)*(e^(1/3) - (e*Sec[c + d*x])^(1
/3)))/((1 + Sqrt[3])*e^(1/3) - (e*Sec[c + d*x])^(1/3))^2])))/(2*e^2))*Tan[
c + d*x])/(d*Sqrt[a - a*Sec[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]))
```

3.278.3.1 Defintions of rubi rules used

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

3.278. $\int \frac{\sqrt{a+a\sec(c+dx)}}{\sqrt[3]{e\sec(c+dx)}}\,dx$

rule 832
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 - Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && PosQ[a]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4293
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]
*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(d*x)^(n - 1)/Sqrt[a - b*x], x], x,
 Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0]
```

3.278.4 Maple [F]

$$\int \frac{\sqrt{a+a\sec\left(dx+c\right)}}{\left(e\sec\left(dx+c\right)\right)^{\frac{1}{3}}}dx$$

input
```
int((a+a*sec(d*x+c))^(1/2)/(e*sec(d*x+c))^(1/3),x)
```

output
```
int((a+a*sec(d*x+c))^(1/2)/(e*sec(d*x+c))^(1/3),x)
```

3.278.5 Fricas [F]

$$\int \frac{\sqrt{a+a\sec(c+dx)}}{\sqrt[3]{e\sec(c+dx)}}\,dx=\int \frac{\sqrt{a\sec(dx+c)+a}}{(e\sec(dx+c))^{\frac{1}{3}}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))^(1/2)/(e*sec(d*x+c))^(1/3),x, algorithm="fricas
")
```

output
```
integral(sqrt(a*sec(d*x + c) + a)*(e*sec(d*x + c))^(2/3)/(e*sec(d*x + c)),
 x)
```

3.278.6 Sympy [F]

$$\int \frac{\sqrt{a+a\sec(c+dx)}}{\sqrt[3]{e\sec(c+dx)}}\,dx=\int \frac{\sqrt{a(\sec(c+dx)+1)}}{\sqrt[3]{e\sec(c+dx)}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))**(1/2)/(e*sec(d*x+c))**(1/3),x)
```

output
```
Integral(sqrt(a*(sec(c + d*x) + 1))/(e*sec(c + d*x))**(1/3), x)
```

3.278.7 Maxima [F]

$$\int \frac{\sqrt{a+a\sec(c+dx)}}{\sqrt[3]{e\sec(c+dx)}}\,dx=\int \frac{\sqrt{a\sec(dx+c)+a}}{(e\sec(dx+c))^{\frac{1}{3}}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))^(1/2)/(e*sec(d*x+c))^(1/3),x, algorithm="maxima
")
```

output
```
integrate(sqrt(a*sec(d*x + c) + a)/(e*sec(d*x + c))^(1/3), x)
```

3.278.8 Giac [F]

$$\int \frac{\sqrt{a+a\sec(c+dx)}}{\sqrt[3]{e\sec(c+dx)}}\,dx=\int \frac{\sqrt{a\sec(dx+c)+a}}{(e\sec(dx+c))^{\frac{1}{3}}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))^(1/2)/(e*sec(d*x+c))^(1/3),x, algorithm="giac")
```

output
```
sage0*x
```

3.278.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+a\sec(c+dx)}}{\sqrt[3]{e\sec(c+dx)}}\,dx=\int \frac{\sqrt{a+\frac{a}{\cos(c+d\,x)}}}{\left(\frac{e}{\cos(c+d\,x)}\right)^{1/3}}\,dx$$

input
```
int((a + a/cos(c + d*x))^(1/2)/(e/cos(c + d*x))^(1/3),x)
```

output
```
int((a + a/cos(c + d*x))^(1/2)/(e/cos(c + d*x))^(1/3), x)
```

3.279 $\int \frac{\sqrt{a+a\sec(c+dx)}}{(e\sec(c+dx))^{4/3}}\,dx$

3.279.1 Optimal result 2198
3.279.2 Mathematica [C] (verified) 2199
3.279.3 Rubi [A] (warning: unable to verify) 2199
3.279.4 Maple [F] 2204
3.279.5 Fricas [F] 2205
3.279.6 Sympy [F] 2205
3.279.7 Maxima [F] 2205
3.279.8 Giac [F] 2206
3.279.9 Mupad [F(-1)] 2206

3.279.1 Optimal result

Integrand size = 27, antiderivative size = 715

$$\int \frac{\sqrt{a+a\sec(c+dx)}}{(e\sec(c+dx))^{4/3}}\,dx = \frac{3a\tan(c+dx)}{4d(e\sec(c+dx))^{4/3}\sqrt{a+a\sec(c+dx)}} + \frac{15a\tan(c+dx)}{8de\sqrt[3]{e\sec(c+dx)}\sqrt{a+a\sec(c+dx)}} + \frac{15a\tan(c+dx)}{8de\sqrt{a+a\sec(c+dx)}\left(\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)}$$

$$-\frac{15\sqrt[4]{3}\sqrt{2-\sqrt{3}}a^2E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}}{\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}}\right)|-7-4\sqrt{3}\right)\left(\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)\sqrt{\frac{e^{2/3}+\sqrt[3]{\cdots}}{\left(\cdots\right.}}}{16de^{5/3}(a-a\sec(c+dx))\sqrt{a+a\sec(c+dx)}\sqrt{\frac{\sqrt[3]{e}\left(\sqrt[3]{e}-\sqrt[3]{e\sec(c\cdots}\right.}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c\cdots}\right.}}}$$

$$+\frac{5\,3^{3/4}a^2\,\mathrm{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}}{\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}}\right),-7-4\sqrt{3}\right)\left(\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)\sqrt{\frac{e^{2/3}+\sqrt[3]{\cdots}}{\left(\left(1\cdots\right.}}}{4\sqrt{2}de^{5/3}(a-a\sec(c+dx))\sqrt{a+a\sec(c+dx)}\sqrt{\frac{\sqrt[3]{e}\left(\sqrt[3]{e}-\sqrt[3]{e\sec(c\cdots}\right.}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c\cdots}\right.}}}$$

output
```
3/4*a*tan(d*x+c)/d/(e*sec(d*x+c))^(4/3)/(a+a*sec(d*x+c))^(1/2)+15/8*a*tan(
d*x+c)/d/e/(e*sec(d*x+c))^(1/3)/(a+a*sec(d*x+c))^(1/2)+15/8*a*tan(d*x+c)/d
/e/(-(e*sec(d*x+c))^(1/3)+e^(1/3)*(1+3^(1/2)))/(a+a*sec(d*x+c))^(1/2)+5/8*
3^(3/4)*a^2*EllipticF((-(e*sec(d*x+c))^(1/3)+e^(1/3)*(1-3^(1/2)))/(-(e*sec
(d*x+c))^(1/3)+e^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(e^(1/3)-(e*sec(d*x+c))
^(1/3))*((e^(2/3)+e^(1/3)*(e*sec(d*x+c))^(1/3)+(e*sec(d*x+c))^(2/3))/(-(e*
sec(d*x+c))^(1/3)+e^(1/3)*(1+3^(1/2)))^2)^(1/2)*tan(d*x+c)/d/e^(5/3)/(a-a*
sec(d*x+c))*2^(1/2)/(a+a*sec(d*x+c))^(1/2)/(e^(1/3)*(e^(1/3)-(e*sec(d*x+c)
)^(1/3))/(-(e*sec(d*x+c))^(1/3)+e^(1/3)*(1+3^(1/2)))^2)^(1/2)-15/16*3^(1/4
)*a^2*EllipticE((-(e*sec(d*x+c))^(1/3)+e^(1/3)*(1-3^(1/2)))/(-(e*sec(d*x+c
))^(1/3)+e^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(e^(1/3)-(e*sec(d*x+c))^(1/3)
)*(1/2*6^(1/2)-1/2*2^(1/2))*((e^(2/3)+e^(1/3)*(e*sec(d*x+c))^(1/3)+(e*sec(
d*x+c))^(2/3))/(-(e*sec(d*x+c))^(1/3)+e^(1/3)*(1+3^(1/2)))^2)^(1/2)*tan(d*
x+c)/d/e^(5/3)/(a-a*sec(d*x+c))/(a+a*sec(d*x+c))^(1/2)/(e^(1/3)*(e^(1/3)-(
e*sec(d*x+c))^(1/3))/(-(e*sec(d*x+c))^(1/3)+e^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

3.279.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.19 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.10

$$\int \frac{\sqrt{a+a\sec(c+dx)}}{(e\sec(c+dx))^{4/3}}\,dx = \frac{2\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{7}{3},\frac{3}{2},1-\sec(c+dx)\right)\sec^{\frac{4}{3}}(c+dx)\sqrt{a(1+\sec(c+dx}}{d(e\sec(c+dx))^{4/3}}$$

input
```
Integrate[Sqrt[a + a*Sec[c + d*x]]/(e*Sec[c + d*x])^(4/3),x]
```

output
```
(2*Hypergeometric2F1[1/2, 7/3, 3/2, 1 - Sec[c + d*x]]*Sec[c + d*x]^(4/3)*S
qrt[a*(1 + Sec[c + d*x])]*Tan[(c + d*x)/2])/(d*(e*Sec[c + d*x])^(4/3))
```

3.279.3 Rubi [A] (warning: unable to verify)

Time = 0.64 (sec) , antiderivative size = 717, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$, Rules used = {3042, 4293, 61, 61, 73, 832, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a\sec(c+dx)+a}}{(e\sec(c+dx))^{4/3}}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\sqrt{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}}{\left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{4/3}}dx$$

$$\Big\downarrow \; 4293$$

$$-\frac{a^2e\tan(c+dx)\int \frac{1}{(e\sec(c+dx))^{7/3}\sqrt{a-a\sec(c+dx)}}d\sec(c+dx)}{d\sqrt{a-a\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \; 61$$

$$-\frac{a^2e\tan(c+dx)\left(\frac{5\int \frac{1}{(e\sec(c+dx))^{4/3}\sqrt{a-a\sec(c+dx)}}d\sec(c+dx)}{8e}-\frac{3\sqrt{a-a\sec(c+dx)}}{4ae(e\sec(c+dx))^{4/3}}\right)}{d\sqrt{a-a\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \; 61$$

$$-\frac{a^2e\tan(c+dx)\left(\frac{5\left(-\frac{\int \frac{1}{\sqrt[3]{e\sec(c+dx)}\sqrt{a-a\sec(c+dx)}}d\sec(c+dx)}{2e}-\frac{3\sqrt{a-a\sec(c+dx)}}{ae\sqrt[3]{e\sec(c+dx)}}\right)}{8e}-\frac{3\sqrt{a-a\sec(c+dx)}}{4ae(e\sec(c+dx))^{4/3}}\right)}{d\sqrt{a-a\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \; 73$$

$$-\frac{a^2e\tan(c+dx)\left(\frac{5\left(-\frac{3\int \frac{\sqrt[3]{e\sec(c+dx)}}{\sqrt{a-a\sec(c+dx)}}d\sqrt[3]{e\sec(c+dx)}}{2e^2}-\frac{3\sqrt{a-a\sec(c+dx)}}{ae\sqrt[3]{e\sec(c+dx)}}\right)}{8e}-\frac{3\sqrt{a-a\sec(c+dx)}}{4ae(e\sec(c+dx))^{4/3}}\right)}{d\sqrt{a-a\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \; 832$$

3.279. $\int \frac{\sqrt{a+a\sec(c+dx)}}{(e\sec(c+dx))^{4/3}}\,dx$

$$-\frac{a^2 e\tan(c+dx)\left(\frac{5\left(-\frac{3\left(\left(1-\sqrt{3}\right)\sqrt[3]{e}\int\frac{1}{\sqrt{a-a\sec(c+dx)}}d\sqrt[3]{e\sec(c+dx)}-\int\frac{\left(1-\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}}{\sqrt{a-a\sec(c+dx)}}d\sqrt[3]{e\sec(c+dx)}\right)}{2e^2}-\right.}{8e}\right.}{d\sqrt{a-a\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}$$

$\downarrow$ 759

$$-\frac{a^2 e\tan(c+dx)\left(\frac{5\left(-\frac{3\left(-\int\frac{\left(1-\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}}{\sqrt{a-a\sec(c+dx)}}d\sqrt[3]{e\sec(c+dx)}-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}\sqrt[3]{e}\sqrt{\frac{\sqrt[3]{e}\sqrt[3]{e\sec(c+dx)}+(e\,\mathrm{se}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\,\mathrm{sec}}\right.}}}{}\right.}{2e^2}\right.}{d\sqrt{a-a\,\mathrm{se}}}$$

$\downarrow$ 2416

3.279. $\int\frac{\sqrt{a+a\sec(c+dx)}}{(e\sec(c+dx))^{4/3}}\,dx$

$$-\frac{a^2 e \tan(c+dx)\left(5\left(-\frac{3\left(-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}\sqrt[3]{e}\left(\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)\sqrt{\frac{\sqrt[3]{e}\sqrt[3]{e\sec(c+dx)}+(e\sec(c+dx))^{2/3}+e^{2/3}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)^2}}\,\text{EllipticF}\left(\text{ar}\right.}{\sqrt[4]{3}\sqrt{a-a\sec(c+dx)}\sqrt{\frac{\sqrt[3]{e}\left(\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)^2}}}\right.\right.\right.}{}$$

input
```
Int[Sqrt[a + a*Sec[c + d*x]]/(e*Sec[c + d*x])^(4/3),x]
```

3.279. $\int \frac{\sqrt{a+a\sec(c+dx)}}{(e\sec(c+dx))^{4/3}}\,dx$

output
```
-((a^2*e*((-3*Sqrt[a - a*Sec[c + d*x]])/(4*a*e*(e*Sec[c + d*x])^(4/3)) + (
5*((-3*Sqrt[a - a*Sec[c + d*x]])/(a*e*(e*Sec[c + d*x])^(1/3)) - (3*((2*e*S
qrt[a - a*Sec[c + d*x]])/(a*((1 + Sqrt[3])*e^(1/3) - (e*Sec[c + d*x])^(1/3
))) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*e^(1/3)*EllipticE[ArcSin[((1 - Sqrt[3])*e
^(1/3) - (e*Sec[c + d*x])^(1/3))/((1 + Sqrt[3])*e^(1/3) - (e*Sec[c + d*x])
^(1/3))], -7 - 4*Sqrt[3]]*(e^(1/3) - (e*Sec[c + d*x])^(1/3))*Sqrt[(e^(2/3)
 + e^(1/3)*(e*Sec[c + d*x])^(1/3) + (e*Sec[c + d*x])^(2/3))/((1 + Sqrt[3])
*e^(1/3) - (e*Sec[c + d*x])^(1/3))^2])/(Sqrt[a - a*Sec[c + d*x]]*Sqrt[(e^(
1/3)*(e^(1/3) - (e*Sec[c + d*x])^(1/3)))/((1 + Sqrt[3])*e^(1/3) - (e*Sec[c
 + d*x])^(1/3))^2]) - (2*(1 - Sqrt[3])*Sqrt[2 + Sqrt[3]]*e^(1/3)*EllipticF
[ArcSin[((1 - Sqrt[3])*e^(1/3) - (e*Sec[c + d*x])^(1/3))/((1 + Sqrt[3])*e^
(1/3) - (e*Sec[c + d*x])^(1/3))], -7 - 4*Sqrt[3]]*(e^(1/3) - (e*Sec[c + d*
x])^(1/3))*Sqrt[(e^(2/3) + e^(1/3)*(e*Sec[c + d*x])^(1/3) + (e*Sec[c + d*x
])^(2/3))/((1 + Sqrt[3])*e^(1/3) - (e*Sec[c + d*x])^(1/3))^2])/(3^(1/4)*Sq
rt[a - a*Sec[c + d*x]]*Sqrt[(e^(1/3)*(e^(1/3) - (e*Sec[c + d*x])^(1/3)))/(
(1 + Sqrt[3])*e^(1/3) - (e*Sec[c + d*x])^(1/3))^2])))/(2*e^2)))/(8*e))*Tan
[c + d*x])/(d*Sqrt[a - a*Sec[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]))
```

3.279.3.1 Defintions of rubi rules used

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 832
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 - Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && PosQ[a]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4293
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]
*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(d*x)^(n - 1)/Sqrt[a - b*x], x], x,
 Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0]
```

3.279.4 Maple [F]

$$\int \frac{\sqrt{a+a\sec\left(dx+c\right)}}{\left(e\sec\left(dx+c\right)\right)^{\frac{4}{3}}}dx$$

input
```
int((a+a*sec(d*x+c))^(1/2)/(e*sec(d*x+c))^(4/3),x)
```

output
```
int((a+a*sec(d*x+c))^(1/2)/(e*sec(d*x+c))^(4/3),x)
```

3.279.5 Fricas [F]

$$\int \frac{\sqrt{a+a\sec(c+dx)}}{(e\sec(c+dx))^{4/3}}\,dx=\int \frac{\sqrt{a\sec(dx+c)+a}}{(e\sec(dx+c))^{\frac{4}{3}}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))^(1/2)/(e*sec(d*x+c))^(4/3),x, algorithm="fricas
")
```

output
```
integral(sqrt(a*sec(d*x + c) + a)*(e*sec(d*x + c))^(2/3)/(e^2*sec(d*x + c)
^2), x)
```

3.279.6 Sympy [F]

$$\int \frac{\sqrt{a+a\sec(c+dx)}}{(e\sec(c+dx))^{4/3}}\,dx=\int \frac{\sqrt{a(\sec(c+dx)+1)}}{(e\sec(c+dx))^{\frac{4}{3}}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))**(1/2)/(e*sec(d*x+c))**(4/3),x)
```

output
```
Integral(sqrt(a*(sec(c + d*x) + 1))/(e*sec(c + d*x))**(4/3), x)
```

3.279.7 Maxima [F]

$$\int \frac{\sqrt{a+a\sec(c+dx)}}{(e\sec(c+dx))^{4/3}}\,dx=\int \frac{\sqrt{a\sec(dx+c)+a}}{(e\sec(dx+c))^{\frac{4}{3}}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))^(1/2)/(e*sec(d*x+c))^(4/3),x, algorithm="maxima
")
```

output
```
integrate(sqrt(a*sec(d*x + c) + a)/(e*sec(d*x + c))^(4/3), x)
```

3.279.8 Giac [F]

$$\int \frac{\sqrt{a+a\sec(c+dx)}}{(e\sec(c+dx))^{4/3}}\,dx=\int \frac{\sqrt{a\sec(dx+c)+a}}{(e\sec(dx+c))^{\frac{4}{3}}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))^(1/2)/(e*sec(d*x+c))^(4/3),x, algorithm="giac")
```

output
```
sage0*x
```

3.279.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+a\sec(c+dx)}}{(e\sec(c+dx))^{4/3}}\,dx=\int \frac{\sqrt{a+\frac{a}{\cos(c+d\,x)}}}{\left(\frac{e}{\cos(c+d\,x)}\right)^{4/3}}\,dx$$

input
```
int((a + a/cos(c + d*x))^(1/2)/(e/cos(c + d*x))^(4/3),x)
```

output
```
int((a + a/cos(c + d*x))^(1/2)/(e/cos(c + d*x))^(4/3), x)
```

3.280 $\int \frac{(e\sec(c+dx))^{2/3}}{\sqrt{a+a\sec(c+dx)}}\,dx$

3.280.1 Optimal result . . . 2207
3.280.2 Mathematica [B] (warning: unable to verify) . . . 2207
3.280.3 Rubi [A] (verified) . . . 2208
3.280.4 Maple [F] . . . 2211
3.280.5 Fricas [F(-1)] . . . 2211
3.280.6 Sympy [F] . . . 2211
3.280.7 Maxima [F] . . . 2212
3.280.8 Giac [F] . . . 2212
3.280.9 Mupad [F(-1)] . . . 2212

3.280.1 Optimal result

Integrand size = 27, antiderivative size = 78

$$\int \frac{(e\sec(c+dx))^{2/3}}{\sqrt{a+a\sec(c+dx)}}\,dx = -\frac{3\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},\sec(c+dx),-\sec(c+dx)\right)(e\sec(c+dx))^{2/3}\tan(c+dx)}{2d\sqrt{1-\sec(c+dx)}\sqrt{a+a\sec(c+dx)}}$$

output
```
-3/2*AppellF1(2/3,1,1/2,5/3,-sec(d*x+c),sec(d*x+c))*(e*sec(d*x+c))^(2/3)*t
an(d*x+c)/d/(1-sec(d*x+c))^(1/2)/(a+a*sec(d*x+c))^(1/2)
```

3.280.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 760 vs. 2(78) = 156.

Time = 7.12 (sec) , antiderivative size = 760, normalized size of antiderivative = 9.74

$$\int \frac{(e\sec(c+dx))^{2/3}}{\sqrt{a+a\sec(c+dx)}}\,dx = \frac{}{ad\left(270\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{6},\frac{1}{3},\frac{3}{2},\tan^2\left(\frac{1}{2}(c+dx)\right),-\tan^2\left(\frac{1}{2}(c+dx)\right)\right)^2\cos^4\left(\frac{1}{2}(c+\right.\right.}$$

input
```
Integrate[(e*Sec[c + d*x])^(2/3)/Sqrt[a + a*Sec[c + d*x]],x]
```

output
```
(90*AppellF1[1/2, 1/6, 1/3, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*
Cos[(c + d*x)/2]*Cos[c + d*x]^2*(e*Sec[c + d*x])^(2/3)*Sqrt[a*(1 + Sec[c +
 d*x])]*Sin[(c + d*x)/2]*(9*AppellF1[1/2, 1/6, 1/3, 3/2, Tan[(c + d*x)/2]^
2, -Tan[(c + d*x)/2]^2] + (-2*AppellF1[3/2, 1/6, 4/3, 5/2, Tan[(c + d*x)/2
]^2, -Tan[(c + d*x)/2]^2] + AppellF1[3/2, 7/6, 1/3, 5/2, Tan[(c + d*x)/2]^
2, -Tan[(c + d*x)/2]^2])*Tan[(c + d*x)/2]^2))/(a*d*(270*AppellF1[1/2, 1/6,
 1/3, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]^2*Cos[(c + d*x)/2]^4*(
1 + 2*Cos[c + d*x]) + 10*(-2*AppellF1[3/2, 1/6, 4/3, 5/2, Tan[(c + d*x)/2]
^2, -Tan[(c + d*x)/2]^2] + AppellF1[3/2, 7/6, 1/3, 5/2, Tan[(c + d*x)/2]^2
, -Tan[(c + d*x)/2]^2])^2*Cos[c + d*x]*Sin[(c + d*x)/2]^2*Tan[(c + d*x)/2]
^2 - 3*AppellF1[1/2, 1/6, 1/3, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^
2]*(10*AppellF1[3/2, 1/6, 4/3, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^
2]*Cos[(c + d*x)/2]^2*(2 - 9*Cos[c + d*x] + Cos[2*(c + d*x)]) - 5*AppellF1
[3/2, 7/6, 1/3, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Cos[(c + d*x
)/2]^2*(2 - 9*Cos[c + d*x] + Cos[2*(c + d*x)]) + 6*(16*AppellF1[5/2, 1/6,
7/3, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - 4*AppellF1[5/2, 7/6,
4/3, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + 7*AppellF1[5/2, 13/6,
 1/3, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Cos[c + d*x]*Sin[(c +
 d*x)/2]^2)*Tan[(c + d*x)/2]^2))
```

3.280.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {3042, 4315, 3042, 4314, 148, 27, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e\sec(c+dx))^{2/3}}{\sqrt{a\sec(c+dx)+a}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{2/3}}{\sqrt{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}}dx$$

$$\Big\downarrow \text{4315}$$

$$\frac{\sqrt{\sec(c+dx)+1}\int \frac{(e\sec(c+dx))^{2/3}}{\sqrt{\sec(c+dx)+1}}dx}{\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \; 3042$$

$$\frac{\sqrt{\sec(c+dx)+1}\int \frac{\left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{2/3}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)+1}}dx}{\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \; 4314$$

$$-\frac{e\tan(c+dx)\int \frac{1}{\sqrt{1-\sec(c+dx)}\sqrt[3]{e\sec(c+dx)}(\sec(c+dx)+1)}d\sec(c+dx)}{d\sqrt{1-\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \; 148$$

$$-\frac{3\tan(c+dx)\int \frac{e\sqrt[3]{e\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}(\sec(c+dx)e+e)}d\sqrt[3]{e\sec(c+dx)}}{d\sqrt{1-\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \; 27$$

$$-\frac{3e\tan(c+dx)\int \frac{\sqrt[3]{e\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}(\sec(c+dx)e+e)}d\sqrt[3]{e\sec(c+dx)}}{d\sqrt{1-\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \; 1012$$

$$-\frac{3\tan(c+dx)\,\text{AppellF1}\left(\frac{2}{3},1,\frac{1}{2},\frac{5}{3},-\sec(c+dx),\sec(c+dx)\right)(e\sec(c+dx))^{2/3}}{2d\sqrt{1-\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}$$

input
```
Int[(e*Sec[c + d*x])^(2/3)/Sqrt[a + a*Sec[c + d*x]],x]
```

output
```
(-3*AppellF1[2/3, 1, 1/2, 5/3, -Sec[c + d*x], Sec[c + d*x]]*(e*Sec[c + d*x
])^(2/3)*Tan[c + d*x])/(2*d*Sqrt[1 - Sec[c + d*x]]*Sqrt[a + a*Sec[c + d*x]
])
```

3.280. $\int \frac{(e\sec(c+dx))^{2/3}}{\sqrt{a+a\sec(c+dx)}}\,dx$

3.280.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 148
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_))^(p_.),
x_] :> With[{k = Denominator[m]}, Simp[k/b   Subst[Int[x^(k*(m + 1) - 1)*(c
 + d*(x^k/b))^n*(e + f*(x^k/b))^p, x], x, (b*x)^(1/k)], x]] /; FreeQ[{b, c,
 d, e, f, n, p}, x] && FractionQ[m] && IntegerQ[p]
```

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4314
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Simp[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x
]]*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(d*x)^(n - 1)*((a + b*x)^(m - 1/2
)/Sqrt[a - b*x]), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n},
x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

rule 4315
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Simp[a^IntPart[m]*((a + b*Csc[e + f*x])^FracPart[m
]/(1 + (b/a)*Csc[e + f*x])^FracPart[m])   Int[(1 + (b/a)*Csc[e + f*x])^m*(d
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^
2, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

3.280. $\int \frac{(e\sec(c+dx))^{2/3}}{\sqrt{a+a\sec(c+dx)}}\,dx$

3.280.4 Maple [F]

$$\int \frac{(e\sec(dx+c))^{\frac{2}{3}}}{\sqrt{a+a\sec(dx+c)}}dx$$

input
```
int((e*sec(d*x+c))^(2/3)/(a+a*sec(d*x+c))^(1/2),x)
```

output
```
int((e*sec(d*x+c))^(2/3)/(a+a*sec(d*x+c))^(1/2),x)
```

3.280.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^{2/3}}{\sqrt{a+a\sec(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate((e*sec(d*x+c))^(2/3)/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas
")
```

output
```
Timed out
```

3.280.6 Sympy [F]

$$\int \frac{(e\sec(c+dx))^{2/3}}{\sqrt{a+a\sec(c+dx)}}\,dx = \int \frac{(e\sec(c+dx))^{\frac{2}{3}}}{\sqrt{a(\sec(c+dx)+1)}}\,dx$$

input
```
integrate((e*sec(d*x+c))**(2/3)/(a+a*sec(d*x+c))**(1/2),x)
```

output
```
Integral((e*sec(c + d*x))**(2/3)/sqrt(a*(sec(c + d*x) + 1)), x)
```

3.280.7 Maxima [F]

$$\int \frac{(e\sec(c+dx))^{2/3}}{\sqrt{a+a\sec(c+dx)}}\,dx = \int \frac{(e\sec(dx+c))^{\frac{2}{3}}}{\sqrt{a\sec(dx+c)+a}}\,dx$$

input
```
integrate((e*sec(d*x+c))^(2/3)/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima
")
```

output
```
integrate((e*sec(d*x + c))^(2/3)/sqrt(a*sec(d*x + c) + a), x)
```

3.280.8 Giac [F]

$$\int \frac{(e\sec(c+dx))^{2/3}}{\sqrt{a+a\sec(c+dx)}}\,dx = \int \frac{(e\sec(dx+c))^{\frac{2}{3}}}{\sqrt{a\sec(dx+c)+a}}\,dx$$

input
```
integrate((e*sec(d*x+c))^(2/3)/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate((e*sec(d*x + c))^(2/3)/sqrt(a*sec(d*x + c) + a), x)
```

3.280.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^{2/3}}{\sqrt{a+a\sec(c+dx)}}\,dx = \int \frac{\left(\frac{e}{\cos(c+d\,x)}\right)^{2/3}}{\sqrt{a+\frac{a}{\cos(c+d\,x)}}}\,dx$$

input
```
int((e/cos(c + d*x))^(2/3)/(a + a/cos(c + d*x))^(1/2),x)
```

output
```
int((e/cos(c + d*x))^(2/3)/(a + a/cos(c + d*x))^(1/2), x)
```

3.281 $\int \frac{\sqrt[3]{e\sec(c+dx)}}{\sqrt{a+a\sec(c+dx)}}\,dx$

3.281.1 Optimal result . . . 2213
3.281.2 Mathematica [B] (warning: unable to verify) . . . 2213
3.281.3 Rubi [A] (verified) . . . 2214
3.281.4 Maple [F] . . . 2217
3.281.5 Fricas [F(-1)] . . . 2217
3.281.6 Sympy [F] . . . 2217
3.281.7 Maxima [F] . . . 2218
3.281.8 Giac [F] . . . 2218
3.281.9 Mupad [F(-1)] . . . 2218

3.281.1 Optimal result

Integrand size = 27, antiderivative size = 76

$$\int \frac{\sqrt[3]{e\sec(c+dx)}}{\sqrt{a+a\sec(c+dx)}}\,dx = -\frac{3\,\text{AppellF1}\left(\frac{1}{3},\frac{1}{2},1,\frac{4}{3},\sec(c+dx),-\sec(c+dx)\right)\sqrt[3]{e\sec(c+dx)}\tan(c+dx)}{d\sqrt{1-\sec(c+dx)}\sqrt{a+a\sec(c+dx)}}$$

output
```
-3*AppellF1(1/3,1,1/2,4/3,-sec(d*x+c),sec(d*x+c))*(e*sec(d*x+c))^(1/3)*tan
(d*x+c)/d/(1-sec(d*x+c))^(1/2)/(a+a*sec(d*x+c))^(1/2)
```

3.281.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 749 vs. 2(76) = 152.

Time = 7.80 (sec) , antiderivative size = 749, normalized size of antiderivative = 9.86

$$\int \frac{\sqrt[3]{e\sec(c+dx)}}{\sqrt{a+a\sec(c+dx)}}\,dx$$

$$= \frac{}{d(e\sec(c+dx))^{2/3}\sqrt{a(1+\sec(c+dx))}\left(4320\,\text{AppellF1}\left(\frac{1}{2},-\frac{1}{6},\frac{2}{3},\frac{3}{2},\tan^2\left(\frac{1}{2}(c+dx)\right),-\tan^2\left(\frac{1}{2}(c+dx)\right)\right.\right.}$$

input
```
Integrate[(e*Sec[c + d*x])^(1/3)/Sqrt[a + a*Sec[c + d*x]],x]
```

output
```
(720*e*AppellF1[1/2, -1/6, 2/3, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]
^2]*Cos[(c + d*x)/2]*(1 + Cos[c + d*x])^2*Sin[(c + d*x)/2]*(9*AppellF1[1/2
, -1/6, 2/3, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - (4*AppellF1[3
/2, -1/6, 5/3, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + AppellF1[3/
2, 5/6, 2/3, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Tan[(c + d*x)/
2]^2))/(d*(e*Sec[c + d*x])^(2/3)*Sqrt[a*(1 + Sec[c + d*x])]*(4320*AppellF1
[1/2, -1/6, 2/3, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]^2*Cos[(c +
d*x)/2]^6*(-1 + 4*Cos[c + d*x]) + 160*(4*AppellF1[3/2, -1/6, 5/3, 5/2, Tan
[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + AppellF1[3/2, 5/6, 2/3, 5/2, Tan[(
c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])^2*Cos[c + d*x]*Sin[(c + d*x)/2]^4 + 1
2*AppellF1[1/2, -1/6, 2/3, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*S
in[(c + d*x)/2]^2*(20*AppellF1[3/2, -1/6, 5/3, 5/2, Tan[(c + d*x)/2]^2, -T
an[(c + d*x)/2]^2]*(7 + 14*Cos[c + d*x] + 5*Cos[2*(c + d*x)] - 2*Cos[3*(c
+ d*x)]) + 5*AppellF1[3/2, 5/6, 2/3, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*
x)/2]^2]*(7 + 14*Cos[c + d*x] + 5*Cos[2*(c + d*x)] - 2*Cos[3*(c + d*x)]) -
 24*(40*AppellF1[5/2, -1/6, 8/3, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2
]^2] + 8*AppellF1[5/2, 5/6, 5/3, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2
]^2] - 5*AppellF1[5/2, 11/6, 2/3, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/
2]^2])*Cos[c + d*x]*Sin[(c + d*x)/2]^2)))
```

3.281.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {3042, 4315, 3042, 4314, 148, 27, 936}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{e\sec(c+dx)}}{\sqrt{a\sec(c+dx)+a}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt[3]{e\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}}dx$$

$$\Big\downarrow \text{4315}$$

$$\frac{\sqrt{\sec(c+dx)+1}\int \frac{\sqrt[3]{e\sec(c+dx)}}{\sqrt{\sec(c+dx)+1}}dx}{\sqrt{a\sec(c+dx)+a}}$$

$\Big\downarrow$ 3042

$$\frac{\sqrt{\sec(c+dx)+1}\int \frac{\sqrt[3]{e\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc(c+dx+\frac{\pi}{2})+1}}dx}{\sqrt{a\sec(c+dx)+a}}$$

$\Big\downarrow$ 4314

$$-\frac{e\tan(c+dx)\int \frac{1}{\sqrt{1-\sec(c+dx)}(e\sec(c+dx))^{2/3}(\sec(c+dx)+1)}d\sec(c+dx)}{d\sqrt{1-\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}$$

$\Big\downarrow$ 148

$$-\frac{3\tan(c+dx)\int \frac{e}{\sqrt{1-\sec(c+dx)}(\sec(c+dx)e+e)}d\sqrt[3]{e\sec(c+dx)}}{d\sqrt{1-\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}$$

$\Big\downarrow$ 27

$$-\frac{3e\tan(c+dx)\int \frac{1}{\sqrt{1-\sec(c+dx)}(\sec(c+dx)e+e)}d\sqrt[3]{e\sec(c+dx)}}{d\sqrt{1-\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}$$

$\Big\downarrow$ 936

$$-\frac{3\tan(c+dx)\,\text{AppellF1}\left(\frac{1}{3},1,\frac{1}{2},\frac{4}{3},-\sec(c+dx),\sec(c+dx)\right)\sqrt[3]{e\sec(c+dx)}}{d\sqrt{1-\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}$$

input
```
Int[(e*Sec[c + d*x])^(1/3)/Sqrt[a + a*Sec[c + d*x]],x]
```

output
```
(-3*AppellF1[1/3, 1, 1/2, 4/3, -Sec[c + d*x], Sec[c + d*x]]*(e*Sec[c + d*x
])^(1/3)*Tan[c + d*x])/(d*Sqrt[1 - Sec[c + d*x]]*Sqrt[a + a*Sec[c + d*x]])
```

3.281. $\int \frac{\sqrt[3]{e\sec(c+dx)}}{\sqrt{a+a\sec(c+dx)}}\,dx$

3.281.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 148
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_))^(p_.),
x_] :> With[{k = Denominator[m]}, Simp[k/b   Subst[Int[x^(k*(m + 1) - 1)*(c
 + d*(x^k/b))^n*(e + f*(x^k/b))^p, x], x, (b*x)^(1/k)], x]] /; FreeQ[{b, c,
 d, e, f, n, p}, x] && FractionQ[m] && IntegerQ[p]
```

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4314
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Simp[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x
]]*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(d*x)^(n - 1)*((a + b*x)^(m - 1/2
)/Sqrt[a - b*x]), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n},
x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

rule 4315
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Simp[a^IntPart[m]*((a + b*Csc[e + f*x])^FracPart[m
]/(1 + (b/a)*Csc[e + f*x])^FracPart[m])   Int[(1 + (b/a)*Csc[e + f*x])^m*(d
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^
2, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

3.281. $\int \frac{\sqrt[3]{e\sec(c+dx)}}{\sqrt{a+a\sec(c+dx)}}\,dx$

3.281.4 Maple [F]

$$\int \frac{(e\sec(dx+c))^{\frac{1}{3}}}{\sqrt{a+a\sec(dx+c)}}dx$$

input
```
int((e*sec(d*x+c))^(1/3)/(a+a*sec(d*x+c))^(1/2),x)
```

output
```
int((e*sec(d*x+c))^(1/3)/(a+a*sec(d*x+c))^(1/2),x)
```

3.281.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{e\sec(c+dx)}}{\sqrt{a+a\sec(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate((e*sec(d*x+c))^(1/3)/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas
")
```

output
```
Timed out
```

3.281.6 Sympy [F]

$$\int \frac{\sqrt[3]{e\sec(c+dx)}}{\sqrt{a+a\sec(c+dx)}}\,dx = \int \frac{\sqrt[3]{e\sec(c+dx)}}{\sqrt{a(\sec(c+dx)+1)}}\,dx$$

input
```
integrate((e*sec(d*x+c))**(1/3)/(a+a*sec(d*x+c))**(1/2),x)
```

output
```
Integral((e*sec(c + d*x))**(1/3)/sqrt(a*(sec(c + d*x) + 1)), x)
```

3.281.7 Maxima [F]

$$\int \frac{\sqrt[3]{e\sec(c+dx)}}{\sqrt{a+a\sec(c+dx)}}\,dx = \int \frac{(e\sec(dx+c))^{\frac{1}{3}}}{\sqrt{a\sec(dx+c)+a}}\,dx$$

input
```
integrate((e*sec(d*x+c))^(1/3)/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima
")
```

output
```
integrate((e*sec(d*x + c))^(1/3)/sqrt(a*sec(d*x + c) + a), x)
```

3.281.8 Giac [F]

$$\int \frac{\sqrt[3]{e\sec(c+dx)}}{\sqrt{a+a\sec(c+dx)}}\,dx = \int \frac{(e\sec(dx+c))^{\frac{1}{3}}}{\sqrt{a\sec(dx+c)+a}}\,dx$$

input
```
integrate((e*sec(d*x+c))^(1/3)/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate((e*sec(d*x + c))^(1/3)/sqrt(a*sec(d*x + c) + a), x)
```

3.281.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{e\sec(c+dx)}}{\sqrt{a+a\sec(c+dx)}}\,dx = \int \frac{\left(\frac{e}{\cos(c+d\,x)}\right)^{1/3}}{\sqrt{a+\frac{a}{\cos(c+d\,x)}}}\,dx$$

input
```
int((e/cos(c + d*x))^(1/3)/(a + a/cos(c + d*x))^(1/2),x)
```

output
```
int((e/cos(c + d*x))^(1/3)/(a + a/cos(c + d*x))^(1/2), x)
```

3.282 $\int \frac{1}{\sqrt[3]{e\sec(c+dx)}\sqrt{a+a\sec(c+dx)}}\,dx$

3.282.1 Optimal result 2219
3.282.2 Mathematica [B] (warning: unable to verify) 2219
3.282.3 Rubi [A] (warning: unable to verify) 2220
3.282.4 Maple [F] 2222
3.282.5 Fricas [F(-1)] 2223
3.282.6 Sympy [F] 2223
3.282.7 Maxima [F] 2223
3.282.8 Giac [F] 2224
3.282.9 Mupad [F(-1)] 2224

3.282.1 Optimal result

Integrand size = 27, antiderivative size = 76

$$\int \frac{1}{\sqrt[3]{e\sec(c+dx)}\sqrt{a+a\sec(c+dx)}}\,dx = \frac{3\,\text{AppellF1}\left(-\frac{1}{3},\frac{1}{2},1,\frac{2}{3},\sec(c+dx),-\sec(c+dx)\right)\tan(c+dx)}{d\sqrt{1-\sec(c+dx)}\sqrt[3]{e\sec(c+dx)}\sqrt{a+a\sec(c+dx)}}$$

output
```
3*AppellF1(-1/3,1,1/2,2/3,-sec(d*x+c),sec(d*x+c))*tan(d*x+c)/d/(e*sec(d*x+
c))^(1/3)/(1-sec(d*x+c))^(1/2)/(a+a*sec(d*x+c))^(1/2)
```

3.282.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 3346 vs. 2(76) = 152.

Time = 19.02 (sec) , antiderivative size = 3346, normalized size of antiderivative = 44.03

$$\int \frac{1}{\sqrt[3]{e\sec(c+dx)}\sqrt{a+a\sec(c+dx)}}\,dx = \text{Result too large to show}$$

input
```
Integrate[1/((e*Sec[c + d*x])^(1/3)*Sqrt[a + a*Sec[c + d*x]]),x]
```

output
```
-(((Cos[(c + d*x)/2]^2*Sec[c + d*x])^(1/6)*Tan[(c + d*x)/2]*(-1 + Tan[(c +
 d*x)/2]^2)*((2*AppellF1[3/2, 1/6, 1/3, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c +
 d*x)/2]^2]*Tan[(c + d*x)/2]^2)/(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(5/6) +
(3*(1 + (3*AppellF1[1/2, 1/6, 1/3, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)
/2]^2])/((-1 + Tan[(c + d*x)/2]^2)*(9*AppellF1[1/2, 1/6, 1/3, 3/2, Tan[(c
+ d*x)/2]^2, -Tan[(c + d*x)/2]^2] + (-2*AppellF1[3/2, 1/6, 4/3, 5/2, Tan[(
c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + AppellF1[3/2, 7/6, 1/3, 5/2, Tan[(c
+ d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Tan[(c + d*x)/2]^2))))/(Sec[(c + d*x)/2
]^2)^(1/3)))/(d*(e*Sec[c + d*x])^(1/3)*Sqrt[a*(1 + Sec[c + d*x])]*(-(Sec[(
c + d*x)/2]^2*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(1/6)*Tan[(c + d*x)/2]^2*(
(2*AppellF1[3/2, 1/6, 1/3, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*T
an[(c + d*x)/2]^2)/(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(5/6) + (3*(1 + (3*Ap
pellF1[1/2, 1/6, 1/3, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])/((-1
+ Tan[(c + d*x)/2]^2)*(9*AppellF1[1/2, 1/6, 1/3, 3/2, Tan[(c + d*x)/2]^2,
-Tan[(c + d*x)/2]^2] + (-2*AppellF1[3/2, 1/6, 4/3, 5/2, Tan[(c + d*x)/2]^2
, -Tan[(c + d*x)/2]^2] + AppellF1[3/2, 7/6, 1/3, 5/2, Tan[(c + d*x)/2]^2,
-Tan[(c + d*x)/2]^2])*Tan[(c + d*x)/2]^2))))/(Sec[(c + d*x)/2]^2)^(1/3)))
- (Sec[(c + d*x)/2]^2*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(1/6)*(-1 + Tan[(c
 + d*x)/2]^2)*((2*AppellF1[3/2, 1/6, 1/3, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c
 + d*x)/2]^2]*Tan[(c + d*x)/2]^2)/(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(5/...
```

3.282.3 Rubi [A] (warning: unable to verify)

Time = 0.45 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.84, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {3042, 4315, 3042, 4314, 148, 27, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a\sec(c+dx)+a}\sqrt[3]{e\sec(c+dx)}}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{\sqrt{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}\sqrt[3]{e\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \; 4315$$

$$\frac{\sqrt{\sec(c+dx)+1}\int \frac{1}{\sqrt[3]{e\sec(c+dx)}\sqrt{\sec(c+dx)+1}}dx}{\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\sec(c+dx)+1}\int \frac{1}{\sqrt[3]{e\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)+1}}dx}{\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{4314}$$

$$-\frac{e\tan(c+dx)\int \frac{1}{\sqrt{1-\sec(c+dx)}(e\sec(c+dx))^{4/3}(\sec(c+dx)+1)}d\sec(c+dx)}{d\sqrt{1-\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{148}$$

$$-\frac{3\tan(c+dx)\int \frac{e\cos^2(c+dx)}{\sqrt{1-\sec(c+dx)}(\sec(c+dx)e+e)}d\sqrt[3]{e\sec(c+dx)}}{d\sqrt{1-\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{3e\tan(c+dx)\int \frac{\cos^2(c+dx)}{\sqrt{1-\sec(c+dx)}(\sec(c+dx)e+e)}d\sqrt[3]{e\sec(c+dx)}}{d\sqrt{1-\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{1012}$$

$$\frac{3\sin(c+dx)\,\text{AppellF1}\left(-\frac{1}{3},1,\frac{1}{2},\frac{2}{3},-\sec(c+dx),\sec(c+dx)\right)}{d\sqrt{1-\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}$$

input
```
Int[1/((e*Sec[c + d*x])^(1/3)*Sqrt[a + a*Sec[c + d*x]]),x]
```

output
```
(3*AppellF1[-1/3, 1, 1/2, 2/3, -Sec[c + d*x], Sec[c + d*x]]*Sin[c + d*x])/
(d*Sqrt[1 - Sec[c + d*x]]*Sqrt[a + a*Sec[c + d*x]])
```

3.282.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 148
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_))^(p_.),
x_] :> With[{k = Denominator[m]}, Simp[k/b   Subst[Int[x^(k*(m + 1) - 1)*(c
 + d*(x^k/b))^n*(e + f*(x^k/b))^p, x], x, (b*x)^(1/k)], x]] /; FreeQ[{b, c,
 d, e, f, n, p}, x] && FractionQ[m] && IntegerQ[p]
```

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4314
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Simp[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x
]]*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(d*x)^(n - 1)*((a + b*x)^(m - 1/2
)/Sqrt[a - b*x]), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n},
x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

rule 4315
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Simp[a^IntPart[m]*((a + b*Csc[e + f*x])^FracPart[m
]/(1 + (b/a)*Csc[e + f*x])^FracPart[m])   Int[(1 + (b/a)*Csc[e + f*x])^m*(d
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^
2, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

3.282.4 Maple [F]

$$\int \frac{1}{(e\sec(dx+c))^{\frac{1}{3}}\sqrt{a+a\sec(dx+c)}}dx$$

input
```
int(1/(e*sec(d*x+c))^(1/3)/(a+a*sec(d*x+c))^(1/2),x)
```

output
```
int(1/(e*sec(d*x+c))^(1/3)/(a+a*sec(d*x+c))^(1/2),x)
```

3.282.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{e\sec(c+dx)}\sqrt{a+a\sec(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*sec(d*x+c))^(1/3)/(a+a*sec(d*x+c))^(1/2),x, algorithm="fric
as")
```

output
```
Timed out
```

3.282.6 Sympy [F]

$$\int \frac{1}{\sqrt[3]{e\sec(c+dx)}\sqrt{a+a\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{a\left(\sec\left(c+dx\right)+1\right)}\sqrt[3]{e\sec\left(c+dx\right)}}\,dx$$

input
```
integrate(1/(e*sec(d*x+c))**(1/3)/(a+a*sec(d*x+c))**(1/2),x)
```

output
```
Integral(1/(sqrt(a*(sec(c + d*x) + 1))*(e*sec(c + d*x))**(1/3)), x)
```

3.282.7 Maxima [F]

$$\int \frac{1}{\sqrt[3]{e\sec(c+dx)}\sqrt{a+a\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{a\sec\left(dx+c\right)+a}\left(e\sec\left(dx+c\right)\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/(e*sec(d*x+c))^(1/3)/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxi
ma")
```

output
```
integrate(1/(sqrt(a*sec(d*x + c) + a)*(e*sec(d*x + c))^(1/3)), x)
```

3.282.8 Giac [F]

$$\int \frac{1}{\sqrt[3]{e\sec(c+dx)}\sqrt{a+a\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{a\sec(dx+c)+a}\,(e\sec(dx+c))^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/(e*sec(d*x+c))^(1/3)/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac
")
```

output
```
integrate(1/(sqrt(a*sec(d*x + c) + a)*(e*sec(d*x + c))^(1/3)), x)
```

3.282.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{e\sec(c+dx)}\sqrt{a+a\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{a+\frac{a}{\cos(c+d\,x)}}\,\left(\frac{e}{\cos(c+d\,x)}\right)^{1/3}}\,dx$$

input
```
int(1/((a + a/cos(c + d*x))^(1/2)*(e/cos(c + d*x))^(1/3)),x)
```

output
```
int(1/((a + a/cos(c + d*x))^(1/2)*(e/cos(c + d*x))^(1/3)), x)
```

3.283 $\int \frac{1}{(e\sec(c+dx))^{2/3}\sqrt{a+a\sec(c+dx)}}\,dx$

3.283.1 Optimal result 2225
3.283.2 Mathematica [B] (warning: unable to verify) 2225
3.283.3 Rubi [A] (warning: unable to verify) 2226
3.283.4 Maple [F] 2228
3.283.5 Fricas [F(-1)] 2229
3.283.6 Sympy [F] 2229
3.283.7 Maxima [F] 2229
3.283.8 Giac [F] 2230
3.283.9 Mupad [F(-1)] 2230

3.283.1 Optimal result

Integrand size = 27, antiderivative size = 78

$$\int \frac{1}{(e\sec(c+dx))^{2/3}\sqrt{a+a\sec(c+dx)}}\,dx = \frac{3\,\text{AppellF1}\left(-\frac{2}{3},\frac{1}{2},1,\frac{1}{3},\sec(c+dx),-\sec(c+dx)\right)\tan(c+d}{2d\sqrt{1-\sec(c+dx)}(e\sec(c+dx))^{2/3}\sqrt{a+a\sec(c+dx)}}$$

output
```
3/2*AppellF1(-2/3,1,1/2,1/3,-sec(d*x+c),sec(d*x+c))*tan(d*x+c)/d/(e*sec(d*
x+c))^(2/3)/(1-sec(d*x+c))^(1/2)/(a+a*sec(d*x+c))^(1/2)
```

3.283.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 585 vs. 2(78) = 156.

Time = 7.85 (sec) , antiderivative size = 585, normalized size of antiderivative = 7.50

$$\int \frac{1}{(e\sec(c+dx))^{2/3}\sqrt{a+a\sec(c+dx)}}\,dx = \frac{\sec^{\frac{7}{6}}(c+dx)\left(-\frac{3}{2}\cos\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{5}{6}}(c+dx)\left(\sin\left(\frac{1}{2}(c+d\right.\right.\right.}{}$$

input
```
Integrate[1/((e*Sec[c + d*x])^(2/3)*Sqrt[a + a*Sec[c + d*x]]),x]
```

output
```
(Sec[c + d*x]^(7/6)*((-3*Cos[(c + d*x)/2]*Sec[c + d*x]^(5/6)*(Sin[(c + d*x
)/2] - Sin[(3*(c + d*x))/2]))/2 + (5*Sqrt[(1 + Cos[c + d*x])^(-1)]*(-1 + 3
*Cos[c + d*x])*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(5/6)*Sin[(c + d*x)/2]*(-
3*Cos[c + d*x]^(5/6)*Hypergeometric2F1[1/2, 5/6, 3/2, 2*Sin[(c + d*x)/2]^2
]*(Sec[(c + d*x)/2]^2)^(1/3) + 2*AppellF1[3/2, 5/6, 2/3, 5/2, Tan[(c + d*x
)/2]^2, -Tan[(c + d*x)/2]^2]*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(5/6)*Tan[(
c + d*x)/2]^2))/(-120*AppellF1[3/2, 5/6, 2/3, 5/2, Tan[(c + d*x)/2]^2, -Ta
n[(c + d*x)/2]^2]*((1 + Cos[c + d*x])^(-1))^(2/3)*(Cos[c + d*x]/(1 + Cos[c
 + d*x]))^(5/6)*Sin[(c + d*x)/2]*Tan[(c + d*x)/2] + 32*AppellF1[5/2, 5/6,
5/3, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*((1 + Cos[c + d*x])^(-1
))^(2/3)*(Cos[c + d*x]/(1 + Cos[c + d*x]))^(5/6)*Sin[(c + d*x)/2]*Tan[(c +
 d*x)/2]^3 + 5*Sqrt[2]*Cos[(c + d*x)/2]*(3 - 4*Sqrt[2]*AppellF1[5/2, 11/6,
 2/3, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*((1 + Cos[c + d*x])^(-
1))^(2/3)*(Cos[c + d*x]/(1 + Cos[c + d*x]))^(5/6)*Tan[(c + d*x)/2]^4))))/(
d*(e*Sec[c + d*x])^(2/3)*Sqrt[a*(1 + Sec[c + d*x])])
```

3.283.3 Rubi [A] (warning: unable to verify)

Time = 0.46 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.92, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {3042, 4315, 3042, 4314, 148, 27, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a\sec(c+dx)+a}(e\sec(c+dx))^{2/3}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}\left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{2/3}}dx$$

$$\Big\downarrow \text{4315}$$

$$\frac{\sqrt{\sec(c+dx)+1}\int \frac{1}{(e\sec(c+dx))^{2/3}\sqrt{\sec(c+dx)+1}}dx}{\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\sec(c+dx)+1}\int \frac{1}{\left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{2/3}\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)+1}}dx}{\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{4314}$$

$$-\frac{e\tan(c+dx)\int \frac{1}{\sqrt{1-\sec(c+dx)}(e\sec(c+dx))^{5/3}(\sec(c+dx)+1)}d\sec(c+dx)}{d\sqrt{1-\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{148}$$

$$-\frac{3\tan(c+dx)\int \frac{e\cos^3(c+dx)}{\sqrt{1-\sec(c+dx)}(\sec(c+dx)e+e)}d\sqrt[3]{e\sec(c+dx)}}{d\sqrt{1-\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{3e\tan(c+dx)\int \frac{\cos^3(c+dx)}{\sqrt{1-\sec(c+dx)}(\sec(c+dx)e+e)}d\sqrt[3]{e\sec(c+dx)}}{d\sqrt{1-\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}$$

$$\Big\downarrow \text{1012}$$

$$\frac{3\sin(c+dx)\cos(c+dx)\,\text{AppellF1}\left(-\frac{2}{3},1,\frac{1}{2},\frac{1}{3},-\sec(c+dx),\sec(c+dx)\right)}{2d\sqrt{1-\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}$$

input
```
Int[1/((e*Sec[c + d*x])^(2/3)*Sqrt[a + a*Sec[c + d*x]]),x]
```

output
```
(3*AppellF1[-2/3, 1, 1/2, 1/3, -Sec[c + d*x], Sec[c + d*x]]*Cos[c + d*x]*S
in[c + d*x])/(2*d*Sqrt[1 - Sec[c + d*x]]*Sqrt[a + a*Sec[c + d*x]])
```

3.283.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 148
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_))^(p_.),
x_] :> With[{k = Denominator[m]}, Simp[k/b   Subst[Int[x^(k*(m + 1) - 1)*(c
 + d*(x^k/b))^n*(e + f*(x^k/b))^p, x], x, (b*x)^(1/k)], x]] /; FreeQ[{b, c,
 d, e, f, n, p}, x] && FractionQ[m] && IntegerQ[p]
```

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4314
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Simp[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x
]]*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(d*x)^(n - 1)*((a + b*x)^(m - 1/2
)/Sqrt[a - b*x]), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n},
x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

rule 4315
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Simp[a^IntPart[m]*((a + b*Csc[e + f*x])^FracPart[m
]/(1 + (b/a)*Csc[e + f*x])^FracPart[m])   Int[(1 + (b/a)*Csc[e + f*x])^m*(d
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^
2, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

3.283.4 Maple [F]

$$\int \frac{1}{(e\sec(dx+c))^{\frac{2}{3}}\sqrt{a+a\sec(dx+c)}}dx$$

input
```
int(1/(e*sec(d*x+c))^(2/3)/(a+a*sec(d*x+c))^(1/2),x)
```

output
```
int(1/(e*sec(d*x+c))^(2/3)/(a+a*sec(d*x+c))^(1/2),x)
```

3.283.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(e\sec(c+dx))^{2/3}\sqrt{a+a\sec(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*sec(d*x+c))^(2/3)/(a+a*sec(d*x+c))^(1/2),x, algorithm="fric
as")
```

output
```
Timed out
```

3.283.6 Sympy [F]

$$\int \frac{1}{(e\sec(c+dx))^{2/3}\sqrt{a+a\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{a\left(\sec\left(c+dx\right)+1\right)}\left(e\sec\left(c+dx\right)\right)^{\frac{2}{3}}}\,dx$$

input
```
integrate(1/(e*sec(d*x+c))**(2/3)/(a+a*sec(d*x+c))**(1/2),x)
```

output
```
Integral(1/(sqrt(a*(sec(c + d*x) + 1))*(e*sec(c + d*x))**(2/3)), x)
```

3.283.7 Maxima [F]

$$\int \frac{1}{(e\sec(c+dx))^{2/3}\sqrt{a+a\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{a\sec\left(dx+c\right)+a}\left(e\sec\left(dx+c\right)\right)^{\frac{2}{3}}}\,dx$$

input
```
integrate(1/(e*sec(d*x+c))^(2/3)/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxi
ma")
```

output
```
integrate(1/(sqrt(a*sec(d*x + c) + a)*(e*sec(d*x + c))^(2/3)), x)
```

3.283.8 Giac [F]

$$\int \frac{1}{(e\sec(c+dx))^{2/3}\sqrt{a+a\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{a\sec(dx+c)+a}\,(e\sec(dx+c))^{\frac{2}{3}}}\,dx$$

input
```
integrate(1/(e*sec(d*x+c))^(2/3)/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac
")
```

output
```
integrate(1/(sqrt(a*sec(d*x + c) + a)*(e*sec(d*x + c))^(2/3)), x)
```

3.283.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(e\sec(c+dx))^{2/3}\sqrt{a+a\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{a+\frac{a}{\cos(c+d\,x)}}\,\left(\frac{e}{\cos(c+d\,x)}\right)^{2/3}}\,dx$$

input
```
int(1/((a + a/cos(c + d*x))^(1/2)*(e/cos(c + d*x))^(2/3)),x)
```

output
```
int(1/((a + a/cos(c + d*x))^(1/2)*(e/cos(c + d*x))^(2/3)), x)
```

3.284 $\int \sec^{\frac{4}{3}}(c+dx)\sqrt[3]{a+a\sec(c+dx)}\,dx$

3.284.1 Optimal result 2231
3.284.2 Mathematica [B] (warning: unable to verify) 2231
3.284.3 Rubi [A] (verified) 2232
3.284.4 Maple [F] 2234
3.284.5 Fricas [F] 2234
3.284.6 Sympy [F(-1)] 2234
3.284.7 Maxima [F] 2235
3.284.8 Giac [F] 2235
3.284.9 Mupad [F(-1)] 2235

3.284.1 Optimal result

Integrand size = 25, antiderivative size = 78

$$\int \sec^{\frac{4}{3}}(c+dx)\sqrt[3]{a+a\sec(c+dx)}\,dx = \frac{2^{5/6}\,\text{AppellF1}\left(\frac{1}{2},-\frac{1}{3},\frac{1}{6},\frac{3}{2},1-\sec(c+dx),\frac{1}{2}(1-\sec(c+dx))\right)\sqrt[3]{a+a\sec(c+dx)}\tan(c+dx)}{d(1+\sec(c+dx))^{5/6}}$$

output
```
2^(5/6)*AppellF1(1/2,-1/3,1/6,3/2,1-sec(d*x+c),1/2-1/2*sec(d*x+c))*(a+a*se
c(d*x+c))^(1/3)*tan(d*x+c)/d/(1+sec(d*x+c))^(5/6)
```

3.284.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 1982 vs. 2(78) = 156.

Time = 23.47 (sec) , antiderivative size = 1982, normalized size of antiderivative = 25.41

$$\int \sec^{\frac{4}{3}}(c+dx)\sqrt[3]{a+a\sec(c+dx)}\,dx = \text{Too large to display}$$

input
```
Integrate[Sec[c + d*x]^(4/3)*(a + a*Sec[c + d*x])^(1/3),x]
```

output
```
(3*Sec[c + d*x]^(1/3)*((1 + Cos[c + d*x])*Sec[c + d*x])^(1/3)*(a*(1 + Sec[
c + d*x]))^(1/3)*Sin[c + d*x])/(2*d*(1 + Sec[c + d*x])^(1/3)) + (3*(a*(1 +
 Sec[c + d*x]))^(1/3)*(-((1 + Sec[c + d*x])^(1/3)/Sec[c + d*x]^(2/3)) + (S
ec[c + d*x]^(1/3)*(1 + Sec[c + d*x])^(1/3))/2)*Tan[(c + d*x)/2]*(-1 + (3*A
ppellF1[1/2, -1/3, 2/3, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])/(9*
AppellF1[1/2, -1/3, 2/3, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - 2
*(2*AppellF1[3/2, -1/3, 5/3, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]
 + HypergeometricPFQ[{2/3, 3/4}, {7/4}, Tan[(c + d*x)/2]^4])*Tan[(c + d*x)
/2]^2)))/(2^(2/3)*d*(Sec[(c + d*x)/2]^2)^(2/3)*(Cos[(c + d*x)/2]^2*Sec[c +
 d*x])^(1/3)*(1 + Sec[c + d*x])^(1/3)*((3*(Sec[(c + d*x)/2]^2)^(1/3)*(-1 +
 (3*AppellF1[1/2, -1/3, 2/3, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]
)/(9*AppellF1[1/2, -1/3, 2/3, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2
] - 2*(2*AppellF1[3/2, -1/3, 5/3, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/
2]^2] + HypergeometricPFQ[{2/3, 3/4}, {7/4}, Tan[(c + d*x)/2]^4])*Tan[(c +
 d*x)/2]^2)))/(2*2^(2/3)*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(1/3)) - (2^(1/
3)*Tan[(c + d*x)/2]^2*(-1 + (3*AppellF1[1/2, -1/3, 2/3, 3/2, Tan[(c + d*x)
/2]^2, -Tan[(c + d*x)/2]^2])/(9*AppellF1[1/2, -1/3, 2/3, 3/2, Tan[(c + d*x
)/2]^2, -Tan[(c + d*x)/2]^2] - 2*(2*AppellF1[3/2, -1/3, 5/3, 5/2, Tan[(c +
 d*x)/2]^2, -Tan[(c + d*x)/2]^2] + HypergeometricPFQ[{2/3, 3/4}, {7/4}, Ta
n[(c + d*x)/2]^4])*Tan[(c + d*x)/2]^2)))/((Sec[(c + d*x)/2]^2)^(2/3)*(C...
```

3.284.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 4315, 3042, 4312, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{\frac{4}{3}}(c+dx)\sqrt[3]{a\sec(c+dx)+a}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{4/3}\sqrt[3]{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}dx$$

$$\Big\downarrow \; 4315$$

$$\frac{\sqrt[3]{a\sec(c+dx)+a}\int \sec^{\frac{4}{3}}(c+dx)\sqrt[3]{\sec(c+dx)+1}dx}{\sqrt[3]{\sec(c+dx)+1}}$$

$$\Big\downarrow \; 3042$$

$$\frac{\sqrt[3]{a\sec(c+dx)+a}\int \csc\left(c+dx+\frac{\pi}{2}\right)^{4/3}\sqrt[3]{\csc\left(c+dx+\frac{\pi}{2}\right)+1}dx}{\sqrt[3]{\sec(c+dx)+1}}$$

$$\Big\downarrow \; 4312$$

$$\frac{\tan(c+dx)\sqrt[3]{a\sec(c+dx)+a}\int \frac{\sqrt[3]{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}\sqrt[6]{\sec(c+dx)+1}}d(1-\sec(c+dx))}{d\sqrt{1-\sec(c+dx)}(\sec(c+dx)+1)^{5/6}}$$

$$\Big\downarrow \; 150$$

$$\frac{2^{5/6}\tan(c+dx)\sqrt[3]{a\sec(c+dx)+a}\,\text{AppellF1}\left(\frac{1}{2},-\frac{1}{3},\frac{1}{6},\frac{3}{2},1-\sec(c+dx),\frac{1}{2}(1-\sec(c+dx))\right)}{d(\sec(c+dx)+1)^{5/6}}$$

input
```
Int[Sec[c + d*x]^(4/3)*(a + a*Sec[c + d*x])^(1/3),x]
```

output
```
(2^(5/6)*AppellF1[1/2, -1/3, 1/6, 3/2, 1 - Sec[c + d*x], (1 - Sec[c + d*x]
)/2]*(a + a*Sec[c + d*x])^(1/3)*Tan[c + d*x])/(d*(1 + Sec[c + d*x])^(5/6))
```

3.284.3.1 Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] && !In
tegerQ[m] && !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4312
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-(a*(d/b))^n)*(Cot[e + f*x]/(a^(n - 2)*f*Sqrt
[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]])) Subst[Int[(a - x)^(n - 1)
*((2*a - x)^(m - 1/2)/Sqrt[x]), x], x, a - b*Csc[e + f*x]], x] /; FreeQ[{a,
 b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && !IntegerQ[m] && GtQ[a, 0] &
& !IntegerQ[n] && GtQ[a*(d/b), 0]
```

rule 4315
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Simp[a^IntPart[m]*((a + b*Csc[e + f*x])^FracPart[m
]/(1 + (b/a)*Csc[e + f*x])^FracPart[m])   Int[(1 + (b/a)*Csc[e + f*x])^m*(d
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^
2, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

3.284.4 Maple [F]

$$\int \sec(dx+c)^{\frac{4}{3}}(a+a\sec(dx+c))^{\frac{1}{3}}\,dx$$

input
```
int(sec(d*x+c)^(4/3)*(a+a*sec(d*x+c))^(1/3),x)
```

output
```
int(sec(d*x+c)^(4/3)*(a+a*sec(d*x+c))^(1/3),x)
```

3.284.5 Fricas [F]

$$\int \sec^{\frac{4}{3}}(c+dx)\sqrt[3]{a+a\sec(c+dx)}\,dx=\int (a\sec(dx+c)+a)^{\frac{1}{3}}\sec(dx+c)^{\frac{4}{3}}\,dx$$

input
```
integrate(sec(d*x+c)^(4/3)*(a+a*sec(d*x+c))^(1/3),x, algorithm="fricas")
```

output
```
integral((a*sec(d*x + c) + a)^(1/3)*sec(d*x + c)^(4/3), x)
```

3.284.6 Sympy [F(-1)]

Timed out.

$$\int \sec^{\frac{4}{3}}(c+dx)\sqrt[3]{a+a\sec(c+dx)}\,dx=\text{Timed out}$$

input
```
integrate(sec(d*x+c)**(4/3)*(a+a*sec(d*x+c))**(1/3),x)
```

output
```
Timed out
```

3.284.7 Maxima [F]

$$\int \sec^{\frac{4}{3}}(c+dx)\sqrt[3]{a+a\sec(c+dx)}\,dx=\int (a\sec(dx+c)+a)^{\frac{1}{3}}\sec(dx+c)^{\frac{4}{3}}\,dx$$

input
```
integrate(sec(d*x+c)^(4/3)*(a+a*sec(d*x+c))^(1/3),x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)^(1/3)*sec(d*x + c)^(4/3), x)
```

3.284.8 Giac [F]

$$\int \sec^{\frac{4}{3}}(c+dx)\sqrt[3]{a+a\sec(c+dx)}\,dx=\int (a\sec(dx+c)+a)^{\frac{1}{3}}\sec(dx+c)^{\frac{4}{3}}\,dx$$

input
```
integrate(sec(d*x+c)^(4/3)*(a+a*sec(d*x+c))^(1/3),x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)^(1/3)*sec(d*x + c)^(4/3), x)
```

3.284.9 Mupad [F(-1)]

Timed out.

$$\int \sec^{\frac{4}{3}}(c+dx)\sqrt[3]{a+a\sec(c+dx)}\,dx=\int \left(a+\frac{a}{\cos(c+d\,x)}\right)^{1/3}\left(\frac{1}{\cos(c+d\,x)}\right)^{4/3}dx$$

input
```
int((a + a/cos(c + d*x))^(1/3)*(1/cos(c + d*x))^(4/3),x)
```

output
```
int((a + a/cos(c + d*x))^(1/3)*(1/cos(c + d*x))^(4/3), x)
```

3.285 $\int \sec^{\frac{4}{3}}(c+dx)(a+a\sec(c+dx))^{2/3}\,dx$

3.285.1 Optimal result 2236
3.285.2 Mathematica [C] (warning: unable to verify) 2236
3.285.3 Rubi [A] (verified) 2237
3.285.4 Maple [F] 2239
3.285.5 Fricas [F(-1)] 2239
3.285.6 Sympy [F(-1)] 2239
3.285.7 Maxima [F] 2240
3.285.8 Giac [F] 2240
3.285.9 Mupad [F(-1)] 2240

3.285.1 Optimal result

Integrand size = 25, antiderivative size = 79

$$\int \sec^{\frac{4}{3}}(c+dx)(a+a\sec(c+dx))^{2/3}\,dx=\frac{2\sqrt[6]{2}\,\text{AppellF1}\left(\frac{1}{2},-\frac{1}{3},-\frac{1}{6},\frac{3}{2},1-\sec(c+dx),\frac{1}{2}(1-\sec(c+dx))\right)(a+a\sec(c+dx))^{2/3}\tan(c}{d(1+\sec(c+dx))^{7/6}}$$

output
```
2*2^(1/6)*AppellF1(1/2,-1/3,-1/6,3/2,1-sec(d*x+c),1/2-1/2*sec(d*x+c))*(a+a
*sec(d*x+c))^(2/3)*tan(d*x+c)/d/(1+sec(d*x+c))^(7/6)
```

3.285.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 27.27 (sec) , antiderivative size = 1965, normalized size of antiderivative = 24.87

$$\int \sec^{\frac{4}{3}}(c+dx)(a+a\sec(c+dx))^{2/3}\,dx=\text{Too large to display}$$

input
```
Integrate[Sec[c + d*x]^(4/3)*(a + a*Sec[c + d*x])^(2/3),x]
```

output
```
(2^(2/3)*Hypergeometric2F1[1/2, 2/3, 3/2, -Tan[(c + d*x)/2]^2]*(a*(1 + Sec
[c + d*x]))^(2/3)*Tan[(c + d*x)/2])/(3*d*(1 + Sec[c + d*x])^(2/3)) + (Sec[
c + d*x]^(1/3)*((1 + Cos[c + d*x])*Sec[c + d*x])^(2/3)*(a*(1 + Sec[c + d*x
]))^(2/3)*Tan[(c + d*x)/2])/(d*(1 + Sec[c + d*x])^(2/3)) - ((Sec[(c + d*x)
/2]^2)^(4/3)*Sec[c + d*x]^(1/3)*(a*(1 + Sec[c + d*x]))^(2/3)*(AppellF1[-2/
3, -1/3, -1/3, 1/3, (-1 - I)/(-1 + Tan[(c + d*x)/2]), (-1 + I)/(-1 + Tan[(
c + d*x)/2])]/(((-I + Tan[(c + d*x)/2])/(-1 + Tan[(c + d*x)/2]))^(1/3)*((I
 + Tan[(c + d*x)/2])/(-1 + Tan[(c + d*x)/2]))^(1/3)) - AppellF1[-2/3, -1/3
, -1/3, 1/3, (1 - I)/(1 + Tan[(c + d*x)/2]), (1 + I)/(1 + Tan[(c + d*x)/2]
)]/(((-I + Tan[(c + d*x)/2])/(1 + Tan[(c + d*x)/2]))^(1/3)*((I + Tan[(c +
d*x)/2])/(1 + Tan[(c + d*x)/2]))^(1/3))))/(2^(1/3)*d*(-(((Sec[(c + d*x)/2]
^2)^(1/3)*Tan[(c + d*x)/2]*(AppellF1[-2/3, -1/3, -1/3, 1/3, (-1 - I)/(-1 +
 Tan[(c + d*x)/2]), (-1 + I)/(-1 + Tan[(c + d*x)/2])]/(((-I + Tan[(c + d*x
)/2])/(-1 + Tan[(c + d*x)/2]))^(1/3)*((I + Tan[(c + d*x)/2])/(-1 + Tan[(c
+ d*x)/2]))^(1/3)) - AppellF1[-2/3, -1/3, -1/3, 1/3, (1 - I)/(1 + Tan[(c +
 d*x)/2]), (1 + I)/(1 + Tan[(c + d*x)/2])]/(((-I + Tan[(c + d*x)/2])/(1 +
Tan[(c + d*x)/2]))^(1/3)*((I + Tan[(c + d*x)/2])/(1 + Tan[(c + d*x)/2]))^(
1/3))))/2^(1/3)) - (3*(Sec[(c + d*x)/2]^2)^(1/3)*((((1/3 - I/3)*AppellF1[1
/3, -1/3, 2/3, 4/3, (-1 - I)/(-1 + Tan[(c + d*x)/2]), (-1 + I)/(-1 + Tan[(
c + d*x)/2])]*Sec[(c + d*x)/2]^2)/(-1 + Tan[(c + d*x)/2])^2 + ((1/3 + I...
```

3.285.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 4315, 3042, 4312, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{\frac{4}{3}}(c+dx)(a\sec(c+dx)+a)^{2/3}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{4/3}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{2/3}dx$$

$$\Big\downarrow \; 4315$$

$$\frac{(a\sec(c+dx)+a)^{2/3}\int \sec^{\frac{4}{3}}(c+dx)(\sec(c+dx)+1)^{2/3}dx}{(\sec(c+dx)+1)^{2/3}}$$

$$\Big\downarrow \; 3042$$

$$\frac{(a\sec(c+dx)+a)^{2/3}\int \csc\left(c+dx+\frac{\pi}{2}\right)^{4/3}\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)^{2/3}dx}{(\sec(c+dx)+1)^{2/3}}$$

$$\Big\downarrow \text{4312}$$

$$\frac{\tan(c+dx)(a\sec(c+dx)+a)^{2/3}\int \frac{\sqrt[3]{\sec(c+dx)}\sqrt[6]{\sec(c+dx)+1}}{\sqrt{1-\sec(c+dx)}}d(1-\sec(c+dx))}{d\sqrt{1-\sec(c+dx)}(\sec(c+dx)+1)^{7/6}}$$

$$\Big\downarrow \text{150}$$

$$\frac{2\sqrt[6]{2}\tan(c+dx)(a\sec(c+dx)+a)^{2/3}\text{AppellF1}\left(\frac{1}{2},-\frac{1}{3},-\frac{1}{6},\frac{3}{2},1-\sec(c+dx),\frac{1}{2}(1-\sec(c+dx))\right)}{d(\sec(c+dx)+1)^{7/6}}$$

input
```
Int[Sec[c + d*x]^(4/3)*(a + a*Sec[c + d*x])^(2/3),x]
```

output
```
(2*2^(1/6)*AppellF1[1/2, -1/3, -1/6, 3/2, 1 - Sec[c + d*x], (1 - Sec[c + d
*x])/2]*(a + a*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(d*(1 + Sec[c + d*x])^(7/
6))
```

3.285.3.1 Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] && !In
tegerQ[m] && !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4312
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-(a*(d/b))^n)*(Cot[e + f*x]/(a^(n - 2)*f*Sqrt
[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]])) Subst[Int[(a - x)^(n - 1)
*((2*a - x)^(m - 1/2)/Sqrt[x]), x], x, a - b*Csc[e + f*x]], x] /; FreeQ[{a,
 b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && !IntegerQ[m] && GtQ[a, 0] &
& !IntegerQ[n] && GtQ[a*(d/b), 0]
```

rule 4315
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Simp[a^IntPart[m]*((a + b*Csc[e + f*x])^FracPart[m
]/(1 + (b/a)*Csc[e + f*x])^FracPart[m])   Int[(1 + (b/a)*Csc[e + f*x])^m*(d
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^
2, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

3.285.4 Maple [F]

$$\int \sec (dx+c)^{\frac{4}{3}} (a+a\sec (dx+c))^{\frac{2}{3}} dx$$

input
```
int(sec(d*x+c)^(4/3)*(a+a*sec(d*x+c))^(2/3),x)
```

output
```
int(sec(d*x+c)^(4/3)*(a+a*sec(d*x+c))^(2/3),x)
```

3.285.5 Fricas [F(-1)]

Timed out.

$$\int \sec^{\frac{4}{3}}(c+dx)(a+a\sec(c+dx))^{2/3}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)^(4/3)*(a+a*sec(d*x+c))^(2/3),x, algorithm="fricas")
```

output
```
Timed out
```

3.285.6 Sympy [F(-1)]

Timed out.

$$\int \sec^{\frac{4}{3}}(c+dx)(a+a\sec(c+dx))^{2/3}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**(4/3)*(a+a*sec(d*x+c))**(2/3),x)
```

output
```
Timed out
```

3.285.7 Maxima [F]

$$\int \sec^{\frac{4}{3}}(c+dx)(a+a\sec(c+dx))^{2/3}\,dx = \int (a\sec(dx+c)+a)^{\frac{2}{3}}\sec(dx+c)^{\frac{4}{3}}\,dx$$

input
```
integrate(sec(d*x+c)^(4/3)*(a+a*sec(d*x+c))^(2/3),x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)^(2/3)*sec(d*x + c)^(4/3), x)
```

3.285.8 Giac [F]

$$\int \sec^{\frac{4}{3}}(c+dx)(a+a\sec(c+dx))^{2/3}\,dx = \int (a\sec(dx+c)+a)^{\frac{2}{3}}\sec(dx+c)^{\frac{4}{3}}\,dx$$

input
```
integrate(sec(d*x+c)^(4/3)*(a+a*sec(d*x+c))^(2/3),x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)^(2/3)*sec(d*x + c)^(4/3), x)
```

3.285.9 Mupad [F(-1)]

Timed out.

$$\int \sec^{\frac{4}{3}}(c+dx)(a+a\sec(c+dx))^{2/3}\,dx = \int \left(a+\frac{a}{\cos(c+d\,x)}\right)^{2/3}\left(\frac{1}{\cos(c+d\,x)}\right)^{4/3}dx$$

input
```
int((a + a/cos(c + d*x))^(2/3)*(1/cos(c + d*x))^(4/3),x)
```

output
```
int((a + a/cos(c + d*x))^(2/3)*(1/cos(c + d*x))^(4/3), x)
```

3.286 $\int \sec^{\frac{5}{3}}(c+dx)(a+a\sec(c+dx))^{2/3}\,dx$

3.286.1 Optimal result 2241
3.286.2 Mathematica [A] (verified) 2242
3.286.3 Rubi [C] (verified) 2243
3.286.4 Maple [F] 2244
3.286.5 Fricas [F] 2245
3.286.6 Sympy [F(-1)] 2245
3.286.7 Maxima [F] 2245
3.286.8 Giac [F] 2246
3.286.9 Mupad [F(-1)] 2246

3.286.1 Optimal result

Integrand size = 25, antiderivative size = 327

$$\int \sec^{\frac{5}{3}}(c+dx)(a+a\sec(c+dx))^{2/3}\,dx = -\frac{3a\sec^{\frac{5}{3}}(c+dx)\sin(c+dx)}{2d\sqrt[3]{a(1+\sec(c+dx))}} + \frac{9\sec^{\frac{2}{3}}(c+dx)(a(1+\sec(c+dx)))^{2/3}\sin(c+dx)}{4d} - \frac{9(a(1+\sec(c+dx)))^{2/3}\tan(c+dx)}{4d\sqrt[3]{\frac{1}{1+\cos(c+dx)}}(1+\sec(c+dx))^{7/3}} + \frac{\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{3},\frac{5}{4},\tan^4\left(\frac{1}{2}(c+dx)\right)\right)\sqrt[3]{\cos(c+dx)\sec^4\left(\frac{1}{2}(c+dx)\right)}(a(1+\sec(c+dx)))^{2/3}\tan}{8d\sqrt[3]{\frac{1}{1+\cos(c+dx)}}(1+\sec(c+dx))^{4/3}} - \frac{5\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{3}{4},\frac{7}{4},\tan^4\left(\frac{1}{2}(c+dx)\right)\right)\sqrt[3]{\cos(c+dx)\sec^4\left(\frac{1}{2}(c+dx)\right)}(a(1+\sec(c+dx)))^{2/3}\,\mathrm{t}}{8d\sqrt[3]{\frac{1}{1+\cos(c+dx)}}(1+\sec(c+dx))^{10/3}}$$

output
```
-3/2*a*sec(d*x+c)^(5/3)*sin(d*x+c)/d/(a*(1+sec(d*x+c)))^(1/3)+9/4*sec(d*x+
c)^(2/3)*(a*(1+sec(d*x+c)))^(2/3)*sin(d*x+c)/d-9/4*(a*(1+sec(d*x+c)))^(2/3
)*tan(d*x+c)/d/(1/(1+cos(d*x+c)))^(1/3)/(1+sec(d*x+c))^(7/3)+1/8*hypergeom
([1/4, 1/3],[5/4],tan(1/2*d*x+1/2*c)^4)*(cos(d*x+c)*sec(1/2*d*x+1/2*c)^4)^
(1/3)*(a*(1+sec(d*x+c)))^(2/3)*tan(d*x+c)/d/(1/(1+cos(d*x+c)))^(1/3)/(1+se
c(d*x+c))^(4/3)-5/8*hypergeom([1/3, 3/4],[7/4],tan(1/2*d*x+1/2*c)^4)*(cos(
d*x+c)*sec(1/2*d*x+1/2*c)^4)^(1/3)*(a*(1+sec(d*x+c)))^(2/3)*tan(d*x+c)^3/d
/(1/(1+cos(d*x+c)))^(1/3)/(1+sec(d*x+c))^(10/3)
```

3.286.2 Mathematica [A] (verified)

Time = 9.27 (sec) , antiderivative size = 274, normalized size of antiderivative = 0.84

$$\int \sec^{\frac{5}{3}}(c+dx)(a+a\sec(c+dx))^{2/3}\,dx = \frac{(a(1+\sec(c+dx)))^{2/3}\left(-3\sec^3\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\sqrt[3]{1+\sec(c+dx)}\left(\sin\left(\frac{1}{2}(c+dx)\right)-\right.\right.}{}$$

input
```
Integrate[Sec[c + d*x]^(5/3)*(a + a*Sec[c + d*x])^(2/3),x]
```

output
```
((a*(1 + Sec[c + d*x]))^(2/3)*(-3*Sec[(c + d*x)/2]^3*Sec[c + d*x]*(1 + Sec
[c + d*x])^(1/3)*(Sin[(c + d*x)/2] - 2*Sin[(3*(c + d*x))/2]) + 2^(1/3)*Hyp
ergeometric2F1[1/4, 1/3, 5/4, Tan[(c + d*x)/2]^4]*(Cos[c + d*x]*Sec[(c + d
*x)/2]^4)^(1/3)*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(1/3)*Tan[(c + d*x)/2] -
 5*2^(1/3)*Hypergeometric2F1[1/3, 3/4, 7/4, Tan[(c + d*x)/2]^4]*(Cos[c + d
*x]*Sec[(c + d*x)/2]^4)^(1/3)*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(1/3)*Tan[
(c + d*x)/2]^3))/(8*d*((1 + Cos[c + d*x])^(-1))^(1/3)*(1 + Sec[c + d*x])^(
2/3))
```

3.286.3 Rubi [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 0.39 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.24, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 4315, 3042, 4312, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{\frac{5}{3}}(c+dx)(a\sec(c+dx)+a)^{2/3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{5/3}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{2/3}dx$$

$$\Big\downarrow \text{4315}$$

$$\frac{(a\sec(c+dx)+a)^{2/3}\int \sec^{\frac{5}{3}}(c+dx)(\sec(c+dx)+1)^{2/3}dx}{(\sec(c+dx)+1)^{2/3}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(a\sec(c+dx)+a)^{2/3}\int \csc\left(c+dx+\frac{\pi}{2}\right)^{5/3}\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)^{2/3}dx}{(\sec(c+dx)+1)^{2/3}}$$

$$\Big\downarrow \text{4312}$$

$$\frac{\tan(c+dx)(a\sec(c+dx)+a)^{2/3}\int \frac{\sec^{\frac{2}{3}}(c+dx)\sqrt[6]{\sec(c+dx)+1}}{\sqrt{1-\sec(c+dx)}}d(1-\sec(c+dx))}{d\sqrt{1-\sec(c+dx)}(\sec(c+dx)+1)^{7/6}}$$

$$\Big\downarrow \text{150}$$

$$\frac{2\sqrt[6]{2}\tan(c+dx)(a\sec(c+dx)+a)^{2/3}\operatorname{AppellF1}\left(\frac{1}{2},-\frac{2}{3},-\frac{1}{6},\frac{3}{2},1-\sec(c+dx),\frac{1}{2}(1-\sec(c+dx))\right)}{d(\sec(c+dx)+1)^{7/6}}$$

input
```
Int[Sec[c + d*x]^(5/3)*(a + a*Sec[c + d*x])^(2/3),x]
```

output
```
(2*2^(1/6)*AppellF1[1/2, -2/3, -1/6, 3/2, 1 - Sec[c + d*x], (1 - Sec[c + d
*x])/2]*(a + a*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(d*(1 + Sec[c + d*x])^(7/
6))
```

3.286.3.1 Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4312
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-(a*(d/b))^n)*(Cot[e + f*x]/(a^(n - 2)*f*Sqrt
[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(a - x)^(n - 1)
*((2*a - x)^(m - 1/2)/Sqrt[x]), x], x, a - b*Csc[e + f*x]], x] /; FreeQ[{a,
 b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0] &
&  !IntegerQ[n] && GtQ[a*(d/b), 0]
```

rule 4315
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Simp[a^IntPart[m]*((a + b*Csc[e + f*x])^FracPart[m
]/(1 + (b/a)*Csc[e + f*x])^FracPart[m])   Int[(1 + (b/a)*Csc[e + f*x])^m*(d
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^
2, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

3.286.4 Maple [F]

$$\int \sec (dx+c)^{\frac{5}{3}} (a+a\sec (dx+c))^{\frac{2}{3}} dx$$

input
```
int(sec(d*x+c)^(5/3)*(a+a*sec(d*x+c))^(2/3),x)
```

output
```
int(sec(d*x+c)^(5/3)*(a+a*sec(d*x+c))^(2/3),x)
```

3.286.5 Fricas [F]

$$\int \sec^{\frac{5}{3}}(c+dx)(a+a\sec(c+dx))^{2/3}\,dx = \int (a\sec(dx+c)+a)^{\frac{2}{3}}\sec(dx+c)^{\frac{5}{3}}\,dx$$

input
```
integrate(sec(d*x+c)^(5/3)*(a+a*sec(d*x+c))^(2/3),x, algorithm="fricas")
```

output
```
integral((a*sec(d*x + c) + a)^(2/3)*sec(d*x + c)^(5/3), x)
```

3.286.6 Sympy [F(-1)]

Timed out.

$$\int \sec^{\frac{5}{3}}(c+dx)(a+a\sec(c+dx))^{2/3}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**(5/3)*(a+a*sec(d*x+c))**(2/3),x)
```

output
```
Timed out
```

3.286.7 Maxima [F]

$$\int \sec^{\frac{5}{3}}(c+dx)(a+a\sec(c+dx))^{2/3}\,dx = \int (a\sec(dx+c)+a)^{\frac{2}{3}}\sec(dx+c)^{\frac{5}{3}}\,dx$$

input
```
integrate(sec(d*x+c)^(5/3)*(a+a*sec(d*x+c))^(2/3),x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)^(2/3)*sec(d*x + c)^(5/3), x)
```

3.286.8 Giac [F]

$$\int \sec^{\frac{5}{3}}(c+dx)(a+a\sec(c+dx))^{2/3}\,dx = \int (a\sec(dx+c)+a)^{\frac{2}{3}}\sec(dx+c)^{\frac{5}{3}}\,dx$$

input
```
integrate(sec(d*x+c)^(5/3)*(a+a*sec(d*x+c))^(2/3),x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)^(2/3)*sec(d*x + c)^(5/3), x)
```

3.286.9 Mupad [F(-1)]

Timed out.

$$\int \sec^{\frac{5}{3}}(c+dx)(a+a\sec(c+dx))^{2/3}\,dx = \int \left(a+\frac{a}{\cos(c+d\,x)}\right)^{2/3}\left(\frac{1}{\cos(c+d\,x)}\right)^{5/3}dx$$

input
```
int((a + a/cos(c + d*x))^(2/3)*(1/cos(c + d*x))^(5/3),x)
```

output
```
int((a + a/cos(c + d*x))^(2/3)*(1/cos(c + d*x))^(5/3), x)
```

3.287 $\int \frac{(a+a\sec(c+dx))^{4/3}}{\sqrt[3]{\sec(c+dx)}}\,dx$

3.287.1 Optimal result . . . 2247
3.287.2 Mathematica [C] (warning: unable to verify) . . . 2247
3.287.3 Rubi [A] (verified) . . . 2248
3.287.4 Maple [F] . . . 2250
3.287.5 Fricas [F(-1)] . . . 2250
3.287.6 Sympy [F] . . . 2251
3.287.7 Maxima [F] . . . 2251
3.287.8 Giac [F] . . . 2251
3.287.9 Mupad [F(-1)] . . . 2252

3.287.1 Optimal result

Integrand size = 25, antiderivative size = 80

$$\int \frac{(a+a\sec(c+dx))^{4/3}}{\sqrt[3]{\sec(c+dx)}}\,dx = \frac{2\,2^{5/6}a\,\text{AppellF1}\left(\frac{1}{2},\frac{4}{3},-\frac{5}{6},\frac{3}{2},1-\sec(c+dx),\frac{1}{2}(1-\sec(c+dx))\right)\sqrt[3]{a+a\,\text{se}}}{d(1+\sec(c+dx))^{5/6}}$$

output
```
2*2^(5/6)*a*AppellF1(1/2,4/3,-5/6,3/2,1-sec(d*x+c),1/2-1/2*sec(d*x+c))*(a+
a*sec(d*x+c))^(1/3)*tan(d*x+c)/d/(1+sec(d*x+c))^(5/6)
```

3.287.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 39.45 (sec) , antiderivative size = 2325, normalized size of antiderivative = 29.06

$$\int \frac{(a+a\sec(c+dx))^{4/3}}{\sqrt[3]{\sec(c+dx)}}\,dx = \text{Result too large to show}$$

input
```
Integrate[(a + a*Sec[c + d*x])^(4/3)/Sec[c + d*x]^(1/3),x]
```

output
```
(-3*(a*(1 + Sec[c + d*x]))^(4/3)*((1 + Sec[c + d*x])^(1/3)/Sec[c + d*x]^(1
/3) + Sec[c + d*x]^(2/3)*(1 + Sec[c + d*x])^(1/3))*(-8*Tan[(c + d*x)/2] +
(AppellF1[-4/3, -2/3, -2/3, -1/3, (-1 - I)/(-1 + Tan[(c + d*x)/2]), (-1 +
I)/(-1 + Tan[(c + d*x)/2])]*Sec[(c + d*x)/2]^2)/(((-I + Tan[(c + d*x)/2])/
(-1 + Tan[(c + d*x)/2]))^(2/3)*((I + Tan[(c + d*x)/2])/(-1 + Tan[(c + d*x)
/2]))^(2/3)) - AppellF1[-4/3, -2/3, -2/3, -1/3, (1 - I)/(1 + Tan[(c + d*x)
/2]), (1 + I)/(1 + Tan[(c + d*x)/2])]*((-I + Tan[(c + d*x)/2])/(1 + Tan[(c
 + d*x)/2]))^(1/3)*((I + Tan[(c + d*x)/2])/(1 + Tan[(c + d*x)/2]))^(1/3)*(
1 + Tan[(c + d*x)/2])^2))/(4*2^(2/3)*d*(Sec[(c + d*x)/2]^2)^(1/3)*(1 + Sec
[c + d*x])^(4/3)*((Tan[(c + d*x)/2]*(-8*Tan[(c + d*x)/2] + (AppellF1[-4/3,
 -2/3, -2/3, -1/3, (-1 - I)/(-1 + Tan[(c + d*x)/2]), (-1 + I)/(-1 + Tan[(c
 + d*x)/2])]*Sec[(c + d*x)/2]^2)/(((-I + Tan[(c + d*x)/2])/(-1 + Tan[(c +
d*x)/2]))^(2/3)*((I + Tan[(c + d*x)/2])/(-1 + Tan[(c + d*x)/2]))^(2/3)) -
AppellF1[-4/3, -2/3, -2/3, -1/3, (1 - I)/(1 + Tan[(c + d*x)/2]), (1 + I)/(
1 + Tan[(c + d*x)/2])]*((-I + Tan[(c + d*x)/2])/(1 + Tan[(c + d*x)/2]))^(1
/3)*((I + Tan[(c + d*x)/2])/(1 + Tan[(c + d*x)/2]))^(1/3)*(1 + Tan[(c + d*
x)/2])^2))/(4*2^(2/3)*(Sec[(c + d*x)/2]^2)^(1/3)) - (3*(-4*Sec[(c + d*x)/2
]^2 + (Sec[(c + d*x)/2]^2*(((-4/3 + (4*I)/3)*AppellF1[-1/3, -2/3, 1/3, 2/3
, (-1 - I)/(-1 + Tan[(c + d*x)/2]), (-1 + I)/(-1 + Tan[(c + d*x)/2])]*Sec[
(c + d*x)/2]^2)/(-1 + Tan[(c + d*x)/2])^2 - ((4/3 + (4*I)/3)*AppellF1[-...
```

3.287.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 4315, 3042, 4312, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sec(c+dx)+a)^{4/3}}{\sqrt[3]{\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{4/3}}{\sqrt[3]{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4315}$$

$$\frac{a\sqrt[3]{a\sec(c+dx)+a}\int\frac{(\sec(c+dx)+1)^{4/3}}{\sqrt[3]{\sec(c+dx)}}dx}{\sqrt[3]{\sec(c+dx)+1}}$$

$\Big\downarrow$ 3042

$$\frac{a\sqrt[3]{a\sec(c+dx)+a}\int\frac{\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)^{4/3}}{\sqrt[3]{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{\sqrt[3]{\sec(c+dx)+1}}$$

$\Big\downarrow$ 4312

$$\frac{a\tan(c+dx)\sqrt[3]{a\sec(c+dx)+a}\int\frac{(\sec(c+dx)+1)^{5/6}}{\sqrt{1-\sec(c+dx)}\sec^{\frac{4}{3}}(c+dx)}d(1-\sec(c+dx))}{d\sqrt{1-\sec(c+dx)}(\sec(c+dx)+1)^{5/6}}$$

$\Big\downarrow$ 150

$$\frac{2\,2^{5/6}a\tan(c+dx)\sqrt[3]{a\sec(c+dx)+a}\,\text{AppellF1}\left(\frac{1}{2},\frac{4}{3},-\frac{5}{6},\frac{3}{2},1-\sec(c+dx),\frac{1}{2}(1-\sec(c+dx))\right)}{d(\sec(c+dx)+1)^{5/6}}$$

input
```
Int[(a + a*Sec[c + d*x])^(4/3)/Sec[c + d*x]^(1/3),x]
```

output
```
(2*2^(5/6)*a*AppellF1[1/2, 4/3, -5/6, 3/2, 1 - Sec[c + d*x], (1 - Sec[c +
d*x])/2]*(a + a*Sec[c + d*x])^(1/3)*Tan[c + d*x])/(d*(1 + Sec[c + d*x])^(5
/6))
```

3.287.3.1 Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] && !In
tegerQ[m] && !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4312
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-(a*(d/b))^n)*(Cot[e + f*x]/(a^(n - 2)*f*Sqrt
[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(a - x)^(n - 1)
*((2*a - x)^(m - 1/2)/Sqrt[x]), x], x, a - b*Csc[e + f*x]], x] /; FreeQ[{a,
 b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0] &
&  !IntegerQ[n] && GtQ[a*(d/b), 0]
```

rule 4315
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Simp[a^IntPart[m]*((a + b*Csc[e + f*x])^FracPart[m
]/(1 + (b/a)*Csc[e + f*x])^FracPart[m])   Int[(1 + (b/a)*Csc[e + f*x])^m*(d
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^
2, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

3.287.4 Maple [F]

$$\int \frac{(a+a\sec(dx+c))^{\frac{4}{3}}}{\sec(dx+c)^{\frac{1}{3}}}dx$$

input
```
int((a+a*sec(d*x+c))^(4/3)/sec(d*x+c)^(1/3),x)
```

output
```
int((a+a*sec(d*x+c))^(4/3)/sec(d*x+c)^(1/3),x)
```

3.287.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(c+dx))^{4/3}}{\sqrt[3]{\sec(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sec(d*x+c))^(4/3)/sec(d*x+c)^(1/3),x, algorithm="fricas")
```

output
```
Timed out
```

3.287.6 Sympy [F]

$$\int \frac{(a+a\sec(c+dx))^{4/3}}{\sqrt[3]{\sec(c+dx)}}\,dx=\int \frac{(a(\sec(c+dx)+1))^{\frac{4}{3}}}{\sqrt[3]{\sec(c+dx)}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))**(4/3)/sec(d*x+c)**(1/3),x)
```

output
```
Integral((a*(sec(c + d*x) + 1))**(4/3)/sec(c + d*x)**(1/3), x)
```

3.287.7 Maxima [F]

$$\int \frac{(a+a\sec(c+dx))^{4/3}}{\sqrt[3]{\sec(c+dx)}}\,dx=\int \frac{(a\sec(dx+c)+a)^{\frac{4}{3}}}{\sec(dx+c)^{\frac{1}{3}}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))^(4/3)/sec(d*x+c)^(1/3),x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)^(4/3)/sec(d*x + c)^(1/3), x)
```

3.287.8 Giac [F]

$$\int \frac{(a+a\sec(c+dx))^{4/3}}{\sqrt[3]{\sec(c+dx)}}\,dx=\int \frac{(a\sec(dx+c)+a)^{\frac{4}{3}}}{\sec(dx+c)^{\frac{1}{3}}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))^(4/3)/sec(d*x+c)^(1/3),x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)^(4/3)/sec(d*x + c)^(1/3), x)
```

3.287.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(c+dx))^{4/3}}{\sqrt[3]{\sec(c+dx)}}\,dx=\int \frac{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{4/3}}{\left(\frac{1}{\cos(c+d\,x)}\right)^{1/3}}\,dx$$

input
```
int((a + a/cos(c + d*x))^(4/3)/(1/cos(c + d*x))^(1/3),x)
```

output
```
int((a + a/cos(c + d*x))^(4/3)/(1/cos(c + d*x))^(1/3), x)
```

3.288 $\int \sec^n(e+fx)(a+a\sec(e+fx))^4\,dx$

3.288.1 Optimal result . . . 2253
3.288.2 Mathematica [A] (verified) . . . 2254
3.288.3 Rubi [A] (verified) . . . 2254
3.288.4 Maple [F] . . . 2258
3.288.5 Fricas [F] . . . 2258
3.288.6 Sympy [F] . . . 2259
3.288.7 Maxima [F] . . . 2259
3.288.8 Giac [F] . . . 2260
3.288.9 Mupad [F(-1)] . . . 2260

3.288.1 Optimal result

Integrand size = 21, antiderivative size = 304

$$\begin{aligned}\int \sec^n(e+fx)(a+a\sec(e+fx))^4\,dx &= \frac{a^4(30+21n+4n^2)\sec^{1+n}(e+fx)\sin(e+fx)}{f(1+n)(2+n)(3+n)}\\ &+\frac{\sec^{1+n}(e+fx)\left(a^2+a^2\sec(e+fx)\right)^2\sin(e+fx)}{f(3+n)}\\ &+\frac{2(4+n)\sec^{1+n}(e+fx)\left(a^4+a^4\sec(e+fx)\right)\sin(e+fx)}{f(2+n)(3+n)}\\ &-\frac{a^4(3+24n+8n^2)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1-n}{2},\frac{3-n}{2},\cos^2(e+fx)\right)\sec^{-1+n}(e+fx)\sin(e+fx)}{f(1-n)(1+n)(3+n)\sqrt{\sin^2(e+fx)}}\\ &+\frac{4a^4(3+2n)\,\text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{n}{2},\frac{2-n}{2},\cos^2(e+fx)\right)\sec^n(e+fx)\sin(e+fx)}{fn(2+n)\sqrt{\sin^2(e+fx)}}\end{aligned}$$

output
```
a^4*(4*n^2+21*n+30)*sec(f*x+e)^(1+n)*sin(f*x+e)/f/(n^3+6*n^2+11*n+6)+sec(f
*x+e)^(1+n)*(a^2+a^2*sec(f*x+e))^2*sin(f*x+e)/f/(3+n)+2*(4+n)*sec(f*x+e)^(
1+n)*(a^4+a^4*sec(f*x+e))*sin(f*x+e)/f/(2+n)/(3+n)-a^4*(8*n^2+24*n+3)*hype
rgeom([1/2, 1/2-1/2*n],[3/2-1/2*n],cos(f*x+e)^2)*sec(f*x+e)^(-1+n)*sin(f*x
+e)/f/(3+n)/(-n^2+1)/(sin(f*x+e)^2)^(1/2)+4*a^4*(3+2*n)*hypergeom([1/2, -1
/2*n],[1-1/2*n],cos(f*x+e)^2)*sec(f*x+e)^n*sin(f*x+e)/f/n/(2+n)/(sin(f*x+e
)^2)^(1/2)
```

3.288.2 Mathematica [A] (verified)

Time = 5.72 (sec) , antiderivative size = 208, normalized size of antiderivative = 0.68

$$\int \sec^n(e+fx)(a+a\sec(e+fx))^4\,dx$$
$$= \frac{a^4\csc(e+fx)\sec^{-1+n}(e+fx)\left(n(40+34n+7n^2+4(3+4n+n^2)\sec(e+fx)+(2+3n+n^2)\sec^2(e+\right.}{}$$

input
```
Integrate[Sec[e + f*x]^n*(a + a*Sec[e + f*x])^4,x]
```

output
```
(a^4*Csc[e + f*x]*Sec[e + f*x]^(-1 + n)*(n*(40 + 34*n + 7*n^2 + 4*(3 + 4*n
 + n^2)*Sec[e + f*x] + (2 + 3*n + n^2)*Sec[e + f*x]^2)*Tan[e + f*x]^2 + (6
 + 51*n + 40*n^2 + 8*n^3)*Hypergeometric2F1[1/2, n/2, (2 + n)/2, Sec[e + f
*x]^2]*Sqrt[-Tan[e + f*x]^2] + 4*n*(9 + 9*n + 2*n^2)*Hypergeometric2F1[1/2
, (1 + n)/2, (3 + n)/2, Sec[e + f*x]^2]*Sec[e + f*x]*Sqrt[-Tan[e + f*x]^2]
))/(f*n*(1 + n)*(2 + n)*(3 + n))
```

3.288.3 Rubi [A] (verified)

Time = 1.40 (sec) , antiderivative size = 305, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {3042, 4301, 3042, 4506, 3042, 4485, 3042, 4274, 3042, 4259, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sec(e+fx)+a)^4\sec^n(e+fx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^4\csc\left(e+fx+\frac{\pi}{2}\right)^n dx$$

$$\Big\downarrow \text{4301}$$

$$\frac{a\int \sec^n(e+fx)(\sec(e+fx)a+a)^2(a(2n+3)+2a(n+4)\sec(e+fx))dx}{n+3}+\frac{\sin(e+fx)\left(a^2\sec(e+fx)+a^2\right)^2\sec^{n+1}(e+fx)}{f(n+3)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a\int \csc\left(e+fx+\frac{\pi}{2}\right)^n\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)^2\left(a(2n+3)+2a(n+4)\csc\left(e+fx+\frac{\pi}{2}\right)\right)dx}{n+3}+\frac{\sin(e+fx)\left(a^2\sec(e+fx)+a^2\right)^2\sec^{n+1}(e+fx)}{f(n+3)}$$

$\Big\downarrow$ 4506

$$\frac{a\left(\frac{\int \sec^n(e+fx)(\sec(e+fx)a+a)\left((4n^2+15n+6)a^2+(4n^2+21n+30)\sec(e+fx)a^2\right)dx}{n+2}+\frac{2(n+4)\sin(e+fx)\left(a^3\sec(e+fx)+a^3\right)\sec^{n+1}(e+fx)}{f(n+2)}\right)}{n+3}+\frac{\sin(e+fx)\left(a^2\sec(e+fx)+a^2\right)^2\sec^{n+1}(e+fx)}{f(n+3)}$$

$\Big\downarrow$ 3042

$$\frac{a\left(\frac{\int \csc\left(e+fx+\frac{\pi}{2}\right)^n\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)\left((4n^2+15n+6)a^2+(4n^2+21n+30)\csc\left(e+fx+\frac{\pi}{2}\right)a^2\right)dx}{n+2}+\frac{2(n+4)\sin(e+fx)\left(a^3\sec(e+fx)+a^3\right)\sec^{n}}{f(n+2)}\right.}{n+3}+\frac{\sin(e+fx)\left(a^2\sec(e+fx)+a^2\right)^2\sec^{n+1}(e+fx)}{f(n+3)}$$

$\Big\downarrow$ 4485

$$\frac{a\left(\frac{\frac{\int \sec^n(e+fx)\left((n+2)\left(8n^2+24n+3\right)a^3+4(n+1)(n+3)(2n+3)\sec(e+fx)a^3\right)dx}{n+1}+\frac{a^3\left(4n^2+21n+30\right)\sin(e+fx)\sec^{n+1}(e+fx)}{f(n+1)}}{n+2}+\frac{2(n+4)\sin(e+fx)\left(a^3\sec\right.}{f(n}\right.}{n+3}+\frac{\sin(e+fx)\left(a^2\sec(e+fx)+a^2\right)^2\sec^{n+1}(e+fx)}{f(n+3)}$$

$\Big\downarrow$ 3042

$$\frac{a\left(\frac{\frac{\int \csc\left(e+fx+\frac{\pi}{2}\right)^n\left((n+2)\left(8n^2+24n+3\right)a^3+4(n+1)(n+3)(2n+3)\csc\left(e+fx+\frac{\pi}{2}\right)a^3\right)dx}{n+1}+\frac{a^3\left(4n^2+21n+30\right)\sin(e+fx)\sec^{n+1}(e+fx)}{f(n+1)}}{n+2}+\frac{2(n+4)\sin(e+fx}{}\right.}{n+3}+\frac{\sin(e+fx)\left(a^2\sec(e+fx)+a^2\right)^2\sec^{n+1}(e+fx)}{f(n+3)}$$

$\Big\downarrow$ 4274

3.288. $\int \sec^n(e+fx)(a+a\sec(e+fx))^4\,dx$

$$\frac{a\left(\frac{\frac{a^3(n+2)\left(8n^2+24n+3\right)\int \sec^n(e+fx)dx+4a^3(n+1)(n+3)(2n+3)\int \sec^{n+1}(e+fx)dx}{n+1}+\frac{a^3\left(4n^2+21n+30\right)\sin(e+fx)\sec^{n+1}(e+fx)}{f(n+1)}}{n+2}+\frac{2(n+4)\sin(e+fx)(}{}\right.}{\frac{\sin(e+fx)\left(a^2\sec(e+fx)+a^2\right)^2\sec^{n+1}(e+fx)}{f(n+3)}}\quad n+3$$

$\downarrow$ 3042

$$\frac{a\left(\frac{\frac{a^3(n+2)\left(8n^2+24n+3\right)\int \csc\left(e+fx+\frac{\pi}{2}\right)^n dx+4a^3(n+1)(n+3)(2n+3)\int \csc\left(e+fx+\frac{\pi}{2}\right)^{n+1}dx}{n+1}+\frac{a^3\left(4n^2+21n+30\right)\sin(e+fx)\sec^{n+1}(e+fx)}{f(n+1)}}{n+2}+\frac{2(n+4)\sin(}{}\right.}{\frac{\sin(e+fx)\left(a^2\sec(e+fx)+a^2\right)^2\sec^{n+1}(e+fx)}{f(n+3)}}\quad n+3$$

$\downarrow$ 4259

$$\frac{a\left(\frac{\frac{a^3(n+2)\left(8n^2+24n+3\right)\cos^n(e+fx)\sec^n(e+fx)\int \cos^{-n}(e+fx)dx+4a^3(n+1)(n+3)(2n+3)\cos^n(e+fx)\sec^n(e+fx)\int \cos^{-n-1}(e+fx)dx}{n+1}+\frac{a^3\left(4n^2+21n+30\right)}{}}{n+2}\right.}{\frac{\sin(e+fx)\left(a^2\sec(e+fx)+a^2\right)^2\sec^{n+1}(e+fx)}{f(n+3)}}\quad n+3$$

$\downarrow$ 3042

$$\frac{a\left(\frac{\frac{a^3(n+2)\left(8n^2+24n+3\right)\cos^n(e+fx)\sec^n(e+fx)\int \sin\left(e+fx+\frac{\pi}{2}\right)^{-n}dx+4a^3(n+1)(n+3)(2n+3)\cos^n(e+fx)\sec^n(e+fx)\int \sin\left(e+fx+\frac{\pi}{2}\right)^{-n-1}dx}{n+1}+\frac{a^3\left(4n^2+\right.}{}}{n+2}\right.}{\frac{\sin(e+fx)\left(a^2\sec(e+fx)+a^2\right)^2\sec^{n+1}(e+fx)}{f(n+3)}}\quad n+3$$

$\downarrow$ 3122

$$\frac{a\left(\frac{\frac{\frac{4a^3(n+1)(n+3)(2n+3)\sin(e+fx)\sec^n(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{n}{2},\frac{2-n}{2},\cos^2(e+fx)\right)}{fn\sqrt{\sin^2(e+fx)}}-\frac{a^3(n+2)\left(8n^2+24n+3\right)\sin(e+fx)\sec^{n-1}(e+fx)\,\text{Hypergeomet}}{f(1-n)\sqrt{\sin^2(e+fx)}}}{n+1}}{n+2}\right.}{\frac{\sin(e+fx)\left(a^2\sec(e+fx)+a^2\right)^2\sec^{n+1}(e+fx)}{f(n+3)}}\quad n+3$$

3.288. $\int \sec^n(e+fx)(a+a\sec(e+fx))^4\,dx$

input
```
Int[Sec[e + f*x]^n*(a + a*Sec[e + f*x])^4,x]
```

output
```
(Sec[e + f*x]^(1 + n)*(a^2 + a^2*Sec[e + f*x])^2*Sin[e + f*x])/(f*(3 + n))
 + (a*((2*(4 + n)*Sec[e + f*x]^(1 + n)*(a^3 + a^3*Sec[e + f*x])*Sin[e + f*
x])/(f*(2 + n)) + ((a^3*(30 + 21*n + 4*n^2)*Sec[e + f*x]^(1 + n)*Sin[e + f
*x])/(f*(1 + n)) + (-((a^3*(2 + n)*(3 + 24*n + 8*n^2)*Hypergeometric2F1[1/
2, (1 - n)/2, (3 - n)/2, Cos[e + f*x]^2]*Sec[e + f*x]^(-1 + n)*Sin[e + f*x
])/(f*(1 - n)*Sqrt[Sin[e + f*x]^2])) + (4*a^3*(1 + n)*(3 + n)*(3 + 2*n)*Hy
pergeometric2F1[1/2, -1/2*n, (2 - n)/2, Cos[e + f*x]^2]*Sec[e + f*x]^n*Sin
[e + f*x])/(f*n*Sqrt[Sin[e + f*x]^2]))/(1 + n))/(2 + n)))/(3 + n)
```

3.288.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

rule 4259
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)   Int[1/(Sin[c + d*x]/b)^n, x]), x] /;
FreeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4301
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
2)*((d*Csc[e + f*x])^n/(f*(m + n - 1))), x] + Simp[b/(m + n - 1)   Int[(a +
 b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*(b*(m + 2*n - 1) + a*(3*m + 2*n
 - 4)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^
2, 0] && GtQ[m, 1] && NeQ[m + n - 1, 0] && IntegerQ[2*m]
```

rule 4485
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*Cot[
e + f*x]*((d*Csc[e + f*x])^n/(f*(n + 1))), x] + Simp[1/(n + 1)   Int[(d*Csc
[e + f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x
], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[
n, -1]
```

rule 4506
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*
Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*((d*Csc[e + f*x])^n/(f*(m + n))),
 x] + Simp[1/(d*(m + n))   Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x]
)^n*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*
Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b -
 a*B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
```

3.288.4 Maple [F]

$$\int \sec (fx+e)^n (a+a\sec (fx+e))^4 \, dx$$

input
```
int(sec(f*x+e)^n*(a+a*sec(f*x+e))^4,x)
```

output
```
int(sec(f*x+e)^n*(a+a*sec(f*x+e))^4,x)
```

3.288.5 Fricas [F]

$$\int \sec^n(e+fx)(a+a\sec(e+fx))^4 \, dx = \int (a\sec (fx+e)+a)^4 \sec (fx+e)^n \, dx$$

input
```
integrate(sec(f*x+e)^n*(a+a*sec(f*x+e))^4,x, algorithm="fricas")
```

output
```
integral((a^4*sec(f*x + e)^4 + 4*a^4*sec(f*x + e)^3 + 6*a^4*sec(f*x + e)^2
 + 4*a^4*sec(f*x + e) + a^4)*sec(f*x + e)^n, x)
```

3.288.6 Sympy [F]

$$\int \sec^n(e+fx)(a+a\sec(e+fx))^4\,dx = a^4\left(\int 4\sec(e+fx)\sec^n(e+fx)\,dx + \int 6\sec^2(e+fx)\sec^n(e+fx)\,dx + \int 4\sec^3(e+fx)\sec^n(e+fx)\,dx + \int \sec^4(e+fx)\sec^n(e+fx)\,dx + \int \sec^n(e+fx)\,dx\right)$$

input
```
integrate(sec(f*x+e)**n*(a+a*sec(f*x+e))**4,x)
```

output
```
a**4*(Integral(4*sec(e + f*x)*sec(e + f*x)**n, x) + Integral(6*sec(e + f*x
)**2*sec(e + f*x)**n, x) + Integral(4*sec(e + f*x)**3*sec(e + f*x)**n, x)
+ Integral(sec(e + f*x)**4*sec(e + f*x)**n, x) + Integral(sec(e + f*x)**n,
 x))
```

3.288.7 Maxima [F]

$$\int \sec^n(e+fx)(a+a\sec(e+fx))^4\,dx = \int (a\sec(fx+e)+a)^4\sec(fx+e)^n\,dx$$

input
```
integrate(sec(f*x+e)^n*(a+a*sec(f*x+e))^4,x, algorithm="maxima")
```

output
```
integrate((a*sec(f*x + e) + a)^4*sec(f*x + e)^n, x)
```

3.288.8 Giac [F]

$$\int \sec^n(e+fx)(a+a\sec(e+fx))^4\,dx = \int (a\sec(fx+e)+a)^4\sec(fx+e)^n\,dx$$

input
```
integrate(sec(f*x+e)^n*(a+a*sec(f*x+e))^4,x, algorithm="giac")
```

output
```
integrate((a*sec(f*x + e) + a)^4*sec(f*x + e)^n, x)
```

3.288.9 Mupad [F(-1)]

Timed out.

$$\int \sec^n(e+fx)(a+a\sec(e+fx))^4\,dx = \int \left(a+\frac{a}{\cos(e+f\,x)}\right)^4\left(\frac{1}{\cos(e+f\,x)}\right)^n dx$$

input
```
int((a + a/cos(e + f*x))^4*(1/cos(e + f*x))^n,x)
```

output
```
int((a + a/cos(e + f*x))^4*(1/cos(e + f*x))^n, x)
```

3.289 $\int \sec^n(e+fx)(a+a\sec(e+fx))^3\,dx$

3.289.1 Optimal result . . . 2261
3.289.2 Mathematica [A] (verified) . . . 2262
3.289.3 Rubi [A] (verified) . . . 2262
3.289.4 Maple [F] . . . 2265
3.289.5 Fricas [F] . . . 2265
3.289.6 Sympy [F] . . . 2266
3.289.7 Maxima [F] . . . 2266
3.289.8 Giac [F] . . . 2266
3.289.9 Mupad [F(-1)] . . . 2267

3.289.1 Optimal result

Integrand size = 21, antiderivative size = 230

$$\begin{aligned}\int \sec^n(e+fx)(a+a\sec(e+fx))^3\,dx &= \frac{a^3(5+2n)\sec^{1+n}(e+fx)\sin(e+fx)}{f(1+n)(2+n)} \\ &\quad+\frac{\sec^{1+n}(e+fx)\left(a^3+a^3\sec(e+fx)\right)\sin(e+fx)}{f(2+n)} \\ &\quad-\frac{a^3(1+4n)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1-n}{2},\frac{3-n}{2},\cos^2(e+fx)\right)\sec^{-1+n}(e+fx)\sin(e+fx)}{f\left(1-n^2\right)\sqrt{\sin^2(e+fx)}} \\ &\quad+\frac{a^3(7+4n)\,\text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{n}{2},\frac{2-n}{2},\cos^2(e+fx)\right)\sec^n(e+fx)\sin(e+fx)}{fn(2+n)\sqrt{\sin^2(e+fx)}}\end{aligned}$$

output
```
a^3*(5+2*n)*sec(f*x+e)^(1+n)*sin(f*x+e)/f/(1+n)/(2+n)+sec(f*x+e)^(1+n)*(a^
3+a^3*sec(f*x+e))*sin(f*x+e)/f/(2+n)-a^3*(1+4*n)*hypergeom([1/2, 1/2-1/2*n
],[3/2-1/2*n],cos(f*x+e)^2)*sec(f*x+e)^(-1+n)*sin(f*x+e)/f/(-n^2+1)/(sin(f
*x+e)^2)^(1/2)+a^3*(7+4*n)*hypergeom([1/2, -1/2*n],[1-1/2*n],cos(f*x+e)^2)
*sec(f*x+e)^n*sin(f*x+e)/f/n/(2+n)/(sin(f*x+e)^2)^(1/2)
```

3.289.2 Mathematica [A] (verified)

Time = 1.50 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.72

$$\int \sec^n(e+fx)(a+a\sec(e+fx))^3\,dx$$
$$= \frac{a^3\csc(e+fx)\sec^{-1+n}(e+fx)\left(n(3(2+n)+(1+n)\sec(e+fx))\tan^2(e+fx)+(2+9n+4n^2)\,\text{Hyper}\right.}{}$$

input
```
Integrate[Sec[e + f*x]^n*(a + a*Sec[e + f*x])^3,x]
```

output
```
(a^3*Csc[e + f*x]*Sec[e + f*x]^(-1 + n)*(n*(3*(2 + n) + (1 + n)*Sec[e + f*
x])*Tan[e + f*x]^2 + (2 + 9*n + 4*n^2)*Hypergeometric2F1[1/2, n/2, (2 + n)
/2, Sec[e + f*x]^2]*Sqrt[-Tan[e + f*x]^2] + n*(7 + 4*n)*Hypergeometric2F1[
1/2, (1 + n)/2, (3 + n)/2, Sec[e + f*x]^2]*Sec[e + f*x]*Sqrt[-Tan[e + f*x]
^2]))/(f*n*(1 + n)*(2 + n))
```

3.289.3 Rubi [A] (verified)

Time = 0.93 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.04, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {3042, 4301, 3042, 4485, 3042, 4274, 3042, 4259, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sec(e+fx)+a)^3\sec^n(e+fx)\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^3\csc\left(e+fx+\frac{\pi}{2}\right)^n dx$$

$$\Big\downarrow \; 4301$$

$$\frac{a\int \sec^n(e+fx)(\sec(e+fx)a+a)(2a(n+1)+a(2n+5)\sec(e+fx))dx}{n+2}+\frac{\sin(e+fx)\left(a^3\sec(e+fx)+a^3\right)\sec^{n+1}(e+fx)}{f(n+2)}$$

$$\Big\downarrow \; 3042$$

$$\frac{a\int \csc\left(e+fx+\frac{\pi}{2}\right)^n\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)\left(2a(n+1)+a(2n+5)\csc\left(e+fx+\frac{\pi}{2}\right)\right)dx}{n+2}+\frac{\sin(e+fx)\left(a^3\sec(e+fx)+a^3\right)\sec^{n+1}(e+fx)}{f(n+2)}$$

$\Big\downarrow$ 4485

$$\frac{a\left(\frac{\int \sec^n(e+fx)\left((n+2)(4n+1)a^2+(n+1)(4n+7)\sec(e+fx)a^2\right)dx}{n+1}+\frac{a^2(2n+5)\sin(e+fx)\sec^{n+1}(e+fx)}{f(n+1)}\right)}{n+2}+\frac{\sin(e+fx)\left(a^3\sec(e+fx)+a^3\right)\sec^{n+1}(e+fx)}{f(n+2)}$$

$\Big\downarrow$ 3042

$$\frac{a\left(\frac{\int \csc\left(e+fx+\frac{\pi}{2}\right)^n\left((n+2)(4n+1)a^2+(n+1)(4n+7)\csc\left(e+fx+\frac{\pi}{2}\right)a^2\right)dx}{n+1}+\frac{a^2(2n+5)\sin(e+fx)\sec^{n+1}(e+fx)}{f(n+1)}\right)}{n+2}+\frac{\sin(e+fx)\left(a^3\sec(e+fx)+a^3\right)\sec^{n+1}(e+fx)}{f(n+2)}$$

$\Big\downarrow$ 4274

$$\frac{a\left(\frac{a^2(n+1)(4n+7)\int \sec^{n+1}(e+fx)dx+a^2(n+2)(4n+1)\int \sec^n(e+fx)dx}{n+1}+\frac{a^2(2n+5)\sin(e+fx)\sec^{n+1}(e+fx)}{f(n+1)}\right)}{n+2}+\frac{\sin(e+fx)\left(a^3\sec(e+fx)+a^3\right)\sec^{n+1}(e+fx)}{f(n+2)}$$

$\Big\downarrow$ 3042

$$\frac{a\left(\frac{a^2(n+2)(4n+1)\int \csc\left(e+fx+\frac{\pi}{2}\right)^n dx+a^2(n+1)(4n+7)\int \csc\left(e+fx+\frac{\pi}{2}\right)^{n+1}dx}{n+1}+\frac{a^2(2n+5)\sin(e+fx)\sec^{n+1}(e+fx)}{f(n+1)}\right)}{n+2}+\frac{\sin(e+fx)\left(a^3\sec(e+fx)+a^3\right)\sec^{n+1}(e+fx)}{f(n+2)}$$

$\Big\downarrow$ 4259

$$\frac{a\left(\frac{a^2(n+1)(4n+7)\cos^n(e+fx)\sec^n(e+fx)\int \cos^{-n-1}(e+fx)dx+a^2(n+2)(4n+1)\cos^n(e+fx)\sec^n(e+fx)\int \cos^{-n}(e+fx)dx}{n+1}+\frac{a^2(2n+5)\sin(e}{f}\right.}{n+2}$$

$$\frac{\sin(e+fx)\left(a^3\sec(e+fx)+a^3\right)\sec^{n+1}(e+fx)}{f(n+2)}$$

$\Big\downarrow$ 3042

3.289. $\int \sec^n(e+fx)(a+a\sec(e+fx))^3\,dx$

$$\frac{a\left(\frac{a^2(n+1)(4n+7)\cos^n(e+fx)\sec^n(e+fx)\int\sin\left(e+fx+\frac{\pi}{2}\right)^{-n-1}dx+a^2(n+2)(4n+1)\cos^n(e+fx)\sec^n(e+fx)\int\sin\left(e+fx+\frac{\pi}{2}\right)^{-n}dx}{n+1}+\frac{a^2(2n+}{}\right.}{n+2}$$

$$\frac{\sin(e+fx)\left(a^3\sec(e+fx)+a^3\right)\sec^{n+1}(e+fx)}{f(n+2)}$$

$$\Big\downarrow \; 3122$$

$$\frac{\sin(e+fx)\left(a^3\sec(e+fx)+a^3\right)\sec^{n+1}(e+fx)}{f(n+2)}+$$

$$\frac{a\left(\frac{\frac{a^2(n+1)(4n+7)\sin(e+fx)\sec^n(e+fx)\text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{n}{2},\frac{2-n}{2},\cos^2(e+fx)\right)}{fn\sqrt{\sin^2(e+fx)}}-\frac{a^2(n+2)(4n+1)\sin(e+fx)\sec^{n-1}(e+fx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1-n}{2}\right.}{f(1-n)\sqrt{\sin^2(e+fx)}}}{n+1}\right.}{n+2}$$

input
```
Int[Sec[e + f*x]^n*(a + a*Sec[e + f*x])^3,x]
```

output
```
(Sec[e + f*x]^(1 + n)*(a^3 + a^3*Sec[e + f*x])*Sin[e + f*x])/(f*(2 + n)) +
 (a*((a^2*(5 + 2*n)*Sec[e + f*x]^(1 + n)*Sin[e + f*x])/(f*(1 + n)) + (-((a
^2*(2 + n)*(1 + 4*n)*Hypergeometric2F1[1/2, (1 - n)/2, (3 - n)/2, Cos[e +
f*x]^2]*Sec[e + f*x]^(-1 + n)*Sin[e + f*x])/(f*(1 - n)*Sqrt[Sin[e + f*x]^2
])) + (a^2*(1 + n)*(7 + 4*n)*Hypergeometric2F1[1/2, -1/2*n, (2 - n)/2, Cos
[e + f*x]^2]*Sec[e + f*x]^n*Sin[e + f*x])/(f*n*Sqrt[Sin[e + f*x]^2]))/(1 +
 n)))/(2 + n)
```

3.289.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

rule 4259
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)   Int[1/(Sin[c + d*x]/b)^n, x]), x] /;
FreeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4301
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
2)*((d*Csc[e + f*x])^n/(f*(m + n - 1))), x] + Simp[b/(m + n - 1)   Int[(a +
 b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*(b*(m + 2*n - 1) + a*(3*m + 2*n
 - 4)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^
2, 0] && GtQ[m, 1] && NeQ[m + n - 1, 0] && IntegerQ[2*m]
```

rule 4485
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*Cot[
e + f*x]*((d*Csc[e + f*x])^n/(f*(n + 1))), x] + Simp[1/(n + 1)   Int[(d*Csc
[e + f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x
], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[
n, -1]
```

3.289.4 Maple [F]

$$\int \sec(fx+e)^n (a+a\sec(fx+e))^3 \, dx$$

input
```
int(sec(f*x+e)^n*(a+a*sec(f*x+e))^3,x)
```

output
```
int(sec(f*x+e)^n*(a+a*sec(f*x+e))^3,x)
```

3.289.5 Fricas [F]

$$\int \sec^n(e+fx)(a+a\sec(e+fx))^3 \, dx = \int (a\sec(fx+e)+a)^3 \sec(fx+e)^n \, dx$$

input
```
integrate(sec(f*x+e)^n*(a+a*sec(f*x+e))^3,x, algorithm="fricas")
```

output
```
integral((a^3*sec(f*x + e)^3 + 3*a^3*sec(f*x + e)^2 + 3*a^3*sec(f*x + e) +
 a^3)*sec(f*x + e)^n, x)
```

3.289.6 Sympy [F]

$$\int \sec^n(e+fx)(a+a\sec(e+fx))^3\,dx = a^3\left(\int 3\sec(e+fx)\sec^n(e+fx)\,dx + \int 3\sec^2(e+fx)\sec^n(e+fx)\,dx + \int \sec^3(e+fx)\sec^n(e+fx)\,dx + \int \sec^n(e+fx)\,dx\right)$$

input
```
integrate(sec(f*x+e)**n*(a+a*sec(f*x+e))**3,x)
```

output
```
a**3*(Integral(3*sec(e + f*x)*sec(e + f*x)**n, x) + Integral(3*sec(e + f*x
)**2*sec(e + f*x)**n, x) + Integral(sec(e + f*x)**3*sec(e + f*x)**n, x) +
Integral(sec(e + f*x)**n, x))
```

3.289.7 Maxima [F]

$$\int \sec^n(e+fx)(a+a\sec(e+fx))^3\,dx = \int (a\sec(fx+e)+a)^3\sec(fx+e)^n\,dx$$

input
```
integrate(sec(f*x+e)^n*(a+a*sec(f*x+e))^3,x, algorithm="maxima")
```

output
```
integrate((a*sec(f*x + e) + a)^3*sec(f*x + e)^n, x)
```

3.289.8 Giac [F]

$$\int \sec^n(e+fx)(a+a\sec(e+fx))^3\,dx = \int (a\sec(fx+e)+a)^3\sec(fx+e)^n\,dx$$

input
```
integrate(sec(f*x+e)^n*(a+a*sec(f*x+e))^3,x, algorithm="giac")
```

output
```
integrate((a*sec(f*x + e) + a)^3*sec(f*x + e)^n, x)
```

3.289.9 Mupad [F(-1)]

Timed out.

$$\int \sec^n(e+fx)(a+a\sec(e+fx))^3\,dx = \int \left(a+\frac{a}{\cos\left(e+f\,x\right)}\right)^3\left(\frac{1}{\cos\left(e+f\,x\right)}\right)^n dx$$

input
```
int((a + a/cos(e + f*x))^3*(1/cos(e + f*x))^n,x)
```

output
```
int((a + a/cos(e + f*x))^3*(1/cos(e + f*x))^n, x)
```

3.290 $\int \sec^n(e+fx)(a+a\sec(e+fx))^2\,dx$

3.290.1 Optimal result 2268
3.290.2 Mathematica [A] (verified) 2269
3.290.3 Rubi [A] (verified) 2269
3.290.4 Maple [F] 2272
3.290.5 Fricas [F] 2272
3.290.6 Sympy [F] 2273
3.290.7 Maxima [F] 2273
3.290.8 Giac [F] 2273
3.290.9 Mupad [F(-1)] 2274

3.290.1 Optimal result

Integrand size = 21, antiderivative size = 172

$$\begin{aligned}\int \sec^n(e+fx)(a+a\sec(e+fx))^2\,dx &= \frac{a^2\sec^{1+n}(e+fx)\sin(e+fx)}{f(1+n)} \\ &\quad -\frac{a^2(1+2n)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1-n}{2},\frac{3-n}{2},\cos^2(e+fx)\right)\sec^{-1+n}(e+fx)\sin(e+fx)}{f\left(1-n^2\right)\sqrt{\sin^2(e+fx)}} \\ &\quad +\frac{2a^2\,\text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{n}{2},\frac{2-n}{2},\cos^2(e+fx)\right)\sec^n(e+fx)\sin(e+fx)}{fn\sqrt{\sin^2(e+fx)}}\end{aligned}$$

output
```
a^2*sec(f*x+e)^(1+n)*sin(f*x+e)/f/(1+n)-a^2*(1+2*n)*hypergeom([1/2, 1/2-1/
2*n],[3/2-1/2*n],cos(f*x+e)^2)*sec(f*x+e)^(-1+n)*sin(f*x+e)/f/(-n^2+1)/(si
n(f*x+e)^2)^(1/2)+2*a^2*hypergeom([1/2, -1/2*n],[1-1/2*n],cos(f*x+e)^2)*se
c(f*x+e)^n*sin(f*x+e)/f/n/(sin(f*x+e)^2)^(1/2)
```

3.290.2 Mathematica [A] (verified)

Time = 0.74 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.78

$$\int \sec^n(e+fx)(a+a\sec(e+fx))^2\,dx$$
$$= \frac{a^2\csc(e+fx)\sec^{-1+n}(e+fx)\left((1+2n)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n}{2},\frac{2+n}{2},\sec^2(e+fx)\right)\sqrt{-\tan^2(e+fx)}\right.}{fn(1-}$$

input
```
Integrate[Sec[e + f*x]^n*(a + a*Sec[e + f*x])^2,x]
```

output
```
(a^2*Csc[e + f*x]*Sec[e + f*x]^(-1 + n)*((1 + 2*n)*Hypergeometric2F1[1/2,
n/2, (2 + n)/2, Sec[e + f*x]^2]*Sqrt[-Tan[e + f*x]^2] + n*(Tan[e + f*x]^2
+ 2*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2, Sec[e + f*x]^2]*Sec[e + f
*x]*Sqrt[-Tan[e + f*x]^2])))/(f*n*(1 + n))
```

3.290.3 Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.02, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.524$, Rules used = {3042, 4275, 3042, 4259, 3042, 3122, 4534, 3042, 4259, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sec(e+fx)+a)^2\sec^n(e+fx)\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^2\csc\left(e+fx+\frac{\pi}{2}\right)^n dx$$
$$\Big\downarrow \text{4275}$$
$$2a^2\int \sec^{n+1}(e+fx)dx+\int \sec^n(e+fx)\left(\sec^2(e+fx)a^2+a^2\right)dx$$
$$\Big\downarrow \text{3042}$$
$$2a^2\int \csc\left(e+fx+\frac{\pi}{2}\right)^{n+1}dx+\int \csc\left(e+fx+\frac{\pi}{2}\right)^n\left(\csc\left(e+fx+\frac{\pi}{2}\right)^2a^2+a^2\right)dx$$
$$\Big\downarrow \text{4259}$$

$$\int \csc\left(e+fx+\frac{\pi}{2}\right)^n\left(\csc\left(e+fx+\frac{\pi}{2}\right)^2a^2+a^2\right)dx+2a^2\cos^n(e+fx)\sec^n(e+fx)\int\cos^{-n-1}(e+fx)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(e+fx+\frac{\pi}{2}\right)^n\left(\csc\left(e+fx+\frac{\pi}{2}\right)^2a^2+a^2\right)dx+2a^2\cos^n(e+fx)\sec^n(e+fx)\int\sin\left(e+fx+\frac{\pi}{2}\right)^{-n-1}dx$$

$$\Big\downarrow \text{3122}$$

$$\int \csc\left(e+fx+\frac{\pi}{2}\right)^n\left(\csc\left(e+fx+\frac{\pi}{2}\right)^2a^2+a^2\right)dx+\frac{2a^2\sin(e+fx)\sec^n(e+fx)\text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{n}{2},\frac{2-n}{2},\cos^2(e+fx)\right)}{fn\sqrt{\sin^2(e+fx)}}$$

$$\Big\downarrow \text{4534}$$

$$\frac{a^2(2n+1)\int\sec^n(e+fx)dx}{n+1}+\frac{2a^2\sin(e+fx)\sec^n(e+fx)\text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{n}{2},\frac{2-n}{2},\cos^2(e+fx)\right)}{fn\sqrt{\sin^2(e+fx)}}+\frac{a^2\sin(e+fx)\sec^{n+1}(e+fx)}{f(n+1)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a^2(2n+1)\int\csc\left(e+fx+\frac{\pi}{2}\right)^n dx}{n+1}+\frac{2a^2\sin(e+fx)\sec^n(e+fx)\text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{n}{2},\frac{2-n}{2},\cos^2(e+fx)\right)}{fn\sqrt{\sin^2(e+fx)}}+\frac{a^2\sin(e+fx)\sec^{n+1}(e+fx)}{f(n+1)}$$

$$\Big\downarrow \text{4259}$$

$$\frac{a^2(2n+1)\cos^n(e+fx)\sec^n(e+fx)\int\cos^{-n}(e+fx)dx}{n+1}+\frac{2a^2\sin(e+fx)\sec^n(e+fx)\text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{n}{2},\frac{2-n}{2},\cos^2(e+fx)\right)}{fn\sqrt{\sin^2(e+fx)}}+\frac{a^2\sin(e+fx)\sec^{n+1}(e+fx)}{f(n+1)}$$

$$\Big\downarrow \text{3042}$$

3.290. $\int \sec^n(e+fx)(a+a\sec(e+fx))^2\,dx$

$$\frac{a^2(2n+1)\cos^n(e+fx)\sec^n(e+fx)\int \sin\left(e+fx+\frac{\pi}{2}\right)^{-n}dx}{n+1}+\frac{2a^2\sin(e+fx)\sec^n(e+fx)\text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{n}{2},\frac{2-n}{2},\cos^2(e+fx)\right)}{fn\sqrt{\sin^2(e+fx)}}+\frac{a^2\sin(e+fx)\sec^{n+1}(e+fx)}{f(n+1)}$$

$$\Big\downarrow \; 3122$$

$$-\frac{a^2(2n+1)\sin(e+fx)\sec^{n-1}(e+fx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1-n}{2},\frac{3-n}{2},\cos^2(e+fx)\right)}{f(1-n)(n+1)\sqrt{\sin^2(e+fx)}}+\frac{2a^2\sin(e+fx)\sec^n(e+fx)\text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{n}{2},\frac{2-n}{2},\cos^2(e+fx)\right)}{fn\sqrt{\sin^2(e+fx)}}+\frac{a^2\sin(e+fx)\sec^{n+1}(e+fx)}{f(n+1)}$$

input
```
Int[Sec[e + f*x]^n*(a + a*Sec[e + f*x])^2,x]
```

output
```
(a^2*Sec[e + f*x]^(1 + n)*Sin[e + f*x])/(f*(1 + n)) - (a^2*(1 + 2*n)*Hyper
geometric2F1[1/2, (1 - n)/2, (3 - n)/2, Cos[e + f*x]^2]*Sec[e + f*x]^(-1 +
 n)*Sin[e + f*x])/(f*(1 - n)*(1 + n)*Sqrt[Sin[e + f*x]^2]) + (2*a^2*Hyperg
eometric2F1[1/2, -1/2*n, (2 - n)/2, Cos[e + f*x]^2]*Sec[e + f*x]^n*Sin[e +
 f*x])/(f*n*Sqrt[Sin[e + f*x]^2])
```

3.290.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

rule 4259
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)   Int[1/(Sin[c + d*x]/b)^n, x]), x] /;
FreeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

rule 4275
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Simp[2*a*(b/d)   Int[(d*Csc[e + f*x])^(n + 1), x], x]
 + Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
 e, f, n}, x]
```

rule 4534
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Simp[(C*m + A*(m + 1))/(m + 1)   Int[(b*Csc[e + f*x])^m, x], x] /;
 FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

3.290.4 Maple [F]

$$\int \sec (fx+e)^n (a+a\sec (fx+e))^2 \, dx$$

input
```
int(sec(f*x+e)^n*(a+a*sec(f*x+e))^2,x)
```

output
```
int(sec(f*x+e)^n*(a+a*sec(f*x+e))^2,x)
```

3.290.5 Fricas [F]

$$\int \sec^n(e+fx)(a+a\sec(e+fx))^2 \, dx = \int (a\sec (fx+e)+a)^2 \sec (fx+e)^n \, dx$$

input
```
integrate(sec(f*x+e)^n*(a+a*sec(f*x+e))^2,x, algorithm="fricas")
```

output
```
integral((a^2*sec(f*x + e)^2 + 2*a^2*sec(f*x + e) + a^2)*sec(f*x + e)^n, x
)
```

3.290.6 Sympy [F]

$$\int \sec^n(e+fx)(a+a\sec(e+fx))^2\,dx = a^2\left(\int 2\sec(e+fx)\sec^n(e+fx)\,dx + \int \sec^2(e+fx)\sec^n(e+fx)\,dx + \int \sec^n(e+fx)\,dx\right)$$

input
```
integrate(sec(f*x+e)**n*(a+a*sec(f*x+e))**2,x)
```

output
```
a**2*(Integral(2*sec(e + f*x)*sec(e + f*x)**n, x) + Integral(sec(e + f*x)*
*2*sec(e + f*x)**n, x) + Integral(sec(e + f*x)**n, x))
```

3.290.7 Maxima [F]

$$\int \sec^n(e+fx)(a+a\sec(e+fx))^2\,dx = \int (a\sec(fx+e)+a)^2\sec(fx+e)^n\,dx$$

input
```
integrate(sec(f*x+e)^n*(a+a*sec(f*x+e))^2,x, algorithm="maxima")
```

output
```
integrate((a*sec(f*x + e) + a)^2*sec(f*x + e)^n, x)
```

3.290.8 Giac [F]

$$\int \sec^n(e+fx)(a+a\sec(e+fx))^2\,dx = \int (a\sec(fx+e)+a)^2\sec(fx+e)^n\,dx$$

input
```
integrate(sec(f*x+e)^n*(a+a*sec(f*x+e))^2,x, algorithm="giac")
```

output
```
integrate((a*sec(f*x + e) + a)^2*sec(f*x + e)^n, x)
```

3.290.9 Mupad [F(-1)]

Timed out.

$$\int \sec^n(e+fx)(a+a\sec(e+fx))^2\,dx = \int \left(a+\frac{a}{\cos\left(e+f\,x\right)}\right)^2\left(\frac{1}{\cos\left(e+f\,x\right)}\right)^n dx$$

input
```
int((a + a/cos(e + f*x))^2*(1/cos(e + f*x))^n,x)
```

output
```
int((a + a/cos(e + f*x))^2*(1/cos(e + f*x))^n, x)
```

3.291 $\int \sec^n(e+fx)(a+a\sec(e+fx))\,dx$

3.291.1 Optimal result . . . 2275
3.291.2 Mathematica [A] (verified) . . . 2275
3.291.3 Rubi [A] (verified) . . . 2276
3.291.4 Maple [F] . . . 2277
3.291.5 Fricas [F] . . . 2278
3.291.6 Sympy [F] . . . 2278
3.291.7 Maxima [F] . . . 2278
3.291.8 Giac [F] . . . 2279
3.291.9 Mupad [F(-1)] . . . 2279

3.291.1 Optimal result

Integrand size = 19, antiderivative size = 132

$$\int \sec^n(e+fx)(a+a\sec(e+fx))\,dx = -\frac{a\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1-n}{2},\frac{3-n}{2},\cos^2(e+fx)\right)\sec^{-1+n}(e+fx)\sin(e+fx)}{f(1-n)\sqrt{\sin^2(e+fx)}} + \frac{a\,\text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{n}{2},\frac{2-n}{2},\cos^2(e+fx)\right)\sec^n(e+fx)\sin(e+fx)}{fn\sqrt{\sin^2(e+fx)}}$$

output
```
-a*hypergeom([1/2, 1/2-1/2*n],[3/2-1/2*n],cos(f*x+e)^2)*sec(f*x+e)^(-1+n)*
sin(f*x+e)/f/(1-n)/(sin(f*x+e)^2)^(1/2)+a*hypergeom([1/2, -1/2*n],[1-1/2*n
],cos(f*x+e)^2)*sec(f*x+e)^n*sin(f*x+e)/f/n/(sin(f*x+e)^2)^(1/2)
```

3.291.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.80

$$\int \sec^n(e+fx)(a+a\sec(e+fx))\,dx = \frac{a\csc(e+fx)\sec^{-1+n}(e+fx)\left((1+n)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n}{2},\frac{2+n}{2},\sec^2(e+fx)\right)+n\,\text{Hypergeometric}\right.}{fn(1+n)}$$

input
```
Integrate[Sec[e + f*x]^n*(a + a*Sec[e + f*x]),x]
```

output
```
(a*Csc[e + f*x]*Sec[e + f*x]^(-1 + n)*((1 + n)*Hypergeometric2F1[1/2, n/2,
 (2 + n)/2, Sec[e + f*x]^2] + n*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/
2, Sec[e + f*x]^2]*Sec[e + f*x])*Sqrt[-Tan[e + f*x]^2])/(f*n*(1 + n))
```

3.291.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {3042, 4274, 3042, 4259, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sec(e+fx)+a)\sec^n(e+fx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)\csc\left(e+fx+\frac{\pi}{2}\right)^n dx$$

$$\Big\downarrow \text{4274}$$

$$a\int \sec^{n+1}(e+fx)dx+a\int \sec^n(e+fx)dx$$

$$\Big\downarrow \text{3042}$$

$$a\int \csc\left(e+fx+\frac{\pi}{2}\right)^n dx+a\int \csc\left(e+fx+\frac{\pi}{2}\right)^{n+1} dx$$

$$\Big\downarrow \text{4259}$$

$$a\cos^n(e+fx)\sec^n(e+fx)\int \cos^{-n-1}(e+fx)dx+a\cos^n(e+fx)\sec^n(e+fx)\int \cos^{-n}(e+fx)dx$$

$$\Big\downarrow \text{3042}$$

$$a\cos^n(e+fx)\sec^n(e+fx)\int \sin\left(e+fx+\frac{\pi}{2}\right)^{-n-1} dx+a\cos^n(e+fx)\sec^n(e+fx)\int \sin\left(e+fx+\frac{\pi}{2}\right)^{-n} dx$$

$$\Big\downarrow \text{3122}$$

$$\frac{a\sin(e+fx)\sec^n(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{n}{2},\frac{2-n}{2},\cos^2(e+fx)\right)}{fn\sqrt{\sin^2(e+fx)}}-\frac{a\sin(e+fx)\sec^{n-1}(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1-n}{2},\frac{3-n}{2},\cos^2(e+fx)\right)}{f(1-n)\sqrt{\sin^2(e+fx)}}$$

input
```
Int[Sec[e + f*x]^n*(a + a*Sec[e + f*x]),x]
```

output
```
-((a*Hypergeometric2F1[1/2, (1 - n)/2, (3 - n)/2, Cos[e + f*x]^2]*Sec[e +
f*x]^(-1 + n)*Sin[e + f*x])/(f*(1 - n)*Sqrt[Sin[e + f*x]^2])) + (a*Hyperge
ometric2F1[1/2, -1/2*n, (2 - n)/2, Cos[e + f*x]^2]*Sec[e + f*x]^n*Sin[e +
f*x])/(f*n*Sqrt[Sin[e + f*x]^2])
```

3.291.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

rule 4259
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)   Int[1/(Sin[c + d*x]/b)^n, x]), x] /;
FreeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

3.291.4 Maple [F]

$$\int \sec(fx+e)^n (a + a\sec(fx+e))\,dx$$

input
```
int(sec(f*x+e)^n*(a+a*sec(f*x+e)),x)
```

output
```
int(sec(f*x+e)^n*(a+a*sec(f*x+e)),x)
```

3.291.5 Fricas [F]

$$\int \sec^n(e+fx)(a+a\sec(e+fx))\,dx = \int (a\sec(fx+e)+a)\sec(fx+e)^n\,dx$$

input
```
integrate(sec(f*x+e)^n*(a+a*sec(f*x+e)),x, algorithm="fricas")
```

output
```
integral((a*sec(f*x + e) + a)*sec(f*x + e)^n, x)
```

3.291.6 Sympy [F]

$$\int \sec^n(e+fx)(a+a\sec(e+fx))\,dx = a\left(\int \sec(e+fx)\sec^n(e+fx)\,dx + \int \sec^n(e+fx)\,dx\right)$$

input
```
integrate(sec(f*x+e)**n*(a+a*sec(f*x+e)),x)
```

output
```
a*(Integral(sec(e + f*x)*sec(e + f*x)**n, x) + Integral(sec(e + f*x)**n, x
))
```

3.291.7 Maxima [F]

$$\int \sec^n(e+fx)(a+a\sec(e+fx))\,dx = \int (a\sec(fx+e)+a)\sec(fx+e)^n\,dx$$

input
```
integrate(sec(f*x+e)^n*(a+a*sec(f*x+e)),x, algorithm="maxima")
```

output
```
integrate((a*sec(f*x + e) + a)*sec(f*x + e)^n, x)
```

3.291.8 Giac [F]

$$\int \sec^n(e+fx)(a+a\sec(e+fx))\,dx = \int (a\sec(fx+e)+a)\sec(fx+e)^n\,dx$$

input
```
integrate(sec(f*x+e)^n*(a+a*sec(f*x+e)),x, algorithm="giac")
```

output
```
integrate((a*sec(f*x + e) + a)*sec(f*x + e)^n, x)
```

3.291.9 Mupad [F(-1)]

Timed out.

$$\int \sec^n(e+fx)(a+a\sec(e+fx))\,dx = \int \left(a+\frac{a}{\cos(e+f\,x)}\right)\left(\frac{1}{\cos(e+f\,x)}\right)^n dx$$

input
```
int((a + a/cos(e + f*x))*(1/cos(e + f*x))^n,x)
```

output
```
int((a + a/cos(e + f*x))*(1/cos(e + f*x))^n, x)
```

3.292 $\int \frac{\sec^n(e+fx)}{a+a\sec(e+fx)}\,dx$

3.292.1 Optimal result . . . 2280
3.292.2 Mathematica [A] (verified) . . . 2281
3.292.3 Rubi [A] (verified) . . . 2281
3.292.4 Maple [F] . . . 2283
3.292.5 Fricas [F] . . . 2284
3.292.6 Sympy [F] . . . 2284
3.292.7 Maxima [F] . . . 2284
3.292.8 Giac [F] . . . 2285
3.292.9 Mupad [F(-1)] . . . 2285

3.292.1 Optimal result

Integrand size = 21, antiderivative size = 174

$$\begin{aligned}\int \frac{\sec^n(e+fx)}{a+a\sec(e+fx)}\,dx &= \frac{\sec^n(e+fx)\sin(e+fx)}{f(a+a\sec(e+fx))} \\ &\quad + \frac{(1-n)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2-n}{2},\frac{4-n}{2},\cos^2(e+fx)\right)\sec^{-2+n}(e+fx)\sin(e+fx)}{af(2-n)\sqrt{\sin^2(e+fx)}} \\ &\quad - \frac{\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1-n}{2},\frac{3-n}{2},\cos^2(e+fx)\right)\sec^{-1+n}(e+fx)\sin(e+fx)}{af\sqrt{\sin^2(e+fx)}}\end{aligned}$$

output
```
sec(f*x+e)^n*sin(f*x+e)/f/(a+a*sec(f*x+e))+(1-n)*hypergeom([1/2, 1-1/2*n],
[2-1/2*n],cos(f*x+e)^2)*sec(f*x+e)^(-2+n)*sin(f*x+e)/a/f/(2-n)/(sin(f*x+e)
^2)^(1/2)-hypergeom([1/2, 1/2-1/2*n],[3/2-1/2*n],cos(f*x+e)^2)*sec(f*x+e)^
(-1+n)*sin(f*x+e)/a/f/(sin(f*x+e)^2)^(1/2)
```

3.292.2 Mathematica [A] (verified)

Time = 0.80 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.80

$$\int \frac{\sec^n(e+fx)}{a+a\sec(e+fx)}\,dx = \frac{\cot\left(\frac{1}{2}(e+fx)\right)\sec^n(e+fx)\left(n-n\cos(e+fx)+n\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(-1+n),\frac{1+n}{2}\right.\right.}{afn(1+}$$

input
```
Integrate[Sec[e + f*x]^n/(a + a*Sec[e + f*x]),x]
```

output
```
(Cot[(e + f*x)/2]*Sec[e + f*x]^n*(n - n*Cos[e + f*x] + n*Cos[e + f*x]*Hype
rgeometric2F1[1/2, (-1 + n)/2, (1 + n)/2, Sec[e + f*x]^2]*Sqrt[-Tan[e + f*
x]^2] - (-1 + n)*Hypergeometric2F1[1/2, n/2, (2 + n)/2, Sec[e + f*x]^2]*Sq
rt[-Tan[e + f*x]^2]))/(a*f*n*(1 + Sec[e + f*x]))
```

3.292.3 Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {3042, 4307, 3042, 4274, 3042, 4259, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^n(e+fx)}{a\sec(e+fx)+a}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)^n}{a\csc\left(e+fx+\frac{\pi}{2}\right)+a}dx$$

$$\Big\downarrow \; 4307$$

$$\frac{\sin(e+fx)\sec^n(e+fx)}{f(a\sec(e+fx)+a)} - \frac{(1-n)\int \sec^{n-1}(e+fx)(a-a\sec(e+fx))dx}{a^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{\sin(e+fx)\sec^n(e+fx)}{f(a\sec(e+fx)+a)} - \frac{(1-n)\int \csc\left(e+fx+\frac{\pi}{2}\right)^{n-1}\left(a-a\csc\left(e+fx+\frac{\pi}{2}\right)\right)dx}{a^2}$$

$$\Big\downarrow \text{4274}$$

$$\frac{\sin(e+fx)\sec^n(e+fx)}{f(a\sec(e+fx)+a)}-\frac{(1-n)\left(a\int\sec^{n-1}(e+fx)dx-a\int\sec^n(e+fx)dx\right)}{a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sin(e+fx)\sec^n(e+fx)}{f(a\sec(e+fx)+a)}-\frac{(1-n)\left(a\int\csc\left(e+fx+\frac{\pi}{2}\right)^{n-1}dx-a\int\csc\left(e+fx+\frac{\pi}{2}\right)^n dx\right)}{a^2}$$

$$\Big\downarrow \text{4259}$$

$$\frac{\sin(e+fx)\sec^n(e+fx)}{f(a\sec(e+fx)+a)}-\frac{(1-n)\left(a\cos^n(e+fx)\sec^n(e+fx)\int\cos^{1-n}(e+fx)dx-a\cos^n(e+fx)\sec^n(e+fx)\int\cos^{-n}(e+fx)dx\right)}{a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sin(e+fx)\sec^n(e+fx)}{f(a\sec(e+fx)+a)}-\frac{(1-n)\left(a\cos^n(e+fx)\sec^n(e+fx)\int\sin\left(e+fx+\frac{\pi}{2}\right)^{1-n}dx-a\cos^n(e+fx)\sec^n(e+fx)\int\sin\left(e+fx+\frac{\pi}{2}\right)^{-}\right.}{a^2}$$

$$\Big\downarrow \text{3122}$$

$$\frac{\sin(e+fx)\sec^n(e+fx)}{f(a\sec(e+fx)+a)}-\frac{(1-n)\left(\frac{a\sin(e+fx)\sec^{n-1}(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1-n}{2},\frac{3-n}{2},\cos^2(e+fx)\right)}{f(1-n)\sqrt{\sin^2(e+fx)}}-\frac{a\sin(e+fx)\sec^{n-2}(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2-n}{2}\right.}{f(2-n)\sqrt{\sin^2(e+fx)}}\right.}{a^2}$$

input
```
Int[Sec[e + f*x]^n/(a + a*Sec[e + f*x]),x]
```

output
```
(Sec[e + f*x]^n*Sin[e + f*x])/(f*(a + a*Sec[e + f*x])) - ((1 - n)*(-((a*Hy
pergeometric2F1[1/2, (2 - n)/2, (4 - n)/2, Cos[e + f*x]^2]*Sec[e + f*x]^(-
2 + n)*Sin[e + f*x])/(f*(2 - n)*Sqrt[Sin[e + f*x]^2])) + (a*Hypergeometric
2F1[1/2, (1 - n)/2, (3 - n)/2, Cos[e + f*x]^2]*Sec[e + f*x]^(-1 + n)*Sin[e
 + f*x])/(f*(1 - n)*Sqrt[Sin[e + f*x]^2])))/a^2
```

3.292. $\int \frac{\sec^n(e+fx)}{a+a\sec(e+fx)}\,dx$

3.292.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 4259
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)   Int[1/(Sin[c + d*x]/b)^n, x]), x] /;
FreeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4307
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-b)*d*Cot[e + f*x]*((d*Csc[e + f*x])^(n - 1)/(a*f*
(a + b*Csc[e + f*x]))), x] + Simp[d*((n - 1)/(a*b))   Int[(d*Csc[e + f*x])^
(n - 1)*(a - b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ
[a^2 - b^2, 0]
```

3.292.4 Maple [F]

$$\int \frac{\sec(fx+e)^n}{a+a\sec(fx+e)}dx$$

input
```
int(sec(f*x+e)^n/(a+a*sec(f*x+e)),x)
```

output
```
int(sec(f*x+e)^n/(a+a*sec(f*x+e)),x)
```

3.292.5 Fricas [F]

$$\int \frac{\sec^n(e+fx)}{a+a\sec(e+fx)}\,dx = \int \frac{\sec(fx+e)^n}{a\sec(fx+e)+a}\,dx$$

input
```
integrate(sec(f*x+e)^n/(a+a*sec(f*x+e)),x, algorithm="fricas")
```

output
```
integral(sec(f*x + e)^n/(a*sec(f*x + e) + a), x)
```

3.292.6 Sympy [F]

$$\int \frac{\sec^n(e+fx)}{a+a\sec(e+fx)}\,dx = \frac{\int \frac{\sec^n(e+fx)}{\sec(e+fx)+1}\,dx}{a}$$

input
```
integrate(sec(f*x+e)**n/(a+a*sec(f*x+e)),x)
```

output
```
Integral(sec(e + f*x)**n/(sec(e + f*x) + 1), x)/a
```

3.292.7 Maxima [F]

$$\int \frac{\sec^n(e+fx)}{a+a\sec(e+fx)}\,dx = \int \frac{\sec(fx+e)^n}{a\sec(fx+e)+a}\,dx$$

input
```
integrate(sec(f*x+e)^n/(a+a*sec(f*x+e)),x, algorithm="maxima")
```

output
```
integrate(sec(f*x + e)^n/(a*sec(f*x + e) + a), x)
```

3.292.8 Giac [F]

$$\int \frac{\sec^n(e+fx)}{a+a\sec(e+fx)}\,dx = \int \frac{\sec(fx+e)^n}{a\sec(fx+e)+a}\,dx$$

input
```
integrate(sec(f*x+e)^n/(a+a*sec(f*x+e)),x, algorithm="giac")
```

output
```
integrate(sec(f*x + e)^n/(a*sec(f*x + e) + a), x)
```

3.292.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^n(e+fx)}{a+a\sec(e+fx)}\,dx = \int \frac{\left(\frac{1}{\cos(e+f\,x)}\right)^n}{a+\frac{a}{\cos(e+f\,x)}}\,dx$$

input
```
int((1/cos(e + f*x))^n/(a + a/cos(e + f*x)),x)
```

output
```
int((1/cos(e + f*x))^n/(a + a/cos(e + f*x)), x)
```

3.293 $\int \frac{\sec^n(e+fx)}{(a+a\sec(e+fx))^2}\,dx$

3.293.1 Optimal result . . . 2286
3.293.2 Mathematica [A] (verified) . . . 2287
3.293.3 Rubi [A] (verified) . . . 2287
3.293.4 Maple [F] . . . 2290
3.293.5 Fricas [F] . . . 2291
3.293.6 Sympy [F] . . . 2291
3.293.7 Maxima [F] . . . 2291
3.293.8 Giac [F] . . . 2292
3.293.9 Mupad [F(-1)] . . . 2292

3.293.1 Optimal result

Integrand size = 21, antiderivative size = 217

$$\begin{aligned}\int \frac{\sec^n(e+fx)}{(a+a\sec(e+fx))^2}\,dx &= -\frac{2(2-n)\sec^{1+n}(e+fx)\sin(e+fx)}{3a^2f(1+\sec(e+fx))}-\frac{\sec^{1+n}(e+fx)\sin(e+fx)}{3f(a+a\sec(e+fx))^2}\\ &\quad-\frac{(3-2n)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1-n}{2},\frac{3-n}{2},\cos^2(e+fx)\right)\sec^{-1+n}(e+fx)\sin(e+fx)}{3a^2f\sqrt{\sin^2(e+fx)}}\\ &\quad+\frac{2(2-n)\,\text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{n}{2},\frac{2-n}{2},\cos^2(e+fx)\right)\sec^{n}(e+fx)\sin(e+fx)}{3a^2f\sqrt{\sin^2(e+fx)}}\end{aligned}$$

output
```
-2/3*(2-n)*sec(f*x+e)^(1+n)*sin(f*x+e)/a^2/f/(1+sec(f*x+e))-1/3*sec(f*x+e)
^(1+n)*sin(f*x+e)/f/(a+a*sec(f*x+e))^2-1/3*(3-2*n)*hypergeom([1/2, 1/2-1/2
*n],[3/2-1/2*n],cos(f*x+e)^2)*sec(f*x+e)^(-1+n)*sin(f*x+e)/a^2/f/(sin(f*x+
e)^2)^(1/2)+2/3*(2-n)*hypergeom([1/2, -1/2*n],[1-1/2*n],cos(f*x+e)^2)*sec(
f*x+e)^n*sin(f*x+e)/a^2/f/(sin(f*x+e)^2)^(1/2)
```

3.293.2 Mathematica [A] (verified)

Time = 1.83 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.88

$$\int \frac{\sec^n(e+fx)}{(a+a\sec(e+fx))^2}\,dx$$
$$= \frac{\csc(e+fx)\sec^n(e+fx)\left(-n(1+n)\sin(e+fx)\tan(e+fx)+(1+\sec(e+fx))\left(2(-2+n)n(1+n)\,\mathrm{si}\right.\right.}{}$$

input
```
Integrate[Sec[e + f*x]^n/(a + a*Sec[e + f*x])^2,x]
```

output
```
(Csc[e + f*x]*Sec[e + f*x]^n*(-(n*(1 + n)*Sin[e + f*x]*Tan[e + f*x]) + (1
+ Sec[e + f*x])*(2*(-2 + n)*n*(1 + n)*Sin[e + f*x]*Tan[e + f*x] + ((-1 + n
)*(1 + n)*(-3 + 2*n)*Cos[e + f*x]*Hypergeometric2F1[1/2, n/2, (2 + n)/2, S
ec[e + f*x]^2] - 2*(-2 + n)*n^2*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/
2, Sec[e + f*x]^2])*(1 + Sec[e + f*x])*Sqrt[-Tan[e + f*x]^2])))/(3*a^2*f*n
*(1 + n)*(1 + Sec[e + f*x])^2)
```

3.293.3 Rubi [A] (verified)

Time = 0.96 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.02, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.524$, Rules used = {3042, 4304, 25, 3042, 4508, 3042, 4274, 3042, 4259, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^n(e+fx)}{(a\sec(e+fx)+a)^2}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)^n}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^2}dx$$
$$\Big\downarrow \; 4304$$
$$-\frac{\int -\frac{\sec^n(e+fx)(a(3-n)-a(1-n)\sec(e+fx))}{\sec(e+fx)a+a}dx}{3a^2}-\frac{\sin(e+fx)\sec^{n+1}(e+fx)}{3f(a\sec(e+fx)+a)^2}$$
$$\Big\downarrow \; 25$$

$$\frac{\int \frac{\sec^n(e+fx)(a(3-n)-a(1-n)\sec(e+fx))}{\sec(e+fx)a+a}dx}{3a^2}-\frac{\sin(e+fx)\sec^{n+1}(e+fx)}{3f(a\sec(e+fx)+a)^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)^n\left(a(3-n)-a(1-n)\csc\left(e+fx+\frac{\pi}{2}\right)\right)}{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}dx}{3a^2}-\frac{\sin(e+fx)\sec^{n+1}(e+fx)}{3f(a\sec(e+fx)+a)^2}$$

$$\Big\downarrow \; 4508$$

$$\frac{\frac{\int \sec^n(e+fx)\left((3-2n)(1-n)a^2+2(2-n)n\sec(e+fx)a^2\right)dx}{a^2}-\frac{2(2-n)\sin(e+fx)\sec^{n+1}(e+fx)}{f(\sec(e+fx)+1)}}{3a^2}-\frac{\sin(e+fx)\sec^{n+1}(e+fx)}{3f(a\sec(e+fx)+a)^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{\int \csc\left(e+fx+\frac{\pi}{2}\right)^n\left((3-2n)(1-n)a^2+2(2-n)n\csc\left(e+fx+\frac{\pi}{2}\right)a^2\right)dx}{a^2}-\frac{2(2-n)\sin(e+fx)\sec^{n+1}(e+fx)}{f(\sec(e+fx)+1)}}{3a^2}-\frac{\sin(e+fx)\sec^{n+1}(e+fx)}{3f(a\sec(e+fx)+a)^2}$$

$$\Big\downarrow \; 4274$$

$$\frac{\frac{2a^2(2-n)n\int \sec^{n+1}(e+fx)dx+a^2(3-2n)(1-n)\int \sec^n(e+fx)dx}{a^2}-\frac{2(2-n)\sin(e+fx)\sec^{n+1}(e+fx)}{f(\sec(e+fx)+1)}}{3a^2}-\frac{\sin(e+fx)\sec^{n+1}(e+fx)}{3f(a\sec(e+fx)+a)^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{a^2(3-2n)(1-n)\int \csc\left(e+fx+\frac{\pi}{2}\right)^n dx+2a^2(2-n)n\int \csc\left(e+fx+\frac{\pi}{2}\right)^{n+1}dx}{a^2}-\frac{2(2-n)\sin(e+fx)\sec^{n+1}(e+fx)}{f(\sec(e+fx)+1)}}{3a^2}-\frac{\sin(e+fx)\sec^{n+1}(e+fx)}{3f(a\sec(e+fx)+a)^2}$$

$$\Big\downarrow \; 4259$$

$$\frac{\frac{2a^2(2-n)n\cos^n(e+fx)\sec^n(e+fx)\int \cos^{-n-1}(e+fx)dx+a^2(3-2n)(1-n)\cos^n(e+fx)\sec^n(e+fx)\int \cos^{-n}(e+fx)dx}{a^2}-\frac{2(2-n)\sin(e+fx)\sec^{n}}{f(\sec(e+fx)+}}{3a^2}-\frac{\sin(e+fx)\sec^{n+1}(e+fx)}{3f(a\sec(e+fx)+a)^2}$$

$$\Big\downarrow \; 3042$$

3.293. $\int \frac{\sec^n(e+fx)}{(a+a\sec(e+fx))^2}\,dx$

$$\frac{\frac{\frac{2a^2(2-n)n\cos^n(e+fx)\sec^n(e+fx)\int\sin\left(e+fx+\frac{\pi}{2}\right)^{-n-1}dx+a^2(3-2n)(1-n)\cos^n(e+fx)\sec^n(e+fx)\int\sin\left(e+fx+\frac{\pi}{2}\right)^{-n}dx}{a^2}-\frac{2(2-n)\sin(e+f}{f(\sec(e}}{3a^2}}{\frac{\sin(e+fx)\sec^{n+1}(e+fx)}{3f(a\sec(e+fx)+a)^2}}$$

$$\Big\downarrow \; 3122$$

$$\frac{\frac{\frac{\frac{2a^2(2-n)\sin(e+fx)\sec^n(e+fx)\text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{n}{2},\frac{2-n}{2},\cos^2(e+fx)\right)}{f\sqrt{\sin^2(e+fx)}}-\frac{a^2(3-2n)\sin(e+fx)\sec^{n-1}(e+fx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1-n}{2},\frac{3-n}{2},\cos^2(e+f\right.}{f\sqrt{\sin^2(e+fx)}}}{a^2}}{3a^2}}{\frac{\sin(e+fx)\sec^{n+1}(e+fx)}{3f(a\sec(e+fx)+a)^2}}$$

input
```
Int[Sec[e + f*x]^n/(a + a*Sec[e + f*x])^2,x]
```

output
```
-1/3*(Sec[e + f*x]^(1 + n)*Sin[e + f*x])/(f*(a + a*Sec[e + f*x])^2) + ((-2
*(2 - n)*Sec[e + f*x]^(1 + n)*Sin[e + f*x])/(f*(1 + Sec[e + f*x])) + (-((a
^2*(3 - 2*n)*Hypergeometric2F1[1/2, (1 - n)/2, (3 - n)/2, Cos[e + f*x]^2]*
Sec[e + f*x]^(-1 + n)*Sin[e + f*x])/(f*Sqrt[Sin[e + f*x]^2])) + (2*a^2*(2
- n)*Hypergeometric2F1[1/2, -1/2*n, (2 - n)/2, Cos[e + f*x]^2]*Sec[e + f*x
]^n*Sin[e + f*x])/(f*Sqrt[Sin[e + f*x]^2]))/a^2)/(3*a^2)
```

3.293.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 4259
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)   Int[1/(Sin[c + d*x]/b)^n, x]), x] /;
FreeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4304
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-Cot[e + f*x])*(a + b*Csc[e + f*x])^m*((d*Csc
[e + f*x])^n/(f*(2*m + 1))), x] + Simp[1/(a^2*(2*m + 1))   Int[(a + b*Csc[e
 + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e
+ f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ
[m, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

rule 4508
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
- a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
 1))), x] - Simp[1/(a^2*(2*m + 1))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Cs
c[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[
e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B
, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

3.293.4 Maple [F]

$$\int \frac{\sec(fx+e)^n}{(a+a\sec(fx+e))^2}dx$$

input
```
int(sec(f*x+e)^n/(a+a*sec(f*x+e))^2,x)
```

output
```
int(sec(f*x+e)^n/(a+a*sec(f*x+e))^2,x)
```

3.293.5 Fricas [F]

$$\int \frac{\sec^n(e+fx)}{(a+a\sec(e+fx))^2}\,dx = \int \frac{\sec(fx+e)^n}{(a\sec(fx+e)+a)^2}\,dx$$

input
```
integrate(sec(f*x+e)^n/(a+a*sec(f*x+e))^2,x, algorithm="fricas")
```

output
```
integral(sec(f*x + e)^n/(a^2*sec(f*x + e)^2 + 2*a^2*sec(f*x + e) + a^2), x
)
```

3.293.6 Sympy [F]

$$\int \frac{\sec^n(e+fx)}{(a+a\sec(e+fx))^2}\,dx = \frac{\int \frac{\sec^n(e+fx)}{\sec^2(e+fx)+2\sec(e+fx)+1}\,dx}{a^2}$$

input
```
integrate(sec(f*x+e)**n/(a+a*sec(f*x+e))**2,x)
```

output
```
Integral(sec(e + f*x)**n/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x)/a**2
```

3.293.7 Maxima [F]

$$\int \frac{\sec^n(e+fx)}{(a+a\sec(e+fx))^2}\,dx = \int \frac{\sec(fx+e)^n}{(a\sec(fx+e)+a)^2}\,dx$$

input
```
integrate(sec(f*x+e)^n/(a+a*sec(f*x+e))^2,x, algorithm="maxima")
```

output
```
integrate(sec(f*x + e)^n/(a*sec(f*x + e) + a)^2, x)
```

3.293.8 Giac [F]

$$\int \frac{\sec^n(e+fx)}{(a+a\sec(e+fx))^2}\,dx = \int \frac{\sec(fx+e)^n}{(a\sec(fx+e)+a)^2}\,dx$$

input
```
integrate(sec(f*x+e)^n/(a+a*sec(f*x+e))^2,x, algorithm="giac")
```

output
```
integrate(sec(f*x + e)^n/(a*sec(f*x + e) + a)^2, x)
```

3.293.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^n(e+fx)}{(a+a\sec(e+fx))^2}\,dx = \int \frac{\left(\frac{1}{\cos(e+f\,x)}\right)^n}{\left(a+\frac{a}{\cos(e+f\,x)}\right)^2}\,dx$$

input
```
int((1/cos(e + f*x))^n/(a + a/cos(e + f*x))^2,x)
```

output
```
int((1/cos(e + f*x))^n/(a + a/cos(e + f*x))^2, x)
```

3.294 $\int \sec^n(e+fx)(1+\sec(e+fx))^{5/2}\,dx$

3.294.1 Optimal result 2293
3.294.2 Mathematica [C] (warning: unable to verify) 2293
3.294.3 Rubi [A] (verified) 2294
3.294.4 Maple [F] 2297
3.294.5 Fricas [F] 2297
3.294.6 Sympy [F(-1)] 2297
3.294.7 Maxima [F] 2298
3.294.8 Giac [F] 2298
3.294.9 Mupad [F(-1)] 2298

3.294.1 Optimal result

Integrand size = 21, antiderivative size = 162

$$\int \sec^n(e+fx)(1+\sec(e+fx))^{5/2}\,dx = \frac{2(7+4n)\sec^{1+n}(e+fx)\sin(e+fx)}{f(1+2n)(3+2n)\sqrt{1+\sec(e+fx)}} + \frac{2\sec^{1+n}(e+fx)\sqrt{1+\sec(e+fx)}\sin(e+fx)}{f(3+2n)} + \frac{2(3+24n+16n^2)\,\text{Hypergeometric2F1}\left(\frac{1}{2},1-n,\frac{3}{2},1-\sec(e+fx)\right)\tan(e+fx)}{f(1+2n)(3+2n)\sqrt{1+\sec(e+fx)}}$$

output
```
2*(7+4*n)*sec(f*x+e)^(1+n)*sin(f*x+e)/f/(4*n^2+8*n+3)/(1+sec(f*x+e))^(1/2)
+2*sec(f*x+e)^(1+n)*sin(f*x+e)*(1+sec(f*x+e))^(1/2)/f/(3+2*n)+2*(16*n^2+24
*n+3)*hypergeom([1/2, 1-n],[3/2],1-sec(f*x+e))*tan(f*x+e)/f/(4*n^2+8*n+3)/
(1+sec(f*x+e))^(1/2)
```

3.294.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 31.83 (sec) , antiderivative size = 398, normalized size of antiderivative = 2.46

$$\int \sec^n(e+fx)(1+\sec(e+fx))^{5/2}\,dx = -\frac{i2^{-\frac{5}{2}+n}e^{-\frac{1}{2}i(3+2n)(e+fx)}\left(\frac{e^{i(e+fx)}}{1+e^{2i(e+fx)}}\right)^{\frac{3}{2}+n}\left(\frac{10e^{i(2+n)(e+fx)}\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}(-1-n),\frac{4+n}{2},-e^{2i(e+fx)}\right)}{2+n}+\frac{5e^{i(4+n)(e+fx)}\,\text{H}}{}\right.}{}$$

input
```
Integrate[Sec[e + f*x]^n*(1 + Sec[e + f*x])^(5/2),x]
```

output
```
((-I)*2^(-5/2 + n)*(E^(I*(e + f*x))/(1 + E^((2*I)*(e + f*x))))^(3/2 + n)*(
(10*E^(I*(2 + n)*(e + f*x))*Hypergeometric2F1[1, (-1 - n)/2, (4 + n)/2, -E
^((2*I)*(e + f*x))])/(2 + n) + (5*E^(I*(4 + n)*(e + f*x))*Hypergeometric2F
1[1, (1 - n)/2, (6 + n)/2, -E^((2*I)*(e + f*x))])/(4 + n) + (E^(I*n*(e + f
*x))*Hypergeometric2F1[1, -3/2 - n/2, 1 + n/2, -E^((2*I)*(e + f*x))])/n +
(5*E^(I*(1 + n)*(e + f*x))*Hypergeometric2F1[1, -1 - n/2, (3 + n)/2, -E^((
2*I)*(e + f*x))])/(1 + n) + (E^(I*(5 + n)*(e + f*x))*Hypergeometric2F1[1,
1 - n/2, (7 + n)/2, -E^((2*I)*(e + f*x))])/(5 + n) + (10*E^(I*(3 + n)*(e +
 f*x))*Hypergeometric2F1[1, -1/2*n, (5 + n)/2, -E^((2*I)*(e + f*x))])/(3 +
 n))*Sec[(e + f*x)/2]^5*(1 + Sec[e + f*x])^(5/2))/(E^((I/2)*(3 + 2*n)*(e +
 f*x))*f*Sec[e + f*x]^(5/2))
```

3.294.3 Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.97, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {3042, 4301, 27, 3042, 4504, 3042, 4293, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (\sec(e+fx)+1)^{5/2}\sec^n(e+fx)\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \left(\csc\left(e+fx+\frac{\pi}{2}\right)+1\right)^{5/2}\csc\left(e+fx+\frac{\pi}{2}\right)^n dx$$

$$\Big\downarrow \; 4301$$

$$\frac{2\int \frac{1}{2}\sec^n(e+fx)\sqrt{\sec(e+fx)+1}(4n+(4n+7)\sec(e+fx)+3)dx}{2n+3}+\frac{2\sin(e+fx)\sqrt{\sec(e+fx)+1}\sec^{n+1}(e+fx)}{f(2n+3)}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \sec^n(e+fx)\sqrt{\sec(e+fx)+1}(4n+(4n+7)\sec(e+fx)+3)dx}{2n+3}+\frac{2\sin(e+fx)\sqrt{\sec(e+fx)+1}\sec^{n+1}(e+fx)}{f(2n+3)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \csc\left(e+fx+\frac{\pi}{2}\right)^n\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)+1}\left(4n+(4n+7)\csc\left(e+fx+\frac{\pi}{2}\right)+3\right)dx}{\frac{2n+3}{f(2n+3)}}+\frac{2\sin(e+fx)\sqrt{\sec(e+fx)+1}\sec^{n+1}(e+fx)}{\frac{2n+3}{f(2n+3)}}$$

$$\Big\downarrow \text{4504}$$

$$\frac{\frac{(16n^2+24n+3)\int\sec^n(e+fx)\sqrt{\sec(e+fx)+1}dx}{2n+1}+\frac{2(4n+7)\sin(e+fx)\sec^{n+1}(e+fx)}{f(2n+1)\sqrt{\sec(e+fx)+1}}}{\frac{2n+3}{f(2n+3)}}+\frac{2\sin(e+fx)\sqrt{\sec(e+fx)+1}\sec^{n+1}(e+fx)}{\frac{2n+3}{f(2n+3)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{(16n^2+24n+3)\int\csc\left(e+fx+\frac{\pi}{2}\right)^n\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)+1}dx}{2n+1}+\frac{2(4n+7)\sin(e+fx)\sec^{n+1}(e+fx)}{f(2n+1)\sqrt{\sec(e+fx)+1}}}{\frac{2n+3}{f(2n+3)}}+\frac{2\sin(e+fx)\sqrt{\sec(e+fx)+1}\sec^{n+1}(e+fx)}{\frac{2n+3}{f(2n+3)}}$$

$$\Big\downarrow \text{4293}$$

$$\frac{\frac{2(4n+7)\sin(e+fx)\sec^{n+1}(e+fx)}{f(2n+1)\sqrt{\sec(e+fx)+1}}-\frac{(16n^2+24n+3)\tan(e+fx)\int\frac{\sec^{n-1}(e+fx)}{\sqrt{1-\sec(e+fx)}}d\sec(e+fx)}{f(2n+1)\sqrt{1-\sec(e+fx)}\sqrt{\sec(e+fx)+1}}}{\frac{2n+3}{f(2n+3)}}+\frac{2\sin(e+fx)\sqrt{\sec(e+fx)+1}\sec^{n+1}(e+fx)}{\frac{2n+3}{f(2n+3)}}$$

$$\Big\downarrow \text{75}$$

$$\frac{\frac{2(16n^2+24n+3)\tan(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},1-n,\frac{3}{2},1-\sec(e+fx)\right)}{f(2n+1)\sqrt{\sec(e+fx)+1}}+\frac{2(4n+7)\sin(e+fx)\sec^{n+1}(e+fx)}{f(2n+1)\sqrt{\sec(e+fx)+1}}}{\frac{2n+3}{f(2n+3)}}+\frac{2\sin(e+fx)\sqrt{\sec(e+fx)+1}\sec^{n+1}(e+fx)}{\frac{2n+3}{f(2n+3)}}$$

input
```
Int[Sec[e + f*x]^n*(1 + Sec[e + f*x])^(5/2),x]
```

output
```
(2*Sec[e + f*x]^(1 + n)*Sqrt[1 + Sec[e + f*x]]*Sin[e + f*x])/(f*(3 + 2*n))
 + ((2*(7 + 4*n)*Sec[e + f*x]^(1 + n)*Sin[e + f*x])/(f*(1 + 2*n)*Sqrt[1 +
Sec[e + f*x]]) + (2*(3 + 24*n + 16*n^2)*Hypergeometric2F1[1/2, 1 - n, 3/2,
 1 - Sec[e + f*x]]*Tan[e + f*x])/(f*(1 + 2*n)*Sqrt[1 + Sec[e + f*x]]))/(3
+ 2*n)
```

3.294. $\int \sec^n(e+fx)(1+\sec(e+fx))^{5/2}\,dx$

3.294.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4293
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]
*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(d*x)^(n - 1)/Sqrt[a - b*x], x], x,
 Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0]
```

rule 4301
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
2)*((d*Csc[e + f*x])^n/(f*(m + n - 1))), x] + Simp[b/(m + n - 1)   Int[(a +
 b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*(b*(m + 2*n - 1) + a*(3*m + 2*n
 - 4)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^
2, 0] && GtQ[m, 1] && NeQ[m + n - 1, 0] && IntegerQ[2*m]
```

rule 4504
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[-2*b*B*C
ot[e + f*x]*((d*Csc[e + f*x])^n/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]])), x]
 + Simp[(A*b*(2*n + 1) + 2*a*B*n)/(b*(2*n + 1))   Int[Sqrt[a + b*Csc[e + f*
x]]*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ
[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] &&
!LtQ[n, 0]
```

3.294. $\int \sec^n(e+fx)(1+\sec(e+fx))^{5/2}\,dx$

3.294.4 Maple [F]

$$\int \sec(fx+e)^n (\sec(fx+e)+1)^{\frac{5}{2}} \, dx$$

input
```
int(sec(f*x+e)^n*(sec(f*x+e)+1)^(5/2),x)
```

output
```
int(sec(f*x+e)^n*(sec(f*x+e)+1)^(5/2),x)
```

3.294.5 Fricas [F]

$$\int \sec^n(e+fx)(1+\sec(e+fx))^{5/2} \, dx = \int \sec(fx+e)^n (\sec(fx+e)+1)^{\frac{5}{2}} \, dx$$

input
```
integrate(sec(f*x+e)^n*(1+sec(f*x+e))^(5/2),x, algorithm="fricas")
```

output
```
integral((sec(f*x + e)^2 + 2*sec(f*x + e) + 1)*sec(f*x + e)^n*sqrt(sec(f*x
 + e) + 1), x)
```

3.294.6 Sympy [F(-1)]

Timed out.

$$\int \sec^n(e+fx)(1+\sec(e+fx))^{5/2} \, dx = \text{Timed out}$$

input
```
integrate(sec(f*x+e)**n*(1+sec(f*x+e))**(5/2),x)
```

output
```
Timed out
```

3.294.7 Maxima [F]

$$\int \sec^n(e+fx)(1+\sec(e+fx))^{5/2}\,dx = \int \sec{(fx+e)}^n\,(\sec{(fx+e)}+1)^{\frac{5}{2}}\,dx$$

input
```
integrate(sec(f*x+e)^n*(1+sec(f*x+e))^(5/2),x, algorithm="maxima")
```

output
```
integrate(sec(f*x + e)^n*(sec(f*x + e) + 1)^(5/2), x)
```

3.294.8 Giac [F]

$$\int \sec^n(e+fx)(1+\sec(e+fx))^{5/2}\,dx = \int \sec{(fx+e)}^n\,(\sec{(fx+e)}+1)^{\frac{5}{2}}\,dx$$

input
```
integrate(sec(f*x+e)^n*(1+sec(f*x+e))^(5/2),x, algorithm="giac")
```

output
```
integrate(sec(f*x + e)^n*(sec(f*x + e) + 1)^(5/2), x)
```

3.294.9 Mupad [F(-1)]

Timed out.

$$\int \sec^n(e+fx)(1+\sec(e+fx))^{5/2}\,dx = \int \left(\frac{1}{\cos{(e+f\,x)}}+1\right)^{5/2}\left(\frac{1}{\cos{(e+f\,x)}}\right)^n\,dx$$

input
```
int((1/cos(e + f*x) + 1)^(5/2)*(1/cos(e + f*x))^n,x)
```

output
```
int((1/cos(e + f*x) + 1)^(5/2)*(1/cos(e + f*x))^n, x)
```

3.295 $\int \sec^n(e+fx)(1+\sec(e+fx))^{3/2}\,dx$

3.295.1 Optimal result 2299
3.295.2 Mathematica [A] (verified) 2299
3.295.3 Rubi [A] (verified) 2300
3.295.4 Maple [F] 2302
3.295.5 Fricas [F] 2302
3.295.6 Sympy [F] 2302
3.295.7 Maxima [F] 2303
3.295.8 Giac [F] 2303
3.295.9 Mupad [F(-1)] 2303

3.295.1 Optimal result

Integrand size = 21, antiderivative size = 98

$$\int \sec^n(e+fx)(1+\sec(e+fx))^{3/2}\,dx = \frac{2\sec^{1+n}(e+fx)\sin(e+fx)}{f(1+2n)\sqrt{1+\sec(e+fx)}} + \frac{2(1+4n)\,\text{Hypergeometric2F1}\left(\frac{1}{2},1-n,\frac{3}{2},1-\sec(e+fx)\right)\tan(e+fx)}{f(1+2n)\sqrt{1+\sec(e+fx)}}$$

output
```
2*sec(f*x+e)^(1+n)*sin(f*x+e)/f/(1+2*n)/(1+sec(f*x+e))^(1/2)+2*(1+4*n)*hyp
ergeom([1/2, 1-n],[3/2],1-sec(f*x+e))*tan(f*x+e)/f/(1+2*n)/(1+sec(f*x+e))^
(1/2)
```

3.295.2 Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.85

$$\int \sec^n(e+fx)(1+\sec(e+fx))^{3/2}\,dx = \frac{\left(-1+(1+4n)\cos^{\frac{1}{2}+n}(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{2}+n,\frac{3}{2},2\sin^2\left(\frac{1}{2}(e+fx)\right)\right)\right)\sec^n(e}{fn}$$

input
```
Integrate[Sec[e + f*x]^n*(1 + Sec[e + f*x])^(3/2),x]
```

output
```
((-1 + (1 + 4*n)*Cos[e + f*x]^(1/2 + n)*Hypergeometric2F1[1/2, 3/2 + n, 3/
2, 2*Sin[(e + f*x)/2]^2])*Sec[e + f*x]^n*Sqrt[1 + Sec[e + f*x]]*Tan[(e + f
*x)/2])/(f*n)
```

3.295.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 4301, 27, 2011, 3042, 4293, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (\sec(e+fx)+1)^{3/2}\sec^n(e+fx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(\csc\left(e+fx+\frac{\pi}{2}\right)+1\right)^{3/2}\csc\left(e+fx+\frac{\pi}{2}\right)^n dx$$

$$\Big\downarrow \text{4301}$$

$$\frac{2\int \frac{\sec^n(e+fx)(4n+(4n+1)\sec(e+fx)+1)}{2\sqrt{\sec(e+fx)+1}}dx}{2n+1}+\frac{2\sin(e+fx)\sec^{n+1}(e+fx)}{f(2n+1)\sqrt{\sec(e+fx)+1}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\sec^n(e+fx)(4n+(4n+1)\sec(e+fx)+1)}{\sqrt{\sec(e+fx)+1}}dx}{2n+1}+\frac{2\sin(e+fx)\sec^{n+1}(e+fx)}{f(2n+1)\sqrt{\sec(e+fx)+1}}$$

$$\Big\downarrow \text{2011}$$

$$\frac{(4n+1)\int \sec^n(e+fx)\sqrt{\sec(e+fx)+1}dx}{2n+1}+\frac{2\sin(e+fx)\sec^{n+1}(e+fx)}{f(2n+1)\sqrt{\sec(e+fx)+1}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(4n+1)\int \csc\left(e+fx+\frac{\pi}{2}\right)^n\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)+1}dx}{2n+1}+\frac{2\sin(e+fx)\sec^{n+1}(e+fx)}{f(2n+1)\sqrt{\sec(e+fx)+1}}$$

$$\Big\downarrow \text{4293}$$

$$\frac{2\sin(e+fx)\sec^{n+1}(e+fx)}{f(2n+1)\sqrt{\sec(e+fx)+1}}-\frac{(4n+1)\tan(e+fx)\int \frac{\sec^{n-1}(e+fx)}{\sqrt{1-\sec(e+fx)}}d\sec(e+fx)}{f(2n+1)\sqrt{1-\sec(e+fx)}\sqrt{\sec(e+fx)+1}}$$

$$\Big\downarrow \; 75$$

$$\frac{2(4n+1)\tan(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},1-n,\frac{3}{2},1-\sec(e+fx)\right)}{f(2n+1)\sqrt{\sec(e+fx)+1}}+\frac{2\sin(e+fx)\sec^{n+1}(e+fx)}{f(2n+1)\sqrt{\sec(e+fx)+1}}$$

input
```
Int[Sec[e + f*x]^n*(1 + Sec[e + f*x])^(3/2),x]
```

output
```
(2*Sec[e + f*x]^(1 + n)*Sin[e + f*x])/(f*(1 + 2*n)*Sqrt[1 + Sec[e + f*x]])
 + (2*(1 + 4*n)*Hypergeometric2F1[1/2, 1 - n, 3/2, 1 - Sec[e + f*x]]*Tan[e
 + f*x])/(f*(1 + 2*n)*Sqrt[1 + Sec[e + f*x]])
```

3.295.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 2011
```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x
] && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x
, a + b*x])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4293
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]
*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(d*x)^(n - 1)/Sqrt[a - b*x], x], x,
 Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0]
```

rule 4301
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
2)*((d*Csc[e + f*x])^n/(f*(m + n - 1))), x] + Simp[b/(m + n - 1)   Int[(a +
 b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*(b*(m + 2*n - 1) + a*(3*m + 2*n
 - 4)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^
2, 0] && GtQ[m, 1] && NeQ[m + n - 1, 0] && IntegerQ[2*m]
```

3.295.4 Maple [F]

$$\int \sec(fx+e)^n (\sec(fx+e)+1)^{\frac{3}{2}}\,dx$$

input
```
int(sec(f*x+e)^n*(sec(f*x+e)+1)^(3/2),x)
```

output
```
int(sec(f*x+e)^n*(sec(f*x+e)+1)^(3/2),x)
```

3.295.5 Fricas [F]

$$\int \sec^n(e+fx)(1+\sec(e+fx))^{3/2}\,dx = \int \sec(fx+e)^n (\sec(fx+e)+1)^{\frac{3}{2}}\,dx$$

input
```
integrate(sec(f*x+e)^n*(1+sec(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
integral(sec(f*x + e)^n*(sec(f*x + e) + 1)^(3/2), x)
```

3.295.6 Sympy [F]

$$\int \sec^n(e+fx)(1+\sec(e+fx))^{3/2}\,dx = \int (\sec(e+fx)+1)^{\frac{3}{2}} \sec^n(e+fx)\,dx$$

input
```
integrate(sec(f*x+e)**n*(1+sec(f*x+e))**(3/2),x)
```

output
```
Integral((sec(e + f*x) + 1)**(3/2)*sec(e + f*x)**n, x)
```

3.295.7 Maxima [F]

$$\int \sec^n(e+fx)(1+\sec(e+fx))^{3/2}\,dx = \int \sec(fx+e)^n\,(\sec(fx+e)+1)^{\frac{3}{2}}\,dx$$

input
```
integrate(sec(f*x+e)^n*(1+sec(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
integrate(sec(f*x + e)^n*(sec(f*x + e) + 1)^(3/2), x)
```

3.295.8 Giac [F]

$$\int \sec^n(e+fx)(1+\sec(e+fx))^{3/2}\,dx = \int \sec(fx+e)^n\,(\sec(fx+e)+1)^{\frac{3}{2}}\,dx$$

input
```
integrate(sec(f*x+e)^n*(1+sec(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
integrate(sec(f*x + e)^n*(sec(f*x + e) + 1)^(3/2), x)
```

3.295.9 Mupad [F(-1)]

Timed out.

$$\int \sec^n(e+fx)(1+\sec(e+fx))^{3/2}\,dx = \int \left(\frac{1}{\cos(e+f\,x)}+1\right)^{3/2}\left(\frac{1}{\cos(e+f\,x)}\right)^n dx$$

input
```
int((1/cos(e + f*x) + 1)^(3/2)*(1/cos(e + f*x))^n,x)
```

output
```
int((1/cos(e + f*x) + 1)^(3/2)*(1/cos(e + f*x))^n, x)
```

3.296 $\int \sec^n(e+fx)\sqrt{1+\sec(e+fx)}\,dx$

3.296.1 Optimal result 2304
3.296.2 Mathematica [A] (verified) 2304
3.296.3 Rubi [A] (verified) 2305
3.296.4 Maple [F] 2306
3.296.5 Fricas [F] 2306
3.296.6 Sympy [F] 2306
3.296.7 Maxima [F] 2307
3.296.8 Giac [F] 2307
3.296.9 Mupad [F(-1)] 2307

3.296.1 Optimal result

Integrand size = 21, antiderivative size = 45

$$\int \sec^n(e+fx)\sqrt{1+\sec(e+fx)}\,dx = \frac{2\,\text{Hypergeometric2F1}\left(\frac{1}{2},1-n,\frac{3}{2},1-\sec(e+fx)\right)\tan(e+fx)}{f\sqrt{1+\sec(e+fx)}}$$

output
```
2*hypergeom([1/2, 1-n],[3/2],1-sec(f*x+e))*tan(f*x+e)/f/(1+sec(f*x+e))^(1/
2)
```

3.296.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00

$$\int \sec^n(e+fx)\sqrt{1+\sec(e+fx)}\,dx = \frac{2\,\text{Hypergeometric2F1}\left(\frac{1}{2},1-n,\frac{3}{2},1-\sec(e+fx)\right)\tan(e+fx)}{f\sqrt{1+\sec(e+fx)}}$$

input
```
Integrate[Sec[e + f*x]^n*Sqrt[1 + Sec[e + f*x]],x]
```

output
```
(2*Hypergeometric2F1[1/2, 1 - n, 3/2, 1 - Sec[e + f*x]]*Tan[e + f*x])/(f*S
qrt[1 + Sec[e + f*x]])
```

3.296.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3042, 4293, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\sec(e+fx)+1}\sec^n(e+fx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)+1}\csc\left(e+fx+\frac{\pi}{2}\right)^n dx$$

$$\Big\downarrow \text{4293}$$

$$-\frac{\tan(e+fx)\int \frac{\sec^{n-1}(e+fx)}{\sqrt{1-\sec(e+fx)}}d\sec(e+fx)}{f\sqrt{1-\sec(e+fx)}\sqrt{\sec(e+fx)+1}}$$

$$\Big\downarrow \text{75}$$

$$\frac{2\tan(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},1-n,\frac{3}{2},1-\sec(e+fx)\right)}{f\sqrt{\sec(e+fx)+1}}$$

input
```
Int[Sec[e + f*x]^n*Sqrt[1 + Sec[e + f*x]],x]
```

output
```
(2*Hypergeometric2F1[1/2, 1 - n, 3/2, 1 - Sec[e + f*x]]*Tan[e + f*x])/(f*S
qrt[1 + Sec[e + f*x]])
```

3.296.3.1 Defintions of rubi rules used

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4293
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]
*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(d*x)^(n - 1)/Sqrt[a - b*x], x], x,
 Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0]
```

3.296.4 Maple [F]

$$\int \sec(fx+e)^n \sqrt{\sec(fx+e)+1}dx$$

input
```
int(sec(f*x+e)^n*(sec(f*x+e)+1)^(1/2),x)
```

output
```
int(sec(f*x+e)^n*(sec(f*x+e)+1)^(1/2),x)
```

3.296.5 Fricas [F]

$$\int \sec^n(e+fx)\sqrt{1+\sec(e+fx)}\,dx = \int \sec(fx+e)^n \sqrt{\sec(fx+e)+1}\,dx$$

input
```
integrate(sec(f*x+e)^n*(1+sec(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
integral(sec(f*x + e)^n*sqrt(sec(f*x + e) + 1), x)
```

3.296.6 Sympy [F]

$$\int \sec^n(e+fx)\sqrt{1+\sec(e+fx)}\,dx = \int \sqrt{\sec(e+fx)+1}\sec^n(e+fx)\,dx$$

input
```
integrate(sec(f*x+e)**n*(1+sec(f*x+e))**(1/2),x)
```

output
```
Integral(sqrt(sec(e + f*x) + 1)*sec(e + f*x)**n, x)
```

3.296.7 Maxima [F]

$$\int \sec^n(e+fx)\sqrt{1+\sec(e+fx)}\,dx = \int \sec(fx+e)^n\sqrt{\sec(fx+e)+1}\,dx$$

input
```
integrate(sec(f*x+e)^n*(1+sec(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sec(f*x + e)^n*sqrt(sec(f*x + e) + 1), x)
```

3.296.8 Giac [F]

$$\int \sec^n(e+fx)\sqrt{1+\sec(e+fx)}\,dx = \int \sec(fx+e)^n\sqrt{\sec(fx+e)+1}\,dx$$

input
```
integrate(sec(f*x+e)^n*(1+sec(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate(sec(f*x + e)^n*sqrt(sec(f*x + e) + 1), x)
```

3.296.9 Mupad [F(-1)]

Timed out.

$$\int \sec^n(e+fx)\sqrt{1+\sec(e+fx)}\,dx = \int \sqrt{\frac{1}{\cos(e+f\,x)}+1}\left(\frac{1}{\cos(e+f\,x)}\right)^n dx$$

input
```
int((1/cos(e + f*x) + 1)^(1/2)*(1/cos(e + f*x))^n,x)
```

output
```
int((1/cos(e + f*x) + 1)^(1/2)*(1/cos(e + f*x))^n, x)
```

3.297 $\int \frac{\sec^n(e+fx)}{\sqrt{1+\sec(e+fx)}}\,dx$

3.297.1 Optimal result 2308
3.297.2 Mathematica [B] (warning: unable to verify) 2308
3.297.3 Rubi [A] (verified) 2309
3.297.4 Maple [F] 2311
3.297.5 Fricas [F] 2311
3.297.6 Sympy [F] 2311
3.297.7 Maxima [F] 2312
3.297.8 Giac [F] 2312
3.297.9 Mupad [F(-1)] 2312

3.297.1 Optimal result

Integrand size = 21, antiderivative size = 59

$$\int \frac{\sec^n(e+fx)}{\sqrt{1+\sec(e+fx)}}\,dx = \frac{\text{AppellF1}\left(\frac{1}{2},1-n,1,\frac{3}{2},1-\sec(e+fx),\frac{1}{2}(1-\sec(e+fx))\right)\tan(e+fx)}{f\sqrt{1+\sec(e+fx)}}$$

output
```
AppellF1(1/2,1-n,1,3/2,1-sec(f*x+e),1/2-1/2*sec(f*x+e))*tan(f*x+e)/f/(1+se
c(f*x+e))^(1/2)
```

3.297.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 2938 vs. 2(59) = 118.

Time = 15.80 (sec) , antiderivative size = 2938, normalized size of antiderivative = 49.80

$$\int \frac{\sec^n(e+fx)}{\sqrt{1+\sec(e+fx)}}\,dx = \text{Result too large to show}$$

input
```
Integrate[Sec[e + f*x]^n/Sqrt[1 + Sec[e + f*x]],x]
```

output
```
(3*Sqrt[2]*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e
 + f*x)/2]^2]*(Sec[(e + f*x)/2]^2)^n*Sec[e + f*x]^(-1/2 + (-1 + 2*n)/2)*(C
os[(e + f*x)/2]^2*Sec[e + f*x])^n*Tan[(e + f*x)/2])/(f*(3*AppellF1[1/2, -1
/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)
*AppellF1[3/2, -1/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]
^2] + (-1 + 2*n)*AppellF1[3/2, 1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -T
an[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)*((3*AppellF1[1/2, -1/2 + n, 1 - n,
 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[e + f*x]*(Sec[(e + f*x)
/2]^2)^(1 + n)*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^n*Sqrt[1 + Sec[e + f*x]])
/(Sqrt[2]*(3*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[
(e + f*x)/2]^2] + (2*(-1 + n)*AppellF1[3/2, -1/2 + n, 2 - n, 5/2, Tan[(e +
 f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-1 + 2*n)*AppellF1[3/2, 1/2 + n, 1 - n
, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)) - (3
*Sqrt[2]*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e +
 f*x)/2]^2]*(Sec[(e + f*x)/2]^2)^n*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^n*Sqr
t[1 + Sec[e + f*x]]*Sin[e + f*x]*Tan[(e + f*x)/2])/(3*AppellF1[1/2, -1/2 +
 n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)*App
ellF1[3/2, -1/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]
+ (-1 + 2*n)*AppellF1[3/2, 1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(
e + f*x)/2]^2])*Tan[(e + f*x)/2]^2) + (3*Sqrt[2]*n*AppellF1[1/2, -1/2 +...
```

3.297.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3042, 4312, 148, 333}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^n(e+fx)}{\sqrt{\sec(e+fx)+1}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)^n}{\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)+1}}dx$$

$$\Big\downarrow \text{4312}$$

$$\frac{\tan(e+fx)\int \frac{\sec^{n-1}(e+fx)}{\sqrt{1-\sec(e+fx)}(\sec(e+fx)+1)}d(1-\sec(e+fx))}{f\sqrt{1-\sec(e+fx)}\sqrt{\sec(e+fx)+1}}$$

$$\Big\downarrow \; 148$$

$$\frac{2\tan(e+fx)\int \frac{\sec^{n-1}(e+fx)}{\sec(e+fx)+1}d\sqrt{1-\sec(e+fx)}}{f\sqrt{1-\sec(e+fx)}\sqrt{\sec(e+fx)+1}}$$

$$\Big\downarrow \; 333$$

$$\frac{\tan(e+fx)\,\text{AppellF1}\left(\frac{1}{2},1-n,1,\frac{3}{2},1-\sec(e+fx),\frac{1}{2}(1-\sec(e+fx))\right)}{f\sqrt{\sec(e+fx)+1}}$$

input
```
Int[Sec[e + f*x]^n/Sqrt[1 + Sec[e + f*x]],x]
```

output
```
(AppellF1[1/2, 1 - n, 1, 3/2, 1 - Sec[e + f*x], (1 - Sec[e + f*x])/2]*Tan[
e + f*x])/(f*Sqrt[1 + Sec[e + f*x]])
```

3.297.3.1 Defintions of rubi rules used

rule 148
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_))^(p_.),
x_] :> With[{k = Denominator[m]}, Simp[k/b   Subst[Int[x^(k*(m + 1) - 1)*(c
 + d*(x^k/b))^n*(e + f*(x^k/b))^p, x], x, (b*x)^(1/k)], x]] /; FreeQ[{b, c,
 d, e, f, n, p}, x] && FractionQ[m] && IntegerQ[p]
```

rule 333
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^p*c^q*x*AppellF1[1/2, -p, -q, 3/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; F
reeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[p] || GtQ[a,
0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4312
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-(a*(d/b))^n)*(Cot[e + f*x]/(a^(n - 2)*f*Sqrt
[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(a - x)^(n - 1)
*((2*a - x)^(m - 1/2)/Sqrt[x]), x], x, a - b*Csc[e + f*x]], x] /; FreeQ[{a,
 b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0] &
&  !IntegerQ[n] && GtQ[a*(d/b), 0]
```

3.297.4 Maple [F]

$$\int \frac{\sec(fx+e)^n}{\sqrt{\sec(fx+e)+1}}dx$$

input
```
int(sec(f*x+e)^n/(sec(f*x+e)+1)^(1/2),x)
```

output
```
int(sec(f*x+e)^n/(sec(f*x+e)+1)^(1/2),x)
```

3.297.5 Fricas [F]

$$\int \frac{\sec^n(e+fx)}{\sqrt{1+\sec(e+fx)}}\,dx = \int \frac{\sec(fx+e)^n}{\sqrt{\sec(fx+e)+1}}\,dx$$

input
```
integrate(sec(f*x+e)^n/(1+sec(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
integral(sec(f*x + e)^n/sqrt(sec(f*x + e) + 1), x)
```

3.297.6 Sympy [F]

$$\int \frac{\sec^n(e+fx)}{\sqrt{1+\sec(e+fx)}}\,dx = \int \frac{\sec^n(e+fx)}{\sqrt{\sec(e+fx)+1}}\,dx$$

input
```
integrate(sec(f*x+e)**n/(1+sec(f*x+e))**(1/2),x)
```

output
```
Integral(sec(e + f*x)**n/sqrt(sec(e + f*x) + 1), x)
```

3.297.7 Maxima [F]

$$\int \frac{\sec^n(e+fx)}{\sqrt{1+\sec(e+fx)}}\,dx = \int \frac{\sec(fx+e)^n}{\sqrt{\sec(fx+e)+1}}\,dx$$

input
```
integrate(sec(f*x+e)^n/(1+sec(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sec(f*x + e)^n/sqrt(sec(f*x + e) + 1), x)
```

3.297.8 Giac [F]

$$\int \frac{\sec^n(e+fx)}{\sqrt{1+\sec(e+fx)}}\,dx = \int \frac{\sec(fx+e)^n}{\sqrt{\sec(fx+e)+1}}\,dx$$

input
```
integrate(sec(f*x+e)^n/(1+sec(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate(sec(f*x + e)^n/sqrt(sec(f*x + e) + 1), x)
```

3.297.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^n(e+fx)}{\sqrt{1+\sec(e+fx)}}\,dx = \int \frac{\left(\frac{1}{\cos(e+f\,x)}\right)^n}{\sqrt{\frac{1}{\cos(e+f\,x)}+1}}\,dx$$

input
```
int((1/cos(e + f*x))^n/(1/cos(e + f*x) + 1)^(1/2),x)
```

output
```
int((1/cos(e + f*x))^n/(1/cos(e + f*x) + 1)^(1/2), x)
```

3.298 $\int \frac{\sec^n(e+fx)}{(1+\sec(e+fx))^{3/2}}\,dx$

3.298.1 Optimal result 2313
3.298.2 Mathematica [B] (warning: unable to verify) 2313
3.298.3 Rubi [A] (verified) 2314
3.298.4 Maple [F] 2316
3.298.5 Fricas [F] 2316
3.298.6 Sympy [F] 2316
3.298.7 Maxima [F] 2317
3.298.8 Giac [F] 2317
3.298.9 Mupad [F(-1)] 2317

3.298.1 Optimal result

Integrand size = 21, antiderivative size = 62

$$\int \frac{\sec^n(e+fx)}{(1+\sec(e+fx))^{3/2}}\,dx=\frac{\text{AppellF1}\left(\frac{1}{2},1-n,2,\frac{3}{2},1-\sec(e+fx),\frac{1}{2}(1-\sec(e+fx))\right)\tan(e+fx)}{2f\sqrt{1+\sec(e+fx)}}$$

output
```
1/2*AppellF1(1/2,1-n,2,3/2,1-sec(f*x+e),1/2-1/2*sec(f*x+e))*tan(f*x+e)/f/(
1+sec(f*x+e))^(1/2)
```

3.298.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 2990 vs. 2(62) = 124.

Time = 16.38 (sec) , antiderivative size = 2990, normalized size of antiderivative = 48.23

$$\int \frac{\sec^n(e+fx)}{(1+\sec(e+fx))^{3/2}}\,dx=\text{Result too large to show}$$

input
```
Integrate[Sec[e + f*x]^n/(1 + Sec[e + f*x])^(3/2),x]
```

output
```
(6*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/
2]^2]*(Sec[(e + f*x)/2]^2)^n*Sec[e + f*x]^(1/2 + (-3 + 2*n)/2)*(Cos[(e + f
*x)/2]^2*Sec[e + f*x])^(3/2 + n)*Tan[(e + f*x)/2]*(-1 + Tan[(e + f*x)/2]^2
)^2)/(f*(1 + Sec[e + f*x])^(3/2)*(3*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Ta
n[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)*AppellF1[3/2, -3/2 +
n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-3 + 2*n)*Appel
lF1[3/2, -1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*T
an[(e + f*x)/2]^2)*((12*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e + f*x)/
2]^2, -Tan[(e + f*x)/2]^2]*Cos[e + f*x]*(Sec[(e + f*x)/2]^2)^(1 + n)*(Cos[
(e + f*x)/2]^2*Sec[e + f*x])^(3/2 + n)*Tan[(e + f*x)/2]^2*(-1 + Tan[(e + f
*x)/2]^2))/(3*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan
[(e + f*x)/2]^2] + (2*(-1 + n)*AppellF1[3/2, -3/2 + n, 2 - n, 5/2, Tan[(e
+ f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-3 + 2*n)*AppellF1[3/2, -1/2 + n, 1 -
 n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2) + (
3*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2
]^2]*Cos[e + f*x]*(Sec[(e + f*x)/2]^2)^(1 + n)*(Cos[(e + f*x)/2]^2*Sec[e +
 f*x])^(3/2 + n)*(-1 + Tan[(e + f*x)/2]^2)^2)/(3*AppellF1[1/2, -3/2 + n, 1
 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)*AppellF1
[3/2, -3/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-3
 + 2*n)*AppellF1[3/2, -1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e...
```

3.298.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3042, 4312, 148, 333}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^n(e+fx)}{(\sec(e+fx)+1)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)^n}{\left(\csc\left(e+fx+\frac{\pi}{2}\right)+1\right)^{3/2}}dx$$

$$\Big\downarrow \text{4312}$$

$$\frac{\tan(e+fx)\int \frac{\sec^{n-1}(e+fx)}{\sqrt{1-\sec(e+fx)}(\sec(e+fx)+1)^2}d(1-\sec(e+fx))}{f\sqrt{1-\sec(e+fx)}\sqrt{\sec(e+fx)+1}}$$

$$\Big\downarrow \; 148$$

$$\frac{2\tan(e+fx)\int \frac{\sec^{n-1}(e+fx)}{(\sec(e+fx)+1)^2}d\sqrt{1-\sec(e+fx)}}{f\sqrt{1-\sec(e+fx)}\sqrt{\sec(e+fx)+1}}$$

$$\Big\downarrow \; 333$$

$$\frac{\tan(e+fx)\,\text{AppellF1}\left(\frac{1}{2},1-n,2,\frac{3}{2},1-\sec(e+fx),\frac{1}{2}(1-\sec(e+fx))\right)}{2f\sqrt{\sec(e+fx)+1}}$$

input
```
Int[Sec[e + f*x]^n/(1 + Sec[e + f*x])^(3/2),x]
```

output
```
(AppellF1[1/2, 1 - n, 2, 3/2, 1 - Sec[e + f*x], (1 - Sec[e + f*x])/2]*Tan[
e + f*x])/(2*f*Sqrt[1 + Sec[e + f*x]])
```

3.298.3.1 Defintions of rubi rules used

rule 148
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_))^(p_.),
x_] :> With[{k = Denominator[m]}, Simp[k/b   Subst[Int[x^(k*(m + 1) - 1)*(c
 + d*(x^k/b))^n*(e + f*(x^k/b))^p, x], x, (b*x)^(1/k)], x]] /; FreeQ[{b, c,
 d, e, f, n, p}, x] && FractionQ[m] && IntegerQ[p]
```

rule 333
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^p*c^q*x*AppellF1[1/2, -p, -q, 3/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; F
reeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[p] || GtQ[a,
0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4312
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-(a*(d/b))^n)*(Cot[e + f*x]/(a^(n - 2)*f*Sqrt
[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(a - x)^(n - 1)
*((2*a - x)^(m - 1/2)/Sqrt[x]), x], x, a - b*Csc[e + f*x]], x] /; FreeQ[{a,
 b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0] &
&  !IntegerQ[n] && GtQ[a*(d/b), 0]
```

3.298.4 Maple [F]

$$\int \frac{\sec(fx+e)^n}{(\sec(fx+e)+1)^{\frac{3}{2}}}dx$$

input
```
int(sec(f*x+e)^n/(sec(f*x+e)+1)^(3/2),x)
```

output
```
int(sec(f*x+e)^n/(sec(f*x+e)+1)^(3/2),x)
```

3.298.5 Fricas [F]

$$\int \frac{\sec^n(e+fx)}{(1+\sec(e+fx))^{3/2}}\,dx = \int \frac{\sec(fx+e)^n}{(\sec(fx+e)+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(f*x+e)^n/(1+sec(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
integral(sec(f*x + e)^n*sqrt(sec(f*x + e) + 1)/(sec(f*x + e)^2 + 2*sec(f*x
 + e) + 1), x)
```

3.298.6 Sympy [F]

$$\int \frac{\sec^n(e+fx)}{(1+\sec(e+fx))^{3/2}}\,dx = \int \frac{\sec^n(e+fx)}{(\sec(e+fx)+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(f*x+e)**n/(1+sec(f*x+e))**(3/2),x)
```

output
```
Integral(sec(e + f*x)**n/(sec(e + f*x) + 1)**(3/2), x)
```

3.298.7 Maxima [F]

$$\int \frac{\sec^n(e+fx)}{(1+\sec(e+fx))^{3/2}}\,dx = \int \frac{\sec(fx+e)^n}{(\sec(fx+e)+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(f*x+e)^n/(1+sec(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
integrate(sec(f*x + e)^n/(sec(f*x + e) + 1)^(3/2), x)
```

3.298.8 Giac [F]

$$\int \frac{\sec^n(e+fx)}{(1+\sec(e+fx))^{3/2}}\,dx = \int \frac{\sec(fx+e)^n}{(\sec(fx+e)+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(f*x+e)^n/(1+sec(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
integrate(sec(f*x + e)^n/(sec(f*x + e) + 1)^(3/2), x)
```

3.298.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^n(e+fx)}{(1+\sec(e+fx))^{3/2}}\,dx = \int \frac{\left(\frac{1}{\cos(e+f\,x)}\right)^n}{\left(\frac{1}{\cos(e+f\,x)}+1\right)^{3/2}}\,dx$$

input
```
int((1/cos(e + f*x))^n/(1/cos(e + f*x) + 1)^(3/2),x)
```

output
```
int((1/cos(e + f*x))^n/(1/cos(e + f*x) + 1)^(3/2), x)
```

3.299 $\int (-\sec(e+fx))^n(1+\sec(e+fx))^{3/2}\,dx$

3.299.1 Optimal result 2318
3.299.2 Mathematica [A] (verified) 2318
3.299.3 Rubi [A] (verified) 2319
3.299.4 Maple [F] 2321
3.299.5 Fricas [F] 2321
3.299.6 Sympy [F] 2321
3.299.7 Maxima [F] 2322
3.299.8 Giac [F] 2322
3.299.9 Mupad [F(-1)] 2322

3.299.1 Optimal result

Integrand size = 23, antiderivative size = 117

$$\int (-\sec(e+fx))^n(1+\sec(e+fx))^{3/2}\,dx = \frac{2(-\sec(e+fx))^n\tan(e+fx)}{f(1+2n)\sqrt{1+\sec(e+fx)}} - \frac{(1+4n)\,\text{Hypergeometric2F1}\left(\frac{1}{2},n,1+n,\sec(e+fx)\right)(-\sec(e+fx))^n\tan(e+fx)}{fn(1+2n)\sqrt{1-\sec(e+fx)}\sqrt{1+\sec(e+fx)}}$$

output
```
2*(-sec(f*x+e))^n*tan(f*x+e)/f/(1+2*n)/(1+sec(f*x+e))^(1/2)-(1+4*n)*hyperg
eom([1/2, n],[1+n],sec(f*x+e))*(-sec(f*x+e))^n*tan(f*x+e)/f/n/(1+2*n)/(1-s
ec(f*x+e))^(1/2)/(1+sec(f*x+e))^(1/2)
```

3.299.2 Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.73

$$\int (-\sec(e+fx))^n(1+\sec(e+fx))^{3/2}\,dx = \frac{\left(-1+(1+4n)\cos^{\frac{1}{2}+n}(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{2}+n,\frac{3}{2},2\sin^2\left(\frac{1}{2}(e+fx)\right)\right)\right)(-\sec(e+fx))}{fn}$$

input
```
Integrate[(-Sec[e + f*x])^n*(1 + Sec[e + f*x])^(3/2),x]
```

output
```
((-1 + (1 + 4*n)*Cos[e + f*x]^(1/2 + n)*Hypergeometric2F1[1/2, 3/2 + n, 3/
2, 2*Sin[(e + f*x)/2]^2])*(-Sec[e + f*x])^n*Sqrt[1 + Sec[e + f*x]]*Tan[(e
+ f*x)/2])/(f*n)
```

3.299.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3042, 4301, 27, 2011, 3042, 4293, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (\sec(e+fx)+1)^{3/2}(-\sec(e+fx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(\csc\left(e+fx+\frac{\pi}{2}\right)+1\right)^{3/2}\left(-\csc\left(e+fx+\frac{\pi}{2}\right)\right)^n dx$$

$$\Big\downarrow \text{4301}$$

$$\frac{2\int \frac{(-\sec(e+fx))^n(4n+(4n+1)\sec(e+fx)+1)}{2\sqrt{\sec(e+fx)+1}}dx}{2n+1}+\frac{2\tan(e+fx)(-\sec(e+fx))^n}{f(2n+1)\sqrt{\sec(e+fx)+1}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{(-\sec(e+fx))^n(4n+(4n+1)\sec(e+fx)+1)}{\sqrt{\sec(e+fx)+1}}dx}{2n+1}+\frac{2\tan(e+fx)(-\sec(e+fx))^n}{f(2n+1)\sqrt{\sec(e+fx)+1}}$$

$$\Big\downarrow \text{2011}$$

$$\frac{(4n+1)\int(-\sec(e+fx))^n\sqrt{\sec(e+fx)+1}dx}{2n+1}+\frac{2\tan(e+fx)(-\sec(e+fx))^n}{f(2n+1)\sqrt{\sec(e+fx)+1}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(4n+1)\int\left(-\csc\left(e+fx+\frac{\pi}{2}\right)\right)^n\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)+1}dx}{2n+1}+\frac{2\tan(e+fx)(-\sec(e+fx))^n}{f(2n+1)\sqrt{\sec(e+fx)+1}}$$

$$\Big\downarrow \text{4293}$$

$$\frac{(4n+1)\tan(e+fx)\int\frac{(-\sec(e+fx))^{n-1}}{\sqrt{1-\sec(e+fx)}}d\sec(e+fx)}{f(2n+1)\sqrt{1-\sec(e+fx)}\sqrt{\sec(e+fx)+1}}+\frac{2\tan(e+fx)(-\sec(e+fx))^n}{f(2n+1)\sqrt{\sec(e+fx)+1}}$$

$$\Big\downarrow \; 74$$

$$\frac{2\tan(e+fx)(-\sec(e+fx))^n}{f(2n+1)\sqrt{\sec(e+fx)+1}} - \frac{(4n+1)\tan(e+fx)(-\sec(e+fx))^n \text{Hypergeometric2F1}\left(\frac{1}{2},n,n+1,\sec(e+fx)\right)}{fn(2n+1)\sqrt{1-\sec(e+fx)}\sqrt{\sec(e+fx)+1}}$$

input
```
Int[(-Sec[e + f*x])^n*(1 + Sec[e + f*x])^(3/2),x]
```

output
```
(2*(-Sec[e + f*x])^n*Tan[e + f*x])/(f*(1 + 2*n)*Sqrt[1 + Sec[e + f*x]]) -
((1 + 4*n)*Hypergeometric2F1[1/2, n, 1 + n, Sec[e + f*x]]*(-Sec[e + f*x])^
n*Tan[e + f*x])/(f*n*(1 + 2*n)*Sqrt[1 - Sec[e + f*x]]*Sqrt[1 + Sec[e + f*x
]])
```

3.299.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 74
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

rule 2011
```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x
] && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x
, a + b*x])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4293
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]
*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(d*x)^(n - 1)/Sqrt[a - b*x], x], x,
 Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0]
```

rule 4301
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
2)*((d*Csc[e + f*x])^n/(f*(m + n - 1))), x] + Simp[b/(m + n - 1)   Int[(a +
 b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*(b*(m + 2*n - 1) + a*(3*m + 2*n
 - 4)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^
2, 0] && GtQ[m, 1] && NeQ[m + n - 1, 0] && IntegerQ[2*m]
```

3.299.4 Maple [F]

$$\int (-\sec(fx+e))^n (\sec(fx+e)+1)^{\frac{3}{2}}\,dx$$

input
```
int((-sec(f*x+e))^n*(sec(f*x+e)+1)^(3/2),x)
```

output
```
int((-sec(f*x+e))^n*(sec(f*x+e)+1)^(3/2),x)
```

3.299.5 Fricas [F]

$$\int (-\sec(e+fx))^n(1+\sec(e+fx))^{3/2}\,dx = \int (-\sec(fx+e))^n (\sec(fx+e)+1)^{\frac{3}{2}}\,dx$$

input
```
integrate((-sec(f*x+e))^n*(1+sec(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
integral((-sec(f*x + e))^n*(sec(f*x + e) + 1)^(3/2), x)
```

3.299.6 Sympy [F]

$$\int (-\sec(e+fx))^n(1+\sec(e+fx))^{3/2}\,dx = \int (-\sec(e+fx))^n (\sec(e+fx)+1)^{\frac{3}{2}}\,dx$$

input
```
integrate((-sec(f*x+e))**n*(1+sec(f*x+e))**(3/2),x)
```

output
```
Integral((-sec(e + f*x))**n*(sec(e + f*x) + 1)**(3/2), x)
```

3.299.7 Maxima [F]

$$\int (-\sec(e+fx))^n(1+\sec(e+fx))^{3/2}\,dx = \int (-\sec(fx+e))^n(\sec(fx+e)+1)^{\frac{3}{2}}\,dx$$

input
```
integrate((-sec(f*x+e))^n*(1+sec(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
integrate((-sec(f*x + e))^n*(sec(f*x + e) + 1)^(3/2), x)
```

3.299.8 Giac [F]

$$\int (-\sec(e+fx))^n(1+\sec(e+fx))^{3/2}\,dx = \int (-\sec(fx+e))^n(\sec(fx+e)+1)^{\frac{3}{2}}\,dx$$

input
```
integrate((-sec(f*x+e))^n*(1+sec(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
integrate((-sec(f*x + e))^n*(sec(f*x + e) + 1)^(3/2), x)
```

3.299.9 Mupad [F(-1)]

Timed out.

$$\int (-\sec(e+fx))^n(1+\sec(e+fx))^{3/2}\,dx = \int \left(\frac{1}{\cos(e+f\,x)}+1\right)^{3/2}\left(-\frac{1}{\cos(e+f\,x)}\right)^n dx$$

input
```
int((1/cos(e + f*x) + 1)^(3/2)*(-1/cos(e + f*x))^n,x)
```

output
```
int((1/cos(e + f*x) + 1)^(3/2)*(-1/cos(e + f*x))^n, x)
```

3.300 $\int (-\sec(e+fx))^n \sqrt{1+\sec(e+fx)}\,dx$

3.300.1 Optimal result 2323
3.300.2 Mathematica [A] (verified) 2323
3.300.3 Rubi [A] (verified) 2324
3.300.4 Maple [F] 2325
3.300.5 Fricas [F] 2325
3.300.6 Sympy [F] 2325
3.300.7 Maxima [F] 2326
3.300.8 Giac [F] 2326
3.300.9 Mupad [F(-1)] 2326

3.300.1 Optimal result

Integrand size = 23, antiderivative size = 64

$$\int (-\sec(e+fx))^n \sqrt{1+\sec(e+fx)}\,dx = -\frac{\text{Hypergeometric2F1}\left(\frac{1}{2},n,1+n,\sec(e+fx)\right)(-\sec(e+fx))^n\tan(e+fx)}{fn\sqrt{1-\sec(e+fx)}\sqrt{1+\sec(e+fx)}}$$

output
```
-hypergeom([1/2, n],[1+n],sec(f*x+e))*(-sec(f*x+e))^n*tan(f*x+e)/f/n/(1-se
c(f*x+e))^(1/2)/(1+sec(f*x+e))^(1/2)
```

3.300.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.05

$$\int (-\sec(e+fx))^n \sqrt{1+\sec(e+fx)}\,dx = \frac{2\,\text{Hypergeometric2F1}\left(\frac{1}{2},1-n,\frac{3}{2},1-\sec(e+fx)\right)(-\sec(e+fx))^n\sec^{1-n}(e+fx)\sin(e+fx)}{f\sqrt{1+\sec(e+fx)}}$$

input
```
Integrate[(-Sec[e + f*x])^n*Sqrt[1 + Sec[e + f*x]],x]
```

output
```
(2*Hypergeometric2F1[1/2, 1 - n, 3/2, 1 - Sec[e + f*x]]*(-Sec[e + f*x])^n*
Sec[e + f*x]^(1 - n)*Sin[e + f*x])/(f*Sqrt[1 + Sec[e + f*x]])
```

3.300.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3042, 4293, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\sec(e+fx)+1}(-\sec(e+fx))^n \, dx$$

$$\Big\downarrow \; 3042$$

$$\int \sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)+1}\left(-\csc\left(e+fx+\frac{\pi}{2}\right)\right)^n dx$$

$$\Big\downarrow \; 4293$$

$$\frac{\tan(e+fx)\int \frac{(-\sec(e+fx))^{n-1}}{\sqrt{1-\sec(e+fx)}}d\sec(e+fx)}{f\sqrt{1-\sec(e+fx)}\sqrt{\sec(e+fx)+1}}$$

$$\Big\downarrow \; 74$$

$$-\frac{\tan(e+fx)(-\sec(e+fx))^n \operatorname{Hypergeometric2F1}\left(\frac{1}{2},n,n+1,\sec(e+fx)\right)}{fn\sqrt{1-\sec(e+fx)}\sqrt{\sec(e+fx)+1}}$$

input
```
Int[(-Sec[e + f*x])^n*Sqrt[1 + Sec[e + f*x]],x]
```

output
```
-((Hypergeometric2F1[1/2, n, 1 + n, Sec[e + f*x]]*(-Sec[e + f*x])^n*Tan[e
+ f*x])/(f*n*Sqrt[1 - Sec[e + f*x]]*Sqrt[1 + Sec[e + f*x]]))
```

3.300.3.1 Defintions of rubi rules used

rule 74
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4293
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]
*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(d*x)^(n - 1)/Sqrt[a - b*x], x], x,
 Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0]
```

3.300.4 Maple [F]

$$\int (-\sec(fx+e))^n \sqrt{\sec(fx+e)+1}dx$$

input
```
int((-sec(f*x+e))^n*(sec(f*x+e)+1)^(1/2),x)
```

output
```
int((-sec(f*x+e))^n*(sec(f*x+e)+1)^(1/2),x)
```

3.300.5 Fricas [F]

$$\int (-\sec(e+fx))^n \sqrt{1+\sec(e+fx)}\,dx = \int (-\sec(fx+e))^n \sqrt{\sec(fx+e)+1}\,dx$$

input
```
integrate((-sec(f*x+e))^n*(1+sec(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
integral((-sec(f*x + e))^n*sqrt(sec(f*x + e) + 1), x)
```

3.300.6 Sympy [F]

$$\int (-\sec(e+fx))^n \sqrt{1+\sec(e+fx)}\,dx = \int (-\sec(e+fx))^n \sqrt{\sec(e+fx)+1}\,dx$$

input
```
integrate((-sec(f*x+e))**n*(1+sec(f*x+e))**(1/2),x)
```

output
```
Integral((-sec(e + f*x))**n*sqrt(sec(e + f*x) + 1), x)
```

3.300.7 Maxima [F]

$$\int (-\sec(e+fx))^n \sqrt{1+\sec(e+fx)}\,dx = \int (-\sec(fx+e))^n \sqrt{\sec(fx+e)+1}\,dx$$

input
```
integrate((-sec(f*x+e))^n*(1+sec(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate((-sec(f*x + e))^n*sqrt(sec(f*x + e) + 1), x)
```

3.300.8 Giac [F]

$$\int (-\sec(e+fx))^n \sqrt{1+\sec(e+fx)}\,dx = \int (-\sec(fx+e))^n \sqrt{\sec(fx+e)+1}\,dx$$

input
```
integrate((-sec(f*x+e))^n*(1+sec(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate((-sec(f*x + e))^n*sqrt(sec(f*x + e) + 1), x)
```

3.300.9 Mupad [F(-1)]

Timed out.

$$\int (-\sec(e+fx))^n \sqrt{1+\sec(e+fx)}\,dx = \int \sqrt{\frac{1}{\cos(e+f\,x)}+1}\left(-\frac{1}{\cos(e+f\,x)}\right)^n dx$$

input
```
int((1/cos(e + f*x) + 1)^(1/2)*(-1/cos(e + f*x))^n,x)
```

output
```
int((1/cos(e + f*x) + 1)^(1/2)*(-1/cos(e + f*x))^n, x)
```

3.301 $\int \frac{(-\sec(e+fx))^n}{\sqrt{1+\sec(e+fx)}}\,dx$

3.301.1 Optimal result . . . 2327
3.301.2 Mathematica [B] (warning: unable to verify) . . . 2327
3.301.3 Rubi [A] (verified) . . . 2328
3.301.4 Maple [F] . . . 2330
3.301.5 Fricas [F] . . . 2330
3.301.6 Sympy [F] . . . 2330
3.301.7 Maxima [F] . . . 2331
3.301.8 Giac [F] . . . 2331
3.301.9 Mupad [F(-1)] . . . 2331

3.301.1 Optimal result

Integrand size = 23, antiderivative size = 73

$$\int \frac{(-\sec(e+fx))^n}{\sqrt{1+\sec(e+fx)}}\,dx = -\frac{\text{AppellF1}\left(n,\frac{1}{2},1,1+n,\sec(e+fx),-\sec(e+fx)\right)(-\sec(e+fx))^n\tan(e+fx)}{fn\sqrt{1-\sec(e+fx)}\sqrt{1+\sec(e+fx)}}$$

output
```
-AppellF1(n,1,1/2,1+n,-sec(f*x+e),sec(f*x+e))*(-sec(f*x+e))^n*tan(f*x+e)/f
/n/(1-sec(f*x+e))^(1/2)/(1+sec(f*x+e))^(1/2)
```

3.301.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 2951 vs. 2(73) = 146.

Time = 6.18 (sec) , antiderivative size = 2951, normalized size of antiderivative = 40.42

$$\int \frac{(-\sec(e+fx))^n}{\sqrt{1+\sec(e+fx)}}\,dx = \text{Result too large to show}$$

input
```
Integrate[(-Sec[e + f*x])^n/Sqrt[1 + Sec[e + f*x]],x]
```

output
```
(3*Sqrt[2]*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e
 + f*x)/2]^2]*(Sec[(e + f*x)/2]^2)^n*(-Sec[e + f*x])^n*Sec[e + f*x]^(-1/2
- n + (-1 + 2*n)/2)*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^n*Tan[(e + f*x)/2])/
(f*(3*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*
x)/2]^2] + (2*(-1 + n)*AppellF1[3/2, -1/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2
]^2, -Tan[(e + f*x)/2]^2] + (-1 + 2*n)*AppellF1[3/2, 1/2 + n, 1 - n, 5/2,
Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)*((3*AppellF1
[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[e
 + f*x]*(Sec[(e + f*x)/2]^2)^(1 + n)*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^n*S
qrt[1 + Sec[e + f*x]])/(Sqrt[2]*(3*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan
[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)*AppellF1[3/2, -1/2 + n
, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-1 + 2*n)*Appell
F1[3/2, 1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan
[(e + f*x)/2]^2)) - (3*Sqrt[2]*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e
+ f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(Sec[(e + f*x)/2]^2)^n*(Cos[(e + f*x)/2]
^2*Sec[e + f*x])^n*Sqrt[1 + Sec[e + f*x]]*Sin[e + f*x]*Tan[(e + f*x)/2])/(
3*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2
]^2] + (2*(-1 + n)*AppellF1[3/2, -1/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2,
 -Tan[(e + f*x)/2]^2] + (-1 + 2*n)*AppellF1[3/2, 1/2 + n, 1 - n, 5/2, Tan[
(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2) + (3*Sqrt[2]*...
```

3.301.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3042, 4313, 153}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(-\sec(e+fx))^n}{\sqrt{\sec(e+fx)+1}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(-\csc\left(e+fx+\frac{\pi}{2}\right)\right)^n}{\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)+1}}dx$$

$$\Big\downarrow \text{4313}$$

$$\frac{\tan(e+fx)\int \frac{(-\sec(e+fx))^{n-1}}{\sqrt{1-\sec(e+fx)}(\sec(e+fx)+1)}d(\sec(e+fx)+1)}{f\sqrt{1-\sec(e+fx)}\sqrt{\sec(e+fx)+1}}$$

$$\Big\downarrow \; 153$$

$$-\frac{\tan(e+fx)(-\sec(e+fx))^n \operatorname{AppellF1}\left(n,\frac{1}{2},1,n+1,\sec(e+fx),-\sec(e+fx)\right)}{fn\sqrt{1-\sec(e+fx)}\sqrt{\sec(e+fx)+1}}$$

input
```
Int[(-Sec[e + f*x])^n/Sqrt[1 + Sec[e + f*x]],x]
```

output
```
-((AppellF1[n, 1/2, 1, 1 + n, Sec[e + f*x], -Sec[e + f*x]]*(-Sec[e + f*x])
^n*Tan[e + f*x])/(f*n*Sqrt[1 - Sec[e + f*x]]*Sqrt[1 + Sec[e + f*x]]))
```

3.301.3.1 Defintions of rubi rules used

rule 153
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(b*e - a*f)^p*((a + b*x)^(m + 1)/(b^(p + 1)*(m + 1)*Simp
lify[b/(b*c - a*d)]^n))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c
 - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] && !IntegerQ[m] && !IntegerQ[n] && IntegerQ[p] && GtQ[Simplify[b/(
b*c - a*d)], 0] && !(GtQ[Simplify[d/(d*a - c*b)], 0] && SimplerQ[c + d*x,
a + b*x])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4313
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-((-a)*(d/b))^n)*(Cot[e + f*x]/(a^(n - 1)*f*S
qrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[x^(m - 1/2)*
((a - x)^(n - 1)/Sqrt[2*a - x]), x], x, a + b*Csc[e + f*x]], x] /; FreeQ[{a
, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && !IntegerQ[m] && GtQ[a, 0]
&& !IntegerQ[n] && LtQ[a*(d/b), 0]
```

3.301.4 Maple [F]

$$\int \frac{(-\sec(fx+e))^n}{\sqrt{\sec(fx+e)+1}}dx$$

input
```
int((-sec(f*x+e))^n/(sec(f*x+e)+1)^(1/2),x)
```

output
```
int((-sec(f*x+e))^n/(sec(f*x+e)+1)^(1/2),x)
```

3.301.5 Fricas [F]

$$\int \frac{(-\sec(e+fx))^n}{\sqrt{1+\sec(e+fx)}}\,dx = \int \frac{(-\sec(fx+e))^n}{\sqrt{\sec(fx+e)+1}}\,dx$$

input
```
integrate((-sec(f*x+e))^n/(1+sec(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
integral((-sec(f*x + e))^n/sqrt(sec(f*x + e) + 1), x)
```

3.301.6 Sympy [F]

$$\int \frac{(-\sec(e+fx))^n}{\sqrt{1+\sec(e+fx)}}\,dx = \int \frac{(-\sec(e+fx))^n}{\sqrt{\sec(e+fx)+1}}\,dx$$

input
```
integrate((-sec(f*x+e))**n/(1+sec(f*x+e))**(1/2),x)
```

output
```
Integral((-sec(e + f*x))**n/sqrt(sec(e + f*x) + 1), x)
```

3.301.7 Maxima [F]

$$\int \frac{(-\sec(e+fx))^n}{\sqrt{1+\sec(e+fx)}}\,dx = \int \frac{(-\sec(fx+e))^n}{\sqrt{\sec(fx+e)+1}}\,dx$$

input
```
integrate((-sec(f*x+e))^n/(1+sec(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate((-sec(f*x + e))^n/sqrt(sec(f*x + e) + 1), x)
```

3.301.8 Giac [F]

$$\int \frac{(-\sec(e+fx))^n}{\sqrt{1+\sec(e+fx)}}\,dx = \int \frac{(-\sec(fx+e))^n}{\sqrt{\sec(fx+e)+1}}\,dx$$

input
```
integrate((-sec(f*x+e))^n/(1+sec(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate((-sec(f*x + e))^n/sqrt(sec(f*x + e) + 1), x)
```

3.301.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(-\sec(e+fx))^n}{\sqrt{1+\sec(e+fx)}}\,dx = \int \frac{\left(-\frac{1}{\cos(e+f\,x)}\right)^n}{\sqrt{\frac{1}{\cos(e+f\,x)}+1}}\,dx$$

input
```
int((-1/cos(e + f*x))^n/(1/cos(e + f*x) + 1)^(1/2),x)
```

output
```
int((-1/cos(e + f*x))^n/(1/cos(e + f*x) + 1)^(1/2), x)
```

3.302 $\int \frac{(-\sec(e+fx))^n}{(1+\sec(e+fx))^{3/2}}\,dx$

3.302.1 Optimal result 2332
3.302.2 Mathematica [B] (warning: unable to verify) 2332
3.302.3 Rubi [A] (verified) 2333
3.302.4 Maple [F] 2335
3.302.5 Fricas [F] 2335
3.302.6 Sympy [F] 2335
3.302.7 Maxima [F] 2336
3.302.8 Giac [F] 2336
3.302.9 Mupad [F(-1)] 2336

3.302.1 Optimal result

Integrand size = 23, antiderivative size = 73

$$\int \frac{(-\sec(e+fx))^n}{(1+\sec(e+fx))^{3/2}}\,dx = -\frac{\text{AppellF1}\left(n,\frac{1}{2},2,1+n,\sec(e+fx),-\sec(e+fx)\right)(-\sec(e+fx))^n\tan(e+fx)}{fn\sqrt{1-\sec(e+fx)}\sqrt{1+\sec(e+fx)}}$$

output
```
-AppellF1(n,2,1/2,1+n,-sec(f*x+e),sec(f*x+e))*(-sec(f*x+e))^n*tan(f*x+e)/f
/n/(1-sec(f*x+e))^(1/2)/(1+sec(f*x+e))^(1/2)
```

3.302.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 3003 vs. 2(73) = 146.

Time = 6.20 (sec) , antiderivative size = 3003, normalized size of antiderivative = 41.14

$$\int \frac{(-\sec(e+fx))^n}{(1+\sec(e+fx))^{3/2}}\,dx = \text{Result too large to show}$$

input
```
Integrate[(-Sec[e + f*x])^n/(1 + Sec[e + f*x])^(3/2),x]
```

output
```
(6*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/
2]^2]*(Sec[(e + f*x)/2]^2)^n*(-Sec[e + f*x])^n*Sec[e + f*x]^(1/2 - n + (-3
 + 2*n)/2)*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(3/2 + n)*Tan[(e + f*x)/2]*(-
1 + Tan[(e + f*x)/2]^2)^2)/(f*(1 + Sec[e + f*x])^(3/2)*(3*AppellF1[1/2, -3
/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)
*AppellF1[3/2, -3/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]
^2] + (-3 + 2*n)*AppellF1[3/2, -1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -
Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)*((12*AppellF1[1/2, -3/2 + n, 1 -
n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[e + f*x]*(Sec[(e + f*
x)/2]^2)^(1 + n)*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(3/2 + n)*Tan[(e + f*x)
/2]^2*(-1 + Tan[(e + f*x)/2]^2))/(3*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Ta
n[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)*AppellF1[3/2, -3/2 +
n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-3 + 2*n)*Appel
lF1[3/2, -1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*T
an[(e + f*x)/2]^2) + (3*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e + f*x)/
2]^2, -Tan[(e + f*x)/2]^2]*Cos[e + f*x]*(Sec[(e + f*x)/2]^2)^(1 + n)*(Cos[
(e + f*x)/2]^2*Sec[e + f*x])^(3/2 + n)*(-1 + Tan[(e + f*x)/2]^2)^2)/(3*App
ellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]
+ (2*(-1 + n)*AppellF1[3/2, -3/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan
[(e + f*x)/2]^2] + (-3 + 2*n)*AppellF1[3/2, -1/2 + n, 1 - n, 5/2, Tan[(...
```

3.302.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3042, 4313, 153}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(-\sec(e+fx))^n}{(\sec(e+fx)+1)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(-\csc\left(e+fx+\frac{\pi}{2}\right)\right)^n}{\left(\csc\left(e+fx+\frac{\pi}{2}\right)+1\right)^{3/2}}dx$$

$$\Big\downarrow \text{4313}$$

$$\frac{\tan(e+fx)\int \frac{(-\sec(e+fx))^{n-1}}{\sqrt{1-\sec(e+fx)}(\sec(e+fx)+1)^2}d(\sec(e+fx)+1)}{f\sqrt{1-\sec(e+fx)}\sqrt{\sec(e+fx)+1}}$$

$$\Big\downarrow \; 153$$

$$-\frac{\tan(e+fx)(-\sec(e+fx))^n \operatorname{AppellF1}\left(n,\frac{1}{2},2,n+1,\sec(e+fx),-\sec(e+fx)\right)}{fn\sqrt{1-\sec(e+fx)}\sqrt{\sec(e+fx)+1}}$$

input
```
Int[(-Sec[e + f*x])^n/(1 + Sec[e + f*x])^(3/2),x]
```

output
```
-((AppellF1[n, 1/2, 2, 1 + n, Sec[e + f*x], -Sec[e + f*x]]*(-Sec[e + f*x])
^n*Tan[e + f*x])/(f*n*Sqrt[1 - Sec[e + f*x]]*Sqrt[1 + Sec[e + f*x]]))
```

3.302.3.1 Defintions of rubi rules used

rule 153
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(b*e - a*f)^p*((a + b*x)^(m + 1)/(b^(p + 1)*(m + 1)*Simp
lify[b/(b*c - a*d)]^n))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c
 - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] && !IntegerQ[m] && !IntegerQ[n] && IntegerQ[p] && GtQ[Simplify[b/(
b*c - a*d)], 0] && !(GtQ[Simplify[d/(d*a - c*b)], 0] && SimplerQ[c + d*x,
a + b*x])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4313
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-((-a)*(d/b))^n)*(Cot[e + f*x]/(a^(n - 1)*f*S
qrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[x^(m - 1/2)*
((a - x)^(n - 1)/Sqrt[2*a - x]), x], x, a + b*Csc[e + f*x]], x] /; FreeQ[{a
, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && !IntegerQ[m] && GtQ[a, 0]
&& !IntegerQ[n] && LtQ[a*(d/b), 0]
```

3.302.4 Maple [F]

$$\int \frac{(-\sec(fx+e))^n}{(\sec(fx+e)+1)^{\frac{3}{2}}}dx$$

input
```
int((-sec(f*x+e))^n/(sec(f*x+e)+1)^(3/2),x)
```

output
```
int((-sec(f*x+e))^n/(sec(f*x+e)+1)^(3/2),x)
```

3.302.5 Fricas [F]

$$\int \frac{(-\sec(e+fx))^n}{(1+\sec(e+fx))^{3/2}}\,dx = \int \frac{(-\sec(fx+e))^n}{(\sec(fx+e)+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate((-sec(f*x+e))^n/(1+sec(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
integral((-sec(f*x + e))^n*sqrt(sec(f*x + e) + 1)/(sec(f*x + e)^2 + 2*sec(
f*x + e) + 1), x)
```

3.302.6 Sympy [F]

$$\int \frac{(-\sec(e+fx))^n}{(1+\sec(e+fx))^{3/2}}\,dx = \int \frac{(-\sec(e+fx))^n}{(\sec(e+fx)+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate((-sec(f*x+e))**n/(1+sec(f*x+e))**(3/2),x)
```

output
```
Integral((-sec(e + f*x))**n/(sec(e + f*x) + 1)**(3/2), x)
```

3.302.7 Maxima [F]

$$\int \frac{(-\sec(e+fx))^n}{(1+\sec(e+fx))^{3/2}}\,dx = \int \frac{(-\sec(fx+e))^n}{(\sec(fx+e)+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate((-sec(f*x+e))^n/(1+sec(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
integrate((-sec(f*x + e))^n/(sec(f*x + e) + 1)^(3/2), x)
```

3.302.8 Giac [F]

$$\int \frac{(-\sec(e+fx))^n}{(1+\sec(e+fx))^{3/2}}\,dx = \int \frac{(-\sec(fx+e))^n}{(\sec(fx+e)+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate((-sec(f*x+e))^n/(1+sec(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
integrate((-sec(f*x + e))^n/(sec(f*x + e) + 1)^(3/2), x)
```

3.302.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(-\sec(e+fx))^n}{(1+\sec(e+fx))^{3/2}}\,dx = \int \frac{\left(-\frac{1}{\cos(e+f\,x)}\right)^n}{\left(\frac{1}{\cos(e+f\,x)}+1\right)^{3/2}}\,dx$$

input
```
int((-1/cos(e + f*x))^n/(1/cos(e + f*x) + 1)^(3/2),x)
```

output
```
int((-1/cos(e + f*x))^n/(1/cos(e + f*x) + 1)^(3/2), x)
```

3.303 $\int (d\sec(e+fx))^n(1+\sec(e+fx))^{3/2}\,dx$

3.303.1 Optimal result . . . 2337
3.303.2 Mathematica [A] (verified) . . . 2337
3.303.3 Rubi [A] (verified) . . . 2338
3.303.4 Maple [F] . . . 2340
3.303.5 Fricas [F] . . . 2340
3.303.6 Sympy [F] . . . 2340
3.303.7 Maxima [F] . . . 2341
3.303.8 Giac [F] . . . 2341
3.303.9 Mupad [F(-1)] . . . 2341

3.303.1 Optimal result

Integrand size = 23, antiderivative size = 117

$$\int (d\sec(e+fx))^n(1+\sec(e+fx))^{3/2}\,dx = \frac{2(d\sec(e+fx))^n\tan(e+fx)}{f(1+2n)\sqrt{1+\sec(e+fx)}} - \frac{(1+4n)\,\text{Hypergeometric2F1}\left(\frac{1}{2},n,1+n,\sec(e+fx)\right)(d\sec(e+fx))^n\tan(e+fx)}{fn(1+2n)\sqrt{1-\sec(e+fx)}\sqrt{1+\sec(e+fx)}}$$

output
```
2*(d*sec(f*x+e))^n*tan(f*x+e)/f/(1+2*n)/(1+sec(f*x+e))^(1/2)-(1+4*n)*hyper
geom([1/2, n],[1+n],sec(f*x+e))*(d*sec(f*x+e))^n*tan(f*x+e)/f/n/(1+2*n)/(1
-sec(f*x+e))^(1/2)/(1+sec(f*x+e))^(1/2)
```

3.303.2 Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.73

$$\int (d\sec(e+fx))^n(1+\sec(e+fx))^{3/2}\,dx = \frac{\left(-1+(1+4n)\cos^{\frac{1}{2}+n}(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{2}+n,\frac{3}{2},2\sin^2\left(\frac{1}{2}(e+fx)\right)\right)\right)(d\sec(}{fn}$$

input
```
Integrate[(d*Sec[e + f*x])^n*(1 + Sec[e + f*x])^(3/2),x]
```

output
```
((-1 + (1 + 4*n)*Cos[e + f*x]^(1/2 + n)*Hypergeometric2F1[1/2, 3/2 + n, 3/
2, 2*Sin[(e + f*x)/2]^2])*(d*Sec[e + f*x])^n*Sqrt[1 + Sec[e + f*x]]*Tan[(e
 + f*x)/2])/(f*n)
```

3.303.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3042, 4301, 27, 2011, 3042, 4293, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (\sec(e+fx)+1)^{3/2}(d\sec(e+fx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(\csc\left(e+fx+\frac{\pi}{2}\right)+1\right)^{3/2}\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^n dx$$

$$\Big\downarrow \text{4301}$$

$$\frac{2\int \frac{(d\sec(e+fx))^n(4n+(4n+1)\sec(e+fx)+1)}{2\sqrt{\sec(e+fx)+1}}dx}{2n+1}+\frac{2\tan(e+fx)(d\sec(e+fx))^n}{f(2n+1)\sqrt{\sec(e+fx)+1}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{(d\sec(e+fx))^n(4n+(4n+1)\sec(e+fx)+1)}{\sqrt{\sec(e+fx)+1}}dx}{2n+1}+\frac{2\tan(e+fx)(d\sec(e+fx))^n}{f(2n+1)\sqrt{\sec(e+fx)+1}}$$

$$\Big\downarrow \text{2011}$$

$$\frac{(4n+1)\int (d\sec(e+fx))^n\sqrt{\sec(e+fx)+1}dx}{2n+1}+\frac{2\tan(e+fx)(d\sec(e+fx))^n}{f(2n+1)\sqrt{\sec(e+fx)+1}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(4n+1)\int \left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^n\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)+1}dx}{2n+1}+\frac{2\tan(e+fx)(d\sec(e+fx))^n}{f(2n+1)\sqrt{\sec(e+fx)+1}}$$

$$\Big\downarrow \text{4293}$$

$$\frac{2\tan(e+fx)(d\sec(e+fx))^n}{f(2n+1)\sqrt{\sec(e+fx)+1}}-\frac{d(4n+1)\tan(e+fx)\int \frac{(d\sec(e+fx))^{n-1}}{\sqrt{1-\sec(e+fx)}}d\sec(e+fx)}{f(2n+1)\sqrt{1-\sec(e+fx)}\sqrt{\sec(e+fx)+1}}$$

$$\Big\downarrow \; 74$$

$$\frac{2\tan(e+fx)(d\sec(e+fx))^n}{f(2n+1)\sqrt{\sec(e+fx)+1}} - \frac{(4n+1)\tan(e+fx)(d\sec(e+fx))^n \text{Hypergeometric2F1}\left(\frac{1}{2},n,n+1,\sec(e+fx)\right)}{fn(2n+1)\sqrt{1-\sec(e+fx)}\sqrt{\sec(e+fx)+1}}$$

input
```
Int[(d*Sec[e + f*x])^n*(1 + Sec[e + f*x])^(3/2),x]
```

output
```
(2*(d*Sec[e + f*x])^n*Tan[e + f*x])/(f*(1 + 2*n)*Sqrt[1 + Sec[e + f*x]]) -
 ((1 + 4*n)*Hypergeometric2F1[1/2, n, 1 + n, Sec[e + f*x]]*(d*Sec[e + f*x]
)^n*Tan[e + f*x])/(f*n*(1 + 2*n)*Sqrt[1 - Sec[e + f*x]]*Sqrt[1 + Sec[e + f
*x]])
```

3.303.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 74
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

rule 2011
```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x
] && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x
, a + b*x])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4293
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]
*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(d*x)^(n - 1)/Sqrt[a - b*x], x], x,
 Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0]
```

rule 4301
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
2)*((d*Csc[e + f*x])^n/(f*(m + n - 1))), x] + Simp[b/(m + n - 1)   Int[(a +
 b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*(b*(m + 2*n - 1) + a*(3*m + 2*n
 - 4)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^
2, 0] && GtQ[m, 1] && NeQ[m + n - 1, 0] && IntegerQ[2*m]
```

3.303.4 Maple [F]

$$\int (d\sec(fx+e))^n (\sec(fx+e)+1)^{\frac{3}{2}}\,dx$$

input
```
int((d*sec(f*x+e))^n*(sec(f*x+e)+1)^(3/2),x)
```

output
```
int((d*sec(f*x+e))^n*(sec(f*x+e)+1)^(3/2),x)
```

3.303.5 Fricas [F]

$$\int (d\sec(e+fx))^n(1+\sec(e+fx))^{3/2}\,dx = \int (d\sec(fx+e))^n (\sec(fx+e)+1)^{\frac{3}{2}}\,dx$$

input
```
integrate((d*sec(f*x+e))^n*(1+sec(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
integral((d*sec(f*x + e))^n*(sec(f*x + e) + 1)^(3/2), x)
```

3.303.6 Sympy [F]

$$\int (d\sec(e+fx))^n(1+\sec(e+fx))^{3/2}\,dx = \int (d\sec(e+fx))^n (\sec(e+fx)+1)^{\frac{3}{2}}\,dx$$

input
```
integrate((d*sec(f*x+e))**n*(1+sec(f*x+e))**(3/2),x)
```

output
```
Integral((d*sec(e + f*x))**n*(sec(e + f*x) + 1)**(3/2), x)
```

3.303.7 Maxima [F]

$$\int (d\sec(e+fx))^n(1+\sec(e+fx))^{3/2}\,dx = \int (d\sec(fx+e))^n(\sec(fx+e)+1)^{\frac{3}{2}}\,dx$$

input
```
integrate((d*sec(f*x+e))^n*(1+sec(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
integrate((d*sec(f*x + e))^n*(sec(f*x + e) + 1)^(3/2), x)
```

3.303.8 Giac [F]

$$\int (d\sec(e+fx))^n(1+\sec(e+fx))^{3/2}\,dx = \int (d\sec(fx+e))^n(\sec(fx+e)+1)^{\frac{3}{2}}\,dx$$

input
```
integrate((d*sec(f*x+e))^n*(1+sec(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
integrate((d*sec(f*x + e))^n*(sec(f*x + e) + 1)^(3/2), x)
```

3.303.9 Mupad [F(-1)]

Timed out.

$$\int (d\sec(e+fx))^n(1+\sec(e+fx))^{3/2}\,dx = \int \left(\frac{1}{\cos(e+f\,x)}+1\right)^{3/2}\left(\frac{d}{\cos(e+f\,x)}\right)^n dx$$

input
```
int((1/cos(e + f*x) + 1)^(3/2)*(d/cos(e + f*x))^n,x)
```

output
```
int((1/cos(e + f*x) + 1)^(3/2)*(d/cos(e + f*x))^n, x)
```

3.304 $\int (d\sec(e+fx))^n \sqrt{1+\sec(e+fx)}\,dx$

3.304.1 Optimal result 2342
3.304.2 Mathematica [A] (verified) 2342
3.304.3 Rubi [A] (verified) 2343
3.304.4 Maple [F] 2344
3.304.5 Fricas [F] 2344
3.304.6 Sympy [F] 2344
3.304.7 Maxima [F] 2345
3.304.8 Giac [F] 2345
3.304.9 Mupad [F(-1)] 2345

3.304.1 Optimal result

Integrand size = 23, antiderivative size = 64

$$\int (d\sec(e+fx))^n \sqrt{1+\sec(e+fx)}\,dx = -\frac{\text{Hypergeometric2F1}\left(\frac{1}{2},n,1+n,\sec(e+fx)\right)(d\sec(e+fx))^n\tan(e+fx)}{fn\sqrt{1-\sec(e+fx)}\sqrt{1+\sec(e+fx)}}$$

output
```
-hypergeom([1/2, n],[1+n],sec(f*x+e))*(d*sec(f*x+e))^n*tan(f*x+e)/f/n/(1-s
ec(f*x+e))^(1/2)/(1+sec(f*x+e))^(1/2)
```

3.304.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.05

$$\int (d\sec(e+fx))^n \sqrt{1+\sec(e+fx)}\,dx = \frac{2\,\text{Hypergeometric2F1}\left(\frac{1}{2},1-n,\frac{3}{2},1-\sec(e+fx)\right)\sec^{1-n}(e+fx)(d\sec(e+fx))^n\sin(e+fx)}{f\sqrt{1+\sec(e+fx)}}$$

input
```
Integrate[(d*Sec[e + f*x])^n*Sqrt[1 + Sec[e + f*x]],x]
```

output
```
(2*Hypergeometric2F1[1/2, 1 - n, 3/2, 1 - Sec[e + f*x]]*Sec[e + f*x]^(1 -
n)*(d*Sec[e + f*x])^n*Sin[e + f*x])/(f*Sqrt[1 + Sec[e + f*x]])
```

3.304.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3042, 4293, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\sec(e+fx)+1}(d\sec(e+fx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)+1}\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^n dx$$

$$\Big\downarrow \text{4293}$$

$$-\frac{d\tan(e+fx)\int \frac{(d\sec(e+fx))^{n-1}}{\sqrt{1-\sec(e+fx)}}d\sec(e+fx)}{f\sqrt{1-\sec(e+fx)}\sqrt{\sec(e+fx)+1}}$$

$$\Big\downarrow \text{74}$$

$$-\frac{\tan(e+fx)(d\sec(e+fx))^n\,\text{Hypergeometric2F1}\left(\frac{1}{2},n,n+1,\sec(e+fx)\right)}{fn\sqrt{1-\sec(e+fx)}\sqrt{\sec(e+fx)+1}}$$

input
```
Int[(d*Sec[e + f*x])^n*Sqrt[1 + Sec[e + f*x]],x]
```

output
```
-((Hypergeometric2F1[1/2, n, 1 + n, Sec[e + f*x]]*(d*Sec[e + f*x])^n*Tan[e
 + f*x])/(f*n*Sqrt[1 - Sec[e + f*x]]*Sqrt[1 + Sec[e + f*x]]))
```

3.304.3.1 Defintions of rubi rules used

rule 74
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4293
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]
*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(d*x)^(n - 1)/Sqrt[a - b*x], x], x,
 Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0]
```

3.304.4 Maple [F]

$$\int (d\sec(fx+e))^n \sqrt{\sec(fx+e)+1}dx$$

input
```
int((d*sec(f*x+e))^n*(sec(f*x+e)+1)^(1/2),x)
```

output
```
int((d*sec(f*x+e))^n*(sec(f*x+e)+1)^(1/2),x)
```

3.304.5 Fricas [F]

$$\int (d\sec(e+fx))^n \sqrt{1+\sec(e+fx)}\,dx = \int (d\sec(fx+e))^n \sqrt{\sec(fx+e)+1}\,dx$$

input
```
integrate((d*sec(f*x+e))^n*(1+sec(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
integral((d*sec(f*x + e))^n*sqrt(sec(f*x + e) + 1), x)
```

3.304.6 Sympy [F]

$$\int (d\sec(e+fx))^n \sqrt{1+\sec(e+fx)}\,dx = \int (d\sec(e+fx))^n \sqrt{\sec(e+fx)+1}\,dx$$

input
```
integrate((d*sec(f*x+e))**n*(1+sec(f*x+e))**(1/2),x)
```

output
```
Integral((d*sec(e + f*x))**n*sqrt(sec(e + f*x) + 1), x)
```

3.304.7 Maxima [F]

$$\int (d\sec(e+fx))^n\sqrt{1+\sec(e+fx)}\,dx=\int (d\sec(fx+e))^n\sqrt{\sec(fx+e)+1}\,dx$$

input
```
integrate((d*sec(f*x+e))^n*(1+sec(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate((d*sec(f*x + e))^n*sqrt(sec(f*x + e) + 1), x)
```

3.304.8 Giac [F]

$$\int (d\sec(e+fx))^n\sqrt{1+\sec(e+fx)}\,dx=\int (d\sec(fx+e))^n\sqrt{\sec(fx+e)+1}\,dx$$

input
```
integrate((d*sec(f*x+e))^n*(1+sec(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate((d*sec(f*x + e))^n*sqrt(sec(f*x + e) + 1), x)
```

3.304.9 Mupad [F(-1)]

Timed out.

$$\int (d\sec(e+fx))^n\sqrt{1+\sec(e+fx)}\,dx=\int \sqrt{\frac{1}{\cos(e+f\,x)}+1}\,\left(\frac{d}{\cos(e+f\,x)}\right)^n\,dx$$

input
```
int((1/cos(e + f*x) + 1)^(1/2)*(d/cos(e + f*x))^n,x)
```

output
```
int((1/cos(e + f*x) + 1)^(1/2)*(d/cos(e + f*x))^n, x)
```

3.305 $\int \frac{(d\sec(e+fx))^n}{\sqrt{1+\sec(e+fx)}}\,dx$

3.305.1 Optimal result 2346
3.305.2 Mathematica [B] (warning: unable to verify) 2346
3.305.3 Rubi [A] (verified) 2347
3.305.4 Maple [F] 2348
3.305.5 Fricas [F] 2349
3.305.6 Sympy [F] 2349
3.305.7 Maxima [F] 2349
3.305.8 Giac [F] 2350
3.305.9 Mupad [F(-1)] 2350

3.305.1 Optimal result

Integrand size = 23, antiderivative size = 73

$$\int \frac{(d\sec(e+fx))^n}{\sqrt{1+\sec(e+fx)}}\,dx = -\frac{\text{AppellF1}\left(n,\frac{1}{2},1,1+n,\sec(e+fx),-\sec(e+fx)\right)(d\sec(e+fx))^n\tan(e+fx)}{fn\sqrt{1-\sec(e+fx)}\sqrt{1+\sec(e+fx)}}$$

output
```
-AppellF1(n,1,1/2,1+n,-sec(f*x+e),sec(f*x+e))*(d*sec(f*x+e))^n*tan(f*x+e)/
f/n/(1-sec(f*x+e))^(1/2)/(1+sec(f*x+e))^(1/2)
```

3.305.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 2951 vs. 2(73) = 146.

Time = 6.18 (sec) , antiderivative size = 2951, normalized size of antiderivative = 40.42

$$\int \frac{(d\sec(e+fx))^n}{\sqrt{1+\sec(e+fx)}}\,dx = \text{Result too large to show}$$

input
```
Integrate[(d*Sec[e + f*x])^n/Sqrt[1 + Sec[e + f*x]],x]
```

output
```
(3*Sqrt[2]*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e
 + f*x)/2]^2]*(Sec[(e + f*x)/2]^2)^n*Sec[e + f*x]^(-1/2 - n + (-1 + 2*n)/2
)*(d*Sec[e + f*x])^n*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^n*Tan[(e + f*x)/2])
/(f*(3*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f
*x)/2]^2] + (2*(-1 + n)*AppellF1[3/2, -1/2 + n, 2 - n, 5/2, Tan[(e + f*x)/
2]^2, -Tan[(e + f*x)/2]^2] + (-1 + 2*n)*AppellF1[3/2, 1/2 + n, 1 - n, 5/2,
 Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)*((3*AppellF
1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[
e + f*x]*(Sec[(e + f*x)/2]^2)^(1 + n)*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^n*
Sqrt[1 + Sec[e + f*x]])/(Sqrt[2]*(3*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Ta
n[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)*AppellF1[3/2, -1/2 +
n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-1 + 2*n)*Appel
lF1[3/2, 1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Ta
n[(e + f*x)/2]^2)) - (3*Sqrt[2]*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e
 + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(Sec[(e + f*x)/2]^2)^n*(Cos[(e + f*x)/2
]^2*Sec[e + f*x])^n*Sqrt[1 + Sec[e + f*x]]*Sin[e + f*x]*Tan[(e + f*x)/2])/
(3*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/
2]^2] + (2*(-1 + n)*AppellF1[3/2, -1/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2
, -Tan[(e + f*x)/2]^2] + (-1 + 2*n)*AppellF1[3/2, 1/2 + n, 1 - n, 5/2, Tan
[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2) + (3*Sqrt[2]...
```

3.305.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3042, 4314, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d\sec(e+fx))^n}{\sqrt{\sec(e+fx)+1}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^n}{\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)+1}}dx$$

$$\Big\downarrow \text{4314}$$

$$-\frac{d\tan(e+fx)\int \frac{(d\sec(e+fx))^{n-1}}{\sqrt{1-\sec(e+fx)}(\sec(e+fx)+1)}d\sec(e+fx)}{f\sqrt{1-\sec(e+fx)}\sqrt{\sec(e+fx)+1}}$$

$$\Big\downarrow \text{150}$$

$$-\frac{\tan(e+fx)\,\text{AppellF1}\left(n,\frac{1}{2},1,n+1,\sec(e+fx),-\sec(e+fx)\right)(d\sec(e+fx))^n}{fn\sqrt{1-\sec(e+fx)}\sqrt{\sec(e+fx)+1}}$$

input
```
Int[(d*Sec[e + f*x])^n/Sqrt[1 + Sec[e + f*x]],x]
```

output
```
-((AppellF1[n, 1/2, 1, 1 + n, Sec[e + f*x], -Sec[e + f*x]]*(d*Sec[e + f*x]
)^n*Tan[e + f*x])/(f*n*Sqrt[1 - Sec[e + f*x]]*Sqrt[1 + Sec[e + f*x]]))
```

3.305.3.1 Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] && !In
tegerQ[m] && !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4314
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Simp[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x
]]*Sqrt[a - b*Csc[e + f*x]])) Subst[Int[(d*x)^(n - 1)*((a + b*x)^(m - 1/2
)/Sqrt[a - b*x]), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n},
x] && EqQ[a^2 - b^2, 0] && !IntegerQ[m] && GtQ[a, 0]
```

3.305.4 Maple [F]

$$\int \frac{(d\sec(fx+e))^n}{\sqrt{\sec(fx+e)+1}}dx$$

input
```
int((d*sec(f*x+e))^n/(sec(f*x+e)+1)^(1/2),x)
```

output
```
int((d*sec(f*x+e))^n/(sec(f*x+e)+1)^(1/2),x)
```

3.305.5 Fricas [F]

$$\int \frac{(d\sec(e+fx))^n}{\sqrt{1+\sec(e+fx)}}\,dx = \int \frac{(d\sec(fx+e))^n}{\sqrt{\sec(fx+e)+1}}\,dx$$

input
```
integrate((d*sec(f*x+e))^n/(1+sec(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
integral((d*sec(f*x + e))^n/sqrt(sec(f*x + e) + 1), x)
```

3.305.6 Sympy [F]

$$\int \frac{(d\sec(e+fx))^n}{\sqrt{1+\sec(e+fx)}}\,dx = \int \frac{(d\sec(e+fx))^n}{\sqrt{\sec(e+fx)+1}}\,dx$$

input
```
integrate((d*sec(f*x+e))**n/(1+sec(f*x+e))**(1/2),x)
```

output
```
Integral((d*sec(e + f*x))**n/sqrt(sec(e + f*x) + 1), x)
```

3.305.7 Maxima [F]

$$\int \frac{(d\sec(e+fx))^n}{\sqrt{1+\sec(e+fx)}}\,dx = \int \frac{(d\sec(fx+e))^n}{\sqrt{\sec(fx+e)+1}}\,dx$$

input
```
integrate((d*sec(f*x+e))^n/(1+sec(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate((d*sec(f*x + e))^n/sqrt(sec(f*x + e) + 1), x)
```

3.305.8 Giac [F]

$$\int \frac{(d\sec(e+fx))^n}{\sqrt{1+\sec(e+fx)}}\,dx = \int \frac{(d\sec(fx+e))^n}{\sqrt{\sec(fx+e)+1}}\,dx$$

input
```
integrate((d*sec(f*x+e))^n/(1+sec(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate((d*sec(f*x + e))^n/sqrt(sec(f*x + e) + 1), x)
```

3.305.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d\sec(e+fx))^n}{\sqrt{1+\sec(e+fx)}}\,dx = \int \frac{\left(\frac{d}{\cos(e+f\,x)}\right)^n}{\sqrt{\frac{1}{\cos(e+f\,x)}+1}}\,dx$$

input
```
int((d/cos(e + f*x))^n/(1/cos(e + f*x) + 1)^(1/2),x)
```

output
```
int((d/cos(e + f*x))^n/(1/cos(e + f*x) + 1)^(1/2), x)
```

3.306 $\int \frac{(d\sec(e+fx))^n}{(1+\sec(e+fx))^{3/2}}\,dx$

3.306.1 Optimal result 2351
3.306.2 Mathematica [B] (warning: unable to verify) 2351
3.306.3 Rubi [A] (verified) 2352
3.306.4 Maple [F] 2353
3.306.5 Fricas [F] 2354
3.306.6 Sympy [F] 2354
3.306.7 Maxima [F] 2354
3.306.8 Giac [F] 2355
3.306.9 Mupad [F(-1)] 2355

3.306.1 Optimal result

Integrand size = 23, antiderivative size = 73

$$\int \frac{(d\sec(e+fx))^n}{(1+\sec(e+fx))^{3/2}}\,dx = -\frac{\text{AppellF1}\left(n,\frac{1}{2},2,1+n,\sec(e+fx),-\sec(e+fx)\right)(d\sec(e+fx))^n\tan(e+fx)}{fn\sqrt{1-\sec(e+fx)}\sqrt{1+\sec(e+fx)}}$$

output
```
-AppellF1(n,2,1/2,1+n,-sec(f*x+e),sec(f*x+e))*(d*sec(f*x+e))^n*tan(f*x+e)/
f/n/(1-sec(f*x+e))^(1/2)/(1+sec(f*x+e))^(1/2)
```

3.306.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 3003 vs. 2(73) = 146.

Time = 6.18 (sec) , antiderivative size = 3003, normalized size of antiderivative = 41.14

$$\int \frac{(d\sec(e+fx))^n}{(1+\sec(e+fx))^{3/2}}\,dx = \text{Result too large to show}$$

input
```
Integrate[(d*Sec[e + f*x])^n/(1 + Sec[e + f*x])^(3/2),x]
```

output
```
(6*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/
2]^2]*(Sec[(e + f*x)/2]^2)^n*Sec[e + f*x]^(1/2 - n + (-3 + 2*n)/2)*(d*Sec[
e + f*x])^n*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(3/2 + n)*Tan[(e + f*x)/2]*(
-1 + Tan[(e + f*x)/2]^2)^2)/(f*(1 + Sec[e + f*x])^(3/2)*(3*AppellF1[1/2, -
3/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n
)*AppellF1[3/2, -3/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2
]^2] + (-3 + 2*n)*AppellF1[3/2, -1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2,
-Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)*((12*AppellF1[1/2, -3/2 + n, 1 -
 n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[e + f*x]*(Sec[(e + f
*x)/2]^2)^(1 + n)*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(3/2 + n)*Tan[(e + f*x
)/2]^2*(-1 + Tan[(e + f*x)/2]^2))/(3*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, T
an[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)*AppellF1[3/2, -3/2 +
 n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-3 + 2*n)*Appe
llF1[3/2, -1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*
Tan[(e + f*x)/2]^2) + (3*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e + f*x)
/2]^2, -Tan[(e + f*x)/2]^2]*Cos[e + f*x]*(Sec[(e + f*x)/2]^2)^(1 + n)*(Cos
[(e + f*x)/2]^2*Sec[e + f*x])^(3/2 + n)*(-1 + Tan[(e + f*x)/2]^2)^2)/(3*Ap
pellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]
 + (2*(-1 + n)*AppellF1[3/2, -3/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Ta
n[(e + f*x)/2]^2] + (-3 + 2*n)*AppellF1[3/2, -1/2 + n, 1 - n, 5/2, Tan[...
```

3.306.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3042, 4314, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d\sec(e+fx))^n}{(\sec(e+fx)+1)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^n}{\left(\csc\left(e+fx+\frac{\pi}{2}\right)+1\right)^{3/2}}dx$$

$$\Big\downarrow \text{4314}$$

$$-\frac{d\tan(e+fx)\int \frac{(d\sec(e+fx))^{n-1}}{\sqrt{1-\sec(e+fx)}(\sec(e+fx)+1)^2}d\sec(e+fx)}{f\sqrt{1-\sec(e+fx)}\sqrt{\sec(e+fx)+1}}$$

$$\Big\downarrow \text{150}$$

$$-\frac{\tan(e+fx)\,\text{AppellF1}\left(n,\frac{1}{2},2,n+1,\sec(e+fx),-\sec(e+fx)\right)(d\sec(e+fx))^n}{fn\sqrt{1-\sec(e+fx)}\sqrt{\sec(e+fx)+1}}$$

input
```
Int[(d*Sec[e + f*x])^n/(1 + Sec[e + f*x])^(3/2),x]
```

output
```
-((AppellF1[n, 1/2, 2, 1 + n, Sec[e + f*x], -Sec[e + f*x]]*(d*Sec[e + f*x]
)^n*Tan[e + f*x])/(f*n*Sqrt[1 - Sec[e + f*x]]*Sqrt[1 + Sec[e + f*x]]))
```

3.306.3.1 Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] && !In
tegerQ[m] && !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4314
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Simp[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x
]]*Sqrt[a - b*Csc[e + f*x]])) Subst[Int[(d*x)^(n - 1)*((a + b*x)^(m - 1/2
)/Sqrt[a - b*x]), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n},
x] && EqQ[a^2 - b^2, 0] && !IntegerQ[m] && GtQ[a, 0]
```

3.306.4 Maple [F]

$$\int \frac{(d\sec(fx+e))^n}{(\sec(fx+e)+1)^{\frac{3}{2}}}dx$$

input
```
int((d*sec(f*x+e))^n/(sec(f*x+e)+1)^(3/2),x)
```

output
```
int((d*sec(f*x+e))^n/(sec(f*x+e)+1)^(3/2),x)
```

3.306.5 Fricas [F]

$$\int \frac{(d\sec(e+fx))^n}{(1+\sec(e+fx))^{3/2}}\,dx = \int \frac{(d\sec(fx+e))^n}{(\sec(fx+e)+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*sec(f*x+e))^n/(1+sec(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
integral((d*sec(f*x + e))^n*sqrt(sec(f*x + e) + 1)/(sec(f*x + e)^2 + 2*sec
(f*x + e) + 1), x)
```

3.306.6 Sympy [F]

$$\int \frac{(d\sec(e+fx))^n}{(1+\sec(e+fx))^{3/2}}\,dx = \int \frac{(d\sec(e+fx))^n}{(\sec(e+fx)+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*sec(f*x+e))**n/(1+sec(f*x+e))**(3/2),x)
```

output
```
Integral((d*sec(e + f*x))**n/(sec(e + f*x) + 1)**(3/2), x)
```

3.306.7 Maxima [F]

$$\int \frac{(d\sec(e+fx))^n}{(1+\sec(e+fx))^{3/2}}\,dx = \int \frac{(d\sec(fx+e))^n}{(\sec(fx+e)+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*sec(f*x+e))^n/(1+sec(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
integrate((d*sec(f*x + e))^n/(sec(f*x + e) + 1)^(3/2), x)
```

3.306.8 Giac [F]

$$\int \frac{(d\sec(e+fx))^n}{(1+\sec(e+fx))^{3/2}}\,dx = \int \frac{(d\sec(fx+e))^n}{(\sec(fx+e)+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*sec(f*x+e))^n/(1+sec(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
integrate((d*sec(f*x + e))^n/(sec(f*x + e) + 1)^(3/2), x)
```

3.306.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d\sec(e+fx))^n}{(1+\sec(e+fx))^{3/2}}\,dx = \int \frac{\left(\frac{d}{\cos(e+f\,x)}\right)^n}{\left(\frac{1}{\cos(e+f\,x)}+1\right)^{3/2}}\,dx$$

input
```
int((d/cos(e + f*x))^n/(1/cos(e + f*x) + 1)^(3/2),x)
```

output
```
int((d/cos(e + f*x))^n/(1/cos(e + f*x) + 1)^(3/2), x)
```

3.307 $\int \sec^n(e+fx)(a+a\sec(e+fx))^{5/2}\,dx$

3.307.1 Optimal result . . . 2356
3.307.2 Mathematica [C] (warning: unable to verify) . . . 2356
3.307.3 Rubi [A] (verified) . . . 2357
3.307.4 Maple [F] . . . 2360
3.307.5 Fricas [F] . . . 2360
3.307.6 Sympy [F(-1)] . . . 2360
3.307.7 Maxima [F] . . . 2361
3.307.8 Giac [F] . . . 2361
3.307.9 Mupad [F(-1)] . . . 2361

3.307.1 Optimal result

Integrand size = 23, antiderivative size = 177

$$\int \sec^n(e+fx)(a+a\sec(e+fx))^{5/2}\,dx = \frac{2a^3(7+4n)\sec^{1+n}(e+fx)\sin(e+fx)}{f(1+2n)(3+2n)\sqrt{a+a\sec(e+fx)}} + \frac{2a^2\sec^{1+n}(e+fx)\sqrt{a+a\sec(e+fx)}\sin(e+fx)}{f(3+2n)} + \frac{2a^3(3+24n+16n^2)\,\text{Hypergeometric2F1}\left(\frac{1}{2},1-n,\frac{3}{2},1-\sec(e+fx)\right)\tan(e+fx)}{f(1+2n)(3+2n)\sqrt{a+a\sec(e+fx)}}$$

output
```
2*a^3*(7+4*n)*sec(f*x+e)^(1+n)*sin(f*x+e)/f/(4*n^2+8*n+3)/(a+a*sec(f*x+e))
^(1/2)+2*a^2*sec(f*x+e)^(1+n)*sin(f*x+e)*(a+a*sec(f*x+e))^(1/2)/f/(3+2*n)+
2*a^3*(16*n^2+24*n+3)*hypergeom([1/2, 1-n],[3/2],1-sec(f*x+e))*tan(f*x+e)/
f/(4*n^2+8*n+3)/(a+a*sec(f*x+e))^(1/2)
```

3.307.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 3.58 (sec) , antiderivative size = 400, normalized size of antiderivative = 2.26

$$\int \sec^n(e+fx)(a+a\sec(e+fx))^{5/2}\,dx = -\frac{i2^{-\frac{5}{2}+n}e^{-\frac{1}{2}i(3+2n)(e+fx)}\left(\frac{e^{i(e+fx)}}{1+e^{2i(e+fx)}}\right)^{\frac{3}{2}+n}\left(\frac{10e^{i(2+n)(e+fx)}\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}(-1-n),\frac{4+n}{2},-e^{2i(e+fx)}\right)}{2+n}+\frac{5e^{i(4+n)(e+fx)}\,\text{H}}{}\right.}{}$$

input
```
Integrate[Sec[e + f*x]^n*(a + a*Sec[e + f*x])^(5/2),x]
```

output
```
((-I)*2^(-5/2 + n)*(E^(I*(e + f*x))/(1 + E^((2*I)*(e + f*x))))^(3/2 + n)*(
(10*E^(I*(2 + n)*(e + f*x))*Hypergeometric2F1[1, (-1 - n)/2, (4 + n)/2, -E
^((2*I)*(e + f*x))])/(2 + n) + (5*E^(I*(4 + n)*(e + f*x))*Hypergeometric2F
1[1, (1 - n)/2, (6 + n)/2, -E^((2*I)*(e + f*x))])/(4 + n) + (E^(I*n*(e + f
*x))*Hypergeometric2F1[1, -3/2 - n/2, 1 + n/2, -E^((2*I)*(e + f*x))])/n +
(5*E^(I*(1 + n)*(e + f*x))*Hypergeometric2F1[1, -1 - n/2, (3 + n)/2, -E^((
2*I)*(e + f*x))])/(1 + n) + (E^(I*(5 + n)*(e + f*x))*Hypergeometric2F1[1,
1 - n/2, (7 + n)/2, -E^((2*I)*(e + f*x))])/(5 + n) + (10*E^(I*(3 + n)*(e +
 f*x))*Hypergeometric2F1[1, -1/2*n, (5 + n)/2, -E^((2*I)*(e + f*x))])/(3 +
 n))*Sec[(e + f*x)/2]^5*(a*(1 + Sec[e + f*x]))^(5/2))/(E^((I/2)*(3 + 2*n)*
(e + f*x))*f*Sec[e + f*x]^(5/2))
```

3.307.3 Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.98, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {3042, 4301, 27, 3042, 4504, 3042, 4293, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sec(e+fx)+a)^{5/2}\sec^n(e+fx)\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{5/2}\csc\left(e+fx+\frac{\pi}{2}\right)^n dx$$

$$\Big\downarrow \; 4301$$

$$\frac{2a\int \frac{1}{2}\sec^n(e+fx)\sqrt{\sec(e+fx)a+a}(a(4n+3)+a(4n+7)\sec(e+fx))dx}{2n+3}+\frac{2a^2\sin(e+fx)\sqrt{a\sec(e+fx)+a}\sec^{n+1}(e+fx)}{f(2n+3)}$$

$$\Big\downarrow \; 27$$

$$\frac{a\int \sec^n(e+fx)\sqrt{\sec(e+fx)a+a}(a(4n+3)+a(4n+7)\sec(e+fx))dx}{2n+3}+\frac{2a^2\sin(e+fx)\sqrt{a\sec(e+fx)+a}\sec^{n+1}(e+fx)}{f(2n+3)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a\int \csc\left(e+fx+\frac{\pi}{2}\right)^n\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}\left(a(4n+3)+a(4n+7)\csc\left(e+fx+\frac{\pi}{2}\right)\right)dx}{2n+3}+\frac{2a^2\sin(e+fx)\sqrt{a\sec(e+fx)+a}\sec^{n+1}(e+fx)}{f(2n+3)}$$

$$\Big\downarrow \text{4504}$$

$$\frac{a\left(\frac{a(16n^2+24n+3)\int\sec^n(e+fx)\sqrt{\sec(e+fx)a+a}dx}{2n+1}+\frac{2a^2(4n+7)\sin(e+fx)\sec^{n+1}(e+fx)}{f(2n+1)\sqrt{a\sec(e+fx)+a}}\right)}{2n+3}+\frac{2a^2\sin(e+fx)\sqrt{a\sec(e+fx)+a}\sec^{n+1}(e+fx)}{f(2n+3)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a\left(\frac{a(16n^2+24n+3)\int\csc\left(e+fx+\frac{\pi}{2}\right)^n\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}dx}{2n+1}+\frac{2a^2(4n+7)\sin(e+fx)\sec^{n+1}(e+fx)}{f(2n+1)\sqrt{a\sec(e+fx)+a}}\right)}{2n+3}+\frac{2a^2\sin(e+fx)\sqrt{a\sec(e+fx)+a}\sec^{n+1}(e+fx)}{f(2n+3)}$$

$$\Big\downarrow \text{4293}$$

$$\frac{a\left(\frac{2a^2(4n+7)\sin(e+fx)\sec^{n+1}(e+fx)}{f(2n+1)\sqrt{a\sec(e+fx)+a}}-\frac{a^3(16n^2+24n+3)\tan(e+fx)\int\frac{\sec^{n-1}(e+fx)}{\sqrt{a-a\sec(e+fx)}}d\sec(e+fx)}{f(2n+1)\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}\right)}{2n+3}+\frac{2a^2\sin(e+fx)\sqrt{a\sec(e+fx)+a}\sec^{n+1}(e+fx)}{f(2n+3)}$$

$$\Big\downarrow \text{75}$$

$$\frac{a\left(\frac{2a^2(16n^2+24n+3)\tan(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},1-n,\frac{3}{2},1-\sec(e+fx)\right)}{f(2n+1)\sqrt{a\sec(e+fx)+a}}+\frac{2a^2(4n+7)\sin(e+fx)\sec^{n+1}(e+fx)}{f(2n+1)\sqrt{a\sec(e+fx)+a}}\right)}{2n+3}+\frac{2a^2\sin(e+fx)\sqrt{a\sec(e+fx)+a}\sec^{n+1}(e+fx)}{f(2n+3)}$$

input
```
Int[Sec[e + f*x]^n*(a + a*Sec[e + f*x])^(5/2),x]
```

3.307. $\int \sec^n(e+fx)(a+a\sec(e+fx))^{5/2}\,dx$

output
```
(2*a^2*Sec[e + f*x]^(1 + n)*Sqrt[a + a*Sec[e + f*x]]*Sin[e + f*x])/(f*(3 +
 2*n)) + (a*((2*a^2*(7 + 4*n)*Sec[e + f*x]^(1 + n)*Sin[e + f*x])/(f*(1 + 2
*n)*Sqrt[a + a*Sec[e + f*x]]) + (2*a^2*(3 + 24*n + 16*n^2)*Hypergeometric2
F1[1/2, 1 - n, 3/2, 1 - Sec[e + f*x]]*Tan[e + f*x])/(f*(1 + 2*n)*Sqrt[a +
a*Sec[e + f*x]])))/(3 + 2*n)
```

3.307.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4293
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]
*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(d*x)^(n - 1)/Sqrt[a - b*x], x], x,
 Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0]
```

rule 4301
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
2)*((d*Csc[e + f*x])^n/(f*(m + n - 1))), x] + Simp[b/(m + n - 1)   Int[(a +
 b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*(b*(m + 2*n - 1) + a*(3*m + 2*n
 - 4)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^
2, 0] && GtQ[m, 1] && NeQ[m + n - 1, 0] && IntegerQ[2*m]
```

rule 4504
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[-2*b*B*C
ot[e + f*x]*((d*Csc[e + f*x])^n/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]])), x]
 + Simp[(A*b*(2*n + 1) + 2*a*B*n)/(b*(2*n + 1))   Int[Sqrt[a + b*Csc[e + f*
x]]*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ
[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] &&
!LtQ[n, 0]
```

3.307.4 Maple [F]

$$\int \sec(fx+e)^n (a+a\sec(fx+e))^{\frac{5}{2}}\,dx$$

input
```
int(sec(f*x+e)^n*(a+a*sec(f*x+e))^(5/2),x)
```

output
```
int(sec(f*x+e)^n*(a+a*sec(f*x+e))^(5/2),x)
```

3.307.5 Fricas [F]

$$\int \sec^n(e+fx)(a+a\sec(e+fx))^{5/2}\,dx = \int (a\sec(fx+e)+a)^{\frac{5}{2}}\sec(fx+e)^n\,dx$$

input
```
integrate(sec(f*x+e)^n*(a+a*sec(f*x+e))^(5/2),x, algorithm="fricas")
```

output
```
integral((a^2*sec(f*x + e)^2 + 2*a^2*sec(f*x + e) + a^2)*sqrt(a*sec(f*x +
e) + a)*sec(f*x + e)^n, x)
```

3.307.6 Sympy [F(-1)]

Timed out.

$$\int \sec^n(e+fx)(a+a\sec(e+fx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate(sec(f*x+e)**n*(a+a*sec(f*x+e))**(5/2),x)
```

output
```
Timed out
```

3.307.7 Maxima [F]

$$\int \sec^n(e+fx)(a+a\sec(e+fx))^{5/2}\,dx = \int (a\sec(fx+e)+a)^{\frac{5}{2}}\sec(fx+e)^n\,dx$$

input
```
integrate(sec(f*x+e)^n*(a+a*sec(f*x+e))^(5/2),x, algorithm="maxima")
```

output
```
integrate((a*sec(f*x + e) + a)^(5/2)*sec(f*x + e)^n, x)
```

3.307.8 Giac [F]

$$\int \sec^n(e+fx)(a+a\sec(e+fx))^{5/2}\,dx = \int (a\sec(fx+e)+a)^{\frac{5}{2}}\sec(fx+e)^n\,dx$$

input
```
integrate(sec(f*x+e)^n*(a+a*sec(f*x+e))^(5/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.307.9 Mupad [F(-1)]

Timed out.

$$\int \sec^n(e+fx)(a+a\sec(e+fx))^{5/2}\,dx = \int \left(a+\frac{a}{\cos(e+f\,x)}\right)^{5/2}\left(\frac{1}{\cos(e+f\,x)}\right)^n dx$$

input
```
int((a + a/cos(e + f*x))^(5/2)*(1/cos(e + f*x))^n,x)
```

output
```
int((a + a/cos(e + f*x))^(5/2)*(1/cos(e + f*x))^n, x)
```

3.308 $\int \sec^n(e+fx)(a+a\sec(e+fx))^{3/2}\,dx$

3.308.1 Optimal result 2362
3.308.2 Mathematica [A] (verified) 2362
3.308.3 Rubi [A] (verified) 2363
3.308.4 Maple [F] 2365
3.308.5 Fricas [F] 2365
3.308.6 Sympy [F] 2365
3.308.7 Maxima [F] 2366
3.308.8 Giac [F] 2366
3.308.9 Mupad [F(-1)] 2366

3.308.1 Optimal result

Integrand size = 23, antiderivative size = 108

$$\int \sec^n(e+fx)(a+a\sec(e+fx))^{3/2}\,dx = \frac{2a^2\sec^{1+n}(e+fx)\sin(e+fx)}{f(1+2n)\sqrt{a+a\sec(e+fx)}} + \frac{2a^2(1+4n)\,\text{Hypergeometric2F1}\left(\frac{1}{2},1-n,\frac{3}{2},1-\sec(e+fx)\right)\tan(e+fx)}{f(1+2n)\sqrt{a+a\sec(e+fx)}}$$

output
```
2*a^2*sec(f*x+e)^(1+n)*sin(f*x+e)/f/(1+2*n)/(a+a*sec(f*x+e))^(1/2)+2*a^2*(
1+4*n)*hypergeom([1/2, 1-n],[3/2],1-sec(f*x+e))*tan(f*x+e)/f/(1+2*n)/(a+a*
sec(f*x+e))^(1/2)
```

3.308.2 Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.80

$$\int \sec^n(e+fx)(a+a\sec(e+fx))^{3/2}\,dx = \frac{a\left(-1+(1+4n)\cos^{\frac{1}{2}+n}(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{2}+n,\frac{3}{2},2\sin^2\left(\frac{1}{2}(e+fx)\right)\right)\right)\sec^n(e+fx)}{fn}$$

input
```
Integrate[Sec[e + f*x]^n*(a + a*Sec[e + f*x])^(3/2),x]
```

output
```
(a*(-1 + (1 + 4*n)*Cos[e + f*x]^(1/2 + n)*Hypergeometric2F1[1/2, 3/2 + n,
3/2, 2*Sin[(e + f*x)/2]^2])*Sec[e + f*x]^n*Sqrt[a*(1 + Sec[e + f*x])]*Tan[
(e + f*x)/2])/(f*n)
```

3.308.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3042, 4301, 27, 2011, 3042, 4293, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sec(e+fx)+a)^{3/2}\sec^n(e+fx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{3/2}\csc\left(e+fx+\frac{\pi}{2}\right)^n dx$$

$$\Big\downarrow \text{4301}$$

$$\frac{2a\int \frac{\sec^n(e+fx)(a(4n+1)+a\sec(e+fx)(4n+1))}{2\sqrt{\sec(e+fx)a+a}}dx}{2n+1}+\frac{2a^2\sin(e+fx)\sec^{n+1}(e+fx)}{f(2n+1)\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{27}$$

$$\frac{a\int \frac{\sec^n(e+fx)(a(4n+1)+a\sec(e+fx)(4n+1))}{\sqrt{\sec(e+fx)a+a}}dx}{2n+1}+\frac{2a^2\sin(e+fx)\sec^{n+1}(e+fx)}{f(2n+1)\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{2011}$$

$$\frac{a(4n+1)\int \sec^n(e+fx)\sqrt{\sec(e+fx)a+a}dx}{2n+1}+\frac{2a^2\sin(e+fx)\sec^{n+1}(e+fx)}{f(2n+1)\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a(4n+1)\int \csc\left(e+fx+\frac{\pi}{2}\right)^n\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}dx}{2n+1}+\frac{2a^2\sin(e+fx)\sec^{n+1}(e+fx)}{f(2n+1)\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{4293}$$

$$\frac{2a^2\sin(e+fx)\sec^{n+1}(e+fx)}{f(2n+1)\sqrt{a\sec(e+fx)+a}}-\frac{a^3(4n+1)\tan(e+fx)\int \frac{\sec^{n-1}(e+fx)}{\sqrt{a-a\sec(e+fx)}}d\sec(e+fx)}{f(2n+1)\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \; 75$$

$$\frac{2a^2(4n+1)\tan(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},1-n,\frac{3}{2},1-\sec(e+fx)\right)}{f(2n+1)\sqrt{a\sec(e+fx)+a}}+\frac{2a^2\sin(e+fx)\sec^{n+1}(e+fx)}{f(2n+1)\sqrt{a\sec(e+fx)+a}}$$

input
```
Int[Sec[e + f*x]^n*(a + a*Sec[e + f*x])^(3/2),x]
```

output
```
(2*a^2*Sec[e + f*x]^(1 + n)*Sin[e + f*x])/(f*(1 + 2*n)*Sqrt[a + a*Sec[e +
f*x]]) + (2*a^2*(1 + 4*n)*Hypergeometric2F1[1/2, 1 - n, 3/2, 1 - Sec[e + f
*x]]*Tan[e + f*x])/(f*(1 + 2*n)*Sqrt[a + a*Sec[e + f*x]])
```

3.308.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 2011
```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x
] && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x
, a + b*x])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4293
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]
*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(d*x)^(n - 1)/Sqrt[a - b*x], x], x,
 Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0]
```

rule 4301
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
2)*((d*Csc[e + f*x])^n/(f*(m + n - 1))), x] + Simp[b/(m + n - 1)   Int[(a +
 b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*(b*(m + 2*n - 1) + a*(3*m + 2*n
 - 4)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^
2, 0] && GtQ[m, 1] && NeQ[m + n - 1, 0] && IntegerQ[2*m]
```

3.308.4 Maple [F]

$$\int \sec (fx+e)^n (a+a\sec (fx+e))^{\frac{3}{2}} dx$$

input
```
int(sec(f*x+e)^n*(a+a*sec(f*x+e))^(3/2),x)
```

output
```
int(sec(f*x+e)^n*(a+a*sec(f*x+e))^(3/2),x)
```

3.308.5 Fricas [F]

$$\int \sec^n(e+fx)(a+a\sec(e+fx))^{3/2}\,dx=\int (a\sec (fx+e)+a)^{\frac{3}{2}}\sec (fx+e)^n\,dx$$

input
```
integrate(sec(f*x+e)^n*(a+a*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
integral((a*sec(f*x + e) + a)^(3/2)*sec(f*x + e)^n, x)
```

3.308.6 Sympy [F]

$$\int \sec^n(e+fx)(a+a\sec(e+fx))^{3/2}\,dx=\int (a(\sec (e+fx)+1))^{\frac{3}{2}}\sec^n (e+fx)\,dx$$

input
```
integrate(sec(f*x+e)**n*(a+a*sec(f*x+e))**(3/2),x)
```

output
```
Integral((a*(sec(e + f*x) + 1))**(3/2)*sec(e + f*x)**n, x)
```

3.308.7 Maxima [F]

$$\int \sec^n(e+fx)(a+a\sec(e+fx))^{3/2}\,dx = \int (a\sec(fx+e)+a)^{\frac{3}{2}}\sec(fx+e)^n\,dx$$

input
```
integrate(sec(f*x+e)^n*(a+a*sec(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
integrate((a*sec(f*x + e) + a)^(3/2)*sec(f*x + e)^n, x)
```

3.308.8 Giac [F]

$$\int \sec^n(e+fx)(a+a\sec(e+fx))^{3/2}\,dx = \int (a\sec(fx+e)+a)^{\frac{3}{2}}\sec(fx+e)^n\,dx$$

input
```
integrate(sec(f*x+e)^n*(a+a*sec(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.308.9 Mupad [F(-1)]

Timed out.

$$\int \sec^n(e+fx)(a+a\sec(e+fx))^{3/2}\,dx = \int \left(a+\frac{a}{\cos(e+f\,x)}\right)^{3/2}\left(\frac{1}{\cos(e+f\,x)}\right)^n dx$$

input
```
int((a + a/cos(e + f*x))^(3/2)*(1/cos(e + f*x))^n,x)
```

output
```
int((a + a/cos(e + f*x))^(3/2)*(1/cos(e + f*x))^n, x)
```

3.309 $\int \sec^n(e+fx)\sqrt{a+a\sec(e+fx)}\,dx$

3.309.1 Optimal result 2367
3.309.2 Mathematica [A] (verified) 2367
3.309.3 Rubi [A] (verified) 2368
3.309.4 Maple [F] 2369
3.309.5 Fricas [F] 2369
3.309.6 Sympy [F] 2369
3.309.7 Maxima [F] 2370
3.309.8 Giac [F] 2370
3.309.9 Mupad [F(-1)] 2370

3.309.1 Optimal result

Integrand size = 23, antiderivative size = 48

$$\int \sec^n(e+fx)\sqrt{a+a\sec(e+fx)}\,dx = \frac{2a\,\text{Hypergeometric2F1}\left(\frac{1}{2},1-n,\frac{3}{2},1-\sec(e+fx)\right)\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}}$$

output
```
2*a*hypergeom([1/2, 1-n],[3/2],1-sec(f*x+e))*tan(f*x+e)/f/(a+a*sec(f*x+e))
^(1/2)
```

3.309.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.06

$$\int \sec^n(e+fx)\sqrt{a+a\sec(e+fx)}\,dx = \frac{2\,\text{Hypergeometric2F1}\left(\frac{1}{2},1-n,\frac{3}{2},1-\sec(e+fx)\right)\sqrt{a(1+\sec(e+fx))}\tan\left(\frac{1}{2}(e+fx)\right)}{f}$$

input
```
Integrate[Sec[e + f*x]^n*Sqrt[a + a*Sec[e + f*x]],x]
```

output
```
(2*Hypergeometric2F1[1/2, 1 - n, 3/2, 1 - Sec[e + f*x]]*Sqrt[a*(1 + Sec[e
+ f*x])]*Tan[(e + f*x)/2])/f
```

3.309.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3042, 4293, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a\sec(e+fx)+a}\sec^n(e+fx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{a\csc\left(e+fx+\frac{\pi}{2}\right)+a}\csc\left(e+fx+\frac{\pi}{2}\right)^n dx$$

$$\Big\downarrow \text{4293}$$

$$-\frac{a^2\tan(e+fx)\int \frac{\sec^{n-1}(e+fx)}{\sqrt{a-a\sec(e+fx)}}d\sec(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{75}$$

$$\frac{2a\tan(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},1-n,\frac{3}{2},1-\sec(e+fx)\right)}{f\sqrt{a\sec(e+fx)+a}}$$

input
```
Int[Sec[e + f*x]^n*Sqrt[a + a*Sec[e + f*x]],x]
```

output
```
(2*a*Hypergeometric2F1[1/2, 1 - n, 3/2, 1 - Sec[e + f*x]]*Tan[e + f*x])/(f
*Sqrt[a + a*Sec[e + f*x]])
```

3.309.3.1 Defintions of rubi rules used

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4293
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]
*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(d*x)^(n - 1)/Sqrt[a - b*x], x], x,
 Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0]
```

3.309.4 Maple [F]

$$\int \sec (fx+e)^{n} \sqrt{a+a\sec (fx+e)}dx$$

input
```
int(sec(f*x+e)^n*(a+a*sec(f*x+e))^(1/2),x)
```

output
```
int(sec(f*x+e)^n*(a+a*sec(f*x+e))^(1/2),x)
```

3.309.5 Fricas [F]

$$\int \sec^n(e+fx)\sqrt{a+a\sec(e+fx)}\,dx=\int \sqrt{a\sec (fx+e)+a}\sec (fx+e)^{n}\,dx$$

input
```
integrate(sec(f*x+e)^n*(a+a*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(a*sec(f*x + e) + a)*sec(f*x + e)^n, x)
```

3.309.6 Sympy [F]

$$\int \sec^n(e+fx)\sqrt{a+a\sec(e+fx)}\,dx=\int \sqrt{a\left(\sec \left(e+fx\right)+1\right)}\sec^{n}\left(e+fx\right)dx$$

input
```
integrate(sec(f*x+e)**n*(a+a*sec(f*x+e))**(1/2),x)
```

output
```
Integral(sqrt(a*(sec(e + f*x) + 1))*sec(e + f*x)**n, x)
```

3.309.7 Maxima [F]

$$\int \sec^n(e+fx)\sqrt{a+a\sec(e+fx)}\,dx = \int \sqrt{a\sec(fx+e)+a}\sec(fx+e)^n\,dx$$

input
```
integrate(sec(f*x+e)^n*(a+a*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(a*sec(f*x + e) + a)*sec(f*x + e)^n, x)
```

3.309.8 Giac [F]

$$\int \sec^n(e+fx)\sqrt{a+a\sec(e+fx)}\,dx = \int \sqrt{a\sec(fx+e)+a}\sec(fx+e)^n\,dx$$

input
```
integrate(sec(f*x+e)^n*(a+a*sec(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.309.9 Mupad [F(-1)]

Timed out.

$$\int \sec^n(e+fx)\sqrt{a+a\sec(e+fx)}\,dx = \int \sqrt{a+\frac{a}{\cos(e+f\,x)}}\left(\frac{1}{\cos(e+f\,x)}\right)^n dx$$

input
```
int((a + a/cos(e + f*x))^(1/2)*(1/cos(e + f*x))^n,x)
```

output
```
int((a + a/cos(e + f*x))^(1/2)*(1/cos(e + f*x))^n, x)
```

3.310 $\int \frac{\sec^n(e+fx)}{\sqrt{a+a\sec(e+fx)}}\,dx$

3.310.1 Optimal result 2371
3.310.2 Mathematica [B] (warning: unable to verify) 2371
3.310.3 Rubi [A] (verified) 2372
3.310.4 Maple [F] 2374
3.310.5 Fricas [F] 2374
3.310.6 Sympy [F] 2375
3.310.7 Maxima [F] 2375
3.310.8 Giac [F] 2375
3.310.9 Mupad [F(-1)] 2376

3.310.1 Optimal result

Integrand size = 23, antiderivative size = 61

$$\int \frac{\sec^n(e+fx)}{\sqrt{a+a\sec(e+fx)}}\,dx = \frac{\text{AppellF1}\left(\frac{1}{2},1-n,1,\frac{3}{2},1-\sec(e+fx),\frac{1}{2}(1-\sec(e+fx))\right)\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}}$$

output
```
AppellF1(1/2,1-n,1,3/2,1-sec(f*x+e),1/2-1/2*sec(f*x+e))*tan(f*x+e)/f/(a+a*
sec(f*x+e))^(1/2)
```

3.310.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 2964 vs. 2(61) = 122.

Time = 6.21 (sec) , antiderivative size = 2964, normalized size of antiderivative = 48.59

$$\int \frac{\sec^n(e+fx)}{\sqrt{a+a\sec(e+fx)}}\,dx = \text{Result too large to show}$$

input
```
Integrate[Sec[e + f*x]^n/Sqrt[a + a*Sec[e + f*x]],x]
```

output
```
(3*Sqrt[2]*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e
 + f*x)/2]^2]*(Sec[(e + f*x)/2]^2)^n*Sec[e + f*x]^(-1/2 + (-1 + 2*n)/2)*(C
os[(e + f*x)/2]^2*Sec[e + f*x])^n*Sqrt[1 + Sec[e + f*x]]*Tan[(e + f*x)/2])
/(f*Sqrt[a*(1 + Sec[e + f*x])]*(3*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[
(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)*AppellF1[3/2, -1/2 + n,
 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-1 + 2*n)*AppellF
1[3/2, 1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[
(e + f*x)/2]^2)*((3*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2
, -Tan[(e + f*x)/2]^2]*Cos[e + f*x]*(Sec[(e + f*x)/2]^2)^(1 + n)*(Cos[(e +
 f*x)/2]^2*Sec[e + f*x])^n*Sqrt[1 + Sec[e + f*x]])/(Sqrt[2]*(3*AppellF1[1/
2, -1/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1
 + n)*AppellF1[3/2, -1/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*
x)/2]^2] + (-1 + 2*n)*AppellF1[3/2, 1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^
2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)) - (3*Sqrt[2]*AppellF1[1/2, -
1/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(Sec[(e + f*
x)/2]^2)^n*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^n*Sqrt[1 + Sec[e + f*x]]*Sin[
e + f*x]*Tan[(e + f*x)/2])/(3*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e +
 f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)*AppellF1[3/2, -1/2 + n, 2 -
 n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-1 + 2*n)*AppellF1[3/
2, 1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(...
```

3.310.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3042, 4315, 3042, 4312, 148, 333}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^n(e+fx)}{\sqrt{a\sec(e+fx)+a}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)^n}{\sqrt{a\csc\left(e+fx+\frac{\pi}{2}\right)+a}}dx$$

$$\Big\downarrow \text{4315}$$

$$\frac{\sqrt{\sec(e+fx)+1}\int \frac{\sec^n(e+fx)}{\sqrt{\sec(e+fx)+1}}dx}{\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \; 3042$$

$$\frac{\sqrt{\sec(e+fx)+1}\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)^n}{\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)+1}}dx}{\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \; 4312$$

$$\frac{\tan(e+fx)\int \frac{\sec^{n-1}(e+fx)}{\sqrt{1-\sec(e+fx)}(\sec(e+fx)+1)}d(1-\sec(e+fx))}{f\sqrt{1-\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \; 148$$

$$\frac{2\tan(e+fx)\int \frac{\sec^{n-1}(e+fx)}{\sec(e+fx)+1}d\sqrt{1-\sec(e+fx)}}{f\sqrt{1-\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \; 333$$

$$\frac{\tan(e+fx)\,\text{AppellF1}\left(\frac{1}{2},1-n,1,\frac{3}{2},1-\sec(e+fx),\frac{1}{2}(1-\sec(e+fx))\right)}{f\sqrt{a\sec(e+fx)+a}}$$

input
```
Int[Sec[e + f*x]^n/Sqrt[a + a*Sec[e + f*x]],x]
```

output
```
(AppellF1[1/2, 1 - n, 1, 3/2, 1 - Sec[e + f*x], (1 - Sec[e + f*x])/2]*Tan[
e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]])
```

3.310.3.1 Defintions of rubi rules used

rule 148
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_))^(p_.),
x_] :> With[{k = Denominator[m]}, Simp[k/b   Subst[Int[x^(k*(m + 1) - 1)*(c
 + d*(x^k/b))^n*(e + f*(x^k/b))^p, x], x, (b*x)^(1/k)], x]] /; FreeQ[{b, c,
 d, e, f, n, p}, x] && FractionQ[m] && IntegerQ[p]
```

rule 333
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^p*c^q*x*AppellF1[1/2, -p, -q, 3/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; F
reeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[p] || GtQ[a,
0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4312
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-(a*(d/b))^n)*(Cot[e + f*x]/(a^(n - 2)*f*Sqrt
[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(a - x)^(n - 1)
*((2*a - x)^(m - 1/2)/Sqrt[x]), x], x, a - b*Csc[e + f*x]], x] /; FreeQ[{a,
 b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0] &
&  !IntegerQ[n] && GtQ[a*(d/b), 0]
```

rule 4315
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Simp[a^IntPart[m]*((a + b*Csc[e + f*x])^FracPart[m
]/(1 + (b/a)*Csc[e + f*x])^FracPart[m])   Int[(1 + (b/a)*Csc[e + f*x])^m*(d
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^
2, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

3.310.4 Maple [F]

$$\int \frac{\sec(fx+e)^n}{\sqrt{a+a\sec(fx+e)}}dx$$

input
```
int(sec(f*x+e)^n/(a+a*sec(f*x+e))^(1/2),x)
```

output
```
int(sec(f*x+e)^n/(a+a*sec(f*x+e))^(1/2),x)
```

3.310.5 Fricas [F]

$$\int \frac{\sec^n(e+fx)}{\sqrt{a+a\sec(e+fx)}}\,dx = \int \frac{\sec(fx+e)^n}{\sqrt{a\sec(fx+e)+a}}\,dx$$

input
```
integrate(sec(f*x+e)^n/(a+a*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
integral(sec(f*x + e)^n/sqrt(a*sec(f*x + e) + a), x)
```

3.310.6 Sympy [F]

$$\int \frac{\sec^n(e+fx)}{\sqrt{a+a\sec(e+fx)}}\,dx = \int \frac{\sec^n(e+fx)}{\sqrt{a(\sec(e+fx)+1)}}\,dx$$

input
```
integrate(sec(f*x+e)**n/(a+a*sec(f*x+e))**(1/2),x)
```

output
```
Integral(sec(e + f*x)**n/sqrt(a*(sec(e + f*x) + 1)), x)
```

3.310.7 Maxima [F]

$$\int \frac{\sec^n(e+fx)}{\sqrt{a+a\sec(e+fx)}}\,dx = \int \frac{\sec(fx+e)^n}{\sqrt{a\sec(fx+e)+a}}\,dx$$

input
```
integrate(sec(f*x+e)^n/(a+a*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sec(f*x + e)^n/sqrt(a*sec(f*x + e) + a), x)
```

3.310.8 Giac [F]

$$\int \frac{\sec^n(e+fx)}{\sqrt{a+a\sec(e+fx)}}\,dx = \int \frac{\sec(fx+e)^n}{\sqrt{a\sec(fx+e)+a}}\,dx$$

input
```
integrate(sec(f*x+e)^n/(a+a*sec(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate(sec(f*x + e)^n/sqrt(a*sec(f*x + e) + a), x)
```

3.310.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^n(e+fx)}{\sqrt{a+a\sec(e+fx)}}\,dx = \int \frac{\left(\frac{1}{\cos(e+f\,x)}\right)^n}{\sqrt{a+\frac{a}{\cos(e+f\,x)}}}\,dx$$

input
```
int((1/cos(e + f*x))^n/(a + a/cos(e + f*x))^(1/2),x)
```

output
```
int((1/cos(e + f*x))^n/(a + a/cos(e + f*x))^(1/2), x)
```

3.311 $\int \frac{\sec^n(e+fx)}{(a+a\sec(e+fx))^{3/2}}\,dx$

3.311.1 Optimal result 2377
3.311.2 Mathematica [B] (warning: unable to verify) 2377
3.311.3 Rubi [A] (verified) 2378
3.311.4 Maple [F] 2380
3.311.5 Fricas [F] 2380
3.311.6 Sympy [F] 2381
3.311.7 Maxima [F] 2381
3.311.8 Giac [F] 2381
3.311.9 Mupad [F(-1)] 2382

3.311.1 Optimal result

Integrand size = 23, antiderivative size = 67

$$\int \frac{\sec^n(e+fx)}{(a+a\sec(e+fx))^{3/2}}\,dx=\frac{\text{AppellF1}\left(\frac{1}{2},1-n,2,\frac{3}{2},1-\sec(e+fx),\frac{1}{2}(1-\sec(e+fx))\right)\tan(e+fx)}{2af\sqrt{a+a\sec(e+fx)}}$$

output
```
1/2*AppellF1(1/2,1-n,2,3/2,1-sec(f*x+e),1/2-1/2*sec(f*x+e))*tan(f*x+e)/a/f
/(a+a*sec(f*x+e))^(1/2)
```

3.311.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 2992 vs. 2(67) = 134.

Time = 6.25 (sec) , antiderivative size = 2992, normalized size of antiderivative = 44.66

$$\int \frac{\sec^n(e+fx)}{(a+a\sec(e+fx))^{3/2}}\,dx=\text{Result too large to show}$$

input
```
Integrate[Sec[e + f*x]^n/(a + a*Sec[e + f*x])^(3/2),x]
```

output
```
(6*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/
2]^2]*(Sec[(e + f*x)/2]^2)^n*Sec[e + f*x]^(1/2 + (-3 + 2*n)/2)*(Cos[(e + f
*x)/2]^2*Sec[e + f*x])^(3/2 + n)*Tan[(e + f*x)/2]*(-1 + Tan[(e + f*x)/2]^2
)^2)/(f*(a*(1 + Sec[e + f*x]))^(3/2)*(3*AppellF1[1/2, -3/2 + n, 1 - n, 3/2
, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)*AppellF1[3/2, -3/
2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-3 + 2*n)*A
ppellF1[3/2, -1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2
])*Tan[(e + f*x)/2]^2)*((12*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e + f
*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[e + f*x]*(Sec[(e + f*x)/2]^2)^(1 + n)*(
Cos[(e + f*x)/2]^2*Sec[e + f*x])^(3/2 + n)*Tan[(e + f*x)/2]^2*(-1 + Tan[(e
 + f*x)/2]^2))/(3*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2,
-Tan[(e + f*x)/2]^2] + (2*(-1 + n)*AppellF1[3/2, -3/2 + n, 2 - n, 5/2, Tan
[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-3 + 2*n)*AppellF1[3/2, -1/2 + n,
 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)
 + (3*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*
x)/2]^2]*Cos[e + f*x]*(Sec[(e + f*x)/2]^2)^(1 + n)*(Cos[(e + f*x)/2]^2*Sec
[e + f*x])^(3/2 + n)*(-1 + Tan[(e + f*x)/2]^2)^2)/(3*AppellF1[1/2, -3/2 +
n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)*Appe
llF1[3/2, -3/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] +
 (-3 + 2*n)*AppellF1[3/2, -1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Ta...
```

3.311.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3042, 4315, 3042, 4312, 148, 333}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^n(e+fx)}{(a\sec(e+fx)+a)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)^n}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{3/2}}dx$$

$$\Big\downarrow \text{4315}$$

$$\frac{\sqrt{\sec(e+fx)+1}\int \frac{\sec^n(e+fx)}{(\sec(e+fx)+1)^{3/2}}dx}{a\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \; 3042$$

$$\frac{\sqrt{\sec(e+fx)+1}\int \frac{\csc\left(e+fx+\frac{\pi}{2}\right)^n}{\left(\csc\left(e+fx+\frac{\pi}{2}\right)+1\right)^{3/2}}dx}{a\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \; 4312$$

$$\frac{\tan(e+fx)\int \frac{\sec^{n-1}(e+fx)}{\sqrt{1-\sec(e+fx)}(\sec(e+fx)+1)^2}d(1-\sec(e+fx))}{af\sqrt{1-\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \; 148$$

$$\frac{2\tan(e+fx)\int \frac{\sec^{n-1}(e+fx)}{(\sec(e+fx)+1)^2}d\sqrt{1-\sec(e+fx)}}{af\sqrt{1-\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \; 333$$

$$\frac{\tan(e+fx)\,\text{AppellF1}\left(\frac{1}{2},1-n,2,\frac{3}{2},1-\sec(e+fx),\frac{1}{2}(1-\sec(e+fx))\right)}{2af\sqrt{a\sec(e+fx)+a}}$$

input
```
Int[Sec[e + f*x]^n/(a + a*Sec[e + f*x])^(3/2),x]
```

output
```
(AppellF1[1/2, 1 - n, 2, 3/2, 1 - Sec[e + f*x], (1 - Sec[e + f*x])/2]*Tan[
e + f*x])/(2*a*f*Sqrt[a + a*Sec[e + f*x]])
```

3.311.3.1 Defintions of rubi rules used

rule 148
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_))^(p_.),
x_] :> With[{k = Denominator[m]}, Simp[k/b   Subst[Int[x^(k*(m + 1) - 1)*(c
 + d*(x^k/b))^n*(e + f*(x^k/b))^p, x], x, (b*x)^(1/k)], x]] /; FreeQ[{b, c,
 d, e, f, n, p}, x] && FractionQ[m] && IntegerQ[p]
```

rule 333
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^p*c^q*x*AppellF1[1/2, -p, -q, 3/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; F
reeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[p] || GtQ[a,
0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4312
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-(a*(d/b))^n)*(Cot[e + f*x]/(a^(n - 2)*f*Sqrt
[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(a - x)^(n - 1)
*((2*a - x)^(m - 1/2)/Sqrt[x]), x], x, a - b*Csc[e + f*x]], x] /; FreeQ[{a,
 b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0] &
&  !IntegerQ[n] && GtQ[a*(d/b), 0]
```

rule 4315
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Simp[a^IntPart[m]*((a + b*Csc[e + f*x])^FracPart[m
]/(1 + (b/a)*Csc[e + f*x])^FracPart[m])   Int[(1 + (b/a)*Csc[e + f*x])^m*(d
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^
2, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

3.311.4 Maple [F]

$$\int \frac{\sec(fx+e)^n}{(a+a\sec(fx+e))^{\frac{3}{2}}}dx$$

input
```
int(sec(f*x+e)^n/(a+a*sec(f*x+e))^(3/2),x)
```

output
```
int(sec(f*x+e)^n/(a+a*sec(f*x+e))^(3/2),x)
```

3.311.5 Fricas [F]

$$\int \frac{\sec^n(e+fx)}{(a+a\sec(e+fx))^{3/2}}\,dx = \int \frac{\sec(fx+e)^n}{(a\sec(fx+e)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(f*x+e)^n/(a+a*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(a*sec(f*x + e) + a)*sec(f*x + e)^n/(a^2*sec(f*x + e)^2 + 2*a
^2*sec(f*x + e) + a^2), x)
```

3.311.6 Sympy [F]

$$\int \frac{\sec^n(e+fx)}{(a+a\sec(e+fx))^{3/2}}\,dx = \int \frac{\sec^n(e+fx)}{(a(\sec(e+fx)+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(f*x+e)**n/(a+a*sec(f*x+e))**(3/2),x)
```

output
```
Integral(sec(e + f*x)**n/(a*(sec(e + f*x) + 1))**(3/2), x)
```

3.311.7 Maxima [F]

$$\int \frac{\sec^n(e+fx)}{(a+a\sec(e+fx))^{3/2}}\,dx = \int \frac{\sec(fx+e)^n}{(a\sec(fx+e)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(f*x+e)^n/(a+a*sec(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
integrate(sec(f*x + e)^n/(a*sec(f*x + e) + a)^(3/2), x)
```

3.311.8 Giac [F]

$$\int \frac{\sec^n(e+fx)}{(a+a\sec(e+fx))^{3/2}}\,dx = \int \frac{\sec(fx+e)^n}{(a\sec(fx+e)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(f*x+e)^n/(a+a*sec(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
integrate(sec(f*x + e)^n/(a*sec(f*x + e) + a)^(3/2), x)
```

3.311.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^n(e+fx)}{(a+a\sec(e+fx))^{3/2}}\,dx = \int \frac{\left(\frac{1}{\cos(e+f\,x)}\right)^n}{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{3/2}}\,dx$$

input
```
int((1/cos(e + f*x))^n/(a + a/cos(e + f*x))^(3/2),x)
```

output
```
int((1/cos(e + f*x))^n/(a + a/cos(e + f*x))^(3/2), x)
```

3.312 $\int (-\sec(e+fx))^n (a+a\sec(e+fx))^{3/2}\,dx$

3.312.1 Optimal result 2383
3.312.2 Mathematica [A] (verified) 2383
3.312.3 Rubi [A] (verified) 2384
3.312.4 Maple [F] 2386
3.312.5 Fricas [F] 2387
3.312.6 Sympy [F] 2387
3.312.7 Maxima [F] 2387
3.312.8 Giac [F] 2388
3.312.9 Mupad [F(-1)] 2388

3.312.1 Optimal result

Integrand size = 25, antiderivative size = 130

$$\int (-\sec(e+fx))^n (a+a\sec(e+fx))^{3/2}\,dx = \frac{2a^2(1+4n)\,\text{Hypergeometric2F1}\left(\frac{1}{2},1-n,\frac{3}{2},1-\sec(e+fx)\right)(-\sec(e+fx))^n\sec^{1-n}(e+f}{f(1+2n)\sqrt{a+a\sec(e+fx)}} + \frac{2a^2(-\sec(e+fx))^n\tan(e+fx)}{f(1+2n)\sqrt{a+a\sec(e+fx)}}$$

output
```
2*a^2*(1+4*n)*hypergeom([1/2, 1-n],[3/2],1-sec(f*x+e))*(-sec(f*x+e))^n*sec
(f*x+e)^(1-n)*sin(f*x+e)/f/(1+2*n)/(a+a*sec(f*x+e))^(1/2)+2*a^2*(-sec(f*x+
e))^n*tan(f*x+e)/f/(1+2*n)/(a+a*sec(f*x+e))^(1/2)
```

3.312.2 Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.68

$$\int (-\sec(e+fx))^n (a+a\sec(e+fx))^{3/2}\,dx = \frac{a\left(-1+(1+4n)\cos^{\frac{1}{2}+n}(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{2}+n,\frac{3}{2},2\sin^2\left(\frac{1}{2}(e+fx)\right)\right)\right)(-\text{se}}{fn}$$

input
```
Integrate[(-Sec[e + f*x])^n*(a + a*Sec[e + f*x])^(3/2),x]
```

output
```
(a*(-1 + (1 + 4*n)*Cos[e + f*x]^(1/2 + n)*Hypergeometric2F1[1/2, 3/2 + n,
3/2, 2*Sin[(e + f*x)/2]^2])*(-Sec[e + f*x])^n*Sqrt[a*(1 + Sec[e + f*x])]*T
an[(e + f*x)/2])/(f*n)
```

3.312.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3042, 4301, 27, 2011, 3042, 4293, 77, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sec(e+fx)+a)^{3/2}(-\sec(e+fx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{3/2}\left(-\csc\left(e+fx+\frac{\pi}{2}\right)\right)^n dx$$

$$\Big\downarrow \text{4301}$$

$$\frac{2a\int \frac{(-\sec(e+fx))^n(a(4n+1)+a\sec(e+fx)(4n+1))}{2\sqrt{\sec(e+fx)a+a}}dx}{2n+1}+\frac{2a^2\tan(e+fx)(-\sec(e+fx))^n}{f(2n+1)\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{27}$$

$$\frac{a\int \frac{(-\sec(e+fx))^n(a(4n+1)+a\sec(e+fx)(4n+1))}{\sqrt{\sec(e+fx)a+a}}dx}{2n+1}+\frac{2a^2\tan(e+fx)(-\sec(e+fx))^n}{f(2n+1)\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{2011}$$

$$\frac{a(4n+1)\int(-\sec(e+fx))^n\sqrt{\sec(e+fx)a+a}dx}{2n+1}+\frac{2a^2\tan(e+fx)(-\sec(e+fx))^n}{f(2n+1)\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a(4n+1)\int\left(-\csc\left(e+fx+\frac{\pi}{2}\right)\right)^n\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}dx}{2n+1}+\frac{2a^2\tan(e+fx)(-\sec(e+fx))^n}{f(2n+1)\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{4293}$$

$$\frac{a^3(4n+1)\tan(e+fx)\int\frac{(-\sec(e+fx))^{n-1}}{\sqrt{a-a\sec(e+fx)}}d\sec(e+fx)}{f(2n+1)\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}+\frac{2a^2\tan(e+fx)(-\sec(e+fx))^n}{f(2n+1)\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \; 77$$

$$\frac{2a^2\tan(e+fx)(-\sec(e+fx))^n}{f(2n+1)\sqrt{a\sec(e+fx)+a}}-\frac{a^3(4n+1)\sin(e+fx)(-\sec(e+fx))^n\sec^{1-n}(e+fx)\int\frac{\sec^{n-1}(e+fx)}{\sqrt{a-a\sec(e+fx)}}d\sec(e+fx)}{f(2n+1)\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \; 75$$

$$\frac{2a^2(4n+1)\sin(e+fx)(-\sec(e+fx))^n\sec^{1-n}(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},1-n,\frac{3}{2},1-\sec(e+fx)\right)}{f(2n+1)\sqrt{a\sec(e+fx)+a}}+\frac{2a^2\tan(e+fx)(-\sec(e+fx))^n}{f(2n+1)\sqrt{a\sec(e+fx)+a}}$$

input
```
Int[(-Sec[e + f*x])^n*(a + a*Sec[e + f*x])^(3/2),x]
```

output
```
(2*a^2*(1 + 4*n)*Hypergeometric2F1[1/2, 1 - n, 3/2, 1 - Sec[e + f*x]]*(-Se
c[e + f*x])^n*Sec[e + f*x]^(1 - n)*Sin[e + f*x])/(f*(1 + 2*n)*Sqrt[a + a*S
ec[e + f*x]]) + (2*a^2*(-Sec[e + f*x])^n*Tan[e + f*x])/(f*(1 + 2*n)*Sqrt[a
 + a*Sec[e + f*x]])
```

3.312.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 77
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((-b)*(c/
d))^IntPart[m]*((b*x)^FracPart[m]/((-d)*(x/c))^FracPart[m])   Int[((-d)*(x/
c))^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&
 !IntegerQ[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0]
```

rule 2011
```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x
] && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x
, a + b*x])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4293
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]
*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(d*x)^(n - 1)/Sqrt[a - b*x], x], x,
 Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0]
```

rule 4301
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
2)*((d*Csc[e + f*x])^n/(f*(m + n - 1))), x] + Simp[b/(m + n - 1)   Int[(a +
 b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*(b*(m + 2*n - 1) + a*(3*m + 2*n
 - 4)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^
2, 0] && GtQ[m, 1] && NeQ[m + n - 1, 0] && IntegerQ[2*m]
```

3.312.4 Maple [F]

$$\int (-\sec(fx+e))^n (a+a\sec(fx+e))^{\frac{3}{2}}\,dx$$

input
```
int((-sec(f*x+e))^n*(a+a*sec(f*x+e))^(3/2),x)
```

output
```
int((-sec(f*x+e))^n*(a+a*sec(f*x+e))^(3/2),x)
```

3.312.5 Fricas [F]

$$\int (-\sec(e+fx))^n(a+a\sec(e+fx))^{3/2}\,dx=\int (a\sec(fx+e)+a)^{\frac{3}{2}}(-\sec(fx+e))^n\,dx$$

input
```
integrate((-sec(f*x+e))^n*(a+a*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
integral((a*sec(f*x + e) + a)^(3/2)*(-sec(f*x + e))^n, x)
```

3.312.6 Sympy [F]

$$\int (-\sec(e+fx))^n(a+a\sec(e+fx))^{3/2}\,dx=\int (-\sec(e+fx))^n(a(\sec(e+fx)+1))^{\frac{3}{2}}\,dx$$

input
```
integrate((-sec(f*x+e))**n*(a+a*sec(f*x+e))**(3/2),x)
```

output
```
Integral((-sec(e + f*x))**n*(a*(sec(e + f*x) + 1))**(3/2), x)
```

3.312.7 Maxima [F]

$$\int (-\sec(e+fx))^n(a+a\sec(e+fx))^{3/2}\,dx=\int (a\sec(fx+e)+a)^{\frac{3}{2}}(-\sec(fx+e))^n\,dx$$

input
```
integrate((-sec(f*x+e))^n*(a+a*sec(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
integrate((a*sec(f*x + e) + a)^(3/2)*(-sec(f*x + e))^n, x)
```

3.312.8 Giac [F]

$$\int (-\sec(e+fx))^n(a+a\sec(e+fx))^{3/2}\,dx = \int (a\sec(fx+e)+a)^{\frac{3}{2}}(-\sec(fx+e))^n\,dx$$

input
```
integrate((-sec(f*x+e))^n*(a+a*sec(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.312.9 Mupad [F(-1)]

Timed out.

$$\int (-\sec(e+fx))^n(a+a\sec(e+fx))^{3/2}\,dx = \int \left(a+\frac{a}{\cos(e+f\,x)}\right)^{3/2}\left(-\frac{1}{\cos(e+f\,x)}\right)^n dx$$

input
```
int((a + a/cos(e + f*x))^(3/2)*(-1/cos(e + f*x))^n,x)
```

output
```
int((a + a/cos(e + f*x))^(3/2)*(-1/cos(e + f*x))^n, x)
```

3.313 $\int (-\sec(e+fx))^n \sqrt{a+a\sec(e+fx)}\,dx$

3.313.1 Optimal result 2389
3.313.2 Mathematica [A] (verified) 2389
3.313.3 Rubi [A] (verified) 2390
3.313.4 Maple [F] 2391
3.313.5 Fricas [F] 2392
3.313.6 Sympy [F] 2392
3.313.7 Maxima [F] 2392
3.313.8 Giac [F] 2393
3.313.9 Mupad [F(-1)] 2393

3.313.1 Optimal result

Integrand size = 25, antiderivative size = 70

$$\int (-\sec(e+fx))^n \sqrt{a+a\sec(e+fx)}\,dx = \frac{2a\,\text{Hypergeometric2F1}\left(\frac{1}{2},1-n,\frac{3}{2},1-\sec(e+fx)\right)(-\sec(e+fx))^n\sec^{1-n}(e+fx)\sin(e+fx)}{f\sqrt{a+a\sec(e+fx)}}$$

output
```
2*a*hypergeom([1/2, 1-n],[3/2],1-sec(f*x+e))*(-sec(f*x+e))^n*sec(f*x+e)^(1
-n)*sin(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)
```

3.313.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.01

$$\int (-\sec(e+fx))^n \sqrt{a+a\sec(e+fx)}\,dx = \frac{2\,\text{Hypergeometric2F1}\left(\frac{1}{2},1-n,\frac{3}{2},1-\sec(e+fx)\right)(-\sec(e+fx))^n\sec^{-n}(e+fx)\sqrt{a(1+\sec(e+fx))}}{f}$$

input
```
Integrate[(-Sec[e + f*x])^n*Sqrt[a + a*Sec[e + f*x]],x]
```

output
```
(2*Hypergeometric2F1[1/2, 1 - n, 3/2, 1 - Sec[e + f*x]]*(-Sec[e + f*x])^n*
Sqrt[a*(1 + Sec[e + f*x])]*Tan[(e + f*x)/2])/(f*Sec[e + f*x]^n)
```

3.313.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {3042, 4293, 77, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a\sec(e+fx)+a}(-\sec(e+fx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{a\csc\left(e+fx+\frac{\pi}{2}\right)+a}\left(-\csc\left(e+fx+\frac{\pi}{2}\right)\right)^n dx$$

$$\Big\downarrow \text{4293}$$

$$\frac{a^2\tan(e+fx)\int \frac{(-\sec(e+fx))^{n-1}}{\sqrt{a-a\sec(e+fx)}}d\sec(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{77}$$

$$-\frac{a^2\sin(e+fx)(-\sec(e+fx))^n\sec^{1-n}(e+fx)\int \frac{\sec^{n-1}(e+fx)}{\sqrt{a-a\sec(e+fx)}}d\sec(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{75}$$

$$\frac{2a\sin(e+fx)(-\sec(e+fx))^n\sec^{1-n}(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},1-n,\frac{3}{2},1-\sec(e+fx)\right)}{f\sqrt{a\sec(e+fx)+a}}$$

input
```
Int[(-Sec[e + f*x])^n*Sqrt[a + a*Sec[e + f*x]],x]
```

output
```
(2*a*Hypergeometric2F1[1/2, 1 - n, 3/2, 1 - Sec[e + f*x]]*(-Sec[e + f*x])^
n*Sec[e + f*x]^(1 - n)*Sin[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]])
```

3.313.3.1 Defintions of rubi rules used

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 77
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((-b)*(c/
d))^IntPart[m]*((b*x)^FracPart[m]/((-d)*(x/c))^FracPart[m])   Int[((-d)*(x/
c))^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&
 !IntegerQ[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4293
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]
*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(d*x)^(n - 1)/Sqrt[a - b*x], x], x,
 Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0]
```

3.313.4 Maple [F]

$$\int (-\sec(fx+e))^n \sqrt{a+a\sec(fx+e)}dx$$

input
```
int((-sec(f*x+e))^n*(a+a*sec(f*x+e))^(1/2),x)
```

output
```
int((-sec(f*x+e))^n*(a+a*sec(f*x+e))^(1/2),x)
```

3.313.5 Fricas [F]

$$\int (-\sec(e+fx))^n \sqrt{a+a\sec(e+fx)}\,dx = \int \sqrt{a\sec(fx+e)+a}(-\sec(fx+e))^n\,dx$$

input
```
integrate((-sec(f*x+e))^n*(a+a*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(a*sec(f*x + e) + a)*(-sec(f*x + e))^n, x)
```

3.313.6 Sympy [F]

$$\int (-\sec(e+fx))^n \sqrt{a+a\sec(e+fx)}\,dx = \int (-\sec(e+fx))^n \sqrt{a(\sec(e+fx)+1)}\,dx$$

input
```
integrate((-sec(f*x+e))**n*(a+a*sec(f*x+e))**(1/2),x)
```

output
```
Integral((-sec(e + f*x))**n*sqrt(a*(sec(e + f*x) + 1)), x)
```

3.313.7 Maxima [F]

$$\int (-\sec(e+fx))^n \sqrt{a+a\sec(e+fx)}\,dx = \int \sqrt{a\sec(fx+e)+a}(-\sec(fx+e))^n\,dx$$

input
```
integrate((-sec(f*x+e))^n*(a+a*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(a*sec(f*x + e) + a)*(-sec(f*x + e))^n, x)
```

3.313.8 Giac [F]

$$\int (-\sec(e+fx))^n \sqrt{a+a\sec(e+fx)}\,dx = \int \sqrt{a\sec(fx+e)+a}(-\sec(fx+e))^n\,dx$$

input
```
integrate((-sec(f*x+e))^n*(a+a*sec(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.313.9 Mupad [F(-1)]

Timed out.

$$\int (-\sec(e+fx))^n \sqrt{a+a\sec(e+fx)}\,dx = \int \sqrt{a+\frac{a}{\cos(e+f\,x)}}\left(-\frac{1}{\cos(e+f\,x)}\right)^n dx$$

input
```
int((a + a/cos(e + f*x))^(1/2)*(-1/cos(e + f*x))^n,x)
```

output
```
int((a + a/cos(e + f*x))^(1/2)*(-1/cos(e + f*x))^n, x)
```

3.314 $\int \frac{(-\sec(e+fx))^n}{\sqrt{a+a\sec(e+fx)}}\,dx$

3.314.1 Optimal result 2394
3.314.2 Mathematica [B] (warning: unable to verify) 2394
3.314.3 Rubi [A] (verified) 2395
3.314.4 Maple [F] 2397
3.314.5 Fricas [F] 2397
3.314.6 Sympy [F] 2398
3.314.7 Maxima [F] 2398
3.314.8 Giac [F] 2398
3.314.9 Mupad [F(-1)] 2399

3.314.1 Optimal result

Integrand size = 25, antiderivative size = 75

$$\int \frac{(-\sec(e+fx))^n}{\sqrt{a+a\sec(e+fx)}}\,dx = -\frac{\text{AppellF1}\left(n,\frac{1}{2},1,1+n,\sec(e+fx),-\sec(e+fx)\right)(-\sec(e+fx))^n\tan(e+fx)}{fn\sqrt{1-\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

output
```
-AppellF1(n,1,1/2,1+n,-sec(f*x+e),sec(f*x+e))*(-sec(f*x+e))^n*tan(f*x+e)/f
/n/(1-sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2)
```

3.314.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 2977 vs. 2(75) = 150.

Time = 6.23 (sec) , antiderivative size = 2977, normalized size of antiderivative = 39.69

$$\int \frac{(-\sec(e+fx))^n}{\sqrt{a+a\sec(e+fx)}}\,dx = \text{Result too large to show}$$

input
```
Integrate[(-Sec[e + f*x])^n/Sqrt[a + a*Sec[e + f*x]],x]
```

output
```
(3*Sqrt[2]*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e
 + f*x)/2]^2]*(Sec[(e + f*x)/2]^2)^n*(-Sec[e + f*x])^n*Sec[e + f*x]^(-1/2
- n + (-1 + 2*n)/2)*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^n*Sqrt[1 + Sec[e + f
*x]]*Tan[(e + f*x)/2])/(f*Sqrt[a*(1 + Sec[e + f*x])]*(3*AppellF1[1/2, -1/2
 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)*A
ppellF1[3/2, -1/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2
] + (-1 + 2*n)*AppellF1[3/2, 1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan
[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)*((3*AppellF1[1/2, -1/2 + n, 1 - n, 3
/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[e + f*x]*(Sec[(e + f*x)/2
]^2)^(1 + n)*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^n*Sqrt[1 + Sec[e + f*x]])/(
Sqrt[2]*(3*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e
 + f*x)/2]^2] + (2*(-1 + n)*AppellF1[3/2, -1/2 + n, 2 - n, 5/2, Tan[(e + f
*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-1 + 2*n)*AppellF1[3/2, 1/2 + n, 1 - n,
5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)) - (3*S
qrt[2]*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f
*x)/2]^2]*(Sec[(e + f*x)/2]^2)^n*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^n*Sqrt[
1 + Sec[e + f*x]]*Sin[e + f*x]*Tan[(e + f*x)/2])/(3*AppellF1[1/2, -1/2 + n
, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)*Appel
lF1[3/2, -1/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] +
(-1 + 2*n)*AppellF1[3/2, 1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[...
```

3.314.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 4315, 3042, 4313, 153}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(-\sec(e+fx))^n}{\sqrt{a\sec(e+fx)+a}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(-\csc\left(e+fx+\frac{\pi}{2}\right)\right)^n}{\sqrt{a\csc\left(e+fx+\frac{\pi}{2}\right)+a}}dx$$

$$\Big\downarrow \text{4315}$$

$$\frac{\sqrt{\sec(e+fx)+1}\int \frac{(-\sec(e+fx))^n}{\sqrt{\sec(e+fx)+1}}dx}{\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\sec(e+fx)+1}\int \frac{\left(-\csc\left(e+fx+\frac{\pi}{2}\right)\right)^n}{\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)+1}}dx}{\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{4313}$$

$$\frac{\tan(e+fx)\int \frac{(-\sec(e+fx))^{n-1}}{\sqrt{1-\sec(e+fx)}(\sec(e+fx)+1)}d(\sec(e+fx)+1)}{f\sqrt{1-\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{153}$$

$$-\frac{\tan(e+fx)(-\sec(e+fx))^n\,\text{AppellF1}\left(n,\frac{1}{2},1,n+1,\sec(e+fx),-\sec(e+fx)\right)}{fn\sqrt{1-\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

input
```
Int[(-Sec[e + f*x])^n/Sqrt[a + a*Sec[e + f*x]],x]
```

output
```
-((AppellF1[n, 1/2, 1, 1 + n, Sec[e + f*x], -Sec[e + f*x]]*(-Sec[e + f*x])
^n*Tan[e + f*x])/(f*n*Sqrt[1 - Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]))
```

3.314.3.1 Defintions of rubi rules used

rule 153
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(b*e - a*f)^p*((a + b*x)^(m + 1)/(b^(p + 1)*(m + 1)*Simp
lify[b/(b*c - a*d)]^n))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c
 - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[Simplify[b/(
b*c - a*d)], 0] &&  !(GtQ[Simplify[d/(d*a - c*b)], 0] && SimplerQ[c + d*x,
a + b*x])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4313
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-((-a)*(d/b))^n)*(Cot[e + f*x]/(a^(n - 1)*f*S
qrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[x^(m - 1/2)*
((a - x)^(n - 1)/Sqrt[2*a - x]), x], x, a + b*Csc[e + f*x]], x] /; FreeQ[{a
, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
&&  !IntegerQ[n] && LtQ[a*(d/b), 0]
```

rule 4315
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Simp[a^IntPart[m]*((a + b*Csc[e + f*x])^FracPart[m
]/(1 + (b/a)*Csc[e + f*x])^FracPart[m])   Int[(1 + (b/a)*Csc[e + f*x])^m*(d
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^
2, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

3.314.4 Maple [F]

$$\int \frac{(-\sec(fx+e))^n}{\sqrt{a+a\sec(fx+e)}}dx$$

input
```
int((-sec(f*x+e))^n/(a+a*sec(f*x+e))^(1/2),x)
```

output
```
int((-sec(f*x+e))^n/(a+a*sec(f*x+e))^(1/2),x)
```

3.314.5 Fricas [F]

$$\int \frac{(-\sec(e+fx))^n}{\sqrt{a+a\sec(e+fx)}}\,dx = \int \frac{(-\sec(fx+e))^n}{\sqrt{a\sec(fx+e)+a}}\,dx$$

input
```
integrate((-sec(f*x+e))^n/(a+a*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
integral((-sec(f*x + e))^n/sqrt(a*sec(f*x + e) + a), x)
```

3.314.6 Sympy [F]

$$\int \frac{(-\sec(e+fx))^n}{\sqrt{a+a\sec(e+fx)}}\,dx=\int \frac{(-\sec(e+fx))^n}{\sqrt{a(\sec(e+fx)+1)}}\,dx$$

input
```
integrate((-sec(f*x+e))**n/(a+a*sec(f*x+e))**(1/2),x)
```

output
```
Integral((-sec(e + f*x))**n/sqrt(a*(sec(e + f*x) + 1)), x)
```

3.314.7 Maxima [F]

$$\int \frac{(-\sec(e+fx))^n}{\sqrt{a+a\sec(e+fx)}}\,dx=\int \frac{(-\sec(fx+e))^n}{\sqrt{a\sec(fx+e)+a}}\,dx$$

input
```
integrate((-sec(f*x+e))^n/(a+a*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate((-sec(f*x + e))^n/sqrt(a*sec(f*x + e) + a), x)
```

3.314.8 Giac [F]

$$\int \frac{(-\sec(e+fx))^n}{\sqrt{a+a\sec(e+fx)}}\,dx=\int \frac{(-\sec(fx+e))^n}{\sqrt{a\sec(fx+e)+a}}\,dx$$

input
```
integrate((-sec(f*x+e))^n/(a+a*sec(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate((-sec(f*x + e))^n/sqrt(a*sec(f*x + e) + a), x)
```

3.314.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(-\sec(e+fx))^n}{\sqrt{a+a\sec(e+fx)}}\,dx=\int \frac{\left(-\frac{1}{\cos(e+f\,x)}\right)^n}{\sqrt{a+\frac{a}{\cos(e+f\,x)}}}\,dx$$

input
```
int((-1/cos(e + f*x))^n/(a + a/cos(e + f*x))^(1/2),x)
```

output
```
int((-1/cos(e + f*x))^n/(a + a/cos(e + f*x))^(1/2), x)
```

3.315 $\int \frac{(-\sec(e+fx))^n}{(a+a\sec(e+fx))^{3/2}}\,dx$

3.315.1 Optimal result 2400
3.315.2 Mathematica [B] (warning: unable to verify) 2400
3.315.3 Rubi [A] (verified) 2401
3.315.4 Maple [F] 2403
3.315.5 Fricas [F] 2403
3.315.6 Sympy [F] 2404
3.315.7 Maxima [F] 2404
3.315.8 Giac [F] 2404
3.315.9 Mupad [F(-1)] 2405

3.315.1 Optimal result

Integrand size = 25, antiderivative size = 78

$$\int \frac{(-\sec(e+fx))^n}{(a+a\sec(e+fx))^{3/2}}\,dx = -\frac{\text{AppellF1}\left(n,\frac{1}{2},2,1+n,\sec(e+fx),-\sec(e+fx)\right)(-\sec(e+fx))^n\tan(e+fx)}{afn\sqrt{1-\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

output
```
-AppellF1(n,2,1/2,1+n,-sec(f*x+e),sec(f*x+e))*(-sec(f*x+e))^n*tan(f*x+e)/a
/f/n/(1-sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2)
```

3.315.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 3005 vs. 2(78) = 156.

Time = 6.25 (sec) , antiderivative size = 3005, normalized size of antiderivative = 38.53

$$\int \frac{(-\sec(e+fx))^n}{(a+a\sec(e+fx))^{3/2}}\,dx = \text{Result too large to show}$$

input
```
Integrate[(-Sec[e + f*x])^n/(a + a*Sec[e + f*x])^(3/2),x]
```

output
```
(6*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/
2]^2]*(Sec[(e + f*x)/2]^2)^n*(-Sec[e + f*x])^n*Sec[e + f*x]^(1/2 - n + (-3
 + 2*n)/2)*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(3/2 + n)*Tan[(e + f*x)/2]*(-
1 + Tan[(e + f*x)/2]^2)^2)/(f*(a*(1 + Sec[e + f*x]))^(3/2)*(3*AppellF1[1/2
, -3/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1
+ n)*AppellF1[3/2, -3/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x
)/2]^2] + (-3 + 2*n)*AppellF1[3/2, -1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^
2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)*((12*AppellF1[1/2, -3/2 + n,
1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[e + f*x]*(Sec[(e
+ f*x)/2]^2)^(1 + n)*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(3/2 + n)*Tan[(e +
f*x)/2]^2*(-1 + Tan[(e + f*x)/2]^2))/(3*AppellF1[1/2, -3/2 + n, 1 - n, 3/2
, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)*AppellF1[3/2, -3/
2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-3 + 2*n)*A
ppellF1[3/2, -1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2
])*Tan[(e + f*x)/2]^2) + (3*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e + f
*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[e + f*x]*(Sec[(e + f*x)/2]^2)^(1 + n)*(
Cos[(e + f*x)/2]^2*Sec[e + f*x])^(3/2 + n)*(-1 + Tan[(e + f*x)/2]^2)^2)/(3
*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]
^2] + (2*(-1 + n)*AppellF1[3/2, -3/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2,
-Tan[(e + f*x)/2]^2] + (-3 + 2*n)*AppellF1[3/2, -1/2 + n, 1 - n, 5/2, T...
```

3.315.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 4315, 3042, 4313, 153}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(-\sec(e+fx))^n}{(a\sec(e+fx)+a)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(-\csc\left(e+fx+\frac{\pi}{2}\right)\right)^n}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{3/2}}dx$$

$$\Big\downarrow \text{4315}$$

$$\frac{\sqrt{\sec(e+fx)+1}\int \frac{(-\sec(e+fx))^n}{(\sec(e+fx)+1)^{3/2}}dx}{a\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \; 3042$$

$$\frac{\sqrt{\sec(e+fx)+1}\int \frac{\left(-\csc\left(e+fx+\frac{\pi}{2}\right)\right)^n}{\left(\csc\left(e+fx+\frac{\pi}{2}\right)+1\right)^{3/2}}dx}{a\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \; 4313$$

$$\frac{\tan(e+fx)\int \frac{(-\sec(e+fx))^{n-1}}{\sqrt{1-\sec(e+fx)}(\sec(e+fx)+1)^2}d(\sec(e+fx)+1)}{af\sqrt{1-\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \; 153$$

$$-\frac{\tan(e+fx)(-\sec(e+fx))^n\,\text{AppellF1}\left(n,\frac{1}{2},2,n+1,\sec(e+fx),-\sec(e+fx)\right)}{afn\sqrt{1-\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

input
```
Int[(-Sec[e + f*x])^n/(a + a*Sec[e + f*x])^(3/2),x]
```

output
```
-((AppellF1[n, 1/2, 2, 1 + n, Sec[e + f*x], -Sec[e + f*x]]*(-Sec[e + f*x])
^n*Tan[e + f*x])/(a*f*n*Sqrt[1 - Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]))
```

3.315.3.1 Defintions of rubi rules used

rule 153
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(b*e - a*f)^p*((a + b*x)^(m + 1)/(b^(p + 1)*(m + 1)*Simp
lify[b/(b*c - a*d)]^n))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c
 - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] && !IntegerQ[m] && !IntegerQ[n] && IntegerQ[p] && GtQ[Simplify[b/(
b*c - a*d)], 0] && !(GtQ[Simplify[d/(d*a - c*b)], 0] && SimplerQ[c + d*x,
a + b*x])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4313
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-((-a)*(d/b))^n)*(Cot[e + f*x]/(a^(n - 1)*f*S
qrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[x^(m - 1/2)*
((a - x)^(n - 1)/Sqrt[2*a - x]), x], x, a + b*Csc[e + f*x]], x] /; FreeQ[{a
, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
&&  !IntegerQ[n] && LtQ[a*(d/b), 0]
```

rule 4315
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Simp[a^IntPart[m]*((a + b*Csc[e + f*x])^FracPart[m
]/(1 + (b/a)*Csc[e + f*x])^FracPart[m])   Int[(1 + (b/a)*Csc[e + f*x])^m*(d
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^
2, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

3.315.4 Maple [F]

$$\int \frac{(-\sec(fx+e))^n}{(a+a\sec(fx+e))^{\frac{3}{2}}}dx$$

input
```
int((-sec(f*x+e))^n/(a+a*sec(f*x+e))^(3/2),x)
```

output
```
int((-sec(f*x+e))^n/(a+a*sec(f*x+e))^(3/2),x)
```

3.315.5 Fricas [F]

$$\int \frac{(-\sec(e+fx))^n}{(a+a\sec(e+fx))^{3/2}}\,dx = \int \frac{(-\sec(fx+e))^n}{(a\sec(fx+e)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((-sec(f*x+e))^n/(a+a*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(a*sec(f*x + e) + a)*(-sec(f*x + e))^n/(a^2*sec(f*x + e)^2 +
2*a^2*sec(f*x + e) + a^2), x)
```

3.315.6 Sympy [F]

$$\int \frac{(-\sec(e+fx))^n}{(a+a\sec(e+fx))^{3/2}}\,dx=\int \frac{(-\sec(e+fx))^n}{(a(\sec(e+fx)+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate((-sec(f*x+e))**n/(a+a*sec(f*x+e))**(3/2),x)
```

output
```
Integral((-sec(e + f*x))**n/(a*(sec(e + f*x) + 1))**(3/2), x)
```

3.315.7 Maxima [F]

$$\int \frac{(-\sec(e+fx))^n}{(a+a\sec(e+fx))^{3/2}}\,dx=\int \frac{(-\sec(fx+e))^n}{(a\sec(fx+e)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((-sec(f*x+e))^n/(a+a*sec(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
integrate((-sec(f*x + e))^n/(a*sec(f*x + e) + a)^(3/2), x)
```

3.315.8 Giac [F]

$$\int \frac{(-\sec(e+fx))^n}{(a+a\sec(e+fx))^{3/2}}\,dx=\int \frac{(-\sec(fx+e))^n}{(a\sec(fx+e)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((-sec(f*x+e))^n/(a+a*sec(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
integrate((-sec(f*x + e))^n/(a*sec(f*x + e) + a)^(3/2), x)
```

3.315.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(-\sec(e+fx))^n}{(a+a\sec(e+fx))^{3/2}}\,dx=\int \frac{\left(-\frac{1}{\cos(e+f\,x)}\right)^n}{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{3/2}}\,dx$$

input
```
int((-1/cos(e + f*x))^n/(a + a/cos(e + f*x))^(3/2),x)
```

output
```
int((-1/cos(e + f*x))^n/(a + a/cos(e + f*x))^(3/2), x)
```

3.316 $\int (d\sec(e+fx))^n(a+a\sec(e+fx))^{3/2}\,dx$

3.316.1 Optimal result 2406
3.316.2 Mathematica [A] (verified) 2406
3.316.3 Rubi [A] (verified) 2407
3.316.4 Maple [F] 2409
3.316.5 Fricas [F] 2410
3.316.6 Sympy [F] 2410
3.316.7 Maxima [F] 2410
3.316.8 Giac [F] 2411
3.316.9 Mupad [F(-1)] 2411

3.316.1 Optimal result

Integrand size = 25, antiderivative size = 130

$$\int (d\sec(e+fx))^n(a+a\sec(e+fx))^{3/2}\,dx=\frac{2a^2(1+4n)\,\text{Hypergeometric2F1}\left(\frac{1}{2},1-n,\frac{3}{2},1-\sec(e+fx)\right)\sec^{1-n}(e+fx)(d\sec(e+fx))^{n}}{f(1+2n)\sqrt{a+a\sec(e+fx)}}+\frac{2a^2(d\sec(e+fx))^n\tan(e+fx)}{f(1+2n)\sqrt{a+a\sec(e+fx)}}$$

output
```
2*a^2*(1+4*n)*hypergeom([1/2, 1-n],[3/2],1-sec(f*x+e))*sec(f*x+e)^(1-n)*(d
*sec(f*x+e))^n*sin(f*x+e)/f/(1+2*n)/(a+a*sec(f*x+e))^(1/2)+2*a^2*(d*sec(f*
x+e))^n*tan(f*x+e)/f/(1+2*n)/(a+a*sec(f*x+e))^(1/2)
```

3.316.2 Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.68

$$\int (d\sec(e+fx))^n(a+a\sec(e+fx))^{3/2}\,dx=\frac{a\left(-1+(1+4n)\cos^{\frac{1}{2}+n}(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{2}+n,\frac{3}{2},2\sin^2\left(\frac{1}{2}(e+fx)\right)\right)\right)(d\sec$$

input
```
Integrate[(d*Sec[e + f*x])^n*(a + a*Sec[e + f*x])^(3/2),x]
```

output
```
(a*(-1 + (1 + 4*n)*Cos[e + f*x]^(1/2 + n)*Hypergeometric2F1[1/2, 3/2 + n,
3/2, 2*Sin[(e + f*x)/2]^2])*(d*Sec[e + f*x])^n*Sqrt[a*(1 + Sec[e + f*x])]*
Tan[(e + f*x)/2])/(f*n)
```

3.316.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3042, 4301, 27, 2011, 3042, 4293, 77, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sec(e+fx)+a)^{3/2}(d\sec(e+fx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{3/2}\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^n dx$$

$$\Big\downarrow \text{4301}$$

$$\frac{2a\int \frac{(d\sec(e+fx))^n(a(4n+1)+a\sec(e+fx)(4n+1))}{2\sqrt{\sec(e+fx)a+a}}dx}{2n+1}+\frac{2a^2\tan(e+fx)(d\sec(e+fx))^n}{f(2n+1)\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{27}$$

$$\frac{a\int \frac{(d\sec(e+fx))^n(a(4n+1)+a\sec(e+fx)(4n+1))}{\sqrt{\sec(e+fx)a+a}}dx}{2n+1}+\frac{2a^2\tan(e+fx)(d\sec(e+fx))^n}{f(2n+1)\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{2011}$$

$$\frac{a(4n+1)\int (d\sec(e+fx))^n\sqrt{\sec(e+fx)a+a}dx}{2n+1}+\frac{2a^2\tan(e+fx)(d\sec(e+fx))^n}{f(2n+1)\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a(4n+1)\int \left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^n\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}dx}{2n+1}+\frac{2a^2\tan(e+fx)(d\sec(e+fx))^n}{f(2n+1)\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{4293}$$

$$\frac{2a^2\tan(e+fx)(d\sec(e+fx))^n}{f(2n+1)\sqrt{a\sec(e+fx)+a}}-\frac{a^3d(4n+1)\tan(e+fx)\int \frac{(d\sec(e+fx))^{n-1}}{\sqrt{a-a\sec(e+fx)}}d\sec(e+fx)}{f(2n+1)\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{77}$$

$$\frac{2a^2\tan(e+fx)(d\sec(e+fx))^n}{f(2n+1)\sqrt{a\sec(e+fx)+a}}-\frac{a^3(4n+1)\sin(e+fx)\sec^{1-n}(e+fx)(d\sec(e+fx))^n\int\frac{\sec^{n-1}(e+fx)}{\sqrt{a-a\sec(e+fx)}}d\sec(e+fx)}{f(2n+1)\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{75}$$

$$\frac{2a^2(4n+1)\sin(e+fx)\sec^{1-n}(e+fx)(d\sec(e+fx))^n\,\text{Hypergeometric2F1}\left(\frac{1}{2},1-n,\frac{3}{2},1-\sec(e+fx)\right)}{f(2n+1)\sqrt{a\sec(e+fx)+a}}+\frac{2a^2\tan(e+fx)(d\sec(e+fx))^n}{f(2n+1)\sqrt{a\sec(e+fx)+a}}$$

input
```
Int[(d*Sec[e + f*x])^n*(a + a*Sec[e + f*x])^(3/2),x]
```

output
```
(2*a^2*(1 + 4*n)*Hypergeometric2F1[1/2, 1 - n, 3/2, 1 - Sec[e + f*x]]*Sec[
e + f*x]^(1 - n)*(d*Sec[e + f*x])^n*Sin[e + f*x])/(f*(1 + 2*n)*Sqrt[a + a*
Sec[e + f*x]]) + (2*a^2*(d*Sec[e + f*x])^n*Tan[e + f*x])/(f*(1 + 2*n)*Sqrt
[a + a*Sec[e + f*x]])
```

3.316.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 77
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((-b)*(c/
d))^IntPart[m]*((b*x)^FracPart[m]/((-d)*(x/c))^FracPart[m])   Int[((-d)*(x/
c))^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&
 !IntegerQ[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0]
```

rule 2011
```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x
] && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x
, a + b*x])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4293
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]
*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(d*x)^(n - 1)/Sqrt[a - b*x], x], x,
 Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0]
```

rule 4301
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
2)*((d*Csc[e + f*x])^n/(f*(m + n - 1))), x] + Simp[b/(m + n - 1)   Int[(a +
 b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*(b*(m + 2*n - 1) + a*(3*m + 2*n
 - 4)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^
2, 0] && GtQ[m, 1] && NeQ[m + n - 1, 0] && IntegerQ[2*m]
```

3.316.4 Maple [F]

$$\int (d\sec(fx+e))^n (a+a\sec(fx+e))^{\frac{3}{2}}\,dx$$

input
```
int((d*sec(f*x+e))^n*(a+a*sec(f*x+e))^(3/2),x)
```

output
```
int((d*sec(f*x+e))^n*(a+a*sec(f*x+e))^(3/2),x)
```

3.316.5 Fricas [F]

$$\int (d\sec(e+fx))^n(a+a\sec(e+fx))^{3/2}\,dx=\int (a\sec(fx+e)+a)^{\frac{3}{2}}(d\sec(fx+e))^n\,dx$$

input
```
integrate((d*sec(f*x+e))^n*(a+a*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
integral((a*sec(f*x + e) + a)^(3/2)*(d*sec(f*x + e))^n, x)
```

3.316.6 Sympy [F]

$$\int (d\sec(e+fx))^n(a+a\sec(e+fx))^{3/2}\,dx=\int (a(\sec(e+fx)+1))^{\frac{3}{2}}(d\sec(e+fx))^n\,dx$$

input
```
integrate((d*sec(f*x+e))**n*(a+a*sec(f*x+e))**(3/2),x)
```

output
```
Integral((a*(sec(e + f*x) + 1))**(3/2)*(d*sec(e + f*x))**n, x)
```

3.316.7 Maxima [F]

$$\int (d\sec(e+fx))^n(a+a\sec(e+fx))^{3/2}\,dx=\int (a\sec(fx+e)+a)^{\frac{3}{2}}(d\sec(fx+e))^n\,dx$$

input
```
integrate((d*sec(f*x+e))^n*(a+a*sec(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
integrate((a*sec(f*x + e) + a)^(3/2)*(d*sec(f*x + e))^n, x)
```

3.316.8 Giac [F]

$$\int (d\sec(e+fx))^n(a+a\sec(e+fx))^{3/2}\,dx = \int (a\sec(fx+e)+a)^{\frac{3}{2}}(d\sec(fx+e))^n\,dx$$

input
```
integrate((d*sec(f*x+e))^n*(a+a*sec(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.316.9 Mupad [F(-1)]

Timed out.

$$\int (d\sec(e+fx))^n(a+a\sec(e+fx))^{3/2}\,dx = \int \left(a+\frac{a}{\cos(e+f\,x)}\right)^{3/2}\left(\frac{d}{\cos(e+f\,x)}\right)^n dx$$

input
```
int((a + a/cos(e + f*x))^(3/2)*(d/cos(e + f*x))^n,x)
```

output
```
int((a + a/cos(e + f*x))^(3/2)*(d/cos(e + f*x))^n, x)
```

3.317 $\int (d\sec(e+fx))^n \sqrt{a+a\sec(e+fx)}\,dx$

3.317.1 Optimal result 2412
3.317.2 Mathematica [A] (verified) 2412
3.317.3 Rubi [A] (verified) 2413
3.317.4 Maple [F] 2414
3.317.5 Fricas [F] 2415
3.317.6 Sympy [F] 2415
3.317.7 Maxima [F] 2415
3.317.8 Giac [F] 2416
3.317.9 Mupad [F(-1)] 2416

3.317.1 Optimal result

Integrand size = 25, antiderivative size = 70

$$\int (d\sec(e+fx))^n \sqrt{a+a\sec(e+fx)}\,dx = \frac{2a\,\text{Hypergeometric2F1}\left(\frac{1}{2},1-n,\frac{3}{2},1-\sec(e+fx)\right)\sec^{1-n}(e+fx)(d\sec(e+fx))^n\sin(e+fx)}{f\sqrt{a+a\sec(e+fx)}}$$

output
```
2*a*hypergeom([1/2, 1-n],[3/2],1-sec(f*x+e))*sec(f*x+e)^(1-n)*(d*sec(f*x+e
))^n*sin(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)
```

3.317.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.01

$$\int (d\sec(e+fx))^n \sqrt{a+a\sec(e+fx)}\,dx = \frac{2\,\text{Hypergeometric2F1}\left(\frac{1}{2},1-n,\frac{3}{2},1-\sec(e+fx)\right)\sec^{-n}(e+fx)(d\sec(e+fx))^n\sqrt{a(1+\sec(e+fx))}\,\text{ta}}{f}$$

input
```
Integrate[(d*Sec[e + f*x])^n*Sqrt[a + a*Sec[e + f*x]],x]
```

output
```
(2*Hypergeometric2F1[1/2, 1 - n, 3/2, 1 - Sec[e + f*x]]*(d*Sec[e + f*x])^n
*Sqrt[a*(1 + Sec[e + f*x])]*Tan[(e + f*x)/2])/(f*Sec[e + f*x]^n)
```

3.317.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {3042, 4293, 77, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a\sec(e+fx)+a}(d\sec(e+fx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{a\csc\left(e+fx+\frac{\pi}{2}\right)+a}\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^n dx$$

$$\Big\downarrow \text{4293}$$

$$-\frac{a^2 d\tan(e+fx)\int \frac{(d\sec(e+fx))^{n-1}}{\sqrt{a-a\sec(e+fx)}}d\sec(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{77}$$

$$-\frac{a^2\sin(e+fx)\sec^{1-n}(e+fx)(d\sec(e+fx))^n\int \frac{\sec^{n-1}(e+fx)}{\sqrt{a-a\sec(e+fx)}}d\sec(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{75}$$

$$\frac{2a\sin(e+fx)\sec^{1-n}(e+fx)(d\sec(e+fx))^n\,\text{Hypergeometric2F1}\left(\frac{1}{2},1-n,\frac{3}{2},1-\sec(e+fx)\right)}{f\sqrt{a\sec(e+fx)+a}}$$

input
```
Int[(d*Sec[e + f*x])^n*Sqrt[a + a*Sec[e + f*x]],x]
```

output
```
(2*a*Hypergeometric2F1[1/2, 1 - n, 3/2, 1 - Sec[e + f*x]]*Sec[e + f*x]^(1
- n)*(d*Sec[e + f*x])^n*Sin[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]])
```

3.317.3.1 Defintions of rubi rules used

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 77
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((-b)*(c/
d))^IntPart[m]*((b*x)^FracPart[m]/((-d)*(x/c))^FracPart[m])   Int[((-d)*(x/
c))^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&
 !IntegerQ[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4293
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]
*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(d*x)^(n - 1)/Sqrt[a - b*x], x], x,
 Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0]
```

3.317.4 Maple [F]

$$\int (d\sec(fx+e))^n \sqrt{a+a\sec(fx+e)}dx$$

input
```
int((d*sec(f*x+e))^n*(a+a*sec(f*x+e))^(1/2),x)
```

output
```
int((d*sec(f*x+e))^n*(a+a*sec(f*x+e))^(1/2),x)
```

3.317.5 Fricas [F]

$$\int (d\sec(e+fx))^n\sqrt{a+a\sec(e+fx)}\,dx=\int \sqrt{a\sec(fx+e)+a}(d\sec(fx+e))^n\,dx$$

input
```
integrate((d*sec(f*x+e))^n*(a+a*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(a*sec(f*x + e) + a)*(d*sec(f*x + e))^n, x)
```

3.317.6 Sympy [F]

$$\int (d\sec(e+fx))^n\sqrt{a+a\sec(e+fx)}\,dx=\int \sqrt{a(\sec(e+fx)+1)}(d\sec(e+fx))^n\,dx$$

input
```
integrate((d*sec(f*x+e))**n*(a+a*sec(f*x+e))**(1/2),x)
```

output
```
Integral(sqrt(a*(sec(e + f*x) + 1))*(d*sec(e + f*x))**n, x)
```

3.317.7 Maxima [F]

$$\int (d\sec(e+fx))^n\sqrt{a+a\sec(e+fx)}\,dx=\int \sqrt{a\sec(fx+e)+a}(d\sec(fx+e))^n\,dx$$

input
```
integrate((d*sec(f*x+e))^n*(a+a*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(a*sec(f*x + e) + a)*(d*sec(f*x + e))^n, x)
```

3.317.8 Giac [F]

$$\int (d\sec(e+fx))^n\sqrt{a+a\sec(e+fx)}\,dx=\int \sqrt{a\sec(fx+e)+a}(d\sec(fx+e))^n\,dx$$

input
```
integrate((d*sec(f*x+e))^n*(a+a*sec(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.317.9 Mupad [F(-1)]

Timed out.

$$\int (d\sec(e+fx))^n\sqrt{a+a\sec(e+fx)}\,dx=\int \sqrt{a+\frac{a}{\cos(e+f\,x)}}\,\left(\frac{d}{\cos(e+f\,x)}\right)^n\,dx$$

input
```
int((a + a/cos(e + f*x))^(1/2)*(d/cos(e + f*x))^n,x)
```

output
```
int((a + a/cos(e + f*x))^(1/2)*(d/cos(e + f*x))^n, x)
```

3.318 $\int \frac{(d\sec(e+fx))^n}{\sqrt{a+a\sec(e+fx)}}\,dx$

3.318.1 Optimal result 2417
3.318.2 Mathematica [B] (warning: unable to verify) 2417
3.318.3 Rubi [A] (verified) 2418
3.318.4 Maple [F] 2420
3.318.5 Fricas [F] 2420
3.318.6 Sympy [F] 2420
3.318.7 Maxima [F] 2421
3.318.8 Giac [F] 2421
3.318.9 Mupad [F(-1)] 2421

3.318.1 Optimal result

Integrand size = 25, antiderivative size = 75

$$\int \frac{(d\sec(e+fx))^n}{\sqrt{a+a\sec(e+fx)}}\,dx = -\frac{\text{AppellF1}\left(n,\frac{1}{2},1,1+n,\sec(e+fx),-\sec(e+fx)\right)(d\sec(e+fx))^n\tan(e+fx)}{fn\sqrt{1-\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

output
```
-AppellF1(n,1,1/2,1+n,-sec(f*x+e),sec(f*x+e))*(d*sec(f*x+e))^n*tan(f*x+e)/
f/n/(1-sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2)
```

3.318.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 2977 vs. 2(75) = 150.

Time = 6.23 (sec) , antiderivative size = 2977, normalized size of antiderivative = 39.69

$$\int \frac{(d\sec(e+fx))^n}{\sqrt{a+a\sec(e+fx)}}\,dx = \text{Result too large to show}$$

input
```
Integrate[(d*Sec[e + f*x])^n/Sqrt[a + a*Sec[e + f*x]],x]
```

output
```
(3*Sqrt[2]*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e
 + f*x)/2]^2]*(Sec[(e + f*x)/2]^2)^n*Sec[e + f*x]^(-1/2 - n + (-1 + 2*n)/2
)*(d*Sec[e + f*x])^n*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^n*Sqrt[1 + Sec[e +
f*x]]*Tan[(e + f*x)/2])/(f*Sqrt[a*(1 + Sec[e + f*x])]*(3*AppellF1[1/2, -1/
2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)*
AppellF1[3/2, -1/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^
2] + (-1 + 2*n)*AppellF1[3/2, 1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Ta
n[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)*((3*AppellF1[1/2, -1/2 + n, 1 - n,
3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[e + f*x]*(Sec[(e + f*x)/
2]^2)^(1 + n)*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^n*Sqrt[1 + Sec[e + f*x]])/
(Sqrt[2]*(3*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(
e + f*x)/2]^2] + (2*(-1 + n)*AppellF1[3/2, -1/2 + n, 2 - n, 5/2, Tan[(e +
f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-1 + 2*n)*AppellF1[3/2, 1/2 + n, 1 - n,
 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)) - (3*
Sqrt[2]*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e +
f*x)/2]^2]*(Sec[(e + f*x)/2]^2)^n*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^n*Sqrt
[1 + Sec[e + f*x]]*Sin[e + f*x]*Tan[(e + f*x)/2])/(3*AppellF1[1/2, -1/2 +
n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)*Appe
llF1[3/2, -1/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] +
 (-1 + 2*n)*AppellF1[3/2, 1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan...
```

3.318.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 4315, 3042, 4314, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d\sec(e+fx))^n}{\sqrt{a\sec(e+fx)+a}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^n}{\sqrt{a\csc\left(e+fx+\frac{\pi}{2}\right)+a}}dx$$

$$\Big\downarrow \text{4315}$$

$$\frac{\sqrt{\sec(e+fx)+1}\int \frac{(d\sec(e+fx))^n}{\sqrt{\sec(e+fx)+1}}dx}{\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\sec(e+fx)+1}\int \frac{\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^n}{\sqrt{\csc\left(e+fx+\frac{\pi}{2}\right)+1}}dx}{\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{4314}$$

$$-\frac{d\tan(e+fx)\int \frac{(d\sec(e+fx))^{n-1}}{\sqrt{1-\sec(e+fx)}(\sec(e+fx)+1)}d\sec(e+fx)}{f\sqrt{1-\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{150}$$

$$-\frac{\tan(e+fx)\,\text{AppellF1}\left(n,\frac{1}{2},1,n+1,\sec(e+fx),-\sec(e+fx)\right)(d\sec(e+fx))^n}{fn\sqrt{1-\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

input
```
Int[(d*Sec[e + f*x])^n/Sqrt[a + a*Sec[e + f*x]],x]
```

output
```
-((AppellF1[n, 1/2, 1, 1 + n, Sec[e + f*x], -Sec[e + f*x]]*(d*Sec[e + f*x]
)^n*Tan[e + f*x])/(f*n*Sqrt[1 - Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]))
```

3.318.3.1 Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] && !In
tegerQ[m] && !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4314
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Simp[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x
]]*Sqrt[a - b*Csc[e + f*x]])) Subst[Int[(d*x)^(n - 1)*((a + b*x)^(m - 1/2
)/Sqrt[a - b*x]), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n},
x] && EqQ[a^2 - b^2, 0] && !IntegerQ[m] && GtQ[a, 0]
```

rule 4315
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Simp[a^IntPart[m]*((a + b*Csc[e + f*x])^FracPart[m
]/(1 + (b/a)*Csc[e + f*x])^FracPart[m])   Int[(1 + (b/a)*Csc[e + f*x])^m*(d
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^
2, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

3.318.4 Maple [F]

$$\int \frac{(d\sec(fx+e))^n}{\sqrt{a+a\sec(fx+e)}}dx$$

input
```
int((d*sec(f*x+e))^n/(a+a*sec(f*x+e))^(1/2),x)
```

output
```
int((d*sec(f*x+e))^n/(a+a*sec(f*x+e))^(1/2),x)
```

3.318.5 Fricas [F]

$$\int \frac{(d\sec(e+fx))^n}{\sqrt{a+a\sec(e+fx)}}\,dx = \int \frac{(d\sec(fx+e))^n}{\sqrt{a\sec(fx+e)+a}}\,dx$$

input
```
integrate((d*sec(f*x+e))^n/(a+a*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
integral((d*sec(f*x + e))^n/sqrt(a*sec(f*x + e) + a), x)
```

3.318.6 Sympy [F]

$$\int \frac{(d\sec(e+fx))^n}{\sqrt{a+a\sec(e+fx)}}\,dx = \int \frac{(d\sec(e+fx))^n}{\sqrt{a(\sec(e+fx)+1)}}\,dx$$

input
```
integrate((d*sec(f*x+e))**n/(a+a*sec(f*x+e))**(1/2),x)
```

output
```
Integral((d*sec(e + f*x))**n/sqrt(a*(sec(e + f*x) + 1)), x)
```

3.318.7 Maxima [F]

$$\int \frac{(d\sec(e+fx))^n}{\sqrt{a+a\sec(e+fx)}}\,dx = \int \frac{(d\sec(fx+e))^n}{\sqrt{a\sec(fx+e)+a}}\,dx$$

input
```
integrate((d*sec(f*x+e))^n/(a+a*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate((d*sec(f*x + e))^n/sqrt(a*sec(f*x + e) + a), x)
```

3.318.8 Giac [F]

$$\int \frac{(d\sec(e+fx))^n}{\sqrt{a+a\sec(e+fx)}}\,dx = \int \frac{(d\sec(fx+e))^n}{\sqrt{a\sec(fx+e)+a}}\,dx$$

input
```
integrate((d*sec(f*x+e))^n/(a+a*sec(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate((d*sec(f*x + e))^n/sqrt(a*sec(f*x + e) + a), x)
```

3.318.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d\sec(e+fx))^n}{\sqrt{a+a\sec(e+fx)}}\,dx = \int \frac{\left(\frac{d}{\cos(e+f\,x)}\right)^n}{\sqrt{a+\frac{a}{\cos(e+f\,x)}}}\,dx$$

input
```
int((d/cos(e + f*x))^n/(a + a/cos(e + f*x))^(1/2),x)
```

output
```
int((d/cos(e + f*x))^n/(a + a/cos(e + f*x))^(1/2), x)
```

3.319 $\int \frac{(d\sec(e+fx))^n}{(a+a\sec(e+fx))^{3/2}}\,dx$

3.319.1 Optimal result 2422
3.319.2 Mathematica [B] (warning: unable to verify) 2422
3.319.3 Rubi [A] (verified) 2423
3.319.4 Maple [F] 2425
3.319.5 Fricas [F] 2425
3.319.6 Sympy [F] 2425
3.319.7 Maxima [F] 2426
3.319.8 Giac [F] 2426
3.319.9 Mupad [F(-1)] 2426

3.319.1 Optimal result

Integrand size = 25, antiderivative size = 78

$$\int \frac{(d\sec(e+fx))^n}{(a+a\sec(e+fx))^{3/2}}\,dx = -\frac{\text{AppellF1}\left(n,\frac{1}{2},2,1+n,\sec(e+fx),-\sec(e+fx)\right)(d\sec(e+fx))^n\tan(e+fx)}{afn\sqrt{1-\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

output
```
-AppellF1(n,2,1/2,1+n,-sec(f*x+e),sec(f*x+e))*(d*sec(f*x+e))^n*tan(f*x+e)/
a/f/n/(1-sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2)
```

3.319.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 3005 vs. 2(78) = 156.

Time = 6.25 (sec) , antiderivative size = 3005, normalized size of antiderivative = 38.53

$$\int \frac{(d\sec(e+fx))^n}{(a+a\sec(e+fx))^{3/2}}\,dx = \text{Result too large to show}$$

input
```
Integrate[(d*Sec[e + f*x])^n/(a + a*Sec[e + f*x])^(3/2),x]
```

output
```
(6*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/
2]^2]*(Sec[(e + f*x)/2]^2)^n*Sec[e + f*x]^(1/2 - n + (-3 + 2*n)/2)*(d*Sec[
e + f*x])^n*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(3/2 + n)*Tan[(e + f*x)/2]*(
-1 + Tan[(e + f*x)/2]^2)^2)/(f*(a*(1 + Sec[e + f*x]))^(3/2)*(3*AppellF1[1/
2, -3/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1
 + n)*AppellF1[3/2, -3/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*
x)/2]^2] + (-3 + 2*n)*AppellF1[3/2, -1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]
^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)*((12*AppellF1[1/2, -3/2 + n,
 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[e + f*x]*(Sec[(e
 + f*x)/2]^2)^(1 + n)*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(3/2 + n)*Tan[(e +
 f*x)/2]^2*(-1 + Tan[(e + f*x)/2]^2))/(3*AppellF1[1/2, -3/2 + n, 1 - n, 3/
2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)*AppellF1[3/2, -3
/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-3 + 2*n)*
AppellF1[3/2, -1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^
2])*Tan[(e + f*x)/2]^2) + (3*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e +
f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[e + f*x]*(Sec[(e + f*x)/2]^2)^(1 + n)*
(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(3/2 + n)*(-1 + Tan[(e + f*x)/2]^2)^2)/(
3*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2
]^2] + (2*(-1 + n)*AppellF1[3/2, -3/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2,
 -Tan[(e + f*x)/2]^2] + (-3 + 2*n)*AppellF1[3/2, -1/2 + n, 1 - n, 5/2, ...
```

3.319.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 4315, 3042, 4314, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d\sec(e+fx))^n}{(a\sec(e+fx)+a)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^n}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^{3/2}}dx$$

$$\Big\downarrow \text{4315}$$

$$\frac{\sqrt{\sec(e+fx)+1}\int \frac{(d\sec(e+fx))^n}{(\sec(e+fx)+1)^{3/2}}dx}{a\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \; 3042$$

$$\frac{\sqrt{\sec(e+fx)+1}\int \frac{\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^n}{\left(\csc\left(e+fx+\frac{\pi}{2}\right)+1\right)^{3/2}}dx}{a\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \; 4314$$

$$-\frac{d\tan(e+fx)\int \frac{(d\sec(e+fx))^{n-1}}{\sqrt{1-\sec(e+fx)}(\sec(e+fx)+1)^2}d\sec(e+fx)}{af\sqrt{1-\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \; 150$$

$$-\frac{\tan(e+fx)\,\text{AppellF1}\left(n,\frac{1}{2},2,n+1,\sec(e+fx),-\sec(e+fx)\right)(d\sec(e+fx))^n}{afn\sqrt{1-\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

input
```
Int[(d*Sec[e + f*x])^n/(a + a*Sec[e + f*x])^(3/2),x]
```

output
```
-((AppellF1[n, 1/2, 2, 1 + n, Sec[e + f*x], -Sec[e + f*x]]*(d*Sec[e + f*x]
)^n*Tan[e + f*x])/(a*f*n*Sqrt[1 - Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]))
```

3.319.3.1 Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] && !In
tegerQ[m] && !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4314
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Simp[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x
]]*Sqrt[a - b*Csc[e + f*x]])) Subst[Int[(d*x)^(n - 1)*((a + b*x)^(m - 1/2
)/Sqrt[a - b*x]), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n},
x] && EqQ[a^2 - b^2, 0] && !IntegerQ[m] && GtQ[a, 0]
```

rule 4315
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Simp[a^IntPart[m]*((a + b*Csc[e + f*x])^FracPart[m
]/(1 + (b/a)*Csc[e + f*x])^FracPart[m])   Int[(1 + (b/a)*Csc[e + f*x])^m*(d
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^
2, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

3.319.4 Maple [F]

$$\int \frac{(d\sec(fx+e))^n}{(a+a\sec(fx+e))^{\frac{3}{2}}}dx$$

input
```
int((d*sec(f*x+e))^n/(a+a*sec(f*x+e))^(3/2),x)
```

output
```
int((d*sec(f*x+e))^n/(a+a*sec(f*x+e))^(3/2),x)
```

3.319.5 Fricas [F]

$$\int \frac{(d\sec(e+fx))^n}{(a+a\sec(e+fx))^{3/2}}\,dx=\int \frac{(d\sec(fx+e))^n}{(a\sec(fx+e)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*sec(f*x+e))^n/(a+a*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(a*sec(f*x + e) + a)*(d*sec(f*x + e))^n/(a^2*sec(f*x + e)^2 +
 2*a^2*sec(f*x + e) + a^2), x)
```

3.319.6 Sympy [F]

$$\int \frac{(d\sec(e+fx))^n}{(a+a\sec(e+fx))^{3/2}}\,dx=\int \frac{(d\sec(e+fx))^n}{(a(\sec(e+fx)+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*sec(f*x+e))**n/(a+a*sec(f*x+e))**(3/2),x)
```

output
```
Integral((d*sec(e + f*x))**n/(a*(sec(e + f*x) + 1))**(3/2), x)
```

3.319.7 Maxima [F]

$$\int \frac{(d\sec(e+fx))^n}{(a+a\sec(e+fx))^{3/2}}\,dx = \int \frac{(d\sec(fx+e))^n}{(a\sec(fx+e)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*sec(f*x+e))^n/(a+a*sec(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
integrate((d*sec(f*x + e))^n/(a*sec(f*x + e) + a)^(3/2), x)
```

3.319.8 Giac [F]

$$\int \frac{(d\sec(e+fx))^n}{(a+a\sec(e+fx))^{3/2}}\,dx = \int \frac{(d\sec(fx+e))^n}{(a\sec(fx+e)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*sec(f*x+e))^n/(a+a*sec(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
integrate((d*sec(f*x + e))^n/(a*sec(f*x + e) + a)^(3/2), x)
```

3.319.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d\sec(e+fx))^n}{(a+a\sec(e+fx))^{3/2}}\,dx = \int \frac{\left(\frac{d}{\cos(e+f\,x)}\right)^n}{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{3/2}}\,dx$$

input
```
int((d/cos(e + f*x))^n/(a + a/cos(e + f*x))^(3/2),x)
```

output
```
int((d/cos(e + f*x))^n/(a + a/cos(e + f*x))^(3/2), x)
```

3.320 $\int (-\sec(e+fx))^n (a-a\sec(e+fx))^{5/2}\,dx$

3.320.1 Optimal result . . . 2427
3.320.2 Mathematica [C] (verified) . . . 2428
3.320.3 Rubi [A] (verified) . . . 2428
3.320.4 Maple [F] . . . 2431
3.320.5 Fricas [F] . . . 2431
3.320.6 Sympy [F(-1)] . . . 2432
3.320.7 Maxima [F] . . . 2432
3.320.8 Giac [F] . . . 2432
3.320.9 Mupad [F(-1)] . . . 2433

3.320.1 Optimal result

Integrand size = 26, antiderivative size = 178

$$\int (-\sec(e+fx))^n (a-a\sec(e+fx))^{5/2}\,dx = \frac{2a^3(3+24n+16n^2)\,\text{Hypergeometric2F1}\left(\frac{1}{2},1-n,\frac{3}{2},1+\sec(e+fx)\right)\tan(e+fx)}{f(1+2n)(3+2n)\sqrt{a-a\sec(e+fx)}} + \frac{2a^3(7+4n)(-\sec(e+fx))^n\tan(e+fx)}{f(1+2n)(3+2n)\sqrt{a-a\sec(e+fx)}} + \frac{2a^2(-\sec(e+fx))^n\sqrt{a-a\sec(e+fx)}\tan(e+fx)}{f(3+2n)}$$

output
```
2*a^3*(16*n^2+24*n+3)*hypergeom([1/2, 1-n],[3/2],1+sec(f*x+e))*tan(f*x+e)/
f/(4*n^2+8*n+3)/(a-a*sec(f*x+e))^(1/2)+2*a^3*(7+4*n)*(-sec(f*x+e))^n*tan(f
*x+e)/f/(4*n^2+8*n+3)/(a-a*sec(f*x+e))^(1/2)+2*a^2*(-sec(f*x+e))^n*(a-a*se
c(f*x+e))^(1/2)*tan(f*x+e)/f/(3+2*n)
```

3.320.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 8.96 (sec) , antiderivative size = 419, normalized size of antiderivative = 2.35

$$\int (-\sec(e+fx))^n (a-a\sec(e+fx))^{5/2}\,dx =$$
$$-\frac{2^{-\frac{5}{2}+n}e^{-\frac{1}{2}i(e+fx)}\left(\frac{e^{i(e+fx)}}{1+e^{2i(e+fx)}}\right)^{\frac{1}{2}+n}\left(1+e^{2i(e+fx)}\right)^{\frac{1}{2}+n}\csc^5\left(\frac{1}{2}(e+fx)\right)\left(-\frac{\text{Hypergeometric2F1}\left(\frac{n}{2},\frac{5}{2}+n,\frac{2+n}{2},-e^{2i(e+fx)}\right)}{n}\right.}{}$$

input
```
Integrate[(-Sec[e + f*x])^n*(a - a*Sec[e + f*x])^(5/2),x]
```

output
```
-((2^(-5/2 + n)*(E^(I*(e + f*x))/(1 + E^((2*I)*(e + f*x))))^(1/2 + n)*(1 +
 E^((2*I)*(e + f*x)))^(1/2 + n)*Csc[(e + f*x)/2]^5*(-(Hypergeometric2F1[n/
2, 5/2 + n, (2 + n)/2, -E^((2*I)*(e + f*x))]/n) + (5*E^(I*(e + f*x))*Hyper
geometric2F1[(1 + n)/2, 5/2 + n, (3 + n)/2, -E^((2*I)*(e + f*x))])/(1 + n)
 - (10*E^((2*I)*(e + f*x))*Hypergeometric2F1[(2 + n)/2, 5/2 + n, (4 + n)/2
, -E^((2*I)*(e + f*x))])/(2 + n) + (10*E^((3*I)*(e + f*x))*Hypergeometric2
F1[5/2 + n, (3 + n)/2, (5 + n)/2, -E^((2*I)*(e + f*x))])/(3 + n) - (5*E^((
4*I)*(e + f*x))*Hypergeometric2F1[5/2 + n, (4 + n)/2, (6 + n)/2, -E^((2*I)
*(e + f*x))])/(4 + n) + (E^((5*I)*(e + f*x))*Hypergeometric2F1[5/2 + n, (5
 + n)/2, (7 + n)/2, -E^((2*I)*(e + f*x))])/(5 + n))*(-Sec[e + f*x])^n*Sec[
e + f*x]^(-5/2 - n)*(a - a*Sec[e + f*x])^(5/2))/(E^((I/2)*(e + f*x))*f))
```

3.320.3 Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.98, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {3042, 4301, 27, 3042, 4504, 3042, 4293, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a-a\sec(e+fx))^{5/2}(-\sec(e+fx))^n\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \left(a-a\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{5/2}\left(-\csc\left(e+fx+\frac{\pi}{2}\right)\right)^n dx$$

$$\Big\downarrow \text{4301}$$

$$\frac{2a^2\tan(e+fx)\sqrt{a-a\sec(e+fx)}(-\sec(e+fx))^n}{f(2n+3)}-\frac{2a\int -\frac{1}{2}(-\sec(e+fx))^n\sqrt{a-a\sec(e+fx)}(a(4n+3)-a(4n+7)\sec(e+fx))dx}{2n+3}$$

$$\Big\downarrow \text{27}$$

$$\frac{a\int(-\sec(e+fx))^n\sqrt{a-a\sec(e+fx)}(a(4n+3)-a(4n+7)\sec(e+fx))dx}{2n+3}+\frac{2a^2\tan(e+fx)\sqrt{a-a\sec(e+fx)}(-\sec(e+fx))^n}{f(2n+3)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a\int\left(-\csc\left(e+fx+\frac{\pi}{2}\right)\right)^n\sqrt{a-a\csc\left(e+fx+\frac{\pi}{2}\right)}\left(a(4n+3)-a(4n+7)\csc\left(e+fx+\frac{\pi}{2}\right)\right)dx}{2n+3}+\frac{2a^2\tan(e+fx)\sqrt{a-a\sec(e+fx)}(-\sec(e+fx))^n}{f(2n+3)}$$

$$\Big\downarrow \text{4504}$$

$$\frac{a\left(\frac{a(16n^2+24n+3)\int(-\sec(e+fx))^n\sqrt{a-a\sec(e+fx)}dx}{2n+1}+\frac{2a^2(4n+7)\tan(e+fx)(-\sec(e+fx))^n}{f(2n+1)\sqrt{a-a\sec(e+fx)}}\right)}{2n+3}+\frac{2a^2\tan(e+fx)\sqrt{a-a\sec(e+fx)}(-\sec(e+fx))^n}{f(2n+3)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a\left(\frac{a(16n^2+24n+3)\int\left(-\csc\left(e+fx+\frac{\pi}{2}\right)\right)^n\sqrt{a-a\csc\left(e+fx+\frac{\pi}{2}\right)}dx}{2n+1}+\frac{2a^2(4n+7)\tan(e+fx)(-\sec(e+fx))^n}{f(2n+1)\sqrt{a-a\sec(e+fx)}}\right)}{2n+3}+\frac{2a^2\tan(e+fx)\sqrt{a-a\sec(e+fx)}(-\sec(e+fx))^n}{f(2n+3)}$$

$$\Big\downarrow \text{4293}$$

$$\frac{a\left(\frac{a^3(16n^2+24n+3)\tan(e+fx)\int\frac{(-\sec(e+fx))^{n-1}}{\sqrt{\sec(e+fx)a+a}}d\sec(e+fx)}{f(2n+1)\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}+\frac{2a^2(4n+7)\tan(e+fx)(-\sec(e+fx))^n}{f(2n+1)\sqrt{a-a\sec(e+fx)}}\right)}{2n+3}+\frac{2a^2\tan(e+fx)\sqrt{a-a\sec(e+fx)}(-\sec(e+fx))^n}{f(2n+3)}$$

$$\Big\downarrow \text{75}$$

3.320. $\int(-\sec(e+fx))^n(a-a\sec(e+fx))^{5/2}\,dx$

$$\frac{a\left(\frac{2a^2(16n^2+24n+3)\tan(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},1-n,\frac{3}{2},\sec(e+fx)+1\right)}{f(2n+1)\sqrt{a-a\sec(e+fx)}}+\frac{2a^2(4n+7)\tan(e+fx)(-\sec(e+fx))^n}{f(2n+1)\sqrt{a-a\sec(e+fx)}}\right)}{2n+3}+\frac{2a^2\tan(e+fx)\sqrt{a-a\sec(e+fx)}(-\sec(e+fx))^n}{f(2n+3)}$$

input
```
Int[(-Sec[e + f*x])^n*(a - a*Sec[e + f*x])^(5/2),x]
```

output
```
(2*a^2*(-Sec[e + f*x])^n*Sqrt[a - a*Sec[e + f*x]]*Tan[e + f*x])/(f*(3 + 2*
n)) + (a*((2*a^2*(3 + 24*n + 16*n^2)*Hypergeometric2F1[1/2, 1 - n, 3/2, 1
+ Sec[e + f*x]]*Tan[e + f*x])/(f*(1 + 2*n)*Sqrt[a - a*Sec[e + f*x]]) + (2*
a^2*(7 + 4*n)*(-Sec[e + f*x])^n*Tan[e + f*x])/(f*(1 + 2*n)*Sqrt[a - a*Sec[
e + f*x]])))/(3 + 2*n)
```

3.320.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4293
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]
*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(d*x)^(n - 1)/Sqrt[a - b*x], x], x,
 Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0]
```

rule 4301
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
2)*((d*Csc[e + f*x])^n/(f*(m + n - 1))), x] + Simp[b/(m + n - 1)   Int[(a +
 b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*(b*(m + 2*n - 1) + a*(3*m + 2*n
 - 4)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^
2, 0] && GtQ[m, 1] && NeQ[m + n - 1, 0] && IntegerQ[2*m]
```

rule 4504
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[-2*b*B*C
ot[e + f*x]*((d*Csc[e + f*x])^n/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]])), x]
 + Simp[(A*b*(2*n + 1) + 2*a*B*n)/(b*(2*n + 1))   Int[Sqrt[a + b*Csc[e + f*
x]]*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ
[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] &&
!LtQ[n, 0]
```

3.320.4 Maple [F]

$$\int (-\sec(fx+e))^n (a-a\sec(fx+e))^{\frac{5}{2}}\,dx$$

input
```
int((-sec(f*x+e))^n*(a-a*sec(f*x+e))^(5/2),x)
```

output
```
int((-sec(f*x+e))^n*(a-a*sec(f*x+e))^(5/2),x)
```

3.320.5 Fricas [F]

$$\int (-\sec(e+fx))^n(a-a\sec(e+fx))^{5/2}\,dx=\int (-a\sec(fx+e)+a)^{\frac{5}{2}}(-\sec(fx+e))^n\,dx$$

input
```
integrate((-sec(f*x+e))^n*(a-a*sec(f*x+e))^(5/2),x, algorithm="fricas")
```

output
```
integral((a^2*sec(f*x + e)^2 - 2*a^2*sec(f*x + e) + a^2)*sqrt(-a*sec(f*x +
 e) + a)*(-sec(f*x + e))^n, x)
```

3.320.6 Sympy [F(-1)]

Timed out.

$$\int (-\sec(e+fx))^n(a-a\sec(e+fx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate((-sec(f*x+e))**n*(a-a*sec(f*x+e))**(5/2),x)
```

output
```
Timed out
```

3.320.7 Maxima [F]

$$\int (-\sec(e+fx))^n(a-a\sec(e+fx))^{5/2}\,dx = \int (-a\sec(fx+e)+a)^{\frac{5}{2}}(-\sec(fx+e))^n\,dx$$

input
```
integrate((-sec(f*x+e))^n*(a-a*sec(f*x+e))^(5/2),x, algorithm="maxima")
```

output
```
integrate((-a*sec(f*x + e) + a)^(5/2)*(-sec(f*x + e))^n, x)
```

3.320.8 Giac [F]

$$\int (-\sec(e+fx))^n(a-a\sec(e+fx))^{5/2}\,dx = \int (-a\sec(fx+e)+a)^{\frac{5}{2}}(-\sec(fx+e))^n\,dx$$

input
```
integrate((-sec(f*x+e))^n*(a-a*sec(f*x+e))^(5/2),x, algorithm="giac")
```

output
```
integrate((-a*sec(f*x + e) + a)^(5/2)*(-sec(f*x + e))^n, x)
```

3.320.9 Mupad [F(-1)]

Timed out.

$$\int (-\sec(e+fx))^n(a-a\sec(e+fx))^{5/2}\,dx=\int \left(a-\frac{a}{\cos\left(e+f\,x\right)}\right)^{5/2}\left(-\frac{1}{\cos\left(e+f\,x\right)}\right)^n\,dx$$

input
```
int((a - a/cos(e + f*x))^(5/2)*(-1/cos(e + f*x))^n,x)
```

output
```
int((a - a/cos(e + f*x))^(5/2)*(-1/cos(e + f*x))^n, x)
```

3.321 $\int (-\sec(e+fx))^n (a-a\sec(e+fx))^{3/2}\,dx$

3.321.1 Optimal result . . . 2434
3.321.2 Mathematica [C] (verified) . . . 2434
3.321.3 Rubi [A] (verified) . . . 2435
3.321.4 Maple [F] . . . 2437
3.321.5 Fricas [F] . . . 2437
3.321.6 Sympy [F] . . . 2438
3.321.7 Maxima [F] . . . 2438
3.321.8 Giac [F] . . . 2438
3.321.9 Mupad [F(-1)] . . . 2439

3.321.1 Optimal result

Integrand size = 26, antiderivative size = 108

$$\int (-\sec(e+fx))^n (a-a\sec(e+fx))^{3/2}\,dx = \frac{2a^2(1+4n)\,\text{Hypergeometric2F1}\left(\frac{1}{2},1-n,\frac{3}{2},1+\sec(e+fx)\right)\tan(e+fx)}{f(1+2n)\sqrt{a-a\sec(e+fx)}} + \frac{2a^2(-\sec(e+fx))^n\tan(e+fx)}{f(1+2n)\sqrt{a-a\sec(e+fx)}}$$

output
```
2*a^2*(1+4*n)*hypergeom([1/2, 1-n],[3/2],1+sec(f*x+e))*tan(f*x+e)/f/(1+2*n
)/(a-a*sec(f*x+e))^(1/2)+2*a^2*(-sec(f*x+e))^n*tan(f*x+e)/f/(1+2*n)/(a-a*s
ec(f*x+e))^(1/2)
```

3.321.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.32 (sec) , antiderivative size = 316, normalized size of antiderivative = 2.93

$$\int (-\sec(e+fx))^n (a-a\sec(e+fx))^{3/2}\,dx = \frac{2^{-\frac{3}{2}+n}e^{-\frac{1}{2}i(e+fx)}\left(\frac{e^{i(e+fx)}}{1+e^{2i(e+fx)}}\right)^{\frac{1}{2}+n}\left(1+e^{2i(e+fx)}\right)^{\frac{1}{2}+n}\csc^3\left(\frac{1}{2}(e+fx)\right)\left(-\frac{\text{Hypergeometric2F1}\left(\frac{n}{2},\frac{3}{2}+n,\right.}{n}\right.}{}$$

input
```
Integrate[(-Sec[e + f*x])^n*(a - a*Sec[e + f*x])^(3/2),x]
```

output
```
(2^(-3/2 + n)*(E^(I*(e + f*x))/(1 + E^((2*I)*(e + f*x))))^(1/2 + n)*(1 + E
^((2*I)*(e + f*x)))^(1/2 + n)*Csc[(e + f*x)/2]^3*(-(Hypergeometric2F1[n/2,
 3/2 + n, (2 + n)/2, -E^((2*I)*(e + f*x))]/n) + (3*E^(I*(e + f*x))*Hyperge
ometric2F1[(1 + n)/2, 3/2 + n, (3 + n)/2, -E^((2*I)*(e + f*x))])/(1 + n) -
 (3*E^((2*I)*(e + f*x))*Hypergeometric2F1[3/2 + n, (2 + n)/2, (4 + n)/2, -
E^((2*I)*(e + f*x))])/(2 + n) + (E^((3*I)*(e + f*x))*Hypergeometric2F1[3/2
 + n, (3 + n)/2, (5 + n)/2, -E^((2*I)*(e + f*x))])/(3 + n))*(-Sec[e + f*x]
)^n*Sec[e + f*x]^(-3/2 - n)*(a - a*Sec[e + f*x])^(3/2))/(E^((I/2)*(e + f*x
))*f)
```

3.321.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {3042, 4301, 27, 2011, 3042, 4293, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a-a\sec(e+fx))^{3/2}(-\sec(e+fx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a-a\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}\left(-\csc\left(e+fx+\frac{\pi}{2}\right)\right)^n dx$$

$$\Big\downarrow \text{4301}$$

$$\frac{2a^2\tan(e+fx)(-\sec(e+fx))^n}{f(2n+1)\sqrt{a-a\sec(e+fx)}}-\frac{2a\int -\frac{(-\sec(e+fx))^n(a(4n+1)-a(4n+1)\sec(e+fx))}{2\sqrt{a-a\sec(e+fx)}}dx}{2n+1}$$

$$\Big\downarrow \text{27}$$

$$\frac{a\int \frac{(-\sec(e+fx))^n(a(4n+1)-a(4n+1)\sec(e+fx))}{\sqrt{a-a\sec(e+fx)}}dx}{2n+1}+\frac{2a^2\tan(e+fx)(-\sec(e+fx))^n}{f(2n+1)\sqrt{a-a\sec(e+fx)}}$$

$$\Big\downarrow \text{2011}$$

$$\frac{a(4n+1)\int(-\sec(e+fx))^n\sqrt{a-a\sec(e+fx)}dx}{2n+1}+\frac{2a^2\tan(e+fx)(-\sec(e+fx))^n}{f(2n+1)\sqrt{a-a\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a(4n+1)\int \left(-\csc\left(e+fx+\frac{\pi}{2}\right)\right)^n \sqrt{a-a\csc\left(e+fx+\frac{\pi}{2}\right)}dx}{2n+1}+\frac{2a^2\tan(e+fx)(-\sec(e+fx))^n}{f(2n+1)\sqrt{a-a\sec(e+fx)}}$$

$$\Big\downarrow \; 4293$$

$$\frac{a^3(4n+1)\tan(e+fx)\int \frac{(-\sec(e+fx))^{n-1}}{\sqrt{\sec(e+fx)a+a}}d\sec(e+fx)}{f(2n+1)\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}+\frac{2a^2\tan(e+fx)(-\sec(e+fx))^n}{f(2n+1)\sqrt{a-a\sec(e+fx)}}$$

$$\Big\downarrow \; 75$$

$$\frac{2a^2(4n+1)\tan(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},1-n,\frac{3}{2},\sec(e+fx)+1\right)}{f(2n+1)\sqrt{a-a\sec(e+fx)}}+\frac{2a^2\tan(e+fx)(-\sec(e+fx))^n}{f(2n+1)\sqrt{a-a\sec(e+fx)}}$$

input
```
Int[(-Sec[e + f*x])^n*(a - a*Sec[e + f*x])^(3/2),x]
```

output
```
(2*a^2*(1 + 4*n)*Hypergeometric2F1[1/2, 1 - n, 3/2, 1 + Sec[e + f*x]]*Tan[
e + f*x])/(f*(1 + 2*n)*Sqrt[a - a*Sec[e + f*x]]) + (2*a^2*(-Sec[e + f*x])^
n*Tan[e + f*x])/(f*(1 + 2*n)*Sqrt[a - a*Sec[e + f*x]])
```

3.321.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 2011
```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x
] && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x
, a + b*x])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4293
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]
*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(d*x)^(n - 1)/Sqrt[a - b*x], x], x,
 Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0]
```

rule 4301
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
2)*((d*Csc[e + f*x])^n/(f*(m + n - 1))), x] + Simp[b/(m + n - 1)   Int[(a +
 b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*(b*(m + 2*n - 1) + a*(3*m + 2*n
 - 4)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^
2, 0] && GtQ[m, 1] && NeQ[m + n - 1, 0] && IntegerQ[2*m]
```

3.321.4 Maple [F]

$$\int (-\sec(fx+e))^n (a-a\sec(fx+e))^{\frac{3}{2}}\,dx$$

input
```
int((-sec(f*x+e))^n*(a-a*sec(f*x+e))^(3/2),x)
```

output
```
int((-sec(f*x+e))^n*(a-a*sec(f*x+e))^(3/2),x)
```

3.321.5 Fricas [F]

$$\int (-\sec(e+fx))^n(a-a\sec(e+fx))^{3/2}\,dx=\int (-a\sec(fx+e)+a)^{\frac{3}{2}}(-\sec(fx+e))^n\,dx$$

input
```
integrate((-sec(f*x+e))^n*(a-a*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
integral(-(a*sec(f*x + e) - a)*sqrt(-a*sec(f*x + e) + a)*(-sec(f*x + e))^n
, x)
```

3.321.6 Sympy [F]

$$\int (-\sec(e+fx))^n(a-a\sec(e+fx))^{3/2}\,dx=\int (-\sec(e+fx))^n\left(-a(\sec(e+fx)-1)\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((-sec(f*x+e))**n*(a-a*sec(f*x+e))**(3/2),x)
```

output
```
Integral((-sec(e + f*x))**n*(-a*(sec(e + f*x) - 1))**(3/2), x)
```

3.321.7 Maxima [F]

$$\int (-\sec(e+fx))^n(a-a\sec(e+fx))^{3/2}\,dx=\int (-a\sec(fx+e)+a)^{\frac{3}{2}}(-\sec(fx+e))^n\,dx$$

input
```
integrate((-sec(f*x+e))^n*(a-a*sec(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
integrate((-a*sec(f*x + e) + a)^(3/2)*(-sec(f*x + e))^n, x)
```

3.321.8 Giac [F]

$$\int (-\sec(e+fx))^n(a-a\sec(e+fx))^{3/2}\,dx=\int (-a\sec(fx+e)+a)^{\frac{3}{2}}(-\sec(fx+e))^n\,dx$$

input
```
integrate((-sec(f*x+e))^n*(a-a*sec(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
integrate((-a*sec(f*x + e) + a)^(3/2)*(-sec(f*x + e))^n, x)
```

3.321.9 Mupad [F(-1)]

Timed out.

$$\int (-\sec(e+fx))^n(a-a\sec(e+fx))^{3/2}\,dx=\int \left(a-\frac{a}{\cos\left(e+f\,x\right)}\right)^{3/2}\left(-\frac{1}{\cos\left(e+f\,x\right)}\right)^n\,dx$$

input
```
int((a - a/cos(e + f*x))^(3/2)*(-1/cos(e + f*x))^n,x)
```

output
```
int((a - a/cos(e + f*x))^(3/2)*(-1/cos(e + f*x))^n, x)
```

3.322 $\int (-\sec(e+fx))^n \sqrt{a-a\sec(e+fx)}\,dx$

3.322.1 Optimal result 2440
3.322.2 Mathematica [A] (verified) 2440
3.322.3 Rubi [A] (verified) 2441
3.322.4 Maple [F] 2442
3.322.5 Fricas [F] 2442
3.322.6 Sympy [F] 2442
3.322.7 Maxima [F] 2443
3.322.8 Giac [F] 2443
3.322.9 Mupad [F(-1)] 2443

3.322.1 Optimal result

Integrand size = 26, antiderivative size = 47

$$\int (-\sec(e+fx))^n \sqrt{a-a\sec(e+fx)}\,dx = \frac{2a\,\text{Hypergeometric2F1}\left(\frac{1}{2},1-n,\frac{3}{2},1+\sec(e+fx)\right)\tan(e+fx)}{f\sqrt{a-a\sec(e+fx)}}$$

output
```
2*a*hypergeom([1/2, 1-n],[3/2],1+sec(f*x+e))*tan(f*x+e)/f/(a-a*sec(f*x+e))
^(1/2)
```

3.322.2 Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.06

$$\int (-\sec(e+fx))^n \sqrt{a-a\sec(e+fx)}\,dx = -\frac{2\cot\left(\frac{1}{2}(e+fx)\right)\text{Hypergeometric2F1}\left(\frac{1}{2},1-n,\frac{3}{2},1+\sec(e+fx)\right)\sqrt{a-a\sec(e+fx)}}{f}$$

input
```
Integrate[(-Sec[e + f*x])^n*Sqrt[a - a*Sec[e + f*x]],x]
```

output
```
(-2*Cot[(e + f*x)/2]*Hypergeometric2F1[1/2, 1 - n, 3/2, 1 + Sec[e + f*x]]*
Sqrt[a - a*Sec[e + f*x]])/f
```

3.322.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {3042, 4293, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a-a\sec(e+fx)}(-\sec(e+fx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{a-a\csc\left(e+fx+\frac{\pi}{2}\right)}\left(-\csc\left(e+fx+\frac{\pi}{2}\right)\right)^n dx$$

$$\Big\downarrow \text{4293}$$

$$\frac{a^2\tan(e+fx)\int \frac{(-\sec(e+fx))^{n-1}}{\sqrt{\sec(e+fx)a+a}}d\sec(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{75}$$

$$\frac{2a\tan(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},1-n,\frac{3}{2},\sec(e+fx)+1\right)}{f\sqrt{a-a\sec(e+fx)}}$$

input
```
Int[(-Sec[e + f*x])^n*Sqrt[a - a*Sec[e + f*x]],x]
```

output
```
(2*a*Hypergeometric2F1[1/2, 1 - n, 3/2, 1 + Sec[e + f*x]]*Tan[e + f*x])/(f
*Sqrt[a - a*Sec[e + f*x]])
```

3.322.3.1 Defintions of rubi rules used

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4293
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]
*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(d*x)^(n - 1)/Sqrt[a - b*x], x], x,
 Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0]
```

3.322.4 Maple [F]

$$\int (-\sec(fx+e))^n \sqrt{a-a\sec(fx+e)}dx$$

input
```
int((-sec(f*x+e))^n*(a-a*sec(f*x+e))^(1/2),x)
```

output
```
int((-sec(f*x+e))^n*(a-a*sec(f*x+e))^(1/2),x)
```

3.322.5 Fricas [F]

$$\int (-\sec(e+fx))^n \sqrt{a-a\sec(e+fx)}\,dx = \int \sqrt{-a\sec(fx+e)+a}(-\sec(fx+e))^n\,dx$$

input
```
integrate((-sec(f*x+e))^n*(a-a*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(-a*sec(f*x + e) + a)*(-sec(f*x + e))^n, x)
```

3.322.6 Sympy [F]

$$\int (-\sec(e+fx))^n \sqrt{a-a\sec(e+fx)}\,dx = \int (-\sec(e+fx))^n \sqrt{-a(\sec(e+fx)-1)}\,dx$$

input
```
integrate((-sec(f*x+e))**n*(a-a*sec(f*x+e))**(1/2),x)
```

output
```
Integral((-sec(e + f*x))**n*sqrt(-a*(sec(e + f*x) - 1)), x)
```

3.322.7 Maxima [F]

$$\int (-\sec(e+fx))^n \sqrt{a-a\sec(e+fx)}\,dx = \int \sqrt{-a\sec(fx+e)+a}(-\sec(fx+e))^n\,dx$$

input
```
integrate((-sec(f*x+e))^n*(a-a*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(-a*sec(f*x + e) + a)*(-sec(f*x + e))^n, x)
```

3.322.8 Giac [F]

$$\int (-\sec(e+fx))^n \sqrt{a-a\sec(e+fx)}\,dx = \int \sqrt{-a\sec(fx+e)+a}(-\sec(fx+e))^n\,dx$$

input
```
integrate((-sec(f*x+e))^n*(a-a*sec(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(-a*sec(f*x + e) + a)*(-sec(f*x + e))^n, x)
```

3.322.9 Mupad [F(-1)]

Timed out.

$$\int (-\sec(e+fx))^n \sqrt{a-a\sec(e+fx)}\,dx = \int \sqrt{a-\frac{a}{\cos(e+f\,x)}}\left(-\frac{1}{\cos(e+f\,x)}\right)^n dx$$

input
```
int((a - a/cos(e + f*x))^(1/2)*(-1/cos(e + f*x))^n,x)
```

output
```
int((a - a/cos(e + f*x))^(1/2)*(-1/cos(e + f*x))^n, x)
```

3.323 $\int \frac{(-\sec(e+fx))^n}{\sqrt{a-a\sec(e+fx)}}\,dx$

3.323.1 Optimal result 2444
3.323.2 Mathematica [F] 2444
3.323.3 Rubi [A] (verified) 2445
3.323.4 Maple [F] 2447
3.323.5 Fricas [F] 2447
3.323.6 Sympy [F] 2447
3.323.7 Maxima [F] 2448
3.323.8 Giac [F] 2448
3.323.9 Mupad [F(-1)] 2448

3.323.1 Optimal result

Integrand size = 26, antiderivative size = 58

$$\int \frac{(-\sec(e+fx))^n}{\sqrt{a-a\sec(e+fx)}}\,dx = \frac{\text{AppellF1}\left(\frac{1}{2},1-n,1,\frac{3}{2},1+\sec(e+fx),\frac{1}{2}(1+\sec(e+fx))\right)\tan(e+fx)}{f\sqrt{a-a\sec(e+fx)}}$$

output
```
AppellF1(1/2,1-n,1,3/2,1+sec(f*x+e),1/2+1/2*sec(f*x+e))*tan(f*x+e)/f/(a-a*
sec(f*x+e))^(1/2)
```

3.323.2 Mathematica [F]

$$\int \frac{(-\sec(e+fx))^n}{\sqrt{a-a\sec(e+fx)}}\,dx = \int \frac{(-\sec(e+fx))^n}{\sqrt{a-a\sec(e+fx)}}\,dx$$

input
```
Integrate[(-Sec[e + f*x])^n/Sqrt[a - a*Sec[e + f*x]],x]
```

output
```
Integrate[(-Sec[e + f*x])^n/Sqrt[a - a*Sec[e + f*x]], x]
```

3.323.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {3042, 4315, 3042, 4312, 148, 333}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(-\sec(e+fx))^n}{\sqrt{a-a\sec(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(-\csc\left(e+fx+\frac{\pi}{2}\right)\right)^n}{\sqrt{a-a\csc\left(e+fx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4315}$$

$$\frac{\sqrt{1-\sec(e+fx)}\int \frac{(-\sec(e+fx))^n}{\sqrt{1-\sec(e+fx)}}dx}{\sqrt{a-a\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{1-\sec(e+fx)}\int \frac{\left(-\csc\left(e+fx+\frac{\pi}{2}\right)\right)^n}{\sqrt{1-\csc\left(e+fx+\frac{\pi}{2}\right)}}dx}{\sqrt{a-a\sec(e+fx)}}$$

$$\Big\downarrow \text{4312}$$

$$\frac{\tan(e+fx)\int \frac{(-\sec(e+fx))^{n-1}}{(1-\sec(e+fx))\sqrt{\sec(e+fx)+1}}d(\sec(e+fx)+1)}{f\sqrt{\sec(e+fx)+1}\sqrt{a-a\sec(e+fx)}}$$

$$\Big\downarrow \text{148}$$

$$\frac{2\tan(e+fx)\int \frac{(-\sec(e+fx))^{n-1}}{1-\sec(e+fx)}d\sqrt{\sec(e+fx)+1}}{f\sqrt{\sec(e+fx)+1}\sqrt{a-a\sec(e+fx)}}$$

$$\Big\downarrow \text{333}$$

$$\frac{\tan(e+fx)\,\text{AppellF1}\left(\frac{1}{2},1-n,1,\frac{3}{2},\sec(e+fx)+1,\frac{1}{2}(\sec(e+fx)+1)\right)}{f\sqrt{a-a\sec(e+fx)}}$$

input
```
Int[(-Sec[e + f*x])^n/Sqrt[a - a*Sec[e + f*x]],x]
```

output
```
(AppellF1[1/2, 1 - n, 1, 3/2, 1 + Sec[e + f*x], (1 + Sec[e + f*x])/2]*Tan[
e + f*x])/(f*Sqrt[a - a*Sec[e + f*x]])
```

3.323.3.1 Defintions of rubi rules used

rule 148
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_))^(p_.),
x_] :> With[{k = Denominator[m]}, Simp[k/b   Subst[Int[x^(k*(m + 1) - 1)*(c
 + d*(x^k/b))^n*(e + f*(x^k/b))^p, x], x, (b*x)^(1/k)], x]] /; FreeQ[{b, c,
 d, e, f, n, p}, x] && FractionQ[m] && IntegerQ[p]
```

rule 333
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^p*c^q*x*AppellF1[1/2, -p, -q, 3/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; F
reeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[p] || GtQ[a,
0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4312
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-(a*(d/b))^n)*(Cot[e + f*x]/(a^(n - 2)*f*Sqrt
[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(a - x)^(n - 1)
*((2*a - x)^(m - 1/2)/Sqrt[x]), x], x, a - b*Csc[e + f*x]], x] /; FreeQ[{a,
 b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0] &
&  !IntegerQ[n] && GtQ[a*(d/b), 0]
```

rule 4315
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Simp[a^IntPart[m]*((a + b*Csc[e + f*x])^FracPart[m
]/(1 + (b/a)*Csc[e + f*x])^FracPart[m])   Int[(1 + (b/a)*Csc[e + f*x])^m*(d
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^
2, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

3.323.4 Maple [F]

$$\int \frac{(-\sec(fx+e))^n}{\sqrt{a-a\sec(fx+e)}}dx$$

input
```
int((-sec(f*x+e))^n/(a-a*sec(f*x+e))^(1/2),x)
```

output
```
int((-sec(f*x+e))^n/(a-a*sec(f*x+e))^(1/2),x)
```

3.323.5 Fricas [F]

$$\int \frac{(-\sec(e+fx))^n}{\sqrt{a-a\sec(e+fx)}}\,dx = \int \frac{(-\sec(fx+e))^n}{\sqrt{-a\sec(fx+e)+a}}\,dx$$

input
```
integrate((-sec(f*x+e))^n/(a-a*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(-a*sec(f*x + e) + a)*(-sec(f*x + e))^n/(a*sec(f*x + e) - a)
, x)
```

3.323.6 Sympy [F]

$$\int \frac{(-\sec(e+fx))^n}{\sqrt{a-a\sec(e+fx)}}\,dx = \int \frac{(-\sec(e+fx))^n}{\sqrt{-a(\sec(e+fx)-1)}}\,dx$$

input
```
integrate((-sec(f*x+e))**n/(a-a*sec(f*x+e))**(1/2),x)
```

output
```
Integral((-sec(e + f*x))**n/sqrt(-a*(sec(e + f*x) - 1)), x)
```

3.323.7 Maxima [F]

$$\int \frac{(-\sec(e+fx))^n}{\sqrt{a-a\sec(e+fx)}}\,dx = \int \frac{(-\sec(fx+e))^n}{\sqrt{-a\sec(fx+e)+a}}\,dx$$

input
```
integrate((-sec(f*x+e))^n/(a-a*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate((-sec(f*x + e))^n/sqrt(-a*sec(f*x + e) + a), x)
```

3.323.8 Giac [F]

$$\int \frac{(-\sec(e+fx))^n}{\sqrt{a-a\sec(e+fx)}}\,dx = \int \frac{(-\sec(fx+e))^n}{\sqrt{-a\sec(fx+e)+a}}\,dx$$

input
```
integrate((-sec(f*x+e))^n/(a-a*sec(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate((-sec(f*x + e))^n/sqrt(-a*sec(f*x + e) + a), x)
```

3.323.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(-\sec(e+fx))^n}{\sqrt{a-a\sec(e+fx)}}\,dx = \int \frac{\left(-\frac{1}{\cos(e+f\,x)}\right)^n}{\sqrt{a-\frac{a}{\cos(e+f\,x)}}}\,dx$$

input
```
int((-1/cos(e + f*x))^n/(a - a/cos(e + f*x))^(1/2),x)
```

output
```
int((-1/cos(e + f*x))^n/(a - a/cos(e + f*x))^(1/2), x)
```

3.324 $\int \frac{(-\sec(e+fx))^n}{(a-a\sec(e+fx))^{3/2}}\,dx$

3.324.1 Optimal result 2449
3.324.2 Mathematica [F] 2449
3.324.3 Rubi [A] (verified) 2450
3.324.4 Maple [F] 2452
3.324.5 Fricas [F] 2452
3.324.6 Sympy [F] 2452
3.324.7 Maxima [F] 2453
3.324.8 Giac [F(-2)] 2453
3.324.9 Mupad [F(-1)] 2453

3.324.1 Optimal result

Integrand size = 26, antiderivative size = 64

$$\int \frac{(-\sec(e+fx))^n}{(a-a\sec(e+fx))^{3/2}}\,dx=\frac{\text{AppellF1}\left(\frac{1}{2},1-n,2,\frac{3}{2},1+\sec(e+fx),\frac{1}{2}(1+\sec(e+fx))\right)\tan(e+fx)}{2af\sqrt{a-a\sec(e+fx)}}$$

output
```
1/2*AppellF1(1/2,1-n,2,3/2,1+sec(f*x+e),1/2+1/2*sec(f*x+e))*tan(f*x+e)/a/f
/(a-a*sec(f*x+e))^(1/2)
```

3.324.2 Mathematica [F]

$$\int \frac{(-\sec(e+fx))^n}{(a-a\sec(e+fx))^{3/2}}\,dx=\int \frac{(-\sec(e+fx))^n}{(a-a\sec(e+fx))^{3/2}}\,dx$$

input
```
Integrate[(-Sec[e + f*x])^n/(a - a*Sec[e + f*x])^(3/2),x]
```

output
```
Integrate[(-Sec[e + f*x])^n/(a - a*Sec[e + f*x])^(3/2), x]
```

3.324.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {3042, 4315, 3042, 4312, 148, 333}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(-\sec(e+fx))^n}{(a-a\sec(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(-\csc\left(e+fx+\frac{\pi}{2}\right)\right)^n}{\left(a-a\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{4315}$$

$$\frac{\sqrt{1-\sec(e+fx)}\int \frac{(-\sec(e+fx))^n}{(1-\sec(e+fx))^{3/2}}dx}{a\sqrt{a-a\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{1-\sec(e+fx)}\int \frac{\left(-\csc\left(e+fx+\frac{\pi}{2}\right)\right)^n}{\left(1-\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{a\sqrt{a-a\sec(e+fx)}}$$

$$\Big\downarrow \text{4312}$$

$$\frac{\tan(e+fx)\int \frac{(-\sec(e+fx))^{n-1}}{(1-\sec(e+fx))^2\sqrt{\sec(e+fx)+1}}d(\sec(e+fx)+1)}{af\sqrt{\sec(e+fx)+1}\sqrt{a-a\sec(e+fx)}}$$

$$\Big\downarrow \text{148}$$

$$\frac{2\tan(e+fx)\int \frac{(-\sec(e+fx))^{n-1}}{(1-\sec(e+fx))^2}d\sqrt{\sec(e+fx)+1}}{af\sqrt{\sec(e+fx)+1}\sqrt{a-a\sec(e+fx)}}$$

$$\Big\downarrow \text{333}$$

$$\frac{\tan(e+fx)\,\text{AppellF1}\left(\frac{1}{2},1-n,2,\frac{3}{2},\sec(e+fx)+1,\frac{1}{2}(\sec(e+fx)+1)\right)}{2af\sqrt{a-a\sec(e+fx)}}$$

input
```
Int[(-Sec[e + f*x])^n/(a - a*Sec[e + f*x])^(3/2),x]
```

output
```
(AppellF1[1/2, 1 - n, 2, 3/2, 1 + Sec[e + f*x], (1 + Sec[e + f*x])/2]*Tan[
e + f*x])/(2*a*f*Sqrt[a - a*Sec[e + f*x]])
```

3.324.3.1 Defintions of rubi rules used

rule 148
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_))^(p_.),
x_] :> With[{k = Denominator[m]}, Simp[k/b   Subst[Int[x^(k*(m + 1) - 1)*(c
 + d*(x^k/b))^n*(e + f*(x^k/b))^p, x], x, (b*x)^(1/k)], x]] /; FreeQ[{b, c,
 d, e, f, n, p}, x] && FractionQ[m] && IntegerQ[p]
```

rule 333
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^p*c^q*x*AppellF1[1/2, -p, -q, 3/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; F
reeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[p] || GtQ[a,
0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4312
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-(a*(d/b))^n)*(Cot[e + f*x]/(a^(n - 2)*f*Sqrt
[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(a - x)^(n - 1)
*((2*a - x)^(m - 1/2)/Sqrt[x]), x], x, a - b*Csc[e + f*x]], x] /; FreeQ[{a,
 b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0] &
&  !IntegerQ[n] && GtQ[a*(d/b), 0]
```

rule 4315
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Simp[a^IntPart[m]*((a + b*Csc[e + f*x])^FracPart[m
]/(1 + (b/a)*Csc[e + f*x])^FracPart[m])   Int[(1 + (b/a)*Csc[e + f*x])^m*(d
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^
2, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

3.324.4 Maple [F]

$$\int \frac{(-\sec(fx+e))^n}{(a-a\sec(fx+e))^{\frac{3}{2}}}dx$$

input
```
int((-sec(f*x+e))^n/(a-a*sec(f*x+e))^(3/2),x)
```

output
```
int((-sec(f*x+e))^n/(a-a*sec(f*x+e))^(3/2),x)
```

3.324.5 Fricas [F]

$$\int \frac{(-\sec(e+fx))^n}{(a-a\sec(e+fx))^{3/2}}\,dx = \int \frac{(-\sec(fx+e))^n}{(-a\sec(fx+e)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((-sec(f*x+e))^n/(a-a*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(-a*sec(f*x + e) + a)*(-sec(f*x + e))^n/(a^2*sec(f*x + e)^2 -
 2*a^2*sec(f*x + e) + a^2), x)
```

3.324.6 Sympy [F]

$$\int \frac{(-\sec(e+fx))^n}{(a-a\sec(e+fx))^{3/2}}\,dx = \int \frac{(-\sec(e+fx))^n}{(-a(\sec(e+fx)-1))^{\frac{3}{2}}}\,dx$$

input
```
integrate((-sec(f*x+e))**n/(a-a*sec(f*x+e))**(3/2),x)
```

output
```
Integral((-sec(e + f*x))**n/(-a*(sec(e + f*x) - 1))**(3/2), x)
```

3.324.7 Maxima [F]

$$\int \frac{(-\sec(e+fx))^n}{(a-a\sec(e+fx))^{3/2}}\,dx = \int \frac{(-\sec(fx+e))^n}{(-a\sec(fx+e)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((-sec(f*x+e))^n/(a-a*sec(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
integrate((-sec(f*x + e))^n/(-a*sec(f*x + e) + a)^(3/2), x)
```

3.324.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(-\sec(e+fx))^n}{(a-a\sec(e+fx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((-sec(f*x+e))^n/(a-a*sec(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{1,[0,4,1,0]%%%}+%%%{2,[0,2,1,1]%%%}+%%%{1,[0,0,1,2]%%%} /
%%%{1,[0,
```

3.324.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(-\sec(e+fx))^n}{(a-a\sec(e+fx))^{3/2}}\,dx = \int \frac{\left(-\frac{1}{\cos(e+f\,x)}\right)^n}{\left(a-\frac{a}{\cos(e+f\,x)}\right)^{3/2}}\,dx$$

input
```
int((-1/cos(e + f*x))^n/(a - a/cos(e + f*x))^(3/2),x)
```

output
```
int((-1/cos(e + f*x))^n/(a - a/cos(e + f*x))^(3/2), x)
```

3.325 $\int \sec^n(e+fx)(a-a\sec(e+fx))^{3/2}\,dx$

3.325.1 Optimal result . . . 2454
3.325.2 Mathematica [C] (verified) . . . 2454
3.325.3 Rubi [A] (verified) . . . 2455
3.325.4 Maple [F] . . . 2457
3.325.5 Fricas [F] . . . 2458
3.325.6 Sympy [F] . . . 2458
3.325.7 Maxima [F] . . . 2458
3.325.8 Giac [F] . . . 2459
3.325.9 Mupad [F(-1)] . . . 2459

3.325.1 Optimal result

Integrand size = 24, antiderivative size = 130

$$\int \sec^n(e+fx)(a-a\sec(e+fx))^{3/2}\,dx = \frac{2a^2\sec^{1+n}(e+fx)\sin(e+fx)}{f(1+2n)\sqrt{a-a\sec(e+fx)}} + \frac{2a^2(1+4n)\,\text{Hypergeometric2F1}\left(\frac{1}{2},1-n,\frac{3}{2},1+\sec(e+fx)\right)(-\sec(e+fx))^{-n}\sec^{1+n}(e+fx)\sin(e+f}{f(1+2n)\sqrt{a-a\sec(e+fx)}}$$

output
```
2*a^2*sec(f*x+e)^(1+n)*sin(f*x+e)/f/(1+2*n)/(a-a*sec(f*x+e))^(1/2)+2*a^2*(
1+4*n)*hypergeom([1/2, 1-n],[3/2],1+sec(f*x+e))*sec(f*x+e)^(1+n)*sin(f*x+e
)/f/(1+2*n)/((-sec(f*x+e))^n)/(a-a*sec(f*x+e))^(1/2)
```

3.325.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.73 (sec) , antiderivative size = 302, normalized size of antiderivative = 2.32

$$\int \sec^n(e+fx)(a-a\sec(e+fx))^{3/2}\,dx = \frac{2^{-\frac{3}{2}+n}e^{-\frac{1}{2}i(e+fx)}\left(\frac{e^{i(e+fx)}}{1+e^{2i(e+fx)}}\right)^{\frac{1}{2}+n}\left(1+e^{2i(e+fx)}\right)^{\frac{1}{2}+n}\csc^3\left(\frac{1}{2}(e+fx)\right)\left(-\frac{\text{Hypergeometric2F1}\left(\frac{n}{2},\frac{3}{2}+n,\right.}{n}\right.}{}$$

input
```
Integrate[Sec[e + f*x]^n*(a - a*Sec[e + f*x])^(3/2),x]
```

output
```
(2^(-3/2 + n)*(E^(I*(e + f*x))/(1 + E^((2*I)*(e + f*x))))^(1/2 + n)*(1 + E
^((2*I)*(e + f*x)))^(1/2 + n)*Csc[(e + f*x)/2]^3*(-(Hypergeometric2F1[n/2,
 3/2 + n, (2 + n)/2, -E^((2*I)*(e + f*x))]/n) + (3*E^(I*(e + f*x))*Hyperge
ometric2F1[(1 + n)/2, 3/2 + n, (3 + n)/2, -E^((2*I)*(e + f*x))])/(1 + n) -
 (3*E^((2*I)*(e + f*x))*Hypergeometric2F1[3/2 + n, (2 + n)/2, (4 + n)/2, -
E^((2*I)*(e + f*x))])/(2 + n) + (E^((3*I)*(e + f*x))*Hypergeometric2F1[3/2
 + n, (3 + n)/2, (5 + n)/2, -E^((2*I)*(e + f*x))])/(3 + n))*(a - a*Sec[e +
 f*x])^(3/2))/(E^((I/2)*(e + f*x))*f*Sec[e + f*x]^(3/2))
```

3.325.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 4301, 27, 2011, 3042, 4293, 77, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a-a\sec(e+fx))^{3/2}\sec^n(e+fx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a-a\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}\csc\left(e+fx+\frac{\pi}{2}\right)^n dx$$

$$\Big\downarrow \text{4301}$$

$$\frac{2a^2\sin(e+fx)\sec^{n+1}(e+fx)}{f(2n+1)\sqrt{a-a\sec(e+fx)}}-\frac{2a\int -\frac{\sec^n(e+fx)(a(4n+1)-a(4n+1)\sec(e+fx))}{2\sqrt{a-a\sec(e+fx)}}dx}{2n+1}$$

$$\Big\downarrow \text{27}$$

$$\frac{a\int \frac{\sec^n(e+fx)(a(4n+1)-a(4n+1)\sec(e+fx))}{\sqrt{a-a\sec(e+fx)}}dx}{2n+1}+\frac{2a^2\sin(e+fx)\sec^{n+1}(e+fx)}{f(2n+1)\sqrt{a-a\sec(e+fx)}}$$

$$\Big\downarrow \text{2011}$$

$$\frac{a(4n+1)\int \sec^n(e+fx)\sqrt{a-a\sec(e+fx)}dx}{2n+1}+\frac{2a^2\sin(e+fx)\sec^{n+1}(e+fx)}{f(2n+1)\sqrt{a-a\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a(4n+1)\int \csc\left(e+fx+\frac{\pi}{2}\right)^n\sqrt{a-a\csc\left(e+fx+\frac{\pi}{2}\right)}dx}{2n+1}+\frac{2a^2\sin(e+fx)\sec^{n+1}(e+fx)}{f(2n+1)\sqrt{a-a\sec(e+fx)}}$$

$$\Big\downarrow \text{4293}$$

$$\frac{2a^2\sin(e+fx)\sec^{n+1}(e+fx)}{f(2n+1)\sqrt{a-a\sec(e+fx)}}-\frac{a^3(4n+1)\tan(e+fx)\int\frac{\sec^{n-1}(e+fx)}{\sqrt{\sec(e+fx)a+a}}d\sec(e+fx)}{f(2n+1)\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{77}$$

$$\frac{a^3(4n+1)\sin(e+fx)\sec^{n+1}(e+fx)(-\sec(e+fx))^{-n}\int\frac{(-\sec(e+fx))^{n-1}}{\sqrt{\sec(e+fx)a+a}}d\sec(e+fx)}{f(2n+1)\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}+\frac{2a^2\sin(e+fx)\sec^{n+1}(e+fx)}{f(2n+1)\sqrt{a-a\sec(e+fx)}}$$

$$\Big\downarrow \text{75}$$

$$\frac{2a^2(4n+1)\sin(e+fx)\sec^{n+1}(e+fx)(-\sec(e+fx))^{-n}\text{Hypergeometric2F1}\left(\frac{1}{2},1-n,\frac{3}{2},\sec(e+fx)+1\right)}{f(2n+1)\sqrt{a-a\sec(e+fx)}}+\frac{2a^2\sin(e+fx)\sec^{n+1}(e+fx)}{f(2n+1)\sqrt{a-a\sec(e+fx)}}$$

input
```
Int[Sec[e + f*x]^n*(a - a*Sec[e + f*x])^(3/2),x]
```

output
```
(2*a^2*Sec[e + f*x]^(1 + n)*Sin[e + f*x])/(f*(1 + 2*n)*Sqrt[a - a*Sec[e +
f*x]]) + (2*a^2*(1 + 4*n)*Hypergeometric2F1[1/2, 1 - n, 3/2, 1 + Sec[e + f
*x]]*Sec[e + f*x]^(1 + n)*Sin[e + f*x])/(f*(1 + 2*n)*(-Sec[e + f*x])^n*Sqr
t[a - a*Sec[e + f*x]])
```

3.325.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 77
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((-b)*(c/
d))^IntPart[m]*((b*x)^FracPart[m]/((-d)*(x/c))^FracPart[m])   Int[((-d)*(x/
c))^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&
 !IntegerQ[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0]
```

rule 2011
```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x
] && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x
, a + b*x])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4293
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]
*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(d*x)^(n - 1)/Sqrt[a - b*x], x], x,
 Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0]
```

rule 4301
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
2)*((d*Csc[e + f*x])^n/(f*(m + n - 1))), x] + Simp[b/(m + n - 1)   Int[(a +
 b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*(b*(m + 2*n - 1) + a*(3*m + 2*n
 - 4)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^
2, 0] && GtQ[m, 1] && NeQ[m + n - 1, 0] && IntegerQ[2*m]
```

3.325.4 Maple [F]

$$\int \sec(fx+e)^n (a - a\sec(fx+e))^{\frac{3}{2}}\,dx$$

input
```
int(sec(f*x+e)^n*(a-a*sec(f*x+e))^(3/2),x)
```

output
```
int(sec(f*x+e)^n*(a-a*sec(f*x+e))^(3/2),x)
```

3.325.5 Fricas [F]

$$\int \sec^n(e+fx)(a-a\sec(e+fx))^{3/2}\,dx=\int (-a\sec(fx+e)+a)^{\frac{3}{2}}\sec(fx+e)^n\,dx$$

input
```
integrate(sec(f*x+e)^n*(a-a*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
integral(-(a*sec(f*x + e) - a)*sqrt(-a*sec(f*x + e) + a)*sec(f*x + e)^n, x
)
```

3.325.6 Sympy [F]

$$\int \sec^n(e+fx)(a-a\sec(e+fx))^{3/2}\,dx=\int (-a(\sec(e+fx)-1))^{\frac{3}{2}}\sec^n(e+fx)\,dx$$

input
```
integrate(sec(f*x+e)**n*(a-a*sec(f*x+e))**(3/2),x)
```

output
```
Integral((-a*(sec(e + f*x) - 1))**(3/2)*sec(e + f*x)**n, x)
```

3.325.7 Maxima [F]

$$\int \sec^n(e+fx)(a-a\sec(e+fx))^{3/2}\,dx=\int (-a\sec(fx+e)+a)^{\frac{3}{2}}\sec(fx+e)^n\,dx$$

input
```
integrate(sec(f*x+e)^n*(a-a*sec(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
integrate((-a*sec(f*x + e) + a)^(3/2)*sec(f*x + e)^n, x)
```

3.325.8 Giac [F]

$$\int \sec^n(e+fx)(a-a\sec(e+fx))^{3/2}\,dx = \int (-a\sec(fx+e)+a)^{\frac{3}{2}}\sec(fx+e)^n\,dx$$

input
```
integrate(sec(f*x+e)^n*(a-a*sec(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
integrate((-a*sec(f*x + e) + a)^(3/2)*sec(f*x + e)^n, x)
```

3.325.9 Mupad [F(-1)]

Timed out.

$$\int \sec^n(e+fx)(a-a\sec(e+fx))^{3/2}\,dx = \int \left(a-\frac{a}{\cos(e+f\,x)}\right)^{3/2}\left(\frac{1}{\cos(e+f\,x)}\right)^n dx$$

input
```
int((a - a/cos(e + f*x))^(3/2)*(1/cos(e + f*x))^n,x)
```

output
```
int((a - a/cos(e + f*x))^(3/2)*(1/cos(e + f*x))^n, x)
```

3.326 $\int \sec^n(e+fx)\sqrt{a-a\sec(e+fx)}\,dx$

3.326.1 Optimal result 2460
3.326.2 Mathematica [A] (verified) 2460
3.326.3 Rubi [A] (verified) 2461
3.326.4 Maple [F] 2462
3.326.5 Fricas [F] 2463
3.326.6 Sympy [F] 2463
3.326.7 Maxima [F] 2463
3.326.8 Giac [F] 2464
3.326.9 Mupad [F(-1)] 2464

3.326.1 Optimal result

Integrand size = 24, antiderivative size = 69

$$\int \sec^n(e+fx)\sqrt{a-a\sec(e+fx)}\,dx = \frac{2a\,\text{Hypergeometric2F1}\left(\frac{1}{2},1-n,\frac{3}{2},1+\sec(e+fx)\right)(-\sec(e+fx))^{-n}\sec^{1+n}(e+fx)\sin(e+fx)}{f\sqrt{a-a\sec(e+fx)}}$$

output
```
2*a*hypergeom([1/2, 1-n],[3/2],1+sec(f*x+e))*sec(f*x+e)^(1+n)*sin(f*x+e)/f
/((-sec(f*x+e))^n)/(a-a*sec(f*x+e))^(1/2)
```

3.326.2 Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.99

$$\int \sec^n(e+fx)\sqrt{a-a\sec(e+fx)}\,dx = \frac{\cot\left(\frac{1}{2}(e+fx)\right)\text{Hypergeometric2F1}\left(\frac{1}{2},n,1+n,-\sec(e+fx)\right)\sec^n(e+fx)\sqrt{a-a\sec(e+fx)}}{fn\sqrt{1+\sec(e+fx)}}$$

input
```
Integrate[Sec[e + f*x]^n*Sqrt[a - a*Sec[e + f*x]],x]
```

output
```
(Cot[(e + f*x)/2]*Hypergeometric2F1[1/2, n, 1 + n, -Sec[e + f*x]]*Sec[e +
f*x]^n*Sqrt[a - a*Sec[e + f*x]])/(f*n*Sqrt[1 + Sec[e + f*x]])
```

3.326.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 4293, 77, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a-a\sec(e+fx)}\sec^n(e+fx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{a-a\csc\left(e+fx+\frac{\pi}{2}\right)}\csc\left(e+fx+\frac{\pi}{2}\right)^n dx$$

$$\Big\downarrow \text{4293}$$

$$-\frac{a^2\tan(e+fx)\int \frac{\sec^{n-1}(e+fx)}{\sqrt{\sec(e+fx)a+a}}d\sec(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{77}$$

$$\frac{a^2\sin(e+fx)(-\sec(e+fx))^{-n}\sec^{n+1}(e+fx)\int \frac{(-\sec(e+fx))^{n-1}}{\sqrt{\sec(e+fx)a+a}}d\sec(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{75}$$

$$\frac{2a\sin(e+fx)(-\sec(e+fx))^{-n}\sec^{n+1}(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},1-n,\frac{3}{2},\sec(e+fx)+1\right)}{f\sqrt{a-a\sec(e+fx)}}$$

input
```
Int[Sec[e + f*x]^n*Sqrt[a - a*Sec[e + f*x]],x]
```

output
```
(2*a*Hypergeometric2F1[1/2, 1 - n, 3/2, 1 + Sec[e + f*x]]*Sec[e + f*x]^(1
+ n)*Sin[e + f*x])/(f*(-Sec[e + f*x])^n*Sqrt[a - a*Sec[e + f*x]])
```

3.326.3.1 Defintions of rubi rules used

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 77
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((-b)*(c/
d))^IntPart[m]*((b*x)^FracPart[m]/((-d)*(x/c))^FracPart[m])   Int[((-d)*(x/
c))^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&
 !IntegerQ[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4293
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]
*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(d*x)^(n - 1)/Sqrt[a - b*x], x], x,
 Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0]
```

3.326.4 Maple [F]

$$\int \sec (fx+e)^n \sqrt{a-a\sec (fx+e)}dx$$

input
```
int(sec(f*x+e)^n*(a-a*sec(f*x+e))^(1/2),x)
```

output
```
int(sec(f*x+e)^n*(a-a*sec(f*x+e))^(1/2),x)
```

3.326.5 Fricas [F]

$$\int \sec^n(e+fx)\sqrt{a-a\sec(e+fx)}\,dx = \int \sqrt{-a\sec(fx+e)+a}\sec(fx+e)^n\,dx$$

input
```
integrate(sec(f*x+e)^n*(a-a*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(-a*sec(f*x + e) + a)*sec(f*x + e)^n, x)
```

3.326.6 Sympy [F]

$$\int \sec^n(e+fx)\sqrt{a-a\sec(e+fx)}\,dx = \int \sqrt{-a(\sec(e+fx)-1)}\sec^n(e+fx)\,dx$$

input
```
integrate(sec(f*x+e)**n*(a-a*sec(f*x+e))**(1/2),x)
```

output
```
Integral(sqrt(-a*(sec(e + f*x) - 1))*sec(e + f*x)**n, x)
```

3.326.7 Maxima [F]

$$\int \sec^n(e+fx)\sqrt{a-a\sec(e+fx)}\,dx = \int \sqrt{-a\sec(fx+e)+a}\sec(fx+e)^n\,dx$$

input
```
integrate(sec(f*x+e)^n*(a-a*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(-a*sec(f*x + e) + a)*sec(f*x + e)^n, x)
```

3.326.8 Giac [F]

$$\int \sec^n(e+fx)\sqrt{a-a\sec(e+fx)}\,dx = \int \sqrt{-a\sec(fx+e)+a}\sec(fx+e)^n\,dx$$

input
```
integrate(sec(f*x+e)^n*(a-a*sec(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(-a*sec(f*x + e) + a)*sec(f*x + e)^n, x)
```

3.326.9 Mupad [F(-1)]

Timed out.

$$\int \sec^n(e+fx)\sqrt{a-a\sec(e+fx)}\,dx = \int \sqrt{a-\frac{a}{\cos(e+f\,x)}}\left(\frac{1}{\cos(e+f\,x)}\right)^n dx$$

input
```
int((a - a/cos(e + f*x))^(1/2)*(1/cos(e + f*x))^n,x)
```

output
```
int((a - a/cos(e + f*x))^(1/2)*(1/cos(e + f*x))^n, x)
```

3.327 $\int (d\sec(e+fx))^n(a-a\sec(e+fx))^{3/2}\,dx$

3.327.1 Optimal result 2465
3.327.2 Mathematica [C] (verified) 2465
3.327.3 Rubi [A] (verified) 2466
3.327.4 Maple [F] 2468
3.327.5 Fricas [F] 2469
3.327.6 Sympy [F] 2469
3.327.7 Maxima [F] 2469
3.327.8 Giac [F] 2470
3.327.9 Mupad [F(-1)] 2470

3.327.1 Optimal result

Integrand size = 26, antiderivative size = 130

$$\int (d\sec(e+fx))^n(a-a\sec(e+fx))^{3/2}\,dx=\frac{2a^2(d\sec(e+fx))^n\tan(e+fx)}{f(1+2n)\sqrt{a-a\sec(e+fx)}}+\frac{2a^2(1+4n)\,\text{Hypergeometric2F1}\left(\frac{1}{2},1-n,\frac{3}{2},1+\sec(e+fx)\right)(-\sec(e+fx))^{-n}(d\sec(e+fx))^n\tan(e+}{f(1+2n)\sqrt{a-a\sec(e+fx)}}$$

output
```
2*a^2*(d*sec(f*x+e))^n*tan(f*x+e)/f/(1+2*n)/(a-a*sec(f*x+e))^(1/2)+2*a^2*(
1+4*n)*hypergeom([1/2, 1-n],[3/2],1+sec(f*x+e))*(d*sec(f*x+e))^n*tan(f*x+e
)/f/(1+2*n)/((-sec(f*x+e))^n)/(a-a*sec(f*x+e))^(1/2)
```

3.327.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.46 (sec) , antiderivative size = 316, normalized size of antiderivative = 2.43

$$\int (d\sec(e+fx))^n(a-a\sec(e+fx))^{3/2}\,dx=\frac{2^{-\frac{3}{2}+n}e^{-\frac{1}{2}i(e+fx)}\left(\frac{e^{i(e+fx)}}{1+e^{2i(e+fx)}}\right)^{\frac{1}{2}+n}\left(1+e^{2i(e+fx)}\right)^{\frac{1}{2}+n}\csc^3\left(\frac{1}{2}(e+fx)\right)\left(-\frac{\text{Hypergeometric2F1}\left(\frac{n}{2},\frac{3}{2}+n,\right.}{n}\right.}{}$$

input
```
Integrate[(d*Sec[e + f*x])^n*(a - a*Sec[e + f*x])^(3/2),x]
```

output
```
(2^(-3/2 + n)*(E^(I*(e + f*x))/(1 + E^((2*I)*(e + f*x))))^(1/2 + n)*(1 + E
^((2*I)*(e + f*x)))^(1/2 + n)*Csc[(e + f*x)/2]^3*(-(Hypergeometric2F1[n/2,
 3/2 + n, (2 + n)/2, -E^((2*I)*(e + f*x))]/n) + (3*E^(I*(e + f*x))*Hyperge
ometric2F1[(1 + n)/2, 3/2 + n, (3 + n)/2, -E^((2*I)*(e + f*x))])/(1 + n) -
 (3*E^((2*I)*(e + f*x))*Hypergeometric2F1[3/2 + n, (2 + n)/2, (4 + n)/2, -
E^((2*I)*(e + f*x))])/(2 + n) + (E^((3*I)*(e + f*x))*Hypergeometric2F1[3/2
 + n, (3 + n)/2, (5 + n)/2, -E^((2*I)*(e + f*x))])/(3 + n))*Sec[e + f*x]^(
-3/2 - n)*(d*Sec[e + f*x])^n*(a - a*Sec[e + f*x])^(3/2))/(E^((I/2)*(e + f*
x))*f)
```

3.327.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {3042, 4301, 27, 2011, 3042, 4293, 77, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a-a\sec(e+fx))^{3/2}(d\sec(e+fx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a-a\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^n dx$$

$$\Big\downarrow \text{4301}$$

$$\frac{2a^2\tan(e+fx)(d\sec(e+fx))^n}{f(2n+1)\sqrt{a-a\sec(e+fx)}}-\frac{2a\int -\frac{(d\sec(e+fx))^n(a(4n+1)-a(4n+1)\sec(e+fx))}{2\sqrt{a-a\sec(e+fx)}}dx}{2n+1}$$

$$\Big\downarrow \text{27}$$

$$\frac{a\int \frac{(d\sec(e+fx))^n(a(4n+1)-a(4n+1)\sec(e+fx))}{\sqrt{a-a\sec(e+fx)}}dx}{2n+1}+\frac{2a^2\tan(e+fx)(d\sec(e+fx))^n}{f(2n+1)\sqrt{a-a\sec(e+fx)}}$$

$$\Big\downarrow \text{2011}$$

$$\frac{a(4n+1)\int (d\sec(e+fx))^n\sqrt{a-a\sec(e+fx)}dx}{2n+1}+\frac{2a^2\tan(e+fx)(d\sec(e+fx))^n}{f(2n+1)\sqrt{a-a\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a(4n+1)\int \left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^n\sqrt{a-a\csc\left(e+fx+\frac{\pi}{2}\right)}dx}{2n+1}+\frac{2a^2\tan(e+fx)(d\sec(e+fx))^n}{f(2n+1)\sqrt{a-a\sec(e+fx)}}$$

$$\Big\downarrow \text{4293}$$

$$\frac{2a^2\tan(e+fx)(d\sec(e+fx))^n}{f(2n+1)\sqrt{a-a\sec(e+fx)}}-\frac{a^3d(4n+1)\tan(e+fx)\int\frac{(d\sec(e+fx))^{n-1}}{\sqrt{\sec(e+fx)a+a}}d\sec(e+fx)}{f(2n+1)\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{77}$$

$$\frac{a^3(4n+1)\tan(e+fx)(-\sec(e+fx))^{-n}(d\sec(e+fx))^n\int\frac{(-\sec(e+fx))^{n-1}}{\sqrt{\sec(e+fx)a+a}}d\sec(e+fx)}{f(2n+1)\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}+\frac{2a^2\tan(e+fx)(d\sec(e+fx))^n}{f(2n+1)\sqrt{a-a\sec(e+fx)}}$$

$$\Big\downarrow \text{75}$$

$$\frac{2a^2(4n+1)\tan(e+fx)(-\sec(e+fx))^{-n}(d\sec(e+fx))^n\text{Hypergeometric2F1}\left(\frac{1}{2},1-n,\frac{3}{2},\sec(e+fx)+1\right)}{f(2n+1)\sqrt{a-a\sec(e+fx)}}+\frac{2a^2\tan(e+fx)(d\sec(e+fx))^n}{f(2n+1)\sqrt{a-a\sec(e+fx)}}$$

input
```
Int[(d*Sec[e + f*x])^n*(a - a*Sec[e + f*x])^(3/2),x]
```

output
```
(2*a^2*(d*Sec[e + f*x])^n*Tan[e + f*x])/(f*(1 + 2*n)*Sqrt[a - a*Sec[e + f*
x]]) + (2*a^2*(1 + 4*n)*Hypergeometric2F1[1/2, 1 - n, 3/2, 1 + Sec[e + f*x
]]*(d*Sec[e + f*x])^n*Tan[e + f*x])/(f*(1 + 2*n)*(-Sec[e + f*x])^n*Sqrt[a
- a*Sec[e + f*x]])
```

3.327.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 77
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((-b)*(c/
d))^IntPart[m]*((b*x)^FracPart[m]/((-d)*(x/c))^FracPart[m])   Int[((-d)*(x/
c))^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&
 !IntegerQ[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0]
```

rule 2011
```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x
] && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x
, a + b*x])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4293
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]
*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(d*x)^(n - 1)/Sqrt[a - b*x], x], x,
 Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0]
```

rule 4301
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
2)*((d*Csc[e + f*x])^n/(f*(m + n - 1))), x] + Simp[b/(m + n - 1)   Int[(a +
 b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*(b*(m + 2*n - 1) + a*(3*m + 2*n
 - 4)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^
2, 0] && GtQ[m, 1] && NeQ[m + n - 1, 0] && IntegerQ[2*m]
```

3.327.4 Maple [F]

$$\int (d\sec(fx+e))^n (a-a\sec(fx+e))^{\frac{3}{2}}\,dx$$

input
```
int((d*sec(f*x+e))^n*(a-a*sec(f*x+e))^(3/2),x)
```

output
```
int((d*sec(f*x+e))^n*(a-a*sec(f*x+e))^(3/2),x)
```

3.327.5 Fricas [F]

$$\int (d\sec(e+fx))^n(a-a\sec(e+fx))^{3/2}\,dx=\int (-a\sec(fx+e)+a)^{\frac{3}{2}}(d\sec(fx+e))^n\,dx$$

input
```
integrate((d*sec(f*x+e))^n*(a-a*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
integral(-(a*sec(f*x + e) - a)*sqrt(-a*sec(f*x + e) + a)*(d*sec(f*x + e))^
n, x)
```

3.327.6 Sympy [F]

$$\int (d\sec(e+fx))^n(a-a\sec(e+fx))^{3/2}\,dx=\int (d\sec(e+fx))^n(-a(\sec(e+fx)-1))^{\frac{3}{2}}\,dx$$

input
```
integrate((d*sec(f*x+e))**n*(a-a*sec(f*x+e))**(3/2),x)
```

output
```
Integral((d*sec(e + f*x))**n*(-a*(sec(e + f*x) - 1))**(3/2), x)
```

3.327.7 Maxima [F]

$$\int (d\sec(e+fx))^n(a-a\sec(e+fx))^{3/2}\,dx=\int (-a\sec(fx+e)+a)^{\frac{3}{2}}(d\sec(fx+e))^n\,dx$$

input
```
integrate((d*sec(f*x+e))^n*(a-a*sec(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
integrate((-a*sec(f*x + e) + a)^(3/2)*(d*sec(f*x + e))^n, x)
```

3.327.8 Giac [F]

$$\int (d\sec(e+fx))^n(a-a\sec(e+fx))^{3/2}\,dx = \int (-a\sec(fx+e)+a)^{\frac{3}{2}}(d\sec(fx+e))^n\,dx$$

input
```
integrate((d*sec(f*x+e))^n*(a-a*sec(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
integrate((-a*sec(f*x + e) + a)^(3/2)*(d*sec(f*x + e))^n, x)
```

3.327.9 Mupad [F(-1)]

Timed out.

$$\int (d\sec(e+fx))^n(a-a\sec(e+fx))^{3/2}\,dx = \int \left(a-\frac{a}{\cos(e+f\,x)}\right)^{3/2}\left(\frac{d}{\cos(e+f\,x)}\right)^n dx$$

input
```
int((a - a/cos(e + f*x))^(3/2)*(d/cos(e + f*x))^n,x)
```

output
```
int((a - a/cos(e + f*x))^(3/2)*(d/cos(e + f*x))^n, x)
```

3.328 $\int (d\sec(e+fx))^n \sqrt{a-a\sec(e+fx)}\,dx$

3.328.1 Optimal result 2471
3.328.2 Mathematica [A] (verified) 2471
3.328.3 Rubi [A] (verified) 2472
3.328.4 Maple [F] 2473
3.328.5 Fricas [F] 2474
3.328.6 Sympy [F] 2474
3.328.7 Maxima [F] 2474
3.328.8 Giac [F] 2475
3.328.9 Mupad [F(-1)] 2475

3.328.1 Optimal result

Integrand size = 26, antiderivative size = 69

$$\int (d\sec(e+fx))^n \sqrt{a-a\sec(e+fx)}\,dx = \frac{2a\,\text{Hypergeometric2F1}\left(\frac{1}{2},1-n,\frac{3}{2},1+\sec(e+fx)\right)(-\sec(e+fx))^{-n}(d\sec(e+fx))^n\tan(e+fx)}{f\sqrt{a-a\sec(e+fx)}}$$

output
```
2*a*hypergeom([1/2, 1-n],[3/2],1+sec(f*x+e))*(d*sec(f*x+e))^n*tan(f*x+e)/f
/((-sec(f*x+e))^n)/(a-a*sec(f*x+e))^(1/2)
```

3.328.2 Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.01

$$\int (d\sec(e+fx))^n \sqrt{a-a\sec(e+fx)}\,dx = \frac{\cot\left(\frac{1}{2}(e+fx)\right)\text{Hypergeometric2F1}\left(\frac{1}{2},n,1+n,-\sec(e+fx)\right)(d\sec(e+fx))^n\sqrt{a-a\sec(e+fx)}}{fn\sqrt{1+\sec(e+fx)}}$$

input
```
Integrate[(d*Sec[e + f*x])^n*Sqrt[a - a*Sec[e + f*x]],x]
```

output
```
(Cot[(e + f*x)/2]*Hypergeometric2F1[1/2, n, 1 + n, -Sec[e + f*x]]*(d*Sec[e
 + f*x])^n*Sqrt[a - a*Sec[e + f*x]])/(f*n*Sqrt[1 + Sec[e + f*x]])
```

3.328.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {3042, 4293, 77, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a-a\sec(e+fx)}(d\sec(e+fx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{a-a\csc\left(e+fx+\frac{\pi}{2}\right)}\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^n dx$$

$$\Big\downarrow \text{4293}$$

$$-\frac{a^2d\tan(e+fx)\int \frac{(d\sec(e+fx))^{n-1}}{\sqrt{\sec(e+fx)a+a}}d\sec(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{77}$$

$$\frac{a^2\tan(e+fx)(-\sec(e+fx))^{-n}(d\sec(e+fx))^n\int \frac{(-\sec(e+fx))^{n-1}}{\sqrt{\sec(e+fx)a+a}}d\sec(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

$$\Big\downarrow \text{75}$$

$$\frac{2a\tan(e+fx)(-\sec(e+fx))^{-n}(d\sec(e+fx))^n\,\text{Hypergeometric2F1}\left(\frac{1}{2},1-n,\frac{3}{2},\sec(e+fx)+1\right)}{f\sqrt{a-a\sec(e+fx)}}$$

input
```
Int[(d*Sec[e + f*x])^n*Sqrt[a - a*Sec[e + f*x]],x]
```

output
```
(2*a*Hypergeometric2F1[1/2, 1 - n, 3/2, 1 + Sec[e + f*x]]*(d*Sec[e + f*x])
^n*Tan[e + f*x])/(f*(-Sec[e + f*x])^n*Sqrt[a - a*Sec[e + f*x]])
```

3.328.3.1 Defintions of rubi rules used

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 77
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((-b)*(c/
d))^IntPart[m]*((b*x)^FracPart[m]/((-d)*(x/c))^FracPart[m])   Int[((-d)*(x/
c))^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&
 !IntegerQ[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4293
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]
*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(d*x)^(n - 1)/Sqrt[a - b*x], x], x,
 Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0]
```

3.328.4 Maple [F]

$$\int (d\sec(fx+e))^n \sqrt{a-a\sec(fx+e)}dx$$

input
```
int((d*sec(f*x+e))^n*(a-a*sec(f*x+e))^(1/2),x)
```

output
```
int((d*sec(f*x+e))^n*(a-a*sec(f*x+e))^(1/2),x)
```

3.328.5 Fricas [F]

$$\int (d\sec(e+fx))^n\sqrt{a-a\sec(e+fx)}\,dx=\int \sqrt{-a\sec(fx+e)+a}(d\sec(fx+e))^n\,dx$$

input
```
integrate((d*sec(f*x+e))^n*(a-a*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(-a*sec(f*x + e) + a)*(d*sec(f*x + e))^n, x)
```

3.328.6 Sympy [F]

$$\int (d\sec(e+fx))^n\sqrt{a-a\sec(e+fx)}\,dx=\int (d\sec(e+fx))^n\sqrt{-a(\sec(e+fx)-1)}\,dx$$

input
```
integrate((d*sec(f*x+e))**n*(a-a*sec(f*x+e))**(1/2),x)
```

output
```
Integral((d*sec(e + f*x))**n*sqrt(-a*(sec(e + f*x) - 1)), x)
```

3.328.7 Maxima [F]

$$\int (d\sec(e+fx))^n\sqrt{a-a\sec(e+fx)}\,dx=\int \sqrt{-a\sec(fx+e)+a}(d\sec(fx+e))^n\,dx$$

input
```
integrate((d*sec(f*x+e))^n*(a-a*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(-a*sec(f*x + e) + a)*(d*sec(f*x + e))^n, x)
```

3.328.8 Giac [F]

$$\int (d\sec(e+fx))^n\sqrt{a-a\sec(e+fx)}\,dx=\int \sqrt{-a\sec(fx+e)+a}(d\sec(fx+e))^n\,dx$$

input
```
integrate((d*sec(f*x+e))^n*(a-a*sec(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(-a*sec(f*x + e) + a)*(d*sec(f*x + e))^n, x)
```

3.328.9 Mupad [F(-1)]

Timed out.

$$\int (d\sec(e+fx))^n\sqrt{a-a\sec(e+fx)}\,dx=\int \sqrt{a-\frac{a}{\cos(e+f\,x)}}\left(\frac{d}{\cos(e+f\,x)}\right)^n\,dx$$

input
```
int((a - a/cos(e + f*x))^(1/2)*(d/cos(e + f*x))^n,x)
```

output
```
int((a - a/cos(e + f*x))^(1/2)*(d/cos(e + f*x))^n, x)
```

3.329 $\int \sec^n(e+fx)(1+\sec(e+fx))^m\,dx$

3.329.1 Optimal result . . . 2476
3.329.2 Mathematica [B] (warning: unable to verify) . . . 2476
3.329.3 Rubi [A] (verified) . . . 2477
3.329.4 Maple [F] . . . 2478
3.329.5 Fricas [F] . . . 2479
3.329.6 Sympy [F] . . . 2479
3.329.7 Maxima [F] . . . 2479
3.329.8 Giac [F] . . . 2480
3.329.9 Mupad [F(-1)] . . . 2480

3.329.1 Optimal result

Integrand size = 19, antiderivative size = 72

$$\int \sec^n(e+fx)(1+\sec(e+fx))^m\,dx = \frac{2^{\frac{1}{2}+m}\,\text{AppellF1}\left(\frac{1}{2},1-n,\frac{1}{2}-m,\frac{3}{2},1-\sec(e+fx),\frac{1}{2}(1-\sec(e+fx))\right)\tan(e+fx)}{f\sqrt{1+\sec(e+fx)}}$$

output
```
2^(1/2+m)*AppellF1(1/2,1-n,1/2-m,3/2,1-sec(f*x+e),1/2-1/2*sec(f*x+e))*tan(
f*x+e)/f/(1+sec(f*x+e))^(1/2)
```

3.329.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 2246 vs. 2(72) = 144.

Time = 14.55 (sec) , antiderivative size = 2246, normalized size of antiderivative = 31.19

$$\int \sec^n(e+fx)(1+\sec(e+fx))^m\,dx = \text{Result too large to show}$$

input
```
Integrate[Sec[e + f*x]^n*(1 + Sec[e + f*x])^m,x]
```

output
```
(3*2^(1 + m)*AppellF1[1/2, m + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e
+ f*x)/2]^2]*(Sec[(e + f*x)/2]^2)^(-1 + n)*Sec[e + f*x]^n*(Cos[(e + f*x)/2
]^2*Sec[e + f*x])^(m + n)*(1 + Sec[e + f*x])^m*Tan[(e + f*x)/2])/(f*(3*App
ellF1[1/2, m + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2
*((-1 + n)*AppellF1[3/2, m + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e +
f*x)/2]^2] + (m + n)*AppellF1[3/2, 1 + m + n, 1 - n, 5/2, Tan[(e + f*x)/2]
^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)*((3*2^m*AppellF1[1/2, m + n,
 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(Sec[(e + f*x)/2]^2)
^n*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(m + n))/(3*AppellF1[1/2, m + n, 1 -
n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*((-1 + n)*AppellF1[3/
2, m + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (m + n)*A
ppellF1[3/2, 1 + m + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^
2])*Tan[(e + f*x)/2]^2) + (3*2^(1 + m)*(-1 + n)*AppellF1[1/2, m + n, 1 - n
, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(Sec[(e + f*x)/2]^2)^(-1 +
 n)*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(m + n)*Tan[(e + f*x)/2]^2)/(3*Appel
lF1[1/2, m + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*(
(-1 + n)*AppellF1[3/2, m + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*
x)/2]^2] + (m + n)*AppellF1[3/2, 1 + m + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2
, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2) + (3*2^(1 + m)*(Sec[(e + f*x)/
2]^2)^(-1 + n)*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(m + n)*Tan[(e + f*x)/...
```

3.329.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {3042, 4312, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (\sec(e+fx)+1)^m \sec^n(e+fx)\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \left(\csc\left(e+fx+\frac{\pi}{2}\right)+1\right)^m \csc\left(e+fx+\frac{\pi}{2}\right)^n dx$$

$$\Big\downarrow \; 4312$$

$$\frac{\tan(e+fx)\int \frac{\sec^{n-1}(e+fx)(\sec(e+fx)+1)^{m-\frac{1}{2}}}{\sqrt{1-\sec(e+fx)}}d(1-\sec(e+fx))}{f\sqrt{1-\sec(e+fx)}\sqrt{\sec(e+fx)+1}}$$

$$\Big\downarrow \; 150$$

$$\frac{2^{m+\frac{1}{2}}\tan(e+fx)\,\text{AppellF1}\left(\frac{1}{2},1-n,\frac{1}{2}-m,\frac{3}{2},1-\sec(e+fx),\frac{1}{2}(1-\sec(e+fx))\right)}{f\sqrt{\sec(e+fx)+1}}$$

input
```
Int[Sec[e + f*x]^n*(1 + Sec[e + f*x])^m,x]
```

output
```
(2^(1/2 + m)*AppellF1[1/2, 1 - n, 1/2 - m, 3/2, 1 - Sec[e + f*x], (1 - Sec
[e + f*x])/2]*Tan[e + f*x])/(f*Sqrt[1 + Sec[e + f*x]])
```

3.329.3.1 Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4312
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-(a*(d/b))^n)*(Cot[e + f*x]/(a^(n - 2)*f*Sqrt
[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(a - x)^(n - 1)
*((2*a - x)^(m - 1/2)/Sqrt[x]), x], x, a - b*Csc[e + f*x]], x] /; FreeQ[{a,
 b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0] &
&  !IntegerQ[n] && GtQ[a*(d/b), 0]
```

3.329.4 Maple [F]

$$\int \sec(fx+e)^n(\sec(fx+e)+1)^m\,dx$$

input
```
int(sec(f*x+e)^n*(sec(f*x+e)+1)^m,x)
```

output
```
int(sec(f*x+e)^n*(sec(f*x+e)+1)^m,x)
```

3.329.5 Fricas [F]

$$\int \sec^n(e+fx)(1+\sec(e+fx))^m\,dx = \int (\sec(fx+e)+1)^m \sec(fx+e)^n\,dx$$

input
```
integrate(sec(f*x+e)^n*(1+sec(f*x+e))^m,x, algorithm="fricas")
```

output
```
integral((sec(f*x + e) + 1)^m*sec(f*x + e)^n, x)
```

3.329.6 Sympy [F]

$$\int \sec^n(e+fx)(1+\sec(e+fx))^m\,dx = \int (\sec(e+fx)+1)^m \sec^n(e+fx)\,dx$$

input
```
integrate(sec(f*x+e)**n*(1+sec(f*x+e))**m,x)
```

output
```
Integral((sec(e + f*x) + 1)**m*sec(e + f*x)**n, x)
```

3.329.7 Maxima [F]

$$\int \sec^n(e+fx)(1+\sec(e+fx))^m\,dx = \int (\sec(fx+e)+1)^m \sec(fx+e)^n\,dx$$

input
```
integrate(sec(f*x+e)^n*(1+sec(f*x+e))^m,x, algorithm="maxima")
```

output
```
integrate((sec(f*x + e) + 1)^m*sec(f*x + e)^n, x)
```

3.329.8 Giac [F]

$$\int \sec^n(e+fx)(1+\sec(e+fx))^m\,dx = \int (\sec(fx+e)+1)^m \sec(fx+e)^n\,dx$$

input
```
integrate(sec(f*x+e)^n*(1+sec(f*x+e))^m,x, algorithm="giac")
```

output
```
integrate((sec(f*x + e) + 1)^m*sec(f*x + e)^n, x)
```

3.329.9 Mupad [F(-1)]

Timed out.

$$\int \sec^n(e+fx)(1+\sec(e+fx))^m\,dx = \int \left(\frac{1}{\cos(e+f\,x)}+1\right)^m \left(\frac{1}{\cos(e+f\,x)}\right)^n dx$$

input
```
int((1/cos(e + f*x) + 1)^m*(1/cos(e + f*x))^n,x)
```

output
```
int((1/cos(e + f*x) + 1)^m*(1/cos(e + f*x))^n, x)
```

3.330 $\int (1-\sec(e+fx))^m \sec^n(e+fx)\,dx$

3.330.1 Optimal result 2481
3.330.2 Mathematica [B] (warning: unable to verify) 2481
3.330.3 Rubi [A] (verified) 2482
3.330.4 Maple [F] 2483
3.330.5 Fricas [F] 2483
3.330.6 Sympy [F] 2484
3.330.7 Maxima [F] 2484
3.330.8 Giac [F] 2484
3.330.9 Mupad [F(-1)] 2485

3.330.1 Optimal result

Integrand size = 21, antiderivative size = 89

$$\int (1-\sec(e+fx))^m \sec^n(e+fx)\,dx = \frac{\sqrt{2}\,\text{AppellF1}\left(\frac{1}{2}+m,1-n,\frac{1}{2},\frac{3}{2}+m,1-\sec(e+fx),\frac{1}{2}(1-\sec(e+fx))\right)(1-\sec(e+fx))^m\tan(e+f}{f(1+2m)\sqrt{1+\sec(e+fx)}}$$

output
```
AppellF1(1/2+m,1-n,1/2,3/2+m,1-sec(f*x+e),1/2-1/2*sec(f*x+e))*(1-sec(f*x+e
))^m*2^(1/2)*tan(f*x+e)/f/(1+2*m)/(1+sec(f*x+e))^(1/2)
```

3.330.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 255 vs. 2(89) = 178.

Time = 3.22 (sec) , antiderivative size = 255, normalized size of antiderivative = 2.87

$$\int (1-\sec(e+fx))^m \sec^n(e+fx)\,dx = \frac{(3+2m)\,\text{AppellF}}{f(1+2m)\left((3+2m)\,\text{AppellF1}\left(\frac{1}{2}+m,m+n,1-n,\frac{3}{2}+m,\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)+2\right.}$$

input
```
Integrate[(1 - Sec[e + f*x])^m*Sec[e + f*x]^n,x]
```

output
```
((3 + 2*m)*AppellF1[1/2 + m, m + n, 1 - n, 3/2 + m, Tan[(e + f*x)/2]^2, -T
an[(e + f*x)/2]^2]*(1 - Sec[e + f*x])^m*Sec[e + f*x]^n*Sin[e + f*x])/(f*(1
 + 2*m)*((3 + 2*m)*AppellF1[1/2 + m, m + n, 1 - n, 3/2 + m, Tan[(e + f*x)/
2]^2, -Tan[(e + f*x)/2]^2] + 2*((-1 + n)*AppellF1[3/2 + m, m + n, 2 - n, 5
/2 + m, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (m + n)*AppellF1[3/2 +
m, 1 + m + n, 1 - n, 5/2 + m, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Ta
n[(e + f*x)/2]^2))
```

3.330.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3042, 4313, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (1-\sec(e+fx))^m \sec^n(e+fx)\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \left(1-\csc\left(e+fx+\frac{\pi}{2}\right)\right)^m \csc\left(e+fx+\frac{\pi}{2}\right)^n dx$$

$$\Big\downarrow \; 4313$$

$$\frac{\tan(e+fx)\int \frac{(1-\sec(e+fx))^{m-\frac{1}{2}}\sec^{n-1}(e+fx)}{\sqrt{\sec(e+fx)+1}}d(1-\sec(e+fx))}{f\sqrt{1-\sec(e+fx)}\sqrt{\sec(e+fx)+1}}$$

$$\Big\downarrow \; 150$$

$$\frac{\sqrt{2}\tan(e+fx)(1-\sec(e+fx))^m \operatorname{AppellF1}\left(m+\frac{1}{2},1-n,\frac{1}{2},m+\frac{3}{2},1-\sec(e+fx),\frac{1}{2}(1-\sec(e+fx))\right)}{f(2m+1)\sqrt{\sec(e+fx)+1}}$$

input
```
Int[(1 - Sec[e + f*x])^m*Sec[e + f*x]^n,x]
```

output
```
(Sqrt[2]*AppellF1[1/2 + m, 1 - n, 1/2, 3/2 + m, 1 - Sec[e + f*x], (1 - Sec
[e + f*x])/2]*(1 - Sec[e + f*x])^m*Tan[e + f*x])/(f*(1 + 2*m)*Sqrt[1 + Sec
[e + f*x]])
```

3.330.3.1 Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4313
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-((-a)*(d/b))^n)*(Cot[e + f*x]/(a^(n - 1)*f*S
qrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[x^(m - 1/2)*
((a - x)^(n - 1)/Sqrt[2*a - x]), x], x, a + b*Csc[e + f*x]], x] /; FreeQ[{a
, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
&&  !IntegerQ[n] && LtQ[a*(d/b), 0]
```

3.330.4 Maple [F]

$$\int (1-\sec(fx+e))^m \sec(fx+e)^n \, dx$$

input
```
int((1-sec(f*x+e))^m*sec(f*x+e)^n,x)
```

output
```
int((1-sec(f*x+e))^m*sec(f*x+e)^n,x)
```

3.330.5 Fricas [F]

$$\int (1-\sec(e+fx))^m \sec^n(e+fx)\, dx = \int (-\sec(fx+e)+1)^m \sec(fx+e)^n \, dx$$

input
```
integrate((1-sec(f*x+e))^m*sec(f*x+e)^n,x, algorithm="fricas")
```

output
```
integral((-sec(f*x + e) + 1)^m*sec(f*x + e)^n, x)
```

3.330.6 Sympy [F]

$$\int (1-\sec(e+fx))^m \sec^n(e+fx)\,dx = \int (1-\sec{(e+fx)})^m \sec^n{(e+fx)}\,dx$$

input
```
integrate((1-sec(f*x+e))**m*sec(f*x+e)**n,x)
```

output
```
Integral((1 - sec(e + f*x))**m*sec(e + f*x)**n, x)
```

3.330.7 Maxima [F]

$$\int (1-\sec(e+fx))^m \sec^n(e+fx)\,dx = \int (-\sec{(fx+e)}+1)^m \sec{(fx+e)}^n\,dx$$

input
```
integrate((1-sec(f*x+e))^m*sec(f*x+e)^n,x, algorithm="maxima")
```

output
```
integrate((-sec(f*x + e) + 1)^m*sec(f*x + e)^n, x)
```

3.330.8 Giac [F]

$$\int (1-\sec(e+fx))^m \sec^n(e+fx)\,dx = \int (-\sec{(fx+e)}+1)^m \sec{(fx+e)}^n\,dx$$

input
```
integrate((1-sec(f*x+e))^m*sec(f*x+e)^n,x, algorithm="giac")
```

output
```
integrate((-sec(f*x + e) + 1)^m*sec(f*x + e)^n, x)
```

3.330.9 Mupad [F(-1)]

Timed out.

$$\int (1-\sec(e+fx))^m \sec^n(e+fx)\,dx = \int \left(1-\frac{1}{\cos\left(e+f\,x\right)}\right)^m \left(\frac{1}{\cos\left(e+f\,x\right)}\right)^n dx$$

input
```
int((1 - 1/cos(e + f*x))^m*(1/cos(e + f*x))^n,x)
```

output
```
int((1 - 1/cos(e + f*x))^m*(1/cos(e + f*x))^n, x)
```

3.331 $\int \sec^n(e+fx)(a+a\sec(e+fx))^m\,dx$

3.331.1 Optimal result 2486
3.331.2 Mathematica [B] (warning: unable to verify) 2486
3.331.3 Rubi [A] (verified) 2487
3.331.4 Maple [F] 2489
3.331.5 Fricas [F] 2489
3.331.6 Sympy [F] 2489
3.331.7 Maxima [F] 2490
3.331.8 Giac [F] 2490
3.331.9 Mupad [F(-1)] 2490

3.331.1 Optimal result

Integrand size = 21, antiderivative size = 88

$$\int \sec^n(e+fx)(a+a\sec(e+fx))^m\,dx = \frac{2^{\frac{1}{2}+m}\operatorname{AppellF1}\left(\frac{1}{2},1-n,\frac{1}{2}-m,\frac{3}{2},1-\sec(e+fx),\frac{1}{2}(1-\sec(e+fx))\right)(1+\sec(e+fx))^{-\frac{1}{2}-m}(a+a\sec$$

output
```
2^(1/2+m)*AppellF1(1/2,1-n,1/2-m,3/2,1-sec(f*x+e),1/2-1/2*sec(f*x+e))*(1+s
ec(f*x+e))^(-1/2-m)*(a+a*sec(f*x+e))^m*tan(f*x+e)/f
```

3.331.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 2248 vs. 2(88) = 176.

Time = 6.30 (sec) , antiderivative size = 2248, normalized size of antiderivative = 25.55

$$\int \sec^n(e+fx)(a+a\sec(e+fx))^m\,dx = \text{Result too large to show}$$

input
```
Integrate[Sec[e + f*x]^n*(a + a*Sec[e + f*x])^m,x]
```

output
```
(3*2^(1 + m)*AppellF1[1/2, m + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e
+ f*x)/2]^2]*(Sec[(e + f*x)/2]^2)^(-1 + n)*Sec[e + f*x]^n*(Cos[(e + f*x)/2
]^2*Sec[e + f*x])^(m + n)*(a*(1 + Sec[e + f*x]))^m*Tan[(e + f*x)/2])/(f*(3
*AppellF1[1/2, m + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]
 + 2*((-1 + n)*AppellF1[3/2, m + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(
e + f*x)/2]^2] + (m + n)*AppellF1[3/2, 1 + m + n, 1 - n, 5/2, Tan[(e + f*x
)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)*((3*2^m*AppellF1[1/2, m
+ n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(Sec[(e + f*x)/2
]^2)^n*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(m + n))/(3*AppellF1[1/2, m + n,
1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*((-1 + n)*AppellF
1[3/2, m + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (m +
n)*AppellF1[3/2, 1 + m + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)
/2]^2])*Tan[(e + f*x)/2]^2) + (3*2^(1 + m)*(-1 + n)*AppellF1[1/2, m + n, 1
 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(Sec[(e + f*x)/2]^2)^(
-1 + n)*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(m + n)*Tan[(e + f*x)/2]^2)/(3*A
ppellF1[1/2, m + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] +
 2*((-1 + n)*AppellF1[3/2, m + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e
+ f*x)/2]^2] + (m + n)*AppellF1[3/2, 1 + m + n, 1 - n, 5/2, Tan[(e + f*x)/
2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2) + (3*2^(1 + m)*(Sec[(e + f
*x)/2]^2)^(-1 + n)*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(m + n)*Tan[(e + f...
```

3.331.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3042, 4315, 3042, 4312, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^n(e+fx)(a\sec(e+fx)+a)^m\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \csc\left(e+fx+\frac{\pi}{2}\right)^n\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^m dx$$

$$\Big\downarrow \; 4315$$

$$(\sec(e+fx)+1)^{-m}(a\sec(e+fx)+a)^m\int \sec^n(e+fx)(\sec(e+fx)+1)^m dx$$

$$\Big\downarrow \; 3042$$

$$(\sec(e+fx)+1)^{-m}(a\sec(e+fx)+a)^m\int \csc\left(e+fx+\frac{\pi}{2}\right)^n\left(\csc\left(e+fx+\frac{\pi}{2}\right)+1\right)^m dx$$

$$\Big\downarrow \text{4312}$$

$$\frac{\tan(e+fx)(\sec(e+fx)+1)^{-m-\frac{1}{2}}(a\sec(e+fx)+a)^m\int \frac{\sec^{n-1}(e+fx)(\sec(e+fx)+1)^{m-\frac{1}{2}}}{\sqrt{1-\sec(e+fx)}}d(1-\sec(e+fx))}{f\sqrt{1-\sec(e+fx)}}$$

$$\Big\downarrow \text{150}$$

$$\frac{2^{m+\frac{1}{2}}\tan(e+fx)(\sec(e+fx)+1)^{-m-\frac{1}{2}}(a\sec(e+fx)+a)^m\operatorname{AppellF1}\left(\frac{1}{2},1-n,\frac{1}{2}-m,\frac{3}{2},1-\sec(e+fx),\frac{1}{2}(1\right.}{f}$$

input
```
Int[Sec[e + f*x]^n*(a + a*Sec[e + f*x])^m,x]
```

output
```
(2^(1/2 + m)*AppellF1[1/2, 1 - n, 1/2 - m, 3/2, 1 - Sec[e + f*x], (1 - Sec
[e + f*x])/2]*(1 + Sec[e + f*x])^(-1/2 - m)*(a + a*Sec[e + f*x])^m*Tan[e +
 f*x])/f
```

3.331.3.1 Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4312
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-(a*(d/b))^n)*(Cot[e + f*x]/(a^(n - 2)*f*Sqrt
[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(a - x)^(n - 1)
*((2*a - x)^(m - 1/2)/Sqrt[x]), x], x, a - b*Csc[e + f*x]], x] /; FreeQ[{a,
 b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0] &
&  !IntegerQ[n] && GtQ[a*(d/b), 0]
```

rule 4315
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Simp[a^IntPart[m]*((a + b*Csc[e + f*x])^FracPart[m
]/(1 + (b/a)*Csc[e + f*x])^FracPart[m])   Int[(1 + (b/a)*Csc[e + f*x])^m*(d
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^
2, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

3.331.4 Maple [F]

$$\int \sec(fx+e)^n (a+a\sec(fx+e))^m \, dx$$

input
```
int(sec(f*x+e)^n*(a+a*sec(f*x+e))^m,x)
```

output
```
int(sec(f*x+e)^n*(a+a*sec(f*x+e))^m,x)
```

3.331.5 Fricas [F]

$$\int \sec^n(e+fx)(a+a\sec(e+fx))^m \, dx = \int (a\sec(fx+e)+a)^m \sec(fx+e)^n \, dx$$

input
```
integrate(sec(f*x+e)^n*(a+a*sec(f*x+e))^m,x, algorithm="fricas")
```

output
```
integral((a*sec(f*x + e) + a)^m*sec(f*x + e)^n, x)
```

3.331.6 Sympy [F]

$$\int \sec^n(e+fx)(a+a\sec(e+fx))^m \, dx = \int (a(\sec(e+fx)+1))^m \sec^n(e+fx) \, dx$$

input
```
integrate(sec(f*x+e)**n*(a+a*sec(f*x+e))**m,x)
```

output
```
Integral((a*(sec(e + f*x) + 1))**m*sec(e + f*x)**n, x)
```

3.331.7 Maxima [F]

$$\int \sec^n(e+fx)(a+a\sec(e+fx))^m\,dx = \int (a\sec(fx+e)+a)^m\sec(fx+e)^n\,dx$$

input
```
integrate(sec(f*x+e)^n*(a+a*sec(f*x+e))^m,x, algorithm="maxima")
```

output
```
integrate((a*sec(f*x + e) + a)^m*sec(f*x + e)^n, x)
```

3.331.8 Giac [F]

$$\int \sec^n(e+fx)(a+a\sec(e+fx))^m\,dx = \int (a\sec(fx+e)+a)^m\sec(fx+e)^n\,dx$$

input
```
integrate(sec(f*x+e)^n*(a+a*sec(f*x+e))^m,x, algorithm="giac")
```

output
```
integrate((a*sec(f*x + e) + a)^m*sec(f*x + e)^n, x)
```

3.331.9 Mupad [F(-1)]

Timed out.

$$\int \sec^n(e+fx)(a+a\sec(e+fx))^m\,dx = \int \left(a+\frac{a}{\cos(e+f\,x)}\right)^m\left(\frac{1}{\cos(e+f\,x)}\right)^n dx$$

input
```
int((a + a/cos(e + f*x))^m*(1/cos(e + f*x))^n,x)
```

output
```
int((a + a/cos(e + f*x))^m*(1/cos(e + f*x))^n, x)
```

3.332 $\int \sec^n(e+fx)(a-a\sec(e+fx))^m\,dx$

3.332.1 Optimal result 2491
3.332.2 Mathematica [F] 2491
3.332.3 Rubi [A] (verified) 2492
3.332.4 Maple [F] 2493
3.332.5 Fricas [F] 2494
3.332.6 Sympy [F] 2494
3.332.7 Maxima [F] 2494
3.332.8 Giac [F] 2495
3.332.9 Mupad [F(-1)] 2495

3.332.1 Optimal result

Integrand size = 22, antiderivative size = 90

$$\int \sec^n(e+fx)(a-a\sec(e+fx))^m\,dx = \frac{\sqrt{2}\,\text{AppellF1}\left(\frac{1}{2}+m,1-n,\frac{1}{2},\frac{3}{2}+m,1-\sec(e+fx),\frac{1}{2}(1-\sec(e+fx))\right)(a-a\sec(e+fx))^m\tan(e+}{f(1+2m)\sqrt{1+\sec(e+fx)}}$$

output
```
AppellF1(1/2+m,1-n,1/2,3/2+m,1-sec(f*x+e),1/2-1/2*sec(f*x+e))*(a-a*sec(f*x
+e))^m*2^(1/2)*tan(f*x+e)/f/(1+2*m)/(1+sec(f*x+e))^(1/2)
```

3.332.2 Mathematica [F]

$$\int \sec^n(e+fx)(a-a\sec(e+fx))^m\,dx = \int \sec^n(e+fx)(a-a\sec(e+fx))^m\,dx$$

input
```
Integrate[Sec[e + f*x]^n*(a - a*Sec[e + f*x])^m,x]
```

output
```
Integrate[Sec[e + f*x]^n*(a - a*Sec[e + f*x])^m, x]
```

3.332.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {3042, 4315, 3042, 4313, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^n(e+fx)(a-a\sec(e+fx))^m\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(e+fx+\frac{\pi}{2}\right)^n\left(a-a\csc\left(e+fx+\frac{\pi}{2}\right)\right)^m dx$$

$$\Big\downarrow \text{4315}$$

$$(1-\sec(e+fx))^{-m}(a-a\sec(e+fx))^m\int(1-\sec(e+fx))^m\sec^n(e+fx)dx$$

$$\Big\downarrow \text{3042}$$

$$(1-\sec(e+fx))^{-m}(a-a\sec(e+fx))^m\int\left(1-\csc\left(e+fx+\frac{\pi}{2}\right)\right)^m\csc\left(e+fx+\frac{\pi}{2}\right)^n dx$$

$$\Big\downarrow \text{4313}$$

$$\frac{\tan(e+fx)(1-\sec(e+fx))^{-m-\frac{1}{2}}(a-a\sec(e+fx))^m\int\frac{(1-\sec(e+fx))^{m-\frac{1}{2}}\sec^{n-1}(e+fx)}{\sqrt{\sec(e+fx)+1}}d(1-\sec(e+fx))}{f\sqrt{\sec(e+fx)+1}}$$

$$\Big\downarrow \text{150}$$

$$\frac{\sqrt{2}\tan(e+fx)(a-a\sec(e+fx))^m\,\text{AppellF1}\left(m+\frac{1}{2},1-n,\frac{1}{2},m+\frac{3}{2},1-\sec(e+fx),\frac{1}{2}(1-\sec(e+fx))\right)}{f(2m+1)\sqrt{\sec(e+fx)+1}}$$

input
```
Int[Sec[e + f*x]^n*(a - a*Sec[e + f*x])^m,x]
```

output
```
(Sqrt[2]*AppellF1[1/2 + m, 1 - n, 1/2, 3/2 + m, 1 - Sec[e + f*x], (1 - Sec
[e + f*x])/2]*(a - a*Sec[e + f*x])^m*Tan[e + f*x])/(f*(1 + 2*m)*Sqrt[1 + S
ec[e + f*x]])
```

3.332.3.1 Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4313
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-((-a)*(d/b))^n)*(Cot[e + f*x]/(a^(n - 1)*f*S
qrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[x^(m - 1/2)*
((a - x)^(n - 1)/Sqrt[2*a - x]), x], x, a + b*Csc[e + f*x]], x] /; FreeQ[{a
, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
&&  !IntegerQ[n] && LtQ[a*(d/b), 0]
```

rule 4315
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Simp[a^IntPart[m]*((a + b*Csc[e + f*x])^FracPart[m
]/(1 + (b/a)*Csc[e + f*x])^FracPart[m])   Int[(1 + (b/a)*Csc[e + f*x])^m*(d
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^
2, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

3.332.4 Maple [F]

$$\int \sec (fx+e)^n (a-a\sec (fx+e))^m \, dx$$

input
```
int(sec(f*x+e)^n*(a-a*sec(f*x+e))^m,x)
```

output
```
int(sec(f*x+e)^n*(a-a*sec(f*x+e))^m,x)
```

3.332.5 Fricas [F]

$$\int \sec^n(e+fx)(a-a\sec(e+fx))^m\,dx=\int (-a\sec(fx+e)+a)^m\sec(fx+e)^n\,dx$$

input
```
integrate(sec(f*x+e)^n*(a-a*sec(f*x+e))^m,x, algorithm="fricas")
```

output
```
integral((-a*sec(f*x + e) + a)^m*sec(f*x + e)^n, x)
```

3.332.6 Sympy [F]

$$\int \sec^n(e+fx)(a-a\sec(e+fx))^m\,dx=\int (-a(\sec(e+fx)-1))^m\sec^n(e+fx)\,dx$$

input
```
integrate(sec(f*x+e)**n*(a-a*sec(f*x+e))**m,x)
```

output
```
Integral((-a*(sec(e + f*x) - 1))**m*sec(e + f*x)**n, x)
```

3.332.7 Maxima [F]

$$\int \sec^n(e+fx)(a-a\sec(e+fx))^m\,dx=\int (-a\sec(fx+e)+a)^m\sec(fx+e)^n\,dx$$

input
```
integrate(sec(f*x+e)^n*(a-a*sec(f*x+e))^m,x, algorithm="maxima")
```

output
```
integrate((-a*sec(f*x + e) + a)^m*sec(f*x + e)^n, x)
```

3.332.8 Giac [F]

$$\int \sec^n(e+fx)(a-a\sec(e+fx))^m\,dx=\int (-a\sec(fx+e)+a)^m\sec(fx+e)^n\,dx$$

input
```
integrate(sec(f*x+e)^n*(a-a*sec(f*x+e))^m,x, algorithm="giac")
```

output
```
integrate((-a*sec(f*x + e) + a)^m*sec(f*x + e)^n, x)
```

3.332.9 Mupad [F(-1)]

Timed out.

$$\int \sec^n(e+fx)(a-a\sec(e+fx))^m\,dx=\int \left(a-\frac{a}{\cos(e+f\,x)}\right)^m\left(\frac{1}{\cos(e+f\,x)}\right)^n\,dx$$

input
```
int((a - a/cos(e + f*x))^m*(1/cos(e + f*x))^n,x)
```

output
```
int((a - a/cos(e + f*x))^m*(1/cos(e + f*x))^n, x)
```

3.333 $\int (-\sec(e+fx))^n(1+\sec(e+fx))^m\,dx$

3.333.1 Optimal result 2496
3.333.2 Mathematica [B] (warning: unable to verify) 2496
3.333.3 Rubi [A] (verified) 2497
3.333.4 Maple [F] 2498
3.333.5 Fricas [F] 2499
3.333.6 Sympy [F] 2499
3.333.7 Maxima [F] 2499
3.333.8 Giac [F] 2500
3.333.9 Mupad [F(-1)] 2500

3.333.1 Optimal result

Integrand size = 21, antiderivative size = 85

$$\int (-\sec(e+fx))^n(1+\sec(e+fx))^m\,dx = \frac{\sqrt{2}\,\text{AppellF1}\left(\frac{1}{2}+m,1-n,\frac{1}{2},\frac{3}{2}+m,1+\sec(e+fx),\frac{1}{2}(1+\sec(e+fx))\right)(1+\sec(e+fx))^m\tan(e+f}{f(1+2m)\sqrt{1-\sec(e+fx)}}$$

output
```
AppellF1(1/2+m,1-n,1/2,3/2+m,1+sec(f*x+e),1/2+1/2*sec(f*x+e))*(1+sec(f*x+e
))^m*2^(1/2)*tan(f*x+e)/f/(1+2*m)/(1-sec(f*x+e))^(1/2)
```

3.333.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 2248 vs. 2(85) = 170.

Time = 6.24 (sec) , antiderivative size = 2248, normalized size of antiderivative = 26.45

$$\int (-\sec(e+fx))^n(1+\sec(e+fx))^m\,dx = \text{Result too large to show}$$

input
```
Integrate[(-Sec[e + f*x])^n*(1 + Sec[e + f*x])^m,x]
```

output
```
(3*2^(1 + m)*AppellF1[1/2, m + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e
+ f*x)/2]^2]*(Sec[(e + f*x)/2]^2)^(-1 + n)*(-Sec[e + f*x])^n*(Cos[(e + f*x
)/2]^2*Sec[e + f*x])^(m + n)*(1 + Sec[e + f*x])^m*Tan[(e + f*x)/2])/(f*(3*
AppellF1[1/2, m + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]
+ 2*((-1 + n)*AppellF1[3/2, m + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e
 + f*x)/2]^2] + (m + n)*AppellF1[3/2, 1 + m + n, 1 - n, 5/2, Tan[(e + f*x)
/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)*((3*2^m*AppellF1[1/2, m +
 n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(Sec[(e + f*x)/2]
^2)^n*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(m + n))/(3*AppellF1[1/2, m + n, 1
 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*((-1 + n)*AppellF1
[3/2, m + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (m + n
)*AppellF1[3/2, 1 + m + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/
2]^2])*Tan[(e + f*x)/2]^2) + (3*2^(1 + m)*(-1 + n)*AppellF1[1/2, m + n, 1
- n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(Sec[(e + f*x)/2]^2)^(-
1 + n)*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(m + n)*Tan[(e + f*x)/2]^2)/(3*Ap
pellF1[1/2, m + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] +
2*((-1 + n)*AppellF1[3/2, m + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e +
 f*x)/2]^2] + (m + n)*AppellF1[3/2, 1 + m + n, 1 - n, 5/2, Tan[(e + f*x)/2
]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2) + (3*2^(1 + m)*(Sec[(e + f*
x)/2]^2)^(-1 + n)*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(m + n)*Tan[(e + f*...
```

3.333.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3042, 4313, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (\sec(e+fx)+1)^m(-\sec(e+fx))^n\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \left(\csc\left(e+fx+\frac{\pi}{2}\right)+1\right)^m\left(-\csc\left(e+fx+\frac{\pi}{2}\right)\right)^n dx$$

$$\Big\downarrow \; 4313$$

$$\frac{\tan(e+fx)\int \frac{(-\sec(e+fx))^{n-1}(\sec(e+fx)+1)^{m-\frac{1}{2}}}{\sqrt{1-\sec(e+fx)}}d(\sec(e+fx)+1)}{f\sqrt{1-\sec(e+fx)}\sqrt{\sec(e+fx)+1}}$$

$$\Big\downarrow \; 150$$

$$\frac{\sqrt{2}\tan(e+fx)(\sec(e+fx)+1)^m \operatorname{AppellF1}\left(m+\frac{1}{2},1-n,\frac{1}{2},m+\frac{3}{2},\sec(e+fx)+1,\frac{1}{2}(\sec(e+fx)+1)\right)}{f(2m+1)\sqrt{1-\sec(e+fx)}}$$

input
```
Int[(-Sec[e + f*x])^n*(1 + Sec[e + f*x])^m,x]
```

output
```
(Sqrt[2]*AppellF1[1/2 + m, 1 - n, 1/2, 3/2 + m, 1 + Sec[e + f*x], (1 + Sec
[e + f*x])/2]*(1 + Sec[e + f*x])^m*Tan[e + f*x])/(f*(1 + 2*m)*Sqrt[1 - Sec
[e + f*x]])
```

3.333.3.1 Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] && !In
tegerQ[m] && !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4313
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-((-a)*(d/b))^n)*(Cot[e + f*x]/(a^(n - 1)*f*S
qrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[x^(m - 1/2)*
((a - x)^(n - 1)/Sqrt[2*a - x]), x], x, a + b*Csc[e + f*x]], x] /; FreeQ[{a
, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && !IntegerQ[m] && GtQ[a, 0]
&& !IntegerQ[n] && LtQ[a*(d/b), 0]
```

3.333.4 Maple [F]

$$\int (-\sec(fx+e))^n (\sec(fx+e)+1)^m \, dx$$

input
```
int((-sec(f*x+e))^n*(sec(f*x+e)+1)^m,x)
```

output
```
int((-sec(f*x+e))^n*(sec(f*x+e)+1)^m,x)
```

3.333.5 Fricas [F]

$$\int (-\sec(e+fx))^n(1+\sec(e+fx))^m\,dx = \int (-\sec(fx+e))^n(\sec(fx+e)+1)^m\,dx$$

input
```
integrate((-sec(f*x+e))^n*(1+sec(f*x+e))^m,x, algorithm="fricas")
```

output
```
integral((-sec(f*x + e))^n*(sec(f*x + e) + 1)^m, x)
```

3.333.6 Sympy [F]

$$\int (-\sec(e+fx))^n(1+\sec(e+fx))^m\,dx = \int (-\sec(e+fx))^n(\sec(e+fx)+1)^m\,dx$$

input
```
integrate((-sec(f*x+e))**n*(1+sec(f*x+e))**m,x)
```

output
```
Integral((-sec(e + f*x))**n*(sec(e + f*x) + 1)**m, x)
```

3.333.7 Maxima [F]

$$\int (-\sec(e+fx))^n(1+\sec(e+fx))^m\,dx = \int (-\sec(fx+e))^n(\sec(fx+e)+1)^m\,dx$$

input
```
integrate((-sec(f*x+e))^n*(1+sec(f*x+e))^m,x, algorithm="maxima")
```

output
```
integrate((-sec(f*x + e))^n*(sec(f*x + e) + 1)^m, x)
```

3.333.8 Giac [F]

$$\int (-\sec(e+fx))^n(1+\sec(e+fx))^m\,dx = \int (-\sec(fx+e))^n(\sec(fx+e)+1)^m\,dx$$

input
```
integrate((-sec(f*x+e))^n*(1+sec(f*x+e))^m,x, algorithm="giac")
```

output
```
integrate((-sec(f*x + e))^n*(sec(f*x + e) + 1)^m, x)
```

3.333.9 Mupad [F(-1)]

Timed out.

$$\int (-\sec(e+fx))^n(1+\sec(e+fx))^m\,dx = \int \left(\frac{1}{\cos(e+f\,x)}+1\right)^m\left(-\frac{1}{\cos(e+f\,x)}\right)^n dx$$

input
```
int((1/cos(e + f*x) + 1)^m*(-1/cos(e + f*x))^n,x)
```

output
```
int((1/cos(e + f*x) + 1)^m*(-1/cos(e + f*x))^n, x)
```

3.334 $\int (1-\sec(e+fx))^m(-\sec(e+fx))^n\,dx$

3.334.1 Optimal result 2501
3.334.2 Mathematica [B] (warning: unable to verify) 2501
3.334.3 Rubi [A] (verified) 2502
3.334.4 Maple [F] 2503
3.334.5 Fricas [F] 2503
3.334.6 Sympy [F] 2504
3.334.7 Maxima [F] 2504
3.334.8 Giac [F] 2504
3.334.9 Mupad [F(-1)] 2505

3.334.1 Optimal result

Integrand size = 23, antiderivative size = 70

$$\int (1-\sec(e+fx))^m(-\sec(e+fx))^n\,dx = \frac{2^{\frac{1}{2}+m}\operatorname{AppellF1}\left(\frac{1}{2},1-n,\frac{1}{2}-m,\frac{3}{2},1+\sec(e+fx),\frac{1}{2}(1+\sec(e+fx))\right)\tan(e+fx)}{f\sqrt{1-\sec(e+fx)}}$$

output
```
2^(1/2+m)*AppellF1(1/2,1-n,1/2-m,3/2,1+sec(f*x+e),1/2+1/2*sec(f*x+e))*tan(
f*x+e)/f/(1-sec(f*x+e))^(1/2)
```

3.334.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 257 vs. 2(70) = 140.

Time = 0.34 (sec) , antiderivative size = 257, normalized size of antiderivative = 3.67

$$\int (1-\sec(e+fx))^m(-\sec(e+fx))^n\,dx = \frac{(3+2m)\operatorname{AppellF1}}{f(1+2m)\left((3+2m)\operatorname{AppellF1}\left(\frac{1}{2}+m,m+n,1-n,\frac{3}{2}+m,\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)+2\right.}$$

input
```
Integrate[(1 - Sec[e + f*x])^m*(-Sec[e + f*x])^n,x]
```

output
```
((3 + 2*m)*AppellF1[1/2 + m, m + n, 1 - n, 3/2 + m, Tan[(e + f*x)/2]^2, -T
an[(e + f*x)/2]^2]*(1 - Sec[e + f*x])^m*(-Sec[e + f*x])^n*Sin[e + f*x])/(f
*(1 + 2*m)*((3 + 2*m)*AppellF1[1/2 + m, m + n, 1 - n, 3/2 + m, Tan[(e + f*
x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*((-1 + n)*AppellF1[3/2 + m, m + n, 2 - n
, 5/2 + m, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (m + n)*AppellF1[3/2
 + m, 1 + m + n, 1 - n, 5/2 + m, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])
*Tan[(e + f*x)/2]^2))
```

3.334.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3042, 4312, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (1-\sec(e+fx))^m(-\sec(e+fx))^n\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \left(1-\csc\left(e+fx+\frac{\pi}{2}\right)\right)^m\left(-\csc\left(e+fx+\frac{\pi}{2}\right)\right)^n dx$$

$$\Big\downarrow \text{ 4312}$$

$$\frac{\tan(e+fx)\int \frac{(1-\sec(e+fx))^{m-\frac{1}{2}}(-\sec(e+fx))^{n-1}}{\sqrt{\sec(e+fx)+1}}d(\sec(e+fx)+1)}{f\sqrt{1-\sec(e+fx)}\sqrt{\sec(e+fx)+1}}$$

$$\Big\downarrow \text{ 150}$$

$$\frac{2^{m+\frac{1}{2}}\tan(e+fx)\,\text{AppellF1}\left(\frac{1}{2},1-n,\frac{1}{2}-m,\frac{3}{2},\sec(e+fx)+1,\frac{1}{2}(\sec(e+fx)+1)\right)}{f\sqrt{1-\sec(e+fx)}}$$

input
```
Int[(1 - Sec[e + f*x])^m*(-Sec[e + f*x])^n,x]
```

output
```
(2^(1/2 + m)*AppellF1[1/2, 1 - n, 1/2 - m, 3/2, 1 + Sec[e + f*x], (1 + Sec
[e + f*x])/2]*Tan[e + f*x])/(f*Sqrt[1 - Sec[e + f*x]])
```

3.334.3.1 Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4312
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-(a*(d/b))^n)*(Cot[e + f*x]/(a^(n - 2)*f*Sqrt
[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(a - x)^(n - 1)
*((2*a - x)^(m - 1/2)/Sqrt[x]), x], x, a - b*Csc[e + f*x]], x] /; FreeQ[{a,
 b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0] &
&  !IntegerQ[n] && GtQ[a*(d/b), 0]
```

3.334.4 Maple [F]

$$\int (1-\sec(fx+e))^m (-\sec(fx+e))^n \, dx$$

input
```
int((1-sec(f*x+e))^m*(-sec(f*x+e))^n,x)
```

output
```
int((1-sec(f*x+e))^m*(-sec(f*x+e))^n,x)
```

3.334.5 Fricas [F]

$$\int (1-\sec(e+fx))^m(-\sec(e+fx))^n \, dx = \int (-\sec(fx+e))^n (-\sec(fx+e)+1)^m \, dx$$

input
```
integrate((1-sec(f*x+e))^m*(-sec(f*x+e))^n,x, algorithm="fricas")
```

output
```
integral((-sec(f*x + e))^n*(-sec(f*x + e) + 1)^m, x)
```

3.334.6 Sympy [F]

$$\int (1-\sec(e+fx))^m(-\sec(e+fx))^n\,dx=\int (-\sec(e+fx))^n(1-\sec(e+fx))^m\,dx$$

input
```
integrate((1-sec(f*x+e))**m*(-sec(f*x+e))**n,x)
```

output
```
Integral((-sec(e + f*x))**n*(1 - sec(e + f*x))**m, x)
```

3.334.7 Maxima [F]

$$\int (1-\sec(e+fx))^m(-\sec(e+fx))^n\,dx=\int (-\sec(fx+e))^n(-\sec(fx+e)+1)^m\,dx$$

input
```
integrate((1-sec(f*x+e))^m*(-sec(f*x+e))^n,x, algorithm="maxima")
```

output
```
integrate((-sec(f*x + e))^n*(-sec(f*x + e) + 1)^m, x)
```

3.334.8 Giac [F]

$$\int (1-\sec(e+fx))^m(-\sec(e+fx))^n\,dx=\int (-\sec(fx+e))^n(-\sec(fx+e)+1)^m\,dx$$

input
```
integrate((1-sec(f*x+e))^m*(-sec(f*x+e))^n,x, algorithm="giac")
```

output
```
integrate((-sec(f*x + e))^n*(-sec(f*x + e) + 1)^m, x)
```

3.334.9 Mupad [F(-1)]

Timed out.

$$\int (1-\sec(e+fx))^m(-\sec(e+fx))^n\,dx=\int \left(1-\frac{1}{\cos\left(e+f\,x\right)}\right)^m\,\left(-\frac{1}{\cos\left(e+f\,x\right)}\right)^n\,dx$$

input
```
int((1 - 1/cos(e + f*x))^m*(-1/cos(e + f*x))^n,x)
```

output
```
int((1 - 1/cos(e + f*x))^m*(-1/cos(e + f*x))^n, x)
```

3.335 $\int (-\sec(e+fx))^n (a+a\sec(e+fx))^m \, dx$

3.335.1 Optimal result 2506
3.335.2 Mathematica [B] (warning: unable to verify) 2506
3.335.3 Rubi [A] (verified) 2507
3.335.4 Maple [F] 2509
3.335.5 Fricas [F] 2509
3.335.6 Sympy [F] 2509
3.335.7 Maxima [F] 2510
3.335.8 Giac [F] 2510
3.335.9 Mupad [F(-1)] 2510

3.335.1 Optimal result

Integrand size = 23, antiderivative size = 87

$$\int (-\sec(e+fx))^n (a+a\sec(e+fx))^m \, dx = \frac{\sqrt{2}\,\text{AppellF1}\left(\frac{1}{2}+m,1-n,\frac{1}{2},\frac{3}{2}+m,1+\sec(e+fx),\frac{1}{2}(1+\sec(e+fx))\right)(a+a\sec(e+fx))^m\tan(e+}{f(1+2m)\sqrt{1-\sec(e+fx)}}$$

output
```
AppellF1(1/2+m,1-n,1/2,3/2+m,1+sec(f*x+e),1/2+1/2*sec(f*x+e))*(a+a*sec(f*x
+e))^m*2^(1/2)*tan(f*x+e)/f/(1+2*m)/(1-sec(f*x+e))^(1/2)
```

3.335.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 2250 vs. 2(87) = 174.

Time = 6.31 (sec) , antiderivative size = 2250, normalized size of antiderivative = 25.86

$$\int (-\sec(e+fx))^n (a+a\sec(e+fx))^m \, dx = \text{Result too large to show}$$

input
```
Integrate[(-Sec[e + f*x])^n*(a + a*Sec[e + f*x])^m,x]
```

output
```
(3*2^(1 + m)*AppellF1[1/2, m + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e
+ f*x)/2]^2]*(Sec[(e + f*x)/2]^2)^(-1 + n)*(-Sec[e + f*x])^n*(Cos[(e + f*x
)/2]^2*Sec[e + f*x])^(m + n)*(a*(1 + Sec[e + f*x]))^m*Tan[(e + f*x)/2])/(f
*(3*AppellF1[1/2, m + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]
^2] + 2*((-1 + n)*AppellF1[3/2, m + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Ta
n[(e + f*x)/2]^2] + (m + n)*AppellF1[3/2, 1 + m + n, 1 - n, 5/2, Tan[(e +
f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)*((3*2^m*AppellF1[1/2,
 m + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(Sec[(e + f*x
)/2]^2)^n*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(m + n))/(3*AppellF1[1/2, m +
n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*((-1 + n)*Appe
llF1[3/2, m + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (m
 + n)*AppellF1[3/2, 1 + m + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f
*x)/2]^2])*Tan[(e + f*x)/2]^2) + (3*2^(1 + m)*(-1 + n)*AppellF1[1/2, m + n
, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(Sec[(e + f*x)/2]^2
)^(-1 + n)*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(m + n)*Tan[(e + f*x)/2]^2)/(
3*AppellF1[1/2, m + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2
] + 2*((-1 + n)*AppellF1[3/2, m + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[
(e + f*x)/2]^2] + (m + n)*AppellF1[3/2, 1 + m + n, 1 - n, 5/2, Tan[(e + f*
x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2) + (3*2^(1 + m)*(Sec[(e
+ f*x)/2]^2)^(-1 + n)*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(m + n)*Tan[(e ...
```

3.335.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 4315, 3042, 4313, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (-\sec(e+fx))^n (a\sec(e+fx)+a)^m \, dx$$

$$\Big\downarrow \; 3042$$

$$\int \left(-\csc\left(e+fx+\frac{\pi}{2}\right)\right)^n \left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^m dx$$

$$\Big\downarrow \; 4315$$

$$(\sec(e+fx)+1)^{-m}(a\sec(e+fx)+a)^m \int (-\sec(e+fx))^n (\sec(e+fx)+1)^m dx$$

$$\Big\downarrow \; 3042$$

$$(\sec(e+fx)+1)^{-m}(a\sec(e+fx)+a)^m\int\left(-\csc\left(e+fx+\frac{\pi}{2}\right)\right)^n\left(\csc\left(e+fx+\frac{\pi}{2}\right)+1\right)^m dx$$

$$\Big\downarrow \text{4313}$$

$$\frac{\tan(e+fx)(\sec(e+fx)+1)^{-m-\frac{1}{2}}(a\sec(e+fx)+a)^m\int\frac{(-\sec(e+fx))^{n-1}(\sec(e+fx)+1)^{m-\frac{1}{2}}}{\sqrt{1-\sec(e+fx)}}d(\sec(e+fx)+1)}{f\sqrt{1-\sec(e+fx)}}$$

$$\Big\downarrow \text{150}$$

$$\frac{\sqrt{2}\tan(e+fx)(a\sec(e+fx)+a)^m\operatorname{AppellF1}\left(m+\frac{1}{2},1-n,\frac{1}{2},m+\frac{3}{2},\sec(e+fx)+1,\frac{1}{2}(\sec(e+fx)+1)\right)}{f(2m+1)\sqrt{1-\sec(e+fx)}}$$

input
```
Int[(-Sec[e + f*x])^n*(a + a*Sec[e + f*x])^m,x]
```

output
```
(Sqrt[2]*AppellF1[1/2 + m, 1 - n, 1/2, 3/2 + m, 1 + Sec[e + f*x], (1 + Sec
[e + f*x])/2]*(a + a*Sec[e + f*x])^m*Tan[e + f*x])/(f*(1 + 2*m)*Sqrt[1 - S
ec[e + f*x]])
```

3.335.3.1 Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] && !In
tegerQ[m] && !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4313
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-((-a)*(d/b))^n)*(Cot[e + f*x]/(a^(n - 1)*f*S
qrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[x^(m - 1/2)*
((a - x)^(n - 1)/Sqrt[2*a - x]), x], x, a + b*Csc[e + f*x]], x] /; FreeQ[{a
, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && !IntegerQ[m] && GtQ[a, 0]
&& !IntegerQ[n] && LtQ[a*(d/b), 0]
```

rule 4315
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Simp[a^IntPart[m]*((a + b*Csc[e + f*x])^FracPart[m
]/(1 + (b/a)*Csc[e + f*x])^FracPart[m])   Int[(1 + (b/a)*Csc[e + f*x])^m*(d
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^
2, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

3.335.4 Maple [F]

$$\int (-\sec(fx+e))^n (a+a\sec(fx+e))^m \, dx$$

input
```
int((-sec(f*x+e))^n*(a+a*sec(f*x+e))^m,x)
```

output
```
int((-sec(f*x+e))^n*(a+a*sec(f*x+e))^m,x)
```

3.335.5 Fricas [F]

$$\int (-\sec(e+fx))^n (a+a\sec(e+fx))^m \, dx = \int (a\sec(fx+e)+a)^m (-\sec(fx+e))^n \, dx$$

input
```
integrate((-sec(f*x+e))^n*(a+a*sec(f*x+e))^m,x, algorithm="fricas")
```

output
```
integral((a*sec(f*x + e) + a)^m*(-sec(f*x + e))^n, x)
```

3.335.6 Sympy [F]

$$\int (-\sec(e+fx))^n (a+a\sec(e+fx))^m \, dx = \int (-\sec(e+fx))^n (a(\sec(e+fx)+1))^m \, dx$$

input
```
integrate((-sec(f*x+e))**n*(a+a*sec(f*x+e))**m,x)
```

output
```
Integral((-sec(e + f*x))**n*(a*(sec(e + f*x) + 1))**m, x)
```

3.335.7 Maxima [F]

$$\int (-\sec(e+fx))^n(a+a\sec(e+fx))^m\,dx=\int (a\sec(fx+e)+a)^m(-\sec(fx+e))^n\,dx$$

input
```
integrate((-sec(f*x+e))^n*(a+a*sec(f*x+e))^m,x, algorithm="maxima")
```

output
```
integrate((a*sec(f*x + e) + a)^m*(-sec(f*x + e))^n, x)
```

3.335.8 Giac [F]

$$\int (-\sec(e+fx))^n(a+a\sec(e+fx))^m\,dx=\int (a\sec(fx+e)+a)^m(-\sec(fx+e))^n\,dx$$

input
```
integrate((-sec(f*x+e))^n*(a+a*sec(f*x+e))^m,x, algorithm="giac")
```

output
```
integrate((a*sec(f*x + e) + a)^m*(-sec(f*x + e))^n, x)
```

3.335.9 Mupad [F(-1)]

Timed out.

$$\int (-\sec(e+fx))^n(a+a\sec(e+fx))^m\,dx=\int \left(a+\frac{a}{\cos(e+f\,x)}\right)^m\left(-\frac{1}{\cos(e+f\,x)}\right)^n\,dx$$

input
```
int((a + a/cos(e + f*x))^m*(-1/cos(e + f*x))^n,x)
```

output
```
int((a + a/cos(e + f*x))^m*(-1/cos(e + f*x))^n, x)
```

3.336 $\int (-\sec(e+fx))^n (a-a\sec(e+fx))^m \, dx$

3.336.1 Optimal result 2511
3.336.2 Mathematica [F] 2511
3.336.3 Rubi [A] (verified) 2512
3.336.4 Maple [F] 2513
3.336.5 Fricas [F] 2514
3.336.6 Sympy [F] 2514
3.336.7 Maxima [F] 2514
3.336.8 Giac [F] 2515
3.336.9 Mupad [F(-1)] 2515

3.336.1 Optimal result

Integrand size = 24, antiderivative size = 87

$$\int (-\sec(e+fx))^n (a-a\sec(e+fx))^m \, dx = \frac{2^{\frac{1}{2}+m} \operatorname{AppellF1}\left(\frac{1}{2},1-n,\frac{1}{2}-m,\frac{3}{2},1+\sec(e+fx),\frac{1}{2}(1+\sec(e+fx))\right)(1-\sec(e+fx))^{-\frac{1}{2}-m}(a-a\sec}{f}$$

output
```
2^(1/2+m)*AppellF1(1/2,1-n,1/2-m,3/2,1+sec(f*x+e),1/2+1/2*sec(f*x+e))*(1-s
ec(f*x+e))^(-1/2-m)*(a-a*sec(f*x+e))^m*tan(f*x+e)/f
```

3.336.2 Mathematica [F]

$$\int (-\sec(e+fx))^n (a-a\sec(e+fx))^m \, dx = \int (-\sec(e+fx))^n (a-a\sec(e+fx))^m \, dx$$

input
```
Integrate[(-Sec[e + f*x])^n*(a - a*Sec[e + f*x])^m,x]
```

output
```
Integrate[(-Sec[e + f*x])^n*(a - a*Sec[e + f*x])^m, x]
```

3.336.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {3042, 4315, 3042, 4312, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (-\sec(e+fx))^n(a-a\sec(e+fx))^m\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(-\csc\left(e+fx+\frac{\pi}{2}\right)\right)^n\left(a-a\csc\left(e+fx+\frac{\pi}{2}\right)\right)^m dx$$

$$\Big\downarrow \text{4315}$$

$$(1-\sec(e+fx))^{-m}(a-a\sec(e+fx))^m\int (1-\sec(e+fx))^m(-\sec(e+fx))^n dx$$

$$\Big\downarrow \text{3042}$$

$$(1-\sec(e+fx))^{-m}(a-a\sec(e+fx))^m\int \left(1-\csc\left(e+fx+\frac{\pi}{2}\right)\right)^m\left(-\csc\left(e+fx+\frac{\pi}{2}\right)\right)^n dx$$

$$\Big\downarrow \text{4312}$$

$$\frac{\tan(e+fx)(1-\sec(e+fx))^{-m-\frac{1}{2}}(a-a\sec(e+fx))^m\int \frac{(1-\sec(e+fx))^{m-\frac{1}{2}}(-\sec(e+fx))^{n-1}}{\sqrt{\sec(e+fx)+1}}d(\sec(e+fx)+1)}{f\sqrt{\sec(e+fx)+1}}$$

$$\Big\downarrow \text{150}$$

$$\frac{2^{m+\frac{1}{2}}\tan(e+fx)(1-\sec(e+fx))^{-m-\frac{1}{2}}(a-a\sec(e+fx))^m\,\text{AppellF1}\left(\frac{1}{2},1-n,\frac{1}{2}-m,\frac{3}{2},\sec(e+fx)+1,\frac{1}{2}(\text{se}\right.}{f}$$

input
```
Int[(-Sec[e + f*x])^n*(a - a*Sec[e + f*x])^m,x]
```

output
```
(2^(1/2 + m)*AppellF1[1/2, 1 - n, 1/2 - m, 3/2, 1 + Sec[e + f*x], (1 + Sec
[e + f*x])/2]*(1 - Sec[e + f*x])^(-1/2 - m)*(a - a*Sec[e + f*x])^m*Tan[e +
 f*x])/f
```

3.336.3.1 Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4312
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-(a*(d/b))^n)*(Cot[e + f*x]/(a^(n - 2)*f*Sqrt
[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(a - x)^(n - 1)
*((2*a - x)^(m - 1/2)/Sqrt[x]), x], x, a - b*Csc[e + f*x]], x] /; FreeQ[{a,
 b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0] &
&  !IntegerQ[n] && GtQ[a*(d/b), 0]
```

rule 4315
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Simp[a^IntPart[m]*((a + b*Csc[e + f*x])^FracPart[m
]/(1 + (b/a)*Csc[e + f*x])^FracPart[m])   Int[(1 + (b/a)*Csc[e + f*x])^m*(d
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^
2, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

3.336.4 Maple [F]

$$\int (-\sec(fx+e))^n (a - a\sec(fx+e))^m \, dx$$

input
```
int((-sec(f*x+e))^n*(a-a*sec(f*x+e))^m,x)
```

output
```
int((-sec(f*x+e))^n*(a-a*sec(f*x+e))^m,x)
```

3.336.5 Fricas [F]

$$\int (-\sec(e+fx))^n(a-a\sec(e+fx))^m\,dx=\int (-a\sec(fx+e)+a)^m(-\sec(fx+e))^n\,dx$$

input
```
integrate((-sec(f*x+e))^n*(a-a*sec(f*x+e))^m,x, algorithm="fricas")
```

output
```
integral((-a*sec(f*x + e) + a)^m*(-sec(f*x + e))^n, x)
```

3.336.6 Sympy [F]

$$\int (-\sec(e+fx))^n(a-a\sec(e+fx))^m\,dx=\int (-\sec(e+fx))^n(-a(\sec(e+fx)-1))^m\,dx$$

input
```
integrate((-sec(f*x+e))**n*(a-a*sec(f*x+e))**m,x)
```

output
```
Integral((-sec(e + f*x))**n*(-a*(sec(e + f*x) - 1))**m, x)
```

3.336.7 Maxima [F]

$$\int (-\sec(e+fx))^n(a-a\sec(e+fx))^m\,dx=\int (-a\sec(fx+e)+a)^m(-\sec(fx+e))^n\,dx$$

input
```
integrate((-sec(f*x+e))^n*(a-a*sec(f*x+e))^m,x, algorithm="maxima")
```

output
```
integrate((-a*sec(f*x + e) + a)^m*(-sec(f*x + e))^n, x)
```

3.336.8 Giac [F]

$$\int (-\sec(e+fx))^n(a-a\sec(e+fx))^m\,dx=\int (-a\sec(fx+e)+a)^m(-\sec(fx+e))^n\,dx$$

input
```
integrate((-sec(f*x+e))^n*(a-a*sec(f*x+e))^m,x, algorithm="giac")
```

output
```
integrate((-a*sec(f*x + e) + a)^m*(-sec(f*x + e))^n, x)
```

3.336.9 Mupad [F(-1)]

Timed out.

$$\int (-\sec(e+fx))^n(a-a\sec(e+fx))^m\,dx=\int \left(a-\frac{a}{\cos(e+f\,x)}\right)^m\left(-\frac{1}{\cos(e+f\,x)}\right)^n\,dx$$

input
```
int((a - a/cos(e + f*x))^m*(-1/cos(e + f*x))^n,x)
```

output
```
int((a - a/cos(e + f*x))^m*(-1/cos(e + f*x))^n, x)
```

3.337 $\int (d\sec(e+fx))^n(1+\sec(e+fx))^m\,dx$

3.337.1 Optimal result 2516
3.337.2 Mathematica [B] (warning: unable to verify) 2516
3.337.3 Rubi [A] (verified) 2517
3.337.4 Maple [F] 2518
3.337.5 Fricas [F] 2519
3.337.6 Sympy [F] 2519
3.337.7 Maxima [F] 2519
3.337.8 Giac [F] 2520
3.337.9 Mupad [F(-1)] 2520

3.337.1 Optimal result

Integrand size = 21, antiderivative size = 79

$$\int (d\sec(e+fx))^n(1+\sec(e+fx))^m\,dx = -\frac{\text{AppellF1}\left(n,\frac{1}{2},\frac{1}{2}-m,1+n,\sec(e+fx),-\sec(e+fx)\right)(d\sec(e+fx))^n\tan(e+fx)}{fn\sqrt{1-\sec(e+fx)}\sqrt{1+\sec(e+fx)}}$$

output
```
-AppellF1(n,1/2-m,1/2,1+n,-sec(f*x+e),sec(f*x+e))*(d*sec(f*x+e))^n*tan(f*x
+e)/f/n/(1-sec(f*x+e))^(1/2)/(1+sec(f*x+e))^(1/2)
```

3.337.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 2248 vs. 2(79) = 158.

Time = 6.24 (sec) , antiderivative size = 2248, normalized size of antiderivative = 28.46

$$\int (d\sec(e+fx))^n(1+\sec(e+fx))^m\,dx = \text{Result too large to show}$$

input
```
Integrate[(d*Sec[e + f*x])^n*(1 + Sec[e + f*x])^m,x]
```

output
```
(3*2^(1 + m)*AppellF1[1/2, m + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e
+ f*x)/2]^2]*(Sec[(e + f*x)/2]^2)^(-1 + n)*(d*Sec[e + f*x])^n*(Cos[(e + f*
x)/2]^2*Sec[e + f*x])^(m + n)*(1 + Sec[e + f*x])^m*Tan[(e + f*x)/2])/(f*(3
*AppellF1[1/2, m + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]
 + 2*((-1 + n)*AppellF1[3/2, m + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(
e + f*x)/2]^2] + (m + n)*AppellF1[3/2, 1 + m + n, 1 - n, 5/2, Tan[(e + f*x
)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)*((3*2^m*AppellF1[1/2, m
+ n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(Sec[(e + f*x)/2
]^2)^n*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(m + n))/(3*AppellF1[1/2, m + n,
1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*((-1 + n)*AppellF
1[3/2, m + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (m +
n)*AppellF1[3/2, 1 + m + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)
/2]^2])*Tan[(e + f*x)/2]^2) + (3*2^(1 + m)*(-1 + n)*AppellF1[1/2, m + n, 1
 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(Sec[(e + f*x)/2]^2)^(
-1 + n)*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(m + n)*Tan[(e + f*x)/2]^2)/(3*A
ppellF1[1/2, m + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] +
 2*((-1 + n)*AppellF1[3/2, m + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e
+ f*x)/2]^2] + (m + n)*AppellF1[3/2, 1 + m + n, 1 - n, 5/2, Tan[(e + f*x)/
2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2) + (3*2^(1 + m)*(Sec[(e + f
*x)/2]^2)^(-1 + n)*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(m + n)*Tan[(e + f...
```

3.337.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3042, 4314, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (\sec(e+fx)+1)^m (d\sec(e+fx))^n \, dx$$

$$\Big\downarrow \; 3042$$

$$\int \left(\csc\left(e+fx+\frac{\pi}{2}\right)+1\right)^m \left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^n dx$$

$$\Big\downarrow \; 4314$$

$$-\frac{d\tan(e+fx)\int \frac{(d\sec(e+fx))^{n-1}(\sec(e+fx)+1)^{m-\frac{1}{2}}}{\sqrt{1-\sec(e+fx)}}d\sec(e+fx)}{f\sqrt{1-\sec(e+fx)}\sqrt{\sec(e+fx)+1}}$$

$$\Big\downarrow \; 150$$

$$-\frac{\tan(e+fx)(d\sec(e+fx))^n \operatorname{AppellF1}\left(n,\frac{1}{2},\frac{1}{2}-m,n+1,\sec(e+fx),-\sec(e+fx)\right)}{fn\sqrt{1-\sec(e+fx)}\sqrt{\sec(e+fx)+1}}$$

input
```
Int[(d*Sec[e + f*x])^n*(1 + Sec[e + f*x])^m,x]
```

output
```
-((AppellF1[n, 1/2, 1/2 - m, 1 + n, Sec[e + f*x], -Sec[e + f*x]]*(d*Sec[e
+ f*x])^n*Tan[e + f*x])/(f*n*Sqrt[1 - Sec[e + f*x]]*Sqrt[1 + Sec[e + f*x]]
))
```

3.337.3.1 Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4314
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Simp[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x
]]*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(d*x)^(n - 1)*((a + b*x)^(m - 1/2
)/Sqrt[a - b*x]), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n},
x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

3.337.4 Maple [F]

$$\int (d\sec(fx+e))^n (\sec(fx+e)+1)^m \, dx$$

input
```
int((d*sec(f*x+e))^n*(sec(f*x+e)+1)^m,x)
```

output
```
int((d*sec(f*x+e))^n*(sec(f*x+e)+1)^m,x)
```

3.337.5 Fricas [F]

$$\int (d\sec(e+fx))^n(1+\sec(e+fx))^m\,dx=\int (d\sec(fx+e))^n(\sec(fx+e)+1)^m\,dx$$

input
```
integrate((d*sec(f*x+e))^n*(1+sec(f*x+e))^m,x, algorithm="fricas")
```

output
```
integral((d*sec(f*x + e))^n*(sec(f*x + e) + 1)^m, x)
```

3.337.6 Sympy [F]

$$\int (d\sec(e+fx))^n(1+\sec(e+fx))^m\,dx=\int (d\sec(e+fx))^n(\sec(e+fx)+1)^m\,dx$$

input
```
integrate((d*sec(f*x+e))**n*(1+sec(f*x+e))**m,x)
```

output
```
Integral((d*sec(e + f*x))**n*(sec(e + f*x) + 1)**m, x)
```

3.337.7 Maxima [F]

$$\int (d\sec(e+fx))^n(1+\sec(e+fx))^m\,dx=\int (d\sec(fx+e))^n(\sec(fx+e)+1)^m\,dx$$

input
```
integrate((d*sec(f*x+e))^n*(1+sec(f*x+e))^m,x, algorithm="maxima")
```

output
```
integrate((d*sec(f*x + e))^n*(sec(f*x + e) + 1)^m, x)
```

3.337.8 Giac [F]

$$\int (d\sec(e+fx))^n(1+\sec(e+fx))^m\,dx=\int (d\sec(fx+e))^n(\sec(fx+e)+1)^m\,dx$$

input
```
integrate((d*sec(f*x+e))^n*(1+sec(f*x+e))^m,x, algorithm="giac")
```

output
```
integrate((d*sec(f*x + e))^n*(sec(f*x + e) + 1)^m, x)
```

3.337.9 Mupad [F(-1)]

Timed out.

$$\int (d\sec(e+fx))^n(1+\sec(e+fx))^m\,dx=\int \left(\frac{1}{\cos(e+f\,x)}+1\right)^m\left(\frac{d}{\cos(e+f\,x)}\right)^n\,dx$$

input
```
int((1/cos(e + f*x) + 1)^m*(d/cos(e + f*x))^n,x)
```

output
```
int((1/cos(e + f*x) + 1)^m*(d/cos(e + f*x))^n, x)
```

3.338 $\int (1-\sec(e+fx))^m (d\sec(e+fx))^n \, dx$

3.338.1 Optimal result 2521
3.338.2 Mathematica [B] (warning: unable to verify) 2521
3.338.3 Rubi [A] (verified) 2522
3.338.4 Maple [F] 2523
3.338.5 Fricas [F] 2523
3.338.6 Sympy [F] 2524
3.338.7 Maxima [F] 2524
3.338.8 Giac [F] 2524
3.338.9 Mupad [F(-1)] 2525

3.338.1 Optimal result

Integrand size = 23, antiderivative size = 79

$$\int (1-\sec(e+fx))^m (d\sec(e+fx))^n \, dx = -\frac{\text{AppellF1}\left(n,\frac{1}{2}-m,\frac{1}{2},1+n,\sec(e+fx),-\sec(e+fx)\right)(d\sec(e+fx))^n\tan(e+fx)}{fn\sqrt{1-\sec(e+fx)}\sqrt{1+\sec(e+fx)}}$$

output
```
-AppellF1(n,1/2,1/2-m,1+n,-sec(f*x+e),sec(f*x+e))*(d*sec(f*x+e))^n*tan(f*x
+e)/f/n/(1-sec(f*x+e))^(1/2)/(1+sec(f*x+e))^(1/2)
```

3.338.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 257 vs. 2(79) = 158.

Time = 0.33 (sec) , antiderivative size = 257, normalized size of antiderivative = 3.25

$$\int (1-\sec(e+fx))^m (d\sec(e+fx))^n \, dx = \frac{(3+2m)\,\text{AppellF1}}{f(1+2m)\left((3+2m)\,\text{AppellF1}\left(\frac{1}{2}+m,m+n,1-n,\frac{3}{2}+m,\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)+2\right.}$$

input
```
Integrate[(1 - Sec[e + f*x])^m*(d*Sec[e + f*x])^n,x]
```

output
```
((3 + 2*m)*AppellF1[1/2 + m, m + n, 1 - n, 3/2 + m, Tan[(e + f*x)/2]^2, -T
an[(e + f*x)/2]^2]*(1 - Sec[e + f*x])^m*(d*Sec[e + f*x])^n*Sin[e + f*x])/(
f*(1 + 2*m)*((3 + 2*m)*AppellF1[1/2 + m, m + n, 1 - n, 3/2 + m, Tan[(e + f
*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*((-1 + n)*AppellF1[3/2 + m, m + n, 2 -
n, 5/2 + m, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (m + n)*AppellF1[3/
2 + m, 1 + m + n, 1 - n, 5/2 + m, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]
)*Tan[(e + f*x)/2]^2))
```

3.338.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3042, 4314, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (1-\sec(e+fx))^m (d\sec(e+fx))^n \, dx$$

$$\Big\downarrow \; 3042$$

$$\int \left(1-\csc\left(e+fx+\frac{\pi}{2}\right)\right)^m \left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^n dx$$

$$\Big\downarrow \; 4314$$

$$-\frac{d\tan(e+fx)\int \frac{(1-\sec(e+fx))^{m-\frac{1}{2}}(d\sec(e+fx))^{n-1}}{\sqrt{\sec(e+fx)+1}}d\sec(e+fx)}{f\sqrt{1-\sec(e+fx)}\sqrt{\sec(e+fx)+1}}$$

$$\Big\downarrow \; 150$$

$$-\frac{\tan(e+fx)(d\sec(e+fx))^n \operatorname{AppellF1}\left(n,\frac{1}{2}-m,\frac{1}{2},n+1,\sec(e+fx),-\sec(e+fx)\right)}{fn\sqrt{1-\sec(e+fx)}\sqrt{\sec(e+fx)+1}}$$

input
```
Int[(1 - Sec[e + f*x])^m*(d*Sec[e + f*x])^n,x]
```

output
```
-((AppellF1[n, 1/2 - m, 1/2, 1 + n, Sec[e + f*x], -Sec[e + f*x]]*(d*Sec[e
+ f*x])^n*Tan[e + f*x])/(f*n*Sqrt[1 - Sec[e + f*x]]*Sqrt[1 + Sec[e + f*x]]
))
```

3.338.3.1 Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] && !In
tegerQ[m] && !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4314
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Simp[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x
]]*Sqrt[a - b*Csc[e + f*x]])) Subst[Int[(d*x)^(n - 1)*((a + b*x)^(m - 1/2
)/Sqrt[a - b*x]), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n},
x] && EqQ[a^2 - b^2, 0] && !IntegerQ[m] && GtQ[a, 0]
```

3.338.4 Maple [F]

$$\int (1-\sec(fx+e))^m (d\sec(fx+e))^n \, dx$$

input
```
int((1-sec(f*x+e))^m*(d*sec(f*x+e))^n,x)
```

output
```
int((1-sec(f*x+e))^m*(d*sec(f*x+e))^n,x)
```

3.338.5 Fricas [F]

$$\int (1-\sec(e+fx))^m (d\sec(e+fx))^n \, dx = \int (d\sec(fx+e))^n (-\sec(fx+e)+1)^m \, dx$$

input
```
integrate((1-sec(f*x+e))^m*(d*sec(f*x+e))^n,x, algorithm="fricas")
```

output
```
integral((d*sec(f*x + e))^n*(-sec(f*x + e) + 1)^m, x)
```

3.338.6 Sympy [F]

$$\int (1-\sec(e+fx))^m(d\sec(e+fx))^n\,dx=\int (d\sec(e+fx))^n(1-\sec(e+fx))^m\,dx$$

input
```
integrate((1-sec(f*x+e))**m*(d*sec(f*x+e))**n,x)
```

output
```
Integral((d*sec(e + f*x))**n*(1 - sec(e + f*x))**m, x)
```

3.338.7 Maxima [F]

$$\int (1-\sec(e+fx))^m(d\sec(e+fx))^n\,dx=\int (d\sec(fx+e))^n(-\sec(fx+e)+1)^m\,dx$$

input
```
integrate((1-sec(f*x+e))^m*(d*sec(f*x+e))^n,x, algorithm="maxima")
```

output
```
integrate((d*sec(f*x + e))^n*(-sec(f*x + e) + 1)^m, x)
```

3.338.8 Giac [F]

$$\int (1-\sec(e+fx))^m(d\sec(e+fx))^n\,dx=\int (d\sec(fx+e))^n(-\sec(fx+e)+1)^m\,dx$$

input
```
integrate((1-sec(f*x+e))^m*(d*sec(f*x+e))^n,x, algorithm="giac")
```

output
```
integrate((d*sec(f*x + e))^n*(-sec(f*x + e) + 1)^m, x)
```

3.338.9 Mupad [F(-1)]

Timed out.

$$\int (1-\sec(e+fx))^m (d\sec(e+fx))^n \, dx = \int \left(1-\frac{1}{\cos(e+f\,x)}\right)^m \left(\frac{d}{\cos(e+f\,x)}\right)^n dx$$

input
```
int((1 - 1/cos(e + f*x))^m*(d/cos(e + f*x))^n,x)
```

output
```
int((1 - 1/cos(e + f*x))^m*(d/cos(e + f*x))^n, x)
```

3.339 $\int (d\sec(e+fx))^n(a+a\sec(e+fx))^m\,dx$

3.339.1 Optimal result 2526
3.339.2 Mathematica [B] (warning: unable to verify) 2526
3.339.3 Rubi [A] (verified) 2527
3.339.4 Maple [F] 2529
3.339.5 Fricas [F] 2529
3.339.6 Sympy [F] 2529
3.339.7 Maxima [F] 2530
3.339.8 Giac [F] 2530
3.339.9 Mupad [F(-1)] 2530

3.339.1 Optimal result

Integrand size = 23, antiderivative size = 95

$$\int (d\sec(e+fx))^n(a+a\sec(e+fx))^m\,dx = -\frac{\text{AppellF1}\left(n,\frac{1}{2},\frac{1}{2}-m,1+n,\sec(e+fx),-\sec(e+fx)\right)(d\sec(e+fx))^n(1+\sec(e+fx))^{-\frac{1}{2}-m}(a+}{fn\sqrt{1-\sec(e+fx)}}$$

output
```
-AppellF1(n,1/2-m,1/2,1+n,-sec(f*x+e),sec(f*x+e))*(d*sec(f*x+e))^n*(1+sec(
f*x+e))^(-1/2-m)*(a+a*sec(f*x+e))^m*tan(f*x+e)/f/n/(1-sec(f*x+e))^(1/2)
```

3.339.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 2250 vs. 2(95) = 190.

Time = 6.32 (sec) , antiderivative size = 2250, normalized size of antiderivative = 23.68

$$\int (d\sec(e+fx))^n(a+a\sec(e+fx))^m\,dx = \text{Result too large to show}$$

input
```
Integrate[(d*Sec[e + f*x])^n*(a + a*Sec[e + f*x])^m,x]
```

output
```
(3*2^(1 + m)*AppellF1[1/2, m + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e
+ f*x)/2]^2]*(Sec[(e + f*x)/2]^2)^(-1 + n)*(d*Sec[e + f*x])^n*(Cos[(e + f*
x)/2]^2*Sec[e + f*x])^(m + n)*(a*(1 + Sec[e + f*x]))^m*Tan[(e + f*x)/2])/(
f*(3*AppellF1[1/2, m + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2
]^2] + 2*((-1 + n)*AppellF1[3/2, m + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -T
an[(e + f*x)/2]^2] + (m + n)*AppellF1[3/2, 1 + m + n, 1 - n, 5/2, Tan[(e +
 f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)*((3*2^m*AppellF1[1/2
, m + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(Sec[(e + f*
x)/2]^2)^n*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(m + n))/(3*AppellF1[1/2, m +
 n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*((-1 + n)*App
ellF1[3/2, m + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (
m + n)*AppellF1[3/2, 1 + m + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e +
f*x)/2]^2])*Tan[(e + f*x)/2]^2) + (3*2^(1 + m)*(-1 + n)*AppellF1[1/2, m +
n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(Sec[(e + f*x)/2]^
2)^(-1 + n)*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(m + n)*Tan[(e + f*x)/2]^2)/
(3*AppellF1[1/2, m + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^
2] + 2*((-1 + n)*AppellF1[3/2, m + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan
[(e + f*x)/2]^2] + (m + n)*AppellF1[3/2, 1 + m + n, 1 - n, 5/2, Tan[(e + f
*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2) + (3*2^(1 + m)*(Sec[(e
 + f*x)/2]^2)^(-1 + n)*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(m + n)*Tan[(e...
```

3.339.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 4315, 3042, 4314, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sec(e+fx)+a)^m(d\sec(e+fx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^m\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^n dx$$

$$\Big\downarrow \text{4315}$$

$$(\sec(e+fx)+1)^{-m}(a\sec(e+fx)+a)^m\int (d\sec(e+fx))^n(\sec(e+fx)+1)^m dx$$

$$\Big\downarrow \text{3042}$$

$$(\sec(e+fx)+1)^{-m}(a\sec(e+fx)+a)^m\int\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^n\left(\csc\left(e+fx+\frac{\pi}{2}\right)+1\right)^m dx$$

$$\Big\downarrow \text{4314}$$

$$-\frac{d\tan(e+fx)(\sec(e+fx)+1)^{-m-\frac{1}{2}}(a\sec(e+fx)+a)^m\int\frac{(d\sec(e+fx))^{n-1}(\sec(e+fx)+1)^{m-\frac{1}{2}}}{\sqrt{1-\sec(e+fx)}}d\sec(e+fx)}{f\sqrt{1-\sec(e+fx)}}$$

$$\Big\downarrow \text{150}$$

$$-\frac{\tan(e+fx)(\sec(e+fx)+1)^{-m-\frac{1}{2}}(a\sec(e+fx)+a)^m(d\sec(e+fx))^n\operatorname{AppellF1}\left(n,\frac{1}{2},\frac{1}{2}-m,n+1,\sec(e+f\right.}{fn\sqrt{1-\sec(e+fx)}}$$

input
```
Int[(d*Sec[e + f*x])^n*(a + a*Sec[e + f*x])^m,x]
```

output
```
-((AppellF1[n, 1/2, 1/2 - m, 1 + n, Sec[e + f*x], -Sec[e + f*x]]*(d*Sec[e
+ f*x])^n*(1 + Sec[e + f*x])^(-1/2 - m)*(a + a*Sec[e + f*x])^m*Tan[e + f*x
])/(f*n*Sqrt[1 - Sec[e + f*x]]))
```

3.339.3.1 Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] && !In
tegerQ[m] && !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4314
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Simp[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x
]]*Sqrt[a - b*Csc[e + f*x]])) Subst[Int[(d*x)^(n - 1)*((a + b*x)^(m - 1/2
)/Sqrt[a - b*x]), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n},
x] && EqQ[a^2 - b^2, 0] && !IntegerQ[m] && GtQ[a, 0]
```

rule 4315
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Simp[a^IntPart[m]*((a + b*Csc[e + f*x])^FracPart[m
]/(1 + (b/a)*Csc[e + f*x])^FracPart[m])   Int[(1 + (b/a)*Csc[e + f*x])^m*(d
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^
2, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

3.339.4 Maple [F]

$$\int (d\sec(fx+e))^n (a+a\sec(fx+e))^m \, dx$$

input
```
int((d*sec(f*x+e))^n*(a+a*sec(f*x+e))^m,x)
```

output
```
int((d*sec(f*x+e))^n*(a+a*sec(f*x+e))^m,x)
```

3.339.5 Fricas [F]

$$\int (d\sec(e+fx))^n(a+a\sec(e+fx))^m \, dx = \int (a\sec(fx+e)+a)^m(d\sec(fx+e))^n \, dx$$

input
```
integrate((d*sec(f*x+e))^n*(a+a*sec(f*x+e))^m,x, algorithm="fricas")
```

output
```
integral((a*sec(f*x + e) + a)^m*(d*sec(f*x + e))^n, x)
```

3.339.6 Sympy [F]

$$\int (d\sec(e+fx))^n(a+a\sec(e+fx))^m \, dx = \int (a(\sec(e+fx)+1))^m (d\sec(e+fx))^n \, dx$$

input
```
integrate((d*sec(f*x+e))**n*(a+a*sec(f*x+e))**m,x)
```

output
```
Integral((a*(sec(e + f*x) + 1))**m*(d*sec(e + f*x))**n, x)
```

3.339.7 Maxima [F]

$$\int (d\sec(e+fx))^n(a+a\sec(e+fx))^m\,dx=\int (a\sec(fx+e)+a)^m(d\sec(fx+e))^n\,dx$$

input
```
integrate((d*sec(f*x+e))^n*(a+a*sec(f*x+e))^m,x, algorithm="maxima")
```

output
```
integrate((a*sec(f*x + e) + a)^m*(d*sec(f*x + e))^n, x)
```

3.339.8 Giac [F]

$$\int (d\sec(e+fx))^n(a+a\sec(e+fx))^m\,dx=\int (a\sec(fx+e)+a)^m(d\sec(fx+e))^n\,dx$$

input
```
integrate((d*sec(f*x+e))^n*(a+a*sec(f*x+e))^m,x, algorithm="giac")
```

output
```
integrate((a*sec(f*x + e) + a)^m*(d*sec(f*x + e))^n, x)
```

3.339.9 Mupad [F(-1)]

Timed out.

$$\int (d\sec(e+fx))^n(a+a\sec(e+fx))^m\,dx=\int \left(a+\frac{a}{\cos(e+f\,x)}\right)^m\left(\frac{d}{\cos(e+f\,x)}\right)^n\,dx$$

input
```
int((a + a/cos(e + f*x))^m*(d/cos(e + f*x))^n,x)
```

output
```
int((a + a/cos(e + f*x))^m*(d/cos(e + f*x))^n, x)
```

3.340 $\int (d\sec(e+fx))^n(a-a\sec(e+fx))^m\,dx$

3.340.1 Optimal result 2531
3.340.2 Mathematica [F] 2531
3.340.3 Rubi [A] (verified) 2532
3.340.4 Maple [F] 2533
3.340.5 Fricas [F] 2534
3.340.6 Sympy [F] 2534
3.340.7 Maxima [F] 2534
3.340.8 Giac [F] 2535
3.340.9 Mupad [F(-1)] 2535

3.340.1 Optimal result

Integrand size = 24, antiderivative size = 96

$$\int (d\sec(e+fx))^n(a-a\sec(e+fx))^m\,dx = -\frac{\text{AppellF1}\left(n,\frac{1}{2}-m,\frac{1}{2},1+n,\sec(e+fx),-\sec(e+fx)\right)(1-\sec(e+fx))^{-\frac{1}{2}-m}(d\sec(e+fx))^n(a-}{fn\sqrt{1+\sec(e+fx)}}$$

output
```
-AppellF1(n,1/2,1/2-m,1+n,-sec(f*x+e),sec(f*x+e))*(1-sec(f*x+e))^(-1/2-m)*
(d*sec(f*x+e))^n*(a-a*sec(f*x+e))^m*tan(f*x+e)/f/n/(1+sec(f*x+e))^(1/2)
```

3.340.2 Mathematica [F]

$$\int (d\sec(e+fx))^n(a-a\sec(e+fx))^m\,dx = \int (d\sec(e+fx))^n(a-a\sec(e+fx))^m\,dx$$

input
```
Integrate[(d*Sec[e + f*x])^n*(a - a*Sec[e + f*x])^m,x]
```

output
```
Integrate[(d*Sec[e + f*x])^n*(a - a*Sec[e + f*x])^m, x]
```

3.340.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {3042, 4315, 3042, 4314, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a-a\sec(e+fx))^m(d\sec(e+fx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a-a\csc\left(e+fx+\frac{\pi}{2}\right)\right)^m\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^n dx$$

$$\Big\downarrow \text{4315}$$

$$(1-\sec(e+fx))^{-m}(a-a\sec(e+fx))^m\int (1-\sec(e+fx))^m(d\sec(e+fx))^n dx$$

$$\Big\downarrow \text{3042}$$

$$(1-\sec(e+fx))^{-m}(a-a\sec(e+fx))^m\int \left(1-\csc\left(e+fx+\frac{\pi}{2}\right)\right)^m\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^n dx$$

$$\Big\downarrow \text{4314}$$

$$-\frac{d\tan(e+fx)(1-\sec(e+fx))^{-m-\frac{1}{2}}(a-a\sec(e+fx))^m\int \frac{(1-\sec(e+fx))^{m-\frac{1}{2}}(d\sec(e+fx))^{n-1}}{\sqrt{\sec(e+fx)+1}}d\sec(e+fx)}{f\sqrt{\sec(e+fx)+1}}$$

$$\Big\downarrow \text{150}$$

$$-\frac{\tan(e+fx)(1-\sec(e+fx))^{-m-\frac{1}{2}}(a-a\sec(e+fx))^m(d\sec(e+fx))^n\,\text{AppellF1}\left(n,\frac{1}{2}-m,\frac{1}{2},n+1,\sec(e+f\right.}{fn\sqrt{\sec(e+fx)+1}}$$

input
```
Int[(d*Sec[e + f*x])^n*(a - a*Sec[e + f*x])^m,x]
```

output
```
-((AppellF1[n, 1/2 - m, 1/2, 1 + n, Sec[e + f*x], -Sec[e + f*x]]*(1 - Sec[
e + f*x])^(-1/2 - m)*(d*Sec[e + f*x])^n*(a - a*Sec[e + f*x])^m*Tan[e + f*x
])/(f*n*Sqrt[1 + Sec[e + f*x]]))
```

3.340.3.1 Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4314
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Simp[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x
]]*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(d*x)^(n - 1)*((a + b*x)^(m - 1/2
)/Sqrt[a - b*x]), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n},
x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

rule 4315
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Simp[a^IntPart[m]*((a + b*Csc[e + f*x])^FracPart[m
]/(1 + (b/a)*Csc[e + f*x])^FracPart[m])   Int[(1 + (b/a)*Csc[e + f*x])^m*(d
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^
2, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

3.340.4 Maple [F]

$$\int (d\sec(fx+e))^n (a-a\sec(fx+e))^m \, dx$$

input
```
int((d*sec(f*x+e))^n*(a-a*sec(f*x+e))^m,x)
```

output
```
int((d*sec(f*x+e))^n*(a-a*sec(f*x+e))^m,x)
```

3.340.5 Fricas [F]

$$\int (d\sec(e+fx))^n(a-a\sec(e+fx))^m\,dx=\int (-a\sec(fx+e)+a)^m(d\sec(fx+e))^n\,dx$$

input
```
integrate((d*sec(f*x+e))^n*(a-a*sec(f*x+e))^m,x, algorithm="fricas")
```

output
```
integral((-a*sec(f*x + e) + a)^m*(d*sec(f*x + e))^n, x)
```

3.340.6 Sympy [F]

$$\int (d\sec(e+fx))^n(a-a\sec(e+fx))^m\,dx=\int (d\sec(e+fx))^n(-a(\sec(e+fx)-1))^m\,dx$$

input
```
integrate((d*sec(f*x+e))**n*(a-a*sec(f*x+e))**m,x)
```

output
```
Integral((d*sec(e + f*x))**n*(-a*(sec(e + f*x) - 1))**m, x)
```

3.340.7 Maxima [F]

$$\int (d\sec(e+fx))^n(a-a\sec(e+fx))^m\,dx=\int (-a\sec(fx+e)+a)^m(d\sec(fx+e))^n\,dx$$

input
```
integrate((d*sec(f*x+e))^n*(a-a*sec(f*x+e))^m,x, algorithm="maxima")
```

output
```
integrate((-a*sec(f*x + e) + a)^m*(d*sec(f*x + e))^n, x)
```

3.340.8 Giac [F]

$$\int (d\sec(e+fx))^n(a-a\sec(e+fx))^m\,dx=\int (-a\sec(fx+e)+a)^m(d\sec(fx+e))^n\,dx$$

input
```
integrate((d*sec(f*x+e))^n*(a-a*sec(f*x+e))^m,x, algorithm="giac")
```

output
```
integrate((-a*sec(f*x + e) + a)^m*(d*sec(f*x + e))^n, x)
```

3.340.9 Mupad [F(-1)]

Timed out.

$$\int (d\sec(e+fx))^n(a-a\sec(e+fx))^m\,dx=\int \left(a-\frac{a}{\cos(e+f\,x)}\right)^m\left(\frac{d}{\cos(e+f\,x)}\right)^n\,dx$$

input
```
int((a - a/cos(e + f*x))^m*(d/cos(e + f*x))^n,x)
```

output
```
int((a - a/cos(e + f*x))^m*(d/cos(e + f*x))^n, x)
```

3.341 $\int \sec^4(e+fx)(a+a\sec(e+fx))^m\,dx$

3.341.1 Optimal result 2536
3.341.2 Mathematica [A] (verified) 2537
3.341.3 Rubi [A] (verified) 2537
3.341.4 Maple [F] 2541
3.341.5 Fricas [F] 2541
3.341.6 Sympy [F] 2541
3.341.7 Maxima [F] 2542
3.341.8 Giac [F] 2542
3.341.9 Mupad [F(-1)] 2542

3.341.1 Optimal result

Integrand size = 21, antiderivative size = 211

$$\int \sec^4(e+fx)(a+a\sec(e+fx))^m\,dx$$
$$= \frac{(4+m)(a+a\sec(e+fx))^m\tan(e+fx)}{f(1+m)(2+m)(3+m)}+\frac{\sec^2(e+fx)(a+a\sec(e+fx))^m\tan(e+fx)}{f(3+m)}$$
$$+\frac{2^{\frac{1}{2}+m}m(5+3m+m^2)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}-m,\frac{3}{2},\frac{1}{2}(1-\sec(e+fx))\right)(1+\sec(e+fx))^{-\frac{1}{2}-m}(a}{f(1+m)(2+m)(3+m)}$$
$$+\frac{m(a+a\sec(e+fx))^{1+m}\tan(e+fx)}{af\,(6+5m+m^2)}$$

output
```
(4+m)*(a+a*sec(f*x+e))^m*tan(f*x+e)/f/(m^3+6*m^2+11*m+6)+sec(f*x+e)^2*(a+a
*sec(f*x+e))^m*tan(f*x+e)/f/(3+m)+2^(1/2+m)*m*(m^2+3*m+5)*hypergeom([1/2,
1/2-m],[3/2],1/2-1/2*sec(f*x+e))*(1+sec(f*x+e))^(-1/2-m)*(a+a*sec(f*x+e))^
m*tan(f*x+e)/f/(m^3+6*m^2+11*m+6)+m*(a+a*sec(f*x+e))^(1+m)*tan(f*x+e)/a/f/
(m^2+5*m+6)
```

3.341.2 Mathematica [A] (verified)

Time = 1.49 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.73

$$\int \sec^4(e+fx)(a+a\sec(e+fx))^m\,dx$$
$$= \frac{(1+\sec(e+fx))^{-\frac{1}{2}-m}(a(1+\sec(e+fx)))^m\left(2^{\frac{3}{2}+m}m(5+3m+m^2)\,\text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{1}{2}-m,\frac{3}{2}\right.\right.}{}$$

input
```
Integrate[Sec[e + f*x]^4*(a + a*Sec[e + f*x])^m,x]
```

output
```
((1 + Sec[e + f*x])^(-1/2 - m)*(a*(1 + Sec[e + f*x]))^m*(2^(3/2 + m)*m*(5
+ 3*m + m^2)*Hypergeometric2F1[1/2, -1/2 - m, 3/2, (1 - Sec[e + f*x])/2] +
 (1 + Sec[e + f*x])^(1/2 + m)*(4 + m + m^2 + m*(1 + 2*m)*Sec[e + f*x] + (2
 + 5*m + 2*m^2)*Sec[e + f*x]^2))*Tan[e + f*x])/(f*(2 + m)*(3 + m)*(1 + 2*m
))
```

3.341.3 Rubi [A] (verified)

Time = 1.03 (sec) , antiderivative size = 209, normalized size of antiderivative = 0.99, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.524$, Rules used = {3042, 4311, 3042, 4498, 3042, 4489, 3042, 4315, 3042, 4314, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^4(e+fx)(a\sec(e+fx)+a)^m\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \csc\left(e+fx+\frac{\pi}{2}\right)^4\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^m dx$$
$$\Big\downarrow \; 4311$$
$$\frac{\int \sec^2(e+fx)(\sec(e+fx)a+a)^m(m\sec(e+fx)a+2a)dx}{a(m+3)}+\frac{\tan(e+fx)\sec^2(e+fx)(a\sec(e+fx)+a)^m}{f(m+3)}$$
$$\Big\downarrow \; 3042$$

$$\frac{\int \csc\left(e+fx+\frac{\pi}{2}\right)^2\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)^m\left(m\csc\left(e+fx+\frac{\pi}{2}\right)a+2a\right)dx}{a(m+3)}+\frac{\tan(e+fx)\sec^2(e+fx)(a\sec(e+fx)+a)^m}{f(m+3)}$$

$\Big\downarrow$ 4498

$$\frac{\frac{\int \sec(e+fx)(\sec(e+fx)a+a)^m\left(m(m+1)a^2+(m+4)\sec(e+fx)a^2\right)dx}{a(m+2)}+\frac{m\tan(e+fx)(a\sec(e+fx)+a)^{m+1}}{f(m+2)}}{a(m+3)}+\frac{\tan(e+fx)\sec^2(e+fx)(a\sec(e+fx)+a)^m}{f(m+3)}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\int \csc\left(e+fx+\frac{\pi}{2}\right)\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)^m\left(m(m+1)a^2+(m+4)\csc\left(e+fx+\frac{\pi}{2}\right)a^2\right)dx}{a(m+2)}+\frac{m\tan(e+fx)(a\sec(e+fx)+a)^{m+1}}{f(m+2)}}{a(m+3)}+\frac{\tan(e+fx)\sec^2(e+fx)(a\sec(e+fx)+a)^m}{f(m+3)}$$

$\Big\downarrow$ 4489

$$\frac{\frac{\frac{a^2m\left(m^2+3m+5\right)\int \sec(e+fx)(\sec(e+fx)a+a)^m dx}{m+1}+\frac{a^2(m+4)\tan(e+fx)(a\sec(e+fx)+a)^m}{f(m+1)}}{a(m+2)}+\frac{m\tan(e+fx)(a\sec(e+fx)+a)^{m+1}}{f(m+2)}}{a(m+3)}+\frac{\tan(e+fx)\sec^2(e+fx)(a\sec(e+fx)+a)^m}{f(m+3)}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\frac{a^2m\left(m^2+3m+5\right)\int \csc\left(e+fx+\frac{\pi}{2}\right)\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)^m dx}{m+1}+\frac{a^2(m+4)\tan(e+fx)(a\sec(e+fx)+a)^m}{f(m+1)}}{a(m+2)}+\frac{m\tan(e+fx)(a\sec(e+fx)+a)^{m+1}}{f(m+2)}}{a(m+3)}+\frac{\tan(e+fx)\sec^2(e+fx)(a\sec(e+fx)+a)^m}{f(m+3)}$$

$\Big\downarrow$ 4315

$$\frac{\frac{\frac{a^2m\left(m^2+3m+5\right)(\sec(e+fx)+1)^{-m}(a\sec(e+fx)+a)^m\int \sec(e+fx)(\sec(e+fx)+1)^m dx}{m+1}+\frac{a^2(m+4)\tan(e+fx)(a\sec(e+fx)+a)^m}{f(m+1)}}{a(m+2)}+\frac{m\tan(e+fx)(a\sec(e+fx}{f(m+2)}}{a(m+3)}+\frac{\tan(e+fx)\sec^2(e+fx)(a\sec(e+fx)+a)^m}{f(m+3)}$$

$\Big\downarrow$ 3042

3.341. $\int \sec^4(e+fx)(a+a\sec(e+fx))^m\,dx$

$$\frac{\frac{\frac{a^2 m\left(m^2+3m+5\right)(\sec(e+fx)+1)^{-m}(a\sec(e+fx)+a)^m\int \csc\left(e+fx+\frac{\pi}{2}\right)\left(\csc\left(e+fx+\frac{\pi}{2}\right)+1\right)^m dx}{m+1}+\frac{a^2(m+4)\tan(e+fx)(a\sec(e+fx)+a)^m}{f(m+1)}}{a(m+2)}+\frac{m\tan(e+fx)(a}{f(}}{a(m+3)}$$

$$\frac{\tan(e+fx)\sec^2(e+fx)(a\sec(e+fx)+a)^m}{f(m+3)}$$

$$\Big\downarrow \; 4314$$

$$\frac{\frac{\frac{a^2(m+4)\tan(e+fx)(a\sec(e+fx)+a)^m}{f(m+1)}-\frac{a^2 m\left(m^2+3m+5\right)\tan(e+fx)(\sec(e+fx)+1)^{-m-\frac{1}{2}}(a\sec(e+fx)+a)^m\int\frac{(\sec(e+fx)+1)^{m-\frac{1}{2}}}{\sqrt{1-\sec(e+fx)}}d\sec(e+fx)}{f(m+1)\sqrt{1-\sec(e+fx)}}}{a(m+2)}+\frac{m\tan(e-}{}}{a(m+3)}$$

$$\frac{\tan(e+fx)\sec^2(e+fx)(a\sec(e+fx)+a)^m}{f(m+3)}$$

$$\Big\downarrow \; 79$$

$$\frac{\frac{\frac{a^2 2^{m+\frac{1}{2}} m\left(m^2+3m+5\right)\tan(e+fx)(\sec(e+fx)+1)^{-m-\frac{1}{2}}(a\sec(e+fx)+a)^m\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}-m,\frac{3}{2},\frac{1}{2}(1-\sec(e+fx))\right)}{f(m+1)}+\frac{a^2(m+4)\tan(e+fx)(a\sec(e+fx}{f(m+1)}}{a(m+2)}}{a(m+3)}$$

$$\frac{\tan(e+fx)\sec^2(e+fx)(a\sec(e+fx)+a)^m}{f(m+3)}$$

input
```
Int[Sec[e + f*x]^4*(a + a*Sec[e + f*x])^m,x]
```

output
```
(Sec[e + f*x]^2*(a + a*Sec[e + f*x])^m*Tan[e + f*x])/(f*(3 + m)) + ((m*(a
+ a*Sec[e + f*x])^(1 + m)*Tan[e + f*x])/(f*(2 + m)) + ((a^2*(4 + m)*(a + a
*Sec[e + f*x])^m*Tan[e + f*x])/(f*(1 + m)) + (2^(1/2 + m)*a^2*m*(5 + 3*m +
 m^2)*Hypergeometric2F1[1/2, 1/2 - m, 3/2, (1 - Sec[e + f*x])/2]*(1 + Sec[
e + f*x])^(-1/2 - m)*(a + a*Sec[e + f*x])^m*Tan[e + f*x])/(f*(1 + m)))/(a*
(2 + m)))/(a*(3 + m))
```

3.341.3.1 Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4311
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-d^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d
*Csc[e + f*x])^(n - 2)/(f*(m + n - 1))), x] + Simp[d^2/(b*(m + n - 1))   In
t[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*m*Csc[e +
f*x]), x], x] /; FreeQ[{a, b, d, e, f, m}, x] && EqQ[a^2 - b^2, 0] && GtQ[n
, 2] && NeQ[m + n - 1, 0] && IntegerQ[n]
```

rule 4314
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Simp[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x
]]*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(d*x)^(n - 1)*((a + b*x)^(m - 1/2
)/Sqrt[a - b*x]), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n},
x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

rule 4315
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Simp[a^IntPart[m]*((a + b*Csc[e + f*x])^FracPart[m
]/(1 + (b/a)*Csc[e + f*x])^FracPart[m])   Int[(1 + (b/a)*Csc[e + f*x])^m*(d
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^
2, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

rule 4489
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*((
a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Simp[(a*B*m + A*b*(m + 1))/(b*(m +
 1))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B
, e, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b
*(m + 1), 0] &&  !LtQ[m, -2^(-1)]
```

rule 4498
```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*
((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m + 2))   Int
[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*B*(m + 1) + (A*b*(m + 2) - a*B)
*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, m}, x] && NeQ[A*b - a
*B, 0] &&  !LtQ[m, -1]
```

3.341. $\int \sec^4(e+fx)(a+a\sec(e+fx))^m\,dx$

3.341.4 Maple [F]

$$\int \sec (fx+e)^4 (a+a\sec (fx+e))^m \, dx$$

input
```
int(sec(f*x+e)^4*(a+a*sec(f*x+e))^m,x)
```

output
```
int(sec(f*x+e)^4*(a+a*sec(f*x+e))^m,x)
```

3.341.5 Fricas [F]

$$\int \sec^4(e+fx)(a+a\sec(e+fx))^m \, dx = \int (a\sec (fx+e)+a)^m \sec (fx+e)^4 \, dx$$

input
```
integrate(sec(f*x+e)^4*(a+a*sec(f*x+e))^m,x, algorithm="fricas")
```

output
```
integral((a*sec(f*x + e) + a)^m*sec(f*x + e)^4, x)
```

3.341.6 Sympy [F]

$$\int \sec^4(e+fx)(a+a\sec(e+fx))^m \, dx = \int (a(\sec (e+fx)+1))^m \sec^4 (e+fx) \, dx$$

input
```
integrate(sec(f*x+e)**4*(a+a*sec(f*x+e))**m,x)
```

output
```
Integral((a*(sec(e + f*x) + 1))**m*sec(e + f*x)**4, x)
```

3.341.7 Maxima [F]

$$\int \sec^4(e+fx)(a+a\sec(e+fx))^m\,dx = \int (a\sec(fx+e)+a)^m\sec(fx+e)^4\,dx$$

input
```
integrate(sec(f*x+e)^4*(a+a*sec(f*x+e))^m,x, algorithm="maxima")
```

output
```
integrate((a*sec(f*x + e) + a)^m*sec(f*x + e)^4, x)
```

3.341.8 Giac [F]

$$\int \sec^4(e+fx)(a+a\sec(e+fx))^m\,dx = \int (a\sec(fx+e)+a)^m\sec(fx+e)^4\,dx$$

input
```
integrate(sec(f*x+e)^4*(a+a*sec(f*x+e))^m,x, algorithm="giac")
```

output
```
integrate((a*sec(f*x + e) + a)^m*sec(f*x + e)^4, x)
```

3.341.9 Mupad [F(-1)]

Timed out.

$$\int \sec^4(e+fx)(a+a\sec(e+fx))^m\,dx = \int \frac{\left(a+\frac{a}{\cos(e+f\,x)}\right)^m}{\cos(e+f\,x)^4}\,dx$$

input
```
int((a + a/cos(e + f*x))^m/cos(e + f*x)^4,x)
```

output
```
int((a + a/cos(e + f*x))^m/cos(e + f*x)^4, x)
```

3.342 $\int \sec^3(e+fx)(a+a\sec(e+fx))^m\,dx$

3.342.1 Optimal result . . . 2543
3.342.2 Mathematica [A] (verified) . . . 2543
3.342.3 Rubi [A] (verified) . . . 2544
3.342.4 Maple [F] . . . 2547
3.342.5 Fricas [F] . . . 2547
3.342.6 Sympy [F] . . . 2547
3.342.7 Maxima [F] . . . 2548
3.342.8 Giac [F] . . . 2548
3.342.9 Mupad [F(-1)] . . . 2548

3.342.1 Optimal result

Integrand size = 21, antiderivative size = 155

$$\int \sec^3(e+fx)(a+a\sec(e+fx))^m\,dx = -\frac{(a+a\sec(e+fx))^m\tan(e+fx)}{f\,(2+3m+m^2)} + \frac{2^{\frac{1}{2}+m}(1+m+m^2)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}-m,\frac{3}{2},\frac{1}{2}(1-\sec(e+fx))\right)(1+\sec(e+fx))^{-\frac{1}{2}-m}(a+a}{f(1+m)(2+m)} + \frac{(a+a\sec(e+fx))^{1+m}\tan(e+fx)}{af(2+m)}$$

output
```
-(a+a*sec(f*x+e))^m*tan(f*x+e)/f/(m^2+3*m+2)+2^(1/2+m)*(m^2+m+1)*hypergeom
([1/2, 1/2-m],[3/2],1/2-1/2*sec(f*x+e))*(1+sec(f*x+e))^(-1/2-m)*(a+a*sec(f
*x+e))^m*tan(f*x+e)/f/(m^2+3*m+2)+(a+a*sec(f*x+e))^(1+m)*tan(f*x+e)/a/f/(2
+m)
```

3.342.2 Mathematica [A] (verified)

Time = 0.68 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.79

$$\int \sec^3(e+fx)(a+a\sec(e+fx))^m\,dx = \frac{(1+\sec(e+fx))^{-\frac{1}{2}-m}(a(1+\sec(e+fx)))^m\left(2^{\frac{3}{2}+m}(1+m+m^2)\,\text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{1}{2}-m,\frac{3}{2},\frac{1}{2}(\right.\right.}{f(2+m)(1+2m}$$

input
```
Integrate[Sec[e + f*x]^3*(a + a*Sec[e + f*x])^m,x]
```

output
```
((1 + Sec[e + f*x])^(-1/2 - m)*(a*(1 + Sec[e + f*x]))^m*(2^(3/2 + m)*(1 +
m + m^2)*Hypergeometric2F1[1/2, -1/2 - m, 3/2, (1 - Sec[e + f*x])/2] + (1
+ Sec[e + f*x])^(1/2 + m)*(-1 + m + (1 + 2*m)*Sec[e + f*x]))*Tan[e + f*x])
/(f*(2 + m)*(1 + 2*m))
```

3.342.3 Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.01, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 4287, 3042, 4489, 3042, 4315, 3042, 4314, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^3(e+fx)(a\sec(e+fx)+a)^m\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(e+fx+\frac{\pi}{2}\right)^3\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^m dx$$

$$\Big\downarrow \text{4287}$$

$$\frac{\int \sec(e+fx)(a(m+1)-a\sec(e+fx))(\sec(e+fx)a+a)^m dx}{a(m+2)}+\frac{\tan(e+fx)(a\sec(e+fx)+a)^{m+1}}{af(m+2)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \csc\left(e+fx+\frac{\pi}{2}\right)\left(a(m+1)-a\csc\left(e+fx+\frac{\pi}{2}\right)\right)\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)^m dx}{a(m+2)}+\frac{\tan(e+fx)(a\sec(e+fx)+a)^{m+1}}{af(m+2)}$$

$$\Big\downarrow \text{4489}$$

$$\frac{\frac{a(m^2+m+1)\int \sec(e+fx)(\sec(e+fx)a+a)^m dx}{m+1}-\frac{a\tan(e+fx)(a\sec(e+fx)+a)^m}{f(m+1)}}{a(m+2)}+\frac{\tan(e+fx)(a\sec(e+fx)+a)^{m+1}}{af(m+2)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{a(m^2+m+1)\int \csc\left(e+fx+\frac{\pi}{2}\right)\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)^m dx}{m+1}-\frac{a\tan(e+fx)(a\sec(e+fx)+a)^m}{f(m+1)}}{a(m+2)}+\frac{\tan(e+fx)(a\sec(e+fx)+a)^{m+1}}{af(m+2)}$$

$\Big\downarrow$ 4315

$$\frac{\frac{a(m^2+m+1)(\sec(e+fx)+1)^{-m}(a\sec(e+fx)+a)^m\int \sec(e+fx)(\sec(e+fx)+1)^m dx}{m+1}-\frac{a\tan(e+fx)(a\sec(e+fx)+a)^m}{f(m+1)}}{a(m+2)}+\frac{\tan(e+fx)(a\sec(e+fx)+a)^{m+1}}{af(m+2)}$$

$\Big\downarrow$ 3042

$$\frac{\frac{a(m^2+m+1)(\sec(e+fx)+1)^{-m}(a\sec(e+fx)+a)^m\int \csc\left(e+fx+\frac{\pi}{2}\right)\left(\csc\left(e+fx+\frac{\pi}{2}\right)+1\right)^m dx}{m+1}-\frac{a\tan(e+fx)(a\sec(e+fx)+a)^m}{f(m+1)}}{a(m+2)}+\frac{\tan(e+fx)(a\sec(e+fx)+a)^{m+1}}{af(m+2)}$$

$\Big\downarrow$ 4314

$$\frac{-\frac{a(m^2+m+1)\tan(e+fx)(\sec(e+fx)+1)^{-m-\frac{1}{2}}(a\sec(e+fx)+a)^m\int \frac{(\sec(e+fx)+1)^{m-\frac{1}{2}}}{\sqrt{1-\sec(e+fx)}}d\sec(e+fx)}{f(m+1)\sqrt{1-\sec(e+fx)}}-\frac{a\tan(e+fx)(a\sec(e+fx)+a)^m}{f(m+1)}}{a(m+2)}+\frac{\tan(e+fx)(a\sec(e+fx)+a)^{m+1}}{af(m+2)}$$

$\Big\downarrow$ 79

$$\frac{\frac{a2^{m+\frac{1}{2}}(m^2+m+1)\tan(e+fx)(\sec(e+fx)+1)^{-m-\frac{1}{2}}(a\sec(e+fx)+a)^m\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}-m,\frac{3}{2},\frac{1}{2}(1-\sec(e+fx))\right)}{f(m+1)}-\frac{a\tan(e+fx)(a\sec(}{f(m+1}}{a(m+2)}+\frac{\tan(e+fx)(a\sec(e+fx)+a)^{m+1}}{af(m+2)}$$

input
```
Int[Sec[e + f*x]^3*(a + a*Sec[e + f*x])^m,x]
```

output
```
((a + a*Sec[e + f*x])^(1 + m)*Tan[e + f*x])/(a*f*(2 + m)) + (-((a*(a + a*S
ec[e + f*x])^m*Tan[e + f*x])/(f*(1 + m))) + (2^(1/2 + m)*a*(1 + m + m^2)*H
ypergeometric2F1[1/2, 1/2 - m, 3/2, (1 - Sec[e + f*x])/2]*(1 + Sec[e + f*x
])^(-1/2 - m)*(a + a*Sec[e + f*x])^m*Tan[e + f*x])/(f*(1 + m)))/(a*(2 + m)
)
```

3.342. $\int \sec^3(e+fx)(a+a\sec(e+fx))^m\,dx$

3.342.3.1 Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4287
```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(-Cot[e + f*x])*((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2
))), x] + Simp[1/(b*(m + 2))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(b*(
m + 1) - a*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 -
b^2, 0] &&  !LtQ[m, -2^(-1)]
```

rule 4314
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Simp[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x
]]*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(d*x)^(n - 1)*((a + b*x)^(m - 1/2
)/Sqrt[a - b*x]), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n},
x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

rule 4315
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Simp[a^IntPart[m]*((a + b*Csc[e + f*x])^FracPart[m
]/(1 + (b/a)*Csc[e + f*x])^FracPart[m])   Int[(1 + (b/a)*Csc[e + f*x])^m*(d
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^
2, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

rule 4489
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*((
a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Simp[(a*B*m + A*b*(m + 1))/(b*(m +
 1))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B
, e, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b
*(m + 1), 0] &&  !LtQ[m, -2^(-1)]
```

3.342. $\int \sec^3(e+fx)(a+a\sec(e+fx))^m\,dx$

3.342.4 Maple [F]

$$\int \sec (fx+e)^3 (a+a\sec (fx+e))^m \, dx$$

input
```
int(sec(f*x+e)^3*(a+a*sec(f*x+e))^m,x)
```

output
```
int(sec(f*x+e)^3*(a+a*sec(f*x+e))^m,x)
```

3.342.5 Fricas [F]

$$\int \sec^3(e+fx)(a+a\sec(e+fx))^m \, dx = \int (a\sec (fx+e)+a)^m \sec (fx+e)^3 \, dx$$

input
```
integrate(sec(f*x+e)^3*(a+a*sec(f*x+e))^m,x, algorithm="fricas")
```

output
```
integral((a*sec(f*x + e) + a)^m*sec(f*x + e)^3, x)
```

3.342.6 Sympy [F]

$$\int \sec^3(e+fx)(a+a\sec(e+fx))^m \, dx = \int (a(\sec (e+fx)+1))^m \sec^3 (e+fx) \, dx$$

input
```
integrate(sec(f*x+e)**3*(a+a*sec(f*x+e))**m,x)
```

output
```
Integral((a*(sec(e + f*x) + 1))**m*sec(e + f*x)**3, x)
```

3.342.7 Maxima [F]

$$\int \sec^3(e+fx)(a+a\sec(e+fx))^m\,dx = \int (a\sec(fx+e)+a)^m\sec(fx+e)^3\,dx$$

input
```
integrate(sec(f*x+e)^3*(a+a*sec(f*x+e))^m,x, algorithm="maxima")
```

output
```
integrate((a*sec(f*x + e) + a)^m*sec(f*x + e)^3, x)
```

3.342.8 Giac [F]

$$\int \sec^3(e+fx)(a+a\sec(e+fx))^m\,dx = \int (a\sec(fx+e)+a)^m\sec(fx+e)^3\,dx$$

input
```
integrate(sec(f*x+e)^3*(a+a*sec(f*x+e))^m,x, algorithm="giac")
```

output
```
integrate((a*sec(f*x + e) + a)^m*sec(f*x + e)^3, x)
```

3.342.9 Mupad [F(-1)]

Timed out.

$$\int \sec^3(e+fx)(a+a\sec(e+fx))^m\,dx = \int \frac{\left(a+\frac{a}{\cos(e+f\,x)}\right)^m}{\cos(e+f\,x)^3}\,dx$$

input
```
int((a + a/cos(e + f*x))^m/cos(e + f*x)^3,x)
```

output
```
int((a + a/cos(e + f*x))^m/cos(e + f*x)^3, x)
```

3.343 $\int \sec^2(e+fx)(a+a\sec(e+fx))^m\,dx$

3.343.1 Optimal result 2549
3.343.2 Mathematica [A] (verified) 2549
3.343.3 Rubi [A] (verified) 2550
3.343.4 Maple [F] 2552
3.343.5 Fricas [F] 2552
3.343.6 Sympy [F] 2553
3.343.7 Maxima [F] 2553
3.343.8 Giac [F] 2553
3.343.9 Mupad [F(-1)] 2554

3.343.1 Optimal result

Integrand size = 21, antiderivative size = 107

$$\int \sec^2(e+fx)(a+a\sec(e+fx))^m\,dx=\frac{(a+a\sec(e+fx))^m\tan(e+fx)}{f(1+m)}+\frac{2^{\frac{1}{2}+m}m\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}-m,\frac{3}{2},\frac{1}{2}(1-\sec(e+fx))\right)(1+\sec(e+fx))^{-\frac{1}{2}-m}(a+a\sec(e+fx}{f(1+m)}$$

output
```
(a+a*sec(f*x+e))^m*tan(f*x+e)/f/(1+m)+2^(1/2+m)*m*hypergeom([1/2, 1/2-m],[
3/2],1/2-1/2*sec(f*x+e))*(1+sec(f*x+e))^(-1/2-m)*(a+a*sec(f*x+e))^m*tan(f*
x+e)/f/(1+m)
```

3.343.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.89

$$\int \sec^2(e+fx)(a+a\sec(e+fx))^m\,dx=\frac{(1+\sec(e+fx))^{-\frac{1}{2}-m}(a(1+\sec(e+fx)))^m\left(2^{\frac{3}{2}+m}m\,\text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{1}{2}-m,\frac{3}{2},\frac{1}{2}(1-\sec(e+\right.\right.}{f+2fm}$$

input
```
Integrate[Sec[e + f*x]^2*(a + a*Sec[e + f*x])^m,x]
```

output
```
((1 + Sec[e + f*x])^(-1/2 - m)*(a*(1 + Sec[e + f*x]))^m*(2^(3/2 + m)*m*Hyp
ergeometric2F1[1/2, -1/2 - m, 3/2, (1 - Sec[e + f*x])/2] + (1 + Sec[e + f*
x])^(1/2 + m))*Tan[e + f*x])/(f + 2*f*m)
```

3.343.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 4285, 3042, 4315, 3042, 4314, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(e+fx)(a\sec(e+fx)+a)^m\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(e+fx+\frac{\pi}{2}\right)^2\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^m dx$$

$$\Big\downarrow \text{4285}$$

$$\frac{m\int \sec(e+fx)(\sec(e+fx)a+a)^m dx}{m+1}+\frac{\tan(e+fx)(a\sec(e+fx)+a)^m}{f(m+1)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{m\int \csc\left(e+fx+\frac{\pi}{2}\right)\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)^m dx}{m+1}+\frac{\tan(e+fx)(a\sec(e+fx)+a)^m}{f(m+1)}$$

$$\Big\downarrow \text{4315}$$

$$\frac{m(\sec(e+fx)+1)^{-m}(a\sec(e+fx)+a)^m\int \sec(e+fx)(\sec(e+fx)+1)^m dx}{m+1}+\frac{\tan(e+fx)(a\sec(e+fx)+a)^m}{f(m+1)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{m(\sec(e+fx)+1)^{-m}(a\sec(e+fx)+a)^m\int \csc\left(e+fx+\frac{\pi}{2}\right)\left(\csc\left(e+fx+\frac{\pi}{2}\right)+1\right)^m dx}{m+1}+\frac{\tan(e+fx)(a\sec(e+fx)+a)^m}{f(m+1)}$$

$$\Big\downarrow \text{4314}$$

$$\frac{\tan(e+fx)(a\sec(e+fx)+a)^m}{f(m+1)} - \frac{m\tan(e+fx)(\sec(e+fx)+1)^{-m-\frac{1}{2}}(a\sec(e+fx)+a)^m\int\frac{(\sec(e+fx)+1)^{m-\frac{1}{2}}}{\sqrt{1-\sec(e+fx)}}d\sec(e+fx)}{f(m+1)\sqrt{1-\sec(e+fx)}}$$

$$\Big\downarrow \; 79$$

$$\frac{2^{m+\frac{1}{2}}m\tan(e+fx)(\sec(e+fx)+1)^{-m-\frac{1}{2}}(a\sec(e+fx)+a)^m\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}-m,\frac{3}{2},\frac{1}{2}(1-\sec(e+\right.}{f(m+1)} - \frac{\tan(e+fx)(a\sec(e+fx)+a)^m}{f(m+1)}$$

input
```
Int[Sec[e + f*x]^2*(a + a*Sec[e + f*x])^m,x]
```

output
```
((a + a*Sec[e + f*x])^m*Tan[e + f*x])/(f*(1 + m)) + (2^(1/2 + m)*m*Hyperge
ometric2F1[1/2, 1/2 - m, 3/2, (1 - Sec[e + f*x])/2]*(1 + Sec[e + f*x])^(-1
/2 - m)*(a + a*Sec[e + f*x])^m*Tan[e + f*x])/(f*(1 + m))
```

3.343.3.1 Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4285
```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(-Cot[e + f*x])*((a + b*Csc[e + f*x])^m/(f*(m + 1))), x]
+ Simp[a*(m/(b*(m + 1)))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x]
/; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

rule 4314
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Simp[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x
]]*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(d*x)^(n - 1)*((a + b*x)^(m - 1/2
)/Sqrt[a - b*x]), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n},
x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

rule 4315
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Simp[a^IntPart[m]*((a + b*Csc[e + f*x])^FracPart[m
]/(1 + (b/a)*Csc[e + f*x])^FracPart[m])   Int[(1 + (b/a)*Csc[e + f*x])^m*(d
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^
2, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

3.343.4 Maple [F]

$$\int \sec (fx+e)^2 (a+a\sec (fx+e))^m \, dx$$

input
```
int(sec(f*x+e)^2*(a+a*sec(f*x+e))^m,x)
```

output
```
int(sec(f*x+e)^2*(a+a*sec(f*x+e))^m,x)
```

3.343.5 Fricas [F]

$$\int \sec^2(e+fx)(a+a\sec(e+fx))^m \, dx = \int (a\sec (fx+e)+a)^m \sec (fx+e)^2 \, dx$$

input
```
integrate(sec(f*x+e)^2*(a+a*sec(f*x+e))^m,x, algorithm="fricas")
```

output
```
integral((a*sec(f*x + e) + a)^m*sec(f*x + e)^2, x)
```

3.343.6 Sympy [F]

$$\int \sec^2(e+fx)(a+a\sec(e+fx))^m\,dx=\int (a(\sec(e+fx)+1))^m\sec^2(e+fx)\,dx$$

input
```
integrate(sec(f*x+e)**2*(a+a*sec(f*x+e))**m,x)
```

output
```
Integral((a*(sec(e + f*x) + 1))**m*sec(e + f*x)**2, x)
```

3.343.7 Maxima [F]

$$\int \sec^2(e+fx)(a+a\sec(e+fx))^m\,dx=\int (a\sec(fx+e)+a)^m\sec(fx+e)^2\,dx$$

input
```
integrate(sec(f*x+e)^2*(a+a*sec(f*x+e))^m,x, algorithm="maxima")
```

output
```
integrate((a*sec(f*x + e) + a)^m*sec(f*x + e)^2, x)
```

3.343.8 Giac [F]

$$\int \sec^2(e+fx)(a+a\sec(e+fx))^m\,dx=\int (a\sec(fx+e)+a)^m\sec(fx+e)^2\,dx$$

input
```
integrate(sec(f*x+e)^2*(a+a*sec(f*x+e))^m,x, algorithm="giac")
```

output
```
integrate((a*sec(f*x + e) + a)^m*sec(f*x + e)^2, x)
```

3.343.9 Mupad [F(-1)]

Timed out.

$$\int \sec^2(e+fx)(a+a\sec(e+fx))^m\,dx=\int \frac{\left(a+\frac{a}{\cos(e+f\,x)}\right)^m}{\cos\left(e+f\,x\right)^2}\,dx$$

input
```
int((a + a/cos(e + f*x))^m/cos(e + f*x)^2,x)
```

output
```
int((a + a/cos(e + f*x))^m/cos(e + f*x)^2, x)
```

3.344 $\int \sec(e+fx)(a+a\sec(e+fx))^m\,dx$

3.344.1 Optimal result 2555
3.344.2 Mathematica [A] (verified) 2555
3.344.3 Rubi [A] (verified) 2556
3.344.4 Maple [F] 2557
3.344.5 Fricas [F] 2558
3.344.6 Sympy [F] 2558
3.344.7 Maxima [F] 2558
3.344.8 Giac [F] 2559
3.344.9 Mupad [F(-1)] 2559

3.344.1 Optimal result

Integrand size = 19, antiderivative size = 73

$$\int \sec(e+fx)(a+a\sec(e+fx))^m\,dx = \frac{2^{\frac{1}{2}+m}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}-m,\frac{3}{2},\frac{1}{2}(1-\sec(e+fx))\right)(1+\sec(e+fx))^{-\frac{1}{2}-m}(a+a\sec(e+fx))^m\,\text{t}}{f}$$

output
```
2^(1/2+m)*hypergeom([1/2, 1/2-m],[3/2],1/2-1/2*sec(f*x+e))*(1+sec(f*x+e))^
(-1/2-m)*(a+a*sec(f*x+e))^m*tan(f*x+e)/f
```

3.344.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00

$$\int \sec(e+fx)(a+a\sec(e+fx))^m\,dx = \frac{2^{\frac{1}{2}+m}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}-m,\frac{3}{2},\frac{1}{2}(1-\sec(e+fx))\right)(1+\sec(e+fx))^{-\frac{1}{2}-m}(a(1+\sec(e+fx)))^m}{f}$$

input
```
Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^m,x]
```

output
```
(2^(1/2 + m)*Hypergeometric2F1[1/2, 1/2 - m, 3/2, (1 - Sec[e + f*x])/2]*(1
 + Sec[e + f*x])^(-1/2 - m)*(a*(1 + Sec[e + f*x]))^m*Tan[e + f*x])/f
```

3.344.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {3042, 4315, 3042, 4314, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(e+fx)(a\sec(e+fx)+a)^m\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(e+fx+\frac{\pi}{2}\right)\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^m dx$$

$$\Big\downarrow \text{4315}$$

$$(\sec(e+fx)+1)^{-m}(a\sec(e+fx)+a)^m\int \sec(e+fx)(\sec(e+fx)+1)^m dx$$

$$\Big\downarrow \text{3042}$$

$$(\sec(e+fx)+1)^{-m}(a\sec(e+fx)+a)^m\int \csc\left(e+fx+\frac{\pi}{2}\right)\left(\csc\left(e+fx+\frac{\pi}{2}\right)+1\right)^m dx$$

$$\Big\downarrow \text{4314}$$

$$-\frac{\tan(e+fx)(\sec(e+fx)+1)^{-m-\frac{1}{2}}(a\sec(e+fx)+a)^m\int \frac{(\sec(e+fx)+1)^{m-\frac{1}{2}}}{\sqrt{1-\sec(e+fx)}}d\sec(e+fx)}{f\sqrt{1-\sec(e+fx)}}$$

$$\Big\downarrow \text{79}$$

$$\frac{2^{m+\frac{1}{2}}\tan(e+fx)(\sec(e+fx)+1)^{-m-\frac{1}{2}}(a\sec(e+fx)+a)^m\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}-m,\frac{3}{2},\frac{1}{2}(1-\sec(e+fx\right.}{f}$$

input
```
Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^m,x]
```

output
```
(2^(1/2 + m)*Hypergeometric2F1[1/2, 1/2 - m, 3/2, (1 - Sec[e + f*x])/2]*(1
 + Sec[e + f*x])^(-1/2 - m)*(a + a*Sec[e + f*x])^m*Tan[e + f*x])/f
```

3.344.3.1 Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4314
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Simp[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x
]]*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(d*x)^(n - 1)*((a + b*x)^(m - 1/2
)/Sqrt[a - b*x]), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n},
x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

rule 4315
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Simp[a^IntPart[m]*((a + b*Csc[e + f*x])^FracPart[m
]/(1 + (b/a)*Csc[e + f*x])^FracPart[m])   Int[(1 + (b/a)*Csc[e + f*x])^m*(d
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^
2, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

3.344.4 Maple [F]

$$\int \sec(fx+e)\,(a+a\sec(fx+e))^m\,dx$$

input
```
int(sec(f*x+e)*(a+a*sec(f*x+e))^m,x)
```

output
```
int(sec(f*x+e)*(a+a*sec(f*x+e))^m,x)
```

3.344.5 Fricas [F]

$$\int \sec(e+fx)(a+a\sec(e+fx))^m\,dx=\int (a\sec(fx+e)+a)^m\sec(fx+e)\,dx$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m,x, algorithm="fricas")
```

output
```
integral((a*sec(f*x + e) + a)^m*sec(f*x + e), x)
```

3.344.6 Sympy [F]

$$\int \sec(e+fx)(a+a\sec(e+fx))^m\,dx=\int (a(\sec(e+fx)+1))^m\sec(e+fx)\,dx$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))**m,x)
```

output
```
Integral((a*(sec(e + f*x) + 1))**m*sec(e + f*x), x)
```

3.344.7 Maxima [F]

$$\int \sec(e+fx)(a+a\sec(e+fx))^m\,dx=\int (a\sec(fx+e)+a)^m\sec(fx+e)\,dx$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m,x, algorithm="maxima")
```

output
```
integrate((a*sec(f*x + e) + a)^m*sec(f*x + e), x)
```

3.344.8 Giac [F]

$$\int \sec(e+fx)(a+a\sec(e+fx))^m\,dx=\int (a\sec(fx+e)+a)^m\sec(fx+e)\,dx$$

input
```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m,x, algorithm="giac")
```

output
```
integrate((a*sec(f*x + e) + a)^m*sec(f*x + e), x)
```

3.344.9 Mupad [F(-1)]

Timed out.

$$\int \sec(e+fx)(a+a\sec(e+fx))^m\,dx=\int \frac{\left(a+\frac{a}{\cos(e+f\,x)}\right)^m}{\cos(e+f\,x)}\,dx$$

input
```
int((a + a/cos(e + f*x))^m/cos(e + f*x),x)
```

output
```
int((a + a/cos(e + f*x))^m/cos(e + f*x), x)
```

3.345 $\int (a+a\sec(e+fx))^m\,dx$

3.345.1 Optimal result 2560
3.345.2 Mathematica [B] (warning: unable to verify) 2560
3.345.3 Rubi [A] (verified) 2561
3.345.4 Maple [F] 2563
3.345.5 Fricas [F] 2563
3.345.6 Sympy [F] 2563
3.345.7 Maxima [F] 2564
3.345.8 Giac [F] 2564
3.345.9 Mupad [F(-1)] 2564

3.345.1 Optimal result

Integrand size = 12, antiderivative size = 83

$$\int (a+a\sec(e+fx))^m\,dx = \frac{\sqrt{2}\,\text{AppellF1}\left(\frac{1}{2}+m,\frac{1}{2},1,\frac{3}{2}+m,\frac{1}{2}(1+\sec(e+fx)),1+\sec(e+fx)\right)(a+a\sec(e+fx))^m\tan(e+fx)}{f(1+2m)\sqrt{1-\sec(e+fx)}}$$

output
```
AppellF1(1/2+m,1,1/2,3/2+m,1+sec(f*x+e),1/2+1/2*sec(f*x+e))*(a+a*sec(f*x+e
))^m*2^(1/2)*tan(f*x+e)/f/(1+2*m)/(1-sec(f*x+e))^(1/2)
```

3.345.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 711 vs. 2(83) = 166.

Time = 7.36 (sec) , antiderivative size = 711, normalized size of antiderivative = 8.57

$$\int (a+a\sec(e+fx))^m\,dx = \frac{}{f\left(45\,\text{AppellF1}\left(\frac{1}{2},m,1,\frac{3}{2},\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)^2\cos^2\left(\frac{1}{2}(e+fx)\right)(1+2m-2m\cos(e\right.}$$

input
```
Integrate[(a + a*Sec[e + f*x])^m,x]
```

output
```
(30*AppellF1[1/2, m, 1, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[
(e + f*x)/2]^2*Cos[e + f*x]*(a*(1 + Sec[e + f*x]))^m*Sin[e + f*x]*(3*Appel
lF1[1/2, m, 1, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - 2*(AppellF1
[3/2, m, 2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - m*AppellF1[3/2
, 1 + m, 1, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2
]^2))/(f*(45*AppellF1[1/2, m, 1, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2
]^2]^2*Cos[(e + f*x)/2]^2*(1 + 2*m - 2*m*Cos[e + f*x] + Cos[2*(e + f*x)])
+ 6*AppellF1[1/2, m, 1, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sin[
(e + f*x)/2]^2*(-5*AppellF1[3/2, m, 2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e +
f*x)/2]^2]*(1 + 2*m - 2*(2 + m)*Cos[e + f*x] + Cos[2*(e + f*x)]) + 5*m*App
ellF1[3/2, 1 + m, 1, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(1 + 2*
m - 2*(2 + m)*Cos[e + f*x] + Cos[2*(e + f*x)]) - 48*(2*AppellF1[5/2, m, 3,
 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - 2*m*AppellF1[5/2, 1 + m,
2, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + m*(1 + m)*AppellF1[5/2,
 2 + m, 1, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Cot[e + f*x]*Csc
[e + f*x]*Sin[(e + f*x)/2]^4) + 40*(AppellF1[3/2, m, 2, 5/2, Tan[(e + f*x)
/2]^2, -Tan[(e + f*x)/2]^2] - m*AppellF1[3/2, 1 + m, 1, 5/2, Tan[(e + f*x)
/2]^2, -Tan[(e + f*x)/2]^2])^2*Cos[e + f*x]*Sin[(e + f*x)/2]^2*Tan[(e + f*
x)/2]^2))
```

3.345.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {3042, 4266, 3042, 4265, 153}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sec(e+fx)+a)^m\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^m dx$$

$$\Big\downarrow \text{4266}$$

$$(\sec(e+fx)+1)^{-m}(a\sec(e+fx)+a)^m\int (\sec(e+fx)+1)^m dx$$

$$\Big\downarrow \text{3042}$$

$$(\sec(e+fx)+1)^{-m}(a\sec(e+fx)+a)^m\int \left(\csc\left(e+fx+\frac{\pi}{2}\right)+1\right)^m dx$$

$$\Big\downarrow \text{4265}$$

$$-\frac{\tan(e+fx)(\sec(e+fx)+1)^{-m-\frac{1}{2}}(a\sec(e+fx)+a)^m\int\frac{\cos(e+fx)(\sec(e+fx)+1)^{m-\frac{1}{2}}}{\sqrt{1-\sec(e+fx)}}d\sec(e+fx)}{f\sqrt{1-\sec(e+fx)}}$$

$$\Big\downarrow \text{153}$$

$$\frac{\sqrt{2}\tan(e+fx)(a\sec(e+fx)+a)^m\operatorname{AppellF1}\left(m+\frac{1}{2},\frac{1}{2},1,m+\frac{3}{2},\frac{1}{2}(\sec(e+fx)+1),\sec(e+fx)+1\right)}{f(2m+1)\sqrt{1-\sec(e+fx)}}$$

input
```
Int[(a + a*Sec[e + f*x])^m,x]
```

output
```
(Sqrt[2]*AppellF1[1/2 + m, 1/2, 1, 3/2 + m, (1 + Sec[e + f*x])/2, 1 + Sec[
e + f*x]]*(a + a*Sec[e + f*x])^m*Tan[e + f*x])/(f*(1 + 2*m)*Sqrt[1 - Sec[e
 + f*x]])
```

3.345.3.1 Defintions of rubi rules used

rule 153
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(b*e - a*f)^p*((a + b*x)^(m + 1)/(b^(p + 1)*(m + 1)*Simp
lify[b/(b*c - a*d)]^n))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c
 - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] && !IntegerQ[m] && !IntegerQ[n] && IntegerQ[p] && GtQ[Simplify[b/(
b*c - a*d)], 0] && !(GtQ[Simplify[d/(d*a - c*b)], 0] && SimplerQ[c + d*x,
a + b*x])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4265
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[a^n*(Cot
[c + d*x]/(d*Sqrt[1 + Csc[c + d*x]]*Sqrt[1 - Csc[c + d*x]]))   Subst[Int[(1
 + b*(x/a))^(n - 1/2)/(x*Sqrt[1 - b*(x/a)]), x], x, Csc[c + d*x]], x] /; Fr
eeQ[{a, b, c, d, n}, x] && EqQ[a^2 - b^2, 0] && !IntegerQ[2*n] && GtQ[a, 0
]
```

rule 4266
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[a^IntPar
t[n]*((a + b*Csc[c + d*x])^FracPart[n]/(1 + (b/a)*Csc[c + d*x])^FracPart[n]
)   Int[(1 + (b/a)*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] &&
EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

3.345.4 Maple [F]

$$\int (a + a\sec(fx + e))^m \, dx$$

input
```
int((a+a*sec(f*x+e))^m,x)
```

output
```
int((a+a*sec(f*x+e))^m,x)
```

3.345.5 Fricas [F]

$$\int (a + a\sec(e + fx))^m \, dx = \int (a\sec(fx + e) + a)^m \, dx$$

input
```
integrate((a+a*sec(f*x+e))^m,x, algorithm="fricas")
```

output
```
integral((a*sec(f*x + e) + a)^m, x)
```

3.345.6 Sympy [F]

$$\int (a + a\sec(e + fx))^m \, dx = \int (a\sec(e + fx) + a)^m \, dx$$

input
```
integrate((a+a*sec(f*x+e))**m,x)
```

output
```
Integral((a*sec(e + f*x) + a)**m, x)
```

3.345.7 Maxima [F]

$$\int (a+a\sec(e+fx))^m\,dx=\int (a\sec(fx+e)+a)^m\,dx$$

input
```
integrate((a+a*sec(f*x+e))^m,x, algorithm="maxima")
```

output
```
integrate((a*sec(f*x + e) + a)^m, x)
```

3.345.8 Giac [F]

$$\int (a+a\sec(e+fx))^m\,dx=\int (a\sec(fx+e)+a)^m\,dx$$

input
```
integrate((a+a*sec(f*x+e))^m,x, algorithm="giac")
```

output
```
integrate((a*sec(f*x + e) + a)^m, x)
```

3.345.9 Mupad [F(-1)]

Timed out.

$$\int (a+a\sec(e+fx))^m\,dx=\int \left(a+\frac{a}{\cos(e+f\,x)}\right)^m\,dx$$

input
```
int((a + a/cos(e + f*x))^m,x)
```

output
```
int((a + a/cos(e + f*x))^m, x)
```

3.346 $\int \cos(e+fx)(a+a\sec(e+fx))^m\,dx$

3.346.1 Optimal result 2565
3.346.2 Mathematica [B] (warning: unable to verify) 2565
3.346.3 Rubi [A] (verified) 2566
3.346.4 Maple [F] 2568
3.346.5 Fricas [F] 2568
3.346.6 Sympy [F] 2568
3.346.7 Maxima [F] 2569
3.346.8 Giac [F] 2569
3.346.9 Mupad [F(-1)] 2569

3.346.1 Optimal result

Integrand size = 19, antiderivative size = 84

$$\int \cos(e+fx)(a+a\sec(e+fx))^m\,dx = -\frac{\sqrt{2}\,\text{AppellF1}\left(\frac{1}{2}+m,\frac{1}{2},2,\frac{3}{2}+m,\frac{1}{2}(1+\sec(e+fx)),1+\sec(e+fx)\right)(a+a\sec(e+fx))^m\tan(e+f$$
$$f(1+2m)\sqrt{1-\sec(e+fx)}$$

output
```
-AppellF1(1/2+m,2,1/2,3/2+m,1+sec(f*x+e),1/2+1/2*sec(f*x+e))*(a+a*sec(f*x+
e))^m*2^(1/2)*tan(f*x+e)/f/(1+2*m)/(1-sec(f*x+e))^(1/2)
```

3.346.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 3781 vs. 2(84) = 168.

Time = 17.47 (sec) , antiderivative size = 3781, normalized size of antiderivative = 45.01

$$\int \cos(e+fx)(a+a\sec(e+fx))^m\,dx = \text{Result too large to show}$$

input
```
Integrate[Cos[e + f*x]*(a + a*Sec[e + f*x])^m,x]
```

output
```
(2^(1 + m)*Cos[(e + f*x)/2]^3*Cos[e + f*x]*(Cos[(e + f*x)/2]^2*Sec[e + f*x
])^m*(a*(1 + Sec[e + f*x]))^m*Sin[(e + f*x)/2]*((-3*AppellF1[1/2, m, 1, 3/
2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2)/(3*AppellF
1[1/2, m, 1, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - 2*(AppellF1[3
/2, m, 2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - m*AppellF1[3/2,
1 + m, 1, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^
2) + (2*AppellF1[1/2, m, 2, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])
/(AppellF1[1/2, m, 2, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(
-2*AppellF1[3/2, m, 3, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + m*A
ppellF1[3/2, 1 + m, 2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[
(e + f*x)/2]^2)/3)))/(f*(2^m*Cos[(e + f*x)/2]^4*(Cos[(e + f*x)/2]^2*Sec[e
+ f*x])^m*((-3*AppellF1[1/2, m, 1, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)
/2]^2]*Sec[(e + f*x)/2]^2)/(3*AppellF1[1/2, m, 1, 3/2, Tan[(e + f*x)/2]^2,
 -Tan[(e + f*x)/2]^2] - 2*(AppellF1[3/2, m, 2, 5/2, Tan[(e + f*x)/2]^2, -T
an[(e + f*x)/2]^2] - m*AppellF1[3/2, 1 + m, 1, 5/2, Tan[(e + f*x)/2]^2, -T
an[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2) + (2*AppellF1[1/2, m, 2, 3/2, Tan[
(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])/(AppellF1[1/2, m, 2, 3/2, Tan[(e + f
*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-2*AppellF1[3/2, m, 3, 5/2, Tan[(e +
f*x)/2]^2, -Tan[(e + f*x)/2]^2] + m*AppellF1[3/2, 1 + m, 2, 5/2, Tan[(e +
f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)/3)) - 3*2^m*Cos[(e...
```

3.346.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {3042, 4315, 3042, 4314, 153}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(e+fx)(a\sec(e+fx)+a)^m\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^m}{\csc\left(e+fx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \; 4315$$

$$(\sec(e+fx)+1)^{-m}(a\sec(e+fx)+a)^m\int \cos(e+fx)(\sec(e+fx)+1)^m dx$$

$$\Big\downarrow \; 3042$$

$$(\sec(e+fx)+1)^{-m}(a\sec(e+fx)+a)^m\int\frac{\left(\csc\left(e+fx+\frac{\pi}{2}\right)+1\right)^m}{\csc\left(e+fx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{4314}$$

$$-\frac{\tan(e+fx)(\sec(e+fx)+1)^{-m-\frac{1}{2}}(a\sec(e+fx)+a)^m\int\frac{\cos^2(e+fx)(\sec(e+fx)+1)^{m-\frac{1}{2}}}{\sqrt{1-\sec(e+fx)}}d\sec(e+fx)}{f\sqrt{1-\sec(e+fx)}}$$

$$\Big\downarrow \text{153}$$

$$-\frac{\sqrt{2}\tan(e+fx)(a\sec(e+fx)+a)^m\operatorname{AppellF1}\left(m+\frac{1}{2},\frac{1}{2},2,m+\frac{3}{2},\frac{1}{2}(\sec(e+fx)+1),\sec(e+fx)+1\right)}{f(2m+1)\sqrt{1-\sec(e+fx)}}$$

input
```
Int[Cos[e + f*x]*(a + a*Sec[e + f*x])^m,x]
```

output
```
-((Sqrt[2]*AppellF1[1/2 + m, 1/2, 2, 3/2 + m, (1 + Sec[e + f*x])/2, 1 + Se
c[e + f*x]]*(a + a*Sec[e + f*x])^m*Tan[e + f*x])/(f*(1 + 2*m)*Sqrt[1 - Sec
[e + f*x]]))
```

3.346.3.1 Defintions of rubi rules used

rule 153
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(b*e - a*f)^p*((a + b*x)^(m + 1)/(b^(p + 1)*(m + 1)*Simp
lify[b/(b*c - a*d)]^n))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c
 - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] && !IntegerQ[m] && !IntegerQ[n] && IntegerQ[p] && GtQ[Simplify[b/(
b*c - a*d)], 0] && !(GtQ[Simplify[d/(d*a - c*b)], 0] && SimplerQ[c + d*x,
a + b*x])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4314
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Simp[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x
]]*Sqrt[a - b*Csc[e + f*x]])) Subst[Int[(d*x)^(n - 1)*((a + b*x)^(m - 1/2
)/Sqrt[a - b*x]), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n},
x] && EqQ[a^2 - b^2, 0] && !IntegerQ[m] && GtQ[a, 0]
```

rule 4315
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Simp[a^IntPart[m]*((a + b*Csc[e + f*x])^FracPart[m
]/(1 + (b/a)*Csc[e + f*x])^FracPart[m])   Int[(1 + (b/a)*Csc[e + f*x])^m*(d
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^
2, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

3.346.4 Maple [F]

$$\int \cos(fx+e)(a+a\sec(fx+e))^m\,dx$$

input
```
int(cos(f*x+e)*(a+a*sec(f*x+e))^m,x)
```

output
```
int(cos(f*x+e)*(a+a*sec(f*x+e))^m,x)
```

3.346.5 Fricas [F]

$$\int \cos(e+fx)(a+a\sec(e+fx))^m\,dx=\int (a\sec(fx+e)+a)^m\cos(fx+e)\,dx$$

input
```
integrate(cos(f*x+e)*(a+a*sec(f*x+e))^m,x, algorithm="fricas")
```

output
```
integral((a*sec(f*x + e) + a)^m*cos(f*x + e), x)
```

3.346.6 Sympy [F]

$$\int \cos(e+fx)(a+a\sec(e+fx))^m\,dx=\int (a(\sec(e+fx)+1))^m\cos(e+fx)\,dx$$

input
```
integrate(cos(f*x+e)*(a+a*sec(f*x+e))**m,x)
```

output
```
Integral((a*(sec(e + f*x) + 1))**m*cos(e + f*x), x)
```

3.346.7 Maxima [F]

$$\int \cos(e+fx)(a+a\sec(e+fx))^m\,dx=\int (a\sec(fx+e)+a)^m\cos(fx+e)\,dx$$

input
```
integrate(cos(f*x+e)*(a+a*sec(f*x+e))^m,x, algorithm="maxima")
```

output
```
integrate((a*sec(f*x + e) + a)^m*cos(f*x + e), x)
```

3.346.8 Giac [F]

$$\int \cos(e+fx)(a+a\sec(e+fx))^m\,dx=\int (a\sec(fx+e)+a)^m\cos(fx+e)\,dx$$

input
```
integrate(cos(f*x+e)*(a+a*sec(f*x+e))^m,x, algorithm="giac")
```

output
```
integrate((a*sec(f*x + e) + a)^m*cos(f*x + e), x)
```

3.346.9 Mupad [F(-1)]

Timed out.

$$\int \cos(e+fx)(a+a\sec(e+fx))^m\,dx=\int \cos(e+f\,x)\,\left(a+\frac{a}{\cos(e+f\,x)}\right)^m\,dx$$

input
```
int(cos(e + f*x)*(a + a/cos(e + f*x))^m,x)
```

output
```
int(cos(e + f*x)*(a + a/cos(e + f*x))^m, x)
```

3.347 $\int (d\sec(e+fx))^{3/2}(a+a\sec(e+fx))^m\,dx$

3.347.1 Optimal result 2570
3.347.2 Mathematica [B] (warning: unable to verify) 2570
3.347.3 Rubi [A] (verified) 2571
3.347.4 Maple [F] 2573
3.347.5 Fricas [F] 2573
3.347.6 Sympy [F(-1)] 2573
3.347.7 Maxima [F] 2574
3.347.8 Giac [F] 2574
3.347.9 Mupad [F(-1)] 2574

3.347.1 Optimal result

Integrand size = 25, antiderivative size = 98

$$\int (d\sec(e+fx))^{3/2}(a+a\sec(e+fx))^m\,dx = -\frac{2\,\text{AppellF1}\left(\frac{3}{2},\frac{1}{2},\frac{1}{2}-m,\frac{5}{2},\sec(e+fx),-\sec(e+fx)\right)(d\sec(e+fx))^{3/2}(1+\sec(e+fx))^{-\frac{1}{2}-m}(a+a\sec}{3f\sqrt{1-\sec(e+fx)}}$$

output
```
-2/3*AppellF1(3/2,1/2-m,1/2,5/2,-sec(f*x+e),sec(f*x+e))*(d*sec(f*x+e))^(3/
2)*(1+sec(f*x+e))^(-1/2-m)*(a+a*sec(f*x+e))^m*tan(f*x+e)/f/(1-sec(f*x+e))^
(1/2)
```

3.347.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 2529 vs. 2(98) = 196.

Time = 17.67 (sec) , antiderivative size = 2529, normalized size of antiderivative = 25.81

$$\int (d\sec(e+fx))^{3/2}(a+a\sec(e+fx))^m\,dx = \text{Result too large to show}$$

input
```
Integrate[(d*Sec[e + f*x])^(3/2)*(a + a*Sec[e + f*x])^m,x]
```

output
```
(-3*2^(1 + m)*AppellF1[1/2, 3/2 + m, -1/2, 3/2, Tan[(e + f*x)/2]^2, -Tan[(
e + f*x)/2]^2]*Sqrt[Sec[e + f*x]]*(d*Sec[e + f*x])^(3/2)*(Cos[(e + f*x)/2]
^2*Sec[e + f*x])^m*(a*(1 + Sec[e + f*x]))^m*Tan[(e + f*x)/2])/(f*(-1 + Tan
[(e + f*x)/2]^2)*(3*AppellF1[1/2, 3/2 + m, -1/2, 3/2, Tan[(e + f*x)/2]^2,
-Tan[(e + f*x)/2]^2] + (AppellF1[3/2, 3/2 + m, 1/2, 5/2, Tan[(e + f*x)/2]^
2, -Tan[(e + f*x)/2]^2] + (3 + 2*m)*AppellF1[3/2, 5/2 + m, -1/2, 5/2, Tan[
(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)*((3*2^(1 + m)*Ap
pellF1[1/2, 3/2 + m, -1/2, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*S
ec[(e + f*x)/2]^2*Sqrt[Sec[e + f*x]]*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^m*T
an[(e + f*x)/2]^2)/((-1 + Tan[(e + f*x)/2]^2)^2*(3*AppellF1[1/2, 3/2 + m,
-1/2, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (AppellF1[3/2, 3/2 +
 m, 1/2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (3 + 2*m)*AppellF
1[3/2, 5/2 + m, -1/2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(
e + f*x)/2]^2)) - (3*2^m*AppellF1[1/2, 3/2 + m, -1/2, 3/2, Tan[(e + f*x)/2
]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Sqrt[Sec[e + f*x]]*(Cos[(e +
f*x)/2]^2*Sec[e + f*x])^m)/((-1 + Tan[(e + f*x)/2]^2)*(3*AppellF1[1/2, 3/2
 + m, -1/2, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (AppellF1[3/2,
 3/2 + m, 1/2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (3 + 2*m)*A
ppellF1[3/2, 5/2 + m, -1/2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])
*Tan[(e + f*x)/2]^2)) - (3*2^m*AppellF1[1/2, 3/2 + m, -1/2, 3/2, Tan[(e...
```

3.347.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 4315, 3042, 4314, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d\sec(e+fx))^{3/2}(a\sec(e+fx)+a)^m\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^m dx$$

$$\Big\downarrow \text{4315}$$

$$(\sec(e+fx)+1)^{-m}(a\sec(e+fx)+a)^m\int (d\sec(e+fx))^{3/2}(\sec(e+fx)+1)^m dx$$

$$\Big\downarrow \text{3042}$$

$$(\sec(e+fx)+1)^{-m}(a\sec(e+fx)+a)^m\int\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}\left(\csc\left(e+fx+\frac{\pi}{2}\right)+1\right)^m dx$$

$$\Big\downarrow \text{4314}$$

$$-\frac{d\tan(e+fx)(\sec(e+fx)+1)^{-m-\frac{1}{2}}(a\sec(e+fx)+a)^m\int\frac{\sqrt{d\sec(e+fx)}(\sec(e+fx)+1)^{m-\frac{1}{2}}}{\sqrt{1-\sec(e+fx)}}d\sec(e+fx)}{f\sqrt{1-\sec(e+fx)}}$$

$$\Big\downarrow \text{150}$$

$$-\frac{2\tan(e+fx)(d\sec(e+fx))^{3/2}(\sec(e+fx)+1)^{-m-\frac{1}{2}}(a\sec(e+fx)+a)^m\operatorname{AppellF1}\left(\frac{3}{2},\frac{1}{2},\frac{1}{2}-m,\frac{5}{2},\sec(e+f\right.}{3f\sqrt{1-\sec(e+fx)}}$$

input
```
Int[(d*Sec[e + f*x])^(3/2)*(a + a*Sec[e + f*x])^m,x]
```

output
```
(-2*AppellF1[3/2, 1/2, 1/2 - m, 5/2, Sec[e + f*x], -Sec[e + f*x]]*(d*Sec[e
 + f*x])^(3/2)*(1 + Sec[e + f*x])^(-1/2 - m)*(a + a*Sec[e + f*x])^m*Tan[e
+ f*x])/(3*f*Sqrt[1 - Sec[e + f*x]])
```

3.347.3.1 Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4314
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Simp[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x
]]*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(d*x)^(n - 1)*((a + b*x)^(m - 1/2
)/Sqrt[a - b*x]), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n},
x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

rule 4315
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Simp[a^IntPart[m]*((a + b*Csc[e + f*x])^FracPart[m
]/(1 + (b/a)*Csc[e + f*x])^FracPart[m])   Int[(1 + (b/a)*Csc[e + f*x])^m*(d
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^
2, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

3.347.4 Maple [F]

$$\int (d\sec(fx+e))^{\frac{3}{2}} (a+a\sec(fx+e))^m \, dx$$

input
```
int((d*sec(f*x+e))^(3/2)*(a+a*sec(f*x+e))^m,x)
```

output
```
int((d*sec(f*x+e))^(3/2)*(a+a*sec(f*x+e))^m,x)
```

3.347.5 Fricas [F]

$$\int (d\sec(e+fx))^{3/2}(a+a\sec(e+fx))^m \, dx = \int (d\sec(fx+e))^{\frac{3}{2}} (a\sec(fx+e)+a)^m \, dx$$

input
```
integrate((d*sec(f*x+e))^(3/2)*(a+a*sec(f*x+e))^m,x, algorithm="fricas")
```

output
```
integral(sqrt(d*sec(f*x + e))*(a*sec(f*x + e) + a)^m*d*sec(f*x + e), x)
```

3.347.6 Sympy [F(-1)]

Timed out.

$$\int (d\sec(e+fx))^{3/2}(a+a\sec(e+fx))^m \, dx = \text{Timed out}$$

input
```
integrate((d*sec(f*x+e))**(3/2)*(a+a*sec(f*x+e))**m,x)
```

output
```
Timed out
```

3.347.7 Maxima [F]

$$\int (d\sec(e+fx))^{3/2}(a+a\sec(e+fx))^m\,dx=\int (d\sec(fx+e))^{\frac{3}{2}}(a\sec(fx+e)+a)^m\,dx$$

input
```
integrate((d*sec(f*x+e))^(3/2)*(a+a*sec(f*x+e))^m,x, algorithm="maxima")
```

output
```
integrate((d*sec(f*x + e))^(3/2)*(a*sec(f*x + e) + a)^m, x)
```

3.347.8 Giac [F]

$$\int (d\sec(e+fx))^{3/2}(a+a\sec(e+fx))^m\,dx=\int (d\sec(fx+e))^{\frac{3}{2}}(a\sec(fx+e)+a)^m\,dx$$

input
```
integrate((d*sec(f*x+e))^(3/2)*(a+a*sec(f*x+e))^m,x, algorithm="giac")
```

output
```
integrate((d*sec(f*x + e))^(3/2)*(a*sec(f*x + e) + a)^m, x)
```

3.347.9 Mupad [F(-1)]

Timed out.

$$\int (d\sec(e+fx))^{3/2}(a+a\sec(e+fx))^m\,dx=\int \left(a+\frac{a}{\cos(e+f\,x)}\right)^m\left(\frac{d}{\cos(e+f\,x)}\right)^{3/2}dx$$

input
```
int((a + a/cos(e + f*x))^m*(d/cos(e + f*x))^(3/2),x)
```

output
```
int((a + a/cos(e + f*x))^m*(d/cos(e + f*x))^(3/2), x)
```

3.348 $\int \sqrt{d\sec(e+fx)}(a+a\sec(e+fx))^m\,dx$

3.348.1 Optimal result 2575
3.348.2 Mathematica [B] (warning: unable to verify) 2575
3.348.3 Rubi [A] (verified) 2576
3.348.4 Maple [F] 2578
3.348.5 Fricas [F] 2578
3.348.6 Sympy [F] 2578
3.348.7 Maxima [F] 2579
3.348.8 Giac [F] 2579
3.348.9 Mupad [F(-1)] 2579

3.348.1 Optimal result

Integrand size = 25, antiderivative size = 96

$$\int \sqrt{d\sec(e+fx)}(a+a\sec(e+fx))^m\,dx = -\frac{2\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},\frac{1}{2}-m,\frac{3}{2},\sec(e+fx),-\sec(e+fx)\right)\sqrt{d\sec(e+fx)}(1+\sec(e+fx))^{-\frac{1}{2}-m}(a+a\,\text{se}}{f\sqrt{1-\sec(e+fx)}}$$

output
```
-2*AppellF1(1/2,1/2-m,1/2,3/2,-sec(f*x+e),sec(f*x+e))*(1+sec(f*x+e))^(-1/2
-m)*(a+a*sec(f*x+e))^m*(d*sec(f*x+e))^(1/2)*tan(f*x+e)/f/(1-sec(f*x+e))^(1
/2)
```

3.348.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 2225 vs. 2(96) = 192.

Time = 16.98 (sec) , antiderivative size = 2225, normalized size of antiderivative = 23.18

$$\int \sqrt{d\sec(e+fx)}(a+a\sec(e+fx))^m\,dx = \text{Result too large to show}$$

input
```
Integrate[Sqrt[d*Sec[e + f*x]]*(a + a*Sec[e + f*x])^m,x]
```

output
```
(2^(1 + m)*AppellF1[1/2, 1/2 + m, 1/2, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e +
f*x)/2]^2]*Sqrt[d*Sec[e + f*x]]*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(1/2 + m
)*(a*(1 + Sec[e + f*x]))^m*Tan[(e + f*x)/2])/(f*Sqrt[Sec[(e + f*x)/2]^2]*(
AppellF1[1/2, 1/2 + m, 1/2, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]
- ((AppellF1[3/2, 1/2 + m, 3/2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]
^2] - (1 + 2*m)*AppellF1[3/2, 3/2 + m, 1/2, 5/2, Tan[(e + f*x)/2]^2, -Tan[
(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)/3)*((2^m*AppellF1[1/2, 1/2 + m, 1/2,
3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sqrt[Sec[(e + f*x)/2]^2]*(Co
s[(e + f*x)/2]^2*Sec[e + f*x])^(1/2 + m))/(AppellF1[1/2, 1/2 + m, 1/2, 3/2
, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - ((AppellF1[3/2, 1/2 + m, 3/2,
 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - (1 + 2*m)*AppellF1[3/2, 3
/2 + m, 1/2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/
2]^2)/3) - (2^m*AppellF1[1/2, 1/2 + m, 1/2, 3/2, Tan[(e + f*x)/2]^2, -Tan[
(e + f*x)/2]^2]*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(1/2 + m)*Tan[(e + f*x)/
2]^2)/(Sqrt[Sec[(e + f*x)/2]^2]*(AppellF1[1/2, 1/2 + m, 1/2, 3/2, Tan[(e +
 f*x)/2]^2, -Tan[(e + f*x)/2]^2] - ((AppellF1[3/2, 1/2 + m, 3/2, 5/2, Tan[
(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - (1 + 2*m)*AppellF1[3/2, 3/2 + m, 1/
2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)/3))
+ (2^(1 + m)*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(1/2 + m)*Tan[(e + f*x)/2]*
(-1/6*(AppellF1[3/2, 1/2 + m, 3/2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f...
```

3.348.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 4315, 3042, 4314, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{d\sec(e+fx)}(a\sec(e+fx)+a)^m\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{d\csc\left(e+fx+\frac{\pi}{2}\right)}\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^m dx$$

$$\Big\downarrow \text{4315}$$

$$(\sec(e+fx)+1)^{-m}(a\sec(e+fx)+a)^m\int \sqrt{d\sec(e+fx)}(\sec(e+fx)+1)^m dx$$

$$\Big\downarrow \text{3042}$$

$$(\sec(e+fx)+1)^{-m}(a\sec(e+fx)+a)^m\int\sqrt{d\csc\left(e+fx+\frac{\pi}{2}\right)}\left(\csc\left(e+fx+\frac{\pi}{2}\right)+1\right)^m dx$$

$$\Big\downarrow \text{4314}$$

$$-\frac{d\tan(e+fx)(\sec(e+fx)+1)^{-m-\frac{1}{2}}(a\sec(e+fx)+a)^m\int\frac{(\sec(e+fx)+1)^{m-\frac{1}{2}}}{\sqrt{1-\sec(e+fx)}\sqrt{d\sec(e+fx)}}d\sec(e+fx)}{f\sqrt{1-\sec(e+fx)}}$$

$$\Big\downarrow \text{150}$$

$$-\frac{2\tan(e+fx)\sqrt{d\sec(e+fx)}(\sec(e+fx)+1)^{-m-\frac{1}{2}}(a\sec(e+fx)+a)^m\operatorname{AppellF1}\left(\frac{1}{2},\frac{1}{2},\frac{1}{2}-m,\frac{3}{2},\sec(e+fx),\right.}{f\sqrt{1-\sec(e+fx)}}$$

input
```
Int[Sqrt[d*Sec[e + f*x]]*(a + a*Sec[e + f*x])^m,x]
```

output
```
(-2*AppellF1[1/2, 1/2, 1/2 - m, 3/2, Sec[e + f*x], -Sec[e + f*x]]*Sqrt[d*S
ec[e + f*x]]*(1 + Sec[e + f*x])^(-1/2 - m)*(a + a*Sec[e + f*x])^m*Tan[e +
f*x])/(f*Sqrt[1 - Sec[e + f*x]])
```

3.348.3.1 Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4314
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Simp[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x
]]*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(d*x)^(n - 1)*((a + b*x)^(m - 1/2
)/Sqrt[a - b*x]), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n},
x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

rule 4315
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Simp[a^IntPart[m]*((a + b*Csc[e + f*x])^FracPart[m
]/(1 + (b/a)*Csc[e + f*x])^FracPart[m])   Int[(1 + (b/a)*Csc[e + f*x])^m*(d
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^
2, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

3.348.4 Maple [F]

$$\int \sqrt{d\sec(fx+e)}\,(a+a\sec(fx+e))^m\,dx$$

input
```
int((d*sec(f*x+e))^(1/2)*(a+a*sec(f*x+e))^m,x)
```

output
```
int((d*sec(f*x+e))^(1/2)*(a+a*sec(f*x+e))^m,x)
```

3.348.5 Fricas [F]

$$\int \sqrt{d\sec(e+fx)}(a+a\sec(e+fx))^m\,dx=\int \sqrt{d\sec(fx+e)}(a\sec(fx+e)+a)^m\,dx$$

input
```
integrate((d*sec(f*x+e))^(1/2)*(a+a*sec(f*x+e))^m,x, algorithm="fricas")
```

output
```
integral(sqrt(d*sec(f*x + e))*(a*sec(f*x + e) + a)^m, x)
```

3.348.6 Sympy [F]

$$\int \sqrt{d\sec(e+fx)}(a+a\sec(e+fx))^m\,dx=\int (a(\sec(e+fx)+1))^m\sqrt{d\sec(e+fx)}\,dx$$

input
```
integrate((d*sec(f*x+e))**(1/2)*(a+a*sec(f*x+e))**m,x)
```

output
```
Integral((a*(sec(e + f*x) + 1))**m*sqrt(d*sec(e + f*x)), x)
```

3.348.7 Maxima [F]

$$\int \sqrt{d\sec(e+fx)}(a+a\sec(e+fx))^m\,dx = \int \sqrt{d\sec(fx+e)}(a\sec(fx+e)+a)^m\,dx$$

input
```
integrate((d*sec(f*x+e))^(1/2)*(a+a*sec(f*x+e))^m,x, algorithm="maxima")
```

output
```
integrate(sqrt(d*sec(f*x + e))*(a*sec(f*x + e) + a)^m, x)
```

3.348.8 Giac [F]

$$\int \sqrt{d\sec(e+fx)}(a+a\sec(e+fx))^m\,dx = \int \sqrt{d\sec(fx+e)}(a\sec(fx+e)+a)^m\,dx$$

input
```
integrate((d*sec(f*x+e))^(1/2)*(a+a*sec(f*x+e))^m,x, algorithm="giac")
```

output
```
integrate(sqrt(d*sec(f*x + e))*(a*sec(f*x + e) + a)^m, x)
```

3.348.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{d\sec(e+fx)}(a+a\sec(e+fx))^m\,dx = \int \left(a+\frac{a}{\cos(e+f\,x)}\right)^m\sqrt{\frac{d}{\cos(e+f\,x)}}\,dx$$

input
```
int((a + a/cos(e + f*x))^m*(d/cos(e + f*x))^(1/2),x)
```

output
```
int((a + a/cos(e + f*x))^m*(d/cos(e + f*x))^(1/2), x)
```

3.349 $\int \frac{(a+a\sec(e+fx))^m}{\sqrt{d\sec(e+fx)}}\,dx$

3.349.1 Optimal result 2580
3.349.2 Mathematica [C] (warning: unable to verify) 2580
3.349.3 Rubi [A] (verified) 2581
3.349.4 Maple [F] 2583
3.349.5 Fricas [F] 2583
3.349.6 Sympy [F] 2583
3.349.7 Maxima [F] 2584
3.349.8 Giac [F] 2584
3.349.9 Mupad [F(-1)] 2584

3.349.1 Optimal result

Integrand size = 25, antiderivative size = 96

$$\int \frac{(a+a\sec(e+fx))^m}{\sqrt{d\sec(e+fx)}}\,dx = \frac{2\,\text{AppellF1}\left(-\frac{1}{2},\frac{1}{2},\frac{1}{2}-m,\frac{1}{2},\sec(e+fx),-\sec(e+fx)\right)(1+\sec(e+fx))^{-\frac{1}{2}-m}(a+a\sec(e+fx))^m\tan}{f\sqrt{1-\sec(e+fx)}\sqrt{d\sec(e+fx)}}$$

output
```
2*AppellF1(-1/2,1/2-m,1/2,1/2,-sec(f*x+e),sec(f*x+e))*(1+sec(f*x+e))^(-1/2
-m)*(a+a*sec(f*x+e))^m*tan(f*x+e)/f/(1-sec(f*x+e))^(1/2)/(d*sec(f*x+e))^(1
/2)
```

3.349.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 19.21 (sec) , antiderivative size = 2424, normalized size of antiderivative = 25.25

$$\int \frac{(a+a\sec(e+fx))^m}{\sqrt{d\sec(e+fx)}}\,dx = \text{Result too large to show}$$

input
```
Integrate[(a + a*Sec[e + f*x])^m/Sqrt[d*Sec[e + f*x]],x]
```

output
```
(-3*2^(1 + m)*AppellF1[1/2, -1/2 + m, 3/2, 3/2, Tan[(e + f*x)/2]^2, -Tan[(
e + f*x)/2]^2]*Sqrt[Sec[e + f*x]]*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(-1/2
+ m)*(a*(1 + Sec[e + f*x]))^m*(Cos[2*(e + f*x)]*((1 + Sec[e + f*x])^m/(2*S
qrt[Sec[e + f*x]]) - (I/2)*Sqrt[Sec[e + f*x]]*(1 + Sec[e + f*x])^m*Sin[e +
 f*x]) + ((1 + Sec[e + f*x])^m/2 + (I/2)*(1 + Sec[e + f*x])^m*Sin[2*(e + f
*x)])/Sqrt[Sec[e + f*x]] + Sqrt[Sec[e + f*x]]*Sin[e + f*x]*((-1/2*I)*(1 +
Sec[e + f*x])^m + ((1 + Sec[e + f*x])^m*Sin[2*(e + f*x)])/2))*Tan[(e + f*x
)/2])/(f*(Sec[(e + f*x)/2]^2)^(3/2)*Sqrt[d*Sec[e + f*x]]*(1 + Sec[e + f*x]
)^m*(-3*AppellF1[1/2, -1/2 + m, 3/2, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*
x)/2]^2] + (3*AppellF1[3/2, -1/2 + m, 5/2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(
e + f*x)/2]^2] + (1 - 2*m)*AppellF1[3/2, 1/2 + m, 3/2, 5/2, Tan[(e + f*x)/
2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)*((-3*2^m*AppellF1[1/2, -1/
2 + m, 3/2, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(Cos[(e + f*x)/2
]^2*Sec[e + f*x])^(-1/2 + m))/(Sqrt[Sec[(e + f*x)/2]^2]*(-3*AppellF1[1/2,
-1/2 + m, 3/2, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (3*AppellF1
[3/2, -1/2 + m, 5/2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (1 -
2*m)*AppellF1[3/2, 1/2 + m, 3/2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2
]^2])*Tan[(e + f*x)/2]^2)) + (9*2^m*AppellF1[1/2, -1/2 + m, 3/2, 3/2, Tan[
(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(-1
/2 + m)*Tan[(e + f*x)/2]^2)/((Sec[(e + f*x)/2]^2)^(3/2)*(-3*AppellF1[1/...
```

3.349.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 4315, 3042, 4314, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sec(e+fx)+a)^m}{\sqrt{d\sec(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^m}{\sqrt{d\csc\left(e+fx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4315}$$

$$(\sec(e+fx)+1)^{-m}(a\sec(e+fx)+a)^m\int \frac{(\sec(e+fx)+1)^m}{\sqrt{d\sec(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$(\sec(e+fx)+1)^{-m}(a\sec(e+fx)+a)^m\int\frac{\left(\csc\left(e+fx+\frac{\pi}{2}\right)+1\right)^m}{\sqrt{d\csc\left(e+fx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4314}$$

$$-\frac{d\tan(e+fx)(\sec(e+fx)+1)^{-m-\frac{1}{2}}(a\sec(e+fx)+a)^m\int\frac{(\sec(e+fx)+1)^{m-\frac{1}{2}}}{\sqrt{1-\sec(e+fx)}(d\sec(e+fx))^{3/2}}d\sec(e+fx)}{f\sqrt{1-\sec(e+fx)}}$$

$$\Big\downarrow \text{150}$$

$$\frac{2\tan(e+fx)(\sec(e+fx)+1)^{-m-\frac{1}{2}}(a\sec(e+fx)+a)^m\operatorname{AppellF1}\left(-\frac{1}{2},\frac{1}{2},\frac{1}{2}-m,\frac{1}{2},\sec(e+fx),-\sec(e+fx)\right)}{f\sqrt{1-\sec(e+fx)}\sqrt{d\sec(e+fx)}}$$

input
```
Int[(a + a*Sec[e + f*x])^m/Sqrt[d*Sec[e + f*x]],x]
```

output
```
(2*AppellF1[-1/2, 1/2, 1/2 - m, 1/2, Sec[e + f*x], -Sec[e + f*x]]*(1 + Sec
[e + f*x])^(-1/2 - m)*(a + a*Sec[e + f*x])^m*Tan[e + f*x])/(f*Sqrt[1 - Sec
[e + f*x]]*Sqrt[d*Sec[e + f*x]])
```

3.349.3.1 Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4314
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Simp[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x
]]*Sqrt[a - b*Csc[e + f*x]]))   Subst[Int[(d*x)^(n - 1)*((a + b*x)^(m - 1/2
)/Sqrt[a - b*x]), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n},
x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

rule 4315
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Simp[a^IntPart[m]*((a + b*Csc[e + f*x])^FracPart[m
]/(1 + (b/a)*Csc[e + f*x])^FracPart[m])   Int[(1 + (b/a)*Csc[e + f*x])^m*(d
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^
2, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

3.349.4 Maple [F]

$$\int \frac{(a+a\sec(fx+e))^m}{\sqrt{d\sec(fx+e)}}dx$$

input
```
int((a+a*sec(f*x+e))^m/(d*sec(f*x+e))^(1/2),x)
```

output
```
int((a+a*sec(f*x+e))^m/(d*sec(f*x+e))^(1/2),x)
```

3.349.5 Fricas [F]

$$\int \frac{(a+a\sec(e+fx))^m}{\sqrt{d\sec(e+fx)}}\,dx=\int \frac{(a\sec(fx+e)+a)^m}{\sqrt{d\sec(fx+e)}}\,dx$$

input
```
integrate((a+a*sec(f*x+e))^m/(d*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(d*sec(f*x + e))*(a*sec(f*x + e) + a)^m/(d*sec(f*x + e)), x)
```

3.349.6 Sympy [F]

$$\int \frac{(a+a\sec(e+fx))^m}{\sqrt{d\sec(e+fx)}}\,dx=\int \frac{(a(\sec(e+fx)+1))^m}{\sqrt{d\sec(e+fx)}}\,dx$$

input
```
integrate((a+a*sec(f*x+e))**m/(d*sec(f*x+e))**(1/2),x)
```

output
```
Integral((a*(sec(e + f*x) + 1))**m/sqrt(d*sec(e + f*x)), x)
```

3.349.7 Maxima [F]

$$\int \frac{(a+a\sec(e+fx))^m}{\sqrt{d\sec(e+fx)}}\,dx = \int \frac{(a\sec(fx+e)+a)^m}{\sqrt{d\sec(fx+e)}}\,dx$$

input
```
integrate((a+a*sec(f*x+e))^m/(d*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate((a*sec(f*x + e) + a)^m/sqrt(d*sec(f*x + e)), x)
```

3.349.8 Giac [F]

$$\int \frac{(a+a\sec(e+fx))^m}{\sqrt{d\sec(e+fx)}}\,dx = \int \frac{(a\sec(fx+e)+a)^m}{\sqrt{d\sec(fx+e)}}\,dx$$

input
```
integrate((a+a*sec(f*x+e))^m/(d*sec(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate((a*sec(f*x + e) + a)^m/sqrt(d*sec(f*x + e)), x)
```

3.349.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(e+fx))^m}{\sqrt{d\sec(e+fx)}}\,dx = \int \frac{\left(a+\frac{a}{\cos(e+f\,x)}\right)^m}{\sqrt{\frac{d}{\cos(e+f\,x)}}}\,dx$$

input
```
int((a + a/cos(e + f*x))^m/(d/cos(e + f*x))^(1/2),x)
```

output
```
int((a + a/cos(e + f*x))^m/(d/cos(e + f*x))^(1/2), x)
```

3.350 $\int \frac{(a+a\sec(e+fx))^m}{(d\sec(e+fx))^{3/2}}\,dx$

3.350.1 Optimal result 2585
3.350.2 Mathematica [C] (warning: unable to verify) 2585
3.350.3 Rubi [A] (verified) 2586
3.350.4 Maple [F] 2588
3.350.5 Fricas [F] 2588
3.350.6 Sympy [F] 2588
3.350.7 Maxima [F] 2589
3.350.8 Giac [F] 2589
3.350.9 Mupad [F(-1)] 2589

3.350.1 Optimal result

Integrand size = 25, antiderivative size = 98

$$\int \frac{(a+a\sec(e+fx))^m}{(d\sec(e+fx))^{3/2}}\,dx = \frac{2\,\text{AppellF1}\left(-\frac{3}{2},\frac{1}{2},\frac{1}{2}-m,-\frac{1}{2},\sec(e+fx),-\sec(e+fx)\right)(1+\sec(e+fx))^{-}}{3f\sqrt{1-\sec(e+fx)}(d\sec(e+fx))^{3/2}}$$

output
```
2/3*AppellF1(-3/2,1/2-m,1/2,-1/2,-sec(f*x+e),sec(f*x+e))*(1+sec(f*x+e))^(-
1/2-m)*(a+a*sec(f*x+e))^m*tan(f*x+e)/f/(d*sec(f*x+e))^(3/2)/(1-sec(f*x+e))
^(1/2)
```

3.350.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 22.33 (sec) , antiderivative size = 3349, normalized size of antiderivative = 34.17

$$\int \frac{(a+a\sec(e+fx))^m}{(d\sec(e+fx))^{3/2}}\,dx = \text{Result too large to show}$$

input
```
Integrate[(a + a*Sec[e + f*x])^m/(d*Sec[e + f*x])^(3/2),x]
```

output
```
(2^(1 + m)*Sec[e + f*x]^(3/2)*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(1/2 + m)*
(a*(1 + Sec[e + f*x]))^m*((Cos[2*(e + f*x)]^3*Sqrt[Sec[e + f*x]]*(1 + Sec[
e + f*x])^m)/4 + Cos[2*(e + f*x)]^2*Sqrt[Sec[e + f*x]]*((1 + Sec[e + f*x])
^m/2 + (I/4)*(1 + Sec[e + f*x])^m*Sin[2*(e + f*x)]) + Cos[2*(e + f*x)]*Sqr
t[Sec[e + f*x]]*((1 + Sec[e + f*x])^m/4 + ((1 + Sec[e + f*x])^m*Sin[2*(e +
 f*x)]^2)/4) + Sqrt[Sec[e + f*x]]*((-1/4*I)*(1 + Sec[e + f*x])^m*Sin[2*(e
+ f*x)] + ((1 + Sec[e + f*x])^m*Sin[2*(e + f*x)]^2)/2 + (I/4)*(1 + Sec[e +
 f*x])^m*Sin[2*(e + f*x)]^3))*Tan[(e + f*x)/2]*(-(AppellF1[3/2, -1/2 + m,
5/2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(Cos[e + f*x]*Sec[(e +
f*x)/2]^2)^(1/2 + m)*Tan[(e + f*x)/2]^2) - (9*AppellF1[1/2, -1/2 + m, 5/2,
 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[e + f*x])/((Sec[(e + f*
x)/2]^2)^(3/2)*(-3*AppellF1[1/2, -1/2 + m, 5/2, 3/2, Tan[(e + f*x)/2]^2, -
Tan[(e + f*x)/2]^2] + (5*AppellF1[3/2, -1/2 + m, 7/2, 5/2, Tan[(e + f*x)/2
]^2, -Tan[(e + f*x)/2]^2] + (1 - 2*m)*AppellF1[3/2, 1/2 + m, 5/2, 5/2, Tan
[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2))))/(3*f*(d*Sec[
e + f*x])^(3/2)*(1 + Sec[e + f*x])^m*((2^m*Sec[(e + f*x)/2]^2*(Cos[(e + f*
x)/2]^2*Sec[e + f*x])^(1/2 + m)*(-(AppellF1[3/2, -1/2 + m, 5/2, 5/2, Tan[(
e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(Cos[e + f*x]*Sec[(e + f*x)/2]^2)^(1/2
 + m)*Tan[(e + f*x)/2]^2) - (9*AppellF1[1/2, -1/2 + m, 5/2, 3/2, Tan[(e +
f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[e + f*x])/((Sec[(e + f*x)/2]^2)^(3/...
```

3.350.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 4315, 3042, 4314, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sec(e+fx)+a)^m}{(d\sec(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^m}{\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{4315}$$

$$(\sec(e+fx)+1)^{-m}(a\sec(e+fx)+a)^m\int \frac{(\sec(e+fx)+1)^m}{(d\sec(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$(\sec(e+fx)+1)^{-m}(a\sec(e+fx)+a)^m\int\frac{\left(\csc\left(e+fx+\frac{\pi}{2}\right)+1\right)^m}{\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{4314}$$

$$-\frac{d\tan(e+fx)(\sec(e+fx)+1)^{-m-\frac{1}{2}}(a\sec(e+fx)+a)^m\int\frac{(\sec(e+fx)+1)^{m-\frac{1}{2}}}{\sqrt{1-\sec(e+fx)}(d\sec(e+fx))^{5/2}}d\sec(e+fx)}{f\sqrt{1-\sec(e+fx)}}$$

$$\Big\downarrow \text{150}$$

$$\frac{2\tan(e+fx)(\sec(e+fx)+1)^{-m-\frac{1}{2}}(a\sec(e+fx)+a)^m\operatorname{AppellF1}\left(-\frac{3}{2},\frac{1}{2},\frac{1}{2}-m,-\frac{1}{2},\sec(e+fx),-\sec(e+fx\right.}{3f\sqrt{1-\sec(e+fx)}(d\sec(e+fx))^{3/2}}$$

input
```
Int[(a + a*Sec[e + f*x])^m/(d*Sec[e + f*x])^(3/2),x]
```

output
```
(2*AppellF1[-3/2, 1/2, 1/2 - m, -1/2, Sec[e + f*x], -Sec[e + f*x]]*(1 + Se
c[e + f*x])^(-1/2 - m)*(a + a*Sec[e + f*x])^m*Tan[e + f*x])/(3*f*Sqrt[1 -
Sec[e + f*x]]*(d*Sec[e + f*x])^(3/2))
```

3.350.3.1 Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] && !In
tegerQ[m] && !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4314
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Simp[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x
]]*Sqrt[a - b*Csc[e + f*x]])) Subst[Int[(d*x)^(n - 1)*((a + b*x)^(m - 1/2
)/Sqrt[a - b*x]), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n},
x] && EqQ[a^2 - b^2, 0] && !IntegerQ[m] && GtQ[a, 0]
```

rule 4315
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Simp[a^IntPart[m]*((a + b*Csc[e + f*x])^FracPart[m
]/(1 + (b/a)*Csc[e + f*x])^FracPart[m])   Int[(1 + (b/a)*Csc[e + f*x])^m*(d
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^
2, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

3.350.4 Maple [F]

$$\int \frac{(a+a\sec(fx+e))^m}{(d\sec(fx+e))^{\frac{3}{2}}}dx$$

input
```
int((a+a*sec(f*x+e))^m/(d*sec(f*x+e))^(3/2),x)
```

output
```
int((a+a*sec(f*x+e))^m/(d*sec(f*x+e))^(3/2),x)
```

3.350.5 Fricas [F]

$$\int \frac{(a+a\sec(e+fx))^m}{(d\sec(e+fx))^{3/2}}\,dx=\int \frac{(a\sec(fx+e)+a)^m}{(d\sec(fx+e))^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+a*sec(f*x+e))^m/(d*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(d*sec(f*x + e))*(a*sec(f*x + e) + a)^m/(d^2*sec(f*x + e)^2),
 x)
```

3.350.6 Sympy [F]

$$\int \frac{(a+a\sec(e+fx))^m}{(d\sec(e+fx))^{3/2}}\,dx=\int \frac{(a(\sec(e+fx)+1))^m}{(d\sec(e+fx))^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+a*sec(f*x+e))**m/(d*sec(f*x+e))**(3/2),x)
```

output
```
Integral((a*(sec(e + f*x) + 1))**m/(d*sec(e + f*x))**(3/2), x)
```

3.350.7 Maxima [F]

$$\int \frac{(a+a\sec(e+fx))^m}{(d\sec(e+fx))^{3/2}}\,dx = \int \frac{(a\sec(fx+e)+a)^m}{(d\sec(fx+e))^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+a*sec(f*x+e))^m/(d*sec(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
integrate((a*sec(f*x + e) + a)^m/(d*sec(f*x + e))^(3/2), x)
```

3.350.8 Giac [F]

$$\int \frac{(a+a\sec(e+fx))^m}{(d\sec(e+fx))^{3/2}}\,dx = \int \frac{(a\sec(fx+e)+a)^m}{(d\sec(fx+e))^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+a*sec(f*x+e))^m/(d*sec(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
integrate((a*sec(f*x + e) + a)^m/(d*sec(f*x + e))^(3/2), x)
```

3.350.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(e+fx))^m}{(d\sec(e+fx))^{3/2}}\,dx = \int \frac{\left(a+\frac{a}{\cos(e+f\,x)}\right)^m}{\left(\frac{d}{\cos(e+f\,x)}\right)^{3/2}}\,dx$$

input
```
int((a + a/cos(e + f*x))^m/(d/cos(e + f*x))^(3/2),x)
```

output
```
int((a + a/cos(e + f*x))^m/(d/cos(e + f*x))^(3/2), x)
```

3.351 $\int \cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))\,dx$

3.351.1 Optimal result 2590
3.351.2 Mathematica [C] (warning: unable to verify) 2591
3.351.3 Rubi [A] (verified) 2592
3.351.4 Maple [A] (verified) 2595
3.351.5 Fricas [C] (verification not implemented) 2595
3.351.6 Sympy [F(-1)] 2596
3.351.7 Maxima [F] 2596
3.351.8 Giac [F] 2596
3.351.9 Mupad [B] (verification not implemented) 2597

3.351.1 Optimal result

Integrand size = 21, antiderivative size = 111

$$\int \cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))\,dx = \frac{6aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{10a\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d}+\frac{10a\sqrt{\cos(c+dx)}\sin(c+dx)}{21d}+\frac{2a\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d}+\frac{2a\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{7d}$$

output
```
6/5*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*
x+1/2*c),2^(1/2))/d+10/21*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c
)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/5*a*cos(d*x+c)^(3/2)*sin(d*x+c
)/d+2/7*a*cos(d*x+c)^(5/2)*sin(d*x+c)/d+10/21*a*sin(d*x+c)*cos(d*x+c)^(1/2
)/d
```

3.351.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 6.09 (sec) , antiderivative size = 490, normalized size of antiderivative = 4.41

$$\int \cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))\,dx = a\left(\sqrt{\cos(c+dx)}(1+\cos(c+dx))\sec^2\left(\frac{c}{2}+\frac{dx}{2}\right)\left(-\frac{3\cot(c)}{5d}+\frac{23\cos(dx)\sin(c)}{84d}+\frac{\cos(2dx)\sin(2c)}{10d}+\frac{\cos(3dx)\sin(3c)}{28d}+\frac{23\cos(c)\sin(dx)}{84d}+\frac{\cos(2c)\sin(2dx)}{10d}+\frac{\cos(3c)\sin(3dx)}{28d}\right)\right.$$

$$-\frac{5(1+\cos(c+dx))\csc(c)\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};\sin^2(dx-\arctan(\cot(c)))\right)\sec^2\left(\frac{c}{2}+\frac{dx}{2}\right)\sec(dx-\arctan(\cot(c)))}{21d}$$

$$-\frac{3(1+\cos(c+dx))\csc(c)\sec^2\left(\frac{c}{2}+\frac{dx}{2}\right)\left(\frac{{}_2F_1\left(-\frac{1}{2},-\frac{1}{4};\frac{3}{4};\cos^2(dx+\arctan(\tan(c)))\right)\sin(dx+\arctan(\tan(}{\sqrt{1-\cos(dx+\arctan(\tan(c)))}\sqrt{1+\cos(dx+\arctan(\tan(c)))}\sqrt{\cos(c)\cos(dx+\arctan(t}}\right.}{10d}$$

input
```
Integrate[Cos[c + d*x]^(7/2)*(a + a*Sec[c + d*x]),x]
```

output
```
a*(Sqrt[Cos[c + d*x]]*(1 + Cos[c + d*x])*Sec[c/2 + (d*x)/2]^2*((-3*Cot[c])
/(5*d) + (23*Cos[d*x]*Sin[c])/(84*d) + (Cos[2*d*x]*Sin[2*c])/(10*d) + (Cos
[3*d*x]*Sin[3*c])/(28*d) + (23*Cos[c]*Sin[d*x])/(84*d) + (Cos[2*c]*Sin[2*d
*x])/(10*d) + (Cos[3*c]*Sin[3*d*x])/(28*d)) - (5*(1 + Cos[c + d*x])*Csc[c]
*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2
 + (d*x)/2]^2*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]
]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Si
n[d*x - ArcTan[Cot[c]]]])/(21*d*Sqrt[1 + Cot[c]^2]) - (3*(1 + Cos[c + d*x]
)*Csc[c]*Sec[c/2 + (d*x)/2]^2*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos
[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[
d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos
[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x
 + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcT
an[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x
 + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(10*d))
```

3.351.3 Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.05, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.524$, Rules used = {3042, 4713, 3042, 3227, 3042, 3115, 3042, 3115, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^{\frac{7}{2}}(c+dx)(a\sec(c+dx)+a)\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^{7/2}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)dx$$

$$\Big\downarrow \; 4713$$

$$\int \cos^{\frac{5}{2}}(c+dx)(a\cos(c+dx)+a)dx$$

$$\Big\downarrow \; 3042$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)dx$$

$$\Big\downarrow \; 3227$$

$$a\int \cos^{\frac{5}{2}}(c+dx)dx + a\int \cos^{\frac{7}{2}}(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$a\int \sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}dx+a\int \sin\left(c+dx+\frac{\pi}{2}\right)^{7/2}dx$$

$$\Big\downarrow \text{3115}$$

$$a\left(\frac{3}{5}\int \sqrt{\cos(c+dx)}dx+\frac{2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}\right)+a\left(\frac{5}{7}\int \cos^{\frac{3}{2}}(c+dx)dx+\frac{2\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d}\right)$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{3}{5}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}\right)+a\left(\frac{5}{7}\int \sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}dx+\frac{2\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d}\right)$$

$$\Big\downarrow \text{3115}$$

$$a\left(\frac{5}{7}\left(\frac{1}{3}\int \frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}\right)+\frac{2\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d}\right)+a\left(\frac{3}{5}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}\right)$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{5}{7}\left(\frac{1}{3}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}\right)+\frac{2\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d}\right)+a\left(\frac{3}{5}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}\right)$$

$$\Big\downarrow \text{3119}$$

$$a\left(\frac{5}{7}\left(\frac{1}{3}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}\right)+\frac{2\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d}\right)+a\left(\frac{6E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}\right)$$

$$\Big\downarrow \text{3120}$$

3.351. $\int \cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))\,dx$

$$a\left(\frac{2\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d}+\frac{5}{7}\left(\frac{2\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}\right)\right)+a\left(\frac{6E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}\right)$$

input
```
Int[Cos[c + d*x]^(7/2)*(a + a*Sec[c + d*x]),x]
```

output
```
a*((6*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*Cos[c + d*x]^(3/2)*Sin[c + d*x
])/(5*d)) + a*((2*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(7*d) + (5*((2*Elliptic
F[(c + d*x)/2, 2])/(3*d) + (2*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d)))/7)
```

3.351.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 4713
```
Int[(csc[(a_.) + (b_.)*(x_)]*(B_.) + (A_))*(u_), x_Symbol] :> Int[ActivateT
rig[u]*((B + A*Sin[a + b*x])/Sin[a + b*x]), x] /; FreeQ[{a, b, A, B}, x] &&
 KnownSineIntegrandQ[u, x]
```

3.351.4 Maple [A] (verified)

Time = 9.44 (sec) , antiderivative size = 270, normalized size of antiderivative = 2.43

method	result
default	$-\frac{2\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,a\left(240\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{8}-528\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+448\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}\cos\left(\frac{d}{2}\right.\right.}{105\sqrt{-2\sin}}$

input
```
int(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-2/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a*(240*cos(
1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8-528*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1
/2*c)+448*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-122*cos(1/2*d*x+1/2*c)*s
in(1/2*d*x+1/2*c)^2+25*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^
2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-63*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1
/2))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*
c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

3.351.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.33

$$\int \cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))\,dx$$
$$=\frac{2\left(15\,a\cos\left(dx+c\right)^{2}+21\,a\cos\left(dx+c\right)+25\,a\right)\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)-25i\,\sqrt{2}a\text{weierstrassPInverse}(}{}$$

input
```
integrate(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c)),x, algorithm="fricas")
```

output
```
1/105*(2*(15*a*cos(d*x + c)^2 + 21*a*cos(d*x + c) + 25*a)*sqrt(cos(d*x + c
))*sin(d*x + c) - 25*I*sqrt(2)*a*weierstrassPInverse(-4, 0, cos(d*x + c) +
 I*sin(d*x + c)) + 25*I*sqrt(2)*a*weierstrassPInverse(-4, 0, cos(d*x + c)
- I*sin(d*x + c)) + 63*I*sqrt(2)*a*weierstrassZeta(-4, 0, weierstrassPInve
rse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 63*I*sqrt(2)*a*weierstrassZet
a(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/d
```

3.351.6 Sympy [F(-1)]

Timed out.

$$\int \cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(7/2)*(a+a*sec(d*x+c)),x)
```

output
```
Timed out
```

3.351.7 Maxima [F]

$$\int \cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))\,dx = \int (a\sec(dx+c)+a)\cos(dx+c)^{\frac{7}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c)),x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)*cos(d*x + c)^(7/2), x)
```

3.351.8 Giac [F]

$$\int \cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))\,dx = \int (a\sec(dx+c)+a)\cos(dx+c)^{\frac{7}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c)),x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)*cos(d*x + c)^(7/2), x)
```

3.351.9 Mupad [B] (verification not implemented)

Time = 13.78 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.78

$$\int \cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))\,dx$$
$$= -\frac{2\,a\cos\left(c+d\,x\right)^{7/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{7}{4};\ \frac{11}{4};\ \cos\left(c+d\,x\right)^2\right)}{7\,d\,\sqrt{\sin\left(c+d\,x\right)^2}} - \frac{2\,a\cos\left(c+d\,x\right)^{9/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{9}{4};\ \frac{13}{4};\ \cos\left(c+d\,x\right)^2\right)}{9\,d\,\sqrt{\sin\left(c+d\,x\right)^2}}$$

input
```
int(cos(c + d*x)^(7/2)*(a + a/cos(c + d*x)),x)
```

output
```
- (2*a*cos(c + d*x)^(7/2)*sin(c + d*x)*hypergeom([1/2, 7/4], 11/4, cos(c +
 d*x)^2))/(7*d*(sin(c + d*x)^2)^(1/2)) - (2*a*cos(c + d*x)^(9/2)*sin(c + d
*x)*hypergeom([1/2, 9/4], 13/4, cos(c + d*x)^2))/(9*d*(sin(c + d*x)^2)^(1/
2))
```

3.352 $\int \cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))\,dx$

3.352.1 Optimal result 2598
3.352.2 Mathematica [C] (warning: unable to verify) 2598
3.352.3 Rubi [A] (verified) 2599
3.352.4 Maple [A] (verified) 2601
3.352.5 Fricas [C] (verification not implemented) 2602
3.352.6 Sympy [F(-1)] 2602
3.352.7 Maxima [F] 2603
3.352.8 Giac [F] 2603
3.352.9 Mupad [B] (verification not implemented) 2603

3.352.1 Optimal result

Integrand size = 21, antiderivative size = 87

$$\int \cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))\,dx = \frac{6aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2a\sqrt{\cos(c+dx)}\sin(c+dx)}{3d}+\frac{2a\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d}$$

output
```
6/5*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*
x+1/2*c),2^(1/2))/d+2/3*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*
EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/5*a*cos(d*x+c)^(3/2)*sin(d*x+c)/
d+2/3*a*sin(d*x+c)*cos(d*x+c)^(1/2)/d
```

3.352.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 5.65 (sec) , antiderivative size = 232, normalized size of antiderivative = 2.67

$$\int \cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))\,dx = \frac{a(1+\cos(c+dx))\sec^2\left(\frac{1}{2}(c+dx)\right)\left(\frac{9(3\cos(c-dx-\arctan(\tan(c)))+\cos(c+dx+\arctan(\tan(c))))\csc(c)\sec(c)}{\sqrt{\sec^2(c)}}-20\cos(c+d\right.}{}$$

input
```
Integrate[Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x]),x]
```

output
```
(a*(1 + Cos[c + d*x])*Sec[(c + d*x)/2]^2*((9*(3*Cos[c - d*x - ArcTan[Tan[c
]]] + Cos[c + d*x + ArcTan[Tan[c]]])*Csc[c]*Sec[c])/Sqrt[Sec[c]^2] - 20*Co
s[c + d*x]*Sqrt[Cos[d*x - ArcTan[Cot[c]]]^2]*Sqrt[Csc[c]^2]*Hypergeometric
PFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[d*x - ArcTan[Cot[c
]]]*Sin[c] + 2*Cos[c + d*x]*(-18*Cot[c] + 10*Sin[c + d*x] + 3*Sin[2*(c + d
*x)]) - 18*Cos[c]*Csc[d*x + ArcTan[Tan[c]]]*HypergeometricPFQ[{-1/2, -1/4}
, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sqrt[Sec[c]^2]*Sqrt[Sin[d*x + ArcTan
[Tan[c]]]^2]))/(60*d*Sqrt[Cos[c + d*x]])
```

3.352.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.02, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 4713, 3042, 3227, 3042, 3115, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^{\frac{5}{2}}(c+dx)(a\sec(c+dx)+a)\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)dx$$

$$\Big\downarrow \; 4713$$

$$\int \cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)dx$$

$$\Big\downarrow \; 3042$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)dx$$

$$\Big\downarrow \; 3227$$

$$a\int \cos^{\frac{3}{2}}(c+dx)dx+a\int \cos^{\frac{5}{2}}(c+dx)dx$$

$$\Big\downarrow \; 3042$$

$$a\int \sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}dx+a\int \sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}dx$$

$$\Big\downarrow \text{3115}$$

$$a\left(\frac{3}{5}\int \sqrt{\cos(c+dx)}dx+\frac{2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}\right)+a\left(\frac{1}{3}\int \frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}\right)$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{3}{5}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}\right)+a\left(\frac{1}{3}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}\right)$$

$$\Big\downarrow \text{3119}$$

$$a\left(\frac{1}{3}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}\right)+a\left(\frac{6E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}\right)$$

$$\Big\downarrow \text{3120}$$

$$a\left(\frac{6E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}\right)+a\left(\frac{2\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}\right)$$

input
```
Int[Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x]),x]
```

output
```
a*((2*EllipticF[(c + d*x)/2, 2])/(3*d) + (2*Sqrt[Cos[c + d*x]]*Sin[c + d*x
])/(3*d)) + a*((6*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*Cos[c + d*x]^(3/2)
*Sin[c + d*x])/(5*d))
```

3.352. $\int \cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))\,dx$

3.352.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 4713
```
Int[(csc[(a_.) + (b_.)*(x_)]*(B_.) + (A_))*(u_), x_Symbol] :> Int[ActivateT
rig[u]*((B + A*Sin[a + b*x])/Sin[a + b*x]), x] /; FreeQ[{a, b, A, B}, x] &&
 KnownSineIntegrandQ[u, x]
```

3.352.4 Maple [A] (verified)

Time = 6.56 (sec) , antiderivative size = 219, normalized size of antiderivative = 2.52

method	result
default	$-\frac{2\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,a\left(24\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}-28\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}+5\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+1}\,\mathrm{EllipticF}\left(\right.\right.}{15\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right.}$

input
```
int(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-2/15*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a*(24*cos(1/
2*d*x+1/2*c)^7-28*cos(1/2*d*x+1/2*c)^5+5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*
cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-9*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1
/2*d*x+1/2*c),2^(1/2))+4*cos(1/2*d*x+1/2*c))/(-2*sin(1/2*d*x+1/2*c)^4+sin(
1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2
)/d
```

3.352.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.57

$$\int \cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))\,dx$$
$$= \frac{2\,(3\,a\cos(dx+c)+5\,a)\sqrt{\cos(dx+c)}\sin(dx+c)-5i\,\sqrt{2}a\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\ \text{s}}{}$$

input
```
integrate(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c)),x, algorithm="fricas")
```

output
```
1/15*(2*(3*a*cos(d*x + c) + 5*a)*sqrt(cos(d*x + c))*sin(d*x + c) - 5*I*sqr
t(2)*a*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 5*I*sqr
t(2)*a*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 9*I*sqr
t(2)*a*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*
sin(d*x + c))) - 9*I*sqrt(2)*a*weierstrassZeta(-4, 0, weierstrassPInverse(
-4, 0, cos(d*x + c) - I*sin(d*x + c))))/d
```

3.352.6 Sympy [F(-1)]

Timed out.

$$\int \cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(5/2)*(a+a*sec(d*x+c)),x)
```

output
```
Timed out
```

3.352.7 Maxima [F]

$$\int \cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))\,dx = \int (a\sec(dx+c)+a)\cos(dx+c)^{\frac{5}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c)),x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)*cos(d*x + c)^(5/2), x)
```

3.352.8 Giac [F]

$$\int \cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))\,dx = \int (a\sec(dx+c)+a)\cos(dx+c)^{\frac{5}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c)),x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)*cos(d*x + c)^(5/2), x)
```

3.352.9 Mupad [B] (verification not implemented)

Time = 13.51 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.92

$$\int \cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))\,dx = \frac{2\,a\,\mathrm{F}\left(\frac{c}{2}+\frac{dx}{2}\middle|2\right)}{3\,d} + \frac{2\,a\,\sqrt{\cos(c+d\,x)}\,\sin(c+d\,x)}{3\,d} - \frac{2\,a\cos(c+d\,x)^{7/2}\,\sin(c+d\,x)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{7}{4};\ \frac{11}{4};\ \cos(c+d\,x)^2\right)}{7\,d\,\sqrt{\sin(c+d\,x)^2}}$$

input
```
int(cos(c + d*x)^(5/2)*(a + a/cos(c + d*x)),x)
```

output
```
(2*a*ellipticF(c/2 + (d*x)/2, 2))/(3*d) + (2*a*cos(c + d*x)^(1/2)*sin(c +
d*x))/(3*d) - (2*a*cos(c + d*x)^(7/2)*sin(c + d*x)*hypergeom([1/2, 7/4], 1
1/4, cos(c + d*x)^2))/(7*d*(sin(c + d*x)^2)^(1/2))
```

3.353 $\int \cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))\,dx$

3.353.1 Optimal result 2604
3.353.2 Mathematica [C] (warning: unable to verify) 2604
3.353.3 Rubi [A] (verified) 2605
3.353.4 Maple [B] (verified) 2607
3.353.5 Fricas [C] (verification not implemented) 2608
3.353.6 Sympy [F(-1)] 2608
3.353.7 Maxima [F] 2608
3.353.8 Giac [F] 2609
3.353.9 Mupad [B] (verification not implemented) 2609

3.353.1 Optimal result

Integrand size = 21, antiderivative size = 61

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))\,dx = \frac{2aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2a\sqrt{\cos(c+dx)}\sin(c+dx)}{3d}$$

output
```
2*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+
1/2*c),2^(1/2))/d+2/3*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*El
lipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/3*a*sin(d*x+c)*cos(d*x+c)^(1/2)/d
```

3.353.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 4.89 (sec) , antiderivative size = 222, normalized size of antiderivative = 3.64

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))\,dx = \frac{a(1+\cos(c+dx))\sec^2\left(\frac{1}{2}(c+dx)\right)\left(\frac{3(3\cos(c-dx-\arctan(\tan(c)))+\cos(c+dx+\arctan(\tan(c))))\csc(c)\sec(c)}{\sqrt{\sec^2(c)}}-4\cos(c+dx\right.}{}$$

input
```
Integrate[Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x]),x]
```

output
```
(a*(1 + Cos[c + d*x])*Sec[(c + d*x)/2]^2*((3*(3*Cos[c - d*x - ArcTan[Tan[c
]]] + Cos[c + d*x + ArcTan[Tan[c]]])*Csc[c]*Sec[c])/Sqrt[Sec[c]^2] - 4*Cos
[c + d*x]*Sqrt[Cos[d*x - ArcTan[Cot[c]]]^2]*Sqrt[Csc[c]^2]*HypergeometricP
FQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[d*x - ArcTan[Cot[c]
]]*Sin[c] - 4*Cos[c + d*x]*(3*Cot[c] - Sin[c + d*x]) - 6*Cos[c]*Csc[d*x +
ArcTan[Tan[c]]]*HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Ta
n[c]]]^2]*Sqrt[Sec[c]^2]*Sqrt[Sin[d*x + ArcTan[Tan[c]]]^2]))/(12*d*Sqrt[Co
s[c + d*x]])
```

3.353.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.02, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 4713, 3042, 3227, 3042, 3115, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)dx$$

$$\Big\downarrow \text{4713}$$

$$\int \sqrt{\cos(c+dx)}(a\cos(c+dx)+a)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+a\right)dx$$

$$\Big\downarrow \text{3227}$$

$$a\int \cos^{\frac{3}{2}}(c+dx)dx+a\int \sqrt{\cos(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$a\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+a\int \sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}dx$$

$$\Big\downarrow \text{3115}$$

$$a\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+a\left(\frac{1}{3}\int \frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}\right)$$

$$\Big\downarrow \text{3042}$$

$$a\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+a\left(\frac{1}{3}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}\right)$$

$$\Big\downarrow \text{3119}$$

$$a\left(\frac{1}{3}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}\right)+\frac{2aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

$$\Big\downarrow \text{3120}$$

$$\frac{2aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+a\left(\frac{2\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}\right)$$

input
```
Int[Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x]),x]
```

output
```
(2*a*EllipticE[(c + d*x)/2, 2])/d + a*((2*EllipticF[(c + d*x)/2, 2])/(3*d)
 + (2*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d))
```

3.353.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 4713
```
Int[(csc[(a_.) + (b_.)*(x_)]*(B_.) + (A_))*(u_), x_Symbol] :> Int[ActivateT
rig[u]*((B + A*Sin[a + b*x])/Sin[a + b*x]), x] /; FreeQ[{a, b, A, B}, x] &&
 KnownSineIntegrandQ[u, x]
```

3.353.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 224 vs. $2(107) = 214$.

Time = 6.24 (sec) , antiderivative size = 225, normalized size of antiderivative = 3.69

method	result
default	$-\frac{2\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\,a\left(4\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\sin\left(\frac{dx}{2}+\right.}}{3\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right.}$

input
```
int(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-2/3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a*(4*sin(1/2*
d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(co
s(1/2*d*x+1/2*c),2^(1/2))-3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2
*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2))/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)
^2-1)^(1/2)/d
```

3.353.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 125, normalized size of antiderivative = 2.05

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))\,dx$$
$$= \frac{2\,a\sqrt{\cos(dx+c)}\sin(dx+c) - i\sqrt{2}a\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c)) + i\sqrt{2}a\text{we}}{}$$

input
```
integrate(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c)),x, algorithm="fricas")
```

output
```
1/3*(2*a*sqrt(cos(d*x + c))*sin(d*x + c) - I*sqrt(2)*a*weierstrassPInverse
(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + I*sqrt(2)*a*weierstrassPInverse(-
4, 0, cos(d*x + c) - I*sin(d*x + c)) + 3*I*sqrt(2)*a*weierstrassZeta(-4, 0
, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 3*I*sqrt(2)
*a*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(
d*x + c))))/d
```

3.353.6 Sympy [F(-1)]

Timed out.

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(3/2)*(a+a*sec(d*x+c)),x)
```

output
```
Timed out
```

3.353.7 Maxima [F]

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))\,dx = \int (a\sec(dx+c)+a)\cos(dx+c)^{\frac{3}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c)),x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)*cos(d*x + c)^(3/2), x)
```

3.353.8 Giac [F]

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))\,dx = \int (a\sec(dx+c)+a)\cos(dx+c)^{\frac{3}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c)),x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)*cos(d*x + c)^(3/2), x)
```

3.353.9 Mupad [B] (verification not implemented)

Time = 13.74 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.87

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))\,dx = \frac{2\,a\,\mathrm{E}\left(\frac{c}{2}+\frac{dx}{2}\middle|2\right)}{d} + \frac{2\,a\,\mathrm{F}\left(\frac{c}{2}+\frac{dx}{2}\middle|2\right)}{3\,d} + \frac{2\,a\,\sqrt{\cos(c+d\,x)}\,\sin(c+d\,x)}{3\,d}$$

input
```
int(cos(c + d*x)^(3/2)*(a + a/cos(c + d*x)),x)
```

output
```
(2*a*ellipticE(c/2 + (d*x)/2, 2))/d + (2*a*ellipticF(c/2 + (d*x)/2, 2))/(3
*d) + (2*a*cos(c + d*x)^(1/2)*sin(c + d*x))/(3*d)
```

3.354 $\int \sqrt{\cos(c+dx)}(a+a\sec(c+dx))\,dx$

3.354.1 Optimal result . . . 2610
3.354.2 Mathematica [C] (verified) . . . 2610
3.354.3 Rubi [A] (verified) . . . 2611
3.354.4 Maple [A] (verified) . . . 2613
3.354.5 Fricas [C] (verification not implemented) . . . 2613
3.354.6 Sympy [F] . . . 2614
3.354.7 Maxima [F] . . . 2614
3.354.8 Giac [F] . . . 2614
3.354.9 Mupad [B] (verification not implemented) . . . 2615

3.354.1 Optimal result

Integrand size = 21, antiderivative size = 35

$$\int \sqrt{\cos(c+dx)}(a+a\sec(c+dx))\,dx=\frac{2aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}$$

output
```
2*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+
1/2*c),2^(1/2))/d+2*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elli
pticF(sin(1/2*d*x+1/2*c),2^(1/2))/d
```

3.354.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.90 (sec) , antiderivative size = 155, normalized size of antiderivative = 4.43

$$\int \sqrt{\cos(c+dx)}(a+a\sec(c+dx))\,dx$$
$$=\frac{a\sqrt{\cos(c+dx)}(1+\cos(c+dx))\sec^2\left(\frac{1}{2}(c+dx)\right)\left(-2\sqrt{\cos^2(dx-\arctan(\cot(c)))}\sqrt{\csc^2(c)}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};\text{si}\right.\right.}{}$$

input
```
Integrate[Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x]),x]
```

output
```
(a*Sqrt[Cos[c + d*x]]*(1 + Cos[c + d*x])*Sec[(c + d*x)/2]^2*(-2*Sqrt[Cos[d
*x - ArcTan[Cot[c]]]^2]*Sqrt[Csc[c]^2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}
, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[d*x - ArcTan[Cot[c]]]*Sin[c] + Tan[d*x
+ ArcTan[Tan[c]]] - (HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcT
an[Tan[c]]]^2]*Tan[d*x + ArcTan[Tan[c]]])/Sqrt[Sin[d*x + ArcTan[Tan[c]]]^2
]))/(2*d)
```

3.354.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 4713, 3042, 3227, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\cos(c+dx)}(a\sec(c+dx)+a)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)dx$$

$$\Big\downarrow \text{4713}$$

$$\int \frac{a\cos(c+dx)+a}{\sqrt{\cos(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a\sin\left(c+dx+\frac{\pi}{2}\right)+a}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{3227}$$

$$a\int \frac{1}{\sqrt{\cos(c+dx)}}dx+a\int \sqrt{\cos(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$a\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+a\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3119}$$

$$a\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

$$\Big\downarrow \text{3120}$$

$$\frac{2a\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+\frac{2aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

input
```
Int[Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x]),x]
```

output
```
(2*a*EllipticE[(c + d*x)/2, 2])/d + (2*a*EllipticF[(c + d*x)/2, 2])/d
```

3.354.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 4713
```
Int[(csc[(a_.) + (b_.)*(x_)]*(B_.) + (A_))*(u_), x_Symbol] :> Int[ActivateT
rig[u]*((B + A*Sin[a + b*x])/Sin[a + b*x]), x] /; FreeQ[{a, b, A, B}, x] &&
 KnownSineIntegrandQ[u, x]
```

3.354.4 Maple [A] (verified)

Time = 4.41 (sec) , antiderivative size = 150, normalized size of antiderivative = 4.29

method	result
default	$-\frac{2\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,a\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+1}\,\left(\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)-\mathrm{EllipticE}\left(\cos\left(\frac{dx}{2}+\right.\right.}{\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1}\,d}$
risch	$-\frac{ia\sqrt{2}\,\sqrt{\left(\mathrm{e}^{2i(dx+c)}+1\right)\mathrm{e}^{-i(dx+c)}}}{d}-\frac{i\left(\frac{i\sqrt{-i\left(\mathrm{e}^{i(dx+c)}+i\right)}\,\sqrt{2}\,\sqrt{i\left(\mathrm{e}^{i(dx+c)}-i\right)}\,\sqrt{i\mathrm{e}^{i(dx+c)}}\,\mathrm{EllipticF}\left(\sqrt{-i\left(\mathrm{e}^{i(dx+c)}+i\right)},\frac{\sqrt{2}}{2}\right)}{\sqrt{\mathrm{e}^{3i(dx+c)}+\mathrm{e}^{i(dx+c)}}}-\frac{2(}{\sqrt{\mathrm{e}^{i(dx+}}}\right.}{}$

input
```
int(cos(d*x+c)^(1/2)*(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-2*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-Ellipt
icE(cos(1/2*d*x+1/2*c),2^(1/2)))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^
2-1)^(1/2)/d
```

3.354.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 107, normalized size of antiderivative = 3.06

$$\int \sqrt{\cos(c+dx)}(a+a\sec(c+dx))\,dx$$
$$=\frac{-i\sqrt{2}a\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+i\sqrt{2}a\mathrm{weierstrassPInverse}(-4,0,\cos(d}{}$$

input
```
integrate(cos(d*x+c)^(1/2)*(a+a*sec(d*x+c)),x, algorithm="fricas")
```

output
```
(-I*sqrt(2)*a*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) +
I*sqrt(2)*a*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + I*
sqrt(2)*a*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) +
 I*sin(d*x + c))) - I*sqrt(2)*a*weierstrassZeta(-4, 0, weierstrassPInverse
(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/d
```

3.354.6 Sympy [F]

$$\int \sqrt{\cos(c+dx)}(a+a\sec(c+dx))\,dx = a\left(\int \sqrt{\cos(c+dx)}\sec(c+dx)\,dx + \int \sqrt{\cos(c+dx)}\,dx\right)$$

input
```
integrate(cos(d*x+c)**(1/2)*(a+a*sec(d*x+c)),x)
```

output
```
a*(Integral(sqrt(cos(c + d*x))*sec(c + d*x), x) + Integral(sqrt(cos(c + d*
x)), x))
```

3.354.7 Maxima [F]

$$\int \sqrt{\cos(c+dx)}(a+a\sec(c+dx))\,dx = \int (a\sec(dx+c)+a)\sqrt{\cos(dx+c)}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)*(a+a*sec(d*x+c)),x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)*sqrt(cos(d*x + c)), x)
```

3.354.8 Giac [F]

$$\int \sqrt{\cos(c+dx)}(a+a\sec(c+dx))\,dx = \int (a\sec(dx+c)+a)\sqrt{\cos(dx+c)}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)*(a+a*sec(d*x+c)),x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)*sqrt(cos(d*x + c)), x)
```

3.354.9 Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.77

$$\int \sqrt{\cos(c+dx)}(a+a\sec(c+dx))\,dx = \frac{2\,a\,\left(\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)+\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)\right)}{d}$$

input
```
int(cos(c + d*x)^(1/2)*(a + a/cos(c + d*x)),x)
```

output
```
(2*a*(ellipticE(c/2 + (d*x)/2, 2) + ellipticF(c/2 + (d*x)/2, 2)))/d
```

3.355 $\int \frac{a+a\sec(c+dx)}{\sqrt{\cos(c+dx)}}\,dx$

3.355.1 Optimal result 2616
3.355.2 Mathematica [C] (warning: unable to verify) 2616
3.355.3 Rubi [A] (verified) 2617
3.355.4 Maple [A] (verified) 2619
3.355.5 Fricas [C] (verification not implemented) 2619
3.355.6 Sympy [F] 2620
3.355.7 Maxima [F] 2620
3.355.8 Giac [F] 2621
3.355.9 Mupad [B] (verification not implemented) 2621

3.355.1 Optimal result

Integrand size = 21, antiderivative size = 57

$$\int \frac{a+a\sec(c+dx)}{\sqrt{\cos(c+dx)}}\,dx=-\frac{2aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+\frac{2a\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

output
```
-2*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x
+1/2*c),2^(1/2))/d+2*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ell
ipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2*a*sin(d*x+c)/d/cos(d*x+c)^(1/2)
```

3.355.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 4.98 (sec) , antiderivative size = 209, normalized size of antiderivative = 3.67

$$\int \frac{a+a\sec(c+dx)}{\sqrt{\cos(c+dx)}}\,dx$$
$$=\frac{a(1+\cos(c+dx))\sec^2\left(\frac{1}{2}(c+dx)\right)\left(4\cos(dx)\csc(c)-\frac{(3\cos(c-dx-\arctan(\tan(c)))+\cos(c+dx+\arctan(\tan(c))))\csc(c)\sec}{\sqrt{\sec^2(c)}}\right.}{}$$

input
```
Integrate[(a + a*Sec[c + d*x])/Sqrt[Cos[c + d*x]],x]
```

output
```
(a*(1 + Cos[c + d*x])*Sec[(c + d*x)/2]^2*(4*Cos[d*x]*Csc[c] - ((3*Cos[c -
d*x - ArcTan[Tan[c]]] + Cos[c + d*x + ArcTan[Tan[c]]])*Csc[c]*Sec[c])/Sqrt
[Sec[c]^2] - 4*Cos[c + d*x]*Sqrt[Cos[d*x - ArcTan[Cot[c]]]^2]*Sqrt[Csc[c]^
2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[d
*x - ArcTan[Cot[c]]]*Sin[c] + 2*Cos[c]*Csc[d*x + ArcTan[Tan[c]]]*Hypergeom
etricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sqrt[Sec[c]^2]*
Sqrt[Sin[d*x + ArcTan[Tan[c]]]^2]))/(4*d*Sqrt[Cos[c + d*x]])
```

3.355.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.02, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 4713, 3042, 3227, 3042, 3116, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a\sec(c+dx)+a}{\sqrt{\cos(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4713}$$

$$\int \frac{a\cos(c+dx)+a}{\cos^{\frac{3}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a\sin\left(c+dx+\frac{\pi}{2}\right)+a}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{3227}$$

$$a\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)}dx+a\int \frac{1}{\sqrt{\cos(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$a\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+a\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{3116}$$

$$a\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+a\left(\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\int \sqrt{\cos(c+dx)}dx\right)$$

$\Big\downarrow$ 3042

$$a\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+a\left(\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx\right)$$

$\Big\downarrow$ 3119

$$a\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+a\left(\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)$$

$\Big\downarrow$ 3120

$$\frac{2a\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+a\left(\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)$$

input
```
Int[(a + a*Sec[c + d*x])/Sqrt[Cos[c + d*x]],x]
```

output
```
(2*a*EllipticF[(c + d*x)/2, 2])/d + a*((-2*EllipticE[(c + d*x)/2, 2])/d +
(2*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]))
```

3.355.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 4713
```
Int[(csc[(a_.) + (b_.)*(x_)]*(B_.) + (A_))*(u_), x_Symbol] :> Int[ActivateT
rig[u]*((B + A*Sin[a + b*x])/Sin[a + b*x]), x] /; FreeQ[{a, b, A, B}, x] &&
 KnownSineIntegrandQ[u, x]
```

3.355.4 Maple [A] (verified)

Time = 5.28 (sec) , antiderivative size = 148, normalized size of antiderivative = 2.60

method	result
default	$\frac{2a\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1}\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)-\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\mathrm{EllipticE}\left(\cos\left(\right.\right.}{\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1}\,d}$

input
```
int((a+a*sec(d*x+c))/cos(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*a*(2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-(
sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1
/2*d*x+1/2*c)^2-1)^(1/2))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1
/2)/d
```

3.355.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 156, normalized size of antiderivative = 2.74

$$\int \frac{a+a\sec(c+dx)}{\sqrt{\cos(c+dx)}}\,dx$$
$$= \frac{-i\sqrt{2}a\cos(dx+c)\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+i\sqrt{2}a\cos(dx+c)\,\mathrm{weierst}}{}$$

input
```
integrate((a+a*sec(d*x+c))/cos(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
(-I*sqrt(2)*a*cos(d*x + c)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin
(d*x + c)) + I*sqrt(2)*a*cos(d*x + c)*weierstrassPInverse(-4, 0, cos(d*x +
 c) - I*sin(d*x + c)) - I*sqrt(2)*a*cos(d*x + c)*weierstrassZeta(-4, 0, we
ierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + I*sqrt(2)*a*cos
(d*x + c)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) -
 I*sin(d*x + c))) + 2*a*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c))
```

3.355.6 Sympy [F]

$$\int \frac{a + a\sec(c + dx)}{\sqrt{\cos(c + dx)}}\,dx = a\left(\int \frac{\sec(c + dx)}{\sqrt{\cos(c + dx)}}\,dx + \int \frac{1}{\sqrt{\cos(c + dx)}}\,dx\right)$$

input
```
integrate((a+a*sec(d*x+c))/cos(d*x+c)**(1/2),x)
```

output
```
a*(Integral(sec(c + d*x)/sqrt(cos(c + d*x)), x) + Integral(1/sqrt(cos(c +
d*x)), x))
```

3.355.7 Maxima [F]

$$\int \frac{a + a\sec(c + dx)}{\sqrt{\cos(c + dx)}}\,dx = \int \frac{a\sec(dx + c) + a}{\sqrt{\cos(dx + c)}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))/cos(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)/sqrt(cos(d*x + c)), x)
```

3.355.8 Giac [F]

$$\int \frac{a+a\sec(c+dx)}{\sqrt{\cos(c+dx)}}\,dx=\int \frac{a\sec(dx+c)+a}{\sqrt{\cos(dx+c)}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))/cos(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)/sqrt(cos(d*x + c)), x)
```

3.355.9 Mupad [B] (verification not implemented)

Time = 13.27 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.05

$$\int \frac{a+a\sec(c+dx)}{\sqrt{\cos(c+dx)}}\,dx=\frac{2\,a\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{2\,a\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{1}{2};\ \frac{3}{4};\ \cos\left(c+d\,x\right)^2\right)}{d\,\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{\sin\left(c+d\,x\right)^2}}$$

input
```
int((a + a/cos(c + d*x))/cos(c + d*x)^(1/2),x)
```

output
```
(2*a*ellipticF(c/2 + (d*x)/2, 2))/d + (2*a*sin(c + d*x)*hypergeom([-1/4, 1
/2], 3/4, cos(c + d*x)^2))/(d*cos(c + d*x)^(1/2)*(sin(c + d*x)^2)^(1/2))
```

3.356 $\int \frac{a+a\sec(c+dx)}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

3.356.1 Optimal result . . . 2622
3.356.2 Mathematica [C] (verified) . . . 2623
3.356.3 Rubi [A] (verified) . . . 2624
3.356.4 Maple [B] (verified) . . . 2626
3.356.5 Fricas [C] (verification not implemented) . . . 2626
3.356.6 Sympy [F] . . . 2627
3.356.7 Maxima [F] . . . 2627
3.356.8 Giac [F] . . . 2628
3.356.9 Mupad [B] (verification not implemented) . . . 2628

3.356.1 Optimal result

Integrand size = 21, antiderivative size = 83

$$\int \frac{a+a\sec(c+dx)}{\cos^{\frac{3}{2}}(c+dx)}\,dx = -\frac{2aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{2a\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

output
```
-2*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x
+1/2*c),2^(1/2))/d+2/3*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*E
llipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/3*a*sin(d*x+c)/d/cos(d*x+c)^(3/2)
+2*a*sin(d*x+c)/d/cos(d*x+c)^(1/2)
```

3.356.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 6.13 (sec) , antiderivative size = 444, normalized size of antiderivative = 5.35

$$\int \frac{a+a\sec(c+dx)}{\cos^{\frac{3}{2}}(c+dx)}\,dx = a\left(\sqrt{\cos(c+dx)}(1+\cos(c+dx))\sec^2\left(\frac{c}{2}+\frac{dx}{2}\right)\left(\frac{\csc(c)\sec(c)}{d}+\frac{\sec(c)\sec^2(c+dx)\sin(dx)}{3d}+\frac{\sec(c)\sec(c+dx)(\sin(c)+3\sin(dx))}{3d}\right)\right.$$
$$-\frac{(1+\cos(c+dx))\csc(c)\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};\sin^2(dx-\arctan(\cot(c)))\right)\sec^2\left(\frac{c}{2}+\frac{dx}{2}\right)\sec(dx-\arctan(\cot(c)))\sqrt{}}{3d\sqrt{}}$$
$$+\frac{(1+\cos(c+dx))\csc(c)\sec^2\left(\frac{c}{2}+\frac{dx}{2}\right)\left(\frac{{}_2F_1\left(-\frac{1}{2},-\frac{1}{4};\frac{3}{4};\cos^2(dx+\arctan(\tan(c)))\right)\sin(dx+\arctan(\tan(c}{\sqrt{1-\cos(dx+\arctan(\tan(c)))}\sqrt{1+\cos(dx+\arctan(\tan(c)))}\sqrt{\cos(c)\cos(dx+\arctan(\tan}}\right.}{2d}$$

input
```
Integrate[(a + a*Sec[c + d*x])/Cos[c + d*x]^(3/2),x]
```

output
```
a*(Sqrt[Cos[c + d*x]]*(1 + Cos[c + d*x])*Sec[c/2 + (d*x)/2]^2*((Csc[c]*Sec
[c])/d + (Sec[c]*Sec[c + d*x]^2*Sin[d*x])/(3*d) + (Sec[c]*Sec[c + d*x]*(Si
n[c] + 3*Sin[d*x]))/(3*d)) - ((1 + Cos[c + d*x])*Csc[c]*HypergeometricPFQ[
{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^2*Sec[d
*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + C
ot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[
c]]]])/(3*d*Sqrt[1 + Cot[c]^2]) + ((1 + Cos[c + d*x])*Csc[c]*Sec[c/2 + (d*
x)/2]^2*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]
^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]
*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]
*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan
[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + T
an[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqr
t[1 + Tan[c]^2]]))/(2*d))
```

3.356.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.02, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3042, 4713, 3042, 3227, 3042, 3116, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a\sec(c+dx)+a}{\cos^{\frac{3}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{4713}$$

$$\int \frac{a\cos(c+dx)+a}{\cos^{\frac{5}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a\sin\left(c+dx+\frac{\pi}{2}\right)+a}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx$$

$$\Big\downarrow \text{3227}$$

$$a\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)}dx+a\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$a\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+a\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{3116}$$

$$a\left(\frac{1}{3}\int \frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}\right)+a\left(\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\int \sqrt{\cos(c+dx)}dx\right)$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{1}{3}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}\right)+ a\left(\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx\right)$$

$$\Big\downarrow \text{3119}$$

$$a\left(\frac{1}{3}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}\right)+a\left(\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)$$

$$\Big\downarrow \text{3120}$$

$$a\left(\frac{2\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}\right)+a\left(\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)$$

input
```
Int[(a + a*Sec[c + d*x])/Cos[c + d*x]^(3/2),x]
```

output
```
a*((2*EllipticF[(c + d*x)/2, 2])/(3*d) + (2*Sin[c + d*x])/(3*d*Cos[c + d*x
]^(3/2))) + a*((-2*EllipticE[(c + d*x)/2, 2])/d + (2*Sin[c + d*x])/(d*Sqrt
[Cos[c + d*x]]))
```

3.356.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 4713
```
Int[(csc[(a_.) + (b_.)*(x_)]*(B_.) + (A_))*(u_), x_Symbol] :> Int[ActivateT
rig[u]*((B + A*Sin[a + b*x])/Sin[a + b*x]), x] /; FreeQ[{a, b, A, B}, x] &&
 KnownSineIntegrandQ[u, x]
```

3.356.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 367 vs. $2(127) = 254$.

Time = 8.10 (sec) , antiderivative size = 368, normalized size of antiderivative = 4.43

method	result
default	$-\frac{2a\sqrt{-\left(-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(12\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1}\,\mathrm{EllipticF}\left(\cos\left(\right.\right.\right.}{}$

input
```
int((a+a*sec(d*x+c))/cos(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*a*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(4*sin(1/
2*d*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)/sin(1/2*d*x+1/2*c)^3*(12*sin(1/2*
d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/
2*c)^2-6*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)
)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2-8*cos(1/2*d*x+1/2*
c)*sin(1/2*d*x+1/2*c)^2+(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)
^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1
/2))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1
/2*c)^2-1)^(1/2)/d
```

3.356.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 175, normalized size of antiderivative = 2.11

$$\int \frac{a+a\sec(c+dx)}{\cos^{\frac{3}{2}}(c+dx)}\,dx$$
$$= \frac{-i\sqrt{2}a\cos(dx+c)^2\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+i\sqrt{2}a\cos(dx+c)^2\,\mathrm{weier}}{}$$

input
```
integrate((a+a*sec(d*x+c))/cos(d*x+c)^(3/2),x, algorithm="fricas")
```

output
```
1/3*(-I*sqrt(2)*a*cos(d*x + c)^2*weierstrassPInverse(-4, 0, cos(d*x + c) +
 I*sin(d*x + c)) + I*sqrt(2)*a*cos(d*x + c)^2*weierstrassPInverse(-4, 0, c
os(d*x + c) - I*sin(d*x + c)) - 3*I*sqrt(2)*a*cos(d*x + c)^2*weierstrassZe
ta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 3*I
*sqrt(2)*a*cos(d*x + c)^2*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0
, cos(d*x + c) - I*sin(d*x + c))) + 2*(3*a*cos(d*x + c) + a)*sqrt(cos(d*x
+ c))*sin(d*x + c))/(d*cos(d*x + c)^2)
```

3.356.6 Sympy [F]

$$\int \frac{a + a\sec(c + dx)}{\cos^{\frac{3}{2}}(c + dx)}\,dx = a\left(\int \frac{\sec(c + dx)}{\cos^{\frac{3}{2}}(c + dx)}\,dx + \int \frac{1}{\cos^{\frac{3}{2}}(c + dx)}\,dx\right)$$

input
```
integrate((a+a*sec(d*x+c))/cos(d*x+c)**(3/2),x)
```

output
```
a*(Integral(sec(c + d*x)/cos(c + d*x)**(3/2), x) + Integral(cos(c + d*x)**
(-3/2), x))
```

3.356.7 Maxima [F]

$$\int \frac{a + a\sec(c + dx)}{\cos^{\frac{3}{2}}(c + dx)}\,dx = \int \frac{a\sec(dx + c) + a}{\cos(dx + c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))/cos(d*x+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)/cos(d*x + c)^(3/2), x)
```

3.356.8 Giac [F]

$$\int \frac{a+a\sec(c+dx)}{\cos^{\frac{3}{2}}(c+dx)}\,dx=\int \frac{a\sec(dx+c)+a}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))/cos(d*x+c)^(3/2),x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)/cos(d*x + c)^(3/2), x)
```

3.356.9 Mupad [B] (verification not implemented)

Time = 14.23 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.05

$$\int \frac{a+a\sec(c+dx)}{\cos^{\frac{3}{2}}(c+dx)}\,dx=\frac{2\,a\,\sin(c+d\,x)\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{1}{2};\ \frac{3}{4};\ \cos(c+d\,x)^2\right)}{d\,\sqrt{\cos(c+d\,x)}\,\sqrt{\sin(c+d\,x)^2}}+\frac{2\,a\,\sin(c+d\,x)\,{}_2\mathrm{F}_1\left(-\frac{3}{4},\frac{1}{2};\ \frac{1}{4};\ \cos(c+d\,x)^2\right)}{3\,d\cos(c+d\,x)^{3/2}\,\sqrt{\sin(c+d\,x)^2}}$$

input
```
int((a + a/cos(c + d*x))/cos(c + d*x)^(3/2),x)
```

output
```
(2*a*sin(c + d*x)*hypergeom([-1/4, 1/2], 3/4, cos(c + d*x)^2))/(d*cos(c +
d*x)^(1/2)*(sin(c + d*x)^2)^(1/2)) + (2*a*sin(c + d*x)*hypergeom([-3/4, 1/
2], 1/4, cos(c + d*x)^2))/(3*d*cos(c + d*x)^(3/2)*(sin(c + d*x)^2)^(1/2))
```

3.357 $\int \frac{a+a\sec(c+dx)}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

3.357.1 Optimal result 2629
3.357.2 Mathematica [C] (warning: unable to verify) 2630
3.357.3 Rubi [A] (verified) 2631
3.357.4 Maple [B] (verified) 2634
3.357.5 Fricas [C] (verification not implemented) 2634
3.357.6 Sympy [F(-1)] 2635
3.357.7 Maxima [F] 2635
3.357.8 Giac [F] 2636
3.357.9 Mupad [B] (verification not implemented) 2636

3.357.1 Optimal result

Integrand size = 21, antiderivative size = 111

$$\int \frac{a+a\sec(c+dx)}{\cos^{\frac{5}{2}}(c+dx)}\,dx = -\frac{6aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2a\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{2a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{6a\sin(c+dx)}{5d\sqrt{\cos(c+dx)}}$$

output
```
-6/5*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d
*x+1/2*c),2^(1/2))/d+2/3*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)
*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/5*a*sin(d*x+c)/d/cos(d*x+c)^(5/
2)+2/3*a*sin(d*x+c)/d/cos(d*x+c)^(3/2)+6/5*a*sin(d*x+c)/d/cos(d*x+c)^(1/2)
```

3.357.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 6.18 (sec) , antiderivative size = 477, normalized size of antiderivative = 4.30

$$\int \frac{a+a\sec(c+dx)}{\cos^{\frac{5}{2}}(c+dx)}\,dx = a\left(\sqrt{\cos(c+dx)}(1+\cos(c+dx))\sec^2\left(\frac{c}{2}+\frac{dx}{2}\right)\left(\frac{3\csc(c)\sec(c)}{5d}+\frac{\sec(c)\sec^3(c+dx)\sin(dx)}{5d}+\frac{\sec(c)\sec^2(c+dx)(3\sin(c)+5\sin(dx))}{15d}+\frac{\sec(c)\sec(c+dx)(5\sin(c)+9\sin(dx))}{15d}\right)\right.$$

$$-\frac{(1+\cos(c+dx))\csc(c)\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};\sin^2(dx-\arctan(\cot(c)))\right)\sec^2\left(\frac{c}{2}+\frac{dx}{2}\right)\sec(dx-\arctan(\cot(c)))\sqrt{}}{3d\sqrt{}}$$

$$+\frac{3(1+\cos(c+dx))\csc(c)\sec^2\left(\frac{c}{2}+\frac{dx}{2}\right)\left(\frac{{}_2F_1\left(-\frac{1}{2},-\frac{1}{4};\frac{3}{4};\cos^2(dx+\arctan(\tan(c)))\right)\sin(dx+\arctan(\tan(}{\sqrt{1-\cos(dx+\arctan(\tan(c)))}\sqrt{1+\cos(dx+\arctan(\tan(c)))}\sqrt{\cos(c)\cos(dx+\arctan(t}}\right.}{10d}$$

input
```
Integrate[(a + a*Sec[c + d*x])/Cos[c + d*x]^(5/2),x]
```

output
```
a*(Sqrt[Cos[c + d*x]]*(1 + Cos[c + d*x])*Sec[c/2 + (d*x)/2]^2*((3*Csc[c]*S
ec[c])/(5*d) + (Sec[c]*Sec[c + d*x]^3*Sin[d*x])/(5*d) + (Sec[c]*Sec[c + d*
x]^2*(3*Sin[c] + 5*Sin[d*x]))/(15*d) + (Sec[c]*Sec[c + d*x]*(5*Sin[c] + 9*
Sin[d*x]))/(15*d)) - ((1 + Cos[c + d*x])*Csc[c]*HypergeometricPFQ[{1/4, 1/
2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^2*Sec[d*x - Arc
Tan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]
*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(
3*d*Sqrt[1 + Cot[c]^2]) + (3*(1 + Cos[c + d*x])*Csc[c]*Sec[c/2 + (d*x)/2]^
2*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Si
n[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[
1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[
1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/S
qrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^
2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 +
Tan[c]^2]]))/(10*d))
```

3.357.3 Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.02, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.524$, Rules used = {3042, 4713, 3042, 3227, 3042, 3116, 3042, 3116, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a\sec(c+dx)+a}{\cos^{\frac{5}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx$$

$$\Big\downarrow \text{4713}$$

$$\int \frac{a\cos(c+dx)+a}{\cos^{\frac{7}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a\sin\left(c+dx+\frac{\pi}{2}\right)+a}{\sin\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx$$

$$\Big\downarrow \text{3227}$$

$$a\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)}dx+a\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$a\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx+a\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx$$

$$\Big\downarrow \text{3116}$$

$$a\left(\frac{3}{5}\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)}dx+\frac{2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}\right)+a\left(\frac{1}{3}\int \frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}\right)$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{3}{5}\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+\frac{2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}\right)+$$
$$a\left(\frac{1}{3}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}\right)$$

$$\Big\downarrow \text{3116}$$

$$a\left(\frac{3}{5}\left(\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\int \sqrt{\cos(c+dx)}dx\right)+\frac{2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}\right)+$$
$$a\left(\frac{1}{3}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}\right)$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{1}{3}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}\right)+$$
$$a\left(\frac{3}{5}\left(\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx\right)+\frac{2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}\right)$$

$$\Big\downarrow \text{3119}$$

$$a\left(\frac{1}{3}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}\right)+$$
$$a\left(\frac{2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{3}{5}\left(\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)\right)$$

$$\Big\downarrow \text{3120}$$

3.357. $\int \frac{a+a\sec(c+dx)}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

$$a\left(\frac{2\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}\right)+a\left(\frac{2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{3}{5}\left(\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)\right)$$

input
```
Int[(a + a*Sec[c + d*x])/Cos[c + d*x]^(5/2),x]
```

output
```
a*((2*EllipticF[(c + d*x)/2, 2])/(3*d) + (2*Sin[c + d*x])/(3*d*Cos[c + d*x
]^(3/2))) + a*((2*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2)) + (3*((-2*Ellipti
cE[(c + d*x)/2, 2])/d + (2*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])))/5)
```

3.357.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 4713
```
Int[(csc[(a_.) + (b_.)*(x_)]*(B_.) + (A_))*(u_), x_Symbol] :> Int[ActivateT
rig[u]*((B + A*Sin[a + b*x])/Sin[a + b*x]), x] /; FreeQ[{a, b, A, B}, x] &&
 KnownSineIntegrandQ[u, x]
```

3.357. $\int \frac{a+a\sec(c+dx)}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

3.357.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 383 vs. $2(147) = 294$.

Time = 10.77 (sec) , antiderivative size = 384, normalized size of antiderivative = 3.46

method	result
default	$-\frac{4\sqrt{-\left(-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,a\left(-\frac{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}}{40\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-\frac{1}{2}\right)^{3}}-\frac{3\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)}{5\sqrt{-\left(-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}}\right.}{}$

input
```
int((a+a*sec(d*x+c))/cos(d*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-4*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a*(-1/40*cos(
1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1
/2*d*x+1/2*c)^2-1/2)^3-3/5*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)/(-(-2*c
os(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)+7/15*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(
1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3/10*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-
EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))-1/12*cos(1/2*d*x+1/2*c)*(-2*sin(1/2
*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2)/si
n(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

3.357.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.69

$$\int \frac{a+a\sec(c+dx)}{\cos^{\frac{5}{2}}(c+dx)}\,dx$$
$$= \frac{-5i\sqrt{2}a\cos(dx+c)^{3}\,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+5i\sqrt{2}a\cos(dx+c)^{3}\,\text{wei}}{}$$

input
```
integrate((a+a*sec(d*x+c))/cos(d*x+c)^(5/2),x, algorithm="fricas")
```

output
```
1/15*(-5*I*sqrt(2)*a*cos(d*x + c)^3*weierstrassPInverse(-4, 0, cos(d*x + c
) + I*sin(d*x + c)) + 5*I*sqrt(2)*a*cos(d*x + c)^3*weierstrassPInverse(-4,
 0, cos(d*x + c) - I*sin(d*x + c)) - 9*I*sqrt(2)*a*cos(d*x + c)^3*weierstr
assZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)))
+ 9*I*sqrt(2)*a*cos(d*x + c)^3*weierstrassZeta(-4, 0, weierstrassPInverse(
-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*(9*a*cos(d*x + c)^2 + 5*a*cos(d
*x + c) + 3*a)*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^3)
```

3.357.6 Sympy [F(-1)]

Timed out.

$$\int \frac{a + a\sec(c + dx)}{\cos^{\frac{5}{2}}(c + dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sec(d*x+c))/cos(d*x+c)**(5/2),x)
```

output
```
Timed out
```

3.357.7 Maxima [F]

$$\int \frac{a + a\sec(c + dx)}{\cos^{\frac{5}{2}}(c + dx)}\,dx = \int \frac{a\sec(dx + c) + a}{\cos(dx + c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))/cos(d*x+c)^(5/2),x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)/cos(d*x + c)^(5/2), x)
```

3.357.8 Giac [F]

$$\int \frac{a+a\sec(c+dx)}{\cos^{\frac{5}{2}}(c+dx)}\,dx=\int \frac{a\sec(dx+c)+a}{\cos(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))/cos(d*x+c)^(5/2),x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)/cos(d*x + c)^(5/2), x)
```

3.357.9 Mupad [B] (verification not implemented)

Time = 13.99 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.78

$$\int \frac{a+a\sec(c+dx)}{\cos^{\frac{5}{2}}(c+dx)}\,dx=\frac{2\,a\,\sin(c+d\,x)\,{}_2\mathrm{F}_1\left(-\frac{3}{4},\frac{1}{2};\ \frac{1}{4};\ \cos(c+d\,x)^2\right)}{3\,d\cos(c+d\,x)^{3/2}\,\sqrt{\sin(c+d\,x)^2}}+\frac{2\,a\,\sin(c+d\,x)\,{}_2\mathrm{F}_1\left(-\frac{5}{4},\frac{1}{2};\ -\frac{1}{4};\ \cos(c+d\,x)^2\right)}{5\,d\cos(c+d\,x)^{5/2}\,\sqrt{\sin(c+d\,x)^2}}$$

input
```
int((a + a/cos(c + d*x))/cos(c + d*x)^(5/2),x)
```

output
```
(2*a*sin(c + d*x)*hypergeom([-3/4, 1/2], 1/4, cos(c + d*x)^2))/(3*d*cos(c
+ d*x)^(3/2)*(sin(c + d*x)^2)^(1/2)) + (2*a*sin(c + d*x)*hypergeom([-5/4,
1/2], -1/4, cos(c + d*x)^2))/(5*d*cos(c + d*x)^(5/2)*(sin(c + d*x)^2)^(1/2
))
```

3.358 $\int \frac{a+a\sec(c+dx)}{\cos^{\frac{7}{2}}(c+dx)}\,dx$

3.358.1 Optimal result 2637
3.358.2 Mathematica [C] (warning: unable to verify) 2638
3.358.3 Rubi [A] (verified) 2638
3.358.4 Maple [B] (verified) 2641
3.358.5 Fricas [C] (verification not implemented) 2642
3.358.6 Sympy [F(-1)] 2642
3.358.7 Maxima [F] 2643
3.358.8 Giac [F] 2643
3.358.9 Mupad [B] (verification not implemented) 2643

3.358.1 Optimal result

Integrand size = 21, antiderivative size = 135

$$\int \frac{a+a\sec(c+dx)}{\cos^{\frac{7}{2}}(c+dx)}\,dx = -\frac{6aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{10a\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d}+\frac{2a\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)}+\frac{2a\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{10a\sin(c+dx)}{21d\cos^{\frac{3}{2}}(c+dx)}+\frac{6a\sin(c+dx)}{5d\sqrt{\cos(c+dx)}}$$

output
```
-6/5*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d
*x+1/2*c),2^(1/2))/d+10/21*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*
c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/7*a*sin(d*x+c)/d/cos(d*x+c)^(
7/2)+2/5*a*sin(d*x+c)/d/cos(d*x+c)^(5/2)+10/21*a*sin(d*x+c)/d/cos(d*x+c)^(
3/2)+6/5*a*sin(d*x+c)/d/cos(d*x+c)^(1/2)
```

3.358.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 5.21 (sec) , antiderivative size = 294, normalized size of antiderivative = 2.18

$$\int \frac{a+a\sec(c+dx)}{\cos^{\frac{7}{2}}(c+dx)}\,dx$$
$$= \frac{a(1+\cos(c+dx))\sec^2\left(\frac{1}{2}(c+dx)\right)\left((189\cos(c)+85\cos(dx)-85\cos(2c+dx)+231\cos(c+2dx)+21\right.}{}$$

input
```
Integrate[(a + a*Sec[c + d*x])/Cos[c + d*x]^(7/2),x]
```

output
```
(a*(1 + Cos[c + d*x])*Sec[(c + d*x)/2]^2*((189*Cos[c] + 85*Cos[d*x] - 85*C
os[2*c + d*x] + 231*Cos[c + 2*d*x] + 21*Cos[3*c + 2*d*x] + 25*Cos[2*c + 3*
d*x] - 25*Cos[4*c + 3*d*x] + 63*Cos[3*c + 4*d*x])*Csc[c] - 200*Cos[c + d*x
]^4*Sqrt[Cos[d*x - ArcTan[Cot[c]]]^2]*Sqrt[Csc[c]^2]*HypergeometricPFQ[{1/
4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[d*x - ArcTan[Cot[c]]]*Sin
[c] - (126*Cos[c + d*x]^3*Sec[c]*(-2*HypergeometricPFQ[{-1/2, -1/4}, {3/4}
, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]] + (3*Cos[c - d*x
- ArcTan[Tan[c]]] + Cos[c + d*x + ArcTan[Tan[c]]])*Csc[c]*Sqrt[Sin[d*x + A
rcTan[Tan[c]]]^2]))/(Sqrt[Sec[c]^2]*Sqrt[Sin[d*x + ArcTan[Tan[c]]]^2])))/(
840*d*Cos[c + d*x]^(7/2))
```

3.358.3 Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.04, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.524$, Rules used = {3042, 4713, 3042, 3227, 3042, 3116, 3042, 3116, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a\sec(c+dx)+a}{\cos^{\frac{7}{2}}(c+dx)}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}{\sin\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx$$

$$\Big\downarrow \text{4713}$$

$$\int \frac{a\cos(c+dx)+a}{\cos^{\frac{9}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a\sin\left(c+dx+\frac{\pi}{2}\right)+a}{\sin\left(c+dx+\frac{\pi}{2}\right)^{9/2}}dx$$

$$\Big\downarrow \text{3227}$$

$$a\int \frac{1}{\cos^{\frac{9}{2}}(c+dx)}dx+a\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$a\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{9/2}}dx+a\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx$$

$$\Big\downarrow \text{3116}$$

$$a\left(\frac{5}{7}\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)}dx+\frac{2\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)}\right)+a\left(\frac{3}{5}\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)}dx+\frac{2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}\right)$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{5}{7}\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+\frac{2\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)}\right)+ a\left(\frac{3}{5}\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+\frac{2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}\right)$$

$$\Big\downarrow \text{3116}$$

$$a\left(\frac{5}{7}\left(\frac{1}{3}\int \frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}\right)+\frac{2\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)}\right)+ a\left(\frac{3}{5}\left(\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\int \sqrt{\cos(c+dx)}dx\right)+\frac{2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}\right)$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{5}{7}\left(\frac{1}{3}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}\right)+\frac{2\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)}\right)+ a\left(\frac{3}{5}\left(\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx\right)+\frac{2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}\right)$$

$$\Big\downarrow \text{3119}$$

3.358. $\int \frac{a+a\sec(c+dx)}{\cos^{\frac{7}{2}}(c+dx)}\,dx$

$$a\left(\frac{5}{7}\left(\frac{1}{3}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}\right)+\frac{2\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)}\right)+a\left(\frac{2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{3}{5}\left(\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)\right)$$

$$\Big\downarrow \; 3120$$

$$a\left(\frac{2\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)}+\frac{5}{7}\left(\frac{2\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}\right)\right)+a\left(\frac{2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{3}{5}\left(\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)\right)$$

input
```
Int[(a + a*Sec[c + d*x])/Cos[c + d*x]^(7/2),x]
```

output
```
a*((2*Sin[c + d*x])/(7*d*Cos[c + d*x]^(7/2)) + (5*((2*EllipticF[(c + d*x)/
2, 2])/(3*d) + (2*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2))))/7) + a*((2*Sin[
c + d*x])/(5*d*Cos[c + d*x]^(5/2)) + (3*((-2*EllipticE[(c + d*x)/2, 2])/d
+ (2*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])))/5)
```

3.358.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 4713
```
Int[(csc[(a_.) + (b_.)*(x_)]*(B_.) + (A_))*(u_), x_Symbol] :> Int[ActivateT
rig[u]*((B + A*Sin[a + b*x])/Sin[a + b*x]), x] /; FreeQ[{a, b, A, B}, x] &&
 KnownSineIntegrandQ[u, x]
```

3.358.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 436 vs. $2(167) = 334$.

Time = 12.17 (sec) , antiderivative size = 437, normalized size of antiderivative = 3.24

method	result
default	$-\frac{4\sqrt{-\left(-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,a\left(-\frac{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}}{40\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-\frac{1}{2}\right)^{3}}-\frac{3\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)}{5\sqrt{-\left(-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)}}\right.}{}$

input
```
int((a+a*sec(d*x+c))/cos(d*x+c)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-4*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a*(-1/40*cos(
1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1
/2*d*x+1/2*c)^2-1/2)^3-3/5*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)/(-(-2*c
os(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)+44/105*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+si
n(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3/10*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+
1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)
)-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))-1/112*cos(1/2*d*x+1/2*c)*(-2*sin(
1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^4-
5/84*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/
2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^
2-1)^(1/2)/d
```

3.358.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.47

$$\int \frac{a+a\sec(c+dx)}{\cos^{\frac{7}{2}}(c+dx)}\,dx$$
$$= \frac{-25i\sqrt{2}a\cos(dx+c)^4\,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+25i\sqrt{2}a\cos(dx+c)^4\,\text{w}}{}$$

input
```
integrate((a+a*sec(d*x+c))/cos(d*x+c)^(7/2),x, algorithm="fricas")
```

output
```
1/105*(-25*I*sqrt(2)*a*cos(d*x + c)^4*weierstrassPInverse(-4, 0, cos(d*x +
 c) + I*sin(d*x + c)) + 25*I*sqrt(2)*a*cos(d*x + c)^4*weierstrassPInverse(
-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 63*I*sqrt(2)*a*cos(d*x + c)^4*weie
rstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c
))) + 63*I*sqrt(2)*a*cos(d*x + c)^4*weierstrassZeta(-4, 0, weierstrassPInv
erse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*(63*a*cos(d*x + c)^3 + 25*
a*cos(d*x + c)^2 + 21*a*cos(d*x + c) + 15*a)*sqrt(cos(d*x + c))*sin(d*x +
c))/(d*cos(d*x + c)^4)
```

3.358.6 Sympy [F(-1)]

Timed out.

$$\int \frac{a+a\sec(c+dx)}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sec(d*x+c))/cos(d*x+c)**(7/2),x)
```

output
```
Timed out
```

3.358.7 Maxima [F]

$$\int \frac{a+a\sec(c+dx)}{\cos^{\frac{7}{2}}(c+dx)}\,dx=\int \frac{a\sec(dx+c)+a}{\cos(dx+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))/cos(d*x+c)^(7/2),x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)/cos(d*x + c)^(7/2), x)
```

3.358.8 Giac [F]

$$\int \frac{a+a\sec(c+dx)}{\cos^{\frac{7}{2}}(c+dx)}\,dx=\int \frac{a\sec(dx+c)+a}{\cos(dx+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))/cos(d*x+c)^(7/2),x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)/cos(d*x + c)^(7/2), x)
```

3.358.9 Mupad [B] (verification not implemented)

Time = 14.08 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.64

$$\int \frac{a+a\sec(c+dx)}{\cos^{\frac{7}{2}}(c+dx)}\,dx=\frac{2\,a\,\sin(c+d\,x)\,{}_2\mathrm{F}_1\left(-\frac{5}{4},\frac{1}{2};\ -\frac{1}{4};\ \cos(c+d\,x)^2\right)}{5\,d\cos(c+d\,x)^{5/2}\,\sqrt{\sin(c+d\,x)^2}}+\frac{2\,a\,\sin(c+d\,x)\,{}_2\mathrm{F}_1\left(-\frac{7}{4},\frac{1}{2};\ -\frac{3}{4};\ \cos(c+d\,x)^2\right)}{7\,d\cos(c+d\,x)^{7/2}\,\sqrt{\sin(c+d\,x)^2}}$$

input
```
int((a + a/cos(c + d*x))/cos(c + d*x)^(7/2),x)
```

output
```
(2*a*sin(c + d*x)*hypergeom([-5/4, 1/2], -1/4, cos(c + d*x)^2))/(5*d*cos(c
 + d*x)^(5/2)*(sin(c + d*x)^2)^(1/2)) + (2*a*sin(c + d*x)*hypergeom([-7/4,
 1/2], -3/4, cos(c + d*x)^2))/(7*d*cos(c + d*x)^(7/2)*(sin(c + d*x)^2)^(1/
2))
```

3.359 $\int \cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^2\,dx$

3.359.1 Optimal result 2644
3.359.2 Mathematica [C] (warning: unable to verify) 2645
3.359.3 Rubi [A] (verified) 2646
3.359.4 Maple [A] (verified) 2649
3.359.5 Fricas [C] (verification not implemented) 2650
3.359.6 Sympy [F(-1)] 2651
3.359.7 Maxima [F] 2651
3.359.8 Giac [F] 2651
3.359.9 Mupad [B] (verification not implemented) 2652

3.359.1 Optimal result

Integrand size = 23, antiderivative size = 147

$$\begin{aligned}\int \cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^2\,dx = {} & \frac{32a^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d} \\ & + \frac{20a^2\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d} \\ & + \frac{20a^2\sqrt{\cos(c+dx)}\sin(c+dx)}{21d} \\ & + \frac{32a^2\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{45d} \\ & + \frac{4a^2\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{7d} \\ & + \frac{2a^2\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{9d}\end{aligned}$$

output
```
32/15*a^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/
2*d*x+1/2*c),2^(1/2))/d+20/21*a^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x
+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+32/45*a^2*cos(d*x+c)^(3/2)
*sin(d*x+c)/d+4/7*a^2*cos(d*x+c)^(5/2)*sin(d*x+c)/d+2/9*a^2*cos(d*x+c)^(7/
2)*sin(d*x+c)/d+20/21*a^2*sin(d*x+c)*cos(d*x+c)^(1/2)/d
```

3.359.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 7.19 (sec) , antiderivative size = 548, normalized size of antiderivative = 3.73

$$\int \cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^2\,dx$$
$$= \cos^{\frac{5}{2}}(c+dx)\sec^4\left(\frac{c}{2}+\frac{dx}{2}\right)(a+a\sec(c+dx))^2\left(-\frac{8\cot(c)}{15d}+\frac{23\cos(dx)\sin(c)}{84d}+\frac{37\cos(2dx)\sin(2c)}{360d}+\frac{\cos(3dx)\sin(3c)}{28d}+\frac{\cos(4dx)\sin(4c)}{144d}+\frac{23\cos(c)\sin(dx)}{84d}+\frac{37\cos(2c)\sin(2dx)}{360d}+\frac{\cos(3c)\sin(3dx)}{28d}+\frac{\cos(4c)\sin(4dx)}{144d}\right)$$
$$-\frac{5\cos^2(c+dx)\csc(c)\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};\sin^2(dx-\arctan(\cot(c)))\right)\sec^4\left(\frac{c}{2}+\frac{dx}{2}\right)(a+a\sec(c+dx))^2\sec(dx-}{}$$
$$-\frac{4\cos^2(c+dx)\csc(c)\sec^4\left(\frac{c}{2}+\frac{dx}{2}\right)(a+a\sec(c+dx))^2\left(\frac{{}_2F_1\left(-\frac{1}{2},-\frac{1}{4};\frac{3}{4};\cos^2(dx+\arctan(\tan(c)))\right.}{\sqrt{1-\cos(dx+\arctan(\tan(c)))}\sqrt{1+\cos(dx+\arctan(\tan(c)))}\sqrt{\mathrm{co}}}\right.}{15d}$$

input
```
Integrate[Cos[c + d*x]^(9/2)*(a + a*Sec[c + d*x])^2,x]
```

output
```
Cos[c + d*x]^(5/2)*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*((-8*Cot[c]
)/(15*d) + (23*Cos[d*x]*Sin[c])/(84*d) + (37*Cos[2*d*x]*Sin[2*c])/(360*d)
+ (Cos[3*d*x]*Sin[3*c])/(28*d) + (Cos[4*d*x]*Sin[4*c])/(144*d) + (23*Cos[c
]*Sin[d*x])/(84*d) + (37*Cos[2*c]*Sin[2*d*x])/(360*d) + (Cos[3*c]*Sin[3*d*
x])/(28*d) + (Cos[4*c]*Sin[4*d*x])/(144*d)) - (5*Cos[c + d*x]^2*Csc[c]*Hyp
ergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (
d*x)/2]^4*(a + a*Sec[c + d*x])^2*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*
x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot
[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(21*d*Sqrt[1 + Cot[c]^2]) - (
4*Cos[c + d*x]^2*Csc[c]*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*((Hype
rgeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x +
ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[
d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[
c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 +
Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos
[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2
]]))/(15*d)
```

3.359.3 Rubi [A] (verified)

Time = 1.29 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.44, number of steps used = 19, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.826$, Rules used = {3042, 4752, 3042, 4275, 3042, 4256, 3042, 4256, 3042, 4258, 3042, 3120, 4533, 3042, 4256, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^{\frac{9}{2}}(c+dx)(a\sec(c+dx)+a)^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^{9/2}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^2dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{(\sec(c+dx)a+a)^2}{\sec^{\frac{9}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{9/2}}dx$$

$$\Big\downarrow \text{4275}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(2a^2\int \frac{1}{\sec^{\frac{7}{2}}(c+dx)}dx+\int \frac{\sec^2(c+dx)a^2+a^2}{\sec^{\frac{9}{2}}(c+dx)}dx\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(2a^2\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx+\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{9/2}}dx\right)$$

$$\Big\downarrow \text{4256}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{9/2}}dx+2a^2\left(\frac{5}{7}\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}dx+\frac{2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}\right)\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{9/2}}dx+2a^2\left(\frac{5}{7}\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+\frac{2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx}\right.\right.$$

$$\Big\downarrow \; 4256$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2 a^2+a^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{9/2}}dx+2a^2\left(\frac{5}{7}\left(\frac{1}{3}\int \sqrt{\sec(c+dx)}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right.\right.\right.$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2 a^2+a^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{9/2}}dx+2a^2\left(\frac{5}{7}\left(\frac{1}{3}\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+\right.\right.\right.$$

$$\Big\downarrow \; 4258$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2 a^2+a^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{9/2}}dx+2a^2\left(\frac{5}{7}\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+}}\right.\right.\right.$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2 a^2+a^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{9/2}}dx+2a^2\left(\frac{5}{7}\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{}{\sqrt{\sin(c}}\right.\right.\right.$$

$$\Big\downarrow \; 3120$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2 a^2+a^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{9/2}}dx+2a^2\left(\frac{2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+\frac{5}{7}\left(\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2\sqrt{}}{}\right.\right.\right.$$

$$\Big\downarrow \; 4533$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{16}{9}a^2\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)}dx+\frac{2a^2\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)}+2a^2\left(\frac{2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+\frac{5}{7}\left(\frac{2\sin(c+}{3d\sqrt{\sec(c}}\right.\right.\right.$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{16}{9}a^2\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+\frac{2a^2\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)}+2a^2\left(\frac{2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+\frac{5}{7}\left(\frac{2s}{3d\sqrt{}}\right.\right.\right.$$

$$\Big\downarrow \; 4256$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{16}{9}a^2\left(\frac{3}{5}\int \frac{1}{\sqrt{\sec(c+dx)}}dx+\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}\right)+\frac{2a^2\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)}+2a^2\left(\frac{2\sin(c}{7d\sec^{\frac{5}{2}}(}\right.\right.$$

$$\Big\downarrow \; 3042$$

3.359. $\int \cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^2\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{16}{9}a^2\left(\frac{3}{5}\int\frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}\right)+\frac{2a^2\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)}+2a^2\left(\frac{2}{7d}\right.\right.$$

$\Big\downarrow$ 4258

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{16}{9}a^2\left(\frac{3}{5}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\cos(c+dx)}dx+\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}\right)+\frac{2a^2\sin}{9d\sec}\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{16}{9}a^2\left(\frac{3}{5}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}\right)+\right.$$

$\Big\downarrow$ 3119

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2a^2\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)}+2a^2\left(\frac{2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+\frac{5}{7}\left(\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2\sqrt{\cos(c+dx)}\sqrt{s}}{}\right.\right.\right.$$

input
```
Int[Cos[c + d*x]^(9/2)*(a + a*Sec[c + d*x])^2,x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*a^2*Sin[c + d*x])/(9*d*Sec[c + d
*x]^(7/2)) + (16*a^2*((6*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt
[Sec[c + d*x]])/(5*d) + (2*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2))))/9 + 2*
a^2*((2*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/2)) + (5*((2*Sqrt[Cos[c + d*x]]
*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (2*Sin[c + d*x])/(3
*d*Sqrt[Sec[c + d*x]])))/7))
```

3.359.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4275
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Simp[2*a*(b/d)   Int[(d*Csc[e + f*x])^(n + 1), x], x]
 + Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
 e, f, n}, x]
```

rule 4533
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] +
Simp[(C*m + A*(m + 1))/(b^2*m)   Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fr
eeQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

3.359.4 Maple [A] (verified)

Time = 12.28 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.77

method	result
default	$-\frac{4\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,a^{2}\left(560\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{11}-960\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{9}+608\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}-96\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}-205\cos\left(\right.\right.}{315\sqrt{-2\sin\left(\frac{d}{2}\right.}}$

input
```
int(cos(d*x+c)^(9/2)*(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
-4/315*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*(560*co
s(1/2*d*x+1/2*c)^11-960*cos(1/2*d*x+1/2*c)^9+608*cos(1/2*d*x+1/2*c)^7-96*c
os(1/2*d*x+1/2*c)^5-205*cos(1/2*d*x+1/2*c)^3+75*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))
-168*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*Ellipt
icE(cos(1/2*d*x+1/2*c),2^(1/2))+93*cos(1/2*d*x+1/2*c))/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)
^2-1)^(1/2)/d
```

3.359.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.19

$$\int \cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^2\,dx = -\frac{2\left(75i\sqrt{2}a^2\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))-75i\sqrt{2}a^2\text{weierstrassPInverse}(-\right.}{}$$

input
```
integrate(cos(d*x+c)^(9/2)*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
-2/315*(75*I*sqrt(2)*a^2*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d
*x + c)) - 75*I*sqrt(2)*a^2*weierstrassPInverse(-4, 0, cos(d*x + c) - I*si
n(d*x + c)) - 168*I*sqrt(2)*a^2*weierstrassZeta(-4, 0, weierstrassPInverse
(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 168*I*sqrt(2)*a^2*weierstrassZet
a(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - (35*
a^2*cos(d*x + c)^3 + 90*a^2*cos(d*x + c)^2 + 112*a^2*cos(d*x + c) + 150*a^
2)*sqrt(cos(d*x + c))*sin(d*x + c))/d
```

3.359.6 Sympy [F(-1)]

Timed out.

$$\int \cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^2\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(9/2)*(a+a*sec(d*x+c))**2,x)
```

output
```
Timed out
```

3.359.7 Maxima [F]

$$\int \cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^2\,dx = \int (a\sec(dx+c)+a)^2\cos(dx+c)^{\frac{9}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(9/2)*(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)^2*cos(d*x + c)^(9/2), x)
```

3.359.8 Giac [F]

$$\int \cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^2\,dx = \int (a\sec(dx+c)+a)^2\cos(dx+c)^{\frac{9}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(9/2)*(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)^2*cos(d*x + c)^(9/2), x)
```

3.359.9 Mupad [B] (verification not implemented)

Time = 14.28 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.93

$$\int \cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^2\,dx$$
$$= -\frac{2\,a^2\cos\left(c+d\,x\right)^{7/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{7}{4};\ \frac{11}{4};\ \cos\left(c+d\,x\right)^2\right)}{7\,d\,\sqrt{\sin\left(c+d\,x\right)^2}} - \frac{4\,a^2\cos\left(c+d\,x\right)^{9/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{9}{4};\ \frac{13}{4};\ \cos\left(c+d\,x\right)^2\right)}{9\,d\,\sqrt{\sin\left(c+d\,x\right)^2}} - \frac{2\,a^2\cos\left(c+d\,x\right)^{11/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{11}{4};\ \frac{15}{4};\ \cos\left(c+d\,x\right)^2\right)}{11\,d\,\sqrt{\sin\left(c+d\,x\right)^2}}$$

input
```
int(cos(c + d*x)^(9/2)*(a + a/cos(c + d*x))^2,x)
```

output
```
- (2*a^2*cos(c + d*x)^(7/2)*sin(c + d*x)*hypergeom([1/2, 7/4], 11/4, cos(c
 + d*x)^2))/(7*d*(sin(c + d*x)^2)^(1/2)) - (4*a^2*cos(c + d*x)^(9/2)*sin(c
 + d*x)*hypergeom([1/2, 9/4], 13/4, cos(c + d*x)^2))/(9*d*(sin(c + d*x)^2)
^(1/2)) - (2*a^2*cos(c + d*x)^(11/2)*sin(c + d*x)*hypergeom([1/2, 11/4], 1
5/4, cos(c + d*x)^2))/(11*d*(sin(c + d*x)^2)^(1/2))
```

3.360 $\int \cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^2\,dx$

3.360.1 Optimal result 2653
3.360.2 Mathematica [C] (warning: unable to verify) 2654
3.360.3 Rubi [A] (verified) 2655
3.360.4 Maple [A] (verified) 2658
3.360.5 Fricas [C] (verification not implemented) 2659
3.360.6 Sympy [F(-1)] 2659
3.360.7 Maxima [F] 2660
3.360.8 Giac [F] 2660
3.360.9 Mupad [B] (verification not implemented) 2660

3.360.1 Optimal result

Integrand size = 23, antiderivative size = 121

$$\int \cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^2\,dx = \frac{12a^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{8a^2\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{7d}+\frac{8a^2\sqrt{\cos(c+dx)}\sin(c+dx)}{7d}+\frac{4a^2\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d}+\frac{2a^2\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{7d}$$

output
```
12/5*a^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2
*d*x+1/2*c),2^(1/2))/d+8/7*a^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/
2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+4/5*a^2*cos(d*x+c)^(3/2)*sin(
d*x+c)/d+2/7*a^2*cos(d*x+c)^(5/2)*sin(d*x+c)/d+8/7*a^2*sin(d*x+c)*cos(d*x+
c)^(1/2)/d
```

3.360.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 6.94 (sec) , antiderivative size = 516, normalized size of antiderivative = 4.26

$$\int \cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^2\,dx = \cos^{\frac{5}{2}}(c+dx)\sec^4\left(\frac{c}{2}+\frac{dx}{2}\right)(a + a\sec(c+dx))^2\left(-\frac{3\cot(c)}{5d}+\frac{17\cos(dx)\sin(c)}{56d}+\frac{\cos(2dx)\sin(2c)}{10d}+\frac{\cos(3dx)\sin(3c)}{56d} + \frac{17\cos(c)\sin(dx)}{56d}+\frac{\cos(2c)\sin(2dx)}{10d}+\frac{\cos(3c)\sin(3dx)}{56d}\right) - \frac{2\cos^2(c+dx)\csc(c)\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};\sin^2(dx-\arctan(\cot(c)))\right)\sec^4\left(\frac{c}{2}+\frac{dx}{2}\right)(a+a\sec(c+dx))^2\sec(dx-}{} - \frac{3\cos^2(c+dx)\csc(c)\sec^4\left(\frac{c}{2}+\frac{dx}{2}\right)(a+a\sec(c+dx))^2\left(\frac{{}_2F_1\left(-\frac{1}{2},-\frac{1}{4};\frac{3}{4};\cos^2(dx+\arctan(\tan(c)))\right.}{\sqrt{1-\cos(dx+\arctan(\tan(c)))}\sqrt{1+\cos(dx+\arctan(\tan(c)))}\sqrt{\mathrm{co}}}\right.}{10d}$$

input
```
Integrate[Cos[c + d*x]^(7/2)*(a + a*Sec[c + d*x])^2,x]
```

output
```
Cos[c + d*x]^(5/2)*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*((-3*Cot[c]
)/(5*d) + (17*Cos[d*x]*Sin[c])/(56*d) + (Cos[2*d*x]*Sin[2*c])/(10*d) + (Co
s[3*d*x]*Sin[3*c])/(56*d) + (17*Cos[c]*Sin[d*x])/(56*d) + (Cos[2*c]*Sin[2*
d*x])/(10*d) + (Cos[3*c]*Sin[3*d*x])/(56*d)) - (2*Cos[c + d*x]^2*Csc[c]*Hy
pergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 +
(d*x)/2]^4*(a + a*Sec[c + d*x])^2*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d
*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Co
t[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(7*d*Sqrt[1 + Cot[c]^2]) - (
3*Cos[c + d*x]^2*Csc[c]*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*((Hype
rgeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x +
ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[
d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[
c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 +
Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos
[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2
]]))/(10*d)
```

3.360.3 Rubi [A] (verified)

Time = 1.14 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.52, number of steps used = 17, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.739, Rules used = {3042, 4752, 3042, 4275, 3042, 4256, 3042, 4258, 3042, 3119, 4533, 3042, 4256, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^{\frac{7}{2}}(c+dx)(a\sec(c+dx)+a)^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^{7/2}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^2dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{(\sec(c+dx)a+a)^2}{\sec^{\frac{7}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx$$

$$\Big\downarrow \text{4275}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(2a^2\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)}dx+\int \frac{\sec^2(c+dx)a^2+a^2}{\sec^{\frac{7}{2}}(c+dx)}dx\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(2a^2\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx\right)$$

$$\Big\downarrow \text{4256}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx+2a^2\left(\frac{3}{5}\int \frac{1}{\sqrt{\sec(c+dx)}}dx+\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}\right)\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx+2a^2\left(\frac{3}{5}\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx}\right.\right.$$

$$\Big\downarrow \text{4258}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2 a^2+a^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx+2a^2\left(\frac{3}{5}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\cos(c+dx}\right.\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2 a^2+a^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx+2a^2\left(\frac{3}{5}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\sin\left(c+d\right.}\right.\right.$$

$$\Big\downarrow \text{3119}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2 a^2+a^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx+2a^2\left(\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{6\sqrt{\cos(c+dx)}\sqrt{\sec(c+c}}{5d}\right.\right.$$

$$\Big\downarrow \text{4533}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{12}{7}a^2\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}dx+\frac{2a^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+2a^2\left(\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{6\sqrt{\cos(c+dx}}{}\right.\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{12}{7}a^2\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+\frac{2a^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+2a^2\left(\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{6\sqrt{\cos(}}{}\right.\right.$$

$$\Big\downarrow \text{4256}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{12}{7}a^2\left(\frac{1}{3}\int \sqrt{\sec(c+dx)}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)+\frac{2a^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+2a^2\left(\frac{2\sin(c}{5d\sec^{\frac{3}{2}}(}\right.\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{12}{7}a^2\left(\frac{1}{3}\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)+\frac{2a^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+2a^2\left(\frac{}{5}\right.\right.$$

$$\Big\downarrow \text{4258}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{12}{7}a^2\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)+\frac{2a^2}{7d\,\mathrm{s}}\right.$$

$$\Big\downarrow \text{3042}$$

3.360. $\int \cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^2\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{12}{7}a^2\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)+$$

$$\Big\downarrow \text{3120}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2a^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+\frac{12}{7}a^2\left(\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}}{3d}\right.\right.$$

input
```
Int[Cos[c + d*x]^(7/2)*(a + a*Sec[c + d*x])^2,x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*a^2*Sin[c + d*x])/(7*d*Sec[c + d
*x]^(5/2)) + 2*a^2*((6*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[S
ec[c + d*x]])/(5*d) + (2*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2))) + (12*a^2
*((2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d
) + (2*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])))/7)
```

3.360.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4275
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Simp[2*a*(b/d)   Int[(d*Csc[e + f*x])^(n + 1), x], x]
 + Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
 e, f, n}, x]
```

rule 4533
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] +
Simp[(C*m + A*(m + 1))/(b^2*m)   Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fr
eeQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

3.360.4 Maple [A] (verified)

Time = 10.94 (sec) , antiderivative size = 272, normalized size of antiderivative = 2.25

method	result
default	$-\frac{4\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\,a^2\left(40\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^8-116\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^6\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+126\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4\cos\left(\frac{d}{}\right.\right.}{35\sqrt{-2\sin\left(\right.}}$

input
```
int(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
-4/35*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*(40*cos(
1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8-116*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1
/2*c)+126*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-39*cos(1/2*d*x+1/2*c)*si
n(1/2*d*x+1/2*c)^2+10*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2
-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-21*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/
2))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c
)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

3.360.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.34

$$\int \cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^2\,dx = -\frac{2\left(10i\sqrt{2}a^2\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))-10i\sqrt{2}a^2\text{weierstrassPInverse}(-\right.}{}$$

input
```
integrate(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
-2/35*(10*I*sqrt(2)*a^2*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*
x + c)) - 10*I*sqrt(2)*a^2*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin
(d*x + c)) - 21*I*sqrt(2)*a^2*weierstrassZeta(-4, 0, weierstrassPInverse(-
4, 0, cos(d*x + c) + I*sin(d*x + c))) + 21*I*sqrt(2)*a^2*weierstrassZeta(-
4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - (5*a^2*
cos(d*x + c)^2 + 14*a^2*cos(d*x + c) + 20*a^2)*sqrt(cos(d*x + c))*sin(d*x
+ c))/d
```

3.360.6 Sympy [F(-1)]

Timed out.

$$\int \cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^2\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(7/2)*(a+a*sec(d*x+c))**2,x)
```

output
```
Timed out
```

3.360.7 Maxima [F]

$$\int \cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^2\,dx = \int (a\sec(dx+c)+a)^2\cos(dx+c)^{\frac{7}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)^2*cos(d*x + c)^(7/2), x)
```

3.360.8 Giac [F]

$$\int \cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^2\,dx = \int (a\sec(dx+c)+a)^2\cos(dx+c)^{\frac{7}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)^2*cos(d*x + c)^(7/2), x)
```

3.360.9 Mupad [B] (verification not implemented)

Time = 13.61 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.07

$$\begin{aligned}\int \cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^2\,dx &= \frac{2\left(a^2\,\mathrm{F}\left(\frac{c}{2}+\frac{dx}{2}\middle|2\right)+a^2\sqrt{\cos(c+dx)}\,\sin(c+dx)\right)}{3\,d} \\ &\quad - \frac{4\,a^2\cos(c+dx)^{7/2}\,\sin(c+dx)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{7}{4};\ \frac{11}{4};\ \cos(c+dx)^2\right)}{7\,d\,\sqrt{\sin(c+dx)^2}} \\ &\quad - \frac{2\,a^2\cos(c+dx)^{9/2}\,\sin(c+dx)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{9}{4};\ \frac{13}{4};\ \cos(c+dx)^2\right)}{9\,d\,\sqrt{\sin(c+dx)^2}}\end{aligned}$$

input
```
int(cos(c + d*x)^(7/2)*(a + a/cos(c + d*x))^2,x)
```

output
```
(2*(a^2*ellipticF(c/2 + (d*x)/2, 2) + a^2*cos(c + d*x)^(1/2)*sin(c + d*x))
)/(3*d) - (4*a^2*cos(c + d*x)^(7/2)*sin(c + d*x)*hypergeom([1/2, 7/4], 11/
4, cos(c + d*x)^2))/(7*d*(sin(c + d*x)^2)^(1/2)) - (2*a^2*cos(c + d*x)^(9/
2)*sin(c + d*x)*hypergeom([1/2, 9/4], 13/4, cos(c + d*x)^2))/(9*d*(sin(c +
 d*x)^2)^(1/2))
```

3.360. $\int \cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^2\,dx$

3.361 $\int \cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^2\,dx$

3.361.1 Optimal result 2662
3.361.2 Mathematica [C] (warning: unable to verify) 2662
3.361.3 Rubi [A] (verified) 2663
3.361.4 Maple [A] (verified) 2666
3.361.5 Fricas [C] (verification not implemented) 2667
3.361.6 Sympy [F(-1)] 2667
3.361.7 Maxima [F] 2668
3.361.8 Giac [F] 2668
3.361.9 Mupad [B] (verification not implemented) 2668

3.361.1 Optimal result

Integrand size = 23, antiderivative size = 95

$$\int \cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^2\,dx = \frac{16a^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{4a^2\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{4a^2\sqrt{\cos(c+dx)}\sin(c+dx)}{3d}+\frac{2a^2\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d}$$

output
```
16/5*a^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2
*d*x+1/2*c),2^(1/2))/d+4/3*a^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/
2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/5*a^2*cos(d*x+c)^(3/2)*sin(
d*x+c)/d+4/3*a^2*sin(d*x+c)*cos(d*x+c)^(1/2)/d
```

3.361.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 6.73 (sec) , antiderivative size = 235, normalized size of antiderivative = 2.47

$$\int \cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^2\,dx = \frac{a^2(1+\cos(c+dx))^2\sec^4\left(\frac{1}{2}(c+dx)\right)\left(\frac{12(3\cos(c-dx-\arctan(\tan(c)))+\cos(c+dx+\arctan(\tan(c))))\csc(c)\sec(c)}{\sqrt{\sec^2(c)}}-20\cos(c\ldots\right.}{}$$

input
```
Integrate[Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^2,x]
```

output
```
(a^2*(1 + Cos[c + d*x])^2*Sec[(c + d*x)/2]^4*((12*(3*Cos[c - d*x - ArcTan[
Tan[c]]] + Cos[c + d*x + ArcTan[Tan[c]]])*Csc[c]*Sec[c])/Sqrt[Sec[c]^2] -
20*Cos[c + d*x]*Sqrt[Cos[d*x - ArcTan[Cot[c]]]^2]*Sqrt[Csc[c]^2]*Hypergeom
etricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[d*x - ArcTan[
Cot[c]]]*Sin[c] + Cos[c + d*x]*(-48*Cot[c] + 20*Sin[c + d*x] + 3*Sin[2*(c
+ d*x)]) - 24*Cos[c]*Csc[d*x + ArcTan[Tan[c]]]*HypergeometricPFQ[{-1/2, -1
/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sqrt[Sec[c]^2]*Sqrt[Sin[d*x + Arc
Tan[Tan[c]]]^2]))/(60*d*Sqrt[Cos[c + d*x]])
```

3.361.3 Rubi [A] (verified)

Time = 0.98 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.64, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.652$, Rules used = {3042, 4752, 3042, 4275, 3042, 4256, 3042, 4258, 3042, 3120, 4533, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^{\frac{5}{2}}(c+dx)(a\sec(c+dx)+a)^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^2dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{(\sec(c+dx)a+a)^2}{\sec^{\frac{5}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx$$

$$\Big\downarrow \text{4275}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(2a^2\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}dx+\int \frac{\sec^2(c+dx)a^2+a^2}{\sec^{\frac{5}{2}}(c+dx)}dx\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(2a^2\int\frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx\right)$$

$\downarrow$ 4256

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+2a^2\left(\frac{1}{3}\int\sqrt{\sec(c+dx)}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)\right)$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+2a^2\left(\frac{1}{3}\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx}}\right.\right.$$

$\downarrow$ 4258

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+2a^2\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx}}\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+2a^2\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+d\right.}}\right.\right.$$

$\downarrow$ 3120

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+2a^2\left(\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+}}{}\right.\right.$$

$\downarrow$ 4533

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{8}{5}a^2\int\frac{1}{\sqrt{\sec(c+dx)}}dx+\frac{2a^2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+2a^2\left(\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2\sqrt{\cos(c+dx}}{}\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{8}{5}a^2\int\frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a^2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+2a^2\left(\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2\sqrt{\cos(c}}{}\right.\right.$$

$\downarrow$ 4258

3.361. $\int\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^2\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{8}{5}a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\cos(c+dx)}dx+\frac{2a^2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+2a^2\left(\frac{2\sin}{3d\sqrt{\mathrm{se}}}\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{8}{5}a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2a^2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+2a^2\left(\right.\right.$$

$\downarrow$ 3119

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2a^2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{16a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+2a^2\left(\frac{2\sin(c}{3d\sqrt{\sec}}\right.\right.$$

input
```
Int[Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^2,x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((16*a^2*Sqrt[Cos[c + d*x]]*Elliptic
E[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d) + (2*a^2*Sin[c + d*x])/(5*d*Se
c[c + d*x]^(3/2)) + 2*a^2*((2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]
*Sqrt[Sec[c + d*x]])/(3*d) + (2*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])))
```

3.361.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4275
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Simp[2*a*(b/d)   Int[(d*Csc[e + f*x])^(n + 1), x], x]
 + Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
 e, f, n}, x]
```

rule 4533
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] +
Simp[(C*m + A*(m + 1))/(b^2*m)   Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fr
eeQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

3.361.4 Maple [A] (verified)

Time = 8.38 (sec) , antiderivative size = 250, normalized size of antiderivative = 2.63

method	result
default	$-\frac{4\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\,a^2\left(-12\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^6\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+32\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-13\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right.\right.}{15\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\sin\left(\right.}}$

input
```
int(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
-4/15*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*(-12*sin
(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+32*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1
/2*c)-13*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+5*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2
))-12*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(
2*sin(1/2*d*x+1/2*c)^2-1)^(1/2))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*
c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

3.361.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.57

$$\int \cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^2\,dx = -\frac{2\left(5i\sqrt{2}a^2\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))-5i\sqrt{2}a^2\text{weierstrassPInverse}(-4,\right.}{}$$

input
```
integrate(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
-2/15*(5*I*sqrt(2)*a^2*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x
 + c)) - 5*I*sqrt(2)*a^2*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d
*x + c)) - 12*I*sqrt(2)*a^2*weierstrassZeta(-4, 0, weierstrassPInverse(-4,
 0, cos(d*x + c) + I*sin(d*x + c))) + 12*I*sqrt(2)*a^2*weierstrassZeta(-4,
 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - (3*a^2*co
s(d*x + c) + 10*a^2)*sqrt(cos(d*x + c))*sin(d*x + c))/d
```

3.361.6 Sympy [F(-1)]

Timed out.

$$\int \cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^2\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(5/2)*(a+a*sec(d*x+c))**2,x)
```

output
```
Timed out
```

3.361.7 Maxima [F]

$$\int \cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^2\,dx = \int (a\sec(dx+c)+a)^2\cos(dx+c)^{\frac{5}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)^2*cos(d*x + c)^(5/2), x)
```

3.361.8 Giac [F]

$$\int \cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^2\,dx = \int (a\sec(dx+c)+a)^2\cos(dx+c)^{\frac{5}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)^2*cos(d*x + c)^(5/2), x)
```

3.361.9 Mupad [B] (verification not implemented)

Time = 13.47 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.09

$$\int \cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^2\,dx = \frac{2\,a^2\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{4\,a^2\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{3\,d}+\frac{4\,a^2\,\sqrt{\cos(c+d\,x)}\,\sin(c+d\,x)}{3\,d} - \frac{2\,a^2\cos(c+d\,x)^{7/2}\,\sin(c+d\,x)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{7}{4};\ \frac{11}{4};\ \cos(c+d\,x)^2\right)}{7\,d\,\sqrt{\sin(c+d\,x)^2}}$$

input
```
int(cos(c + d*x)^(5/2)*(a + a/cos(c + d*x))^2,x)
```

output
```
(2*a^2*ellipticE(c/2 + (d*x)/2, 2))/d + (4*a^2*ellipticF(c/2 + (d*x)/2, 2)
)/(3*d) + (4*a^2*cos(c + d*x)^(1/2)*sin(c + d*x))/(3*d) - (2*a^2*cos(c + d
*x)^(7/2)*sin(c + d*x)*hypergeom([1/2, 7/4], 11/4, cos(c + d*x)^2))/(7*d*(
sin(c + d*x)^2)^(1/2))
```

3.362 $\int \cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2\,dx$

3.362.1 Optimal result 2669
3.362.2 Mathematica [C] (warning: unable to verify) 2669
3.362.3 Rubi [A] (verified) 2670
3.362.4 Maple [B] (verified) 2673
3.362.5 Fricas [C] (verification not implemented) 2673
3.362.6 Sympy [F(-1)] 2674
3.362.7 Maxima [F] 2674
3.362.8 Giac [F] 2674
3.362.9 Mupad [B] (verification not implemented) 2675

3.362.1 Optimal result

Integrand size = 23, antiderivative size = 67

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2\,dx=\frac{4a^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{8a^2\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2a^2\sqrt{\cos(c+dx)}\sin(c+dx)}{3d}$$

output
```
4*a^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*
x+1/2*c),2^(1/2))/d+8/3*a^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c
)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/3*a^2*sin(d*x+c)*cos(d*x+c)^(1
/2)/d
```

3.362.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 6.23 (sec) , antiderivative size = 224, normalized size of antiderivative = 3.34

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2\,dx$$
$$=\frac{a^2(1+\cos(c+dx))^2\sec^4\left(\frac{1}{2}(c+dx)\right)\left(\frac{3(3\cos(c-dx-\arctan(\tan(c)))+\cos(c+dx+\arctan(\tan(c))))\csc(c)\sec(c)}{\sqrt{\sec^2(c)}}-8\cos(c+\right.}{}$$

input
```
Integrate[Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^2,x]
```

output
```
(a^2*(1 + Cos[c + d*x])^2*Sec[(c + d*x)/2]^4*((3*(3*Cos[c - d*x - ArcTan[T
an[c]]] + Cos[c + d*x + ArcTan[Tan[c]]])*Csc[c]*Sec[c])/Sqrt[Sec[c]^2] - 8
*Cos[c + d*x]*Sqrt[Cos[d*x - ArcTan[Cot[c]]]^2]*Sqrt[Csc[c]^2]*Hypergeomet
ricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[d*x - ArcTan[Co
t[c]]]*Sin[c] + 2*Cos[c + d*x]*(-6*Cot[c] + Sin[c + d*x]) - 6*Cos[c]*Csc[d
*x + ArcTan[Tan[c]]]*HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcT
an[Tan[c]]]^2]*Sqrt[Sec[c]^2]*Sqrt[Sin[d*x + ArcTan[Tan[c]]]^2]))/(12*d*Sq
rt[Cos[c + d*x]])
```

3.362.3 Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.91, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.565$, Rules used = {3042, 4752, 3042, 4275, 3042, 4258, 3042, 3119, 4533, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^2\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^2dx$$

$$\Big\downarrow \; 4752$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{(\sec(c+dx)a+a)^2}{\sec^{\frac{3}{2}}(c+dx)}dx$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx$$

$$\Big\downarrow \; 4275$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \frac{\sec^2(c+dx)a^2+a^2}{\sec^{\frac{3}{2}}(c+dx)}dx+2a^2\int \frac{1}{\sqrt{\sec(c+dx)}}dx\right)$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(2a^2\int\frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx\right)$$

$\downarrow$ 4258

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+2a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\cos(c+dx)}dx\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+2a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\right.}\right.$$

$\downarrow$ 3119

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+\frac{4a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)$$

$\downarrow$ 4533

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{4}{3}a^2\int\sqrt{\sec(c+dx)}dx+\frac{2a^2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{4a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c\right.}{d}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{4}{3}a^2\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2a^2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{4a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E}{d}\right.$$

$\downarrow$ 4258

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{4}{3}a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2a^2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{4a^2\sqrt{\cos(c}}{}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{4}{3}a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a^2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{4a^2\sqrt{}}{}\right.$$

$\downarrow$ 3120

3.362. $\int\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2a^2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{8a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{4a^2\sqrt{c}}{}\right.$$

input
```
Int[Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^2,x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((4*a^2*Sqrt[Cos[c + d*x]]*EllipticE
[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (8*a^2*Sqrt[Cos[c + d*x]]*Ellipti
cF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (2*a^2*Sin[c + d*x])/(3*d*S
qrt[Sec[c + d*x]]))
```

3.362.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4275
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Simp[2*a*(b/d)   Int[(d*Csc[e + f*x])^(n + 1), x], x]
 + Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
 e, f, n}, x]
```

rule 4533
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] +
Simp[(C*m + A*(m + 1))/(b^2*m)   Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fr
eeQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

3.362. $\int \cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2\,dx$

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

3.362.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 227 vs. $2(113) = 226$.

Time = 4.19 (sec) , antiderivative size = 228, normalized size of antiderivative = 3.40

method	result
default	$-\frac{4\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\,a^2\left(2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2+2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\sin\left(\frac{dx}{2}\right.}\right.}{3\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\sin\left(\frac{dx}{2}+\frac{c}{2}\right.}$

input
```
int(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
-4/3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*(2*sin(1/
2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+
2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(
cos(1/2*d*x+1/2*c),2^(1/2))-3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1
/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2))/(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*
c)^2-1)^(1/2)/d
```

3.362.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 134, normalized size of antiderivative = 2.00

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2\,dx$$
$$= \frac{2\left(a^2\sqrt{\cos(dx+c)}\sin(dx+c) - 2i\sqrt{2}a^2\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)) + 2i\sqrt{}\right.}{}$$

input
```
integrate(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
2/3*(a^2*sqrt(cos(d*x + c))*sin(d*x + c) - 2*I*sqrt(2)*a^2*weierstrassPInv
erse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 2*I*sqrt(2)*a^2*weierstrassPI
nverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 3*I*sqrt(2)*a^2*weierstrass
Zeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 3
*I*sqrt(2)*a^2*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x +
 c) - I*sin(d*x + c))))/d
```

3.362.6 Sympy [F(-1)]

Timed out.

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(3/2)*(a+a*sec(d*x+c))**2,x)
```

output
```
Timed out
```

3.362.7 Maxima [F]

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2\,dx = \int (a\sec(dx+c)+a)^2\cos(dx+c)^{\frac{3}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)^2*cos(d*x + c)^(3/2), x)
```

3.362.8 Giac [F]

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2\,dx = \int (a\sec(dx+c)+a)^2\cos(dx+c)^{\frac{3}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)^2*cos(d*x + c)^(3/2), x)
```

3.362.9 Mupad [B] (verification not implemented)

Time = 13.53 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.88

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2\,dx = \frac{4\,a^2\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{8\,a^2\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{3\,d}+\frac{2\,a^2\,\sqrt{\cos\left(c+d\,x\right)}\,\sin\left(c+d\,x\right)}{3\,d}$$

input
```
int(cos(c + d*x)^(3/2)*(a + a/cos(c + d*x))^2,x)
```

output
```
(4*a^2*ellipticE(c/2 + (d*x)/2, 2))/d + (8*a^2*ellipticF(c/2 + (d*x)/2, 2)
)/(3*d) + (2*a^2*cos(c + d*x)^(1/2)*sin(c + d*x))/(3*d)
```

3.363 $\int \sqrt{\cos(c+dx)}(a+a\sec(c+dx))^2\,dx$

3.363.1 Optimal result 2676
3.363.2 Mathematica [C] (verified) 2676
3.363.3 Rubi [A] (verified) 2677
3.363.4 Maple [B] (verified) 2679
3.363.5 Fricas [C] (verification not implemented) 2680
3.363.6 Sympy [F] 2680
3.363.7 Maxima [F] 2681
3.363.8 Giac [F] 2681
3.363.9 Mupad [B] (verification not implemented) 2681

3.363.1 Optimal result

Integrand size = 23, antiderivative size = 44

$$\int \sqrt{\cos(c+dx)}(a+a\sec(c+dx))^2\,dx=\frac{4a^2\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+\frac{2a^2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

output
```
4*a^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*
x+1/2*c),2^(1/2))/d+2*a^2*sin(d*x+c)/d/cos(d*x+c)^(1/2)
```

3.363.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.42 (sec) , antiderivative size = 112, normalized size of antiderivative = 2.55

$$\int \sqrt{\cos(c+dx)}(a+a\sec(c+dx))^2\,dx=$$
$$-\frac{2a^2\csc(c+dx)\left(-3\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},\cos^2(c+dx)\right)+\cos(c+dx)\left(6\,\text{Hypergeometric2F1}\right.\right.}{3d\sqrt{\cos(c-}}$$

input
```
Integrate[Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])^2,x]
```

output
```
(-2*a^2*Csc[c + d*x]*(-3*Hypergeometric2F1[-1/4, 1/2, 3/4, Cos[c + d*x]^2]
 + Cos[c + d*x]*(6*Hypergeometric2F1[1/4, 1/2, 5/4, Cos[c + d*x]^2] + Cos[
c + d*x]*Hypergeometric2F1[1/2, 3/4, 7/4, Cos[c + d*x]^2]))*Sqrt[Sin[c + d
*x]^2])/(3*d*Sqrt[Cos[c + d*x]])
```

3.363.3 Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.93, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {3042, 4752, 3042, 4275, 3042, 4258, 3042, 3120, 4531}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\cos(c+dx)}(a\sec(c+dx)+a)^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^2dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{(\sec(c+dx)a+a)^2}{\sqrt{\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4275}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \frac{\sec^2(c+dx)a^2+a^2}{\sqrt{\sec(c+dx)}}dx+2a^2\int \sqrt{\sec(c+dx)}dx\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(2a^2\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)$$

$$\Big\downarrow \text{4258}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+2a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}d\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2 a^2+a^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+2a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\right.}}\right.$$

$$\Big\downarrow \; 3120$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2 a^2+a^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{4a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx\right.}{d}\right.$$

$$\Big\downarrow \; 4531$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}+\frac{4a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\right.}{d}\right.$$

input
```
Int[Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])^2,x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((4*a^2*Sqrt[Cos[c + d*x]]*EllipticF
[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (2*a^2*Sqrt[Sec[c + d*x]]*Sin[c +
 d*x])/d)
```

3.363.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4275
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Simp[2*a*(b/d)   Int[(d*Csc[e + f*x])^(n + 1), x], x]
 + Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
 e, f, n}, x]
```

rule 4531
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] /;
 FreeQ[{b, e, f, A, C, m}, x] && EqQ[C*m + A*(m + 1), 0]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

3.363.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 184 vs. $2(68) = 136$.

Time = 5.46 (sec) , antiderivative size = 185, normalized size of antiderivative = 4.20

method	result
default	$-\frac{4a^2\left(-\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1}\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\right.\right.}{\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1}\,d}$

input
```
int(cos(d*x+c)^(1/2)*(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
-4*a^2*(-(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+
1/2*c)*sin(1/2*d*x+1/2*c)^2+(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/
2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(-2*sin(1/2*d*x+1/2*
c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2
*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

3.363.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 97, normalized size of antiderivative = 2.20

$$\int \sqrt{\cos(c+dx)}(a+a\sec(c+dx))^2\,dx = -\frac{2\left(i\sqrt{2}a^2\cos(dx+c)\,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))-i\sqrt{2}a^2\cos(dx+c)\,\text{w}\right.}{d\cos(dx+c)}$$

input
```
integrate(cos(d*x+c)^(1/2)*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
-2*(I*sqrt(2)*a^2*cos(d*x + c)*weierstrassPInverse(-4, 0, cos(d*x + c) + I
*sin(d*x + c)) - I*sqrt(2)*a^2*cos(d*x + c)*weierstrassPInverse(-4, 0, cos
(d*x + c) - I*sin(d*x + c)) - a^2*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(
d*x + c))
```

3.363.6 Sympy [F]

$$\int \sqrt{\cos(c+dx)}(a+a\sec(c+dx))^2\,dx = a^2\left(\int 2\sqrt{\cos(c+dx)}\sec(c+dx)\,dx + \int \sqrt{\cos(c+dx)}\sec^2(c+dx)\,dx + \int \sqrt{\cos(c+dx)}\,dx\right)$$

input
```
integrate(cos(d*x+c)**(1/2)*(a+a*sec(d*x+c))**2,x)
```

output
```
a**2*(Integral(2*sqrt(cos(c + d*x))*sec(c + d*x), x) + Integral(sqrt(cos(c
 + d*x))*sec(c + d*x)**2, x) + Integral(sqrt(cos(c + d*x)), x))
```

3.363.7 Maxima [F]

$$\int \sqrt{\cos(c+dx)}(a+a\sec(c+dx))^2\,dx = \int (a\sec(dx+c)+a)^2\sqrt{\cos(dx+c)}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)*(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)^2*sqrt(cos(d*x + c)), x)
```

3.363.8 Giac [F]

$$\int \sqrt{\cos(c+dx)}(a+a\sec(c+dx))^2\,dx = \int (a\sec(dx+c)+a)^2\sqrt{\cos(dx+c)}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)*(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)^2*sqrt(cos(d*x + c)), x)
```

3.363.9 Mupad [B] (verification not implemented)

Time = 14.29 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.86

$$\int \sqrt{\cos(c+dx)}(a+a\sec(c+dx))^2\,dx = \frac{2\,a^2\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d} + \frac{4\,a^2\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d} + \frac{2\,a^2\,\sin(c+d\,x)\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{1}{2};\ \frac{3}{4};\ \cos(c+d\,x)^2\right)}{d\,\sqrt{\cos(c+d\,x)}\,\sqrt{\sin(c+d\,x)^2}}$$

input
```
int(cos(c + d*x)^(1/2)*(a + a/cos(c + d*x))^2,x)
```

output
```
(2*a^2*ellipticE(c/2 + (d*x)/2, 2))/d + (4*a^2*ellipticF(c/2 + (d*x)/2, 2)
)/d + (2*a^2*sin(c + d*x)*hypergeom([-1/4, 1/2], 3/4, cos(c + d*x)^2))/(d*
cos(c + d*x)^(1/2)*(sin(c + d*x)^2)^(1/2))
```

3.364 $\int \frac{(a+a\sec(c+dx))^2}{\sqrt{\cos(c+dx)}}\,dx$

3.364.1 Optimal result . . . 2682
3.364.2 Mathematica [C] (verified) . . . 2682
3.364.3 Rubi [A] (verified) . . . 2683
3.364.4 Maple [B] (verified) . . . 2686
3.364.5 Fricas [C] (verification not implemented) . . . 2687
3.364.6 Sympy [F] . . . 2688
3.364.7 Maxima [F] . . . 2688
3.364.8 Giac [F] . . . 2688
3.364.9 Mupad [B] (verification not implemented) . . . 2689

3.364.1 Optimal result

Integrand size = 23, antiderivative size = 91

$$\int \frac{(a+a\sec(c+dx))^2}{\sqrt{\cos(c+dx)}}\,dx = -\frac{4a^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{8a^2\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2a^2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{4a^2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

output
```
-4*a^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d
*x+1/2*c),2^(1/2))/d+8/3*a^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*
c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/3*a^2*sin(d*x+c)/d/cos(d*x+c)
^(3/2)+4*a^2*sin(d*x+c)/d/cos(d*x+c)^(1/2)
```

3.364.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.37 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.22

$$\int \frac{(a+a\sec(c+dx))^2}{\sqrt{\cos(c+dx)}}\,dx = \frac{2a^2\csc(c+dx)\left(\text{Hypergeometric2F1}\left(-\frac{3}{4},\frac{1}{2},\frac{1}{4},\cos^2(c+dx)\right)+6\cos(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2}\right.\right.}{3d\cos^{\frac{3}{2}}(c+dx)}$$

input
```
Integrate[(a + a*Sec[c + d*x])^2/Sqrt[Cos[c + d*x]],x]
```

output
```
(2*a^2*Csc[c + d*x]*(Hypergeometric2F1[-3/4, 1/2, 1/4, Cos[c + d*x]^2] + 6
*Cos[c + d*x]*Hypergeometric2F1[-1/4, 1/2, 3/4, Cos[c + d*x]^2] - 3*Cos[c
+ d*x]^2*Hypergeometric2F1[1/4, 1/2, 5/4, Cos[c + d*x]^2])*Sqrt[Sin[c + d*
x]^2])/(3*d*Cos[c + d*x]^(3/2))
```

3.364.3 Rubi [A] (verified)

Time = 0.91 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.67, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.652$, Rules used = {3042, 4752, 3042, 4275, 3042, 4255, 3042, 4258, 3042, 3119, 4534, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sec(c+dx)+a)^2}{\sqrt{\cos(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^2}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\sec(c+dx)}(\sec(c+dx)a+a)^2dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2dx$$

$$\Big\downarrow \text{4275}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(2a^2\int \sec^{\frac{3}{2}}(c+dx)dx+\int \sqrt{\sec(c+dx)}\left(\sec^2(c+dx)a^2+a^2\right)dx\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(2a^2\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}dx+\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2\right)\right.$$

$$\Big\downarrow \text{4255}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(\csc\left(c+dx+\frac{\pi}{2}\right)^2 a^2+a^2\right)dx+2a^2\left(\frac{2\sin(c+dx)\sqrt{\sec(c}}{d}\right.\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(\csc\left(c+dx+\frac{\pi}{2}\right)^2 a^2+a^2\right)dx+2a^2\left(\frac{2\sin(c+dx)\sqrt{\sec(c}}{d}\right.\right.$$

$$\Big\downarrow \text{4258}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(\csc\left(c+dx+\frac{\pi}{2}\right)^2 a^2+a^2\right)dx+2a^2\left(\frac{2\sin(c+dx)\sqrt{\sec(c}}{d}\right.\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(\csc\left(c+dx+\frac{\pi}{2}\right)^2 a^2+a^2\right)dx+2a^2\left(\frac{2\sin(c+dx)\sqrt{\sec(c}}{d}\right.\right.$$

$$\Big\downarrow \text{3119}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(\csc\left(c+dx+\frac{\pi}{2}\right)^2 a^2+a^2\right)dx+2a^2\left(\frac{2\sin(c+dx)\sqrt{\sec(c}}{d}\right.\right.$$

$$\Big\downarrow \text{4534}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{4}{3}a^2\int \sqrt{\sec(c+dx)}dx+\frac{2a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+2a^2\left(\frac{2\sin(c+dx)\sqrt{\sec(c}}{d}\right.\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{4}{3}a^2\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+2a^2\left(\frac{2\sin(c+dx)\sqrt{}}{d}\right.\right.$$

$$\Big\downarrow \text{4258}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{4}{3}a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}\right.$$

$$\Big\downarrow \text{3042}$$

3.364. $\int \frac{(a+a\sec(c+dx))^2}{\sqrt{\cos(c+dx)}}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{4}{3}a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+\right.$$

$$\Big\downarrow \text{3120}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{8a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\right.}{3d}\right.$$

input
```
Int[(a + a*Sec[c + d*x])^2/Sqrt[Cos[c + d*x]],x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((8*a^2*Sqrt[Cos[c + d*x]]*EllipticF
[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (2*a^2*Sec[c + d*x]^(3/2)*Sin
[c + d*x])/(3*d) + 2*a^2*((-2*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]
*Sqrt[Sec[c + d*x]])/d + (2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d))
```

3.364.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4275
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Simp[2*a*(b/d)   Int[(d*Csc[e + f*x])^(n + 1), x], x]
 + Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
 e, f, n}, x]
```

rule 4534
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Simp[(C*m + A*(m + 1))/(m + 1)   Int[(b*Csc[e + f*x])^m, x], x] /;
 FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

3.364.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 370 vs. $2(135) = 270$.

Time = 8.50 (sec) , antiderivative size = 371, normalized size of antiderivative = 4.08

method	result
default	$-\frac{4\sqrt{-\left(-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\,a^2\left(12\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-4\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1}\,\mathrm{EllipticF}\left(\cos\right.\right.}{}$

input
```
int((a+a*sec(d*x+c))^2/cos(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-4/3*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2/(4*sin(
1/2*d*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)/sin(1/2*d*x+1/2*c)^3*(12*sin(1/
2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/
2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+
1/2*c)^2-6*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/
2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2-7*cos(1/2*d*x+1/
2*c)*sin(1/2*d*x+1/2*c)^2+2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/
2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1
)^(1/2))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d
*x+1/2*c)^2-1)^(1/2)/d
```

3.364.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 187, normalized size of antiderivative = 2.05

$$\int \frac{(a+a\sec(c+dx))^2}{\sqrt{\cos(c+dx)}}\,dx = -\frac{2\left(2i\sqrt{2}a^2\cos(dx+c)^2\,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))-2i\sqrt{2}a^2\cos(dx+c\right.}{}$$

input
```
integrate((a+a*sec(d*x+c))^2/cos(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
-2/3*(2*I*sqrt(2)*a^2*cos(d*x + c)^2*weierstrassPInverse(-4, 0, cos(d*x +
c) + I*sin(d*x + c)) - 2*I*sqrt(2)*a^2*cos(d*x + c)^2*weierstrassPInverse(
-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 3*I*sqrt(2)*a^2*cos(d*x + c)^2*wei
erstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x +
c))) - 3*I*sqrt(2)*a^2*cos(d*x + c)^2*weierstrassZeta(-4, 0, weierstrassPI
nverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - (6*a^2*cos(d*x + c) + a^2)
*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^2)
```

3.364.6 Sympy [F]

$$\int \frac{(a+a\sec(c+dx))^2}{\sqrt{\cos(c+dx)}}\,dx = a^2\left(\int \frac{2\sec(c+dx)}{\sqrt{\cos(c+dx)}}\,dx + \int \frac{\sec^2(c+dx)}{\sqrt{\cos(c+dx)}}\,dx + \int \frac{1}{\sqrt{\cos(c+dx)}}\,dx\right)$$

input
```
integrate((a+a*sec(d*x+c))**2/cos(d*x+c)**(1/2),x)
```

output
```
a**2*(Integral(2*sec(c + d*x)/sqrt(cos(c + d*x)), x) + Integral(sec(c + d*
x)**2/sqrt(cos(c + d*x)), x) + Integral(1/sqrt(cos(c + d*x)), x))
```

3.364.7 Maxima [F]

$$\int \frac{(a+a\sec(c+dx))^2}{\sqrt{\cos(c+dx)}}\,dx = \int \frac{(a\sec(dx+c)+a)^2}{\sqrt{\cos(dx+c)}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))^2/cos(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)^2/sqrt(cos(d*x + c)), x)
```

3.364.8 Giac [F]

$$\int \frac{(a+a\sec(c+dx))^2}{\sqrt{\cos(c+dx)}}\,dx = \int \frac{(a\sec(dx+c)+a)^2}{\sqrt{\cos(dx+c)}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))^2/cos(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)^2/sqrt(cos(d*x + c)), x)
```

3.364.9 Mupad [B] (verification not implemented)

Time = 14.35 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.20

$$\int \frac{(a+a\sec(c+dx))^2}{\sqrt{\cos(c+dx)}}\,dx = \frac{2\,a^2\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d} + \frac{4\,a^2\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{1}{2};\ \frac{3}{4};\ \cos\left(c+d\,x\right)^2\right)}{d\,\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{\sin\left(c+d\,x\right)^2}} + \frac{2\,a^2\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{3}{4},\frac{1}{2};\ \frac{1}{4};\ \cos\left(c+d\,x\right)^2\right)}{3\,d\,\cos\left(c+d\,x\right)^{3/2}\,\sqrt{\sin\left(c+d\,x\right)^2}}$$

input
```
int((a + a/cos(c + d*x))^2/cos(c + d*x)^(1/2),x)
```

output
```
(2*a^2*ellipticF(c/2 + (d*x)/2, 2))/d + (4*a^2*sin(c + d*x)*hypergeom([-1/
4, 1/2], 3/4, cos(c + d*x)^2))/(d*cos(c + d*x)^(1/2)*(sin(c + d*x)^2)^(1/2
)) + (2*a^2*sin(c + d*x)*hypergeom([-3/4, 1/2], 1/4, cos(c + d*x)^2))/(3*d
*cos(c + d*x)^(3/2)*(sin(c + d*x)^2)^(1/2))
```

3.365 $\int \frac{(a+a\sec(c+dx))^2}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

3.365.1 Optimal result 2690
3.365.2 Mathematica [C] (verified) 2690
3.365.3 Rubi [A] (verified) 2691
3.365.4 Maple [B] (verified) 2694
3.365.5 Fricas [C] (verification not implemented) 2695
3.365.6 Sympy [F] 2696
3.365.7 Maxima [F] 2696
3.365.8 Giac [F] 2696
3.365.9 Mupad [B] (verification not implemented) 2697

3.365.1 Optimal result

Integrand size = 23, antiderivative size = 121

$$\int \frac{(a+a\sec(c+dx))^2}{\cos^{\frac{3}{2}}(c+dx)}\,dx = -\frac{16a^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{4a^2\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2a^2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{4a^2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{16a^2\sin(c+dx)}{5d\sqrt{\cos(c+dx)}}$$

output
```
-16/5*a^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/
2*d*x+1/2*c),2^(1/2))/d+4/3*a^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1
/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/5*a^2*sin(d*x+c)/d/cos(d*x
+c)^(5/2)+4/3*a^2*sin(d*x+c)/d/cos(d*x+c)^(3/2)+16/5*a^2*sin(d*x+c)/d/cos(
d*x+c)^(1/2)
```

3.365.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.48 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.94

$$\int \frac{(a+a\sec(c+dx))^2}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{2a^2\csc(c+dx)\left(3\operatorname{Hypergeometric2F1}\left(-\frac{5}{4},\frac{1}{2},-\frac{1}{4},\cos^2(c+dx)\right)+5\cos(c+dx)\left(2\operatorname{Hypergeometric2F1}\left(\right.\right.\right.}{15d\cos^{\frac{5}{2}}(c}$$

input
```
Integrate[(a + a*Sec[c + d*x])^2/Cos[c + d*x]^(3/2),x]
```

output
```
(2*a^2*Csc[c + d*x]*(3*Hypergeometric2F1[-5/4, 1/2, -1/4, Cos[c + d*x]^2]
+ 5*Cos[c + d*x]*(2*Hypergeometric2F1[-3/4, 1/2, 1/4, Cos[c + d*x]^2] + 3*
Cos[c + d*x]*Hypergeometric2F1[-1/4, 1/2, 3/4, Cos[c + d*x]^2]))*Sqrt[Sin[
c + d*x]^2])/(15*d*Cos[c + d*x]^(5/2))
```

3.365.3 Rubi [A] (verified)

Time = 1.03 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.49, number of steps used = 17, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.739$, Rules used = {3042, 4752, 3042, 4275, 3042, 4255, 3042, 4258, 3042, 3120, 4534, 3042, 4255, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sec(c+dx)+a)^2}{\cos^{\frac{3}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sec^{\frac{3}{2}}(c+dx)(\sec(c+dx)a+a)^2dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2dx$$

$$\Big\downarrow \text{4275}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(2a^2\int \sec^{\frac{5}{2}}(c+dx)dx+\int \sec^{\frac{3}{2}}(c+dx)\left(\sec^2(c+dx)a^2+a^2\right)dx\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(2a^2\int \csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}dx+\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2\right.\right.$$

$$\Big\downarrow \text{4255}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(\csc\left(c+dx+\frac{\pi}{2}\right)^2 a^2+a^2\right)dx+2a^2\left(\frac{1}{3}\int \sqrt{\sec(c+dx)}d\right.\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(\csc\left(c+dx+\frac{\pi}{2}\right)^2 a^2+a^2\right)dx+2a^2\left(\frac{1}{3}\int \sqrt{\csc\left(c+dx+\right.}\right.\right.$$

$\Big\downarrow$ 4258

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(\csc\left(c+dx+\frac{\pi}{2}\right)^2 a^2+a^2\right)dx+2a^2\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\text{se}}\right.\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(\csc\left(c+dx+\frac{\pi}{2}\right)^2 a^2+a^2\right)dx+2a^2\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\text{se}}\right.\right.$$

$\Big\downarrow$ 3120

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(\csc\left(c+dx+\frac{\pi}{2}\right)^2 a^2+a^2\right)dx+2a^2\left(\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c}{3d}\right.\right.$$

$\Big\downarrow$ 4534

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{8}{5}a^2\int \sec^{\frac{3}{2}}(c+dx)dx+\frac{2a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}+2a^2\left(\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+}{3d}\right.\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{8}{5}a^2\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}dx+\frac{2a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}+2a^2\left(\frac{2\sin(c+dx)\text{s}}{3d}\right.\right.$$

$\Big\downarrow$ 4255

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{8}{5}a^2\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\int \frac{1}{\sqrt{\sec(c+dx)}}dx\right)+\frac{2a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+}{5d}\right.$$

$\Big\downarrow$ 3042

3.365. $\int \frac{(a+a\sec(c+dx))^2}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{8}{5}a^2\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\int\frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)+\frac{2a^2\sin(c+dx)\sec}{5d}\right.$$

$$\Big\downarrow \text{4258}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{8}{5}a^2\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\cos(c+dx)}dx\right)\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{8}{5}a^2\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right.}\right.\right.$$

$$\Big\downarrow \text{3119}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}+2a^2\left(\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{2\sqrt{\cos(c+dx)}\sqrt{s}}{}\right.\right.$$

input
```
Int[(a + a*Sec[c + d*x])^2/Cos[c + d*x]^(3/2),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*a^2*Sec[c + d*x]^(5/2)*Sin[c + d
*x])/(5*d) + (8*a^2*((-2*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt
[Sec[c + d*x]])/d + (2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d))/5 + 2*a^2*((2*
Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (
2*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d)))
```

3.365.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4275
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Simp[2*a*(b/d)   Int[(d*Csc[e + f*x])^(n + 1), x], x]
 + Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
 e, f, n}, x]
```

rule 4534
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Simp[(C*m + A*(m + 1))/(m + 1)   Int[(b*Csc[e + f*x])^m, x], x] /;
 FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

3.365.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 385 vs. $2(157) = 314$.

Time = 9.69 (sec) , antiderivative size = 386, normalized size of antiderivative = 3.19

method	result
default	$-\frac{8\sqrt{-\left(-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\,a^2\left(-\frac{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}}{12\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-\frac{1}{2}\right)^2}+\frac{17\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2+}}{30\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+}}\right.}{}$

input
```
int((a+a*sec(d*x+c))^2/cos(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-8*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*(-1/12*co
s(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos
(1/2*d*x+1/2*c)^2-1/2)^2+17/30*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*
x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*E
llipticF(cos(1/2*d*x+1/2*c),2^(1/2))-4/5*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+
1/2*c)/(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)-2/5*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+
1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)
)-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))-1/80*cos(1/2*d*x+1/2*c)*(-2*sin(1
/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^3)/
sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

3.365.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.67

$$\int \frac{(a+a\sec(c+dx))^2}{\cos^{\frac{3}{2}}(c+dx)}\,dx = -\frac{2\left(5i\sqrt{2}a^2\cos(dx+c)^3\,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))-5i\sqrt{2}a^2\cos(dx+c\right.}{}$$

input
```
integrate((a+a*sec(d*x+c))^2/cos(d*x+c)^(3/2),x, algorithm="fricas")
```

output
```
-2/15*(5*I*sqrt(2)*a^2*cos(d*x + c)^3*weierstrassPInverse(-4, 0, cos(d*x +
 c) + I*sin(d*x + c)) - 5*I*sqrt(2)*a^2*cos(d*x + c)^3*weierstrassPInverse
(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 12*I*sqrt(2)*a^2*cos(d*x + c)^3*w
eierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x
+ c))) - 12*I*sqrt(2)*a^2*cos(d*x + c)^3*weierstrassZeta(-4, 0, weierstras
sPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - (24*a^2*cos(d*x + c)^2
+ 10*a^2*cos(d*x + c) + 3*a^2)*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x
 + c)^3)
```

3.365.6 Sympy [F]

$$\int \frac{(a+a\sec(c+dx))^2}{\cos^{\frac{3}{2}}(c+dx)}\,dx = a^2\left(\int \frac{2\sec(c+dx)}{\cos^{\frac{3}{2}}(c+dx)}\,dx + \int \frac{\sec^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)}\,dx + \int \frac{1}{\cos^{\frac{3}{2}}(c+dx)}\,dx\right)$$

input
```
integrate((a+a*sec(d*x+c))**2/cos(d*x+c)**(3/2),x)
```

output
```
a**2*(Integral(2*sec(c + d*x)/cos(c + d*x)**(3/2), x) + Integral(sec(c + d
*x)**2/cos(c + d*x)**(3/2), x) + Integral(cos(c + d*x)**(-3/2), x))
```

3.365.7 Maxima [F]

$$\int \frac{(a+a\sec(c+dx))^2}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(a\sec(dx+c)+a)^2}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))^2/cos(d*x+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)^2/cos(d*x + c)^(3/2), x)
```

3.365.8 Giac [F]

$$\int \frac{(a+a\sec(c+dx))^2}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(a\sec(dx+c)+a)^2}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))^2/cos(d*x+c)^(3/2),x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)^2/cos(d*x + c)^(3/2), x)
```

3.365.9 Mupad [B] (verification not implemented)

Time = 14.15 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.94

$$\int \frac{(a+a\sec(c+dx))^2}{\cos^{\frac{3}{2}}(c+dx)}\,dx$$
$$= \frac{6\,a^2\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{5}{4},\frac{1}{2};\ -\frac{1}{4};\ \cos\left(c+d\,x\right)^2\right)+20\,a^2\,\cos\left(c+d\,x\right)\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{3}{4},\frac{1}{2};\ \frac{1}{4};\ \cos\left(c\right.\right.}{15\,d\cos\left(c+d\,x\right)^{5/2}\,\sqrt{1-\cos\left(c+d\right.}}$$

input
```
int((a + a/cos(c + d*x))^2/cos(c + d*x)^(3/2),x)
```

output
```
(6*a^2*sin(c + d*x)*hypergeom([-5/4, 1/2], -1/4, cos(c + d*x)^2) + 20*a^2*
cos(c + d*x)*sin(c + d*x)*hypergeom([-3/4, 1/2], 1/4, cos(c + d*x)^2) + 30
*a^2*cos(c + d*x)^2*sin(c + d*x)*hypergeom([-1/4, 1/2], 3/4, cos(c + d*x)^
2))/(15*d*cos(c + d*x)^(5/2)*(1 - cos(c + d*x)^2)^(1/2))
```

3.366 $\int \frac{(a+a\sec(c+dx))^2}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

3.366.1 Optimal result 2698
3.366.2 Mathematica [C] (verified) 2699
3.366.3 Rubi [A] (verified) 2699
3.366.4 Maple [B] (verified) 2703
3.366.5 Fricas [C] (verification not implemented) 2704
3.366.6 Sympy [F(-1)] 2704
3.366.7 Maxima [F] 2705
3.366.8 Giac [F] 2705
3.366.9 Mupad [B] (verification not implemented) 2705

3.366.1 Optimal result

Integrand size = 23, antiderivative size = 147

$$\int \frac{(a+a\sec(c+dx))^2}{\cos^{\frac{5}{2}}(c+dx)}\,dx = -\frac{12a^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{8a^2\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{7d}+\frac{2a^2\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)}+\frac{4a^2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{8a^2\sin(c+dx)}{7d\cos^{\frac{3}{2}}(c+dx)}+\frac{12a^2\sin(c+dx)}{5d\sqrt{\cos(c+dx)}}$$

output
```
-12/5*a^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/
2*d*x+1/2*c),2^(1/2))/d+8/7*a^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1
/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/7*a^2*sin(d*x+c)/d/cos(d*x
+c)^(7/2)+4/5*a^2*sin(d*x+c)/d/cos(d*x+c)^(5/2)+8/7*a^2*sin(d*x+c)/d/cos(d
*x+c)^(3/2)+12/5*a^2*sin(d*x+c)/d/cos(d*x+c)^(1/2)
```

3.366.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.64 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.78

$$\int \frac{(a+a\sec(c+dx))^2}{\cos^{\frac{5}{2}}(c+dx)}\,dx$$
$$= \frac{2a^2\csc(c+dx)\left(15\,\text{Hypergeometric2F1}\left(-\frac{7}{4},\frac{1}{2},-\frac{3}{4},\cos^2(c+dx)\right)+7\cos(c+dx)\left(6\,\text{Hypergeometric2F1}\right.\right.}{105d\cos^{\frac{7}{2}}}$$

input
```
Integrate[(a + a*Sec[c + d*x])^2/Cos[c + d*x]^(5/2),x]
```

output
```
(2*a^2*Csc[c + d*x]*(15*Hypergeometric2F1[-7/4, 1/2, -3/4, Cos[c + d*x]^2]
 + 7*Cos[c + d*x]*(6*Hypergeometric2F1[-5/4, 1/2, -1/4, Cos[c + d*x]^2] +
5*Cos[c + d*x]*Hypergeometric2F1[-3/4, 1/2, 1/4, Cos[c + d*x]^2]))*Sqrt[Si
n[c + d*x]^2])/(105*d*Cos[c + d*x]^(7/2))
```

3.366.3 Rubi [A] (verified)

Time = 1.25 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.41, number of steps used = 19, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.826$, Rules used = {3042, 4752, 3042, 4275, 3042, 4255, 3042, 4255, 3042, 4258, 3042, 3119, 4534, 3042, 4255, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sec(c+dx)+a)^2}{\cos^{\frac{5}{2}}(c+dx)}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx$$
$$\Big\downarrow \; 4752$$
$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sec^{\frac{5}{2}}(c+dx)(\sec(c+dx)a+a)^2dx$$
$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2dx$$

$\Big\downarrow$ 4275

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(2a^2\int \sec^{\frac{7}{2}}(c+dx)dx+\int \sec^{\frac{5}{2}}(c+dx)\left(\sec^2(c+dx)a^2+a^2\right)dx\right)$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(2a^2\int \csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}dx+\int \csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2\right.\right.$$

$\Big\downarrow$ 4255

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2\right)dx+2a^2\left(\frac{3}{5}\int \sec^{\frac{3}{2}}(c+dx)dx\right.\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2\right)dx+2a^2\left(\frac{3}{5}\int \csc\left(c+dx+\frac{\pi}{2}\right.\right.\right.$$

$\Big\downarrow$ 4255

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2\right)dx+2a^2\left(\frac{3}{5}\left(\frac{2\sin(c+dx)\sqrt{s}}{d}\right.\right.\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2\right)dx+2a^2\left(\frac{3}{5}\left(\frac{2\sin(c+dx)\sqrt{s}}{d}\right.\right.\right.$$

$\Big\downarrow$ 4258

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2\right)dx+2a^2\left(\frac{3}{5}\left(\frac{2\sin(c+dx)\sqrt{s}}{d}\right.\right.\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+a^2\right)dx+2a^2\left(\frac{3}{5}\left(\frac{2\sin(c+dx)\sqrt{s}}{d}\right.\right.\right.$$

$\Big\downarrow$ 3119

3.366. $\int \frac{(a+a\sec(c+dx))^2}{\cos^{\frac{5}{2}}(c+dx)}dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(\csc\left(c+dx+\frac{\pi}{2}\right)^2 a^2+a^2\right)dx+2a^2\left(\frac{2\sin(c+dx)\sec^{\frac{5}{2}}(c\right.\right.$$

$\downarrow$ 4534

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{12}{7}a^2\int \sec^{\frac{5}{2}}(c+dx)dx+\frac{2a^2\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{7d}+2a^2\left(\frac{2\sin(c+dx)\sec^{\frac{5}{2}}(c\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{12}{7}a^2\int \csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}dx+\frac{2a^2\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{7d}+2a^2\left(\frac{2\sin(c+dx)}{5}\right.\right.$$

$\downarrow$ 4255

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{12}{7}a^2\left(\frac{1}{3}\int \sqrt{\sec(c+dx)}dx+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}\right)+\frac{2a^2\sin(c+dx)\sec^{\frac{7}{2}}(c\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{12}{7}a^2\left(\frac{1}{3}\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}\right)+\frac{2a^2\sin(c+dx)}{7d}\right.$$

$\downarrow$ 4258

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{12}{7}a^2\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{12}{7}a^2\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c\right.\right.$$

$\downarrow$ 3120

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2a^2\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{7d}+\frac{12}{7}a^2\left(\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{2\sqrt{\cos(c+dx)}\sqrt{}}{}\right.\right.$$

input
```
Int[(a + a*Sec[c + d*x])^2/Cos[c + d*x]^(5/2),x]
```

3.366. $\int \frac{(a+a\sec(c+dx))^2}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*a^2*Sec[c + d*x]^(7/2)*Sin[c + d
*x])/(7*d) + (12*a^2*((2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt
[Sec[c + d*x]])/(3*d) + (2*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d)))/7 + 2*
a^2*((2*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5*d) + (3*((-2*Sqrt[Cos[c + d*x]
]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (2*Sqrt[Sec[c + d*x]]*
Sin[c + d*x])/d))/5))
```

3.366.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4275
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Simp[2*a*(b/d)   Int[(d*Csc[e + f*x])^(n + 1), x], x]
 + Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
 e, f, n}, x]
```

3.366. $\int \frac{(a+a\sec(c+dx))^2}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

rule 4534
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Simp[(C*m + A*(m + 1))/(m + 1)   Int[(b*Csc[e + f*x])^m, x], x] /;
 FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

3.366.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 438 vs. $2(179) = 358$.

Time = 12.84 (sec) , antiderivative size = 439, normalized size of antiderivative = 2.99

method	result
default	$-\frac{8\sqrt{-\left(-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\,a^2\left(-\frac{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}}{40\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-\frac{1}{2}\right)^3}-\frac{3\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)}{5\sqrt{-\left(-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)}}\right.}{}$

input
```
int((a+a*sec(d*x+c))^2/cos(d*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-8*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*(-1/40*co
s(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos
(1/2*d*x+1/2*c)^2-1/2)^3-3/5*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)/(-(-2
*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)+31/70*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+s
in(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3/10*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x
+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2
))-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))-1/14*cos(1/2*d*x+1/2*c)*(-2*sin(
1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2-
1/224*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1
/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^4)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)
^2-1)^(1/2)/d
```

3.366.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.46

$$\int \frac{(a+a\sec(c+dx))^2}{\cos^{\frac{5}{2}}(c+dx)}\,dx = -\frac{2\left(10i\sqrt{2}a^2\cos(dx+c)^4\,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))-10i\sqrt{2}a^2\cos(dx+\right.}{}$$

input
```
integrate((a+a*sec(d*x+c))^2/cos(d*x+c)^(5/2),x, algorithm="fricas")
```

output
```
-2/35*(10*I*sqrt(2)*a^2*cos(d*x + c)^4*weierstrassPInverse(-4, 0, cos(d*x
+ c) + I*sin(d*x + c)) - 10*I*sqrt(2)*a^2*cos(d*x + c)^4*weierstrassPInver
se(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 21*I*sqrt(2)*a^2*cos(d*x + c)^4
*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*
x + c))) - 21*I*sqrt(2)*a^2*cos(d*x + c)^4*weierstrassZeta(-4, 0, weierstr
assPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - (42*a^2*cos(d*x + c)^
3 + 20*a^2*cos(d*x + c)^2 + 14*a^2*cos(d*x + c) + 5*a^2)*sqrt(cos(d*x + c)
)*sin(d*x + c))/(d*cos(d*x + c)^4)
```

3.366.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(c+dx))^2}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sec(d*x+c))**2/cos(d*x+c)**(5/2),x)
```

output
```
Timed out
```

3.366.7 Maxima [F]

$$\int \frac{(a+a\sec(c+dx))^2}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{(a\sec(dx+c)+a)^2}{\cos(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))^2/cos(d*x+c)^(5/2),x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)^2/cos(d*x + c)^(5/2), x)
```

3.366.8 Giac [F]

$$\int \frac{(a+a\sec(c+dx))^2}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{(a\sec(dx+c)+a)^2}{\cos(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))^2/cos(d*x+c)^(5/2),x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)^2/cos(d*x + c)^(5/2), x)
```

3.366.9 Mupad [B] (verification not implemented)

Time = 14.35 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.78

$$\int \frac{(a+a\sec(c+dx))^2}{\cos^{\frac{5}{2}}(c+dx)}\,dx$$
$$= \frac{30\,a^2\,\sin(c+d\,x)\,{}_2\mathrm{F}_1\left(-\frac{7}{4},\frac{1}{2};\ -\frac{3}{4};\ \cos(c+d\,x)^2\right)+84\,a^2\,\cos(c+d\,x)\,\sin(c+d\,x)\,{}_2\mathrm{F}_1\left(-\frac{5}{4},\frac{1}{2};\ -\frac{1}{4};\ \cos\right.}{105\,d\cos(c+d\,x)^{7/2}\,\sqrt{1-\cos(c+}}$$

input
```
int((a + a/cos(c + d*x))^2/cos(c + d*x)^(5/2),x)
```

output
```
(30*a^2*sin(c + d*x)*hypergeom([-7/4, 1/2], -3/4, cos(c + d*x)^2) + 84*a^2
*cos(c + d*x)*sin(c + d*x)*hypergeom([-5/4, 1/2], -1/4, cos(c + d*x)^2) +
70*a^2*cos(c + d*x)^2*sin(c + d*x)*hypergeom([-3/4, 1/2], 1/4, cos(c + d*x
)^2))/(105*d*cos(c + d*x)^(7/2)*(1 - cos(c + d*x)^2)^(1/2))
```

3.367 $\int \cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^3\,dx$

3.367.1 Optimal result . . . 2706
3.367.2 Mathematica [C] (verified) . . . 2707
3.367.3 Rubi [A] (verified) . . . 2708
3.367.4 Maple [A] (verified) . . . 2709
3.367.5 Fricas [C] (verification not implemented) . . . 2710
3.367.6 Sympy [F(-1)] . . . 2710
3.367.7 Maxima [F] . . . 2711
3.367.8 Giac [F] . . . 2711
3.367.9 Mupad [B] (verification not implemented) . . . 2711

3.367.1 Optimal result

Integrand size = 23, antiderivative size = 147

$$\begin{aligned}\int \cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^3\,dx = {} & \frac{68a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d} \\ & + \frac{44a^3\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d} \\ & + \frac{44a^3\sqrt{\cos(c+dx)}\sin(c+dx)}{21d} \\ & + \frac{68a^3\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{45d} \\ & + \frac{6a^3\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{7d} \\ & + \frac{2a^3\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{9d}\end{aligned}$$

output
```
68/15*a^3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/
2*d*x+1/2*c),2^(1/2))/d+44/21*a^3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x
+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+68/45*a^3*cos(d*x+c)^(3/2)
*sin(d*x+c)/d+6/7*a^3*cos(d*x+c)^(5/2)*sin(d*x+c)/d+2/9*a^3*cos(d*x+c)^(7/
2)*sin(d*x+c)/d+44/21*a^3*sin(d*x+c)*cos(d*x+c)^(1/2)/d
```

3.367.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 7.34 (sec) , antiderivative size = 548, normalized size of antiderivative = 3.73

$$\int \cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^3\,dx$$
$$= \cos^{\frac{7}{2}}(c+dx)\sec^6\left(\frac{c}{2}+\frac{dx}{2}\right)(a+a\sec(c+dx))^3\left(-\frac{17\cot(c)}{30d}+\frac{97\cos(dx)\sin(c)}{336d}+\frac{73\cos(2dx)\sin(2c)}{720d}+\frac{3\cos(3dx)\sin(3c)}{112d}+\frac{\cos(4dx)\sin(4c)}{288d}+\frac{97\cos(c)\sin(dx)}{336d}+\frac{73\cos(2c)\sin(2dx)}{720d}+\frac{3\cos(3c)\sin(3dx)}{112d}+\frac{\cos(4c)\sin(4dx)}{288d}\right)$$
$$-\frac{11\cos^3(c+dx)\csc(c)\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};\sin^2(dx-\arctan(\cot(c)))\right)\sec^6\left(\frac{c}{2}+\frac{dx}{2}\right)(a+a\sec(c+dx))^3\sec(dx-}{}$$
$$-\frac{17\cos^3(c+dx)\csc(c)\sec^6\left(\frac{c}{2}+\frac{dx}{2}\right)(a+a\sec(c+dx))^3\left(\frac{{}_2F_1\left(-\frac{1}{2},-\frac{1}{4};\frac{3}{4};\cos^2(dx+\arctan(\tan(c)\right.}{\sqrt{1-\cos(dx+\arctan(\tan(c)))}\sqrt{1+\cos(dx+\arctan(\tan(c)))}\sqrt{}}\right.}{60d}$$

input
```
Integrate[Cos[c + d*x]^(9/2)*(a + a*Sec[c + d*x])^3,x]
```

output
```
Cos[c + d*x]^(7/2)*Sec[c/2 + (d*x)/2]^6*(a + a*Sec[c + d*x])^3*((-17*Cot[c
])/(30*d) + (97*Cos[d*x]*Sin[c])/(336*d) + (73*Cos[2*d*x]*Sin[2*c])/(720*d
) + (3*Cos[3*d*x]*Sin[3*c])/(112*d) + (Cos[4*d*x]*Sin[4*c])/(288*d) + (97*
Cos[c]*Sin[d*x])/(336*d) + (73*Cos[2*c]*Sin[2*d*x])/(720*d) + (3*Cos[3*c]*
Sin[3*d*x])/(112*d) + (Cos[4*c]*Sin[4*d*x])/(288*d)) - (11*Cos[c + d*x]^3*
Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*S
ec[c/2 + (d*x)/2]^6*(a + a*Sec[c + d*x])^3*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[
1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x -
ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(42*d*Sqrt[1 + Cot[
c]^2]) - (17*Cos[c + d*x]^3*Csc[c]*Sec[c/2 + (d*x)/2]^6*(a + a*Sec[c + d*x
])^3*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]
*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sq
rt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sq
rt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c]
)/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[
c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1
 + Tan[c]^2]]))/(60*d)
```

3.367.3 Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.41, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 4752, 3042, 4278, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^{\frac{9}{2}}(c+dx)(a\sec(c+dx)+a)^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^{9/2}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^3dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{(\sec(c+dx)a+a)^3}{\sec^{\frac{9}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^3}{\csc\left(c+dx+\frac{\pi}{2}\right)^{9/2}}dx$$

$$\Big\downarrow \text{4278}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \left(\frac{a^3}{\sec^{\frac{3}{2}}(c+dx)}+\frac{3a^3}{\sec^{\frac{5}{2}}(c+dx)}+\frac{3a^3}{\sec^{\frac{7}{2}}(c+dx)}+\frac{a^3}{\sec^{\frac{9}{2}}(c+dx)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{68a^3\sin(c+dx)}{45d\sec^{\frac{3}{2}}(c+dx)}+\frac{6a^3\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+\frac{2a^3\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)}+\frac{44a^3\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}+\frac{44a^3}{}\right.$$

input
```
Int[Cos[c + d*x]^(9/2)*(a + a*Sec[c + d*x])^3,x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((68*a^3*Sqrt[Cos[c + d*x]]*Elliptic
E[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(15*d) + (44*a^3*Sqrt[Cos[c + d*x]]*
EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (2*a^3*Sin[c + d*x]
)/(9*d*Sec[c + d*x]^(7/2)) + (6*a^3*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/2))
 + (68*a^3*Sin[c + d*x])/(45*d*Sec[c + d*x]^(3/2)) + (44*a^3*Sin[c + d*x])
/(21*d*Sqrt[Sec[c + d*x]]))
```

3.367.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4278
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

3.367.4 Maple [A] (verified)

Time = 22.34 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.77

method	result
default	$-\frac{4\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,a^{3}\left(560\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{11}-600\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{9}+212\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}+66\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}-430\cos\left(\right.\right.}{315\sqrt{-2\sin\left(\right.}}$

input
```
int(cos(d*x+c)^(9/2)*(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
-4/315*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3*(560*co
s(1/2*d*x+1/2*c)^11-600*cos(1/2*d*x+1/2*c)^9+212*cos(1/2*d*x+1/2*c)^7+66*c
os(1/2*d*x+1/2*c)^5-430*cos(1/2*d*x+1/2*c)^3+165*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)
)-357*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*Ellip
ticE(cos(1/2*d*x+1/2*c),2^(1/2))+192*cos(1/2*d*x+1/2*c))/(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*
c)^2-1)^(1/2)/d
```

3.367.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.19

$$\int \cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^3\,dx = -\frac{2\left(165i\sqrt{2}a^3\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))-165i\sqrt{2}a^3\text{weierstrassPInverse}\right.}{}$$

input
```
integrate(cos(d*x+c)^(9/2)*(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
-2/315*(165*I*sqrt(2)*a^3*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(
d*x + c)) - 165*I*sqrt(2)*a^3*weierstrassPInverse(-4, 0, cos(d*x + c) - I*
sin(d*x + c)) - 357*I*sqrt(2)*a^3*weierstrassZeta(-4, 0, weierstrassPInver
se(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 357*I*sqrt(2)*a^3*weierstrassZ
eta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - (3
5*a^3*cos(d*x + c)^3 + 135*a^3*cos(d*x + c)^2 + 238*a^3*cos(d*x + c) + 330
*a^3)*sqrt(cos(d*x + c))*sin(d*x + c))/d
```

3.367.6 Sympy [F(-1)]

Timed out.

$$\int \cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^3\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(9/2)*(a+a*sec(d*x+c))**3,x)
```

output
```
Timed out
```

3.367.7 Maxima [F]

$$\int \cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^3\,dx = \int (a\sec(dx+c)+a)^3\cos(dx+c)^{\frac{9}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(9/2)*(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)^3*cos(d*x + c)^(9/2), x)
```

3.367.8 Giac [F]

$$\int \cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^3\,dx = \int (a\sec(dx+c)+a)^3\cos(dx+c)^{\frac{9}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(9/2)*(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)^3*cos(d*x + c)^(9/2), x)
```

3.367.9 Mupad [B] (verification not implemented)

Time = 14.62 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.40

$$\begin{aligned}
&\int \cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^3\,dx \\
&= \frac{2\left(a^3\,\mathrm{F}\left(\frac{c}{2}+\frac{dx}{2}\middle|2\right)+a^3\sqrt{\cos(c+dx)}\,\sin(c+dx)\right)}{3\,d} \\
&\quad - \frac{2\left(\frac{33\,a^3\cos(c+dx)^{7/2}\,\sin(c+dx)}{\sqrt{\sin(c+dx)^2}}-\frac{5\,a^3\cos(c+dx)^{11/2}\,\sin(c+dx)}{\sqrt{\sin(c+dx)^2}}\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{11}{4};\ \frac{15}{4};\ \cos(c+dx)^2\right)}{77\,d} \\
&\quad - \frac{2\,a^3\cos(c+dx)^{9/2}\,\sin(c+dx)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{9}{4};\ \frac{13}{4};\ \cos(c+dx)^2\right)}{3\,d\sqrt{\sin(c+dx)^2}} \\
&\quad - \frac{104\,a^3\cos(c+dx)^{11/2}\,\sin(c+dx)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{11}{4};\ \frac{19}{4};\ \cos(c+dx)^2\right)}{385\,d\sqrt{\sin(c+dx)^2}}
\end{aligned}$$

input
```
int(cos(c + d*x)^(9/2)*(a + a/cos(c + d*x))^3,x)
```

output
```
(2*(a^3*ellipticF(c/2 + (d*x)/2, 2) + a^3*cos(c + d*x)^(1/2)*sin(c + d*x))
)/(3*d) - (2*((33*a^3*cos(c + d*x)^(7/2)*sin(c + d*x))/(sin(c + d*x)^2)^(1
/2) - (5*a^3*cos(c + d*x)^(11/2)*sin(c + d*x))/(sin(c + d*x)^2)^(1/2))*hyp
ergeom([1/2, 11/4], 15/4, cos(c + d*x)^2))/(77*d) - (2*a^3*cos(c + d*x)^(9
/2)*sin(c + d*x)*hypergeom([1/2, 9/4], 13/4, cos(c + d*x)^2))/(3*d*(sin(c
+ d*x)^2)^(1/2)) - (104*a^3*cos(c + d*x)^(11/2)*sin(c + d*x)*hypergeom([1/
2, 11/4], 19/4, cos(c + d*x)^2))/(385*d*(sin(c + d*x)^2)^(1/2))
```

3.367. $\int \cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^3\,dx$

3.368 $\int \cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^3\,dx$

3.368.1 Optimal result 2713
3.368.2 Mathematica [C] (warning: unable to verify) 2714
3.368.3 Rubi [A] (verified) 2714
3.368.4 Maple [A] (verified) 2716
3.368.5 Fricas [C] (verification not implemented) 2716
3.368.6 Sympy [F(-1)] 2717
3.368.7 Maxima [F] 2717
3.368.8 Giac [F] 2717
3.368.9 Mupad [B] (verification not implemented) 2718

3.368.1 Optimal result

Integrand size = 23, antiderivative size = 121

$$\begin{aligned}\int \cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^3\,dx = {} & \frac{28a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} \\ & + \frac{52a^3\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d} \\ & + \frac{52a^3\sqrt{\cos(c+dx)}\sin(c+dx)}{21d} \\ & + \frac{6a^3\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d} \\ & + \frac{2a^3\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{7d}\end{aligned}$$

output
```
28/5*a^3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2
*d*x+1/2*c),2^(1/2))/d+52/21*a^3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+
1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+6/5*a^3*cos(d*x+c)^(3/2)*si
n(d*x+c)/d+2/7*a^3*cos(d*x+c)^(5/2)*sin(d*x+c)/d+52/21*a^3*sin(d*x+c)*cos(
d*x+c)^(1/2)/d
```

3.368.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 5.78 (sec) , antiderivative size = 245, normalized size of antiderivative = 2.02

$$\int \cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^3\,dx$$
$$= \frac{a^3(1+\cos(c+dx))^3\sec^6\left(\frac{1}{2}(c+dx)\right)\left(\frac{294(3\cos(c-dx-\arctan(\tan(c)))+\cos(c+dx+\arctan(\tan(c))))\csc(c)\sec(c)}{\sqrt{\sec^2(c)}}-520\cos(\right.}{}$$

input
```
Integrate[Cos[c + d*x]^(7/2)*(a + a*Sec[c + d*x])^3,x]
```

output
```
(a^3*(1 + Cos[c + d*x])^3*Sec[(c + d*x)/2]^6*((294*(3*Cos[c - d*x - ArcTan
[Tan[c]]] + Cos[c + d*x + ArcTan[Tan[c]]])*Csc[c]*Sec[c])/Sqrt[Sec[c]^2] -
 520*Cos[c + d*x]*Sqrt[Cos[d*x - ArcTan[Cot[c]]]^2]*Sqrt[Csc[c]^2]*Hyperge
ometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[d*x - ArcTa
n[Cot[c]]]*Sin[c] + Cos[c + d*x]*(-1176*Cot[c] + 535*Sin[c + d*x] + 126*Si
n[2*(c + d*x)] + 15*Sin[3*(c + d*x)]) - 588*Cos[c]*Csc[d*x + ArcTan[Tan[c]
]]*HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sqr
t[Sec[c]^2]*Sqrt[Sin[d*x + ArcTan[Tan[c]]]^2]))/(1680*d*Sqrt[Cos[c + d*x]]
)
```

3.368.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.50, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 4752, 3042, 4278, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^{\frac{7}{2}}(c+dx)(a\sec(c+dx)+a)^3\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^{7/2}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^3dx$$
$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{(\sec(c+dx)a+a)^3}{\sec^{\frac{7}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^3}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx$$

$$\Big\downarrow \text{4278}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \left(\frac{a^3}{\sqrt{\sec(c+dx)}}+\frac{3a^3}{\sec^{\frac{3}{2}}(c+dx)}+\frac{3a^3}{\sec^{\frac{5}{2}}(c+dx)}+\frac{a^3}{\sec^{\frac{7}{2}}(c+dx)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{6a^3\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{2a^3\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+\frac{52a^3\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}+\frac{52a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c}}{}\right.$$

input
```
Int[Cos[c + d*x]^(7/2)*(a + a*Sec[c + d*x])^3,x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((28*a^3*Sqrt[Cos[c + d*x]]*Elliptic
E[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d) + (52*a^3*Sqrt[Cos[c + d*x]]*E
llipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (2*a^3*Sin[c + d*x])
/(7*d*Sec[c + d*x]^(5/2)) + (6*a^3*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2))
+ (52*a^3*Sin[c + d*x])/(21*d*Sqrt[Sec[c + d*x]]))
```

3.368.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4278
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

3.368.4 Maple [A] (verified)

Time = 22.64 (sec) , antiderivative size = 272, normalized size of antiderivative = 2.25

method	result
default	$-\frac{4\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,a^{3}\left(120\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{8}-432\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+602\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}\cos\left(\right.\right.}{105\sqrt{-2\,\mathrm{si}}}$

input
```
int(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
-4/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3*(120*co
s(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8-432*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x
+1/2*c)+602*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-208*cos(1/2*d*x+1/2*c)
*sin(1/2*d*x+1/2*c)^2+65*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c
)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-147*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)
^(1/2))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1
/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

3.368.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.34

$$\int \cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^3\,dx =$$
$$-\frac{2\left(65i\sqrt{2}a^3\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))-65i\sqrt{2}a^3\text{weierstrassPInverse}(-\right.}{}$$

input
```
integrate(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
-2/105*(65*I*sqrt(2)*a^3*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d
*x + c)) - 65*I*sqrt(2)*a^3*weierstrassPInverse(-4, 0, cos(d*x + c) - I*si
n(d*x + c)) - 147*I*sqrt(2)*a^3*weierstrassZeta(-4, 0, weierstrassPInverse
(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 147*I*sqrt(2)*a^3*weierstrassZet
a(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - (15*
a^3*cos(d*x + c)^2 + 63*a^3*cos(d*x + c) + 130*a^3)*sqrt(cos(d*x + c))*sin
(d*x + c))/d
```

3.368.6 Sympy [F(-1)]

Timed out.

$$\int \cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^3\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(7/2)*(a+a*sec(d*x+c))**3,x)
```

output
```
Timed out
```

3.368.7 Maxima [F]

$$\int \cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^3\,dx = \int (a\sec(dx+c)+a)^3\cos(dx+c)^{\frac{7}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)^3*cos(d*x + c)^(7/2), x)
```

3.368.8 Giac [F]

$$\int \cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^3\,dx = \int (a\sec(dx+c)+a)^3\cos(dx+c)^{\frac{7}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)^3*cos(d*x + c)^(7/2), x)
```

3.368.9 Mupad [B] (verification not implemented)

Time = 14.81 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.18

$$\int \cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^3\,dx$$
$$= \frac{2\left(a^3\,\mathrm{E}\left(\frac{c}{2}+\frac{dx}{2}\middle|2\right)+a^3\,\mathrm{F}\left(\frac{c}{2}+\frac{dx}{2}\middle|2\right)+a^3\,\sqrt{\cos\left(c+d\,x\right)}\,\sin\left(c+d\,x\right)\right)}{d}$$
$$-\frac{6\,a^3\cos\left(c+d\,x\right)^{7/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{7}{4};\ \frac{11}{4};\ \cos\left(c+d\,x\right)^2\right)}{7\,d\,\sqrt{\sin\left(c+d\,x\right)^2}}$$
$$-\frac{2\,a^3\cos\left(c+d\,x\right)^{9/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{9}{4};\ \frac{13}{4};\ \cos\left(c+d\,x\right)^2\right)}{9\,d\,\sqrt{\sin\left(c+d\,x\right)^2}}$$

input
```
int(cos(c + d*x)^(7/2)*(a + a/cos(c + d*x))^3,x)
```

output
```
(2*(a^3*ellipticE(c/2 + (d*x)/2, 2) + a^3*ellipticF(c/2 + (d*x)/2, 2) + a^
3*cos(c + d*x)^(1/2)*sin(c + d*x)))/d - (6*a^3*cos(c + d*x)^(7/2)*sin(c +
d*x)*hypergeom([1/2, 7/4], 11/4, cos(c + d*x)^2))/(7*d*(sin(c + d*x)^2)^(1
/2)) - (2*a^3*cos(c + d*x)^(9/2)*sin(c + d*x)*hypergeom([1/2, 9/4], 13/4,
cos(c + d*x)^2))/(9*d*(sin(c + d*x)^2)^(1/2))
```

3.369 $\int \cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^3\,dx$

3.369.1 Optimal result 2719
3.369.2 Mathematica [C] (warning: unable to verify) 2719
3.369.3 Rubi [A] (verified) 2720
3.369.4 Maple [A] (verified) 2722
3.369.5 Fricas [C] (verification not implemented) 2722
3.369.6 Sympy [F(-1)] 2723
3.369.7 Maxima [F] 2723
3.369.8 Giac [F] 2723
3.369.9 Mupad [B] (verification not implemented) 2724

3.369.1 Optimal result

Integrand size = 23, antiderivative size = 91

$$\int \cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^3\,dx=\frac{36a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{4a^3\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+\frac{2a^3\sqrt{\cos(c+dx)}\sin(c+dx)}{d}+\frac{2a^3\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d}$$

output
```
36/5*a^3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2
*d*x+1/2*c),2^(1/2))/d+4*a^3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*
c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/5*a^3*cos(d*x+c)^(3/2)*sin(d*
x+c)/d+2*a^3*sin(d*x+c)*cos(d*x+c)^(1/2)/d
```

3.369.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 5.35 (sec) , antiderivative size = 233, normalized size of antiderivative = 2.56

$$\int \cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^3\,dx$$
$$=\frac{a^3(1+\cos(c+dx))^3\sec^6\left(\frac{1}{2}(c+dx)\right)\left(\frac{9(3\cos(c-dx-\arctan(\tan(c)))+\cos(c+dx+\arctan(\tan(c))))\csc(c)\sec(c)}{\sqrt{\sec^2(c)}}-20\cos(c+\right.}{}$$

input
```
Integrate[Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^3,x]
```

output
```
(a^3*(1 + Cos[c + d*x])^3*Sec[(c + d*x)/2]^6*((9*(3*Cos[c - d*x - ArcTan[T
an[c]]] + Cos[c + d*x + ArcTan[Tan[c]]])*Csc[c]*Sec[c])/Sqrt[Sec[c]^2] - 2
0*Cos[c + d*x]*Sqrt[Cos[d*x - ArcTan[Cot[c]]]^2]*Sqrt[Csc[c]^2]*Hypergeome
tricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[d*x - ArcTan[C
ot[c]]]*Sin[c] + Cos[c + d*x]*(-36*Cot[c] + 10*Sin[c + d*x] + Sin[2*(c + d
*x)]) - 18*Cos[c]*Csc[d*x + ArcTan[Tan[c]]]*HypergeometricPFQ[{-1/2, -1/4}
, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sqrt[Sec[c]^2]*Sqrt[Sin[d*x + ArcTan
[Tan[c]]]^2]))/(40*d*Sqrt[Cos[c + d*x]])
```

3.369.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.67, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 4752, 3042, 4278, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^{\frac{5}{2}}(c+dx)(a\sec(c+dx)+a)^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^3dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{(\sec(c+dx)a+a)^3}{\sec^{\frac{5}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^3}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx$$

$$\Big\downarrow \text{4278}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \left(\sqrt{\sec(c+dx)}a^3+\frac{3a^3}{\sqrt{\sec(c+dx)}}+\frac{3a^3}{\sec^{\frac{3}{2}}(c+dx)}+\frac{a^3}{\sec^{\frac{5}{2}}(c+dx)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2a^3\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{2a^3\sin(c+dx)}{d\sqrt{\sec(c+dx)}}+\frac{4a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+\right.}{d}\right.$$

input
```
Int[Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^3,x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((36*a^3*Sqrt[Cos[c + d*x]]*Elliptic
E[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d) + (4*a^3*Sqrt[Cos[c + d*x]]*El
lipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (2*a^3*Sin[c + d*x])/(5*d*
Sec[c + d*x]^(3/2)) + (2*a^3*Sin[c + d*x])/(d*Sqrt[Sec[c + d*x]]))
```

3.369.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4278
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

3.369.4 Maple [A] (verified)

Time = 19.90 (sec) , antiderivative size = 250, normalized size of antiderivative = 2.75

method	result
default	$-\frac{4\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,a^{3}\left(-4\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+14\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-6\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\right.\right.}{5\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\sin\left(\frac{dx}{2}\right.}}$

input
```
int(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
-4/5*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3*(-4*sin(1
/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+14*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2
*c)-6*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+5*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-
9*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*si
n(1/2*d*x+1/2*c)^2-1)^(1/2))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2
)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

3.369.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.63

$$\int \cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^3\,dx = -\frac{2\left(5i\sqrt{2}a^3\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))-5i\sqrt{2}a^3\text{weierstrassPInverse}(-4,\right.}{}$$

input
```
integrate(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
-2/5*(5*I*sqrt(2)*a^3*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x
+ c)) - 5*I*sqrt(2)*a^3*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*
x + c)) - 9*I*sqrt(2)*a^3*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0
, cos(d*x + c) + I*sin(d*x + c))) + 9*I*sqrt(2)*a^3*weierstrassZeta(-4, 0,
 weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - (a^3*cos(d*x
 + c) + 5*a^3)*sqrt(cos(d*x + c))*sin(d*x + c))/d
```

3.369.6 Sympy [F(-1)]

Timed out.

$$\int \cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^3\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(5/2)*(a+a*sec(d*x+c))**3,x)
```

output
```
Timed out
```

3.369.7 Maxima [F]

$$\int \cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^3\,dx = \int (a\sec(dx+c)+a)^3\cos(dx+c)^{\frac{5}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)^3*cos(d*x + c)^(5/2), x)
```

3.369.8 Giac [F]

$$\int \cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^3\,dx = \int (a\sec(dx+c)+a)^3\cos(dx+c)^{\frac{5}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)^3*cos(d*x + c)^(5/2), x)
```

3.369.9 Mupad [B] (verification not implemented)

Time = 14.27 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.14

$$\int \cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^3\,dx$$
$$= \frac{6\,a^3\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{4\,a^3\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{2\,a^3\,\sqrt{\cos\left(c+d\,x\right)}\,\sin\left(c+d\,x\right)}{d}-\frac{2\,a^3\,\cos\left(c+d\,x\right)^{7/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{7}{4};\ \frac{11}{4};\ \cos\left(c+d\,x\right)^2\right)}{7\,d\,\sqrt{\sin\left(c+d\,x\right)^2}}$$

input
```
int(cos(c + d*x)^(5/2)*(a + a/cos(c + d*x))^3,x)
```

output
```
(6*a^3*ellipticE(c/2 + (d*x)/2, 2))/d + (4*a^3*ellipticF(c/2 + (d*x)/2, 2)
)/d + (2*a^3*cos(c + d*x)^(1/2)*sin(c + d*x))/d - (2*a^3*cos(c + d*x)^(7/2
)*sin(c + d*x)*hypergeom([1/2, 7/4], 11/4, cos(c + d*x)^2))/(7*d*(sin(c +
d*x)^2)^(1/2))
```

3.370 $\int \cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3\,dx$

3.370.1 Optimal result 2725
3.370.2 Mathematica [C] (verified) 2725
3.370.3 Rubi [A] (verified) 2726
3.370.4 Maple [A] (verified) 2727
3.370.5 Fricas [C] (verification not implemented) 2728
3.370.6 Sympy [F(-1)] 2728
3.370.7 Maxima [F] 2729
3.370.8 Giac [F] 2729
3.370.9 Mupad [B] (verification not implemented) 2729

3.370.1 Optimal result

Integrand size = 23, antiderivative size = 91

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3\,dx=\frac{4a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{20a^3\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2a^3\sin(c+dx)}{d\sqrt{\cos(c+dx)}}+\frac{2a^3\sqrt{\cos(c+dx)}\sin(c+dx)}{3d}$$

output
```
4*a^3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*
x+1/2*c),2^(1/2))/d+20/3*a^3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*
c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2*a^3*sin(d*x+c)/d/cos(d*x+c)^(
1/2)+2/3*a^3*sin(d*x+c)*cos(d*x+c)^(1/2)/d
```

3.370.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.39 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.60

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3\,dx=$$
$$-\frac{2a^3\csc(c+dx)\left(-3\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},\cos^2(c+dx)\right)\sqrt{\sin^2(c+dx)}+\cos(c+dx)\left(-1+c\right.\right.}{}$$

input
```
Integrate[Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^3,x]
```

output
```
(-2*a^3*Csc[c + d*x]*(-3*Hypergeometric2F1[-1/4, 1/2, 3/4, Cos[c + d*x]^2]
*Sqrt[Sin[c + d*x]^2] + Cos[c + d*x]*(-1 + Cos[c + d*x]^2 + 10*Hypergeomet
ric2F1[1/4, 1/2, 5/4, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2] + 3*Cos[c + d*x
]*Hypergeometric2F1[1/2, 3/4, 7/4, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2])))
/(3*d*Sqrt[Cos[c + d*x]])
```

3.370.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.67, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 4752, 3042, 4278, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^3dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{(\sec(c+dx)a+a)^3}{\sec^{\frac{3}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^3}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{4278}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \left(\sec^{\frac{3}{2}}(c+dx)a^3+3\sqrt{\sec(c+dx)}a^3+\frac{3a^3}{\sqrt{\sec(c+dx)}}+\frac{a^3}{\sec^{\frac{3}{2}}(c+dx)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2a^3\sin(c+dx)\sqrt{\sec(c+dx)}}{d}+\frac{2a^3\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{20a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{3d}\right.$$

input
```
Int[Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^3,x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((4*a^3*Sqrt[Cos[c + d*x]]*EllipticE
[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (20*a^3*Sqrt[Cos[c + d*x]]*Ellipt
icF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (2*a^3*Sin[c + d*x])/(3*d*
Sqrt[Sec[c + d*x]]) + (2*a^3*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d)
```

3.370.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4278
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

3.370.4 Maple [A] (verified)

Time = 6.94 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.89

method	result
default	$-\frac{4a^3\left(2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2+5\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1}\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right.}{3\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1}\,d}$

input
```
int(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
-4/3*a^3*(2*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-4*cos(1/2*d*x+1/2*c)*s
in(1/2*d*x+1/2*c)^2+5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2
-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2
))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

3.370.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.98

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3\,dx = -\frac{2\left(5i\sqrt{2}a^3\cos(dx+c)\,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))-5i\sqrt{2}a^3\cos(dx+c)\right.}{}$$

input
```
integrate(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
-2/3*(5*I*sqrt(2)*a^3*cos(d*x + c)*weierstrassPInverse(-4, 0, cos(d*x + c)
 + I*sin(d*x + c)) - 5*I*sqrt(2)*a^3*cos(d*x + c)*weierstrassPInverse(-4,
0, cos(d*x + c) - I*sin(d*x + c)) - 3*I*sqrt(2)*a^3*cos(d*x + c)*weierstra
ssZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) +
 3*I*sqrt(2)*a^3*cos(d*x + c)*weierstrassZeta(-4, 0, weierstrassPInverse(-
4, 0, cos(d*x + c) - I*sin(d*x + c))) - (a^3*cos(d*x + c) + 3*a^3)*sqrt(co
s(d*x + c))*sin(d*x + c))/(d*cos(d*x + c))
```

3.370.6 Sympy [F(-1)]

Timed out.

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(3/2)*(a+a*sec(d*x+c))**3,x)
```

output
```
Timed out
```

3.370.7 Maxima [F]

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3\,dx = \int (a\sec(dx+c)+a)^3\cos(dx+c)^{\frac{3}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)^3*cos(d*x + c)^(3/2), x)
```

3.370.8 Giac [F]

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3\,dx = \int (a\sec(dx+c)+a)^3\cos(dx+c)^{\frac{3}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)^3*cos(d*x + c)^(3/2), x)
```

3.370.9 Mupad [B] (verification not implemented)

Time = 13.79 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.14

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3\,dx = \frac{6\,a^3\,\mathrm{E}\left(\frac{c}{2}+\frac{dx}{2}\middle|2\right)}{d} + \frac{20\,a^3\,\mathrm{F}\left(\frac{c}{2}+\frac{dx}{2}\middle|2\right)}{3\,d} + \frac{2\,a^3\,\sqrt{\cos(c+d\,x)}\,\sin(c+d\,x)}{3\,d} + \frac{2\,a^3\,\sin(c+d\,x)\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{1}{2};\ \frac{3}{4};\ \cos(c+d\,x)^2\right)}{d\,\sqrt{\cos(c+d\,x)}\,\sqrt{\sin(c+d\,x)^2}}$$

input
```
int(cos(c + d*x)^(3/2)*(a + a/cos(c + d*x))^3,x)
```

output
```
(6*a^3*ellipticE(c/2 + (d*x)/2, 2))/d + (20*a^3*ellipticF(c/2 + (d*x)/2, 2
))/(3*d) + (2*a^3*cos(c + d*x)^(1/2)*sin(c + d*x))/(3*d) + (2*a^3*sin(c +
d*x)*hypergeom([-1/4, 1/2], 3/4, cos(c + d*x)^2))/(d*cos(c + d*x)^(1/2)*(s
in(c + d*x)^2)^(1/2))
```

3.371 $\int \sqrt{\cos(c+dx)}(a+a\sec(c+dx))^3\,dx$

3.371.1 Optimal result . . . 2730
3.371.2 Mathematica [C] (verified) . . . 2730
3.371.3 Rubi [A] (verified) . . . 2731
3.371.4 Maple [B] (verified) . . . 2733
3.371.5 Fricas [C] (verification not implemented) . . . 2733
3.371.6 Sympy [F] . . . 2734
3.371.7 Maxima [F] . . . 2734
3.371.8 Giac [F] . . . 2735
3.371.9 Mupad [B] (verification not implemented) . . . 2735

3.371.1 Optimal result

Integrand size = 23, antiderivative size = 91

$$\int \sqrt{\cos(c+dx)}(a+a\sec(c+dx))^3\,dx = -\frac{4a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{20a^3\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d} + \frac{2a^3\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{6a^3\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

output
```
-4*a^3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d
*x+1/2*c),2^(1/2))/d+20/3*a^3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2
*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/3*a^3*sin(d*x+c)/d/cos(d*x+c
)^(3/2)+6*a^3*sin(d*x+c)/d/cos(d*x+c)^(1/2)
```

3.371.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.35 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.53

$$\int \sqrt{\cos(c+dx)}(a+a\sec(c+dx))^3\,dx$$
$$= \frac{2a^3\csc(c+dx)\left(\text{Hypergeometric2F1}\left(-\frac{3}{4},\frac{1}{2},\frac{1}{4},\cos^2(c+dx)\right)+9\cos(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\right.\right.}{}$$

input
```
Integrate[Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])^3,x]
```

output
```
(2*a^3*Csc[c + d*x]*(Hypergeometric2F1[-3/4, 1/2, 1/4, Cos[c + d*x]^2] + 9
*Cos[c + d*x]*Hypergeometric2F1[-1/4, 1/2, 3/4, Cos[c + d*x]^2] - Cos[c +
d*x]^2*(9*Hypergeometric2F1[1/4, 1/2, 5/4, Cos[c + d*x]^2] + Cos[c + d*x]*
Hypergeometric2F1[1/2, 3/4, 7/4, Cos[c + d*x]^2]))*Sqrt[Sin[c + d*x]^2])/(
3*d*Cos[c + d*x]^(3/2))
```

3.371.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.67, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 4752, 3042, 4278, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\cos(c+dx)}(a\sec(c+dx)+a)^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^3dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{(\sec(c+dx)a+a)^3}{\sqrt{\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^3}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4278}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \left(\sec^{\frac{5}{2}}(c+dx)a^3+3\sec^{\frac{3}{2}}(c+dx)a^3+3\sqrt{\sec(c+dx)}a^3+\frac{a^3}{\sqrt{\sec(c+dx)}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2a^3\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{6a^3\sin(c+dx)\sqrt{\sec(c+dx)}}{d}+\frac{20a^3\sqrt{\cos(c+dx)}\sqrt{}}{}\right.$$

input
```
Int[Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])^3,x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((-4*a^3*Sqrt[Cos[c + d*x]]*Elliptic
E[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (20*a^3*Sqrt[Cos[c + d*x]]*Ellip
ticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (6*a^3*Sqrt[Sec[c + d*x]]
*Sin[c + d*x])/d + (2*a^3*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d))
```

3.371.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4278
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

3.371.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 370 vs. $2(135) = 270$.

Time = 8.38 (sec) , antiderivative size = 371, normalized size of antiderivative = 4.08

method	result
default	$-\frac{4\sqrt{-\left(-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,a^{3}\left(18\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-10\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1}\,\mathrm{EllipticF}\left(\mathrm{co}\right.\right.}{}$

input
```
int(cos(d*x+c)^(1/2)*(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
-4/3*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3/(4*sin(
1/2*d*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)/sin(1/2*d*x+1/2*c)^3*(18*sin(1/
2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-10*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1
/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x
+1/2*c)^2-6*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1
/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2-10*cos(1/2*d*x+
1/2*c)*sin(1/2*d*x+1/2*c)^2+5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+
1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2
-1)^(1/2))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2
*d*x+1/2*c)^2-1)^(1/2)/d
```

3.371.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 187, normalized size of antiderivative = 2.05

$$\int \sqrt{\cos(c+dx)}(a+a\sec(c+dx))^3\,dx =$$
$$-\frac{2\left(5i\sqrt{2}a^3\cos\left(dx+c\right)^2\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right))-5i\sqrt{2}a^3\cos\left(dx+c\right.\right.}{}$$

input
```
integrate(cos(d*x+c)^(1/2)*(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
-2/3*(5*I*sqrt(2)*a^3*cos(d*x + c)^2*weierstrassPInverse(-4, 0, cos(d*x +
c) + I*sin(d*x + c)) - 5*I*sqrt(2)*a^3*cos(d*x + c)^2*weierstrassPInverse(
-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 3*I*sqrt(2)*a^3*cos(d*x + c)^2*wei
erstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x +
c))) - 3*I*sqrt(2)*a^3*cos(d*x + c)^2*weierstrassZeta(-4, 0, weierstrassPI
nverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - (9*a^3*cos(d*x + c) + a^3)
*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^2)
```

3.371.6 Sympy [F]

$$\int \sqrt{\cos(c+dx)}(a+a\sec(c+dx))^3\,dx = a^3\left(\int 3\sqrt{\cos(c+dx)}\sec(c+dx)\,dx + \int 3\sqrt{\cos(c+dx)}\sec^2(c+dx)\,dx + \int \sqrt{\cos(c+dx)}\sec^3(c+dx)\,dx + \int \sqrt{\cos(c+dx)}\,dx\right)$$

input
```
integrate(cos(d*x+c)**(1/2)*(a+a*sec(d*x+c))**3,x)
```

output
```
a**3*(Integral(3*sqrt(cos(c + d*x))*sec(c + d*x), x) + Integral(3*sqrt(cos
(c + d*x))*sec(c + d*x)**2, x) + Integral(sqrt(cos(c + d*x))*sec(c + d*x)*
*3, x) + Integral(sqrt(cos(c + d*x)), x))
```

3.371.7 Maxima [F]

$$\int \sqrt{\cos(c+dx)}(a+a\sec(c+dx))^3\,dx = \int (a\sec(dx+c)+a)^3\sqrt{\cos(dx+c)}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)*(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)^3*sqrt(cos(d*x + c)), x)
```

3.371.8 Giac [F]

$$\int \sqrt{\cos(c+dx)}(a+a\sec(c+dx))^3\,dx = \int (a\sec(dx+c)+a)^3\sqrt{\cos(dx+c)}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)*(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)^3*sqrt(cos(d*x + c)), x)
```

3.371.9 Mupad [B] (verification not implemented)

Time = 14.24 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.38

$$\int \sqrt{\cos(c+dx)}(a+a\sec(c+dx))^3\,dx = \frac{2\left(a^3\,\mathrm{E}\left(\frac{c}{2}+\frac{dx}{2}\middle|2\right)+3\,a^3\,\mathrm{F}\left(\frac{c}{2}+\frac{dx}{2}\middle|2\right)\right)}{d} + \frac{6\,a^3\,\sin(c+d\,x)\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{1}{2};\ \frac{3}{4};\ \cos(c+d\,x)^2\right)}{d\,\sqrt{\cos(c+d\,x)}\,\sqrt{\sin(c+d\,x)^2}} + \frac{2\,a^3\,\sin(c+d\,x)\,{}_2\mathrm{F}_1\left(-\frac{3}{4},\frac{1}{2};\ \frac{1}{4};\ \cos(c+d\,x)^2\right)}{3\,d\cos(c+d\,x)^{3/2}\,\sqrt{\sin(c+d\,x)^2}}$$

input
```
int(cos(c + d*x)^(1/2)*(a + a/cos(c + d*x))^3,x)
```

output
```
(2*(a^3*ellipticE(c/2 + (d*x)/2, 2) + 3*a^3*ellipticF(c/2 + (d*x)/2, 2)))/
d + (6*a^3*sin(c + d*x)*hypergeom([-1/4, 1/2], 3/4, cos(c + d*x)^2))/(d*co
s(c + d*x)^(1/2)*(sin(c + d*x)^2)^(1/2)) + (2*a^3*sin(c + d*x)*hypergeom([
-3/4, 1/2], 1/4, cos(c + d*x)^2))/(3*d*cos(c + d*x)^(3/2)*(sin(c + d*x)^2)
^(1/2))
```

3.372 $\int \frac{(a+a\sec(c+dx))^3}{\sqrt{\cos(c+dx)}}\,dx$

3.372.1 Optimal result . . . 2736
3.372.2 Mathematica [C] (verified) . . . 2736
3.372.3 Rubi [A] (verified) . . . 2737
3.372.4 Maple [B] (verified) . . . 2738
3.372.5 Fricas [C] (verification not implemented) . . . 2739
3.372.6 Sympy [F] . . . 2740
3.372.7 Maxima [F] . . . 2740
3.372.8 Giac [F] . . . 2740
3.372.9 Mupad [B] (verification not implemented) . . . 2741

3.372.1 Optimal result

Integrand size = 23, antiderivative size = 117

$$\int \frac{(a+a\sec(c+dx))^3}{\sqrt{\cos(c+dx)}}\,dx = -\frac{36a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{4a^3\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+\frac{2a^3\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{2a^3\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)}+\frac{36a^3\sin(c+dx)}{5d\sqrt{\cos(c+dx)}}$$

output
```
-36/5*a^3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/
2*d*x+1/2*c),2^(1/2))/d+4*a^3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2
*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/5*a^3*sin(d*x+c)/d/cos(d*x+c
)^(5/2)+2*a^3*sin(d*x+c)/d/cos(d*x+c)^(3/2)+36/5*a^3*sin(d*x+c)/d/cos(d*x+
c)^(1/2)
```

3.372.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.48 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.18

$$\int \frac{(a+a\sec(c+dx))^3}{\sqrt{\cos(c+dx)}}\,dx = \frac{2a^3\csc(c+dx)\left(\text{Hypergeometric2F1}\left(-\frac{5}{4},\frac{1}{2},-\frac{1}{4},\cos^2(c+dx)\right)+5\cos(c+dx)\left(\text{Hypergeometric2F1}\left(-\frac{3}{4}\right.\right.\right.}{}$$

input
```
Integrate[(a + a*Sec[c + d*x])^3/Sqrt[Cos[c + d*x]],x]
```

output
```
(2*a^3*Csc[c + d*x]*(Hypergeometric2F1[-5/4, 1/2, -1/4, Cos[c + d*x]^2] +
5*Cos[c + d*x]*(Hypergeometric2F1[-3/4, 1/2, 1/4, Cos[c + d*x]^2] + Cos[c
+ d*x]*(3*Hypergeometric2F1[-1/4, 1/2, 3/4, Cos[c + d*x]^2] - Cos[c + d*x]
*Hypergeometric2F1[1/4, 1/2, 5/4, Cos[c + d*x]^2])))*Sqrt[Sin[c + d*x]^2])
/(5*d*Cos[c + d*x]^(5/2))
```

3.372.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.52, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 4752, 3042, 4278, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sec(c+dx)+a)^3}{\sqrt{\cos(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\sec(c+dx)}(\sec(c+dx)a+a)^3dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^3dx$$

$$\Big\downarrow \text{4278}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \left(a^3\sec^{\frac{7}{2}}(c+dx)+3a^3\sec^{\frac{5}{2}}(c+dx)+3a^3\sec^{\frac{3}{2}}(c+dx)+a^3\sqrt{\sec(c+dx)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2a^3\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}+\frac{2a^3\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d}+\frac{36a^3\sin(c+dx)\sqrt{\sec}}{5d}\right.$$

input
```
Int[(a + a*Sec[c + d*x])^3/Sqrt[Cos[c + d*x]],x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((-36*a^3*Sqrt[Cos[c + d*x]]*Ellipti
cE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d) + (4*a^3*Sqrt[Cos[c + d*x]]*E
llipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (36*a^3*Sqrt[Sec[c + d*x]
]*Sin[c + d*x])/(5*d) + (2*a^3*Sec[c + d*x]^(3/2)*Sin[c + d*x])/d + (2*a^3
*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5*d))
```

3.372.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4278
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

3.372.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 385 vs. $2(157) = 314$.

Time = 10.60 (sec) , antiderivative size = 386, normalized size of antiderivative = 3.30

method	result
default	$-\frac{16\sqrt{-\left(-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\,a^3\left(\frac{7\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1}\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{10\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}}-\frac{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\,\mathrm{s}}}{160\left(\cos\right.}\right.}{}$

input
```
int((a+a*sec(d*x+c))^3/cos(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-16*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3*(7/10*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d
*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/
2))-1/160*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2
)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^3-9/10*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x
+1/2*c)/(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)-9/20*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*
x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/
2))-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))-1/16*cos(1/2*d*x+1/2*c)*(-2*sin
(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2
)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

3.372.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.71

$$\int \frac{(a+a\sec(c+dx))^3}{\sqrt{\cos(c+dx)}}\,dx = -\frac{2\left(5i\sqrt{2}a^3\cos(dx+c)^3\,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c))-5i\sqrt{2}a^3\cos(dx+c\right.}{}$$

input
```
integrate((a+a*sec(d*x+c))^3/cos(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
-2/5*(5*I*sqrt(2)*a^3*cos(d*x + c)^3*weierstrassPInverse(-4, 0, cos(d*x +
c) + I*sin(d*x + c)) - 5*I*sqrt(2)*a^3*cos(d*x + c)^3*weierstrassPInverse(
-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 9*I*sqrt(2)*a^3*cos(d*x + c)^3*wei
erstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x +
c))) - 9*I*sqrt(2)*a^3*cos(d*x + c)^3*weierstrassZeta(-4, 0, weierstrassPI
nverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - (18*a^3*cos(d*x + c)^2 + 5
*a^3*cos(d*x + c) + a^3)*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^
3)
```

3.372.6 Sympy [F]

$$\int \frac{(a+a\sec(c+dx))^3}{\sqrt{\cos(c+dx)}}\,dx = a^3\left(\int \frac{3\sec(c+dx)}{\sqrt{\cos(c+dx)}}\,dx + \int \frac{3\sec^2(c+dx)}{\sqrt{\cos(c+dx)}}\,dx + \int \frac{\sec^3(c+dx)}{\sqrt{\cos(c+dx)}}\,dx + \int \frac{1}{\sqrt{\cos(c+dx)}}\,dx\right)$$

input
```
integrate((a+a*sec(d*x+c))**3/cos(d*x+c)**(1/2),x)
```

output
```
a**3*(Integral(3*sec(c + d*x)/sqrt(cos(c + d*x)), x) + Integral(3*sec(c +
d*x)**2/sqrt(cos(c + d*x)), x) + Integral(sec(c + d*x)**3/sqrt(cos(c + d*x
)), x) + Integral(1/sqrt(cos(c + d*x)), x))
```

3.372.7 Maxima [F]

$$\int \frac{(a+a\sec(c+dx))^3}{\sqrt{\cos(c+dx)}}\,dx = \int \frac{(a\sec(dx+c)+a)^3}{\sqrt{\cos(dx+c)}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))^3/cos(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)^3/sqrt(cos(d*x + c)), x)
```

3.372.8 Giac [F]

$$\int \frac{(a+a\sec(c+dx))^3}{\sqrt{\cos(c+dx)}}\,dx = \int \frac{(a\sec(dx+c)+a)^3}{\sqrt{\cos(dx+c)}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))^3/cos(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)^3/sqrt(cos(d*x + c)), x)
```

3.372.9 Mupad [B] (verification not implemented)

Time = 14.13 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.32

$$\int \frac{(a+a\sec(c+dx))^3}{\sqrt{\cos(c+dx)}}\,dx = \frac{2\,a^3\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d} + \frac{6\,a^3\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{1}{2};\ \frac{3}{4};\ \cos\left(c+d\,x\right)^2\right)}{d\,\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{\sin\left(c+d\,x\right)^2}} + \frac{2\,a^3\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{3}{4},\frac{1}{2};\ \frac{1}{4};\ \cos\left(c+d\,x\right)^2\right)}{d\,\cos\left(c+d\,x\right)^{3/2}\,\sqrt{\sin\left(c+d\,x\right)^2}} + \frac{2\,a^3\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{5}{4},\frac{1}{2};\ -\frac{1}{4};\ \cos\left(c+d\,x\right)^2\right)}{5\,d\,\cos\left(c+d\,x\right)^{5/2}\,\sqrt{\sin\left(c+d\,x\right)^2}}$$

input
```
int((a + a/cos(c + d*x))^3/cos(c + d*x)^(1/2),x)
```

output
```
(2*a^3*ellipticF(c/2 + (d*x)/2, 2))/d + (6*a^3*sin(c + d*x)*hypergeom([-1/
4, 1/2], 3/4, cos(c + d*x)^2))/(d*cos(c + d*x)^(1/2)*(sin(c + d*x)^2)^(1/2
)) + (2*a^3*sin(c + d*x)*hypergeom([-3/4, 1/2], 1/4, cos(c + d*x)^2))/(d*c
os(c + d*x)^(3/2)*(sin(c + d*x)^2)^(1/2)) + (2*a^3*sin(c + d*x)*hypergeom(
[-5/4, 1/2], -1/4, cos(c + d*x)^2))/(5*d*cos(c + d*x)^(5/2)*(sin(c + d*x)^
2)^(1/2))
```

3.373 $\int \frac{(a+a\sec(c+dx))^3}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

3.373.1 Optimal result . . . 2742
3.373.2 Mathematica [C] (verified) . . . 2743
3.373.3 Rubi [A] (verified) . . . 2743
3.373.4 Maple [B] (verified) . . . 2745
3.373.5 Fricas [C] (verification not implemented) . . . 2745
3.373.6 Sympy [F] . . . 2746
3.373.7 Maxima [F] . . . 2746
3.373.8 Giac [F] . . . 2747
3.373.9 Mupad [B] (verification not implemented) . . . 2747

3.373.1 Optimal result

Integrand size = 23, antiderivative size = 147

$$\int \frac{(a+a\sec(c+dx))^3}{\cos^{\frac{3}{2}}(c+dx)}\,dx = -\frac{28a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{52a^3\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d}+\frac{2a^3\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)}+\frac{6a^3\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{52a^3\sin(c+dx)}{21d\cos^{\frac{3}{2}}(c+dx)}+\frac{28a^3\sin(c+dx)}{5d\sqrt{\cos(c+dx)}}$$

output
```
-28/5*a^3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/
2*d*x+1/2*c),2^(1/2))/d+52/21*a^3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x
+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/7*a^3*sin(d*x+c)/d/cos(d
*x+c)^(7/2)+6/5*a^3*sin(d*x+c)/d/cos(d*x+c)^(5/2)+52/21*a^3*sin(d*x+c)/d/c
os(d*x+c)^(3/2)+28/5*a^3*sin(d*x+c)/d/cos(d*x+c)^(1/2)
```

3.373.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.67 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.95

$$\int \frac{(a+a\sec(c+dx))^3}{\cos^{\frac{3}{2}}(c+dx)}\,dx$$
$$= \frac{2a^3\csc(c+dx)\left(5\,\text{Hypergeometric2F1}\left(-\frac{7}{4},\frac{1}{2},-\frac{3}{4},\cos^2(c+dx)\right)+7\cos(c+dx)\left(3\,\text{Hypergeometric2F1}\left(\right.\right.\right.}{}$$

input
```
Integrate[(a + a*Sec[c + d*x])^3/Cos[c + d*x]^(3/2),x]
```

output
```
(2*a^3*Csc[c + d*x]*(5*Hypergeometric2F1[-7/4, 1/2, -3/4, Cos[c + d*x]^2]
+ 7*Cos[c + d*x]*(3*Hypergeometric2F1[-5/4, 1/2, -1/4, Cos[c + d*x]^2] + 5
*Cos[c + d*x]*(Hypergeometric2F1[-3/4, 1/2, 1/4, Cos[c + d*x]^2] + Cos[c +
 d*x]*Hypergeometric2F1[-1/4, 1/2, 3/4, Cos[c + d*x]^2])))*Sqrt[Sin[c + d*
x]^2])/(35*d*Cos[c + d*x]^(7/2))
```

3.373.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.41, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 4752, 3042, 4278, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sec(c+dx)+a)^3}{\cos^{\frac{3}{2}}(c+dx)}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx$$
$$\Big\downarrow \; 4752$$
$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sec^{\frac{3}{2}}(c+dx)(\sec(c+dx)a+a)^3dx$$
$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^3dx$$

$$\Big\downarrow \text{4278}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \left(a^3\sec^{\frac{9}{2}}(c+dx)+3a^3\sec^{\frac{7}{2}}(c+dx)+3a^3\sec^{\frac{5}{2}}(c+dx)+a^3\sec^{\frac{3}{2}}(c+dx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2a^3\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{7d}+\frac{6a^3\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}+\frac{52a^3\sin(c+dx)\sec^{\frac{3}{2}}}{21d}\right.$$

input
```
Int[(a + a*Sec[c + d*x])^3/Cos[c + d*x]^(3/2),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((-28*a^3*Sqrt[Cos[c + d*x]]*Ellipti
cE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d) + (52*a^3*Sqrt[Cos[c + d*x]]*
EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (28*a^3*Sqrt[Sec[c
+ d*x]]*Sin[c + d*x])/(5*d) + (52*a^3*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(21
*d) + (6*a^3*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5*d) + (2*a^3*Sec[c + d*x]^
(7/2)*Sin[c + d*x])/(7*d))
```

3.373.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4278
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

3.373.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 438 vs. $2(179) = 358$.

Time = 12.78 (sec) , antiderivative size = 439, normalized size of antiderivative = 2.99

method	result
default	$-\frac{16\sqrt{-\left(-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,a^{3}\left(-\frac{3\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}}{160\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-\frac{1}{2}\right)^{3}}-\frac{7\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)}{10\sqrt{-\left(-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+1\right)\sin\left(\frac{dx}{2}\right.}}\right.}{}$

input
```
int((a+a*sec(d*x+c))^3/cos(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-16*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3*(-3/160*
cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(c
os(1/2*d*x+1/2*c)^2-1/2)^3-7/10*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)/(-
(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)+53/105*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)
^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-7/20*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2
*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^
(1/2))-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))-13/168*cos(1/2*d*x+1/2*c)*(-
2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1
/2)^2-1/448*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)
^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^4)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+
1/2*c)^2-1)^(1/2)/d
```

3.373.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.46

$$\int \frac{(a+a\sec(c+dx))^3}{\cos^{\frac{3}{2}}(c+dx)}\,dx = -\frac{2\left(65i\sqrt{2}a^3\cos(dx+c)^4\,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)) - 65i\sqrt{2}a^3\cos(dx+\right.}{}$$

input
```
integrate((a+a*sec(d*x+c))^3/cos(d*x+c)^(3/2),x, algorithm="fricas")
```

output
```
-2/105*(65*I*sqrt(2)*a^3*cos(d*x + c)^4*weierstrassPInverse(-4, 0, cos(d*x
 + c) + I*sin(d*x + c)) - 65*I*sqrt(2)*a^3*cos(d*x + c)^4*weierstrassPInve
rse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 147*I*sqrt(2)*a^3*cos(d*x + c)
^4*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(
d*x + c))) - 147*I*sqrt(2)*a^3*cos(d*x + c)^4*weierstrassZeta(-4, 0, weier
strassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - (294*a^3*cos(d*x +
 c)^3 + 130*a^3*cos(d*x + c)^2 + 63*a^3*cos(d*x + c) + 15*a^3)*sqrt(cos(d*
x + c))*sin(d*x + c))/(d*cos(d*x + c)^4)
```

3.373.6 Sympy [F]

$$\int \frac{(a+a\sec(c+dx))^3}{\cos^{\frac{3}{2}}(c+dx)}\,dx = a^3\left(\int \frac{3\sec(c+dx)}{\cos^{\frac{3}{2}}(c+dx)}\,dx + \int \frac{3\sec^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)}\,dx + \int \frac{\sec^3(c+dx)}{\cos^{\frac{3}{2}}(c+dx)}\,dx + \int \frac{1}{\cos^{\frac{3}{2}}(c+dx)}\,dx\right)$$

input
```
integrate((a+a*sec(d*x+c))**3/cos(d*x+c)**(3/2),x)
```

output
```
a**3*(Integral(3*sec(c + d*x)/cos(c + d*x)**(3/2), x) + Integral(3*sec(c +
 d*x)**2/cos(c + d*x)**(3/2), x) + Integral(sec(c + d*x)**3/cos(c + d*x)**
(3/2), x) + Integral(cos(c + d*x)**(-3/2), x))
```

3.373.7 Maxima [F]

$$\int \frac{(a+a\sec(c+dx))^3}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(a\sec(dx+c)+a)^3}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))^3/cos(d*x+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate((a*sec(d*x + c) + a)^3/cos(d*x + c)^(3/2), x)
```

3.373.8 Giac [F]

$$\int \frac{(a+a\sec(c+dx))^3}{\cos^{\frac{3}{2}}(c+dx)}\,dx=\int \frac{(a\sec(dx+c)+a)^3}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))^3/cos(d*x+c)^(3/2),x, algorithm="giac")
```

output
```
integrate((a*sec(d*x + c) + a)^3/cos(d*x + c)^(3/2), x)
```

3.373.9 Mupad [B] (verification not implemented)

Time = 14.24 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.99

$$\int \frac{(a+a\sec(c+dx))^3}{\cos^{\frac{3}{2}}(c+dx)}\,dx$$
$$=\frac{\frac{2\,a^3\,\sin(c+d\,x)\,{}_2\mathrm{F}_1\left(-\frac{7}{4},\frac{1}{2};\,-\frac{3}{4};\,\cos(c+d\,x)^2\right)}{7}+\frac{6\,a^3\,\cos(c+d\,x)\,\sin(c+d\,x)\,{}_2\mathrm{F}_1\left(-\frac{5}{4},\frac{1}{2};\,-\frac{1}{4};\,\cos(c+d\,x)^2\right)}{5}+2\,a^3\cos(c+d\,x)^2\,\sin}{d\cos(c+d\,x)^{7/2}\,\sqrt{}}$$

input
```
int((a + a/cos(c + d*x))^3/cos(c + d*x)^(3/2),x)
```

output
```
((2*a^3*sin(c + d*x)*hypergeom([-7/4, 1/2], -3/4, cos(c + d*x)^2))/7 + (6*
a^3*cos(c + d*x)*sin(c + d*x)*hypergeom([-5/4, 1/2], -1/4, cos(c + d*x)^2)
)/5 + 2*a^3*cos(c + d*x)^2*sin(c + d*x)*hypergeom([-3/4, 1/2], 1/4, cos(c
+ d*x)^2) + 2*a^3*cos(c + d*x)^3*sin(c + d*x)*hypergeom([-1/4, 1/2], 3/4,
cos(c + d*x)^2))/(d*cos(c + d*x)^(7/2)*(1 - cos(c + d*x)^2)^(1/2))
```

3.374 $\int \frac{\cos^{\frac{5}{2}}(c+dx)}{a+a\sec(c+dx)}\,dx$

3.374.1 Optimal result 2748
3.374.2 Mathematica [C] (verified) 2748
3.374.3 Rubi [A] (verified) 2749
3.374.4 Maple [A] (verified) 2753
3.374.5 Fricas [C] (verification not implemented) 2753
3.374.6 Sympy [F(-1)] 2754
3.374.7 Maxima [F] 2754
3.374.8 Giac [F] 2754
3.374.9 Mupad [F(-1)] 2755

3.374.1 Optimal result

Integrand size = 23, antiderivative size = 128

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{21E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5ad} - \frac{5\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3ad} - \frac{5\sqrt{\cos(c+dx)}\sin(c+dx)}{3ad} + \frac{7\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5ad} - \frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d(a+a\sec(c+dx))}$$

output
```
21/5*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x
+1/2*c),2^(1/2))/a/d-5/3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*E
llipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a/d+7/5*cos(d*x+c)^(3/2)*sin(d*x+c)/a
/d-cos(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))-5/3*sin(d*x+c)*cos(d*x+c
)^(1/2)/a/d
```

3.374.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 2.10 (sec) , antiderivative size = 314, normalized size of antiderivative = 2.45

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\cos^2\left(\frac{1}{2}(c+dx)\right)\left(\frac{2i\sqrt{2}e^{-i(c+dx)}\left(63\left(1+e^{2i(c+dx)}\right)+63\left(-1+e^{2ic}\right)\sqrt{1+e^{2i(c+dx)}}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},-e^{2i(c+dx)}\right)+25e^{i(c+dx)}(}{d\left(-1+e^{2ic}\right)\sqrt{e^{-i(c+dx)}\left(1+e^{2i(c+dx)}\right)}}\right.}{}$$

input
```
Integrate[Cos[c + d*x]^(5/2)/(a + a*Sec[c + d*x]),x]
```

output
```
(Cos[(c + d*x)/2]^2*(((2*I)*Sqrt[2]*(63*(1 + E^((2*I)*(c + d*x))) + 63*(-1
 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2,
 3/4, -E^((2*I)*(c + d*x))] + 25*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1
 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*
x))])*Sec[c + d*x])/(d*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[(1 + E^((2*
I)*(c + d*x)))/E^(I*(c + d*x))]) + (-96*Cot[c] - 30*Csc[c] - 20*Cos[d*x]*S
in[c] + 6*Cos[2*d*x]*Sin[2*c] - 30*Sec[c/2]*Sec[(c + d*x)/2]*Sin[(d*x)/2]
- 20*Cos[c]*Sin[d*x] + 6*Cos[2*c]*Sin[2*d*x])/(d*Sqrt[Cos[c + d*x]])))/(15
*a*(1 + Sec[c + d*x]))
```

3.374.3 Rubi [A] (verified)

Time = 0.87 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.51, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.609$, Rules used = {3042, 4752, 3042, 4306, 27, 3042, 4274, 3042, 4256, 3042, 4258, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{a\sec(c+dx)+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}}{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(\sec(c+dx)a+a)}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)}dx$$

$$\Big\downarrow \text{4306}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\int -\frac{7a-5a\sec(c+dx)}{2\sec^{\frac{5}{2}}(c+dx)}dx}{a^2}-\frac{\sin(c+dx)}{d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)}\right)$$

$$\Big\downarrow \; 27$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{7a-5a\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)}dx}{2a^2}-\frac{\sin(c+dx)}{d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)}\right)$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{7a-5a\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx}{2a^2}-\frac{\sin(c+dx)}{d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)}\right)$$

$$\Big\downarrow \; 4274$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{7a\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)}dx-5a\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}dx}{2a^2}-\frac{\sin(c+dx)}{d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)}\right)$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{7a\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx-5a\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx}{2a^2}-\frac{\sin(c+dx)}{d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)}\right)$$

$$\Big\downarrow \; 4256$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{7a\left(\frac{3}{5}\int \frac{1}{\sqrt{\sec(c+dx)}}dx+\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}\right)-5a\left(\frac{1}{3}\int \sqrt{\sec(c+dx)}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)}{2a^2}-\right.$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{7a\left(\frac{3}{5}\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}\right)-5a\left(\frac{1}{3}\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+}{3d\sqrt{\sec(c-}}\right.}{2a^2}\right.$$

$$\Big\downarrow \; 4258$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{7a\left(\frac{3}{5}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\cos(c+dx)}dx+\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}\right)-5a\left(\frac{1}{3}\sqrt{\cos(c}\right.}{2a^2}\right.$$

3.374. $\int \frac{\cos^{\frac{5}{2}}(c+dx)}{a+a\sec(c+dx)}\,dx$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{7a\left(\frac{3}{5}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}\right)-5a\left(\frac{1}{3}\sqrt{\right.}{2a^2}\right.$$

$$\Big\downarrow \text{3119}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{7a\left(\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{6\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|2\right)}{5d}\right)-5a\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c}{2a^2}\right.$$

$$\Big\downarrow \text{3120}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{7a\left(\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{6\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|2\right)}{5d}\right)-5a\left(\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2\sqrt{\cos(c+}}{2a^2}\right.$$

input
```
Int[Cos[c + d*x]^(5/2)/(a + a*Sec[c + d*x]),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(-(Sin[c + d*x]/(d*Sec[c + d*x]^(3/2
)*(a + a*Sec[c + d*x]))) + (7*a*((6*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)
/2, 2]*Sqrt[Sec[c + d*x]])/(5*d) + (2*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2
))) - 5*a*((2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*
x]])/(3*d) + (2*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])))/(2*a^2))
```

3.374.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4306
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*(a + b*Csc[e +
f*x]))), x] - Simp[1/a^2   Int[(d*Csc[e + f*x])^n*(a*(n - 1) - b*n*Csc[e +
f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, 0
]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

3.374. $\int \frac{\cos^{\frac{5}{2}}(c+dx)}{a+a\sec(c+dx)}\,dx$

3.374.4 Maple [A] (verified)

Time = 8.21 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.79

method	result
default	$-\frac{\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(-\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1}\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\left(25\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)+63\,\mathrm{E}\right.\right.}{15a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\sin\left(\frac{d}{2}\right.}$

input
```
int(cos(d*x+c)^(5/2)/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-1/15*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-cos(1/2*d*
x+1/2*c)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(25
*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+63*EllipticE(cos(1/2*d*x+1/2*c),2^(
1/2)))+48*sin(1/2*d*x+1/2*c)^8-56*sin(1/2*d*x+1/2*c)^6-30*sin(1/2*d*x+1/2*
c)^4+23*sin(1/2*d*x+1/2*c)^2)/a/cos(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1
)^(1/2)/d
```

3.374.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.62

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{a+a\sec(c+dx)}\,dx$$
$$= \frac{2\left(6\,\cos\left(dx+c\right)^2-4\,\cos\left(dx+c\right)-25\right)\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)-25\left(-i\,\sqrt{2}\cos\left(dx+c\right)-i\,\sqrt{2}\right)\mathrm{weie}}{}$$

input
```
integrate(cos(d*x+c)^(5/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

output
```
1/30*(2*(6*cos(d*x + c)^2 - 4*cos(d*x + c) - 25)*sqrt(cos(d*x + c))*sin(d*
x + c) - 25*(-I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassPInverse(-4,
0, cos(d*x + c) + I*sin(d*x + c)) - 25*(I*sqrt(2)*cos(d*x + c) + I*sqrt(2)
)*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 63*(-I*sqrt(
2)*cos(d*x + c) - I*sqrt(2))*weierstrassZeta(-4, 0, weierstrassPInverse(-4
, 0, cos(d*x + c) + I*sin(d*x + c))) - 63*(I*sqrt(2)*cos(d*x + c) + I*sqrt
(2))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*si
n(d*x + c))))/(a*d*cos(d*x + c) + a*d)
```

3.374.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{a+a\sec(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(5/2)/(a+a*sec(d*x+c)),x)
```

output
```
Timed out
```

3.374.7 Maxima [F]

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{a+a\sec(c+dx)}\,dx = \int \frac{\cos(dx+c)^{\frac{5}{2}}}{a\sec(dx+c)+a}\,dx$$

input
```
integrate(cos(d*x+c)^(5/2)/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

output
```
integrate(cos(d*x + c)^(5/2)/(a*sec(d*x + c) + a), x)
```

3.374.8 Giac [F]

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{a+a\sec(c+dx)}\,dx = \int \frac{\cos(dx+c)^{\frac{5}{2}}}{a\sec(dx+c)+a}\,dx$$

input
```
integrate(cos(d*x+c)^(5/2)/(a+a*sec(d*x+c)),x, algorithm="giac")
```

output
```
integrate(cos(d*x + c)^(5/2)/(a*sec(d*x + c) + a), x)
```

3.374.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{a+a\sec(c+dx)}\,dx=\int \frac{\cos\left(c+d\,x\right)^{5/2}}{a+\frac{a}{\cos\left(c+d\,x\right)}}\,dx$$

input
```
int(cos(c + d*x)^(5/2)/(a + a/cos(c + d*x)),x)
```

output
```
int(cos(c + d*x)^(5/2)/(a + a/cos(c + d*x)), x)
```

3.375 $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{a+a\sec(c+dx)}\,dx$

3.375.1 Optimal result . . . 2756
3.375.2 Mathematica [C] (verified) . . . 2756
3.375.3 Rubi [A] (verified) . . . 2757
3.375.4 Maple [A] (verified) . . . 2761
3.375.5 Fricas [C] (verification not implemented) . . . 2761
3.375.6 Sympy [F] . . . 2762
3.375.7 Maxima [F] . . . 2762
3.375.8 Giac [F] . . . 2762
3.375.9 Mupad [F(-1)] . . . 2763

3.375.1 Optimal result

Integrand size = 23, antiderivative size = 100

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}+\frac{5\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3ad}+\frac{5\sqrt{\cos(c+dx)}\sin(c+dx)}{3ad}-\frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{d(a+a\sec(c+dx))}$$

output
```
-3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1
/2*c),2^(1/2))/a/d+5/3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ell
ipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a/d+5/3*sin(d*x+c)*cos(d*x+c)^(1/2)/a/d
-sin(d*x+c)*cos(d*x+c)^(1/2)/d/(a+a*sec(d*x+c))
```

3.375.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 2.05 (sec) , antiderivative size = 292, normalized size of antiderivative = 2.92

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\cos^2\left(\frac{1}{2}(c+dx)\right)\left(-\frac{2i\sqrt{2}e^{-i(c+dx)}\left(9\left(1+e^{2i(c+dx)}\right)+9\left(-1+e^{2ic}\right)\sqrt{1+e^{2i(c+dx)}}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},-e^{2i(c+dx)}\right)+5e^{i(c+dx)}(\cdots}{d\left(-1+e^{2ic}\right)\sqrt{e^{-i(c+dx)}\left(1+e^{2i(c+dx)}\right)}}\right)}{\cdots}$$

input
```
Integrate[Cos[c + d*x]^(3/2)/(a + a*Sec[c + d*x]),x]
```

output
```
(Cos[(c + d*x)/2]^2*(((-2*I)*Sqrt[2]*(9*(1 + E^((2*I)*(c + d*x))) + 9*(-1
+ E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2,
3/4, -E^((2*I)*(c + d*x))] + 5*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 +
 E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x)
)])*Sec[c + d*x])/(d*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[(1 + E^((2*I)
*(c + d*x)))/E^(I*(c + d*x))]) + (12*Cot[c] + 6*Csc[c] + 4*Cos[d*x]*Sin[c]
 + 6*Sec[c/2]*Sec[(c + d*x)/2]*Sin[(d*x)/2] + 4*Cos[c]*Sin[d*x])/(d*Sqrt[C
os[c + d*x]])))/(3*a*(1 + Sec[c + d*x]))
```

3.375.3 Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.65, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.609$, Rules used = {3042, 4752, 3042, 4306, 27, 3042, 4274, 3042, 4256, 3042, 4258, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{a\sec(c+dx)+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(\sec(c+dx)a+a)}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)}dx$$

$$\Big\downarrow \text{4306}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\int -\frac{5a-3a\sec(c+dx)}{2\sec^{\frac{3}{2}}(c+dx)}dx}{a^2}-\frac{\sin(c+dx)}{d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)}\right)$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{5a-3a\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)}dx}{2a^2}-\frac{\sin(c+dx)}{d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)}\right)$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{5a-3a\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx}{2a^2}-\frac{\sin(c+dx)}{d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)}\right)$$

$\Big\downarrow$ 4274

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{5a\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}dx-3a\int \frac{1}{\sqrt{\sec(c+dx)}}dx}{2a^2}-\frac{\sin(c+dx)}{d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)}\right)$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{5a\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx-3a\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{2a^2}-\frac{\sin(c+dx)}{d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)}\right)$$

$\Big\downarrow$ 4256

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{5a\left(\frac{1}{3}\int \sqrt{\sec(c+dx)}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)-3a\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{2a^2}-\frac{\sin(c}{d\sqrt{\sec(c+dx)}(a}\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{5a\left(\frac{1}{3}\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)-3a\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{2a^2}-\frac{}{d\sqrt{\sec(c+}}\right.$$

$\Big\downarrow$ 4258

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{5a\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)-3a\sqrt{\cos(c+dx)}}{2a^2}\right.$$

$\Big\downarrow$ 3042

3.375. $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{a+a\sec(c+dx)}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{5a\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)-3a\sqrt{\cos(c+}}{2a^2}\right.$$

$\Big\downarrow$ 3119

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{5a\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)-\frac{6a\sqrt{\cos(c+dx)}}{}}{2a^2}\right.$$

$\Big\downarrow$ 3120

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{5a\left(\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}\right)-\frac{6a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{d}}{2a^2}\right.$$

input
```
Int[Cos[c + d*x]^(3/2)/(a + a*Sec[c + d*x]),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(-(Sin[c + d*x]/(d*Sqrt[Sec[c + d*x]
]*(a + a*Sec[c + d*x]))) + ((-6*a*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2
, 2]*Sqrt[Sec[c + d*x]])/d + 5*a*((2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x
)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (2*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x
]])))/(2*a^2))
```

3.375.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4306
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*(a + b*Csc[e +
f*x]))), x] - Simp[1/a^2   Int[(d*Csc[e + f*x])^n*(a*(n - 1) - b*n*Csc[e +
f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, 0
]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

3.375. $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{a+a\sec(c+dx)}\,dx$

3.375.4 Maple [A] (verified)

Time = 6.87 (sec) , antiderivative size = 215, normalized size of antiderivative = 2.15

method	result
default	$-\frac{\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1}\left(5\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)+9\,\mathrm{Ellipti}\right.\right.}{3a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\cos}}$

input
```
int(cos(d*x+c)^(3/2)/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-1/3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(cos(1/2*d*x+
1/2*c)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(5*El
lipticF(cos(1/2*d*x+1/2*c),2^(1/2))+9*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)
))-8*sin(1/2*d*x+1/2*c)^6+18*sin(1/2*d*x+1/2*c)^4-7*sin(1/2*d*x+1/2*c)^2)/
a/cos(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/
sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

3.375.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.98

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{a+a\sec(c+dx)}\,dx$$
$$= \frac{2\left(2\cos\left(dx+c\right)+5\right)\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)-5\left(i\sqrt{2}\cos\left(dx+c\right)+i\sqrt{2}\right)\mathrm{weierstrassPInverse}(-4,0,}{}$$

input
```
integrate(cos(d*x+c)^(3/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

output
```
1/6*(2*(2*cos(d*x + c) + 5)*sqrt(cos(d*x + c))*sin(d*x + c) - 5*(I*sqrt(2)
*cos(d*x + c) + I*sqrt(2))*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin
(d*x + c)) - 5*(-I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassPInverse(-
4, 0, cos(d*x + c) - I*sin(d*x + c)) - 9*(I*sqrt(2)*cos(d*x + c) + I*sqrt(
2))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin
(d*x + c))) - 9*(-I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassZeta(-4,
0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/(a*d*cos(d*
x + c) + a*d)
```

3.375.6 Sympy [F]

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{a+a\sec(c+dx)}\,dx=\frac{\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sec(c+dx)+1}\,dx}{a}$$

input
```
integrate(cos(d*x+c)**(3/2)/(a+a*sec(d*x+c)),x)
```

output
```
Integral(cos(c + d*x)**(3/2)/(sec(c + d*x) + 1), x)/a
```

3.375.7 Maxima [F]

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{a+a\sec(c+dx)}\,dx=\int \frac{\cos(dx+c)^{\frac{3}{2}}}{a\sec(dx+c)+a}\,dx$$

input
```
integrate(cos(d*x+c)^(3/2)/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

output
```
integrate(cos(d*x + c)^(3/2)/(a*sec(d*x + c) + a), x)
```

3.375.8 Giac [F]

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{a+a\sec(c+dx)}\,dx=\int \frac{\cos(dx+c)^{\frac{3}{2}}}{a\sec(dx+c)+a}\,dx$$

input
```
integrate(cos(d*x+c)^(3/2)/(a+a*sec(d*x+c)),x, algorithm="giac")
```

output
```
integrate(cos(d*x + c)^(3/2)/(a*sec(d*x + c) + a), x)
```

3.375.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{a+a\sec(c+dx)}\,dx=\int \frac{\cos\left(c+d\,x\right)^{3/2}}{a+\frac{a}{\cos\left(c+d\,x\right)}}\,dx$$

input
```
int(cos(c + d*x)^(3/2)/(a + a/cos(c + d*x)),x)
```

output
```
int(cos(c + d*x)^(3/2)/(a + a/cos(c + d*x)), x)
```

3.376 $\int \frac{\sqrt{\cos(c+dx)}}{a+a\sec(c+dx)}\,dx$

3.376.1 Optimal result 2764
3.376.2 Mathematica [C] (verified) 2764
3.376.3 Rubi [A] (verified) 2765
3.376.4 Maple [A] (verified) 2768
3.376.5 Fricas [C] (verification not implemented) 2768
3.376.6 Sympy [F] 2769
3.376.7 Maxima [F] 2769
3.376.8 Giac [F] 2770
3.376.9 Mupad [F(-1)] 2770

3.376.1 Optimal result

Integrand size = 23, antiderivative size = 72

$$\int \frac{\sqrt{\cos(c+dx)}}{a+a\sec(c+dx)}\,dx = \frac{3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad} - \frac{\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{ad} - \frac{\sin(c+dx)}{d\sqrt{\cos(c+dx)}(a+a\sec(c+dx))}$$

output
```
3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/
2*c),2^(1/2))/a/d-(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elliptic
F(sin(1/2*d*x+1/2*c),2^(1/2))/a/d-sin(d*x+c)/d/(a+a*sec(d*x+c))/cos(d*x+c)
^(1/2)
```

3.376.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.73 (sec) , antiderivative size = 270, normalized size of antiderivative = 3.75

$$\int \frac{\sqrt{\cos(c+dx)}}{a+a\sec(c+dx)}\,dx = \frac{\cos^2\left(\frac{1}{2}(c+dx)\right)\left(\frac{2i\sqrt{2}e^{-i(c+dx)}\left(3\left(1+e^{2i(c+dx)}\right)+3\left(-1+e^{2ic}\right)\sqrt{1+e^{2i(c+dx)}}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},-e^{2i(c+dx)}\right)+e^{i(c+dx)}(-1+}{d(-1+e^{2ic})\sqrt{e^{-i(c+dx)}\left(1+e^{2i(c+dx)}\right)}}\right.}{a(1+\sec(c+}$$

input
```
Integrate[Sqrt[Cos[c + d*x]]/(a + a*Sec[c + d*x]),x]
```

output
```
(Cos[(c + d*x)/2]^2*(((2*I)*Sqrt[2]*(3*(1 + E^((2*I)*(c + d*x))) + 3*(-1 +
 E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3
/4, -E^((2*I)*(c + d*x))] + E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 + E^
((2*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))])
*Sec[c + d*x])/(d*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[(1 + E^((2*I)*(c
 + d*x)))/E^(I*(c + d*x))]) - (2*(2*Cot[c] + Csc[c] + Sec[c/2]*Sec[(c + d*
x)/2]*Sin[(d*x)/2]))/(d*Sqrt[Cos[c + d*x]])))/(a*(1 + Sec[c + d*x]))
```

3.376.3 Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.90, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.522$, Rules used = {3042, 4752, 3042, 4306, 27, 3042, 4274, 3042, 4258, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\cos(c+dx)}}{a\sec(c+dx)+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sec(c+dx)}(\sec(c+dx)a+a)}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)}dx$$

$$\Big\downarrow \text{4306}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\int -\frac{3a-a\sec(c+dx)}{2\sqrt{\sec(c+dx)}}dx}{a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}\right)$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{3a-a\sec(c+dx)}{\sqrt{\sec(c+dx)}}dx}{2a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}\right)$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{3a-a\csc(c+dx+\frac{\pi}{2})}{\sqrt{\csc(c+dx+\frac{\pi}{2})}}dx}{2a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}\right)$$

$\Big\downarrow$ 4274

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{3a\int \frac{1}{\sqrt{\sec(c+dx)}}dx-a\int \sqrt{\sec(c+dx)}dx}{2a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}\right)$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{3a\int \frac{1}{\sqrt{\csc(c+dx+\frac{\pi}{2})}}dx-a\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{2a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}\right)$$

$\Big\downarrow$ 4258

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{3a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\cos(c+dx)}dx-a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{}{\sqrt{}}}{2a^2}\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{3a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx-a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx}}{2a^2}\right.$$

$\Big\downarrow$ 3119

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{6a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|2\right)}{d}-a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin(c+dx+\frac{\pi}{2})}}dx}{2a^2}\right.$$

$\Big\downarrow$ 3120

3.376. $\int \frac{\sqrt{\cos(c+dx)}}{a+a\sec(c+dx)}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{6a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|2\right)}{d}-\frac{2a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}}{2a^2}-\frac{\sin(c}{d(}\right.$$

input
```
Int[Sqrt[Cos[c + d*x]]/(a + a*Sec[c + d*x]),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(((6*a*Sqrt[Cos[c + d*x]]*EllipticE[
(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d - (2*a*Sqrt[Cos[c + d*x]]*EllipticF[
(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d)/(2*a^2) - (Sqrt[Sec[c + d*x]]*Sin[c
 + d*x])/(d*(a + a*Sec[c + d*x])))
```

3.376.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

3.376. $\int \frac{\sqrt{\cos(c+dx)}}{a+a\sec(c+dx)}\,dx$

rule 4306
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*(a + b*Csc[e +
f*x]))), x] - Simp[1/a^2   Int[(d*Csc[e + f*x])^n*(a*(n - 1) - b*n*Csc[e +
f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, 0
]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

3.376.4 Maple [A] (verified)

Time = 5.08 (sec) , antiderivative size = 199, normalized size of antiderivative = 2.76

method	result
default	$\frac{\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1}\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\left(\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)+3\,\mathrm{EllipticE}\left(\right.\right.\right.}{a\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1}}$

input
```
int(cos(d*x+c)^(1/2)/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(cos(1/2*d*x+1/2*c
)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF
(cos(1/2*d*x+1/2*c),2^(1/2))+3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))+2*si
n(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2)/a/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/
2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x
+1/2*c)^2-1)^(1/2)/d
```

3.376.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 186, normalized size of antiderivative = 2.58

$$\int \frac{\sqrt{\cos(c+dx)}}{a+a\sec(c+dx)}\,dx$$
$$= \frac{\left(i\sqrt{2}\cos\left(dx+c\right)+i\sqrt{2}\right)\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right))+\left(-i\sqrt{2}\cos\left(dx+c\right)\right.}{}$$

input
```
integrate(cos(d*x+c)^(1/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

output
```
1/2*((I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierstrassPInverse(-4, 0, cos(d
*x + c) + I*sin(d*x + c)) + (-I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstr
assPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 3*(-I*sqrt(2)*cos(d*x
+ c) - I*sqrt(2))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*
x + c) + I*sin(d*x + c))) - 3*(I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierst
rassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)))
 - 2*sqrt(cos(d*x + c))*sin(d*x + c))/(a*d*cos(d*x + c) + a*d)
```

3.376.6 Sympy [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{a+a\sec(c+dx)}\,dx = \frac{\int \frac{\sqrt{\cos(c+dx)}}{\sec(c+dx)+1}\,dx}{a}$$

input
```
integrate(cos(d*x+c)**(1/2)/(a+a*sec(d*x+c)),x)
```

output
```
Integral(sqrt(cos(c + d*x))/(sec(c + d*x) + 1), x)/a
```

3.376.7 Maxima [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{a+a\sec(c+dx)}\,dx = \int \frac{\sqrt{\cos(dx+c)}}{a\sec(dx+c)+a}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

output
```
integrate(sqrt(cos(d*x + c))/(a*sec(d*x + c) + a), x)
```

3.376.8 Giac [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{a+a\sec(c+dx)}\,dx=\int \frac{\sqrt{\cos{(dx+c)}}}{a\sec{(dx+c)}+a}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)/(a+a*sec(d*x+c)),x, algorithm="giac")
```

output
```
integrate(sqrt(cos(d*x + c))/(a*sec(d*x + c) + a), x)
```

3.376.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\cos(c+dx)}}{a+a\sec(c+dx)}\,dx=\int \frac{\sqrt{\cos{(c+d\,x)}}}{a+\frac{a}{\cos(c+d\,x)}}\,dx$$

input
```
int(cos(c + d*x)^(1/2)/(a + a/cos(c + d*x)),x)
```

output
```
int(cos(c + d*x)^(1/2)/(a + a/cos(c + d*x)), x)
```

3.377 $\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\sec(c+dx))}\,dx$

3.377.1 Optimal result 2771
3.377.2 Mathematica [C] (verified) 2771
3.377.3 Rubi [A] (verified) 2772
3.377.4 Maple [A] (verified) 2775
3.377.5 Fricas [C] (verification not implemented) 2775
3.377.6 Sympy [F] 2776
3.377.7 Maxima [F] 2776
3.377.8 Giac [F] 2777
3.377.9 Mupad [F(-1)] 2777

3.377.1 Optimal result

Integrand size = 23, antiderivative size = 70

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\sec(c+dx))}\,dx = -\frac{E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}+\frac{\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{ad}+\frac{\sin(c+dx)}{d\sqrt{\cos(c+dx)}(a+a\sec(c+dx))}$$

output
```
-(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2
*c),2^(1/2))/a/d+(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF
(sin(1/2*d*x+1/2*c),2^(1/2))/a/d+sin(d*x+c)/d/(a+a*sec(d*x+c))/cos(d*x+c)^
(1/2)
```

3.377.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.12 (sec) , antiderivative size = 262, normalized size of antiderivative = 3.74

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\sec(c+dx))}\,dx = \frac{\cos^2\left(\frac{1}{2}(c+dx)\right)\left(-\frac{2i\sqrt{2}e^{-i(c+dx)}\left(1+e^{2i(c+dx)}+(-1+e^{2ic})\sqrt{1+e^{2i(c+dx)}}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},-e^{2i(c+dx)}\right)+e^{i(c+dx)}(-1+e^{\cdots}\right.}{d(-1+e^{2ic})\sqrt{e^{-i(c+dx)}\left(1+e^{2i(c+dx)}\right)}}\cdots\right.}{a(1+\sec(c+dx))\cdots}$$

input
```
Integrate[1/(Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])),x]
```

output
```
(Cos[(c + d*x)/2]^2*(((-2*I)*Sqrt[2]*(1 + E^((2*I)*(c + d*x)) + (-1 + E^((
2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3/4, -
E^((2*I)*(c + d*x))] + E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I
)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))])*Sec[
c + d*x])/(d*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[(1 + E^((2*I)*(c + d*
x)))/E^(I*(c + d*x))]) + (2*(Csc[c] + Sec[c/2]*Sec[(c + d*x)/2]*Sin[(d*x)/
2]))/(d*Sqrt[Cos[c + d*x]])))/(a*(1 + Sec[c + d*x]))
```

3.377.3 Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.94, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules used = {3042, 4752, 3042, 4307, 3042, 4274, 3042, 4258, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a\sec(c+dx)+a)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)}dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sqrt{\sec(c+dx)}}{\sec(c+dx)a+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx$$

$$\Big\downarrow \text{4307}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}-\frac{\int \frac{a-a\sec(c+dx)}{\sqrt{\sec(c+dx)}}dx}{2a^2}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}-\frac{\int\frac{a-a\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{2a^2}\right)$$

$\Big\downarrow$ 4274

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}-\frac{a\int\frac{1}{\sqrt{\sec(c+dx)}}dx-a\int\sqrt{\sec(c+dx)}dx}{2a^2}\right)$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}-\frac{a\int\frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-a\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{2a^2}\right)$$

$\Big\downarrow$ 4258

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}-\frac{a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\cos(c+dx)}dx-a\sqrt{\mathrm{co}}}{2a^2}\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}-\frac{a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx-}{2a^2}\right.$$

$\Big\downarrow$ 3119

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}-\frac{\frac{2a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|2\right)}{d}-a\sqrt{\cos(c+dx)}\sqrt{\mathrm{se}}}{2a^2}\right.$$

$\Big\downarrow$ 3120

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}-\frac{\frac{2a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|2\right)}{d}-\frac{2a\sqrt{\cos(c+dx)}\sqrt{\sec(c+}}{}}{2a^2}\right.$$

3.377. $\int\frac{1}{\sqrt{\cos(c+dx)}(a+a\sec(c+dx))}dx$

input
```
Int[1/(Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(-1/2*((2*a*Sqrt[Cos[c + d*x]]*Ellip
ticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d - (2*a*Sqrt[Cos[c + d*x]]*Ellip
ticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d)/a^2 + (Sqrt[Sec[c + d*x]]*Sin[
c + d*x])/(d*(a + a*Sec[c + d*x])))
```

3.377.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4307
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-b)*d*Cot[e + f*x]*((d*Csc[e + f*x])^(n - 1)/(a*f*
(a + b*Csc[e + f*x]))), x] + Simp[d*((n - 1)/(a*b))   Int[(d*Csc[e + f*x])^
(n - 1)*(a - b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ
[a^2 - b^2, 0]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

3.377.4 Maple [A] (verified)

Time = 4.90 (sec) , antiderivative size = 198, normalized size of antiderivative = 2.83

method	result
default	$-\frac{\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1}\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\left(\text{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)+\text{EllipticE}\right.\right.}{a\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-}}$

input
```
int(1/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(cos(1/2*d*x+1/2*
c)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(Elliptic
F(cos(1/2*d*x+1/2*c),2^(1/2))+EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))+2*sin
(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2)/a/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2
*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+
1/2*c)^2-1)^(1/2)/d
```

3.377.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 184, normalized size of antiderivative = 2.63

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\sec(c+dx))}\,dx$$
$$= \frac{\left(-i\sqrt{2}\cos(dx+c)-i\sqrt{2}\right)\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+\left(i\sqrt{2}\cos(dx+c)\right.}{}$$

input
```
integrate(1/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

output
```
1/2*((-I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassPInverse(-4, 0, cos(
d*x + c) + I*sin(d*x + c)) + (I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierstr
assPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + (-I*sqrt(2)*cos(d*x +
c) - I*sqrt(2))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x
+ c) + I*sin(d*x + c))) + (I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierstrass
Zeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2
*sqrt(cos(d*x + c))*sin(d*x + c))/(a*d*cos(d*x + c) + a*d)
```

3.377.6 Sympy [F]

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\sec(c+dx))}\,dx = \frac{\int \frac{1}{\sqrt{\cos(c+dx)}\sec(c+dx)+\sqrt{\cos(c+dx)}}\,dx}{a}$$

input
```
integrate(1/cos(d*x+c)**(1/2)/(a+a*sec(d*x+c)),x)
```

output
```
Integral(1/(sqrt(cos(c + d*x))*sec(c + d*x) + sqrt(cos(c + d*x))), x)/a
```

3.377.7 Maxima [F]

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\sec(c+dx))}\,dx = \int \frac{1}{(a\sec(dx+c)+a)\sqrt{\cos(dx+c)}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

output
```
integrate(1/((a*sec(d*x + c) + a)*sqrt(cos(d*x + c))), x)
```

3.377.8 Giac [F]

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\sec(c+dx))}\,dx = \int \frac{1}{(a\sec(dx+c)+a)\sqrt{\cos(dx+c)}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c)),x, algorithm="giac")
```

output
```
integrate(1/((a*sec(d*x + c) + a)*sqrt(cos(d*x + c))), x)
```

3.377.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\sec(c+dx))}\,dx = \int \frac{1}{\sqrt{\cos(c+d\,x)}\,\left(a+\frac{a}{\cos(c+d\,x)}\right)}\,dx$$

input
```
int(1/(cos(c + d*x)^(1/2)*(a + a/cos(c + d*x))),x)
```

output
```
int(1/(cos(c + d*x)^(1/2)*(a + a/cos(c + d*x))), x)
```

3.378 $\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))}\,dx$

3.378.1 Optimal result . . . 2778
3.378.2 Mathematica [C] (verified) . . . 2778
3.378.3 Rubi [A] (verified) . . . 2779
3.378.4 Maple [A] (verified) . . . 2782
3.378.5 Fricas [C] (verification not implemented) . . . 2782
3.378.6 Sympy [F] . . . 2783
3.378.7 Maxima [F] . . . 2783
3.378.8 Giac [F] . . . 2784
3.378.9 Mupad [F(-1)] . . . 2784

3.378.1 Optimal result

Integrand size = 23, antiderivative size = 70

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))}\,dx = \frac{E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}+\frac{\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{ad}-\frac{\sin(c+dx)}{d\sqrt{\cos(c+dx)}(a+a\sec(c+dx))}$$

output
```
(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*
c),2^(1/2))/a/d+(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(
sin(1/2*d*x+1/2*c),2^(1/2))/a/d-sin(d*x+c)/d/(a+a*sec(d*x+c))/cos(d*x+c)^(
1/2)
```

3.378.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.28 (sec) , antiderivative size = 263, normalized size of antiderivative = 3.76

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))}\,dx = \frac{\cos^2\left(\frac{1}{2}(c+dx)\right)\left(\frac{2i\sqrt{2}e^{-i(c+dx)}\left(1+e^{2i(c+dx)}+(-1+e^{2ic})\sqrt{1+e^{2i(c+dx)}}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},-e^{2i(c+dx)}\right)-e^{i(c+dx)}(-1+e^{2ic}\right.}{d(-1+e^{2ic})\sqrt{e^{-i(c+dx)}(1+e^{2i(c+dx)})}}\right.}{a(1+\sec(c+dx))}$$

input
```
Integrate[1/(Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])),x]
```

output
```
(Cos[(c + d*x)/2]^2*(((2*I)*Sqrt[2]*(1 + E^((2*I)*(c + d*x)) + (-1 + E^((2
*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3/4, -E
^((2*I)*(c + d*x))] - E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)
*(c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))])*Sec[c
 + d*x])/(d*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[(1 + E^((2*I)*(c + d*x
)))/E^(I*(c + d*x))]) - (2*(Csc[c] + Sec[c/2]*Sec[(c + d*x)/2]*Sin[(d*x)/2
]))/(d*Sqrt[Cos[c + d*x]])))/(a*(1 + Sec[c + d*x]))
```

3.378.3 Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.96, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.522$, Rules used = {3042, 4752, 3042, 4305, 27, 3042, 4274, 3042, 4258, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)}dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sec(c+dx)a+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx$$

$$\Big\downarrow \text{4305}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\int -\frac{\sec(c+dx)a+a}{2\sqrt{\sec(c+dx)}}dx}{a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}\right)$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{\sec(c+dx)a+a}{\sqrt{\sec(c+dx)}}dx}{2a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}\right)$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{\csc(c+dx+\frac{\pi}{2})a+a}{\sqrt{\csc(c+dx+\frac{\pi}{2})}}dx}{2a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}\right)$$

$\downarrow$ 4274

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{a\int \frac{1}{\sqrt{\sec(c+dx)}}dx+a\int \sqrt{\sec(c+dx)}dx}{2a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}\right)$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{a\int \frac{1}{\sqrt{\csc(c+dx+\frac{\pi}{2})}}dx+a\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{2a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}\right)$$

$\downarrow$ 4258

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx+a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\cos(}}{2a^2}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin(c+dx+\frac{\pi}{2})}}dx+a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{s}}{2a^2}\right.$$

$\downarrow$ 3119

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin(c+dx+\frac{\pi}{2})}}dx+\frac{2a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E(\frac{1}{2}(c+dx)|2)}{d}}{2a^2}\right.$$

$\downarrow$ 3120

3.378. $\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))}dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+\frac{2a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|2\right)}{d}}{2a^2}-\frac{\sin(c}{d(}\right.$$

input
```
Int[1/(Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(((2*a*Sqrt[Cos[c + d*x]]*EllipticE[
(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (2*a*Sqrt[Cos[c + d*x]]*EllipticF[
(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d)/(2*a^2) - (Sqrt[Sec[c + d*x]]*Sin[c
 + d*x])/(d*(a + a*Sec[c + d*x])))
```

3.378.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

3.378. $\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))}\,dx$

rule 4305
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[d^2*Cot[e + f*x]*((d*Csc[e + f*x])^(n - 2)/(f*(a +
b*Csc[e + f*x]))), x] - Simp[d^2/(a*b)   Int[(d*Csc[e + f*x])^(n - 2)*(b*(n
 - 2) - a*(n - 1)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ
[a^2 - b^2, 0] && GtQ[n, 1]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

3.378.4 Maple [A] (verified)

Time = 3.94 (sec) , antiderivative size = 200, normalized size of antiderivative = 2.86

method	result
default	$\frac{\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(-\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1}\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\left(\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)-\mathrm{EllipticE}\right.\right.}{a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1}}$

input
```
int(1/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-cos(1/2*d*x+1/2*
c)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(Elliptic
F(cos(1/2*d*x+1/2*c),2^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))+2*sin
(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2)/a/cos(1/2*d*x+1/2*c)/(-2*sin(1/2*d
*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+
1/2*c)^2-1)^(1/2)/d
```

3.378.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 184, normalized size of antiderivative = 2.63

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))}\,dx$$
$$= \frac{\left(-i\sqrt{2}\cos(dx+c)-i\sqrt{2}\right)\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+\left(i\sqrt{2}\cos(dx+c)\right.}{}$$

input
```
integrate(1/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

output
```
1/2*((-I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassPInverse(-4, 0, cos(
d*x + c) + I*sin(d*x + c)) + (I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierstr
assPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + (I*sqrt(2)*cos(d*x + c
) + I*sqrt(2))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x +
 c) + I*sin(d*x + c))) + (-I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrass
Zeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - 2
*sqrt(cos(d*x + c))*sin(d*x + c))/(a*d*cos(d*x + c) + a*d)
```

3.378.6 Sympy [F]

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))}\,dx = \frac{\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)\sec(c+dx)+\cos^{\frac{3}{2}}(c+dx)}\,dx}{a}$$

input
```
integrate(1/cos(d*x+c)**(3/2)/(a+a*sec(d*x+c)),x)
```

output
```
Integral(1/(cos(c + d*x)**(3/2)*sec(c + d*x) + cos(c + d*x)**(3/2)), x)/a
```

3.378.7 Maxima [F]

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))}\,dx = \int \frac{1}{(a\sec(dx+c)+a)\cos(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

output
```
integrate(1/((a*sec(d*x + c) + a)*cos(d*x + c)^(3/2)), x)
```

3.378.8 Giac [F]

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))}\,dx = \int \frac{1}{(a\sec(dx+c)+a)\cos(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c)),x, algorithm="giac")
```

output
```
integrate(1/((a*sec(d*x + c) + a)*cos(d*x + c)^(3/2)), x)
```

3.378.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))}\,dx = \int \frac{1}{\cos(c+d\,x)^{3/2}\,\left(a+\frac{a}{\cos(c+d\,x)}\right)}\,dx$$

input
```
int(1/(cos(c + d*x)^(3/2)*(a + a/cos(c + d*x))),x)
```

output
```
int(1/(cos(c + d*x)^(3/2)*(a + a/cos(c + d*x))), x)
```

3.379 $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))}\,dx$

3.379.1 Optimal result 2785
3.379.2 Mathematica [C] (verified) 2785
3.379.3 Rubi [A] (verified) 2786
3.379.4 Maple [A] (verified) 2789
3.379.5 Fricas [C] (verification not implemented) 2790
3.379.6 Sympy [F(-1)] 2791
3.379.7 Maxima [F] 2791
3.379.8 Giac [F] 2791
3.379.9 Mupad [F(-1)] 2792

3.379.1 Optimal result

Integrand size = 23, antiderivative size = 96

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))}\,dx = -\frac{3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad} - \frac{\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{ad} + \frac{3\sin(c+dx)}{ad\sqrt{\cos(c+dx)}} - \frac{\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))}$$

output
```
-3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1
/2*c),2^(1/2))/a/d-(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellipti
cF(sin(1/2*d*x+1/2*c),2^(1/2))/a/d-sin(d*x+c)/d/cos(d*x+c)^(3/2)/(a+a*sec(
d*x+c))+3*sin(d*x+c)/a/d/cos(d*x+c)^(1/2)
```

3.379.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.70 (sec) , antiderivative size = 303, normalized size of antiderivative = 3.16

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))}\,dx = \frac{\cos^2\left(\frac{1}{2}(c+dx)\right)\left(\frac{\left(2\cos\left(\frac{1}{2}(c-dx)\right)+\cos\left(\frac{1}{2}(3c+dx)\right)+3\cos\left(\frac{1}{2}(c+3dx)\right)\right)\csc\left(\frac{c}{2}\right)\sec\left(\frac{c}{2}\right)\sec\left(\frac{1}{2}(c+dx)\right)}{2d\cos^{\frac{3}{2}}(c+dx)} - \frac{2i\sqrt{2}e^{-i(c+dx)}\left(3\left(1+e^{2i(c+dx)}\right.\right.}{}\right.}{a(}$$

input
```
Integrate[1/(Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])),x]
```

output
```
(Cos[(c + d*x)/2]^2*(((2*Cos[(c - d*x)/2] + Cos[(3*c + d*x)/2] + 3*Cos[(c
+ 3*d*x)/2])*Csc[c/2]*Sec[c/2]*Sec[(c + d*x)/2])/(2*d*Cos[c + d*x]^(3/2))
- ((2*I)*Sqrt[2]*(3*(1 + E^((2*I)*(c + d*x))) + 3*(-1 + E^((2*I)*c))*Sqrt[
1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c +
d*x))] - E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*
Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))])*Sec[c + d*x])/(d*E
^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[(1 + E^((2*I)*(c + d*x)))/E^(I*(c +
 d*x))])))/(a*(1 + Sec[c + d*x]))
```

3.379.3 Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.68, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.609$, Rules used = {3042, 4752, 3042, 4305, 27, 3042, 4274, 3042, 4255, 3042, 4258, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a\sec(c+dx)+a)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)}dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sec^{\frac{5}{2}}(c+dx)}{\sec(c+dx)a+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx$$

$$\Big\downarrow \text{4305}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\int \frac{1}{2}\sqrt{\sec(c+dx)}(a-3a\sec(c+dx))dx}{a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d(a\sec(c+dx)+a)}\right)$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\int\sqrt{\sec(c+dx)}(a-3a\sec(c+dx))dx}{2a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d(a\sec(c+dx)+a)}\right)$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a-3a\csc\left(c+dx+\frac{\pi}{2}\right)\right)dx}{2a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d(a\sec(c+dx)+a)}\right)$$

$\Big\downarrow$ 4274

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{a\int\sqrt{\sec(c+dx)}dx-3a\int\sec^{\frac{3}{2}}(c+dx)dx}{2a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d(a\sec(c+dx)+a)}\right)$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{a\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx-3a\int\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}dx}{2a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx}{d(a\sec(c+dx)+a)}\right.$$

$\Big\downarrow$ 4255

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{a\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx-3a\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\int\frac{1}{\sqrt{\sec(c+dx)}}dx\right)}{2a^2}-\frac{\sin(c+d}{d(a\sec}\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{a\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx-3a\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\int\frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)}{2a^2}-\frac{\sin(}{d(}\right.$$

$\Big\downarrow$ 4258

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx-3a\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\sqrt{\cos(c+}\right.}{2a^2}\right.$$

$\Big\downarrow$ 3042

3.379. $\int\frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))}dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-3a\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\sqrt{\cos(}\right.}{2a^2}\right.$$

$\downarrow$ 3119

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-3a\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(}}{}\right.}{2a^2}\right.$$

$\downarrow$ 3120

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{2a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}-3a\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{s}}{}\right.}{2a^2}\right.$$

input
```
Int[1/(Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(-((Sec[c + d*x]^(3/2)*Sin[c + d*x])
/(d*(a + a*Sec[c + d*x]))) - ((2*a*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/
2, 2]*Sqrt[Sec[c + d*x]])/d - 3*a*((-2*Sqrt[Cos[c + d*x]]*EllipticE[(c + d
*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d))/
(2*a^2))
```

3.379.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4305
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[d^2*Cot[e + f*x]*((d*Csc[e + f*x])^(n - 2)/(f*(a +
b*Csc[e + f*x]))), x] - Simp[d^2/(a*b)   Int[(d*Csc[e + f*x])^(n - 2)*(b*(n
 - 2) - a*(n - 1)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ
[a^2 - b^2, 0] && GtQ[n, 1]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

3.379.4 Maple [A] (verified)

Time = 6.28 (sec) , antiderivative size = 253, normalized size of antiderivative = 2.64

method	result
default	$-\frac{-\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1}\left(\operatorname{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)-3\operatorname{EllipticE}\left(\right.\right.}{a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\sin\left(\frac{dx}{2}+\frac{c}{2}\right.}}$

input
```
int(1/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-(-cos(1/2*d*x+1/2*c)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(EllipticF(
cos(1/2*d*x+1/2*c),2^(1/2))-3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))+6*(-2
*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-5*(
-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2)/
a/cos(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/
sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

3.379.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 236, normalized size of antiderivative = 2.46

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))}\,dx$$
$$= \frac{2\,(3\,\cos\,(dx+c)+2)\sqrt{\cos\,(dx+c)}\sin\,(dx+c)+\left(i\,\sqrt{2}\cos\,(dx+c)^2+i\,\sqrt{2}\cos\,(dx+c)\right)\text{weierstrassPIn}}{}$$

input
```
integrate(1/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

output
```
1/2*(2*(3*cos(d*x + c) + 2)*sqrt(cos(d*x + c))*sin(d*x + c) + (I*sqrt(2)*c
os(d*x + c)^2 + I*sqrt(2)*cos(d*x + c))*weierstrassPInverse(-4, 0, cos(d*x
 + c) + I*sin(d*x + c)) + (-I*sqrt(2)*cos(d*x + c)^2 - I*sqrt(2)*cos(d*x +
 c))*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 3*(I*sqrt
(2)*cos(d*x + c)^2 + I*sqrt(2)*cos(d*x + c))*weierstrassZeta(-4, 0, weiers
trassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 3*(-I*sqrt(2)*cos(d
*x + c)^2 - I*sqrt(2)*cos(d*x + c))*weierstrassZeta(-4, 0, weierstrassPInv
erse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/(a*d*cos(d*x + c)^2 + a*d*cos
(d*x + c))
```

3.379.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))}\,dx = \text{Timed out}$$

input
```
integrate(1/cos(d*x+c)**(5/2)/(a+a*sec(d*x+c)),x)
```

output
```
Timed out
```

3.379.7 Maxima [F]

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))}\,dx = \int \frac{1}{(a\sec(dx+c)+a)\cos(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

output
```
integrate(1/((a*sec(d*x + c) + a)*cos(d*x + c)^(5/2)), x)
```

3.379.8 Giac [F]

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))}\,dx = \int \frac{1}{(a\sec(dx+c)+a)\cos(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c)),x, algorithm="giac")
```

output
```
integrate(1/((a*sec(d*x + c) + a)*cos(d*x + c)^(5/2)), x)
```

3.379.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))}\,dx=\int \frac{1}{\cos\left(c+d\,x\right)^{5/2}\,\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}\,dx$$

input
```
int(1/(cos(c + d*x)^(5/2)*(a + a/cos(c + d*x))),x)
```

output
```
int(1/(cos(c + d*x)^(5/2)*(a + a/cos(c + d*x))), x)
```

3.380 $\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))}\,dx$

3.380.1 Optimal result 2793
3.380.2 Mathematica [C] (verified) 2793
3.380.3 Rubi [A] (verified) 2794
3.380.4 Maple [B] (verified) 2798
3.380.5 Fricas [C] (verification not implemented) 2798
3.380.6 Sympy [F(-1)] 2799
3.380.7 Maxima [F] 2799
3.380.8 Giac [F] 2800
3.380.9 Mupad [F(-1)] 2800

3.380.1 Optimal result

Integrand size = 23, antiderivative size = 124

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))}\,dx = \frac{3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}+\frac{5\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3ad}+\frac{5\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)}-\frac{3\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}-\frac{\sin(c+dx)}{d\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))}$$

output
```
3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/
2*c),2^(1/2))/a/d+5/3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elli
pticF(sin(1/2*d*x+1/2*c),2^(1/2))/a/d+5/3*sin(d*x+c)/a/d/cos(d*x+c)^(3/2)-
sin(d*x+c)/d/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))-3*sin(d*x+c)/a/d/cos(d*x+c)
^(1/2)
```

3.380.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 3.09 (sec) , antiderivative size = 338, normalized size of antiderivative = 2.73

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))}\,dx = \frac{\cos^2\left(\frac{1}{2}(c+dx)\right)\left(-\frac{(10\cos(\frac{1}{2}(c-dx))+8\cos(\frac{1}{2}(3c+dx))+4\cos(\frac{1}{2}(c+3dx))+5\cos(\frac{1}{2}(5c+3dx))+9\cos(\frac{1}{2}(3c+5dx)))\csc(\frac{c}{2})\sec(\frac{c}{2})\sec(\cdots}{4d\cos^{\frac{5}{2}}(c+dx)}\right.}{\cdots}$$

input
```
Integrate[1/(Cos[c + d*x]^(7/2)*(a + a*Sec[c + d*x])),x]
```

output
```
(Cos[(c + d*x)/2]^2*(-1/4*((10*Cos[(c - d*x)/2] + 8*Cos[(3*c + d*x)/2] + 4
*Cos[(c + 3*d*x)/2] + 5*Cos[(5*c + 3*d*x)/2] + 9*Cos[(3*c + 5*d*x)/2])*Csc
[c/2]*Sec[c/2]*Sec[(c + d*x)/2])/(d*Cos[c + d*x]^(5/2)) + ((2*I)*Sqrt[2]*(
9*(1 + E^((2*I)*(c + d*x))) + 9*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c +
d*x))]*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))] - 5*E^(I*(c
 + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F
1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))])*Sec[c + d*x])/(d*E^(I*(c + d*x))*(
-1 + E^((2*I)*c))*Sqrt[(1 + E^((2*I)*(c + d*x)))/E^(I*(c + d*x))])))/(3*a*
(1 + Sec[c + d*x]))
```

3.380.3 Rubi [A] (verified)

Time = 0.88 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.52, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.609$, Rules used = {3042, 4752, 3042, 4305, 27, 3042, 4274, 3042, 4255, 3042, 4258, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a\sec(c+dx)+a)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{7/2}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)}dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sec^{\frac{7}{2}}(c+dx)}{\sec(c+dx)a+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx$$

$$\Big\downarrow \text{4305}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\int \frac{1}{2}\sec^{\frac{3}{2}}(c+dx)(3a-5a\sec(c+dx))dx}{a^2}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{d(a\sec(c+dx)+a)}\right)$$

$$\Big\downarrow \; 27$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\int \sec^{\frac{3}{2}}(c+dx)(3a-5a\sec(c+dx))dx}{2a^2}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{d(a\sec(c+dx)+a)}\right)$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(3a-5a\csc\left(c+dx+\frac{\pi}{2}\right)\right)dx}{2a^2}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{d(a\sec(c+dx)+a)}\right)$$

$$\Big\downarrow \; 4274$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{3a\int \sec^{\frac{3}{2}}(c+dx)dx-5a\int \sec^{\frac{5}{2}}(c+dx)dx}{2a^2}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{d(a\sec(c+dx)+a)}\right)$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{3a\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}dx-5a\int \csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}dx}{2a^2}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+d}{d(a\sec(c+dx)+a)}\right.$$

$$\Big\downarrow \; 4255$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{3a\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\int \frac{1}{\sqrt{\sec(c+dx)}}dx\right)-5a\left(\frac{1}{3}\int \sqrt{\sec(c+dx)}dx+\frac{2\sin(c+dx)}{3}\right.}{2a^2}\right.$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{3a\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)-5a\left(\frac{1}{3}\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\right.}{2a^2}\right.$$

$$\Big\downarrow \; 4258$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{3a\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\cos(c+dx)}dx\right)-5a\left(\frac{1}{3}\right.}{2a^2}\right.$$

$$\Big\downarrow \; 3042$$

3.380. $\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))}dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{3a\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx\right)-5}{2a^2}\right.$$

$\downarrow$ 3119

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{3a\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|2\right)}{d}\right)-5a\left(\frac{1}{3}\sqrt{\cos(c+dx}\right.}{2a^2}\right.$$

$\downarrow$ 3120

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{3a\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|2\right)}{d}\right)-5a\left(\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(}{3d}\right.}{2a^2}\right.$$

input
```
Int[1/(Cos[c + d*x]^(7/2)*(a + a*Sec[c + d*x])),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(-((Sec[c + d*x]^(5/2)*Sin[c + d*x])
/(d*(a + a*Sec[c + d*x]))) - (3*a*((-2*Sqrt[Cos[c + d*x]]*EllipticE[(c + d
*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d) -
 5*a*((2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/
(3*d) + (2*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d)))/(2*a^2))
```

3.380.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4305
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[d^2*Cot[e + f*x]*((d*Csc[e + f*x])^(n - 2)/(f*(a +
b*Csc[e + f*x]))), x] - Simp[d^2/(a*b)   Int[(d*Csc[e + f*x])^(n - 2)*(b*(n
 - 2) - a*(n - 1)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ
[a^2 - b^2, 0] && GtQ[n, 1]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

3.380. $\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))}\,dx$

3.380.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 412 vs. $2(166) = 332$.

Time = 9.17 (sec) , antiderivative size = 413, normalized size of antiderivative = 3.33

method	result
default	$\frac{\sqrt{-\left(-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\left(10\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\operatorname{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sqrt{2\sin\left(\frac{dx}{2}\right.}\right.}{}$

input
```
int(1/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/3*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/a/sin(1/2*d*
x+1/2*c)^3/cos(1/2*d*x+1/2*c)/(4*sin(1/2*d*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)
^2+1)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(10*cos(1/2*d*x
+1/2*c)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(
1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2-18*cos(1/2*d*x+1/2*
c)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2-36*sin(1/2*d*x+1/2*c)^6-
5*cos(1/2*d*x+1/2*c)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+
1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)+9*cos(1/2*d*x+1/2*c)*Ellipt
icE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*
d*x+1/2*c)^2)^(1/2)+44*sin(1/2*d*x+1/2*c)^4-11*sin(1/2*d*x+1/2*c)^2)/(2*co
s(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

3.380.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 258, normalized size of antiderivative = 2.08

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))}\,dx = -\frac{2\left(9\cos\left(dx+c\right)^{2}+4\cos\left(dx+c\right)-2\right)\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)+5\left(i\sqrt{2}\cos\left(dx+c\right)^{3}+i\sqrt{2}\cos\left(d\right.\right.}{}$$

input
```
integrate(1/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

output
```
-1/6*(2*(9*cos(d*x + c)^2 + 4*cos(d*x + c) - 2)*sqrt(cos(d*x + c))*sin(d*x
 + c) + 5*(I*sqrt(2)*cos(d*x + c)^3 + I*sqrt(2)*cos(d*x + c)^2)*weierstras
sPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 5*(-I*sqrt(2)*cos(d*x +
c)^3 - I*sqrt(2)*cos(d*x + c)^2)*weierstrassPInverse(-4, 0, cos(d*x + c) -
 I*sin(d*x + c)) + 9*(-I*sqrt(2)*cos(d*x + c)^3 - I*sqrt(2)*cos(d*x + c)^2
)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d
*x + c))) + 9*(I*sqrt(2)*cos(d*x + c)^3 + I*sqrt(2)*cos(d*x + c)^2)*weiers
trassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))
))/(a*d*cos(d*x + c)^3 + a*d*cos(d*x + c)^2)
```

3.380.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))}\,dx = \text{Timed out}$$

input
```
integrate(1/cos(d*x+c)**(7/2)/(a+a*sec(d*x+c)),x)
```

output
```
Timed out
```

3.380.7 Maxima [F]

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))}\,dx = \int \frac{1}{(a\sec(dx+c)+a)\cos(dx+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

output
```
integrate(1/((a*sec(d*x + c) + a)*cos(d*x + c)^(7/2)), x)
```

3.380.8 Giac [F]

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))}\,dx = \int \frac{1}{(a\sec(dx+c)+a)\cos(dx+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c)),x, algorithm="giac")
```

output
```
integrate(1/((a*sec(d*x + c) + a)*cos(d*x + c)^(7/2)), x)
```

3.380.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))}\,dx = \int \frac{1}{\cos(c+d\,x)^{7/2}\,\left(a+\frac{a}{\cos(c+d\,x)}\right)}\,dx$$

input
```
int(1/(cos(c + d*x)^(7/2)*(a + a/cos(c + d*x))),x)
```

output
```
int(1/(cos(c + d*x)^(7/2)*(a + a/cos(c + d*x))), x)
```

3.381 $\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

3.381.1 Optimal result 2801
3.381.2 Mathematica [C] (verified) 2801
3.381.3 Rubi [A] (verified) 2802
3.381.4 Maple [A] (verified) 2806
3.381.5 Fricas [C] (verification not implemented) 2807
3.381.6 Sympy [F(-1)] 2808
3.381.7 Maxima [F] 2808
3.381.8 Giac [F] 2808
3.381.9 Mupad [F(-1)] 2809

3.381.1 Optimal result

Integrand size = 23, antiderivative size = 160

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{56E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5a^2d} - \frac{5\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{a^2d} - \frac{5\sqrt{\cos(c+dx)}\sin(c+dx)}{a^2d} + \frac{56\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15a^2d} - \frac{3\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{a^2d(1+\sec(c+dx))} - \frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2}$$

output
```
56/5*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x
+1/2*c),2^(1/2))/a^2/d-5*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*E
llipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a^2/d+56/15*cos(d*x+c)^(3/2)*sin(d*x+
c)/a^2/d-3*cos(d*x+c)^(3/2)*sin(d*x+c)/a^2/d/(1+sec(d*x+c))-1/3*cos(d*x+c)
^(3/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^2-5*sin(d*x+c)*cos(d*x+c)^(1/2)/a^2/d
```

3.381.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.02 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.92

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{\sqrt{\cos(c+dx)}\csc^3(c+dx)\left(-240-1186\cos(c+dx)+340\cos(2(c+dx))+207\cos(3(c+dx))-20\cos(\right.}{}$$

input
```
Integrate[Cos[c + d*x]^(5/2)/(a + a*Sec[c + d*x])^2,x]
```

output
```
(Sqrt[Cos[c + d*x]]*Csc[c + d*x]^3*(-240 - 1186*Cos[c + d*x] + 340*Cos[2*(
c + d*x)] + 207*Cos[3*(c + d*x)] - 20*Cos[4*(c + d*x)] + 3*Cos[5*(c + d*x)
] + 600*Hypergeometric2F1[1/4, 1/2, 5/4, Cos[c + d*x]^2]*(Sin[c + d*x]^2)^
(3/2) + 1792*Cos[c + d*x]*Hypergeometric2F1[3/4, 5/2, 7/4, Cos[c + d*x]^2]
*(Sin[c + d*x]^2)^(3/2)))/(120*a^2*d)
```

3.381.3 Rubi [A] (verified)

Time = 1.20 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.47, number of steps used = 16, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.696$, Rules used = {3042, 4752, 3042, 4304, 27, 3042, 4508, 3042, 4274, 3042, 4256, 3042, 4258, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a\sec(c+dx)+a)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^2}dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(\sec(c+dx)a+a)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}dx$$

$$\Big\downarrow \text{4304}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\int -\frac{11a-7a\sec(c+dx)}{2\sec^{\frac{5}{2}}(c+dx)(\sec(c+dx)a+a)}dx}{3a^2}-\frac{\sin(c+dx)}{3d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^2}\right)$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{11a-7a\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)(\sec(c+dx)a+a)}dx}{6a^2}-\frac{\sin(c+dx)}{3d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^2}\right)$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{11a-7a\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)}dx}{6a^2}-\frac{\sin(c+dx)}{3d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^2}\right)$$

$\downarrow$ 4508

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\int \frac{56a^2-45a^2\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)}dx}{a^2}-\frac{18\sin(c+dx)}{d\sec^{\frac{3}{2}}(c+dx)(\sec(c+dx)+1)}}{6a^2}-\frac{\sin(c+dx)}{3d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^2}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\int \frac{56a^2-45a^2\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx}{a^2}-\frac{18\sin(c+dx)}{d\sec^{\frac{3}{2}}(c+dx)(\sec(c+dx)+1)}}{6a^2}-\frac{\sin(c+dx)}{3d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+\right.$$

$\downarrow$ 4274

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{56a^2\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)}dx-45a^2\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}dx}{a^2}-\frac{18\sin(c+dx)}{d\sec^{\frac{3}{2}}(c+dx)(\sec(c+dx)+1)}}{6a^2}-\frac{\sin(c+}{3d\sec^{\frac{3}{2}}(c+dx)(a\,\mathrm{se}}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{56a^2\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx-45a^2\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx}{a^2}-\frac{18\sin(c+dx)}{d\sec^{\frac{3}{2}}(c+dx)(\sec(c+dx)+1)}}{6a^2}-\frac{}{3d\sec^{\frac{3}{2}}(c+}\right.$$

$\downarrow$ 4256

3.381. $\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{56a^2\left(\frac{3}{5}\int\frac{1}{\sqrt{\sec(c+dx)}}dx+\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}\right)-45a^2\left(\frac{1}{3}\int\sqrt{\sec(c+dx)}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)}{a^2}-\frac{18\sin(c+}{d\sec^{\frac{3}{2}}(c+dx)(se}}{6a^2}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{56a^2\left(\frac{3}{5}\int\frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}\right)-45a^2\left(\frac{1}{3}\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)}{a^2}-\frac{}{d\sec^{\frac{3}{2}}(c}}{6a^2}\right.$$

$\downarrow$ 4258

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{56a^2\left(\frac{3}{5}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\cos(c+dx)}dx+\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}\right)-45a^2\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\right.}{a^2}}{6a^2}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{56a^2\left(\frac{3}{5}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}\right)-45a^2\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right.}{a^2}}{6a^2}\right.$$

$\downarrow$ 3119

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{56a^2\left(\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{6\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}\right)-45a^2\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin(c+}}\right.}{a^2}}{6a^2}\right.$$

$\downarrow$ 3120

3.381. $\int\frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{56a^2\left(\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{6\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}\right)-45a^2\left(\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{3d}\right.}{a^2}}{6a^2}\right.$$

input
```
Int[Cos[c + d*x]^(5/2)/(a + a*Sec[c + d*x])^2,x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(-1/3*Sin[c + d*x]/(d*Sec[c + d*x]^(
3/2)*(a + a*Sec[c + d*x])^2) + ((-18*Sin[c + d*x])/(d*Sec[c + d*x]^(3/2)*(
1 + Sec[c + d*x])) + (56*a^2*((6*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2,
 2]*Sqrt[Sec[c + d*x]])/(5*d) + (2*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)))
 - 45*a^2*((2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*
x]])/(3*d) + (2*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])))/a^2)/(6*a^2))
```

3.381.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4304
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-Cot[e + f*x])*(a + b*Csc[e + f*x])^m*((d*Csc
[e + f*x])^n/(f*(2*m + 1))), x] + Simp[1/(a^2*(2*m + 1))   Int[(a + b*Csc[e
 + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e
+ f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ
[m, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

rule 4508
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
- a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
 1))), x] - Simp[1/(a^2*(2*m + 1))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Cs
c[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[
e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B
, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

3.381.4 Maple [A] (verified)

Time = 10.16 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.77

method	result
default	$-\frac{\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(96\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}-352\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^8+120\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^6-150\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\cos\left(\frac{dx}{2}\right.}\right.}{30a^2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)}$

input
```
int(cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
-1/30*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(96*cos(1/2*
d*x+1/2*c)^10-352*cos(1/2*d*x+1/2*c)^8+120*cos(1/2*d*x+1/2*c)^6-150*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2
*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^3-336*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*cos(1/2*d*x+1/2*c)^3*EllipticE(cos(1/2*
d*x+1/2*c),2^(1/2))+266*cos(1/2*d*x+1/2*c)^4-135*cos(1/2*d*x+1/2*c)^2+5)/a
^2/cos(1/2*d*x+1/2*c)^3/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/
2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

3.381.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.13 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.80

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^2}\,dx$$
$$= \frac{2\left(6\cos\left(dx+c\right)^3-8\cos\left(dx+c\right)^2-94\cos\left(dx+c\right)-75\right)\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)-75\left(-i\sqrt{2}\cos\left(d\right.\right.}{}$$

input
```
integrate(cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/30*(2*(6*cos(d*x + c)^3 - 8*cos(d*x + c)^2 - 94*cos(d*x + c) - 75)*sqrt(
cos(d*x + c))*sin(d*x + c) - 75*(-I*sqrt(2)*cos(d*x + c)^2 - 2*I*sqrt(2)*c
os(d*x + c) - I*sqrt(2))*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d
*x + c)) - 75*(I*sqrt(2)*cos(d*x + c)^2 + 2*I*sqrt(2)*cos(d*x + c) + I*sqr
t(2))*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 168*(-I*
sqrt(2)*cos(d*x + c)^2 - 2*I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrass
Zeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 1
68*(I*sqrt(2)*cos(d*x + c)^2 + 2*I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*weier
strassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)
)))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)
```

3.381.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(5/2)/(a+a*sec(d*x+c))**2,x)
```

output
```
Timed out
```

3.381.7 Maxima [F]

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \int \frac{\cos(dx+c)^{\frac{5}{2}}}{(a\sec(dx+c)+a)^2}\,dx$$

input
```
integrate(cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
integrate(cos(d*x + c)^(5/2)/(a*sec(d*x + c) + a)^2, x)
```

3.381.8 Giac [F]

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \int \frac{\cos(dx+c)^{\frac{5}{2}}}{(a\sec(dx+c)+a)^2}\,dx$$

input
```
integrate(cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate(cos(d*x + c)^(5/2)/(a*sec(d*x + c) + a)^2, x)
```

3.381.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^2}\,dx=\int \frac{\cos\left(c+d\,x\right)^{5/2}}{\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)^2}\,dx$$

input
```
int(cos(c + d*x)^(5/2)/(a + a/cos(c + d*x))^2,x)
```

output
```
int(cos(c + d*x)^(5/2)/(a + a/cos(c + d*x))^2, x)
```

3.382 $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

3.382.1 Optimal result 2810
3.382.2 Mathematica [C] (verified) 2810
3.382.3 Rubi [A] (verified) 2811
3.382.4 Maple [A] (verified) 2816
3.382.5 Fricas [C] (verification not implemented) 2816
3.382.6 Sympy [F] 2817
3.382.7 Maxima [F] 2817
3.382.8 Giac [F] 2818
3.382.9 Mupad [F(-1)] 2818

3.382.1 Optimal result

Integrand size = 23, antiderivative size = 138

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{7E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}+\frac{10\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3a^2d}+\frac{10\sqrt{\cos(c+dx)}\sin(c+dx)}{3a^2d}-\frac{7\sqrt{\cos(c+dx)}\sin(c+dx)}{3a^2d(1+\sec(c+dx))}-\frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{3d(a+a\sec(c+dx))^2}$$

output
```
-7*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1
/2*c),2^(1/2))/a^2/d+10/3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*
EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a^2/d+10/3*sin(d*x+c)*cos(d*x+c)^(1/
2)/a^2/d-7/3*sin(d*x+c)*cos(d*x+c)^(1/2)/a^2/d/(1+sec(d*x+c))-1/3*sin(d*x+
c)*cos(d*x+c)^(1/2)/d/(a+a*sec(d*x+c))^2
```

3.382.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.66 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.99

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{\sqrt{\cos(c+dx)}\csc^3(c+dx)\left(15+76\cos(c+dx)-24\cos(2(c+dx))-12\cos(3(c+dx))+\cos(4(c+dx)\right.}{}$$

input
```
Integrate[Cos[c + d*x]^(3/2)/(a + a*Sec[c + d*x])^2,x]
```

output
```
(Sqrt[Cos[c + d*x]]*Csc[c + d*x]^3*(15 + 76*Cos[c + d*x] - 24*Cos[2*(c + d
*x)] - 12*Cos[3*(c + d*x)] + Cos[4*(c + d*x)] - 40*Hypergeometric2F1[1/4,
1/2, 5/4, Cos[c + d*x]^2]*(Sin[c + d*x]^2)^(3/2) - 112*Cos[c + d*x]*Hyperg
eometric2F1[3/4, 5/2, 7/4, Cos[c + d*x]^2]*(Sin[c + d*x]^2)^(3/2)))/(12*a^
2*d)
```

3.382.3 Rubi [A] (verified)

Time = 1.13 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.51, number of steps used = 17, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.739$, Rules used = {3042, 4752, 3042, 4304, 27, 3042, 4508, 27, 3042, 4274, 3042, 4256, 3042, 4258, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a\sec(c+dx)+a)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^2}dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(\sec(c+dx)a+a)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}dx$$

$$\Big\downarrow \text{4304}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\int -\frac{9a-5a\sec(c+dx)}{2\sec^{\frac{3}{2}}(c+dx)(\sec(c+dx)a+a)}dx}{3a^2}-\frac{\sin(c+dx)}{3d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^2}\right)$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{9a-5a\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(\sec(c+dx)a+a)}dx}{6a^2}-\frac{\sin(c+dx)}{3d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^2}\right)$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{9a-5a\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)}dx}{6a^2}-\frac{\sin(c+dx)}{3d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^2}\right)$$

$\downarrow$ 4508

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\int \frac{3\left(10a^2-7a^2\sec(c+dx)\right)}{\sec^{\frac{3}{2}}(c+dx)}dx}{a^2}-\frac{14\sin(c+dx)}{d\sqrt{\sec(c+dx)}(\sec(c+dx)+1)}}{6a^2}-\frac{\sin(c+dx)}{3d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a}\right.$$

$\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{3\int \frac{10a^2-7a^2\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)}dx}{a^2}-\frac{14\sin(c+dx)}{d\sqrt{\sec(c+dx)}(\sec(c+dx)+1)}}{6a^2}-\frac{\sin(c+dx)}{3d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{3\int \frac{10a^2-7a^2\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx}{a^2}-\frac{14\sin(c+dx)}{d\sqrt{\sec(c+dx)}(\sec(c+dx)+1)}}{6a^2}-\frac{\sin(c+dx)}{3d\sqrt{\sec(c+dx)}(a\sec(c+dx)+}\right.$$

$\downarrow$ 4274

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{3\left(10a^2\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}dx-7a^2\int \frac{1}{\sqrt{\sec(c+dx)}}dx\right)}{a^2}-\frac{14\sin(c+dx)}{d\sqrt{\sec(c+dx)}(\sec(c+dx)+1)}}{6a^2}-\frac{\sin(c}{3d\sqrt{\sec(c+dx)}(}\right.$$

$\downarrow$ 3042

3.382. $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{3\left(10a^2\int\frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx-7a^2\int\frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)}{a^2}-\frac{14\sin(c+dx)}{d\sqrt{\sec(c+dx)}(\sec(c+dx)+1)}}{6a^2}-\frac{}{3d\sqrt{\sec(c}}\right.$$

$\downarrow$ 4256

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{3\left(10a^2\left(\frac{1}{3}\int\sqrt{\sec(c+dx)}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)-7a^2\int\frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)}{a^2}-\frac{14\sin(c+dx)}{d\sqrt{\sec(c+dx)}(\sec(c+dx)+1)}}{6a^2}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{3\left(10a^2\left(\frac{1}{3}\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)-7a^2\int\frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)}{a^2}-\frac{14\sin(c+dx)}{d\sqrt{\sec(c+dx)}(\sec(c+dx)}}{6a^2}\right.$$

$\downarrow$ 4258

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{3\left(10a^2\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)-7a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\cos}\right.}{a^2}}{6a^2}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{3\left(10a^2\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)-7a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\right.}{a^2}}{6a^2}\right.$$

$\downarrow$ 3119

3.382. $\int\frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{3\left(10a^2\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)-\frac{14a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}\right.}{d}}{a^2}}{6a^2}\right.$$

$$\Big\downarrow \; 3120$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{3\left(10a^2\left(\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}\right)-\frac{14a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+d\right.}{d}}{a^2}}{6a^2}\right.$$

input
```
Int[Cos[c + d*x]^(3/2)/(a + a*Sec[c + d*x])^2,x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(-1/3*Sin[c + d*x]/(d*Sqrt[Sec[c + d
*x]]*(a + a*Sec[c + d*x])^2) + ((-14*Sin[c + d*x])/(d*Sqrt[Sec[c + d*x]]*(
1 + Sec[c + d*x])) + (3*((-14*a^2*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2
, 2]*Sqrt[Sec[c + d*x]])/d + 10*a^2*((2*Sqrt[Cos[c + d*x]]*EllipticF[(c +
d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (2*Sin[c + d*x])/(3*d*Sqrt[Sec[c +
d*x]])))))/a^2)/(6*a^2))
```

3.382.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4304
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-Cot[e + f*x])*(a + b*Csc[e + f*x])^m*((d*Csc
[e + f*x])^n/(f*(2*m + 1))), x] + Simp[1/(a^2*(2*m + 1))   Int[(a + b*Csc[e
 + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e
+ f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ
[m, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

rule 4508
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
- a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
 1))), x] - Simp[1/(a^2*(2*m + 1))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Cs
c[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[
e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B
, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

3.382. $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

3.382.4 Maple [A] (verified)

Time = 8.59 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.96

method	result
default	$-\frac{\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\left(16\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{8}+12\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}+20\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+1}\,\mathrm{EllipticF}\left(\mathrm{c}\right.\right.}{6a^{2}\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\sin\left(\right.}}$

input
```
int(cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
-1/6*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(16*cos(1/2*d
*x+1/2*c)^8+12*cos(1/2*d*x+1/2*c)^6+20*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*co
s(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*
d*x+1/2*c)^3+42*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(
1/2)*cos(1/2*d*x+1/2*c)^3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-48*cos(1/2
*d*x+1/2*c)^4+21*cos(1/2*d*x+1/2*c)^2-1)/a^2/(-2*sin(1/2*d*x+1/2*c)^4+sin(
1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)^3/sin(1/2*d*x+1/2*c)/(2*cos(1/2
*d*x+1/2*c)^2-1)^(1/2)/d
```

3.382.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 278, normalized size of antiderivative = 2.01

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^{2}}\,dx$$
$$= \frac{2\left(2\,\cos\left(dx+c\right)^{2}+13\,\cos\left(dx+c\right)+10\right)\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)-10\left(i\,\sqrt{2}\cos\left(dx+c\right)^{2}+2i\,\sqrt{2}\cos\right.}{}$$

input
```
integrate(cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/6*(2*(2*cos(d*x + c)^2 + 13*cos(d*x + c) + 10)*sqrt(cos(d*x + c))*sin(d*
x + c) - 10*(I*sqrt(2)*cos(d*x + c)^2 + 2*I*sqrt(2)*cos(d*x + c) + I*sqrt(
2))*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) - 10*(-I*sqr
t(2)*cos(d*x + c)^2 - 2*I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassPIn
verse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 21*(I*sqrt(2)*cos(d*x + c)^2
 + 2*I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierstrassZeta(-4, 0, weierstras
sPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 21*(-I*sqrt(2)*cos(d*x
+ c)^2 - 2*I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassZeta(-4, 0, weie
rstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/(a^2*d*cos(d*x + c
)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)
```

3.382.6 Sympy [F]

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx}{a^2}$$

input
```
integrate(cos(d*x+c)**(3/2)/(a+a*sec(d*x+c))**2,x)
```

output
```
Integral(cos(c + d*x)**(3/2)/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a*
*2
```

3.382.7 Maxima [F]

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \int \frac{\cos(dx+c)^{\frac{3}{2}}}{(a\sec(dx+c)+a)^2}\,dx$$

input
```
integrate(cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
integrate(cos(d*x + c)^(3/2)/(a*sec(d*x + c) + a)^2, x)
```

3.382.8 Giac [F]

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \int \frac{\cos{(dx+c)}^{\frac{3}{2}}}{(a\sec{(dx+c)}+a)^2}\,dx$$

input
```
integrate(cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate(cos(d*x + c)^(3/2)/(a*sec(d*x + c) + a)^2, x)
```

3.382.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \int \frac{\cos{(c+d\,x)}^{3/2}}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^2}\,dx$$

input
```
int(cos(c + d*x)^(3/2)/(a + a/cos(c + d*x))^2,x)
```

output
```
int(cos(c + d*x)^(3/2)/(a + a/cos(c + d*x))^2, x)
```

3.383 $\int \frac{\sqrt{\cos(c+dx)}}{(a+a\sec(c+dx))^2}\,dx$

3.383.1 Optimal result . . . 2819
3.383.2 Mathematica [C] (verified) . . . 2819
3.383.3 Rubi [A] (verified) . . . 2820
3.383.4 Maple [A] (verified) . . . 2824
3.383.5 Fricas [C] (verification not implemented) . . . 2824
3.383.6 Sympy [F] . . . 2825
3.383.7 Maxima [F] . . . 2825
3.383.8 Giac [F] . . . 2826
3.383.9 Mupad [F(-1)] . . . 2826

3.383.1 Optimal result

Integrand size = 23, antiderivative size = 112

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+a\sec(c+dx))^2}\,dx = \frac{4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d} - \frac{5\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3a^2d} - \frac{5\sin(c+dx)}{3a^2d\sqrt{\cos(c+dx)}(1+\sec(c+dx))} - \frac{\sin(c+dx)}{3d\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^2}$$

output
```
4*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/
2*c),2^(1/2))/a^2/d-5/3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*El
lipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a^2/d-5/3*sin(d*x+c)/a^2/d/(1+sec(d*x+
c))/cos(d*x+c)^(1/2)-1/3*sin(d*x+c)/d/(a+a*sec(d*x+c))^2/cos(d*x+c)^(1/2)
```

3.383.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.53 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.14

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+a\sec(c+dx))^2}\,dx = \frac{\sqrt{\cos(c+dx)}\csc^3(c+dx)\left(-6-46\cos(c+dx)+14\cos(2(c+dx))+6\cos(3(c+dx))+20\,\text{Hypergeom}\right.}{}$$

input
```
Integrate[Sqrt[Cos[c + d*x]]/(a + a*Sec[c + d*x])^2,x]
```

output
```
(Sqrt[Cos[c + d*x]]*Csc[c + d*x]^3*(-6 - 46*Cos[c + d*x] + 14*Cos[2*(c + d
*x)] + 6*Cos[3*(c + d*x)] + 20*Hypergeometric2F1[1/4, 1/2, 5/4, Cos[c + d*
x]^2]*(Sin[c + d*x]^2)^(3/2) + 64*Cos[c + d*x]*Hypergeometric2F1[3/4, 5/2,
 7/4, Cos[c + d*x]^2]*(Sin[c + d*x]^2)^(3/2)))/(12*a^2*d)
```

3.383.3 Rubi [A] (verified)

Time = 0.96 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.60, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.609$, Rules used = {3042, 4752, 3042, 4304, 27, 3042, 4508, 3042, 4274, 3042, 4258, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\cos(c+dx)}}{(a\sec(c+dx)+a)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^2}dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sec(c+dx)}(\sec(c+dx)a+a)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}dx$$

$$\Big\downarrow \text{4304}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\int -\frac{7a-3a\sec(c+dx)}{2\sqrt{\sec(c+dx)}(\sec(c+dx)a+a)}dx}{3a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}\right)$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{7a-3a\sec(c+dx)}{\sqrt{\sec(c+dx)}(\sec(c+dx)a+a)}dx}{6a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}\right)$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{7a-3a\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)}dx}{6a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}\right)$$

$$\Big\downarrow \; 4508$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\int \frac{12a^2-5a^2\sec(c+dx)}{\sqrt{\sec(c+dx)}}dx}{a^2}-\frac{10\sin(c+dx)\sqrt{\sec(c+dx)}}{d(\sec(c+dx)+1)}}{6a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}\right)$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\int \frac{12a^2-5a^2\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2}-\frac{10\sin(c+dx)\sqrt{\sec(c+dx)}}{d(\sec(c+dx)+1)}}{6a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}\right)$$

$$\Big\downarrow \; 4274$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{12a^2\int \frac{1}{\sqrt{\sec(c+dx)}}dx-5a^2\int \sqrt{\sec(c+dx)}dx}{a^2}-\frac{10\sin(c+dx)\sqrt{\sec(c+dx)}}{d(\sec(c+dx)+1)}}{6a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+d}}{3d(a\sec(c+dx)+a)}\right.$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{12a^2\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-5a^2\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}-\frac{10\sin(c+dx)\sqrt{\sec(c+dx)}}{d(\sec(c+dx)+1)}}{6a^2}-\frac{\sin(c+dx)\sqrt{\sec}}{3d(a\sec(c+dx}\right.$$

$$\Big\downarrow \; 4258$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{12a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\cos(c+dx)}dx-5a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{a^2}-\frac{10\sin(c+}{d(se}}{6a^2}\right.$$

$$\Big\downarrow \; 3042$$

3.383. $\int \frac{\sqrt{\cos(c+dx)}}{(a+a\sec(c+dx))^2}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{12a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx-5a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2}-1}{6a^2}\right.$$

$\downarrow$ 3119

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{24a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-5a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2}-\frac{10\sin(c+dx}{d(\sec(c}}{6a^2}\right.$$

$\downarrow$ 3120

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{24a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-\frac{10a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}}{a^2}-\frac{10\sin(c+dx)\sqrt{}}{d(\sec(c+}}{6a^2}\right.$$

input
```
Int[Sqrt[Cos[c + d*x]]/(a + a*Sec[c + d*x])^2,x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(-1/3*(Sqrt[Sec[c + d*x]]*Sin[c + d*
x])/(d*(a + a*Sec[c + d*x])^2) + (((24*a^2*Sqrt[Cos[c + d*x]]*EllipticE[(c
 + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d - (10*a^2*Sqrt[Cos[c + d*x]]*EllipticF
[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d)/a^2 - (10*Sqrt[Sec[c + d*x]]*Sin[c
 + d*x])/(d*(1 + Sec[c + d*x])))/(6*a^2))
```

3.383.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4304
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-Cot[e + f*x])*(a + b*Csc[e + f*x])^m*((d*Csc
[e + f*x])^n/(f*(2*m + 1))), x] + Simp[1/(a^2*(2*m + 1))   Int[(a + b*Csc[e
 + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e
+ f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ
[m, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

rule 4508
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
- a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
 1))), x] - Simp[1/(a^2*(2*m + 1))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Cs
c[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[
e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B
, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

3.383. $\int \frac{\sqrt{\cos(c+dx)}}{(a+a\sec(c+dx))^2}\,dx$

3.383.4 Maple [A] (verified)

Time = 7.61 (sec) , antiderivative size = 257, normalized size of antiderivative = 2.29

method	result
default	$\frac{\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\left(24\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}+10\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+1}\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\cos\left(\right.\right.}{6a^{2}\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\cos}$

input
```
int(cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/6*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(24*cos(1/2*d*
x+1/2*c)^6+10*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/
2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^3+24*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*cos(1/2*d*x+1/2*c)^3
*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-38*cos(1/2*d*x+1/2*c)^4+15*cos(1/2*
d*x+1/2*c)^2-1)/a^2/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/c
os(1/2*d*x+1/2*c)^3/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

3.383.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 268, normalized size of antiderivative = 2.39

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+a\sec(c+dx))^2}\,dx = -\frac{2\left(6\cos\left(dx+c\right)+5\right)\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)+5\left(-i\sqrt{2}\cos\left(dx+c\right)^{2}-2i\sqrt{2}\cos\left(dx+c\right)-i\sqrt{2}\right)}{}$$

input
```
integrate(cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
-1/6*(2*(6*cos(d*x + c) + 5)*sqrt(cos(d*x + c))*sin(d*x + c) + 5*(-I*sqrt(
2)*cos(d*x + c)^2 - 2*I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassPInve
rse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 5*(I*sqrt(2)*cos(d*x + c)^2 +
2*I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierstrassPInverse(-4, 0, cos(d*x +
 c) - I*sin(d*x + c)) + 12*(-I*sqrt(2)*cos(d*x + c)^2 - 2*I*sqrt(2)*cos(d*
x + c) - I*sqrt(2))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(
d*x + c) + I*sin(d*x + c))) + 12*(I*sqrt(2)*cos(d*x + c)^2 + 2*I*sqrt(2)*c
os(d*x + c) + I*sqrt(2))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0,
 cos(d*x + c) - I*sin(d*x + c))))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x
+ c) + a^2*d)
```

3.383.6 Sympy [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+a\sec(c+dx))^2}\,dx = \frac{\int \frac{\sqrt{\cos(c+dx)}}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx}{a^2}$$

input
```
integrate(cos(d*x+c)**(1/2)/(a+a*sec(d*x+c))**2,x)
```

output
```
Integral(sqrt(cos(c + d*x))/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a**
2
```

3.383.7 Maxima [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+a\sec(c+dx))^2}\,dx = \int \frac{\sqrt{\cos(dx+c)}}{(a\sec(dx+c)+a)^2}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(cos(d*x + c))/(a*sec(d*x + c) + a)^2, x)
```

3.383.8 Giac [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+a\sec(c+dx))^2}\,dx = \int \frac{\sqrt{\cos(dx+c)}}{(a\sec(dx+c)+a)^2}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate(sqrt(cos(d*x + c))/(a*sec(d*x + c) + a)^2, x)
```

3.383.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+a\sec(c+dx))^2}\,dx = \int \frac{\sqrt{\cos(c+d\,x)}}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^2}\,dx$$

input
```
int(cos(c + d*x)^(1/2)/(a + a/cos(c + d*x))^2,x)
```

output
```
int(cos(c + d*x)^(1/2)/(a + a/cos(c + d*x))^2, x)
```

3.384 $\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^2}\,dx$

3.384.1 Optimal result 2827
3.384.2 Mathematica [C] (verified) 2827
3.384.3 Rubi [A] (verified) 2828
3.384.4 Maple [A] (verified) 2832
3.384.5 Fricas [C] (verification not implemented) 2833
3.384.6 Sympy [F] 2833
3.384.7 Maxima [F] 2834
3.384.8 Giac [F] 2834
3.384.9 Mupad [F(-1)] 2834

3.384.1 Optimal result

Integrand size = 23, antiderivative size = 109

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^2}\,dx = -\frac{E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}+\frac{2\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3a^2d}+\frac{\sin(c+dx)}{a^2d\sqrt{\cos(c+dx)}(1+\sec(c+dx))}-\frac{\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2}$$

output
```
-(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2
*c),2^(1/2))/a^2/d+2/3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ell
ipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a^2/d-1/3*sin(d*x+c)/d/cos(d*x+c)^(3/2)
/(a+a*sec(d*x+c))^2+sin(d*x+c)/a^2/d/(1+sec(d*x+c))/cos(d*x+c)^(1/2)
```

3.384.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.39 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.05

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^2}\,dx = -\frac{2\sqrt{\cos(c+dx)}\csc^3(c+dx)\left(-2\cos(c+dx)+\cos(2(c+dx))+\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},\cos^2(c+\right.\right.}{3a^2d}$$

input
```
Integrate[1/(Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])^2),x]
```

output
```
(-2*Sqrt[Cos[c + d*x]]*Csc[c + d*x]^3*(-2*Cos[c + d*x] + Cos[2*(c + d*x)]
+ Hypergeometric2F1[1/4, 1/2, 5/4, Cos[c + d*x]^2]*(Sin[c + d*x]^2)^(3/2)
+ 2*Cos[c + d*x]*Hypergeometric2F1[3/4, 5/2, 7/4, Cos[c + d*x]^2]*(Sin[c +
 d*x]^2)^(3/2)))/(3*a^2*d)
```

3.384.3 Rubi [A] (verified)

Time = 0.97 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.65, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.652$, Rules used = {3042, 4752, 3042, 4304, 27, 3042, 4507, 25, 3042, 4274, 3042, 4258, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a\sec(c+dx)+a)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^2}dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sqrt{\sec(c+dx)}}{(\sec(c+dx)a+a)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}dx$$

$$\Big\downarrow \text{4304}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\int -\frac{\sqrt{\sec(c+dx)}(5a-a\sec(c+dx))}{2(\sec(c+dx)a+a)}dx}{3a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}\right)$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{\sqrt{\sec(c+dx)}(5a-a\sec(c+dx))}{\sec(c+dx)a+a}dx}{6a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(5a-a\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{6a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}\right)$$

$$\Big\downarrow \text{4507}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\int -\frac{3a^2-2a^2\sec(c+dx)}{\sqrt{\sec(c+dx)}}dx}{a^2}+\frac{6\sin(c+dx)\sqrt{\sec(c+dx)}}{d(\sec(c+dx)+1)}}{6a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}\right)$$

$$\Big\downarrow \text{25}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{6\sin(c+dx)\sqrt{\sec(c+dx)}}{d(\sec(c+dx)+1)}-\frac{\int \frac{3a^2-2a^2\sec(c+dx)}{\sqrt{\sec(c+dx)}}dx}{a^2}}{6a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{6\sin(c+dx)\sqrt{\sec(c+dx)}}{d(\sec(c+dx)+1)}-\frac{\int \frac{3a^2-2a^2\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2}}{6a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}\right)$$

$$\Big\downarrow \text{4274}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{6\sin(c+dx)\sqrt{\sec(c+dx)}}{d(\sec(c+dx)+1)}-\frac{3a^2\int \frac{1}{\sqrt{\sec(c+dx)}}dx-2a^2\int \sqrt{\sec(c+dx)}dx}{a^2}}{6a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{6\sin(c+dx)\sqrt{\sec(c+dx)}}{d(\sec(c+dx)+1)}-\frac{3a^2\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-2a^2\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}}{6a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c}{3d(a\sec(c+dx)}\right.$$

$$\Big\downarrow \text{4258}$$

3.384. $\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^2}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{6\sin(c+dx)\sqrt{\sec(c+dx)}}{d(\sec(c+dx)+1)}-\frac{3a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\cos(c+dx)}dx-2a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int}{a^2}}{6a^2}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{6\sin(c+dx)\sqrt{\sec(c+dx)}}{d(\sec(c+dx)+1)}-\frac{3a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx-2a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+d}}{a^2}}{6a^2}\right.$$

$\downarrow$ 3119

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{6\sin(c+dx)\sqrt{\sec(c+dx)}}{d(\sec(c+dx)+1)}-\frac{\frac{6a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-2a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{}{\sqrt{\sin(}}}{a^2}}{6a^2}\right.$$

$\downarrow$ 3120

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{6\sin(c+dx)\sqrt{\sec(c+dx)}}{d(\sec(c+dx)+1)}-\frac{\frac{6a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-\frac{4a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c\right.}{d}}{a^2}}{6a^2}\right.$$

input
```
Int[1/(Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])^2),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(-1/3*(Sec[c + d*x]^(3/2)*Sin[c + d*
x])/(d*(a + a*Sec[c + d*x])^2) + (-(((6*a^2*Sqrt[Cos[c + d*x]]*EllipticE[(
c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d - (4*a^2*Sqrt[Cos[c + d*x]]*EllipticF
[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d)/a^2) + (6*Sqrt[Sec[c + d*x]]*Sin[c
 + d*x])/(d*(1 + Sec[c + d*x])))/(6*a^2))
```

3.384. $\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^2}\,dx$

3.384.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4304
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-Cot[e + f*x])*(a + b*Csc[e + f*x])^m*((d*Csc
[e + f*x])^n/(f*(2*m + 1))), x] + Simp[1/(a^2*(2*m + 1))   Int[(a + b*Csc[e
 + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e
+ f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ
[m, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

3.384. $\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^2}\,dx$

rule 4507
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[d*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(a*f*(
2*m + 1))), x] - Simp[1/(a*b*(2*m + 1))   Int[(a + b*Csc[e + f*x])^(m + 1)*
(d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
 - n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f,
A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && G
tQ[n, 0]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

3.384.4 Maple [A] (verified)

Time = 6.33 (sec) , antiderivative size = 257, normalized size of antiderivative = 2.36

method	result
default	$-\frac{\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(12\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^6+4\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1}\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\cos}{6a^2\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\,\mathrm{co}}$

input
```
int(1/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
-1/6*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(12*cos(1/2*d
*x+1/2*c)^6+4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/
2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^3+6*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*cos(1/2*d*x+1/2*c)^3*
EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-20*cos(1/2*d*x+1/2*c)^4+9*cos(1/2*d*
x+1/2*c)^2-1)/a^2/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/cos
(1/2*d*x+1/2*c)^3/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

3.384.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 268, normalized size of antiderivative = 2.46

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^2}\,dx = \frac{2\left(3\cos\left(dx+c\right)+2\right)\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)-2\left(i\sqrt{2}\cos\left(dx+c\right)^2+2i\sqrt{2}\cos\left(dx+c\right)+i\sqrt{2}\right)\text{weier}}{}$$

input
```
integrate(1/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/6*(2*(3*cos(d*x + c) + 2)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*(I*sqrt(2)
*cos(d*x + c)^2 + 2*I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierstrassPInvers
e(-4, 0, cos(d*x + c) + I*sin(d*x + c)) - 2*(-I*sqrt(2)*cos(d*x + c)^2 - 2
*I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassPInverse(-4, 0, cos(d*x +
c) - I*sin(d*x + c)) - 3*(I*sqrt(2)*cos(d*x + c)^2 + 2*I*sqrt(2)*cos(d*x +
 c) + I*sqrt(2))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x
 + c) + I*sin(d*x + c))) - 3*(-I*sqrt(2)*cos(d*x + c)^2 - 2*I*sqrt(2)*cos(
d*x + c) - I*sqrt(2))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, co
s(d*x + c) - I*sin(d*x + c))))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c
) + a^2*d)
```

3.384.6 Sympy [F]

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^2}\,dx = \frac{\int \frac{1}{\sqrt{\cos\left(c+dx\right)}\sec^2\left(c+dx\right)+2\sqrt{\cos\left(c+dx\right)}\sec\left(c+dx\right)+\sqrt{\cos\left(c+dx\right)}}\,dx}{a^2}$$

input
```
integrate(1/cos(d*x+c)**(1/2)/(a+a*sec(d*x+c))**2,x)
```

output
```
Integral(1/(sqrt(cos(c + d*x))*sec(c + d*x)**2 + 2*sqrt(cos(c + d*x))*sec(
c + d*x) + sqrt(cos(c + d*x))), x)/a**2
```

3.384.7 Maxima [F]

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^2}\,dx = \int \frac{1}{(a\sec(dx+c)+a)^2\sqrt{\cos(dx+c)}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
integrate(1/((a*sec(d*x + c) + a)^2*sqrt(cos(d*x + c))), x)
```

3.384.8 Giac [F]

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^2}\,dx = \int \frac{1}{(a\sec(dx+c)+a)^2\sqrt{\cos(dx+c)}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate(1/((a*sec(d*x + c) + a)^2*sqrt(cos(d*x + c))), x)
```

3.384.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^2}\,dx = \int \frac{1}{\sqrt{\cos(c+d\,x)}\left(a+\frac{a}{\cos(c+d\,x)}\right)^2}\,dx$$

input
```
int(1/(cos(c + d*x)^(1/2)*(a + a/cos(c + d*x))^2),x)
```

output
```
int(1/(cos(c + d*x)^(1/2)*(a + a/cos(c + d*x))^2), x)
```

3.385 $\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx$

3.385.1 Optimal result 2835
3.385.2 Mathematica [C] (verified) 2835
3.385.3 Rubi [A] (verified) 2836
3.385.4 Maple [B] (verified) 2838
3.385.5 Fricas [C] (verification not implemented) 2839
3.385.6 Sympy [F] 2839
3.385.7 Maxima [F] 2840
3.385.8 Giac [F] 2840
3.385.9 Mupad [F(-1)] 2840

3.385.1 Optimal result

Integrand size = 23, antiderivative size = 57

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx = \frac{\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3a^2d} + \frac{\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2}$$

output
```
1/3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+
1/2*c),2^(1/2))/a^2/d+1/3*sin(d*x+c)/d/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^2
```

3.385.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.31 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.25

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx = \frac{\sqrt{\cos(c+dx)}\csc(c+dx)\left(-\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},\cos^2(c+dx)\right)\sqrt{\sin^2(c+dx)}+\tan^2\left(\frac{1}{2}(c+dx\right.\right.}{3a^2d}$$

input
```
Integrate[1/(Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^2),x]
```

output
```
(Sqrt[Cos[c + d*x]]*Csc[c + d*x]*(-(Hypergeometric2F1[1/4, 1/2, 5/4, Cos[c
 + d*x]^2]*Sqrt[Sin[c + d*x]^2]) + Tan[(c + d*x)/2]^2))/(3*a^2*d)
```

3.385.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.72, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {3042, 4752, 3042, 4302, 27, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^2}dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(\sec(c+dx)a+a)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}dx$$

$$\Big\downarrow \text{4302}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{1}{2}\sqrt{\sec(c+dx)}dx}{3a^2}+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}\right)$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \sqrt{\sec(c+dx)}dx}{6a^2}+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{6a^2}+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}\right)$$

$$\Big\downarrow \text{4258}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx}{6a^2}+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx+\frac{\pi}{2})}}dx}{6a^2}+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}\right)$$

$$\Big\downarrow \text{3120}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3a^2d}+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}\right)$$

input
```
Int[1/(Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^2),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((Sqrt[Cos[c + d*x]]*EllipticF[(c +
d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*a^2*d) + (Sec[c + d*x]^(3/2)*Sin[c + d*x
])/(3*d*(a + a*Sec[c + d*x])^2))
```

3.385.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4302
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[b*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Cs
c[e + f*x])^(n - 1)/(a*f*(2*m + 1))), x] - Simp[d/(a*b*(2*m + 1))   Int[(a
+ b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1)*(a*(n - 1) - b*(m + n)*C
sc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && L
tQ[m, -1] && LtQ[1, n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

3.385.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 187 vs. $2(77) = 154$.

Time = 5.42 (sec) , antiderivative size = 188, normalized size of antiderivative = 3.30

method	result
default	$-\frac{\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1}\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^3+2\cos(\right.}{6a^2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^3\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1}\,d}$

input
```
int(1/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
-1/6*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x
+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^3+2*cos(1/2*d*x+1/2*c)^4-3*cos(1/2*d*x
+1/2*c)^2+1)/a^2/cos(1/2*d*x+1/2*c)^3/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x
+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

3.385.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 150, normalized size of antiderivative = 2.63

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx = \frac{\left(-i\sqrt{2}\cos(dx+c)^2-2i\sqrt{2}\cos(dx+c)-i\sqrt{2}\right)\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)}{6\left(a^2d\right.}$$

input
```
integrate(1/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/6*((-I*sqrt(2)*cos(d*x + c)^2 - 2*I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*we
ierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + (I*sqrt(2)*cos(d
*x + c)^2 + 2*I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierstrassPInverse(-4,
0, cos(d*x + c) - I*sin(d*x + c)) + 2*sqrt(cos(d*x + c))*sin(d*x + c))/(a^
2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)
```

3.385.6 Sympy [F]

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx = \frac{\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)\sec^2(c+dx)+2\cos^{\frac{3}{2}}(c+dx)\sec(c+dx)+\cos^{\frac{3}{2}}(c+dx)}\,dx}{a^2}$$

input
```
integrate(1/cos(d*x+c)**(3/2)/(a+a*sec(d*x+c))**2,x)
```

output
```
Integral(1/(cos(c + d*x)**(3/2)*sec(c + d*x)**2 + 2*cos(c + d*x)**(3/2)*se
c(c + d*x) + cos(c + d*x)**(3/2)), x)/a**2
```

3.385.7 Maxima [F]

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx = \int \frac{1}{(a\sec(dx+c)+a)^2\cos(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
integrate(1/((a*sec(d*x + c) + a)^2*cos(d*x + c)^(3/2)), x)
```

3.385.8 Giac [F]

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx = \int \frac{1}{(a\sec(dx+c)+a)^2\cos(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate(1/((a*sec(d*x + c) + a)^2*cos(d*x + c)^(3/2)), x)
```

3.385.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx = \int \frac{1}{\cos(c+d\,x)^{3/2}\left(a+\frac{a}{\cos(c+d\,x)}\right)^2}\,dx$$

input
```
int(1/(cos(c + d*x)^(3/2)*(a + a/cos(c + d*x))^2),x)
```

output
```
int(1/(cos(c + d*x)^(3/2)*(a + a/cos(c + d*x))^2), x)
```

3.386 $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx$

3.386.1 Optimal result 2841
3.386.2 Mathematica [C] (verified) 2841
3.386.3 Rubi [A] (verified) 2842
3.386.4 Maple [A] (verified) 2846
3.386.5 Fricas [C] (verification not implemented) 2847
3.386.6 Sympy [F(-1)] 2847
3.386.7 Maxima [F] 2848
3.386.8 Giac [F] 2848
3.386.9 Mupad [F(-1)] 2848

3.386.1 Optimal result

Integrand size = 23, antiderivative size = 109

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx = \frac{E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}+\frac{2\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3a^2d}-\frac{\sin(c+dx)}{a^2d\sqrt{\cos(c+dx)}(1+\sec(c+dx))}-\frac{\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2}$$

output
```
(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*
c),2^(1/2))/a^2/d+2/3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elli
pticF(sin(1/2*d*x+1/2*c),2^(1/2))/a^2/d-1/3*sin(d*x+c)/d/cos(d*x+c)^(3/2)/
(a+a*sec(d*x+c))^2-sin(d*x+c)/a^2/d/(1+sec(d*x+c))/cos(d*x+c)^(1/2)
```

3.386.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 4.43 (sec) , antiderivative size = 312, normalized size of antiderivative = 2.86

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx = \frac{\cos^4\left(\frac{1}{2}(c+dx)\right)\left(-\frac{\left(7\cos\left(\frac{1}{2}(c-dx)\right)+2\cos\left(\frac{1}{2}(3c+dx)\right)+3\cos\left(\frac{1}{2}(c+3dx)\right)\right)\csc\left(\frac{c}{2}\right)\sec\left(\frac{c}{2}\right)\sec^3\left(\frac{1}{2}(c+dx)\right)}{2d\cos^{\frac{3}{2}}(c+dx)}+\frac{4i\sqrt{2}e^{-i(c+dx)}\left(3\left(1+e^{2i(c}\right.\right.}{}\right.}{}$$

input
```
Integrate[1/(Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^2),x]
```

output
```
(Cos[(c + d*x)/2]^4*(-1/2*((7*Cos[(c - d*x)/2] + 2*Cos[(3*c + d*x)/2] + 3*
Cos[(c + 3*d*x)/2])*Csc[c/2]*Sec[c/2]*Sec[(c + d*x)/2]^3)/(d*Cos[c + d*x]^
(3/2)) + ((4*I)*Sqrt[2]*(3*(1 + E^((2*I)*(c + d*x))) + 3*(-1 + E^((2*I)*c)
)*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I
)*(c + d*x))] - 2*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c
+ d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))])*Sec[c + d
*x]^2)/(d*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[(1 + E^((2*I)*(c + d*x))
)/E^(I*(c + d*x))])))/(3*a^2*(1 + Sec[c + d*x])^2)
```

3.386.3 Rubi [A] (verified)

Time = 0.96 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.65, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.652$, Rules used = {3042, 4752, 3042, 4303, 27, 3042, 4507, 25, 3042, 4274, 3042, 4258, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a\sec(c+dx)+a)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^2}dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(\sec(c+dx)a+a)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}dx$$

$$\Big\downarrow \text{4303}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\int \frac{\sqrt{\sec(c+dx)}(a-5a\sec(c+dx))}{2(\sec(c+dx)a+a)}dx}{3a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}\right)$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\int \frac{\sqrt{\sec(c+dx)}(a-5a\sec(c+dx))}{\sec(c+dx)a+a}dx}{6a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}\right)$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\int \frac{\sqrt{\csc(c+dx+\frac{\pi}{2})}(a-5a\csc(c+dx+\frac{\pi}{2}))}{\csc(c+dx+\frac{\pi}{2})a+a}dx}{6a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}\right)$$

$\downarrow$ 4507

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{\int -\frac{2\sec(c+dx)a^2+3a^2}{\sqrt{\sec(c+dx)}}dx}{a^2}+\frac{6\sin(c+dx)\sqrt{\sec(c+dx)}}{d(\sec(c+dx)+1)}}{6a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}\right)$$

$\downarrow$ 25

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{6\sin(c+dx)\sqrt{\sec(c+dx)}}{d(\sec(c+dx)+1)}-\frac{\int \frac{2\sec(c+dx)a^2+3a^2}{\sqrt{\sec(c+dx)}}dx}{a^2}}{6a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}\right)$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{6\sin(c+dx)\sqrt{\sec(c+dx)}}{d(\sec(c+dx)+1)}-\frac{\int \frac{2\csc\left(c+dx+\frac{\pi}{2}\right)a^2+3a^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2}}{6a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}\right)$$

$\downarrow$ 4274

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{6\sin(c+dx)\sqrt{\sec(c+dx)}}{d(\sec(c+dx)+1)}-\frac{3a^2\int \frac{1}{\sqrt{\sec(c+dx)}}dx+2a^2\int \sqrt{\sec(c+dx)}dx}{a^2}}{6a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}\right)$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{6\sin(c+dx)\sqrt{\sec(c+dx)}}{d(\sec(c+dx)+1)}-\frac{3a^2\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+2a^2\int \sqrt{\csc(c+dx+\frac{\pi}{2})}dx}{a^2}}{6a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}}{3d(a\sec(c+dx}\right.$$

3.386. $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^2}dx$

$$\Big\downarrow \; 4258$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{6\sin(c+dx)\sqrt{\sec(c+dx)}}{d(\sec(c+dx)+1)}-\frac{2a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx+3a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{a^2}}{6a^2}\right.$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{6\sin(c+dx)\sqrt{\sec(c+dx)}}{d(\sec(c+dx)+1)}-\frac{2a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+3a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+}}{a^2}}{6a^2}\right.$$

$$\Big\downarrow \; 3119$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{6\sin(c+dx)\sqrt{\sec(c+dx)}}{d(\sec(c+dx)+1)}-\frac{2a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{6a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{d}}{a^2}}{6a^2}\right.$$

$$\Big\downarrow \; 3120$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{6\sin(c+dx)\sqrt{\sec(c+dx)}}{d(\sec(c+dx)+1)}-\frac{\frac{4a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+\frac{6a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\right.}{d}}{a^2}}{6a^2}\right.$$

input
```
Int[1/(Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^2),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(-1/3*(Sec[c + d*x]^(3/2)*Sin[c + d*
x])/(d*(a + a*Sec[c + d*x])^2) - (-(((6*a^2*Sqrt[Cos[c + d*x]]*EllipticE[(
c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (4*a^2*Sqrt[Cos[c + d*x]]*EllipticF
[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d)/a^2) + (6*Sqrt[Sec[c + d*x]]*Sin[c
 + d*x])/(d*(1 + Sec[c + d*x])))/(6*a^2))
```

3.386. $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx$

3.386.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4303
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-d^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d
*Csc[e + f*x])^(n - 2)/(f*(2*m + 1))), x] + Simp[d^2/(a*b*(2*m + 1))   Int[
(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n
 + 2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2,
0] && LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

3.386. $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx$

rule 4507
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[d*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(a*f*(
2*m + 1))), x] - Simp[1/(a*b*(2*m + 1))   Int[(a + b*Csc[e + f*x])^(m + 1)*
(d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
 - n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f,
A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && G
tQ[n, 0]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

3.386.4 Maple [A] (verified)

Time = 5.58 (sec) , antiderivative size = 257, normalized size of antiderivative = 2.36

method	result
default	$\frac{\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(12\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^6-4\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1}\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\cos\left(\right.\right.}{6a^2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^3\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\sin\left(\right.}}$

input
```
int(1/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/6*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(12*cos(1/2*d*
x+1/2*c)^6-4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2
)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^3+6*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*cos(1/2*d*x+1/2*c)^3*E
llipticE(cos(1/2*d*x+1/2*c),2^(1/2))-16*cos(1/2*d*x+1/2*c)^4+3*cos(1/2*d*x
+1/2*c)^2+1)/a^2/cos(1/2*d*x+1/2*c)^3/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x
+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

3.386.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 268, normalized size of antiderivative = 2.46

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx =$$
$$-\frac{2\left(3\cos\left(dx+c\right)+4\right)\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)+2\left(i\sqrt{2}\cos\left(dx+c\right)^2+2i\sqrt{2}\cos\left(dx+c\right)+i\sqrt{2}\right)\mathrm{we}}{}$$

input
```
integrate(1/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
-1/6*(2*(3*cos(d*x + c) + 4)*sqrt(cos(d*x + c))*sin(d*x + c) + 2*(I*sqrt(2
)*cos(d*x + c)^2 + 2*I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierstrassPInver
se(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 2*(-I*sqrt(2)*cos(d*x + c)^2 -
2*I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassPInverse(-4, 0, cos(d*x +
 c) - I*sin(d*x + c)) + 3*(-I*sqrt(2)*cos(d*x + c)^2 - 2*I*sqrt(2)*cos(d*x
 + c) - I*sqrt(2))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d
*x + c) + I*sin(d*x + c))) + 3*(I*sqrt(2)*cos(d*x + c)^2 + 2*I*sqrt(2)*cos
(d*x + c) + I*sqrt(2))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, c
os(d*x + c) - I*sin(d*x + c))))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x +
c) + a^2*d)
```

3.386.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(1/cos(d*x+c)**(5/2)/(a+a*sec(d*x+c))**2,x)
```

output
```
Timed out
```

3.386.7 Maxima [F]

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx = \int \frac{1}{(a\sec(dx+c)+a)^2\cos(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
integrate(1/((a*sec(d*x + c) + a)^2*cos(d*x + c)^(5/2)), x)
```

3.386.8 Giac [F]

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx = \int \frac{1}{(a\sec(dx+c)+a)^2\cos(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate(1/((a*sec(d*x + c) + a)^2*cos(d*x + c)^(5/2)), x)
```

3.386.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx = \int \frac{1}{\cos(c+d\,x)^{5/2}\left(a+\frac{a}{\cos(c+d\,x)}\right)^2}\,dx$$

input
```
int(1/(cos(c + d*x)^(5/2)*(a + a/cos(c + d*x))^2),x)
```

output
```
int(1/(cos(c + d*x)^(5/2)*(a + a/cos(c + d*x))^2), x)
```

3.387 $\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx$

3.387.1 Optimal result . . . 2849
3.387.2 Mathematica [C] (verified) . . . 2850
3.387.3 Rubi [A] (verified) . . . 2850
3.387.4 Maple [B] (verified) . . . 2855
3.387.5 Fricas [C] (verification not implemented) . . . 2855
3.387.6 Sympy [F(-1)] . . . 2856
3.387.7 Maxima [F(-1)] . . . 2856
3.387.8 Giac [F] . . . 2857
3.387.9 Mupad [F(-1)] . . . 2857

3.387.1 Optimal result

Integrand size = 23, antiderivative size = 136

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx = -\frac{4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d} - \frac{5\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3a^2d} + \frac{4\sin(c+dx)}{a^2d\sqrt{\cos(c+dx)}} - \frac{5\sin(c+dx)}{3a^2d\cos^{\frac{3}{2}}(c+dx)(1+\sec(c+dx))} - \frac{\sin(c+dx)}{3d\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^2}$$

output
```
-4*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1
/2*c),2^(1/2))/a^2/d-5/3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*E
llipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a^2/d-5/3*sin(d*x+c)/a^2/d/cos(d*x+c)
^(3/2)/(1+sec(d*x+c))-1/3*sin(d*x+c)/d/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^2
+4*sin(d*x+c)/a^2/d/cos(d*x+c)^(1/2)
```

3.387.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 6.49 (sec) , antiderivative size = 342, normalized size of antiderivative = 2.51

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx$$
$$= \frac{\cos^4\left(\frac{1}{2}(c+dx)\right)\left(\frac{\left(29\cos\left(\frac{1}{2}(c-dx)\right)+19\cos\left(\frac{1}{2}(3c+dx)\right)+31\cos\left(\frac{1}{2}(c+3dx)\right)+5\cos\left(\frac{1}{2}(5c+3dx)\right)+12\cos\left(\frac{1}{2}(3c+5dx)\right)\right)\csc\left(\frac{c}{2}\right)\sec\left(\frac{c}{2}\right)\sec^3}{4d\cos^{\frac{5}{2}}(c+dx)}\right.}{}$$

input
```
Integrate[1/(Cos[c + d*x]^(7/2)*(a + a*Sec[c + d*x])^2),x]
```

output
```
(Cos[(c + d*x)/2]^4*(((29*Cos[(c - d*x)/2] + 19*Cos[(3*c + d*x)/2] + 31*Co
s[(c + 3*d*x)/2] + 5*Cos[(5*c + 3*d*x)/2] + 12*Cos[(3*c + 5*d*x)/2])*Csc[c
/2]*Sec[c/2]*Sec[(c + d*x)/2]^3)/(4*d*Cos[c + d*x]^(5/2)) - ((4*I)*Sqrt[2]
*(12*(1 + E^((2*I)*(c + d*x))) + 12*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(
c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))] - 5*E^(
I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometr
ic2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))])*Sec[c + d*x]^2)/(d*E^(I*(c + d
*x))*(-1 + E^((2*I)*c))*Sqrt[(1 + E^((2*I)*(c + d*x)))/E^(I*(c + d*x))])))
/(3*a^2*(1 + Sec[c + d*x])^2)
```

3.387.3 Rubi [A] (verified)

Time = 1.12 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.49, number of steps used = 16, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.696$, Rules used = {3042, 4752, 3042, 4303, 27, 3042, 4507, 3042, 4274, 3042, 4255, 3042, 4258, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a\sec(c+dx)+a)^2}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{7/2}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^2}dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(\sec(c+dx)a+a)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}dx$$

$$\Big\downarrow \text{4303}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\int \frac{\sec^{\frac{3}{2}}(c+dx)(3a-7a\sec(c+dx))}{2(\sec(c+dx)a+a)}dx}{3a^2}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}\right)$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\int \frac{\sec^{\frac{3}{2}}(c+dx)(3a-7a\sec(c+dx))}{\sec(c+dx)a+a}dx}{6a^2}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(3a-7a\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{6a^2}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}\right)$$

$$\Big\downarrow \text{4507}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{\int \sqrt{\sec(c+dx)}\left(5a^2-12a^2\sec(c+dx)\right)dx}{a^2}+\frac{10\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d(\sec(c+dx)+1)}}{6a^2}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(5a^2-12a^2\csc\left(c+dx+\frac{\pi}{2}\right)\right)dx}{a^2}+\frac{10\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d(\sec(c+dx)+1)}}{6a^2}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c}{3d(a\sec(c+dx)}\right.$$

$$\Big\downarrow \text{4274}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{5a^2\int \sqrt{\sec(c+dx)}dx-12a^2\int \sec^{\frac{3}{2}}(c+dx)dx}{a^2}+\frac{10\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d(\sec(c+dx)+1)}}{6a^2}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+}{3d(a\sec(c+dx)+}\right.$$

3.387. $\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^2}dx$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{5a^2\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx-12a^2\int\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}dx}{a^2}+\frac{10\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d(\sec(c+dx)+1)}}{6a^2}-\frac{\sin(c+dx)\,\mathrm{se}}{3d(a\sec(c+}\right.$$

$$\Big\downarrow \; 4255$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{5a^2\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx-12a^2\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\int\frac{1}{\sqrt{\sec(c+dx)}}dx\right)}{a^2}+\frac{10\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d(\sec(c+dx)+1)}}{6a^2}\right.$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{5a^2\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx-12a^2\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\int\frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)}{a^2}+\frac{10\sin(c+dx)\sec^{\frac{3}{2}}(c+}{d(\sec(c+dx)+1)}}{6a^2}\right.$$

$$\Big\downarrow \; 4258$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{5a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx-12a^2\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\right.}{a^2}}{6a^2}\right.$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{5a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-12a^2\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx}\right.}{a^2}}{6a^2}\right.$$

$$\Big\downarrow \; 3119$$

3.387. $\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{5a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-12a^2\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E}{d}\right.}{a^2}}{6a^2}\right.$$

$$\Big\downarrow \; 3120$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{\frac{10a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}-12a^2\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\right.}{d}\right.}{a^2}}{6a^2}\right.$$

input
```
Int[1/(Cos[c + d*x]^(7/2)*(a + a*Sec[c + d*x])^2),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(-1/3*(Sec[c + d*x]^(5/2)*Sin[c + d*
x])/(d*(a + a*Sec[c + d*x])^2) - ((10*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(d*
(1 + Sec[c + d*x])) + ((10*a^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2
]*Sqrt[Sec[c + d*x]])/d - 12*a^2*((-2*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*
x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d))/a
^2)/(6*a^2))
```

3.387.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4303
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-d^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d
*Csc[e + f*x])^(n - 2)/(f*(2*m + 1))), x] + Simp[d^2/(a*b*(2*m + 1))   Int[
(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n
 + 2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2,
0] && LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

rule 4507
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[d*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(a*f*(
2*m + 1))), x] - Simp[1/(a*b*(2*m + 1))   Int[(a + b*Csc[e + f*x])^(m + 1)*
(d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
 - n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f,
A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && G
tQ[n, 0]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

3.387. $\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx$

3.387.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 404 vs. $2(176) = 352$.

Time = 7.25 (sec) , antiderivative size = 405, normalized size of antiderivative = 2.98

method	result
default	$-\frac{2\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(5\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)-12\,\mathrm{EllipticE}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right.\right.\right.}{}$

input
```
int(1/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
-1/6*(2*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*
sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(5*EllipticF(cos(1/2*d*x+
1/2*c),2^(1/2))-12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*cos(1/2*d*x+1/2*
c)*sin(1/2*d*x+1/2*c)^2-2*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(5*Elli
pticF(cos(1/2*d*x+1/2*c),2^(1/2))-12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))
)*cos(1/2*d*x+1/2*c)-48*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/
2)*sin(1/2*d*x+1/2*c)^6+86*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^
(1/2)*sin(1/2*d*x+1/2*c)^4-37*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^
2)^(1/2)*sin(1/2*d*x+1/2*c)^2)/a^2/cos(1/2*d*x+1/2*c)^3/(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c
)^2-1)^(1/2)/d
```

3.387.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 318, normalized size of antiderivative = 2.34

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx$$
$$= \frac{2\left(12\,\cos\left(dx+c\right)^2+19\,\cos\left(dx+c\right)+6\right)\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)-5\left(-i\sqrt{2}\cos\left(dx+c\right)^3-2i\sqrt{2}\cos\right.}{}$$

input
```
integrate(1/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/6*(2*(12*cos(d*x + c)^2 + 19*cos(d*x + c) + 6)*sqrt(cos(d*x + c))*sin(d*
x + c) - 5*(-I*sqrt(2)*cos(d*x + c)^3 - 2*I*sqrt(2)*cos(d*x + c)^2 - I*sqr
t(2)*cos(d*x + c))*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c
)) - 5*(I*sqrt(2)*cos(d*x + c)^3 + 2*I*sqrt(2)*cos(d*x + c)^2 + I*sqrt(2)*
cos(d*x + c))*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) -
12*(I*sqrt(2)*cos(d*x + c)^3 + 2*I*sqrt(2)*cos(d*x + c)^2 + I*sqrt(2)*cos(
d*x + c))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) +
 I*sin(d*x + c))) - 12*(-I*sqrt(2)*cos(d*x + c)^3 - 2*I*sqrt(2)*cos(d*x +
c)^2 - I*sqrt(2)*cos(d*x + c))*weierstrassZeta(-4, 0, weierstrassPInverse(
-4, 0, cos(d*x + c) - I*sin(d*x + c))))/(a^2*d*cos(d*x + c)^3 + 2*a^2*d*co
s(d*x + c)^2 + a^2*d*cos(d*x + c))
```

3.387.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(1/cos(d*x+c)**(7/2)/(a+a*sec(d*x+c))**2,x)
```

output
```
Timed out
```

3.387.7 Maxima [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(1/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
Timed out
```

3.387.8 Giac [F]

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx = \int \frac{1}{(a\sec(dx+c)+a)^2\cos(dx+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate(1/((a*sec(d*x + c) + a)^2*cos(d*x + c)^(7/2)), x)
```

3.387.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx = \int \frac{1}{\cos(c+d\,x)^{7/2}\left(a+\frac{a}{\cos(c+d\,x)}\right)^2}\,dx$$

input
```
int(1/(cos(c + d*x)^(7/2)*(a + a/cos(c + d*x))^2),x)
```

output
```
int(1/(cos(c + d*x)^(7/2)*(a + a/cos(c + d*x))^2), x)
```

3.388 $\int \frac{1}{\cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx$

3.388.1 Optimal result . . . 2858
3.388.2 Mathematica [C] (verified) . . . 2859
3.388.3 Rubi [A] (verified) . . . 2859
3.388.4 Maple [B] (verified) . . . 2864
3.388.5 Fricas [C] (verification not implemented) . . . 2865
3.388.6 Sympy [F(-1)] . . . 2865
3.388.7 Maxima [F(-1)] . . . 2866
3.388.8 Giac [F] . . . 2866
3.388.9 Mupad [F(-1)] . . . 2866

3.388.1 Optimal result

Integrand size = 23, antiderivative size = 162

$$\int \frac{1}{\cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx = \frac{7E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d} + \frac{10\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3a^2d} + \frac{10\sin(c+dx)}{3a^2d\cos^{\frac{3}{2}}(c+dx)} - \frac{7\sin(c+dx)}{a^2d\sqrt{\cos(c+dx)}} - \frac{7\sin(c+dx)}{3a^2d\cos^{\frac{5}{2}}(c+dx)(1+\sec(c+dx))} - \frac{\sin(c+dx)}{3d\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^2}$$

output
```
7*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/
2*c),2^(1/2))/a^2/d+10/3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*E
llipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a^2/d+10/3*sin(d*x+c)/a^2/d/cos(d*x+c
)^(3/2)-7/3*sin(d*x+c)/a^2/d/cos(d*x+c)^(5/2)/(1+sec(d*x+c))-1/3*sin(d*x+c
)/d/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c))^2-7*sin(d*x+c)/a^2/d/cos(d*x+c)^(1/2
)
```

3.388.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 2.46 (sec) , antiderivative size = 372, normalized size of antiderivative = 2.30

$$\int \frac{1}{\cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx$$
$$= \frac{\cos^4\left(\frac{1}{2}(c+dx)\right)\left(-\frac{(82\cos(\frac{1}{2}(c-dx))+65\cos(\frac{1}{2}(3c+dx))+68\cos(\frac{1}{2}(c+3dx))+37\cos(\frac{1}{2}(5c+3dx))+53\cos(\frac{1}{2}(3c+5dx))+10\cos(\frac{1}{2}(7c+5}{8d\cos^{\frac{7}{2}}(c+dx)}\right.}{}$$

input
```
Integrate[1/(Cos[c + d*x]^(9/2)*(a + a*Sec[c + d*x])^2),x]
```

output
```
(Cos[(c + d*x)/2]^4*(-1/8*((82*Cos[(c - d*x)/2] + 65*Cos[(3*c + d*x)/2] +
68*Cos[(c + 3*d*x)/2] + 37*Cos[(5*c + 3*d*x)/2] + 53*Cos[(3*c + 5*d*x)/2]
+ 10*Cos[(7*c + 5*d*x)/2] + 21*Cos[(5*c + 7*d*x)/2])*Csc[c/2]*Sec[c/2]*Sec
[(c + d*x)/2]^3)/(d*Cos[c + d*x]^(7/2)) + ((4*I)*Sqrt[2]*(21*(1 + E^((2*I)
*(c + d*x))) + 21*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hyperge
ometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))] - 10*E^(I*(c + d*x))*(-1
+ E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5
/4, -E^((2*I)*(c + d*x))])*Sec[c + d*x]^2)/(d*E^(I*(c + d*x))*(-1 + E^((2*
I)*c))*Sqrt[(1 + E^((2*I)*(c + d*x)))/E^(I*(c + d*x))])))/(3*a^2*(1 + Sec[
c + d*x])^2)
```

3.388.3 Rubi [A] (verified)

Time = 1.20 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.43, number of steps used = 17, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.739$, Rules used = {3042, 4752, 3042, 4303, 27, 3042, 4507, 27, 3042, 4274, 3042, 4255, 3042, 4258, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\cos^{\frac{9}{2}}(c+dx)(a\sec(c+dx)+a)^2}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{9/2}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^2}dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(\sec(c+dx)a+a)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{9/2}}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}dx$$

$$\Big\downarrow \text{4303}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\int \frac{\sec^{\frac{5}{2}}(c+dx)(5a-9a\sec(c+dx))}{2(\sec(c+dx)a+a)}dx}{3a^2}-\frac{\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}\right)$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\int \frac{\sec^{\frac{5}{2}}(c+dx)(5a-9a\sec(c+dx))}{\sec(c+dx)a+a}dx}{6a^2}-\frac{\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(5a-9a\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{6a^2}-\frac{\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}\right)$$

$$\Big\downarrow \text{4507}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\int 3\sec^{\frac{3}{2}}(c+dx)\left(7a^2-10a^2\sec(c+dx)\right)dx}{a^2}+\frac{14\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{d(\sec(c+dx)+1)}}{6a^2}-\frac{\sin(c+dx)\sec^{\frac{7}{2}}(c+dx}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{3\int \sec^{\frac{3}{2}}(c+dx)\left(7a^2-10a^2\sec(c+dx)\right)dx}{a^2}+\frac{14\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{d(\sec(c+dx)+1)}}{6a^2}-\frac{\sin(c+dx)\sec^{\frac{7}{2}}(c+dx}{3d(a\sec(c+dx)+a)^2}$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{3\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(7a^2-10a^2\csc\left(c+dx+\frac{\pi}{2}\right)\right)dx}{a^2}+\frac{14\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{d(\sec(c+dx)+1)}}{6a^2}-\frac{\sin(c+dx)\sec^{\frac{7}{2}}}{3d(a\sec(c+dx}$$

3.388. $\int \frac{1}{\cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^2}dx$

$$\Big\downarrow \; 4274$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{3\left(7a^2\int \sec^{\frac{3}{2}}(c+dx)dx-10a^2\int \sec^{\frac{5}{2}}(c+dx)dx\right)}{a^2}+\frac{14\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{d(\sec(c+dx)+1)}}{6a^2}-\frac{\sin(c+dx)\sec^{\frac{7}{2}}(c}{3d(a\sec(c+dx)}\right.$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{3\left(7a^2\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}dx-10a^2\int \csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}dx\right)}{a^2}+\frac{14\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{d(\sec(c+dx)+1)}}{6a^2}-\frac{\sin(c+d}{3d(a\sec}\right.$$

$$\Big\downarrow \; 4255$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{3\left(7a^2\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\int \frac{1}{\sqrt{\sec(c+dx)}}dx\right)-10a^2\left(\frac{1}{3}\int \sqrt{\sec(c+dx)}dx+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}\right)\right)}{a^2}}{6a^2}\right.$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{3\left(7a^2\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)-10a^2\left(\frac{1}{3}\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c}{3d}\right.\right.}{a^2}}{6a^2}\right.$$

$$\Big\downarrow \; 4258$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{3\left(7a^2\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\cos(c+dx)}dx\right)-10a^2\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(}\right.\right.}{a^2}}{6a^2}\right.$$

$$\Big\downarrow \; 3042$$

3.388. $\int \frac{1}{\cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{3\left(7a^2\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin(c+dx+\frac{\pi}{2})}dx\right)-10a^2\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\right.\right.}{a^2}}{6a^2}\right.$$

$\downarrow$ 3119

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{3\left(7a^2\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)-10a^2\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx}\right.\right.}{a^2}}{6a^2}\right.$$

$\downarrow$ 3120

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{3\left(7a^2\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)-10a^2\left(\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{2}{}\right.\right.}{a^2}}{6a^2}\right.$$

input
```
Int[1/(Cos[c + d*x]^(9/2)*(a + a*Sec[c + d*x])^2),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(-1/3*(Sec[c + d*x]^(7/2)*Sin[c + d*
x])/(d*(a + a*Sec[c + d*x])^2) - ((14*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(d*
(1 + Sec[c + d*x])) + (3*(7*a^2*((-2*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x
)/2, 2]*Sqrt[Sec[c + d*x]])/d + (2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d) - 1
0*a^2*((2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])
/(3*d) + (2*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d))))/a^2)/(6*a^2))
```

3.388.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4303
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-d^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d
*Csc[e + f*x])^(n - 2)/(f*(2*m + 1))), x] + Simp[d^2/(a*b*(2*m + 1))   Int[
(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n
 + 2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2,
0] && LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

rule 4507
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[d*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(a*f*(
2*m + 1))), x] - Simp[1/(a*b*(2*m + 1))   Int[(a + b*Csc[e + f*x])^(m + 1)*
(d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
 - n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f,
A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && G
tQ[n, 0]
```

3.388. $\int \frac{1}{\cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx$

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

3.388.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 412 vs. $2(198) = 396$.

Time = 11.27 (sec) , antiderivative size = 413, normalized size of antiderivative = 2.55

method	result
default	$-\frac{\sqrt{-\left(-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(\frac{\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}}{3\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^3}+\frac{6\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}}{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{22\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}}{3\sqrt{}}\right.}{}$

input
```
int(1/cos(d*x+c)^(9/2)/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/a^2*(1/3*(-
2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)^3+6*
(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)-22
/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(
1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),
2^(1/2))+14*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)
/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d
*x+1/2*c),2^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))+16*sin(1/2*d*x+1
/2*c)^2*cos(1/2*d*x+1/2*c)/(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c
)^2)^(1/2)-2/3*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2
*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d
*x+1/2*c)^2-1)^(1/2)/d
```

3.388.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 338, normalized size of antiderivative = 2.09

$$\int \frac{1}{\cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx = -\frac{2\left(21\cos\left(dx+c\right)^3+32\cos\left(dx+c\right)^2+8\cos\left(dx+c\right)-2\right)\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)+10\left(i\sqrt{2}\cos\left(c\right.\right.}{}$$

input
```
integrate(1/cos(d*x+c)^(9/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
-1/6*(2*(21*cos(d*x + c)^3 + 32*cos(d*x + c)^2 + 8*cos(d*x + c) - 2)*sqrt(
cos(d*x + c))*sin(d*x + c) + 10*(I*sqrt(2)*cos(d*x + c)^4 + 2*I*sqrt(2)*co
s(d*x + c)^3 + I*sqrt(2)*cos(d*x + c)^2)*weierstrassPInverse(-4, 0, cos(d*
x + c) + I*sin(d*x + c)) + 10*(-I*sqrt(2)*cos(d*x + c)^4 - 2*I*sqrt(2)*cos
(d*x + c)^3 - I*sqrt(2)*cos(d*x + c)^2)*weierstrassPInverse(-4, 0, cos(d*x
 + c) - I*sin(d*x + c)) + 21*(-I*sqrt(2)*cos(d*x + c)^4 - 2*I*sqrt(2)*cos(
d*x + c)^3 - I*sqrt(2)*cos(d*x + c)^2)*weierstrassZeta(-4, 0, weierstrassP
Inverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 21*(I*sqrt(2)*cos(d*x + c
)^4 + 2*I*sqrt(2)*cos(d*x + c)^3 + I*sqrt(2)*cos(d*x + c)^2)*weierstrassZe
ta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/(a^2
*d*cos(d*x + c)^4 + 2*a^2*d*cos(d*x + c)^3 + a^2*d*cos(d*x + c)^2)
```

3.388.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(1/cos(d*x+c)**(9/2)/(a+a*sec(d*x+c))**2,x)
```

output
```
Timed out
```

3.388.7 Maxima [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(1/cos(d*x+c)^(9/2)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
Timed out
```

3.388.8 Giac [F]

$$\int \frac{1}{\cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx = \int \frac{1}{(a\sec(dx+c)+a)^2\cos(dx+c)^{\frac{9}{2}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(9/2)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate(1/((a*sec(d*x + c) + a)^2*cos(d*x + c)^(9/2)), x)
```

3.388.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx = \int \frac{1}{\cos(c+d\,x)^{9/2}\left(a+\frac{a}{\cos(c+d\,x)}\right)^2}\,dx$$

input
```
int(1/(cos(c + d*x)^(9/2)*(a + a/cos(c + d*x))^2),x)
```

output
```
int(1/(cos(c + d*x)^(9/2)*(a + a/cos(c + d*x))^2), x)
```

3.389 $\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

3.389.1 Optimal result 2867
3.389.2 Mathematica [C] (verified) 2868
3.389.3 Rubi [A] (verified) 2868
3.389.4 Maple [A] (verified) 2877
3.389.5 Fricas [C] (verification not implemented) 2877
3.389.6 Sympy [F(-1)] 2878
3.389.7 Maxima [F] 2878
3.389.8 Giac [F] 2879
3.389.9 Mupad [F(-1)] 2879

3.389.1 Optimal result

Integrand size = 23, antiderivative size = 207

$$\begin{aligned}\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx = {} & \frac{231E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d} - \frac{21\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{2a^3d} \\ & - \frac{21\sqrt{\cos(c+dx)}\sin(c+dx)}{2a^3d} + \frac{77\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{10a^3d} \\ & - \frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{4\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5ad(a+a\sec(c+dx))^2} \\ & - \frac{63\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{10d\left(a^3+a^3\sec(c+dx)\right)}\end{aligned}$$

output
```
231/10*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d
*x+1/2*c),2^(1/2))/a^3/d-21/2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2
*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a^3/d+77/10*cos(d*x+c)^(3/2)*sin
(d*x+c)/a^3/d-1/5*cos(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^3-4/5*cos
(d*x+c)^(3/2)*sin(d*x+c)/a/d/(a+a*sec(d*x+c))^2-63/10*cos(d*x+c)^(3/2)*sin
(d*x+c)/d/(a^3+a^3*sec(d*x+c))-21/2*sin(d*x+c)*cos(d*x+c)^(1/2)/a^3/d
```

3.389.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 2.22 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.85

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx$$
$$= \frac{\sqrt{\cos(c+dx)}\csc(c+dx)\left(-((614+2995\cos(c+dx)-766\cos(2(c+dx))-1139\cos(3(c+dx))+290\right.}{}$$

input
```
Integrate[Cos[c + d*x]^(5/2)/(a + a*Sec[c + d*x])^3,x]
```

output
```
(Sqrt[Cos[c + d*x]]*Csc[c + d*x]*(-((614 + 2995*Cos[c + d*x] - 766*Cos[2*(
c + d*x)] - 1139*Cos[3*(c + d*x)] + 290*Cos[4*(c + d*x)] + 127*Cos[5*(c +
d*x)] - 10*Cos[6*(c + d*x)] + Cos[7*(c + d*x)])*Csc[c + d*x]^4) + 1680*Hyp
ergeometric2F1[1/4, 1/2, 5/4, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2] + 7040*
Cos[c + d*x]*Hypergeometric2F1[3/4, 7/2, 7/4, Cos[c + d*x]^2]*Sqrt[Sin[c +
 d*x]^2]))/(160*a^3*d)
```

3.389.3 Rubi [A] (verified)

Time = 1.52 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.39, number of steps used = 20, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.870$, Rules used = {3042, 4752, 3042, 4304, 27, 3042, 4508, 27, 3042, 4508, 27, 3042, 4274, 3042, 4256, 3042, 4258, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a\sec(c+dx)+a)^3}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}dx$$
$$\Big\downarrow \; 4752$$
$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(\sec(c+dx)a+a)^3}dx$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^3}dx$$

$$\Big\downarrow \; 4304$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\int -\frac{3(5a-3a\sec(c+dx))}{2\sec^{\frac{5}{2}}(c+dx)(\sec(c+dx)a+a)^2}dx}{5a^2}-\frac{\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^3}\right)$$

$$\Big\downarrow \; 27$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{3\int \frac{5a-3a\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)(\sec(c+dx)a+a)^2}dx}{10a^2}-\frac{\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^3}\right)$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{3\int \frac{5a-3a\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}dx}{10a^2}-\frac{\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^3}\right)$$

$$\Big\downarrow \; 4508$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{3\left(\frac{\int \frac{7\left(5a^2-4a^2\sec(c+dx)\right)}{\sec^{\frac{5}{2}}(c+dx)(\sec(c+dx)a+a)}dx}{3a^2}-\frac{8a\sin(c+dx)}{3d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^2}\right)}{10a^2}-\frac{\sin(c+dx}{5d\sec^{\frac{3}{2}}(c+dx)(a\sec(}\right.$$

$$\Big\downarrow \; 27$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{3\left(\frac{7\int \frac{5a^2-4a^2\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)(\sec(c+dx)a+a)}dx}{3a^2}-\frac{8a\sin(c+dx)}{3d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^2}\right)}{10a^2}-\frac{\sin(c+d}{5d\sec^{\frac{3}{2}}(c+dx)(a\sec}\right.$$

$$\Big\downarrow \; 3042$$

3.389. $\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{3\left(\frac{7\int\frac{5a^2-4a^2\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)}dx}{3a^2}-\frac{8a\sin(c+dx)}{3d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^2}\right)}{10a^2}-\frac{s}{5d\sec^{\frac{3}{2}}(c+d}\right.$$

$\downarrow$ 4508

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{3\left(\frac{7\left(\frac{\int\frac{5\left(11a^3-9a^3\sec(c+dx)\right)}{2\sec^{\frac{5}{2}}(c+dx)}dx}{a^2}-\frac{9a^2\sin(c+dx)}{d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)}\right)}{3a^2}-\frac{8a\sin(c+dx)}{3d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^2}\right)}{10a^2}-\right.$$

$\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{3\left(\frac{7\left(\frac{5\int\frac{11a^3-9a^3\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)}dx}{2a^2}-\frac{9a^2\sin(c+dx)}{d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)}\right)}{3a^2}-\frac{8a\sin(c+dx)}{3d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^2}\right)}{10a^2}-\right.$$

$\downarrow$ 3042

3.389. $\int\frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{3\left(\frac{7\left(\frac{5\int\frac{11a^3-9a^3\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx}{2a^2}-\frac{9a^2\sin(c+dx)}{d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)}\right)}{3a^2}-\frac{8a\sin(c+dx)}{3d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^2}\right)}{10a^2}\right.$$

$\downarrow$ 4274

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{3\left(\frac{7\left(\frac{5\left(11a^3\int\frac{1}{\sec^{\frac{5}{2}}(c+dx)}dx-9a^3\int\frac{1}{\sec^{\frac{3}{2}}(c+dx)}dx\right)}{2a^2}-\frac{9a^2\sin(c+dx)}{d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)}\right)}{3a^2}-\frac{8a\sin(c+}{3d\sec^{\frac{3}{2}}(c+dx)(a\,\mathrm{se}}\right.}{10a^2}\right.$$

$\downarrow$ 3042

3.389. $\int\frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{3\left(\frac{7\left(\frac{5\left(11a^3\int\frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx-9a^3\int\frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx\right)}{2a^2}-\frac{9a^2\sin(c+dx)}{d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)}\right)}{3a^2}-\frac{}{3d\sec^{\frac{3}{2}}}\right.}{10a^2}\right.$$

$\downarrow$ 4256

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{3\left(\frac{7\left(\frac{5\left(11a^3\left(\frac{3}{5}\int\frac{1}{\sqrt{\sec(c+dx)}}dx+\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}\right)-9a^3\left(\frac{1}{3}\int\sqrt{\sec(c+dx)}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)\right)}{2a^2}-\frac{9a^2\sin}{d\sec^{\frac{3}{2}}(c+dx)(}\right)}{3a^2}\right.}{10a^2}\right.$$

$\downarrow$ 3042

3.389. $\int\frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{3\left(\frac{7\left(\frac{5\left(11a^3\left(\frac{3}{5}\int\frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}\right)-9a^3\left(\frac{1}{3}\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)\right)}{2a^2}-\frac{}{d\sec^{\frac{3}{2}}}}{3a^2}}{10a^2}\right.$$

$\downarrow$ 4258

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{3\left(\frac{7\left(\frac{5\left(11a^3\left(\frac{3}{5}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\cos(c+dx)}dx+\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}\right)-9a^3\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{}{\sqrt{\cos}}\right.\right.}{2a^2}}{3a^2}}{10a^2}\right.$$

$\downarrow$ 3042

3.389. $\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{3\left(\frac{7\left(\frac{5\left(11a^3\left(\frac{3}{5}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}\right)-9a^3\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\right.}{2a^2}\right.}{3a^2}\right.}{10}\right.$$

$\downarrow$ 3119

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{3\left(\frac{7\left(\frac{5\left(11a^3\left(\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{6\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}\right)-9a^3\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{}{\sqrt{\sin}}\right.}{2a^2}\right.}{3a^2}\right.}{10a^2}\right.$$

$\downarrow$ 3120

3.389. $\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{3\left(\frac{7\left(\frac{5\left(11a^3\left(\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{6\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}\right)-9a^3\left(\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec}}{}\right.}{2a^2}\right.}{3a^2}\right.}{10a^2}\right.$$

input
```
Int[Cos[c + d*x]^(5/2)/(a + a*Sec[c + d*x])^3,x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(-1/5*Sin[c + d*x]/(d*Sec[c + d*x]^(
3/2)*(a + a*Sec[c + d*x])^3) + (3*((-8*a*Sin[c + d*x])/(3*d*Sec[c + d*x]^(
3/2)*(a + a*Sec[c + d*x])^2) + (7*((-9*a^2*Sin[c + d*x])/(d*Sec[c + d*x]^(
3/2)*(a + a*Sec[c + d*x])) + (5*(11*a^3*((6*Sqrt[Cos[c + d*x]]*EllipticE[(
c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d) + (2*Sin[c + d*x])/(5*d*Sec[c + d
*x]^(3/2))) - 9*a^3*((2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[
Sec[c + d*x]])/(3*d) + (2*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])))/(2*a^2
)))/(3*a^2)))/(10*a^2))
```

3.389.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4304
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-Cot[e + f*x])*(a + b*Csc[e + f*x])^m*((d*Csc
[e + f*x])^n/(f*(2*m + 1))), x] + Simp[1/(a^2*(2*m + 1))   Int[(a + b*Csc[e
 + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e
+ f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ
[m, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

rule 4508
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
- a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
 1))), x] - Simp[1/(a^2*(2*m + 1))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Cs
c[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[
e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B
, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

3.389. $\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

3.389.4 Maple [A] (verified)

Time = 15.88 (sec) , antiderivative size = 296, normalized size of antiderivative = 1.43

method	result
default	$-\frac{\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\left(64\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{12}-288\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}-76\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{8}-210\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\cos\left(\frac{dx}{2}\right.}\right.}{20a^{3}\sqrt{-}}$

input
```
int(cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
-1/20*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(64*cos(1/2*
d*x+1/2*c)^12-288*cos(1/2*d*x+1/2*c)^10-76*cos(1/2*d*x+1/2*c)^8-210*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2
*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5-462*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*cos(1/2*d*x+1/2*c)^5*EllipticE(cos(1/2*
d*x+1/2*c),2^(1/2))+530*cos(1/2*d*x+1/2*c)^6-248*cos(1/2*d*x+1/2*c)^4+19*c
os(1/2*d*x+1/2*c)^2-1)/a^3/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^
(1/2)/cos(1/2*d*x+1/2*c)^5/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(
1/2)/d
```

3.389.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 364, normalized size of antiderivative = 1.76

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^{3}}\,dx$$
$$= \frac{2\left(4\cos\left(dx+c\right)^{4}-8\cos\left(dx+c\right)^{3}-147\cos\left(dx+c\right)^{2}-238\cos\left(dx+c\right)-105\right)\sqrt{\cos\left(dx+c\right)}\sin\left(dx+\right.}{}$$

input
```
integrate(cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/20*(2*(4*cos(d*x + c)^4 - 8*cos(d*x + c)^3 - 147*cos(d*x + c)^2 - 238*co
s(d*x + c) - 105)*sqrt(cos(d*x + c))*sin(d*x + c) - 105*(-I*sqrt(2)*cos(d*
x + c)^3 - 3*I*sqrt(2)*cos(d*x + c)^2 - 3*I*sqrt(2)*cos(d*x + c) - I*sqrt(
2))*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) - 105*(I*sqr
t(2)*cos(d*x + c)^3 + 3*I*sqrt(2)*cos(d*x + c)^2 + 3*I*sqrt(2)*cos(d*x + c
) + I*sqrt(2))*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) -
 231*(-I*sqrt(2)*cos(d*x + c)^3 - 3*I*sqrt(2)*cos(d*x + c)^2 - 3*I*sqrt(2)
*cos(d*x + c) - I*sqrt(2))*weierstrassZeta(-4, 0, weierstrassPInverse(-4,
0, cos(d*x + c) + I*sin(d*x + c))) - 231*(I*sqrt(2)*cos(d*x + c)^3 + 3*I*s
qrt(2)*cos(d*x + c)^2 + 3*I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierstrassZ
eta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/(a^
3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d
)
```

3.389.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(5/2)/(a+a*sec(d*x+c))**3,x)
```

output
```
Timed out
```

3.389.7 Maxima [F]

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \int \frac{\cos\left(dx+c\right)^{\frac{5}{2}}}{\left(a\sec\left(dx+c\right)+a\right)^3}\,dx$$

input
```
integrate(cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
integrate(cos(d*x + c)^(5/2)/(a*sec(d*x + c) + a)^3, x)
```

3.389.8 Giac [F]

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \int \frac{\cos{(dx+c)}^{\frac{5}{2}}}{(a\sec{(dx+c)}+a)^3}\,dx$$

input
```
integrate(cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate(cos(d*x + c)^(5/2)/(a*sec(d*x + c) + a)^3, x)
```

3.389.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \int \frac{\cos{(c+d\,x)}^{5/2}}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^3}\,dx$$

input
```
int(cos(c + d*x)^(5/2)/(a + a/cos(c + d*x))^3,x)
```

output
```
int(cos(c + d*x)^(5/2)/(a + a/cos(c + d*x))^3, x)
```

3.390 $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

3.390.1 Optimal result 2880
3.390.2 Mathematica [C] (verified) 2881
3.390.3 Rubi [A] (verified) 2881
3.390.4 Maple [A] (verified) 2887
3.390.5 Fricas [C] (verification not implemented) 2887
3.390.6 Sympy [F] 2888
3.390.7 Maxima [F] 2888
3.390.8 Giac [F] 2889
3.390.9 Mupad [F(-1)] 2889

3.390.1 Optimal result

Integrand size = 23, antiderivative size = 181

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{119E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}+\frac{11\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{2a^3d}+\frac{11\sqrt{\cos(c+dx)}\sin(c+dx)}{2a^3d}-\frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{5d(a+a\sec(c+dx))^3}-\frac{2\sqrt{\cos(c+dx)}\sin(c+dx)}{3ad(a+a\sec(c+dx))^2}-\frac{119\sqrt{\cos(c+dx)}\sin(c+dx)}{30d\left(a^3+a^3\sec(c+dx)\right)}$$

output
```
-119/10*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*
d*x+1/2*c),2^(1/2))/a^3/d+11/2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/
2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a^3/d+11/2*sin(d*x+c)*cos(d*x+c
)^(1/2)/a^3/d-1/5*sin(d*x+c)*cos(d*x+c)^(1/2)/d/(a+a*sec(d*x+c))^3-2/3*sin
(d*x+c)*cos(d*x+c)^(1/2)/a/d/(a+a*sec(d*x+c))^2-119/30*sin(d*x+c)*cos(d*x+
c)^(1/2)/d/(a^3+a^3*sec(d*x+c))
```

3.390.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.44 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.92

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx$$
$$= \frac{\sqrt{\cos(c+dx)}\csc(c+dx)\left((511+2260\cos(c+dx)-559\cos(2(c+dx))-910\cos(3(c+dx))+245\cos(\right.}{}$$

input
```
Integrate[Cos[c + d*x]^(3/2)/(a + a*Sec[c + d*x])^3,x]
```

output
```
(Sqrt[Cos[c + d*x]]*Csc[c + d*x]*((511 + 2260*Cos[c + d*x] - 559*Cos[2*(c
+ d*x)] - 910*Cos[3*(c + d*x)] + 245*Cos[4*(c + d*x)] + 90*Cos[5*(c + d*x)
] - 5*Cos[6*(c + d*x)])*Csc[c + d*x]^4 - 1320*Hypergeometric2F1[1/4, 1/2,
5/4, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2] - 5440*Cos[c + d*x]*Hypergeometr
ic2F1[3/4, 7/2, 7/4, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2]))/(240*a^3*d)
```

3.390.3 Rubi [A] (verified)

Time = 1.49 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.43, number of steps used = 19, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.826$, Rules used = {3042, 4752, 3042, 4304, 27, 3042, 4508, 3042, 4508, 27, 3042, 4274, 3042, 4256, 3042, 4258, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a\sec(c+dx)+a)^3}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}dx$$
$$\Big\downarrow \; 4752$$
$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(\sec(c+dx)a+a)^3}dx$$
$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^3}dx$$

$$\Big\downarrow \; 4304$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\int -\frac{13a-7a\sec(c+dx)}{2\sec^{\frac{3}{2}}(c+dx)(\sec(c+dx)a+a)^2}dx}{5a^2}-\frac{\sin(c+dx)}{5d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^3}\right)$$

$$\Big\downarrow \; 27$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{13a-7a\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(\sec(c+dx)a+a)^2}dx}{10a^2}-\frac{\sin(c+dx)}{5d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^3}\right)$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{13a-7a\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}dx}{10a^2}-\frac{\sin(c+dx)}{5d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^3}\right)$$

$$\Big\downarrow \; 4508$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\int \frac{69a^2-50a^2\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(\sec(c+dx)a+a)}dx}{3a^2}-\frac{20a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^2}}{10a^2}-\frac{\sin(c+dx)}{5d\sqrt{\sec(c+dx)}(a\sec(c+d}\right.$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\int \frac{69a^2-50a^2\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)}dx}{3a^2}-\frac{20a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^2}}{10a^2}-\frac{\sin(c+}{5d\sqrt{\sec(c+dx)}(a\,\mathrm{s}}\right.$$

$$\Big\downarrow \; 4508$$

3.390. $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{\int \frac{3\left(165a^3-119a^3\sec(c+dx)\right)}{\sec^{\frac{3}{2}}(c+dx)}dx}{2a^2}}{3a^2}-\frac{119a^2\sin(c+dx)}{d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)}}{10a^2}-\frac{20a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^2}-\frac{}{5d\sqrt{s}}\right.$$

$\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{3\int \frac{165a^3-119a^3\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)}dx}{2a^2}}{3a^2}-\frac{119a^2\sin(c+dx)}{d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)}}{10a^2}-\frac{20a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^2}-\frac{}{5d\sqrt{se}}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{3\int \frac{165a^3-119a^3\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx}{2a^2}}{3a^2}-\frac{119a^2\sin(c+dx)}{d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)}}{10a^2}-\frac{20a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^2}-\frac{}{5d\sqrt{}}\right.$$

$\downarrow$ 4274

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{3\left(165a^3\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}dx-119a^3\int \frac{1}{\sqrt{\sec(c+dx)}}dx\right)}{2a^2}}{3a^2}-\frac{119a^2\sin(c+dx)}{d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)}}{10a^2}-\frac{20a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}(a\sec(c+d}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{3\left(165a^3\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx-119a^3\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)}{2a^2}}{3a^2}-\frac{119a^2\sin(c+dx)}{d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)}}{10a^2}-\frac{20a\sin}{3d\sqrt{\sec(c+dx)}(}\right.$$

3.390. $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

$$\Big\downarrow \; 4256$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{3\left(165a^3\left(\frac{1}{3}\int\sqrt{\sec(c+dx)}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)-119a^3\int\frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)}{2a^2}-\frac{119a^2\sin(c+dx)}{d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)}}{3a^2}-\frac{}{3d}}{10a^2}\right.$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{3\left(165a^3\left(\frac{1}{3}\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)-119a^3\int\frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)}{2a^2}-\frac{119a^2\sin(c+dx)}{d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)}}{3a^2}-}{10a^2}\right.$$

$$\Big\downarrow \; 4258$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{3\left(165a^3\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)-119a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\cos(c+dx)}d}{2a^2}}{3a^2}}{10a^2}\right.$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{3\left(165a^3\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)-119a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+\right.}}{2a^2}}{3a^2}}{10a^2}\right.$$

$$\Big\downarrow \; 3119$$

3.390. $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{3\left(165a^3\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)-\frac{238a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+d\right.}{d}}{2a^2}}{3a^2}}{10a^2}\right.$$

$$\Big\downarrow \; 3120$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{3\left(165a^3\left(\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}\right)-\frac{238a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+d\right.}{d}}{2a^2}}{3a^2}}{10a^2}\right.$$

input
```
Int[Cos[c + d*x]^(3/2)/(a + a*Sec[c + d*x])^3,x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(-1/5*Sin[c + d*x]/(d*Sqrt[Sec[c + d
*x]]*(a + a*Sec[c + d*x])^3) + ((-20*a*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x
]]*(a + a*Sec[c + d*x])^2) + ((-119*a^2*Sin[c + d*x])/(d*Sqrt[Sec[c + d*x]
]*(a + a*Sec[c + d*x])) + (3*((-238*a^3*Sqrt[Cos[c + d*x]]*EllipticE[(c +
d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + 165*a^3*((2*Sqrt[Cos[c + d*x]]*Elliptic
F[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (2*Sin[c + d*x])/(3*d*Sqrt[S
ec[c + d*x]])))/(2*a^2))/(3*a^2))/(10*a^2))
```

3.390.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4304
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-Cot[e + f*x])*(a + b*Csc[e + f*x])^m*((d*Csc
[e + f*x])^n/(f*(2*m + 1))), x] + Simp[1/(a^2*(2*m + 1))   Int[(a + b*Csc[e
 + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e
+ f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ
[m, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

rule 4508
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
- a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
 1))), x] - Simp[1/(a^2*(2*m + 1))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Cs
c[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[
e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B
, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

3.390. $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

3.390.4 Maple [A] (verified)

Time = 8.74 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.56

method	result
default	$-\frac{\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\left(160\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}+468\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{8}+330\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+1}\ \text{Elliptic}\right.}{60a^{3}\sqrt{-2\sin\left(\frac{d}{2}\right.}}$

input
```
int(cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
-1/60*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(160*cos(1/2
*d*x+1/2*c)^10+468*cos(1/2*d*x+1/2*c)^8+330*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos
(1/2*d*x+1/2*c)^5+714*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^
2+1)^(1/2)*cos(1/2*d*x+1/2*c)^5*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-1058
*cos(1/2*d*x+1/2*c)^6+474*cos(1/2*d*x+1/2*c)^4-47*cos(1/2*d*x+1/2*c)^2+3)/
a^3/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c
)^5/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

3.390.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 354, normalized size of antiderivative = 1.96

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^{3}}\,dx$$
$$= \frac{2\left(20\,\cos\left(dx+c\right)^{3}+237\,\cos\left(dx+c\right)^{2}+376\,\cos\left(dx+c\right)+165\right)\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)-165\left(i\sqrt{2}\right.}{}$$

input
```
integrate(cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/60*(2*(20*cos(d*x + c)^3 + 237*cos(d*x + c)^2 + 376*cos(d*x + c) + 165)*
sqrt(cos(d*x + c))*sin(d*x + c) - 165*(I*sqrt(2)*cos(d*x + c)^3 + 3*I*sqrt
(2)*cos(d*x + c)^2 + 3*I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierstrassPInv
erse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) - 165*(-I*sqrt(2)*cos(d*x + c)^
3 - 3*I*sqrt(2)*cos(d*x + c)^2 - 3*I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*wei
erstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 357*(I*sqrt(2)*co
s(d*x + c)^3 + 3*I*sqrt(2)*cos(d*x + c)^2 + 3*I*sqrt(2)*cos(d*x + c) + I*s
qrt(2))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I
*sin(d*x + c))) - 357*(-I*sqrt(2)*cos(d*x + c)^3 - 3*I*sqrt(2)*cos(d*x + c
)^2 - 3*I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassZeta(-4, 0, weierst
rassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/(a^3*d*cos(d*x + c)^3
 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

3.390.6 Sympy [F]

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx}{a^3}$$

input
```
integrate(cos(d*x+c)**(3/2)/(a+a*sec(d*x+c))**3,x)
```

output
```
Integral(cos(c + d*x)**(3/2)/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(
c + d*x) + 1), x)/a**3
```

3.390.7 Maxima [F]

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \int \frac{\cos(dx+c)^{\frac{3}{2}}}{(a\sec(dx+c)+a)^3}\,dx$$

input
```
integrate(cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
integrate(cos(d*x + c)^(3/2)/(a*sec(d*x + c) + a)^3, x)
```

3.390.8 Giac [F]

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \int \frac{\cos{(dx+c)}^{\frac{3}{2}}}{(a\sec{(dx+c)}+a)^3}\,dx$$

input
```
integrate(cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate(cos(d*x + c)^(3/2)/(a*sec(d*x + c) + a)^3, x)
```

3.390.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \int \frac{\cos{(c+d\,x)}^{3/2}}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^3}\,dx$$

input
```
int(cos(c + d*x)^(3/2)/(a + a/cos(c + d*x))^3,x)
```

output
```
int(cos(c + d*x)^(3/2)/(a + a/cos(c + d*x))^3, x)
```

3.391 $\int \frac{\sqrt{\cos(c+dx)}}{(a+a\sec(c+dx))^3}\,dx$

3.391.1 Optimal result . . . 2890
3.391.2 Mathematica [C] (verified) . . . 2891
3.391.3 Rubi [A] (verified) . . . 2891
3.391.4 Maple [A] (verified) . . . 2896
3.391.5 Fricas [C] (verification not implemented) . . . 2896
3.391.6 Sympy [F] . . . 2897
3.391.7 Maxima [F] . . . 2897
3.391.8 Giac [F] . . . 2898
3.391.9 Mupad [F(-1)] . . . 2898

3.391.1 Optimal result

Integrand size = 23, antiderivative size = 155

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+a\sec(c+dx))^3}\,dx = \frac{49E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d} - \frac{13\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{6a^3d} - \frac{\sin(c+dx)}{5d\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^3} - \frac{8\sin(c+dx)}{15ad\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^2} - \frac{13\sin(c+dx)}{6d\sqrt{\cos(c+dx)}\,(a^3+a^3\sec(c+dx))}$$

output
```
49/10*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*
x+1/2*c),2^(1/2))/a^3/d-13/6*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*
c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a^3/d-1/5*sin(d*x+c)/d/(a+a*sec(d
*x+c))^3/cos(d*x+c)^(1/2)-8/15*sin(d*x+c)/a/d/(a+a*sec(d*x+c))^2/cos(d*x+c
)^(1/2)-13/6*sin(d*x+c)/d/(a^3+a^3*sec(d*x+c))/cos(d*x+c)^(1/2)
```

3.391.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.41 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.01

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+a\sec(c+dx))^3}\,dx$$
$$= \frac{\sqrt{\cos(c+dx)}\csc(c+dx)\left(-((241+860\cos(c+dx)-164\cos(2(c+dx))-410\cos(3(c+dx))+115\,\mathrm{co}}{}$$

input
```
Integrate[Sqrt[Cos[c + d*x]]/(a + a*Sec[c + d*x])^3,x]
```

output
```
(Sqrt[Cos[c + d*x]]*Csc[c + d*x]*(-((241 + 860*Cos[c + d*x] - 164*Cos[2*(c
 + d*x)] - 410*Cos[3*(c + d*x)] + 115*Cos[4*(c + d*x)] + 30*Cos[5*(c + d*x
)])*Csc[c + d*x]^4) + 520*Hypergeometric2F1[1/4, 1/2, 5/4, Cos[c + d*x]^2]
*Sqrt[Sin[c + d*x]^2] + 2240*Cos[c + d*x]*Hypergeometric2F1[3/4, 7/2, 7/4,
 Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2]))/(240*a^3*d)
```

3.391.3 Rubi [A] (verified)

Time = 1.26 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.49, number of steps used = 17, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.739$, Rules used = {3042, 4752, 3042, 4304, 27, 3042, 4508, 3042, 4508, 27, 3042, 4274, 3042, 4258, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\cos(c+dx)}}{(a\sec(c+dx)+a)^3}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}dx$$
$$\Big\downarrow \; 4752$$
$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sec(c+dx)}(\sec(c+dx)a+a)^3}dx$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^3}dx$$

$$\Big\downarrow \; 4304$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\int -\frac{11a-5a\sec(c+dx)}{2\sqrt{\sec(c+dx)}(\sec(c+dx)a+a)^2}dx}{5a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{5d(a\sec(c+dx)+a)^3}\right)$$

$$\Big\downarrow \; 27$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{11a-5a\sec(c+dx)}{\sqrt{\sec(c+dx)}(\sec(c+dx)a+a)^2}dx}{10a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{5d(a\sec(c+dx)+a)^3}\right)$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{11a-5a\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}dx}{10a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{5d(a\sec(c+dx)+a)^3}\right)$$

$$\Big\downarrow \; 4508$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\int \frac{41a^2-24a^2\sec(c+dx)}{\sqrt{\sec(c+dx)}(\sec(c+dx)a+a)}dx}{3a^2}-\frac{16a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}}{10a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{5d(a\sec(c+dx)+a)^3}\right)$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\int \frac{41a^2-24a^2\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)}dx}{3a^2}-\frac{16a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}}{10a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{5d(a\sec(c+dx)+a)^3}\right.$$

$$\Big\downarrow \; 4508$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{\int \frac{147a^3-65a^3\sec(c+dx)}{2\sqrt{\sec(c+dx)}}dx}{a^2}-\frac{65a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}}{3a^2}-\frac{16a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}}{10a^2}-\frac{\sin(c+dx)\sqrt{\mathrm{se}}}{5d(a\sec(c+d}\right.$$

$$\Big\downarrow \; 27$$

3.391. $\int \frac{\sqrt{\cos(c+dx)}}{(a+a\sec(c+dx))^3}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{\int \frac{147a^3-65a^3\sec(c+dx)}{\sqrt{\sec(c+dx)}}dx}{2a^2}-\frac{65a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}}{3a^2}-\frac{16a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}}{10a^2}-\frac{\sin(c+dx)\sqrt{\mathrm{se}}}{5d(a\sec(c+d}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{\int \frac{147a^3-65a^3\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{2a^2}-\frac{65a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}}{3a^2}-\frac{16a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}}{10a^2}-\frac{\sin(c+dx)}{5d(a\sec(c}\right.$$

$\downarrow$ 4274

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{147a^3\int \frac{1}{\sqrt{\sec(c+dx)}}dx-65a^3\int \sqrt{\sec(c+dx)}dx}{2a^2}-\frac{65a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}}{3a^2}-\frac{16a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}}{10a^2}-\frac{\mathrm{si}}{5}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{147a^3\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-65a^3\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{2a^2}-\frac{65a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}}{3a^2}-\frac{16a\sin(c+dx)\sqrt{\sec(c+dx}}{3d(a\sec(c+dx)+a)^2}}{10a^2}\right.$$

$\downarrow$ 4258

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{147a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\cos(c+dx)}dx-65a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{2a^2}-\frac{65a^2\sin(c+dx)\sqrt{\sec(c}}{d(a\sec(c+dx)+a}}{3a^2}}{10a^2}\right.$$

$\downarrow$ 3042

3.391. $\int \frac{\sqrt{\cos(c+dx)}}{(a+a\sec(c+dx))^3}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{147a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx-65a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{2a^2}-\frac{65a^2\sin(c+d}{d(a\sec(}}{3a^2}}{10a^2}\right.$$

$\downarrow$ 3119

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{294a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-65a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{2a^2}-\frac{65a^2\sin(c+dx)\sqrt{s}}{d(a\sec(c+dx}}{3a^2}}{10a^2}\right.$$

$\downarrow$ 3120

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{294a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-\frac{130a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}}{2a^2}-\frac{65a^2\sin(c+dx)}{d(a\sec(c+}}{3a^2}}{10a^2}\right.$$

input
```
Int[Sqrt[Cos[c + d*x]]/(a + a*Sec[c + d*x])^3,x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(-1/5*(Sqrt[Sec[c + d*x]]*Sin[c + d*
x])/(d*(a + a*Sec[c + d*x])^3) + ((-16*a*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/
(3*d*(a + a*Sec[c + d*x])^2) + (((294*a^3*Sqrt[Cos[c + d*x]]*EllipticE[(c
+ d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d - (130*a^3*Sqrt[Cos[c + d*x]]*EllipticF
[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d)/(2*a^2) - (65*a^2*Sqrt[Sec[c + d*x
]]*Sin[c + d*x])/(d*(a + a*Sec[c + d*x])))/(3*a^2))/(10*a^2))
```

3.391. $\int \frac{\sqrt{\cos(c+dx)}}{(a+a\sec(c+dx))^3}\,dx$

3.391.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4304
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-Cot[e + f*x])*(a + b*Csc[e + f*x])^m*((d*Csc
[e + f*x])^n/(f*(2*m + 1))), x] + Simp[1/(a^2*(2*m + 1))   Int[(a + b*Csc[e
 + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e
+ f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ
[m, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

rule 4508
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
- a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
 1))), x] - Simp[1/(a^2*(2*m + 1))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Cs
c[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[
e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B
, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

3.391. $\int \frac{\sqrt{\cos(c+dx)}}{(a+a\sec(c+dx))^3}\,dx$

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

3.391.4 Maple [A] (verified)

Time = 9.36 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.74

method	result
default	$\frac{\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(348\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^8+130\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1}\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\mathrm{co}\right.}{60a^3\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\sin\left(\right.}}$

input
```
int(cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/60*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(348*cos(1/2*
d*x+1/2*c)^8+130*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^
(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5+294*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*cos(1/2*d*x+1/2*
c)^5*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-578*cos(1/2*d*x+1/2*c)^6+264*co
s(1/2*d*x+1/2*c)^4-37*cos(1/2*d*x+1/2*c)^2+3)/a^3/(-2*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)^5/sin(1/2*d*x+1/2*c)/(2*co
s(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

3.391.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 344, normalized size of antiderivative = 2.22

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+a\sec(c+dx))^3}\,dx = -\frac{2\left(87\cos(dx+c)^2+146\cos(dx+c)+65\right)\sqrt{\cos(dx+c)}\sin(dx+c)+65\left(-i\sqrt{2}\cos(dx+c)^3-3i\right.}{}$$

input
```
integrate(cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
-1/60*(2*(87*cos(d*x + c)^2 + 146*cos(d*x + c) + 65)*sqrt(cos(d*x + c))*si
n(d*x + c) + 65*(-I*sqrt(2)*cos(d*x + c)^3 - 3*I*sqrt(2)*cos(d*x + c)^2 -
3*I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassPInverse(-4, 0, cos(d*x +
 c) + I*sin(d*x + c)) + 65*(I*sqrt(2)*cos(d*x + c)^3 + 3*I*sqrt(2)*cos(d*x
 + c)^2 + 3*I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierstrassPInverse(-4, 0,
 cos(d*x + c) - I*sin(d*x + c)) + 147*(-I*sqrt(2)*cos(d*x + c)^3 - 3*I*sqr
t(2)*cos(d*x + c)^2 - 3*I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassZet
a(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 147*
(I*sqrt(2)*cos(d*x + c)^3 + 3*I*sqrt(2)*cos(d*x + c)^2 + 3*I*sqrt(2)*cos(d
*x + c) + I*sqrt(2))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos
(d*x + c) - I*sin(d*x + c))))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)
^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

3.391.6 Sympy [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+a\sec(c+dx))^3}\,dx = \frac{\int \frac{\sqrt{\cos(c+dx)}}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx}{a^3}$$

input
```
integrate(cos(d*x+c)**(1/2)/(a+a*sec(d*x+c))**3,x)
```

output
```
Integral(sqrt(cos(c + d*x))/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c
 + d*x) + 1), x)/a**3
```

3.391.7 Maxima [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+a\sec(c+dx))^3}\,dx = \int \frac{\sqrt{\cos(dx+c)}}{(a\sec(dx+c)+a)^3}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
integrate(sqrt(cos(d*x + c))/(a*sec(d*x + c) + a)^3, x)
```

3.391.8 Giac [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+a\sec(c+dx))^3}\,dx = \int \frac{\sqrt{\cos(dx+c)}}{(a\sec(dx+c)+a)^3}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate(sqrt(cos(d*x + c))/(a*sec(d*x + c) + a)^3, x)
```

3.391.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+a\sec(c+dx))^3}\,dx = \int \frac{\sqrt{\cos(c+d\,x)}}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^3}\,dx$$

input
```
int(cos(c + d*x)^(1/2)/(a + a/cos(c + d*x))^3,x)
```

output
```
int(cos(c + d*x)^(1/2)/(a + a/cos(c + d*x))^3, x)
```

3.392 $\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^3}\,dx$

3.392.1 Optimal result . . . 2899
3.392.2 Mathematica [C] (verified) . . . 2900
3.392.3 Rubi [A] (verified) . . . 2900
3.392.4 Maple [A] (verified) . . . 2906
3.392.5 Fricas [C] (verification not implemented) . . . 2907
3.392.6 Sympy [F] . . . 2907
3.392.7 Maxima [F] . . . 2908
3.392.8 Giac [F] . . . 2908
3.392.9 Mupad [F(-1)] . . . 2908

3.392.1 Optimal result

Integrand size = 23, antiderivative size = 155

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^3}\,dx = -\frac{9E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}+\frac{\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{2a^3d} - \frac{\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3} + \frac{2\sin(c+dx)}{5ad\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^2} + \frac{\sin(c+dx)}{2d\sqrt{\cos(c+dx)}\,(a^3+a^3\sec(c+dx))}$$

output
```
-9/10*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*
x+1/2*c),2^(1/2))/a^3/d+1/2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c
)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a^3/d-1/5*sin(d*x+c)/d/cos(d*x+c)^
(3/2)/(a+a*sec(d*x+c))^3+2/5*sin(d*x+c)/a/d/(a+a*sec(d*x+c))^2/cos(d*x+c)^
(1/2)+1/2*sin(d*x+c)/d/(a^3+a^3*sec(d*x+c))/cos(d*x+c)^(1/2)
```

3.392.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.11 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.94

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^3}\,dx$$
$$= \frac{\sqrt{\cos(c+dx)}\csc(c+dx)\left((259+120\cos(c+dx)+84\cos(2(c+dx))-280\cos(3(c+dx))+105\cos(4(\right.}{}$$

input
```
Integrate[1/(Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])^3),x]
```

output
```
(Sqrt[Cos[c + d*x]]*Csc[c + d*x]*((259 + 120*Cos[c + d*x] + 84*Cos[2*(c +
d*x)] - 280*Cos[3*(c + d*x)] + 105*Cos[4*(c + d*x)])*Csc[c + d*x]^4 - 280*
Hypergeometric2F1[1/4, 1/2, 5/4, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2] - 96
0*Cos[c + d*x]*Hypergeometric2F1[3/4, 7/2, 7/4, Cos[c + d*x]^2]*Sqrt[Sin[c
 + d*x]^2]))/(560*a^3*d)
```

3.392.3 Rubi [A] (verified)

Time = 1.30 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.49, number of steps used = 18, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.783$, Rules used = {3042, 4752, 3042, 4304, 27, 3042, 4507, 25, 3042, 4508, 27, 3042, 4274, 3042, 4258, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a\sec(c+dx)+a)^3}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}dx$$
$$\Big\downarrow \; 4752$$
$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sqrt{\sec(c+dx)}}{(\sec(c+dx)a+a)^3}dx$$
$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^3}dx$$

$\Big\downarrow$ 4304

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\int -\frac{3\sqrt{\sec(c+dx)}(3a-a\sec(c+dx))}{2(\sec(c+dx)a+a)^2}dx}{5a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}\right)$$

$\Big\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{3\int \frac{\sqrt{\sec(c+dx)}(3a-a\sec(c+dx))}{(\sec(c+dx)a+a)^2}dx}{10a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}\right)$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{3\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(3a-a\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}dx}{10a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}\right)$$

$\Big\downarrow$ 4507

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{3\left(\frac{\int -\frac{2a^2-3a^2\sec(c+dx)}{\sqrt{\sec(c+dx)}(\sec(c+dx)a+a)}dx}{3a^2}+\frac{4a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}\right)}{10a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}\right)$$

$\Big\downarrow$ 25

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{3\left(\frac{4a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}-\frac{\int \frac{2a^2-3a^2\sec(c+dx)}{\sqrt{\sec(c+dx)}(\sec(c+dx)a+a)}dx}{3a^2}\right)}{10a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}\right)$$

$\Big\downarrow$ 3042

3.392. $\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^3}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{3\left(\frac{4a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}-\frac{\int\frac{2a^2-3a^2\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)}dx}{3a^2}\right)}{10a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+}{5d(a\sec(c+dx)+a}\right.$$

$\downarrow$ 4508

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{3\left(\frac{4a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}-\frac{\frac{\int\frac{9a^3-5a^3\sec(c+dx)}{2\sqrt{\sec(c+dx)}}dx}{a^2}-\frac{5a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}}{3a^2}\right)}{10a^2}-\frac{\sin(c+dx)\sec}{5d(a\sec(c+}\right.$$

$\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{3\left(\frac{4a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}-\frac{\frac{\int\frac{9a^3-5a^3\sec(c+dx)}{\sqrt{\sec(c+dx)}}dx}{2a^2}-\frac{5a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}}{3a^2}\right)}{10a^2}-\frac{\sin(c+dx)\sec}{5d(a\sec(c+}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{3\left(\frac{4a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}-\frac{\frac{\int\frac{9a^3-5a^3\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{2a^2}-\frac{5a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}}{3a^2}\right)}{10a^2}-\frac{\sin(c+dx)}{5d(a\sec(c}\right.$$

$\downarrow$ 4274

3.392. $\int\frac{1}{\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^3}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{3\left(\frac{4a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}-\frac{\frac{9a^3\int\frac{1}{\sqrt{\sec(c+dx)}}dx-5a^3\int\sqrt{\sec(c+dx)}dx}{2a^2}-\frac{5a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}}{3a^2}\right)}{10a^2}-\frac{\mathrm{s}}{}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{3\left(\frac{4a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}-\frac{\frac{9a^3\int\frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-5a^3\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{2a^2}-\frac{5a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}}{3a^2}\right)}{10a^2}\right.$$

$\downarrow$ 4258

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{3\left(\frac{4a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}-\frac{\frac{9a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\cos(c+dx)}dx-5a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{}{\sqrt{\cos}}}{2a^2}}{3a^2}\right)}{10a^2}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{3\left(\frac{4a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}-\frac{\frac{9a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx-5a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int}{2a^2}}{3a^2}\right)}{10a^2}\right.$$

$\downarrow$ 3119

3.392. $\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^3}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{3\left(\frac{4a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}-\frac{\frac{\frac{18a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-5a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{}{\sqrt{\sin}}}{2a^2}}{3a^2}}{}\right.}{10a^2}\right.$$

$$\Big\downarrow \; 3120$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{3\left(\frac{4a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}-\frac{\frac{\frac{18a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-\frac{10a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\mathrm{Ellip}}{d}}{2a^2}}{3a^2}}{}\right.}{10a^2}\right.$$

input
```
Int[1/(Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])^3),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(-1/5*(Sec[c + d*x]^(3/2)*Sin[c + d*
x])/(d*(a + a*Sec[c + d*x])^3) + (3*((4*a*Sqrt[Sec[c + d*x]]*Sin[c + d*x])
/(3*d*(a + a*Sec[c + d*x])^2) - (((18*a^3*Sqrt[Cos[c + d*x]]*EllipticE[(c
+ d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d - (10*a^3*Sqrt[Cos[c + d*x]]*EllipticF[
(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d)/(2*a^2) - (5*a^2*Sqrt[Sec[c + d*x]]
*Sin[c + d*x])/(d*(a + a*Sec[c + d*x])))/(3*a^2)))/(10*a^2))
```

3.392.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4304
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-Cot[e + f*x])*(a + b*Csc[e + f*x])^m*((d*Csc
[e + f*x])^n/(f*(2*m + 1))), x] + Simp[1/(a^2*(2*m + 1))   Int[(a + b*Csc[e
 + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e
+ f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ
[m, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

rule 4507
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[d*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(a*f*(
2*m + 1))), x] - Simp[1/(a*b*(2*m + 1))   Int[(a + b*Csc[e + f*x])^(m + 1)*
(d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
 - n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f,
A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && G
tQ[n, 0]
```

3.392. $\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^3}\,dx$

rule 4508
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
- a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
 1))), x] - Simp[1/(a^2*(2*m + 1))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Cs
c[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[
e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B
, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

3.392.4 Maple [A] (verified)

Time = 8.44 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.74

method	result
default	$-\frac{\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(36\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^8+10\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1}\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\mathrm{co}\right.}{20a^3\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\sin\left(\right.}}$

input
```
int(1/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
-1/20*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(36*cos(1/2*
d*x+1/2*c)^8+10*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(
1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5+18*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*cos(1/2*d*x+1/2*c)
^5*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-66*cos(1/2*d*x+1/2*c)^6+38*cos(1/
2*d*x+1/2*c)^4-9*cos(1/2*d*x+1/2*c)^2+1)/a^3/(-2*sin(1/2*d*x+1/2*c)^4+sin(
1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)^5/sin(1/2*d*x+1/2*c)/(2*cos(1/2
*d*x+1/2*c)^2-1)^(1/2)/d
```

3.392.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 344, normalized size of antiderivative = 2.22

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^3}\,dx$$
$$= \frac{2\left(9\cos\left(dx+c\right)^2+12\cos\left(dx+c\right)+5\right)\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)-5\left(i\sqrt{2}\cos\left(dx+c\right)^3+3i\sqrt{2}\cos\left(d\right.\right.}{}$$

input
```
integrate(1/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/20*(2*(9*cos(d*x + c)^2 + 12*cos(d*x + c) + 5)*sqrt(cos(d*x + c))*sin(d*
x + c) - 5*(I*sqrt(2)*cos(d*x + c)^3 + 3*I*sqrt(2)*cos(d*x + c)^2 + 3*I*sq
rt(2)*cos(d*x + c) + I*sqrt(2))*weierstrassPInverse(-4, 0, cos(d*x + c) +
I*sin(d*x + c)) - 5*(-I*sqrt(2)*cos(d*x + c)^3 - 3*I*sqrt(2)*cos(d*x + c)^
2 - 3*I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassPInverse(-4, 0, cos(d
*x + c) - I*sin(d*x + c)) - 9*(I*sqrt(2)*cos(d*x + c)^3 + 3*I*sqrt(2)*cos(
d*x + c)^2 + 3*I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierstrassZeta(-4, 0,
weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 9*(-I*sqrt(2)
*cos(d*x + c)^3 - 3*I*sqrt(2)*cos(d*x + c)^2 - 3*I*sqrt(2)*cos(d*x + c) -
I*sqrt(2))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c)
- I*sin(d*x + c))))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3
*d*cos(d*x + c) + a^3*d)
```

3.392.6 Sympy [F]

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^3}\,dx$$
$$= \frac{\int \frac{1}{\sqrt{\cos\left(c+dx\right)}\sec^3\left(c+dx\right)+3\sqrt{\cos\left(c+dx\right)}\sec^2\left(c+dx\right)+3\sqrt{\cos\left(c+dx\right)}\sec\left(c+dx\right)+\sqrt{\cos\left(c+dx\right)}}\,dx}{a^3}$$

input
```
integrate(1/cos(d*x+c)**(1/2)/(a+a*sec(d*x+c))**3,x)
```

output
```
Integral(1/(sqrt(cos(c + d*x))*sec(c + d*x)**3 + 3*sqrt(cos(c + d*x))*sec(
c + d*x)**2 + 3*sqrt(cos(c + d*x))*sec(c + d*x) + sqrt(cos(c + d*x))), x)/
a**3
```

3.392.7 Maxima [F]

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^3}\,dx = \int \frac{1}{(a\sec(dx+c)+a)^3\sqrt{\cos(dx+c)}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
integrate(1/((a*sec(d*x + c) + a)^3*sqrt(cos(d*x + c))), x)
```

3.392.8 Giac [F]

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^3}\,dx = \int \frac{1}{(a\sec(dx+c)+a)^3\sqrt{\cos(dx+c)}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate(1/((a*sec(d*x + c) + a)^3*sqrt(cos(d*x + c))), x)
```

3.392.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^3}\,dx = \int \frac{1}{\sqrt{\cos(c+d\,x)}\,\left(a+\frac{a}{\cos(c+d\,x)}\right)^3}\,dx$$

input
```
int(1/(cos(c + d*x)^(1/2)*(a + a/cos(c + d*x))^3),x)
```

output
```
int(1/(cos(c + d*x)^(1/2)*(a + a/cos(c + d*x))^3), x)
```

3.393 $\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx$

3.393.1 Optimal result . . . 2909
3.393.2 Mathematica [C] (verified) . . . 2910
3.393.3 Rubi [A] (verified) . . . 2910
3.393.4 Maple [A] (verified) . . . 2915
3.393.5 Fricas [C] (verification not implemented) . . . 2916
3.393.6 Sympy [F(-1)] . . . 2917
3.393.7 Maxima [F(-1)] . . . 2917
3.393.8 Giac [F] . . . 2917
3.393.9 Mupad [F(-1)] . . . 2918

3.393.1 Optimal result

Integrand size = 23, antiderivative size = 155

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx = -\frac{E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}+\frac{\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{6a^3d}+\frac{\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3}-\frac{\sin(c+dx)}{15ad\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^2}+\frac{\sin(c+dx)}{6d\sqrt{\cos(c+dx)}\,(a^3+a^3\sec(c+dx))}$$

output
```
-1/10*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*
x+1/2*c),2^(1/2))/a^3/d+1/6*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c
)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a^3/d+1/5*sin(d*x+c)/d/cos(d*x+c)^
(3/2)/(a+a*sec(d*x+c))^3-1/15*sin(d*x+c)/a/d/(a+a*sec(d*x+c))^2/cos(d*x+c)
^(1/2)+1/6*sin(d*x+c)/d/(a^3+a^3*sec(d*x+c))/cos(d*x+c)^(1/2)
```

3.393.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.12 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.94

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx = -\frac{\sqrt{\cos(c+dx)}\csc(c+dx)\left((497-1160\cos(c+dx)+812\cos(2(c+dx))-280\cos(3(c+dx))+35\cos\right.}{}$$

input
```
Integrate[1/(Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^3),x]
```

output
```
-1/1680*(Sqrt[Cos[c + d*x]]*Csc[c + d*x]*((497 - 1160*Cos[c + d*x] + 812*C
os[2*(c + d*x)] - 280*Cos[3*(c + d*x)] + 35*Cos[4*(c + d*x)])*Csc[c + d*x]
^4 + 280*Hypergeometric2F1[1/4, 1/2, 5/4, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x
]^2] + 320*Cos[c + d*x]*Hypergeometric2F1[3/4, 7/2, 7/4, Cos[c + d*x]^2]*S
qrt[Sin[c + d*x]^2]))/(a^3*d)
```

3.393.3 Rubi [A] (verified)

Time = 1.25 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.49, number of steps used = 17, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.739$, Rules used = {3042, 4752, 3042, 4302, 27, 3042, 4507, 3042, 4508, 27, 3042, 4274, 3042, 4258, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(\sec(c+dx)a+a)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^3}dx$$

$$\Big\downarrow \; 4302$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{\sqrt{\sec(c+dx)}(3\sec(c+dx)a+a)}{2(\sec(c+dx)a+a)^2}dx}{5a^2}+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}\right)$$

$$\Big\downarrow \; 27$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{\sqrt{\sec(c+dx)}(3\sec(c+dx)a+a)}{(\sec(c+dx)a+a)^2}dx}{10a^2}+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}\right)$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(3\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}dx}{10a^2}+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}\right)$$

$$\Big\downarrow \; 4507$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\int \frac{6\sec(c+dx)a^2+a^2}{\sqrt{\sec(c+dx)}(\sec(c+dx)a+a)}dx}{3a^2}-\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}}{10a^2}+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}\right)$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\int \frac{6\csc\left(c+dx+\frac{\pi}{2}\right)a^2+a^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)}dx}{3a^2}-\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}}{10a^2}+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}\right)$$

$$\Big\downarrow \; 4508$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{\int -\frac{3a^3-5a^3\sec(c+dx)}{2\sqrt{\sec(c+dx)}}dx}{a^2}+\frac{5a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}}{3a^2}-\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}}{10a^2}+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c}{5d(a\sec(c+dx)}\right.$$

$$\Big\downarrow \; 27$$

3.393. $\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3}dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{5a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}-\frac{\int\frac{3a^3-5a^3\sec(c+dx)}{\sqrt{\sec(c+dx)}}dx}{2a^2}}{3a^2}-\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}}{10a^2}+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{5a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}-\frac{\int\frac{3a^3-5a^3\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{2a^2}}{3a^2}-\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}}{10a^2}+\frac{\sin(c+dx)\sec^{\frac{3}{2}}}{5d(a\sec(c+dx}\right.$$

$\downarrow$ 4274

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{5a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}-\frac{3a^3\int\frac{1}{\sqrt{\sec(c+dx)}}dx-5a^3\int\sqrt{\sec(c+dx)}dx}{2a^2}}{3a^2}-\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}}{10a^2}+\frac{\sin(c+}{5d(a}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{5a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}-\frac{3a^3\int\frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-5a^3\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{2a^2}}{3a^2}-\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}}{10a^2}+\right.$$

$\downarrow$ 4258

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{5a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}-\frac{3a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\cos(c+dx)}dx-5a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}d}{2a^2}}{3a^2}}{10a^2}\right.$$

$\downarrow$ 3042

3.393. $\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{5a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}-\frac{3a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx-5a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+d\right.}}}{2a^2}}{3a^2}}{10a^2}\right.$$

$\downarrow$ 3119

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{5a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}-\frac{\frac{6a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-5a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right.}}}{2a^2}}{3a^2}}{10a^2}\right.$$

$\downarrow$ 3120

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{5a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}-\frac{\frac{6a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-\frac{10a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+\right.}{d}}{2a^2}}{3a^2}}{10a^2}\right.$$

input
```
Int[1/(Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^3),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((Sec[c + d*x]^(3/2)*Sin[c + d*x])/(
5*d*(a + a*Sec[c + d*x])^3) + ((-2*a*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*d
*(a + a*Sec[c + d*x])^2) + (-1/2*((6*a^3*Sqrt[Cos[c + d*x]]*EllipticE[(c +
 d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d - (10*a^3*Sqrt[Cos[c + d*x]]*EllipticF[(
c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d)/a^2 + (5*a^2*Sqrt[Sec[c + d*x]]*Sin[
c + d*x])/(d*(a + a*Sec[c + d*x])))/(3*a^2))/(10*a^2))
```

3.393. $\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx$

3.393.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4302
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[b*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Cs
c[e + f*x])^(n - 1)/(a*f*(2*m + 1))), x] - Simp[d/(a*b*(2*m + 1))   Int[(a
+ b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1)*(a*(n - 1) - b*(m + n)*C
sc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && L
tQ[m, -1] && LtQ[1, n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

3.393. $\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx$

rule 4507
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[d*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(a*f*(
2*m + 1))), x] - Simp[1/(a*b*(2*m + 1))   Int[(a + b*Csc[e + f*x])^(m + 1)*
(d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
 - n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f,
A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && G
tQ[n, 0]
```

rule 4508
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
- a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
 1))), x] - Simp[1/(a^2*(2*m + 1))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Cs
c[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[
e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B
, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

3.393.4 Maple [A] (verified)

Time = 7.22 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.74

method	result
default	$-\frac{\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(12\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^8+10\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1}\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\mathrm{co}\right.}{60a^3\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^5\sqrt{-2\sin\left(\frac{dx}{2}\right.}}$

input
```
int(1/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
-1/60*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(12*cos(1/2*
d*x+1/2*c)^8+10*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(
1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5+6*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*cos(1/2*d*x+1/2*c)^
5*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-2*cos(1/2*d*x+1/2*c)^6-24*cos(1/2*
d*x+1/2*c)^4+17*cos(1/2*d*x+1/2*c)^2-3)/a^3/cos(1/2*d*x+1/2*c)^5/(-2*sin(1
/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*
d*x+1/2*c)^2-1)^(1/2)/d
```

3.393.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 344, normalized size of antiderivative = 2.22

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx$$
$$= \frac{2\left(3\cos\left(dx+c\right)^2+14\cos\left(dx+c\right)+5\right)\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)-5\left(i\sqrt{2}\cos\left(dx+c\right)^3+3i\sqrt{2}\cos\left(d\right.\right.}{}$$

input
```
integrate(1/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/60*(2*(3*cos(d*x + c)^2 + 14*cos(d*x + c) + 5)*sqrt(cos(d*x + c))*sin(d*
x + c) - 5*(I*sqrt(2)*cos(d*x + c)^3 + 3*I*sqrt(2)*cos(d*x + c)^2 + 3*I*sq
rt(2)*cos(d*x + c) + I*sqrt(2))*weierstrassPInverse(-4, 0, cos(d*x + c) +
I*sin(d*x + c)) - 5*(-I*sqrt(2)*cos(d*x + c)^3 - 3*I*sqrt(2)*cos(d*x + c)^
2 - 3*I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassPInverse(-4, 0, cos(d
*x + c) - I*sin(d*x + c)) - 3*(I*sqrt(2)*cos(d*x + c)^3 + 3*I*sqrt(2)*cos(
d*x + c)^2 + 3*I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierstrassZeta(-4, 0,
weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 3*(-I*sqrt(2)
*cos(d*x + c)^3 - 3*I*sqrt(2)*cos(d*x + c)^2 - 3*I*sqrt(2)*cos(d*x + c) -
I*sqrt(2))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c)
- I*sin(d*x + c))))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3
*d*cos(d*x + c) + a^3*d)
```

3.393.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(1/cos(d*x+c)**(3/2)/(a+a*sec(d*x+c))**3,x)
```

output
```
Timed out
```

3.393.7 Maxima [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(1/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
Timed out
```

3.393.8 Giac [F]

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx = \int \frac{1}{(a\sec(dx+c)+a)^3\cos(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate(1/((a*sec(d*x + c) + a)^3*cos(d*x + c)^(3/2)), x)
```

3.393.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx = \int \frac{1}{\cos\left(c+d\,x\right)^{3/2}\left(a+\frac{a}{\cos(c+d\,x)}\right)^3}\,dx$$

input
```
int(1/(cos(c + d*x)^(3/2)*(a + a/cos(c + d*x))^3),x)
```

output
```
int(1/(cos(c + d*x)^(3/2)*(a + a/cos(c + d*x))^3), x)
```

3.394 $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx$

3.394.1 Optimal result . . . 2919
3.394.2 Mathematica [C] (verified) . . . 2920
3.394.3 Rubi [A] (verified) . . . 2920
3.394.4 Maple [A] (verified) . . . 2925
3.394.5 Fricas [C] (verification not implemented) . . . 2926
3.394.6 Sympy [F(-1)] . . . 2927
3.394.7 Maxima [F(-1)] . . . 2927
3.394.8 Giac [F] . . . 2927
3.394.9 Mupad [F(-1)] . . . 2928

3.394.1 Optimal result

Integrand size = 23, antiderivative size = 155

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx = \frac{E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d} + \frac{\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{6a^3d} - \frac{\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3} - \frac{4\sin(c+dx)}{15ad\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^2} + \frac{\sin(c+dx)}{6d\sqrt{\cos(c+dx)}\,(a^3+a^3\sec(c+dx))}$$

output
```
1/10*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x
+1/2*c),2^(1/2))/a^3/d+1/6*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)
*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a^3/d-1/5*sin(d*x+c)/d/cos(d*x+c)^(
3/2)/(a+a*sec(d*x+c))^3-4/15*sin(d*x+c)/a/d/(a+a*sec(d*x+c))^2/cos(d*x+c)^
(1/2)+1/6*sin(d*x+c)/d/(a^3+a^3*sec(d*x+c))/cos(d*x+c)^(1/2)
```

3.394.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 2.39 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.90

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx = -\frac{\sqrt{\cos(c+dx)}\csc(c+dx)\left(\frac{1}{8}(-847+1440\cos(c+dx)-532\cos(2(c+dx))+35\cos(4(c+dx)))\csc^4(\right.}{}$$

input
```
Integrate[1/(Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^3),x]
```

output
```
-1/210*(Sqrt[Cos[c + d*x]]*Csc[c + d*x]*(((-847 + 1440*Cos[c + d*x] - 532*
Cos[2*(c + d*x)] + 35*Cos[4*(c + d*x)])*Csc[c + d*x]^4)/8 + 35*Hypergeomet
ric2F1[1/4, 1/2, 5/4, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2] - 40*Cos[c + d*
x]*Hypergeometric2F1[3/4, 7/2, 7/4, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2]))
/(a^3*d)
```

3.394.3 Rubi [A] (verified)

Time = 1.27 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.49, number of steps used = 18, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.783$, Rules used = {3042, 4752, 3042, 4303, 27, 3042, 4507, 25, 3042, 4508, 27, 3042, 4274, 3042, 4258, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a\sec(c+dx)+a)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(\sec(c+dx)a+a)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^3}dx$$

$\Big\downarrow$ 4303

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\int \frac{\sqrt{\sec(c+dx)}(a-7a\sec(c+dx))}{2(\sec(c+dx)a+a)^2}dx}{5a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}\right)$$

$\Big\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\int \frac{\sqrt{\sec(c+dx)}(a-7a\sec(c+dx))}{(\sec(c+dx)a+a)^2}dx}{10a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}\right)$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a-7a\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}dx}{10a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}\right)$$

$\Big\downarrow$ 4507

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{\int -\frac{9\sec(c+dx)a^2+4a^2}{\sqrt{\sec(c+dx)}(\sec(c+dx)a+a)}dx}{3a^2}+\frac{8a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}}{10a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}\right)$$

$\Big\downarrow$ 25

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{8a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}-\frac{\int \frac{9\sec(c+dx)a^2+4a^2}{\sqrt{\sec(c+dx)}(\sec(c+dx)a+a)}dx}{3a^2}}{10a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}\right)$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{8a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}-\frac{\int \frac{9\csc\left(c+dx+\frac{\pi}{2}\right)a^2+4a^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)}dx}{3a^2}}{10a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}\right)$$

$\Big\downarrow$ 4508

3.394. $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^3}dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{8a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}-\frac{\frac{\int\frac{5\sec(c+dx)a^3+3a^3}{2\sqrt{\sec(c+dx)}}dx}{a^2}+\frac{5a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}}{3a^2}}{10a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c}{5d(a\sec(c+dx)}\right.$$

$\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{8a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}-\frac{\frac{\int\frac{5\sec(c+dx)a^3+3a^3}{\sqrt{\sec(c+dx)}}dx}{2a^2}+\frac{5a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}}{3a^2}}{10a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c}{5d(a\sec(c+dx)}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{8a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}-\frac{\frac{\int\frac{5\csc\left(c+dx+\frac{\pi}{2}\right)a^3+3a^3}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{2a^2}+\frac{5a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}}{3a^2}}{10a^2}-\frac{\sin(c+dx)\sec}{5d(a\sec(c+}\right.$$

$\downarrow$ 4274

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{8a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}-\frac{\frac{3a^3\int\frac{1}{\sqrt{\sec(c+dx)}}dx+5a^3\int\sqrt{\sec(c+dx)}dx}{2a^2}+\frac{5a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}}{3a^2}}{10a^2}-\frac{\sin(}{5d(}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{8a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}-\frac{\frac{3a^3\int\frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+5a^3\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{2a^2}+\frac{5a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}}{3a^2}}{10a^2}\right.$$

$\downarrow$ 4258

3.394. $\int\frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^3}dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{8a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}-\frac{\frac{5a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx+3a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\cos(c}}{2a^2}}{3a^2}}{10a^2}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{8a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}-\frac{\frac{5a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+3a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{}}{2a^2}}{3a^2}}{10a^2}\right.$$

$\downarrow$ 3119

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{8a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}-\frac{\frac{5a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{6a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\right.}{d}}{2a^2}}{3a^2}}{10a^2}\right.$$

$\downarrow$ 3120

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{8a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}-\frac{\frac{5a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}+\frac{\frac{10a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+}{2a^2}}{3a^2}}{10a^2}\right.$$

input
```
Int[1/(Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^3),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(-1/5*(Sec[c + d*x]^(3/2)*Sin[c + d*
x])/(d*(a + a*Sec[c + d*x])^3) - ((8*a*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3
*d*(a + a*Sec[c + d*x])^2) - (((6*a^3*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*
x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (10*a^3*Sqrt[Cos[c + d*x]]*EllipticF[(c +
 d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d)/(2*a^2) + (5*a^2*Sqrt[Sec[c + d*x]]*Sin
[c + d*x])/(d*(a + a*Sec[c + d*x])))/(3*a^2))/(10*a^2))
```

3.394. $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx$

3.394.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4303
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-d^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d
*Csc[e + f*x])^(n - 2)/(f*(2*m + 1))), x] + Simp[d^2/(a*b*(2*m + 1))   Int[
(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n
 + 2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2,
0] && LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

3.394. $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx$

rule 4507
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[d*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(a*f*(
2*m + 1))), x] - Simp[1/(a*b*(2*m + 1))   Int[(a + b*Csc[e + f*x])^(m + 1)*
(d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
 - n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f,
A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && G
tQ[n, 0]
```

rule 4508
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
- a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
 1))), x] - Simp[1/(a^2*(2*m + 1))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Cs
c[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[
e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B
, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

3.394.4 Maple [A] (verified)

Time = 7.97 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.74

method	result
default	$\frac{\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(12\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^8-10\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1}\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\cos\left(\right.\right.}{60a^3\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^5\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right.}}$

input
```
int(1/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/60*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(12*cos(1/2*d
*x+1/2*c)^8-10*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1
/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5+6*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*cos(1/2*d*x+1/2*c)^5
*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-22*cos(1/2*d*x+1/2*c)^6+6*cos(1/2*d
*x+1/2*c)^4+7*cos(1/2*d*x+1/2*c)^2-3)/a^3/cos(1/2*d*x+1/2*c)^5/(-2*sin(1/2
*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*
x+1/2*c)^2-1)^(1/2)/d
```

3.394.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 344, normalized size of antiderivative = 2.22

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx = -\frac{2\left(3\cos\left(dx+c\right)^2+4\cos\left(dx+c\right)-5\right)\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)+5\left(i\sqrt{2}\cos\left(dx+c\right)^3+3i\sqrt{2}\cos\left(\right.\right.}{}$$

input
```
integrate(1/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
-1/60*(2*(3*cos(d*x + c)^2 + 4*cos(d*x + c) - 5)*sqrt(cos(d*x + c))*sin(d*
x + c) + 5*(I*sqrt(2)*cos(d*x + c)^3 + 3*I*sqrt(2)*cos(d*x + c)^2 + 3*I*sq
rt(2)*cos(d*x + c) + I*sqrt(2))*weierstrassPInverse(-4, 0, cos(d*x + c) +
I*sin(d*x + c)) + 5*(-I*sqrt(2)*cos(d*x + c)^3 - 3*I*sqrt(2)*cos(d*x + c)^
2 - 3*I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassPInverse(-4, 0, cos(d
*x + c) - I*sin(d*x + c)) + 3*(-I*sqrt(2)*cos(d*x + c)^3 - 3*I*sqrt(2)*cos
(d*x + c)^2 - 3*I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassZeta(-4, 0,
 weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 3*(I*sqrt(2)
*cos(d*x + c)^3 + 3*I*sqrt(2)*cos(d*x + c)^2 + 3*I*sqrt(2)*cos(d*x + c) +
I*sqrt(2))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c)
- I*sin(d*x + c))))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3
*d*cos(d*x + c) + a^3*d)
```

3.394.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(1/cos(d*x+c)**(5/2)/(a+a*sec(d*x+c))**3,x)
```

output
```
Timed out
```

3.394.7 Maxima [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(1/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
Timed out
```

3.394.8 Giac [F]

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx = \int \frac{1}{(a\sec(dx+c)+a)^3\cos(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate(1/((a*sec(d*x + c) + a)^3*cos(d*x + c)^(5/2)), x)
```

3.394.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx=\int \frac{1}{\cos\left(c+d\,x\right)^{5/2}\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)^3}\,dx$$

input
```
int(1/(cos(c + d*x)^(5/2)*(a + a/cos(c + d*x))^3),x)
```

output
```
int(1/(cos(c + d*x)^(5/2)*(a + a/cos(c + d*x))^3), x)
```

3.395 $\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx$

3.395.1 Optimal result . . . 2929
3.395.2 Mathematica [C] (warning: unable to verify) . . . 2930
3.395.3 Rubi [A] (verified) . . . 2931
3.395.4 Maple [A] (verified) . . . 2936
3.395.5 Fricas [C] (verification not implemented) . . . 2937
3.395.6 Sympy [F(-1)] . . . 2937
3.395.7 Maxima [F(-1)] . . . 2938
3.395.8 Giac [F] . . . 2938
3.395.9 Mupad [F(-1)] . . . 2938

3.395.1 Optimal result

Integrand size = 23, antiderivative size = 155

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx = \frac{9E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d} + \frac{\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{2a^3d} - \frac{\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^3} - \frac{2\sin(c+dx)}{5ad\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2} - \frac{9\sin(c+dx)}{10d\sqrt{\cos(c+dx)}\,(a^3+a^3\sec(c+dx))}$$

output
```
9/10*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x
+1/2*c),2^(1/2))/a^3/d+1/2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)
*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a^3/d-1/5*sin(d*x+c)/d/cos(d*x+c)^(
5/2)/(a+a*sec(d*x+c))^3-2/5*sin(d*x+c)/a/d/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c
))^2-9/10*sin(d*x+c)/d/(a^3+a^3*sec(d*x+c))/cos(d*x+c)^(1/2)
```

3.395.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 7.23 (sec) , antiderivative size = 603, normalized size of antiderivative = 3.89

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx =$$

$$-\frac{2\cos^6\left(\frac{c}{2}+\frac{dx}{2}\right)\csc\left(\frac{c}{2}\right){}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};\sin^2(dx-\arctan(\cot(c)))\right)\sec\left(\frac{c}{2}\right)\sec^3(c+dx)\sec(dx-\arctan(\cot(}{d\sqrt{1+\cot}}$$

$$+\frac{\cos^6\left(\frac{c}{2}+\frac{dx}{2}\right)\left(-\frac{36\csc(c)}{5d}-\frac{36\sec\left(\frac{c}{2}\right)\sec\left(\frac{c}{2}+\frac{dx}{2}\right)\sin\left(\frac{dx}{2}\right)}{5d}-\frac{8\sec\left(\frac{c}{2}\right)\sec^3\left(\frac{c}{2}+\frac{dx}{2}\right)\sin\left(\frac{dx}{2}\right)}{5d}-\frac{2\sec\left(\frac{c}{2}\right)\sec^5\left(\frac{c}{2}+\frac{dx}{2}\right)\sin\left(\frac{dx}{2}\right)}{5d}\right.}{\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^3}$$

$$-\frac{9\cos^6\left(\frac{c}{2}+\frac{dx}{2}\right)\csc\left(\frac{c}{2}\right)\sec\left(\frac{c}{2}\right)\sec^3(c+dx)\left(\frac{{}_2F_1\left(-\frac{1}{2},-\frac{1}{4};\frac{3}{4};\cos^2(dx+\arctan(\tan(c)))\right)\sin(dx+\arctan}{\sqrt{1-\cos(dx+\arctan(\tan(c)))}\sqrt{1+\cos(dx+\arctan(\tan(c)))}\sqrt{\cos(c)\cos(dx+\arc}}\right.}{5d(a+a\sec(c+dx)}$$

input
```
Integrate[1/(Cos[c + d*x]^(7/2)*(a + a*Sec[c + d*x])^3),x]
```

output
```
(-2*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin
[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[c + d*x]^3*Sec[d*x - ArcTan[Cot[c]]
]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin
[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*Sqrt[1 +
Cot[c]^2]*(a + a*Sec[c + d*x])^3) + (Cos[c/2 + (d*x)/2]^6*((-36*Csc[c])/(5
*d) - (36*Sec[c/2]*Sec[c/2 + (d*x)/2]*Sin[(d*x)/2])/(5*d) - (8*Sec[c/2]*Se
c[c/2 + (d*x)/2]^3*Sin[(d*x)/2])/(5*d) - (2*Sec[c/2]*Sec[c/2 + (d*x)/2]^5*
Sin[(d*x)/2])/(5*d) - (8*Sec[c/2 + (d*x)/2]^2*Tan[c/2])/(5*d) - (2*Sec[c/2
 + (d*x)/2]^4*Tan[c/2])/(5*d)))/(Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^3
) - (9*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*Sec[c/2]*Sec[c + d*x]^3*((Hypergeomet
ricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[
Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + A
rcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*
Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^
2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 +
 Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(5
*d*(a + a*Sec[c + d*x])^3)
```

3.395.3 Rubi [A] (verified)

Time = 1.27 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.49, number of steps used = 17, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.739, Rules used = {3042, 4752, 3042, 4303, 27, 3042, 4507, 3042, 4507, 27, 3042, 4274, 3042, 4258, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a\sec(c+dx)+a)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{7/2}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(\sec(c+dx)a+a)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^3}dx$$

$$\Big\downarrow \text{4303}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\int \frac{3\sec^{\frac{3}{2}}(c+dx)(a-3a\sec(c+dx))}{2(\sec(c+dx)a+a)^2}dx}{5a^2}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}\right)$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{3\int \frac{\sec^{\frac{3}{2}}(c+dx)(a-3a\sec(c+dx))}{(\sec(c+dx)a+a)^2}dx}{10a^2}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{3\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a-3a\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}dx}{10a^2}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}\right)$$

$$\Big\downarrow \text{4507}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{3\left(\frac{\int \frac{\sqrt{\sec(c+dx)}\left(2a^2-7a^2\sec(c+dx)\right)}{\sec(c+dx)a+a}dx}{3a^2}+\frac{4a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}\right)}{10a^2}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+d}{5d(a\sec(c+dx)+a)}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{3\left(\frac{\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(2a^2-7a^2\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{3a^2}+\frac{4a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}\right)}{10a^2}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}}{5d(a\sec(c+d}\right.$$

$\downarrow$ 4507

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{3\left(\frac{\frac{\int -\frac{5\sec(c+dx)a^3+9a^3}{2\sqrt{\sec(c+dx)}}dx}{a^2}+\frac{9a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}}{3a^2}+\frac{4a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}\right)}{10a^2}-\frac{\sin(c+dx)}{5d(a\sec(c}\right.$$

$\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{3\left(\frac{\frac{9a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}-\frac{\int \frac{5\sec(c+dx)a^3+9a^3}{\sqrt{\sec(c+dx)}}dx}{2a^2}}{3a^2}+\frac{4a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}\right)}{10a^2}-\frac{\sin(c+dx)s}{5d(a\sec(c-}\right.$$

$\downarrow$ 3042

3.395. $\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^3}dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{3\left(\frac{\frac{9a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}-\frac{\int\frac{5\csc\left(c+dx+\frac{\pi}{2}\right)a^3+9a^3}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{2a^2}}{3a^2}+\frac{4a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}\right)}{10a^2}-\frac{\sin(c+d}{5d(a\sec}\right.$$

$\downarrow$ 4274

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{3\left(\frac{\frac{9a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}-\frac{9a^3\int\frac{1}{\sqrt{\sec(c+dx)}}dx+5a^3\int\sqrt{\sec(c+dx)}dx}{2a^2}}{3a^2}+\frac{4a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}\right)}{10a^2}-\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{3\left(\frac{\frac{9a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}-\frac{9a^3\int\frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+5a^3\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{2a^2}}{3a^2}+\frac{4a\sin(c+dx)\sec^{\frac{3}{2}}(c+d}{3d(a\sec(c+dx)+a)^2}\right.}{10a^2}\right.$$

$\downarrow$ 4258

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{3\left(\frac{\frac{9a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}-\frac{5a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx+9a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\cos(}}{2a^2}}{3a^2}\right.}{10a^2}\right.$$

$\downarrow$ 3042

3.395. $\int\frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^3}dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{3\left(\frac{\frac{9a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}-\frac{5a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+9a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{}}{2a^2}}{3a^2}\right.}{10a^2}\right.$$

$\downarrow$ 3119

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{3\left(\frac{\frac{9a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}-\frac{5a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{18a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E(}{d}}{2a^2}}{3a^2}\right.}{10a^2}\right.$$

$\downarrow$ 3120

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{3\left(\frac{\frac{9a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}-\frac{\frac{10a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+\frac{18a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{d}}{2a^2}}{3a^2}\right.}{10a^2}\right.$$

input
```
Int[1/(Cos[c + d*x]^(7/2)*(a + a*Sec[c + d*x])^3),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(-1/5*(Sec[c + d*x]^(5/2)*Sin[c + d*
x])/(d*(a + a*Sec[c + d*x])^3) - (3*((4*a*Sec[c + d*x]^(3/2)*Sin[c + d*x])
/(3*d*(a + a*Sec[c + d*x])^2) + (-1/2*((18*a^3*Sqrt[Cos[c + d*x]]*Elliptic
E[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (10*a^3*Sqrt[Cos[c + d*x]]*Ellip
ticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d)/a^2 + (9*a^2*Sqrt[Sec[c + d*x]
]*Sin[c + d*x])/(d*(a + a*Sec[c + d*x])))/(3*a^2)))/(10*a^2))
```

3.395. $\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx$

3.395.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4303
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-d^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d
*Csc[e + f*x])^(n - 2)/(f*(2*m + 1))), x] + Simp[d^2/(a*b*(2*m + 1))   Int[
(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n
 + 2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2,
0] && LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

3.395. $\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx$

rule 4507
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[d*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(a*f*(
2*m + 1))), x] - Simp[1/(a*b*(2*m + 1))   Int[(a + b*Csc[e + f*x])^(m + 1)*
(d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
 - n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f,
A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && G
tQ[n, 0]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

3.395.4 Maple [A] (verified)

Time = 5.80 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.73

method	result
default	$\frac{\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(36\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^8-10\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1}\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\cos\left(\right.\right.}{20a^3\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^5\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right.}}$

input
```
int(1/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/20*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(36*cos(1/2*d
*x+1/2*c)^8-10*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1
/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5+18*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*cos(1/2*d*x+1/2*c)^
5*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-46*cos(1/2*d*x+1/2*c)^6+8*cos(1/2*
d*x+1/2*c)^4+cos(1/2*d*x+1/2*c)^2+1)/a^3/cos(1/2*d*x+1/2*c)^5/(-2*sin(1/2*
d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x
+1/2*c)^2-1)^(1/2)/d
```

3.395.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 344, normalized size of antiderivative = 2.22

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx = -\frac{2\left(9\cos\left(dx+c\right)^2+22\cos\left(dx+c\right)+15\right)\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)+5\left(i\sqrt{2}\cos\left(dx+c\right)^3+3i\sqrt{2}\,\mathrm{co}\right.}{}$$

input
```
integrate(1/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
-1/20*(2*(9*cos(d*x + c)^2 + 22*cos(d*x + c) + 15)*sqrt(cos(d*x + c))*sin(
d*x + c) + 5*(I*sqrt(2)*cos(d*x + c)^3 + 3*I*sqrt(2)*cos(d*x + c)^2 + 3*I*
sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierstrassPInverse(-4, 0, cos(d*x + c)
+ I*sin(d*x + c)) + 5*(-I*sqrt(2)*cos(d*x + c)^3 - 3*I*sqrt(2)*cos(d*x + c
)^2 - 3*I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassPInverse(-4, 0, cos
(d*x + c) - I*sin(d*x + c)) + 9*(-I*sqrt(2)*cos(d*x + c)^3 - 3*I*sqrt(2)*c
os(d*x + c)^2 - 3*I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassZeta(-4,
0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 9*(I*sqrt(
2)*cos(d*x + c)^3 + 3*I*sqrt(2)*cos(d*x + c)^2 + 3*I*sqrt(2)*cos(d*x + c)
+ I*sqrt(2))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c
) - I*sin(d*x + c))))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a
^3*d*cos(d*x + c) + a^3*d)
```

3.395.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(1/cos(d*x+c)**(7/2)/(a+a*sec(d*x+c))**3,x)
```

output
```
Timed out
```

3.395.7 Maxima [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(1/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
Timed out
```

3.395.8 Giac [F]

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx = \int \frac{1}{(a\sec(dx+c)+a)^3\cos(dx+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate(1/((a*sec(d*x + c) + a)^3*cos(d*x + c)^(7/2)), x)
```

3.395.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx = \int \frac{1}{\cos(c+d\,x)^{7/2}\left(a+\frac{a}{\cos(c+d\,x)}\right)^3}\,dx$$

input
```
int(1/(cos(c + d*x)^(7/2)*(a + a/cos(c + d*x))^3),x)
```

output
```
int(1/(cos(c + d*x)^(7/2)*(a + a/cos(c + d*x))^3), x)
```

3.396 $\int \frac{1}{\cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx$

3.396.1 Optimal result . . . 2939
3.396.2 Mathematica [C] (verified) . . . 2940
3.396.3 Rubi [A] (verified) . . . 2940
3.396.4 Maple [B] (verified) . . . 2945
3.396.5 Fricas [C] (verification not implemented) . . . 2946
3.396.6 Sympy [F(-1)] . . . 2947
3.396.7 Maxima [F(-1)] . . . 2947
3.396.8 Giac [F] . . . 2948
3.396.9 Mupad [F(-1)] . . . 2948

3.396.1 Optimal result

Integrand size = 23, antiderivative size = 181

$$\begin{aligned}\int \frac{1}{\cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx = &-\frac{49E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}-\frac{13\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{6a^3d}\\ &+\frac{49\sin(c+dx)}{10a^3d\sqrt{\cos(c+dx)}}\\ &-\frac{\sin(c+dx)}{5d\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^3}\\ &-\frac{8\sin(c+dx)}{15ad\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^2}\\ &-\frac{13\sin(c+dx)}{6d\cos^{\frac{3}{2}}(c+dx)\left(a^3+a^3\sec(c+dx)\right)}\end{aligned}$$

output
```
-49/10*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d
*x+1/2*c),2^(1/2))/a^3/d-13/6*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2
*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a^3/d-1/5*sin(d*x+c)/d/cos(d*x+c
)^(7/2)/(a+a*sec(d*x+c))^3-8/15*sin(d*x+c)/a/d/cos(d*x+c)^(5/2)/(a+a*sec(d
*x+c))^2-13/6*sin(d*x+c)/d/cos(d*x+c)^(3/2)/(a^3+a^3*sec(d*x+c))+49/10*sin
(d*x+c)/a^3/d/cos(d*x+c)^(1/2)
```

3.396.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 2.50 (sec) , antiderivative size = 372, normalized size of antiderivative = 2.06

$$\int \frac{1}{\cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx$$
$$= \frac{\cos^6\left(\frac{1}{2}(c+dx)\right)\left(\frac{(1284\cos(\frac{1}{2}(c-dx))+921\cos(\frac{1}{2}(3c+dx))+1243\cos(\frac{1}{2}(c+3dx))+374\cos(\frac{1}{2}(5c+3dx))+670\cos(\frac{1}{2}(3c+5dx))+65\cos(\frac{1}{2}}{16d\cos^{\frac{7}{2}}(c+dx)}\right.}{}$$

input
```
Integrate[1/(Cos[c + d*x]^(9/2)*(a + a*Sec[c + d*x])^3),x]
```

output
```
(Cos[(c + d*x)/2]^6*(((1284*Cos[(c - d*x)/2] + 921*Cos[(3*c + d*x)/2] + 12
43*Cos[(c + 3*d*x)/2] + 374*Cos[(5*c + 3*d*x)/2] + 670*Cos[(3*c + 5*d*x)/2
] + 65*Cos[(7*c + 5*d*x)/2] + 147*Cos[(5*c + 7*d*x)/2])*Csc[c/2]*Sec[c/2]*
Sec[(c + d*x)/2]^5)/(16*d*Cos[c + d*x]^(7/2)) - ((4*I)*Sqrt[2]*(147*(1 + E
^((2*I)*(c + d*x))) + 147*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]
*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))] - 65*E^(I*(c + d*
x))*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/4
, 1/2, 5/4, -E^((2*I)*(c + d*x))])*Sec[c + d*x]^3)/(d*E^(I*(c + d*x))*(-1
+ E^((2*I)*c))*Sqrt[(1 + E^((2*I)*(c + d*x)))/E^(I*(c + d*x))])))/(15*a^3*
(1 + Sec[c + d*x])^3)
```

3.396.3 Rubi [A] (verified)

Time = 1.53 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.41, number of steps used = 19, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.826$, Rules used = {3042, 4752, 3042, 4303, 27, 3042, 4507, 3042, 4507, 27, 3042, 4274, 3042, 4255, 3042, 4258, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\cos^{\frac{9}{2}}(c+dx)(a\sec(c+dx)+a)^3}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{9/2}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(\sec(c+dx)a+a)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{9/2}}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^3}dx$$

$$\Big\downarrow \text{4303}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\int \frac{\sec^{\frac{5}{2}}(c+dx)(5a-11a\sec(c+dx))}{2(\sec(c+dx)a+a)^2}dx}{5a^2}-\frac{\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}\right)$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\int \frac{\sec^{\frac{5}{2}}(c+dx)(5a-11a\sec(c+dx))}{(\sec(c+dx)a+a)^2}dx}{10a^2}-\frac{\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(5a-11a\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}dx}{10a^2}-\frac{\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}\right)$$

$$\Big\downarrow \text{4507}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{\int \frac{\sec^{\frac{3}{2}}(c+dx)\left(24a^2-41a^2\sec(c+dx)\right)}{\sec(c+dx)a+a}dx}{3a^2}+\frac{16a\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}}{10a^2}-\frac{\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(24a^2-41a^2\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{3a^2}+\frac{16a\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}}{10a^2}-\frac{\sin(c+dx)\sec^{\frac{7}{2}}(\right.$$

$$\Big\downarrow \text{4507}$$

3.396. $\int \frac{1}{\cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^3}dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{\frac{\int \frac{1}{2}\sqrt{\sec(c+dx)}\left(65a^3-147a^3\sec(c+dx)\right)dx}{a^2}+\frac{65a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d(a\sec(c+dx)+a)}}{3a^2}+\frac{16a\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}}{10a^2}-\frac{\sin}{5}\right.$$

$\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{\frac{\int \sqrt{\sec(c+dx)}\left(65a^3-147a^3\sec(c+dx)\right)dx}{2a^2}+\frac{65a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d(a\sec(c+dx)+a)}}{3a^2}+\frac{16a\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}}{10a^2}-\frac{\sin(}{5d}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{\frac{\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(65a^3-147a^3\csc\left(c+dx+\frac{\pi}{2}\right)\right)dx}{2a^2}+\frac{65a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d(a\sec(c+dx)+a)}}{3a^2}+\frac{16a\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}}{10a^2}\right.$$

$\downarrow$ 4274

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{\frac{65a^3\int \sqrt{\sec(c+dx)}dx-147a^3\int \sec^{\frac{3}{2}}(c+dx)dx}{2a^2}+\frac{65a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d(a\sec(c+dx)+a)}}{3a^2}+\frac{16a\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}}{10a^2}-\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{\frac{65a^3\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx-147a^3\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}dx}{2a^2}+\frac{65a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d(a\sec(c+dx)+a)}}{3a^2}+\frac{16a\sin(c+dx)\sec^{\frac{5}{2}}(c+}{3d(a\sec(c+dx)+a)}}{10a^2}\right.$$

$\downarrow$ 4255

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{\frac{65a^3\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx-147a^3\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\int \frac{1}{\sqrt{\sec(c+dx)}}dx\right)}{2a^2}+\frac{65a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d(a\sec(c+dx)+a)}}{3a^2}+\frac{1}{}}{10a^2}\right.$$

$\downarrow$ 3042

3.396. $\int \frac{1}{\cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{\frac{65a^3\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx-147a^3\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\int\frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)}{2a^2}+\frac{65a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d(a\sec(c+dx)+a)}}{3a^2}}{10a^2}\right.$$

$\downarrow$ 4258

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{\frac{65a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx-147a^3\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\cos(c+}\right.}{2a^2}}{3a^2}}{10a^2}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{\frac{65a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-147a^3\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin}\right.}{2a^2}}{3a^2}}{10a^2}\right.$$

$\downarrow$ 3119

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{\frac{65a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx-147a^3\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}\right.}{d}\right.}{2a^2}}{3a^2}}{10a^2}\right.$$

$\downarrow$ 3120

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{\frac{65a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d(a\sec(c+dx)+a)}+\frac{\frac{130a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}-147a^3\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}\right.}{2a^2}}{3a^2}}{10a^2}\right.$$

input
```
Int[1/(Cos[c + d*x]^(9/2)*(a + a*Sec[c + d*x])^3),x]
```

3.396. $\int\frac{1}{\cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx$

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(-1/5*(Sec[c + d*x]^(7/2)*Sin[c + d*
x])/(d*(a + a*Sec[c + d*x])^3) - ((16*a*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(
3*d*(a + a*Sec[c + d*x])^2) + ((65*a^2*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(d
*(a + a*Sec[c + d*x])) + ((130*a^3*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/
2, 2]*Sqrt[Sec[c + d*x]])/d - 147*a^3*((-2*Sqrt[Cos[c + d*x]]*EllipticE[(c
 + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/
d))/(2*a^2))/(3*a^2))/(10*a^2))
```

3.396.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

3.396. $\int \frac{1}{\cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx$

rule 4303
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-d^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d
*Csc[e + f*x])^(n - 2)/(f*(2*m + 1))), x] + Simp[d^2/(a*b*(2*m + 1))   Int[
(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n
 + 2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2,
0] && LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

rule 4507
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[d*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(a*f*(
2*m + 1))), x] - Simp[1/(a*b*(2*m + 1))   Int[(a + b*Csc[e + f*x])^(m + 1)*
(d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
 - n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f,
A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && G
tQ[n, 0]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

3.396.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 554 vs. $2(213) = 426$.

Time = 9.62 (sec) , antiderivative size = 555, normalized size of antiderivative = 3.07

method	result
default	$-\frac{-2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1}\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(65\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)-147\,\mathrm{EllipticE}\left(\cos\left(\frac{dx}{2}\right.\right.\right.}{}$

input
```
int(1/cos(d*x+c)^(9/2)/(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
-1/60*(-2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-
2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(65*EllipticF(cos(1/2*d
*x+1/2*c),2^(1/2))-147*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*cos(1/2*d*x+
1/2*c)*sin(1/2*d*x+1/2*c)^4+4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+
1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(65
*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-147*EllipticE(cos(1/2*d*x+1/2*c),2^
(1/2)))*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-2*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x
+1/2*c)^2)^(1/2)*(65*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-147*EllipticE(c
os(1/2*d*x+1/2*c),2^(1/2)))*cos(1/2*d*x+1/2*c)+588*(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^8-1634*(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^6+1488*(-2*sin(1/2*d
*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-439*(-2*sin(1
/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2)/a^3/cos(
1/2*d*x+1/2*c)^5/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(
1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

3.396.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 394, normalized size of antiderivative = 2.18

$$\int \frac{1}{\cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx$$
$$= \frac{2\left(147\cos\left(dx+c\right)^3+376\cos\left(dx+c\right)^2+295\cos\left(dx+c\right)+60\right)\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)-65\left(-i\sqrt{2}\right.}{}$$

input
```
integrate(1/cos(d*x+c)^(9/2)/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/60*(2*(147*cos(d*x + c)^3 + 376*cos(d*x + c)^2 + 295*cos(d*x + c) + 60)*
sqrt(cos(d*x + c))*sin(d*x + c) - 65*(-I*sqrt(2)*cos(d*x + c)^4 - 3*I*sqrt
(2)*cos(d*x + c)^3 - 3*I*sqrt(2)*cos(d*x + c)^2 - I*sqrt(2)*cos(d*x + c))*
weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) - 65*(I*sqrt(2)*
cos(d*x + c)^4 + 3*I*sqrt(2)*cos(d*x + c)^3 + 3*I*sqrt(2)*cos(d*x + c)^2 +
 I*sqrt(2)*cos(d*x + c))*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d
*x + c)) - 147*(I*sqrt(2)*cos(d*x + c)^4 + 3*I*sqrt(2)*cos(d*x + c)^3 + 3*
I*sqrt(2)*cos(d*x + c)^2 + I*sqrt(2)*cos(d*x + c))*weierstrassZeta(-4, 0,
weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 147*(-I*sqrt(
2)*cos(d*x + c)^4 - 3*I*sqrt(2)*cos(d*x + c)^3 - 3*I*sqrt(2)*cos(d*x + c)^
2 - I*sqrt(2)*cos(d*x + c))*weierstrassZeta(-4, 0, weierstrassPInverse(-4,
 0, cos(d*x + c) - I*sin(d*x + c))))/(a^3*d*cos(d*x + c)^4 + 3*a^3*d*cos(d
*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + a^3*d*cos(d*x + c))
```

3.396.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(1/cos(d*x+c)**(9/2)/(a+a*sec(d*x+c))**3,x)
```

output
```
Timed out
```

3.396.7 Maxima [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(1/cos(d*x+c)^(9/2)/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
Timed out
```

3.396.8 Giac [F]

$$\int \frac{1}{\cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx = \int \frac{1}{(a\sec(dx+c)+a)^3\cos(dx+c)^{\frac{9}{2}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(9/2)/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate(1/((a*sec(d*x + c) + a)^3*cos(d*x + c)^(9/2)), x)
```

3.396.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx = \int \frac{1}{\cos(c+d\,x)^{9/2}\left(a+\frac{a}{\cos(c+d\,x)}\right)^3}\,dx$$

input
```
int(1/(cos(c + d*x)^(9/2)*(a + a/cos(c + d*x))^3),x)
```

output
```
int(1/(cos(c + d*x)^(9/2)*(a + a/cos(c + d*x))^3), x)
```

3.397 $\int \frac{1}{\cos^{\frac{11}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx$

3.397.1 Optimal result . . . 2949
3.397.2 Mathematica [C] (verified) . . . 2950
3.397.3 Rubi [A] (verified) . . . 2950
3.397.4 Maple [A] (verified) . . . 2956
3.397.5 Fricas [C] (verification not implemented) . . . 2956
3.397.6 Sympy [F(-1)] . . . 2957
3.397.7 Maxima [F(-1)] . . . 2957
3.397.8 Giac [F] . . . 2958
3.397.9 Mupad [F(-1)] . . . 2958

3.397.1 Optimal result

Integrand size = 23, antiderivative size = 207

$$\begin{aligned}\int \frac{1}{\cos^{\frac{11}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx &= \frac{119E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}+\frac{11\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{2a^3d}\\ &\quad+\frac{11\sin(c+dx)}{2a^3d\cos^{\frac{3}{2}}(c+dx)}-\frac{119\sin(c+dx)}{10a^3d\sqrt{\cos(c+dx)}}\\ &\quad-\frac{\sin(c+dx)}{5d\cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^3}\\ &\quad-\frac{2\sin(c+dx)}{3ad\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^2}\\ &\quad-\frac{119\sin(c+dx)}{30d\cos^{\frac{5}{2}}(c+dx)\left(a^3+a^3\sec(c+dx)\right)}\end{aligned}$$

output
```
119/10*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d
*x+1/2*c),2^(1/2))/a^3/d+11/2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2
*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a^3/d+11/2*sin(d*x+c)/a^3/d/cos(
d*x+c)^(3/2)-1/5*sin(d*x+c)/d/cos(d*x+c)^(9/2)/(a+a*sec(d*x+c))^3-2/3*sin(
d*x+c)/a/d/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c))^2-119/30*sin(d*x+c)/d/cos(d*x
+c)^(5/2)/(a^3+a^3*sec(d*x+c))-119/10*sin(d*x+c)/a^3/d/cos(d*x+c)^(1/2)
```

3.397.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 2.92 (sec) , antiderivative size = 402, normalized size of antiderivative = 1.94

$$\int \frac{1}{\cos^{\frac{11}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx$$
$$= \frac{\cos^6\left(\frac{1}{2}(c+dx)\right)\left(-\frac{(5134\cos(\frac{1}{2}(c-dx))+4148\cos(\frac{1}{2}(3c+dx))+4664\cos(\frac{1}{2}(c+3dx))+2476\cos(\frac{1}{2}(5c+3dx))+3340\cos(\frac{1}{2}(3c+5dx))+94}{96d\cos}\right.}{}$$

input
```
Integrate[1/(Cos[c + d*x]^(11/2)*(a + a*Sec[c + d*x])^3),x]
```

output
```
(Cos[(c + d*x)/2]^6*(-1/96*((5134*Cos[(c - d*x)/2] + 4148*Cos[(3*c + d*x)/
2] + 4664*Cos[(c + 3*d*x)/2] + 2476*Cos[(5*c + 3*d*x)/2] + 3340*Cos[(3*c +
 5*d*x)/2] + 944*Cos[(7*c + 5*d*x)/2] + 1620*Cos[(5*c + 7*d*x)/2] + 165*Co
s[(9*c + 7*d*x)/2] + 357*Cos[(7*c + 9*d*x)/2])*Csc[c/2]*Sec[c/2]*Sec[(c +
d*x)/2]^5)/(d*Cos[c + d*x]^(9/2)) + ((4*I)*Sqrt[2]*(119*(1 + E^((2*I)*(c +
 d*x))) + 119*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeomet
ric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))] - 55*E^(I*(c + d*x))*(-1 + E^
((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4,
-E^((2*I)*(c + d*x))])*Sec[c + d*x]^3)/(d*E^(I*(c + d*x))*(-1 + E^((2*I)*c
))*Sqrt[(1 + E^((2*I)*(c + d*x)))/E^(I*(c + d*x))])))/(5*a^3*(1 + Sec[c +
d*x])^3)
```

3.397.3 Rubi [A] (verified)

Time = 1.54 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.37, number of steps used = 19, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.826$, Rules used = {3042, 4752, 3042, 4303, 27, 3042, 4507, 3042, 4507, 27, 3042, 4274, 3042, 4255, 3042, 4258, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\cos^{\frac{11}{2}}(c+dx)(a\sec(c+dx)+a)^3}\,dx$$
$$\Big\downarrow \; 3042$$

$$\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{11/2}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^3}dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sec^{\frac{11}{2}}(c+dx)}{(\sec(c+dx)a+a)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{11/2}}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^3}dx$$

$$\Big\downarrow \text{4303}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\int \frac{\sec^{\frac{7}{2}}(c+dx)(7a-13a\sec(c+dx))}{2(\sec(c+dx)a+a)^2}dx}{5a^2}-\frac{\sin(c+dx)\sec^{\frac{9}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}\right)$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\int \frac{\sec^{\frac{7}{2}}(c+dx)(7a-13a\sec(c+dx))}{(\sec(c+dx)a+a)^2}dx}{10a^2}-\frac{\sin(c+dx)\sec^{\frac{9}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}\left(7a-13a\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^2}dx}{10a^2}-\frac{\sin(c+dx)\sec^{\frac{9}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}\right)$$

$$\Big\downarrow \text{4507}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{\int \frac{\sec^{\frac{5}{2}}(c+dx)\left(50a^2-69a^2\sec(c+dx)\right)}{\sec(c+dx)a+a}dx}{3a^2}+\frac{20a\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}}{10a^2}-\frac{\sin(c+dx)\sec^{\frac{9}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(50a^2-69a^2\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{3a^2}+\frac{20a\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}}{10a^2}-\frac{\sin(c+dx)\sec^{\frac{9}{2}}(c}{5d(a\sec(c+dx)}\right.$$

3.397. $\int \frac{1}{\cos^{\frac{11}{2}}(c+dx)(a+a\sec(c+dx))^3}dx$

$$\Big\downarrow \; 4507$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{\frac{\int \frac{3}{2}\sec^{\frac{3}{2}}(c+dx)\left(119a^3-165a^3\sec(c+dx)\right)dx}{a^2}}{3a^2}+\frac{119a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{d(a\sec(c+dx)+a)}}{10a^2}+\frac{20a\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}-\right.$$

$$\Big\downarrow \; 27$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{\frac{3\int \sec^{\frac{3}{2}}(c+dx)\left(119a^3-165a^3\sec(c+dx)\right)dx}{2a^2}}{3a^2}+\frac{119a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{d(a\sec(c+dx)+a)}}{10a^2}+\frac{20a\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}-\right.$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{\frac{3\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(119a^3-165a^3\csc\left(c+dx+\frac{\pi}{2}\right)\right)dx}{2a^2}}{3a^2}+\frac{119a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{d(a\sec(c+dx)+a)}}{10a^2}+\frac{20a\sin(c+dx)\sec^{\frac{7}{2}}(c+\right.$$

$$\Big\downarrow \; 4274$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{\frac{3\left(119a^3\int \sec^{\frac{3}{2}}(c+dx)dx-165a^3\int \sec^{\frac{5}{2}}(c+dx)dx\right)}{2a^2}}{3a^2}+\frac{119a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{d(a\sec(c+dx)+a)}}{10a^2}+\frac{20a\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}\right.$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{\frac{3\left(119a^3\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}dx-165a^3\int \csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}dx\right)}{2a^2}}{3a^2}+\frac{119a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{d(a\sec(c+dx)+a)}}{10a^2}+\frac{20a\sin(c+dx)\,s}{3d(a\sec(c+\right.$$

$$\Big\downarrow \; 4255$$

3.397. $\int \frac{1}{\cos^{\frac{11}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{\frac{3\left(119a^3\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\int\frac{1}{\sqrt{\sec(c+dx)}}dx\right)-165a^3\left(\frac{1}{3}\int\sqrt{\sec(c+dx)}dx+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}\right)\right)}{2a^2}}{3a^2}+\frac{1}{}}{10a^2}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{\frac{3\left(119a^3\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\int\frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)-165a^3\left(\frac{1}{3}\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+d}{3d}\right.}{2a^2}}{3a^2}}{10a^2}\right.$$

$\downarrow$ 4258

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{\frac{3\left(119a^3\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\cos(c+dx)}dx\right)-165a^3\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\right.}{2a^2}}{3a^2}}{10a^2}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{\frac{3\left(119a^3\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx\right)-165a^3\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+d}\right.}{2a^2}}{3a^2}}{1}\right.$$

$\downarrow$ 3119

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{\frac{3\left(119a^3\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)-165a^3\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\right.}{2a^2}}{3a^2}}{10}\right.$$

3.397. $\int\frac{1}{\cos^{\frac{11}{2}}(c+dx)(a+a\sec(c+dx))^3}dx$

$$\Big\downarrow \; 3120$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{\frac{119a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{d(a\sec(c+dx)+a)}+\frac{3\left(119a^3\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)-165a}{2a^2}}{3a^2}}{10}\right.$$

input
```
Int[1/(Cos[c + d*x]^(11/2)*(a + a*Sec[c + d*x])^3),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(-1/5*(Sec[c + d*x]^(9/2)*Sin[c + d*
x])/(d*(a + a*Sec[c + d*x])^3) - ((20*a*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(
3*d*(a + a*Sec[c + d*x])^2) + ((119*a^2*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(
d*(a + a*Sec[c + d*x])) + (3*(119*a^3*((-2*Sqrt[Cos[c + d*x]]*EllipticE[(c
 + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/
d) - 165*a^3*((2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c +
 d*x]])/(3*d) + (2*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d))))/(2*a^2))/(3*a
^2))/(10*a^2))
```

3.397.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4303
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-d^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d
*Csc[e + f*x])^(n - 2)/(f*(2*m + 1))), x] + Simp[d^2/(a*b*(2*m + 1))   Int[
(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n
 + 2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2,
0] && LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

rule 4507
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[d*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(a*f*(
2*m + 1))), x] - Simp[1/(a*b*(2*m + 1))   Int[(a + b*Csc[e + f*x])^(m + 1)*
(d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
 - n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f,
A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && G
tQ[n, 0]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

3.397. $\int \frac{1}{\cos^{\frac{11}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx$

3.397.4 Maple [A] (verified)

Time = 13.14 (sec) , antiderivative size = 453, normalized size of antiderivative = 2.19

method	result
default	$-\frac{\sqrt{-\left(-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\left(\frac{\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}}{5\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}+\frac{32\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}}{15\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}+\frac{118\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right.}}{5\cos\left(\frac{d}{}\right.}\right.}{}$

input
```
int(1/cos(d*x+c)^(11/2)/(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
-1/4*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/a^3*(1/5*(-
2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)^5+32
/15*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c
)^3+118/5*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x
+1/2*c)-128/5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/
2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*
d*x+1/2*c),2^(1/2))+238/5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2
*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(Ellip
ticF(cos(1/2*d*x+1/2*c),2^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))-4/
3*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/
(cos(1/2*d*x+1/2*c)^2-1/2)^2+48*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)/(-
(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2))/sin(1/2*d*x+1/2*c
)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

3.397.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 414, normalized size of antiderivative = 2.00

$$\int \frac{1}{\cos^{\frac{11}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx =$$
$$-\frac{2\left(357\cos\left(dx+c\right)^4+906\cos\left(dx+c\right)^3+695\cos\left(dx+c\right)^2+120\cos\left(dx+c\right)-20\right)\sqrt{\cos\left(dx+c\right)}\sin}{}$$

input
```
integrate(1/cos(d*x+c)^(11/2)/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
-1/60*(2*(357*cos(d*x + c)^4 + 906*cos(d*x + c)^3 + 695*cos(d*x + c)^2 + 1
20*cos(d*x + c) - 20)*sqrt(cos(d*x + c))*sin(d*x + c) + 165*(I*sqrt(2)*cos
(d*x + c)^5 + 3*I*sqrt(2)*cos(d*x + c)^4 + 3*I*sqrt(2)*cos(d*x + c)^3 + I*
sqrt(2)*cos(d*x + c)^2)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*
x + c)) + 165*(-I*sqrt(2)*cos(d*x + c)^5 - 3*I*sqrt(2)*cos(d*x + c)^4 - 3*
I*sqrt(2)*cos(d*x + c)^3 - I*sqrt(2)*cos(d*x + c)^2)*weierstrassPInverse(-
4, 0, cos(d*x + c) - I*sin(d*x + c)) + 357*(-I*sqrt(2)*cos(d*x + c)^5 - 3*
I*sqrt(2)*cos(d*x + c)^4 - 3*I*sqrt(2)*cos(d*x + c)^3 - I*sqrt(2)*cos(d*x
+ c)^2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I
*sin(d*x + c))) + 357*(I*sqrt(2)*cos(d*x + c)^5 + 3*I*sqrt(2)*cos(d*x + c)
^4 + 3*I*sqrt(2)*cos(d*x + c)^3 + I*sqrt(2)*cos(d*x + c)^2)*weierstrassZet
a(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/(a^3*
d*cos(d*x + c)^5 + 3*a^3*d*cos(d*x + c)^4 + 3*a^3*d*cos(d*x + c)^3 + a^3*d
*cos(d*x + c)^2)
```

3.397.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{11}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(1/cos(d*x+c)**(11/2)/(a+a*sec(d*x+c))**3,x)
```

output
```
Timed out
```

3.397.7 Maxima [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{11}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(1/cos(d*x+c)^(11/2)/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
Timed out
```

3.397.8 Giac [F]

$$\int \frac{1}{\cos^{\frac{11}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx = \int \frac{1}{(a\sec(dx+c)+a)^3\cos(dx+c)^{\frac{11}{2}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(11/2)/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate(1/((a*sec(d*x + c) + a)^3*cos(d*x + c)^(11/2)), x)
```

3.397.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{11}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx = \int \frac{1}{\cos(c+d\,x)^{11/2}\left(a+\frac{a}{\cos(c+d\,x)}\right)^3}\,dx$$

input
```
int(1/(cos(c + d*x)^(11/2)*(a + a/cos(c + d*x))^3),x)
```

output
```
int(1/(cos(c + d*x)^(11/2)*(a + a/cos(c + d*x))^3), x)
```

3.398 $\int \cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\,dx$

3.398.1 Optimal result 2959
3.398.2 Mathematica [A] (verified) 2960
3.398.3 Rubi [A] (verified) 2960
3.398.4 Maple [A] (verified) 2962
3.398.5 Fricas [A] (verification not implemented) 2963
3.398.6 Sympy [F(-1)] 2963
3.398.7 Maxima [B] (verification not implemented) 2964
3.398.8 Giac [F] 2964
3.398.9 Mupad [F(-1)] 2965

3.398.1 Optimal result

Integrand size = 25, antiderivative size = 153

$$\int \cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \frac{32a\sin(c+dx)}{35d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} + \frac{16a\sqrt{\cos(c+dx)}\sin(c+dx)}{35d\sqrt{a+a\sec(c+dx)}} + \frac{12a\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{35d\sqrt{a+a\sec(c+dx)}} + \frac{2a\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{7d\sqrt{a+a\sec(c+dx)}}$$

output
```
12/35*a*cos(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+2/7*a*cos(d*x
+c)^(5/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+32/35*a*sin(d*x+c)/d/cos(d*x
+c)^(1/2)/(a+a*sec(d*x+c))^(1/2)+16/35*a*sin(d*x+c)*cos(d*x+c)^(1/2)/d/(a+
a*sec(d*x+c))^(1/2)
```

3.398.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.52

$$\int \cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\,dx$$
$$=\frac{\sqrt{\cos(c+dx)}\sqrt{a(1+\sec(c+dx))}(140\sin(c+dx)+42\sin(2(c+dx))+12\sin(3(c+dx))+5\sin(4(c+}{140d(1+\cos(c+dx))}$$

input
```
Integrate[Cos[c + d*x]^(7/2)*Sqrt[a + a*Sec[c + d*x]],x]
```

output
```
(Sqrt[Cos[c + d*x]]*Sqrt[a*(1 + Sec[c + d*x])]*(140*Sin[c + d*x] + 42*Sin[
2*(c + d*x)] + 12*Sin[3*(c + d*x)] + 5*Sin[4*(c + d*x)]))/(140*d*(1 + Cos[
c + d*x]))
```

3.398.3 Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.20, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3042, 4752, 3042, 4292, 3042, 4292, 3042, 4292, 3042, 4291}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^{\frac{7}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^{7/2}\sqrt{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}dx$$
$$\Big\downarrow \text{4752}$$
$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sqrt{\sec(c+dx)a+a}}{\sec^{\frac{7}{2}}(c+dx)}dx$$
$$\Big\downarrow \text{3042}$$
$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx$$
$$\Big\downarrow \text{4292}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{6}{7}\int\frac{\sqrt{\sec(c+dx)a+a}}{\sec^{\frac{5}{2}}(c+dx)}dx+\frac{2a\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}\right)$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{6}{7}\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+\frac{2a\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}\right)$$

$\Big\downarrow$ 4292

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{6}{7}\left(\frac{4}{5}\int\frac{\sqrt{\sec(c+dx)a+a}}{\sec^{\frac{3}{2}}(c+dx)}dx+\frac{2a\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}\right)+\frac{}{7d\sec^{\frac{5}{2}}(c-}\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{6}{7}\left(\frac{4}{5}\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+\frac{2a\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}\right)+\frac{}{7d\,\mathrm{se}}\right.$$

$\Big\downarrow$ 4292

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{6}{7}\left(\frac{4}{5}\left(\frac{2}{3}\int\frac{\sqrt{\sec(c+dx)a+a}}{\sqrt{\sec(c+dx)}}dx+\frac{2a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}\right)+\frac{}{5d\sec^{\frac{}{}}}\right.\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{6}{7}\left(\frac{4}{5}\left(\frac{2}{3}\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}\right)+\right.\right.$$

$\Big\downarrow$ 4291

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2a\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{6}{7}\left(\frac{2a\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{4}{5}\left(\right.\right.\right.$$

input
```
Int[Cos[c + d*x]^(7/2)*Sqrt[a + a*Sec[c + d*x]],x]
```

3.398. $\int \cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\,dx$

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*a*Sin[c + d*x])/(7*d*Sec[c + d*x
]^(5/2)*Sqrt[a + a*Sec[c + d*x]]) + (6*((2*a*Sin[c + d*x])/(5*d*Sec[c + d*
x]^(3/2)*Sqrt[a + a*Sec[c + d*x]]) + (4*((2*a*Sin[c + d*x])/(3*d*Sqrt[Sec[
c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) + (4*a*Sqrt[Sec[c + d*x]]*Sin[c + d*x]
)/(3*d*Sqrt[a + a*Sec[c + d*x]])))/5))/7)
```

3.398.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4291
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[-2*a*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*S
qrt[d*Csc[e + f*x]])), x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4292
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[a*Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*n*Sqrt[a
+ b*Csc[e + f*x]])), x] + Simp[a*((2*n + 1)/(2*b*d*n))   Int[Sqrt[a + b*Csc
[e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
 EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

3.398.4 Maple [A] (verified)

Time = 1.77 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.46

method	result	size
default	$-\frac{2\sqrt{a(1+\sec(dx+c))}\,\sqrt{\cos(dx+c)}\,\left(5\cos(dx+c)^4+\cos(dx+c)^3+2\cos(dx+c)^2+8\cos(dx+c)-16\right)\csc(dx+c)}{35d}$	70

input
```
int(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/35/d*(a*(1+sec(d*x+c)))^(1/2)*cos(d*x+c)^(1/2)*(5*cos(d*x+c)^4+cos(d*x+
c)^3+2*cos(d*x+c)^2+8*cos(d*x+c)-16)*csc(d*x+c)
```

3.398.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.52

$$\int \cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\,dx$$
$$=\frac{2\left(5\cos\left(dx+c\right)^{3}+6\cos\left(dx+c\right)^{2}+8\cos\left(dx+c\right)+16\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)}{35\left(d\cos\left(dx+c\right)+d\right)}$$

input
```
integrate(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
2/35*(5*cos(d*x + c)^3 + 6*cos(d*x + c)^2 + 8*cos(d*x + c) + 16)*sqrt((a*c
os(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(d*cos(d*x
+ c) + d)
```

3.398.6 Sympy [F(-1)]

Timed out.

$$\int \cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(7/2)*(a+a*sec(d*x+c))**(1/2),x)
```

output
```
Timed out
```

3.398.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 293 vs. $2(129) = 258$.

Time = 0.39 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.92

$$\int \cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\,dx$$
$$= \frac{\sqrt{2}\left(105\cos\left(\frac{6}{7}\arctan\left(\sin\left(\frac{7}{2}dx+\frac{7}{2}c\right),\cos\left(\frac{7}{2}dx+\frac{7}{2}c\right)\right)\right)\sin\left(\frac{7}{2}dx+\frac{7}{2}c\right)+35\cos\left(\frac{4}{7}\arctan\left(\sin\left(\frac{7}{2}dx\right.\right.\right.\right.}{}$$

input
```
integrate(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
1/280*sqrt(2)*(105*cos(6/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2
*c)))*sin(7/2*d*x + 7/2*c) + 35*cos(4/7*arctan2(sin(7/2*d*x + 7/2*c), cos(
7/2*d*x + 7/2*c)))*sin(7/2*d*x + 7/2*c) + 7*cos(2/7*arctan2(sin(7/2*d*x +
7/2*c), cos(7/2*d*x + 7/2*c)))*sin(7/2*d*x + 7/2*c) - 105*cos(7/2*d*x + 7/
2*c)*sin(6/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) - 35*cos
(7/2*d*x + 7/2*c)*sin(4/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*
c))) - 7*cos(7/2*d*x + 7/2*c)*sin(2/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/
2*d*x + 7/2*c))) + 10*sin(7/2*d*x + 7/2*c) + 7*sin(5/7*arctan2(sin(7/2*d*x
 + 7/2*c), cos(7/2*d*x + 7/2*c))) + 35*sin(3/7*arctan2(sin(7/2*d*x + 7/2*c
), cos(7/2*d*x + 7/2*c))) + 105*sin(1/7*arctan2(sin(7/2*d*x + 7/2*c), cos(
7/2*d*x + 7/2*c))))*sqrt(a)/d
```

3.398.8 Giac [F]

$$\int \cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \int \sqrt{a\sec(dx+c)+a}\cos(dx+c)^{\frac{7}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.398.9 Mupad [F(-1)]

Timed out.

$$\int \cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\,dx=\int \cos\left(c+d\,x\right)^{7/2}\,\sqrt{a+\frac{a}{\cos\left(c+d\,x\right)}}\,dx$$

input
```
int(cos(c + d*x)^(7/2)*(a + a/cos(c + d*x))^(1/2),x)
```

output
```
int(cos(c + d*x)^(7/2)*(a + a/cos(c + d*x))^(1/2), x)
```

3.399 $\int \cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\,dx$

3.399.1 Optimal result 2966
3.399.2 Mathematica [A] (verified) 2966
3.399.3 Rubi [A] (verified) 2967
3.399.4 Maple [A] (verified) 2969
3.399.5 Fricas [A] (verification not implemented) 2969
3.399.6 Sympy [F(-1)] 2970
3.399.7 Maxima [B] (verification not implemented) 2970
3.399.8 Giac [F] 2971
3.399.9 Mupad [F(-1)] 2971

3.399.1 Optimal result

Integrand size = 25, antiderivative size = 115

$$\int \cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \frac{16a\sin(c+dx)}{15d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} + \frac{8a\sqrt{\cos(c+dx)}\sin(c+dx)}{15d\sqrt{a+a\sec(c+dx)}} + \frac{2a\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d\sqrt{a+a\sec(c+dx)}}$$

output
```
2/5*a*cos(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+16/15*a*sin(d*x
+c)/d/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2)+8/15*a*sin(d*x+c)*cos(d*x+c)
^(1/2)/d/(a+a*sec(d*x+c))^(1/2)
```

3.399.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.53

$$\int \cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \frac{\sqrt{\cos(c+dx)}(19+8\cos(c+dx)+3\cos(2(c+dx)))\sqrt{a(1+\sec(c+dx))}\tan\left(\frac{1}{2}(c+dx)\right)}{15d}$$

input
```
Integrate[Cos[c + d*x]^(5/2)*Sqrt[a + a*Sec[c + d*x]],x]
```

output
```
(Sqrt[Cos[c + d*x]]*(19 + 8*Cos[c + d*x] + 3*Cos[2*(c + d*x)])*Sqrt[a*(1 +
 Sec[c + d*x])]*Tan[(c + d*x)/2])/(15*d)
```

3.399.3 Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.23, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3042, 4752, 3042, 4292, 3042, 4292, 3042, 4291}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\sqrt{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sqrt{\sec(c+dx)a+a}}{\sec^{\frac{5}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx$$

$$\Big\downarrow \text{4292}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{4}{5}\int \frac{\sqrt{\sec(c+dx)a+a}}{\sec^{\frac{3}{2}}(c+dx)}dx+\frac{2a\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{4}{5}\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+\frac{2a\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}\right)$$

$$\Big\downarrow \text{4292}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{4}{5}\left(\frac{2}{3}\int \frac{\sqrt{\sec(c+dx)a+a}}{\sqrt{\sec(c+dx)}}dx+\frac{2a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}\right)+\frac{}{5d\sec^{\frac{3}{2}}(c}\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{4}{5}\left(\frac{2}{3}\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}\right)+\frac{}{5d\,\mathrm{s}}\right.$$

$$\Big\downarrow \text{4291}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2a\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{4}{5}\left(\frac{4a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\sqrt{a\sec(c+dx)+a}}+\frac{}{3d\sqrt{\sec(c+}}\right.\right.$$

input
```
Int[Cos[c + d*x]^(5/2)*Sqrt[a + a*Sec[c + d*x]],x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*a*Sin[c + d*x])/(5*d*Sec[c + d*x
]^(3/2)*Sqrt[a + a*Sec[c + d*x]]) + (4*((2*a*Sin[c + d*x])/(3*d*Sqrt[Sec[c
 + d*x]]*Sqrt[a + a*Sec[c + d*x]]) + (4*a*Sqrt[Sec[c + d*x]]*Sin[c + d*x])
/(3*d*Sqrt[a + a*Sec[c + d*x]])))/5)
```

3.399.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4291
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[-2*a*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*S
qrt[d*Csc[e + f*x]])), x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4292
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[a*Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*n*Sqrt[a
+ b*Csc[e + f*x]])), x] + Simp[a*((2*n + 1)/(2*b*d*n))   Int[Sqrt[a + b*Csc
[e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
 EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

3.399.4 Maple [A] (verified)

Time = 1.81 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.52

method	result	size
default	$-\frac{2\sqrt{a(1+\sec(dx+c))}\,\sqrt{\cos(dx+c)}\,\left(3\cos(dx+c)^{3}+\cos(dx+c)^{2}+4\cos(dx+c)-8\right)\csc(dx+c)}{15d}$	60

input
```
int(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/15/d*(a*(1+sec(d*x+c)))^(1/2)*cos(d*x+c)^(1/2)*(3*cos(d*x+c)^3+cos(d*x+
c)^2+4*cos(d*x+c)-8)*csc(d*x+c)
```

3.399.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.60

$$\int \cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \frac{2\left(3\,\cos\left(dx+c\right)^{2}+4\,\cos\left(dx+c\right)+8\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)}{15\left(d\cos\left(dx+c\right)+d\right)}$$

input
```
integrate(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
2/15*(3*cos(d*x + c)^2 + 4*cos(d*x + c) + 8)*sqrt((a*cos(d*x + c) + a)/cos
(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c) + d)
```

3.399.6 Sympy [F(-1)]

Timed out.

$$\int \cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(5/2)*(a+a*sec(d*x+c))**(1/2),x)
```

output
```
Timed out
```

3.399.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 203 vs. $2(97) = 194$.

Time = 0.40 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.77

$$\int \cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\,dx$$
$$= \frac{\sqrt{2}\left(30\cos\left(\frac{4}{5}\arctan\left(\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right),\cos\left(\frac{5}{2}dx+\frac{5}{2}c\right)\right)\right)\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right)+5\cos\left(\frac{2}{5}\arctan\left(\sin\left(\frac{5}{2}dx+\right.\right.\right.}{}$$

input
```
integrate(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
1/60*sqrt(2)*(30*cos(4/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c
)))*sin(5/2*d*x + 5/2*c) + 5*cos(2/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2
*d*x + 5/2*c)))*sin(5/2*d*x + 5/2*c) - 30*cos(5/2*d*x + 5/2*c)*sin(4/5*arc
tan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) - 5*cos(5/2*d*x + 5/2*c)
*sin(2/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 6*sin(5/2*
d*x + 5/2*c) + 5*sin(3/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c
))) + 30*sin(1/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))))*sqr
t(a)/d
```

3.399.8 Giac [F]

$$\int \cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \int \sqrt{a\sec(dx+c)+a}\cos(dx+c)^{\frac{5}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.399.9 Mupad [F(-1)]

Timed out.

$$\int \cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \int \cos(c+d\,x)^{5/2}\,\sqrt{a+\frac{a}{\cos(c+d\,x)}}\,dx$$

input
```
int(cos(c + d*x)^(5/2)*(a + a/cos(c + d*x))^(1/2),x)
```

output
```
int(cos(c + d*x)^(5/2)*(a + a/cos(c + d*x))^(1/2), x)
```

3.400 $\int \cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\,dx$

3.400.1 Optimal result . . . 2972
3.400.2 Mathematica [A] (verified) . . . 2972
3.400.3 Rubi [A] (verified) . . . 2973
3.400.4 Maple [A] (verified) . . . 2974
3.400.5 Fricas [A] (verification not implemented) . . . 2975
3.400.6 Sympy [F(-1)] . . . 2975
3.400.7 Maxima [A] (verification not implemented) . . . 2975
3.400.8 Giac [F] . . . 2976
3.400.9 Mupad [F(-1)] . . . 2976

3.400.1 Optimal result

Integrand size = 25, antiderivative size = 77

$$\int \cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \frac{4a\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} + \frac{2a\sqrt{\cos(c+dx)}\sin(c+dx)}{3d\sqrt{a+a\sec(c+dx)}}$$

output
```
4/3*a*sin(d*x+c)/d/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2)+2/3*a*sin(d*x+c
)*cos(d*x+c)^(1/2)/d/(a+a*sec(d*x+c))^(1/2)
```

3.400.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.64

$$\int \cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \frac{2\sqrt{\cos(c+dx)}(2+\cos(c+dx))\sqrt{a(1+\sec(c+dx))}\tan\left(\frac{1}{2}(c+dx)\right)}{3d}$$

input
```
Integrate[Cos[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]],x]
```

output
```
(2*Sqrt[Cos[c + d*x]]*(2 + Cos[c + d*x])*Sqrt[a*(1 + Sec[c + d*x])]*Tan[(c
 + d*x)/2])/(3*d)
```

3.400.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.27, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3042, 4752, 3042, 4292, 3042, 4291}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sqrt{\sec(c+dx)a+a}}{\sec^{\frac{3}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{4292}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2}{3}\int \frac{\sqrt{\sec(c+dx)a+a}}{\sqrt{\sec(c+dx)}}dx+\frac{2a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2}{3}\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}\right)$$

$$\Big\downarrow \text{4291}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{4a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\sqrt{a\sec(c+dx)+a}}+\frac{2a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}\right)$$

input
```
Int[Cos[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]],x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*a*Sin[c + d*x])/(3*d*Sqrt[Sec[c
+ d*x]]*Sqrt[a + a*Sec[c + d*x]]) + (4*a*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/
(3*d*Sqrt[a + a*Sec[c + d*x]]))
```

3.400.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4291
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[-2*a*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*S
qrt[d*Csc[e + f*x]])), x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4292
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[a*Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*n*Sqrt[a
+ b*Csc[e + f*x]])), x] + Simp[a*((2*n + 1)/(2*b*d*n))   Int[Sqrt[a + b*Csc
[e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
 EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

3.400.4 Maple [A] (verified)

Time = 1.79 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.62

method	result	size
default	$-\frac{2\sqrt{a(1+\sec(dx+c))}\,\sqrt{\cos(dx+c)}\,\left(\cos(dx+c)^2+\cos(dx+c)-2\right)\csc(dx+c)}{3d}$	48

input
```
int(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3/d*(a*(1+sec(d*x+c)))^(1/2)*cos(d*x+c)^(1/2)*(cos(d*x+c)^2+cos(d*x+c)-
2)*csc(d*x+c)
```

3.400.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.74

$$\int \cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \frac{2\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}(\cos(dx+c)+2)\sqrt{\cos(dx+c)}\sin(dx+c)}{3(d\cos(dx+c)+d)}$$

input
```
integrate(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
2/3*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(cos(d*x + c) + 2)*sqrt(cos(d*
x + c))*sin(d*x + c)/(d*cos(d*x + c) + d)
```

3.400.6 Sympy [F(-1)]

Timed out.

$$\int \cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(3/2)*(a+a*sec(d*x+c))**(1/2),x)
```

output
```
Timed out
```

3.400.7 Maxima [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.47

$$\int \cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \frac{\sqrt{2}\left(3\cos\left(\frac{2}{3}\arctan\left(\sin\left(\frac{3}{2}dx+\frac{3}{2}c\right),\cos\left(\frac{3}{2}dx+\frac{3}{2}c\right)\right)\right)\sin\left(\frac{3}{2}dx+\frac{3}{2}c\right)-3\cos\left(\frac{3}{2}dx+\frac{3}{2}c\right)\sin\left(\frac{2}{3}\text{arcta}\right.\right.}{}$$

input
```
integrate(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
1/6*sqrt(2)*(3*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))
)*sin(3/2*d*x + 3/2*c) - 3*cos(3/2*d*x + 3/2*c)*sin(2/3*arctan2(sin(3/2*d*
x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sin(3/2*d*x + 3/2*c) + 3*sin(1/3*ar
ctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sqrt(a)/d
```

3.400.8 Giac [F]

$$\int \cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \int \sqrt{a\sec\left(dx+c\right)+a}\cos\left(dx+c\right)^{\frac{3}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.400.9 Mupad [F(-1)]

Timed out.

$$\int \cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \int \cos\left(c+d\,x\right)^{3/2}\,\sqrt{a+\frac{a}{\cos\left(c+d\,x\right)}}\,dx$$

input
```
int(cos(c + d*x)^(3/2)*(a + a/cos(c + d*x))^(1/2),x)
```

output
```
int(cos(c + d*x)^(3/2)*(a + a/cos(c + d*x))^(1/2), x)
```

3.401 $\int \sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}\,dx$

3.401.1 Optimal result 2977
3.401.2 Mathematica [A] (verified) 2977
3.401.3 Rubi [A] (verified) 2978
3.401.4 Maple [A] (verified) 2979
3.401.5 Fricas [A] (verification not implemented) 2979
3.401.6 Sympy [F] 2980
3.401.7 Maxima [A] (verification not implemented) 2980
3.401.8 Giac [F] 2980
3.401.9 Mupad [F(-1)] 2981

3.401.1 Optimal result

Integrand size = 25, antiderivative size = 36

$$\int \sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}\,dx=\frac{2a\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}}$$

output
```
2*a*sin(d*x+c)/d/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2)
```

3.401.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.08

$$\int \sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}\,dx=\frac{2\sqrt{\cos(c+dx)}\sqrt{a(1+\sec(c+dx))}\tan\left(\frac{1}{2}(c+dx)\right)}{d}$$

input
```
Integrate[Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]],x]
```

output
```
(2*Sqrt[Cos[c + d*x]]*Sqrt[a*(1 + Sec[c + d*x])]*Tan[(c + d*x)/2])/d
```

3.401.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3042, 4752, 3042, 4291}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sqrt{\sec(c+dx)a+a}}{\sqrt{\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4291}$$

$$\frac{2a\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}$$

input
```
Int[Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]],x]
```

output
```
(2*a*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]])
```

3.401.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4291
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[-2*a*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*S
qrt[d*Csc[e + f*x]])), x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

3.401.4 Maple [A] (verified)

Time = 1.44 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.14

method	result	size
default	$-\frac{2\sqrt{\cos(dx+c)}\,\sqrt{a(1+\sec(dx+c))}\,(\cot(dx+c)-\csc(dx+c))}{d}$	41
risch	$-\frac{2i\sqrt{\frac{a\left(e^{i(dx+c)}+1\right)^2}{e^{2i(dx+c)}+1}}\,\sqrt{\cos(dx+c)}\,\left(e^{i(dx+c)}-1\right)}{\left(e^{i(dx+c)}+1\right)d}$	69

input
```
int(cos(d*x+c)^(1/2)*(a+a*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/d*cos(d*x+c)^(1/2)*(a*(1+sec(d*x+c)))^(1/2)*(cot(d*x+c)-csc(d*x+c))
```

3.401.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.36

$$\int \sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}\,dx = \frac{2\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)}{d\cos(dx+c)+d}$$

input
```
integrate(cos(d*x+c)^(1/2)*(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
2*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/
(d*cos(d*x + c) + d)
```

3.401.6 Sympy [F]

$$\int \sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}\,dx = \int \sqrt{a\left(\sec\left(c+dx\right)+1\right)}\sqrt{\cos\left(c+dx\right)}\,dx$$

input
```
integrate(cos(d*x+c)**(1/2)*(a+a*sec(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(a*(sec(c + d*x) + 1))*sqrt(cos(c + d*x)), x)
```

3.401.7 Maxima [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.56

$$\int \sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}\,dx = \frac{2\sqrt{2}\sqrt{a}\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{d}$$

input
```
integrate(cos(d*x+c)^(1/2)*(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
2*sqrt(2)*sqrt(a)*sin(1/2*d*x + 1/2*c)/d
```

3.401.8 Giac [F]

$$\int \sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}\,dx = \int \sqrt{a\sec\left(dx+c\right)+a}\sqrt{\cos\left(dx+c\right)}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)*(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.401.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}\,dx=\int \sqrt{\cos\left(c+d\,x\right)}\,\sqrt{a+\frac{a}{\cos\left(c+d\,x\right)}}\,dx$$

input
```
int(cos(c + d*x)^(1/2)*(a + a/cos(c + d*x))^(1/2),x)
```

output
```
int(cos(c + d*x)^(1/2)*(a + a/cos(c + d*x))^(1/2), x)
```

3.402 $\int \frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{\cos(c+dx)}}\,dx$

3.402.1 Optimal result 2982
3.402.2 Mathematica [A] (verified) 2982
3.402.3 Rubi [A] (verified) 2983
3.402.4 Maple [B] (verified) 2984
3.402.5 Fricas [A] (verification not implemented) 2985
3.402.6 Sympy [F] 2985
3.402.7 Maxima [B] (verification not implemented) 2985
3.402.8 Giac [F] 2986
3.402.9 Mupad [F(-1)] 2986

3.402.1 Optimal result

Integrand size = 25, antiderivative size = 57

$$\int \frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{\cos(c+dx)}}\,dx=\frac{2\sqrt{a}\operatorname{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{d}$$

output
```
2*arcsinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))*a^(1/2)*cos(d*x+c)^(1
/2)*sec(d*x+c)^(1/2)/d
```

3.402.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.30

$$\int \frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{\cos(c+dx)}}\,dx=-\frac{2\arcsin\left(\sqrt{\sec(c+dx)}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sqrt{a(1+\sec(c+dx))}\tan\left(\frac{1}{2}(c+dx)\right)}{d\sqrt{1-\sec(c+dx)}}$$

input
```
Integrate[Sqrt[a + a*Sec[c + d*x]]/Sqrt[Cos[c + d*x]],x]
```

output
```
(-2*ArcSin[Sqrt[Sec[c + d*x]]]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sqrt[
a*(1 + Sec[c + d*x])]*Tan[(c + d*x)/2])/(d*Sqrt[1 - Sec[c + d*x]])
```

3.402.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3042, 4752, 3042, 4288, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a\sec(c+dx)+a}}{\sqrt{\cos(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\sec(c+dx)}\sqrt{\sec(c+dx)a+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx$$

$$\Big\downarrow \text{4288}$$

$$-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1}}d\left(-\frac{a\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}$$

$$\Big\downarrow \text{222}$$

$$\frac{2\sqrt{a}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}$$

input
```
Int[Sqrt[a + a*Sec[c + d*x]]/Sqrt[Cos[c + d*x]],x]
```

output
```
(2*Sqrt[a]*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]*Sqrt[C
os[c + d*x]]*Sqrt[Sec[c + d*x]])/d
```

3.402.3.1 Defintions of rubi rules used

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4288
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[-2*(a/(b*f))*Sqrt[a*(d/b)]   Subst[Int[1/Sqrt[1
+ x^2/a], x], x, b*(Cot[e + f*x]/Sqrt[a + b*Csc[e + f*x]])], x] /; FreeQ[{a
, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[a*(d/b), 0]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

3.402.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(47) = 94$.

Time = 2.22 (sec) , antiderivative size = 135, normalized size of antiderivative = 2.37

method	result	size
default	$-\frac{\sqrt{\cos(dx+c)}\left(\arctan\left(\frac{-\cos(dx+c)+\sin(dx+c)-1}{2(\cos(dx+c)+1)\sqrt{-\frac{1}{\cos(dx+c)+1}}}\right)+\arctan\left(\frac{\cos(dx+c)+\sin(dx+c)+1}{2(\cos(dx+c)+1)\sqrt{-\frac{1}{\cos(dx+c)+1}}}\right)\right)\sqrt{a(1+\sec(dx+c))}}{d(\cos(dx+c)+1)\sqrt{-\frac{1}{\cos(dx+c)+1}}}$	135

input
```
int((a+a*sec(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/d*cos(d*x+c)^(1/2)*(arctan(1/2*(-cos(d*x+c)+sin(d*x+c)-1)/(cos(d*x+c)+1
)/(-1/(cos(d*x+c)+1))^(1/2))+arctan(1/2*(cos(d*x+c)+sin(d*x+c)+1)/(cos(d*x
+c)+1)/(-1/(cos(d*x+c)+1))^(1/2)))*(a*(1+sec(d*x+c)))^(1/2)/(cos(d*x+c)+1)
/(-1/(cos(d*x+c)+1))^(1/2)
```

3.402.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 180, normalized size of antiderivative = 3.16

$$\int \frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{\cos(c+dx)}}\,dx$$
$$= \left[\frac{\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-4\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}(\cos(dx+c)-2)\sqrt{\cos(dx+c)}\sin(dx+c)-7\,a\cos(dx+c)^2+8\,a}{\cos(dx+c)^3+\cos(dx+c)^2}\right)}{2\,d}, \frac{\sqrt{-a}\arctan\left(\frac{2\sqrt{-a}\sqrt{}}{a\,c}\right.}{}\right.$$

input
```
integrate((a+a*sec(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
[1/2*sqrt(a)*log((a*cos(d*x + c)^3 - 4*sqrt(a)*sqrt((a*cos(d*x + c) + a)/c
os(d*x + c))*(cos(d*x + c) - 2)*sqrt(cos(d*x + c))*sin(d*x + c) - 7*a*cos(
d*x + c)^2 + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2))/d, sqrt(-a)*arctan(2*
sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*
x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a))/d]
```

3.402.6 Sympy [F]

$$\int \frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{\cos(c+dx)}}\,dx = \int \frac{\sqrt{a\left(\sec\left(c+dx\right)+1\right)}}{\sqrt{\cos\left(c+dx\right)}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))**(1/2)/cos(d*x+c)**(1/2),x)
```

output
```
Integral(sqrt(a*(sec(c + d*x) + 1))/sqrt(cos(c + d*x)), x)
```

3.402.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 241 vs. 2(47) = 94.

Time = 0.39 (sec) , antiderivative size = 241, normalized size of antiderivative = 4.23

$$\int \frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{\cos(c+dx)}}\,dx$$
$$= \frac{\sqrt{a}\left(\log\left(2\,\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+2\,\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+2\,\sqrt{2}\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+2\,\sqrt{2}\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+2\right)-}{}$$

input
```
integrate((a+a*sec(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
1/2*sqrt(a)*(log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*s
qrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - log(2*
cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x
+ 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + log(2*cos(1/2*d*x + 1/2*c
)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2
)*sin(1/2*d*x + 1/2*c) + 2) - log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x
 + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2
*c) + 2))/d
```

3.402.8 Giac [F]

$$\int \frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{\cos(c+dx)}}\,dx=\int \frac{\sqrt{a\sec{(dx+c)}+a}}{\sqrt{\cos{(dx+c)}}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.402.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{\cos(c+dx)}}\,dx=\int \frac{\sqrt{a+\frac{a}{\cos(c+d\,x)}}}{\sqrt{\cos{(c+d\,x)}}}\,dx$$

input
```
int((a + a/cos(c + d*x))^(1/2)/cos(c + d*x)^(1/2),x)
```

output
```
int((a + a/cos(c + d*x))^(1/2)/cos(c + d*x)^(1/2), x)
```

3.403 $\int \frac{\sqrt{a+a\sec(c+dx)}}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

3.403.1 Optimal result 2987
3.403.2 Mathematica [A] (verified) 2987
3.403.3 Rubi [A] (verified) 2988
3.403.4 Maple [B] (verified) 2990
3.403.5 Fricas [A] (verification not implemented) 2990
3.403.6 Sympy [F] 2991
3.403.7 Maxima [B] (verification not implemented) 2991
3.403.8 Giac [F] 2992
3.403.9 Mupad [F(-1)] 2993

3.403.1 Optimal result

Integrand size = 25, antiderivative size = 92

$$\int \frac{\sqrt{a+a\sec(c+dx)}}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{\sqrt{a}\operatorname{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{d} + \frac{a\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}$$

output
```
arcsinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))*a^(1/2)*cos(d*x+c)^(1/2
)*sec(d*x+c)^(1/2)/d+a*sin(d*x+c)/d/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2
)
```

3.403.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.03

$$\int \frac{\sqrt{a+a\sec(c+dx)}}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{a\sqrt{\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)\left(\arcsin\left(\sqrt{1-\sec(c+dx)}\right)+\sqrt{-((-1+\sec(c+dx))\sec(c+dx))}\right)\sin(c+}{d\sqrt{1-\sec(c+dx)}\sqrt{a(1+\sec(c+dx))}}$$

input
```
Integrate[Sqrt[a + a*Sec[c + d*x]]/Cos[c + d*x]^(3/2),x]
```

output
```
(a*Sqrt[Cos[c + d*x]]*Sec[c + d*x]^(3/2)*(ArcSin[Sqrt[1 - Sec[c + d*x]]] +
 Sqrt[-((-1 + Sec[c + d*x])*Sec[c + d*x])])*Sin[c + d*x])/(d*Sqrt[1 - Sec[
c + d*x]]*Sqrt[a*(1 + Sec[c + d*x])])
```

3.403.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3042, 4752, 3042, 4290, 3042, 4288, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a\sec(c+dx)+a}}{\cos^{\frac{3}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sec^{\frac{3}{2}}(c+dx)\sqrt{\sec(c+dx)a+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx$$

$$\Big\downarrow \text{4290}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{2}\int \sqrt{\sec(c+dx)}\sqrt{\sec(c+dx)a+a}dx+\frac{a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{2}\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx+\frac{a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}\right)$$

$$\Big\downarrow \text{4288}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}-\frac{\int\frac{1}{\sqrt{\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1}}d\left(-\frac{a\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}\right)$$

$$\Big\downarrow \text{222}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\sqrt{a}\text{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}\right)$$

input
```
Int[Sqrt[a + a*Sec[c + d*x]]/Cos[c + d*x]^(3/2),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((Sqrt[a]*ArcSinh[(Sqrt[a]*Tan[c + d
*x])/Sqrt[a + a*Sec[c + d*x]]])/d + (a*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(d
*Sqrt[a + a*Sec[c + d*x]]))
```

3.403.3.1 Defintions of rubi rules used

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4288
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[-2*(a/(b*f))*Sqrt[a*(d/b)]   Subst[Int[1/Sqrt[1
+ x^2/a], x], x, b*(Cot[e + f*x]/Sqrt[a + b*Csc[e + f*x]])], x] /; FreeQ[{a
, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[a*(d/b), 0]
```

rule 4290
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[-2*b*d*Cot[e + f*x]*((d*Csc[e + f*x])^(n - 1)/(
f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]])), x] + Simp[2*a*d*((n - 1)/(b*(2*n -
1)))   Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e + f*x])^(n - 1), x], x] /; Fre
eQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

3.403.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 172 vs. $2(78) = 156$.

Time = 2.47 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.88

method	result
default	$\frac{\sqrt{a(1+\sec(dx+c))}\left(2\sin(dx+c)\sqrt{-\frac{1}{\cos(dx+c)+1}}-\cos(dx+c)\arctan\left(\frac{-\cos(dx+c)+\sin(dx+c)-1}{2(\cos(dx+c)+1)\sqrt{-\frac{1}{\cos(dx+c)+1}}}\right)-\cos(dx+c)\arctan\left(\frac{\cos(}{2(\cos(dx}\right.\right.}{2d(\cos(dx+c)+1)\sqrt{-\frac{1}{\cos(dx+c)+1}}\sqrt{\cos(dx+c)}}$

input
```
int((a+a*sec(d*x+c))^(1/2)/cos(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/d*(a*(1+sec(d*x+c)))^(1/2)*(2*sin(d*x+c)*(-1/(cos(d*x+c)+1))^(1/2)-cos
(d*x+c)*arctan(1/2*(-cos(d*x+c)+sin(d*x+c)-1)/(cos(d*x+c)+1)/(-1/(cos(d*x+
c)+1))^(1/2))-cos(d*x+c)*arctan(1/2*(cos(d*x+c)+sin(d*x+c)+1)/(cos(d*x+c)+
1)/(-1/(cos(d*x+c)+1))^(1/2)))/(cos(d*x+c)+1)/(-1/(cos(d*x+c)+1))^(1/2)/co
s(d*x+c)^(1/2)
```

3.403.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 325, normalized size of antiderivative = 3.53

$$\int \frac{\sqrt{a+a\sec(c+dx)}}{\cos^{\frac{3}{2}}(c+dx)}\,dx$$
$$= \left[\frac{\left(\cos\left(dx+c\right)^2+\cos\left(dx+c\right)\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-4\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}(\cos(dx+c)-2)\sqrt{\cos(dx+c)}\sin(dx+c)-7\,a\cos(dx+c}{\cos(dx+c)^3+\cos(dx+c)^2}\right.}{4\left(d\cos\left(dx+c\right)^2+d\cos\left(dx+c\right)\right)}\right.$$

input
```
integrate((a+a*sec(d*x+c))^(1/2)/cos(d*x+c)^(3/2),x, algorithm="fricas")
```

output
```
[1/4*((cos(d*x + c)^2 + cos(d*x + c))*sqrt(a)*log((a*cos(d*x + c)^3 - 4*sq
rt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(cos(d*x + c) - 2)*sqrt(cos(
d*x + c))*sin(d*x + c) - 7*a*cos(d*x + c)^2 + 8*a)/(cos(d*x + c)^3 + cos(d
*x + c)^2)) + 4*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))
*sin(d*x + c))/(d*cos(d*x + c)^2 + d*cos(d*x + c)), 1/2*((cos(d*x + c)^2 +
 cos(d*x + c))*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*
x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c)
 - 2*a)) + 2*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*si
n(d*x + c))/(d*cos(d*x + c)^2 + d*cos(d*x + c))]
```

3.403.6 Sympy [F]

$$\int \frac{\sqrt{a+a\sec(c+dx)}}{\cos^{\frac{3}{2}}(c+dx)}\,dx=\int \frac{\sqrt{a\left(\sec\left(c+dx\right)+1\right)}}{\cos^{\frac{3}{2}}\left(c+dx\right)}\,dx$$

input
```
integrate((a+a*sec(d*x+c))**(1/2)/cos(d*x+c)**(3/2),x)
```

output
```
Integral(sqrt(a*(sec(c + d*x) + 1))/cos(c + d*x)**(3/2), x)
```

3.403.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 662 vs. $2(78) = 156$.

Time = 0.42 (sec) , antiderivative size = 662, normalized size of antiderivative = 7.20

$$\int \frac{\sqrt{a+a\sec(c+dx)}}{\cos^{\frac{3}{2}}(c+dx)}\,dx=\text{Too large to display}$$

input
```
integrate((a+a*sec(d*x+c))^(1/2)/cos(d*x+c)^(3/2),x, algorithm="maxima")
```

output
```
-1/4*(4*sqrt(2)*cos(3/2*arctan2(sin(d*x + c), cos(d*x + c)))*sin(2*d*x + 2
*c) - 4*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))*sin(2*d*x + 2
*c) - (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*l
og(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(si
n(d*x + c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos
(d*x + c))) + 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2)
+ (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2
*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*
x + c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x
 + c))) - 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) - (c
os(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2*cos
(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x +
c), cos(d*x + c)))^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c
))) + 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) + (cos(2
*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2
*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c),
cos(d*x + c)))^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))
- 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) - 4*(sqrt(2)
*cos(2*d*x + 2*c) + sqrt(2))*sin(3/2*arctan2(sin(d*x + c), cos(d*x + c)))
+ 4*(sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*sin(1/2*arctan2(sin(d*x + c), ...
```

3.403.8 Giac [F]

$$\int \frac{\sqrt{a+a\sec(c+dx)}}{\cos^{\frac{3}{2}}(c+dx)}\,dx=\int \frac{\sqrt{a\sec{(dx+c)}+a}}{\cos{(dx+c)}^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))^(1/2)/cos(d*x+c)^(3/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.403.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+a\sec(c+dx)}}{\cos^{\frac{3}{2}}(c+dx)}\,dx=\int \frac{\sqrt{a+\frac{a}{\cos(c+d\,x)}}}{\cos\left(c+d\,x\right)^{3/2}}\,dx$$

input
```
int((a + a/cos(c + d*x))^(1/2)/cos(c + d*x)^(3/2),x)
```

output
```
int((a + a/cos(c + d*x))^(1/2)/cos(c + d*x)^(3/2), x)
```

3.404 $\int \frac{\sqrt{a+a\sec(c+dx)}}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

3.404.1 Optimal result 2994
3.404.2 Mathematica [A] (verified) 2994
3.404.3 Rubi [A] (verified) 2995
3.404.4 Maple [A] (verified) 2997
3.404.5 Fricas [A] (verification not implemented) 2998
3.404.6 Sympy [F(-1)] 2998
3.404.7 Maxima [B] (verification not implemented) 2999
3.404.8 Giac [F] 2999
3.404.9 Mupad [F(-1)] 3000

3.404.1 Optimal result

Integrand size = 25, antiderivative size = 136

$$\int \frac{\sqrt{a+a\sec(c+dx)}}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{3\sqrt{a}\operatorname{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{4d} + \frac{a\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{3a\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}$$

output
```
3/4*arcsinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))*a^(1/2)*cos(d*x+c)^
(1/2)*sec(d*x+c)^(1/2)/d+1/2*a*sin(d*x+c)/d/cos(d*x+c)^(5/2)/(a+a*sec(d*x+
c))^(1/2)+3/4*a*sin(d*x+c)/d/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2)
```

3.404.2 Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.94

$$\int \frac{\sqrt{a+a\sec(c+dx)}}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{a\sqrt{\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)\left(3\arcsin\left(\sqrt{1-\sec(c+dx)}\right)+2\sqrt{1-\sec(c+dx)}\sec^{\frac{3}{2}}(c+dx)+3\sqrt{-((-}\right.}{4d\sqrt{1-\sec(c+dx)}\sqrt{a(1+\sec(c+dx))}}$$

input
```
Integrate[Sqrt[a + a*Sec[c + d*x]]/Cos[c + d*x]^(5/2),x]
```

output
```
(a*Sqrt[Cos[c + d*x]]*Sec[c + d*x]^(3/2)*(3*ArcSin[Sqrt[1 - Sec[c + d*x]]]
 + 2*Sqrt[1 - Sec[c + d*x]]*Sec[c + d*x]^(3/2) + 3*Sqrt[-((-1 + Sec[c + d*
x])*Sec[c + d*x])])*Sin[c + d*x])/(4*d*Sqrt[1 - Sec[c + d*x]]*Sqrt[a*(1 +
Sec[c + d*x])])
```

3.404.3 Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {3042, 4752, 3042, 4290, 3042, 4290, 3042, 4288, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a\sec(c+dx)+a}}{\cos^{\frac{5}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sec^{\frac{5}{2}}(c+dx)\sqrt{\sec(c+dx)a+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx$$

$$\Big\downarrow \text{4290}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{3}{4}\int \sec^{\frac{3}{2}}(c+dx)\sqrt{\sec(c+dx)a+a}dx+\frac{a\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{3}{4}\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx+\frac{a\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}\right)$$

$$\Big\downarrow \text{4290}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{3}{4}\left(\frac{1}{2}\int\sqrt{\sec(c+dx)}\sqrt{\sec(c+dx)a+a}dx+\frac{a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}\right)+\frac{a\sin(c}{2d\sqrt{}}\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{3}{4}\left(\frac{1}{2}\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx+\frac{a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx}{d\sqrt{a\sec(c+dx)+a}}\right.\right.$$

$$\Big\downarrow \text{4288}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{3}{4}\left(\frac{a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}-\frac{\int\frac{1}{\sqrt{\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1}}d\left(-\frac{a\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}\right)+\frac{a\sin(c+}{2d\sqrt{a}}\right.$$

$$\Big\downarrow \text{222}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{3}{4}\left(\frac{\sqrt{a}\text{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}\right)+\frac{a\sin(c+dx)\sec^{\frac{5}{2}}(}{2d\sqrt{a\sec(c+d}}\right.$$

input
```
Int[Sqrt[a + a*Sec[c + d*x]]/Cos[c + d*x]^(5/2),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((a*Sec[c + d*x]^(5/2)*Sin[c + d*x])
/(2*d*Sqrt[a + a*Sec[c + d*x]]) + (3*((Sqrt[a]*ArcSinh[(Sqrt[a]*Tan[c + d*
x])/Sqrt[a + a*Sec[c + d*x]]])/d + (a*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(d*
Sqrt[a + a*Sec[c + d*x]])))/4)
```

3.404.3.1 Defintions of rubi rules used

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4288
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[-2*(a/(b*f))*Sqrt[a*(d/b)]   Subst[Int[1/Sqrt[1
+ x^2/a], x], x, b*(Cot[e + f*x]/Sqrt[a + b*Csc[e + f*x]])], x] /; FreeQ[{a
, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[a*(d/b), 0]
```

rule 4290
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[-2*b*d*Cot[e + f*x]*((d*Csc[e + f*x])^(n - 1)/(
f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]])), x] + Simp[2*a*d*((n - 1)/(b*(2*n -
1)))   Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e + f*x])^(n - 1), x], x] /; Fre
eQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

3.404.4 Maple [A] (verified)

Time = 2.16 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.51

method	result
default	$\frac{\left(6\sqrt{-\frac{1}{\cos(dx+c)+1}}\,\sin(dx+c)\cos(dx+c)-3\cos(dx+c)^2\arctan\left(\frac{-\cos(dx+c)+\sin(dx+c)-1}{2(\cos(dx+c)+1)\sqrt{-\frac{1}{\cos(dx+c)+1}}}\right)-3\arctan\left(\frac{\cos(dx+c)+\sin(dx+}{2(\cos(dx+c)+1)\sqrt{-\frac{}{\cos(}}}\right.\right.}{8d(\cos(dx+c)+1)\sqrt{-\frac{1}{\cos(dx+c)+1}}\,\cos(dx+c)^{\frac{3}{2}}}$

input
```
int((a+a*sec(d*x+c))^(1/2)/cos(d*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/8/d*(6*(-1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*cos(d*x+c)-3*cos(d*x+c)^2*ar
ctan(1/2*(-cos(d*x+c)+sin(d*x+c)-1)/(cos(d*x+c)+1)/(-1/(cos(d*x+c)+1))^(1/
2))-3*arctan(1/2*(cos(d*x+c)+sin(d*x+c)+1)/(cos(d*x+c)+1)/(-1/(cos(d*x+c)+
1))^(1/2))*cos(d*x+c)^2+4*sin(d*x+c)*(-1/(cos(d*x+c)+1))^(1/2))*(a*(1+sec(
d*x+c)))^(1/2)/(cos(d*x+c)+1)/(-1/(cos(d*x+c)+1))^(1/2)/cos(d*x+c)^(3/2)
```

3.404.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 355, normalized size of antiderivative = 2.61

$$\int \frac{\sqrt{a+a\sec(c+dx)}}{\cos^{\frac{5}{2}}(c+dx)}\,dx$$
$$= \left[\frac{4\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}(3\cos(dx+c)+2)\sqrt{\cos(dx+c)}\sin(dx+c)+3\left(\cos(dx+c)^3+\cos(dx+c)^2\right)\sqrt{a}\log}{16\left(d\cos(dx+c)^3+d\cos(dx+\right.}\right.$$

input
```
integrate((a+a*sec(d*x+c))^(1/2)/cos(d*x+c)^(5/2),x, algorithm="fricas")
```

output
```
[1/16*(4*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(3*cos(d*x + c) + 2)*sqrt
(cos(d*x + c))*sin(d*x + c) + 3*(cos(d*x + c)^3 + cos(d*x + c)^2)*sqrt(a)*
log((a*cos(d*x + c)^3 - 4*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*
(cos(d*x + c) - 2)*sqrt(cos(d*x + c))*sin(d*x + c) - 7*a*cos(d*x + c)^2 +
8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)))/(d*cos(d*x + c)^3 + d*cos(d*x + c
)^2), 1/8*(2*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(3*cos(d*x + c) + 2)*
sqrt(cos(d*x + c))*sin(d*x + c) + 3*(cos(d*x + c)^3 + cos(d*x + c)^2)*sqrt
(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*
x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)))/(d*cos(d*
x + c)^3 + d*cos(d*x + c)^2)]
```

3.404.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+a\sec(c+dx)}}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sec(d*x+c))**(1/2)/cos(d*x+c)**(5/2),x)
```

output
```
Timed out
```

3.404.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1264 vs. $2(112) = 224$.

Time = 0.41 (sec) , antiderivative size = 1264, normalized size of antiderivative = 9.29

$$\int \frac{\sqrt{a + a\sec(c + dx)}}{\cos^{\frac{5}{2}}(c + dx)}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sec(d*x+c))^(1/2)/cos(d*x+c)^(5/2),x, algorithm="maxima")
```

output
```
-1/16*(12*(sqrt(2)*sin(4*d*x + 4*c) + 2*sqrt(2)*sin(2*d*x + 2*c))*cos(7/2*
arctan2(sin(d*x + c), cos(d*x + c))) + 4*(sqrt(2)*sin(4*d*x + 4*c) + 2*sqr
t(2)*sin(2*d*x + 2*c))*cos(5/2*arctan2(sin(d*x + c), cos(d*x + c))) - 4*(s
qrt(2)*sin(4*d*x + 4*c) + 2*sqrt(2)*sin(2*d*x + 2*c))*cos(3/2*arctan2(sin(
d*x + c), cos(d*x + c))) - 12*(sqrt(2)*sin(4*d*x + 4*c) + 2*sqrt(2)*sin(2*
d*x + 2*c))*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) - 3*(2*(2*cos(2*d
*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^
2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x
 + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*x + c), co
s(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sqrt
(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2*sqrt(2)*sin(1/2*arcta
n2(sin(d*x + c), cos(d*x + c))) + 2) + 3*(2*(2*cos(2*d*x + 2*c) + 1)*cos(4
*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)
^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*
d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*s
in(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(
sin(d*x + c), cos(d*x + c))) - 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos
(d*x + c))) + 2) - 3*(2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*
d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4
*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*l...
```

3.404.8 Giac [F]

$$\int \frac{\sqrt{a + a\sec(c + dx)}}{\cos^{\frac{5}{2}}(c + dx)}\,dx = \int \frac{\sqrt{a\sec(dx + c) + a}}{\cos(dx + c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))^(1/2)/cos(d*x+c)^(5/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.404.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+a\sec(c+dx)}}{\cos^{\frac{5}{2}}(c+dx)}\,dx=\int \frac{\sqrt{a+\frac{a}{\cos(c+d\,x)}}}{\cos\left(c+d\,x\right)^{5/2}}\,dx$$

input
```
int((a + a/cos(c + d*x))^(1/2)/cos(c + d*x)^(5/2),x)
```

output
```
int((a + a/cos(c + d*x))^(1/2)/cos(c + d*x)^(5/2), x)
```

3.405 $\int \cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\,dx$

3.405.1 Optimal result . . . 3001
3.405.2 Mathematica [A] (verified) . . . 3001
3.405.3 Rubi [A] (verified) . . . 3002
3.405.4 Maple [A] (verified) . . . 3005
3.405.5 Fricas [A] (verification not implemented) . . . 3005
3.405.6 Sympy [F(-1)] . . . 3005
3.405.7 Maxima [B] (verification not implemented) . . . 3006
3.405.8 Giac [F] . . . 3006
3.405.9 Mupad [F(-1)] . . . 3007

3.405.1 Optimal result

Integrand size = 25, antiderivative size = 161

$$\int \cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \frac{208a^2\sin(c+dx)}{105d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} + \frac{104a^2\sqrt{\cos(c+dx)}\sin(c+dx)}{105d\sqrt{a+a\sec(c+dx)}} + \frac{26a^2\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{35d\sqrt{a+a\sec(c+dx)}} + \frac{2a^2\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{7d\sqrt{a+a\sec(c+dx)}}$$

output
```
26/35*a^2*cos(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+2/7*a^2*cos
(d*x+c)^(5/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+208/105*a^2*sin(d*x+c)/d
/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2)+104/105*a^2*sin(d*x+c)*cos(d*x+c)
^(1/2)/d/(a+a*sec(d*x+c))^(1/2)
```

3.405.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.45

$$\int \cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \frac{a\sqrt{\cos(c+dx)}(494+253\cos(c+dx)+78\cos(2(c+dx))+15\cos(3(c+dx)))\sqrt{a(1+\sec(c}}{210d}$$

input
```
Integrate[Cos[c + d*x]^(7/2)*(a + a*Sec[c + d*x])^(3/2),x]
```

output
```
(a*Sqrt[Cos[c + d*x]]*(494 + 253*Cos[c + d*x] + 78*Cos[2*(c + d*x)] + 15*C
os[3*(c + d*x)])*Sqrt[a*(1 + Sec[c + d*x])]*Tan[(c + d*x)/2])/(210*d)
```

3.405.3 Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.16, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {3042, 4752, 3042, 4300, 27, 3042, 4292, 3042, 4292, 3042, 4291}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^{\frac{7}{2}}(c+dx)(a\sec(c+dx)+a)^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^{7/2}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{(\sec(c+dx)a+a)^{3/2}}{\sec^{\frac{7}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx$$

$$\Big\downarrow \text{4300}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2}{7}a\int \frac{13\sqrt{\sec(c+dx)a+a}}{2\sec^{\frac{5}{2}}(c+dx)}dx+\frac{2a^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}\right)$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{13}{7}a\int \frac{\sqrt{\sec(c+dx)a+a}}{\sec^{\frac{5}{2}}(c+dx)}dx+\frac{2a^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{13}{7}a\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+\frac{2a^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}\right)$$

$\Big\downarrow$ 4292

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{13}{7}a\left(\frac{4}{5}\int\frac{\sqrt{\sec(c+dx)a+a}}{\sec^{\frac{3}{2}}(c+dx)}dx+\frac{2a\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}\right)+\frac{}{7d\sec^{\frac{5}{2}}(}\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{13}{7}a\left(\frac{4}{5}\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+\frac{2a\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}\right)+\frac{}{7d}\right.$$

$\Big\downarrow$ 4292

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{13}{7}a\left(\frac{4}{5}\left(\frac{2}{3}\int\frac{\sqrt{\sec(c+dx)a+a}}{\sqrt{\sec(c+dx)}}dx+\frac{2a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}\right)+\frac{}{5d\sec}\right.\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{13}{7}a\left(\frac{4}{5}\left(\frac{2}{3}\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}\right)\right.\right.$$

$\Big\downarrow$ 4291

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2a^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{13}{7}a\left(\frac{2a\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{4}{5}\right.\right.$$

input
```
Int[Cos[c + d*x]^(7/2)*(a + a*Sec[c + d*x])^(3/2),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*a^2*Sin[c + d*x])/(7*d*Sec[c + d
*x]^(5/2)*Sqrt[a + a*Sec[c + d*x]]) + (13*a*((2*a*Sin[c + d*x])/(5*d*Sec[c
 + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]]) + (4*((2*a*Sin[c + d*x])/(3*d*Sqrt
[Sec[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) + (4*a*Sqrt[Sec[c + d*x]]*Sin[c +
 d*x])/(3*d*Sqrt[a + a*Sec[c + d*x]])))/5))/7)
```

3.405. $\int \cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\,dx$

3.405.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4291
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[-2*a*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*S
qrt[d*Csc[e + f*x]])), x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4292
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[a*Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*n*Sqrt[a
+ b*Csc[e + f*x]])), x] + Simp[a*((2*n + 1)/(2*b*d*n))   Int[Sqrt[a + b*Csc
[e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
 EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

rule 4300
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)*
((d*Csc[e + f*x])^n/(f*n)), x] - Simp[a/(d*n)   Int[(a + b*Csc[e + f*x])^(m
 - 2)*(d*Csc[e + f*x])^(n + 1)*(b*(m - 2*n - 2) - a*(m + 2*n - 1)*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1]
 && (LtQ[n, -1] || (EqQ[m, 3/2] && EqQ[n, -2^(-1)])) && IntegerQ[2*m]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

3.405.4 Maple [A] (verified)

Time = 1.58 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.45

method	result	size
default	$-\frac{2a\sqrt{a(1+\sec(dx+c))}\,\sqrt{\cos(dx+c)}\,\left(15\cos(dx+c)^4+24\cos(dx+c)^3+13\cos(dx+c)^2+52\cos(dx+c)-104\right)\csc(dx+c)}{105d}$	73

input
```
int(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/105/d*a*(a*(1+sec(d*x+c)))^(1/2)*cos(d*x+c)^(1/2)*(15*cos(d*x+c)^4+24*c
os(d*x+c)^3+13*cos(d*x+c)^2+52*cos(d*x+c)-104)*csc(d*x+c)
```

3.405.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.52

$$\int \cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \frac{2\left(15\,a\cos\left(dx+c\right)^{3}+39\,a\cos\left(dx+c\right)^{2}+52\,a\cos\left(dx+c\right)+104\,a\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos\left(dx+\right.}}{105\left(d\cos\left(dx+c\right)+d\right)}$$

input
```
integrate(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
2/105*(15*a*cos(d*x + c)^3 + 39*a*cos(d*x + c)^2 + 52*a*cos(d*x + c) + 104
*a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c
)/(d*cos(d*x + c) + d)
```

3.405.6 Sympy [F(-1)]

Timed out.

$$\int \cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(7/2)*(a+a*sec(d*x+c))**(3/2),x)
```

output
```
Timed out
```

3.405.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 303 vs. $2(137) = 274$.

Time = 0.39 (sec) , antiderivative size = 303, normalized size of antiderivative = 1.88

$$\int \cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \frac{\sqrt{2}\left(735\,a\cos\left(\frac{6}{7}\arctan\left(\sin\left(\frac{7}{2}\,dx+\frac{7}{2}\,c\right),\cos\left(\frac{7}{2}\,dx+\frac{7}{2}\,c\right)\right)\right)\sin\left(\frac{7}{2}\,dx+\frac{7}{2}\,c\right)+175\,a\cos\left(\frac{4}{7}\,\mathrm{ar}\right.\right.}{}$$

input
```
integrate(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
1/840*sqrt(2)*(735*a*cos(6/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7
/2*c)))*sin(7/2*d*x + 7/2*c) + 175*a*cos(4/7*arctan2(sin(7/2*d*x + 7/2*c),
 cos(7/2*d*x + 7/2*c)))*sin(7/2*d*x + 7/2*c) + 63*a*cos(2/7*arctan2(sin(7/
2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c)))*sin(7/2*d*x + 7/2*c) - 735*a*cos(7/
2*d*x + 7/2*c)*sin(6/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))
) - 175*a*cos(7/2*d*x + 7/2*c)*sin(4/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7
/2*d*x + 7/2*c))) - 63*a*cos(7/2*d*x + 7/2*c)*sin(2/7*arctan2(sin(7/2*d*x
+ 7/2*c), cos(7/2*d*x + 7/2*c))) + 30*a*sin(7/2*d*x + 7/2*c) + 63*a*sin(5/
7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) + 175*a*sin(3/7*arc
tan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) + 735*a*sin(1/7*arctan2(
sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))))*sqrt(a)/d
```

3.405.8 Giac [F]

$$\int \cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \int (a\sec(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^{\frac{7}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.405.9 Mupad [F(-1)]

Timed out.

$$\int \cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\,dx=\int \cos\left(c+d\,x\right)^{7/2}\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)^{3/2}dx$$

input
```
int(cos(c + d*x)^(7/2)*(a + a/cos(c + d*x))^(3/2),x)
```

output
```
int(cos(c + d*x)^(7/2)*(a + a/cos(c + d*x))^(3/2), x)
```

3.406 $\int \cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\,dx$

3.406.1 Optimal result . . . 3008
3.406.2 Mathematica [A] (verified) . . . 3008
3.406.3 Rubi [A] (verified) . . . 3009
3.406.4 Maple [A] (verified) . . . 3011
3.406.5 Fricas [A] (verification not implemented) . . . 3011
3.406.6 Sympy [F(-1)] . . . 3012
3.406.7 Maxima [B] (verification not implemented) . . . 3012
3.406.8 Giac [F] . . . 3013
3.406.9 Mupad [F(-1)] . . . 3013

3.406.1 Optimal result

Integrand size = 25, antiderivative size = 116

$$\int \cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \frac{8a^2\sin(c+dx)}{5d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} + \frac{2a\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{5d} + \frac{2\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\sin(c+dx)}{5d}$$

output
```
2/5*cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(3/2)*sin(d*x+c)/d+8/5*a^2*sin(d*x+c
)/d/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2)+2/5*a*sin(d*x+c)*cos(d*x+c)^(1
/2)*(a+a*sec(d*x+c))^(1/2)/d
```

3.406.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.52

$$\int \cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \frac{a\sqrt{\cos(c+dx)}(13+6\cos(c+dx)+\cos(2(c+dx)))\sqrt{a(1+\sec(c+dx))}\tan\left(\frac{1}{2}(c+dx)\right)}{5d}$$

input
```
Integrate[Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^(3/2),x]
```

output
```
(a*Sqrt[Cos[c + d*x]]*(13 + 6*Cos[c + d*x] + Cos[2*(c + d*x)])*Sqrt[a*(1 +
 Sec[c + d*x])]*Tan[(c + d*x)/2])/(5*d)
```

3.406.3 Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.22, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3042, 4752, 3042, 4299, 3042, 4296, 3042, 4291}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^{\frac{5}{2}}(c+dx)(a\sec(c+dx)+a)^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{(\sec(c+dx)a+a)^{3/2}}{\sec^{\frac{5}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx$$

$$\Big\downarrow \text{4299}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{3}{5}\int \frac{(\sec(c+dx)a+a)^{3/2}}{\sec^{\frac{3}{2}}(c+dx)}dx+\frac{2\sin(c+dx)(a\sec(c+dx)+a)^{3/2}}{5d\sec^{\frac{3}{2}}(c+dx)}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{3}{5}\int \frac{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+\frac{2\sin(c+dx)(a\sec(c+dx)+a)^{3/2}}{5d\sec^{\frac{3}{2}}(c+dx)}\right)$$

$$\Big\downarrow \text{4296}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{3}{5}\left(\frac{4}{3}a\int \frac{\sqrt{\sec(c+dx)a+a}}{\sqrt{\sec(c+dx)}}dx+\frac{2a\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{3d\sqrt{\sec(c+dx)}}\right)+\frac{2\sin(c+d}{5}\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{3}{5}\left(\frac{4}{3}a\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{3d\sqrt{\sec(c+dx)}}\right)+\frac{2\sin}{}$$

$$\Big\downarrow \text{4291}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{3}{5}\left(\frac{8a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\sqrt{a\sec(c+dx)+a}}+\frac{2a\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{3d\sqrt{\sec(c+dx)}}\right)+\frac{2\sin(c+}{}$$

input
```
Int[Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^(3/2),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*(a + a*Sec[c + d*x])^(3/2)*Sin[c
 + d*x])/(5*d*Sec[c + d*x]^(3/2)) + (3*((8*a^2*Sqrt[Sec[c + d*x]]*Sin[c +
d*x])/(3*d*Sqrt[a + a*Sec[c + d*x]]) + (2*a*Sqrt[a + a*Sec[c + d*x]]*Sin[c
 + d*x])/(3*d*Sqrt[Sec[c + d*x]])))/5)
```

3.406.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4291
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[-2*a*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*S
qrt[d*Csc[e + f*x]])), x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4296
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-a)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)
*((d*Csc[e + f*x])^n/(f*m)), x] + Simp[b*((2*m - 1)/(d*m))   Int[(a + b*Csc
[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f
, m, n}, x] && EqQ[a^2 - b^2, 0] && EqQ[m + n, 0] && GtQ[m, 1/2] && Integer
Q[2*m]
```

rule 4299
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-Cot[e + f*x])*(a + b*Csc[e + f*x])^m*((d*Csc
[e + f*x])^n/(f*(m + 1))), x] + Simp[a*(m/(b*d*(m + 1)))   Int[(a + b*Csc[e
 + f*x])^m*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m, n},
 x] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LtQ[m, -2^(-1)]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

3.406.4 Maple [A] (verified)

Time = 1.74 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.53

method	result	size
default	$-\frac{2a\sqrt{a(1+\sec(dx+c))}\,\sqrt{\cos(dx+c)}\,\left(\cos(dx+c)^3+2\cos(dx+c)^2+3\cos(dx+c)-6\right)\csc(dx+c)}{5d}$	61

input
```
int(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/5/d*a*(a*(1+sec(d*x+c)))^(1/2)*cos(d*x+c)^(1/2)*(cos(d*x+c)^3+2*cos(d*x
+c)^2+3*cos(d*x+c)-6)*csc(d*x+c)
```

3.406.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.62

$$\int \cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \frac{2\left(a\cos\left(dx+c\right)^2+3\,a\cos\left(dx+c\right)+6\,a\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)}{5\left(d\cos\left(dx+c\right)+d\right)}$$

input
```
integrate(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
2/5*(a*cos(d*x + c)^2 + 3*a*cos(d*x + c) + 6*a)*sqrt((a*cos(d*x + c) + a)/
cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c) + d)
```

3.406.6 Sympy [F(-1)]

Timed out.

$$\int \cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(5/2)*(a+a*sec(d*x+c))**(3/2),x)
```

output
```
Timed out
```

3.406.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 210 vs. $2(98) = 196$.

Time = 0.36 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.81

$$\int \cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \frac{\sqrt{2}\left(20\,a\cos\left(\frac{4}{5}\arctan\left(\sin\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right),\cos\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right)\right)\right)\sin\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right)+5\,a\cos\left(\frac{2}{5}\,\text{arctan}\right.\right.}{}$$

input
```
integrate(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
1/20*sqrt(2)*(20*a*cos(4/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2
*c)))*sin(5/2*d*x + 5/2*c) + 5*a*cos(2/5*arctan2(sin(5/2*d*x + 5/2*c), cos
(5/2*d*x + 5/2*c)))*sin(5/2*d*x + 5/2*c) - 20*a*cos(5/2*d*x + 5/2*c)*sin(4
/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) - 5*a*cos(5/2*d*x
+ 5/2*c)*sin(2/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 2*
a*sin(5/2*d*x + 5/2*c) + 5*a*sin(3/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2
*d*x + 5/2*c))) + 20*a*sin(1/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x +
 5/2*c))))*sqrt(a)/d
```

3.406.8 Giac [F]

$$\int \cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \int (a\sec(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^{\frac{5}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.406.9 Mupad [F(-1)]

Timed out.

$$\int \cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \int \cos(c+d\,x)^{5/2}\left(a+\frac{a}{\cos(c+d\,x)}\right)^{3/2}dx$$

input
```
int(cos(c + d*x)^(5/2)*(a + a/cos(c + d*x))^(3/2),x)
```

output
```
int(cos(c + d*x)^(5/2)*(a + a/cos(c + d*x))^(3/2), x)
```

3.407 $\int \cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\,dx$

3.407.1 Optimal result . . . 3014
3.407.2 Mathematica [A] (verified) . . . 3014
3.407.3 Rubi [A] (verified) . . . 3015
3.407.4 Maple [A] (verified) . . . 3016
3.407.5 Fricas [A] (verification not implemented) . . . 3017
3.407.6 Sympy [F(-1)] . . . 3017
3.407.7 Maxima [A] (verification not implemented) . . . 3017
3.407.8 Giac [F] . . . 3018
3.407.9 Mupad [F(-1)] . . . 3018

3.407.1 Optimal result

Integrand size = 25, antiderivative size = 79

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\,dx=\frac{8a^2\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}}+\frac{2a\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{3d}$$

output
```
8/3*a^2*sin(d*x+c)/d/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2)+2/3*a*sin(d*x
+c)*cos(d*x+c)^(1/2)*(a+a*sec(d*x+c))^(1/2)/d
```

3.407.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.63

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\,dx=\frac{2a\sqrt{\cos(c+dx)}(5+\cos(c+dx))\sqrt{a(1+\sec(c+dx))}\tan\left(\frac{1}{2}(c+dx)\right)}{3d}$$

input
```
Integrate[Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(3/2),x]
```

output
```
(2*a*Sqrt[Cos[c + d*x]]*(5 + Cos[c + d*x])*Sqrt[a*(1 + Sec[c + d*x])]*Tan[
(c + d*x)/2])/(3*d)
```

3.407.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.27, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3042, 4752, 3042, 4296, 3042, 4291}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{(\sec(c+dx)a+a)^{3/2}}{\sec^{\frac{3}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{4296}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{4}{3}a\int \frac{\sqrt{\sec(c+dx)a+a}}{\sqrt{\sec(c+dx)}}dx+\frac{2a\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{3d\sqrt{\sec(c+dx)}}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{4}{3}a\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{3d\sqrt{\sec(c+dx)}}\right)$$

$$\Big\downarrow \text{4291}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{8a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\sqrt{a\sec(c+dx)+a}}+\frac{2a\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{3d\sqrt{\sec(c+dx)}}\right)$$

input
```
Int[Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(3/2),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((8*a^2*Sqrt[Sec[c + d*x]]*Sin[c + d
*x])/(3*d*Sqrt[a + a*Sec[c + d*x]]) + (2*a*Sqrt[a + a*Sec[c + d*x]]*Sin[c
+ d*x])/(3*d*Sqrt[Sec[c + d*x]]))
```

3.407.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4291
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[-2*a*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*S
qrt[d*Csc[e + f*x]])), x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4296
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-a)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)
*((d*Csc[e + f*x])^n/(f*m)), x] + Simp[b*((2*m - 1)/(d*m))   Int[(a + b*Csc
[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f
, m, n}, x] && EqQ[a^2 - b^2, 0] && EqQ[m + n, 0] && GtQ[m, 1/2] && Integer
Q[2*m]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

3.407.4 Maple [A] (verified)

Time = 1.70 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.65

method	result	size
default	$-\frac{2a\sqrt{a(1+\sec(dx+c))}\,\sqrt{\cos(dx+c)}\,\left(\cos(dx+c)^2+4\cos(dx+c)-5\right)\csc(dx+c)}{3d}$	51

input
```
int(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3/d*a*(a*(1+sec(d*x+c)))^(1/2)*cos(d*x+c)^(1/2)*(cos(d*x+c)^2+4*cos(d*x
+c)-5)*csc(d*x+c)
```

3.407.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.77

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \frac{2\,(a\cos(dx+c)+5\,a)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)}{3\,(d\cos(dx+c)+d)}$$

input
```
integrate(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
2/3*(a*cos(d*x + c) + 5*a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(co
s(d*x + c))*sin(d*x + c)/(d*cos(d*x + c) + d)
```

3.407.6 Sympy [F(-1)]

Timed out.

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(3/2)*(a+a*sec(d*x+c))**(3/2),x)
```

output
```
Timed out
```

3.407.7 Maxima [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.48

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \frac{\left(\sqrt{2}a\sin\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)+9\,\sqrt{2}a\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sqrt{a}}{3\,d}$$

input
```
integrate(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
1/3*(sqrt(2)*a*sin(3/2*d*x + 3/2*c) + 9*sqrt(2)*a*sin(1/2*d*x + 1/2*c))*sq
rt(a)/d
```

3.407.8 Giac [F]

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \int (a\sec(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^{\frac{3}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.407.9 Mupad [F(-1)]

Timed out.

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \int \cos(c+d\,x)^{3/2}\left(a+\frac{a}{\cos(c+d\,x)}\right)^{3/2}dx$$

input
```
int(cos(c + d*x)^(3/2)*(a + a/cos(c + d*x))^(3/2),x)
```

output
```
int(cos(c + d*x)^(3/2)*(a + a/cos(c + d*x))^(3/2), x)
```

3.408 $\int \sqrt{\cos(c+dx)}(a+a\sec(c+dx))^{3/2}\,dx$

3.408.1 Optimal result . . . 3019
3.408.2 Mathematica [A] (verified) . . . 3019
3.408.3 Rubi [A] (verified) . . . 3020
3.408.4 Maple [A] (verified) . . . 3022
3.408.5 Fricas [A] (verification not implemented) . . . 3022
3.408.6 Sympy [F(-1)] . . . 3023
3.408.7 Maxima [B] (verification not implemented) . . . 3023
3.408.8 Giac [F] . . . 3024
3.408.9 Mupad [F(-1)] . . . 3024

3.408.1 Optimal result

Integrand size = 25, antiderivative size = 96

$$\int \sqrt{\cos(c+dx)}(a+a\sec(c+dx))^{3/2}\,dx = \frac{2a^{3/2}\operatorname{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{d} + \frac{2a^2\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}}$$

output
```
2*a^(3/2)*arcsinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))*cos(d*x+c)^(1
/2)*sec(d*x+c)^(1/2)/d+2*a^2*sin(d*x+c)/d/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c)
)^(1/2)
```

3.408.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.84

$$\int \sqrt{\cos(c+dx)}(a+a\sec(c+dx))^{3/2}\,dx = \frac{2a^2\left(\sqrt{1-\sec(c+dx)}+\arcsin\left(\sqrt{1-\sec(c+dx)}\right)\sqrt{\sec(c+dx)}\right)\sin(c+dx)}{d\sqrt{-1+\cos(c+dx)}\sqrt{a(1+\sec(c+dx))}}$$

input
```
Integrate[Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])^(3/2),x]
```

output
```
(2*a^2*(Sqrt[1 - Sec[c + d*x]] + ArcSin[Sqrt[1 - Sec[c + d*x]]]*Sqrt[Sec[c
 + d*x]])*Sin[c + d*x])/(d*Sqrt[-1 + Cos[c + d*x]]*Sqrt[a*(1 + Sec[c + d*x
])])
```

3.408.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.01, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3042, 4752, 3042, 4300, 27, 3042, 4288, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\cos(c+dx)}(a\sec(c+dx)+a)^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{(\sec(c+dx)a+a)^{3/2}}{\sqrt{\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4300}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(2a\int \frac{1}{2}\sqrt{\sec(c+dx)}\sqrt{\sec(c+dx)a+a}dx+\frac{2a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\sec(c+dx)+a}}\right)$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(a\int \sqrt{\sec(c+dx)}\sqrt{\sec(c+dx)a+a}dx+\frac{2a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\sec(c+dx)+a}}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(a\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx+\frac{2a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\sec(c+dx)+a}}\right.$$

$$\Big\downarrow \text{4288}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\sec(c+dx)+a}}-\frac{2a\int\frac{1}{\sqrt{\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1}}d\left(-\frac{a\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}\right)$$

$$\Big\downarrow \text{222}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2a^{3/2}\text{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{2a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\sec(c+dx)+a}}\right)$$

input
```
Int[Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])^(3/2),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*a^(3/2)*ArcSinh[(Sqrt[a]*Tan[c +
 d*x])/Sqrt[a + a*Sec[c + d*x]]])/d + (2*a^2*Sqrt[Sec[c + d*x]]*Sin[c + d*
x])/(d*Sqrt[a + a*Sec[c + d*x]]))
```

3.408.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4288
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[-2*(a/(b*f))*Sqrt[a*(d/b)]   Subst[Int[1/Sqrt[1
+ x^2/a], x], x, b*(Cot[e + f*x]/Sqrt[a + b*Csc[e + f*x]])], x] /; FreeQ[{a
, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[a*(d/b), 0]
```

rule 4300
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)*
((d*Csc[e + f*x])^n/(f*n)), x] - Simp[a/(d*n)   Int[(a + b*Csc[e + f*x])^(m
 - 2)*(d*Csc[e + f*x])^(n + 1)*(b*(m - 2*n - 2) - a*(m + 2*n - 1)*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1]
 && (LtQ[n, -1] || (EqQ[m, 3/2] && EqQ[n, -2^(-1)])) && IntegerQ[2*m]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

3.408.4 Maple [A] (verified)

Time = 2.10 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.64

method	result
default	$\frac{a\sqrt{\cos(dx+c)}\,\sqrt{a(1+\sec(dx+c))}\left(\arctan\left(\frac{-\cos(dx+c)+\sin(dx+c)-1}{2(\cos(dx+c)+1)\sqrt{-\frac{1}{\cos(dx+c)+1}}}\right)\sqrt{-\frac{1}{\cos(dx+c)+1}}+\sqrt{-\frac{1}{\cos(dx+c)+1}}\arctan\left(\frac{\cos(dx+}{2(\cos(dx+c)}\right.\right.}{d}$

input
```
int((a+a*sec(d*x+c))^(3/2)*cos(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/d*a*cos(d*x+c)^(1/2)*(a*(1+sec(d*x+c)))^(1/2)*(arctan(1/2*(-cos(d*x+c)+s
in(d*x+c)-1)/(cos(d*x+c)+1)/(-1/(cos(d*x+c)+1))^(1/2))*(-1/(cos(d*x+c)+1))
^(1/2)+(-1/(cos(d*x+c)+1))^(1/2)*arctan(1/2*(cos(d*x+c)+sin(d*x+c)+1)/(cos
(d*x+c)+1)/(-1/(cos(d*x+c)+1))^(1/2))-2*cot(d*x+c)+2*csc(d*x+c))
```

3.408.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 298, normalized size of antiderivative = 3.10

$$\int \sqrt{\cos(c+dx)}(a+a\sec(c+dx))^{3/2}\,dx = \left[\frac{4\,a\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)+(a\cos(dx+c)+a)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-4\sqrt{a}\sqrt{a}}{}\right.}{2\,(d\cos(dx+c)+d)}\right.$$

input
```
integrate((a+a*sec(d*x+c))^(3/2)*cos(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
[1/2*(4*a*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d
*x + c) + (a*cos(d*x + c) + a)*sqrt(a)*log((a*cos(d*x + c)^3 - 4*sqrt(a)*s
qrt((a*cos(d*x + c) + a)/cos(d*x + c))*(cos(d*x + c) - 2)*sqrt(cos(d*x + c
))*sin(d*x + c) - 7*a*cos(d*x + c)^2 + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)
^2)))/(d*cos(d*x + c) + d), (2*a*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*s
qrt(cos(d*x + c))*sin(d*x + c) + (a*cos(d*x + c) + a)*sqrt(-a)*arctan(2*sq
rt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x
+ c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)))/(d*cos(d*x + c) + d)]
```

3.408.6 Sympy [F(-1)]

Timed out.

$$\int \sqrt{\cos(c+dx)}(a+a\sec(c+dx))^{3/2}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sec(d*x+c))**(3/2)*cos(d*x+c)**(1/2),x)
```

output
```
Timed out
```

3.408.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 274 vs. $2(82) = 164$.

Time = 0.39 (sec) , antiderivative size = 274, normalized size of antiderivative = 2.85

$$\int \sqrt{\cos(c+dx)}(a+a\sec(c+dx))^{3/2}\,dx = \frac{\sqrt{2}\left(\sqrt{2}a\log\left(2\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+2\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+2\sqrt{2}\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)+2\sqrt{2}\sin\left(\frac{1}{2}d\right.\right.\right.}{}$$

input
```
integrate((a+a*sec(d*x+c))^(3/2)*cos(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
1/4*sqrt(2)*(sqrt(2)*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*
c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2
) - sqrt(2)*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*
sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + sqrt(
2)*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*c
os(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - sqrt(2)*a*log(
2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*
x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 8*a*sin(1/2*d*x + 1/2*c
))*sqrt(a)/d
```

3.408.8 Giac [F]

$$\int \sqrt{\cos(c+dx)}(a+a\sec(c+dx))^{3/2}\,dx = \int (a\sec(dx+c)+a)^{\frac{3}{2}}\sqrt{\cos(dx+c)}\,dx$$

input
```
integrate((a+a*sec(d*x+c))^(3/2)*cos(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.408.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{\cos(c+dx)}(a+a\sec(c+dx))^{3/2}\,dx = \int \sqrt{\cos(c+d\,x)}\left(a+\frac{a}{\cos(c+d\,x)}\right)^{3/2}dx$$

input
```
int(cos(c + d*x)^(1/2)*(a + a/cos(c + d*x))^(3/2),x)
```

output
```
int(cos(c + d*x)^(1/2)*(a + a/cos(c + d*x))^(3/2), x)
```

3.409 $\int \frac{(a+a\sec(c+dx))^{3/2}}{\sqrt{\cos(c+dx)}}\,dx$

3.409.1 Optimal result . . . 3025
3.409.2 Mathematica [A] (verified) . . . 3025
3.409.3 Rubi [A] (verified) . . . 3026
3.409.4 Maple [B] (verified) . . . 3028
3.409.5 Fricas [A] (verification not implemented) . . . 3029
3.409.6 Sympy [F] . . . 3029
3.409.7 Maxima [B] (verification not implemented) . . . 3030
3.409.8 Giac [F] . . . 3030
3.409.9 Mupad [F(-1)] . . . 3031

3.409.1 Optimal result

Integrand size = 25, antiderivative size = 95

$$\int \frac{(a+a\sec(c+dx))^{3/2}}{\sqrt{\cos(c+dx)}}\,dx = \frac{3a^{3/2}\text{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{d} + \frac{a^2\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}$$

output
```
3*a^(3/2)*arcsinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))*cos(d*x+c)^(1
/2)*sec(d*x+c)^(1/2)/d+a^2*sin(d*x+c)/d/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^
(1/2)
```

3.409.2 Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.97

$$\int \frac{(a+a\sec(c+dx))^{3/2}}{\sqrt{\cos(c+dx)}}\,dx = -\frac{a^2\left(-\sqrt{1-\sec(c+dx)}+\frac{3\arcsin\left(\sqrt{\sec(c+dx)}\right)}{\sqrt{\sec(c+dx)}}\right)\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)\sqrt{1-\sec(c+dx)}\sqrt{a(1+\sec(c+dx))}}$$

input
```
Integrate[(a + a*Sec[c + d*x])^(3/2)/Sqrt[Cos[c + d*x]],x]
```

output
```
-((a^2*(-Sqrt[1 - Sec[c + d*x]] + (3*ArcSin[Sqrt[Sec[c + d*x]]])/Sqrt[Sec[
c + d*x]])*Sin[c + d*x])/(d*Cos[c + d*x]^(3/2)*Sqrt[1 - Sec[c + d*x]]*Sqrt
[a*(1 + Sec[c + d*x])]))
```

3.409.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.01, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3042, 4752, 3042, 4301, 27, 3042, 4288, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sec(c+dx)+a)^{3/2}}{\sqrt{\cos(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\sec(c+dx)}(\sec(c+dx)a+a)^{3/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}dx$$

$$\Big\downarrow \text{4301}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(a\int \frac{3}{2}\sqrt{\sec(c+dx)}\sqrt{\sec(c+dx)a+a}dx+\frac{a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}\right)$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{3}{2}a\int \sqrt{\sec(c+dx)}\sqrt{\sec(c+dx)a+a}dx+\frac{a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{3}{2}a\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx+\frac{a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}\right.$$

$$\Big\downarrow \text{4288}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}-\frac{3a\int\frac{1}{\sqrt{\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1}}d\left(-\frac{a\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}\right)$$

$$\Big\downarrow \text{222}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{3a^{3/2}\text{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}\right)$$

input
```
Int[(a + a*Sec[c + d*x])^(3/2)/Sqrt[Cos[c + d*x]],x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((3*a^(3/2)*ArcSinh[(Sqrt[a]*Tan[c +
 d*x])/Sqrt[a + a*Sec[c + d*x]]])/d + (a^2*Sec[c + d*x]^(3/2)*Sin[c + d*x]
)/(d*Sqrt[a + a*Sec[c + d*x]]))
```

3.409.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4288
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[-2*(a/(b*f))*Sqrt[a*(d/b)]   Subst[Int[1/Sqrt[1
+ x^2/a], x], x, b*(Cot[e + f*x]/Sqrt[a + b*Csc[e + f*x]])], x] /; FreeQ[{a
, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[a*(d/b), 0]
```

rule 4301
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
2)*((d*Csc[e + f*x])^n/(f*(m + n - 1))), x] + Simp[b/(m + n - 1)   Int[(a +
 b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*(b*(m + 2*n - 1) + a*(3*m + 2*n
 - 4)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^
2, 0] && GtQ[m, 1] && NeQ[m + n - 1, 0] && IntegerQ[2*m]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

3.409.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 173 vs. $2(81) = 162$.

Time = 2.21 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.83

method	result
default	$\frac{a\sqrt{a(1+\sec(dx+c))}\left(3\cos(dx+c)\arctan\left(\frac{\cos(dx+c)-\sin(dx+c)+1}{2(\cos(dx+c)+1)\sqrt{-\frac{1}{\cos(dx+c)+1}}}\right)-3\cos(dx+c)\arctan\left(\frac{\cos(dx+c)+\sin(dx+c)+1}{2(\cos(dx+c)+1)\sqrt{-\frac{1}{\cos(dx+c)+1}}}\right)\right.}{2d(\cos(dx+c)+1)\sqrt{-\frac{1}{\cos(dx+c)+1}}\sqrt{\cos(dx+c)}}$

input
```
int((a+a*sec(d*x+c))^(3/2)/cos(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/d*a*(a*(1+sec(d*x+c)))^(1/2)*(3*cos(d*x+c)*arctan(1/2*(cos(d*x+c)-sin(
d*x+c)+1)/(cos(d*x+c)+1)/(-1/(cos(d*x+c)+1))^(1/2))-3*cos(d*x+c)*arctan(1/
2*(cos(d*x+c)+sin(d*x+c)+1)/(cos(d*x+c)+1)/(-1/(cos(d*x+c)+1))^(1/2))+2*si
n(d*x+c)*(-1/(cos(d*x+c)+1))^(1/2))/(cos(d*x+c)+1)/(-1/(cos(d*x+c)+1))^(1/
2)/cos(d*x+c)^(1/2)
```

3.409.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 337, normalized size of antiderivative = 3.55

$$\int \frac{(a+a\sec(c+dx))^{3/2}}{\sqrt{\cos(c+dx)}}\,dx = \left[\frac{4\,a\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)+3\left(a\cos\left(dx+c\right)^{2}+a\cos\left(dx\right.\right.}{4\left(d\cos\left(dx\right.\right.}\right.$$

input
```
integrate((a+a*sec(d*x+c))^(3/2)/cos(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
[1/4*(4*a*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d
*x + c) + 3*(a*cos(d*x + c)^2 + a*cos(d*x + c))*sqrt(a)*log((a*cos(d*x + c
)^3 - 4*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(cos(d*x + c) - 2)
*sqrt(cos(d*x + c))*sin(d*x + c) - 7*a*cos(d*x + c)^2 + 8*a)/(cos(d*x + c)
^3 + cos(d*x + c)^2)))/(d*cos(d*x + c)^2 + d*cos(d*x + c)), 1/2*(2*a*sqrt(
(a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) + 3*(a*
cos(d*x + c)^2 + a*cos(d*x + c))*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*
x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^
2 - a*cos(d*x + c) - 2*a)))/(d*cos(d*x + c)^2 + d*cos(d*x + c))]
```

3.409.6 Sympy [F]

$$\int \frac{(a+a\sec(c+dx))^{3/2}}{\sqrt{\cos(c+dx)}}\,dx = \int \frac{(a(\sec\left(c+dx\right)+1))^{\frac{3}{2}}}{\sqrt{\cos\left(c+dx\right)}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))**(3/2)/cos(d*x+c)**(1/2),x)
```

output
```
Integral((a*(sec(c + d*x) + 1))**(3/2)/sqrt(cos(c + d*x)), x)
```

3.409.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1143 vs. $2(81) = 162$.

Time = 0.40 (sec) , antiderivative size = 1143, normalized size of antiderivative = 12.03

$$\int \frac{(a + a\sec(c + dx))^{3/2}}{\sqrt{\cos(c + dx)}}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sec(d*x+c))^(3/2)/cos(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
1/4*(3*(a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt
(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - a*log(2*c
os(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x +
 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + a*log(2*cos(1/2*d*x + 1/2*
c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(
2)*sin(1/2*d*x + 1/2*c) + 2) - a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*
d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x +
1/2*c) + 2))*cos(2*d*x + 2*c)^2 + 3*(a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*si
n(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*
d*x + 1/2*c) + 2) - a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c
)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2)
 + a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*c
os(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - a*log(2*cos(1/
2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*
c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*sin(2*d*x + 2*c)^2 + 4*sqrt(2)*a
*sin(3/2*d*x + 3/2*c) - 4*sqrt(2)*a*sin(1/2*d*x + 1/2*c) + 2*(2*sqrt(2)*a*
sin(3/2*d*x + 3/2*c) - 2*sqrt(2)*a*sin(1/2*d*x + 1/2*c) + 3*a*log(2*cos(1/
2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*
c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 3*a*log(2*cos(1/2*d*x + 1/2*c)^
2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(...
```

3.409.8 Giac [F]

$$\int \frac{(a + a\sec(c + dx))^{3/2}}{\sqrt{\cos(c + dx)}}\,dx = \int \frac{(a\sec(dx + c) + a)^{\frac{3}{2}}}{\sqrt{\cos(dx + c)}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))^(3/2)/cos(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.409.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(c+dx))^{3/2}}{\sqrt{\cos(c+dx)}}\,dx=\int \frac{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{3/2}}{\sqrt{\cos\left(c+d\,x\right)}}\,dx$$

input
```
int((a + a/cos(c + d*x))^(3/2)/cos(c + d*x)^(1/2),x)
```

output
```
int((a + a/cos(c + d*x))^(3/2)/cos(c + d*x)^(1/2), x)
```

3.410 $\int \frac{(a+a\sec(c+dx))^{3/2}}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

3.410.1 Optimal result . . . 3032
3.410.2 Mathematica [A] (verified) . . . 3032
3.410.3 Rubi [A] (verified) . . . 3033
3.410.4 Maple [A] (verified) . . . 3036
3.410.5 Fricas [A] (verification not implemented) . . . 3036
3.410.6 Sympy [F] . . . 3037
3.410.7 Maxima [B] (verification not implemented) . . . 3037
3.410.8 Giac [F] . . . 3038
3.410.9 Mupad [F(-1)] . . . 3038

3.410.1 Optimal result

Integrand size = 25, antiderivative size = 140

$$\int \frac{(a+a\sec(c+dx))^{3/2}}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{7a^{3/2}\operatorname{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{4d} + \frac{a^2\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{7a^2\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}$$

output
```
7/4*a^(3/2)*arcsinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))*cos(d*x+c)^
(1/2)*sec(d*x+c)^(1/2)/d+1/2*a^2*sin(d*x+c)/d/cos(d*x+c)^(5/2)/(a+a*sec(d*
x+c))^(1/2)+7/4*a^2*sin(d*x+c)/d/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2)
```

3.410.2 Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.76

$$\int \frac{(a+a\sec(c+dx))^{3/2}}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{a^2\left((2+7\cos(c+dx))\sqrt{1-\sec(c+dx)} + \frac{7\arcsin\left(\sqrt{1-\sec(c+dx)}\right)}{\sec^{\frac{3}{2}}(c+dx)}\right)\sin(c+dx)}{4d\cos^{\frac{5}{2}}(c+dx)\sqrt{1-\sec(c+dx)}\sqrt{a(1+\sec(c+dx))}}$$

input
```
Integrate[(a + a*Sec[c + d*x])^(3/2)/Cos[c + d*x]^(3/2),x]
```

output
```
(a^2*((2 + 7*Cos[c + d*x])*Sqrt[1 - Sec[c + d*x]] + (7*ArcSin[Sqrt[1 - Sec
[c + d*x]]])/Sec[c + d*x]^(3/2))*Sin[c + d*x])/(4*d*Cos[c + d*x]^(5/2)*Sqr
t[1 - Sec[c + d*x]]*Sqrt[a*(1 + Sec[c + d*x])])
```

3.410.3 Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.99, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 4752, 3042, 4301, 27, 3042, 4290, 3042, 4288, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sec(c+dx)+a)^{3/2}}{\cos^{\frac{3}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sec^{\frac{3}{2}}(c+dx)(\sec(c+dx)a+a)^{3/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}dx$$

$$\Big\downarrow \text{4301}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{2}a\int \frac{7}{2}\sec^{\frac{3}{2}}(c+dx)\sqrt{\sec(c+dx)a+a}dx+\frac{a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}\right)$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{7}{4}a\int \sec^{\frac{3}{2}}(c+dx)\sqrt{\sec(c+dx)a+a}dx+\frac{a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{7}{4}a\int\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx+\frac{a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx}{2d\sqrt{a\sec(c+dx)+a}}\right.$$

$\downarrow$ 4290

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{7}{4}a\left(\frac{1}{2}\int\sqrt{\sec(c+dx)}\sqrt{\sec(c+dx)a+a}dx+\frac{a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}\right)+\frac{a^2\sin}{2d}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{7}{4}a\left(\frac{1}{2}\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx+\frac{a\sin(c+dx)\sec^{\frac{3}{2}}(c+d}{d\sqrt{a\sec(c+dx)+a}}\right.\right.$$

$\downarrow$ 4288

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{7}{4}a\left(\frac{a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}-\frac{\int\frac{1}{\sqrt{\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}}+1}d\left(-\frac{a\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}\right)+\frac{a^2\sin(c}{2d\sqrt{}}\right.$$

$\downarrow$ 222

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}+\frac{7}{4}a\left(\frac{\sqrt{a}\operatorname{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{a\sin(c+dx)\sec^{\frac{3}{2}}}{d\sqrt{a\sec(c+dx}}\right.\right.$$

input
```
Int[(a + a*Sec[c + d*x])^(3/2)/Cos[c + d*x]^(3/2),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((a^2*Sec[c + d*x]^(5/2)*Sin[c + d*x
])/(2*d*Sqrt[a + a*Sec[c + d*x]]) + (7*a*((Sqrt[a]*ArcSinh[(Sqrt[a]*Tan[c
+ d*x])/Sqrt[a + a*Sec[c + d*x]]])/d + (a*Sec[c + d*x]^(3/2)*Sin[c + d*x])
/(d*Sqrt[a + a*Sec[c + d*x]])))/4)
```

3.410. $\int \frac{(a+a\sec(c+dx))^{3/2}}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

3.410.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4288
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[-2*(a/(b*f))*Sqrt[a*(d/b)]   Subst[Int[1/Sqrt[1
+ x^2/a], x], x, b*(Cot[e + f*x]/Sqrt[a + b*Csc[e + f*x]])], x] /; FreeQ[{a
, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[a*(d/b), 0]
```

rule 4290
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[-2*b*d*Cot[e + f*x]*((d*Csc[e + f*x])^(n - 1)/(
f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]])), x] + Simp[2*a*d*((n - 1)/(b*(2*n -
1)))   Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e + f*x])^(n - 1), x], x] /; Fre
eQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

rule 4301
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
2)*((d*Csc[e + f*x])^n/(f*(m + n - 1))), x] + Simp[b/(m + n - 1)   Int[(a +
 b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*(b*(m + 2*n - 1) + a*(3*m + 2*n
 - 4)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^
2, 0] && GtQ[m, 1] && NeQ[m + n - 1, 0] && IntegerQ[2*m]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

3.410. $\int \frac{(a+a\sec(c+dx))^{3/2}}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

3.410.4 Maple [A] (verified)

Time = 2.11 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.47

method	result
default	$\frac{a\left(14\sqrt{-\frac{1}{\cos(dx+c)+1}}\,\sin(dx+c)\cos(dx+c)-7\cos(dx+c)^2\arctan\left(\frac{-\cos(dx+c)+\sin(dx+c)-1}{2(\cos(dx+c)+1)\sqrt{-\frac{1}{\cos(dx+c)+1}}}\right)-7\arctan\left(\frac{\cos(dx+c)+\sin(d}{2(\cos(dx+c)+1)\sqrt{-\frac{1}{c}}}\right.\right.}{8d(\cos(dx+c)+1)\cos(dx+c)^{\frac{3}{2}}\sqrt{-\frac{1}{\cos(dx+c)+1}}}$

input
```
int((a+a*sec(d*x+c))^(3/2)/cos(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/8/d*a*(14*(-1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*cos(d*x+c)-7*cos(d*x+c)^2
*arctan(1/2*(-cos(d*x+c)+sin(d*x+c)-1)/(cos(d*x+c)+1)/(-1/(cos(d*x+c)+1))^
(1/2))-7*arctan(1/2*(cos(d*x+c)+sin(d*x+c)+1)/(cos(d*x+c)+1)/(-1/(cos(d*x+
c)+1))^(1/2))*cos(d*x+c)^2+4*sin(d*x+c)*(-1/(cos(d*x+c)+1))^(1/2))*(a*(1+s
ec(d*x+c)))^(1/2)/(cos(d*x+c)+1)/cos(d*x+c)^(3/2)/(-1/(cos(d*x+c)+1))^(1/2
)
```

3.410.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 369, normalized size of antiderivative = 2.64

$$\int \frac{(a+a\sec(c+dx))^{3/2}}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \left[\frac{4\,(7\,a\cos(dx+c)+2\,a)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)+7\,(a\cos}{16}\right.$$

input
```
integrate((a+a*sec(d*x+c))^(3/2)/cos(d*x+c)^(3/2),x, algorithm="fricas")
```

output
```
[1/16*(4*(7*a*cos(d*x + c) + 2*a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*
sqrt(cos(d*x + c))*sin(d*x + c) + 7*(a*cos(d*x + c)^3 + a*cos(d*x + c)^2)*
sqrt(a)*log((a*cos(d*x + c)^3 - 4*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*
x + c))*(cos(d*x + c) - 2)*sqrt(cos(d*x + c))*sin(d*x + c) - 7*a*cos(d*x +
 c)^2 + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)))/(d*cos(d*x + c)^3 + d*cos
(d*x + c)^2), 1/8*(2*(7*a*cos(d*x + c) + 2*a)*sqrt((a*cos(d*x + c) + a)/co
s(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) + 7*(a*cos(d*x + c)^3 + a*cos(
d*x + c)^2)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x +
 c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) -
2*a)))/(d*cos(d*x + c)^3 + d*cos(d*x + c)^2)]
```

3.410.6 Sympy [F]

$$\int \frac{(a+a\sec(c+dx))^{3/2}}{\cos^{\frac{3}{2}}(c+dx)}\,dx=\int \frac{(a(\sec(c+dx)+1))^{\frac{3}{2}}}{\cos^{\frac{3}{2}}(c+dx)}\,dx$$

input
```
integrate((a+a*sec(d*x+c))**(3/2)/cos(d*x+c)**(3/2),x)
```

output
```
Integral((a*(sec(c + d*x) + 1))**(3/2)/cos(c + d*x)**(3/2), x)
```

3.410.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2244 vs. $2(116) = 232$.

Time = 0.48 (sec) , antiderivative size = 2244, normalized size of antiderivative = 16.03

$$\int \frac{(a+a\sec(c+dx))^{3/2}}{\cos^{\frac{3}{2}}(c+dx)}\,dx=\text{Too large to display}$$

input
```
integrate((a+a*sec(d*x+c))^(3/2)/cos(d*x+c)^(3/2),x, algorithm="maxima")
```

output
```
-1/16*(56*sqrt(2)*a*cos(7/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/
2*c)))*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 24*s
qrt(2)*a*cos(5/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))*sin(
4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 12*sqrt(2)*a*si
n(3/2*d*x + 3/2*c) + 28*sqrt(2)*a*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), co
s(3/2*d*x + 3/2*c))) - 4*(3*sqrt(2)*a*sin(3/2*d*x + 3/2*c) + 7*sqrt(2)*a*s
in(7/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 3*sqrt(2)*a*
sin(5/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 7*sqrt(2)*a
*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*cos(8/3*arc
tan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 8*(3*sqrt(2)*a*sin(3/2
*d*x + 3/2*c) - 7*sqrt(2)*a*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*
d*x + 3/2*c))))*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)
)) - 7*(a*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 +
 4*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + a*si
n(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*a*sin(8/3
*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))*sin(4/3*arctan2(sin(
3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 4*a*sin(4/3*arctan2(sin(3/2*d*x
 + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*(2*a*cos(4/3*arctan2(sin(3/2*d*x +
 3/2*c), cos(3/2*d*x + 3/2*c))) + a)*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c),
 cos(3/2*d*x + 3/2*c))) + 4*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos...
```

3.410.8 Giac [F]

$$\int \frac{(a+a\sec(c+dx))^{3/2}}{\cos^{\frac{3}{2}}(c+dx)}\,dx=\int \frac{(a\sec{(dx+c)}+a)^{\frac{3}{2}}}{\cos{(dx+c)}^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))^(3/2)/cos(d*x+c)^(3/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.410.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(c+dx))^{3/2}}{\cos^{\frac{3}{2}}(c+dx)}\,dx=\int \frac{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{3/2}}{\cos{(c+d\,x)}^{3/2}}\,dx$$

input
```
int((a + a/cos(c + d*x))^(3/2)/cos(c + d*x)^(3/2),x)
```

output
```
int((a + a/cos(c + d*x))^(3/2)/cos(c + d*x)^(3/2), x)
```

3.411 $\int \frac{(a+a\sec(c+dx))^{3/2}}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

3.411.1 Optimal result . . . 3039
3.411.2 Mathematica [A] (verified) . . . 3039
3.411.3 Rubi [A] (verified) . . . 3040
3.411.4 Maple [A] (verified) . . . 3043
3.411.5 Fricas [A] (verification not implemented) . . . 3043
3.411.6 Sympy [F(-1)] . . . 3044
3.411.7 Maxima [B] (verification not implemented) . . . 3044
3.411.8 Giac [F] . . . 3045
3.411.9 Mupad [F(-1)] . . . 3046

3.411.1 Optimal result

Integrand size = 25, antiderivative size = 180

$$\int \frac{(a+a\sec(c+dx))^{3/2}}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{11a^{3/2}\operatorname{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{8d} + \frac{a^2\sin(c+dx)}{3d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{11a^2\sin(c+dx)}{12d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{11a^2\sin(c+dx)}{8d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}$$

output
```
11/8*a^(3/2)*arcsinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))*cos(d*x+c)
^(1/2)*sec(d*x+c)^(1/2)/d+1/3*a^2*sin(d*x+c)/d/cos(d*x+c)^(7/2)/(a+a*sec(d
*x+c))^(1/2)+11/12*a^2*sin(d*x+c)/d/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(1/2
)+11/8*a^2*sin(d*x+c)/d/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2)
```

3.411.2 Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.65

$$\int \frac{(a+a\sec(c+dx))^{3/2}}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{a^2\left((8+22\cos(c+dx)+33\cos^2(c+dx))\sqrt{1-\sec(c+dx)} + \frac{33\arcsin\left(\sqrt{1-\sec}\right.}{\sec^{\frac{5}{2}}(c+d}\right.}{24d\cos^{\frac{7}{2}}(c+dx)\sqrt{1-\sec(c+dx)}\sqrt{a(1+\sec(c+dx))}}$$

input
```
Integrate[(a + a*Sec[c + d*x])^(3/2)/Cos[c + d*x]^(5/2),x]
```

output
```
(a^2*((8 + 22*Cos[c + d*x] + 33*Cos[c + d*x]^2)*Sqrt[1 - Sec[c + d*x]] + (
33*ArcSin[Sqrt[1 - Sec[c + d*x]]])/Sec[c + d*x]^(5/2))*Sin[c + d*x])/(24*d
*Cos[c + d*x]^(7/2)*Sqrt[1 - Sec[c + d*x]]*Sqrt[a*(1 + Sec[c + d*x])])
```

3.411.3 Rubi [A] (verified)

Time = 0.87 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.01, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {3042, 4752, 3042, 4301, 27, 3042, 4290, 3042, 4290, 3042, 4288, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sec(c+dx)+a)^{3/2}}{\cos^{\frac{5}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sec^{\frac{5}{2}}(c+dx)(\sec(c+dx)a+a)^{3/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}dx$$

$$\Big\downarrow \text{4301}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}a\int \frac{11}{2}\sec^{\frac{5}{2}}(c+dx)\sqrt{\sec(c+dx)a+a}dx+\frac{a^2\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{3d\sqrt{a\sec(c+dx)+a}}\right)$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{11}{6}a\int \sec^{\frac{5}{2}}(c+dx)\sqrt{\sec(c+dx)a+a}dx+\frac{a^2\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{3d\sqrt{a\sec(c+dx)+a}}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{11}{6}a\int\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx+\frac{a^2\sin(c+dx)\sec^{\frac{7}{2}}(c+d}{3d\sqrt{a\sec(c+dx)+a}}\right.$$

$\downarrow$ 4290

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{11}{6}a\left(\frac{3}{4}\int\sec^{\frac{3}{2}}(c+dx)\sqrt{\sec(c+dx)a+a}dx+\frac{a\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}\right)+\frac{a^2\sin}{3d}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{11}{6}a\left(\frac{3}{4}\int\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx+\frac{a\sin(c+dx)\sec^{\frac{5}{2}}(c+}{2d\sqrt{a\sec(c+dx)+}}\right.\right.$$

$\downarrow$ 4290

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{11}{6}a\left(\frac{3}{4}\left(\frac{1}{2}\int\sqrt{\sec(c+dx)}\sqrt{\sec(c+dx)a+a}dx+\frac{a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}\right)+\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{11}{6}a\left(\frac{3}{4}\left(\frac{1}{2}\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx+\frac{a\sin(c+dx)\sec^{\frac{3}{2}}(}{d\sqrt{a\sec(c+dx)}}\right.\right.\right.$$

$\downarrow$ 4288

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{11}{6}a\left(\frac{3}{4}\left(\frac{a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}-\frac{\int\frac{1}{\sqrt{\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1}}d\left(-\frac{a\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}\right)+\frac{a\,s}{2}\right.\right.$$

$\downarrow$ 222

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{a^2\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{3d\sqrt{a\sec(c+dx)+a}}+\frac{11}{6}a\left(\frac{3}{4}\left(\frac{\sqrt{a}\operatorname{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{a\sin(c+dx)}{d\sqrt{a\sec(c}}\right.\right.\right.$$

input
```
Int[(a + a*Sec[c + d*x])^(3/2)/Cos[c + d*x]^(5/2),x]
```

3.411. $\int\frac{(a+a\sec(c+dx))^{3/2}}{\cos^{\frac{5}{2}}(c+dx)}dx$

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((a^2*Sec[c + d*x]^(7/2)*Sin[c + d*x
])/(3*d*Sqrt[a + a*Sec[c + d*x]]) + (11*a*((a*Sec[c + d*x]^(5/2)*Sin[c + d
*x])/(2*d*Sqrt[a + a*Sec[c + d*x]]) + (3*((Sqrt[a]*ArcSinh[(Sqrt[a]*Tan[c
+ d*x])/Sqrt[a + a*Sec[c + d*x]]])/d + (a*Sec[c + d*x]^(3/2)*Sin[c + d*x])
/(d*Sqrt[a + a*Sec[c + d*x]])))/4))/6)
```

3.411.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4288
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[-2*(a/(b*f))*Sqrt[a*(d/b)]   Subst[Int[1/Sqrt[1
+ x^2/a], x], x, b*(Cot[e + f*x]/Sqrt[a + b*Csc[e + f*x]])], x] /; FreeQ[{a
, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[a*(d/b), 0]
```

rule 4290
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[-2*b*d*Cot[e + f*x]*((d*Csc[e + f*x])^(n - 1)/(
f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]])), x] + Simp[2*a*d*((n - 1)/(b*(2*n -
1)))   Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e + f*x])^(n - 1), x], x] /; Fre
eQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

rule 4301
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
2)*((d*Csc[e + f*x])^n/(f*(m + n - 1))), x] + Simp[b/(m + n - 1)   Int[(a +
 b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*(b*(m + 2*n - 1) + a*(3*m + 2*n
 - 4)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^
2, 0] && GtQ[m, 1] && NeQ[m + n - 1, 0] && IntegerQ[2*m]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

3.411.4 Maple [A] (verified)

Time = 2.39 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.31

method	result
default	$\frac{a\left(33\cos(dx+c)^3\arctan\left(\frac{\cos(dx+c)-\sin(dx+c)+1}{2(\cos(dx+c)+1)\sqrt{-\frac{1}{\cos(dx+c)+1}}}\right)-33\cos(dx+c)^3\arctan\left(\frac{\cos(dx+c)+\sin(dx+c)+1}{2(\cos(dx+c)+1)\sqrt{-\frac{1}{\cos(dx+c)+1}}}\right)+66\sqrt{-\frac{1}{\cos(dx+}}\right.}{48d(\cos(dx+c)+1)\sqrt{-\frac{}{\cos(dx}}}$

input
```
int((a+a*sec(d*x+c))^(3/2)/cos(d*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/48/d*a*(33*cos(d*x+c)^3*arctan(1/2*(cos(d*x+c)-sin(d*x+c)+1)/(cos(d*x+c)
+1)/(-1/(cos(d*x+c)+1))^(1/2))-33*cos(d*x+c)^3*arctan(1/2*(cos(d*x+c)+sin(
d*x+c)+1)/(cos(d*x+c)+1)/(-1/(cos(d*x+c)+1))^(1/2))+66*(-1/(cos(d*x+c)+1))
^(1/2)*sin(d*x+c)*cos(d*x+c)^2+44*(-1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*cos
(d*x+c)+16*sin(d*x+c)*(-1/(cos(d*x+c)+1))^(1/2))*(a*(1+sec(d*x+c)))^(1/2)/
(cos(d*x+c)+1)/(-1/(cos(d*x+c)+1))^(1/2)/cos(d*x+c)^(5/2)
```

3.411.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 391, normalized size of antiderivative = 2.17

$$\int \frac{(a+a\sec(c+dx))^{3/2}}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \left[\frac{4\left(33\,a\cos\left(dx+c\right)^2+22\,a\cos\left(dx+c\right)+8\,a\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos\left(dx+c\right)}\,s}{}\right.$$

input
```
integrate((a+a*sec(d*x+c))^(3/2)/cos(d*x+c)^(5/2),x, algorithm="fricas")
```

output
```
[1/96*(4*(33*a*cos(d*x + c)^2 + 22*a*cos(d*x + c) + 8*a)*sqrt((a*cos(d*x +
 c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) + 33*(a*cos(d*x + c
)^4 + a*cos(d*x + c)^3)*sqrt(a)*log((a*cos(d*x + c)^3 - 4*sqrt(a)*sqrt((a*
cos(d*x + c) + a)/cos(d*x + c))*(cos(d*x + c) - 2)*sqrt(cos(d*x + c))*sin(
d*x + c) - 7*a*cos(d*x + c)^2 + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)))/(
d*cos(d*x + c)^4 + d*cos(d*x + c)^3), 1/48*(2*(33*a*cos(d*x + c)^2 + 22*a*
cos(d*x + c) + 8*a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x +
 c))*sin(d*x + c) + 33*(a*cos(d*x + c)^4 + a*cos(d*x + c)^3)*sqrt(-a)*arct
an(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*s
in(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)))/(d*cos(d*x + c)^4
+ d*cos(d*x + c)^3)]
```

3.411.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(c+dx))^{3/2}}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sec(d*x+c))**(3/2)/cos(d*x+c)**(5/2),x)
```

output
```
Timed out
```

3.411.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2361 vs. $2(150) = 300$.

Time = 0.47 (sec) , antiderivative size = 2361, normalized size of antiderivative = 13.12

$$\int \frac{(a+a\sec(c+dx))^{3/2}}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sec(d*x+c))^(3/2)/cos(d*x+c)^(5/2),x, algorithm="maxima")
```

output
```
-1/96*(132*(sqrt(2)*a*sin(6*d*x + 6*c) + 3*sqrt(2)*a*sin(4*d*x + 4*c) + 3*
sqrt(2)*a*sin(2*d*x + 2*c))*cos(11/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c))) + 44*(sqrt(2)*a*sin(6*d*x + 6*c) + 3*sqrt(2)*a*sin(4*d*x + 4*c) +
3*sqrt(2)*a*sin(2*d*x + 2*c))*cos(9/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c))) + 216*(sqrt(2)*a*sin(6*d*x + 6*c) + 3*sqrt(2)*a*sin(4*d*x + 4*c)
+ 3*sqrt(2)*a*sin(2*d*x + 2*c))*cos(7/4*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c))) - 216*(sqrt(2)*a*sin(6*d*x + 6*c) + 3*sqrt(2)*a*sin(4*d*x + 4*c
) + 3*sqrt(2)*a*sin(2*d*x + 2*c))*cos(5/4*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c))) - 44*(sqrt(2)*a*sin(6*d*x + 6*c) + 3*sqrt(2)*a*sin(4*d*x + 4*
c) + 3*sqrt(2)*a*sin(2*d*x + 2*c))*cos(3/4*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c))) - 132*(sqrt(2)*a*sin(6*d*x + 6*c) + 3*sqrt(2)*a*sin(4*d*x +
4*c) + 3*sqrt(2)*a*sin(2*d*x + 2*c))*cos(1/4*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c))) - 33*(a*cos(6*d*x + 6*c)^2 + 9*a*cos(4*d*x + 4*c)^2 + 9*a*
cos(2*d*x + 2*c)^2 + a*sin(6*d*x + 6*c)^2 + 9*a*sin(4*d*x + 4*c)^2 + 18*a*
sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*a*sin(2*d*x + 2*c)^2 + 2*(3*a*cos(4*
d*x + 4*c) + 3*a*cos(2*d*x + 2*c) + a)*cos(6*d*x + 6*c) + 6*(3*a*cos(2*d*x
 + 2*c) + a)*cos(4*d*x + 4*c) + 6*a*cos(2*d*x + 2*c) + 6*(a*sin(4*d*x + 4*
c) + a*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + a)*log(2*cos(1/4*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c)))^2 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*...
```

3.411.8 Giac [F]

$$\int \frac{(a+a\sec(c+dx))^{3/2}}{\cos^{\frac{5}{2}}(c+dx)}\,dx=\int \frac{(a\sec(dx+c)+a)^{\frac{3}{2}}}{\cos(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))^(3/2)/cos(d*x+c)^(5/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.411.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(c+dx))^{3/2}}{\cos^{\frac{5}{2}}(c+dx)}\,dx=\int \frac{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{3/2}}{\cos\left(c+d\,x\right)^{5/2}}\,dx$$

input
```
int((a + a/cos(c + d*x))^(3/2)/cos(c + d*x)^(5/2),x)
```

output
```
int((a + a/cos(c + d*x))^(3/2)/cos(c + d*x)^(5/2), x)
```

3.412 $\int \cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\,dx$

3.412.1 Optimal result . . . 3047
3.412.2 Mathematica [A] (verified) . . . 3047
3.412.3 Rubi [A] (verified) . . . 3048
3.412.4 Maple [A] (verified) . . . 3051
3.412.5 Fricas [A] (verification not implemented) . . . 3051
3.412.6 Sympy [F(-1)] . . . 3052
3.412.7 Maxima [B] (verification not implemented) . . . 3052
3.412.8 Giac [F] . . . 3053
3.412.9 Mupad [F(-1)] . . . 3053

3.412.1 Optimal result

Integrand size = 25, antiderivative size = 201

$$\int \cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \frac{1168a^3\sin(c+dx)}{315d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} + \frac{584a^3\sqrt{\cos(c+dx)}\sin(c+dx)}{315d\sqrt{a+a\sec(c+dx)}} + \frac{146a^3\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{105d\sqrt{a+a\sec(c+dx)}} + \frac{38a^3\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{63d\sqrt{a+a\sec(c+dx)}} + \frac{2a^2\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{9d}$$

output
```
146/105*a^3*cos(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+38/63*a^3
*cos(d*x+c)^(5/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+1168/315*a^3*sin(d*x
+c)/d/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2)+584/315*a^3*sin(d*x+c)*cos(d
*x+c)^(1/2)/d/(a+a*sec(d*x+c))^(1/2)+2/9*a^2*cos(d*x+c)^(7/2)*sin(d*x+c)*(
a+a*sec(d*x+c))^(1/2)/d
```

3.412.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.45

$$\int \cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \frac{2a^2\sqrt{\cos(c+dx)}(584+292\cos(c+dx)+219\cos^2(c+dx)+130\cos^3(c+dx)+35\cos^4(c+}{315d(1+\cos(c+dx))}$$

input
```
Integrate[Cos[c + d*x]^(9/2)*(a + a*Sec[c + d*x])^(5/2),x]
```

output
```
(2*a^2*Sqrt[Cos[c + d*x]]*(584 + 292*Cos[c + d*x] + 219*Cos[c + d*x]^2 + 1
30*Cos[c + d*x]^3 + 35*Cos[c + d*x]^4)*Sqrt[a*(1 + Sec[c + d*x])]*Sin[c +
d*x])/(315*d*(1 + Cos[c + d*x]))
```

3.412.3 Rubi [A] (verified)

Time = 1.11 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.16, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.520$, Rules used = {3042, 4752, 3042, 4300, 27, 3042, 4503, 3042, 4292, 3042, 4292, 3042, 4291}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^{\frac{9}{2}}(c+dx)(a\sec(c+dx)+a)^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^{9/2}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{(\sec(c+dx)a+a)^{5/2}}{\sec^{\frac{9}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{5/2}}{\csc\left(c+dx+\frac{\pi}{2}\right)^{9/2}}dx$$

$$\Big\downarrow \text{4300}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2}{9}a\int \frac{\sqrt{\sec(c+dx)a+a}(15\sec(c+dx)a+19a)}{2\sec^{\frac{7}{2}}(c+dx)}dx+\frac{2a^2\sin(c+dx)\sqrt{a\sec(c+dx}}{9d\sec^{\frac{7}{2}}(c+dx)}\right.$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{9}a\int \frac{\sqrt{\sec(c+dx)a+a}(15\sec(c+dx)a+19a)}{\sec^{\frac{7}{2}}(c+dx)}dx+\frac{2a^2\sin(c+dx)\sqrt{a\sec(c+dx}}{9d\sec^{\frac{7}{2}}(c+dx)}\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{9}a\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}\left(15\csc\left(c+dx+\frac{\pi}{2}\right)a+19a\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx+\frac{2a^2\sin(c+dx)\sqrt{}}{9d\sec^{\frac{7}{2}}(}\right.$$

$\downarrow$ 4503

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{9}a\left(\frac{219}{7}a\int\frac{\sqrt{\sec(c+dx)a+a}}{\sec^{\frac{5}{2}}(c+dx)}dx+\frac{38a^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}\right)+\frac{2a^2\sin}{}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{9}a\left(\frac{219}{7}a\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+\frac{38a^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}\right)+\right.$$

$\downarrow$ 4292

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{9}a\left(\frac{219}{7}a\left(\frac{4}{5}\int\frac{\sqrt{\sec(c+dx)a+a}}{\sec^{\frac{3}{2}}(c+dx)}dx+\frac{2a\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}\right)+\frac{}{7d}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{9}a\left(\frac{219}{7}a\left(\frac{4}{5}\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+\frac{2a\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}\right)\right.$$

$\downarrow$ 4292

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{9}a\left(\frac{219}{7}a\left(\frac{4}{5}\left(\frac{2}{3}\int\frac{\sqrt{\sec(c+dx)a+a}}{\sqrt{\sec(c+dx)}}dx+\frac{2a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}\right)\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{9}a\left(\frac{219}{7}a\left(\frac{4}{5}\left(\frac{2}{3}\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)}}\right.\right.\right.$$

$\downarrow$ 4291

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2a^2\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{9d\sec^{\frac{7}{2}}(c+dx)}+\frac{1}{9}a\left(\frac{38a^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{219}{7}\right.\right.$$

3.412. $\int\cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\,dx$

input
```
Int[Cos[c + d*x]^(9/2)*(a + a*Sec[c + d*x])^(5/2),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*a^2*Sqrt[a + a*Sec[c + d*x]]*Sin
[c + d*x])/(9*d*Sec[c + d*x]^(7/2)) + (a*((38*a^2*Sin[c + d*x])/(7*d*Sec[c
 + d*x]^(5/2)*Sqrt[a + a*Sec[c + d*x]]) + (219*a*((2*a*Sin[c + d*x])/(5*d*
Sec[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]]) + (4*((2*a*Sin[c + d*x])/(3*d
*Sqrt[Sec[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) + (4*a*Sqrt[Sec[c + d*x]]*Si
n[c + d*x])/(3*d*Sqrt[a + a*Sec[c + d*x]])))/5))/7))/9)
```

3.412.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4291
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[-2*a*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*S
qrt[d*Csc[e + f*x]])), x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4292
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[a*Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*n*Sqrt[a
+ b*Csc[e + f*x]])), x] + Simp[a*((2*n + 1)/(2*b*d*n))   Int[Sqrt[a + b*Csc
[e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
 EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

rule 4300
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)*
((d*Csc[e + f*x])^n/(f*n)), x] - Simp[a/(d*n)   Int[(a + b*Csc[e + f*x])^(m
 - 2)*(d*Csc[e + f*x])^(n + 1)*(b*(m - 2*n - 2) - a*(m + 2*n - 1)*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1]
 && (LtQ[n, -1] || (EqQ[m, 3/2] && EqQ[n, -2^(-1)])) && IntegerQ[2*m]
```

rule 4503
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*b^2*Co
t[e + f*x]*((d*Csc[e + f*x])^n/(a*f*n*Sqrt[a + b*Csc[e + f*x]])), x] + Simp
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n)   Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[
e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a
*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

3.412.4 Maple [A] (verified)

Time = 1.23 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.42

$$-\frac{2a^2\sqrt{a\left(1+\sec\left(dx+c\right)\right)}\,\sqrt{\cos\left(dx+c\right)}\,\left(35\cos\left(dx+c\right)^5+95\cos\left(dx+c\right)^4+89\cos\left(dx+c\right)^3+73\cos\left(d\right.\right.}{315d}$$

input
```
int(cos(d*x+c)^(9/2)*(a+a*sec(d*x+c))^(5/2),x)
```

output
```
-2/315/d*a^2*(a*(1+sec(d*x+c)))^(1/2)*cos(d*x+c)^(1/2)*(35*cos(d*x+c)^5+95
*cos(d*x+c)^4+89*cos(d*x+c)^3+73*cos(d*x+c)^2+292*cos(d*x+c)-584)*csc(d*x+
c)
```

3.412.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.52

$$\int \cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\,dx=\frac{2\left(35\,a^2\cos\left(dx+c\right)^4+130\,a^2\cos\left(dx+c\right)^3+219\,a^2\cos\left(dx+c\right)^2+292\,a^2\cos\left(dx+c\right)+584\right.}{315\left(d\cos\left(dx+c\right)+d\right)}$$

input
```
integrate(cos(d*x+c)^(9/2)*(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
2/315*(35*a^2*cos(d*x + c)^4 + 130*a^2*cos(d*x + c)^3 + 219*a^2*cos(d*x +
c)^2 + 292*a^2*cos(d*x + c) + 584*a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x +
 c))*sqrt(cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c) + d)
```

3.412.6 Sympy [F(-1)]

Timed out.

$$\int \cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(9/2)*(a+a*sec(d*x+c))**(5/2),x)
```

output
```
Timed out
```

3.412.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 422 vs. $2(171) = 342$.

Time = 0.39 (sec) , antiderivative size = 422, normalized size of antiderivative = 2.10

$$\int \cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \frac{\sqrt{2}\left(8190\,a^2\cos\left(\frac{8}{9}\arctan\left(\sin\left(\frac{9}{2}\,dx+\frac{9}{2}\,c\right),\cos\left(\frac{9}{2}\,dx+\frac{9}{2}\,c\right)\right)\right)\sin\left(\frac{9}{2}\,dx+\frac{9}{2}\,c\right)+2100\,a^2\cos\left(\right.\right.}{}$$

input
```
integrate(cos(d*x+c)^(9/2)*(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
1/5040*sqrt(2)*(8190*a^2*cos(8/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x
 + 9/2*c)))*sin(9/2*d*x + 9/2*c) + 2100*a^2*cos(2/3*arctan2(sin(9/2*d*x +
9/2*c), cos(9/2*d*x + 9/2*c)))*sin(9/2*d*x + 9/2*c) + 756*a^2*cos(4/9*arct
an2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c)))*sin(9/2*d*x + 9/2*c) + 22
5*a^2*cos(2/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c)))*sin(9/2
*d*x + 9/2*c) - 8190*a^2*cos(9/2*d*x + 9/2*c)*sin(8/9*arctan2(sin(9/2*d*x
+ 9/2*c), cos(9/2*d*x + 9/2*c))) - 2100*a^2*cos(9/2*d*x + 9/2*c)*sin(2/3*a
rctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c))) - 756*a^2*cos(9/2*d*x
+ 9/2*c)*sin(4/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c))) - 22
5*a^2*cos(9/2*d*x + 9/2*c)*sin(2/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d
*x + 9/2*c))) + 70*a^2*sin(9/2*d*x + 9/2*c) + 225*a^2*sin(7/9*arctan2(sin(
9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c))) + 756*a^2*sin(5/9*arctan2(sin(9/2
*d*x + 9/2*c), cos(9/2*d*x + 9/2*c))) + 2100*a^2*sin(1/3*arctan2(sin(9/2*d
*x + 9/2*c), cos(9/2*d*x + 9/2*c))) + 8190*a^2*sin(1/9*arctan2(sin(9/2*d*x
 + 9/2*c), cos(9/2*d*x + 9/2*c))))*sqrt(a)/d
```

3.412.8 Giac [F]

$$\int \cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \int (a\sec(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^{\frac{9}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(9/2)*(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.412.9 Mupad [F(-1)]

Timed out.

$$\int \cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \int \cos(c+d\,x)^{9/2}\left(a+\frac{a}{\cos(c+d\,x)}\right)^{5/2}dx$$

input
```
int(cos(c + d*x)^(9/2)*(a + a/cos(c + d*x))^(5/2),x)
```

output
```
int(cos(c + d*x)^(9/2)*(a + a/cos(c + d*x))^(5/2), x)
```

3.413 $\int \cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\,dx$

3.413.1 Optimal result . . . 3054
3.413.2 Mathematica [A] (verified) . . . 3055
3.413.3 Rubi [A] (verified) . . . 3055
3.413.4 Maple [A] (verified) . . . 3058
3.413.5 Fricas [A] (verification not implemented) . . . 3058
3.413.6 Sympy [F(-1)] . . . 3058
3.413.7 Maxima [B] (verification not implemented) . . . 3059
3.413.8 Giac [F] . . . 3059
3.413.9 Mupad [F(-1)] . . . 3060

3.413.1 Optimal result

Integrand size = 25, antiderivative size = 156

$$\int \cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \frac{64a^3\sin(c+dx)}{21d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} + \frac{16a^2\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{21d} + \frac{2a\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\sin(c+dx)}{7d} + \frac{2\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\sin(c+dx)}{7d}$$

output
```
2/7*a*cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(3/2)*sin(d*x+c)/d+2/7*cos(d*x+c)^
(5/2)*(a+a*sec(d*x+c))^(5/2)*sin(d*x+c)/d+64/21*a^3*sin(d*x+c)/d/cos(d*x+c
)^(1/2)/(a+a*sec(d*x+c))^(1/2)+16/21*a^2*sin(d*x+c)*cos(d*x+c)^(1/2)*(a+a*
sec(d*x+c))^(1/2)/d
```

3.413.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.47

$$\int \cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \frac{a^2\sqrt{\cos(c+dx)}(208+101\cos(c+dx)+24\cos(2(c+dx))+3\cos(3(c+dx)))\sqrt{a(1+\sec(c}}{42d}$$

input
```
Integrate[Cos[c + d*x]^(7/2)*(a + a*Sec[c + d*x])^(5/2),x]
```

output
```
(a^2*Sqrt[Cos[c + d*x]]*(208 + 101*Cos[c + d*x] + 24*Cos[2*(c + d*x)] + 3*
Cos[3*(c + d*x)])*Sqrt[a*(1 + Sec[c + d*x])]*Tan[(c + d*x)/2])/(42*d)
```

3.413.3 Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.19, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 4752, 3042, 4299, 3042, 4296, 3042, 4296, 3042, 4291}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^{\frac{7}{2}}(c+dx)(a\sec(c+dx)+a)^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^{7/2}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{(\sec(c+dx)a+a)^{5/2}}{\sec^{\frac{7}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{5/2}}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx$$

$$\Big\downarrow \text{4299}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{5}{7}\int\frac{(\sec(c+dx)a+a)^{5/2}}{\sec^{\frac{5}{2}}(c+dx)}dx+\frac{2\sin(c+dx)(a\sec(c+dx)+a)^{5/2}}{7d\sec^{\frac{5}{2}}(c+dx)}\right)$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{5}{7}\int\frac{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{5/2}}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+\frac{2\sin(c+dx)(a\sec(c+dx)+a)^{5/2}}{7d\sec^{\frac{5}{2}}(c+dx)}\right)$$

$\Big\downarrow$ 4296

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{5}{7}\left(\frac{8}{5}a\int\frac{(\sec(c+dx)a+a)^{3/2}}{\sec^{\frac{3}{2}}(c+dx)}dx+\frac{2a\sin(c+dx)(a\sec(c+dx)+a)^{3/2}}{5d\sec^{\frac{3}{2}}(c+dx)}\right)+\frac{2\sin(c}{}\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{5}{7}\left(\frac{8}{5}a\int\frac{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+\frac{2a\sin(c+dx)(a\sec(c+dx)+a)^{3/2}}{5d\sec^{\frac{3}{2}}(c+dx)}\right)+\right.$$

$\Big\downarrow$ 4296

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{5}{7}\left(\frac{8}{5}a\left(\frac{4}{3}a\int\frac{\sqrt{\sec(c+dx)a+a}}{\sqrt{\sec(c+dx)}}dx+\frac{2a\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{3d\sqrt{\sec(c+dx)}}\right)+\frac{2a\sin}{}\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{5}{7}\left(\frac{8}{5}a\left(\frac{4}{3}a\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{3d\sqrt{\sec(c+dx)}}\right)+\right.$$

$\Big\downarrow$ 4291

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{5}{7}\left(\frac{8}{5}a\left(\frac{8a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\sqrt{a\sec(c+dx)+a}}+\frac{2a\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{3d\sqrt{\sec(c+dx)}}\right)+\frac{2a\sin}{}\right.$$

input
```
Int[Cos[c + d*x]^(7/2)*(a + a*Sec[c + d*x])^(5/2),x]
```

3.413. $\int \cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\,dx$

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*(a + a*Sec[c + d*x])^(5/2)*Sin[c
 + d*x])/(7*d*Sec[c + d*x]^(5/2)) + (5*((2*a*(a + a*Sec[c + d*x])^(3/2)*Si
n[c + d*x])/(5*d*Sec[c + d*x]^(3/2)) + (8*a*((8*a^2*Sqrt[Sec[c + d*x]]*Sin
[c + d*x])/(3*d*Sqrt[a + a*Sec[c + d*x]]) + (2*a*Sqrt[a + a*Sec[c + d*x]]*
Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])))/5))/7)
```

3.413.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4291
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[-2*a*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*S
qrt[d*Csc[e + f*x]])), x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4296
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-a)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)
*((d*Csc[e + f*x])^n/(f*m)), x] + Simp[b*((2*m - 1)/(d*m))   Int[(a + b*Csc
[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f
, m, n}, x] && EqQ[a^2 - b^2, 0] && EqQ[m + n, 0] && GtQ[m, 1/2] && Integer
Q[2*m]
```

rule 4299
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-Cot[e + f*x])*(a + b*Csc[e + f*x])^m*((d*Csc
[e + f*x])^n/(f*(m + 1))), x] + Simp[a*(m/(b*d*(m + 1)))   Int[(a + b*Csc[e
 + f*x])^m*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m, n},
 x] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LtQ[m, -2^(-1)]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

3.413.4 Maple [A] (verified)

Time = 0.98 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.48

$$-\frac{2a^2\sqrt{a\left(1+\sec\left(dx+c\right)\right)}\,\sqrt{\cos\left(dx+c\right)}\,\left(3\cos\left(dx+c\right)^4+9\cos\left(dx+c\right)^3+11\cos\left(dx+c\right)^2+23\cos\left(dx+\right.\right.}{21d}$$

input
```
int(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))^(5/2),x)
```

output
```
-2/21/d*a^2*(a*(1+sec(d*x+c)))^(1/2)*cos(d*x+c)^(1/2)*(3*cos(d*x+c)^4+9*co
s(d*x+c)^3+11*cos(d*x+c)^2+23*cos(d*x+c)-46)*csc(d*x+c)
```

3.413.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.59

$$\int \cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\,dx=\frac{2\left(3\,a^2\cos\left(dx+c\right)^3+12\,a^2\cos\left(dx+c\right)^2+23\,a^2\cos\left(dx+c\right)+46\,a^2\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos\left(dx\right.}}{21\left(d\cos\left(dx+c\right)+d\right)}$$

input
```
integrate(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
2/21*(3*a^2*cos(d*x + c)^3 + 12*a^2*cos(d*x + c)^2 + 23*a^2*cos(d*x + c) +
 46*a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*
x + c)/(d*cos(d*x + c) + d)
```

3.413.6 Sympy [F(-1)]

Timed out.

$$\int \cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\,dx=\text{Timed out}$$

input
```
integrate(cos(d*x+c)**(7/2)*(a+a*sec(d*x+c))**(5/2),x)
```

output
```
Timed out
```

3.413.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 323 vs. $2(132) = 264$.

Time = 0.39 (sec) , antiderivative size = 323, normalized size of antiderivative = 2.07

$$\int \cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \frac{\sqrt{2}\left(315\,a^2\cos\left(\frac{6}{7}\arctan\left(\sin\left(\frac{7}{2}\,dx+\frac{7}{2}\,c\right),\cos\left(\frac{7}{2}\,dx+\frac{7}{2}\,c\right)\right)\right)\sin\left(\frac{7}{2}\,dx+\frac{7}{2}\,c\right)+77\,a^2\cos\left(\frac{4}{7}\,\mathrm{ar}\right.\right.}{}$$

input
```
integrate(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
1/168*sqrt(2)*(315*a^2*cos(6/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x +
 7/2*c)))*sin(7/2*d*x + 7/2*c) + 77*a^2*cos(4/7*arctan2(sin(7/2*d*x + 7/2*
c), cos(7/2*d*x + 7/2*c)))*sin(7/2*d*x + 7/2*c) + 21*a^2*cos(2/7*arctan2(s
in(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c)))*sin(7/2*d*x + 7/2*c) - 315*a^2
*cos(7/2*d*x + 7/2*c)*sin(6/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x +
7/2*c))) - 77*a^2*cos(7/2*d*x + 7/2*c)*sin(4/7*arctan2(sin(7/2*d*x + 7/2*c
), cos(7/2*d*x + 7/2*c))) - 21*a^2*cos(7/2*d*x + 7/2*c)*sin(2/7*arctan2(si
n(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) + 6*a^2*sin(7/2*d*x + 7/2*c) +
21*a^2*sin(5/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) + 77*a
^2*sin(3/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) + 315*a^2*
sin(1/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))))*sqrt(a)/d
```

3.413.8 Giac [F]

$$\int \cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \int (a\sec(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^{\frac{7}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.413.9 Mupad [F(-1)]

Timed out.

$$\int \cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\,dx=\int \cos\left(c+d\,x\right)^{7/2}\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)^{5/2}dx$$

input
```
int(cos(c + d*x)^(7/2)*(a + a/cos(c + d*x))^(5/2),x)
```

output
```
int(cos(c + d*x)^(7/2)*(a + a/cos(c + d*x))^(5/2), x)
```

3.414 $\int \cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\,dx$

3.414.1 Optimal result . . . 3061
3.414.2 Mathematica [A] (verified) . . . 3061
3.414.3 Rubi [A] (verified) . . . 3062
3.414.4 Maple [A] (verified) . . . 3064
3.414.5 Fricas [A] (verification not implemented) . . . 3064
3.414.6 Sympy [F(-1)] . . . 3065
3.414.7 Maxima [A] (verification not implemented) . . . 3065
3.414.8 Giac [F] . . . 3065
3.414.9 Mupad [F(-1)] . . . 3066

3.414.1 Optimal result

Integrand size = 25, antiderivative size = 119

$$\int \cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \frac{64a^3\sin(c+dx)}{15d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} + \frac{16a^2\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{15d} + \frac{2a\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\sin(c+dx)}{5d}$$

output
```
2/5*a*cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(3/2)*sin(d*x+c)/d+64/15*a^3*sin(d
*x+c)/d/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2)+16/15*a^2*sin(d*x+c)*cos(d
*x+c)^(1/2)*(a+a*sec(d*x+c))^(1/2)/d
```

3.414.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.54

$$\int \cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \frac{a^2\sqrt{\cos(c+dx)}(89+28\cos(c+dx)+3\cos(2(c+dx)))\sqrt{a(1+\sec(c+dx))}\tan\left(\frac{1}{2}(c+dx)\right)}{15d}$$

input
```
Integrate[Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^(5/2),x]
```

output
```
(a^2*Sqrt[Cos[c + d*x]]*(89 + 28*Cos[c + d*x] + 3*Cos[2*(c + d*x)])*Sqrt[a
*(1 + Sec[c + d*x])]*Tan[(c + d*x)/2])/(15*d)
```

3.414.3 Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.21, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3042, 4752, 3042, 4296, 3042, 4296, 3042, 4291}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^{\frac{5}{2}}(c+dx)(a\sec(c+dx)+a)^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{(\sec(c+dx)a+a)^{5/2}}{\sec^{\frac{5}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{5/2}}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx$$

$$\Big\downarrow \text{4296}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{8}{5}a\int \frac{(\sec(c+dx)a+a)^{3/2}}{\sec^{\frac{3}{2}}(c+dx)}dx+\frac{2a\sin(c+dx)(a\sec(c+dx)+a)^{3/2}}{5d\sec^{\frac{3}{2}}(c+dx)}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{8}{5}a\int \frac{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+\frac{2a\sin(c+dx)(a\sec(c+dx)+a)^{3/2}}{5d\sec^{\frac{3}{2}}(c+dx)}\right)$$

$$\Big\downarrow \text{4296}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{8}{5}a\left(\frac{4}{3}a\int \frac{\sqrt{\sec(c+dx)a+a}}{\sqrt{\sec(c+dx)}}dx+\frac{2a\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{3d\sqrt{\sec(c+dx)}}\right)+\frac{2a\sin(c+}{}\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{8}{5}a\left(\frac{4}{3}a\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{3d\sqrt{\sec(c+dx)}}\right)+\frac{2a\,s}{}$$

$$\Big\downarrow \text{4291}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{8}{5}a\left(\frac{8a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\sqrt{a\sec(c+dx)+a}}+\frac{2a\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{3d\sqrt{\sec(c+dx)}}\right)+\frac{2a\sin(c}{}$$

input
```
Int[Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^(5/2),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*a*(a + a*Sec[c + d*x])^(3/2)*Sin
[c + d*x])/(5*d*Sec[c + d*x]^(3/2)) + (8*a*((8*a^2*Sqrt[Sec[c + d*x]]*Sin[
c + d*x])/(3*d*Sqrt[a + a*Sec[c + d*x]]) + (2*a*Sqrt[a + a*Sec[c + d*x]]*S
in[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])))/5)
```

3.414.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4291
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[-2*a*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*S
qrt[d*Csc[e + f*x]])), x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4296
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-a)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)
*((d*Csc[e + f*x])^n/(f*m)), x] + Simp[b*((2*m - 1)/(d*m))   Int[(a + b*Csc
[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f
, m, n}, x] && EqQ[a^2 - b^2, 0] && EqQ[m + n, 0] && GtQ[m, 1/2] && Integer
Q[2*m]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

3.414.4 Maple [A] (verified)

Time = 1.03 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.55

$$-\frac{2a^2\sqrt{a\left(1+\sec\left(dx+c\right)\right)}\,\sqrt{\cos\left(dx+c\right)}\,\left(3\cos\left(dx+c\right)^3+11\cos\left(dx+c\right)^2+29\cos\left(dx+c\right)-43\right)\csc\left(dx\right.}{15d}$$

input
```
int(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(5/2),x)
```

output
```
-2/15/d*a^2*(a*(1+sec(d*x+c)))^(1/2)*cos(d*x+c)^(1/2)*(3*cos(d*x+c)^3+11*c
os(d*x+c)^2+29*cos(d*x+c)-43)*csc(d*x+c)
```

3.414.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.66

$$\int \cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\,dx=\frac{2\left(3\,a^2\cos\left(dx+c\right)^2+14\,a^2\cos\left(dx+c\right)+43\,a^2\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)}{15\left(d\cos\left(dx+c\right)+d\right)}$$

input
```
integrate(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
2/15*(3*a^2*cos(d*x + c)^2 + 14*a^2*cos(d*x + c) + 43*a^2)*sqrt((a*cos(d*x
 + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c) +
 d)
```

3.414.6 Sympy [F(-1)]

Timed out.

$$\int \cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(5/2)*(a+a*sec(d*x+c))**(5/2),x)
```

output
```
Timed out
```

3.414.7 Maxima [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.50

$$\int \cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \frac{\left(3\sqrt{2}a^2\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right)+25\sqrt{2}a^2\sin\left(\frac{3}{2}dx+\frac{3}{2}c\right)+150\sqrt{2}a^2\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)\sqrt{a}}{30\,d}$$

input
```
integrate(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
1/30*(3*sqrt(2)*a^2*sin(5/2*d*x + 5/2*c) + 25*sqrt(2)*a^2*sin(3/2*d*x + 3/
2*c) + 150*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c))*sqrt(a)/d
```

3.414.8 Giac [F]

$$\int \cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \int (a\sec(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^{\frac{5}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.414.9 Mupad [F(-1)]

Timed out.

$$\int \cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\,dx=\int \cos{(c+d\,x)}^{5/2}\left(a+\frac{a}{\cos{(c+d\,x)}}\right)^{5/2}dx$$

input
```
int(cos(c + d*x)^(5/2)*(a + a/cos(c + d*x))^(5/2),x)
```

output
```
int(cos(c + d*x)^(5/2)*(a + a/cos(c + d*x))^(5/2), x)
```

3.415 $\int \cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\,dx$

3.415.1 Optimal result . . . 3067
3.415.2 Mathematica [A] (verified) . . . 3067
3.415.3 Rubi [A] (verified) . . . 3068
3.415.4 Maple [B] (verified) . . . 3071
3.415.5 Fricas [A] (verification not implemented) . . . 3071
3.415.6 Sympy [F(-1)] . . . 3072
3.415.7 Maxima [B] (verification not implemented) . . . 3072
3.415.8 Giac [F] . . . 3073
3.415.9 Mupad [F(-1)] . . . 3074

3.415.1 Optimal result

Integrand size = 25, antiderivative size = 138

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \frac{2a^{5/2}\operatorname{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{d} + \frac{14a^3\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} + \frac{2a^2\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{3d}$$

output
```
2*a^(5/2)*arcsinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))*cos(d*x+c)^(1
/2)*sec(d*x+c)^(1/2)/d+14/3*a^3*sin(d*x+c)/d/cos(d*x+c)^(1/2)/(a+a*sec(d*x
+c))^(1/2)+2/3*a^2*sin(d*x+c)*cos(d*x+c)^(1/2)*(a+a*sec(d*x+c))^(1/2)/d
```

3.415.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.67

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \frac{2a^3\left((8+\cos(c+dx))\sqrt{1-\sec(c+dx)}+3\arcsin\left(\sqrt{1-\sec(c+dx)}\right)\sqrt{\sec(c+dx)}\right)\sin(c}{3d\sqrt{-1+\cos(c+dx)}\sqrt{a(1+\sec(c+dx))}}$$

input
```
Integrate[Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(5/2),x]
```

output
```
(2*a^3*((8 + Cos[c + d*x])*Sqrt[1 - Sec[c + d*x]] + 3*ArcSin[Sqrt[1 - Sec[
c + d*x]]]*Sqrt[Sec[c + d*x]])*Sin[c + d*x])/(3*d*Sqrt[-1 + Cos[c + d*x]]*
Sqrt[a*(1 + Sec[c + d*x])])
```

3.415.3 Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.04, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 4752, 3042, 4300, 27, 3042, 4503, 3042, 4288, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^{5/2}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}dx$$

$$\Big\downarrow \; 4752$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{(\sec(c+dx)a+a)^{5/2}}{\sec^{\frac{3}{2}}(c+dx)}dx$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{5/2}}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx$$

$$\Big\downarrow \; 4300$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2}{3}a\int \frac{\sqrt{\sec(c+dx)a+a}(3\sec(c+dx)a+7a)}{2\sqrt{\sec(c+dx)}}dx+\frac{2a^2\sin(c+dx)\sqrt{a\sec(c+dx)+}}{3d\sqrt{\sec(c+dx)}}\right.$$

$$\Big\downarrow \; 27$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}a\int \frac{\sqrt{\sec(c+dx)a+a}(3\sec(c+dx)a+7a)}{\sqrt{\sec(c+dx)}}dx+\frac{2a^2\sin(c+dx)\sqrt{a\sec(c+dx)+}}{3d\sqrt{\sec(c+dx)}}\right.$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}a\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}\left(3\csc\left(c+dx+\frac{\pi}{2}\right)a+7a\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a^2\sin(c+dx)\sqrt{a\,\mathrm{s}}}{3d\sqrt{\sec(c}}\right.$$

$\downarrow$ 4503

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}a\left(3a\int\sqrt{\sec(c+dx)}\sqrt{\sec(c+dx)a+a}dx+\frac{14a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\sec(c+dx)+a}}\right)+\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}a\left(3a\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx+\frac{14a^2\sin(c+dx)\sqrt{\sec(}}{d\sqrt{a\sec(c+dx)+}}\right.\right.$$

$\downarrow$ 4288

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}a\left(\frac{14a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\sec(c+dx)+a}}-\frac{6a\int\frac{1}{\sqrt{\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}}+1}d\left(-\frac{a\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}\right)+\right.$$

$\downarrow$ 222

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2a^2\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{3d\sqrt{\sec(c+dx)}}+\frac{1}{3}a\left(\frac{6a^{3/2}\text{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{14a^2\sin(}{d\sqrt{}}\right.\right.$$

input
```
Int[Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(5/2),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*a^2*Sqrt[a + a*Sec[c + d*x]]*Sin
[c + d*x])/(3*d*Sqrt[Sec[c + d*x]]) + (a*((6*a^(3/2)*ArcSinh[(Sqrt[a]*Tan[
c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d + (14*a^2*Sqrt[Sec[c + d*x]]*Sin[c
+ d*x])/(d*Sqrt[a + a*Sec[c + d*x]])))/3)
```

3.415. $\int \cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\,dx$

3.415.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4288
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[-2*(a/(b*f))*Sqrt[a*(d/b)]   Subst[Int[1/Sqrt[1
+ x^2/a], x], x, b*(Cot[e + f*x]/Sqrt[a + b*Csc[e + f*x]])], x] /; FreeQ[{a
, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[a*(d/b), 0]
```

rule 4300
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)*
((d*Csc[e + f*x])^n/(f*n)), x] - Simp[a/(d*n)   Int[(a + b*Csc[e + f*x])^(m
 - 2)*(d*Csc[e + f*x])^(n + 1)*(b*(m - 2*n - 2) - a*(m + 2*n - 1)*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1]
 && (LtQ[n, -1] || (EqQ[m, 3/2] && EqQ[n, -2^(-1)])) && IntegerQ[2*m]
```

rule 4503
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*b^2*Co
t[e + f*x]*((d*Csc[e + f*x])^n/(a*f*n*Sqrt[a + b*Csc[e + f*x]])), x] + Simp
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n)   Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[
e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a
*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

3.415.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 305 vs. $2(116) = 232$.

Time = 2.11 (sec) , antiderivative size = 306, normalized size of antiderivative = 2.22

method	result
default	$\frac{a^2\left(3\arctan\left(\frac{-\cos(dx+c)+\sin(dx+c)-1}{2(\cos(dx+c)+1)\sqrt{-\frac{1}{\cos(dx+c)+1}}}\right)\sqrt{-\frac{1}{\cos(dx+c)+1}}\cos(dx+c)+3\sqrt{-\frac{1}{\cos(dx+c)+1}}\arctan\left(\frac{\cos(dx+c)+\sin(dx+c)+1}{2(\cos(dx+c)+1)\sqrt{-\frac{1}{\cos(dx+c)-}}}\right.\right.}{}$

input
```
int(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/3/d*a^2*(3*arctan(1/2*(-cos(d*x+c)+sin(d*x+c)-1)/(cos(d*x+c)+1)/(-1/(cos
(d*x+c)+1))^(1/2))*(-1/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)+3*(-1/(cos(d*x+c)+
1))^(1/2)*arctan(1/2*(cos(d*x+c)+sin(d*x+c)+1)/(cos(d*x+c)+1)/(-1/(cos(d*x
+c)+1))^(1/2))*cos(d*x+c)+3*arctan(1/2*(-cos(d*x+c)+sin(d*x+c)-1)/(cos(d*x
+c)+1)/(-1/(cos(d*x+c)+1))^(1/2))*(-1/(cos(d*x+c)+1))^(1/2)+3*(-1/(cos(d*x
+c)+1))^(1/2)*arctan(1/2*(cos(d*x+c)+sin(d*x+c)+1)/(cos(d*x+c)+1)/(-1/(cos
(d*x+c)+1))^(1/2))+2*cos(d*x+c)*sin(d*x+c)+16*sin(d*x+c))*(a*(1+sec(d*x+c)
))^(1/2)*cos(d*x+c)^(1/2)/(cos(d*x+c)+1)
```

3.415.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 339, normalized size of antiderivative = 2.46

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \left[\frac{4\left(a^2\cos\left(dx+c\right)+8\,a^2\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)+3\left(a^2\cos\left(dx+c\right)+a^2\right)\sqrt{}}{6\left(d\cos\left(dx+c\right)\right.}\right.$$

input
```
integrate(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
[1/6*(4*(a^2*cos(d*x + c) + 8*a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))
*sqrt(cos(d*x + c))*sin(d*x + c) + 3*(a^2*cos(d*x + c) + a^2)*sqrt(a)*log(
(a*cos(d*x + c)^3 - 4*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(cos
(d*x + c) - 2)*sqrt(cos(d*x + c))*sin(d*x + c) - 7*a*cos(d*x + c)^2 + 8*a)
/(cos(d*x + c)^3 + cos(d*x + c)^2)))/(d*cos(d*x + c) + d), 1/3*(2*(a^2*cos
(d*x + c) + 8*a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x +
c))*sin(d*x + c) + 3*(a^2*cos(d*x + c) + a^2)*sqrt(-a)*arctan(2*sqrt(-a)*s
qrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*
cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)))/(d*cos(d*x + c) + d)]
```

3.415.6 Sympy [F(-1)]

Timed out.

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(3/2)*(a+a*sec(d*x+c))**(5/2),x)
```

output
```
Timed out
```

3.415.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 593 vs. $2(116) = 232$.

Time = 0.40 (sec) , antiderivative size = 593, normalized size of antiderivative = 4.30

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
1/12*sqrt(2)*(30*a^2*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3
/2*c)))*sin(3/2*d*x + 3/2*c) - 30*a^2*cos(3/2*d*x + 3/2*c)*sin(2/3*arctan2
(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 3*sqrt(2)*a^2*log(2*cos(1/
3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arcta
n2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sqrt(2)*cos(1/3*arct
an2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sqrt(2)*sin(1/3*arcta
n2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) - 3*sqrt(2)*a^2*log(2
*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/
3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sqrt(2)*cos(1
/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 2*sqrt(2)*sin(1/
3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) + 3*sqrt(2)*a^
2*log(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2
*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 - 2*sqrt(2
)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sqrt(2)
*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) - 3*sqr
t(2)*a^2*log(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))
)^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 - 2
*sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 2*
sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2)
+ 4*a^2*sin(3/2*d*x + 3/2*c) + 30*a^2*sin(1/3*arctan2(sin(3/2*d*x + 3/2...
```

3.415.8 Giac [F]

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \int (a\sec(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^{\frac{3}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.415.9 Mupad [F(-1)]

Timed out.

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\,dx=\int \cos{(c+d\,x)}^{3/2}\left(a+\frac{a}{\cos{(c+d\,x)}}\right)^{5/2}dx$$

input
```
int(cos(c + d*x)^(3/2)*(a + a/cos(c + d*x))^(5/2),x)
```

output
```
int(cos(c + d*x)^(3/2)*(a + a/cos(c + d*x))^(5/2), x)
```

3.416 $\int \sqrt{\cos(c+dx)}(a+a\sec(c+dx))^{5/2}\,dx$

3.416.1 Optimal result . . . 3075
3.416.2 Mathematica [A] (verified) . . . 3075
3.416.3 Rubi [A] (verified) . . . 3076
3.416.4 Maple [B] (verified) . . . 3079
3.416.5 Fricas [A] (verification not implemented) . . . 3079
3.416.6 Sympy [F(-1)] . . . 3080
3.416.7 Maxima [B] (verification not implemented) . . . 3080
3.416.8 Giac [F] . . . 3081
3.416.9 Mupad [F(-1)] . . . 3082

3.416.1 Optimal result

Integrand size = 25, antiderivative size = 132

$$\int \sqrt{\cos(c+dx)}(a+a\sec(c+dx))^{5/2}\,dx = \frac{5a^{5/2}\operatorname{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{d} + \frac{a^3\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} + \frac{a^2\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

output
```
5*a^(5/2)*arcsinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))*cos(d*x+c)^(1
/2)*sec(d*x+c)^(1/2)/d+a^3*sin(d*x+c)/d/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^
(1/2)+a^2*sin(d*x+c)*(a+a*sec(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)
```

3.416.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.68

$$\int \sqrt{\cos(c+dx)}(a+a\sec(c+dx))^{5/2}\,dx = \frac{a^3\left(5\arcsin\left(\sqrt{1-\sec(c+dx)}\right)\sqrt{\sec(c+dx)}+\sqrt{1-\sec(c+dx)}(2+\sec(c+dx))\right)\sin(c+}{d\sqrt{-1+\cos(c+dx)}\sqrt{a(1+\sec(c+dx))}}$$

input
```
Integrate[Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])^(5/2),x]
```

output
```
(a^3*(5*ArcSin[Sqrt[1 - Sec[c + d*x]]]*Sqrt[Sec[c + d*x]] + Sqrt[1 - Sec[c
 + d*x]]*(2 + Sec[c + d*x]))*Sin[c + d*x])/(d*Sqrt[-1 + Cos[c + d*x]]*Sqrt
[a*(1 + Sec[c + d*x])])
```

3.416.3 Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.06, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 4752, 3042, 4301, 27, 3042, 4503, 3042, 4288, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\cos(c+dx)}(a\sec(c+dx)+a)^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{(\sec(c+dx)a+a)^{5/2}}{\sqrt{\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{5/2}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4301}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(a\int \frac{\sqrt{\sec(c+dx)a+a}(5\sec(c+dx)a+a)}{2\sqrt{\sec(c+dx)}}dx+\frac{a^2\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a\sec}}{d}\right.$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{2}a\int \frac{\sqrt{\sec(c+dx)a+a}(5\sec(c+dx)a+a)}{\sqrt{\sec(c+dx)}}dx+\frac{a^2\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a\,\mathrm{se}}}{d}\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{2}a\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}\left(5\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{a^2\sin(c+dx)\sqrt{\sec(c}}{}\right.$$

$\Big\downarrow$ 4503

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{2}a\left(5a\int\sqrt{\sec(c+dx)}\sqrt{\sec(c+dx)a+a}dx+\frac{2a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\sec(c+dx)+a}}\right)+\frac{a^2}{}\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{2}a\left(5a\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx+\frac{2a^2\sin(c+dx)\sqrt{\sec(c}}{d\sqrt{a\sec(c+dx)+}}\right.\right.$$

$\Big\downarrow$ 4288

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{2}a\left(\frac{2a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\sec(c+dx)+a}}-\frac{10a\int\frac{1}{\sqrt{\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}}+1}d\left(-\frac{a\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}\right)+\frac{a}{}\right.$$

$\Big\downarrow$ 222

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{a^2\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}{d}+\frac{1}{2}a\left(\frac{10a^{3/2}\text{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right.}{d}\right.\right.$$

input
```
Int[Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])^(5/2),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((a^2*Sqrt[Sec[c + d*x]]*Sqrt[a + a*
Sec[c + d*x]]*Sin[c + d*x])/d + (a*((10*a^(3/2)*ArcSinh[(Sqrt[a]*Tan[c + d
*x])/Sqrt[a + a*Sec[c + d*x]]])/d + (2*a^2*Sqrt[Sec[c + d*x]]*Sin[c + d*x]
)/(d*Sqrt[a + a*Sec[c + d*x]])))/2)
```

3.416. $\int\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^{5/2}\,dx$

3.416.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4288
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[-2*(a/(b*f))*Sqrt[a*(d/b)]   Subst[Int[1/Sqrt[1
+ x^2/a], x], x, b*(Cot[e + f*x]/Sqrt[a + b*Csc[e + f*x]])], x] /; FreeQ[{a
, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[a*(d/b), 0]
```

rule 4301
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
2)*((d*Csc[e + f*x])^n/(f*(m + n - 1))), x] + Simp[b/(m + n - 1)   Int[(a +
 b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*(b*(m + 2*n - 1) + a*(3*m + 2*n
 - 4)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^
2, 0] && GtQ[m, 1] && NeQ[m + n - 1, 0] && IntegerQ[2*m]
```

rule 4503
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*b^2*Co
t[e + f*x]*((d*Csc[e + f*x])^n/(a*f*n*Sqrt[a + b*Csc[e + f*x]])), x] + Simp
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n)   Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[
e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a
*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

3.416.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 321 vs. $2(114) = 228$.

Time = 2.16 (sec) , antiderivative size = 322, normalized size of antiderivative = 2.44

method	result
default	$\frac{a^2\left(5\sqrt{-\frac{1}{\cos(dx+c)+1}}\,\arctan\left(\frac{-\cos(dx+c)+\sin(dx+c)-1}{2(\cos(dx+c)+1)\sqrt{-\frac{1}{\cos(dx+c)+1}}}\right)\cos(dx+c)^2+5\sqrt{-\frac{1}{\cos(dx+c)+1}}\,\arctan\left(\frac{\cos(dx+c)+\sin(dx+c)+1}{2(\cos(dx+c)+1)\sqrt{-\frac{1}{\cos(dx+c}}}\right.\right.}{}$

input
```
int((a+a*sec(d*x+c))^(5/2)*cos(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/d*a^2*(5*(-1/(cos(d*x+c)+1))^(1/2)*arctan(1/2*(-cos(d*x+c)+sin(d*x+c)-
1)/(cos(d*x+c)+1)/(-1/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)^2+5*(-1/(cos(d*x+c
)+1))^(1/2)*arctan(1/2*(cos(d*x+c)+sin(d*x+c)+1)/(cos(d*x+c)+1)/(-1/(cos(d
*x+c)+1))^(1/2))*cos(d*x+c)^2+5*arctan(1/2*(-cos(d*x+c)+sin(d*x+c)-1)/(cos
(d*x+c)+1)/(-1/(cos(d*x+c)+1))^(1/2))*(-1/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)
+5*(-1/(cos(d*x+c)+1))^(1/2)*arctan(1/2*(cos(d*x+c)+sin(d*x+c)+1)/(cos(d*x
+c)+1)/(-1/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)+4*cos(d*x+c)*sin(d*x+c)+2*sin
(d*x+c))*(a*(1+sec(d*x+c)))^(1/2)/(cos(d*x+c)+1)/cos(d*x+c)^(1/2)
```

3.416.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 373, normalized size of antiderivative = 2.83

$$\int \sqrt{\cos(c+dx)}(a+a\sec(c+dx))^{5/2}\,dx = \left[\frac{4\left(2\,a^2\cos\left(dx+c\right)+a^2\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)+5\left(a^2\cos\left(dx+c\right)^2+a^2\right.}{4\left(d\cos\left(dx+c\right)\right.}\right.$$

input
```
integrate((a+a*sec(d*x+c))^(5/2)*cos(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
[1/4*(4*(2*a^2*cos(d*x + c) + a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))
*sqrt(cos(d*x + c))*sin(d*x + c) + 5*(a^2*cos(d*x + c)^2 + a^2*cos(d*x + c
))*sqrt(a)*log((a*cos(d*x + c)^3 - 4*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos
(d*x + c))*(cos(d*x + c) - 2)*sqrt(cos(d*x + c))*sin(d*x + c) - 7*a*cos(d*
x + c)^2 + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)))/(d*cos(d*x + c)^2 + d*
cos(d*x + c)), 1/2*(2*(2*a^2*cos(d*x + c) + a^2)*sqrt((a*cos(d*x + c) + a)
/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) + 5*(a^2*cos(d*x + c)^2 + a
^2*cos(d*x + c))*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(
d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x +
c) - 2*a)))/(d*cos(d*x + c)^2 + d*cos(d*x + c))]
```

3.416.6 Sympy [F(-1)]

Timed out.

$$\int \sqrt{\cos(c+dx)}(a+a\sec(c+dx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sec(d*x+c))**(5/2)*cos(d*x+c)**(1/2),x)
```

output
```
Timed out
```

3.416.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 11494 vs. $2(114) = 228$.

Time = 0.49 (sec) , antiderivative size = 11494, normalized size of antiderivative = 87.08

$$\int \sqrt{\cos(c+dx)}(a+a\sec(c+dx))^{5/2}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sec(d*x+c))^(5/2)*cos(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
1/4*(8*a^2*cos(1/2*d*x + 1/2*c)^4*sin(1/2*d*x + 1/2*c) + 16*a^2*cos(1/2*d*
x + 1/2*c)^2*sin(1/2*d*x + 1/2*c)^3 + 8*a^2*sin(1/2*d*x + 1/2*c)^5 + 5*(sq
rt(2)*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt
(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - sqrt(2)*a
^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos
(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + sqrt(2)*a^2*log(
2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*
x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - sqrt(2)*a^2*log(2*cos(1
/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2
*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*cos(1/2*d*x + 1/2*c)^4 + 10*(sq
rt(2)*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt
(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - sqrt(2)*a
^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos
(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + sqrt(2)*a^2*log(
2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*
x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - sqrt(2)*a^2*log(2*cos(1
/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2
*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*cos(1/2*d*x + 1/2*c)^2*sin(1/2*
d*x + 1/2*c)^2 + 5*(sqrt(2)*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d
*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x ...
```

3.416.8 Giac [F]

$$\int \sqrt{\cos(c+dx)}(a+a\sec(c+dx))^{5/2}\,dx = \int (a\sec(dx+c)+a)^{\frac{5}{2}}\sqrt{\cos(dx+c)}\,dx$$

input
```
integrate((a+a*sec(d*x+c))^(5/2)*cos(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.416.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{\cos(c+dx)}(a+a\sec(c+dx))^{5/2}\,dx=\int \sqrt{\cos\left(c+d\,x\right)}\,\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)^{5/2}\,dx$$

input
```
int(cos(c + d*x)^(1/2)*(a + a/cos(c + d*x))^(5/2),x)
```

output
```
int(cos(c + d*x)^(1/2)*(a + a/cos(c + d*x))^(5/2), x)
```

3.417 $\int \frac{(a+a\sec(c+dx))^{5/2}}{\sqrt{\cos(c+dx)}}\,dx$

3.417.1 Optimal result . . . 3083
3.417.2 Mathematica [A] (verified) . . . 3083
3.417.3 Rubi [A] (verified) . . . 3084
3.417.4 Maple [A] (verified) . . . 3087
3.417.5 Fricas [A] (verification not implemented) . . . 3087
3.417.6 Sympy [F(-1)] . . . 3088
3.417.7 Maxima [B] (verification not implemented) . . . 3088
3.417.8 Giac [F] . . . 3089
3.417.9 Mupad [F(-1)] . . . 3089

3.417.1 Optimal result

Integrand size = 25, antiderivative size = 140

$$\int \frac{(a+a\sec(c+dx))^{5/2}}{\sqrt{\cos(c+dx)}}\,dx = \frac{19a^{5/2}\operatorname{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{4d} + \frac{9a^3\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{a^2\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)}$$

output
```
19/4*a^(5/2)*arcsinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))*cos(d*x+c)
^(1/2)*sec(d*x+c)^(1/2)/d+9/4*a^3*sin(d*x+c)/d/cos(d*x+c)^(3/2)/(a+a*sec(d
*x+c))^(1/2)+1/2*a^2*sin(d*x+c)*(a+a*sec(d*x+c))^(1/2)/d/cos(d*x+c)^(3/2)
```

3.417.2 Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.68

$$\int \frac{(a+a\sec(c+dx))^{5/2}}{\sqrt{\cos(c+dx)}}\,dx = \frac{a^2\sqrt{a(1+\sec(c+dx))}\left(-\frac{19\arcsin\left(\sqrt{\sec(c+dx)}\right)}{\sqrt{\sec(c+dx)}} + \sqrt{1-\sec(c+dx)}(11+2\sec\right.}{4d\sqrt{-1+\cos(c+dx)}}$$

input
```
Integrate[(a + a*Sec[c + d*x])^(5/2)/Sqrt[Cos[c + d*x]],x]
```

output
```
(a^2*Sqrt[a*(1 + Sec[c + d*x])]*((-19*ArcSin[Sqrt[Sec[c + d*x]]])/Sqrt[Sec
[c + d*x]] + Sqrt[1 - Sec[c + d*x]]*(11 + 2*Sec[c + d*x]))*Tan[(c + d*x)/2
])/(4*d*Sqrt[-1 + Cos[c + d*x]])
```

3.417.3 Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.02, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 4752, 3042, 4301, 27, 3042, 4504, 3042, 4288, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sec(c+dx)+a)^{5/2}}{\sqrt{\cos(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\sec(c+dx)}(\sec(c+dx)a+a)^{5/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{5/2}dx$$

$$\Big\downarrow \text{4301}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{2}a\int \frac{1}{2}\sqrt{\sec(c+dx)}\sqrt{\sec(c+dx)a+a}(9\sec(c+dx)a+5a)dx+\frac{a^2\sin(c+dx)\sec}{}\right.$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{4}a\int \sqrt{\sec(c+dx)}\sqrt{\sec(c+dx)a+a}(9\sec(c+dx)a+5a)dx+\frac{a^2\sin(c+dx)\sec^{\frac{3}{2}}}{}\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{4}a\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}\left(9\csc\left(c+dx+\frac{\pi}{2}\right)a+5a\right)dx+\right.$$

$$\Big\downarrow \; 4504$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{4}a\left(\frac{19}{2}a\int\sqrt{\sec(c+dx)}\sqrt{\sec(c+dx)a+a}dx+\frac{9a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}\right)+\right.$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{4}a\left(\frac{19}{2}a\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx+\frac{9a^2\sin(c+dx)\sec^{\frac{3}{2}}(\right.\right.$$

$$\Big\downarrow \; 4288$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{4}a\left(\frac{9a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}-\frac{19a\int\frac{1}{\sqrt{\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1}}d\left(-\frac{a\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}\right)+\right.$$

$$\Big\downarrow \; 222$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}{2d}+\frac{1}{4}a\left(\frac{19a^{3/2}\text{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right.}{d}\right.\right.$$

input
```
Int[(a + a*Sec[c + d*x])^(5/2)/Sqrt[Cos[c + d*x]],x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((a^2*Sec[c + d*x]^(3/2)*Sqrt[a + a*
Sec[c + d*x]]*Sin[c + d*x])/(2*d) + (a*((19*a^(3/2)*ArcSinh[(Sqrt[a]*Tan[c
 + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d + (9*a^2*Sec[c + d*x]^(3/2)*Sin[c +
d*x])/(d*Sqrt[a + a*Sec[c + d*x]])))/4)
```

3.417. $\int \frac{(a+a\sec(c+dx))^{5/2}}{\sqrt{\cos(c+dx)}}\,dx$

3.417.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4288
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[-2*(a/(b*f))*Sqrt[a*(d/b)]   Subst[Int[1/Sqrt[1
+ x^2/a], x], x, b*(Cot[e + f*x]/Sqrt[a + b*Csc[e + f*x]])], x] /; FreeQ[{a
, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[a*(d/b), 0]
```

rule 4301
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
2)*((d*Csc[e + f*x])^n/(f*(m + n - 1))), x] + Simp[b/(m + n - 1)   Int[(a +
 b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*(b*(m + 2*n - 1) + a*(3*m + 2*n
 - 4)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^
2, 0] && GtQ[m, 1] && NeQ[m + n - 1, 0] && IntegerQ[2*m]
```

rule 4504
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[-2*b*B*C
ot[e + f*x]*((d*Csc[e + f*x])^n/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]])), x]
 + Simp[(A*b*(2*n + 1) + 2*a*B*n)/(b*(2*n + 1))   Int[Sqrt[a + b*Csc[e + f*
x]]*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ
[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] &&
!LtQ[n, 0]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

3.417. $\int \frac{(a+a\sec(c+dx))^{5/2}}{\sqrt{\cos(c+dx)}}\,dx$

3.417.4 Maple [A] (verified)

Time = 2.33 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.49

method	result
default	$\frac{a^2\left(22\sqrt{-\frac{1}{\cos(dx+c)+1}}\,\sin(dx+c)\cos(dx+c)-19\cos(dx+c)^2\arctan\left(\frac{-\cos(dx+c)+\sin(dx+c)-1}{2(\cos(dx+c)+1)\sqrt{-\frac{1}{\cos(dx+c)+1}}}\right)-19\arctan\left(\frac{\cos(dx+c)+\sin}{2(\cos(dx+c)+1)\sqrt{}}\right.\right.}{8d(\cos(dx+c)+1)\cos(dx+c)^{\frac{3}{2}}\sqrt{-\frac{1}{\cos(dx+c)+1}}}$

input
```
int((a+a*sec(d*x+c))^(5/2)/cos(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/8/d*a^2*(22*(-1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*cos(d*x+c)-19*cos(d*x+c
)^2*arctan(1/2*(-cos(d*x+c)+sin(d*x+c)-1)/(cos(d*x+c)+1)/(-1/(cos(d*x+c)+1
))^(1/2))-19*arctan(1/2*(cos(d*x+c)+sin(d*x+c)+1)/(cos(d*x+c)+1)/(-1/(cos(
d*x+c)+1))^(1/2))*cos(d*x+c)^2+4*sin(d*x+c)*(-1/(cos(d*x+c)+1))^(1/2))*(a*
(1+sec(d*x+c)))^(1/2)/(cos(d*x+c)+1)/cos(d*x+c)^(3/2)/(-1/(cos(d*x+c)+1))^
(1/2)
```

3.417.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 385, normalized size of antiderivative = 2.75

$$\int \frac{(a+a\sec(c+dx))^{5/2}}{\sqrt{\cos(c+dx)}}\,dx = \left[\frac{4\,(11\,a^2\cos(dx+c)+2\,a^2)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)+19\,(a}{}\right.$$

input
```
integrate((a+a*sec(d*x+c))^(5/2)/cos(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
[1/16*(4*(11*a^2*cos(d*x + c) + 2*a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x +
 c))*sqrt(cos(d*x + c))*sin(d*x + c) + 19*(a^2*cos(d*x + c)^3 + a^2*cos(d*
x + c)^2)*sqrt(a)*log((a*cos(d*x + c)^3 - 4*sqrt(a)*sqrt((a*cos(d*x + c) +
 a)/cos(d*x + c))*(cos(d*x + c) - 2)*sqrt(cos(d*x + c))*sin(d*x + c) - 7*a
*cos(d*x + c)^2 + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)))/(d*cos(d*x + c)
^3 + d*cos(d*x + c)^2), 1/8*(2*(11*a^2*cos(d*x + c) + 2*a^2)*sqrt((a*cos(d
*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) + 19*(a^2*cos(d
*x + c)^3 + a^2*cos(d*x + c)^2)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x
 + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2
 - a*cos(d*x + c) - 2*a)))/(d*cos(d*x + c)^3 + d*cos(d*x + c)^2)]
```

3.417.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(c+dx))^{5/2}}{\sqrt{\cos(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sec(d*x+c))**(5/2)/cos(d*x+c)**(1/2),x)
```

output
```
Timed out
```

3.417.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2826 vs. $2(116) = 232$.

Time = 3.08 (sec) , antiderivative size = 2826, normalized size of antiderivative = 20.19

$$\int \frac{(a+a\sec(c+dx))^{5/2}}{\sqrt{\cos(c+dx)}}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sec(d*x+c))^(5/2)/cos(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
-1/16*(88*sqrt(2)*a^2*cos(7/2*d*x + 7/2*c)*sin(2*d*x + 2*c) - 56*sqrt(2)*a
^2*cos(5/2*d*x + 5/2*c)*sin(2*d*x + 2*c) - 28*sqrt(2)*a^2*sin(3/2*d*x + 3/
2*c) + 44*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c) - 19*(a^2*log(2*cos(1/2*d*x + 1
/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sq
rt(2)*sin(1/2*d*x + 1/2*c) + 2) - a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin
(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d
*x + 1/2*c) + 2) + a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*
c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2
) - a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2
)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*cos(4*d*x +
4*c)^2 - 76*(a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 +
 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - a^
2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(
1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + a^2*log(2*cos(1/2
*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c
) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - a^2*log(2*cos(1/2*d*x + 1/2*c)^2
 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*s
in(1/2*d*x + 1/2*c) + 2))*cos(2*d*x + 2*c)^2 - 19*a^2*log(2*cos(1/2*d*x +
1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*s
qrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 ...
```

3.417.8 Giac [F]

$$\int \frac{(a+a\sec(c+dx))^{5/2}}{\sqrt{\cos(c+dx)}}\,dx=\int \frac{(a\sec(dx+c)+a)^{\frac{5}{2}}}{\sqrt{\cos(dx+c)}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))^(5/2)/cos(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.417.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(c+dx))^{5/2}}{\sqrt{\cos(c+dx)}}\,dx=\int \frac{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{5/2}}{\sqrt{\cos(c+d\,x)}}\,dx$$

input
```
int((a + a/cos(c + d*x))^(5/2)/cos(c + d*x)^(1/2),x)
```

output
```
int((a + a/cos(c + d*x))^(5/2)/cos(c + d*x)^(1/2), x)
```

3.418 $\int \frac{(a+a\sec(c+dx))^{5/2}}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

3.418.1 Optimal result . . . 3090
3.418.2 Mathematica [A] (verified) . . . 3090
3.418.3 Rubi [A] (verified) . . . 3091
3.418.4 Maple [A] (verified) . . . 3094
3.418.5 Fricas [A] (verification not implemented) . . . 3095
3.418.6 Sympy [F(-1)] . . . 3095
3.418.7 Maxima [B] (verification not implemented) . . . 3096
3.418.8 Giac [F] . . . 3096
3.418.9 Mupad [F(-1)] . . . 3097

3.418.1 Optimal result

Integrand size = 25, antiderivative size = 180

$$\int \frac{(a+a\sec(c+dx))^{5/2}}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{25a^{5/2}\operatorname{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{8d} + \frac{13a^3\sin(c+dx)}{12d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{25a^3\sin(c+dx)}{8d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{a^2\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{3d\cos^{\frac{5}{2}}(c+dx)}$$

output
```
25/8*a^(5/2)*arcsinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))*cos(d*x+c)
^(1/2)*sec(d*x+c)^(1/2)/d+13/12*a^3*sin(d*x+c)/d/cos(d*x+c)^(5/2)/(a+a*sec
(d*x+c))^(1/2)+25/8*a^3*sin(d*x+c)/d/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/
2)+1/3*a^2*sin(d*x+c)*(a+a*sec(d*x+c))^(1/2)/d/cos(d*x+c)^(5/2)
```

3.418.2 Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.64

$$\int \frac{(a+a\sec(c+dx))^{5/2}}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{a^3\left(\frac{75\arcsin\left(\sqrt{1-\sec(c+dx)}\right)}{\sec^{\frac{3}{2}}(c+dx)} + \sqrt{1-\sec(c+dx)}(34+75\cos(c+dx)+8\sec(c+\right.}{24d\cos^{\frac{5}{2}}(c+dx)\sqrt{1-\sec(c+dx)}\sqrt{a(1+\sec(c+dx))}}$$

input
```
Integrate[(a + a*Sec[c + d*x])^(5/2)/Cos[c + d*x]^(3/2),x]
```

output
```
(a^3*((75*ArcSin[Sqrt[1 - Sec[c + d*x]]])/Sec[c + d*x]^(3/2) + Sqrt[1 - Se
c[c + d*x]]*(34 + 75*Cos[c + d*x] + 8*Sec[c + d*x]))*Sin[c + d*x])/(24*d*C
os[c + d*x]^(5/2)*Sqrt[1 - Sec[c + d*x]]*Sqrt[a*(1 + Sec[c + d*x])])
```

3.418.3 Rubi [A] (verified)

Time = 0.93 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.03, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {3042, 4752, 3042, 4301, 27, 3042, 4504, 3042, 4290, 3042, 4288, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sec(c+dx)+a)^{5/2}}{\cos^{\frac{3}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sec^{\frac{3}{2}}(c+dx)(\sec(c+dx)a+a)^{5/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{5/2}dx$$

$$\Big\downarrow \text{4301}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}a\int \frac{1}{2}\sec^{\frac{3}{2}}(c+dx)\sqrt{\sec(c+dx)a+a}(13\sec(c+dx)a+9a)dx+\frac{a^2\sin(c+dx)\,\mathrm{se}}{}\right.$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{6}a\int \sec^{\frac{3}{2}}(c+dx)\sqrt{\sec(c+dx)a+a}(13\sec(c+dx)a+9a)dx+\frac{a^2\sin(c+dx)\sec^{\frac{5}{2}}}{}\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{6}a\int\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}\left(13\csc\left(c+dx+\frac{\pi}{2}\right)a+9a\right)d\right.$$

$\downarrow$ 4504

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{6}a\left(\frac{75}{4}a\int\sec^{\frac{3}{2}}(c+dx)\sqrt{\sec(c+dx)a+a}dx+\frac{13a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}\right)+\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{6}a\left(\frac{75}{4}a\int\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx+\frac{13a^2\sin(c+dx)\sec^{\frac{5}{2}}}{2d\sqrt{a\sec(c+dx}}\right.\right.$$

$\downarrow$ 4290

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{6}a\left(\frac{75}{4}a\left(\frac{1}{2}\int\sqrt{\sec(c+dx)}\sqrt{\sec(c+dx)a+a}dx+\frac{a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}\right)+\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{6}a\left(\frac{75}{4}a\left(\frac{1}{2}\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx+\frac{a\sin(c+dx)\sec^{\frac{3}{2}}}{d\sqrt{a\sec(c+dx}}\right.\right.\right.$$

$\downarrow$ 4288

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{6}a\left(\frac{75}{4}a\left(\frac{a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}-\frac{\int\frac{1}{\sqrt{\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1}}d\left(-\frac{a\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}\right)+\right.\right.$$

$\downarrow$ 222

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{6}a\left(\frac{13a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}+\frac{75}{4}a\left(\frac{\sqrt{a}\operatorname{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{a\sin(c+\right.\right.\right.$$

input
```
Int[(a + a*Sec[c + d*x])^(5/2)/Cos[c + d*x]^(3/2),x]
```

3.418. $\int\frac{(a+a\sec(c+dx))^{5/2}}{\cos^{\frac{3}{2}}(c+dx)}dx$

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((a^2*Sec[c + d*x]^(5/2)*Sqrt[a + a*
Sec[c + d*x]]*Sin[c + d*x])/(3*d) + (a*((13*a^2*Sec[c + d*x]^(5/2)*Sin[c +
 d*x])/(2*d*Sqrt[a + a*Sec[c + d*x]]) + (75*a*((Sqrt[a]*ArcSinh[(Sqrt[a]*T
an[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d + (a*Sec[c + d*x]^(3/2)*Sin[c +
d*x])/(d*Sqrt[a + a*Sec[c + d*x]])))/4))/6)
```

3.418.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4288
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[-2*(a/(b*f))*Sqrt[a*(d/b)]   Subst[Int[1/Sqrt[1
+ x^2/a], x], x, b*(Cot[e + f*x]/Sqrt[a + b*Csc[e + f*x]])], x] /; FreeQ[{a
, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[a*(d/b), 0]
```

rule 4290
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[-2*b*d*Cot[e + f*x]*((d*Csc[e + f*x])^(n - 1)/(
f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]])), x] + Simp[2*a*d*((n - 1)/(b*(2*n -
1)))   Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e + f*x])^(n - 1), x], x] /; Fre
eQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

rule 4301
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
2)*((d*Csc[e + f*x])^n/(f*(m + n - 1))), x] + Simp[b/(m + n - 1)   Int[(a +
 b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*(b*(m + 2*n - 1) + a*(3*m + 2*n
 - 4)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^
2, 0] && GtQ[m, 1] && NeQ[m + n - 1, 0] && IntegerQ[2*m]
```

rule 4504
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[-2*b*B*C
ot[e + f*x]*((d*Csc[e + f*x])^n/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]])), x]
 + Simp[(A*b*(2*n + 1) + 2*a*B*n)/(b*(2*n + 1))   Int[Sqrt[a + b*Csc[e + f*
x]]*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ
[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] &&
!LtQ[n, 0]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

3.418.4 Maple [A] (verified)

Time = 2.11 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.32

method	result
default	$-\frac{a^2\left(75\cos(dx+c)^3\arctan\left(\frac{\cos(dx+c)+\sin(dx+c)+1}{2(\cos(dx+c)+1)\sqrt{-\frac{1}{\cos(dx+c)+1}}}\right)-75\cos(dx+c)^3\arctan\left(\frac{\cos(dx+c)-\sin(dx+c)+1}{2(\cos(dx+c)+1)\sqrt{-\frac{1}{\cos(dx+c)+1}}}\right)-150\sqrt{-\frac{}{c}}}{48d(\cos(dx+c)+1)\cos(d}$

input
```
int((a+a*sec(d*x+c))^(5/2)/cos(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/48/d*a^2*(75*cos(d*x+c)^3*arctan(1/2*(cos(d*x+c)+sin(d*x+c)+1)/(cos(d*x
+c)+1)/(-1/(cos(d*x+c)+1))^(1/2))-75*cos(d*x+c)^3*arctan(1/2*(cos(d*x+c)-s
in(d*x+c)+1)/(cos(d*x+c)+1)/(-1/(cos(d*x+c)+1))^(1/2))-150*(-1/(cos(d*x+c)
+1))^(1/2)*sin(d*x+c)*cos(d*x+c)^2-68*(-1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)
*cos(d*x+c)-16*sin(d*x+c)*(-1/(cos(d*x+c)+1))^(1/2))*(a*(1+sec(d*x+c)))^(1
/2)/(cos(d*x+c)+1)/cos(d*x+c)^(5/2)/(-1/(cos(d*x+c)+1))^(1/2)
```

3.418.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 411, normalized size of antiderivative = 2.28

$$\int \frac{(a+a\sec(c+dx))^{5/2}}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \left[\frac{4\left(75\,a^2\cos\left(dx+c\right)^2+34\,a^2\cos\left(dx+c\right)+8\,a^2\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos\left(dx+\right.}}{}\right.$$

input
```
integrate((a+a*sec(d*x+c))^(5/2)/cos(d*x+c)^(3/2),x, algorithm="fricas")
```

output
```
[1/96*(4*(75*a^2*cos(d*x + c)^2 + 34*a^2*cos(d*x + c) + 8*a^2)*sqrt((a*cos
(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) + 75*(a^2*cos
(d*x + c)^4 + a^2*cos(d*x + c)^3)*sqrt(a)*log((a*cos(d*x + c)^3 - 4*sqrt(a
)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(cos(d*x + c) - 2)*sqrt(cos(d*x
+ c))*sin(d*x + c) - 7*a*cos(d*x + c)^2 + 8*a)/(cos(d*x + c)^3 + cos(d*x +
 c)^2)))/(d*cos(d*x + c)^4 + d*cos(d*x + c)^3), 1/48*(2*(75*a^2*cos(d*x +
c)^2 + 34*a^2*cos(d*x + c) + 8*a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)
)*sqrt(cos(d*x + c))*sin(d*x + c) + 75*(a^2*cos(d*x + c)^4 + a^2*cos(d*x +
 c)^3)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*
sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a))
)/(d*cos(d*x + c)^4 + d*cos(d*x + c)^3)]
```

3.418.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(c+dx))^{5/2}}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sec(d*x+c))**(5/2)/cos(d*x+c)**(3/2),x)
```

output
```
Timed out
```

3.418.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3469 vs. $2(150) = 300$.

Time = 0.55 (sec) , antiderivative size = 3469, normalized size of antiderivative = 19.27

$$\int \frac{(a + a\sec(c + dx))^{5/2}}{\cos^{\frac{3}{2}}(c + dx)}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sec(d*x+c))^(5/2)/cos(d*x+c)^(3/2),x, algorithm="maxima")
```

output
```
1/96*(300*sqrt(2)*a^2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x +
3/2*c)))*sin(6*d*x + 6*c) - 28*sqrt(2)*a^2*sin(9/2*d*x + 9/2*c) + 28*sqrt(
2)*a^2*sin(3/2*d*x + 3/2*c) - 28*(sqrt(2)*a^2*sin(9/2*d*x + 9/2*c) - sqrt(
2)*a^2*sin(3/2*d*x + 3/2*c))*cos(6*d*x + 6*c) - 300*(sqrt(2)*a^2*sin(6*d*x
 + 6*c) + 3*sqrt(2)*a^2*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x
+ 3/2*c))) + 3*sqrt(2)*a^2*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d
*x + 3/2*c))))*cos(11/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)
)) - 12*(7*sqrt(2)*a^2*sin(9/2*d*x + 9/2*c) - 7*sqrt(2)*a^2*sin(3/2*d*x +
3/2*c) - 114*sqrt(2)*a^2*sin(7/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x
 + 3/2*c))) + 114*sqrt(2)*a^2*sin(5/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/
2*d*x + 3/2*c))) + 75*sqrt(2)*a^2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), co
s(3/2*d*x + 3/2*c))))*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x +
3/2*c))) - 456*(sqrt(2)*a^2*sin(6*d*x + 6*c) + 3*sqrt(2)*a^2*sin(4/3*arcta
n2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*cos(7/3*arctan2(sin(3/2*d
*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 456*(sqrt(2)*a^2*sin(6*d*x + 6*c) +
3*sqrt(2)*a^2*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))
)*cos(5/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 12*(7*sqr
t(2)*a^2*sin(9/2*d*x + 9/2*c) - 7*sqrt(2)*a^2*sin(3/2*d*x + 3/2*c) + 75*sq
rt(2)*a^2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*co
s(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 75*(a^2*co...
```

3.418.8 Giac [F]

$$\int \frac{(a + a\sec(c + dx))^{5/2}}{\cos^{\frac{3}{2}}(c + dx)}\,dx = \int \frac{(a\sec(dx + c) + a)^{\frac{5}{2}}}{\cos(dx + c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))^(5/2)/cos(d*x+c)^(3/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.418.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(c+dx))^{5/2}}{\cos^{\frac{3}{2}}(c+dx)}\,dx=\int \frac{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{5/2}}{\cos\left(c+d\,x\right)^{3/2}}\,dx$$

input
```
int((a + a/cos(c + d*x))^(5/2)/cos(c + d*x)^(3/2),x)
```

output
```
int((a + a/cos(c + d*x))^(5/2)/cos(c + d*x)^(3/2), x)
```

3.419 $\int \frac{(a+a\sec(c+dx))^{5/2}}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

3.419.1 Optimal result . . . 3098
3.419.2 Mathematica [A] (verified) . . . 3099
3.419.3 Rubi [A] (verified) . . . 3099
3.419.4 Maple [A] (verified) . . . 3103
3.419.5 Fricas [A] (verification not implemented) . . . 3103
3.419.6 Sympy [F(-1)] . . . 3104
3.419.7 Maxima [B] (verification not implemented) . . . 3104
3.419.8 Giac [F] . . . 3105
3.419.9 Mupad [F(-1)] . . . 3106

3.419.1 Optimal result

Integrand size = 25, antiderivative size = 220

$$\int \frac{(a+a\sec(c+dx))^{5/2}}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{163a^{5/2}\operatorname{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{64d} + \frac{17a^3\sin(c+dx)}{24d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{163a^3\sin(c+dx)}{96d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{163a^3\sin(c+dx)}{64d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{a^2\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{4d\cos^{\frac{7}{2}}(c+dx)}$$

output
```
163/64*a^(5/2)*arcsinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))*cos(d*x+
c)^(1/2)*sec(d*x+c)^(1/2)/d+17/24*a^3*sin(d*x+c)/d/cos(d*x+c)^(7/2)/(a+a*s
ec(d*x+c))^(1/2)+163/96*a^3*sin(d*x+c)/d/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))
^(1/2)+163/64*a^3*sin(d*x+c)/d/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2)+1/4
*a^2*sin(d*x+c)*(a+a*sec(d*x+c))^(1/2)/d/cos(d*x+c)^(7/2)
```

3.419.2 Mathematica [A] (verified)

Time = 0.62 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.57

$$\int \frac{(a+a\sec(c+dx))^{5/2}}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{a^3\left(\frac{489\arcsin\left(\sqrt{1-\sec(c+dx)}\right)}{\sec^{\frac{5}{2}}(c+dx)}+\sqrt{1-\sec(c+dx)}(184+326\cos(c+dx)+489\,\mathrm{c}\right.}{192d\cos^{\frac{7}{2}}(c+dx)\sqrt{1-\sec(c+dx)}\sqrt{a(1+\mathrm{s}}}$$

input
```
Integrate[(a + a*Sec[c + d*x])^(5/2)/Cos[c + d*x]^(5/2),x]
```

output
```
(a^3*((489*ArcSin[Sqrt[1 - Sec[c + d*x]]])/Sec[c + d*x]^(5/2) + Sqrt[1 - S
ec[c + d*x]]*(184 + 326*Cos[c + d*x] + 489*Cos[c + d*x]^2 + 48*Sec[c + d*x
]))*Sin[c + d*x])/(192*d*Cos[c + d*x]^(7/2)*Sqrt[1 - Sec[c + d*x]]*Sqrt[a*
(1 + Sec[c + d*x])])
```

3.419.3 Rubi [A] (verified)

Time = 1.16 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.04, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.560$, Rules used = {3042, 4752, 3042, 4301, 27, 3042, 4504, 3042, 4290, 3042, 4290, 3042, 4288, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sec(c+dx)+a)^{5/2}}{\cos^{\frac{5}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sec^{\frac{5}{2}}(c+dx)(\sec(c+dx)a+a)^{5/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{5/2}dx$$

$$\Big\downarrow \text{4301}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{4}a\int\frac{1}{2}\sec^{\frac{5}{2}}(c+dx)\sqrt{\sec(c+dx)a+a}(17\sec(c+dx)a+13a)dx+\frac{a^2\sin(c+dx)\,\mathrm{s}}{}\right.$$

$\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{8}a\int\sec^{\frac{5}{2}}(c+dx)\sqrt{\sec(c+dx)a+a}(17\sec(c+dx)a+13a)dx+\frac{a^2\sin(c+dx)\sec}{}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{8}a\int\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}\left(17\csc\left(c+dx+\frac{\pi}{2}\right)a+13a\right)\right.$$

$\downarrow$ 4504

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{8}a\left(\frac{163}{6}a\int\sec^{\frac{5}{2}}(c+dx)\sqrt{\sec(c+dx)a+a}dx+\frac{17a^2\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{3d\sqrt{a\sec(c+dx)+a}}\right)+\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{8}a\left(\frac{163}{6}a\int\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx+\frac{17a^2\sin(c+dx)\sec}{3d\sqrt{a\sec(c+d}}\right.\right.$$

$\downarrow$ 4290

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{8}a\left(\frac{163}{6}a\left(\frac{3}{4}\int\sec^{\frac{3}{2}}(c+dx)\sqrt{\sec(c+dx)a+a}dx+\frac{a\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}\right)+\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{8}a\left(\frac{163}{6}a\left(\frac{3}{4}\int\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx+\frac{a\sin(c+dx)\sec}{2d\sqrt{a\sec(c+}}\right.\right.$$

$\downarrow$ 4290

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{8}a\left(\frac{163}{6}a\left(\frac{3}{4}\left(\frac{1}{2}\int\sqrt{\sec(c+dx)}\sqrt{\sec(c+dx)a+a}dx+\frac{a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}\right.\right.\right.$$

$\downarrow$ 3042

3.419. $\int\frac{(a+a\sec(c+dx))^{5/2}}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{8}a\left(\frac{163}{6}a\left(\frac{3}{4}\left(\frac{1}{2}\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx+\frac{a\sin(c+dx)}{d\sqrt{a\sec(c}}\right.\right.\right.\right.$$

$$\Big\downarrow \text{4288}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{8}a\left(\frac{163}{6}a\left(\frac{3}{4}\left(\frac{a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}-\frac{\int\frac{1}{\sqrt{\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1}}d\left(-\frac{a\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}\right)\right)\right)$$

$$\Big\downarrow \text{222}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{8}a\left(\frac{17a^2\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{3d\sqrt{a\sec(c+dx)+a}}+\frac{163}{6}a\left(\frac{3}{4}\left(\frac{\sqrt{a}\text{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{a\sin}{d}\right.\right.\right.\right.$$

input
```
Int[(a + a*Sec[c + d*x])^(5/2)/Cos[c + d*x]^(5/2),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((a^2*Sec[c + d*x]^(7/2)*Sqrt[a + a*
Sec[c + d*x]]*Sin[c + d*x])/(4*d) + (a*((17*a^2*Sec[c + d*x]^(7/2)*Sin[c +
 d*x])/(3*d*Sqrt[a + a*Sec[c + d*x]]) + (163*a*((a*Sec[c + d*x]^(5/2)*Sin[
c + d*x])/(2*d*Sqrt[a + a*Sec[c + d*x]]) + (3*((Sqrt[a]*ArcSinh[(Sqrt[a]*T
an[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d + (a*Sec[c + d*x]^(3/2)*Sin[c +
d*x])/(d*Sqrt[a + a*Sec[c + d*x]])))/4))/6))/8)
```

3.419.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4288
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[-2*(a/(b*f))*Sqrt[a*(d/b)]   Subst[Int[1/Sqrt[1
+ x^2/a], x], x, b*(Cot[e + f*x]/Sqrt[a + b*Csc[e + f*x]])], x] /; FreeQ[{a
, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[a*(d/b), 0]
```

rule 4290
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[-2*b*d*Cot[e + f*x]*((d*Csc[e + f*x])^(n - 1)/(
f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]])), x] + Simp[2*a*d*((n - 1)/(b*(2*n -
1)))   Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e + f*x])^(n - 1), x], x] /; Fre
eQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

rule 4301
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
2)*((d*Csc[e + f*x])^n/(f*(m + n - 1))), x] + Simp[b/(m + n - 1)   Int[(a +
 b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*(b*(m + 2*n - 1) + a*(3*m + 2*n
 - 4)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^
2, 0] && GtQ[m, 1] && NeQ[m + n - 1, 0] && IntegerQ[2*m]
```

rule 4504
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[-2*b*B*C
ot[e + f*x]*((d*Csc[e + f*x])^n/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]])), x]
 + Simp[(A*b*(2*n + 1) + 2*a*B*n)/(b*(2*n + 1))   Int[Sqrt[a + b*Csc[e + f*
x]]*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ
[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] &&
!LtQ[n, 0]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

3.419. $\int \frac{(a+a\sec(c+dx))^{5/2}}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

3.419.4 Maple [A] (verified)

Time = 2.27 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.22

method	result
default	$\frac{a^2\left(489\arctan\left(\frac{\cos(dx+c)-\sin(dx+c)+1}{2(\cos(dx+c)+1)\sqrt{-\frac{1}{\cos(dx+c)+1}}}\right)\cos(dx+c)^4-489\arctan\left(\frac{\cos(dx+c)+\sin(dx+c)+1}{2(\cos(dx+c)+1)\sqrt{-\frac{1}{\cos(dx+c)+1}}}\right)\cos(dx+c)^4+978\sin(dx}{384}$

input
```
int((a+a*sec(d*x+c))^(5/2)/cos(d*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/384/d*a^2*(489*arctan(1/2*(cos(d*x+c)-sin(d*x+c)+1)/(cos(d*x+c)+1)/(-1/(
cos(d*x+c)+1))^(1/2))*cos(d*x+c)^4-489*arctan(1/2*(cos(d*x+c)+sin(d*x+c)+1
)/(cos(d*x+c)+1)/(-1/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)^4+978*sin(d*x+c)*(-
1/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^3+652*(-1/(cos(d*x+c)+1))^(1/2)*sin(d*x
+c)*cos(d*x+c)^2+368*(-1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*cos(d*x+c)+96*si
n(d*x+c)*(-1/(cos(d*x+c)+1))^(1/2))*(a*(1+sec(d*x+c)))^(1/2)/(cos(d*x+c)+1
)/cos(d*x+c)^(7/2)/(-1/(cos(d*x+c)+1))^(1/2)
```

3.419.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 437, normalized size of antiderivative = 1.99

$$\int \frac{(a+a\sec(c+dx))^{5/2}}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \left[\frac{4\left(489\,a^2\cos\left(dx+c\right)^3+326\,a^2\cos\left(dx+c\right)^2+184\,a^2\cos\left(dx+c\right)+48\,a^2\right)}{}\right.$$

input
```
integrate((a+a*sec(d*x+c))^(5/2)/cos(d*x+c)^(5/2),x, algorithm="fricas")
```

output
```
[1/768*(4*(489*a^2*cos(d*x + c)^3 + 326*a^2*cos(d*x + c)^2 + 184*a^2*cos(d
*x + c) + 48*a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c
))*sin(d*x + c) + 489*(a^2*cos(d*x + c)^5 + a^2*cos(d*x + c)^4)*sqrt(a)*lo
g((a*cos(d*x + c)^3 - 4*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(c
os(d*x + c) - 2)*sqrt(cos(d*x + c))*sin(d*x + c) - 7*a*cos(d*x + c)^2 + 8*
a)/(cos(d*x + c)^3 + cos(d*x + c)^2)))/(d*cos(d*x + c)^5 + d*cos(d*x + c)^
4), 1/384*(2*(489*a^2*cos(d*x + c)^3 + 326*a^2*cos(d*x + c)^2 + 184*a^2*co
s(d*x + c) + 48*a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x
+ c))*sin(d*x + c) + 489*(a^2*cos(d*x + c)^5 + a^2*cos(d*x + c)^4)*sqrt(-a
)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x +
 c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)))/(d*cos(d*x +
 c)^5 + d*cos(d*x + c)^4)]
```

3.419.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(c+dx))^{5/2}}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sec(d*x+c))**(5/2)/cos(d*x+c)**(5/2),x)
```

output
```
Timed out
```

3.419.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3860 vs. $2(184) = 368$.

Time = 0.62 (sec) , antiderivative size = 3860, normalized size of antiderivative = 17.55

$$\int \frac{(a+a\sec(c+dx))^{5/2}}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sec(d*x+c))^(5/2)/cos(d*x+c)^(5/2),x, algorithm="maxima")
```

output
```
-1/768*(1956*(sqrt(2)*a^2*sin(8*d*x + 8*c) + 4*sqrt(2)*a^2*sin(6*d*x + 6*c
) + 6*sqrt(2)*a^2*sin(4*d*x + 4*c) + 4*sqrt(2)*a^2*sin(2*d*x + 2*c))*cos(1
5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 652*(sqrt(2)*a^2*sin(8*
d*x + 8*c) + 4*sqrt(2)*a^2*sin(6*d*x + 6*c) + 6*sqrt(2)*a^2*sin(4*d*x + 4*
c) + 4*sqrt(2)*a^2*sin(2*d*x + 2*c))*cos(13/4*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c))) + 6204*(sqrt(2)*a^2*sin(8*d*x + 8*c) + 4*sqrt(2)*a^2*sin(
6*d*x + 6*c) + 6*sqrt(2)*a^2*sin(4*d*x + 4*c) + 4*sqrt(2)*a^2*sin(2*d*x +
2*c))*cos(11/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2060*(sqrt(2
)*a^2*sin(8*d*x + 8*c) + 4*sqrt(2)*a^2*sin(6*d*x + 6*c) + 6*sqrt(2)*a^2*si
n(4*d*x + 4*c) + 4*sqrt(2)*a^2*sin(2*d*x + 2*c))*cos(9/4*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c))) + 2060*(sqrt(2)*a^2*sin(8*d*x + 8*c) + 4*sqrt(
2)*a^2*sin(6*d*x + 6*c) + 6*sqrt(2)*a^2*sin(4*d*x + 4*c) + 4*sqrt(2)*a^2*s
in(2*d*x + 2*c))*cos(7/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 62
04*(sqrt(2)*a^2*sin(8*d*x + 8*c) + 4*sqrt(2)*a^2*sin(6*d*x + 6*c) + 6*sqrt
(2)*a^2*sin(4*d*x + 4*c) + 4*sqrt(2)*a^2*sin(2*d*x + 2*c))*cos(5/4*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 652*(sqrt(2)*a^2*sin(8*d*x + 8*c)
+ 4*sqrt(2)*a^2*sin(6*d*x + 6*c) + 6*sqrt(2)*a^2*sin(4*d*x + 4*c) + 4*sqrt
(2)*a^2*sin(2*d*x + 2*c))*cos(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c))) - 1956*(sqrt(2)*a^2*sin(8*d*x + 8*c) + 4*sqrt(2)*a^2*sin(6*d*x + 6*c)
 + 6*sqrt(2)*a^2*sin(4*d*x + 4*c) + 4*sqrt(2)*a^2*sin(2*d*x + 2*c))*cos...
```

3.419.8 Giac [F]

$$\int \frac{(a+a\sec(c+dx))^{5/2}}{\cos^{\frac{5}{2}}(c+dx)}\,dx=\int \frac{(a\sec(dx+c)+a)^{\frac{5}{2}}}{\cos(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+a*sec(d*x+c))^(5/2)/cos(d*x+c)^(5/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.419.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sec(c+dx))^{5/2}}{\cos^{\frac{5}{2}}(c+dx)}\,dx=\int \frac{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{5/2}}{\cos\left(c+d\,x\right)^{5/2}}\,dx$$

input
```
int((a + a/cos(c + d*x))^(5/2)/cos(c + d*x)^(5/2),x)
```

output
```
int((a + a/cos(c + d*x))^(5/2)/cos(c + d*x)^(5/2), x)
```

3.420 $\int \frac{\cos^{\frac{5}{2}}(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$

3.420.1 Optimal result . . . 3107
3.420.2 Mathematica [A] (verified) . . . 3108
3.420.3 Rubi [A] (verified) . . . 3108
3.420.4 Maple [A] (verified) . . . 3112
3.420.5 Fricas [A] (verification not implemented) . . . 3112
3.420.6 Sympy [F(-1)] . . . 3113
3.420.7 Maxima [B] (verification not implemented) . . . 3113
3.420.8 Giac [F] . . . 3114
3.420.9 Mupad [F(-1)] . . . 3114

3.420.1 Optimal result

Integrand size = 25, antiderivative size = 189

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = -\frac{\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{\sqrt{a}d} + \frac{26\sin(c+dx)}{15d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} - \frac{2\sqrt{\cos(c+dx)}\sin(c+dx)}{15d\sqrt{a+a\sec(c+dx)}} + \frac{2\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d\sqrt{a+a\sec(c+dx)}}$$

output
```
-arctanh(1/2*sin(d*x+c)*a^(1/2)*sec(d*x+c)^(1/2)*2^(1/2)/(a+a*sec(d*x+c))^
(1/2))*2^(1/2)*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d/a^(1/2)+2/5*cos(d*x+c)^
(3/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+26/15*sin(d*x+c)/d/cos(d*x+c)^(1
/2)/(a+a*sec(d*x+c))^(1/2)-2/15*sin(d*x+c)*cos(d*x+c)^(1/2)/d/(a+a*sec(d*x
+c))^(1/2)
```

3.420.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.72

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$$
$$= \frac{\cos^{\frac{3}{2}}(c+dx)\left(15\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)\sec^{\frac{5}{2}}(c+dx)+2\sqrt{1-\sec(c+dx)}(3-\sec(c+dx)+13\sec^2(c\right.}{15d\sqrt{1-\sec(c+dx)}\sqrt{a(1+\sec(c+dx))}}$$

input
```
Integrate[Cos[c + d*x]^(5/2)/Sqrt[a + a*Sec[c + d*x]],x]
```

output
```
(Cos[c + d*x]^(3/2)*(15*Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1
 - Sec[c + d*x]]]*Sec[c + d*x]^(5/2) + 2*Sqrt[1 - Sec[c + d*x]]*(3 - Sec[c
 + d*x] + 13*Sec[c + d*x]^2))*Sin[c + d*x])/(15*d*Sqrt[1 - Sec[c + d*x]]*S
qrt[a*(1 + Sec[c + d*x])])
```

3.420.3 Rubi [A] (verified)

Time = 1.00 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.10, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {3042, 4752, 3042, 4310, 3042, 4510, 27, 3042, 4501, 3042, 4295, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{\sqrt{a\sec(c+dx)+a}}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}}{\sqrt{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}}dx$$
$$\Big\downarrow \; 4752$$
$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)\sqrt{\sec(c+dx)a+a}}dx$$
$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx$$

$\Big\downarrow$ 4310

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}-\frac{\int \frac{a-4a\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)\sqrt{\sec(c+dx)a+a}}dx}{5a}\right)$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}-\frac{\int \frac{a-4a\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{5a}\right)$$

$\Big\downarrow$ 4510

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}-\frac{\frac{2\int -\frac{13a^2-2a^2\sec(c+dx)}{2\sqrt{\sec(c+dx)}\sqrt{\sec(c+dx)a+a}}dx}{3a}+\frac{2a\sin(c+}{3d\sqrt{\sec(c+dx)}\sqrt{a}}}{5a}\right.$$

$\Big\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}-\frac{\frac{2a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}-\frac{\int \frac{13a^2-2a^2\sec(c+}{\sqrt{\sec(c+dx)}\sqrt{\sec(c+}}}{3a}}{5a}\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}-\frac{\frac{2a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}-\frac{\int \frac{13a^2-2a^2\csc(}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\csc}}}{3a}}{5a}\right.$$

$\Big\downarrow$ 4501

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}-\frac{\frac{2a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}-\frac{\frac{26a^2\sin(c+dx)\sqrt{\sec(c+}}{d\sqrt{a\sec(c+dx)+a}}}{3a}}{5a}\right.$$

3.420. $\int \frac{\cos^{\frac{5}{2}}(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}-\frac{\frac{2a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}-\frac{\frac{26a^2\sin(c+dx)\sqrt{\sec(c+}}{d\sqrt{a\sec(c+dx)+a}}}{}}{5a}\right.$$

$$\Big\downarrow \; 4295$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}-\frac{\frac{2a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}-\frac{\frac{30a^2\int\frac{1}{2a-\frac{a^2\sin(c+dx}{\sec(c+c}}}{}}{}}{}\right.$$

$$\Big\downarrow \; 219$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}-\frac{\frac{2a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}-\frac{\frac{26a^2\sin(c+dx)\sqrt{\sec(c+}}{d\sqrt{a\sec(c+dx)+a}}}{}}{5a}\right.$$

input
```
Int[Cos[c + d*x]^(5/2)/Sqrt[a + a*Sec[c + d*x]],x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*Sin[c + d*x])/(5*d*Sec[c + d*x]^
(3/2)*Sqrt[a + a*Sec[c + d*x]]) - ((2*a*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*
x]]*Sqrt[a + a*Sec[c + d*x]]) - ((-15*Sqrt[2]*a^(3/2)*ArcTanh[(Sqrt[a]*Sqr
t[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/d + (26
*a^2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]]))/(3*a))
/(5*a))
```

3.420. $\int \frac{\cos^{\frac{5}{2}}(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$

3.420.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4295
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[-2*b*(d/(a*f))   Subst[Int[1/(2*b - d*x^2), x],
x, b*(Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]]))], x] /;
 FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4310
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*n*Sqrt[a +
b*Csc[e + f*x]])), x] + Simp[1/(2*b*d*n)   Int[(d*Csc[e + f*x])^(n + 1)*((a
 + b*(2*n + 1)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, 0] && IntegerQ[2*n]
```

rule 4501
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*Cot[e
 + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(f*n)), x] - Simp[(a*A*m
 - b*B*n)/(b*d*n)   Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x]
, x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a
^2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LeQ[m, -1]
```

rule 4510
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*Cot[e
 + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(f*n)), x] - Simp[1/(b*d
*n)   Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*
n - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

3.420. $\int \frac{\cos^{\frac{5}{2}}(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

3.420.4 Maple [A] (verified)

Time = 1.65 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.03

method	result
default	$-\frac{\left(15\sqrt{2}\,\arctan\left(\frac{\sin(dx+c)\sqrt{2}}{2(\cos(dx+c)+1)\sqrt{-\frac{1}{\cos(dx+c)+1}}}\right)\sqrt{-\frac{1}{\cos(dx+c)+1}}\,\cos(dx+c)+15\sqrt{2}\,\arctan\left(\frac{\sin(dx+c)\sqrt{2}}{2(\cos(dx+c)+1)\sqrt{-\frac{1}{\cos(dx+c)+1}}}\right)\sqrt{}\right.}{15da(\cos(dx+c)+1}$

input
```
int(cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/15/d/a*(15*2^(1/2)*arctan(1/2*sin(d*x+c)*2^(1/2)/(cos(d*x+c)+1)/(-1/(co
s(d*x+c)+1))^(1/2))*(-1/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)+15*2^(1/2)*arctan
(1/2*sin(d*x+c)*2^(1/2)/(cos(d*x+c)+1)/(-1/(cos(d*x+c)+1))^(1/2))*(-1/(cos
(d*x+c)+1))^(1/2)-6*cos(d*x+c)^2*sin(d*x+c)+2*cos(d*x+c)*sin(d*x+c)-26*sin
(d*x+c))*(a*(1+sec(d*x+c)))^(1/2)*cos(d*x+c)^(1/2)/(cos(d*x+c)+1)
```

3.420.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 324, normalized size of antiderivative = 1.71

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$$
$$= \left[\frac{4\left(3\cos\left(dx+c\right)^2-\cos\left(dx+c\right)+13\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)+\frac{15\sqrt{2}(a\cos(dx+c)+a)\log\left(-\right.}{}}{30\left(ad\cos\left(dx+c\right)+ad\right)}\right.$$

input
```
integrate(cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
[1/30*(4*(3*cos(d*x + c)^2 - cos(d*x + c) + 13)*sqrt((a*cos(d*x + c) + a)/
cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) + 15*sqrt(2)*(a*cos(d*x + c)
 + a)*log(-(cos(d*x + c)^2 + 2*sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x +
 c))*sqrt(cos(d*x + c))*sin(d*x + c)/sqrt(a) - 2*cos(d*x + c) - 3)/(cos(d*
x + c)^2 + 2*cos(d*x + c) + 1))/sqrt(a))/(a*d*cos(d*x + c) + a*d), 1/15*(1
5*sqrt(2)*(a*cos(d*x + c) + a)*sqrt(-1/a)*arctan(sqrt(2)*sqrt((a*cos(d*x +
 c) + a)/cos(d*x + c))*sqrt(-1/a)*sqrt(cos(d*x + c))/sin(d*x + c)) + 2*(3*
cos(d*x + c)^2 - cos(d*x + c) + 13)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)
)*sqrt(cos(d*x + c))*sin(d*x + c))/(a*d*cos(d*x + c) + a*d)]
```

3.420.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(5/2)/(a+a*sec(d*x+c))**(1/2),x)
```

output
```
Timed out
```

3.420.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 357 vs. $2(156) = 312$.

Time = 0.37 (sec) , antiderivative size = 357, normalized size of antiderivative = 1.89

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$$
$$= \frac{\sqrt{2}\Big(60\cos\left(\frac{4}{5}\arctan\left(\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right),\cos\left(\frac{5}{2}dx+\frac{5}{2}c\right)\right)\right)\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right)-5\cos\left(\frac{2}{5}\arctan\left(\sin\left(\frac{5}{2}dx+\right.\right.\right.}{}$$

input
```
integrate(cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
1/60*sqrt(2)*(60*cos(4/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c
)))*sin(5/2*d*x + 5/2*c) - 5*cos(2/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2
*d*x + 5/2*c)))*sin(5/2*d*x + 5/2*c) - 60*cos(5/2*d*x + 5/2*c)*sin(4/5*arc
tan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 5*cos(5/2*d*x + 5/2*c)
*sin(2/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) - 30*log(cos
(1/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c)))^2 + sin(1/5*arct
an2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c)))^2 + 2*sin(1/5*arctan2(sin
(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 1) + 30*log(cos(1/5*arctan2(si
n(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c)))^2 + sin(1/5*arctan2(sin(5/2*d*x
 + 5/2*c), cos(5/2*d*x + 5/2*c)))^2 - 2*sin(1/5*arctan2(sin(5/2*d*x + 5/2*
c), cos(5/2*d*x + 5/2*c))) + 1) + 6*sin(5/2*d*x + 5/2*c) - 5*sin(3/5*arcta
n2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 60*sin(1/5*arctan2(sin(5
/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))))/(sqrt(a)*d)
```

3.420.8 Giac [F]

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \int \frac{\cos\left(dx+c\right)^{\frac{5}{2}}}{\sqrt{a\sec\left(dx+c\right)+a}}\,dx$$

input
```
integrate(cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(cos(d*x + c)^(5/2)/sqrt(a*sec(d*x + c) + a), x)
```

3.420.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \int \frac{\cos\left(c+d\,x\right)^{5/2}}{\sqrt{a+\frac{a}{\cos(c+d\,x)}}}\,dx$$

input
```
int(cos(c + d*x)^(5/2)/(a + a/cos(c + d*x))^(1/2),x)
```

output
```
int(cos(c + d*x)^(5/2)/(a + a/cos(c + d*x))^(1/2), x)
```

3.421 $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$

3.421.1 Optimal result 3115
3.421.2 Mathematica [A] (verified) 3115
3.421.3 Rubi [A] (verified) 3116
3.421.4 Maple [A] (verified) 3119
3.421.5 Fricas [A] (verification not implemented) 3119
3.421.6 Sympy [F] 3120
3.421.7 Maxima [B] (verification not implemented) 3120
3.421.8 Giac [F] 3121
3.421.9 Mupad [F(-1)] 3121

3.421.1 Optimal result

Integrand size = 25, antiderivative size = 151

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \frac{\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{\sqrt{a}d} - \frac{2\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} + \frac{2\sqrt{\cos(c+dx)}\sin(c+dx)}{3d\sqrt{a+a\sec(c+dx)}}$$

output
```
arctanh(1/2*sin(d*x+c)*a^(1/2)*sec(d*x+c)^(1/2)*2^(1/2)/(a+a*sec(d*x+c))^(
1/2))*2^(1/2)*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d/a^(1/2)-2/3*sin(d*x+c)/d
/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2)+2/3*sin(d*x+c)*cos(d*x+c)^(1/2)/d
/(a+a*sec(d*x+c))^(1/2)
```

3.421.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.77

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \frac{\sqrt{\cos(c+dx)}\left(2(1-\sec(c+dx))^{3/2} - 3\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)\sec^{\frac{3}{2}}(c+dx)\right)\sin(c+dx)}{3d\sqrt{1-\sec(c+dx)}\sqrt{a(1+\sec(c+dx))}}$$

input
```
Integrate[Cos[c + d*x]^(3/2)/Sqrt[a + a*Sec[c + d*x]],x]
```

output
```
(Sqrt[Cos[c + d*x]]*(2*(1 - Sec[c + d*x])^(3/2) - 3*Sqrt[2]*ArcTan[(Sqrt[2
]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*Sec[c + d*x]^(3/2))*Sin[c +
d*x])/(3*d*Sqrt[1 - Sec[c + d*x]]*Sqrt[a*(1 + Sec[c + d*x])])
```

3.421.3 Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.06, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {3042, 4752, 3042, 4310, 3042, 4501, 3042, 4295, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{a\sec(c+dx)+a}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\sqrt{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}}dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)\sqrt{\sec(c+dx)a+a}}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx$$

$$\Big\downarrow \text{4310}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}-\frac{\int \frac{a-2a\sec(c+dx)}{\sqrt{\sec(c+dx)}\sqrt{\sec(c+dx)a+a}}dx}{3a}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}-\frac{\int\frac{a-2a\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{3a}\right)$$

$\Big\downarrow$ 4501

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}-\frac{\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\sec(c+dx)+a}}-3a\int\frac{\sqrt{\sec(c+dx)}}{\sqrt{\sec(c+dx)a+a}}dx}{3a}\right)$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}-\frac{\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\sec(c+dx)+a}}-3a\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}}{3a}\right)$$

$\Big\downarrow$ 4295

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}-\frac{\frac{6a\int\frac{1}{2a-\frac{a^2\sin(c+dx)\tan(c+dx)}{\sec(c+dx)a+a}}d\left(-\frac{a\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right.}{d}}{3a}\right)$$

$\Big\downarrow$ 219

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}-\frac{\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\sec(c+dx)+a}}-\frac{3\sqrt{2}\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a}\sin(c-}{\sqrt{2}\sqrt{a}}\right.}{d}}{3a}\right)$$

input
```
Int[Cos[c + d*x]^(3/2)/Sqrt[a + a*Sec[c + d*x]],x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*Sin[c + d*x])/(3*d*Sqrt[Sec[c +
d*x]]*Sqrt[a + a*Sec[c + d*x]]) - ((-3*Sqrt[2]*Sqrt[a]*ArcTanh[(Sqrt[a]*Sq
rt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/d + (2
*a*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]]))/(3*a))
```

3.421. $\int\frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$

3.421.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4295
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[-2*b*(d/(a*f))   Subst[Int[1/(2*b - d*x^2), x],
x, b*(Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]]))], x] /;
 FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4310
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*n*Sqrt[a +
b*Csc[e + f*x]])), x] + Simp[1/(2*b*d*n)   Int[(d*Csc[e + f*x])^(n + 1)*((a
 + b*(2*n + 1)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, 0] && IntegerQ[2*n]
```

rule 4501
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*Cot[e
 + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(f*n)), x] - Simp[(a*A*m
 - b*B*n)/(b*d*n)   Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x]
, x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a
^2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LeQ[m, -1]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

3.421. $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$

3.421.4 Maple [A] (verified)

Time = 1.87 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.18

method	result
default	$\frac{\left(3\sqrt{2}\,\arctan\left(\frac{\sin(dx+c)\sqrt{2}}{2(\cos(dx+c)+1)\sqrt{-\frac{1}{\cos(dx+c)+1}}}\right)\sqrt{-\frac{1}{\cos(dx+c)+1}}\,\cos(dx+c)+3\sqrt{2}\,\arctan\left(\frac{\sin(dx+c)\sqrt{2}}{2(\cos(dx+c)+1)\sqrt{-\frac{1}{\cos(dx+c)+1}}}\right)\sqrt{-\frac{}{\cos(}}\right.}{3da(\cos(dx+c)+1)}$

input
```
int(cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3/d/a*(3*2^(1/2)*arctan(1/2*sin(d*x+c)*2^(1/2)/(cos(d*x+c)+1)/(-1/(cos(d
*x+c)+1))^(1/2))*(-1/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)+3*2^(1/2)*arctan(1/2
*sin(d*x+c)*2^(1/2)/(cos(d*x+c)+1)/(-1/(cos(d*x+c)+1))^(1/2))*(-1/(cos(d*x
+c)+1))^(1/2)+2*cos(d*x+c)*sin(d*x+c)-2*sin(d*x+c))*(a*(1+sec(d*x+c)))^(1/
2)*cos(d*x+c)^(1/2)/(cos(d*x+c)+1)
```

3.421.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 300, normalized size of antiderivative = 1.99

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$$

$$= \left[\frac{4\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}(\cos(dx+c)-1)\sqrt{\cos(dx+c)}\sin(dx+c)+\frac{3\sqrt{2}(a\cos(dx+c)+a)\log\left(-\frac{\cos(dx+c)^2-\frac{2\sqrt{2}\sqrt{\frac{a\cos(dx+}{\cos(dx-}}}{\cos(dx}}{\sqrt{a}}\right.}{6(ad\cos(dx+c)+ad)}\right.$$

$$-\frac{3\sqrt{2}(a\cos(dx+c)+a)\sqrt{-\frac{1}{a}}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{-\frac{1}{a}}\sqrt{\cos(dx+c)}}{\sin(dx+c)}\right)-2\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}(\cos(dx+c)-}{3(ad\cos(dx+c)+ad)}$$

input
```
integrate(cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
[1/6*(4*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(cos(d*x + c) - 1)*sqrt(co
s(d*x + c))*sin(d*x + c) + 3*sqrt(2)*(a*cos(d*x + c) + a)*log(-(cos(d*x +
c)^2 - 2*sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c)
)*sin(d*x + c)/sqrt(a) - 2*cos(d*x + c) - 3)/(cos(d*x + c)^2 + 2*cos(d*x +
 c) + 1))/sqrt(a))/(a*d*cos(d*x + c) + a*d), -1/3*(3*sqrt(2)*(a*cos(d*x +
c) + a)*sqrt(-1/a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*
sqrt(-1/a)*sqrt(cos(d*x + c))/sin(d*x + c)) - 2*sqrt((a*cos(d*x + c) + a)/
cos(d*x + c))*(cos(d*x + c) - 1)*sqrt(cos(d*x + c))*sin(d*x + c))/(a*d*cos
(d*x + c) + a*d)]
```

3.421.6 Sympy [F]

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{a\left(\sec(c+dx)+1\right)}}\,dx$$

input
```
integrate(cos(d*x+c)**(3/2)/(a+a*sec(d*x+c))**(1/2),x)
```

output
```
Integral(cos(c + d*x)**(3/2)/sqrt(a*(sec(c + d*x) + 1)), x)
```

3.421.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 282 vs. $2(124) = 248$.

Time = 0.40 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.87

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = -\frac{3\sqrt{2}\cos\left(\frac{2}{3}\arctan\left(\sin\left(\frac{3}{2}dx+\frac{3}{2}c\right),\cos\left(\frac{3}{2}dx+\frac{3}{2}c\right)\right)\right)\sin\left(\frac{3}{2}dx+\frac{3}{2}c\right)-3\sqrt{2}\cos\left(\frac{3}{2}dx+\frac{3}{2}c\right)\sin\left(\frac{2}{3}\,a\right.}{}$$

input
```
integrate(cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
-1/6*(3*sqrt(2)*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)
))*sin(3/2*d*x + 3/2*c) - 3*sqrt(2)*cos(3/2*d*x + 3/2*c)*sin(2/3*arctan2(s
in(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 3*sqrt(2)*log(cos(1/3*arctan
2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + sin(1/3*arctan2(sin(3/2
*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x +
3/2*c), cos(3/2*d*x + 3/2*c))) + 1) + 3*sqrt(2)*log(cos(1/3*arctan2(sin(3/
2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + sin(1/3*arctan2(sin(3/2*d*x + 3
/2*c), cos(3/2*d*x + 3/2*c)))^2 - 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c),
cos(3/2*d*x + 3/2*c))) + 1) - 2*sqrt(2)*sin(3/2*d*x + 3/2*c) + 3*sqrt(2)*s
in(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))/(sqrt(a)*d)
```

3.421.8 Giac [F]

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \int \frac{\cos\left(dx+c\right)^{\frac{3}{2}}}{\sqrt{a\sec\left(dx+c\right)+a}}\,dx$$

input
```
integrate(cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(cos(d*x + c)^(3/2)/sqrt(a*sec(d*x + c) + a), x)
```

3.421.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \int \frac{\cos\left(c+d\,x\right)^{3/2}}{\sqrt{a+\frac{a}{\cos(c+d\,x)}}}\,dx$$

input
```
int(cos(c + d*x)^(3/2)/(a + a/cos(c + d*x))^(1/2),x)
```

output
```
int(cos(c + d*x)^(3/2)/(a + a/cos(c + d*x))^(1/2), x)
```

3.422 $\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+a\sec(c+dx)}}\,dx$

3.422.1 Optimal result . . . 3122
3.422.2 Mathematica [A] (verified) . . . 3122
3.422.3 Rubi [A] (verified) . . . 3123
3.422.4 Maple [A] (verified) . . . 3125
3.422.5 Fricas [A] (verification not implemented) . . . 3125
3.422.6 Sympy [F] . . . 3126
3.422.7 Maxima [A] (verification not implemented) . . . 3126
3.422.8 Giac [F] . . . 3127
3.422.9 Mupad [F(-1)] . . . 3127

3.422.1 Optimal result

Integrand size = 25, antiderivative size = 113

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+a\sec(c+dx)}}\,dx = -\frac{\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{\sqrt{a}d} + \frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}}$$

output
```
-arctanh(1/2*sin(d*x+c)*a^(1/2)*sec(d*x+c)^(1/2)*2^(1/2)/(a+a*sec(d*x+c))^
(1/2))*2^(1/2)*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d/a^(1/2)+2*sin(d*x+c)/d/
cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2)
```

3.422.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.88

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+a\sec(c+dx)}}\,dx = \frac{\left(2\sqrt{1-\sec(c+dx)}+\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)\sqrt{\sec(c+dx)}\right)\sin(c+dx)}{d\sqrt{-1+\cos(c+dx)}\sqrt{a(1+\sec(c+dx))}}$$

input
```
Integrate[Sqrt[Cos[c + d*x]]/Sqrt[a + a*Sec[c + d*x]],x]
```

output
```
((2*Sqrt[1 - Sec[c + d*x]] + Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/S
qrt[1 - Sec[c + d*x]]]*Sqrt[Sec[c + d*x]])*Sin[c + d*x])/(d*Sqrt[-1 + Cos[
c + d*x]]*Sqrt[a*(1 + Sec[c + d*x])])
```

3.422.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3042, 4752, 3042, 4299, 3042, 4295, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a\sec(c+dx)+a}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}}dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{\sec(c+dx)a+a}}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx$$

$$\Big\downarrow \text{4299}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\sec(c+dx)+a}}-\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{\sec(c+dx)a+a}}dx\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\sec(c+dx)+a}}-\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx\right)$$

$$\Big\downarrow \text{4295}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\int \frac{1}{2a-\frac{a^2\sin(c+dx)\tan(c+dx)}{\sec(c+dx)a+a}}d\left(-\frac{a\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}+\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\sec(c+dx)+a}}\right)$$

$$\Big\downarrow \; 219$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\sec(c+dx)+a}}-\frac{\sqrt{2}\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}\right)$$

input
```
Int[Sqrt[Cos[c + d*x]]/Sqrt[a + a*Sec[c + d*x]],x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(-((Sqrt[2]*ArcTanh[(Sqrt[a]*Sqrt[Se
c[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/(Sqrt[a]*d)
) + (2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]]))
```

3.422.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4295
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[-2*b*(d/(a*f))   Subst[Int[1/(2*b - d*x^2), x],
x, b*(Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]]))], x] /;
 FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4299
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-Cot[e + f*x])*(a + b*Csc[e + f*x])^m*((d*Csc
[e + f*x])^n/(f*(m + 1))), x] + Simp[a*(m/(b*d*(m + 1)))   Int[(a + b*Csc[e
 + f*x])^m*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m, n},
 x] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LtQ[m, -2^(-1)]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

3.422.4 Maple [A] (verified)

Time = 1.43 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.88

method	result
default	$-\frac{\sqrt{\cos(dx+c)}\,\sqrt{a(1+\sec(dx+c))}\left(\sqrt{2}\,\arctan\left(\frac{\sin(dx+c)\sqrt{2}}{2(\cos(dx+c)+1)\sqrt{-\frac{1}{\cos(dx+c)+1}}}\right)\sqrt{-\frac{1}{\cos(dx+c)+1}}+2\cot(dx+c)-2\csc(dx+c)\right)}{da}$

input
```
int(cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/d/a*cos(d*x+c)^(1/2)*(a*(1+sec(d*x+c)))^(1/2)*(2^(1/2)*arctan(1/2*sin(d
*x+c)*2^(1/2)/(cos(d*x+c)+1)/(-1/(cos(d*x+c)+1))^(1/2))*(-1/(cos(d*x+c)+1)
)^(1/2)+2*cot(d*x+c)-2*csc(d*x+c))
```

3.422.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 281, normalized size of antiderivative = 2.49

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+a\sec(c+dx)}}\,dx = \left[\frac{\frac{\sqrt{2}(a\cos(dx+c)+a)\log\left(-\frac{\cos(dx+c)^2+\frac{2\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)}{\sqrt{a}}-2\cos(dx+c)-3}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)}{\sqrt{a}}+4\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}}{2(ad\cos(dx+c)+ad)}\right.$$

input
```
integrate(cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
[1/2*(sqrt(2)*(a*cos(d*x + c) + a)*log(-(cos(d*x + c)^2 + 2*sqrt(2)*sqrt((
a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/sqrt(a)
- 2*cos(d*x + c) - 3)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1))/sqrt(a) + 4*s
qrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(a
*d*cos(d*x + c) + a*d), (sqrt(2)*(a*cos(d*x + c) + a)*sqrt(-1/a)*arctan(sq
rt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(-1/a)*sqrt(cos(d*x + c)
)/sin(d*x + c)) + 2*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x +
 c))*sin(d*x + c))/(a*d*cos(d*x + c) + a*d)]
```

3.422.6 Sympy [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+a\sec(c+dx)}}\,dx = \int \frac{\sqrt{\cos{(c+dx)}}}{\sqrt{a\left(\sec{(c+dx)}+1\right)}}\,dx$$

input
```
integrate(cos(d*x+c)**(1/2)/(a+a*sec(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(cos(c + d*x))/sqrt(a*(sec(c + d*x) + 1)), x)
```

3.422.7 Maxima [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+a\sec(c+dx)}}\,dx = -\frac{\sqrt{2}\log\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+2\,\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)-\sqrt{2}\log\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+\mathrm{s}\right.}{2\,\sqrt{a}d}$$

input
```
integrate(cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
-1/2*(sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(
1/2*d*x + 1/2*c) + 1) - sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x +
 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) - 4*sqrt(2)*sin(1/2*d*x + 1/2*c))/
(sqrt(a)*d)
```

3.422.8 Giac [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+a\sec(c+dx)}}\,dx = \int \frac{\sqrt{\cos(dx+c)}}{\sqrt{a\sec(dx+c)+a}}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(cos(d*x + c))/sqrt(a*sec(d*x + c) + a), x)
```

3.422.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+a\sec(c+dx)}}\,dx = \int \frac{\sqrt{\cos(c+d\,x)}}{\sqrt{a+\frac{a}{\cos(c+d\,x)}}}\,dx$$

input
```
int(cos(c + d*x)^(1/2)/(a + a/cos(c + d*x))^(1/2),x)
```

output
```
int(cos(c + d*x)^(1/2)/(a + a/cos(c + d*x))^(1/2), x)
```

3.423 $\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}}\,dx$

3.423.1 Optimal result . . . 3128
3.423.2 Mathematica [A] (verified) . . . 3128
3.423.3 Rubi [A] (verified) . . . 3129
3.423.4 Maple [B] (verified) . . . 3130
3.423.5 Fricas [A] (verification not implemented) . . . 3131
3.423.6 Sympy [F] . . . 3131
3.423.7 Maxima [A] (verification not implemented) . . . 3132
3.423.8 Giac [F] . . . 3132
3.423.9 Mupad [F(-1)] . . . 3132

3.423.1 Optimal result

Integrand size = 25, antiderivative size = 56

$$\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}}\,dx=\frac{\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}d}$$

output
```
arctanh(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(
1/2))*2^(1/2)/d/a^(1/2)
```

3.423.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.70

$$\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}}\,dx = -\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d\sqrt{1-\sec(c+dx)}\sqrt{a(1+\sec(c+dx))}}$$

input
```
Integrate[1/(Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]),x]
```

output
```
-((Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*Sqr
t[Cos[c + d*x]]*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(d*Sqrt[1 - Sec[c + d*x]]
*Sqrt[a*(1 + Sec[c + d*x])]))
```

3.423.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.36, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3042, 4752, 3042, 4295, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}}dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{\sec(c+dx)a+a}}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx$$

$$\Big\downarrow \text{4295}$$

$$-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{2a-\frac{a^2\sin(c+dx)\tan(c+dx)}{\sec(c+dx)a+a}}d\left(-\frac{a\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}$$

$$\Big\downarrow \text{219}$$

$$\frac{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}$$

input
```
Int[1/(Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]),x]
```

output
```
(Sqrt[2]*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a
 + a*Sec[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(Sqrt[a]*d)
```

3.423.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4295
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[-2*b*(d/(a*f))   Subst[Int[1/(2*b - d*x^2), x],
x, b*(Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]]))], x] /;
 FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

3.423.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 91 vs. $2(45) = 90$.

Time = 1.87 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.64

method	result	size
default	$-\frac{\sqrt{2}\,\sqrt{a(1+\sec(dx+c))}\,\sqrt{\cos(dx+c)}\,\arctan\left(\frac{\sin(dx+c)\sqrt{2}}{2(\cos(dx+c)+1)\sqrt{-\frac{1}{\cos(dx+c)+1}}}\right)}{da(\cos(dx+c)+1)\sqrt{-\frac{1}{\cos(dx+c)+1}}}$	92

input
```
int(1/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/d/a*2^(1/2)*(a*(1+sec(d*x+c)))^(1/2)*cos(d*x+c)^(1/2)*arctan(1/2*sin(d*
x+c)*2^(1/2)/(cos(d*x+c)+1)/(-1/(cos(d*x+c)+1))^(1/2))/(cos(d*x+c)+1)/(-1/
(cos(d*x+c)+1))^(1/2)
```

3.423.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 160, normalized size of antiderivative = 2.86

$$\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}}\,dx = \left[\frac{\sqrt{2}\log\left(-\frac{\cos(dx+c)^2-\frac{2\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)}{\sqrt{a}}-2\cos(dx+c)-3}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)}{2\sqrt{a}d}, -\frac{\sqrt{2}\sqrt{-\frac{1}{a}}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{-\frac{1}{a}}\sqrt{\cos(dx+c)}}{\sin(dx+c)}\right)}{d}\right]$$

input
```
integrate(1/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
[1/2*sqrt(2)*log(-(cos(d*x + c)^2 - 2*sqrt(2)*sqrt((a*cos(d*x + c) + a)/co
s(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/sqrt(a) - 2*cos(d*x + c) - 3)/
(cos(d*x + c)^2 + 2*cos(d*x + c) + 1))/(sqrt(a)*d), -sqrt(2)*sqrt(-1/a)*ar
ctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(-1/a)*sqrt(cos(d
*x + c))/sin(d*x + c))/d]
```

3.423.6 Sympy [F]

$$\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{a\left(\sec\left(c+dx\right)+1\right)}\sqrt{\cos\left(c+dx\right)}}\,dx$$

input
```
integrate(1/cos(d*x+c)**(1/2)/(a+a*sec(d*x+c))**(1/2),x)
```

output
```
Integral(1/(sqrt(a*(sec(c + d*x) + 1))*sqrt(cos(c + d*x))), x)
```

3.423.7 Maxima [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.61

$$\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}}\,dx = \frac{\sqrt{2}\log\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+2\,\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)-\sqrt{2}\log\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+\sin\right.}{2\,\sqrt{a}d}$$

input
```
integrate(1/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
1/2*(sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1
/2*d*x + 1/2*c) + 1) - sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x +
1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))/(sqrt(a)*d)
```

3.423.8 Giac [F]

$$\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{a\sec(dx+c)+a}\sqrt{\cos(dx+c)}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(a*sec(d*x + c) + a)*sqrt(cos(d*x + c))), x)
```

3.423.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{\cos(c+d\,x)}\,\sqrt{a+\frac{a}{\cos(c+d\,x)}}}\,dx$$

input
```
int(1/(cos(c + d*x)^(1/2)*(a + a/cos(c + d*x))^(1/2)),x)
```

output
```
int(1/(cos(c + d*x)^(1/2)*(a + a/cos(c + d*x))^(1/2)), x)
```

3.424 $\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\,dx$

3.424.1 Optimal result . . . 3133
3.424.2 Mathematica [A] (verified) . . . 3133
3.424.3 Rubi [A] (verified) . . . 3134
3.424.4 Maple [A] (verified) . . . 3136
3.424.5 Fricas [A] (verification not implemented) . . . 3137
3.424.6 Sympy [F] . . . 3138
3.424.7 Maxima [B] (verification not implemented) . . . 3138
3.424.8 Giac [F] . . . 3139
3.424.9 Mupad [F(-1)] . . . 3139

3.424.1 Optimal result

Integrand size = 25, antiderivative size = 135

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\,dx = \frac{2\operatorname{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{\sqrt{a}d} - \frac{\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{\sqrt{a}d}$$

output
```
2*arcsinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(
d*x+c)^(1/2)/d/a^(1/2)-arctanh(1/2*sin(d*x+c)*a^(1/2)*sec(d*x+c)^(1/2)*2^(
1/2)/(a+a*sec(d*x+c))^(1/2))*2^(1/2)*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d/a
^(1/2)
```

3.424.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.81

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\,dx = \frac{\left(-2\arcsin\left(\sqrt{\sec(c+dx)}\right)+\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)\right)\sqrt{\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d\sqrt{1-\sec(c+dx)}\sqrt{a(1+\sec(c+dx))}}$$

input
```
Integrate[1/(Cos[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]]),x]
```

output
```
((-2*ArcSin[Sqrt[Sec[c + d*x]]] + Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x
]])/Sqrt[1 - Sec[c + d*x]]])*Sqrt[Cos[c + d*x]]*Sec[c + d*x]^(3/2)*Sin[c +
 d*x])/(d*Sqrt[1 - Sec[c + d*x]]*Sqrt[a*(1 + Sec[c + d*x])])
```

3.424.3 Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.86, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {3042, 4752, 3042, 4308, 3042, 4288, 222, 4295, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}}dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{\sec(c+dx)a+a}}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx$$

$$\Big\downarrow \text{4308}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \sqrt{\sec(c+dx)}\sqrt{\sec(c+dx)a+a}dx}{a}-\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{\sec(c+dx)a+a}}dx\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{a}-\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx\right)$$

$$\Big\downarrow \text{4288}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx-\frac{2\int \frac{1}{\sqrt{\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1}}d\left(-\frac{a\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{ad}\right)$$

$$\Big\downarrow \text{222}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\text{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}-\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx\right)$$

$$\Big\downarrow \text{4295}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\int \frac{1}{2a-\frac{a^2\sin(c+dx)\tan(c+dx)}{\sec(c+dx)a+a}}d\left(-\frac{a\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}+\frac{2\text{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}\right)$$

$$\Big\downarrow \text{219}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\text{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}-\frac{\sqrt{2}\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}\right)$$

input
```
Int[1/(Cos[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]]),x]
```

output
```
((2*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(Sqrt[a]*d)
- (Sqrt[2]*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt
[a + a*Sec[c + d*x]])])/(Sqrt[a]*d))*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]
```

3.424.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4288
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[-2*(a/(b*f))*Sqrt[a*(d/b)]   Subst[Int[1/Sqrt[1
+ x^2/a], x], x, b*(Cot[e + f*x]/Sqrt[a + b*Csc[e + f*x]])], x] /; FreeQ[{a
, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[a*(d/b), 0]
```

rule 4295
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[-2*b*(d/(a*f))   Subst[Int[1/(2*b - d*x^2), x],
x, b*(Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]]))], x] /;
 FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4308
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Simp[d/b   Int[Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[
e + f*x]], x], x] - Simp[a*(d/b)   Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[
e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

3.424.4 Maple [A] (verified)

Time = 2.13 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.34

method	result
default	$\frac{\sqrt{\cos(dx+c)}\left(\arctan\left(\frac{\sin(dx+c)\sqrt{2}}{2(\cos(dx+c)+1)\sqrt{-\frac{1}{\cos(dx+c)+1}}}\right)\sqrt{2}-\arctan\left(\frac{\cos(dx+c)+\sin(dx+c)+1}{2(\cos(dx+c)+1)\sqrt{-\frac{1}{\cos(dx+c)+1}}}\right)-\arctan\left(\frac{-\cos(dx+c)+\sin(}{2(\cos(dx+c)+1)\sqrt{-}}\right.\right.}{da(\cos(dx+c)+1)\sqrt{-\frac{1}{\cos(dx+c)+1}}}$

input
```
int(1/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/d/a*cos(d*x+c)^(1/2)*(arctan(1/2*sin(d*x+c)*2^(1/2)/(cos(d*x+c)+1)/(-1/(
cos(d*x+c)+1))^(1/2))*2^(1/2)-arctan(1/2*(cos(d*x+c)+sin(d*x+c)+1)/(cos(d*
x+c)+1)/(-1/(cos(d*x+c)+1))^(1/2))-arctan(1/2*(-cos(d*x+c)+sin(d*x+c)-1)/(
cos(d*x+c)+1)/(-1/(cos(d*x+c)+1))^(1/2)))*(a*(1+sec(d*x+c)))^(1/2)/(cos(d*
x+c)+1)/(-1/(cos(d*x+c)+1))^(1/2)
```

3.424.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 342, normalized size of antiderivative = 2.53

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\,dx$$
$$= \left[\frac{\sqrt{2}\sqrt{a}\log\left(-\frac{\cos(dx+c)^2+\frac{2\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)}{\sqrt{a}}-2\cos(dx+c)-3}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)+\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-4\sqrt{a}\sqrt{\frac{a\cos(dx+c}{\cos(dx+}}}{}\right.}{2\,ad}\right.$$

input
```
integrate(1/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
[1/2*(sqrt(2)*sqrt(a)*log(-(cos(d*x + c)^2 + 2*sqrt(2)*sqrt((a*cos(d*x + c
) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/sqrt(a) - 2*cos(d*x +
 c) - 3)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + sqrt(a)*log((a*cos(d*x +
 c)^3 - 4*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(cos(d*x + c) -
2)*sqrt(cos(d*x + c))*sin(d*x + c) - 7*a*cos(d*x + c)^2 + 8*a)/(cos(d*x +
c)^3 + cos(d*x + c)^2)))/(a*d), (sqrt(2)*a*sqrt(-1/a)*arctan(sqrt(2)*sqrt(
(a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(-1/a)*sqrt(cos(d*x + c))/sin(d*x +
 c)) + sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*
sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a))
)/(a*d)]
```

3.424.6 Sympy [F]

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{a\left(\sec\left(c+dx\right)+1\right)}\cos^{\frac{3}{2}}\left(c+dx\right)}\,dx$$

input
```
integrate(1/cos(d*x+c)**(3/2)/(a+a*sec(d*x+c))**(1/2),x)
```

output
```
Integral(1/(sqrt(a*(sec(c + d*x) + 1))*cos(c + d*x)**(3/2)), x)
```

3.424.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 476 vs. $2(110) = 220$.

Time = 0.39 (sec) , antiderivative size = 476, normalized size of antiderivative = 3.53

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\,dx =$$
$$-\frac{\sqrt{2}\log\left(\cos\left(\frac{1}{2}\arctan\left(\sin\left(dx+c\right),\cos\left(dx+c\right)\right)\right)^2+\sin\left(\frac{1}{2}\arctan\left(\sin\left(dx+c\right),\cos\left(dx+c\right)\right)\right)^2+2\,\mathrm{si}\right.}{}$$

input
```
integrate(1/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
-1/2*(sqrt(2)*log(cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + sin(1/2
*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c),
cos(d*x + c))) + 1) - sqrt(2)*log(cos(1/2*arctan2(sin(d*x + c), cos(d*x +
c)))^2 + sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 - 2*sin(1/2*arctan
2(sin(d*x + c), cos(d*x + c))) + 1) - log(2*cos(1/2*arctan2(sin(d*x + c),
cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sq
rt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2*sqrt(2)*sin(1/2*arc
tan2(sin(d*x + c), cos(d*x + c))) + 2) + log(2*cos(1/2*arctan2(sin(d*x + c
), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2
*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) - 2*sqrt(2)*sin(1/2*
arctan2(sin(d*x + c), cos(d*x + c))) + 2) - log(2*cos(1/2*arctan2(sin(d*x
+ c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2
- 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2*sqrt(2)*sin(1
/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) + log(2*cos(1/2*arctan2(sin(d
*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))
^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) - 2*sqrt(2)*si
n(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2))/(sqrt(a)*d)
```

3.424.8 Giac [F]

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{a\sec(dx+c)+a}\cos(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(a*sec(d*x + c) + a)*cos(d*x + c)^(3/2)), x)
```

3.424.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\,dx = \int \frac{1}{\cos(c+d\,x)^{3/2}\,\sqrt{a+\frac{a}{\cos(c+d\,x)}}}\,dx$$

input
```
int(1/(cos(c + d*x)^(3/2)*(a + a/cos(c + d*x))^(1/2)),x)
```

output
```
int(1/(cos(c + d*x)^(3/2)*(a + a/cos(c + d*x))^(1/2)), x)
```

3.425 $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\,dx$

3.425.1 Optimal result . . . 3140
3.425.2 Mathematica [A] (verified) . . . 3141
3.425.3 Rubi [A] (verified) . . . 3141
3.425.4 Maple [A] (verified) . . . 3144
3.425.5 Fricas [A] (verification not implemented) . . . 3145
3.425.6 Sympy [F(-1)] . . . 3146
3.425.7 Maxima [B] (verification not implemented) . . . 3146
3.425.8 Giac [F] . . . 3147
3.425.9 Mupad [F(-1)] . . . 3148

3.425.1 Optimal result

Integrand size = 25, antiderivative size = 168

$$\begin{aligned}\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\,dx &= -\frac{\operatorname{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{\sqrt{a}d} \\ &\quad + \frac{\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{\sqrt{a}d} \\ &\quad + \frac{\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\end{aligned}$$

output
```
-arcsinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d
*x+c)^(1/2)/d/a^(1/2)+arctanh(1/2*sin(d*x+c)*a^(1/2)*sec(d*x+c)^(1/2)*2^(1
/2)/(a+a*sec(d*x+c))^(1/2))*2^(1/2)*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d/a^
(1/2)+sin(d*x+c)/d/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2)
```

3.425.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.86

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\,dx$$
$$= \frac{\sqrt{\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)\left(\arcsin\left(\sqrt{1-\sec(c+dx)}\right)+2\arcsin\left(\sqrt{\sec(c+dx)}\right)-\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{\sec}}{\sqrt{1-\sec}}\right.\right.}{d\sqrt{1-\sec(c+dx)}\sqrt{a(1+\sec(c+dx))}}$$

input
```
Integrate[1/(Cos[c + d*x]^(5/2)*Sqrt[a + a*Sec[c + d*x]]),x]
```

output
```
(Sqrt[Cos[c + d*x]]*Sec[c + d*x]^(3/2)*(ArcSin[Sqrt[1 - Sec[c + d*x]]] + 2
*ArcSin[Sqrt[Sec[c + d*x]]] - Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/
Sqrt[1 - Sec[c + d*x]]] + Sqrt[-((-1 + Sec[c + d*x])*Sec[c + d*x])])*Sin[c
 + d*x])/(d*Sqrt[1 - Sec[c + d*x]]*Sqrt[a*(1 + Sec[c + d*x])])
```

3.425.3 Rubi [A] (verified)

Time = 0.91 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.94, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {3042, 4752, 3042, 4309, 3042, 4511, 3042, 4288, 222, 4295, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\sqrt{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}}dx$$
$$\Big\downarrow \; 4752$$
$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sec^{\frac{5}{2}}(c+dx)}{\sqrt{\sec(c+dx)a+a}}dx$$
$$\Big\downarrow \; 3042$$
$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx$$

$$\Big\downarrow \text{4309}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{\sqrt{\sec(c+dx)}(a-a\sec(c+dx))}{\sqrt{\sec(c+dx)a+a}}dx}{2a}+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{\sqrt{\csc(c+dx+\frac{\pi}{2})}(a-a\csc(c+dx+\frac{\pi}{2}))}{\sqrt{\csc(c+dx+\frac{\pi}{2})a+a}}dx}{2a}+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}\right)$$

$$\Big\downarrow \text{4511}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2a\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{\sec(c+dx)a+a}}dx-\int \sqrt{\sec(c+dx)}\sqrt{\sec(c+dx)a+a}dx}{2a}+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2a\int \frac{\sqrt{\csc(c+dx+\frac{\pi}{2})}}{\sqrt{\csc(c+dx+\frac{\pi}{2})a+a}}dx-\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{2a}+\frac{\sin(c}{d\sqrt{}}\right.$$

$$\Big\downarrow \text{4288}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2a\int \frac{\sqrt{\csc(c+dx+\frac{\pi}{2})}}{\sqrt{\csc(c+dx+\frac{\pi}{2})a+a}}dx+\frac{2\int \frac{1}{\sqrt{\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1}}d\left(-\frac{a\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}}{2a}+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c-}{d\sqrt{a\sec(c+dx)}}\right.$$

$$\Big\downarrow \text{222}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2a\int \frac{\sqrt{\csc(c+dx+\frac{\pi}{2})}}{\sqrt{\csc(c+dx+\frac{\pi}{2})a+a}}dx-\frac{2\sqrt{a}\text{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}}{2a}+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}\right)$$

3.425. $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}dx$

$$\Big\downarrow \; 4295$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{-\frac{4a\int \frac{1}{2a-\frac{a^2\sin(c+dx)\tan(c+dx)}{\sec(c+dx)a+a}}d\left(-\frac{a\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}-\frac{2\sqrt{a}\text{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}}{2a}+\frac{\sin(}{d}\right.$$

$$\Big\downarrow \; 219$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2\sqrt{2}\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{d}-\frac{2\sqrt{a}\text{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}}{2a}+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(}{d\sqrt{a\sec(c+d}}\right.$$

input
```
Int[1/(Cos[c + d*x]^(5/2)*Sqrt[a + a*Sec[c + d*x]]),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(((-2*Sqrt[a]*ArcSinh[(Sqrt[a]*Tan[c
 + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d + (2*Sqrt[2]*Sqrt[a]*ArcTanh[(Sqrt[a
]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/d)
/(2*a) + (Sec[c + d*x]^(3/2)*Sin[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]]))
```

3.425.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4288
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[-2*(a/(b*f))*Sqrt[a*(d/b)]   Subst[Int[1/Sqrt[1
+ x^2/a], x], x, b*(Cot[e + f*x]/Sqrt[a + b*Csc[e + f*x]])], x] /; FreeQ[{a
, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[a*(d/b), 0]
```

rule 4295
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[-2*b*(d/(a*f))   Subst[Int[1/(2*b - d*x^2), x],
x, b*(Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]]))], x] /;
 FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4309
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[-2*d^2*Cot[e + f*x]*((d*Csc[e + f*x])^(n - 2)/(
f*(2*n - 3)*Sqrt[a + b*Csc[e + f*x]])), x] + Simp[d^2/(b*(2*n - 3))   Int[(
d*Csc[e + f*x])^(n - 2)*((2*b*(n - 2) - a*Csc[e + f*x])/Sqrt[a + b*Csc[e +
f*x]]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[n,
2] && IntegerQ[2*n]
```

rule 4511
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b -
a*B)/b   Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n, x], x] + Simp[B/b
 Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b
, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

3.425.4 Maple [A] (verified)

Time = 2.47 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.32

method	result
default	$-\frac{\sqrt{a(1+\sec(dx+c))}\left(2\arctan\left(\frac{\sin(dx+c)\sqrt{2}}{2(\cos(dx+c)+1)\sqrt{-\frac{1}{\cos(dx+c)+1}}}\right)\sqrt{2}\cos(dx+c)+\cos(dx+c)\arctan\left(\frac{\cos(dx+c)-\sin(dx+c)+1}{2(\cos(dx+c)+1)\sqrt{-\frac{1}{\cos(dx+c)+1}}}\right.\right.}{2da(\cos(dx+c)+1)\sqrt{-\frac{1}{\cos(dx+c)+1}}\sqrt{\cos(dx}}$

input
```
int(1/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2/d/a*(a*(1+sec(d*x+c)))^(1/2)*(2*arctan(1/2*sin(d*x+c)*2^(1/2)/(cos(d*
x+c)+1)/(-1/(cos(d*x+c)+1))^(1/2))*2^(1/2)*cos(d*x+c)+cos(d*x+c)*arctan(1/
2*(cos(d*x+c)-sin(d*x+c)+1)/(cos(d*x+c)+1)/(-1/(cos(d*x+c)+1))^(1/2))-cos(
d*x+c)*arctan(1/2*(cos(d*x+c)+sin(d*x+c)+1)/(cos(d*x+c)+1)/(-1/(cos(d*x+c)
+1))^(1/2))-2*sin(d*x+c)*(-1/(cos(d*x+c)+1))^(1/2))/(cos(d*x+c)+1)/(-1/(co
s(d*x+c)+1))^(1/2)/cos(d*x+c)^(1/2)
```

3.425.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 520, normalized size of antiderivative = 3.10

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\,dx$$

$$= \left[\frac{\left(\cos\left(dx+c\right)^2+\cos\left(dx+c\right)\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3+4\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}(\cos(dx+c)-2)\sqrt{\cos(dx+c)}\sin(dx+c)-7\,a\cos(dx+c}{\cos(dx+c)^3+\cos(dx+c)^2}\right.}{4\,(} \right.$$

$$-\frac{2\sqrt{2}\left(a\cos\left(dx+c\right)^2+a\cos\left(dx+c\right)\right)\sqrt{-\frac{1}{a}}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{-\frac{1}{a}}\sqrt{\cos(dx+c)}}{\sin(dx+c)}\right)+\left(\cos\left(dx+c\right)^2+\right.}{2\left(ad\cos\left(dx+\right.\right.}$$

input
```
integrate(1/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
[1/4*((cos(d*x + c)^2 + cos(d*x + c))*sqrt(a)*log((a*cos(d*x + c)^3 + 4*sq
rt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(cos(d*x + c) - 2)*sqrt(cos(
d*x + c))*sin(d*x + c) - 7*a*cos(d*x + c)^2 + 8*a)/(cos(d*x + c)^3 + cos(d
*x + c)^2)) + 2*sqrt(2)*(a*cos(d*x + c)^2 + a*cos(d*x + c))*log(-(cos(d*x
+ c)^2 - 2*sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x +
c))*sin(d*x + c)/sqrt(a) - 2*cos(d*x + c) - 3)/(cos(d*x + c)^2 + 2*cos(d*x
 + c) + 1))/sqrt(a) + 4*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d
*x + c))*sin(d*x + c))/(a*d*cos(d*x + c)^2 + a*d*cos(d*x + c)), -1/2*(2*sq
rt(2)*(a*cos(d*x + c)^2 + a*cos(d*x + c))*sqrt(-1/a)*arctan(sqrt(2)*sqrt((
a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(-1/a)*sqrt(cos(d*x + c))/sin(d*x +
c)) + (cos(d*x + c)^2 + cos(d*x + c))*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*c
os(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x
+ c)^2 - a*cos(d*x + c) - 2*a)) - 2*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)
)*sqrt(cos(d*x + c))*sin(d*x + c))/(a*d*cos(d*x + c)^2 + a*d*cos(d*x + c))
]
```

3.425.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate(1/cos(d*x+c)**(5/2)/(a+a*sec(d*x+c))**(1/2),x)
```

output
```
Timed out
```

3.425.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 876 vs. $2(139) = 278$.

Time = 0.41 (sec) , antiderivative size = 876, normalized size of antiderivative = 5.21

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\,dx = \text{Too large to display}$$

input
```
integrate(1/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
-1/4*(4*sqrt(2)*cos(3/2*arctan2(sin(d*x + c), cos(d*x + c)))*sin(2*d*x + 2
*c) - 4*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))*sin(2*d*x + 2
*c) + (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*l
og(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(si
n(d*x + c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos
(d*x + c))) + 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2)
- (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2
*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*
x + c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x
 + c))) - 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) + (c
os(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2*cos
(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x +
c), cos(d*x + c)))^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c
))) + 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) - (cos(2
*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2
*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c),
cos(d*x + c)))^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))
- 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) - 2*(sqrt(2)
*cos(2*d*x + 2*c)^2 + sqrt(2)*sin(2*d*x + 2*c)^2 + 2*sqrt(2)*cos(2*d*x + 2
*c) + sqrt(2))*log(cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + sin...
```

3.425.8 Giac [F]

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{a\sec(dx+c)+a}\cos(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(a*sec(d*x + c) + a)*cos(d*x + c)^(5/2)), x)
```

3.425.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\,dx=\int \frac{1}{\cos\left(c+d\,x\right)^{5/2}\,\sqrt{a+\frac{a}{\cos\left(c+d\,x\right)}}}\,dx$$

input
```
int(1/(cos(c + d*x)^(5/2)*(a + a/cos(c + d*x))^(1/2)),x)
```

output
```
int(1/(cos(c + d*x)^(5/2)*(a + a/cos(c + d*x))^(1/2)), x)
```

3.426 $\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\,dx$

3.426.1 Optimal result . . . 3149
3.426.2 Mathematica [A] (verified) . . . 3150
3.426.3 Rubi [A] (verified) . . . 3150
3.426.4 Maple [A] (verified) . . . 3155
3.426.5 Fricas [A] (verification not implemented) . . . 3155
3.426.6 Sympy [F(-1)] . . . 3156
3.426.7 Maxima [B] (verification not implemented) . . . 3156
3.426.8 Giac [F] . . . 3157
3.426.9 Mupad [F(-1)] . . . 3158

3.426.1 Optimal result

Integrand size = 25, antiderivative size = 211

$$\begin{aligned}\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\,dx &= \frac{7\operatorname{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{4\sqrt{a}d} \\ &\quad - \frac{\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{\sqrt{a}d} \\ &\quad + \frac{\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} - \frac{\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\end{aligned}$$

output
```
7/4*arcsinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*se
c(d*x+c)^(1/2)/d/a^(1/2)-arctanh(1/2*sin(d*x+c)*a^(1/2)*sec(d*x+c)^(1/2)*2
^(1/2)/(a+a*sec(d*x+c))^(1/2))*2^(1/2)*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
/a^(1/2)+1/2*sin(d*x+c)/d/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(1/2)-1/4*sin(
d*x+c)/d/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2)
```

3.426.2 Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.84

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\,dx$$
$$= \frac{\sqrt{\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)\left(-\arcsin\left(\sqrt{1-\sec(c+dx)}\right)-8\arcsin\left(\sqrt{\sec(c+dx)}\right)+4\sqrt{2}\arctan\left(\frac{\sqrt{2}}{\sqrt{1}}\right.\right.}{4d\sqrt{1-\sec(c+dx)}\sqrt{a}}$$

input
```
Integrate[1/(Cos[c + d*x]^(7/2)*Sqrt[a + a*Sec[c + d*x]]),x]
```

output
```
(Sqrt[Cos[c + d*x]]*Sec[c + d*x]^(3/2)*(-ArcSin[Sqrt[1 - Sec[c + d*x]]] -
8*ArcSin[Sqrt[Sec[c + d*x]]] + 4*Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]
])/Sqrt[1 - Sec[c + d*x]]] + 2*Sqrt[1 - Sec[c + d*x]]*Sec[c + d*x]^(3/2) -
 Sqrt[-((-1 + Sec[c + d*x])*Sec[c + d*x])])*Sin[c + d*x])/(4*d*Sqrt[1 - Se
c[c + d*x]]*Sqrt[a*(1 + Sec[c + d*x])])
```

3.426.3 Rubi [A] (verified)

Time = 1.19 (sec) , antiderivative size = 205, normalized size of antiderivative = 0.97, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.560$, Rules used = {3042, 4752, 3042, 4309, 3042, 4509, 27, 3042, 4511, 3042, 4288, 222, 4295, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{7/2}\sqrt{a\csc\left(c+dx+\frac{\pi}{2}\right)+a}}dx$$
$$\Big\downarrow \; 4752$$
$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sec^{\frac{7}{2}}(c+dx)}{\sqrt{\sec(c+dx)a+a}}dx$$
$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx$$

$\Big\downarrow$ 4309

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{\sec^{\frac{3}{2}}(c+dx)(3a-a\sec(c+dx))}{\sqrt{\sec(c+dx)a+a}}dx}{4a}+\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}\right)$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(3a-a\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a}+\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}\right)$$

$\Big\downarrow$ 4509

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\int -\frac{\sqrt{\sec(c+dx)}\left(a^2-7a^2\sec(c+dx)\right)}{2\sqrt{\sec(c+dx)a+a}}dx}{a}-\frac{a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}}{4a}+\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}\right)$$

$\Big\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{-\frac{\int \frac{\sqrt{\sec(c+dx)}\left(a^2-7a^2\sec(c+dx)\right)}{\sqrt{\sec(c+dx)a+a}}dx}{2a}-\frac{a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}}{4a}+\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}\right)$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{-\frac{\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a^2-7a^2\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{2a}-\frac{a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}}{4a}+\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+d}{2d\sqrt{a\sec(c+dx)+a}}\right.$$

$\Big\downarrow$ 4511

3.426. $\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{-\frac{8a^2\int\frac{\sqrt{\sec(c+dx)}}{\sqrt{\sec(c+dx)a+a}}dx-7a\int\sqrt{\sec(c+dx)}\sqrt{\sec(c+dx)a+a}dx}{2a}-\frac{a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}}{4a}+\frac{\sin(c+}{2d\sqrt{a}}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{-\frac{8a^2\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx-7a\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{2a}-\frac{a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}}{4a}\right.$$

$\downarrow$ 4288

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{-\frac{8a^2\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx+\frac{14a\int\frac{1}{\sqrt{\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1}}d\left(-\frac{a\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}}{2a}-\frac{a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}}{4a}\right.$$

$\downarrow$ 222

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{-\frac{8a^2\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx-\frac{14a^{3/2}\text{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}}{2a}-\frac{a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}}{4a}+\frac{\sin(c}{2d\sqrt{}}\right.$$

$\downarrow$ 4295

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{-\frac{-\frac{16a^2\int\frac{1}{2a-\frac{a^2\sin(c+dx)\tan(c+dx)}{\sec(c+dx)a+a}}d\left(-\frac{a\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}-\frac{14a^{3/2}\text{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}}{2a}-\frac{a\,s}{}}{4a}\right.$$

$\downarrow$ 219

3.426. $\int\frac{1}{\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{-\frac{\frac{8\sqrt{2}a^{3/2}\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{d}-\frac{14a^{3/2}\text{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}}{2a}-\frac{a\sin(c+dx)\sec^{\frac{3}{2}}(c}{d\sqrt{a\sec(c+dx)+}}}{4a}\right.$$

input
```
Int[1/(Cos[c + d*x]^(7/2)*Sqrt[a + a*Sec[c + d*x]]),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((Sec[c + d*x]^(5/2)*Sin[c + d*x])/(
2*d*Sqrt[a + a*Sec[c + d*x]]) + (-1/2*((-14*a^(3/2)*ArcSinh[(Sqrt[a]*Tan[c
 + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d + (8*Sqrt[2]*a^(3/2)*ArcTanh[(Sqrt[a
]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/d)
/a - (a*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]]))/(4*
a))
```

3.426.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4288
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[-2*(a/(b*f))*Sqrt[a*(d/b)]   Subst[Int[1/Sqrt[1
+ x^2/a], x], x, b*(Cot[e + f*x]/Sqrt[a + b*Csc[e + f*x]])], x] /; FreeQ[{a
, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[a*(d/b), 0]
```

rule 4295
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[-2*b*(d/(a*f))   Subst[Int[1/(2*b - d*x^2), x],
x, b*(Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]]))], x] /;
 FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4309
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[-2*d^2*Cot[e + f*x]*((d*Csc[e + f*x])^(n - 2)/(
f*(2*n - 3)*Sqrt[a + b*Csc[e + f*x]])), x] + Simp[d^2/(b*(2*n - 3))   Int[(
d*Csc[e + f*x])^(n - 2)*((2*b*(n - 2) - a*Csc[e + f*x])/Sqrt[a + b*Csc[e +
f*x]]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[n,
2] && IntegerQ[2*n]
```

rule 4509
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*d*
Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(f*(m + n))),
 x] + Simp[d/(b*(m + n))   Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n -
 1)*Simp[b*B*(n - 1) + (A*b*(m + n) + a*B*m)*Csc[e + f*x], x], x], x] /; Fr
eeQ[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
&& GtQ[n, 1]
```

rule 4511
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b -
a*B)/b   Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n, x], x] + Simp[B/b
 Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b
, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

3.426. $\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\,dx$

3.426.4 Maple [A] (verified)

Time = 2.49 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.22

method	result
default	$\frac{\left(8\sqrt{2}\,\cos(dx+c)^2\arctan\left(\frac{\sin(dx+c)\sqrt{2}}{2(\cos(dx+c)+1)\sqrt{-\frac{1}{\cos(dx+c)+1}}}\right)+7\cos(dx+c)^2\arctan\left(\frac{\cos(dx+c)-\sin(dx+c)+1}{2(\cos(dx+c)+1)\sqrt{-\frac{1}{\cos(dx+c)+1}}}\right)-7\arctan\left(\frac{}{2(c}\right.\right.}{8da(\cos(dx+c)+1)\sqrt{-}}$

input
```
int(1/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/8/d/a*(8*2^(1/2)*cos(d*x+c)^2*arctan(1/2*sin(d*x+c)*2^(1/2)/(cos(d*x+c)+
1)/(-1/(cos(d*x+c)+1))^(1/2))+7*cos(d*x+c)^2*arctan(1/2*(cos(d*x+c)-sin(d*
x+c)+1)/(cos(d*x+c)+1)/(-1/(cos(d*x+c)+1))^(1/2))-7*arctan(1/2*(cos(d*x+c)
+sin(d*x+c)+1)/(cos(d*x+c)+1)/(-1/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)^2-2*(-
1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*cos(d*x+c)+4*sin(d*x+c)*(-1/(cos(d*x+c)
+1))^(1/2))*(a*(1+sec(d*x+c)))^(1/2)/(cos(d*x+c)+1)/(-1/(cos(d*x+c)+1))^(1
/2)/cos(d*x+c)^(3/2)
```

3.426.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 550, normalized size of antiderivative = 2.61

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\,dx$$
$$= \left[-\frac{4\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}(\cos(dx+c)-2)\sqrt{\cos(dx+c)}\sin(dx+c)-7\left(\cos(dx+c)^3+\cos(dx+c)^2\right)\sqrt{a}\log}{} \right.$$

input
```
integrate(1/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
[-1/16*(4*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(cos(d*x + c) - 2)*sqrt(
cos(d*x + c))*sin(d*x + c) - 7*(cos(d*x + c)^3 + cos(d*x + c)^2)*sqrt(a)*l
og((a*cos(d*x + c)^3 - 4*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(
cos(d*x + c) - 2)*sqrt(cos(d*x + c))*sin(d*x + c) - 7*a*cos(d*x + c)^2 + 8
*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) - 8*sqrt(2)*(a*cos(d*x + c)^3 + a*c
os(d*x + c)^2)*log(-(cos(d*x + c)^2 + 2*sqrt(2)*sqrt((a*cos(d*x + c) + a)/
cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/sqrt(a) - 2*cos(d*x + c) - 3
)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1))/sqrt(a))/(a*d*cos(d*x + c)^3 + a*
d*cos(d*x + c)^2), 1/8*(8*sqrt(2)*(a*cos(d*x + c)^3 + a*cos(d*x + c)^2)*sq
rt(-1/a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(-1/a)
*sqrt(cos(d*x + c))/sin(d*x + c)) - 2*sqrt((a*cos(d*x + c) + a)/cos(d*x +
c))*(cos(d*x + c) - 2)*sqrt(cos(d*x + c))*sin(d*x + c) + 7*(cos(d*x + c)^3
 + cos(d*x + c)^2)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/co
s(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x
+ c) - 2*a)))/(a*d*cos(d*x + c)^3 + a*d*cos(d*x + c)^2)]
```

3.426.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate(1/cos(d*x+c)**(7/2)/(a+a*sec(d*x+c))**(1/2),x)
```

output
```
Timed out
```

3.426.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1646 vs. $2(172) = 344$.

Time = 0.43 (sec) , antiderivative size = 1646, normalized size of antiderivative = 7.80

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\,dx = \text{Too large to display}$$

input
```
integrate(1/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
1/16*(4*(sqrt(2)*sin(4*d*x + 4*c) + 2*sqrt(2)*sin(2*d*x + 2*c))*cos(7/2*ar
ctan2(sin(d*x + c), cos(d*x + c))) - 20*(sqrt(2)*sin(4*d*x + 4*c) + 2*sqrt
(2)*sin(2*d*x + 2*c))*cos(5/2*arctan2(sin(d*x + c), cos(d*x + c))) + 20*(s
qrt(2)*sin(4*d*x + 4*c) + 2*sqrt(2)*sin(2*d*x + 2*c))*cos(3/2*arctan2(sin(
d*x + c), cos(d*x + c))) - 4*(sqrt(2)*sin(4*d*x + 4*c) + 2*sqrt(2)*sin(2*d
*x + 2*c))*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 7*(2*(2*cos(2*d*
x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2
 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x
+ 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*x + c), cos
(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sqrt(
2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2*sqrt(2)*sin(1/2*arctan
2(sin(d*x + c), cos(d*x + c))) + 2) - 7*(2*(2*cos(2*d*x + 2*c) + 1)*cos(4*
d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^
2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d
*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*si
n(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(s
in(d*x + c), cos(d*x + c))) - 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(
d*x + c))) + 2) + 7*(2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d
*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*
c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*lo...
```

3.426.8 Giac [F]

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{a\sec(dx+c)+a}\cos(dx+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(a*sec(d*x + c) + a)*cos(d*x + c)^(7/2)), x)
```

3.426.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\,dx=\int \frac{1}{\cos\left(c+d\,x\right)^{7/2}\,\sqrt{a+\frac{a}{\cos\left(c+d\,x\right)}}}\,dx$$

input
```
int(1/(cos(c + d*x)^(7/2)*(a + a/cos(c + d*x))^(1/2)),x)
```

output
```
int(1/(cos(c + d*x)^(7/2)*(a + a/cos(c + d*x))^(1/2)), x)
```

3.427 $\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx$

3.427.1 Optimal result . . . 3159
3.427.2 Mathematica [A] (verified) . . . 3160
3.427.3 Rubi [A] (verified) . . . 3160
3.427.4 Maple [A] (verified) . . . 3165
3.427.5 Fricas [A] (verification not implemented) . . . 3166
3.427.6 Sympy [F(-1)] . . . 3167
3.427.7 Maxima [F(-2)] . . . 3167
3.427.8 Giac [F] . . . 3167
3.427.9 Mupad [F(-1)] . . . 3168

3.427.1 Optimal result

Integrand size = 25, antiderivative size = 237

$$\begin{aligned}\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx &= \\ &-\frac{15\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{2\sqrt{2}a^{3/2}d} \\ &-\frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}}+\frac{49\sin(c+dx)}{10ad\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} \\ &-\frac{13\sqrt{\cos(c+dx)}\sin(c+dx)}{10ad\sqrt{a+a\sec(c+dx)}}+\frac{9\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{10ad\sqrt{a+a\sec(c+dx)}}\end{aligned}$$

output
```
-1/2*cos(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(3/2)-15/4*arctanh(1/2
*sin(d*x+c)*a^(1/2)*sec(d*x+c)^(1/2)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))*cos(d
*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^(3/2)/d*2^(1/2)+9/10*cos(d*x+c)^(3/2)*sin(d
*x+c)/a/d/(a+a*sec(d*x+c))^(1/2)+49/10*sin(d*x+c)/a/d/cos(d*x+c)^(1/2)/(a+
a*sec(d*x+c))^(1/2)-13/10*sin(d*x+c)*cos(d*x+c)^(1/2)/a/d/(a+a*sec(d*x+c))
^(1/2)
```

3.427.2 Mathematica [A] (verified)

Time = 0.71 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.64

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \frac{75\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)\cos^2\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)+\sqrt{1-}}{10d\sqrt{-1+\cos(c+}}$$

input
```
Integrate[Cos[c + d*x]^(5/2)/(a + a*Sec[c + d*x])^(3/2),x]
```

output
```
(75*Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*Co
s[(c + d*x)/2]^2*Sec[c + d*x]^(3/2)*Sin[c + d*x] + Sqrt[1 - Sec[c + d*x]]*
(4*(9 + Cos[c + d*x]^2)*Sin[c + d*x] - 2*Sin[2*(c + d*x)] + 49*Tan[c + d*x
]))/(10*d*Sqrt[-1 + Cos[c + d*x]]*(a*(1 + Sec[c + d*x]))^(3/2))
```

3.427.3 Rubi [A] (verified)

Time = 1.36 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.08, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.640$, Rules used = {3042, 4752, 3042, 4304, 27, 3042, 4510, 27, 3042, 4510, 27, 3042, 4501, 3042, 4295, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a\sec(c+dx)+a)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}}dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(\sec(c+dx)a+a)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}}dx$$

$$\Big\downarrow \text{4304}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\int -\frac{3(3a-2a\sec(c+dx))}{2\sec^{\frac{5}{2}}(c+dx)\sqrt{\sec(c+dx)a+a}}dx}{2a^2}-\frac{\sin(c+dx)}{2d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^{3/2}}\right)$$

$\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{3\int \frac{3a-2a\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)\sqrt{\sec(c+dx)a+a}}dx}{4a^2}-\frac{\sin(c+dx)}{2d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^{3/2}}\right)$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{3\int \frac{3a-2a\csc(c+dx+\frac{\pi}{2})}{\csc(c+dx+\frac{\pi}{2})^{5/2}\sqrt{\csc(c+dx+\frac{\pi}{2})a+a}}dx}{4a^2}-\frac{\sin(c+dx)}{2d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^{3/2}}\right)$$

$\downarrow$ 4510

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{3\left(\frac{2\int -\frac{13a^2-12a^2\sec(c+dx)}{2\sec^{\frac{3}{2}}(c+dx)\sqrt{\sec(c+dx)a+a}}dx}{5a}+\frac{6a\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}\right)}{4a^2}-\frac{\sin(c\cdots}{2d\sec^{\frac{3}{2}}(c+dx)(a\,\mathrm{s}\cdots}\right.$$

$\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{3\left(\frac{6a\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}-\frac{\int \frac{13a^2-12a^2\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)\sqrt{\sec(c+dx)a+a}}dx}{5a}\right)}{4a^2}-\frac{\sin(c+dx\cdots}{2d\sec^{\frac{3}{2}}(c+dx)(a\sec(\cdots}\right.$$

$\downarrow$ 3042

3.427. $\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{3\left(\frac{6a\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}-\frac{\int\frac{13a^2-12a^2\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{5a}\right)}{4a^2}-\frac{\mathrm{si}}{2d\sec^{\frac{3}{2}}(c+dx)}\right.$$

$\downarrow$ 4510

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{3\left(\frac{6a\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}-\frac{\frac{2\int-\frac{49a^3-26a^3\sec(c+dx)}{2\sqrt{\sec(c+dx)}\sqrt{\sec(c+dx)a+a}}dx}{3a}+\frac{26a^2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}}{5a}\right)}{4a^2}\right.$$

$\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{3\left(\frac{6a\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}-\frac{\frac{26a^2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}-\frac{\int\frac{49a^3-26a^3\sec(c+dx)}{\sqrt{\sec(c+dx)}\sqrt{\sec(c+dx)a+a}}dx}{3a}}{5a}\right)}{4a^2}-\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{3\left(\frac{6a\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}-\frac{\frac{26a^2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}-\frac{\int\frac{49a^3-26a^3\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}}{3a}}{5a}\right)}{4a^2}\right.$$

$\downarrow$ 4501

3.427. $\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{3\left(\frac{6a\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}-\frac{\frac{26a^2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}-\frac{\frac{98a^3\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\sec(c+dx)+a}}-75a^3\int\frac{\sqrt{}}{\sqrt{\sec}}}{3a}}{5a}\right.}{4a^2}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{3\left(\frac{6a\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}-\frac{\frac{26a^2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}-\frac{\frac{98a^3\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\sec(c+dx)+a}}-75a^3\int\frac{\sqrt{}}{\sqrt{\mathrm{cs}}}}{3a}}{5a}\right.}{4a^2}\right.$$

$\downarrow$ 4295

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{3\left(\frac{6a\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}-\frac{\frac{26a^2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}-\frac{\frac{150a^3\int\frac{1}{2a-\frac{a^2\sin(c+dx)\tan(c+dx)}{\sec(c+dx)a+a}}d\left(\right.}{d}}{5a}\right.}{4a^2}\right.$$

$\downarrow$ 219

3.427. $\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{3\left(\frac{6a\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}-\frac{\frac{26a^2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}-\frac{\frac{98a^3\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\sec(c+dx)+a}}-\frac{75\sqrt{2}a^{5/2}}{}}{3}}{5a}\right.}{4a^2}\right.$$

input
```
Int[Cos[c + d*x]^(5/2)/(a + a*Sec[c + d*x])^(3/2),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(-1/2*Sin[c + d*x]/(d*Sec[c + d*x]^(
3/2)*(a + a*Sec[c + d*x])^(3/2)) + (3*((6*a*Sin[c + d*x])/(5*d*Sec[c + d*x
]^(3/2)*Sqrt[a + a*Sec[c + d*x]]) - ((26*a^2*Sin[c + d*x])/(3*d*Sqrt[Sec[c
 + d*x]]*Sqrt[a + a*Sec[c + d*x]]) - ((-75*Sqrt[2]*a^(5/2)*ArcTanh[(Sqrt[a
]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/d
+ (98*a^3*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]]))/(
3*a))/(5*a)))/(4*a^2))
```

3.427.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4295
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[-2*b*(d/(a*f))   Subst[Int[1/(2*b - d*x^2), x],
x, b*(Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]]))], x] /;
 FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4304
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-Cot[e + f*x])*(a + b*Csc[e + f*x])^m*((d*Csc
[e + f*x])^n/(f*(2*m + 1))), x] + Simp[1/(a^2*(2*m + 1))   Int[(a + b*Csc[e
 + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e
+ f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ
[m, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

rule 4501
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*Cot[e
 + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(f*n)), x] - Simp[(a*A*m
 - b*B*n)/(b*d*n)   Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x]
, x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a
^2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LeQ[m, -1]
```

rule 4510
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*Cot[e
 + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(f*n)), x] - Simp[1/(b*d
*n)   Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*
n - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

3.427.4 Maple [A] (verified)

Time = 1.82 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.15

method	result
default	$-\frac{\left(75\sqrt{2}\,\arctan\left(\frac{\sin(dx+c)\sqrt{2}}{2(\cos(dx+c)+1)\sqrt{-\frac{1}{\cos(dx+c)+1}}}\right)\sqrt{-\frac{1}{\cos(dx+c)+1}}\,\cos(dx+c)^2+150\sqrt{2}\,\arctan\left(\frac{\sin(dx+c)\sqrt{2}}{2(\cos(dx+c)+1)\sqrt{-\frac{1}{\cos(dx+c)+1}}}\right)\right.}{}$

input
```
int(cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/20/d/a^2*(75*2^(1/2)*arctan(1/2*sin(d*x+c)*2^(1/2)/(cos(d*x+c)+1)/(-1/(
cos(d*x+c)+1))^(1/2))*(-1/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^2+150*2^(1/2)*a
rctan(1/2*sin(d*x+c)*2^(1/2)/(cos(d*x+c)+1)/(-1/(cos(d*x+c)+1))^(1/2))*(-1
/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)-8*cos(d*x+c)^3*sin(d*x+c)+75*2^(1/2)*arc
tan(1/2*sin(d*x+c)*2^(1/2)/(cos(d*x+c)+1)/(-1/(cos(d*x+c)+1))^(1/2))*(-1/(
cos(d*x+c)+1))^(1/2)+8*cos(d*x+c)^2*sin(d*x+c)-72*cos(d*x+c)*sin(d*x+c)-98
*sin(d*x+c))*cos(d*x+c)^(1/2)*(a*(1+sec(d*x+c)))^(1/2)/(cos(d*x+c)+1)^2
```

3.427.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 400, normalized size of antiderivative = 1.69

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \left[\frac{75\sqrt{2}\left(\cos\left(dx+c\right)^2+2\,\cos\left(dx+c\right)+1\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^2+2\sqrt{2}\sqrt{a}\sqrt{\frac{a\cos(}{\cos}}}{\cos(}\right.}{}\right.$$

input
```
integrate(cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
[1/40*(75*sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*log(-(a*co
s(d*x + c)^2 + 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*s
qrt(cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 +
 2*cos(d*x + c) + 1)) + 4*(4*cos(d*x + c)^3 - 4*cos(d*x + c)^2 + 36*cos(d*
x + c) + 49)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*si
n(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d), 1/20*(7
5*sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(-a)*arctan(sqrt(2)*sq
rt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))/(a*sin(d
*x + c))) + 2*(4*cos(d*x + c)^3 - 4*cos(d*x + c)^2 + 36*cos(d*x + c) + 49)
*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/
(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)]
```

3.427.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(5/2)/(a+a*sec(d*x+c))**(3/2),x)
```

output
```
Timed out
```

3.427.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

3.427.8 Giac [F]

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \int \frac{\cos(dx+c)^{\frac{5}{2}}}{(a\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate(cos(d*x + c)^(5/2)/(a*sec(d*x + c) + a)^(3/2), x)
```

3.427.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx=\int \frac{\cos\left(c+d\,x\right)^{5/2}}{\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)^{3/2}}\,dx$$

input
```
int(cos(c + d*x)^(5/2)/(a + a/cos(c + d*x))^(3/2),x)
```

output
```
int(cos(c + d*x)^(5/2)/(a + a/cos(c + d*x))^(3/2), x)
```

3.428 $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx$

3.428.1 Optimal result . . . 3169
3.428.2 Mathematica [A] (verified) . . . 3170
3.428.3 Rubi [A] (verified) . . . 3170
3.428.4 Maple [A] (verified) . . . 3174
3.428.5 Fricas [A] (verification not implemented) . . . 3174
3.428.6 Sympy [F] . . . 3175
3.428.7 Maxima [B] (verification not implemented) . . . 3175
3.428.8 Giac [F] . . . 3176
3.428.9 Mupad [F(-1)] . . . 3177

3.428.1 Optimal result

Integrand size = 25, antiderivative size = 197

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \frac{11\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{2\sqrt{2}a^{3/2}d} - \frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} - \frac{19\sin(c+dx)}{6ad\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} + \frac{7\sqrt{\cos(c+dx)}\sin(c+dx)}{6ad\sqrt{a+a\sec(c+dx)}}$$

output
```
-1/2*sin(d*x+c)*cos(d*x+c)^(1/2)/d/(a+a*sec(d*x+c))^(3/2)+11/4*arctanh(1/2
*sin(d*x+c)*a^(1/2)*sec(d*x+c)^(1/2)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))*cos(d
*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^(3/2)/d*2^(1/2)-19/6*sin(d*x+c)/a/d/cos(d*x
+c)^(1/2)/(a+a*sec(d*x+c))^(1/2)+7/6*sin(d*x+c)*cos(d*x+c)^(1/2)/a/d/(a+a*
sec(d*x+c))^(1/2)
```

3.428.2 Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.68

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \frac{\left((-12+4\cos(c+dx)-19\sec(c+dx))\sqrt{1-\sec(c+dx)}-33\sqrt{2}\arctan\left(\right.\right.}{6d\sqrt{-1+\cos(c+dx)}(a(1+\sec($$

input
```
Integrate[Cos[c + d*x]^(3/2)/(a + a*Sec[c + d*x])^(3/2),x]
```

output
```
(((-12 + 4*Cos[c + d*x] - 19*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]] - 33*Sqr
t[2]*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*Cos[(c +
d*x)/2]^2*Sec[c + d*x]^(3/2))*Sin[c + d*x])/(6*d*Sqrt[-1 + Cos[c + d*x]]*(
a*(1 + Sec[c + d*x]))^(3/2))
```

3.428.3 Rubi [A] (verified)

Time = 1.06 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.06, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.520$, Rules used = {3042, 4752, 3042, 4304, 27, 3042, 4510, 27, 3042, 4501, 3042, 4295, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a\sec(c+dx)+a)^{3/2}}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}}dx$$

$$\Big\downarrow \; 4752$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(\sec(c+dx)a+a)^{3/2}}dx$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}}dx$$

$$\Big\downarrow \; 4304$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\int -\frac{7a-4a\sec(c+dx)}{2\sec^{\frac{3}{2}}(c+dx)\sqrt{\sec(c+dx)a+a}}dx}{2a^2}-\frac{\sin(c+dx)}{2d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^{3/2}}\right)$$

$\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{7a-4a\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)\sqrt{\sec(c+dx)a+a}}dx}{4a^2}-\frac{\sin(c+dx)}{2d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^{3/2}}\right)$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{7a-4a\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a^2}-\frac{\sin(c+dx)}{2d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^{3/2}}\right)$$

$\downarrow$ 4510

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2\int -\frac{19a^2-14a^2\sec(c+dx)}{2\sqrt{\sec(c+dx)}\sqrt{\sec(c+dx)a+a}}dx}{3a}+\frac{14a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}}{4a^2}-\frac{\sin(c+dx)}{2d\sqrt{\sec(c+dx)}(a\sec(c+}\right.$$

$\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{14a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}-\frac{\int \frac{19a^2-14a^2\sec(c+dx)}{\sqrt{\sec(c+dx)}\sqrt{\sec(c+dx)a+a}}dx}{3a}}{4a^2}-\frac{\sin(c+dx)}{2d\sqrt{\sec(c+dx)}(a\sec(c+d}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{14a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}-\frac{\int \frac{19a^2-14a^2\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{3a}}{4a^2}-\frac{\sin(c+}{2d\sqrt{\sec(c+dx)}(a\sec}\right.$$

$\downarrow$ 4501

3.428. $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{14a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}-\frac{\frac{38a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\sec(c+dx)+a}}-33a^2\int\frac{\sqrt{\sec(c+dx)}}{\sqrt{\sec(c+dx)a+a}}dx}{3a}}{4a^2}-\frac{}{2d\sqrt{\sec(c+}}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{14a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}-\frac{\frac{38a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\sec(c+dx)+a}}-33a^2\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{3a}}{4a^2}-\frac{}{2d\sqrt{\sec(}}\right.$$

$\downarrow$ 4295

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{14a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}-\frac{\frac{\frac{66a^2\int\frac{1}{2a-\frac{a^2\sin(c+dx)\tan(c+dx)}{\sec(c+dx)a+a}}d\left(-\frac{a\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}}{3a}+\frac{38a^2\sin(c}{d\sqrt{a}}}{4a^2}\right.$$

$\downarrow$ 219

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{14a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}-\frac{\frac{38a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\sec(c+dx)+a}}-\frac{33\sqrt{2}a^{3/2}\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right.}{d}}{3a}}{4a^2}\right.$$

input
```
Int[Cos[c + d*x]^(3/2)/(a + a*Sec[c + d*x])^(3/2),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(-1/2*Sin[c + d*x]/(d*Sqrt[Sec[c + d
*x]]*(a + a*Sec[c + d*x])^(3/2)) + ((14*a*Sin[c + d*x])/(3*d*Sqrt[Sec[c +
d*x]]*Sqrt[a + a*Sec[c + d*x]]) - ((-33*Sqrt[2]*a^(3/2)*ArcTanh[(Sqrt[a]*S
qrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/d + (
38*a^2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]]))/(3*a
))/(4*a^2))
```

3.428. $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx$

3.428.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4295
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[-2*b*(d/(a*f))   Subst[Int[1/(2*b - d*x^2), x],
x, b*(Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]]))], x] /;
 FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4304
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-Cot[e + f*x])*(a + b*Csc[e + f*x])^m*((d*Csc
[e + f*x])^n/(f*(2*m + 1))), x] + Simp[1/(a^2*(2*m + 1))   Int[(a + b*Csc[e
 + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e
+ f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ
[m, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

rule 4501
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*Cot[e
 + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(f*n)), x] - Simp[(a*A*m
 - b*B*n)/(b*d*n)   Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x]
, x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a
^2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LeQ[m, -1]
```

rule 4510
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*Cot[e
 + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(f*n)), x] - Simp[1/(b*d
*n)   Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*
n - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

3.428. $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx$

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

3.428.4 Maple [A] (verified)

Time = 1.60 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.30

method	result
default	$\frac{\left(33\sqrt{2}\,\arctan\left(\frac{\sin(dx+c)\sqrt{2}}{2(\cos(dx+c)+1)\sqrt{-\frac{1}{\cos(dx+c)+1}}}\right)\sqrt{-\frac{1}{\cos(dx+c)+1}}\,\cos(dx+c)^2+66\sqrt{2}\,\arctan\left(\frac{\sin(dx+c)\sqrt{2}}{2(\cos(dx+c)+1)\sqrt{-\frac{1}{\cos(dx+c)+1}}}\right)\sqrt{-}\right.}{}$

input
```
int(cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/12/d/a^2*(33*2^(1/2)*arctan(1/2*sin(d*x+c)*2^(1/2)/(cos(d*x+c)+1)/(-1/(c
os(d*x+c)+1))^(1/2))*(-1/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^2+66*2^(1/2)*arc
tan(1/2*sin(d*x+c)*2^(1/2)/(cos(d*x+c)+1)/(-1/(cos(d*x+c)+1))^(1/2))*(-1/(
cos(d*x+c)+1))^(1/2)*cos(d*x+c)+33*2^(1/2)*arctan(1/2*sin(d*x+c)*2^(1/2)/(
cos(d*x+c)+1)/(-1/(cos(d*x+c)+1))^(1/2))*(-1/(cos(d*x+c)+1))^(1/2)+8*cos(d
*x+c)^2*sin(d*x+c)-24*cos(d*x+c)*sin(d*x+c)-38*sin(d*x+c))*cos(d*x+c)^(1/2
)*(a*(1+sec(d*x+c)))^(1/2)/(cos(d*x+c)+1)^2
```

3.428.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 380, normalized size of antiderivative = 1.93

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \left[\frac{33\sqrt{2}\left(\cos(dx+c)^2+2\cos(dx+c)+1\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^2-2\sqrt{2}\sqrt{a}\sqrt{\frac{a\cos(}{\cos}}}{\cos(}\right.}{}\right.$$

$$-\frac{33\sqrt{2}\left(\cos(dx+c)^2+2\cos(dx+c)+1\right)\sqrt{-a}\arctan\left(\frac{\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}}{a\sin(dx+c)}\right)-2\left(4\cos(dx+c)^2\right.}{12\left(a^2d\cos(dx+c)^2+2a^2d\cos(dx+c)+a\right.}$$

input
```
integrate(cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
[1/24*(33*sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*log(-(a*co
s(d*x + c)^2 - 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*s
qrt(cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 +
 2*cos(d*x + c) + 1)) + 4*(4*cos(d*x + c)^2 - 12*cos(d*x + c) - 19)*sqrt((
a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(a^2*d*
cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d), -1/12*(33*sqrt(2)*(cos(d*x
 + c)^2 + 2*cos(d*x + c) + 1)*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt((a*cos
(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))/(a*sin(d*x + c))) - 2*(4*c
os(d*x + c)^2 - 12*cos(d*x + c) - 19)*sqrt((a*cos(d*x + c) + a)/cos(d*x +
c))*sqrt(cos(d*x + c))*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d
*x + c) + a^2*d)]
```

3.428.6 Sympy [F]

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a(\sec(c+dx)+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)**(3/2)/(a+a*sec(d*x+c))**(3/2),x)
```

output
```
Integral(cos(c + d*x)**(3/2)/(a*(sec(c + d*x) + 1))**(3/2), x)
```

3.428.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 33960 vs. $2(162) = 324$.

Time = 0.79 (sec) , antiderivative size = 33960, normalized size of antiderivative = 172.39

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
1/12*(4*(cos(3*d*x + 3*c)^2*sin(3/2*d*x + 3/2*c) + sin(3*d*x + 3*c)^2*sin(
3/2*d*x + 3/2*c) - 9*(cos(3*d*x + 3*c)^2 + sin(3*d*x + 3*c)^2)*sin(1/3*arc
tan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*cos(7/3*arctan2(sin(3/2
*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^4 + 64*(cos(3*d*x + 3*c)^2*sin(3/2*d
*x + 3/2*c) + sin(3*d*x + 3*c)^2*sin(3/2*d*x + 3/2*c) - 9*(cos(3*d*x + 3*c
)^2 + sin(3*d*x + 3*c)^2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*
x + 3/2*c))))*cos(5/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))
^4 + 4*sin(3/2*d*x + 3/2*c)^5 + 4*(cos(3*d*x + 3*c)^2*sin(3/2*d*x + 3/2*c)
 + sin(3*d*x + 3*c)^2*sin(3/2*d*x + 3/2*c) - 9*(cos(3*d*x + 3*c)^2 + sin(3
*d*x + 3*c)^2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))
))*sin(7/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^4 + 64*(co
s(3*d*x + 3*c)^2*sin(3/2*d*x + 3/2*c) + sin(3*d*x + 3*c)^2*sin(3/2*d*x + 3
/2*c) - 9*(cos(3*d*x + 3*c)^2 + sin(3*d*x + 3*c)^2)*sin(1/3*arctan2(sin(3/
2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sin(5/3*arctan2(sin(3/2*d*x + 3/2*
c), cos(3/2*d*x + 3/2*c)))^4 + 4*(2*cos(3*d*x + 3*c)^2*cos(3/2*d*x + 3/2*c
)*sin(3/2*d*x + 3/2*c) + 2*cos(3/2*d*x + 3/2*c)*sin(3*d*x + 3*c)^2*sin(3/2
*d*x + 3/2*c) + 2*cos(3*d*x + 3*c)*cos(3/2*d*x + 3/2*c)*sin(3/2*d*x + 3/2*
c) + 8*(cos(3*d*x + 3*c)^2*sin(3/2*d*x + 3/2*c) + sin(3*d*x + 3*c)^2*sin(3
/2*d*x + 3/2*c) - 9*(cos(3*d*x + 3*c)^2 + sin(3*d*x + 3*c)^2)*sin(1/3*arct
an2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*cos(5/3*arctan2(sin(3...
```

3.428.8 Giac [F]

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \int \frac{\cos\left(dx+c\right)^{\frac{3}{2}}}{\left(a\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate(cos(d*x + c)^(3/2)/(a*sec(d*x + c) + a)^(3/2), x)
```

3.428.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx=\int \frac{\cos\left(c+d\,x\right)^{3/2}}{\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)^{3/2}}\,dx$$

input
```
int(cos(c + d*x)^(3/2)/(a + a/cos(c + d*x))^(3/2),x)
```

output
```
int(cos(c + d*x)^(3/2)/(a + a/cos(c + d*x))^(3/2), x)
```

3.429 $\int \frac{\sqrt{\cos(c+dx)}}{(a+a\sec(c+dx))^{3/2}}\,dx$

3.429.1 Optimal result . . . 3178
3.429.2 Mathematica [A] (verified) . . . 3178
3.429.3 Rubi [A] (verified) . . . 3179
3.429.4 Maple [A] (verified) . . . 3182
3.429.5 Fricas [A] (verification not implemented) . . . 3182
3.429.6 Sympy [F] . . . 3183
3.429.7 Maxima [B] (verification not implemented) . . . 3183
3.429.8 Giac [F] . . . 3184
3.429.9 Mupad [F(-1)] . . . 3185

3.429.1 Optimal result

Integrand size = 25, antiderivative size = 157

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+a\sec(c+dx))^{3/2}}\,dx = -\frac{7\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{2\sqrt{2}a^{3/2}d} - \frac{\sin(c+dx)}{2d\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^{3/2}} + \frac{5\sin(c+dx)}{2ad\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}}$$

output
```
-1/2*sin(d*x+c)/d/(a+a*sec(d*x+c))^(3/2)/cos(d*x+c)^(1/2)-7/4*arctanh(1/2*
sin(d*x+c)*a^(1/2)*sec(d*x+c)^(1/2)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))*cos(d*
x+c)^(1/2)*sec(d*x+c)^(1/2)/a^(3/2)/d*2^(1/2)+5/2*sin(d*x+c)/a/d/cos(d*x+c
)^(1/2)/(a+a*sec(d*x+c))^(1/2)
```

3.429.2 Mathematica [A] (verified)

Time = 0.67 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.88

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+a\sec(c+dx))^{3/2}}\,dx = \frac{\sqrt{\sec(c+dx)}\left(7\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)+(5+\right.}{2d\sqrt{-1+\cos(c+dx)}(a(1+}$$

input
```
Integrate[Sqrt[Cos[c + d*x]]/(a + a*Sec[c + d*x])^(3/2),x]
```

output
```
(Sqrt[Sec[c + d*x]]*(7*Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1
- Sec[c + d*x]]]*Cos[(c + d*x)/2]^2*Sec[c + d*x] + (5 + 4*Cos[c + d*x])*Sq
rt[(-1 + Cos[c + d*x])*Sec[c + d*x]^2])*Sin[c + d*x])/(2*d*Sqrt[-1 + Cos[c
 + d*x]]*(a*(1 + Sec[c + d*x]))^(3/2))
```

3.429.3 Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.02, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 4752, 3042, 4304, 27, 3042, 4501, 3042, 4295, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\cos(c+dx)}}{(a\sec(c+dx)+a)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}}dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sec(c+dx)}(\sec(c+dx)a+a)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}}dx$$

$$\Big\downarrow \text{4304}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\int -\frac{5a-2a\sec(c+dx)}{2\sqrt{\sec(c+dx)}\sqrt{\sec(c+dx)a+a}}dx}{2a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{2d(a\sec(c+dx)+a)^{3/2}}\right)$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{5a-2a\sec(c+dx)}{\sqrt{\sec(c+dx)}\sqrt{\sec(c+dx)a+a}}dx}{4a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{2d(a\sec(c+dx)+a)^{3/2}}\right)$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{5a-2a\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{2d(a\sec(c+dx)+a)^{3/2}}\right)$$

$$\Big\downarrow \; 4501$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{10a\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\sec(c+dx)+a}}-7a\int\frac{\sqrt{\sec(c+dx)}}{\sqrt{\sec(c+dx)a+a}}dx}{4a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{2d(a\sec(c+dx)+a)^{3/2}}\right)$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{10a\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\sec(c+dx)+a}}-7a\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{2d(a\sec(c+dx)+a)^{3/2}}\right)$$

$$\Big\downarrow \; 4295$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{14a\int\frac{1}{2a-\frac{a^2\sin(c+dx)\tan(c+dx)}{\sec(c+dx)a+a}}d\left(-\frac{a\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}+\frac{10a\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\sec(c+dx)+a}}}{4a^2}-\frac{\sin(c+d}{2d(a\sec(}\right.$$

$$\Big\downarrow \; 219$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{10a\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\sec(c+dx)+a}}-\frac{7\sqrt{2}\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{d}}{4a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+}}{2d(a\sec(c+dx)+a)}\right.$$

input
```
Int[Sqrt[Cos[c + d*x]]/(a + a*Sec[c + d*x])^(3/2),x]
```

3.429. $\int \frac{\sqrt{\cos(c+dx)}}{(a+a\sec(c+dx))^{3/2}}\,dx$

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(-1/2*(Sqrt[Sec[c + d*x]]*Sin[c + d*
x])/(d*(a + a*Sec[c + d*x])^(3/2)) + ((-7*Sqrt[2]*Sqrt[a]*ArcTanh[(Sqrt[a]
*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/d +
 (10*a*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]]))/(4*a
^2))
```

3.429.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4295
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[-2*b*(d/(a*f))   Subst[Int[1/(2*b - d*x^2), x],
x, b*(Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]]))], x] /;
 FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4304
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-Cot[e + f*x])*(a + b*Csc[e + f*x])^m*((d*Csc
[e + f*x])^n/(f*(2*m + 1))), x] + Simp[1/(a^2*(2*m + 1))   Int[(a + b*Csc[e
 + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e
+ f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ
[m, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

rule 4501
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*Cot[e
 + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(f*n)), x] - Simp[(a*A*m
 - b*B*n)/(b*d*n)   Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x]
, x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a
^2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LeQ[m, -1]
```

3.429. $\int \frac{\sqrt{\cos(c+dx)}}{(a+a\sec(c+dx))^{3/2}}\,dx$

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

3.429.4 Maple [A] (verified)

Time = 1.81 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.28

method	result
default	$-\frac{\sqrt{-\frac{(1-\cos(dx+c))^2\csc(dx+c)^2-1}{(1-\cos(dx+c))^2\csc(dx+c)^2+1}}\sqrt{-\frac{2a}{(1-\cos(dx+c))^2\csc(dx+c)^2-1}}\left(-(1-\cos(dx+c))^3\csc(dx+c)^3+7\arctan\left(\frac{-\cot(dx+c)+\csc}{\sqrt{-(1-\cos(dx+c))^2\cos}}\right.\right.}{4d\,a^2}$

input
```
int(cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4/d/a^2*(-((1-cos(d*x+c))^2*csc(d*x+c)^2-1)/((1-cos(d*x+c))^2*csc(d*x+c
)^2+1))^(1/2)*(-2*a/((1-cos(d*x+c))^2*csc(d*x+c)^2-1))^(1/2)*(-(1-cos(d*x+
c))^3*csc(d*x+c)^3+7*arctan(1/(-(1-cos(d*x+c))^2*csc(d*x+c)^2-1)^(1/2)*(-c
ot(d*x+c)+csc(d*x+c)))*(-(1-cos(d*x+c))^2*csc(d*x+c)^2-1)^(1/2)-9*csc(d*x+
c)+9*cot(d*x+c))
```

3.429.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 360, normalized size of antiderivative = 2.29

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+a\sec(c+dx))^{3/2}}\,dx = \left[\frac{7\sqrt{2}\left(\cos(dx+c)^2+2\cos(dx+c)+1\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^2+2\sqrt{2}\sqrt{a}\sqrt{\frac{a\cos(d}{\cos(d}}}{\cos(d}\right.}{8\left(a^2d\cos\right.}\right.$$

input
```
integrate(cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
[1/8*(7*sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*log(-(a*cos(
d*x + c)^2 + 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqr
t(cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2
*cos(d*x + c) + 1)) + 4*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(4*cos(d*x
 + c) + 5)*sqrt(cos(d*x + c))*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*
d*cos(d*x + c) + a^2*d), 1/4*(7*sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) +
 1)*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c
))*sqrt(cos(d*x + c))/(a*sin(d*x + c))) + 2*sqrt((a*cos(d*x + c) + a)/cos(
d*x + c))*(4*cos(d*x + c) + 5)*sqrt(cos(d*x + c))*sin(d*x + c))/(a^2*d*cos
(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)]
```

3.429.6 Sympy [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+a\sec(c+dx))^{3/2}}\,dx=\int \frac{\sqrt{\cos{(c+dx)}}}{(a\left(\sec{(c+dx)}+1\right))^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)**(1/2)/(a+a*sec(d*x+c))**(3/2),x)
```

output
```
Integral(sqrt(cos(c + d*x))/(a*(sec(c + d*x) + 1))**(3/2), x)
```

3.429.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7176 vs. $2(128) = 256$.

Time = 0.43 (sec) , antiderivative size = 7176, normalized size of antiderivative = 45.71

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+a\sec(c+dx))^{3/2}}\,dx=\text{Too large to display}$$

input
```
integrate(cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
-1/4*(4*(7*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2
*d*x + 1/2*c) + 1) - 7*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2
 - 2*sin(1/2*d*x + 1/2*c) + 1) - 8*sin(1/2*d*x + 1/2*c))*cos(3/2*d*x + 3/2
*c)^4 + 63*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/
2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2
- 2*sin(1/2*d*x + 1/2*c) + 1))*cos(1/2*d*x + 1/2*c)^4 + 4*(7*log(cos(1/2*d
*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - 7*l
og(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c
) + 1) - 8*sin(1/2*d*x + 1/2*c))*sin(3/2*d*x + 3/2*c)^4 + 70*(log(cos(1/2*
d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - lo
g(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c)
 + 1))*cos(1/2*d*x + 1/2*c)^2*sin(1/2*d*x + 1/2*c)^2 + 7*(log(cos(1/2*d*x
+ 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(co
s(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1
))*sin(1/2*d*x + 1/2*c)^4 - 8*sin(1/2*d*x + 1/2*c)^5 + 28*(7*(log(cos(1/2*
d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - lo
g(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c)
 + 1))*cos(1/2*d*x + 1/2*c) - 8*cos(1/2*d*x + 1/2*c)*sin(1/2*d*x + 1/2*c))
*cos(3/2*d*x + 3/2*c)^3 + 4*(21*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x
+ 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 +...
```

3.429.8 Giac [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+a\sec(c+dx))^{3/2}}\,dx = \int \frac{\sqrt{\cos{(dx+c)}}}{(a\sec{(dx+c)}+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate(sqrt(cos(d*x + c))/(a*sec(d*x + c) + a)^(3/2), x)
```

3.429.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+a\sec(c+dx))^{3/2}}\,dx=\int \frac{\sqrt{\cos\left(c+d\,x\right)}}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

input
```
int(cos(c + d*x)^(1/2)/(a + a/cos(c + d*x))^(3/2),x)
```

output
```
int(cos(c + d*x)^(1/2)/(a + a/cos(c + d*x))^(3/2), x)
```

3.430 $\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^{3/2}}\,dx$

3.430.1 Optimal result . . . 3186
3.430.2 Mathematica [A] (verified) . . . 3186
3.430.3 Rubi [A] (verified) . . . 3187
3.430.4 Maple [A] (verified) . . . 3189
3.430.5 Fricas [A] (verification not implemented) . . . 3190
3.430.6 Sympy [F] . . . 3190
3.430.7 Maxima [B] (verification not implemented) . . . 3191
3.430.8 Giac [F] . . . 3191
3.430.9 Mupad [F(-1)] . . . 3192

3.430.1 Optimal result

Integrand size = 25, antiderivative size = 117

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^{3/2}}\,dx = \frac{3\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{2\sqrt{2}a^{3/2}d} - \frac{\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}$$

output
```
-1/2*sin(d*x+c)/d/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(3/2)+3/4*arctanh(1/2*
sin(d*x+c)*a^(1/2)*sec(d*x+c)^(1/2)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))*cos(d*
x+c)^(1/2)*sec(d*x+c)^(1/2)/a^(3/2)/d*2^(1/2)
```

3.430.2 Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.12

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^{3/2}}\,dx = -\frac{\left(2\sqrt{-((-1+\sec(c+dx))\sec(c+dx))}+3\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)(1+\sec(c+dx))\right)\sin(c+dx)}{4ad\sqrt{-1+\cos(c+dx)}(1+\cos(c+dx))\sqrt{\sec(c+dx)}\sqrt{a(1+\sec(c+dx))}}$$

input
```
Integrate[1/(Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])^(3/2)),x]
```

output
```
-1/4*((2*Sqrt[-((-1 + Sec[c + d*x])*Sec[c + d*x])] + 3*Sqrt[2]*ArcTan[(Sqr
t[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*(1 + Sec[c + d*x]))*Sin[c
 + d*x])/(a*d*Sqrt[-1 + Cos[c + d*x]]*(1 + Cos[c + d*x])*Sqrt[Sec[c + d*x]
]*Sqrt[a*(1 + Sec[c + d*x])])
```

3.430.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3042, 4752, 3042, 4298, 3042, 4295, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a\sec(c+dx)+a)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}}dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sqrt{\sec(c+dx)}}{(\sec(c+dx)a+a)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}}dx$$

$$\Big\downarrow \text{4298}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{3\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{\sec(c+dx)a+a}}dx}{4a}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{3\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}\right)$$

$$\Big\downarrow \text{4295}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{3\int \frac{1}{2a-\frac{a^2\sin(c+dx)\tan(c+dx)}{\sec(c+dx)a+a}}d\left(-\frac{a\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{2ad}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}\right)$$

$$\Big\downarrow \text{219}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{3\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}\right)$$

input
```
Int[1/(Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])^(3/2)),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((3*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*
x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/(2*Sqrt[2]*a^(3/2)*
d) - (Sec[c + d*x]^(3/2)*Sin[c + d*x])/(2*d*(a + a*Sec[c + d*x])^(3/2)))
```

3.430.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4295
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[-2*b*(d/(a*f))   Subst[Int[1/(2*b - d*x^2), x],
x, b*(Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]]))], x] /;
 FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4298
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-Cot[e + f*x])*(a + b*Csc[e + f*x])^m*((d*Csc
[e + f*x])^n/(f*(2*m + 1))), x] + Simp[m/(a*(2*m + 1))   Int[(a + b*Csc[e +
 f*x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f}, x] && E
qQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && LtQ[m, -2^(-1)]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

3.430.4 Maple [A] (verified)

Time = 1.80 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.40

method	result
default	$-\frac{\left(3\arctan\left(\frac{\sin(dx+c)\sqrt{2}}{2(\cos(dx+c)+1)\sqrt{-\frac{1}{\cos(dx+c)+1}}}\right)\sqrt{2}\cos(dx+c)+3\arctan\left(\frac{\sin(dx+c)\sqrt{2}}{2(\cos(dx+c)+1)\sqrt{-\frac{1}{\cos(dx+c)+1}}}\right)\sqrt{2}+2\sin(dx+c)\sqrt{-\frac{}{\cos(d}}\right.}{4d\,a^2(\cos(dx+c)+1)^2\sqrt{-\frac{1}{\cos(dx+c)+1}}}$

input
```
int(1/(a+a*sec(d*x+c))^(3/2)/cos(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4/d/a^2*(3*arctan(1/2*sin(d*x+c)*2^(1/2)/(cos(d*x+c)+1)/(-1/(cos(d*x+c)
+1))^(1/2))*2^(1/2)*cos(d*x+c)+3*arctan(1/2*sin(d*x+c)*2^(1/2)/(cos(d*x+c)
+1)/(-1/(cos(d*x+c)+1))^(1/2))*2^(1/2)+2*sin(d*x+c)*(-1/(cos(d*x+c)+1))^(1
/2))*cos(d*x+c)^(1/2)*(a*(1+sec(d*x+c)))^(1/2)/(cos(d*x+c)+1)^2/(-1/(cos(d
*x+c)+1))^(1/2)
```

3.430.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 340, normalized size of antiderivative = 2.91

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^{3/2}}\,dx = \left[\frac{3\sqrt{2}\left(\cos\left(dx+c\right)^2+2\cos\left(dx+c\right)+1\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^2}{}\right.}{8\left(a^2d\right.}\right.$$

$$-\frac{3\sqrt{2}\left(\cos\left(dx+c\right)^2+2\cos\left(dx+c\right)+1\right)\sqrt{-a}\arctan\left(\frac{\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}}{a\sin(dx+c)}\right)+2\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{c}}{4\left(a^2d\cos\left(dx+c\right)^2+2a^2d\cos\left(dx+c\right)+a^2d\right)}$$

input
```
integrate(1/(a+a*sec(d*x+c))^(3/2)/cos(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
[1/8*(3*sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*log(-(a*cos(
d*x + c)^2 - 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqr
t(cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2
*cos(d*x + c) + 1)) - 4*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d
*x + c))*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*
d), -1/4*(3*sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(-a)*arctan(
sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c)
)/(a*sin(d*x + c))) + 2*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d
*x + c))*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*
d)]
```

3.430.6 Sympy [F]

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^{3/2}}\,dx = \int \frac{1}{\left(a\left(\sec\left(c+dx\right)+1\right)\right)^{\frac{3}{2}}\sqrt{\cos\left(c+dx\right)}}\,dx$$

input
```
integrate(1/(a+a*sec(d*x+c))**(3/2)/cos(d*x+c)**(1/2),x)
```

output
```
Integral(1/((a*(sec(c + d*x) + 1))**(3/2)*sqrt(cos(c + d*x))), x)
```

3.430.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1031 vs. $2(94) = 188$.

Time = 0.37 (sec) , antiderivative size = 1031, normalized size of antiderivative = 8.81

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+a*sec(d*x+c))^(3/2)/cos(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
1/4*(3*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*
x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*
sin(1/2*d*x + 1/2*c) + 1))*cos(2*d*x + 2*c)^2 + 12*(log(cos(1/2*d*x + 1/2*
c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*
d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos
(d*x + c)^2 + 3*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*s
in(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*
c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(2*d*x + 2*c)^2 + 12*(log(cos(1/2*d
*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log
(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c)
+ 1))*sin(d*x + c)^2 + 2*(6*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/
2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/
2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(d*x + c) + 3*log(cos(1
/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) -
 3*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1
/2*c) + 1) - 2*sin(3/2*d*x + 3/2*c) + 2*sin(1/2*d*x + 1/2*c))*cos(2*d*x +
2*c) + 4*(3*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/
2*d*x + 1/2*c) + 1) - 3*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^
2 - 2*sin(1/2*d*x + 1/2*c) + 1) + 2*sin(1/2*d*x + 1/2*c))*cos(d*x + c) + 4
*(3*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*...
```

3.430.8 Giac [F]

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^{3/2}}\,dx = \int \frac{1}{(a\sec(dx+c)+a)^{\frac{3}{2}}\sqrt{\cos(dx+c)}}\,dx$$

input
```
integrate(1/(a+a*sec(d*x+c))^(3/2)/cos(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/((a*sec(d*x + c) + a)^(3/2)*sqrt(cos(d*x + c))), x)
```

3.430.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^{3/2}}\,dx=\int \frac{1}{\sqrt{\cos\left(c+d\,x\right)}\,\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)^{3/2}}\,dx$$

input
```
int(1/(cos(c + d*x)^(1/2)*(a + a/cos(c + d*x))^(3/2)),x)
```

output
```
int(1/(cos(c + d*x)^(1/2)*(a + a/cos(c + d*x))^(3/2)), x)
```

3.431 $\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}\,dx$

3.431.1 Optimal result 3193
3.431.2 Mathematica [B] (verified) 3193
3.431.3 Rubi [A] (verified) 3194
3.431.4 Maple [A] (verified) 3196
3.431.5 Fricas [A] (verification not implemented) 3197
3.431.6 Sympy [F(-1)] 3197
3.431.7 Maxima [B] (verification not implemented) 3198
3.431.8 Giac [F] 3198
3.431.9 Mupad [F(-1)] 3199

3.431.1 Optimal result

Integrand size = 25, antiderivative size = 117

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}\,dx=\frac{\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{2\sqrt{2}a^{3/2}d}+\frac{\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}$$

output
```
1/2*sin(d*x+c)/d/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(3/2)+1/4*arctanh(1/2*s
in(d*x+c)*a^(1/2)*sec(d*x+c)^(1/2)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))*cos(d*x
+c)^(1/2)*sec(d*x+c)^(1/2)/a^(3/2)/d*2^(1/2)
```

3.431.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 248 vs. 2(117) = 234.

Time = 0.65 (sec) , antiderivative size = 248, normalized size of antiderivative = 2.12

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}\,dx=\frac{\sqrt{\cos(c+dx)}\sec^{\frac{5}{2}}(c+dx)\left(-\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)-\sqrt{2}\,\mathrm{a}\right.}{}$$

input
```
Integrate[1/(Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(3/2)),x]
```

output
```
(Sqrt[Cos[c + d*x]]*Sec[c + d*x]^(5/2)*(-(Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[Sec
[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]) - Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[Sec[c
+ d*x]])/Sqrt[1 - Sec[c + d*x]]]*Cos[c + d*x] + 2*ArcSin[Sqrt[1 - Sec[c +
d*x]]]*(1 + Cos[c + d*x]) + 2*ArcSin[Sqrt[Sec[c + d*x]]]*(1 + Cos[c + d*x]
) + Sqrt[1 - Sec[c + d*x]]*Sec[c + d*x]^(3/2) + Cos[2*(c + d*x)]*Sqrt[1 -
Sec[c + d*x]]*Sec[c + d*x]^(3/2))*Sin[c + d*x])/(4*d*Sqrt[1 - Sec[c + d*x]
]*(a*(1 + Sec[c + d*x]))^(3/2))
```

3.431.3 Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3042, 4752, 3042, 4297, 3042, 4295, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}}dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(\sec(c+dx)a+a)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}}dx$$

$$\Big\downarrow \text{4297}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{\sec(c+dx)}a+a}dx}{4a}+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{\sqrt{\csc(c+dx+\frac{\pi}{2})}}{\sqrt{\csc(c+dx+\frac{\pi}{2})a+a}}dx}{4a}+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}\right)$$

$$\Big\downarrow \text{4295}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}-\frac{\int \frac{1}{2a-\frac{a^2\sin(c+dx)\tan(c+dx)}{\sec(c+dx)a+a}}d\left(-\frac{a\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{2ad}\right)$$

$$\Big\downarrow \text{219}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}\right)$$

input
```
Int[1/(Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(3/2)),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]
*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])]/(2*Sqrt[2]*a^(3/2)*d) +
 (Sec[c + d*x]^(3/2)*Sin[c + d*x])/(2*d*(a + a*Sec[c + d*x])^(3/2)))
```

3.431.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4295
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[-2*b*(d/(a*f))   Subst[Int[1/(2*b - d*x^2), x],
x, b*(Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]]))], x] /;
 FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4297
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[b*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Cs
c[e + f*x])^(n - 1)/(a*f*(2*m + 1))), x] + Simp[d*((m + 1)/(b*(2*m + 1)))
 Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ
[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && EqQ[m + n, 0] && LtQ[m,
-2^(-1)] && IntegerQ[2*m]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

3.431.4 Maple [A] (verified)

Time = 1.70 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.40

method	result
default	$\frac{\left(-\arctan\left(\frac{\sin(dx+c)\sqrt{2}}{2(\cos(dx+c)+1)\sqrt{-\frac{1}{\cos(dx+c)+1}}}\right)\sqrt{2}\,\cos(dx+c)+2\sin(dx+c)\sqrt{-\frac{1}{\cos(dx+c)+1}}-\arctan\left(\frac{\sin(dx+c)\sqrt{2}}{2(\cos(dx+c)+1)\sqrt{-\frac{1}{\cos(dx+c)+1}}}\right)\right.}{4d\,a^2(\cos(dx+c)+1)^2\sqrt{-\frac{1}{\cos(dx+c)+1}}}$

input
```
int(1/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/4/d/a^2*(-arctan(1/2*sin(d*x+c)*2^(1/2)/(cos(d*x+c)+1)/(-1/(cos(d*x+c)+1
))^(1/2))*2^(1/2)*cos(d*x+c)+2*sin(d*x+c)*(-1/(cos(d*x+c)+1))^(1/2)-arctan
(1/2*sin(d*x+c)*2^(1/2)/(cos(d*x+c)+1)/(-1/(cos(d*x+c)+1))^(1/2))*2^(1/2))
*cos(d*x+c)^(1/2)*(a*(1+sec(d*x+c)))^(1/2)/(cos(d*x+c)+1)^2/(-1/(cos(d*x+c
)+1))^(1/2)
```

3.431.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 338, normalized size of antiderivative = 2.89

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}\,dx = \left[\frac{\sqrt{2}\left(\cos\left(dx+c\right)^2+2\,\cos\left(dx+c\right)+1\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^2-2}{}\right.}{8\left(a^2d\,c\right.}\right.$$

$$-\frac{\sqrt{2}\left(\cos\left(dx+c\right)^2+2\,\cos\left(dx+c\right)+1\right)\sqrt{-a}\arctan\left(\frac{\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}}{a\sin(dx+c)}\right)-2\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\mathrm{co}}}{4\left(a^2d\cos\left(dx+c\right)^2+2\,a^2d\cos\left(dx+c\right)+a^2d\right)}$$

input
```
integrate(1/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
[1/8*(sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*log(-(a*cos(d*
x + c)^2 - 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(
cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*c
os(d*x + c) + 1)) + 4*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x
 + c))*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)
, -1/4*(sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(-a)*arctan(sqrt
(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))/(a
*sin(d*x + c))) - 2*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x +
 c))*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)]
```

3.431.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/cos(d*x+c)**(3/2)/(a+a*sec(d*x+c))**(3/2),x)
```

output
```
Timed out
```

3.431.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 15721 vs. $2(94) = 188$.

Time = 0.86 (sec) , antiderivative size = 15721, normalized size of antiderivative = 134.37

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
1/4*(32*(cos(3/2*d*x + 3/2*c)*sin(2*d*x + 2*c) + cos(2*d*x + 2*c)*sin(3/2*
d*x + 3/2*c) + cos(d*x + c)*sin(3/2*d*x + 3/2*c) + cos(3/2*d*x + 3/2*c)*si
n(d*x + c))*cos(3*d*x + 3*c)^2 + 96*(cos(3/2*d*x + 3/2*c)*sin(3*d*x + 3*c)
 + 3*cos(3/2*d*x + 3/2*c)*sin(2*d*x + 2*c) - (3*cos(d*x + c) + 1)*sin(3/2*
d*x + 3/2*c) - cos(3*d*x + 3*c)*sin(3/2*d*x + 3/2*c) - 3*cos(2*d*x + 2*c)*
sin(3/2*d*x + 3/2*c) + 3*cos(3/2*d*x + 3/2*c)*sin(d*x + c))*cos(4/3*arctan
2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 96*(cos(3/2*d*x + 3/2*c
)*sin(3*d*x + 3*c) + 3*cos(3/2*d*x + 3/2*c)*sin(2*d*x + 2*c) - (3*cos(d*x
+ c) + 1)*sin(3/2*d*x + 3/2*c) - cos(3*d*x + 3*c)*sin(3/2*d*x + 3/2*c) - 3
*cos(2*d*x + 2*c)*sin(3/2*d*x + 3/2*c) + 3*cos(3/2*d*x + 3/2*c)*sin(d*x +
c))*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 - 32*(c
os(3/2*d*x + 3/2*c)*sin(2*d*x + 2*c) + cos(2*d*x + 2*c)*sin(3/2*d*x + 3/2*
c) + cos(d*x + c)*sin(3/2*d*x + 3/2*c) + cos(3/2*d*x + 3/2*c)*sin(d*x + c)
)*sin(3*d*x + 3*c)^2 + 32*(6*cos(d*x + c) + 1)*cos(2*d*x + 2*c)*sin(3/2*d*
x + 3/2*c) + 96*cos(2*d*x + 2*c)^2*sin(3/2*d*x + 3/2*c) + 96*sin(2*d*x + 2
*c)^2*sin(3/2*d*x + 3/2*c) + 96*(cos(3/2*d*x + 3/2*c)*sin(3*d*x + 3*c) + 3
*cos(3/2*d*x + 3/2*c)*sin(2*d*x + 2*c) - (3*cos(d*x + c) + 1)*sin(3/2*d*x
+ 3/2*c) - cos(3*d*x + 3*c)*sin(3/2*d*x + 3/2*c) - 3*cos(2*d*x + 2*c)*sin(
3/2*d*x + 3/2*c) + 3*cos(3/2*d*x + 3/2*c)*sin(d*x + c))*sin(4/3*arctan2(si
n(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 96*(cos(3/2*d*x + 3/2*c)...
```

3.431.8 Giac [F]

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}\,dx = \int \frac{1}{(a\sec(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((a*sec(d*x + c) + a)^(3/2)*cos(d*x + c)^(3/2)), x)
```

3.431.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}\,dx=\int \frac{1}{\cos\left(c+d\,x\right)^{3/2}\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)^{3/2}}\,dx$$

input
```
int(1/(cos(c + d*x)^(3/2)*(a + a/cos(c + d*x))^(3/2)),x)
```

output
```
int(1/(cos(c + d*x)^(3/2)*(a + a/cos(c + d*x))^(3/2)), x)
```

3.432 $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}\,dx$

3.432.1 Optimal result . . . 3200
3.432.2 Mathematica [A] (verified) . . . 3200
3.432.3 Rubi [A] (verified) . . . 3201
3.432.4 Maple [B] (warning: unable to verify) . . . 3205
3.432.5 Fricas [A] (verification not implemented) . . . 3205
3.432.6 Sympy [F(-1)] . . . 3206
3.432.7 Maxima [B] (verification not implemented) . . . 3206
3.432.8 Giac [F] . . . 3207
3.432.9 Mupad [F(-1)] . . . 3208

3.432.1 Optimal result

Integrand size = 25, antiderivative size = 174

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}\,dx = \frac{2\operatorname{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{a^{3/2}d} - \frac{5\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{2\sqrt{2}a^{3/2}d} - \frac{\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}$$

output
```
-1/2*sin(d*x+c)/d/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(3/2)+2*arcsinh(a^(1/2
)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^(
3/2)/d-5/4*arctanh(1/2*sin(d*x+c)*a^(1/2)*sec(d*x+c)^(1/2)*2^(1/2)/(a+a*se
c(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^(3/2)/d*2^(1/2)
```

3.432.2 Mathematica [A] (verified)

Time = 0.57 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.43

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}\,dx = -\frac{\sqrt{\cos(c+dx)}\sec^{\frac{5}{2}}(c+dx)\left(-5\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)-5\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)\cos(c+dx)+2\,\mathrm{ar}\right.}{}$$

input
```
Integrate[1/(Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^(3/2)),x]
```

output
```
-1/4*(Sqrt[Cos[c + d*x]]*Sec[c + d*x]^(5/2)*(-5*Sqrt[2]*ArcTan[(Sqrt[2]*Sq
rt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]] - 5*Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt
[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*Cos[c + d*x] + 2*ArcSin[Sqrt[1 - S
ec[c + d*x]]]*(1 + Cos[c + d*x]) + 10*ArcSin[Sqrt[Sec[c + d*x]]]*(1 + Cos[
c + d*x]) + Sqrt[1 - Sec[c + d*x]]*Sec[c + d*x]^(3/2) + Cos[2*(c + d*x)]*S
qrt[1 - Sec[c + d*x]]*Sec[c + d*x]^(3/2))*Sin[c + d*x])/(d*Sqrt[1 - Sec[c
+ d*x]]*(a*(1 + Sec[c + d*x]))^(3/2))
```

3.432.3 Rubi [A] (verified)

Time = 0.91 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.93, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {3042, 4752, 3042, 4303, 27, 3042, 4511, 3042, 4288, 222, 4295, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a\sec(c+dx)+a)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}}dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(\sec(c+dx)a+a)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}}dx$$

$$\Big\downarrow \text{4303}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\int \frac{\sqrt{\sec(c+dx)}(a-4a\sec(c+dx))}{2\sqrt{\sec(c+dx)a+a}}dx}{2a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}\right)$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\int \frac{\sqrt{\sec(c+dx)}(a-4a\sec(c+dx))}{\sqrt{\sec(c+dx)a+a}}dx}{4a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}\right)$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\int \frac{\sqrt{\csc(c+dx+\frac{\pi}{2})}(a-4a\csc(c+dx+\frac{\pi}{2}))}{\sqrt{\csc(c+dx+\frac{\pi}{2})a+a}}dx}{4a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}\right)$$

$\downarrow$ 4511

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{5a\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{\sec(c+dx)a+a}}dx-4\int \sqrt{\sec(c+dx)}\sqrt{\sec(c+dx)a+a}dx}{4a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}}{2d(a\sec(c+dx)}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{5a\int \frac{\sqrt{\csc(c+dx+\frac{\pi}{2})}}{\sqrt{\csc(c+dx+\frac{\pi}{2})a+a}}dx-4\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{4a^2}-\frac{\sin}{2d}\right.$$

$\downarrow$ 4288

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{5a\int \frac{\sqrt{\csc(c+dx+\frac{\pi}{2})}}{\sqrt{\csc(c+dx+\frac{\pi}{2})a+a}}dx+\frac{8\int \frac{1}{\sqrt{\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1}}d\left(-\frac{a\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}}{4a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(}{2d(a\sec(c+dx)}\right.$$

$\downarrow$ 222

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{5a\int \frac{\sqrt{\csc(c+dx+\frac{\pi}{2})}}{\sqrt{\csc(c+dx+\frac{\pi}{2})a+a}}dx-\frac{8\sqrt{a}\text{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}}{4a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}\right)$$

$\downarrow$ 4295

3.432. $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{-\frac{10a\int\frac{1}{2a-\frac{a^2\sin(c+dx)\tan(c+dx)}{\sec(c+dx)a+a}}d\left(-\frac{a\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}-\frac{8\sqrt{a}\text{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}}{4a^2}-\frac{s}{2}\right.$$

$$\Big\downarrow \; 219$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{5\sqrt{2}\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{d}-\frac{8\sqrt{a}\text{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}}{4a^2}-\frac{\sin(c+dx)\sec}{2d(a\sec(c+d}\right.$$

input
```
Int[1/(Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^(3/2)),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(-1/4*((-8*Sqrt[a]*ArcSinh[(Sqrt[a]*
Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d + (5*Sqrt[2]*Sqrt[a]*ArcTanh[(S
qrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])
])/d)/a^2 - (Sec[c + d*x]^(3/2)*Sin[c + d*x])/(2*d*(a + a*Sec[c + d*x])^(3
/2)))
```

3.432.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4288
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[-2*(a/(b*f))*Sqrt[a*(d/b)]   Subst[Int[1/Sqrt[1
+ x^2/a], x], x, b*(Cot[e + f*x]/Sqrt[a + b*Csc[e + f*x]])], x] /; FreeQ[{a
, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[a*(d/b), 0]
```

rule 4295
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[-2*b*(d/(a*f))   Subst[Int[1/(2*b - d*x^2), x],
x, b*(Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]]))], x] /;
 FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4303
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-d^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d
*Csc[e + f*x])^(n - 2)/(f*(2*m + 1))), x] + Simp[d^2/(a*b*(2*m + 1))   Int[
(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n
 + 2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2,
0] && LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

rule 4511
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b -
a*B)/b   Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n, x], x] + Simp[B/b
 Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b
, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

3.432. $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}\,dx$

3.432.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 360 vs. $2(141) = 282$.

Time = 2.30 (sec) , antiderivative size = 361, normalized size of antiderivative = 2.07

method	result
default	$\frac{\left((1-\cos(dx+c))^2\csc(dx+c)^2-1\right)^3\sqrt{-\frac{2a}{(1-\cos(dx+c))^2\csc(dx+c)^2-1}}\left(2\sqrt{2}\arctan\left(\frac{(\csc(dx+c)-\cot(dx+c)+1)\sqrt{2}}{2\sqrt{-(1-\cos(dx+c))^2\csc(dx+c)^2-1}}\right)+2\sqrt{2}\arctan\right.}{4d\,a^2\left(-\frac{(1-\cos(dx+c))^2\csc(dx+c)^2-1}{(1-\cos(dx+c))^2\csc(dx+c)^2+1}\right)^{\frac{5}{2}}\left((1-\cos\right.}$

input
```
int(1/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/4/d/a^2*((1-cos(d*x+c))^2*csc(d*x+c)^2-1)^3*(-2*a/((1-cos(d*x+c))^2*csc(
d*x+c)^2-1))^(1/2)*(2*2^(1/2)*arctan(1/2*(csc(d*x+c)-cot(d*x+c)+1)*2^(1/2)
/(-(1-cos(d*x+c))^2*csc(d*x+c)^2-1)^(1/2))+2*2^(1/2)*arctan(1/2*(-cot(d*x+
c)+csc(d*x+c)-1)*2^(1/2)/(-(1-cos(d*x+c))^2*csc(d*x+c)^2-1)^(1/2))+(-(1-co
s(d*x+c))^2*csc(d*x+c)^2-1)^(1/2)*(-cot(d*x+c)+csc(d*x+c))-5*arctan(1/(-(1
-cos(d*x+c))^2*csc(d*x+c)^2-1)^(1/2)*(-cot(d*x+c)+csc(d*x+c))))/(-((1-cos(
d*x+c))^2*csc(d*x+c)^2-1)/((1-cos(d*x+c))^2*csc(d*x+c)^2+1))^(5/2)/((1-cos
(d*x+c))^2*csc(d*x+c)^2+1)^2/(-(1-cos(d*x+c))^2*csc(d*x+c)^2-1)^(1/2)
```

3.432.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 550, normalized size of antiderivative = 3.16

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}\,dx = \left[\frac{5\sqrt{2}\left(\cos(dx+c)^2+2\cos(dx+c)+1\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^2+}{}\right.}{}\right.$$

input
```
integrate(1/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
[1/8*(5*sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*log(-(a*cos(
d*x + c)^2 + 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqr
t(cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2
*cos(d*x + c) + 1)) + 4*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*log(
(a*cos(d*x + c)^3 - 4*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(cos
(d*x + c) - 2)*sqrt(cos(d*x + c))*sin(d*x + c) - 7*a*cos(d*x + c)^2 + 8*a)
/(cos(d*x + c)^3 + cos(d*x + c)^2)) - 4*sqrt((a*cos(d*x + c) + a)/cos(d*x
+ c))*sqrt(cos(d*x + c))*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos
(d*x + c) + a^2*d), 1/4*(5*sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*s
qrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sq
rt(cos(d*x + c))/(a*sin(d*x + c))) + 4*(cos(d*x + c)^2 + 2*cos(d*x + c) +
1)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt
(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)) - 2
*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/
(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)]
```

3.432.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/cos(d*x+c)**(5/2)/(a+a*sec(d*x+c))**(3/2),x)
```

output
```
Timed out
```

3.432.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2122 vs. $2(141) = 282$.

Time = 0.43 (sec) , antiderivative size = 2122, normalized size of antiderivative = 12.20

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
1/4*(4*(sin(2*d*x + 2*c) + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c))))*cos(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*(sqrt(2)
*cos(2*d*x + 2*c)^2 + 4*sqrt(2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c)))^2 + sqrt(2)*sin(2*d*x + 2*c)^2 + 4*sqrt(2)*sin(2*d*x + 2*c)*sin
(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*sqrt(2)*sin(1/2*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 4*(sqrt(2)*cos(2*d*x + 2*c) +
 sqrt(2))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)
*cos(2*d*x + 2*c) + sqrt(2))*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2
 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt
(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) - 2*(sqrt(2)
*cos(2*d*x + 2*c)^2 + 4*sqrt(2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c)))^2 + sqrt(2)*sin(2*d*x + 2*c)^2 + 4*sqrt(2)*sin(2*d*x + 2*c)*sin
(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*sqrt(2)*sin(1/2*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 4*(sqrt(2)*cos(2*d*x + 2*c) +
 sqrt(2))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)
*cos(2*d*x + 2*c) + sqrt(2))*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2
 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2*sqrt
(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) + 2*(sqrt...
```

3.432.8 Giac [F]

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}\,dx = \int \frac{1}{(a\sec(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((a*sec(d*x + c) + a)^(3/2)*cos(d*x + c)^(5/2)), x)
```

3.432.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}\,dx=\int \frac{1}{\cos\left(c+d\,x\right)^{5/2}\,\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)^{3/2}}\,dx$$

input
```
int(1/(cos(c + d*x)^(5/2)*(a + a/cos(c + d*x))^(3/2)),x)
```

output
```
int(1/(cos(c + d*x)^(5/2)*(a + a/cos(c + d*x))^(3/2)), x)
```

3.433 $\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}\,dx$

3.433.1 Optimal result . . . 3209
3.433.2 Mathematica [A] (verified) . . . 3210
3.433.3 Rubi [A] (verified) . . . 3210
3.433.4 Maple [B] (verified) . . . 3215
3.433.5 Fricas [A] (verification not implemented) . . . 3216
3.433.6 Sympy [F(-1)] . . . 3217
3.433.7 Maxima [B] (verification not implemented) . . . 3217
3.433.8 Giac [F] . . . 3218
3.433.9 Mupad [F(-1)] . . . 3219

3.433.1 Optimal result

Integrand size = 25, antiderivative size = 214

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}\,dx = -\frac{3\operatorname{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{a^{3/2}d} + \frac{9\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{2\sqrt{2}a^{3/2}d} - \frac{\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}} + \frac{3\sin(c+dx)}{2ad\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}$$

output
```
-1/2*sin(d*x+c)/d/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(3/2)-3*arcsinh(a^(1/2
)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^(
3/2)/d+9/4*arctanh(1/2*sin(d*x+c)*a^(1/2)*sec(d*x+c)^(1/2)*2^(1/2)/(a+a*se
c(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^(3/2)/d*2^(1/2)+3/2*s
in(d*x+c)/a/d/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2)
```

3.433.2 Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.27

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}\,dx = \frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(6\sqrt{1-\sec(c+dx)}\sec^{\frac{3}{2}}(c+dx)\sin\right.}{}$$

input
```
Integrate[1/(Cos[c + d*x]^(7/2)*(a + a*Sec[c + d*x])^(3/2)),x]
```

output
```
(Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(6*Sqrt[1 - Sec[c + d*x]]*Sec[c + d
*x]^(3/2)*Sin[c + d*x] + 4*Sqrt[1 - Sec[c + d*x]]*Sec[c + d*x]^(5/2)*Sin[c
 + d*x] - 9*Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d
*x]]]*Tan[c + d*x] - 9*Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1
- Sec[c + d*x]]]*Sec[c + d*x]*Tan[c + d*x] + 6*ArcSin[Sqrt[1 - Sec[c + d*x
]]]*(1 + Sec[c + d*x])*Tan[c + d*x] + 18*ArcSin[Sqrt[Sec[c + d*x]]]*(1 + S
ec[c + d*x])*Tan[c + d*x]))/(4*d*Sqrt[1 - Sec[c + d*x]]*(a*(1 + Sec[c + d*
x]))^(3/2))
```

3.433.3 Rubi [A] (verified)

Time = 1.19 (sec) , antiderivative size = 203, normalized size of antiderivative = 0.95, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {3042, 4752, 3042, 4303, 27, 3042, 4509, 25, 3042, 4511, 3042, 4288, 222, 4295, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a\sec(c+dx)+a)^{3/2}}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{7/2}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{3/2}}dx$$

$$\Big\downarrow \; 4752$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(\sec(c+dx)a+a)^{3/2}}dx$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}}dx$$

$\Big\downarrow$ 4303

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\int \frac{3\sec^{\frac{3}{2}}(c+dx)(a-2a\sec(c+dx))}{2\sqrt{\sec(c+dx)a+a}}dx}{2a^2}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}\right)$$

$\Big\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{3\int \frac{\sec^{\frac{3}{2}}(c+dx)(a-2a\sec(c+dx))}{\sqrt{\sec(c+dx)a+a}}dx}{4a^2}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}\right)$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{3\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a-2a\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a^2}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}\right)$$

$\Big\downarrow$ 4509

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{3\left(\frac{\int -\frac{\sqrt{\sec(c+dx)}\left(a^2-2a^2\sec(c+dx)\right)}{\sqrt{\sec(c+dx)a+a}}dx}{a}-\frac{2a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}\right)}{4a^2}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+d}{2d(a\sec(c+dx)+a)}\right.$$

$\Big\downarrow$ 25

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{3\left(-\frac{\int \frac{\sqrt{\sec(c+dx)}\left(a^2-2a^2\sec(c+dx)\right)}{\sqrt{\sec(c+dx)a+a}}dx}{a}-\frac{2a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}\right)}{4a^2}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+}{2d(a\sec(c+dx)+a)}\right.$$

$\Big\downarrow$ 3042

3.433. $\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{3\left(-\frac{\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a^2-2a^2\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{a}-\frac{2a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}\right)}{4a^2}-\frac{\sin(c+dx)\sec}{2d(a\sec(c+d}\right.$$

$\downarrow$ 4511

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{3\left(-\frac{3a^2\int\frac{\sqrt{\sec(c+dx)}}{\sqrt{\sec(c+dx)a+a}}dx-2a\int\sqrt{\sec(c+dx)}\sqrt{\sec(c+dx)a+a}dx}{a}-\frac{2a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}\right)}{4a^2}-\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{3\left(-\frac{3a^2\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx-2a\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{a}-\frac{2a\sin(c+dx)\sec^{\frac{3}{2}}(c}{d\sqrt{a\sec(c+dx)+}}\right.}{4a^2}\right.$$

$\downarrow$ 4288

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{3\left(-\frac{3a^2\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx+\frac{4a\int\frac{1}{\sqrt{\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1}}d\left(-\frac{a\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}}{a}-\frac{2a\sin(c+dx)\sec^{\frac{3}{2}}(c-}{d\sqrt{a\sec(c+dx)+}}\right.}{4a^2}\right.$$

3.433. $\int\frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}\,dx$

$$\Big\downarrow \; 222$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{3\left(-\frac{3a^2\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx-\frac{4a^{3/2}\text{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}}{a}-\frac{2a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}\right)}{4a^2}-\right.$$

$$\Big\downarrow \; 4295$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{3\left(-\frac{-\frac{6a^2\int\frac{1}{2a-\frac{a^2\sin(c+dx)\tan(c+dx)}{\sec(c+dx)a+a}}d\left(-\frac{a\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}}{a}-\frac{4a^{3/2}\text{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}}{}-\right.}{4a^2}$$

$$\Big\downarrow \; 219$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{3\left(-\frac{\frac{3\sqrt{2}a^{3/2}\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{d}-\frac{4a^{3/2}\text{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}}{a}-\frac{2a\sin(c+dx)\,s}{d\sqrt{a\sec(c}}\right.}{4a^2}$$

input
```
Int[1/(Cos[c + d*x]^(7/2)*(a + a*Sec[c + d*x])^(3/2)),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(-1/2*(Sec[c + d*x]^(5/2)*Sin[c + d*
x])/(d*(a + a*Sec[c + d*x])^(3/2)) - (3*(-(((-4*a^(3/2)*ArcSinh[(Sqrt[a]*T
an[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d + (3*Sqrt[2]*a^(3/2)*ArcTanh[(Sq
rt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])]
)/d)/a) - (2*a*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]
])))/(4*a^2))
```

3.433. $\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}\,dx$

3.433.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4288
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[-2*(a/(b*f))*Sqrt[a*(d/b)]   Subst[Int[1/Sqrt[1
+ x^2/a], x], x, b*(Cot[e + f*x]/Sqrt[a + b*Csc[e + f*x]])], x] /; FreeQ[{a
, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[a*(d/b), 0]
```

rule 4295
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[-2*b*(d/(a*f))   Subst[Int[1/(2*b - d*x^2), x],
x, b*(Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]]))], x] /;
 FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4303
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-d^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d
*Csc[e + f*x])^(n - 2)/(f*(2*m + 1))), x] + Simp[d^2/(a*b*(2*m + 1))   Int[
(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n
 + 2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2,
0] && LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

3.433. $\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}\,dx$

rule 4509
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*d*
Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(f*(m + n))),
 x] + Simp[d/(b*(m + n))   Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n -
 1)*Simp[b*B*(n - 1) + (A*b*(m + n) + a*B*m)*Csc[e + f*x], x], x], x] /; Fr
eeQ[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
&& GtQ[n, 1]
```

rule 4511
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b -
a*B)/b   Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n, x], x] + Simp[B/b
 Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b
, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

3.433.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 403 vs. $2(175) = 350$.

Time = 2.48 (sec) , antiderivative size = 404, normalized size of antiderivative = 1.89

method	result
default	$\frac{\left(-9\sqrt{2}\,\cos(dx+c)^2\arctan\left(\frac{\sin(dx+c)\sqrt{2}}{2(\cos(dx+c)+1)\sqrt{-\frac{1}{\cos(dx+c)+1}}}\right)+6\sqrt{-\frac{1}{\cos(dx+c)+1}}\,\sin(dx+c)\cos(dx+c)-6\cos(dx+c)^2\arctan\left(\frac{}{2(\cos}\right.\right.}{}$

input
```
int(1/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/4/d/a^2*(-9*2^(1/2)*cos(d*x+c)^2*arctan(1/2*sin(d*x+c)*2^(1/2)/(cos(d*x+
c)+1)/(-1/(cos(d*x+c)+1))^(1/2))+6*(-1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*co
s(d*x+c)-6*cos(d*x+c)^2*arctan(1/2*(cos(d*x+c)-sin(d*x+c)+1)/(cos(d*x+c)+1
)/(-1/(cos(d*x+c)+1))^(1/2))+6*arctan(1/2*(cos(d*x+c)+sin(d*x+c)+1)/(cos(d
*x+c)+1)/(-1/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)^2-9*arctan(1/2*sin(d*x+c)*2
^(1/2)/(cos(d*x+c)+1)/(-1/(cos(d*x+c)+1))^(1/2))*2^(1/2)*cos(d*x+c)+4*sin(
d*x+c)*(-1/(cos(d*x+c)+1))^(1/2)-6*cos(d*x+c)*arctan(1/2*(cos(d*x+c)-sin(d
*x+c)+1)/(cos(d*x+c)+1)/(-1/(cos(d*x+c)+1))^(1/2))+6*cos(d*x+c)*arctan(1/2
*(cos(d*x+c)+sin(d*x+c)+1)/(cos(d*x+c)+1)/(-1/(cos(d*x+c)+1))^(1/2)))*(a*(
1+sec(d*x+c)))^(1/2)/(cos(d*x+c)+1)^2/(-1/(cos(d*x+c)+1))^(1/2)/cos(d*x+c)
^(1/2)
```

3.433.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 614, normalized size of antiderivative = 2.87

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}\,dx = \left[\frac{9\sqrt{2}\left(\cos\left(dx+c\right)^{3}+2\,\cos\left(dx+c\right)^{2}+\cos\left(dx+c\right)\right)\sqrt{a}\log\left(-\right.}{}\right.$$

$$-\frac{9\sqrt{2}\left(\cos\left(dx+c\right)^{3}+2\,\cos\left(dx+c\right)^{2}+\cos\left(dx+c\right)\right)\sqrt{-a}\arctan\left(\frac{\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}}{a\sin(dx+c)}\right)-2\sqrt{\frac{a\,\mathrm{co}}{\mathrm{c}}}}{4\left(a^{2}\right.}$$

input
```
integrate(1/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
[1/8*(9*sqrt(2)*(cos(d*x + c)^3 + 2*cos(d*x + c)^2 + cos(d*x + c))*sqrt(a)
*log(-(a*cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(
d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d
*x + c)^2 + 2*cos(d*x + c) + 1)) + 4*sqrt((a*cos(d*x + c) + a)/cos(d*x + c
))*(3*cos(d*x + c) + 2)*sqrt(cos(d*x + c))*sin(d*x + c) + 6*(cos(d*x + c)^
3 + 2*cos(d*x + c)^2 + cos(d*x + c))*sqrt(a)*log((a*cos(d*x + c)^3 + 4*sqr
t(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(cos(d*x + c) - 2)*sqrt(cos(d
*x + c))*sin(d*x + c) - 7*a*cos(d*x + c)^2 + 8*a)/(cos(d*x + c)^3 + cos(d*
x + c)^2)))/(a^2*d*cos(d*x + c)^3 + 2*a^2*d*cos(d*x + c)^2 + a^2*d*cos(d*x
 + c)), -1/4*(9*sqrt(2)*(cos(d*x + c)^3 + 2*cos(d*x + c)^2 + cos(d*x + c))
*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*
sqrt(cos(d*x + c))/(a*sin(d*x + c))) - 2*sqrt((a*cos(d*x + c) + a)/cos(d*x
 + c))*(3*cos(d*x + c) + 2)*sqrt(cos(d*x + c))*sin(d*x + c) + 6*(cos(d*x +
 c)^3 + 2*cos(d*x + c)^2 + cos(d*x + c))*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((
a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d
*x + c)^2 - a*cos(d*x + c) - 2*a)))/(a^2*d*cos(d*x + c)^3 + 2*a^2*d*cos(d*
x + c)^2 + a^2*d*cos(d*x + c))]
```

3.433.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/cos(d*x+c)**(7/2)/(a+a*sec(d*x+c))**(3/2),x)
```

output
```
Timed out
```

3.433.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4934 vs. $2(175) = 350$.

Time = 0.61 (sec) , antiderivative size = 4934, normalized size of antiderivative = 23.06

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
-1/4*(12*(sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c) + 2*sin(3/2*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c))) + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c))))*cos(7/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 8*(s
in(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - sin(3/4*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c))) - 3*sin(1/4*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c))))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*
(sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c) + 2*sin(1/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c))))*cos(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))
) - 4*(sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c) + 2*sin(1/2*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c))))*cos(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c))) - 12*(sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*cos(1/4*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c))) + 3*(sqrt(2)*cos(4*d*x + 4*c)^2 + 4*sqrt(2
)*cos(2*d*x + 2*c)^2 + 4*sqrt(2)*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c)))^2 + 4*sqrt(2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c)))^2 + sqrt(2)*sin(4*d*x + 4*c)^2 + 4*sqrt(2)*sin(4*d*x + 4*c)*sin(2*d*x
 + 2*c) + 4*sqrt(2)*sin(2*d*x + 2*c)^2 + 4*sqrt(2)*sin(3/2*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c)))^2 + 4*sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c)))^2 + 2*(2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(4*
d*x + 4*c) + 4*(sqrt(2)*cos(4*d*x + 4*c) + 2*sqrt(2)*cos(2*d*x + 2*c) + 2*
sqrt(2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sqrt(2))...
```

3.433.8 Giac [F]

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}\,dx = \int \frac{1}{(a\sec(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((a*sec(d*x + c) + a)^(3/2)*cos(d*x + c)^(7/2)), x)
```

3.433.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}\,dx=\int \frac{1}{\cos\left(c+d\,x\right)^{7/2}\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)^{3/2}}\,dx$$

input
```
int(1/(cos(c + d*x)^(7/2)*(a + a/cos(c + d*x))^(3/2)),x)
```

output
```
int(1/(cos(c + d*x)^(7/2)*(a + a/cos(c + d*x))^(3/2)), x)
```

3.434 $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx$

3.434.1 Optimal result . . . 3220
3.434.2 Mathematica [A] (verified) . . . 3221
3.434.3 Rubi [A] (verified) . . . 3221
3.434.4 Maple [A] (warning: unable to verify) . . . 3226
3.434.5 Fricas [A] (verification not implemented) . . . 3226
3.434.6 Sympy [F(-1)] . . . 3227
3.434.7 Maxima [B] (verification not implemented) . . . 3227
3.434.8 Giac [F] . . . 3228
3.434.9 Mupad [F(-1)] . . . 3229

3.434.1 Optimal result

Integrand size = 25, antiderivative size = 237

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \frac{163\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{16\sqrt{2}a^{5/2}d} - \frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{17\sqrt{\cos(c+dx)}\sin(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} - \frac{299\sin(c+dx)}{48a^2d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} + \frac{95\sqrt{\cos(c+dx)}\sin(c+dx)}{48a^2d\sqrt{a+a\sec(c+dx)}}$$

output
```
-1/4*sin(d*x+c)*cos(d*x+c)^(1/2)/d/(a+a*sec(d*x+c))^(5/2)-17/16*sin(d*x+c)
*cos(d*x+c)^(1/2)/a/d/(a+a*sec(d*x+c))^(3/2)+163/32*arctanh(1/2*sin(d*x+c)
*a^(1/2)*sec(d*x+c)^(1/2)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))*cos(d*x+c)^(1/2)
*sec(d*x+c)^(1/2)/a^(5/2)/d*2^(1/2)-299/48*sin(d*x+c)/a^2/d/cos(d*x+c)^(1/
2)/(a+a*sec(d*x+c))^(1/2)+95/48*sin(d*x+c)*cos(d*x+c)^(1/2)/a^2/d/(a+a*sec
(d*x+c))^(1/2)
```

3.434.2 Mathematica [A] (verified)

Time = 0.78 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.61

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = -\frac{\left(1956\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)\cos^4\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{5}{2}}(c+dx)+2\sqrt{1-\sec(c+dx)}(160-32\cos(c+dx)\right.}{96d\sqrt{-1+\cos(c+dx)}(a(1+\sec(c+dx)))^{5/2}}$$

input
```
Integrate[Cos[c + d*x]^(3/2)/(a + a*Sec[c + d*x])^(5/2),x]
```

output
```
-1/96*((1956*Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c +
d*x]]]*Cos[(c + d*x)/2]^4*Sec[c + d*x]^(5/2) + 2*Sqrt[1 - Sec[c + d*x]]*(1
60 - 32*Cos[c + d*x] + 503*Sec[c + d*x] + 299*Sec[c + d*x]^2))*Sin[c + d*x
])/(d*Sqrt[-1 + Cos[c + d*x]]*(a*(1 + Sec[c + d*x]))^(5/2))
```

3.434.3 Rubi [A] (verified)

Time = 1.33 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.08, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.640$, Rules used = {3042, 4752, 3042, 4304, 27, 3042, 4508, 27, 3042, 4510, 27, 3042, 4501, 3042, 4295, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a\sec(c+dx)+a)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}}dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(\sec(c+dx)a+a)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{5/2}}dx$$

$$\Big\downarrow \; 4304$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\int -\frac{11a-6a\sec(c+dx)}{2\sec^{\frac{3}{2}}(c+dx)(\sec(c+dx)a+a)^{3/2}}dx}{4a^2}-\frac{\sin(c+dx)}{4d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^{5/2}}\right)$$

$$\Big\downarrow \; 27$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{11a-6a\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(\sec(c+dx)a+a)^{3/2}}dx}{8a^2}-\frac{\sin(c+dx)}{4d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^{5/2}}\right)$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{11a-6a\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}}dx}{8a^2}-\frac{\sin(c+dx)}{4d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^{5/2}}\right)$$

$$\Big\downarrow \; 4508$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\int \frac{95a^2-68a^2\sec(c+dx)}{2\sec^{\frac{3}{2}}(c+dx)\sqrt{\sec(c+dx)a+a}}dx}{2a^2}-\frac{17a\sin(c+dx)}{2d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^{3/2}}}{8a^2}-\frac{\sin(c+dx)}{4d\sqrt{\sec(c+dx)}(a\sec(c+\right.$$

$$\Big\downarrow \; 27$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\int \frac{95a^2-68a^2\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)\sqrt{\sec(c+dx)a+a}}dx}{4a^2}-\frac{17a\sin(c+dx)}{2d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^{3/2}}}{8a^2}-\frac{\sin(c+dx)}{4d\sqrt{\sec(c+dx)}(a\sec(c+\right.$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\int \frac{95a^2-68a^2\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a^2}-\frac{17a\sin(c+dx)}{2d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^{3/2}}}{8a^2}-\frac{\sin(c}{4d\sqrt{\sec(c+dx)}(a}\right.$$

3.434. $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx$

$$\Big\downarrow \; 4510$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{2\int -\frac{299a^3-190a^3\sec(c+dx)}{2\sqrt{\sec(c+dx)}\sqrt{\sec(c+dx)a+a}}dx}{3a}+\frac{190a^2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}}{4a^2}-\frac{17a\sin(c+dx)}{2d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^{3/2}}}{8a^2}-\right.$$

$$\Big\downarrow \; 27$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{190a^2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}-\frac{\int\frac{299a^3-190a^3\sec(c+dx)}{\sqrt{\sec(c+dx)}\sqrt{\sec(c+dx)a+a}}dx}{3a}}{4a^2}-\frac{17a\sin(c+dx)}{2d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^{3/2}}}{8a^2}-\frac{}{4a}\right.$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{190a^2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}-\frac{\int\frac{299a^3-190a^3\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{3a}}{4a^2}-\frac{17a\sin(c+dx)}{2d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^{3/}}}{8a^2}\right.$$

$$\Big\downarrow \; 4501$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{190a^2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}-\frac{\frac{598a^3\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\sec(c+dx)+a}}-489a^3\int\frac{\sqrt{\sec(c+dx)}}{\sqrt{\sec(c+dx)a+a}}dx}{3a}}{4a^2}-\frac{17a\sin(c-}{2d\sqrt{\sec(c+dx)}(a\sec}}{8a^2}\right.$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{190a^2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}-\frac{\frac{598a^3\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\sec(c+dx)+a}}-489a^3\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{3a}}{4a^2}-\frac{17a\,\mathrm{si}}{2d\sqrt{\sec(c+dx)}(}}{8a^2}\right.$$

3.434. $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx$

$$\Big\downarrow \; 4295$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{190a^2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}-\frac{\frac{978a^3\int\frac{1}{2a-\frac{a^2\sin(c+dx)\tan(c+dx)}{\sec(c+dx)a+a}}d\left(-\frac{a\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}}{3a}+\frac{598a^3\sin(c+dx}{d\sqrt{a\sec(c-}}}{4a^2}}{8a^2}\right.$$

$$\Big\downarrow \; 219$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{190a^2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}-\frac{\frac{598a^3\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\sec(c+dx)+a}}-\frac{489\sqrt{2}a^{5/2}\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{d}}{3a}}{4a^2}-}{8a^2}\right.$$

input
```
Int[Cos[c + d*x]^(3/2)/(a + a*Sec[c + d*x])^(5/2),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(-1/4*Sin[c + d*x]/(d*Sqrt[Sec[c + d
*x]]*(a + a*Sec[c + d*x])^(5/2)) + ((-17*a*Sin[c + d*x])/(2*d*Sqrt[Sec[c +
 d*x]]*(a + a*Sec[c + d*x])^(3/2)) + ((190*a^2*Sin[c + d*x])/(3*d*Sqrt[Sec
[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) - ((-489*Sqrt[2]*a^(5/2)*ArcTanh[(Sqr
t[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])
/d + (598*a^3*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]]
))/(3*a))/(4*a^2))/(8*a^2))
```

3.434.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4295
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[-2*b*(d/(a*f))   Subst[Int[1/(2*b - d*x^2), x],
x, b*(Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]]))], x] /;
 FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4304
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-Cot[e + f*x])*(a + b*Csc[e + f*x])^m*((d*Csc
[e + f*x])^n/(f*(2*m + 1))), x] + Simp[1/(a^2*(2*m + 1))   Int[(a + b*Csc[e
 + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e
+ f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ
[m, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

rule 4501
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*Cot[e
 + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(f*n)), x] - Simp[(a*A*m
 - b*B*n)/(b*d*n)   Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x]
, x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a
^2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LeQ[m, -1]
```

rule 4508
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
- a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
 1))), x] - Simp[1/(a^2*(2*m + 1))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Cs
c[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[
e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B
, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

rule 4510
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*Cot[e
 + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(f*n)), x] - Simp[1/(b*d
*n)   Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*
n - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

3.434. $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx$

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

3.434.4 Maple [A] (warning: unable to verify)

Time = 1.68 (sec) , antiderivative size = 361, normalized size of antiderivative = 1.52

method	result
default	$-\frac{\left(-\frac{(1-\cos(dx+c))^2\csc(dx+c)^2-1}{(1-\cos(dx+c))^2\csc(dx+c)^2+1}\right)^{\frac{3}{2}}\sqrt{-\frac{2a}{(1-\cos(dx+c))^2\csc(dx+c)^2-1}}\left(6(1-\cos(dx+c))^7\csc(dx+c)^7-69(1-\cos(dx+c))^5\csc(dx+c\right.}{}$

input
```
int(cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/96/d/a^3*(-((1-cos(d*x+c))^2*csc(d*x+c)^2-1)/((1-cos(d*x+c))^2*csc(d*x+
c)^2+1))^(3/2)*(-2*a/((1-cos(d*x+c))^2*csc(d*x+c)^2-1))^(1/2)*(6*(1-cos(d*
x+c))^7*csc(d*x+c)^7-69*(1-cos(d*x+c))^5*csc(d*x+c)^5+489*arctan(1/(-(1-co
s(d*x+c))^2*csc(d*x+c)^2-1)^(1/2)*(-cot(d*x+c)+csc(d*x+c)))*(-(1-cos(d*x+c
))^2*csc(d*x+c)^2-1)^(1/2)*(1-cos(d*x+c))^2*csc(d*x+c)^2-668*(1-cos(d*x+c)
)^3*csc(d*x+c)^3+489*arctan(1/(-(1-cos(d*x+c))^2*csc(d*x+c)^2-1)^(1/2)*(-c
ot(d*x+c)+csc(d*x+c)))*(-(1-cos(d*x+c))^2*csc(d*x+c)^2-1)^(1/2)-465*csc(d*
x+c)+465*cot(d*x+c))/((1-cos(d*x+c))^2*csc(d*x+c)^2-1)
```

3.434.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 448, normalized size of antiderivative = 1.89

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \left[\frac{489\sqrt{2}\left(\cos(dx+c)^3+3\cos(dx+c)^2+3\cos(dx+c)+1\right)\sqrt{a}\log\left(-\frac{a\cos}{}\right.}{}\right.$$

$$-\frac{489\sqrt{2}\left(\cos(dx+c)^3+3\cos(dx+c)^2+3\cos(dx+c)+1\right)\sqrt{-a}\arctan\left(\frac{\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}}{a\sin(dx+c)}\right)}{96\left(a^3d\cos(dx+c)^3+3a^3d\cos(\right.}$$

input
```
integrate(cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
[1/192*(489*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) +
1)*sqrt(a)*log(-(a*cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c)
 + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3
*a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + 4*(32*cos(d*x + c)^3 - 160*co
s(d*x + c)^2 - 503*cos(d*x + c) - 299)*sqrt((a*cos(d*x + c) + a)/cos(d*x +
 c))*sqrt(cos(d*x + c))*sin(d*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(
d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d), -1/96*(489*sqrt(2)*(cos(d*x +
c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*sqrt(-a)*arctan(sqrt(2)*sqrt
(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))/(a*sin(d*x
 + c))) - 2*(32*cos(d*x + c)^3 - 160*cos(d*x + c)^2 - 503*cos(d*x + c) - 2
99)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c
))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) +
 a^3*d)]
```

3.434.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(3/2)/(a+a*sec(d*x+c))**(5/2),x)
```

output
```
Timed out
```

3.434.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 148823 vs. $2(196) = 392$.

Time = 2.71 (sec) , antiderivative size = 148823, normalized size of antiderivative = 627.95

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
1/96*(32*(cos(3*d*x + 3*c)^2*sin(3/2*d*x + 3/2*c) + sin(3*d*x + 3*c)^2*sin
(3/2*d*x + 3/2*c) - 15*(cos(3*d*x + 3*c)^2 + sin(3*d*x + 3*c)^2)*sin(1/3*a
rctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*cos(11/3*arctan2(sin(
3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^4 + 41472*(cos(3*d*x + 3*c)^2*sin
(3/2*d*x + 3/2*c) + sin(3*d*x + 3*c)^2*sin(3/2*d*x + 3/2*c) - 15*(cos(3*d*
x + 3*c)^2 + sin(3*d*x + 3*c)^2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos
(3/2*d*x + 3/2*c))))*cos(7/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3
/2*c)))^4 + 8192*(cos(3*d*x + 3*c)^2*sin(3/2*d*x + 3/2*c) + sin(3*d*x + 3*
c)^2*sin(3/2*d*x + 3/2*c) - 15*(cos(3*d*x + 3*c)^2 + sin(3*d*x + 3*c)^2)*s
in(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*cos(5/3*arcta
n2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^4 + 288*sin(3/2*d*x + 3/2*
c)^5 + 32*(cos(3*d*x + 3*c)^2*sin(3/2*d*x + 3/2*c) + sin(3*d*x + 3*c)^2*si
n(3/2*d*x + 3/2*c) - 15*(cos(3*d*x + 3*c)^2 + sin(3*d*x + 3*c)^2)*sin(1/3*
arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sin(11/3*arctan2(sin
(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^4 + 41472*(cos(3*d*x + 3*c)^2*si
n(3/2*d*x + 3/2*c) + sin(3*d*x + 3*c)^2*sin(3/2*d*x + 3/2*c) - 15*(cos(3*d
*x + 3*c)^2 + sin(3*d*x + 3*c)^2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), co
s(3/2*d*x + 3/2*c))))*sin(7/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x +
3/2*c)))^4 + 8192*(cos(3*d*x + 3*c)^2*sin(3/2*d*x + 3/2*c) + sin(3*d*x + 3
*c)^2*sin(3/2*d*x + 3/2*c) - 15*(cos(3*d*x + 3*c)^2 + sin(3*d*x + 3*c)^...
```

3.434.8 Giac [F]

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \int \frac{\cos\left(dx+c\right)^{\frac{3}{2}}}{\left(a\sec\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate(cos(d*x + c)^(3/2)/(a*sec(d*x + c) + a)^(5/2), x)
```

3.434.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx=\int \frac{\cos\left(c+d\,x\right)^{3/2}}{\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)^{5/2}}\,dx$$

input
```
int(cos(c + d*x)^(3/2)/(a + a/cos(c + d*x))^(5/2),x)
```

output
```
int(cos(c + d*x)^(3/2)/(a + a/cos(c + d*x))^(5/2), x)
```

3.435 $\int \frac{\sqrt{\cos(c+dx)}}{(a+a\sec(c+dx))^{5/2}}\,dx$

3.435.1 Optimal result . . . 3230
3.435.2 Mathematica [A] (verified) . . . 3231
3.435.3 Rubi [A] (verified) . . . 3231
3.435.4 Maple [A] (verified) . . . 3235
3.435.5 Fricas [A] (verification not implemented) . . . 3235
3.435.6 Sympy [F(-1)] . . . 3236
3.435.7 Maxima [B] (verification not implemented) . . . 3236
3.435.8 Giac [F] . . . 3237
3.435.9 Mupad [F(-1)] . . . 3238

3.435.1 Optimal result

Integrand size = 25, antiderivative size = 197

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+a\sec(c+dx))^{5/2}}\,dx = -\frac{75\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{16\sqrt{2}a^{5/2}d} - \frac{\sin(c+dx)}{4d\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^{5/2}} - \frac{13\sin(c+dx)}{16ad\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^{3/2}} + \frac{49\sin(c+dx)}{16a^2d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}}$$

output
```
-1/4*sin(d*x+c)/d/(a+a*sec(d*x+c))^(5/2)/cos(d*x+c)^(1/2)-13/16*sin(d*x+c)
/a/d/(a+a*sec(d*x+c))^(3/2)/cos(d*x+c)^(1/2)-75/32*arctanh(1/2*sin(d*x+c)*
a^(1/2)*sec(d*x+c)^(1/2)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*
sec(d*x+c)^(1/2)/a^(5/2)/d*2^(1/2)+49/16*sin(d*x+c)/a^2/d/cos(d*x+c)^(1/2)
/(a+a*sec(d*x+c))^(1/2)
```

3.435.2 Mathematica [A] (verified)

Time = 0.70 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.72

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+a\sec(c+dx))^{5/2}}\,dx = \frac{150\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)\cos^4\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)+\sqrt{1}}{16d\sqrt{-1+\cos(c+dx)}(a(1}$$

input
```
Integrate[Sqrt[Cos[c + d*x]]/(a + a*Sec[c + d*x])^(5/2),x]
```

output
```
(150*Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*C
os[(c + d*x)/2]^4*Sec[c + d*x]^(5/2)*Sin[c + d*x] + Sqrt[1 - Sec[c + d*x]]
*(32*Sin[c + d*x] + (85 + 49*Sec[c + d*x])*Tan[c + d*x]))/(16*d*Sqrt[-1 +
Cos[c + d*x]]*(a*(1 + Sec[c + d*x]))^(5/2))
```

3.435.3 Rubi [A] (verified)

Time = 1.05 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.06, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.520$, Rules used = {3042, 4752, 3042, 4304, 27, 3042, 4508, 27, 3042, 4501, 3042, 4295, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\cos(c+dx)}}{(a\sec(c+dx)+a)^{5/2}}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}{\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}}dx$$

$$\Big\downarrow \; 4752$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sec(c+dx)}(\sec(c+dx)a+a)^{5/2}}dx$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{5/2}}dx$$

$$\Big\downarrow \; 4304$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\int -\frac{9a-4a\sec(c+dx)}{2\sqrt{\sec(c+dx)}(\sec(c+dx)a+a)^{3/2}}dx}{4a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{4d(a\sec(c+dx)+a)^{5/2}}\right)$$

$\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{9a-4a\sec(c+dx)}{\sqrt{\sec(c+dx)}(\sec(c+dx)a+a)^{3/2}}dx}{8a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{4d(a\sec(c+dx)+a)^{5/2}}\right)$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{9a-4a\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}}dx}{8a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{4d(a\sec(c+dx)+a)^{5/2}}\right)$$

$\downarrow$ 4508

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\int \frac{49a^2-26a^2\sec(c+dx)}{2\sqrt{\sec(c+dx)}\sqrt{\sec(c+dx)a+a}}dx}{2a^2}-\frac{13a\sin(c+dx)\sqrt{\sec(c+dx)}}{2d(a\sec(c+dx)+a)^{3/2}}}{8a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{4d(a\sec(c+dx)+a)^{5/2}}\right)$$

$\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\int \frac{49a^2-26a^2\sec(c+dx)}{\sqrt{\sec(c+dx)}\sqrt{\sec(c+dx)a+a}}dx}{4a^2}-\frac{13a\sin(c+dx)\sqrt{\sec(c+dx)}}{2d(a\sec(c+dx)+a)^{3/2}}}{8a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{4d(a\sec(c+dx)+a)^{5/2}}\right)$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\int \frac{49a^2-26a^2\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a^2}-\frac{13a\sin(c+dx)\sqrt{\sec(c+dx)}}{2d(a\sec(c+dx)+a)^{3/2}}}{8a^2}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{4d(a\sec(c+dx)+a)^{5/2}}\right)$$

$\downarrow$ 4501

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{98a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\sec(c+dx)+a}}-75a^2\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{\sec(c+dx)a+a}}dx}{4a^2}-\frac{13a\sin(c+dx)\sqrt{\sec(c+dx)}}{2d(a\sec(c+dx)+a)^{3/2}}}{8a^2}-\frac{\sin(c+dx)\sqrt{\sec}}{4d(a\sec(c+dx}\right.$$

$\downarrow$ 3042

3.435. $\int \frac{\sqrt{\cos(c+dx)}}{(a+a\sec(c+dx))^{5/2}}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{98a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\sec(c+dx)+a}}-75a^2\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a^2}-\frac{13a\sin(c+dx)\sqrt{\sec(c+dx)}}{2d(a\sec(c+dx)+a)^{3/2}}}{8a^2}-\frac{\sin(c+dx)\sqrt{}}{4d(a\sec(c+}\right.$$

$\downarrow$ 4295

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{150a^2\int\frac{1}{2a-\frac{a^2\sin(c+dx)\tan(c+dx)}{\sec(c+dx)a+a}}d\left(-\frac{a\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}+\frac{98a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\sec(c+dx)+a}}}{4a^2}-\frac{13a\sin(c+dx)\sqrt{}}{2d(a\sec(c+d}}{8a^2}\right.$$

$\downarrow$ 219

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{98a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\sec(c+dx)+a}}-\frac{75\sqrt{2}a^{3/2}\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{d}}{4a^2}-\frac{13a\sin(c+dx)\sqrt{\sec(c+dx)}}{2d(a\sec(c+dx)+a)^{3/2}}}{8a^2}-\right.$$

input
```
Int[Sqrt[Cos[c + d*x]]/(a + a*Sec[c + d*x])^(5/2),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(-1/4*(Sqrt[Sec[c + d*x]]*Sin[c + d*
x])/(d*(a + a*Sec[c + d*x])^(5/2)) + ((-13*a*Sqrt[Sec[c + d*x]]*Sin[c + d*
x])/(2*d*(a + a*Sec[c + d*x])^(3/2)) + ((-75*Sqrt[2]*a^(3/2)*ArcTanh[(Sqrt
[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/
d + (98*a^2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]]))
/(4*a^2))/(8*a^2))
```

3.435. $\int \frac{\sqrt{\cos(c+dx)}}{(a+a\sec(c+dx))^{5/2}}\,dx$

3.435.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4295
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[-2*b*(d/(a*f))   Subst[Int[1/(2*b - d*x^2), x],
x, b*(Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]]))], x] /;
 FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4304
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-Cot[e + f*x])*(a + b*Csc[e + f*x])^m*((d*Csc
[e + f*x])^n/(f*(2*m + 1))), x] + Simp[1/(a^2*(2*m + 1))   Int[(a + b*Csc[e
 + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e
+ f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ
[m, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

rule 4501
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*Cot[e
 + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(f*n)), x] - Simp[(a*A*m
 - b*B*n)/(b*d*n)   Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x]
, x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a
^2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LeQ[m, -1]
```

3.435. $\int \frac{\sqrt{\cos(c+dx)}}{(a+a\sec(c+dx))^{5/2}}\,dx$

rule 4508
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
- a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
 1))), x] - Simp[1/(a^2*(2*m + 1))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Cs
c[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[
e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B
, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

3.435.4 Maple [A] (verified)

Time = 1.65 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.13

method	result
default	$-\frac{\sqrt{-\frac{(1-\cos(dx+c))^2\csc(dx+c)^2-1}{(1-\cos(dx+c))^2\csc(dx+c)^2+1}}\sqrt{-\frac{2a}{(1-\cos(dx+c))^2\csc(dx+c)^2-1}}\left(2(1-\cos(dx+c))^5\csc(dx+c)^5-17(1-\cos(dx+c))^3\csc(dx+c)^3\right.}{32da^3}$

input
```
int(cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/32/d/a^3*(-((1-cos(d*x+c))^2*csc(d*x+c)^2-1)/((1-cos(d*x+c))^2*csc(d*x+
c)^2+1))^(1/2)*(-2*a/((1-cos(d*x+c))^2*csc(d*x+c)^2-1))^(1/2)*(2*(1-cos(d*
x+c))^5*csc(d*x+c)^5-17*(1-cos(d*x+c))^3*csc(d*x+c)^3+75*arctan(1/(-(1-cos
(d*x+c))^2*csc(d*x+c)^2-1)^(1/2)*(-cot(d*x+c)+csc(d*x+c)))*(-(1-cos(d*x+c)
)^2*csc(d*x+c)^2-1)^(1/2)-83*csc(d*x+c)+83*cot(d*x+c))
```

3.435.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 428, normalized size of antiderivative = 2.17

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+a\sec(c+dx))^{5/2}}\,dx = \left[\frac{75\sqrt{2}\left(\cos(dx+c)^3+3\cos(dx+c)^2+3\cos(dx+c)+1\right)\sqrt{a}\log\left(-\frac{a\cos(}{}\right.}{6}\right.$$

input
```
integrate(cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
[1/64*(75*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)
*sqrt(a)*log(-(a*cos(d*x + c)^2 + 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) +
 a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a
)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + 4*(32*cos(d*x + c)^2 + 85*cos(d
*x + c) + 49)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*s
in(d*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(
d*x + c) + a^3*d), 1/32*(75*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3
*cos(d*x + c) + 1)*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) +
 a)/cos(d*x + c))*sqrt(cos(d*x + c))/(a*sin(d*x + c))) + 2*(32*cos(d*x + c
)^2 + 85*cos(d*x + c) + 49)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(c
os(d*x + c))*sin(d*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2
+ 3*a^3*d*cos(d*x + c) + a^3*d)]
```

3.435.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+a\sec(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(1/2)/(a+a*sec(d*x+c))**(5/2),x)
```

output
```
Timed out
```

3.435.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 258456 vs. $2(162) = 324$.

Time = 2.75 (sec) , antiderivative size = 258456, normalized size of antiderivative = 1311.96

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+a\sec(c+dx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
-1/32*(576*(75*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin
(1/2*d*x + 1/2*c) + 1) - 75*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2
*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) - 64*sin(1/2*d*x + 1/2*c))*cos(5/2*d*x
 + 5/2*c)^6 + 14400*(75*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^
2 + 2*sin(1/2*d*x + 1/2*c) + 1) - 75*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*
d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) - 64*sin(1/2*d*x + 1/2*c))*co
s(3/2*d*x + 3/2*c)^6 + 187500*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x +
1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(
1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(1/2*d*x + 1/2*c)^6 +
 576*(75*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d
*x + 1/2*c) + 1) - 75*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2
- 2*sin(1/2*d*x + 1/2*c) + 1) - 64*sin(1/2*d*x + 1/2*c))*sin(5/2*d*x + 5/2
*c)^6 + 5184*(75*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*s
in(1/2*d*x + 1/2*c) + 1) - 75*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1
/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) - 64*sin(1/2*d*x + 1/2*c))*sin(3/2*d
*x + 3/2*c)^6 + 262500*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^
2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x
 + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(1/2*d*x + 1/2*c)^4*sin(1/2*
d*x + 1/2*c)^2 + 77700*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^
2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*...
```

3.435.8 Giac [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+a\sec(c+dx))^{5/2}}\,dx = \int \frac{\sqrt{\cos{(dx+c)}}}{(a\sec{(dx+c)}+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate(sqrt(cos(d*x + c))/(a*sec(d*x + c) + a)^(5/2), x)
```

3.435.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+a\sec(c+dx))^{5/2}}\,dx=\int \frac{\sqrt{\cos\left(c+d\,x\right)}}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

input
```
int(cos(c + d*x)^(1/2)/(a + a/cos(c + d*x))^(5/2),x)
```

output
```
int(cos(c + d*x)^(1/2)/(a + a/cos(c + d*x))^(5/2), x)
```

3.436 $\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^{5/2}}\,dx$

3.436.1 Optimal result . . . 3239
3.436.2 Mathematica [A] (verified) . . . 3239
3.436.3 Rubi [A] (verified) . . . 3240
3.436.4 Maple [B] (verified) . . . 3243
3.436.5 Fricas [A] (verification not implemented) . . . 3243
3.436.6 Sympy [F(-1)] . . . 3244
3.436.7 Maxima [B] (verification not implemented) . . . 3244
3.436.8 Giac [F] . . . 3245
3.436.9 Mupad [F(-1)] . . . 3246

3.436.1 Optimal result

Integrand size = 25, antiderivative size = 157

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^{5/2}}\,dx = \frac{19\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{16\sqrt{2}a^{5/2}d} - \frac{\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}} - \frac{9\sin(c+dx)}{16ad\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}$$

output
```
-1/4*sin(d*x+c)/d/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(5/2)-9/16*sin(d*x+c)/
a/d/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(3/2)+19/32*arctanh(1/2*sin(d*x+c)*a
^(1/2)*sec(d*x+c)^(1/2)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*s
ec(d*x+c)^(1/2)/a^(5/2)/d*2^(1/2)
```

3.436.2 Mathematica [A] (verified)

Time = 0.93 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.07

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^{5/2}}\,dx = -\frac{\sqrt{\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)\left(9\sqrt{1-\sec(c+dx)}\sec^{\frac{3}{2}}(c+dx)+38\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)\cos^4\left(\frac{1}{2}(c+d\right.\right.}{16d\sqrt{1-\sec(c+dx)}(a(1+\sec(c+dx)}$$

input
```
Integrate[1/(Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])^(5/2)),x]
```

output
```
-1/16*(Sqrt[Cos[c + d*x]]*Sec[c + d*x]^(3/2)*(9*Sqrt[1 - Sec[c + d*x]]*Sec
[c + d*x]^(3/2) + 38*Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 -
Sec[c + d*x]]]*Cos[(c + d*x)/2]^4*Sec[c + d*x]^2 + 13*Sqrt[-((-1 + Sec[c +
 d*x])*Sec[c + d*x])])*Sin[c + d*x])/(d*Sqrt[1 - Sec[c + d*x]]*(a*(1 + Sec
[c + d*x]))^(5/2))
```

3.436.3 Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.04, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 4752, 3042, 4304, 27, 3042, 4500, 3042, 4295, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a\sec(c+dx)+a)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}}dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sqrt{\sec(c+dx)}}{(\sec(c+dx)a+a)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{5/2}}dx$$

$$\Big\downarrow \text{4304}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\int -\frac{\sqrt{\sec(c+dx)}(7a-2a\sec(c+dx))}{2(\sec(c+dx)a+a)^{3/2}}dx}{4a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}\right)$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{\sqrt{\sec(c+dx)}(7a-2a\sec(c+dx))}{(\sec(c+dx)a+a)^{3/2}}dx}{8a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(7a-2a\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}}dx}{8a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}\right)$$

$\Big\downarrow$ 4500

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{19}{4}\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{\sec(c+dx)}a+a}dx-\frac{9a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}}{8a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}\right)$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{19}{4}\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}a+a}dx-\frac{9a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}}{8a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}\right)$$

$\Big\downarrow$ 4295

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{-\frac{19\int \frac{1}{2a-\frac{a^2\sin(c+dx)\tan(c+dx)}{\sec(c+dx)a+a}}d\left(-\frac{a\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{\sec(c+dx)}a+a}\right)}{2d}-\frac{9a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}}{8a^2}-\frac{\sin(c+d}{4d(a\sec(}\right.$$

$\Big\downarrow$ 219

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{19\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}\sqrt{a}d}-\frac{9a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}}{8a^2}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}\right)$$

input
```
Int[1/(Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])^(5/2)),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(-1/4*(Sec[c + d*x]^(3/2)*Sin[c + d*
x])/(d*(a + a*Sec[c + d*x])^(5/2)) + ((19*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*
x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/(2*Sqrt[2]*Sqrt[a]*
d) - (9*a*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(2*d*(a + a*Sec[c + d*x])^(3/2)
))/(8*a^2))
```

3.436. $\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^{5/2}}\,dx$

3.436.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4295
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[-2*b*(d/(a*f))   Subst[Int[1/(2*b - d*x^2), x],
x, b*(Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]]))], x] /;
 FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4304
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-Cot[e + f*x])*(a + b*Csc[e + f*x])^m*((d*Csc
[e + f*x])^n/(f*(2*m + 1))), x] + Simp[1/(a^2*(2*m + 1))   Int[(a + b*Csc[e
 + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e
+ f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ
[m, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

rule 4500
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
- a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
 1))), x] + Simp[(a*A*m + b*B*(m + 1))/(a^2*(2*m + 1))   Int[(a + b*Csc[e +
 f*x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, n}
, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && LeQ[
m, -1]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

3.436. $\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^{5/2}}\,dx$

3.436.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 276 vs. $2(128) = 256$.

Time = 1.56 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.76

method	result
default	$\frac{\left((1-\cos(dx+c))^2\csc(dx+c)^2-1\right)\sqrt{-\frac{2a}{(1-\cos(dx+c))^2\csc(dx+c)^2-1}}\left(-2\sqrt{-(1-\cos(dx+c))^2\csc(dx+c)^2-1}\,(1-\cos(dx+c))^3\csc(dx+\right.}{32d\,a^3\sqrt{-\frac{(1-\cos(dx+c))^2\csc(dx+c)^2-1}{(1-\cos(dx+c))^2\csc(dx+c)^2+1}}\sqrt{-(}}$

input
```
int(1/(a+a*sec(d*x+c))^(5/2)/cos(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/32/d/a^3*((1-cos(d*x+c))^2*csc(d*x+c)^2-1)*(-2*a/((1-cos(d*x+c))^2*csc(d
*x+c)^2-1))^(1/2)*(-2*(-(1-cos(d*x+c))^2*csc(d*x+c)^2-1)^(1/2)*(1-cos(d*x+
c))^3*csc(d*x+c)^3+11*(-(1-cos(d*x+c))^2*csc(d*x+c)^2-1)^(1/2)*(-cot(d*x+c
)+csc(d*x+c))+19*arctan(1/(-(1-cos(d*x+c))^2*csc(d*x+c)^2-1)^(1/2)*(-cot(d
*x+c)+csc(d*x+c))))/(-((1-cos(d*x+c))^2*csc(d*x+c)^2-1)/((1-cos(d*x+c))^2*
csc(d*x+c)^2+1))^(1/2)/(-(1-cos(d*x+c))^2*csc(d*x+c)^2-1)^(1/2)
```

3.436.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 408, normalized size of antiderivative = 2.60

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^{5/2}}\,dx = \left[\frac{19\sqrt{2}\left(\cos(dx+c)^3+3\cos(dx+c)^2+3\cos(dx+c)+1\right)\sqrt{}}{}\right.$$

$$-\frac{19\sqrt{2}\left(\cos(dx+c)^3+3\cos(dx+c)^2+3\cos(dx+c)+1\right)\sqrt{-a}\arctan\left(\frac{\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}}{a\sin(dx+c)}\right)+}{32\left(a^3d\cos(dx+c)^3+3a^3d\cos(dx+c)^2+3a^3d\cos(d\right.}$$

input
```
integrate(1/(a+a*sec(d*x+c))^(5/2)/cos(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
[1/64*(19*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)
*sqrt(a)*log(-(a*cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) +
 a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a
)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) - 4*sqrt((a*cos(d*x + c) + a)/cos
(d*x + c))*(13*cos(d*x + c) + 9)*sqrt(cos(d*x + c))*sin(d*x + c))/(a^3*d*c
os(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d), -1
/32*(19*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*s
qrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sq
rt(cos(d*x + c))/(a*sin(d*x + c))) + 2*sqrt((a*cos(d*x + c) + a)/cos(d*x +
 c))*(13*cos(d*x + c) + 9)*sqrt(cos(d*x + c))*sin(d*x + c))/(a^3*d*cos(d*x
 + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)]
```

3.436.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+a*sec(d*x+c))**(5/2)/cos(d*x+c)**(1/2),x)
```

output
```
Timed out
```

3.436.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3049 vs. $2(128) = 256$.

Time = 0.64 (sec) , antiderivative size = 3049, normalized size of antiderivative = 19.42

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+a*sec(d*x+c))^(5/2)/cos(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
1/32*(19*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*
d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 -
2*sin(1/2*d*x + 1/2*c) + 1))*cos(4*d*x + 4*c)^2 + 304*(log(cos(1/2*d*x + 1
/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1
/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*
cos(3*d*x + 3*c)^2 + 684*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c
)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d
*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(2*d*x + 2*c)^2 + 304*(log
(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)
+ 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x
 + 1/2*c) + 1))*cos(d*x + c)^2 + 19*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*
d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2
+ sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(4*d*x + 4*c)^2
 + 304*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*
x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*
sin(1/2*d*x + 1/2*c) + 1))*sin(3*d*x + 3*c)^2 + 684*(log(cos(1/2*d*x + 1/2
*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2
*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*si
n(2*d*x + 2*c)^2 + 304*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^
2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*...
```

3.436.8 Giac [F]

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^{5/2}}\,dx = \int \frac{1}{(a\sec(dx+c)+a)^{\frac{5}{2}}\sqrt{\cos(dx+c)}}\,dx$$

input
```
integrate(1/(a+a*sec(d*x+c))^(5/2)/cos(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/((a*sec(d*x + c) + a)^(5/2)*sqrt(cos(d*x + c))), x)
```

3.436.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^{5/2}}\,dx=\int \frac{1}{\sqrt{\cos\left(c+d\,x\right)}\,\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)^{5/2}}\,dx$$

input
```
int(1/(cos(c + d*x)^(1/2)*(a + a/cos(c + d*x))^(5/2)),x)
```

output
```
int(1/(cos(c + d*x)^(1/2)*(a + a/cos(c + d*x))^(5/2)), x)
```

3.437 $\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}}\,dx$

3.437.1 Optimal result . . . 3247
3.437.2 Mathematica [A] (verified) . . . 3247
3.437.3 Rubi [A] (verified) . . . 3248
3.437.4 Maple [B] (verified) . . . 3251
3.437.5 Fricas [A] (verification not implemented) . . . 3251
3.437.6 Sympy [F(-1)] . . . 3252
3.437.7 Maxima [B] (verification not implemented) . . . 3252
3.437.8 Giac [F] . . . 3253
3.437.9 Mupad [F(-1)] . . . 3254

3.437.1 Optimal result

Integrand size = 25, antiderivative size = 157

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}}\,dx = \frac{5\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{16\sqrt{2}a^{5/2}d} - \frac{\sin(c+dx)}{4d\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}} + \frac{5\sin(c+dx)}{16ad\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}$$

output
```
-1/4*sin(d*x+c)/d/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(5/2)+5/16*sin(d*x+c)/
a/d/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(3/2)+5/32*arctanh(1/2*sin(d*x+c)*a^
(1/2)*sec(d*x+c)^(1/2)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*se
c(d*x+c)^(1/2)/a^(5/2)/d*2^(1/2)
```

3.437.2 Mathematica [A] (verified)

Time = 2.68 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.43

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}}\,dx = \frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-8\sec^{\frac{5}{2}}(c+dx)\sin(c+dx) - \frac{5(1+}{}\right.}{}$$

input
```
Integrate[1/(Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(5/2)),x]
```

output
```
(Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(-8*Sec[c + d*x]^(5/2)*Sin[c + d*x]
 - (5*(1 + Sec[c + d*x])*(2*Sqrt[1 - Sec[c + d*x]]*Sec[c + d*x]^(5/2)*Sin[
c + d*x] - (1 + Sec[c + d*x])*(2*ArcSin[Sqrt[1 - Sec[c + d*x]]] + 2*ArcSin
[Sqrt[Sec[c + d*x]]] - Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1
- Sec[c + d*x]]] + 2*Sqrt[-((-1 + Sec[c + d*x])*Sec[c + d*x])])*Tan[c + d*
x]))/Sqrt[1 - Sec[c + d*x]]))/(32*d*(a*(1 + Sec[c + d*x]))^(5/2))
```

3.437.3 Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.04, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {3042, 4752, 3042, 4298, 3042, 4297, 3042, 4295, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}}dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(\sec(c+dx)a+a)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{5/2}}dx$$

$$\Big\downarrow \text{4298}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{5\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(\sec(c+dx)a+a)^{3/2}}dx}{8a}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{5\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}}dx}{8a}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}\right)$$

$$\Big\downarrow \; 4297$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{5\left(\frac{\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{\sec(c+dx)}a+a}dx}{4a}+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}\right)}{8a}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}\right)$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{5\left(\frac{\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}a+a}dx}{4a}+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}\right)}{8a}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}\right)$$

$$\Big\downarrow \; 4295$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{5\left(\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}-\frac{\int \frac{1}{2a-\frac{a^2\sin(c+dx)\tan(c+dx)}{\sec(c+dx)a+a}}d\left(-\frac{a\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{2ad}\right)}{8a}-\frac{\sin(c+dx}{4d(a\sec(c}\right.$$

$$\Big\downarrow \; 219$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{5\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}\right)}{8a}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}\right)$$

input
```
Int[1/(Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(5/2)),x]
```

3.437. $\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}}\,dx$

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(-1/4*(Sec[c + d*x]^(5/2)*Sin[c + d*
x])/(d*(a + a*Sec[c + d*x])^(5/2)) + (5*(ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x
]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])]/(2*Sqrt[2]*a^(3/2)*d)
 + (Sec[c + d*x]^(3/2)*Sin[c + d*x])/(2*d*(a + a*Sec[c + d*x])^(3/2))))/(8
*a))
```

3.437.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4295
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[-2*b*(d/(a*f))   Subst[Int[1/(2*b - d*x^2), x],
x, b*(Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]]))], x] /;
 FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4297
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[b*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Cs
c[e + f*x])^(n - 1)/(a*f*(2*m + 1))), x] + Simp[d*((m + 1)/(b*(2*m + 1)))
 Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ
[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && EqQ[m + n, 0] && LtQ[m,
-2^(-1)] && IntegerQ[2*m]
```

rule 4298
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-Cot[e + f*x])*(a + b*Csc[e + f*x])^m*((d*Csc
[e + f*x])^n/(f*(2*m + 1))), x] + Simp[m/(a*(2*m + 1))   Int[(a + b*Csc[e +
 f*x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f}, x] && E
qQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && LtQ[m, -2^(-1)]
```

3.437. $\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}}\,dx$

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

3.437.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 303 vs. $2(128) = 256$.

Time = 1.58 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.94

method	result
default	$-\frac{\left((1-\cos(dx+c))^2\csc(dx+c)^2-1\right)^2\sqrt{-\frac{2a}{(1-\cos(dx+c))^2\csc(dx+c)^2-1}}\left(2\sqrt{-(1-\cos(dx+c))^2\csc(dx+c)^2-1}\,(1-\cos(dx+c))^3\csc(dx\right.}{32d\,a^3\left(-\frac{(1-\cos(dx+c))^2\csc(dx+c)^2-1}{(1-\cos(dx+c))^2\csc(dx+c)^2+1}\right)^{\frac{3}{2}}\left((1-\cos(dx+c))^2\,\mathrm{cs}\right.}$

input
```
int(1/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/32/d/a^3*((1-cos(d*x+c))^2*csc(d*x+c)^2-1)^2*(-2*a/((1-cos(d*x+c))^2*cs
c(d*x+c)^2-1))^(1/2)*(2*(-(1-cos(d*x+c))^2*csc(d*x+c)^2-1)^(1/2)*(1-cos(d*
x+c))^3*csc(d*x+c)^3-3*(-(1-cos(d*x+c))^2*csc(d*x+c)^2-1)^(1/2)*(-cot(d*x+
c)+csc(d*x+c))+5*arctan(1/(-(1-cos(d*x+c))^2*csc(d*x+c)^2-1)^(1/2)*(-cot(d
*x+c)+csc(d*x+c))))/(-((1-cos(d*x+c))^2*csc(d*x+c)^2-1)/((1-cos(d*x+c))^2*
csc(d*x+c)^2+1))^(3/2)/((1-cos(d*x+c))^2*csc(d*x+c)^2+1)/(-(1-cos(d*x+c))^
2*csc(d*x+c)^2-1)^(1/2)
```

3.437.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 408, normalized size of antiderivative = 2.60

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}}\,dx = \left[\frac{5\sqrt{2}\left(\cos(dx+c)^3+3\cos(dx+c)^2+3\cos(dx+c)+1\right)\sqrt{a}}{}\right.$$

$$-\frac{5\sqrt{2}\left(\cos(dx+c)^3+3\cos(dx+c)^2+3\cos(dx+c)+1\right)\sqrt{-a}\arctan\left(\frac{\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}}{a\sin(dx+c)}\right)-2}{32\left(a^3d\cos(dx+c)^3+3a^3d\cos(dx+c)^2+3a^3d\cos(dx\right.}$$

input
```
integrate(1/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
[1/64*(5*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*
sqrt(a)*log(-(a*cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) +
a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)
/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + 4*sqrt((a*cos(d*x + c) + a)/cos(
d*x + c))*(5*cos(d*x + c) + 1)*sqrt(cos(d*x + c))*sin(d*x + c))/(a^3*d*cos
(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d), -1/3
2*(5*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*sqrt
(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(
cos(d*x + c))/(a*sin(d*x + c))) - 2*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)
)*(5*cos(d*x + c) + 1)*sqrt(cos(d*x + c))*sin(d*x + c))/(a^3*d*cos(d*x + c
)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)]
```

3.437.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/cos(d*x+c)**(3/2)/(a+a*sec(d*x+c))**(5/2),x)
```

output
```
Timed out
```

3.437.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2875 vs. $2(128) = 256$.

Time = 0.58 (sec) , antiderivative size = 2875, normalized size of antiderivative = 18.31

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
1/32*(4*(3*sin(3/2*d*x + 3/2*c) + 5*sin(7/3*arctan2(sin(3/2*d*x + 3/2*c),
cos(3/2*d*x + 3/2*c))) - 3*sin(5/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d
*x + 3/2*c))) - 5*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*
c))))*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 40*(2
*sin(3*d*x + 3*c) + 3*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x +
3/2*c))) + 2*sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))
*cos(7/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 24*(2*sin(
3*d*x + 3*c) + 3*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c
))) + 2*sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*cos(
5/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 24*(3*sin(3/2*d
*x + 3/2*c) - 5*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)
)))*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 16*(3*s
in(3/2*d*x + 3/2*c) - 5*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x
+ 3/2*c))))*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) +
 5*(16*cos(3*d*x + 3*c)^2 + 2*(4*cos(3*d*x + 3*c) + 6*cos(4/3*arctan2(sin(
3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 4*cos(2/3*arctan2(sin(3/2*d*x +
 3/2*c), cos(3/2*d*x + 3/2*c))) + 1)*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c),
 cos(3/2*d*x + 3/2*c))) + cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*
x + 3/2*c)))^2 + 12*(4*cos(3*d*x + 3*c) + 4*cos(2/3*arctan2(sin(3/2*d*x +
3/2*c), cos(3/2*d*x + 3/2*c))) + 1)*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c...
```

3.437.8 Giac [F]

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}}\,dx = \int \frac{1}{(a\sec(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate(1/((a*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^(3/2)), x)
```

3.437.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}}\,dx=\int \frac{1}{\cos\left(c+d\,x\right)^{3/2}\,\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)^{5/2}}\,dx$$

input
```
int(1/(cos(c + d*x)^(3/2)*(a + a/cos(c + d*x))^(5/2)),x)
```

output
```
int(1/(cos(c + d*x)^(3/2)*(a + a/cos(c + d*x))^(5/2)), x)
```

3.438 $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}}\,dx$

3.438.1 Optimal result . . . 3255
3.438.2 Mathematica [B] (verified) . . . 3255
3.438.3 Rubi [A] (verified) . . . 3256
3.438.4 Maple [B] (verified) . . . 3259
3.438.5 Fricas [A] (verification not implemented) . . . 3259
3.438.6 Sympy [F(-1)] . . . 3260
3.438.7 Maxima [B] (verification not implemented) . . . 3260
3.438.8 Giac [F] . . . 3261
3.438.9 Mupad [F(-1)] . . . 3262

3.438.1 Optimal result

Integrand size = 25, antiderivative size = 157

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}}\,dx=\frac{3\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{16\sqrt{2}a^{5/2}d}+\frac{\sin(c+dx)}{4d\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}}+\frac{3\sin(c+dx)}{16ad\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}$$

output
```
1/4*sin(d*x+c)/d/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(5/2)+3/16*sin(d*x+c)/a
/d/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(3/2)+3/32*arctanh(1/2*sin(d*x+c)*a^(
1/2)*sec(d*x+c)^(1/2)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec
(d*x+c)^(1/2)/a^(5/2)/d*2^(1/2)
```

3.438.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 328 vs. 2(157) = 314.

Time = 0.86 (sec) , antiderivative size = 328, normalized size of antiderivative = 2.09

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}}\,dx=\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(6\sqrt{1-\sec(c+dx)}\sec^{\frac{3}{2}}(c+dx)\sin\right.}{}$$

input
```
Integrate[1/(Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^(5/2)),x]
```

output
```
(Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(6*Sqrt[1 - Sec[c + d*x]]*Sec[c + d
*x]^(3/2)*Sin[c + d*x] + 14*Sqrt[1 - Sec[c + d*x]]*Sec[c + d*x]^(5/2)*Sin[
c + d*x] - 3*Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c +
d*x]]]*Tan[c + d*x] - 6*Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1
 - Sec[c + d*x]]]*Sec[c + d*x]*Tan[c + d*x] - 3*Sqrt[2]*ArcTan[(Sqrt[2]*Sq
rt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*Sec[c + d*x]^2*Tan[c + d*x] + 6*
ArcSin[Sqrt[1 - Sec[c + d*x]]]*(1 + Sec[c + d*x])^2*Tan[c + d*x] + 6*ArcSi
n[Sqrt[Sec[c + d*x]]]*(1 + Sec[c + d*x])^2*Tan[c + d*x]))/(32*d*Sqrt[1 - S
ec[c + d*x]]*(a*(1 + Sec[c + d*x]))^(5/2))
```

3.438.3 Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.04, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {3042, 4752, 3042, 4297, 3042, 4297, 3042, 4295, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a\sec(c+dx)+a)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}}dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(\sec(c+dx)a+a)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{5/2}}dx$$

$$\Big\downarrow \text{4297}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{3\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(\sec(c+dx)a+a)^{3/2}}dx}{8a}+\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{3\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}}dx}{8a}+\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}\right)$$

$\Big\downarrow$ 4297

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{3\left(\frac{\int\frac{\sqrt{\sec(c+dx)}}{\sqrt{\sec(c+dx)a+a}}dx}{4a}+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}\right)}{8a}+\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}\right)$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{3\left(\frac{\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a}+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}\right)}{8a}+\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}\right)$$

$\Big\downarrow$ 4295

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{3\left(\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}-\frac{\int\frac{1}{2a-\frac{a^2\sin(c+dx)\tan(c+dx)}{\sec(c+dx)a+a}}d\left(-\frac{a\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{2ad}\right)}{8a}+\frac{\sin(c+dx}{4d(a\sec(c}\right.$$

$\Big\downarrow$ 219

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{3\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}\right)}{8a}+\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}\right)$$

3.438. $\int\frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}}dx$

input
```
Int[1/(Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^(5/2)),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((Sec[c + d*x]^(5/2)*Sin[c + d*x])/(
4*d*(a + a*Sec[c + d*x])^(5/2)) + (3*(ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*
Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])]/(2*Sqrt[2]*a^(3/2)*d) +
(Sec[c + d*x]^(3/2)*Sin[c + d*x])/(2*d*(a + a*Sec[c + d*x])^(3/2))))/(8*a)
)
```

3.438.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4295
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[-2*b*(d/(a*f))   Subst[Int[1/(2*b - d*x^2), x],
x, b*(Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]]))], x] /;
 FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4297
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[b*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Cs
c[e + f*x])^(n - 1)/(a*f*(2*m + 1))), x] + Simp[d*((m + 1)/(b*(2*m + 1)))
 Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ
[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && EqQ[m + n, 0] && LtQ[m,
-2^(-1)] && IntegerQ[2*m]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

3.438. $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}}\,dx$

3.438.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 303 vs. $2(128) = 256$.

Time = 1.66 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.94

method	result
default	$-\frac{\left((1-\cos(dx+c))^2\csc(dx+c)^2-1\right)^3\sqrt{-\frac{2a}{(1-\cos(dx+c))^2\csc(dx+c)^2-1}}\left(2\sqrt{-(1-\cos(dx+c))^2\csc(dx+c)^2-1}\,(1-\cos(dx+c))^3\csc(dx\right.}{32d\,a^3\left(-\frac{(1-\cos(dx+c))^2\csc(dx+c)^2-1}{(1-\cos(dx+c))^2\csc(dx+c)^2+1}\right)^{\frac{5}{2}}\left((1-\cos(dx+c))^2\,\mathrm{cs}\right.}$

input
```
int(1/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/32/d/a^3*((1-cos(d*x+c))^2*csc(d*x+c)^2-1)^3*(-2*a/((1-cos(d*x+c))^2*cs
c(d*x+c)^2-1))^(1/2)*(2*(-(1-cos(d*x+c))^2*csc(d*x+c)^2-1)^(1/2)*(1-cos(d*
x+c))^3*csc(d*x+c)^3+5*(-(1-cos(d*x+c))^2*csc(d*x+c)^2-1)^(1/2)*(-cot(d*x+
c)+csc(d*x+c))-3*arctan(1/(-(1-cos(d*x+c))^2*csc(d*x+c)^2-1)^(1/2)*(-cot(d
*x+c)+csc(d*x+c))))/(-((1-cos(d*x+c))^2*csc(d*x+c)^2-1)/((1-cos(d*x+c))^2*
csc(d*x+c)^2+1))^(5/2)/((1-cos(d*x+c))^2*csc(d*x+c)^2+1)^2/(-(1-cos(d*x+c)
)^2*csc(d*x+c)^2-1)^(1/2)
```

3.438.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 408, normalized size of antiderivative = 2.60

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}}\,dx = \left[\frac{3\sqrt{2}\left(\cos(dx+c)^3+3\cos(dx+c)^2+3\cos(dx+c)+1\right)\sqrt{a}}{}\right.$$

$$-\frac{3\sqrt{2}\left(\cos(dx+c)^3+3\cos(dx+c)^2+3\cos(dx+c)+1\right)\sqrt{-a}\arctan\left(\frac{\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}}{a\sin(dx+c)}\right)-2}{32\left(a^3d\cos(dx+c)^3+3a^3d\cos(dx+c)^2+3a^3d\cos(dx\right.}$$

input
```
integrate(1/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
[1/64*(3*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*
sqrt(a)*log(-(a*cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) +
a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)
/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + 4*sqrt((a*cos(d*x + c) + a)/cos(
d*x + c))*(3*cos(d*x + c) + 7)*sqrt(cos(d*x + c))*sin(d*x + c))/(a^3*d*cos
(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d), -1/3
2*(3*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*sqrt
(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(
cos(d*x + c))/(a*sin(d*x + c))) - 2*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)
)*(3*cos(d*x + c) + 7)*sqrt(cos(d*x + c))*sin(d*x + c))/(a^3*d*cos(d*x + c
)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)]
```

3.438.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/cos(d*x+c)**(5/2)/(a+a*sec(d*x+c))**(5/2),x)
```

output
```
Timed out
```

3.438.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 84332 vs. $2(128) = 256$.

Time = 14.83 (sec) , antiderivative size = 84332, normalized size of antiderivative = 537.15

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
1/32*(512*((2*sin(2*d*x + 2*c) + sin(d*x + c))*cos(5/2*d*x + 5/2*c) + cos(
5/2*d*x + 5/2*c)*sin(4*d*x + 4*c) + 2*cos(5/2*d*x + 5/2*c)*sin(3*d*x + 3*c
) + (2*cos(2*d*x + 2*c) + cos(d*x + c))*sin(5/2*d*x + 5/2*c) + cos(4*d*x +
 4*c)*sin(5/2*d*x + 5/2*c) + 2*cos(3*d*x + 3*c)*sin(5/2*d*x + 5/2*c))*cos(
5*d*x + 5*c)^2 + 2560*(5*(2*sin(2*d*x + 2*c) + sin(d*x + c))*cos(5/2*d*x +
 5/2*c) + cos(5/2*d*x + 5/2*c)*sin(5*d*x + 5*c) + 5*cos(5/2*d*x + 5/2*c)*s
in(4*d*x + 4*c) + 10*cos(5/2*d*x + 5/2*c)*sin(3*d*x + 3*c) - (10*cos(2*d*x
 + 2*c) + 5*cos(d*x + c) + 1)*sin(5/2*d*x + 5/2*c) - cos(5*d*x + 5*c)*sin(
5/2*d*x + 5/2*c) - 5*cos(4*d*x + 4*c)*sin(5/2*d*x + 5/2*c) - 10*cos(3*d*x
+ 3*c)*sin(5/2*d*x + 5/2*c))*cos(8/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2
*d*x + 5/2*c)))^2 + 10240*(5*(2*sin(2*d*x + 2*c) + sin(d*x + c))*cos(5/2*d
*x + 5/2*c) + cos(5/2*d*x + 5/2*c)*sin(5*d*x + 5*c) + 5*cos(5/2*d*x + 5/2*
c)*sin(4*d*x + 4*c) + 10*cos(5/2*d*x + 5/2*c)*sin(3*d*x + 3*c) - (10*cos(2
*d*x + 2*c) + 5*cos(d*x + c) + 1)*sin(5/2*d*x + 5/2*c) - cos(5*d*x + 5*c)*
sin(5/2*d*x + 5/2*c) - 5*cos(4*d*x + 4*c)*sin(5/2*d*x + 5/2*c) - 10*cos(3*
d*x + 3*c)*sin(5/2*d*x + 5/2*c))*cos(6/5*arctan2(sin(5/2*d*x + 5/2*c), cos
(5/2*d*x + 5/2*c)))^2 + 10240*(5*(2*sin(2*d*x + 2*c) + sin(d*x + c))*cos(5
/2*d*x + 5/2*c) + cos(5/2*d*x + 5/2*c)*sin(5*d*x + 5*c) + 5*cos(5/2*d*x +
5/2*c)*sin(4*d*x + 4*c) + 10*cos(5/2*d*x + 5/2*c)*sin(3*d*x + 3*c) - (10*c
os(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*sin(5/2*d*x + 5/2*c) - cos(5*d*x ...
```

3.438.8 Giac [F]

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}}\,dx = \int \frac{1}{(a\sec(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate(1/((a*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^(5/2)), x)
```

3.438.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}}\,dx=\int \frac{1}{\cos\left(c+d\,x\right)^{5/2}\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)^{5/2}}\,dx$$

input
```
int(1/(cos(c + d*x)^(5/2)*(a + a/cos(c + d*x))^(5/2)),x)
```

output
```
int(1/(cos(c + d*x)^(5/2)*(a + a/cos(c + d*x))^(5/2)), x)
```

3.439 $\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}}\,dx$

3.439.1 Optimal result . . . 3263
3.439.2 Mathematica [A] (verified) . . . 3264
3.439.3 Rubi [A] (verified) . . . 3264
3.439.4 Maple [B] (warning: unable to verify) . . . 3269
3.439.5 Fricas [A] (verification not implemented) . . . 3269
3.439.6 Sympy [F(-1)] . . . 3270
3.439.7 Maxima [B] (verification not implemented) . . . 3270
3.439.8 Giac [F] . . . 3271
3.439.9 Mupad [F(-1)] . . . 3272

3.439.1 Optimal result

Integrand size = 25, antiderivative size = 214

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}}\,dx = \frac{2\operatorname{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{a^{5/2}d} - \frac{43\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{16\sqrt{2}a^{5/2}d} - \frac{\sin(c+dx)}{4d\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}} - \frac{11\sin(c+dx)}{16ad\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}$$

output
```
-1/4*sin(d*x+c)/d/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(5/2)-11/16*sin(d*x+c)
/a/d/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(3/2)+2*arcsinh(a^(1/2)*tan(d*x+c)/
(a+a*sec(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^(5/2)/d-43/32*
arctanh(1/2*sin(d*x+c)*a^(1/2)*sec(d*x+c)^(1/2)*2^(1/2)/(a+a*sec(d*x+c))^(
1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^(5/2)/d*2^(1/2)
```

3.439.2 Mathematica [A] (verified)

Time = 1.03 (sec) , antiderivative size = 328, normalized size of antiderivative = 1.53

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}}\,dx = \frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-22\sqrt{1-\sec(c+dx)}\sec^{\frac{3}{2}}(c+dx)\right.}{}$$

input
```
Integrate[1/(Cos[c + d*x]^(7/2)*(a + a*Sec[c + d*x])^(5/2)),x]
```

output
```
(Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(-22*Sqrt[1 - Sec[c + d*x]]*Sec[c +
 d*x]^(3/2)*Sin[c + d*x] - 30*Sqrt[1 - Sec[c + d*x]]*Sec[c + d*x]^(5/2)*Si
n[c + d*x] + 43*Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c
 + d*x]]]*Tan[c + d*x] + 86*Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sq
rt[1 - Sec[c + d*x]]]*Sec[c + d*x]*Tan[c + d*x] + 43*Sqrt[2]*ArcTan[(Sqrt[
2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*Sec[c + d*x]^2*Tan[c + d*x]
 - 22*ArcSin[Sqrt[1 - Sec[c + d*x]]]*(1 + Sec[c + d*x])^2*Tan[c + d*x] - 8
6*ArcSin[Sqrt[Sec[c + d*x]]]*(1 + Sec[c + d*x])^2*Tan[c + d*x]))/(32*d*Sqr
t[1 - Sec[c + d*x]]*(a*(1 + Sec[c + d*x]))^(5/2))
```

3.439.3 Rubi [A] (verified)

Time = 1.19 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.97, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {3042, 4752, 3042, 4303, 27, 3042, 4507, 27, 3042, 4511, 3042, 4288, 222, 4295, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a\sec(c+dx)+a)^{5/2}}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{7/2}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}}dx$$

$$\Big\downarrow \; 4752$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(\sec(c+dx)a+a)^{5/2}}dx$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{5/2}}dx$$

$\Big\downarrow$ 4303

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\int \frac{\sec^{\frac{3}{2}}(c+dx)(3a-8a\sec(c+dx))}{2(\sec(c+dx)a+a)^{3/2}}dx}{4a^2}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}\right)$$

$\Big\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\int \frac{\sec^{\frac{3}{2}}(c+dx)(3a-8a\sec(c+dx))}{(\sec(c+dx)a+a)^{3/2}}dx}{8a^2}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}\right)$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(3a-8a\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}}dx}{8a^2}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}\right)$$

$\Big\downarrow$ 4507

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{\int \frac{\sqrt{\sec(c+dx)}\left(11a^2-32a^2\sec(c+dx)\right)}{2\sqrt{\sec(c+dx)a+a}}dx}{2a^2}+\frac{11a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}}{8a^2}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}\right.$$

$\Big\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{\int \frac{\sqrt{\sec(c+dx)}\left(11a^2-32a^2\sec(c+dx)\right)}{\sqrt{\sec(c+dx)a+a}}dx}{4a^2}+\frac{11a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}}{8a^2}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(11a^2-32a^2\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a^2}+\frac{11a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}}{8a^2}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c}{4d(a\sec(c+dx)+}\right.$$

$\Big\downarrow$ 4511

3.439. $\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{43a^2\int\frac{\sqrt{\sec(c+dx)}}{\sqrt{\sec(c+dx)a+a}}dx-32a\int\sqrt{\sec(c+dx)}\sqrt{\sec(c+dx)a+a}dx}{4a^2}+\frac{11a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}}{8a^2}-\frac{\sin(}{4d(}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{43a^2\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx-32a\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{4a^2}+\frac{11a\sin(c+dx)\sec^{\frac{3}{2}}(c+d}{2d(a\sec(c+dx)+a)^{3/2}}}{8a^2}\right.$$

$\downarrow$ 4288

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{43a^2\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx+\frac{64a\int\frac{1}{\sqrt{\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1}}d\left(-\frac{a\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}}{4a^2}+\frac{11a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx}{2d(a\sec(c+dx)+a)^{3/2}}}{8a^2}\right.$$

$\downarrow$ 222

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{43a^2\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx-\frac{64a^{3/2}\text{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}}{4a^2}+\frac{11a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}}{8a^2}-\frac{\sin}{4d(}\right.$$

$\downarrow$ 4295

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{-\frac{86a^2\int\frac{1}{2a-\frac{a^2\sin(c+dx)\tan(c+dx)}{\sec(c+dx)a+a}}d\left(-\frac{a\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}-\frac{64a^{3/2}\text{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}}{4a^2}+\frac{11}{}}{8a^2}\right.$$

$\downarrow$ 219

3.439. $\int\frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{\frac{43\sqrt{2}a^{3/2}\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{d}-\frac{64a^{3/2}\text{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}}{4a^2}+\frac{11a\sin(c+dx)\sec^{\frac{3}{2}}}{2d(a\sec(c+dx)+}}{8a^2}\right.$$

input
```
Int[1/(Cos[c + d*x]^(7/2)*(a + a*Sec[c + d*x])^(5/2)),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(-1/4*(Sec[c + d*x]^(5/2)*Sin[c + d*
x])/(d*(a + a*Sec[c + d*x])^(5/2)) - (((-64*a^(3/2)*ArcSinh[(Sqrt[a]*Tan[c
 + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d + (43*Sqrt[2]*a^(3/2)*ArcTanh[(Sqrt[
a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/d
)/(4*a^2) + (11*a*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(2*d*(a + a*Sec[c + d*x
])^(3/2)))/(8*a^2))
```

3.439.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4288
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[-2*(a/(b*f))*Sqrt[a*(d/b)]   Subst[Int[1/Sqrt[1
+ x^2/a], x], x, b*(Cot[e + f*x]/Sqrt[a + b*Csc[e + f*x]])], x] /; FreeQ[{a
, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[a*(d/b), 0]
```

3.439. $\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}}\,dx$

rule 4295
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[-2*b*(d/(a*f))   Subst[Int[1/(2*b - d*x^2), x],
x, b*(Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]]))], x] /;
 FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4303
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-d^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d
*Csc[e + f*x])^(n - 2)/(f*(2*m + 1))), x] + Simp[d^2/(a*b*(2*m + 1))   Int[
(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n
 + 2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2,
0] && LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

rule 4507
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[d*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(a*f*(
2*m + 1))), x] - Simp[1/(a*b*(2*m + 1))   Int[(a + b*Csc[e + f*x])^(m + 1)*
(d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
 - n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f,
A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && G
tQ[n, 0]
```

rule 4511
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b -
a*B)/b   Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n, x], x] + Simp[B/b
 Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b
, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

3.439. $\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}}\,dx$

3.439.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 409 vs. $2(175) = 350$.

Time = 2.46 (sec) , antiderivative size = 410, normalized size of antiderivative = 1.92

method	result
default	$-\frac{\left((1-\cos(dx+c))^2\csc(dx+c)^2-1\right)^4\sqrt{-\frac{2a}{(1-\cos(dx+c))^2\csc(dx+c)^2-1}}\left(2\sqrt{-(1-\cos(dx+c))^2\csc(dx+c)^2-1}\,(1-\cos(dx+c))^3\csc(dx\right.}{32d\,a^3\left(-\right.}$

input
```
int(1/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/32/d/a^3*((1-cos(d*x+c))^2*csc(d*x+c)^2-1)^4*(-2*a/((1-cos(d*x+c))^2*cs
c(d*x+c)^2-1))^(1/2)*(2*(-(1-cos(d*x+c))^2*csc(d*x+c)^2-1)^(1/2)*(1-cos(d*
x+c))^3*csc(d*x+c)^3+16*2^(1/2)*arctan(1/2*(csc(d*x+c)-cot(d*x+c)+1)*2^(1/
2)/(-(1-cos(d*x+c))^2*csc(d*x+c)^2-1)^(1/2))+16*2^(1/2)*arctan(1/2*(-cot(d
*x+c)+csc(d*x+c)-1)*2^(1/2)/(-(1-cos(d*x+c))^2*csc(d*x+c)^2-1)^(1/2))+13*(
-(1-cos(d*x+c))^2*csc(d*x+c)^2-1)^(1/2)*(-cot(d*x+c)+csc(d*x+c))-43*arctan
(1/(-(1-cos(d*x+c))^2*csc(d*x+c)^2-1)^(1/2)*(-cot(d*x+c)+csc(d*x+c))))/(-(
(1-cos(d*x+c))^2*csc(d*x+c)^2-1)/((1-cos(d*x+c))^2*csc(d*x+c)^2+1))^(7/2)/
((1-cos(d*x+c))^2*csc(d*x+c)^2+1)^3/(-(1-cos(d*x+c))^2*csc(d*x+c)^2-1)^(1/
2)
```

3.439.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 638, normalized size of antiderivative = 2.98

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}}\,dx = \left[\frac{43\sqrt{2}\left(\cos(dx+c)^3+3\cos(dx+c)^2+3\cos(dx+c)+1\right)\sqrt{c}}{}\right.$$

input
```
integrate(1/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
[1/64*(43*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)
*sqrt(a)*log(-(a*cos(d*x + c)^2 + 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) +
 a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a
)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) - 4*sqrt((a*cos(d*x + c) + a)/cos
(d*x + c))*(11*cos(d*x + c) + 15)*sqrt(cos(d*x + c))*sin(d*x + c) + 32*(co
s(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*sqrt(a)*log((a*cos(d
*x + c)^3 - 4*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(cos(d*x + c
) - 2)*sqrt(cos(d*x + c))*sin(d*x + c) - 7*a*cos(d*x + c)^2 + 8*a)/(cos(d*
x + c)^3 + cos(d*x + c)^2)))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^
2 + 3*a^3*d*cos(d*x + c) + a^3*d), 1/32*(43*sqrt(2)*(cos(d*x + c)^3 + 3*co
s(d*x + c)^2 + 3*cos(d*x + c) + 1)*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt((
a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))/(a*sin(d*x + c))) - 2
*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(11*cos(d*x + c) + 15)*sqrt(cos(d
*x + c))*sin(d*x + c) + 32*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x
+ c) + 1)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c
))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*
a)))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c)
 + a^3*d)]
```

3.439.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/cos(d*x+c)**(7/2)/(a+a*sec(d*x+c))**(5/2),x)
```

output
```
Timed out
```

3.439.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4988 vs. $2(175) = 350$.

Time = 0.61 (sec) , antiderivative size = 4988, normalized size of antiderivative = 23.31

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
1/32*(44*(sin(4*d*x + 4*c) + 6*sin(2*d*x + 2*c) + 4*sin(3/2*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c))) + 4*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c))))*cos(7/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 16*(
19*sin(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 19*sin(3/4*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 11*sin(1/4*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c))))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
))) + 76*(sin(4*d*x + 4*c) + 6*sin(2*d*x + 2*c) + 4*sin(1/2*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c))))*cos(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c))) - 76*(sin(4*d*x + 4*c) + 6*sin(2*d*x + 2*c) + 4*sin(1/2*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(3/4*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c))) - 44*(sin(4*d*x + 4*c) + 6*sin(2*d*x + 2*c))*cos(1/4*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 16*(sqrt(2)*cos(4*d*x + 4*c)^2
 + 36*sqrt(2)*cos(2*d*x + 2*c)^2 + 16*sqrt(2)*cos(3/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c)))^2 + 16*sqrt(2)*cos(1/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c)))^2 + sqrt(2)*sin(4*d*x + 4*c)^2 + 12*sqrt(2)*sin(4*d*x +
 4*c)*sin(2*d*x + 2*c) + 36*sqrt(2)*sin(2*d*x + 2*c)^2 + 16*sqrt(2)*sin(3/
2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 16*sqrt(2)*sin(1/2*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*(6*sqrt(2)*cos(2*d*x + 2*c)
 + sqrt(2))*cos(4*d*x + 4*c) + 8*(sqrt(2)*cos(4*d*x + 4*c) + 6*sqrt(2)*cos
(2*d*x + 2*c) + 4*sqrt(2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +...
```

3.439.8 Giac [F]

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}}\,dx = \int \frac{1}{(a\sec(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate(1/((a*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^(7/2)), x)
```

3.439.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}}\,dx=\int \frac{1}{\cos\left(c+d\,x\right)^{7/2}\,\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)^{5/2}}\,dx$$

input
```
int(1/(cos(c + d*x)^(7/2)*(a + a/cos(c + d*x))^(5/2)),x)
```

output
```
int(1/(cos(c + d*x)^(7/2)*(a + a/cos(c + d*x))^(5/2)), x)
```

3.440 $\int \frac{1}{\cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}}\,dx$

3.440.1 Optimal result . . . 3273
3.440.2 Mathematica [A] (verified) . . . 3274
3.440.3 Rubi [A] (verified) . . . 3274
3.440.4 Maple [B] (warning: unable to verify) . . . 3281
3.440.5 Fricas [A] (verification not implemented) . . . 3282
3.440.6 Sympy [F(-1)] . . . 3283
3.440.7 Maxima [B] (verification not implemented) . . . 3283
3.440.8 Giac [F] . . . 3284
3.440.9 Mupad [F(-1)] . . . 3284

3.440.1 Optimal result

Integrand size = 25, antiderivative size = 254

$$\begin{aligned}\int \frac{1}{\cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}}\,dx &= \\ &-\frac{5\operatorname{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{a^{5/2}d} \\ &+\frac{115\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{16\sqrt{2}a^{5/2}d} \\ &-\frac{\sin(c+dx)}{4d\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}}-\frac{15\sin(c+dx)}{16ad\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}} \\ &+\frac{35\sin(c+dx)}{16a^2d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\end{aligned}$$

output
```
-1/4*sin(d*x+c)/d/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c))^(5/2)-15/16*sin(d*x+c)
/a/d/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(3/2)-5*arcsinh(a^(1/2)*tan(d*x+c)/
(a+a*sec(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^(5/2)/d+115/32
*arctanh(1/2*sin(d*x+c)*a^(1/2)*sec(d*x+c)^(1/2)*2^(1/2)/(a+a*sec(d*x+c))^
(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^(5/2)/d*2^(1/2)+35/16*sin(d*x+c
)/a^2/d/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2)
```

3.440.2 Mathematica [A] (verified)

Time = 1.19 (sec) , antiderivative size = 360, normalized size of antiderivative = 1.42

$$\int \frac{1}{\cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}}\,dx = \frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(70\sqrt{1-\sec(c+dx)}\sec^{\frac{3}{2}}(c+dx)\,\mathrm{s}\right.}{}$$

input
```
Integrate[1/(Cos[c + d*x]^(9/2)*(a + a*Sec[c + d*x])^(5/2)),x]
```

output
```
(Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(70*Sqrt[1 - Sec[c + d*x]]*Sec[c +
d*x]^(3/2)*Sin[c + d*x] + 110*Sqrt[1 - Sec[c + d*x]]*Sec[c + d*x]^(5/2)*Si
n[c + d*x] + 32*Sqrt[1 - Sec[c + d*x]]*Sec[c + d*x]^(7/2)*Sin[c + d*x] - 1
15*Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*Tan
[c + d*x] - 230*Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c
 + d*x]]]*Sec[c + d*x]*Tan[c + d*x] - 115*Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[Sec
[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*Sec[c + d*x]^2*Tan[c + d*x] + 70*ArcSi
n[Sqrt[1 - Sec[c + d*x]]]*(1 + Sec[c + d*x])^2*Tan[c + d*x] + 230*ArcSin[S
qrt[Sec[c + d*x]]]*(1 + Sec[c + d*x])^2*Tan[c + d*x]))/(32*d*Sqrt[1 - Sec[
c + d*x]]*(a*(1 + Sec[c + d*x]))^(5/2))
```

3.440.3 Rubi [A] (verified)

Time = 1.53 (sec) , antiderivative size = 251, normalized size of antiderivative = 0.99, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.720$, Rules used = {3042, 4752, 3042, 4303, 27, 3042, 4507, 27, 3042, 4509, 25, 3042, 4511, 3042, 4288, 222, 4295, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\cos^{\frac{9}{2}}(c+dx)(a\sec(c+dx)+a)^{5/2}}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{9/2}\left(a\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)^{5/2}}dx$$

$$\Big\downarrow \; 4752$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(\sec(c+dx)a+a)^{5/2}}dx$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{9/2}}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{5/2}}dx$$

$$\Big\downarrow \; 4303$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\int \frac{5\sec^{\frac{5}{2}}(c+dx)(a-2a\sec(c+dx))}{2(\sec(c+dx)a+a)^{3/2}}dx}{4a^2}-\frac{\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}\right)$$

$$\Big\downarrow \; 27$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{5\int \frac{\sec^{\frac{5}{2}}(c+dx)(a-2a\sec(c+dx))}{(\sec(c+dx)a+a)^{3/2}}dx}{8a^2}-\frac{\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}\right)$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{5\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a-2a\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(\csc\left(c+dx+\frac{\pi}{2}\right)a+a\right)^{3/2}}dx}{8a^2}-\frac{\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}\right)$$

$$\Big\downarrow \; 4507$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{5\left(\frac{\int \frac{\sec^{\frac{3}{2}}(c+dx)\left(9a^2-14a^2\sec(c+dx)\right)}{2\sqrt{\sec(c+dx)a+a}}dx}{2a^2}+\frac{3a\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}\right)}{8a^2}-\frac{\sin(c+dx)\sec^{\frac{7}{2}}(c+\cdots}{4d(a\sec(c+dx)+a)\cdots}\right.$$

$$\Big\downarrow \; 27$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{5\left(\frac{\int \frac{\sec^{\frac{3}{2}}(c+dx)\left(9a^2-14a^2\sec(c+dx)\right)}{\sqrt{\sec(c+dx)a+a}}dx}{4a^2}+\frac{3a\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}\right)}{8a^2}-\frac{\sin(c+dx)\sec^{\frac{7}{2}}(c+\cdots}{4d(a\sec(c+dx)+a)\cdots}\right.$$

$$\Big\downarrow \; 3042$$

3.440. $\int \frac{1}{\cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{5\left(\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(9a^2-14a^2\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{4a^2}+\frac{3a\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}\right)}{8a^2}-\frac{\sin(c+dx)\sec}{4d(a\sec(c+a}\right.$$

$\downarrow$ 4509

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{5\left(\frac{\int -\frac{\sqrt{\sec(c+dx)}\left(7a^3-16a^3\sec(c+dx)\right)}{\sqrt{\sec(c+dx)a+a}}dx}{a}-\frac{14a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}}{4a^2}+\frac{3a\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}\right)}{8a^2}-\right.$$

$\downarrow$ 25

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{5\left(\frac{-\int \frac{\sqrt{\sec(c+dx)}\left(7a^3-16a^3\sec(c+dx)\right)}{\sqrt{\sec(c+dx)a+a}}dx}{a}-\frac{14a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}}{4a^2}+\frac{3a\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}\right)}{8a^2}-\right.$$

$\downarrow$ 3042

3.440. $\int \frac{1}{\cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{5\left(\frac{-\frac{\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(7a^3-16a^3\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx}{a}-\frac{14a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}}{4a^2}+\frac{3a\sin(c+dx)\sec^{\frac{5}{2}}(c+d}{2d(a\sec(c+dx)+a)^{3/}}\right.}{8a^2}\right.$$

$\downarrow$ 4511

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{5\left(\frac{-\frac{23a^3\int\frac{\sqrt{\sec(c+dx)}}{\sqrt{\sec(c+dx)a+a}}dx-16a^2\int\sqrt{\sec(c+dx)}\sqrt{\sec(c+dx)a+a}dx}{a}-\frac{14a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}}{4a^2}+\frac{3a\sin(c+}{2d(a\,se}\right.}{8a^2}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{5\left(\frac{-\frac{23a^3\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx-16a^2\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}dx}{a}-\frac{14a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}}{4a^2}\right.}{8a^2}\right.$$

$\downarrow$ 4288

3.440. $\int \frac{1}{\cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{5\left(\frac{-\frac{23a^3\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx+\frac{32a^2\int\frac{1}{\sqrt{\frac{a\tan^2(c+dx)}{\sec(c+dx)a+a}+1}}d\left(-\frac{a\tan(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}}{a}-\frac{14a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+}{d\sqrt{a\sec(c+dx)+a}}}{4a^2}\right.}{8a^2}\right.$$

$\downarrow$ 222

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{5\left(\frac{-\frac{23a^3\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)a+a}}dx-\frac{32a^{5/2}\text{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}}{a}-\frac{14a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}}{4a^2}+\frac{3a}{2}\right.}{8a^2}\right.$$

$\downarrow$ 4295

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{5\left(\frac{-\frac{-\frac{46a^3\int\frac{1}{2a-\frac{a^2\sin(c+dx)\tan(c+dx)}{\sec(c+dx)a+a}}d\left(-\frac{a\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{\sec(c+dx)a+a}}\right)}{d}}{a}-\frac{32a^{5/2}\text{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}}{}}{4a^2}\right.}{8a^2}\right.$$

$\downarrow$ 219

3.440. $\int \frac{1}{\cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{5\left(\frac{-\frac{\frac{23\sqrt{2}a^{5/2}\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{d}-\frac{32a^{5/2}\text{arcsinh}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}}{a}-\frac{14a^2\sin(c+dx)}{d\sqrt{a\sec(c+}}}{4a^2}\right.}{8a^2}\right.$$

input
```
Int[1/(Cos[c + d*x]^(9/2)*(a + a*Sec[c + d*x])^(5/2)),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(-1/4*(Sec[c + d*x]^(7/2)*Sin[c + d*
x])/(d*(a + a*Sec[c + d*x])^(5/2)) - (5*((3*a*Sec[c + d*x]^(5/2)*Sin[c + d
*x])/(2*d*(a + a*Sec[c + d*x])^(3/2)) + (-(((-32*a^(5/2)*ArcSinh[(Sqrt[a]*
Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d + (23*Sqrt[2]*a^(5/2)*ArcTanh[(
Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]]
)])/d)/a) - (14*a^2*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(d*Sqrt[a + a*Sec[c +
 d*x]]))/(4*a^2)))/(8*a^2))
```

3.440.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4288
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[-2*(a/(b*f))*Sqrt[a*(d/b)]   Subst[Int[1/Sqrt[1
+ x^2/a], x], x, b*(Cot[e + f*x]/Sqrt[a + b*Csc[e + f*x]])], x] /; FreeQ[{a
, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[a*(d/b), 0]
```

rule 4295
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[-2*b*(d/(a*f))   Subst[Int[1/(2*b - d*x^2), x],
x, b*(Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]]))], x] /;
 FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4303
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-d^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d
*Csc[e + f*x])^(n - 2)/(f*(2*m + 1))), x] + Simp[d^2/(a*b*(2*m + 1))   Int[
(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n
 + 2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2,
0] && LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

rule 4507
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[d*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(a*f*(
2*m + 1))), x] - Simp[1/(a*b*(2*m + 1))   Int[(a + b*Csc[e + f*x])^(m + 1)*
(d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
 - n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f,
A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && G
tQ[n, 0]
```

rule 4509
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*d*
Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(f*(m + n))),
 x] + Simp[d/(b*(m + n))   Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n -
 1)*Simp[b*B*(n - 1) + (A*b*(m + n) + a*B*m)*Csc[e + f*x], x], x], x] /; Fr
eeQ[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
&& GtQ[n, 1]
```

3.440. $\int \frac{1}{\cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}}\,dx$

rule 4511
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b -
a*B)/b   Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n, x], x] + Simp[B/b
 Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b
, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

3.440.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 668 vs. $2(209) = 418$.

Time = 2.51 (sec) , antiderivative size = 669, normalized size of antiderivative = 2.63

method	result
default	$-\frac{\left((1-\cos(dx+c))^2\csc(dx+c)^2-1\right)^4\sqrt{-\frac{2a}{(1-\cos(dx+c))^2\csc(dx+c)^2-1}}\left(2\sqrt{-(1-\cos(dx+c))^2\csc(dx+c)^2-1}\,(1-\cos(dx+c))^5\csc(dx\right.}{}$

input
```
int(1/cos(d*x+c)^(9/2)/(a+a*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/32/d/a^3*((1-cos(d*x+c))^2*csc(d*x+c)^2-1)^4*(-2*a/((1-cos(d*x+c))^2*cs
c(d*x+c)^2-1))^(1/2)*(2*(-(1-cos(d*x+c))^2*csc(d*x+c)^2-1)^(1/2)*(1-cos(d*
x+c))^5*csc(d*x+c)^5+40*2^(1/2)*arctan(1/2*(-cot(d*x+c)+csc(d*x+c)-1)*2^(1
/2)/(-(1-cos(d*x+c))^2*csc(d*x+c)^2-1)^(1/2))*(1-cos(d*x+c))^2*csc(d*x+c)^
2+40*2^(1/2)*arctan(1/2*(csc(d*x+c)-cot(d*x+c)+1)*2^(1/2)/(-(1-cos(d*x+c))
^2*csc(d*x+c)^2-1)^(1/2))*(1-cos(d*x+c))^2*csc(d*x+c)^2+19*(-(1-cos(d*x+c)
)^2*csc(d*x+c)^2-1)^(1/2)*(1-cos(d*x+c))^3*csc(d*x+c)^3-115*arctan(1/(-(1-
cos(d*x+c))^2*csc(d*x+c)^2-1)^(1/2)*(-cot(d*x+c)+csc(d*x+c)))*(1-cos(d*x+c
))^2*csc(d*x+c)^2-40*2^(1/2)*arctan(1/2*(-cot(d*x+c)+csc(d*x+c)-1)*2^(1/2)
/(-(1-cos(d*x+c))^2*csc(d*x+c)^2-1)^(1/2))-40*2^(1/2)*arctan(1/2*(csc(d*x+
c)-cot(d*x+c)+1)*2^(1/2)/(-(1-cos(d*x+c))^2*csc(d*x+c)^2-1)^(1/2))-53*(-(1
-cos(d*x+c))^2*csc(d*x+c)^2-1)^(1/2)*(-cot(d*x+c)+csc(d*x+c))+115*arctan(1
/(-(1-cos(d*x+c))^2*csc(d*x+c)^2-1)^(1/2)*(-cot(d*x+c)+csc(d*x+c))))/(-(1-
cos(d*x+c))^2*csc(d*x+c)^2-1)^(1/2)/((1-cos(d*x+c))^2*csc(d*x+c)^2+1)^4/(-
((1-cos(d*x+c))^2*csc(d*x+c)^2-1)/((1-cos(d*x+c))^2*csc(d*x+c)^2+1))^(9/2)
```

3.440.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 702, normalized size of antiderivative = 2.76

$$\int \frac{1}{\cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}}\,dx = \left[\frac{115\sqrt{2}\left(\cos\left(dx+c\right)^4+3\cos\left(dx+c\right)^3+3\cos\left(dx+c\right)^2+\cos\right.}{}\right.$$

$$-\frac{115\sqrt{2}\left(\cos\left(dx+c\right)^4+3\cos\left(dx+c\right)^3+3\cos\left(dx+c\right)^2+\cos\left(dx+c\right)\right)\sqrt{-a}\arctan\left(\frac{\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}}{a\sin(dx+c)}\right.}{}$$

input
```
integrate(1/cos(d*x+c)^(9/2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
[1/64*(115*sqrt(2)*(cos(d*x + c)^4 + 3*cos(d*x + c)^3 + 3*cos(d*x + c)^2 +
 cos(d*x + c))*sqrt(a)*log(-(a*cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a)*sqrt((a*
cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 2*a*cos(
d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + 4*(35*cos(d*x + c
)^2 + 55*cos(d*x + c) + 16)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(c
os(d*x + c))*sin(d*x + c) + 80*(cos(d*x + c)^4 + 3*cos(d*x + c)^3 + 3*cos(
d*x + c)^2 + cos(d*x + c))*sqrt(a)*log((a*cos(d*x + c)^3 + 4*sqrt(a)*sqrt(
(a*cos(d*x + c) + a)/cos(d*x + c))*(cos(d*x + c) - 2)*sqrt(cos(d*x + c))*s
in(d*x + c) - 7*a*cos(d*x + c)^2 + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2))
)/(a^3*d*cos(d*x + c)^4 + 3*a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2
+ a^3*d*cos(d*x + c)), -1/32*(115*sqrt(2)*(cos(d*x + c)^4 + 3*cos(d*x + c)
^3 + 3*cos(d*x + c)^2 + cos(d*x + c))*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqr
t((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))/(a*sin(d*x + c)))
- 2*(35*cos(d*x + c)^2 + 55*cos(d*x + c) + 16)*sqrt((a*cos(d*x + c) + a)/c
os(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) + 80*(cos(d*x + c)^4 + 3*cos(
d*x + c)^3 + 3*cos(d*x + c)^2 + cos(d*x + c))*sqrt(-a)*arctan(2*sqrt(-a)*s
qrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*
cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)))/(a^3*d*cos(d*x + c)^4 + 3*a^3*d*c
os(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + a^3*d*cos(d*x + c))]
```

3.440.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/cos(d*x+c)**(9/2)/(a+a*sec(d*x+c))**(5/2),x)
```

output
```
Timed out
```

3.440.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 9048 vs. $2(209) = 418$.

Time = 2.56 (sec) , antiderivative size = 9048, normalized size of antiderivative = 35.62

$$\int \frac{1}{\cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/cos(d*x+c)^(9/2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
-1/32*(140*(sin(6*d*x + 6*c) + 7*sin(4*d*x + 4*c) + 7*sin(2*d*x + 2*c) + 4
*sin(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 8*sin(3/2*arctan2(
sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*sin(1/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c))))*cos(11/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
 - 16*(75*sin(9/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 24*sin(7/
4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 24*sin(5/4*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c))) - 75*sin(3/4*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c))) - 35*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*
cos(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 300*(sin(6*d*x + 6*
c) + 7*sin(4*d*x + 4*c) + 7*sin(2*d*x + 2*c) + 8*sin(3/2*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c))) + 4*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c))))*cos(9/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 96*(sin
(6*d*x + 6*c) + 7*sin(4*d*x + 4*c) + 7*sin(2*d*x + 2*c) + 8*sin(3/2*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*sin(1/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c))))*cos(7/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))
) + 32*(24*sin(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 75*sin(3
/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 35*sin(1/4*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c))))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c))) - 96*(sin(6*d*x + 6*c) + 7*sin(4*d*x + 4*c) + 7*sin(2*d*x + 2*
c) + 4*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(5/4*ar...
```

3.440.8 Giac [F]

$$\int \frac{1}{\cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}}\,dx = \int \frac{1}{(a\sec(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^{\frac{9}{2}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(9/2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate(1/((a*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^(9/2)), x)
```

3.440.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}}\,dx = \int \frac{1}{\cos(c+d\,x)^{9/2}\left(a+\frac{a}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

input
```
int(1/(cos(c + d*x)^(9/2)*(a + a/cos(c + d*x))^(5/2)),x)
```

output
```
int(1/(cos(c + d*x)^(9/2)*(a + a/cos(c + d*x))^(5/2)), x)
```

3.441 $\int (d\cos(e+fx))^n(a+a\sec(e+fx))^3\,dx$

3.441.1 Optimal result . . . 3285
3.441.2 Mathematica [A] (verified) . . . 3286
3.441.3 Rubi [A] (verified) . . . 3286
3.441.4 Maple [F] . . . 3290
3.441.5 Fricas [F] . . . 3290
3.441.6 Sympy [F] . . . 3290
3.441.7 Maxima [F] . . . 3291
3.441.8 Giac [F] . . . 3291
3.441.9 Mupad [F(-1)] . . . 3291

3.441.1 Optimal result

Integrand size = 23, antiderivative size = 244

$$\begin{aligned}
&\int (d\cos(e+fx))^n(a+a\sec(e+fx))^3\,dx \\
&= -\frac{a^3(7-4n)(d\cos(e+fx))^n\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n}{2},\frac{2+n}{2},\cos^2(e+fx)\right)\sin(e+fx)}{f(2-n)n\sqrt{\sin^2(e+fx)}} \\
&\quad -\frac{a^3(1-4n)\cos(e+fx)(d\cos(e+fx))^n\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+n}{2},\frac{3+n}{2},\cos^2(e+fx)\right)\sin(e+fx)}{f(1-n)(1+n)\sqrt{\sin^2(e+fx)}} \\
&\quad +\frac{a^3(5-2n)(d\cos(e+fx))^n\tan(e+fx)}{f(1-n)(2-n)} \\
&\quad +\frac{(d\cos(e+fx))^n\left(a^3+a^3\sec(e+fx)\right)\tan(e+fx)}{f(2-n)}
\end{aligned}$$

output
```
-a^3*(7-4*n)*(d*cos(f*x+e))^n*hypergeom([1/2, 1/2*n],[1+1/2*n],cos(f*x+e)^
2)*sin(f*x+e)/f/(2-n)/n/(sin(f*x+e)^2)^(1/2)-a^3*(1-4*n)*cos(f*x+e)*(d*cos
(f*x+e))^n*hypergeom([1/2, 1/2+1/2*n],[3/2+1/2*n],cos(f*x+e)^2)*sin(f*x+e)
/f/(-n^2+1)/(sin(f*x+e)^2)^(1/2)+a^3*(5-2*n)*(d*cos(f*x+e))^n*tan(f*x+e)/f
/(n^2-3*n+2)+(d*cos(f*x+e))^n*(a^3+a^3*sec(f*x+e))*tan(f*x+e)/f/(2-n)
```

3.441.2 Mathematica [A] (verified)

Time = 0.96 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.64

$$\int (d\cos(e+fx))^n(a+a\sec(e+fx))^3\,dx$$
$$= \frac{a^3d(d\cos(e+fx))^{-1+n}\left(-n(-7+4n)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(-2+n),\frac{n}{2},\cos^2(e+fx)\right)+(-2+n)\left(\right.\right.}{}$$

input
```
Integrate[(d*Cos[e + f*x])^n*(a + a*Sec[e + f*x])^3,x]
```

output
```
(a^3*d*(d*Cos[e + f*x])^(-1 + n)*(-(n*(-7 + 4*n)*Hypergeometric2F1[1/2, (-
2 + n)/2, n/2, Cos[e + f*x]^2]) + (-2 + n)*(-((-1 + 4*n)*Cos[e + f*x]*Hype
rgeometric2F1[1/2, (-1 + n)/2, (1 + n)/2, Cos[e + f*x]^2]) + (3*n + (-1 +
n)*Cos[e + f*x])*Sqrt[Sin[e + f*x]^2]))*Tan[e + f*x])/(f*(-2 + n)*(-1 + n)
*n*Sqrt[Sin[e + f*x]^2])
```

3.441.3 Rubi [A] (verified)

Time = 1.25 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.14, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.522$, Rules used = {3042, 4752, 3042, 4301, 3042, 4485, 3042, 4274, 3042, 4259, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sec(e+fx)+a)^3(d\cos(e+fx))^n\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^3\left(d\sin\left(e+fx+\frac{\pi}{2}\right)\right)^n dx$$
$$\Big\downarrow \text{4752}$$
$$(d\cos(e+fx))^n(d\sec(e+fx))^n\int (d\sec(e+fx))^{-n}(\sec(e+fx)a+a)^3dx$$
$$\Big\downarrow \text{3042}$$
$$(d\cos(e+fx))^n(d\sec(e+fx))^n\int \left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{-n}\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)^3dx$$

$$\Big\downarrow \; 4301$$

$$(d\cos(e+fx))^n(d\sec(e+fx))^n\left(\frac{a\int(d\sec(e+fx))^{-n}(\sec(e+fx)a+a)(2a(1-n)+a(5-2n)\sec(e+fx))dx}{2-n}+\frac{\tan(e+fx)\left(a^3\sec(e+\right.}{f(}\right.$$

$$\Big\downarrow \; 3042$$

$$(d\cos(e+fx))^n(d\sec(e+fx))^n\left(\frac{a\int\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{-n}\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)\left(2a(1-n)+a(5-2n)\csc\left(e+fx+\frac{\pi}{2}\right)\right)dx}{2-n}+\frac{\tan(e}{}\right.$$

$$\Big\downarrow \; 4485$$

$$(d\cos(e+fx))^n(d\sec(e+fx))^n\left(\frac{a\left(\frac{\int(d\sec(e+fx))^{-n}\left((1-4n)(2-n)a^2+(7-4n)(1-n)\sec(e+fx)a^2\right)dx}{1-n}+\frac{a^2(5-2n)\tan(e+fx)(d\sec(e+fx))^{-n}}{f(1-n)}\right)}{2-n}+\frac{\tan(e+fx)}{}\right.$$

$$\Big\downarrow \; 3042$$

$$(d\cos(e+fx))^n(d\sec(e+fx))^n\left(\frac{a\left(\frac{\int\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{-n}\left((1-4n)(2-n)a^2+(7-4n)(1-n)\csc\left(e+fx+\frac{\pi}{2}\right)a^2\right)dx}{1-n}+\frac{a^2(5-2n)\tan(e+fx)(d\sec(e+fx))^{-n}}{f(1-n)}\right)}{2-n}+\frac{\tan(}{}\right.$$

$$\Big\downarrow \; 4274$$

$$(d\cos(e+fx))^n(d\sec(e+fx))^n\left(\frac{a\left(\frac{\frac{a^2(7-4n)(1-n)\int(d\sec(e+fx))^{1-n}dx}{d}+a^2(1-4n)(2-n)\int(d\sec(e+fx))^{-n}dx}{1-n}+\frac{a^2(5-2n)\tan(e+fx)(d\sec(e+fx))^{-n}}{f(1-n)}\right)}{2-n}+\frac{\tan(e}{}\right.$$

$$\Big\downarrow \; 3042$$

$$(d\cos(e+fx))^n(d\sec(e+fx))^n\left(\frac{a\left(\frac{\frac{a^2(7-4n)(1-n)\int\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{1-n}dx}{d}+a^2(1-4n)(2-n)\int\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{-n}dx}{1-n}+\frac{a^2(5-2n)\tan(e+fx)(d\sec(e+fx))^{-n}}{f(1-n)}\right)}{2-n}\right.$$

$$\Big\downarrow \; 4259$$

3.441. $\int(d\cos(e+fx))^n(a+a\sec(e+fx))^3\,dx$

$$(d\cos(e+fx))^n(d\sec(e+fx))^n\left(\frac{a\left(\frac{\frac{a^2(7-4n)(1-n)\left(\frac{\cos(e+fx)}{d}\right)^{-n}(d\sec(e+fx))^{-n}\int\left(\frac{\cos(e+fx)}{d}\right)^{n-1}dx}{d}+a^2(1-4n)(2-n)\left(\frac{\cos(e+fx)}{d}\right)^{-n}(d\sec(e+fx))^{-n}\int\left(\frac{\cos(e+fx)}{d}\right.}{1-n}\right.}{2-n}\right.$$

$\downarrow$ 3042

$$(d\cos(e+fx))^n(d\sec(e+fx))^n\left(\frac{a\left(\frac{\frac{a^2(7-4n)(1-n)\left(\frac{\cos(e+fx)}{d}\right)^{-n}(d\sec(e+fx))^{-n}\int\left(\frac{\sin\left(e+fx+\frac{\pi}{2}\right)}{d}\right)^{n-1}dx}{d}+a^2(1-4n)(2-n)\left(\frac{\cos(e+fx)}{d}\right)^{-n}(d\sec(e+fx))^{-n}\int\left(\frac{\sin(e-}{}\right.}{1-n}\right.}{2-n}\right.$$

$\downarrow$ 3122

$$(d\cos(e+fx))^n(d\sec(e+fx))^n\left(\frac{\tan(e+fx)\left(a^3\sec(e+fx)+a^3\right)(d\sec(e+fx))^{-n}}{f(2-n)}+\frac{a\left(-\frac{a^2d(1-4n)(2-n)\sin(e+fx)(d\sec(e+fx))^{-n-1}\text{Hypergeometric2}}{f(n+1)\sqrt{\sin^2(e+fx)}}\right.}{}\right.$$

input
```
Int[(d*Cos[e + f*x])^n*(a + a*Sec[e + f*x])^3,x]
```

output
```
(d*Cos[e + f*x])^n*(d*Sec[e + f*x])^n*(((a^3 + a^3*Sec[e + f*x])*Tan[e + f
*x])/(f*(2 - n)*(d*Sec[e + f*x])^n) + (a*((-((a^2*d*(1 - 4*n)*(2 - n)*Hype
rgeometric2F1[1/2, (1 + n)/2, (3 + n)/2, Cos[e + f*x]^2]*(d*Sec[e + f*x])^
(-1 - n)*Sin[e + f*x])/(f*(1 + n)*Sqrt[Sin[e + f*x]^2])) - (a^2*(7 - 4*n)*
(1 - n)*Hypergeometric2F1[1/2, n/2, (2 + n)/2, Cos[e + f*x]^2]*Sin[e + f*x
])/(f*n*(d*Sec[e + f*x])^n*Sqrt[Sin[e + f*x]^2]))/(1 - n) + (a^2*(5 - 2*n)
*Tan[e + f*x])/(f*(1 - n)*(d*Sec[e + f*x])^n)))/(2 - n))
```

3.441. $\int (d\cos(e+fx))^n(a+a\sec(e+fx))^3\,dx$

3.441.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

rule 4259
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)   Int[1/(Sin[c + d*x]/b)^n, x]), x] /;
FreeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4301
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
2)*((d*Csc[e + f*x])^n/(f*(m + n - 1))), x] + Simp[b/(m + n - 1)   Int[(a +
 b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*(b*(m + 2*n - 1) + a*(3*m + 2*n
 - 4)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^
2, 0] && GtQ[m, 1] && NeQ[m + n - 1, 0] && IntegerQ[2*m]
```

rule 4485
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*Cot[
e + f*x]*((d*Csc[e + f*x])^n/(f*(n + 1))), x] + Simp[1/(n + 1)   Int[(d*Csc
[e + f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x
], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && !LeQ[
n, -1]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

3.441.4 Maple [F]

$$\int (d\cos(fx+e))^n (a+a\sec(fx+e))^3 \, dx$$

input
```
int((d*cos(f*x+e))^n*(a+a*sec(f*x+e))^3,x)
```

output
```
int((d*cos(f*x+e))^n*(a+a*sec(f*x+e))^3,x)
```

3.441.5 Fricas [F]

$$\int (d\cos(e+fx))^n(a+a\sec(e+fx))^3 \, dx = \int (a\sec(fx+e)+a)^3(d\cos(fx+e))^n \, dx$$

input
```
integrate((d*cos(f*x+e))^n*(a+a*sec(f*x+e))^3,x, algorithm="fricas")
```

output
```
integral((a^3*sec(f*x + e)^3 + 3*a^3*sec(f*x + e)^2 + 3*a^3*sec(f*x + e) +
 a^3)*(d*cos(f*x + e))^n, x)
```

3.441.6 Sympy [F]

$$\int (d\cos(e+fx))^n(a+a\sec(e+fx))^3 \, dx = a^3\left(\int (d\cos(e+fx))^n \, dx + \int 3(d\cos(e+fx))^n \sec(e+fx) \, dx + \int 3(d\cos(e+fx))^n \sec^2(e+fx) \, dx + \int (d\cos(e+fx))^n \sec^3(e+fx) \, dx\right)$$

input
```
integrate((d*cos(f*x+e))**n*(a+a*sec(f*x+e))**3,x)
```

output
```
a**3*(Integral((d*cos(e + f*x))**n, x) + Integral(3*(d*cos(e + f*x))**n*se
c(e + f*x), x) + Integral(3*(d*cos(e + f*x))**n*sec(e + f*x)**2, x) + Inte
gral((d*cos(e + f*x))**n*sec(e + f*x)**3, x))
```

3.441.7 Maxima [F]

$$\int (d\cos(e+fx))^n(a+a\sec(e+fx))^3\,dx = \int (a\sec(fx+e)+a)^3(d\cos(fx+e))^n\,dx$$

input
```
integrate((d*cos(f*x+e))^n*(a+a*sec(f*x+e))^3,x, algorithm="maxima")
```

output
```
integrate((a*sec(f*x + e) + a)^3*(d*cos(f*x + e))^n, x)
```

3.441.8 Giac [F]

$$\int (d\cos(e+fx))^n(a+a\sec(e+fx))^3\,dx = \int (a\sec(fx+e)+a)^3(d\cos(fx+e))^n\,dx$$

input
```
integrate((d*cos(f*x+e))^n*(a+a*sec(f*x+e))^3,x, algorithm="giac")
```

output
```
integrate((a*sec(f*x + e) + a)^3*(d*cos(f*x + e))^n, x)
```

3.441.9 Mupad [F(-1)]

Timed out.

$$\int (d\cos(e+fx))^n(a+a\sec(e+fx))^3\,dx = \int (d\,\cos(e+f\,x))^n\left(a+\frac{a}{\cos(e+f\,x)}\right)^3\,dx$$

input
```
int((d*cos(e + f*x))^n*(a + a/cos(e + f*x))^3,x)
```

output
```
int((d*cos(e + f*x))^n*(a + a/cos(e + f*x))^3, x)
```

3.442 $\int (d\cos(e+fx))^n(a+a\sec(e+fx))^2\,dx$

3.442.1 Optimal result . . . 3292
3.442.2 Mathematica [A] (verified) . . . 3293
3.442.3 Rubi [A] (verified) . . . 3293
3.442.4 Maple [F] . . . 3296
3.442.5 Fricas [F] . . . 3296
3.442.6 Sympy [F] . . . 3297
3.442.7 Maxima [F] . . . 3297
3.442.8 Giac [F] . . . 3297
3.442.9 Mupad [F(-1)] . . . 3298

3.442.1 Optimal result

Integrand size = 23, antiderivative size = 179

$$\begin{aligned}\int (d\cos(e+fx))^n(a+a\sec(e+fx))^2\,dx &= -\frac{2a^2(d\cos(e+fx))^n\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n}{2},\frac{2+n}{2},\cos^2(e+fx)\right)\sin(e+fx)}{fn\sqrt{\sin^2(e+fx)}}\\ &\quad-\frac{a^2(1-2n)\cos(e+fx)(d\cos(e+fx))^n\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+n}{2},\frac{3+n}{2},\cos^2(e+fx)\right)\sin(e+fx)}{f(1-n)(1+n)\sqrt{\sin^2(e+fx)}}\\ &\quad+\frac{a^2(d\cos(e+fx))^n\tan(e+fx)}{f(1-n)}\end{aligned}$$

output
```
-2*a^2*(d*cos(f*x+e))^n*hypergeom([1/2, 1/2*n],[1+1/2*n],cos(f*x+e)^2)*sin
(f*x+e)/f/n/(sin(f*x+e)^2)^(1/2)-a^2*(1-2*n)*cos(f*x+e)*(d*cos(f*x+e))^n*h
ypergeom([1/2, 1/2+1/2*n],[3/2+1/2*n],cos(f*x+e)^2)*sin(f*x+e)/f/(-n^2+1)/
(sin(f*x+e)^2)^(1/2)+a^2*(d*cos(f*x+e))^n*tan(f*x+e)/f/(1-n)
```

3.442.2 Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.70

$$\int (d\cos(e+fx))^n(a+a\sec(e+fx))^2\,dx$$
$$= \frac{a^2(d\cos(e+fx))^n\left((1-2n)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(-1+n),\frac{1+n}{2},\cos^2(e+fx)\right)+(-1+n)\left(-2\cos(\right.\right.}{f(-1+n)n\sqrt{\sin^2(e+}}$$

input
```
Integrate[(d*Cos[e + f*x])^n*(a + a*Sec[e + f*x])^2,x]
```

output
```
(a^2*(d*Cos[e + f*x])^n*((1 - 2*n)*Hypergeometric2F1[1/2, (-1 + n)/2, (1 +
 n)/2, Cos[e + f*x]^2] + (-1 + n)*(-2*Cos[e + f*x]*Hypergeometric2F1[1/2,
n/2, (2 + n)/2, Cos[e + f*x]^2] + Sqrt[Sin[e + f*x]^2]))*Tan[e + f*x])/(f*
(-1 + n)*n*Sqrt[Sin[e + f*x]^2])
```

3.442.3 Rubi [A] (verified)

Time = 0.98 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.13, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.565$, Rules used = {3042, 4752, 3042, 4275, 3042, 4259, 3042, 3122, 4534, 3042, 4259, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sec(e+fx)+a)^2(d\cos(e+fx))^n\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^2\left(d\sin\left(e+fx+\frac{\pi}{2}\right)\right)^n dx$$
$$\Big\downarrow \text{4752}$$
$$(d\cos(e+fx))^n(d\sec(e+fx))^n\int (d\sec(e+fx))^{-n}(\sec(e+fx)a+a)^2dx$$
$$\Big\downarrow \text{3042}$$
$$(d\cos(e+fx))^n(d\sec(e+fx))^n\int \left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{-n}\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)^2dx$$
$$\Big\downarrow \text{4275}$$

$$(d\cos(e+fx))^n(d\sec(e+fx))^n\left(\int(d\sec(e+fx))^{-n}\left(\sec^2(e+fx)a^2+a^2\right)dx+\frac{2a^2\int(d\sec(e+fx))^{1-n}dx}{d}\right)$$

$\Big\downarrow$ 3042

$$(d\cos(e+fx))^n(d\sec(e+fx))^n\left(\frac{2a^2\int\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{1-n}dx}{d}+\int\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{-n}\left(\csc\left(e+fx+\frac{\pi}{2}\right)^2a^2+a^2\right)dx\right)$$

$\Big\downarrow$ 4259

$$(d\cos(e+fx))^n(d\sec(e+fx))^n\left(\int\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{-n}\left(\csc\left(e+fx+\frac{\pi}{2}\right)^2a^2+a^2\right)dx+\frac{2a^2\left(\frac{\cos(e+fx)}{d}\right)^{-n}(d\sec(e+fx))^{-n}\int\left(\cos\right.}{d}\right.$$

$\Big\downarrow$ 3042

$$(d\cos(e+fx))^n(d\sec(e+fx))^n\left(\int\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{-n}\left(\csc\left(e+fx+\frac{\pi}{2}\right)^2a^2+a^2\right)dx+\frac{2a^2\left(\frac{\cos(e+fx)}{d}\right)^{-n}(d\sec(e+fx))^{-n}\int\left(\sin\right.}{d}\right.$$

$\Big\downarrow$ 3122

$$(d\cos(e+fx))^n(d\sec(e+fx))^n\left(\int\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{-n}\left(\csc\left(e+fx+\frac{\pi}{2}\right)^2a^2+a^2\right)dx-\frac{2a^2\sin(e+fx)(d\sec(e+fx))^{-n}\text{Hyperge}}{fn\sqrt{\sin^2(e}}\right.$$

$\Big\downarrow$ 4534

$$(d\cos(e+fx))^n(d\sec(e+fx))^n\left(\frac{a^2(1-2n)\int(d\sec(e+fx))^{-n}dx}{1-n}-\frac{2a^2\sin(e+fx)(d\sec(e+fx))^{-n}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n}{2},\frac{n+2}{2},\cos^2\right.}{fn\sqrt{\sin^2(e+fx)}}\right.$$

$\Big\downarrow$ 3042

$$(d\cos(e+fx))^n(d\sec(e+fx))^n\left(\frac{a^2(1-2n)\int\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{-n}dx}{1-n}-\frac{2a^2\sin(e+fx)(d\sec(e+fx))^{-n}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n}{2},\frac{n+}{2}\right.}{fn\sqrt{\sin^2(e+fx)}}\right.$$

$\Big\downarrow$ 4259

$$(d\cos(e+fx))^n(d\sec(e+fx))^n\left(\frac{a^2(1-2n)\left(\frac{\cos(e+fx)}{d}\right)^{-n}(d\sec(e+fx))^{-n}\int\left(\frac{\cos(e+fx)}{d}\right)^n dx}{1-n}-\frac{2a^2\sin(e+fx)(d\sec(e+fx))^{-n}\text{Hyperg}}{fn\sqrt{\sin^2(}}\right.$$

3.442. $\int(d\cos(e+fx))^n(a+a\sec(e+fx))^2\,dx$

$$\Big\downarrow \text{3042}$$

$$(d\cos(e+fx))^n(d\sec(e+fx))^n\left(\frac{a^2(1-2n)\left(\frac{\cos(e+fx)}{d}\right)^{-n}(d\sec(e+fx))^{-n}\int\left(\frac{\sin\left(e+fx+\frac{\pi}{2}\right)}{d}\right)^n dx}{1-n}-\frac{2a^2\sin(e+fx)(d\sec(e+fx))^{-n}\text{Hyp}}{fn\sqrt{\text{si}}}\right.$$

$$\Big\downarrow \text{3122}$$

$$(d\cos(e+fx))^n(d\sec(e+fx))^n\left(-\frac{a^2d(1-2n)\sin(e+fx)(d\sec(e+fx))^{-n-1}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n+1}{2},\frac{n+3}{2},\cos^2(e+fx)\right)}{f(1-n)(n+1)\sqrt{\sin^2(e+fx)}}-\frac{2a^2\sin(}{}\right.$$

input
```
Int[(d*Cos[e + f*x])^n*(a + a*Sec[e + f*x])^2,x]
```

output
```
(d*Cos[e + f*x])^n*(d*Sec[e + f*x])^n*(-((a^2*d*(1 - 2*n)*Hypergeometric2F
1[1/2, (1 + n)/2, (3 + n)/2, Cos[e + f*x]^2]*(d*Sec[e + f*x])^(-1 - n)*Sin
[e + f*x])/(f*(1 - n)*(1 + n)*Sqrt[Sin[e + f*x]^2])) - (2*a^2*Hypergeometr
ic2F1[1/2, n/2, (2 + n)/2, Cos[e + f*x]^2]*Sin[e + f*x])/(f*n*(d*Sec[e + f
*x])^n*Sqrt[Sin[e + f*x]^2]) + (a^2*Tan[e + f*x])/(f*(1 - n)*(d*Sec[e + f*
x])^n))
```

3.442.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

rule 4259
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)   Int[1/(Sin[c + d*x]/b)^n, x]), x] /;
FreeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

rule 4275
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Simp[2*a*(b/d)   Int[(d*Csc[e + f*x])^(n + 1), x], x]
 + Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
 e, f, n}, x]
```

rule 4534
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Simp[(C*m + A*(m + 1))/(m + 1)   Int[(b*Csc[e + f*x])^m, x], x] /;
 FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

3.442.4 Maple [F]

$$\int (d\cos(fx+e))^n (a+a\sec(fx+e))^2 \, dx$$

input
```
int((d*cos(f*x+e))^n*(a+a*sec(f*x+e))^2,x)
```

output
```
int((d*cos(f*x+e))^n*(a+a*sec(f*x+e))^2,x)
```

3.442.5 Fricas [F]

$$\int (d\cos(e+fx))^n(a+a\sec(e+fx))^2 \, dx = \int (a\sec(fx+e)+a)^2(d\cos(fx+e))^n \, dx$$

input
```
integrate((d*cos(f*x+e))^n*(a+a*sec(f*x+e))^2,x, algorithm="fricas")
```

output
```
integral((a^2*sec(f*x + e)^2 + 2*a^2*sec(f*x + e) + a^2)*(d*cos(f*x + e))^
n, x)
```

3.442.6 Sympy [F]

$$\int (d\cos(e+fx))^n(a+a\sec(e+fx))^2\,dx = a^2\left(\int (d\cos(e+fx))^n\,dx + \int 2(d\cos(e+fx))^n\sec(e+fx)\,dx + \int (d\cos(e+fx))^n\sec^2(e+fx)\,dx\right)$$

input
```
integrate((d*cos(f*x+e))**n*(a+a*sec(f*x+e))**2,x)
```

output
```
a**2*(Integral((d*cos(e + f*x))**n, x) + Integral(2*(d*cos(e + f*x))**n*se
c(e + f*x), x) + Integral((d*cos(e + f*x))**n*sec(e + f*x)**2, x))
```

3.442.7 Maxima [F]

$$\int (d\cos(e+fx))^n(a+a\sec(e+fx))^2\,dx = \int (a\sec(fx+e)+a)^2(d\cos(fx+e))^n\,dx$$

input
```
integrate((d*cos(f*x+e))^n*(a+a*sec(f*x+e))^2,x, algorithm="maxima")
```

output
```
integrate((a*sec(f*x + e) + a)^2*(d*cos(f*x + e))^n, x)
```

3.442.8 Giac [F]

$$\int (d\cos(e+fx))^n(a+a\sec(e+fx))^2\,dx = \int (a\sec(fx+e)+a)^2(d\cos(fx+e))^n\,dx$$

input
```
integrate((d*cos(f*x+e))^n*(a+a*sec(f*x+e))^2,x, algorithm="giac")
```

output
```
integrate((a*sec(f*x + e) + a)^2*(d*cos(f*x + e))^n, x)
```

3.442.9 Mupad [F(-1)]

Timed out.

$$\int (d\cos(e+fx))^n(a+a\sec(e+fx))^2\,dx=\int (d\,\cos\left(e+f\,x\right))^n\,\left(a+\frac{a}{\cos\left(e+f\,x\right)}\right)^2\,dx$$

input
```
int((d*cos(e + f*x))^n*(a + a/cos(e + f*x))^2,x)
```

output
```
int((d*cos(e + f*x))^n*(a + a/cos(e + f*x))^2, x)
```

3.443 $\int (d\cos(e+fx))^n(a+a\sec(e+fx))\,dx$

3.443.1 Optimal result . . . 3299
3.443.2 Mathematica [A] (verified) . . . 3299
3.443.3 Rubi [A] (verified) . . . 3300
3.443.4 Maple [F] . . . 3302
3.443.5 Fricas [F] . . . 3302
3.443.6 Sympy [F] . . . 3302
3.443.7 Maxima [F] . . . 3303
3.443.8 Giac [F] . . . 3303
3.443.9 Mupad [F(-1)] . . . 3303

3.443.1 Optimal result

Integrand size = 21, antiderivative size = 132

$$\int (d\cos(e+fx))^n(a+a\sec(e+fx))\,dx = -\frac{a(d\cos(e+fx))^n\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n}{2},\frac{2+n}{2},\cos^2(e+fx)\right)\sin(e+fx)}{fn\sqrt{\sin^2(e+fx)}} - \frac{a(d\cos(e+fx))^{1+n}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+n}{2},\frac{3+n}{2},\cos^2(e+fx)\right)\sin(e+fx)}{df(1+n)\sqrt{\sin^2(e+fx)}}$$

output
```
-a*(d*cos(f*x+e))^n*hypergeom([1/2, 1/2*n],[1+1/2*n],cos(f*x+e)^2)*sin(f*x
+e)/f/n/(sin(f*x+e)^2)^(1/2)-a*(d*cos(f*x+e))^(1+n)*hypergeom([1/2, 1/2+1/
2*n],[3/2+1/2*n],cos(f*x+e)^2)*sin(f*x+e)/d/f/(1+n)/(sin(f*x+e)^2)^(1/2)
```

3.443.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.80

$$\int (d\cos(e+fx))^n(a+a\sec(e+fx))\,dx = -\frac{a(d\cos(e+fx))^n\left((1+n)\csc(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n}{2},\frac{2+n}{2},\cos^2(e+fx)\right)+n\cot(e+fx)\,\text{H}}{fn(1+n)}$$

input
```
Integrate[(d*Cos[e + f*x])^n*(a + a*Sec[e + f*x]),x]
```

output
```
-((a*(d*Cos[e + f*x])^n*((1 + n)*Csc[e + f*x]*Hypergeometric2F1[1/2, n/2,
(2 + n)/2, Cos[e + f*x]^2] + n*Cot[e + f*x]*Hypergeometric2F1[1/2, (1 + n)
/2, (3 + n)/2, Cos[e + f*x]^2])*Sqrt[Sin[e + f*x]^2])/(f*n*(1 + n)))
```

3.443.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.04, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 4713, 3042, 2030, 3227, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sec(e+fx)+a)(d\cos(e+fx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)\left(d\sin\left(e+fx+\frac{\pi}{2}\right)\right)^n dx$$

$$\Big\downarrow \text{4713}$$

$$\int \sec(e+fx)(a\cos(e+fx)+a)(d\cos(e+fx))^n dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\sin\left(e+fx+\frac{\pi}{2}\right)+a\right)\left(d\sin\left(e+fx+\frac{\pi}{2}\right)\right)^n}{\sin\left(e+fx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{2030}$$

$$d\int \left(d\sin\left(\frac{1}{2}(2e+\pi)+fx\right)\right)^{n-1}\left(\sin\left(\frac{1}{2}(2e+\pi)+fx\right)a+a\right)dx$$

$$\Big\downarrow \text{3227}$$

$$d\left(a\int (d\cos(e+fx))^{n-1}dx+\frac{a\int (d\cos(e+fx))^n dx}{d}\right)$$

$$\Big\downarrow \text{3042}$$

$$d\left(a\int \left(d\sin\left(e+fx+\frac{\pi}{2}\right)\right)^{n-1}dx+\frac{a\int \left(d\sin\left(e+fx+\frac{\pi}{2}\right)\right)^n dx}{d}\right)$$

$$\Big\downarrow \text{3122}$$

$$d\left(-\frac{a\sin(e+fx)(d\cos(e+fx))^{n+1}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n+1}{2},\frac{n+3}{2},\cos^2(e+fx)\right)}{d^2f(n+1)\sqrt{\sin^2(e+fx)}}-\frac{a\sin(e+fx)(d\cos(e+f}{}\right.$$

input
```
Int[(d*Cos[e + f*x])^n*(a + a*Sec[e + f*x]),x]
```

output
```
d*(-((a*(d*Cos[e + f*x])^n*Hypergeometric2F1[1/2, n/2, (2 + n)/2, Cos[e +
f*x]^2]*Sin[e + f*x])/(d*f*n*Sqrt[Sin[e + f*x]^2])) - (a*(d*Cos[e + f*x])^
(1 + n)*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2, Cos[e + f*x]^2]*Sin[e
 + f*x])/(d^2*f*(1 + n)*Sqrt[Sin[e + f*x]^2]))
```

3.443.3.1 Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 4713
```
Int[(csc[(a_.) + (b_.)*(x_)]*(B_.) + (A_))*(u_), x_Symbol] :> Int[ActivateT
rig[u]*((B + A*Sin[a + b*x])/Sin[a + b*x]), x] /; FreeQ[{a, b, A, B}, x] &&
 KnownSineIntegrandQ[u, x]
```

3.443.4 Maple [F]

$$\int (d\cos(fx+e))^n (a+a\sec(fx+e))\,dx$$

input
```
int((d*cos(f*x+e))^n*(a+a*sec(f*x+e)),x)
```

output
```
int((d*cos(f*x+e))^n*(a+a*sec(f*x+e)),x)
```

3.443.5 Fricas [F]

$$\int (d\cos(e+fx))^n(a+a\sec(e+fx))\,dx = \int (a\sec(fx+e)+a)(d\cos(fx+e))^n\,dx$$

input
```
integrate((d*cos(f*x+e))^n*(a+a*sec(f*x+e)),x, algorithm="fricas")
```

output
```
integral((a*sec(f*x + e) + a)*(d*cos(f*x + e))^n, x)
```

3.443.6 Sympy [F]

$$\int (d\cos(e+fx))^n(a+a\sec(e+fx))\,dx = a\left(\int (d\cos(e+fx))^n\,dx + \int (d\cos(e+fx))^n\sec(e+fx)\,dx\right)$$

input
```
integrate((d*cos(f*x+e))**n*(a+a*sec(f*x+e)),x)
```

output
```
a*(Integral((d*cos(e + f*x))**n, x) + Integral((d*cos(e + f*x))**n*sec(e +
 f*x), x))
```

3.443.7 Maxima [F]

$$\int (d\cos(e+fx))^n(a+a\sec(e+fx))\,dx = \int (a\sec(fx+e)+a)(d\cos(fx+e))^n\,dx$$

input
```
integrate((d*cos(f*x+e))^n*(a+a*sec(f*x+e)),x, algorithm="maxima")
```

output
```
integrate((a*sec(f*x + e) + a)*(d*cos(f*x + e))^n, x)
```

3.443.8 Giac [F]

$$\int (d\cos(e+fx))^n(a+a\sec(e+fx))\,dx = \int (a\sec(fx+e)+a)(d\cos(fx+e))^n\,dx$$

input
```
integrate((d*cos(f*x+e))^n*(a+a*sec(f*x+e)),x, algorithm="giac")
```

output
```
integrate((a*sec(f*x + e) + a)*(d*cos(f*x + e))^n, x)
```

3.443.9 Mupad [F(-1)]

Timed out.

$$\int (d\cos(e+fx))^n(a+a\sec(e+fx))\,dx = \int (d\,\cos(e+f\,x))^n\left(a+\frac{a}{\cos(e+f\,x)}\right)dx$$

input
```
int((d*cos(e + f*x))^n*(a + a/cos(e + f*x)),x)
```

output
```
int((d*cos(e + f*x))^n*(a + a/cos(e + f*x)), x)
```

3.444 $\int \frac{(d\cos(e+fx))^n}{a+a\sec(e+fx)}\,dx$

3.444.1 Optimal result . . . 3304
3.444.2 Mathematica [A] (verified) . . . 3304
3.444.3 Rubi [A] (verified) . . . 3305
3.444.4 Maple [F] . . . 3308
3.444.5 Fricas [F] . . . 3308
3.444.6 Sympy [F] . . . 3308
3.444.7 Maxima [F] . . . 3309
3.444.8 Giac [F] . . . 3309
3.444.9 Mupad [F(-1)] . . . 3309

3.444.1 Optimal result

Integrand size = 23, antiderivative size = 178

$$\int \frac{(d\cos(e+fx))^n}{a+a\sec(e+fx)}\,dx = \frac{(d\cos(e+fx))^n\sin(e+fx)}{f(a+a\sec(e+fx))} - \frac{\cos(e+fx)(d\cos(e+fx))^n\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+n}{2},\frac{3+n}{2},\cos^2(e+fx)\right)\sin(e+fx)}{af\sqrt{\sin^2(e+fx)}} + \frac{(1+n)\cos^2(e+fx)(d\cos(e+fx))^n\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2+n}{2},\frac{4+n}{2},\cos^2(e+fx)\right)\sin(e+fx)}{af(2+n)\sqrt{\sin^2(e+fx)}}$$

output
```
(d*cos(f*x+e))^n*sin(f*x+e)/f/(a+a*sec(f*x+e))-cos(f*x+e)*(d*cos(f*x+e))^n
*hypergeom([1/2, 1/2+1/2*n],[3/2+1/2*n],cos(f*x+e)^2)*sin(f*x+e)/a/f/(sin(
f*x+e)^2)^(1/2)+(1+n)*cos(f*x+e)^2*(d*cos(f*x+e))^n*hypergeom([1/2, 1+1/2*
n],[2+1/2*n],cos(f*x+e)^2)*sin(f*x+e)/a/f/(2+n)/(sin(f*x+e)^2)^(1/2)
```

3.444.2 Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.93

$$\int \frac{(d\cos(e+fx))^n}{a+a\sec(e+fx)}\,dx = \frac{\cos(e+fx)(d\cos(e+fx))^n\sin(e+fx)\left(-2(2+n)\cos^2\left(\frac{1}{2}(e+fx)\right)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+n}{2},\frac{3+n}{2},\text{co}\right.\right.}{af(2+n)}$$

input
```
Integrate[(d*Cos[e + f*x])^n/(a + a*Sec[e + f*x]),x]
```

output
```
(Cos[e + f*x]*(d*Cos[e + f*x])^n*Sin[e + f*x]*(-2*(2 + n)*Cos[(e + f*x)/2]
^2*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2, Cos[e + f*x]^2] + 2*(1 + n
)*Cos[(e + f*x)/2]^2*Cos[e + f*x]*Hypergeometric2F1[1/2, (2 + n)/2, (4 + n
)/2, Cos[e + f*x]^2] + (2 + n)*Sqrt[Sin[e + f*x]^2]))/(a*f*(2 + n)*(1 + Co
s[e + f*x])*Sqrt[Sin[e + f*x]^2])
```

3.444.3 Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.16, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {3042, 4752, 3042, 4307, 3042, 4274, 3042, 4259, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d\cos(e+fx))^n}{a\sec(e+fx)+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(d\sin\left(e+fx+\frac{\pi}{2}\right)\right)^n}{a\csc\left(e+fx+\frac{\pi}{2}\right)+a}dx$$

$$\Big\downarrow \text{4752}$$

$$(d\cos(e+fx))^n(d\sec(e+fx))^n\int \frac{(d\sec(e+fx))^{-n}}{\sec(e+fx)a+a}dx$$

$$\Big\downarrow \text{3042}$$

$$(d\cos(e+fx))^n(d\sec(e+fx))^n\int \frac{\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{-n}}{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}dx$$

$$\Big\downarrow \text{4307}$$

$$(d\cos(e+fx))^n(d\sec(e+fx))^n\left(\frac{d\tan(e+fx)(d\sec(e+fx))^{-n-1}}{f(a\sec(e+fx)+a)}-\frac{d(n+1)\int(d\sec(e+fx))^{-n-1}(a-a\sec(e+fx))dx}{a^2}\right)$$

$$\Big\downarrow \text{3042}$$

$$(d\cos(e+fx))^n(d\sec(e+fx))^n\left(\frac{d\tan(e+fx)(d\sec(e+fx))^{-n-1}}{f(a\sec(e+fx)+a)}-\frac{d(n+1)\int\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{-n-1}\left(a-a\csc\left(e+fx+\frac{\pi}{2}\right)\right)dx}{a^2}\right)$$

$$\Big\downarrow \; 4274$$

$$(d\cos(e+fx))^n(d\sec(e+fx))^n\left(\frac{d\tan(e+fx)(d\sec(e+fx))^{-n-1}}{f(a\sec(e+fx)+a)}-\frac{d(n+1)\left(a\int(d\sec(e+fx))^{-n-1}dx-\frac{a\int(d\sec(e+fx))^{-n}dx}{d}\right)}{a^2}\right)$$

$$\Big\downarrow \; 3042$$

$$(d\cos(e+fx))^n(d\sec(e+fx))^n\left(\frac{d\tan(e+fx)(d\sec(e+fx))^{-n-1}}{f(a\sec(e+fx)+a)}-\frac{d(n+1)\left(a\int\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{-n-1}dx-\frac{a\int\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{-n}dx}{d}\right)}{a^2}\right.$$

$$\Big\downarrow \; 4259$$

$$(d\cos(e+fx))^n(d\sec(e+fx))^n\left(\frac{d\tan(e+fx)(d\sec(e+fx))^{-n-1}}{f(a\sec(e+fx)+a)}-\frac{d(n+1)\left(a\left(\frac{\cos(e+fx)}{d}\right)^{-n}(d\sec(e+fx))^{-n}\int\left(\frac{\cos(e+fx)}{d}\right)^{n+1}dx-\right.}{a^2}\right.$$

$$\Big\downarrow \; 3042$$

$$(d\cos(e+fx))^n(d\sec(e+fx))^n\left(\frac{d\tan(e+fx)(d\sec(e+fx))^{-n-1}}{f(a\sec(e+fx)+a)}-\frac{d(n+1)\left(a\left(\frac{\cos(e+fx)}{d}\right)^{-n}(d\sec(e+fx))^{-n}\int\left(\frac{\sin\left(e+fx+\frac{\pi}{2}\right)}{d}\right)^{n+1}dx\right.}{a^2}\right.$$

$$\Big\downarrow \; 3122$$

$$(d\cos(e+fx))^n(d\sec(e+fx))^n\left(\frac{d\tan(e+fx)(d\sec(e+fx))^{-n-1}}{f(a\sec(e+fx)+a)}-\frac{d(n+1)\left(\frac{a\sin(e+fx)(d\sec(e+fx))^{-n-1}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n+1}{2},\frac{n+3}{2},\cos^2(e+\right.}{f(n+1)\sqrt{\sin^2(e+fx)}}\right.}{}\right.$$

input
```
Int[(d*Cos[e + f*x])^n/(a + a*Sec[e + f*x]),x]
```

3.444. $\int \frac{(d\cos(e+fx))^n}{a+a\sec(e+fx)}\,dx$

output
```
(d*Cos[e + f*x])^n*(d*Sec[e + f*x])^n*(-((d*(1 + n)*(-((a*d*Hypergeometric
2F1[1/2, (2 + n)/2, (4 + n)/2, Cos[e + f*x]^2]*(d*Sec[e + f*x])^(-2 - n)*S
in[e + f*x])/(f*(2 + n)*Sqrt[Sin[e + f*x]^2])) + (a*Hypergeometric2F1[1/2,
 (1 + n)/2, (3 + n)/2, Cos[e + f*x]^2]*(d*Sec[e + f*x])^(-1 - n)*Sin[e + f
*x])/(f*(1 + n)*Sqrt[Sin[e + f*x]^2])))/a^2) + (d*(d*Sec[e + f*x])^(-1 - n
)*Tan[e + f*x])/(f*(a + a*Sec[e + f*x])))
```

3.444.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

rule 4259
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)   Int[1/(Sin[c + d*x]/b)^n, x]), x] /;
FreeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4307
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-b)*d*Cot[e + f*x]*((d*Csc[e + f*x])^(n - 1)/(a*f*
(a + b*Csc[e + f*x]))), x] + Simp[d*((n - 1)/(a*b))   Int[(d*Csc[e + f*x])^
(n - 1)*(a - b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ
[a^2 - b^2, 0]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

3.444.4 Maple [F]

$$\int \frac{(d\cos(fx+e))^n}{a+a\sec(fx+e)}dx$$

input
```
int((d*cos(f*x+e))^n/(a+a*sec(f*x+e)),x)
```

output
```
int((d*cos(f*x+e))^n/(a+a*sec(f*x+e)),x)
```

3.444.5 Fricas [F]

$$\int \frac{(d\cos(e+fx))^n}{a+a\sec(e+fx)}\,dx = \int \frac{(d\cos(fx+e))^n}{a\sec(fx+e)+a}\,dx$$

input
```
integrate((d*cos(f*x+e))^n/(a+a*sec(f*x+e)),x, algorithm="fricas")
```

output
```
integral((d*cos(f*x + e))^n/(a*sec(f*x + e) + a), x)
```

3.444.6 Sympy [F]

$$\int \frac{(d\cos(e+fx))^n}{a+a\sec(e+fx)}\,dx = \frac{\int \frac{(d\cos(e+fx))^n}{\sec(e+fx)+1}\,dx}{a}$$

input
```
integrate((d*cos(f*x+e))**n/(a+a*sec(f*x+e)),x)
```

output
```
Integral((d*cos(e + f*x))**n/(sec(e + f*x) + 1), x)/a
```

3.444.7 Maxima [F]

$$\int \frac{(d\cos(e+fx))^n}{a+a\sec(e+fx)}\,dx = \int \frac{(d\cos(fx+e))^n}{a\sec(fx+e)+a}\,dx$$

input
```
integrate((d*cos(f*x+e))^n/(a+a*sec(f*x+e)),x, algorithm="maxima")
```

output
```
integrate((d*cos(f*x + e))^n/(a*sec(f*x + e) + a), x)
```

3.444.8 Giac [F]

$$\int \frac{(d\cos(e+fx))^n}{a+a\sec(e+fx)}\,dx = \int \frac{(d\cos(fx+e))^n}{a\sec(fx+e)+a}\,dx$$

input
```
integrate((d*cos(f*x+e))^n/(a+a*sec(f*x+e)),x, algorithm="giac")
```

output
```
integrate((d*cos(f*x + e))^n/(a*sec(f*x + e) + a), x)
```

3.444.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d\cos(e+fx))^n}{a+a\sec(e+fx)}\,dx = \int \frac{(d\,\cos(e+f\,x))^n}{a+\frac{a}{\cos(e+f\,x)}}\,dx$$

input
```
int((d*cos(e + f*x))^n/(a + a/cos(e + f*x)),x)
```

output
```
int((d*cos(e + f*x))^n/(a + a/cos(e + f*x)), x)
```

3.445 $\int \frac{(d\cos(e+fx))^n}{(a+a\sec(e+fx))^2}\,dx$

3.445.1 Optimal result . . . 3310
3.445.2 Mathematica [A] (verified) . . . 3311
3.445.3 Rubi [A] (verified) . . . 3311
3.445.4 Maple [F] . . . 3315
3.445.5 Fricas [F] . . . 3315
3.445.6 Sympy [F] . . . 3315
3.445.7 Maxima [F] . . . 3316
3.445.8 Giac [F] . . . 3316
3.445.9 Mupad [F(-1)] . . . 3316

3.445.1 Optimal result

Integrand size = 23, antiderivative size = 215

$$\begin{aligned}\int \frac{(d\cos(e+fx))^n}{(a+a\sec(e+fx))^2}\,dx &= \frac{2(2+n)(d\cos(e+fx))^n\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n}{2},\frac{2+n}{2},\cos^2(e+fx)\right)\sin(e+fx)}{3a^2f\sqrt{\sin^2(e+fx)}}\\ &\quad-\frac{(3+2n)\cos(e+fx)(d\cos(e+fx))^n\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+n}{2},\frac{3+n}{2},\cos^2(e+fx)\right)\sin(e+fx)}{3a^2f\sqrt{\sin^2(e+fx)}}\\ &\quad-\frac{2(2+n)(d\cos(e+fx))^n\tan(e+fx)}{3a^2f(1+\sec(e+fx))}-\frac{(d\cos(e+fx))^n\tan(e+fx)}{3f(a+a\sec(e+fx))^2}\end{aligned}$$

output
```
2/3*(2+n)*(d*cos(f*x+e))^n*hypergeom([1/2, 1/2*n],[1+1/2*n],cos(f*x+e)^2)*
sin(f*x+e)/a^2/f/(sin(f*x+e)^2)^(1/2)-1/3*(3+2*n)*cos(f*x+e)*(d*cos(f*x+e)
)^n*hypergeom([1/2, 1/2+1/2*n],[3/2+1/2*n],cos(f*x+e)^2)*sin(f*x+e)/a^2/f/
(sin(f*x+e)^2)^(1/2)-2/3*(2+n)*(d*cos(f*x+e))^n*tan(f*x+e)/a^2/f/(1+sec(f*
x+e))-1/3*(d*cos(f*x+e))^n*tan(f*x+e)/f/(a+a*sec(f*x+e))^2
```

3.445.2 Mathematica [A] (verified)

Time = 0.88 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.04

$$\int \frac{(d\cos(e+fx))^n}{(a+a\sec(e+fx))^2}\,dx = -\frac{(d\cos(e+fx))^{3+n}\sin(e+fx)}{3d^3f(a+a\cos(e+fx))^2} - \frac{\left((d(2+n)(4+n)(3+2n)(d\cos(e+fx))^{3+n}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3+n}{2},\frac{5+n}{2},\cos^2(e+fx)\right)-2(1+n\right.}{3a^2d^4f(3+n)(4+n)\sqrt{\text{si}}} + \frac{2(1+n)\cos^3(e+fx)(d\cos(e+fx))^n\tan\left(\frac{1}{2}(e+fx)\right)}{3a^2f}$$

input
```
Integrate[(d*Cos[e + f*x])^n/(a + a*Sec[e + f*x])^2,x]
```

output
```
-1/3*((d*Cos[e + f*x])^(3 + n)*Sin[e + f*x])/(d^3*f*(a + a*Cos[e + f*x])^2
) - ((d*(2 + n)*(4 + n)*(3 + 2*n)*(d*Cos[e + f*x])^(3 + n)*Hypergeometric2
F1[1/2, (3 + n)/2, (5 + n)/2, Cos[e + f*x]^2] - 2*(1 + n)*(3 + n)^2*(d*Cos
[e + f*x])^(4 + n)*Hypergeometric2F1[1/2, (4 + n)/2, (6 + n)/2, Cos[e + f*
x]^2])*Sin[e + f*x])/(3*a^2*d^4*f*(3 + n)*(4 + n)*Sqrt[Sin[e + f*x]^2]) +
(2*(1 + n)*Cos[e + f*x]^3*(d*Cos[e + f*x])^n*Tan[(e + f*x)/2])/(3*a^2*f)
```

3.445.3 Rubi [A] (verified)

Time = 1.14 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.14, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.565$, Rules used = {3042, 4752, 3042, 4304, 25, 3042, 4508, 3042, 4274, 3042, 4259, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d\cos(e+fx))^n}{(a\sec(e+fx)+a)^2}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\left(d\sin\left(e+fx+\frac{\pi}{2}\right)\right)^n}{\left(a\csc\left(e+fx+\frac{\pi}{2}\right)+a\right)^2}dx$$

$$\Big\downarrow \; 4752$$

$$(d\cos(e+fx))^n(d\sec(e+fx))^n\int \frac{(d\sec(e+fx))^{-n}}{(\sec(e+fx)a+a)^2}dx$$

$$\Big\downarrow \; 3042$$

$$(d\cos(e+fx))^n(d\sec(e+fx))^n\int\frac{\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{-n}}{\left(\csc\left(e+fx+\frac{\pi}{2}\right)a+a\right)^2}dx$$

$\Big\downarrow$ 4304

$$(d\cos(e+fx))^n(d\sec(e+fx))^n\left(-\frac{\int-\frac{(d\sec(e+fx))^{-n}(a(n+3)-a(n+1)\sec(e+fx))}{\sec(e+fx)a+a}dx}{3a^2}-\frac{\tan(e+fx)(d\sec(e+fx))^{-n}}{3f(a\sec(e+fx)+a)^2}\right)$$

$\Big\downarrow$ 25

$$(d\cos(e+fx))^n(d\sec(e+fx))^n\left(\frac{\int\frac{(d\sec(e+fx))^{-n}(a(n+3)-a(n+1)\sec(e+fx))}{\sec(e+fx)a+a}dx}{3a^2}-\frac{\tan(e+fx)(d\sec(e+fx))^{-n}}{3f(a\sec(e+fx)+a)^2}\right)$$

$\Big\downarrow$ 3042

$$(d\cos(e+fx))^n(d\sec(e+fx))^n\left(\frac{\int\frac{\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{-n}\left(a(n+3)-a(n+1)\csc\left(e+fx+\frac{\pi}{2}\right)\right)}{\csc\left(e+fx+\frac{\pi}{2}\right)a+a}dx}{3a^2}-\frac{\tan(e+fx)(d\sec(e+fx))^{-n}}{3f(a\sec(e+fx)+a)^2}\right)$$

$\Big\downarrow$ 4508

$$(d\cos(e+fx))^n(d\sec(e+fx))^n\left(\frac{\frac{\int(d\sec(e+fx))^{-n}\left(a^2(n+1)(2n+3)-2a^2n(n+2)\sec(e+fx)\right)dx}{a^2}-\frac{2(n+2)\tan(e+fx)(d\sec(e+fx))^{-n}}{f(\sec(e+fx)+1)}}{3a^2}-\frac{\tan(e+fx)(d\sec(e+}{3f(a\sec(e+fx)}\right.$$

$\Big\downarrow$ 3042

$$(d\cos(e+fx))^n(d\sec(e+fx))^n\left(\frac{\frac{\int\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{-n}\left(a^2(n+1)(2n+3)-2a^2n(n+2)\csc\left(e+fx+\frac{\pi}{2}\right)\right)dx}{a^2}-\frac{2(n+2)\tan(e+fx)(d\sec(e+fx))^{-n}}{f(\sec(e+fx)+1)}}{3a^2}-\frac{\tan(e+fx)(d}{3f(a\sec(e}\right.$$

$\Big\downarrow$ 4274

$$(d\cos(e+fx))^n(d\sec(e+fx))^n\left(\frac{\frac{a^2(n+1)(2n+3)\int(d\sec(e+fx))^{-n}dx-\frac{2a^2n(n+2)\int(d\sec(e+fx))^{1-n}dx}{d}}{a^2}-\frac{2(n+2)\tan(e+fx)(d\sec(e+fx))^{-n}}{f(\sec(e+fx)+1)}}{3a^2}-\frac{\tan(e+fx)(d\sec}{3f(a\sec(e+}\right.$$

$\Big\downarrow$ 3042

3.445. $\int\frac{(d\cos(e+fx))^n}{(a+a\sec(e+fx))^2}dx$

$$(d\cos(e+fx))^n(d\sec(e+fx))^n\left(\frac{\frac{a^2(n+1)(2n+3)\int\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{-n}dx-\frac{2a^2n(n+2)\int\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{1-n}dx}{d}}{a^2}-\frac{2(n+2)\tan(e+fx)(d\sec(e+fx))^{-n}}{f(\sec(e+fx)+1)}}{3a^2}-\frac{\tan(e+}{3f(a}\right.$$

$\Big\downarrow$ 4259

$$(d\cos(e+fx))^n(d\sec(e+fx))^n\left(\frac{\frac{a^2(n+1)(2n+3)\left(\frac{\cos(e+fx)}{d}\right)^{-n}(d\sec(e+fx))^{-n}\int\left(\frac{\cos(e+fx)}{d}\right)^{n}dx-\frac{2a^2n(n+2)\left(\frac{\cos(e+fx)}{d}\right)^{-n}(d\sec(e+fx))^{-n}\int\left(\frac{\cos(e+fx)}{d}\right)^{n-1}dx}{d}}{a^2}}{3a^2}-\right.$$

$\Big\downarrow$ 3042

$$(d\cos(e+fx))^n(d\sec(e+fx))^n\left(\frac{\frac{a^2(n+1)(2n+3)\left(\frac{\cos(e+fx)}{d}\right)^{-n}(d\sec(e+fx))^{-n}\int\left(\frac{\sin\left(e+fx+\frac{\pi}{2}\right)}{d}\right)^{n}dx-\frac{2a^2n(n+2)\left(\frac{\cos(e+fx)}{d}\right)^{-n}(d\sec(e+fx))^{-n}\int\left(\frac{\sin\left(e+fx+\frac{\pi}{2}\right)}{d}\right)^{n}}{d}}{a^2}}{3a^2}\right.$$

$\Big\downarrow$ 3122

$$(d\cos(e+fx))^n(d\sec(e+fx))^n\left(\frac{\frac{\frac{2a^2(n+2)\sin(e+fx)(d\sec(e+fx))^{-n}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n}{2},\frac{n+2}{2},\cos^2(e+fx)\right)}{f\sqrt{\sin^2(e+fx)}}-\frac{a^2d(2n+3)\sin(e+fx)(d\sec(e+fx))^{-n-1}\text{Hypergeometric2F1}\left(\frac{1}{2},\right.}{f\sqrt{\sin^2(e+fx)}}}{a^2}}{3a^2}\right.$$

input
```
Int[(d*Cos[e + f*x])^n/(a + a*Sec[e + f*x])^2,x]
```

output
```
(d*Cos[e + f*x])^n*(d*Sec[e + f*x])^n*(-1/3*Tan[e + f*x]/(f*(d*Sec[e + f*x
])^n*(a + a*Sec[e + f*x])^2) + ((-((a^2*d*(3 + 2*n)*Hypergeometric2F1[1/2,
 (1 + n)/2, (3 + n)/2, Cos[e + f*x]^2]*(d*Sec[e + f*x])^(-1 - n)*Sin[e + f
*x])/(f*Sqrt[Sin[e + f*x]^2])) + (2*a^2*(2 + n)*Hypergeometric2F1[1/2, n/2
, (2 + n)/2, Cos[e + f*x]^2]*Sin[e + f*x])/(f*(d*Sec[e + f*x])^n*Sqrt[Sin[
e + f*x]^2]))/a^2 - (2*(2 + n)*Tan[e + f*x])/(f*(d*Sec[e + f*x])^n*(1 + Se
c[e + f*x])))/(3*a^2))
```

3.445. $\int \frac{(d\cos(e+fx))^n}{(a+a\sec(e+fx))^2}\,dx$

3.445.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 4259
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)   Int[1/(Sin[c + d*x]/b)^n, x]), x] /;
FreeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4304
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-Cot[e + f*x])*(a + b*Csc[e + f*x])^m*((d*Csc
[e + f*x])^n/(f*(2*m + 1))), x] + Simp[1/(a^2*(2*m + 1))   Int[(a + b*Csc[e
 + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e
+ f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ
[m, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

rule 4508
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
- a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
 1))), x] - Simp[1/(a^2*(2*m + 1))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Cs
c[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[
e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B
, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

3.445.4 Maple [F]

$$\int \frac{(d\cos(fx+e))^n}{(a+a\sec(fx+e))^2}dx$$

input
```
int((d*cos(f*x+e))^n/(a+a*sec(f*x+e))^2,x)
```

output
```
int((d*cos(f*x+e))^n/(a+a*sec(f*x+e))^2,x)
```

3.445.5 Fricas [F]

$$\int \frac{(d\cos(e+fx))^n}{(a+a\sec(e+fx))^2}\,dx = \int \frac{(d\cos(fx+e))^n}{(a\sec(fx+e)+a)^2}\,dx$$

input
```
integrate((d*cos(f*x+e))^n/(a+a*sec(f*x+e))^2,x, algorithm="fricas")
```

output
```
integral((d*cos(f*x + e))^n/(a^2*sec(f*x + e)^2 + 2*a^2*sec(f*x + e) + a^2
), x)
```

3.445.6 Sympy [F]

$$\int \frac{(d\cos(e+fx))^n}{(a+a\sec(e+fx))^2}\,dx = \frac{\int \frac{(d\cos(e+fx))^n}{\sec^2(e+fx)+2\sec(e+fx)+1}\,dx}{a^2}$$

input
```
integrate((d*cos(f*x+e))**n/(a+a*sec(f*x+e))**2,x)
```

output
```
Integral((d*cos(e + f*x))**n/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x)/a*
*2
```

3.445.7 Maxima [F]

$$\int \frac{(d\cos(e+fx))^n}{(a+a\sec(e+fx))^2}\,dx = \int \frac{(d\cos(fx+e))^n}{(a\sec(fx+e)+a)^2}\,dx$$

input
```
integrate((d*cos(f*x+e))^n/(a+a*sec(f*x+e))^2,x, algorithm="maxima")
```

output
```
integrate((d*cos(f*x + e))^n/(a*sec(f*x + e) + a)^2, x)
```

3.445.8 Giac [F]

$$\int \frac{(d\cos(e+fx))^n}{(a+a\sec(e+fx))^2}\,dx = \int \frac{(d\cos(fx+e))^n}{(a\sec(fx+e)+a)^2}\,dx$$

input
```
integrate((d*cos(f*x+e))^n/(a+a*sec(f*x+e))^2,x, algorithm="giac")
```

output
```
integrate((d*cos(f*x + e))^n/(a*sec(f*x + e) + a)^2, x)
```

3.445.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d\cos(e+fx))^n}{(a+a\sec(e+fx))^2}\,dx = \int \frac{(d\,\cos(e+f\,x))^n}{\left(a+\frac{a}{\cos(e+f\,x)}\right)^2}\,dx$$

input
```
int((d*cos(e + f*x))^n/(a + a/cos(e + f*x))^2,x)
```

output
```
int((d*cos(e + f*x))^n/(a + a/cos(e + f*x))^2, x)
```

3.446 $\int \sec^4(c+dx)(a+b\sec(c+dx))\,dx$

3.446.1 Optimal result . . . 3317
3.446.2 Mathematica [A] (verified) . . . 3317
3.446.3 Rubi [A] (verified) . . . 3318
3.446.4 Maple [A] (verified) . . . 3320
3.446.5 Fricas [A] (verification not implemented) . . . 3320
3.446.6 Sympy [F] . . . 3321
3.446.7 Maxima [A] (verification not implemented) . . . 3321
3.446.8 Giac [B] (verification not implemented) . . . 3322
3.446.9 Mupad [B] (verification not implemented) . . . 3322

3.446.1 Optimal result

Integrand size = 19, antiderivative size = 85

$$\int \sec^4(c+dx)(a+b\sec(c+dx))\,dx = \frac{3b\text{arctanh}(\sin(c+dx))}{8d}+\frac{a\tan(c+dx)}{d}+\frac{3b\sec(c+dx)\tan(c+dx)}{8d}+\frac{b\sec^3(c+dx)\tan(c+dx)}{4d}+\frac{a\tan^3(c+dx)}{3d}$$

output
```
3/8*b*arctanh(sin(d*x+c))/d+a*tan(d*x+c)/d+3/8*b*sec(d*x+c)*tan(d*x+c)/d+1
/4*b*sec(d*x+c)^3*tan(d*x+c)/d+1/3*a*tan(d*x+c)^3/d
```

3.446.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.69

$$\int \sec^4(c+dx)(a+b\sec(c+dx))\,dx = \frac{9b\text{arctanh}(\sin(c+dx))+\tan(c+dx)\left(9b\sec(c+dx)+6b\sec^3(c+dx)+8a(3+\tan^2(c+dx))\right)}{24d}$$

input
```
Integrate[Sec[c + d*x]^4*(a + b*Sec[c + d*x]),x]
```

output
```
(9*b*ArcTanh[Sin[c + d*x]] + Tan[c + d*x]*(9*b*Sec[c + d*x] + 6*b*Sec[c +
d*x]^3 + 8*a*(3 + Tan[c + d*x]^2)))/(24*d)
```

3.446.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.06, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.526, Rules used = {3042, 4274, 3042, 4254, 2009, 4255, 3042, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^4(c+dx)(a+b\sec(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^4\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{4274}$$

$$a\int \sec^4(c+dx)dx+b\int \sec^5(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$a\int \csc\left(c+dx+\frac{\pi}{2}\right)^4dx+b\int \csc\left(c+dx+\frac{\pi}{2}\right)^5dx$$

$$\Big\downarrow \text{4254}$$

$$b\int \csc\left(c+dx+\frac{\pi}{2}\right)^5dx-\frac{a\int\left(\tan^2(c+dx)+1\right)d(-\tan(c+dx))}{d}$$

$$\Big\downarrow \text{2009}$$

$$b\int \csc\left(c+dx+\frac{\pi}{2}\right)^5dx-\frac{a\left(-\frac{1}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{d}$$

$$\Big\downarrow \text{4255}$$

$$b\left(\frac{3}{4}\int \sec^3(c+dx)dx+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)-\frac{a\left(-\frac{1}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{d}$$

$$\Big\downarrow \text{3042}$$

$$b\left(\frac{3}{4}\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)-\frac{a\left(-\frac{1}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{d}$$

$$\Big\downarrow \text{4255}$$

$$b\left(\frac{3}{4}\left(\frac{1}{2}\int \sec(c+dx)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)-\frac{a\left(-\frac{1}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{d}$$

$$\Big\downarrow \text{3042}$$

$$b\left(\frac{3}{4}\left(\frac{1}{2}\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)-\frac{a\left(-\frac{1}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{d}$$

$$\Big\downarrow \text{4257}$$

$$b\left(\frac{3}{4}\left(\frac{\text{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)-\frac{a\left(-\frac{1}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{d}$$

input
```
Int[Sec[c + d*x]^4*(a + b*Sec[c + d*x]),x]
```

output
```
-((a*(-Tan[c + d*x] - Tan[c + d*x]^3/3))/d) + b*((Sec[c + d*x]^3*Tan[c + d
*x])/(4*d) + (3*(ArcTanh[Sin[c + d*x]]/(2*d) + (Sec[c + d*x]*Tan[c + d*x])
/(2*d)))/4)
```

3.446.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

3.446.4 Maple [A] (verified)

Time = 1.10 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.86

method	result
derivativedivides	$\frac{-a\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right)\tan(dx+c)+b\left(-\left(-\frac{\sec(dx+c)^3}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)}{d}$
default	$\frac{-a\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right)\tan(dx+c)+b\left(-\left(-\frac{\sec(dx+c)^3}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)}{d}$
parts	$-\frac{a\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right)\tan(dx+c)}{d}+\frac{b\left(-\left(-\frac{\sec(dx+c)^3}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)}{d}$
risch	$-\frac{i\left(9b\,e^{7i(dx+c)}+33b\,e^{5i(dx+c)}-48a\,e^{4i(dx+c)}-33b\,e^{3i(dx+c)}-64a\,e^{2i(dx+c)}-9b\,e^{i(dx+c)}-16a\right)}{12d\left(e^{2i(dx+c)}+1\right)^4}-\frac{3b\ln\left(e^{i(dx+c)}-i\right)}{8d}$
norman	$\frac{-\frac{(8a-5b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{4d}+\frac{(8a+5b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d}-\frac{(40a-9b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{12d}+\frac{(40a+9b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{12d}}{\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^4}-\frac{3b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{8d}$
parallelrisch	$\frac{-18\left(\frac{3}{4}+\frac{\cos(4dx+4c)}{4}+\cos(2dx+2c)\right)b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+18\left(\frac{3}{4}+\frac{\cos(4dx+4c)}{4}+\cos(2dx+2c)\right)b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+}{12d(3+\cos(4dx+4c)+4\cos(2dx+2c))}$

input
```
int(sec(d*x+c)^4*(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-a*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c)+b*(-(-1/4*sec(d*x+c)^3-3/8*sec(
d*x+c))*tan(d*x+c)+3/8*ln(sec(d*x+c)+tan(d*x+c))))
```

3.446.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.16

$$\int \sec^4(c+dx)(a+b\sec(c+dx))\,dx$$
$$=\frac{9\,b\cos(dx+c)^4\log(\sin(dx+c)+1)-9\,b\cos(dx+c)^4\log(-\sin(dx+c)+1)+2\left(16\,a\cos(dx+c)^3+\right.}{48\,d\cos(dx+c)^4}$$

input
```
integrate(sec(d*x+c)^4*(a+b*sec(d*x+c)),x, algorithm="fricas")
```

output
```
1/48*(9*b*cos(d*x + c)^4*log(sin(d*x + c) + 1) - 9*b*cos(d*x + c)^4*log(-s
in(d*x + c) + 1) + 2*(16*a*cos(d*x + c)^3 + 9*b*cos(d*x + c)^2 + 8*a*cos(d
*x + c) + 6*b)*sin(d*x + c))/(d*cos(d*x + c)^4)
```

3.446.6 Sympy [F]

$$\int \sec^4(c+dx)(a+b\sec(c+dx))\,dx = \int (a+b\sec(c+dx))\sec^4(c+dx)\,dx$$

input
```
integrate(sec(d*x+c)**4*(a+b*sec(d*x+c)),x)
```

output
```
Integral((a + b*sec(c + d*x))*sec(c + d*x)**4, x)
```

3.446.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.12

$$\int \sec^4(c+dx)(a+b\sec(c+dx))\,dx$$
$$= \frac{16\left(\tan(dx+c)^3+3\tan(dx+c)\right)a-3\,b\left(\frac{2\left(3\sin(dx+c)^3-5\sin(dx+c)\right)}{\sin(dx+c)^4-2\sin(dx+c)^2+1}-3\log(\sin(dx+c)+1)+3\log(\sin\right.}{48\,d}$$

input
```
integrate(sec(d*x+c)^4*(a+b*sec(d*x+c)),x, algorithm="maxima")
```

output
```
1/48*(16*(tan(d*x + c)^3 + 3*tan(d*x + c))*a - 3*b*(2*(3*sin(d*x + c)^3 -
5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x +
c) + 1) + 3*log(sin(d*x + c) - 1)))/d
```

3.446.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 164 vs. $2(77) = 154$.

Time = 0.30 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.93

$$\int \sec^4(c+dx)(a+b\sec(c+dx))\,dx$$
$$= \frac{9\,b\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-9\,b\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(24\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-15\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-40\,a\,t}{}}{24\,d}$$

input
```
integrate(sec(d*x+c)^4*(a+b*sec(d*x+c)),x, algorithm="giac")
```

output
```
1/24*(9*b*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 9*b*log(abs(tan(1/2*d*x + 1
/2*c) - 1)) - 2*(24*a*tan(1/2*d*x + 1/2*c)^7 - 15*b*tan(1/2*d*x + 1/2*c)^7
 - 40*a*tan(1/2*d*x + 1/2*c)^5 - 9*b*tan(1/2*d*x + 1/2*c)^5 + 40*a*tan(1/2
*d*x + 1/2*c)^3 - 9*b*tan(1/2*d*x + 1/2*c)^3 - 24*a*tan(1/2*d*x + 1/2*c) -
 15*b*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^4)/d
```

3.446.9 Mupad [B] (verification not implemented)

Time = 17.07 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.79

$$\int \sec^4(c+dx)(a+b\sec(c+dx))\,dx$$
$$= \frac{\left(\frac{5\,b}{4}-2\,a\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+\left(\frac{10\,a}{3}+\frac{3\,b}{4}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+\left(\frac{3\,b}{4}-\frac{10\,a}{3}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+\left(2\,a+\frac{5\,b}{4}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right.}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8-4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}$$
$$+\frac{3\,b\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{4\,d}$$

input
```
int((a + b/cos(c + d*x))/cos(c + d*x)^4,x)
```

output
```
(tan(c/2 + (d*x)/2)*(2*a + (5*b)/4) - tan(c/2 + (d*x)/2)^7*(2*a - (5*b)/4)
 - tan(c/2 + (d*x)/2)^3*((10*a)/3 - (3*b)/4) + tan(c/2 + (d*x)/2)^5*((10*a
)/3 + (3*b)/4))/(d*(6*tan(c/2 + (d*x)/2)^4 - 4*tan(c/2 + (d*x)/2)^2 - 4*ta
n(c/2 + (d*x)/2)^6 + tan(c/2 + (d*x)/2)^8 + 1)) + (3*b*atanh(tan(c/2 + (d*
x)/2)))/(4*d)
```

3.447 $\int \sec^3(c+dx)(a+b\sec(c+dx))\,dx$

3.447.1 Optimal result . . . 3323
3.447.2 Mathematica [A] (verified) . . . 3323
3.447.3 Rubi [A] (verified) . . . 3324
3.447.4 Maple [A] (verified) . . . 3326
3.447.5 Fricas [A] (verification not implemented) . . . 3326
3.447.6 Sympy [F] . . . 3327
3.447.7 Maxima [A] (verification not implemented) . . . 3327
3.447.8 Giac [B] (verification not implemented) . . . 3327
3.447.9 Mupad [B] (verification not implemented) . . . 3328

3.447.1 Optimal result

Integrand size = 19, antiderivative size = 63

$$\int \sec^3(c+dx)(a+b\sec(c+dx))\,dx = \frac{a\operatorname{arctanh}(\sin(c+dx))}{2d}+\frac{b\tan(c+dx)}{d}+\frac{a\sec(c+dx)\tan(c+dx)}{2d}+\frac{b\tan^3(c+dx)}{3d}$$

output
```
1/2*a*arctanh(sin(d*x+c))/d+b*tan(d*x+c)/d+1/2*a*sec(d*x+c)*tan(d*x+c)/d+1
/3*b*tan(d*x+c)^3/d
```

3.447.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.95

$$\int \sec^3(c+dx)(a+b\sec(c+dx))\,dx = \frac{a\operatorname{arctanh}(\sin(c+dx))}{2d}+\frac{a\sec(c+dx)\tan(c+dx)}{2d}+\frac{b\left(\tan(c+dx)+\frac{1}{3}\tan^3(c+dx)\right)}{d}$$

input
```
Integrate[Sec[c + d*x]^3*(a + b*Sec[c + d*x]),x]
```

output
```
(a*ArcTanh[Sin[c + d*x]])/(2*d) + (a*Sec[c + d*x]*Tan[c + d*x])/(2*d) + (b
*(Tan[c + d*x] + Tan[c + d*x]^3/3))/d
```

3.447.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.02, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {3042, 4274, 3042, 4254, 2009, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^3(c+dx)(a+b\sec(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^3\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{4274}$$

$$a\int \sec^3(c+dx)dx+b\int \sec^4(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$a\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx+b\int \csc\left(c+dx+\frac{\pi}{2}\right)^4dx$$

$$\Big\downarrow \text{4254}$$

$$a\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx-\frac{b\int\left(\tan^2(c+dx)+1\right)d(-\tan(c+dx))}{d}$$

$$\Big\downarrow \text{2009}$$

$$a\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx-\frac{b\left(-\frac{1}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{d}$$

$$\Big\downarrow \text{4255}$$

$$a\left(\frac{1}{2}\int \sec(c+dx)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)-\frac{b\left(-\frac{1}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{d}$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{1}{2}\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)-\frac{b\left(-\frac{1}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{d}$$

$$\Big\downarrow \text{4257}$$

$$a\left(\frac{\operatorname{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)-\frac{b\left(-\frac{1}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{d}$$

input
```
Int[Sec[c + d*x]^3*(a + b*Sec[c + d*x]),x]
```

output
```
a*(ArcTanh[Sin[c + d*x]]/(2*d) + (Sec[c + d*x]*Tan[c + d*x])/(2*d)) - (b*(
-Tan[c + d*x] - Tan[c + d*x]^3/3))/d
```

3.447.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

3.447.4 Maple [A] (verified)

Time = 0.96 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.95

method	result
derivativedivides	$\frac{a\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)-b\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right)\tan(dx+c)}{d}$
default	$\frac{a\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)-b\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right)\tan(dx+c)}{d}$
parts	$\frac{a\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)}{d}-\frac{b\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right)\tan(dx+c)}{d}$
risch	$-\frac{i\left(3a\,e^{5i(dx+c)}-12b\,e^{2i(dx+c)}-3\,e^{i(dx+c)}a-4b\right)}{3d\left(e^{2i(dx+c)}+1\right)^3}+\frac{a\ln\left(e^{i(dx+c)}+i\right)}{2d}-\frac{a\ln\left(e^{i(dx+c)}-i\right)}{2d}$
norman	$\frac{\frac{(a-2b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{d}+\frac{4b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3d}-\frac{(a+2b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}}{\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^3}-\frac{a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2d}+\frac{a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{2d}$
parallelrisch	$\frac{-9\left(\cos(dx+c)+\frac{\cos(3dx+3c)}{3}\right)a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+9\left(\cos(dx+c)+\frac{\cos(3dx+3c)}{3}\right)a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+6a\sin(2dx+2}{6d(\cos(3dx+3c)+3\cos(dx+c))}$

input
```
int(sec(d*x+c)^3*(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(a*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c)))-b*(-2/3-1
/3*sec(d*x+c)^2)*tan(d*x+c))
```

3.447.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.40

$$\int \sec^3(c+dx)(a+b\sec(c+dx))\,dx$$
$$=\frac{3\,a\cos\left(dx+c\right)^3\log\left(\sin\left(dx+c\right)+1\right)-3\,a\cos\left(dx+c\right)^3\log\left(-\sin\left(dx+c\right)+1\right)+2\left(4\,b\cos\left(dx+c\right)^2+3\right.}{12\,d\cos\left(dx+c\right)^3}$$

input
```
integrate(sec(d*x+c)^3*(a+b*sec(d*x+c)),x, algorithm="fricas")
```

output
```
1/12*(3*a*cos(d*x + c)^3*log(sin(d*x + c) + 1) - 3*a*cos(d*x + c)^3*log(-s
in(d*x + c) + 1) + 2*(4*b*cos(d*x + c)^2 + 3*a*cos(d*x + c) + 2*b)*sin(d*x
 + c))/(d*cos(d*x + c)^3)
```

3.447.6 Sympy [F]

$$\int \sec^3(c+dx)(a+b\sec(c+dx))\,dx = \int (a+b\sec(c+dx))\sec^3(c+dx)\,dx$$

input
```
integrate(sec(d*x+c)**3*(a+b*sec(d*x+c)),x)
```

output
```
Integral((a + b*sec(c + d*x))*sec(c + d*x)**3, x)
```

3.447.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.11

$$\int \sec^3(c+dx)(a+b\sec(c+dx))\,dx = \frac{4\left(\tan(dx+c)^3+3\tan(dx+c)\right)b-3a\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1}-\log(\sin(dx+c)+1)+\log(\sin(dx+c)-1)\right)}{12d}$$

input
```
integrate(sec(d*x+c)^3*(a+b*sec(d*x+c)),x, algorithm="maxima")
```

output
```
1/12*(4*(tan(d*x + c)^3 + 3*tan(d*x + c))*b - 3*a*(2*sin(d*x + c)/(sin(d*x
 + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)))/d
```

3.447.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 122 vs. 2(57) = 114.

Time = 0.32 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.94

$$\int \sec^3(c+dx)(a+b\sec(c+dx))\,dx = \frac{3a\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)-3a\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)+\frac{2\left(3a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-6b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+4b\tan\right.}{\left(\tan\left(\frac{1}{2}dx\right.\right.}}{6d}$$

input
```
integrate(sec(d*x+c)^3*(a+b*sec(d*x+c)),x, algorithm="giac")
```

output
```
1/6*(3*a*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*a*log(abs(tan(1/2*d*x + 1/
2*c) - 1)) + 2*(3*a*tan(1/2*d*x + 1/2*c)^5 - 6*b*tan(1/2*d*x + 1/2*c)^5 +
4*b*tan(1/2*d*x + 1/2*c)^3 - 3*a*tan(1/2*d*x + 1/2*c) - 6*b*tan(1/2*d*x +
1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^3)/d
```

3.447.9 Mupad [B] (verification not implemented)

Time = 15.39 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.73

$$\int \sec^3(c+dx)(a+b\sec(c+dx))\,dx$$
$$= \frac{(a-2\,b)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+\frac{4\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{3}+(-a-2\,b)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)}$$
$$+\frac{a\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}$$

input
```
int((a + b/cos(c + d*x))/cos(c + d*x)^3,x)
```

output
```
(tan(c/2 + (d*x)/2)^5*(a - 2*b) - tan(c/2 + (d*x)/2)*(a + 2*b) + (4*b*tan(
c/2 + (d*x)/2)^3)/3)/(d*(3*tan(c/2 + (d*x)/2)^2 - 3*tan(c/2 + (d*x)/2)^4 +
 tan(c/2 + (d*x)/2)^6 - 1)) + (a*atanh(tan(c/2 + (d*x)/2)))/d
```

3.448 $\int \sec^2(c+dx)(a+b\sec(c+dx))\,dx$

3.448.1 Optimal result . . . 3329
3.448.2 Mathematica [A] (verified) . . . 3329
3.448.3 Rubi [A] (verified) . . . 3330
3.448.4 Maple [A] (verified) . . . 3332
3.448.5 Fricas [A] (verification not implemented) . . . 3332
3.448.6 Sympy [F] . . . 3333
3.448.7 Maxima [A] (verification not implemented) . . . 3333
3.448.8 Giac [B] (verification not implemented) . . . 3333
3.448.9 Mupad [B] (verification not implemented) . . . 3334

3.448.1 Optimal result

Integrand size = 19, antiderivative size = 47

$$\int \sec^2(c+dx)(a+b\sec(c+dx))\,dx=\frac{b\operatorname{arctanh}(\sin(c+dx))}{2d}+\frac{a\tan(c+dx)}{d}+\frac{b\sec(c+dx)\tan(c+dx)}{2d}$$

output
```
1/2*b*arctanh(sin(d*x+c))/d+a*tan(d*x+c)/d+1/2*b*sec(d*x+c)*tan(d*x+c)/d
```

3.448.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00

$$\int \sec^2(c+dx)(a+b\sec(c+dx))\,dx=\frac{b\operatorname{arctanh}(\sin(c+dx))}{2d}+\frac{a\tan(c+dx)}{d}+\frac{b\sec(c+dx)\tan(c+dx)}{2d}$$

input
```
Integrate[Sec[c + d*x]^2*(a + b*Sec[c + d*x]),x]
```

output
```
(b*ArcTanh[Sin[c + d*x]])/(2*d) + (a*Tan[c + d*x])/d + (b*Sec[c + d*x]*Tan
[c + d*x])/(2*d)
```

3.448.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.02, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {3042, 4274, 3042, 4254, 24, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(c+dx)(a+b\sec(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^2\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{4274}$$

$$a\int \sec^2(c+dx)dx+b\int \sec^3(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$a\int \csc\left(c+dx+\frac{\pi}{2}\right)^2dx+b\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx$$

$$\Big\downarrow \text{4254}$$

$$b\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx-\frac{a\int 1d(-\tan(c+dx))}{d}$$

$$\Big\downarrow \text{24}$$

$$b\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx+\frac{a\tan(c+dx)}{d}$$

$$\Big\downarrow \text{4255}$$

$$b\left(\frac{1}{2}\int \sec(c+dx)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{a\tan(c+dx)}{d}$$

$$\Big\downarrow \text{3042}$$

$$b\left(\frac{1}{2}\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{a\tan(c+dx)}{d}$$

$$\Big\downarrow \text{4257}$$

$$\frac{a\tan(c+dx)}{d}+b\left(\frac{\text{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)$$

input
```
Int[Sec[c + d*x]^2*(a + b*Sec[c + d*x]),x]
```

output
```
(a*Tan[c + d*x])/d + b*(ArcTanh[Sin[c + d*x]]/(2*d) + (Sec[c + d*x]*Tan[c
+ d*x])/(2*d))
```

3.448.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

3.448.4 Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00

method	result	s
derivativedivides	$\frac{\tan(dx+c)a+b\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)}{d}$	4
default	$\frac{\tan(dx+c)a+b\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)}{d}$	4
parts	$\frac{a\tan(dx+c)}{d}+\frac{b\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)}{d}$	4
parallelrisch	$\frac{-b(1+\cos(2dx+2c))\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+b(1+\cos(2dx+2c))\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+2a\sin(2dx+2c)+2\sin(dx+c)b}{2d(1+\cos(2dx+2c))}$	9
norman	$\frac{\frac{(2a+b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}-\frac{(2a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{d}}{\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^2}-\frac{b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2d}+\frac{b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{2d}$	9
risch	$-\frac{i\left(b\,e^{3i(dx+c)}-2a\,e^{2i(dx+c)}-b\,e^{i(dx+c)}-2a\right)}{d\left(e^{2i(dx+c)}+1\right)^2}+\frac{b\ln\left(e^{i(dx+c)}+i\right)}{2d}-\frac{b\ln\left(e^{i(dx+c)}-i\right)}{2d}$	9

input
```
int(sec(d*x+c)^2*(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(tan(d*x+c)*a+b*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c
))))
```

3.448.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.57

$$\int \sec^2(c+dx)(a+b\sec(c+dx))\,dx$$
$$= \frac{b\cos(dx+c)^2\log(\sin(dx+c)+1)-b\cos(dx+c)^2\log(-\sin(dx+c)+1)+2(2a\cos(dx+c)+b)\sin}{4d\cos(dx+c)^2}$$

input
```
integrate(sec(d*x+c)^2*(a+b*sec(d*x+c)),x, algorithm="fricas")
```

output
```
1/4*(b*cos(d*x + c)^2*log(sin(d*x + c) + 1) - b*cos(d*x + c)^2*log(-sin(d*
x + c) + 1) + 2*(2*a*cos(d*x + c) + b)*sin(d*x + c))/(d*cos(d*x + c)^2)
```

3.448.6 Sympy [F]

$$\int \sec^2(c+dx)(a+b\sec(c+dx))\,dx = \int (a+b\sec(c+dx))\sec^2(c+dx)\,dx$$

input
```
integrate(sec(d*x+c)**2*(a+b*sec(d*x+c)),x)
```

output
```
Integral((a + b*sec(c + d*x))*sec(c + d*x)**2, x)
```

3.448.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.23

$$\int \sec^2(c+dx)(a+b\sec(c+dx))\,dx = -\frac{b\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1} - \log(\sin(dx+c)+1) + \log(\sin(dx+c)-1)\right) - 4\,a\tan(dx+c)}{4\,d}$$

input
```
integrate(sec(d*x+c)^2*(a+b*sec(d*x+c)),x, algorithm="maxima")
```

output
```
-1/4*(b*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log
(sin(d*x + c) - 1)) - 4*a*tan(d*x + c))/d
```

3.448.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 107 vs. 2(43) = 86.

Time = 0.30 (sec) , antiderivative size = 107, normalized size of antiderivative = 2.28

$$\int \sec^2(c+dx)(a+b\sec(c+dx))\,dx = \frac{b\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right) - b\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right) - \frac{2\left(2\,a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3 - b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3 - 2\,a\tan\left(\frac{1}{2}dx+\right.\right.}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)^2}}{2\,d}$$

input
```
integrate(sec(d*x+c)^2*(a+b*sec(d*x+c)),x, algorithm="giac")
```

output
```
1/2*(b*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - b*log(abs(tan(1/2*d*x + 1/2*c)
 - 1)) - 2*(2*a*tan(1/2*d*x + 1/2*c)^3 - b*tan(1/2*d*x + 1/2*c)^3 - 2*a*ta
n(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^
2)/d
```

3.448.9 Mupad [B] (verification not implemented)

Time = 14.16 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.81

$$\int \sec^2(c+dx)(a+b\sec(c+dx))\,dx = \frac{b\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d} - \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3(2\,a-b)-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\,(2\,a+b)}{d\,\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4-2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)}$$

input
```
int((a + b/cos(c + d*x))/cos(c + d*x)^2,x)
```

output
```
(b*atanh(tan(c/2 + (d*x)/2)))/d - (tan(c/2 + (d*x)/2)^3*(2*a - b) - tan(c/
2 + (d*x)/2)*(2*a + b))/(d*(tan(c/2 + (d*x)/2)^4 - 2*tan(c/2 + (d*x)/2)^2
+ 1))
```

3.449 $\int \sec(c+dx)(a+b\sec(c+dx))\,dx$

3.449.1 Optimal result . . . 3335
3.449.2 Mathematica [A] (verified) . . . 3335
3.449.3 Rubi [A] (verified) . . . 3336
3.449.4 Maple [A] (verified) . . . 3337
3.449.5 Fricas [B] (verification not implemented) . . . 3338
3.449.6 Sympy [A] (verification not implemented) . . . 3338
3.449.7 Maxima [A] (verification not implemented) . . . 3339
3.449.8 Giac [B] (verification not implemented) . . . 3339
3.449.9 Mupad [B] (verification not implemented) . . . 3339

3.449.1 Optimal result

Integrand size = 17, antiderivative size = 24

$$\int \sec(c+dx)(a+b\sec(c+dx))\,dx=\frac{a\operatorname{arctanh}(\sin(c+dx))}{d}+\frac{b\tan(c+dx)}{d}$$

output
```
a*arctanh(sin(d*x+c))/d+b*tan(d*x+c)/d
```

3.449.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \sec(c+dx)(a+b\sec(c+dx))\,dx=\frac{a\operatorname{arctanh}(\sin(c+dx))}{d}+\frac{b\tan(c+dx)}{d}$$

input
```
Integrate[Sec[c + d*x]*(a + b*Sec[c + d*x]),x]
```

output
```
(a*ArcTanh[Sin[c + d*x]])/d + (b*Tan[c + d*x])/d
```

3.449.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {3042, 4274, 3042, 4254, 24, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(c+dx)(a+b\sec(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{4274}$$

$$a\int \sec(c+dx)dx+b\int \sec^2(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$a\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+b\int \csc\left(c+dx+\frac{\pi}{2}\right)^2dx$$

$$\Big\downarrow \text{4254}$$

$$a\int \csc\left(c+dx+\frac{\pi}{2}\right)dx-\frac{b\int 1d(-\tan(c+dx))}{d}$$

$$\Big\downarrow \text{24}$$

$$a\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{b\tan(c+dx)}{d}$$

$$\Big\downarrow \text{4257}$$

$$\frac{a\text{arctanh}(\sin(c+dx))}{d}+\frac{b\tan(c+dx)}{d}$$

input
```
Int[Sec[c + d*x]*(a + b*Sec[c + d*x]),x]
```

output
```
(a*ArcTanh[Sin[c + d*x]])/d + (b*Tan[c + d*x])/d
```

3.449.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

3.449.4 Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.25

method	result	size
derivativedivides	$\frac{a\ln(\sec(dx+c)+\tan(dx+c))+\tan(dx+c)b}{d}$	30
default	$\frac{a\ln(\sec(dx+c)+\tan(dx+c))+\tan(dx+c)b}{d}$	30
parts	$\frac{a\ln(\sec(dx+c)+\tan(dx+c))}{d}+\frac{b\tan(dx+c)}{d}$	32
risch	$\frac{2ib}{d(e^{2i(dx+c)}+1)}+\frac{a\ln(e^{i(dx+c)}+i)}{d}-\frac{a\ln(e^{i(dx+c)}-i)}{d}$	59
parallelrisch	$\frac{-a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\cos(dx+c)+a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)\cos(dx+c)+\sin(dx+c)b}{d\cos(dx+c)}$	63
norman	$-\frac{2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)}+\frac{a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{d}-\frac{a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{d}$	67

input
```
int(sec(d*x+c)*(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(a*ln(sec(d*x+c)+tan(d*x+c))+tan(d*x+c)*b)
```

3.449.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 60 vs. $2(24) = 48$.

Time = 0.27 (sec) , antiderivative size = 60, normalized size of antiderivative = 2.50

$$\int \sec(c+dx)(a+b\sec(c+dx))\,dx = \frac{a\cos(dx+c)\log(\sin(dx+c)+1) - a\cos(dx+c)\log(-\sin(dx+c)+1) + 2\,b\sin(dx+c)}{2\,d\cos(dx+c)}$$

input
```
integrate(sec(d*x+c)*(a+b*sec(d*x+c)),x, algorithm="fricas")
```

output
```
1/2*(a*cos(d*x + c)*log(sin(d*x + c) + 1) - a*cos(d*x + c)*log(-sin(d*x +
c) + 1) + 2*b*sin(d*x + c))/(d*cos(d*x + c))
```

3.449.6 Sympy [A] (verification not implemented)

Time = 2.19 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.54

$$\int \sec(c+dx)(a+b\sec(c+dx))\,dx = \begin{cases} \frac{a\log(\tan(c+dx)+\sec(c+dx))+b\tan(c+dx)}{d} & \text{for } d \neq 0 \\ x(a+b\sec(c))\sec(c) & \text{otherwise} \end{cases}$$

input
```
integrate(sec(d*x+c)*(a+b*sec(d*x+c)),x)
```

output
```
Piecewise(((a*log(tan(c + d*x) + sec(c + d*x)) + b*tan(c + d*x))/d, Ne(d,
0)), (x*(a + b*sec(c))*sec(c), True))
```

3.449.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.21

$$\int \sec(c+dx)(a+b\sec(c+dx))\,dx = \frac{a\log\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)+b\tan\left(dx+c\right)}{d}$$

input
```
integrate(sec(d*x+c)*(a+b*sec(d*x+c)),x, algorithm="maxima")
```

output
```
(a*log(sec(d*x + c) + tan(d*x + c)) + b*tan(d*x + c))/d
```

3.449.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 63 vs. 2(24) = 48.

Time = 0.30 (sec) , antiderivative size = 63, normalized size of antiderivative = 2.62

$$\int \sec(c+dx)(a+b\sec(c+dx))\,dx = \frac{a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1}}{d}$$

input
```
integrate(sec(d*x+c)*(a+b*sec(d*x+c)),x, algorithm="giac")
```

output
```
(a*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - a*log(abs(tan(1/2*d*x + 1/2*c) - 1
)) - 2*b*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^2 - 1))/d
```

3.449.9 Mupad [B] (verification not implemented)

Time = 13.67 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.96

$$\int \sec(c+dx)(a+b\sec(c+dx))\,dx = \frac{2\,a\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)}$$

input
```
int((a + b/cos(c + d*x))/cos(c + d*x),x)
```

output
```
(2*a*atanh(tan(c/2 + (d*x)/2)))/d - (2*b*tan(c/2 + (d*x)/2))/(d*(tan(c/2 +
 (d*x)/2)^2 - 1))
```

3.450 $\int (a + b\sec(c + dx))\,dx$

3.450.1 Optimal result 3340
3.450.2 Mathematica [A] (verified) 3340
3.450.3 Rubi [A] (verified) 3341
3.450.4 Maple [A] (verified) 3341
3.450.5 Fricas [B] (verification not implemented) 3342
3.450.6 Sympy [A] (verification not implemented) 3342
3.450.7 Maxima [A] (verification not implemented) 3343
3.450.8 Giac [B] (verification not implemented) 3343
3.450.9 Mupad [B] (verification not implemented) 3343

3.450.1 Optimal result

Integrand size = 10, antiderivative size = 16

$$\int (a + b\sec(c + dx))\,dx = ax + \frac{b\operatorname{arctanh}(\sin(c + dx))}{d}$$

output
```
a*x+b*arctanh(sin(d*x+c))/d
```

3.450.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int (a + b\sec(c + dx))\,dx = ax + \frac{b\operatorname{arctanh}(\sin(c + dx))}{d}$$

input
```
Integrate[a + b*Sec[c + d*x],x]
```

output
```
a*x + (b*ArcTanh[Sin[c + d*x]])/d
```

3.450.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a + b\sec(c + dx))\,dx$$

$$\Big\downarrow \text{ 2009}$$

$$ax + \frac{b\text{arctanh}(\sin(c + dx))}{d}$$

input
```
Int[a + b*Sec[c + d*x],x]
```

output
```
a*x + (b*ArcTanh[Sin[c + d*x]])/d
```

3.450.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.450.4 Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.50

method	result	size
default	$ax + \frac{b\ln(\sec(dx+c)+\tan(dx+c))}{d}$	24
parts	$ax + \frac{b\ln(\sec(dx+c)+\tan(dx+c))}{d}$	24
derivativedivides	$\frac{a(dx+c)+b\ln(\sec(dx+c)+\tan(dx+c))}{d}$	29
parallelrisch	$\frac{b\left(-\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)\right)}{d} + ax$	37
norman	$ax + \frac{b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{d} - \frac{b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{d}$	40
risch	$ax - \frac{b\ln\left(e^{i(dx+c)}-i\right)}{d} + \frac{b\ln\left(e^{i(dx+c)}+i\right)}{d}$	42

input
```
int(a+b*sec(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
a*x+b/d*ln(sec(d*x+c)+tan(d*x+c))
```

3.450.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 36 vs. $2(16) = 32$.

Time = 0.28 (sec) , antiderivative size = 36, normalized size of antiderivative = 2.25

$$\int (a + b\sec(c + dx))\,dx = \frac{2\,adx + b\log\left(\sin\left(dx + c\right) + 1\right) - b\log\left(-\sin\left(dx + c\right) + 1\right)}{2\,d}$$

input
```
integrate(a+b*sec(d*x+c),x, algorithm="fricas")
```

output
```
1/2*(2*a*d*x + b*log(sin(d*x + c) + 1) - b*log(-sin(d*x + c) + 1))/d
```

3.450.6 Sympy [A] (verification not implemented)

Time = 0.82 (sec) , antiderivative size = 41, normalized size of antiderivative = 2.56

$$\int (a + b\sec(c + dx))\,dx = ax + b\left(\begin{cases} \frac{\log\left(\tan\left(c+dx\right)+\sec\left(c+dx\right)\right)}{d} & \text{for}\ d \neq 0 \\ \frac{x\left(\tan\left(c\right)\sec\left(c\right)+\sec^{2}\left(c\right)\right)}{\tan\left(c\right)+\sec\left(c\right)} & \text{otherwise} \end{cases}\right)$$

input
```
integrate(a+b*sec(d*x+c),x)
```

output
```
a*x + b*Piecewise((log(tan(c + d*x) + sec(c + d*x))/d, Ne(d, 0)), (x*(tan(
c)*sec(c) + sec(c)**2)/(tan(c) + sec(c)), True))
```

3.450.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.44

$$\int (a + b\sec(c + dx))\,dx = ax + \frac{b\log\left(\sec\left(dx + c\right) + \tan\left(dx + c\right)\right)}{d}$$

input
```
integrate(a+b*sec(d*x+c),x, algorithm="maxima")
```

output
```
a*x + b*log(sec(d*x + c) + tan(d*x + c))/d
```

3.450.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 49 vs. 2(16) = 32.

Time = 0.28 (sec) , antiderivative size = 49, normalized size of antiderivative = 3.06

$$\int (a + b\sec(c + dx))\,dx = ax + \frac{b\left(\log\left(\left|\frac{1}{\sin(dx+c)} + \sin\left(dx + c\right) + 2\right|\right) - \log\left(\left|\frac{1}{\sin(dx+c)} + \sin\left(dx + c\right) - 2\right|\right)\right)}{4\,d}$$

input
```
integrate(a+b*sec(d*x+c),x, algorithm="giac")
```

output
```
a*x + 1/4*b*(log(abs(1/sin(d*x + c) + sin(d*x + c) + 2)) - log(abs(1/sin(d
*x + c) + sin(d*x + c) - 2)))/d
```

3.450.9 Mupad [B] (verification not implemented)

Time = 14.55 (sec) , antiderivative size = 57, normalized size of antiderivative = 3.56

$$\int (a + b\sec(c + dx))\,dx = \frac{2\,a\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d} + \frac{2\,b\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}$$

input
```
int(a + b/cos(c + d*x),x)
```

output
```
(2*a*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (2*b*atanh(sin(c/2 +
 (d*x)/2)/cos(c/2 + (d*x)/2)))/d
```

3.451 $\int \cos(c+dx)(a+b\sec(c+dx))\,dx$

3.451.1 Optimal result . . . 3344
3.451.2 Mathematica [A] (verified) . . . 3344
3.451.3 Rubi [A] (verified) . . . 3345
3.451.4 Maple [A] (verified) . . . 3346
3.451.5 Fricas [A] (verification not implemented) . . . 3347
3.451.6 Sympy [A] (verification not implemented) . . . 3347
3.451.7 Maxima [A] (verification not implemented) . . . 3347
3.451.8 Giac [B] (verification not implemented) . . . 3348
3.451.9 Mupad [B] (verification not implemented) . . . 3348

3.451.1 Optimal result

Integrand size = 17, antiderivative size = 15

$$\int \cos(c+dx)(a+b\sec(c+dx))\,dx = bx+\frac{a\sin(c+dx)}{d}$$

output
```
b*x+a*sin(d*x+c)/d
```

3.451.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.73

$$\int \cos(c+dx)(a+b\sec(c+dx))\,dx = bx+\frac{a\cos(dx)\sin(c)}{d}+\frac{a\cos(c)\sin(dx)}{d}$$

input
```
Integrate[Cos[c + d*x]*(a + b*Sec[c + d*x]),x]
```

output
```
b*x + (a*Cos[d*x]*Sin[c])/d + (a*Cos[c]*Sin[d*x])/d
```

3.451.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {3042, 4274, 24, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(c+dx)(a+b\sec(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{4274}$$

$$a\int \cos(c+dx)dx+b\int 1dx$$

$$\Big\downarrow \text{24}$$

$$a\int \cos(c+dx)dx+bx$$

$$\Big\downarrow \text{3042}$$

$$a\int \sin\left(c+dx+\frac{\pi}{2}\right)dx+bx$$

$$\Big\downarrow \text{3117}$$

$$\frac{a\sin(c+dx)}{d}+bx$$

input
```
Int[Cos[c + d*x]*(a + b*Sec[c + d*x]),x]
```

output
```
b*x + (a*Sin[c + d*x])/d
```

3.451.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

3.451.4 Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.07

method	result	size
risch	$bx+\frac{a\sin(dx+c)}{d}$	16
parallelrisch	$\frac{bxd+a\sin(dx+c)}{d}$	18
derivativedivides	$\frac{a\sin(dx+c)+b(dx+c)}{d}$	21
default	$\frac{a\sin(dx+c)+b(dx+c)}{d}$	21
norman	$\frac{bx+bx\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}}{1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}$	50

input
```
int(cos(d*x+c)*(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
b*x+a*sin(d*x+c)/d
```

3.451.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \cos(c+dx)(a+b\sec(c+dx))\,dx = \frac{bdx + a\sin(dx+c)}{d}$$

input
```
integrate(cos(d*x+c)*(a+b*sec(d*x+c)),x, algorithm="fricas")
```

output
```
(b*d*x + a*sin(d*x + c))/d
```

3.451.6 Sympy [A] (verification not implemented)

Time = 0.78 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \cos(c+dx)(a+b\sec(c+dx))\,dx = a\left(\begin{cases} x\cos(c) & \text{for } d=0 \\ \frac{\sin(c+dx)}{d} & \text{otherwise} \end{cases}\right) + bx$$

input
```
integrate(cos(d*x+c)*(a+b*sec(d*x+c)),x)
```

output
```
a*Piecewise((x*cos(c), Eq(d, 0)), (sin(c + d*x)/d, True)) + b*x
```

3.451.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.33

$$\int \cos(c+dx)(a+b\sec(c+dx))\,dx = \frac{(dx+c)b + a\sin(dx+c)}{d}$$

input
```
integrate(cos(d*x+c)*(a+b*sec(d*x+c)),x, algorithm="maxima")
```

output
```
((d*x + c)*b + a*sin(d*x + c))/d
```

3.451.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 39 vs. $2(15) = 30$.

Time = 0.28 (sec) , antiderivative size = 39, normalized size of antiderivative = 2.60

$$\int \cos(c+dx)(a+b\sec(c+dx))\,dx = \frac{(dx+c)b + \frac{2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1}}{d}$$

input
```
integrate(cos(d*x+c)*(a+b*sec(d*x+c)),x, algorithm="giac")
```

output
```
((d*x + c)*b + 2*a*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^2 + 1))/d
```

3.451.9 Mupad [B] (verification not implemented)

Time = 14.23 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \cos(c+dx)(a+b\sec(c+dx))\,dx = \frac{a\,\sin\left(c+d\,x\right)+b\,d\,x}{d}$$

input
```
int(cos(c + d*x)*(a + b/cos(c + d*x)),x)
```

output
```
(a*sin(c + d*x) + b*d*x)/d
```

3.452 $\int \cos^2(c+dx)(a+b\sec(c+dx))\,dx$

3.452.1 Optimal result . . . 3349
3.452.2 Mathematica [A] (verified) . . . 3349
3.452.3 Rubi [A] (verified) . . . 3350
3.452.4 Maple [A] (verified) . . . 3351
3.452.5 Fricas [A] (verification not implemented) . . . 3352
3.452.6 Sympy [F] . . . 3352
3.452.7 Maxima [A] (verification not implemented) . . . 3352
3.452.8 Giac [B] (verification not implemented) . . . 3353
3.452.9 Mupad [B] (verification not implemented) . . . 3353

3.452.1 Optimal result

Integrand size = 19, antiderivative size = 38

$$\int \cos^2(c+dx)(a+b\sec(c+dx))\,dx=\frac{ax}{2}+\frac{b\sin(c+dx)}{d}+\frac{a\cos(c+dx)\sin(c+dx)}{2d}$$

output
```
1/2*a*x+b*sin(d*x+c)/d+1/2*a*cos(d*x+c)*sin(d*x+c)/d
```

3.452.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.92

$$\int \cos^2(c+dx)(a+b\sec(c+dx))\,dx=\frac{4b\sin(c+dx)+a(2(c+dx)+\sin(2(c+dx)))}{4d}$$

input
```
Integrate[Cos[c + d*x]^2*(a + b*Sec[c + d*x]),x]
```

output
```
(4*b*Sin[c + d*x] + a*(2*(c + d*x) + Sin[2*(c + d*x)]))/(4*d)
```

3.452.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {3042, 4274, 3042, 3115, 24, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(c+dx)(a+b\sec(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^2}dx$$

$$\Big\downarrow \text{4274}$$

$$a\int \cos^2(c+dx)dx+b\int \cos(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$a\int \sin\left(c+dx+\frac{\pi}{2}\right)^2dx+b\int \sin\left(c+dx+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{3115}$$

$$a\left(\frac{\int 1dx}{2}+\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)+b\int \sin\left(c+dx+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{24}$$

$$b\int \sin\left(c+dx+\frac{\pi}{2}\right)dx+a\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)$$

$$\Big\downarrow \text{3117}$$

$$a\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)+\frac{b\sin(c+dx)}{d}$$

input
```
Int[Cos[c + d*x]^2*(a + b*Sec[c + d*x]),x]
```

output
```
(b*Sin[c + d*x])/d + a*(x/2 + (Cos[c + d*x]*Sin[c + d*x])/(2*d))
```

3.452.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

3.452.4 Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.84

method	result	size
risch	$\frac{ax}{2}+\frac{b\sin(dx+c)}{d}+\frac{a\sin(2dx+2c)}{4d}$	32
parallelrisch	$\frac{2axd+4\sin(dx+c)b+a\sin(2dx+2c)}{4d}$	32
derivativedivides	$\frac{a\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+\sin(dx+c)b}{d}$	38
default	$\frac{a\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+\sin(dx+c)b}{d}$	38
norman	$\frac{ax\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+\frac{(a+2b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{ax}{2}+\frac{ax\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{2}-\frac{(a-2b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{d}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^2}$	90

input
```
int(cos(d*x+c)^2*(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/2*a*x+b*sin(d*x+c)/d+1/4*a/d*sin(2*d*x+2*c)
```

3.452.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.76

$$\int \cos^2(c+dx)(a+b\sec(c+dx))\,dx = \frac{adx + (a\cos(dx+c)+2\,b)\sin(dx+c)}{2\,d}$$

input
```
integrate(cos(d*x+c)^2*(a+b*sec(d*x+c)),x, algorithm="fricas")
```

output
```
1/2*(a*d*x + (a*cos(d*x + c) + 2*b)*sin(d*x + c))/d
```

3.452.6 Sympy [F]

$$\int \cos^2(c+dx)(a+b\sec(c+dx))\,dx = \int (a+b\sec(c+dx))\cos^2(c+dx)\,dx$$

input
```
integrate(cos(d*x+c)**2*(a+b*sec(d*x+c)),x)
```

output
```
Integral((a + b*sec(c + d*x))*cos(c + d*x)**2, x)
```

3.452.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.89

$$\int \cos^2(c+dx)(a+b\sec(c+dx))\,dx = \frac{(2\,dx+2\,c+\sin(2\,dx+2\,c))a+4\,b\sin(dx+c)}{4\,d}$$

input
```
integrate(cos(d*x+c)^2*(a+b*sec(d*x+c)),x, algorithm="maxima")
```

output
```
1/4*((2*d*x + 2*c + sin(2*d*x + 2*c))*a + 4*b*sin(d*x + c))/d
```

3.452.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 82 vs. $2(34) = 68$.

Time = 0.30 (sec) , antiderivative size = 82, normalized size of antiderivative = 2.16

$$\int \cos^2(c+dx)(a+b\sec(c+dx))\,dx = \frac{(dx+c)a - \frac{2\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3 - 2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3 - a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right) - 2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)^2}}{2d}$$

input
```
integrate(cos(d*x+c)^2*(a+b*sec(d*x+c)),x, algorithm="giac")
```

output
```
1/2*((d*x + c)*a - 2*(a*tan(1/2*d*x + 1/2*c)^3 - 2*b*tan(1/2*d*x + 1/2*c)^
3 - a*tan(1/2*d*x + 1/2*c) - 2*b*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*
c)^2 + 1)^2)/d
```

3.452.9 Mupad [B] (verification not implemented)

Time = 13.57 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.82

$$\int \cos^2(c+dx)(a+b\sec(c+dx))\,dx = \frac{a\,x}{2} + \frac{a\,\sin\left(2\,c+2\,d\,x\right)}{4\,d} + \frac{b\,\sin\left(c+d\,x\right)}{d}$$

input
```
int(cos(c + d*x)^2*(a + b/cos(c + d*x)),x)
```

output
```
(a*x)/2 + (a*sin(2*c + 2*d*x))/(4*d) + (b*sin(c + d*x))/d
```

3.453 $\int \cos^3(c+dx)(a+b\sec(c+dx))\,dx$

3.453.1 Optimal result . . . 3354
3.453.2 Mathematica [A] (verified) . . . 3354
3.453.3 Rubi [A] (verified) . . . 3355
3.453.4 Maple [A] (verified) . . . 3357
3.453.5 Fricas [A] (verification not implemented) . . . 3357
3.453.6 Sympy [F] . . . 3358
3.453.7 Maxima [A] (verification not implemented) . . . 3358
3.453.8 Giac [B] (verification not implemented) . . . 3358
3.453.9 Mupad [B] (verification not implemented) . . . 3359

3.453.1 Optimal result

Integrand size = 19, antiderivative size = 54

$$\int \cos^3(c+dx)(a+b\sec(c+dx))\,dx = \frac{bx}{2}+\frac{a\sin(c+dx)}{d}+\frac{b\cos(c+dx)\sin(c+dx)}{2d}-\frac{a\sin^3(c+dx)}{3d}$$

output
```
1/2*b*x+a*sin(d*x+c)/d+1/2*b*cos(d*x+c)*sin(d*x+c)/d-1/3*a*sin(d*x+c)^3/d
```

3.453.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.06

$$\int \cos^3(c+dx)(a+b\sec(c+dx))\,dx = \frac{b(c+dx)}{2d}+\frac{a\sin(c+dx)}{d}-\frac{a\sin^3(c+dx)}{3d}+\frac{b\sin(2(c+dx))}{4d}$$

input
```
Integrate[Cos[c + d*x]^3*(a + b*Sec[c + d*x]),x]
```

output
```
(b*(c + d*x))/(2*d) + (a*Sin[c + d*x])/d - (a*Sin[c + d*x]^3)/(3*d) + (b*S
in[2*(c + d*x)])/(4*d)
```

3.453.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.02, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {3042, 4274, 3042, 3113, 2009, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^3(c+dx)(a+b\sec(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^3}dx$$

$$\Big\downarrow \text{4274}$$

$$a\int \cos^3(c+dx)dx+b\int \cos^2(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$a\int \sin\left(c+dx+\frac{\pi}{2}\right)^3dx+b\int \sin\left(c+dx+\frac{\pi}{2}\right)^2dx$$

$$\Big\downarrow \text{3113}$$

$$b\int \sin\left(c+dx+\frac{\pi}{2}\right)^2dx-\frac{a\int \left(1-\sin^2(c+dx)\right)d(-\sin(c+dx))}{d}$$

$$\Big\downarrow \text{2009}$$

$$b\int \sin\left(c+dx+\frac{\pi}{2}\right)^2dx-\frac{a\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}$$

$$\Big\downarrow \text{3115}$$

$$b\left(\frac{\int 1dx}{2}+\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)-\frac{a\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}$$

$$\Big\downarrow \text{24}$$

$$b\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)-\frac{a\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}$$

input
```
Int[Cos[c + d*x]^3*(a + b*Sec[c + d*x]),x]
```

output
```
b*(x/2 + (Cos[c + d*x]*Sin[c + d*x])/(2*d)) - (a*(-Sin[c + d*x] + Sin[c +
d*x]^3/3))/d
```

3.453.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

3.453.4 Maple [A] (verified)

Time = 0.69 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.81

method	result
parallelrisch	$\frac{6bxd+9a\sin(dx+c)+a\sin(3dx+3c)+3b\sin(2dx+2c)}{12d}$
risch	$\frac{bx}{2}+\frac{3a\sin(dx+c)}{4d}+\frac{a\sin(3dx+3c)}{12d}+\frac{b\sin(2dx+2c)}{4d}$
derivativedivides	$\frac{\frac{a\left(2+\cos(dx+c)^2\right)\sin(dx+c)}{3}+b\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}$
default	$\frac{\frac{a\left(2+\cos(dx+c)^2\right)\sin(dx+c)}{3}+b\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}$
norman	$\frac{\frac{(2a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{d}+\frac{(2a+b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{bx}{2}+\frac{4a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3d}+\frac{3bx\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{2}+\frac{3bx\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{2}+\frac{bx\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{2}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^3}$

input
```
int(cos(d*x+c)^3*(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/12*(6*b*x*d+9*a*sin(d*x+c)+a*sin(3*d*x+3*c)+3*b*sin(2*d*x+2*c))/d
```

3.453.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.78

$$\int \cos^3(c+dx)(a+b\sec(c+dx))\,dx = \frac{3\,bdx+\left(2\,a\cos\left(dx+c\right)^2+3\,b\cos\left(dx+c\right)+4\,a\right)\sin\left(dx+c\right)}{6\,d}$$

input
```
integrate(cos(d*x+c)^3*(a+b*sec(d*x+c)),x, algorithm="fricas")
```

output
```
1/6*(3*b*d*x + (2*a*cos(d*x + c)^2 + 3*b*cos(d*x + c) + 4*a)*sin(d*x + c))
/d
```

3.453.6 Sympy [F]

$$\int \cos^3(c+dx)(a+b\sec(c+dx))\,dx = \int (a+b\sec(c+dx))\cos^3(c+dx)\,dx$$

input
```
integrate(cos(d*x+c)**3*(a+b*sec(d*x+c)),x)
```

output
```
Integral((a + b*sec(c + d*x))*cos(c + d*x)**3, x)
```

3.453.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.85

$$\int \cos^3(c+dx)(a+b\sec(c+dx))\,dx = -\frac{4\left(\sin(dx+c)^3-3\sin(dx+c)\right)a-3\left(2\,dx+2\,c+\sin(2\,dx+2\,c)\right)b}{12\,d}$$

input
```
integrate(cos(d*x+c)^3*(a+b*sec(d*x+c)),x, algorithm="maxima")
```

output
```
-1/12*(4*(sin(d*x + c)^3 - 3*sin(d*x + c))*a - 3*(2*d*x + 2*c + sin(2*d*x
+ 2*c))*b)/d
```

3.453.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 98 vs. 2(48) = 96.

Time = 0.28 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.81

$$\int \cos^3(c+dx)(a+b\sec(c+dx))\,dx = \frac{3\,(dx+c)b+\frac{2\left(6\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-3\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+4\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+6\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+3\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^3}}{6\,d}$$

input
```
integrate(cos(d*x+c)^3*(a+b*sec(d*x+c)),x, algorithm="giac")
```

output
```
1/6*(3*(d*x + c)*b + 2*(6*a*tan(1/2*d*x + 1/2*c)^5 - 3*b*tan(1/2*d*x + 1/2
*c)^5 + 4*a*tan(1/2*d*x + 1/2*c)^3 + 6*a*tan(1/2*d*x + 1/2*c) + 3*b*tan(1/
2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^3)/d
```

3.453.9 Mupad [B] (verification not implemented)

Time = 13.43 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.02

$$\int \cos^3(c+dx)(a+b\sec(c+dx))\,dx = \frac{b\,x}{2} + \frac{2\,a\,\sin\left(c+d\,x\right)}{3\,d} + \frac{b\,\cos\left(c+d\,x\right)\,\sin\left(c+d\,x\right)}{2\,d} + \frac{a\cos\left(c+d\,x\right)^2\,\sin\left(c+d\,x\right)}{3\,d}$$

input
```
int(cos(c + d*x)^3*(a + b/cos(c + d*x)),x)
```

output
```
(b*x)/2 + (2*a*sin(c + d*x))/(3*d) + (b*cos(c + d*x)*sin(c + d*x))/(2*d) +
 (a*cos(c + d*x)^2*sin(c + d*x))/(3*d)
```

3.454 $\int \cos^4(c+dx)(a+b\sec(c+dx))\,dx$

3.454.1 Optimal result . . . 3360
3.454.2 Mathematica [A] (verified) . . . 3360
3.454.3 Rubi [A] (verified) . . . 3361
3.454.4 Maple [A] (verified) . . . 3363
3.454.5 Fricas [A] (verification not implemented) . . . 3363
3.454.6 Sympy [F] . . . 3364
3.454.7 Maxima [A] (verification not implemented) . . . 3364
3.454.8 Giac [B] (verification not implemented) . . . 3364
3.454.9 Mupad [B] (verification not implemented) . . . 3365

3.454.1 Optimal result

Integrand size = 19, antiderivative size = 76

$$\int \cos^4(c+dx)(a+b\sec(c+dx))\,dx = \frac{3ax}{8}+\frac{b\sin(c+dx)}{d}+\frac{3a\cos(c+dx)\sin(c+dx)}{8d}+\frac{a\cos^3(c+dx)\sin(c+dx)}{4d}-\frac{b\sin^3(c+dx)}{3d}$$

output
```
3/8*a*x+b*sin(d*x+c)/d+3/8*a*cos(d*x+c)*sin(d*x+c)/d+1/4*a*cos(d*x+c)^3*si
n(d*x+c)/d-1/3*b*sin(d*x+c)^3/d
```

3.454.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.96

$$\int \cos^4(c+dx)(a+b\sec(c+dx))\,dx = \frac{3a(c+dx)}{8d}+\frac{b\sin(c+dx)}{d}-\frac{b\sin^3(c+dx)}{3d}+\frac{a\sin(2(c+dx))}{4d}+\frac{a\sin(4(c+dx))}{32d}$$

input
```
Integrate[Cos[c + d*x]^4*(a + b*Sec[c + d*x]),x]
```

output
```
(3*a*(c + d*x))/(8*d) + (b*Sin[c + d*x])/d - (b*Sin[c + d*x]^3)/(3*d) + (a
*Sin[2*(c + d*x)])/(4*d) + (a*Sin[4*(c + d*x)])/(32*d)
```

3.454.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.07, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474, Rules used = {3042, 4274, 3042, 3113, 2009, 3115, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^4(c+dx)(a+b\sec(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^4}dx$$

$$\Big\downarrow \text{4274}$$

$$a\int \cos^4(c+dx)dx+b\int \cos^3(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$a\int \sin\left(c+dx+\frac{\pi}{2}\right)^4dx+b\int \sin\left(c+dx+\frac{\pi}{2}\right)^3dx$$

$$\Big\downarrow \text{3113}$$

$$a\int \sin\left(c+dx+\frac{\pi}{2}\right)^4dx-\frac{b\int \left(1-\sin^2(c+dx)\right)d(-\sin(c+dx))}{d}$$

$$\Big\downarrow \text{2009}$$

$$a\int \sin\left(c+dx+\frac{\pi}{2}\right)^4dx-\frac{b\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}$$

$$\Big\downarrow \text{3115}$$

$$a\left(\frac{3}{4}\int \cos^2(c+dx)dx+\frac{\sin(c+dx)\cos^3(c+dx)}{4d}\right)-\frac{b\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{3}{4}\int \sin\left(c+dx+\frac{\pi}{2}\right)^2dx+\frac{\sin(c+dx)\cos^3(c+dx)}{4d}\right)-\frac{b\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}$$

$$\Big\downarrow \text{3115}$$

$$a\left(\frac{3}{4}\left(\frac{\int 1dx}{2}+\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)+\frac{\sin(c+dx)\cos^3(c+dx)}{4d}\right)-\frac{b\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}$$

$$\Big\downarrow \; 24$$

$$a\left(\frac{\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3}{4}\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)\right)-\frac{b\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}$$

input
```
Int[Cos[c + d*x]^4*(a + b*Sec[c + d*x]),x]
```

output
```
-((b*(-Sin[c + d*x] + Sin[c + d*x]^3/3))/d) + a*((Cos[c + d*x]^3*Sin[c + d
*x])/(4*d) + (3*(x/2 + (Cos[c + d*x]*Sin[c + d*x])/(2*d)))/4)
```

3.454.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

3.454.4 Maple [A] (verified)

Time = 0.89 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.75

method	result
parallelrisch	$\frac{36axd+72\sin(dx+c)b+3a\sin(4dx+4c)+8b\sin(3dx+3c)+24a\sin(2dx+2c)}{96d}$
derivativedivides	$\frac{a\left(\frac{\left(\cos(dx+c)^3+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+\frac{b\left(2+\cos(dx+c)^2\right)\sin(dx+c)}{3}}{d}$
default	$\frac{a\left(\frac{\left(\cos(dx+c)^3+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+\frac{b\left(2+\cos(dx+c)^2\right)\sin(dx+c)}{3}}{d}$
risch	$\frac{3ax}{8}+\frac{3b\sin(dx+c)}{4d}+\frac{a\sin(4dx+4c)}{32d}+\frac{b\sin(3dx+3c)}{12d}+\frac{a\sin(2dx+2c)}{4d}$
norman	$\frac{\frac{3ax}{8}+\frac{3ax\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{2}+\frac{9ax\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{4}+\frac{3ax\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{2}+\frac{3ax\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8}{8}-\frac{(5a-8b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{4d}+\frac{(5a+8b)\tan\left(\frac{dx}{2}+\right.}{4d}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^4}$

input
```
int(cos(d*x+c)^4*(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/96*(36*a*x*d+72*sin(d*x+c)*b+3*a*sin(4*d*x+4*c)+8*b*sin(3*d*x+3*c)+24*a*
sin(2*d*x+2*c))/d
```

3.454.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.70

$$\int \cos^4(c+dx)(a+b\sec(c+dx))\,dx = \frac{9\,adx+\left(6\,a\cos\left(dx+c\right)^3+8\,b\cos\left(dx+c\right)^2+9\,a\cos\left(dx+c\right)+16\,b\right)\sin\left(dx+c\right)}{24\,d}$$

input
```
integrate(cos(d*x+c)^4*(a+b*sec(d*x+c)),x, algorithm="fricas")
```

output
```
1/24*(9*a*d*x + (6*a*cos(d*x + c)^3 + 8*b*cos(d*x + c)^2 + 9*a*cos(d*x + c
) + 16*b)*sin(d*x + c))/d
```

3.454.6 Sympy [F]

$$\int \cos^4(c+dx)(a+b\sec(c+dx))\,dx = \int (a+b\sec(c+dx))\cos^4(c+dx)\,dx$$

input
```
integrate(cos(d*x+c)**4*(a+b*sec(d*x+c)),x)
```

output
```
Integral((a + b*sec(c + d*x))*cos(c + d*x)**4, x)
```

3.454.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.75

$$\int \cos^4(c+dx)(a+b\sec(c+dx))\,dx = \frac{3\,(12\,dx+12\,c+\sin(4\,dx+4\,c)+8\,\sin(2\,dx+2\,c))a-32\left(\sin(dx+c)^3-3\,\sin(dx+c)\right)b}{96\,d}$$

input
```
integrate(cos(d*x+c)^4*(a+b*sec(d*x+c)),x, algorithm="maxima")
```

output
```
1/96*(3*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*a - 32*(si
n(d*x + c)^3 - 3*sin(d*x + c))*b)/d
```

3.454.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 140 vs. 2(68) = 136.

Time = 0.29 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.84

$$\int \cos^4(c+dx)(a+b\sec(c+dx))\,dx = \frac{9\,(dx+c)a-\frac{2\left(15\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-24\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-9\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-40\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+9\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-40\,b\tan\left(\frac{1}{2}\,dx\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^4}}{24\,d}$$

input
```
integrate(cos(d*x+c)^4*(a+b*sec(d*x+c)),x, algorithm="giac")
```

output
```
1/24*(9*(d*x + c)*a - 2*(15*a*tan(1/2*d*x + 1/2*c)^7 - 24*b*tan(1/2*d*x +
1/2*c)^7 - 9*a*tan(1/2*d*x + 1/2*c)^5 - 40*b*tan(1/2*d*x + 1/2*c)^5 + 9*a*
tan(1/2*d*x + 1/2*c)^3 - 40*b*tan(1/2*d*x + 1/2*c)^3 - 15*a*tan(1/2*d*x +
1/2*c) - 24*b*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^4)/d
```

3.454.9 Mupad [B] (verification not implemented)

Time = 14.02 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.99

$$\int \cos^4(c+dx)(a+b\sec(c+dx))\,dx = \frac{3\,a\,x}{8}+\frac{2\,b\,\sin\left(c+d\,x\right)}{3\,d}+\frac{3\,a\,\cos\left(c+d\,x\right)\,\sin\left(c+d\,x\right)}{8\,d}+\frac{a\,\cos\left(c+d\,x\right)^3\,\sin\left(c+d\,x\right)}{4\,d}+\frac{b\,\cos\left(c+d\,x\right)^2\,\sin\left(c+d\,x\right)}{3\,d}$$

input
```
int(cos(c + d*x)^4*(a + b/cos(c + d*x)),x)
```

output
```
(3*a*x)/8 + (2*b*sin(c + d*x))/(3*d) + (3*a*cos(c + d*x)*sin(c + d*x))/(8*
d) + (a*cos(c + d*x)^3*sin(c + d*x))/(4*d) + (b*cos(c + d*x)^2*sin(c + d*x
))/(3*d)
```

3.455 $\int \cos^5(c+dx)(a+b\sec(c+dx))\,dx$

3.455.1 Optimal result . . . 3366
3.455.2 Mathematica [A] (verified) . . . 3366
3.455.3 Rubi [A] (verified) . . . 3367
3.455.4 Maple [A] (verified) . . . 3369
3.455.5 Fricas [A] (verification not implemented) . . . 3370
3.455.6 Sympy [F] . . . 3370
3.455.7 Maxima [A] (verification not implemented) . . . 3370
3.455.8 Giac [A] (verification not implemented) . . . 3371
3.455.9 Mupad [B] (verification not implemented) . . . 3371

3.455.1 Optimal result

Integrand size = 19, antiderivative size = 92

$$\int \cos^5(c+dx)(a+b\sec(c+dx))\,dx = \frac{3bx}{8}+\frac{a\sin(c+dx)}{d}+\frac{3b\cos(c+dx)\sin(c+dx)}{8d}+\frac{b\cos^3(c+dx)\sin(c+dx)}{4d}-\frac{2a\sin^3(c+dx)}{3d}+\frac{a\sin^5(c+dx)}{5d}$$

output
```
3/8*b*x+a*sin(d*x+c)/d+3/8*b*cos(d*x+c)*sin(d*x+c)/d+1/4*b*cos(d*x+c)^3*si
n(d*x+c)/d-2/3*a*sin(d*x+c)^3/d+1/5*a*sin(d*x+c)^5/d
```

3.455.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.97

$$\int \cos^5(c+dx)(a+b\sec(c+dx))\,dx = \frac{3b(c+dx)}{8d}+\frac{a\sin(c+dx)}{d}-\frac{2a\sin^3(c+dx)}{3d}+\frac{a\sin^5(c+dx)}{5d}+\frac{b\sin(2(c+dx))}{4d}+\frac{b\sin(4(c+dx))}{32d}$$

input
```
Integrate[Cos[c + d*x]^5*(a + b*Sec[c + d*x]),x]
```

output
```
(3*b*(c + d*x))/(8*d) + (a*Sin[c + d*x])/d - (2*a*Sin[c + d*x]^3)/(3*d) +
(a*Sin[c + d*x]^5)/(5*d) + (b*Sin[2*(c + d*x)])/(4*d) + (b*Sin[4*(c + d*x)
])/(32*d)
```

3.455.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.01, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {3042, 4274, 3042, 3113, 2009, 3115, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^5(c+dx)(a+b\sec(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^5}dx$$

$$\Big\downarrow \text{4274}$$

$$a\int \cos^5(c+dx)dx+b\int \cos^4(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$a\int \sin\left(c+dx+\frac{\pi}{2}\right)^5dx+b\int \sin\left(c+dx+\frac{\pi}{2}\right)^4dx$$

$$\Big\downarrow \text{3113}$$

$$b\int \sin\left(c+dx+\frac{\pi}{2}\right)^4dx-\frac{a\int \left(\sin^4(c+dx)-2\sin^2(c+dx)+1\right)d(-\sin(c+dx))}{d}$$

$$\Big\downarrow \text{2009}$$

$$b\int \sin\left(c+dx+\frac{\pi}{2}\right)^4dx-\frac{a\left(-\frac{1}{5}\sin^5(c+dx)+\frac{2}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}$$

$$\Big\downarrow \text{3115}$$

$$b\left(\frac{3}{4}\int \cos^2(c+dx)dx+\frac{\sin(c+dx)\cos^3(c+dx)}{4d}\right)-\frac{a\left(-\frac{1}{5}\sin^5(c+dx)+\frac{2}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}$$

$$\Big\downarrow \text{3042}$$

$$b\left(\frac{3}{4}\int \sin\left(c+dx+\frac{\pi}{2}\right)^2 dx+\frac{\sin(c+dx)\cos^3(c+dx)}{4d}\right)-\frac{a\left(-\frac{1}{5}\sin^5(c+dx)+\frac{2}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}$$

$$\Big\downarrow \; 3115$$

$$b\left(\frac{3}{4}\left(\frac{\int 1dx}{2}+\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)+\frac{\sin(c+dx)\cos^3(c+dx)}{4d}\right)-\frac{a\left(-\frac{1}{5}\sin^5(c+dx)+\frac{2}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}$$

$$\Big\downarrow \; 24$$

$$b\left(\frac{\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3}{4}\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)\right)-\frac{a\left(-\frac{1}{5}\sin^5(c+dx)+\frac{2}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}$$

input
```
Int[Cos[c + d*x]^5*(a + b*Sec[c + d*x]),x]
```

output
```
-((a*(-Sin[c + d*x] + (2*Sin[c + d*x]^3)/3 - Sin[c + d*x]^5/5))/d) + b*((C
os[c + d*x]^3*Sin[c + d*x])/(4*d) + (3*(x/2 + (Cos[c + d*x]*Sin[c + d*x])/
(2*d)))/4)
```

3.455.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

3.455.4 Maple [A] (verified)

Time = 0.83 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.75

method	result
parallelrisch	$\frac{180bxd+6a\sin(5dx+5c)+15b\sin(4dx+4c)+50a\sin(3dx+3c)+120b\sin(2dx+2c)+300a\sin(dx+c)}{480d}$
derivativedivides	$\frac{\frac{a\left(\frac{8}{3}+\cos(dx+c)^4+\frac{4\cos(dx+c)^2}{3}\right)\sin(dx+c)}{5}+b\left(\frac{\left(\cos(dx+c)^3+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)}{d}$
default	$\frac{\frac{a\left(\frac{8}{3}+\cos(dx+c)^4+\frac{4\cos(dx+c)^2}{3}\right)\sin(dx+c)}{5}+b\left(\frac{\left(\cos(dx+c)^3+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)}{d}$
risch	$\frac{3bx}{8}+\frac{5a\sin(dx+c)}{8d}+\frac{a\sin(5dx+5c)}{80d}+\frac{b\sin(4dx+4c)}{32d}+\frac{5a\sin(3dx+3c)}{48d}+\frac{b\sin(2dx+2c)}{4d}$
norman	$\frac{\frac{3bx}{8}+\frac{116a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{15d}+\frac{15bx\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{8}+\frac{15bx\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{4}+\frac{15bx\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{4}+\frac{15bx\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8}{8}+\frac{3bx\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{1}}{8}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^5}$

input
```
int(cos(d*x+c)^5*(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/480*(180*b*x*d+6*a*sin(5*d*x+5*c)+15*b*sin(4*d*x+4*c)+50*a*sin(3*d*x+3*c
)+120*b*sin(2*d*x+2*c)+300*a*sin(d*x+c))/d
```

3.455.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.70

$$\int \cos^5(c+dx)(a+b\sec(c+dx))\,dx$$
$$= \frac{45\,bdx + \left(24\,a\cos\left(dx+c\right)^4 + 30\,b\cos\left(dx+c\right)^3 + 32\,a\cos\left(dx+c\right)^2 + 45\,b\cos\left(dx+c\right) + 64\,a\right)\sin\left(dx+c\right)}{120\,d}$$

input
```
integrate(cos(d*x+c)^5*(a+b*sec(d*x+c)),x, algorithm="fricas")
```

output
```
1/120*(45*b*d*x + (24*a*cos(d*x + c)^4 + 30*b*cos(d*x + c)^3 + 32*a*cos(d*
x + c)^2 + 45*b*cos(d*x + c) + 64*a)*sin(d*x + c))/d
```

3.455.6 Sympy [F]

$$\int \cos^5(c+dx)(a+b\sec(c+dx))\,dx = \int (a+b\sec(c+dx))\cos^5(c+dx)\,dx$$

input
```
integrate(cos(d*x+c)**5*(a+b*sec(d*x+c)),x)
```

output
```
Integral((a + b*sec(c + d*x))*cos(c + d*x)**5, x)
```

3.455.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.75

$$\int \cos^5(c+dx)(a+b\sec(c+dx))\,dx$$
$$= \frac{32\left(3\,\sin\left(dx+c\right)^5 - 10\,\sin\left(dx+c\right)^3 + 15\,\sin\left(dx+c\right)\right)a + 15\left(12\,dx + 12\,c + \sin\left(4\,dx+4\,c\right) + 8\,\sin\left(2\,d\right.\right.}{480\,d}$$

input
```
integrate(cos(d*x+c)^5*(a+b*sec(d*x+c)),x, algorithm="maxima")
```

output
```
1/480*(32*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*a + 15*
(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*b)/d
```

3.455.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.67

$$\int \cos^5(c+dx)(a+b\sec(c+dx))\,dx$$
$$= \frac{45\,(dx+c)b + \frac{2\left(120\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9 - 75\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9 + 160\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7 - 30\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7 + 464\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5 + 160\,a}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^5}}{120\,d}$$

input
```
integrate(cos(d*x+c)^5*(a+b*sec(d*x+c)),x, algorithm="giac")
```

output
```
1/120*(45*(d*x + c)*b + 2*(120*a*tan(1/2*d*x + 1/2*c)^9 - 75*b*tan(1/2*d*x
 + 1/2*c)^9 + 160*a*tan(1/2*d*x + 1/2*c)^7 - 30*b*tan(1/2*d*x + 1/2*c)^7 +
 464*a*tan(1/2*d*x + 1/2*c)^5 + 160*a*tan(1/2*d*x + 1/2*c)^3 + 30*b*tan(1/
2*d*x + 1/2*c)^3 + 120*a*tan(1/2*d*x + 1/2*c) + 75*b*tan(1/2*d*x + 1/2*c))
/(tan(1/2*d*x + 1/2*c)^2 + 1)^5)/d
```

3.455.9 Mupad [B] (verification not implemented)

Time = 17.84 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.23

$$\int \cos^5(c+dx)(a+b\sec(c+dx))\,dx = \frac{3\,b\,x}{8} + \frac{\left(2\,a-\frac{5\,b}{4}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9 + \left(\frac{8\,a}{3}-\frac{b}{2}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7 + \frac{116\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{15} + \left(\frac{8\,a}{3}+\frac{b}{2}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3 + (2\,a -}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)^5}$$

input
```
int(cos(c + d*x)^5*(a + b/cos(c + d*x)),x)
```

output
```
(3*b*x)/8 + (tan(c/2 + (d*x)/2)*(2*a + (5*b)/4) + tan(c/2 + (d*x)/2)^3*((8
*a)/3 + b/2) + tan(c/2 + (d*x)/2)^9*(2*a - (5*b)/4) + tan(c/2 + (d*x)/2)^7
*((8*a)/3 - b/2) + (116*a*tan(c/2 + (d*x)/2)^5)/15)/(d*(tan(c/2 + (d*x)/2)
^2 + 1)^5)
```

3.456 $\int \sec^4(c+dx)(a+b\sec(c+dx))^2\,dx$

3.456.1 Optimal result . . . 3372
3.456.2 Mathematica [A] (verified) . . . 3373
3.456.3 Rubi [A] (verified) . . . 3373
3.456.4 Maple [A] (verified) . . . 3376
3.456.5 Fricas [A] (verification not implemented) . . . 3377
3.456.6 Sympy [F] . . . 3377
3.456.7 Maxima [A] (verification not implemented) . . . 3377
3.456.8 Giac [B] (verification not implemented) . . . 3378
3.456.9 Mupad [B] (verification not implemented) . . . 3378

3.456.1 Optimal result

Integrand size = 21, antiderivative size = 135

$$\int \sec^4(c+dx)(a+b\sec(c+dx))^2\,dx = \frac{3ab\operatorname{arctanh}(\sin(c+dx))}{4d}+\frac{(5a^2+4b^2)\tan(c+dx)}{5d}+\frac{3ab\sec(c+dx)\tan(c+dx)}{4d}+\frac{ab\sec^3(c+dx)\tan(c+dx)}{2d}+\frac{b^2\sec^4(c+dx)\tan(c+dx)}{5d}+\frac{(5a^2+4b^2)\tan^3(c+dx)}{15d}$$

output
```
3/4*a*b*arctanh(sin(d*x+c))/d+1/5*(5*a^2+4*b^2)*tan(d*x+c)/d+3/4*a*b*sec(d
*x+c)*tan(d*x+c)/d+1/2*a*b*sec(d*x+c)^3*tan(d*x+c)/d+1/5*b^2*sec(d*x+c)^4*
tan(d*x+c)/d+1/15*(5*a^2+4*b^2)*tan(d*x+c)^3/d
```

3.456.2 Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.67

$$\int \sec^4(c+dx)(a+b\sec(c+dx))^2\,dx$$
$$= \frac{45ab\text{arctanh}(\sin(c+dx))+\tan(c+dx)\left(60(a^2+b^2)+45ab\sec(c+dx)+30ab\sec^3(c+dx)+20(a^2+2\right.}{60d}$$

input
```
Integrate[Sec[c + d*x]^4*(a + b*Sec[c + d*x])^2,x]
```

output
```
(45*a*b*ArcTanh[Sin[c + d*x]] + Tan[c + d*x]*(60*(a^2 + b^2) + 45*a*b*Sec[
c + d*x] + 30*a*b*Sec[c + d*x]^3 + 20*(a^2 + 2*b^2)*Tan[c + d*x]^2 + 12*b^
2*Tan[c + d*x]^4))/(60*d)
```

3.456.3 Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.95, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {3042, 4275, 3042, 4255, 3042, 4255, 3042, 4257, 4534, 3042, 4254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^4(c+dx)(a+b\sec(c+dx))^2\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^4\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2dx$$
$$\Big\downarrow \text{4275}$$
$$\int \sec^4(c+dx)\left(a^2+b^2\sec^2(c+dx)\right)dx+2ab\int \sec^5(c+dx)dx$$
$$\Big\downarrow \text{3042}$$
$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^4\left(a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)dx+2ab\int \csc\left(c+dx+\frac{\pi}{2}\right)^5dx$$
$$\Big\downarrow \text{4255}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^4\left(a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)dx+2ab\left(\frac{3}{4}\int \sec^3(c+dx)dx+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)$$

$\Big\downarrow$ 3042

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^4\left(a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)dx+2ab\left(\frac{3}{4}\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)$$

$\Big\downarrow$ 4255

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^4\left(a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)dx+2ab\left(\frac{3}{4}\left(\frac{1}{2}\int \sec(c+dx)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)$$

$\Big\downarrow$ 3042

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^4\left(a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)dx+2ab\left(\frac{3}{4}\left(\frac{1}{2}\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)$$

$\Big\downarrow$ 4257

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^4\left(a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)dx+2ab\left(\frac{3}{4}\left(\frac{\text{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)$$

$\Big\downarrow$ 4534

$$\frac{1}{5}(5a^2+4b^2)\int \sec^4(c+dx)dx+2ab\left(\frac{3}{4}\left(\frac{\text{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)+\frac{b^2\tan(c+dx)\sec^4(c+dx)}{5d}$$

$\Big\downarrow$ 3042

$$\frac{1}{5}(5a^2+4b^2)\int \csc\left(c+dx+\frac{\pi}{2}\right)^4dx+2ab\left(\frac{3}{4}\left(\frac{\text{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)+\frac{b^2\tan(c+dx)\sec^4(c+dx)}{5d}$$

$\Big\downarrow$ 4254

3.456. $\int \sec^4(c+dx)(a+b\sec(c+dx))^2\,dx$

$$-\frac{(5a^2+4b^2)\int(\tan^2(c+dx)+1)\,d(-\tan(c+dx))}{5d}+2ab\left(\frac{3}{4}\left(\frac{\text{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)+\frac{b^2\tan(c+dx)\sec^4(c+dx)}{5d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{(5a^2+4b^2)\left(-\frac{1}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{5d}+2ab\left(\frac{3}{4}\left(\frac{\text{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)+\frac{b^2\tan(c+dx)\sec^4(c+dx)}{5d}$$

input
```
Int[Sec[c + d*x]^4*(a + b*Sec[c + d*x])^2,x]
```

output
```
(b^2*Sec[c + d*x]^4*Tan[c + d*x])/(5*d) - ((5*a^2 + 4*b^2)*(-Tan[c + d*x]
- Tan[c + d*x]^3/3))/(5*d) + 2*a*b*((Sec[c + d*x]^3*Tan[c + d*x])/(4*d) +
(3*(ArcTanh[Sin[c + d*x]]/(2*d) + (Sec[c + d*x]*Tan[c + d*x])/(2*d)))/4)
```

3.456.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4275
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Simp[2*a*(b/d)   Int[(d*Csc[e + f*x])^(n + 1), x], x]
 + Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
 e, f, n}, x]
```

rule 4534
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Simp[(C*m + A*(m + 1))/(m + 1)   Int[(b*Csc[e + f*x])^m, x], x] /;
 FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

3.456.4 Maple [A] (verified)

Time = 1.40 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.81

method	result
derivativedivides	$\frac{-a^2\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right)\tan(dx+c)+2ab\left(-\left(-\frac{\sec(dx+c)^3}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)-b^2\left(-\right.}{d}$
default	$\frac{-a^2\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right)\tan(dx+c)+2ab\left(-\left(-\frac{\sec(dx+c)^3}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)-b^2\left(-\right.}{d}$
parts	$-\frac{a^2\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right)\tan(dx+c)}{d}-\frac{b^2\left(-\frac{8}{15}-\frac{\sec(dx+c)^4}{5}-\frac{4\sec(dx+c)^2}{15}\right)\tan(dx+c)}{d}+\frac{2ab\left(-\left(-\frac{\sec(dx+c)^3}{4}-\frac{3\sec}{}\right.\right.}{}$
risch	$-\frac{i\left(45ab\,e^{9i(dx+c)}+210ab\,e^{7i(dx+c)}-120a^2e^{6i(dx+c)}-280a^2e^{4i(dx+c)}-320b^2e^{4i(dx+c)}-210ab\,e^{3i(dx+c)}-200a^2e^{2i(dx+}\right.}{30d\left(e^{2i(dx+c)}+1\right)^5}$
norman	$\frac{-\frac{4\left(25a^2+29b^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{15d}-\frac{\left(4a^2-5ab+4b^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^9}{2d}-\frac{\left(4a^2+5ab+4b^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2d}+\frac{\left(16a^2-3ab+8b^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{3d}}{\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^5}$
parallelrisch	$\frac{-450ab\left(\frac{\cos(5dx+5c)}{10}+\frac{\cos(3dx+3c)}{2}+\cos(dx+c)\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+450ab\left(\frac{\cos(5dx+5c)}{10}+\frac{\cos(3dx+3c)}{2}+\cos(dx+c)\right)\ln}{60d(\cos(5dx-}$

input
```
int(sec(d*x+c)^4*(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-a^2*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c)+2*a*b*(-(-1/4*sec(d*x+c)^3-3/
8*sec(d*x+c))*tan(d*x+c)+3/8*ln(sec(d*x+c)+tan(d*x+c)))-b^2*(-8/15-1/5*sec
(d*x+c)^4-4/15*sec(d*x+c)^2)*tan(d*x+c))
```

3.456.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.01

$$\int \sec^4(c+dx)(a+b\sec(c+dx))^2\,dx$$
$$= \frac{45\,ab\cos(dx+c)^5\log(\sin(dx+c)+1) - 45\,ab\cos(dx+c)^5\log(-\sin(dx+c)+1) + 2\,(45\,ab\cos(dx+}{120\,d\cos(}$$

input
```
integrate(sec(d*x+c)^4*(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/120*(45*a*b*cos(d*x + c)^5*log(sin(d*x + c) + 1) - 45*a*b*cos(d*x + c)^5
*log(-sin(d*x + c) + 1) + 2*(45*a*b*cos(d*x + c)^3 + 8*(5*a^2 + 4*b^2)*cos
(d*x + c)^4 + 30*a*b*cos(d*x + c) + 4*(5*a^2 + 4*b^2)*cos(d*x + c)^2 + 12*
b^2)*sin(d*x + c))/(d*cos(d*x + c)^5)
```

3.456.6 Sympy [F]

$$\int \sec^4(c+dx)(a+b\sec(c+dx))^2\,dx = \int (a+b\sec(c+dx))^2\sec^4(c+dx)\,dx$$

input
```
integrate(sec(d*x+c)**4*(a+b*sec(d*x+c))**2,x)
```

output
```
Integral((a + b*sec(c + d*x))**2*sec(c + d*x)**4, x)
```

3.456.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.98

$$\int \sec^4(c+dx)(a+b\sec(c+dx))^2\,dx$$
$$= \frac{40\left(\tan(dx+c)^3 + 3\tan(dx+c)\right)a^2 + 8\left(3\tan(dx+c)^5 + 10\tan(dx+c)^3 + 15\tan(dx+c)\right)b^2 - 15\,a}{120\,d}$$

input
```
integrate(sec(d*x+c)^4*(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
1/120*(40*(tan(d*x + c)^3 + 3*tan(d*x + c))*a^2 + 8*(3*tan(d*x + c)^5 + 10
*tan(d*x + c)^3 + 15*tan(d*x + c))*b^2 - 15*a*b*(2*(3*sin(d*x + c)^3 - 5*s
in(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c)
+ 1) + 3*log(sin(d*x + c) - 1)))/d
```

3.456.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 272 vs. $2(123) = 246$.

Time = 0.31 (sec) , antiderivative size = 272, normalized size of antiderivative = 2.01

$$\int \sec^4(c+dx)(a+b\sec(c+dx))^2\,dx$$
$$= \frac{45\,ab\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-45\,ab\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(60\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9-75\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right.\right.}{}}{}$$

input
```
integrate(sec(d*x+c)^4*(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
1/60*(45*a*b*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 45*a*b*log(abs(tan(1/2*d
*x + 1/2*c) - 1)) - 2*(60*a^2*tan(1/2*d*x + 1/2*c)^9 - 75*a*b*tan(1/2*d*x
+ 1/2*c)^9 + 60*b^2*tan(1/2*d*x + 1/2*c)^9 - 160*a^2*tan(1/2*d*x + 1/2*c)^
7 + 30*a*b*tan(1/2*d*x + 1/2*c)^7 - 80*b^2*tan(1/2*d*x + 1/2*c)^7 + 200*a^
2*tan(1/2*d*x + 1/2*c)^5 + 232*b^2*tan(1/2*d*x + 1/2*c)^5 - 160*a^2*tan(1/
2*d*x + 1/2*c)^3 - 30*a*b*tan(1/2*d*x + 1/2*c)^3 - 80*b^2*tan(1/2*d*x + 1/
2*c)^3 + 60*a^2*tan(1/2*d*x + 1/2*c) + 75*a*b*tan(1/2*d*x + 1/2*c) + 60*b^
2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^5)/d
```

3.456.9 Mupad [B] (verification not implemented)

Time = 17.08 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.64

$$\int \sec^4(c+dx)(a+b\sec(c+dx))^2\,dx = \frac{3\,a\,b\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{2\,d}$$
$$-\frac{\left(2\,a^2-\frac{5\,a\,b}{2}+2\,b^2\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9+\left(-\frac{16\,a^2}{3}+a\,b-\frac{8\,b^2}{3}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+\left(\frac{20\,a^2}{3}+\frac{116\,b^2}{15}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}-5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+10\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-10\tan\right.}$$

input
```
int((a + b/cos(c + d*x))^2/cos(c + d*x)^4,x)
```

output
```
(3*a*b*atanh(tan(c/2 + (d*x)/2)))/(2*d) - (tan(c/2 + (d*x)/2)^5*((20*a^2)/
3 + (116*b^2)/15) + tan(c/2 + (d*x)/2)^9*(2*a^2 - (5*a*b)/2 + 2*b^2) - tan
(c/2 + (d*x)/2)^3*(a*b + (16*a^2)/3 + (8*b^2)/3) - tan(c/2 + (d*x)/2)^7*((
16*a^2)/3 - a*b + (8*b^2)/3) + tan(c/2 + (d*x)/2)*((5*a*b)/2 + 2*a^2 + 2*b
^2))/(d*(5*tan(c/2 + (d*x)/2)^2 - 10*tan(c/2 + (d*x)/2)^4 + 10*tan(c/2 + (
d*x)/2)^6 - 5*tan(c/2 + (d*x)/2)^8 + tan(c/2 + (d*x)/2)^10 - 1))
```

3.456. $\int \sec^4(c+dx)(a+b\sec(c+dx))^2\,dx$

3.457 $\int \sec^3(c+dx)(a+b\sec(c+dx))^2\,dx$

3.457.1 Optimal result . . . 3380
3.457.2 Mathematica [A] (verified) . . . 3380
3.457.3 Rubi [A] (verified) . . . 3381
3.457.4 Maple [A] (verified) . . . 3383
3.457.5 Fricas [A] (verification not implemented) . . . 3384
3.457.6 Sympy [F] . . . 3384
3.457.7 Maxima [A] (verification not implemented) . . . 3385
3.457.8 Giac [B] (verification not implemented) . . . 3385
3.457.9 Mupad [B] (verification not implemented) . . . 3386

3.457.1 Optimal result

Integrand size = 21, antiderivative size = 110

$$\int \sec^3(c+dx)(a+b\sec(c+dx))^2\,dx = \frac{(4a^2+3b^2)\operatorname{arctanh}(\sin(c+dx))}{8d}+\frac{2ab\tan(c+dx)}{d}+\frac{(4a^2+3b^2)\sec(c+dx)\tan(c+dx)}{8d}+\frac{b^2\sec^3(c+dx)\tan(c+dx)}{4d}+\frac{2ab\tan^3(c+dx)}{3d}$$

output
```
1/8*(4*a^2+3*b^2)*arctanh(sin(d*x+c))/d+2*a*b*tan(d*x+c)/d+1/8*(4*a^2+3*b^
2)*sec(d*x+c)*tan(d*x+c)/d+1/4*b^2*sec(d*x+c)^3*tan(d*x+c)/d+2/3*a*b*tan(d
*x+c)^3/d
```

3.457.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.75

$$\int \sec^3(c+dx)(a+b\sec(c+dx))^2\,dx = \frac{3(4a^2+3b^2)\operatorname{arctanh}(\sin(c+dx))+\tan(c+dx)\left(3(4a^2+3b^2)\sec(c+dx)+6b^2\sec^3(c+dx)+16ab(3+\right.}{24d}$$

input
```
Integrate[Sec[c + d*x]^3*(a + b*Sec[c + d*x])^2,x]
```

output
```
(3*(4*a^2 + 3*b^2)*ArcTanh[Sin[c + d*x]] + Tan[c + d*x]*(3*(4*a^2 + 3*b^2)
*Sec[c + d*x] + 6*b^2*Sec[c + d*x]^3 + 16*a*b*(3 + Tan[c + d*x]^2)))/(24*d
)
```

3.457.3 Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.93, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {3042, 4275, 3042, 4254, 2009, 4534, 3042, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^3(c+dx)(a+b\sec(c+dx))^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^3\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2dx$$

$$\Big\downarrow \text{4275}$$

$$\int \sec^3(c+dx)\left(a^2+b^2\sec^2(c+dx)\right)dx+2ab\int \sec^4(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^3\left(a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)dx+2ab\int \csc\left(c+dx+\frac{\pi}{2}\right)^4dx$$

$$\Big\downarrow \text{4254}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^3\left(a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)dx-\frac{2ab\int\left(\tan^2(c+dx)+1\right)d(-\tan(c+dx))}{d}$$

$$\Big\downarrow \text{2009}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^3\left(a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)dx-\frac{2ab\left(-\frac{1}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{d}$$

$$\Big\downarrow \text{4534}$$

$$\frac{1}{4}\left(4a^2+3b^2\right)\int \sec^3(c+dx)dx-\frac{2ab\left(-\frac{1}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{d}+\frac{b^2\tan(c+dx)\sec^3(c+dx)}{4d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}(4a^2+3b^2)\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx-\frac{2ab\left(-\frac{1}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{d}+\frac{b^2\tan(c+dx)\sec^3(c+dx)}{4d}$$

$$\Big\downarrow \text{4255}$$

$$\frac{1}{4}(4a^2+3b^2)\left(\frac{1}{2}\int \sec(c+dx)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)-\frac{2ab\left(-\frac{1}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{d}+\frac{b^2\tan(c+dx)\sec^3(c+dx)}{4d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}(4a^2+3b^2)\left(\frac{1}{2}\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)-\frac{2ab\left(-\frac{1}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{d}+\frac{b^2\tan(c+dx)\sec^3(c+dx)}{4d}$$

$$\Big\downarrow \text{4257}$$

$$\frac{1}{4}(4a^2+3b^2)\left(\frac{\text{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)-\frac{2ab\left(-\frac{1}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{d}+\frac{b^2\tan(c+dx)\sec^3(c+dx)}{4d}$$

input
```
Int[Sec[c + d*x]^3*(a + b*Sec[c + d*x])^2,x]
```

output
```
(b^2*Sec[c + d*x]^3*Tan[c + d*x])/(4*d) + ((4*a^2 + 3*b^2)*(ArcTanh[Sin[c
+ d*x]]/(2*d) + (Sec[c + d*x]*Tan[c + d*x])/(2*d)))/4 - (2*a*b*(-Tan[c + d
*x] - Tan[c + d*x]^3/3))/d
```

3.457.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4275
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Simp[2*a*(b/d)   Int[(d*Csc[e + f*x])^(n + 1), x], x]
 + Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
 e, f, n}, x]
```

rule 4534
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Simp[(C*m + A*(m + 1))/(m + 1)   Int[(b*Csc[e + f*x])^m, x], x] /;
 FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

3.457.4 Maple [A] (verified)

Time = 1.20 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.01

method	result
derivativedivides	$\frac{a^2\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)-2ab\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right)\tan(dx+c)+b^2\left(-\left(-\frac{\sec(dx+c)^3}{4}-\frac{3\sec(dx+c}{8}\right.\right.}{d}$
default	$\frac{a^2\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)-2ab\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right)\tan(dx+c)+b^2\left(-\left(-\frac{\sec(dx+c)^3}{4}-\frac{3\sec(dx+c}{8}\right.\right.}{d}$
parts	$\frac{a^2\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)}{d}+\frac{b^2\left(-\left(-\frac{\sec(dx+c)^3}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan}{8}\right.}{d}$
parallelrisch	$\frac{-48\left(\frac{3}{4}+\frac{\cos(4dx+4c)}{4}+\cos(2dx+2c)\right)\left(a^2+\frac{3b^2}{4}\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+48\left(\frac{3}{4}+\frac{\cos(4dx+4c)}{4}+\cos(2dx+2c)\right)\left(a^2+\frac{3b^2}{4}\right)\ln(}{24d(3+\cos(4dx+4c)+}$
norman	$\frac{\frac{\left(4a^2-16ab+5b^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{4d}+\frac{\left(4a^2+16ab+5b^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d}-\frac{\left(12a^2-80ab-9b^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{12d}-\frac{\left(12a^2+80ab-9b^2\right)\tan\left(\frac{dx}{2}\right.}{12d}}{\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^4}$
risch	$-\frac{i\left(12a^2e^{7i(dx+c)}+9b^2e^{7i(dx+c)}+12a^2e^{5i(dx+c)}+33b^2e^{5i(dx+c)}-96ab\,e^{4i(dx+c)}-12a^2e^{3i(dx+c)}-33b^2e^{3i(dx+c)}-128ab\right.}{12d\left(e^{2i(dx+c)}+1\right)^4}$

input
```
int(sec(d*x+c)^3*(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^2*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c)))-2*a*b*(
-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c)+b^2*(-(-1/4*sec(d*x+c)^3-3/8*sec(d*x+c))
*tan(d*x+c)+3/8*ln(sec(d*x+c)+tan(d*x+c))))
```

3.457.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.21

$$\int \sec^3(c+dx)(a+b\sec(c+dx))^2\,dx$$
$$= \frac{3\,(4\,a^2+3\,b^2)\cos(dx+c)^4\log(\sin(dx+c)+1)-3\,(4\,a^2+3\,b^2)\cos(dx+c)^4\log(-\sin(dx+c)+1)+2}{48\,d\cos(dx+c)}$$

input
```
integrate(sec(d*x+c)^3*(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/48*(3*(4*a^2 + 3*b^2)*cos(d*x + c)^4*log(sin(d*x + c) + 1) - 3*(4*a^2 +
3*b^2)*cos(d*x + c)^4*log(-sin(d*x + c) + 1) + 2*(32*a*b*cos(d*x + c)^3 +
16*a*b*cos(d*x + c) + 3*(4*a^2 + 3*b^2)*cos(d*x + c)^2 + 6*b^2)*sin(d*x +
c))/(d*cos(d*x + c)^4)
```

3.457.6 Sympy [F]

$$\int \sec^3(c+dx)(a+b\sec(c+dx))^2\,dx = \int (a+b\sec(c+dx))^2\sec^3(c+dx)\,dx$$

input
```
integrate(sec(d*x+c)**3*(a+b*sec(d*x+c))**2,x)
```

output
```
Integral((a + b*sec(c + d*x))**2*sec(c + d*x)**3, x)
```

3.457.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.31

$$\int \sec^3(c+dx)(a+b\sec(c+dx))^2\,dx$$
$$= \frac{32\left(\tan\left(dx+c\right)^3+3\,\tan\left(dx+c\right)\right)ab-3\,b^2\left(\frac{2\left(3\,\sin(dx+c)^3-5\,\sin(dx+c)\right)}{\sin(dx+c)^4-2\,\sin(dx+c)^2+1}-3\,\log\left(\sin\left(dx+c\right)+1\right)+3\,\log\left(s\right.\right.}{48\,d}$$

input
```
integrate(sec(d*x+c)^3*(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
1/48*(32*(tan(d*x + c)^3 + 3*tan(d*x + c))*a*b - 3*b^2*(2*(3*sin(d*x + c)^
3 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*
x + c) + 1) + 3*log(sin(d*x + c) - 1)) - 12*a^2*(2*sin(d*x + c)/(sin(d*x +
 c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)))/d
```

3.457.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 258 vs. 2(102) = 204.

Time = 0.33 (sec) , antiderivative size = 258, normalized size of antiderivative = 2.35

$$\int \sec^3(c+dx)(a+b\sec(c+dx))^2\,dx$$
$$= \frac{3\left(4\,a^2+3\,b^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\left(4\,a^2+3\,b^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{2\left(12\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right.\right.}{}}{}$$

input
```
integrate(sec(d*x+c)^3*(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
1/24*(3*(4*a^2 + 3*b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*(4*a^2 + 3*
b^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(12*a^2*tan(1/2*d*x + 1/2*c)^7
 - 48*a*b*tan(1/2*d*x + 1/2*c)^7 + 15*b^2*tan(1/2*d*x + 1/2*c)^7 - 12*a^2*
tan(1/2*d*x + 1/2*c)^5 + 80*a*b*tan(1/2*d*x + 1/2*c)^5 + 9*b^2*tan(1/2*d*x
 + 1/2*c)^5 - 12*a^2*tan(1/2*d*x + 1/2*c)^3 - 80*a*b*tan(1/2*d*x + 1/2*c)^
3 + 9*b^2*tan(1/2*d*x + 1/2*c)^3 + 12*a^2*tan(1/2*d*x + 1/2*c) + 48*a*b*ta
n(1/2*d*x + 1/2*c) + 15*b^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2
- 1)^4)/d
```

3.457.9 Mupad [B] (verification not implemented)

Time = 16.18 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.67

$$\int \sec^3(c+dx)(a+b\sec(c+dx))^2\,dx = \frac{\operatorname{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\left(a^2+\frac{3b^2}{4}\right)}{d} + \frac{\left(a^2-4ab+\frac{5b^2}{4}\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7+\left(-a^2+\frac{20ab}{3}+\frac{3b^2}{4}\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5+\left(-a^2-\frac{20ab}{3}+\frac{3b^2}{4}\right)\tan\left(\frac{c}{2}+\right.}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8-4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6+6\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4-4\tan\left(\frac{c}{2}+\frac{dx}{2}\right.\right.}$$

input
```
int((a + b/cos(c + d*x))^2/cos(c + d*x)^3,x)
```

output
```
(atanh(tan(c/2 + (d*x)/2))*(a^2 + (3*b^2)/4))/d + (tan(c/2 + (d*x)/2)^5*((
20*a*b)/3 - a^2 + (3*b^2)/4) + tan(c/2 + (d*x)/2)*(4*a*b + a^2 + (5*b^2)/4
) + tan(c/2 + (d*x)/2)^7*(a^2 - 4*a*b + (5*b^2)/4) - tan(c/2 + (d*x)/2)^3*
((20*a*b)/3 + a^2 - (3*b^2)/4))/(d*(6*tan(c/2 + (d*x)/2)^4 - 4*tan(c/2 + (
d*x)/2)^2 - 4*tan(c/2 + (d*x)/2)^6 + tan(c/2 + (d*x)/2)^8 + 1))
```

3.458 $\int \sec^2(c+dx)(a+b\sec(c+dx))^2\,dx$

3.458.1 Optimal result . . . 3387
3.458.2 Mathematica [A] (verified) . . . 3387
3.458.3 Rubi [A] (verified) . . . 3388
3.458.4 Maple [A] (verified) . . . 3391
3.458.5 Fricas [A] (verification not implemented) . . . 3391
3.458.6 Sympy [F] . . . 3392
3.458.7 Maxima [A] (verification not implemented) . . . 3392
3.458.8 Giac [B] (verification not implemented) . . . 3392
3.458.9 Mupad [B] (verification not implemented) . . . 3393

3.458.1 Optimal result

Integrand size = 21, antiderivative size = 80

$$\int \sec^2(c+dx)(a+b\sec(c+dx))^2\,dx = \frac{ab\operatorname{arctanh}(\sin(c+dx))}{d}+\frac{(3a^2+2b^2)\tan(c+dx)}{3d}+\frac{ab\sec(c+dx)\tan(c+dx)}{d}+\frac{b^2\sec^2(c+dx)\tan(c+dx)}{3d}$$

output
```
a*b*arctanh(sin(d*x+c))/d+1/3*(3*a^2+2*b^2)*tan(d*x+c)/d+a*b*sec(d*x+c)*ta
n(d*x+c)/d+1/3*b^2*sec(d*x+c)^2*tan(d*x+c)/d
```

3.458.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.89

$$\int \sec^2(c+dx)(a+b\sec(c+dx))^2\,dx = \frac{ab\operatorname{arctanh}(\sin(c+dx))}{d}+\frac{a^2\tan(c+dx)}{d}+\frac{ab\sec(c+dx)\tan(c+dx)}{d}+\frac{b^2\left(\tan(c+dx)+\frac{1}{3}\tan^3(c+dx)\right)}{d}$$

input
```
Integrate[Sec[c + d*x]^2*(a + b*Sec[c + d*x])^2,x]
```

output
```
(a*b*ArcTanh[Sin[c + d*x]])/d + (a^2*Tan[c + d*x])/d + (a*b*Sec[c + d*x]*T
an[c + d*x])/d + (b^2*(Tan[c + d*x] + Tan[c + d*x]^3/3))/d
```

3.458.3 Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.09, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {3042, 4275, 3042, 4255, 3042, 4257, 4534, 3042, 4254, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(c+dx)(a+b\sec(c+dx))^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^2\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2dx$$

$$\Big\downarrow \text{4275}$$

$$\int \sec^2(c+dx)\left(a^2+b^2\sec^2(c+dx)\right)dx+2ab\int \sec^3(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^2\left(a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)dx+2ab\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx$$

$$\Big\downarrow \text{4255}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^2\left(a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)dx+ 2ab\left(\frac{1}{2}\int \sec(c+dx)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^2\left(a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)dx+ 2ab\left(\frac{1}{2}\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)$$

$$\Big\downarrow \text{4257}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^2\left(a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)dx+2ab\left(\frac{\text{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)$$

$$\Big\downarrow \text{4534}$$

$$\frac{1}{3}(3a^2+2b^2)\int \sec^2(c+dx)dx+2ab\left(\frac{\text{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{b^2\tan(c+dx)\sec^2(c+dx)}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}(3a^2+2b^2)\int \csc\left(c+dx+\frac{\pi}{2}\right)^2dx+2ab\left(\frac{\text{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{b^2\tan(c+dx)\sec^2(c+dx)}{3d}$$

$$\Big\downarrow \text{4254}$$

$$-\frac{(3a^2+2b^2)\int 1d(-\tan(c+dx))}{3d}+2ab\left(\frac{\text{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{b^2\tan(c+dx)\sec^2(c+dx)}{3d}$$

$$\Big\downarrow \text{24}$$

$$\frac{(3a^2+2b^2)\tan(c+dx)}{3d}+2ab\left(\frac{\text{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{b^2\tan(c+dx)\sec^2(c+dx)}{3d}$$

input
```
Int[Sec[c + d*x]^2*(a + b*Sec[c + d*x])^2,x]
```

output
```
((3*a^2 + 2*b^2)*Tan[c + d*x])/(3*d) + (b^2*Sec[c + d*x]^2*Tan[c + d*x])/(
3*d) + 2*a*b*(ArcTanh[Sin[c + d*x]]/(2*d) + (Sec[c + d*x]*Tan[c + d*x])/(2
*d))
```

3.458. $\int \sec^2(c+dx)(a+b\sec(c+dx))^2\,dx$

3.458.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4275
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Simp[2*a*(b/d)   Int[(d*Csc[e + f*x])^(n + 1), x], x]
 + Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
 e, f, n}, x]
```

rule 4534
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Simp[(C*m + A*(m + 1))/(m + 1)   Int[(b*Csc[e + f*x])^m, x], x] /;
 FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

3.458.4 Maple [A] (verified)

Time = 1.22 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.92

method	result
derivativedivides	$\frac{\tan(dx+c)a^2+2ab\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)-b^2\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right)\tan(dx+c)}{d}$
default	$\frac{\tan(dx+c)a^2+2ab\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)-b^2\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right)\tan(dx+c)}{d}$
parts	$\frac{a^2\tan(dx+c)}{d}-\frac{b^2\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right)\tan(dx+c)}{d}+\frac{ab\sec(dx+c)\tan(dx+c)}{d}+\frac{ab\ln(\sec(dx+c)+\tan(dx+c))}{d}$
norman	$\frac{\frac{4\left(3a^2+b^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3d}-\frac{2\left(a^2-ab+b^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{d}-\frac{2\left(a^2+ab+b^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}}{\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^3}+\frac{ab\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{d}-\frac{ab\ln(\mathrm{ta}}{}$
risch	$-\frac{2i\left(3ab\,\mathrm{e}^{5i(dx+c)}-3a^2\mathrm{e}^{4i(dx+c)}-6a^2\mathrm{e}^{2i(dx+c)}-6b^2\mathrm{e}^{2i(dx+c)}-3ab\,\mathrm{e}^{i(dx+c)}-3a^2-2b^2\right)}{3d\left(\mathrm{e}^{2i(dx+c)}+1\right)^3}+\frac{ab\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{d}-\frac{ab\ln}{}$
parallelrisch	$\frac{-9\left(\cos(dx+c)+\frac{\cos(3dx+3c)}{3}\right)ab\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+9\left(\cos(dx+c)+\frac{\cos(3dx+3c)}{3}\right)ab\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+\left(3a^2+2b^2\right)}{3d(\cos(3dx+3c)+3\cos(dx+c))}$

input
```
int(sec(d*x+c)^2*(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(tan(d*x+c)*a^2+2*a*b*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan
(d*x+c)))-b^2*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c))
```

3.458.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.25

$$\int \sec^2(c+dx)(a+b\sec(c+dx))^2\,dx$$
$$=\frac{3\,ab\cos\left(dx+c\right)^3\log\left(\sin\left(dx+c\right)+1\right)-3\,ab\cos\left(dx+c\right)^3\log\left(-\sin\left(dx+c\right)+1\right)+2\left(3\,ab\cos\left(dx+c\right)\right.}{6\,d\cos\left(dx+c\right)^3}$$

input
```
integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/6*(3*a*b*cos(d*x + c)^3*log(sin(d*x + c) + 1) - 3*a*b*cos(d*x + c)^3*log
(-sin(d*x + c) + 1) + 2*(3*a*b*cos(d*x + c) + (3*a^2 + 2*b^2)*cos(d*x + c)
^2 + b^2)*sin(d*x + c))/(d*cos(d*x + c)^3)
```

3.458.6 Sympy [F]

$$\int \sec^2(c+dx)(a+b\sec(c+dx))^2\,dx = \int (a+b\sec(c+dx))^2\sec^2(c+dx)\,dx$$

input
```
integrate(sec(d*x+c)**2*(a+b*sec(d*x+c))**2,x)
```

output
```
Integral((a + b*sec(c + d*x))**2*sec(c + d*x)**2, x)
```

3.458.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.05

$$\int \sec^2(c+dx)(a+b\sec(c+dx))^2\,dx = \frac{2\left(\tan(dx+c)^3+3\tan(dx+c)\right)b^2-3ab\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1}-\log(\sin(dx+c)+1)+\log(\sin(dx+c)-1)\right)}{6d}$$

input
```
integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
1/6*(2*(tan(d*x + c)^3 + 3*tan(d*x + c))*b^2 - 3*a*b*(2*sin(d*x + c)/(sin(
d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 6*a^2*t
an(d*x + c))/d
```

3.458.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 178 vs. 2(76) = 152.

Time = 0.31 (sec) , antiderivative size = 178, normalized size of antiderivative = 2.22

$$\int \sec^2(c+dx)(a+b\sec(c+dx))^2\,dx = \frac{3ab\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)-3ab\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)-\frac{2\left(3a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-3ab\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+3\right.}{}}{3d}$$

input
```
integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
1/3*(3*a*b*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*a*b*log(abs(tan(1/2*d*x
+ 1/2*c) - 1)) - 2*(3*a^2*tan(1/2*d*x + 1/2*c)^5 - 3*a*b*tan(1/2*d*x + 1/2
*c)^5 + 3*b^2*tan(1/2*d*x + 1/2*c)^5 - 6*a^2*tan(1/2*d*x + 1/2*c)^3 - 2*b^
2*tan(1/2*d*x + 1/2*c)^3 + 3*a^2*tan(1/2*d*x + 1/2*c) + 3*a*b*tan(1/2*d*x
+ 1/2*c) + 3*b^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^3)/d
```

3.458.9 Mupad [B] (verification not implemented)

Time = 15.11 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.76

$$\int \sec^2(c+dx)(a+b\sec(c+dx))^2\,dx = \frac{2\,a\,b\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d} - \frac{\left(2\,a^2-2\,a\,b+2\,b^2\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+\left(-4\,a^2-\frac{4\,b^2}{3}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+\left(2\,a^2+2\,a\,b+2\,b^2\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)}$$

input
```
int((a + b/cos(c + d*x))^2/cos(c + d*x)^2,x)
```

output
```
(2*a*b*atanh(tan(c/2 + (d*x)/2)))/d - (tan(c/2 + (d*x)/2)^5*(2*a^2 - 2*a*b
 + 2*b^2) - tan(c/2 + (d*x)/2)^3*(4*a^2 + (4*b^2)/3) + tan(c/2 + (d*x)/2)*
(2*a*b + 2*a^2 + 2*b^2))/(d*(3*tan(c/2 + (d*x)/2)^2 - 3*tan(c/2 + (d*x)/2)
^4 + tan(c/2 + (d*x)/2)^6 - 1))
```

3.459 $\int \sec(c+dx)(a+b\sec(c+dx))^2\,dx$

3.459.1 Optimal result . . . 3394
3.459.2 Mathematica [A] (verified) . . . 3394
3.459.3 Rubi [A] (verified) . . . 3395
3.459.4 Maple [A] (verified) . . . 3397
3.459.5 Fricas [A] (verification not implemented) . . . 3397
3.459.6 Sympy [F] . . . 3398
3.459.7 Maxima [A] (verification not implemented) . . . 3398
3.459.8 Giac [B] (verification not implemented) . . . 3398
3.459.9 Mupad [B] (verification not implemented) . . . 3399

3.459.1 Optimal result

Integrand size = 19, antiderivative size = 59

$$\int \sec(c+dx)(a+b\sec(c+dx))^2\,dx = \frac{(2a^2+b^2)\operatorname{arctanh}(\sin(c+dx))}{2d} + \frac{2ab\tan(c+dx)}{d} + \frac{b^2\sec(c+dx)\tan(c+dx)}{2d}$$

output
```
1/2*(2*a^2+b^2)*arctanh(sin(d*x+c))/d+2*a*b*tan(d*x+c)/d+1/2*b^2*sec(d*x+c
)*tan(d*x+c)/d
```

3.459.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.76

$$\int \sec(c+dx)(a+b\sec(c+dx))^2\,dx = \frac{(2a^2+b^2)\operatorname{arctanh}(\sin(c+dx)) + b(4a+b\sec(c+dx))\tan(c+dx)}{2d}$$

input
```
Integrate[Sec[c + d*x]*(a + b*Sec[c + d*x])^2,x]
```

output
```
((2*a^2 + b^2)*ArcTanh[Sin[c + d*x]] + b*(4*a + b*Sec[c + d*x])*Tan[c + d*
x])/(2*d)
```

3.459.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {3042, 4275, 3042, 4254, 24, 4534, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(c+dx)(a+b\sec(c+dx))^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2dx$$

$$\Big\downarrow \text{4275}$$

$$\int \sec(c+dx)\left(a^2+b^2\sec^2(c+dx)\right)dx+2ab\int \sec^2(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)\left(a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)dx+2ab\int \csc\left(c+dx+\frac{\pi}{2}\right)^2dx$$

$$\Big\downarrow \text{4254}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)\left(a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)dx-\frac{2ab\int 1d(-\tan(c+dx))}{d}$$

$$\Big\downarrow \text{24}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)\left(a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)dx+\frac{2ab\tan(c+dx)}{d}$$

$$\Big\downarrow \text{4534}$$

$$\frac{1}{2}\left(2a^2+b^2\right)\int \sec(c+dx)dx+\frac{2ab\tan(c+dx)}{d}+\frac{b^2\tan(c+dx)\sec(c+dx)}{2d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\left(2a^2+b^2\right)\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{2ab\tan(c+dx)}{d}+\frac{b^2\tan(c+dx)\sec(c+dx)}{2d}$$

$$\Big\downarrow \text{4257}$$

$$\frac{\left(2a^2+b^2\right)\operatorname{arctanh}(\sin(c+dx))}{2d}+\frac{2ab\tan(c+dx)}{d}+\frac{b^2\tan(c+dx)\sec(c+dx)}{2d}$$

input
```
Int[Sec[c + d*x]*(a + b*Sec[c + d*x])^2,x]
```

output
```
((2*a^2 + b^2)*ArcTanh[Sin[c + d*x]])/(2*d) + (2*a*b*Tan[c + d*x])/d + (b^
2*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

3.459.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4275
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Simp[2*a*(b/d)   Int[(d*Csc[e + f*x])^(n + 1), x], x]
 + Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
 e, f, n}, x]
```

rule 4534
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Simp[(C*m + A*(m + 1))/(m + 1)   Int[(b*Csc[e + f*x])^m, x], x] /;
 FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

3.459.4 Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.17

method	result
derivativedivides	$\frac{a^2 \ln(\sec(dx+c)+\tan(dx+c))+2\tan(dx+c)ab+b^2\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)}{d}$
default	$\frac{a^2 \ln(\sec(dx+c)+\tan(dx+c))+2\tan(dx+c)ab+b^2\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)}{d}$
parts	$\frac{a^2 \ln(\sec(dx+c)+\tan(dx+c))}{d}+\frac{b^2\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)}{d}+\frac{2ab\tan(dx+c)}{d}$
parallelrisch	$\frac{-\left(a^2+\frac{b^2}{2}\right)(1+\cos(2dx+2c))\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+\left(a^2+\frac{b^2}{2}\right)(1+\cos(2dx+2c))\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+2ab\sin(2dx+2c)+}{d(1+\cos(2dx+2c))}$
norman	$\frac{\frac{b(4a+b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}-\frac{b(4a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{d}}{\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^2}-\frac{(2a^2+b^2)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2d}+\frac{(2a^2+b^2)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{2d}$
risch	$-\frac{ib\left(b\,e^{3i(dx+c)}-4a\,e^{2i(dx+c)}-b\,e^{i(dx+c)}-4a\right)}{d\left(e^{2i(dx+c)}+1\right)^2}-\frac{a^2\ln\left(e^{i(dx+c)}-i\right)}{d}-\frac{\ln\left(e^{i(dx+c)}-i\right)b^2}{2d}+\frac{a^2\ln\left(e^{i(dx+c)}+i\right)}{d}+$

input
```
int(sec(d*x+c)*(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^2*ln(sec(d*x+c)+tan(d*x+c))+2*tan(d*x+c)*a*b+b^2*(1/2*sec(d*x+c)*ta
n(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c))))
```

3.459.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.58

$$\int \sec(c+dx)(a+b\sec(c+dx))^2\,dx$$
$$=\frac{(2\,a^2+b^2)\cos(dx+c)^2\log(\sin(dx+c)+1)-(2\,a^2+b^2)\cos(dx+c)^2\log(-\sin(dx+c)+1)+2\,(4\,ab}{4\,d\cos(dx+c)^2}$$

input
```
integrate(sec(d*x+c)*(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/4*((2*a^2 + b^2)*cos(d*x + c)^2*log(sin(d*x + c) + 1) - (2*a^2 + b^2)*co
s(d*x + c)^2*log(-sin(d*x + c) + 1) + 2*(4*a*b*cos(d*x + c) + b^2)*sin(d*x
 + c))/(d*cos(d*x + c)^2)
```

3.459.6 Sympy [F]

$$\int \sec(c+dx)(a+b\sec(c+dx))^2\,dx = \int (a+b\sec(c+dx))^2\sec(c+dx)\,dx$$

input
```
integrate(sec(d*x+c)*(a+b*sec(d*x+c))**2,x)
```

output
```
Integral((a + b*sec(c + d*x))**2*sec(c + d*x), x)
```

3.459.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.36

$$\int \sec(c+dx)(a+b\sec(c+dx))^2\,dx = -\frac{b^2\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1}-\log(\sin(dx+c)+1)+\log(\sin(dx+c)-1)\right)-4a^2\log(\sec(dx+c)+\tan(dx+c))}{4d}$$

input
```
integrate(sec(d*x+c)*(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
-1/4*(b^2*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + l
og(sin(d*x + c) - 1)) - 4*a^2*log(sec(d*x + c) + tan(d*x + c)) - 8*a*b*tan
(d*x + c))/d
```

3.459.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 129 vs. 2(55) = 110.

Time = 0.31 (sec) , antiderivative size = 129, normalized size of antiderivative = 2.19

$$\int \sec(c+dx)(a+b\sec(c+dx))^2\,dx = \frac{(2a^2+b^2)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)-(2a^2+b^2)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)-\frac{2\left(4ab\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-b^2\tan\right.}{}}{2d}$$

input
```
integrate(sec(d*x+c)*(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
1/2*((2*a^2 + b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - (2*a^2 + b^2)*log(
abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(4*a*b*tan(1/2*d*x + 1/2*c)^3 - b^2*tan
(1/2*d*x + 1/2*c)^3 - 4*a*b*tan(1/2*d*x + 1/2*c) - b^2*tan(1/2*d*x + 1/2*c
))/(tan(1/2*d*x + 1/2*c)^2 - 1)^2)/d
```

3.459.9 Mupad [B] (verification not implemented)

Time = 13.50 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.68

$$\int \sec(c+dx)(a+b\sec(c+dx))^2\,dx = \frac{\operatorname{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\,(2\,a^2+b^2)}{d} - \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3(4\,a\,b-b^2)-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\,(b^2+4\,a\,b)}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4-2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)}$$

input
```
int((a + b/cos(c + d*x))^2/cos(c + d*x),x)
```

output
```
(atanh(tan(c/2 + (d*x)/2))*(2*a^2 + b^2))/d - (tan(c/2 + (d*x)/2)^3*(4*a*b
 - b^2) - tan(c/2 + (d*x)/2)*(4*a*b + b^2))/(d*(tan(c/2 + (d*x)/2)^4 - 2*t
an(c/2 + (d*x)/2)^2 + 1))
```

3.460 $\int (a+b\sec(c+dx))^2\,dx$

3.460.1 Optimal result . . . 3400
3.460.2 Mathematica [A] (verified) . . . 3400
3.460.3 Rubi [A] (verified) . . . 3401
3.460.4 Maple [A] (verified) . . . 3402
3.460.5 Fricas [B] (verification not implemented) . . . 3403
3.460.6 Sympy [F] . . . 3403
3.460.7 Maxima [A] (verification not implemented) . . . 3403
3.460.8 Giac [B] (verification not implemented) . . . 3404
3.460.9 Mupad [B] (verification not implemented) . . . 3404

3.460.1 Optimal result

Integrand size = 12, antiderivative size = 33

$$\int (a+b\sec(c+dx))^2\,dx = a^2x+\frac{2ab\operatorname{arctanh}(\sin(c+dx))}{d}+\frac{b^2\tan(c+dx)}{d}$$

output
```
a^2*x+2*a*b*arctanh(sin(d*x+c))/d+b^2*tan(d*x+c)/d
```

3.460.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.97

$$\int (a+b\sec(c+dx))^2\,dx = \frac{a^2dx+2ab\operatorname{arctanh}(\sin(c+dx))+b^2\tan(c+dx)}{d}$$

input
```
Integrate[(a + b*Sec[c + d*x])^2,x]
```

output
```
(a^2*d*x + 2*a*b*ArcTanh[Sin[c + d*x]] + b^2*Tan[c + d*x])/d
```

3.460.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3042, 4260, 3042, 4254, 24, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\sec(c+dx))^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2dx$$

$$\Big\downarrow \text{4260}$$

$$2ab\int \sec(c+dx)dx+b^2\int \sec^2(c+dx)dx+a^2x$$

$$\Big\downarrow \text{3042}$$

$$2ab\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+b^2\int \csc\left(c+dx+\frac{\pi}{2}\right)^2dx+a^2x$$

$$\Big\downarrow \text{4254}$$

$$2ab\int \csc\left(c+dx+\frac{\pi}{2}\right)dx-\frac{b^2\int 1d(-\tan(c+dx))}{d}+a^2x$$

$$\Big\downarrow \text{24}$$

$$2ab\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+a^2x+\frac{b^2\tan(c+dx)}{d}$$

$$\Big\downarrow \text{4257}$$

$$a^2x+\frac{2ab\text{arctanh}(\sin(c+dx))}{d}+\frac{b^2\tan(c+dx)}{d}$$

input
```
Int[(a + b*Sec[c + d*x])^2,x]
```

output
```
a^2*x + (2*a*b*ArcTanh[Sin[c + d*x]])/d + (b^2*Tan[c + d*x])/d
```

3.460.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4260
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^2, x_Symbol] :> Simp[a^2*x, x] +
 (Simp[2*a*b   Int[Csc[c + d*x], x], x] + Simp[b^2   Int[Csc[c + d*x]^2, x]
, x]) /; FreeQ[{a, b, c, d}, x]
```

3.460.4 Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.24

method	result	size
parts	$a^2x+\frac{b^2\tan(dx+c)}{d}+\frac{2ab\ln(\sec(dx+c)+\tan(dx+c))}{d}$	41
derivativedivides	$\frac{a^2(dx+c)+2ab\ln(\sec(dx+c)+\tan(dx+c))+\tan(dx+c)b^2}{d}$	43
default	$\frac{a^2(dx+c)+2ab\ln(\sec(dx+c)+\tan(dx+c))+\tan(dx+c)b^2}{d}$	43
risch	$a^2x+\frac{2ib^2}{d(e^{2i(dx+c)}+1)}-\frac{2ab\ln(e^{i(dx+c)}-i)}{d}+\frac{2ab\ln(e^{i(dx+c)}+i)}{d}$	69
parallelrisch	$\frac{a^2xd\cos(dx+c)-2ab\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\cos(dx+c)+2ab\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)\cos(dx+c)+b^2\sin(dx+c)}{d\cos(dx+c)}$	80
norman	$\frac{a^2x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2-a^2x-\frac{2b^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}}{-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}-\frac{2ab\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{d}+\frac{2ab\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{d}$	96

input
```
int((a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
a^2*x+b^2*tan(d*x+c)/d+2*a*b/d*ln(sec(d*x+c)+tan(d*x+c))
```

3.460.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 74 vs. $2(33) = 66$.

Time = 0.26 (sec) , antiderivative size = 74, normalized size of antiderivative = 2.24

$$\int (a+b\sec(c+dx))^2\,dx = \frac{a^2dx\cos(dx+c)+ab\cos(dx+c)\log(\sin(dx+c)+1)-ab\cos(dx+c)\log(-\sin(dx+c)+1)+b^2\sin}{d\cos(dx+c)}$$

input
```
integrate((a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
(a^2*d*x*cos(d*x + c) + a*b*cos(d*x + c)*log(sin(d*x + c) + 1) - a*b*cos(d
*x + c)*log(-sin(d*x + c) + 1) + b^2*sin(d*x + c))/(d*cos(d*x + c))
```

3.460.6 Sympy [F]

$$\int (a+b\sec(c+dx))^2\,dx = \int (a+b\sec(c+dx))^2\,dx$$

input
```
integrate((a+b*sec(d*x+c))**2,x)
```

output
```
Integral((a + b*sec(c + d*x))**2, x)
```

3.460.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.21

$$\int (a+b\sec(c+dx))^2\,dx = a^2x+\frac{2\,ab\log(\sec(dx+c)+\tan(dx+c))}{d}+\frac{b^2\tan(dx+c)}{d}$$

input
```
integrate((a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
a^2*x + 2*a*b*log(sec(d*x + c) + tan(d*x + c))/d + b^2*tan(d*x + c)/d
```

3.460.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 77 vs. $2(33) = 66$.

Time = 0.32 (sec) , antiderivative size = 77, normalized size of antiderivative = 2.33

$$\int (a+b\sec(c+dx))^2\,dx = \frac{(dx+c)a^2+2\,ab\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-2\,ab\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1}}{d}$$

input
```
integrate((a+b*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
((d*x + c)*a^2 + 2*a*b*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 2*a*b*log(abs(
tan(1/2*d*x + 1/2*c) - 1)) - 2*b^2*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2
*c)^2 - 1))/d
```

3.460.9 Mupad [B] (verification not implemented)

Time = 13.50 (sec) , antiderivative size = 181, normalized size of antiderivative = 5.48

$$\int (a+b\sec(c+dx))^2\,dx = \frac{2\,a^2\,\mathrm{atan}\left(\frac{64\,a^6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{64\,a^6+256\,a^4\,b^2}+\frac{256\,a^4\,b^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{64\,a^6+256\,a^4\,b^2}\right)}{d} - \frac{2\,b^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)} + \frac{4\,a\,b\,\mathrm{atanh}\left(\frac{128\,a^5\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{128\,a^5\,b+512\,a^3\,b^3}+\frac{512\,a^3\,b^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{128\,a^5\,b+512\,a^3\,b^3}\right)}{d}$$

input
```
int((a + b/cos(c + d*x))^2,x)
```

output
```
(2*a^2*atan((64*a^6*tan(c/2 + (d*x)/2))/(64*a^6 + 256*a^4*b^2) + (256*a^4*
b^2*tan(c/2 + (d*x)/2))/(64*a^6 + 256*a^4*b^2)))/d - (2*b^2*tan(c/2 + (d*x
)/2))/(d*(tan(c/2 + (d*x)/2)^2 - 1)) + (4*a*b*atanh((128*a^5*b*tan(c/2 + (
d*x)/2))/(128*a^5*b + 512*a^3*b^3) + (512*a^3*b^3*tan(c/2 + (d*x)/2))/(128
*a^5*b + 512*a^3*b^3)))/d
```

3.461 $\int \cos(c+dx)(a+b\sec(c+dx))^2\,dx$

3.461.1 Optimal result . . . 3405
3.461.2 Mathematica [A] (verified) . . . 3405
3.461.3 Rubi [A] (verified) . . . 3406
3.461.4 Maple [A] (verified) . . . 3407
3.461.5 Fricas [A] (verification not implemented) . . . 3408
3.461.6 Sympy [F] . . . 3408
3.461.7 Maxima [A] (verification not implemented) . . . 3408
3.461.8 Giac [B] (verification not implemented) . . . 3409
3.461.9 Mupad [B] (verification not implemented) . . . 3409

3.461.1 Optimal result

Integrand size = 19, antiderivative size = 33

$$\int \cos(c+dx)(a+b\sec(c+dx))^2\,dx = 2abx+\frac{b^2\text{arctanh}(\sin(c+dx))}{d}+\frac{a^2\sin(c+dx)}{d}$$

output
```
2*a*b*x+b^2*arctanh(sin(d*x+c))/d+a^2*sin(d*x+c)/d
```

3.461.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.39

$$\int \cos(c+dx)(a+b\sec(c+dx))^2\,dx = 2abx+\frac{b^2\text{arctanh}(\sin(c+dx))}{d}+\frac{a^2\cos(dx)\sin(c)}{d}+\frac{a^2\cos(c)\sin(dx)}{d}$$

input
```
Integrate[Cos[c + d*x]*(a + b*Sec[c + d*x])^2,x]
```

output
```
2*a*b*x + (b^2*ArcTanh[Sin[c + d*x]])/d + (a^2*Cos[d*x]*Sin[c])/d + (a^2*C
os[c]*Sin[d*x])/d
```

3.461.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {3042, 4275, 24, 3042, 4533, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(c+dx)(a+b\sec(c+dx))^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{4275}$$

$$\int \cos(c+dx)\left(a^2+b^2\sec^2(c+dx)\right)dx+2ab\int 1dx$$

$$\Big\downarrow \text{24}$$

$$\int \cos(c+dx)\left(a^2+b^2\sec^2(c+dx)\right)dx+2abx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+2abx$$

$$\Big\downarrow \text{4533}$$

$$b^2\int \sec(c+dx)dx+\frac{a^2\sin(c+dx)}{d}+2abx$$

$$\Big\downarrow \text{3042}$$

$$b^2\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{a^2\sin(c+dx)}{d}+2abx$$

$$\Big\downarrow \text{4257}$$

$$\frac{a^2\sin(c+dx)}{d}+2abx+\frac{b^2\text{arctanh}(\sin(c+dx))}{d}$$

input
```
Int[Cos[c + d*x]*(a + b*Sec[c + d*x])^2,x]
```

output
```
2*a*b*x + (b^2*ArcTanh[Sin[c + d*x]])/d + (a^2*Sin[c + d*x])/d
```

3.461.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4275
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Simp[2*a*(b/d)   Int[(d*Csc[e + f*x])^(n + 1), x], x]
 + Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
 e, f, n}, x]
```

rule 4533
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] +
Simp[(C*m + A*(m + 1))/(b^2*m)   Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fr
eeQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

3.461.4 Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.30

method	result
derivativedivides	$\frac{a^2 \sin(dx+c)+2ab(dx+c)+b^2 \ln(\sec(dx+c)+\tan(dx+c))}{d}$
default	$\frac{a^2 \sin(dx+c)+2ab(dx+c)+b^2 \ln(\sec(dx+c)+\tan(dx+c))}{d}$
parallelrisch	$\frac{2abxd+a^2 \sin(dx+c)-\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)b^2+\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)b^2}{d}$
risch	$2abx-\frac{ia^2 e^{i(dx+c)}}{2d}+\frac{ia^2 e^{-i(dx+c)}}{2d}+\frac{\ln\left(e^{i(dx+c)}+i\right)b^2}{d}-\frac{\ln\left(e^{i(dx+c)}-i\right)b^2}{d}$
norman	$\frac{-2abx-\frac{2a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{2a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{d}+2abx\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)}+\frac{b^2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{d}-\frac{b^2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{d}$

input
```
int(cos(d*x+c)*(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^2*sin(d*x+c)+2*a*b*(d*x+c)+b^2*ln(sec(d*x+c)+tan(d*x+c)))
```

3.461.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.58

$$\int \cos(c+dx)(a+b\sec(c+dx))^2\,dx = \frac{4\,abdx + b^2\log\left(\sin\left(dx+c\right)+1\right) - b^2\log\left(-\sin\left(dx+c\right)+1\right) + 2\,a^2\sin\left(dx+c\right)}{2\,d}$$

input
```
integrate(cos(d*x+c)*(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/2*(4*a*b*d*x + b^2*log(sin(d*x + c) + 1) - b^2*log(-sin(d*x + c) + 1) +
2*a^2*sin(d*x + c))/d
```

3.461.6 Sympy [F]

$$\int \cos(c+dx)(a+b\sec(c+dx))^2\,dx = \int \left(a+b\sec\left(c+dx\right)\right)^2\cos\left(c+dx\right)dx$$

input
```
integrate(cos(d*x+c)*(a+b*sec(d*x+c))**2,x)
```

output
```
Integral((a + b*sec(c + d*x))**2*cos(c + d*x), x)
```

3.461.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.55

$$\int \cos(c+dx)(a+b\sec(c+dx))^2\,dx = \frac{4\,(dx+c)ab + b^2(\log\left(\sin\left(dx+c\right)+1\right) - \log\left(\sin\left(dx+c\right)-1\right)) + 2\,a^2\sin\left(dx+c\right)}{2\,d}$$

input
```
integrate(cos(d*x+c)*(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
1/2*(4*(d*x + c)*a*b + b^2*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1))
 + 2*a^2*sin(d*x + c))/d
```

3.461.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 78 vs. $2(33) = 66$.

Time = 0.29 (sec) , antiderivative size = 78, normalized size of antiderivative = 2.36

$$\int \cos(c+dx)(a+b\sec(c+dx))^2\,dx = \frac{2\,(dx+c)ab + b^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right) - b^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right) + \frac{2\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1}}{d}$$

input
```
integrate(cos(d*x+c)*(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
(2*(d*x + c)*a*b + b^2*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - b^2*log(abs(ta
n(1/2*d*x + 1/2*c) - 1)) + 2*a^2*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c
)^2 + 1))/d
```

3.461.9 Mupad [B] (verification not implemented)

Time = 13.44 (sec) , antiderivative size = 73, normalized size of antiderivative = 2.21

$$\int \cos(c+dx)(a+b\sec(c+dx))^2\,dx = \frac{a^2\,\sin(c+d\,x)}{d} + \frac{2\,b^2\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d} + \frac{4\,a\,b\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}$$

input
```
int(cos(c + d*x)*(a + b/cos(c + d*x))^2,x)
```

output
```
(a^2*sin(c + d*x))/d + (2*b^2*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))
)/d + (4*a*b*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d
```

3.462 $\int \cos^2(c+dx)(a+b\sec(c+dx))^2\,dx$

3.462.1 Optimal result . . . 3410
3.462.2 Mathematica [A] (verified) . . . 3410
3.462.3 Rubi [A] (verified) . . . 3411
3.462.4 Maple [A] (verified) . . . 3412
3.462.5 Fricas [A] (verification not implemented) . . . 3413
3.462.6 Sympy [F] . . . 3413
3.462.7 Maxima [A] (verification not implemented) . . . 3413
3.462.8 Giac [B] (verification not implemented) . . . 3414
3.462.9 Mupad [B] (verification not implemented) . . . 3414

3.462.1 Optimal result

Integrand size = 21, antiderivative size = 50

$$\int \cos^2(c+dx)(a+b\sec(c+dx))^2\,dx=\frac{1}{2}\left(a^2+2b^2\right)x+\frac{2ab\sin(c+dx)}{d}+\frac{a^2\cos(c+dx)\sin(c+dx)}{2d}$$

output
```
1/2*(a^2+2*b^2)*x+2*a*b*sin(d*x+c)/d+1/2*a^2*cos(d*x+c)*sin(d*x+c)/d
```

3.462.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.92

$$\int \cos^2(c+dx)(a+b\sec(c+dx))^2\,dx=\frac{2(a^2+2b^2)(c+dx)+8ab\sin(c+dx)+a^2\sin(2(c+dx))}{4d}$$

input
```
Integrate[Cos[c + d*x]^2*(a + b*Sec[c + d*x])^2,x]
```

output
```
(2*(a^2 + 2*b^2)*(c + d*x) + 8*a*b*Sin[c + d*x] + a^2*Sin[2*(c + d*x)])/(4
*d)
```

3.462.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 4275, 3042, 3117, 4533, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(c+dx)(a+b\sec(c+dx))^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^2}dx$$

$$\Big\downarrow \text{4275}$$

$$\int \cos^2(c+dx)\left(a^2+b^2\sec^2(c+dx)\right)dx+2ab\int \cos(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^2}dx+2ab\int \sin\left(c+dx+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{3117}$$

$$\int \frac{a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^2}dx+\frac{2ab\sin(c+dx)}{d}$$

$$\Big\downarrow \text{4533}$$

$$\frac{1}{2}\left(a^2+2b^2\right)\int 1dx+\frac{a^2\sin(c+dx)\cos(c+dx)}{2d}+\frac{2ab\sin(c+dx)}{d}$$

$$\Big\downarrow \text{24}$$

$$\frac{1}{2}x\left(a^2+2b^2\right)+\frac{a^2\sin(c+dx)\cos(c+dx)}{2d}+\frac{2ab\sin(c+dx)}{d}$$

input
```
Int[Cos[c + d*x]^2*(a + b*Sec[c + d*x])^2,x]
```

output
```
((a^2 + 2*b^2)*x)/2 + (2*a*b*Sin[c + d*x])/d + (a^2*Cos[c + d*x]*Sin[c + d
*x])/(2*d)
```

3.462.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 4275
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Simp[2*a*(b/d)   Int[(d*Csc[e + f*x])^(n + 1), x], x]
 + Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
 e, f, n}, x]
```

rule 4533
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] +
Simp[(C*m + A*(m + 1))/(b^2*m)   Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fr
eeQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

3.462.4 Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.86

method	result
risch	$\frac{a^2 x}{2}+x\,b^2+\frac{2ab\sin(dx+c)}{d}+\frac{a^2\sin(2dx+2c)}{4d}$
parallelrisch	$\frac{a^2\sin(2dx+2c)+8ab\sin(dx+c)+2\left(a^2+2b^2\right)xd}{4d}$
derivativedivides	$\frac{a^2\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+2ab\sin(dx+c)+b^2(dx+c)}{d}$
default	$\frac{a^2\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+2ab\sin(dx+c)+b^2(dx+c)}{d}$
norman	$\frac{\left(-\frac{a^2}{2}-b^2\right)x+\left(-\frac{a^2}{2}-b^2\right)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+\left(\frac{a^2}{2}+b^2\right)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\left(\frac{a^2}{2}+b^2\right)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6+\frac{2a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{d}-\frac{a(}{}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^2\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)}$

input
```
int(cos(d*x+c)^2*(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*a^2*x+x*b^2+2*a*b*sin(d*x+c)/d+1/4*a^2/d*sin(2*d*x+2*c)
```

3.462.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.80

$$\int \cos^2(c+dx)(a+b\sec(c+dx))^2\,dx = \frac{(a^2+2\,b^2)dx+(a^2\cos(dx+c)+4\,ab)\sin(dx+c)}{2\,d}$$

input
```
integrate(cos(d*x+c)^2*(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/2*((a^2 + 2*b^2)*d*x + (a^2*cos(d*x + c) + 4*a*b)*sin(d*x + c))/d
```

3.462.6 Sympy [F]

$$\int \cos^2(c+dx)(a+b\sec(c+dx))^2\,dx = \int (a+b\sec(c+dx))^2\cos^2(c+dx)\,dx$$

input
```
integrate(cos(d*x+c)**2*(a+b*sec(d*x+c))**2,x)
```

output
```
Integral((a + b*sec(c + d*x))**2*cos(c + d*x)**2, x)
```

3.462.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.94

$$\int \cos^2(c+dx)(a+b\sec(c+dx))^2\,dx = \frac{(2\,dx+2\,c+\sin(2\,dx+2\,c))a^2+4\,(dx+c)b^2+8\,ab\sin(dx+c)}{4\,d}$$

input
```
integrate(cos(d*x+c)^2*(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
1/4*((2*d*x + 2*c + sin(2*d*x + 2*c))*a^2 + 4*(d*x + c)*b^2 + 8*a*b*sin(d*
x + c))/d
```

3.462.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 96 vs. $2(46) = 92$.

Time = 0.28 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.92

$$\int \cos^2(c+dx)(a+b\sec(c+dx))^2\,dx = \frac{(a^2+2\,b^2)(dx+c) - \frac{2\left(a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-4\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-4\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^2}}{2\,d}$$

input
```
integrate(cos(d*x+c)^2*(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
1/2*((a^2 + 2*b^2)*(d*x + c) - 2*(a^2*tan(1/2*d*x + 1/2*c)^3 - 4*a*b*tan(1
/2*d*x + 1/2*c)^3 - a^2*tan(1/2*d*x + 1/2*c) - 4*a*b*tan(1/2*d*x + 1/2*c))
/(tan(1/2*d*x + 1/2*c)^2 + 1)^2)/d
```

3.462.9 Mupad [B] (verification not implemented)

Time = 13.48 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.84

$$\int \cos^2(c+dx)(a+b\sec(c+dx))^2\,dx = \frac{a^2\,x}{2} + b^2\,x + \frac{a^2\,\sin\left(2\,c+2\,d\,x\right)}{4\,d} + \frac{2\,a\,b\,\sin\left(c+d\,x\right)}{d}$$

input
```
int(cos(c + d*x)^2*(a + b/cos(c + d*x))^2,x)
```

output
```
(a^2*x)/2 + b^2*x + (a^2*sin(2*c + 2*d*x))/(4*d) + (2*a*b*sin(c + d*x))/d
```

3.463 $\int \cos^3(c+dx)(a+b\sec(c+dx))^2\,dx$

3.463.1 Optimal result . . . 3415
3.463.2 Mathematica [A] (verified) . . . 3415
3.463.3 Rubi [A] (verified) . . . 3416
3.463.4 Maple [A] (verified) . . . 3418
3.463.5 Fricas [A] (verification not implemented) . . . 3418
3.463.6 Sympy [F] . . . 3419
3.463.7 Maxima [A] (verification not implemented) . . . 3419
3.463.8 Giac [B] (verification not implemented) . . . 3419
3.463.9 Mupad [B] (verification not implemented) . . . 3420

3.463.1 Optimal result

Integrand size = 21, antiderivative size = 58

$$\int \cos^3(c+dx)(a+b\sec(c+dx))^2\,dx = abx+\frac{(a^2+b^2)\sin(c+dx)}{d}+\frac{ab\cos(c+dx)\sin(c+dx)}{d}-\frac{a^2\sin^3(c+dx)}{3d}$$

output
```
a*b*x+(a^2+b^2)*sin(d*x+c)/d+a*b*cos(d*x+c)*sin(d*x+c)/d-1/3*a^2*sin(d*x+c
)^3/d
```

3.463.2 Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.02

$$\int \cos^3(c+dx)(a+b\sec(c+dx))^2\,dx = \frac{3(3a^2+4b^2)\sin(c+dx)+a(12b(c+dx)+6b\sin(2(c+dx))+a\sin(3(c+dx)))}{12d}$$

input
```
Integrate[Cos[c + d*x]^3*(a + b*Sec[c + d*x])^2,x]
```

output
```
(3*(3*a^2 + 4*b^2)*Sin[c + d*x] + a*(12*b*(c + d*x) + 6*b*Sin[2*(c + d*x)]
 + a*Sin[3*(c + d*x)]))/(12*d)
```

3.463.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.14, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 4275, 3042, 3115, 24, 4532, 3042, 3492, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^3(c+dx)(a+b\sec(c+dx))^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^3}dx$$

$$\Big\downarrow \text{4275}$$

$$\int \cos^3(c+dx)\left(a^2+b^2\sec^2(c+dx)\right)dx+2ab\int \cos^2(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^3}dx+2ab\int \sin\left(c+dx+\frac{\pi}{2}\right)^2dx$$

$$\Big\downarrow \text{3115}$$

$$\int \frac{a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^3}dx+2ab\left(\frac{\int 1dx}{2}+\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)$$

$$\Big\downarrow \text{24}$$

$$\int \frac{a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^3}dx+2ab\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)$$

$$\Big\downarrow \text{4532}$$

$$\int \cos(c+dx)\left(b^2+a^2\cos^2(c+dx)\right)dx+2ab\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)\left(b^2+a^2\sin\left(c+dx+\frac{\pi}{2}\right)^2\right)dx+2ab\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)$$

$$\Big\downarrow \text{3492}$$

$$2ab\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)-\frac{\int\left(-\sin^2(c+dx)a^2+a^2+b^2\right)d(-\sin(c+dx))}{d}$$

$$\Big\downarrow \text{2009}$$

$$2ab\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)-\frac{\frac{1}{3}a^2\sin^3(c+dx)-\left(a^2+b^2\right)\sin(c+dx)}{d}$$

input
```
Int[Cos[c + d*x]^3*(a + b*Sec[c + d*x])^2,x]
```

output
```
2*a*b*(x/2 + (Cos[c + d*x]*Sin[c + d*x])/(2*d)) - (-((a^2 + b^2)*Sin[c + d
*x]) + (a^2*Sin[c + d*x]^3)/3)/d
```

3.463.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3492
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2),
 x_Symbol] :> Simp[-f^(-1)   Subst[Int[(1 - x^2)^((m - 1)/2)*(A + C - C*x^2
), x], x, Cos[e + f*x]], x] /; FreeQ[{e, f, A, C}, x] && IGtQ[(m + 1)/2, 0]
```

rule 4275
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Simp[2*a*(b/d)   Int[(d*Csc[e + f*x])^(n + 1), x], x]
 + Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
 e, f, n}, x]
```

rule 4532
```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.) + (A_)),
 x_Symbol] :> Int[(C + A*Sin[e + f*x]^2)/Sin[e + f*x]^(m + 2), x] /; FreeQ[
{e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && ILtQ[(m + 1)/2, 0]
```

3.463.4 Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.05

method	result
parallelrisch	$\frac{12abxd+9a^2\sin(dx+c)+12b^2\sin(dx+c)+a^2\sin(3dx+3c)+6ab\sin(2dx+2c)}{12d}$
derivativedivides	$\frac{\frac{a^2\left(2+\cos(dx+c)^2\right)\sin(dx+c)}{3}+2ab\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+b^2\sin(dx+c)}{d}$
default	$\frac{\frac{a^2\left(2+\cos(dx+c)^2\right)\sin(dx+c)}{3}+2ab\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+b^2\sin(dx+c)}{d}$
risch	$abx+\frac{3a^2\sin(dx+c)}{4d}+\frac{\sin(dx+c)b^2}{d}+\frac{a^2\sin(3dx+3c)}{12d}+\frac{ab\sin(2dx+2c)}{2d}$
norman	$\frac{abx\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8-abx-\frac{2\left(a^2-3ab-3b^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{3d}+\frac{2\left(a^2-ab+b^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{d}-\frac{2\left(a^2+ab+b^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{2\left(a^2+3a\right.}{}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^3\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)}$

input
```
int(cos(d*x+c)^3*(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/12*(12*a*b*x*d+9*a^2*sin(d*x+c)+12*b^2*sin(d*x+c)+a^2*sin(3*d*x+3*c)+6*a
*b*sin(2*d*x+2*c))/d
```

3.463.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.90

$$\int \cos^3(c+dx)(a+b\sec(c+dx))^2\,dx = \frac{3\,abdx+\left(a^2\cos\left(dx+c\right)^2+3\,ab\cos\left(dx+c\right)+2\,a^2+3\,b^2\right)\sin\left(dx+c\right)}{3\,d}$$

input
```
integrate(cos(d*x+c)^3*(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/3*(3*a*b*d*x + (a^2*cos(d*x + c)^2 + 3*a*b*cos(d*x + c) + 2*a^2 + 3*b^2)
*sin(d*x + c))/d
```

3.463.6 Sympy [F]

$$\int \cos^3(c+dx)(a+b\sec(c+dx))^2\,dx = \int (a+b\sec(c+dx))^2\cos^3(c+dx)\,dx$$

input
```
integrate(cos(d*x+c)**3*(a+b*sec(d*x+c))**2,x)
```

output
```
Integral((a + b*sec(c + d*x))**2*cos(c + d*x)**3, x)
```

3.463.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.03

$$\int \cos^3(c+dx)(a+b\sec(c+dx))^2\,dx = -\frac{2\left(\sin(dx+c)^3-3\sin(dx+c)\right)a^2-3\left(2\,dx+2\,c+\sin(2\,dx+2\,c)\right)ab-6\,b^2\sin(dx+c)}{6\,d}$$

input
```
integrate(cos(d*x+c)^3*(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
-1/6*(2*(sin(d*x + c)^3 - 3*sin(d*x + c))*a^2 - 3*(2*d*x + 2*c + sin(2*d*x
 + 2*c))*a*b - 6*b^2*sin(d*x + c))/d
```

3.463.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 153 vs. $2(56) = 112$.

Time = 0.30 (sec) , antiderivative size = 153, normalized size of antiderivative = 2.64

$$\int \cos^3(c+dx)(a+b\sec(c+dx))^2\,dx = \frac{3\,(dx+c)ab+\frac{2\left(3\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-3\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+3\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+2\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+6\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+3\,a^2\tan(}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^3}}{3\,d}$$

input
```
integrate(cos(d*x+c)^3*(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
1/3*(3*(d*x + c)*a*b + 2*(3*a^2*tan(1/2*d*x + 1/2*c)^5 - 3*a*b*tan(1/2*d*x
 + 1/2*c)^5 + 3*b^2*tan(1/2*d*x + 1/2*c)^5 + 2*a^2*tan(1/2*d*x + 1/2*c)^3
+ 6*b^2*tan(1/2*d*x + 1/2*c)^3 + 3*a^2*tan(1/2*d*x + 1/2*c) + 3*a*b*tan(1/
2*d*x + 1/2*c) + 3*b^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^
3)/d
```

3.463.9 Mupad [B] (verification not implemented)

Time = 13.44 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.24

$$\int \cos^3(c+dx)(a+b\sec(c+dx))^2\,dx = \frac{2\,a^2\,\sin\left(c+d\,x\right)}{3\,d}+\frac{b^2\,\sin\left(c+d\,x\right)}{d}+a\,b\,x+\frac{a^2\cos\left(c+d\,x\right)^2\,\sin\left(c+d\,x\right)}{3\,d}+\frac{a\,b\,\cos\left(c+d\,x\right)\,\sin\left(c+d\,x\right)}{d}$$

input
```
int(cos(c + d*x)^3*(a + b/cos(c + d*x))^2,x)
```

output
```
(2*a^2*sin(c + d*x))/(3*d) + (b^2*sin(c + d*x))/d + a*b*x + (a^2*cos(c + d
*x)^2*sin(c + d*x))/(3*d) + (a*b*cos(c + d*x)*sin(c + d*x))/d
```

3.464 $\int \cos^4(c+dx)(a+b\sec(c+dx))^2\,dx$

3.464.1 Optimal result . . . 3421
3.464.2 Mathematica [A] (verified) . . . 3421
3.464.3 Rubi [A] (verified) . . . 3422
3.464.4 Maple [A] (verified) . . . 3424
3.464.5 Fricas [A] (verification not implemented) . . . 3425
3.464.6 Sympy [F] . . . 3425
3.464.7 Maxima [A] (verification not implemented) . . . 3425
3.464.8 Giac [B] (verification not implemented) . . . 3426
3.464.9 Mupad [B] (verification not implemented) . . . 3426

3.464.1 Optimal result

Integrand size = 21, antiderivative size = 101

$$\int \cos^4(c+dx)(a+b\sec(c+dx))^2\,dx = \frac{1}{8}(3a^2+4b^2)\,x+\frac{2ab\sin(c+dx)}{d}+\frac{(3a^2+4b^2)\cos(c+dx)\sin(c+dx)}{8d}+\frac{a^2\cos^3(c+dx)\sin(c+dx)}{4d}-\frac{2ab\sin^3(c+dx)}{3d}$$

output
```
1/8*(3*a^2+4*b^2)*x+2*a*b*sin(d*x+c)/d+1/8*(3*a^2+4*b^2)*cos(d*x+c)*sin(d*
x+c)/d+1/4*a^2*cos(d*x+c)^3*sin(d*x+c)/d-2/3*a*b*sin(d*x+c)^3/d
```

3.464.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.85

$$\int \cos^4(c+dx)(a+b\sec(c+dx))^2\,dx = \frac{36a^2c+48b^2c+36a^2dx+48b^2dx+192ab\sin(c+dx)-64ab\sin^3(c+dx)+24(a^2+b^2)\sin(2(c+dx))+}{96d}$$

input
```
Integrate[Cos[c + d*x]^4*(a + b*Sec[c + d*x])^2,x]
```

output
```
(36*a^2*c + 48*b^2*c + 36*a^2*d*x + 48*b^2*d*x + 192*a*b*Sin[c + d*x] - 64
*a*b*Sin[c + d*x]^3 + 24*(a^2 + b^2)*Sin[2*(c + d*x)] + 3*a^2*Sin[4*(c + d
*x)])/(96*d)
```

3.464.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.92, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3042, 4275, 3042, 3113, 2009, 4533, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^4(c+dx)(a+b\sec(c+dx))^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^4}dx$$

$$\Big\downarrow \text{4275}$$

$$\int \cos^4(c+dx)\left(a^2+b^2\sec^2(c+dx)\right)dx+2ab\int \cos^3(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^4}dx+2ab\int \sin\left(c+dx+\frac{\pi}{2}\right)^3dx$$

$$\Big\downarrow \text{3113}$$

$$\int \frac{a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^4}dx-\frac{2ab\int\left(1-\sin^2(c+dx)\right)d(-\sin(c+dx))}{d}$$

$$\Big\downarrow \text{2009}$$

$$\int \frac{a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^4}dx-\frac{2ab\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}$$

$$\Big\downarrow \text{4533}$$

$$\frac{1}{4}\left(3a^2+4b^2\right)\int \cos^2(c+dx)dx+\frac{a^2\sin(c+dx)\cos^3(c+dx)}{4d}-\frac{2ab\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}\left(3a^2+4b^2\right)\int \sin\left(c+dx+\frac{\pi}{2}\right)^2dx+\frac{a^2\sin(c+dx)\cos^3(c+dx)}{4d}-\frac{2ab\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}$$

$$\Big\downarrow \text{3115}$$

$$\frac{1}{4}(3a^2+4b^2)\left(\frac{\int 1dx}{2}+\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)+\frac{a^2\sin(c+dx)\cos^3(c+dx)}{4d}-\frac{2ab\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}$$

$$\Big\downarrow \; 24$$

$$\frac{1}{4}(3a^2+4b^2)\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)+\frac{a^2\sin(c+dx)\cos^3(c+dx)}{4d}-\frac{2ab\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}$$

input
```
Int[Cos[c + d*x]^4*(a + b*Sec[c + d*x])^2,x]
```

output
```
(a^2*Cos[c + d*x]^3*Sin[c + d*x])/(4*d) + ((3*a^2 + 4*b^2)*(x/2 + (Cos[c +
 d*x]*Sin[c + d*x])/(2*d)))/4 - (2*a*b*(-Sin[c + d*x] + Sin[c + d*x]^3/3))
/d
```

3.464.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 4275
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Simp[2*a*(b/d)   Int[(d*Csc[e + f*x])^(n + 1), x], x]
 + Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
 e, f, n}, x]
```

rule 4533
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] +
Simp[(C*m + A*(m + 1))/(b^2*m)   Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fr
eeQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

3.464.4 Maple [A] (verified)

Time = 0.88 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.74

method	result
parallelrisch	$\frac{24\left(a^{2}+b^{2}\right)\sin\left(2dx+2c\right)+16ab\sin\left(3dx+3c\right)+3a^{2}\sin\left(4dx+4c\right)+144ab\sin\left(dx+c\right)+36d\left(a^{2}+\frac{4b^{2}}{3}\right)x}{96d}$
derivativedivides	$\frac{a^{2}\left(\frac{\left(\cos\left(dx+c\right)^{3}+\frac{3\cos\left(dx+c\right)}{2}\right)\sin\left(dx+c\right)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+\frac{2ab\left(2+\cos\left(dx+c\right)^{2}\right)\sin\left(dx+c\right)}{3}+b^{2}\left(\frac{\cos\left(dx+c\right)\sin\left(dx+c\right)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}$
default	$\frac{a^{2}\left(\frac{\left(\cos\left(dx+c\right)^{3}+\frac{3\cos\left(dx+c\right)}{2}\right)\sin\left(dx+c\right)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+\frac{2ab\left(2+\cos\left(dx+c\right)^{2}\right)\sin\left(dx+c\right)}{3}+b^{2}\left(\frac{\cos\left(dx+c\right)\sin\left(dx+c\right)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}$
risch	$\frac{3a^{2}x}{8}+\frac{x\,b^{2}}{2}+\frac{3ab\sin\left(dx+c\right)}{2d}+\frac{a^{2}\sin\left(4dx+4c\right)}{32d}+\frac{ab\sin\left(3dx+3c\right)}{6d}+\frac{a^{2}\sin\left(2dx+2c\right)}{4d}+\frac{\sin\left(2dx+2c\right)b^{2}}{4d}$
norman	$\frac{\left(-\frac{3a^{2}}{8}-\frac{b^{2}}{2}\right)x+\left(-\frac{9a^{2}}{8}-\frac{3b^{2}}{2}\right)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+\left(-\frac{3a^{2}}{4}-b^{2}\right)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\left(\frac{3a^{2}}{4}+b^{2}\right)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}+\left(\frac{3a^{2}}{8}+\frac{b^{2}}{2}\right)x}{}$

input
```
int(cos(d*x+c)^4*(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/96*(24*(a^2+b^2)*sin(2*d*x+2*c)+16*a*b*sin(3*d*x+3*c)+3*a^2*sin(4*d*x+4*
c)+144*a*b*sin(d*x+c)+36*d*(a^2+4/3*b^2)*x)/d
```

3.464.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.76

$$\int \cos^4(c+dx)(a+b\sec(c+dx))^2\,dx$$
$$= \frac{3\,(3\,a^2+4\,b^2)dx+\left(6\,a^2\cos\left(dx+c\right)^3+16\,ab\cos\left(dx+c\right)^2+32\,ab+3\,(3\,a^2+4\,b^2)\cos\left(dx+c\right)\right)\sin\left(dx+\right.}{24\,d}$$

input
```
integrate(cos(d*x+c)^4*(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/24*(3*(3*a^2 + 4*b^2)*d*x + (6*a^2*cos(d*x + c)^3 + 16*a*b*cos(d*x + c)^
2 + 32*a*b + 3*(3*a^2 + 4*b^2)*cos(d*x + c))*sin(d*x + c))/d
```

3.464.6 Sympy [F]

$$\int \cos^4(c+dx)(a+b\sec(c+dx))^2\,dx = \int (a+b\sec(c+dx))^2\cos^4(c+dx)\,dx$$

input
```
integrate(cos(d*x+c)**4*(a+b*sec(d*x+c))**2,x)
```

output
```
Integral((a + b*sec(c + d*x))**2*cos(c + d*x)**4, x)
```

3.464.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.81

$$\int \cos^4(c+dx)(a+b\sec(c+dx))^2\,dx$$
$$= \frac{3\,(12\,dx+12\,c+\sin\left(4\,dx+4\,c\right)+8\,\sin\left(2\,dx+2\,c\right))a^2-64\left(\sin\left(dx+c\right)^3-3\,\sin\left(dx+c\right)\right)ab+24\,(2\,dx}{96\,d}$$

input
```
integrate(cos(d*x+c)^4*(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
1/96*(3*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*a^2 - 64*(
sin(d*x + c)^3 - 3*sin(d*x + c))*a*b + 24*(2*d*x + 2*c + sin(2*d*x + 2*c))
*b^2)/d
```

3.464.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 224 vs. $2(93) = 186$.

Time = 0.30 (sec) , antiderivative size = 224, normalized size of antiderivative = 2.22

$$\int \cos^4(c+dx)(a+b\sec(c+dx))^2\,dx$$
$$= \frac{3\,(3\,a^2+4\,b^2)(dx+c) - \frac{2\left(15\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-48\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+12\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-9\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-80\,ab\tan\left(\frac{1}{2}\,dx\right.}{}}{}$$

input
```
integrate(cos(d*x+c)^4*(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
1/24*(3*(3*a^2 + 4*b^2)*(d*x + c) - 2*(15*a^2*tan(1/2*d*x + 1/2*c)^7 - 48*
a*b*tan(1/2*d*x + 1/2*c)^7 + 12*b^2*tan(1/2*d*x + 1/2*c)^7 - 9*a^2*tan(1/2
*d*x + 1/2*c)^5 - 80*a*b*tan(1/2*d*x + 1/2*c)^5 + 12*b^2*tan(1/2*d*x + 1/2
*c)^5 + 9*a^2*tan(1/2*d*x + 1/2*c)^3 - 80*a*b*tan(1/2*d*x + 1/2*c)^3 - 12*
b^2*tan(1/2*d*x + 1/2*c)^3 - 15*a^2*tan(1/2*d*x + 1/2*c) - 48*a*b*tan(1/2*
d*x + 1/2*c) - 12*b^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^4
)/d
```

3.464.9 Mupad [B] (verification not implemented)

Time = 13.56 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.92

$$\int \cos^4(c+dx)(a+b\sec(c+dx))^2\,dx = \frac{3\,a^2\,x}{8}+\frac{b^2\,x}{2}+\frac{a^2\,\sin\left(2\,c+2\,d\,x\right)}{4\,d}+\frac{a^2\,\sin\left(4\,c+4\,d\,x\right)}{32\,d}+\frac{b^2\,\sin\left(2\,c+2\,d\,x\right)}{4\,d}+\frac{3\,a\,b\,\sin\left(c+d\,x\right)}{2\,d}+\frac{a\,b\,\sin\left(3\,c+3\,d\,x\right)}{6\,d}$$

input
```
int(cos(c + d*x)^4*(a + b/cos(c + d*x))^2,x)
```

output
```
(3*a^2*x)/8 + (b^2*x)/2 + (a^2*sin(2*c + 2*d*x))/(4*d) + (a^2*sin(4*c + 4*
d*x))/(32*d) + (b^2*sin(2*c + 2*d*x))/(4*d) + (3*a*b*sin(c + d*x))/(2*d) +
 (a*b*sin(3*c + 3*d*x))/(6*d)
```

3.465 $\int \cos^5(c+dx)(a+b\sec(c+dx))^2\,dx$

3.465.1 Optimal result . . . 3427
3.465.2 Mathematica [A] (verified) . . . 3427
3.465.3 Rubi [A] (verified) . . . 3428
3.465.4 Maple [A] (verified) . . . 3431
3.465.5 Fricas [A] (verification not implemented) . . . 3431
3.465.6 Sympy [F] . . . 3432
3.465.7 Maxima [A] (verification not implemented) . . . 3432
3.465.8 Giac [B] (verification not implemented) . . . 3432
3.465.9 Mupad [B] (verification not implemented) . . . 3433

3.465.1 Optimal result

Integrand size = 21, antiderivative size = 111

$$\int \cos^5(c+dx)(a+b\sec(c+dx))^2\,dx = \frac{3abx}{4}+\frac{(a^2+b^2)\sin(c+dx)}{d}+\frac{3ab\cos(c+dx)\sin(c+dx)}{4d}+\frac{ab\cos^3(c+dx)\sin(c+dx)}{2d}-\frac{(2a^2+b^2)\sin^3(c+dx)}{3d}+\frac{a^2\sin^5(c+dx)}{5d}$$

output
```
3/4*a*b*x+(a^2+b^2)*sin(d*x+c)/d+3/4*a*b*cos(d*x+c)*sin(d*x+c)/d+1/2*a*b*c
os(d*x+c)^3*sin(d*x+c)/d-1/3*(2*a^2+b^2)*sin(d*x+c)^3/d+1/5*a^2*sin(d*x+c)
^5/d
```

3.465.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.77

$$\int \cos^5(c+dx)(a+b\sec(c+dx))^2\,dx = \frac{240(a^2+b^2)\sin(c+dx)-80(2a^2+b^2)\sin^3(c+dx)+48a^2\sin^5(c+dx)+15ab(12(c+dx)+8\sin(2(c+}{240d}$$

input
```
Integrate[Cos[c + d*x]^5*(a + b*Sec[c + d*x])^2,x]
```

output
```
(240*(a^2 + b^2)*Sin[c + d*x] - 80*(2*a^2 + b^2)*Sin[c + d*x]^3 + 48*a^2*S
in[c + d*x]^5 + 15*a*b*(12*(c + d*x) + 8*Sin[2*(c + d*x)] + Sin[4*(c + d*x
)]))/(240*d)
```

3.465.3 Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.02, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {3042, 4275, 3042, 3115, 3042, 3115, 24, 4532, 3042, 3492, 290, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^5(c+dx)(a+b\sec(c+dx))^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^5}dx$$

$$\Big\downarrow \text{4275}$$

$$\int \cos^5(c+dx)\left(a^2+b^2\sec^2(c+dx)\right)dx+2ab\int \cos^4(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^5}dx+2ab\int \sin\left(c+dx+\frac{\pi}{2}\right)^4dx$$

$$\Big\downarrow \text{3115}$$

$$\int \frac{a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^5}dx+2ab\left(\frac{3}{4}\int \cos^2(c+dx)dx+\frac{\sin(c+dx)\cos^3(c+dx)}{4d}\right)$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^5}dx+2ab\left(\frac{3}{4}\int \sin\left(c+dx+\frac{\pi}{2}\right)^2dx+\frac{\sin(c+dx)\cos^3(c+dx)}{4d}\right)$$

$$\Big\downarrow \text{3115}$$

$$\int \frac{a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^5}dx+2ab\left(\frac{3}{4}\left(\frac{\int 1dx}{2}+\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)+\frac{\sin(c+dx)\cos^3(c+dx)}{4d}\right)$$

$$\Big\downarrow \; 24$$

$$\int \frac{a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^5}dx+2ab\left(\frac{\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3}{4}\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)\right)$$

$$\Big\downarrow \; 4532$$

$$\int \cos^3(c+dx)\left(b^2+a^2\cos^2(c+dx)\right)dx+2ab\left(\frac{\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3}{4}\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)\right)$$

$$\Big\downarrow \; 3042$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^3\left(b^2+a^2\sin\left(c+dx+\frac{\pi}{2}\right)^2\right)dx+2ab\left(\frac{\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3}{4}\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)\right)$$

$$\Big\downarrow \; 3492$$

$$2ab\left(\frac{\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3}{4}\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)\right)-\frac{\int\left(1-\sin^2(c+dx)\right)\left(-\sin^2(c+dx)a^2+a^2+b^2\right)d(-\sin(c+dx))}{d}$$

$$\Big\downarrow \; 290$$

$$2ab\left(\frac{\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3}{4}\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)\right)-\frac{\int\left(a^2\sin^4(c+dx)-\left(2a^2+b^2\right)\sin^2(c+dx)+a^2\left(\frac{b^2}{a^2}+1\right)\right)d(-\sin(c+dx))}{d}$$

$$\Big\downarrow \; 2009$$

$$2ab\left(\frac{\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3}{4}\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)\right)-\frac{\frac{1}{3}\left(2a^2+b^2\right)\sin^3(c+dx)-\left(a^2+b^2\right)\sin(c+dx)-\frac{1}{5}a^2\sin^5(c+dx)}{d}$$

input
```
Int[Cos[c + d*x]^5*(a + b*Sec[c + d*x])^2,x]
```

3.465. $\int \cos^5(c+dx)(a+b\sec(c+dx))^2\,dx$

output
```
-((-((a^2 + b^2)*Sin[c + d*x]) + ((2*a^2 + b^2)*Sin[c + d*x]^3)/3 - (a^2*S
in[c + d*x]^5)/5)/d) + 2*a*b*((Cos[c + d*x]^3*Sin[c + d*x])/(4*d) + (3*(x/
2 + (Cos[c + d*x]*Sin[c + d*x])/(2*d)))/4)
```

3.465.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 290
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol] :> I
nt[ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d
}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3492
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2),
 x_Symbol] :> Simp[-f^(-1)   Subst[Int[(1 - x^2)^((m - 1)/2)*(A + C - C*x^2
), x], x, Cos[e + f*x]], x] /; FreeQ[{e, f, A, C}, x] && IGtQ[(m + 1)/2, 0]
```

rule 4275
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Simp[2*a*(b/d)   Int[(d*Csc[e + f*x])^(n + 1), x], x]
 + Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
 e, f, n}, x]
```

rule 4532
```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.) + (A_)),
 x_Symbol] :> Int[(C + A*Sin[e + f*x]^2)/Sin[e + f*x]^(m + 2), x] /; FreeQ[
{e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && ILtQ[(m + 1)/2, 0]
```

3.465.4 Maple [A] (verified)

Time = 1.17 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.86

method	result
derivativedivides	$\frac{\frac{a^2\left(\frac{8}{3}+\cos(dx+c)^4+\frac{4\cos(dx+c)^2}{3}\right)\sin(dx+c)}{5}+2ab\left(\frac{\left(\cos(dx+c)^3+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+\frac{b^2\left(2+\cos(dx+c)^2\right)\sin}{3}}{d}$
default	$\frac{\frac{a^2\left(\frac{8}{3}+\cos(dx+c)^4+\frac{4\cos(dx+c)^2}{3}\right)\sin(dx+c)}{5}+2ab\left(\frac{\left(\cos(dx+c)^3+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+\frac{b^2\left(2+\cos(dx+c)^2\right)\sin}{3}}{d}$
parallelrisch	$\frac{180abxd+3a^2\sin(5dx+5c)+15ab\sin(4dx+4c)+25a^2\sin(3dx+3c)+20b^2\sin(3dx+3c)+120ab\sin(2dx+2c)+150a^2\sin(dx}{240d}$
risch	$\frac{3abx}{4}+\frac{5a^2\sin(dx+c)}{8d}+\frac{3\sin(dx+c)b^2}{4d}+\frac{a^2\sin(5dx+5c)}{80d}+\frac{ab\sin(4dx+4c)}{16d}+\frac{5a^2\sin(3dx+3c)}{48d}+\frac{\sin(3dx+3c}{12d}$
norman	$\frac{-\frac{3abx}{4}-\frac{\left(4a^2-9ab+20b^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{6d}+\frac{\left(4a^2-5ab+4b^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{11}}{2d}-\frac{\left(4a^2+5ab+4b^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2d}+\frac{\left(4a^2+9ab+20b^2\right)\tan}{6d}}{}$

input
```
int(cos(d*x+c)^5*(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/5*a^2*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+2*a*b*(1/4*(co
s(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+1/3*b^2*(2+cos(d*x+c)
^2)*sin(d*x+c))
```

3.465.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.77

$$\int \cos^5(c+dx)(a+b\sec(c+dx))^2\,dx$$
$$=\frac{45\,abdx+\left(12\,a^2\cos\left(dx+c\right)^4+30\,ab\cos\left(dx+c\right)^3+45\,ab\cos\left(dx+c\right)+4\left(4\,a^2+5\,b^2\right)\cos\left(dx+c\right)^2+3}{60\,d}$$

input
```
integrate(cos(d*x+c)^5*(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/60*(45*a*b*d*x + (12*a^2*cos(d*x + c)^4 + 30*a*b*cos(d*x + c)^3 + 45*a*b
*cos(d*x + c) + 4*(4*a^2 + 5*b^2)*cos(d*x + c)^2 + 32*a^2 + 40*b^2)*sin(d*
x + c))/d
```

3.465.6 Sympy [F]

$$\int \cos^5(c+dx)(a+b\sec(c+dx))^2\,dx = \int (a+b\sec(c+dx))^2\cos^5(c+dx)\,dx$$

input
```
integrate(cos(d*x+c)**5*(a+b*sec(d*x+c))**2,x)
```

output
```
Integral((a + b*sec(c + d*x))**2*cos(c + d*x)**5, x)
```

3.465.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.85

$$\int \cos^5(c+dx)(a+b\sec(c+dx))^2\,dx = \frac{16\left(3\sin(dx+c)^5-10\sin(dx+c)^3+15\sin(dx+c)\right)a^2+15\left(12\,dx+12\,c+\sin(4\,dx+4\,c)+8\sin(2\right.}{240\,d}$$

input
```
integrate(cos(d*x+c)^5*(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
1/240*(16*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*a^2 + 1
5*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*a*b - 80*(sin(d*
x + c)^3 - 3*sin(d*x + c))*b^2)/d
```

3.465.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 247 vs. 2(101) = 202.

Time = 0.31 (sec) , antiderivative size = 247, normalized size of antiderivative = 2.23

$$\int \cos^5(c+dx)(a+b\sec(c+dx))^2\,dx = 45\,(dx+c)ab+\frac{2\left(60\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9-75\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9+60\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9+80\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-30\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+1\right.}{}$$

input
```
integrate(cos(d*x+c)^5*(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
1/60*(45*(d*x + c)*a*b + 2*(60*a^2*tan(1/2*d*x + 1/2*c)^9 - 75*a*b*tan(1/2
*d*x + 1/2*c)^9 + 60*b^2*tan(1/2*d*x + 1/2*c)^9 + 80*a^2*tan(1/2*d*x + 1/2
*c)^7 - 30*a*b*tan(1/2*d*x + 1/2*c)^7 + 160*b^2*tan(1/2*d*x + 1/2*c)^7 + 2
32*a^2*tan(1/2*d*x + 1/2*c)^5 + 200*b^2*tan(1/2*d*x + 1/2*c)^5 + 80*a^2*ta
n(1/2*d*x + 1/2*c)^3 + 30*a*b*tan(1/2*d*x + 1/2*c)^3 + 160*b^2*tan(1/2*d*x
 + 1/2*c)^3 + 60*a^2*tan(1/2*d*x + 1/2*c) + 75*a*b*tan(1/2*d*x + 1/2*c) +
60*b^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^5)/d
```

3.465.9 Mupad [B] (verification not implemented)

Time = 13.46 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.05

$$\int \cos^5(c+dx)(a+b\sec(c+dx))^2\,dx = \frac{5\,a^2\,\sin\left(c+d\,x\right)}{8\,d}+\frac{3\,b^2\,\sin\left(c+d\,x\right)}{4\,d}+\frac{3\,a\,b\,x}{4}+\frac{5\,a^2\,\sin\left(3\,c+3\,d\,x\right)}{48\,d}+\frac{a^2\,\sin\left(5\,c+5\,d\,x\right)}{80\,d}+\frac{b^2\,\sin\left(3\,c+3\,d\,x\right)}{12\,d}+\frac{a\,b\,\sin\left(2\,c+2\,d\,x\right)}{2\,d}+\frac{a\,b\,\sin\left(4\,c+4\,d\,x\right)}{16\,d}$$

input
```
int(cos(c + d*x)^5*(a + b/cos(c + d*x))^2,x)
```

output
```
(5*a^2*sin(c + d*x))/(8*d) + (3*b^2*sin(c + d*x))/(4*d) + (3*a*b*x)/4 + (5
*a^2*sin(3*c + 3*d*x))/(48*d) + (a^2*sin(5*c + 5*d*x))/(80*d) + (b^2*sin(3
*c + 3*d*x))/(12*d) + (a*b*sin(2*c + 2*d*x))/(2*d) + (a*b*sin(4*c + 4*d*x)
)/(16*d)
```

3.466 $\int \sec^3(c+dx)(a+b\sec(c+dx))^3\,dx$

3.466.1 Optimal result . . . 3434
3.466.2 Mathematica [A] (verified) . . . 3435
3.466.3 Rubi [A] (verified) . . . 3435
3.466.4 Maple [A] (verified) . . . 3439
3.466.5 Fricas [A] (verification not implemented) . . . 3440
3.466.6 Sympy [F] . . . 3440
3.466.7 Maxima [A] (verification not implemented) . . . 3440
3.466.8 Giac [B] (verification not implemented) . . . 3441
3.466.9 Mupad [B] (verification not implemented) . . . 3442

3.466.1 Optimal result

Integrand size = 21, antiderivative size = 189

$$\begin{aligned}\int \sec^3(c+dx)(a+b\sec(c+dx))^3\,dx = {} & \frac{a(4a^2+9b^2)\operatorname{arctanh}(\sin(c+dx))}{8d} \\ & - \frac{(3a^4-52a^2b^2-16b^4)\tan(c+dx)}{30bd} \\ & - \frac{a(6a^2-71b^2)\sec(c+dx)\tan(c+dx)}{120d} \\ & - \frac{(3a^2-16b^2)(a+b\sec(c+dx))^2\tan(c+dx)}{60bd} \\ & - \frac{a(a+b\sec(c+dx))^3\tan(c+dx)}{20bd} \\ & + \frac{(a+b\sec(c+dx))^4\tan(c+dx)}{5bd}\end{aligned}$$

output
```
1/8*a*(4*a^2+9*b^2)*arctanh(sin(d*x+c))/d-1/30*(3*a^4-52*a^2*b^2-16*b^4)*t
an(d*x+c)/b/d-1/120*a*(6*a^2-71*b^2)*sec(d*x+c)*tan(d*x+c)/d-1/60*(3*a^2-1
6*b^2)*(a+b*sec(d*x+c))^2*tan(d*x+c)/b/d-1/20*a*(a+b*sec(d*x+c))^3*tan(d*x
+c)/b/d+1/5*(a+b*sec(d*x+c))^4*tan(d*x+c)/b/d
```

3.466.2 Mathematica [A] (verified)

Time = 0.65 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.63

$$\int \sec^3(c+dx)(a+b\sec(c+dx))^3\,dx$$
$$= \frac{15a(4a^2+9b^2)\operatorname{arctanh}(\sin(c+dx))+\tan(c+dx)\left(15a(4a^2+9b^2)\sec(c+dx)+90ab^2\sec^3(c+dx)+8b\right.}{120d}$$

input
```
Integrate[Sec[c + d*x]^3*(a + b*Sec[c + d*x])^3,x]
```

output
```
(15*a*(4*a^2 + 9*b^2)*ArcTanh[Sin[c + d*x]] + Tan[c + d*x]*(15*a*(4*a^2 +
9*b^2)*Sec[c + d*x] + 90*a*b^2*Sec[c + d*x]^3 + 8*b*(15*(3*a^2 + b^2) + 5*
(3*a^2 + 2*b^2)*Tan[c + d*x]^2 + 3*b^2*Tan[c + d*x]^4)))/(120*d)
```

3.466.3 Rubi [A] (verified)

Time = 1.18 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.06, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {3042, 4327, 3042, 4490, 3042, 4490, 3042, 4485, 3042, 4274, 3042, 4254, 24, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^3(c+dx)(a+b\sec(c+dx))^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^3\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^3dx$$

$$\Big\downarrow \text{4327}$$

$$\frac{\int \sec(c+dx)(4b-a\sec(c+dx))(a+b\sec(c+dx))^3dx}{5b}+\frac{\tan(c+dx)(a+b\sec(c+dx))^4}{5bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \csc\left(c+dx+\frac{\pi}{2}\right)\left(4b-a\csc\left(c+dx+\frac{\pi}{2}\right)\right)\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^3dx}{5b}+\frac{\tan(c+dx)(a+b\sec(c+dx))^4}{5bd}$$

$$\Big\downarrow \text{4490}$$

$$\frac{\frac{1}{4}\int \sec(c+dx)(a+b\sec(c+dx))^2\left(13ab-\left(3a^2-16b^2\right)\sec(c+dx)\right)dx-\frac{a\tan(c+dx)(a+b\sec(c+dx))^3}{4d}}{5b}+\frac{\tan(c+dx)(a+b\sec(c+dx))^4}{5bd}$$

$\Big\downarrow$ 3042

$$\frac{\frac{1}{4}\int \csc\left(c+dx+\frac{\pi}{2}\right)\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2\left(13ab+\left(16b^2-3a^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)\right)dx-\frac{a\tan(c+dx)(a+b\sec(c+dx)}{4d}}{5b}+\frac{\tan(c+dx)(a+b\sec(c+dx))^4}{5bd}$$

$\Big\downarrow$ 4490

$$\frac{\frac{1}{4}\left(\frac{1}{3}\int \sec(c+dx)(a+b\sec(c+dx))\left(b\left(33a^2+32b^2\right)-a\left(6a^2-71b^2\right)\sec(c+dx)\right)dx-\frac{\left(3a^2-16b^2\right)\tan(c+dx)(a+b\sec}{3d}\right.}{5b}+\frac{\tan(c+dx)(a+b\sec(c+dx))^4}{5bd}$$

$\Big\downarrow$ 3042

$$\frac{\frac{1}{4}\left(\frac{1}{3}\int \csc\left(c+dx+\frac{\pi}{2}\right)\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)\left(b\left(33a^2+32b^2\right)-a\left(6a^2-71b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)\right)dx-\frac{\left(3a^2-16b^2\right.}{}\right.}{5b}+\frac{\tan(c+dx)(a+b\sec(c+dx))^4}{5bd}$$

$\Big\downarrow$ 4485

$$\frac{\frac{1}{4}\left(\frac{1}{3}\left(\frac{1}{2}\int \sec(c+dx)\left(15ab\left(4a^2+9b^2\right)-4\left(3a^4-52b^2a^2-16b^4\right)\sec(c+dx)\right)dx-\frac{ab\left(6a^2-71b^2\right)\tan(c+dx)\sec(c+dx)}{2d}\right)\right.}{5b}+\frac{\tan(c+dx)(a+b\sec(c+dx))^4}{5bd}$$

$\Big\downarrow$ 3042

$$\frac{\frac{1}{4}\left(\frac{1}{3}\left(\frac{1}{2}\int \csc\left(c+dx+\frac{\pi}{2}\right)\left(15ab\left(4a^2+9b^2\right)-4\left(3a^4-52b^2a^2-16b^4\right)\csc\left(c+dx+\frac{\pi}{2}\right)\right)dx-\frac{ab\left(6a^2-71b^2\right)\tan(c+dx}{2d}\right.\right.}{5b}+\frac{\tan(c+dx)(a+b\sec(c+dx))^4}{5bd}$$

$\Big\downarrow$ 4274

3.466. $\int \sec^3(c+dx)(a+b\sec(c+dx))^3\,dx$

$$\frac{\frac{1}{4}\left(\frac{1}{3}\left(\frac{1}{2}\left(15ab\left(4a^2+9b^2\right)\int \sec(c+dx)dx-4\left(3a^4-52a^2b^2-16b^4\right)\int \sec^2(c+dx)dx\right)-\frac{ab\left(6a^2-71b^2\right)\tan(c+dx)\sec(c+}{2d}\right.\right.}{5b}$$
$$\frac{\tan(c+dx)(a+b\sec(c+dx))^4}{5bd}$$

$\Big\downarrow$ 3042

$$\frac{\frac{1}{4}\left(\frac{1}{3}\left(\frac{1}{2}\left(15ab\left(4a^2+9b^2\right)\int \csc\left(c+dx+\frac{\pi}{2}\right)dx-4\left(3a^4-52a^2b^2-16b^4\right)\int \csc\left(c+dx+\frac{\pi}{2}\right)^2dx\right)-\frac{ab\left(6a^2-71b^2\right)\text{ta}}{}\right.\right.}{5b}$$
$$\frac{\tan(c+dx)(a+b\sec(c+dx))^4}{5bd}$$

$\Big\downarrow$ 4254

$$\frac{\frac{1}{4}\left(\frac{1}{3}\left(\frac{1}{2}\left(15ab\left(4a^2+9b^2\right)\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{4\left(3a^4-52a^2b^2-16b^4\right)\int 1d(-\tan(c+dx))}{d}\right)-\frac{ab\left(6a^2-71b^2\right)\tan(c+dx)\sec(c+dx}{2d}\right.\right.}{5b}$$
$$\frac{\tan(c+dx)(a+b\sec(c+dx))^4}{5bd}$$

$\Big\downarrow$ 24

$$\frac{\frac{1}{4}\left(\frac{1}{3}\left(\frac{1}{2}\left(15ab\left(4a^2+9b^2\right)\int \csc\left(c+dx+\frac{\pi}{2}\right)dx-\frac{4\left(3a^4-52a^2b^2-16b^4\right)\tan(c+dx)}{d}\right)-\frac{ab\left(6a^2-71b^2\right)\tan(c+dx)\sec(c+dx)}{2d}\right)-\right.}{5b}$$
$$\frac{\tan(c+dx)(a+b\sec(c+dx))^4}{5bd}$$

$\Big\downarrow$ 4257

$$\frac{\frac{1}{4}\left(\frac{1}{3}\left(\frac{1}{2}\left(\frac{15ab\left(4a^2+9b^2\right)\text{arctanh}(\sin(c+dx))}{d}-\frac{4\left(3a^4-52a^2b^2-16b^4\right)\tan(c+dx)}{d}\right)-\frac{ab\left(6a^2-71b^2\right)\tan(c+dx)\sec(c+dx)}{2d}\right)-\frac{(3a^2-16b}{}\right.}{5b}$$
$$\frac{\tan(c+dx)(a+b\sec(c+dx))^4}{5bd}$$

input
```
Int[Sec[c + d*x]^3*(a + b*Sec[c + d*x])^3,x]
```

output
```
((a + b*Sec[c + d*x])^4*Tan[c + d*x])/(5*b*d) + (-1/4*(a*(a + b*Sec[c + d*
x])^3*Tan[c + d*x])/d + (-1/3*((3*a^2 - 16*b^2)*(a + b*Sec[c + d*x])^2*Tan
[c + d*x])/d + (-1/2*(a*b*(6*a^2 - 71*b^2)*Sec[c + d*x]*Tan[c + d*x])/d +
((15*a*b*(4*a^2 + 9*b^2)*ArcTanh[Sin[c + d*x]])/d - (4*(3*a^4 - 52*a^2*b^2
 - 16*b^4)*Tan[c + d*x])/d)/2)/3)/4)/(5*b)
```

3.466. $\int \sec^3(c+dx)(a+b\sec(c+dx))^3\,dx$

3.466.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4327
```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(-Cot[e + f*x])*((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2
))), x] + Simp[1/(b*(m + 2))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(b*(
m + 1) - a*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 -
b^2, 0] &&  !LtQ[m, -1]
```

rule 4485
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*Cot[
e + f*x]*((d*Csc[e + f*x])^n/(f*(n + 1))), x] + Simp[1/(n + 1)   Int[(d*Csc
[e + f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x
], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[
n, -1]
```

rule 4490
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*((
a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Simp[1/(m + 1)   Int[Csc[e + f*x]*
(a + b*Csc[e + f*x])^(m - 1)*Simp[b*B*m + a*A*(m + 1) + (a*B*m + A*b*(m + 1
))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*
B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

3.466.4 Maple [A] (verified)

Time = 1.89 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.78

method	result
derivativedivides	$\frac{a^3\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)-3a^2b\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right)\tan(dx+c)+3a\,b^2\left(-\left(-\frac{\sec(dx+c)^3}{4}-\frac{3\sec(}{}\right.\right.}{d}$
default	$\frac{a^3\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)-3a^2b\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right)\tan(dx+c)+3a\,b^2\left(-\left(-\frac{\sec(dx+c)^3}{4}-\frac{3\sec(}{}\right.\right.}{d}$
parts	$\frac{a^3\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)}{d}-\frac{b^3\left(-\frac{8}{15}-\frac{\sec(dx+c)^4}{5}-\frac{4\sec(dx+c)^2}{15}\right)\tan(dx+c)}{d}+\frac{3a\,b^2\left(-\left(-\right.\right.}{}$
parallelrisch	$\frac{-600a\left(\frac{\cos(5dx+5c)}{10}+\frac{\cos(3dx+3c)}{2}+\cos(dx+c)\right)\left(a^2+\frac{9b^2}{4}\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+600a\left(\frac{\cos(5dx+5c)}{10}+\frac{\cos(3dx+3c)}{2}+\cos(d}{}$
norman	$\frac{\frac{\left(4a^3-24a^2b+15a\,b^2-8b^3\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^9}{4d}-\frac{\left(4a^3+24a^2b+15a\,b^2+8b^3\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d}-\frac{\left(12a^3-96a^2b+9a\,b^2-16b^3\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{6d}+}{\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^5}$
risch	$-\frac{i\left(60a^3e^{9i(dx+c)}+135a\,b^2e^{9i(dx+c)}+120a^3e^{7i(dx+c)}+630a\,b^2e^{7i(dx+c)}-720a^2b\,e^{6i(dx+c)}-1680a^2b\,e^{4i(dx+c)}-640b^3e\right.}{60d}$

input
```
int(sec(d*x+c)^3*(a+b*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^3*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c)))-3*a^2*b
*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c)+3*a*b^2*(-(-1/4*sec(d*x+c)^3-3/8*sec(d
*x+c))*tan(d*x+c)+3/8*ln(sec(d*x+c)+tan(d*x+c)))-b^3*(-8/15-1/5*sec(d*x+c)
^4-4/15*sec(d*x+c)^2)*tan(d*x+c))
```

3.466.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.90

$$\int \sec^3(c+dx)(a+b\sec(c+dx))^3\,dx$$
$$= \frac{15\,(4\,a^3+9\,ab^2)\cos\,(dx+c)^5\log\,(\sin\,(dx+c)+1)-15\,(4\,a^3+9\,ab^2)\cos\,(dx+c)^5\log\,(-\sin\,(dx+c)+1}{}$$

input
```
integrate(sec(d*x+c)^3*(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/240*(15*(4*a^3 + 9*a*b^2)*cos(d*x + c)^5*log(sin(d*x + c) + 1) - 15*(4*a
^3 + 9*a*b^2)*cos(d*x + c)^5*log(-sin(d*x + c) + 1) + 2*(16*(15*a^2*b + 4*
b^3)*cos(d*x + c)^4 + 90*a*b^2*cos(d*x + c) + 15*(4*a^3 + 9*a*b^2)*cos(d*x
 + c)^3 + 24*b^3 + 8*(15*a^2*b + 4*b^3)*cos(d*x + c)^2)*sin(d*x + c))/(d*c
os(d*x + c)^5)
```

3.466.6 Sympy [F]

$$\int \sec^3(c+dx)(a+b\sec(c+dx))^3\,dx = \int (a+b\sec\,(c+dx))^3\sec^3\,(c+dx)\,dx$$

input
```
integrate(sec(d*x+c)**3*(a+b*sec(d*x+c))**3,x)
```

output
```
Integral((a + b*sec(c + d*x))**3*sec(c + d*x)**3, x)
```

3.466.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.96

$$\int \sec^3(c+dx)(a+b\sec(c+dx))^3\,dx$$
$$= \frac{240\,\left(\tan\,(dx+c)^3+3\,\tan\,(dx+c)\right)a^2b+16\,\left(3\,\tan\,(dx+c)^5+10\,\tan\,(dx+c)^3+15\,\tan\,(dx+c)\right)b^3-}{}$$

input
```
integrate(sec(d*x+c)^3*(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/240*(240*(tan(d*x + c)^3 + 3*tan(d*x + c))*a^2*b + 16*(3*tan(d*x + c)^5
+ 10*tan(d*x + c)^3 + 15*tan(d*x + c))*b^3 - 45*a*b^2*(2*(3*sin(d*x + c)^3
 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x
 + c) + 1) + 3*log(sin(d*x + c) - 1)) - 60*a^3*(2*sin(d*x + c)/(sin(d*x +
c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)))/d
```

3.466.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 367 vs. $2(177) = 354$.

Time = 0.35 (sec) , antiderivative size = 367, normalized size of antiderivative = 1.94

$$\int \sec^3(c+dx)(a+b\sec(c+dx))^3\,dx$$
$$= \frac{15\,(4\,a^3+9\,ab^2)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-15\,(4\,a^3+9\,ab^2)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{2\left(60\,a^3\tan\left(\frac{1}{2}\right.\right.}{}}{}$$

input
```
integrate(sec(d*x+c)^3*(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
1/120*(15*(4*a^3 + 9*a*b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 15*(4*a^3
 + 9*a*b^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(60*a^3*tan(1/2*d*x + 1
/2*c)^9 - 360*a^2*b*tan(1/2*d*x + 1/2*c)^9 + 225*a*b^2*tan(1/2*d*x + 1/2*c
)^9 - 120*b^3*tan(1/2*d*x + 1/2*c)^9 - 120*a^3*tan(1/2*d*x + 1/2*c)^7 + 96
0*a^2*b*tan(1/2*d*x + 1/2*c)^7 - 90*a*b^2*tan(1/2*d*x + 1/2*c)^7 + 160*b^3
*tan(1/2*d*x + 1/2*c)^7 - 1200*a^2*b*tan(1/2*d*x + 1/2*c)^5 - 464*b^3*tan(
1/2*d*x + 1/2*c)^5 + 120*a^3*tan(1/2*d*x + 1/2*c)^3 + 960*a^2*b*tan(1/2*d*
x + 1/2*c)^3 + 90*a*b^2*tan(1/2*d*x + 1/2*c)^3 + 160*b^3*tan(1/2*d*x + 1/2
*c)^3 - 60*a^3*tan(1/2*d*x + 1/2*c) - 360*a^2*b*tan(1/2*d*x + 1/2*c) - 225
*a*b^2*tan(1/2*d*x + 1/2*c) - 120*b^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x +
 1/2*c)^2 - 1)^5)/d
```

3.466.9 Mupad [B] (verification not implemented)

Time = 17.17 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.37

$$\int \sec^3(c+dx)(a+b\sec(c+dx))^3\,dx = \frac{\operatorname{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\left(a^3+\frac{9\,a\,b^2}{4}\right)}{d} - \frac{\left(-a^3+6\,a^2\,b-\frac{15\,a\,b^2}{4}+2\,b^3\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^9+\left(2\,a^3-16\,a^2\,b+\frac{3\,a\,b^2}{2}-\frac{8\,b^3}{3}\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7+\left(20\,a^2\,b+\right.}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}-5\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8+10\,\mathrm{ta}\right.}$$

input
```
int((a + b/cos(c + d*x))^3/cos(c + d*x)^3,x)
```

output
```
(atanh(tan(c/2 + (d*x)/2))*((9*a*b^2)/4 + a^3))/d - (tan(c/2 + (d*x)/2)^7*
((3*a*b^2)/2 - 16*a^2*b + 2*a^3 - (8*b^3)/3) - tan(c/2 + (d*x)/2)^3*((3*a*
b^2)/2 + 16*a^2*b + 2*a^3 + (8*b^3)/3) + tan(c/2 + (d*x)/2)*((15*a*b^2)/4
+ 6*a^2*b + a^3 + 2*b^3) + tan(c/2 + (d*x)/2)^5*(20*a^2*b + (116*b^3)/15)
- tan(c/2 + (d*x)/2)^9*((15*a*b^2)/4 - 6*a^2*b + a^3 - 2*b^3))/(d*(5*tan(c
/2 + (d*x)/2)^2 - 10*tan(c/2 + (d*x)/2)^4 + 10*tan(c/2 + (d*x)/2)^6 - 5*ta
n(c/2 + (d*x)/2)^8 + tan(c/2 + (d*x)/2)^10 - 1))
```

3.467 $\int \sec^2(c+dx)(a+b\sec(c+dx))^3\,dx$

3.467.1 Optimal result . . . 3443
3.467.2 Mathematica [A] (verified) . . . 3444
3.467.3 Rubi [A] (verified) . . . 3444
3.467.4 Maple [A] (verified) . . . 3448
3.467.5 Fricas [A] (verification not implemented) . . . 3448
3.467.6 Sympy [F] . . . 3449
3.467.7 Maxima [A] (verification not implemented) . . . 3449
3.467.8 Giac [B] (verification not implemented) . . . 3450
3.467.9 Mupad [B] (verification not implemented) . . . 3450

3.467.1 Optimal result

Integrand size = 21, antiderivative size = 130

$$\begin{aligned}\int \sec^2(c+dx)(a+b\sec(c+dx))^3\,dx = {} & \frac{3b(4a^2+b^2)\operatorname{arctanh}(\sin(c+dx))}{8d} \\ & + \frac{a(a^2+4b^2)\tan(c+dx)}{2d} \\ & + \frac{b(2a^2+3b^2)\sec(c+dx)\tan(c+dx)}{8d} \\ & + \frac{a(a+b\sec(c+dx))^2\tan(c+dx)}{4d} \\ & + \frac{(a+b\sec(c+dx))^3\tan(c+dx)}{4d}\end{aligned}$$

output
```
3/8*b*(4*a^2+b^2)*arctanh(sin(d*x+c))/d+1/2*a*(a^2+4*b^2)*tan(d*x+c)/d+1/8
*b*(2*a^2+3*b^2)*sec(d*x+c)*tan(d*x+c)/d+1/4*a*(a+b*sec(d*x+c))^2*tan(d*x+
c)/d+1/4*(a+b*sec(d*x+c))^3*tan(d*x+c)/d
```

3.467.2 Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.69

$$\int \sec^2(c+dx)(a+b\sec(c+dx))^3\,dx$$
$$= \frac{3b(4a^2+b^2)\operatorname{arctanh}(\sin(c+dx))+\tan(c+dx)\left(3b(4a^2+b^2)\sec(c+dx)+2b^3\sec^3(c+dx)+8a(a^2+3\right.}{8d}$$

input
```
Integrate[Sec[c + d*x]^2*(a + b*Sec[c + d*x])^3,x]
```

output
```
(3*b*(4*a^2 + b^2)*ArcTanh[Sin[c + d*x]] + Tan[c + d*x]*(3*b*(4*a^2 + b^2)
*Sec[c + d*x] + 2*b^3*Sec[c + d*x]^3 + 8*a*(a^2 + 3*b^2 + b^2*Tan[c + d*x]
^2)))/(8*d)
```

3.467.3 Rubi [A] (verified)

Time = 0.87 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.08, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {3042, 4322, 3042, 4490, 3042, 4485, 3042, 4274, 3042, 4254, 24, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(c+dx)(a+b\sec(c+dx))^3\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^2\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^3dx$$
$$\Big\downarrow \text{4322}$$
$$\frac{3}{4}\int \sec(c+dx)(b+a\sec(c+dx))(a+b\sec(c+dx))^2dx+\frac{\tan(c+dx)(a+b\sec(c+dx))^3}{4d}$$
$$\Big\downarrow \text{3042}$$
$$\frac{3}{4}\int \csc\left(c+dx+\frac{\pi}{2}\right)\left(b+a\csc\left(c+dx+\frac{\pi}{2}\right)\right)\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2dx+\frac{\tan(c+dx)(a+b\sec(c+dx))^3}{4d}$$
$$\Big\downarrow \text{4490}$$

$$\frac{3}{4}\left(\frac{1}{3}\int \sec(c+dx)(a+b\sec(c+dx))\left(5ab+\left(2a^2+3b^2\right)\sec(c+dx)\right)dx+\frac{a\tan(c+dx)(a+b\sec(c+dx))^2}{3d}\right)+\frac{\tan(c+dx)(a+b\sec(c+dx))^3}{4d}$$

$\downarrow$ 3042

$$\frac{3}{4}\left(\frac{1}{3}\int \csc\left(c+dx+\frac{\pi}{2}\right)\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)\left(5ab+\left(2a^2+3b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)\right)dx+\frac{a\tan(c+dx)(a}{3}\right.$$
$$\frac{\tan(c+dx)(a+b\sec(c+dx))^3}{4d}$$

$\downarrow$ 4485

$$\frac{3}{4}\left(\frac{1}{3}\left(\frac{1}{2}\int \sec(c+dx)\left(3b\left(4a^2+b^2\right)+4a\left(a^2+4b^2\right)\sec(c+dx)\right)dx+\frac{b\left(2a^2+3b^2\right)\tan(c+dx)\sec(c+dx)}{2d}\right)+\right.$$
$$\frac{\tan(c+dx)(a+b\sec(c+dx))^3}{4d}$$

$\downarrow$ 3042

$$\frac{3}{4}\left(\frac{1}{3}\left(\frac{1}{2}\int \csc\left(c+dx+\frac{\pi}{2}\right)\left(3b\left(4a^2+b^2\right)+4a\left(a^2+4b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)\right)dx+\frac{b\left(2a^2+3b^2\right)\tan(c+dx)\sec(\right.}{2d}\right.$$
$$\frac{\tan(c+dx)(a+b\sec(c+dx))^3}{4d}$$

$\downarrow$ 4274

$$\frac{3}{4}\left(\frac{1}{3}\left(\frac{1}{2}\left(4a\left(a^2+4b^2\right)\int \sec^2(c+dx)dx+3b\left(4a^2+b^2\right)\int \sec(c+dx)dx\right)+\frac{b\left(2a^2+3b^2\right)\tan(c+dx)\sec(c+d}{2d}\right.\right.$$
$$\frac{\tan(c+dx)(a+b\sec(c+dx))^3}{4d}$$

$\downarrow$ 3042

$$\frac{3}{4}\left(\frac{1}{3}\left(\frac{1}{2}\left(3b\left(4a^2+b^2\right)\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+4a\left(a^2+4b^2\right)\int \csc\left(c+dx+\frac{\pi}{2}\right)^2dx\right)+\frac{b\left(2a^2+3b^2\right)\tan(c+}{2d}\right.\right.$$
$$\frac{\tan(c+dx)(a+b\sec(c+dx))^3}{4d}$$

$\downarrow$ 4254

3.467. $\int \sec^2(c+dx)(a+b\sec(c+dx))^3\,dx$

$$\frac{3}{4}\left(\frac{1}{3}\left(\frac{1}{2}\left(3b(4a^2+b^2)\int \csc\left(c+dx+\frac{\pi}{2}\right)dx-\frac{4a(a^2+4b^2)\int 1d(-\tan(c+dx))}{d}\right)+\frac{b(2a^2+3b^2)\tan(c+dx)}{2d}\right.\right.$$
$$\frac{\tan(c+dx)(a+b\sec(c+dx))^3}{4d}$$

$$\Big\downarrow \; 24$$

$$\frac{3}{4}\left(\frac{1}{3}\left(\frac{1}{2}\left(3b(4a^2+b^2)\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{4a(a^2+4b^2)\tan(c+dx)}{d}\right)+\frac{b(2a^2+3b^2)\tan(c+dx)\sec(c+\right.\right.\right.$$
$$\frac{\tan(c+dx)(a+b\sec(c+dx))^3}{4d}$$

$$\Big\downarrow \; 4257$$

$$\frac{3}{4}\left(\frac{1}{3}\left(\frac{1}{2}\left(\frac{3b(4a^2+b^2)\operatorname{arctanh}(\sin(c+dx))}{d}+\frac{4a(a^2+4b^2)\tan(c+dx)}{d}\right)+\frac{b(2a^2+3b^2)\tan(c+dx)\sec(c+dx}{2d}\right.\right.$$
$$\frac{\tan(c+dx)(a+b\sec(c+dx))^3}{4d}$$

input
```
Int[Sec[c + d*x]^2*(a + b*Sec[c + d*x])^3,x]
```

output
```
((a + b*Sec[c + d*x])^3*Tan[c + d*x])/(4*d) + (3*((a*(a + b*Sec[c + d*x])^
2*Tan[c + d*x])/(3*d) + ((b*(2*a^2 + 3*b^2)*Sec[c + d*x]*Tan[c + d*x])/(2*
d) + ((3*b*(4*a^2 + b^2)*ArcTanh[Sin[c + d*x]])/d + (4*a*(a^2 + 4*b^2)*Tan
[c + d*x])/d)/2)/3))/4
```

3.467.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4322
```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(-Cot[e + f*x])*((a + b*Csc[e + f*x])^m/(f*(m + 1))), x]
+ Simp[m/(m + 1)   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(b + a*Csc
[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m,
 0]
```

rule 4485
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*Cot[
e + f*x]*((d*Csc[e + f*x])^n/(f*(n + 1))), x] + Simp[1/(n + 1)   Int[(d*Csc
[e + f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x
], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[
n, -1]
```

rule 4490
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*((
a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Simp[1/(m + 1)   Int[Csc[e + f*x]*
(a + b*Csc[e + f*x])^(m - 1)*Simp[b*B*m + a*A*(m + 1) + (a*B*m + A*b*(m + 1
))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*
B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

3.467. $\int \sec^2(c+dx)(a+b\sec(c+dx))^3\,dx$

3.467.4 Maple [A] (verified)

Time = 1.23 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.96

method	result
derivativedivides	$\frac{a^3\tan(dx+c)+3a^2b\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)-3a\,b^2\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right)\tan(dx+c)+b^3\left(-\left(-\sec\right.\right.}{d}$
default	$\frac{a^3\tan(dx+c)+3a^2b\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)-3a\,b^2\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right)\tan(dx+c)+b^3\left(-\left(-\sec\right.\right.}{d}$
parts	$\frac{a^3\tan(dx+c)}{d}+\frac{b^3\left(-\left(-\frac{\sec(dx+c)^3}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)}{d}+\frac{3a^2b\left(\frac{\sec(dx+c)\tan(dx}{2}\right.}{}$
parallelrisch	$\frac{-24\left(\frac{3}{4}+\frac{\cos(4dx+4c)}{4}+\cos(2dx+2c)\right)b\left(a^2+\frac{b^2}{4}\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+24\left(\frac{3}{4}+\frac{\cos(4dx+4c)}{4}+\cos(2dx+2c)\right)b\left(a^2+\frac{b^2}{4}\right)\ln(}{4d(3+\cos(4dx-}$
norman	$\frac{-\frac{\left(8a^3-12a^2b+24a\,b^2-5b^3\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{4d}+\frac{\left(8a^3+12a^2b+24a\,b^2+5b^3\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d}+\frac{\left(24a^3-12a^2b+40a\,b^2+3b^3\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{4d}}{\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^4}$
risch	$-\frac{i\left(12a^2b\,e^{7i(dx+c)}+3b^3e^{7i(dx+c)}-8a^3e^{6i(dx+c)}+12a^2b\,e^{5i(dx+c)}+11b^3e^{5i(dx+c)}-24a^3e^{4i(dx+c)}-48a\,b^2e^{4i(dx+c)}-12\right.}{4d\left(e^{2i(dx+c)}+\right.}$

input
```
int(sec(d*x+c)^2*(a+b*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^3*tan(d*x+c)+3*a^2*b*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+t
an(d*x+c)))-3*a*b^2*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c)+b^3*(-(-1/4*sec(d*x
+c)^3-3/8*sec(d*x+c))*tan(d*x+c)+3/8*ln(sec(d*x+c)+tan(d*x+c))))
```

3.467.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.08

$$\int \sec^2(c+dx)(a+b\sec(c+dx))^3\,dx$$
$$=\frac{3\left(4\,a^2b+b^3\right)\cos\left(dx+c\right)^4\log\left(\sin\left(dx+c\right)+1\right)-3\left(4\,a^2b+b^3\right)\cos\left(dx+c\right)^4\log\left(-\sin\left(dx+c\right)+1\right)+2}{16\,d\cos\left(dx-\right.}$$

input
```
integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/16*(3*(4*a^2*b + b^3)*cos(d*x + c)^4*log(sin(d*x + c) + 1) - 3*(4*a^2*b
+ b^3)*cos(d*x + c)^4*log(-sin(d*x + c) + 1) + 2*(8*a*b^2*cos(d*x + c) + 8
*(a^3 + 2*a*b^2)*cos(d*x + c)^3 + 2*b^3 + 3*(4*a^2*b + b^3)*cos(d*x + c)^2
)*sin(d*x + c))/(d*cos(d*x + c)^4)
```

3.467.6 Sympy [F]

$$\int \sec^2(c+dx)(a+b\sec(c+dx))^3\,dx = \int (a+b\sec(c+dx))^3\sec^2(c+dx)\,dx$$

input
```
integrate(sec(d*x+c)**2*(a+b*sec(d*x+c))**3,x)
```

output
```
Integral((a + b*sec(c + d*x))**3*sec(c + d*x)**2, x)
```

3.467.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.22

$$\int \sec^2(c+dx)(a+b\sec(c+dx))^3\,dx$$
$$= \frac{16\left(\tan(dx+c)^3+3\tan(dx+c)\right)ab^2 - b^3\left(\frac{2\left(3\sin(dx+c)^3-5\sin(dx+c)\right)}{\sin(dx+c)^4-2\sin(dx+c)^2+1} - 3\log(\sin(dx+c)+1) + 3\log(\mathrm{si}\right.}{}$$

input
```
integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/16*(16*(tan(d*x + c)^3 + 3*tan(d*x + c))*a*b^2 - b^3*(2*(3*sin(d*x + c)^
3 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*
x + c) + 1) + 3*log(sin(d*x + c) - 1)) - 12*a^2*b*(2*sin(d*x + c)/(sin(d*x
 + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 16*a^3*tan
(d*x + c))/d
```

3.467.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 330 vs. $2(120) = 240$.

Time = 0.34 (sec) , antiderivative size = 330, normalized size of antiderivative = 2.54

$$\int \sec^2(c+dx)(a+b\sec(c+dx))^3\,dx$$
$$= \frac{3\,(4\,a^2b+b^3)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\,(4\,a^2b+b^3)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(8\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7\right.}{}}{}$$

input
```
integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
1/8*(3*(4*a^2*b + b^3)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*(4*a^2*b + b
^3)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(8*a^3*tan(1/2*d*x + 1/2*c)^7 -
 12*a^2*b*tan(1/2*d*x + 1/2*c)^7 + 24*a*b^2*tan(1/2*d*x + 1/2*c)^7 - 5*b^3
*tan(1/2*d*x + 1/2*c)^7 - 24*a^3*tan(1/2*d*x + 1/2*c)^5 + 12*a^2*b*tan(1/2
*d*x + 1/2*c)^5 - 40*a*b^2*tan(1/2*d*x + 1/2*c)^5 - 3*b^3*tan(1/2*d*x + 1/
2*c)^5 + 24*a^3*tan(1/2*d*x + 1/2*c)^3 + 12*a^2*b*tan(1/2*d*x + 1/2*c)^3 +
 40*a*b^2*tan(1/2*d*x + 1/2*c)^3 - 3*b^3*tan(1/2*d*x + 1/2*c)^3 - 8*a^3*ta
n(1/2*d*x + 1/2*c) - 12*a^2*b*tan(1/2*d*x + 1/2*c) - 24*a*b^2*tan(1/2*d*x
+ 1/2*c) - 5*b^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^4)/d
```

3.467.9 Mupad [B] (verification not implemented)

Time = 17.08 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.74

$$\int \sec^2(c+dx)(a+b\sec(c+dx))^3\,dx = \frac{3\,b\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\,(4\,a^2+b^2)}{4\,d}$$
$$-\frac{\left(2\,a^3-3\,a^2\,b+6\,a\,b^2-\frac{5\,b^3}{4}\right)\,\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7+\left(-6\,a^3+3\,a^2\,b-10\,a\,b^2-\frac{3\,b^3}{4}\right)\,\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5+\left(6\,a^3+\right.}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8-4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6+6\tan\left(\frac{c}{2}+\right.\right.}$$

input
```
int((a + b/cos(c + d*x))^3/cos(c + d*x)^2,x)
```

output
```
(3*b*atanh(tan(c/2 + (d*x)/2))*(4*a^2 + b^2))/(4*d) - (tan(c/2 + (d*x)/2)^
7*(6*a*b^2 - 3*a^2*b + 2*a^3 - (5*b^3)/4) + tan(c/2 + (d*x)/2)^3*(10*a*b^2
 + 3*a^2*b + 6*a^3 - (3*b^3)/4) - tan(c/2 + (d*x)/2)^5*(10*a*b^2 - 3*a^2*b
 + 6*a^3 + (3*b^3)/4) - tan(c/2 + (d*x)/2)*(6*a*b^2 + 3*a^2*b + 2*a^3 + (5
*b^3)/4))/(d*(6*tan(c/2 + (d*x)/2)^4 - 4*tan(c/2 + (d*x)/2)^2 - 4*tan(c/2
+ (d*x)/2)^6 + tan(c/2 + (d*x)/2)^8 + 1))
```

3.467. $\int \sec^2(c+dx)(a+b\sec(c+dx))^3\,dx$

3.468 $\int \sec(c+dx)(a+b\sec(c+dx))^3\,dx$

3.468.1 Optimal result . . . 3452
3.468.2 Mathematica [A] (verified) . . . 3452
3.468.3 Rubi [A] (verified) . . . 3453
3.468.4 Maple [A] (verified) . . . 3456
3.468.5 Fricas [A] (verification not implemented) . . . 3456
3.468.6 Sympy [F] . . . 3457
3.468.7 Maxima [A] (verification not implemented) . . . 3457
3.468.8 Giac [B] (verification not implemented) . . . 3458
3.468.9 Mupad [B] (verification not implemented) . . . 3458

3.468.1 Optimal result

Integrand size = 19, antiderivative size = 99

$$\int \sec(c+dx)(a+b\sec(c+dx))^3\,dx = \frac{a(2a^2+3b^2)\operatorname{arctanh}(\sin(c+dx))}{2d} + \frac{2b(4a^2+b^2)\tan(c+dx)}{3d} + \frac{5ab^2\sec(c+dx)\tan(c+dx)}{6d} + \frac{b(a+b\sec(c+dx))^2\tan(c+dx)}{3d}$$

output
```
1/2*a*(2*a^2+3*b^2)*arctanh(sin(d*x+c))/d+2/3*b*(4*a^2+b^2)*tan(d*x+c)/d+5
/6*a*b^2*sec(d*x+c)*tan(d*x+c)/d+1/3*b*(a+b*sec(d*x+c))^2*tan(d*x+c)/d
```

3.468.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.71

$$\int \sec(c+dx)(a+b\sec(c+dx))^3\,dx = \frac{(6a^3+9ab^2)\operatorname{arctanh}(\sin(c+dx)) + b\tan(c+dx)\left(18a^2+6b^2+9ab\sec(c+dx)+2b^2\tan^2(c+dx)\right)}{6d}$$

input
```
Integrate[Sec[c + d*x]*(a + b*Sec[c + d*x])^3,x]
```

output
```
((6*a^3 + 9*a*b^2)*ArcTanh[Sin[c + d*x]] + b*Tan[c + d*x]*(18*a^2 + 6*b^2
+ 9*a*b*Sec[c + d*x] + 2*b^2*Tan[c + d*x]^2))/(6*d)
```

3.468.3 Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.06, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.526$, Rules used = {3042, 4317, 3042, 4485, 3042, 4274, 3042, 4254, 24, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(c+dx)(a+b\sec(c+dx))^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^3dx$$

$$\Big\downarrow \text{4317}$$

$$\frac{1}{3}\int \sec(c+dx)(a+b\sec(c+dx))\left(3a^2+5b\sec(c+dx)a+2b^2\right)dx+\frac{b\tan(c+dx)(a+b\sec(c+dx))^2}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\int \csc\left(c+dx+\frac{\pi}{2}\right)\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)\left(3a^2+5b\csc\left(c+dx+\frac{\pi}{2}\right)a+2b^2\right)dx+\frac{b\tan(c+dx)(a+b\sec(c+dx))^2}{3d}$$

$$\Big\downarrow \text{4485}$$

$$\frac{1}{3}\left(\frac{1}{2}\int \sec(c+dx)\left(3a\left(2a^2+3b^2\right)+4b\left(4a^2+b^2\right)\sec(c+dx)\right)dx+\frac{5ab^2\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{b\tan(c+dx)(a+b\sec(c+dx))^2}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\left(\frac{1}{2}\int \csc\left(c+dx+\frac{\pi}{2}\right)\left(3a\left(2a^2+3b^2\right)+4b\left(4a^2+b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)\right)dx+\frac{5ab^2\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{b\tan(c+dx)(a+b\sec(c+dx))^2}{3d}$$

$$\Big\downarrow \text{4274}$$

$$\frac{1}{3}\left(\frac{1}{2}\left(4b(4a^2+b^2)\int \sec^2(c+dx)dx+3a(2a^2+3b^2)\int \sec(c+dx)dx\right)+\frac{5ab^2\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{b\tan(c+dx)(a+b\sec(c+dx))^2}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\left(\frac{1}{2}\left(3a(2a^2+3b^2)\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+4b(4a^2+b^2)\int \csc\left(c+dx+\frac{\pi}{2}\right)^2dx\right)+\frac{5ab^2\tan(c+dx)\sec(c+}{2d}\right.$$
$$\frac{b\tan(c+dx)(a+b\sec(c+dx))^2}{3d}$$

$$\Big\downarrow \text{4254}$$

$$\frac{1}{3}\left(\frac{1}{2}\left(3a(2a^2+3b^2)\int \csc\left(c+dx+\frac{\pi}{2}\right)dx-\frac{4b(4a^2+b^2)\int 1d(-\tan(c+dx))}{d}\right)+\frac{5ab^2\tan(c+dx)\sec(c+dx}{2d}\right.$$
$$\frac{b\tan(c+dx)(a+b\sec(c+dx))^2}{3d}$$

$$\Big\downarrow \text{24}$$

$$\frac{1}{3}\left(\frac{1}{2}\left(3a(2a^2+3b^2)\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{4b(4a^2+b^2)\tan(c+dx)}{d}\right)+\frac{5ab^2\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{b\tan(c+dx)(a+b\sec(c+dx))^2}{3d}$$

$$\Big\downarrow \text{4257}$$

$$\frac{1}{3}\left(\frac{1}{2}\left(\frac{3a(2a^2+3b^2)\operatorname{arctanh}(\sin(c+dx))}{d}+\frac{4b(4a^2+b^2)\tan(c+dx)}{d}\right)+\frac{5ab^2\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{b\tan(c+dx)(a+b\sec(c+dx))^2}{3d}$$

input
```
Int[Sec[c + d*x]*(a + b*Sec[c + d*x])^3,x]
```

output
```
(b*(a + b*Sec[c + d*x])^2*Tan[c + d*x])/(3*d) + ((5*a*b^2*Sec[c + d*x]*Tan
[c + d*x])/(2*d) + ((3*a*(2*a^2 + 3*b^2)*ArcTanh[Sin[c + d*x]])/d + (4*b*(
4*a^2 + b^2)*Tan[c + d*x])/d)/2)/3
```

3.468. $\int \sec(c+dx)(a+b\sec(c+dx))^3\,dx$

3.468.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4317
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[(-b)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m - 1)/(f*m)), x]
+ Simp[1/m   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)*(b^2*(m - 1) + a
^2*m + a*b*(2*m - 1)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x] && NeQ
[a^2 - b^2, 0] && GtQ[m, 1] && IntegerQ[2*m]
```

rule 4485
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*Cot[
e + f*x]*((d*Csc[e + f*x])^n/(f*(n + 1))), x] + Simp[1/(n + 1)   Int[(d*Csc
[e + f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x
], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[
n, -1]
```

3.468.4 Maple [A] (verified)

Time = 1.14 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.97

method	result
derivativedivides	$\frac{a^3 \ln(\sec(dx+c)+\tan(dx+c))+3\tan(dx+c)a^2b+3a\,b^2\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)-b^3\left(-\frac{2}{3}-\frac{\sec(dx+}{3}\right.}{d}$
default	$\frac{a^3 \ln(\sec(dx+c)+\tan(dx+c))+3\tan(dx+c)a^2b+3a\,b^2\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)-b^3\left(-\frac{2}{3}-\frac{\sec(dx+}{3}\right.}{d}$
parts	$\frac{a^3 \ln(\sec(dx+c)+\tan(dx+c))}{d}-\frac{b^3\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right)\tan(dx+c)}{d}+\frac{3a^2b\tan(dx+c)}{d}+\frac{3a\,b^2\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\right.}{d}$
parallelrisch	$\frac{-3\left(\cos(dx+c)+\frac{\cos(3dx+3c)}{3}\right)a\left(a^2+\frac{3b^2}{2}\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+3\left(\cos(dx+c)+\frac{\cos(3dx+3c)}{3}\right)a\left(a^2+\frac{3b^2}{2}\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}{d(\cos(3dx+3c)+3\cos(dx+c))}$
norman	$\frac{\frac{4b\left(9a^2+b^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3d}-\frac{b\left(6a^2-3ab+2b^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{d}-\frac{b\left(6a^2+3ab+2b^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}}{\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^3}-\frac{a\left(2a^2+3b^2\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}{2d}$
risch	$-\frac{ib\left(9ab\,e^{5i(dx+c)}-18a^2e^{4i(dx+c)}-36a^2e^{2i(dx+c)}-12b^2e^{2i(dx+c)}-9ab\,e^{i(dx+c)}-18a^2-4b^2\right)}{3d\left(e^{2i(dx+c)}+1\right)^3}-\frac{a^3\ln\left(e^{i(dx+c)}-i\right)}{d}-$

input
```
int(sec(d*x+c)*(a+b*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^3*ln(sec(d*x+c)+tan(d*x+c))+3*tan(d*x+c)*a^2*b+3*a*b^2*(1/2*sec(d*x
+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c)))-b^3*(-2/3-1/3*sec(d*x+c)^2)*
tan(d*x+c))
```

3.468.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.27

$$\int \sec(c+dx)(a+b\sec(c+dx))^3\,dx$$
$$=\frac{3\left(2\,a^3+3\,ab^2\right)\cos\left(dx+c\right)^3\log\left(\sin\left(dx+c\right)+1\right)-3\left(2\,a^3+3\,ab^2\right)\cos\left(dx+c\right)^3\log\left(-\sin\left(dx+c\right)+1\right)}{12\,d\cos\left(dx+c\right)^3}$$

input
```
integrate(sec(d*x+c)*(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/12*(3*(2*a^3 + 3*a*b^2)*cos(d*x + c)^3*log(sin(d*x + c) + 1) - 3*(2*a^3
+ 3*a*b^2)*cos(d*x + c)^3*log(-sin(d*x + c) + 1) + 2*(9*a*b^2*cos(d*x + c)
 + 2*b^3 + 2*(9*a^2*b + 2*b^3)*cos(d*x + c)^2)*sin(d*x + c))/(d*cos(d*x +
c)^3)
```

3.468.6 Sympy [F]

$$\int \sec(c+dx)(a+b\sec(c+dx))^3\,dx = \int (a+b\sec(c+dx))^3\sec(c+dx)\,dx$$

input
```
integrate(sec(d*x+c)*(a+b*sec(d*x+c))**3,x)
```

output
```
Integral((a + b*sec(c + d*x))**3*sec(c + d*x), x)
```

3.468.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.07

$$\int \sec(c+dx)(a+b\sec(c+dx))^3\,dx = \frac{4\left(\tan(dx+c)^3+3\tan(dx+c)\right)b^3-9ab^2\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1}-\log(\sin(dx+c)+1)+\log(\sin(dx+c)-1)\right)}{12d}$$

input
```
integrate(sec(d*x+c)*(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/12*(4*(tan(d*x + c)^3 + 3*tan(d*x + c))*b^3 - 9*a*b^2*(2*sin(d*x + c)/(s
in(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 12*a
^3*log(sec(d*x + c) + tan(d*x + c)) + 36*a^2*b*tan(d*x + c))/d
```

3.468.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 205 vs. $2(91) = 182$.

Time = 0.32 (sec) , antiderivative size = 205, normalized size of antiderivative = 2.07

$$\int \sec(c+dx)(a+b\sec(c+dx))^3\,dx$$
$$= \frac{3\,(2\,a^3+3\,ab^2)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\,(2\,a^3+3\,ab^2)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(18\,a^2b\tan\left(\frac{1}{2}\,dx\right.\right.}{}}{}$$

input
```
integrate(sec(d*x+c)*(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
1/6*(3*(2*a^3 + 3*a*b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*(2*a^3 + 3
*a*b^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(18*a^2*b*tan(1/2*d*x + 1/2
*c)^5 - 9*a*b^2*tan(1/2*d*x + 1/2*c)^5 + 6*b^3*tan(1/2*d*x + 1/2*c)^5 - 36
*a^2*b*tan(1/2*d*x + 1/2*c)^3 - 4*b^3*tan(1/2*d*x + 1/2*c)^3 + 18*a^2*b*ta
n(1/2*d*x + 1/2*c) + 9*a*b^2*tan(1/2*d*x + 1/2*c) + 6*b^3*tan(1/2*d*x + 1/
2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^3)/d
```

3.468.9 Mupad [B] (verification not implemented)

Time = 16.33 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.59

$$\int \sec(c+dx)(a+b\sec(c+dx))^3\,dx = \frac{\operatorname{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\,(2\,a^3+3\,a\,b^2)}{d} - \frac{(6\,a^2\,b-3\,a\,b^2+2\,b^3)\,\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5+\left(-12\,a^2\,b-\frac{4\,b^3}{3}\right)\,\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3+(6\,a^2\,b+3\,a\,b^2+2\,b^3)\,\tan\left(\frac{c}{2}+\right.}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6-3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2-1\right)}$$

input
```
int((a + b/cos(c + d*x))^3/cos(c + d*x),x)
```

output
```
(atanh(tan(c/2 + (d*x)/2))*(3*a*b^2 + 2*a^3))/d - (tan(c/2 + (d*x)/2)^5*(6
*a^2*b - 3*a*b^2 + 2*b^3) - tan(c/2 + (d*x)/2)^3*(12*a^2*b + (4*b^3)/3) +
tan(c/2 + (d*x)/2)*(3*a*b^2 + 6*a^2*b + 2*b^3))/(d*(3*tan(c/2 + (d*x)/2)^2
 - 3*tan(c/2 + (d*x)/2)^4 + tan(c/2 + (d*x)/2)^6 - 1))
```

3.469 $\int (a+b\sec(c+dx))^3\,dx$

3.469.1 Optimal result . . . 3459
3.469.2 Mathematica [A] (verified) . . . 3459
3.469.3 Rubi [A] (verified) . . . 3460
3.469.4 Maple [A] (verified) . . . 3461
3.469.5 Fricas [A] (verification not implemented) . . . 3461
3.469.6 Sympy [F] . . . 3462
3.469.7 Maxima [A] (verification not implemented) . . . 3462
3.469.8 Giac [B] (verification not implemented) . . . 3463
3.469.9 Mupad [B] (verification not implemented) . . . 3463

3.469.1 Optimal result

Integrand size = 12, antiderivative size = 73

$$\int (a+b\sec(c+dx))^3\,dx = a^3x+\frac{b(6a^2+b^2)\operatorname{arctanh}(\sin(c+dx))}{2d}+\frac{5ab^2\tan(c+dx)}{2d}+\frac{b^2(a+b\sec(c+dx))\tan(c+dx)}{2d}$$

output
```
a^3*x+1/2*b*(6*a^2+b^2)*arctanh(sin(d*x+c))/d+5/2*a*b^2*tan(d*x+c)/d+1/2*b
^2*(a+b*sec(d*x+c))*tan(d*x+c)/d
```

3.469.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.75

$$\int (a+b\sec(c+dx))^3\,dx = \frac{2a^3dx+b(6a^2+b^2)\operatorname{arctanh}(\sin(c+dx))+b^2(6a+b\sec(c+dx))\tan(c+dx)}{2d}$$

input
```
Integrate[(a + b*Sec[c + d*x])^3,x]
```

output
```
(2*a^3*d*x + b*(6*a^2 + b^2)*ArcTanh[Sin[c + d*x]] + b^2*(6*a + b*Sec[c +
d*x])*Tan[c + d*x])/(2*d)
```

3.469.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.01, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3042, 4269, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\sec(c+dx))^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^3 dx$$

$$\Big\downarrow \text{4269}$$

$$\frac{1}{2}\int \left(2a^3+5b^2\sec^2(c+dx)a+b\left(6a^2+b^2\right)\sec(c+dx)\right)dx+\frac{b^2\tan(c+dx)(a+b\sec(c+dx))}{2d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(2a^3x+\frac{b\left(6a^2+b^2\right)\operatorname{arctanh}(\sin(c+dx))}{d}+\frac{5ab^2\tan(c+dx)}{d}\right)+\frac{b^2\tan(c+dx)(a+b\sec(c+dx))}{2d}$$

input
```
Int[(a + b*Sec[c + d*x])^3,x]
```

output
```
(b^2*(a + b*Sec[c + d*x])*Tan[c + d*x])/(2*d) + (2*a^3*x + (b*(6*a^2 + b^2
)*ArcTanh[Sin[c + d*x]])/d + (5*a*b^2*Tan[c + d*x])/d)/2
```

3.469.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4269
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(-b^2)*C
ot[c + d*x]*((a + b*Csc[c + d*x])^(n - 2)/(d*(n - 1))), x] + Simp[1/(n - 1)
   Int[(a + b*Csc[c + d*x])^(n - 3)*Simp[a^3*(n - 1) + (b*(b^2*(n - 2) + 3*
a^2*(n - 1)))*Csc[c + d*x] + (a*b^2*(3*n - 4))*Csc[c + d*x]^2, x], x], x] /
; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 2] && IntegerQ[2*n]
```

3.469.4 Maple [A] (verified)

Time = 0.88 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.12

method	result
derivativedivides	$\frac{a^3(dx+c)+3a^2b\ln(\sec(dx+c)+\tan(dx+c))+3\tan(dx+c)a\,b^2+b^3\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)}{d}$
default	$\frac{a^3(dx+c)+3a^2b\ln(\sec(dx+c)+\tan(dx+c))+3\tan(dx+c)a\,b^2+b^3\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)}{d}$
parts	$a^3x+\frac{b^3\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)}{d}+\frac{3a^2b\ln(\sec(dx+c)+\tan(dx+c))}{d}+\frac{3a\,b^2\tan(dx+c)}{d}$
parallelrisch	$\frac{-3\left(a^2+\frac{b^2}{6}\right)b(1+\cos(2dx+2c))\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+3\left(a^2+\frac{b^2}{6}\right)b(1+\cos(2dx+2c))\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+a^3xd\cos(2dx}{d(1+\cos(2dx+2c))}$
risch	$a^3x-\frac{ib^2\left(b\,\mathrm{e}^{3i(dx+c)}-6a\,\mathrm{e}^{2i(dx+c)}-b\,\mathrm{e}^{i(dx+c)}-6a\right)}{d\left(\mathrm{e}^{2i(dx+c)}+1\right)^2}+\frac{3b\ln\left(\mathrm{e}^{i(dx+c)}+i\right)a^2}{d}+\frac{b^3\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{2d}-\frac{3b\ln\left(\mathrm{e}^{i(dx+c}\right.}{d}$
norman	$\frac{a^3x+a^3x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\frac{b^2(b+6a)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}-2a^3x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2-\frac{b^2(6a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{d}}{\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^2}-\frac{b(6a^2+b^2)\ln\left(\tan\left(\frac{dx}{2}+\right.\right.}{2d}$

input
```
int((a+b*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^3*(d*x+c)+3*a^2*b*ln(sec(d*x+c)+tan(d*x+c))+3*tan(d*x+c)*a*b^2+b^3*
(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c))))
```

3.469.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.53

$$\int (a+b\sec(c+dx))^3\,dx$$
$$=\frac{4\,a^3dx\cos(dx+c)^2+(6\,a^2b+b^3)\cos(dx+c)^2\log(\sin(dx+c)+1)-(6\,a^2b+b^3)\cos(dx+c)^2\log(-\mathrm{si}}{4\,d\cos(dx+c)^2}$$

input
```
integrate((a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/4*(4*a^3*d*x*cos(d*x + c)^2 + (6*a^2*b + b^3)*cos(d*x + c)^2*log(sin(d*x
 + c) + 1) - (6*a^2*b + b^3)*cos(d*x + c)^2*log(-sin(d*x + c) + 1) + 2*(6*
a*b^2*cos(d*x + c) + b^3)*sin(d*x + c))/(d*cos(d*x + c)^2)
```

3.469.6 Sympy [F]

$$\int (a+b\sec(c+dx))^3\,dx=\int (a+b\sec{(c+dx)})^3\,dx$$

input
```
integrate((a+b*sec(d*x+c))**3,x)
```

output
```
Integral((a + b*sec(c + d*x))**3, x)
```

3.469.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.27

$$\begin{aligned}\int (a+b\sec(c+dx))^3\,dx &= a^3x-\frac{b^3\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1}-\log\left(\sin\left(dx+c\right)+1\right)+\log\left(\sin\left(dx+c\right)-1\right)\right)}{4\,d}\\ &\quad+\frac{3\,a^2b\log\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)}{d}+\frac{3\,ab^2\tan\left(dx+c\right)}{d}\end{aligned}$$

input
```
integrate((a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
a^3*x - 1/4*b^3*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) +
1) + log(sin(d*x + c) - 1))/d + 3*a^2*b*log(sec(d*x + c) + tan(d*x + c))/d
 + 3*a*b^2*tan(d*x + c)/d
```

3.469.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 145 vs. $2(67) = 134$.

Time = 0.29 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.99

$$\int (a+b\sec(c+dx))^3\,dx$$
$$= \frac{2\,(dx+c)a^3 + (6\,a^2b+b^3)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right) - (6\,a^2b+b^3)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right) - \frac{2\left(6\,ab^2\right.}{}}{2\,d}$$

input
```
integrate((a+b*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
1/2*(2*(d*x + c)*a^3 + (6*a^2*b + b^3)*log(abs(tan(1/2*d*x + 1/2*c) + 1))
- (6*a^2*b + b^3)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(6*a*b^2*tan(1/2*
d*x + 1/2*c)^3 - b^3*tan(1/2*d*x + 1/2*c)^3 - 6*a*b^2*tan(1/2*d*x + 1/2*c)
 - b^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^2)/d
```

3.469.9 Mupad [B] (verification not implemented)

Time = 14.01 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.86

$$\int (a+b\sec(c+dx))^3\,dx = \frac{2\,a^3\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d} + \frac{b^3\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d} + \frac{b^3\,\sin\left(c+d\,x\right)}{2\,d\cos\left(c+d\,x\right)^2} + \frac{6\,a^2\,b\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d} + \frac{3\,a\,b^2\,\sin\left(c+d\,x\right)}{d\,\cos\left(c+d\,x\right)}$$

input
```
int((a + b/cos(c + d*x))^3,x)
```

output
```
(2*a^3*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (b^3*atanh(sin(c/2
 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (b^3*sin(c + d*x))/(2*d*cos(c + d*x)^
2) + (6*a^2*b*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (3*a*b^2*s
in(c + d*x))/(d*cos(c + d*x))
```

3.470 $\int \cos(c+dx)(a+b\sec(c+dx))^3\,dx$

3.470.1 Optimal result . . . 3464
3.470.2 Mathematica [A] (verified) . . . 3464
3.470.3 Rubi [A] (verified) . . . 3465
3.470.4 Maple [A] (verified) . . . 3467
3.470.5 Fricas [A] (verification not implemented) . . . 3468
3.470.6 Sympy [F] . . . 3468
3.470.7 Maxima [A] (verification not implemented) . . . 3468
3.470.8 Giac [A] (verification not implemented) . . . 3469
3.470.9 Mupad [B] (verification not implemented) . . . 3469

3.470.1 Optimal result

Integrand size = 19, antiderivative size = 67

$$\int \cos(c+dx)(a+b\sec(c+dx))^3\,dx = 3a^2bx+\frac{3ab^2\text{arctanh}(\sin(c+dx))}{d}+\frac{a(a^2-b^2)\sin(c+dx)}{d}+\frac{b^2(a+b\sec(c+dx))\sin(c+dx)}{d}$$

output
```
3*a^2*b*x+3*a*b^2*arctanh(sin(d*x+c))/d+a*(a^2-b^2)*sin(d*x+c)/d+b^2*(a+b*
sec(d*x+c))*sin(d*x+c)/d
```

3.470.2 Mathematica [A] (verified)

Time = 0.86 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.31

$$\int \cos(c+dx)(a+b\sec(c+dx))^3\,dx = \frac{3ab\left(ac+adx-b\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+b\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)+}{d}$$

input
```
Integrate[Cos[c + d*x]*(a + b*Sec[c + d*x])^3,x]
```

output
```
(3*a*b*(a*c + a*d*x - b*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + b*Log[C
os[(c + d*x)/2] + Sin[(c + d*x)/2]]) + a^3*Sin[c + d*x] + b^3*Tan[c + d*x]
)/d
```

3.470.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {3042, 4329, 3042, 4535, 24, 3042, 4533, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(c+dx)(a+b\sec(c+dx))^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^3}{\csc\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{4329}$$

$$\int \cos(c+dx)\left(3b\sec(c+dx)a^2+3b^2\sec^2(c+dx)a+\left(a^2-b^2\right)a\right)dx+\frac{b^2\sin(c+dx)(a+b\sec(c+dx))}{d}$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{3b\csc\left(c+dx+\frac{\pi}{2}\right)a^2+3b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2a+\left(a^2-b^2\right)a}{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{b^2\sin(c+dx)(a+b\sec(c+dx))}{d}$$

$$\Big\downarrow \text{4535}$$

$$\int \cos(c+dx)\left(3ab^2\sec^2(c+dx)+a\left(a^2-b^2\right)\right)dx+3a^2b\int 1dx+\frac{b^2\sin(c+dx)(a+b\sec(c+dx))}{d}$$

$$\Big\downarrow \text{24}$$

$$\int \cos(c+dx)\left(3ab^2\sec^2(c+dx)+a\left(a^2-b^2\right)\right)dx+3a^2bx+\frac{b^2\sin(c+dx)(a+b\sec(c+dx))}{d}$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{3ab^2\csc\left(c+dx+\frac{\pi}{2}\right)^2+a\left(a^2-b^2\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+3a^2bx+\frac{b^2\sin(c+dx)(a+b\sec(c+dx))}{d}$$

$$\Big\downarrow \text{4533}$$

$$3ab^2\int \sec(c+dx)dx+\frac{a\left(a^2-b^2\right)\sin(c+dx)}{d}+3a^2bx+\frac{b^2\sin(c+dx)(a+b\sec(c+dx))}{d}$$

$$\Big\downarrow \text{3042}$$

$$3ab^2\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{a\left(a^2-b^2\right)\sin(c+dx)}{d}+3a^2bx+\frac{b^2\sin(c+dx)(a+b\sec(c+dx))}{d}$$

$$\Big\downarrow \text{4257}$$

$$\frac{a\left(a^2-b^2\right)\sin(c+dx)}{d}+3a^2bx+\frac{3ab^2\text{arctanh}(\sin(c+dx))}{d}+\frac{b^2\sin(c+dx)(a+b\sec(c+dx))}{d}$$

input
```
Int[Cos[c + d*x]*(a + b*Sec[c + d*x])^3,x]
```

output
```
3*a^2*b*x + (3*a*b^2*ArcTanh[Sin[c + d*x]])/d + (a*(a^2 - b^2)*Sin[c + d*x
])/d + (b^2*(a + b*Sec[c + d*x])*Sin[c + d*x])/d
```

3.470.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4329
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
2)*((d*Csc[e + f*x])^n/(f*(m + n - 1))), x] + Simp[1/(d*(m + n - 1))   Int[
(a + b*Csc[e + f*x])^(m - 3)*(d*Csc[e + f*x])^n*Simp[a^3*d*(m + n - 1) + a*
b^2*d*n + b*(b^2*d*(m + n - 2) + 3*a^2*d*(m + n - 1))*Csc[e + f*x] + a*b^2*
d*(3*m + 2*n - 4)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, n}, x
] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
 &&  !(IGtQ[n, 2] &&  !IntegerQ[m])
```

rule 4533
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] +
Simp[(C*m + A*(m + 1))/(b^2*m)   Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fr
eeQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

rule 4535
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Simp[B/b   Int[(b*Cs
c[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2)
, x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

3.470.4 Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.85

method	result
derivativedivides	$\frac{a^3\sin(dx+c)+3a^2b(dx+c)+3a\,b^2\ln(\sec(dx+c)+\tan(dx+c))+\tan(dx+c)b^3}{d}$
default	$\frac{a^3\sin(dx+c)+3a^2b(dx+c)+3a\,b^2\ln(\sec(dx+c)+\tan(dx+c))+\tan(dx+c)b^3}{d}$
parallelrisch	$\frac{6a^2bxd\cos(dx+c)-6\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)a\,b^2\cos(dx+c)+6\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)a\,b^2\cos(dx+c)+2b^3\sin(dx+c)+a^3\sin(}{2d\cos(dx+c)}$
risch	$3a^2bx-\frac{ia^3e^{i(dx+c)}}{2d}+\frac{ia^3e^{-i(dx+c)}}{2d}+\frac{2ib^3}{d\left(e^{2i(dx+c)}+1\right)}-\frac{3a\ln\left(e^{i(dx+c)}-i\right)b^2}{d}+\frac{3a\ln\left(e^{i(dx+c)}+i\right)b^2}{d}$
norman	$\frac{3a^2bx-\frac{4a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{d}+\frac{2\left(a^3-b^3\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{d}+\frac{2\left(a^3+b^3\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}-3a^2bx\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2-3a^2bx\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4+3}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^2}$

input
```
int(cos(d*x+c)*(a+b*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^3*sin(d*x+c)+3*a^2*b*(d*x+c)+3*a*b^2*ln(sec(d*x+c)+tan(d*x+c))+tan(
d*x+c)*b^3)
```

3.470.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.40

$$\int \cos(c+dx)(a+b\sec(c+dx))^3\,dx$$
$$= \frac{6\,a^2bdx\cos(dx+c)+3\,ab^2\cos(dx+c)\log(\sin(dx+c)+1)-3\,ab^2\cos(dx+c)\log(-\sin(dx+c)+1)}{2\,d\cos(dx+c)}$$

input
```
integrate(cos(d*x+c)*(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/2*(6*a^2*b*d*x*cos(d*x + c) + 3*a*b^2*cos(d*x + c)*log(sin(d*x + c) + 1)
 - 3*a*b^2*cos(d*x + c)*log(-sin(d*x + c) + 1) + 2*(a^3*cos(d*x + c) + b^3
)*sin(d*x + c))/(d*cos(d*x + c))
```

3.470.6 Sympy [F]

$$\int \cos(c+dx)(a+b\sec(c+dx))^3\,dx = \int (a+b\sec(c+dx))^3\cos(c+dx)\,dx$$

input
```
integrate(cos(d*x+c)*(a+b*sec(d*x+c))**3,x)
```

output
```
Integral((a + b*sec(c + d*x))**3*cos(c + d*x), x)
```

3.470.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.99

$$\int \cos(c+dx)(a+b\sec(c+dx))^3\,dx$$
$$= \frac{6\,(dx+c)a^2b+3\,ab^2(\log(\sin(dx+c)+1)-\log(\sin(dx+c)-1))+2\,a^3\sin(dx+c)+2\,b^3\tan(dx+c)}{2\,d}$$

input
```
integrate(cos(d*x+c)*(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/2*(6*(d*x + c)*a^2*b + 3*a*b^2*(log(sin(d*x + c) + 1) - log(sin(d*x + c)
 - 1)) + 2*a^3*sin(d*x + c) + 2*b^3*tan(d*x + c))/d
```

3.470.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.96

$$\int \cos(c+dx)(a+b\sec(c+dx))^3\,dx$$
$$= \frac{3\,(dx+c)a^2b + 3\,ab^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right) - 3\,ab^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right) + \frac{2\left(a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-b\right.}{}}{d}$$

input
```
integrate(cos(d*x+c)*(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
(3*(d*x + c)*a^2*b + 3*a*b^2*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*a*b^2*
log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(a^3*tan(1/2*d*x + 1/2*c)^3 - b^3*t
an(1/2*d*x + 1/2*c)^3 - a^3*tan(1/2*d*x + 1/2*c) - b^3*tan(1/2*d*x + 1/2*c
))/(tan(1/2*d*x + 1/2*c)^4 - 1))/d
```

3.470.9 Mupad [B] (verification not implemented)

Time = 13.91 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.45

$$\int \cos(c+dx)(a+b\sec(c+dx))^3\,dx = \frac{a^3\,\sin(c+d\,x)}{d} + \frac{b^3\,\sin(c+d\,x)}{d\,\cos(c+d\,x)} + \frac{6\,a^2\,b\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d} + \frac{6\,a\,b^2\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}$$

input
```
int(cos(c + d*x)*(a + b/cos(c + d*x))^3,x)
```

output
```
(a^3*sin(c + d*x))/d + (b^3*sin(c + d*x))/(d*cos(c + d*x)) + (6*a^2*b*atan
(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (6*a*b^2*atanh(sin(c/2 + (d*x
)/2)/cos(c/2 + (d*x)/2)))/d
```

3.470. $\int \cos(c+dx)(a+b\sec(c+dx))^3\,dx$

3.471 $\int \cos^2(c+dx)(a+b\sec(c+dx))^3\,dx$

3.471.1 Optimal result 3471
3.471.2 Mathematica [A] (verified) 3471
3.471.3 Rubi [A] (verified) 3472
3.471.4 Maple [A] (verified) 3474
3.471.5 Fricas [A] (verification not implemented) 3475
3.471.6 Sympy [F] 3475
3.471.7 Maxima [A] (verification not implemented) 3475
3.471.8 Giac [A] (verification not implemented) 3476
3.471.9 Mupad [B] (verification not implemented) 3476

3.471.1 Optimal result

Integrand size = 21, antiderivative size = 79

$$\int \cos^2(c+dx)(a+b\sec(c+dx))^3\,dx = \frac{1}{2}a\left(a^2+6b^2\right)x+\frac{b^3\operatorname{arctanh}(\sin(c+dx))}{d}+\frac{5a^2b\sin(c+dx)}{2d}+\frac{a^2\cos(c+dx)(a+b\sec(c+dx))\sin(c+dx)}{2d}$$

output
```
1/2*a*(a^2+6*b^2)*x+b^3*arctanh(sin(d*x+c))/d+5/2*a^2*b*sin(d*x+c)/d+1/2*a
^2*cos(d*x+c)*(a+b*sec(d*x+c))*sin(d*x+c)/d
```

3.471.2 Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.33

$$\int \cos^2(c+dx)(a+b\sec(c+dx))^3\,dx = \frac{2a(a^2+6b^2)(c+dx)-4b^3\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+4b^3\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c\right.\right.}{4d}$$

input
```
Integrate[Cos[c + d*x]^2*(a + b*Sec[c + d*x])^3,x]
```

output
```
(2*a*(a^2 + 6*b^2)*(c + d*x) - 4*b^3*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/
2]] + 4*b^3*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + 12*a^2*b*Sin[c + d*
x] + a^3*Sin[2*(c + d*x)])/(4*d)
```

3.471.3 Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.01, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 4328, 3042, 4535, 24, 3042, 4533, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(c+dx)(a+b\sec(c+dx))^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^3}{\csc\left(c+dx+\frac{\pi}{2}\right)^2}dx$$

$$\Big\downarrow \text{4328}$$

$$\frac{1}{2}\int \cos(c+dx)\left(2\sec^2(c+dx)b^3+5a^2b+a\left(a^2+6b^2\right)\sec(c+dx)\right)dx+\frac{a^2\sin(c+dx)\cos(c+dx)(a+b\sec(c+dx))}{2d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\int \frac{2\csc\left(c+dx+\frac{\pi}{2}\right)^2b^3+5a^2b+a\left(a^2+6b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{a^2\sin(c+dx)\cos(c+dx)(a+b\sec(c+dx))}{2d}$$

$$\Big\downarrow \text{4535}$$

$$\frac{1}{2}\left(\int \cos(c+dx)\left(2\sec^2(c+dx)b^3+5a^2b\right)dx+a\left(a^2+6b^2\right)\int 1dx\right)+\frac{a^2\sin(c+dx)\cos(c+dx)(a+b\sec(c+dx))}{2d}$$

$$\Big\downarrow \text{24}$$

$$\frac{1}{2}\left(\int \cos(c+dx)\left(2\sec^2(c+dx)b^3+5a^2b\right)dx+ax\left(a^2+6b^2\right)\right)+\frac{a^2\sin(c+dx)\cos(c+dx)(a+b\sec(c+dx))}{2d}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{2}\left(\int \frac{2\csc\left(c+dx+\frac{\pi}{2}\right)^2 b^3+5a^2b}{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+ax\left(a^2+6b^2\right)\right)+\frac{a^2\sin(c+dx)\cos(c+dx)(a+b\sec(c+dx))}{2d}$$

$$\Big\downarrow \; 4533$$

$$\frac{1}{2}\left(2b^3\int \sec(c+dx)dx+ax\left(a^2+6b^2\right)+\frac{5a^2b\sin(c+dx)}{d}\right)+\frac{a^2\sin(c+dx)\cos(c+dx)(a+b\sec(c+dx))}{2d}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{2}\left(2b^3\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+ax\left(a^2+6b^2\right)+\frac{5a^2b\sin(c+dx)}{d}\right)+\frac{a^2\sin(c+dx)\cos(c+dx)(a+b\sec(c+dx))}{2d}$$

$$\Big\downarrow \; 4257$$

$$\frac{1}{2}\left(ax\left(a^2+6b^2\right)+\frac{5a^2b\sin(c+dx)}{d}+\frac{2b^3\text{arctanh}(\sin(c+dx))}{d}\right)+\frac{a^2\sin(c+dx)\cos(c+dx)(a+b\sec(c+dx))}{2d}$$

input
```
Int[Cos[c + d*x]^2*(a + b*Sec[c + d*x])^3,x]
```

output
```
(a^2*Cos[c + d*x]*(a + b*Sec[c + d*x])*Sin[c + d*x])/(2*d) + (a*(a^2 + 6*b
^2)*x + (2*b^3*ArcTanh[Sin[c + d*x]])/d + (5*a^2*b*Sin[c + d*x])/d)/2
```

3.471.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4328
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)*
((d*Csc[e + f*x])^n/(f*n)), x] - Simp[1/(d*n)   Int[(a + b*Csc[e + f*x])^(m
 - 3)*(d*Csc[e + f*x])^(n + 1)*Simp[a^2*b*(m - 2*n - 2) - a*(3*b^2*n + a^2*
(n + 1))*Csc[e + f*x] - b*(b^2*n + a^2*(m + n - 1))*Csc[e + f*x]^2, x], x],
 x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && ((Int
egerQ[m] && LtQ[n, -1]) || (IntegersQ[m + 1/2, 2*n] && LeQ[n, -1]))
```

rule 4533
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] +
Simp[(C*m + A*(m + 1))/(b^2*m)   Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fr
eeQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

rule 4535
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Simp[B/b   Int[(b*Cs
c[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2)
, x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

3.471.4 Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.92

method	result
derivativedivides	$\frac{a^3\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+3a^2b\sin(dx+c)+3a\,b^2(dx+c)+b^3\ln(\sec(dx+c)+\tan(dx+c))}{d}$
default	$\frac{a^3\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+3a^2b\sin(dx+c)+3a\,b^2(dx+c)+b^3\ln(\sec(dx+c)+\tan(dx+c))}{d}$
parallelrisch	$\frac{2a^3xd+12a\,b^2dx+12a^2b\sin(dx+c)+a^3\sin(2dx+2c)+4\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)b^3-4\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)b^3}{4d}$
risch	$\frac{a^3x}{2}+3a\,b^2x-\frac{3ia^2b\,\mathrm{e}^{i(dx+c)}}{2d}+\frac{3ia^2b\,\mathrm{e}^{-i(dx+c)}}{2d}+\frac{b^3\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{d}-\frac{b^3\ln\left(\mathrm{e}^{i(dx+c)}-i\right)}{d}+\frac{a^3\sin(2dx+2c}{4d}$
norman	$\frac{\left(\frac{1}{2}a^3+3a\,b^2\right)x+\left(\frac{1}{2}a^3+3a\,b^2\right)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8+\left(-a^3-6a\,b^2\right)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\frac{a^2(a+6b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}-\frac{a^2(a-6b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^2\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^2}$

input
```
int(cos(d*x+c)^2*(a+b*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^3*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+3*a^2*b*sin(d*x+c)+3*a*
b^2*(d*x+c)+b^3*ln(sec(d*x+c)+tan(d*x+c)))
```

3.471.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.91

$$\int \cos^2(c+dx)(a+b\sec(c+dx))^3\,dx$$
$$= \frac{b^3\log\left(\sin\left(dx+c\right)+1\right)-b^3\log\left(-\sin\left(dx+c\right)+1\right)+(a^3+6\,ab^2)dx+(a^3\cos\left(dx+c\right)+6\,a^2b)\sin\left(dx+\right.}{2\,d}$$

input
```
integrate(cos(d*x+c)^2*(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/2*(b^3*log(sin(d*x + c) + 1) - b^3*log(-sin(d*x + c) + 1) + (a^3 + 6*a*b
^2)*d*x + (a^3*cos(d*x + c) + 6*a^2*b)*sin(d*x + c))/d
```

3.471.6 Sympy [F]

$$\int \cos^2(c+dx)(a+b\sec(c+dx))^3\,dx = \int (a+b\sec\left(c+dx\right))^3\cos^2\left(c+dx\right)dx$$

input
```
integrate(cos(d*x+c)**2*(a+b*sec(d*x+c))**3,x)
```

output
```
Integral((a + b*sec(c + d*x))**3*cos(c + d*x)**2, x)
```

3.471.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.96

$$\int \cos^2(c+dx)(a+b\sec(c+dx))^3\,dx$$
$$= \frac{(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right))a^3+12\,(dx+c)ab^2+2\,b^3(\log\left(\sin\left(dx+c\right)+1\right)-\log\left(\sin\left(dx+c\right)-1\right))+}{4\,d}$$

input
```
integrate(cos(d*x+c)^2*(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/4*((2*d*x + 2*c + sin(2*d*x + 2*c))*a^3 + 12*(d*x + c)*a*b^2 + 2*b^3*(lo
g(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 12*a^2*b*sin(d*x + c))/d
```

3.471.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.73

$$\int \cos^2(c+dx)(a+b\sec(c+dx))^3\,dx$$
$$= \frac{2\,b^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-2\,b^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+(a^3+6\,ab^2)(dx+c)-\frac{2\left(a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\right.\right.}{}}{2\,d}$$

input
```
integrate(cos(d*x+c)^2*(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
1/2*(2*b^3*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 2*b^3*log(abs(tan(1/2*d*x
+ 1/2*c) - 1)) + (a^3 + 6*a*b^2)*(d*x + c) - 2*(a^3*tan(1/2*d*x + 1/2*c)^3
 - 6*a^2*b*tan(1/2*d*x + 1/2*c)^3 - a^3*tan(1/2*d*x + 1/2*c) - 6*a^2*b*tan
(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^2)/d
```

3.471.9 Mupad [B] (verification not implemented)

Time = 13.59 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.56

$$\int \cos^2(c+dx)(a+b\sec(c+dx))^3\,dx = \frac{a^3\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{2\,b^3\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{a^3\,\sin\left(2\,c+2\,d\,x\right)}{4\,d}+\frac{3\,a^2\,b\,\sin\left(c+d\,x\right)}{d}+\frac{6\,a\,b^2\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}$$

input
```
int(cos(c + d*x)^2*(a + b/cos(c + d*x))^3,x)
```

output
```
(a^3*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (2*b^3*atanh(sin(c/2
 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (a^3*sin(2*c + 2*d*x))/(4*d) + (3*a^2
*b*sin(c + d*x))/d + (6*a*b^2*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))
/d
```

3.471. $\int \cos^2(c+dx)(a+b\sec(c+dx))^3\,dx$

3.472 $\int \cos^3(c+dx)(a+b\sec(c+dx))^3\,dx$

3.472.1 Optimal result . . . 3478
3.472.2 Mathematica [A] (verified) . . . 3478
3.472.3 Rubi [A] (verified) . . . 3479
3.472.4 Maple [A] (verified) . . . 3481
3.472.5 Fricas [A] (verification not implemented) . . . 3482
3.472.6 Sympy [F] . . . 3482
3.472.7 Maxima [A] (verification not implemented) . . . 3482
3.472.8 Giac [A] (verification not implemented) . . . 3483
3.472.9 Mupad [B] (verification not implemented) . . . 3483

3.472.1 Optimal result

Integrand size = 21, antiderivative size = 100

$$\int \cos^3(c+dx)(a+b\sec(c+dx))^3\,dx = \frac{1}{2}b(3a^2+2b^2)\,x + \frac{a(2a^2+9b^2)\sin(c+dx)}{3d} + \frac{7a^2b\cos(c+dx)\sin(c+dx)}{6d} + \frac{a^2\cos^2(c+dx)(a+b\sec(c+dx))\sin(c+dx)}{3d}$$

output
```
1/2*b*(3*a^2+2*b^2)*x+1/3*a*(2*a^2+9*b^2)*sin(d*x+c)/d+7/6*a^2*b*cos(d*x+c
)*sin(d*x+c)/d+1/3*a^2*cos(d*x+c)^2*(a+b*sec(d*x+c))*sin(d*x+c)/d
```

3.472.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.80

$$\int \cos^3(c+dx)(a+b\sec(c+dx))^3\,dx = \frac{18a^2bc+12b^3c+18a^2bdx+12b^3dx+9a(a^2+4b^2)\sin(c+dx)+9a^2b\sin(2(c+dx))+a^3\sin(3(c+dx))}{12d}$$

input
```
Integrate[Cos[c + d*x]^3*(a + b*Sec[c + d*x])^3,x]
```

output
```
(18*a^2*b*c + 12*b^3*c + 18*a^2*b*d*x + 12*b^3*d*x + 9*a*(a^2 + 4*b^2)*Sin
[c + d*x] + 9*a^2*b*Sin[2*(c + d*x)] + a^3*Sin[3*(c + d*x)])/(12*d)
```

3.472.3 Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.02, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {3042, 4328, 3042, 4535, 3042, 3117, 4533, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^3(c+dx)(a+b\sec(c+dx))^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^3}{\csc\left(c+dx+\frac{\pi}{2}\right)^3}dx$$

$$\Big\downarrow \text{4328}$$

$$\frac{1}{3}\int \cos^2(c+dx)\left(7ba^2+\left(2a^2+9b^2\right)\sec(c+dx)a+b\left(a^2+3b^2\right)\sec^2(c+dx)\right)dx+\frac{a^2\sin(c+dx)\cos^2(c+dx)(a+b\sec(c+dx))}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\int \frac{7ba^2+\left(2a^2+9b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)a+b\left(a^2+3b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^2}dx+\frac{a^2\sin(c+dx)\cos^2(c+dx)(a+b\sec(c+dx))}{3d}$$

$$\Big\downarrow \text{4535}$$

$$\frac{1}{3}\left(a\left(2a^2+9b^2\right)\int \cos(c+dx)dx+\int \cos^2(c+dx)\left(7ba^2+b\left(a^2+3b^2\right)\sec^2(c+dx)\right)dx\right)+\frac{a^2\sin(c+dx)\cos^2(c+dx)(a+b\sec(c+dx))}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\left(a\left(2a^2+9b^2\right)\int \sin\left(c+dx+\frac{\pi}{2}\right)dx+\int \frac{7ba^2+b\left(a^2+3b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^2}dx\right)+\frac{a^2\sin(c+dx)\cos^2(c+dx)(a+b\sec(c+dx))}{3d}$$

$$\Big\downarrow \text{3117}$$

$$\frac{1}{3}\left(\int \frac{7ba^2+b(a^2+3b^2)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^2}dx+\frac{a(2a^2+9b^2)\sin(c+dx)}{d}\right)+\frac{a^2\sin(c+dx)\cos^2(c+dx)(a+b\sec(c+dx))}{3d}$$

$$\Big\downarrow \; 4533$$

$$\frac{1}{3}\left(\frac{3}{2}b(3a^2+2b^2)\int 1dx+\frac{a(2a^2+9b^2)\sin(c+dx)}{d}+\frac{7a^2b\sin(c+dx)\cos(c+dx)}{2d}\right)+\frac{a^2\sin(c+dx)\cos^2(c+dx)(a+b\sec(c+dx))}{3d}$$

$$\Big\downarrow \; 24$$

$$\frac{1}{3}\left(\frac{a(2a^2+9b^2)\sin(c+dx)}{d}+\frac{3}{2}bx(3a^2+2b^2)+\frac{7a^2b\sin(c+dx)\cos(c+dx)}{2d}\right)+\frac{a^2\sin(c+dx)\cos^2(c+dx)(a+b\sec(c+dx))}{3d}$$

input
```
Int[Cos[c + d*x]^3*(a + b*Sec[c + d*x])^3,x]
```

output
```
(a^2*Cos[c + d*x]^2*(a + b*Sec[c + d*x])*Sin[c + d*x])/(3*d) + ((3*b*(3*a^
2 + 2*b^2)*x)/2 + (a*(2*a^2 + 9*b^2)*Sin[c + d*x])/d + (7*a^2*b*Cos[c + d*
x]*Sin[c + d*x])/(2*d))/3
```

3.472.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 4328
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)*
((d*Csc[e + f*x])^n/(f*n)), x] - Simp[1/(d*n)   Int[(a + b*Csc[e + f*x])^(m
 - 3)*(d*Csc[e + f*x])^(n + 1)*Simp[a^2*b*(m - 2*n - 2) - a*(3*b^2*n + a^2*
(n + 1))*Csc[e + f*x] - b*(b^2*n + a^2*(m + n - 1))*Csc[e + f*x]^2, x], x],
 x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && ((Int
egerQ[m] && LtQ[n, -1]) || (IntegersQ[m + 1/2, 2*n] && LeQ[n, -1]))
```

rule 4533
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] +
Simp[(C*m + A*(m + 1))/(b^2*m)   Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fr
eeQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

rule 4535
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Simp[B/b   Int[(b*Cs
c[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2)
, x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

3.472.4 Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.67

method	result
parallelrisch	$\frac{9a^2b\sin(2dx+2c)+a^3\sin(3dx+3c)+9(a^3+4a\,b^2)\sin(dx+c)+18d\left(a^2+\frac{2b^2}{3}\right)xb}{12d}$
derivativedivides	$\frac{\frac{a^3\left(2+\cos(dx+c)^2\right)\sin(dx+c)}{3}+3a^2b\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+3a\,b^2\sin(dx+c)+b^3(dx+c)}{d}$
default	$\frac{\frac{a^3\left(2+\cos(dx+c)^2\right)\sin(dx+c)}{3}+3a^2b\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+3a\,b^2\sin(dx+c)+b^3(dx+c)}{d}$
risch	$\frac{3a^2bx}{2}+b^3x+\frac{3a^3\sin(dx+c)}{4d}+\frac{3\sin(dx+c)a\,b^2}{d}+\frac{a^3\sin(3dx+3c)}{12d}+\frac{3a^2b\sin(2dx+2c)}{4d}$
norman	$\frac{(\frac{3}{2}a^2b+b^3)x+(\frac{3}{2}a^2b+b^3)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+(\frac{3}{2}a^2b+b^3)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8+(\frac{3}{2}a^2b+b^3)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}+(-3a^2b-2b^3)x\,\mathrm{ta}}{}$

input
```
int(cos(d*x+c)^3*(a+b*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/12*(9*a^2*b*sin(2*d*x+2*c)+a^3*sin(3*d*x+3*c)+9*(a^3+4*a*b^2)*sin(d*x+c)
+18*d*(a^2+2/3*b^2)*x*b)/d
```

3.472.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.66

$$\int \cos^3(c+dx)(a+b\sec(c+dx))^3\,dx = \frac{3\,(3\,a^2b+2\,b^3)dx+\left(2\,a^3\cos\left(dx+c\right)^2+9\,a^2b\cos\left(dx+c\right)+4\,a^3+18\,ab^2\right)\sin\left(dx+c\right)}{6\,d}$$

input
```
integrate(cos(d*x+c)^3*(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/6*(3*(3*a^2*b + 2*b^3)*d*x + (2*a^3*cos(d*x + c)^2 + 9*a^2*b*cos(d*x + c
) + 4*a^3 + 18*a*b^2)*sin(d*x + c))/d
```

3.472.6 Sympy [F]

$$\int \cos^3(c+dx)(a+b\sec(c+dx))^3\,dx = \int \left(a+b\sec\left(c+dx\right)\right)^3\cos^3\left(c+dx\right)dx$$

input
```
integrate(cos(d*x+c)**3*(a+b*sec(d*x+c))**3,x)
```

output
```
Integral((a + b*sec(c + d*x))**3*cos(c + d*x)**3, x)
```

3.472.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.73

$$\int \cos^3(c+dx)(a+b\sec(c+dx))^3\,dx = -\frac{4\left(\sin\left(dx+c\right)^3-3\,\sin\left(dx+c\right)\right)a^3-9\left(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right)\right)a^2b-12\left(dx+c\right)b^3-36\,ab^2\sin\left(d\right.}{12\,d}$$

input
```
integrate(cos(d*x+c)^3*(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
-1/12*(4*(sin(d*x + c)^3 - 3*sin(d*x + c))*a^3 - 9*(2*d*x + 2*c + sin(2*d*
x + 2*c))*a^2*b - 12*(d*x + c)*b^3 - 36*a*b^2*sin(d*x + c))/d
```

3.472.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.70

$$\int \cos^3(c+dx)(a+b\sec(c+dx))^3\,dx = \frac{3\,(3\,a^2b+2\,b^3)(dx+c)+\frac{2\left(6\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-9\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+18\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+4\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+36\,ab^2\tan\left(\frac{1}{2}\right.\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^3}}{6\,d}$$

input
```
integrate(cos(d*x+c)^3*(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
1/6*(3*(3*a^2*b + 2*b^3)*(d*x + c) + 2*(6*a^3*tan(1/2*d*x + 1/2*c)^5 - 9*a
^2*b*tan(1/2*d*x + 1/2*c)^5 + 18*a*b^2*tan(1/2*d*x + 1/2*c)^5 + 4*a^3*tan(
1/2*d*x + 1/2*c)^3 + 36*a*b^2*tan(1/2*d*x + 1/2*c)^3 + 6*a^3*tan(1/2*d*x +
 1/2*c) + 9*a^2*b*tan(1/2*d*x + 1/2*c) + 18*a*b^2*tan(1/2*d*x + 1/2*c))/(t
an(1/2*d*x + 1/2*c)^2 + 1)^3)/d
```

3.472.9 Mupad [B] (verification not implemented)

Time = 13.54 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.77

$$\int \cos^3(c+dx)(a+b\sec(c+dx))^3\,dx = b^3\,x+\frac{3\,a^3\,\sin\left(c+d\,x\right)}{4\,d}+\frac{a^3\,\sin\left(3\,c+3\,d\,x\right)}{12\,d}+\frac{3\,a^2\,b\,x}{2}+\frac{3\,a^2\,b\,\sin\left(2\,c+2\,d\,x\right)}{4\,d}+\frac{3\,a\,b^2\,\sin\left(c+d\,x\right)}{d}$$

input
```
int(cos(c + d*x)^3*(a + b/cos(c + d*x))^3,x)
```

output
```
b^3*x + (3*a^3*sin(c + d*x))/(4*d) + (a^3*sin(3*c + 3*d*x))/(12*d) + (3*a^
2*b*x)/2 + (3*a^2*b*sin(2*c + 2*d*x))/(4*d) + (3*a*b^2*sin(c + d*x))/d
```

3.473 $\int \cos^4(c+dx)(a+b\sec(c+dx))^3\,dx$

3.473.1 Optimal result . . . 3484
3.473.2 Mathematica [A] (verified) . . . 3484
3.473.3 Rubi [A] (verified) . . . 3485
3.473.4 Maple [A] (verified) . . . 3488
3.473.5 Fricas [A] (verification not implemented) . . . 3489
3.473.6 Sympy [F] . . . 3489
3.473.7 Maxima [A] (verification not implemented) . . . 3489
3.473.8 Giac [B] (verification not implemented) . . . 3490
3.473.9 Mupad [B] (verification not implemented) . . . 3490

3.473.1 Optimal result

Integrand size = 21, antiderivative size = 123

$$\int \cos^4(c+dx)(a+b\sec(c+dx))^3\,dx = \frac{3}{8}a\left(a^2+4b^2\right)x+\frac{b(11a^2+4b^2)\sin(c+dx)}{4d}+\frac{3a(a^2+4b^2)\cos(c+dx)\sin(c+dx)}{8d}+\frac{a^2\cos^3(c+dx)(a+b\sec(c+dx))\sin(c+dx)}{4d}-\frac{3a^2b\sin^3(c+dx)}{4d}$$

output
```
3/8*a*(a^2+4*b^2)*x+1/4*b*(11*a^2+4*b^2)*sin(d*x+c)/d+3/8*a*(a^2+4*b^2)*co
s(d*x+c)*sin(d*x+c)/d+1/4*a^2*cos(d*x+c)^3*(a+b*sec(d*x+c))*sin(d*x+c)/d-3
/4*a^2*b*sin(d*x+c)^3/d
```

3.473.2 Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.81

$$\int \cos^4(c+dx)(a+b\sec(c+dx))^3\,dx = \frac{8b(9a^2+4b^2)\sin(c+dx)+a(12a^2c+48b^2c+12a^2dx+48b^2dx+8(a^2+3b^2)\sin(2(c+dx))+8ab\sin(3($$

input
```
Integrate[Cos[c + d*x]^4*(a + b*Sec[c + d*x])^3,x]
```

output
```
(8*b*(9*a^2 + 4*b^2)*Sin[c + d*x] + a*(12*a^2*c + 48*b^2*c + 12*a^2*d*x +
48*b^2*d*x + 8*(a^2 + 3*b^2)*Sin[2*(c + d*x)] + 8*a*b*Sin[3*(c + d*x)] + a
^2*Sin[4*(c + d*x)]))/(32*d)
```

3.473.3 Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.95, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.524$, Rules used = {3042, 4328, 3042, 4535, 3042, 3115, 24, 4532, 3042, 3492, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^4(c+dx)(a+b\sec(c+dx))^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^3}{\csc\left(c+dx+\frac{\pi}{2}\right)^4}dx$$

$$\Big\downarrow \text{4328}$$

$$\frac{1}{4}\int \cos^3(c+dx)\left(9ba^2+3\left(a^2+4b^2\right)\sec(c+dx)a+2b\left(a^2+2b^2\right)\sec^2(c+dx)\right)dx+\frac{a^2\sin(c+dx)\cos^3(c+dx)(a+b\sec(c+dx))}{4d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}\int \frac{9ba^2+3\left(a^2+4b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)a+2b\left(a^2+2b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^3}dx+\frac{a^2\sin(c+dx)\cos^3(c+dx)(a+b\sec(c+dx))}{4d}$$

$$\Big\downarrow \text{4535}$$

$$\frac{1}{4}\left(3a\left(a^2+4b^2\right)\int \cos^2(c+dx)dx+\int \cos^3(c+dx)\left(9ba^2+2b\left(a^2+2b^2\right)\sec^2(c+dx)\right)dx\right)+\frac{a^2\sin(c+dx)\cos^3(c+dx)(a+b\sec(c+dx))}{4d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}\left(3a\left(a^2+4b^2\right)\int \sin\left(c+dx+\frac{\pi}{2}\right)^2dx+\int \frac{9ba^2+2b\left(a^2+2b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^3}dx\right)+\frac{a^2\sin(c+dx)\cos^3(c+dx)(a+b\sec(c+dx))}{4d}$$

$\Big\downarrow$ 3115

$$\frac{1}{4}\left(\int \frac{9ba^2+2b\left(a^2+2b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^3}dx+3a\left(a^2+4b^2\right)\left(\frac{\int 1dx}{2}+\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)\right)+\frac{a^2\sin(c+dx)\cos^3(c+dx)(a+b\sec(c+dx))}{4d}$$

$\Big\downarrow$ 24

$$\frac{1}{4}\left(\int \frac{9ba^2+2b\left(a^2+2b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^3}dx+3a\left(a^2+4b^2\right)\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)\right)+\frac{a^2\sin(c+dx)\cos^3(c+dx)(a+b\sec(c+dx))}{4d}$$

$\Big\downarrow$ 4532

$$\frac{1}{4}\left(\int \cos(c+dx)\left(9a^2b\cos^2(c+dx)+2b\left(a^2+2b^2\right)\right)dx+3a\left(a^2+4b^2\right)\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)\right)+\frac{a^2\sin(c+dx)\cos^3(c+dx)(a+b\sec(c+dx))}{4d}$$

$\Big\downarrow$ 3042

$$\frac{1}{4}\left(\int \sin\left(c+dx+\frac{\pi}{2}\right)\left(9a^2b\sin\left(c+dx+\frac{\pi}{2}\right)^2+2b\left(a^2+2b^2\right)\right)dx+3a\left(a^2+4b^2\right)\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\right.\right.\frac{a^2\sin(c+dx)\cos^3(c+dx)(a+b\sec(c+dx))}{4d}$$

$\Big\downarrow$ 3492

$$\frac{1}{4}\left(3a\left(a^2+4b^2\right)\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)-\frac{\int\left(b\left(11a^2+4b^2\right)-9a^2b\sin^2(c+dx)\right)d(-\sin(c+dx))}{d}\right)+\frac{a^2\sin(c+dx)\cos^3(c+dx)(a+b\sec(c+dx))}{4d}$$

$\Big\downarrow$ 2009

$$\frac{1}{4}\left(3a\left(a^2+4b^2\right)\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)-\frac{3a^2b\sin^3(c+dx)-b\left(11a^2+4b^2\right)\sin(c+dx)}{d}\right)+\frac{a^2\sin(c+dx)\cos^3(c+dx)(a+b\sec(c+dx))}{4d}$$

3.473. $\int \cos^4(c+dx)(a+b\sec(c+dx))^3\,dx$

input
```
Int[Cos[c + d*x]^4*(a + b*Sec[c + d*x])^3,x]
```

output
```
(a^2*Cos[c + d*x]^3*(a + b*Sec[c + d*x])*Sin[c + d*x])/(4*d) + (3*a*(a^2 +
 4*b^2)*(x/2 + (Cos[c + d*x]*Sin[c + d*x])/(2*d)) - (-(b*(11*a^2 + 4*b^2)*
Sin[c + d*x]) + 3*a^2*b*Sin[c + d*x]^3)/d)/4
```

3.473.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3492
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2),
 x_Symbol] :> Simp[-f^(-1)   Subst[Int[(1 - x^2)^((m - 1)/2)*(A + C - C*x^2
), x], x, Cos[e + f*x]], x] /; FreeQ[{e, f, A, C}, x] && IGtQ[(m + 1)/2, 0]
```

rule 4328
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)*
((d*Csc[e + f*x])^n/(f*n)), x] - Simp[1/(d*n)   Int[(a + b*Csc[e + f*x])^(m
 - 3)*(d*Csc[e + f*x])^(n + 1)*Simp[a^2*b*(m - 2*n - 2) - a*(3*b^2*n + a^2*
(n + 1))*Csc[e + f*x] - b*(b^2*n + a^2*(m + n - 1))*Csc[e + f*x]^2, x], x],
 x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && ((Int
egerQ[m] && LtQ[n, -1]) || (IntegersQ[m + 1/2, 2*n] && LeQ[n, -1]))
```

rule 4532
```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.) + (A_)),
 x_Symbol] :> Int[(C + A*Sin[e + f*x]^2)/Sin[e + f*x]^(m + 2), x] /; FreeQ[
{e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && ILtQ[(m + 1)/2, 0]
```

rule 4535
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Simp[B/b   Int[(b*Cs
c[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2)
, x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

3.473.4 Maple [A] (verified)

Time = 0.85 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.73

method	result
parallelrisch	$\frac{8(a^3+3a\,b^2)\sin(2dx+2c)+8a^2b\sin(3dx+3c)+a^3\sin(4dx+4c)+8(9a^2b+4b^3)\sin(dx+c)+12a(a^2+4b^2)xd}{32d}$
derivativedivides	$\frac{a^3\left(\frac{\left(\cos(dx+c)^3+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+a^2b\left(2+\cos(dx+c)^2\right)\sin(dx+c)+3a\,b^2\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\right.}{d}$
default	$\frac{a^3\left(\frac{\left(\cos(dx+c)^3+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+a^2b\left(2+\cos(dx+c)^2\right)\sin(dx+c)+3a\,b^2\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\right.}{d}$
risch	$\frac{3a^3x}{8}+\frac{3a\,b^2x}{2}+\frac{9a^2b\sin(dx+c)}{4d}+\frac{\sin(dx+c)b^3}{d}+\frac{a^3\sin(4dx+4c)}{32d}+\frac{a^2b\sin(3dx+3c)}{4d}+\frac{a^3\sin(2dx+2c)}{4d}+$
norman	$\frac{(\frac{3}{8}a^3+\frac{3}{2}a\,b^2)x+(-\frac{3}{2}a^3-6a\,b^2)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6+(-\frac{3}{8}a^3-\frac{3}{2}a\,b^2)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4+(-\frac{3}{8}a^3-\frac{3}{2}a\,b^2)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8+(\frac{3}{4}a^3}{}$

input
```
int(cos(d*x+c)^4*(a+b*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/32*(8*(a^3+3*a*b^2)*sin(2*d*x+2*c)+8*a^2*b*sin(3*d*x+3*c)+a^3*sin(4*d*x+
4*c)+8*(9*a^2*b+4*b^3)*sin(d*x+c)+12*a*(a^2+4*b^2)*x*d)/d
```

3.473.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.68

$$\int \cos^4(c+dx)(a+b\sec(c+dx))^3\,dx$$
$$= \frac{3\,(a^3+4\,ab^2)dx + \left(2\,a^3\cos\left(dx+c\right)^3 + 8\,a^2b\cos\left(dx+c\right)^2 + 16\,a^2b + 8\,b^3 + 3\,(a^3+4\,ab^2)\cos\left(dx+c\right)\right)\sin}{8\,d}$$

input
```
integrate(cos(d*x+c)^4*(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/8*(3*(a^3 + 4*a*b^2)*d*x + (2*a^3*cos(d*x + c)^3 + 8*a^2*b*cos(d*x + c)^
2 + 16*a^2*b + 8*b^3 + 3*(a^3 + 4*a*b^2)*cos(d*x + c))*sin(d*x + c))/d
```

3.473.6 Sympy [F]

$$\int \cos^4(c+dx)(a+b\sec(c+dx))^3\,dx = \int (a+b\sec(c+dx))^3\cos^4(c+dx)\,dx$$

input
```
integrate(cos(d*x+c)**4*(a+b*sec(d*x+c))**3,x)
```

output
```
Integral((a + b*sec(c + d*x))**3*cos(c + d*x)**4, x)
```

3.473.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.77

$$\int \cos^4(c+dx)(a+b\sec(c+dx))^3\,dx$$
$$= \frac{(12\,dx+12\,c+\sin\left(4\,dx+4\,c\right)+8\,\sin\left(2\,dx+2\,c\right))a^3 - 32\left(\sin\left(dx+c\right)^3 - 3\,\sin\left(dx+c\right)\right)a^2b + 24\,(2\,dx}{32\,d}$$

input
```
integrate(cos(d*x+c)^4*(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/32*((12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*a^3 - 32*(si
n(d*x + c)^3 - 3*sin(d*x + c))*a^2*b + 24*(2*d*x + 2*c + sin(2*d*x + 2*c))
*a*b^2 + 32*b^3*sin(d*x + c))/d
```

3.473.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 297 vs. $2(113) = 226$.

Time = 0.32 (sec) , antiderivative size = 297, normalized size of antiderivative = 2.41

$$\int \cos^4(c+dx)(a+b\sec(c+dx))^3\,dx$$
$$= \frac{3\,(a^3+4\,ab^2)(dx+c) - \frac{2\left(5\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-24\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+12\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-8\,b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-3\,a^3\tan\left(\frac{1}{2}\,dx\right.}{}}{}$$

input
```
integrate(cos(d*x+c)^4*(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
1/8*(3*(a^3 + 4*a*b^2)*(d*x + c) - 2*(5*a^3*tan(1/2*d*x + 1/2*c)^7 - 24*a^
2*b*tan(1/2*d*x + 1/2*c)^7 + 12*a*b^2*tan(1/2*d*x + 1/2*c)^7 - 8*b^3*tan(1
/2*d*x + 1/2*c)^7 - 3*a^3*tan(1/2*d*x + 1/2*c)^5 - 40*a^2*b*tan(1/2*d*x +
1/2*c)^5 + 12*a*b^2*tan(1/2*d*x + 1/2*c)^5 - 24*b^3*tan(1/2*d*x + 1/2*c)^5
 + 3*a^3*tan(1/2*d*x + 1/2*c)^3 - 40*a^2*b*tan(1/2*d*x + 1/2*c)^3 - 12*a*b
^2*tan(1/2*d*x + 1/2*c)^3 - 24*b^3*tan(1/2*d*x + 1/2*c)^3 - 5*a^3*tan(1/2*
d*x + 1/2*c) - 24*a^2*b*tan(1/2*d*x + 1/2*c) - 12*a*b^2*tan(1/2*d*x + 1/2*
c) - 8*b^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^4)/d
```

3.473.9 Mupad [B] (verification not implemented)

Time = 16.57 (sec) , antiderivative size = 250, normalized size of antiderivative = 2.03

$$\int \cos^4(c+dx)(a+b\sec(c+dx))^3\,dx$$
$$= \frac{\left(-\frac{5\,a^3}{4}+6\,a^2\,b-3\,a\,b^2+2\,b^3\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+\left(\frac{3\,a^3}{4}+10\,a^2\,b-3\,a\,b^2+6\,b^3\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+\left(-\frac{3\,a^3}{4}+\right.}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+6\tan\left(\frac{c}{2}+\frac{d}{2}\right.\right.}$$
$$+\frac{3\,a\,\mathrm{atan}\left(\frac{3\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,(a^2+4\,b^2)}{4\,\left(\frac{3\,a^3}{4}+3\,a\,b^2\right)}\right)\,(a^2+4\,b^2)}{4\,d}$$

input
```
int(cos(c + d*x)^4*(a + b/cos(c + d*x))^3,x)
```

output
```
(tan(c/2 + (d*x)/2)^3*(3*a*b^2 + 10*a^2*b - (3*a^3)/4 + 6*b^3) - tan(c/2 +
 (d*x)/2)^7*(3*a*b^2 - 6*a^2*b + (5*a^3)/4 - 2*b^3) + tan(c/2 + (d*x)/2)^5
*(10*a^2*b - 3*a*b^2 + (3*a^3)/4 + 6*b^3) + tan(c/2 + (d*x)/2)*(3*a*b^2 +
6*a^2*b + (5*a^3)/4 + 2*b^3))/(d*(4*tan(c/2 + (d*x)/2)^2 + 6*tan(c/2 + (d*
x)/2)^4 + 4*tan(c/2 + (d*x)/2)^6 + tan(c/2 + (d*x)/2)^8 + 1)) + (3*a*atan(
(3*a*tan(c/2 + (d*x)/2)*(a^2 + 4*b^2))/(4*(3*a*b^2 + (3*a^3)/4)))*(a^2 + 4
*b^2))/(4*d)
```

3.473. $\int \cos^4(c+dx)(a+b\sec(c+dx))^3\,dx$

3.474 $\int \cos^5(c+dx)(a+b\sec(c+dx))^3\,dx$

3.474.1 Optimal result . . . 3492
3.474.2 Mathematica [A] (verified) . . . 3493
3.474.3 Rubi [A] (verified) . . . 3493
3.474.4 Maple [A] (verified) . . . 3496
3.474.5 Fricas [A] (verification not implemented) . . . 3497
3.474.6 Sympy [F(-1)] . . . 3498
3.474.7 Maxima [A] (verification not implemented) . . . 3498
3.474.8 Giac [B] (verification not implemented) . . . 3498
3.474.9 Mupad [B] (verification not implemented) . . . 3499

3.474.1 Optimal result

Integrand size = 21, antiderivative size = 160

$$\begin{aligned}\int \cos^5(c+dx)(a+b\sec(c+dx))^3\,dx &= \frac{1}{8}b(9a^2+4b^2)\,x+\frac{a(4a^2+15b^2)\sin(c+dx)}{5d}\\&+\frac{b(9a^2+4b^2)\cos(c+dx)\sin(c+dx)}{8d}\\&+\frac{11a^2b\cos^3(c+dx)\sin(c+dx)}{20d}\\&+\frac{a^2\cos^4(c+dx)(a+b\sec(c+dx))\sin(c+dx)}{5d}\\&-\frac{a(4a^2+15b^2)\sin^3(c+dx)}{15d}\end{aligned}$$

output
```
1/8*b*(9*a^2+4*b^2)*x+1/5*a*(4*a^2+15*b^2)*sin(d*x+c)/d+1/8*b*(9*a^2+4*b^2
)*cos(d*x+c)*sin(d*x+c)/d+11/20*a^2*b*cos(d*x+c)^3*sin(d*x+c)/d+1/5*a^2*co
s(d*x+c)^4*(a+b*sec(d*x+c))*sin(d*x+c)/d-1/15*a*(4*a^2+15*b^2)*sin(d*x+c)^
3/d
```

3.474.2 Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.81

$$\int \cos^5(c+dx)(a+b\sec(c+dx))^3\,dx = \frac{540a^2bc+240b^3c+540a^2bdx+240b^3dx+60a(5a^2+18b^2)\sin(c+dx)+120(3a^2b+b^3)\sin(2(c+dx))+}{480d}$$

input
```
Integrate[Cos[c + d*x]^5*(a + b*Sec[c + d*x])^3,x]
```

output
```
(540*a^2*b*c + 240*b^3*c + 540*a^2*b*d*x + 240*b^3*d*x + 60*a*(5*a^2 + 18*
b^2)*Sin[c + d*x] + 120*(3*a^2*b + b^3)*Sin[2*(c + d*x)] + 50*a^3*Sin[3*(c
 + d*x)] + 120*a*b^2*Sin[3*(c + d*x)] + 45*a^2*b*Sin[4*(c + d*x)] + 6*a^3*
Sin[5*(c + d*x)])/(480*d)
```

3.474.3 Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.90, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.524$, Rules used = {3042, 4328, 3042, 4535, 3042, 3113, 2009, 4533, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^5(c+dx)(a+b\sec(c+dx))^3\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^3}{\csc\left(c+dx+\frac{\pi}{2}\right)^5}dx$$

$$\Big\downarrow \; 4328$$

$$\frac{1}{5}\int \cos^4(c+dx)\left(11ba^2+\left(4a^2+15b^2\right)\sec(c+dx)a+b\left(3a^2+5b^2\right)\sec^2(c+dx)\right)dx+\frac{a^2\sin(c+dx)\cos^4(c+dx)(a+b\sec(c+dx))}{5d}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{5}\int \frac{11ba^2+\left(4a^2+15b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)a+b\left(3a^2+5b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^4}dx+\frac{a^2\sin(c+dx)\cos^4(c+dx)(a+b\sec(c+dx))}{5d}$$

$$\Big\downarrow \text{4535}$$

$$\frac{1}{5}\left(a\left(4a^2+15b^2\right)\int\cos^3(c+dx)dx+\int\cos^4(c+dx)\left(11ba^2+b\left(3a^2+5b^2\right)\sec^2(c+dx)\right)dx\right)+\frac{a^2\sin(c+dx)\cos^4(c+dx)(a+b\sec(c+dx))}{5d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}\left(a\left(4a^2+15b^2\right)\int\sin\left(c+dx+\frac{\pi}{2}\right)^3dx+\int\frac{11ba^2+b\left(3a^2+5b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^4}dx\right)+\frac{a^2\sin(c+dx)\cos^4(c+dx)(a+b\sec(c+dx))}{5d}$$

$$\Big\downarrow \text{3113}$$

$$\frac{1}{5}\left(\int\frac{11ba^2+b\left(3a^2+5b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^4}dx-\frac{a\left(4a^2+15b^2\right)\int\left(1-\sin^2(c+dx)\right)d(-\sin(c+dx))}{d}\right)+\frac{a^2\sin(c+dx)\cos^4(c+dx)(a+b\sec(c+dx))}{5d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{5}\left(\int\frac{11ba^2+b\left(3a^2+5b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^4}dx-\frac{a\left(4a^2+15b^2\right)\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}\right)+\frac{a^2\sin(c+dx)\cos^4(c+dx)(a+b\sec(c+dx))}{5d}$$

$$\Big\downarrow \text{4533}$$

$$\frac{1}{5}\left(\frac{5}{4}b\left(9a^2+4b^2\right)\int\cos^2(c+dx)dx-\frac{a\left(4a^2+15b^2\right)\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}+\frac{11a^2b\sin(c+dx)\cos^3(c+\right.$$
$$\frac{a^2\sin(c+dx)\cos^4(c+dx)(a+b\sec(c+dx))}{5d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}\left(\frac{5}{4}b\left(9a^2+4b^2\right)\int\sin\left(c+dx+\frac{\pi}{2}\right)^2dx-\frac{a\left(4a^2+15b^2\right)\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}+\frac{11a^2b\sin(c+dx)\cos}{4d}\right.$$
$$\frac{a^2\sin(c+dx)\cos^4(c+dx)(a+b\sec(c+dx))}{5d}$$

3.474. $\int\cos^5(c+dx)(a+b\sec(c+dx))^3\,dx$

$$\Big\downarrow \text{3115}$$

$$\frac{1}{5}\left(\frac{5}{4}b\left(9a^2+4b^2\right)\left(\frac{\int 1dx}{2}+\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)-\frac{a\left(4a^2+15b^2\right)\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}+\frac{11a^2b\sin}{}\right.$$
$$\frac{a^2\sin(c+dx)\cos^4(c+dx)(a+b\sec(c+dx))}{5d}$$

$$\Big\downarrow \text{24}$$

$$\frac{1}{5}\left(-\frac{a\left(4a^2+15b^2\right)\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}+\frac{5}{4}b\left(9a^2+4b^2\right)\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)+\frac{11a^2b\sin(}{}\right.$$
$$\frac{a^2\sin(c+dx)\cos^4(c+dx)(a+b\sec(c+dx))}{5d}$$

input
```
Int[Cos[c + d*x]^5*(a + b*Sec[c + d*x])^3,x]
```

output
```
(a^2*Cos[c + d*x]^4*(a + b*Sec[c + d*x])*Sin[c + d*x])/(5*d) + ((11*a^2*b*
Cos[c + d*x]^3*Sin[c + d*x])/(4*d) + (5*b*(9*a^2 + 4*b^2)*(x/2 + (Cos[c +
d*x]*Sin[c + d*x])/(2*d)))/4 - (a*(4*a^2 + 15*b^2)*(-Sin[c + d*x] + Sin[c
+ d*x]^3/3))/d)/5
```

3.474.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 4328
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)*
((d*Csc[e + f*x])^n/(f*n)), x] - Simp[1/(d*n)   Int[(a + b*Csc[e + f*x])^(m
 - 3)*(d*Csc[e + f*x])^(n + 1)*Simp[a^2*b*(m - 2*n - 2) - a*(3*b^2*n + a^2*
(n + 1))*Csc[e + f*x] - b*(b^2*n + a^2*(m + n - 1))*Csc[e + f*x]^2, x], x],
 x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && ((Int
egerQ[m] && LtQ[n, -1]) || (IntegersQ[m + 1/2, 2*n] && LeQ[n, -1]))
```

rule 4533
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] +
Simp[(C*m + A*(m + 1))/(b^2*m)   Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fr
eeQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

rule 4535
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Simp[B/b   Int[(b*Cs
c[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2)
, x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

3.474.4 Maple [A] (verified)

Time = 1.08 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.71

method	result
parallelrisch	$\frac{(360a^2b+120b^3)\sin(2dx+2c)+(50a^3+120a\,b^2)\sin(3dx+3c)+45a^2b\sin(4dx+4c)+6a^3\sin(5dx+5c)+(300a^3+1080a\,b^2)}{480d}$
derivativedivides	$\frac{\frac{a^3\left(\frac{8}{3}+\cos(dx+c)^4+\frac{4\cos(dx+c)^2}{3}\right)\sin(dx+c)}{5}+3a^2b\left(\frac{\left(\cos(dx+c)^3+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+a\,b^2\left(2+\cos(dx+c)\right.}{d}$
default	$\frac{\frac{a^3\left(\frac{8}{3}+\cos(dx+c)^4+\frac{4\cos(dx+c)^2}{3}\right)\sin(dx+c)}{5}+3a^2b\left(\frac{\left(\cos(dx+c)^3+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+a\,b^2\left(2+\cos(dx+c)\right.}{d}$
risch	$\frac{9a^2bx}{8}+\frac{b^3x}{2}+\frac{5a^3\sin(dx+c)}{8d}+\frac{9\sin(dx+c)a\,b^2}{4d}+\frac{a^3\sin(5dx+5c)}{80d}+\frac{3a^2b\sin(4dx+4c)}{32d}+\frac{5a^3\sin(3dx+3c)}{48d}+$
norman	$\frac{(\frac{9}{8}a^2b+\frac{1}{2}b^3)x+(-\frac{45}{8}a^2b-\frac{5}{2}b^3)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6+(-\frac{45}{8}a^2b-\frac{5}{2}b^3)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8+(\frac{9}{8}a^2b+\frac{1}{2}b^3)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4+(\frac{9}{8}a^2b}{}$

input
```
int(cos(d*x+c)^5*(a+b*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/480*((360*a^2*b+120*b^3)*sin(2*d*x+2*c)+(50*a^3+120*a*b^2)*sin(3*d*x+3*c
)+45*a^2*b*sin(4*d*x+4*c)+6*a^3*sin(5*d*x+5*c)+(300*a^3+1080*a*b^2)*sin(d*
x+c)+540*d*x*b*(a^2+4/9*b^2))/d
```

3.474.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.69

$$\int \cos^5(c+dx)(a+b\sec(c+dx))^3\,dx$$
$$=\frac{15\,(9\,a^2b+4\,b^3)dx+\left(24\,a^3\cos\left(dx+c\right)^4+90\,a^2b\cos\left(dx+c\right)^3+64\,a^3+240\,ab^2+8\,(4\,a^3+15\,ab^2)\cos\left(c\right.\right.}{120\,d}$$

input
```
integrate(cos(d*x+c)^5*(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/120*(15*(9*a^2*b + 4*b^3)*d*x + (24*a^3*cos(d*x + c)^4 + 90*a^2*b*cos(d*
x + c)^3 + 64*a^3 + 240*a*b^2 + 8*(4*a^3 + 15*a*b^2)*cos(d*x + c)^2 + 15*(
9*a^2*b + 4*b^3)*cos(d*x + c))*sin(d*x + c))/d
```

3.474.6 Sympy [F(-1)]

Timed out.

$$\int \cos^5(c+dx)(a+b\sec(c+dx))^3\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**5*(a+b*sec(d*x+c))**3,x)
```

output
```
Timed out
```

3.474.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.74

$$\int \cos^5(c+dx)(a+b\sec(c+dx))^3\,dx$$
$$= \frac{32\left(3\sin(dx+c)^5 - 10\sin(dx+c)^3 + 15\sin(dx+c)\right)a^3 + 45\left(12\,dx + 12\,c + \sin(4\,dx+4\,c) + 8\sin(2\right.}{480\,d}$$

input
```
integrate(cos(d*x+c)^5*(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/480*(32*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*a^3 + 4
5*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*a^2*b - 480*(sin
(d*x + c)^3 - 3*sin(d*x + c))*a*b^2 + 120*(2*d*x + 2*c + sin(2*d*x + 2*c))
*b^3)/d
```

3.474.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 332 vs. 2(148) = 296.

Time = 0.32 (sec) , antiderivative size = 332, normalized size of antiderivative = 2.08

$$\int \cos^5(c+dx)(a+b\sec(c+dx))^3\,dx$$
$$= 15\left(9\,a^2b + 4\,b^3\right)(dx+c) + \frac{2\left(120\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9 - 225\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9 + 360\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9 - 60\,b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9 + 160\right.}{}$$

input
```
integrate(cos(d*x+c)^5*(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
1/120*(15*(9*a^2*b + 4*b^3)*(d*x + c) + 2*(120*a^3*tan(1/2*d*x + 1/2*c)^9
- 225*a^2*b*tan(1/2*d*x + 1/2*c)^9 + 360*a*b^2*tan(1/2*d*x + 1/2*c)^9 - 60
*b^3*tan(1/2*d*x + 1/2*c)^9 + 160*a^3*tan(1/2*d*x + 1/2*c)^7 - 90*a^2*b*ta
n(1/2*d*x + 1/2*c)^7 + 960*a*b^2*tan(1/2*d*x + 1/2*c)^7 - 120*b^3*tan(1/2*
d*x + 1/2*c)^7 + 464*a^3*tan(1/2*d*x + 1/2*c)^5 + 1200*a*b^2*tan(1/2*d*x +
 1/2*c)^5 + 160*a^3*tan(1/2*d*x + 1/2*c)^3 + 90*a^2*b*tan(1/2*d*x + 1/2*c)
^3 + 960*a*b^2*tan(1/2*d*x + 1/2*c)^3 + 120*b^3*tan(1/2*d*x + 1/2*c)^3 + 1
20*a^3*tan(1/2*d*x + 1/2*c) + 225*a^2*b*tan(1/2*d*x + 1/2*c) + 360*a*b^2*t
an(1/2*d*x + 1/2*c) + 60*b^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2
 + 1)^5)/d
```

3.474.9 Mupad [B] (verification not implemented)

Time = 16.28 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.79

$$\int \cos^5(c+dx)(a+b\sec(c+dx))^3\,dx$$
$$= \frac{\left(2\,a^3-\frac{15\,a^2\,b}{4}+6\,a\,b^2-b^3\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9+\left(\frac{8\,a^3}{3}-\frac{3\,a^2\,b}{2}+16\,a\,b^2-2\,b^3\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+\left(\frac{116\,a^3}{15}+20\,a\right.}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}+5\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+10\,\tan\left(\frac{c}{2}\right.\right.}$$
$$+\frac{b\,\mathrm{atan}\left(\frac{b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(9\,a^2+4\,b^2\right)}{4\,\left(\frac{9\,a^2\,b}{4}+b^3\right)}\right)\,\left(9\,a^2+4\,b^2\right)}{4\,d}$$

input
```
int(cos(c + d*x)^5*(a + b/cos(c + d*x))^3,x)
```

output
```
(tan(c/2 + (d*x)/2)^3*(16*a*b^2 + (3*a^2*b)/2 + (8*a^3)/3 + 2*b^3) + tan(c
/2 + (d*x)/2)^9*(6*a*b^2 - (15*a^2*b)/4 + 2*a^3 - b^3) + tan(c/2 + (d*x)/2
)^7*(16*a*b^2 - (3*a^2*b)/2 + (8*a^3)/3 - 2*b^3) + tan(c/2 + (d*x)/2)*(6*a
*b^2 + (15*a^2*b)/4 + 2*a^3 + b^3) + tan(c/2 + (d*x)/2)^5*(20*a*b^2 + (116
*a^3)/15))/(d*(5*tan(c/2 + (d*x)/2)^2 + 10*tan(c/2 + (d*x)/2)^4 + 10*tan(c
/2 + (d*x)/2)^6 + 5*tan(c/2 + (d*x)/2)^8 + tan(c/2 + (d*x)/2)^10 + 1)) + (
b*atan((b*tan(c/2 + (d*x)/2)*(9*a^2 + 4*b^2))/(4*((9*a^2*b)/4 + b^3)))*(9*
a^2 + 4*b^2))/(4*d)
```

3.475 $\int \cos^6(c+dx)(a+b\sec(c+dx))^3\,dx$

3.475.1 Optimal result . . . 3500
3.475.2 Mathematica [A] (verified) . . . 3501
3.475.3 Rubi [A] (verified) . . . 3501
3.475.4 Maple [A] (verified) . . . 3505
3.475.5 Fricas [A] (verification not implemented) . . . 3506
3.475.6 Sympy [F(-1)] . . . 3506
3.475.7 Maxima [A] (verification not implemented) . . . 3507
3.475.8 Giac [B] (verification not implemented) . . . 3507
3.475.9 Mupad [B] (verification not implemented) . . . 3508

3.475.1 Optimal result

Integrand size = 21, antiderivative size = 185

$$\begin{aligned}\int \cos^6(c+dx)(a+b\sec(c+dx))^3\,dx &= \frac{1}{16}a\left(5a^2+18b^2\right)x+\frac{b(17a^2+6b^2)\sin(c+dx)}{6d}\\&+\frac{a(5a^2+18b^2)\cos(c+dx)\sin(c+dx)}{16d}\\&+\frac{a(5a^2+18b^2)\cos^3(c+dx)\sin(c+dx)}{24d}\\&+\frac{a^2\cos^5(c+dx)(a+b\sec(c+dx))\sin(c+dx)}{6d}\\&-\frac{b(5a^2+b^2)\sin^3(c+dx)}{3d}+\frac{13a^2b\sin^5(c+dx)}{30d}\end{aligned}$$

output
```
1/16*a*(5*a^2+18*b^2)*x+1/6*b*(17*a^2+6*b^2)*sin(d*x+c)/d+1/16*a*(5*a^2+18
*b^2)*cos(d*x+c)*sin(d*x+c)/d+1/24*a*(5*a^2+18*b^2)*cos(d*x+c)^3*sin(d*x+c
)/d+1/6*a^2*cos(d*x+c)^5*(a+b*sec(d*x+c))*sin(d*x+c)/d-1/3*b*(5*a^2+b^2)*s
in(d*x+c)^3/d+13/30*a^2*b*sin(d*x+c)^5/d
```

3.475.2 Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.86

$$\int \cos^6(c+dx)(a+b\sec(c+dx))^3\,dx$$
$$= \frac{300a^3c+1080ab^2c+300a^3dx+1080ab^2dx+360b(5a^2+2b^2)\sin(c+dx)+45(5a^3+16ab^2)\sin(2(c+dx}{}$$

input
```
Integrate[Cos[c + d*x]^6*(a + b*Sec[c + d*x])^3,x]
```

output
```
(300*a^3*c + 1080*a*b^2*c + 300*a^3*d*x + 1080*a*b^2*d*x + 360*b*(5*a^2 +
2*b^2)*Sin[c + d*x] + 45*(5*a^3 + 16*a*b^2)*Sin[2*(c + d*x)] + 300*a^2*b*S
in[3*(c + d*x)] + 80*b^3*Sin[3*(c + d*x)] + 45*a^3*Sin[4*(c + d*x)] + 90*a
*b^2*Sin[4*(c + d*x)] + 36*a^2*b*Sin[5*(c + d*x)] + 5*a^3*Sin[6*(c + d*x)]
)/(960*d)
```

3.475.3 Rubi [A] (verified)

Time = 0.96 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.89, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {3042, 4328, 3042, 4535, 3042, 3115, 3042, 3115, 24, 4532, 3042, 3492, 27, 290, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^6(c+dx)(a+b\sec(c+dx))^3\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^3}{\csc\left(c+dx+\frac{\pi}{2}\right)^6}dx$$

$$\Big\downarrow \; 4328$$

$$\frac{1}{6}\int \cos^5(c+dx)\left(13ba^2+\left(5a^2+18b^2\right)\sec(c+dx)a+2b\left(2a^2+3b^2\right)\sec^2(c+dx)\right)dx+\frac{a^2\sin(c+dx)\cos^5(c+dx)(a+b\sec(c+dx))}{6d}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{6}\int \frac{13ba^2+\left(5a^2+18b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)a+2b\left(2a^2+3b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^5}dx+\frac{a^2\sin(c+dx)\cos^5(c+dx)(a+b\sec(c+dx))}{6d}$$

$\Big\downarrow$ 4535

$$\frac{1}{6}\left(a\left(5a^2+18b^2\right)\int\cos^4(c+dx)dx+\int\cos^5(c+dx)\left(13ba^2+2b\left(2a^2+3b^2\right)\sec^2(c+dx)\right)dx\right)+\frac{a^2\sin(c+dx)\cos^5(c+dx)(a+b\sec(c+dx))}{6d}$$

$\Big\downarrow$ 3042

$$\frac{1}{6}\left(a\left(5a^2+18b^2\right)\int\sin\left(c+dx+\frac{\pi}{2}\right)^4dx+\int\frac{13ba^2+2b\left(2a^2+3b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^5}dx\right)+\frac{a^2\sin(c+dx)\cos^5(c+dx)(a+b\sec(c+dx))}{6d}$$

$\Big\downarrow$ 3115

$$\frac{1}{6}\left(\int\frac{13ba^2+2b\left(2a^2+3b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^5}dx+a\left(5a^2+18b^2\right)\left(\frac{3}{4}\int\cos^2(c+dx)dx+\frac{\sin(c+dx)\cos^3(c+d}{4d}\right.\right.\frac{a^2\sin(c+dx)\cos^5(c+dx)(a+b\sec(c+dx))}{6d}$$

$\Big\downarrow$ 3042

$$\frac{1}{6}\left(\int\frac{13ba^2+2b\left(2a^2+3b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^5}dx+a\left(5a^2+18b^2\right)\left(\frac{3}{4}\int\sin\left(c+dx+\frac{\pi}{2}\right)^2dx+\frac{\sin(c+dx)\cos^3}{4d}\right.\right.\frac{a^2\sin(c+dx)\cos^5(c+dx)(a+b\sec(c+dx))}{6d}$$

$\Big\downarrow$ 3115

$$\frac{1}{6}\left(\int\frac{13ba^2+2b\left(2a^2+3b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^5}dx+a\left(5a^2+18b^2\right)\left(\frac{3}{4}\left(\frac{\int 1dx}{2}+\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)+\frac{\sin(}{}\right.\right.\frac{a^2\sin(c+dx)\cos^5(c+dx)(a+b\sec(c+dx))}{6d}$$

$\Big\downarrow$ 24

$$\frac{1}{6}\left(\int\frac{13ba^2+2b\left(2a^2+3b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^5}dx+a\left(5a^2+18b^2\right)\left(\frac{\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3}{4}\left(\frac{\sin(c+dx)\,\mathrm{co}}{2d}\right.\right.\right.\frac{a^2\sin(c+dx)\cos^5(c+dx)(a+b\sec(c+dx))}{6d}$$

3.475. $\int\cos^6(c+dx)(a+b\sec(c+dx))^3\,dx$

$$\Big\downarrow \; 4532$$

$$\frac{1}{6}\left(\int \cos^3(c+dx)\left(13a^2b\cos^2(c+dx)+2b\left(2a^2+3b^2\right)\right)dx+a\left(5a^2+18b^2\right)\left(\frac{\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3}{4}\left(\frac{\sin(\right.\right.\right.$$
$$\frac{a^2\sin(c+dx)\cos^5(c+dx)(a+b\sec(c+dx))}{6d}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{6}\left(\int \sin\left(c+dx+\frac{\pi}{2}\right)^3\left(13a^2b\sin\left(c+dx+\frac{\pi}{2}\right)^2+2b\left(2a^2+3b^2\right)\right)dx+a\left(5a^2+18b^2\right)\left(\frac{\sin(c+dx)\cos^3(c+d\right.\right.$$
$$\frac{a^2\sin(c+dx)\cos^5(c+dx)(a+b\sec(c+dx))}{6d}$$

$$\Big\downarrow \; 3492$$

$$\frac{1}{6}\left(a\left(5a^2+18b^2\right)\left(\frac{\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3}{4}\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)\right)-\frac{\int b\left(1-\sin^2(c+dx)\right)\left(-13\,s\right.}{}\right.$$
$$\frac{a^2\sin(c+dx)\cos^5(c+dx)(a+b\sec(c+dx))}{6d}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{6}\left(a\left(5a^2+18b^2\right)\left(\frac{\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3}{4}\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)\right)-\frac{b\int\left(1-\sin^2(c+dx)\right)\left(-13\,s\right.}{}\right.$$
$$\frac{a^2\sin(c+dx)\cos^5(c+dx)(a+b\sec(c+dx))}{6d}$$

$$\Big\downarrow \; 290$$

$$\frac{1}{6}\left(a\left(5a^2+18b^2\right)\left(\frac{\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3}{4}\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)\right)-\frac{b\int\left(13a^2\sin^4(c+dx)-6\left(5\right.\right.}{}\right.$$
$$\frac{a^2\sin(c+dx)\cos^5(c+dx)(a+b\sec(c+dx))}{6d}$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{6}\left(a\left(5a^2+18b^2\right)\left(\frac{\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3}{4}\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)\right)-\frac{b\left(2\left(5a^2+b^2\right)\sin^3(c+dx)-\right.}{}\right.$$
$$\frac{a^2\sin(c+dx)\cos^5(c+dx)(a+b\sec(c+dx))}{6d}$$

3.475. $\int \cos^6(c+dx)(a+b\sec(c+dx))^3\,dx$

input
```
Int[Cos[c + d*x]^6*(a + b*Sec[c + d*x])^3,x]
```

output
```
(a^2*Cos[c + d*x]^5*(a + b*Sec[c + d*x])*Sin[c + d*x])/(6*d) + (-((b*(-((1
7*a^2 + 6*b^2)*Sin[c + d*x]) + 2*(5*a^2 + b^2)*Sin[c + d*x]^3 - (13*a^2*Si
n[c + d*x]^5)/5))/d) + a*(5*a^2 + 18*b^2)*((Cos[c + d*x]^3*Sin[c + d*x])/(
4*d) + (3*(x/2 + (Cos[c + d*x]*Sin[c + d*x])/(2*d)))/4))/6
```

3.475.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 290
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol] :> I
nt[ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d
}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3492
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2),
 x_Symbol] :> Simp[-f^(-1)   Subst[Int[(1 - x^2)^((m - 1)/2)*(A + C - C*x^2
), x], x, Cos[e + f*x]], x] /; FreeQ[{e, f, A, C}, x] && IGtQ[(m + 1)/2, 0]
```

rule 4328
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)*
((d*Csc[e + f*x])^n/(f*n)), x] - Simp[1/(d*n)   Int[(a + b*Csc[e + f*x])^(m
 - 3)*(d*Csc[e + f*x])^(n + 1)*Simp[a^2*b*(m - 2*n - 2) - a*(3*b^2*n + a^2*
(n + 1))*Csc[e + f*x] - b*(b^2*n + a^2*(m + n - 1))*Csc[e + f*x]^2, x], x],
 x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && ((Int
egerQ[m] && LtQ[n, -1]) || (IntegersQ[m + 1/2, 2*n] && LeQ[n, -1]))
```

rule 4532
```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.) + (A_)),
 x_Symbol] :> Int[(C + A*Sin[e + f*x]^2)/Sin[e + f*x]^(m + 2), x] /; FreeQ[
{e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && ILtQ[(m + 1)/2, 0]
```

rule 4535
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Simp[B/b   Int[(b*Cs
c[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2)
, x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

3.475.4 Maple [A] (verified)

Time = 1.53 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.73

method	result
parallelrisch	$\frac{(225a^3+720a\,b^2)\sin(2dx+2c)+(300a^2b+80b^3)\sin(3dx+3c)+(45a^3+90a\,b^2)\sin(4dx+4c)+36a^2b\sin(5dx+5c)+5a^3\sin}{960d}$
derivativedivides	$\frac{a^3\left(\frac{\left(\cos(dx+c)^5+\frac{5\cos(dx+c)^3}{4}+\frac{15\cos(dx+c)}{8}\right)\sin(dx+c)}{6}+\frac{5dx}{16}+\frac{5c}{16}\right)+\frac{3a^2b\left(\frac{8}{3}+\cos(dx+c)^4+\frac{4\cos(dx+c)^2}{3}\right)\sin(dx+c)}{5}+3a\,b}{d}$
default	$\frac{a^3\left(\frac{\left(\cos(dx+c)^5+\frac{5\cos(dx+c)^3}{4}+\frac{15\cos(dx+c)}{8}\right)\sin(dx+c)}{6}+\frac{5dx}{16}+\frac{5c}{16}\right)+\frac{3a^2b\left(\frac{8}{3}+\cos(dx+c)^4+\frac{4\cos(dx+c)^2}{3}\right)\sin(dx+c)}{5}+3a\,b}{d}$
risch	$\frac{5a^3x}{16}+\frac{9a\,b^2x}{8}+\frac{15a^2b\sin(dx+c)}{8d}+\frac{3\sin(dx+c)b^3}{4d}+\frac{a^3\sin(6dx+6c)}{192d}+\frac{3a^2b\sin(5dx+5c)}{80d}+\frac{3a^3\sin(4dx+4c)}{64d}$
norman	$\frac{(\frac{5}{16}a^3+\frac{9}{8}a\,b^2)x+(-\frac{25}{8}a^3-\frac{45}{4}a\,b^2)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8+(-\frac{5}{4}a^3-\frac{9}{2}a\,b^2)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6+(-\frac{5}{4}a^3-\frac{9}{2}a\,b^2)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}+}{}$

input
```
int(cos(d*x+c)^6*(a+b*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/960*((225*a^3+720*a*b^2)*sin(2*d*x+2*c)+(300*a^2*b+80*b^3)*sin(3*d*x+3*c
)+(45*a^3+90*a*b^2)*sin(4*d*x+4*c)+36*a^2*b*sin(5*d*x+5*c)+5*a^3*sin(6*d*x
+6*c)+(1800*a^2*b+720*b^3)*sin(d*x+c)+300*d*(a^2+18/5*b^2)*x*a)/d
```

3.475.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.71

$$\int \cos^6(c+dx)(a+b\sec(c+dx))^3\,dx$$
$$= \frac{15\,(5\,a^3+18\,ab^2)dx+\left(40\,a^3\cos\left(dx+c\right)^5+144\,a^2b\cos\left(dx+c\right)^4+10\,(5\,a^3+18\,ab^2)\cos\left(dx+c\right)^3+384\right.}{240\,d}$$

input
```
integrate(cos(d*x+c)^6*(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/240*(15*(5*a^3 + 18*a*b^2)*d*x + (40*a^3*cos(d*x + c)^5 + 144*a^2*b*cos(
d*x + c)^4 + 10*(5*a^3 + 18*a*b^2)*cos(d*x + c)^3 + 384*a^2*b + 160*b^3 +
16*(12*a^2*b + 5*b^3)*cos(d*x + c)^2 + 15*(5*a^3 + 18*a*b^2)*cos(d*x + c))
*sin(d*x + c))/d
```

3.475.6 Sympy [F(-1)]

Timed out.

$$\int \cos^6(c+dx)(a+b\sec(c+dx))^3\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**6*(a+b*sec(d*x+c))**3,x)
```

output
```
Timed out
```

3.475.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.78

$$\int \cos^6(c+dx)(a+b\sec(c+dx))^3\,dx = -\frac{5\left(4\sin\left(2\,dx+2\,c\right)^3-60\,dx-60\,c-9\sin\left(4\,dx+4\,c\right)-48\sin\left(2\,dx+2\,c\right)\right)a^3-192\left(3\sin\left(dx+c\right)^5\right.}{}$$

input
```
integrate(cos(d*x+c)^6*(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
-1/960*(5*(4*sin(2*d*x + 2*c)^3 - 60*d*x - 60*c - 9*sin(4*d*x + 4*c) - 48*
sin(2*d*x + 2*c))*a^3 - 192*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin
(d*x + c))*a^2*b - 90*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*
c))*a*b^2 + 320*(sin(d*x + c)^3 - 3*sin(d*x + c))*b^3)/d
```

3.475.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 431 vs. 2(171) = 342.

Time = 0.32 (sec) , antiderivative size = 431, normalized size of antiderivative = 2.33

$$\int \cos^6(c+dx)(a+b\sec(c+dx))^3\,dx = 15\left(5\,a^3+18\,ab^2\right)(dx+c)-\frac{2\left(165\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}-720\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}+450\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}-240\,b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{1}\right.}{}$$

input
```
integrate(cos(d*x+c)^6*(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
1/240*(15*(5*a^3 + 18*a*b^2)*(d*x + c) - 2*(165*a^3*tan(1/2*d*x + 1/2*c)^1
1 - 720*a^2*b*tan(1/2*d*x + 1/2*c)^11 + 450*a*b^2*tan(1/2*d*x + 1/2*c)^11
- 240*b^3*tan(1/2*d*x + 1/2*c)^11 - 25*a^3*tan(1/2*d*x + 1/2*c)^9 - 1680*a
^2*b*tan(1/2*d*x + 1/2*c)^9 + 630*a*b^2*tan(1/2*d*x + 1/2*c)^9 - 880*b^3*t
an(1/2*d*x + 1/2*c)^9 + 450*a^3*tan(1/2*d*x + 1/2*c)^7 - 3744*a^2*b*tan(1/
2*d*x + 1/2*c)^7 + 180*a*b^2*tan(1/2*d*x + 1/2*c)^7 - 1440*b^3*tan(1/2*d*x
 + 1/2*c)^7 - 450*a^3*tan(1/2*d*x + 1/2*c)^5 - 3744*a^2*b*tan(1/2*d*x + 1/
2*c)^5 - 180*a*b^2*tan(1/2*d*x + 1/2*c)^5 - 1440*b^3*tan(1/2*d*x + 1/2*c)^
5 + 25*a^3*tan(1/2*d*x + 1/2*c)^3 - 1680*a^2*b*tan(1/2*d*x + 1/2*c)^3 - 63
0*a*b^2*tan(1/2*d*x + 1/2*c)^3 - 880*b^3*tan(1/2*d*x + 1/2*c)^3 - 165*a^3*
tan(1/2*d*x + 1/2*c) - 720*a^2*b*tan(1/2*d*x + 1/2*c) - 450*a*b^2*tan(1/2*
d*x + 1/2*c) - 240*b^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^
6)/d
```

3.475.9 Mupad [B] (verification not implemented)

Time = 15.67 (sec) , antiderivative size = 350, normalized size of antiderivative = 1.89

$$\int \cos^6(c+dx)(a+b\sec(c+dx))^3\,dx$$
$$= \frac{\left(-\frac{11\,a^3}{8}+6\,a^2\,b-\frac{15\,a\,b^2}{4}+2\,b^3\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}+\left(\frac{5\,a^3}{24}+14\,a^2\,b-\frac{21\,a\,b^2}{4}+\frac{22\,b^3}{3}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9+\left(-\frac{15\,a^3}{4}\right.}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}+\right.}$$
$$+\frac{a\,\mathrm{atan}\left(\frac{a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(5\,a^2+18\,b^2\right)}{8\,\left(\frac{5\,a^3}{8}+\frac{9\,a\,b^2}{4}\right)}\right)\,\left(5\,a^2+18\,b^2\right)}{8\,d}$$

input
```
int(cos(c + d*x)^6*(a + b/cos(c + d*x))^3,x)
```

output
```
(tan(c/2 + (d*x)/2)^3*((21*a*b^2)/4 + 14*a^2*b - (5*a^3)/24 + (22*b^3)/3)
- tan(c/2 + (d*x)/2)^11*((15*a*b^2)/4 - 6*a^2*b + (11*a^3)/8 - 2*b^3) + ta
n(c/2 + (d*x)/2)^9*(14*a^2*b - (21*a*b^2)/4 + (5*a^3)/24 + (22*b^3)/3) + t
an(c/2 + (d*x)/2)^5*((3*a*b^2)/2 + (156*a^2*b)/5 + (15*a^3)/4 + 12*b^3) -
tan(c/2 + (d*x)/2)^7*((3*a*b^2)/2 - (156*a^2*b)/5 + (15*a^3)/4 - 12*b^3) +
 tan(c/2 + (d*x)/2)*((15*a*b^2)/4 + 6*a^2*b + (11*a^3)/8 + 2*b^3))/(d*(6*t
an(c/2 + (d*x)/2)^2 + 15*tan(c/2 + (d*x)/2)^4 + 20*tan(c/2 + (d*x)/2)^6 +
15*tan(c/2 + (d*x)/2)^8 + 6*tan(c/2 + (d*x)/2)^10 + tan(c/2 + (d*x)/2)^12
+ 1)) + (a*atan((a*tan(c/2 + (d*x)/2)*(5*a^2 + 18*b^2))/(8*((9*a*b^2)/4 +
(5*a^3)/8)))*(5*a^2 + 18*b^2))/(8*d)
```

3.476 $\int \sec^3(c+dx)(a+b\sec(c+dx))^4\,dx$

3.476.1 Optimal result . . . 3509
3.476.2 Mathematica [A] (verified) . . . 3510
3.476.3 Rubi [A] (verified) . . . 3510
3.476.4 Maple [A] (verified) . . . 3515
3.476.5 Fricas [A] (verification not implemented) . . . 3515
3.476.6 Sympy [F] . . . 3516
3.476.7 Maxima [A] (verification not implemented) . . . 3516
3.476.8 Giac [B] (verification not implemented) . . . 3517
3.476.9 Mupad [B] (verification not implemented) . . . 3517

3.476.1 Optimal result

Integrand size = 21, antiderivative size = 244

$$\begin{aligned}\int \sec^3(c+dx)(a+b\sec(c+dx))^4\,dx = {} & \frac{(8a^4+36a^2b^2+5b^4)\operatorname{arctanh}(\sin(c+dx))}{16d} \\ & - \frac{a(4a^4-121a^2b^2-128b^4)\tan(c+dx)}{60bd} \\ & - \frac{(8a^4-178a^2b^2-75b^4)\sec(c+dx)\tan(c+dx)}{240d} \\ & - \frac{a(4a^2-53b^2)\,(a+b\sec(c+dx))^2\tan(c+dx)}{120bd} \\ & - \frac{(4a^2-25b^2)\,(a+b\sec(c+dx))^3\tan(c+dx)}{120bd} \\ & - \frac{a(a+b\sec(c+dx))^4\tan(c+dx)}{30bd} \\ & + \frac{(a+b\sec(c+dx))^5\tan(c+dx)}{6bd}\end{aligned}$$

output
```
1/16*(8*a^4+36*a^2*b^2+5*b^4)*arctanh(sin(d*x+c))/d-1/60*a*(4*a^4-121*a^2*
b^2-128*b^4)*tan(d*x+c)/b/d-1/240*(8*a^4-178*a^2*b^2-75*b^4)*sec(d*x+c)*ta
n(d*x+c)/d-1/120*a*(4*a^2-53*b^2)*(a+b*sec(d*x+c))^2*tan(d*x+c)/b/d-1/120*
(4*a^2-25*b^2)*(a+b*sec(d*x+c))^3*tan(d*x+c)/b/d-1/30*a*(a+b*sec(d*x+c))^4
*tan(d*x+c)/b/d+1/6*(a+b*sec(d*x+c))^5*tan(d*x+c)/b/d
```

3.476.2 Mathematica [A] (verified)

Time = 0.68 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.63

$$\int \sec^3(c+dx)(a+b\sec(c+dx))^4\,dx$$
$$= \frac{15(8a^4+36a^2b^2+5b^4)\operatorname{arctanh}(\sin(c+dx))+\tan(c+dx)\left(15(8a^4+36a^2b^2+5b^4)\sec(c+dx)+10b^2(36\right.}{}$$

input
```
Integrate[Sec[c + d*x]^3*(a + b*Sec[c + d*x])^4,x]
```

output
```
(15*(8*a^4 + 36*a^2*b^2 + 5*b^4)*ArcTanh[Sin[c + d*x]] + Tan[c + d*x]*(15*
(8*a^4 + 36*a^2*b^2 + 5*b^4)*Sec[c + d*x] + 10*b^2*(36*a^2 + 5*b^2)*Sec[c
+ d*x]^3 + 40*b^4*Sec[c + d*x]^5 + 64*a*b*(15*(a^2 + b^2) + 5*(a^2 + 2*b^2
)*Tan[c + d*x]^2 + 3*b^2*Tan[c + d*x]^4)))/(240*d)
```

3.476.3 Rubi [A] (verified)

Time = 1.55 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.06, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.810$, Rules used = {3042, 4327, 3042, 4490, 3042, 4490, 27, 3042, 4490, 3042, 4485, 3042, 4274, 3042, 4254, 24, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^3(c+dx)(a+b\sec(c+dx))^4\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^3\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^4dx$$

$$\Big\downarrow \text{4327}$$

$$\frac{\int \sec(c+dx)(5b-a\sec(c+dx))(a+b\sec(c+dx))^4dx}{6b}+\frac{\tan(c+dx)(a+b\sec(c+dx))^5}{6bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \csc\left(c+dx+\frac{\pi}{2}\right)\left(5b-a\csc\left(c+dx+\frac{\pi}{2}\right)\right)\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^4dx}{6b}+\frac{\tan(c+dx)(a+b\sec(c+dx))^5}{6bd}$$

$$\Big\downarrow \; 4490$$

$$\frac{\frac{1}{5}\int \sec(c+dx)(a+b\sec(c+dx))^3\left(21ab-\left(4a^2-25b^2\right)\sec(c+dx)\right)dx-\frac{a\tan(c+dx)(a+b\sec(c+dx))^4}{5d}}{6b}+\frac{\tan(c+dx)(a+b\sec(c+dx))^5}{6bd}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{1}{5}\int \csc\left(c+dx+\frac{\pi}{2}\right)\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^3\left(21ab+\left(25b^2-4a^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)\right)dx-\frac{a\tan(c+dx)(a+b\sec(c+dx)}{5d}}{6b}\frac{\tan(c+dx)(a+b\sec(c+dx))^5}{6bd}$$

$$\Big\downarrow \; 4490$$

$$\frac{\frac{1}{5}\left(\frac{1}{4}\int 3\sec(c+dx)(a+b\sec(c+dx))^2\left(b\left(24a^2+25b^2\right)-a\left(4a^2-53b^2\right)\sec(c+dx)\right)dx-\frac{\left(4a^2-25b^2\right)\tan(c+dx)(a+b}{4d}\right.}{6b}\frac{\tan(c+dx)(a+b\sec(c+dx))^5}{6bd}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{1}{5}\left(\frac{3}{4}\int \sec(c+dx)(a+b\sec(c+dx))^2\left(b\left(24a^2+25b^2\right)-a\left(4a^2-53b^2\right)\sec(c+dx)\right)dx-\frac{\left(4a^2-25b^2\right)\tan(c+dx)(a+b\,\mathrm{se}}{4d}\right.}{6b}\frac{\tan(c+dx)(a+b\sec(c+dx))^5}{6bd}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{1}{5}\left(\frac{3}{4}\int \csc\left(c+dx+\frac{\pi}{2}\right)\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2\left(b\left(24a^2+25b^2\right)-a\left(4a^2-53b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)\right)dx-\frac{\left(4a^2-25\right.}{}\right.}{6b}\frac{\tan(c+dx)(a+b\sec(c+dx))^5}{6bd}$$

$$\Big\downarrow \; 4490$$

$$\frac{\frac{1}{5}\left(\frac{3}{4}\left(\frac{1}{3}\int \sec(c+dx)(a+b\sec(c+dx))\left(ab\left(64a^2+181b^2\right)-\left(8a^4-178b^2a^2-75b^4\right)\sec(c+dx)\right)dx-\frac{a\left(4a^2-53b^2\right.}{}\right.\right.}{6b}\frac{\tan(c+dx)(a+b\sec(c+dx))^5}{6bd}$$

$$\Big\downarrow \; 3042$$

3.476. $\int \sec^3(c+dx)(a+b\sec(c+dx))^4\,dx$

$$\frac{\frac{1}{5}\left(\frac{3}{4}\left(\frac{1}{3}\int \csc\left(c+dx+\frac{\pi}{2}\right)\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)\left(ab\left(64a^2+181b^2\right)+\left(-8a^4+178b^2a^2+75b^4\right)\csc\left(c+dx+\frac{\pi}{2}\right.\right.\right.\right.}{6b}$$

$$\frac{\tan(c+dx)(a+b\sec(c+dx))^5}{6bd}$$

$\downarrow$ 4485

$$\frac{\frac{1}{5}\left(\frac{3}{4}\left(\frac{1}{3}\left(\frac{1}{2}\int \sec(c+dx)\left(15b\left(8a^4+36b^2a^2+5b^4\right)-4a\left(4a^4-121b^2a^2-128b^4\right)\sec(c+dx)\right)dx-\frac{b\left(8a^4-178a^2b^2-75\right.}{}\right.\right.\right.}{}$$

$$\frac{\tan(c+dx)(a+b\sec(c+dx))^5}{6bd}$$

$\downarrow$ 3042

$$\frac{\frac{1}{5}\left(\frac{3}{4}\left(\frac{1}{3}\left(\frac{1}{2}\int \csc\left(c+dx+\frac{\pi}{2}\right)\left(15b\left(8a^4+36b^2a^2+5b^4\right)-4a\left(4a^4-121b^2a^2-128b^4\right)\csc\left(c+dx+\frac{\pi}{2}\right)\right)dx-\frac{b\left(8a^4\right.}{}\right.\right.\right.}{}$$

$$\frac{\tan(c+dx)(a+b\sec(c+dx))^5}{6bd}$$

$\downarrow$ 4274

$$\frac{\frac{1}{5}\left(\frac{3}{4}\left(\frac{1}{3}\left(\frac{1}{2}\left(15b\left(8a^4+36a^2b^2+5b^4\right)\int \sec(c+dx)dx-4a\left(4a^4-121a^2b^2-128b^4\right)\int \sec^2(c+dx)dx\right)-\frac{b\left(8a^4-178a^2\right.}{}\right.\right.\right.}{}$$

$$\frac{\tan(c+dx)(a+b\sec(c+dx))^5}{6bd}$$

$\downarrow$ 3042

$$\frac{\frac{1}{5}\left(\frac{3}{4}\left(\frac{1}{3}\left(\frac{1}{2}\left(15b\left(8a^4+36a^2b^2+5b^4\right)\int \csc\left(c+dx+\frac{\pi}{2}\right)dx-4a\left(4a^4-121a^2b^2-128b^4\right)\int \csc\left(c+dx+\frac{\pi}{2}\right)^2dx\right)-\right.\right.\right.}{}$$

$$\frac{\tan(c+dx)(a+b\sec(c+dx))^5}{6bd}$$

$\downarrow$ 4254

$$\frac{\frac{1}{5}\left(\frac{3}{4}\left(\frac{1}{3}\left(\frac{1}{2}\left(\frac{4a\left(4a^4-121a^2b^2-128b^4\right)\int 1d(-\tan(c+dx))}{d}+15b\left(8a^4+36a^2b^2+5b^4\right)\int \csc\left(c+dx+\frac{\pi}{2}\right)dx\right)-\frac{b\left(8a^4-178a^2b^2-\right.}{}\right.\right.\right.}{}$$

$$\frac{\tan(c+dx)(a+b\sec(c+dx))^5}{6bd}$$

$\downarrow$ 24

3.476. $\int \sec^3(c+dx)(a+b\sec(c+dx))^4\,dx$

$$\frac{\frac{1}{5}\left(\frac{3}{4}\left(\frac{1}{3}\left(\frac{1}{2}\left(15b\left(8a^4+36a^2b^2+5b^4\right)\int \csc\left(c+dx+\frac{\pi}{2}\right)dx-\frac{4a\left(4a^4-121a^2b^2-128b^4\right)\tan(c+dx)}{d}\right)-\frac{b\left(8a^4-178a^2b^2-75b^4\right)\text{ta}}{2d}\right.\right.\right.}{6bd}$$

$$\frac{\tan(c+dx)(a+b\sec(c+dx))^5}{6bd}$$

$$\Big\downarrow \; 4257$$

$$\frac{\frac{1}{5}\left(\frac{3}{4}\left(\frac{1}{3}\left(\frac{1}{2}\left(\frac{15b\left(8a^4+36a^2b^2+5b^4\right)\text{arctanh}(\sin(c+dx))}{d}-\frac{4a\left(4a^4-121a^2b^2-128b^4\right)\tan(c+dx)}{d}\right)-\frac{b\left(8a^4-178a^2b^2-75b^4\right)\tan(c+dx)\sec}{2d}\right.\right.\right.}{6b}$$

$$\frac{\tan(c+dx)(a+b\sec(c+dx))^5}{6bd}$$

input
```
Int[Sec[c + d*x]^3*(a + b*Sec[c + d*x])^4,x]
```

output
```
((a + b*Sec[c + d*x])^5*Tan[c + d*x])/(6*b*d) + (-1/5*(a*(a + b*Sec[c + d*
x])^4*Tan[c + d*x])/d + (-1/4*((4*a^2 - 25*b^2)*(a + b*Sec[c + d*x])^3*Tan
[c + d*x])/d + (3*(-1/3*(a*(4*a^2 - 53*b^2)*(a + b*Sec[c + d*x])^2*Tan[c +
 d*x])/d + (-1/2*(b*(8*a^4 - 178*a^2*b^2 - 75*b^4)*Sec[c + d*x]*Tan[c + d*
x])/d + ((15*b*(8*a^4 + 36*a^2*b^2 + 5*b^4)*ArcTanh[Sin[c + d*x]])/d - (4*
a*(4*a^4 - 121*a^2*b^2 - 128*b^4)*Tan[c + d*x])/d)/2)/3))/4)/5)/(6*b)
```

3.476.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4327
```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(-Cot[e + f*x])*((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2
))), x] + Simp[1/(b*(m + 2))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(b*(
m + 1) - a*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 -
b^2, 0] &&  !LtQ[m, -1]
```

rule 4485
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*Cot[
e + f*x]*((d*Csc[e + f*x])^n/(f*(n + 1))), x] + Simp[1/(n + 1)   Int[(d*Csc
[e + f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x
], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[
n, -1]
```

rule 4490
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*((
a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Simp[1/(m + 1)   Int[Csc[e + f*x]*
(a + b*Csc[e + f*x])^(m - 1)*Simp[b*B*m + a*A*(m + 1) + (a*B*m + A*b*(m + 1
))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*
B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

3.476. $\int \sec^3(c+dx)(a+b\sec(c+dx))^4\,dx$

3.476.4 Maple [A] (verified)

Time = 4.21 (sec) , antiderivative size = 209, normalized size of antiderivative = 0.86

method	result
derivativedivides	$\frac{a^4\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)-4a^3b\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right)\tan(dx+c)+6a^2b^2\left(-\left(-\frac{\sec(dx+c)^3}{4}-\frac{3\sec(}{}\right.\right.}{}$
default	$\frac{a^4\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)-4a^3b\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right)\tan(dx+c)+6a^2b^2\left(-\left(-\frac{\sec(dx+c)^3}{4}-\frac{3\sec(}{}\right.\right.}{}$
parts	$\frac{a^4\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)}{d}+\frac{b^4\left(-\left(-\frac{\sec(dx+c)^5}{6}-\frac{5\sec(dx+c)^3}{24}-\frac{5\sec(dx+c)}{16}\right)\tan(dx+c)+\frac{5\ln}{}\right.}{d}$
parallelrisch	$\frac{-1800\left(\frac{2}{3}+\frac{\cos(6dx+6c)}{15}+\frac{2\cos(4dx+4c)}{5}+\cos(2dx+2c)\right)\left(a^4+\frac{9}{2}a^2b^2+\frac{5}{8}b^4\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+1800\left(\frac{2}{3}+\frac{\cos(6dx+6c)}{15}+\right.}{}$
norman	$\frac{\left(8a^4-64a^3b+60a^2b^2-64a\,b^3+11b^4\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{11}}{8d}+\frac{\left(8a^4+64a^3b+60a^2b^2+64a\,b^3+11b^4\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8d}+\frac{\left(40a^4-960a^3b+60a^2b^2\right.}{}$
risch	$-\frac{i\left(-640a^3b-512a\,b^3-2520a^2b^2e^{5i(dx+c)}-7680a^3b\,e^{4i(dx+c)}-3840a^3b\,e^{2i(dx+c)}-3060a^2b^2e^{3i(dx+c)}-7680a\,b^3e^{4i(dx}\right.}{}$

input
```
int(sec(d*x+c)^3*(a+b*sec(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^4*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c)))-4*a^3*b
*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c)+6*a^2*b^2*(-(-1/4*sec(d*x+c)^3-3/8*sec
(d*x+c))*tan(d*x+c)+3/8*ln(sec(d*x+c)+tan(d*x+c)))-4*a*b^3*(-8/15-1/5*sec(
d*x+c)^4-4/15*sec(d*x+c)^2)*tan(d*x+c)+b^4*(-(-1/6*sec(d*x+c)^5-5/24*sec(d
*x+c)^3-5/16*sec(d*x+c))*tan(d*x+c)+5/16*ln(sec(d*x+c)+tan(d*x+c))))
```

3.476.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 217, normalized size of antiderivative = 0.89

$$\int \sec^3(c+dx)(a+b\sec(c+dx))^4\,dx$$
$$=\frac{15\left(8\,a^4+36\,a^2b^2+5\,b^4\right)\cos\left(dx+c\right)^6\log\left(\sin\left(dx+c\right)+1\right)-15\left(8\,a^4+36\,a^2b^2+5\,b^4\right)\cos\left(dx+c\right)^6\log\left(\right.}{}$$

input
```
integrate(sec(d*x+c)^3*(a+b*sec(d*x+c))^4,x, algorithm="fricas")
```

output
```
1/480*(15*(8*a^4 + 36*a^2*b^2 + 5*b^4)*cos(d*x + c)^6*log(sin(d*x + c) + 1
) - 15*(8*a^4 + 36*a^2*b^2 + 5*b^4)*cos(d*x + c)^6*log(-sin(d*x + c) + 1)
+ 2*(128*(5*a^3*b + 4*a*b^3)*cos(d*x + c)^5 + 192*a*b^3*cos(d*x + c) + 15*
(8*a^4 + 36*a^2*b^2 + 5*b^4)*cos(d*x + c)^4 + 40*b^4 + 64*(5*a^3*b + 4*a*b
^3)*cos(d*x + c)^3 + 10*(36*a^2*b^2 + 5*b^4)*cos(d*x + c)^2)*sin(d*x + c))
/(d*cos(d*x + c)^6)
```

3.476.6 Sympy [F]

$$\int \sec^3(c+dx)(a+b\sec(c+dx))^4\,dx = \int (a+b\sec(c+dx))^4\sec^3(c+dx)\,dx$$

input
```
integrate(sec(d*x+c)**3*(a+b*sec(d*x+c))**4,x)
```

output
```
Integral((a + b*sec(c + d*x))**4*sec(c + d*x)**3, x)
```

3.476.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.13

$$\int \sec^3(c+dx)(a+b\sec(c+dx))^4\,dx$$
$$= \frac{640\left(\tan(dx+c)^3+3\tan(dx+c)\right)a^3b+128\left(3\tan(dx+c)^5+10\tan(dx+c)^3+15\tan(dx+c)\right)ab^3}{}$$

input
```
integrate(sec(d*x+c)^3*(a+b*sec(d*x+c))^4,x, algorithm="maxima")
```

output
```
1/480*(640*(tan(d*x + c)^3 + 3*tan(d*x + c))*a^3*b + 128*(3*tan(d*x + c)^5
 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))*a*b^3 - 5*b^4*(2*(15*sin(d*x + c)^
5 - 40*sin(d*x + c)^3 + 33*sin(d*x + c))/(sin(d*x + c)^6 - 3*sin(d*x + c)^
4 + 3*sin(d*x + c)^2 - 1) - 15*log(sin(d*x + c) + 1) + 15*log(sin(d*x + c)
 - 1)) - 180*a^2*b^2*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c)^
4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) -
 1)) - 120*a^4*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1
) + log(sin(d*x + c) - 1)))/d
```

3.476.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 592 vs. $2(230) = 460$.

Time = 0.35 (sec) , antiderivative size = 592, normalized size of antiderivative = 2.43

$$\int \sec^3(c+dx)(a+b\sec(c+dx))^4\,dx$$
$$= \frac{15\,(8\,a^4+36\,a^2b^2+5\,b^4)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-15\,(8\,a^4+36\,a^2b^2+5\,b^4)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\right.\right.}{}$$

input
```
integrate(sec(d*x+c)^3*(a+b*sec(d*x+c))^4,x, algorithm="giac")
```

output
```
1/240*(15*(8*a^4 + 36*a^2*b^2 + 5*b^4)*log(abs(tan(1/2*d*x + 1/2*c) + 1))
- 15*(8*a^4 + 36*a^2*b^2 + 5*b^4)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(
120*a^4*tan(1/2*d*x + 1/2*c)^11 - 960*a^3*b*tan(1/2*d*x + 1/2*c)^11 + 900*
a^2*b^2*tan(1/2*d*x + 1/2*c)^11 - 960*a*b^3*tan(1/2*d*x + 1/2*c)^11 + 165*
b^4*tan(1/2*d*x + 1/2*c)^11 - 360*a^4*tan(1/2*d*x + 1/2*c)^9 + 3520*a^3*b*
tan(1/2*d*x + 1/2*c)^9 - 1260*a^2*b^2*tan(1/2*d*x + 1/2*c)^9 + 2240*a*b^3*
tan(1/2*d*x + 1/2*c)^9 + 25*b^4*tan(1/2*d*x + 1/2*c)^9 + 240*a^4*tan(1/2*d
*x + 1/2*c)^7 - 5760*a^3*b*tan(1/2*d*x + 1/2*c)^7 + 360*a^2*b^2*tan(1/2*d*
x + 1/2*c)^7 - 4992*a*b^3*tan(1/2*d*x + 1/2*c)^7 + 450*b^4*tan(1/2*d*x + 1
/2*c)^7 + 240*a^4*tan(1/2*d*x + 1/2*c)^5 + 5760*a^3*b*tan(1/2*d*x + 1/2*c)
^5 + 360*a^2*b^2*tan(1/2*d*x + 1/2*c)^5 + 4992*a*b^3*tan(1/2*d*x + 1/2*c)^
5 + 450*b^4*tan(1/2*d*x + 1/2*c)^5 - 360*a^4*tan(1/2*d*x + 1/2*c)^3 - 3520
*a^3*b*tan(1/2*d*x + 1/2*c)^3 - 1260*a^2*b^2*tan(1/2*d*x + 1/2*c)^3 - 2240
*a*b^3*tan(1/2*d*x + 1/2*c)^3 + 25*b^4*tan(1/2*d*x + 1/2*c)^3 + 120*a^4*ta
n(1/2*d*x + 1/2*c) + 960*a^3*b*tan(1/2*d*x + 1/2*c) + 900*a^2*b^2*tan(1/2*
d*x + 1/2*c) + 960*a*b^3*tan(1/2*d*x + 1/2*c) + 165*b^4*tan(1/2*d*x + 1/2*
c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^6)/d
```

3.476.9 Mupad [B] (verification not implemented)

Time = 18.30 (sec) , antiderivative size = 370, normalized size of antiderivative = 1.52

$$\int \sec^3(c+dx)(a+b\sec(c+dx))^4\,dx = \frac{\operatorname{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(a^4+\frac{9\,a^2\,b^2}{2}+\frac{5\,b^4}{8}\right)}{d}$$
$$+\frac{\left(a^4-8\,a^3\,b+\frac{15\,a^2\,b^2}{2}-8\,a\,b^3+\frac{11\,b^4}{8}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}+\left(-3\,a^4+\frac{88\,a^3\,b}{3}-\frac{21\,a^2\,b^2}{2}+\frac{56\,a\,b^3}{3}+\frac{5\,b^4}{24}\right)\,\tan\left(\frac{c}{2}\right.}{}$$

input
```
int((a + b/cos(c + d*x))^4/cos(c + d*x)^3,x)
```

output
```
(atanh(tan(c/2 + (d*x)/2))*(a^4 + (5*b^4)/8 + (9*a^2*b^2)/2))/d + (tan(c/2
 + (d*x)/2)^9*((56*a*b^3)/3 + (88*a^3*b)/3 - 3*a^4 + (5*b^4)/24 - (21*a^2*
b^2)/2) - tan(c/2 + (d*x)/2)^3*((56*a*b^3)/3 + (88*a^3*b)/3 + 3*a^4 - (5*b
^4)/24 + (21*a^2*b^2)/2) + tan(c/2 + (d*x)/2)^5*((208*a*b^3)/5 + 48*a^3*b
+ 2*a^4 + (15*b^4)/4 + 3*a^2*b^2) + tan(c/2 + (d*x)/2)^7*(2*a^4 - 48*a^3*b
 - (208*a*b^3)/5 + (15*b^4)/4 + 3*a^2*b^2) + tan(c/2 + (d*x)/2)*(8*a*b^3 +
 8*a^3*b + a^4 + (11*b^4)/8 + (15*a^2*b^2)/2) + tan(c/2 + (d*x)/2)^11*(a^4
 - 8*a^3*b - 8*a*b^3 + (11*b^4)/8 + (15*a^2*b^2)/2))/(d*(15*tan(c/2 + (d*x
)/2)^4 - 6*tan(c/2 + (d*x)/2)^2 - 20*tan(c/2 + (d*x)/2)^6 + 15*tan(c/2 + (
d*x)/2)^8 - 6*tan(c/2 + (d*x)/2)^10 + tan(c/2 + (d*x)/2)^12 + 1))
```

3.476. $\int \sec^3(c+dx)(a+b\sec(c+dx))^4\,dx$

3.477 $\int \sec^2(c+dx)(a+b\sec(c+dx))^4\,dx$

3.477.1 Optimal result . . . 3519
3.477.2 Mathematica [A] (verified) . . . 3520
3.477.3 Rubi [A] (verified) . . . 3520
3.477.4 Maple [A] (verified) . . . 3524
3.477.5 Fricas [A] (verification not implemented) . . . 3525
3.477.6 Sympy [F] . . . 3525
3.477.7 Maxima [A] (verification not implemented) . . . 3525
3.477.8 Giac [B] (verification not implemented) . . . 3526
3.477.9 Mupad [B] (verification not implemented) . . . 3527

3.477.1 Optimal result

Integrand size = 21, antiderivative size = 179

$$\begin{aligned}\int \sec^2(c+dx)(a+b\sec(c+dx))^4\,dx = {} & \frac{ab(4a^2+3b^2)\operatorname{arctanh}(\sin(c+dx))}{2d} \\ & + \frac{2(3a^4+28a^2b^2+4b^4)\tan(c+dx)}{15d} \\ & + \frac{ab(6a^2+29b^2)\sec(c+dx)\tan(c+dx)}{30d} \\ & + \frac{(3a^2+4b^2)(a+b\sec(c+dx))^2\tan(c+dx)}{15d} \\ & + \frac{a(a+b\sec(c+dx))^3\tan(c+dx)}{5d} \\ & + \frac{(a+b\sec(c+dx))^4\tan(c+dx)}{5d}\end{aligned}$$

output
```
1/2*a*b*(4*a^2+3*b^2)*arctanh(sin(d*x+c))/d+2/15*(3*a^4+28*a^2*b^2+4*b^4)*
tan(d*x+c)/d+1/30*a*b*(6*a^2+29*b^2)*sec(d*x+c)*tan(d*x+c)/d+1/15*(3*a^2+4
*b^2)*(a+b*sec(d*x+c))^2*tan(d*x+c)/d+1/5*a*(a+b*sec(d*x+c))^3*tan(d*x+c)/
d+1/5*(a+b*sec(d*x+c))^4*tan(d*x+c)/d
```

3.477.2 Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.70

$$\int \sec^2(c+dx)(a+b\sec(c+dx))^4\,dx$$
$$= \frac{15ab(4a^2+3b^2)\operatorname{arctanh}(\sin(c+dx))+\tan(c+dx)\left(30(a^4+6a^2b^2+b^4)+15ab(4a^2+3b^2)\sec(c+dx)+\right.}{30d}$$

input
```
Integrate[Sec[c + d*x]^2*(a + b*Sec[c + d*x])^4,x]
```

output
```
(15*a*b*(4*a^2 + 3*b^2)*ArcTanh[Sin[c + d*x]] + Tan[c + d*x]*(30*(a^4 + 6*
a^2*b^2 + b^4) + 15*a*b*(4*a^2 + 3*b^2)*Sec[c + d*x] + 30*a*b^3*Sec[c + d*
x]^3 + 20*b^2*(3*a^2 + b^2)*Tan[c + d*x]^2 + 6*b^4*Tan[c + d*x]^4))/(30*d)
```

3.477.3 Rubi [A] (verified)

Time = 1.18 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.09, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {3042, 4322, 3042, 4490, 3042, 4490, 3042, 4485, 3042, 4274, 3042, 4254, 24, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(c+dx)(a+b\sec(c+dx))^4\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^2\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^4dx$$
$$\Big\downarrow \text{4322}$$
$$\frac{4}{5}\int \sec(c+dx)(b+a\sec(c+dx))(a+b\sec(c+dx))^3dx+\frac{\tan(c+dx)(a+b\sec(c+dx))^4}{5d}$$
$$\Big\downarrow \text{3042}$$
$$\frac{4}{5}\int \csc\left(c+dx+\frac{\pi}{2}\right)\left(b+a\csc\left(c+dx+\frac{\pi}{2}\right)\right)\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^3dx+\frac{\tan(c+dx)(a+b\sec(c+dx))^4}{5d}$$
$$\Big\downarrow \text{4490}$$

$$\frac{4}{5}\left(\frac{1}{4}\int \sec(c+dx)(a+b\sec(c+dx))^2\left(7ab+\left(3a^2+4b^2\right)\sec(c+dx)\right)dx+\frac{a\tan(c+dx)(a+b\sec(c+dx))^3}{4d}\right)+\frac{\tan(c+dx)(a+b\sec(c+dx))^4}{5d}$$

$\Big\downarrow$ 3042

$$\frac{4}{5}\left(\frac{1}{4}\int \csc\left(c+dx+\frac{\pi}{2}\right)\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2\left(7ab+\left(3a^2+4b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)\right)dx+\frac{a\tan(c+dx)(a}{}\right.$$
$$\frac{\tan(c+dx)(a+b\sec(c+dx))^4}{5d}$$

$\Big\downarrow$ 4490

$$\frac{4}{5}\left(\frac{1}{4}\left(\frac{1}{3}\int \sec(c+dx)(a+b\sec(c+dx))\left(b\left(27a^2+8b^2\right)+a\left(6a^2+29b^2\right)\sec(c+dx)\right)dx+\frac{\left(3a^2+4b^2\right)\tan(c+}{}\right.\right.$$
$$\frac{\tan(c+dx)(a+b\sec(c+dx))^4}{5d}$$

$\Big\downarrow$ 3042

$$\frac{4}{5}\left(\frac{1}{4}\left(\frac{1}{3}\int \csc\left(c+dx+\frac{\pi}{2}\right)\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)\left(b\left(27a^2+8b^2\right)+a\left(6a^2+29b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)\right)dx+\right.\right.$$
$$\frac{\tan(c+dx)(a+b\sec(c+dx))^4}{5d}$$

$\Big\downarrow$ 4485

$$\frac{4}{5}\left(\frac{1}{4}\left(\frac{1}{3}\left(\frac{1}{2}\int \sec(c+dx)\left(15ab\left(4a^2+3b^2\right)+4\left(3a^4+28b^2a^2+4b^4\right)\sec(c+dx)\right)dx+\frac{ab\left(6a^2+29b^2\right)\tan(c+d}{2d}\right.\right.\right.$$
$$\frac{\tan(c+dx)(a+b\sec(c+dx))^4}{5d}$$

$\Big\downarrow$ 3042

$$\frac{4}{5}\left(\frac{1}{4}\left(\frac{1}{3}\left(\frac{1}{2}\int \csc\left(c+dx+\frac{\pi}{2}\right)\left(15ab\left(4a^2+3b^2\right)+4\left(3a^4+28b^2a^2+4b^4\right)\csc\left(c+dx+\frac{\pi}{2}\right)\right)dx+\frac{ab\left(6a^2+29}{}\right.\right.\right.$$
$$\frac{\tan(c+dx)(a+b\sec(c+dx))^4}{5d}$$

$\Big\downarrow$ 4274

3.477. $\int \sec^2(c+dx)(a+b\sec(c+dx))^4\,dx$

$$\frac{4}{5}\left(\frac{1}{4}\left(\frac{1}{3}\left(\frac{1}{2}\left(15ab(4a^2+3b^2)\int \sec(c+dx)dx+4(3a^4+28a^2b^2+4b^4)\int \sec^2(c+dx)dx\right)+\frac{ab(6a^2+29b^2)\tan}{}\right.\right.\right.$$
$$\frac{\tan(c+dx)(a+b\sec(c+dx))^4}{5d}$$

$\Big\downarrow$ 3042

$$\frac{4}{5}\left(\frac{1}{4}\left(\frac{1}{3}\left(\frac{1}{2}\left(15ab(4a^2+3b^2)\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+4(3a^4+28a^2b^2+4b^4)\int \csc\left(c+dx+\frac{\pi}{2}\right)^2dx\right)+\frac{ab(6}{}\right.\right.\right.$$
$$\frac{\tan(c+dx)(a+b\sec(c+dx))^4}{5d}$$

$\Big\downarrow$ 4254

$$\frac{4}{5}\left(\frac{1}{4}\left(\frac{1}{3}\left(\frac{1}{2}\left(15ab(4a^2+3b^2)\int \csc\left(c+dx+\frac{\pi}{2}\right)dx-\frac{4(3a^4+28a^2b^2+4b^4)\int 1d(-\tan(c+dx))}{d}\right)+\frac{ab(6a^2+}{}\right.\right.\right.$$
$$\frac{\tan(c+dx)(a+b\sec(c+dx))^4}{5d}$$

$\Big\downarrow$ 24

$$\frac{4}{5}\left(\frac{1}{4}\left(\frac{1}{3}\left(\frac{1}{2}\left(15ab(4a^2+3b^2)\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{4(3a^4+28a^2b^2+4b^4)\tan(c+dx)}{d}\right)+\frac{ab(6a^2+29b^2)\,t}{}\right.\right.\right.$$
$$\frac{\tan(c+dx)(a+b\sec(c+dx))^4}{5d}$$

$\Big\downarrow$ 4257

$$\frac{4}{5}\left(\frac{1}{4}\left(\frac{(3a^2+4b^2)\tan(c+dx)(a+b\sec(c+dx))^2}{3d}+\frac{1}{3}\left(\frac{ab(6a^2+29b^2)\tan(c+dx)\sec(c+dx)}{2d}+\frac{1}{2}\left(\frac{15ab(4a^2}{}\right.\right.\right.\right.$$
$$\frac{\tan(c+dx)(a+b\sec(c+dx))^4}{5d}$$

input
```
Int[Sec[c + d*x]^2*(a + b*Sec[c + d*x])^4,x]
```

output
```
((a + b*Sec[c + d*x])^4*Tan[c + d*x])/(5*d) + (4*((a*(a + b*Sec[c + d*x])^
3*Tan[c + d*x])/(4*d) + (((3*a^2 + 4*b^2)*(a + b*Sec[c + d*x])^2*Tan[c + d
*x])/(3*d) + ((a*b*(6*a^2 + 29*b^2)*Sec[c + d*x]*Tan[c + d*x])/(2*d) + ((1
5*a*b*(4*a^2 + 3*b^2)*ArcTanh[Sin[c + d*x]])/d + (4*(3*a^4 + 28*a^2*b^2 +
4*b^4)*Tan[c + d*x])/d)/2)/3)/4))/5
```

3.477. $\int \sec^2(c+dx)(a+b\sec(c+dx))^4\,dx$

3.477.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4322
```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(-Cot[e + f*x])*((a + b*Csc[e + f*x])^m/(f*(m + 1))), x]
+ Simp[m/(m + 1)   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(b + a*Csc
[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m,
 0]
```

rule 4485
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*Cot[
e + f*x]*((d*Csc[e + f*x])^n/(f*(n + 1))), x] + Simp[1/(n + 1)   Int[(d*Csc
[e + f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x
], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[
n, -1]
```

3.477. $\int \sec^2(c+dx)(a+b\sec(c+dx))^4\,dx$

rule 4490
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*((
a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Simp[1/(m + 1)   Int[Csc[e + f*x]*
(a + b*Csc[e + f*x])^(m - 1)*Simp[b*B*m + a*A*(m + 1) + (a*B*m + A*b*(m + 1
))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*
B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

3.477.4 Maple [A] (verified)

Time = 3.67 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.91

method	result
derivativedivides	$\frac{a^4\tan(dx+c)+4a^3b\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)-6a^2b^2\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right)\tan(dx+c)+4a\,b^3\left(-\left(-\right.\right.}{d}$
default	$\frac{a^4\tan(dx+c)+4a^3b\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)-6a^2b^2\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right)\tan(dx+c)+4a\,b^3\left(-\left(-\right.\right.}{d}$
parts	$\frac{a^4\tan(dx+c)}{d}-\frac{b^4\left(-\frac{8}{15}-\frac{\sec(dx+c)^4}{5}-\frac{4\sec(dx+c)^2}{15}\right)\tan(dx+c)}{d}+\frac{2a^3b\sec(dx+c)\tan(dx+c)}{d}+\frac{2a^3b\ln(\sec(dx+c}{d}$
parallelrisch	$\frac{-600a\left(\frac{\cos(5dx+5c)}{10}+\frac{\cos(3dx+3c)}{2}+\cos(dx+c)\right)b\left(a^2+\frac{3b^2}{4}\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+600a\left(\frac{\cos(5dx+5c)}{10}+\frac{\cos(3dx+3c)}{2}+\cos(\right.}{}$
norman	$\frac{-\frac{4\left(45a^4+150a^2b^2+29b^4\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{15d}-\frac{\left(2a^4-4a^3b+12a^2b^2-5a\,b^3+2b^4\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^9}{d}-\frac{\left(2a^4+4a^3b+12a^2b^2+5a\,b^3+2b^4\right)\tan}{d}}{\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}$
risch	$\frac{i\left(-60a^3b\,e^{9i(dx+c)}-45a\,b^3e^{9i(dx+c)}+30a^4e^{8i(dx+c)}-120a^3b\,e^{7i(dx+c)}-210a\,b^3e^{7i(dx+c)}+120a^4e^{6i(dx+c)}+360a^2b^2e^{6i}\right.}{}$

input
```
int(sec(d*x+c)^2*(a+b*sec(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^4*tan(d*x+c)+4*a^3*b*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+t
an(d*x+c)))-6*a^2*b^2*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c)+4*a*b^3*(-(-1/4*s
ec(d*x+c)^3-3/8*sec(d*x+c))*tan(d*x+c)+3/8*ln(sec(d*x+c)+tan(d*x+c)))-b^4*
(-8/15-1/5*sec(d*x+c)^4-4/15*sec(d*x+c)^2)*tan(d*x+c))
```

3.477.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.02

$$\int \sec^2(c+dx)(a+b\sec(c+dx))^4\,dx$$
$$= \frac{15\,(4\,a^3b+3\,ab^3)\cos\,(dx+c)^5\log\,(\sin\,(dx+c)+1)-15\,(4\,a^3b+3\,ab^3)\cos\,(dx+c)^5\log\,(-\sin\,(dx+c)\,-}{}$$

input
```
integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^4,x, algorithm="fricas")
```

output
```
1/60*(15*(4*a^3*b + 3*a*b^3)*cos(d*x + c)^5*log(sin(d*x + c) + 1) - 15*(4*
a^3*b + 3*a*b^3)*cos(d*x + c)^5*log(-sin(d*x + c) + 1) + 2*(30*a*b^3*cos(d
*x + c) + 2*(15*a^4 + 60*a^2*b^2 + 8*b^4)*cos(d*x + c)^4 + 6*b^4 + 15*(4*a
^3*b + 3*a*b^3)*cos(d*x + c)^3 + 4*(15*a^2*b^2 + 2*b^4)*cos(d*x + c)^2)*si
n(d*x + c))/(d*cos(d*x + c)^5)
```

3.477.6 Sympy [F]

$$\int \sec^2(c+dx)(a+b\sec(c+dx))^4\,dx = \int (a+b\sec\,(c+dx))^4\sec^2\,(c+dx)\,dx$$

input
```
integrate(sec(d*x+c)**2*(a+b*sec(d*x+c))**4,x)
```

output
```
Integral((a + b*sec(c + d*x))**4*sec(c + d*x)**2, x)
```

3.477.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.09

$$\int \sec^2(c+dx)(a+b\sec(c+dx))^4\,dx$$
$$= \frac{120\,\left(\tan\,(dx+c)^3+3\,\tan\,(dx+c)\right)a^2b^2+4\,\left(3\,\tan\,(dx+c)^5+10\,\tan\,(dx+c)^3+15\,\tan\,(dx+c)\right)b^4-}{}$$

input
```
integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^4,x, algorithm="maxima")
```

output
```
1/60*(120*(tan(d*x + c)^3 + 3*tan(d*x + c))*a^2*b^2 + 4*(3*tan(d*x + c)^5
+ 10*tan(d*x + c)^3 + 15*tan(d*x + c))*b^4 - 15*a*b^3*(2*(3*sin(d*x + c)^3
 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x
 + c) + 1) + 3*log(sin(d*x + c) - 1)) - 60*a^3*b*(2*sin(d*x + c)/(sin(d*x
+ c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 60*a^4*tan(
d*x + c))/d
```

3.477.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 461 vs. $2(167) = 334$.

Time = 0.33 (sec) , antiderivative size = 461, normalized size of antiderivative = 2.58

$$\int \sec^2(c+dx)(a+b\sec(c+dx))^4\,dx$$

$$= \frac{15\,(4\,a^3b+3\,ab^3)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-15\,(4\,a^3b+3\,ab^3)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(30\,a^4\tan(\right.}{}}{}$$

input
```
integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^4,x, algorithm="giac")
```

output
```
1/30*(15*(4*a^3*b + 3*a*b^3)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 15*(4*a^
3*b + 3*a*b^3)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(30*a^4*tan(1/2*d*x
+ 1/2*c)^9 - 60*a^3*b*tan(1/2*d*x + 1/2*c)^9 + 180*a^2*b^2*tan(1/2*d*x + 1
/2*c)^9 - 75*a*b^3*tan(1/2*d*x + 1/2*c)^9 + 30*b^4*tan(1/2*d*x + 1/2*c)^9
- 120*a^4*tan(1/2*d*x + 1/2*c)^7 + 120*a^3*b*tan(1/2*d*x + 1/2*c)^7 - 480*
a^2*b^2*tan(1/2*d*x + 1/2*c)^7 + 30*a*b^3*tan(1/2*d*x + 1/2*c)^7 - 40*b^4*
tan(1/2*d*x + 1/2*c)^7 + 180*a^4*tan(1/2*d*x + 1/2*c)^5 + 600*a^2*b^2*tan(
1/2*d*x + 1/2*c)^5 + 116*b^4*tan(1/2*d*x + 1/2*c)^5 - 120*a^4*tan(1/2*d*x
+ 1/2*c)^3 - 120*a^3*b*tan(1/2*d*x + 1/2*c)^3 - 480*a^2*b^2*tan(1/2*d*x +
1/2*c)^3 - 30*a*b^3*tan(1/2*d*x + 1/2*c)^3 - 40*b^4*tan(1/2*d*x + 1/2*c)^3
 + 30*a^4*tan(1/2*d*x + 1/2*c) + 60*a^3*b*tan(1/2*d*x + 1/2*c) + 180*a^2*b
^2*tan(1/2*d*x + 1/2*c) + 75*a*b^3*tan(1/2*d*x + 1/2*c) + 30*b^4*tan(1/2*d
*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^5)/d
```

3.477.9 Mupad [B] (verification not implemented)

Time = 18.15 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.70

$$\int \sec^2(c+dx)(a+b\sec(c+dx))^4\,dx = \frac{\operatorname{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\,(4\,a^3\,b+3\,a\,b^3)}{d} - \frac{(2\,a^4-4\,a^3\,b+12\,a^2\,b^2-5\,a\,b^3+2\,b^4)\,\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^9+\left(-8\,a^4+8\,a^3\,b-32\,a^2\,b^2+2\,a\,b^3-\frac{8\,b^4}{3}\right)\,\tan\left(\frac{c}{2}\right.}{d\,\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}\right.}$$

input
```
int((a + b/cos(c + d*x))^4/cos(c + d*x)^2,x)
```

output
```
(atanh(tan(c/2 + (d*x)/2))*(3*a*b^3 + 4*a^3*b))/d - (tan(c/2 + (d*x)/2)^5*
(12*a^4 + (116*b^4)/15 + 40*a^2*b^2) + tan(c/2 + (d*x)/2)^9*(2*a^4 - 4*a^3
*b - 5*a*b^3 + 2*b^4 + 12*a^2*b^2) - tan(c/2 + (d*x)/2)^3*(2*a*b^3 + 8*a^3
*b + 8*a^4 + (8*b^4)/3 + 32*a^2*b^2) - tan(c/2 + (d*x)/2)^7*(8*a^4 - 8*a^3
*b - 2*a*b^3 + (8*b^4)/3 + 32*a^2*b^2) + tan(c/2 + (d*x)/2)*(5*a*b^3 + 4*a
^3*b + 2*a^4 + 2*b^4 + 12*a^2*b^2))/(d*(5*tan(c/2 + (d*x)/2)^2 - 10*tan(c/
2 + (d*x)/2)^4 + 10*tan(c/2 + (d*x)/2)^6 - 5*tan(c/2 + (d*x)/2)^8 + tan(c/
2 + (d*x)/2)^10 - 1))
```

3.478 $\int \sec(c+dx)(a+b\sec(c+dx))^4\,dx$

3.478.1 Optimal result . . . 3528
3.478.2 Mathematica [A] (verified) . . . 3529
3.478.3 Rubi [A] (verified) . . . 3529
3.478.4 Maple [A] (verified) . . . 3533
3.478.5 Fricas [A] (verification not implemented) . . . 3533
3.478.6 Sympy [F] . . . 3534
3.478.7 Maxima [A] (verification not implemented) . . . 3534
3.478.8 Giac [B] (verification not implemented) . . . 3535
3.478.9 Mupad [B] (verification not implemented) . . . 3535

3.478.1 Optimal result

Integrand size = 19, antiderivative size = 146

$$\begin{aligned}\int \sec(c+dx)(a+b\sec(c+dx))^4\,dx = {} & \frac{(8a^4+24a^2b^2+3b^4)\operatorname{arctanh}(\sin(c+dx))}{8d} \\ & + \frac{ab(19a^2+16b^2)\tan(c+dx)}{6d} \\ & + \frac{b^2(26a^2+9b^2)\sec(c+dx)\tan(c+dx)}{24d} \\ & + \frac{7ab(a+b\sec(c+dx))^2\tan(c+dx)}{12d} \\ & + \frac{b(a+b\sec(c+dx))^3\tan(c+dx)}{4d}\end{aligned}$$

output
```
1/8*(8*a^4+24*a^2*b^2+3*b^4)*arctanh(sin(d*x+c))/d+1/6*a*b*(19*a^2+16*b^2)
*tan(d*x+c)/d+1/24*b^2*(26*a^2+9*b^2)*sec(d*x+c)*tan(d*x+c)/d+7/12*a*b*(a+
b*sec(d*x+c))^2*tan(d*x+c)/d+1/4*b*(a+b*sec(d*x+c))^3*tan(d*x+c)/d
```

3.478.2 Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.69

$$\int \sec(c+dx)(a+b\sec(c+dx))^4\,dx = \frac{3(8a^4+24a^2b^2+3b^4)\operatorname{arctanh}(\sin(c+dx))+b\tan(c+dx)\left(9b(8a^2+b^2)\sec(c+dx)+6b^3\sec^3(c+dx)+\right.}{24d}$$

input
```
Integrate[Sec[c + d*x]*(a + b*Sec[c + d*x])^4,x]
```

output
```
(3*(8*a^4 + 24*a^2*b^2 + 3*b^4)*ArcTanh[Sin[c + d*x]] + b*Tan[c + d*x]*(9*
b*(8*a^2 + b^2)*Sec[c + d*x] + 6*b^3*Sec[c + d*x]^3 + 32*a*(3*(a^2 + b^2)
+ b^2*Tan[c + d*x]^2)))/(24*d)
```

3.478.3 Rubi [A] (verified)

Time = 0.95 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.08, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.632$, Rules used = {3042, 4317, 3042, 4490, 3042, 4485, 3042, 4274, 3042, 4254, 24, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(c+dx)(a+b\sec(c+dx))^4\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^4dx$$

$$\Big\downarrow \text{4317}$$

$$\frac{1}{4}\int \sec(c+dx)(a+b\sec(c+dx))^2\left(4a^2+7b\sec(c+dx)a+3b^2\right)dx+\frac{b\tan(c+dx)(a+b\sec(c+dx))^3}{4d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}\int \csc\left(c+dx+\frac{\pi}{2}\right)\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2\left(4a^2+7b\csc\left(c+dx+\frac{\pi}{2}\right)a+3b^2\right)dx+\frac{b\tan(c+dx)(a+b\sec(c+dx))^3}{4d}$$

$$\Big\downarrow \text{4490}$$

$$\frac{1}{4}\left(\frac{1}{3}\int \sec(c+dx)(a+b\sec(c+dx))\left(a\left(12a^2+23b^2\right)+b\left(26a^2+9b^2\right)\sec(c+dx)\right)dx+\frac{7ab\tan(c+dx)(a+b\sec}{3d}\right.$$
$$\frac{b\tan(c+dx)(a+b\sec(c+dx))^3}{4d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}\left(\frac{1}{3}\int \csc\left(c+dx+\frac{\pi}{2}\right)\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)\left(a\left(12a^2+23b^2\right)+b\left(26a^2+9b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)\right)dx+\frac{7a}{}\right.$$
$$\frac{b\tan(c+dx)(a+b\sec(c+dx))^3}{4d}$$

$$\Big\downarrow \text{4485}$$

$$\frac{1}{4}\left(\frac{1}{3}\left(\frac{1}{2}\int \sec(c+dx)\left(3\left(8a^4+24b^2a^2+3b^4\right)+4ab\left(19a^2+16b^2\right)\sec(c+dx)\right)dx+\frac{b^2\left(26a^2+9b^2\right)\tan(c+dx)}{2d}\right.\right.$$
$$\frac{b\tan(c+dx)(a+b\sec(c+dx))^3}{4d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}\left(\frac{1}{3}\left(\frac{1}{2}\int \csc\left(c+dx+\frac{\pi}{2}\right)\left(3\left(8a^4+24b^2a^2+3b^4\right)+4ab\left(19a^2+16b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)\right)dx+\frac{b^2\left(26a^2+9b^2\right)}{}\right.\right.$$
$$\frac{b\tan(c+dx)(a+b\sec(c+dx))^3}{4d}$$

$$\Big\downarrow \text{4274}$$

$$\frac{1}{4}\left(\frac{1}{3}\left(\frac{1}{2}\left(4ab\left(19a^2+16b^2\right)\int \sec^2(c+dx)dx+3\left(8a^4+24a^2b^2+3b^4\right)\int \sec(c+dx)dx\right)+\frac{b^2\left(26a^2+9b^2\right)\tan(c}{2d}\right.\right.$$
$$\frac{b\tan(c+dx)(a+b\sec(c+dx))^3}{4d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}\left(\frac{1}{3}\left(\frac{1}{2}\left(4ab\left(19a^2+16b^2\right)\int \csc\left(c+dx+\frac{\pi}{2}\right)^2dx+3\left(8a^4+24a^2b^2+3b^4\right)\int \csc\left(c+dx+\frac{\pi}{2}\right)dx\right)+\frac{b^2\left(26a^2\right.}{}\right.\right.$$
$$\frac{b\tan(c+dx)(a+b\sec(c+dx))^3}{4d}$$

$$\Big\downarrow \text{4254}$$

3.478. $\int \sec(c+dx)(a+b\sec(c+dx))^4\,dx$

$$\frac{1}{4}\left(\frac{1}{3}\left(\frac{1}{2}\left(3\left(8a^4+24a^2b^2+3b^4\right)\int \csc\left(c+dx+\frac{\pi}{2}\right)dx-\frac{4ab\left(19a^2+16b^2\right)\int 1d(-\tan(c+dx))}{d}\right)+\frac{b^2\left(26a^2+\right.}{}\right.\right.$$

$$\frac{b\tan(c+dx)(a+b\sec(c+dx))^3}{4d}$$

$$\Big\downarrow \; 24$$

$$\frac{1}{4}\left(\frac{1}{3}\left(\frac{1}{2}\left(3\left(8a^4+24a^2b^2+3b^4\right)\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{4ab\left(19a^2+16b^2\right)\tan(c+dx)}{d}\right)+\frac{b^2\left(26a^2+9b^2\right)\tan(}{2}\right.\right.$$

$$\frac{b\tan(c+dx)(a+b\sec(c+dx))^3}{4d}$$

$$\Big\downarrow \; 4257$$

$$\frac{1}{4}\left(\frac{1}{3}\left(\frac{b^2\left(26a^2+9b^2\right)\tan(c+dx)\sec(c+dx)}{2d}+\frac{1}{2}\left(\frac{4ab\left(19a^2+16b^2\right)\tan(c+dx)}{d}+\frac{3\left(8a^4+24a^2b^2+3b^4\right)\arctan}{d}\right.\right.\right.$$

$$\frac{b\tan(c+dx)(a+b\sec(c+dx))^3}{4d}$$

input
```
Int[Sec[c + d*x]*(a + b*Sec[c + d*x])^4,x]
```

output
```
(b*(a + b*Sec[c + d*x])^3*Tan[c + d*x])/(4*d) + ((7*a*b*(a + b*Sec[c + d*x
])^2*Tan[c + d*x])/(3*d) + ((b^2*(26*a^2 + 9*b^2)*Sec[c + d*x]*Tan[c + d*x
])/(2*d) + ((3*(8*a^4 + 24*a^2*b^2 + 3*b^4)*ArcTanh[Sin[c + d*x]])/d + (4*
a*b*(19*a^2 + 16*b^2)*Tan[c + d*x])/d)/2)/3)/4
```

3.478.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4317
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[(-b)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m - 1)/(f*m)), x]
+ Simp[1/m   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)*(b^2*(m - 1) + a
^2*m + a*b*(2*m - 1)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x] && NeQ
[a^2 - b^2, 0] && GtQ[m, 1] && IntegerQ[2*m]
```

rule 4485
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*Cot[
e + f*x]*((d*Csc[e + f*x])^n/(f*(n + 1))), x] + Simp[1/(n + 1)   Int[(d*Csc
[e + f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x
], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[
n, -1]
```

rule 4490
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*((
a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Simp[1/(m + 1)   Int[Csc[e + f*x]*
(a + b*Csc[e + f*x])^(m - 1)*Simp[b*B*m + a*A*(m + 1) + (a*B*m + A*b*(m + 1
))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*
B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

3.478. $\int \sec(c+dx)(a+b\sec(c+dx))^4\,dx$

3.478.4 Maple [A] (verified)

Time = 1.46 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.01

method	result
derivativedivides	$\frac{a^4 \ln(\sec(dx+c)+\tan(dx+c))+4\tan(dx+c)a^3b+6a^2b^2\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)-4a\,b^3\left(-\frac{2}{3}-\frac{\sec}{}\right.}{d}$
default	$\frac{a^4 \ln(\sec(dx+c)+\tan(dx+c))+4\tan(dx+c)a^3b+6a^2b^2\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)-4a\,b^3\left(-\frac{2}{3}-\frac{\sec}{}\right.}{d}$
parts	$\frac{a^4 \ln(\sec(dx+c)+\tan(dx+c))}{d}+\frac{b^4\left(-\left(-\frac{\sec(dx+c)^3}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)}{d}+\frac{4a^3b\,t}{}$
parallelrisch	$\frac{-96\left(a^4+3a^2b^2+\frac{3}{8}b^4\right)\left(\frac{3}{4}+\frac{\cos(4dx+4c)}{4}+\cos(2dx+2c)\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+96\left(a^4+3a^2b^2+\frac{3}{8}b^4\right)\left(\frac{3}{4}+\frac{\cos(4dx+4c)}{4}+\cos\right.}{}$
norman	$\frac{-\frac{b\left(32a^3-24a^2b+32a\,b^2-5b^3\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{4d}+\frac{b\left(32a^3+24a^2b+32a\,b^2+5b^3\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d}+\frac{b\left(288a^3-72a^2b+160a\,b^2+9b^3\right)\tan\left(\frac{dx}{2}\right.}{12d}}{\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^4}$
risch	$-\frac{ib\left(72a^2b\,e^{7i(dx+c)}+9b^3e^{7i(dx+c)}-96a^3e^{6i(dx+c)}+72a^2b\,e^{5i(dx+c)}+33b^3e^{5i(dx+c)}-288a^3e^{4i(dx+c)}-192a\,b^2e^{4i(dx+c)}\right.}{12d\left(e^{2i(dx}\right.}$

input
```
int(sec(d*x+c)*(a+b*sec(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^4*ln(sec(d*x+c)+tan(d*x+c))+4*tan(d*x+c)*a^3*b+6*a^2*b^2*(1/2*sec(d
*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c)))-4*a*b^3*(-2/3-1/3*sec(d*x+
c)^2)*tan(d*x+c)+b^4*(-(-1/4*sec(d*x+c)^3-3/8*sec(d*x+c))*tan(d*x+c)+3/8*l
n(sec(d*x+c)+tan(d*x+c))))
```

3.478.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.12

$$\int \sec(c+dx)(a+b\sec(c+dx))^4\,dx$$
$$=\frac{3\left(8\,a^4+24\,a^2b^2+3\,b^4\right)\cos\left(dx+c\right)^4\log\left(\sin\left(dx+c\right)+1\right)-3\left(8\,a^4+24\,a^2b^2+3\,b^4\right)\cos\left(dx+c\right)^4\log\left(-\right.}{}$$

input
```
integrate(sec(d*x+c)*(a+b*sec(d*x+c))^4,x, algorithm="fricas")
```

output
```
1/48*(3*(8*a^4 + 24*a^2*b^2 + 3*b^4)*cos(d*x + c)^4*log(sin(d*x + c) + 1)
- 3*(8*a^4 + 24*a^2*b^2 + 3*b^4)*cos(d*x + c)^4*log(-sin(d*x + c) + 1) + 2
*(32*a*b^3*cos(d*x + c) + 6*b^4 + 32*(3*a^3*b + 2*a*b^3)*cos(d*x + c)^3 +
9*(8*a^2*b^2 + b^4)*cos(d*x + c)^2)*sin(d*x + c))/(d*cos(d*x + c)^4)
```

3.478.6 Sympy [F]

$$\int \sec(c+dx)(a+b\sec(c+dx))^4\,dx = \int (a+b\sec(c+dx))^4\sec(c+dx)\,dx$$

input
```
integrate(sec(d*x+c)*(a+b*sec(d*x+c))**4,x)
```

output
```
Integral((a + b*sec(c + d*x))**4*sec(c + d*x), x)
```

3.478.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.23

$$\int \sec(c+dx)(a+b\sec(c+dx))^4\,dx$$
$$= \frac{64\left(\tan(dx+c)^3+3\tan(dx+c)\right)ab^3-3b^4\left(\frac{2\left(3\sin(dx+c)^3-5\sin(dx+c)\right)}{\sin(dx+c)^4-2\sin(dx+c)^2+1}-3\log(\sin(dx+c)+1)+3\log(\right.}{}$$

input
```
integrate(sec(d*x+c)*(a+b*sec(d*x+c))^4,x, algorithm="maxima")
```

output
```
1/48*(64*(tan(d*x + c)^3 + 3*tan(d*x + c))*a*b^3 - 3*b^4*(2*(3*sin(d*x + c
)^3 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(
d*x + c) + 1) + 3*log(sin(d*x + c) - 1)) - 72*a^2*b^2*(2*sin(d*x + c)/(sin
(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 48*a^4
*log(sec(d*x + c) + tan(d*x + c)) + 192*a^3*b*tan(d*x + c))/d
```

3.478.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 360 vs. $2(136) = 272$.

Time = 0.34 (sec) , antiderivative size = 360, normalized size of antiderivative = 2.47

$$\int \sec(c+dx)(a+b\sec(c+dx))^4\,dx$$
$$= \frac{3\,(8\,a^4+24\,a^2b^2+3\,b^4)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\,(8\,a^4+24\,a^2b^2+3\,b^4)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{}$$

input
```
integrate(sec(d*x+c)*(a+b*sec(d*x+c))^4,x, algorithm="giac")
```

output
```
1/24*(3*(8*a^4 + 24*a^2*b^2 + 3*b^4)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) -
3*(8*a^4 + 24*a^2*b^2 + 3*b^4)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(96*
a^3*b*tan(1/2*d*x + 1/2*c)^7 - 72*a^2*b^2*tan(1/2*d*x + 1/2*c)^7 + 96*a*b^
3*tan(1/2*d*x + 1/2*c)^7 - 15*b^4*tan(1/2*d*x + 1/2*c)^7 - 288*a^3*b*tan(1
/2*d*x + 1/2*c)^5 + 72*a^2*b^2*tan(1/2*d*x + 1/2*c)^5 - 160*a*b^3*tan(1/2*
d*x + 1/2*c)^5 - 9*b^4*tan(1/2*d*x + 1/2*c)^5 + 288*a^3*b*tan(1/2*d*x + 1/
2*c)^3 + 72*a^2*b^2*tan(1/2*d*x + 1/2*c)^3 + 160*a*b^3*tan(1/2*d*x + 1/2*c
)^3 - 9*b^4*tan(1/2*d*x + 1/2*c)^3 - 96*a^3*b*tan(1/2*d*x + 1/2*c) - 72*a^
2*b^2*tan(1/2*d*x + 1/2*c) - 96*a*b^3*tan(1/2*d*x + 1/2*c) - 15*b^4*tan(1/
2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^4)/d
```

3.478.9 Mupad [B] (verification not implemented)

Time = 17.17 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.68

$$\int \sec(c+dx)(a+b\sec(c+dx))^4\,dx$$
$$= \frac{\left(-8\,a^3\,b+6\,a^2\,b^2-8\,a\,b^3+\frac{5\,b^4}{4}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+\left(24\,a^3\,b-6\,a^2\,b^2+\frac{40\,a\,b^3}{3}+\frac{3\,b^4}{4}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+\left(-24\right.}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8-4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+6\,\tan\left(\frac{c}{2}\right.\right.}$$
$$+\frac{\operatorname{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(2\,a^4+6\,a^2\,b^2+\frac{3\,b^4}{4}\right)}{d}$$

input
```
int((a + b/cos(c + d*x))^4/cos(c + d*x),x)
```

output
```
(tan(c/2 + (d*x)/2)*(8*a*b^3 + 8*a^3*b + (5*b^4)/4 + 6*a^2*b^2) - tan(c/2
+ (d*x)/2)^7*(8*a*b^3 + 8*a^3*b - (5*b^4)/4 - 6*a^2*b^2) - tan(c/2 + (d*x)
/2)^3*((40*a*b^3)/3 + 24*a^3*b - (3*b^4)/4 + 6*a^2*b^2) + tan(c/2 + (d*x)/
2)^5*((40*a*b^3)/3 + 24*a^3*b + (3*b^4)/4 - 6*a^2*b^2))/(d*(6*tan(c/2 + (d
*x)/2)^4 - 4*tan(c/2 + (d*x)/2)^2 - 4*tan(c/2 + (d*x)/2)^6 + tan(c/2 + (d*
x)/2)^8 + 1)) + (atanh(tan(c/2 + (d*x)/2))*(2*a^4 + (3*b^4)/4 + 6*a^2*b^2)
)/d
```

3.478. $\int \sec(c+dx)(a+b\sec(c+dx))^4\,dx$

3.479 $\int (a+b\sec(c+dx))^4\,dx$

3.479.1 Optimal result . . . 3537
3.479.2 Mathematica [A] (verified) . . . 3537
3.479.3 Rubi [A] (verified) . . . 3538
3.479.4 Maple [A] (verified) . . . 3539
3.479.5 Fricas [A] (verification not implemented) . . . 3540
3.479.6 Sympy [F] . . . 3541
3.479.7 Maxima [A] (verification not implemented) . . . 3541
3.479.8 Giac [B] (verification not implemented) . . . 3541
3.479.9 Mupad [B] (verification not implemented) . . . 3542

3.479.1 Optimal result

Integrand size = 12, antiderivative size = 107

$$\int (a+b\sec(c+dx))^4\,dx = a^4x+\frac{2ab(2a^2+b^2)\operatorname{arctanh}(\sin(c+dx))}{d}+\frac{b^2(17a^2+2b^2)\tan(c+dx)}{3d}+\frac{4ab^3\sec(c+dx)\tan(c+dx)}{3d}+\frac{b^2(a+b\sec(c+dx))^2\tan(c+dx)}{3d}$$

output
```
a^4*x+2*a*b*(2*a^2+b^2)*arctanh(sin(d*x+c))/d+1/3*b^2*(17*a^2+2*b^2)*tan(d
*x+c)/d+4/3*a*b^3*sec(d*x+c)*tan(d*x+c)/d+1/3*b^2*(a+b*sec(d*x+c))^2*tan(d
*x+c)/d
```

3.479.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.72

$$\int (a+b\sec(c+dx))^4\,dx = \frac{3a^4dx+6ab(2a^2+b^2)\operatorname{arctanh}(\sin(c+dx))+3b^2(6a^2+b^2+2ab\sec(c+dx))\tan(c+dx)+b^4\tan^3(c+d}{3d}$$

input
```
Integrate[(a + b*Sec[c + d*x])^4,x]
```

output
```
(3*a^4*d*x + 6*a*b*(2*a^2 + b^2)*ArcTanh[Sin[c + d*x]] + 3*b^2*(6*a^2 + b^
2 + 2*a*b*Sec[c + d*x])*Tan[c + d*x] + b^4*Tan[c + d*x]^3)/(3*d)
```

3.479.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.07, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {3042, 4269, 3042, 4536, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\sec(c+dx))^4\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^4 dx$$

$$\Big\downarrow \text{4269}$$

$$\frac{1}{3}\int (a+b\sec(c+dx))\left(3a^3+8b^2\sec^2(c+dx)a+b\left(9a^2+2b^2\right)\sec(c+dx)\right)dx+\frac{b^2\tan(c+dx)(a+b\sec(c+dx))^2}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\int \left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)\left(3a^3+8b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2a+b\left(9a^2+2b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)\right)dx+\frac{b^2\tan(c+dx)(a+b\sec(c+dx))^2}{3d}$$

$$\Big\downarrow \text{4536}$$

$$\frac{1}{3}\left(\frac{1}{2}\int \left(6a^4+12b\left(2a^2+b^2\right)\sec(c+dx)a+2b^2\left(17a^2+2b^2\right)\sec^2(c+dx)\right)dx+\frac{4ab^3\tan(c+dx)\sec(c+dx)}{d}\right)+\frac{b^2\tan(c+dx)(a+b\sec(c+dx))^2}{3d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(\frac{1}{2}\left(6a^4x+\frac{12ab\left(2a^2+b^2\right)\operatorname{arctanh}(\sin(c+dx))}{d}+\frac{2b^2\left(17a^2+2b^2\right)\tan(c+dx)}{d}\right)+\frac{4ab^3\tan(c+dx)\sec(c+d}{d}\right.$$
$$\frac{b^2\tan(c+dx)(a+b\sec(c+dx))^2}{3d}$$

input
```
Int[(a + b*Sec[c + d*x])^4,x]
```

output
```
(b^2*(a + b*Sec[c + d*x])^2*Tan[c + d*x])/(3*d) + ((4*a*b^3*Sec[c + d*x]*T
an[c + d*x])/d + (6*a^4*x + (12*a*b*(2*a^2 + b^2)*ArcTanh[Sin[c + d*x]])/d
 + (2*b^2*(17*a^2 + 2*b^2)*Tan[c + d*x])/d)/2)/3
```

3.479.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4269
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(-b^2)*C
ot[c + d*x]*((a + b*Csc[c + d*x])^(n - 2)/(d*(n - 1))), x] + Simp[1/(n - 1)
   Int[(a + b*Csc[c + d*x])^(n - 3)*Simp[a^3*(n - 1) + (b*(b^2*(n - 2) + 3*
a^2*(n - 1)))*Csc[c + d*x] + (a*b^2*(3*n - 4))*Csc[c + d*x]^2, x], x], x] /
; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 2] && IntegerQ[2*n]
```

rule 4536
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[(-b)*C*Csc[e +
 f*x]*(Cot[e + f*x]/(2*f)), x] + Simp[1/2   Int[Simp[2*A*a + (2*B*a + b*(2*
A + C))*Csc[e + f*x] + 2*(a*C + B*b)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a
, b, e, f, A, B, C}, x]
```

3.479.4 Maple [A] (verified)

Time = 1.23 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.02

method	result
derivativedivides	$\frac{a^4(dx+c)+4a^3b\ln(\sec(dx+c)+\tan(dx+c))+6\tan(dx+c)a^2b^2+4a\,b^3\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)-b}{d}$
default	$\frac{a^4(dx+c)+4a^3b\ln(\sec(dx+c)+\tan(dx+c))+6\tan(dx+c)a^2b^2+4a\,b^3\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)-b}{d}$
parts	$a^4x-\frac{b^4\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right)\tan(dx+c)}{d}+\frac{4a^3b\ln(\sec(dx+c)+\tan(dx+c))}{d}+\frac{6a^2b^2\tan(dx+c)}{d}+\frac{2a\,b^3\sec(dx+c}{d}$
risch	$a^4x-\frac{4ib^2\left(3ab\,e^{5i(dx+c)}-9a^2e^{4i(dx+c)}-18a^2e^{2i(dx+c)}-3b^2e^{2i(dx+c)}-3ab\,e^{i(dx+c)}-9a^2-b^2\right)}{3d\left(e^{2i(dx+c)}+1\right)^3}-\frac{4a^3b\ln\left(e^{i(dx+c)}-\right.}{d}$
parallelrisch	$\frac{-36\left(a^2+\frac{b^2}{2}\right)\left(\cos(dx+c)+\frac{\cos(3dx+3c)}{3}\right)ab\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+36\left(a^2+\frac{b^2}{2}\right)\left(\cos(dx+c)+\frac{\cos(3dx+3c)}{3}\right)ab\ln\left(\tan\left(\frac{dx}{2}\right.\right.}{3d(\cos(3dx+3}$
norman	$\frac{a^4x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6-a^4x+3a^4x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2-3a^4x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\frac{4b^2\left(18a^2+b^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3d}-\frac{2b^2\left(6a^2-2ab+b^2\right)\tan\left(\frac{d}{2}\right.}{d}}{\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^3}$

input
```
int((a+b*sec(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^4*(d*x+c)+4*a^3*b*ln(sec(d*x+c)+tan(d*x+c))+6*tan(d*x+c)*a^2*b^2+4*
a*b^3*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c)))-b^4*(-2/3-
1/3*sec(d*x+c)^2)*tan(d*x+c))
```

3.479.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.29

$$\int (a+b\sec(c+dx))^4\,dx$$
$$=\frac{3\,a^4dx\cos(dx+c)^3+3\,(2\,a^3b+ab^3)\cos(dx+c)^3\log(\sin(dx+c)+1)-3\,(2\,a^3b+ab^3)\cos(dx+c)^3\log}{3\,d\cos(dx+c)^3}$$

input
```
integrate((a+b*sec(d*x+c))^4,x, algorithm="fricas")
```

output
```
1/3*(3*a^4*d*x*cos(d*x + c)^3 + 3*(2*a^3*b + a*b^3)*cos(d*x + c)^3*log(sin
(d*x + c) + 1) - 3*(2*a^3*b + a*b^3)*cos(d*x + c)^3*log(-sin(d*x + c) + 1)
 + (6*a*b^3*cos(d*x + c) + b^4 + 2*(9*a^2*b^2 + b^4)*cos(d*x + c)^2)*sin(d
*x + c))/(d*cos(d*x + c)^3)
```

3.479.6 Sympy [F]

$$\int (a+b\sec(c+dx))^4\,dx = \int (a+b\sec{(c+dx)})^4\,dx$$

input
```
integrate((a+b*sec(d*x+c))**4,x)
```

output
```
Integral((a + b*sec(c + d*x))**4, x)
```

3.479.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.13

$$\begin{aligned}\int (a+b\sec(c+dx))^4\,dx &= a^4x + \frac{\left(\tan{(dx+c)}^3 + 3\,\tan{(dx+c)}\right)b^4}{3\,d} \\ &\quad - \frac{ab^3\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1} - \log{(\sin{(dx+c)}+1)} + \log{(\sin{(dx+c)}-1)}\right)}{d} \\ &\quad + \frac{4\,a^3b\log{(\sec{(dx+c)}+\tan{(dx+c)})}}{d} + \frac{6\,a^2b^2\tan{(dx+c)}}{d}\end{aligned}$$

input
```
integrate((a+b*sec(d*x+c))^4,x, algorithm="maxima")
```

output
```
a^4*x + 1/3*(tan(d*x + c)^3 + 3*tan(d*x + c))*b^4/d - a*b^3*(2*sin(d*x + c
)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1))/d
+ 4*a^3*b*log(sec(d*x + c) + tan(d*x + c))/d + 6*a^2*b^2*tan(d*x + c)/d
```

3.479.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 221 vs. 2(101) = 202.

Time = 0.33 (sec) , antiderivative size = 221, normalized size of antiderivative = 2.07

$$\int (a+b\sec(c+dx))^4\,dx$$

$$= \frac{3\,(dx+c)a^4 + 6\,(2\,a^3b+ab^3)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right) - 6\,(2\,a^3b+ab^3)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right) -}{}$$

input
```
integrate((a+b*sec(d*x+c))^4,x, algorithm="giac")
```

output
```
1/3*(3*(d*x + c)*a^4 + 6*(2*a^3*b + a*b^3)*log(abs(tan(1/2*d*x + 1/2*c) +
1)) - 6*(2*a^3*b + a*b^3)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(18*a^2*b
^2*tan(1/2*d*x + 1/2*c)^5 - 6*a*b^3*tan(1/2*d*x + 1/2*c)^5 + 3*b^4*tan(1/2
*d*x + 1/2*c)^5 - 36*a^2*b^2*tan(1/2*d*x + 1/2*c)^3 - 2*b^4*tan(1/2*d*x +
1/2*c)^3 + 18*a^2*b^2*tan(1/2*d*x + 1/2*c) + 6*a*b^3*tan(1/2*d*x + 1/2*c)
+ 3*b^4*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^3)/d
```

3.479.9 Mupad [B] (verification not implemented)

Time = 13.76 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.73

$$\int (a+b\sec(c+dx))^4\,dx = \frac{2\,a^4\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d} + \frac{2\,b^4\,\sin\left(c+d\,x\right)}{3\,d\,\cos\left(c+d\,x\right)} + \frac{b^4\,\sin\left(c+d\,x\right)}{3\,d\,\cos\left(c+d\,x\right)^3} + \frac{4\,a\,b^3\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d} + \frac{8\,a^3\,b\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d} + \frac{2\,a\,b^3\,\sin\left(c+d\,x\right)}{d\,\cos\left(c+d\,x\right)^2} + \frac{6\,a^2\,b^2\,\sin\left(c+d\,x\right)}{d\,\cos\left(c+d\,x\right)}$$

input
```
int((a + b/cos(c + d*x))^4,x)
```

output
```
(2*a^4*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (2*b^4*sin(c + d*x
))/(3*d*cos(c + d*x)) + (b^4*sin(c + d*x))/(3*d*cos(c + d*x)^3) + (4*a*b^3
*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (8*a^3*b*atanh(sin(c/2
+ (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (2*a*b^3*sin(c + d*x))/(d*cos(c + d*x)
^2) + (6*a^2*b^2*sin(c + d*x))/(d*cos(c + d*x))
```

3.480 $\int \cos(c+dx)(a+b\sec(c+dx))^4\,dx$

3.480.1 Optimal result . . . 3543
3.480.2 Mathematica [B] (verified) . . . 3543
3.480.3 Rubi [A] (verified) . . . 3544
3.480.4 Maple [A] (verified) . . . 3547
3.480.5 Fricas [A] (verification not implemented) . . . 3548
3.480.6 Sympy [F] . . . 3548
3.480.7 Maxima [A] (verification not implemented) . . . 3548
3.480.8 Giac [A] (verification not implemented) . . . 3549
3.480.9 Mupad [B] (verification not implemented) . . . 3549

3.480.1 Optimal result

Integrand size = 19, antiderivative size = 104

$$\int \cos(c+dx)(a+b\sec(c+dx))^4\,dx = 4a^3bx + \frac{b^2(12a^2+b^2)\operatorname{arctanh}(\sin(c+dx))}{2d} + \frac{a^2(2a^2-b^2)\sin(c+dx)}{2d} + \frac{b^2(a+b\sec(c+dx))^2\sin(c+dx)}{2d} + \frac{3ab^3\tan(c+dx)}{d}$$

output
```
4*a^3*b*x+1/2*b^2*(12*a^2+b^2)*arctanh(sin(d*x+c))/d+1/2*a^2*(2*a^2-b^2)*s
in(d*x+c)/d+1/2*b^2*(a+b*sec(d*x+c))^2*sin(d*x+c)/d+3*a*b^3*tan(d*x+c)/d
```

3.480.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 280 vs. 2(104) = 208.

Time = 1.23 (sec) , antiderivative size = 280, normalized size of antiderivative = 2.69

$$\int \cos(c+dx)(a+b\sec(c+dx))^4\,dx$$
$$= \frac{\sec^2(c+dx)\left(8a^3bc+8a^3bdx-12a^2b^2\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-b^4\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\right.\right.}{}$$

input
```
Integrate[Cos[c + d*x]*(a + b*Sec[c + d*x])^4,x]
```

output
```
(Sec[c + d*x]^2*(8*a^3*b*c + 8*a^3*b*d*x - 12*a^2*b^2*Log[Cos[(c + d*x)/2]
 - Sin[(c + d*x)/2]] - b^4*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 12*a
^2*b^2*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + b^4*Log[Cos[(c + d*x)/2]
 + Sin[(c + d*x)/2]] + b*Cos[2*(c + d*x)]*(8*a^3*(c + d*x) - b*(12*a^2 + b
^2)*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + b*(12*a^2 + b^2)*Log[Cos[(c
 + d*x)/2] + Sin[(c + d*x)/2]]) + (a^4 + 2*b^4)*Sin[c + d*x] + 8*a*b^3*Sin
[2*(c + d*x)] + a^4*Sin[3*(c + d*x)]))/(4*d)
```

3.480.3 Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.99, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.579$, Rules used = {3042, 4329, 3042, 4564, 3042, 4535, 24, 3042, 4533, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(c+dx)(a+b\sec(c+dx))^4\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^4}{\csc\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \; 4329$$

$$\frac{1}{2}\int \cos(c+dx)(a+b\sec(c+dx))\left(6ab^2\sec^2(c+dx)+b\left(6a^2+b^2\right)\sec(c+dx)+a\left(2a^2-b^2\right)\right)dx+\frac{b^2\sin(c+dx)(a+b\sec(c+dx))^2}{2d}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{2}\int \frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)\left(6ab^2\csc\left(c+dx+\frac{\pi}{2}\right)^2+b\left(6a^2+b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)+a\left(2a^2-b^2\right)\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{b^2\sin(c+dx)(a+b\sec(c+dx))^2}{2d}$$

$$\Big\downarrow \; 4564$$

$$\frac{1}{2}\left(\int \cos(c+dx)\left(8b\sec(c+dx)a^3+\left(2a^2-b^2\right)a^2+b^2\left(12a^2+b^2\right)\sec^2(c+dx)\right)dx+\frac{6ab^3\tan(c+dx)}{d}\right)+\frac{b^2\sin(c+dx)(a+b\sec(c+dx))^2}{2d}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{2}\left(\int \frac{8b\csc\left(c+dx+\frac{\pi}{2}\right)a^3+\left(2a^2-b^2\right)a^2+b^2\left(12a^2+b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{6ab^3\tan(c+dx)}{d}\right)+\frac{b^2\sin(c+dx)(a+b\sec(c+dx))^2}{2d}$$

$$\Big\downarrow \; 4535$$

$$\frac{1}{2}\left(8a^3b\int 1dx+\int \cos(c+dx)\left(\left(2a^2-b^2\right)a^2+b^2\left(12a^2+b^2\right)\sec^2(c+dx)\right)dx+\frac{6ab^3\tan(c+dx)}{d}\right)+\frac{b^2\sin(c+dx)(a+b\sec(c+dx))^2}{2d}$$

$$\Big\downarrow \; 24$$

$$\frac{1}{2}\left(\int \cos(c+dx)\left(\left(2a^2-b^2\right)a^2+b^2\left(12a^2+b^2\right)\sec^2(c+dx)\right)dx+8a^3bx+\frac{6ab^3\tan(c+dx)}{d}\right)+\frac{b^2\sin(c+dx)(a+b\sec(c+dx))^2}{2d}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{2}\left(\int \frac{\left(2a^2-b^2\right)a^2+b^2\left(12a^2+b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+8a^3bx+\frac{6ab^3\tan(c+dx)}{d}\right)+\frac{b^2\sin(c+dx)(a+b\sec(c+dx))^2}{2d}$$

$$\Big\downarrow \; 4533$$

$$\frac{1}{2}\left(b^2\left(12a^2+b^2\right)\int \sec(c+dx)dx+8a^3bx+\frac{a^2\left(2a^2-b^2\right)\sin(c+dx)}{d}+\frac{6ab^3\tan(c+dx)}{d}\right)+\frac{b^2\sin(c+dx)(a+b\sec(c+dx))^2}{2d}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{2}\left(b^2\left(12a^2+b^2\right)\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+8a^3bx+\frac{a^2\left(2a^2-b^2\right)\sin(c+dx)}{d}+\frac{6ab^3\tan(c+dx)}{d}\right)+\frac{b^2\sin(c+dx)(a+b\sec(c+dx))^2}{2d}$$

3.480. $\int \cos(c+dx)(a+b\sec(c+dx))^4\,dx$

$$\Big\downarrow \text{4257}$$

$$\frac{1}{2}\left(8a^3bx+\frac{b^2\left(12a^2+b^2\right)\text{arctanh}(\sin(c+dx))}{d}+\frac{a^2\left(2a^2-b^2\right)\sin(c+dx)}{d}+\frac{6ab^3\tan(c+dx)}{d}\right)+\frac{b^2\sin(c+dx)(a+b\sec(c+dx))^2}{2d}$$

input
```
Int[Cos[c + d*x]*(a + b*Sec[c + d*x])^4,x]
```

output
```
(b^2*(a + b*Sec[c + d*x])^2*Sin[c + d*x])/(2*d) + (8*a^3*b*x + (b^2*(12*a^
2 + b^2)*ArcTanh[Sin[c + d*x]])/d + (a^2*(2*a^2 - b^2)*Sin[c + d*x])/d + (
6*a*b^3*Tan[c + d*x])/d)/2
```

3.480.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4329
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
2)*((d*Csc[e + f*x])^n/(f*(m + n - 1))), x] + Simp[1/(d*(m + n - 1))   Int[
(a + b*Csc[e + f*x])^(m - 3)*(d*Csc[e + f*x])^n*Simp[a^3*d*(m + n - 1) + a*
b^2*d*n + b*(b^2*d*(m + n - 2) + 3*a^2*d*(m + n - 1))*Csc[e + f*x] + a*b^2*
d*(3*m + 2*n - 4)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, n}, x
] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
 &&  !(IGtQ[n, 2] &&  !IntegerQ[m])
```

rule 4533
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] +
Simp[(C*m + A*(m + 1))/(b^2*m)   Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fr
eeQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

rule 4535
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Simp[B/b   Int[(b*Cs
c[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2)
, x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

rule 4564
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-b)*C*Csc[e + f*x]*Cot[e + f*x]*((d*Csc[e + f*x])^
n/(f*(n + 2))), x] + Simp[1/(n + 2)   Int[(d*Csc[e + f*x])^n*Simp[A*a*(n +
2) + (B*a*(n + 2) + b*(C*(n + 1) + A*(n + 2)))*Csc[e + f*x] + (a*C + B*b)*(
n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x]
&&  !LtQ[n, -1]
```

3.480.4 Maple [A] (verified)

Time = 1.00 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.92

method	result
derivativedivides	$\frac{a^4\sin(dx+c)+4a^3b(dx+c)+6a^2b^2\ln(\sec(dx+c)+\tan(dx+c))+4\tan(dx+c)a\,b^3+b^4\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+}{2}\right.}{d}$
default	$\frac{a^4\sin(dx+c)+4a^3b(dx+c)+6a^2b^2\ln(\sec(dx+c)+\tan(dx+c))+4\tan(dx+c)a\,b^3+b^4\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+}{2}\right.}{d}$
parallelrisch	$\frac{-12\left(a^2+\frac{b^2}{12}\right)(1+\cos(2dx+2c))b^2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+12\left(a^2+\frac{b^2}{12}\right)(1+\cos(2dx+2c))b^2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+8a^3bxd}{2d(1+\cos(2dx+2c))}$
risch	$4a^3bx-\frac{ia^4e^{i(dx+c)}}{2d}+\frac{ia^4e^{-i(dx+c)}}{2d}-\frac{ib^3\left(b\,e^{3i(dx+c)}-8a\,e^{2i(dx+c)}-b\,e^{i(dx+c)}-8a\right)}{d\left(e^{2i(dx+c)}+1\right)^2}-\frac{6\ln\left(e^{i(dx+c)}-i\right)a^2b^2}{d}$
norman	$\frac{\frac{\left(2a^4-8a\,b^3+b^4\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{d}+\frac{\left(6a^4+8a\,b^3-b^4\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{d}-4a^3bx-\frac{\left(2a^4+8a\,b^3+b^4\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}-\frac{\left(6a^4-8a\,b^3-b^4\right)\tan(}{d}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)\left(-1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^3}$

input
```
int(cos(d*x+c)*(a+b*sec(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^4*sin(d*x+c)+4*a^3*b*(d*x+c)+6*a^2*b^2*ln(sec(d*x+c)+tan(d*x+c))+4*
tan(d*x+c)*a*b^3+b^4*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+
c))))
```

3.480.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.25

$$\int \cos(c+dx)(a+b\sec(c+dx))^4\,dx$$
$$= \frac{16\,a^3bdx\cos\left(dx+c\right)^2+\left(12\,a^2b^2+b^4\right)\cos\left(dx+c\right)^2\log\left(\sin\left(dx+c\right)+1\right)-\left(12\,a^2b^2+b^4\right)\cos\left(dx+c\right)^2\text{lo}}{4\,d\cos\left(dx+c\right)^2}$$

input
```
integrate(cos(d*x+c)*(a+b*sec(d*x+c))^4,x, algorithm="fricas")
```

output
```
1/4*(16*a^3*b*d*x*cos(d*x + c)^2 + (12*a^2*b^2 + b^4)*cos(d*x + c)^2*log(s
in(d*x + c) + 1) - (12*a^2*b^2 + b^4)*cos(d*x + c)^2*log(-sin(d*x + c) + 1
) + 2*(2*a^4*cos(d*x + c)^2 + 8*a*b^3*cos(d*x + c) + b^4)*sin(d*x + c))/(d
*cos(d*x + c)^2)
```

3.480.6 Sympy [F]

$$\int \cos(c+dx)(a+b\sec(c+dx))^4\,dx = \int \left(a+b\sec\left(c+dx\right)\right)^4\cos\left(c+dx\right)dx$$

input
```
integrate(cos(d*x+c)*(a+b*sec(d*x+c))**4,x)
```

output
```
Integral((a + b*sec(c + d*x))**4*cos(c + d*x), x)
```

3.480.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.11

$$\int \cos(c+dx)(a+b\sec(c+dx))^4\,dx$$
$$= \frac{16\,(dx+c)a^3b-b^4\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1}-\log\left(\sin\left(dx+c\right)+1\right)+\log\left(\sin\left(dx+c\right)-1\right)\right)+12\,a^2b^2(\log\left(\sin\left(dx+\right.\right.}{4\,d}$$

input
```
integrate(cos(d*x+c)*(a+b*sec(d*x+c))^4,x, algorithm="maxima")
```

output
```
1/4*(16*(d*x + c)*a^3*b - b^4*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(s
in(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 12*a^2*b^2*(log(sin(d*x + c) +
 1) - log(sin(d*x + c) - 1)) + 4*a^4*sin(d*x + c) + 16*a*b^3*tan(d*x + c))
/d
```

3.480.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.72

$$\int \cos(c+dx)(a+b\sec(c+dx))^4\,dx$$
$$= \frac{8\,(dx+c)a^3b + \frac{4\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1} + (12\,a^2b^2+b^4)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right) - (12\,a^2b^2+b^4)\log\left(\left|\tan\left(\frac{1}{2}\,d\right.\right.\right.}{2\,d}$$

input
```
integrate(cos(d*x+c)*(a+b*sec(d*x+c))^4,x, algorithm="giac")
```

output
```
1/2*(8*(d*x + c)*a^3*b + 4*a^4*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^
2 + 1) + (12*a^2*b^2 + b^4)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - (12*a^2*b
^2 + b^4)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(8*a*b^3*tan(1/2*d*x + 1/
2*c)^3 - b^4*tan(1/2*d*x + 1/2*c)^3 - 8*a*b^3*tan(1/2*d*x + 1/2*c) - b^4*t
an(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^2)/d
```

3.480.9 Mupad [B] (verification not implemented)

Time = 13.80 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.46

$$\int \cos(c+dx)(a+b\sec(c+dx))^4\,dx = \frac{a^4\,\sin(c+d\,x)}{d} + \frac{b^4\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d} + \frac{b^4\,\sin(c+d\,x)}{2\,d\cos(c+d\,x)^2} + \frac{12\,a^2\,b^2\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d} + \frac{8\,a^3\,b\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d} + \frac{4\,a\,b^3\,\sin(c+d\,x)}{d\,\cos(c+d\,x)}$$

input
```
int(cos(c + d*x)*(a + b/cos(c + d*x))^4,x)
```

output
```
(a^4*sin(c + d*x))/d + (b^4*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/
d + (b^4*sin(c + d*x))/(2*d*cos(c + d*x)^2) + (12*a^2*b^2*atanh(sin(c/2 +
(d*x)/2)/cos(c/2 + (d*x)/2)))/d + (8*a^3*b*atan(sin(c/2 + (d*x)/2)/cos(c/2
 + (d*x)/2)))/d + (4*a*b^3*sin(c + d*x))/(d*cos(c + d*x))
```

3.480. $\int \cos(c+dx)(a+b\sec(c+dx))^4\,dx$

3.481 $\int \cos^2(c+dx)(a+b\sec(c+dx))^4\,dx$

3.481.1 Optimal result . . . 3551
3.481.2 Mathematica [A] (verified) . . . 3551
3.481.3 Rubi [A] (verified) . . . 3552
3.481.4 Maple [A] (verified) . . . 3555
3.481.5 Fricas [A] (verification not implemented) . . . 3556
3.481.6 Sympy [F] . . . 3556
3.481.7 Maxima [A] (verification not implemented) . . . 3556
3.481.8 Giac [A] (verification not implemented) . . . 3557
3.481.9 Mupad [B] (verification not implemented) . . . 3557

3.481.1 Optimal result

Integrand size = 21, antiderivative size = 108

$$\int \cos^2(c+dx)(a+b\sec(c+dx))^4\,dx = \frac{1}{2}a^2\left(a^2+12b^2\right)x+\frac{4ab^3\operatorname{arctanh}(\sin(c+dx))}{d}+\frac{3a^3b\sin(c+dx)}{d}+\frac{a^2\cos(c+dx)(a+b\sec(c+dx))^2\sin(c+dx)}{2d}-\frac{b^2(a^2-2b^2)\tan(c+dx)}{2d}$$

output
```
1/2*a^2*(a^2+12*b^2)*x+4*a*b^3*arctanh(sin(d*x+c))/d+3*a^3*b*sin(d*x+c)/d+
1/2*a^2*cos(d*x+c)*(a+b*sec(d*x+c))^2*sin(d*x+c)/d-1/2*b^2*(a^2-2*b^2)*tan
(d*x+c)/d
```

3.481.2 Mathematica [A] (verified)

Time = 0.98 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.10

$$\int \cos^2(c+dx)(a+b\sec(c+dx))^4\,dx = \frac{2a\left(a(a^2+12b^2)(c+dx)-8b^3\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+8b^3\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}\right.\right.\right.}{4d}$$

input
```
Integrate[Cos[c + d*x]^2*(a + b*Sec[c + d*x])^4,x]
```

output
```
(2*a*(a*(a^2 + 12*b^2)*(c + d*x) - 8*b^3*Log[Cos[(c + d*x)/2] - Sin[(c + d
*x)/2]] + 8*b^3*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + 16*a^3*b*Sin[c
 + d*x] + a^4*Sin[2*(c + d*x)] + 4*b^4*Tan[c + d*x])/(4*d)
```

3.481.3 Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.524$, Rules used = {3042, 4328, 3042, 4564, 3042, 4535, 24, 3042, 4533, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(c+dx)(a+b\sec(c+dx))^4\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^4}{\csc\left(c+dx+\frac{\pi}{2}\right)^2}dx$$

$$\Big\downarrow \text{4328}$$

$$\frac{1}{2}\int \cos(c+dx)(a+b\sec(c+dx))\left(6ba^2+\left(a^2+6b^2\right)\sec(c+dx)a-b\left(a^2-2b^2\right)\sec^2(c+dx)\right)dx+\frac{a^2\sin(c+dx)\cos(c+dx)(a+b\sec(c+dx))^2}{2d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\int \frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)\left(6ba^2+\left(a^2+6b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)a-b\left(a^2-2b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{a^2\sin(c+dx)\cos(c+dx)(a+b\sec(c+dx))^2}{2d}$$

$$\Big\downarrow \text{4564}$$

$$\frac{1}{2}\left(\int \cos(c+dx)\left(6ba^3+\left(a^2+12b^2\right)\sec(c+dx)a^2+8b^3\sec^2(c+dx)a\right)dx-\frac{b^2\left(a^2-2b^2\right)\tan(c+dx)}{d}\right)+\frac{a^2\sin(c+dx)\cos(c+dx)(a+b\sec(c+dx))^2}{2d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\left(\int \frac{6ba^3+\left(a^2+12b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)a^2+8b^3\csc\left(c+dx+\frac{\pi}{2}\right)^2a}{\csc\left(c+dx+\frac{\pi}{2}\right)}dx-\frac{b^2\left(a^2-2b^2\right)\tan(c+dx)}{d}\right)+\frac{a^2\sin(c+dx)\cos(c+dx)(a+b\sec(c+dx))^2}{2d}$$

$$\Big\downarrow \text{4535}$$

$$\frac{1}{2}\left(\int \cos(c+dx)\left(6ba^3+8b^3\sec^2(c+dx)a\right)dx+a^2\left(a^2+12b^2\right)\int 1dx-\frac{b^2\left(a^2-2b^2\right)\tan(c+dx)}{d}\right)+\frac{a^2\sin(c+dx)\cos(c+dx)(a+b\sec(c+dx))^2}{2d}$$

$$\Big\downarrow \text{24}$$

$$\frac{1}{2}\left(\int \cos(c+dx)\left(6ba^3+8b^3\sec^2(c+dx)a\right)dx-\frac{b^2\left(a^2-2b^2\right)\tan(c+dx)}{d}+a^2x\left(a^2+12b^2\right)\right)+\frac{a^2\sin(c+dx)\cos(c+dx)(a+b\sec(c+dx))^2}{2d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\left(\int \frac{6ba^3+8b^3\csc\left(c+dx+\frac{\pi}{2}\right)^2a}{\csc\left(c+dx+\frac{\pi}{2}\right)}dx-\frac{b^2\left(a^2-2b^2\right)\tan(c+dx)}{d}+a^2x\left(a^2+12b^2\right)\right)+\frac{a^2\sin(c+dx)\cos(c+dx)(a+b\sec(c+dx))^2}{2d}$$

$$\Big\downarrow \text{4533}$$

$$\frac{1}{2}\left(8ab^3\int \sec(c+dx)dx+\frac{6a^3b\sin(c+dx)}{d}-\frac{b^2\left(a^2-2b^2\right)\tan(c+dx)}{d}+a^2x\left(a^2+12b^2\right)\right)+\frac{a^2\sin(c+dx)\cos(c+dx)(a+b\sec(c+dx))^2}{2d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\left(8ab^3\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{6a^3b\sin(c+dx)}{d}-\frac{b^2\left(a^2-2b^2\right)\tan(c+dx)}{d}+a^2x\left(a^2+12b^2\right)\right)+\frac{a^2\sin(c+dx)\cos(c+dx)(a+b\sec(c+dx))^2}{2d}$$

$$\Big\downarrow \text{4257}$$

3.481. $\int \cos^2(c+dx)(a+b\sec(c+dx))^4\,dx$

$$\frac{a^2\sin(c+dx)\cos(c+dx)(a+b\sec(c+dx))^2}{2d}+\frac{1}{2}\left(\frac{6a^3b\sin(c+dx)}{d}-\frac{b^2(a^2-2b^2)\tan(c+dx)}{d}+a^2x(a^2+12b^2)+\frac{8ab^3\text{arctanh}(\sin(c+dx))}{d}\right)$$

input
```
Int[Cos[c + d*x]^2*(a + b*Sec[c + d*x])^4,x]
```

output
```
(a^2*Cos[c + d*x]*(a + b*Sec[c + d*x])^2*Sin[c + d*x])/(2*d) + (a^2*(a^2 +
 12*b^2)*x + (8*a*b^3*ArcTanh[Sin[c + d*x]])/d + (6*a^3*b*Sin[c + d*x])/d
- (b^2*(a^2 - 2*b^2)*Tan[c + d*x])/d)/2
```

3.481.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4328
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)*
((d*Csc[e + f*x])^n/(f*n)), x] - Simp[1/(d*n)   Int[(a + b*Csc[e + f*x])^(m
 - 3)*(d*Csc[e + f*x])^(n + 1)*Simp[a^2*b*(m - 2*n - 2) - a*(3*b^2*n + a^2*
(n + 1))*Csc[e + f*x] - b*(b^2*n + a^2*(m + n - 1))*Csc[e + f*x]^2, x], x],
 x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && ((Int
egerQ[m] && LtQ[n, -1]) || (IntegersQ[m + 1/2, 2*n] && LeQ[n, -1]))
```

rule 4533
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] +
Simp[(C*m + A*(m + 1))/(b^2*m)   Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fr
eeQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

rule 4535
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Simp[B/b   Int[(b*Cs
c[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2)
, x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

rule 4564
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-b)*C*Csc[e + f*x]*Cot[e + f*x]*((d*Csc[e + f*x])^
n/(f*(n + 2))), x] + Simp[1/(n + 2)   Int[(d*Csc[e + f*x])^n*Simp[A*a*(n +
2) + (B*a*(n + 2) + b*(C*(n + 1) + A*(n + 2)))*Csc[e + f*x] + (a*C + B*b)*(
n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x]
&&  !LtQ[n, -1]
```

3.481.4 Maple [A] (verified)

Time = 0.90 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.81

method	result
derivativedivides	$\frac{a^4\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+4a^3b\sin(dx+c)+6a^2b^2(dx+c)+4a\,b^3\ln(\sec(dx+c)+\tan(dx+c))+\tan(dx+c)b^4}{d}$
default	$\frac{a^4\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+4a^3b\sin(dx+c)+6a^2b^2(dx+c)+4a\,b^3\ln(\sec(dx+c)+\tan(dx+c))+\tan(dx+c)b^4}{d}$
parallelrisch	$\frac{-32\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)a\,b^3\cos(dx+c)+32\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)a\,b^3\cos(dx+c)+16a^3b\sin(2dx+2c)+a^4\sin(3dx+3c)+}{8d\cos(dx+c)}$
risch	$\frac{a^4x}{2}+6a^2b^2x-\frac{ia^4e^{2i(dx+c)}}{8d}-\frac{2ia^3b\,e^{i(dx+c)}}{d}+\frac{2ia^3b\,e^{-i(dx+c)}}{d}+\frac{ia^4e^{-2i(dx+c)}}{8d}+\frac{2ib^4}{d(e^{2i(dx+c)}+1)}-\frac{4a}{}$
norman	$\frac{(-\frac{1}{2}a^4-6a^2b^2)x+(-\frac{1}{2}a^4-6a^2b^2)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8+(\frac{1}{2}a^4+6a^2b^2)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+(\frac{1}{2}a^4+6a^2b^2)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}+(-a}{}$

input
```
int(cos(d*x+c)^2*(a+b*sec(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^4*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+4*a^3*b*sin(d*x+c)+6*a^
2*b^2*(d*x+c)+4*a*b^3*ln(sec(d*x+c)+tan(d*x+c))+tan(d*x+c)*b^4)
```

3.481.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.07

$$\int \cos^2(c+dx)(a+b\sec(c+dx))^4\,dx$$
$$= \frac{4\,ab^3\cos(dx+c)\log(\sin(dx+c)+1) - 4\,ab^3\cos(dx+c)\log(-\sin(dx+c)+1) + (a^4+12\,a^2b^2)dx\cos}{2\,d\cos(dx+c)}$$

input
```
integrate(cos(d*x+c)^2*(a+b*sec(d*x+c))^4,x, algorithm="fricas")
```

output
```
1/2*(4*a*b^3*cos(d*x + c)*log(sin(d*x + c) + 1) - 4*a*b^3*cos(d*x + c)*log
(-sin(d*x + c) + 1) + (a^4 + 12*a^2*b^2)*d*x*cos(d*x + c) + (a^4*cos(d*x +
 c)^2 + 8*a^3*b*cos(d*x + c) + 2*b^4)*sin(d*x + c))/(d*cos(d*x + c))
```

3.481.6 Sympy [F]

$$\int \cos^2(c+dx)(a+b\sec(c+dx))^4\,dx = \int (a+b\sec(c+dx))^4\cos^2(c+dx)\,dx$$

input
```
integrate(cos(d*x+c)**2*(a+b*sec(d*x+c))**4,x)
```

output
```
Integral((a + b*sec(c + d*x))**4*cos(c + d*x)**2, x)
```

3.481.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.83

$$\int \cos^2(c+dx)(a+b\sec(c+dx))^4\,dx$$
$$= \frac{(2\,dx+2\,c+\sin(2\,dx+2\,c))a^4 + 24\,(dx+c)a^2b^2 + 8\,ab^3(\log(\sin(dx+c)+1) - \log(\sin(dx+c)-1)) +}{4\,d}$$

input
```
integrate(cos(d*x+c)^2*(a+b*sec(d*x+c))^4,x, algorithm="maxima")
```

output
```
1/4*((2*d*x + 2*c + sin(2*d*x + 2*c))*a^4 + 24*(d*x + c)*a^2*b^2 + 8*a*b^3
*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 16*a^3*b*sin(d*x + c) +
 4*b^4*tan(d*x + c))/d
```

3.481.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.57

$$\int \cos^2(c+dx)(a+b\sec(c+dx))^4\,dx$$
$$= \frac{8\,ab^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-8\,ab^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{4\,b^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1}+(a^4+12\,a^2b^2)(dx}{2\,d}$$

input
```
integrate(cos(d*x+c)^2*(a+b*sec(d*x+c))^4,x, algorithm="giac")
```

output
```
1/2*(8*a*b^3*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 8*a*b^3*log(abs(tan(1/2*
d*x + 1/2*c) - 1)) - 4*b^4*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^2 -
1) + (a^4 + 12*a^2*b^2)*(d*x + c) - 2*(a^4*tan(1/2*d*x + 1/2*c)^3 - 8*a^3*
b*tan(1/2*d*x + 1/2*c)^3 - a^4*tan(1/2*d*x + 1/2*c) - 8*a^3*b*tan(1/2*d*x
+ 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^2)/d
```

3.481.9 Mupad [B] (verification not implemented)

Time = 14.20 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.39

$$\int \cos^2(c+dx)(a+b\sec(c+dx))^4\,dx = \frac{a^4\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{b^4\,\sin\left(c+d\,x\right)}{d\,\cos\left(c+d\,x\right)}+\frac{12\,a^2\,b^2\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{a^4\,\cos\left(c+d\,x\right)\,\sin\left(c+d\,x\right)}{2\,d}+\frac{4\,a^3\,b\,\sin\left(c+d\,x\right)}{d}+\frac{8\,a\,b^3\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}$$

input
```
int(cos(c + d*x)^2*(a + b/cos(c + d*x))^4,x)
```

output
```
(a^4*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (b^4*sin(c + d*x))/(
d*cos(c + d*x)) + (12*a^2*b^2*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))
/d + (a^4*cos(c + d*x)*sin(c + d*x))/(2*d) + (4*a^3*b*sin(c + d*x))/d + (8
*a*b^3*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d
```

3.482 $\int \cos^3(c+dx)(a+b\sec(c+dx))^4\,dx$

3.482.1 Optimal result . . . 3558
3.482.2 Mathematica [A] (verified) . . . 3558
3.482.3 Rubi [A] (verified) . . . 3559
3.482.4 Maple [A] (verified) . . . 3562
3.482.5 Fricas [A] (verification not implemented) . . . 3563
3.482.6 Sympy [F] . . . 3564
3.482.7 Maxima [A] (verification not implemented) . . . 3564
3.482.8 Giac [A] (verification not implemented) . . . 3564
3.482.9 Mupad [B] (verification not implemented) . . . 3565

3.482.1 Optimal result

Integrand size = 21, antiderivative size = 115

$$\int \cos^3(c+dx)(a+b\sec(c+dx))^4\,dx = 2ab(a^2+2b^2)\,x+\frac{b^4\text{arctanh}(\sin(c+dx))}{d}+\frac{a^2(2a^2+17b^2)\sin(c+dx)}{3d}+\frac{4a^3b\cos(c+dx)\sin(c+dx)}{3d}+\frac{a^2\cos^2(c+dx)(a+b\sec(c+dx))^2\sin(c+dx)}{3d}$$

output
```
2*a*b*(a^2+2*b^2)*x+b^4*arctanh(sin(d*x+c))/d+1/3*a^2*(2*a^2+17*b^2)*sin(d
*x+c)/d+4/3*a^3*b*cos(d*x+c)*sin(d*x+c)/d+1/3*a^2*cos(d*x+c)^2*(a+b*sec(d*
x+c))^2*sin(d*x+c)/d
```

3.482.2 Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.11

$$\int \cos^3(c+dx)(a+b\sec(c+dx))^4\,dx = \frac{24ab(a^2+2b^2)(c+dx)-12b^4\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+12b^4\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin(\right.}{12d}$$

input
```
Integrate[Cos[c + d*x]^3*(a + b*Sec[c + d*x])^4,x]
```

output
```
(24*a*b*(a^2 + 2*b^2)*(c + d*x) - 12*b^4*Log[Cos[(c + d*x)/2] - Sin[(c + d
*x)/2]] + 12*b^4*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + 9*a^2*(a^2 + 8
*b^2)*Sin[c + d*x] + 12*a^3*b*Sin[2*(c + d*x)] + a^4*Sin[3*(c + d*x)])/(12
*d)
```

3.482.3 Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.01, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {3042, 4328, 3042, 4562, 27, 3042, 4535, 24, 3042, 4533, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^3(c+dx)(a+b\sec(c+dx))^4\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^4}{\csc\left(c+dx+\frac{\pi}{2}\right)^3}dx$$

$$\Big\downarrow \text{4328}$$

$$\frac{1}{3}\int \cos^2(c+dx)(a+b\sec(c+dx))\left(3\sec^2(c+dx)b^3+8a^2b+a\left(2a^2+9b^2\right)\sec(c+dx)\right)dx+\frac{a^2\sin(c+dx)\cos^2(c+dx)(a+b\sec(c+dx))^2}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\int \frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)\left(3\csc\left(c+dx+\frac{\pi}{2}\right)^2b^3+8a^2b+a\left(2a^2+9b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^2}dx+\frac{a^2\sin(c+dx)\cos^2(c+dx)(a+b\sec(c+dx))^2}{3d}$$

$$\Big\downarrow \text{4562}$$

$$\frac{1}{3}\left(\frac{4a^3b\sin(c+dx)\cos(c+dx)}{d}-\frac{1}{2}\int -2\cos(c+dx)\left(3\sec^2(c+dx)b^4+6a\left(a^2+2b^2\right)\sec(c+dx)b+a^2\left(2a^2+1\right.\right.\right.$$

$$\frac{a^2\sin(c+dx)\cos^2(c+dx)(a+b\sec(c+dx))^2}{3d}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{3}\left(\int \cos(c+dx)\left(3\sec^2(c+dx)b^4+6a\left(a^2+2b^2\right)\sec(c+dx)b+a^2\left(2a^2+17b^2\right)\right)dx+\frac{4a^3b\sin(c+dx)\cos(c+d}{d}\right.$$
$$\frac{a^2\sin(c+dx)\cos^2(c+dx)(a+b\sec(c+dx))^2}{3d}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{3}\left(\int \frac{3\csc\left(c+dx+\frac{\pi}{2}\right)^2b^4+6a\left(a^2+2b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)b+a^2\left(2a^2+17b^2\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{4a^3b\sin(c+dx)\cos(c+dx)}{d}\right.$$
$$\frac{a^2\sin(c+dx)\cos^2(c+dx)(a+b\sec(c+dx))^2}{3d}$$

$$\Big\downarrow \; 4535$$

$$\frac{1}{3}\left(6ab\left(a^2+2b^2\right)\int 1dx+\int \cos(c+dx)\left(3\sec^2(c+dx)b^4+a^2\left(2a^2+17b^2\right)\right)dx+\frac{4a^3b\sin(c+dx)\cos(c+dx)}{d}\right)$$
$$\frac{a^2\sin(c+dx)\cos^2(c+dx)(a+b\sec(c+dx))^2}{3d}$$

$$\Big\downarrow \; 24$$

$$\frac{1}{3}\left(\int \cos(c+dx)\left(3\sec^2(c+dx)b^4+a^2\left(2a^2+17b^2\right)\right)dx+\frac{4a^3b\sin(c+dx)\cos(c+dx)}{d}+6abx\left(a^2+2b^2\right)\right)+$$
$$\frac{a^2\sin(c+dx)\cos^2(c+dx)(a+b\sec(c+dx))^2}{3d}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{3}\left(\int \frac{3\csc\left(c+dx+\frac{\pi}{2}\right)^2b^4+a^2\left(2a^2+17b^2\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{4a^3b\sin(c+dx)\cos(c+dx)}{d}+6abx\left(a^2+2b^2\right)\right)+$$
$$\frac{a^2\sin(c+dx)\cos^2(c+dx)(a+b\sec(c+dx))^2}{3d}$$

$$\Big\downarrow \; 4533$$

$$\frac{1}{3}\left(3b^4\int \sec(c+dx)dx+\frac{4a^3b\sin(c+dx)\cos(c+dx)}{d}+\frac{a^2\left(2a^2+17b^2\right)\sin(c+dx)}{d}+6abx\left(a^2+2b^2\right)\right)+$$
$$\frac{a^2\sin(c+dx)\cos^2(c+dx)(a+b\sec(c+dx))^2}{3d}$$

$$\Big\downarrow \; 3042$$

3.482. $\int \cos^3(c+dx)(a+b\sec(c+dx))^4\,dx$

$$\frac{1}{3}\left(3b^4\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{4a^3b\sin(c+dx)\cos(c+dx)}{d}+\frac{a^2(2a^2+17b^2)\sin(c+dx)}{d}+6abx(a^2+2b^2)\right)+\frac{a^2\sin(c+dx)\cos^2(c+dx)(a+b\sec(c+dx))^2}{3d}$$

$$\Big\downarrow \; 4257$$

$$\frac{a^2\sin(c+dx)\cos^2(c+dx)(a+b\sec(c+dx))^2}{3d}+\frac{1}{3}\left(\frac{4a^3b\sin(c+dx)\cos(c+dx)}{d}+\frac{a^2(2a^2+17b^2)\sin(c+dx)}{d}+6abx(a^2+2b^2)+\frac{3b^4\text{arctanh}(\sin(c+dx))}{d}\right)$$

input
```
Int[Cos[c + d*x]^3*(a + b*Sec[c + d*x])^4,x]
```

output
```
(a^2*Cos[c + d*x]^2*(a + b*Sec[c + d*x])^2*Sin[c + d*x])/(3*d) + (6*a*b*(a
^2 + 2*b^2)*x + (3*b^4*ArcTanh[Sin[c + d*x]])/d + (a^2*(2*a^2 + 17*b^2)*Si
n[c + d*x])/d + (4*a^3*b*Cos[c + d*x]*Sin[c + d*x])/d)/3
```

3.482.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4328
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)*
((d*Csc[e + f*x])^n/(f*n)), x] - Simp[1/(d*n)   Int[(a + b*Csc[e + f*x])^(m
 - 3)*(d*Csc[e + f*x])^(n + 1)*Simp[a^2*b*(m - 2*n - 2) - a*(3*b^2*n + a^2*
(n + 1))*Csc[e + f*x] - b*(b^2*n + a^2*(m + n - 1))*Csc[e + f*x]^2, x], x],
 x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && ((Int
egerQ[m] && LtQ[n, -1]) || (IntegersQ[m + 1/2, 2*n] && LeQ[n, -1]))
```

rule 4533
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] +
Simp[(C*m + A*(m + 1))/(b^2*m)   Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fr
eeQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

rule 4535
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Simp[B/b   Int[(b*Cs
c[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2)
, x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

rule 4562
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_)), x_Symbol] :> Simp[A*a*Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*n)), x] + Si
mp[1/(d*n)   Int[(d*Csc[e + f*x])^(n + 1)*Simp[n*(B*a + A*b) + (n*(a*C + B*
b) + A*a*(n + 1))*Csc[e + f*x] + b*C*n*Csc[e + f*x]^2, x], x], x] /; FreeQ[
{a, b, d, e, f, A, B, C}, x] && LtQ[n, -1]
```

3.482.4 Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.85

method	result
derivativedivides	$\frac{\frac{a^4\left(2+\cos(dx+c)^2\right)\sin(dx+c)}{3}+4a^3b\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+6a^2b^2\sin(dx+c)+4a\,b^3(dx+c)+b^4\ln(\sec(dx+c)+\tan(}{d}$
default	$\frac{\frac{a^4\left(2+\cos(dx+c)^2\right)\sin(dx+c)}{3}+4a^3b\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+6a^2b^2\sin(dx+c)+4a\,b^3(dx+c)+b^4\ln(\sec(dx+c)+\tan(}{d}$
parallelrisch	$\frac{24a^3bxd+48a\,b^3dx+12\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)b^4-12\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)b^4+a^4\sin(3dx+3c)+12a^3b\sin(2dx+2c)+9a^4\sin}{12d}$
risch	$2a^3bx+4a\,b^3x-\frac{3ia^4e^{i(dx+c)}}{8d}-\frac{3ie^{i(dx+c)}a^2b^2}{d}+\frac{3ia^4e^{-i(dx+c)}}{8d}+\frac{3ie^{-i(dx+c)}a^2b^2}{d}+\frac{\ln\left(e^{i(dx+c)}+i\right)b^4}{d}$
norman	$\frac{(-2a^3b-4a\,b^3)x+(-6a^3b-12a\,b^3)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8+(2a^3b+4a\,b^3)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{12}+(6a^3b+12a\,b^3)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\frac{4a}{}}{}$

input
```
int(cos(d*x+c)^3*(a+b*sec(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/3*a^4*(2+cos(d*x+c)^2)*sin(d*x+c)+4*a^3*b*(1/2*cos(d*x+c)*sin(d*x+c
)+1/2*d*x+1/2*c)+6*a^2*b^2*sin(d*x+c)+4*a*b^3*(d*x+c)+b^4*ln(sec(d*x+c)+ta
n(d*x+c)))
```

3.482.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.85

$$\int \cos^3(c+dx)(a+b\sec(c+dx))^4\,dx$$
$$=\frac{3\,b^4\log\left(\sin\left(dx+c\right)+1\right)-3\,b^4\log\left(-\sin\left(dx+c\right)+1\right)+12\left(a^3b+2\,ab^3\right)dx+2\left(a^4\cos\left(dx+c\right)^2+6\,a^3b\right.}{6\,d}$$

input
```
integrate(cos(d*x+c)^3*(a+b*sec(d*x+c))^4,x, algorithm="fricas")
```

output
```
1/6*(3*b^4*log(sin(d*x + c) + 1) - 3*b^4*log(-sin(d*x + c) + 1) + 12*(a^3*
b + 2*a*b^3)*d*x + 2*(a^4*cos(d*x + c)^2 + 6*a^3*b*cos(d*x + c) + 2*a^4 +
18*a^2*b^2)*sin(d*x + c))/d
```

3.482.6 Sympy [F]

$$\int \cos^3(c+dx)(a+b\sec(c+dx))^4\,dx = \int (a+b\sec(c+dx))^4\cos^3(c+dx)\,dx$$

input
```
integrate(cos(d*x+c)**3*(a+b*sec(d*x+c))**4,x)
```

output
```
Integral((a + b*sec(c + d*x))**4*cos(c + d*x)**3, x)
```

3.482.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.89

$$\int \cos^3(c+dx)(a+b\sec(c+dx))^4\,dx = -\frac{2\left(\sin(dx+c)^3-3\sin(dx+c)\right)a^4-6\left(2\,dx+2\,c+\sin(2\,dx+2\,c)\right)a^3b-24\,(dx+c)ab^3-3\,b^4(\log(\mathrm{si}}{6\,d}$$

input
```
integrate(cos(d*x+c)^3*(a+b*sec(d*x+c))^4,x, algorithm="maxima")
```

output
```
-1/6*(2*(sin(d*x + c)^3 - 3*sin(d*x + c))*a^4 - 6*(2*d*x + 2*c + sin(2*d*x
 + 2*c))*a^3*b - 24*(d*x + c)*a*b^3 - 3*b^4*(log(sin(d*x + c) + 1) - log(s
in(d*x + c) - 1)) - 36*a^2*b^2*sin(d*x + c))/d
```

3.482.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.84

$$\int \cos^3(c+dx)(a+b\sec(c+dx))^4\,dx = \frac{3\,b^4\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\,b^4\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+6\,(a^3b+2\,ab^3)(dx+c)+\frac{2\left(3\,a^4\tan\left(\frac{1}{2}\,d\right.\right.}{}}{}$$

input
```
integrate(cos(d*x+c)^3*(a+b*sec(d*x+c))^4,x, algorithm="giac")
```

output
```
1/3*(3*b^4*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*b^4*log(abs(tan(1/2*d*x
+ 1/2*c) - 1)) + 6*(a^3*b + 2*a*b^3)*(d*x + c) + 2*(3*a^4*tan(1/2*d*x + 1/
2*c)^5 - 6*a^3*b*tan(1/2*d*x + 1/2*c)^5 + 18*a^2*b^2*tan(1/2*d*x + 1/2*c)^
5 + 2*a^4*tan(1/2*d*x + 1/2*c)^3 + 36*a^2*b^2*tan(1/2*d*x + 1/2*c)^3 + 3*a
^4*tan(1/2*d*x + 1/2*c) + 6*a^3*b*tan(1/2*d*x + 1/2*c) + 18*a^2*b^2*tan(1/
2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^3)/d
```

3.482.9 Mupad [B] (verification not implemented)

Time = 13.14 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.37

$$\int \cos^3(c+dx)(a+b\sec(c+dx))^4\,dx = \frac{3\,a^4\,\sin\left(c+d\,x\right)}{4\,d}+\frac{2\,b^4\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{a^4\,\sin\left(3\,c+3\,d\,x\right)}{12\,d}+\frac{a^3\,b\,\sin\left(2\,c+2\,d\,x\right)}{d}+\frac{6\,a^2\,b^2\,\sin\left(c+d\,x\right)}{d}+\frac{8\,a\,b^3\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{4\,a^3\,b\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}$$

input
```
int(cos(c + d*x)^3*(a + b/cos(c + d*x))^4,x)
```

output
```
(3*a^4*sin(c + d*x))/(4*d) + (2*b^4*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*
x)/2)))/d + (a^4*sin(3*c + 3*d*x))/(12*d) + (a^3*b*sin(2*c + 2*d*x))/d + (
6*a^2*b^2*sin(c + d*x))/d + (8*a*b^3*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*
x)/2)))/d + (4*a^3*b*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d
```

3.483 $\int \cos^4(c+dx)(a+b\sec(c+dx))^4\,dx$

3.483.1 Optimal result . . . 3566
3.483.2 Mathematica [A] (verified) . . . 3566
3.483.3 Rubi [A] (verified) . . . 3567
3.483.4 Maple [A] (verified) . . . 3570
3.483.5 Fricas [A] (verification not implemented) . . . 3571
3.483.6 Sympy [F] . . . 3571
3.483.7 Maxima [A] (verification not implemented) . . . 3572
3.483.8 Giac [B] (verification not implemented) . . . 3572
3.483.9 Mupad [B] (verification not implemented) . . . 3573

3.483.1 Optimal result

Integrand size = 21, antiderivative size = 145

$$\int \cos^4(c+dx)(a+b\sec(c+dx))^4\,dx = \frac{1}{8}\left(3a^4+24a^2b^2+8b^4\right)x+\frac{4ab(2a^2+3b^2)\sin(c+dx)}{3d}+\frac{a^2(3a^2+22b^2)\cos(c+dx)\sin(c+dx)}{8d}+\frac{5a^3b\cos^2(c+dx)\sin(c+dx)}{6d}+\frac{a^2\cos^3(c+dx)(a+b\sec(c+dx))^2\sin(c+dx)}{4d}$$

output
```
1/8*(3*a^4+24*a^2*b^2+8*b^4)*x+4/3*a*b*(2*a^2+3*b^2)*sin(d*x+c)/d+1/8*a^2*
(3*a^2+22*b^2)*cos(d*x+c)*sin(d*x+c)/d+5/6*a^3*b*cos(d*x+c)^2*sin(d*x+c)/d
+1/4*a^2*cos(d*x+c)^3*(a+b*sec(d*x+c))^2*sin(d*x+c)/d
```

3.483.2 Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.72

$$\int \cos^4(c+dx)(a+b\sec(c+dx))^4\,dx = \frac{12(3a^4+24a^2b^2+8b^4)(c+dx)+96ab(3a^2+4b^2)\sin(c+dx)+24a^2(a^2+6b^2)\sin(2(c+dx))+32a^3b\,\mathrm{si}}{96d}$$

input
```
Integrate[Cos[c + d*x]^4*(a + b*Sec[c + d*x])^4,x]
```

output
```
(12*(3*a^4 + 24*a^2*b^2 + 8*b^4)*(c + d*x) + 96*a*b*(3*a^2 + 4*b^2)*Sin[c
+ d*x] + 24*a^2*(a^2 + 6*b^2)*Sin[2*(c + d*x)] + 32*a^3*b*Sin[3*(c + d*x)]
 + 3*a^4*Sin[4*(c + d*x)])/(96*d)
```

3.483.3 Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.06, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.524$, Rules used = {3042, 4328, 3042, 4562, 25, 3042, 4535, 3042, 3117, 4533, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^4(c+dx)(a+b\sec(c+dx))^4\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^4}{\csc\left(c+dx+\frac{\pi}{2}\right)^4}dx$$

$$\Big\downarrow \; 4328$$

$$\frac{1}{4}\int \cos^3(c+dx)(a+b\sec(c+dx))\left(10ba^2+3\left(a^2+4b^2\right)\sec(c+dx)a+b\left(a^2+4b^2\right)\sec^2(c+dx)\right)dx+\frac{a^2\sin(c+dx)\cos^3(c+dx)(a+b\sec(c+dx))^2}{4d}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{4}\int \frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)\left(10ba^2+3\left(a^2+4b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)a+b\left(a^2+4b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^3}dx+\frac{a^2\sin(c+dx)\cos^3(c+dx)(a+b\sec(c+dx))^2}{4d}$$

$$\Big\downarrow \; 4562$$

$$\frac{1}{4}\left(\frac{10a^3b\sin(c+dx)\cos^2(c+dx)}{3d}-\frac{1}{3}\int -\cos^2(c+dx)\left(3\left(3a^2+22b^2\right)a^2+16b\left(2a^2+3b^2\right)\sec(c+dx)a+3b^2(a\right.\right.$$
$$\frac{a^2\sin(c+dx)\cos^3(c+dx)(a+b\sec(c+dx))^2}{4d}$$

$$\Big\downarrow \; 25$$

$$\frac{1}{4}\left(\frac{1}{3}\int \cos^2(c+dx)\left(3\left(3a^2+22b^2\right)a^2+16b\left(2a^2+3b^2\right)\sec(c+dx)a+3b^2\left(a^2+4b^2\right)\sec^2(c+dx)\right)dx+\frac{10a^3b\,\mathrm{si}}{}\right.$$
$$\frac{a^2\sin(c+dx)\cos^3(c+dx)(a+b\sec(c+dx))^2}{4d}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{4}\left(\frac{1}{3}\int \frac{3\left(3a^2+22b^2\right)a^2+16b\left(2a^2+3b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)a+3b^2\left(a^2+4b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^2}dx+\frac{10a^3b\sin(c}{}\right.$$
$$\frac{a^2\sin(c+dx)\cos^3(c+dx)(a+b\sec(c+dx))^2}{4d}$$

$$\Big\downarrow \; 4535$$

$$\frac{1}{4}\left(\frac{1}{3}\left(16ab\left(2a^2+3b^2\right)\int \cos(c+dx)dx+\int \cos^2(c+dx)\left(3\left(3a^2+22b^2\right)a^2+3b^2\left(a^2+4b^2\right)\sec^2(c+dx)\right)dx\right)+\right.$$
$$\frac{a^2\sin(c+dx)\cos^3(c+dx)(a+b\sec(c+dx))^2}{4d}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{4}\left(\frac{1}{3}\left(16ab\left(2a^2+3b^2\right)\int \sin\left(c+dx+\frac{\pi}{2}\right)dx+\int \frac{3\left(3a^2+22b^2\right)a^2+3b^2\left(a^2+4b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^2}dx\right)+\frac{10}{}\right.$$
$$\frac{a^2\sin(c+dx)\cos^3(c+dx)(a+b\sec(c+dx))^2}{4d}$$

$$\Big\downarrow \; 3117$$

$$\frac{1}{4}\left(\frac{1}{3}\left(\int \frac{3\left(3a^2+22b^2\right)a^2+3b^2\left(a^2+4b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^2}dx+\frac{16ab\left(2a^2+3b^2\right)\sin(c+dx)}{d}\right)+\frac{10a^3b\sin(c+}{}\right.$$
$$\frac{a^2\sin(c+dx)\cos^3(c+dx)(a+b\sec(c+dx))^2}{4d}$$

$$\Big\downarrow \; 4533$$

$$\frac{1}{4}\left(\frac{1}{3}\left(\frac{3}{2}\left(3a^4+24a^2b^2+8b^4\right)\int 1dx+\frac{16ab\left(2a^2+3b^2\right)\sin(c+dx)}{d}+\frac{3a^2\left(3a^2+22b^2\right)\sin(c+dx)\cos(c+dx)}{2d}\right)\right.$$
$$\frac{a^2\sin(c+dx)\cos^3(c+dx)(a+b\sec(c+dx))^2}{4d}$$

$$\Big\downarrow \; 24$$

3.483. $\int \cos^4(c+dx)(a+b\sec(c+dx))^4\,dx$

$$\frac{a^2\sin(c+dx)\cos^3(c+dx)(a+b\sec(c+dx))^2}{4d}+\frac{1}{4}\left(\frac{10a^3b\sin(c+dx)\cos^2(c+dx)}{3d}+\frac{1}{3}\left(\frac{16ab(2a^2+3b^2)\sin(c+dx)}{d}+\frac{3a^2(3a^2+22b^2)\sin(c+dx)\cos(c+dx)}{2d}+\right.\right.$$

input
```
Int[Cos[c + d*x]^4*(a + b*Sec[c + d*x])^4,x]
```

output
```
(a^2*Cos[c + d*x]^3*(a + b*Sec[c + d*x])^2*Sin[c + d*x])/(4*d) + ((10*a^3*
b*Cos[c + d*x]^2*Sin[c + d*x])/(3*d) + ((3*(3*a^4 + 24*a^2*b^2 + 8*b^4)*x)
/2 + (16*a*b*(2*a^2 + 3*b^2)*Sin[c + d*x])/d + (3*a^2*(3*a^2 + 22*b^2)*Cos
[c + d*x]*Sin[c + d*x])/(2*d))/3)/4
```

3.483.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 4328
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)*
((d*Csc[e + f*x])^n/(f*n)), x] - Simp[1/(d*n)   Int[(a + b*Csc[e + f*x])^(m
 - 3)*(d*Csc[e + f*x])^(n + 1)*Simp[a^2*b*(m - 2*n - 2) - a*(3*b^2*n + a^2*
(n + 1))*Csc[e + f*x] - b*(b^2*n + a^2*(m + n - 1))*Csc[e + f*x]^2, x], x],
 x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && ((Int
egerQ[m] && LtQ[n, -1]) || (IntegersQ[m + 1/2, 2*n] && LeQ[n, -1]))
```

rule 4533
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] +
Simp[(C*m + A*(m + 1))/(b^2*m)   Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fr
eeQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

rule 4535
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Simp[B/b   Int[(b*Cs
c[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2)
, x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

rule 4562
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_)), x_Symbol] :> Simp[A*a*Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*n)), x] + Si
mp[1/(d*n)   Int[(d*Csc[e + f*x])^(n + 1)*Simp[n*(B*a + A*b) + (n*(a*C + B*
b) + A*a*(n + 1))*Csc[e + f*x] + b*C*n*Csc[e + f*x]^2, x], x], x] /; FreeQ[
{a, b, d, e, f, A, B, C}, x] && LtQ[n, -1]
```

3.483.4 Maple [A] (verified)

Time = 0.90 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.70

method	result
parallelrisch	$\frac{24(a^4+6a^2b^2)\sin(2dx+2c)+32a^3b\sin(3dx+3c)+3a^4\sin(4dx+4c)+96(3a^3b+4ab^3)\sin(dx+c)+36dx(a^4+8a^2b^2+\frac{8}{3}b^4)}{96d}$
derivativedivides	$\frac{a^4\left(\frac{\left(\cos(dx+c)^3+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+\frac{4a^3b\left(2+\cos(dx+c)^2\right)\sin(dx+c)}{3}+6a^2b^2\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right.}{d}$
default	$\frac{a^4\left(\frac{\left(\cos(dx+c)^3+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+\frac{4a^3b\left(2+\cos(dx+c)^2\right)\sin(dx+c)}{3}+6a^2b^2\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right.}{d}$
risch	$\frac{3a^4x}{8}+3a^2b^2x+x\,b^4+\frac{3a^3b\sin(dx+c)}{d}+\frac{4\sin(dx+c)a\,b^3}{d}+\frac{a^4\sin(4dx+4c)}{32d}+\frac{a^3b\sin(3dx+3c)}{3d}+\frac{\sin(2d}{}$
norman	$\frac{(-\frac{3}{8}a^4-3a^2b^2-b^4)x+(-\frac{9}{8}a^4-9a^2b^2-3b^4)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8+(-\frac{9}{8}a^4-9a^2b^2-3b^4)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}+(-\frac{3}{8}a^4-3a^2b^2-b^4}{}$

input
```
int(cos(d*x+c)^4*(a+b*sec(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
1/96*(24*(a^4+6*a^2*b^2)*sin(2*d*x+2*c)+32*a^3*b*sin(3*d*x+3*c)+3*a^4*sin(
4*d*x+4*c)+96*(3*a^3*b+4*a*b^3)*sin(d*x+c)+36*d*x*(a^4+8*a^2*b^2+8/3*b^4))
/d
```

3.483.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.66

$$\int \cos^4(c+dx)(a+b\sec(c+dx))^4\,dx$$
$$= \frac{3\,(3\,a^4+24\,a^2b^2+8\,b^4)dx+\left(6\,a^4\cos\left(dx+c\right)^3+32\,a^3b\cos\left(dx+c\right)^2+64\,a^3b+96\,ab^3+9\,(a^4+8\,a^2b^2)\,c\right.}{24\,d}$$

input
```
integrate(cos(d*x+c)^4*(a+b*sec(d*x+c))^4,x, algorithm="fricas")
```

output
```
1/24*(3*(3*a^4 + 24*a^2*b^2 + 8*b^4)*d*x + (6*a^4*cos(d*x + c)^3 + 32*a^3*
b*cos(d*x + c)^2 + 64*a^3*b + 96*a*b^3 + 9*(a^4 + 8*a^2*b^2)*cos(d*x + c))
*sin(d*x + c))/d
```

3.483.6 Sympy [F]

$$\int \cos^4(c+dx)(a+b\sec(c+dx))^4\,dx = \int (a+b\sec(c+dx))^4\cos^4(c+dx)\,dx$$

input
```
integrate(cos(d*x+c)**4*(a+b*sec(d*x+c))**4,x)
```

output
```
Integral((a + b*sec(c + d*x))**4*cos(c + d*x)**4, x)
```

3.483.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.75

$$\int \cos^4(c+dx)(a+b\sec(c+dx))^4\,dx$$
$$= \frac{3\,(12\,dx+12\,c+\sin\left(4\,dx+4\,c\right)+8\,\sin\left(2\,dx+2\,c\right))a^4-128\left(\sin\left(dx+c\right)^3-3\,\sin\left(dx+c\right)\right)a^3b+144\,(2}{96\,d}$$

input
```
integrate(cos(d*x+c)^4*(a+b*sec(d*x+c))^4,x, algorithm="maxima")
```

output
```
1/96*(3*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*a^4 - 128*
(sin(d*x + c)^3 - 3*sin(d*x + c))*a^3*b + 144*(2*d*x + 2*c + sin(2*d*x + 2
*c))*a^2*b^2 + 96*(d*x + c)*b^4 + 384*a*b^3*sin(d*x + c))/d
```

3.483.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 318 vs. 2(135) = 270.

Time = 0.31 (sec) , antiderivative size = 318, normalized size of antiderivative = 2.19

$$\int \cos^4(c+dx)(a+b\sec(c+dx))^4\,dx$$
$$= \frac{3\,(3\,a^4+24\,a^2b^2+8\,b^4)(dx+c)-\frac{2\left(15\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-96\,a^3b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+72\,a^2b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-96\,ab^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\right.}{}}{}$$

input
```
integrate(cos(d*x+c)^4*(a+b*sec(d*x+c))^4,x, algorithm="giac")
```

output
```
1/24*(3*(3*a^4 + 24*a^2*b^2 + 8*b^4)*(d*x + c) - 2*(15*a^4*tan(1/2*d*x + 1
/2*c)^7 - 96*a^3*b*tan(1/2*d*x + 1/2*c)^7 + 72*a^2*b^2*tan(1/2*d*x + 1/2*c
)^7 - 96*a*b^3*tan(1/2*d*x + 1/2*c)^7 - 9*a^4*tan(1/2*d*x + 1/2*c)^5 - 160
*a^3*b*tan(1/2*d*x + 1/2*c)^5 + 72*a^2*b^2*tan(1/2*d*x + 1/2*c)^5 - 288*a*
b^3*tan(1/2*d*x + 1/2*c)^5 + 9*a^4*tan(1/2*d*x + 1/2*c)^3 - 160*a^3*b*tan(
1/2*d*x + 1/2*c)^3 - 72*a^2*b^2*tan(1/2*d*x + 1/2*c)^3 - 288*a*b^3*tan(1/2
*d*x + 1/2*c)^3 - 15*a^4*tan(1/2*d*x + 1/2*c) - 96*a^3*b*tan(1/2*d*x + 1/2
*c) - 72*a^2*b^2*tan(1/2*d*x + 1/2*c) - 96*a*b^3*tan(1/2*d*x + 1/2*c))/(ta
n(1/2*d*x + 1/2*c)^2 + 1)^4)/d
```

3.483.9 Mupad [B] (verification not implemented)

Time = 12.87 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.85

$$\int \cos^4(c+dx)(a+b\sec(c+dx))^4\,dx = \frac{3\,a^4\,x}{8}+b^4\,x+3\,a^2\,b^2\,x+\frac{a^4\,\sin\left(2\,c+2\,d\,x\right)}{4\,d}+\frac{a^4\,\sin\left(4\,c+4\,d\,x\right)}{32\,d}+\frac{a^3\,b\,\sin\left(3\,c+3\,d\,x\right)}{3\,d}+\frac{3\,a^2\,b^2\,\sin\left(2\,c+2\,d\,x\right)}{2\,d}+\frac{4\,a\,b^3\,\sin\left(c+d\,x\right)}{d}+\frac{3\,a^3\,b\,\sin\left(c+d\,x\right)}{d}$$

input
```
int(cos(c + d*x)^4*(a + b/cos(c + d*x))^4,x)
```

output
```
(3*a^4*x)/8 + b^4*x + 3*a^2*b^2*x + (a^4*sin(2*c + 2*d*x))/(4*d) + (a^4*si
n(4*c + 4*d*x))/(32*d) + (a^3*b*sin(3*c + 3*d*x))/(3*d) + (3*a^2*b^2*sin(2
*c + 2*d*x))/(2*d) + (4*a*b^3*sin(c + d*x))/d + (3*a^3*b*sin(c + d*x))/d
```

3.484 $\int \cos^5(c+dx)(a+b\sec(c+dx))^4\,dx$

3.484.1 Optimal result . . . 3574
3.484.2 Mathematica [A] (verified) . . . 3575
3.484.3 Rubi [A] (verified) . . . 3575
3.484.4 Maple [A] (verified) . . . 3579
3.484.5 Fricas [A] (verification not implemented) . . . 3580
3.484.6 Sympy [F(-1)] . . . 3580
3.484.7 Maxima [A] (verification not implemented) . . . 3581
3.484.8 Giac [B] (verification not implemented) . . . 3581
3.484.9 Mupad [B] (verification not implemented) . . . 3582

3.484.1 Optimal result

Integrand size = 21, antiderivative size = 173

$$\begin{aligned}\int \cos^5(c+dx)(a+b\sec(c+dx))^4\,dx &= \frac{1}{2}ab\left(3a^2+4b^2\right)x+\frac{(4a^4+29a^2b^2+5b^4)\sin(c+dx)}{5d}\\&+\frac{ab(3a^2+4b^2)\cos(c+dx)\sin(c+dx)}{2d}\\&+\frac{3a^3b\cos^3(c+dx)\sin(c+dx)}{5d}\\&+\frac{a^2\cos^4(c+dx)(a+b\sec(c+dx))^2\sin(c+dx)}{5d}\\&-\frac{a^2(4a^2+27b^2)\sin^3(c+dx)}{15d}\end{aligned}$$

output
```
1/2*a*b*(3*a^2+4*b^2)*x+1/5*(4*a^4+29*a^2*b^2+5*b^4)*sin(d*x+c)/d+1/2*a*b*
(3*a^2+4*b^2)*cos(d*x+c)*sin(d*x+c)/d+3/5*a^3*b*cos(d*x+c)^3*sin(d*x+c)/d+
1/5*a^2*cos(d*x+c)^4*(a+b*sec(d*x+c))^2*sin(d*x+c)/d-1/15*a^2*(4*a^2+27*b^
2)*sin(d*x+c)^3/d
```

3.484.2 Mathematica [A] (verified)

Time = 0.66 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.77

$$\int \cos^5(c+dx)(a+b\sec(c+dx))^4\,dx = \frac{30(5a^4+36a^2b^2+8b^4)\sin(c+dx)+a(360a^2bc+480b^3c+360a^2bdx+480b^3dx+240b(a^2+b^2)\sin(2(c+}{240d}$$

input
```
Integrate[Cos[c + d*x]^5*(a + b*Sec[c + d*x])^4,x]
```

output
```
(30*(5*a^4 + 36*a^2*b^2 + 8*b^4)*Sin[c + d*x] + a*(360*a^2*b*c + 480*b^3*c
 + 360*a^2*b*d*x + 480*b^3*d*x + 240*b*(a^2 + b^2)*Sin[2*(c + d*x)] + 5*(5
*a^3 + 24*a*b^2)*Sin[3*(c + d*x)] + 30*a^2*b*Sin[4*(c + d*x)] + 3*a^3*Sin[
5*(c + d*x)]))/(240*d)
```

3.484.3 Rubi [A] (verified)

Time = 1.08 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.95, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {3042, 4328, 3042, 4562, 27, 3042, 4535, 3042, 3115, 24, 4532, 3042, 3492, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^5(c+dx)(a+b\sec(c+dx))^4\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^4}{\csc\left(c+dx+\frac{\pi}{2}\right)^5}dx$$

$$\Big\downarrow \; 4328$$

$$\frac{1}{5}\int \cos^4(c+dx)(a+b\sec(c+dx))\left(12ba^2+\left(4a^2+15b^2\right)\sec(c+dx)a+b\left(2a^2+5b^2\right)\sec^2(c+dx)\right)dx+\frac{a^2\sin(c+dx)\cos^4(c+dx)(a+b\sec(c+dx))^2}{5d}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{5}\int \frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)\left(12ba^2+\left(4a^2+15b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)a+b\left(2a^2+5b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^4}dx+\frac{a^2\sin(c+dx)\cos^4(c+dx)(a+b\sec(c+dx))^2}{5d}$$

$\downarrow$ 4562

$$\frac{1}{5}\left(\frac{3a^3b\sin(c+dx)\cos^3(c+dx)}{d}-\frac{1}{4}\int -4\cos^3(c+dx)\left(\left(4a^2+27b^2\right)a^2+5b\left(3a^2+4b^2\right)\sec(c+dx)a+b^2\left(2a^2\right.\right.\right.\frac{a^2\sin(c+dx)\cos^4(c+dx)(a+b\sec(c+dx))^2}{5d}$$

$\downarrow$ 27

$$\frac{1}{5}\left(\int \cos^3(c+dx)\left(\left(4a^2+27b^2\right)a^2+5b\left(3a^2+4b^2\right)\sec(c+dx)a+b^2\left(2a^2+5b^2\right)\sec^2(c+dx)\right)dx+\frac{3a^3b\sin(c+\frac{a^2\sin(c+dx)\cos^4(c+dx)(a+b\sec(c+dx))^2}{5d}}{}\right.$$

$\downarrow$ 3042

$$\frac{1}{5}\left(\int \frac{\left(4a^2+27b^2\right)a^2+5b\left(3a^2+4b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)a+b^2\left(2a^2+5b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^3}dx+\frac{3a^3b\sin(c+dx)}{d}\right.\frac{a^2\sin(c+dx)\cos^4(c+dx)(a+b\sec(c+dx))^2}{5d}$$

$\downarrow$ 4535

$$\frac{1}{5}\left(5ab\left(3a^2+4b^2\right)\int \cos^2(c+dx)dx+\int \cos^3(c+dx)\left(\left(4a^2+27b^2\right)a^2+b^2\left(2a^2+5b^2\right)\sec^2(c+dx)\right)dx+\frac{3a^3b\,s}{}\right.\frac{a^2\sin(c+dx)\cos^4(c+dx)(a+b\sec(c+dx))^2}{5d}$$

$\downarrow$ 3042

$$\frac{1}{5}\left(5ab\left(3a^2+4b^2\right)\int \sin\left(c+dx+\frac{\pi}{2}\right)^2dx+\int \frac{\left(4a^2+27b^2\right)a^2+b^2\left(2a^2+5b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^3}dx+\frac{3a^3b\sin(}{}\right.\frac{a^2\sin(c+dx)\cos^4(c+dx)(a+b\sec(c+dx))^2}{5d}$$

$\downarrow$ 3115

3.484. $\int \cos^5(c+dx)(a+b\sec(c+dx))^4\,dx$

$$\frac{1}{5}\left(\int \frac{\left(4a^2+27b^2\right)a^2+b^2\left(2a^2+5b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^3}dx+5ab\left(3a^2+4b^2\right)\left(\frac{\int 1dx}{2}+\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)\right.$$
$$\frac{a^2\sin(c+dx)\cos^4(c+dx)(a+b\sec(c+dx))^2}{5d}$$

$\Big\downarrow$ 24

$$\frac{1}{5}\left(\int \frac{\left(4a^2+27b^2\right)a^2+b^2\left(2a^2+5b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^3}dx+\frac{3a^3b\sin(c+dx)\cos^3(c+dx)}{d}+5ab\left(3a^2+4b^2\right)\left(\text{si}\right.\right.$$
$$\frac{a^2\sin(c+dx)\cos^4(c+dx)(a+b\sec(c+dx))^2}{5d}$$

$\Big\downarrow$ 4532

$$\frac{1}{5}\left(\int \cos(c+dx)\left(\left(2a^2+5b^2\right)b^2+a^2\left(4a^2+27b^2\right)\cos^2(c+dx)\right)dx+\frac{3a^3b\sin(c+dx)\cos^3(c+dx)}{d}+5ab\left(3a^2+\right.\right.$$
$$\frac{a^2\sin(c+dx)\cos^4(c+dx)(a+b\sec(c+dx))^2}{5d}$$

$\Big\downarrow$ 3042

$$\frac{1}{5}\left(\int \sin\left(c+dx+\frac{\pi}{2}\right)\left(\left(2a^2+5b^2\right)b^2+a^2\left(4a^2+27b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2\right)dx+\frac{3a^3b\sin(c+dx)\cos^3(c+dx)}{d}\right.$$
$$\frac{a^2\sin(c+dx)\cos^4(c+dx)(a+b\sec(c+dx))^2}{5d}$$

$\Big\downarrow$ 3492

$$\frac{1}{5}\left(-\frac{\int\left(4a^4+29b^2a^2-\left(4a^2+27b^2\right)\sin^2(c+dx)a^2+5b^4\right)d(-\sin(c+dx))}{d}+\frac{3a^3b\sin(c+dx)\cos^3(c+dx)}{d}+5a\right.$$
$$\frac{a^2\sin(c+dx)\cos^4(c+dx)(a+b\sec(c+dx))^2}{5d}$$

$\Big\downarrow$ 2009

$$\frac{a^2\sin(c+dx)\cos^4(c+dx)(a+b\sec(c+dx))^2}{5d}+$$
$$\frac{1}{5}\left(\frac{3a^3b\sin(c+dx)\cos^3(c+dx)}{d}+5ab\left(3a^2+4b^2\right)\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)-\frac{\frac{1}{3}a^2\left(4a^2+27b^2\right)\sin^3(c+d\right.$$

input
```
Int[Cos[c + d*x]^5*(a + b*Sec[c + d*x])^4,x]
```

3.484. $\int \cos^5(c+dx)(a+b\sec(c+dx))^4\,dx$

output
```
(a^2*Cos[c + d*x]^4*(a + b*Sec[c + d*x])^2*Sin[c + d*x])/(5*d) + ((3*a^3*b
*Cos[c + d*x]^3*Sin[c + d*x])/d + 5*a*b*(3*a^2 + 4*b^2)*(x/2 + (Cos[c + d*
x]*Sin[c + d*x])/(2*d)) - (-((4*a^4 + 29*a^2*b^2 + 5*b^4)*Sin[c + d*x]) +
(a^2*(4*a^2 + 27*b^2)*Sin[c + d*x]^3)/3)/d)/5
```

3.484.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3492
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2),
 x_Symbol] :> Simp[-f^(-1)   Subst[Int[(1 - x^2)^((m - 1)/2)*(A + C - C*x^2
), x], x, Cos[e + f*x]], x] /; FreeQ[{e, f, A, C}, x] && IGtQ[(m + 1)/2, 0]
```

rule 4328
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)*
((d*Csc[e + f*x])^n/(f*n)), x] - Simp[1/(d*n)   Int[(a + b*Csc[e + f*x])^(m
 - 3)*(d*Csc[e + f*x])^(n + 1)*Simp[a^2*b*(m - 2*n - 2) - a*(3*b^2*n + a^2*
(n + 1))*Csc[e + f*x] - b*(b^2*n + a^2*(m + n - 1))*Csc[e + f*x]^2, x], x],
 x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && ((Int
egerQ[m] && LtQ[n, -1]) || (IntegersQ[m + 1/2, 2*n] && LeQ[n, -1]))
```

rule 4532
```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.) + (A_)),
 x_Symbol] :> Int[(C + A*Sin[e + f*x]^2)/Sin[e + f*x]^(m + 2), x] /; FreeQ[
{e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && ILtQ[(m + 1)/2, 0]
```

rule 4535
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Simp[B/b   Int[(b*Cs
c[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2)
, x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

rule 4562
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_)), x_Symbol] :> Simp[A*a*Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*n)), x] + Si
mp[1/(d*n)   Int[(d*Csc[e + f*x])^(n + 1)*Simp[n*(B*a + A*b) + (n*(a*C + B*
b) + A*a*(n + 1))*Csc[e + f*x] + b*C*n*Csc[e + f*x]^2, x], x], x] /; FreeQ[
{a, b, d, e, f, A, B, C}, x] && LtQ[n, -1]
```

3.484.4 Maple [A] (verified)

Time = 1.27 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.72

method	result
parallelrisch	$\frac{240\left(a^3b+a\,b^3\right)\sin(2dx+2c)+5\left(5a^4+24a^2b^2\right)\sin(3dx+3c)+30a^3b\sin(4dx+4c)+3a^4\sin(5dx+5c)+30\left(5a^4+36a^2b^2+8b^4\right.}{240d}$
derivativedivides	$\frac{\frac{a^4\left(\frac{8}{3}+\cos(dx+c)^4+\frac{4\cos(dx+c)^2}{3}\right)\sin(dx+c)}{5}+4a^3b\left(\frac{\left(\cos(dx+c)^3+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+2a^2b^2\left(2+\cos(dx+\right.}{d}$
default	$\frac{\frac{a^4\left(\frac{8}{3}+\cos(dx+c)^4+\frac{4\cos(dx+c)^2}{3}\right)\sin(dx+c)}{5}+4a^3b\left(\frac{\left(\cos(dx+c)^3+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+2a^2b^2\left(2+\cos(dx+\right.}{d}$
risch	$\frac{3a^3bx}{2}+2a\,b^3x+\frac{5a^4\sin(dx+c)}{8d}+\frac{9\sin(dx+c)a^2b^2}{2d}+\frac{\sin(dx+c)b^4}{d}+\frac{a^4\sin(5dx+5c)}{80d}+\frac{a^3b\sin(4dx+4c)}{8d}+$
norman	$\frac{\left(-\frac{3}{2}a^3b-2a\,b^3\right)x+\left(\frac{3}{2}a^3b+2a\,b^3\right)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{16}+\left(-9a^3b-12a\,b^3\right)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}+\left(-3a^3b-4a\,b^3\right)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+}{}$

input
```
int(cos(d*x+c)^5*(a+b*sec(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
1/240*(240*(a^3*b+a*b^3)*sin(2*d*x+2*c)+5*(5*a^4+24*a^2*b^2)*sin(3*d*x+3*c
)+30*a^3*b*sin(4*d*x+4*c)+3*a^4*sin(5*d*x+5*c)+30*(5*a^4+36*a^2*b^2+8*b^4)
*sin(d*x+c)+360*d*(a^2+4/3*b^2)*x*a*b)/d
```

3.484.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.70

$$\int \cos^5(c+dx)(a+b\sec(c+dx))^4\,dx$$
$$= \frac{15\,(3\,a^3b+4\,ab^3)dx+\left(6\,a^4\cos\left(dx+c\right)^4+30\,a^3b\cos\left(dx+c\right)^3+16\,a^4+120\,a^2b^2+30\,b^4+4\,(2\,a^4+15\,a}{30\,d}$$

input
```
integrate(cos(d*x+c)^5*(a+b*sec(d*x+c))^4,x, algorithm="fricas")
```

output
```
1/30*(15*(3*a^3*b + 4*a*b^3)*d*x + (6*a^4*cos(d*x + c)^4 + 30*a^3*b*cos(d*
x + c)^3 + 16*a^4 + 120*a^2*b^2 + 30*b^4 + 4*(2*a^4 + 15*a^2*b^2)*cos(d*x
+ c)^2 + 15*(3*a^3*b + 4*a*b^3)*cos(d*x + c))*sin(d*x + c))/d
```

3.484.6 Sympy [F(-1)]

Timed out.

$$\int \cos^5(c+dx)(a+b\sec(c+dx))^4\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**5*(a+b*sec(d*x+c))**4,x)
```

output
```
Timed out
```

3.484.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.77

$$\int \cos^5(c+dx)(a+b\sec(c+dx))^4\,dx$$
$$= \frac{8\left(3\,\sin\left(dx+c\right)^5-10\,\sin\left(dx+c\right)^3+15\,\sin\left(dx+c\right)\right)a^4+15\left(12\,dx+12\,c+\sin\left(4\,dx+4\,c\right)+8\,\sin\left(2\,d\right.\right.}{}$$

input
```
integrate(cos(d*x+c)^5*(a+b*sec(d*x+c))^4,x, algorithm="maxima")
```

output
```
1/120*(8*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*a^4 + 15
*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*a^3*b - 240*(sin(
d*x + c)^3 - 3*sin(d*x + c))*a^2*b^2 + 120*(2*d*x + 2*c + sin(2*d*x + 2*c)
)*a*b^3 + 120*b^4*sin(d*x + c))/d
```

3.484.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 425 vs. 2(161) = 322.

Time = 0.33 (sec) , antiderivative size = 425, normalized size of antiderivative = 2.46

$$\int \cos^5(c+dx)(a+b\sec(c+dx))^4\,dx$$
$$= 15\left(3\,a^3b+4\,ab^3\right)(dx+c)+\frac{2\left(30\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9-75\,a^3b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9+180\,a^2b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9-60\,ab^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9+30\right.}{}$$

input
```
integrate(cos(d*x+c)^5*(a+b*sec(d*x+c))^4,x, algorithm="giac")
```

output
```
1/30*(15*(3*a^3*b + 4*a*b^3)*(d*x + c) + 2*(30*a^4*tan(1/2*d*x + 1/2*c)^9
- 75*a^3*b*tan(1/2*d*x + 1/2*c)^9 + 180*a^2*b^2*tan(1/2*d*x + 1/2*c)^9 - 6
0*a*b^3*tan(1/2*d*x + 1/2*c)^9 + 30*b^4*tan(1/2*d*x + 1/2*c)^9 + 40*a^4*ta
n(1/2*d*x + 1/2*c)^7 - 30*a^3*b*tan(1/2*d*x + 1/2*c)^7 + 480*a^2*b^2*tan(1
/2*d*x + 1/2*c)^7 - 120*a*b^3*tan(1/2*d*x + 1/2*c)^7 + 120*b^4*tan(1/2*d*x
 + 1/2*c)^7 + 116*a^4*tan(1/2*d*x + 1/2*c)^5 + 600*a^2*b^2*tan(1/2*d*x + 1
/2*c)^5 + 180*b^4*tan(1/2*d*x + 1/2*c)^5 + 40*a^4*tan(1/2*d*x + 1/2*c)^3 +
 30*a^3*b*tan(1/2*d*x + 1/2*c)^3 + 480*a^2*b^2*tan(1/2*d*x + 1/2*c)^3 + 12
0*a*b^3*tan(1/2*d*x + 1/2*c)^3 + 120*b^4*tan(1/2*d*x + 1/2*c)^3 + 30*a^4*t
an(1/2*d*x + 1/2*c) + 75*a^3*b*tan(1/2*d*x + 1/2*c) + 180*a^2*b^2*tan(1/2*
d*x + 1/2*c) + 60*a*b^3*tan(1/2*d*x + 1/2*c) + 30*b^4*tan(1/2*d*x + 1/2*c)
)/(tan(1/2*d*x + 1/2*c)^2 + 1)^5)/d
```

3.484.9 Mupad [B] (verification not implemented)

Time = 15.81 (sec) , antiderivative size = 330, normalized size of antiderivative = 1.91

$$\int \cos^5(c+dx)(a+b\sec(c+dx))^4\,dx$$
$$= \frac{(2a^4-5a^3b+12a^2b^2-4ab^3+2b^4)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^9+\left(\frac{8a^4}{3}-2a^3b+32a^2b^2-8ab^3+8b^4\right)\tan\left(\frac{c}{2}+\right.}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}+5\,\mathrm{t}\right.}$$
$$+\frac{ab\,\mathrm{atan}\left(\frac{ab\tan\left(\frac{c}{2}+\frac{dx}{2}\right)(3a^2+4b^2)}{3a^3b+4ab^3}\right)(3a^2+4b^2)}{d}$$

input
```
int(cos(c + d*x)^5*(a + b/cos(c + d*x))^4,x)
```

output
```
(tan(c/2 + (d*x)/2)^5*((116*a^4)/15 + 12*b^4 + 40*a^2*b^2) + tan(c/2 + (d*
x)/2)^9*(2*a^4 - 5*a^3*b - 4*a*b^3 + 2*b^4 + 12*a^2*b^2) + tan(c/2 + (d*x)
/2)^3*(8*a*b^3 + 2*a^3*b + (8*a^4)/3 + 8*b^4 + 32*a^2*b^2) + tan(c/2 + (d*
x)/2)^7*((8*a^4)/3 - 2*a^3*b - 8*a*b^3 + 8*b^4 + 32*a^2*b^2) + tan(c/2 + (
d*x)/2)*(4*a*b^3 + 5*a^3*b + 2*a^4 + 2*b^4 + 12*a^2*b^2))/(d*(5*tan(c/2 +
(d*x)/2)^2 + 10*tan(c/2 + (d*x)/2)^4 + 10*tan(c/2 + (d*x)/2)^6 + 5*tan(c/2
 + (d*x)/2)^8 + tan(c/2 + (d*x)/2)^10 + 1)) + (a*b*atan((a*b*tan(c/2 + (d*
x)/2)*(3*a^2 + 4*b^2))/(4*a*b^3 + 3*a^3*b))*(3*a^2 + 4*b^2))/d
```

3.485 $\int \cos^6(c+dx)(a+b\sec(c+dx))^4\,dx$

3.485.1 Optimal result . . . 3583
3.485.2 Mathematica [A] (verified) . . . 3584
3.485.3 Rubi [A] (verified) . . . 3584
3.485.4 Maple [A] (verified) . . . 3588
3.485.5 Fricas [A] (verification not implemented) . . . 3589
3.485.6 Sympy [F(-1)] . . . 3589
3.485.7 Maxima [A] (verification not implemented) . . . 3590
3.485.8 Giac [B] (verification not implemented) . . . 3590
3.485.9 Mupad [B] (verification not implemented) . . . 3591

3.485.1 Optimal result

Integrand size = 21, antiderivative size = 213

$$\begin{aligned}\int \cos^6(c+dx)(a+b\sec(c+dx))^4\,dx &= \frac{1}{16}\left(5a^4+36a^2b^2+8b^4\right)x \\ &+ \frac{4ab(4a^2+5b^2)\sin(c+dx)}{5d} \\ &+ \frac{(5a^4+36a^2b^2+8b^4)\cos(c+dx)\sin(c+dx)}{16d} \\ &+ \frac{a^2(5a^2+32b^2)\cos^3(c+dx)\sin(c+dx)}{24d} \\ &+ \frac{7a^3b\cos^4(c+dx)\sin(c+dx)}{15d} \\ &+ \frac{a^2\cos^5(c+dx)(a+b\sec(c+dx))^2\sin(c+dx)}{6d} \\ &- \frac{4ab(4a^2+5b^2)\sin^3(c+dx)}{15d}\end{aligned}$$

output
```
1/16*(5*a^4+36*a^2*b^2+8*b^4)*x+4/5*a*b*(4*a^2+5*b^2)*sin(d*x+c)/d+1/16*(5
*a^4+36*a^2*b^2+8*b^4)*cos(d*x+c)*sin(d*x+c)/d+1/24*a^2*(5*a^2+32*b^2)*cos
(d*x+c)^3*sin(d*x+c)/d+7/15*a^3*b*cos(d*x+c)^4*sin(d*x+c)/d+1/6*a^2*cos(d*
x+c)^5*(a+b*sec(d*x+c))^2*sin(d*x+c)/d-4/15*a*b*(4*a^2+5*b^2)*sin(d*x+c)^3
/d
```

3.485.2 Mathematica [A] (verified)

Time = 0.74 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.73

$$\int \cos^6(c+dx)(a+b\sec(c+dx))^4\,dx$$
$$= \frac{60(5a^4+36a^2b^2+8b^4)(c+dx)+480ab(5a^2+6b^2)\sin(c+dx)+15(15a^4+96a^2b^2+16b^4)\sin(2(c+dx)}{}$$

input
```
Integrate[Cos[c + d*x]^6*(a + b*Sec[c + d*x])^4,x]
```

output
```
(60*(5*a^4 + 36*a^2*b^2 + 8*b^4)*(c + d*x) + 480*a*b*(5*a^2 + 6*b^2)*Sin[c
 + d*x] + 15*(15*a^4 + 96*a^2*b^2 + 16*b^4)*Sin[2*(c + d*x)] + 80*a*b*(5*a
^2 + 4*b^2)*Sin[3*(c + d*x)] + 45*a^2*(a^2 + 4*b^2)*Sin[4*(c + d*x)] + 48*
a^3*b*Sin[5*(c + d*x)] + 5*a^4*Sin[6*(c + d*x)])/(960*d)
```

3.485.3 Rubi [A] (verified)

Time = 1.12 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.91, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {3042, 4328, 3042, 4562, 25, 3042, 4535, 3042, 3113, 2009, 4533, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^6(c+dx)(a+b\sec(c+dx))^4\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^4}{\csc\left(c+dx+\frac{\pi}{2}\right)^6}dx$$

$$\Big\downarrow \; 4328$$

$$\frac{1}{6}\int \cos^5(c+dx)(a+b\sec(c+dx))\left(14ba^2+\left(5a^2+18b^2\right)\sec(c+dx)a+3b\left(a^2+2b^2\right)\sec^2(c+dx)\right)dx+\frac{a^2\sin(c+dx)\cos^5(c+dx)(a+b\sec(c+dx))^2}{6d}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{6}\int \frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)\left(14ba^2+\left(5a^2+18b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)a+3b\left(a^2+2b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^5}dx+\frac{a^2\sin(c+dx)\cos^5(c+dx)(a+b\sec(c+dx))^2}{6d}$$

$\downarrow$ 4562

$$\frac{1}{6}\left(\frac{14a^3b\sin(c+dx)\cos^4(c+dx)}{5d}-\frac{1}{5}\int -\cos^4(c+dx)\left(5\left(5a^2+32b^2\right)a^2+24b\left(4a^2+5b^2\right)\sec(c+dx)a+15b^2\left(\right.\right.\right.\frac{a^2\sin(c+dx)\cos^5(c+dx)(a+b\sec(c+dx))^2}{6d}$$

$\downarrow$ 25

$$\frac{1}{6}\left(\frac{1}{5}\int \cos^4(c+dx)\left(5\left(5a^2+32b^2\right)a^2+24b\left(4a^2+5b^2\right)\sec(c+dx)a+15b^2\left(a^2+2b^2\right)\sec^2(c+dx)\right)dx+\frac{14a^3b\right.\frac{a^2\sin(c+dx)\cos^5(c+dx)(a+b\sec(c+dx))^2}{6d}$$

$\downarrow$ 3042

$$\frac{1}{6}\left(\frac{1}{5}\int \frac{5\left(5a^2+32b^2\right)a^2+24b\left(4a^2+5b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)a+15b^2\left(a^2+2b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^4}dx+\frac{14a^3b\sin(\right.\frac{a^2\sin(c+dx)\cos^5(c+dx)(a+b\sec(c+dx))^2}{6d}$$

$\downarrow$ 4535

$$\frac{1}{6}\left(\frac{1}{5}\left(24ab\left(4a^2+5b^2\right)\int \cos^3(c+dx)dx+\int \cos^4(c+dx)\left(5\left(5a^2+32b^2\right)a^2+15b^2\left(a^2+2b^2\right)\sec^2(c+dx)\right)dx\right)\right.\frac{a^2\sin(c+dx)\cos^5(c+dx)(a+b\sec(c+dx))^2}{6d}$$

$\downarrow$ 3042

$$\frac{1}{6}\left(\frac{1}{5}\left(24ab\left(4a^2+5b^2\right)\int \sin\left(c+dx+\frac{\pi}{2}\right)^3dx+\int \frac{5\left(5a^2+32b^2\right)a^2+15b^2\left(a^2+2b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^4}dx\right)+\right.\frac{a^2\sin(c+dx)\cos^5(c+dx)(a+b\sec(c+dx))^2}{6d}$$

$\downarrow$ 3113

3.485. $\int \cos^6(c+dx)(a+b\sec(c+dx))^4\,dx$

$$\frac{1}{6}\left(\frac{1}{5}\left(\int \frac{5\left(5a^2+32b^2\right)a^2+15b^2\left(a^2+2b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^4}dx-\frac{24ab\left(4a^2+5b^2\right)\int\left(1-\sin^2(c+dx)\right)d(-\sin(\right.}{d}\right.\right.$$
$$\frac{a^2\sin(c+dx)\cos^5(c+dx)(a+b\sec(c+dx))^2}{6d}$$

$\Big\downarrow$ 2009

$$\frac{1}{6}\left(\frac{1}{5}\left(\int \frac{5\left(5a^2+32b^2\right)a^2+15b^2\left(a^2+2b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^4}dx-\frac{24ab\left(4a^2+5b^2\right)\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx\right.}{d}\right.\right.$$
$$\frac{a^2\sin(c+dx)\cos^5(c+dx)(a+b\sec(c+dx))^2}{6d}$$

$\Big\downarrow$ 4533

$$\frac{1}{6}\left(\frac{1}{5}\left(\frac{15}{4}\left(5a^4+36a^2b^2+8b^4\right)\int\cos^2(c+dx)dx-\frac{24ab\left(4a^2+5b^2\right)\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}+\frac{5a^2\left(5a^2+\right.}{}\right.\right.$$
$$\frac{a^2\sin(c+dx)\cos^5(c+dx)(a+b\sec(c+dx))^2}{6d}$$

$\Big\downarrow$ 3042

$$\frac{1}{6}\left(\frac{1}{5}\left(\frac{15}{4}\left(5a^4+36a^2b^2+8b^4\right)\int\sin\left(c+dx+\frac{\pi}{2}\right)^2dx-\frac{24ab\left(4a^2+5b^2\right)\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}+\frac{5a^2(}{}\right.\right.$$
$$\frac{a^2\sin(c+dx)\cos^5(c+dx)(a+b\sec(c+dx))^2}{6d}$$

$\Big\downarrow$ 3115

$$\frac{1}{6}\left(\frac{1}{5}\left(\frac{15}{4}\left(5a^4+36a^2b^2+8b^4\right)\left(\frac{\int 1dx}{2}+\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)-\frac{24ab\left(4a^2+5b^2\right)\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+\right.}{d}\right.\right.$$
$$\frac{a^2\sin(c+dx)\cos^5(c+dx)(a+b\sec(c+dx))^2}{6d}$$

$\Big\downarrow$ 24

$$\frac{a^2\sin(c+dx)\cos^5(c+dx)(a+b\sec(c+dx))^2}{6d}+$$
$$\frac{1}{6}\left(\frac{14a^3b\sin(c+dx)\cos^4(c+dx)}{5d}+\frac{1}{5}\left(-\frac{24ab\left(4a^2+5b^2\right)\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}+\frac{5a^2\left(5a^2+32b^2\right)\sin(}{4}\right.\right.$$

input
```
Int[Cos[c + d*x]^6*(a + b*Sec[c + d*x])^4,x]
```

3.485. $\int \cos^6(c+dx)(a+b\sec(c+dx))^4\,dx$

output
```
(a^2*Cos[c + d*x]^5*(a + b*Sec[c + d*x])^2*Sin[c + d*x])/(6*d) + ((14*a^3*
b*Cos[c + d*x]^4*Sin[c + d*x])/(5*d) + ((5*a^2*(5*a^2 + 32*b^2)*Cos[c + d*
x]^3*Sin[c + d*x])/(4*d) + (15*(5*a^4 + 36*a^2*b^2 + 8*b^4)*(x/2 + (Cos[c
+ d*x]*Sin[c + d*x])/(2*d)))/4 - (24*a*b*(4*a^2 + 5*b^2)*(-Sin[c + d*x] +
Sin[c + d*x]^3/3))/d)/5)/6
```

3.485.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 4328
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)*
((d*Csc[e + f*x])^n/(f*n)), x] - Simp[1/(d*n)   Int[(a + b*Csc[e + f*x])^(m
 - 3)*(d*Csc[e + f*x])^(n + 1)*Simp[a^2*b*(m - 2*n - 2) - a*(3*b^2*n + a^2*
(n + 1))*Csc[e + f*x] - b*(b^2*n + a^2*(m + n - 1))*Csc[e + f*x]^2, x], x],
 x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && ((Int
egerQ[m] && LtQ[n, -1]) || (IntegersQ[m + 1/2, 2*n] && LeQ[n, -1]))
```

rule 4533
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] +
Simp[(C*m + A*(m + 1))/(b^2*m)   Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fr
eeQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

rule 4535
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Simp[B/b   Int[(b*Cs
c[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2)
, x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

rule 4562
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_)), x_Symbol] :> Simp[A*a*Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*n)), x] + Si
mp[1/(d*n)   Int[(d*Csc[e + f*x])^(n + 1)*Simp[n*(B*a + A*b) + (n*(a*C + B*
b) + A*a*(n + 1))*Csc[e + f*x] + b*C*n*Csc[e + f*x]^2, x], x], x] /; FreeQ[
{a, b, d, e, f, A, B, C}, x] && LtQ[n, -1]
```

3.485.4 Maple [A] (verified)

Time = 1.26 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.72

method	result
parallelrisch	$\frac{(225a^4+1440a^2b^2+240b^4)\sin(2dx+2c)+(400a^3b+320a\,b^3)\sin(3dx+3c)+(45a^4+180a^2b^2)\sin(4dx+4c)+48a^3b\sin(5dx}{960d}$
derivativedivides	$\frac{a^4\left(\frac{\left(\cos(dx+c)^5+\frac{5\cos(dx+c)^3}{4}+\frac{15\cos(dx+c)}{8}\right)\sin(dx+c)}{6}+\frac{5dx}{16}+\frac{5c}{16}\right)+\frac{4a^3b\left(\frac{8}{3}+\cos(dx+c)^4+\frac{4\cos(dx+c)^2}{3}\right)\sin(dx+c)}{5}+6a^2}{d}$
default	$\frac{a^4\left(\frac{\left(\cos(dx+c)^5+\frac{5\cos(dx+c)^3}{4}+\frac{15\cos(dx+c)}{8}\right)\sin(dx+c)}{6}+\frac{5dx}{16}+\frac{5c}{16}\right)+\frac{4a^3b\left(\frac{8}{3}+\cos(dx+c)^4+\frac{4\cos(dx+c)^2}{3}\right)\sin(dx+c)}{5}+6a^2}{d}$
risch	$\frac{5a^4x}{16}+\frac{9a^2b^2x}{4}+\frac{x\,b^4}{2}+\frac{5a^3b\sin(dx+c)}{2d}+\frac{3\sin(dx+c)a\,b^3}{d}+\frac{a^4\sin(6dx+6c)}{192d}+\frac{a^3b\sin(5dx+5c)}{20d}+\frac{3a^4\sin(}{64}$

input
```
int(cos(d*x+c)^6*(a+b*sec(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
1/960*((225*a^4+1440*a^2*b^2+240*b^4)*sin(2*d*x+2*c)+(400*a^3*b+320*a*b^3)
*sin(3*d*x+3*c)+(45*a^4+180*a^2*b^2)*sin(4*d*x+4*c)+48*a^3*b*sin(5*d*x+5*c
)+5*a^4*sin(6*d*x+6*c)+(2400*a^3*b+2880*a*b^3)*sin(d*x+c)+300*d*(a^4+36/5*
a^2*b^2+8/5*b^4)*x)/d
```

3.485.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.70

$$\int \cos^6(c+dx)(a+b\sec(c+dx))^4\,dx$$
$$= \frac{15\,(5\,a^4+36\,a^2b^2+8\,b^4)dx+\left(40\,a^4\cos\left(dx+c\right)^5+192\,a^3b\cos\left(dx+c\right)^4+512\,a^3b+640\,ab^3+10\,(5\,a^4+}{2}$$

input
```
integrate(cos(d*x+c)^6*(a+b*sec(d*x+c))^4,x, algorithm="fricas")
```

output
```
1/240*(15*(5*a^4 + 36*a^2*b^2 + 8*b^4)*d*x + (40*a^4*cos(d*x + c)^5 + 192*
a^3*b*cos(d*x + c)^4 + 512*a^3*b + 640*a*b^3 + 10*(5*a^4 + 36*a^2*b^2)*cos
(d*x + c)^3 + 64*(4*a^3*b + 5*a*b^3)*cos(d*x + c)^2 + 15*(5*a^4 + 36*a^2*b
^2 + 8*b^4)*cos(d*x + c))*sin(d*x + c))/d
```

3.485.6 Sympy [F(-1)]

Timed out.

$$\int \cos^6(c+dx)(a+b\sec(c+dx))^4\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**6*(a+b*sec(d*x+c))**4,x)
```

output
```
Timed out
```

3.485.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.80

$$\int \cos^6(c+dx)(a+b\sec(c+dx))^4\,dx = -\frac{5\left(4\sin\left(2\,dx+2\,c\right)^3-60\,dx-60\,c-9\sin\left(4\,dx+4\,c\right)-48\sin\left(2\,dx+2\,c\right)\right)a^4-256\left(3\sin\left(dx+c\right)^5\right.}{}$$

input
```
integrate(cos(d*x+c)^6*(a+b*sec(d*x+c))^4,x, algorithm="maxima")
```

output
```
-1/960*(5*(4*sin(2*d*x + 2*c)^3 - 60*d*x - 60*c - 9*sin(4*d*x + 4*c) - 48*
sin(2*d*x + 2*c))*a^4 - 256*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin
(d*x + c))*a^3*b - 180*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2
*c))*a^2*b^2 + 1280*(sin(d*x + c)^3 - 3*sin(d*x + c))*a*b^3 - 240*(2*d*x +
 2*c + sin(2*d*x + 2*c))*b^4)/d
```

3.485.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 550 vs. 2(199) = 398.

Time = 0.34 (sec) , antiderivative size = 550, normalized size of antiderivative = 2.58

$$\int \cos^6(c+dx)(a+b\sec(c+dx))^4\,dx = \frac{15\left(5\,a^4+36\,a^2b^2+8\,b^4\right)(dx+c)-\frac{2\left(165\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}-960\,a^3b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}+900\,a^2b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}-960\,ab^3\tan\right.}{}}{}$$

input
```
integrate(cos(d*x+c)^6*(a+b*sec(d*x+c))^4,x, algorithm="giac")
```

output
```
1/240*(15*(5*a^4 + 36*a^2*b^2 + 8*b^4)*(d*x + c) - 2*(165*a^4*tan(1/2*d*x
+ 1/2*c)^11 - 960*a^3*b*tan(1/2*d*x + 1/2*c)^11 + 900*a^2*b^2*tan(1/2*d*x
+ 1/2*c)^11 - 960*a*b^3*tan(1/2*d*x + 1/2*c)^11 + 120*b^4*tan(1/2*d*x + 1/
2*c)^11 - 25*a^4*tan(1/2*d*x + 1/2*c)^9 - 2240*a^3*b*tan(1/2*d*x + 1/2*c)^
9 + 1260*a^2*b^2*tan(1/2*d*x + 1/2*c)^9 - 3520*a*b^3*tan(1/2*d*x + 1/2*c)^
9 + 360*b^4*tan(1/2*d*x + 1/2*c)^9 + 450*a^4*tan(1/2*d*x + 1/2*c)^7 - 4992
*a^3*b*tan(1/2*d*x + 1/2*c)^7 + 360*a^2*b^2*tan(1/2*d*x + 1/2*c)^7 - 5760*
a*b^3*tan(1/2*d*x + 1/2*c)^7 + 240*b^4*tan(1/2*d*x + 1/2*c)^7 - 450*a^4*ta
n(1/2*d*x + 1/2*c)^5 - 4992*a^3*b*tan(1/2*d*x + 1/2*c)^5 - 360*a^2*b^2*tan
(1/2*d*x + 1/2*c)^5 - 5760*a*b^3*tan(1/2*d*x + 1/2*c)^5 - 240*b^4*tan(1/2*
d*x + 1/2*c)^5 + 25*a^4*tan(1/2*d*x + 1/2*c)^3 - 2240*a^3*b*tan(1/2*d*x +
1/2*c)^3 - 1260*a^2*b^2*tan(1/2*d*x + 1/2*c)^3 - 3520*a*b^3*tan(1/2*d*x +
1/2*c)^3 - 360*b^4*tan(1/2*d*x + 1/2*c)^3 - 165*a^4*tan(1/2*d*x + 1/2*c) -
 960*a^3*b*tan(1/2*d*x + 1/2*c) - 900*a^2*b^2*tan(1/2*d*x + 1/2*c) - 960*a
*b^3*tan(1/2*d*x + 1/2*c) - 120*b^4*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1
/2*c)^2 + 1)^6)/d
```

3.485.9 Mupad [B] (verification not implemented)

Time = 13.08 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.00

$$\int \cos^6(c+dx)(a+b\sec(c+dx))^4\,dx = \frac{5\,a^4\,x}{16}+\frac{b^4\,x}{2}+\frac{9\,a^2\,b^2\,x}{4}+\frac{15\,a^4\,\sin\left(2\,c+2\,d\,x\right)}{64\,d}+\frac{3\,a^4\,\sin\left(4\,c+4\,d\,x\right)}{64\,d}+\frac{a^4\,\sin\left(6\,c+6\,d\,x\right)}{192\,d}+\frac{b^4\,\sin\left(2\,c+2\,d\,x\right)}{4\,d}+\frac{a\,b^3\,\sin\left(3\,c+3\,d\,x\right)}{3\,d}+\frac{5\,a^3\,b\,\sin\left(3\,c+3\,d\,x\right)}{12\,d}+\frac{a^3\,b\,\sin\left(5\,c+5\,d\,x\right)}{20\,d}+\frac{3\,a^2\,b^2\,\sin\left(2\,c+2\,d\,x\right)}{2\,d}+\frac{3\,a^2\,b^2\,\sin\left(4\,c+4\,d\,x\right)}{16\,d}+\frac{3\,a\,b^3\,\sin\left(c+d\,x\right)}{d}+\frac{5\,a^3\,b\,\sin\left(c+d\,x\right)}{2\,d}$$

input
```
int(cos(c + d*x)^6*(a + b/cos(c + d*x))^4,x)
```

output
```
(5*a^4*x)/16 + (b^4*x)/2 + (9*a^2*b^2*x)/4 + (15*a^4*sin(2*c + 2*d*x))/(64
*d) + (3*a^4*sin(4*c + 4*d*x))/(64*d) + (a^4*sin(6*c + 6*d*x))/(192*d) + (
b^4*sin(2*c + 2*d*x))/(4*d) + (a*b^3*sin(3*c + 3*d*x))/(3*d) + (5*a^3*b*si
n(3*c + 3*d*x))/(12*d) + (a^3*b*sin(5*c + 5*d*x))/(20*d) + (3*a^2*b^2*sin(
2*c + 2*d*x))/(2*d) + (3*a^2*b^2*sin(4*c + 4*d*x))/(16*d) + (3*a*b^3*sin(c
 + d*x))/d + (5*a^3*b*sin(c + d*x))/(2*d)
```

3.485. $\int \cos^6(c+dx)(a+b\sec(c+dx))^4\,dx$

3.486 $\int (a+b\sec(c+dx))^5\,dx$

3.486.1 Optimal result . . . 3593
3.486.2 Mathematica [A] (verified) . . . 3594
3.486.3 Rubi [A] (verified) . . . 3594
3.486.4 Maple [A] (verified) . . . 3597
3.486.5 Fricas [A] (verification not implemented) . . . 3597
3.486.6 Sympy [F] . . . 3598
3.486.7 Maxima [A] (verification not implemented) . . . 3598
3.486.8 Giac [B] (verification not implemented) . . . 3599
3.486.9 Mupad [B] (verification not implemented) . . . 3600

3.486.1 Optimal result

Integrand size = 12, antiderivative size = 158

$$\begin{aligned}\int (a+b\sec(c+dx))^5\,dx &= a^5x+\frac{b(40a^4+40a^2b^2+3b^4)\operatorname{arctanh}(\sin(c+dx))}{8d}\\ &\quad+\frac{ab^2(53a^2+20b^2)\tan(c+dx)}{6d}\\ &\quad+\frac{b^3(58a^2+9b^2)\sec(c+dx)\tan(c+dx)}{24d}\\ &\quad+\frac{11ab^2(a+b\sec(c+dx))^2\tan(c+dx)}{12d}\\ &\quad+\frac{b^2(a+b\sec(c+dx))^3\tan(c+dx)}{4d}\end{aligned}$$

output
```
a^5*x+1/8*b*(40*a^4+40*a^2*b^2+3*b^4)*arctanh(sin(d*x+c))/d+1/6*a*b^2*(53*
a^2+20*b^2)*tan(d*x+c)/d+1/24*b^3*(58*a^2+9*b^2)*sec(d*x+c)*tan(d*x+c)/d+1
1/12*a*b^2*(a+b*sec(d*x+c))^2*tan(d*x+c)/d+1/4*b^2*(a+b*sec(d*x+c))^3*tan(
d*x+c)/d
```

3.486.2 Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.72

$$\int (a+b\sec(c+dx))^5\,dx = \frac{24a^5dx+3b(40a^4+40a^2b^2+3b^4)\operatorname{arctanh}(\sin(c+dx))+3b^2(40a(2a^2+b^2)+b(40a^2+3b^2)\sec(c+dx)}{24d}$$

input
```
Integrate[(a + b*Sec[c + d*x])^5,x]
```

output
```
(24*a^5*d*x + 3*b*(40*a^4 + 40*a^2*b^2 + 3*b^4)*ArcTanh[Sin[c + d*x]] + 3*
b^2*(40*a*(2*a^2 + b^2) + b*(40*a^2 + 3*b^2)*Sec[c + d*x] + 2*b^3*Sec[c +
d*x]^3)*Tan[c + d*x] + 40*a*b^4*Tan[c + d*x]^3)/(24*d)
```

3.486.3 Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.08, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.583$, Rules used = {3042, 4269, 3042, 4544, 3042, 4536, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\sec(c+dx))^5\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^5 dx$$

$$\Big\downarrow \text{4269}$$

$$\frac{1}{4}\int (a+b\sec(c+dx))^2\left(4a^3+11b^2\sec^2(c+dx)a+3b(4a^2+b^2)\sec(c+dx)\right)dx+\frac{b^2\tan(c+dx)(a+b\sec(c+dx))^3}{4d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}\int \left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2\left(4a^3+11b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2a+3b(4a^2+b^2)\csc\left(c+dx+\frac{\pi}{2}\right)\right)dx+\frac{b^2\tan(c+dx)(a+b\sec(c+dx))^3}{4d}$$

$$\Big\downarrow \text{4544}$$

$$\frac{1}{4}\left(\frac{1}{3}\int (a+b\sec(c+dx))\left(12a^4+b\left(48a^2+31b^2\right)\sec(c+dx)a+b^2\left(58a^2+9b^2\right)\sec^2(c+dx)\right)dx+\frac{11ab^2\tan(c+\right.$$
$$\frac{b^2\tan(c+dx)(a+b\sec(c+dx))^3}{4d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}\left(\frac{1}{3}\int \left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)\left(12a^4+b\left(48a^2+31b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)a+b^2\left(58a^2+9b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)\right.\right.$$
$$\frac{b^2\tan(c+dx)(a+b\sec(c+dx))^3}{4d}$$

$$\Big\downarrow \text{4536}$$

$$\frac{1}{4}\left(\frac{1}{3}\left(\frac{1}{2}\int \left(24a^5+4b^2\left(53a^2+20b^2\right)\sec^2(c+dx)a+3b\left(40a^4+40b^2a^2+3b^4\right)\sec(c+dx)\right)dx+\frac{b^3\left(58a^2+9b^2\right)\right.}{}\right.$$
$$\frac{b^2\tan(c+dx)(a+b\sec(c+dx))^3}{4d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}\left(\frac{1}{3}\left(\frac{b^3\left(58a^2+9b^2\right)\tan(c+dx)\sec(c+dx)}{2d}+\frac{1}{2}\left(24a^5x+\frac{4ab^2\left(53a^2+20b^2\right)\tan(c+dx)}{d}+\frac{3b\left(40a^4+40a^2b^2\right.}{}\right.\right.\right.$$
$$\frac{b^2\tan(c+dx)(a+b\sec(c+dx))^3}{4d}$$

input
```
Int[(a + b*Sec[c + d*x])^5,x]
```

output
```
(b^2*(a + b*Sec[c + d*x])^3*Tan[c + d*x])/(4*d) + ((11*a*b^2*(a + b*Sec[c
+ d*x])^2*Tan[c + d*x])/(3*d) + ((b^3*(58*a^2 + 9*b^2)*Sec[c + d*x]*Tan[c
+ d*x])/(2*d) + (24*a^5*x + (3*b*(40*a^4 + 40*a^2*b^2 + 3*b^4)*ArcTanh[Sin
[c + d*x]])/d + (4*a*b^2*(53*a^2 + 20*b^2)*Tan[c + d*x])/d)/2)/3)/4
```

3.486. $\int (a+b\sec(c+dx))^5\,dx$

3.486.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4269
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(-b^2)*C
ot[c + d*x]*((a + b*Csc[c + d*x])^(n - 2)/(d*(n - 1))), x] + Simp[1/(n - 1)
   Int[(a + b*Csc[c + d*x])^(n - 3)*Simp[a^3*(n - 1) + (b*(b^2*(n - 2) + 3*
a^2*(n - 1)))*Csc[c + d*x] + (a*b^2*(3*n - 4))*Csc[c + d*x]^2, x], x], x] /
; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 2] && IntegerQ[2*n]
```

rule 4536
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[(-b)*C*Csc[e +
 f*x]*(Cot[e + f*x]/(2*f)), x] + Simp[1/2   Int[Simp[2*A*a + (2*B*a + b*(2*
A + C))*Csc[e + f*x] + 2*(a*C + B*b)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a
, b, e, f, A, B, C}, x]
```

rule 4544
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.), x_Symbol] :> Simp[(-C)*Cot
[e + f*x]*((a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Simp[1/(m + 1)   Int[(
a + b*Csc[e + f*x])^(m - 1)*Simp[a*A*(m + 1) + ((A*b + a*B)*(m + 1) + b*C*m
)*Csc[e + f*x] + (b*B*(m + 1) + a*C*m)*Csc[e + f*x]^2, x], x], x] /; FreeQ[
{a, b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && IGtQ[2*m, 0]
```

3.486. $\int (a+b\sec(c+dx))^5\,dx$

3.486.4 Maple [A] (verified)

Time = 1.66 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.01

method	result
derivativedivides	$\frac{a^5(dx+c)+5a^4b\ln(\sec(dx+c)+\tan(dx+c))+10a^3b^2\tan(dx+c)+10a^2b^3\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)}{d}$
default	$\frac{a^5(dx+c)+5a^4b\ln(\sec(dx+c)+\tan(dx+c))+10a^3b^2\tan(dx+c)+10a^2b^3\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)}{d}$
parts	$a^5x+\frac{b^5\left(-\left(-\frac{\sec(dx+c)^3}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)}{d}-\frac{5a\,b^4\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right)\tan(dx}{d}$
parallelrisch	$\frac{-480\left(a^4+a^2b^2+\frac{3}{40}b^4\right)b\left(\frac{3}{4}+\frac{\cos(4dx+4c)}{4}+\cos(2dx+2c)\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+480\left(a^4+a^2b^2+\frac{3}{40}b^4\right)b\left(\frac{3}{4}+\frac{\cos(4dx+4c)}{4}\right.}{}$
norman	$\frac{a^5x+a^5x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8-4a^5x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+6a^5x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4-4a^5x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6-\frac{5b^2\left(16a^3-8a^2b+8a\,b^2-b^3\right)\tan\left(\right.}{4d}}{\left(\right.}$
risch	$a^5x-\frac{ib^2\left(120a^2b\,e^{7i(dx+c)}+9b^3e^{7i(dx+c)}-240a^3e^{6i(dx+c)}+120a^2b\,e^{5i(dx+c)}+33b^3e^{5i(dx+c)}-720a^3e^{4i(dx+c)}-240a\right.}{}$

input
```
int((a+b*sec(d*x+c))^5,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^5*(d*x+c)+5*a^4*b*ln(sec(d*x+c)+tan(d*x+c))+10*a^3*b^2*tan(d*x+c)+1
0*a^2*b^3*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c)))-5*a*b^
4*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c)+b^5*(-(-1/4*sec(d*x+c)^3-3/8*sec(d*x+
c))*tan(d*x+c)+3/8*ln(sec(d*x+c)+tan(d*x+c))))
```

3.486.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.16

$$\int (a+b\sec(c+dx))^5\,dx$$
$$=\frac{48\,a^5dx\cos(dx+c)^4+3\,(40\,a^4b+40\,a^2b^3+3\,b^5)\cos(dx+c)^4\log(\sin(dx+c)+1)-3\,(40\,a^4b+40\,a^2b^3}{}$$

input
```
integrate((a+b*sec(d*x+c))^5,x, algorithm="fricas")
```

output
```
1/48*(48*a^5*d*x*cos(d*x + c)^4 + 3*(40*a^4*b + 40*a^2*b^3 + 3*b^5)*cos(d*
x + c)^4*log(sin(d*x + c) + 1) - 3*(40*a^4*b + 40*a^2*b^3 + 3*b^5)*cos(d*x
 + c)^4*log(-sin(d*x + c) + 1) + 2*(40*a*b^4*cos(d*x + c) + 6*b^5 + 80*(3*
a^3*b^2 + a*b^4)*cos(d*x + c)^3 + 3*(40*a^2*b^3 + 3*b^5)*cos(d*x + c)^2)*s
in(d*x + c))/(d*cos(d*x + c)^4)
```

3.486.6 Sympy [F]

$$\int (a+b\sec(c+dx))^5\,dx=\int (a+b\sec{(c+dx)})^5\,dx$$

input
```
integrate((a+b*sec(d*x+c))**5,x)
```

output
```
Integral((a + b*sec(c + d*x))**5, x)
```

3.486.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.25

$$\begin{aligned}\int (a+b\sec(c+dx))^5\,dx &= a^5x+\frac{5\left(\tan{(dx+c)}^3+3\,\tan{(dx+c)}\right)ab^4}{3\,d}\\ &\quad-\frac{b^5\left(\frac{2\left(3\,\sin(dx+c)^3-5\,\sin(dx+c)\right)}{\sin(dx+c)^4-2\,\sin(dx+c)^2+1}-3\,\log{(\sin{(dx+c)}+1)}+3\,\log{(\sin{(dx+c)}-1)}\right)}{16\,d}\\ &\quad-\frac{5\,a^2b^3\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1}-\log{(\sin{(dx+c)}+1)}+\log{(\sin{(dx+c)}-1)}\right)}{2\,d}\\ &\quad+\frac{5\,a^4b\log{(\sec{(dx+c)}+\tan{(dx+c)})}}{d}+\frac{10\,a^3b^2\tan{(dx+c)}}{d}\end{aligned}$$

input
```
integrate((a+b*sec(d*x+c))^5,x, algorithm="maxima")
```

output
```
a^5*x + 5/3*(tan(d*x + c)^3 + 3*tan(d*x + c))*a*b^4/d - 1/16*b^5*(2*(3*sin
(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*
log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1))/d - 5/2*a^2*b^3*(2*sin(d*
x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1
))/d + 5*a^4*b*log(sec(d*x + c) + tan(d*x + c))/d + 10*a^3*b^2*tan(d*x + c
)/d
```

3.486.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 380 vs. $2(148) = 296$.

Time = 0.31 (sec) , antiderivative size = 380, normalized size of antiderivative = 2.41

$$\int (a + b\sec(c + dx))^5 \, dx$$

$$= \frac{24\,(dx + c)a^5 + 3\,(40\,a^4b + 40\,a^2b^3 + 3\,b^5)\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + 1\right|\right) - 3\,(40\,a^4b + 40\,a^2b^3 + 3\,b^5)\log\left(\left|t\right.\right.}{}$$

input
```
integrate((a+b*sec(d*x+c))^5,x, algorithm="giac")
```

output
```
1/24*(24*(d*x + c)*a^5 + 3*(40*a^4*b + 40*a^2*b^3 + 3*b^5)*log(abs(tan(1/2
*d*x + 1/2*c) + 1)) - 3*(40*a^4*b + 40*a^2*b^3 + 3*b^5)*log(abs(tan(1/2*d*
x + 1/2*c) - 1)) - 2*(240*a^3*b^2*tan(1/2*d*x + 1/2*c)^7 - 120*a^2*b^3*tan
(1/2*d*x + 1/2*c)^7 + 120*a*b^4*tan(1/2*d*x + 1/2*c)^7 - 15*b^5*tan(1/2*d*
x + 1/2*c)^7 - 720*a^3*b^2*tan(1/2*d*x + 1/2*c)^5 + 120*a^2*b^3*tan(1/2*d*
x + 1/2*c)^5 - 200*a*b^4*tan(1/2*d*x + 1/2*c)^5 - 9*b^5*tan(1/2*d*x + 1/2*
c)^5 + 720*a^3*b^2*tan(1/2*d*x + 1/2*c)^3 + 120*a^2*b^3*tan(1/2*d*x + 1/2*
c)^3 + 200*a*b^4*tan(1/2*d*x + 1/2*c)^3 - 9*b^5*tan(1/2*d*x + 1/2*c)^3 - 2
40*a^3*b^2*tan(1/2*d*x + 1/2*c) - 120*a^2*b^3*tan(1/2*d*x + 1/2*c) - 120*a
*b^4*tan(1/2*d*x + 1/2*c) - 15*b^5*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/
2*c)^2 - 1)^4)/d
```

3.486.9 Mupad [B] (verification not implemented)

Time = 13.95 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.73

$$\int (a+b\sec(c+dx))^5\,dx = \frac{2\,a^5\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d} + \frac{3\,b^5\,\sin\left(c+d\,x\right)}{8\,d\cos\left(c+d\,x\right)^2} + \frac{b^5\,\sin\left(c+d\,x\right)}{4\,d\cos\left(c+d\,x\right)^4} + \frac{10\,a\,b^4\,\sin\left(c+d\,x\right)}{3\,d\,\cos\left(c+d\,x\right)} + \frac{5\,a\,b^4\,\sin\left(c+d\,x\right)}{3\,d\cos\left(c+d\,x\right)^3} + \frac{10\,a^3\,b^2\,\sin\left(c+d\,x\right)}{d\,\cos\left(c+d\,x\right)} + \frac{5\,a^2\,b^3\,\sin\left(c+d\,x\right)}{d\cos\left(c+d\,x\right)^2} - \frac{b^5\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,1\mathrm{i}}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)\,3\mathrm{i}}{4\,d} - \frac{a^2\,b^3\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,1\mathrm{i}}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)\,10\mathrm{i}}{d} - \frac{a^4\,b\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,1\mathrm{i}}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)\,10\mathrm{i}}{d}$$

input
```
int((a + b/cos(c + d*x))^5,x)
```

output
```
(2*a^5*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d - (b^5*atan((sin(c/2
 + (d*x)/2)*1i)/cos(c/2 + (d*x)/2))*3i)/(4*d) + (3*b^5*sin(c + d*x))/(8*d*
cos(c + d*x)^2) + (b^5*sin(c + d*x))/(4*d*cos(c + d*x)^4) - (a^2*b^3*atan(
(sin(c/2 + (d*x)/2)*1i)/cos(c/2 + (d*x)/2))*10i)/d - (a^4*b*atan((sin(c/2
+ (d*x)/2)*1i)/cos(c/2 + (d*x)/2))*10i)/d + (10*a*b^4*sin(c + d*x))/(3*d*c
os(c + d*x)) + (5*a*b^4*sin(c + d*x))/(3*d*cos(c + d*x)^3) + (10*a^3*b^2*s
in(c + d*x))/(d*cos(c + d*x)) + (5*a^2*b^3*sin(c + d*x))/(d*cos(c + d*x)^2
)
```

3.487 $\int \frac{\sec^5(c+dx)}{a+b\sec(c+dx)}\,dx$

3.487.1 Optimal result . . . 3601
3.487.2 Mathematica [A] (verified) . . . 3601
3.487.3 Rubi [A] (verified) . . . 3602
3.487.4 Maple [A] (verified) . . . 3607
3.487.5 Fricas [A] (verification not implemented) . . . 3607
3.487.6 Sympy [F] . . . 3608
3.487.7 Maxima [F(-2)] . . . 3608
3.487.8 Giac [B] (verification not implemented) . . . 3609
3.487.9 Mupad [B] (verification not implemented) . . . 3609

3.487.1 Optimal result

Integrand size = 21, antiderivative size = 157

$$\int \frac{\sec^5(c+dx)}{a+b\sec(c+dx)}\,dx = -\frac{a(2a^2+b^2)\operatorname{arctanh}(\sin(c+dx))}{2b^4d} + \frac{2a^4\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}b^4\sqrt{a+b}d} + \frac{(3a^2+2b^2)\tan(c+dx)}{3b^3d} - \frac{a\sec(c+dx)\tan(c+dx)}{2b^2d} + \frac{\sec^2(c+dx)\tan(c+dx)}{3bd}$$

output
```
-1/2*a*(2*a^2+b^2)*arctanh(sin(d*x+c))/b^4/d+2*a^4*arctanh((a-b)^(1/2)*tan
(1/2*d*x+1/2*c)/(a+b)^(1/2))/b^4/d/(a-b)^(1/2)/(a+b)^(1/2)+1/3*(3*a^2+2*b^
2)*tan(d*x+c)/b^3/d-1/2*a*sec(d*x+c)*tan(d*x+c)/b^2/d+1/3*sec(d*x+c)^2*tan
(d*x+c)/b/d
```

3.487.2 Mathematica [A] (verified)

Time = 2.05 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.64

$$\int \frac{\sec^5(c+dx)}{a+b\sec(c+dx)}\,dx = \frac{-\frac{24a^4\operatorname{arctanh}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} + \frac{1}{2}\sec^3(c+dx)\left(9a(2a^2+b^2)\cos(c+dx)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right) - \sin\left(\frac{1}{2}\right.\right.\right.}{}$$

input
```
Integrate[Sec[c + d*x]^5/(a + b*Sec[c + d*x]),x]
```

output
```
((-24*a^4*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 -
 b^2] + (Sec[c + d*x]^3*(9*a*(2*a^2 + b^2)*Cos[c + d*x]*(Log[Cos[(c + d*x)
/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + 3*a*
(2*a^2 + b^2)*Cos[3*(c + d*x)]*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] -
 Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + 4*b*(3*a^2 + 4*b^2 - 3*a*b*Co
s[c + d*x] + (3*a^2 + 2*b^2)*Cos[2*(c + d*x)])*Sin[c + d*x]))/2)/(12*b^4*d
)
```

3.487.3 Rubi [A] (verified)

Time = 1.28 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.11, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.762$, Rules used = {3042, 4338, 3042, 4580, 25, 3042, 4570, 27, 3042, 4486, 3042, 4257, 4318, 3042, 3138, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^5(c+dx)}{a+b\sec(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^5}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{4338}$$

$$\frac{\int \frac{\sec^2(c+dx)\left(-3a\sec^2(c+dx)+2b\sec(c+dx)+2a\right)}{a+b\sec(c+dx)}dx}{3b}+\frac{\tan(c+dx)\sec^2(c+dx)}{3bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2\left(-3a\csc\left(c+dx+\frac{\pi}{2}\right)^2+2b\csc\left(c+dx+\frac{\pi}{2}\right)+2a\right)}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{3b}+\frac{\tan(c+dx)\sec^2(c+dx)}{3bd}$$

$$\Big\downarrow \text{4580}$$

$$\frac{\frac{\int -\frac{\sec(c+dx)\left(3a^2-b\sec(c+dx)a-2\left(3a^2+2b^2\right)\sec^2(c+dx)\right)}{a+b\sec(c+dx)}dx}{2b}-\frac{3a\tan(c+dx)\sec(c+dx)}{2bd}}{3b}+\frac{\tan(c+dx)\sec^2(c+dx)}{3bd}$$

$$\Big\downarrow \text{25}$$

$$\frac{-\frac{\int \frac{\sec(c+dx)\left(3a^2-b\sec(c+dx)a-2\left(3a^2+2b^2\right)\sec^2(c+dx)\right)}{a+b\sec(c+dx)}dx}{2b}-\frac{3a\tan(c+dx)\sec(c+dx)}{2bd}}{3b}+\frac{\tan(c+dx)\sec^2(c+dx)}{3bd}$$

$\Big\downarrow$ 3042

$$\frac{-\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(3a^2-b\csc\left(c+dx+\frac{\pi}{2}\right)a-2\left(3a^2+2b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{2b}-\frac{3a\tan(c+dx)\sec(c+dx)}{2bd}}{3b}+\frac{\tan(c+dx)\sec^2(c+dx)}{3bd}$$

$\Big\downarrow$ 4570

$$\frac{-\frac{\frac{\int \frac{3\sec(c+dx)\left(ba^2+\left(2a^2+b^2\right)\sec(c+dx)a\right)}{a+b\sec(c+dx)}dx}{b}-\frac{2\left(3a^2+2b^2\right)\tan(c+dx)}{bd}}{2b}-\frac{3a\tan(c+dx)\sec(c+dx)}{2bd}}{3b}+\frac{\tan(c+dx)\sec^2(c+dx)}{3bd}$$

$\Big\downarrow$ 27

$$\frac{-\frac{\frac{3\int \frac{\sec(c+dx)\left(ba^2+\left(2a^2+b^2\right)\sec(c+dx)a\right)}{a+b\sec(c+dx)}dx}{b}-\frac{2\left(3a^2+2b^2\right)\tan(c+dx)}{bd}}{2b}-\frac{3a\tan(c+dx)\sec(c+dx)}{2bd}}{3b}+\frac{\tan(c+dx)\sec^2(c+dx)}{3bd}$$

$\Big\downarrow$ 3042

$$\frac{-\frac{\frac{3\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(ba^2+\left(2a^2+b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)a\right)}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b}-\frac{2\left(3a^2+2b^2\right)\tan(c+dx)}{bd}}{2b}-\frac{3a\tan(c+dx)\sec(c+dx)}{2bd}}{3b}+\frac{\tan(c+dx)\sec^2(c+dx)}{3bd}$$

$\Big\downarrow$ 4486

$$\frac{-\frac{\frac{3\left(\frac{a\left(2a^2+b^2\right)\int \sec(c+dx)dx}{b}-\frac{2a^4\int \frac{\sec(c+dx)}{a+b\sec(c+dx)}dx}{b}\right)}{b}-\frac{2\left(3a^2+2b^2\right)\tan(c+dx)}{bd}}{2b}-\frac{3a\tan(c+dx)\sec(c+dx)}{2bd}}{3b}+\frac{\tan(c+dx)\sec^2(c+dx)}{3bd}$$

$\Big\downarrow$ 3042

3.487. $\int \frac{\sec^5(c+dx)}{a+b\sec(c+dx)}\,dx$

$$\frac{-\frac{\frac{3\left(\frac{a\left(2a^2+b^2\right)\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{b}-\frac{2a^4\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b}\right)}{b}-\frac{2\left(3a^2+2b^2\right)\tan(c+dx)}{bd}}{2b}-\frac{3a\tan(c+dx)\sec(c+dx)}{2bd}}{3b}+\frac{\tan(c+dx)\sec^2(c+dx)}{3bd}$$

$\downarrow$ 4257

$$\frac{-\frac{\frac{3\left(\frac{a\left(2a^2+b^2\right)\text{arctanh}(\sin(c+dx))}{bd}-\frac{2a^4\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b}\right)}{b}-\frac{2\left(3a^2+2b^2\right)\tan(c+dx)}{bd}}{2b}-\frac{3a\tan(c+dx)\sec(c+dx)}{2bd}}{3b}+\frac{\tan(c+dx)\sec^2(c+dx)}{3bd}$$

$\downarrow$ 4318

$$\frac{-\frac{\frac{3\left(\frac{a\left(2a^2+b^2\right)\text{arctanh}(\sin(c+dx))}{bd}-\frac{2a^4\int \frac{1}{\frac{a\cos(c+dx)}{b}+1}dx}{b^2}\right)}{b}-\frac{2\left(3a^2+2b^2\right)\tan(c+dx)}{bd}}{2b}-\frac{3a\tan(c+dx)\sec(c+dx)}{2bd}}{3b}+\frac{\tan(c+dx)\sec^2(c+dx)}{3bd}$$

$\downarrow$ 3042

$$\frac{-\frac{\frac{3\left(\frac{a\left(2a^2+b^2\right)\text{arctanh}(\sin(c+dx))}{bd}-\frac{2a^4\int \frac{1}{\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{b}+1}dx}{b^2}\right)}{b}-\frac{2\left(3a^2+2b^2\right)\tan(c+dx)}{bd}}{2b}-\frac{3a\tan(c+dx)\sec(c+dx)}{2bd}}{3b}+\frac{\tan(c+dx)\sec^2(c+dx)}{3bd}$$

$\downarrow$ 3138

$$\frac{-\frac{\frac{3\left(\frac{a\left(2a^2+b^2\right)\text{arctanh}(\sin(c+dx))}{bd}-\frac{4a^4\int \frac{1}{\left(1-\frac{a}{b}\right)\tan^2\left(\frac{1}{2}(c+dx)\right)+\frac{a+b}{b}}d\tan\left(\frac{1}{2}(c+dx)\right)}{b^2d}\right)}{b}-\frac{2\left(3a^2+2b^2\right)\tan(c+dx)}{bd}}{2b}-\frac{3a\tan(c+dx)\sec(c+dx)}{2bd}}{3b}+\frac{\tan(c+dx)\sec^2(c+dx)}{3bd}$$

$\downarrow$ 221

3.487. $\int \frac{\sec^5(c+dx)}{a+b\sec(c+dx)}\,dx$

$$\frac{\frac{\frac{3\left(\frac{a\left(2a^2+b^2\right)\text{arctanh}(\sin(c+dx))}{bd}-\frac{4a^4\text{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{bd\sqrt{a-b}\sqrt{a+b}}\right)}{b}-\frac{2\left(3a^2+2b^2\right)\tan(c+dx)}{bd}}{2b}-\frac{3a\tan(c+dx)\sec(c+dx)}{2bd}}{3b}+\frac{\tan(c+dx)\sec^2(c+dx)}{3bd}$$

input
```
Int[Sec[c + d*x]^5/(a + b*Sec[c + d*x]),x]
```

output
```
(Sec[c + d*x]^2*Tan[c + d*x])/(3*b*d) + ((-3*a*Sec[c + d*x]*Tan[c + d*x])/
(2*b*d) - ((3*((a*(2*a^2 + b^2)*ArcTanh[Sin[c + d*x]])/(b*d) - (4*a^4*ArcT
anh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(Sqrt[a - b]*b*Sqrt[a + b
]*d)))/b - (2*(3*a^2 + 2*b^2)*Tan[c + d*x])/(b*d))/(2*b))/(3*b)
```

3.487.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4318
```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[1/b   Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4338
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-d^3)*Cot[e + f*x]*((d*Csc[e + f*x])^(n - 3)/(b*f*
(n - 2))), x] + Simp[d^3/(b*(n - 2))   Int[(d*Csc[e + f*x])^(n - 3)*(Simp[a
*(n - 3) + b*(n - 3)*Csc[e + f*x] - a*(n - 2)*Csc[e + f*x]^2, x]/(a + b*Csc
[e + f*x])), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && Gt
Q[n, 3]
```

rule 4486
```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/(csc[(
e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[B/b   Int[Csc[e + f*x],
 x], x] + Simp[(A*b - a*B)/b   Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x
] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0]
```

rule 4570
```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> Simp[(-C)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2)
)), x] + Simp[1/(b*(m + 2))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[
b*A*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /;
 FreeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

rule 4580
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[
(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x
_Symbol] :> Simp[(-C)*Csc[e + f*x]*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m +
1)/(b*f*(m + 3))), x] + Simp[1/(b*(m + 3))   Int[Csc[e + f*x]*(a + b*Csc[e
+ f*x])^m*Simp[a*C + b*(C*(m + 2) + A*(m + 3))*Csc[e + f*x] - (2*a*C - b*B*
(m + 3))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &
& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

3.487. $\int \frac{\sec^5(c+dx)}{a+b\sec(c+dx)}\,dx$

3.487.4 Maple [A] (verified)

Time = 0.71 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.61

method	result
derivativedivides	$\frac{\frac{2a^4 \operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{b^4\sqrt{(a-b)(a+b)}} - \frac{1}{3b\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3} - \frac{-a-b}{2b^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2} - \frac{2a^2+ab+2b^2}{2b^3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)} - \frac{a\left(2a^2+b^2\right)\ln\left(\tan\left(\right.\right.}{2b^4}}{d}$
default	$\frac{\frac{2a^4 \operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{b^4\sqrt{(a-b)(a+b)}} - \frac{1}{3b\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3} - \frac{-a-b}{2b^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2} - \frac{2a^2+ab+2b^2}{2b^3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)} - \frac{a\left(2a^2+b^2\right)\ln\left(\tan\left(\right.\right.}{2b^4}}{d}$
risch	$\frac{i\left(3ab\,e^{5i(dx+c)}+6a^2e^{4i(dx+c)}+12a^2e^{2i(dx+c)}+12b^2e^{2i(dx+c)}-3ab\,e^{i(dx+c)}+6a^2+4b^2\right)}{3b^3d\left(e^{2i(dx+c)}+1\right)^3} + \frac{a^4\ln\left(e^{i(dx+c)}+\frac{ia^2-ib^2+b\sqrt{}}{\sqrt{a^2-b}}\right.}{\sqrt{a^2-b^2}\,d\,b^4}$

input
```
int(sec(d*x+c)^5/(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(2*a^4/b^4/((a-b)*(a+b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a-b)
*(a+b))^(1/2))-1/3/b/(tan(1/2*d*x+1/2*c)+1)^3-1/2*(-a-b)/b^2/(tan(1/2*d*x+
1/2*c)+1)^2-1/2*(2*a^2+a*b+2*b^2)/b^3/(tan(1/2*d*x+1/2*c)+1)-1/2*a*(2*a^2+
b^2)/b^4*ln(tan(1/2*d*x+1/2*c)+1)-1/3/b/(tan(1/2*d*x+1/2*c)-1)^3-1/2*(a+b)
/b^2/(tan(1/2*d*x+1/2*c)-1)^2-1/2*(2*a^2+a*b+2*b^2)/b^3/(tan(1/2*d*x+1/2*c
)-1)+1/2*a*(2*a^2+b^2)/b^4*ln(tan(1/2*d*x+1/2*c)-1))
```

3.487.5 Fricas [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 557, normalized size of antiderivative = 3.55

$$\int \frac{\sec^5(c+dx)}{a+b\sec(c+dx)}\,dx$$
$$= \left[\frac{6\sqrt{a^2-b^2}a^4\cos(dx+c)^3\log\left(\frac{2\,ab\cos(dx+c)-(a^2-2\,b^2)\cos(dx+c)^2+2\sqrt{a^2-b^2}(b\cos(dx+c)+a)\sin(dx+c)+2\,a^2-b^2}{a^2\cos(dx+c)^2+2\,ab\cos(dx+c)+b^2}\right)-3\,(2\right.$$

input
```
integrate(sec(d*x+c)^5/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

output
```
[1/12*(6*sqrt(a^2 - b^2)*a^4*cos(d*x + c)^3*log((2*a*b*cos(d*x + c) - (a^2
 - 2*b^2)*cos(d*x + c)^2 + 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x
+ c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) - 3*(
2*a^5 - a^3*b^2 - a*b^4)*cos(d*x + c)^3*log(sin(d*x + c) + 1) + 3*(2*a^5 -
 a^3*b^2 - a*b^4)*cos(d*x + c)^3*log(-sin(d*x + c) + 1) + 2*(2*a^2*b^3 - 2
*b^5 + 2*(3*a^4*b - a^2*b^3 - 2*b^5)*cos(d*x + c)^2 - 3*(a^3*b^2 - a*b^4)*
cos(d*x + c))*sin(d*x + c))/((a^2*b^4 - b^6)*d*cos(d*x + c)^3), 1/12*(12*s
qrt(-a^2 + b^2)*a^4*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 -
b^2)*sin(d*x + c)))*cos(d*x + c)^3 - 3*(2*a^5 - a^3*b^2 - a*b^4)*cos(d*x +
 c)^3*log(sin(d*x + c) + 1) + 3*(2*a^5 - a^3*b^2 - a*b^4)*cos(d*x + c)^3*l
og(-sin(d*x + c) + 1) + 2*(2*a^2*b^3 - 2*b^5 + 2*(3*a^4*b - a^2*b^3 - 2*b^
5)*cos(d*x + c)^2 - 3*(a^3*b^2 - a*b^4)*cos(d*x + c))*sin(d*x + c))/((a^2*
b^4 - b^6)*d*cos(d*x + c)^3)]
```

3.487.6 Sympy [F]

$$\int \frac{\sec^5(c+dx)}{a+b\sec(c+dx)}\,dx = \int \frac{\sec^5{(c+dx)}}{a+b\sec{(c+dx)}}\,dx$$

input
```
integrate(sec(d*x+c)**5/(a+b*sec(d*x+c)),x)
```

output
```
Integral(sec(c + d*x)**5/(a + b*sec(c + d*x)), x)
```

3.487.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\sec^5(c+dx)}{a+b\sec(c+dx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(sec(d*x+c)^5/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

3.487.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 286 vs. $2(140) = 280$.

Time = 0.33 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.82

$$\int \frac{\sec^5(c+dx)}{a+b\sec(c+dx)}\,dx = \frac{\frac{12\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{-a^2+b^2}}\right)\right)a^4}{\sqrt{-a^2+b^2}b^4} - \frac{3(2a^3+ab^2)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{b^4} + \frac{3(2a^3+ab^2)\,\mathrm{l}}{}}{}$$

input
```
integrate(sec(d*x+c)^5/(a+b*sec(d*x+c)),x, algorithm="giac")
```

output
```
1/6*(12*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan
(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))*a^4/(sqrt(-
a^2 + b^2)*b^4) - 3*(2*a^3 + a*b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/b^4
 + 3*(2*a^3 + a*b^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1))/b^4 - 2*(6*a^2*tan
(1/2*d*x + 1/2*c)^5 + 3*a*b*tan(1/2*d*x + 1/2*c)^5 + 6*b^2*tan(1/2*d*x + 1
/2*c)^5 - 12*a^2*tan(1/2*d*x + 1/2*c)^3 - 4*b^2*tan(1/2*d*x + 1/2*c)^3 + 6
*a^2*tan(1/2*d*x + 1/2*c) - 3*a*b*tan(1/2*d*x + 1/2*c) + 6*b^2*tan(1/2*d*x
 + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 - 1)^3*b^3))/d
```

3.487.9 Mupad [B] (verification not implemented)

Time = 15.23 (sec) , antiderivative size = 1021, normalized size of antiderivative = 6.50

$$\int \frac{\sec^5(c+dx)}{a+b\sec(c+dx)}\,dx = \text{Too large to display}$$

input
```
int(1/(cos(c + d*x)^5*(a + b/cos(c + d*x))),x)
```

output
```
-((9*a^4*cos(c + d*x)*atanh((8*a^6*sin(c/2 + (d*x)/2)*(a^2 - b^2) - 8*a^8*
sin(c/2 + (d*x)/2) + b^6*sin(c/2 + (d*x)/2)*(a^2 - b^2) + 8*a^7*b*sin(c/2
+ (d*x)/2) - 2*a*b^5*sin(c/2 + (d*x)/2)*(a^2 - b^2) - 8*a^5*b*sin(c/2 + (d
*x)/2)*(a^2 - b^2) + 5*a^2*b^4*sin(c/2 + (d*x)/2)*(a^2 - b^2) - 8*a^3*b^3*
sin(c/2 + (d*x)/2)*(a^2 - b^2) + 8*a^4*b^2*sin(c/2 + (d*x)/2)*(a^2 - b^2))
/(b*cos(c/2 + (d*x)/2)*(a^2 - b^2)^(1/2)*(4*a^4*(a^2 - b^2) + b^4*(a^2 - b
^2) + 2*a^5*b - 4*a^6 + 2*a^3*b^3 + 4*a^2*b^2*(a^2 - b^2) - a*b^3*(a^2 - b
^2) - 2*a^3*b*(a^2 - b^2)))))/2 - (3*b^3*sin(c + d*x)*(a^2 - b^2)^(1/2))/2
 - (b^3*sin(3*c + 3*d*x)*(a^2 - b^2)^(1/2))/2 + (3*a^4*cos(3*c + 3*d*x)*at
anh((8*a^6*sin(c/2 + (d*x)/2)*(a^2 - b^2) - 8*a^8*sin(c/2 + (d*x)/2) + b^6
*sin(c/2 + (d*x)/2)*(a^2 - b^2) + 8*a^7*b*sin(c/2 + (d*x)/2) - 2*a*b^5*sin
(c/2 + (d*x)/2)*(a^2 - b^2) - 8*a^5*b*sin(c/2 + (d*x)/2)*(a^2 - b^2) + 5*a
^2*b^4*sin(c/2 + (d*x)/2)*(a^2 - b^2) - 8*a^3*b^3*sin(c/2 + (d*x)/2)*(a^2
- b^2) + 8*a^4*b^2*sin(c/2 + (d*x)/2)*(a^2 - b^2))/(b*cos(c/2 + (d*x)/2)*(
a^2 - b^2)^(1/2)*(4*a^4*(a^2 - b^2) + b^4*(a^2 - b^2) + 2*a^5*b - 4*a^6 +
2*a^3*b^3 + 4*a^2*b^2*(a^2 - b^2) - a*b^3*(a^2 - b^2) - 2*a^3*b*(a^2 - b^2
)))))/2 + (3*a^3*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(3*c + 3*
d*x)*(a^2 - b^2)^(1/2))/2 - (3*a^2*b*sin(c + d*x)*(a^2 - b^2)^(1/2))/4 + (
9*a^3*cos(c + d*x)*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*(a^2 - b^2
)^(1/2))/2 + (3*a*b^2*sin(2*c + 2*d*x)*(a^2 - b^2)^(1/2))/4 - (3*a^2*b*...
```

3.487. $\int \frac{\sec^5(c+dx)}{a+b\sec(c+dx)}\,dx$

3.488 $\int \frac{\sec^4(c+dx)}{a+b\sec(c+dx)}\,dx$

3.488.1 Optimal result 3611
3.488.2 Mathematica [A] (verified) 3611
3.488.3 Rubi [A] (verified) 3612
3.488.4 Maple [A] (verified) 3615
3.488.5 Fricas [B] (verification not implemented) 3616
3.488.6 Sympy [F] 3616
3.488.7 Maxima [F(-2)] 3617
3.488.8 Giac [A] (verification not implemented) 3617
3.488.9 Mupad [B] (verification not implemented) 3618

3.488.1 Optimal result

Integrand size = 21, antiderivative size = 119

$$\int \frac{\sec^4(c+dx)}{a+b\sec(c+dx)}\,dx = \frac{(2a^2+b^2)\operatorname{arctanh}(\sin(c+dx))}{2b^3d} - \frac{2a^3\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}b^3\sqrt{a+b}d} - \frac{a\tan(c+dx)}{b^2d} + \frac{\sec(c+dx)\tan(c+dx)}{2bd}$$

output
```
1/2*(2*a^2+b^2)*arctanh(sin(d*x+c))/b^3/d-2*a^3*arctanh((a-b)^(1/2)*tan(1/
2*d*x+1/2*c)/(a+b)^(1/2))/b^3/d/(a-b)^(1/2)/(a+b)^(1/2)-a*tan(d*x+c)/b^2/d
+1/2*sec(d*x+c)*tan(d*x+c)/b/d
```

3.488.2 Mathematica [A] (verified)

Time = 0.99 (sec) , antiderivative size = 238, normalized size of antiderivative = 2.00

$$\int \frac{\sec^4(c+dx)}{a+b\sec(c+dx)}\,dx = \frac{\frac{8a^3\operatorname{arctanh}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} - 4a^2\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right) - 2b^2\log\left(\cos\left(\frac{1}{2}(c+dx)\right) - s}{}$$

input
```
Integrate[Sec[c + d*x]^4/(a + b*Sec[c + d*x]),x]
```

output
```
((8*a^3*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b
^2] - 4*a^2*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - 2*b^2*Log[Cos[(c +
d*x)/2] - Sin[(c + d*x)/2]] + 4*a^2*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2
]] + 2*b^2*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + b^2/(Cos[(c + d*x)/2
] - Sin[(c + d*x)/2])^2 - b^2/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2 - 4*
a*b*Tan[c + d*x])/(4*b^3*d)
```

3.488.3 Rubi [A] (verified)

Time = 0.87 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.08, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {3042, 4338, 3042, 4570, 3042, 4486, 3042, 4257, 4318, 3042, 3138, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^4(c+dx)}{a+b\sec(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^4}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{4338}$$

$$\frac{\int \frac{\sec(c+dx)\left(-2a\sec^2(c+dx)+b\sec(c+dx)+a\right)}{a+b\sec(c+dx)}dx}{2b}+\frac{\tan(c+dx)\sec(c+dx)}{2bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(-2a\csc\left(c+dx+\frac{\pi}{2}\right)^2+b\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{2b}+\frac{\tan(c+dx)\sec(c+dx)}{2bd}$$

$$\Big\downarrow \text{4570}$$

$$\frac{\frac{\int \frac{\sec(c+dx)\left(ab+\left(2a^2+b^2\right)\sec(c+dx)\right)}{a+b\sec(c+dx)}dx}{b}-\frac{2a\tan(c+dx)}{bd}}{2b}+\frac{\tan(c+dx)\sec(c+dx)}{2bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(ab+\left(2a^2+b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b}-\frac{2a\tan(c+dx)}{bd}}{2b}+\frac{\tan(c+dx)\sec(c+dx)}{2bd}$$

$$\Big\downarrow \; 4486$$

$$\frac{\frac{\frac{\left(2a^2+b^2\right)\int \sec(c+dx)dx}{b}-\frac{2a^3\int \frac{\sec(c+dx)}{a+b\sec(c+dx)}dx}{b}}{b}-\frac{2a\tan(c+dx)}{bd}}{2b}+\frac{\tan(c+dx)\sec(c+dx)}{2bd}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{\frac{\left(2a^2+b^2\right)\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{b}-\frac{2a^3\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b}}{b}-\frac{2a\tan(c+dx)}{bd}}{2b}+\frac{\tan(c+dx)\sec(c+dx)}{2bd}$$

$$\Big\downarrow \; 4257$$

$$\frac{\frac{\frac{\left(2a^2+b^2\right)\text{arctanh}(\sin(c+dx))}{bd}-\frac{2a^3\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b}}{b}-\frac{2a\tan(c+dx)}{bd}}{2b}+\frac{\tan(c+dx)\sec(c+dx)}{2bd}$$

$$\Big\downarrow \; 4318$$

$$\frac{\frac{\frac{\left(2a^2+b^2\right)\text{arctanh}(\sin(c+dx))}{bd}-\frac{2a^3\int \frac{1}{\frac{a\cos(c+dx)}{b}+1}dx}{b^2}}{b}-\frac{2a\tan(c+dx)}{bd}}{2b}+\frac{\tan(c+dx)\sec(c+dx)}{2bd}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{\frac{\left(2a^2+b^2\right)\text{arctanh}(\sin(c+dx))}{bd}-\frac{2a^3\int \frac{1}{\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{b}+1}dx}{b^2}}{b}-\frac{2a\tan(c+dx)}{bd}}{2b}+\frac{\tan(c+dx)\sec(c+dx)}{2bd}$$

$$\Big\downarrow \; 3138$$

$$\frac{\frac{\frac{\left(2a^2+b^2\right)\text{arctanh}(\sin(c+dx))}{bd}-\frac{4a^3\int \frac{1}{\left(1-\frac{a}{b}\right)\tan^2\left(\frac{1}{2}(c+dx)\right)+\frac{a+b}{b}}d\tan\left(\frac{1}{2}(c+dx)\right)}{b^2d}}{b}-\frac{2a\tan(c+dx)}{bd}}{2b}+\frac{\tan(c+dx)\sec(c+dx)}{2bd}$$

$$\Big\downarrow \; 221$$

$$\frac{\frac{\frac{\left(2a^2+b^2\right)\text{arctanh}(\sin(c+dx))}{bd}-\frac{4a^3\text{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{bd\sqrt{a-b}\sqrt{a+b}}}{b}-\frac{2a\tan(c+dx)}{bd}}{2b}+\frac{\tan(c+dx)\sec(c+dx)}{2bd}$$

input
```
Int[Sec[c + d*x]^4/(a + b*Sec[c + d*x]),x]
```

3.488. $\int \frac{\sec^4(c+dx)}{a+b\sec(c+dx)}\,dx$

output
```
(Sec[c + d*x]*Tan[c + d*x])/(2*b*d) + ((((2*a^2 + b^2)*ArcTanh[Sin[c + d*x
]])/(b*d) - (4*a^3*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(S
qrt[a - b]*b*Sqrt[a + b]*d))/b - (2*a*Tan[c + d*x])/(b*d))/(2*b)
```

3.488.3.1 Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4318
```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[1/b   Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4338
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-d^3)*Cot[e + f*x]*((d*Csc[e + f*x])^(n - 3)/(b*f*
(n - 2))), x] + Simp[d^3/(b*(n - 2))   Int[(d*Csc[e + f*x])^(n - 3)*(Simp[a
*(n - 3) + b*(n - 3)*Csc[e + f*x] - a*(n - 2)*Csc[e + f*x]^2, x]/(a + b*Csc
[e + f*x])), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && Gt
Q[n, 3]
```

rule 4486
```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/(csc[(
e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[B/b   Int[Csc[e + f*x],
 x], x] + Simp[(A*b - a*B)/b   Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x
] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0]
```

3.488. $\int \frac{\sec^4(c+dx)}{a+b\sec(c+dx)}\,dx$

rule 4570
```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> Simp[(-C)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2)
)), x] + Simp[1/(b*(m + 2))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[
b*A*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /;
 FreeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

3.488.4 Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.61

method	result
derivativedivides	$\frac{-\frac{2a^3 \operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{b^3\sqrt{(a-b)(a+b)}}+\frac{1}{2b\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}-\frac{-2a-b}{2b^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{\left(-2a^2-b^2\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2b^3}-\frac{}{2b\left(\tan\left(\right.\right.}}{d}$
default	$\frac{-\frac{2a^3 \operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{b^3\sqrt{(a-b)(a+b)}}+\frac{1}{2b\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}-\frac{-2a-b}{2b^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{\left(-2a^2-b^2\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2b^3}-\frac{}{2b\left(\tan\left(\right.\right.}}{d}$
risch	$-\frac{i\left(b\,\mathrm{e}^{3i(dx+c)}+2a\,\mathrm{e}^{2i(dx+c)}-b\,\mathrm{e}^{i(dx+c)}+2a\right)}{d\,b^2\left(\mathrm{e}^{2i(dx+c)}+1\right)^2}+\frac{\ln\left(\mathrm{e}^{i(dx+c)}+i\right)a^2}{d\,b^3}+\frac{\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{2db}-\frac{\ln\left(\mathrm{e}^{i(dx+c)}-i\right)a^2}{d\,b^3}-\frac{\ln\left(\mathrm{e}^{i}\right.}{}$

input
```
int(sec(d*x+c)^4/(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-2*a^3/b^3/((a-b)*(a+b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a-b
)*(a+b))^(1/2))+1/2/b/(tan(1/2*d*x+1/2*c)-1)^2-1/2*(-2*a-b)/b^2/(tan(1/2*d
*x+1/2*c)-1)+1/2/b^3*(-2*a^2-b^2)*ln(tan(1/2*d*x+1/2*c)-1)-1/2/b/(tan(1/2*
d*x+1/2*c)+1)^2-1/2*(-2*a-b)/b^2/(tan(1/2*d*x+1/2*c)+1)+1/2*(2*a^2+b^2)/b^
3*ln(tan(1/2*d*x+1/2*c)+1))
```

3.488.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 214 vs. $2(106) = 212$.

Time = 0.39 (sec) , antiderivative size = 485, normalized size of antiderivative = 4.08

$$\int \frac{\sec^4(c+dx)}{a+b\sec(c+dx)}\,dx$$
$$= \left[\frac{2\sqrt{a^2-b^2}a^3\cos(dx+c)^2\log\left(\frac{2\,ab\cos(dx+c)-(a^2-2\,b^2)\cos(dx+c)^2-2\sqrt{a^2-b^2}(b\cos(dx+c)+a)\sin(dx+c)+2\,a^2-b^2}{a^2\cos(dx+c)^2+2\,ab\cos(dx+c)+b^2}\right)+(2\,a}{}\right.$$
$$-\frac{4\sqrt{-a^2+b^2}a^3\arctan\left(-\frac{\sqrt{-a^2+b^2}(b\cos(dx+c)+a)}{(a^2-b^2)\sin(dx+c)}\right)\cos(dx+c)^2-(2\,a^4-a^2b^2-b^4)\cos(dx+c)^2\log(\sin}{4}$$

input
```
integrate(sec(d*x+c)^4/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

output
```
[1/4*(2*sqrt(a^2 - b^2)*a^3*cos(d*x + c)^2*log((2*a*b*cos(d*x + c) - (a^2
- 2*b^2)*cos(d*x + c)^2 - 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x +
 c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) + (2*a
^4 - a^2*b^2 - b^4)*cos(d*x + c)^2*log(sin(d*x + c) + 1) - (2*a^4 - a^2*b^
2 - b^4)*cos(d*x + c)^2*log(-sin(d*x + c) + 1) + 2*(a^2*b^2 - b^4 - 2*(a^3
*b - a*b^3)*cos(d*x + c))*sin(d*x + c))/((a^2*b^3 - b^5)*d*cos(d*x + c)^2)
, -1/4*(4*sqrt(-a^2 + b^2)*a^3*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) +
a)/((a^2 - b^2)*sin(d*x + c)))*cos(d*x + c)^2 - (2*a^4 - a^2*b^2 - b^4)*co
s(d*x + c)^2*log(sin(d*x + c) + 1) + (2*a^4 - a^2*b^2 - b^4)*cos(d*x + c)^
2*log(-sin(d*x + c) + 1) - 2*(a^2*b^2 - b^4 - 2*(a^3*b - a*b^3)*cos(d*x +
c))*sin(d*x + c))/((a^2*b^3 - b^5)*d*cos(d*x + c)^2)]
```

3.488.6 Sympy [F]

$$\int \frac{\sec^4(c+dx)}{a+b\sec(c+dx)}\,dx = \int \frac{\sec^4(c+dx)}{a+b\sec(c+dx)}\,dx$$

input
```
integrate(sec(d*x+c)**4/(a+b*sec(d*x+c)),x)
```

output
```
Integral(sec(c + d*x)**4/(a + b*sec(c + d*x)), x)
```

3.488.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\sec^4(c+dx)}{a+b\sec(c+dx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(sec(d*x+c)^4/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

3.488.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.77

$$\int \frac{\sec^4(c+dx)}{a+b\sec(c+dx)}\,dx = -\frac{\frac{4\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{-a^2+b^2}}\right)\right)a^3}{\sqrt{-a^2+b^2}b^3} - \frac{(2a^2+b^2)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{b^3} + \frac{(2a^2+b^2)\log(}{}}{2d}$$

input
```
integrate(sec(d*x+c)^4/(a+b*sec(d*x+c)),x, algorithm="giac")
```

output
```
-1/2*(4*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan
(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))*a^3/(sqrt(-
a^2 + b^2)*b^3) - (2*a^2 + b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/b^3 + (
2*a^2 + b^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1))/b^3 - 2*(2*a*tan(1/2*d*x +
 1/2*c)^3 + b*tan(1/2*d*x + 1/2*c)^3 - 2*a*tan(1/2*d*x + 1/2*c) + b*tan(1/
2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 - 1)^2*b^2))/d
```

3.488.9 Mupad [B] (verification not implemented)

Time = 14.12 (sec) , antiderivative size = 1002, normalized size of antiderivative = 8.42

$$\int \frac{\sec^4(c+dx)}{a+b\sec(c+dx)}\,dx = \text{Too large to display}$$

input
```
int(1/(cos(c + d*x)^4*(a + b/cos(c + d*x))),x)
```

output
```
sin(c + d*x)/(2*b*d*(cos(2*c + 2*d*x)/2 + 1/2)) + atanh(sin(c/2 + (d*x)/2)
/cos(c/2 + (d*x)/2))/(2*b*d*(cos(2*c + 2*d*x)/2 + 1/2)) + (a^2*atanh(sin(c
/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/(b^3*d*(cos(2*c + 2*d*x)/2 + 1/2)) + (a
tanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(2*c + 2*d*x))/(2*b*d*(cos(
2*c + 2*d*x)/2 + 1/2)) - (a*sin(2*c + 2*d*x))/(2*b^2*d*(cos(2*c + 2*d*x)/2
 + 1/2)) - (a^3*atan(((8*a^6*sin(c/2 + (d*x)/2)*(a^2 - b^2) - 8*a^8*sin(c/
2 + (d*x)/2) + b^6*sin(c/2 + (d*x)/2)*(a^2 - b^2) + 8*a^7*b*sin(c/2 + (d*x
)/2) - 2*a*b^5*sin(c/2 + (d*x)/2)*(a^2 - b^2) - 8*a^5*b*sin(c/2 + (d*x)/2)
*(a^2 - b^2) + 5*a^2*b^4*sin(c/2 + (d*x)/2)*(a^2 - b^2) - 8*a^3*b^3*sin(c/
2 + (d*x)/2)*(a^2 - b^2) + 8*a^4*b^2*sin(c/2 + (d*x)/2)*(a^2 - b^2))*1i)/(
b*cos(c/2 + (d*x)/2)*(a^2 - b^2)^(1/2)*(4*a^4*(a^2 - b^2) + b^4*(a^2 - b^2
) + 2*a^5*b - 4*a^6 + 2*a^3*b^3 + 4*a^2*b^2*(a^2 - b^2) - a*b^3*(a^2 - b^2
) - 2*a^3*b*(a^2 - b^2))))*1i)/(b^3*d*(a^2 - b^2)^(1/2)*(cos(2*c + 2*d*x)/
2 + 1/2)) + (a^2*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(2*c + 2*
d*x))/(b^3*d*(cos(2*c + 2*d*x)/2 + 1/2)) - (a^3*cos(2*c + 2*d*x)*atan(((8*
a^6*sin(c/2 + (d*x)/2)*(a^2 - b^2) - 8*a^8*sin(c/2 + (d*x)/2) + b^6*sin(c/
2 + (d*x)/2)*(a^2 - b^2) + 8*a^7*b*sin(c/2 + (d*x)/2) - 2*a*b^5*sin(c/2 +
(d*x)/2)*(a^2 - b^2) - 8*a^5*b*sin(c/2 + (d*x)/2)*(a^2 - b^2) + 5*a^2*b^4*
sin(c/2 + (d*x)/2)*(a^2 - b^2) - 8*a^3*b^3*sin(c/2 + (d*x)/2)*(a^2 - b^2)
+ 8*a^4*b^2*sin(c/2 + (d*x)/2)*(a^2 - b^2))*1i)/(b*cos(c/2 + (d*x)/2)*(...
```

3.489 $\int \frac{\sec^3(c+dx)}{a+b\sec(c+dx)}\,dx$

3.489.1 Optimal result . . . 3619
3.489.2 Mathematica [A] (verified) . . . 3619
3.489.3 Rubi [A] (verified) . . . 3620
3.489.4 Maple [A] (verified) . . . 3623
3.489.5 Fricas [B] (verification not implemented) . . . 3623
3.489.6 Sympy [F] . . . 3624
3.489.7 Maxima [F(-2)] . . . 3624
3.489.8 Giac [A] (verification not implemented) . . . 3625
3.489.9 Mupad [B] (verification not implemented) . . . 3625

3.489.1 Optimal result

Integrand size = 21, antiderivative size = 85

$$\int \frac{\sec^3(c+dx)}{a+b\sec(c+dx)}\,dx = -\frac{a\operatorname{arctanh}(\sin(c+dx))}{b^2d} + \frac{2a^2\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}b^2\sqrt{a+b}d} + \frac{\tan(c+dx)}{bd}$$

output
```
-a*arctanh(sin(d*x+c))/b^2/d+2*a^2*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/
(a+b)^(1/2))/b^2/d/(a-b)^(1/2)/(a+b)^(1/2)+tan(d*x+c)/b/d
```

3.489.2 Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.35

$$\int \frac{\sec^3(c+dx)}{a+b\sec(c+dx)}\,dx = \frac{-\frac{2a^2\operatorname{arctanh}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} + a\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\right.\right.}{b^2d}$$

input
```
Integrate[Sec[c + d*x]^3/(a + b*Sec[c + d*x]),x]
```

output
```
((-2*a^2*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 -
b^2] + a*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2]
+ Sin[(c + d*x)/2]]) + b*Tan[c + d*x])/(b^2*d)
```

3.489.3 Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.04, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {3042, 4277, 3042, 4276, 3042, 4257, 4318, 3042, 3138, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^3(c+dx)}{a+b\sec(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^3}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{4277}$$

$$\frac{\tan(c+dx)}{bd}-\frac{a\int \frac{\sec^2(c+dx)}{a+b\sec(c+dx)}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\tan(c+dx)}{bd}-\frac{a\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b}$$

$$\Big\downarrow \text{4276}$$

$$\frac{\tan(c+dx)}{bd}-\frac{a\left(\frac{\int \sec(c+dx)dx}{b}-\frac{a\int \frac{\sec(c+dx)}{a+b\sec(c+dx)}dx}{b}\right)}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\tan(c+dx)}{bd}-\frac{a\left(\frac{\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{b}-\frac{a\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b}\right)}{b}$$

$$\Big\downarrow \text{4257}$$

$$\frac{\tan(c+dx)}{bd}-\frac{a\left(\frac{\text{arctanh}(\sin(c+dx))}{bd}-\frac{a\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b}\right)}{b}$$

$\Big\downarrow$ 4318

$$\frac{\tan(c+dx)}{bd}-\frac{a\left(\frac{\text{arctanh}(\sin(c+dx))}{bd}-\frac{a\int\frac{1}{\frac{a\cos(c+dx)}{b}+1}dx}{b^2}\right)}{b}$$

$\Big\downarrow$ 3042

$$\frac{\tan(c+dx)}{bd}-\frac{a\left(\frac{\text{arctanh}(\sin(c+dx))}{bd}-\frac{a\int\frac{1}{\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{b}+1}dx}{b^2}\right)}{b}$$

$\Big\downarrow$ 3138

$$\frac{\tan(c+dx)}{bd}-\frac{a\left(\frac{\text{arctanh}(\sin(c+dx))}{bd}-\frac{2a\int\frac{1}{\left(1-\frac{a}{b}\right)\tan^2\left(\frac{1}{2}(c+dx)\right)+\frac{a+b}{b}}d\tan\left(\frac{1}{2}(c+dx)\right)}{b^2d}\right)}{b}$$

$\Big\downarrow$ 221

$$\frac{\tan(c+dx)}{bd}-\frac{a\left(\frac{\text{arctanh}(\sin(c+dx))}{bd}-\frac{2a\text{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{bd\sqrt{a-b}\sqrt{a+b}}\right)}{b}$$

input
```
Int[Sec[c + d*x]^3/(a + b*Sec[c + d*x]),x]
```

output
```
-((a*(ArcTanh[Sin[c + d*x]]/(b*d) - (2*a*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x
)/2])/Sqrt[a + b]])/(Sqrt[a - b]*b*Sqrt[a + b]*d)))/b) + Tan[c + d*x]/(b*d
)
```

3.489. $\int \frac{\sec^3(c+dx)}{a+b\sec(c+dx)}\,dx$

3.489.3.1 Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4276
```
Int[csc[(e_.) + (f_.)*(x_)]^2/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Sym
bol] :> Simp[1/b   Int[Csc[e + f*x], x], x] - Simp[a/b   Int[Csc[e + f*x]/(
a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x]
```

rule 4277
```
Int[csc[(e_.) + (f_.)*(x_)]^3/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Sym
bol] :> Simp[-Cot[e + f*x]/(b*f), x] - Simp[a/b   Int[Csc[e + f*x]^2/(a + b
*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x]
```

rule 4318
```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[1/b   Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

3.489. $\int \frac{\sec^3(c+dx)}{a+b\sec(c+dx)}\,dx$

3.489.4 Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.45

method	result
derivativedivides	$\frac{-\frac{1}{b\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{b^2}-\frac{1}{b\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{b^2}+\frac{2a^2\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{b^2\sqrt{(a-b)(a+b)}}}{d}$
default	$\frac{-\frac{1}{b\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{b^2}-\frac{1}{b\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{b^2}+\frac{2a^2\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{b^2\sqrt{(a-b)(a+b)}}}{d}$
risch	$\frac{2i}{db\left(e^{2i(dx+c)}+1\right)}+\frac{a^2\ln\left(e^{i(dx+c)}+\frac{ia^2-ib^2+b\sqrt{a^2-b^2}}{\sqrt{a^2-b^2}\,a}\right)}{\sqrt{a^2-b^2}\,d\,b^2}-\frac{a^2\ln\left(e^{i(dx+c)}+\frac{-ia^2+ib^2+b\sqrt{a^2-b^2}}{a\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,d\,b^2}-\frac{a\ln\left(e^{i(dx+c)}\right.}{b^2d}$

input
```
int(sec(d*x+c)^3/(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/b/(tan(1/2*d*x+1/2*c)+1)-a/b^2*ln(tan(1/2*d*x+1/2*c)+1)-1/b/(tan(1
/2*d*x+1/2*c)-1)+a/b^2*ln(tan(1/2*d*x+1/2*c)-1)+2*a^2/b^2/((a-b)*(a+b))^(1
/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a-b)*(a+b))^(1/2)))
```

3.489.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 168 vs. 2(76) = 152.

Time = 0.31 (sec) , antiderivative size = 392, normalized size of antiderivative = 4.61

$$\int \frac{\sec^3(c+dx)}{a+b\sec(c+dx)}\,dx$$
$$= \left[\frac{\sqrt{a^2-b^2}a^2\cos(dx+c)\log\left(\frac{2\,ab\cos(dx+c)-(a^2-2\,b^2)\cos(dx+c)^2+2\sqrt{a^2-b^2}(b\cos(dx+c)+a)\sin(dx+c)+2\,a^2-b^2}{a^2\cos(dx+c)^2+2\,ab\cos(dx+c)+b^2}\right)-(a^3-}{2\,(a^2b^2-}\right.$$

input
```
integrate(sec(d*x+c)^3/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

output
```
[1/2*(sqrt(a^2 - b^2)*a^2*cos(d*x + c)*log((2*a*b*cos(d*x + c) - (a^2 - 2*
b^2)*cos(d*x + c)^2 + 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c)
+ 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) - (a^3 - a
*b^2)*cos(d*x + c)*log(sin(d*x + c) + 1) + (a^3 - a*b^2)*cos(d*x + c)*log(
-sin(d*x + c) + 1) + 2*(a^2*b - b^3)*sin(d*x + c))/((a^2*b^2 - b^4)*d*cos(
d*x + c)), 1/2*(2*sqrt(-a^2 + b^2)*a^2*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x
 + c) + a)/((a^2 - b^2)*sin(d*x + c)))*cos(d*x + c) - (a^3 - a*b^2)*cos(d*
x + c)*log(sin(d*x + c) + 1) + (a^3 - a*b^2)*cos(d*x + c)*log(-sin(d*x + c
) + 1) + 2*(a^2*b - b^3)*sin(d*x + c))/((a^2*b^2 - b^4)*d*cos(d*x + c))]
```

3.489.6 Sympy [F]

$$\int \frac{\sec^3(c+dx)}{a+b\sec(c+dx)}\,dx = \int \frac{\sec^3{(c+dx)}}{a+b\sec{(c+dx)}}\,dx$$

input
```
integrate(sec(d*x+c)**3/(a+b*sec(d*x+c)),x)
```

output
```
Integral(sec(c + d*x)**3/(a + b*sec(c + d*x)), x)
```

3.489.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\sec^3(c+dx)}{a+b\sec(c+dx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(sec(d*x+c)^3/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

3.489.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.79

$$\int \frac{\sec^3(c+dx)}{a+b\sec(c+dx)}\,dx = \frac{\frac{2\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{-a^2+b^2}}\right)\right)a^2}{\sqrt{-a^2+b^2}b^2} - \frac{a\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{b^2} + \frac{a\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{b^2}}{d}$$

input
```
integrate(sec(d*x+c)^3/(a+b*sec(d*x+c)),x, algorithm="giac")
```

output
```
(2*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*
d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))*a^2/(sqrt(-a^2 +
 b^2)*b^2) - a*log(abs(tan(1/2*d*x + 1/2*c) + 1))/b^2 + a*log(abs(tan(1/2*
d*x + 1/2*c) - 1))/b^2 - 2*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/2*c)^2 -
 1)*b))/d
```

3.489.9 Mupad [B] (verification not implemented)

Time = 14.05 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.40

$$\int \frac{\sec^3(c+dx)}{a+b\sec(c+dx)}\,dx = \frac{\tan(c+d\,x)}{b\,d} - \frac{2\,a\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{b^2\,d} - \frac{a^2\,\mathrm{atan}\left(\frac{a\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,1\mathrm{i}-b\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,1\mathrm{i}}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sqrt{a^2-b^2}}\right)\,2\mathrm{i}}{b^2\,d\,\sqrt{a^2-b^2}}$$

input
```
int(1/(cos(c + d*x)^3*(a + b/cos(c + d*x))),x)
```

output
```
tan(c + d*x)/(b*d) - (2*a*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/(b
^2*d) - (a^2*atan((a*sin(c/2 + (d*x)/2)*1i - b*sin(c/2 + (d*x)/2)*1i)/(cos
(c/2 + (d*x)/2)*(a^2 - b^2)^(1/2)))*2i)/(b^2*d*(a^2 - b^2)^(1/2))
```

3.490 $\int \frac{\sec^2(c+dx)}{a+b\sec(c+dx)}\,dx$

3.490.1 Optimal result . . . 3626
3.490.2 Mathematica [A] (verified) . . . 3626
3.490.3 Rubi [A] (verified) . . . 3627
3.490.4 Maple [A] (verified) . . . 3629
3.490.5 Fricas [A] (verification not implemented) . . . 3629
3.490.6 Sympy [F] . . . 3630
3.490.7 Maxima [F(-2)] . . . 3630
3.490.8 Giac [B] (verification not implemented) . . . 3631
3.490.9 Mupad [B] (verification not implemented) . . . 3631

3.490.1 Optimal result

Integrand size = 21, antiderivative size = 68

$$\int \frac{\sec^2(c+dx)}{a+b\sec(c+dx)}\,dx=\frac{\operatorname{arctanh}(\sin(c+dx))}{bd}-\frac{2a\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}b\sqrt{a+b}d}$$

output
```
arctanh(sin(d*x+c))/b/d-2*a*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(
1/2))/b/d/(a-b)^(1/2)/(a+b)^(1/2)
```

3.490.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.50

$$\int \frac{\sec^2(c+dx)}{a+b\sec(c+dx)}\,dx=\frac{\frac{2a\operatorname{arctanh}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}-\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c\right.\right.}{bd}$$

input
```
Integrate[Sec[c + d*x]^2/(a + b*Sec[c + d*x]),x]
```

output
```
((2*a*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2
] - Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + Log[Cos[(c + d*x)/2] + Sin[
(c + d*x)/2]])/(b*d)
```

3.490.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {3042, 4276, 3042, 4257, 4318, 3042, 3138, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(c+dx)}{a+b\sec(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{4276}$$

$$\frac{\int \sec(c+dx)dx}{b}-\frac{a\int \frac{\sec(c+dx)}{a+b\sec(c+dx)}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{b}-\frac{a\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b}$$

$$\Big\downarrow \text{4257}$$

$$\frac{\text{arctanh}(\sin(c+dx))}{bd}-\frac{a\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b}$$

$$\Big\downarrow \text{4318}$$

$$\frac{\text{arctanh}(\sin(c+dx))}{bd}-\frac{a\int \frac{1}{\frac{a\cos(c+dx)}{b}+1}dx}{b^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\text{arctanh}(\sin(c+dx))}{bd}-\frac{a\int \frac{1}{\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{b}+1}dx}{b^2}$$

$$\Big\downarrow \text{3138}$$

$$\frac{\text{arctanh}(\sin(c+dx))}{bd}-\frac{2a\int \frac{1}{\left(1-\frac{a}{b}\right)\tan^2\left(\frac{1}{2}(c+dx)\right)+\frac{a+b}{b}}d\tan\left(\frac{1}{2}(c+dx)\right)}{b^2d}$$

$$\Big\downarrow \text{221}$$

$$\frac{\text{arctanh}(\sin(c+dx))}{bd}-\frac{2a\text{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{bd\sqrt{a-b}\sqrt{a+b}}$$

input
```
Int[Sec[c + d*x]^2/(a + b*Sec[c + d*x]),x]
```

output
```
ArcTanh[Sin[c + d*x]]/(b*d) - (2*a*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/
Sqrt[a + b]])/(Sqrt[a - b]*b*Sqrt[a + b]*d)
```

3.490.3.1 Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4276
```
Int[csc[(e_.) + (f_.)*(x_)]^2/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Sym
bol] :> Simp[1/b   Int[Csc[e + f*x], x], x] - Simp[a/b   Int[Csc[e + f*x]/(
a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x]
```

rule 4318
```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[1/b   Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

3.490.4 Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.22

method	result
derivativedivides	$\frac{\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{b}-\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{b}-\frac{2a\,\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{b\sqrt{(a-b)(a+b)}}}{d}$
default	$\frac{\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{b}-\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{b}-\frac{2a\,\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{b\sqrt{(a-b)(a+b)}}}{d}$
risch	$\frac{a\ln\left(e^{i(dx+c)}+\frac{-ia^2+ib^2+b\sqrt{a^2-b^2}}{a\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,db}-\frac{a\ln\left(e^{i(dx+c)}+\frac{ia^2-ib^2+b\sqrt{a^2-b^2}}{\sqrt{a^2-b^2}\,a}\right)}{\sqrt{a^2-b^2}\,db}+\frac{\ln\left(e^{i(dx+c)}+i\right)}{db}-\frac{\ln\left(e^{i(dx+c)}-i\right)}{db}$

input
```
int(sec(d*x+c)^2/(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/b*ln(tan(1/2*d*x+1/2*c)+1)-1/b*ln(tan(1/2*d*x+1/2*c)-1)-2*a/b/((a-b
)*(a+b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a-b)*(a+b))^(1/2)))
```

3.490.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 290, normalized size of antiderivative = 4.26

$$\int \frac{\sec^2(c+dx)}{a+b\sec(c+dx)}\,dx$$
$$= \left[\frac{\sqrt{a^2-b^2}a\log\left(\frac{2\,ab\cos(dx+c)-(a^2-2\,b^2)\cos(dx+c)^2-2\sqrt{a^2-b^2}(b\cos(dx+c)+a)\sin(dx+c)+2\,a^2-b^2}{a^2\cos(dx+c)^2+2\,ab\cos(dx+c)+b^2}\right)+(a^2-b^2)\log(\sin(dx}{2\,(a^2b-b^3)d}\right.$$
$$-\frac{2\sqrt{-a^2+b^2}a\arctan\left(-\frac{\sqrt{-a^2+b^2}(b\cos(dx+c)+a)}{(a^2-b^2)\sin(dx+c)}\right)-(a^2-b^2)\log(\sin(dx+c)+1)+(a^2-b^2)\log(-\sin(}{2\,(a^2b-b^3)d}$$

input
```
integrate(sec(d*x+c)^2/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

output
```
[1/2*(sqrt(a^2 - b^2)*a*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x +
c)^2 - 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/
(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) + (a^2 - b^2)*log(sin(d*x
 + c) + 1) - (a^2 - b^2)*log(-sin(d*x + c) + 1))/((a^2*b - b^3)*d), -1/2*(
2*sqrt(-a^2 + b^2)*a*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 -
 b^2)*sin(d*x + c))) - (a^2 - b^2)*log(sin(d*x + c) + 1) + (a^2 - b^2)*log
(-sin(d*x + c) + 1))/((a^2*b - b^3)*d)]
```

3.490.6 Sympy [F]

$$\int \frac{\sec^2(c+dx)}{a+b\sec(c+dx)}\,dx = \int \frac{\sec^2{(c+dx)}}{a+b\sec{(c+dx)}}\,dx$$

input
```
integrate(sec(d*x+c)**2/(a+b*sec(d*x+c)),x)
```

output
```
Integral(sec(c + d*x)**2/(a + b*sec(c + d*x)), x)
```

3.490.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\sec^2(c+dx)}{a+b\sec(c+dx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(sec(d*x+c)^2/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

3.490.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 120 vs. $2(59) = 118$.

Time = 0.32 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.76

$$\int \frac{\sec^2(c+dx)}{a+b\sec(c+dx)}\,dx = -\frac{\frac{2\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{-a^2+b^2}}\right)\right)a}{\sqrt{-a^2+b^2}b} - \frac{\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{b} + \frac{\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{b}}{d}$$

input
```
integrate(sec(d*x+c)^2/(a+b*sec(d*x+c)),x, algorithm="giac")
```

output
```
-(2*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2
*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))*a/(sqrt(-a^2 +
b^2)*b) - log(abs(tan(1/2*d*x + 1/2*c) + 1))/b + log(abs(tan(1/2*d*x + 1/2
*c) - 1))/b)/d
```

3.490.9 Mupad [B] (verification not implemented)

Time = 13.88 (sec) , antiderivative size = 186, normalized size of antiderivative = 2.74

$$\int \frac{\sec^2(c+dx)}{a+b\sec(c+dx)}\,dx = \frac{2\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{b\,d} + \frac{2\,a\,\mathrm{atanh}\left(\frac{2\,a^2\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,(a^2-b^2)-2\,a^4\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)+b^2\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,(a^2-b^2)+2\,a^3\,b\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)-2\,a\,b\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,(a^2-b^2)}{b\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sqrt{a^2-b^2}\,(a\,b-b^2)}\right)}{b\,d\,\sqrt{a^2-b^2}}$$

input
```
int(1/(cos(c + d*x)^2*(a + b/cos(c + d*x))),x)
```

output
```
(2*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/(b*d) + (2*a*atanh((2*a^2
*sin(c/2 + (d*x)/2)*(a^2 - b^2) - 2*a^4*sin(c/2 + (d*x)/2) + b^2*sin(c/2 +
 (d*x)/2)*(a^2 - b^2) + 2*a^3*b*sin(c/2 + (d*x)/2) - 2*a*b*sin(c/2 + (d*x)
/2)*(a^2 - b^2))/(b*cos(c/2 + (d*x)/2)*(a^2 - b^2)^(1/2)*(a*b - b^2))))/(b
*d*(a^2 - b^2)^(1/2))
```

3.491 $\int \frac{\sec(c+dx)}{a+b\sec(c+dx)}\,dx$

3.491.1 Optimal result . . . 3632
3.491.2 Mathematica [A] (verified) . . . 3632
3.491.3 Rubi [A] (verified) . . . 3633
3.491.4 Maple [A] (verified) . . . 3634
3.491.5 Fricas [A] (verification not implemented) . . . 3635
3.491.6 Sympy [F] . . . 3635
3.491.7 Maxima [F(-2)] . . . 3635
3.491.8 Giac [A] (verification not implemented) . . . 3636
3.491.9 Mupad [B] (verification not implemented) . . . 3636

3.491.1 Optimal result

Integrand size = 19, antiderivative size = 49

$$\int \frac{\sec(c+dx)}{a+b\sec(c+dx)}\,dx=\frac{2\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\sqrt{a+b}d}$$

output
```
2*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/d/(a-b)^(1/2)/(a+b)^
(1/2)
```

3.491.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.98

$$\int \frac{\sec(c+dx)}{a+b\sec(c+dx)}\,dx=-\frac{2\operatorname{arctanh}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}d}$$

input
```
Integrate[Sec[c + d*x]/(a + b*Sec[c + d*x]),x]
```

output
```
(-2*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(Sqrt[a^2 - b^2]
*d)
```

3.491.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {3042, 4318, 3042, 3138, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(c+dx)}{a+b\sec(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{4318}$$

$$\frac{\int \frac{1}{\frac{a\cos(c+dx)}{b}+1}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{1}{\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{b}+1}dx}{b}$$

$$\Big\downarrow \text{3138}$$

$$\frac{2\int \frac{1}{\left(1-\frac{a}{b}\right)\tan^2\left(\frac{1}{2}(c+dx)\right)+\frac{a+b}{b}}d\tan\left(\frac{1}{2}(c+dx)\right)}{bd}$$

$$\Big\downarrow \text{221}$$

$$\frac{2\text{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d\sqrt{a-b}\sqrt{a+b}}$$

input
```
Int[Sec[c + d*x]/(a + b*Sec[c + d*x]),x]
```

output
```
(2*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(Sqrt[a - b]*Sqrt[
a + b]*d)
```

3.491.3.1 Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 4318
```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[1/b   Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

3.491.4 Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.90

method	result	size
derivativedivides	$\frac{2\,\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{d\sqrt{(a-b)(a+b)}}$	44
default	$\frac{2\,\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{d\sqrt{(a-b)(a+b)}}$	44
risch	$\frac{\ln\left(e^{i(dx+c)}+\frac{ia^2-ib^2+b\sqrt{a^2-b^2}}{\sqrt{a^2-b^2}\,a}\right)}{\sqrt{a^2-b^2}\,d}-\frac{\ln\left(e^{i(dx+c)}+\frac{-ia^2+ib^2+b\sqrt{a^2-b^2}}{a\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,d}$	137

input
```
int(sec(d*x+c)/(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
2/d/((a-b)*(a+b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a-b)*(a+b))^(1/
2))
```

3.491.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 185, normalized size of antiderivative = 3.78

$$\int \frac{\sec(c+dx)}{a+b\sec(c+dx)}\,dx = \left[\frac{\log\left(\frac{2\,ab\cos(dx+c)-(a^2-2\,b^2)\cos(dx+c)^2+2\,\sqrt{a^2-b^2}(b\cos(dx+c)+a)\sin(dx+c)+2\,a^2-b^2}{a^2\cos(dx+c)^2+2\,ab\cos(dx+c)+b^2}\right)}{2\,\sqrt{a^2-b^2}d}, \frac{\sqrt{-a^2+b^2}\arctan\left(-\frac{\sqrt{-a^2+b^2}(b\,c}{(a^2-b^2)\,s}\right.}{(a^2-b^2)d}\right.$$

input
```
integrate(sec(d*x+c)/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

output
```
[1/2*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 + 2*sqrt(a^2 -
 b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2
 + 2*a*b*cos(d*x + c) + b^2))/(sqrt(a^2 - b^2)*d), sqrt(-a^2 + b^2)*arctan
(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c)))/((a^2
- b^2)*d)]
```

3.491.6 Sympy [F]

$$\int \frac{\sec(c+dx)}{a+b\sec(c+dx)}\,dx = \int \frac{\sec(c+dx)}{a+b\sec(c+dx)}\,dx$$

input
```
integrate(sec(d*x+c)/(a+b*sec(d*x+c)),x)
```

output
```
Integral(sec(c + d*x)/(a + b*sec(c + d*x)), x)
```

3.491.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\sec(c+dx)}{a+b\sec(c+dx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(sec(d*x+c)/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

3.491.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.57

$$\int \frac{\sec(c+dx)}{a+b\sec(c+dx)}\,dx = -\frac{2\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(2a-2b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{-a^2+b^2}}\right)\right)}{\sqrt{-a^2+b^2}d}$$

input
```
integrate(sec(d*x+c)/(a+b*sec(d*x+c)),x, algorithm="giac")
```

output
```
-2*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(2*a - 2*b) + arctan((a*tan(1/2*d*
x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))/(sqrt(-a^2 + b^2)*
d)
```

3.491.9 Mupad [B] (verification not implemented)

Time = 14.06 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.82

$$\int \frac{\sec(c+dx)}{a+b\sec(c+dx)}\,dx = \frac{2\,\mathrm{atanh}\left(\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\sqrt{a-b}}{\sqrt{a+b}}\right)}{d\,\sqrt{a+b}\,\sqrt{a-b}}$$

input
```
int(1/(cos(c + d*x)*(a + b/cos(c + d*x))),x)
```

output
```
(2*atanh((tan(c/2 + (d*x)/2)*(a - b)^(1/2))/(a + b)^(1/2)))/(d*(a + b)^(1/
2)*(a - b)^(1/2))
```

3.492 $\int \frac{1}{a+b\sec(c+dx)}\,dx$

3.492.1 Optimal result . . . 3637
3.492.2 Mathematica [A] (verified) . . . 3637
3.492.3 Rubi [A] (verified) . . . 3638
3.492.4 Maple [A] (verified) . . . 3639
3.492.5 Fricas [A] (verification not implemented) . . . 3640
3.492.6 Sympy [F] . . . 3640
3.492.7 Maxima [F(-2)] . . . 3640
3.492.8 Giac [B] (verification not implemented) . . . 3641
3.492.9 Mupad [B] (verification not implemented) . . . 3641

3.492.1 Optimal result

Integrand size = 12, antiderivative size = 59

$$\int \frac{1}{a+b\sec(c+dx)}\,dx=\frac{x}{a}-\frac{2b\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a\sqrt{a-b}\sqrt{a+b}d}$$

output
```
x/a-2*b*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/a/d/(a-b)^(1/2
)/(a+b)^(1/2)
```

3.492.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.02

$$\int \frac{1}{a+b\sec(c+dx)}\,dx=\frac{\frac{c}{d}+x+\frac{2b\operatorname{arctanh}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}d}}{a}$$

input
```
Integrate[(a + b*Sec[c + d*x])^(-1),x]
```

output
```
(c/d + x + (2*b*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(Sqr
t[a^2 - b^2]*d))/a
```

3.492.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {3042, 4270, 3042, 3138, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a+b\sec(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{4270}$$

$$\frac{x}{a}-\frac{\int \frac{1}{\frac{a\cos(c+dx)}{b}+1}dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{x}{a}-\frac{\int \frac{1}{\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{b}+1}dx}{a}$$

$$\Big\downarrow \text{3138}$$

$$\frac{x}{a}-\frac{2\int \frac{1}{\left(1-\frac{a}{b}\right)\tan^2\left(\frac{1}{2}(c+dx)\right)+\frac{a+b}{b}}d\tan\left(\frac{1}{2}(c+dx)\right)}{ad}$$

$$\Big\downarrow \text{221}$$

$$\frac{x}{a}-\frac{2b\text{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{ad\sqrt{a-b}\sqrt{a+b}}$$

input
```
Int[(a + b*Sec[c + d*x])^(-1),x]
```

output
```
x/a - (2*b*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a*Sqrt[a
- b]*Sqrt[a + b]*d)
```

3.492.3.1 Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 4270
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(-1), x_Symbol] :> Simp[x/a, x]
- Simp[1/a   Int[1/(1 + (a/b)*Sin[c + d*x]), x], x] /; FreeQ[{a, b, c, d},
x] && NeQ[a^2 - b^2, 0]
```

3.492.4 Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.10

method	result	size
derivativedivides	$\frac{\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}-\frac{2b\,\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{a\sqrt{(a-b)(a+b)}}}{d}$	65
default	$\frac{\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}-\frac{2b\,\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{a\sqrt{(a-b)(a+b)}}}{d}$	65
risch	$\frac{x}{a}+\frac{b\ln\left(e^{i(dx+c)}-\frac{ia^2-ib^2-b\sqrt{a^2-b^2}}{\sqrt{a^2-b^2}\,a}\right)}{\sqrt{a^2-b^2}\,da}-\frac{b\ln\left(e^{i(dx+c)}+\frac{ia^2-ib^2+b\sqrt{a^2-b^2}}{\sqrt{a^2-b^2}\,a}\right)}{\sqrt{a^2-b^2}\,da}$	152

input
```
int(1/(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(2/a*arctan(tan(1/2*d*x+1/2*c))-2*b/a/((a-b)*(a+b))^(1/2)*arctanh((a-b
)*tan(1/2*d*x+1/2*c)/((a-b)*(a+b))^(1/2)))
```

3.492.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 230, normalized size of antiderivative = 3.90

$$\int \frac{1}{a+b\sec(c+dx)}\,dx = \left[\frac{2\,(a^2-b^2)dx+\sqrt{a^2-b^2}b\log\left(\frac{2\,ab\cos(dx+c)-(a^2-2\,b^2)\cos(dx+c)^2-2\,\sqrt{a^2-b^2}(b\cos(dx+c)+a)\sin(dx+c)+2\,a^2-b^2}{a^2\cos(dx+c)^2+2\,ab\cos(dx+c)+b^2}\right)}{2\,(a^3-ab^2)d}, \frac{(a^2-}{}\right.$$

input
```
integrate(1/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

output
```
[1/2*(2*(a^2 - b^2)*d*x + sqrt(a^2 - b^2)*b*log((2*a*b*cos(d*x + c) - (a^2
 - 2*b^2)*cos(d*x + c)^2 - 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x
+ c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)))/((a^
3 - a*b^2)*d), ((a^2 - b^2)*d*x - sqrt(-a^2 + b^2)*b*arctan(-sqrt(-a^2 + b
^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c))))/((a^3 - a*b^2)*d)]
```

3.492.6 Sympy [F]

$$\int \frac{1}{a+b\sec(c+dx)}\,dx = \int \frac{1}{a+b\sec(c+dx)}\,dx$$

input
```
integrate(1/(a+b*sec(d*x+c)),x)
```

output
```
Integral(1/(a + b*sec(c + d*x)), x)
```

3.492.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{a+b\sec(c+dx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

3.492.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 218 vs. $2(50) = 100$.

Time = 0.31 (sec) , antiderivative size = 218, normalized size of antiderivative = 3.69

$$\int \frac{1}{a+b\sec(c+dx)}\,dx = \frac{\frac{\left(\sqrt{-a^2+b^2}(a-2\,b)|-a+b|-\sqrt{-a^2+b^2}|a||-a+b|\right)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor+\arctan\left(\frac{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{-\frac{b+\sqrt{(a+b)(a-b)+b^2}}{a-b}}}\right)\right)}{(a^2-2\,ab+b^2)a^2+(a^2b-2\,ab^2+b^3)|a|} + \frac{\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor+\arctan\left(\frac{\tan\left(\frac{1}{2}\,dx\right.}{\sqrt{-\frac{b-\sqrt{(a+b}}{a}}}\right.\right.}{a^2-b|a|}}{d}$$

input
```
integrate(1/(a+b*sec(d*x+c)),x, algorithm="giac")
```

output
```
((sqrt(-a^2 + b^2)*(a - 2*b)*abs(-a + b) - sqrt(-a^2 + b^2)*abs(a)*abs(-a
+ b))*(pi*floor(1/2*(d*x + c)/pi + 1/2) + arctan(tan(1/2*d*x + 1/2*c)/sqrt
(-(b + sqrt((a + b)*(a - b) + b^2))/(a - b))))/((a^2 - 2*a*b + b^2)*a^2 +
(a^2*b - 2*a*b^2 + b^3)*abs(a)) + (pi*floor(1/2*(d*x + c)/pi + 1/2) + arct
an(tan(1/2*d*x + 1/2*c)/sqrt(-(b - sqrt((a + b)*(a - b) + b^2))/(a - b))))
*(a - 2*b + abs(a))/(a^2 - b*abs(a)))/d
```

3.492.9 Mupad [B] (verification not implemented)

Time = 14.23 (sec) , antiderivative size = 186, normalized size of antiderivative = 3.15

$$\int \frac{1}{a+b\sec(c+dx)}\,dx = \frac{2\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{a\,d} + \frac{2\,b\,\mathrm{atanh}\left(\frac{2\,b^4\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)+a^2\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,(a^2-b^2)+2\,b^2\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,(a^2-b^2)-2\,a\,b^3\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)-2\,a\,b\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,(a^2-b^2)}{a\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sqrt{a^2-b^2}\,(a\,b-a^2)}\right)}{a\,d\,\sqrt{a^2-b^2}}$$

input
```
int(1/(a + b/cos(c + d*x)),x)
```

output
```
(2*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/(a*d) + (2*b*atanh((2*b^4*
sin(c/2 + (d*x)/2) + a^2*sin(c/2 + (d*x)/2)*(a^2 - b^2) + 2*b^2*sin(c/2 +
(d*x)/2)*(a^2 - b^2) - 2*a*b^3*sin(c/2 + (d*x)/2) - 2*a*b*sin(c/2 + (d*x)/
2)*(a^2 - b^2))/(a*cos(c/2 + (d*x)/2)*(a^2 - b^2)^(1/2)*(a*b - a^2))))/(a*
d*(a^2 - b^2)^(1/2))
```

3.492. $\int \frac{1}{a+b\sec(c+dx)}\,dx$

3.493 $\int \frac{\cos(c+dx)}{a+b\sec(c+dx)}\,dx$

3.493.1 Optimal result . . . 3643
3.493.2 Mathematica [A] (verified) . . . 3643
3.493.3 Rubi [A] (verified) . . . 3644
3.493.4 Maple [A] (verified) . . . 3646
3.493.5 Fricas [A] (verification not implemented) . . . 3647
3.493.6 Sympy [F] . . . 3647
3.493.7 Maxima [F(-2)] . . . 3647
3.493.8 Giac [A] (verification not implemented) . . . 3648
3.493.9 Mupad [B] (verification not implemented) . . . 3648

3.493.1 Optimal result

Integrand size = 19, antiderivative size = 76

$$\int \frac{\cos(c+dx)}{a+b\sec(c+dx)}\,dx=-\frac{bx}{a^2}+\frac{2b^2\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2\sqrt{a-b}\sqrt{a+b}d}+\frac{\sin(c+dx)}{ad}$$

output
```
-b*x/a^2+sin(d*x+c)/a/d+2*b^2*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)
^(1/2))/a^2/d/(a-b)^(1/2)/(a+b)^(1/2)
```

3.493.2 Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.95

$$\int \frac{\cos(c+dx)}{a+b\sec(c+dx)}\,dx=\frac{-b(c+dx)-\frac{2b^2\operatorname{arctanh}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}+a\sin(c+dx)}{a^2d}$$

input
```
Integrate[Cos[c + d*x]/(a + b*Sec[c + d*x]),x]
```

output
```
(-(b*(c + d*x)) - (2*b^2*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^
2]])/Sqrt[a^2 - b^2] + a*Sin[c + d*x])/(a^2*d)
```

3.493.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.04, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474, Rules used = {3042, 4340, 25, 27, 3042, 4270, 3042, 3138, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(c+dx)}{a+b\sec(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)}dx$$

$$\Big\downarrow \text{4340}$$

$$\frac{\int -\frac{b}{a+b\sec(c+dx)}dx}{a}+\frac{\sin(c+dx)}{ad}$$

$$\Big\downarrow \text{25}$$

$$\frac{\sin(c+dx)}{ad}-\frac{\int \frac{b}{a+b\sec(c+dx)}dx}{a}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sin(c+dx)}{ad}-\frac{b\int \frac{1}{a+b\sec(c+dx)}dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sin(c+dx)}{ad}-\frac{b\int \frac{1}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a}$$

$$\Big\downarrow \text{4270}$$

$$\frac{\sin(c+dx)}{ad}-\frac{b\left(\frac{x}{a}-\frac{\int \frac{1}{\frac{a\cos(c+dx)}{b}+1}dx}{a}\right)}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sin(c+dx)}{ad}-\frac{b\left(\frac{x}{a}-\frac{\int \frac{1}{\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{b}+1}dx}{a}\right)}{a}$$

$$\Big\downarrow \text{3138}$$

$$\frac{\sin(c+dx)}{ad}-\frac{b\left(\frac{x}{a}-\frac{2\int\frac{1}{\left(1-\frac{a}{b}\right)\tan^2\left(\frac{1}{2}(c+dx)\right)+\frac{a+b}{b}}d\tan\left(\frac{1}{2}(c+dx)\right)}{ad}\right)}{a}$$

$$\Big\downarrow \; 221$$

$$\frac{\sin(c+dx)}{ad}-\frac{b\left(\frac{x}{a}-\frac{2b\text{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{ad\sqrt{a-b}\sqrt{a+b}}\right)}{a}$$

input
```
Int[Cos[c + d*x]/(a + b*Sec[c + d*x]),x]
```

output
```
-((b*(x/a - (2*b*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a*S
qrt[a - b]*Sqrt[a + b]*d)))/a) + Sin[c + d*x]/(a*d)
```

3.493.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 4270
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(-1), x_Symbol] :> Simp[x/a, x]
- Simp[1/a   Int[1/(1 + (a/b)*Sin[c + d*x]), x], x] /; FreeQ[{a, b, c, d},
x] && NeQ[a^2 - b^2, 0]
```

rule 4340
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[Cot[e + f*x]*((d*Csc[e + f*x])^n/(a*f*n)), x] - Sim
p[1/(a*d*n)   Int[((d*Csc[e + f*x])^(n + 1)/(a + b*Csc[e + f*x]))*Simp[b*n
- a*(n + 1)*Csc[e + f*x] - b*(n + 1)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a
, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1] && IntegerQ[2*n]
```

3.493.4 Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.28

method	result
derivativedivides	$\frac{-\frac{2\left(-\frac{a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}+b\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}{a^2}+\frac{2b^2\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{a^2\sqrt{(a-b)(a+b)}}}{d}$
default	$\frac{-\frac{2\left(-\frac{a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}+b\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}{a^2}+\frac{2b^2\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{a^2\sqrt{(a-b)(a+b)}}}{d}$
risch	$-\frac{bx}{a^2}-\frac{i\mathrm{e}^{i(dx+c)}}{2da}+\frac{i\mathrm{e}^{-i(dx+c)}}{2da}+\frac{b^2\ln\left(\mathrm{e}^{i(dx+c)}+\frac{ia^2-ib^2+b\sqrt{a^2-b^2}}{\sqrt{a^2-b^2}\,a}\right)}{\sqrt{a^2-b^2}\,d\,a^2}-\frac{b^2\ln\left(\mathrm{e}^{i(dx+c)}+\frac{-ia^2+ib^2+b\sqrt{a^2-b^2}}{a\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,d\,a^2}$

input
```
int(cos(d*x+c)/(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-2/a^2*(-a*tan(1/2*d*x+1/2*c)/(1+tan(1/2*d*x+1/2*c)^2)+b*arctan(tan(1
/2*d*x+1/2*c)))+2*b^2/a^2/((a-b)*(a+b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/
2*c)/((a-b)*(a+b))^(1/2)))
```

3.493.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 277, normalized size of antiderivative = 3.64

$$\int \frac{\cos(c+dx)}{a+b\sec(c+dx)}\,dx$$
$$= \left[\frac{\sqrt{a^2-b^2}b^2\log\left(\frac{2\,ab\cos(dx+c)-(a^2-2\,b^2)\cos(dx+c)^2+2\,\sqrt{a^2-b^2}(b\cos(dx+c)+a)\sin(dx+c)+2\,a^2-b^2}{a^2\cos(dx+c)^2+2\,ab\cos(dx+c)+b^2}\right)-2\,(a^2b-b^3)dx+2\,(}{2\,(a^4-a^2b^2)d}\right.$$

input
```
integrate(cos(d*x+c)/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

output
```
[1/2*(sqrt(a^2 - b^2)*b^2*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x
+ c)^2 + 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2
)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) - 2*(a^2*b - b^3)*d*x +
 2*(a^3 - a*b^2)*sin(d*x + c))/((a^4 - a^2*b^2)*d), (sqrt(-a^2 + b^2)*b^2*
arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c)))
- (a^2*b - b^3)*d*x + (a^3 - a*b^2)*sin(d*x + c))/((a^4 - a^2*b^2)*d)]
```

3.493.6 Sympy [F]

$$\int \frac{\cos(c+dx)}{a+b\sec(c+dx)}\,dx = \int \frac{\cos(c+dx)}{a+b\sec(c+dx)}\,dx$$

input
```
integrate(cos(d*x+c)/(a+b*sec(d*x+c)),x)
```

output
```
Integral(cos(c + d*x)/(a + b*sec(c + d*x)), x)
```

3.493.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\cos(c+dx)}{a+b\sec(c+dx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(cos(d*x+c)/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

3.493.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.66

$$\int \frac{\cos(c+dx)}{a+b\sec(c+dx)}\,dx = \frac{\frac{2\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{-a^2+b^2}}\right)\right)b^2}{\sqrt{-a^2+b^2}a^2}-\frac{(dx+c)b}{a^2}+\frac{2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)a}}{d}$$

input
```
integrate(cos(d*x+c)/(a+b*sec(d*x+c)),x, algorithm="giac")
```

output
```
(2*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*
d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))*b^2/(sqrt(-a^2 +
 b^2)*a^2) - (d*x + c)*b/a^2 + 2*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/2*
c)^2 + 1)*a))/d
```

3.493.9 Mupad [B] (verification not implemented)

Time = 13.78 (sec) , antiderivative size = 395, normalized size of antiderivative = 5.20

$$\int \frac{\cos(c+dx)}{a+b\sec(c+dx)}\,dx = \frac{a^3\sin(c+dx)}{d\,(a^4-a^2b^2)}+\frac{2b^3\operatorname{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)}{d\,(a^4-a^2b^2)}-\frac{ab^2\sin(c+dx)}{d\,(a^4-a^2b^2)}-\frac{2a^2b\operatorname{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)}{d\,(a^4-a^2b^2)}+\frac{b^2\operatorname{atan}\left(\frac{-a^5\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a^2-b^2}\,1\mathrm{i}+b^3\sin\left(\frac{c}{2}+\frac{dx}{2}\right)(a^2-b^2)^{3/2}\,2\mathrm{i}+b^5\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a^2-b^2}\,2\mathrm{i}-a^2b^3\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a^2-b^2}\,3\mathrm{i}+a^3b^2\,\mathrm{sin}}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)a^6-2\cos\left(\frac{c}{2}+\frac{dx}{2}\right)a^4b^2+\cos\left(\frac{c}{2}+\frac{dx}{2}\right)a^2b^4}\right)}{d\,(a^4-a^2b^2)}$$

input
```
int(cos(c + d*x)/(a + b/cos(c + d*x)),x)
```

output
```
(a^3*sin(c + d*x))/(d*(a^4 - a^2*b^2)) + (2*b^3*atan(sin(c/2 + (d*x)/2)/co
s(c/2 + (d*x)/2)))/(d*(a^4 - a^2*b^2)) - (a*b^2*sin(c + d*x))/(d*(a^4 - a^
2*b^2)) + (b^2*atan((b^3*sin(c/2 + (d*x)/2)*(a^2 - b^2)^(3/2)*2i - a^5*sin
(c/2 + (d*x)/2)*(a^2 - b^2)^(1/2)*1i + b^5*sin(c/2 + (d*x)/2)*(a^2 - b^2)^
(1/2)*2i - a^2*b^3*sin(c/2 + (d*x)/2)*(a^2 - b^2)^(1/2)*3i + a^3*b^2*sin(c
/2 + (d*x)/2)*(a^2 - b^2)^(1/2)*1i + a^4*b*sin(c/2 + (d*x)/2)*(a^2 - b^2)^
(1/2)*1i)/(a^6*cos(c/2 + (d*x)/2) + a^2*b^4*cos(c/2 + (d*x)/2) - 2*a^4*b^2
*cos(c/2 + (d*x)/2)))*(a^2 - b^2)^(1/2)*2i)/(d*(a^4 - a^2*b^2)) - (2*a^2*b
*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/(d*(a^4 - a^2*b^2))
```

3.493. $\int \frac{\cos(c+dx)}{a+b\sec(c+dx)}\,dx$

3.494 $\int \frac{\cos^2(c+dx)}{a+b\sec(c+dx)}\,dx$

3.494.1 Optimal result . . . 3650
3.494.2 Mathematica [A] (verified) . . . 3650
3.494.3 Rubi [A] (verified) . . . 3651
3.494.4 Maple [A] (verified) . . . 3654
3.494.5 Fricas [A] (verification not implemented) . . . 3655
3.494.6 Sympy [F] . . . 3655
3.494.7 Maxima [F(-2)] . . . 3656
3.494.8 Giac [A] (verification not implemented) . . . 3656
3.494.9 Mupad [B] (verification not implemented) . . . 3657

3.494.1 Optimal result

Integrand size = 21, antiderivative size = 110

$$\int \frac{\cos^2(c+dx)}{a+b\sec(c+dx)}\,dx = \frac{(a^2+2b^2)\,x}{2a^3} - \frac{2b^3\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3\sqrt{a-b}\sqrt{a+b}\,d} - \frac{b\sin(c+dx)}{a^2d} + \frac{\cos(c+dx)\sin(c+dx)}{2ad}$$

output
```
1/2*(a^2+2*b^2)*x/a^3-b*sin(d*x+c)/a^2/d+1/2*cos(d*x+c)*sin(d*x+c)/a/d-2*b
^3*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/a^3/d/(a-b)^(1/2)/(
a+b)^(1/2)
```

3.494.2 Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.88

$$\int \frac{\cos^2(c+dx)}{a+b\sec(c+dx)}\,dx = \frac{2(a^2+2b^2)(c+dx) + \frac{8b^3\operatorname{arctanh}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} - 4ab\sin(c+dx) + a^2\sin(2(c+dx))}{4a^3d}$$

input
```
Integrate[Cos[c + d*x]^2/(a + b*Sec[c + d*x]),x]
```

output
```
(2*(a^2 + 2*b^2)*(c + d*x) + (8*b^3*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sq
rt[a^2 - b^2]])/Sqrt[a^2 - b^2] - 4*a*b*Sin[c + d*x] + a^2*Sin[2*(c + d*x)
])/(4*a^3*d)
```

3.494.3 Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.10, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {3042, 4340, 25, 3042, 4592, 3042, 4407, 3042, 4318, 3042, 3138, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^2(c+dx)}{a+b\sec(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^2\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)}dx$$

$$\Big\downarrow \text{4340}$$

$$\frac{\int -\frac{\cos(c+dx)\left(-b\sec^2(c+dx)-a\sec(c+dx)+2b\right)}{a+b\sec(c+dx)}dx}{2a}+\frac{\sin(c+dx)\cos(c+dx)}{2ad}$$

$$\Big\downarrow \text{25}$$

$$\frac{\sin(c+dx)\cos(c+dx)}{2ad}-\frac{\int \frac{\cos(c+dx)\left(-b\sec^2(c+dx)-a\sec(c+dx)+2b\right)}{a+b\sec(c+dx)}dx}{2a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sin(c+dx)\cos(c+dx)}{2ad}-\frac{\int \frac{-b\csc\left(c+dx+\frac{\pi}{2}\right)^2-a\csc\left(c+dx+\frac{\pi}{2}\right)+2b}{\csc\left(c+dx+\frac{\pi}{2}\right)\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{2a}$$

$$\Big\downarrow \text{4592}$$

$$\frac{\sin(c+dx)\cos(c+dx)}{2ad}-\frac{\frac{2b\sin(c+dx)}{ad}-\frac{\int \frac{a^2+b\sec(c+dx)a+2b^2}{a+b\sec(c+dx)}dx}{a}}{2a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sin(c+dx)\cos(c+dx)}{2ad}-\frac{\frac{2b\sin(c+dx)}{ad}-\frac{\int \frac{a^2+b\csc\left(c+dx+\frac{\pi}{2}\right)a+2b^2}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a}}{2a}$$

$$\Big\downarrow \text{4407}$$

$$\frac{\sin(c+dx)\cos(c+dx)}{2ad}-\frac{\frac{2b\sin(c+dx)}{ad}-\frac{\frac{x\left(a^2+2b^2\right)}{a}-\frac{2b^3\int\frac{\sec(c+dx)}{a+b\sec(c+dx)}dx}{a}}{a}}{2a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sin(c+dx)\cos(c+dx)}{2ad}-\frac{\frac{2b\sin(c+dx)}{ad}-\frac{\frac{x\left(a^2+2b^2\right)}{a}-\frac{2b^3\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a}}{a}}{2a}$$

$$\Big\downarrow \text{4318}$$

$$\frac{\sin(c+dx)\cos(c+dx)}{2ad}-\frac{\frac{2b\sin(c+dx)}{ad}-\frac{\frac{x\left(a^2+2b^2\right)}{a}-\frac{2b^2\int\frac{1}{\frac{a\cos(c+dx)}{b}+1}dx}{a}}{a}}{2a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sin(c+dx)\cos(c+dx)}{2ad}-\frac{\frac{2b\sin(c+dx)}{ad}-\frac{\frac{x\left(a^2+2b^2\right)}{a}-\frac{2b^2\int\frac{1}{\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{b}+1}dx}{a}}{a}}{2a}$$

$$\Big\downarrow \text{3138}$$

$$\frac{\sin(c+dx)\cos(c+dx)}{2ad}-\frac{\frac{2b\sin(c+dx)}{ad}-\frac{\frac{x\left(a^2+2b^2\right)}{a}-\frac{4b^2\int\frac{1}{\left(1-\frac{a}{b}\right)\tan^2\left(\frac{1}{2}(c+dx)\right)+\frac{a+b}{b}}d\tan\left(\frac{1}{2}(c+dx)\right)}{ad}}{a}}{2a}$$

$$\Big\downarrow \text{221}$$

$$\frac{\sin(c+dx)\cos(c+dx)}{2ad}-\frac{\frac{2b\sin(c+dx)}{ad}-\frac{\frac{x\left(a^2+2b^2\right)}{a}-\frac{4b^3\text{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{ad\sqrt{a-b}\sqrt{a+b}}}{a}}{2a}$$

input
```
Int[Cos[c + d*x]^2/(a + b*Sec[c + d*x]),x]
```

output
```
(Cos[c + d*x]*Sin[c + d*x])/(2*a*d) - (-((((a^2 + 2*b^2)*x)/a - (4*b^3*Arc
Tanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a*Sqrt[a - b]*Sqrt[a +
b]*d))/a) + (2*b*Sin[c + d*x])/(a*d))/(2*a)
```

3.494. $\int \frac{\cos^2(c+dx)}{a+b\sec(c+dx)}\,dx$

3.494.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 4318
```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[1/b   Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4340
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[Cot[e + f*x]*((d*Csc[e + f*x])^n/(a*f*n)), x] - Sim
p[1/(a*d*n)   Int[((d*Csc[e + f*x])^(n + 1)/(a + b*Csc[e + f*x]))*Simp[b*n
- a*(n + 1)*Csc[e + f*x] - b*(n + 1)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a
, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1] && IntegerQ[2*n]
```

rule 4407
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Simp[(b*c - a*d)/a   Int[Csc[e + f*
x]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
- a*d, 0]
```

3.494. $\int \frac{\cos^2(c+dx)}{a+b\sec(c+dx)}\,dx$

rule 4592
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Simp[1/(a*d*n)   Int[(a + b*Csc[e + f*x])^m
*(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*
Csc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

3.494.4 Maple [A] (verified)

Time = 0.69 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.25

method	result
derivativedivides	$\frac{-\frac{2b^3 \operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{a^3\sqrt{(a-b)(a+b)}}+\frac{\frac{2\left(\left(-\frac{1}{2}a^2-ab\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3+\left(\frac{1}{2}a^2-ab\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^2}+\left(a^2+2b^2\right)\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^3}}{d}$
default	$\frac{-\frac{2b^3 \operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{a^3\sqrt{(a-b)(a+b)}}+\frac{\frac{2\left(\left(-\frac{1}{2}a^2-ab\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3+\left(\frac{1}{2}a^2-ab\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^2}+\left(a^2+2b^2\right)\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^3}}{d}$
risch	$\frac{x}{2a}+\frac{x\,b^2}{a^3}+\frac{ib\,\mathrm{e}^{i(dx+c)}}{2a^2d}-\frac{ib\,\mathrm{e}^{-i(dx+c)}}{2a^2d}+\frac{b^3\ln\left(\mathrm{e}^{i(dx+c)}+\frac{-ia^2+ib^2+b\sqrt{a^2-b^2}}{a\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,d\,a^3}-\frac{b^3\ln\left(\mathrm{e}^{i(dx+c)}+\frac{ia^2-ib^2+b\sqrt{}}{\sqrt{a^2-b^2}}\right.}{\sqrt{a^2-b^2}\,d\,a^3}$

input
```
int(cos(d*x+c)^2/(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-2*b^3/a^3/((a-b)*(a+b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a-b
)*(a+b))^(1/2))+2/a^3*(((-1/2*a^2-a*b)*tan(1/2*d*x+1/2*c)^3+(1/2*a^2-a*b)*
tan(1/2*d*x+1/2*c))/(1+tan(1/2*d*x+1/2*c)^2)^2+1/2*(a^2+2*b^2)*arctan(tan(
1/2*d*x+1/2*c))))
```

3.494.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 334, normalized size of antiderivative = 3.04

$$\int \frac{\cos^2(c+dx)}{a+b\sec(c+dx)}\,dx$$
$$= \left[\frac{\sqrt{a^2-b^2}b^3\log\left(\frac{2\,ab\cos(dx+c)-(a^2-2\,b^2)\cos(dx+c)^2-2\,\sqrt{a^2-b^2}(b\cos(dx+c)+a)\sin(dx+c)+2\,a^2-b^2}{a^2\cos(dx+c)^2+2\,ab\cos(dx+c)+b^2}\right)+(a^4+a^2b^2-2\,b^4)d}{2\,(a^5-a^3b^2)d}\right.$$
$$-\frac{2\,\sqrt{-a^2+b^2}b^3\arctan\left(-\frac{\sqrt{-a^2+b^2}(b\cos(dx+c)+a)}{(a^2-b^2)\sin(dx+c)}\right)-(a^4+a^2b^2-2\,b^4)dx+(2\,a^3b-2\,ab^3-(a^4-a^2b^2)\,\mathrm{co}}{2\,(a^5-a^3b^2)d}$$

input
```
integrate(cos(d*x+c)^2/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

output
```
[1/2*(sqrt(a^2 - b^2)*b^3*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x
+ c)^2 - 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2
)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) + (a^4 + a^2*b^2 - 2*b^
4)*d*x - (2*a^3*b - 2*a*b^3 - (a^4 - a^2*b^2)*cos(d*x + c))*sin(d*x + c))/
((a^5 - a^3*b^2)*d), -1/2*(2*sqrt(-a^2 + b^2)*b^3*arctan(-sqrt(-a^2 + b^2)
*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c))) - (a^4 + a^2*b^2 - 2*b^4
)*d*x + (2*a^3*b - 2*a*b^3 - (a^4 - a^2*b^2)*cos(d*x + c))*sin(d*x + c))/(
(a^5 - a^3*b^2)*d)]
```

3.494.6 Sympy [F]

$$\int \frac{\cos^2(c+dx)}{a+b\sec(c+dx)}\,dx = \int \frac{\cos^2(c+dx)}{a+b\sec(c+dx)}\,dx$$

input
```
integrate(cos(d*x+c)**2/(a+b*sec(d*x+c)),x)
```

output
```
Integral(cos(c + d*x)**2/(a + b*sec(c + d*x)), x)
```

3.494.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\cos^2(c+dx)}{a+b\sec(c+dx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(cos(d*x+c)^2/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

3.494.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.62

$$\int \frac{\cos^2(c+dx)}{a+b\sec(c+dx)}\,dx = -\frac{\frac{4\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{-a^2+b^2}}\right)\right)b^3}{\sqrt{-a^2+b^2}a^3} - \frac{(a^2+2b^2)(dx+c)}{a^3} + \frac{2\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+2b\tan\left(\frac{1}{2}c\right.\right.}{\left(\tan\right.}}{2d}$$

input
```
integrate(cos(d*x+c)^2/(a+b*sec(d*x+c)),x, algorithm="giac")
```

output
```
-1/2*(4*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan
(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))*b^3/(sqrt(-
a^2 + b^2)*a^3) - (a^2 + 2*b^2)*(d*x + c)/a^3 + 2*(a*tan(1/2*d*x + 1/2*c)^
3 + 2*b*tan(1/2*d*x + 1/2*c)^3 - a*tan(1/2*d*x + 1/2*c) + 2*b*tan(1/2*d*x
+ 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^2*a^2))/d
```

3.494.9 Mupad [B] (verification not implemented)

Time = 14.29 (sec) , antiderivative size = 592, normalized size of antiderivative = 5.38

$$\int \frac{\cos^2(c+dx)}{a+b\sec(c+dx)}\,dx = \frac{a\left(\operatorname{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)+\frac{\sin(2\,c+2\,d\,x)}{4}\right)}{d\,(a^2-b^2)} - \frac{b\,\sin(c+d\,x)}{d\,(a^2-b^2)} + \frac{b^2\operatorname{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)-\frac{b^2\,\sin(2\,c+2\,d\,x)}{4}}{a\,d\,(a^2-b^2)} + \frac{b^3\,\sin(c+d\,x)}{a^2\,d\,(a^2-b^2)} - \frac{2\,b^4\operatorname{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{a^3\,d\,(a^2-b^2)} - \frac{b^3\operatorname{atan}\left(\frac{\left(8\,b^7\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)(a^2-b^2)^{3/2}-a^9\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\sqrt{a^2-b^2}+8\,b^9\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\sqrt{a^2-b^2}-8\,a^2\,b^7\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\sqrt{a^2-b^2}-3\,a^4\,b^5\,\sin\left(\frac{c}{2}\right.\right.}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)(a\,b^2-a^3)\,(4\,b^5\,(a^2-b^2)+2\,a\,b^6-a^7+4\,b^7-}\right.}{a^3\,d\,\sqrt{}}$$

input
```
int(cos(c + d*x)^2/(a + b/cos(c + d*x)),x)
```

output
```
(a*(atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)) + sin(2*c + 2*d*x)/4))/(d*
(a^2 - b^2)) - (b*sin(c + d*x))/(d*(a^2 - b^2)) + (b^2*atan(sin(c/2 + (d*x
)/2)/cos(c/2 + (d*x)/2)) - (b^2*sin(2*c + 2*d*x))/4)/(a*d*(a^2 - b^2)) - (
b^3*atan(((8*b^7*sin(c/2 + (d*x)/2)*(a^2 - b^2)^(3/2) - a^9*sin(c/2 + (d*x
)/2)*(a^2 - b^2)^(1/2) + 8*b^9*sin(c/2 + (d*x)/2)*(a^2 - b^2)^(1/2) - 8*a^
2*b^7*sin(c/2 + (d*x)/2)*(a^2 - b^2)^(1/2) - 3*a^4*b^5*sin(c/2 + (d*x)/2)*
(a^2 - b^2)^(1/2) + 3*a^5*b^4*sin(c/2 + (d*x)/2)*(a^2 - b^2)^(1/2) + 2*a^6
*b^3*sin(c/2 + (d*x)/2)*(a^2 - b^2)^(1/2) - 2*a^7*b^2*sin(c/2 + (d*x)/2)*(
a^2 - b^2)^(1/2) + a^8*b*sin(c/2 + (d*x)/2)*(a^2 - b^2)^(1/2))*1i)/(cos(c/
2 + (d*x)/2)*(a*b^2 - a^3)*(4*b^5*(a^2 - b^2) + 2*a*b^6 - a^7 + 4*b^7 - 2*
a^2*b^5 + a^3*b^4 - 2*a^4*b^3 - 2*a^5*b^2 + 2*a^2*b^3*(a^2 - b^2) + 2*a*b^
4*(a^2 - b^2))))*2i)/(a^3*d*(a^2 - b^2)^(1/2)) + (b^3*sin(c + d*x))/(a^2*d
*(a^2 - b^2)) - (2*b^4*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/(a^3*d
*(a^2 - b^2))
```

3.495 $\int \frac{\cos^3(c+dx)}{a+b\sec(c+dx)}\,dx$

3.495.1 Optimal result . . . 3658
3.495.2 Mathematica [A] (verified) . . . 3658
3.495.3 Rubi [A] (verified) . . . 3659
3.495.4 Maple [A] (verified) . . . 3663
3.495.5 Fricas [A] (verification not implemented) . . . 3663
3.495.6 Sympy [F(-1)] . . . 3664
3.495.7 Maxima [F(-2)] . . . 3664
3.495.8 Giac [A] (verification not implemented) . . . 3665
3.495.9 Mupad [B] (verification not implemented) . . . 3665

3.495.1 Optimal result

Integrand size = 21, antiderivative size = 148

$$\int \frac{\cos^3(c+dx)}{a+b\sec(c+dx)}\,dx = -\frac{b(a^2+2b^2)\,x}{2a^4} + \frac{2b^4\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^4\sqrt{a-b}\sqrt{a+b}d} + \frac{(2a^2+3b^2)\sin(c+dx)}{3a^3d} - \frac{b\cos(c+dx)\sin(c+dx)}{2a^2d} + \frac{\cos^2(c+dx)\sin(c+dx)}{3ad}$$

output
```
-1/2*b*(a^2+2*b^2)*x/a^4+1/3*(2*a^2+3*b^2)*sin(d*x+c)/a^3/d-1/2*b*cos(d*x+
c)*sin(d*x+c)/a^2/d+1/3*cos(d*x+c)^2*sin(d*x+c)/a/d+2*b^4*arctanh((a-b)^(1
/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/a^4/d/(a-b)^(1/2)/(a+b)^(1/2)
```

3.495.2 Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.82

$$\int \frac{\cos^3(c+dx)}{a+b\sec(c+dx)}\,dx = \frac{-6b(a^2+2b^2)(c+dx) - \frac{24b^4\operatorname{arctanh}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} + 3a(3a^2+4b^2)\sin(c+dx) - 3a^2b\sin(2(c+dx))}{12a^4d}$$

input
```
Integrate[Cos[c + d*x]^3/(a + b*Sec[c + d*x]),x]
```

output
```
(-6*b*(a^2 + 2*b^2)*(c + d*x) - (24*b^4*ArcTanh[((-a + b)*Tan[(c + d*x)/2]
)/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2] + 3*a*(3*a^2 + 4*b^2)*Sin[c + d*x] - 3
*a^2*b*Sin[2*(c + d*x)] + a^3*Sin[3*(c + d*x)])/(12*a^4*d)
```

3.495.3 Rubi [A] (verified)

Time = 1.12 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.11, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {3042, 4340, 25, 3042, 4592, 3042, 4592, 27, 3042, 4407, 3042, 4318, 3042, 3138, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^3(c+dx)}{a+b\sec(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^3\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)}dx$$

$$\Big\downarrow \text{4340}$$

$$\frac{\int -\frac{\cos^2(c+dx)\left(-2b\sec^2(c+dx)-2a\sec(c+dx)+3b\right)}{a+b\sec(c+dx)}dx}{3a}+\frac{\sin(c+dx)\cos^2(c+dx)}{3ad}$$

$$\Big\downarrow \text{25}$$

$$\frac{\sin(c+dx)\cos^2(c+dx)}{3ad}-\frac{\int \frac{\cos^2(c+dx)\left(-2b\sec^2(c+dx)-2a\sec(c+dx)+3b\right)}{a+b\sec(c+dx)}dx}{3a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sin(c+dx)\cos^2(c+dx)}{3ad}-\frac{\int \frac{-2b\csc\left(c+dx+\frac{\pi}{2}\right)^2-2a\csc\left(c+dx+\frac{\pi}{2}\right)+3b}{\csc\left(c+dx+\frac{\pi}{2}\right)^2\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{3a}$$

$$\Big\downarrow \text{4592}$$

$$\frac{\sin(c+dx)\cos^2(c+dx)}{3ad}-\frac{\frac{3b\sin(c+dx)\cos(c+dx)}{2ad}-\frac{\int \frac{\cos(c+dx)\left(-3b^2\sec^2(c+dx)+ab\sec(c+dx)+2\left(2a^2+3b^2\right)\right)}{a+b\sec(c+dx)}dx}{2a}}{3a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sin(c+dx)\cos^2(c+dx)}{3ad}-\frac{\frac{3b\sin(c+dx)\cos(c+dx)}{2ad}-\frac{\int\frac{-3b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2+ab\csc\left(c+dx+\frac{\pi}{2}\right)+2\left(2a^2+3b^2\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{2a}}{3a}$$

$\downarrow$ 4592

$$\frac{\sin(c+dx)\cos^2(c+dx)}{3ad}-\frac{\frac{3b\sin(c+dx)\cos(c+dx)}{2ad}-\frac{\frac{2\left(2a^2+3b^2\right)\sin(c+dx)}{ad}-\frac{\int\frac{3\left(a\sec(c+dx)b^2+\left(a^2+2b^2\right)b\right)}{a+b\sec(c+dx)}dx}{a}}{2a}}{3a}$$

$\downarrow$ 27

$$\frac{\sin(c+dx)\cos^2(c+dx)}{3ad}-\frac{\frac{3b\sin(c+dx)\cos(c+dx)}{2ad}-\frac{\frac{2\left(2a^2+3b^2\right)\sin(c+dx)}{ad}-\frac{3\int\frac{a\sec(c+dx)b^2+\left(a^2+2b^2\right)b}{a+b\sec(c+dx)}dx}{a}}{2a}}{3a}$$

$\downarrow$ 3042

$$\frac{\sin(c+dx)\cos^2(c+dx)}{3ad}-\frac{\frac{3b\sin(c+dx)\cos(c+dx)}{2ad}-\frac{\frac{2\left(2a^2+3b^2\right)\sin(c+dx)}{ad}-\frac{3\int\frac{a\csc\left(c+dx+\frac{\pi}{2}\right)b^2+\left(a^2+2b^2\right)b}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a}}{2a}}{3a}$$

$\downarrow$ 4407

$$\frac{\sin(c+dx)\cos^2(c+dx)}{3ad}-\frac{\frac{3b\sin(c+dx)\cos(c+dx)}{2ad}-\frac{\frac{2\left(2a^2+3b^2\right)\sin(c+dx)}{ad}-\frac{3\left(\frac{bx\left(a^2+2b^2\right)}{a}-\frac{2b^4\int\frac{\sec(c+dx)}{a+b\sec(c+dx)}dx}{a}\right)}{a}}{2a}}{3a}$$

$\downarrow$ 3042

$$\frac{\sin(c+dx)\cos^2(c+dx)}{3ad}-\frac{\frac{3b\sin(c+dx)\cos(c+dx)}{2ad}-\frac{\frac{2\left(2a^2+3b^2\right)\sin(c+dx)}{ad}-\frac{3\left(\frac{bx\left(a^2+2b^2\right)}{a}-\frac{2b^4\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a}\right)}{a}}{2a}}{3a}$$

$\downarrow$ 4318

$$\frac{\sin(c+dx)\cos^2(c+dx)}{3ad}-\frac{\frac{3b\sin(c+dx)\cos(c+dx)}{2ad}-\frac{\frac{2\left(2a^2+3b^2\right)\sin(c+dx)}{ad}-\frac{3\left(\frac{bx\left(a^2+2b^2\right)}{a}-\frac{2b^3\int\frac{1}{\frac{a\cos(c+dx)}{b}+1}dx}{a}\right)}{a}}{2a}}{3a}$$

$\downarrow$ 3042

3.495. $\int\frac{\cos^3(c+dx)}{a+b\sec(c+dx)}\,dx$

$$\frac{\sin(c+dx)\cos^2(c+dx)}{3ad}-\frac{\frac{3b\sin(c+dx)\cos(c+dx)}{2ad}-\frac{\frac{2\left(2a^2+3b^2\right)\sin(c+dx)}{ad}-\frac{3\left(\frac{bx\left(a^2+2b^2\right)}{a}-\frac{2b^3\int\frac{1}{a\sin\left(c+dx+\frac{\pi}{2}\right)+1}dx}{\frac{b}{a}}\right)}{a}}{2a}}{3a}$$

$\Big\downarrow$ 3138

$$\frac{\sin(c+dx)\cos^2(c+dx)}{3ad}-\frac{\frac{3b\sin(c+dx)\cos(c+dx)}{2ad}-\frac{\frac{2\left(2a^2+3b^2\right)\sin(c+dx)}{ad}-\frac{3\left(\frac{bx\left(a^2+2b^2\right)}{a}-\frac{4b^3\int\frac{1}{\left(1-\frac{a}{b}\right)\tan^2\left(\frac{1}{2}(c+dx)\right)+\frac{a+b}{b}}d\tan\left(\frac{1}{2}(c+dx)\right)}{ad}\right)}{a}}{2a}}{3a}$$

$\Big\downarrow$ 221

$$\frac{\sin(c+dx)\cos^2(c+dx)}{3ad}-\frac{\frac{3b\sin(c+dx)\cos(c+dx)}{2ad}-\frac{\frac{2\left(2a^2+3b^2\right)\sin(c+dx)}{ad}-\frac{3\left(\frac{bx\left(a^2+2b^2\right)}{a}-\frac{4b^4\text{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{ad\sqrt{a-b}\sqrt{a+b}}\right)}{a}}{2a}}{3a}$$

input
```
Int[Cos[c + d*x]^3/(a + b*Sec[c + d*x]),x]
```

output
```
(Cos[c + d*x]^2*Sin[c + d*x])/(3*a*d) - ((3*b*Cos[c + d*x]*Sin[c + d*x])/(
2*a*d) - ((-3*((b*(a^2 + 2*b^2)*x)/a - (4*b^4*ArcTanh[(Sqrt[a - b]*Tan[(c
+ d*x)/2])/Sqrt[a + b]])/(a*Sqrt[a - b]*Sqrt[a + b]*d)))/a + (2*(2*a^2 + 3
*b^2)*Sin[c + d*x])/(a*d))/(2*a))/(3*a)
```

3.495.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 4318
```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[1/b   Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4340
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[Cot[e + f*x]*((d*Csc[e + f*x])^n/(a*f*n)), x] - Sim
p[1/(a*d*n)   Int[((d*Csc[e + f*x])^(n + 1)/(a + b*Csc[e + f*x]))*Simp[b*n
- a*(n + 1)*Csc[e + f*x] - b*(n + 1)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a
, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1] && IntegerQ[2*n]
```

rule 4407
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Simp[(b*c - a*d)/a   Int[Csc[e + f*
x]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
- a*d, 0]
```

rule 4592
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Simp[1/(a*d*n)   Int[(a + b*Csc[e + f*x])^m
*(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*
Csc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

3.495. $\int \frac{\cos^3(c+dx)}{a+b\sec(c+dx)}\,dx$

3.495.4 Maple [A] (verified)

Time = 0.71 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.21

method	result
derivativedivides	$\frac{\frac{2b^4 \operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{a^4\sqrt{(a-b)(a+b)}}-\frac{2\left(\frac{\left(-a^3-\frac{1}{2}a^2b-a\,b^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5+\left(-\frac{2}{3}a^3-2a\,b^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3+\left(-a^3-a\,b^2+\frac{1}{2}a^2b\right)\tan}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^3}\right.}{a^4}}{d}$
default	$\frac{\frac{2b^4 \operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{a^4\sqrt{(a-b)(a+b)}}-\frac{2\left(\frac{\left(-a^3-\frac{1}{2}a^2b-a\,b^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5+\left(-\frac{2}{3}a^3-2a\,b^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3+\left(-a^3-a\,b^2+\frac{1}{2}a^2b\right)\tan}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^3}\right.}{a^4}}{d}$
risch	$-\frac{bx}{2a^2}-\frac{b^3x}{a^4}-\frac{3ie^{i(dx+c)}}{8da}-\frac{ie^{i(dx+c)}b^2}{2a^3d}+\frac{3ie^{-i(dx+c)}}{8da}+\frac{ie^{-i(dx+c)}b^2}{2a^3d}+\frac{b^4\ln\left(e^{i(dx+c)}+\frac{ia^2-ib^2+b\sqrt{a^2-b^2}}{\sqrt{a^2-b^2}\,a}\right.}{\sqrt{a^2-b^2}\,d\,a^4}$

input
```
int(cos(d*x+c)^3/(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(2*b^4/a^4/((a-b)*(a+b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a-b)
*(a+b))^(1/2))-2/a^4*(((-a^3-1/2*a^2*b-a*b^2)*tan(1/2*d*x+1/2*c)^5+(-2/3*a
^3-2*a*b^2)*tan(1/2*d*x+1/2*c)^3+(-a^3-a*b^2+1/2*a^2*b)*tan(1/2*d*x+1/2*c)
)/(1+tan(1/2*d*x+1/2*c)^2)^3+1/2*b*(a^2+2*b^2)*arctan(tan(1/2*d*x+1/2*c)))
)
```

3.495.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 401, normalized size of antiderivative = 2.71

$$\int \frac{\cos^3(c+dx)}{a+b\sec(c+dx)}\,dx$$
$$= \left[\frac{3\sqrt{a^2-b^2}b^4\log\left(\frac{2\,ab\cos(dx+c)-(a^2-2\,b^2)\cos(dx+c)^2+2\sqrt{a^2-b^2}(b\cos(dx+c)+a)\sin(dx+c)+2\,a^2-b^2}{a^2\cos(dx+c)^2+2\,ab\cos(dx+c)+b^2}\right)-3\,(a^4b+a^2b^3-2}{6\,(a^6-a}\right.$$

input
```
integrate(cos(d*x+c)^3/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

output
```
[1/6*(3*sqrt(a^2 - b^2)*b^4*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*
x + c)^2 + 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b
^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) - 3*(a^4*b + a^2*b^3
- 2*b^5)*d*x + (4*a^5 + 2*a^3*b^2 - 6*a*b^4 + 2*(a^5 - a^3*b^2)*cos(d*x +
c)^2 - 3*(a^4*b - a^2*b^3)*cos(d*x + c))*sin(d*x + c))/((a^6 - a^4*b^2)*d)
, 1/6*(6*sqrt(-a^2 + b^2)*b^4*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a
)/((a^2 - b^2)*sin(d*x + c))) - 3*(a^4*b + a^2*b^3 - 2*b^5)*d*x + (4*a^5 +
 2*a^3*b^2 - 6*a*b^4 + 2*(a^5 - a^3*b^2)*cos(d*x + c)^2 - 3*(a^4*b - a^2*b
^3)*cos(d*x + c))*sin(d*x + c))/((a^6 - a^4*b^2)*d)]
```

3.495.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^3(c+dx)}{a+b\sec(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**3/(a+b*sec(d*x+c)),x)
```

output
```
Timed out
```

3.495.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\cos^3(c+dx)}{a+b\sec(c+dx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(cos(d*x+c)^3/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

3.495.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.68

$$\int \frac{\cos^3(c+dx)}{a+b\sec(c+dx)}\,dx$$
$$= \frac{\frac{12\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{-a^2+b^2}}\right)\right)b^4}{\sqrt{-a^2+b^2}a^4} - \frac{3(a^2b+2b^3)(dx+c)}{a^4} + \frac{2\left(6a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+3ab\tan\right.}{}}{6}$$

input
```
integrate(cos(d*x+c)^3/(a+b*sec(d*x+c)),x, algorithm="giac")
```

output
```
1/6*(12*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan
(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))*b^4/(sqrt(-
a^2 + b^2)*a^4) - 3*(a^2*b + 2*b^3)*(d*x + c)/a^4 + 2*(6*a^2*tan(1/2*d*x +
 1/2*c)^5 + 3*a*b*tan(1/2*d*x + 1/2*c)^5 + 6*b^2*tan(1/2*d*x + 1/2*c)^5 +
4*a^2*tan(1/2*d*x + 1/2*c)^3 + 12*b^2*tan(1/2*d*x + 1/2*c)^3 + 6*a^2*tan(1
/2*d*x + 1/2*c) - 3*a*b*tan(1/2*d*x + 1/2*c) + 6*b^2*tan(1/2*d*x + 1/2*c))
/((tan(1/2*d*x + 1/2*c)^2 + 1)^3*a^3))/d
```

3.495.9 Mupad [B] (verification not implemented)

Time = 14.67 (sec) , antiderivative size = 654, normalized size of antiderivative = 4.42

$$\int \frac{\cos^3(c+dx)}{a+b\sec(c+dx)}\,dx = \frac{\frac{b^2\sin(c+dx)}{4} - \frac{b^2\sin(3c+3dx)}{12}}{a\,d\,(a^2-b^2)}$$
$$- \frac{b\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}\right) + \frac{b\sin(2c+2dx)}{4}}{d\,(a^2-b^2)} + \frac{a\left(\frac{3\sin(c+dx)}{4} + \frac{\sin(3c+3dx)}{12}\right)}{d\,(a^2-b^2)}$$
$$- \frac{b^3\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}\right) - \frac{b^3\sin(2c+2dx)}{4}}{a^2\,d\,(a^2-b^2)} - \frac{b^4\sin(c+dx)}{a^3\,d\,(a^2-b^2)} + \frac{2b^5\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)}{a^4\,d\,(a^2-b^2)}$$
$$+ \frac{b^4\,\mathrm{atan}\left(\frac{\left(8b^7\sin\left(\frac{c}{2}+\frac{dx}{2}\right)(a^2-b^2)^{3/2} - a^9\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a^2-b^2} + 8b^9\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a^2-b^2} - 8a^2b^7\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a^2-b^2} - 3a^4b^5\sin\left(\frac{c}{2}\right.\right.}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)(ab^2-a^3)(4b^5(a^2-b^2)+2ab^6-a^7+4b^7-}\right.}{a^4\,d\,\sqrt{}}$$

input
```
int(cos(c + d*x)^3/(a + b/cos(c + d*x)),x)
```

output
```
((b^2*sin(c + d*x))/4 - (b^2*sin(3*c + 3*d*x))/12)/(a*d*(a^2 - b^2)) - (b*
atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)) + (b*sin(2*c + 2*d*x))/4)/(d*(
a^2 - b^2)) + (a*((3*sin(c + d*x))/4 + sin(3*c + 3*d*x)/12))/(d*(a^2 - b^2
)) - (b^3*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)) - (b^3*sin(2*c + 2*d
*x))/4)/(a^2*d*(a^2 - b^2)) + (b^4*atan(((8*b^7*sin(c/2 + (d*x)/2)*(a^2 -
b^2)^(3/2) - a^9*sin(c/2 + (d*x)/2)*(a^2 - b^2)^(1/2) + 8*b^9*sin(c/2 + (d
*x)/2)*(a^2 - b^2)^(1/2) - 8*a^2*b^7*sin(c/2 + (d*x)/2)*(a^2 - b^2)^(1/2)
- 3*a^4*b^5*sin(c/2 + (d*x)/2)*(a^2 - b^2)^(1/2) + 3*a^5*b^4*sin(c/2 + (d*
x)/2)*(a^2 - b^2)^(1/2) + 2*a^6*b^3*sin(c/2 + (d*x)/2)*(a^2 - b^2)^(1/2) -
 2*a^7*b^2*sin(c/2 + (d*x)/2)*(a^2 - b^2)^(1/2) + a^8*b*sin(c/2 + (d*x)/2)
*(a^2 - b^2)^(1/2))*1i)/(cos(c/2 + (d*x)/2)*(a*b^2 - a^3)*(4*b^5*(a^2 - b^
2) + 2*a*b^6 - a^7 + 4*b^7 - 2*a^2*b^5 + a^3*b^4 - 2*a^4*b^3 - 2*a^5*b^2 +
 2*a^2*b^3*(a^2 - b^2) + 2*a*b^4*(a^2 - b^2))))*2i)/(a^4*d*(a^2 - b^2)^(1/
2)) - (b^4*sin(c + d*x))/(a^3*d*(a^2 - b^2)) + (2*b^5*atan(sin(c/2 + (d*x)
/2)/cos(c/2 + (d*x)/2)))/(a^4*d*(a^2 - b^2))
```

3.495. $\int \frac{\cos^3(c+dx)}{a+b\sec(c+dx)}\,dx$

3.496 $\int \frac{\cos^4(c+dx)}{a+b\sec(c+dx)}\,dx$

3.496.1 Optimal result . . . 3667
3.496.2 Mathematica [A] (verified) . . . 3667
3.496.3 Rubi [A] (verified) . . . 3668
3.496.4 Maple [A] (verified) . . . 3672
3.496.5 Fricas [A] (verification not implemented) . . . 3673
3.496.6 Sympy [F] . . . 3674
3.496.7 Maxima [F(-2)] . . . 3674
3.496.8 Giac [B] (verification not implemented) . . . 3674
3.496.9 Mupad [B] (verification not implemented) . . . 3675

3.496.1 Optimal result

Integrand size = 21, antiderivative size = 193

$$\int \frac{\cos^4(c+dx)}{a+b\sec(c+dx)}\,dx = \frac{(3a^4+4a^2b^2+8b^4)\,x}{8a^5} - \frac{2b^5\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^5\sqrt{a-b}\sqrt{a+b}d} - \frac{b(2a^2+3b^2)\sin(c+dx)}{3a^4d} + \frac{(3a^2+4b^2)\cos(c+dx)\sin(c+dx)}{8a^3d} - \frac{b\cos^2(c+dx)\sin(c+dx)}{3a^2d} + \frac{\cos^3(c+dx)\sin(c+dx)}{4ad}$$

output
```
1/8*(3*a^4+4*a^2*b^2+8*b^4)*x/a^5-1/3*b*(2*a^2+3*b^2)*sin(d*x+c)/a^4/d+1/8
*(3*a^2+4*b^2)*cos(d*x+c)*sin(d*x+c)/a^3/d-1/3*b*cos(d*x+c)^2*sin(d*x+c)/a
^2/d+1/4*cos(d*x+c)^3*sin(d*x+c)/a/d-2*b^5*arctanh((a-b)^(1/2)*tan(1/2*d*x
+1/2*c)/(a+b)^(1/2))/a^5/d/(a-b)^(1/2)/(a+b)^(1/2)
```

3.496.2 Mathematica [A] (verified)

Time = 0.60 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.79

$$\int \frac{\cos^4(c+dx)}{a+b\sec(c+dx)}\,dx = \frac{12(3a^4+4a^2b^2+8b^4)(c+dx) + \frac{192b^5\operatorname{arctanh}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} - 24ab(3a^2+4b^2)\sin(c+dx) + 24a^2(a^2}{96a^5d}$$

input
```
Integrate[Cos[c + d*x]^4/(a + b*Sec[c + d*x]),x]
```

output
```
(12*(3*a^4 + 4*a^2*b^2 + 8*b^4)*(c + d*x) + (192*b^5*ArcTanh[((-a + b)*Tan
[(c + d*x)/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2] - 24*a*b*(3*a^2 + 4*b^2)*
Sin[c + d*x] + 24*a^2*(a^2 + b^2)*Sin[2*(c + d*x)] - 8*a^3*b*Sin[3*(c + d*
x)] + 3*a^4*Sin[4*(c + d*x)])/(96*a^5*d)
```

3.496.3 Rubi [A] (verified)

Time = 1.59 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.13, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.810$, Rules used = {3042, 4340, 25, 3042, 4592, 3042, 4592, 3042, 4592, 27, 3042, 4407, 3042, 4318, 3042, 3138, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^4(c+dx)}{a+b\sec(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^4\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)}dx$$

$$\Big\downarrow \text{4340}$$

$$\frac{\int -\frac{\cos^3(c+dx)\left(-3b\sec^2(c+dx)-3a\sec(c+dx)+4b\right)}{a+b\sec(c+dx)}dx}{4a}+\frac{\sin(c+dx)\cos^3(c+dx)}{4ad}$$

$$\Big\downarrow \text{25}$$

$$\frac{\sin(c+dx)\cos^3(c+dx)}{4ad}-\frac{\int \frac{\cos^3(c+dx)\left(-3b\sec^2(c+dx)-3a\sec(c+dx)+4b\right)}{a+b\sec(c+dx)}dx}{4a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sin(c+dx)\cos^3(c+dx)}{4ad}-\frac{\int \frac{-3b\csc\left(c+dx+\frac{\pi}{2}\right)^2-3a\csc\left(c+dx+\frac{\pi}{2}\right)+4b}{\csc\left(c+dx+\frac{\pi}{2}\right)^3\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{4a}$$

$$\Big\downarrow \text{4592}$$

$$\frac{\sin(c+dx)\cos^3(c+dx)}{4ad}-\frac{\frac{4b\sin(c+dx)\cos^2(c+dx)}{3ad}-\frac{\int \frac{\cos^2(c+dx)\left(-8b^2\sec^2(c+dx)+ab\sec(c+dx)+3\left(3a^2+4b^2\right)\right)}{a+b\sec(c+dx)}dx}{3a}}{4a}$$

$$\Big\downarrow \; 3042$$

$$\frac{\sin(c+dx)\cos^3(c+dx)}{4ad}-\frac{\frac{4b\sin(c+dx)\cos^2(c+dx)}{3ad}-\frac{\int \frac{-8b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2+ab\csc\left(c+dx+\frac{\pi}{2}\right)+3\left(3a^2+4b^2\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^2\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{3a}}{4a}$$

$$\Big\downarrow \; 4592$$

$$\frac{\sin(c+dx)\cos^3(c+dx)}{4ad}-\frac{\frac{4b\sin(c+dx)\cos^2(c+dx)}{3ad}-\frac{\frac{3\left(3a^2+4b^2\right)\sin(c+dx)\cos(c+dx)}{2ad}-\frac{\int \frac{\cos(c+dx)\left(-3b\left(3a^2+4b^2\right)\sec^2(c+dx)-a\left(9a^2-4b^2\right)\sec(c+dx)+8b\left(2a^2+3b^2\right)\right)}{a+b\sec(c+dx)}dx}{2a}}{3a}}{4a}$$

$$\Big\downarrow \; 3042$$

$$\frac{\sin(c+dx)\cos^3(c+dx)}{4ad}-\frac{\frac{4b\sin(c+dx)\cos^2(c+dx)}{3ad}-\frac{\frac{3\left(3a^2+4b^2\right)\sin(c+dx)\cos(c+dx)}{2ad}-\frac{\int \frac{-3b\left(3a^2+4b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2-a\left(9a^2-4b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)+8b\left(2a^2+3b^2\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{2a}}{3a}}{4a}$$

$$\Big\downarrow \; 4592$$

$$\frac{\sin(c+dx)\cos^3(c+dx)}{4ad}-\frac{\frac{4b\sin(c+dx)\cos^2(c+dx)}{3ad}-\frac{\frac{3\left(3a^2+4b^2\right)\sin(c+dx)\cos(c+dx)}{2ad}-\frac{\frac{8b\left(2a^2+3b^2\right)\sin(c+dx)}{ad}-\frac{\int \frac{3\left(3a^4+4b^2a^2+b\left(3a^2+4b^2\right)\sec(c+dx)a+8b^4\right)}{a+b\sec(c+dx)}dx}{a}}{2a}}{3a}}{4a}$$

$$\Big\downarrow \; 27$$

$$\frac{\sin(c+dx)\cos^3(c+dx)}{4ad}-\frac{\frac{4b\sin(c+dx)\cos^2(c+dx)}{3ad}-\frac{\frac{3\left(3a^2+4b^2\right)\sin(c+dx)\cos(c+dx)}{2ad}-\frac{\frac{8b\left(2a^2+3b^2\right)\sin(c+dx)}{ad}-\frac{3\int \frac{3a^4+4b^2a^2+b\left(3a^2+4b^2\right)\sec(c+dx)a+8b^4}{a+b\sec(c+dx)}dx}{a}}{2a}}{3a}}{4a}$$

$$\Big\downarrow \; 3042$$

$$\frac{\sin(c+dx)\cos^3(c+dx)}{4ad}-\frac{\frac{4b\sin(c+dx)\cos^2(c+dx)}{3ad}-\frac{\frac{3\left(3a^2+4b^2\right)\sin(c+dx)\cos(c+dx)}{2ad}-\frac{\frac{8b\left(2a^2+3b^2\right)\sin(c+dx)}{ad}-\frac{3\int \frac{3a^4+4b^2a^2+b\left(3a^2+4b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)a+8b^4}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a}}{2a}}{3a}}{4a}$$

$$\Big\downarrow \; 4407$$

3.496. $\int \frac{\cos^4(c+dx)}{a+b\sec(c+dx)}\,dx$

$$\frac{\sin(c+dx)\cos^3(c+dx)}{4ad}-\frac{\frac{4b\sin(c+dx)\cos^2(c+dx)}{3ad}-\frac{\frac{3\left(3a^2+4b^2\right)\sin(c+dx)\cos(c+dx)}{2ad}-\frac{8b\left(2a^2+3b^2\right)\sin(c+dx)}{ad}-\frac{3\left(\frac{x\left(3a^4+4a^2b^2+8b^4\right)}{a}-\frac{8b^5\int\frac{\sec(c+dx)}{a+b\sec(c+dx)}dx}{a}\right)}{a}}{2a}}{3a}}{4a}$$

$$\Big\downarrow \; 3042$$

$$\frac{\sin(c+dx)\cos^3(c+dx)}{4ad}-\frac{\frac{4b\sin(c+dx)\cos^2(c+dx)}{3ad}-\frac{\frac{3\left(3a^2+4b^2\right)\sin(c+dx)\cos(c+dx)}{2ad}-\frac{8b\left(2a^2+3b^2\right)\sin(c+dx)}{ad}-\frac{3\left(\frac{x\left(3a^4+4a^2b^2+8b^4\right)}{a}-\frac{8b^5\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a}\right)}{a}}{2a}}{3a}}{4a}$$

$$\Big\downarrow \; 4318$$

$$\frac{\sin(c+dx)\cos^3(c+dx)}{4ad}-\frac{\frac{4b\sin(c+dx)\cos^2(c+dx)}{3ad}-\frac{\frac{3\left(3a^2+4b^2\right)\sin(c+dx)\cos(c+dx)}{2ad}-\frac{8b\left(2a^2+3b^2\right)\sin(c+dx)}{ad}-\frac{3\left(\frac{x\left(3a^4+4a^2b^2+8b^4\right)}{a}-\frac{8b^4\int\frac{1}{\frac{a\cos(c+dx)}{b}+1}dx}{a}\right)}{a}}{2a}}{3a}}{4a}$$

$$\Big\downarrow \; 3042$$

$$\frac{\sin(c+dx)\cos^3(c+dx)}{4ad}-\frac{\frac{4b\sin(c+dx)\cos^2(c+dx)}{3ad}-\frac{\frac{3\left(3a^2+4b^2\right)\sin(c+dx)\cos(c+dx)}{2ad}-\frac{8b\left(2a^2+3b^2\right)\sin(c+dx)}{ad}-\frac{3\left(\frac{x\left(3a^4+4a^2b^2+8b^4\right)}{a}-\frac{8b^4\int\frac{1}{\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{b}+1}dx}{a}\right)}{a}}{2a}}{3a}}{4a}$$

$$\Big\downarrow \; 3138$$

$$\frac{\sin(c+dx)\cos^3(c+dx)}{4ad}-\frac{\frac{4b\sin(c+dx)\cos^2(c+dx)}{3ad}-\frac{\frac{3\left(3a^2+4b^2\right)\sin(c+dx)\cos(c+dx)}{2ad}-\frac{8b\left(2a^2+3b^2\right)\sin(c+dx)}{ad}-\frac{3\left(\frac{x\left(3a^4+4a^2b^2+8b^4\right)}{a}-\frac{16b^4\int\frac{1}{\left(1-\frac{a}{b}\right)\tan^2\left(\frac{1}{2}(c+dx)\right)+\frac{a+b}{b}}d\,\mathrm{ta}}{ad}\right.}{a}}{2a}}{3a}}{4a}$$

$$\Big\downarrow \; 221$$

3.496. $\int \frac{\cos^4(c+dx)}{a+b\sec(c+dx)}\,dx$

$$\frac{\sin(c+dx)\cos^3(c+dx)}{4ad} - \frac{\frac{4b\sin(c+dx)\cos^2(c+dx)}{3ad} - \frac{\frac{3\left(3a^2+4b^2\right)\sin(c+dx)\cos(c+dx)}{2ad} - \frac{\frac{8b\left(2a^2+3b^2\right)\sin(c+dx)}{ad} - \frac{3\left(\frac{x\left(3a^4+4a^2b^2+8b^4\right)}{a} - \frac{16b^5\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{ad\sqrt{a-b}\sqrt{a+b}}\right)}{a}}{2a}}{3a}}{4a}$$

input
```
Int[Cos[c + d*x]^4/(a + b*Sec[c + d*x]),x]
```

output
```
(Cos[c + d*x]^3*Sin[c + d*x])/(4*a*d) - ((4*b*Cos[c + d*x]^2*Sin[c + d*x])
/(3*a*d) - ((3*(3*a^2 + 4*b^2)*Cos[c + d*x]*Sin[c + d*x])/(2*a*d) - ((-3*(
((3*a^4 + 4*a^2*b^2 + 8*b^4)*x)/a - (16*b^5*ArcTanh[(Sqrt[a - b]*Tan[(c +
d*x)/2])/Sqrt[a + b]])/(a*Sqrt[a - b]*Sqrt[a + b]*d)))/a + (8*b*(2*a^2 + 3
*b^2)*Sin[c + d*x])/(a*d))/(2*a))/(3*a))/(4*a)
```

3.496.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 4318
```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[1/b   Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4340
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[Cot[e + f*x]*((d*Csc[e + f*x])^n/(a*f*n)), x] - Sim
p[1/(a*d*n)   Int[((d*Csc[e + f*x])^(n + 1)/(a + b*Csc[e + f*x]))*Simp[b*n
- a*(n + 1)*Csc[e + f*x] - b*(n + 1)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a
, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1] && IntegerQ[2*n]
```

rule 4407
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Simp[(b*c - a*d)/a   Int[Csc[e + f*
x]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
- a*d, 0]
```

rule 4592
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Simp[1/(a*d*n)   Int[(a + b*Csc[e + f*x])^m
*(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*
Csc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

3.496.4 Maple [A] (verified)

Time = 0.90 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.33

method	result
derivativedivides	$\frac{\frac{\frac{2\left(\left(-\frac{5}{8}a^4-a^3b-\frac{1}{2}a^2b^2-a\,b^3\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7+\left(\frac{3}{8}a^4-\frac{5}{3}a^3b-3a\,b^3-\frac{1}{2}a^2b^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5+\left(-\frac{3}{8}a^4+\frac{1}{2}a^2b^2-\frac{5}{3}a^3b-3a\,b^3\right)\tan\left(\frac{dx}{2}\right.}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^4}}{a^5}}{d}$
default	$\frac{\frac{\frac{2\left(\left(-\frac{5}{8}a^4-a^3b-\frac{1}{2}a^2b^2-a\,b^3\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7+\left(\frac{3}{8}a^4-\frac{5}{3}a^3b-3a\,b^3-\frac{1}{2}a^2b^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5+\left(-\frac{3}{8}a^4+\frac{1}{2}a^2b^2-\frac{5}{3}a^3b-3a\,b^3\right)\tan\left(\frac{dx}{2}\right.}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^4}}{a^5}}{d}$
risch	$\frac{3x}{8a}+\frac{x\,b^2}{2a^3}+\frac{x\,b^4}{a^5}+\frac{3ib\,\mathrm{e}^{i(dx+c)}}{8a^2d}+\frac{ib^3\mathrm{e}^{i(dx+c)}}{2a^4d}-\frac{3ib\,\mathrm{e}^{-i(dx+c)}}{8a^2d}-\frac{ib^3\mathrm{e}^{-i(dx+c)}}{2a^4d}+\frac{b^5\ln\left(\mathrm{e}^{i(dx+c)}-\frac{ia^2-ib^2-b}{\sqrt{a^2-b}}\right.}{\sqrt{a^2-b^2}\,d\,a^5}$

input
```
int(cos(d*x+c)^4/(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(2/a^5*(((-5/8*a^4-a^3*b-1/2*a^2*b^2-a*b^3)*tan(1/2*d*x+1/2*c)^7+(3/8*
a^4-5/3*a^3*b-3*a*b^3-1/2*a^2*b^2)*tan(1/2*d*x+1/2*c)^5+(-3/8*a^4+1/2*a^2*
b^2-5/3*a^3*b-3*a*b^3)*tan(1/2*d*x+1/2*c)^3+(5/8*a^4+1/2*a^2*b^2-a^3*b-a*b
^3)*tan(1/2*d*x+1/2*c))/(1+tan(1/2*d*x+1/2*c)^2)^4+1/8*(3*a^4+4*a^2*b^2+8*
b^4)*arctan(tan(1/2*d*x+1/2*c)))-2*b^5/a^5/((a-b)*(a+b))^(1/2)*arctanh((a-
b)*tan(1/2*d*x+1/2*c)/((a-b)*(a+b))^(1/2)))
```

3.496.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 482, normalized size of antiderivative = 2.50

$$\int \frac{\cos^4(c+dx)}{a+b\sec(c+dx)}\,dx$$
$$= \left[\frac{12\sqrt{a^2-b^2}b^5\log\left(\frac{2\,ab\cos(dx+c)-(a^2-2\,b^2)\cos(dx+c)^2-2\sqrt{a^2-b^2}(b\cos(dx+c)+a)\sin(dx+c)+2\,a^2-b^2}{a^2\cos(dx+c)^2+2\,ab\cos(dx+c)+b^2}\right)+3\,(3\,a^6+a^4b^2+}{}\right.$$
$$-\frac{24\sqrt{-a^2+b^2}b^5\arctan\left(-\frac{\sqrt{-a^2+b^2}(b\cos(dx+c)+a)}{(a^2-b^2)\sin(dx+c)}\right)-3\,(3\,a^6+a^4b^2+4\,a^2b^4-8\,b^6)dx+\left(16\,a^5b+8\,a^3b^3\right.}{}$$

input
```
integrate(cos(d*x+c)^4/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

output
```
[1/24*(12*sqrt(a^2 - b^2)*b^5*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(
d*x + c)^2 - 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 -
 b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) + 3*(3*a^6 + a^4*b^
2 + 4*a^2*b^4 - 8*b^6)*d*x - (16*a^5*b + 8*a^3*b^3 - 24*a*b^5 - 6*(a^6 - a
^4*b^2)*cos(d*x + c)^3 + 8*(a^5*b - a^3*b^3)*cos(d*x + c)^2 - 3*(3*a^6 + a
^4*b^2 - 4*a^2*b^4)*cos(d*x + c))*sin(d*x + c))/((a^7 - a^5*b^2)*d), -1/24
*(24*sqrt(-a^2 + b^2)*b^5*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((
a^2 - b^2)*sin(d*x + c))) - 3*(3*a^6 + a^4*b^2 + 4*a^2*b^4 - 8*b^6)*d*x +
(16*a^5*b + 8*a^3*b^3 - 24*a*b^5 - 6*(a^6 - a^4*b^2)*cos(d*x + c)^3 + 8*(a
^5*b - a^3*b^3)*cos(d*x + c)^2 - 3*(3*a^6 + a^4*b^2 - 4*a^2*b^4)*cos(d*x +
 c))*sin(d*x + c))/((a^7 - a^5*b^2)*d)]
```

3.496.6 Sympy [F]

$$\int \frac{\cos^4(c+dx)}{a+b\sec(c+dx)}\,dx = \int \frac{\cos^4{(c+dx)}}{a+b\sec{(c+dx)}}\,dx$$

input
```
integrate(cos(d*x+c)**4/(a+b*sec(d*x+c)),x)
```

output
```
Integral(cos(c + d*x)**4/(a + b*sec(c + d*x)), x)
```

3.496.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\cos^4(c+dx)}{a+b\sec(c+dx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(cos(d*x+c)^4/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

3.496.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 393 vs. $2(174) = 348$.

Time = 0.32 (sec) , antiderivative size = 393, normalized size of antiderivative = 2.04

$$\int \frac{\cos^4(c+dx)}{a+b\sec(c+dx)}\,dx = -\frac{\frac{48\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{-a^2+b^2}}\right)\right)b^5}{\sqrt{-a^2+b^2}a^5} - \frac{3\left(3a^4+4a^2b^2+8b^4\right)(dx+c)}{a^5} + \frac{2\left(15a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}\right.\right.}{}$$

input
```
integrate(cos(d*x+c)^4/(a+b*sec(d*x+c)),x, algorithm="giac")
```

output
```
-1/24*(48*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*t
an(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))*b^5/(sqrt
(-a^2 + b^2)*a^5) - 3*(3*a^4 + 4*a^2*b^2 + 8*b^4)*(d*x + c)/a^5 + 2*(15*a^
3*tan(1/2*d*x + 1/2*c)^7 + 24*a^2*b*tan(1/2*d*x + 1/2*c)^7 + 12*a*b^2*tan(
1/2*d*x + 1/2*c)^7 + 24*b^3*tan(1/2*d*x + 1/2*c)^7 - 9*a^3*tan(1/2*d*x + 1
/2*c)^5 + 40*a^2*b*tan(1/2*d*x + 1/2*c)^5 + 12*a*b^2*tan(1/2*d*x + 1/2*c)^
5 + 72*b^3*tan(1/2*d*x + 1/2*c)^5 + 9*a^3*tan(1/2*d*x + 1/2*c)^3 + 40*a^2*
b*tan(1/2*d*x + 1/2*c)^3 - 12*a*b^2*tan(1/2*d*x + 1/2*c)^3 + 72*b^3*tan(1/
2*d*x + 1/2*c)^3 - 15*a^3*tan(1/2*d*x + 1/2*c) + 24*a^2*b*tan(1/2*d*x + 1/
2*c) - 12*a*b^2*tan(1/2*d*x + 1/2*c) + 24*b^3*tan(1/2*d*x + 1/2*c))/((tan(
1/2*d*x + 1/2*c)^2 + 1)^4*a^4))/d
```

3.496.9 Mupad [B] (verification not implemented)

Time = 16.37 (sec) , antiderivative size = 2678, normalized size of antiderivative = 13.88

$$\int \frac{\cos^4(c+dx)}{a+b\sec(c+dx)}\,dx = \text{Too large to display}$$

input
```
int(cos(c + d*x)^4/(a + b/cos(c + d*x)),x)
```

output
```
(b^5*atan(((b^5*(a^2 - b^2)^(1/2)*((tan(c/2 + (d*x)/2)*(256*a*b^10 - 27*a^
10*b + 9*a^11 - 128*b^11 - 256*a^2*b^9 + 256*a^3*b^8 - 256*a^4*b^7 + 256*a
^5*b^6 - 216*a^6*b^5 + 136*a^7*b^4 - 81*a^8*b^3 + 51*a^9*b^2))/(2*a^8) + (
b^5*((12*a^16 - 12*a^15*b + 32*a^10*b^6 - 48*a^11*b^5 + 16*a^12*b^4 - 4*a^
13*b^3 + 4*a^14*b^2)/a^12 - (b^5*tan(c/2 + (d*x)/2)*(a^2 - b^2)^(1/2)*(128
*a^12*b + 128*a^10*b^3 - 256*a^11*b^2))/(2*a^8*(a^7 - a^5*b^2)))*(a^2 - b^
2)^(1/2))/(a^7 - a^5*b^2))*1i)/(a^7 - a^5*b^2) + (b^5*(a^2 - b^2)^(1/2)*((
tan(c/2 + (d*x)/2)*(256*a*b^10 - 27*a^10*b + 9*a^11 - 128*b^11 - 256*a^2*b
^9 + 256*a^3*b^8 - 256*a^4*b^7 + 256*a^5*b^6 - 216*a^6*b^5 + 136*a^7*b^4 -
 81*a^8*b^3 + 51*a^9*b^2))/(2*a^8) - (b^5*((12*a^16 - 12*a^15*b + 32*a^10*
b^6 - 48*a^11*b^5 + 16*a^12*b^4 - 4*a^13*b^3 + 4*a^14*b^2)/a^12 + (b^5*tan
(c/2 + (d*x)/2)*(a^2 - b^2)^(1/2)*(128*a^12*b + 128*a^10*b^3 - 256*a^11*b^
2))/(2*a^8*(a^7 - a^5*b^2)))*(a^2 - b^2)^(1/2))/(a^7 - a^5*b^2))*1i)/(a^7
- a^5*b^2))/((96*a*b^13 - 64*b^14 - 96*a^2*b^12 + 104*a^3*b^11 - 104*a^4*b
^10 + 88*a^5*b^9 - 48*a^6*b^8 + 33*a^7*b^7 - 18*a^8*b^6 + 9*a^9*b^5)/a^12
- (b^5*(a^2 - b^2)^(1/2)*((tan(c/2 + (d*x)/2)*(256*a*b^10 - 27*a^10*b + 9*
a^11 - 128*b^11 - 256*a^2*b^9 + 256*a^3*b^8 - 256*a^4*b^7 + 256*a^5*b^6 -
216*a^6*b^5 + 136*a^7*b^4 - 81*a^8*b^3 + 51*a^9*b^2))/(2*a^8) + (b^5*((12*
a^16 - 12*a^15*b + 32*a^10*b^6 - 48*a^11*b^5 + 16*a^12*b^4 - 4*a^13*b^3 +
4*a^14*b^2)/a^12 - (b^5*tan(c/2 + (d*x)/2)*(a^2 - b^2)^(1/2)*(128*a^12*...
```

3.496. $\int \frac{\cos^4(c+dx)}{a+b\sec(c+dx)}\,dx$

3.497 $\int \frac{\sec^5(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

3.497.1 Optimal result . . . 3677
3.497.2 Mathematica [A] (verified) . . . 3678
3.497.3 Rubi [A] (verified) . . . 3678
3.497.4 Maple [A] (verified) . . . 3683
3.497.5 Fricas [B] (verification not implemented) . . . 3683
3.497.6 Sympy [F] . . . 3684
3.497.7 Maxima [F(-2)] . . . 3685
3.497.8 Giac [A] (verification not implemented) . . . 3685
3.497.9 Mupad [B] (verification not implemented) . . . 3686

3.497.1 Optimal result

Integrand size = 21, antiderivative size = 222

$$\begin{aligned}\int \frac{\sec^5(c+dx)}{(a+b\sec(c+dx))^2}\,dx = &\frac{(6a^2+b^2)\operatorname{arctanh}(\sin(c+dx))}{2b^4d}\\ &-\frac{2a^3(3a^2-4b^2)\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{3/2}b^4(a+b)^{3/2}d}\\ &-\frac{a(3a^2-2b^2)\tan(c+dx)}{b^3\left(a^2-b^2\right)d}\\ &+\frac{(3a^2-b^2)\sec(c+dx)\tan(c+dx)}{2b^2\left(a^2-b^2\right)d}\\ &-\frac{a^2\sec^2(c+dx)\tan(c+dx)}{b\left(a^2-b^2\right)d(a+b\sec(c+dx))}\end{aligned}$$

output
```
1/2*(6*a^2+b^2)*arctanh(sin(d*x+c))/b^4/d-2*a^3*(3*a^2-4*b^2)*arctanh((a-b
)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/(a-b)^(3/2)/b^4/(a+b)^(3/2)/d-a*(3
*a^2-2*b^2)*tan(d*x+c)/b^3/(a^2-b^2)/d+1/2*(3*a^2-b^2)*sec(d*x+c)*tan(d*x+
c)/b^2/(a^2-b^2)/d-a^2*sec(d*x+c)^2*tan(d*x+c)/b/(a^2-b^2)/d/(a+b*sec(d*x+
c))
```

3.497.2 Mathematica [A] (verified)

Time = 5.18 (sec) , antiderivative size = 285, normalized size of antiderivative = 1.28

$$\int \frac{\sec^5(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \frac{\frac{8a^3(3a^2-4b^2)\operatorname{arctanh}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}} - 12a^2\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right) - 2b^2\log\left(\cos\left(\frac{1}{2}(c+\right.\right.}{}$$

input
```
Integrate[Sec[c + d*x]^5/(a + b*Sec[c + d*x])^2,x]
```

output
```
((8*a^3*(3*a^2 - 4*b^2)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2
]])/(a^2 - b^2)^(3/2) - 12*a^2*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] -
2*b^2*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 12*a^2*Log[Cos[(c + d*x)/
2] + Sin[(c + d*x)/2]] + 2*b^2*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] +
b^2/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2 - b^2/(Cos[(c + d*x)/2] + Sin[
(c + d*x)/2])^2 + (4*a^4*b*Sin[c + d*x])/((-a + b)*(a + b)*(b + a*Cos[c +
d*x])) - 8*a*b*Tan[c + d*x])/(4*b^4*d)
```

3.497.3 Rubi [A] (verified)

Time = 1.56 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.05, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {3042, 4332, 3042, 4580, 25, 3042, 4570, 3042, 4486, 3042, 4257, 4318, 3042, 3138, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^5(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^5}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \; 4332$$

$$-\frac{\int \frac{\sec^2(c+dx)\left(2a^2-b\sec(c+dx)a-\left(3a^2-b^2\right)\sec^2(c+dx)\right)}{a+b\sec(c+dx)}dx}{b\left(a^2-b^2\right)} - \frac{a^2\tan(c+dx)\sec^2(c+dx)}{bd\left(a^2-b^2\right)(a+b\sec(c+dx))}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2\left(2a^2-b\csc\left(c+dx+\frac{\pi}{2}\right)a+\left(b^2-3a^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b\left(a^2-b^2\right)}-\frac{a^2\tan(c+dx)\sec^2(c+dx)}{bd\left(a^2-b^2\right)(a+b\sec(c+dx))}$$

$$\Big\downarrow \; 4580$$

$$-\frac{\frac{\int -\frac{\sec(c+dx)\left(-2a\left(3a^2-2b^2\right)\sec^2(c+dx)-b\left(a^2+b^2\right)\sec(c+dx)+a\left(3a^2-b^2\right)\right)}{a+b\sec(c+dx)}dx}{2b}-\frac{\left(3a^2-b^2\right)\tan(c+dx)\sec(c+dx)}{2bd}}{b\left(a^2-b^2\right)}-\frac{a^2\tan(c+dx)\sec^2(c+dx)}{bd\left(a^2-b^2\right)(a+b\sec(c+dx))}$$

$$\Big\downarrow \; 25$$

$$-\frac{-\frac{\int \frac{\sec(c+dx)\left(-2a\left(3a^2-2b^2\right)\sec^2(c+dx)-b\left(a^2+b^2\right)\sec(c+dx)+a\left(3a^2-b^2\right)\right)}{a+b\sec(c+dx)}dx}{2b}-\frac{\left(3a^2-b^2\right)\tan(c+dx)\sec(c+dx)}{2bd}}{b\left(a^2-b^2\right)}-\frac{a^2\tan(c+dx)\sec^2(c+dx)}{bd\left(a^2-b^2\right)(a+b\sec(c+dx))}$$

$$\Big\downarrow \; 3042$$

$$-\frac{-\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(-2a\left(3a^2-2b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2-b\left(a^2+b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)+a\left(3a^2-b^2\right)\right)}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{2b}-\frac{\left(3a^2-b^2\right)\tan(c+dx)\sec(c+dx)}{2bd}}{b\left(a^2-b^2\right)}-\frac{a^2\tan(c+dx)\sec^2(c+dx)}{bd\left(a^2-b^2\right)(a+b\sec(c+dx))}$$

$$\Big\downarrow \; 4570$$

$$-\frac{-\frac{\frac{\int \frac{\sec(c+dx)\left(ab\left(3a^2-b^2\right)+\left(a^2-b^2\right)\left(6a^2+b^2\right)\sec(c+dx)\right)}{a+b\sec(c+dx)}dx}{b}-\frac{2a\left(3a^2-2b^2\right)\tan(c+dx)}{bd}}{2b}-\frac{\left(3a^2-b^2\right)\tan(c+dx)\sec(c+dx)}{2bd}}{b\left(a^2-b^2\right)}-\frac{a^2\tan(c+dx)\sec^2(c+dx)}{bd\left(a^2-b^2\right)(a+b\sec(c+dx))}$$

$$\Big\downarrow \; 3042$$

$$-\frac{-\frac{\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(ab\left(3a^2-b^2\right)+\left(a^2-b^2\right)\left(6a^2+b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b}-\frac{2a\left(3a^2-2b^2\right)\tan(c+dx)}{bd}}{2b}-\frac{\left(3a^2-b^2\right)\tan(c+dx)\sec(c+dx)}{2bd}}{b\left(a^2-b^2\right)}-\frac{a^2\tan(c+dx)\sec^2(c+dx)}{bd\left(a^2-b^2\right)(a+b\sec(c+dx))}$$

$$\Big\downarrow \; 4486$$

3.497. $\int \frac{\sec^5(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

$$-\frac{-\frac{\frac{\left(a^2-b^2\right)\left(6a^2+b^2\right)\int \sec(c+dx)dx}{b}-2a^3\left(\frac{3a^2}{b}-4b\right)\int \frac{\sec(c+dx)}{a+b\sec(c+dx)}dx}{b}-\frac{2a\left(3a^2-2b^2\right)\tan(c+dx)}{bd}}{2b}-\frac{\left(3a^2-b^2\right)\tan(c+dx)\sec(c+dx)}{2bd}}{b\left(a^2-b^2\right)}-\frac{a^2\tan(c+dx)\sec^2(c+dx)}{bd\left(a^2-b^2\right)(a+b\sec(c+dx))}$$

$\downarrow$ 3042

$$-\frac{-\frac{\frac{\left(a^2-b^2\right)\left(6a^2+b^2\right)\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{b}-2a^3\left(\frac{3a^2}{b}-4b\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b}-\frac{2a\left(3a^2-2b^2\right)\tan(c+dx)}{bd}}{2b}-\frac{\left(3a^2-b^2\right)\tan(c+dx)\sec(c+dx)}{2bd}}{b\left(a^2-b^2\right)}-\frac{a^2\tan(c+dx)\sec^2(c+dx)}{bd\left(a^2-b^2\right)(a+b\sec(c+dx))}$$

$\downarrow$ 4257

$$-\frac{-\frac{\frac{\left(a^2-b^2\right)\left(6a^2+b^2\right)\text{arctanh}(\sin(c+dx))}{bd}-2a^3\left(\frac{3a^2}{b}-4b\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b}-\frac{2a\left(3a^2-2b^2\right)\tan(c+dx)}{bd}}{2b}-\frac{\left(3a^2-b^2\right)\tan(c+dx)\sec(c+dx)}{2bd}}{b\left(a^2-b^2\right)}-\frac{a^2\tan(c+dx)\sec^2(c+dx)}{bd\left(a^2-b^2\right)(a+b\sec(c+dx))}$$

$\downarrow$ 4318

$$-\frac{-\frac{\frac{\left(a^2-b^2\right)\left(6a^2+b^2\right)\text{arctanh}(\sin(c+dx))}{bd}-\frac{2a^3\left(\frac{3a^2}{b}-4b\right)\int \frac{1}{\frac{a\cos(c+dx)}{b}+1}dx}{b}}{b}-\frac{2a\left(3a^2-2b^2\right)\tan(c+dx)}{bd}}{2b}-\frac{\left(3a^2-b^2\right)\tan(c+dx)\sec(c+dx)}{2bd}}{b\left(a^2-b^2\right)}-\frac{a^2\tan(c+dx)\sec^2(c+dx)}{bd\left(a^2-b^2\right)(a+b\sec(c+dx))}$$

$\downarrow$ 3042

$$-\frac{-\frac{\frac{\left(a^2-b^2\right)\left(6a^2+b^2\right)\text{arctanh}(\sin(c+dx))}{bd}-\frac{2a^3\left(\frac{3a^2}{b}-4b\right)\int \frac{1}{\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{b}+1}dx}{b}}{b}-\frac{2a\left(3a^2-2b^2\right)\tan(c+dx)}{bd}}{2b}-\frac{\left(3a^2-b^2\right)\tan(c+dx)\sec(c+dx)}{2bd}}{b\left(a^2-b^2\right)}-\frac{a^2\tan(c+dx)\sec^2(c+dx)}{bd\left(a^2-b^2\right)(a+b\sec(c+dx))}$$

$\downarrow$ 3138

3.497. $\int \frac{\sec^5(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

$$-\frac{-\frac{\frac{\left(a^2-b^2\right)\left(6a^2+b^2\right)\text{arctanh}(\sin(c+dx))}{bd}-\frac{4a^3\left(\frac{3a^2}{b}-4b\right)\int\frac{1}{\left(1-\frac{a}{b}\right)\tan^2\left(\frac{1}{2}(c+dx)\right)+\frac{a+b}{b}}d\tan\left(\frac{1}{2}(c+dx)\right)}{bd}}{b}-\frac{2a\left(3a^2-2b^2\right)\tan(c+dx)}{bd}}{2b}-\frac{(3a^2-b^2)\tan}{}}{b\left(a^2-b^2\right)}$$

$$\frac{a^2\tan(c+dx)\sec^2(c+dx)}{bd\left(a^2-b^2\right)(a+b\sec(c+dx))}$$

$$\Big\downarrow \; 221$$

$$-\frac{a^2\tan(c+dx)\sec^2(c+dx)}{bd\left(a^2-b^2\right)(a+b\sec(c+dx))}-$$

$$\frac{-\frac{(3a^2-b^2)\tan(c+dx)\sec(c+dx)}{2bd}-\frac{\frac{\frac{\left(a^2-b^2\right)\left(6a^2+b^2\right)\text{arctanh}(\sin(c+dx))}{bd}-\frac{4a^3\left(\frac{3a^2}{b}-4b\right)\text{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d\sqrt{a-b}\sqrt{a+b}}}{b}-\frac{2a\left(3a^2-2b^2\right)\tan(c}{bd}}{2b}}{b\left(a^2-b^2\right)}$$

input
```
Int[Sec[c + d*x]^5/(a + b*Sec[c + d*x])^2,x]
```

output
```
-((a^2*Sec[c + d*x]^2*Tan[c + d*x])/(b*(a^2 - b^2)*d*(a + b*Sec[c + d*x]))
) - (-1/2*((3*a^2 - b^2)*Sec[c + d*x]*Tan[c + d*x])/(b*d) - ((((a^2 - b^2)
*(6*a^2 + b^2)*ArcTanh[Sin[c + d*x]])/(b*d) - (4*a^3*((3*a^2)/b - 4*b)*Arc
Tanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(Sqrt[a - b]*Sqrt[a + b]
*d))/b - (2*a*(3*a^2 - 2*b^2)*Tan[c + d*x])/(b*d))/(2*b))/(b*(a^2 - b^2))
```

3.497.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4318
```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[1/b   Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4332
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-a^2)*d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(
m + 1)*((d*Csc[e + f*x])^(n - 3)/(b*f*(m + 1)*(a^2 - b^2))), x] + Simp[d^3/
(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])
^(n - 3)*Simp[a^2*(n - 3) + a*b*(m + 1)*Csc[e + f*x] - (a^2*(n - 2) + b^2*(
m + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2
- b^2, 0] && LtQ[m, -1] && (IGtQ[n, 3] || (IntegersQ[n + 1/2, 2*m] && GtQ[n
, 2]))
```

rule 4486
```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/(csc[(
e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[B/b   Int[Csc[e + f*x],
 x], x] + Simp[(A*b - a*B)/b   Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x
] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0]
```

rule 4570
```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> Simp[(-C)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2)
)), x] + Simp[1/(b*(m + 2))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[
b*A*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /;
 FreeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

rule 4580
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[
(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x
_Symbol] :> Simp[(-C)*Csc[e + f*x]*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m +
1)/(b*f*(m + 3))), x] + Simp[1/(b*(m + 3))   Int[Csc[e + f*x]*(a + b*Csc[e
+ f*x])^m*Simp[a*C + b*(C*(m + 2) + A*(m + 3))*Csc[e + f*x] - (2*a*C - b*B*
(m + 3))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &
& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

3.497. $\int \frac{\sec^5(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

3.497.4 Maple [A] (verified)

Time = 0.95 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.25

method	result
derivativedivides	$\frac{\frac{2a^3\left(\frac{ab\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\left(a^2-b^2\right)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2b-a-b\right)}-\frac{\left(3a^2-4b^2\right)\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{(a-b)(a+b)\sqrt{(a-b)(a+b)}}\right)}{b^4}+\frac{1}{2b^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}-}{}$
default	$\frac{\frac{2a^3\left(\frac{ab\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\left(a^2-b^2\right)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2b-a-b\right)}-\frac{\left(3a^2-4b^2\right)\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{(a-b)(a+b)\sqrt{(a-b)(a+b)}}\right)}{b^4}+\frac{1}{2b^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}-}{}$
risch	$-\frac{i\left(-3a^3b\,e^{5i(dx+c)}+a\,b^3e^{5i(dx+c)}-6a^4e^{4i(dx+c)}+2a^2b^2e^{4i(dx+c)}+2b^4e^{4i(dx+c)}-12a^3b\,e^{3i(dx+c)}+8a\,b^3e^{3i(dx+c)}-12\right.}{d\,b^3\left(e^{2i(dx+c)}+1\right)^2\left(-a^2+b^2\right)\left(a\,e^{2i(dx+c)}+\right.}$

input
```
int(sec(d*x+c)^5/(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(2*a^3/b^4*(a*b/(a^2-b^2)*tan(1/2*d*x+1/2*c)/(tan(1/2*d*x+1/2*c)^2*a-t
an(1/2*d*x+1/2*c)^2*b-a-b)-(3*a^2-4*b^2)/(a-b)/(a+b)/((a-b)*(a+b))^(1/2)*a
rctanh((a-b)*tan(1/2*d*x+1/2*c)/((a-b)*(a+b))^(1/2)))+1/2/b^2/(tan(1/2*d*x
+1/2*c)-1)^2-1/2*(-4*a-b)/b^3/(tan(1/2*d*x+1/2*c)-1)+1/2/b^4*(-6*a^2-b^2)*
ln(tan(1/2*d*x+1/2*c)-1)-1/2/b^2/(tan(1/2*d*x+1/2*c)+1)^2-1/2*(-4*a-b)/b^3
/(tan(1/2*d*x+1/2*c)+1)+1/2*(6*a^2+b^2)/b^4*ln(tan(1/2*d*x+1/2*c)+1))
```

3.497.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 426 vs. 2(209) = 418.

Time = 0.73 (sec) , antiderivative size = 909, normalized size of antiderivative = 4.09

$$\int \frac{\sec^5(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$
$$= \left[\frac{2\left(\left(3\,a^6-4\,a^4b^2\right)\cos\left(dx+c\right)^3+\left(3\,a^5b-4\,a^3b^3\right)\cos\left(dx+c\right)^2\right)\sqrt{a^2-b^2}\log\left(\frac{2\,ab\cos(dx+c)-\left(a^2-2\,b^2\right)\cos(dx+c}{a^2\cos(dx}\right.}{}\right.$$
$$-\frac{4\left(\left(3\,a^6-4\,a^4b^2\right)\cos\left(dx+c\right)^3+\left(3\,a^5b-4\,a^3b^3\right)\cos\left(dx+c\right)^2\right)\sqrt{-a^2+b^2}\arctan\left(-\frac{\sqrt{-a^2+b^2}(b\cos(dx+c)+}{\left(a^2-b^2\right)\sin(dx+c)}\right.}{}$$

input
```
integrate(sec(d*x+c)^5/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
[1/4*(2*((3*a^6 - 4*a^4*b^2)*cos(d*x + c)^3 + (3*a^5*b - 4*a^3*b^3)*cos(d*
x + c)^2)*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x
+ c)^2 - 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2
)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) + ((6*a^7 - 11*a^5*b^2
+ 4*a^3*b^4 + a*b^6)*cos(d*x + c)^3 + (6*a^6*b - 11*a^4*b^3 + 4*a^2*b^5 +
b^7)*cos(d*x + c)^2)*log(sin(d*x + c) + 1) - ((6*a^7 - 11*a^5*b^2 + 4*a^3*
b^4 + a*b^6)*cos(d*x + c)^3 + (6*a^6*b - 11*a^4*b^3 + 4*a^2*b^5 + b^7)*cos
(d*x + c)^2)*log(-sin(d*x + c) + 1) + 2*(a^4*b^3 - 2*a^2*b^5 + b^7 - 2*(3*
a^6*b - 5*a^4*b^3 + 2*a^2*b^5)*cos(d*x + c)^2 - 3*(a^5*b^2 - 2*a^3*b^4 + a
*b^6)*cos(d*x + c))*sin(d*x + c))/((a^5*b^4 - 2*a^3*b^6 + a*b^8)*d*cos(d*x
 + c)^3 + (a^4*b^5 - 2*a^2*b^7 + b^9)*d*cos(d*x + c)^2), -1/4*(4*((3*a^6 -
 4*a^4*b^2)*cos(d*x + c)^3 + (3*a^5*b - 4*a^3*b^3)*cos(d*x + c)^2)*sqrt(-a
^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d
*x + c))) - ((6*a^7 - 11*a^5*b^2 + 4*a^3*b^4 + a*b^6)*cos(d*x + c)^3 + (6*
a^6*b - 11*a^4*b^3 + 4*a^2*b^5 + b^7)*cos(d*x + c)^2)*log(sin(d*x + c) + 1
) + ((6*a^7 - 11*a^5*b^2 + 4*a^3*b^4 + a*b^6)*cos(d*x + c)^3 + (6*a^6*b -
11*a^4*b^3 + 4*a^2*b^5 + b^7)*cos(d*x + c)^2)*log(-sin(d*x + c) + 1) - 2*(
a^4*b^3 - 2*a^2*b^5 + b^7 - 2*(3*a^6*b - 5*a^4*b^3 + 2*a^2*b^5)*cos(d*x +
c)^2 - 3*(a^5*b^2 - 2*a^3*b^4 + a*b^6)*cos(d*x + c))*sin(d*x + c))/((a^5*b
^4 - 2*a^3*b^6 + a*b^8)*d*cos(d*x + c)^3 + (a^4*b^5 - 2*a^2*b^7 + b^9)*...
```

3.497.6 Sympy [F]

$$\int \frac{\sec^5(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \int \frac{\sec^5{(c+dx)}}{(a+b\sec{(c+dx)})^2}\,dx$$

input
```
integrate(sec(d*x+c)**5/(a+b*sec(d*x+c))**2,x)
```

output
```
Integral(sec(c + d*x)**5/(a + b*sec(c + d*x))**2, x)
```

3.497.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\sec^5(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(sec(d*x+c)^5/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

3.497.8 Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.35

$$\int \frac{\sec^5(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$
$$= \frac{\frac{4\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{(a^2b^3-b^5)\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a-b\right)} - \frac{4\,(3\,a^5-4\,a^3b^2)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx\right.}{\sqrt{-a^2+b^2}}\right.\right.}{(a^2b^4-b^6)\sqrt{-a^2+b^2}}}{}$$

input
```
integrate(sec(d*x+c)^5/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
1/2*(4*a^4*tan(1/2*d*x + 1/2*c)/((a^2*b^3 - b^5)*(a*tan(1/2*d*x + 1/2*c)^2
 - b*tan(1/2*d*x + 1/2*c)^2 - a - b)) - 4*(3*a^5 - 4*a^3*b^2)*(pi*floor(1/
2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) -
b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))/((a^2*b^4 - b^6)*sqrt(-a^2 + b^
2)) + (6*a^2 + b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/b^4 - (6*a^2 + b^2)
*log(abs(tan(1/2*d*x + 1/2*c) - 1))/b^4 + 2*(4*a*tan(1/2*d*x + 1/2*c)^3 +
b*tan(1/2*d*x + 1/2*c)^3 - 4*a*tan(1/2*d*x + 1/2*c) + b*tan(1/2*d*x + 1/2*
c))/((tan(1/2*d*x + 1/2*c)^2 - 1)^2*b^3))/d
```

3.497.9 Mupad [B] (verification not implemented)

Time = 19.94 (sec) , antiderivative size = 3685, normalized size of antiderivative = 16.60

$$\int \frac{\sec^5(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
int(1/(cos(c + d*x)^5*(a + b/cos(c + d*x))^2),x)
```

output
```
- ((tan(c/2 + (d*x)/2)^5*(3*a*b^3 - 3*a^3*b + 6*a^4 + b^4 - 5*a^2*b^2))/((
a*b^3 - b^4)*(a + b)) + (2*tan(c/2 + (d*x)/2)^3*(b^4 - 6*a^4 + 3*a^2*b^2))
/(b*(a*b^2 - b^3)*(a + b)) + (tan(c/2 + (d*x)/2)*(3*a^3*b - 3*a*b^3 + 6*a^
4 + b^4 - 5*a^2*b^2))/(b^3*(a + b)*(a - b)))/(d*(a + b - tan(c/2 + (d*x)/2
)^2*(3*a + b) - tan(c/2 + (d*x)/2)^6*(a - b) + tan(c/2 + (d*x)/2)^4*(3*a -
 b))) - (atan((((6*a^2 + b^2)*((8*tan(c/2 + (d*x)/2)*(72*a^10 - 72*a^9*b -
 2*a*b^9 + b^10 + 11*a^2*b^8 - 20*a^3*b^7 + 23*a^4*b^6 - 26*a^5*b^5 + 17*a
^6*b^4 + 120*a^7*b^3 - 120*a^8*b^2))/(a*b^8 + b^9 - a^2*b^7 - a^3*b^6) - (
(6*a^2 + b^2)*((8*(2*b^15 + 6*a^2*b^13 - 16*a^3*b^12 - 14*a^4*b^11 + 28*a^
5*b^10 + 6*a^6*b^9 - 12*a^7*b^8))/(a*b^11 + b^12 - a^2*b^10 - a^3*b^9) - (
4*tan(c/2 + (d*x)/2)*(6*a^2 + b^2)*(8*a*b^13 - 8*a^2*b^12 - 16*a^3*b^11 +
16*a^4*b^10 + 8*a^5*b^9 - 8*a^6*b^8))/(b^4*(a*b^8 + b^9 - a^2*b^7 - a^3*b^
6))))/(2*b^4))*1i)/(2*b^4) + ((6*a^2 + b^2)*((8*tan(c/2 + (d*x)/2)*(72*a^1
0 - 72*a^9*b - 2*a*b^9 + b^10 + 11*a^2*b^8 - 20*a^3*b^7 + 23*a^4*b^6 - 26*
a^5*b^5 + 17*a^6*b^4 + 120*a^7*b^3 - 120*a^8*b^2))/(a*b^8 + b^9 - a^2*b^7
- a^3*b^6) + ((6*a^2 + b^2)*((8*(2*b^15 + 6*a^2*b^13 - 16*a^3*b^12 - 14*a^
4*b^11 + 28*a^5*b^10 + 6*a^6*b^9 - 12*a^7*b^8))/(a*b^11 + b^12 - a^2*b^10
- a^3*b^9) + (4*tan(c/2 + (d*x)/2)*(6*a^2 + b^2)*(8*a*b^13 - 8*a^2*b^12 -
16*a^3*b^11 + 16*a^4*b^10 + 8*a^5*b^9 - 8*a^6*b^8))/(b^4*(a*b^8 + b^9 - a^
2*b^7 - a^3*b^6))))/(2*b^4))*1i)/(2*b^4))/((16*(108*a^11 - 54*a^10*b + ...
```

3.498 $\int \frac{\sec^4(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

3.498.1 Optimal result . . . 3687
3.498.2 Mathematica [A] (verified) . . . 3687
3.498.3 Rubi [A] (verified) . . . 3688
3.498.4 Maple [A] (verified) . . . 3691
3.498.5 Fricas [B] (verification not implemented) . . . 3692
3.498.6 Sympy [F] . . . 3693
3.498.7 Maxima [F(-2)] . . . 3693
3.498.8 Giac [B] (verification not implemented) . . . 3693
3.498.9 Mupad [B] (verification not implemented) . . . 3694

3.498.1 Optimal result

Integrand size = 21, antiderivative size = 164

$$\int \frac{\sec^4(c+dx)}{(a+b\sec(c+dx))^2}\,dx = -\frac{2a\operatorname{arctanh}(\sin(c+dx))}{b^3d} + \frac{2a^2(2a^2-3b^2)\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{3/2}b^3(a+b)^{3/2}d} + \frac{(2a^2-b^2)\tan(c+dx)}{b^2(a^2-b^2)d} - \frac{a^2\sec(c+dx)\tan(c+dx)}{b(a^2-b^2)d(a+b\sec(c+dx))}$$

output
```
-2*a*arctanh(sin(d*x+c))/b^3/d+2*a^2*(2*a^2-3*b^2)*arctanh((a-b)^(1/2)*tan
(1/2*d*x+1/2*c)/(a+b)^(1/2))/(a-b)^(3/2)/b^3/(a+b)^(3/2)/d+(2*a^2-b^2)*tan
(d*x+c)/b^2/(a^2-b^2)/d-a^2*sec(d*x+c)*tan(d*x+c)/b/(a^2-b^2)/d/(a+b*sec(d
*x+c))
```

3.498.2 Mathematica [A] (verified)

Time = 0.99 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.99

$$\int \frac{\sec^4(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \frac{-\frac{2a^2(2a^2-3b^2)\operatorname{arctanh}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}} + 2a\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right) - 2a\log\left(\cos\left(\frac{1}{2}(c+\right.\right.}{b^3d}$$

input
```
Integrate[Sec[c + d*x]^4/(a + b*Sec[c + d*x])^2,x]
```

output
```
((-2*a^2*(2*a^2 - 3*b^2)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^
2]])/(a^2 - b^2)^(3/2) + 2*a*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - 2*
a*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + (a^3*b*Sin[c + d*x])/((a - b)
*(a + b)*(b + a*Cos[c + d*x])) + b*Tan[c + d*x])/(b^3*d)
```

3.498.3 Rubi [A] (verified)

Time = 1.09 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.12, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {3042, 4332, 3042, 4570, 3042, 4486, 3042, 4257, 4318, 3042, 3138, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^4(c+dx)}{(a+b\sec(c+dx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^4}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \text{4332}$$

$$-\frac{\int \frac{\sec(c+dx)\left(a^2-b\sec(c+dx)a-\left(2a^2-b^2\right)\sec^2(c+dx)\right)}{a+b\sec(c+dx)}dx}{b\left(a^2-b^2\right)}-\frac{a^2\tan(c+dx)\sec(c+dx)}{bd\left(a^2-b^2\right)(a+b\sec(c+dx))}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(a^2-b\csc\left(c+dx+\frac{\pi}{2}\right)a+\left(b^2-2a^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b\left(a^2-b^2\right)}-\frac{a^2\tan(c+dx)\sec(c+dx)}{bd\left(a^2-b^2\right)(a+b\sec(c+dx))}$$

$$\Big\downarrow \text{4570}$$

$$-\frac{\frac{\int \frac{\sec(c+dx)\left(ba^2+2\left(a^2-b^2\right)\sec(c+dx)a\right)}{a+b\sec(c+dx)}dx}{b}-\frac{\left(2a^2-b^2\right)\tan(c+dx)}{bd}}{b\left(a^2-b^2\right)}-\frac{a^2\tan(c+dx)\sec(c+dx)}{bd\left(a^2-b^2\right)(a+b\sec(c+dx))}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(ba^2+2\left(a^2-b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)a\right)}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b}-\frac{\left(2a^2-b^2\right)\tan(c+dx)}{bd}}{b\left(a^2-b^2\right)}-\frac{a^2\tan(c+dx)\sec(c+dx)}{bd\left(a^2-b^2\right)(a+b\sec(c+dx))}$$

$$\Big\downarrow \; 4486$$

$$-\frac{\frac{\frac{2a\left(a^2-b^2\right)\int \sec(c+dx)dx}{b}-a^2\left(\frac{2a^2}{b}-3b\right)\int \frac{\sec(c+dx)}{a+b\sec(c+dx)}dx}{b}-\frac{\left(2a^2-b^2\right)\tan(c+dx)}{bd}}{b\left(a^2-b^2\right)}-\frac{a^2\tan(c+dx)\sec(c+dx)}{bd\left(a^2-b^2\right)(a+b\sec(c+dx))}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\frac{\frac{2a\left(a^2-b^2\right)\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{b}-a^2\left(\frac{2a^2}{b}-3b\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b}-\frac{\left(2a^2-b^2\right)\tan(c+dx)}{bd}}{b\left(a^2-b^2\right)}-\frac{a^2\tan(c+dx)\sec(c+dx)}{bd\left(a^2-b^2\right)(a+b\sec(c+dx))}$$

$$\Big\downarrow \; 4257$$

$$-\frac{\frac{\frac{2a\left(a^2-b^2\right)\text{arctanh}(\sin(c+dx))}{bd}-a^2\left(\frac{2a^2}{b}-3b\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b}-\frac{\left(2a^2-b^2\right)\tan(c+dx)}{bd}}{b\left(a^2-b^2\right)}-\frac{a^2\tan(c+dx)\sec(c+dx)}{bd\left(a^2-b^2\right)(a+b\sec(c+dx))}$$

$$\Big\downarrow \; 4318$$

$$-\frac{\frac{\frac{2a\left(a^2-b^2\right)\text{arctanh}(\sin(c+dx))}{bd}-\frac{a^2\left(\frac{2a^2}{b}-3b\right)\int \frac{1}{\frac{a\cos(c+dx)}{b}+1}dx}{b}}{b}-\frac{\left(2a^2-b^2\right)\tan(c+dx)}{bd}}{b\left(a^2-b^2\right)}-\frac{a^2\tan(c+dx)\sec(c+dx)}{bd\left(a^2-b^2\right)(a+b\sec(c+dx))}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\frac{\frac{2a\left(a^2-b^2\right)\text{arctanh}(\sin(c+dx))}{bd}-\frac{a^2\left(\frac{2a^2}{b}-3b\right)\int \frac{1}{\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{b}+1}dx}{b}}{b}-\frac{\left(2a^2-b^2\right)\tan(c+dx)}{bd}}{b\left(a^2-b^2\right)}-\frac{a^2\tan(c+dx)\sec(c+dx)}{bd\left(a^2-b^2\right)(a+b\sec(c+dx))}$$

$$\Big\downarrow \; 3138$$

$$-\frac{\frac{\frac{2a\left(a^2-b^2\right)\text{arctanh}(\sin(c+dx))}{bd}-\frac{2a^2\left(\frac{2a^2}{b}-3b\right)\int \frac{1}{\left(1-\frac{a}{b}\right)\tan^2\left(\frac{1}{2}(c+dx)\right)+\frac{a+b}{b}}d\tan\left(\frac{1}{2}(c+dx)\right)}{bd}}{b}-\frac{\left(2a^2-b^2\right)\tan(c+dx)}{bd}}{b\left(a^2-b^2\right)}-\frac{a^2\tan(c+dx)\sec(c+dx)}{bd\left(a^2-b^2\right)(a+b\sec(c+dx))}$$

3.498. $\int \frac{\sec^4(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

$$\Big\downarrow \text{221}$$

$$-\frac{\frac{\frac{2a\left(a^2-b^2\right)\text{arctanh}(\sin(c+dx))}{bd}-\frac{2a^2\left(\frac{2a^2}{b}-3b\right)\text{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d\sqrt{a-b}\sqrt{a+b}}}{b}-\frac{\left(2a^2-b^2\right)\tan(c+dx)}{bd}}{b\left(a^2-b^2\right)}-\frac{a^2\tan(c+dx)\sec(c+dx)}{bd\left(a^2-b^2\right)(a+b\sec(c+dx))}$$

input
```
Int[Sec[c + d*x]^4/(a + b*Sec[c + d*x])^2,x]
```

output
```
-((a^2*Sec[c + d*x]*Tan[c + d*x])/(b*(a^2 - b^2)*d*(a + b*Sec[c + d*x])))
- (((2*a*(a^2 - b^2)*ArcTanh[Sin[c + d*x]])/(b*d) - (2*a^2*((2*a^2)/b - 3*
b)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(Sqrt[a - b]*Sqrt[
a + b]*d))/b - ((2*a^2 - b^2)*Tan[c + d*x])/(b*d))/(b*(a^2 - b^2))
```

3.498.3.1 Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4318
```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[1/b   Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4332
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-a^2)*d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(
m + 1)*((d*Csc[e + f*x])^(n - 3)/(b*f*(m + 1)*(a^2 - b^2))), x] + Simp[d^3/
(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])
^(n - 3)*Simp[a^2*(n - 3) + a*b*(m + 1)*Csc[e + f*x] - (a^2*(n - 2) + b^2*(
m + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2
- b^2, 0] && LtQ[m, -1] && (IGtQ[n, 3] || (IntegersQ[n + 1/2, 2*m] && GtQ[n
, 2]))
```

rule 4486
```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/(csc[(
e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[B/b   Int[Csc[e + f*x],
 x], x] + Simp[(A*b - a*B)/b   Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x
] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0]
```

rule 4570
```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> Simp[(-C)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2)
)), x] + Simp[1/(b*(m + 2))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[
b*A*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /;
 FreeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

3.498.4 Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.27

method	result
derivativedivides	$\frac{-\frac{2a^2\left(\frac{ab\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\left(a^2-b^2\right)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2b-a-b\right)}-\frac{\left(2a^2-3b^2\right)\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{(a-b)(a+b)\sqrt{(a-b)(a+b)}}\right)}{b^3}+\frac{2a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{b^3}}{d}$
default	$\frac{-\frac{2a^2\left(\frac{ab\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\left(a^2-b^2\right)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2b-a-b\right)}-\frac{\left(2a^2-3b^2\right)\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{(a-b)(a+b)\sqrt{(a-b)(a+b)}}\right)}{b^3}+\frac{2a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{b^3}}{d}$
risch	$\frac{2i\left(-a^2b\,e^{3i(dx+c)}-2a^3e^{2i(dx+c)}+a\,b^2e^{2i(dx+c)}-3a^2b\,e^{i(dx+c)}+2b^3e^{i(dx+c)}-2a^3+a\,b^2\right)}{(-a^2+b^2)d\,b^2\left(a\,e^{2i(dx+c)}+2b\,e^{i(dx+c)}+a\right)\left(e^{2i(dx+c)}+1\right)}+\frac{2a\ln\left(e^{i(dx+c)}-i\right)}{b^3d}-\frac{2a\,\mathrm{l}}{}$

input
```
int(sec(d*x+c)^4/(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-2*a^2/b^3*(a*b/(a^2-b^2)*tan(1/2*d*x+1/2*c)/(tan(1/2*d*x+1/2*c)^2*a-
tan(1/2*d*x+1/2*c)^2*b-a-b)-(2*a^2-3*b^2)/(a-b)/(a+b)/((a-b)*(a+b))^(1/2)*
arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a-b)*(a+b))^(1/2)))+2*a/b^3*ln(tan(1/2*
d*x+1/2*c)-1)-1/b^2/(tan(1/2*d*x+1/2*c)-1)-1/b^2/(tan(1/2*d*x+1/2*c)+1)-2*
a/b^3*ln(tan(1/2*d*x+1/2*c)+1))
```

3.498.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 350 vs. $2(155) = 310$.

Time = 0.47 (sec) , antiderivative size = 760, normalized size of antiderivative = 4.63

$$\int \frac{\sec^4(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$
$$= \left[\frac{\left(\left(2\,a^5-3\,a^3b^2\right)\cos\left(dx+c\right)^2+\left(2\,a^4b-3\,a^2b^3\right)\cos\left(dx+c\right)\right)\sqrt{a^2-b^2}\log\left(\frac{2\,ab\cos(dx+c)-\left(a^2-2\,b^2\right)\cos(dx+c)^2-}{a^2\cos(dx+c)}\right.}{}\right.$$

input
```
integrate(sec(d*x+c)^4/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
[1/2*(((2*a^5 - 3*a^3*b^2)*cos(d*x + c)^2 + (2*a^4*b - 3*a^2*b^3)*cos(d*x
+ c))*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)
^2 + 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a
^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) - 2*((a^6 - 2*a^4*b^2 + a^2
*b^4)*cos(d*x + c)^2 + (a^5*b - 2*a^3*b^3 + a*b^5)*cos(d*x + c))*log(sin(d
*x + c) + 1) + 2*((a^6 - 2*a^4*b^2 + a^2*b^4)*cos(d*x + c)^2 + (a^5*b - 2*
a^3*b^3 + a*b^5)*cos(d*x + c))*log(-sin(d*x + c) + 1) + 2*(a^4*b^2 - 2*a^2
*b^4 + b^6 + (2*a^5*b - 3*a^3*b^3 + a*b^5)*cos(d*x + c))*sin(d*x + c))/((a
^5*b^3 - 2*a^3*b^5 + a*b^7)*d*cos(d*x + c)^2 + (a^4*b^4 - 2*a^2*b^6 + b^8)
*d*cos(d*x + c)), (((2*a^5 - 3*a^3*b^2)*cos(d*x + c)^2 + (2*a^4*b - 3*a^2*
b^3)*cos(d*x + c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x +
c) + a)/((a^2 - b^2)*sin(d*x + c))) - ((a^6 - 2*a^4*b^2 + a^2*b^4)*cos(d*x
 + c)^2 + (a^5*b - 2*a^3*b^3 + a*b^5)*cos(d*x + c))*log(sin(d*x + c) + 1)
+ ((a^6 - 2*a^4*b^2 + a^2*b^4)*cos(d*x + c)^2 + (a^5*b - 2*a^3*b^3 + a*b^5
)*cos(d*x + c))*log(-sin(d*x + c) + 1) + (a^4*b^2 - 2*a^2*b^4 + b^6 + (2*a
^5*b - 3*a^3*b^3 + a*b^5)*cos(d*x + c))*sin(d*x + c))/((a^5*b^3 - 2*a^3*b^
5 + a*b^7)*d*cos(d*x + c)^2 + (a^4*b^4 - 2*a^2*b^6 + b^8)*d*cos(d*x + c))]
```

3.498.6 Sympy [F]

$$\int \frac{\sec^4(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \int \frac{\sec^4(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$

input
```
integrate(sec(d*x+c)**4/(a+b*sec(d*x+c))**2,x)
```

output
```
Integral(sec(c + d*x)**4/(a + b*sec(c + d*x))**2, x)
```

3.498.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\sec^4(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(sec(d*x+c)^4/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

3.498.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 331 vs. $2(155) = 310$.

Time = 0.31 (sec) , antiderivative size = 331, normalized size of antiderivative = 2.02

$$\int \frac{\sec^4(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \frac{2\left(\frac{(2a^4-3a^2b^2)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{-a^2+b^2}}\right)\right)}{(a^2b^3-b^5)\sqrt{-a^2+b^2}} - \frac{2a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-a^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3}{}\right.}{}$$

input
```
integrate(sec(d*x+c)^4/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
2*((2*a^4 - 3*a^2*b^2)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) +
 arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2
)))/((a^2*b^3 - b^5)*sqrt(-a^2 + b^2)) - (2*a^3*tan(1/2*d*x + 1/2*c)^3 - a
^2*b*tan(1/2*d*x + 1/2*c)^3 - a*b^2*tan(1/2*d*x + 1/2*c)^3 + b^3*tan(1/2*d
*x + 1/2*c)^3 - 2*a^3*tan(1/2*d*x + 1/2*c) - a^2*b*tan(1/2*d*x + 1/2*c) +
a*b^2*tan(1/2*d*x + 1/2*c) + b^3*tan(1/2*d*x + 1/2*c))/((a*tan(1/2*d*x + 1
/2*c)^4 - b*tan(1/2*d*x + 1/2*c)^4 - 2*a*tan(1/2*d*x + 1/2*c)^2 + a + b)*(
a^2*b^2 - b^4)) - a*log(abs(tan(1/2*d*x + 1/2*c) + 1))/b^3 + a*log(abs(tan
(1/2*d*x + 1/2*c) - 1))/b^3)/d
```

3.498.9 Mupad [B] (verification not implemented)

Time = 19.21 (sec) , antiderivative size = 3159, normalized size of antiderivative = 19.26

$$\int \frac{\sec^4(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
int(1/(cos(c + d*x)^4*(a + b/cos(c + d*x))^2),x)
```

output
```
((2*tan(c/2 + (d*x)/2)^3*(a*b^2 + a^2*b - 2*a^3 - b^3))/(b^2*(a + b)*(a -
b)) - (2*tan(c/2 + (d*x)/2)*(a*b^2 - a^2*b - 2*a^3 + b^3))/(b^2*(a + b)*(a
 - b)))/(d*(a + b + tan(c/2 + (d*x)/2)^4*(a - b) - 2*a*tan(c/2 + (d*x)/2)^
2)) + (a*atan(((a*((32*tan(c/2 + (d*x)/2)*(8*a^8 - 8*a^7*b + 4*a^2*b^6 - 8
*a^3*b^5 + 5*a^4*b^4 + 16*a^5*b^3 - 16*a^6*b^2))/(a*b^6 + b^7 - a^2*b^5 -
a^3*b^4) - (2*a*((32*(2*a*b^11 - 3*a^2*b^10 - 3*a^3*b^9 + 5*a^4*b^8 + a^5*
b^7 - 2*a^6*b^6))/(a*b^8 + b^9 - a^2*b^7 - a^3*b^6) - (64*a*tan(c/2 + (d*x
)/2)*(2*a*b^11 - 2*a^2*b^10 - 4*a^3*b^9 + 4*a^4*b^8 + 2*a^5*b^7 - 2*a^6*b^
6))/(b^3*(a*b^6 + b^7 - a^2*b^5 - a^3*b^4))))/b^3)*2i)/b^3 + (a*((32*tan(c
/2 + (d*x)/2)*(8*a^8 - 8*a^7*b + 4*a^2*b^6 - 8*a^3*b^5 + 5*a^4*b^4 + 16*a^
5*b^3 - 16*a^6*b^2))/(a*b^6 + b^7 - a^2*b^5 - a^3*b^4) + (2*a*((32*(2*a*b^
11 - 3*a^2*b^10 - 3*a^3*b^9 + 5*a^4*b^8 + a^5*b^7 - 2*a^6*b^6))/(a*b^8 + b
^9 - a^2*b^7 - a^3*b^6) + (64*a*tan(c/2 + (d*x)/2)*(2*a*b^11 - 2*a^2*b^10
- 4*a^3*b^9 + 4*a^4*b^8 + 2*a^5*b^7 - 2*a^6*b^6))/(b^3*(a*b^6 + b^7 - a^2*
b^5 - a^3*b^4))))/b^3)*2i)/b^3)/((64*(8*a^8 - 4*a^7*b + 12*a^4*b^4 + 6*a^5
*b^3 - 20*a^6*b^2))/(a*b^8 + b^9 - a^2*b^7 - a^3*b^6) - (2*a*((32*tan(c/2
+ (d*x)/2)*(8*a^8 - 8*a^7*b + 4*a^2*b^6 - 8*a^3*b^5 + 5*a^4*b^4 + 16*a^5*b
^3 - 16*a^6*b^2))/(a*b^6 + b^7 - a^2*b^5 - a^3*b^4) - (2*a*((32*(2*a*b^11
- 3*a^2*b^10 - 3*a^3*b^9 + 5*a^4*b^8 + a^5*b^7 - 2*a^6*b^6))/(a*b^8 + b^9
- a^2*b^7 - a^3*b^6) - (64*a*tan(c/2 + (d*x)/2)*(2*a*b^11 - 2*a^2*b^10 ...
```

3.498. $\int \frac{\sec^4(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

3.499 $\int \frac{\sec^3(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

3.499.1 Optimal result . . . 3696
3.499.2 Mathematica [A] (verified) . . . 3696
3.499.3 Rubi [A] (verified) . . . 3697
3.499.4 Maple [A] (verified) . . . 3700
3.499.5 Fricas [B] (verification not implemented) . . . 3700
3.499.6 Sympy [F] . . . 3701
3.499.7 Maxima [F(-2)] . . . 3701
3.499.8 Giac [A] (verification not implemented) . . . 3702
3.499.9 Mupad [B] (verification not implemented) . . . 3702

3.499.1 Optimal result

Integrand size = 21, antiderivative size = 117

$$\int \frac{\sec^3(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \frac{\operatorname{arctanh}(\sin(c+dx))}{b^2d} - \frac{2a(a^2-2b^2)\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{3/2}b^2(a+b)^{3/2}d} - \frac{a^2\tan(c+dx)}{b\,(a^2-b^2)\,d(a+b\sec(c+dx))}$$

output
```
arctanh(sin(d*x+c))/b^2/d-2*a*(a^2-2*b^2)*arctanh((a-b)^(1/2)*tan(1/2*d*x+
1/2*c)/(a+b)^(1/2))/(a-b)^(3/2)/b^2/(a+b)^(3/2)/d-a^2*tan(d*x+c)/b/(a^2-b^
2)/d/(a+b*sec(d*x+c))
```

3.499.2 Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.25

$$\int \frac{\sec^3(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \frac{\frac{2a(a^2-2b^2)\operatorname{arctanh}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}} - \log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\right.}{b^2d}$$

input
```
Integrate[Sec[c + d*x]^3/(a + b*Sec[c + d*x])^2,x]
```

output
```
((2*a*(a^2 - 2*b^2)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/
(a^2 - b^2)^(3/2) - Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + Log[Cos[(c
+ d*x)/2] + Sin[(c + d*x)/2]] + (a^2*b*Sin[c + d*x])/((-a + b)*(a + b)*(b
+ a*Cos[c + d*x])))/(b^2*d)
```

3.499.3 Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.21, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.524$, Rules used = {3042, 4326, 25, 3042, 4486, 3042, 4257, 4318, 3042, 3138, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^3(c+dx)}{(a+b\sec(c+dx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^3}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \text{4326}$$

$$-\frac{\int -\frac{\sec(c+dx)\left(ab+\left(a^2-b^2\right)\sec(c+dx)\right)}{a+b\sec(c+dx)}dx}{b\left(a^2-b^2\right)}-\frac{a^2\tan(c+dx)}{bd\left(a^2-b^2\right)(a+b\sec(c+dx))}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\sec(c+dx)\left(ab+\left(a^2-b^2\right)\sec(c+dx)\right)}{a+b\sec(c+dx)}dx}{b\left(a^2-b^2\right)}-\frac{a^2\tan(c+dx)}{bd\left(a^2-b^2\right)(a+b\sec(c+dx))}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(ab+\left(a^2-b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b\left(a^2-b^2\right)}-\frac{a^2\tan(c+dx)}{bd\left(a^2-b^2\right)(a+b\sec(c+dx))}$$

$$\Big\downarrow \text{4486}$$

$$\frac{\frac{\left(a^2-b^2\right)\int\sec(c+dx)dx}{b}-\frac{a\left(a^2-2b^2\right)\int\frac{\sec(c+dx)}{a+b\sec(c+dx)}dx}{b}}{b\left(a^2-b^2\right)}-\frac{a^2\tan(c+dx)}{bd\left(a^2-b^2\right)(a+b\sec(c+dx))}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{(a^2-b^2)\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{b}-\frac{a(a^2-2b^2)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b}}{b(a^2-b^2)}-\frac{a^2\tan(c+dx)}{bd(a^2-b^2)(a+b\sec(c+dx))}$$

$\Big\downarrow$ 4257

$$\frac{\frac{(a^2-b^2)\text{arctanh}(\sin(c+dx))}{bd}-\frac{a(a^2-2b^2)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b}}{b(a^2-b^2)}-\frac{a^2\tan(c+dx)}{bd(a^2-b^2)(a+b\sec(c+dx))}$$

$\Big\downarrow$ 4318

$$\frac{\frac{(a^2-b^2)\text{arctanh}(\sin(c+dx))}{bd}-\frac{a(a^2-2b^2)\int \frac{1}{\frac{a\cos(c+dx)}{b}+1}dx}{b^2}}{b(a^2-b^2)}-\frac{a^2\tan(c+dx)}{bd(a^2-b^2)(a+b\sec(c+dx))}$$

$\Big\downarrow$ 3042

$$\frac{\frac{(a^2-b^2)\text{arctanh}(\sin(c+dx))}{bd}-\frac{a(a^2-2b^2)\int \frac{1}{\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{b}+1}dx}{b^2}}{b(a^2-b^2)}-\frac{a^2\tan(c+dx)}{bd(a^2-b^2)(a+b\sec(c+dx))}$$

$\Big\downarrow$ 3138

$$\frac{\frac{(a^2-b^2)\text{arctanh}(\sin(c+dx))}{bd}-\frac{2a(a^2-2b^2)\int \frac{1}{\left(1-\frac{a}{b}\right)\tan^2\left(\frac{1}{2}(c+dx)\right)+\frac{a+b}{b}}d\tan\left(\frac{1}{2}(c+dx)\right)}{b^2d}}{b(a^2-b^2)}-\frac{a^2\tan(c+dx)}{bd(a^2-b^2)(a+b\sec(c+dx))}$$

$\Big\downarrow$ 221

$$\frac{\frac{(a^2-b^2)\text{arctanh}(\sin(c+dx))}{bd}-\frac{2a(a^2-2b^2)\text{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{bd\sqrt{a-b}\sqrt{a+b}}}{b(a^2-b^2)}-\frac{a^2\tan(c+dx)}{bd(a^2-b^2)(a+b\sec(c+dx))}$$

input
```
Int[Sec[c + d*x]^3/(a + b*Sec[c + d*x])^2,x]
```

output
```
(((a^2 - b^2)*ArcTanh[Sin[c + d*x]])/(b*d) - (2*a*(a^2 - 2*b^2)*ArcTanh[(S
qrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(Sqrt[a - b]*b*Sqrt[a + b]*d))/
(b*(a^2 - b^2)) - (a^2*Tan[c + d*x])/(b*(a^2 - b^2)*d*(a + b*Sec[c + d*x])
)
```

3.499. $\int \frac{\sec^3(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

3.499.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4318
```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[1/b   Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4326
```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(-a^2)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m
 + 1)*(a^2 - b^2))), x] + Simp[1/(b*(m + 1)*(a^2 - b^2))   Int[Csc[e + f*x]
*(a + b*Csc[e + f*x])^(m + 1)*Simp[a*b*(m + 1) - (a^2 + b^2*(m + 1))*Csc[e
+ f*x], x], x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m,
 -1]
```

rule 4486
```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/(csc[(
e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[B/b   Int[Csc[e + f*x],
 x], x] + Simp[(A*b - a*B)/b   Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x
] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0]
```

3.499. $\int \frac{\sec^3(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

3.499.4 Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.42

method	result
derivativedivides	$\frac{\frac{2a\left(\frac{ab\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\left(a^2-b^2\right)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2b-a-b\right)}-\frac{\left(a^2-2b^2\right)\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{(a-b)(a+b)\sqrt{(a-b)(a+b)}}\right)}{b^2}+\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{b^2}-\frac{\ln(\mathrm{ta}}{}}{d}$
default	$\frac{\frac{2a\left(\frac{ab\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\left(a^2-b^2\right)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2b-a-b\right)}-\frac{\left(a^2-2b^2\right)\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{(a-b)(a+b)\sqrt{(a-b)(a+b)}}\right)}{b^2}+\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{b^2}-\frac{\ln(\mathrm{ta}}{}}{d}$
risch	$-\frac{2ia\left(b\,\mathrm{e}^{i(dx+c)}+a\right)}{(a^2-b^2)db\left(a\,\mathrm{e}^{2i(dx+c)}+2b\,\mathrm{e}^{i(dx+c)}+a\right)}+\frac{a^3\ln\left(\mathrm{e}^{i(dx+c)}+\frac{-ia^2+ib^2+b\sqrt{a^2-b^2}}{a\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,(a+b)(a-b)d\,b^2}-\frac{2a\ln\left(\mathrm{e}^{i(dx+c)}+\frac{-ia^2+ib^2+b\sqrt{}}{a\sqrt{a^2-b}}\right.}{\sqrt{a^2-b^2}\,(a+b)(a-b)d}$

input
```
int(sec(d*x+c)^3/(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(2*a/b^2*(a*b/(a^2-b^2)*tan(1/2*d*x+1/2*c)/(tan(1/2*d*x+1/2*c)^2*a-tan
(1/2*d*x+1/2*c)^2*b-a-b)-(a^2-2*b^2)/(a-b)/(a+b)/((a-b)*(a+b))^(1/2)*arcta
nh((a-b)*tan(1/2*d*x+1/2*c)/((a-b)*(a+b))^(1/2)))+1/b^2*ln(tan(1/2*d*x+1/2
*c)+1)-1/b^2*ln(tan(1/2*d*x+1/2*c)-1))
```

3.499.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 270 vs. 2(108) = 216.

Time = 0.51 (sec) , antiderivative size = 596, normalized size of antiderivative = 5.09

$$\int \frac{\sec^3(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$
$$= \left[\frac{(a^3b-2\,ab^3+(a^4-2\,a^2b^2)\cos(dx+c))\sqrt{a^2-b^2}\log\left(\frac{2\,ab\cos(dx+c)-(a^2-2\,b^2)\cos(dx+c)^2-2\,\sqrt{a^2-b^2}(b\cos(dx+c)+a)}{a^2\cos(dx+c)^2+2\,ab\cos(dx+c)+b^2}\right.}{}\right.$$
$$-\frac{2\,(a^3b-2\,ab^3+(a^4-2\,a^2b^2)\cos(dx+c))\sqrt{-a^2+b^2}\arctan\left(-\frac{\sqrt{-a^2+b^2}(b\cos(dx+c)+a)}{(a^2-b^2)\sin(dx+c)}\right)-(a^4b-2\,a^2b^3-}{}$$

input
```
integrate(sec(d*x+c)^3/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
[1/2*((a^3*b - 2*a*b^3 + (a^4 - 2*a^2*b^2)*cos(d*x + c))*sqrt(a^2 - b^2)*l
og((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 - 2*sqrt(a^2 - b^2)*
(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a
*b*cos(d*x + c) + b^2)) + (a^4*b - 2*a^2*b^3 + b^5 + (a^5 - 2*a^3*b^2 + a*
b^4)*cos(d*x + c))*log(sin(d*x + c) + 1) - (a^4*b - 2*a^2*b^3 + b^5 + (a^5
 - 2*a^3*b^2 + a*b^4)*cos(d*x + c))*log(-sin(d*x + c) + 1) - 2*(a^4*b - a^
2*b^3)*sin(d*x + c))/((a^5*b^2 - 2*a^3*b^4 + a*b^6)*d*cos(d*x + c) + (a^4*
b^3 - 2*a^2*b^5 + b^7)*d), -1/2*(2*(a^3*b - 2*a*b^3 + (a^4 - 2*a^2*b^2)*co
s(d*x + c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)
/((a^2 - b^2)*sin(d*x + c))) - (a^4*b - 2*a^2*b^3 + b^5 + (a^5 - 2*a^3*b^2
 + a*b^4)*cos(d*x + c))*log(sin(d*x + c) + 1) + (a^4*b - 2*a^2*b^3 + b^5 +
 (a^5 - 2*a^3*b^2 + a*b^4)*cos(d*x + c))*log(-sin(d*x + c) + 1) + 2*(a^4*b
 - a^2*b^3)*sin(d*x + c))/((a^5*b^2 - 2*a^3*b^4 + a*b^6)*d*cos(d*x + c) +
(a^4*b^3 - 2*a^2*b^5 + b^7)*d)]
```

3.499.6 Sympy [F]

$$\int \frac{\sec^3(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \int \frac{\sec^3{(c+dx)}}{(a+b\sec{(c+dx)})^2}\,dx$$

input
```
integrate(sec(d*x+c)**3/(a+b*sec(d*x+c))**2,x)
```

output
```
Integral(sec(c + d*x)**3/(a + b*sec(c + d*x))**2, x)
```

3.499.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\sec^3(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(sec(d*x+c)^3/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

3.499.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.74

$$\int \frac{\sec^3(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$
$$= \frac{\frac{2\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{(a^2b-b^3)\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a-b\right)} - \frac{2\,(a^3-2\,ab^2)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\right.}{\sqrt{-a^2+b^2}}\right.\right.}{(a^2b^2-b^4)\sqrt{-a^2+b^2}}}{d}$$

input
```
integrate(sec(d*x+c)^3/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
(2*a^2*tan(1/2*d*x + 1/2*c)/((a^2*b - b^3)*(a*tan(1/2*d*x + 1/2*c)^2 - b*t
an(1/2*d*x + 1/2*c)^2 - a - b)) - 2*(a^3 - 2*a*b^2)*(pi*floor(1/2*(d*x + c
)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*
d*x + 1/2*c))/sqrt(-a^2 + b^2)))/((a^2*b^2 - b^4)*sqrt(-a^2 + b^2)) + log(
abs(tan(1/2*d*x + 1/2*c) + 1))/b^2 - log(abs(tan(1/2*d*x + 1/2*c) - 1))/b^
2)/d
```

3.499.9 Mupad [B] (verification not implemented)

Time = 19.07 (sec) , antiderivative size = 2848, normalized size of antiderivative = 24.34

$$\int \frac{\sec^3(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
int(1/(cos(c + d*x)^3*(a + b/cos(c + d*x))^2),x)
```

output
```
- (atan((((((32*(2*a*b^8 - b^9 + a^2*b^7 - 3*a^3*b^6 + a^5*b^4))/(a*b^5 +
b^6 - a^2*b^4 - a^3*b^3) - (32*tan(c/2 + (d*x)/2)*(2*a*b^9 - 2*a^2*b^8 - 4
*a^3*b^7 + 4*a^4*b^6 + 2*a^5*b^5 - 2*a^6*b^4))/(b^2*(a*b^4 + b^5 - a^2*b^3
 - a^3*b^2)))/b^2 - (32*tan(c/2 + (d*x)/2)*(2*a^6 - 2*a^5*b - 2*a*b^5 + b^
6 + 3*a^2*b^4 + 4*a^3*b^3 - 5*a^4*b^2))/(a*b^4 + b^5 - a^2*b^3 - a^3*b^2))
*1i)/b^2 - ((((32*(2*a*b^8 - b^9 + a^2*b^7 - 3*a^3*b^6 + a^5*b^4))/(a*b^5
+ b^6 - a^2*b^4 - a^3*b^3) + (32*tan(c/2 + (d*x)/2)*(2*a*b^9 - 2*a^2*b^8 -
 4*a^3*b^7 + 4*a^4*b^6 + 2*a^5*b^5 - 2*a^6*b^4))/(b^2*(a*b^4 + b^5 - a^2*b
^3 - a^3*b^2)))/b^2 + (32*tan(c/2 + (d*x)/2)*(2*a^6 - 2*a^5*b - 2*a*b^5 +
b^6 + 3*a^2*b^4 + 4*a^3*b^3 - 5*a^4*b^2))/(a*b^4 + b^5 - a^2*b^3 - a^3*b^2
))*1i)/b^2)/((((32*(2*a*b^8 - b^9 + a^2*b^7 - 3*a^3*b^6 + a^5*b^4))/(a*b^5
 + b^6 - a^2*b^4 - a^3*b^3) - (32*tan(c/2 + (d*x)/2)*(2*a*b^9 - 2*a^2*b^8
- 4*a^3*b^7 + 4*a^4*b^6 + 2*a^5*b^5 - 2*a^6*b^4))/(b^2*(a*b^4 + b^5 - a^2*
b^3 - a^3*b^2)))/b^2 - (32*tan(c/2 + (d*x)/2)*(2*a^6 - 2*a^5*b - 2*a*b^5 +
 b^6 + 3*a^2*b^4 + 4*a^3*b^3 - 5*a^4*b^2))/(a*b^4 + b^5 - a^2*b^3 - a^3*b^
2))/b^2 - (64*(2*a*b^4 - a^4*b + a^5 + 2*a^2*b^3 - 3*a^3*b^2))/(a*b^5 + b^
6 - a^2*b^4 - a^3*b^3) + (((32*(2*a*b^8 - b^9 + a^2*b^7 - 3*a^3*b^6 + a^5*
b^4))/(a*b^5 + b^6 - a^2*b^4 - a^3*b^3) + (32*tan(c/2 + (d*x)/2)*(2*a*b^9
- 2*a^2*b^8 - 4*a^3*b^7 + 4*a^4*b^6 + 2*a^5*b^5 - 2*a^6*b^4))/(b^2*(a*b^4
+ b^5 - a^2*b^3 - a^3*b^2)))/b^2 + (32*tan(c/2 + (d*x)/2)*(2*a^6 - 2*a^...
```

3.499. $\int \frac{\sec^3(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

3.500 $\int \frac{\sec^2(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

3.500.1 Optimal result . . . 3704
3.500.2 Mathematica [A] (verified) . . . 3704
3.500.3 Rubi [A] (verified) . . . 3705
3.500.4 Maple [A] (verified) . . . 3707
3.500.5 Fricas [A] (verification not implemented) . . . 3707
3.500.6 Sympy [F] . . . 3708
3.500.7 Maxima [F(-2)] . . . 3708
3.500.8 Giac [A] (verification not implemented) . . . 3709
3.500.9 Mupad [B] (verification not implemented) . . . 3709

3.500.1 Optimal result

Integrand size = 21, antiderivative size = 85

$$\int \frac{\sec^2(c+dx)}{(a+b\sec(c+dx))^2}\,dx = -\frac{2b\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{3/2}(a+b)^{3/2}d} + \frac{a\tan(c+dx)}{(a^2-b^2)\,d(a+b\sec(c+dx))}$$

output
```
-2*b*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/(a-b)^(3/2)/(a+b)
^(3/2)/d+a*tan(d*x+c)/(a^2-b^2)/d/(a+b*sec(d*x+c))
```

3.500.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.98

$$\int \frac{\sec^2(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \frac{\frac{2b\operatorname{arctanh}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}} + \frac{a\sin(c+dx)}{(a-b)(a+b)(b+a\cos(c+dx))}}{d}$$

input
```
Integrate[Sec[c + d*x]^2/(a + b*Sec[c + d*x])^2,x]
```

output
```
((2*b*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(3
/2) + (a*Sin[c + d*x])/((a - b)*(a + b)*(b + a*Cos[c + d*x])))/d
```

3.500.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.13, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3042, 4323, 25, 27, 3042, 4318, 3042, 3138, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(c+dx)}{(a+b\sec(c+dx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \text{4323}$$

$$\frac{\int -\frac{b\sec(c+dx)}{a+b\sec(c+dx)}dx}{a^2-b^2}+\frac{a\tan(c+dx)}{d\left(a^2-b^2\right)(a+b\sec(c+dx))}$$

$$\Big\downarrow \text{25}$$

$$\frac{a\tan(c+dx)}{d\left(a^2-b^2\right)(a+b\sec(c+dx))}-\frac{\int \frac{b\sec(c+dx)}{a+b\sec(c+dx)}dx}{a^2-b^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{a\tan(c+dx)}{d\left(a^2-b^2\right)(a+b\sec(c+dx))}-\frac{b\int \frac{\sec(c+dx)}{a+b\sec(c+dx)}dx}{a^2-b^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a\tan(c+dx)}{d\left(a^2-b^2\right)(a+b\sec(c+dx))}-\frac{b\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2-b^2}$$

$$\Big\downarrow \text{4318}$$

$$\frac{a\tan(c+dx)}{d\left(a^2-b^2\right)(a+b\sec(c+dx))}-\frac{\int \frac{1}{\frac{a\cos(c+dx)}{b}+1}dx}{a^2-b^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a\tan(c+dx)}{d\left(a^2-b^2\right)(a+b\sec(c+dx))}-\frac{\int \frac{1}{\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{b}+1}dx}{a^2-b^2}$$

$$\Big\downarrow \text{3138}$$

$$\frac{a\tan(c+dx)}{d(a^2-b^2)(a+b\sec(c+dx))}-\frac{2\int\frac{1}{(1-\frac{a}{b})\tan^2(\frac{1}{2}(c+dx))+\frac{a+b}{b}}d\tan\left(\frac{1}{2}(c+dx)\right)}{d(a^2-b^2)}$$

$$\Big\downarrow \text{221}$$

$$\frac{a\tan(c+dx)}{d(a^2-b^2)(a+b\sec(c+dx))}-\frac{2b\text{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d\sqrt{a-b}\sqrt{a+b}(a^2-b^2)}$$

input
```
Int[Sec[c + d*x]^2/(a + b*Sec[c + d*x])^2,x]
```

output
```
(-2*b*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(Sqrt[a - b]*Sq
rt[a + b]*(a^2 - b^2)*d) + (a*Tan[c + d*x])/((a^2 - b^2)*d*(a + b*Sec[c +
d*x]))
```

3.500.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 4318
```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[1/b   Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4323
```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[a*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(f*(m + 1)*(
a^2 - b^2))), x] - Simp[1/((m + 1)*(a^2 - b^2))   Int[Csc[e + f*x]*(a + b*C
sc[e + f*x])^(m + 1)*(b*(m + 1) - a*(m + 2)*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

3.500.4 Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.39

method	result
derivativedivides	$\frac{-\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\left(a^2-b^2\right)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2b-a-b\right)}-\frac{2b\,\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{(a-b)(a+b)\sqrt{(a-b)(a+b)}}}{d}$
default	$\frac{-\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\left(a^2-b^2\right)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2b-a-b\right)}-\frac{2b\,\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{(a-b)(a+b)\sqrt{(a-b)(a+b)}}}{d}$
risch	$\frac{2i\left(b\,e^{i(dx+c)}+a\right)}{d(a^2-b^2)\left(a\,e^{2i(dx+c)}+2b\,e^{i(dx+c)}+a\right)}+\frac{b\ln\left(e^{i(dx+c)}+\frac{-ia^2+ib^2+b\sqrt{a^2-b^2}}{a\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,(a+b)(a-b)d}-\frac{b\ln\left(e^{i(dx+c)}+\frac{ia^2-ib^2+b\sqrt{a^2-b^2}}{\sqrt{a^2-b^2}\,a}\right)}{\sqrt{a^2-b^2}\,(a+b)(a-b)d}$

input
```
int(sec(d*x+c)^2/(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-2*a/(a^2-b^2)*tan(1/2*d*x+1/2*c)/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x
+1/2*c)^2*b-a-b)-2*b/(a-b)/(a+b)/((a-b)*(a+b))^(1/2)*arctanh((a-b)*tan(1/2
*d*x+1/2*c)/((a-b)*(a+b))^(1/2)))
```

3.500.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 329, normalized size of antiderivative = 3.87

$$\int \frac{\sec^2(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$
$$= \left[-\frac{(ab\cos(dx+c)+b^2)\sqrt{a^2-b^2}\log\left(\frac{2\,ab\cos(dx+c)-(a^2-2\,b^2)\cos(dx+c)^2+2\sqrt{a^2-b^2}(b\cos(dx+c)+a)\sin(dx+c)+2\,a^2-b^2}{a^2\cos(dx+c)^2+2\,ab\cos(dx+c)+b^2}\right)}{2\left((a^5-2\,a^3b^2+ab^4)d\cos(dx+c)+(a^4b-2\,a^2b^3+b^5)d\right)}\right.$$
$$\left.-\frac{(ab\cos(dx+c)+b^2)\sqrt{-a^2+b^2}\arctan\left(-\frac{\sqrt{-a^2+b^2}(b\cos(dx+c)+a)}{(a^2-b^2)\sin(dx+c)}\right)-(a^3-ab^2)\sin(dx+c)}{(a^5-2\,a^3b^2+ab^4)d\cos(dx+c)+(a^4b-2\,a^2b^3+b^5)d}\right]$$

input
```
integrate(sec(d*x+c)^2/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
[-1/2*((a*b*cos(d*x + c) + b^2)*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) -
(a^2 - 2*b^2)*cos(d*x + c)^2 + 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(
d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) -
 2*(a^3 - a*b^2)*sin(d*x + c))/((a^5 - 2*a^3*b^2 + a*b^4)*d*cos(d*x + c) +
 (a^4*b - 2*a^2*b^3 + b^5)*d), -((a*b*cos(d*x + c) + b^2)*sqrt(-a^2 + b^2)
*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c)))
 - (a^3 - a*b^2)*sin(d*x + c))/((a^5 - 2*a^3*b^2 + a*b^4)*d*cos(d*x + c) +
 (a^4*b - 2*a^2*b^3 + b^5)*d)]
```

3.500.6 Sympy [F]

$$\int \frac{\sec^2(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \int \frac{\sec^2{(c+dx)}}{(a+b\sec{(c+dx)})^2}\,dx$$

input
```
integrate(sec(d*x+c)**2/(a+b*sec(d*x+c))**2,x)
```

output
```
Integral(sec(c + d*x)**2/(a + b*sec(c + d*x))**2, x)
```

3.500.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\sec^2(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(sec(d*x+c)^2/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

3.500.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.76

$$\int \frac{\sec^2(c+dx)}{(a+b\sec(c+dx))^2}\,dx = -\frac{2\left(\frac{\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{-a^2+b^2}}\right)\right)b}{(a^2-b^2)\sqrt{-a^2+b^2}}+\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a-b\right)(a^2-b^2)}\right.}{d}$$

input
```
integrate(sec(d*x+c)^2/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
-2*((pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2
*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))*b/((a^2 - b^2)*
sqrt(-a^2 + b^2)) + a*tan(1/2*d*x + 1/2*c)/((a*tan(1/2*d*x + 1/2*c)^2 - b*
tan(1/2*d*x + 1/2*c)^2 - a - b)*(a^2 - b^2)))/d
```

3.500.9 Mupad [B] (verification not implemented)

Time = 14.33 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.08

$$\int \frac{\sec^2(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \frac{2\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,(a+b)\,(a-b)\,\left((b-a)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+a+b\right)} - \frac{2\,b\,\mathrm{atanh}\left(\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\sqrt{a-b}}{\sqrt{a+b}}\right)}{d\,(a+b)^{3/2}\,(a-b)^{3/2}}$$

input
```
int(1/(cos(c + d*x)^2*(a + b/cos(c + d*x))^2),x)
```

output
```
(2*a*tan(c/2 + (d*x)/2))/(d*(a + b)*(a - b)*(a + b - tan(c/2 + (d*x)/2)^2*
(a - b))) - (2*b*atanh((tan(c/2 + (d*x)/2)*(a - b)^(1/2))/(a + b)^(1/2)))/
(d*(a + b)^(3/2)*(a - b)^(3/2))
```

3.501 $\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

3.501.1 Optimal result . . . 3710
3.501.2 Mathematica [A] (verified) . . . 3710
3.501.3 Rubi [A] (verified) . . . 3711
3.501.4 Maple [A] (verified) . . . 3713
3.501.5 Fricas [A] (verification not implemented) . . . 3713
3.501.6 Sympy [F] . . . 3714
3.501.7 Maxima [F(-2)] . . . 3714
3.501.8 Giac [A] (verification not implemented) . . . 3715
3.501.9 Mupad [B] (verification not implemented) . . . 3715

3.501.1 Optimal result

Integrand size = 19, antiderivative size = 86

$$\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^2}\,dx=\frac{2a\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{3/2}(a+b)^{3/2}d}-\frac{b\tan(c+dx)}{(a^2-b^2)\,d(a+b\sec(c+dx))}$$

output
```
2*a*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/(a-b)^(3/2)/(a+b)^
(3/2)/d-b*tan(d*x+c)/(a^2-b^2)/d/(a+b*sec(d*x+c))
```

3.501.2 Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.97

$$\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^2}\,dx=\frac{-\frac{2a\operatorname{arctanh}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}}+\frac{b\sin(c+dx)}{(-a+b)(a+b)(b+a\cos(c+dx))}}{d}$$

input
```
Integrate[Sec[c + d*x]/(a + b*Sec[c + d*x])^2,x]
```

output
```
((-2*a*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(
3/2) + (b*Sin[c + d*x])/((-a + b)*(a + b)*(b + a*Cos[c + d*x])))/d
```

3.501.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.13, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474, Rules used = {3042, 4320, 25, 27, 3042, 4318, 3042, 3138, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \text{4320}$$

$$-\frac{\int -\frac{a\sec(c+dx)}{a+b\sec(c+dx)}dx}{a^2-b^2}-\frac{b\tan(c+dx)}{d\left(a^2-b^2\right)(a+b\sec(c+dx))}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{a\sec(c+dx)}{a+b\sec(c+dx)}dx}{a^2-b^2}-\frac{b\tan(c+dx)}{d\left(a^2-b^2\right)(a+b\sec(c+dx))}$$

$$\Big\downarrow \text{27}$$

$$\frac{a\int \frac{\sec(c+dx)}{a+b\sec(c+dx)}dx}{a^2-b^2}-\frac{b\tan(c+dx)}{d\left(a^2-b^2\right)(a+b\sec(c+dx))}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2-b^2}-\frac{b\tan(c+dx)}{d\left(a^2-b^2\right)(a+b\sec(c+dx))}$$

$$\Big\downarrow \text{4318}$$

$$\frac{a\int \frac{1}{\frac{a\cos(c+dx)}{b}+1}dx}{b\left(a^2-b^2\right)}-\frac{b\tan(c+dx)}{d\left(a^2-b^2\right)(a+b\sec(c+dx))}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a\int \frac{1}{\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{b}+1}dx}{b\left(a^2-b^2\right)}-\frac{b\tan(c+dx)}{d\left(a^2-b^2\right)(a+b\sec(c+dx))}$$

$$\Big\downarrow \text{3138}$$

$$\frac{2a\int \frac{1}{\left(1-\frac{a}{b}\right)\tan^2\left(\frac{1}{2}(c+dx)\right)+\frac{a+b}{b}}d\tan\left(\frac{1}{2}(c+dx)\right)}{bd\left(a^2-b^2\right)}-\frac{b\tan(c+dx)}{d\left(a^2-b^2\right)(a+b\sec(c+dx))}$$

$$\Big\downarrow \; 221$$

$$\frac{2a\text{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d\sqrt{a-b}\sqrt{a+b}\left(a^2-b^2\right)}-\frac{b\tan(c+dx)}{d\left(a^2-b^2\right)(a+b\sec(c+dx))}$$

input
```
Int[Sec[c + d*x]/(a + b*Sec[c + d*x])^2,x]
```

output
```
(2*a*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(Sqrt[a - b]*Sqr
t[a + b]*(a^2 - b^2)*d) - (b*Tan[c + d*x])/((a^2 - b^2)*d*(a + b*Sec[c + d
*x]))
```

3.501.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 4318
```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[1/b   Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4320
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[(-b)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(f*(m + 1)*
(a^2 - b^2))), x] + Simp[1/((m + 1)*(a^2 - b^2))   Int[Csc[e + f*x]*(a + b*
Csc[e + f*x])^(m + 1)*(a*(m + 1) - b*(m + 2)*Csc[e + f*x]), x], x] /; FreeQ
[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

3.501.4 Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.37

method	result
derivativedivides	$\frac{\frac{2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\left(a^2-b^2\right)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2b-a-b\right)}+\frac{2a\ \operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{(a-b)(a+b)\sqrt{(a-b)(a+b)}}}{d}$
default	$\frac{\frac{2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\left(a^2-b^2\right)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2b-a-b\right)}+\frac{2a\ \operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{(a-b)(a+b)\sqrt{(a-b)(a+b)}}}{d}$
risch	$-\frac{2ib\left(b\,\mathrm{e}^{i(dx+c)}+a\right)}{a(a^2-b^2)d\left(a\,\mathrm{e}^{2i(dx+c)}+2b\,\mathrm{e}^{i(dx+c)}+a\right)}+\frac{a\ln\left(\mathrm{e}^{i(dx+c)}+\frac{ia^2-ib^2+b\sqrt{a^2-b^2}}{\sqrt{a^2-b^2}\,a}\right)}{\sqrt{a^2-b^2}\,(a+b)(a-b)d}-\frac{a\ln\left(\mathrm{e}^{i(dx+c)}+\frac{-ia^2+ib^2+b\sqrt{a^2-}}{a\sqrt{a^2-b^2}}\right.}{\sqrt{a^2-b^2}\,(a+b)(a-b)d}$

input
```
int(sec(d*x+c)/(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(2*b/(a^2-b^2)*tan(1/2*d*x+1/2*c)/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+
1/2*c)^2*b-a-b)+2*a/(a-b)/(a+b)/((a-b)*(a+b))^(1/2)*arctanh((a-b)*tan(1/2*
d*x+1/2*c)/((a-b)*(a+b))^(1/2)))
```

3.501.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 332, normalized size of antiderivative = 3.86

$$\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$
$$= \left[-\frac{(a^2\cos(dx+c)+ab)\sqrt{a^2-b^2}\log\left(\frac{2\,ab\cos(dx+c)-(a^2-2\,b^2)\cos(dx+c)^2-2\sqrt{a^2-b^2}(b\cos(dx+c)+a)\sin(dx+c)+2\,a^2-b^2}{a^2\cos(dx+c)^2+2\,ab\cos(dx+c)+b^2}\right)}{2\left((a^5-2\,a^3b^2+ab^4)d\cos(dx+c)+(a^4b-2\,a^2b^3+b^5)d\right)}\right.$$

input
```
integrate(sec(d*x+c)/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
[-1/2*((a^2*cos(d*x + c) + a*b)*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) -
(a^2 - 2*b^2)*cos(d*x + c)^2 - 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(
d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) +
 2*(a^2*b - b^3)*sin(d*x + c))/((a^5 - 2*a^3*b^2 + a*b^4)*d*cos(d*x + c) +
 (a^4*b - 2*a^2*b^3 + b^5)*d), ((a^2*cos(d*x + c) + a*b)*sqrt(-a^2 + b^2)*
arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c)))
- (a^2*b - b^3)*sin(d*x + c))/((a^5 - 2*a^3*b^2 + a*b^4)*d*cos(d*x + c) +
(a^4*b - 2*a^2*b^3 + b^5)*d)]
```

3.501.6 Sympy [F]

$$\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \int \frac{\sec{(c+dx)}}{(a+b\sec{(c+dx)})^2}\,dx$$

input
```
integrate(sec(d*x+c)/(a+b*sec(d*x+c))**2,x)
```

output
```
Integral(sec(c + d*x)/(a + b*sec(c + d*x))**2, x)
```

3.501.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(sec(d*x+c)/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

3.501.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.74

$$\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^2}\,dx = -\frac{2\left(\frac{\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(2a-2b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{-a^2+b^2}}\right)\right)a}{(a^2-b^2)\sqrt{-a^2+b^2}}-\frac{b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a-b\right)(a^2-b^2)}\right)}{d}$$

input
```
integrate(sec(d*x+c)/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
-2*((pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(2*a - 2*b) + arctan((a*tan(1/2*d
*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))*a/((a^2 - b^2)*sq
rt(-a^2 + b^2)) - b*tan(1/2*d*x + 1/2*c)/((a*tan(1/2*d*x + 1/2*c)^2 - b*ta
n(1/2*d*x + 1/2*c)^2 - a - b)*(a^2 - b^2)))/d
```

3.501.9 Mupad [B] (verification not implemented)

Time = 13.58 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.07

$$\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \frac{2\,a\,\mathrm{atanh}\left(\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\sqrt{a-b}}{\sqrt{a+b}}\right)}{d\,(a+b)^{3/2}\,(a-b)^{3/2}} - \frac{2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,(a+b)\,(a-b)\,\left((b-a)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+a+b\right)}$$

input
```
int(1/(cos(c + d*x)*(a + b/cos(c + d*x))^2),x)
```

output
```
(2*a*atanh((tan(c/2 + (d*x)/2)*(a - b)^(1/2))/(a + b)^(1/2)))/(d*(a + b)^(
3/2)*(a - b)^(3/2)) - (2*b*tan(c/2 + (d*x)/2))/(d*(a + b)*(a - b)*(a + b -
 tan(c/2 + (d*x)/2)^2*(a - b)))
```

3.502 $\int \frac{1}{(a+b\sec(c+dx))^2}\,dx$

3.502.1 Optimal result . . . 3716
3.502.2 Mathematica [A] (verified) . . . 3716
3.502.3 Rubi [A] (verified) . . . 3717
3.502.4 Maple [A] (verified) . . . 3719
3.502.5 Fricas [B] (verification not implemented) . . . 3720
3.502.6 Sympy [F] . . . 3721
3.502.7 Maxima [F(-2)] . . . 3721
3.502.8 Giac [A] (verification not implemented) . . . 3722
3.502.9 Mupad [B] (verification not implemented) . . . 3722

3.502.1 Optimal result

Integrand size = 12, antiderivative size = 109

$$\int \frac{1}{(a+b\sec(c+dx))^2}\,dx = \frac{x}{a^2} - \frac{2b(2a^2-b^2)\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2(a-b)^{3/2}(a+b)^{3/2}d} + \frac{b^2\tan(c+dx)}{a\left(a^2-b^2\right)d(a+b\sec(c+dx))}$$

output
```
x/a^2-2*b*(2*a^2-b^2)*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/
a^2/(a-b)^(3/2)/(a+b)^(3/2)/d+b^2*tan(d*x+c)/a/(a^2-b^2)/d/(a+b*sec(d*x+c)
)
```

3.502.2 Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.27

$$\int \frac{1}{(a+b\sec(c+dx))^2}\,dx = \frac{-\frac{2b(-2a^2+b^2)\operatorname{arctanh}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} + \frac{a\left(a^2-b^2\right)(c+dx)\cos(c+dx)+b\left(\left(a^2-b^2\right)(c+dx)+ab\sin(c+dx)\right)}{b+a\cos(c+dx)}}{a^2(a-b)(a+b)d}$$

input
```
Integrate[(a + b*Sec[c + d*x])^(-2),x]
```

output
```
((-2*b*(-2*a^2 + b^2)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]]
)/Sqrt[a^2 - b^2] + (a*(a^2 - b^2)*(c + d*x)*Cos[c + d*x] + b*((a^2 - b^2)
*(c + d*x) + a*b*Sin[c + d*x]))/(b + a*Cos[c + d*x]))/(a^2*(a - b)*(a + b)
*d)
```

3.502.3 Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.23, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.833$, Rules used = {3042, 4272, 25, 3042, 4407, 3042, 4318, 3042, 3138, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+b\sec(c+dx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \text{4272}$$

$$\frac{b^2\tan(c+dx)}{ad\left(a^2-b^2\right)(a+b\sec(c+dx))}-\frac{\int -\frac{a^2-b\sec(c+dx)a-b^2}{a+b\sec(c+dx)}dx}{a\left(a^2-b^2\right)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{a^2-b\sec(c+dx)a-b^2}{a+b\sec(c+dx)}dx}{a\left(a^2-b^2\right)}+\frac{b^2\tan(c+dx)}{ad\left(a^2-b^2\right)(a+b\sec(c+dx))}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{a^2-b\csc\left(c+dx+\frac{\pi}{2}\right)a-b^2}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a\left(a^2-b^2\right)}+\frac{b^2\tan(c+dx)}{ad\left(a^2-b^2\right)(a+b\sec(c+dx))}$$

$$\Big\downarrow \text{4407}$$

$$\frac{\frac{x\left(a^2-b^2\right)}{a}-\frac{b\left(2a^2-b^2\right)\int \frac{\sec(c+dx)}{a+b\sec(c+dx)}dx}{a}}{a\left(a^2-b^2\right)}+\frac{b^2\tan(c+dx)}{ad\left(a^2-b^2\right)(a+b\sec(c+dx))}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{x(a^2-b^2)}{a}-\frac{b(2a^2-b^2)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a}}{a(a^2-b^2)}+\frac{b^2\tan(c+dx)}{ad(a^2-b^2)(a+b\sec(c+dx))}$$

$\Big\downarrow$ 4318

$$\frac{\frac{x(a^2-b^2)}{a}-\frac{(2a^2-b^2)\int \frac{1}{\frac{a\cos(c+dx)}{b}+1}dx}{a}}{a(a^2-b^2)}+\frac{b^2\tan(c+dx)}{ad(a^2-b^2)(a+b\sec(c+dx))}$$

$\Big\downarrow$ 3042

$$\frac{\frac{x(a^2-b^2)}{a}-\frac{(2a^2-b^2)\int \frac{1}{\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{b}+1}dx}{a}}{a(a^2-b^2)}+\frac{b^2\tan(c+dx)}{ad(a^2-b^2)(a+b\sec(c+dx))}$$

$\Big\downarrow$ 3138

$$\frac{\frac{x(a^2-b^2)}{a}-\frac{2(2a^2-b^2)\int \frac{1}{\left(1-\frac{a}{b}\right)\tan^2\left(\frac{1}{2}(c+dx)\right)+\frac{a+b}{b}}d\tan\left(\frac{1}{2}(c+dx)\right)}{ad}}{a(a^2-b^2)}+\frac{b^2\tan(c+dx)}{ad(a^2-b^2)(a+b\sec(c+dx))}$$

$\Big\downarrow$ 221

$$\frac{\frac{x(a^2-b^2)}{a}-\frac{2b(2a^2-b^2)\text{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{ad\sqrt{a-b}\sqrt{a+b}}}{a(a^2-b^2)}+\frac{b^2\tan(c+dx)}{ad(a^2-b^2)(a+b\sec(c+dx))}$$

input
```
Int[(a + b*Sec[c + d*x])^(-2),x]
```

output
```
(((a^2 - b^2)*x)/a - (2*b*(2*a^2 - b^2)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)
/2])/Sqrt[a + b]])/(a*Sqrt[a - b]*Sqrt[a + b]*d))/(a*(a^2 - b^2)) + (b^2*T
an[c + d*x])/(a*(a^2 - b^2)*d*(a + b*Sec[c + d*x]))
```

3.502.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 4272
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[b^2*Cot[
c + d*x]*((a + b*Csc[c + d*x])^(n + 1)/(a*d*(n + 1)*(a^2 - b^2))), x] + Sim
p[1/(a*(n + 1)*(a^2 - b^2))   Int[(a + b*Csc[c + d*x])^(n + 1)*Simp[(a^2 -
b^2)*(n + 1) - a*b*(n + 1)*Csc[c + d*x] + b^2*(n + 2)*Csc[c + d*x]^2, x], x
], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && LtQ[n, -1] && Integ
erQ[2*n]
```

rule 4318
```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[1/b   Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4407
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Simp[(b*c - a*d)/a   Int[Csc[e + f*
x]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
- a*d, 0]
```

3.502.4 Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.39

method	result
derivativedivides	$\frac{\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^2}+\frac{2b\left(-\frac{ab\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\left(a^2-b^2\right)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2b-a-b\right)}-\frac{\left(2a^2-b^2\right)\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{(a-b)(a+b)\sqrt{(a-b)(a+b)}}\right)}{a^2}}{d}$
default	$\frac{\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^2}+\frac{2b\left(-\frac{ab\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\left(a^2-b^2\right)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2b-a-b\right)}-\frac{\left(2a^2-b^2\right)\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{(a-b)(a+b)\sqrt{(a-b)(a+b)}}\right)}{a^2}}{d}$
risch	$\frac{x}{a^2}+\frac{2ib^2\left(b\,e^{i(dx+c)}+a\right)}{a^2(a^2-b^2)d\left(a\,e^{2i(dx+c)}+2b\,e^{i(dx+c)}+a\right)}+\frac{2b\ln\left(e^{i(dx+c)}-\frac{ia^2-ib^2-b\sqrt{a^2-b^2}}{\sqrt{a^2-b^2}\,a}\right)}{\sqrt{a^2-b^2}\,(a+b)(a-b)d}-\frac{b^3\ln\left(e^{i(dx+c)}-\frac{ia^2-ib^2-}{\sqrt{a^2}}\right.}{\sqrt{a^2-b^2}\,(a+b)(a-b}$

input
```
int(1/(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(2/a^2*arctan(tan(1/2*d*x+1/2*c))+2*b/a^2*(-a*b/(a^2-b^2)*tan(1/2*d*x+
1/2*c)/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)-(2*a^2-b^2)/(a-
b)/(a+b)/((a-b)*(a+b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a-b)*(a+b)
)^(1/2))))
```

3.502.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 212 vs. $2(100) = 200$.

Time = 0.31 (sec) , antiderivative size = 484, normalized size of antiderivative = 4.44

$$\int \frac{1}{(a+b\sec(c+dx))^2}\,dx$$
$$= \left[\frac{2\left(a^5-2\,a^3b^2+ab^4\right)dx\cos\left(dx+c\right)+2\left(a^4b-2\,a^2b^3+b^5\right)dx+\left(2\,a^2b^2-b^4+\left(2\,a^3b-ab^3\right)\cos\left(dx+c\right)\right.}{2\left(\left(a^7-2\,a^5b^2+a^3b^4\right)d\cos\left(\right.\right.}\right.$$

input
```
integrate(1/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
[1/2*(2*(a^5 - 2*a^3*b^2 + a*b^4)*d*x*cos(d*x + c) + 2*(a^4*b - 2*a^2*b^3
+ b^5)*d*x + (2*a^2*b^2 - b^4 + (2*a^3*b - a*b^3)*cos(d*x + c))*sqrt(a^2 -
 b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 - 2*sqrt(a^2
- b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^
2 + 2*a*b*cos(d*x + c) + b^2)) + 2*(a^3*b^2 - a*b^4)*sin(d*x + c))/((a^7 -
 2*a^5*b^2 + a^3*b^4)*d*cos(d*x + c) + (a^6*b - 2*a^4*b^3 + a^2*b^5)*d), (
(a^5 - 2*a^3*b^2 + a*b^4)*d*x*cos(d*x + c) + (a^4*b - 2*a^2*b^3 + b^5)*d*x
 - (2*a^2*b^2 - b^4 + (2*a^3*b - a*b^3)*cos(d*x + c))*sqrt(-a^2 + b^2)*arc
tan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c))) + (
a^3*b^2 - a*b^4)*sin(d*x + c))/((a^7 - 2*a^5*b^2 + a^3*b^4)*d*cos(d*x + c)
 + (a^6*b - 2*a^4*b^3 + a^2*b^5)*d)]
```

3.502.6 Sympy [F]

$$\int \frac{1}{(a+b\sec(c+dx))^2}\,dx = \int \frac{1}{\left(a+b\sec\left(c+dx\right)\right)^2}\,dx$$

input
```
integrate(1/(a+b*sec(d*x+c))**2,x)
```

output
```
Integral((a + b*sec(c + d*x))**(-2), x)
```

3.502.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(a+b\sec(c+dx))^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

3.502.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.64

$$\int \frac{1}{(a+b\sec(c+dx))^2}\,dx = -\frac{\frac{2\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{(a^3-ab^2)\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a-b\right)}+\frac{2\,(2\,a^2b-b^3)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx\right.}{\sqrt{-a^2+b^2}}\right.\right.}{(a^4-a^2b^2)\sqrt{-a^2+b^2}}}{d}$$

input
```
integrate(1/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
-(2*b^2*tan(1/2*d*x + 1/2*c)/((a^3 - a*b^2)*(a*tan(1/2*d*x + 1/2*c)^2 - b*
tan(1/2*d*x + 1/2*c)^2 - a - b)) + 2*(2*a^2*b - b^3)*(pi*floor(1/2*(d*x +
c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2
*d*x + 1/2*c))/sqrt(-a^2 + b^2)))/((a^4 - a^2*b^2)*sqrt(-a^2 + b^2)) - (d*
x + c)/a^2)/d
```

3.502.9 Mupad [B] (verification not implemented)

Time = 19.12 (sec) , antiderivative size = 2886, normalized size of antiderivative = 26.48

$$\int \frac{1}{(a+b\sec(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
int(1/(a + b/cos(c + d*x))^2,x)
```

output
```
(2*atan((((((32*(2*a^8*b - a^9 + a^4*b^5 - 3*a^6*b^3 + a^7*b^2))/(a^5*b +
a^6 - a^3*b^3 - a^4*b^2) - (tan(c/2 + (d*x)/2)*(2*a^9*b - 2*a^4*b^6 + 2*a^
5*b^5 + 4*a^6*b^4 - 4*a^7*b^3 - 2*a^8*b^2)*32i)/(a^2*(a^4*b + a^5 - a^2*b^
3 - a^3*b^2)))*1i)/a^2 + (32*tan(c/2 + (d*x)/2)*(a^6 - 2*a^5*b - 2*a*b^5 +
 2*b^6 - 5*a^2*b^4 + 4*a^3*b^3 + 3*a^4*b^2))/(a^4*b + a^5 - a^2*b^3 - a^3*
b^2))/a^2 - ((((32*(2*a^8*b - a^9 + a^4*b^5 - 3*a^6*b^3 + a^7*b^2))/(a^5*b
 + a^6 - a^3*b^3 - a^4*b^2) + (tan(c/2 + (d*x)/2)*(2*a^9*b - 2*a^4*b^6 + 2
*a^5*b^5 + 4*a^6*b^4 - 4*a^7*b^3 - 2*a^8*b^2)*32i)/(a^2*(a^4*b + a^5 - a^2
*b^3 - a^3*b^2)))*1i)/a^2 - (32*tan(c/2 + (d*x)/2)*(a^6 - 2*a^5*b - 2*a*b^
5 + 2*b^6 - 5*a^2*b^4 + 4*a^3*b^3 + 3*a^4*b^2))/(a^4*b + a^5 - a^2*b^3 - a
^3*b^2))/a^2)/((64*(2*a^4*b - a*b^4 + b^5 - 3*a^2*b^3 + 2*a^3*b^2))/(a^5*b
 + a^6 - a^3*b^3 - a^4*b^2) + (((((32*(2*a^8*b - a^9 + a^4*b^5 - 3*a^6*b^3
 + a^7*b^2))/(a^5*b + a^6 - a^3*b^3 - a^4*b^2) - (tan(c/2 + (d*x)/2)*(2*a^
9*b - 2*a^4*b^6 + 2*a^5*b^5 + 4*a^6*b^4 - 4*a^7*b^3 - 2*a^8*b^2)*32i)/(a^2
*(a^4*b + a^5 - a^2*b^3 - a^3*b^2)))*1i)/a^2 + (32*tan(c/2 + (d*x)/2)*(a^6
 - 2*a^5*b - 2*a*b^5 + 2*b^6 - 5*a^2*b^4 + 4*a^3*b^3 + 3*a^4*b^2))/(a^4*b
+ a^5 - a^2*b^3 - a^3*b^2))*1i)/a^2 + (((((32*(2*a^8*b - a^9 + a^4*b^5 - 3
*a^6*b^3 + a^7*b^2))/(a^5*b + a^6 - a^3*b^3 - a^4*b^2) + (tan(c/2 + (d*x)/
2)*(2*a^9*b - 2*a^4*b^6 + 2*a^5*b^5 + 4*a^6*b^4 - 4*a^7*b^3 - 2*a^8*b^2)*3
2i)/(a^2*(a^4*b + a^5 - a^2*b^3 - a^3*b^2)))*1i)/a^2 - (32*tan(c/2 + (d...
```

3.502. $\int \frac{1}{(a+b\sec(c+dx))^2}\,dx$

3.503 $\int \frac{\cos(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

3.503.1 Optimal result . . . 3724
3.503.2 Mathematica [A] (verified) . . . 3724
3.503.3 Rubi [A] (verified) . . . 3725
3.503.4 Maple [A] (verified) . . . 3728
3.503.5 Fricas [A] (verification not implemented) . . . 3729
3.503.6 Sympy [F] . . . 3729
3.503.7 Maxima [F(-2)] . . . 3730
3.503.8 Giac [B] (verification not implemented) . . . 3730
3.503.9 Mupad [B] (verification not implemented) . . . 3731

3.503.1 Optimal result

Integrand size = 19, antiderivative size = 146

$$\int \frac{\cos(c+dx)}{(a+b\sec(c+dx))^2}\,dx = -\frac{2bx}{a^3}+\frac{2b^2(3a^2-2b^2)\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3(a-b)^{3/2}(a+b)^{3/2}d}+\frac{(a^2-2b^2)\sin(c+dx)}{a^2\left(a^2-b^2\right)d}+\frac{b^2\sin(c+dx)}{a\left(a^2-b^2\right)d(a+b\sec(c+dx))}$$

output
```
-2*b*x/a^3+2*b^2*(3*a^2-2*b^2)*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b
)^(1/2))/a^3/(a-b)^(3/2)/(a+b)^(3/2)/d+(a^2-2*b^2)*sin(d*x+c)/a^2/(a^2-b^2
)/d+b^2*sin(d*x+c)/a/(a^2-b^2)/d/(a+b*sec(d*x+c))
```

3.503.2 Mathematica [A] (verified)

Time = 0.83 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.18

$$\int \frac{\cos(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \frac{\frac{4b^2(-3a^2+2b^2)\operatorname{arctanh}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}+\frac{-4ab(a^2-b^2)(c+dx)\cos(c+dx)+2ab(a^2-2b^2)\sin(c+dx)+(a^2-b^2)(-4b^2(c+dx)+a^2\sin}{b+a\cos(c+dx)}}{2a^3(a-b)(a+b)d}$$

input
```
Integrate[Cos[c + d*x]/(a + b*Sec[c + d*x])^2,x]
```

output
```
((4*b^2*(-3*a^2 + 2*b^2)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^
2]])/Sqrt[a^2 - b^2] + (-4*a*b*(a^2 - b^2)*(c + d*x)*Cos[c + d*x] + 2*a*b*
(a^2 - 2*b^2)*Sin[c + d*x] + (a^2 - b^2)*(-4*b^2*(c + d*x) + a^2*Sin[2*(c
+ d*x)]))/(b + a*Cos[c + d*x]))/(2*a^3*(a - b)*(a + b)*d)
```

3.503.3 Rubi [A] (verified)

Time = 0.94 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.14, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.632$, Rules used = {3042, 4334, 25, 3042, 4592, 3042, 4407, 3042, 4318, 3042, 3138, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(c+dx)}{(a+b\sec(c+dx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \text{4334}$$

$$\frac{b^2\sin(c+dx)}{ad\left(a^2-b^2\right)(a+b\sec(c+dx))}-\frac{\int -\frac{\cos(c+dx)\left(a^2-b\sec(c+dx)a-2b^2+b^2\sec^2(c+dx)\right)}{a+b\sec(c+dx)}dx}{a\left(a^2-b^2\right)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\cos(c+dx)\left(a^2-b\sec(c+dx)a-2b^2+b^2\sec^2(c+dx)\right)}{a+b\sec(c+dx)}dx}{a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)}{ad\left(a^2-b^2\right)(a+b\sec(c+dx))}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{a^2-b\csc\left(c+dx+\frac{\pi}{2}\right)a-2b^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)}{ad\left(a^2-b^2\right)(a+b\sec(c+dx))}$$

$$\Big\downarrow \text{4592}$$

$$\frac{\frac{\left(a^2-2b^2\right)\sin(c+dx)}{ad}-\frac{\int \frac{2b\left(a^2-b^2\right)-ab^2\sec(c+dx)}{a+b\sec(c+dx)}dx}{a}}{a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)}{ad\left(a^2-b^2\right)(a+b\sec(c+dx))}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\left(a^2-2b^2\right)\sin(c+dx)}{ad}-\frac{\int \frac{2b\left(a^2-b^2\right)-ab^2\csc\left(c+dx+\frac{\pi}{2}\right)}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a}}{a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)}{ad\left(a^2-b^2\right)(a+b\sec(c+dx))}$$

$\Big\downarrow$ 4407

$$\frac{\frac{\left(a^2-2b^2\right)\sin(c+dx)}{ad}-\frac{\frac{2bx\left(a^2-b^2\right)}{a}-\frac{b^2\left(3a^2-2b^2\right)\int \frac{\sec(c+dx)}{a+b\sec(c+dx)}dx}{a}}{a}}{a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)}{ad\left(a^2-b^2\right)(a+b\sec(c+dx))}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\left(a^2-2b^2\right)\sin(c+dx)}{ad}-\frac{\frac{2bx\left(a^2-b^2\right)}{a}-\frac{b^2\left(3a^2-2b^2\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a}}{a}}{a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)}{ad\left(a^2-b^2\right)(a+b\sec(c+dx))}$$

$\Big\downarrow$ 4318

$$\frac{\frac{\left(a^2-2b^2\right)\sin(c+dx)}{ad}-\frac{\frac{2bx\left(a^2-b^2\right)}{a}-\frac{b\left(3a^2-2b^2\right)\int \frac{1}{\frac{a\cos(c+dx)}{b}+1}dx}{a}}{a}}{a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)}{ad\left(a^2-b^2\right)(a+b\sec(c+dx))}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\left(a^2-2b^2\right)\sin(c+dx)}{ad}-\frac{\frac{2bx\left(a^2-b^2\right)}{a}-\frac{b\left(3a^2-2b^2\right)\int \frac{1}{\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{b}+1}dx}{a}}{a}}{a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)}{ad\left(a^2-b^2\right)(a+b\sec(c+dx))}$$

$\Big\downarrow$ 3138

$$\frac{\frac{\left(a^2-2b^2\right)\sin(c+dx)}{ad}-\frac{\frac{2bx\left(a^2-b^2\right)}{a}-\frac{2b\left(3a^2-2b^2\right)\int \frac{1}{\left(1-\frac{a}{b}\right)\tan^2\left(\frac{1}{2}(c+dx)\right)+\frac{a+b}{b}}d\tan\left(\frac{1}{2}(c+dx)\right)}{ad}}{a}}{a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)}{ad\left(a^2-b^2\right)(a+b\sec(c+dx))}$$

$\Big\downarrow$ 221

$$\frac{\frac{\left(a^2-2b^2\right)\sin(c+dx)}{ad}-\frac{\frac{2bx\left(a^2-b^2\right)}{a}-\frac{2b^2\left(3a^2-2b^2\right)\text{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{ad\sqrt{a-b}\sqrt{a+b}}}{a}}{a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)}{ad\left(a^2-b^2\right)(a+b\sec(c+dx))}$$

input
```
Int[Cos[c + d*x]/(a + b*Sec[c + d*x])^2,x]
```

3.503. $\int \frac{\cos(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

output
```
(b^2*Sin[c + d*x])/(a*(a^2 - b^2)*d*(a + b*Sec[c + d*x])) + (-(((2*b*(a^2
- b^2)*x)/a - (2*b^2*(3*a^2 - 2*b^2)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2]
)/Sqrt[a + b]])/(a*Sqrt[a - b]*Sqrt[a + b]*d))/a) + ((a^2 - 2*b^2)*Sin[c +
 d*x])/(a*d))/(a*(a^2 - b^2))
```

3.503.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 4318
```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[1/b   Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4334
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*
((d*Csc[e + f*x])^n/(a*f*(m + 1)*(a^2 - b^2))), x] + Simp[1/(a*(m + 1)*(a^2
 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a^2*(m + 1)
 - b^2*(m + n + 1) - a*b*(m + 1)*Csc[e + f*x] + b^2*(m + n + 2)*Csc[e + f*x
]^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0] && LtQ[m,
 -1] && IntegersQ[2*m, 2*n]
```

rule 4407
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Simp[(b*c - a*d)/a   Int[Csc[e + f*
x]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
- a*d, 0]
```

rule 4592
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Simp[1/(a*d*n)   Int[(a + b*Csc[e + f*x])^m
*(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*
Csc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

3.503.4 Maple [A] (verified)

Time = 0.75 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.26

method	result
derivativedivides	$\frac{-\frac{2\left(-\frac{a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}+2b\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}{a^3}-\frac{2b^2\left(-\frac{ab\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\left(a^2-b^2\right)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2b-a-b\right)}-\frac{\left(3a^2-2b^2\right)\arctan}{(a-b)(a-}\right.}{a^3}}{d}$
default	$\frac{-\frac{2\left(-\frac{a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}+2b\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}{a^3}-\frac{2b^2\left(-\frac{ab\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\left(a^2-b^2\right)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2b-a-b\right)}-\frac{\left(3a^2-2b^2\right)\arctan}{(a-b)(a-}\right.}{a^3}}{d}$
risch	$-\frac{2bx}{a^3}-\frac{ie^{i(dx+c)}}{2a^2d}+\frac{ie^{-i(dx+c)}}{2a^2d}-\frac{2ib^3\left(b\,e^{i(dx+c)}+a\right)}{a^3(a^2-b^2)d\left(a\,e^{2i(dx+c)}+2b\,e^{i(dx+c)}+a\right)}+\frac{3b^2\ln\left(e^{i(dx+c)}+\frac{ia^2-ib^2+b\sqrt{a^2-}}{\sqrt{a^2-b^2}\,a}\right.}{\sqrt{a^2-b^2}\,(a+b)(a-b)da}$

input
```
int(cos(d*x+c)/(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-2/a^3*(-a*tan(1/2*d*x+1/2*c)/(1+tan(1/2*d*x+1/2*c)^2)+2*b*arctan(tan
(1/2*d*x+1/2*c)))-2*b^2/a^3*(-a*b/(a^2-b^2)*tan(1/2*d*x+1/2*c)/(tan(1/2*d*
x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)-(3*a^2-2*b^2)/(a-b)/(a+b)/((a-b)*
(a+b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a-b)*(a+b))^(1/2))))
```

3.503.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 565, normalized size of antiderivative = 3.87

$$\int \frac{\cos(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$
$$= \left[-\frac{4\,(a^5b-2\,a^3b^3+ab^5)dx\cos\left(dx+c\right)+4\,(a^4b^2-2\,a^2b^4+b^6)dx-(3\,a^2b^3-2\,b^5+(3\,a^3b^2-2\,ab^4)\cos(}{2}\right.$$
$$-\frac{2\,(a^5b-2\,a^3b^3+ab^5)dx\cos\left(dx+c\right)+2\,(a^4b^2-2\,a^2b^4+b^6)dx-(3\,a^2b^3-2\,b^5+(3\,a^3b^2-2\,ab^4)\cos(}{(a^8-2\,a^6b^2+a^4b^4}$$

input
```
integrate(cos(d*x+c)/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
[-1/2*(4*(a^5*b - 2*a^3*b^3 + a*b^5)*d*x*cos(d*x + c) + 4*(a^4*b^2 - 2*a^2
*b^4 + b^6)*d*x - (3*a^2*b^3 - 2*b^5 + (3*a^3*b^2 - 2*a*b^4)*cos(d*x + c))
*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 +
2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*co
s(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) - 2*(a^5*b - 3*a^3*b^3 + 2*a*b^5
 + (a^6 - 2*a^4*b^2 + a^2*b^4)*cos(d*x + c))*sin(d*x + c))/((a^8 - 2*a^6*b
^2 + a^4*b^4)*d*cos(d*x + c) + (a^7*b - 2*a^5*b^3 + a^3*b^5)*d), -(2*(a^5*
b - 2*a^3*b^3 + a*b^5)*d*x*cos(d*x + c) + 2*(a^4*b^2 - 2*a^2*b^4 + b^6)*d*
x - (3*a^2*b^3 - 2*b^5 + (3*a^3*b^2 - 2*a*b^4)*cos(d*x + c))*sqrt(-a^2 + b
^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c
))) - (a^5*b - 3*a^3*b^3 + 2*a*b^5 + (a^6 - 2*a^4*b^2 + a^2*b^4)*cos(d*x +
 c))*sin(d*x + c))/((a^8 - 2*a^6*b^2 + a^4*b^4)*d*cos(d*x + c) + (a^7*b -
2*a^5*b^3 + a^3*b^5)*d)]
```

3.503.6 Sympy [F]

$$\int \frac{\cos(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \int \frac{\cos\left(c+dx\right)}{\left(a+b\sec\left(c+dx\right)\right)^2}\,dx$$

input
```
integrate(cos(d*x+c)/(a+b*sec(d*x+c))**2,x)
```

output
```
Integral(cos(c + d*x)/(a + b*sec(c + d*x))**2, x)
```

3.503.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\cos(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(cos(d*x+c)/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

3.503.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 837 vs. $2(137) = 274$.

Time = 0.37 (sec) , antiderivative size = 837, normalized size of antiderivative = 5.73

$$\int \frac{\cos(c+dx)}{(a+b\sec(c+dx))^2}\,dx =$$

$$-\frac{\frac{(2a^7b-5a^6b^2-4a^5b^3+9a^4b^4+2a^3b^5-4a^2b^6-2a^2b|-a^5+a^3b^2|-ab^2|-a^5+a^3b^2|+2b^3|-a^5+a^3b^2|)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor+\arctan\left(\frac{}{\sqrt{-\frac{a^4b-a^2b^3+}{}}}\right.\right.}{a^4b|-a^5+a^3b^2|-a^2b^3|-a^5+a^3b^2|+(a^5-a^3b^2)^2}}{}$$

input
```
integrate(cos(d*x+c)/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
-((2*a^7*b - 5*a^6*b^2 - 4*a^5*b^3 + 9*a^4*b^4 + 2*a^3*b^5 - 4*a^2*b^6 - 2
*a^2*b*abs(-a^5 + a^3*b^2) - a*b^2*abs(-a^5 + a^3*b^2) + 2*b^3*abs(-a^5 +
a^3*b^2))*(pi*floor(1/2*(d*x + c)/pi + 1/2) + arctan(tan(1/2*d*x + 1/2*c)/
sqrt(-(a^4*b - a^2*b^3 + sqrt((a^5 + a^4*b - a^3*b^2 - a^2*b^3)*(a^5 - a^4
*b - a^3*b^2 + a^2*b^3) + (a^4*b - a^2*b^3)^2))/(a^5 - a^4*b - a^3*b^2 + a
^2*b^3))))/(a^4*b*abs(-a^5 + a^3*b^2) - a^2*b^3*abs(-a^5 + a^3*b^2) + (a^5
 - a^3*b^2)^2) + ((2*a^2*b + a*b^2 - 2*b^3)*sqrt(-a^2 + b^2)*abs(-a^5 + a^
3*b^2)*abs(-a + b) + (2*a^7*b - 5*a^6*b^2 - 4*a^5*b^3 + 9*a^4*b^4 + 2*a^3*
b^5 - 4*a^2*b^6)*sqrt(-a^2 + b^2)*abs(-a + b))*(pi*floor(1/2*(d*x + c)/pi
+ 1/2) + arctan(tan(1/2*d*x + 1/2*c)/sqrt(-(a^4*b - a^2*b^3 - sqrt((a^5 +
a^4*b - a^3*b^2 - a^2*b^3)*(a^5 - a^4*b - a^3*b^2 + a^2*b^3) + (a^4*b - a^
2*b^3)^2))/(a^5 - a^4*b - a^3*b^2 + a^2*b^3))))/((a^5 - a^3*b^2)^2*(a^2 -
2*a*b + b^2) - (a^6*b - 2*a^5*b^2 + 2*a^3*b^4 - a^2*b^5)*abs(-a^5 + a^3*b^
2)) - 2*(a^3*tan(1/2*d*x + 1/2*c)^3 - a^2*b*tan(1/2*d*x + 1/2*c)^3 - a*b^2
*tan(1/2*d*x + 1/2*c)^3 + 2*b^3*tan(1/2*d*x + 1/2*c)^3 - a^3*tan(1/2*d*x +
 1/2*c) - a^2*b*tan(1/2*d*x + 1/2*c) + a*b^2*tan(1/2*d*x + 1/2*c) + 2*b^3*
tan(1/2*d*x + 1/2*c))/((a*tan(1/2*d*x + 1/2*c)^4 - b*tan(1/2*d*x + 1/2*c)^
4 - 2*b*tan(1/2*d*x + 1/2*c)^2 - a - b)*(a^4 - a^2*b^2)))/d
```

3.503.9 Mupad [B] (verification not implemented)

Time = 19.39 (sec) , antiderivative size = 3169, normalized size of antiderivative = 21.71

$$\int \frac{\cos(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
int(cos(c + d*x)/(a + b/cos(c + d*x))^2,x)
```

output
```
((2*tan(c/2 + (d*x)/2)^3*(a*b^2 + a^2*b - a^3 - 2*b^3))/(a^2*(a + b)*(a -
b)) - (2*tan(c/2 + (d*x)/2)*(a*b^2 - a^2*b - a^3 + 2*b^3))/(a^2*(a + b)*(a
 - b)))/(d*(a + b - tan(c/2 + (d*x)/2)^4*(a - b) + 2*b*tan(c/2 + (d*x)/2)^
2)) - (4*b*atan(((2*b*((32*tan(c/2 + (d*x)/2)*(8*b^8 - 8*a*b^7 - 16*a^2*b^
6 + 16*a^3*b^5 + 5*a^4*b^4 - 8*a^5*b^3 + 4*a^6*b^2))/(a^6*b + a^7 - a^4*b^
3 - a^5*b^2) + (b*((32*(2*a^11*b - 2*a^6*b^6 + a^7*b^5 + 5*a^8*b^4 - 3*a^9
*b^3 - 3*a^10*b^2))/(a^8*b + a^9 - a^6*b^3 - a^7*b^2) - (b*tan(c/2 + (d*x)
/2)*(2*a^11*b - 2*a^6*b^6 + 2*a^7*b^5 + 4*a^8*b^4 - 4*a^9*b^3 - 2*a^10*b^2
)*64i)/(a^3*(a^6*b + a^7 - a^4*b^3 - a^5*b^2)))*2i)/a^3))/a^3 + (2*b*((32*
tan(c/2 + (d*x)/2)*(8*b^8 - 8*a*b^7 - 16*a^2*b^6 + 16*a^3*b^5 + 5*a^4*b^4
- 8*a^5*b^3 + 4*a^6*b^2))/(a^6*b + a^7 - a^4*b^3 - a^5*b^2) - (b*((32*(2*a
^11*b - 2*a^6*b^6 + a^7*b^5 + 5*a^8*b^4 - 3*a^9*b^3 - 3*a^10*b^2))/(a^8*b
+ a^9 - a^6*b^3 - a^7*b^2) + (b*tan(c/2 + (d*x)/2)*(2*a^11*b - 2*a^6*b^6 +
 2*a^7*b^5 + 4*a^8*b^4 - 4*a^9*b^3 - 2*a^10*b^2)*64i)/(a^3*(a^6*b + a^7 -
a^4*b^3 - a^5*b^2)))*2i)/a^3))/a^3)/((64*(8*b^8 - 4*a*b^7 - 20*a^2*b^6 + 6
*a^3*b^5 + 12*a^4*b^4))/(a^8*b + a^9 - a^6*b^3 - a^7*b^2) - (b*((32*tan(c/
2 + (d*x)/2)*(8*b^8 - 8*a*b^7 - 16*a^2*b^6 + 16*a^3*b^5 + 5*a^4*b^4 - 8*a^
5*b^3 + 4*a^6*b^2))/(a^6*b + a^7 - a^4*b^3 - a^5*b^2) + (b*((32*(2*a^11*b
- 2*a^6*b^6 + a^7*b^5 + 5*a^8*b^4 - 3*a^9*b^3 - 3*a^10*b^2))/(a^8*b + a^9
- a^6*b^3 - a^7*b^2) - (b*tan(c/2 + (d*x)/2)*(2*a^11*b - 2*a^6*b^6 + 2*...
```

3.503. $\int \frac{\cos(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

3.504 $\int \frac{\cos^2(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

3.504.1 Optimal result . . . 3733
3.504.2 Mathematica [A] (verified) . . . 3733
3.504.3 Rubi [A] (verified) . . . 3734
3.504.4 Maple [A] (verified) . . . 3738
3.504.5 Fricas [A] (verification not implemented) . . . 3739
3.504.6 Sympy [F] . . . 3739
3.504.7 Maxima [F(-2)] . . . 3740
3.504.8 Giac [A] (verification not implemented) . . . 3740
3.504.9 Mupad [B] (verification not implemented) . . . 3741

3.504.1 Optimal result

Integrand size = 21, antiderivative size = 208

$$\int \frac{\cos^2(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \frac{(a^2+6b^2)\,x}{2a^4} - \frac{2b^3(4a^2-3b^2)\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^4(a-b)^{3/2}(a+b)^{3/2}d} - \frac{b(2a^2-3b^2)\sin(c+dx)}{a^3\,(a^2-b^2)\,d} + \frac{(a^2-3b^2)\cos(c+dx)\sin(c+dx)}{2a^2\,(a^2-b^2)\,d} + \frac{b^2\cos(c+dx)\sin(c+dx)}{a\,(a^2-b^2)\,d(a+b\sec(c+dx))}$$

output
```
1/2*(a^2+6*b^2)*x/a^4-2*b^3*(4*a^2-3*b^2)*arctanh((a-b)^(1/2)*tan(1/2*d*x+
1/2*c)/(a+b)^(1/2))/a^4/(a-b)^(3/2)/(a+b)^(3/2)/d-b*(2*a^2-3*b^2)*sin(d*x+
c)/a^3/(a^2-b^2)/d+1/2*(a^2-3*b^2)*cos(d*x+c)*sin(d*x+c)/a^2/(a^2-b^2)/d+b
^2*cos(d*x+c)*sin(d*x+c)/a/(a^2-b^2)/d/(a+b*sec(d*x+c))
```

3.504.2 Mathematica [A] (verified)

Time = 0.81 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.69

$$\int \frac{\cos^2(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \frac{2(a^2+6b^2)\,(c+dx) - \frac{8b^3(-4a^2+3b^2)\operatorname{arctanh}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}} - 8ab\sin(c+dx) + \frac{4ab^4\sin(c+dx)}{(a-b)(a+b)(b+a\cos(c+dx))} +}{4a^4d}$$

input
```
Integrate[Cos[c + d*x]^2/(a + b*Sec[c + d*x])^2,x]
```

output
```
(2*(a^2 + 6*b^2)*(c + d*x) - (8*b^3*(-4*a^2 + 3*b^2)*ArcTanh[((-a + b)*Tan
[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(3/2) - 8*a*b*Sin[c + d*x] +
(4*a*b^4*Sin[c + d*x])/((a - b)*(a + b)*(b + a*Cos[c + d*x])) + a^2*Sin[2*
(c + d*x)])/(4*a^4*d)
```

3.504.3 Rubi [A] (verified)

Time = 1.41 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.05, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {3042, 4334, 25, 3042, 4592, 3042, 4592, 3042, 4407, 3042, 4318, 3042, 3138, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^2(c+dx)}{(a+b\sec(c+dx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^2\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \text{4334}$$

$$\frac{b^2\sin(c+dx)\cos(c+dx)}{ad\left(a^2-b^2\right)(a+b\sec(c+dx))}-\frac{\int -\frac{\cos^2(c+dx)\left(a^2-b\sec(c+dx)a-3b^2+2b^2\sec^2(c+dx)\right)}{a+b\sec(c+dx)}dx}{a\left(a^2-b^2\right)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\cos^2(c+dx)\left(a^2-b\sec(c+dx)a-3b^2+2b^2\sec^2(c+dx)\right)}{a+b\sec(c+dx)}dx}{a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)\cos(c+dx)}{ad\left(a^2-b^2\right)(a+b\sec(c+dx))}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{a^2-b\csc\left(c+dx+\frac{\pi}{2}\right)a-3b^2+2b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^2\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)\cos(c+dx)}{ad\left(a^2-b^2\right)(a+b\sec(c+dx))}$$

$$\Big\downarrow \text{4592}$$

$$\frac{\frac{(a^2-3b^2)\sin(c+dx)\cos(c+dx)}{2ad}-\frac{\int\frac{\cos(c+dx)\left(-b\left(a^2-3b^2\right)\sec^2(c+dx)-a\left(a^2+b^2\right)\sec(c+dx)+2b\left(2a^2-3b^2\right)\right)}{a+b\sec(c+dx)}dx}{2a}}{a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)\cos(c+dx)}{ad\left(a^2-b^2\right)(a+b\sec(c+dx))}$$

$\downarrow$ 3042

$$\frac{\frac{(a^2-3b^2)\sin(c+dx)\cos(c+dx)}{2ad}-\frac{\int\frac{-b\left(a^2-3b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2-a\left(a^2+b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)+2b\left(2a^2-3b^2\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{2a}}{a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)\cos(c+dx)}{ad\left(a^2-b^2\right)(a+b\sec(c+dx))}$$

$\downarrow$ 4592

$$\frac{\frac{(a^2-3b^2)\sin(c+dx)\cos(c+dx)}{2ad}-\frac{\frac{2b\left(2a^2-3b^2\right)\sin(c+dx)}{ad}-\frac{\int\frac{a^4+5b^2a^2+b\left(a^2-3b^2\right)\sec(c+dx)a-6b^4}{a+b\sec(c+dx)}dx}{a}}{2a}}{a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)\cos(c+dx)}{ad\left(a^2-b^2\right)(a+b\sec(c+dx))}$$

$\downarrow$ 3042

$$\frac{\frac{(a^2-3b^2)\sin(c+dx)\cos(c+dx)}{2ad}-\frac{\frac{2b\left(2a^2-3b^2\right)\sin(c+dx)}{ad}-\frac{\int\frac{a^4+5b^2a^2+b\left(a^2-3b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)a-6b^4}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a}}{2a}}{a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)\cos(c+dx)}{ad\left(a^2-b^2\right)(a+b\sec(c+dx))}$$

$\downarrow$ 4407

$$\frac{\frac{(a^2-3b^2)\sin(c+dx)\cos(c+dx)}{2ad}-\frac{\frac{2b\left(2a^2-3b^2\right)\sin(c+dx)}{ad}-\frac{\frac{x\left(a^4+5a^2b^2-6b^4\right)}{a}-2b^3\left(4a-\frac{3b^2}{a}\right)\int\frac{\sec(c+dx)}{a+b\sec(c+dx)}dx}{a}}{2a}}{a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)\cos(c+dx)}{ad\left(a^2-b^2\right)(a+b\sec(c+dx))}$$

$\downarrow$ 3042

$$\frac{\frac{(a^2-3b^2)\sin(c+dx)\cos(c+dx)}{2ad}-\frac{\frac{2b\left(2a^2-3b^2\right)\sin(c+dx)}{ad}-\frac{\frac{x\left(a^4+5a^2b^2-6b^4\right)}{a}-2b^3\left(4a-\frac{3b^2}{a}\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a}}{2a}}{a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)\cos(c+dx)}{ad\left(a^2-b^2\right)(a+b\sec(c+dx))}$$

$\downarrow$ 4318

3.504. $\int \frac{\cos^2(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

$$\frac{\frac{(a^2-3b^2)\sin(c+dx)\cos(c+dx)}{2ad}-\frac{\frac{2b\left(2a^2-3b^2\right)\sin(c+dx)}{ad}-\frac{\frac{x\left(a^4+5a^2b^2-6b^4\right)}{a}-2b^2\left(4a-\frac{3b^2}{a}\right)\int\frac{1}{\frac{a\cos(c+dx)}{b}+1}dx}{a}}{2a}}{a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)\cos(c+dx)}{ad\left(a^2-b^2\right)(a+b\sec(c+dx))}$$

$\Big\downarrow$ 3042

$$\frac{\frac{(a^2-3b^2)\sin(c+dx)\cos(c+dx)}{2ad}-\frac{\frac{2b\left(2a^2-3b^2\right)\sin(c+dx)}{ad}-\frac{\frac{x\left(a^4+5a^2b^2-6b^4\right)}{a}-2b^2\left(4a-\frac{3b^2}{a}\right)\int\frac{1}{\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{b}+1}dx}{a}}{2a}}{a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)\cos(c+dx)}{ad\left(a^2-b^2\right)(a+b\sec(c+dx))}$$

$\Big\downarrow$ 3138

$$\frac{\frac{(a^2-3b^2)\sin(c+dx)\cos(c+dx)}{2ad}-\frac{\frac{2b\left(2a^2-3b^2\right)\sin(c+dx)}{ad}-\frac{\frac{x\left(a^4+5a^2b^2-6b^4\right)}{a}-\frac{4b^2\left(4a-\frac{3b^2}{a}\right)\int\frac{1}{\left(1-\frac{a}{b}\right)\tan^2\left(\frac{1}{2}(c+dx)\right)+\frac{a+b}{b}}d\tan\left(\frac{1}{2}(c+dx)\right)}{d}}{a}}{2a}}{a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)\cos(c+dx)}{ad\left(a^2-b^2\right)(a+b\sec(c+dx))}$$

$\Big\downarrow$ 221

$$\frac{b^2\sin(c+dx)\cos(c+dx)}{ad\left(a^2-b^2\right)(a+b\sec(c+dx))}+\frac{\frac{(a^2-3b^2)\sin(c+dx)\cos(c+dx)}{2ad}-\frac{\frac{2b\left(2a^2-3b^2\right)\sin(c+dx)}{ad}-\frac{\frac{x\left(a^4+5a^2b^2-6b^4\right)}{a}-\frac{4b^3\left(4a-\frac{3b^2}{a}\right)\text{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d\sqrt{a-b}\sqrt{a+b}}}{a}}{2a}}{a\left(a^2-b^2\right)}$$

input
```
Int[Cos[c + d*x]^2/(a + b*Sec[c + d*x])^2,x]
```

output
```
(b^2*Cos[c + d*x]*Sin[c + d*x])/(a*(a^2 - b^2)*d*(a + b*Sec[c + d*x])) + (
((a^2 - 3*b^2)*Cos[c + d*x]*Sin[c + d*x])/(2*a*d) - (-((((a^4 + 5*a^2*b^2
- 6*b^4)*x)/a - (4*b^3*(4*a - (3*b^2)/a)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x
)/2])/Sqrt[a + b]])/(Sqrt[a - b]*Sqrt[a + b]*d))/a) + (2*b*(2*a^2 - 3*b^2)
*Sin[c + d*x])/(a*d))/(2*a))/(a*(a^2 - b^2))
```

3.504. $\int \frac{\cos^2(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

3.504.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 4318
```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[1/b   Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4334
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*
((d*Csc[e + f*x])^n/(a*f*(m + 1)*(a^2 - b^2))), x] + Simp[1/(a*(m + 1)*(a^2
 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a^2*(m + 1)
 - b^2*(m + n + 1) - a*b*(m + 1)*Csc[e + f*x] + b^2*(m + n + 2)*Csc[e + f*x
]^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0] && LtQ[m,
 -1] && IntegersQ[2*m, 2*n]
```

rule 4407
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Simp[(b*c - a*d)/a   Int[Csc[e + f*
x]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
- a*d, 0]
```

3.504. $\int \frac{\cos^2(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

rule 4592
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Simp[1/(a*d*n)   Int[(a + b*Csc[e + f*x])^m
*(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*
Csc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

3.504.4 Maple [A] (verified)

Time = 0.86 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.08

method	result
derivativedivides	$\frac{\frac{2\left(\left(-\frac{1}{2}a^2-2ab\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3+\left(\frac{1}{2}a^2-2ab\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^2}+\left(a^2+6b^2\right)\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^4}+\frac{2b^3\left(-\frac{ab\tan\left(\frac{dx}{2}\right.}{\left(a^2-b^2\right)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-\right.}\right.}{d}$
default	$\frac{\frac{2\left(\left(-\frac{1}{2}a^2-2ab\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3+\left(\frac{1}{2}a^2-2ab\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^2}+\left(a^2+6b^2\right)\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^4}+\frac{2b^3\left(-\frac{ab\tan\left(\frac{dx}{2}\right.}{\left(a^2-b^2\right)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-\right.}\right.}{d}$
risch	$\frac{x}{2a^2}+\frac{3xb^2}{a^4}-\frac{i\mathrm{e}^{2i(dx+c)}}{8a^2d}+\frac{ib\,\mathrm{e}^{i(dx+c)}}{a^3d}-\frac{ib\,\mathrm{e}^{-i(dx+c)}}{a^3d}+\frac{i\mathrm{e}^{-2i(dx+c)}}{8a^2d}+\frac{2ib^4\left(b\,\mathrm{e}^{i(dx+c)}+a\right)}{a^4(a^2-b^2)d\left(a\,\mathrm{e}^{2i(dx+c)}+2b\,\mathrm{e}^{i(dx+c)}+\right.}$

input
```
int(cos(d*x+c)^2/(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(2/a^4*(((-1/2*a^2-2*a*b)*tan(1/2*d*x+1/2*c)^3+(1/2*a^2-2*a*b)*tan(1/2
*d*x+1/2*c))/(1+tan(1/2*d*x+1/2*c)^2)^2+1/2*(a^2+6*b^2)*arctan(tan(1/2*d*x
+1/2*c)))+2*b^3/a^4*(-a*b/(a^2-b^2)*tan(1/2*d*x+1/2*c)/(tan(1/2*d*x+1/2*c)
^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)-(4*a^2-3*b^2)/(a-b)/(a+b)/((a-b)*(a+b))^(
1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a-b)*(a+b))^(1/2))))
```

3.504.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 660, normalized size of antiderivative = 3.17

$$\int \frac{\cos^2(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$
$$= \left[\frac{(a^7+4\,a^5b^2-11\,a^3b^4+6\,ab^6)dx\cos(dx+c)+(a^6b+4\,a^4b^3-11\,a^2b^5+6\,b^7)dx+(4\,a^2b^4-3\,b^6+(4\,a^{\cdots}}{}\right.$$

input
```
integrate(cos(d*x+c)^2/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
[1/2*((a^7 + 4*a^5*b^2 - 11*a^3*b^4 + 6*a*b^6)*d*x*cos(d*x + c) + (a^6*b +
 4*a^4*b^3 - 11*a^2*b^5 + 6*b^7)*d*x + (4*a^2*b^4 - 3*b^6 + (4*a^3*b^3 - 3
*a*b^5)*cos(d*x + c))*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b
^2)*cos(d*x + c)^2 - 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) +
 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) - (4*a^5*b^
2 - 10*a^3*b^4 + 6*a*b^6 - (a^7 - 2*a^5*b^2 + a^3*b^4)*cos(d*x + c)^2 + 3*
(a^6*b - 2*a^4*b^3 + a^2*b^5)*cos(d*x + c))*sin(d*x + c))/((a^9 - 2*a^7*b^
2 + a^5*b^4)*d*cos(d*x + c) + (a^8*b - 2*a^6*b^3 + a^4*b^5)*d), 1/2*((a^7
+ 4*a^5*b^2 - 11*a^3*b^4 + 6*a*b^6)*d*x*cos(d*x + c) + (a^6*b + 4*a^4*b^3
- 11*a^2*b^5 + 6*b^7)*d*x - 2*(4*a^2*b^4 - 3*b^6 + (4*a^3*b^3 - 3*a*b^5)*c
os(d*x + c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a
)/((a^2 - b^2)*sin(d*x + c))) - (4*a^5*b^2 - 10*a^3*b^4 + 6*a*b^6 - (a^7 -
 2*a^5*b^2 + a^3*b^4)*cos(d*x + c)^2 + 3*(a^6*b - 2*a^4*b^3 + a^2*b^5)*cos
(d*x + c))*sin(d*x + c))/((a^9 - 2*a^7*b^2 + a^5*b^4)*d*cos(d*x + c) + (a^
8*b - 2*a^6*b^3 + a^4*b^5)*d)]
```

3.504.6 Sympy [F]

$$\int \frac{\cos^2(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \int \frac{\cos^2(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$

input
```
integrate(cos(d*x+c)**2/(a+b*sec(d*x+c))**2,x)
```

output
```
Integral(cos(c + d*x)**2/(a + b*sec(c + d*x))**2, x)
```

3.504.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\cos^2(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(cos(d*x+c)^2/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

3.504.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.27

$$\int \frac{\cos^2(c+dx)}{(a+b\sec(c+dx))^2}\,dx = -\frac{\frac{4\,b^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(a^5-a^3b^2\right)\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a-b\right)}+\frac{4\left(4\,a^2b^3-3\,b^5\right)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\right.}{\sqrt{-a^2+b^2}}\right.\right.}{\left(a^6-a^4b^2\right)\sqrt{-a^2+b^2}}}{2\,d}$$

input
```
integrate(cos(d*x+c)^2/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
-1/2*(4*b^4*tan(1/2*d*x + 1/2*c)/((a^5 - a^3*b^2)*(a*tan(1/2*d*x + 1/2*c)^
2 - b*tan(1/2*d*x + 1/2*c)^2 - a - b)) + 4*(4*a^2*b^3 - 3*b^5)*(pi*floor(1
/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) -
 b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))/((a^6 - a^4*b^2)*sqrt(-a^2 + b
^2)) - (a^2 + 6*b^2)*(d*x + c)/a^4 + 2*(a*tan(1/2*d*x + 1/2*c)^3 + 4*b*tan
(1/2*d*x + 1/2*c)^3 - a*tan(1/2*d*x + 1/2*c) + 4*b*tan(1/2*d*x + 1/2*c))/(
(tan(1/2*d*x + 1/2*c)^2 + 1)^2*a^3))/d
```

3.504.9 Mupad [B] (verification not implemented)

Time = 20.52 (sec) , antiderivative size = 3738, normalized size of antiderivative = 17.97

$$\int \frac{\cos^2(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
int(cos(c + d*x)^2/(a + b/cos(c + d*x))^2,x)
```

output
```
(atan(((((8*tan(c/2 + (d*x)/2)*(a^10 - 2*a^9*b - 72*a*b^9 + 72*b^10 - 120*
a^2*b^8 + 120*a^3*b^7 + 17*a^4*b^6 - 26*a^5*b^5 + 23*a^6*b^4 - 20*a^7*b^3
+ 11*a^8*b^2))/(a^8*b + a^9 - a^6*b^3 - a^7*b^2) + ((a^2*1i + b^2*6i)*((8*
(2*a^15 - 12*a^8*b^7 + 6*a^9*b^6 + 28*a^10*b^5 - 14*a^11*b^4 - 16*a^12*b^3
 + 6*a^13*b^2))/(a^11*b + a^12 - a^9*b^3 - a^10*b^2) - (4*tan(c/2 + (d*x)/
2)*(a^2*1i + b^2*6i)*(8*a^13*b - 8*a^8*b^6 + 8*a^9*b^5 + 16*a^10*b^4 - 16*
a^11*b^3 - 8*a^12*b^2))/(a^4*(a^8*b + a^9 - a^6*b^3 - a^7*b^2))))/(2*a^4))
*(a^2*1i + b^2*6i)*1i)/(2*a^4) + (((8*tan(c/2 + (d*x)/2)*(a^10 - 2*a^9*b -
 72*a*b^9 + 72*b^10 - 120*a^2*b^8 + 120*a^3*b^7 + 17*a^4*b^6 - 26*a^5*b^5
+ 23*a^6*b^4 - 20*a^7*b^3 + 11*a^8*b^2))/(a^8*b + a^9 - a^6*b^3 - a^7*b^2)
 - ((a^2*1i + b^2*6i)*((8*(2*a^15 - 12*a^8*b^7 + 6*a^9*b^6 + 28*a^10*b^5 -
 14*a^11*b^4 - 16*a^12*b^3 + 6*a^13*b^2))/(a^11*b + a^12 - a^9*b^3 - a^10*
b^2) + (4*tan(c/2 + (d*x)/2)*(a^2*1i + b^2*6i)*(8*a^13*b - 8*a^8*b^6 + 8*a
^9*b^5 + 16*a^10*b^4 - 16*a^11*b^3 - 8*a^12*b^2))/(a^4*(a^8*b + a^9 - a^6*
b^3 - a^7*b^2))))/(2*a^4))*(a^2*1i + b^2*6i)*1i)/(2*a^4))/((16*(108*b^11 -
 54*a*b^10 - 216*a^2*b^9 + 81*a^3*b^8 + 63*a^4*b^7 - 9*a^5*b^6 + 41*a^6*b^
5 - 4*a^7*b^4 + 4*a^8*b^3))/(a^11*b + a^12 - a^9*b^3 - a^10*b^2) - (((8*ta
n(c/2 + (d*x)/2)*(a^10 - 2*a^9*b - 72*a*b^9 + 72*b^10 - 120*a^2*b^8 + 120*
a^3*b^7 + 17*a^4*b^6 - 26*a^5*b^5 + 23*a^6*b^4 - 20*a^7*b^3 + 11*a^8*b^2))
/(a^8*b + a^9 - a^6*b^3 - a^7*b^2) + ((a^2*1i + b^2*6i)*((8*(2*a^15 - 1...
```

3.505 $\int \frac{\cos^3(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

3.505.1 Optimal result . . . 3742
3.505.2 Mathematica [C] (verified) . . . 3743
3.505.3 Rubi [A] (verified) . . . 3743
3.505.4 Maple [A] (verified) . . . 3748
3.505.5 Fricas [A] (verification not implemented) . . . 3749
3.505.6 Sympy [F(-1)] . . . 3750
3.505.7 Maxima [F(-2)] . . . 3750
3.505.8 Giac [A] (verification not implemented) . . . 3751
3.505.9 Mupad [B] (verification not implemented) . . . 3751

3.505.1 Optimal result

Integrand size = 21, antiderivative size = 261

$$\begin{aligned}\int \frac{\cos^3(c+dx)}{(a+b\sec(c+dx))^2}\,dx &= -\frac{b(a^2+4b^2)\,x}{a^5}+\frac{2b^4(5a^2-4b^2)\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^5(a-b)^{3/2}(a+b)^{3/2}d}\\ &\quad+\frac{(2a^4+7a^2b^2-12b^4)\sin(c+dx)}{3a^4\,(a^2-b^2)\,d}\\ &\quad-\frac{b(a^2-2b^2)\cos(c+dx)\sin(c+dx)}{a^3\,(a^2-b^2)\,d}\\ &\quad+\frac{(a^2-4b^2)\cos^2(c+dx)\sin(c+dx)}{3a^2\,(a^2-b^2)\,d}\\ &\quad+\frac{b^2\cos^2(c+dx)\sin(c+dx)}{a\,(a^2-b^2)\,d(a+b\sec(c+dx))}\end{aligned}$$

output
```
-b*(a^2+4*b^2)*x/a^5+2*b^4*(5*a^2-4*b^2)*arctanh((a-b)^(1/2)*tan(1/2*d*x+1
/2*c)/(a+b)^(1/2))/a^5/(a-b)^(3/2)/(a+b)^(3/2)/d+1/3*(2*a^4+7*a^2*b^2-12*b
^4)*sin(d*x+c)/a^4/(a^2-b^2)/d-b*(a^2-2*b^2)*cos(d*x+c)*sin(d*x+c)/a^3/(a^
2-b^2)/d+1/3*(a^2-4*b^2)*cos(d*x+c)^2*sin(d*x+c)/a^2/(a^2-b^2)/d+b^2*cos(d
*x+c)^2*sin(d*x+c)/a/(a^2-b^2)/d/(a+b*sec(d*x+c))
```

3.505.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.06 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.67

$$\int \frac{\cos^3(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \frac{-12b(-ia+2b)(ia+2b)(c+dx)+\frac{24b^4(-5a^2+4b^2)\text{arctanh}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}}+9a(a^2+4b^2)\sin(c+dx)+}{12a^5d}$$

input
```
Integrate[Cos[c + d*x]^3/(a + b*Sec[c + d*x])^2,x]
```

output
```
(-12*b*((-I)*a + 2*b)*(I*a + 2*b)*(c + d*x) + (24*b^4*(-5*a^2 + 4*b^2)*Arc
Tanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(3/2) + 9*a
*(a^2 + 4*b^2)*Sin[c + d*x] + (12*a*b^5*Sin[c + d*x])/((-a + b)*(a + b)*(b
 + a*Cos[c + d*x])) - 6*a^2*b*Sin[2*(c + d*x)] + a^3*Sin[3*(c + d*x)])/(12
*a^5*d)
```

3.505.3 Rubi [A] (verified)

Time = 1.92 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.03, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.857$, Rules used = {3042, 4334, 25, 3042, 4592, 3042, 4592, 27, 3042, 4592, 27, 3042, 4407, 3042, 4318, 3042, 3138, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^3(c+dx)}{(a+b\sec(c+dx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^3\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \text{4334}$$

$$\frac{b^2\sin(c+dx)\cos^2(c+dx)}{ad\left(a^2-b^2\right)(a+b\sec(c+dx))}-\frac{\int -\frac{\cos^3(c+dx)\left(a^2-b\sec(c+dx)a-4b^2+3b^2\sec^2(c+dx)\right)}{a+b\sec(c+dx)}dx}{a\left(a^2-b^2\right)}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{\cos^3(c+dx)\left(a^2-b\sec(c+dx)a-4b^2+3b^2\sec^2(c+dx)\right)}{a+b\sec(c+dx)}dx}{a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)\cos^2(c+dx)}{ad\left(a^2-b^2\right)(a+b\sec(c+dx))}$$

$$\Big\downarrow \; 3042$$

$$\frac{\int \frac{a^2-b\csc\left(c+dx+\frac{\pi}{2}\right)a-4b^2+3b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^3\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)\cos^2(c+dx)}{ad\left(a^2-b^2\right)(a+b\sec(c+dx))}$$

$$\Big\downarrow \; 4592$$

$$\frac{\frac{\left(a^2-4b^2\right)\sin(c+dx)\cos^2(c+dx)}{3ad}-\frac{\int \frac{\cos^2(c+dx)\left(-2b\left(a^2-4b^2\right)\sec^2(c+dx)-a\left(2a^2+b^2\right)\sec(c+dx)+6b\left(a^2-2b^2\right)\right)}{a+b\sec(c+dx)}dx}{3a}}{a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)\cos^2(c+dx)}{ad\left(a^2-b^2\right)(a+b\sec(c+dx))}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{\left(a^2-4b^2\right)\sin(c+dx)\cos^2(c+dx)}{3ad}-\frac{\int \frac{-2b\left(a^2-4b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2-a\left(2a^2+b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)+6b\left(a^2-2b^2\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^2\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{3a}}{a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)\cos^2(c+dx)}{ad\left(a^2-b^2\right)(a+b\sec(c+dx))}$$

$$\Big\downarrow \; 4592$$

$$\frac{\frac{\left(a^2-4b^2\right)\sin(c+dx)\cos^2(c+dx)}{3ad}-\frac{\frac{3b\left(a^2-2b^2\right)\sin(c+dx)\cos(c+dx)}{ad}-\frac{\int \frac{2\cos(c+dx)\left(2a^4+7b^2a^2-b\left(a^2+2b^2\right)\sec(c+dx)a-12b^4-3b^2\left(a^2-2b^2\right)\sec^2(c+dx)\right)}{a+b\sec(c+dx)}}{2a}}{3a}}{a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)\cos^2(c+dx)}{ad\left(a^2-b^2\right)(a+b\sec(c+dx))}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\left(a^2-4b^2\right)\sin(c+dx)\cos^2(c+dx)}{3ad}-\frac{\frac{3b\left(a^2-2b^2\right)\sin(c+dx)\cos(c+dx)}{ad}-\frac{\int \frac{\cos(c+dx)\left(2a^4+7b^2a^2-b\left(a^2+2b^2\right)\sec(c+dx)a-12b^4-3b^2\left(a^2-2b^2\right)\sec^2(c+dx)\right)}{a+b\sec(c+dx)}}{a}}{3a}}{a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)\cos^2(c+dx)}{ad\left(a^2-b^2\right)(a+b\sec(c+dx))}$$

$$\Big\downarrow \; 3042$$

3.505. $\int \frac{\cos^3(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

$$\frac{\frac{\left(a^2-4b^2\right)\sin(c+dx)\cos^2(c+dx)}{3ad}-\frac{\frac{3b\left(a^2-2b^2\right)\sin(c+dx)\cos(c+dx)}{ad}-\frac{\int\frac{2a^4+7b^2a^2-b\left(a^2+2b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)a-12b^4-3b^2\left(a^2-2b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a}}{3a}}{a\left(a^2-b^2\right)}$$

$$\frac{b^2\sin(c+dx)\cos^2(c+dx)}{ad\left(a^2-b^2\right)(a+b\sec(c+dx))}$$

$\downarrow$ 4592

$$\frac{\frac{\left(a^2-4b^2\right)\sin(c+dx)\cos^2(c+dx)}{3ad}-\frac{\frac{3b\left(a^2-2b^2\right)\sin(c+dx)\cos(c+dx)}{ad}-\frac{\frac{\left(2a^4+7a^2b^2-12b^4\right)\sin(c+dx)}{ad}-\frac{\int\frac{3\left(a\left(a^2-2b^2\right)\sec(c+dx)b^2+\left(a^4+3b^2a^2-4b^4\right)b\right)}{a+b\sec(c+dx)}}{a}}{a}}{3a}}{a\left(a^2-b^2\right)}$$

$$\frac{b^2\sin(c+dx)\cos^2(c+dx)}{ad\left(a^2-b^2\right)(a+b\sec(c+dx))}$$

$\downarrow$ 27

$$\frac{\frac{\left(a^2-4b^2\right)\sin(c+dx)\cos^2(c+dx)}{3ad}-\frac{\frac{3b\left(a^2-2b^2\right)\sin(c+dx)\cos(c+dx)}{ad}-\frac{\frac{\left(2a^4+7a^2b^2-12b^4\right)\sin(c+dx)}{ad}-\frac{3\int\frac{a\left(a^2-2b^2\right)\sec(c+dx)b^2+\left(a^4+3b^2a^2-4b^4\right)b}{a+b\sec(c+dx)}dx}{a}}{a}}{3a}}{a\left(a^2-b^2\right)}$$

$$\frac{b^2\sin(c+dx)\cos^2(c+dx)}{ad\left(a^2-b^2\right)(a+b\sec(c+dx))}$$

$\downarrow$ 3042

$$\frac{\frac{\left(a^2-4b^2\right)\sin(c+dx)\cos^2(c+dx)}{3ad}-\frac{\frac{3b\left(a^2-2b^2\right)\sin(c+dx)\cos(c+dx)}{ad}-\frac{\frac{\left(2a^4+7a^2b^2-12b^4\right)\sin(c+dx)}{ad}-\frac{3\int\frac{a\left(a^2-2b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)b^2+\left(a^4+3b^2a^2-4b^4\right)}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}{a}}{a}}{3a}}{a\left(a^2-b^2\right)}$$

$$\frac{b^2\sin(c+dx)\cos^2(c+dx)}{ad\left(a^2-b^2\right)(a+b\sec(c+dx))}$$

$\downarrow$ 4407

$$\frac{\frac{\left(a^2-4b^2\right)\sin(c+dx)\cos^2(c+dx)}{3ad}-\frac{\frac{3b\left(a^2-2b^2\right)\sin(c+dx)\cos(c+dx)}{ad}-\frac{\frac{\left(2a^4+7a^2b^2-12b^4\right)\sin(c+dx)}{ad}-\frac{3\left(\frac{bx\left(a^4+3a^2b^2-4b^4\right)}{a}-\frac{b^4\left(5a^2-4b^2\right)\int\frac{\sec(c+d}{a+b\sec(c}}{a}\right.}{a}}{a}}{3a}}{a\left(a^2-b^2\right)}$$

$$\frac{b^2\sin(c+dx)\cos^2(c+dx)}{ad\left(a^2-b^2\right)(a+b\sec(c+dx))}$$

$\downarrow$ 3042

3.505. $\int\frac{\cos^3(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

$$\frac{\frac{(a^2-4b^2)\sin(c+dx)\cos^2(c+dx)}{3ad}-\frac{\frac{3b\left(a^2-2b^2\right)\sin(c+dx)\cos(c+dx)}{ad}-\frac{\left(2a^4+7a^2b^2-12b^4\right)\sin(c+dx)}{ad}-\frac{3\left(\frac{bx\left(a^4+3a^2b^2-4b^4\right)}{a}-\frac{b^4\left(5a^2-4b^2\right)\int\frac{\csc\left(c+\right.}{a+b\csc\left(c\right.}}{a}\right)}{a}}{3a}}{a\left(a^2-b^2\right)}$$

$$\frac{b^2\sin(c+dx)\cos^2(c+dx)}{ad\left(a^2-b^2\right)(a+b\sec(c+dx))}$$

$\downarrow$ 4318

$$\frac{\frac{(a^2-4b^2)\sin(c+dx)\cos^2(c+dx)}{3ad}-\frac{\frac{3b\left(a^2-2b^2\right)\sin(c+dx)\cos(c+dx)}{ad}-\frac{\left(2a^4+7a^2b^2-12b^4\right)\sin(c+dx)}{ad}-\frac{3\left(\frac{bx\left(a^4+3a^2b^2-4b^4\right)}{a}-\frac{b^3\left(5a^2-4b^2\right)\int\frac{1}{\frac{a\cos(c+d}{b}}}{a}\right)}{a}}{3a}}{a\left(a^2-b^2\right)}$$

$$\frac{b^2\sin(c+dx)\cos^2(c+dx)}{ad\left(a^2-b^2\right)(a+b\sec(c+dx))}$$

$\downarrow$ 3042

$$\frac{\frac{(a^2-4b^2)\sin(c+dx)\cos^2(c+dx)}{3ad}-\frac{\frac{3b\left(a^2-2b^2\right)\sin(c+dx)\cos(c+dx)}{ad}-\frac{\left(2a^4+7a^2b^2-12b^4\right)\sin(c+dx)}{ad}-\frac{3\left(\frac{bx\left(a^4+3a^2b^2-4b^4\right)}{a}-\frac{b^3\left(5a^2-4b^2\right)\int\frac{}{\frac{a\sin\left(c+d\right.}{b}}}{a}\right)}{a}}{3a}}{a\left(a^2-b^2\right)}$$

$$\frac{b^2\sin(c+dx)\cos^2(c+dx)}{ad\left(a^2-b^2\right)(a+b\sec(c+dx))}$$

$\downarrow$ 3138

$$\frac{\frac{(a^2-4b^2)\sin(c+dx)\cos^2(c+dx)}{3ad}-\frac{\frac{3b\left(a^2-2b^2\right)\sin(c+dx)\cos(c+dx)}{ad}-\frac{\left(2a^4+7a^2b^2-12b^4\right)\sin(c+dx)}{ad}-\frac{3\left(\frac{bx\left(a^4+3a^2b^2-4b^4\right)}{a}-\frac{2b^3\left(5a^2-4b^2\right)\int\frac{}{\left(1-\frac{a}{b}\right)t}}{a}\right)}{a}}{3a}}{a\left(a^2-b^2\right)}$$

$$\frac{b^2\sin(c+dx)\cos^2(c+dx)}{ad\left(a^2-b^2\right)(a+b\sec(c+dx))}$$

$\downarrow$ 221

3.505. $\int \frac{\cos^3(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

$$\frac{b^2\sin(c+dx)\cos^2(c+dx)}{ad\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)}+\frac{\frac{\left(a^2-4b^2\right)\sin(c+dx)\cos^2(c+dx)}{3ad}-\frac{\frac{3b\left(a^2-2b^2\right)\sin(c+dx)\cos(c+dx)}{ad}-\frac{\frac{\left(2a^4+7a^2b^2-12b^4\right)\sin(c+dx)}{ad}-\frac{3\left(\frac{bx\left(a^4+3a^2b^2-4b^4\right)}{a}-\frac{2b^4\left(5a^2-4b^2\right)\operatorname{arctanh}}{ad\sqrt{a-}}\right.}{a}}{a}}{3a}}{a\left(a^2-b^2\right)}$$

input
```
Int[Cos[c + d*x]^3/(a + b*Sec[c + d*x])^2,x]
```

output
```
(b^2*Cos[c + d*x]^2*Sin[c + d*x])/(a*(a^2 - b^2)*d*(a + b*Sec[c + d*x])) +
 (((a^2 - 4*b^2)*Cos[c + d*x]^2*Sin[c + d*x])/(3*a*d) - ((3*b*(a^2 - 2*b^2
)*Cos[c + d*x]*Sin[c + d*x])/(a*d) - ((-3*((b*(a^4 + 3*a^2*b^2 - 4*b^4)*x)
/a - (2*b^4*(5*a^2 - 4*b^2)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a
+ b]])/(a*Sqrt[a - b]*Sqrt[a + b]*d)))/a + ((2*a^4 + 7*a^2*b^2 - 12*b^4)*S
in[c + d*x])/(a*d))/a)/(3*a))/(a*(a^2 - b^2))
```

3.505.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 4318
```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[1/b   Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4334
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*
((d*Csc[e + f*x])^n/(a*f*(m + 1)*(a^2 - b^2))), x] + Simp[1/(a*(m + 1)*(a^2
 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a^2*(m + 1)
 - b^2*(m + n + 1) - a*b*(m + 1)*Csc[e + f*x] + b^2*(m + n + 2)*Csc[e + f*x
]^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0] && LtQ[m,
 -1] && IntegersQ[2*m, 2*n]
```

rule 4407
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Simp[(b*c - a*d)/a   Int[Csc[e + f*
x]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
- a*d, 0]
```

rule 4592
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Simp[1/(a*d*n)   Int[(a + b*Csc[e + f*x])^m
*(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*
Csc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

3.505.4 Maple [A] (verified)

Time = 1.06 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.01

method	result
derivativedivides	$\frac{-\frac{2b^4\left(-\frac{ab\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\left(a^2-b^2\right)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2b-a-b\right)}-\frac{\left(5a^2-4b^2\right)\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{(a-b)(a+b)\sqrt{(a-b)(a+b)}}\right)}{a^5}-\frac{2\left(\frac{\left(-a^3-a^2b-3ab^2\right)}{}\right.}{}}{d}$
default	$\frac{-\frac{2b^4\left(-\frac{ab\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\left(a^2-b^2\right)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2b-a-b\right)}-\frac{\left(5a^2-4b^2\right)\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{(a-b)(a+b)\sqrt{(a-b)(a+b)}}\right)}{a^5}-\frac{2\left(\frac{\left(-a^3-a^2b-3ab^2\right)}{}\right.}{}}{d}$
risch	$-\frac{bx}{a^3}-\frac{4b^3x}{a^5}+\frac{ib\,\mathrm{e}^{2i(dx+c)}}{4a^3d}-\frac{3i\mathrm{e}^{i(dx+c)}}{8a^2d}-\frac{3i\mathrm{e}^{i(dx+c)}b^2}{2a^4d}+\frac{3i\mathrm{e}^{-i(dx+c)}}{8a^2d}+\frac{3i\mathrm{e}^{-i(dx+c)}b^2}{2a^4d}-\frac{ib\,\mathrm{e}^{-2i(dx+c)}}{4a^3d}-$

input
```
int(cos(d*x+c)^3/(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-2*b^4/a^5*(-a*b/(a^2-b^2)*tan(1/2*d*x+1/2*c)/(tan(1/2*d*x+1/2*c)^2*a
-tan(1/2*d*x+1/2*c)^2*b-a-b)-(5*a^2-4*b^2)/(a-b)/(a+b)/((a-b)*(a+b))^(1/2)
*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a-b)*(a+b))^(1/2)))-2/a^5*(((-a^3-a^2*
b-3*a*b^2)*tan(1/2*d*x+1/2*c)^5+(-2/3*a^3-6*a*b^2)*tan(1/2*d*x+1/2*c)^3+(-
a^3+a^2*b-3*a*b^2)*tan(1/2*d*x+1/2*c))/(1+tan(1/2*d*x+1/2*c)^2)^3+b*(a^2+4
*b^2)*arctan(tan(1/2*d*x+1/2*c))))
```

3.505.5 Fricas [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 757, normalized size of antiderivative = 2.90

$$\int \frac{\cos^3(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$
$$= \left[-\frac{6\,(a^7b+2\,a^5b^3-7\,a^3b^5+4\,ab^7)dx\cos(dx+c)+6\,(a^6b^2+2\,a^4b^4-7\,a^2b^6+4\,b^8)dx-3\,(5\,a^2b^5-4\,b^7}{}\right.$$
$$-\frac{3\,(a^7b+2\,a^5b^3-7\,a^3b^5+4\,ab^7)dx\cos(dx+c)+3\,(a^6b^2+2\,a^4b^4-7\,a^2b^6+4\,b^8)dx-3\,(5\,a^2b^5-4\,b^7-}{}$$

input
```
integrate(cos(d*x+c)^3/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
[-1/6*(6*(a^7*b + 2*a^5*b^3 - 7*a^3*b^5 + 4*a*b^7)*d*x*cos(d*x + c) + 6*(a
^6*b^2 + 2*a^4*b^4 - 7*a^2*b^6 + 4*b^8)*d*x - 3*(5*a^2*b^5 - 4*b^7 + (5*a^
3*b^4 - 4*a*b^6)*cos(d*x + c))*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (
a^2 - 2*b^2)*cos(d*x + c)^2 + 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d
*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) -
2*(2*a^7*b + 5*a^5*b^3 - 19*a^3*b^5 + 12*a*b^7 + (a^8 - 2*a^6*b^2 + a^4*b^
4)*cos(d*x + c)^3 - 2*(a^7*b - 2*a^5*b^3 + a^3*b^5)*cos(d*x + c)^2 + 2*(a^
8 + a^6*b^2 - 5*a^4*b^4 + 3*a^2*b^6)*cos(d*x + c))*sin(d*x + c))/((a^10 -
2*a^8*b^2 + a^6*b^4)*d*cos(d*x + c) + (a^9*b - 2*a^7*b^3 + a^5*b^5)*d), -1
/3*(3*(a^7*b + 2*a^5*b^3 - 7*a^3*b^5 + 4*a*b^7)*d*x*cos(d*x + c) + 3*(a^6*
b^2 + 2*a^4*b^4 - 7*a^2*b^6 + 4*b^8)*d*x - 3*(5*a^2*b^5 - 4*b^7 + (5*a^3*b
^4 - 4*a*b^6)*cos(d*x + c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*c
os(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c))) - (2*a^7*b + 5*a^5*b^3 - 19*a
^3*b^5 + 12*a*b^7 + (a^8 - 2*a^6*b^2 + a^4*b^4)*cos(d*x + c)^3 - 2*(a^7*b
- 2*a^5*b^3 + a^3*b^5)*cos(d*x + c)^2 + 2*(a^8 + a^6*b^2 - 5*a^4*b^4 + 3*a
^2*b^6)*cos(d*x + c))*sin(d*x + c))/((a^10 - 2*a^8*b^2 + a^6*b^4)*d*cos(d*
x + c) + (a^9*b - 2*a^7*b^3 + a^5*b^5)*d)]
```

3.505.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^3(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**3/(a+b*sec(d*x+c))**2,x)
```

output
```
Timed out
```

3.505.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\cos^3(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(cos(d*x+c)^3/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

3.505.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 335, normalized size of antiderivative = 1.28

$$\int \frac{\cos^3(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$
$$= \frac{\frac{6\,b^5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{(a^6-a^4b^2)\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a-b\right)}+\frac{6\left(5\,a^2b^4-4\,b^6\right)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx\right.}{\sqrt{-a^2+b^2}}\right.\right.}{(a^7-a^5b^2)\sqrt{-a^2+b^2}}}{}$$

input
```
integrate(cos(d*x+c)^3/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
1/3*(6*b^5*tan(1/2*d*x + 1/2*c)/((a^6 - a^4*b^2)*(a*tan(1/2*d*x + 1/2*c)^2
 - b*tan(1/2*d*x + 1/2*c)^2 - a - b)) + 6*(5*a^2*b^4 - 4*b^6)*(pi*floor(1/
2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) -
b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))/((a^7 - a^5*b^2)*sqrt(-a^2 + b^
2)) - 3*(a^2*b + 4*b^3)*(d*x + c)/a^5 + 2*(3*a^2*tan(1/2*d*x + 1/2*c)^5 +
3*a*b*tan(1/2*d*x + 1/2*c)^5 + 9*b^2*tan(1/2*d*x + 1/2*c)^5 + 2*a^2*tan(1/
2*d*x + 1/2*c)^3 + 18*b^2*tan(1/2*d*x + 1/2*c)^3 + 3*a^2*tan(1/2*d*x + 1/2
*c) - 3*a*b*tan(1/2*d*x + 1/2*c) + 9*b^2*tan(1/2*d*x + 1/2*c))/((tan(1/2*d
*x + 1/2*c)^2 + 1)^3*a^4))/d
```

3.505.9 Mupad [B] (verification not implemented)

Time = 20.72 (sec) , antiderivative size = 3839, normalized size of antiderivative = 14.71

$$\int \frac{\cos^3(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
int(cos(c + d*x)^3/(a + b/cos(c + d*x))^2,x)
```

output
```
- ((2*tan(c/2 + (d*x)/2)^7*(a^5 - 2*a*b^4 + 4*b^5 - 3*a^2*b^3 + a^3*b^2))/
(a^4*(a + b)*(a - b)) + (2*tan(c/2 + (d*x)/2)^3*(6*a*b^4 - 8*a^4*b + a^5 +
 36*b^5 - 19*a^2*b^3 - 7*a^3*b^2))/(3*a^4*(a + b)*(a - b)) - (2*tan(c/2 +
(d*x)/2)^5*(6*a*b^4 + 8*a^4*b + a^5 - 36*b^5 + 19*a^2*b^3 - 7*a^3*b^2))/(3
*a^4*(a + b)*(a - b)) - (2*tan(c/2 + (d*x)/2)*(a^5 - 2*a*b^4 - 4*b^5 + 3*a
^2*b^3 + a^3*b^2))/(a^4*(a + b)*(a - b)))/(d*(a + b - tan(c/2 + (d*x)/2)^8
*(a - b) + tan(c/2 + (d*x)/2)^2*(2*a + 4*b) - tan(c/2 + (d*x)/2)^6*(2*a -
4*b) + 6*b*tan(c/2 + (d*x)/2)^4)) - (2*b*atan(((b*(a^2 + 4*b^2)*((32*tan(c
/2 + (d*x)/2)*(32*b^12 - 32*a*b^11 - 48*a^2*b^10 + 48*a^3*b^9 + 2*a^4*b^8
- 2*a^5*b^7 + 7*a^6*b^6 - 12*a^7*b^5 + 7*a^8*b^4 - 2*a^9*b^3 + a^10*b^2))/
(a^10*b + a^11 - a^8*b^3 - a^9*b^2) + (b*(a^2 + 4*b^2)*((32*(a^17*b - 4*a^
10*b^8 + 2*a^11*b^7 + 9*a^12*b^6 - 4*a^13*b^5 - 5*a^14*b^4 + a^15*b^3))/(a
^14*b + a^15 - a^12*b^3 - a^13*b^2) - (b*tan(c/2 + (d*x)/2)*(a^2 + 4*b^2)*
(2*a^15*b - 2*a^10*b^6 + 2*a^11*b^5 + 4*a^12*b^4 - 4*a^13*b^3 - 2*a^14*b^2
)*32i)/(a^5*(a^10*b + a^11 - a^8*b^3 - a^9*b^2)))*1i)/a^5))/a^5 + (b*(a^2
+ 4*b^2)*((32*tan(c/2 + (d*x)/2)*(32*b^12 - 32*a*b^11 - 48*a^2*b^10 + 48*a
^3*b^9 + 2*a^4*b^8 - 2*a^5*b^7 + 7*a^6*b^6 - 12*a^7*b^5 + 7*a^8*b^4 - 2*a^
9*b^3 + a^10*b^2))/(a^10*b + a^11 - a^8*b^3 - a^9*b^2) - (b*(a^2 + 4*b^2)*
((32*(a^17*b - 4*a^10*b^8 + 2*a^11*b^7 + 9*a^12*b^6 - 4*a^13*b^5 - 5*a^14*
b^4 + a^15*b^3))/(a^14*b + a^15 - a^12*b^3 - a^13*b^2) + (b*tan(c/2 + (...
```

3.505. $\int \frac{\cos^3(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

3.506 $\int \frac{\sec^5(c+dx)}{(a+b\sec(c+dx))^3}\,dx$

3.506.1 Optimal result . . . 3753
3.506.2 Mathematica [A] (verified) . . . 3754
3.506.3 Rubi [A] (verified) . . . 3754
3.506.4 Maple [A] (verified) . . . 3759
3.506.5 Fricas [B] (verification not implemented) . . . 3760
3.506.6 Sympy [F] . . . 3760
3.506.7 Maxima [F(-2)] . . . 3761
3.506.8 Giac [A] (verification not implemented) . . . 3761
3.506.9 Mupad [B] (verification not implemented) . . . 3762

3.506.1 Optimal result

Integrand size = 21, antiderivative size = 230

$$\begin{aligned}\int \frac{\sec^5(c+dx)}{(a+b\sec(c+dx))^3}\,dx = &-\frac{3a\operatorname{arctanh}(\sin(c+dx))}{b^4d}\\ &+\frac{3a^2(2a^4-5a^2b^2+4b^4)\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}b^4(a+b)^{5/2}d}\\ &+\frac{(3a^2-2b^2)\tan(c+dx)}{2b^3(a^2-b^2)d}-\frac{a^2\sec^2(c+dx)\tan(c+dx)}{2b(a^2-b^2)d(a+b\sec(c+dx))^2}\\ &+\frac{3a^3(a^2-2b^2)\tan(c+dx)}{2b^3(a^2-b^2)^2d(a+b\sec(c+dx))}\end{aligned}$$

output
```
-3*a*arctanh(sin(d*x+c))/b^4/d+3*a^2*(2*a^4-5*a^2*b^2+4*b^4)*arctanh((a-b)
^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/(a-b)^(5/2)/b^4/(a+b)^(5/2)/d+1/2*(
3*a^2-2*b^2)*tan(d*x+c)/b^3/(a^2-b^2)/d-1/2*a^2*sec(d*x+c)^2*tan(d*x+c)/b/
(a^2-b^2)/d/(a+b*sec(d*x+c))^2+3/2*a^3*(a^2-2*b^2)*tan(d*x+c)/b^3/(a^2-b^2
)^2/d/(a+b*sec(d*x+c))
```

3.506.2 Mathematica [A] (verified)

Time = 3.58 (sec) , antiderivative size = 205, normalized size of antiderivative = 0.89

$$\int \frac{\sec^5(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \frac{-\frac{6a^2(2a^4-5a^2b^2+4b^4)\operatorname{arctanh}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{5/2}}+6a\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-6a\log\left(\cos\left(\frac{1}{2}\right.\right.}{2b^4d}$$

input
```
Integrate[Sec[c + d*x]^5/(a + b*Sec[c + d*x])^3,x]
```

output
```
((-6*a^2*(2*a^4 - 5*a^2*b^2 + 4*b^4)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/S
qrt[a^2 - b^2]])/(a^2 - b^2)^(5/2) + 6*a*Log[Cos[(c + d*x)/2] - Sin[(c + d
*x)/2]] - 6*a*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + (a^3*b*(5*a^2*b -
 8*b^3 + a*(4*a^2 - 7*b^2)*Cos[c + d*x])*Sin[c + d*x])/((a - b)^2*(a + b)^
2*(b + a*Cos[c + d*x])^2) + 2*b*Tan[c + d*x])/(2*b^4*d)
```

3.506.3 Rubi [A] (verified)

Time = 1.68 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.16, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {3042, 4332, 3042, 4578, 3042, 4570, 27, 3042, 4486, 3042, 4257, 4318, 3042, 3138, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^5(c+dx)}{(a+b\sec(c+dx))^3}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^5}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \; 4332$$

$$-\frac{\int \frac{\sec^2(c+dx)\left(2a^2-2b\sec(c+dx)a-\left(3a^2-2b^2\right)\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^2}dx}{2b\left(a^2-b^2\right)}-\frac{a^2\tan(c+dx)\sec^2(c+dx)}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2\left(2a^2-2b\csc\left(c+dx+\frac{\pi}{2}\right)a+\left(2b^2-3a^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx}{2b\left(a^2-b^2\right)}-\frac{a^2\tan(c+dx)\sec^2(c+dx)}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$\Big\downarrow$ 4578

$$-\frac{\frac{\int \frac{\sec(c+dx)\left(3b\left(a^2-2b^2\right)a^2+\left(3a^2-4b^2\right)\left(a^2-b^2\right)\sec(c+dx)a-b\left(3a^2-2b^2\right)\left(a^2-b^2\right)\sec^2(c+dx)\right)}{a+b\sec(c+dx)}dx}{b^2\left(a^2-b^2\right)}-\frac{3a^3\left(a^2-2b^2\right)\tan(c+dx)}{b^2d\left(a^2-b^2\right)(a+b\sec(c+dx))}}{2b\left(a^2-b^2\right)}-\frac{a^2\tan(c+dx)\sec^2(c+dx)}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$\Big\downarrow$ 3042

$$-\frac{\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(3b\left(a^2-2b^2\right)a^2+\left(3a^2-4b^2\right)\left(a^2-b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)a-b\left(3a^2-2b^2\right)\left(a^2-b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2\left(a^2-b^2\right)}-\frac{3a^3\left(a^2-2b^2\right)\tan(c+dx)}{b^2d\left(a^2-b^2\right)(a+b\sec(c+dx))}}{2b\left(a^2-b^2\right)}-\frac{a^2\tan(c+dx)\sec^2(c+dx)}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$\Big\downarrow$ 4570

$$-\frac{\frac{\frac{\int \frac{3\sec(c+dx)\left(a^2\left(a^2-2b^2\right)b^2+2a\left(a^2-b^2\right)^2\sec(c+dx)b\right)}{a+b\sec(c+dx)}dx}{b}-\frac{\left(3a^2-2b^2\right)\left(a^2-b^2\right)\tan(c+dx)}{d}}{b^2\left(a^2-b^2\right)}-\frac{3a^3\left(a^2-2b^2\right)\tan(c+dx)}{b^2d\left(a^2-b^2\right)(a+b\sec(c+dx))}}{2b\left(a^2-b^2\right)}-\frac{a^2\tan(c+dx)\sec^2(c+dx)}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$\Big\downarrow$ 27

$$-\frac{\frac{\frac{3\int \frac{\sec(c+dx)\left(a^2\left(a^2-2b^2\right)b^2+2a\left(a^2-b^2\right)^2\sec(c+dx)b\right)}{a+b\sec(c+dx)}dx}{b}-\frac{\left(3a^2-2b^2\right)\left(a^2-b^2\right)\tan(c+dx)}{d}}{b^2\left(a^2-b^2\right)}-\frac{3a^3\left(a^2-2b^2\right)\tan(c+dx)}{b^2d\left(a^2-b^2\right)(a+b\sec(c+dx))}}{2b\left(a^2-b^2\right)}-\frac{a^2\tan(c+dx)\sec^2(c+dx)}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$\Big\downarrow$ 3042

$$-\frac{\frac{\frac{3\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(a^2\left(a^2-2b^2\right)b^2+2a\left(a^2-b^2\right)^2\csc\left(c+dx+\frac{\pi}{2}\right)b\right)}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b}-\frac{\left(3a^2-2b^2\right)\left(a^2-b^2\right)\tan(c+dx)}{d}}{b^2\left(a^2-b^2\right)}-\frac{3a^3\left(a^2-2b^2\right)\tan(c+dx)}{b^2d\left(a^2-b^2\right)(a+b\sec(c+dx))}}{2b\left(a^2-b^2\right)}-\frac{a^2\tan(c+dx)\sec^2(c+dx)}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

3.506. $\int \frac{\sec^5(c+dx)}{(a+b\sec(c+dx))^3}\,dx$

$$\Big\downarrow \; 4486$$

$$-\frac{\frac{\frac{3\left(2a\left(a^2-b^2\right)^2\int \sec(c+dx)dx-a^2\left(2a^4-5a^2b^2+4b^4\right)\int \frac{\sec(c+dx)}{a+b\sec(c+dx)}dx\right)}{b}-\frac{\left(3a^2-2b^2\right)\left(a^2-b^2\right)\tan(c+dx)}{d}}{b^2(a^2-b^2)}-\frac{3a^3\left(a^2-2b^2\right)\tan(c+dx)}{b^2d(a^2-b^2)(a+b\sec(c+dx))}}{2b\left(a^2-b^2\right)}-\frac{a^2\tan(c+dx)\sec^2(c+dx)}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\frac{\frac{3\left(2a\left(a^2-b^2\right)^2\int \csc\left(c+dx+\frac{\pi}{2}\right)dx-a^2\left(2a^4-5a^2b^2+4b^4\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx\right)}{b}-\frac{\left(3a^2-2b^2\right)\left(a^2-b^2\right)\tan(c+dx)}{d}}{b^2(a^2-b^2)}-\frac{3a^3\left(a^2-2b^2\right)\tan(c+dx)}{b^2d(a^2-b^2)(a+b\sec(c+dx))}}{2b\left(a^2-b^2\right)}-\frac{a^2\tan(c+dx)\sec^2(c+dx)}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$$\Big\downarrow \; 4257$$

$$-\frac{\frac{\frac{3\left(\frac{2a\left(a^2-b^2\right)^2\text{arctanh}(\sin(c+dx))}{d}-a^2\left(2a^4-5a^2b^2+4b^4\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx\right)}{b}-\frac{\left(3a^2-2b^2\right)\left(a^2-b^2\right)\tan(c+dx)}{d}}{b^2(a^2-b^2)}-\frac{3a^3\left(a^2-2b^2\right)\tan(c+dx)}{b^2d(a^2-b^2)(a+b\sec(c+dx))}}{2b\left(a^2-b^2\right)}-\frac{a^2\tan(c+dx)\sec^2(c+dx)}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$$\Big\downarrow \; 4318$$

$$-\frac{\frac{\frac{3\left(\frac{2a\left(a^2-b^2\right)^2\text{arctanh}(\sin(c+dx))}{d}-\frac{a^2\left(2a^4-5a^2b^2+4b^4\right)\int \frac{1}{\frac{a\cos(c+dx)}{b}+1}dx}{b}\right)}{b}-\frac{\left(3a^2-2b^2\right)\left(a^2-b^2\right)\tan(c+dx)}{d}}{b^2(a^2-b^2)}-\frac{3a^3\left(a^2-2b^2\right)\tan(c+dx)}{b^2d(a^2-b^2)(a+b\sec(c+dx))}}{2b\left(a^2-b^2\right)}-\frac{a^2\tan(c+dx)\sec^2(c+dx)}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\frac{\frac{3\left(\frac{2a\left(a^2-b^2\right)^2\text{arctanh}(\sin(c+dx))}{d}-\frac{a^2\left(2a^4-5a^2b^2+4b^4\right)\int \frac{1}{\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{b}+1}dx}{b}\right)}{b}-\frac{\left(3a^2-2b^2\right)\left(a^2-b^2\right)\tan(c+dx)}{d}}{b^2(a^2-b^2)}-\frac{3a^3\left(a^2-2b^2\right)\tan(c+dx)}{b^2d(a^2-b^2)(a+b\sec(c+dx))}}{2b\left(a^2-b^2\right)}-\frac{a^2\tan(c+dx)\sec^2(c+dx)}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

3.506. $\int \frac{\sec^5(c+dx)}{(a+b\sec(c+dx))^3}\,dx$

$$\Big\downarrow \; 3138$$

$$-\frac{\frac{\frac{3\left(\frac{2a\left(a^2-b^2\right)^2\text{arctanh}(\sin(c+dx))}{d}-\frac{2a^2\left(2a^4-5a^2b^2+4b^4\right)\int\frac{1}{\left(1-\frac{a}{b}\right)\tan^2\left(\frac{1}{2}(c+dx)\right)+\frac{a+b}{b}}d\tan\left(\frac{1}{2}(c+dx)\right)}{bd}\right)}{b}-\frac{\left(3a^2-2b^2\right)\left(a^2-b^2\right)\tan(c+dx)}{d}}{b^2(a^2-b^2)}-\frac{3a}{b^2d}}{2b\left(a^2-b^2\right)}$$

$$\frac{a^2\tan(c+dx)\sec^2(c+dx)}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$$\Big\downarrow \; 221$$

$$-\frac{a^2\tan(c+dx)\sec^2(c+dx)}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^2}-\frac{\frac{\frac{3\left(\frac{2a\left(a^2-b^2\right)^2\text{arctanh}(\sin(c+dx))}{d}-\frac{2a^2\left(2a^4-5a^2b^2+4b^4\right)\text{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d\sqrt{a-b}\sqrt{a+b}}\right)}{b}-\frac{\left(3a^2-2b^2\right)\left(a^2-b^2\right)\tan(c+dx)}{d}}{b^2(a^2-b^2)}-\frac{3a^3\left(a^2-2b^2\right)\text{ta}}{b^2d(a^2-b^2)(a+b\text{s}}}{2b\left(a^2-b^2\right)}$$

input
```
Int[Sec[c + d*x]^5/(a + b*Sec[c + d*x])^3,x]
```

output
```
-1/2*(a^2*Sec[c + d*x]^2*Tan[c + d*x])/(b*(a^2 - b^2)*d*(a + b*Sec[c + d*x
])^2) - ((-3*a^3*(a^2 - 2*b^2)*Tan[c + d*x])/(b^2*(a^2 - b^2)*d*(a + b*Sec
[c + d*x])) + ((3*((2*a*(a^2 - b^2)^2*ArcTanh[Sin[c + d*x]])/d - (2*a^2*(2
*a^4 - 5*a^2*b^2 + 4*b^4)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a +
b]])/(Sqrt[a - b]*Sqrt[a + b]*d)))/b - ((3*a^2 - 2*b^2)*(a^2 - b^2)*Tan[c
+ d*x])/d)/(b^2*(a^2 - b^2)))/(2*b*(a^2 - b^2))
```

3.506.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4318
```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[1/b   Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4332
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-a^2)*d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(
m + 1)*((d*Csc[e + f*x])^(n - 3)/(b*f*(m + 1)*(a^2 - b^2))), x] + Simp[d^3/
(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])
^(n - 3)*Simp[a^2*(n - 3) + a*b*(m + 1)*Csc[e + f*x] - (a^2*(n - 2) + b^2*(
m + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2
- b^2, 0] && LtQ[m, -1] && (IGtQ[n, 3] || (IntegersQ[n + 1/2, 2*m] && GtQ[n
, 2]))
```

rule 4486
```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/(csc[(
e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[B/b   Int[Csc[e + f*x],
 x], x] + Simp[(A*b - a*B)/b   Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x
] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0]
```

rule 4570
```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> Simp[(-C)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2)
)), x] + Simp[1/(b*(m + 2))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[
b*A*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /;
 FreeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

3.506. $\int \frac{\sec^5(c+dx)}{(a+b\sec(c+dx))^3}\,dx$

rule 4578
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[
(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x
_Symbol] :> Simp[a*(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*((a + b*Csc[e + f*x
])^(m + 1)/(b^2*f*(m + 1)*(a^2 - b^2))), x] - Simp[1/(b^2*(m + 1)*(a^2 - b^
2))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[b*(m + 1)*((-a)*(b
*B - a*C) + A*b^2) + (b*B*(a^2 + b^2*(m + 1)) - a*(A*b^2*(m + 2) + C*(a^2 +
 b^2*(m + 1))))*Csc[e + f*x] - b*C*(m + 1)*(a^2 - b^2)*Csc[e + f*x]^2, x],
x], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1
]
```

3.506.4 Maple [A] (verified)

Time = 1.15 (sec) , antiderivative size = 296, normalized size of antiderivative = 1.29

method	result
derivativedivides	$\frac{-\frac{1}{b^3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{3a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{b^4}-\frac{2a^2\left(\frac{\frac{\left(4a^2-ab-8b^2\right)ab\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{2(a-b)\left(a^2+2ab+b^2\right)}-\frac{\left(4a^2+ab-8b^2\right)ab\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2(a+b)\left(a^2-2ab+b^2\right)}}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2b-a-b\right)^2}-\frac{3\left(2a^4-\right.}{}\right.}{b^4}}{d}$
default	$\frac{-\frac{1}{b^3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{3a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{b^4}-\frac{2a^2\left(\frac{\frac{\left(4a^2-ab-8b^2\right)ab\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{2(a-b)\left(a^2+2ab+b^2\right)}-\frac{\left(4a^2+ab-8b^2\right)ab\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2(a+b)\left(a^2-2ab+b^2\right)}}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2b-a-b\right)^2}-\frac{3\left(2a^4-\right.}{}\right.}{b^4}}{d}$
risch	$\frac{i\left(3a^5b\,e^{5i(dx+c)}-6a^3b^3e^{5i(dx+c)}+6a^6e^{4i(dx+c)}-3a^4b^2e^{4i(dx+c)}-12a^2b^4e^{4i(dx+c)}+24a^5b\,e^{3i(dx+c)}-44a^3b^3e^{3i(dx+c)}+\right.}{(-a^2+b^2)^2d\,b}$

input
```
int(sec(d*x+c)^5/(a+b*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/b^3/(tan(1/2*d*x+1/2*c)+1)-3*a/b^4*ln(tan(1/2*d*x+1/2*c)+1)-2*a^2/
b^4*((1/2*(4*a^2-a*b-8*b^2)*a*b/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3
-1/2*(4*a^2+a*b-8*b^2)*a*b/(a+b)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c))/(tan(
1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2-3/2*(2*a^4-5*a^2*b^2+4*b^
4)/(a^4-2*a^2*b^2+b^4)/((a-b)*(a+b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c
)/((a-b)*(a+b))^(1/2)))-1/b^3/(tan(1/2*d*x+1/2*c)-1)+3*a/b^4*ln(tan(1/2*d*
x+1/2*c)-1))
```

3.506.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 648 vs. $2(215) = 430$.

Time = 0.85 (sec) , antiderivative size = 1354, normalized size of antiderivative = 5.89

$$\int \frac{\sec^5(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(sec(d*x+c)^5/(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
[1/4*(3*((2*a^8 - 5*a^6*b^2 + 4*a^4*b^4)*cos(d*x + c)^3 + 2*(2*a^7*b - 5*a
^5*b^3 + 4*a^3*b^5)*cos(d*x + c)^2 + (2*a^6*b^2 - 5*a^4*b^4 + 4*a^2*b^6)*c
os(d*x + c))*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d
*x + c)^2 + 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 -
b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) - 6*((a^9 - 3*a^7*b^
2 + 3*a^5*b^4 - a^3*b^6)*cos(d*x + c)^3 + 2*(a^8*b - 3*a^6*b^3 + 3*a^4*b^5
 - a^2*b^7)*cos(d*x + c)^2 + (a^7*b^2 - 3*a^5*b^4 + 3*a^3*b^6 - a*b^8)*cos
(d*x + c))*log(sin(d*x + c) + 1) + 6*((a^9 - 3*a^7*b^2 + 3*a^5*b^4 - a^3*b
^6)*cos(d*x + c)^3 + 2*(a^8*b - 3*a^6*b^3 + 3*a^4*b^5 - a^2*b^7)*cos(d*x +
 c)^2 + (a^7*b^2 - 3*a^5*b^4 + 3*a^3*b^6 - a*b^8)*cos(d*x + c))*log(-sin(d
*x + c) + 1) + 2*(2*a^6*b^3 - 6*a^4*b^5 + 6*a^2*b^7 - 2*b^9 + (6*a^8*b - 1
7*a^6*b^3 + 13*a^4*b^5 - 2*a^2*b^7)*cos(d*x + c)^2 + (9*a^7*b^2 - 25*a^5*b
^4 + 20*a^3*b^6 - 4*a*b^8)*cos(d*x + c))*sin(d*x + c))/((a^8*b^4 - 3*a^6*b
^6 + 3*a^4*b^8 - a^2*b^10)*d*cos(d*x + c)^3 + 2*(a^7*b^5 - 3*a^5*b^7 + 3*a
^3*b^9 - a*b^11)*d*cos(d*x + c)^2 + (a^6*b^6 - 3*a^4*b^8 + 3*a^2*b^10 - b^
12)*d*cos(d*x + c)), 1/2*(3*((2*a^8 - 5*a^6*b^2 + 4*a^4*b^4)*cos(d*x + c)^
3 + 2*(2*a^7*b - 5*a^5*b^3 + 4*a^3*b^5)*cos(d*x + c)^2 + (2*a^6*b^2 - 5*a^
4*b^4 + 4*a^2*b^6)*cos(d*x + c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)
*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c))) - 3*((a^9 - 3*a^7*b^2 +
3*a^5*b^4 - a^3*b^6)*cos(d*x + c)^3 + 2*(a^8*b - 3*a^6*b^3 + 3*a^4*b^5 ...
```

3.506.6 Sympy [F]

$$\int \frac{\sec^5(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \int \frac{\sec^5(c+dx)}{(a+b\sec(c+dx))^3}\,dx$$

input
```
integrate(sec(d*x+c)**5/(a+b*sec(d*x+c))**3,x)
```

output
```
Integral(sec(c + d*x)**5/(a + b*sec(c + d*x))**3, x)
```

3.506.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\sec^5(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(sec(d*x+c)^5/(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

3.506.8 Giac [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 383, normalized size of antiderivative = 1.67

$$\int \frac{\sec^5(c+dx)}{(a+b\sec(c+dx))^3}\,dx = -\frac{\frac{3\,(2\,a^6-5\,a^4b^2+4\,a^2b^4)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(2\,a-2\,b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{-a^2+b^2}}\right)\right)}{(a^4b^4-2\,a^2b^6+b^8)\sqrt{-a^2+b^2}}+\frac{4\,a^6\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-5\,a^5b\tan\left(\frac{1}{2}\,dx+\right.}{}}{}$$

input
```
integrate(sec(d*x+c)^5/(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
-(3*(2*a^6 - 5*a^4*b^2 + 4*a^2*b^4)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(
2*a - 2*b) + arctan((a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt
(-a^2 + b^2)))/((a^4*b^4 - 2*a^2*b^6 + b^8)*sqrt(-a^2 + b^2)) + (4*a^6*tan
(1/2*d*x + 1/2*c)^3 - 5*a^5*b*tan(1/2*d*x + 1/2*c)^3 - 7*a^4*b^2*tan(1/2*d
*x + 1/2*c)^3 + 8*a^3*b^3*tan(1/2*d*x + 1/2*c)^3 - 4*a^6*tan(1/2*d*x + 1/2
*c) - 5*a^5*b*tan(1/2*d*x + 1/2*c) + 7*a^4*b^2*tan(1/2*d*x + 1/2*c) + 8*a^
3*b^3*tan(1/2*d*x + 1/2*c))/((a^4*b^3 - 2*a^2*b^5 + b^7)*(a*tan(1/2*d*x +
1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 - a - b)^2) + 3*a*log(abs(tan(1/2*d*x
+ 1/2*c) + 1))/b^4 - 3*a*log(abs(tan(1/2*d*x + 1/2*c) - 1))/b^4 + 2*tan(1/
2*d*x + 1/2*c)/((tan(1/2*d*x + 1/2*c)^2 - 1)*b^3))/d
```

3.506.9 Mupad [B] (verification not implemented)

Time = 21.88 (sec) , antiderivative size = 5332, normalized size of antiderivative = 23.18

$$\int \frac{\sec^5(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
int(1/(cos(c + d*x)^5*(a + b/cos(c + d*x))^3),x)
```

output
```
((tan(c/2 + (d*x)/2)^5*(2*a*b^4 - 3*a^4*b + 6*a^5 - 2*b^5 + 4*a^2*b^3 - 12
*a^3*b^2))/((a*b^3 - b^4)*(a + b)^2) + (tan(c/2 + (d*x)/2)*(2*a*b^4 + 3*a^
4*b + 6*a^5 + 2*b^5 - 4*a^2*b^3 - 12*a^3*b^2))/((a + b)*(b^5 - 2*a*b^4 + a
^2*b^3)) - (2*tan(c/2 + (d*x)/2)^3*(6*a^6 - 2*b^6 + 6*a^2*b^4 - 13*a^4*b^2
))/(b*(a*b^2 - b^3)*(a + b)^2*(a - b)))/(d*(2*a*b - tan(c/2 + (d*x)/2)^2*(
2*a*b + 3*a^2 - b^2) - tan(c/2 + (d*x)/2)^6*(a^2 - 2*a*b + b^2) + a^2 + b^
2 - tan(c/2 + (d*x)/2)^4*(2*a*b - 3*a^2 + b^2))) + (a*atan(((a*((8*tan(c/2
 + (d*x)/2)*(72*a^12 - 72*a^11*b + 36*a^2*b^10 - 72*a^3*b^9 + 36*a^4*b^8 +
 288*a^5*b^7 - 288*a^6*b^6 - 432*a^7*b^5 + 441*a^8*b^4 + 288*a^9*b^3 - 288
*a^10*b^2))/(a*b^12 + b^13 - 3*a^2*b^11 - 3*a^3*b^10 + 3*a^4*b^9 + 3*a^5*b
^8 - a^6*b^7 - a^7*b^6) - (3*a*((24*(4*a*b^17 - 8*a^2*b^16 - 12*a^3*b^15 +
 26*a^4*b^14 + 14*a^5*b^13 - 32*a^6*b^12 - 8*a^7*b^11 + 18*a^8*b^10 + 2*a^
9*b^9 - 4*a^10*b^8))/(a*b^15 + b^16 - 3*a^2*b^14 - 3*a^3*b^13 + 3*a^4*b^12
 + 3*a^5*b^11 - a^6*b^10 - a^7*b^9) - (24*a*tan(c/2 + (d*x)/2)*(8*a*b^17 -
 8*a^2*b^16 - 32*a^3*b^15 + 32*a^4*b^14 + 48*a^5*b^13 - 48*a^6*b^12 - 32*a
^7*b^11 + 32*a^8*b^10 + 8*a^9*b^9 - 8*a^10*b^8))/(b^4*(a*b^12 + b^13 - 3*a
^2*b^11 - 3*a^3*b^10 + 3*a^4*b^9 + 3*a^5*b^8 - a^6*b^7 - a^7*b^6))))/b^4)*
3i)/b^4 + (a*((8*tan(c/2 + (d*x)/2)*(72*a^12 - 72*a^11*b + 36*a^2*b^10 - 7
2*a^3*b^9 + 36*a^4*b^8 + 288*a^5*b^7 - 288*a^6*b^6 - 432*a^7*b^5 + 441*a^8
*b^4 + 288*a^9*b^3 - 288*a^10*b^2))/(a*b^12 + b^13 - 3*a^2*b^11 - 3*a^3...
```

3.507 $\int \frac{\sec^4(c+dx)}{(a+b\sec(c+dx))^3}\,dx$

3.507.1 Optimal result . . . 3763
3.507.2 Mathematica [A] (verified) . . . 3764
3.507.3 Rubi [A] (verified) . . . 3764
3.507.4 Maple [A] (verified) . . . 3768
3.507.5 Fricas [B] (verification not implemented) . . . 3769
3.507.6 Sympy [F] . . . 3769
3.507.7 Maxima [F(-2)] . . . 3770
3.507.8 Giac [A] (verification not implemented) . . . 3770
3.507.9 Mupad [B] (verification not implemented) . . . 3771

3.507.1 Optimal result

Integrand size = 21, antiderivative size = 188

$$\begin{aligned}\int \frac{\sec^4(c+dx)}{(a+b\sec(c+dx))^3}\,dx = {} & \frac{\operatorname{arctanh}(\sin(c+dx))}{b^3d} \\ & - \frac{a(2a^4-5a^2b^2+6b^4)\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}b^3(a+b)^{5/2}d} \\ & - \frac{a^2\sec(c+dx)\tan(c+dx)}{2b\,(a^2-b^2)\,d(a+b\sec(c+dx))^2} \\ & - \frac{a^2(2a^2-5b^2)\tan(c+dx)}{2b^2\,(a^2-b^2)^2\,d(a+b\sec(c+dx))}\end{aligned}$$

output
```
arctanh(sin(d*x+c))/b^3/d-a*(2*a^4-5*a^2*b^2+6*b^4)*arctanh((a-b)^(1/2)*ta
n(1/2*d*x+1/2*c)/(a+b)^(1/2))/(a-b)^(5/2)/b^3/(a+b)^(5/2)/d-1/2*a^2*sec(d*
x+c)*tan(d*x+c)/b/(a^2-b^2)/d/(a+b*sec(d*x+c))^2-1/2*a^2*(2*a^2-5*b^2)*tan
(d*x+c)/b^2/(a^2-b^2)^2/d/(a+b*sec(d*x+c))
```

3.507.2 Mathematica [A] (verified)

Time = 1.10 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.03

$$\int \frac{\sec^4(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \frac{\frac{2a(2a^4-5a^2b^2+6b^4)\text{arctanh}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{5/2}} - 2\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+2\log\left(\cos\left(\frac{1}{2}(c+\right.\right.}{2b^3d}$$

input
```
Integrate[Sec[c + d*x]^4/(a + b*Sec[c + d*x])^3,x]
```

output
```
((2*a*(2*a^4 - 5*a^2*b^2 + 6*b^4)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt
[a^2 - b^2]])/(a^2 - b^2)^(5/2) - 2*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2
]] + 2*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] - (a^2*b*(3*b*(a^2 - 2*b^2
) + a*(2*a^2 - 5*b^2)*Cos[c + d*x])*Sin[c + d*x])/((a - b)^2*(a + b)^2*(b
+ a*Cos[c + d*x])^2))/(2*b^3*d)
```

3.507.3 Rubi [A] (verified)

Time = 1.22 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.24, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {3042, 4332, 3042, 4568, 3042, 4486, 3042, 4257, 4318, 3042, 3138, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^4(c+dx)}{(a+b\sec(c+dx))^3}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^4}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \; 4332$$

$$-\frac{\int \frac{\sec(c+dx)\left(a^2-2b\sec(c+dx)a-2\left(a^2-b^2\right)\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^2}dx}{2b\left(a^2-b^2\right)} - \frac{a^2\tan(c+dx)\sec(c+dx)}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(a^2-2b\csc\left(c+dx+\frac{\pi}{2}\right)a-2\left(a^2-b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx}{2b\left(a^2-b^2\right)}-\frac{a^2\tan(c+dx)\sec(c+dx)}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$\Big\downarrow$ 4568

$$-\frac{-\frac{a^2\left(2a^2-5b^2\right)\tan(c+dx)}{bd(a^2-b^2)(a+b\sec(c+dx))}-\frac{\int \frac{\sec(c+dx)\left(2\sec(c+dx)\left(a^2-b^2\right)^2+ab\left(a^2-4b^2\right)\right)}{a+b\sec(c+dx)}dx}{b(a^2-b^2)}}{2b\left(a^2-b^2\right)}-\frac{a^2\tan(c+dx)\sec(c+dx)}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$\Big\downarrow$ 3042

$$-\frac{-\frac{a^2\left(2a^2-5b^2\right)\tan(c+dx)}{bd(a^2-b^2)(a+b\sec(c+dx))}-\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(2\csc\left(c+dx+\frac{\pi}{2}\right)\left(a^2-b^2\right)^2+ab\left(a^2-4b^2\right)\right)}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b(a^2-b^2)}}{2b\left(a^2-b^2\right)}-\frac{a^2\tan(c+dx)\sec(c+dx)}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$\Big\downarrow$ 4486

$$-\frac{-\frac{a^2\left(2a^2-5b^2\right)\tan(c+dx)}{bd(a^2-b^2)(a+b\sec(c+dx))}-\frac{\frac{2\left(a^2-b^2\right)^2\int\sec(c+dx)dx}{b}-\frac{a\left(2a^4-5a^2b^2+6b^4\right)\int\frac{\sec(c+dx)}{a+b\sec(c+dx)}dx}{b}}{b(a^2-b^2)}}{2b\left(a^2-b^2\right)}-\frac{a^2\tan(c+dx)\sec(c+dx)}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$\Big\downarrow$ 3042

$$-\frac{-\frac{a^2\left(2a^2-5b^2\right)\tan(c+dx)}{bd(a^2-b^2)(a+b\sec(c+dx))}-\frac{\frac{2\left(a^2-b^2\right)^2\int\csc\left(c+dx+\frac{\pi}{2}\right)dx}{b}-\frac{a\left(2a^4-5a^2b^2+6b^4\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b}}{b(a^2-b^2)}}{2b\left(a^2-b^2\right)}-\frac{a^2\tan(c+dx)\sec(c+dx)}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$\Big\downarrow$ 4257

$$-\frac{-\frac{a^2\left(2a^2-5b^2\right)\tan(c+dx)}{bd(a^2-b^2)(a+b\sec(c+dx))}-\frac{\frac{2\left(a^2-b^2\right)^2\text{arctanh}(\sin(c+dx))}{bd}-\frac{a\left(2a^4-5a^2b^2+6b^4\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b}}{b(a^2-b^2)}}{2b\left(a^2-b^2\right)}-\frac{a^2\tan(c+dx)\sec(c+dx)}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$\Big\downarrow$ 4318

3.507. $\int \frac{\sec^4(c+dx)}{(a+b\sec(c+dx))^3}\,dx$

$$-\frac{\frac{a^2\left(2a^2-5b^2\right)\tan(c+dx)}{bd(a^2-b^2)(a+b\sec(c+dx))}-\frac{\frac{2\left(a^2-b^2\right)^2\text{arctanh}(\sin(c+dx))}{bd}-\frac{a\left(2a^4-5a^2b^2+6b^4\right)\int\frac{1}{\frac{a\cos(c+dx)}{b}+1}dx}{b^2}}{b(a^2-b^2)}}{2b\left(a^2-b^2\right)}-\frac{a^2\tan(c+dx)\sec(c+dx)}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$\Big\downarrow$ 3042

$$-\frac{\frac{a^2\left(2a^2-5b^2\right)\tan(c+dx)}{bd(a^2-b^2)(a+b\sec(c+dx))}-\frac{\frac{2\left(a^2-b^2\right)^2\text{arctanh}(\sin(c+dx))}{bd}-\frac{a\left(2a^4-5a^2b^2+6b^4\right)\int\frac{1}{\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{b}+1}dx}{b^2}}{b(a^2-b^2)}}{2b\left(a^2-b^2\right)}-\frac{a^2\tan(c+dx)\sec(c+dx)}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$\Big\downarrow$ 3138

$$-\frac{\frac{a^2\left(2a^2-5b^2\right)\tan(c+dx)}{bd(a^2-b^2)(a+b\sec(c+dx))}-\frac{\frac{2\left(a^2-b^2\right)^2\text{arctanh}(\sin(c+dx))}{bd}-\frac{2a\left(2a^4-5a^2b^2+6b^4\right)\int\frac{1}{\left(1-\frac{a}{b}\right)\tan^2\left(\frac{1}{2}(c+dx)\right)+\frac{a+b}{b}}d\tan\left(\frac{1}{2}(c+dx)\right)}{b^2d}}{b(a^2-b^2)}}{2b\left(a^2-b^2\right)}-\frac{a^2\tan(c+dx)\sec(c+dx)}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$\Big\downarrow$ 221

$$-\frac{a^2\tan(c+dx)\sec(c+dx)}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^2}-\frac{\frac{\frac{a^2\left(2a^2-5b^2\right)\tan(c+dx)}{bd(a^2-b^2)(a+b\sec(c+dx))}-\frac{\frac{2\left(a^2-b^2\right)^2\text{arctanh}(\sin(c+dx))}{bd}-\frac{2a\left(2a^4-5a^2b^2+6b^4\right)\text{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{bd\sqrt{a-b}\sqrt{a+b}}}{b(a^2-b^2)}}{2b\left(a^2-b^2\right)}}{}$$

input
```
Int[Sec[c + d*x]^4/(a + b*Sec[c + d*x])^3,x]
```

output
```
-1/2*(a^2*Sec[c + d*x]*Tan[c + d*x])/(b*(a^2 - b^2)*d*(a + b*Sec[c + d*x])
^2) - (-(((2*(a^2 - b^2)^2*ArcTanh[Sin[c + d*x]])/(b*d) - (2*a*(2*a^4 - 5*
a^2*b^2 + 6*b^4)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(Sqr
t[a - b]*b*Sqrt[a + b]*d))/(b*(a^2 - b^2))) + (a^2*(2*a^2 - 5*b^2)*Tan[c +
 d*x])/(b*(a^2 - b^2)*d*(a + b*Sec[c + d*x])))/(2*b*(a^2 - b^2))
```

3.507. $\int \frac{\sec^4(c+dx)}{(a+b\sec(c+dx))^3}\,dx$

3.507.3.1 Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4318
```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[1/b   Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4332
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-a^2)*d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(
m + 1)*((d*Csc[e + f*x])^(n - 3)/(b*f*(m + 1)*(a^2 - b^2))), x] + Simp[d^3/
(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])
^(n - 3)*Simp[a^2*(n - 3) + a*b*(m + 1)*Csc[e + f*x] - (a^2*(n - 2) + b^2*(
m + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2
- b^2, 0] && LtQ[m, -1] && (IGtQ[n, 3] || (IntegersQ[n + 1/2, 2*m] && GtQ[n
, 2]))
```

rule 4486
```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/(csc[(
e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[B/b   Int[Csc[e + f*x],
 x], x] + Simp[(A*b - a*B)/b   Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x
] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0]
```

3.507. $\int \frac{\sec^4(c+dx)}{(a+b\sec(c+dx))^3}\,dx$

rule 4568
```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cot[e + f*x]*((a + b*Csc[e + f*x]
)^(m + 1)/(b*f*(m + 1)*(a^2 - b^2))), x] + Simp[1/(b*(m + 1)*(a^2 - b^2))
 Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m
+ 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b - a*B + b*C)*(m + 1))*Csc[e + f*x],
x], x], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^
2, 0]
```

3.507.4 Maple [A] (verified)

Time = 0.92 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.36

method	result
derivativedivides	$\frac{-\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{b^3}+\frac{2a\left(\frac{\frac{\left(2a^2-ab-6b^2\right)ab\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{2(a-b)\left(a^2+2ab+b^2\right)}-\frac{\left(2a^2+ab-6b^2\right)ab\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2(a+b)\left(a^2-2ab+b^2\right)}}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2b-a-b\right)^2}-\frac{\left(2a^4-5a^2b^2+6b^4\right)\operatorname{arctanh}\left(\frac{(a-b}{\sqrt{}}\right.}{2\left(a^4-2a^2b^2+b^4\right)\sqrt{(a-b}}\right.}{b^3}}{d}$
default	$\frac{-\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{b^3}+\frac{2a\left(\frac{\frac{\left(2a^2-ab-6b^2\right)ab\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{2(a-b)\left(a^2+2ab+b^2\right)}-\frac{\left(2a^2+ab-6b^2\right)ab\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2(a+b)\left(a^2-2ab+b^2\right)}}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2b-a-b\right)^2}-\frac{\left(2a^4-5a^2b^2+6b^4\right)\operatorname{arctanh}\left(\frac{(a-b}{\sqrt{}}\right.}{2\left(a^4-2a^2b^2+b^4\right)\sqrt{(a-b}}\right.}{b^3}}{d}$
risch	$\frac{ia\left(-a^3b\,e^{3i(dx+c)}+4a\,b^3e^{3i(dx+c)}-2a^4e^{2i(dx+c)}+a^2b^2e^{2i(dx+c)}+10b^4e^{2i(dx+c)}-7a^3b\,e^{i(dx+c)}+16b^3a\,e^{i(dx+c)}-2a^4+5\right.}{(-a^2+b^2)^2d\,b^2\left(a\,e^{2i(dx+c)}+2b\,e^{i(dx+c)}+a\right)^2}$

input
```
int(sec(d*x+c)^4/(a+b*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/b^3*ln(tan(1/2*d*x+1/2*c)-1)+2*a/b^3*((1/2*(2*a^2-a*b-6*b^2)*a*b/(
a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3-1/2*(2*a^2+a*b-6*b^2)*a*b/(a+b)/
(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c))/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/
2*c)^2*b-a-b)^2-1/2*(2*a^4-5*a^2*b^2+6*b^4)/(a^4-2*a^2*b^2+b^4)/((a-b)*(a+
b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a-b)*(a+b))^(1/2)))+1/b^3*ln(
tan(1/2*d*x+1/2*c)+1))
```

3.507.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 547 vs. $2(175) = 350$.

Time = 0.84 (sec) , antiderivative size = 1153, normalized size of antiderivative = 6.13

$$\int \frac{\sec^4(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(sec(d*x+c)^4/(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
[1/4*((2*a^5*b^2 - 5*a^3*b^4 + 6*a*b^6 + (2*a^7 - 5*a^5*b^2 + 6*a^3*b^4)*c
os(d*x + c)^2 + 2*(2*a^6*b - 5*a^4*b^3 + 6*a^2*b^5)*cos(d*x + c))*sqrt(a^2
 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 - 2*sqrt(a^
2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c
)^2 + 2*a*b*cos(d*x + c) + b^2)) + 2*(a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^
8 + (a^8 - 3*a^6*b^2 + 3*a^4*b^4 - a^2*b^6)*cos(d*x + c)^2 + 2*(a^7*b - 3*
a^5*b^3 + 3*a^3*b^5 - a*b^7)*cos(d*x + c))*log(sin(d*x + c) + 1) - 2*(a^6*
b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8 + (a^8 - 3*a^6*b^2 + 3*a^4*b^4 - a^2*b^6
)*cos(d*x + c)^2 + 2*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*cos(d*x + c))
*log(-sin(d*x + c) + 1) - 2*(3*a^6*b^2 - 9*a^4*b^4 + 6*a^2*b^6 + (2*a^7*b
- 7*a^5*b^3 + 5*a^3*b^5)*cos(d*x + c))*sin(d*x + c))/((a^8*b^3 - 3*a^6*b^5
 + 3*a^4*b^7 - a^2*b^9)*d*cos(d*x + c)^2 + 2*(a^7*b^4 - 3*a^5*b^6 + 3*a^3*
b^8 - a*b^10)*d*cos(d*x + c) + (a^6*b^5 - 3*a^4*b^7 + 3*a^2*b^9 - b^11)*d)
, -1/2*((2*a^5*b^2 - 5*a^3*b^4 + 6*a*b^6 + (2*a^7 - 5*a^5*b^2 + 6*a^3*b^4)
*cos(d*x + c)^2 + 2*(2*a^6*b - 5*a^4*b^3 + 6*a^2*b^5)*cos(d*x + c))*sqrt(-
a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(
d*x + c))) - (a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8 + (a^8 - 3*a^6*b^2 + 3
*a^4*b^4 - a^2*b^6)*cos(d*x + c)^2 + 2*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*
b^7)*cos(d*x + c))*log(sin(d*x + c) + 1) + (a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^
6 - b^8 + (a^8 - 3*a^6*b^2 + 3*a^4*b^4 - a^2*b^6)*cos(d*x + c)^2 + 2*(a...
```

3.507.6 Sympy [F]

$$\int \frac{\sec^4(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \int \frac{\sec^4(c+dx)}{(a+b\sec(c+dx))^3}\,dx$$

input
```
integrate(sec(d*x+c)**4/(a+b*sec(d*x+c))**3,x)
```

output
```
Integral(sec(c + d*x)**4/(a + b*sec(c + d*x))**3, x)
```

3.507.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\sec^4(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(sec(d*x+c)^4/(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

3.507.8 Giac [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 347, normalized size of antiderivative = 1.85

$$\int \frac{\sec^4(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \frac{\frac{(2a^5-5a^3b^2+6ab^4)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(2a-2b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{-a^2+b^2}}\right)\right)}{(a^4b^3-2a^2b^5+b^7)\sqrt{-a^2+b^2}} + \frac{2a^5\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-3a^4b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-}{}}{}$$

input
```
integrate(sec(d*x+c)^4/(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
((2*a^5 - 5*a^3*b^2 + 6*a*b^4)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(2*a -
 2*b) + arctan((a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2
 + b^2)))/((a^4*b^3 - 2*a^2*b^5 + b^7)*sqrt(-a^2 + b^2)) + (2*a^5*tan(1/2*
d*x + 1/2*c)^3 - 3*a^4*b*tan(1/2*d*x + 1/2*c)^3 - 5*a^3*b^2*tan(1/2*d*x +
1/2*c)^3 + 6*a^2*b^3*tan(1/2*d*x + 1/2*c)^3 - 2*a^5*tan(1/2*d*x + 1/2*c) -
 3*a^4*b*tan(1/2*d*x + 1/2*c) + 5*a^3*b^2*tan(1/2*d*x + 1/2*c) + 6*a^2*b^3
*tan(1/2*d*x + 1/2*c))/((a^4*b^2 - 2*a^2*b^4 + b^6)*(a*tan(1/2*d*x + 1/2*c
)^2 - b*tan(1/2*d*x + 1/2*c)^2 - a - b)^2) + log(abs(tan(1/2*d*x + 1/2*c)
+ 1))/b^3 - log(abs(tan(1/2*d*x + 1/2*c) - 1))/b^3)/d
```

3.507.9 Mupad [B] (verification not implemented)

Time = 22.43 (sec) , antiderivative size = 5078, normalized size of antiderivative = 27.01

$$\int \frac{\sec^4(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
int(1/(cos(c + d*x)^4*(a + b/cos(c + d*x))^3),x)
```

output
```
- (atan((((((8*(12*a*b^14 - 4*b^15 + 8*a^2*b^13 - 34*a^3*b^12 - 6*a^4*b^11
 + 36*a^5*b^10 + 4*a^6*b^9 - 18*a^7*b^8 - 2*a^8*b^7 + 4*a^9*b^6))/(a*b^12
+ b^13 - 3*a^2*b^11 - 3*a^3*b^10 + 3*a^4*b^9 + 3*a^5*b^8 - a^6*b^7 - a^7*b
^6) - (8*tan(c/2 + (d*x)/2)*(8*a*b^15 - 8*a^2*b^14 - 32*a^3*b^13 + 32*a^4*
b^12 + 48*a^5*b^11 - 48*a^6*b^10 - 32*a^7*b^9 + 32*a^8*b^8 + 8*a^9*b^7 - 8
*a^10*b^6))/(b^3*(a*b^10 + b^11 - 3*a^2*b^9 - 3*a^3*b^8 + 3*a^4*b^7 + 3*a^
5*b^6 - a^6*b^5 - a^7*b^4)))/b^3 - (8*tan(c/2 + (d*x)/2)*(8*a^10 - 8*a^9*b
 - 8*a*b^9 + 4*b^10 + 24*a^2*b^8 + 32*a^3*b^7 - 52*a^4*b^6 - 48*a^5*b^5 +
57*a^6*b^4 + 32*a^7*b^3 - 32*a^8*b^2))/(a*b^10 + b^11 - 3*a^2*b^9 - 3*a^3*
b^8 + 3*a^4*b^7 + 3*a^5*b^6 - a^6*b^5 - a^7*b^4))*1i)/b^3 - ((((8*(12*a*b^
14 - 4*b^15 + 8*a^2*b^13 - 34*a^3*b^12 - 6*a^4*b^11 + 36*a^5*b^10 + 4*a^6*
b^9 - 18*a^7*b^8 - 2*a^8*b^7 + 4*a^9*b^6))/(a*b^12 + b^13 - 3*a^2*b^11 - 3
*a^3*b^10 + 3*a^4*b^9 + 3*a^5*b^8 - a^6*b^7 - a^7*b^6) + (8*tan(c/2 + (d*x
)/2)*(8*a*b^15 - 8*a^2*b^14 - 32*a^3*b^13 + 32*a^4*b^12 + 48*a^5*b^11 - 48
*a^6*b^10 - 32*a^7*b^9 + 32*a^8*b^8 + 8*a^9*b^7 - 8*a^10*b^6))/(b^3*(a*b^1
0 + b^11 - 3*a^2*b^9 - 3*a^3*b^8 + 3*a^4*b^7 + 3*a^5*b^6 - a^6*b^5 - a^7*b
^4)))/b^3 + (8*tan(c/2 + (d*x)/2)*(8*a^10 - 8*a^9*b - 8*a*b^9 + 4*b^10 + 2
4*a^2*b^8 + 32*a^3*b^7 - 52*a^4*b^6 - 48*a^5*b^5 + 57*a^6*b^4 + 32*a^7*b^3
 - 32*a^8*b^2))/(a*b^10 + b^11 - 3*a^2*b^9 - 3*a^3*b^8 + 3*a^4*b^7 + 3*a^5
*b^6 - a^6*b^5 - a^7*b^4))*1i)/b^3)/((((8*(12*a*b^14 - 4*b^15 + 8*a^2*b...
```

3.508 $\int \frac{\sec^3(c+dx)}{(a+b\sec(c+dx))^3}\,dx$

3.508.1 Optimal result . . . 3772
3.508.2 Mathematica [A] (verified) . . . 3772
3.508.3 Rubi [A] (verified) . . . 3773
3.508.4 Maple [A] (verified) . . . 3776
3.508.5 Fricas [A] (verification not implemented) . . . 3777
3.508.6 Sympy [F] . . . 3777
3.508.7 Maxima [F(-2)] . . . 3778
3.508.8 Giac [A] (verification not implemented) . . . 3778
3.508.9 Mupad [B] (verification not implemented) . . . 3779

3.508.1 Optimal result

Integrand size = 21, antiderivative size = 149

$$\int \frac{\sec^3(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \frac{(a^2+2b^2)\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}d} - \frac{a^2\tan(c+dx)}{2b\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} + \frac{a(a^2-4b^2)\tan(c+dx)}{2b\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))}$$

output
```
(a^2+2*b^2)*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/(a-b)^(5/2
)/(a+b)^(5/2)/d-1/2*a^2*tan(d*x+c)/b/(a^2-b^2)/d/(a+b*sec(d*x+c))^2+1/2*a*
(a^2-4*b^2)*tan(d*x+c)/b/(a^2-b^2)^2/d/(a+b*sec(d*x+c))
```

3.508.2 Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.76

$$\int \frac{\sec^3(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \frac{-\frac{2(a^2+2b^2)\operatorname{arctanh}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{5/2}} + \frac{a\left(a^2-4b^2-3ab\cos(c+dx)\right)\sin(c+dx)}{(a-b)^2(a+b)^2(b+a\cos(c+dx))^2}}{2d}$$

input
```
Integrate[Sec[c + d*x]^3/(a + b*Sec[c + d*x])^3,x]
```

output
```
((-2*(a^2 + 2*b^2)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(
a^2 - b^2)^(5/2) + (a*(a^2 - 4*b^2 - 3*a*b*Cos[c + d*x])*Sin[c + d*x])/((a
 - b)^2*(a + b)^2*(b + a*Cos[c + d*x])^2))/(2*d)
```

3.508.3 Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.17, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {3042, 4326, 25, 3042, 4491, 25, 27, 3042, 4318, 3042, 3138, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^3(c+dx)}{(a+b\sec(c+dx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^3}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \text{4326}$$

$$-\frac{\int -\frac{\sec(c+dx)\left(2ab+\left(a^2-2b^2\right)\sec(c+dx)\right)}{(a+b\sec(c+dx))^2}dx}{2b\left(a^2-b^2\right)}-\frac{a^2\tan(c+dx)}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\sec(c+dx)\left(2ab+\left(a^2-2b^2\right)\sec(c+dx)\right)}{(a+b\sec(c+dx))^2}dx}{2b\left(a^2-b^2\right)}-\frac{a^2\tan(c+dx)}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(2ab+\left(a^2-2b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx}{2b\left(a^2-b^2\right)}-\frac{a^2\tan(c+dx)}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$$\Big\downarrow \text{4491}$$

$$\frac{\frac{a\left(a^2-4b^2\right)\tan(c+dx)}{d\left(a^2-b^2\right)(a+b\sec(c+dx))}-\frac{\int -\frac{b\left(a^2+2b^2\right)\sec(c+dx)}{a+b\sec(c+dx)}dx}{a^2-b^2}}{2b\left(a^2-b^2\right)}-\frac{a^2\tan(c+dx)}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{b\left(a^2+2b^2\right)\sec(c+dx)}{a+b\sec(c+dx)}dx}{a^2-b^2}+\frac{a\left(a^2-4b^2\right)\tan(c+dx)}{d(a^2-b^2)(a+b\sec(c+dx))}}{2b\left(a^2-b^2\right)}-\frac{a^2\tan(c+dx)}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$\Big\downarrow$ 27

$$\frac{\frac{b\left(a^2+2b^2\right)\int \frac{\sec(c+dx)}{a+b\sec(c+dx)}dx}{a^2-b^2}+\frac{a\left(a^2-4b^2\right)\tan(c+dx)}{d(a^2-b^2)(a+b\sec(c+dx))}}{2b\left(a^2-b^2\right)}-\frac{a^2\tan(c+dx)}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$\Big\downarrow$ 3042

$$\frac{\frac{b\left(a^2+2b^2\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2-b^2}+\frac{a\left(a^2-4b^2\right)\tan(c+dx)}{d(a^2-b^2)(a+b\sec(c+dx))}}{2b\left(a^2-b^2\right)}-\frac{a^2\tan(c+dx)}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$\Big\downarrow$ 4318

$$\frac{\frac{\left(a^2+2b^2\right)\int \frac{1}{\frac{a\cos(c+dx)}{b}+1}dx}{a^2-b^2}+\frac{a\left(a^2-4b^2\right)\tan(c+dx)}{d(a^2-b^2)(a+b\sec(c+dx))}}{2b\left(a^2-b^2\right)}-\frac{a^2\tan(c+dx)}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\left(a^2+2b^2\right)\int \frac{1}{\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{b}+1}dx}{a^2-b^2}+\frac{a\left(a^2-4b^2\right)\tan(c+dx)}{d(a^2-b^2)(a+b\sec(c+dx))}}{2b\left(a^2-b^2\right)}-\frac{a^2\tan(c+dx)}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$\Big\downarrow$ 3138

$$\frac{\frac{2\left(a^2+2b^2\right)\int \frac{1}{\left(1-\frac{a}{b}\right)\tan^2\left(\frac{1}{2}(c+dx)\right)+\frac{a+b}{b}}d\tan\left(\frac{1}{2}(c+dx)\right)}{d(a^2-b^2)}+\frac{a\left(a^2-4b^2\right)\tan(c+dx)}{d(a^2-b^2)(a+b\sec(c+dx))}}{2b\left(a^2-b^2\right)}-\frac{a^2\tan(c+dx)}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$\Big\downarrow$ 221

$$\frac{\frac{2b\left(a^2+2b^2\right)\text{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d\sqrt{a-b}\sqrt{a+b}(a^2-b^2)}+\frac{a\left(a^2-4b^2\right)\tan(c+dx)}{d(a^2-b^2)(a+b\sec(c+dx))}}{2b\left(a^2-b^2\right)}-\frac{a^2\tan(c+dx)}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

input
```
Int[Sec[c + d*x]^3/(a + b*Sec[c + d*x])^3,x]
```

3.508. $\int \frac{\sec^3(c+dx)}{(a+b\sec(c+dx))^3}\,dx$

output
```
-1/2*(a^2*Tan[c + d*x])/(b*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^2) + ((2*b*(
a^2 + 2*b^2)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(Sqrt[a
- b]*Sqrt[a + b]*(a^2 - b^2)*d) + (a*(a^2 - 4*b^2)*Tan[c + d*x])/((a^2 - b
^2)*d*(a + b*Sec[c + d*x])))/(2*b*(a^2 - b^2))
```

3.508.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 4318
```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[1/b   Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4326
```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(-a^2)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m
 + 1)*(a^2 - b^2))), x] + Simp[1/(b*(m + 1)*(a^2 - b^2))   Int[Csc[e + f*x]
*(a + b*Csc[e + f*x])^(m + 1)*Simp[a*b*(m + 1) - (a^2 + b^2*(m + 1))*Csc[e
+ f*x], x], x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m,
 -1]
```

3.508. $\int \frac{\sec^3(c+dx)}{(a+b\sec(c+dx))^3}\,dx$

rule 4491
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b - a*B))*Cot[e
 + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Simp[1
/((m + 1)*(a^2 - b^2))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp
[(a*A - b*B)*(m + 1) - (A*b - a*B)*(m + 2)*Csc[e + f*x], x], x], x] /; Free
Q[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m
, -1]
```

3.508.4 Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.23

method	result
derivativedivides	$\frac{-\frac{2\left(-\frac{(4b+a)a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{2(a-b)\left(a^2+2ab+b^2\right)}-\frac{(a-4b)a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2(a+b)\left(a^2-2ab+b^2\right)}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2b-a-b\right)^2}+\frac{\left(a^2+2b^2\right)\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{\left(a^4-2a^2b^2+b^4\right)\sqrt{(a-b)(a+b)}}}{d}$
default	$\frac{-\frac{2\left(-\frac{(4b+a)a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{2(a-b)\left(a^2+2ab+b^2\right)}-\frac{(a-4b)a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2(a+b)\left(a^2-2ab+b^2\right)}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2b-a-b\right)^2}+\frac{\left(a^2+2b^2\right)\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{\left(a^4-2a^2b^2+b^4\right)\sqrt{(a-b)(a+b)}}}{d}$
risch	$-\frac{i\left(a^3e^{3i(dx+c)}+2a\,b^2e^{3i(dx+c)}+3a^2b\,e^{2i(dx+c)}+6b^3e^{2i(dx+c)}-a^3e^{i(dx+c)}+10b^2a\,e^{i(dx+c)}+3a^2b\right)}{d(-a^2+b^2)^2\left(a\,e^{2i(dx+c)}+2b\,e^{i(dx+c)}+a\right)^2}+\frac{a^2\ln\left(e^{i(dx+c)}\right.}{2\sqrt{a^2-b^2}}$

input
```
int(sec(d*x+c)^3/(a+b*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-2*(-1/2*(4*b+a)*a/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3-1/2*(a-
4*b)*a/(a+b)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c))/(tan(1/2*d*x+1/2*c)^2*a-t
an(1/2*d*x+1/2*c)^2*b-a-b)^2+(a^2+2*b^2)/(a^4-2*a^2*b^2+b^4)/((a-b)*(a+b))
^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a-b)*(a+b))^(1/2)))
```

3.508.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 594, normalized size of antiderivative = 3.99

$$\int \frac{\sec^3(c+dx)}{(a+b\sec(c+dx))^3}\,dx$$
$$= \left[\frac{\left(a^2b^2+2\,b^4+\left(a^4+2\,a^2b^2\right)\cos\left(dx+c\right)^2+2\left(a^3b+2\,ab^3\right)\cos\left(dx+c\right)\right)\sqrt{a^2-b^2}\log\left(\frac{2\,ab\cos(dx+c)-(a^2-2\,b}{}\right.}{4\left(\left(a^8-3\,a^6b^2+3\,a^4b^4-a^2b^6\right)d\cos\left(dx+c\right)^2+2\left(a^7b-3\right.\right.}\right.$$

input
```
integrate(sec(d*x+c)^3/(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
[1/4*((a^2*b^2 + 2*b^4 + (a^4 + 2*a^2*b^2)*cos(d*x + c)^2 + 2*(a^3*b + 2*a
*b^3)*cos(d*x + c))*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2
)*cos(d*x + c)^2 + 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2
*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) + 2*(a^5 - 5*
a^3*b^2 + 4*a*b^4 - 3*(a^4*b - a^2*b^3)*cos(d*x + c))*sin(d*x + c))/((a^8
- 3*a^6*b^2 + 3*a^4*b^4 - a^2*b^6)*d*cos(d*x + c)^2 + 2*(a^7*b - 3*a^5*b^3
 + 3*a^3*b^5 - a*b^7)*d*cos(d*x + c) + (a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 -
b^8)*d), 1/2*((a^2*b^2 + 2*b^4 + (a^4 + 2*a^2*b^2)*cos(d*x + c)^2 + 2*(a^3
*b + 2*a*b^3)*cos(d*x + c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*c
os(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c))) + (a^5 - 5*a^3*b^2 + 4*a*b^4
- 3*(a^4*b - a^2*b^3)*cos(d*x + c))*sin(d*x + c))/((a^8 - 3*a^6*b^2 + 3*a^
4*b^4 - a^2*b^6)*d*cos(d*x + c)^2 + 2*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b
^7)*d*cos(d*x + c) + (a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*d)]
```

3.508.6 Sympy [F]

$$\int \frac{\sec^3(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \int \frac{\sec^3\left(c+dx\right)}{\left(a+b\sec\left(c+dx\right)\right)^3}\,dx$$

input
```
integrate(sec(d*x+c)**3/(a+b*sec(d*x+c))**3,x)
```

output
```
Integral(sec(c + d*x)**3/(a + b*sec(c + d*x))**3, x)
```

3.508.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\sec^3(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(sec(d*x+c)^3/(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

3.508.8 Giac [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.70

$$\int \frac{\sec^3(c+dx)}{(a+b\sec(c+dx))^3}\,dx = -\frac{\frac{\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(2a-2b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{-a^2+b^2}}\right)\right)(a^2+2b^2)}{(a^4-2a^2b^2+b^4)\sqrt{-a^2+b^2}} - \frac{a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+3a^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-4ab^2\tan}{(a^4-2a^2b^2+b^4)\left(a\right.}}{d}$$

input
```
integrate(sec(d*x+c)^3/(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
-((pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(2*a - 2*b) + arctan((a*tan(1/2*d*x
 + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))*(a^2 + 2*b^2)/((a^4
 - 2*a^2*b^2 + b^4)*sqrt(-a^2 + b^2)) - (a^3*tan(1/2*d*x + 1/2*c)^3 + 3*a^
2*b*tan(1/2*d*x + 1/2*c)^3 - 4*a*b^2*tan(1/2*d*x + 1/2*c)^3 + a^3*tan(1/2*
d*x + 1/2*c) - 3*a^2*b*tan(1/2*d*x + 1/2*c) - 4*a*b^2*tan(1/2*d*x + 1/2*c)
)/((a^4 - 2*a^2*b^2 + b^4)*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2
*c)^2 - a - b)^2))/d
```

3.508.9 Mupad [B] (verification not implemented)

Time = 16.04 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.37

$$\int \frac{\sec^3(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \frac{\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,(a^2+4\,b\,a)}{(a+b)^2\,(a-b)}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,(4\,a\,b-a^2)}{(a+b)\,(a^2-2\,a\,b+b^2)}}{d\,\left(2\,a\,b-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,(2\,a^2-2\,b^2)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,(a^2-2\,a\,b+b^2)+a^2+b^2\right)} + \frac{\operatorname{atanh}\left(\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,(2\,a-2\,b)\,(a^2-2\,a\,b+b^2)}{2\,\sqrt{a+b}\,(a-b)^{5/2}}\right)\,(a^2+2\,b^2)}{d\,(a+b)^{5/2}\,(a-b)^{5/2}}$$

input
```
int(1/(cos(c + d*x)^3*(a + b/cos(c + d*x))^3),x)
```

output
```
((tan(c/2 + (d*x)/2)^3*(4*a*b + a^2))/((a + b)^2*(a - b)) - (tan(c/2 + (d*
x)/2)*(4*a*b - a^2))/((a + b)*(a^2 - 2*a*b + b^2)))/(d*(2*a*b - tan(c/2 +
(d*x)/2)^2*(2*a^2 - 2*b^2) + tan(c/2 + (d*x)/2)^4*(a^2 - 2*a*b + b^2) + a^
2 + b^2)) + (atanh((tan(c/2 + (d*x)/2)*(2*a - 2*b)*(a^2 - 2*a*b + b^2))/(2
*(a + b)^(1/2)*(a - b)^(5/2)))*(a^2 + 2*b^2))/(d*(a + b)^(5/2)*(a - b)^(5/
2))
```

3.509 $\int \frac{\sec^2(c+dx)}{(a+b\sec(c+dx))^3}\,dx$

3.509.1 Optimal result . . . 3780
3.509.2 Mathematica [A] (verified) . . . 3780
3.509.3 Rubi [A] (verified) . . . 3781
3.509.4 Maple [A] (verified) . . . 3784
3.509.5 Fricas [B] (verification not implemented) . . . 3784
3.509.6 Sympy [F] . . . 3785
3.509.7 Maxima [F(-2)] . . . 3785
3.509.8 Giac [B] (verification not implemented) . . . 3786
3.509.9 Mupad [B] (verification not implemented) . . . 3786

3.509.1 Optimal result

Integrand size = 21, antiderivative size = 134

$$\int \frac{\sec^2(c+dx)}{(a+b\sec(c+dx))^3}\,dx = -\frac{3ab\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}d} + \frac{a\tan(c+dx)}{2\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} + \frac{\left(a^2+2b^2\right)\tan(c+dx)}{2\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))}$$

output
```
-3*a*b*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/(a-b)^(5/2)/(a+
b)^(5/2)/d+1/2*a*tan(d*x+c)/(a^2-b^2)/d/(a+b*sec(d*x+c))^2+1/2*(a^2+2*b^2)
*tan(d*x+c)/(a^2-b^2)^2/d/(a+b*sec(d*x+c))
```

3.509.2 Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.86

$$\int \frac{\sec^2(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \frac{\frac{6ab\operatorname{arctanh}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} + \frac{\left(b\left(a^2+2b^2\right)+a\left(2a^2+b^2\right)\cos(c+dx)\right)\sin(c+dx)}{(b+a\cos(c+dx))^2}}{2(a-b)^2(a+b)^2d}$$

input
```
Integrate[Sec[c + d*x]^2/(a + b*Sec[c + d*x])^3,x]
```

output
```
((6*a*b*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b
^2] + ((b*(a^2 + 2*b^2) + a*(2*a^2 + b^2)*Cos[c + d*x])*Sin[c + d*x])/(b +
 a*Cos[c + d*x])^2)/(2*(a - b)^2*(a + b)^2*d)
```

3.509.3 Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.19, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.524$, Rules used = {3042, 4323, 25, 3042, 4491, 27, 3042, 4318, 3042, 3138, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(c+dx)}{(a+b\sec(c+dx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \text{4323}$$

$$\frac{\int -\frac{\sec(c+dx)(2b-a\sec(c+dx))}{(a+b\sec(c+dx))^2}dx}{2\left(a^2-b^2\right)}+\frac{a\tan(c+dx)}{2d\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{a\tan(c+dx)}{2d\left(a^2-b^2\right)(a+b\sec(c+dx))^2}-\frac{\int \frac{\sec(c+dx)(2b-a\sec(c+dx))}{(a+b\sec(c+dx))^2}dx}{2\left(a^2-b^2\right)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a\tan(c+dx)}{2d\left(a^2-b^2\right)(a+b\sec(c+dx))^2}-\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(2b-a\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx}{2\left(a^2-b^2\right)}$$

$$\Big\downarrow \text{4491}$$

$$\frac{a\tan(c+dx)}{2d\left(a^2-b^2\right)(a+b\sec(c+dx))^2}-\frac{-\frac{\int -\frac{3ab\sec(c+dx)}{a+b\sec(c+dx)}dx}{a^2-b^2}-\frac{\left(a^2+2b^2\right)\tan(c+dx)}{d(a^2-b^2)(a+b\sec(c+dx))}}{2\left(a^2-b^2\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{a\tan(c+dx)}{2d\left(a^2-b^2\right)(a+b\sec(c+dx))^2}-\frac{\frac{3ab\int\frac{\sec(c+dx)}{a+b\sec(c+dx)}dx}{a^2-b^2}-\frac{\left(a^2+2b^2\right)\tan(c+dx)}{d(a^2-b^2)(a+b\sec(c+dx))}}{2\left(a^2-b^2\right)}$$

$\Big\downarrow$ 3042

$$\frac{a\tan(c+dx)}{2d\left(a^2-b^2\right)(a+b\sec(c+dx))^2}-\frac{\frac{3ab\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2-b^2}-\frac{\left(a^2+2b^2\right)\tan(c+dx)}{d(a^2-b^2)(a+b\sec(c+dx))}}{2\left(a^2-b^2\right)}$$

$\Big\downarrow$ 4318

$$\frac{a\tan(c+dx)}{2d\left(a^2-b^2\right)(a+b\sec(c+dx))^2}-\frac{\frac{3a\int\frac{1}{\frac{a\cos(c+dx)}{b}+1}dx}{a^2-b^2}-\frac{\left(a^2+2b^2\right)\tan(c+dx)}{d(a^2-b^2)(a+b\sec(c+dx))}}{2\left(a^2-b^2\right)}$$

$\Big\downarrow$ 3042

$$\frac{a\tan(c+dx)}{2d\left(a^2-b^2\right)(a+b\sec(c+dx))^2}-\frac{\frac{3a\int\frac{1}{\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{b}+1}dx}{a^2-b^2}-\frac{\left(a^2+2b^2\right)\tan(c+dx)}{d(a^2-b^2)(a+b\sec(c+dx))}}{2\left(a^2-b^2\right)}$$

$\Big\downarrow$ 3138

$$\frac{a\tan(c+dx)}{2d\left(a^2-b^2\right)(a+b\sec(c+dx))^2}-\frac{\frac{6a\int\frac{1}{\left(1-\frac{a}{b}\right)\tan^2\left(\frac{1}{2}(c+dx)\right)+\frac{a+b}{b}}d\tan\left(\frac{1}{2}(c+dx)\right)}{d(a^2-b^2)}-\frac{\left(a^2+2b^2\right)\tan(c+dx)}{d(a^2-b^2)(a+b\sec(c+dx))}}{2\left(a^2-b^2\right)}$$

$\Big\downarrow$ 221

$$\frac{a\tan(c+dx)}{2d\left(a^2-b^2\right)(a+b\sec(c+dx))^2}-\frac{\frac{6ab\text{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d\sqrt{a-b}\sqrt{a+b}(a^2-b^2)}-\frac{\left(a^2+2b^2\right)\tan(c+dx)}{d(a^2-b^2)(a+b\sec(c+dx))}}{2\left(a^2-b^2\right)}$$

input
```
Int[Sec[c + d*x]^2/(a + b*Sec[c + d*x])^3,x]
```

output
```
(a*Tan[c + d*x])/(2*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^2) - ((6*a*b*ArcTan
h[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(Sqrt[a - b]*Sqrt[a + b]*(a
^2 - b^2)*d) - ((a^2 + 2*b^2)*Tan[c + d*x])/((a^2 - b^2)*d*(a + b*Sec[c +
d*x])))/(2*(a^2 - b^2))
```

3.509. $\int \frac{\sec^2(c+dx)}{(a+b\sec(c+dx))^3}\,dx$

3.509.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 4318
```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[1/b   Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4323
```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[a*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(f*(m + 1)*(
a^2 - b^2))), x] - Simp[1/((m + 1)*(a^2 - b^2))   Int[Csc[e + f*x]*(a + b*C
sc[e + f*x])^(m + 1)*(b*(m + 1) - a*(m + 2)*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

rule 4491
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b - a*B))*Cot[e
 + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Simp[1
/((m + 1)*(a^2 - b^2))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp
[(a*A - b*B)*(m + 1) - (A*b - a*B)*(m + 2)*Csc[e + f*x], x], x], x] /; Free
Q[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m
, -1]
```

3.509. $\int \frac{\sec^2(c+dx)}{(a+b\sec(c+dx))^3}\,dx$

3.509.4 Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.46

method	result
derivativedivides	$\frac{\frac{-\frac{\left(2a^2+ab+2b^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{(a-b)\left(a^2+2ab+b^2\right)}+\frac{\left(2a^2-ab+2b^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{(a+b)\left(a^2-2ab+b^2\right)}}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2b-a-b\right)^2}-\frac{3ab\,\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{\left(a^4-2a^2b^2+b^4\right)\sqrt{(a-b)(a+b)}}}{d}$
default	$\frac{\frac{-\frac{\left(2a^2+ab+2b^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{(a-b)\left(a^2+2ab+b^2\right)}+\frac{\left(2a^2-ab+2b^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{(a+b)\left(a^2-2ab+b^2\right)}}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2b-a-b\right)^2}-\frac{3ab\,\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{\left(a^4-2a^2b^2+b^4\right)\sqrt{(a-b)(a+b)}}}{d}$
risch	$\frac{i\left(3a^3b\,e^{3i(dx+c)}+2a^4e^{2i(dx+c)}+5a^2b^2e^{2i(dx+c)}+2b^4e^{2i(dx+c)}+5a^3b\,e^{i(dx+c)}+4b^3a\,e^{i(dx+c)}+2a^4+a^2b^2\right)}{a(-a^2+b^2)^2d\left(a\,e^{2i(dx+c)}+2b\,e^{i(dx+c)}+a\right)^2}+\frac{3ab\ln\left(e^{i}\right.}{2\sqrt{}}$

input
```
int(sec(d*x+c)^2/(a+b*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(2*(-1/2*(2*a^2+a*b+2*b^2)/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3+
1/2*(2*a^2-a*b+2*b^2)/(a+b)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c))/(tan(1/2*d
*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2-3*a*b/(a^4-2*a^2*b^2+b^4)/((a-
b)*(a+b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a-b)*(a+b))^(1/2)))
```

3.509.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 254 vs. 2(121) = 242.

Time = 0.31 (sec) , antiderivative size = 565, normalized size of antiderivative = 4.22

$$\int \frac{\sec^2(c+dx)}{(a+b\sec(c+dx))^3}\,dx$$
$$= \left[\frac{3\left(a^3b\cos\left(dx+c\right)^2+2\,a^2b^2\cos\left(dx+c\right)+ab^3\right)\sqrt{a^2-b^2}\log\left(\frac{2\,ab\cos(dx+c)-\left(a^2-2\,b^2\right)\cos(dx+c)^2-2\,\sqrt{a^2-b^2}(b\cos(}{a^2\cos(dx+c)^2+2\,ab\cos(dx+c)}\right.}{4\left(\left(a^8-3\,a^6b^2+3\,a^4b^4-a^2b^6\right)d\cos\left(dx+c\right)^2+2\left(a^7b-3\,a^5b^3+3\,a\right.\right.}\right.$$
$$-\frac{3\left(a^3b\cos\left(dx+c\right)^2+2\,a^2b^2\cos\left(dx+c\right)+ab^3\right)\sqrt{-a^2+b^2}\arctan\left(-\frac{\sqrt{-a^2+b^2}(b\cos(dx+c)+a)}{(a^2-b^2)\sin(dx+c)}\right)-\left(a^4b+a^2\right.}{2\left(\left(a^8-3\,a^6b^2+3\,a^4b^4-a^2b^6\right)d\cos\left(dx+c\right)^2+2\left(a^7b-3\,a^5b^3+3\,a^3b^5-ab^7\right)d\cos\left(d\right.\right.}$$

input
```
integrate(sec(d*x+c)^2/(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
[1/4*(3*(a^3*b*cos(d*x + c)^2 + 2*a^2*b^2*cos(d*x + c) + a*b^3)*sqrt(a^2 -
 b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 - 2*sqrt(a^2
- b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^
2 + 2*a*b*cos(d*x + c) + b^2)) + 2*(a^4*b + a^2*b^3 - 2*b^5 + (2*a^5 - a^3
*b^2 - a*b^4)*cos(d*x + c))*sin(d*x + c))/((a^8 - 3*a^6*b^2 + 3*a^4*b^4 -
a^2*b^6)*d*cos(d*x + c)^2 + 2*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*d*co
s(d*x + c) + (a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*d), -1/2*(3*(a^3*b*co
s(d*x + c)^2 + 2*a^2*b^2*cos(d*x + c) + a*b^3)*sqrt(-a^2 + b^2)*arctan(-sq
rt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c))) - (a^4*b +
 a^2*b^3 - 2*b^5 + (2*a^5 - a^3*b^2 - a*b^4)*cos(d*x + c))*sin(d*x + c))/(
(a^8 - 3*a^6*b^2 + 3*a^4*b^4 - a^2*b^6)*d*cos(d*x + c)^2 + 2*(a^7*b - 3*a^
5*b^3 + 3*a^3*b^5 - a*b^7)*d*cos(d*x + c) + (a^6*b^2 - 3*a^4*b^4 + 3*a^2*b
^6 - b^8)*d)]
```

3.509.6 Sympy [F]

$$\int \frac{\sec^2(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \int \frac{\sec^2{(c+dx)}}{(a+b\sec{(c+dx)})^3}\,dx$$

input
```
integrate(sec(d*x+c)**2/(a+b*sec(d*x+c))**3,x)
```

output
```
Integral(sec(c + d*x)**2/(a + b*sec(c + d*x))**3, x)
```

3.509.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\sec^2(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(sec(d*x+c)^2/(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

3.509.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 277 vs. $2(121) = 242$.

Time = 0.36 (sec) , antiderivative size = 277, normalized size of antiderivative = 2.07

$$\int \frac{\sec^2(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \frac{\frac{3\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(2a-2b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{-a^2+b^2}}\right)\right)ab}{(a^4-2a^2b^2+b^4)\sqrt{-a^2+b^2}} - \frac{2a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-a^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+ab^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right.}{(a^4-2a^2}}{d}$$

input
```
integrate(sec(d*x+c)^2/(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
(3*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(2*a - 2*b) + arctan((a*tan(1/2*d*
x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))*a*b/((a^4 - 2*a^2*
b^2 + b^4)*sqrt(-a^2 + b^2)) - (2*a^3*tan(1/2*d*x + 1/2*c)^3 - a^2*b*tan(1
/2*d*x + 1/2*c)^3 + a*b^2*tan(1/2*d*x + 1/2*c)^3 - 2*b^3*tan(1/2*d*x + 1/2
*c)^3 - 2*a^3*tan(1/2*d*x + 1/2*c) - a^2*b*tan(1/2*d*x + 1/2*c) - a*b^2*ta
n(1/2*d*x + 1/2*c) - 2*b^3*tan(1/2*d*x + 1/2*c))/((a^4 - 2*a^2*b^2 + b^4)*
(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 - a - b)^2))/d
```

3.509.9 Mupad [B] (verification not implemented)

Time = 16.04 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.57

$$\int \frac{\sec^2(c+dx)}{(a+b\sec(c+dx))^3}\,dx = -\frac{\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3(2a^2+ab+2b^2)}{(a+b)^2(a-b)} - \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)(2a^2-ab+2b^2)}{(a+b)(a^2-2ab+b^2)}}{d\left(2ab-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2(2a^2-2b^2)+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4(a^2-2ab+b^2)+a^2+b^2\right)} - \frac{3ab\operatorname{atanh}\left(\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)(2a-2b)(a^2-2ab+b^2)}{2\sqrt{a+b}(a-b)^{5/2}}\right)}{d(a+b)^{5/2}(a-b)^{5/2}}$$

input
```
int(1/(cos(c + d*x)^2*(a + b/cos(c + d*x))^3),x)
```

output
```
- ((tan(c/2 + (d*x)/2)^3*(a*b + 2*a^2 + 2*b^2))/((a + b)^2*(a - b)) - (tan
(c/2 + (d*x)/2)*(2*a^2 - a*b + 2*b^2))/((a + b)*(a^2 - 2*a*b + b^2)))/(d*(
2*a*b - tan(c/2 + (d*x)/2)^2*(2*a^2 - 2*b^2) + tan(c/2 + (d*x)/2)^4*(a^2 -
 2*a*b + b^2) + a^2 + b^2)) - (3*a*b*atanh((tan(c/2 + (d*x)/2)*(2*a - 2*b)
*(a^2 - 2*a*b + b^2))/(2*(a + b)^(1/2)*(a - b)^(5/2))))/(d*(a + b)^(5/2)*(
a - b)^(5/2))
```

3.509. $\int \frac{\sec^2(c+dx)}{(a+b\sec(c+dx))^3}\,dx$

3.510 $\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^3}\,dx$

3.510.1 Optimal result . . . 3788
3.510.2 Mathematica [A] (verified) . . . 3788
3.510.3 Rubi [A] (verified) . . . 3789
3.510.4 Maple [A] (verified) . . . 3792
3.510.5 Fricas [B] (verification not implemented) . . . 3793
3.510.6 Sympy [F] . . . 3793
3.510.7 Maxima [F(-2)] . . . 3794
3.510.8 Giac [B] (verification not implemented) . . . 3794
3.510.9 Mupad [B] (verification not implemented) . . . 3795

3.510.1 Optimal result

Integrand size = 19, antiderivative size = 133

$$\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \frac{(2a^2+b^2)\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}d} - \frac{b\tan(c+dx)}{2\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} - \frac{3ab\tan(c+dx)}{2\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))}$$

output
```
(2*a^2+b^2)*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/(a-b)^(5/2
)/(a+b)^(5/2)/d-1/2*b*tan(d*x+c)/(a^2-b^2)/d/(a+b*sec(d*x+c))^2-3/2*a*b*ta
n(d*x+c)/(a^2-b^2)^2/d/(a+b*sec(d*x+c))
```

3.510.2 Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.86

$$\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \frac{-\frac{2(2a^2+b^2)\operatorname{arctanh}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} + \frac{b\left(-3ab+\left(-4a^2+b^2\right)\cos(c+dx)\right)\sin(c+dx)}{(b+a\cos(c+dx))^2}}{2(a-b)^2(a+b)^2d}$$

input
```
Integrate[Sec[c + d*x]/(a + b*Sec[c + d*x])^3,x]
```

output
```
((-2*(2*a^2 + b^2)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/S
qrt[a^2 - b^2] + (b*(-3*a*b + (-4*a^2 + b^2)*Cos[c + d*x])*Sin[c + d*x])/(
b + a*Cos[c + d*x])^2)/(2*(a - b)^2*(a + b)^2*d)
```

3.510.3 Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.20, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.632$, Rules used = {3042, 4320, 25, 3042, 4491, 25, 27, 3042, 4318, 3042, 3138, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \text{4320}$$

$$-\frac{\int -\frac{\sec(c+dx)(2a-b\sec(c+dx))}{(a+b\sec(c+dx))^2}dx}{2\left(a^2-b^2\right)}-\frac{b\tan(c+dx)}{2d\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\sec(c+dx)(2a-b\sec(c+dx))}{(a+b\sec(c+dx))^2}dx}{2\left(a^2-b^2\right)}-\frac{b\tan(c+dx)}{2d\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(2a-b\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx}{2\left(a^2-b^2\right)}-\frac{b\tan(c+dx)}{2d\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$$\Big\downarrow \text{4491}$$

$$\frac{-\frac{\int -\frac{\left(2a^2+b^2\right)\sec(c+dx)}{a+b\sec(c+dx)}dx}{a^2-b^2}-\frac{3ab\tan(c+dx)}{d(a^2-b^2)(a+b\sec(c+dx))}}{2\left(a^2-b^2\right)}-\frac{b\tan(c+dx)}{2d\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{\left(2a^2+b^2\right)\sec(c+dx)}{a+b\sec(c+dx)}dx}{a^2-b^2}-\frac{3ab\tan(c+dx)}{d(a^2-b^2)(a+b\sec(c+dx))}}{2\left(a^2-b^2\right)}-\frac{b\tan(c+dx)}{2d\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$\Big\downarrow$ 27

$$\frac{\frac{\left(2a^2+b^2\right)\int \frac{\sec(c+dx)}{a+b\sec(c+dx)}dx}{a^2-b^2}-\frac{3ab\tan(c+dx)}{d(a^2-b^2)(a+b\sec(c+dx))}}{2\left(a^2-b^2\right)}-\frac{b\tan(c+dx)}{2d\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\left(2a^2+b^2\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2-b^2}-\frac{3ab\tan(c+dx)}{d(a^2-b^2)(a+b\sec(c+dx))}}{2\left(a^2-b^2\right)}-\frac{b\tan(c+dx)}{2d\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$\Big\downarrow$ 4318

$$\frac{\frac{\left(2a^2+b^2\right)\int \frac{1}{\frac{a\cos(c+dx)}{b}+1}dx}{b(a^2-b^2)}-\frac{3ab\tan(c+dx)}{d(a^2-b^2)(a+b\sec(c+dx))}}{2\left(a^2-b^2\right)}-\frac{b\tan(c+dx)}{2d\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\left(2a^2+b^2\right)\int \frac{1}{\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{b}+1}dx}{b(a^2-b^2)}-\frac{3ab\tan(c+dx)}{d(a^2-b^2)(a+b\sec(c+dx))}}{2\left(a^2-b^2\right)}-\frac{b\tan(c+dx)}{2d\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$\Big\downarrow$ 3138

$$\frac{\frac{2\left(2a^2+b^2\right)\int \frac{1}{\left(1-\frac{a}{b}\right)\tan^2\left(\frac{1}{2}(c+dx)\right)+\frac{a+b}{b}}d\tan\left(\frac{1}{2}(c+dx)\right)}{bd(a^2-b^2)}-\frac{3ab\tan(c+dx)}{d(a^2-b^2)(a+b\sec(c+dx))}}{2\left(a^2-b^2\right)}-\frac{b\tan(c+dx)}{2d\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$\Big\downarrow$ 221

$$\frac{\frac{2\left(2a^2+b^2\right)\text{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d\sqrt{a-b}\sqrt{a+b}(a^2-b^2)}-\frac{3ab\tan(c+dx)}{d(a^2-b^2)(a+b\sec(c+dx))}}{2\left(a^2-b^2\right)}-\frac{b\tan(c+dx)}{2d\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

input
```
Int[Sec[c + d*x]/(a + b*Sec[c + d*x])^3,x]
```

3.510. $\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^3}\,dx$

output
```
-1/2*(b*Tan[c + d*x])/((a^2 - b^2)*d*(a + b*Sec[c + d*x])^2) + ((2*(2*a^2
+ b^2)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(Sqrt[a - b]*S
qrt[a + b]*(a^2 - b^2)*d) - (3*a*b*Tan[c + d*x])/((a^2 - b^2)*d*(a + b*Sec
[c + d*x])))/(2*(a^2 - b^2))
```

3.510.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 4318
```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[1/b   Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4320
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[(-b)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(f*(m + 1)*
(a^2 - b^2))), x] + Simp[1/((m + 1)*(a^2 - b^2))   Int[Csc[e + f*x]*(a + b*
Csc[e + f*x])^(m + 1)*(a*(m + 1) - b*(m + 2)*Csc[e + f*x]), x], x] /; FreeQ
[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

rule 4491
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b - a*B))*Cot[e
 + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Simp[1
/((m + 1)*(a^2 - b^2))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp
[(a*A - b*B)*(m + 1) - (A*b - a*B)*(m + 2)*Csc[e + f*x], x], x], x] /; Free
Q[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m
, -1]
```

3.510.4 Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.40

method	result
derivativedivides	$\frac{-\frac{2\left(-\frac{(4a+b)b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{2(a-b)\left(a^2+2ab+b^2\right)}+\frac{(4a-b)b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2(a+b)\left(a^2-2ab+b^2\right)}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2b-a-b\right)^2}+\frac{\left(2a^2+b^2\right)\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{\left(a^4-2a^2b^2+b^4\right)\sqrt{(a-b)(a+b)}}}{d}$
default	$\frac{-\frac{2\left(-\frac{(4a+b)b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{2(a-b)\left(a^2+2ab+b^2\right)}+\frac{(4a-b)b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2(a+b)\left(a^2-2ab+b^2\right)}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2b-a-b\right)^2}+\frac{\left(2a^2+b^2\right)\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{\left(a^4-2a^2b^2+b^4\right)\sqrt{(a-b)(a+b)}}}{d}$
risch	$\frac{ib\left(-5a^3b\,e^{3i(dx+c)}+2a\,b^3e^{3i(dx+c)}-4a^4e^{2i(dx+c)}-7a^2b^2e^{2i(dx+c)}+2b^4e^{2i(dx+c)}-11a^3b\,e^{i(dx+c)}+2b^3a\,e^{i(dx+c)}-4a^4+\right.}{a^2(-a^2+b^2)^2d\left(a\,e^{2i(dx+c)}+2b\,e^{i(dx+c)}+a\right)^2}$

input
```
int(sec(d*x+c)/(a+b*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-2*(-1/2*(4*a+b)*b/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3+1/2*(4*
a-b)*b/(a+b)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c))/(tan(1/2*d*x+1/2*c)^2*a-t
an(1/2*d*x+1/2*c)^2*b-a-b)^2+(2*a^2+b^2)/(a^4-2*a^2*b^2+b^4)/((a-b)*(a+b))
^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a-b)*(a+b))^(1/2)))
```

3.510.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 269 vs. $2(120) = 240$.

Time = 0.32 (sec) , antiderivative size = 595, normalized size of antiderivative = 4.47

$$\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^3}\,dx$$
$$= \left[\frac{\left(2\,a^2b^2+b^4+\left(2\,a^4+a^2b^2\right)\cos\left(dx+c\right)^2+2\left(2\,a^3b+ab^3\right)\cos\left(dx+c\right)\right)\sqrt{a^2-b^2}\log\left(\frac{2\,ab\cos(dx+c)-(a^2-2\,b}{}\right.}{4\left(\left(a^8-3\,a^6b^2+3\,a^4b^4-a^2b^6\right)d\cos\left(dx+c\right)^2+2\left(a^7b-\right.\right.}\right.$$

input
```
integrate(sec(d*x+c)/(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
[1/4*((2*a^2*b^2 + b^4 + (2*a^4 + a^2*b^2)*cos(d*x + c)^2 + 2*(2*a^3*b + a
*b^3)*cos(d*x + c))*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2
)*cos(d*x + c)^2 + 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2
*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) - 2*(3*a^3*b^
2 - 3*a*b^4 + (4*a^4*b - 5*a^2*b^3 + b^5)*cos(d*x + c))*sin(d*x + c))/((a^
8 - 3*a^6*b^2 + 3*a^4*b^4 - a^2*b^6)*d*cos(d*x + c)^2 + 2*(a^7*b - 3*a^5*b
^3 + 3*a^3*b^5 - a*b^7)*d*cos(d*x + c) + (a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6
- b^8)*d), 1/2*((2*a^2*b^2 + b^4 + (2*a^4 + a^2*b^2)*cos(d*x + c)^2 + 2*(2
*a^3*b + a*b^3)*cos(d*x + c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b
*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c))) - (3*a^3*b^2 - 3*a*b^4 + (4
*a^4*b - 5*a^2*b^3 + b^5)*cos(d*x + c))*sin(d*x + c))/((a^8 - 3*a^6*b^2 +
3*a^4*b^4 - a^2*b^6)*d*cos(d*x + c)^2 + 2*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 -
 a*b^7)*d*cos(d*x + c) + (a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*d)]
```

3.510.6 Sympy [F]

$$\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \int \frac{\sec\left(c+dx\right)}{\left(a+b\sec\left(c+dx\right)\right)^3}\,dx$$

input
```
integrate(sec(d*x+c)/(a+b*sec(d*x+c))**3,x)
```

output
```
Integral(sec(c + d*x)/(a + b*sec(c + d*x))**3, x)
```

3.510.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(sec(d*x+c)/(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

3.510.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 254 vs. $2(120) = 240$.

Time = 0.37 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.91

$$\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^3}\,dx = -\frac{\frac{\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(2a-2b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{-a^2+b^2}}\right)\right)(2a^2+b^2)}{(a^4-2a^2b^2+b^4)\sqrt{-a^2+b^2}} - \frac{4a^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-3ab^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-b^3\tan}{(a^4-2a^2b^2+b^4)(a}}{d}$$

input
```
integrate(sec(d*x+c)/(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
-((pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(2*a - 2*b) + arctan((a*tan(1/2*d*x
 + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))*(2*a^2 + b^2)/((a^4
 - 2*a^2*b^2 + b^4)*sqrt(-a^2 + b^2)) - (4*a^2*b*tan(1/2*d*x + 1/2*c)^3 -
3*a*b^2*tan(1/2*d*x + 1/2*c)^3 - b^3*tan(1/2*d*x + 1/2*c)^3 - 4*a^2*b*tan(
1/2*d*x + 1/2*c) - 3*a*b^2*tan(1/2*d*x + 1/2*c) + b^3*tan(1/2*d*x + 1/2*c)
)/((a^4 - 2*a^2*b^2 + b^4)*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2
*c)^2 - a - b)^2))/d
```

3.510.9 Mupad [B] (verification not implemented)

Time = 16.17 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.53

$$\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \frac{\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,(b^2+4\,a\,b)}{(a+b)^2\,(a-b)}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,(4\,a\,b-b^2)}{(a+b)\,(a^2-2\,a\,b+b^2)}}{d\,\left(2\,a\,b-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,(2\,a^2-2\,b^2)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,(a^2-2\,a\,b+b^2)+a^2+b^2\right)} + \frac{\operatorname{atanh}\left(\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,(2\,a-2\,b)\,(a^2-2\,a\,b+b^2)}{2\,\sqrt{a+b}\,(a-b)^{5/2}}\right)\,(2\,a^2+b^2)}{d\,(a+b)^{5/2}\,(a-b)^{5/2}}$$

input
```
int(1/(cos(c + d*x)*(a + b/cos(c + d*x))^3),x)
```

output
```
((tan(c/2 + (d*x)/2)^3*(4*a*b + b^2))/((a + b)^2*(a - b)) - (tan(c/2 + (d*
x)/2)*(4*a*b - b^2))/((a + b)*(a^2 - 2*a*b + b^2)))/(d*(2*a*b - tan(c/2 +
(d*x)/2)^2*(2*a^2 - 2*b^2) + tan(c/2 + (d*x)/2)^4*(a^2 - 2*a*b + b^2) + a^
2 + b^2)) + (atanh((tan(c/2 + (d*x)/2)*(2*a - 2*b)*(a^2 - 2*a*b + b^2))/(2
*(a + b)^(1/2)*(a - b)^(5/2)))*(2*a^2 + b^2))/(d*(a + b)^(5/2)*(a - b)^(5/
2))
```

3.511 $\int \frac{1}{(a+b\sec(c+dx))^3}\,dx$

3.511.1 Optimal result . . . 3796
3.511.2 Mathematica [A] (verified) . . . 3796
3.511.3 Rubi [A] (verified) . . . 3797
3.511.4 Maple [A] (verified) . . . 3800
3.511.5 Fricas [B] (verification not implemented) . . . 3801
3.511.6 Sympy [F] . . . 3802
3.511.7 Maxima [F(-2)] . . . 3803
3.511.8 Giac [B] (verification not implemented) . . . 3803
3.511.9 Mupad [B] (verification not implemented) . . . 3804

3.511.1 Optimal result

Integrand size = 12, antiderivative size = 173

$$\int \frac{1}{(a+b\sec(c+dx))^3}\,dx = \frac{x}{a^3} - \frac{b(6a^4-5a^2b^2+2b^4)\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3(a-b)^{5/2}(a+b)^{5/2}d} + \frac{b^2\tan(c+dx)}{2a\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} + \frac{b^2(5a^2-2b^2)\tan(c+dx)}{2a^2\left(a^2-b^2\right)^2d(a+b\sec(c+dx))}$$

output
```
x/a^3-b*(6*a^4-5*a^2*b^2+2*b^4)*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+
b)^(1/2))/a^3/(a-b)^(5/2)/(a+b)^(5/2)/d+1/2*b^2*tan(d*x+c)/a/(a^2-b^2)/d/(
a+b*sec(d*x+c))^2+1/2*b^2*(5*a^2-2*b^2)*tan(d*x+c)/a^2/(a^2-b^2)^2/d/(a+b*
sec(d*x+c))
```

3.511.2 Mathematica [A] (verified)

Time = 0.78 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.18

$$\int \frac{1}{(a+b\sec(c+dx))^3}\,dx = \frac{(b+a\cos(c+dx))\sec^3(c+dx)\left(2(c+dx)(b+a\cos(c+dx))^2+\frac{2b(6a^4-5a^2b^2+2b^4)\operatorname{arctanh}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right.}{(a^2-b^2)^{5/2}}\right.}{2a^3d(a+b\sec(c+dx))^3}$$

input
```
Integrate[(a + b*Sec[c + d*x])^(-3),x]
```

output
```
((b + a*Cos[c + d*x])*Sec[c + d*x]^3*(2*(c + d*x)*(b + a*Cos[c + d*x])^2 +
 (2*b*(6*a^4 - 5*a^2*b^2 + 2*b^4)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt
[a^2 - b^2]]*(b + a*Cos[c + d*x])^2)/(a^2 - b^2)^(5/2) + (a*b^3*Sin[c + d*
x])/((-a + b)*(a + b)) + (3*a*b^2*(2*a^2 - b^2)*(b + a*Cos[c + d*x])*Sin[c
 + d*x])/((a - b)^2*(a + b)^2)))/(2*a^3*d*(a + b*Sec[c + d*x])^3)
```

3.511.3 Rubi [A] (verified)

Time = 1.00 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.25, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 1.083$, Rules used = {3042, 4272, 25, 3042, 4548, 25, 3042, 4407, 3042, 4318, 3042, 3138, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+b\sec(c+dx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \text{4272}$$

$$\frac{b^2\tan(c+dx)}{2ad\left(a^2-b^2\right)(a+b\sec(c+dx))^2}-\frac{\int -\frac{b^2\sec^2(c+dx)-2ab\sec(c+dx)+2\left(a^2-b^2\right)}{(a+b\sec(c+dx))^2}dx}{2a\left(a^2-b^2\right)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{b^2\sec^2(c+dx)-2ab\sec(c+dx)+2\left(a^2-b^2\right)}{(a+b\sec(c+dx))^2}dx}{2a\left(a^2-b^2\right)}+\frac{b^2\tan(c+dx)}{2ad\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2-2ab\csc\left(c+dx+\frac{\pi}{2}\right)+2\left(a^2-b^2\right)}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx}{2a\left(a^2-b^2\right)}+\frac{b^2\tan(c+dx)}{2ad\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$$\Big\downarrow \text{4548}$$

$$\frac{\frac{b^2\left(5a^2-2b^2\right)\tan(c+dx)}{ad\left(a^2-b^2\right)(a+b\sec(c+dx))}-\frac{\int -\frac{2\left(a^2-b^2\right)^2-ab\left(4a^2-b^2\right)\sec(c+dx)}{a+b\sec(c+dx)}dx}{a\left(a^2-b^2\right)}}{2a\left(a^2-b^2\right)}+\frac{b^2\tan(c+dx)}{2ad\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\int \frac{2\left(a^2-b^2\right)^2-ab\left(4a^2-b^2\right)\sec(c+dx)}{a+b\sec(c+dx)}dx}{a(a^2-b^2)}+\frac{b^2\left(5a^2-2b^2\right)\tan(c+dx)}{ad(a^2-b^2)(a+b\sec(c+dx))}}{2a\left(a^2-b^2\right)}+\frac{b^2\tan(c+dx)}{2ad\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{\int \frac{2\left(a^2-b^2\right)^2-ab\left(4a^2-b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a(a^2-b^2)}+\frac{b^2\left(5a^2-2b^2\right)\tan(c+dx)}{ad(a^2-b^2)(a+b\sec(c+dx))}}{2a\left(a^2-b^2\right)}+\frac{b^2\tan(c+dx)}{2ad\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$$\Big\downarrow \; 4407$$

$$\frac{\frac{\frac{2x\left(a^2-b^2\right)^2}{a}-\frac{b\left(6a^4-5a^2b^2+2b^4\right)\int \frac{\sec(c+dx)}{a+b\sec(c+dx)}dx}{a}}{a(a^2-b^2)}+\frac{b^2\left(5a^2-2b^2\right)\tan(c+dx)}{ad(a^2-b^2)(a+b\sec(c+dx))}}{2a\left(a^2-b^2\right)}+\frac{b^2\tan(c+dx)}{2ad\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{\frac{2x\left(a^2-b^2\right)^2}{a}-\frac{b\left(6a^4-5a^2b^2+2b^4\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a}}{a(a^2-b^2)}+\frac{b^2\left(5a^2-2b^2\right)\tan(c+dx)}{ad(a^2-b^2)(a+b\sec(c+dx))}}{2a\left(a^2-b^2\right)}+\frac{b^2\tan(c+dx)}{2ad\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$$\Big\downarrow \; 4318$$

$$\frac{\frac{\frac{2x\left(a^2-b^2\right)^2}{a}-\frac{\left(6a^4-5a^2b^2+2b^4\right)\int \frac{1}{\frac{a\cos(c+dx)}{b}+1}dx}{a}}{a(a^2-b^2)}+\frac{b^2\left(5a^2-2b^2\right)\tan(c+dx)}{ad(a^2-b^2)(a+b\sec(c+dx))}}{2a\left(a^2-b^2\right)}+\frac{b^2\tan(c+dx)}{2ad\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{\frac{2x\left(a^2-b^2\right)^2}{a}-\frac{\left(6a^4-5a^2b^2+2b^4\right)\int \frac{1}{\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{b}+1}dx}{a}}{a(a^2-b^2)}+\frac{b^2\left(5a^2-2b^2\right)\tan(c+dx)}{ad(a^2-b^2)(a+b\sec(c+dx))}}{2a\left(a^2-b^2\right)}+\frac{b^2\tan(c+dx)}{2ad\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$$\Big\downarrow \; 3138$$

3.511. $\int \frac{1}{(a+b\sec(c+dx))^3}\,dx$

$$\frac{\frac{\frac{2x\left(a^2-b^2\right)^2}{a}-\frac{2\left(6a^4-5a^2b^2+2b^4\right)\int \frac{1}{\left(1-\frac{a}{b}\right)\tan^2\left(\frac{1}{2}(c+dx)\right)+\frac{a+b}{b}}d\tan\left(\frac{1}{2}(c+dx)\right)}{ad}}{a(a^2-b^2)}+\frac{b^2\left(5a^2-2b^2\right)\tan(c+dx)}{ad(a^2-b^2)(a+b\sec(c+dx))}}{2a\left(a^2-b^2\right)}+\frac{b^2\tan(c+dx)}{2ad\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$$\Big\downarrow \; 221$$

$$\frac{b^2\tan(c+dx)}{2ad\left(a^2-b^2\right)(a+b\sec(c+dx))^2}+\frac{\frac{b^2\left(5a^2-2b^2\right)\tan(c+dx)}{ad(a^2-b^2)(a+b\sec(c+dx))}+\frac{\frac{2x\left(a^2-b^2\right)^2}{a}-\frac{2b\left(6a^4-5a^2b^2+2b^4\right)\text{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{ad\sqrt{a-b}\sqrt{a+b}}}{a(a^2-b^2)}}{2a\left(a^2-b^2\right)}$$

input
```
Int[(a + b*Sec[c + d*x])^(-3),x]
```

output
```
(b^2*Tan[c + d*x])/(2*a*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^2) + (((2*(a^2
- b^2)^2*x)/a - (2*b*(6*a^4 - 5*a^2*b^2 + 2*b^4)*ArcTanh[(Sqrt[a - b]*Tan[
(c + d*x)/2])/Sqrt[a + b]])/(a*Sqrt[a - b]*Sqrt[a + b]*d))/(a*(a^2 - b^2))
 + (b^2*(5*a^2 - 2*b^2)*Tan[c + d*x])/(a*(a^2 - b^2)*d*(a + b*Sec[c + d*x]
)))/(2*a*(a^2 - b^2))
```

3.511.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 4272
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[b^2*Cot[
c + d*x]*((a + b*Csc[c + d*x])^(n + 1)/(a*d*(n + 1)*(a^2 - b^2))), x] + Sim
p[1/(a*(n + 1)*(a^2 - b^2))   Int[(a + b*Csc[c + d*x])^(n + 1)*Simp[(a^2 -
b^2)*(n + 1) - a*b*(n + 1)*Csc[c + d*x] + b^2*(n + 2)*Csc[c + d*x]^2, x], x
], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && LtQ[n, -1] && Integ
erQ[2*n]
```

rule 4318
```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[1/b   Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4407
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Simp[(b*c - a*d)/a   Int[Csc[e + f*
x]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
- a*d, 0]
```

rule 4548
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(A*b^2 -
a*b*B + a^2*C)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(a*f*(m + 1)*(a^2
 - b^2))), x] + Simp[1/(a*(m + 1)*(a^2 - b^2))   Int[(a + b*Csc[e + f*x])^(
m + 1)*Simp[A*(a^2 - b^2)*(m + 1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x
] + (A*b^2 - a*b*B + a^2*C)*(m + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

3.511.4 Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.37

method	result
derivativedivides	$\frac{\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^3}+\frac{2b\left(\frac{-\frac{\left(6a^2+ab-2b^2\right)ab\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{2(a-b)\left(a^2+2ab+b^2\right)}+\frac{\left(6a^2-ab-2b^2\right)ab\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2(a+b)\left(a^2-2ab+b^2\right)}}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2b-a-b\right)^2}-\frac{\left(6a^4-5a^2b^2+2b^4\right)\operatorname{arctanh}\left(\frac{(a}{}\right.}{2\left(a^4-2a^2b^2+b^4\right)\sqrt{(a}}\right.}{a^3}}{d}$
default	$\frac{\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^3}+\frac{2b\left(\frac{-\frac{\left(6a^2+ab-2b^2\right)ab\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{2(a-b)\left(a^2+2ab+b^2\right)}+\frac{\left(6a^2-ab-2b^2\right)ab\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2(a+b)\left(a^2-2ab+b^2\right)}}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2b-a-b\right)^2}-\frac{\left(6a^4-5a^2b^2+2b^4\right)\operatorname{arctanh}\left(\frac{(a}{}\right.}{2\left(a^4-2a^2b^2+b^4\right)\sqrt{(a}}\right.}{a^3}}{d}$
risch	$\frac{x}{a^3}-\frac{ib^2\left(-7a^3b\,e^{3i(dx+c)}+4a\,b^3e^{3i(dx+c)}-6a^4e^{2i(dx+c)}-9a^2b^2e^{2i(dx+c)}+6b^4e^{2i(dx+c)}-17a^3b\,e^{i(dx+c)}+8b^3a\,e^{i(dx+}\right.}{a^3(-a^2+b^2)^2d\left(a\,e^{2i(dx+c)}+2b\,e^{i(dx+c)}+a\right)^2}$

input
```
int(1/(a+b*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(2/a^3*arctan(tan(1/2*d*x+1/2*c))+2*b/a^3*((-1/2*(6*a^2+a*b-2*b^2)*a*b
/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3+1/2*(6*a^2-a*b-2*b^2)*a*b/(a+b
)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c))/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+
1/2*c)^2*b-a-b)^2-1/2*(6*a^4-5*a^2*b^2+2*b^4)/(a^4-2*a^2*b^2+b^4)/((a-b)*(
a+b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a-b)*(a+b))^(1/2))))
```

3.511.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 431 vs. $2(160) = 320$.

Time = 0.31 (sec) , antiderivative size = 919, normalized size of antiderivative = 5.31

$$\int \frac{1}{(a+b\sec(c+dx))^3}\,dx$$
$$= \left[\frac{4\,(a^8-3\,a^6b^2+3\,a^4b^4-a^2b^6)dx\cos(dx+c)^2+8\,(a^7b-3\,a^5b^3+3\,a^3b^5-ab^7)dx\cos(dx+c)+4\,(a^6b^2}{}\right.$$

input
```
integrate(1/(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
[1/4*(4*(a^8 - 3*a^6*b^2 + 3*a^4*b^4 - a^2*b^6)*d*x*cos(d*x + c)^2 + 8*(a^
7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*d*x*cos(d*x + c) + 4*(a^6*b^2 - 3*a^4
*b^4 + 3*a^2*b^6 - b^8)*d*x + (6*a^4*b^3 - 5*a^2*b^5 + 2*b^7 + (6*a^6*b -
5*a^4*b^3 + 2*a^2*b^5)*cos(d*x + c)^2 + 2*(6*a^5*b^2 - 5*a^3*b^4 + 2*a*b^6
)*cos(d*x + c))*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*co
s(d*x + c)^2 - 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2
 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) + 2*(5*a^5*b^3 -
7*a^3*b^5 + 2*a*b^7 + 3*(2*a^6*b^2 - 3*a^4*b^4 + a^2*b^6)*cos(d*x + c))*si
n(d*x + c))/((a^11 - 3*a^9*b^2 + 3*a^7*b^4 - a^5*b^6)*d*cos(d*x + c)^2 + 2
*(a^10*b - 3*a^8*b^3 + 3*a^6*b^5 - a^4*b^7)*d*cos(d*x + c) + (a^9*b^2 - 3*
a^7*b^4 + 3*a^5*b^6 - a^3*b^8)*d), 1/2*(2*(a^8 - 3*a^6*b^2 + 3*a^4*b^4 - a
^2*b^6)*d*x*cos(d*x + c)^2 + 4*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*d*x
*cos(d*x + c) + 2*(a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*d*x - (6*a^4*b^3
 - 5*a^2*b^5 + 2*b^7 + (6*a^6*b - 5*a^4*b^3 + 2*a^2*b^5)*cos(d*x + c)^2 +
2*(6*a^5*b^2 - 5*a^3*b^4 + 2*a*b^6)*cos(d*x + c))*sqrt(-a^2 + b^2)*arctan(
-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c))) + (5*a^
5*b^3 - 7*a^3*b^5 + 2*a*b^7 + 3*(2*a^6*b^2 - 3*a^4*b^4 + a^2*b^6)*cos(d*x
+ c))*sin(d*x + c))/((a^11 - 3*a^9*b^2 + 3*a^7*b^4 - a^5*b^6)*d*cos(d*x +
c)^2 + 2*(a^10*b - 3*a^8*b^3 + 3*a^6*b^5 - a^4*b^7)*d*cos(d*x + c) + (a^9*
b^2 - 3*a^7*b^4 + 3*a^5*b^6 - a^3*b^8)*d)]
```

3.511.6 Sympy [F]

$$\int \frac{1}{(a+b\sec(c+dx))^3}\,dx = \int \frac{1}{\left(a+b\sec\left(c+dx\right)\right)^3}\,dx$$

input
```
integrate(1/(a+b*sec(d*x+c))**3,x)
```

output
```
Integral((a + b*sec(c + d*x))**(-3), x)
```

3.511.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(a+b\sec(c+dx))^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

3.511.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 322 vs. $2(160) = 320$.

Time = 0.29 (sec) , antiderivative size = 322, normalized size of antiderivative = 1.86

$$\int \frac{1}{(a+b\sec(c+dx))^3}\,dx = \frac{\frac{(6a^4b-5a^2b^3+2b^5)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(2a-2b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{-a^2+b^2}}\right)\right)}{(a^7-2a^5b^2+a^3b^4)\sqrt{-a^2+b^2}} - \frac{6a^3b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-5a^2b^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right.}{}}{}$$

input
```
integrate(1/(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
((6*a^4*b - 5*a^2*b^3 + 2*b^5)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(2*a -
 2*b) + arctan((a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2
 + b^2)))/((a^7 - 2*a^5*b^2 + a^3*b^4)*sqrt(-a^2 + b^2)) - (6*a^3*b^2*tan(
1/2*d*x + 1/2*c)^3 - 5*a^2*b^3*tan(1/2*d*x + 1/2*c)^3 - 3*a*b^4*tan(1/2*d*
x + 1/2*c)^3 + 2*b^5*tan(1/2*d*x + 1/2*c)^3 - 6*a^3*b^2*tan(1/2*d*x + 1/2*
c) - 5*a^2*b^3*tan(1/2*d*x + 1/2*c) + 3*a*b^4*tan(1/2*d*x + 1/2*c) + 2*b^5
*tan(1/2*d*x + 1/2*c))/((a^6 - 2*a^4*b^2 + a^2*b^4)*(a*tan(1/2*d*x + 1/2*c
)^2 - b*tan(1/2*d*x + 1/2*c)^2 - a - b)^2) + (d*x + c)/a^3)/d
```

3.511.9 Mupad [B] (verification not implemented)

Time = 22.44 (sec) , antiderivative size = 5090, normalized size of antiderivative = 29.42

$$\int \frac{1}{(a+b\sec(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
int(1/(a + b/cos(c + d*x))^3,x)
```

output
```
(2*atan((((((8*(12*a^14*b - 4*a^15 + 4*a^6*b^9 - 2*a^7*b^8 - 18*a^8*b^7 +
4*a^9*b^6 + 36*a^10*b^5 - 6*a^11*b^4 - 34*a^12*b^3 + 8*a^13*b^2))/(a^12*b
+ a^13 - a^6*b^7 - a^7*b^6 + 3*a^8*b^5 + 3*a^9*b^4 - 3*a^10*b^3 - 3*a^11*b
^2) - (tan(c/2 + (d*x)/2)*(8*a^15*b - 8*a^6*b^10 + 8*a^7*b^9 + 32*a^8*b^8
- 32*a^9*b^7 - 48*a^10*b^6 + 48*a^11*b^5 + 32*a^12*b^4 - 32*a^13*b^3 - 8*a
^14*b^2)*8i)/(a^3*(a^10*b + a^11 - a^4*b^7 - a^5*b^6 + 3*a^6*b^5 + 3*a^7*b
^4 - 3*a^8*b^3 - 3*a^9*b^2)))*1i)/a^3 + (8*tan(c/2 + (d*x)/2)*(4*a^10 - 8*
a^9*b - 8*a*b^9 + 8*b^10 - 32*a^2*b^8 + 32*a^3*b^7 + 57*a^4*b^6 - 48*a^5*b
^5 - 52*a^6*b^4 + 32*a^7*b^3 + 24*a^8*b^2))/(a^10*b + a^11 - a^4*b^7 - a^5
*b^6 + 3*a^6*b^5 + 3*a^7*b^4 - 3*a^8*b^3 - 3*a^9*b^2))/a^3 - ((((8*(12*a^1
4*b - 4*a^15 + 4*a^6*b^9 - 2*a^7*b^8 - 18*a^8*b^7 + 4*a^9*b^6 + 36*a^10*b^
5 - 6*a^11*b^4 - 34*a^12*b^3 + 8*a^13*b^2))/(a^12*b + a^13 - a^6*b^7 - a^7
*b^6 + 3*a^8*b^5 + 3*a^9*b^4 - 3*a^10*b^3 - 3*a^11*b^2) + (tan(c/2 + (d*x)
/2)*(8*a^15*b - 8*a^6*b^10 + 8*a^7*b^9 + 32*a^8*b^8 - 32*a^9*b^7 - 48*a^10
*b^6 + 48*a^11*b^5 + 32*a^12*b^4 - 32*a^13*b^3 - 8*a^14*b^2)*8i)/(a^3*(a^1
0*b + a^11 - a^4*b^7 - a^5*b^6 + 3*a^6*b^5 + 3*a^7*b^4 - 3*a^8*b^3 - 3*a^9
*b^2)))*1i)/a^3 - (8*tan(c/2 + (d*x)/2)*(4*a^10 - 8*a^9*b - 8*a*b^9 + 8*b^
10 - 32*a^2*b^8 + 32*a^3*b^7 + 57*a^4*b^6 - 48*a^5*b^5 - 52*a^6*b^4 + 32*a
^7*b^3 + 24*a^8*b^2))/(a^10*b + a^11 - a^4*b^7 - a^5*b^6 + 3*a^6*b^5 + 3*a
^7*b^4 - 3*a^8*b^3 - 3*a^9*b^2))/a^3)/((((((8*(12*a^14*b - 4*a^15 + 4*a...
```

3.512 $\int \frac{\cos(c+dx)}{(a+b\sec(c+dx))^3}\,dx$

3.512.1 Optimal result . . . 3805
3.512.2 Mathematica [A] (verified) . . . 3806
3.512.3 Rubi [A] (verified) . . . 3806
3.512.4 Maple [A] (verified) . . . 3811
3.512.5 Fricas [B] (verification not implemented) . . . 3812
3.512.6 Sympy [F] . . . 3813
3.512.7 Maxima [F(-2)] . . . 3813
3.512.8 Giac [A] (verification not implemented) . . . 3813
3.512.9 Mupad [B] (verification not implemented) . . . 3814

3.512.1 Optimal result

Integrand size = 19, antiderivative size = 223

$$\begin{aligned}\int \frac{\cos(c+dx)}{(a+b\sec(c+dx))^3}\,dx = &-\frac{3bx}{a^4}+\frac{3b^2(4a^4-5a^2b^2+2b^4)\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^4(a-b)^{5/2}(a+b)^{5/2}d}\\ &+\frac{(2a^4-11a^2b^2+6b^4)\sin(c+dx)}{2a^3\left(a^2-b^2\right)^2d}\\ &+\frac{b^2\sin(c+dx)}{2a\left(a^2-b^2\right)d(a+b\sec(c+dx))^2}\\ &+\frac{3b^2(2a^2-b^2)\sin(c+dx)}{2a^2\left(a^2-b^2\right)^2d(a+b\sec(c+dx))}\end{aligned}$$

output
```
-3*b*x/a^4+3*b^2*(4*a^4-5*a^2*b^2+2*b^4)*arctanh((a-b)^(1/2)*tan(1/2*d*x+1
/2*c)/(a+b)^(1/2))/a^4/(a-b)^(5/2)/(a+b)^(5/2)/d+1/2*(2*a^4-11*a^2*b^2+6*b
^4)*sin(d*x+c)/a^3/(a^2-b^2)^2/d+1/2*b^2*sin(d*x+c)/a/(a^2-b^2)/d/(a+b*sec
(d*x+c))^2+3/2*b^2*(2*a^2-b^2)*sin(d*x+c)/a^2/(a^2-b^2)^2/d/(a+b*sec(d*x+c
))
```

3.512.2 Mathematica [A] (verified)

Time = 1.01 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.03

$$\int \frac{\cos(c+dx)}{(a+b\sec(c+dx))^3}\,dx$$
$$= \frac{(b+a\cos(c+dx))\sec^3(c+dx)\left(-6b(c+dx)(b+a\cos(c+dx))^2 - \frac{6b^2(4a^4-5a^2b^2+2b^4)\operatorname{arctanh}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c\right.}{\sqrt{a^2-b^2}}\right.}{(a^2-b^2)^{5/2}}\right.}{2a^4d(a+b\,\mathrm{s}}$$

input
```
Integrate[Cos[c + d*x]/(a + b*Sec[c + d*x])^3,x]
```

output
```
((b + a*Cos[c + d*x])*Sec[c + d*x]^3*(-6*b*(c + d*x)*(b + a*Cos[c + d*x])^
2 - (6*b^2*(4*a^4 - 5*a^2*b^2 + 2*b^4)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])
/Sqrt[a^2 - b^2]]*(b + a*Cos[c + d*x])^2)/(a^2 - b^2)^(5/2) + (a*b^4*Sin[c
 + d*x])/((a - b)*(a + b)) - (a*b^3*(8*a^2 - 5*b^2)*(b + a*Cos[c + d*x])*S
in[c + d*x])/((a - b)^2*(a + b)^2) + 2*a*(b + a*Cos[c + d*x])^2*Sin[c + d*
x]))/(2*a^4*d*(a + b*Sec[c + d*x])^3)
```

3.512.3 Rubi [A] (verified)

Time = 1.53 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.16, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.842$, Rules used = {3042, 4334, 25, 3042, 4588, 25, 3042, 4592, 27, 3042, 4407, 3042, 4318, 3042, 3138, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(c+dx)}{(a+b\sec(c+dx))^3}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^3}dx$$
$$\Big\downarrow \; 4334$$
$$\frac{b^2\sin(c+dx)}{2ad\left(a^2-b^2\right)(a+b\sec(c+dx))^2} - \frac{\int -\frac{\cos(c+dx)\left(2a^2-2b\sec(c+dx)a-3b^2+2b^2\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^2}dx}{2a\left(a^2-b^2\right)}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{\cos(c+dx)\left(2a^2-2b\sec(c+dx)a-3b^2+2b^2\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^2}dx}{2a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)}{2ad\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{\int \frac{2a^2-2b\csc\left(c+dx+\frac{\pi}{2}\right)a-3b^2+2b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx}{2a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)}{2ad\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$$\Big\downarrow \; 4588$$

$$\frac{\frac{3b^2\left(2a^2-b^2\right)\sin(c+dx)}{ad(a^2-b^2)(a+b\sec(c+dx))}-\frac{\int -\frac{\cos(c+dx)\left(2a^4-11b^2a^2-b\left(4a^2-b^2\right)\sec(c+dx)a+6b^4+3b^2\left(2a^2-b^2\right)\sec^2(c+dx)\right)}{a+b\sec(c+dx)}dx}{a(a^2-b^2)}}{2a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)}{2ad\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\int \frac{\cos(c+dx)\left(2a^4-11b^2a^2-b\left(4a^2-b^2\right)\sec(c+dx)a+6b^4+3b^2\left(2a^2-b^2\right)\sec^2(c+dx)\right)}{a+b\sec(c+dx)}dx}{a(a^2-b^2)}+\frac{3b^2\left(2a^2-b^2\right)\sin(c+dx)}{ad(a^2-b^2)(a+b\sec(c+dx))}}{2a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)}{2ad\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{\int \frac{2a^4-11b^2a^2-b\left(4a^2-b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)a+6b^4+3b^2\left(2a^2-b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a(a^2-b^2)}+\frac{3b^2\left(2a^2-b^2\right)\sin(c+dx)}{ad(a^2-b^2)(a+b\sec(c+dx))}}{2a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)}{2ad\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$$\Big\downarrow \; 4592$$

$$\frac{\frac{\frac{\left(2a^4-11a^2b^2+6b^4\right)\sin(c+dx)}{ad}-\frac{\int \frac{3\left(2b\left(a^2-b^2\right)^2-ab^2\left(2a^2-b^2\right)\sec(c+dx)\right)}{a+b\sec(c+dx)}dx}{a}}{a(a^2-b^2)}+\frac{3b^2\left(2a^2-b^2\right)\sin(c+dx)}{ad(a^2-b^2)(a+b\sec(c+dx))}}{2a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)}{2ad\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$$\Big\downarrow \; 27$$

3.512. $\int \frac{\cos(c+dx)}{(a+b\sec(c+dx))^3}\,dx$

$$\frac{\frac{\frac{\left(2a^4-11a^2b^2+6b^4\right)\sin(c+dx)}{ad}-\frac{3\int\frac{2b\left(a^2-b^2\right)^2-ab^2\left(2a^2-b^2\right)\sec(c+dx)}{a+b\sec(c+dx)}dx}{a}}{a(a^2-b^2)}+\frac{3b^2\left(2a^2-b^2\right)\sin(c+dx)}{ad(a^2-b^2)(a+b\sec(c+dx))}}{2a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)}{2ad\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$\downarrow$ 3042

$$\frac{\frac{\frac{\left(2a^4-11a^2b^2+6b^4\right)\sin(c+dx)}{ad}-\frac{3\int\frac{2b\left(a^2-b^2\right)^2-ab^2\left(2a^2-b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a}}{a(a^2-b^2)}+\frac{3b^2\left(2a^2-b^2\right)\sin(c+dx)}{ad(a^2-b^2)(a+b\sec(c+dx))}}{2a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)}{2ad\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$\downarrow$ 4407

$$\frac{\frac{\frac{\left(2a^4-11a^2b^2+6b^4\right)\sin(c+dx)}{ad}-\frac{3\left(\frac{2bx\left(a^2-b^2\right)^2}{a}-\frac{b^2\left(4a^4-5a^2b^2+2b^4\right)\int\frac{\sec(c+dx)}{a+b\sec(c+dx)}dx}{a}\right)}{a}}{a(a^2-b^2)}+\frac{3b^2\left(2a^2-b^2\right)\sin(c+dx)}{ad(a^2-b^2)(a+b\sec(c+dx))}}{2a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)}{2ad\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$\downarrow$ 3042

$$\frac{\frac{\frac{\left(2a^4-11a^2b^2+6b^4\right)\sin(c+dx)}{ad}-\frac{3\left(\frac{2bx\left(a^2-b^2\right)^2}{a}-\frac{b^2\left(4a^4-5a^2b^2+2b^4\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a}\right)}{a}}{a(a^2-b^2)}+\frac{3b^2\left(2a^2-b^2\right)\sin(c+dx)}{ad(a^2-b^2)(a+b\sec(c+dx))}}{2a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)}{2ad\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$\downarrow$ 4318

$$\frac{\frac{\frac{\left(2a^4-11a^2b^2+6b^4\right)\sin(c+dx)}{ad}-\frac{3\left(\frac{2bx\left(a^2-b^2\right)^2}{a}-\frac{b\left(4a^4-5a^2b^2+2b^4\right)\int\frac{1}{\frac{a\cos(c+dx)}{b}+1}dx}{a}\right)}{a}}{a(a^2-b^2)}+\frac{3b^2\left(2a^2-b^2\right)\sin(c+dx)}{ad(a^2-b^2)(a+b\sec(c+dx))}}{2a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)}{2ad\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$\downarrow$ 3042

3.512. $\int \frac{\cos(c+dx)}{(a+b\sec(c+dx))^3}\,dx$

$$\frac{\frac{\frac{\left(2a^4-11a^2b^2+6b^4\right)\sin(c+dx)}{ad}-\frac{3\left(\frac{2bx\left(a^2-b^2\right)^2}{a}-\frac{b\left(4a^4-5a^2b^2+2b^4\right)\int\frac{1}{\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{b}+1}dx}{a}\right)}{a}}{a(a^2-b^2)}+\frac{3b^2\left(2a^2-b^2\right)\sin(c+dx)}{ad(a^2-b^2)(a+b\sec(c+dx))}}{2a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)}{2ad\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$\Big\downarrow$ 3138

$$\frac{\frac{\frac{\left(2a^4-11a^2b^2+6b^4\right)\sin(c+dx)}{ad}-\frac{3\left(\frac{2bx\left(a^2-b^2\right)^2}{a}-\frac{2b\left(4a^4-5a^2b^2+2b^4\right)\int\frac{1}{\left(1-\frac{a}{b}\right)\tan^2\left(\frac{1}{2}(c+dx)\right)+\frac{a+b}{b}}d\tan\left(\frac{1}{2}(c+dx)\right)}{ad}\right)}{a}}{a(a^2-b^2)}+\frac{3b^2\left(2a^2-b^2\right)\sin(c+dx)}{ad(a^2-b^2)(a+b\sec(c+dx))}}{2a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)}{2ad\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$\Big\downarrow$ 221

$$\frac{b^2\sin(c+dx)}{2ad\left(a^2-b^2\right)(a+b\sec(c+dx))^2}+\frac{\frac{3b^2\left(2a^2-b^2\right)\sin(c+dx)}{ad(a^2-b^2)(a+b\sec(c+dx))}+\frac{\frac{\left(2a^4-11a^2b^2+6b^4\right)\sin(c+dx)}{ad}-\frac{3\left(\frac{2bx\left(a^2-b^2\right)^2}{a}-\frac{2b^2\left(4a^4-5a^2b^2+2b^4\right)\text{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{ad\sqrt{a-b}\sqrt{a+b}}\right)}{a}}{a(a^2-b^2)}}{2a\left(a^2-b^2\right)}$$

input
```
Int[Cos[c + d*x]/(a + b*Sec[c + d*x])^3,x]
```

output
```
(b^2*Sin[c + d*x])/(2*a*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^2) + ((3*b^2*(2
*a^2 - b^2)*Sin[c + d*x])/(a*(a^2 - b^2)*d*(a + b*Sec[c + d*x])) + ((-3*((
2*b*(a^2 - b^2)^2*x)/a - (2*b^2*(4*a^4 - 5*a^2*b^2 + 2*b^4)*ArcTanh[(Sqrt[
a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a*Sqrt[a - b]*Sqrt[a + b]*d)))/a +
 ((2*a^4 - 11*a^2*b^2 + 6*b^4)*Sin[c + d*x])/(a*d))/(a*(a^2 - b^2)))/(2*a*
(a^2 - b^2))
```

3.512. $\int \frac{\cos(c+dx)}{(a+b\sec(c+dx))^3}\,dx$

3.512.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 4318
```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[1/b   Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4334
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*
((d*Csc[e + f*x])^n/(a*f*(m + 1)*(a^2 - b^2))), x] + Simp[1/(a*(m + 1)*(a^2
 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a^2*(m + 1)
 - b^2*(m + n + 1) - a*b*(m + 1)*Csc[e + f*x] + b^2*(m + n + 2)*Csc[e + f*x
]^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0] && LtQ[m,
 -1] && IntegersQ[2*m, 2*n]
```

rule 4407
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Simp[(b*c - a*d)/a   Int[Csc[e + f*
x]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
- a*d, 0]
```

rule 4588
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc
[e + f*x])^(m + 1)*((d*Csc[e + f*x])^n/(a*f*(m + 1)*(a^2 - b^2))), x] + Sim
p[1/(a*(m + 1)*(a^2 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f
*x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n +
1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m
+ n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x
] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

rule 4592
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Simp[1/(a*d*n)   Int[(a + b*Csc[e + f*x])^m
*(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*
Csc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

3.512.4 Maple [A] (verified)

Time = 1.11 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.21

method	result
derivativedivides	$\frac{-\frac{2\left(-\frac{a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}+3b\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}{a^4}-\frac{2b^2\left(\frac{-\frac{\left(8a^2+ab-4b^2\right)ab\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{2(a-b)\left(a^2+2ab+b^2\right)}+\frac{\left(8a^2-ab-4b^2\right)ab\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2(a+b)\left(a^2-2ab+b^2\right)}}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2b-a-b\right)^2}-\cdots\right.}{a^4}}{d}$
default	$\frac{-\frac{2\left(-\frac{a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}+3b\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}{a^4}-\frac{2b^2\left(\frac{-\frac{\left(8a^2+ab-4b^2\right)ab\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{2(a-b)\left(a^2+2ab+b^2\right)}+\frac{\left(8a^2-ab-4b^2\right)ab\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2(a+b)\left(a^2-2ab+b^2\right)}}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2b-a-b\right)^2}-\cdots\right.}{a^4}}{d}$
risch	$-\frac{3bx}{a^4}-\frac{ie^{i(dx+c)}}{2a^3d}+\frac{ie^{-i(dx+c)}}{2a^3d}+\frac{ib^3\left(-9a^3b\,e^{3i(dx+c)}+6a\,b^3e^{3i(dx+c)}-8a^4e^{2i(dx+c)}-11a^2b^2e^{2i(dx+c)}+10b^4e^{2i(\cdots}\right.}{a^4\left(-a^2+b^2\right)^2d\left(a\,e^{2i(dx+c)}+2b\,e^{i(dx\cdots}\right.}$

input
```
int(cos(d*x+c)/(a+b*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-2/a^4*(-a*tan(1/2*d*x+1/2*c)/(1+tan(1/2*d*x+1/2*c)^2)+3*b*arctan(tan
(1/2*d*x+1/2*c)))-2*b^2/a^4*((-1/2*(8*a^2+a*b-4*b^2)*a*b/(a-b)/(a^2+2*a*b+
b^2)*tan(1/2*d*x+1/2*c)^3+1/2*(8*a^2-a*b-4*b^2)*a*b/(a+b)/(a^2-2*a*b+b^2)*
tan(1/2*d*x+1/2*c))/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2-
3/2*(4*a^4-5*a^2*b^2+2*b^4)/(a^4-2*a^2*b^2+b^4)/((a-b)*(a+b))^(1/2)*arctan
h((a-b)*tan(1/2*d*x+1/2*c)/((a-b)*(a+b))^(1/2))))
```

3.512.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 490 vs. $2(208) = 416$.

Time = 0.32 (sec) , antiderivative size = 1037, normalized size of antiderivative = 4.65

$$\int \frac{\cos(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)/(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
[-1/4*(12*(a^8*b - 3*a^6*b^3 + 3*a^4*b^5 - a^2*b^7)*d*x*cos(d*x + c)^2 + 2
4*(a^7*b^2 - 3*a^5*b^4 + 3*a^3*b^6 - a*b^8)*d*x*cos(d*x + c) + 12*(a^6*b^3
 - 3*a^4*b^5 + 3*a^2*b^7 - b^9)*d*x - 3*(4*a^4*b^4 - 5*a^2*b^6 + 2*b^8 + (
4*a^6*b^2 - 5*a^4*b^4 + 2*a^2*b^6)*cos(d*x + c)^2 + 2*(4*a^5*b^3 - 5*a^3*b
^5 + 2*a*b^7)*cos(d*x + c))*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2
 - 2*b^2)*cos(d*x + c)^2 + 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x
+ c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) - 2*(
2*a^7*b^2 - 13*a^5*b^4 + 17*a^3*b^6 - 6*a*b^8 + 2*(a^9 - 3*a^7*b^2 + 3*a^5
*b^4 - a^3*b^6)*cos(d*x + c)^2 + (4*a^8*b - 20*a^6*b^3 + 25*a^4*b^5 - 9*a^
2*b^7)*cos(d*x + c))*sin(d*x + c))/((a^12 - 3*a^10*b^2 + 3*a^8*b^4 - a^6*b
^6)*d*cos(d*x + c)^2 + 2*(a^11*b - 3*a^9*b^3 + 3*a^7*b^5 - a^5*b^7)*d*cos(
d*x + c) + (a^10*b^2 - 3*a^8*b^4 + 3*a^6*b^6 - a^4*b^8)*d), -1/2*(6*(a^8*b
 - 3*a^6*b^3 + 3*a^4*b^5 - a^2*b^7)*d*x*cos(d*x + c)^2 + 12*(a^7*b^2 - 3*a
^5*b^4 + 3*a^3*b^6 - a*b^8)*d*x*cos(d*x + c) + 6*(a^6*b^3 - 3*a^4*b^5 + 3*
a^2*b^7 - b^9)*d*x - 3*(4*a^4*b^4 - 5*a^2*b^6 + 2*b^8 + (4*a^6*b^2 - 5*a^4
*b^4 + 2*a^2*b^6)*cos(d*x + c)^2 + 2*(4*a^5*b^3 - 5*a^3*b^5 + 2*a*b^7)*cos
(d*x + c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/
((a^2 - b^2)*sin(d*x + c))) - (2*a^7*b^2 - 13*a^5*b^4 + 17*a^3*b^6 - 6*a*b
^8 + 2*(a^9 - 3*a^7*b^2 + 3*a^5*b^4 - a^3*b^6)*cos(d*x + c)^2 + (4*a^8*b -
 20*a^6*b^3 + 25*a^4*b^5 - 9*a^2*b^7)*cos(d*x + c))*sin(d*x + c))/((a^1...
```

3.512.6 Sympy [F]

$$\int \frac{\cos(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \int \frac{\cos{(c+dx)}}{(a+b\sec{(c+dx)})^3}\,dx$$

input
```
integrate(cos(d*x+c)/(a+b*sec(d*x+c))**3,x)
```

output
```
Integral(cos(c + d*x)/(a + b*sec(c + d*x))**3, x)
```

3.512.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\cos(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(cos(d*x+c)/(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

3.512.8 Giac [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 357, normalized size of antiderivative = 1.60

$$\int \frac{\cos(c+dx)}{(a+b\sec(c+dx))^3}\,dx = -\frac{\frac{3\,(4\,a^4b^2-5\,a^2b^4+2\,b^6)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2\,a-2\,b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{-a^2+b^2}}\right)\right)}{(a^8-2\,a^6b^2+a^4b^4)\sqrt{-a^2+b^2}} - \frac{8\,a^3b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-7\,a^2b^4\tan\left(\frac{1}{2}\,d\right.}{}}{}$$

input
```
integrate(cos(d*x+c)/(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
-(3*(4*a^4*b^2 - 5*a^2*b^4 + 2*b^6)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(
2*a - 2*b) + arctan((a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt
(-a^2 + b^2)))/((a^8 - 2*a^6*b^2 + a^4*b^4)*sqrt(-a^2 + b^2)) - (8*a^3*b^3
*tan(1/2*d*x + 1/2*c)^3 - 7*a^2*b^4*tan(1/2*d*x + 1/2*c)^3 - 5*a*b^5*tan(1
/2*d*x + 1/2*c)^3 + 4*b^6*tan(1/2*d*x + 1/2*c)^3 - 8*a^3*b^3*tan(1/2*d*x +
 1/2*c) - 7*a^2*b^4*tan(1/2*d*x + 1/2*c) + 5*a*b^5*tan(1/2*d*x + 1/2*c) +
4*b^6*tan(1/2*d*x + 1/2*c))/((a^7 - 2*a^5*b^2 + a^3*b^4)*(a*tan(1/2*d*x +
1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 - a - b)^2) + 3*(d*x + c)*b/a^4 - 2*ta
n(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/2*c)^2 + 1)*a^3))/d
```

3.512.9 Mupad [B] (verification not implemented)

Time = 21.74 (sec) , antiderivative size = 5338, normalized size of antiderivative = 23.94

$$\int \frac{\cos(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
int(cos(c + d*x)/(a + b/cos(c + d*x))^3,x)
```

output
```
((tan(c/2 + (d*x)/2)*(3*a*b^4 + 2*a^4*b + 2*a^5 + 6*b^5 - 12*a^2*b^3 - 4*a
^3*b^2))/((a + b)*(a^5 - 2*a^4*b + a^3*b^2)) - (tan(c/2 + (d*x)/2)^5*(3*a*
b^4 - 2*a^4*b + 2*a^5 - 6*b^5 + 12*a^2*b^3 - 4*a^3*b^2))/((a^3*b - a^4)*(a
 + b)^2) + (2*tan(c/2 + (d*x)/2)^3*(2*a^6 - 6*b^6 + 13*a^2*b^4 - 6*a^4*b^2
))/(a*(a^2*b - a^3)*(a + b)^2*(a - b)))/(d*(2*a*b + tan(c/2 + (d*x)/2)^2*(
2*a*b - a^2 + 3*b^2) + tan(c/2 + (d*x)/2)^6*(a^2 - 2*a*b + b^2) + a^2 + b^
2 - tan(c/2 + (d*x)/2)^4*(2*a*b + a^2 - 3*b^2))) - (6*b*atan(((3*b*((8*tan
(c/2 + (d*x)/2)*(72*b^12 - 72*a*b^11 - 288*a^2*b^10 + 288*a^3*b^9 + 441*a^
4*b^8 - 432*a^5*b^7 - 288*a^6*b^6 + 288*a^7*b^5 + 36*a^8*b^4 - 72*a^9*b^3
+ 36*a^10*b^2))/(a^12*b + a^13 - a^6*b^7 - a^7*b^6 + 3*a^8*b^5 + 3*a^9*b^4
 - 3*a^10*b^3 - 3*a^11*b^2) + (b*((24*(4*a^17*b - 4*a^8*b^10 + 2*a^9*b^9 +
 18*a^10*b^8 - 8*a^11*b^7 - 32*a^12*b^6 + 14*a^13*b^5 + 26*a^14*b^4 - 12*a
^15*b^3 - 8*a^16*b^2))/(a^15*b + a^16 - a^9*b^7 - a^10*b^6 + 3*a^11*b^5 +
3*a^12*b^4 - 3*a^13*b^3 - 3*a^14*b^2) - (b*tan(c/2 + (d*x)/2)*(8*a^17*b -
8*a^8*b^10 + 8*a^9*b^9 + 32*a^10*b^8 - 32*a^11*b^7 - 48*a^12*b^6 + 48*a^13
*b^5 + 32*a^14*b^4 - 32*a^15*b^3 - 8*a^16*b^2)*24i)/(a^4*(a^12*b + a^13 -
a^6*b^7 - a^7*b^6 + 3*a^8*b^5 + 3*a^9*b^4 - 3*a^10*b^3 - 3*a^11*b^2)))*3i)
/a^4))/a^4 + (3*b*((8*tan(c/2 + (d*x)/2)*(72*b^12 - 72*a*b^11 - 288*a^2*b^
10 + 288*a^3*b^9 + 441*a^4*b^8 - 432*a^5*b^7 - 288*a^6*b^6 + 288*a^7*b^5 +
 36*a^8*b^4 - 72*a^9*b^3 + 36*a^10*b^2))/(a^12*b + a^13 - a^6*b^7 - a^7...
```

3.513 $\int \frac{\cos^2(c+dx)}{(a+b\sec(c+dx))^3}\,dx$

3.513.1 Optimal result . . . 3815
3.513.2 Mathematica [A] (verified) . . . 3816
3.513.3 Rubi [A] (verified) . . . 3816
3.513.4 Maple [A] (verified) . . . 3822
3.513.5 Fricas [A] (verification not implemented) . . . 3822
3.513.6 Sympy [F] . . . 3823
3.513.7 Maxima [F(-2)] . . . 3824
3.513.8 Giac [B] (verification not implemented) . . . 3824
3.513.9 Mupad [B] (verification not implemented) . . . 3825

3.513.1 Optimal result

Integrand size = 21, antiderivative size = 296

$$\begin{aligned}\int \frac{\cos^2(c+dx)}{(a+b\sec(c+dx))^3}\,dx = {} & \frac{(a^2+12b^2)\,x}{2a^5} \\ & - \frac{b^3(20a^4-29a^2b^2+12b^4)\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^5(a-b)^{5/2}(a+b)^{5/2}d} \\ & - \frac{3b(2a^4-7a^2b^2+4b^4)\sin(c+dx)}{2a^4\left(a^2-b^2\right)^2 d} \\ & + \frac{(a^4-10a^2b^2+6b^4)\cos(c+dx)\sin(c+dx)}{2a^3\left(a^2-b^2\right)^2 d} \\ & + \frac{b^2\cos(c+dx)\sin(c+dx)}{2a\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} \\ & + \frac{b^2(7a^2-4b^2)\cos(c+dx)\sin(c+dx)}{2a^2\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))}\end{aligned}$$

output
```
1/2*(a^2+12*b^2)*x/a^5-b^3*(20*a^4-29*a^2*b^2+12*b^4)*arctanh((a-b)^(1/2)*
tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/a^5/(a-b)^(5/2)/(a+b)^(5/2)/d-3/2*b*(2*a^4
-7*a^2*b^2+4*b^4)*sin(d*x+c)/a^4/(a^2-b^2)^2/d+1/2*(a^4-10*a^2*b^2+6*b^4)*
cos(d*x+c)*sin(d*x+c)/a^3/(a^2-b^2)^2/d+1/2*b^2*cos(d*x+c)*sin(d*x+c)/a/(a
^2-b^2)/d/(a+b*sec(d*x+c))^2+1/2*b^2*(7*a^2-4*b^2)*cos(d*x+c)*sin(d*x+c)/a
^2/(a^2-b^2)^2/d/(a+b*sec(d*x+c))
```

3.513.2 Mathematica [A] (verified)

Time = 1.84 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.67

$$\int \frac{\cos^2(c+dx)}{(a+b\sec(c+dx))^3}\,dx$$
$$= \frac{2(a^2+12b^2)(c+dx)+\frac{4b^3(20a^4-29a^2b^2+12b^4)\operatorname{arctanh}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{5/2}}-12ab\sin(c+dx)+\frac{2ab^5\sin(c+}{(-a+b)(a+b)(b+a}}{4a^5d}$$

input
```
Integrate[Cos[c + d*x]^2/(a + b*Sec[c + d*x])^3,x]
```

output
```
(2*(a^2 + 12*b^2)*(c + d*x) + (4*b^3*(20*a^4 - 29*a^2*b^2 + 12*b^4)*ArcTan
h[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(5/2) - 12*a*b
*Sin[c + d*x] + (2*a*b^5*Sin[c + d*x])/((-a + b)*(a + b)*(b + a*Cos[c + d*
x])^2) + (2*a*b^4*(10*a^2 - 7*b^2)*Sin[c + d*x])/((a - b)^2*(a + b)^2*(b +
 a*Cos[c + d*x])) + a^2*Sin[2*(c + d*x)])/(4*a^5*d)
```

3.513.3 Rubi [A] (verified)

Time = 2.17 (sec) , antiderivative size = 321, normalized size of antiderivative = 1.08, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.857$, Rules used = {3042, 4334, 25, 3042, 4588, 25, 3042, 4592, 27, 3042, 4592, 3042, 4407, 3042, 4318, 3042, 3138, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^2(c+dx)}{(a+b\sec(c+dx))^3}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^2\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^3}dx$$
$$\Big\downarrow \; 4334$$
$$\frac{b^2\sin(c+dx)\cos(c+dx)}{2ad\left(a^2-b^2\right)(a+b\sec(c+dx))^2}-\frac{\int -\frac{\cos^2(c+dx)\left(3b^2\sec^2(c+dx)-2ab\sec(c+dx)+2\left(a^2-2b^2\right)\right)}{(a+b\sec(c+dx))^2}dx}{2a\left(a^2-b^2\right)}$$
$$\Big\downarrow \; 25$$

$$\frac{\int \frac{\cos^2(c+dx)\left(3b^2\sec^2(c+dx)-2ab\sec(c+dx)+2\left(a^2-2b^2\right)\right)}{(a+b\sec(c+dx))^2}dx}{2a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)\cos(c+dx)}{2ad\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{\int \frac{3b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2-2ab\csc\left(c+dx+\frac{\pi}{2}\right)+2\left(a^2-2b^2\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^2\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx}{2a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)\cos(c+dx)}{2ad\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$$\Big\downarrow \; 4588$$

$$\frac{\frac{b^2\left(7a^2-4b^2\right)\sin(c+dx)\cos(c+dx)}{ad(a^2-b^2)(a+b\sec(c+dx))}-\frac{\int -\frac{\cos^2(c+dx)\left(2b^2\left(7a^2-4b^2\right)\sec^2(c+dx)-ab\left(4a^2-b^2\right)\sec(c+dx)+2\left(a^4-10b^2a^2+6b^4\right)\right)}{a+b\sec(c+dx)}dx}{a(a^2-b^2)}}{2a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)\cos(c+dx)}{2ad\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\int \frac{\cos^2(c+dx)\left(2b^2\left(7a^2-4b^2\right)\sec^2(c+dx)-ab\left(4a^2-b^2\right)\sec(c+dx)+2\left(a^4-10b^2a^2+6b^4\right)\right)}{a+b\sec(c+dx)}dx}{a(a^2-b^2)}+\frac{b^2\left(7a^2-4b^2\right)\sin(c+dx)\cos(c+dx)}{ad(a^2-b^2)(a+b\sec(c+dx))}}{2a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)\cos(c+dx)}{2ad\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{\int \frac{2b^2\left(7a^2-4b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2-ab\left(4a^2-b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)+2\left(a^4-10b^2a^2+6b^4\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^2\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a(a^2-b^2)}+\frac{b^2\left(7a^2-4b^2\right)\sin(c+dx)\cos(c+dx)}{ad(a^2-b^2)(a+b\sec(c+dx))}}{2a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)\cos(c+dx)}{2ad\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$$\Big\downarrow \; 4592$$

$$\frac{\frac{\left(a^4-10a^2b^2+6b^4\right)\sin(c+dx)\cos(c+dx)}{ad}-\frac{\int \frac{2\cos(c+dx)\left(-b\left(a^4-10b^2a^2+6b^4\right)\sec^2(c+dx)-a\left(a^4+4b^2a^2-2b^4\right)\sec(c+dx)+3b\left(2a^4-7b^2a^2+4b^4\right)\right)}{a+b\sec(c+dx)}dx}{2a}}{a(a^2-b^2)}+\frac{b^2(7a^2}{ad}}{2a\left(a^2-b^2\right)}$$

$$\frac{b^2\sin(c+dx)\cos(c+dx)}{2ad\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$$\Big\downarrow \; 27$$

3.513. $\int \frac{\cos^2(c+dx)}{(a+b\sec(c+dx))^3}\,dx$

$$\frac{\frac{\frac{\left(a^4-10a^2b^2+6b^4\right)\sin(c+dx)\cos(c+dx)}{ad}-\frac{\int\frac{\cos(c+dx)\left(-b\left(a^4-10b^2a^2+6b^4\right)\sec^2(c+dx)-a\left(a^4+4b^2a^2-2b^4\right)\sec(c+dx)+3b\left(2a^4-7b^2a^2+4b^4\right)\right)}{a+b\sec(c+dx)}dx}{a}}{a(a^2-b^2)}+\frac{b^2(7a^2-}{ad(a}}{2a\left(a^2-b^2\right)}$$

$$\frac{b^2\sin(c+dx)\cos(c+dx)}{2ad\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$\downarrow$ 3042

$$\frac{\frac{\frac{\left(a^4-10a^2b^2+6b^4\right)\sin(c+dx)\cos(c+dx)}{ad}-\frac{\int\frac{-b\left(a^4-10b^2a^2+6b^4\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2-a\left(a^4+4b^2a^2-2b^4\right)\csc\left(c+dx+\frac{\pi}{2}\right)+3b\left(2a^4-7b^2a^2+4b^4\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a}}{a(a^2-b^2)}+\frac{b^2(7a^2-4b}{ad(a^2-}}{2a\left(a^2-b^2\right)}$$

$$\frac{b^2\sin(c+dx)\cos(c+dx)}{2ad\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$\downarrow$ 4592

$$\frac{\frac{\frac{\left(a^4-10a^2b^2+6b^4\right)\sin(c+dx)\cos(c+dx)}{ad}-\frac{\frac{3b\left(2a^4-7a^2b^2+4b^4\right)\sin(c+dx)}{ad}-\frac{\int\frac{\left(a^2+12b^2\right)\left(a^2-b^2\right)^2+ab\left(a^4-10b^2a^2+6b^4\right)\sec(c+dx)}{a+b\sec(c+dx)}dx}{a}}{a}}{a(a^2-b^2)}+\frac{b^2(7a^2-4b^2)\sin(}{ad(a^2-b^2)(a-}}{2a\left(a^2-b^2\right)}$$

$$\frac{b^2\sin(c+dx)\cos(c+dx)}{2ad\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$\downarrow$ 3042

$$\frac{\frac{\frac{\left(a^4-10a^2b^2+6b^4\right)\sin(c+dx)\cos(c+dx)}{ad}-\frac{\frac{3b\left(2a^4-7a^2b^2+4b^4\right)\sin(c+dx)}{ad}-\frac{\int\frac{\left(a^2+12b^2\right)\left(a^2-b^2\right)^2+ab\left(a^4-10b^2a^2+6b^4\right)\csc\left(c+dx+\frac{\pi}{2}\right)}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a}}{a}}{a(a^2-b^2)}+\frac{b^2(7a^2-4b^2)}{ad(a^2-b^2}}{2a\left(a^2-b^2\right)}$$

$$\frac{b^2\sin(c+dx)\cos(c+dx)}{2ad\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$\downarrow$ 4407

$$\frac{\frac{\frac{\left(a^4-10a^2b^2+6b^4\right)\sin(c+dx)\cos(c+dx)}{ad}-\frac{\frac{3b\left(2a^4-7a^2b^2+4b^4\right)\sin(c+dx)}{ad}-\frac{\frac{x\left(a^2-b^2\right)^2\left(a^2+12b^2\right)}{a}-\frac{b^3\left(20a^4-29a^2b^2+12b^4\right)\int\frac{\sec(c+dx)}{a+b\sec(c+dx)}dx}{a}}{a}}{a}}{a(a^2-b^2)}+\frac{b^2(7a^2-}{ad(}}{2a\left(a^2-b^2\right)}$$

$$\frac{b^2\sin(c+dx)\cos(c+dx)}{2ad\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$\downarrow$ 3042

3.513. $\int \frac{\cos^2(c+dx)}{(a+b\sec(c+dx))^3}\,dx$

$$\frac{\frac{\frac{\left(a^4-10a^2b^2+6b^4\right)\sin(c+dx)\cos(c+dx)}{ad}-\frac{\frac{3b\left(2a^4-7a^2b^2+4b^4\right)\sin(c+dx)}{ad}-\frac{\frac{x\left(a^2-b^2\right)^2\left(a^2+12b^2\right)}{a}-\frac{b^3\left(20a^4-29a^2b^2+12b^4\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a}}{a}}{a(a^2-b^2)}}{2a\left(a^2-b^2\right)}+\frac{b^2(}{}$$

$$\frac{b^2\sin(c+dx)\cos(c+dx)}{2ad\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$$\Big\downarrow \; 4318$$

$$\frac{\frac{\frac{\left(a^4-10a^2b^2+6b^4\right)\sin(c+dx)\cos(c+dx)}{ad}-\frac{\frac{3b\left(2a^4-7a^2b^2+4b^4\right)\sin(c+dx)}{ad}-\frac{\frac{x\left(a^2-b^2\right)^2\left(a^2+12b^2\right)}{a}-\frac{b^2\left(20a^4-29a^2b^2+12b^4\right)\int\frac{1}{\frac{a\cos(c+dx)}{b}+1}dx}{a}}{a}}{a(a^2-b^2)}}{2a\left(a^2-b^2\right)}+\frac{b^2(7a^2}{ad(}$$

$$\frac{b^2\sin(c+dx)\cos(c+dx)}{2ad\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{\frac{\left(a^4-10a^2b^2+6b^4\right)\sin(c+dx)\cos(c+dx)}{ad}-\frac{\frac{3b\left(2a^4-7a^2b^2+4b^4\right)\sin(c+dx)}{ad}-\frac{\frac{x\left(a^2-b^2\right)^2\left(a^2+12b^2\right)}{a}-\frac{b^2\left(20a^4-29a^2b^2+12b^4\right)\int\frac{1}{\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{b}+1}dx}{a}}{a}}{a(a^2-b^2)}}{2a\left(a^2-b^2\right)}+\frac{b^2(}{}$$

$$\frac{b^2\sin(c+dx)\cos(c+dx)}{2ad\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$$\Big\downarrow \; 3138$$

$$\frac{\frac{\frac{\left(a^4-10a^2b^2+6b^4\right)\sin(c+dx)\cos(c+dx)}{ad}-\frac{\frac{3b\left(2a^4-7a^2b^2+4b^4\right)\sin(c+dx)}{ad}-\frac{\frac{x\left(a^2-b^2\right)^2\left(a^2+12b^2\right)}{a}-\frac{2b^2\left(20a^4-29a^2b^2+12b^4\right)\int\frac{1}{\left(1-\frac{a}{b}\right)\tan^2\left(\frac{1}{2}(c+dx)\right)+\frac{a+b}{b}}}{ad}}{a}}{a(a^2-b^2)}}{2a\left(a^2-b^2\right)}$$

$$\frac{b^2\sin(c+dx)\cos(c+dx)}{2ad\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$$\Big\downarrow \; 221$$

$$\frac{b^2\sin(c+dx)\cos(c+dx)}{2ad\left(a^2-b^2\right)(a+b\sec(c+dx))^2}+$$

$$\frac{b^2\left(7a^2-4b^2\right)\sin(c+dx)\cos(c+dx)}{ad(a^2-b^2)(a+b\sec(c+dx))}+\frac{\frac{\frac{\left(a^4-10a^2b^2+6b^4\right)\sin(c+dx)\cos(c+dx)}{ad}-\frac{\frac{3b\left(2a^4-7a^2b^2+4b^4\right)\sin(c+dx)}{ad}-\frac{\frac{x\left(a^2-b^2\right)^2\left(a^2+12b^2\right)}{a}-\frac{2b^3\left(20a^4-29a^2\right.}{}}{a}}{a(a^2-b^2)}}{2a\left(a^2-b^2\right)}$$

3.513. $\int \frac{\cos^2(c+dx)}{(a+b\sec(c+dx))^3}\,dx$

input
```
Int[Cos[c + d*x]^2/(a + b*Sec[c + d*x])^3,x]
```

output
```
(b^2*Cos[c + d*x]*Sin[c + d*x])/(2*a*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^2)
 + ((b^2*(7*a^2 - 4*b^2)*Cos[c + d*x]*Sin[c + d*x])/(a*(a^2 - b^2)*d*(a +
b*Sec[c + d*x])) + (((a^4 - 10*a^2*b^2 + 6*b^4)*Cos[c + d*x]*Sin[c + d*x])
/(a*d) - (-((((a^2 - b^2)^2*(a^2 + 12*b^2)*x)/a - (2*b^3*(20*a^4 - 29*a^2*
b^2 + 12*b^4)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a*Sqrt
[a - b]*Sqrt[a + b]*d))/a) + (3*b*(2*a^4 - 7*a^2*b^2 + 4*b^4)*Sin[c + d*x]
)/(a*d))/a)/(a*(a^2 - b^2)))/(2*a*(a^2 - b^2))
```

3.513.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 4318
```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[1/b   Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4334
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*
((d*Csc[e + f*x])^n/(a*f*(m + 1)*(a^2 - b^2))), x] + Simp[1/(a*(m + 1)*(a^2
 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a^2*(m + 1)
 - b^2*(m + n + 1) - a*b*(m + 1)*Csc[e + f*x] + b^2*(m + n + 2)*Csc[e + f*x
]^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0] && LtQ[m,
 -1] && IntegersQ[2*m, 2*n]
```

rule 4407
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Simp[(b*c - a*d)/a   Int[Csc[e + f*
x]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
- a*d, 0]
```

rule 4588
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc
[e + f*x])^(m + 1)*((d*Csc[e + f*x])^n/(a*f*(m + 1)*(a^2 - b^2))), x] + Sim
p[1/(a*(m + 1)*(a^2 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f
*x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n +
1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m
+ n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x
] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

rule 4592
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Simp[1/(a*d*n)   Int[(a + b*Csc[e + f*x])^m
*(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*
Csc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

3.513. $\int \frac{\cos^2(c+dx)}{(a+b\sec(c+dx))^3}\,dx$

3.513.4 Maple [A] (verified)

Time = 1.20 (sec) , antiderivative size = 310, normalized size of antiderivative = 1.05

method	result
derivativedivides	$\frac{\frac{2b^3\left(\frac{-\frac{\left(10a^2+ab-6b^2\right)ab\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{2(a-b)\left(a^2+2ab+b^2\right)}+\frac{\left(10a^2-ab-6b^2\right)ab\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2(a+b)\left(a^2-2ab+b^2\right)}}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2b-a-b\right)^2}-\frac{\left(20a^4-29a^2b^2+12b^4\right)\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{2\left(a^4-2a^2b^2+b^4\right)\sqrt{(a-b)(a+b)}}\right.}{a^5}}{d}$
default	$\frac{\frac{2b^3\left(\frac{-\frac{\left(10a^2+ab-6b^2\right)ab\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{2(a-b)\left(a^2+2ab+b^2\right)}+\frac{\left(10a^2-ab-6b^2\right)ab\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2(a+b)\left(a^2-2ab+b^2\right)}}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2b-a-b\right)^2}-\frac{\left(20a^4-29a^2b^2+12b^4\right)\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{2\left(a^4-2a^2b^2+b^4\right)\sqrt{(a-b)(a+b)}}\right.}{a^5}}{d}$
risch	$\frac{x}{2a^3}+\frac{6x\,b^2}{a^5}-\frac{i\mathrm{e}^{2i(dx+c)}}{8a^3d}+\frac{3ib\,\mathrm{e}^{i(dx+c)}}{2a^4d}-\frac{3ib\,\mathrm{e}^{-i(dx+c)}}{2a^4d}+\frac{i\mathrm{e}^{-2i(dx+c)}}{8a^3d}-\frac{ib^4\left(-11a^3b\,\mathrm{e}^{3i(dx+c)}+8a\,b^3\mathrm{e}^{3i(dx+}\right.}{}$

input
```
int(cos(d*x+c)^2/(a+b*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(2*b^3/a^5*((-1/2*(10*a^2+a*b-6*b^2)*a*b/(a-b)/(a^2+2*a*b+b^2)*tan(1/2
*d*x+1/2*c)^3+1/2*(10*a^2-a*b-6*b^2)*a*b/(a+b)/(a^2-2*a*b+b^2)*tan(1/2*d*x
+1/2*c))/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2-1/2*(20*a^4
-29*a^2*b^2+12*b^4)/(a^4-2*a^2*b^2+b^4)/((a-b)*(a+b))^(1/2)*arctanh((a-b)*
tan(1/2*d*x+1/2*c)/((a-b)*(a+b))^(1/2)))+2/a^5*(((-1/2*a^2-3*a*b)*tan(1/2*
d*x+1/2*c)^3+(-3*a*b+1/2*a^2)*tan(1/2*d*x+1/2*c))/(1+tan(1/2*d*x+1/2*c)^2)
^2+1/2*(a^2+12*b^2)*arctan(tan(1/2*d*x+1/2*c))))
```

3.513.5 Fricas [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 1158, normalized size of antiderivative = 3.91

$$\int \frac{\cos^2(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^2/(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
[1/4*(2*(a^10 + 9*a^8*b^2 - 33*a^6*b^4 + 35*a^4*b^6 - 12*a^2*b^8)*d*x*cos(
d*x + c)^2 + 4*(a^9*b + 9*a^7*b^3 - 33*a^5*b^5 + 35*a^3*b^7 - 12*a*b^9)*d*
x*cos(d*x + c) + 2*(a^8*b^2 + 9*a^6*b^4 - 33*a^4*b^6 + 35*a^2*b^8 - 12*b^1
0)*d*x + (20*a^4*b^5 - 29*a^2*b^7 + 12*b^9 + (20*a^6*b^3 - 29*a^4*b^5 + 12
*a^2*b^7)*cos(d*x + c)^2 + 2*(20*a^5*b^4 - 29*a^3*b^6 + 12*a*b^8)*cos(d*x
+ c))*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)
^2 - 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a
^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) - 2*(6*a^7*b^3 - 27*a^5*b^5
 + 33*a^3*b^7 - 12*a*b^9 - (a^10 - 3*a^8*b^2 + 3*a^6*b^4 - a^4*b^6)*cos(d*
x + c)^3 + 4*(a^9*b - 3*a^7*b^3 + 3*a^5*b^5 - a^3*b^7)*cos(d*x + c)^2 + (1
1*a^8*b^2 - 43*a^6*b^4 + 50*a^4*b^6 - 18*a^2*b^8)*cos(d*x + c))*sin(d*x +
c))/((a^13 - 3*a^11*b^2 + 3*a^9*b^4 - a^7*b^6)*d*cos(d*x + c)^2 + 2*(a^12*
b - 3*a^10*b^3 + 3*a^8*b^5 - a^6*b^7)*d*cos(d*x + c) + (a^11*b^2 - 3*a^9*b
^4 + 3*a^7*b^6 - a^5*b^8)*d), 1/2*((a^10 + 9*a^8*b^2 - 33*a^6*b^4 + 35*a^4
*b^6 - 12*a^2*b^8)*d*x*cos(d*x + c)^2 + 2*(a^9*b + 9*a^7*b^3 - 33*a^5*b^5
+ 35*a^3*b^7 - 12*a*b^9)*d*x*cos(d*x + c) + (a^8*b^2 + 9*a^6*b^4 - 33*a^4*
b^6 + 35*a^2*b^8 - 12*b^10)*d*x - (20*a^4*b^5 - 29*a^2*b^7 + 12*b^9 + (20*
a^6*b^3 - 29*a^4*b^5 + 12*a^2*b^7)*cos(d*x + c)^2 + 2*(20*a^5*b^4 - 29*a^3
*b^6 + 12*a*b^8)*cos(d*x + c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(
b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c))) - (6*a^7*b^3 - 27*a^5*b...
```

3.513.6 Sympy [F]

$$\int \frac{\cos^2(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \int \frac{\cos^2{(c+dx)}}{(a+b\sec{(c+dx)})^3}\,dx$$

input
```
integrate(cos(d*x+c)**2/(a+b*sec(d*x+c))**3,x)
```

output
```
Integral(cos(c + d*x)**2/(a + b*sec(c + d*x))**3, x)
```

3.513.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\cos^2(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(cos(d*x+c)^2/(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

3.513.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1723 vs. $2(277) = 554$.

Time = 0.54 (sec) , antiderivative size = 1723, normalized size of antiderivative = 5.82

$$\int \frac{\cos^2(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^2/(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
-1/2*(((a^6 - a^5*b + 10*a^4*b^2 + 10*a^3*b^3 - 23*a^2*b^4 - 6*a*b^5 + 12*
b^6)*sqrt(-a^2 + b^2)*abs(a^9 - 2*a^7*b^2 + a^5*b^4)*abs(-a + b) - (a^15 -
 a^14*b + 8*a^13*b^2 - 28*a^12*b^3 - 42*a^11*b^4 + 111*a^10*b^5 + 68*a^9*b
^6 - 158*a^8*b^7 - 47*a^7*b^8 + 100*a^6*b^9 + 12*a^5*b^10 - 24*a^4*b^11)*s
qrt(-a^2 + b^2)*abs(-a + b))*(pi*floor(1/2*(d*x + c)/pi + 1/2) + arctan(ta
n(1/2*d*x + 1/2*c)/sqrt(-(a^8*b - 2*a^6*b^3 + a^4*b^5 + sqrt((a^9 + a^8*b
- 2*a^7*b^2 - 2*a^6*b^3 + a^5*b^4 + a^4*b^5)*(a^9 - a^8*b - 2*a^7*b^2 + 2*
a^6*b^3 + a^5*b^4 - a^4*b^5) + (a^8*b - 2*a^6*b^3 + a^4*b^5)^2))/(a^9 - a^
8*b - 2*a^7*b^2 + 2*a^6*b^3 + a^5*b^4 - a^4*b^5))))/((a^9 - 2*a^7*b^2 + a^
5*b^4)^2*(a^2 - 2*a*b + b^2) + (a^10*b - 2*a^9*b^2 - a^8*b^3 + 4*a^7*b^4 -
 a^6*b^5 - 2*a^5*b^6 + a^4*b^7)*abs(a^9 - 2*a^7*b^2 + a^5*b^4)) + (a^15 -
a^14*b + 8*a^13*b^2 - 28*a^12*b^3 - 42*a^11*b^4 + 111*a^10*b^5 + 68*a^9*b^
6 - 158*a^8*b^7 - 47*a^7*b^8 + 100*a^6*b^9 + 12*a^5*b^10 - 24*a^4*b^11 + a
^6*abs(a^9 - 2*a^7*b^2 + a^5*b^4) - a^5*b*abs(a^9 - 2*a^7*b^2 + a^5*b^4) +
 10*a^4*b^2*abs(a^9 - 2*a^7*b^2 + a^5*b^4) + 10*a^3*b^3*abs(a^9 - 2*a^7*b^
2 + a^5*b^4) - 23*a^2*b^4*abs(a^9 - 2*a^7*b^2 + a^5*b^4) - 6*a*b^5*abs(a^9
 - 2*a^7*b^2 + a^5*b^4) + 12*b^6*abs(a^9 - 2*a^7*b^2 + a^5*b^4))*(pi*floor
(1/2*(d*x + c)/pi + 1/2) + arctan(tan(1/2*d*x + 1/2*c)/sqrt(-(a^8*b - 2*a^
6*b^3 + a^4*b^5 - sqrt((a^9 + a^8*b - 2*a^7*b^2 - 2*a^6*b^3 + a^5*b^4 + a^
4*b^5)*(a^9 - a^8*b - 2*a^7*b^2 + 2*a^6*b^3 + a^5*b^4 - a^4*b^5) + (a^8...
```

3.513.9 Mupad [B] (verification not implemented)

Time = 22.68 (sec) , antiderivative size = 5950, normalized size of antiderivative = 20.10

$$\int \frac{\cos^2(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
int(cos(c + d*x)^2/(a + b/cos(c + d*x))^3,x)
```

output
```
(atan(((((8*tan(c/2 + (d*x)/2)*(a^14 - 2*a^13*b - 288*a*b^13 + 288*b^14 -
1104*a^2*b^12 + 1104*a^3*b^11 + 1538*a^4*b^10 - 1538*a^5*b^9 - 827*a^6*b^8
 + 872*a^7*b^7 + 18*a^8*b^6 - 108*a^9*b^5 + 74*a^10*b^4 - 40*a^11*b^3 + 21
*a^12*b^2))/(a^14*b + a^15 - a^8*b^7 - a^9*b^6 + 3*a^10*b^5 + 3*a^11*b^4 -
 3*a^12*b^3 - 3*a^13*b^2) + ((a^2*1i + b^2*12i)*((4*(4*a^21 - 48*a^10*b^11
 + 24*a^11*b^10 + 212*a^12*b^9 - 100*a^13*b^8 - 360*a^14*b^7 + 164*a^15*b^
6 + 276*a^16*b^5 - 120*a^17*b^4 - 80*a^18*b^3 + 28*a^19*b^2))/(a^18*b + a^
19 - a^12*b^7 - a^13*b^6 + 3*a^14*b^5 + 3*a^15*b^4 - 3*a^16*b^3 - 3*a^17*b
^2) - (4*tan(c/2 + (d*x)/2)*(a^2*1i + b^2*12i)*(8*a^19*b - 8*a^10*b^10 + 8
*a^11*b^9 + 32*a^12*b^8 - 32*a^13*b^7 - 48*a^14*b^6 + 48*a^15*b^5 + 32*a^1
6*b^4 - 32*a^17*b^3 - 8*a^18*b^2))/(a^5*(a^14*b + a^15 - a^8*b^7 - a^9*b^6
 + 3*a^10*b^5 + 3*a^11*b^4 - 3*a^12*b^3 - 3*a^13*b^2))))/(2*a^5))*(a^2*1i
+ b^2*12i)*1i)/(2*a^5) + (((8*tan(c/2 + (d*x)/2)*(a^14 - 2*a^13*b - 288*a*
b^13 + 288*b^14 - 1104*a^2*b^12 + 1104*a^3*b^11 + 1538*a^4*b^10 - 1538*a^5
*b^9 - 827*a^6*b^8 + 872*a^7*b^7 + 18*a^8*b^6 - 108*a^9*b^5 + 74*a^10*b^4
- 40*a^11*b^3 + 21*a^12*b^2))/(a^14*b + a^15 - a^8*b^7 - a^9*b^6 + 3*a^10*
b^5 + 3*a^11*b^4 - 3*a^12*b^3 - 3*a^13*b^2) - ((a^2*1i + b^2*12i)*((4*(4*a
^21 - 48*a^10*b^11 + 24*a^11*b^10 + 212*a^12*b^9 - 100*a^13*b^8 - 360*a^14
*b^7 + 164*a^15*b^6 + 276*a^16*b^5 - 120*a^17*b^4 - 80*a^18*b^3 + 28*a^19*
b^2))/(a^18*b + a^19 - a^12*b^7 - a^13*b^6 + 3*a^14*b^5 + 3*a^15*b^4 - ...
```

3.513. $\int \frac{\cos^2(c+dx)}{(a+b\sec(c+dx))^3}\,dx$

3.514 $\int \frac{\sec^6(c+dx)}{(a+b\sec(c+dx))^4}\,dx$

3.514.1 Optimal result . . . 3827
3.514.2 Mathematica [A] (verified) . . . 3828
3.514.3 Rubi [A] (verified) . . . 3829
3.514.4 Maple [A] (verified) . . . 3835
3.514.5 Fricas [B] (verification not implemented) . . . 3836
3.514.6 Sympy [F] . . . 3836
3.514.7 Maxima [F(-2)] . . . 3837
3.514.8 Giac [A] (verification not implemented) . . . 3837
3.514.9 Mupad [B] (verification not implemented) . . . 3838

3.514.1 Optimal result

Integrand size = 21, antiderivative size = 316

$$\begin{aligned}\int \frac{\sec^6(c+dx)}{(a+b\sec(c+dx))^4}\,dx = &-\frac{4a\operatorname{arctanh}(\sin(c+dx))}{b^5d}\\ &+\frac{a^2(8a^6-28a^4b^2+35a^2b^4-20b^6)\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{7/2}b^5(a+b)^{7/2}d}\\ &+\frac{(12a^4-23a^2b^2+6b^4)\tan(c+dx)}{6b^4\left(a^2-b^2\right)^2d}\\ &-\frac{a^2\sec^3(c+dx)\tan(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^3}\\ &-\frac{a^2(4a^2-9b^2)\sec^2(c+dx)\tan(c+dx)}{6b^2\left(a^2-b^2\right)^2d(a+b\sec(c+dx))^2}\\ &+\frac{a^3(4a^4-11a^2b^2+12b^4)\tan(c+dx)}{2b^4\left(a^2-b^2\right)^3d(a+b\sec(c+dx))}\end{aligned}$$

output
```
-4*a*arctanh(sin(d*x+c))/b^5/d+a^2*(8*a^6-28*a^4*b^2+35*a^2*b^4-20*b^6)*ar
ctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/(a-b)^(7/2)/b^5/(a+b)^(7
/2)/d+1/6*(12*a^4-23*a^2*b^2+6*b^4)*tan(d*x+c)/b^4/(a^2-b^2)^2/d-1/3*a^2*s
ec(d*x+c)^3*tan(d*x+c)/b/(a^2-b^2)/d/(a+b*sec(d*x+c))^3-1/6*a^2*(4*a^2-9*b
^2)*sec(d*x+c)^2*tan(d*x+c)/b^2/(a^2-b^2)^2/d/(a+b*sec(d*x+c))^2+1/2*a^3*(
4*a^4-11*a^2*b^2+12*b^4)*tan(d*x+c)/b^4/(a^2-b^2)^3/d/(a+b*sec(d*x+c))
```

3.514.2 Mathematica [A] (verified)

Time = 6.65 (sec) , antiderivative size = 416, normalized size of antiderivative = 1.32

$$\int \frac{\sec^6(c+dx)}{(a+b\sec(c+dx))^4}\,dx$$
$$= -\frac{a^2(-8a^6+28a^4b^2-35a^2b^4+20b^6)\operatorname{arctanh}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b^5\sqrt{a^2-b^2}\left(-a^2+b^2\right)^3 d}$$
$$+\frac{4a\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}{b^5d}$$
$$-\frac{4a\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}{b^5d}$$
$$+\frac{\sin\left(\frac{1}{2}(c+dx)\right)}{b^4d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}+\frac{\sin\left(\frac{1}{2}(c+dx)\right)}{b^4d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}$$
$$-\frac{a^3\sin(c+dx)}{3b^2(-a+b)(a+b)d(b+a\cos(c+dx))^3}+\frac{6a^5\sin(c+dx)-11a^3b^2\sin(c+dx)}{6b^3(-a+b)^2(a+b)^2d(b+a\cos(c+dx))^2}$$
$$+\frac{-18a^7\sin(c+dx)+50a^5b^2\sin(c+dx)-47a^3b^4\sin(c+dx)}{6b^4(-a+b)^3(a+b)^3d(b+a\cos(c+dx))}$$

input
```
Integrate[Sec[c + d*x]^6/(a + b*Sec[c + d*x])^4,x]
```

output
```
-((a^2*(-8*a^6 + 28*a^4*b^2 - 35*a^2*b^4 + 20*b^6)*ArcTanh[((-a + b)*Tan[(
c + d*x)/2])/Sqrt[a^2 - b^2]])/(b^5*Sqrt[a^2 - b^2]*(-a^2 + b^2)^3*d)) + (
4*a*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]])/(b^5*d) - (4*a*Log[Cos[(c +
d*x)/2] + Sin[(c + d*x)/2]])/(b^5*d) + Sin[(c + d*x)/2]/(b^4*d*(Cos[(c + d
*x)/2] - Sin[(c + d*x)/2])) + Sin[(c + d*x)/2]/(b^4*d*(Cos[(c + d*x)/2] +
Sin[(c + d*x)/2])) - (a^3*Sin[c + d*x])/(3*b^2*(-a + b)*(a + b)*d*(b + a*C
os[c + d*x])^3) + (6*a^5*Sin[c + d*x] - 11*a^3*b^2*Sin[c + d*x])/(6*b^3*(-
a + b)^2*(a + b)^2*d*(b + a*Cos[c + d*x])^2) + (-18*a^7*Sin[c + d*x] + 50*
a^5*b^2*Sin[c + d*x] - 47*a^3*b^4*Sin[c + d*x])/(6*b^4*(-a + b)^3*(a + b)^
3*d*(b + a*Cos[c + d*x]))
```

3.514.3 Rubi [A] (verified)

Time = 2.47 (sec) , antiderivative size = 373, normalized size of antiderivative = 1.18, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {3042, 4332, 3042, 4586, 25, 3042, 4578, 3042, 4570, 27, 3042, 4486, 3042, 4257, 4318, 3042, 3138, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^6(c+dx)}{(a+b\sec(c+dx))^4}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^6}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^4}dx$$

$$\Big\downarrow \text{4332}$$

$$-\frac{\int \frac{\sec^3(c+dx)\left(3a^2-3b\sec(c+dx)a-\left(4a^2-3b^2\right)\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^3}dx}{3b\left(a^2-b^2\right)}-\frac{a^2\tan(c+dx)\sec^3(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^3}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^3\left(3a^2-3b\csc\left(c+dx+\frac{\pi}{2}\right)a+\left(3b^2-4a^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^3}dx}{3b\left(a^2-b^2\right)}-\frac{a^2\tan(c+dx)\sec^3(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^3}$$

$$\Big\downarrow \text{4586}$$

$$-\frac{\frac{a^2\left(4a^2-9b^2\right)\tan(c+dx)\sec^2(c+dx)}{2bd(a^2-b^2)(a+b\sec(c+dx))^2}-\frac{\int -\frac{\sec^2(c+dx)\left(2\left(4a^2-9b^2\right)a^2-2b\left(a^2-6b^2\right)\sec(c+dx)a-\left(12a^4-23b^2a^2+6b^4\right)\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^2}dx}{2b(a^2-b^2)}}{3b\left(a^2-b^2\right)}-\frac{a^2\tan(c+dx)\sec^3(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^3}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\frac{\int \frac{\sec^2(c+dx)\left(2\left(4a^2-9b^2\right)a^2-2b\left(a^2-6b^2\right)\sec(c+dx)a-\left(12a^4-23b^2a^2+6b^4\right)\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^2}dx}{2b(a^2-b^2)}+\frac{a^2\left(4a^2-9b^2\right)\tan(c+dx)\sec^2(c+dx)}{2bd(a^2-b^2)(a+b\sec(c+dx))^2}}{3b\left(a^2-b^2\right)}-\frac{a^2\tan(c+dx)\sec^3(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^3}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2\left(2\left(4a^2-9b^2\right)a^2-2b\left(a^2-6b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)a+\left(-12a^4+23b^2a^2-6b^4\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx}{2b(a^2-b^2)}+\frac{a^2\left(4a^2-9b^2\right)\tan(c+dx)\sec^2(c+dx)}{2bd(a^2-b^2)(a+b\sec(c+dx))^2}}{3b\left(a^2-b^2\right)}-\frac{a^2\tan(c+dx)\sec^3(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^3}$$

$\downarrow$ 4578

$$-\frac{\frac{\int \frac{\sec(c+dx)\left(3b\left(4a^4-11b^2a^2+12b^4\right)a^2+\left(12a^6-37b^2a^4+43b^4a^2-18b^6\right)\sec(c+dx)a-b\left(a^2-b^2\right)\left(12a^4-23b^2a^2+6b^4\right)\sec^2(c+dx)\right)}{a+b\sec(c+dx)}dx}{b^2\left(a^2-b^2\right)}-\frac{3a^3\left(4a^4-11a^2b^2+12b^4\right.}{b^2d\left(a^2-b^2\right)(a+b\,\mathrm{se}}}{2b(a^2-b^2)}}{3b\left(a^2-b^2\right)}-\frac{a^2\tan(c+dx)\sec^3(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^3}$$

$\downarrow$ 3042

$$-\frac{\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(3b\left(4a^4-11b^2a^2+12b^4\right)a^2+\left(12a^6-37b^2a^4+43b^4a^2-18b^6\right)\csc\left(c+dx+\frac{\pi}{2}\right)a-b\left(a^2-b^2\right)\left(12a^4-23b^2a^2+6b^4\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2\left(a^2-b^2\right)}-\frac{3a^3\left(4a^4\right.}{b^2d\left(\right.}}{2b(a^2-b^2)}}{3b\left(a^2-b^2\right)}-\frac{a^2\tan(c+dx)\sec^3(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^3}$$

$\downarrow$ 4570

$$-\frac{\frac{\frac{\int \frac{3\sec(c+dx)\left(8ab\sec(c+dx)\left(a^2-b^2\right)^3+a^2b^2\left(4a^4-11b^2a^2+12b^4\right)\right)}{a+b\sec(c+dx)}dx}{b}-\frac{\left(a^2-b^2\right)\left(12a^4-23a^2b^2+6b^4\right)\tan(c+dx)}{d}}{b^2\left(a^2-b^2\right)}-\frac{3a^3\left(4a^4-11a^2b^2+12b^4\right)\tan(c+dx)}{b^2d\left(a^2-b^2\right)(a+b\sec(c+dx))}}{2b(a^2-b^2)}+}{3b\left(a^2-b^2\right)}-\frac{a^2\tan(c+dx)\sec^3(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^3}$$

$\downarrow$ 27

$$-\frac{\frac{\frac{3\int \frac{\sec(c+dx)\left(8ab\sec(c+dx)\left(a^2-b^2\right)^3+a^2b^2\left(4a^4-11b^2a^2+12b^4\right)\right)}{a+b\sec(c+dx)}dx}{b}-\frac{\left(a^2-b^2\right)\left(12a^4-23a^2b^2+6b^4\right)\tan(c+dx)}{d}}{b^2\left(a^2-b^2\right)}-\frac{3a^3\left(4a^4-11a^2b^2+12b^4\right)\tan(c+dx)}{b^2d\left(a^2-b^2\right)(a+b\sec(c+dx))}}{2b(a^2-b^2)}+}{3b\left(a^2-b^2\right)}-\frac{a^2\tan(c+dx)\sec^3(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^3}$$

$\downarrow$ 3042

3.514. $\int \frac{\sec^6(c+dx)}{(a+b\sec(c+dx))^4}\,dx$

$$-\frac{\frac{\frac{3\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(8ab\csc\left(c+dx+\frac{\pi}{2}\right)\left(a^2-b^2\right)^3+a^2b^2\left(4a^4-11b^2a^2+12b^4\right)\right)}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b}-\frac{\left(a^2-b^2\right)\left(12a^4-23a^2b^2+6b^4\right)\tan(c+dx)}{d}}{b^2\left(a^2-b^2\right)}-\frac{3a^3\left(4a^4-11a^2b^2+12b^4\right)\tan(c}{b^2d\left(a^2-b^2\right)(a+b\sec(c+dx)}}{2b(a^2-b^2)}}{3b\left(a^2-b^2\right)}$$

$$\frac{a^2\tan(c+dx)\sec^3(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^3}$$

$\downarrow$ 4486

$$-\frac{\frac{\frac{3\left(8a\left(a^2-b^2\right)^3\int\sec(c+dx)dx-a^2\left(8a^6-28a^4b^2+35a^2b^4-20b^6\right)\int\frac{\sec(c+dx)}{a+b\sec(c+dx)}dx\right)}{b}-\frac{\left(a^2-b^2\right)\left(12a^4-23a^2b^2+6b^4\right)\tan(c+dx)}{d}}{b^2\left(a^2-b^2\right)}-\frac{3a^3\left(4a^4-11a^2b^2+12b^4\right)t}{b^2d\left(a^2-b^2\right)(a+b\sec(c}}{2b(a^2-b^2)}}{3b\left(a^2-b^2\right)}$$

$$\frac{a^2\tan(c+dx)\sec^3(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^3}$$

$\downarrow$ 3042

$$-\frac{\frac{\frac{3\left(8a\left(a^2-b^2\right)^3\int\csc\left(c+dx+\frac{\pi}{2}\right)dx-a^2\left(8a^6-28a^4b^2+35a^2b^4-20b^6\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx\right)}{b}-\frac{\left(a^2-b^2\right)\left(12a^4-23a^2b^2+6b^4\right)\tan(c+dx)}{d}}{b^2\left(a^2-b^2\right)}-\frac{3a^3\left(4a^4-11a^2b\right.}{b^2d\left(a^2-b^2\right)}}{2b(a^2-b^2)}}{3b\left(a^2-b^2\right)}$$

$$\frac{a^2\tan(c+dx)\sec^3(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^3}$$

$\downarrow$ 4257

$$-\frac{\frac{\frac{3\left(\frac{8a\left(a^2-b^2\right)^3\text{arctanh}(\sin(c+dx))}{d}-a^2\left(8a^6-28a^4b^2+35a^2b^4-20b^6\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx\right)}{b}-\frac{\left(a^2-b^2\right)\left(12a^4-23a^2b^2+6b^4\right)\tan(c+dx)}{d}}{b^2\left(a^2-b^2\right)}-\frac{3a^3\left(4a^4-1\right.}{b^2d\left(a^2-\right.}}{2b(a^2-b^2)}}{3b\left(a^2-b^2\right)}$$

$$\frac{a^2\tan(c+dx)\sec^3(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^3}$$

$\downarrow$ 4318

3.514. $\int \frac{\sec^6(c+dx)}{(a+b\sec(c+dx))^4}\,dx$

$$-\frac{\frac{\frac{3\left(\frac{8a\left(a^2-b^2\right)^3\text{arctanh}(\sin(c+dx))}{d}-\frac{a^2\left(8a^6-28a^4b^2+35a^2b^4-20b^6\right)\int\frac{1}{\frac{a\cos(c+dx)}{b}+1}dx}{b}\right)}{b}-\frac{\left(a^2-b^2\right)\left(12a^4-23a^2b^2+6b^4\right)\tan(c+dx)}{d}}{b^2\left(a^2-b^2\right)}-\frac{3a^3\left(4a^4-11a^2\right.}{b^2d\left(a^2-b^2\right.}}{2b(a^2-b^2)}}{3b\left(a^2-b^2\right)}$$

$$\frac{a^2\tan(c+dx)\sec^3(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^3}$$

$\downarrow$ 3042

$$-\frac{\frac{\frac{3\left(\frac{8a\left(a^2-b^2\right)^3\text{arctanh}(\sin(c+dx))}{d}-\frac{a^2\left(8a^6-28a^4b^2+35a^2b^4-20b^6\right)\int\frac{1}{\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{b}+1}dx}{b}\right)}{b}-\frac{\left(a^2-b^2\right)\left(12a^4-23a^2b^2+6b^4\right)\tan(c+dx)}{d}}{b^2\left(a^2-b^2\right)}-\frac{3a^3\left(4a^4-\right.}{b^2d\left(a^2\right.}}{2b(a^2-b^2)}}{3b\left(a^2-b^2\right)}$$

$$\frac{a^2\tan(c+dx)\sec^3(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^3}$$

$\downarrow$ 3138

$$-\frac{\frac{\frac{3\left(\frac{8a\left(a^2-b^2\right)^3\text{arctanh}(\sin(c+dx))}{d}-\frac{2a^2\left(8a^6-28a^4b^2+35a^2b^4-20b^6\right)\int\frac{1}{\left(1-\frac{a}{b}\right)\tan^2\left(\frac{1}{2}(c+dx)\right)+\frac{a+b}{b}}d\tan\left(\frac{1}{2}(c+dx)\right)}{bd}\right)}{b}-\frac{\left(a^2-b^2\right)\left(12a^4-23a^2b^2+6b^4\right.}{d}}{b^2\left(a^2-b^2\right)}}{2b(a^2-b^2)}}{3b\left(a^2-b^2\right)}$$

$$\frac{a^2\tan(c+dx)\sec^3(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^3}$$

$\downarrow$ 221

$$-\frac{a^2\tan(c+dx)\sec^3(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^3}-\frac{\frac{a^2\left(4a^2-9b^2\right)\tan(c+dx)\sec^2(c+dx)}{2bd(a^2-b^2)(a+b\sec(c+dx))^2}+\frac{\frac{\frac{3\left(\frac{8a\left(a^2-b^2\right)^3\text{arctanh}(\sin(c+dx))}{d}-\frac{2a^2\left(8a^6-28a^4b^2+35a^2b^4-20b^6\right)\text{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d\sqrt{a-b}\sqrt{a+b}}\right)}{b}-}{b^2\left(a^2-b^2\right)}}{2b(a^2-b^2)}}{3b\left(a^2-b^2\right)}$$

input
```
Int[Sec[c + d*x]^6/(a + b*Sec[c + d*x])^4,x]
```

3.514. $\int \frac{\sec^6(c+dx)}{(a+b\sec(c+dx))^4}\,dx$

output
```
-1/3*(a^2*Sec[c + d*x]^3*Tan[c + d*x])/(b*(a^2 - b^2)*d*(a + b*Sec[c + d*x
])^3) - ((a^2*(4*a^2 - 9*b^2)*Sec[c + d*x]^2*Tan[c + d*x])/(2*b*(a^2 - b^2
)*d*(a + b*Sec[c + d*x])^2) + ((-3*a^3*(4*a^4 - 11*a^2*b^2 + 12*b^4)*Tan[c
 + d*x])/(b^2*(a^2 - b^2)*d*(a + b*Sec[c + d*x])) + ((3*((8*a*(a^2 - b^2)^
3*ArcTanh[Sin[c + d*x]])/d - (2*a^2*(8*a^6 - 28*a^4*b^2 + 35*a^2*b^4 - 20*
b^6)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(Sqrt[a - b]*Sqr
t[a + b]*d)))/b - ((a^2 - b^2)*(12*a^4 - 23*a^2*b^2 + 6*b^4)*Tan[c + d*x])
/d)/(b^2*(a^2 - b^2)))/(2*b*(a^2 - b^2)))/(3*b*(a^2 - b^2))
```

3.514.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4318
```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[1/b   Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4332
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-a^2)*d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(
m + 1)*((d*Csc[e + f*x])^(n - 3)/(b*f*(m + 1)*(a^2 - b^2))), x] + Simp[d^3/
(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])
^(n - 3)*Simp[a^2*(n - 3) + a*b*(m + 1)*Csc[e + f*x] - (a^2*(n - 2) + b^2*(
m + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2
- b^2, 0] && LtQ[m, -1] && (IGtQ[n, 3] || (IntegersQ[n + 1/2, 2*m] && GtQ[n
, 2]))
```

rule 4486
```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/(csc[(
e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[B/b   Int[Csc[e + f*x],
 x], x] + Simp[(A*b - a*B)/b   Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x
] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0]
```

rule 4570
```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> Simp[(-C)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2)
)), x] + Simp[1/(b*(m + 2))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[
b*A*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /;
 FreeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

rule 4578
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[
(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x
_Symbol] :> Simp[a*(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*((a + b*Csc[e + f*x
])^(m + 1)/(b^2*f*(m + 1)*(a^2 - b^2))), x] - Simp[1/(b^2*(m + 1)*(a^2 - b^
2))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[b*(m + 1)*((-a)*(b
*B - a*C) + A*b^2) + (b*B*(a^2 + b^2*(m + 1)) - a*(A*b^2*(m + 2) + C*(a^2 +
 b^2*(m + 1))))*Csc[e + f*x] - b*C*(m + 1)*(a^2 - b^2)*Csc[e + f*x]^2, x],
x], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1
]
```

3.514. $\int \frac{\sec^6(c+dx)}{(a+b\sec(c+dx))^4}\,dx$

rule 4586
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(-d)*(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a +
b*Csc[e + f*x])^(m + 1)*((d*Csc[e + f*x])^(n - 1)/(b*f*(a^2 - b^2)*(m + 1))
), x] + Simp[d/(b*(a^2 - b^2)*(m + 1))   Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*b^2*(n - 1) - a*(b*B - a*C)*(n - 1) + b*(a*A
 - b*B + a*C)*(m + 1)*Csc[e + f*x] - (b*(A*b - a*B)*(m + n + 1) + C*(a^2*n
+ b^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C
}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n, 0]
```

3.514.4 Maple [A] (verified)

Time = 1.30 (sec) , antiderivative size = 425, normalized size of antiderivative = 1.34

method	result
derivativedivides	$\frac{-\frac{1}{b^4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{4a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{b^5}-\frac{1}{b^4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{4a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{b^5}-\frac{2a^2\left(\frac{\left(6a^4-2a^3b-18a^2b^2+5a\,b^{\cdots}\right.}{2(a-b)\left(a^3+3a^{\cdots}\right.}\right.}{\cdots}}{\cdots}$
default	$\frac{-\frac{1}{b^4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{4a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{b^5}-\frac{1}{b^4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{4a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{b^5}-\frac{2a^2\left(\frac{\left(6a^4-2a^3b-18a^2b^2+5a\,b^{\cdots}\right.}{2(a-b)\left(a^3+3a^{\cdots}\right.}\right.}{\cdots}}{\cdots}$
risch	Expression too large to display

input
```
int(sec(d*x+c)^6/(a+b*sec(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/b^4/(tan(1/2*d*x+1/2*c)-1)+4*a/b^5*ln(tan(1/2*d*x+1/2*c)-1)-1/b^4/
(tan(1/2*d*x+1/2*c)+1)-4*a/b^5*ln(tan(1/2*d*x+1/2*c)+1)-2*a^2/b^5*((1/2*(6
*a^4-2*a^3*b-18*a^2*b^2+5*a*b^3+20*b^4)*a*b/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3
)*tan(1/2*d*x+1/2*c)^5-2/3*(9*a^4-29*a^2*b^2+30*b^4)*a*b/(a^2+2*a*b+b^2)/(
a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3+1/2*(6*a^4+2*a^3*b-18*a^2*b^2-5*a*b^3+
20*b^4)*a*b/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c))/(tan(1/2*d
*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3-1/2*(8*a^6-28*a^4*b^2+35*a^2*b
^4-20*b^6)/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a-b)*(a+b))^(1/2)*arctanh((a-b)
*tan(1/2*d*x+1/2*c)/((a-b)*(a+b))^(1/2))))
```

3.514.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1000 vs. $2(299) = 598$.

Time = 1.71 (sec) , antiderivative size = 2058, normalized size of antiderivative = 6.51

$$\int \frac{\sec^6(c+dx)}{(a+b\sec(c+dx))^4}\,dx = \text{Too large to display}$$

input
```
integrate(sec(d*x+c)^6/(a+b*sec(d*x+c))^4,x, algorithm="fricas")
```

output
```
[1/12*(3*((8*a^11 - 28*a^9*b^2 + 35*a^7*b^4 - 20*a^5*b^6)*cos(d*x + c)^4 +
 3*(8*a^10*b - 28*a^8*b^3 + 35*a^6*b^5 - 20*a^4*b^7)*cos(d*x + c)^3 + 3*(8
*a^9*b^2 - 28*a^7*b^4 + 35*a^5*b^6 - 20*a^3*b^8)*cos(d*x + c)^2 + (8*a^8*b
^3 - 28*a^6*b^5 + 35*a^4*b^7 - 20*a^2*b^9)*cos(d*x + c))*sqrt(a^2 - b^2)*l
og((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 + 2*sqrt(a^2 - b^2)*
(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a
*b*cos(d*x + c) + b^2)) - 24*((a^12 - 4*a^10*b^2 + 6*a^8*b^4 - 4*a^6*b^6 +
 a^4*b^8)*cos(d*x + c)^4 + 3*(a^11*b - 4*a^9*b^3 + 6*a^7*b^5 - 4*a^5*b^7 +
 a^3*b^9)*cos(d*x + c)^3 + 3*(a^10*b^2 - 4*a^8*b^4 + 6*a^6*b^6 - 4*a^4*b^8
 + a^2*b^10)*cos(d*x + c)^2 + (a^9*b^3 - 4*a^7*b^5 + 6*a^5*b^7 - 4*a^3*b^9
 + a*b^11)*cos(d*x + c))*log(sin(d*x + c) + 1) + 24*((a^12 - 4*a^10*b^2 +
6*a^8*b^4 - 4*a^6*b^6 + a^4*b^8)*cos(d*x + c)^4 + 3*(a^11*b - 4*a^9*b^3 +
6*a^7*b^5 - 4*a^5*b^7 + a^3*b^9)*cos(d*x + c)^3 + 3*(a^10*b^2 - 4*a^8*b^4
+ 6*a^6*b^6 - 4*a^4*b^8 + a^2*b^10)*cos(d*x + c)^2 + (a^9*b^3 - 4*a^7*b^5
+ 6*a^5*b^7 - 4*a^3*b^9 + a*b^11)*cos(d*x + c))*log(-sin(d*x + c) + 1) + 2
*(6*a^8*b^4 - 24*a^6*b^6 + 36*a^4*b^8 - 24*a^2*b^10 + 6*b^12 + (24*a^11*b
- 92*a^9*b^3 + 133*a^7*b^5 - 71*a^5*b^7 + 6*a^3*b^9)*cos(d*x + c)^3 + 3*(2
0*a^10*b^2 - 77*a^8*b^4 + 110*a^6*b^6 - 59*a^4*b^8 + 6*a^2*b^10)*cos(d*x +
 c)^2 + (44*a^9*b^3 - 169*a^7*b^5 + 239*a^5*b^7 - 132*a^3*b^9 + 18*a*b^11)
*cos(d*x + c))*sin(d*x + c))/((a^11*b^5 - 4*a^9*b^7 + 6*a^7*b^9 - 4*a^5...
```

3.514.6 Sympy [F]

$$\int \frac{\sec^6(c+dx)}{(a+b\sec(c+dx))^4}\,dx = \int \frac{\sec^6(c+dx)}{(a+b\sec(c+dx))^4}\,dx$$

input
```
integrate(sec(d*x+c)**6/(a+b*sec(d*x+c))**4,x)
```

output
```
Integral(sec(c + d*x)**6/(a + b*sec(c + d*x))**4, x)
```

3.514.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\sec^6(c+dx)}{(a+b\sec(c+dx))^4}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(sec(d*x+c)^6/(a+b*sec(d*x+c))^4,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

3.514.8 Giac [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 592, normalized size of antiderivative = 1.87

$$\int \frac{\sec^6(c+dx)}{(a+b\sec(c+dx))^4}\,dx =$$

$$-\frac{\frac{3\,(8\,a^8-28\,a^6b^2+35\,a^4b^4-20\,a^2b^6)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2\,a-2\,b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{-a^2+b^2}}\right)\right)}{(a^6b^5-3\,a^4b^7+3\,a^2b^9-b^{11})\sqrt{-a^2+b^2}}+\frac{18\,a^9\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-42\,a}{}}{}$$

input
```
integrate(sec(d*x+c)^6/(a+b*sec(d*x+c))^4,x, algorithm="giac")
```

output
```
-1/3*(3*(8*a^8 - 28*a^6*b^2 + 35*a^4*b^4 - 20*a^2*b^6)*(pi*floor(1/2*(d*x
+ c)/pi + 1/2)*sgn(2*a - 2*b) + arctan((a*tan(1/2*d*x + 1/2*c) - b*tan(1/2
*d*x + 1/2*c))/sqrt(-a^2 + b^2)))/((a^6*b^5 - 3*a^4*b^7 + 3*a^2*b^9 - b^11
)*sqrt(-a^2 + b^2)) + (18*a^9*tan(1/2*d*x + 1/2*c)^5 - 42*a^8*b*tan(1/2*d*
x + 1/2*c)^5 - 24*a^7*b^2*tan(1/2*d*x + 1/2*c)^5 + 117*a^6*b^3*tan(1/2*d*x
 + 1/2*c)^5 - 24*a^5*b^4*tan(1/2*d*x + 1/2*c)^5 - 105*a^4*b^5*tan(1/2*d*x
+ 1/2*c)^5 + 60*a^3*b^6*tan(1/2*d*x + 1/2*c)^5 - 36*a^9*tan(1/2*d*x + 1/2*
c)^3 + 152*a^7*b^2*tan(1/2*d*x + 1/2*c)^3 - 236*a^5*b^4*tan(1/2*d*x + 1/2*
c)^3 + 120*a^3*b^6*tan(1/2*d*x + 1/2*c)^3 + 18*a^9*tan(1/2*d*x + 1/2*c) +
42*a^8*b*tan(1/2*d*x + 1/2*c) - 24*a^7*b^2*tan(1/2*d*x + 1/2*c) - 117*a^6*
b^3*tan(1/2*d*x + 1/2*c) - 24*a^5*b^4*tan(1/2*d*x + 1/2*c) + 105*a^4*b^5*t
an(1/2*d*x + 1/2*c) + 60*a^3*b^6*tan(1/2*d*x + 1/2*c))/((a^6*b^4 - 3*a^4*b
^6 + 3*a^2*b^8 - b^10)*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^
2 - a - b)^3) + 12*a*log(abs(tan(1/2*d*x + 1/2*c) + 1))/b^5 - 12*a*log(abs
(tan(1/2*d*x + 1/2*c) - 1))/b^5 + 6*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1
/2*c)^2 - 1)*b^4))/d
```

3.514.9 Mupad [B] (verification not implemented)

Time = 23.55 (sec) , antiderivative size = 7476, normalized size of antiderivative = 23.66

$$\int \frac{\sec^6(c+dx)}{(a+b\sec(c+dx))^4}\,dx = \text{Too large to display}$$

input
```
int(1/(cos(c + d*x)^6*(a + b/cos(c + d*x))^4),x)
```

output
```
((tan(c/2 + (d*x)/2)^3*(12*a^7*b - 72*a^8 - 18*b^8 + 72*a^2*b^6 + 60*a^3*b
^5 - 273*a^4*b^4 - 47*a^5*b^3 + 236*a^6*b^2))/(3*b^4*(a + b)^2*(a - b)^3)
+ (tan(c/2 + (d*x)/2)^5*(12*a^7*b + 72*a^8 + 18*b^8 - 72*a^2*b^6 + 60*a^3*
b^5 + 273*a^4*b^4 - 47*a^5*b^3 - 236*a^6*b^2))/(3*b^4*(a + b)^3*(a - b)^2)
 - (tan(c/2 + (d*x)/2)*(2*a*b^6 - 4*a^6*b - 8*a^7 + 2*b^7 - 6*a^2*b^5 - 26
*a^3*b^4 + 11*a^4*b^3 + 24*a^5*b^2))/(b^4*(a + b)*(a - b)^3) + (tan(c/2 +
(d*x)/2)^7*(2*a*b^6 + 4*a^6*b - 8*a^7 - 2*b^7 + 6*a^2*b^5 - 26*a^3*b^4 - 1
1*a^4*b^3 + 24*a^5*b^2))/(b^4*(a + b)^3*(a - b)))/(d*(3*a*b^2 + 3*a^2*b -
tan(c/2 + (d*x)/2)^4*(6*a*b^2 - 6*a^3) - tan(c/2 + (d*x)/2)^2*(6*a^2*b + 4
*a^3 - 2*b^3) - tan(c/2 + (d*x)/2)^6*(4*a^3 - 6*a^2*b + 2*b^3) + a^3 + b^3
 + tan(c/2 + (d*x)/2)^8*(3*a*b^2 - 3*a^2*b + a^3 - b^3))) + (a*atan(((a*((
8*tan(c/2 + (d*x)/2)*(128*a^16 - 128*a^15*b + 64*a^2*b^14 - 128*a^3*b^13 +
 80*a^4*b^12 + 768*a^5*b^11 - 824*a^6*b^10 - 1920*a^7*b^9 + 2025*a^8*b^8 +
 2560*a^9*b^7 - 2600*a^10*b^6 - 1920*a^11*b^5 + 1920*a^12*b^4 + 768*a^13*b
^3 - 768*a^14*b^2))/(a*b^18 + b^19 - 5*a^2*b^17 - 5*a^3*b^16 + 10*a^4*b^15
 + 10*a^5*b^14 - 10*a^6*b^13 - 10*a^7*b^12 + 5*a^8*b^11 + 5*a^9*b^10 - a^1
0*b^9 - a^11*b^8) - (4*a*((16*(8*a*b^23 - 20*a^2*b^22 - 36*a^3*b^21 + 95*a
^4*b^20 + 73*a^5*b^19 - 193*a^6*b^18 - 87*a^7*b^17 + 217*a^8*b^16 + 63*a^9
*b^15 - 143*a^10*b^14 - 25*a^11*b^13 + 52*a^12*b^12 + 4*a^13*b^11 - 8*a^14
*b^10))/(a*b^22 + b^23 - 5*a^2*b^21 - 5*a^3*b^20 + 10*a^4*b^19 + 10*a^5...
```

3.514. $\int \frac{\sec^6(c+dx)}{(a+b\sec(c+dx))^4}\,dx$

3.515 $\int \frac{\sec^5(c+dx)}{(a+b\sec(c+dx))^4}\,dx$

3.515.1 Optimal result . . . 3840
3.515.2 Mathematica [A] (verified) . . . 3841
3.515.3 Rubi [A] (verified) . . . 3841
3.515.4 Maple [A] (verified) . . . 3846
3.515.5 Fricas [B] (verification not implemented) . . . 3847
3.515.6 Sympy [F] . . . 3848
3.515.7 Maxima [F(-2)] . . . 3848
3.515.8 Giac [B] (verification not implemented) . . . 3848
3.515.9 Mupad [B] (verification not implemented) . . . 3849

3.515.1 Optimal result

Integrand size = 21, antiderivative size = 259

$$\begin{aligned}\int \frac{\sec^5(c+dx)}{(a+b\sec(c+dx))^4}\,dx = {} & \frac{\operatorname{arctanh}(\sin(c+dx))}{b^4 d} \\ & - \frac{a(2a^6-7a^4b^2+8a^2b^4-8b^6)\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{7/2}b^4(a+b)^{7/2}d} \\ & - \frac{a^2\sec^2(c+dx)\tan(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} \\ & + \frac{a^3(3a^2-8b^2)\tan(c+dx)}{6b^3\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))^2} \\ & - \frac{a^2(9a^4-28a^2b^2+34b^4)\tan(c+dx)}{6b^3\left(a^2-b^2\right)^3 d(a+b\sec(c+dx))}\end{aligned}$$

output
```
arctanh(sin(d*x+c))/b^4/d-a*(2*a^6-7*a^4*b^2+8*a^2*b^4-8*b^6)*arctanh((a-b
)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/(a-b)^(7/2)/b^4/(a+b)^(7/2)/d-1/3*
a^2*sec(d*x+c)^2*tan(d*x+c)/b/(a^2-b^2)/d/(a+b*sec(d*x+c))^3+1/6*a^3*(3*a^
2-8*b^2)*tan(d*x+c)/b^3/(a^2-b^2)^2/d/(a+b*sec(d*x+c))^2-1/6*a^2*(9*a^4-28
*a^2*b^2+34*b^4)*tan(d*x+c)/b^3/(a^2-b^2)^3/d/(a+b*sec(d*x+c))
```

3.515.2 Mathematica [A] (verified)

Time = 3.99 (sec) , antiderivative size = 250, normalized size of antiderivative = 0.97

$$\int \frac{\sec^5(c+dx)}{(a+b\sec(c+dx))^4}\,dx = \frac{\frac{6a(2a^6-7a^4b^2+8a^2b^4-8b^6)\operatorname{arctanh}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{7/2}} - 6\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+6\log\left(\cos\left(\right.\right.}{}$$

input
```
Integrate[Sec[c + d*x]^5/(a + b*Sec[c + d*x])^4,x]
```

output
```
((6*a*(2*a^6 - 7*a^4*b^2 + 8*a^2*b^4 - 8*b^6)*ArcTanh[((-a + b)*Tan[(c + d
*x)/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(7/2) - 6*Log[Cos[(c + d*x)/2] - Sin
[(c + d*x)/2]] + 6*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] - (a^2*b*(11*a
^4*b^2 - 32*a^2*b^4 + 36*b^6 + 15*a*b*(a^4 - 3*a^2*b^2 + 4*b^4)*Cos[c + d*
x] + a^2*(6*a^4 - 17*a^2*b^2 + 26*b^4)*Cos[c + d*x]^2)*Sin[c + d*x])/((a -
 b)^3*(a + b)^3*(b + a*Cos[c + d*x])^3))/(6*b^4*d)
```

3.515.3 Rubi [A] (verified)

Time = 1.83 (sec) , antiderivative size = 314, normalized size of antiderivative = 1.21, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {3042, 4332, 3042, 4578, 3042, 4568, 27, 3042, 4486, 3042, 4257, 4318, 3042, 3138, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^5(c+dx)}{(a+b\sec(c+dx))^4}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^5}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^4}dx$$

$$\Big\downarrow \; 4332$$

$$-\frac{\int \frac{\sec^2(c+dx)\left(2a^2-3b\sec(c+dx)a-3\left(a^2-b^2\right)\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^3}dx}{3b\left(a^2-b^2\right)}-\frac{a^2\tan(c+dx)\sec^2(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^3}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2\left(2a^2-3b\csc\left(c+dx+\frac{\pi}{2}\right)a-3\left(a^2-b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^3}dx}{3b\left(a^2-b^2\right)}-\frac{a^2\tan(c+dx)\sec^2(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^3}$$

$$\Big\downarrow \; 4578$$

$$-\frac{\frac{\int \frac{\sec(c+dx)\left(2b\left(3a^2-8b^2\right)a^2+\left(3a^4-10b^2a^2+12b^4\right)\sec(c+dx)a-6b\left(a^2-b^2\right)^2\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^2}dx}{2b^2\left(a^2-b^2\right)}-\frac{a^3\left(3a^2-8b^2\right)\tan(c+dx)}{2b^2d\left(a^2-b^2\right)(a+b\sec(c+dx))^2}}{3b\left(a^2-b^2\right)}-\frac{a^2\tan(c+dx)\sec^2(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^3}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(2b\left(3a^2-8b^2\right)a^2+\left(3a^4-10b^2a^2+12b^4\right)\csc\left(c+dx+\frac{\pi}{2}\right)a-6b\left(a^2-b^2\right)^2\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx}{2b^2\left(a^2-b^2\right)}-\frac{a^3\left(3a^2-8b^2\right)\tan(c+dx)}{2b^2d\left(a^2-b^2\right)(a+b\sec(c+dx))^2}}{3b\left(a^2-b^2\right)}-\frac{a^2\tan(c+dx)\sec^2(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^3}$$

$$\Big\downarrow \; 4568$$

$$-\frac{\frac{\frac{a^2\left(9a^4-28a^2b^2+34b^4\right)\tan(c+dx)}{d\left(a^2-b^2\right)(a+b\sec(c+dx))}-\frac{\int \frac{3\sec(c+dx)\left(2b\sec(c+dx)\left(a^2-b^2\right)^3+ab^2\left(a^4-2b^2a^2+6b^4\right)\right)}{a+b\sec(c+dx)}dx}{b\left(a^2-b^2\right)}}{2b^2\left(a^2-b^2\right)}-\frac{a^3\left(3a^2-8b^2\right)\tan(c+dx)}{2b^2d\left(a^2-b^2\right)(a+b\sec(c+dx))^2}}{3b\left(a^2-b^2\right)}-\frac{a^2\tan(c+dx)\sec^2(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^3}$$

$$\Big\downarrow \; 27$$

$$-\frac{\frac{\frac{a^2\left(9a^4-28a^2b^2+34b^4\right)\tan(c+dx)}{d\left(a^2-b^2\right)(a+b\sec(c+dx))}-\frac{3\int \frac{\sec(c+dx)\left(2b\sec(c+dx)\left(a^2-b^2\right)^3+ab^2\left(a^4-2b^2a^2+6b^4\right)\right)}{a+b\sec(c+dx)}dx}{b\left(a^2-b^2\right)}}{2b^2\left(a^2-b^2\right)}-\frac{a^3\left(3a^2-8b^2\right)\tan(c+dx)}{2b^2d\left(a^2-b^2\right)(a+b\sec(c+dx))^2}}{3b\left(a^2-b^2\right)}-\frac{a^2\tan(c+dx)\sec^2(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^3}$$

$$\Big\downarrow \; 3042$$

3.515. $\int \frac{\sec^5(c+dx)}{(a+b\sec(c+dx))^4}\,dx$

$$-\frac{\frac{\frac{a^2\left(9a^4-28a^2b^2+34b^4\right)\tan(c+dx)}{d\left(a^2-b^2\right)(a+b\sec(c+dx))}-\frac{3\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(2b\csc\left(c+dx+\frac{\pi}{2}\right)\left(a^2-b^2\right)^3+ab^2\left(a^4-2b^2a^2+6b^4\right)\right)}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b\left(a^2-b^2\right)}}{2b^2(a^2-b^2)}-\frac{a^3\left(3a^2-8b^2\right)\tan(c+dx)}{2b^2d(a^2-b^2)(a+b\sec(c+dx))^2}}{3b\left(a^2-b^2\right)}-\frac{a^2\tan(c+dx)\sec^2(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^3}$$

$\downarrow$ 4486

$$-\frac{\frac{\frac{a^2\left(9a^4-28a^2b^2+34b^4\right)\tan(c+dx)}{d\left(a^2-b^2\right)(a+b\sec(c+dx))}-\frac{3\left(2\left(a^2-b^2\right)^3\int\sec(c+dx)dx-a\left(2a^6-7a^4b^2+8a^2b^4-8b^6\right)\int\frac{\sec(c+dx)}{a+b\sec(c+dx)}dx\right)}{b\left(a^2-b^2\right)}}{2b^2(a^2-b^2)}-\frac{a^3\left(3a^2-8b^2\right)\tan(c+dx)}{2b^2d(a^2-b^2)(a+b\sec(c+dx))^2}}{3b\left(a^2-b^2\right)}-\frac{a^2\tan(c+dx)\sec^2(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^3}$$

$\downarrow$ 3042

$$-\frac{\frac{\frac{a^2\left(9a^4-28a^2b^2+34b^4\right)\tan(c+dx)}{d\left(a^2-b^2\right)(a+b\sec(c+dx))}-\frac{3\left(2\left(a^2-b^2\right)^3\int\csc\left(c+dx+\frac{\pi}{2}\right)dx-a\left(2a^6-7a^4b^2+8a^2b^4-8b^6\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx\right)}{b\left(a^2-b^2\right)}}{2b^2(a^2-b^2)}-\frac{a^3\left(3a^2-8b^2\right)\tan(c+}{2b^2d(a^2-b^2)(a+b\sec(c-}}{3b\left(a^2-b^2\right)}-\frac{a^2\tan(c+dx)\sec^2(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^3}$$

$\downarrow$ 4257

$$-\frac{\frac{\frac{a^2\left(9a^4-28a^2b^2+34b^4\right)\tan(c+dx)}{d\left(a^2-b^2\right)(a+b\sec(c+dx))}-\frac{3\left(\frac{2\left(a^2-b^2\right)^3\text{arctanh}(\sin(c+dx))}{d}-a\left(2a^6-7a^4b^2+8a^2b^4-8b^6\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx\right)}{b\left(a^2-b^2\right)}}{2b^2(a^2-b^2)}-\frac{a^3\left(3a^2-8b^2\right)\tan}{2b^2d(a^2-b^2)(a+b\text{se}}}{3b\left(a^2-b^2\right)}-\frac{a^2\tan(c+dx)\sec^2(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^3}$$

$\downarrow$ 4318

$$-\frac{\frac{\frac{a^2\left(9a^4-28a^2b^2+34b^4\right)\tan(c+dx)}{d\left(a^2-b^2\right)(a+b\sec(c+dx))}-\frac{3\left(\frac{2\left(a^2-b^2\right)^3\text{arctanh}(\sin(c+dx))}{d}-\frac{a\left(2a^6-7a^4b^2+8a^2b^4-8b^6\right)\int\frac{1}{\frac{a\cos(c+dx)}{b}+1}dx}{b}\right)}{b\left(a^2-b^2\right)}}{2b^2(a^2-b^2)}-\frac{a^3\left(3a^2-8b^2\right)\tan(c-}{2b^2d(a^2-b^2)(a+b\sec(}}{3b\left(a^2-b^2\right)}-\frac{a^2\tan(c+dx)\sec^2(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^3}$$

3.515. $\int\frac{\sec^5(c+dx)}{(a+b\sec(c+dx))^4}\,dx$

$$\Big\downarrow \; 3042$$

$$-\frac{\frac{\frac{a^2\left(9a^4-28a^2b^2+34b^4\right)\tan(c+dx)}{d\left(a^2-b^2\right)(a+b\sec(c+dx))}-\frac{3\left(\frac{2\left(a^2-b^2\right)^3\text{arctanh}(\sin(c+dx))}{d}-\frac{a\left(2a^6-7a^4b^2+8a^2b^4-8b^6\right)\int\frac{1}{\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{b}+1}dx}{b}\right)}{b\left(a^2-b^2\right)}}{2b^2(a^2-b^2)}-\frac{a^3(3a^2-8b^2)\,\text{ta}}{2b^2d(a^2-b^2)(a+b\,\text{s}}}{3b\left(a^2-b^2\right)}$$

$$\frac{a^2\tan(c+dx)\sec^2(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^3}$$

$$\Big\downarrow \; 3138$$

$$-\frac{\frac{\frac{a^2\left(9a^4-28a^2b^2+34b^4\right)\tan(c+dx)}{d\left(a^2-b^2\right)(a+b\sec(c+dx))}-\frac{3\left(\frac{2\left(a^2-b^2\right)^3\text{arctanh}(\sin(c+dx))}{d}-\frac{2a\left(2a^6-7a^4b^2+8a^2b^4-8b^6\right)\int\frac{1}{\left(1-\frac{a}{b}\right)\tan^2\left(\frac{1}{2}(c+dx)\right)+\frac{a+b}{b}}d\tan\left(\frac{1}{2}(c+dx)\right)}{bd}\right)}{b\left(a^2-b^2\right)}}{2b^2(a^2-b^2)}}{3b\left(a^2-b^2\right)}$$

$$\frac{a^2\tan(c+dx)\sec^2(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^3}$$

$$\Big\downarrow \; 221$$

$$-\frac{a^2\tan(c+dx)\sec^2(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^3}-\frac{\frac{\frac{a^2\left(9a^4-28a^2b^2+34b^4\right)\tan(c+dx)}{d\left(a^2-b^2\right)(a+b\sec(c+dx))}-\frac{3\left(\frac{2\left(a^2-b^2\right)^3\text{arctanh}(\sin(c+dx))}{d}-\frac{2a\left(2a^6-7a^4b^2+8a^2b^4-8b^6\right)\text{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d\sqrt{a-b}\sqrt{a+b}}\right)}{b\left(a^2-b^2\right)}}{2b^2(a^2-b^2)}-\frac{a^3(3a^2}{2b^2d(a^2-}}{3b\left(a^2-b^2\right)}$$

input
```
Int[Sec[c + d*x]^5/(a + b*Sec[c + d*x])^4,x]
```

output
```
-1/3*(a^2*Sec[c + d*x]^2*Tan[c + d*x])/(b*(a^2 - b^2)*d*(a + b*Sec[c + d*x
])^3) - (-1/2*(a^3*(3*a^2 - 8*b^2)*Tan[c + d*x])/(b^2*(a^2 - b^2)*d*(a + b
*Sec[c + d*x])^2) + ((-3*((2*(a^2 - b^2)^3*ArcTanh[Sin[c + d*x]])/d - (2*a
*(2*a^6 - 7*a^4*b^2 + 8*a^2*b^4 - 8*b^6)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x
)/2])/Sqrt[a + b]])/(Sqrt[a - b]*Sqrt[a + b]*d)))/(b*(a^2 - b^2)) + (a^2*(
9*a^4 - 28*a^2*b^2 + 34*b^4)*Tan[c + d*x])/((a^2 - b^2)*d*(a + b*Sec[c + d
*x])))/(2*b^2*(a^2 - b^2)))/(3*b*(a^2 - b^2))
```

3.515. $\int \frac{\sec^5(c+dx)}{(a+b\sec(c+dx))^4}\,dx$

3.515.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4318
```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[1/b   Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4332
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-a^2)*d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(
m + 1)*((d*Csc[e + f*x])^(n - 3)/(b*f*(m + 1)*(a^2 - b^2))), x] + Simp[d^3/
(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])
^(n - 3)*Simp[a^2*(n - 3) + a*b*(m + 1)*Csc[e + f*x] - (a^2*(n - 2) + b^2*(
m + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2
- b^2, 0] && LtQ[m, -1] && (IGtQ[n, 3] || (IntegersQ[n + 1/2, 2*m] && GtQ[n
, 2]))
```

rule 4486
```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/(csc[(
e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[B/b   Int[Csc[e + f*x],
 x], x] + Simp[(A*b - a*B)/b   Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x
] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0]
```

3.515. $\int \frac{\sec^5(c+dx)}{(a+b\sec(c+dx))^4}\,dx$

rule 4568
```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cot[e + f*x]*((a + b*Csc[e + f*x]
)^(m + 1)/(b*f*(m + 1)*(a^2 - b^2))), x] + Simp[1/(b*(m + 1)*(a^2 - b^2))
 Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m
+ 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b - a*B + b*C)*(m + 1))*Csc[e + f*x],
x], x], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^
2, 0]
```

rule 4578
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[
(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x
_Symbol] :> Simp[a*(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*((a + b*Csc[e + f*x
])^(m + 1)/(b^2*f*(m + 1)*(a^2 - b^2))), x] - Simp[1/(b^2*(m + 1)*(a^2 - b^
2))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[b*(m + 1)*((-a)*(b
*B - a*C) + A*b^2) + (b*B*(a^2 + b^2*(m + 1)) - a*(A*b^2*(m + 2) + C*(a^2 +
 b^2*(m + 1))))*Csc[e + f*x] - b*C*(m + 1)*(a^2 - b^2)*Csc[e + f*x]^2, x],
x], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1
]
```

3.515.4 Maple [A] (verified)

Time = 1.50 (sec) , antiderivative size = 383, normalized size of antiderivative = 1.48

method	result
derivativedivides	$\frac{2a\left(\frac{\left(2a^4-a^3b-6a^2b^2+4a\,b^3+12b^4\right)ab\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{2(a-b)\left(a^3+3a^2b+3a\,b^2+b^3\right)}-\frac{2\left(3a^4-11a^2b^2+18b^4\right)ab\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3\left(a^2+2ab+b^2\right)\left(a^2-2ab+b^2\right)}+\frac{\left(2a^4+a^3b-6a^2b^2-4a\,b^3+12b^4\right)a\cdots}{2(a+b)\left(a^3-3a^2b+3a\,b^2-\cdots\right.}\right.}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2b-a-b\right)^3}}{b^4}}{d}$
default	$\frac{2a\left(\frac{\left(2a^4-a^3b-6a^2b^2+4a\,b^3+12b^4\right)ab\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{2(a-b)\left(a^3+3a^2b+3a\,b^2+b^3\right)}-\frac{2\left(3a^4-11a^2b^2+18b^4\right)ab\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3\left(a^2+2ab+b^2\right)\left(a^2-2ab+b^2\right)}+\frac{\left(2a^4+a^3b-6a^2b^2-4a\,b^3+12b^4\right)a\cdots}{2(a+b)\left(a^3-3a^2b+3a\,b^2-\cdots\right.}\right.}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2b-a-b\right)^3}}{b^4}}{d}$
risch	Expression too large to display

input
```
int(sec(d*x+c)^5/(a+b*sec(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
1/d*(2*a/b^4*((1/2*(2*a^4-a^3*b-6*a^2*b^2+4*a*b^3+12*b^4)*a*b/(a-b)/(a^3+3
*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5-2/3*(3*a^4-11*a^2*b^2+18*b^4)*a*b
/(a^2+2*a*b+b^2)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3+1/2*(2*a^4+a^3*b-6*a
^2*b^2-4*a*b^3+12*b^4)*a*b/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2
*c))/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3-1/2*(2*a^6-7*a^
4*b^2+8*a^2*b^4-8*b^6)/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a-b)*(a+b))^(1/2)*a
rctanh((a-b)*tan(1/2*d*x+1/2*c)/((a-b)*(a+b))^(1/2)))+1/b^4*ln(tan(1/2*d*x
+1/2*c)+1)-1/b^4*ln(tan(1/2*d*x+1/2*c)-1))
```

3.515.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 882 vs. $2(244) = 488$.

Time = 1.84 (sec) , antiderivative size = 1822, normalized size of antiderivative = 7.03

$$\int \frac{\sec^5(c+dx)}{(a+b\sec(c+dx))^4}\,dx = \text{Too large to display}$$

input
```
integrate(sec(d*x+c)^5/(a+b*sec(d*x+c))^4,x, algorithm="fricas")
```

output
```
[1/12*(3*(2*a^7*b^3 - 7*a^5*b^5 + 8*a^3*b^7 - 8*a*b^9 + (2*a^10 - 7*a^8*b^
2 + 8*a^6*b^4 - 8*a^4*b^6)*cos(d*x + c)^3 + 3*(2*a^9*b - 7*a^7*b^3 + 8*a^5
*b^5 - 8*a^3*b^7)*cos(d*x + c)^2 + 3*(2*a^8*b^2 - 7*a^6*b^4 + 8*a^4*b^6 -
8*a^2*b^8)*cos(d*x + c))*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 -
2*b^2)*cos(d*x + c)^2 - 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c
) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) + 6*(a^8
*b^3 - 4*a^6*b^5 + 6*a^4*b^7 - 4*a^2*b^9 + b^11 + (a^11 - 4*a^9*b^2 + 6*a^
7*b^4 - 4*a^5*b^6 + a^3*b^8)*cos(d*x + c)^3 + 3*(a^10*b - 4*a^8*b^3 + 6*a^
6*b^5 - 4*a^4*b^7 + a^2*b^9)*cos(d*x + c)^2 + 3*(a^9*b^2 - 4*a^7*b^4 + 6*a
^5*b^6 - 4*a^3*b^8 + a*b^10)*cos(d*x + c))*log(sin(d*x + c) + 1) - 6*(a^8*
b^3 - 4*a^6*b^5 + 6*a^4*b^7 - 4*a^2*b^9 + b^11 + (a^11 - 4*a^9*b^2 + 6*a^7
*b^4 - 4*a^5*b^6 + a^3*b^8)*cos(d*x + c)^3 + 3*(a^10*b - 4*a^8*b^3 + 6*a^6
*b^5 - 4*a^4*b^7 + a^2*b^9)*cos(d*x + c)^2 + 3*(a^9*b^2 - 4*a^7*b^4 + 6*a^
5*b^6 - 4*a^3*b^8 + a*b^10)*cos(d*x + c))*log(-sin(d*x + c) + 1) - 2*(11*a
^8*b^3 - 43*a^6*b^5 + 68*a^4*b^7 - 36*a^2*b^9 + (6*a^10*b - 23*a^8*b^3 + 4
3*a^6*b^5 - 26*a^4*b^7)*cos(d*x + c)^2 + 15*(a^9*b^2 - 4*a^7*b^4 + 7*a^5*b
^6 - 4*a^3*b^8)*cos(d*x + c))*sin(d*x + c))/((a^11*b^4 - 4*a^9*b^6 + 6*a^7
*b^8 - 4*a^5*b^10 + a^3*b^12)*d*cos(d*x + c)^3 + 3*(a^10*b^5 - 4*a^8*b^7 +
 6*a^6*b^9 - 4*a^4*b^11 + a^2*b^13)*d*cos(d*x + c)^2 + 3*(a^9*b^6 - 4*a^7*
b^8 + 6*a^5*b^10 - 4*a^3*b^12 + a*b^14)*d*cos(d*x + c) + (a^8*b^7 - 4*a...
```

3.515.6 Sympy [F]

$$\int \frac{\sec^5(c+dx)}{(a+b\sec(c+dx))^4}\,dx = \int \frac{\sec^5(c+dx)}{(a+b\sec(c+dx))^4}\,dx$$

input
```
integrate(sec(d*x+c)**5/(a+b*sec(d*x+c))**4,x)
```

output
```
Integral(sec(c + d*x)**5/(a + b*sec(c + d*x))**4, x)
```

3.515.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\sec^5(c+dx)}{(a+b\sec(c+dx))^4}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(sec(d*x+c)^5/(a+b*sec(d*x+c))^4,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

3.515.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 559 vs. $2(244) = 488$.

Time = 0.39 (sec) , antiderivative size = 559, normalized size of antiderivative = 2.16

$$\int \frac{\sec^5(c+dx)}{(a+b\sec(c+dx))^4}\,dx$$
$$= \frac{\frac{3\,(2\,a^7-7\,a^5b^2+8\,a^3b^4-8\,ab^6)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(2\,a-2\,b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{-a^2+b^2}}\right)\right)}{(a^6b^4-3\,a^4b^6+3\,a^2b^8-b^{10})\sqrt{-a^2+b^2}} + \frac{6\,a^8\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-15\,a^7b\tan\left(\frac{1}{2}\right.}{}}{}$$

input
```
integrate(sec(d*x+c)^5/(a+b*sec(d*x+c))^4,x, algorithm="giac")
```

output
```
1/3*(3*(2*a^7 - 7*a^5*b^2 + 8*a^3*b^4 - 8*a*b^6)*(pi*floor(1/2*(d*x + c)/p
i + 1/2)*sgn(2*a - 2*b) + arctan((a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x +
 1/2*c))/sqrt(-a^2 + b^2)))/((a^6*b^4 - 3*a^4*b^6 + 3*a^2*b^8 - b^10)*sqrt
(-a^2 + b^2)) + (6*a^8*tan(1/2*d*x + 1/2*c)^5 - 15*a^7*b*tan(1/2*d*x + 1/2
*c)^5 - 6*a^6*b^2*tan(1/2*d*x + 1/2*c)^5 + 45*a^5*b^3*tan(1/2*d*x + 1/2*c)
^5 - 6*a^4*b^4*tan(1/2*d*x + 1/2*c)^5 - 60*a^3*b^5*tan(1/2*d*x + 1/2*c)^5
+ 36*a^2*b^6*tan(1/2*d*x + 1/2*c)^5 - 12*a^8*tan(1/2*d*x + 1/2*c)^3 + 56*a
^6*b^2*tan(1/2*d*x + 1/2*c)^3 - 116*a^4*b^4*tan(1/2*d*x + 1/2*c)^3 + 72*a^
2*b^6*tan(1/2*d*x + 1/2*c)^3 + 6*a^8*tan(1/2*d*x + 1/2*c) + 15*a^7*b*tan(1
/2*d*x + 1/2*c) - 6*a^6*b^2*tan(1/2*d*x + 1/2*c) - 45*a^5*b^3*tan(1/2*d*x
+ 1/2*c) - 6*a^4*b^4*tan(1/2*d*x + 1/2*c) + 60*a^3*b^5*tan(1/2*d*x + 1/2*c
) + 36*a^2*b^6*tan(1/2*d*x + 1/2*c))/((a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 - b
^9)*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 - a - b)^3) + 3*l
og(abs(tan(1/2*d*x + 1/2*c) + 1))/b^4 - 3*log(abs(tan(1/2*d*x + 1/2*c) - 1
))/b^4)/d
```

3.515.9 Mupad [B] (verification not implemented)

Time = 27.13 (sec) , antiderivative size = 7222, normalized size of antiderivative = 27.88

$$\int \frac{\sec^5(c+dx)}{(a+b\sec(c+dx))^4}\,dx = \text{Too large to display}$$

input
```
int(1/(cos(c + d*x)^5*(a + b/cos(c + d*x))^4),x)
```

output
```
- ((tan(c/2 + (d*x)/2)^5*(2*a^6 - a^5*b + 12*a^2*b^4 + 4*a^3*b^3 - 6*a^4*b
^2))/((a*b^3 - b^4)*(a + b)^3) - (4*tan(c/2 + (d*x)/2)^3*(3*a^6 + 18*a^2*b
^4 - 11*a^4*b^2))/(3*(a + b)^2*(b^5 - 2*a*b^4 + a^2*b^3)) + (tan(c/2 + (d*
x)/2)*(a^5*b + 2*a^6 + 12*a^2*b^4 - 4*a^3*b^3 - 6*a^4*b^2))/((a + b)*(3*a*
b^5 - b^6 - 3*a^2*b^4 + a^3*b^3)))/(d*(tan(c/2 + (d*x)/2)^2*(3*a*b^2 - 3*a
^2*b - 3*a^3 + 3*b^3) - tan(c/2 + (d*x)/2)^4*(3*a*b^2 + 3*a^2*b - 3*a^3 -
3*b^3) + 3*a*b^2 + 3*a^2*b + a^3 + b^3 - tan(c/2 + (d*x)/2)^6*(3*a*b^2 - 3
*a^2*b + a^3 - b^3))) - (atan((((((8*(16*a*b^20 - 4*b^21 + 12*a^2*b^19 - 6
4*a^3*b^18 - 20*a^4*b^17 + 110*a^5*b^16 + 30*a^6*b^15 - 110*a^7*b^14 - 30*
a^8*b^13 + 70*a^9*b^12 + 14*a^10*b^11 - 26*a^11*b^10 - 2*a^12*b^9 + 4*a^13
*b^8))/(a*b^19 + b^20 - 5*a^2*b^18 - 5*a^3*b^17 + 10*a^4*b^16 + 10*a^5*b^1
5 - 10*a^6*b^14 - 10*a^7*b^13 + 5*a^8*b^12 + 5*a^9*b^11 - a^10*b^10 - a^11
*b^9) - (8*tan(c/2 + (d*x)/2)*(8*a*b^21 - 8*a^2*b^20 - 48*a^3*b^19 + 48*a^
4*b^18 + 120*a^5*b^17 - 120*a^6*b^16 - 160*a^7*b^15 + 160*a^8*b^14 + 120*a
^9*b^13 - 120*a^10*b^12 - 48*a^11*b^11 + 48*a^12*b^10 + 8*a^13*b^9 - 8*a^1
4*b^8))/(b^4*(a*b^16 + b^17 - 5*a^2*b^15 - 5*a^3*b^14 + 10*a^4*b^13 + 10*a
^5*b^12 - 10*a^6*b^11 - 10*a^7*b^10 + 5*a^8*b^9 + 5*a^9*b^8 - a^10*b^7 - a
^11*b^6)))/b^4 - (8*tan(c/2 + (d*x)/2)*(8*a^14 - 8*a^13*b - 8*a*b^13 + 4*b
^14 + 44*a^2*b^12 + 48*a^3*b^11 - 92*a^4*b^10 - 120*a^5*b^9 + 156*a^6*b^8
+ 160*a^7*b^7 - 164*a^8*b^6 - 120*a^9*b^5 + 117*a^10*b^4 + 48*a^11*b^3 ...
```

3.515. $\int \frac{\sec^5(c+dx)}{(a+b\sec(c+dx))^4}\,dx$

3.516 $\int \frac{\sec^4(c+dx)}{(a+b\sec(c+dx))^4}\,dx$

3.516.1 Optimal result . . . 3851
3.516.2 Mathematica [A] (verified) . . . 3852
3.516.3 Rubi [A] (verified) . . . 3852
3.516.4 Maple [A] (verified) . . . 3856
3.516.5 Fricas [B] (verification not implemented) . . . 3857
3.516.6 Sympy [F] . . . 3858
3.516.7 Maxima [F(-2)] . . . 3858
3.516.8 Giac [A] (verification not implemented) . . . 3858
3.516.9 Mupad [B] (verification not implemented) . . . 3859

3.516.1 Optimal result

Integrand size = 21, antiderivative size = 222

$$\begin{aligned}\int \frac{\sec^4(c+dx)}{(a+b\sec(c+dx))^4}\,dx = &-\frac{b(3a^2+2b^2)\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{7/2}(a+b)^{7/2}d}\\ &-\frac{a^2\sec(c+dx)\tan(c+dx)}{3b\,(a^2-b^2)\,d(a+b\sec(c+dx))^3}\\ &-\frac{a^2(2a^2-7b^2)\tan(c+dx)}{6b^2\,(a^2-b^2)^2\,d(a+b\sec(c+dx))^2}\\ &+\frac{a(2a^4-5a^2b^2+18b^4)\tan(c+dx)}{6b^2\,(a^2-b^2)^3\,d(a+b\sec(c+dx))}\end{aligned}$$

output
```
-b*(3*a^2+2*b^2)*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/(a-b)
^(7/2)/(a+b)^(7/2)/d-1/3*a^2*sec(d*x+c)*tan(d*x+c)/b/(a^2-b^2)/d/(a+b*sec(
d*x+c))^3-1/6*a^2*(2*a^2-7*b^2)*tan(d*x+c)/b^2/(a^2-b^2)^2/d/(a+b*sec(d*x+
c))^2+1/6*a*(2*a^4-5*a^2*b^2+18*b^4)*tan(d*x+c)/b^2/(a^2-b^2)^3/d/(a+b*sec
(d*x+c))
```

3.516.2 Mathematica [A] (verified)

Time = 0.74 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.71

$$\int \frac{\sec^4(c+dx)}{(a+b\sec(c+dx))^4}\,dx = \frac{\frac{6b(3a^2+2b^2)\text{arctanh}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{7/2}} + \frac{a(2a^4-5a^2b^2+18b^4+3ab(a^2+9b^2)\cos(c+dx)+a^2(4a^2+11b^2)\cos^2(c+dx))\sin(c+dx)}{(a-b)^3(a+b)^3(b+a\cos(c+dx))^3}}{6d}$$

input
```
Integrate[Sec[c + d*x]^4/(a + b*Sec[c + d*x])^4,x]
```

output
```
((6*b*(3*a^2 + 2*b^2)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]]
)/(a^2 - b^2)^(7/2) + (a*(2*a^4 - 5*a^2*b^2 + 18*b^4 + 3*a*b*(a^2 + 9*b^2)
*Cos[c + d*x] + a^2*(4*a^2 + 11*b^2)*Cos[c + d*x]^2)*Sin[c + d*x])/((a - b
)^3*(a + b)^3*(b + a*Cos[c + d*x])^3))/(6*d)
```

3.516.3 Rubi [A] (verified)

Time = 1.18 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.20, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {3042, 4332, 3042, 4568, 3042, 4491, 27, 3042, 4318, 3042, 3138, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^4(c+dx)}{(a+b\sec(c+dx))^4}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^4}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^4}dx$$

$$\Big\downarrow \text{4332}$$

$$-\frac{\int \frac{\sec(c+dx)\left(a^2-3b\sec(c+dx)a-\left(2a^2-3b^2\right)\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^3}dx}{3b\left(a^2-b^2\right)} - \frac{a^2\tan(c+dx)\sec(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^3}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(a^2-3b\csc\left(c+dx+\frac{\pi}{2}\right)a+\left(3b^2-2a^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^3}dx}{3b\left(a^2-b^2\right)}-\frac{a^2\tan(c+dx)\sec(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^3}$$

$\Big\downarrow$ 4568

$$-\frac{\frac{a^2\left(2a^2-7b^2\right)\tan(c+dx)}{2bd(a^2-b^2)(a+b\sec(c+dx))^2}-\frac{\int \frac{\sec(c+dx)\left(2ab\left(a^2-6b^2\right)+\left(2a^4-3b^2a^2+6b^4\right)\sec(c+dx)\right)}{(a+b\sec(c+dx))^2}dx}{2b(a^2-b^2)}}{3b\left(a^2-b^2\right)}-\frac{a^2\tan(c+dx)\sec(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^3}$$

$\Big\downarrow$ 3042

$$-\frac{\frac{a^2\left(2a^2-7b^2\right)\tan(c+dx)}{2bd(a^2-b^2)(a+b\sec(c+dx))^2}-\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(2ab\left(a^2-6b^2\right)+\left(2a^4-3b^2a^2+6b^4\right)\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx}{2b(a^2-b^2)}}{3b\left(a^2-b^2\right)}-\frac{a^2\tan(c+dx)\sec(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^3}$$

$\Big\downarrow$ 4491

$$-\frac{\frac{a^2\left(2a^2-7b^2\right)\tan(c+dx)}{2bd(a^2-b^2)(a+b\sec(c+dx))^2}-\frac{\frac{a\left(2a^4-5a^2b^2+18b^4\right)\tan(c+dx)}{d\left(a^2-b^2\right)(a+b\sec(c+dx))}-\frac{\int \frac{3b^3\left(3a^2+2b^2\right)\sec(c+dx)}{a+b\sec(c+dx)}dx}{a^2-b^2}}{2b(a^2-b^2)}}{3b\left(a^2-b^2\right)}-\frac{a^2\tan(c+dx)\sec(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^3}$$

$\Big\downarrow$ 27

$$-\frac{\frac{a^2\left(2a^2-7b^2\right)\tan(c+dx)}{2bd(a^2-b^2)(a+b\sec(c+dx))^2}-\frac{\frac{a\left(2a^4-5a^2b^2+18b^4\right)\tan(c+dx)}{d\left(a^2-b^2\right)(a+b\sec(c+dx))}-\frac{3b^3\left(3a^2+2b^2\right)\int \frac{\sec(c+dx)}{a+b\sec(c+dx)}dx}{a^2-b^2}}{2b(a^2-b^2)}}{3b\left(a^2-b^2\right)}-\frac{a^2\tan(c+dx)\sec(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^3}$$

$\Big\downarrow$ 3042

$$-\frac{\frac{a^2\left(2a^2-7b^2\right)\tan(c+dx)}{2bd(a^2-b^2)(a+b\sec(c+dx))^2}-\frac{\frac{a\left(2a^4-5a^2b^2+18b^4\right)\tan(c+dx)}{d\left(a^2-b^2\right)(a+b\sec(c+dx))}-\frac{3b^3\left(3a^2+2b^2\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2-b^2}}{2b(a^2-b^2)}}{3b\left(a^2-b^2\right)}-\frac{a^2\tan(c+dx)\sec(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^3}$$

$\Big\downarrow$ 4318

3.516. $\int \frac{\sec^4(c+dx)}{(a+b\sec(c+dx))^4}\,dx$

$$-\frac{\frac{a^2(2a^2-7b^2)\tan(c+dx)}{2bd(a^2-b^2)(a+b\sec(c+dx))^2}-\frac{\frac{a(2a^4-5a^2b^2+18b^4)\tan(c+dx)}{d(a^2-b^2)(a+b\sec(c+dx))}-\frac{3b^2(3a^2+2b^2)\int\frac{1}{\frac{a\cos(c+dx)}{b}+1}dx}{a^2-b^2}}{2b(a^2-b^2)}}{3b(a^2-b^2)}-\frac{a^2\tan(c+dx)\sec(c+dx)}{3bd(a^2-b^2)(a+b\sec(c+dx))^3}$$

$\Big\downarrow$ 3042

$$-\frac{\frac{a^2(2a^2-7b^2)\tan(c+dx)}{2bd(a^2-b^2)(a+b\sec(c+dx))^2}-\frac{\frac{a(2a^4-5a^2b^2+18b^4)\tan(c+dx)}{d(a^2-b^2)(a+b\sec(c+dx))}-\frac{3b^2(3a^2+2b^2)\int\frac{1}{\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{b}+1}dx}{a^2-b^2}}{2b(a^2-b^2)}}{3b(a^2-b^2)}-\frac{a^2\tan(c+dx)\sec(c+dx)}{3bd(a^2-b^2)(a+b\sec(c+dx))^3}$$

$\Big\downarrow$ 3138

$$-\frac{\frac{a^2(2a^2-7b^2)\tan(c+dx)}{2bd(a^2-b^2)(a+b\sec(c+dx))^2}-\frac{\frac{a(2a^4-5a^2b^2+18b^4)\tan(c+dx)}{d(a^2-b^2)(a+b\sec(c+dx))}-\frac{6b^2(3a^2+2b^2)\int\frac{1}{\left(1-\frac{a}{b}\right)\tan^2\left(\frac{1}{2}(c+dx)\right)+\frac{a+b}{b}}d\tan\left(\frac{1}{2}(c+dx)\right)}{d(a^2-b^2)}}{2b(a^2-b^2)}}{3b(a^2-b^2)}-\frac{a^2\tan(c+dx)\sec(c+dx)}{3bd(a^2-b^2)(a+b\sec(c+dx))^3}$$

$\Big\downarrow$ 221

$$-\frac{a^2\tan(c+dx)\sec(c+dx)}{3bd(a^2-b^2)(a+b\sec(c+dx))^3}-\frac{\frac{a^2(2a^2-7b^2)\tan(c+dx)}{2bd(a^2-b^2)(a+b\sec(c+dx))^2}-\frac{\frac{a(2a^4-5a^2b^2+18b^4)\tan(c+dx)}{d(a^2-b^2)(a+b\sec(c+dx))}-\frac{6b^3(3a^2+2b^2)\text{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d\sqrt{a-b}\sqrt{a+b}(a^2-b^2)}}{2b(a^2-b^2)}}{3b(a^2-b^2)}$$

input
```
Int[Sec[c + d*x]^4/(a + b*Sec[c + d*x])^4,x]
```

output
```
-1/3*(a^2*Sec[c + d*x]*Tan[c + d*x])/(b*(a^2 - b^2)*d*(a + b*Sec[c + d*x])
^3) - ((a^2*(2*a^2 - 7*b^2)*Tan[c + d*x])/(2*b*(a^2 - b^2)*d*(a + b*Sec[c
+ d*x])^2) - ((-6*b^3*(3*a^2 + 2*b^2)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2
])/Sqrt[a + b]])/(Sqrt[a - b]*Sqrt[a + b]*(a^2 - b^2)*d) + (a*(2*a^4 - 5*a
^2*b^2 + 18*b^4)*Tan[c + d*x])/((a^2 - b^2)*d*(a + b*Sec[c + d*x])))/(2*b*
(a^2 - b^2)))/(3*b*(a^2 - b^2))
```

3.516. $\int \frac{\sec^4(c+dx)}{(a+b\sec(c+dx))^4}\,dx$

3.516.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 4318
```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[1/b   Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4332
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-a^2)*d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(
m + 1)*((d*Csc[e + f*x])^(n - 3)/(b*f*(m + 1)*(a^2 - b^2))), x] + Simp[d^3/
(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])
^(n - 3)*Simp[a^2*(n - 3) + a*b*(m + 1)*Csc[e + f*x] - (a^2*(n - 2) + b^2*(
m + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2
- b^2, 0] && LtQ[m, -1] && (IGtQ[n, 3] || (IntegersQ[n + 1/2, 2*m] && GtQ[n
, 2]))
```

rule 4491
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b - a*B))*Cot[e
 + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Simp[1
/((m + 1)*(a^2 - b^2))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp
[(a*A - b*B)*(m + 1) - (A*b - a*B)*(m + 2)*Csc[e + f*x], x], x], x] /; Free
Q[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m
, -1]
```

3.516. $\int \frac{\sec^4(c+dx)}{(a+b\sec(c+dx))^4}\,dx$

rule 4568
```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cot[e + f*x]*((a + b*Csc[e + f*x]
)^(m + 1)/(b*f*(m + 1)*(a^2 - b^2))), x] + Simp[1/(b*(m + 1)*(a^2 - b^2))
 Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m
+ 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b - a*B + b*C)*(m + 1))*Csc[e + f*x],
x], x], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^
2, 0]
```

3.516.4 Maple [A] (verified)

Time = 0.94 (sec) , antiderivative size = 285, normalized size of antiderivative = 1.28

method	result
derivativedivides	$\frac{\frac{-\frac{\left(2a^2+3ab+6b^2\right)a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{(a-b)\left(a^3+3a^2b+3a\,b^2+b^3\right)}+\frac{4\left(a^2+9b^2\right)a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3\left(a^2+2ab+b^2\right)\left(a^2-2ab+b^2\right)}-\frac{\left(2a^2-3ab+6b^2\right)a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{(a+b)\left(a^3-3a^2b+3a\,b^2-b^3\right)}}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2b-a-b\right)^3}-\frac{b\left(3a^2+2b^2\right)\operatorname{arctanh}\left(\frac{(a-b)\cdots}{\sqrt{(a\cdots}}\right)}{\left(a^6-3a^4b^2+3a^2b^4-b^6\right)\sqrt{\cdots}}}{d}$
default	$\frac{\frac{-\frac{\left(2a^2+3ab+6b^2\right)a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{(a-b)\left(a^3+3a^2b+3a\,b^2+b^3\right)}+\frac{4\left(a^2+9b^2\right)a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3\left(a^2+2ab+b^2\right)\left(a^2-2ab+b^2\right)}-\frac{\left(2a^2-3ab+6b^2\right)a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{(a+b)\left(a^3-3a^2b+3a\,b^2-b^3\right)}}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2b-a-b\right)^3}-\frac{b\left(3a^2+2b^2\right)\operatorname{arctanh}\left(\frac{(a-b)\cdots}{\sqrt{(a\cdots}}\right)}{\left(a^6-3a^4b^2+3a^2b^4-b^6\right)\sqrt{\cdots}}}{d}$
risch	$-\frac{i\left(9a^4b\,e^{5i(dx+c)}+6a^2b^3e^{5i(dx+c)}+45a^3b^2e^{4i(dx+c)}+30a\,b^4e^{4i(dx+c)}+24a^4b\,e^{3i(dx+c)}+82a^2b^3e^{3i(dx+c)}+44b^5e^{3i(dx\cdots}\right.}{3(-a^2+b^2)^3d\left(a\,e^{2i(dx+c)}+2\cdots\right.}$

input
```
int(sec(d*x+c)^4/(a+b*sec(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
1/d*(2*(-1/2*(2*a^2+3*a*b+6*b^2)*a/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2
*d*x+1/2*c)^5+2/3*(a^2+9*b^2)*a/(a^2+2*a*b+b^2)/(a^2-2*a*b+b^2)*tan(1/2*d*
x+1/2*c)^3-1/2*(2*a^2-3*a*b+6*b^2)*a/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1
/2*d*x+1/2*c))/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3-b*(3*
a^2+2*b^2)/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a-b)*(a+b))^(1/2)*arctanh((a-b)
*tan(1/2*d*x+1/2*c)/((a-b)*(a+b))^(1/2)))
```

3.516.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 423 vs. $2(207) = 414$.

Time = 0.31 (sec) , antiderivative size = 903, normalized size of antiderivative = 4.07

$$\int \frac{\sec^4(c+dx)}{(a+b\sec(c+dx))^4}\,dx$$
$$= \left[-\frac{3\left(3\,a^2b^4+2\,b^6+\left(3\,a^5b+2\,a^3b^3\right)\cos\left(dx+c\right)^3+3\left(3\,a^4b^2+2\,a^2b^4\right)\cos\left(dx+c\right)^2+3\left(3\,a^3b^3+2\,ab^5\right)\mathrm{c}\right.}{12\left(\left(a^{11}-4\,a^9b^2+6\,a^7b^4-4\,a^5b^6+a^3b^8\right)d\cos\left(\right.\right.}\right.$$
$$-\frac{3\left(3\,a^2b^4+2\,b^6+\left(3\,a^5b+2\,a^3b^3\right)\cos\left(dx+c\right)^3+3\left(3\,a^4b^2+2\,a^2b^4\right)\cos\left(dx+c\right)^2+3\left(3\,a^3b^3+2\,ab^5\right)\mathrm{co}\right.}{6\left(\left(a^{11}-4\,a^9b^2+6\,a^7b^4-4\,a^5b^6+a^3b^8\right)d\cos\left(dx+c\right)^3+3\left(a^{10}b-\right.\right.}$$

input
```
integrate(sec(d*x+c)^4/(a+b*sec(d*x+c))^4,x, algorithm="fricas")
```

output
```
[-1/12*(3*(3*a^2*b^4 + 2*b^6 + (3*a^5*b + 2*a^3*b^3)*cos(d*x + c)^3 + 3*(3
*a^4*b^2 + 2*a^2*b^4)*cos(d*x + c)^2 + 3*(3*a^3*b^3 + 2*a*b^5)*cos(d*x + c
))*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2
+ 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*
cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) - 2*(2*a^7 - 7*a^5*b^2 + 23*a^
3*b^4 - 18*a*b^6 + (4*a^7 + 7*a^5*b^2 - 11*a^3*b^4)*cos(d*x + c)^2 + 3*(a^
6*b + 8*a^4*b^3 - 9*a^2*b^5)*cos(d*x + c))*sin(d*x + c))/((a^11 - 4*a^9*b^
2 + 6*a^7*b^4 - 4*a^5*b^6 + a^3*b^8)*d*cos(d*x + c)^3 + 3*(a^10*b - 4*a^8*
b^3 + 6*a^6*b^5 - 4*a^4*b^7 + a^2*b^9)*d*cos(d*x + c)^2 + 3*(a^9*b^2 - 4*a
^7*b^4 + 6*a^5*b^6 - 4*a^3*b^8 + a*b^10)*d*cos(d*x + c) + (a^8*b^3 - 4*a^6
*b^5 + 6*a^4*b^7 - 4*a^2*b^9 + b^11)*d), -1/6*(3*(3*a^2*b^4 + 2*b^6 + (3*a
^5*b + 2*a^3*b^3)*cos(d*x + c)^3 + 3*(3*a^4*b^2 + 2*a^2*b^4)*cos(d*x + c)^
2 + 3*(3*a^3*b^3 + 2*a*b^5)*cos(d*x + c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a
^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c))) - (2*a^7 - 7*a^
5*b^2 + 23*a^3*b^4 - 18*a*b^6 + (4*a^7 + 7*a^5*b^2 - 11*a^3*b^4)*cos(d*x +
 c)^2 + 3*(a^6*b + 8*a^4*b^3 - 9*a^2*b^5)*cos(d*x + c))*sin(d*x + c))/((a^
11 - 4*a^9*b^2 + 6*a^7*b^4 - 4*a^5*b^6 + a^3*b^8)*d*cos(d*x + c)^3 + 3*(a^
10*b - 4*a^8*b^3 + 6*a^6*b^5 - 4*a^4*b^7 + a^2*b^9)*d*cos(d*x + c)^2 + 3*(
a^9*b^2 - 4*a^7*b^4 + 6*a^5*b^6 - 4*a^3*b^8 + a*b^10)*d*cos(d*x + c) + (a^
8*b^3 - 4*a^6*b^5 + 6*a^4*b^7 - 4*a^2*b^9 + b^11)*d)]
```

3.516.6 Sympy [F]

$$\int \frac{\sec^4(c+dx)}{(a+b\sec(c+dx))^4}\,dx = \int \frac{\sec^4(c+dx)}{(a+b\sec(c+dx))^4}\,dx$$

input
```
integrate(sec(d*x+c)**4/(a+b*sec(d*x+c))**4,x)
```

output
```
Integral(sec(c + d*x)**4/(a + b*sec(c + d*x))**4, x)
```

3.516.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\sec^4(c+dx)}{(a+b\sec(c+dx))^4}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(sec(d*x+c)^4/(a+b*sec(d*x+c))^4,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

3.516.8 Giac [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 403, normalized size of antiderivative = 1.82

$$\int \frac{\sec^4(c+dx)}{(a+b\sec(c+dx))^4}\,dx = -\frac{\frac{3\,(3\,a^2b+2\,b^3)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{-a^2+b^2}}\right)\right)}{(a^6-3\,a^4b^2+3\,a^2b^4-b^6)\sqrt{-a^2+b^2}} + \frac{6\,a^5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-3\,a^4b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5}{}}{}$$

input
```
integrate(sec(d*x+c)^4/(a+b*sec(d*x+c))^4,x, algorithm="giac")
```

output
```
-1/3*(3*(3*a^2*b + 2*b^3)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b
) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 +
b^2)))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*sqrt(-a^2 + b^2)) + (6*a^5*tan
(1/2*d*x + 1/2*c)^5 - 3*a^4*b*tan(1/2*d*x + 1/2*c)^5 + 6*a^3*b^2*tan(1/2*d
*x + 1/2*c)^5 - 27*a^2*b^3*tan(1/2*d*x + 1/2*c)^5 + 18*a*b^4*tan(1/2*d*x +
 1/2*c)^5 - 4*a^5*tan(1/2*d*x + 1/2*c)^3 - 32*a^3*b^2*tan(1/2*d*x + 1/2*c)
^3 + 36*a*b^4*tan(1/2*d*x + 1/2*c)^3 + 6*a^5*tan(1/2*d*x + 1/2*c) + 3*a^4*
b*tan(1/2*d*x + 1/2*c) + 6*a^3*b^2*tan(1/2*d*x + 1/2*c) + 27*a^2*b^3*tan(1
/2*d*x + 1/2*c) + 18*a*b^4*tan(1/2*d*x + 1/2*c))/((a^6 - 3*a^4*b^2 + 3*a^2
*b^4 - b^6)*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 - a - b)^
3))/d
```

3.516.9 Mupad [B] (verification not implemented)

Time = 17.89 (sec) , antiderivative size = 378, normalized size of antiderivative = 1.70

$$\int \frac{\sec^4(c+dx)}{(a+b\sec(c+dx))^4}\,dx$$

$$= \frac{\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5(2a^3+3a^2b+6ab^2)}{(a+b)^3(a-b)} - \frac{4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3(a^3+9ab^2)}{3(a+b)^2(a^2-2ab+b^2)} + \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right.}{(a+b)(a}}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2(-3a^3-3a^2b+3ab^2+3b^3) - \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4(-3a^3+3a^2b+3ab^2-3b^3) + 3ab^2 -\right.} - \frac{b\,\mathrm{atanh}\left(\frac{b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)(3a^2+2b^2)(2a-2b)(a^3-3a^2b+3ab^2-b^3)}{2(3a^2b+2b^3)\sqrt{a+b}(a-b)^{7/2}}\right)(3a^2+2b^2)}{d(a+b)^{7/2}(a-b)^{7/2}}$$

input
```
int(1/(cos(c + d*x)^4*(a + b/cos(c + d*x))^4),x)
```

output
```
((tan(c/2 + (d*x)/2)^5*(6*a*b^2 + 3*a^2*b + 2*a^3))/((a + b)^3*(a - b)) -
(4*tan(c/2 + (d*x)/2)^3*(9*a*b^2 + a^3))/(3*(a + b)^2*(a^2 - 2*a*b + b^2))
 + (tan(c/2 + (d*x)/2)*(6*a*b^2 - 3*a^2*b + 2*a^3))/((a + b)*(3*a*b^2 - 3*
a^2*b + a^3 - b^3)))/(d*(tan(c/2 + (d*x)/2)^2*(3*a*b^2 - 3*a^2*b - 3*a^3 +
 3*b^3) - tan(c/2 + (d*x)/2)^4*(3*a*b^2 + 3*a^2*b - 3*a^3 - 3*b^3) + 3*a*b
^2 + 3*a^2*b + a^3 + b^3 - tan(c/2 + (d*x)/2)^6*(3*a*b^2 - 3*a^2*b + a^3 -
 b^3))) - (b*atanh((b*tan(c/2 + (d*x)/2)*(3*a^2 + 2*b^2)*(2*a - 2*b)*(3*a*
b^2 - 3*a^2*b + a^3 - b^3))/(2*(3*a^2*b + 2*b^3)*(a + b)^(1/2)*(a - b)^(7/
2)))*(3*a^2 + 2*b^2))/(d*(a + b)^(7/2)*(a - b)^(7/2))
```

3.517 $\int \frac{\sec^3(c+dx)}{(a+b\sec(c+dx))^4}\,dx$

3.517.1 Optimal result . . . 3860
3.517.2 Mathematica [A] (verified) . . . 3861
3.517.3 Rubi [A] (verified) . . . 3861
3.517.4 Maple [A] (verified) . . . 3865
3.517.5 Fricas [B] (verification not implemented) . . . 3866
3.517.6 Sympy [F] . . . 3867
3.517.7 Maxima [F(-2)] . . . 3867
3.517.8 Giac [B] (verification not implemented) . . . 3867
3.517.9 Mupad [B] (verification not implemented) . . . 3868

3.517.1 Optimal result

Integrand size = 21, antiderivative size = 206

$$\int \frac{\sec^3(c+dx)}{(a+b\sec(c+dx))^4}\,dx = \frac{a(a^2+4b^2)\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{7/2}(a+b)^{7/2}d} - \frac{a^2\tan(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} + \frac{a(a^2-6b^2)\tan(c+dx)}{6b\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))^2} + \frac{(a^4-10a^2b^2-6b^4)\tan(c+dx)}{6b\left(a^2-b^2\right)^3 d(a+b\sec(c+dx))}$$

output
```
a*(a^2+4*b^2)*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/(a-b)^(7
/2)/(a+b)^(7/2)/d-1/3*a^2*tan(d*x+c)/b/(a^2-b^2)/d/(a+b*sec(d*x+c))^3+1/6*
a*(a^2-6*b^2)*tan(d*x+c)/b/(a^2-b^2)^2/d/(a+b*sec(d*x+c))^2+1/6*(a^4-10*a^
2*b^2-6*b^4)*tan(d*x+c)/b/(a^2-b^2)^3/d/(a+b*sec(d*x+c))
```

3.517.2 Mathematica [A] (verified)

Time = 0.95 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.80

$$\int \frac{\sec^3(c+dx)}{(a+b\sec(c+dx))^4}\,dx = \frac{-\frac{6a(a^2+4b^2)\text{arctanh}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{7/2}} + \frac{(b(a^4-10a^2b^2-6b^4)+3a(a^4-9a^2b^2-2b^4)\cos(c+dx)-a^2b(13a^2+2b^2)\cos^2(c+dx))\sin(c+}{(a-b)^3(a+b)^3(b+a\cos(c+dx))^3}}{6d}$$

input
```
Integrate[Sec[c + d*x]^3/(a + b*Sec[c + d*x])^4,x]
```

output
```
((-6*a*(a^2 + 4*b^2)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])
/(a^2 - b^2)^(7/2) + ((b*(a^4 - 10*a^2*b^2 - 6*b^4) + 3*a*(a^4 - 9*a^2*b^2
 - 2*b^4)*Cos[c + d*x] - a^2*b*(13*a^2 + 2*b^2)*Cos[c + d*x]^2)*Sin[c + d*
x])/((a - b)^3*(a + b)^3*(b + a*Cos[c + d*x])^3))/(6*d)
```

3.517.3 Rubi [A] (verified)

Time = 1.06 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.19, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {3042, 4326, 25, 3042, 4491, 25, 3042, 4491, 27, 3042, 4318, 3042, 3138, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^3(c+dx)}{(a+b\sec(c+dx))^4}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^3}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^4}dx$$

$$\Big\downarrow \; 4326$$

$$-\frac{\int -\frac{\sec(c+dx)\left(3ab+\left(a^2-3b^2\right)\sec(c+dx)\right)}{(a+b\sec(c+dx))^3}dx}{3b\left(a^2-b^2\right)} - \frac{a^2\tan(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^3}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{\sec(c+dx)\left(3ab+\left(a^2-3b^2\right)\sec(c+dx)\right)}{(a+b\sec(c+dx))^3}dx}{3b\left(a^2-b^2\right)}-\frac{a^2\tan(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^3}$$

$\Big\downarrow$ 3042

$$\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(3ab+\left(a^2-3b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^3}dx}{3b\left(a^2-b^2\right)}-\frac{a^2\tan(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^3}$$

$\Big\downarrow$ 4491

$$\frac{\frac{a\left(a^2-6b^2\right)\tan(c+dx)}{2d(a^2-b^2)(a+b\sec(c+dx))^2}-\frac{\int -\frac{\sec(c+dx)\left(2b\left(2a^2+3b^2\right)+a\left(a^2-6b^2\right)\sec(c+dx)\right)}{(a+b\sec(c+dx))^2}dx}{2(a^2-b^2)}}{3b\left(a^2-b^2\right)}-\frac{a^2\tan(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^3}$$

$\Big\downarrow$ 25

$$\frac{\frac{\int \frac{\sec(c+dx)\left(2b\left(2a^2+3b^2\right)+a\left(a^2-6b^2\right)\sec(c+dx)\right)}{(a+b\sec(c+dx))^2}dx}{2(a^2-b^2)}+\frac{a\left(a^2-6b^2\right)\tan(c+dx)}{2d(a^2-b^2)(a+b\sec(c+dx))^2}}{3b\left(a^2-b^2\right)}-\frac{a^2\tan(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^3}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(2b\left(2a^2+3b^2\right)+a\left(a^2-6b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx}{2(a^2-b^2)}+\frac{a\left(a^2-6b^2\right)\tan(c+dx)}{2d(a^2-b^2)(a+b\sec(c+dx))^2}}{3b\left(a^2-b^2\right)}-\frac{a^2\tan(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^3}$$

$\Big\downarrow$ 4491

$$\frac{\frac{\frac{\left(a^4-10a^2b^2-6b^4\right)\tan(c+dx)}{d\left(a^2-b^2\right)(a+b\sec(c+dx))}-\frac{\int -\frac{3ab\left(a^2+4b^2\right)\sec(c+dx)}{a+b\sec(c+dx)}dx}{a^2-b^2}}{2(a^2-b^2)}+\frac{a\left(a^2-6b^2\right)\tan(c+dx)}{2d(a^2-b^2)(a+b\sec(c+dx))^2}}{3b\left(a^2-b^2\right)}-\frac{a^2\tan(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^3}$$

$\Big\downarrow$ 27

$$\frac{\frac{\frac{3ab\left(a^2+4b^2\right)\int \frac{\sec(c+dx)}{a+b\sec(c+dx)}dx}{a^2-b^2}+\frac{\left(a^4-10a^2b^2-6b^4\right)\tan(c+dx)}{d\left(a^2-b^2\right)(a+b\sec(c+dx))}}{2(a^2-b^2)}+\frac{a\left(a^2-6b^2\right)\tan(c+dx)}{2d(a^2-b^2)(a+b\sec(c+dx))^2}}{3b\left(a^2-b^2\right)}-\frac{a^2\tan(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^3}$$

3.517. $\int \frac{\sec^3(c+dx)}{(a+b\sec(c+dx))^4}\,dx$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{\frac{3ab\left(a^2+4b^2\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2-b^2}+\frac{\left(a^4-10a^2b^2-6b^4\right)\tan(c+dx)}{d\left(a^2-b^2\right)(a+b\sec(c+dx))}}{2(a^2-b^2)}+\frac{a\left(a^2-6b^2\right)\tan(c+dx)}{2d(a^2-b^2)(a+b\sec(c+dx))^2}}{3b\left(a^2-b^2\right)}-\frac{a^2\tan(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^3}$$

$$\Big\downarrow \; 4318$$

$$\frac{\frac{\frac{3a\left(a^2+4b^2\right)\int \frac{1}{\frac{a\cos(c+dx)}{b}+1}dx}{a^2-b^2}+\frac{\left(a^4-10a^2b^2-6b^4\right)\tan(c+dx)}{d\left(a^2-b^2\right)(a+b\sec(c+dx))}}{2(a^2-b^2)}+\frac{a\left(a^2-6b^2\right)\tan(c+dx)}{2d(a^2-b^2)(a+b\sec(c+dx))^2}}{3b\left(a^2-b^2\right)}-\frac{a^2\tan(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^3}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{\frac{3a\left(a^2+4b^2\right)\int \frac{1}{\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{b}+1}dx}{a^2-b^2}+\frac{\left(a^4-10a^2b^2-6b^4\right)\tan(c+dx)}{d\left(a^2-b^2\right)(a+b\sec(c+dx))}}{2(a^2-b^2)}+\frac{a\left(a^2-6b^2\right)\tan(c+dx)}{2d(a^2-b^2)(a+b\sec(c+dx))^2}}{3b\left(a^2-b^2\right)}-\frac{a^2\tan(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^3}$$

$$\Big\downarrow \; 3138$$

$$\frac{\frac{\frac{6a\left(a^2+4b^2\right)\int \frac{1}{\left(1-\frac{a}{b}\right)\tan^2\left(\frac{1}{2}(c+dx)\right)+\frac{a+b}{b}}d\tan\left(\frac{1}{2}(c+dx)\right)}{d\left(a^2-b^2\right)}+\frac{\left(a^4-10a^2b^2-6b^4\right)\tan(c+dx)}{d\left(a^2-b^2\right)(a+b\sec(c+dx))}}{2(a^2-b^2)}+\frac{a\left(a^2-6b^2\right)\tan(c+dx)}{2d(a^2-b^2)(a+b\sec(c+dx))^2}}{3b\left(a^2-b^2\right)}-\frac{a^2\tan(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^3}$$

$$\Big\downarrow \; 221$$

$$\frac{\frac{a\left(a^2-6b^2\right)\tan(c+dx)}{2d(a^2-b^2)(a+b\sec(c+dx))^2}+\frac{\frac{6ab\left(a^2+4b^2\right)\text{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d\sqrt{a-b}\sqrt{a+b}\left(a^2-b^2\right)}+\frac{\left(a^4-10a^2b^2-6b^4\right)\tan(c+dx)}{d\left(a^2-b^2\right)(a+b\sec(c+dx))}}{2(a^2-b^2)}}{3b\left(a^2-b^2\right)}-\frac{a^2\tan(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^3}$$

input
```
Int[Sec[c + d*x]^3/(a + b*Sec[c + d*x])^4,x]
```

3.517. $\int \frac{\sec^3(c+dx)}{(a+b\sec(c+dx))^4}\,dx$

output
```
-1/3*(a^2*Tan[c + d*x])/(b*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^3) + ((a*(a^
2 - 6*b^2)*Tan[c + d*x])/(2*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^2) + ((6*a*
b*(a^2 + 4*b^2)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(Sqrt
[a - b]*Sqrt[a + b]*(a^2 - b^2)*d) + ((a^4 - 10*a^2*b^2 - 6*b^4)*Tan[c + d
*x])/((a^2 - b^2)*d*(a + b*Sec[c + d*x])))/(2*(a^2 - b^2)))/(3*b*(a^2 - b^
2))
```

3.517.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 4318
```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[1/b   Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4326
```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(-a^2)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m
 + 1)*(a^2 - b^2))), x] + Simp[1/(b*(m + 1)*(a^2 - b^2))   Int[Csc[e + f*x]
*(a + b*Csc[e + f*x])^(m + 1)*Simp[a*b*(m + 1) - (a^2 + b^2*(m + 1))*Csc[e
+ f*x], x], x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m,
 -1]
```

3.517. $\int \frac{\sec^3(c+dx)}{(a+b\sec(c+dx))^4}\,dx$

rule 4491
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b - a*B))*Cot[e
 + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Simp[1
/((m + 1)*(a^2 - b^2))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp
[(a*A - b*B)*(m + 1) - (A*b - a*B)*(m + 2)*Csc[e + f*x], x], x], x] /; Free
Q[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m
, -1]
```

3.517.4 Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.43

method	result
derivativedivides	$\frac{-\frac{2\left(-\frac{\left(a^3+6a^2b+2a\,b^2+2b^3\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{2(a-b)\left(a^3+3a^2b+3a\,b^2+b^3\right)}+\frac{2\left(7a^2+3b^2\right)b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3\left(a^2+2ab+b^2\right)\left(a^2-2ab+b^2\right)}+\frac{\left(a^3-6a^2b+2a\,b^2-2b^3\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2(a+b)\left(a^3-3a^2b+3a\,b^2-b^3\right)}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2b-a-b\right)^3}+\frac{a\left(a^2+4b^2\right)}{\left(a^6-3a^4\right.}}{d}$
default	$\frac{-\frac{2\left(-\frac{\left(a^3+6a^2b+2a\,b^2+2b^3\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{2(a-b)\left(a^3+3a^2b+3a\,b^2+b^3\right)}+\frac{2\left(7a^2+3b^2\right)b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3\left(a^2+2ab+b^2\right)\left(a^2-2ab+b^2\right)}+\frac{\left(a^3-6a^2b+2a\,b^2-2b^3\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2(a+b)\left(a^3-3a^2b+3a\,b^2-b^3\right)}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2b-a-b\right)^3}+\frac{a\left(a^2+4b^2\right)}{\left(a^6-3a^4\right.}}{d}$
risch	$\frac{i\left(3a^6e^{5i(dx+c)}+12a^4b^2e^{5i(dx+c)}+15a^5b\,e^{4i(dx+c)}+60a^3b^3e^{4i(dx+c)}+78a^4b^2e^{3i(dx+c)}+64a^2b^4e^{3i(dx+c)}+8b^6e^{3i(dx+c)}\right.}{3a\left(-a^2+b^2\right)^3d\left(a\,e^{2i(dx\right.}}$

input
```
int(sec(d*x+c)^3/(a+b*sec(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-2*(-1/2*(a^3+6*a^2*b+2*a*b^2+2*b^3)/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*
tan(1/2*d*x+1/2*c)^5+2/3*(7*a^2+3*b^2)*b/(a^2+2*a*b+b^2)/(a^2-2*a*b+b^2)*t
an(1/2*d*x+1/2*c)^3+1/2*(a^3-6*a^2*b+2*a*b^2-2*b^3)/(a+b)/(a^3-3*a^2*b+3*a
*b^2-b^3)*tan(1/2*d*x+1/2*c))/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2
*b-a-b)^3+a*(a^2+4*b^2)/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a-b)*(a+b))^(1/2)*
arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a-b)*(a+b))^(1/2)))
```

3.517.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 422 vs. $2(191) = 382$.

Time = 0.32 (sec) , antiderivative size = 902, normalized size of antiderivative = 4.38

$$\int \frac{\sec^3(c+dx)}{(a+b\sec(c+dx))^4}\,dx$$
$$= \left[-\frac{3\left(a^3b^3+4\,ab^5+\left(a^6+4\,a^4b^2\right)\cos\left(dx+c\right)^3+3\left(a^5b+4\,a^3b^3\right)\cos\left(dx+c\right)^2+3\left(a^4b^2+4\,a^2b^4\right)\cos\left(dx\right.\right.}{12\left(\left(a^{11}-4\,a^9b^2+6\,a^7b^4-4\,a^5b^6+a^3b^8\right)d\cos\right.}\right.$$

input
```
integrate(sec(d*x+c)^3/(a+b*sec(d*x+c))^4,x, algorithm="fricas")
```

output
```
[-1/12*(3*(a^3*b^3 + 4*a*b^5 + (a^6 + 4*a^4*b^2)*cos(d*x + c)^3 + 3*(a^5*b
 + 4*a^3*b^3)*cos(d*x + c)^2 + 3*(a^4*b^2 + 4*a^2*b^4)*cos(d*x + c))*sqrt(
a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 - 2*sqrt
(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x
+ c)^2 + 2*a*b*cos(d*x + c) + b^2)) - 2*(a^6*b - 11*a^4*b^3 + 4*a^2*b^5 +
6*b^7 - (13*a^6*b - 11*a^4*b^3 - 2*a^2*b^5)*cos(d*x + c)^2 + 3*(a^7 - 10*a
^5*b^2 + 7*a^3*b^4 + 2*a*b^6)*cos(d*x + c))*sin(d*x + c))/((a^11 - 4*a^9*b
^2 + 6*a^7*b^4 - 4*a^5*b^6 + a^3*b^8)*d*cos(d*x + c)^3 + 3*(a^10*b - 4*a^8
*b^3 + 6*a^6*b^5 - 4*a^4*b^7 + a^2*b^9)*d*cos(d*x + c)^2 + 3*(a^9*b^2 - 4*
a^7*b^4 + 6*a^5*b^6 - 4*a^3*b^8 + a*b^10)*d*cos(d*x + c) + (a^8*b^3 - 4*a^
6*b^5 + 6*a^4*b^7 - 4*a^2*b^9 + b^11)*d), 1/6*(3*(a^3*b^3 + 4*a*b^5 + (a^6
 + 4*a^4*b^2)*cos(d*x + c)^3 + 3*(a^5*b + 4*a^3*b^3)*cos(d*x + c)^2 + 3*(a
^4*b^2 + 4*a^2*b^4)*cos(d*x + c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2
)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c))) + (a^6*b - 11*a^4*b^3 +
 4*a^2*b^5 + 6*b^7 - (13*a^6*b - 11*a^4*b^3 - 2*a^2*b^5)*cos(d*x + c)^2 +
3*(a^7 - 10*a^5*b^2 + 7*a^3*b^4 + 2*a*b^6)*cos(d*x + c))*sin(d*x + c))/((a
^11 - 4*a^9*b^2 + 6*a^7*b^4 - 4*a^5*b^6 + a^3*b^8)*d*cos(d*x + c)^3 + 3*(a
^10*b - 4*a^8*b^3 + 6*a^6*b^5 - 4*a^4*b^7 + a^2*b^9)*d*cos(d*x + c)^2 + 3*
(a^9*b^2 - 4*a^7*b^4 + 6*a^5*b^6 - 4*a^3*b^8 + a*b^10)*d*cos(d*x + c) + (a
^8*b^3 - 4*a^6*b^5 + 6*a^4*b^7 - 4*a^2*b^9 + b^11)*d)]
```

3.517.6 Sympy [F]

$$\int \frac{\sec^3(c+dx)}{(a+b\sec(c+dx))^4}\,dx = \int \frac{\sec^3(c+dx)}{(a+b\sec(c+dx))^4}\,dx$$

input
```
integrate(sec(d*x+c)**3/(a+b*sec(d*x+c))**4,x)
```

output
```
Integral(sec(c + d*x)**3/(a + b*sec(c + d*x))**4, x)
```

3.517.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\sec^3(c+dx)}{(a+b\sec(c+dx))^4}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(sec(d*x+c)^3/(a+b*sec(d*x+c))^4,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

3.517.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 431 vs. $2(191) = 382$.

Time = 0.36 (sec) , antiderivative size = 431, normalized size of antiderivative = 2.09

$$\int \frac{\sec^3(c+dx)}{(a+b\sec(c+dx))^4}\,dx = -\frac{\frac{3\,(a^3+4\,ab^2)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(2\,a-2\,b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{-a^2+b^2}}\right)\right)}{(a^6-3\,a^4b^2+3\,a^2b^4-b^6)\sqrt{-a^2+b^2}} - \frac{3\,a^5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+12\,a^4b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-27}{}}{}$$

input
```
integrate(sec(d*x+c)^3/(a+b*sec(d*x+c))^4,x, algorithm="giac")
```

output
```
-1/3*(3*(a^3 + 4*a*b^2)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(2*a - 2*b) +
 arctan((a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)
))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*sqrt(-a^2 + b^2)) - (3*a^5*tan(1/2
*d*x + 1/2*c)^5 + 12*a^4*b*tan(1/2*d*x + 1/2*c)^5 - 27*a^3*b^2*tan(1/2*d*x
 + 1/2*c)^5 + 12*a^2*b^3*tan(1/2*d*x + 1/2*c)^5 - 6*a*b^4*tan(1/2*d*x + 1/
2*c)^5 + 6*b^5*tan(1/2*d*x + 1/2*c)^5 - 28*a^4*b*tan(1/2*d*x + 1/2*c)^3 +
16*a^2*b^3*tan(1/2*d*x + 1/2*c)^3 + 12*b^5*tan(1/2*d*x + 1/2*c)^3 - 3*a^5*
tan(1/2*d*x + 1/2*c) + 12*a^4*b*tan(1/2*d*x + 1/2*c) + 27*a^3*b^2*tan(1/2*
d*x + 1/2*c) + 12*a^2*b^3*tan(1/2*d*x + 1/2*c) + 6*a*b^4*tan(1/2*d*x + 1/2
*c) + 6*b^5*tan(1/2*d*x + 1/2*c))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*(a*
tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 - a - b)^3))/d
```

3.517.9 Mupad [B] (verification not implemented)

Time = 18.02 (sec) , antiderivative size = 380, normalized size of antiderivative = 1.84

$$\int \frac{\sec^3(c+dx)}{(a+b\sec(c+dx))^4}\,dx$$

$$= \frac{\frac{4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3\left(7a^2b+3b^3\right)}{3(a+b)^2(a^2-2ab+b^2)} - \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5\left(a^3+6a^2b+2ab^2+2b^3\right)}{(a+b)^3(a-b)} + \frac{\tan\left(\frac{c}{2}+\frac{d}{2}\right.}{(a+b)}}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\left(-3a^3-3a^2b+3ab^2+3b^3\right)-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4\left(-3a^3+3a^2b+3ab^2-3b^3\right)+3ab^2\right.} + \frac{a\,\mathrm{atanh}\left(\frac{a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\left(a^2+4b^2\right)(2a-2b)\left(a^3-3a^2b+3ab^2-b^3\right)}{2\sqrt{a+b}\,(a-b)^{7/2}\left(a^3+4ab^2\right)}\right)\left(a^2+4b^2\right)}{d(a+b)^{7/2}(a-b)^{7/2}}$$

input
```
int(1/(cos(c + d*x)^3*(a + b/cos(c + d*x))^4),x)
```

output
```
((4*tan(c/2 + (d*x)/2)^3*(7*a^2*b + 3*b^3))/(3*(a + b)^2*(a^2 - 2*a*b + b^
2)) - (tan(c/2 + (d*x)/2)^5*(2*a*b^2 + 6*a^2*b + a^3 + 2*b^3))/((a + b)^3*
(a - b)) + (tan(c/2 + (d*x)/2)*(2*a*b^2 - 6*a^2*b + a^3 - 2*b^3))/((a + b)
*(3*a*b^2 - 3*a^2*b + a^3 - b^3)))/(d*(tan(c/2 + (d*x)/2)^2*(3*a*b^2 - 3*a
^2*b - 3*a^3 + 3*b^3) - tan(c/2 + (d*x)/2)^4*(3*a*b^2 + 3*a^2*b - 3*a^3 -
3*b^3) + 3*a*b^2 + 3*a^2*b + a^3 + b^3 - tan(c/2 + (d*x)/2)^6*(3*a*b^2 - 3
*a^2*b + a^3 - b^3))) + (a*atanh((a*tan(c/2 + (d*x)/2)*(a^2 + 4*b^2)*(2*a
- 2*b)*(3*a*b^2 - 3*a^2*b + a^3 - b^3))/(2*(a + b)^(1/2)*(a - b)^(7/2)*(4*
a*b^2 + a^3)))*(a^2 + 4*b^2))/(d*(a + b)^(7/2)*(a - b)^(7/2))
```

3.518 $\int \frac{\sec^2(c+dx)}{(a+b\sec(c+dx))^4}\,dx$

3.518.1 Optimal result . . . 3869
3.518.2 Mathematica [A] (verified) . . . 3870
3.518.3 Rubi [A] (verified) . . . 3870
3.518.4 Maple [A] (verified) . . . 3874
3.518.5 Fricas [B] (verification not implemented) . . . 3875
3.518.6 Sympy [F] . . . 3876
3.518.7 Maxima [F(-2)] . . . 3876
3.518.8 Giac [B] (verification not implemented) . . . 3876
3.518.9 Mupad [B] (verification not implemented) . . . 3877

3.518.1 Optimal result

Integrand size = 21, antiderivative size = 192

$$\int \frac{\sec^2(c+dx)}{(a+b\sec(c+dx))^4}\,dx = -\frac{b(4a^2+b^2)\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{7/2}(a+b)^{7/2}d} + \frac{a\tan(c+dx)}{3\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} + \frac{(2a^2+3b^2)\tan(c+dx)}{6\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))^2} + \frac{a(2a^2+13b^2)\tan(c+dx)}{6\left(a^2-b^2\right)^3 d(a+b\sec(c+dx))}$$

output
```
-b*(4*a^2+b^2)*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/(a-b)^(
7/2)/(a+b)^(7/2)/d+1/3*a*tan(d*x+c)/(a^2-b^2)/d/(a+b*sec(d*x+c))^3+1/6*(2*
a^2+3*b^2)*tan(d*x+c)/(a^2-b^2)^2/d/(a+b*sec(d*x+c))^2+1/6*a*(2*a^2+13*b^2
)*tan(d*x+c)/(a^2-b^2)^3/d/(a+b*sec(d*x+c))
```

3.518.2 Mathematica [A] (verified)

Time = 1.12 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.85

$$\int \frac{\sec^2(c+dx)}{(a+b\sec(c+dx))^4}\,dx = \frac{\frac{6b(4a^2+b^2)\text{arctanh}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{7/2}} + \frac{(2a^3b^2+13ab^4-3b(-2a^4-9a^2b^2+b^4)\cos(c+dx)+a(6a^4+10a^2b^2-b^4)\cos^2(c+dx))\sin(c+dx}{(a-b)^3(a+b)^3(b+a\cos(c+dx))^3}}{6d}$$

input
```
Integrate[Sec[c + d*x]^2/(a + b*Sec[c + d*x])^4,x]
```

output
```
((6*b*(4*a^2 + b^2)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/
(a^2 - b^2)^(7/2) + ((2*a^3*b^2 + 13*a*b^4 - 3*b*(-2*a^4 - 9*a^2*b^2 + b^4
)*Cos[c + d*x] + a*(6*a^4 + 10*a^2*b^2 - b^4)*Cos[c + d*x]^2)*Sin[c + d*x]
)/((a - b)^3*(a + b)^3*(b + a*Cos[c + d*x])^3))/(6*d)
```

3.518.3 Rubi [A] (verified)

Time = 1.01 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.21, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {3042, 4323, 25, 3042, 4491, 25, 3042, 4491, 27, 3042, 4318, 3042, 3138, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(c+dx)}{(a+b\sec(c+dx))^4}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^4}dx$$

$$\Big\downarrow \; 4323$$

$$\frac{\int -\frac{\sec(c+dx)(3b-2a\sec(c+dx))}{(a+b\sec(c+dx))^3}dx}{3\left(a^2-b^2\right)} + \frac{a\tan(c+dx)}{3d\left(a^2-b^2\right)(a+b\sec(c+dx))^3}$$

$$\Big\downarrow \; 25$$

$$\frac{a\tan(c+dx)}{3d\left(a^2-b^2\right)(a+b\sec(c+dx))^3} - \frac{\int \frac{\sec(c+dx)(3b-2a\sec(c+dx))}{(a+b\sec(c+dx))^3}dx}{3\left(a^2-b^2\right)}$$

$$\Big\downarrow \; 3042$$

$$\frac{a\tan(c+dx)}{3d\left(a^2-b^2\right)(a+b\sec(c+dx))^3}-\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(3b-2a\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^3}dx}{3\left(a^2-b^2\right)}$$

$$\Big\downarrow \; 4491$$

$$\frac{a\tan(c+dx)}{3d\left(a^2-b^2\right)(a+b\sec(c+dx))^3}-\frac{-\frac{\int -\frac{\sec(c+dx)\left(10ab-\left(2a^2+3b^2\right)\sec(c+dx)\right)}{(a+b\sec(c+dx))^2}dx}{2(a^2-b^2)}-\frac{\left(2a^2+3b^2\right)\tan(c+dx)}{2d(a^2-b^2)(a+b\sec(c+dx))^2}}{3\left(a^2-b^2\right)}$$

$$\Big\downarrow \; 25$$

$$\frac{a\tan(c+dx)}{3d\left(a^2-b^2\right)(a+b\sec(c+dx))^3}-\frac{\frac{\int \frac{\sec(c+dx)\left(10ab-\left(2a^2+3b^2\right)\sec(c+dx)\right)}{(a+b\sec(c+dx))^2}dx}{2(a^2-b^2)}-\frac{\left(2a^2+3b^2\right)\tan(c+dx)}{2d(a^2-b^2)(a+b\sec(c+dx))^2}}{3\left(a^2-b^2\right)}$$

$$\Big\downarrow \; 3042$$

$$\frac{a\tan(c+dx)}{3d\left(a^2-b^2\right)(a+b\sec(c+dx))^3}-\frac{\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(10ab+\left(-2a^2-3b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx}{2(a^2-b^2)}-\frac{\left(2a^2+3b^2\right)\tan(c+dx)}{2d(a^2-b^2)(a+b\sec(c+dx))^2}}{3\left(a^2-b^2\right)}$$

$$\Big\downarrow \; 4491$$

$$\frac{a\tan(c+dx)}{3d\left(a^2-b^2\right)(a+b\sec(c+dx))^3}-\frac{\frac{-\frac{\int -\frac{3b\left(4a^2+b^2\right)\sec(c+dx)}{a+b\sec(c+dx)}dx}{a^2-b^2}-\frac{a\left(2a^2+13b^2\right)\tan(c+dx)}{d\left(a^2-b^2\right)(a+b\sec(c+dx))}}{2(a^2-b^2)}-\frac{\left(2a^2+3b^2\right)\tan(c+dx)}{2d(a^2-b^2)(a+b\sec(c+dx))^2}}{3\left(a^2-b^2\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{a\tan(c+dx)}{3d\left(a^2-b^2\right)(a+b\sec(c+dx))^3}-\frac{\frac{\frac{3b\left(4a^2+b^2\right)\int \frac{\sec(c+dx)}{a+b\sec(c+dx)}dx}{a^2-b^2}-\frac{a\left(2a^2+13b^2\right)\tan(c+dx)}{d\left(a^2-b^2\right)(a+b\sec(c+dx))}}{2(a^2-b^2)}-\frac{\left(2a^2+3b^2\right)\tan(c+dx)}{2d(a^2-b^2)(a+b\sec(c+dx))^2}}{3\left(a^2-b^2\right)}$$

$$\Big\downarrow \; 3042$$

3.518. $\int \frac{\sec^2(c+dx)}{(a+b\sec(c+dx))^4}\,dx$

$$\frac{a\tan(c+dx)}{3d\left(a^2-b^2\right)(a+b\sec(c+dx))^3}-\frac{\frac{\frac{3b\left(4a^2+b^2\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2-b^2}-\frac{a\left(2a^2+13b^2\right)\tan(c+dx)}{d\left(a^2-b^2\right)(a+b\sec(c+dx))}}{2(a^2-b^2)}-\frac{\left(2a^2+3b^2\right)\tan(c+dx)}{2d(a^2-b^2)(a+b\sec(c+dx))^2}}{3\left(a^2-b^2\right)}$$

$\Big\downarrow$ 4318

$$\frac{a\tan(c+dx)}{3d\left(a^2-b^2\right)(a+b\sec(c+dx))^3}-\frac{\frac{\frac{3\left(4a^2+b^2\right)\int\frac{1}{\frac{a\cos(c+dx)}{b}+1}dx}{a^2-b^2}-\frac{a\left(2a^2+13b^2\right)\tan(c+dx)}{d\left(a^2-b^2\right)(a+b\sec(c+dx))}}{2(a^2-b^2)}-\frac{\left(2a^2+3b^2\right)\tan(c+dx)}{2d(a^2-b^2)(a+b\sec(c+dx))^2}}{3\left(a^2-b^2\right)}$$

$\Big\downarrow$ 3042

$$\frac{a\tan(c+dx)}{3d\left(a^2-b^2\right)(a+b\sec(c+dx))^3}-\frac{\frac{\frac{3\left(4a^2+b^2\right)\int\frac{1}{\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{b}+1}dx}{a^2-b^2}-\frac{a\left(2a^2+13b^2\right)\tan(c+dx)}{d\left(a^2-b^2\right)(a+b\sec(c+dx))}}{2(a^2-b^2)}-\frac{\left(2a^2+3b^2\right)\tan(c+dx)}{2d(a^2-b^2)(a+b\sec(c+dx))^2}}{3\left(a^2-b^2\right)}$$

$\Big\downarrow$ 3138

$$\frac{a\tan(c+dx)}{3d\left(a^2-b^2\right)(a+b\sec(c+dx))^3}-\frac{\frac{\frac{6\left(4a^2+b^2\right)\int\frac{1}{\left(1-\frac{a}{b}\right)\tan^2\left(\frac{1}{2}(c+dx)\right)+\frac{a+b}{b}}d\tan\left(\frac{1}{2}(c+dx)\right)}{d\left(a^2-b^2\right)}-\frac{a\left(2a^2+13b^2\right)\tan(c+dx)}{d\left(a^2-b^2\right)(a+b\sec(c+dx))}}{2(a^2-b^2)}-\frac{\left(2a^2+3b^2\right)\tan(c+dx)}{2d(a^2-b^2)(a+b\sec(c+dx))^2}}{3\left(a^2-b^2\right)}$$

$\Big\downarrow$ 221

$$\frac{a\tan(c+dx)}{3d\left(a^2-b^2\right)(a+b\sec(c+dx))^3}-\frac{\frac{\frac{6b\left(4a^2+b^2\right)\text{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d\sqrt{a-b}\sqrt{a+b}\left(a^2-b^2\right)}-\frac{a\left(2a^2+13b^2\right)\tan(c+dx)}{d\left(a^2-b^2\right)(a+b\sec(c+dx))}}{2(a^2-b^2)}-\frac{\left(2a^2+3b^2\right)\tan(c+dx)}{2d(a^2-b^2)(a+b\sec(c+dx))^2}}{3\left(a^2-b^2\right)}$$

input
```
Int[Sec[c + d*x]^2/(a + b*Sec[c + d*x])^4,x]
```

3.518. $\int \frac{\sec^2(c+dx)}{(a+b\sec(c+dx))^4}\,dx$

output
```
(a*Tan[c + d*x])/(3*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^3) - (-1/2*((2*a^2
+ 3*b^2)*Tan[c + d*x])/((a^2 - b^2)*d*(a + b*Sec[c + d*x])^2) + ((6*b*(4*a
^2 + b^2)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(Sqrt[a - b
]*Sqrt[a + b]*(a^2 - b^2)*d) - (a*(2*a^2 + 13*b^2)*Tan[c + d*x])/((a^2 - b
^2)*d*(a + b*Sec[c + d*x])))/(2*(a^2 - b^2)))/(3*(a^2 - b^2))
```

3.518.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 4318
```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[1/b   Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4323
```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[a*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(f*(m + 1)*(
a^2 - b^2))), x] - Simp[1/((m + 1)*(a^2 - b^2))   Int[Csc[e + f*x]*(a + b*C
sc[e + f*x])^(m + 1)*(b*(m + 1) - a*(m + 2)*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

rule 4491
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b - a*B))*Cot[e
 + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Simp[1
/((m + 1)*(a^2 - b^2))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp
[(a*A - b*B)*(m + 1) - (A*b - a*B)*(m + 2)*Csc[e + f*x], x], x], x] /; Free
Q[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m
, -1]
```

3.518.4 Maple [A] (verified)

Time = 0.84 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.55

method	result
derivativedivides	$\frac{\frac{-\frac{\left(2a^3+2a^2b+6a\,b^2+b^3\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{(a-b)\left(a^3+3a^2b+3a\,b^2+b^3\right)}+\frac{4\left(3a^2+7b^2\right)a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3\left(a^2+2ab+b^2\right)\left(a^2-2ab+b^2\right)}-\frac{\left(2a^3-2a^2b+6a\,b^2-b^3\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{(a+b)\left(a^3-3a^2b+3a\,b^2-b^3\right)}}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2b-a-b\right)^3}-\frac{b\left(4a^2+b^2\right)\arctan}{\left(a^6-3a^4b^2+3a\right.}}{d}$
default	$\frac{\frac{-\frac{\left(2a^3+2a^2b+6a\,b^2+b^3\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{(a-b)\left(a^3+3a^2b+3a\,b^2+b^3\right)}+\frac{4\left(3a^2+7b^2\right)a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3\left(a^2+2ab+b^2\right)\left(a^2-2ab+b^2\right)}-\frac{\left(2a^3-2a^2b+6a\,b^2-b^3\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{(a+b)\left(a^3-3a^2b+3a\,b^2-b^3\right)}}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2b-a-b\right)^3}-\frac{b\left(4a^2+b^2\right)\arctan}{\left(a^6-3a^4b^2+3a\right.}}{d}$
risch	$\frac{i\left(-12a^6b\,e^{5i(dx+c)}-3a^4b^3e^{5i(dx+c)}-6a^7e^{4i(dx+c)}-42a^5b^2e^{4i(dx+c)}-33a^3b^4e^{4i(dx+c)}+6a\,b^6e^{4i(dx+c)}-36a^6b\,e^{3i(dx+c}\right.}{}$

input
```
int(sec(d*x+c)^2/(a+b*sec(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
1/d*(2*(-1/2*(2*a^3+2*a^2*b+6*a*b^2+b^3)/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*t
an(1/2*d*x+1/2*c)^5+2/3*(3*a^2+7*b^2)*a/(a^2+2*a*b+b^2)/(a^2-2*a*b+b^2)*ta
n(1/2*d*x+1/2*c)^3-1/2*(2*a^3-2*a^2*b+6*a*b^2-b^3)/(a+b)/(a^3-3*a^2*b+3*a*
b^2-b^3)*tan(1/2*d*x+1/2*c))/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*
b-a-b)^3-b*(4*a^2+b^2)/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a-b)*(a+b))^(1/2)*a
rctanh((a-b)*tan(1/2*d*x+1/2*c)/((a-b)*(a+b))^(1/2)))
```

3.518.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 422 vs. $2(177) = 354$.

Time = 0.35 (sec) , antiderivative size = 901, normalized size of antiderivative = 4.69

$$\int \frac{\sec^2(c+dx)}{(a+b\sec(c+dx))^4}\,dx$$
$$= \left[-\frac{3\left(4\,a^2b^4+b^6+\left(4\,a^5b+a^3b^3\right)\cos\left(dx+c\right)^3+3\left(4\,a^4b^2+a^2b^4\right)\cos\left(dx+c\right)^2+3\left(4\,a^3b^3+ab^5\right)\cos\left(dx\right.\right.}{12\left(\left(a^{11}-4\,a^9b^2+6\,a^7b^4-4\,a^5b^6+a^3b^8\right)d\cos\right.}\right.$$
$$-\frac{3\left(4\,a^2b^4+b^6+\left(4\,a^5b+a^3b^3\right)\cos\left(dx+c\right)^3+3\left(4\,a^4b^2+a^2b^4\right)\cos\left(dx+c\right)^2+3\left(4\,a^3b^3+ab^5\right)\cos\left(dx\right.\right.}{6\left(\left(a^{11}-4\,a^9b^2+6\,a^7b^4-4\,a^5b^6+a^3b^8\right)d\cos\left(dx+c\right)^3+3\left(a^{10}b-\right.\right.}$$

input
```
integrate(sec(d*x+c)^2/(a+b*sec(d*x+c))^4,x, algorithm="fricas")
```

output
```
[-1/12*(3*(4*a^2*b^4 + b^6 + (4*a^5*b + a^3*b^3)*cos(d*x + c)^3 + 3*(4*a^4
*b^2 + a^2*b^4)*cos(d*x + c)^2 + 3*(4*a^3*b^3 + a*b^5)*cos(d*x + c))*sqrt(
a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 + 2*sqrt
(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x
+ c)^2 + 2*a*b*cos(d*x + c) + b^2)) - 2*(2*a^5*b^2 + 11*a^3*b^4 - 13*a*b^6
 + (6*a^7 + 4*a^5*b^2 - 11*a^3*b^4 + a*b^6)*cos(d*x + c)^2 + 3*(2*a^6*b +
7*a^4*b^3 - 10*a^2*b^5 + b^7)*cos(d*x + c))*sin(d*x + c))/((a^11 - 4*a^9*b
^2 + 6*a^7*b^4 - 4*a^5*b^6 + a^3*b^8)*d*cos(d*x + c)^3 + 3*(a^10*b - 4*a^8
*b^3 + 6*a^6*b^5 - 4*a^4*b^7 + a^2*b^9)*d*cos(d*x + c)^2 + 3*(a^9*b^2 - 4*
a^7*b^4 + 6*a^5*b^6 - 4*a^3*b^8 + a*b^10)*d*cos(d*x + c) + (a^8*b^3 - 4*a^
6*b^5 + 6*a^4*b^7 - 4*a^2*b^9 + b^11)*d), -1/6*(3*(4*a^2*b^4 + b^6 + (4*a^
5*b + a^3*b^3)*cos(d*x + c)^3 + 3*(4*a^4*b^2 + a^2*b^4)*cos(d*x + c)^2 + 3
*(4*a^3*b^3 + a*b^5)*cos(d*x + c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^
2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c))) - (2*a^5*b^2 + 11*a^3*
b^4 - 13*a*b^6 + (6*a^7 + 4*a^5*b^2 - 11*a^3*b^4 + a*b^6)*cos(d*x + c)^2 +
 3*(2*a^6*b + 7*a^4*b^3 - 10*a^2*b^5 + b^7)*cos(d*x + c))*sin(d*x + c))/((
a^11 - 4*a^9*b^2 + 6*a^7*b^4 - 4*a^5*b^6 + a^3*b^8)*d*cos(d*x + c)^3 + 3*(
a^10*b - 4*a^8*b^3 + 6*a^6*b^5 - 4*a^4*b^7 + a^2*b^9)*d*cos(d*x + c)^2 + 3
*(a^9*b^2 - 4*a^7*b^4 + 6*a^5*b^6 - 4*a^3*b^8 + a*b^10)*d*cos(d*x + c) + (
a^8*b^3 - 4*a^6*b^5 + 6*a^4*b^7 - 4*a^2*b^9 + b^11)*d)]
```

3.518.6 Sympy [F]

$$\int \frac{\sec^2(c+dx)}{(a+b\sec(c+dx))^4}\,dx = \int \frac{\sec^2(c+dx)}{(a+b\sec(c+dx))^4}\,dx$$

input
```
integrate(sec(d*x+c)**2/(a+b*sec(d*x+c))**4,x)
```

output
```
Integral(sec(c + d*x)**2/(a + b*sec(c + d*x))**4, x)
```

3.518.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\sec^2(c+dx)}{(a+b\sec(c+dx))^4}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(sec(d*x+c)^2/(a+b*sec(d*x+c))^4,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

3.518.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 431 vs. $2(177) = 354$.

Time = 0.36 (sec) , antiderivative size = 431, normalized size of antiderivative = 2.24

$$\int \frac{\sec^2(c+dx)}{(a+b\sec(c+dx))^4}\,dx = -\frac{\frac{3\,(4\,a^2b+b^3)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{-a^2+b^2}}\right)\right)}{(a^6-3\,a^4b^2+3\,a^2b^4-b^6)\sqrt{-a^2+b^2}} + \frac{6\,a^5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-6\,a^4b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+}{}}{}$$

input
```
integrate(sec(d*x+c)^2/(a+b*sec(d*x+c))^4,x, algorithm="giac")
```

output
```
-1/3*(3*(4*a^2*b + b^3)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b)
+ arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^
2)))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*sqrt(-a^2 + b^2)) + (6*a^5*tan(1
/2*d*x + 1/2*c)^5 - 6*a^4*b*tan(1/2*d*x + 1/2*c)^5 + 12*a^3*b^2*tan(1/2*d*
x + 1/2*c)^5 - 27*a^2*b^3*tan(1/2*d*x + 1/2*c)^5 + 12*a*b^4*tan(1/2*d*x +
1/2*c)^5 + 3*b^5*tan(1/2*d*x + 1/2*c)^5 - 12*a^5*tan(1/2*d*x + 1/2*c)^3 -
16*a^3*b^2*tan(1/2*d*x + 1/2*c)^3 + 28*a*b^4*tan(1/2*d*x + 1/2*c)^3 + 6*a^
5*tan(1/2*d*x + 1/2*c) + 6*a^4*b*tan(1/2*d*x + 1/2*c) + 12*a^3*b^2*tan(1/2
*d*x + 1/2*c) + 27*a^2*b^3*tan(1/2*d*x + 1/2*c) + 12*a*b^4*tan(1/2*d*x + 1
/2*c) - 3*b^5*tan(1/2*d*x + 1/2*c))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*(
a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 - a - b)^3))/d
```

3.518.9 Mupad [B] (verification not implemented)

Time = 17.91 (sec) , antiderivative size = 382, normalized size of antiderivative = 1.99

$$\int \frac{\sec^2(c+dx)}{(a+b\sec(c+dx))^4}\,dx$$

$$= \frac{\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5\left(2a^3+2a^2b+6ab^2+b^3\right)}{(a+b)^3(a-b)} - \frac{4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3\left(3a^3+7ab^2\right)}{3(a+b)^2(a^2-2ab+b^2)} + \frac{\tan\left(\frac{c}{2}+\frac{d}{2}\right.}{(a+b)}}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\left(-3a^3-3a^2b+3ab^2+3b^3\right)-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4\left(-3a^3+3a^2b+3ab^2-3b^3\right)+3ab^2\right.} - \frac{b\,\mathrm{atanh}\left(\frac{b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)(4a^2+b^2)(2a-2b)(a^3-3a^2b+3ab^2-b^3)}{2\sqrt{a+b}\,(a-b)^{7/2}(4a^2b+b^3)}\right)(4a^2+b^2)}{d(a+b)^{7/2}(a-b)^{7/2}}$$

input
```
int(1/(cos(c + d*x)^2*(a + b/cos(c + d*x))^4),x)
```

output
```
((tan(c/2 + (d*x)/2)^5*(6*a*b^2 + 2*a^2*b + 2*a^3 + b^3))/((a + b)^3*(a -
b)) - (4*tan(c/2 + (d*x)/2)^3*(7*a*b^2 + 3*a^3))/(3*(a + b)^2*(a^2 - 2*a*b
 + b^2)) + (tan(c/2 + (d*x)/2)*(6*a*b^2 - 2*a^2*b + 2*a^3 - b^3))/((a + b)
*(3*a*b^2 - 3*a^2*b + a^3 - b^3)))/(d*(tan(c/2 + (d*x)/2)^2*(3*a*b^2 - 3*a
^2*b - 3*a^3 + 3*b^3) - tan(c/2 + (d*x)/2)^4*(3*a*b^2 + 3*a^2*b - 3*a^3 -
3*b^3) + 3*a*b^2 + 3*a^2*b + a^3 + b^3 - tan(c/2 + (d*x)/2)^6*(3*a*b^2 - 3
*a^2*b + a^3 - b^3))) - (b*atanh((b*tan(c/2 + (d*x)/2)*(4*a^2 + b^2)*(2*a
- 2*b)*(3*a*b^2 - 3*a^2*b + a^3 - b^3))/(2*(a + b)^(1/2)*(a - b)^(7/2)*(4*
a^2*b + b^3)))*(4*a^2 + b^2))/(d*(a + b)^(7/2)*(a - b)^(7/2))
```

3.519 $\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^4}\,dx$

3.519.1 Optimal result . . . 3878
3.519.2 Mathematica [A] (verified) . . . 3879
3.519.3 Rubi [A] (verified) . . . 3879
3.519.4 Maple [A] (verified) . . . 3883
3.519.5 Fricas [B] (verification not implemented) . . . 3883
3.519.6 Sympy [F] . . . 3884
3.519.7 Maxima [F(-2)] . . . 3885
3.519.8 Giac [B] (verification not implemented) . . . 3885
3.519.9 Mupad [B] (verification not implemented) . . . 3886

3.519.1 Optimal result

Integrand size = 19, antiderivative size = 184

$$\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^4}\,dx = \frac{a(2a^2+3b^2)\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{7/2}(a+b)^{7/2}d} - \frac{b\tan(c+dx)}{3\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} - \frac{5ab\tan(c+dx)}{6\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))^2} - \frac{b(11a^2+4b^2)\tan(c+dx)}{6\left(a^2-b^2\right)^3 d(a+b\sec(c+dx))}$$

output
```
a*(2*a^2+3*b^2)*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/(a-b)^
(7/2)/(a+b)^(7/2)/d-1/3*b*tan(d*x+c)/(a^2-b^2)/d/(a+b*sec(d*x+c))^3-5/6*a*
b*tan(d*x+c)/(a^2-b^2)^2/d/(a+b*sec(d*x+c))^2-1/6*b*(11*a^2+4*b^2)*tan(d*x
+c)/(a^2-b^2)^3/d/(a+b*sec(d*x+c))
```

3.519.2 Mathematica [A] (verified)

Time = 1.21 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.89

$$\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^4}\,dx = -\frac{\frac{12a(2a^2+3b^2)\text{arctanh}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} + \frac{b(18a^4+17a^2b^2+10b^4+6ab(9a^2+b^2)\cos(c+dx)+(18a^4-5a^2b^2+2b^4)\cos(2(c+dx)))\,\text{si}}{(b+a\cos(c+dx))^3}}{12(a-b)^3(a+b)^3d}$$

input
```
Integrate[Sec[c + d*x]/(a + b*Sec[c + d*x])^4,x]
```

output
```
-1/12*((12*a*(2*a^2 + 3*b^2)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2
- b^2]])/Sqrt[a^2 - b^2] + (b*(18*a^4 + 17*a^2*b^2 + 10*b^4 + 6*a*b*(9*a^2
 + b^2)*Cos[c + d*x] + (18*a^4 - 5*a^2*b^2 + 2*b^4)*Cos[2*(c + d*x)])*Sin[
c + d*x])/(b + a*Cos[c + d*x])^3)/((a - b)^3*(a + b)^3*d)
```

3.519.3 Rubi [A] (verified)

Time = 1.00 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.23, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.737$, Rules used = {3042, 4320, 25, 3042, 4491, 25, 3042, 4491, 27, 3042, 4318, 3042, 3138, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^4}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^4}dx$$

$$\Big\downarrow \; 4320$$

$$-\frac{\int -\frac{\sec(c+dx)(3a-2b\sec(c+dx))}{(a+b\sec(c+dx))^3}dx}{3\left(a^2-b^2\right)} - \frac{b\tan(c+dx)}{3d\left(a^2-b^2\right)(a+b\sec(c+dx))^3}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{\sec(c+dx)(3a-2b\sec(c+dx))}{(a+b\sec(c+dx))^3}dx}{3\left(a^2-b^2\right)} - \frac{b\tan(c+dx)}{3d\left(a^2-b^2\right)(a+b\sec(c+dx))^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(3a-2b\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^3}dx}{3\left(a^2-b^2\right)}-\frac{b\tan(c+dx)}{3d\left(a^2-b^2\right)(a+b\sec(c+dx))^3}$$

$$\Big\downarrow \text{4491}$$

$$\frac{-\frac{\int -\frac{\sec(c+dx)\left(2\left(3a^2+2b^2\right)-5ab\sec(c+dx)\right)}{(a+b\sec(c+dx))^2}dx}{2(a^2-b^2)}-\frac{5ab\tan(c+dx)}{2d(a^2-b^2)(a+b\sec(c+dx))^2}}{3\left(a^2-b^2\right)}-\frac{b\tan(c+dx)}{3d\left(a^2-b^2\right)(a+b\sec(c+dx))^3}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{\sec(c+dx)\left(2\left(3a^2+2b^2\right)-5ab\sec(c+dx)\right)}{(a+b\sec(c+dx))^2}dx}{2(a^2-b^2)}-\frac{5ab\tan(c+dx)}{2d(a^2-b^2)(a+b\sec(c+dx))^2}}{3\left(a^2-b^2\right)}-\frac{b\tan(c+dx)}{3d\left(a^2-b^2\right)(a+b\sec(c+dx))^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(2\left(3a^2+2b^2\right)-5ab\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx}{2(a^2-b^2)}-\frac{5ab\tan(c+dx)}{2d(a^2-b^2)(a+b\sec(c+dx))^2}}{3\left(a^2-b^2\right)}-\frac{b\tan(c+dx)}{3d\left(a^2-b^2\right)(a+b\sec(c+dx))^3}$$

$$\Big\downarrow \text{4491}$$

$$\frac{\frac{-\frac{\int -\frac{3a\left(2a^2+3b^2\right)\sec(c+dx)}{a+b\sec(c+dx)}dx}{a^2-b^2}-\frac{b\left(11a^2+4b^2\right)\tan(c+dx)}{d\left(a^2-b^2\right)(a+b\sec(c+dx))}}{2(a^2-b^2)}-\frac{5ab\tan(c+dx)}{2d(a^2-b^2)(a+b\sec(c+dx))^2}}{3\left(a^2-b^2\right)}-\frac{b\tan(c+dx)}{3d\left(a^2-b^2\right)(a+b\sec(c+dx))^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\frac{3a\left(2a^2+3b^2\right)\int \frac{\sec(c+dx)}{a+b\sec(c+dx)}dx}{a^2-b^2}-\frac{b\left(11a^2+4b^2\right)\tan(c+dx)}{d\left(a^2-b^2\right)(a+b\sec(c+dx))}}{2(a^2-b^2)}-\frac{5ab\tan(c+dx)}{2d(a^2-b^2)(a+b\sec(c+dx))^2}}{3\left(a^2-b^2\right)}-\frac{b\tan(c+dx)}{3d\left(a^2-b^2\right)(a+b\sec(c+dx))^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\frac{3a\left(2a^2+3b^2\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2-b^2}-\frac{b\left(11a^2+4b^2\right)\tan(c+dx)}{d\left(a^2-b^2\right)(a+b\sec(c+dx))}}{2(a^2-b^2)}-\frac{5ab\tan(c+dx)}{2d(a^2-b^2)(a+b\sec(c+dx))^2}}{3\left(a^2-b^2\right)}-\frac{b\tan(c+dx)}{3d\left(a^2-b^2\right)(a+b\sec(c+dx))^3}$$

3.519. $\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^4}\,dx$

$$\Big\downarrow \text{4318}$$

$$\frac{\frac{\frac{3a\left(2a^2+3b^2\right)\int \frac{1}{\frac{a\cos(c+dx)}{b}+1}dx}{b\left(a^2-b^2\right)}-\frac{b\left(11a^2+4b^2\right)\tan(c+dx)}{d\left(a^2-b^2\right)(a+b\sec(c+dx))}}{2(a^2-b^2)}-\frac{5ab\tan(c+dx)}{2d(a^2-b^2)(a+b\sec(c+dx))^2}}{3\left(a^2-b^2\right)}-\frac{b\tan(c+dx)}{3d\left(a^2-b^2\right)(a+b\sec(c+dx))^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\frac{3a\left(2a^2+3b^2\right)\int \frac{1}{\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{b}+1}dx}{b\left(a^2-b^2\right)}-\frac{b\left(11a^2+4b^2\right)\tan(c+dx)}{d\left(a^2-b^2\right)(a+b\sec(c+dx))}}{2(a^2-b^2)}-\frac{5ab\tan(c+dx)}{2d(a^2-b^2)(a+b\sec(c+dx))^2}}{3\left(a^2-b^2\right)}-\frac{b\tan(c+dx)}{3d\left(a^2-b^2\right)(a+b\sec(c+dx))^3}$$

$$\Big\downarrow \text{3138}$$

$$\frac{\frac{\frac{6a\left(2a^2+3b^2\right)\int \frac{1}{\left(1-\frac{a}{b}\right)\tan^2\left(\frac{1}{2}(c+dx)\right)+\frac{a+b}{b}}d\tan\left(\frac{1}{2}(c+dx)\right)}{bd\left(a^2-b^2\right)}-\frac{b\left(11a^2+4b^2\right)\tan(c+dx)}{d\left(a^2-b^2\right)(a+b\sec(c+dx))}}{2(a^2-b^2)}-\frac{5ab\tan(c+dx)}{2d(a^2-b^2)(a+b\sec(c+dx))^2}}{3\left(a^2-b^2\right)}-\frac{b\tan(c+dx)}{3d\left(a^2-b^2\right)(a+b\sec(c+dx))^3}$$

$$\Big\downarrow \text{221}$$

$$\frac{\frac{\frac{6a\left(2a^2+3b^2\right)\text{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d\sqrt{a-b}\sqrt{a+b}\left(a^2-b^2\right)}-\frac{b\left(11a^2+4b^2\right)\tan(c+dx)}{d\left(a^2-b^2\right)(a+b\sec(c+dx))}}{2(a^2-b^2)}-\frac{5ab\tan(c+dx)}{2d(a^2-b^2)(a+b\sec(c+dx))^2}}{3\left(a^2-b^2\right)}-\frac{b\tan(c+dx)}{3d\left(a^2-b^2\right)(a+b\sec(c+dx))^3}$$

input
```
Int[Sec[c + d*x]/(a + b*Sec[c + d*x])^4,x]
```

output
```
-1/3*(b*Tan[c + d*x])/((a^2 - b^2)*d*(a + b*Sec[c + d*x])^3) + ((-5*a*b*Ta
n[c + d*x])/(2*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^2) + ((6*a*(2*a^2 + 3*b^
2)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(Sqrt[a - b]*Sqrt[
a + b]*(a^2 - b^2)*d) - (b*(11*a^2 + 4*b^2)*Tan[c + d*x])/((a^2 - b^2)*d*(
a + b*Sec[c + d*x])))/(2*(a^2 - b^2)))/(3*(a^2 - b^2))
```

3.519. $\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^4}\,dx$

3.519.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 4318
```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[1/b   Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4320
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[(-b)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(f*(m + 1)*
(a^2 - b^2))), x] + Simp[1/((m + 1)*(a^2 - b^2))   Int[Csc[e + f*x]*(a + b*
Csc[e + f*x])^(m + 1)*(a*(m + 1) - b*(m + 2)*Csc[e + f*x]), x], x] /; FreeQ
[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

rule 4491
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b - a*B))*Cot[e
 + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Simp[1
/((m + 1)*(a^2 - b^2))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp
[(a*A - b*B)*(m + 1) - (A*b - a*B)*(m + 2)*Csc[e + f*x], x], x], x] /; Free
Q[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m
, -1]
```

3.519. $\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^4}\,dx$

3.519.4 Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.54

method	result
derivativedivides	$\frac{-\frac{2\left(-\frac{\left(6a^2+3ab+2b^2\right)b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{2(a-b)\left(a^3+3a^2b+3ab^2+b^3\right)}+\frac{2\left(9a^2+b^2\right)b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3\left(a^2+2ab+b^2\right)\left(a^2-2ab+b^2\right)}-\frac{\left(6a^2-3ab+2b^2\right)b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2(a+b)\left(a^3-3a^2b+3ab^2-b^3\right)}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2b-a-b\right)^3}+\frac{a\left(2a^2+3b^2\right)\operatorname{arctanh}\left(\right.}{\left(a^6-3a^4b^2+3a^2b^4\right.}}{d}$
default	$\frac{-\frac{2\left(-\frac{\left(6a^2+3ab+2b^2\right)b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{2(a-b)\left(a^3+3a^2b+3ab^2+b^3\right)}+\frac{2\left(9a^2+b^2\right)b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3\left(a^2+2ab+b^2\right)\left(a^2-2ab+b^2\right)}-\frac{\left(6a^2-3ab+2b^2\right)b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2(a+b)\left(a^3-3a^2b+3ab^2-b^3\right)}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2b-a-b\right)^3}+\frac{a\left(2a^2+3b^2\right)\operatorname{arctanh}\left(\right.}{\left(a^6-3a^4b^2+3a^2b^4\right.}}{d}$
risch	$\frac{ib\left(27a^6b\,e^{5i(dx+c)}-18a^4b^3e^{5i(dx+c)}+6a^2b^5e^{5i(dx+c)}+18a^7e^{4i(dx+c)}+81a^5b^2e^{4i(dx+c)}-36a^3b^4e^{4i(dx+c)}+12ab^6e^{4i(dx}\right.}{}$

input
```
int(sec(d*x+c)/(a+b*sec(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-2*(-1/2*(6*a^2+3*a*b+2*b^2)*b/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/
2*d*x+1/2*c)^5+2/3*(9*a^2+b^2)*b/(a^2+2*a*b+b^2)/(a^2-2*a*b+b^2)*tan(1/2*d
*x+1/2*c)^3-1/2*(6*a^2-3*a*b+2*b^2)*b/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(
1/2*d*x+1/2*c))/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3+a*(2
*a^2+3*b^2)/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a-b)*(a+b))^(1/2)*arctanh((a-b
)*tan(1/2*d*x+1/2*c)/((a-b)*(a+b))^(1/2)))
```

3.519.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 424 vs. 2(169) = 338.

Time = 0.33 (sec) , antiderivative size = 905, normalized size of antiderivative = 4.92

$$\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^4}\,dx$$
$$= \left[-\frac{3\left(2\,a^3b^3+3\,ab^5+\left(2\,a^6+3\,a^4b^2\right)\cos\left(dx+c\right)^3+3\left(2\,a^5b+3\,a^3b^3\right)\cos\left(dx+c\right)^2+3\left(2\,a^4b^2+3\,a^2b^4\right)c\right.}{12\left(\left(a^{11}-4\,a^9b^2+6\,a^7b^4-4\,a^5b^6+a^3b^8\right)d\cos\left(\right.\right.}\right.$$

input
```
integrate(sec(d*x+c)/(a+b*sec(d*x+c))^4,x, algorithm="fricas")
```

output
```
[-1/12*(3*(2*a^3*b^3 + 3*a*b^5 + (2*a^6 + 3*a^4*b^2)*cos(d*x + c)^3 + 3*(2
*a^5*b + 3*a^3*b^3)*cos(d*x + c)^2 + 3*(2*a^4*b^2 + 3*a^2*b^4)*cos(d*x + c
))*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2
- 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*
cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) + 2*(11*a^4*b^3 - 7*a^2*b^5 -
4*b^7 + (18*a^6*b - 23*a^4*b^3 + 7*a^2*b^5 - 2*b^7)*cos(d*x + c)^2 + 3*(9*
a^5*b^2 - 8*a^3*b^4 - a*b^6)*cos(d*x + c))*sin(d*x + c))/((a^11 - 4*a^9*b^
2 + 6*a^7*b^4 - 4*a^5*b^6 + a^3*b^8)*d*cos(d*x + c)^3 + 3*(a^10*b - 4*a^8*
b^3 + 6*a^6*b^5 - 4*a^4*b^7 + a^2*b^9)*d*cos(d*x + c)^2 + 3*(a^9*b^2 - 4*a
^7*b^4 + 6*a^5*b^6 - 4*a^3*b^8 + a*b^10)*d*cos(d*x + c) + (a^8*b^3 - 4*a^6
*b^5 + 6*a^4*b^7 - 4*a^2*b^9 + b^11)*d), 1/6*(3*(2*a^3*b^3 + 3*a*b^5 + (2*
a^6 + 3*a^4*b^2)*cos(d*x + c)^3 + 3*(2*a^5*b + 3*a^3*b^3)*cos(d*x + c)^2 +
 3*(2*a^4*b^2 + 3*a^2*b^4)*cos(d*x + c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^
2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c))) - (11*a^4*b^3 -
7*a^2*b^5 - 4*b^7 + (18*a^6*b - 23*a^4*b^3 + 7*a^2*b^5 - 2*b^7)*cos(d*x +
c)^2 + 3*(9*a^5*b^2 - 8*a^3*b^4 - a*b^6)*cos(d*x + c))*sin(d*x + c))/((a^1
1 - 4*a^9*b^2 + 6*a^7*b^4 - 4*a^5*b^6 + a^3*b^8)*d*cos(d*x + c)^3 + 3*(a^1
0*b - 4*a^8*b^3 + 6*a^6*b^5 - 4*a^4*b^7 + a^2*b^9)*d*cos(d*x + c)^2 + 3*(a
^9*b^2 - 4*a^7*b^4 + 6*a^5*b^6 - 4*a^3*b^8 + a*b^10)*d*cos(d*x + c) + (a^8
*b^3 - 4*a^6*b^5 + 6*a^4*b^7 - 4*a^2*b^9 + b^11)*d)]
```

3.519.6 Sympy [F]

$$\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^4}\,dx = \int \frac{\sec{(c+dx)}}{(a+b\sec{(c+dx)})^4}\,dx$$

input
```
integrate(sec(d*x+c)/(a+b*sec(d*x+c))**4,x)
```

output
```
Integral(sec(c + d*x)/(a + b*sec(c + d*x))**4, x)
```

3.519.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^4}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(sec(d*x+c)/(a+b*sec(d*x+c))^4,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

3.519.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 403 vs. $2(169) = 338$.

Time = 0.36 (sec) , antiderivative size = 403, normalized size of antiderivative = 2.19

$$\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^4}\,dx = -\frac{\frac{3\,(2\,a^3+3\,ab^2)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(2\,a-2\,b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{-a^2+b^2}}\right)\right)}{(a^6-3\,a^4b^2+3\,a^2b^4-b^6)\sqrt{-a^2+b^2}} - \frac{18\,a^4b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-27\,a^3b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{}}{}$$

input
```
integrate(sec(d*x+c)/(a+b*sec(d*x+c))^4,x, algorithm="giac")
```

output
```
-1/3*(3*(2*a^3 + 3*a*b^2)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(2*a - 2*b)
 + arctan((a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^
2)))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*sqrt(-a^2 + b^2)) - (18*a^4*b*ta
n(1/2*d*x + 1/2*c)^5 - 27*a^3*b^2*tan(1/2*d*x + 1/2*c)^5 + 6*a^2*b^3*tan(1
/2*d*x + 1/2*c)^5 - 3*a*b^4*tan(1/2*d*x + 1/2*c)^5 + 6*b^5*tan(1/2*d*x + 1
/2*c)^5 - 36*a^4*b*tan(1/2*d*x + 1/2*c)^3 + 32*a^2*b^3*tan(1/2*d*x + 1/2*c
)^3 + 4*b^5*tan(1/2*d*x + 1/2*c)^3 + 18*a^4*b*tan(1/2*d*x + 1/2*c) + 27*a^
3*b^2*tan(1/2*d*x + 1/2*c) + 6*a^2*b^3*tan(1/2*d*x + 1/2*c) + 3*a*b^4*tan(
1/2*d*x + 1/2*c) + 6*b^5*tan(1/2*d*x + 1/2*c))/((a^6 - 3*a^4*b^2 + 3*a^2*b
^4 - b^6)*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 - a - b)^3)
)/d
```

3.519.9 Mupad [B] (verification not implemented)

Time = 17.86 (sec) , antiderivative size = 378, normalized size of antiderivative = 2.05

$$\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^4}\,dx$$
$$= \frac{a\,\mathrm{atanh}\left(\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,(2\,a^2+3\,b^2)\,(2\,a-2\,b)\,(a^3-3\,a^2\,b+3\,a\,b^2-b^3)}{2\,(2\,a^3+3\,a\,b^2)\,\sqrt{a+b}\,(a-b)^{7/2}}\right)\,(2\,a^2+3\,b^2)}{d\,(a+b)^{7/2}\,(a-b)^{7/2}}$$
$$-\frac{\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\,(6\,a^2\,b+3\,a\,b^2+2\,b^3)}{(a+b)^3\,(a-b)}-\frac{4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,(9\,a^2\,b+b^3)}{3\,(a+b)^2\,(a^2-2\,a\,b+b^2)}+\frac{\tan\left(\frac{c}{2}+\right.}{(a+b)}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,(-3\,a^3-3\,a^2\,b+3\,a\,b^2+3\,b^3)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,(-3\,a^3+3\,a^2\,b+3\,a\,b^2-3\,b^3)+3\,a\,b\right.}$$

input
```
int(1/(cos(c + d*x)*(a + b/cos(c + d*x))^4),x)
```

output
```
(a*atanh((a*tan(c/2 + (d*x)/2)*(2*a^2 + 3*b^2)*(2*a - 2*b)*(3*a*b^2 - 3*a^
2*b + a^3 - b^3))/(2*(3*a*b^2 + 2*a^3)*(a + b)^(1/2)*(a - b)^(7/2)))*(2*a^
2 + 3*b^2))/(d*(a + b)^(7/2)*(a - b)^(7/2)) - ((tan(c/2 + (d*x)/2)^5*(3*a*
b^2 + 6*a^2*b + 2*b^3))/((a + b)^3*(a - b)) - (4*tan(c/2 + (d*x)/2)^3*(9*a
^2*b + b^3))/(3*(a + b)^2*(a^2 - 2*a*b + b^2)) + (tan(c/2 + (d*x)/2)*(6*a^
2*b - 3*a*b^2 + 2*b^3))/((a + b)*(3*a*b^2 - 3*a^2*b + a^3 - b^3)))/(d*(tan
(c/2 + (d*x)/2)^2*(3*a*b^2 - 3*a^2*b - 3*a^3 + 3*b^3) - tan(c/2 + (d*x)/2)
^4*(3*a*b^2 + 3*a^2*b - 3*a^3 - 3*b^3) + 3*a*b^2 + 3*a^2*b + a^3 + b^3 - t
an(c/2 + (d*x)/2)^6*(3*a*b^2 - 3*a^2*b + a^3 - b^3)))
```

3.520 $\int \frac{1}{(a+b\sec(c+dx))^4}\,dx$

3.520.1 Optimal result . . . 3887
3.520.2 Mathematica [A] (verified) . . . 3888
3.520.3 Rubi [A] (verified) . . . 3888
3.520.4 Maple [A] (verified) . . . 3893
3.520.5 Fricas [B] (verification not implemented) . . . 3894
3.520.6 Sympy [F] . . . 3894
3.520.7 Maxima [F(-2)] . . . 3895
3.520.8 Giac [B] (verification not implemented) . . . 3895
3.520.9 Mupad [B] (verification not implemented) . . . 3896

3.520.1 Optimal result

Integrand size = 12, antiderivative size = 242

$$\begin{aligned}\int \frac{1}{(a+b\sec(c+dx))^4}\,dx &= \frac{x}{a^4}-\frac{b(8a^6-8a^4b^2+7a^2b^4-2b^6)\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^4(a-b)^{7/2}(a+b)^{7/2}d}\\ &+\frac{b^2\tan(c+dx)}{3a\left(a^2-b^2\right)d(a+b\sec(c+dx))^3}\\ &+\frac{b^2(8a^2-3b^2)\tan(c+dx)}{6a^2\left(a^2-b^2\right)^2d(a+b\sec(c+dx))^2}\\ &+\frac{b^2(26a^4-17a^2b^2+6b^4)\tan(c+dx)}{6a^3\left(a^2-b^2\right)^3d(a+b\sec(c+dx))}\end{aligned}$$

output
```
x/a^4-b*(8*a^6-8*a^4*b^2+7*a^2*b^4-2*b^6)*arctanh((a-b)^(1/2)*tan(1/2*d*x+
1/2*c)/(a+b)^(1/2))/a^4/(a-b)^(7/2)/(a+b)^(7/2)/d+1/3*b^2*tan(d*x+c)/a/(a^
2-b^2)/d/(a+b*sec(d*x+c))^3+1/6*b^2*(8*a^2-3*b^2)*tan(d*x+c)/a^2/(a^2-b^2)
^2/d/(a+b*sec(d*x+c))^2+1/6*b^2*(26*a^4-17*a^2*b^2+6*b^4)*tan(d*x+c)/a^3/(
a^2-b^2)^3/d/(a+b*sec(d*x+c))
```

3.520.2 Mathematica [A] (verified)

Time = 1.37 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.11

$$\int \frac{1}{(a+b\sec(c+dx))^4}\,dx = \frac{(b+a\cos(c+dx))\sec^4(c+dx)\left(6(c+dx)(b+a\cos(c+dx))^3-\frac{6b(-8a^6+8a^4b^2-7a^2b^4+2b^6)\operatorname{arctanh}\left(\frac{(-a+b)\tan}{\sqrt{a^2}}\right.}{(a^2-b^2)^{7/2}}\right.}{6a^4d(a}$$

input
```
Integrate[(a + b*Sec[c + d*x])^(-4),x]
```

output
```
((b + a*Cos[c + d*x])*Sec[c + d*x]^4*(6*(c + d*x)*(b + a*Cos[c + d*x])^3 -
 (6*b*(-8*a^6 + 8*a^4*b^2 - 7*a^2*b^4 + 2*b^6)*ArcTanh[((-a + b)*Tan[(c +
d*x)/2])/Sqrt[a^2 - b^2]]*(b + a*Cos[c + d*x])^3)/(a^2 - b^2)^(7/2) + (2*a
*b^4*Sin[c + d*x])/((a - b)*(a + b)) - (a*b^3*(12*a^2 - 7*b^2)*(b + a*Cos[
c + d*x])*Sin[c + d*x])/((a - b)^2*(a + b)^2) + (a*b^2*(36*a^4 - 32*a^2*b^
2 + 11*b^4)*(b + a*Cos[c + d*x])^2*Sin[c + d*x])/((a - b)^3*(a + b)^3)))/(
6*a^4*d*(a + b*Sec[c + d*x])^4)
```

3.520.3 Rubi [A] (verified)

Time = 1.47 (sec) , antiderivative size = 306, normalized size of antiderivative = 1.26, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 1.333$, Rules used = {3042, 4272, 25, 3042, 4548, 25, 3042, 4548, 27, 3042, 4407, 3042, 4318, 3042, 3138, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+b\sec(c+dx))^4}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^4}dx$$

$$\Big\downarrow \; 4272$$

$$\frac{b^2\tan(c+dx)}{3ad\left(a^2-b^2\right)(a+b\sec(c+dx))^3}-\frac{\int -\frac{2b^2\sec^2(c+dx)-3ab\sec(c+dx)+3\left(a^2-b^2\right)}{(a+b\sec(c+dx))^3}dx}{3a\left(a^2-b^2\right)}$$

$\Big\downarrow$ 25

$$\frac{\int \frac{2b^2\sec^2(c+dx)-3ab\sec(c+dx)+3\left(a^2-b^2\right)}{(a+b\sec(c+dx))^3}dx}{3a\left(a^2-b^2\right)}+\frac{b^2\tan(c+dx)}{3ad\left(a^2-b^2\right)(a+b\sec(c+dx))^3}$$

$\Big\downarrow$ 3042

$$\frac{\int \frac{2b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2-3ab\csc\left(c+dx+\frac{\pi}{2}\right)+3\left(a^2-b^2\right)}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^3}dx}{3a\left(a^2-b^2\right)}+\frac{b^2\tan(c+dx)}{3ad\left(a^2-b^2\right)(a+b\sec(c+dx))^3}$$

$\Big\downarrow$ 4548

$$\frac{\frac{b^2\left(8a^2-3b^2\right)\tan(c+dx)}{2ad(a^2-b^2)(a+b\sec(c+dx))^2}-\frac{\int -\frac{6\left(a^2-b^2\right)^2+b^2\left(8a^2-3b^2\right)\sec^2(c+dx)-2ab\left(6a^2-b^2\right)\sec(c+dx)}{(a+b\sec(c+dx))^2}dx}{2a(a^2-b^2)}}{3a\left(a^2-b^2\right)}+\frac{b^2\tan(c+dx)}{3ad\left(a^2-b^2\right)(a+b\sec(c+dx))^3}$$

$\Big\downarrow$ 25

$$\frac{\frac{\int \frac{6\left(a^2-b^2\right)^2+b^2\left(8a^2-3b^2\right)\sec^2(c+dx)-2ab\left(6a^2-b^2\right)\sec(c+dx)}{(a+b\sec(c+dx))^2}dx}{2a(a^2-b^2)}+\frac{b^2\left(8a^2-3b^2\right)\tan(c+dx)}{2ad(a^2-b^2)(a+b\sec(c+dx))^2}}{3a\left(a^2-b^2\right)}+\frac{b^2\tan(c+dx)}{3ad\left(a^2-b^2\right)(a+b\sec(c+dx))^3}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\int \frac{6\left(a^2-b^2\right)^2+b^2\left(8a^2-3b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2-2ab\left(6a^2-b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx}{2a(a^2-b^2)}+\frac{b^2\left(8a^2-3b^2\right)\tan(c+dx)}{2ad(a^2-b^2)(a+b\sec(c+dx))^2}}{3a\left(a^2-b^2\right)}+\frac{b^2\tan(c+dx)}{3ad\left(a^2-b^2\right)(a+b\sec(c+dx))^3}$$

$\Big\downarrow$ 4548

$$\frac{\frac{\frac{b^2\left(26a^4-17a^2b^2+6b^4\right)\tan(c+dx)}{ad\left(a^2-b^2\right)(a+b\sec(c+dx))}-\frac{\int -\frac{3\left(2\left(a^2-b^2\right)^3-ab\left(6a^4-2b^2a^2+b^4\right)\sec(c+dx)\right)}{a+b\sec(c+dx)}dx}{a\left(a^2-b^2\right)}}{2a(a^2-b^2)}+\frac{b^2\left(8a^2-3b^2\right)\tan(c+dx)}{2ad(a^2-b^2)(a+b\sec(c+dx))^2}}{3a\left(a^2-b^2\right)}+\frac{b^2\tan(c+dx)}{3ad\left(a^2-b^2\right)(a+b\sec(c+dx))^3}$$

$\Big\downarrow$ 27

3.520. $\int \frac{1}{(a+b\sec(c+dx))^4}\,dx$

$$\frac{\frac{\frac{3\int \frac{2\left(a^2-b^2\right)^3-ab\left(6a^4-2b^2a^2+b^4\right)\sec(c+dx)}{a+b\sec(c+dx)}dx}{a\left(a^2-b^2\right)}+\frac{b^2\left(26a^4-17a^2b^2+6b^4\right)\tan(c+dx)}{ad\left(a^2-b^2\right)(a+b\sec(c+dx))}}{2a(a^2-b^2)}+\frac{b^2\left(8a^2-3b^2\right)\tan(c+dx)}{2ad(a^2-b^2)(a+b\sec(c+dx))^2}}{3a\left(a^2-b^2\right)}+\frac{b^2\tan(c+dx)}{3ad\left(a^2-b^2\right)(a+b\sec(c+dx))^3}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\frac{3\int \frac{2\left(a^2-b^2\right)^3-ab\left(6a^4-2b^2a^2+b^4\right)\csc\left(c+dx+\frac{\pi}{2}\right)}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a\left(a^2-b^2\right)}+\frac{b^2\left(26a^4-17a^2b^2+6b^4\right)\tan(c+dx)}{ad\left(a^2-b^2\right)(a+b\sec(c+dx))}}{2a(a^2-b^2)}+\frac{b^2\left(8a^2-3b^2\right)\tan(c+dx)}{2ad(a^2-b^2)(a+b\sec(c+dx))^2}}{3a\left(a^2-b^2\right)}+\frac{b^2\tan(c+dx)}{3ad\left(a^2-b^2\right)(a+b\sec(c+dx))^3}$$

$\Big\downarrow$ 4407

$$\frac{\frac{\frac{3\left(\frac{2x\left(a^2-b^2\right)^3}{a}-\frac{b\left(8a^6-8a^4b^2+7a^2b^4-2b^6\right)\int \frac{\sec(c+dx)}{a+b\sec(c+dx)}dx}{a}\right)}{a\left(a^2-b^2\right)}+\frac{b^2\left(26a^4-17a^2b^2+6b^4\right)\tan(c+dx)}{ad\left(a^2-b^2\right)(a+b\sec(c+dx))}}{2a(a^2-b^2)}+\frac{b^2\left(8a^2-3b^2\right)\tan(c+dx)}{2ad(a^2-b^2)(a+b\sec(c+dx))^2}}{3a\left(a^2-b^2\right)}+\frac{b^2\tan(c+dx)}{3ad\left(a^2-b^2\right)(a+b\sec(c+dx))^3}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\frac{3\left(\frac{2x\left(a^2-b^2\right)^3}{a}-\frac{b\left(8a^6-8a^4b^2+7a^2b^4-2b^6\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a}\right)}{a\left(a^2-b^2\right)}+\frac{b^2\left(26a^4-17a^2b^2+6b^4\right)\tan(c+dx)}{ad\left(a^2-b^2\right)(a+b\sec(c+dx))}}{2a(a^2-b^2)}+\frac{b^2\left(8a^2-3b^2\right)\tan(c+dx)}{2ad(a^2-b^2)(a+b\sec(c+dx))^2}}{3a\left(a^2-b^2\right)}+\frac{b^2\tan(c+dx)}{3ad\left(a^2-b^2\right)(a+b\sec(c+dx))^3}$$

$\Big\downarrow$ 4318

$$\frac{\frac{\frac{3\left(\frac{2x\left(a^2-b^2\right)^3}{a}-\frac{\left(8a^6-8a^4b^2+7a^2b^4-2b^6\right)\int \frac{1}{\frac{a\cos(c+dx)}{b}+1}dx}{a}\right)}{a\left(a^2-b^2\right)}+\frac{b^2\left(26a^4-17a^2b^2+6b^4\right)\tan(c+dx)}{ad\left(a^2-b^2\right)(a+b\sec(c+dx))}}{2a(a^2-b^2)}+\frac{b^2\left(8a^2-3b^2\right)\tan(c+dx)}{2ad(a^2-b^2)(a+b\sec(c+dx))^2}}{3a\left(a^2-b^2\right)}+\frac{b^2\tan(c+dx)}{3ad\left(a^2-b^2\right)(a+b\sec(c+dx))^3}$$

3.520. $\int \frac{1}{(a+b\sec(c+dx))^4}\,dx$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{\frac{3\left(\frac{2x\left(a^2-b^2\right)^3}{a}-\frac{\left(8a^6-8a^4b^2+7a^2b^4-2b^6\right)\int \frac{1}{\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{b}+1}dx}{a}\right)}{a\left(a^2-b^2\right)}+\frac{b^2\left(26a^4-17a^2b^2+6b^4\right)\tan(c+dx)}{ad\left(a^2-b^2\right)(a+b\sec(c+dx))}}{2a(a^2-b^2)}+\frac{b^2\left(8a^2-3b^2\right)\tan(c+dx)}{2ad(a^2-b^2)(a+b\sec(c+dx))^2}}{3a\left(a^2-b^2\right)}+\frac{b^2\tan(c+dx)}{3ad\left(a^2-b^2\right)(a+b\sec(c+dx))^3}$$

$$\Big\downarrow \; 3138$$

$$\frac{\frac{\frac{3\left(\frac{2x\left(a^2-b^2\right)^3}{a}-\frac{2\left(8a^6-8a^4b^2+7a^2b^4-2b^6\right)\int \frac{1}{\left(1-\frac{a}{b}\right)\tan^2\left(\frac{1}{2}(c+dx)\right)+\frac{a+b}{b}}d\tan\left(\frac{1}{2}(c+dx)\right)}{ad}\right)}{a\left(a^2-b^2\right)}+\frac{b^2\left(26a^4-17a^2b^2+6b^4\right)\tan(c+dx)}{ad\left(a^2-b^2\right)(a+b\sec(c+dx))}}{2a(a^2-b^2)}+\frac{b^2\left(8a^2-3b^2\right)\tan}{2ad(a^2-b^2)(a+b\sec}}{3a\left(a^2-b^2\right)}+\frac{b^2\tan(c+dx)}{3ad\left(a^2-b^2\right)(a+b\sec(c+dx))^3}$$

$$\Big\downarrow \; 221$$

$$\frac{b^2\tan(c+dx)}{3ad\left(a^2-b^2\right)(a+b\sec(c+dx))^3}+\frac{\frac{b^2\left(8a^2-3b^2\right)\tan(c+dx)}{2ad(a^2-b^2)(a+b\sec(c+dx))^2}+\frac{\frac{b^2\left(26a^4-17a^2b^2+6b^4\right)\tan(c+dx)}{ad\left(a^2-b^2\right)(a+b\sec(c+dx))}+\frac{3\left(\frac{2x\left(a^2-b^2\right)^3}{a}-\frac{2b\left(8a^6-8a^4b^2+7a^2b^4-2b^6\right)\text{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{ad\sqrt{a-b}\sqrt{a+b}}\right)}{a\left(a^2-b^2\right)}}{2a(a^2-b^2)}}{3a\left(a^2-b^2\right)}$$

input
```
Int[(a + b*Sec[c + d*x])^(-4),x]
```

output
```
(b^2*Tan[c + d*x])/(3*a*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^3) + ((b^2*(8*a
^2 - 3*b^2)*Tan[c + d*x])/(2*a*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^2) + ((3
*((2*(a^2 - b^2)^3*x)/a - (2*b*(8*a^6 - 8*a^4*b^2 + 7*a^2*b^4 - 2*b^6)*Arc
Tanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a*Sqrt[a - b]*Sqrt[a +
b]*d)))/(a*(a^2 - b^2)) + (b^2*(26*a^4 - 17*a^2*b^2 + 6*b^4)*Tan[c + d*x])
/(a*(a^2 - b^2)*d*(a + b*Sec[c + d*x])))/(2*a*(a^2 - b^2)))/(3*a*(a^2 - b^
2))
```

3.520. $\int \frac{1}{(a+b\sec(c+dx))^4}\,dx$

3.520.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 4272
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[b^2*Cot[
c + d*x]*((a + b*Csc[c + d*x])^(n + 1)/(a*d*(n + 1)*(a^2 - b^2))), x] + Sim
p[1/(a*(n + 1)*(a^2 - b^2))   Int[(a + b*Csc[c + d*x])^(n + 1)*Simp[(a^2 -
b^2)*(n + 1) - a*b*(n + 1)*Csc[c + d*x] + b^2*(n + 2)*Csc[c + d*x]^2, x], x
], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && LtQ[n, -1] && Integ
erQ[2*n]
```

rule 4318
```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[1/b   Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4407
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Simp[(b*c - a*d)/a   Int[Csc[e + f*
x]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
- a*d, 0]
```

3.520. $\int \frac{1}{(a+b\sec(c+dx))^4}\,dx$

rule 4548
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(A*b^2 -
a*b*B + a^2*C)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(a*f*(m + 1)*(a^2
 - b^2))), x] + Simp[1/(a*(m + 1)*(a^2 - b^2))   Int[(a + b*Csc[e + f*x])^(
m + 1)*Simp[A*(a^2 - b^2)*(m + 1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x
] + (A*b^2 - a*b*B + a^2*C)*(m + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

3.520.4 Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 365, normalized size of antiderivative = 1.51

method	result
derivativedivides	$\frac{\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^4}+\frac{2b\left(\frac{-\frac{\left(12a^4+4a^3b-6a^2b^2-a\,b^3+2b^4\right)ab\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{2(a-b)\left(a^3+3a^2b+3a\,b^2+b^3\right)}+\frac{2\left(18a^4-11a^2b^2+3b^4\right)ab\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3\left(a^2+2ab+b^2\right)\left(a^2-2ab+b^2\right)}-\left(12a^4\cdots\right.}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2b-a-b\right)^3}\cdots\right)}{a^4\cdots}}{d}$
default	$\frac{\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^4}+\frac{2b\left(\frac{-\frac{\left(12a^4+4a^3b-6a^2b^2-a\,b^3+2b^4\right)ab\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{2(a-b)\left(a^3+3a^2b+3a\,b^2+b^3\right)}+\frac{2\left(18a^4-11a^2b^2+3b^4\right)ab\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3\left(a^2+2ab+b^2\right)\left(a^2-2ab+b^2\right)}-\left(12a^4\cdots\right.}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2b-a-b\right)^3}\cdots\right)}{a^4\cdots}}{d}$
risch	$\frac{x}{a^4}-\frac{ib^2\left(48a^6b\,e^{5i(dx+c)}-51a^4b^3e^{5i(dx+c)}+18a^2b^5e^{5i(dx+c)}+36a^7e^{4i(dx+c)}+132a^5b^2e^{4i(dx+c)}-147a^3b^4e^{4i(dx+c)}+\cdots\right.}{\cdots}$

input
```
int(1/(a+b*sec(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
1/d*(2/a^4*arctan(tan(1/2*d*x+1/2*c))+2*b/a^4*((-1/2*(12*a^4+4*a^3*b-6*a^2
*b^2-a*b^3+2*b^4)*a*b/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5
+2/3*(18*a^4-11*a^2*b^2+3*b^4)*a*b/(a^2+2*a*b+b^2)/(a^2-2*a*b+b^2)*tan(1/2
*d*x+1/2*c)^3-1/2*(12*a^4-4*a^3*b-6*a^2*b^2+a*b^3+2*b^4)*a*b/(a+b)/(a^3-3*
a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c))/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x
+1/2*c)^2*b-a-b)^3-1/2*(8*a^6-8*a^4*b^2+7*a^2*b^4-2*b^6)/(a^6-3*a^4*b^2+3*
a^2*b^4-b^6)/((a-b)*(a+b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a-b)*(
a+b))^(1/2))))
```

3.520.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 699 vs. $2(227) = 454$.

Time = 0.37 (sec) , antiderivative size = 1456, normalized size of antiderivative = 6.02

$$\int \frac{1}{(a+b\sec(c+dx))^4}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b*sec(d*x+c))^4,x, algorithm="fricas")
```

output
```
[1/12*(12*(a^11 - 4*a^9*b^2 + 6*a^7*b^4 - 4*a^5*b^6 + a^3*b^8)*d*x*cos(d*x
 + c)^3 + 36*(a^10*b - 4*a^8*b^3 + 6*a^6*b^5 - 4*a^4*b^7 + a^2*b^9)*d*x*co
s(d*x + c)^2 + 36*(a^9*b^2 - 4*a^7*b^4 + 6*a^5*b^6 - 4*a^3*b^8 + a*b^10)*d
*x*cos(d*x + c) + 12*(a^8*b^3 - 4*a^6*b^5 + 6*a^4*b^7 - 4*a^2*b^9 + b^11)*
d*x + 3*(8*a^6*b^4 - 8*a^4*b^6 + 7*a^2*b^8 - 2*b^10 + (8*a^9*b - 8*a^7*b^3
 + 7*a^5*b^5 - 2*a^3*b^7)*cos(d*x + c)^3 + 3*(8*a^8*b^2 - 8*a^6*b^4 + 7*a^
4*b^6 - 2*a^2*b^8)*cos(d*x + c)^2 + 3*(8*a^7*b^3 - 8*a^5*b^5 + 7*a^3*b^7 -
 2*a*b^9)*cos(d*x + c))*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2
*b^2)*cos(d*x + c)^2 - 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c)
 + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) + 2*(26*a
^7*b^4 - 43*a^5*b^6 + 23*a^3*b^8 - 6*a*b^10 + (36*a^9*b^2 - 68*a^7*b^4 + 4
3*a^5*b^6 - 11*a^3*b^8)*cos(d*x + c)^2 + 15*(4*a^8*b^3 - 7*a^6*b^5 + 4*a^4
*b^7 - a^2*b^9)*cos(d*x + c))*sin(d*x + c))/((a^15 - 4*a^13*b^2 + 6*a^11*b
^4 - 4*a^9*b^6 + a^7*b^8)*d*cos(d*x + c)^3 + 3*(a^14*b - 4*a^12*b^3 + 6*a^
10*b^5 - 4*a^8*b^7 + a^6*b^9)*d*cos(d*x + c)^2 + 3*(a^13*b^2 - 4*a^11*b^4
+ 6*a^9*b^6 - 4*a^7*b^8 + a^5*b^10)*d*cos(d*x + c) + (a^12*b^3 - 4*a^10*b^
5 + 6*a^8*b^7 - 4*a^6*b^9 + a^4*b^11)*d), 1/6*(6*(a^11 - 4*a^9*b^2 + 6*a^7
*b^4 - 4*a^5*b^6 + a^3*b^8)*d*x*cos(d*x + c)^3 + 18*(a^10*b - 4*a^8*b^3 +
6*a^6*b^5 - 4*a^4*b^7 + a^2*b^9)*d*x*cos(d*x + c)^2 + 18*(a^9*b^2 - 4*a^7*
b^4 + 6*a^5*b^6 - 4*a^3*b^8 + a*b^10)*d*x*cos(d*x + c) + 6*(a^8*b^3 - 4...
```

3.520.6 Sympy [F]

$$\int \frac{1}{(a+b\sec(c+dx))^4}\,dx = \int \frac{1}{(a+b\sec(c+dx))^4}\,dx$$

input
```
integrate(1/(a+b*sec(d*x+c))**4,x)
```

output
```
Integral((a + b*sec(c + d*x))**(-4), x)
```

3.520.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(a+b\sec(c+dx))^4}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(a+b*sec(d*x+c))^4,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

3.520.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 532 vs. $2(227) = 454$.

Time = 0.31 (sec) , antiderivative size = 532, normalized size of antiderivative = 2.20

$$\int \frac{1}{(a+b\sec(c+dx))^4}\,dx = \frac{\frac{3\,(8\,a^6b-8\,a^4b^3+7\,a^2b^5-2\,b^7)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(2\,a-2\,b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{-a^2+b^2}}\right)\right)}{(a^{10}-3\,a^8b^2+3\,a^6b^4-a^4b^6)\sqrt{-a^2+b^2}} - \frac{36\,a^6b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-60\,a^5b^3\,\mathrm{ta}}{}}{}$$

input
```
integrate(1/(a+b*sec(d*x+c))^4,x, algorithm="giac")
```

output
```
1/3*(3*(8*a^6*b - 8*a^4*b^3 + 7*a^2*b^5 - 2*b^7)*(pi*floor(1/2*(d*x + c)/p
i + 1/2)*sgn(2*a - 2*b) + arctan((a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x +
 1/2*c))/sqrt(-a^2 + b^2)))/((a^10 - 3*a^8*b^2 + 3*a^6*b^4 - a^4*b^6)*sqrt
(-a^2 + b^2)) - (36*a^6*b^2*tan(1/2*d*x + 1/2*c)^5 - 60*a^5*b^3*tan(1/2*d*
x + 1/2*c)^5 - 6*a^4*b^4*tan(1/2*d*x + 1/2*c)^5 + 45*a^3*b^5*tan(1/2*d*x +
 1/2*c)^5 - 6*a^2*b^6*tan(1/2*d*x + 1/2*c)^5 - 15*a*b^7*tan(1/2*d*x + 1/2*
c)^5 + 6*b^8*tan(1/2*d*x + 1/2*c)^5 - 72*a^6*b^2*tan(1/2*d*x + 1/2*c)^3 +
116*a^4*b^4*tan(1/2*d*x + 1/2*c)^3 - 56*a^2*b^6*tan(1/2*d*x + 1/2*c)^3 + 1
2*b^8*tan(1/2*d*x + 1/2*c)^3 + 36*a^6*b^2*tan(1/2*d*x + 1/2*c) + 60*a^5*b^
3*tan(1/2*d*x + 1/2*c) - 6*a^4*b^4*tan(1/2*d*x + 1/2*c) - 45*a^3*b^5*tan(1
/2*d*x + 1/2*c) - 6*a^2*b^6*tan(1/2*d*x + 1/2*c) + 15*a*b^7*tan(1/2*d*x +
1/2*c) + 6*b^8*tan(1/2*d*x + 1/2*c))/((a^9 - 3*a^7*b^2 + 3*a^5*b^4 - a^3*b
^6)*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 - a - b)^3) + 3*(
d*x + c)/a^4)/d
```

3.520.9 Mupad [B] (verification not implemented)

Time = 27.00 (sec) , antiderivative size = 7234, normalized size of antiderivative = 29.89

$$\int \frac{1}{(a+b\sec(c+dx))^4}\,dx = \text{Too large to display}$$

input
```
int(1/(a + b/cos(c + d*x))^4,x)
```

output
```
(2*atan((((((8*(16*a^20*b - 4*a^21 + 4*a^8*b^13 - 2*a^9*b^12 - 26*a^10*b^1
1 + 14*a^11*b^10 + 70*a^12*b^9 - 30*a^13*b^8 - 110*a^14*b^7 + 30*a^15*b^6
+ 110*a^16*b^5 - 20*a^17*b^4 - 64*a^18*b^3 + 12*a^19*b^2))/(a^19*b + a^20
- a^9*b^11 - a^10*b^10 + 5*a^11*b^9 + 5*a^12*b^8 - 10*a^13*b^7 - 10*a^14*b
^6 + 10*a^15*b^5 + 10*a^16*b^4 - 5*a^17*b^3 - 5*a^18*b^2) - (tan(c/2 + (d*
x)/2)*(8*a^21*b - 8*a^8*b^14 + 8*a^9*b^13 + 48*a^10*b^12 - 48*a^11*b^11 -
120*a^12*b^10 + 120*a^13*b^9 + 160*a^14*b^8 - 160*a^15*b^7 - 120*a^16*b^6
+ 120*a^17*b^5 + 48*a^18*b^4 - 48*a^19*b^3 - 8*a^20*b^2)*8i)/(a^4*(a^16*b
+ a^17 - a^6*b^11 - a^7*b^10 + 5*a^8*b^9 + 5*a^9*b^8 - 10*a^10*b^7 - 10*a^
11*b^6 + 10*a^12*b^5 + 10*a^13*b^4 - 5*a^14*b^3 - 5*a^15*b^2)))*1i)/a^4 +
(8*tan(c/2 + (d*x)/2)*(4*a^14 - 8*a^13*b - 8*a*b^13 + 8*b^14 - 48*a^2*b^12
 + 48*a^3*b^11 + 117*a^4*b^10 - 120*a^5*b^9 - 164*a^6*b^8 + 160*a^7*b^7 +
156*a^8*b^6 - 120*a^9*b^5 - 92*a^10*b^4 + 48*a^11*b^3 + 44*a^12*b^2))/(a^1
6*b + a^17 - a^6*b^11 - a^7*b^10 + 5*a^8*b^9 + 5*a^9*b^8 - 10*a^10*b^7 - 1
0*a^11*b^6 + 10*a^12*b^5 + 10*a^13*b^4 - 5*a^14*b^3 - 5*a^15*b^2))/a^4 - (
(((8*(16*a^20*b - 4*a^21 + 4*a^8*b^13 - 2*a^9*b^12 - 26*a^10*b^11 + 14*a^1
1*b^10 + 70*a^12*b^9 - 30*a^13*b^8 - 110*a^14*b^7 + 30*a^15*b^6 + 110*a^16
*b^5 - 20*a^17*b^4 - 64*a^18*b^3 + 12*a^19*b^2))/(a^19*b + a^20 - a^9*b^11
 - a^10*b^10 + 5*a^11*b^9 + 5*a^12*b^8 - 10*a^13*b^7 - 10*a^14*b^6 + 10*a^
15*b^5 + 10*a^16*b^4 - 5*a^17*b^3 - 5*a^18*b^2) + (tan(c/2 + (d*x)/2)*(...
```

3.520. $\int \frac{1}{(a+b\sec(c+dx))^4}\,dx$

3.521 $\int \frac{\cos(c+dx)}{(a+b\sec(c+dx))^4}\,dx$

3.521.1 Optimal result . . . 3898
3.521.2 Mathematica [A] (verified) . . . 3899
3.521.3 Rubi [A] (verified) . . . 3899
3.521.4 Maple [A] (verified) . . . 3905
3.521.5 Fricas [B] (verification not implemented) . . . 3906
3.521.6 Sympy [F] . . . 3907
3.521.7 Maxima [F(-2)] . . . 3907
3.521.8 Giac [A] (verification not implemented) . . . 3907
3.521.9 Mupad [B] (verification not implemented) . . . 3908

3.521.1 Optimal result

Integrand size = 19, antiderivative size = 299

$$\begin{aligned}\int \frac{\cos(c+dx)}{(a+b\sec(c+dx))^4}\,dx = &-\frac{4bx}{a^5}\\ &+\frac{b^2(20a^6-35a^4b^2+28a^2b^4-8b^6)\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^5(a-b)^{7/2}(a+b)^{7/2}d}\\ &+\frac{(6a^6-65a^4b^2+68a^2b^4-24b^6)\sin(c+dx)}{6a^4\left(a^2-b^2\right)^3d}\\ &+\frac{b^2\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\sec(c+dx))^3}\\ &+\frac{b^2(9a^2-4b^2)\sin(c+dx)}{6a^2\left(a^2-b^2\right)^2d(a+b\sec(c+dx))^2}\\ &+\frac{b^2(12a^4-11a^2b^2+4b^4)\sin(c+dx)}{2a^3\left(a^2-b^2\right)^3d(a+b\sec(c+dx))}\end{aligned}$$

output
```
-4*b*x/a^5+b^2*(20*a^6-35*a^4*b^2+28*a^2*b^4-8*b^6)*arctanh((a-b)^(1/2)*ta
n(1/2*d*x+1/2*c)/(a+b)^(1/2))/a^5/(a-b)^(7/2)/(a+b)^(7/2)/d+1/6*(6*a^6-65*
a^4*b^2+68*a^2*b^4-24*b^6)*sin(d*x+c)/a^4/(a^2-b^2)^3/d+1/3*b^2*sin(d*x+c)
/a/(a^2-b^2)/d/(a+b*sec(d*x+c))^3+1/6*b^2*(9*a^2-4*b^2)*sin(d*x+c)/a^2/(a^
2-b^2)^2/d/(a+b*sec(d*x+c))^2+1/2*b^2*(12*a^4-11*a^2*b^2+4*b^4)*sin(d*x+c)
/a^3/(a^2-b^2)^3/d/(a+b*sec(d*x+c))
```

3.521.2 Mathematica [A] (verified)

Time = 1.69 (sec) , antiderivative size = 293, normalized size of antiderivative = 0.98

$$\int \frac{\cos(c+dx)}{(a+b\sec(c+dx))^4}\,dx$$
$$= \frac{(b+a\cos(c+dx))\sec^4(c+dx)\left(-24b(c+dx)(b+a\cos(c+dx))^3+\frac{6b^2(-20a^6+35a^4b^2-28a^2b^4+8b^6)\operatorname{arctanh}\left(\right.}{(a^2-b^2)^7}\right.}{}$$

input
```
Integrate[Cos[c + d*x]/(a + b*Sec[c + d*x])^4,x]
```

output
```
((b + a*Cos[c + d*x])*Sec[c + d*x]^4*(-24*b*(c + d*x)*(b + a*Cos[c + d*x])
^3 + (6*b^2*(-20*a^6 + 35*a^4*b^2 - 28*a^2*b^4 + 8*b^6)*ArcTanh[((-a + b)*
Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]]*(b + a*Cos[c + d*x])^3)/(a^2 - b^2)^(7/
2) + (2*a*b^5*Sin[c + d*x])/((-a + b)*(a + b)) + (5*a*b^4*(3*a^2 - 2*b^2)*
(b + a*Cos[c + d*x])*Sin[c + d*x])/((a - b)^2*(a + b)^2) - (a*b^3*(60*a^4
- 71*a^2*b^2 + 26*b^4)*(b + a*Cos[c + d*x])^2*Sin[c + d*x])/((a - b)^3*(a
+ b)^3) + 6*a*(b + a*Cos[c + d*x])^3*Sin[c + d*x]))/(6*a^5*d*(a + b*Sec[c
+ d*x])^4)
```

3.521.3 Rubi [A] (verified)

Time = 2.24 (sec) , antiderivative size = 355, normalized size of antiderivative = 1.19, number of steps used = 20, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {3042, 4334, 25, 3042, 4588, 25, 3042, 4588, 25, 3042, 4592, 27, 3042, 4407, 3042, 4318, 3042, 3138, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(c+dx)}{(a+b\sec(c+dx))^4}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^4}dx$$
$$\Big\downarrow \; 4334$$

$$\frac{b^2\sin(c+dx)}{3ad\left(a^2-b^2\right)(a+b\sec(c+dx))^3}-\frac{\int -\frac{\cos(c+dx)\left(3a^2-3b\sec(c+dx)a-4b^2+3b^2\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^3}dx}{3a\left(a^2-b^2\right)}$$

$\Big\downarrow$ 25

$$\frac{\int \frac{\cos(c+dx)\left(3a^2-3b\sec(c+dx)a-4b^2+3b^2\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^3}dx}{3a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)}{3ad\left(a^2-b^2\right)(a+b\sec(c+dx))^3}$$

$\Big\downarrow$ 3042

$$\frac{\int \frac{3a^2-3b\csc\left(c+dx+\frac{\pi}{2}\right)a-4b^2+3b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^3}dx}{3a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)}{3ad\left(a^2-b^2\right)(a+b\sec(c+dx))^3}$$

$\Big\downarrow$ 4588

$$\frac{\frac{b^2\left(9a^2-4b^2\right)\sin(c+dx)}{2ad(a^2-b^2)(a+b\sec(c+dx))^2}-\frac{\int -\frac{\cos(c+dx)\left(6a^4-23b^2a^2-2b\left(6a^2-b^2\right)\sec(c+dx)a+12b^4+2b^2\left(9a^2-4b^2\right)\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^2}dx}{2a(a^2-b^2)}}{3a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)}{3ad\left(a^2-b^2\right)(a+b\sec(c+dx))^3}$$

$\Big\downarrow$ 25

$$\frac{\frac{\int \frac{\cos(c+dx)\left(6a^4-23b^2a^2-2b\left(6a^2-b^2\right)\sec(c+dx)a+12b^4+2b^2\left(9a^2-4b^2\right)\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^2}dx}{2a(a^2-b^2)}+\frac{b^2\left(9a^2-4b^2\right)\sin(c+dx)}{2ad(a^2-b^2)(a+b\sec(c+dx))^2}}{3a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)}{3ad\left(a^2-b^2\right)(a+b\sec(c+dx))^3}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\int \frac{6a^4-23b^2a^2-2b\left(6a^2-b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)a+12b^4+2b^2\left(9a^2-4b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx}{2a(a^2-b^2)}+\frac{b^2\left(9a^2-4b^2\right)\sin(c+dx)}{2ad(a^2-b^2)(a+b\sec(c+dx))^2}}{3a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)}{3ad\left(a^2-b^2\right)(a+b\sec(c+dx))^3}$$

$\Big\downarrow$ 4588

$$\frac{\frac{\frac{3b^2\left(12a^4-11a^2b^2+4b^4\right)\sin(c+dx)}{ad\left(a^2-b^2\right)(a+b\sec(c+dx))}-\frac{\int -\frac{\cos(c+dx)\left(6a^6-65b^2a^4+68b^4a^2-b\left(18a^4-7b^2a^2+4b^4\right)\sec(c+dx)a-24b^6+3b^2\left(12a^4-11b^2a^2+4b^4\right)\sec^2(c+dx)\right)}{a+b\sec(c+dx)}dx}{a\left(a^2-b^2\right)}}{2a(a^2-b^2)}+\frac{b^2\left(9a^2-4b^2\right)\sin(c+dx)}{2ad(a^2-b^2)(a+b\sec(c+dx))^2}}{3a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)}{3ad\left(a^2-b^2\right)(a+b\sec(c+dx))^3}$$

$\Big\downarrow$ 25

3.521. $\int \frac{\cos(c+dx)}{(a+b\sec(c+dx))^4}\,dx$

$$\frac{\frac{\frac{\int \frac{\cos(c+dx)\left(6a^6-65b^2a^4+68b^4a^2-b\left(18a^4-7b^2a^2+4b^4\right)\sec(c+dx)a-24b^6+3b^2\left(12a^4-11b^2a^2+4b^4\right)\sec^2(c+dx)\right)}{a+b\sec(c+dx)}dx}{a\left(a^2-b^2\right)}+\frac{3b^2\left(12a^4-11a^2b^2+4b^4\right)\sin(c+dx)}{ad\left(a^2-b^2\right)(a+b\sec(c+dx))}}{2a(a^2-b^2)}+\cdots}{3a\left(a^2-b^2\right)}$$

$$\frac{b^2\sin(c+dx)}{3ad\left(a^2-b^2\right)(a+b\sec(c+dx))^3}$$

$\downarrow$ 3042

$$\frac{\frac{\frac{\int \frac{6a^6-65b^2a^4+68b^4a^2-b\left(18a^4-7b^2a^2+4b^4\right)\csc\left(c+dx+\frac{\pi}{2}\right)a-24b^6+3b^2\left(12a^4-11b^2a^2+4b^4\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a\left(a^2-b^2\right)}+\frac{3b^2\left(12a^4-11a^2b^2+4b^4\right)\sin(c+dx)}{ad\left(a^2-b^2\right)(a+b\sec(c+dx))}}{2a(a^2-b^2)}+\frac{b\cdots}{2ad\cdots}}{3a\left(a^2-b^2\right)}$$

$$\frac{b^2\sin(c+dx)}{3ad\left(a^2-b^2\right)(a+b\sec(c+dx))^3}$$

$\downarrow$ 4592

$$\frac{\frac{\frac{\left(6a^6-65a^4b^2+68a^2b^4-24b^6\right)\sin(c+dx)}{ad}-\frac{3\int \frac{3\left(8b\left(a^2-b^2\right)^3-ab^2\left(12a^4-11b^2a^2+4b^4\right)\sec(c+dx)\right)}{a+b\sec(c+dx)}dx}{a}}{a\left(a^2-b^2\right)}+\frac{3b^2\left(12a^4-11a^2b^2+4b^4\right)\sin(c+dx)}{ad\left(a^2-b^2\right)(a+b\sec(c+dx))}}{2a(a^2-b^2)}+\frac{b^2(9a^2-4b^2\cdots}{2ad(a^2-b^2)(a\cdots}}{3a\left(a^2-b^2\right)}$$

$$\frac{b^2\sin(c+dx)}{3ad\left(a^2-b^2\right)(a+b\sec(c+dx))^3}$$

$\downarrow$ 27

$$\frac{\frac{\frac{\left(6a^6-65a^4b^2+68a^2b^4-24b^6\right)\sin(c+dx)}{ad}-\frac{3\int \frac{8b\left(a^2-b^2\right)^3-ab^2\left(12a^4-11b^2a^2+4b^4\right)\sec(c+dx)}{a+b\sec(c+dx)}dx}{a}}{a\left(a^2-b^2\right)}+\frac{3b^2\left(12a^4-11a^2b^2+4b^4\right)\sin(c+dx)}{ad\left(a^2-b^2\right)(a+b\sec(c+dx))}}{2a(a^2-b^2)}+\frac{b^2(9a^2-4b^2)\cdots}{2ad(a^2-b^2)(a+b\cdots}}{3a\left(a^2-b^2\right)}$$

$$\frac{b^2\sin(c+dx)}{3ad\left(a^2-b^2\right)(a+b\sec(c+dx))^3}$$

$\downarrow$ 3042

$$\frac{\frac{\frac{\left(6a^6-65a^4b^2+68a^2b^4-24b^6\right)\sin(c+dx)}{ad}-\frac{3\int \frac{8b\left(a^2-b^2\right)^3-ab^2\left(12a^4-11b^2a^2+4b^4\right)\csc\left(c+dx+\frac{\pi}{2}\right)}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a}}{a\left(a^2-b^2\right)}+\frac{3b^2\left(12a^4-11a^2b^2+4b^4\right)\sin(c+dx)}{ad\left(a^2-b^2\right)(a+b\sec(c+dx))}}{2a(a^2-b^2)}+\frac{b^2(9a^2-4\cdots}{2ad(a^2-b^2)(\cdots}}{3a\left(a^2-b^2\right)}$$

$$\frac{b^2\sin(c+dx)}{3ad\left(a^2-b^2\right)(a+b\sec(c+dx))^3}$$

3.521. $\int \frac{\cos(c+dx)}{(a+b\sec(c+dx))^4}\,dx$

$$\Big\downarrow \; 4407$$

$$\frac{\frac{\frac{\left(6a^6-65a^4b^2+68a^2b^4-24b^6\right)\sin(c+dx)}{ad}-\frac{3\left(\frac{8bx\left(a^2-b^2\right)^3}{a}-\frac{b^2\left(20a^6-35a^4b^2+28a^2b^4-8b^6\right)\int\frac{\sec(c+dx)}{a+b\sec(c+dx)}dx}{a}\right)}{a\left(a^2-b^2\right)}}{2a(a^2-b^2)}+\frac{3b^2\left(12a^4-11a^2b^2+4b^4\right)\sin(c+dx)}{ad\left(a^2-b^2\right)(a+b\sec(c+dx))}}{3a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)}{3ad\left(a^2-b^2\right)(a+b\sec(c+dx))^3}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{\frac{\left(6a^6-65a^4b^2+68a^2b^4-24b^6\right)\sin(c+dx)}{ad}-\frac{3\left(\frac{8bx\left(a^2-b^2\right)^3}{a}-\frac{b^2\left(20a^6-35a^4b^2+28a^2b^4-8b^6\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a}\right)}{a\left(a^2-b^2\right)}}{2a(a^2-b^2)}+\frac{3b^2\left(12a^4-11a^2b^2+4b^4\right)\sin(c+dx)}{ad\left(a^2-b^2\right)(a+b\sec(c+dx))}}{3a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)}{3ad\left(a^2-b^2\right)(a+b\sec(c+dx))^3}$$

$$\Big\downarrow \; 4318$$

$$\frac{\frac{\frac{\left(6a^6-65a^4b^2+68a^2b^4-24b^6\right)\sin(c+dx)}{ad}-\frac{3\left(\frac{8bx\left(a^2-b^2\right)^3}{a}-\frac{b\left(20a^6-35a^4b^2+28a^2b^4-8b^6\right)\int\frac{1}{\frac{a\cos(c+dx)}{b}+1}dx}{a}\right)}{a\left(a^2-b^2\right)}}{2a(a^2-b^2)}+\frac{3b^2\left(12a^4-11a^2b^2+4b^4\right)\sin(c+dx)}{ad\left(a^2-b^2\right)(a+b\sec(c+dx))}}{3a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)}{3ad\left(a^2-b^2\right)(a+b\sec(c+dx))^3}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{\frac{\left(6a^6-65a^4b^2+68a^2b^4-24b^6\right)\sin(c+dx)}{ad}-\frac{3\left(\frac{8bx\left(a^2-b^2\right)^3}{a}-\frac{b\left(20a^6-35a^4b^2+28a^2b^4-8b^6\right)\int\frac{1}{\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{b}+1}dx}{a}\right)}{a\left(a^2-b^2\right)}}{2a(a^2-b^2)}+\frac{3b^2\left(12a^4-11a^2b^2+4b^4\right)\sin(c+dx)}{ad\left(a^2-b^2\right)(a+b\sec(c+dx))}}{3a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)}{3ad\left(a^2-b^2\right)(a+b\sec(c+dx))^3}$$

$$\Big\downarrow \; 3138$$

3.521. $\int \frac{\cos(c+dx)}{(a+b\sec(c+dx))^4}\,dx$

$$\frac{\frac{\frac{\frac{\left(6a^6-65a^4b^2+68a^2b^4-24b^6\right)\sin(c+dx)}{ad}-\frac{3\left(\frac{8bx\left(a^2-b^2\right)^3}{a}-\frac{2b\left(20a^6-35a^4b^2+28a^2b^4-8b^6\right)\int\frac{1}{\left(1-\frac{a}{b}\right)\tan^2\left(\frac{1}{2}(c+dx)\right)+\frac{a+b}{b}}d\tan\left(\frac{1}{2}(c+dx)\right)}{ad}\right)}{a}}{a\left(a^2-b^2\right)}+\frac{3b^2\left(12a^4-\right.}{ad\left(a^2-\right.}}{2a(a^2-b^2)}}{3a\left(a^2-b^2\right)}$$

$$\frac{b^2\sin(c+dx)}{3ad\left(a^2-b^2\right)(a+b\sec(c+dx))^3}$$

$$\Big\downarrow \; 221$$

$$\frac{b^2\sin(c+dx)}{3ad\left(a^2-b^2\right)(a+b\sec(c+dx))^3}+$$

$$\frac{\frac{b^2\left(9a^2-4b^2\right)\sin(c+dx)}{2ad(a^2-b^2)(a+b\sec(c+dx))^2}+\frac{\frac{3b^2\left(12a^4-11a^2b^2+4b^4\right)\sin(c+dx)}{ad\left(a^2-b^2\right)(a+b\sec(c+dx))}+\frac{\frac{\left(6a^6-65a^4b^2+68a^2b^4-24b^6\right)\sin(c+dx)}{ad}-\frac{3\left(\frac{8bx\left(a^2-b^2\right)^3}{a}-\frac{2b^2\left(20a^6-35a^4b^2+28a\right.}{}\right.}{a\left(a^2-b^2\right)}}{2a(a^2-b^2)}}{3a\left(a^2-b^2\right)}$$

input
```
Int[Cos[c + d*x]/(a + b*Sec[c + d*x])^4,x]
```

output
```
(b^2*Sin[c + d*x])/(3*a*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^3) + ((b^2*(9*a
^2 - 4*b^2)*Sin[c + d*x])/(2*a*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^2) + ((3
*b^2*(12*a^4 - 11*a^2*b^2 + 4*b^4)*Sin[c + d*x])/(a*(a^2 - b^2)*d*(a + b*S
ec[c + d*x])) + ((-3*((8*b*(a^2 - b^2)^3*x)/a - (2*b^2*(20*a^6 - 35*a^4*b^
2 + 28*a^2*b^4 - 8*b^6)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]
])/(a*Sqrt[a - b]*Sqrt[a + b]*d)))/a + ((6*a^6 - 65*a^4*b^2 + 68*a^2*b^4 -
 24*b^6)*Sin[c + d*x])/(a*d))/(a*(a^2 - b^2)))/(2*a*(a^2 - b^2)))/(3*a*(a^
2 - b^2))
```

3.521.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 4318
```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[1/b   Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4334
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*
((d*Csc[e + f*x])^n/(a*f*(m + 1)*(a^2 - b^2))), x] + Simp[1/(a*(m + 1)*(a^2
 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a^2*(m + 1)
 - b^2*(m + n + 1) - a*b*(m + 1)*Csc[e + f*x] + b^2*(m + n + 2)*Csc[e + f*x
]^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0] && LtQ[m,
 -1] && IntegersQ[2*m, 2*n]
```

rule 4407
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Simp[(b*c - a*d)/a   Int[Csc[e + f*
x]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
- a*d, 0]
```

3.521. $\int \frac{\cos(c+dx)}{(a+b\sec(c+dx))^4}\,dx$

rule 4588
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc
[e + f*x])^(m + 1)*((d*Csc[e + f*x])^n/(a*f*(m + 1)*(a^2 - b^2))), x] + Sim
p[1/(a*(m + 1)*(a^2 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f
*x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n +
1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m
+ n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x
] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

rule 4592
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Simp[1/(a*d*n)   Int[(a + b*Csc[e + f*x])^m
*(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*
Csc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

3.521.4 Maple [A] (verified)

Time = 1.48 (sec) , antiderivative size = 399, normalized size of antiderivative = 1.33

method	result
derivativedivides	$-\frac{2\left(-\frac{a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}+4b\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}{a^5}-\frac{2b^2\left(\frac{-\frac{\left(20a^4+5a^3b-18a^2b^2-2a\,b^3+6b^4\right)ab\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{2(a-b)\left(a^3+3a^2b+3a\,b^2+b^3\right)}+\frac{2\left(30a^4-29a^2b^2\right.}{3\left(a^2+2ab+\right.}}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-\right.}\right.}{}$
default	$-\frac{2\left(-\frac{a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}+4b\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}{a^5}-\frac{2b^2\left(\frac{-\frac{\left(20a^4+5a^3b-18a^2b^2-2a\,b^3+6b^4\right)ab\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{2(a-b)\left(a^3+3a^2b+3a\,b^2+b^3\right)}+\frac{2\left(30a^4-29a^2b^2\right.}{3\left(a^2+2ab+\right.}}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-\right.}\right.}{}$
risch	Expression too large to display

input
```
int(cos(d*x+c)/(a+b*sec(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-2/a^5*(-a*tan(1/2*d*x+1/2*c)/(1+tan(1/2*d*x+1/2*c)^2)+4*b*arctan(tan
(1/2*d*x+1/2*c)))-2*b^2/a^5*((-1/2*(20*a^4+5*a^3*b-18*a^2*b^2-2*a*b^3+6*b^
4)*a*b/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5+2/3*(30*a^4-29
*a^2*b^2+9*b^4)*a*b/(a^2+2*a*b+b^2)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3-1
/2*(20*a^4-5*a^3*b-18*a^2*b^2+2*a*b^3+6*b^4)*a*b/(a+b)/(a^3-3*a^2*b+3*a*b^
2-b^3)*tan(1/2*d*x+1/2*c))/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-
a-b)^3-1/2*(20*a^6-35*a^4*b^2+28*a^2*b^4-8*b^6)/(a^6-3*a^4*b^2+3*a^2*b^4-b
^6)/((a-b)*(a+b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a-b)*(a+b))^(1/
2))))
```

3.521.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 773 vs. $2(282) = 564$.

Time = 0.39 (sec) , antiderivative size = 1603, normalized size of antiderivative = 5.36

$$\int \frac{\cos(c+dx)}{(a+b\sec(c+dx))^4}\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)/(a+b*sec(d*x+c))^4,x, algorithm="fricas")
```

output
```
[-1/12*(48*(a^11*b - 4*a^9*b^3 + 6*a^7*b^5 - 4*a^5*b^7 + a^3*b^9)*d*x*cos(
d*x + c)^3 + 144*(a^10*b^2 - 4*a^8*b^4 + 6*a^6*b^6 - 4*a^4*b^8 + a^2*b^10)
*d*x*cos(d*x + c)^2 + 144*(a^9*b^3 - 4*a^7*b^5 + 6*a^5*b^7 - 4*a^3*b^9 + a
*b^11)*d*x*cos(d*x + c) + 48*(a^8*b^4 - 4*a^6*b^6 + 6*a^4*b^8 - 4*a^2*b^10
 + b^12)*d*x - 3*(20*a^6*b^5 - 35*a^4*b^7 + 28*a^2*b^9 - 8*b^11 + (20*a^9*
b^2 - 35*a^7*b^4 + 28*a^5*b^6 - 8*a^3*b^8)*cos(d*x + c)^3 + 3*(20*a^8*b^3
- 35*a^6*b^5 + 28*a^4*b^7 - 8*a^2*b^9)*cos(d*x + c)^2 + 3*(20*a^7*b^4 - 35
*a^5*b^6 + 28*a^3*b^8 - 8*a*b^10)*cos(d*x + c))*sqrt(a^2 - b^2)*log((2*a*b
*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 + 2*sqrt(a^2 - b^2)*(b*cos(d*
x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*
x + c) + b^2)) - 2*(6*a^9*b^3 - 71*a^7*b^5 + 133*a^5*b^7 - 92*a^3*b^9 + 24
*a*b^11 + 6*(a^12 - 4*a^10*b^2 + 6*a^8*b^4 - 4*a^6*b^6 + a^4*b^8)*cos(d*x
+ c)^3 + (18*a^11*b - 132*a^9*b^3 + 239*a^7*b^5 - 169*a^5*b^7 + 44*a^3*b^9
)*cos(d*x + c)^2 + 3*(6*a^10*b^2 - 59*a^8*b^4 + 110*a^6*b^6 - 77*a^4*b^8 +
 20*a^2*b^10)*cos(d*x + c))*sin(d*x + c))/((a^16 - 4*a^14*b^2 + 6*a^12*b^4
 - 4*a^10*b^6 + a^8*b^8)*d*cos(d*x + c)^3 + 3*(a^15*b - 4*a^13*b^3 + 6*a^1
1*b^5 - 4*a^9*b^7 + a^7*b^9)*d*cos(d*x + c)^2 + 3*(a^14*b^2 - 4*a^12*b^4 +
 6*a^10*b^6 - 4*a^8*b^8 + a^6*b^10)*d*cos(d*x + c) + (a^13*b^3 - 4*a^11*b^
5 + 6*a^9*b^7 - 4*a^7*b^9 + a^5*b^11)*d), -1/6*(24*(a^11*b - 4*a^9*b^3 + 6
*a^7*b^5 - 4*a^5*b^7 + a^3*b^9)*d*x*cos(d*x + c)^3 + 72*(a^10*b^2 - 4*a...
```

3.521.6 Sympy [F]

$$\int \frac{\cos(c+dx)}{(a+b\sec(c+dx))^4}\,dx = \int \frac{\cos{(c+dx)}}{(a+b\sec{(c+dx)})^4}\,dx$$

input
```
integrate(cos(d*x+c)/(a+b*sec(d*x+c))**4,x)
```

output
```
Integral(cos(c + d*x)/(a + b*sec(c + d*x))**4, x)
```

3.521.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\cos(c+dx)}{(a+b\sec(c+dx))^4}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(cos(d*x+c)/(a+b*sec(d*x+c))^4,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

3.521.8 Giac [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 564, normalized size of antiderivative = 1.89

$$\int \frac{\cos(c+dx)}{(a+b\sec(c+dx))^4}\,dx = -\frac{\frac{3\,(20\,a^6b^2-35\,a^4b^4+28\,a^2b^6-8\,b^8)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(2\,a-2\,b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{-a^2+b^2}}\right)\right)}{(a^{11}-3\,a^9b^2+3\,a^7b^4-a^5b^6)\sqrt{-a^2+b^2}} - \frac{60\,a^6b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-1\cdots}{\cdots}}{\cdots}$$

input
```
integrate(cos(d*x+c)/(a+b*sec(d*x+c))^4,x, algorithm="giac")
```

output
```
-1/3*(3*(20*a^6*b^2 - 35*a^4*b^4 + 28*a^2*b^6 - 8*b^8)*(pi*floor(1/2*(d*x
+ c)/pi + 1/2)*sgn(2*a - 2*b) + arctan((a*tan(1/2*d*x + 1/2*c) - b*tan(1/2
*d*x + 1/2*c))/sqrt(-a^2 + b^2)))/((a^11 - 3*a^9*b^2 + 3*a^7*b^4 - a^5*b^6
)*sqrt(-a^2 + b^2)) - (60*a^6*b^3*tan(1/2*d*x + 1/2*c)^5 - 105*a^5*b^4*tan
(1/2*d*x + 1/2*c)^5 - 24*a^4*b^5*tan(1/2*d*x + 1/2*c)^5 + 117*a^3*b^6*tan(
1/2*d*x + 1/2*c)^5 - 24*a^2*b^7*tan(1/2*d*x + 1/2*c)^5 - 42*a*b^8*tan(1/2*
d*x + 1/2*c)^5 + 18*b^9*tan(1/2*d*x + 1/2*c)^5 - 120*a^6*b^3*tan(1/2*d*x +
 1/2*c)^3 + 236*a^4*b^5*tan(1/2*d*x + 1/2*c)^3 - 152*a^2*b^7*tan(1/2*d*x +
 1/2*c)^3 + 36*b^9*tan(1/2*d*x + 1/2*c)^3 + 60*a^6*b^3*tan(1/2*d*x + 1/2*c
) + 105*a^5*b^4*tan(1/2*d*x + 1/2*c) - 24*a^4*b^5*tan(1/2*d*x + 1/2*c) - 1
17*a^3*b^6*tan(1/2*d*x + 1/2*c) - 24*a^2*b^7*tan(1/2*d*x + 1/2*c) + 42*a*b
^8*tan(1/2*d*x + 1/2*c) + 18*b^9*tan(1/2*d*x + 1/2*c))/((a^10 - 3*a^8*b^2
+ 3*a^6*b^4 - a^4*b^6)*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^
2 - a - b)^3) + 12*(d*x + c)*b/a^5 - 6*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x
+ 1/2*c)^2 + 1)*a^4))/d
```

3.521.9 Mupad [B] (verification not implemented)

Time = 24.10 (sec) , antiderivative size = 7534, normalized size of antiderivative = 25.20

$$\int \frac{\cos(c+dx)}{(a+b\sec(c+dx))^4}\,dx = \text{Too large to display}$$

input
```
int(cos(c + d*x)/(a + b/cos(c + d*x))^4,x)
```

output
```
- ((tan(c/2 + (d*x)/2)^7*(4*a*b^6 + 2*a^6*b - 2*a^7 - 8*b^7 + 24*a^2*b^5 -
 11*a^3*b^4 - 26*a^4*b^3 + 6*a^5*b^2))/((a^4*b - a^5)*(a + b)^3) + (tan(c/
2 + (d*x)/2)^3*(12*a*b^7 - 18*a^8 - 72*b^8 + 236*a^2*b^6 - 47*a^3*b^5 - 27
3*a^4*b^4 + 60*a^5*b^3 + 72*a^6*b^2))/(3*(a + b)^2*(3*a^6*b - a^7 + a^4*b^
3 - 3*a^5*b^2)) - (tan(c/2 + (d*x)/2)*(4*a*b^6 - 2*a^6*b - 2*a^7 + 8*b^7 -
 24*a^2*b^5 - 11*a^3*b^4 + 26*a^4*b^3 + 6*a^5*b^2))/((a + b)*(3*a^6*b - a^
7 + a^4*b^3 - 3*a^5*b^2)) + (tan(c/2 + (d*x)/2)^5*(12*a*b^7 + 18*a^8 + 72*
b^8 - 236*a^2*b^6 - 47*a^3*b^5 + 273*a^4*b^4 + 60*a^5*b^3 - 72*a^6*b^2))/(
3*(a^4*b - a^5)*(a + b)^3*(a - b)))/(d*(3*a*b^2 + 3*a^2*b - tan(c/2 + (d*x
)/2)^4*(6*a^2*b - 6*b^3) + tan(c/2 + (d*x)/2)^2*(6*a*b^2 - 2*a^3 + 4*b^3)
+ tan(c/2 + (d*x)/2)^6*(2*a^3 - 6*a*b^2 + 4*b^3) + a^3 + b^3 - tan(c/2 + (
d*x)/2)^8*(3*a*b^2 - 3*a^2*b + a^3 - b^3))) - (8*b*atan(((4*b*((8*tan(c/2
+ (d*x)/2)*(128*b^16 - 128*a*b^15 - 768*a^2*b^14 + 768*a^3*b^13 + 1920*a^4
*b^12 - 1920*a^5*b^11 - 2600*a^6*b^10 + 2560*a^7*b^9 + 2025*a^8*b^8 - 1920
*a^9*b^7 - 824*a^10*b^6 + 768*a^11*b^5 + 80*a^12*b^4 - 128*a^13*b^3 + 64*a
^14*b^2))/(a^18*b + a^19 - a^8*b^11 - a^9*b^10 + 5*a^10*b^9 + 5*a^11*b^8 -
 10*a^12*b^7 - 10*a^13*b^6 + 10*a^14*b^5 + 10*a^15*b^4 - 5*a^16*b^3 - 5*a^
17*b^2) + (b*((16*(8*a^23*b - 8*a^10*b^14 + 4*a^11*b^13 + 52*a^12*b^12 - 2
5*a^13*b^11 - 143*a^14*b^10 + 63*a^15*b^9 + 217*a^16*b^8 - 87*a^17*b^7 - 1
93*a^18*b^6 + 73*a^19*b^5 + 95*a^20*b^4 - 36*a^21*b^3 - 20*a^22*b^2))/(...
```

3.521. $\int \frac{\cos(c+dx)}{(a+b\sec(c+dx))^4}\,dx$

3.522 $\int \frac{\cos^2(c+dx)}{(a+b\sec(c+dx))^4}\,dx$

3.522.1 Optimal result . . . 3910
3.522.2 Mathematica [A] (verified) . . . 3911
3.522.3 Rubi [A] (verified) . . . 3911
3.522.4 Maple [A] (verified) . . . 3918
3.522.5 Fricas [B] (verification not implemented) . . . 3919
3.522.6 Sympy [F] . . . 3920
3.522.7 Maxima [F(-2)] . . . 3921
3.522.8 Giac [A] (verification not implemented) . . . 3921
3.522.9 Mupad [B] (verification not implemented) . . . 3922

3.522.1 Optimal result

Integrand size = 21, antiderivative size = 387

$$\begin{aligned}\int \frac{\cos^2(c+dx)}{(a+b\sec(c+dx))^4}\,dx = {} & \frac{(a^2+20b^2)\,x}{2a^6} \\ & - \frac{b^3(40a^6-84a^4b^2+69a^2b^4-20b^6)\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^6(a-b)^{7/2}(a+b)^{7/2}d} \\ & - \frac{b(24a^6-146a^4b^2+167a^2b^4-60b^6)\sin(c+dx)}{6a^5\left(a^2-b^2\right)^3 d} \\ & + \frac{(a^6-23a^4b^2+27a^2b^4-10b^6)\cos(c+dx)\sin(c+dx)}{2a^4\left(a^2-b^2\right)^3 d} \\ & + \frac{b^2\cos(c+dx)\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} \\ & + \frac{5b^2(2a^2-b^2)\cos(c+dx)\sin(c+dx)}{6a^2\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))^2} \\ & + \frac{b^2(48a^4-53a^2b^2+20b^4)\cos(c+dx)\sin(c+dx)}{6a^3\left(a^2-b^2\right)^3 d(a+b\sec(c+dx))}\end{aligned}$$

output
```
1/2*(a^2+20*b^2)*x/a^6-b^3*(40*a^6-84*a^4*b^2+69*a^2*b^4-20*b^6)*arctanh((
a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/a^6/(a-b)^(7/2)/(a+b)^(7/2)/d-1
/6*b*(24*a^6-146*a^4*b^2+167*a^2*b^4-60*b^6)*sin(d*x+c)/a^5/(a^2-b^2)^3/d+
1/2*(a^6-23*a^4*b^2+27*a^2*b^4-10*b^6)*cos(d*x+c)*sin(d*x+c)/a^4/(a^2-b^2)
^3/d+1/3*b^2*cos(d*x+c)*sin(d*x+c)/a/(a^2-b^2)/d/(a+b*sec(d*x+c))^3+5/6*b^
2*(2*a^2-b^2)*cos(d*x+c)*sin(d*x+c)/a^2/(a^2-b^2)^2/d/(a+b*sec(d*x+c))^2+1
/6*b^2*(48*a^4-53*a^2*b^2+20*b^4)*cos(d*x+c)*sin(d*x+c)/a^3/(a^2-b^2)^3/d/
(a+b*sec(d*x+c))
```

3.522.2 Mathematica [A] (verified)

Time = 5.74 (sec) , antiderivative size = 263, normalized size of antiderivative = 0.68

$$\int \frac{\cos^2(c+dx)}{(a+b\sec(c+dx))^4}\,dx = \frac{6(a^2+20b^2)(c+dx) - \frac{12b^3(-40a^6+84a^4b^2-69a^2b^4+20b^6)\operatorname{arctanh}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{7/2}} - 48ab\sin(c+dx) + \frac{}{(a-b)(}}{12a^6d}$$

input
```
Integrate[Cos[c + d*x]^2/(a + b*Sec[c + d*x])^4,x]
```

output
```
(6*(a^2 + 20*b^2)*(c + d*x) - (12*b^3*(-40*a^6 + 84*a^4*b^2 - 69*a^2*b^4 +
 20*b^6)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)
^(7/2) - 48*a*b*Sin[c + d*x] + (4*a*b^6*Sin[c + d*x])/((a - b)*(a + b)*(b
+ a*Cos[c + d*x])^3) + (2*a*b^5*(-18*a^2 + 13*b^2)*Sin[c + d*x])/((a - b)^
2*(a + b)^2*(b + a*Cos[c + d*x])^2) + (2*a*b^4*(90*a^4 - 122*a^2*b^2 + 47*
b^4)*Sin[c + d*x])/((a - b)^3*(a + b)^3*(b + a*Cos[c + d*x])) + 3*a^2*Sin[
2*(c + d*x)])/(12*a^6*d)
```

3.522.3 Rubi [A] (verified)

Time = 3.00 (sec) , antiderivative size = 431, normalized size of antiderivative = 1.11, number of steps used = 23, number of rules used = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 1.048$, Rules used = {3042, 4334, 25, 3042, 4588, 25, 3042, 4588, 25, 3042, 4592, 27, 3042, 4592, 27, 3042, 4407, 3042, 4318, 3042, 3138, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^2(c+dx)}{(a+b\sec(c+dx))^4}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^2\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^4}dx$$

$$\Big\downarrow \; 4334$$

$$\frac{b^2\sin(c+dx)\cos(c+dx)}{3ad\left(a^2-b^2\right)(a+b\sec(c+dx))^3}-\frac{\int -\frac{\cos^2(c+dx)\left(3a^2-3b\sec(c+dx)a-5b^2+4b^2\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^3}dx}{3a\left(a^2-b^2\right)}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{\cos^2(c+dx)\left(3a^2-3b\sec(c+dx)a-5b^2+4b^2\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^3}dx}{3a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)\cos(c+dx)}{3ad\left(a^2-b^2\right)(a+b\sec(c+dx))^3}$$

$$\Big\downarrow \; 3042$$

$$\frac{\int \frac{3a^2-3b\csc\left(c+dx+\frac{\pi}{2}\right)a-5b^2+4b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^2\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^3}dx}{3a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)\cos(c+dx)}{3ad\left(a^2-b^2\right)(a+b\sec(c+dx))^3}$$

$$\Big\downarrow \; 4588$$

$$\frac{\frac{5b^2\left(2a^2-b^2\right)\sin(c+dx)\cos(c+dx)}{2ad(a^2-b^2)(a+b\sec(c+dx))^2}-\frac{\int -\frac{\cos^2(c+dx)\left(15b^2\left(2a^2-b^2\right)\sec^2(c+dx)-2ab\left(6a^2-b^2\right)\sec(c+dx)+2\left(3a^4-18b^2a^2+10b^4\right)\right)}{(a+b\sec(c+dx))^2}dx}{2a(a^2-b^2)}}{3a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)\cos(c+dx)}{3ad\left(a^2-b^2\right)(a+b\sec(c+dx))^3}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\int \frac{\cos^2(c+dx)\left(15b^2\left(2a^2-b^2\right)\sec^2(c+dx)-2ab\left(6a^2-b^2\right)\sec(c+dx)+2\left(3a^4-18b^2a^2+10b^4\right)\right)}{(a+b\sec(c+dx))^2}dx}{2a(a^2-b^2)}+\frac{5b^2\left(2a^2-b^2\right)\sin(c+dx)\cos(c+dx)}{2ad(a^2-b^2)(a+b\sec(c+dx))^2}}{3a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)\cos(c+dx)}{3ad\left(a^2-b^2\right)(a+b\sec(c+dx))^3}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{\int \frac{15b^2\left(2a^2-b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2-2ab\left(6a^2-b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)+2\left(3a^4-18b^2a^2+10b^4\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^2\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx}{2a(a^2-b^2)}+\frac{5b^2\left(2a^2-b^2\right)\sin(c+dx)\cos(c+dx)}{2ad(a^2-b^2)(a+b\sec(c+dx))^2}}{3a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)\cos(c+dx)}{3ad\left(a^2-b^2\right)(a+b\sec(c+dx))^3}$$

3.522. $\int \frac{\cos^2(c+dx)}{(a+b\sec(c+dx))^4}\,dx$

$$\Big\downarrow \; 4588$$

$$\frac{\frac{\frac{b^2\left(48a^4-53a^2b^2+20b^4\right)\sin(c+dx)\cos(c+dx)}{ad\left(a^2-b^2\right)(a+b\sec(c+dx))}-\int -\frac{\frac{\cos^2(c+dx)\left(2b^2\left(48a^4-53b^2a^2+20b^4\right)\sec^2(c+dx)-ab\left(18a^4-8b^2a^2+5b^4\right)\sec(c+dx)+6\left(a^6-23b^2a^4+27b^4a^2\right.}{a+b\sec(c+dx)}}{a\left(a^2-b^2\right)}}{2a(a^2-b^2)}}{3a\left(a^2-b^2\right)}$$

$$\frac{b^2\sin(c+dx)\cos(c+dx)}{3ad\left(a^2-b^2\right)(a+b\sec(c+dx))^3}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\frac{\int\frac{\cos^2(c+dx)\left(2b^2\left(48a^4-53b^2a^2+20b^4\right)\sec^2(c+dx)-ab\left(18a^4-8b^2a^2+5b^4\right)\sec(c+dx)+6\left(a^6-23b^2a^4+27b^4a^2-10b^6\right)\right)}{a+b\sec(c+dx)}dx}{a\left(a^2-b^2\right)}+\frac{b^2\left(48a^4-53a^2b^2+20b^4\right)\sin(c+dx)\,c}{ad\left(a^2-b^2\right)(a+b\sec(c+dx)}}{2a(a^2-b^2)}}{3a\left(a^2-b^2\right)}$$

$$\frac{b^2\sin(c+dx)\cos(c+dx)}{3ad\left(a^2-b^2\right)(a+b\sec(c+dx))^3}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{\frac{\int\frac{2b^2\left(48a^4-53b^2a^2+20b^4\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2-ab\left(18a^4-8b^2a^2+5b^4\right)\csc\left(c+dx+\frac{\pi}{2}\right)+6\left(a^6-23b^2a^4+27b^4a^2-10b^6\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^2\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a\left(a^2-b^2\right)}+\frac{b^2\left(48a^4-53a^2b^2+20b^4\right)\sin(c+dx)\cos(c}{ad\left(a^2-b^2\right)(a+b\sec(c+dx))}}{2a(a^2-b^2)}}{3a\left(a^2-b^2\right)}$$

$$\frac{b^2\sin(c+dx)\cos(c+dx)}{3ad\left(a^2-b^2\right)(a+b\sec(c+dx))^3}$$

$$\Big\downarrow \; 4592$$

$$\frac{\frac{\frac{\frac{3\left(a^6-23a^4b^2+27a^2b^4-10b^6\right)\sin(c+dx)\cos(c+dx)}{ad}-\int\frac{\frac{2\cos(c+dx)\left(-3b\left(a^6-23b^2a^4+27b^4a^2-10b^6\right)\sec^2(c+dx)-a\left(3a^6+27b^2a^4-25b^4a^2+10b^6\right)\sec(c+dx)+b\left(2\right.}{a+b\sec(c+dx)}}{2a}}{a\left(a^2-b^2\right)}}{2a(a^2-b^2)}}{3a\left(a^2-b^2\right)}$$

$$\frac{b^2\sin(c+dx)\cos(c+dx)}{3ad\left(a^2-b^2\right)(a+b\sec(c+dx))^3}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\frac{\frac{3\left(a^6-23a^4b^2+27a^2b^4-10b^6\right)\sin(c+dx)\cos(c+dx)}{ad}-\int\frac{\frac{\cos(c+dx)\left(-3b\left(a^6-23b^2a^4+27b^4a^2-10b^6\right)\sec^2(c+dx)-a\left(3a^6+27b^2a^4-25b^4a^2+10b^6\right)\sec(c+dx)+b\left(24\right.}{a+b\sec(c+dx)}}{a}}{a\left(a^2-b^2\right)}}{2a(a^2-b^2)}}{3a\left(a^2-b^2\right)}$$

$$\frac{b^2\sin(c+dx)\cos(c+dx)}{3ad\left(a^2-b^2\right)(a+b\sec(c+dx))^3}$$

3.522. $\int \frac{\cos^2(c+dx)}{(a+b\sec(c+dx))^4}\,dx$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{\frac{3\left(a^6-23a^4b^2+27a^2b^4-10b^6\right)\sin(c+dx)\cos(c+dx)}{ad}-\frac{\int \frac{-3b\left(a^6-23b^2a^4+27b^4a^2-10b^6\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2-a\left(3a^6+27b^2a^4-25b^4a^2+10b^6\right)\csc\left(c+dx+\frac{\pi}{2}\right)+b\left(24a^6\right.}{\csc\left(c+dx+\frac{\pi}{2}\right)\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)}}{a}}{a\left(a^2-b^2\right)}}{2a(a^2-b^2)}}{3a\left(a^2-b^2\right)}$$

$$\frac{b^2\sin(c+dx)\cos(c+dx)}{3ad\left(a^2-b^2\right)(a+b\sec(c+dx))^3}$$

$$\Big\downarrow \; 4592$$

$$\frac{\frac{\frac{3\left(a^6-23a^4b^2+27a^2b^4-10b^6\right)\sin(c+dx)\cos(c+dx)}{ad}-\frac{\frac{b\left(24a^6-146a^4b^2+167a^2b^4-60b^6\right)\sin(c+dx)}{ad}-\frac{\int \frac{3\left(\left(a^2+20b^2\right)\left(a^2-b^2\right)^3+ab\left(a^6-23b^2a^4+27b^4a^2-10b^6\right)\right.}{a+b\sec(c+dx)}}{a}}{a}}{a\left(a^2-b^2\right)}}{2a(a^2-b^2)}}{3a\left(a^2-b^2\right)}$$

$$\frac{b^2\sin(c+dx)\cos(c+dx)}{3ad\left(a^2-b^2\right)(a+b\sec(c+dx))^3}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\frac{3\left(a^6-23a^4b^2+27a^2b^4-10b^6\right)\sin(c+dx)\cos(c+dx)}{ad}-\frac{\frac{b\left(24a^6-146a^4b^2+167a^2b^4-60b^6\right)\sin(c+dx)}{ad}-\frac{3\int \frac{\left(a^2+20b^2\right)\left(a^2-b^2\right)^3+ab\left(a^6-23b^2a^4+27b^4a^2-10b^6\right)\text{se}}{a+b\sec(c+dx)}}{a}}{a}}{a\left(a^2-b^2\right)}}{2a(a^2-b^2)}}{3a\left(a^2-b^2\right)}$$

$$\frac{b^2\sin(c+dx)\cos(c+dx)}{3ad\left(a^2-b^2\right)(a+b\sec(c+dx))^3}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{\frac{3\left(a^6-23a^4b^2+27a^2b^4-10b^6\right)\sin(c+dx)\cos(c+dx)}{ad}-\frac{\frac{b\left(24a^6-146a^4b^2+167a^2b^4-60b^6\right)\sin(c+dx)}{ad}-\frac{3\int \frac{\left(a^2+20b^2\right)\left(a^2-b^2\right)^3+ab\left(a^6-23b^2a^4+27b^4a^2-10b^6\right)\text{c}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}{a}}{a}}{a\left(a^2-b^2\right)}}{2a(a^2-b^2)}}{3a\left(a^2-b^2\right)}$$

$$\frac{b^2\sin(c+dx)\cos(c+dx)}{3ad\left(a^2-b^2\right)(a+b\sec(c+dx))^3}$$

$$\Big\downarrow \; 4407$$

3.522. $\int \frac{\cos^2(c+dx)}{(a+b\sec(c+dx))^4}\,dx$

$$\frac{\frac{\frac{3\left(a^6-23a^4b^2+27a^2b^4-10b^6\right)\sin(c+dx)\cos(c+dx)}{ad}-\frac{b\left(24a^6-146a^4b^2+167a^2b^4-60b^6\right)\sin(c+dx)}{ad}-\frac{3\left(\frac{x\left(a^2-b^2\right)^3\left(a^2+20b^2\right)}{a}-\frac{b^3\left(40a^6-84a^4b^2+69a^2b^4-20b^{\cdots}\right.}{a}\right.}{a}}{a\left(a^2-b^2\right)}}{2a(a^2-b^2)}}{3a\left(a^2-b^2\right)}$$

$$\frac{b^2\sin(c+dx)\cos(c+dx)}{3ad\left(a^2-b^2\right)(a+b\sec(c+dx))^3}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{\frac{3\left(a^6-23a^4b^2+27a^2b^4-10b^6\right)\sin(c+dx)\cos(c+dx)}{ad}-\frac{b\left(24a^6-146a^4b^2+167a^2b^4-60b^6\right)\sin(c+dx)}{ad}-\frac{3\left(\frac{x\left(a^2-b^2\right)^3\left(a^2+20b^2\right)}{a}-\frac{b^3\left(40a^6-84a^4b^2+69a^2b^4-20b^{\cdots}\right.}{a}\right.}{a}}{a\left(a^2-b^2\right)}}{2a(a^2-b^2)}}{3a\left(a^2-b^2\right)}$$

$$\frac{b^2\sin(c+dx)\cos(c+dx)}{3ad\left(a^2-b^2\right)(a+b\sec(c+dx))^3}$$

$$\Big\downarrow \; 4318$$

$$\frac{\frac{\frac{3\left(a^6-23a^4b^2+27a^2b^4-10b^6\right)\sin(c+dx)\cos(c+dx)}{ad}-\frac{b\left(24a^6-146a^4b^2+167a^2b^4-60b^6\right)\sin(c+dx)}{ad}-\frac{3\left(\frac{x\left(a^2-b^2\right)^3\left(a^2+20b^2\right)}{a}-\frac{b^2\left(40a^6-84a^4b^2+69a^2b^4-20b^{\cdots}\right.}{a}\right.}{a}}{a\left(a^2-b^2\right)}}{2a(a^2-b^2)}}{3a\left(a^2-b^2\right)}$$

$$\frac{b^2\sin(c+dx)\cos(c+dx)}{3ad\left(a^2-b^2\right)(a+b\sec(c+dx))^3}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{\frac{3\left(a^6-23a^4b^2+27a^2b^4-10b^6\right)\sin(c+dx)\cos(c+dx)}{ad}-\frac{b\left(24a^6-146a^4b^2+167a^2b^4-60b^6\right)\sin(c+dx)}{ad}-\frac{3\left(\frac{x\left(a^2-b^2\right)^3\left(a^2+20b^2\right)}{a}-\frac{b^2\left(40a^6-84a^4b^2+69a^2b^4-20b^{\cdots}\right.}{a}\right.}{a}}{a\left(a^2-b^2\right)}}{2a(a^2-b^2)}}{3a\left(a^2-b^2\right)}$$

$$\frac{b^2\sin(c+dx)\cos(c+dx)}{3ad\left(a^2-b^2\right)(a+b\sec(c+dx))^3}$$

$$\Big\downarrow \; 3138$$

3.522. $\int \frac{\cos^2(c+dx)}{(a+b\sec(c+dx))^4}\,dx$

$$\frac{\frac{\frac{3\left(a^6-23a^4b^2+27a^2b^4-10b^6\right)\sin(c+dx)\cos(c+dx)}{ad}-\frac{b\left(24a^6-146a^4b^2+167a^2b^4-60b^6\right)\sin(c+dx)}{ad}-\frac{3\left(\frac{x\left(a^2-b^2\right)^3\left(a^2+20b^2\right)}{a}-\frac{2b^2\left(40a^6-84a^4b^2+69a^2b^4-20\cdots\right.}{a}\right.}{a}}{a\left(a^2-b^2\right)}}{2a(a^2-b^2)}}{3a\left(a^2-b^2\right.}$$

$$\frac{b^2\sin(c+dx)\cos(c+dx)}{3ad\left(a^2-b^2\right)(a+b\sec(c+dx))^3}$$

$$\Big\downarrow \; 221$$

$$\frac{b^2\sin(c+dx)\cos(c+dx)}{3ad\left(a^2-b^2\right)(a+b\sec(c+dx))^3}+$$

$$\frac{\frac{5b^2\left(2a^2-b^2\right)\sin(c+dx)\cos(c+dx)}{2ad(a^2-b^2)(a+b\sec(c+dx))^2}+\frac{\frac{b^2\left(48a^4-53a^2b^2+20b^4\right)\sin(c+dx)\cos(c+dx)}{ad\left(a^2-b^2\right)(a+b\sec(c+dx))}+\frac{\frac{3\left(a^6-23a^4b^2+27a^2b^4-10b^6\right)\sin(c+dx)\cos(c+dx)}{ad}-\frac{b\left(24a^6-146a^4b^2\cdots\right.}{\cdots}}{\cdots}}{\cdots}}{3a\left(a^2-b^2\right)}$$

input
```
Int[Cos[c + d*x]^2/(a + b*Sec[c + d*x])^4,x]
```

output
```
(b^2*Cos[c + d*x]*Sin[c + d*x])/(3*a*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^3)
 + ((5*b^2*(2*a^2 - b^2)*Cos[c + d*x]*Sin[c + d*x])/(2*a*(a^2 - b^2)*d*(a
+ b*Sec[c + d*x])^2) + ((b^2*(48*a^4 - 53*a^2*b^2 + 20*b^4)*Cos[c + d*x]*S
in[c + d*x])/(a*(a^2 - b^2)*d*(a + b*Sec[c + d*x])) + ((3*(a^6 - 23*a^4*b^
2 + 27*a^2*b^4 - 10*b^6)*Cos[c + d*x]*Sin[c + d*x])/(a*d) - ((-3*(((a^2 -
b^2)^3*(a^2 + 20*b^2)*x)/a - (2*b^3*(40*a^6 - 84*a^4*b^2 + 69*a^2*b^4 - 20
*b^6)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a*Sqrt[a - b]*
Sqrt[a + b]*d)))/a + (b*(24*a^6 - 146*a^4*b^2 + 167*a^2*b^4 - 60*b^6)*Sin[
c + d*x])/(a*d))/a)/(a*(a^2 - b^2)))/(2*a*(a^2 - b^2)))/(3*a*(a^2 - b^2))
```

3.522.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 4318
```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[1/b   Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4334
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*
((d*Csc[e + f*x])^n/(a*f*(m + 1)*(a^2 - b^2))), x] + Simp[1/(a*(m + 1)*(a^2
 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a^2*(m + 1)
 - b^2*(m + n + 1) - a*b*(m + 1)*Csc[e + f*x] + b^2*(m + n + 2)*Csc[e + f*x
]^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0] && LtQ[m,
 -1] && IntegersQ[2*m, 2*n]
```

rule 4407
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Simp[(b*c - a*d)/a   Int[Csc[e + f*
x]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
- a*d, 0]
```

3.522. $\int \frac{\cos^2(c+dx)}{(a+b\sec(c+dx))^4}\,dx$

rule 4588
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc
[e + f*x])^(m + 1)*((d*Csc[e + f*x])^n/(a*f*(m + 1)*(a^2 - b^2))), x] + Sim
p[1/(a*(m + 1)*(a^2 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f
*x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n +
1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m
+ n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x
] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

rule 4592
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Simp[1/(a*d*n)   Int[(a + b*Csc[e + f*x])^m
*(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*
Csc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

3.522.4 Maple [A] (verified)

Time = 2.14 (sec) , antiderivative size = 439, normalized size of antiderivative = 1.13

method	result
derivativedivides	$\frac{\frac{2\left(\left(-\frac{1}{2}a^2-4ab\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3+\left(\frac{1}{2}a^2-4ab\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^2}+\left(a^2+20b^2\right)\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^6}+\frac{2b^3\left(-\frac{\left(30a^4+6a^3b-34a^2b^2-3a\right.}{2(a-b)\left(a^3+3a\right.}\right.}{}$
default	$\frac{\frac{2\left(\left(-\frac{1}{2}a^2-4ab\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3+\left(\frac{1}{2}a^2-4ab\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^2}+\left(a^2+20b^2\right)\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^6}+\frac{2b^3\left(-\frac{\left(30a^4+6a^3b-34a^2b^2-3a\right.}{2(a-b)\left(a^3+3a\right.}\right.}{}$
risch	Expression too large to display

input
```
int(cos(d*x+c)^2/(a+b*sec(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
1/d*(2/a^6*(((-1/2*a^2-4*a*b)*tan(1/2*d*x+1/2*c)^3+(1/2*a^2-4*a*b)*tan(1/2
*d*x+1/2*c))/(1+tan(1/2*d*x+1/2*c)^2)^2+1/2*(a^2+20*b^2)*arctan(tan(1/2*d*
x+1/2*c)))+2*b^3/a^6*((-1/2*(30*a^4+6*a^3*b-34*a^2*b^2-3*a*b^3+12*b^4)*a*b
/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5+2/3*(45*a^4-53*a^2*b
^2+18*b^4)*a*b/(a^2+2*a*b+b^2)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3-1/2*(3
0*a^4-6*a^3*b-34*a^2*b^2+3*a*b^3+12*b^4)*a*b/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^
3)*tan(1/2*d*x+1/2*c))/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)
^3-1/2*(40*a^6-84*a^4*b^2+69*a^2*b^4-20*b^6)/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)
/((a-b)*(a+b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a-b)*(a+b))^(1/2))
))
```

3.522.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 855 vs. $2(366) = 732$.

Time = 0.45 (sec) , antiderivative size = 1767, normalized size of antiderivative = 4.57

$$\int \frac{\cos^2(c+dx)}{(a+b\sec(c+dx))^4}\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^2/(a+b*sec(d*x+c))^4,x, algorithm="fricas")
```

output
```
[1/12*(6*(a^13 + 16*a^11*b^2 - 74*a^9*b^4 + 116*a^7*b^6 - 79*a^5*b^8 + 20*
a^3*b^10)*d*x*cos(d*x + c)^3 + 18*(a^12*b + 16*a^10*b^3 - 74*a^8*b^5 + 116
*a^6*b^7 - 79*a^4*b^9 + 20*a^2*b^11)*d*x*cos(d*x + c)^2 + 18*(a^11*b^2 + 1
6*a^9*b^4 - 74*a^7*b^6 + 116*a^5*b^8 - 79*a^3*b^10 + 20*a*b^12)*d*x*cos(d*
x + c) + 6*(a^10*b^3 + 16*a^8*b^5 - 74*a^6*b^7 + 116*a^4*b^9 - 79*a^2*b^11
 + 20*b^13)*d*x + 3*(40*a^6*b^6 - 84*a^4*b^8 + 69*a^2*b^10 - 20*b^12 + (40
*a^9*b^3 - 84*a^7*b^5 + 69*a^5*b^7 - 20*a^3*b^9)*cos(d*x + c)^3 + 3*(40*a^
8*b^4 - 84*a^6*b^6 + 69*a^4*b^8 - 20*a^2*b^10)*cos(d*x + c)^2 + 3*(40*a^7*
b^5 - 84*a^5*b^7 + 69*a^3*b^9 - 20*a*b^11)*cos(d*x + c))*sqrt(a^2 - b^2)*l
og((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 - 2*sqrt(a^2 - b^2)*
(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a
*b*cos(d*x + c) + b^2)) - 2*(24*a^9*b^4 - 170*a^7*b^6 + 313*a^5*b^8 - 227*
a^3*b^10 + 60*a*b^12 - 3*(a^13 - 4*a^11*b^2 + 6*a^9*b^4 - 4*a^7*b^6 + a^5*
b^8)*cos(d*x + c)^4 + 15*(a^12*b - 4*a^10*b^3 + 6*a^8*b^5 - 4*a^6*b^7 + a^
4*b^9)*cos(d*x + c)^3 + (63*a^11*b^2 - 342*a^9*b^4 + 590*a^7*b^6 - 421*a^5
*b^8 + 110*a^3*b^10)*cos(d*x + c)^2 + 3*(23*a^10*b^3 - 146*a^8*b^5 + 263*a
^6*b^7 - 190*a^4*b^9 + 50*a^2*b^11)*cos(d*x + c))*sin(d*x + c))/((a^17 - 4
*a^15*b^2 + 6*a^13*b^4 - 4*a^11*b^6 + a^9*b^8)*d*cos(d*x + c)^3 + 3*(a^16*
b - 4*a^14*b^3 + 6*a^12*b^5 - 4*a^10*b^7 + a^8*b^9)*d*cos(d*x + c)^2 + 3*(
a^15*b^2 - 4*a^13*b^4 + 6*a^11*b^6 - 4*a^9*b^8 + a^7*b^10)*d*cos(d*x + ...
```

3.522.6 Sympy [F]

$$\int \frac{\cos^2(c+dx)}{(a+b\sec(c+dx))^4}\,dx = \int \frac{\cos^2{(c+dx)}}{(a+b\sec{(c+dx)})^4}\,dx$$

input
```
integrate(cos(d*x+c)**2/(a+b*sec(d*x+c))**4,x)
```

output
```
Integral(cos(c + d*x)**2/(a + b*sec(c + d*x))**4, x)
```

3.522.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\cos^2(c+dx)}{(a+b\sec(c+dx))^4}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(cos(d*x+c)^2/(a+b*sec(d*x+c))^4,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

3.522.8 Giac [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 615, normalized size of antiderivative = 1.59

$$\int \frac{\cos^2(c+dx)}{(a+b\sec(c+dx))^4}\,dx$$
$$= \frac{\frac{6\,(40\,a^6b^3-84\,a^4b^5+69\,a^2b^7-20\,b^9)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2\,a-2\,b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{-a^2+b^2}}\right)\right)}{(a^{12}-3\,a^{10}b^2+3\,a^8b^4-a^6b^6)\sqrt{-a^2+b^2}} - \frac{2\left(90\,a^6b^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-1\right.}{}}{}$$

input
```
integrate(cos(d*x+c)^2/(a+b*sec(d*x+c))^4,x, algorithm="giac")
```

output
```
1/6*(6*(40*a^6*b^3 - 84*a^4*b^5 + 69*a^2*b^7 - 20*b^9)*(pi*floor(1/2*(d*x
+ c)/pi + 1/2)*sgn(2*a - 2*b) + arctan((a*tan(1/2*d*x + 1/2*c) - b*tan(1/2
*d*x + 1/2*c))/sqrt(-a^2 + b^2)))/((a^12 - 3*a^10*b^2 + 3*a^8*b^4 - a^6*b^
6)*sqrt(-a^2 + b^2)) - 2*(90*a^6*b^4*tan(1/2*d*x + 1/2*c)^5 - 162*a^5*b^5*
tan(1/2*d*x + 1/2*c)^5 - 48*a^4*b^6*tan(1/2*d*x + 1/2*c)^5 + 213*a^3*b^7*t
an(1/2*d*x + 1/2*c)^5 - 48*a^2*b^8*tan(1/2*d*x + 1/2*c)^5 - 81*a*b^9*tan(1
/2*d*x + 1/2*c)^5 + 36*b^10*tan(1/2*d*x + 1/2*c)^5 - 180*a^6*b^4*tan(1/2*d
*x + 1/2*c)^3 + 392*a^4*b^6*tan(1/2*d*x + 1/2*c)^3 - 284*a^2*b^8*tan(1/2*d
*x + 1/2*c)^3 + 72*b^10*tan(1/2*d*x + 1/2*c)^3 + 90*a^6*b^4*tan(1/2*d*x +
1/2*c) + 162*a^5*b^5*tan(1/2*d*x + 1/2*c) - 48*a^4*b^6*tan(1/2*d*x + 1/2*c
) - 213*a^3*b^7*tan(1/2*d*x + 1/2*c) - 48*a^2*b^8*tan(1/2*d*x + 1/2*c) + 8
1*a*b^9*tan(1/2*d*x + 1/2*c) + 36*b^10*tan(1/2*d*x + 1/2*c))/((a^11 - 3*a^
9*b^2 + 3*a^7*b^4 - a^5*b^6)*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1
/2*c)^2 - a - b)^3) + 3*(a^2 + 20*b^2)*(d*x + c)/a^6 - 6*(a*tan(1/2*d*x +
1/2*c)^3 + 8*b*tan(1/2*d*x + 1/2*c)^3 - a*tan(1/2*d*x + 1/2*c) + 8*b*tan(1
/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^2*a^5))/d
```

3.522.9 Mupad [B] (verification not implemented)

Time = 23.03 (sec) , antiderivative size = 8133, normalized size of antiderivative = 21.02

$$\int \frac{\cos^2(c+dx)}{(a+b\sec(c+dx))^4}\,dx = \text{Too large to display}$$

input
```
int(cos(c + d*x)^2/(a + b/cos(c + d*x))^4,x)
```

output
```
((tan(c/2 + (d*x)/2)^9*(7*a^7*b - 10*a*b^7 + a^8 + 20*b^8 - 59*a^2*b^6 + 2
7*a^3*b^5 + 57*a^4*b^4 - 21*a^5*b^3 - 11*a^6*b^2))/(a^5*(a + b)^3*(a - b))
 + (2*tan(c/2 + (d*x)/2)^3*(30*a*b^8 + 21*a^8*b - 6*a^9 + 120*b^9 - 364*a^
2*b^7 - 71*a^3*b^6 + 369*a^4*b^5 + 45*a^5*b^4 - 111*a^6*b^3 - 3*a^7*b^2))/
(3*a^5*(a + b)^2*(a - b)^3) - (2*tan(c/2 + (d*x)/2)^7*(21*a^8*b - 30*a*b^8
 + 6*a^9 + 120*b^9 - 364*a^2*b^7 + 71*a^3*b^6 + 369*a^4*b^5 - 45*a^5*b^4 -
 111*a^6*b^3 + 3*a^7*b^2))/(3*a^5*(a + b)^3*(a - b)^2) + (2*tan(c/2 + (d*x
)/2)^5*(9*a^10 + 180*b^10 - 611*a^2*b^8 + 740*a^4*b^6 - 324*a^6*b^4 + 36*a
^8*b^2))/(3*a^5*(a + b)^3*(a - b)^3) + (tan(c/2 + (d*x)/2)*(10*a*b^7 - 7*a
^7*b + a^8 + 20*b^8 - 59*a^2*b^6 - 27*a^3*b^5 + 57*a^4*b^4 + 21*a^5*b^3 -
11*a^6*b^2))/(a^5*(a + b)*(a - b)^3))/(d*(tan(c/2 + (d*x)/2)^2*(9*a*b^2 +
3*a^2*b - a^3 + 5*b^3) + tan(c/2 + (d*x)/2)^4*(6*a*b^2 - 6*a^2*b - 2*a^3 +
 10*b^3) - tan(c/2 + (d*x)/2)^6*(6*a*b^2 + 6*a^2*b - 2*a^3 - 10*b^3) + 3*a
*b^2 + 3*a^2*b + a^3 + b^3 - tan(c/2 + (d*x)/2)^10*(3*a*b^2 - 3*a^2*b + a^
3 - b^3) + tan(c/2 + (d*x)/2)^8*(3*a^2*b - 9*a*b^2 + a^3 + 5*b^3))) - (ata
n(((((((4*(4*a^27 - 80*a^12*b^15 + 40*a^13*b^14 + 516*a^14*b^13 - 248*a^15
*b^12 - 1404*a^16*b^11 + 640*a^17*b^10 + 2076*a^18*b^9 - 896*a^19*b^8 - 17
64*a^20*b^7 + 724*a^21*b^6 + 816*a^22*b^5 - 316*a^23*b^4 - 160*a^24*b^3 +
52*a^25*b^2))/(a^25*b + a^26 - a^15*b^11 - a^16*b^10 + 5*a^17*b^9 + 5*a^18
*b^8 - 10*a^19*b^7 - 10*a^20*b^6 + 10*a^21*b^5 + 10*a^22*b^4 - 5*a^23*b...
```

3.522. $\int \frac{\cos^2(c+dx)}{(a+b\sec(c+dx))^4}\,dx$

3.523 $\int \frac{1}{3+5\sec(c+dx)}\,dx$

3.523.1 Optimal result . . . 3924
3.523.2 Mathematica [A] (verified) . . . 3924
3.523.3 Rubi [A] (verified) . . . 3925
3.523.4 Maple [A] (verified) . . . 3926
3.523.5 Fricas [A] (verification not implemented) . . . 3926
3.523.6 Sympy [F] . . . 3927
3.523.7 Maxima [A] (verification not implemented) . . . 3927
3.523.8 Giac [A] (verification not implemented) . . . 3927
3.523.9 Mupad [B] (verification not implemented) . . . 3928

3.523.1 Optimal result

Integrand size = 12, antiderivative size = 31

$$\int \frac{1}{3+5\sec(c+dx)}\,dx=-\frac{x}{12}+\frac{5\arctan\left(\frac{\sin(c+dx)}{3+\cos(c+dx)}\right)}{6d}$$

output
```
-1/12*x+5/6*arctan(sin(d*x+c)/(3+cos(d*x+c)))/d
```

3.523.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.97

$$\int \frac{1}{3+5\sec(c+dx)}\,dx=\frac{2(c+dx)+5\arctan\left(2\cot\left(\frac{1}{2}(c+dx)\right)\right)}{6d}$$

input
```
Integrate[(3 + 5*Sec[c + d*x])^(-1),x]
```

output
```
(2*(c + d*x) + 5*ArcTan[2*Cot[(c + d*x)/2]])/(6*d)
```

3.523.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.32, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 4270, 3042, 3136}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{5\sec(c+dx)+3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{5\csc\left(c+dx+\frac{\pi}{2}\right)+3}dx$$

$$\Big\downarrow \text{4270}$$

$$\frac{x}{3}-\frac{1}{3}\int \frac{1}{\frac{3}{5}\cos(c+dx)+1}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{x}{3}-\frac{1}{3}\int \frac{1}{\frac{3}{5}\sin\left(c+dx+\frac{\pi}{2}\right)+1}dx$$

$$\Big\downarrow \text{3136}$$

$$\frac{1}{3}\left(\frac{5\arctan\left(\frac{\sin(c+dx)}{\cos(c+dx)+3}\right)}{2d}-\frac{5x}{4}\right)+\frac{x}{3}$$

input
```
Int[(3 + 5*Sec[c + d*x])^(-1),x]
```

output
```
x/3 + ((-5*x)/4 + (5*ArcTan[Sin[c + d*x]/(3 + Cos[c + d*x])])/(2*d))/3
```

3.523.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3136
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{q = Rt[
a^2 - b^2, 2]}, Simp[x/q, x] + Simp[(2/(d*q))*ArcTan[b*(Cos[c + d*x]/(a + q
 + b*Sin[c + d*x]))], x]] /; FreeQ[{a, b, c, d}, x] && GtQ[a^2 - b^2, 0] &&
 PosQ[a]
```

rule 4270
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(-1), x_Symbol] :> Simp[x/a, x]
- Simp[1/a   Int[1/(1 + (a/b)*Sin[c + d*x]), x], x] /; FreeQ[{a, b, c, d},
x] && NeQ[a^2 - b^2, 0]
```

3.523.4 Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.03

method	result	size
derivativedivides	$\frac{\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-\frac{5\arctan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}\right)}{6}}{d}$	32
default	$\frac{\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-\frac{5\arctan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}\right)}{6}}{d}$	32
risch	$\frac{x}{3}+\frac{5i\ln\left(e^{i(dx+c)}+\frac{1}{3}\right)}{12d}-\frac{5i\ln\left(e^{i(dx+c)}+3\right)}{12d}$	41
parallelrisch	$\frac{5i\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2i\right)-5i\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2i\right)+4dx}{12d}$	43

input
```
int(1/(3+5*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(2/3*arctan(tan(1/2*d*x+1/2*c))-5/6*arctan(1/2*tan(1/2*d*x+1/2*c)))
```

3.523.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int \frac{1}{3+5\sec(c+dx)}\,dx=\frac{4\,dx+5\,\arctan\left(\frac{5\,\cos(dx+c)+3}{4\,\sin(dx+c)}\right)}{12\,d}$$

input
```
integrate(1/(3+5*sec(d*x+c)),x, algorithm="fricas")
```

output
```
1/12*(4*d*x + 5*arctan(1/4*(5*cos(d*x + c) + 3)/sin(d*x + c)))/d
```

3.523.6 Sympy [F]

$$\int \frac{1}{3+5\sec(c+dx)}\,dx = \int \frac{1}{5\sec(c+dx)+3}\,dx$$

input
```
integrate(1/(3+5*sec(d*x+c)),x)
```

output
```
Integral(1/(5*sec(c + d*x) + 3), x)
```

3.523.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.52

$$\int \frac{1}{3+5\sec(c+dx)}\,dx = \frac{4\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right) - 5\arctan\left(\frac{\sin(dx+c)}{2\left(\cos(dx+c)+1\right)}\right)}{6\,d}$$

input
```
integrate(1/(3+5*sec(d*x+c)),x, algorithm="maxima")
```

output
```
1/6*(4*arctan(sin(d*x + c)/(cos(d*x + c) + 1)) - 5*arctan(1/2*sin(d*x + c)
/(cos(d*x + c) + 1)))/d
```

3.523.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.97

$$\int \frac{1}{3+5\sec(c+dx)}\,dx = -\frac{dx+c-10\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+3}\right)}{12\,d}$$

input
```
integrate(1/(3+5*sec(d*x+c)),x, algorithm="giac")
```

output
```
-1/12*(d*x + c - 10*arctan(sin(d*x + c)/(cos(d*x + c) + 3)))/d
```

3.523.9 Mupad [B] (verification not implemented)

Time = 13.71 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.68

$$\int \frac{1}{3+5\sec(c+dx)}\,dx=\frac{x}{3}-\frac{5\,\mathrm{atan}\left(\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2}\right)}{6\,d}$$

input
```
int(1/(5/cos(c + d*x) + 3),x)
```

output
```
x/3 - (5*atan(tan(c/2 + (d*x)/2)/2))/(6*d)
```

3.524 $\int \frac{1}{(3+5\sec(c+dx))^2}\,dx$

3.524.1 Optimal result . . . 3929
3.524.2 Mathematica [A] (verified) . . . 3929
3.524.3 Rubi [A] (verified) . . . 3930
3.524.4 Maple [A] (verified) . . . 3932
3.524.5 Fricas [A] (verification not implemented) . . . 3932
3.524.6 Sympy [F] . . . 3933
3.524.7 Maxima [A] (verification not implemented) . . . 3933
3.524.8 Giac [A] (verification not implemented) . . . 3933
3.524.9 Mupad [B] (verification not implemented) . . . 3934

3.524.1 Optimal result

Integrand size = 12, antiderivative size = 56

$$\int \frac{1}{(3+5\sec(c+dx))^2}\,dx=\frac{29x}{576}+\frac{35\arctan\left(\frac{\sin(c+dx)}{3+\cos(c+dx)}\right)}{288d}-\frac{25\tan(c+dx)}{48d(3+5\sec(c+dx))}$$

output
```
29/576*x+35/288*arctan(sin(d*x+c)/(3+cos(d*x+c)))/d-25/48*tan(d*x+c)/d/(3+
5*sec(d*x+c))
```

3.524.2 Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.30

$$\int \frac{1}{(3+5\sec(c+dx))^2}\,dx$$
$$=\frac{160(c+dx)+96(c+dx)\cos(c+dx)+35\arctan\left(2\cot\left(\frac{1}{2}(c+dx)\right)\right)(5+3\cos(c+dx))-150\sin(c+d}{288d(5+3\cos(c+dx))}$$

input
```
Integrate[(3 + 5*Sec[c + d*x])^(-2),x]
```

output
```
(160*(c + d*x) + 96*(c + d*x)*Cos[c + d*x] + 35*ArcTan[2*Cot[(c + d*x)/2]]
*(5 + 3*Cos[c + d*x]) - 150*Sin[c + d*x])/(288*d*(5 + 3*Cos[c + d*x]))
```

3.524.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.27, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {3042, 4272, 3042, 4407, 3042, 4318, 3042, 3136}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(5\sec(c+dx)+3)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(5\csc\left(c+dx+\frac{\pi}{2}\right)+3\right)^2}dx$$

$$\Big\downarrow \text{4272}$$

$$\frac{1}{48}\int \frac{15\sec(c+dx)+16}{5\sec(c+dx)+3}dx-\frac{25\tan(c+dx)}{48d(5\sec(c+dx)+3)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{48}\int \frac{15\csc\left(c+dx+\frac{\pi}{2}\right)+16}{5\csc\left(c+dx+\frac{\pi}{2}\right)+3}dx-\frac{25\tan(c+dx)}{48d(5\sec(c+dx)+3)}$$

$$\Big\downarrow \text{4407}$$

$$\frac{1}{48}\left(\frac{16x}{3}-\frac{35}{3}\int \frac{\sec(c+dx)}{5\sec(c+dx)+3}dx\right)-\frac{25\tan(c+dx)}{48d(5\sec(c+dx)+3)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{48}\left(\frac{16x}{3}-\frac{35}{3}\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{5\csc\left(c+dx+\frac{\pi}{2}\right)+3}dx\right)-\frac{25\tan(c+dx)}{48d(5\sec(c+dx)+3)}$$

$$\Big\downarrow \text{4318}$$

$$\frac{1}{48}\left(\frac{16x}{3}-\frac{7}{3}\int \frac{1}{\frac{3}{5}\cos(c+dx)+1}dx\right)-\frac{25\tan(c+dx)}{48d(5\sec(c+dx)+3)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{48}\left(\frac{16x}{3}-\frac{7}{3}\int \frac{1}{\frac{3}{5}\sin\left(c+dx+\frac{\pi}{2}\right)+1}dx\right)-\frac{25\tan(c+dx)}{48d(5\sec(c+dx)+3)}$$

$$\Big\downarrow \text{3136}$$

$$\frac{1}{48}\left(\frac{16x}{3}-\frac{7}{3}\left(\frac{5x}{4}-\frac{5\arctan\left(\frac{\sin(c+dx)}{\cos(c+dx)+3}\right)}{2d}\right)\right)-\frac{25\tan(c+dx)}{48d(5\sec(c+dx)+3)}$$

input
```
Int[(3 + 5*Sec[c + d*x])^(-2),x]
```

output
```
((16*x)/3 - (7*((5*x)/4 - (5*ArcTan[Sin[c + d*x]/(3 + Cos[c + d*x])])/(2*d
)))/3)/48 - (25*Tan[c + d*x])/(48*d*(3 + 5*Sec[c + d*x]))
```

3.524.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3136
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{q = Rt[
a^2 - b^2, 2]}, Simp[x/q, x] + Simp[(2/(d*q))*ArcTan[b*(Cos[c + d*x]/(a + q
 + b*Sin[c + d*x]))], x]] /; FreeQ[{a, b, c, d}, x] && GtQ[a^2 - b^2, 0] &&
 PosQ[a]
```

rule 4272
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[b^2*Cot[
c + d*x]*((a + b*Csc[c + d*x])^(n + 1)/(a*d*(n + 1)*(a^2 - b^2))), x] + Sim
p[1/(a*(n + 1)*(a^2 - b^2))   Int[(a + b*Csc[c + d*x])^(n + 1)*Simp[(a^2 -
b^2)*(n + 1) - a*b*(n + 1)*Csc[c + d*x] + b^2*(n + 2)*Csc[c + d*x]^2, x],
], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && LtQ[n, -1] && Integ
erQ[2*n]
```

rule 4318
```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[1/b   Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4407
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Simp[(b*c - a*d)/a   Int[Csc[e + f*
x]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
- a*d, 0]
```

3.524.4 Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.04

method	result
derivativedivides	$\frac{-\frac{25\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{48\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+4\right)}-\frac{35\arctan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}\right)}{288}+\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{9}}{d}$
default	$\frac{-\frac{25\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{48\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+4\right)}-\frac{35\arctan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}\right)}{288}+\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{9}}{d}$
risch	$\frac{x}{9}-\frac{25i\left(5\,\mathrm{e}^{i(dx+c)}+3\right)}{72d\left(3\,\mathrm{e}^{2i(dx+c)}+10\,\mathrm{e}^{i(dx+c)}+3\right)}+\frac{35i\ln\left(\mathrm{e}^{i(dx+c)}+\frac{1}{3}\right)}{576d}-\frac{35i\ln\left(\mathrm{e}^{i(dx+c)}+3\right)}{576d}$
parallelrisch	$\frac{(105i\cos(dx+c)+175i)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2i\right)+(-105i\cos(dx+c)-175i)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2i\right)+192dx\cos(dx+c)+320dx}{1728d\cos(dx+c)+2880d}$

input
```
int(1/(3+5*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-25/48*tan(1/2*d*x+1/2*c)/(tan(1/2*d*x+1/2*c)^2+4)-35/288*arctan(1/2*
tan(1/2*d*x+1/2*c))+2/9*arctan(tan(1/2*d*x+1/2*c)))
```

3.524.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.30

$$\int \frac{1}{(3+5\sec(c+dx))^2}\,dx = \frac{192\,dx\cos(dx+c)+320\,dx+35\,(3\,\cos(dx+c)+5)\arctan\left(\frac{5\,\cos(dx+c)+3}{4\,\sin(dx+c)}\right)-300\,\sin(dx+c)}{576\,(3\,d\cos(dx+c)+5\,d)}$$

input
```
integrate(1/(3+5*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/576*(192*d*x*cos(d*x + c) + 320*d*x + 35*(3*cos(d*x + c) + 5)*arctan(1/4
*(5*cos(d*x + c) + 3)/sin(d*x + c)) - 300*sin(d*x + c))/(3*d*cos(d*x + c)
+ 5*d)
```

3.524.6 Sympy [F]

$$\int \frac{1}{(3+5\sec(c+dx))^2}\,dx = \int \frac{1}{(5\sec(c+dx)+3)^2}\,dx$$

input
```
integrate(1/(3+5*sec(d*x+c))**2,x)
```

output
```
Integral((5*sec(c + d*x) + 3)**(-2), x)
```

3.524.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.57

$$\int \frac{1}{(3+5\sec(c+dx))^2}\,dx = -\frac{\frac{150\sin(dx+c)}{\left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+4\right)(\cos(dx+c)+1)} - 64\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right) + 35\arctan\left(\frac{\sin(dx+c)}{2(\cos(dx+c)+1)}\right)}{288\,d}$$

input
```
integrate(1/(3+5*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
-1/288*(150*sin(d*x + c)/((sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 4)*(cos(d
*x + c) + 1)) - 64*arctan(sin(d*x + c)/(cos(d*x + c) + 1)) + 35*arctan(1/2
*sin(d*x + c)/(cos(d*x + c) + 1)))/d
```

3.524.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.05

$$\int \frac{1}{(3+5\sec(c+dx))^2}\,dx = \frac{29\,dx + 29\,c - \frac{300\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+4} + 70\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+3}\right)}{576\,d}$$

input
```
integrate(1/(3+5*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
1/576*(29*d*x + 29*c - 300*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^2 +
4) + 70*arctan(sin(d*x + c)/(cos(d*x + c) + 3)))/d
```

3.524.9 Mupad [B] (verification not implemented)

Time = 14.25 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.93

$$\int \frac{1}{(3+5\sec(c+dx))^2}\,dx=\frac{x}{9}-\frac{\frac{35\,\mathrm{atan}\left(\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2}\right)}{288}+\frac{25\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{48\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+4\right)}}{d}$$

input
```
int(1/(5/cos(c + d*x) + 3)^2,x)
```

output
```
x/9 - ((35*atan(tan(c/2 + (d*x)/2)/2))/288 + (25*tan(c/2 + (d*x)/2))/(48*(
tan(c/2 + (d*x)/2)^2 + 4)))/d
```

3.525 $\int \frac{1}{(3+5\sec(c+dx))^3}\,dx$

3.525.1 Optimal result . . . 3935
3.525.2 Mathematica [A] (verified) . . . 3935
3.525.3 Rubi [A] (verified) . . . 3936
3.525.4 Maple [A] (verified) . . . 3939
3.525.5 Fricas [A] (verification not implemented) . . . 3939
3.525.6 Sympy [F] . . . 3940
3.525.7 Maxima [A] (verification not implemented) . . . 3940
3.525.8 Giac [A] (verification not implemented) . . . 3940
3.525.9 Mupad [B] (verification not implemented) . . . 3941

3.525.1 Optimal result

Integrand size = 12, antiderivative size = 81

$$\int \frac{1}{(3+5\sec(c+dx))^3}\,dx = -\frac{1007x}{55296}+\frac{3055\arctan\left(\frac{\sin(c+dx)}{3+\cos(c+dx)}\right)}{27648d} - \frac{25\tan(c+dx)}{96d(3+5\sec(c+dx))^2}-\frac{125\tan(c+dx)}{4608d(3+5\sec(c+dx))}$$

output
```
-1007/55296*x+3055/27648*arctan(sin(d*x+c)/(3+cos(d*x+c)))/d-25/96*tan(d*x
+c)/d/(3+5*sec(d*x+c))^2-125/4608*tan(d*x+c)/d/(3+5*sec(d*x+c))
```

3.525.2 Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.33

$$\int \frac{1}{(3+5\sec(c+dx))^3}\,dx = \frac{30208c+30208dx+30720(c+dx)\cos(c+dx)+3055\arctan\left(2\cot\left(\frac{1}{2}(c+dx)\right)\right)(5+3\cos(c+dx))^2+}{27648d(5+3\cos(c+d}$$

input
```
Integrate[(3 + 5*Sec[c + d*x])^(-3),x]
```

output
```
(30208*c + 30208*d*x + 30720*(c + d*x)*Cos[c + d*x] + 3055*ArcTan[2*Cot[(c
 + d*x)/2]]*(5 + 3*Cos[c + d*x])^2 + 4608*c*Cos[2*(c + d*x)] + 4608*d*x*Co
s[2*(c + d*x)] - 3750*Sin[c + d*x] - 4725*Sin[2*(c + d*x)])/(27648*d*(5 +
3*Cos[c + d*x])^2)
```

3.525.3 Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.25, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.833$, Rules used = {3042, 4272, 3042, 4548, 3042, 4407, 3042, 4318, 3042, 3136}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(5\sec(c+dx)+3)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(5\csc\left(c+dx+\frac{\pi}{2}\right)+3\right)^3}dx$$

$$\Big\downarrow \text{4272}$$

$$\frac{1}{96}\int \frac{-25\sec^2(c+dx)+30\sec(c+dx)+32}{(5\sec(c+dx)+3)^2}dx-\frac{25\tan(c+dx)}{96d(5\sec(c+dx)+3)^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{96}\int \frac{-25\csc\left(c+dx+\frac{\pi}{2}\right)^2+30\csc\left(c+dx+\frac{\pi}{2}\right)+32}{\left(5\csc\left(c+dx+\frac{\pi}{2}\right)+3\right)^2}dx-\frac{25\tan(c+dx)}{96d(5\sec(c+dx)+3)^2}$$

$$\Big\downarrow \text{4548}$$

$$\frac{1}{96}\left(\frac{1}{48}\int \frac{512-165\sec(c+dx)}{5\sec(c+dx)+3}dx-\frac{125\tan(c+dx)}{48d(5\sec(c+dx)+3)}\right)-\frac{25\tan(c+dx)}{96d(5\sec(c+dx)+3)^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{96}\left(\frac{1}{48}\int \frac{512-165\csc\left(c+dx+\frac{\pi}{2}\right)}{5\csc\left(c+dx+\frac{\pi}{2}\right)+3}dx-\frac{125\tan(c+dx)}{48d(5\sec(c+dx)+3)}\right)-\frac{25\tan(c+dx)}{96d(5\sec(c+dx)+3)^2}$$

$$\Big\downarrow \text{4407}$$

$$\frac{1}{96}\left(\frac{1}{48}\left(\frac{512x}{3}-\frac{3055}{3}\int \frac{\sec(c+dx)}{5\sec(c+dx)+3}dx\right)-\frac{125\tan(c+dx)}{48d(5\sec(c+dx)+3)}\right)-\frac{25\tan(c+dx)}{96d(5\sec(c+dx)+3)^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{96}\left(\frac{1}{48}\left(\frac{512x}{3}-\frac{3055}{3}\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{5\csc\left(c+dx+\frac{\pi}{2}\right)+3}dx\right)-\frac{125\tan(c+dx)}{48d(5\sec(c+dx)+3)}\right)-\frac{25\tan(c+dx)}{96d(5\sec(c+dx)+3)^2}$$

$$\Big\downarrow \text{4318}$$

$$\frac{1}{96}\left(\frac{1}{48}\left(\frac{512x}{3}-\frac{611}{3}\int\frac{1}{\frac{3}{5}\cos(c+dx)+1}dx\right)-\frac{125\tan(c+dx)}{48d(5\sec(c+dx)+3)}\right)-\frac{25\tan(c+dx)}{96d(5\sec(c+dx)+3)^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{96}\left(\frac{1}{48}\left(\frac{512x}{3}-\frac{611}{3}\int\frac{1}{\frac{3}{5}\sin\left(c+dx+\frac{\pi}{2}\right)+1}dx\right)-\frac{125\tan(c+dx)}{48d(5\sec(c+dx)+3)}\right)-\frac{25\tan(c+dx)}{96d(5\sec(c+dx)+3)^2}$$

$$\Big\downarrow \text{3136}$$

$$\frac{1}{96}\left(\frac{1}{48}\left(\frac{512x}{3}-\frac{611}{3}\left(\frac{5x}{4}-\frac{5\arctan\left(\frac{\sin(c+dx)}{\cos(c+dx)+3}\right)}{2d}\right)\right)-\frac{125\tan(c+dx)}{48d(5\sec(c+dx)+3)}\right)-\frac{25\tan(c+dx)}{96d(5\sec(c+dx)+3)^2}$$

input
```
Int[(3 + 5*Sec[c + d*x])^(-3),x]
```

output
```
(-25*Tan[c + d*x])/(96*d*(3 + 5*Sec[c + d*x])^2) + (((512*x)/3 - (611*((5*
x)/4 - (5*ArcTan[Sin[c + d*x]/(3 + Cos[c + d*x])])/(2*d)))/3)/48 - (125*Ta
n[c + d*x])/(48*d*(3 + 5*Sec[c + d*x])))/96
```

3.525. $\int \frac{1}{(3+5\sec(c+dx))^3}\,dx$

3.525.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3136
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{q = Rt[
a^2 - b^2, 2]}, Simp[x/q, x] + Simp[(2/(d*q))*ArcTan[b*(Cos[c + d*x]/(a + q
 + b*Sin[c + d*x]))], x]] /; FreeQ[{a, b, c, d}, x] && GtQ[a^2 - b^2, 0] &&
 PosQ[a]
```

rule 4272
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[b^2*Cot[
c + d*x]*((a + b*Csc[c + d*x])^(n + 1)/(a*d*(n + 1)*(a^2 - b^2))), x] + Sim
p[1/(a*(n + 1)*(a^2 - b^2))   Int[(a + b*Csc[c + d*x])^(n + 1)*Simp[(a^2 -
b^2)*(n + 1) - a*b*(n + 1)*Csc[c + d*x] + b^2*(n + 2)*Csc[c + d*x]^2, x], x
], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && LtQ[n, -1] && Integ
erQ[2*n]
```

rule 4318
```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[1/b   Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4407
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Simp[(b*c - a*d)/a   Int[Csc[e + f*
x]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
- a*d, 0]
```

rule 4548
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(A*b^2 -
a*b*B + a^2*C)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(a*f*(m + 1)*(a^2
 - b^2))), x] + Simp[1/(a*(m + 1)*(a^2 - b^2))   Int[(a + b*Csc[e + f*x])^(
m + 1)*Simp[A*(a^2 - b^2)*(m + 1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x
] + (A*b^2 - a*b*B + a^2*C)*(m + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

3.525. $\int \frac{1}{(3+5\sec(c+dx))^3}\,dx$

3.525.4 Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.91

method	result
derivativedivides	$\frac{-\frac{5\left(-\frac{285\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{128}+\frac{165\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{32}\right)}{108\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+4\right)^{2}}-\frac{3055\arctan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}\right)}{27648}+\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{27}}{d}$
default	$\frac{-\frac{5\left(-\frac{285\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{128}+\frac{165\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{32}\right)}{108\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+4\right)^{2}}-\frac{3055\arctan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}\right)}{27648}+\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{27}}{d}$
risch	$\frac{x}{27}-\frac{25i\left(185\,\mathrm{e}^{3i(dx+c)}+413\,\mathrm{e}^{2i(dx+c)}+235\,\mathrm{e}^{i(dx+c)}+63\right)}{2304d\left(3\,\mathrm{e}^{2i(dx+c)}+10\,\mathrm{e}^{i(dx+c)}+3\right)^{2}}+\frac{3055i\ln\left(\mathrm{e}^{i(dx+c)}+\frac{1}{3}\right)}{55296d}-\frac{3055i\ln\left(\mathrm{e}^{i(dx+c)}+3\right)}{55296d}$
parallelrisch	$\frac{3055i(59+9\cos(2dx+2c)+60\cos(dx+c))\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2i\right)+3055i(-59-9\cos(2dx+2c)-60\cos(dx+c))\ln\left(\tan\left(\frac{dx}{2}\right.\right.}{55296d(59+9\cos(2dx+2c)+}$

input
```
int(1/(3+5*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-5/108*(-285/128*tan(1/2*d*x+1/2*c)^3+165/32*tan(1/2*d*x+1/2*c))/(tan
(1/2*d*x+1/2*c)^2+4)^2-3055/27648*arctan(1/2*tan(1/2*d*x+1/2*c))+2/27*arct
an(tan(1/2*d*x+1/2*c)))
```

3.525.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.43

$$\int \frac{1}{(3+5\sec(c+dx))^3}\,dx$$
$$= \frac{18432\,dx\cos\left(dx+c\right)^2+61440\,dx\cos\left(dx+c\right)+51200\,dx+3055\left(9\,\cos\left(dx+c\right)^2+30\,\cos\left(dx+c\right)+25\right.}{55296\left(9\,d\cos\left(dx+c\right)^2+30\,d\cos\left(dx+c\right)+\right.}$$

input
```
integrate(1/(3+5*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/55296*(18432*d*x*cos(d*x + c)^2 + 61440*d*x*cos(d*x + c) + 51200*d*x + 3
055*(9*cos(d*x + c)^2 + 30*cos(d*x + c) + 25)*arctan(1/4*(5*cos(d*x + c) +
 3)/sin(d*x + c)) - 300*(63*cos(d*x + c) + 25)*sin(d*x + c))/(9*d*cos(d*x
+ c)^2 + 30*d*cos(d*x + c) + 25*d)
```

3.525.6 Sympy [F]

$$\int \frac{1}{(3+5\sec(c+dx))^3}\,dx = \int \frac{1}{(5\sec(c+dx)+3)^3}\,dx$$

input
```
integrate(1/(3+5*sec(d*x+c))**3,x)
```

output
```
Integral((5*sec(c + d*x) + 3)**(-3), x)
```

3.525.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.62

$$\int \frac{1}{(3+5\sec(c+dx))^3}\,dx = -\frac{\frac{150\left(\frac{44\sin(dx+c)}{\cos(dx+c)+1}-\frac{19\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{\frac{8\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+16}-2048\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)+3055\arctan\left(\frac{\sin(dx+c)}{2(\cos(dx+c)+1)}\right)}{27648\,d}$$

input
```
integrate(1/(3+5*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
-1/27648*(150*(44*sin(d*x + c)/(cos(d*x + c) + 1) - 19*sin(d*x + c)^3/(cos
(d*x + c) + 1)^3)/(8*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + sin(d*x + c)^4/
(cos(d*x + c) + 1)^4 + 16) - 2048*arctan(sin(d*x + c)/(cos(d*x + c) + 1))
+ 3055*arctan(1/2*sin(d*x + c)/(cos(d*x + c) + 1)))/d
```

3.525.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.93

$$\int \frac{1}{(3+5\sec(c+dx))^3}\,dx = -\frac{1007\,dx+1007\,c-\frac{300\left(19\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-44\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+4\right)^2}-6110\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+3}\right)}{55296\,d}$$

input
```
integrate(1/(3+5*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
-1/55296*(1007*d*x + 1007*c - 300*(19*tan(1/2*d*x + 1/2*c)^3 - 44*tan(1/2*
d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 4)^2 - 6110*arctan(sin(d*x + c)/(c
os(d*x + c) + 3)))/d
```

3.525.9 Mupad [B] (verification not implemented)

Time = 14.55 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.98

$$\int \frac{1}{(3+5\sec(c+dx))^3}\,dx = \frac{x}{27} - \frac{3055\,\mathrm{atan}\left(\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2}\right)}{27648\,d} - \frac{\frac{275\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{1152} - \frac{475\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{4608}}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4 + 8\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2 + 16\right)}$$

input
```
int(1/(5/cos(c + d*x) + 3)^3,x)
```

output
```
x/27 - (3055*atan(tan(c/2 + (d*x)/2)/2))/(27648*d) - ((275*tan(c/2 + (d*x)
/2))/1152 - (475*tan(c/2 + (d*x)/2)^3)/4608)/(d*(8*tan(c/2 + (d*x)/2)^2 +
tan(c/2 + (d*x)/2)^4 + 16))
```

3.526 $\int \frac{1}{(3+5\sec(c+dx))^4}\,dx$

3.526.1 Optimal result . . . 3942
3.526.2 Mathematica [A] (verified) . . . 3942
3.526.3 Rubi [A] (verified) . . . 3943
3.526.4 Maple [A] (verified) . . . 3946
3.526.5 Fricas [A] (verification not implemented) . . . 3947
3.526.6 Sympy [F] . . . 3948
3.526.7 Maxima [A] (verification not implemented) . . . 3948
3.526.8 Giac [A] (verification not implemented) . . . 3948
3.526.9 Mupad [B] (verification not implemented) . . . 3949

3.526.1 Optimal result

Integrand size = 12, antiderivative size = 106

$$\int \frac{1}{(3+5\sec(c+dx))^4}\,dx=\frac{21553x}{2654208}+\frac{11215\arctan\left(\frac{\sin(c+dx)}{3+\cos(c+dx)}\right)}{1327104d}-\frac{25\tan(c+dx)}{144d(3+5\sec(c+dx))^3}-\frac{25\tan(c+dx)}{4608d(3+5\sec(c+dx))^2}-\frac{16925\tan(c+dx)}{221184d(3+5\sec(c+dx))}$$

output
```
21553/2654208*x+11215/1327104*arctan(sin(d*x+c)/(3+cos(d*x+c)))/d-25/144*t
an(d*x+c)/d/(3+5*sec(d*x+c))^3-25/4608*tan(d*x+c)/d/(3+5*sec(d*x+c))^2-169
25/221184*tan(d*x+c)/d/(3+5*sec(d*x+c))
```

3.526.2 Mathematica [A] (verified)

Time = 0.61 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.33

$$\int \frac{1}{(3+5\sec(c+dx))^4}\,dx$$
$$=\frac{6307840c+6307840dx+8036352(c+dx)\cos(c+dx)+22430\arctan\left(2\cot\left(\frac{1}{2}(c+dx)\right)\right)(5+3\cos(c+$$

input
```
Integrate[(3 + 5*Sec[c + d*x])^(-4),x]
```

output
```
(6307840*c + 6307840*d*x + 8036352*(c + d*x)*Cos[c + d*x] + 22430*ArcTan[2
*Cot[(c + d*x)/2]]*(5 + 3*Cos[c + d*x])^3 + 2211840*c*Cos[2*(c + d*x)] + 2
211840*d*x*Cos[2*(c + d*x)] + 221184*c*Cos[3*(c + d*x)] + 221184*d*x*Cos[3
*(c + d*x)] - 5660475*Sin[c + d*x] - 3082500*Sin[2*(c + d*x)] - 582975*Sin
[3*(c + d*x)])/(2654208*d*(5 + 3*Cos[c + d*x])^3)
```

3.526.3 Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.24, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 1.083$, Rules used = {3042, 4272, 3042, 4548, 27, 3042, 4548, 3042, 4407, 3042, 4318, 3042, 3136}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(5\sec(c+dx)+3)^4}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(5\csc\left(c+dx+\frac{\pi}{2}\right)+3\right)^4}dx$$

$$\Big\downarrow \text{4272}$$

$$\frac{1}{144}\int \frac{-50\sec^2(c+dx)+45\sec(c+dx)+48}{(5\sec(c+dx)+3)^3}dx-\frac{25\tan(c+dx)}{144d(5\sec(c+dx)+3)^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{144}\int \frac{-50\csc\left(c+dx+\frac{\pi}{2}\right)^2+45\csc\left(c+dx+\frac{\pi}{2}\right)+48}{\left(5\csc\left(c+dx+\frac{\pi}{2}\right)+3\right)^3}dx-\frac{25\tan(c+dx)}{144d(5\sec(c+dx)+3)^3}$$

$$\Big\downarrow \text{4548}$$

$$\frac{1}{144}\left(\frac{1}{96}\int \frac{3\left(-25\sec^2(c+dx)-290\sec(c+dx)+512\right)}{(5\sec(c+dx)+3)^2}dx-\frac{25\tan(c+dx)}{32d(5\sec(c+dx)+3)^2}\right)-\frac{25\tan(c+dx)}{144d(5\sec(c+dx)+3)^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{144}\left(\frac{1}{32}\int \frac{-25\sec^2(c+dx)-290\sec(c+dx)+512}{(5\sec(c+dx)+3)^2}dx-\frac{25\tan(c+dx)}{32d(5\sec(c+dx)+3)^2}\right)-\frac{25\tan(c+dx)}{144d(5\sec(c+dx)+3)^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{144}\left(\frac{1}{32}\int \frac{-25\csc\left(c+dx+\frac{\pi}{2}\right)^2-290\csc\left(c+dx+\frac{\pi}{2}\right)+512}{\left(5\csc\left(c+dx+\frac{\pi}{2}\right)+3\right)^2}dx-\frac{25\tan(c+dx)}{32d(5\sec(c+dx)+3)^2}\right)-\frac{25\tan(c+dx)}{144d(5\sec(c+dx)+3)^3}$$

$$\Big\downarrow \text{4548}$$

$$\frac{1}{144}\left(\frac{1}{32}\left(\frac{1}{48}\int \frac{9915\sec(c+dx)+8192}{5\sec(c+dx)+3}dx-\frac{16925\tan(c+dx)}{48d(5\sec(c+dx)+3)}\right)-\frac{25\tan(c+dx)}{32d(5\sec(c+dx)+3)^2}\right)-\frac{25\tan(c+dx)}{144d(5\sec(c+dx)+3)^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{144}\left(\frac{1}{32}\left(\frac{1}{48}\int \frac{9915\csc\left(c+dx+\frac{\pi}{2}\right)+8192}{5\csc\left(c+dx+\frac{\pi}{2}\right)+3}dx-\frac{16925\tan(c+dx)}{48d(5\sec(c+dx)+3)}\right)-\frac{25\tan(c+dx)}{32d(5\sec(c+dx)+3)^2}\right)-\frac{25\tan(c+dx)}{144d(5\sec(c+dx)+3)^3}$$

$$\Big\downarrow \text{4407}$$

$$\frac{1}{144}\left(\frac{1}{32}\left(\frac{1}{48}\left(\frac{8192x}{3}-\frac{11215}{3}\int \frac{\sec(c+dx)}{5\sec(c+dx)+3}dx\right)-\frac{16925\tan(c+dx)}{48d(5\sec(c+dx)+3)}\right)-\frac{25\tan(c+dx)}{32d(5\sec(c+dx)+3)^2}\right)-\frac{25\tan(c+dx)}{144d(5\sec(c+dx)+3)^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{144}\left(\frac{1}{32}\left(\frac{1}{48}\left(\frac{8192x}{3}-\frac{11215}{3}\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{5\csc\left(c+dx+\frac{\pi}{2}\right)+3}dx\right)-\frac{16925\tan(c+dx)}{48d(5\sec(c+dx)+3)}\right)-\frac{25\tan(c+dx)}{32d(5\sec(c+dx)+3}\right.$$
$$\frac{25\tan(c+dx)}{144d(5\sec(c+dx)+3)^3}$$

$$\Big\downarrow \text{4318}$$

$$\frac{1}{144}\left(\frac{1}{32}\left(\frac{1}{48}\left(\frac{8192x}{3}-\frac{2243}{3}\int \frac{1}{\frac{3}{5}\cos(c+dx)+1}dx\right)-\frac{16925\tan(c+dx)}{48d(5\sec(c+dx)+3)}\right)-\frac{25\tan(c+dx)}{32d(5\sec(c+dx)+3)^2}\right)-\frac{25\tan(c+dx)}{144d(5\sec(c+dx)+3)^3}$$

$$\Big\downarrow \text{3042}$$

3.526. $\int \frac{1}{(3+5\sec(c+dx))^4}\,dx$

$$\frac{1}{144}\left(\frac{1}{32}\left(\frac{1}{48}\left(\frac{8192x}{3}-\frac{2243}{3}\int\frac{1}{\frac{3}{5}\sin\left(c+dx+\frac{\pi}{2}\right)+1}dx\right)-\frac{16925\tan(c+dx)}{48d(5\sec(c+dx)+3)}\right)-\frac{25\tan(c+dx)}{32d(5\sec(c+dx)+3)}\right.$$
$$\frac{25\tan(c+dx)}{144d(5\sec(c+dx)+3)^3}$$

$$\Big\downarrow \; 3136$$

$$\frac{1}{144}\left(\frac{1}{32}\left(\frac{1}{48}\left(\frac{8192x}{3}-\frac{2243}{3}\left(\frac{5x}{4}-\frac{5\arctan\left(\frac{\sin(c+dx)}{\cos(c+dx)+3}\right)}{2d}\right)\right)-\frac{16925\tan(c+dx)}{48d(5\sec(c+dx)+3)}\right)-\frac{25\tan(c+dx)}{32d(5\sec(c+dx)}\right.$$
$$\frac{25\tan(c+dx)}{144d(5\sec(c+dx)+3)^3}$$

input
```
Int[(3 + 5*Sec[c + d*x])^(-4),x]
```

output
```
(-25*Tan[c + d*x])/(144*d*(3 + 5*Sec[c + d*x])^3) + ((-25*Tan[c + d*x])/(3
2*d*(3 + 5*Sec[c + d*x])^2) + (((8192*x)/3 - (2243*((5*x)/4 - (5*ArcTan[Si
n[c + d*x]/(3 + Cos[c + d*x])])/(2*d)))/3)/48 - (16925*Tan[c + d*x])/(48*d
*(3 + 5*Sec[c + d*x])))/32)/144
```

3.526.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3136
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{q = Rt[
a^2 - b^2, 2]}, Simp[x/q, x] + Simp[(2/(d*q))*ArcTan[b*(Cos[c + d*x]/(a + q
 + b*Sin[c + d*x]))], x]] /; FreeQ[{a, b, c, d}, x] && GtQ[a^2 - b^2, 0] &&
 PosQ[a]
```

rule 4272
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[b^2*Cot[
c + d*x]*((a + b*Csc[c + d*x])^(n + 1)/(a*d*(n + 1)*(a^2 - b^2))), x] + Sim
p[1/(a*(n + 1)*(a^2 - b^2))   Int[(a + b*Csc[c + d*x])^(n + 1)*Simp[(a^2 -
b^2)*(n + 1) - a*b*(n + 1)*Csc[c + d*x] + b^2*(n + 2)*Csc[c + d*x]^2, x], x
], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && LtQ[n, -1] && Integ
erQ[2*n]
```

rule 4318
```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[1/b   Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4407
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Simp[(b*c - a*d)/a   Int[Csc[e + f*
x]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
- a*d, 0]
```

rule 4548
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(A*b^2 -
a*b*B + a^2*C)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(a*f*(m + 1)*(a^2
 - b^2))), x] + Simp[1/(a*(m + 1)*(a^2 - b^2))   Int[(a + b*Csc[e + f*x])^(
m + 1)*Simp[A*(a^2 - b^2)*(m + 1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x
] + (A*b^2 - a*b*B + a^2*C)*(m + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

3.526.4 Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.82

method	result
derivativedivides	$\frac{\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{81}+\frac{-\frac{25925\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{221184}-\frac{3575\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{6912}-\frac{17675\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{13824}}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+4\right)^3}-\frac{11215\arctan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}\right)}{1327104}}{d}$
default	$\frac{\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{81}+\frac{-\frac{25925\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{221184}-\frac{3575\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{6912}-\frac{17675\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{13824}}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+4\right)^3}-\frac{11215\arctan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}\right)}{1327104}}{d}$
risch	$\frac{x}{81}-\frac{25i\left(164835\,e^{5i(dx+c)}+931257\,e^{4i(dx+c)}+1995070\,e^{3i(dx+c)}+1610514\,e^{2i(dx+c)}+534735\,e^{i(dx+c)}+69957\right)}{995328d\left(3\,e^{2i(dx+c)}+10\,e^{i(dx+c)}+3\right)^3}-\frac{11}{}$
parallelrisch	$\frac{11215i(770+27\cos(3dx+3c)+981\cos(dx+c)+270\cos(2dx+2c))\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2i\right)+11215i(-770-27\cos(3dx+3c)-}{}$

input
```
int(1/(3+5*sec(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
1/d*(2/81*arctan(tan(1/2*d*x+1/2*c))+5/648*(-15555/1024*tan(1/2*d*x+1/2*c)
^5-2145/32*tan(1/2*d*x+1/2*c)^3-10605/64*tan(1/2*d*x+1/2*c))/(tan(1/2*d*x+
1/2*c)^2+4)^3-11215/1327104*arctan(1/2*tan(1/2*d*x+1/2*c)))
```

3.526.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.50

$$\int \frac{1}{(3+5\sec(c+dx))^4}\,dx$$
$$= \frac{884736\,dx\cos(dx+c)^3+4423680\,dx\cos(dx+c)^2+7372800\,dx\cos(dx+c)+4096000\,dx+11215\,(27\,}{2654208\,(27\,d\,co}$$

input
```
integrate(1/(3+5*sec(d*x+c))^4,x, algorithm="fricas")
```

output
```
1/2654208*(884736*d*x*cos(d*x + c)^3 + 4423680*d*x*cos(d*x + c)^2 + 737280
0*d*x*cos(d*x + c) + 4096000*d*x + 11215*(27*cos(d*x + c)^3 + 135*cos(d*x
+ c)^2 + 225*cos(d*x + c) + 125)*arctan(1/4*(5*cos(d*x + c) + 3)/sin(d*x +
 c)) - 300*(7773*cos(d*x + c)^2 + 20550*cos(d*x + c) + 16925)*sin(d*x + c)
)/(27*d*cos(d*x + c)^3 + 135*d*cos(d*x + c)^2 + 225*d*cos(d*x + c) + 125*d
)
```

3.526.6 Sympy [F]

$$\int \frac{1}{(3+5\sec(c+dx))^4}\,dx = \int \frac{1}{(5\sec(c+dx)+3)^4}\,dx$$

input
```
integrate(1/(3+5*sec(d*x+c))**4,x)
```

output
```
Integral((5*sec(c + d*x) + 3)**(-4), x)
```

3.526.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.61

$$\int \frac{1}{(3+5\sec(c+dx))^4}\,dx = -\frac{\frac{150\left(\frac{11312\sin(dx+c)}{\cos(dx+c)+1}+\frac{4576\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{1037\sin(dx+c)^5}{(\cos(dx+c)+1)^5}\right)}{\frac{48\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{12\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+64} - 32768\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right) + 11215\arctan\left(\frac{\sin(dx+c)}{2(\cos(dx+c)+1)}\right.}{1327104\,d}$$

input
```
integrate(1/(3+5*sec(d*x+c))^4,x, algorithm="maxima")
```

output
```
-1/1327104*(150*(11312*sin(d*x + c)/(cos(d*x + c) + 1) + 4576*sin(d*x + c)
^3/(cos(d*x + c) + 1)^3 + 1037*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/(48*si
n(d*x + c)^2/(cos(d*x + c) + 1)^2 + 12*sin(d*x + c)^4/(cos(d*x + c) + 1)^4
 + sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 64) - 32768*arctan(sin(d*x + c)/(
cos(d*x + c) + 1)) + 11215*arctan(1/2*sin(d*x + c)/(cos(d*x + c) + 1)))/d
```

3.526.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.83

$$\int \frac{1}{(3+5\sec(c+dx))^4}\,dx = \frac{21553\,dx + 21553\,c - \frac{300\left(1037\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+4576\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+11312\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+4\right)^3} + 22430\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+3}\right.}{2654208\,d}$$

input
```
integrate(1/(3+5*sec(d*x+c))^4,x, algorithm="giac")
```

output
```
1/2654208*(21553*d*x + 21553*c - 300*(1037*tan(1/2*d*x + 1/2*c)^5 + 4576*t
an(1/2*d*x + 1/2*c)^3 + 11312*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^
2 + 4)^3 + 22430*arctan(sin(d*x + c)/(cos(d*x + c) + 3)))/d
```

3.526.9 Mupad [B] (verification not implemented)

Time = 14.43 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.99

$$\int \frac{1}{(3+5\sec(c+dx))^4}\,dx = \frac{x}{81} - \frac{11215\,\mathrm{atan}\left(\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2}\right)}{1327104\,d} - \frac{\frac{25925\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{221184}+\frac{3575\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{6912}+\frac{17675\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{13824}}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+12\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+48\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+64\right)}$$

input
```
int(1/(5/cos(c + d*x) + 3)^4,x)
```

output
```
x/81 - (11215*atan(tan(c/2 + (d*x)/2)/2))/(1327104*d) - ((17675*tan(c/2 +
(d*x)/2))/13824 + (3575*tan(c/2 + (d*x)/2)^3)/6912 + (25925*tan(c/2 + (d*x
)/2)^5)/221184)/(d*(48*tan(c/2 + (d*x)/2)^2 + 12*tan(c/2 + (d*x)/2)^4 + ta
n(c/2 + (d*x)/2)^6 + 64))
```

3.527 $\int \frac{1}{5+3\sec(c+dx)}\,dx$

3.527.1 Optimal result . . . 3950
3.527.2 Mathematica [A] (verified) . . . 3950
3.527.3 Rubi [A] (verified) . . . 3951
3.527.4 Maple [A] (verified) . . . 3952
3.527.5 Fricas [A] (verification not implemented) . . . 3953
3.527.6 Sympy [F] . . . 3953
3.527.7 Maxima [A] (verification not implemented) . . . 3953
3.527.8 Giac [A] (verification not implemented) . . . 3954
3.527.9 Mupad [B] (verification not implemented) . . . 3954

3.527.1 Optimal result

Integrand size = 12, antiderivative size = 70

$$\int \frac{1}{5+3\sec(c+dx)}\,dx = \frac{x}{5}+\frac{3\log\left(2\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}{20d} - \frac{3\log\left(2\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}{20d}$$

output
```
1/5*x+3/20*ln(2*cos(1/2*d*x+1/2*c)-sin(1/2*d*x+1/2*c))/d-3/20*ln(2*cos(1/2
*d*x+1/2*c)+sin(1/2*d*x+1/2*c))/d
```

3.527.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.99

$$\int \frac{1}{5+3\sec(c+dx)}\,dx = \frac{4(c+dx)+3\log\left(2\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-3\log\left(2\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}{20d}$$

input
```
Integrate[(5 + 3*Sec[c + d*x])^(-1),x]
```

output
```
(4*(c + d*x) + 3*Log[2*Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - 3*Log[2*Cos[
(c + d*x)/2] + Sin[(c + d*x)/2]])/(20*d)
```

3.527.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.40, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {3042, 4270, 3042, 3138, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{3\sec(c+dx)+5}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{3\csc\left(c+dx+\frac{\pi}{2}\right)+5}dx$$

$$\Big\downarrow \text{4270}$$

$$\frac{x}{5}-\frac{1}{5}\int \frac{1}{\frac{5}{3}\cos(c+dx)+1}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{x}{5}-\frac{1}{5}\int \frac{1}{\frac{5}{3}\sin\left(c+dx+\frac{\pi}{2}\right)+1}dx$$

$$\Big\downarrow \text{3138}$$

$$\frac{x}{5}-\frac{2\int \frac{1}{\frac{8}{3}-\frac{2}{3}\tan^2\left(\frac{1}{2}(c+dx)\right)}d\tan\left(\frac{1}{2}(c+dx)\right)}{5d}$$

$$\Big\downarrow \text{219}$$

$$\frac{x}{5}-\frac{3\text{arctanh}\left(\frac{1}{2}\tan\left(\frac{1}{2}(c+dx)\right)\right)}{10d}$$

input
```
Int[(5 + 3*Sec[c + d*x])^(-1),x]
```

output
```
x/5 - (3*ArcTanh[Tan[(c + d*x)/2]/2])/(10*d)
```

3.527.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 4270
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(-1), x_Symbol] :> Simp[x/a, x]
- Simp[1/a   Int[1/(1 + (a/b)*Sin[c + d*x]), x], x] /; FreeQ[{a, b, c, d},
x] && NeQ[a^2 - b^2, 0]
```

3.527.4 Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.56

method	result	size
norman	$\frac{x}{5}+\frac{3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2\right)}{20d}-\frac{3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2\right)}{20d}$	39
parallelrisch	$\frac{4dx+3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2\right)-3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2\right)}{20d}$	39
risch	$\frac{x}{5}-\frac{3\ln\left(e^{i(dx+c)}+\frac{3}{5}+\frac{4i}{5}\right)}{20d}+\frac{3\ln\left(e^{i(dx+c)}+\frac{3}{5}-\frac{4i}{5}\right)}{20d}$	43
derivativedivides	$\frac{\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}+\frac{3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2\right)}{20}-\frac{3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2\right)}{20}}{d}$	46
default	$\frac{\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}+\frac{3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2\right)}{20}-\frac{3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2\right)}{20}}{d}$	46

input
```
int(1/(5+3*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/5*x+3/20/d*ln(tan(1/2*d*x+1/2*c)-2)-3/20/d*ln(tan(1/2*d*x+1/2*c)+2)
```

3.527.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.74

$$\int \frac{1}{5+3\sec(c+dx)}\,dx = \frac{8\,dx - 3\,\log\left(\frac{3}{2}\,\cos\left(dx+c\right) + 2\,\sin\left(dx+c\right) + \frac{5}{2}\right) + 3\,\log\left(\frac{3}{2}\,\cos\left(dx+c\right) - 2\,\sin\left(dx+c\right) + \frac{5}{2}\right)}{40\,d}$$

input
```
integrate(1/(5+3*sec(d*x+c)),x, algorithm="fricas")
```

output
```
1/40*(8*d*x - 3*log(3/2*cos(d*x + c) + 2*sin(d*x + c) + 5/2) + 3*log(3/2*c
os(d*x + c) - 2*sin(d*x + c) + 5/2))/d
```

3.527.6 Sympy [F]

$$\int \frac{1}{5+3\sec(c+dx)}\,dx = \int \frac{1}{3\sec\left(c+dx\right)+5}\,dx$$

input
```
integrate(1/(5+3*sec(d*x+c)),x)
```

output
```
Integral(1/(3*sec(c + d*x) + 5), x)
```

3.527.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00

$$\int \frac{1}{5+3\sec(c+dx)}\,dx = \frac{8\,\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right) - 3\,\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+2\right) + 3\,\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-2\right)}{20\,d}$$

input
```
integrate(1/(5+3*sec(d*x+c)),x, algorithm="maxima")
```

output
```
1/20*(8*arctan(sin(d*x + c)/(cos(d*x + c) + 1)) - 3*log(sin(d*x + c)/(cos(
d*x + c) + 1) + 2) + 3*log(sin(d*x + c)/(cos(d*x + c) + 1) - 2))/d
```

3.527.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.61

$$\int \frac{1}{5+3\sec(c+dx)}\,dx = \frac{4\,dx+4\,c-3\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+2\right|\right)+3\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\right|\right)}{20\,d}$$

input
```
integrate(1/(5+3*sec(d*x+c)),x, algorithm="giac")
```

output
```
1/20*(4*d*x + 4*c - 3*log(abs(tan(1/2*d*x + 1/2*c) + 2)) + 3*log(abs(tan(1
/2*d*x + 1/2*c) - 2)))/d
```

3.527.9 Mupad [B] (verification not implemented)

Time = 14.22 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.30

$$\int \frac{1}{5+3\sec(c+dx)}\,dx = \frac{x}{5}-\frac{3\,\mathrm{atanh}\left(\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2}\right)}{10\,d}$$

input
```
int(1/(3/cos(c + d*x) + 5),x)
```

output
```
x/5 - (3*atanh(tan(c/2 + (d*x)/2)/2))/(10*d)
```

3.528 $\int \frac{1}{(5+3\sec(c+dx))^2}\,dx$

3.528.1 Optimal result . . . 3955
3.528.2 Mathematica [A] (verified) . . . 3955
3.528.3 Rubi [A] (verified) . . . 3956
3.528.4 Maple [A] (verified) . . . 3958
3.528.5 Fricas [A] (verification not implemented) . . . 3959
3.528.6 Sympy [F] . . . 3959
3.528.7 Maxima [A] (verification not implemented) . . . 3960
3.528.8 Giac [A] (verification not implemented) . . . 3960
3.528.9 Mupad [B] (verification not implemented) . . . 3961

3.528.1 Optimal result

Integrand size = 12, antiderivative size = 95

$$\int \frac{1}{(5+3\sec(c+dx))^2}\,dx = \frac{x}{25}+\frac{123\log\left(2\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}{1600d} - \frac{123\log\left(2\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}{1600d} + \frac{9\tan(c+dx)}{80d(5+3\sec(c+dx))}$$

output
```
1/25*x+123/1600*ln(2*cos(1/2*d*x+1/2*c)-sin(1/2*d*x+1/2*c))/d-123/1600*ln(
2*cos(1/2*d*x+1/2*c)+sin(1/2*d*x+1/2*c))/d+9/80*tan(d*x+c)/d/(5+3*sec(d*x+
c))
```

3.528.2 Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.71

$$\int \frac{1}{(5+3\sec(c+dx))^2}\,dx = \frac{5\cos(c+dx)\left(64(c+dx)+123\log\left(2\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-123\log\left(2\cos\left(\frac{1}{2}(c+dx)\right)+s\right.\right.}{}$$

input
```
Integrate[(5 + 3*Sec[c + d*x])^(-2),x]
```

output
```
(5*Cos[c + d*x]*(64*(c + d*x) + 123*Log[2*Cos[(c + d*x)/2] - Sin[(c + d*x)
/2]] - 123*Log[2*Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + 3*(64*c + 64*d*x
+ 123*Log[2*Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - 123*Log[2*Cos[(c + d*x)
/2] + Sin[(c + d*x)/2]] + 60*Sin[c + d*x]))/(1600*d*(3 + 5*Cos[c + d*x]))
```

3.528.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.61, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.833$, Rules used = {3042, 4272, 25, 3042, 4407, 3042, 4318, 3042, 3138, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(3\sec(c+dx)+5)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(3\csc\left(c+dx+\frac{\pi}{2}\right)+5\right)^2}dx$$

$$\Big\downarrow \text{4272}$$

$$\frac{9\tan(c+dx)}{80d(3\sec(c+dx)+5)}-\frac{1}{80}\int -\frac{16-15\sec(c+dx)}{3\sec(c+dx)+5}dx$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{80}\int \frac{16-15\sec(c+dx)}{3\sec(c+dx)+5}dx+\frac{9\tan(c+dx)}{80d(3\sec(c+dx)+5)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{80}\int \frac{16-15\csc\left(c+dx+\frac{\pi}{2}\right)}{3\csc\left(c+dx+\frac{\pi}{2}\right)+5}dx+\frac{9\tan(c+dx)}{80d(3\sec(c+dx)+5)}$$

$$\Big\downarrow \text{4407}$$

$$\frac{1}{80}\left(\frac{16x}{5}-\frac{123}{5}\int \frac{\sec(c+dx)}{3\sec(c+dx)+5}dx\right)+\frac{9\tan(c+dx)}{80d(3\sec(c+dx)+5)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{80}\left(\frac{16x}{5}-\frac{123}{5}\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{3\csc\left(c+dx+\frac{\pi}{2}\right)+5}dx\right)+\frac{9\tan(c+dx)}{80d(3\sec(c+dx)+5)}$$

$$\Big\downarrow \text{4318}$$

$$\frac{1}{80}\left(\frac{16x}{5}-\frac{41}{5}\int\frac{1}{\frac{5}{3}\cos(c+dx)+1}dx\right)+\frac{9\tan(c+dx)}{80d(3\sec(c+dx)+5)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{80}\left(\frac{16x}{5}-\frac{41}{5}\int\frac{1}{\frac{5}{3}\sin\left(c+dx+\frac{\pi}{2}\right)+1}dx\right)+\frac{9\tan(c+dx)}{80d(3\sec(c+dx)+5)}$$

$$\Big\downarrow \text{3138}$$

$$\frac{1}{80}\left(\frac{16x}{5}-\frac{82\int\frac{1}{\frac{8}{3}-\frac{2}{3}\tan^2\left(\frac{1}{2}(c+dx)\right)}d\tan\left(\frac{1}{2}(c+dx)\right)}{5d}\right)+\frac{9\tan(c+dx)}{80d(3\sec(c+dx)+5)}$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{80}\left(\frac{16x}{5}-\frac{123\text{arctanh}\left(\frac{1}{2}\tan\left(\frac{1}{2}(c+dx)\right)\right)}{10d}\right)+\frac{9\tan(c+dx)}{80d(3\sec(c+dx)+5)}$$

input
```
Int[(5 + 3*Sec[c + d*x])^(-2),x]
```

output
```
((16*x)/5 - (123*ArcTanh[Tan[(c + d*x)/2]/2])/(10*d))/80 + (9*Tan[c + d*x]
)/(80*d*(5 + 3*Sec[c + d*x]))
```

3.528.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 4272
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[b^2*Cot[
c + d*x]*((a + b*Csc[c + d*x])^(n + 1)/(a*d*(n + 1)*(a^2 - b^2))), x] + Sim
p[1/(a*(n + 1)*(a^2 - b^2))   Int[(a + b*Csc[c + d*x])^(n + 1)*Simp[(a^2 -
b^2)*(n + 1) - a*b*(n + 1)*Csc[c + d*x] + b^2*(n + 2)*Csc[c + d*x]^2, x], x
], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && LtQ[n, -1] && Integ
erQ[2*n]
```

rule 4318
```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[1/b   Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4407
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Simp[(b*c - a*d)/a   Int[Csc[e + f*
x]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
- a*d, 0]
```

3.528.4 Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.80

method	result
derivativedivides	$\frac{\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{25}-\frac{9}{160\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2\right)}-\frac{123\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2\right)}{1600}-\frac{9}{160\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2\right)}+\frac{123\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2\right)}{1600}}{d}$
default	$\frac{\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{25}-\frac{9}{160\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2\right)}-\frac{123\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2\right)}{1600}-\frac{9}{160\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2\right)}+\frac{123\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2\right)}{1600}}{d}$
norman	$\frac{-\frac{4x}{25}-\frac{9\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{80d}+\frac{x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{25}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2-4}+\frac{123\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2\right)}{1600d}-\frac{123\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2\right)}{1600d}$
risch	$\frac{x}{25}+\frac{9i\left(3\,e^{i(dx+c)}+5\right)}{200d\left(5\,e^{2i(dx+c)}+6\,e^{i(dx+c)}+5\right)}-\frac{123\ln\left(e^{i(dx+c)}+\frac{3}{5}+\frac{4i}{5}\right)}{1600d}+\frac{123\ln\left(e^{i(dx+c)}+\frac{3}{5}-\frac{4i}{5}\right)}{1600d}$
parallelrisch	$\frac{320dx\cos(dx+c)+615\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2\right)\cos(dx+c)-615\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2\right)\cos(dx+c)+192dx+180\sin(dx+c)+36}{8000d\cos(dx+c)+4800d}$

input
```
int(1/(5+3*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(2/25*arctan(tan(1/2*d*x+1/2*c))-9/160/(tan(1/2*d*x+1/2*c)+2)-123/1600
*ln(tan(1/2*d*x+1/2*c)+2)-9/160/(tan(1/2*d*x+1/2*c)-2)+123/1600*ln(tan(1/2
*d*x+1/2*c)-2))
```

3.528.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.07

$$\int \frac{1}{(5+3\sec(c+dx))^2}\,dx = \frac{640\,dx\cos(dx+c)+384\,dx-123\,(5\,\cos(dx+c)+3)\log\left(\frac{3}{2}\,\cos(dx+c)+2\,\sin(dx+c)+\frac{5}{2}\right)+123\,(5}{3200\,(5\,d\cos(dx+c)+3\,d)}$$

input
```
integrate(1/(5+3*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/3200*(640*d*x*cos(d*x + c) + 384*d*x - 123*(5*cos(d*x + c) + 3)*log(3/2*
cos(d*x + c) + 2*sin(d*x + c) + 5/2) + 123*(5*cos(d*x + c) + 3)*log(3/2*co
s(d*x + c) - 2*sin(d*x + c) + 5/2) + 360*sin(d*x + c))/(5*d*cos(d*x + c) +
 3*d)
```

3.528.6 Sympy [F]

$$\int \frac{1}{(5+3\sec(c+dx))^2}\,dx = \int \frac{1}{(3\sec(c+dx)+5)^2}\,dx$$

input
```
integrate(1/(5+3*sec(d*x+c))**2,x)
```

output
```
Integral((3*sec(c + d*x) + 5)**(-2), x)
```

3.528.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.17

$$\int \frac{1}{(5+3\sec(c+dx))^2}\,dx = -\frac{\frac{180\,\sin(dx+c)}{\left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-4\right)(\cos(dx+c)+1)} - 128\,\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right) + 123\,\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+2\right) - 123\,\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+}\right.}{1600\,d}$$

input
```
integrate(1/(5+3*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
-1/1600*(180*sin(d*x + c)/((sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 4)*(cos(
d*x + c) + 1)) - 128*arctan(sin(d*x + c)/(cos(d*x + c) + 1)) + 123*log(sin
(d*x + c)/(cos(d*x + c) + 1) + 2) - 123*log(sin(d*x + c)/(cos(d*x + c) + 1
) - 2))/d
```

3.528.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.73

$$\int \frac{1}{(5+3\sec(c+dx))^2}\,dx = \frac{64\,dx + 64\,c - \frac{180\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-4} - 123\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+2\right|\right) + 123\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\right|\right)}{1600\,d}$$

input
```
integrate(1/(5+3*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
1/1600*(64*d*x + 64*c - 180*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^2 -
 4) - 123*log(abs(tan(1/2*d*x + 1/2*c) + 2)) + 123*log(abs(tan(1/2*d*x + 1
/2*c) - 2)))/d
```

3.528.9 Mupad [B] (verification not implemented)

Time = 14.31 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.55

$$\int \frac{1}{(5+3\sec(c+dx))^2}\,dx = \frac{x}{25} - \frac{\frac{123\,\mathrm{atanh}\left(\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2}\right)}{800} + \frac{9\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{80\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-4\right)}}{d}$$

input
```
int(1/(3/cos(c + d*x) + 5)^2,x)
```

output
```
x/25 - ((123*atanh(tan(c/2 + (d*x)/2)/2))/800 + (9*tan(c/2 + (d*x)/2))/(80
*(tan(c/2 + (d*x)/2)^2 - 4)))/d
```

3.529 $\int \frac{1}{(5+3\sec(c+dx))^3}\,dx$

3.529.1 Optimal result . . . 3962
3.529.2 Mathematica [B] (verified) . . . 3962
3.529.3 Rubi [A] (verified) . . . 3963
3.529.4 Maple [A] (verified) . . . 3966
3.529.5 Fricas [A] (verification not implemented) . . . 3967
3.529.6 Sympy [F] . . . 3967
3.529.7 Maxima [A] (verification not implemented) . . . 3968
3.529.8 Giac [A] (verification not implemented) . . . 3968
3.529.9 Mupad [B] (verification not implemented) . . . 3969

3.529.1 Optimal result

Integrand size = 12, antiderivative size = 120

$$\int \frac{1}{(5+3\sec(c+dx))^3}\,dx = \frac{x}{125}+\frac{8361\log\left(2\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}{256000d} - \frac{8361\log\left(2\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}{256000d} + \frac{9\tan(c+dx)}{160d(5+3\sec(c+dx))^2}+\frac{963\tan(c+dx)}{12800d(5+3\sec(c+dx))}$$

output
```
1/125*x+8361/256000*ln(2*cos(1/2*d*x+1/2*c)-sin(1/2*d*x+1/2*c))/d-8361/256
000*ln(2*cos(1/2*d*x+1/2*c)+sin(1/2*d*x+1/2*c))/d+9/160*tan(d*x+c)/d/(5+3*
sec(d*x+c))^2+963/12800*tan(d*x+c)/d/(5+3*sec(d*x+c))
```

3.529.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 241 vs. 2(120) = 240.

Time = 0.37 (sec) , antiderivative size = 241, normalized size of antiderivative = 2.01

$$\int \frac{1}{(5+3\sec(c+dx))^3}\,dx = \frac{88064c+88064dx+359523\log\left(2\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+60\cos(c+dx)\left(2048(c+dx)+8\right.}{}$$

input
```
Integrate[(5 + 3*Sec[c + d*x])^(-3),x]
```

output
```
(88064*c + 88064*d*x + 359523*Log[2*Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] +
 60*Cos[c + d*x]*(2048*(c + d*x) + 8361*Log[2*Cos[(c + d*x)/2] - Sin[(c +
d*x)/2]] - 8361*Log[2*Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + 25*Cos[2*(c
+ d*x)]*(2048*(c + d*x) + 8361*Log[2*Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]
- 8361*Log[2*Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) - 359523*Log[2*Cos[(c +
 d*x)/2] + Sin[(c + d*x)/2]] + 115560*Sin[c + d*x] + 110700*Sin[2*(c + d*x
)])/(512000*d*(3 + 5*Cos[c + d*x])^2)
```

3.529.3 Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.73, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 1.083$, Rules used = {3042, 4272, 25, 3042, 4548, 25, 3042, 4407, 3042, 4318, 3042, 3138, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(3\sec(c+dx)+5)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(3\csc\left(c+dx+\frac{\pi}{2}\right)+5\right)^3}dx$$

$$\Big\downarrow \text{4272}$$

$$\frac{9\tan(c+dx)}{160d(3\sec(c+dx)+5)^2}-\frac{1}{160}\int -\frac{9\sec^2(c+dx)-30\sec(c+dx)+32}{(3\sec(c+dx)+5)^2}dx$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{160}\int \frac{9\sec^2(c+dx)-30\sec(c+dx)+32}{(3\sec(c+dx)+5)^2}dx+\frac{9\tan(c+dx)}{160d(3\sec(c+dx)+5)^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{160}\int \frac{9\csc\left(c+dx+\frac{\pi}{2}\right)^2-30\csc\left(c+dx+\frac{\pi}{2}\right)+32}{\left(3\csc\left(c+dx+\frac{\pi}{2}\right)+5\right)^2}dx+\frac{9\tan(c+dx)}{160d(3\sec(c+dx)+5)^2}$$

$$\Big\downarrow \text{4548}$$

$$\frac{1}{160}\left(\frac{963\tan(c+dx)}{80d(3\sec(c+dx)+5)}-\frac{1}{80}\int -\frac{512-1365\sec(c+dx)}{3\sec(c+dx)+5}dx\right)+\frac{9\tan(c+dx)}{160d(3\sec(c+dx)+5)^2}$$

$$\Big\downarrow \; 25$$

$$\frac{1}{160}\left(\frac{1}{80}\int \frac{512-1365\sec(c+dx)}{3\sec(c+dx)+5}dx+\frac{963\tan(c+dx)}{80d(3\sec(c+dx)+5)}\right)+\frac{9\tan(c+dx)}{160d(3\sec(c+dx)+5)^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{160}\left(\frac{1}{80}\int \frac{512-1365\csc\left(c+dx+\frac{\pi}{2}\right)}{3\csc\left(c+dx+\frac{\pi}{2}\right)+5}dx+\frac{963\tan(c+dx)}{80d(3\sec(c+dx)+5)}\right)+\frac{9\tan(c+dx)}{160d(3\sec(c+dx)+5)^2}$$

$$\Big\downarrow \; 4407$$

$$\frac{1}{160}\left(\frac{1}{80}\left(\frac{512x}{5}-\frac{8361}{5}\int \frac{\sec(c+dx)}{3\sec(c+dx)+5}dx\right)+\frac{963\tan(c+dx)}{80d(3\sec(c+dx)+5)}\right)+\frac{9\tan(c+dx)}{160d(3\sec(c+dx)+5)^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{160}\left(\frac{1}{80}\left(\frac{512x}{5}-\frac{8361}{5}\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{3\csc\left(c+dx+\frac{\pi}{2}\right)+5}dx\right)+\frac{963\tan(c+dx)}{80d(3\sec(c+dx)+5)}\right)+\frac{9\tan(c+dx)}{160d(3\sec(c+dx)+5)^2}$$

$$\Big\downarrow \; 4318$$

$$\frac{1}{160}\left(\frac{1}{80}\left(\frac{512x}{5}-\frac{2787}{5}\int \frac{1}{\frac{5}{3}\cos(c+dx)+1}dx\right)+\frac{963\tan(c+dx)}{80d(3\sec(c+dx)+5)}\right)+\frac{9\tan(c+dx)}{160d(3\sec(c+dx)+5)^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{160}\left(\frac{1}{80}\left(\frac{512x}{5}-\frac{2787}{5}\int \frac{1}{\frac{5}{3}\sin\left(c+dx+\frac{\pi}{2}\right)+1}dx\right)+\frac{963\tan(c+dx)}{80d(3\sec(c+dx)+5)}\right)+\frac{9\tan(c+dx)}{160d(3\sec(c+dx)+5)^2}$$

$$\Big\downarrow \; 3138$$

$$\frac{1}{160}\left(\frac{1}{80}\left(\frac{512x}{5}-\frac{5574\int \frac{1}{\frac{8}{3}-\frac{2}{3}\tan^2\left(\frac{1}{2}(c+dx)\right)}d\tan\left(\frac{1}{2}(c+dx)\right)}{5d}\right)+\frac{963\tan(c+dx)}{80d(3\sec(c+dx)+5)}\right)+\frac{9\tan(c+dx)}{160d(3\sec(c+dx)+5)^2}$$

$$\Big\downarrow \; 219$$

3.529. $\int \frac{1}{(5+3\sec(c+dx))^3}\,dx$

$$\frac{1}{160}\left(\frac{1}{80}\left(\frac{512x}{5}-\frac{8361\text{arctanh}\left(\frac{1}{2}\tan\left(\frac{1}{2}(c+dx)\right)\right)}{10d}\right)+\frac{963\tan(c+dx)}{80d(3\sec(c+dx)+5)}\right)+\frac{9\tan(c+dx)}{160d(3\sec(c+dx)+5)^2}$$

input
```
Int[(5 + 3*Sec[c + d*x])^(-3),x]
```

output
```
(9*Tan[c + d*x])/(160*d*(5 + 3*Sec[c + d*x])^2) + (((512*x)/5 - (8361*ArcT
anh[Tan[(c + d*x)/2]/2])/(10*d))/80 + (963*Tan[c + d*x])/(80*d*(5 + 3*Sec[
c + d*x])))/160
```

3.529.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 4272
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[b^2*Cot[
c + d*x]*((a + b*Csc[c + d*x])^(n + 1)/(a*d*(n + 1)*(a^2 - b^2))), x] + Sim
p[1/(a*(n + 1)*(a^2 - b^2))   Int[(a + b*Csc[c + d*x])^(n + 1)*Simp[(a^2 -
b^2)*(n + 1) - a*b*(n + 1)*Csc[c + d*x] + b^2*(n + 2)*Csc[c + d*x]^2, x], x
], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && LtQ[n, -1] && Integ
erQ[2*n]
```

rule 4318
```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[1/b   Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4407
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Simp[(b*c - a*d)/a   Int[Csc[e + f*
x]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
- a*d, 0]
```

rule 4548
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(A*b^2 -
a*b*B + a^2*C)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(a*f*(m + 1)*(a^2
 - b^2))), x] + Simp[1/(a*(m + 1)*(a^2 - b^2))   Int[(a + b*Csc[e + f*x])^(
m + 1)*Simp[A*(a^2 - b^2)*(m + 1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x
] + (A*b^2 - a*b*B + a^2*C)*(m + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

3.529.4 Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.88

method	result
derivativedivides	$\frac{-\frac{27}{2560\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2\right)^2}-\frac{1323}{25600\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2\right)}+\frac{8361\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2\right)}{256000}+\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{125}+\frac{27}{2560\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2\right)^2}}{d}$
default	$\frac{-\frac{27}{2560\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2\right)^2}-\frac{1323}{25600\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2\right)}+\frac{8361\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2\right)}{256000}+\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{125}+\frac{27}{2560\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2\right)^2}}{d}$
risch	$\frac{x}{125}+\frac{27i\left(695\,\mathrm{e}^{3i(dx+c)}+1763\,\mathrm{e}^{2i(dx+c)}+1765\,\mathrm{e}^{i(dx+c)}+1025\right)}{32000d\left(5\,\mathrm{e}^{2i(dx+c)}+6\,\mathrm{e}^{i(dx+c)}+5\right)^2}+\frac{8361\ln\left(\mathrm{e}^{i(dx+c)}+\frac{3}{5}-\frac{4i}{5}\right)}{256000d}-\frac{8361\ln\left(\mathrm{e}^{i(dx+c)}+\frac{3}{5}\right.}{256000d}$
norman	$\frac{\frac{16x}{125}+\frac{1053\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{3200d}-\frac{1323\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{12800d}-\frac{8x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{125}+\frac{x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{125}}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2-4\right)^2}+\frac{8361\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2\right)}{256000d}-\frac{8361\ln\left(\right.}{}$
parallelrisch	$\frac{(501660\cos(dx+c)+209025\cos(2dx+2c)+359523)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2\right)+(-501660\cos(dx+c)-209025\cos(2dx+2c)-3}{256000d(43+25\cos}$

input
```
int(1/(5+3*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-27/2560/(tan(1/2*d*x+1/2*c)-2)^2-1323/25600/(tan(1/2*d*x+1/2*c)-2)+8
361/256000*ln(tan(1/2*d*x+1/2*c)-2)+2/125*arctan(tan(1/2*d*x+1/2*c))+27/25
60/(tan(1/2*d*x+1/2*c)+2)^2-1323/25600/(tan(1/2*d*x+1/2*c)+2)-8361/256000*
ln(tan(1/2*d*x+1/2*c)+2))
```

3.529.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.29

$$\int \frac{1}{(5+3\sec(c+dx))^3}\,dx$$
$$= \frac{102400\,dx\cos(dx+c)^2 + 122880\,dx\cos(dx+c) + 36864\,dx - 8361\left(25\cos(dx+c)^2 + 30\cos(dx+c) +\right.}{}$$

input
```
integrate(1/(5+3*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/512000*(102400*d*x*cos(d*x + c)^2 + 122880*d*x*cos(d*x + c) + 36864*d*x
- 8361*(25*cos(d*x + c)^2 + 30*cos(d*x + c) + 9)*log(3/2*cos(d*x + c) + 2*
sin(d*x + c) + 5/2) + 8361*(25*cos(d*x + c)^2 + 30*cos(d*x + c) + 9)*log(3
/2*cos(d*x + c) - 2*sin(d*x + c) + 5/2) + 1080*(205*cos(d*x + c) + 107)*si
n(d*x + c))/(25*d*cos(d*x + c)^2 + 30*d*cos(d*x + c) + 9*d)
```

3.529.6 Sympy [F]

$$\int \frac{1}{(5+3\sec(c+dx))^3}\,dx = \int \frac{1}{(3\sec(c+dx)+5)^3}\,dx$$

input
```
integrate(1/(5+3*sec(d*x+c))**3,x)
```

output
```
Integral((3*sec(c + d*x) + 5)**(-3), x)
```

3.529.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.29

$$\int \frac{1}{(5+3\sec(c+dx))^3}\,dx = -\frac{\frac{540\left(\frac{156\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{49\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{\frac{8\,\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-16}-4096\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)+8361\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+2\right)-8361\log\left(\frac{\sin(}{\cos(d}\right.}{256000\,d}$$

input
```
integrate(1/(5+3*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
-1/256000*(540*(156*sin(d*x + c)/(cos(d*x + c) + 1) - 49*sin(d*x + c)^3/(c
os(d*x + c) + 1)^3)/(8*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - sin(d*x + c)^
4/(cos(d*x + c) + 1)^4 - 16) - 4096*arctan(sin(d*x + c)/(cos(d*x + c) + 1)
) + 8361*log(sin(d*x + c)/(cos(d*x + c) + 1) + 2) - 8361*log(sin(d*x + c)/
(cos(d*x + c) + 1) - 2))/d
```

3.529.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.71

$$\int \frac{1}{(5+3\sec(c+dx))^3}\,dx = \frac{2048\,dx+2048\,c-\frac{540\left(49\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-156\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-4\right)^2}-8361\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+2\right|\right)+8361\log\left(\left|t\right.\right.}{256000\,d}$$

input
```
integrate(1/(5+3*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
1/256000*(2048*d*x + 2048*c - 540*(49*tan(1/2*d*x + 1/2*c)^3 - 156*tan(1/2
*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 4)^2 - 8361*log(abs(tan(1/2*d*x +
 1/2*c) + 2)) + 8361*log(abs(tan(1/2*d*x + 1/2*c) - 2)))/d
```

3.529.9 Mupad [B] (verification not implemented)

Time = 14.39 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.65

$$\int \frac{1}{(5+3\sec(c+dx))^3}\,dx = \frac{x}{125} - \frac{8361\,\mathrm{atanh}\left(\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2}\right)}{128000\,d} + \frac{\frac{1053\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{3200} - \frac{1323\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{12800}}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4 - 8\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2 + 16\right)}$$

input
```
int(1/(3/cos(c + d*x) + 5)^3,x)
```

output
```
x/125 - (8361*atanh(tan(c/2 + (d*x)/2)/2))/(128000*d) + ((1053*tan(c/2 + (
d*x)/2))/3200 - (1323*tan(c/2 + (d*x)/2)^3)/12800)/(d*(tan(c/2 + (d*x)/2)^
4 - 8*tan(c/2 + (d*x)/2)^2 + 16))
```

3.530 $\int \frac{1}{(5+3\sec(c+dx))^4}\,dx$

3.530.1 Optimal result . . . 3970
3.530.2 Mathematica [B] (verified) . . . 3971
3.530.3 Rubi [A] (verified) . . . 3971
3.530.4 Maple [C] (verified) . . . 3975
3.530.5 Fricas [A] (verification not implemented) . . . 3976
3.530.6 Sympy [F] . . . 3977
3.530.7 Maxima [A] (verification not implemented) . . . 3977
3.530.8 Giac [A] (verification not implemented) . . . 3977
3.530.9 Mupad [B] (verification not implemented) . . . 3978

3.530.1 Optimal result

Integrand size = 12, antiderivative size = 145

$$\begin{aligned}\int \frac{1}{(5+3\sec(c+dx))^4}\,dx = {} & \frac{x}{625}+\frac{278151\log\left(2\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}{20480000d} \\ & -\frac{278151\log\left(2\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}{20480000d} \\ & +\frac{3\tan(c+dx)}{80d(5+3\sec(c+dx))^3}+\frac{519\tan(c+dx)}{12800d(5+3\sec(c+dx))^2} \\ & +\frac{38733\tan(c+dx)}{1024000d(5+3\sec(c+dx))}\end{aligned}$$

output
```
1/625*x+278151/20480000*ln(2*cos(1/2*d*x+1/2*c)-sin(1/2*d*x+1/2*c))/d-2781
51/20480000*ln(2*cos(1/2*d*x+1/2*c)+sin(1/2*d*x+1/2*c))/d+3/80*tan(d*x+c)/
d/(5+3*sec(d*x+c))^3+519/12800*tan(d*x+c)/d/(5+3*sec(d*x+c))^2+38733/10240
00*tan(d*x+c)/d/(5+3*sec(d*x+c))
```

3.530.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 344 vs. $2(145) = 290$.

Time = 0.63 (sec) , antiderivative size = 344, normalized size of antiderivative = 2.37

$$\int \frac{1}{(5+3\sec(c+dx))^4}\,dx$$
$$= \frac{18284544c + 18284544dx + 4096000c\cos(3(c+dx)) + 4096000dx\cos(3(c+dx)) + 155208258\log\left(2\cos\right.}{}$$

input
```
Integrate[(5 + 3*Sec[c + d*x])^(-4),x]
```

output
```
(18284544*c + 18284544*d*x + 4096000*c*Cos[3*(c + d*x)] + 4096000*d*x*Cos[
3*(c + d*x)] + 155208258*Log[2*Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 3476
8875*Cos[3*(c + d*x)]*Log[2*Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 915*Cos
[c + d*x]*(32768*(c + d*x) + 278151*Log[2*Cos[(c + d*x)/2] - Sin[(c + d*x)
/2]] - 278151*Log[2*Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + 450*Cos[2*(c +
 d*x)]*(32768*(c + d*x) + 278151*Log[2*Cos[(c + d*x)/2] - Sin[(c + d*x)/2]
] - 278151*Log[2*Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) - 155208258*Log[2*C
os[(c + d*x)/2] + Sin[(c + d*x)/2]] - 34768875*Cos[3*(c + d*x)]*Log[2*Cos[
(c + d*x)/2] + Sin[(c + d*x)/2]] + 52174260*Sin[c + d*x] + 51462000*Sin[2*
(c + d*x)] + 24286500*Sin[3*(c + d*x)])/(81920000*d*(3 + 5*Cos[c + d*x])^3
)
```

3.530.3 Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.81, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 1.333$, Rules used = {3042, 4272, 27, 3042, 4548, 25, 3042, 4548, 25, 3042, 4407, 3042, 4318, 3042, 3138, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(3\sec(c+dx)+5)^4}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{1}{\left(3\csc\left(c+dx+\frac{\pi}{2}\right)+5\right)^4}dx$$

$$\Big\downarrow \text{4272}$$

$$\frac{3\tan(c+dx)}{80d(3\sec(c+dx)+5)^3}-\frac{1}{240}\int -\frac{3\left(6\sec^2(c+dx)-15\sec(c+dx)+16\right)}{(3\sec(c+dx)+5)^3}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{80}\int \frac{6\sec^2(c+dx)-15\sec(c+dx)+16}{(3\sec(c+dx)+5)^3}dx+\frac{3\tan(c+dx)}{80d(3\sec(c+dx)+5)^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{80}\int \frac{6\csc\left(c+dx+\frac{\pi}{2}\right)^2-15\csc\left(c+dx+\frac{\pi}{2}\right)+16}{\left(3\csc\left(c+dx+\frac{\pi}{2}\right)+5\right)^3}dx+\frac{3\tan(c+dx)}{80d(3\sec(c+dx)+5)^3}$$

$$\Big\downarrow \text{4548}$$

$$\frac{1}{80}\left(\frac{519\tan(c+dx)}{160d(3\sec(c+dx)+5)^2}-\frac{1}{160}\int -\frac{519\sec^2(c+dx)-1410\sec(c+dx)+512}{(3\sec(c+dx)+5)^2}dx\right)+\frac{3\tan(c+dx)}{80d(3\sec(c+dx)+5)^3}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{80}\left(\frac{1}{160}\int \frac{519\sec^2(c+dx)-1410\sec(c+dx)+512}{(3\sec(c+dx)+5)^2}dx+\frac{519\tan(c+dx)}{160d(3\sec(c+dx)+5)^2}\right)+\frac{3\tan(c+dx)}{80d(3\sec(c+dx)+5)^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{80}\left(\frac{1}{160}\int \frac{519\csc\left(c+dx+\frac{\pi}{2}\right)^2-1410\csc\left(c+dx+\frac{\pi}{2}\right)+512}{\left(3\csc\left(c+dx+\frac{\pi}{2}\right)+5\right)^2}dx+\frac{519\tan(c+dx)}{160d(3\sec(c+dx)+5)^2}\right)+\frac{3\tan(c+dx)}{80d(3\sec(c+dx)+5)^3}$$

$$\Big\downarrow \text{4548}$$

$$\frac{1}{80}\left(\frac{1}{160}\left(\frac{38733\tan(c+dx)}{80d(3\sec(c+dx)+5)}-\frac{1}{80}\int -\frac{8192-50715\sec(c+dx)}{3\sec(c+dx)+5}dx\right)+\frac{519\tan(c+dx)}{160d(3\sec(c+dx)+5)^2}\right)+\frac{3\tan(c+dx)}{80d(3\sec(c+dx)+5)^3}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{80}\left(\frac{1}{160}\left(\frac{1}{80}\int \frac{8192-50715\sec(c+dx)}{3\sec(c+dx)+5}dx+\frac{38733\tan(c+dx)}{80d(3\sec(c+dx)+5)}\right)+\frac{519\tan(c+dx)}{160d(3\sec(c+dx)+5)^2}\right)+\frac{3\tan(c+dx)}{80d(3\sec(c+dx)+5)^3}$$

3.530. $\int \frac{1}{(5+3\sec(c+dx))^4}\,dx$

$$\Big\downarrow \; 3042$$

$$\frac{1}{80}\left(\frac{1}{160}\left(\frac{1}{80}\int \frac{8192-50715\csc\left(c+dx+\frac{\pi}{2}\right)}{3\csc\left(c+dx+\frac{\pi}{2}\right)+5}dx+\frac{38733\tan(c+dx)}{80d(3\sec(c+dx)+5)}\right)+\frac{519\tan(c+dx)}{160d(3\sec(c+dx)+5)^2}\right)+\frac{3\tan(c+dx)}{80d(3\sec(c+dx)+5)^3}$$

$$\Big\downarrow \; 4407$$

$$\frac{1}{80}\left(\frac{1}{160}\left(\frac{1}{80}\left(\frac{8192x}{5}-\frac{278151}{5}\int \frac{\sec(c+dx)}{3\sec(c+dx)+5}dx\right)+\frac{38733\tan(c+dx)}{80d(3\sec(c+dx)+5)}\right)+\frac{519\tan(c+dx)}{160d(3\sec(c+dx)+5)^2}\right)\frac{3\tan(c+dx)}{80d(3\sec(c+dx)+5)^3}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{80}\left(\frac{1}{160}\left(\frac{1}{80}\left(\frac{8192x}{5}-\frac{278151}{5}\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{3\csc\left(c+dx+\frac{\pi}{2}\right)+5}dx\right)+\frac{38733\tan(c+dx)}{80d(3\sec(c+dx)+5)}\right)+\frac{519\tan(c+dx)}{160d(3\sec(c+dx)+\right.\frac{3\tan(c+dx)}{80d(3\sec(c+dx)+5)^3}$$

$$\Big\downarrow \; 4318$$

$$\frac{1}{80}\left(\frac{1}{160}\left(\frac{1}{80}\left(\frac{8192x}{5}-\frac{92717}{5}\int \frac{1}{\frac{5}{3}\cos(c+dx)+1}dx\right)+\frac{38733\tan(c+dx)}{80d(3\sec(c+dx)+5)}\right)+\frac{519\tan(c+dx)}{160d(3\sec(c+dx)+5)^2}\right)\frac{3\tan(c+dx)}{80d(3\sec(c+dx)+5)^3}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{80}\left(\frac{1}{160}\left(\frac{1}{80}\left(\frac{8192x}{5}-\frac{92717}{5}\int \frac{1}{\frac{5}{3}\sin\left(c+dx+\frac{\pi}{2}\right)+1}dx\right)+\frac{38733\tan(c+dx)}{80d(3\sec(c+dx)+5)}\right)+\frac{519\tan(c+dx)}{160d(3\sec(c+dx)+\right.\frac{3\tan(c+dx)}{80d(3\sec(c+dx)+5)^3}$$

$$\Big\downarrow \; 3138$$

$$\frac{1}{80}\left(\frac{1}{160}\left(\frac{1}{80}\left(\frac{8192x}{5}-\frac{185434\int \frac{1}{\frac{8}{3}-\frac{2}{3}\tan^2\left(\frac{1}{2}(c+dx)\right)}d\tan\left(\frac{1}{2}(c+dx)\right)}{5d}\right)+\frac{38733\tan(c+dx)}{80d(3\sec(c+dx)+5)}\right)+\frac{519\tan}{160d(3\sec(}\right.\frac{3\tan(c+dx)}{80d(3\sec(c+dx)+5)^3}$$

$$\Big\downarrow \; 219$$

3.530. $\int \frac{1}{(5+3\sec(c+dx))^4}\,dx$

$$\frac{1}{80}\left(\frac{1}{160}\left(\frac{1}{80}\left(\frac{8192x}{5}-\frac{278151\text{arctanh}\left(\frac{1}{2}\tan\left(\frac{1}{2}(c+dx)\right)\right)}{10d}\right)+\frac{38733\tan(c+dx)}{80d(3\sec(c+dx)+5)}\right)+\frac{519\tan(c+dx)}{160d(3\sec(c+dx)+}\right.$$
$$\frac{3\tan(c+dx)}{80d(3\sec(c+dx)+5)^3}$$

input
```
Int[(5 + 3*Sec[c + d*x])^(-4),x]
```

output
```
(3*Tan[c + d*x])/(80*d*(5 + 3*Sec[c + d*x])^3) + ((519*Tan[c + d*x])/(160*
d*(5 + 3*Sec[c + d*x])^2) + (((8192*x)/5 - (278151*ArcTanh[Tan[(c + d*x)/2
]/2])/(10*d))/80 + (38733*Tan[c + d*x])/(80*d*(5 + 3*Sec[c + d*x])))/160)/
80
```

3.530.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 4272
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[b^2*Cot[
c + d*x]*((a + b*Csc[c + d*x])^(n + 1)/(a*d*(n + 1)*(a^2 - b^2))), x] + Sim
p[1/(a*(n + 1)*(a^2 - b^2))   Int[(a + b*Csc[c + d*x])^(n + 1)*Simp[(a^2 -
b^2)*(n + 1) - a*b*(n + 1)*Csc[c + d*x] + b^2*(n + 2)*Csc[c + d*x]^2, x], x
], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && LtQ[n, -1] && Integ
erQ[2*n]
```

rule 4318
```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[1/b   Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4407
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Simp[(b*c - a*d)/a   Int[Csc[e + f*
x]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
- a*d, 0]
```

rule 4548
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(A*b^2 -
a*b*B + a^2*C)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(a*f*(m + 1)*(a^2
 - b^2))), x] + Simp[1/(a*(m + 1)*(a^2 - b^2))   Int[(a + b*Csc[e + f*x])^(
m + 1)*Simp[A*(a^2 - b^2)*(m + 1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x
] + (A*b^2 - a*b*B + a^2*C)*(m + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

3.530.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.49 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.91

method	result
risch	$\frac{x}{625}+\frac{27i\left(166525\,\mathrm{e}^{5i(dx+c)}+581495\,\mathrm{e}^{4i(dx+c)}+1003842\,\mathrm{e}^{3i(dx+c)}+1064590\,\mathrm{e}^{2i(dx+c)}+643025\,\mathrm{e}^{i(dx+c)}+224875\right)}{2560000d\left(5\,\mathrm{e}^{2i(dx+c)}+6\,\mathrm{e}^{i(dx+c)}+5\right)^3}-$
derivativedivides	$\frac{-\frac{27}{10240\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2\right)^3}+\frac{1431}{102400\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2\right)^2}-\frac{69093}{2048000\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2\right)}-\frac{278151\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2\right)}{20480000}+\frac{2\arctan\left(\tan\left(\right.\right.}{625}}{d}$
default	$\frac{-\frac{27}{10240\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2\right)^3}+\frac{1431}{102400\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2\right)^2}-\frac{69093}{2048000\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2\right)}-\frac{278151\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2\right)}{20480000}+\frac{2\arctan\left(\tan\left(\right.\right.}{625}}{d}$
norman	$\frac{-\frac{64x}{625}-\frac{44523\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{64000d}+\frac{13527\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{32000d}-\frac{69093\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{1024000d}+\frac{48x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{625}-\frac{12x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{625}+\frac{x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{625}}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2-4\right)^3}$
parallelrisch	$\frac{(254508165\cos(dx+c)+125167950\cos(2dx+2c)+34768875\cos(3dx+3c)+155208258)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2\right)+(-2545081}{}$

input
```
int(1/(5+3*sec(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
1/625*x+27/2560000*I*(166525*exp(5*I*(d*x+c))+581495*exp(4*I*(d*x+c))+1003
842*exp(3*I*(d*x+c))+1064590*exp(2*I*(d*x+c))+643025*exp(I*(d*x+c))+224875
)/d/(5*exp(2*I*(d*x+c))+6*exp(I*(d*x+c))+5)^3-278151/20480000/d*ln(exp(I*(
d*x+c))+3/5+4/5*I)+278151/20480000/d*ln(exp(I*(d*x+c))+3/5-4/5*I)
```

3.530.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.43

$$\int \frac{1}{(5+3\sec(c+dx))^4}\,dx$$
$$=\frac{8192000\,dx\cos(dx+c)^3+14745600\,dx\cos(dx+c)^2+8847360\,dx\cos(dx+c)+1769472\,dx-278151\,(}{}$$

input
```
integrate(1/(5+3*sec(d*x+c))^4,x, algorithm="fricas")
```

output
```
1/40960000*(8192000*d*x*cos(d*x + c)^3 + 14745600*d*x*cos(d*x + c)^2 + 884
7360*d*x*cos(d*x + c) + 1769472*d*x - 278151*(125*cos(d*x + c)^3 + 225*cos
(d*x + c)^2 + 135*cos(d*x + c) + 27)*log(3/2*cos(d*x + c) + 2*sin(d*x + c)
 + 5/2) + 278151*(125*cos(d*x + c)^3 + 225*cos(d*x + c)^2 + 135*cos(d*x +
c) + 27)*log(3/2*cos(d*x + c) - 2*sin(d*x + c) + 5/2) + 1080*(44975*cos(d*
x + c)^2 + 47650*cos(d*x + c) + 12911)*sin(d*x + c))/(125*d*cos(d*x + c)^3
 + 225*d*cos(d*x + c)^2 + 135*d*cos(d*x + c) + 27*d)
```

3.530.6 Sympy [F]

$$\int \frac{1}{(5+3\sec(c+dx))^4}\,dx = \int \frac{1}{(3\sec(c+dx)+5)^4}\,dx$$

input
```
integrate(1/(5+3*sec(d*x+c))**4,x)
```

output
```
Integral((3*sec(c + d*x) + 5)**(-4), x)
```

3.530.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.34

$$\int \frac{1}{(5+3\sec(c+dx))^4}\,dx = -\frac{\frac{540\left(\frac{26384\sin(dx+c)}{\cos(dx+c)+1}-\frac{16032\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{2559\sin(dx+c)^5}{(\cos(dx+c)+1)^5}\right)}{\frac{48\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{12\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-64}-65536\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)+278151\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+2\right.}{20480000\,d}$$

input
```
integrate(1/(5+3*sec(d*x+c))^4,x, algorithm="maxima")
```

output
```
-1/20480000*(540*(26384*sin(d*x + c)/(cos(d*x + c) + 1) - 16032*sin(d*x +
c)^3/(cos(d*x + c) + 1)^3 + 2559*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/(48*
sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 12*sin(d*x + c)^4/(cos(d*x + c) + 1)
^4 + sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 64) - 65536*arctan(sin(d*x + c)
/(cos(d*x + c) + 1)) + 278151*log(sin(d*x + c)/(cos(d*x + c) + 1) + 2) - 2
78151*log(sin(d*x + c)/(cos(d*x + c) + 1) - 2))/d
```

3.530.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.68

$$\int \frac{1}{(5+3\sec(c+dx))^4}\,dx = \frac{32768\,dx+32768\,c-\frac{540\left(2559\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-16032\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+26384\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-4\right)^3}-278151\log\left(\left|\tan\left(\frac{1}{2}dx-\right.\right.\right.}{20480000\,d}$$

input
```
integrate(1/(5+3*sec(d*x+c))^4,x, algorithm="giac")
```

output
```
1/20480000*(32768*d*x + 32768*c - 540*(2559*tan(1/2*d*x + 1/2*c)^5 - 16032
*tan(1/2*d*x + 1/2*c)^3 + 26384*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c
)^2 - 4)^3 - 278151*log(abs(tan(1/2*d*x + 1/2*c) + 2)) + 278151*log(abs(ta
n(1/2*d*x + 1/2*c) - 2)))/d
```

3.530.9 Mupad [B] (verification not implemented)

Time = 14.47 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.72

$$\int \frac{1}{(5+3\sec(c+dx))^4}\,dx = \frac{x}{625} - \frac{278151\,\mathrm{atanh}\left(\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2}\right)}{10240000\,d} - \frac{\frac{69093\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{1024000} - \frac{13527\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{32000} + \frac{44523\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{64000}}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6 - 12\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4 + 48\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2 - 64\right)}$$

input
```
int(1/(3/cos(c + d*x) + 5)^4,x)
```

output
```
x/625 - (278151*atanh(tan(c/2 + (d*x)/2)/2))/(10240000*d) - ((44523*tan(c/
2 + (d*x)/2))/64000 - (13527*tan(c/2 + (d*x)/2)^3)/32000 + (69093*tan(c/2
+ (d*x)/2)^5)/1024000)/(d*(48*tan(c/2 + (d*x)/2)^2 - 12*tan(c/2 + (d*x)/2)
^4 + tan(c/2 + (d*x)/2)^6 - 64))
```

3.531 $\int \sec^3(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$

3.531.1 Optimal result . . . 3979
3.531.2 Mathematica [A] (warning: unable to verify) . . . 3980
3.531.3 Rubi [A] (verified) . . . 3980
3.531.4 Maple [B] (verified) . . . 3984
3.531.5 Fricas [F] . . . 3985
3.531.6 Sympy [F] . . . 3985
3.531.7 Maxima [F] . . . 3985
3.531.8 Giac [F] . . . 3986
3.531.9 Mupad [F(-1)] . . . 3986

3.531.1 Optimal result

Integrand size = 23, antiderivative size = 292

$$\int \sec^3(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$$
$$= \frac{2(a-b)\sqrt{a+b}(2a^2-9b^2)\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{15b^3d}$$
$$+\frac{2(a-b)\sqrt{a+b}(2a+9b)\cot(c+dx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c}{a-}}}{15b^2d}$$
$$-\frac{4a\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{15bd}+\frac{2(a+b\sec(c+dx))^{3/2}\tan(c+dx)}{5bd}$$

output
```
2/15*(a-b)*(2*a^2-9*b^2)*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(a+b)
^(1/2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b
*(1+sec(d*x+c))/(a-b))^(1/2)/b^3/d+2/15*(a-b)*(2*a+9*b)*cot(d*x+c)*Ellipti
cF((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*
(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/b^2/d+2/5*(a+b
*sec(d*x+c))^(3/2)*tan(d*x+c)/b/d-4/15*a*(a+b*sec(d*x+c))^(1/2)*tan(d*x+c)
/b/d
```

3.531.2 Mathematica [A] (warning: unable to verify)

Time = 27.52 (sec) , antiderivative size = 366, normalized size of antiderivative = 1.25

$$\int \sec^3(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$$
$$= \frac{2\sqrt{a+b\sec(c+dx)}\left((-2a^2+9b^2)\sin(c+dx)+\frac{\sqrt{\sec^2\left(\frac{1}{2}(c+dx)\right)}\left(\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\right)^{3/2}\left(2(2a^3+2a^2b-9ab^2-9b^3)\right.}{}\right.}{}$$

input
```
Integrate[Sec[c + d*x]^3*Sqrt[a + b*Sec[c + d*x]],x]
```

output
```
(2*Sqrt[a + b*Sec[c + d*x]]*((-2*a^2 + 9*b^2)*Sin[c + d*x] + (Sqrt[Sec[(c
+ d*x)/2]^2]*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(3/2)*(2*(2*a^3 + 2*a^2*b -
 9*a*b^2 - 9*b^3)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c
+ d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]],
(a - b)/(a + b)] + 2*b*(-2*a^2 + 7*a*b + 9*b^2)*Sqrt[Cos[c + d*x]/(1 + Cos
[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Ellipt
icF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + (2*a^2 - 9*b^2)*Cos[c + d
*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/((b + a*Cos
[c + d*x])*Sec[c + d*x]^(3/2)) + b*(a + 3*b*Sec[c + d*x])*Tan[c + d*x]))/(
15*b^2*d)
```

3.531.3 Rubi [A] (verified)

Time = 1.08 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.02, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules used = {3042, 4327, 27, 3042, 4490, 27, 3042, 4493, 3042, 4319, 4492}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^3(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^3\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx$$
$$\Big\downarrow \text{4327}$$

$$\frac{2\int \frac{1}{2}\sec(c+dx)(3b-2a\sec(c+dx))\sqrt{a+b\sec(c+dx)}dx}{5b}+\frac{2\tan(c+dx)(a+b\sec(c+dx))^{3/2}}{5bd}$$

$\Big\downarrow$ 27

$$\frac{\int \sec(c+dx)(3b-2a\sec(c+dx))\sqrt{a+b\sec(c+dx)}dx}{5b}+\frac{2\tan(c+dx)(a+b\sec(c+dx))^{3/2}}{5bd}$$

$\Big\downarrow$ 3042

$$\frac{\int \csc\left(c+dx+\frac{\pi}{2}\right)\left(3b-2a\csc\left(c+dx+\frac{\pi}{2}\right)\right)\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{5b}+\frac{2\tan(c+dx)(a+b\sec(c+dx))^{3/2}}{5bd}$$

$\Big\downarrow$ 4490

$$\frac{\frac{2}{3}\int \frac{\sec(c+dx)\left(7ab-\left(2a^2-9b^2\right)\sec(c+dx)\right)}{2\sqrt{a+b\sec(c+dx)}}dx-\frac{4a\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}}{5b}+\frac{2\tan(c+dx)(a+b\sec(c+dx))^{3/2}}{5bd}$$

$\Big\downarrow$ 27

$$\frac{\frac{1}{3}\int \frac{\sec(c+dx)\left(7ab-\left(2a^2-9b^2\right)\sec(c+dx)\right)}{\sqrt{a+b\sec(c+dx)}}dx-\frac{4a\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}}{5b}+\frac{2\tan(c+dx)(a+b\sec(c+dx))^{3/2}}{5bd}$$

$\Big\downarrow$ 3042

$$\frac{\frac{1}{3}\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(7ab+\left(9b^2-2a^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{4a\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}}{5b}+\frac{2\tan(c+dx)(a+b\sec(c+dx))^{3/2}}{5bd}$$

$\Big\downarrow$ 4493

$$\frac{\frac{1}{3}\left((a-b)(2a+9b)\int \frac{\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx-\left(2a^2-9b^2\right)\int \frac{\sec(c+dx)(\sec(c+dx)+1)}{\sqrt{a+b\sec(c+dx)}}dx\right)-\frac{4a\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}}{5b}+\frac{2\tan(c+dx)(a+b\sec(c+dx))^{3/2}}{5bd}$$

$\Big\downarrow$ 3042

3.531. $\int \sec^3(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$

$$\frac{\frac{1}{3}\left((a-b)(2a+9b)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-\left(2a^2-9b^2\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)-\frac{4a\tan(c+dx)\sqrt{a+b\sec(c+}}{3d}}{5b}$$
$$\frac{2\tan(c+dx)(a+b\sec(c+dx))^{3/2}}{5bd}$$

$\downarrow$ 4319

$$\frac{\frac{1}{3}\left(\frac{2(a-b)\sqrt{a+b}(2a+9b)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{bd}-\left(2a^2-9b^2\right)\int \frac{\csc(c+d}{\sqrt{}}\right.}{5b}$$
$$\frac{2\tan(c+dx)(a+b\sec(c+dx))^{3/2}}{5bd}$$

$\downarrow$ 4492

$$\frac{\frac{1}{3}\left(\frac{2(a-b)\sqrt{a+b}(2a^2-9b^2)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,E\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{b^2d}+\frac{2(a-b)\sqrt{a+b}(2a+9b)\cot(c+dx}{}\right.}{5b}$$
$$\frac{2\tan(c+dx)(a+b\sec(c+dx))^{3/2}}{5bd}$$

input
```
Int[Sec[c + d*x]^3*Sqrt[a + b*Sec[c + d*x]],x]
```

output
```
(2*(a + b*Sec[c + d*x])^(3/2)*Tan[c + d*x])/(5*b*d) + (((2*(a - b)*Sqrt[a
+ b]*(2*a^2 - 9*b^2)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]
]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt
[-((b*(1 + Sec[c + d*x]))/(a - b))])/(b^2*d) + (2*(a - b)*Sqrt[a + b]*(2*a
 + 9*b)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]
], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Se
c[c + d*x]))/(a - b))])/(b*d))/3 - (4*a*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d
*x])/(3*d))/(5*b)
```

3.531. $\int \sec^3(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$

3.531.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4319
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4327
```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(-Cot[e + f*x])*((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2
))), x] + Simp[1/(b*(m + 2))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(b*(
m + 1) - a*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 -
b^2, 0] &&  !LtQ[m, -1]
```

rule 4490
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*((
a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Simp[1/(m + 1)   Int[Csc[e + f*x]*
(a + b*Csc[e + f*x])^(m - 1)*Simp[b*B*m + a*A*(m + 1) + (a*B*m + A*b*(m + 1
))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*
B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

rule 4492
```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

rule 4493
```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(A - B)   Int[Csc[e
 + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Simp[B   Int[Csc[e + f*x]*((1 +
Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A, B}
, x] && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

3.531.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1981 vs. $2(262) = 524$.

Time = 11.20 (sec) , antiderivative size = 1982, normalized size of antiderivative = 6.79

method	result	size
default	Expression too large to display	1982

input
```
int(sec(d*x+c)^3*(a+b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/15/d/b^2*(a+b*sec(d*x+c))^(1/2)/(b+a*cos(d*x+c))/(cos(d*x+c)+1)*(-4*a*b
^2*sin(d*x+c)+a^2*b*sin(d*x+c)+2*EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a
+b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(c
os(d*x+c)+1))^(1/2)*a^2*b-9*EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^
(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*
x+c)+1))^(1/2)*a*b^2+2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(co
s(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b
))^(1/2))*a^3*cos(d*x+c)^2-9*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/
2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(cot(d*x+c)-csc(d*x+c),((a-b
)/(a+b))^(1/2))*b^3*cos(d*x+c)^2-2*EllipticF(cot(d*x+c)-csc(d*x+c),((a-b)/
(a+b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/
(cos(d*x+c)+1))^(1/2)*a^2*b+7*EllipticF(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b)
)^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(
d*x+c)+1))^(1/2)*a*b^2+18*EllipticF(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1
/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+
c)+1))^(1/2)*b^3*cos(d*x+c)+4*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1
/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(cot(d*x+c)-csc(d*x+c),((a-
b)/(a+b))^(1/2))*a^3*cos(d*x+c)+9*EllipticF(cot(d*x+c)-csc(d*x+c),((a-b)/(
a+b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(
cos(d*x+c)+1))^(1/2)*b^3*cos(d*x+c)^2-18*(1/(a+b)*(b+a*cos(d*x+c))/(cos...
```

3.531.5 Fricas [F]

$$\int \sec^3(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = \int \sqrt{b\sec(dx+c)+a}\sec(dx+c)^3\,dx$$

input
```
integrate(sec(d*x+c)^3*(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^3, x)
```

3.531.6 Sympy [F]

$$\int \sec^3(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = \int \sqrt{a+b\sec(c+dx)}\sec^3(c+dx)\,dx$$

input
```
integrate(sec(d*x+c)**3*(a+b*sec(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(a + b*sec(c + d*x))*sec(c + d*x)**3, x)
```

3.531.7 Maxima [F]

$$\int \sec^3(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = \int \sqrt{b\sec(dx+c)+a}\sec(dx+c)^3\,dx$$

input
```
integrate(sec(d*x+c)^3*(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^3, x)
```

3.531.8 Giac [F]

$$\int \sec^3(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = \int \sqrt{b\sec(dx+c)+a}\sec(dx+c)^3\,dx$$

input
```
integrate(sec(d*x+c)^3*(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^3, x)
```

3.531.9 Mupad [F(-1)]

Timed out.

$$\int \sec^3(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = \int \frac{\sqrt{a+\frac{b}{\cos(c+d\,x)}}}{\cos(c+d\,x)^3}\,dx$$

input
```
int((a + b/cos(c + d*x))^(1/2)/cos(c + d*x)^3,x)
```

output
```
int((a + b/cos(c + d*x))^(1/2)/cos(c + d*x)^3, x)
```

3.532 $\int \sec^2(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$

3.532.1 Optimal result . . . 3987
3.532.2 Mathematica [A] (verified) . . . 3988
3.532.3 Rubi [A] (verified) . . . 3988
3.532.4 Maple [B] (verified) . . . 3991
3.532.5 Fricas [F] . . . 3991
3.532.6 Sympy [F] . . . 3992
3.532.7 Maxima [F] . . . 3992
3.532.8 Giac [F] . . . 3992
3.532.9 Mupad [F(-1)] . . . 3993

3.532.1 Optimal result

Integrand size = 23, antiderivative size = 241

$$\int \sec^2(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = -\frac{2a(a-b)\sqrt{a+b}\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{3b^2d} - \frac{2(a-b)\sqrt{a+b}\cot(c+dx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{3bd} + \frac{2\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{3d}$$

output
```
-2/3*a*(a-b)*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b
)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+
c))/(a-b))^(1/2)/b^2/d-2/3*(a-b)*cot(d*x+c)*EllipticF((a+b*sec(d*x+c))^(1/
2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(
1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/b/d+2/3*(a+b*sec(d*x+c))^(1/2)*tan(d*
x+c)/d
```

3.532.2 Mathematica [A] (verified)

Time = 15.79 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.22

$$\int \sec^2(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = -\frac{2\cos^2\left(\frac{1}{2}(c+dx)\right)\sqrt{a+b\sec(c+dx)}\left(2a(a+b)\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\sqrt{\frac{b+a\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}E\left(\arcsin\left(\tan\left(\frac{1}{2}(c+d\right.\right.\right.}{}$$

$$+\frac{\sqrt{a+b\sec(c+dx)}\left(\frac{2a\sin(c+dx)}{3b}+\frac{2}{3}\tan(c+dx)\right)}{d}$$

input
```
Integrate[Sec[c + d*x]^2*Sqrt[a + b*Sec[c + d*x]],x]
```

output
```
(-2*Cos[(c + d*x)/2]^2*Sqrt[a + b*Sec[c + d*x]]*(2*a*(a + b)*Sqrt[Cos[c +
d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d
*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] - 2*b*(a + b)*
Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(
1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] +
 a*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))
/(3*b*d*(b + a*Cos[c + d*x])) + (Sqrt[a + b*Sec[c + d*x]]*((2*a*Sin[c + d*
x])/(3*b) + (2*Tan[c + d*x])/3))/d
```

3.532.3 Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3042, 4322, 3042, 4493, 3042, 4319, 4492}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^2\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \; 4322$$

$$\frac{1}{3}\int \frac{\sec(c+dx)(b+a\sec(c+dx))}{\sqrt{a+b\sec(c+dx)}}dx+\frac{2\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}$$

$\Big\downarrow$ 3042

$$\frac{1}{3}\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(b+a\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}$$

$\Big\downarrow$ 4493

$$\frac{1}{3}\left(a\int \frac{\sec(c+dx)(\sec(c+dx)+1)}{\sqrt{a+b\sec(c+dx)}}dx-(a-b)\int \frac{\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx\right)+\frac{2\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}$$

$\Big\downarrow$ 3042

$$\frac{1}{3}\left(a\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-(a-b)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)+\frac{2\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}$$

$\Big\downarrow$ 4319

$$\frac{1}{3}\left(a\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2(a-b)\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}}{bd}\right.$$
$$\frac{2\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}$$

$\Big\downarrow$ 4492

$$\frac{1}{3}\left(-\frac{2a(a-b)\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}E\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{b^2d}-\frac{2(a-b)\sqrt{a}}{}\right.$$
$$\frac{2\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}$$

input
```
Int[Sec[c + d*x]^2*Sqrt[a + b*Sec[c + d*x]],x]
```

3.532. $\int \sec^2(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$

output
```
((-2*a*(a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c
+ d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)
]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(b^2*d) - (2*(a - b)*Sqrt[a + b
]*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a
+ b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c +
d*x]))/(a - b))])/(b*d))/3 + (2*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(3*
d)
```

3.532.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4319
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4322
```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(-Cot[e + f*x])*((a + b*Csc[e + f*x])^m/(f*(m + 1))), x]
+ Simp[m/(m + 1)   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(b + a*Csc
[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m,
 0]
```

rule 4492
```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

rule 4493
```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(A - B)   Int[Csc[e
 + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Simp[B   Int[Csc[e + f*x]*((1 +
Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A, B}
, x] && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

3.532.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1126 vs. 2(215) = 430.

Time = 9.64 (sec) , antiderivative size = 1127, normalized size of antiderivative = 4.68

method	result	size
default	Expression too large to display	1127

input
```
int(sec(d*x+c)^2*(a+b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3/d/b*(a+b*sec(d*x+c))^(1/2)/(b+a*cos(d*x+c))/(cos(d*x+c)+1)*(EllipticF
(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos
(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*a*b*cos(d*x+c)^2+Ellip
ticF(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/
(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*b^2*cos(d*x+c)^2-E
llipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+
c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*a^2*cos(d*x+c)
^2-EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*(1/(a+b)*(b+a*cos(
d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*a*b*cos(d*
x+c)^2+2*EllipticF(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*(1/(a+b)*(b+
a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*a*b*
cos(d*x+c)+2*EllipticF(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*(1/(a+b)
*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*
b^2*cos(d*x+c)-2*EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*(1/(
a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1
/2)*a^2*cos(d*x+c)-2*EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*
(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1)
)^(1/2)*a*b*cos(d*x+c)+(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ell
ipticF(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+
1))^(1/2)*a*b+(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF...
```

3.532.5 Fricas [F]

$$\int \sec^2(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = \int \sqrt{b\sec\left(dx+c\right)+a}\sec\left(dx+c\right)^2\,dx$$

input
```
integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^2, x)
```

3.532.6 Sympy [F]

$$\int \sec^2(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = \int \sqrt{a+b\sec(c+dx)}\sec^2(c+dx)\,dx$$

input
```
integrate(sec(d*x+c)**2*(a+b*sec(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(a + b*sec(c + d*x))*sec(c + d*x)**2, x)
```

3.532.7 Maxima [F]

$$\int \sec^2(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = \int \sqrt{b\sec(dx+c)+a}\sec(dx+c)^2\,dx$$

input
```
integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^2, x)
```

3.532.8 Giac [F]

$$\int \sec^2(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = \int \sqrt{b\sec(dx+c)+a}\sec(dx+c)^2\,dx$$

input
```
integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^2, x)
```

3.532.9 Mupad [F(-1)]

Timed out.

$$\int \sec^2(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = \int \frac{\sqrt{a+\frac{b}{\cos(c+d\,x)}}}{\cos\left(c+d\,x\right)^2}\,dx$$

input
```
int((a + b/cos(c + d*x))^(1/2)/cos(c + d*x)^2,x)
```

output
```
int((a + b/cos(c + d*x))^(1/2)/cos(c + d*x)^2, x)
```

3.533 $\int \sec(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$

3.533.1 Optimal result . . . 3994
3.533.2 Mathematica [A] (verified) . . . 3994
3.533.3 Rubi [A] (verified) . . . 3995
3.533.4 Maple [B] (verified) . . . 3997
3.533.5 Fricas [F] . . . 3998
3.533.6 Sympy [F] . . . 3998
3.533.7 Maxima [F] . . . 3998
3.533.8 Giac [F] . . . 3999
3.533.9 Mupad [F(-1)] . . . 3999

3.533.1 Optimal result

Integrand size = 21, antiderivative size = 209

$$\int \sec(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = -\frac{2(a-b)\sqrt{a+b}\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{bd} + \frac{2(a-b)\sqrt{a+b}\cot(c+dx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{bd}$$

output
```
-2*(a-b)*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a
-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/
(a-b))^(1/2)/b/d+2*(a-b)*cot(d*x+c)*EllipticF((a+b*sec(d*x+c))^(1/2)/(a+b)
^(1/2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b
*(1+sec(d*x+c))/(a-b))^(1/2)/b/d
```

3.533.2 Mathematica [A] (verified)

Time = 8.44 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.00

$$\int \sec(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = \frac{2\sqrt{a+b\sec(c+dx)}\left(\sin(c+dx) - \frac{\sqrt{\sec^2\left(\frac{1}{2}(c+dx)\right)}\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)}\left(\frac{(a+b)\left(E\left(\arcsin\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{a-b}{a+b}\right)-\text{EllipticF}\right.}{\sqrt{1+}}\right.}{(b+a\cos(c+dx))\,\text{se}}\right.}{d}$$

input
```
Integrate[Sec[c + d*x]*Sqrt[a + b*Sec[c + d*x]],x]
```

output
```
(2*Sqrt[a + b*Sec[c + d*x]]*(Sin[c + d*x] - (Sqrt[Sec[(c + d*x)/2]^2]*Sqrt
[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(((a + b)*(EllipticE[ArcSin[Tan[(c + d*x
)/2]], (a - b)/(a + b)] - EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a +
 b)])*Sqrt[(a + b*Sec[c + d*x])/((a + b)*(1 + Sec[c + d*x]))])/Sqrt[(1 + S
ec[c + d*x])^(-1)] + (b + a*Cos[c + d*x])*Tan[(c + d*x)/2]))/((b + a*Cos[c
 + d*x])*Sec[c + d*x]^(3/2))))/d
```

3.533.3 Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3042, 4316, 3042, 4319, 4492}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{4316}$$

$$(a-b)\int \frac{\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx+b\int \frac{\sec(c+dx)(\sec(c+dx)+1)}{\sqrt{a+b\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$(a-b)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+b\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4319}$$

$$b\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2(a-b)\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{bd}$$

$$\Big\downarrow \text{4492}$$

$$\frac{2(a-b)\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{bd}-\frac{2(a-b)\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}E\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{bd}$$

input
```
Int[Sec[c + d*x]*Sqrt[a + b*Sec[c + d*x]],x]
```

output
```
(-2*(a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d
*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*S
qrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(b*d) + (2*(a - b)*Sqrt[a + b]*Cot
[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/
(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x])
)/(a - b))])/(b*d)
```

3.533.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4316
```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(a - b)   Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x]
+ Simp[b   Int[Csc[e + f*x]*((1 + Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]),
x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4319
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4492
```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

3.533.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1062 vs. $2(191) = 382$.

Time = 8.45 (sec) , antiderivative size = 1063, normalized size of antiderivative = 5.09

method	result	size
default	Expression too large to display	1063

input
```
int(sec(d*x+c)*(a+b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/d*(EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos
(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a*cos(d*
x+c)^2+EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(c
os(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*b*cos(
d*x+c)^2-EllipticF(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/
(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a*co
s(d*x+c)^2-EllipticF(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c
)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*b*
cos(d*x+c)^2+2*EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d
*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2
)*a*cos(d*x+c)+2*EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*(cos
(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1
/2)*b*cos(d*x+c)-2*EllipticF(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*(c
os(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^
(1/2)*a*cos(d*x+c)-2*EllipticF(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*
(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1)
)^(1/2)*b*cos(d*x+c)+EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*
(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1)
)^(1/2)*a+EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)
/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*...
```

3.533.5 Fricas [F]

$$\int \sec(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = \int \sqrt{b\sec(dx+c)+a}\sec(dx+c)\,dx$$

input
```
integrate(sec(d*x+c)*(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c), x)
```

3.533.6 Sympy [F]

$$\int \sec(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = \int \sqrt{a+b\sec(c+dx)}\sec(c+dx)\,dx$$

input
```
integrate(sec(d*x+c)*(a+b*sec(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(a + b*sec(c + d*x))*sec(c + d*x), x)
```

3.533.7 Maxima [F]

$$\int \sec(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = \int \sqrt{b\sec(dx+c)+a}\sec(dx+c)\,dx$$

input
```
integrate(sec(d*x+c)*(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sec(d*x + c) + a)*sec(d*x + c), x)
```

3.533.8 Giac [F]

$$\int \sec(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = \int \sqrt{b\sec(dx+c)+a}\sec(dx+c)\,dx$$

input
```
integrate(sec(d*x+c)*(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*sec(d*x + c) + a)*sec(d*x + c), x)
```

3.533.9 Mupad [F(-1)]

Timed out.

$$\int \sec(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = \int \frac{\sqrt{a+\frac{b}{\cos(c+d\,x)}}}{\cos(c+d\,x)}\,dx$$

input
```
int((a + b/cos(c + d*x))^(1/2)/cos(c + d*x),x)
```

output
```
int((a + b/cos(c + d*x))^(1/2)/cos(c + d*x), x)
```

3.534 $\int \sqrt{a+b\sec(c+dx)}\,dx$

3.534.1 Optimal result 4000
3.534.2 Mathematica [A] (verified) 4000
3.534.3 Rubi [A] (verified) 4001
3.534.4 Maple [A] (verified) 4002
3.534.5 Fricas [F] 4002
3.534.6 Sympy [F] 4003
3.534.7 Maxima [F] 4003
3.534.8 Giac [F] 4003
3.534.9 Mupad [F(-1)] 4004

3.534.1 Optimal result

Integrand size = 14, antiderivative size = 125

$$\int \sqrt{a+b\sec(c+dx)}\,dx = -\frac{2\cot(c+dx)\,\text{EllipticPi}\left(\frac{a}{a+b},\arcsin\left(\frac{\sqrt{a+b}}{\sqrt{a+b\sec(c+dx)}}\right),\frac{a-b}{a+b}\right)\sqrt{-\frac{b(1-\sec(c+dx))}{a+b\sec(c+dx)}}\sqrt{\frac{b(1+\sec(c+dx))}{a+b\sec(c+dx)}}(a+b\sec(c+}{\sqrt{a+b}d}$$

output
```
-2*cot(d*x+c)*EllipticPi((a+b)^(1/2)/(a+b*sec(d*x+c))^(1/2),a/(a+b),((a-b)
/(a+b))^(1/2))*(a+b*sec(d*x+c))*(-b*(1-sec(d*x+c))/(a+b*sec(d*x+c)))^(1/2)
*(b*(1+sec(d*x+c))/(a+b*sec(d*x+c)))^(1/2)/d/(a+b)^(1/2)
```

3.534.2 Mathematica [A] (verified)

Time = 1.89 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.21

$$\int \sqrt{a+b\sec(c+dx)}\,dx = \frac{4\cos^2\left(\frac{1}{2}(c+dx)\right)\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\sqrt{\frac{b+a\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}\left((-a+b)\,\text{EllipticF}\left(\arcsin\left(\tan\left(\frac{1}{2}(c+dx)\right)\right),\frac{a-b}{a+b}\right)+2c}{d(b+a\cos(c+dx))}$$

input
```
Integrate[Sqrt[a + b*Sec[c + d*x]],x]
```

output
```
(4*Cos[(c + d*x)/2]^2*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Co
s[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*((-a + b)*EllipticF[ArcSin[Tan[(
c + d*x)/2]], (a - b)/(a + b)] + 2*a*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2
]], (a - b)/(a + b)])*Sqrt[a + b*Sec[c + d*x]])/(d*(b + a*Cos[c + d*x]))
```

3.534.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3042, 4267}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+b\sec(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{4267}$$

$$-\frac{2\cot(c+dx)\sqrt{-\frac{b(1-\sec(c+dx))}{a+b\sec(c+dx)}}\sqrt{\frac{b(\sec(c+dx)+1)}{a+b\sec(c+dx)}}(a+b\sec(c+dx))\,\text{EllipticPi}\left(\frac{a}{a+b},\arcsin\left(\frac{\sqrt{a+b}}{\sqrt{a+b\sec(c+dx)}}\right),\frac{a-b}{a+b}\right)}{d\sqrt{a+b}}$$

input
```
Int[Sqrt[a + b*Sec[c + d*x]],x]
```

output
```
(-2*Cot[c + d*x]*EllipticPi[a/(a + b), ArcSin[Sqrt[a + b]/Sqrt[a + b*Sec[c
 + d*x]]], (a - b)/(a + b)]*Sqrt[-((b*(1 - Sec[c + d*x]))/(a + b*Sec[c + d
*x]))]*Sqrt[(b*(1 + Sec[c + d*x]))/(a + b*Sec[c + d*x])]*(a + b*Sec[c + d*
x]))/(Sqrt[a + b]*d)
```

3.534.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4267
```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*((a + b
*Csc[c + d*x])/(d*Rt[a + b, 2]*Cot[c + d*x]))*Sqrt[b*((1 + Csc[c + d*x])/(a
 + b*Csc[c + d*x]))]*Sqrt[(-b)*((1 - Csc[c + d*x])/(a + b*Csc[c + d*x]))]*E
llipticPi[a/(a + b), ArcSin[Rt[a + b, 2]/Sqrt[a + b*Csc[c + d*x]]], (a - b)
/(a + b)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]
```

3.534.4 Maple [A] (verified)

Time = 6.03 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.46

method	result
default	$\frac{2(\cos(dx+c)+1)\left(\text{EllipticF}\left(\cot(dx+c)-\csc(dx+c),\sqrt{\frac{a-b}{a+b}}\right)a-\text{EllipticF}\left(\cot(dx+c)-\csc(dx+c),\sqrt{\frac{a-b}{a+b}}\right)b-2a\,\text{EllipticPi}\left(\cot(dx+c)-\right.\right.}{d(b+a\cos(dx+c))}$

input
```
int((a+b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/d*(cos(d*x+c)+1)*(EllipticF(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*a
-EllipticF(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*b-2*a*EllipticPi(cot
(d*x+c)-csc(d*x+c),-1,((a-b)/(a+b))^(1/2)))*(1/(a+b)*(b+a*cos(d*x+c))/(cos
(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(a+b*sec(d*x+c))^(1/2)
/(b+a*cos(d*x+c))
```

3.534.5 Fricas [F]

$$\int \sqrt{a+b\sec(c+dx)}\,dx = \int \sqrt{b\sec(dx+c)+a}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sec(d*x + c) + a), x)
```

3.534.6 Sympy [F]

$$\int \sqrt{a+b\sec(c+dx)}\,dx = \int \sqrt{a+b\sec{(c+dx)}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(a + b*sec(c + d*x)), x)
```

3.534.7 Maxima [F]

$$\int \sqrt{a+b\sec(c+dx)}\,dx = \int \sqrt{b\sec{(dx+c)}+a}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sec(d*x + c) + a), x)
```

3.534.8 Giac [F]

$$\int \sqrt{a+b\sec(c+dx)}\,dx = \int \sqrt{b\sec{(dx+c)}+a}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*sec(d*x + c) + a), x)
```

3.534.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+b\sec(c+dx)}\,dx=\int \sqrt{a+\frac{b}{\cos\left(c+d\,x\right)}}\,dx$$

input
```
int((a + b/cos(c + d*x))^(1/2),x)
```

output
```
int((a + b/cos(c + d*x))^(1/2), x)
```

3.535 $\int \cos(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$

3.535.1 Optimal result 4005
3.535.2 Mathematica [A] (verified) 4006
3.535.3 Rubi [A] (verified) 4006
3.535.4 Maple [B] (verified) 4010
3.535.5 Fricas [F] 4010
3.535.6 Sympy [F] 4011
3.535.7 Maxima [F] 4011
3.535.8 Giac [F] 4011
3.535.9 Mupad [F(-1)] 4012

3.535.1 Optimal result

Integrand size = 21, antiderivative size = 330

$$\begin{aligned}\int \cos(c+dx)\sqrt{a+b\sec(c+dx)}\,dx &= \frac{(a-b)\sqrt{a+b}\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{bd} \\ &+ \frac{\sqrt{a+b}\cot(c+dx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{d} \\ &- \frac{b\sqrt{a+b}\cot(c+dx)\,\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{ad} \\ &+ \frac{\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{d}\end{aligned}$$

output
```
(a-b)*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b)
)^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-
b))^(1/2)/b/d+cot(d*x+c)*EllipticF((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+
b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x
+c))/(a-b))^(1/2)/d-b*cot(d*x+c)*EllipticPi((a+b*sec(d*x+c))^(1/2)/(a+b)^(
1/2),(a+b)/a,((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/
2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/a/d+sin(d*x+c)*(a+b*sec(d*x+c))^(1/2)/d
```

3.535.2 Mathematica [A] (verified)

Time = 10.17 (sec) , antiderivative size = 373, normalized size of antiderivative = 1.13

$$\int \cos(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$$
$$= \frac{\cos^2\left(\frac{1}{2}(c+dx)\right)\sqrt{\cos(c+dx)\sec^4\left(\frac{1}{2}(c+dx)\right)}\sqrt{1+\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\left(2(a+b)\sqrt{\frac{b+a\cos(}{(a+b)(1+c}}\right.}{}$$

input
```
Integrate[Cos[c + d*x]*Sqrt[a + b*Sec[c + d*x]],x]
```

output
```
(Cos[(c + d*x)/2]^2*Sqrt[Cos[c + d*x]*Sec[(c + d*x)/2]^4]*Sqrt[1 + Sec[c +
 d*x]]*Sqrt[a + b*Sec[c + d*x]]*(2*(a + b)*Sqrt[(b + a*Cos[c + d*x])/((a +
 b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a +
b)] - 4*b*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Elliptic
F[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + 4*b*Sqrt[(b + a*Cos[c + d*x
])/((a + b)*(1 + Cos[c + d*x]))]*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]],
(a - b)/(a + b)] + a*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sec[(c + d*x)/2
]*Sin[(3*(c + d*x))/2] - a*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Tan[(c +
d*x)/2] + 2*b*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Tan[(c + d*x)/2]))/(2*
d*(b + a*Cos[c + d*x])*Sqrt[Sec[c + d*x]])
```

3.535.3 Rubi [A] (verified)

Time = 1.18 (sec) , antiderivative size = 337, normalized size of antiderivative = 1.02, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {3042, 4344, 3042, 4547, 3042, 4409, 3042, 4271, 4319, 4492}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}{\csc\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{4344}$$

$$\frac{1}{2}\int \frac{b-b\sec^2(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx+\frac{\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\int \frac{b-b\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{d}$$

$$\Big\downarrow \text{4547}$$

$$\frac{1}{2}\left(\int \frac{\sec(c+dx)b+b}{\sqrt{a+b\sec(c+dx)}}dx-b\int \frac{\sec(c+dx)(\sec(c+dx)+1)}{\sqrt{a+b\sec(c+dx)}}dx\right)+\frac{\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\left(\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)b+b}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-b\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)+\frac{\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{d}$$

$$\Big\downarrow \text{4409}$$

$$\frac{1}{2}\left(-b\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+b\int \frac{1}{\sqrt{a+b\sec(c+dx)}}dx+b\int \frac{\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx\right)+\frac{\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\left(b\int \frac{1}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+b\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-b\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+\right.}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}\right.$$
$$\frac{\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{d}$$

$$\Big\downarrow \text{4271}$$

$$\frac{1}{2}\left(b\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-b\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2b\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(}{}}}{}\right.$$
$$\frac{\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{d}$$

3.535. $\int \cos(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$

$$\Big\downarrow \text{4319}$$

$$\frac{1}{2}\left(-b\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{Ellip}}{d}\right.$$

$$\frac{\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{d}$$

$$\Big\downarrow \text{4492}$$

$$\frac{1}{2}\left(\frac{2\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{d}+\frac{2(a-b)\sqrt{a+}}{}\right.$$

$$\frac{\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{d}$$

input
```
Int[Cos[c + d*x]*Sqrt[a + b*Sec[c + d*x]],x]
```

output
```
((2*(a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d
*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*S
qrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(b*d) + (2*Sqrt[a + b]*Cot[c + d*x
]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]
*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b
))])/d - (2*b*Sqrt[a + b]*Cot[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a
 + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x
]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(a*d))/2 + (Sqrt[a +
 b*Sec[c + d*x]]*Sin[c + d*x])/d
```

3.535.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4271
```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*(Rt[a
 + b, 2]/(a*d*Cot[c + d*x]))*Sqrt[b*((1 - Csc[c + d*x])/(a + b))]*Sqrt[(-b)
*((1 + Csc[c + d*x])/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[
c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

rule 4319
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4344
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[Cot[e + f*x]*Sqrt[a + b*Csc[e + f*x]]*((d*Csc[e
 + f*x])^n/(f*n)), x] - Simp[1/(2*d*n)   Int[(d*Csc[e + f*x])^(n + 1)*(Simp
[b - 2*a*(n + 1)*Csc[e + f*x] - b*(2*n + 3)*Csc[e + f*x]^2, x]/Sqrt[a + b*C
sc[e + f*x]]), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] &&
LeQ[n, -1] && IntegerQ[2*n]
```

rule 4409
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Simp[c   Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] +
Simp[d   Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

rule 4492
```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

rule 4547
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))/Sqrt[csc[(e_.) + (f_.)*(x_)]*
(b_.) + (a_)], x_Symbol] :> Int[(A - C*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x
]], x] + Simp[C   Int[Csc[e + f*x]*((1 + Csc[e + f*x])/Sqrt[a + b*Csc[e + f
*x]]), x], x] /; FreeQ[{a, b, e, f, A, C}, x] && NeQ[a^2 - b^2, 0]
```

3.535. $\int \cos(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$

3.535.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1076 vs. 2(301) = 602.

Time = 3.80 (sec) , antiderivative size = 1077, normalized size of antiderivative = 3.26

method	result	size
default	Expression too large to display	1077

input
```
int(cos(d*x+c)*(a+b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(co
s(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a*cos(d
*x+c)^2-EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(
cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*b*cos
(d*x+c)^2-2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(c
os(d*x+c)+1))^(1/2)*EllipticPi(cot(d*x+c)-csc(d*x+c),-1,((a-b)/(a+b))^(1/2
))*b*cos(d*x+c)^2+2*EllipticF(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*(
cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))
^(1/2)*b*cos(d*x+c)^2-2*EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2
))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)
+1))^(1/2)*a*cos(d*x+c)-2*EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1
/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+
c)+1))^(1/2)*b*cos(d*x+c)-4*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2
)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticPi(cot(d*x+c)-csc(d*x+c),-1,((
a-b)/(a+b))^(1/2))*b*cos(d*x+c)+4*EllipticF(cot(d*x+c)-csc(d*x+c),((a-b)/(
a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(
cos(d*x+c)+1))^(1/2)*b*cos(d*x+c)+cos(d*x+c)^2*sin(d*x+c)*a-EllipticE(cot(
d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(
1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a-EllipticE(cot(d*x+c)-csc(
d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*...
```

3.535.5 Fricas [F]

$$\int \cos(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = \int \sqrt{b\sec\left(dx+c\right)+a}\cos\left(dx+c\right)\,dx$$

input
```
integrate(cos(d*x+c)*(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sec(d*x + c) + a)*cos(d*x + c), x)
```

3.535.6 Sympy [F]

$$\int \cos(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = \int \sqrt{a+b\sec(c+dx)}\cos(c+dx)\,dx$$

input
```
integrate(cos(d*x+c)*(a+b*sec(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(a + b*sec(c + d*x))*cos(c + d*x), x)
```

3.535.7 Maxima [F]

$$\int \cos(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = \int \sqrt{b\sec(dx+c)+a}\cos(dx+c)\,dx$$

input
```
integrate(cos(d*x+c)*(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sec(d*x + c) + a)*cos(d*x + c), x)
```

3.535.8 Giac [F]

$$\int \cos(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = \int \sqrt{b\sec(dx+c)+a}\cos(dx+c)\,dx$$

input
```
integrate(cos(d*x+c)*(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*sec(d*x + c) + a)*cos(d*x + c), x)
```

3.535.9 Mupad [F(-1)]

Timed out.

$$\int \cos(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = \int \cos\left(c+d\,x\right)\,\sqrt{a+\frac{b}{\cos\left(c+d\,x\right)}}\,dx$$

input
```
int(cos(c + d*x)*(a + b/cos(c + d*x))^(1/2),x)
```

output
```
int(cos(c + d*x)*(a + b/cos(c + d*x))^(1/2), x)
```

3.536 $\int \cos^2(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$

3.536.1 Optimal result 4013
3.536.2 Mathematica [B] (verified) 4014
3.536.3 Rubi [A] (verified) 4015
3.536.4 Maple [B] (verified) 4019
3.536.5 Fricas [F] 4020
3.536.6 Sympy [F] 4021
3.536.7 Maxima [F] 4021
3.536.8 Giac [F] 4021
3.536.9 Mupad [F(-1)] 4022

3.536.1 Optimal result

Integrand size = 23, antiderivative size = 396

$$\begin{aligned}
&\int \cos^2(c+dx)\sqrt{a+b\sec(c+dx)}\,dx \\
&= \frac{(a-b)\sqrt{a+b}\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{4ad} \\
&\quad+\frac{\sqrt{a+b}(2a+b)\cot(c+dx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{4ad} \\
&\quad-\frac{\sqrt{a+b}(4a^2-b^2)\cot(c+dx)\,\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+d}{a-b}}}{4a^2d} \\
&\quad+\frac{b\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{4ad}+\frac{\cos(c+dx)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{2d}
\end{aligned}$$

output
```
1/4*(a-b)*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(
a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))
/(a-b))^(1/2)/a/d+1/4*(2*a+b)*cot(d*x+c)*EllipticF((a+b*sec(d*x+c))^(1/2)/
(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2
)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/a/d-1/4*(4*a^2-b^2)*cot(d*x+c)*EllipticP
i((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),(a+b)/a,((a+b)/(a-b))^(1/2))*(a+b)^(1
/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/a^2/d+1
/4*b*sin(d*x+c)*(a+b*sec(d*x+c))^(1/2)/a/d+1/2*cos(d*x+c)*sin(d*x+c)*(a+b*
sec(d*x+c))^(1/2)/d
```

3.536.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 897 vs. $2(396) = 792$.

Time = 17.51 (sec) , antiderivative size = 897, normalized size of antiderivative = 2.27

$$\int \cos^2(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = \frac{\sqrt{a+b\sec(c+dx)}\sin(2(c+dx))}{4d} + \frac{\sqrt{a+b\sec(c+dx)}\sqrt{\frac{a+b-a\tan^2\left(\frac{1}{2}(c+dx)\right)+b\tan^2\left(\frac{1}{2}(c+dx)\right)}{1+\tan^2\left(\frac{1}{2}(c+dx)\right)}}\left(ab\tan\left(\frac{1}{2}(c+dx)\right)+b^2\tan\left(\frac{1}{2}(c+dx)\right)-2ab\,\mathrm{ta}\right.}{}$$

input
```
Integrate[Cos[c + d*x]^2*Sqrt[a + b*Sec[c + d*x]],x]
```

output
```
(Sqrt[a + b*Sec[c + d*x]]*Sin[2*(c + d*x)])/(4*d) + (Sqrt[a + b*Sec[c + d*
x]]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c
 + d*x)/2]^2)]*(a*b*Tan[(c + d*x)/2] + b^2*Tan[(c + d*x)/2] - 2*a*b*Tan[(c
 + d*x)/2]^3 + a*b*Tan[(c + d*x)/2]^5 - b^2*Tan[(c + d*x)/2]^5 + 8*a^2*Ell
ipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d
*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a +
b)] - 2*b^2*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt
[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d
*x)/2]^2)/(a + b)] + 8*a^2*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (a - b
)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a
*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - 2*b^2*EllipticPi[-1
, ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - T
an[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]
^2)/(a + b)] + b*(a + b)*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a +
b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b - a*
Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - 2*a*(2*a - b)*Ellipt
icF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2
]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c +
 d*x)/2]^2)/(a + b)]))/(4*a*d*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*
(-1 + Tan[(c + d*x)/2]^2)*Sqrt[(1 + Tan[(c + d*x)/2]^2)/(1 - Tan[(c + d...
```

3.536.3 Rubi [A] (verified)

Time = 1.54 (sec) , antiderivative size = 394, normalized size of antiderivative = 0.99, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.565$, Rules used = {3042, 4344, 3042, 4592, 27, 3042, 4546, 3042, 4409, 3042, 4271, 4319, 4492}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}{\csc\left(c+dx+\frac{\pi}{2}\right)^2}dx$$

$$\Big\downarrow \; 4344$$

$$\frac{1}{4}\int \frac{\cos(c+dx)\left(b\sec^2(c+dx)+2a\sec(c+dx)+b\right)}{\sqrt{a+b\sec(c+dx)}}dx+\frac{\sin(c+dx)\cos(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{4}\int \frac{b\csc\left(c+dx+\frac{\pi}{2}\right)^2+2a\csc\left(c+dx+\frac{\pi}{2}\right)+b}{\csc\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{\sin(c+dx)\cos(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}$$

$$\Big\downarrow \; 4592$$

$$\frac{1}{4}\left(\frac{b\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{ad}-\frac{\int -\frac{4a^2+2b\sec(c+dx)a-b^2-b^2\sec^2(c+dx)}{2\sqrt{a+b\sec(c+dx)}}dx}{a}\right)+\frac{\sin(c+dx)\cos(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{4}\left(\frac{\int \frac{4a^2+2b\sec(c+dx)a-b^2-b^2\sec^2(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx}{2a}+\frac{b\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{ad}\right)+\frac{\sin(c+dx)\cos(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{4}\left(\frac{\int \frac{4a^2+2b\csc\left(c+dx+\frac{\pi}{2}\right)a-b^2-b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{2a}+\frac{b\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{ad}\right)+\frac{\sin(c+dx)\cos(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}$$

$\Big\downarrow$ 4546

$$\frac{1}{4}\left(\frac{\int \frac{4a^2-b^2+(b^2+2ab)\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx-b^2\int \frac{\sec(c+dx)(\sec(c+dx)+1)}{\sqrt{a+b\sec(c+dx)}}dx}{2a}+\frac{b\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{ad}\right)+\frac{\sin(c+dx)\cos(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}$$

$\Big\downarrow$ 3042

$$\frac{1}{4}\left(\frac{\int \frac{4a^2-b^2+(b^2+2ab)\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-b^2\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{2a}+\frac{b\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{ad}\right)+\frac{\sin(c+dx)\cos(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}$$

$\Big\downarrow$ 4409

$$\frac{1}{4}\left(\frac{\left(4a^2-b^2\right)\int \frac{1}{\sqrt{a+b\sec(c+dx)}}dx+b^2\left(-\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)+b(2a+b)\int \frac{\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx}{2a}+\frac{b\sin}{}\right.$$

$$\frac{\sin(c+dx)\cos(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}$$

$\Big\downarrow$ 3042

$$\frac{1}{4}\left(\frac{\left(4a^2-b^2\right)\int \frac{1}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+b^2\left(-\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)+b(2a+b)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{2a}\right.$$

$$\frac{\sin(c+dx)\cos(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}$$

$\Big\downarrow$ 4271

3.536. $\int \cos^2(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$

$$\frac{1}{4}\left(\frac{b^2\left(-\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)+b(2a+b)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2\sqrt{a+b}(4a^2-b^2)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx}{a+b}}}{}}{2a}\right.$$

$$\frac{\sin(c+dx)\cos(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}$$

$\downarrow$ 4319

$$\frac{1}{4}\left(\frac{b^2\left(-\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)-\frac{2\sqrt{a+b}(4a^2-b^2)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,\text{EllipticPi}\left(\frac{a+b}{a},\arcsin}{ad}}{2a}\right.$$

$$\frac{\sin(c+dx)\cos(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}$$

$\downarrow$ 4492

$$\frac{1}{4}\left(\frac{-\frac{2\sqrt{a+b}(4a^2-b^2)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{ad}+\frac{2\sqrt{a+b}(2a+b)\cot(c+d}{}}{}\right.$$

$$\frac{\sin(c+dx)\cos(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}$$

input
```
Int[Cos[c + d*x]^2*Sqrt[a + b*Sec[c + d*x]],x]
```

output
```
(Cos[c + d*x]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(2*d) + (((2*(a - b)*
Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a
+ b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1
+ Sec[c + d*x]))/(a - b))])/d + (2*Sqrt[a + b]*(2*a + b)*Cot[c + d*x]*Elli
pticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[
(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/d
 - (2*Sqrt[a + b]*(4*a^2 - b^2)*Cot[c + d*x]*EllipticPi[(a + b)/a, ArcSin[
Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c
 + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(a*d))/(2*a) +
 (b*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(a*d))/4
```

3.536. $\int \cos^2(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$

3.536.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4271
```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*(Rt[a
 + b, 2]/(a*d*Cot[c + d*x]))*Sqrt[b*((1 - Csc[c + d*x])/(a + b))]*Sqrt[(-b)
*((1 + Csc[c + d*x])/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[
c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

rule 4319
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4344
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[Cot[e + f*x]*Sqrt[a + b*Csc[e + f*x]]*((d*Csc[e
 + f*x])^n/(f*n)), x] - Simp[1/(2*d*n)   Int[(d*Csc[e + f*x])^(n + 1)*(Simp
[b - 2*a*(n + 1)*Csc[e + f*x] - b*(2*n + 3)*Csc[e + f*x]^2, x]/Sqrt[a + b*C
sc[e + f*x]]), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] &&
LeQ[n, -1] && IntegerQ[2*n]
```

rule 4409
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Simp[c   Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] +
Simp[d   Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

rule 4492
```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

rule 4546
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Simp[C   Int[Csc[e + f*x]*((
1 + Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A
, B, C}, x] && NeQ[a^2 - b^2, 0]
```

rule 4592
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Simp[1/(a*d*n)   Int[(a + b*Csc[e + f*x])^m
*(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*
Csc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

3.536.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1673 vs. $2(355) = 710$.

Time = 4.85 (sec) , antiderivative size = 1674, normalized size of antiderivative = 4.23

method	result	size
default	Expression too large to display	1674

input
```
int(cos(d*x+c)^2*(a+b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4/d/a*(-8*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(c
os(d*x+c)+1))^(1/2)*EllipticPi(cot(d*x+c)-csc(d*x+c),-1,((a-b)/(a+b))^(1/2
))*a^2*cos(d*x+c)^2+2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(
d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi(cot(d*x+c)-csc(d*x+c),-1,((a-b)/(
a+b))^(1/2))*b^2*cos(d*x+c)^2-EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b)
)^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(
d*x+c)+1))^(1/2)*a*b*cos(d*x+c)^2-(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+
b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE(cot(d*x+c)-csc(d*x+c),
((a-b)/(a+b))^(1/2))*b^2*cos(d*x+c)^2+4*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*
(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF(cot(d*x+c)-csc(d
*x+c),((a-b)/(a+b))^(1/2))*a^2*cos(d*x+c)^2-2*EllipticF(cot(d*x+c)-csc(d*x
+c),((a-b)/(a+b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(
cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*a*b*cos(d*x+c)^2+2*a^2*cos(d*x+c)^3*sin(d
*x+c)-16*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(
d*x+c)+1))^(1/2)*EllipticPi(cot(d*x+c)-csc(d*x+c),-1,((a-b)/(a+b))^(1/2))*
a^2*cos(d*x+c)+4*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c
))/(cos(d*x+c)+1))^(1/2)*EllipticPi(cot(d*x+c)-csc(d*x+c),-1,((a-b)/(a+b))
^(1/2))*b^2*cos(d*x+c)-2*EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/
2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c
)+1))^(1/2)*a*b*cos(d*x+c)-2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)...
```

3.536.5 Fricas [F]

$$\int \cos^2(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = \int \sqrt{b\sec\left(dx+c\right)+a}\cos\left(dx+c\right)^2\,dx$$

input
```
integrate(cos(d*x+c)^2*(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sec(d*x + c) + a)*cos(d*x + c)^2, x)
```

3.536.6 Sympy [F]

$$\int \cos^2(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = \int \sqrt{a+b\sec(c+dx)}\cos^2(c+dx)\,dx$$

input
```
integrate(cos(d*x+c)**2*(a+b*sec(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(a + b*sec(c + d*x))*cos(c + d*x)**2, x)
```

3.536.7 Maxima [F]

$$\int \cos^2(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = \int \sqrt{b\sec(dx+c)+a}\cos(dx+c)^2\,dx$$

input
```
integrate(cos(d*x+c)^2*(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sec(d*x + c) + a)*cos(d*x + c)^2, x)
```

3.536.8 Giac [F]

$$\int \cos^2(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = \int \sqrt{b\sec(dx+c)+a}\cos(dx+c)^2\,dx$$

input
```
integrate(cos(d*x+c)^2*(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*sec(d*x + c) + a)*cos(d*x + c)^2, x)
```

3.536.9 Mupad [F(-1)]

Timed out.

$$\int \cos^2(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = \int \cos(c+d\,x)^2\,\sqrt{a+\frac{b}{\cos(c+d\,x)}}\,dx$$

input
```
int(cos(c + d*x)^2*(a + b/cos(c + d*x))^(1/2),x)
```

output
```
int(cos(c + d*x)^2*(a + b/cos(c + d*x))^(1/2), x)
```

3.537 $\int \sec^4(c+dx)(a+b\sec(c+dx))^{3/2}\,dx$

3.537.1 Optimal result . . . 4023
3.537.2 Mathematica [A] (warning: unable to verify) . . . 4024
3.537.3 Rubi [A] (verified) . . . 4024
3.537.4 Maple [B] (verified) . . . 4029
3.537.5 Fricas [F] . . . 4030
3.537.6 Sympy [F] . . . 4031
3.537.7 Maxima [F] . . . 4031
3.537.8 Giac [F] . . . 4031
3.537.9 Mupad [F(-1)] . . . 4032

3.537.1 Optimal result

Integrand size = 23, antiderivative size = 405

$$\int \sec^4(c+dx)(a+b\sec(c+dx))^{3/2}\,dx =$$
$$-\frac{2(a-b)\sqrt{a+b}(8a^4+33a^2b^2+147b^4)\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+s}{}}}{315b^4d}$$
$$-\frac{2(a-b)\sqrt{a+b}(8a^3+6a^2b+39ab^2-147b^3)\cot(c+dx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec}{a-}}}{315b^3d}$$
$$+\frac{2a(8a^2+39b^2)\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{315b^2d}$$
$$+\frac{2(8a^2+49b^2)(a+b\sec(c+dx))^{3/2}\tan(c+dx)}{315b^2d}$$
$$-\frac{8a(a+b\sec(c+dx))^{5/2}\tan(c+dx)}{63b^2d}+\frac{2\sec(c+dx)(a+b\sec(c+dx))^{5/2}\tan(c+dx)}{9bd}$$

output
```
-2/315*(a-b)*(8*a^4+33*a^2*b^2+147*b^4)*cot(d*x+c)*EllipticE((a+b*sec(d*x+
c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(
a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/b^4/d-2/315*(a-b)*(8*a^3+6*a^2
*b+39*a*b^2-147*b^3)*cot(d*x+c)*EllipticF((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/
2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+
sec(d*x+c))/(a-b))^(1/2)/b^3/d+2/315*(8*a^2+49*b^2)*(a+b*sec(d*x+c))^(3/2)
*tan(d*x+c)/b^2/d-8/63*a*(a+b*sec(d*x+c))^(5/2)*tan(d*x+c)/b^2/d+2/9*sec(d
*x+c)*(a+b*sec(d*x+c))^(5/2)*tan(d*x+c)/b/d+2/315*a*(8*a^2+39*b^2)*(a+b*se
c(d*x+c))^(1/2)*tan(d*x+c)/b^2/d
```

3.537.2 Mathematica [A] (warning: unable to verify)

Time = 14.07 (sec) , antiderivative size = 550, normalized size of antiderivative = 1.36

$$\int \sec^4(c+dx)(a+b\sec(c+dx))^{3/2}\,dx =$$
$$-\frac{2\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)}(a+b\sec(c+dx))^{3/2}\left(2(8a^5+8a^4b+33a^3b^2+33a^2b^3+147ab^4+147b^5)\right.}{}$$
$$+\frac{\cos(c+dx)(a+b\sec(c+dx))^{3/2}\left(\frac{2(8a^4+33a^2b^2+147b^4)\sin(c+dx)}{315b^3}+\frac{2\sec^2(c+dx)(3a^2\sin(c+dx)+49b^2\sin(c+dx))}{315b}+\frac{8\sec(c+}{}\right.}{d(b+a\cos(c+d}$$

input
```
Integrate[Sec[c + d*x]^4*(a + b*Sec[c + d*x])^(3/2),x]
```

output
```
(-2*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(a + b*Sec[c + d*x])^(3/2)*(2*(8
*a^5 + 8*a^4*b + 33*a^3*b^2 + 33*a^2*b^3 + 147*a*b^4 + 147*b^5)*Sqrt[Cos[c
 + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c
+ d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] - 2*b*(8*a^
4 + 2*a^3*b + 33*a^2*b^2 + 186*a*b^3 + 147*b^4)*Sqrt[Cos[c + d*x]/(1 + Cos
[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Ellipt
icF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + (8*a^4 + 33*a^2*b^2 + 147
*b^4)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2
]))/(315*b^3*d*(b + a*Cos[c + d*x])^2*Sqrt[Sec[(c + d*x)/2]^2]*Sec[c + d*x
]^(3/2)) + (Cos[c + d*x]*(a + b*Sec[c + d*x])^(3/2)*((2*(8*a^4 + 33*a^2*b^
2 + 147*b^4)*Sin[c + d*x])/(315*b^3) + (2*Sec[c + d*x]^2*(3*a^2*Sin[c + d*
x] + 49*b^2*Sin[c + d*x]))/(315*b) + (8*Sec[c + d*x]*(-(a^3*Sin[c + d*x])
+ 22*a*b^2*Sin[c + d*x]))/(315*b^2) + (20*a*Sec[c + d*x]^2*Tan[c + d*x])/6
3 + (2*b*Sec[c + d*x]^3*Tan[c + d*x])/9))/(d*(b + a*Cos[c + d*x]))
```

3.537.3 Rubi [A] (verified)

Time = 1.78 (sec) , antiderivative size = 421, normalized size of antiderivative = 1.04, number of steps used = 17, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.739$, Rules used = {3042, 4352, 27, 3042, 4570, 27, 3042, 4490, 27, 3042, 4490, 27, 3042, 4493, 3042, 4319, 4492}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^4(c+dx)(a+b\sec(c+dx))^{3/2}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^4\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}dx$$

$$\Big\downarrow \; 4352$$

$$\frac{2\int \frac{1}{2}\sec(c+dx)(a+b\sec(c+dx))^{3/2}\left(-4a\sec^2(c+dx)+7b\sec(c+dx)+2a\right)dx}{9b}+\frac{2\tan(c+dx)\sec(c+dx)(a+b\sec(c+dx))^{5/2}}{9bd}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \sec(c+dx)(a+b\sec(c+dx))^{3/2}\left(-4a\sec^2(c+dx)+7b\sec(c+dx)+2a\right)dx}{9b}+\frac{2\tan(c+dx)\sec(c+dx)(a+b\sec(c+dx))^{5/2}}{9bd}$$

$$\Big\downarrow \; 3042$$

$$\frac{\int \csc\left(c+dx+\frac{\pi}{2}\right)\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}\left(-4a\csc\left(c+dx+\frac{\pi}{2}\right)^2+7b\csc\left(c+dx+\frac{\pi}{2}\right)+2a\right)dx}{9b}+\frac{2\tan(c+dx)\sec(c+dx)(a+b\sec(c+dx))^{5/2}}{9bd}$$

$$\Big\downarrow \; 4570$$

$$\frac{\frac{2\int -\frac{1}{2}\sec(c+dx)(a+b\sec(c+dx))^{3/2}\left(6ab-\left(8a^2+49b^2\right)\sec(c+dx)\right)dx}{7b}-\frac{8a\tan(c+dx)(a+b\sec(c+dx))^{5/2}}{7bd}}{9b}+\frac{2\tan(c+dx)\sec(c+dx)(a+b\sec(c+dx))^{5/2}}{9bd}$$

$$\Big\downarrow \; 27$$

$$\frac{-\frac{\int \sec(c+dx)(a+b\sec(c+dx))^{3/2}\left(6ab-\left(8a^2+49b^2\right)\sec(c+dx)\right)dx}{7b}-\frac{8a\tan(c+dx)(a+b\sec(c+dx))^{5/2}}{7bd}}{9b}+\frac{2\tan(c+dx)\sec(c+dx)(a+b\sec(c+dx))^{5/2}}{9bd}$$

$$\Big\downarrow \; 3042$$

$$\frac{-\frac{\int \csc\left(c+dx+\frac{\pi}{2}\right)\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}\left(6ab+\left(-8a^2-49b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)\right)dx}{7b}-\frac{8a\tan(c+dx)(a+b\sec(c+dx))^{5/2}}{7bd}}{9b}+\frac{2\tan(c+dx)\sec(c+dx)(a+b\sec(c+dx))^{5/2}}{9bd}$$

$$\Big\downarrow \; 4490$$

3.537. $\int \sec^4(c+dx)(a+b\sec(c+dx))^{3/2}\,dx$

$$\frac{-\frac{\frac{2}{5}\int \frac{3}{2}\sec(c+dx)\sqrt{a+b\sec(c+dx)}\left(b\left(2a^2-49b^2\right)-a\left(8a^2+39b^2\right)\sec(c+dx)\right)dx-\frac{2\left(8a^2+49b^2\right)\tan(c+dx)(a+b\sec(c+dx))^{3/2}}{5d}}{7b}-\frac{8a\tan(c+dx)(a+b\sec}{7bd}}{9b}$$

$$\frac{2\tan(c+dx)\sec(c+dx)(a+b\sec(c+dx))^{5/2}}{9bd}$$

$\downarrow$ 27

$$\frac{-\frac{\frac{3}{5}\int \sec(c+dx)\sqrt{a+b\sec(c+dx)}\left(b\left(2a^2-49b^2\right)-a\left(8a^2+39b^2\right)\sec(c+dx)\right)dx-\frac{2\left(8a^2+49b^2\right)\tan(c+dx)(a+b\sec(c+dx))^{3/2}}{5d}}{7b}-\frac{8a\tan(c+dx)(a+b\sec}{7bd}}{9b}$$

$$\frac{2\tan(c+dx)\sec(c+dx)(a+b\sec(c+dx))^{5/2}}{9bd}$$

$\downarrow$ 3042

$$\frac{-\frac{\frac{3}{5}\int \csc\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}\left(b\left(2a^2-49b^2\right)-a\left(8a^2+39b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)\right)dx-\frac{2\left(8a^2+49b^2\right)\tan(c+dx)(a+b\sec(c+dx))^{3/2}}{5d}}{7b}-\frac{8a\tan(c+}{}}{9b}$$

$$\frac{2\tan(c+dx)\sec(c+dx)(a+b\sec(c+dx))^{5/2}}{9bd}$$

$\downarrow$ 4490

$$\frac{-\frac{\frac{3}{5}\left(\frac{2}{3}\int -\frac{\sec(c+dx)\left(2ab\left(a^2+93b^2\right)+\left(8a^4+33b^2a^2+147b^4\right)\sec(c+dx)\right)}{2\sqrt{a+b\sec(c+dx)}}dx-\frac{2a\left(8a^2+39b^2\right)\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}\right)-\frac{2\left(8a^2+49b^2\right)\tan(c+dx)(a+b\sec(c}{5d}}{7b}}{9b}$$

$$\frac{2\tan(c+dx)\sec(c+dx)(a+b\sec(c+dx))^{5/2}}{9bd}$$

$\downarrow$ 27

$$\frac{-\frac{\frac{3}{5}\left(-\frac{1}{3}\int \frac{\sec(c+dx)\left(2ab\left(a^2+93b^2\right)+\left(8a^4+33b^2a^2+147b^4\right)\sec(c+dx)\right)}{\sqrt{a+b\sec(c+dx)}}dx-\frac{2a\left(8a^2+39b^2\right)\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}\right)-\frac{2\left(8a^2+49b^2\right)\tan(c+dx)(a+b\sec(c}{5d}}{7b}}{9b}$$

$$\frac{2\tan(c+dx)\sec(c+dx)(a+b\sec(c+dx))^{5/2}}{9bd}$$

$\downarrow$ 3042

$$\frac{-\frac{\frac{3}{5}\left(-\frac{1}{3}\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(2ab\left(a^2+93b^2\right)+\left(8a^4+33b^2a^2+147b^4\right)\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2a\left(8a^2+39b^2\right)\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}\right)-\frac{2\left(8a^2+49b^2\right)\tan(c+dx)(}{5d}}{7b}}{9b}$$

$$\frac{2\tan(c+dx)\sec(c+dx)(a+b\sec(c+dx))^{5/2}}{9bd}$$

3.537. $\int \sec^4(c+dx)(a+b\sec(c+dx))^{3/2}\,dx$

$$\Big\downarrow \; 4493$$

$$-\frac{\frac{3}{5}\left(\frac{1}{3}\left((a-b)\left(8a^3+6a^2b+39ab^2-147b^3\right)\int \frac{\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx-\left(8a^4+33a^2b^2+147b^4\right)\int \frac{\sec(c+dx)(\sec(c+dx)+1)}{\sqrt{a+b\sec(c+dx)}}dx\right)-\frac{2a\left(8a^2+39b^2\right)\tan(c+dx)\sqrt{a+}}{3d}\right.}{7b}}{9b}$$

$$\frac{2\tan(c+dx)\sec(c+dx)(a+b\sec(c+dx))^{5/2}}{9bd}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\frac{3}{5}\left(\frac{1}{3}\left((a-b)\left(8a^3+6a^2b+39ab^2-147b^3\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-\left(8a^4+33a^2b^2+147b^4\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)-\frac{2a\left(8a^2+39b^2\right)}{}\right.}{7b}}{9b}$$

$$\frac{2\tan(c+dx)\sec(c+dx)(a+b\sec(c+dx))^{5/2}}{9bd}$$

$$\Big\downarrow \; 4319$$

$$-\frac{\frac{3}{5}\left(\frac{1}{3}\left(\frac{2(a-b)\sqrt{a+b}\left(8a^3+6a^2b+39ab^2-147b^3\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{bd}-\left(8a^4+33a^2b^2\right.\right.\right.}{7b}}{}$$

$$\frac{2\tan(c+dx)\sec(c+dx)(a+b\sec(c+dx))^{5/2}}{9bd}$$

$$\Big\downarrow \; 4492$$

$$-\frac{\frac{3}{5}\left(\frac{1}{3}\left(\frac{2(a-b)\sqrt{a+b}\left(8a^4+33a^2b^2+147b^4\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,E\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{b^2d}+\frac{2(a-b)\sqrt{a+b}\left(8a^3+6a^2b+39\right.}{}\right.\right.}{}$$

$$\frac{2\tan(c+dx)\sec(c+dx)(a+b\sec(c+dx))^{5/2}}{9bd}$$

input
```
Int[Sec[c + d*x]^4*(a + b*Sec[c + d*x])^(3/2),x]
```

3.537. $\int \sec^4(c+dx)(a+b\sec(c+dx))^{3/2}\,dx$

output
```
(2*Sec[c + d*x]*(a + b*Sec[c + d*x])^(5/2)*Tan[c + d*x])/(9*b*d) + ((-8*a*
(a + b*Sec[c + d*x])^(5/2)*Tan[c + d*x])/(7*b*d) - ((-2*(8*a^2 + 49*b^2)*(
a + b*Sec[c + d*x])^(3/2)*Tan[c + d*x])/(5*d) + (3*(((2*(a - b)*Sqrt[a + b
]*(8*a^4 + 33*a^2*b^2 + 147*b^4)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*
Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(
a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(b^2*d) + (2*(a - b)*Sqrt
[a + b]*(8*a^3 + 6*a^2*b + 39*a*b^2 - 147*b^3)*Cot[c + d*x]*EllipticF[ArcS
in[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Se
c[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(b*d))/3 -
(2*a*(8*a^2 + 39*b^2)*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(3*d)))/5)/(7
*b))/(9*b)
```

3.537.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4319
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4352
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-d^3)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m +
1)*((d*Csc[e + f*x])^(n - 3)/(b*f*(m + n - 1))), x] + Simp[d^3/(b*(m + n -
1))   Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 3)*Simp[a*(n - 3) +
b*(m + n - 2)*Csc[e + f*x] - a*(n - 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[
{a, b, d, e, f, m}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 3] && (IntegerQ[n] ||
 IntegersQ[2*m, 2*n]) &&  !IGtQ[m, 2]
```

rule 4490
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*((
a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Simp[1/(m + 1)   Int[Csc[e + f*x]*
(a + b*Csc[e + f*x])^(m - 1)*Simp[b*B*m + a*A*(m + 1) + (a*B*m + A*b*(m + 1
))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*
B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

rule 4492
```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

rule 4493
```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(A - B)   Int[Csc[e
 + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Simp[B   Int[Csc[e + f*x]*((1 +
Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A, B}
, x] && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

rule 4570
```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> Simp[(-C)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2)
)), x] + Simp[1/(b*(m + 2))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[
b*A*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /;
 FreeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

3.537.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3238 vs. $2(367) = 734$.

Time = 16.94 (sec) , antiderivative size = 3239, normalized size of antiderivative = 8.00

method	result	size
default	Expression too large to display	3239

input
```
int(sec(d*x+c)^4*(a+b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/315/d/b^3*(a+b*sec(d*x+c))^(1/2)/(b+a*cos(d*x+c))/(cos(d*x+c)+1)*(147*b^
5*sin(d*x+c)+121*a^2*b^3*sin(d*x+c)+49*b^5*tan(d*x+c)-a^3*b^2*sin(d*x+c)-3
3*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+
1))^(1/2)*EllipticF(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^3*cos
(d*x+c)^2-186*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/
(cos(d*x+c)+1))^(1/2)*EllipticF(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))
*a*b^4*cos(d*x+c)^2+16*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(co
s(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b
))^(1/2))*a^4*b*cos(d*x+c)+66*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1
/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(cot(d*x+c)-csc(d*x+c),((a-
b)/(a+b))^(1/2))*a^3*b^2*cos(d*x+c)+66*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+
c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(cot(d*x+c)-csc(d*
x+c),((a-b)/(a+b))^(1/2))*a^2*b^3*cos(d*x+c)+8*(1/(a+b)*(b+a*cos(d*x+c))/(
cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(cot(d*x+c
)-csc(d*x+c),((a-b)/(a+b))^(1/2))*a^4*b*cos(d*x+c)^2+294*(1/(a+b)*(b+a*cos
(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE
(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*a*b^4*cos(d*x+c)-16*(1/(a+b)*(
b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*El
lipticF(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*a^4*b*cos(d*x+c)-4*(1/(
a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))...
```

3.537.5 Fricas [F]

$$\int \sec^4(c+dx)(a+b\sec(c+dx))^{3/2}\,dx = \int (b\sec(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^4\,dx$$

input
```
integrate(sec(d*x+c)^4*(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
integral((b*sec(d*x + c)^5 + a*sec(d*x + c)^4)*sqrt(b*sec(d*x + c) + a), x
)
```

3.537.6 Sympy [F]

$$\int \sec^4(c+dx)(a+b\sec(c+dx))^{3/2}\,dx = \int (a+b\sec(c+dx))^{\frac{3}{2}}\sec^4(c+dx)\,dx$$

input
```
integrate(sec(d*x+c)**4*(a+b*sec(d*x+c))**(3/2),x)
```

output
```
Integral((a + b*sec(c + d*x))**(3/2)*sec(c + d*x)**4, x)
```

3.537.7 Maxima [F]

$$\int \sec^4(c+dx)(a+b\sec(c+dx))^{3/2}\,dx = \int (b\sec(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^4\,dx$$

input
```
integrate(sec(d*x+c)^4*(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^(3/2)*sec(d*x + c)^4, x)
```

3.537.8 Giac [F]

$$\int \sec^4(c+dx)(a+b\sec(c+dx))^{3/2}\,dx = \int (b\sec(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^4\,dx$$

input
```
integrate(sec(d*x+c)^4*(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^(3/2)*sec(d*x + c)^4, x)
```

3.537.9 Mupad [F(-1)]

Timed out.

$$\int \sec^4(c+dx)(a+b\sec(c+dx))^{3/2}\,dx = \int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}}{\cos\left(c+d\,x\right)^4}\,dx$$

input
```
int((a + b/cos(c + d*x))^(3/2)/cos(c + d*x)^4,x)
```

output
```
int((a + b/cos(c + d*x))^(3/2)/cos(c + d*x)^4, x)
```

3.538 $\int \sec^3(c+dx)(a+b\sec(c+dx))^{3/2}\,dx$

3.538.1 Optimal result 4033
3.538.2 Mathematica [A] (warning: unable to verify) 4034
3.538.3 Rubi [A] (verified) 4034
3.538.4 Maple [B] (verified) 4038
3.538.5 Fricas [F] 4039
3.538.6 Sympy [F] 4040
3.538.7 Maxima [F] 4040
3.538.8 Giac [F] 4040
3.538.9 Mupad [F(-1)] 4041

3.538.1 Optimal result

Integrand size = 23, antiderivative size = 342

$$\int \sec^3(c+dx)(a+b\sec(c+dx))^{3/2}\,dx = \frac{4a(a-b)\sqrt{a+b}(3a^2-41b^2)\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(}{}}}{105b^3d}$$
$$+\frac{2(a-b)\sqrt{a+b}(6a^2+57ab-25b^2)\cot(c+dx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-}}{105b^2d}$$
$$-\frac{2(6a^2-25b^2)\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{105bd}$$
$$-\frac{4a(a+b\sec(c+dx))^{3/2}\tan(c+dx)}{35bd}+\frac{2(a+b\sec(c+dx))^{5/2}\tan(c+dx)}{7bd}$$

output
```
4/105*a*(a-b)*(3*a^2-41*b^2)*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(
a+b)^(1/2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)
*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/b^3/d+2/105*(a-b)*(6*a^2+57*a*b-25*b^2)*c
ot(d*x+c)*EllipticF((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2)
)*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/
2)/b^2/d-4/35*a*(a+b*sec(d*x+c))^(3/2)*tan(d*x+c)/b/d+2/7*(a+b*sec(d*x+c))
^(5/2)*tan(d*x+c)/b/d-2/105*(6*a^2-25*b^2)*(a+b*sec(d*x+c))^(1/2)*tan(d*x+
c)/b/d
```

3.538.2 Mathematica [A] (warning: unable to verify)

Time = 11.79 (sec) , antiderivative size = 471, normalized size of antiderivative = 1.38

$$\int \sec^3(c+dx)(a+b\sec(c+dx))^{3/2}\,dx = \frac{4\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)}(a+b\sec(c+dx))^{3/2}\left(2a(3a^3+3a^2b-41ab^2-41b^3)\sqrt{\frac{\cos(c}{1+\cos}}\right.}{}$$

$$+\frac{\cos(c+dx)(a+b\sec(c+dx))^{3/2}\left(-\frac{4a(3a^2-41b^2)\sin(c+dx)}{105b^2}+\frac{2\sec(c+dx)(3a^2\sin(c+dx)+25b^2\sin(c+dx))}{105b}+\frac{16}{35}a\sec(c+\right.}{d(b+a\cos(c+dx))}$$

input
```
Integrate[Sec[c + d*x]^3*(a + b*Sec[c + d*x])^(3/2),x]
```

output
```
(4*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(a + b*Sec[c + d*x])^(3/2)*(2*a*(
3*a^3 + 3*a^2*b - 41*a*b^2 - 41*b^3)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]
*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[
Tan[(c + d*x)/2]], (a - b)/(a + b)] + b*(-6*a^3 + 51*a^2*b + 82*a*b^2 + 25
*b^3)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a
+ b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a +
 b)] + a*(3*a^2 - 41*b^2)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/
2]^2*Tan[(c + d*x)/2]))/(105*b^2*d*(b + a*Cos[c + d*x])^2*Sqrt[Sec[(c + d*
x)/2]^2]*Sec[c + d*x]^(3/2)) + (Cos[c + d*x]*(a + b*Sec[c + d*x])^(3/2)*((
-4*a*(3*a^2 - 41*b^2)*Sin[c + d*x])/(105*b^2) + (2*Sec[c + d*x]*(3*a^2*Sin
[c + d*x] + 25*b^2*Sin[c + d*x]))/(105*b) + (16*a*Sec[c + d*x]*Tan[c + d*x
])/35 + (2*b*Sec[c + d*x]^2*Tan[c + d*x])/7))/(d*(b + a*Cos[c + d*x]))
```

3.538.3 Rubi [A] (verified)

Time = 1.35 (sec) , antiderivative size = 350, normalized size of antiderivative = 1.02, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.609$, Rules used = {3042, 4327, 27, 3042, 4490, 27, 3042, 4490, 27, 3042, 4493, 3042, 4319, 4492}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^3(c+dx)(a+b\sec(c+dx))^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^3\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}dx$$

$$\Big\downarrow \; 4327$$

$$\frac{2\int \frac{1}{2}\sec(c+dx)(5b-2a\sec(c+dx))(a+b\sec(c+dx))^{3/2}dx}{7b}+\frac{2\tan(c+dx)(a+b\sec(c+dx))^{5/2}}{7bd}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \sec(c+dx)(5b-2a\sec(c+dx))(a+b\sec(c+dx))^{3/2}dx}{7b}+\frac{2\tan(c+dx)(a+b\sec(c+dx))^{5/2}}{7bd}$$

$$\Big\downarrow \; 3042$$

$$\frac{\int \csc\left(c+dx+\frac{\pi}{2}\right)\left(5b-2a\csc\left(c+dx+\frac{\pi}{2}\right)\right)\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}dx}{7b}+\frac{2\tan(c+dx)(a+b\sec(c+dx))^{5/2}}{7bd}$$

$$\Big\downarrow \; 4490$$

$$\frac{\frac{2}{5}\int \frac{1}{2}\sec(c+dx)\sqrt{a+b\sec(c+dx)}\left(19ab-\left(6a^2-25b^2\right)\sec(c+dx)\right)dx-\frac{4a\tan(c+dx)(a+b\sec(c+dx))^{3/2}}{5d}}{7b}+\frac{2\tan(c+dx)(a+b\sec(c+dx))^{5/2}}{7bd}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{1}{5}\int \sec(c+dx)\sqrt{a+b\sec(c+dx)}\left(19ab-\left(6a^2-25b^2\right)\sec(c+dx)\right)dx-\frac{4a\tan(c+dx)(a+b\sec(c+dx))^{3/2}}{5d}}{7b}+\frac{2\tan(c+dx)(a+b\sec(c+dx))^{5/2}}{7bd}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{1}{5}\int \csc\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}\left(19ab+\left(25b^2-6a^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)\right)dx-\frac{4a\tan(c+dx)(a+b\sec(c+dx)}{5d}}{7b}+\frac{2\tan(c+dx)(a+b\sec(c+dx))^{5/2}}{7bd}$$

$$\Big\downarrow \; 4490$$

3.538. $\int \sec^3(c+dx)(a+b\sec(c+dx))^{3/2}\,dx$

$$\frac{\frac{1}{5}\left(\frac{2}{3}\int \frac{\sec(c+dx)\left(b\left(51a^2+25b^2\right)-2a\left(3a^2-41b^2\right)\sec(c+dx)\right)}{2\sqrt{a+b\sec(c+dx)}}dx-\frac{2\left(6a^2-25b^2\right)\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}\right)-\frac{4a\tan(c+dx)(a+b\sec(c+dx}{5d}}{7b}$$

$$\frac{2\tan(c+dx)(a+b\sec(c+dx))^{5/2}}{7bd}$$

$\downarrow$ 27

$$\frac{\frac{1}{5}\left(\frac{1}{3}\int \frac{\sec(c+dx)\left(b\left(51a^2+25b^2\right)-2a\left(3a^2-41b^2\right)\sec(c+dx)\right)}{\sqrt{a+b\sec(c+dx)}}dx-\frac{2\left(6a^2-25b^2\right)\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}\right)-\frac{4a\tan(c+dx)(a+b\sec(c+dx}{5d}}{7b}$$

$$\frac{2\tan(c+dx)(a+b\sec(c+dx))^{5/2}}{7bd}$$

$\downarrow$ 3042

$$\frac{\frac{1}{5}\left(\frac{1}{3}\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(b\left(51a^2+25b^2\right)-2a\left(3a^2-41b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2\left(6a^2-25b^2\right)\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}\right)-\frac{4a\tan(c+dx)(a+b}{5d}}{7b}$$

$$\frac{2\tan(c+dx)(a+b\sec(c+dx))^{5/2}}{7bd}$$

$\downarrow$ 4493

$$\frac{\frac{1}{5}\left(\frac{1}{3}\left((a-b)\left(6a^2+57ab-25b^2\right)\int \frac{\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx-2a\left(3a^2-41b^2\right)\int \frac{\sec(c+dx)(\sec(c+dx)+1)}{\sqrt{a+b\sec(c+dx)}}dx\right)-\frac{2\left(6a^2-25b^2\right)\tan}{}\right.}{7b}$$

$$\frac{2\tan(c+dx)(a+b\sec(c+dx))^{5/2}}{7bd}$$

$\downarrow$ 3042

$$\frac{\frac{1}{5}\left(\frac{1}{3}\left((a-b)\left(6a^2+57ab-25b^2\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-2a\left(3a^2-41b^2\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)-\frac{2(6}{}\right.}{7b}$$

$$\frac{2\tan(c+dx)(a+b\sec(c+dx))^{5/2}}{7bd}$$

$\downarrow$ 4319

$$\frac{\frac{1}{5}\left(\frac{1}{3}\left(\frac{2(a-b)\sqrt{a+b}\left(6a^2+57ab-25b^2\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{bd}-2a\left(3a^2-\right.\right.\right.}{7b}$$

$$\frac{2\tan(c+dx)(a+b\sec(c+dx))^{5/2}}{7bd}$$

3.538. $\int \sec^3(c+dx)(a+b\sec(c+dx))^{3/2}\,dx$

$$\Big\downarrow \text{4492}$$

$$\frac{\frac{1}{5}\left(\frac{1}{3}\left(\frac{2(a-b)\sqrt{a+b}\left(6a^2+57ab-25b^2\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{bd}+\frac{4a(a-b)\sqrt{a}}{}\right.\right.}{}$$

$$\frac{2\tan(c+dx)(a+b\sec(c+dx))^{5/2}}{7bd}$$

input
```
Int[Sec[c + d*x]^3*(a + b*Sec[c + d*x])^(3/2),x]
```

output
```
(2*(a + b*Sec[c + d*x])^(5/2)*Tan[c + d*x])/(7*b*d) + ((-4*a*(a + b*Sec[c
+ d*x])^(3/2)*Tan[c + d*x])/(5*d) + (((4*a*(a - b)*Sqrt[a + b]*(3*a^2 - 41
*b^2)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]],
 (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[
c + d*x]))/(a - b))])/(b^2*d) + (2*(a - b)*Sqrt[a + b]*(6*a^2 + 57*a*b - 2
5*b^2)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]]
, (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec
[c + d*x]))/(a - b))])/(b*d))/3 - (2*(6*a^2 - 25*b^2)*Sqrt[a + b*Sec[c + d
*x]]*Tan[c + d*x])/(3*d))/5)/(7*b)
```

3.538.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4319
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4327
```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(-Cot[e + f*x])*((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2
))), x] + Simp[1/(b*(m + 2))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(b*(
m + 1) - a*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 -
b^2, 0] &&  !LtQ[m, -1]
```

rule 4490
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*((
a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Simp[1/(m + 1)   Int[Csc[e + f*x]*
(a + b*Csc[e + f*x])^(m - 1)*Simp[b*B*m + a*A*(m + 1) + (a*B*m + A*b*(m + 1
))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*
B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

rule 4492
```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

rule 4493
```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(A - B)   Int[Csc[e
 + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Simp[B   Int[Csc[e + f*x]*((1 +
Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A, B}
, x] && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

3.538.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2342 vs. $2(308) = 616$.

Time = 15.60 (sec) , antiderivative size = 2343, normalized size of antiderivative = 6.85

method	result	size
default	Expression too large to display	2343

input
```
int(sec(d*x+c)^3*(a+b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/105/d/b^2*(a+b*sec(d*x+c))^(1/2)/(b+a*cos(d*x+c))/(cos(d*x+c)+1)*(-3*a^3
*b*sin(d*x+c)+107*a*b^3*sin(d*x+c)+25*b^4*sin(d*x+c)+27*a^2*b^2*sin(d*x+c)
-6*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE(cot(d*x+c)-cs
c(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*a^3*b+82*(
1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE(cot(d*x+c)-csc(d*
x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*a^2*b^2+82*(1/
(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE(cot(d*x+c)-csc(d*x+
c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*a*b^3-25*Ellipti
cF(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(c
os(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*b^4*cos(d*x+c)^2-6*(
1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))
^(1/2)*EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*a^4*cos(d*x+c)
^2-50*EllipticF(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*(1/(a+b)*(b+a*c
os(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*b^4*cos
(d*x+c)-12*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(co
s(d*x+c)+1))^(1/2)*EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*a^
4*cos(d*x+c)+6*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF(c
ot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2
)*a^3*b-51*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF(cot(d
*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)...
```

3.538.5 Fricas [F]

$$\int \sec^3(c+dx)(a+b\sec(c+dx))^{3/2}\,dx = \int (b\sec(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^3\,dx$$

input
```
integrate(sec(d*x+c)^3*(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
integral((b*sec(d*x + c)^4 + a*sec(d*x + c)^3)*sqrt(b*sec(d*x + c) + a), x
)
```

3.538.6 Sympy [F]

$$\int \sec^3(c+dx)(a+b\sec(c+dx))^{3/2}\,dx = \int (a+b\sec(c+dx))^{\frac{3}{2}}\sec^3(c+dx)\,dx$$

input
```
integrate(sec(d*x+c)**3*(a+b*sec(d*x+c))**(3/2),x)
```

output
```
Integral((a + b*sec(c + d*x))**(3/2)*sec(c + d*x)**3, x)
```

3.538.7 Maxima [F]

$$\int \sec^3(c+dx)(a+b\sec(c+dx))^{3/2}\,dx = \int (b\sec(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^3\,dx$$

input
```
integrate(sec(d*x+c)^3*(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^(3/2)*sec(d*x + c)^3, x)
```

3.538.8 Giac [F]

$$\int \sec^3(c+dx)(a+b\sec(c+dx))^{3/2}\,dx = \int (b\sec(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^3\,dx$$

input
```
integrate(sec(d*x+c)^3*(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^(3/2)*sec(d*x + c)^3, x)
```

3.538.9 Mupad [F(-1)]

Timed out.

$$\int \sec^3(c+dx)(a+b\sec(c+dx))^{3/2}\,dx = \int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}}{\cos\left(c+d\,x\right)^3}\,dx$$

input
```
int((a + b/cos(c + d*x))^(3/2)/cos(c + d*x)^3,x)
```

output
```
int((a + b/cos(c + d*x))^(3/2)/cos(c + d*x)^3, x)
```

3.539 $\int \sec^2(c+dx)(a+b\sec(c+dx))^{3/2}\,dx$

3.539.1 Optimal result 4042
3.539.2 Mathematica [A] (warning: unable to verify) 4043
3.539.3 Rubi [A] (verified) 4043
3.539.4 Maple [B] (verified) 4047
3.539.5 Fricas [F] 4048
3.539.6 Sympy [F] 4048
3.539.7 Maxima [F] 4048
3.539.8 Giac [F] 4049
3.539.9 Mupad [F(-1)] 4049

3.539.1 Optimal result

Integrand size = 23, antiderivative size = 282

$$\int \sec^2(c+dx)(a+b\sec(c+dx))^{3/2}\,dx = -\frac{2(a-b)\sqrt{a+b}(a^2+3b^2)\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{5b^2d} - \frac{2(a-3b)(a-b)\sqrt{a+b}\cot(c+dx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx}{a-b}}}{5bd} + \frac{2a\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{5d} + \frac{2(a+b\sec(c+dx))^{3/2}\tan(c+dx)}{5d}$$

output
```
-2/5*(a-b)*(a^2+3*b^2)*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(a+b)^(
1/2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(
1+sec(d*x+c))/(a-b))^(1/2)/b^2/d-2/5*(a-3*b)*(a-b)*cot(d*x+c)*EllipticF((a
+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-se
c(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/b/d+2/5*(a+b*sec(d*
x+c))^(3/2)*tan(d*x+c)/d+2/5*a*(a+b*sec(d*x+c))^(1/2)*tan(d*x+c)/d
```

3.539.2 Mathematica [A] (warning: unable to verify)

Time = 11.24 (sec) , antiderivative size = 408, normalized size of antiderivative = 1.45

$$\int \sec^2(c+dx)(a+b\sec(c+dx))^{3/2}\,dx =$$
$$-\frac{2\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)}(a+b\sec(c+dx))^{3/2}\left(2(a^3+a^2b+3ab^2+3b^3)\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\sqrt{\frac{b+a\cos(c+}{(a+b)(1+\cos(}}\right.}{}$$
$$+\frac{\cos(c+dx)(a+b\sec(c+dx))^{3/2}\left(\frac{2(a^2+3b^2)\sin(c+dx)}{5b}+\frac{4}{5}a\tan(c+dx)+\frac{2}{5}b\sec(c+dx)\tan(c+dx)\right)}{d(b+a\cos(c+dx))}$$

input
```
Integrate[Sec[c + d*x]^2*(a + b*Sec[c + d*x])^(3/2),x]
```

output
```
(-2*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(a + b*Sec[c + d*x])^(3/2)*(2*(a
^3 + a^2*b + 3*a*b^2 + 3*b^3)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(
b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c
+ d*x)/2]], (a - b)/(a + b)] - 2*b*(a^2 + 4*a*b + 3*b^2)*Sqrt[Cos[c + d*x]
/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x])
)]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + (a^2 + 3*b^2)*Co
s[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(5*b
*d*(b + a*Cos[c + d*x])^2*Sqrt[Sec[(c + d*x)/2]^2]*Sec[c + d*x]^(3/2)) + (
Cos[c + d*x]*(a + b*Sec[c + d*x])^(3/2)*((2*(a^2 + 3*b^2)*Sin[c + d*x])/(5
*b) + (4*a*Tan[c + d*x])/5 + (2*b*Sec[c + d*x]*Tan[c + d*x])/5))/(d*(b + a
*Cos[c + d*x]))
```

3.539.3 Rubi [A] (verified)

Time = 1.02 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.02, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {3042, 4322, 3042, 4490, 27, 3042, 4493, 3042, 4319, 4492}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(c+dx)(a+b\sec(c+dx))^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^2\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}dx$$

$$\Big\downarrow \text{4322}$$

$$\frac{3}{5}\int \sec(c+dx)(b+a\sec(c+dx))\sqrt{a+b\sec(c+dx)}dx+\frac{2\tan(c+dx)(a+b\sec(c+dx))^{3/2}}{5d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3}{5}\int \csc\left(c+dx+\frac{\pi}{2}\right)\left(b+a\csc\left(c+dx+\frac{\pi}{2}\right)\right)\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\tan(c+dx)(a+b\sec(c+dx))^{3/2}}{5d}$$

$$\Big\downarrow \text{4490}$$

$$\frac{3}{5}\left(\frac{2}{3}\int \frac{\sec(c+dx)\left(4ab+\left(a^2+3b^2\right)\sec(c+dx)\right)}{2\sqrt{a+b\sec(c+dx)}}dx+\frac{2a\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}\right)+\frac{2\tan(c+dx)(a+b\sec(c+dx))^{3/2}}{5d}$$

$$\Big\downarrow \text{27}$$

$$\frac{3}{5}\left(\frac{1}{3}\int \frac{\sec(c+dx)\left(4ab+\left(a^2+3b^2\right)\sec(c+dx)\right)}{\sqrt{a+b\sec(c+dx)}}dx+\frac{2a\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}\right)+\frac{2\tan(c+dx)(a+b\sec(c+dx))^{3/2}}{5d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3}{5}\left(\frac{1}{3}\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(4ab+\left(a^2+3b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}\right)+\frac{2\tan(c+dx)(a+b\sec(c+dx))^{3/2}}{5d}$$

$$\Big\downarrow \text{4493}$$

$$\frac{3}{5}\left(\frac{1}{3}\left(\left(a^2+3b^2\right)\int \frac{\sec(c+dx)(\sec(c+dx)+1)}{\sqrt{a+b\sec(c+dx)}}dx-(a-3b)(a-b)\int \frac{\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx\right)+\frac{2a\tan(c+dx}{}\right.\frac{2\tan(c+dx)(a+b\sec(c+dx))^{3/2}}{5d}$$

$$\Big\downarrow \text{3042}$$

3.539. $\int \sec^2(c+dx)(a+b\sec(c+dx))^{3/2}\,dx$

$$\frac{3}{5}\left(\frac{1}{3}\left(\left(a^2+3b^2\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-(a-3b)(a-b)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)\right.$$
$$\frac{2\tan(c+dx)(a+b\sec(c+dx))^{3/2}}{5d}$$

$\downarrow$ 4319

$$\frac{3}{5}\left(\frac{1}{3}\left(\left(a^2+3b^2\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2(a-3b)(a-b)\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+}{a+b}}}{}\right.\right.$$
$$\frac{2\tan(c+dx)(a+b\sec(c+dx))^{3/2}}{5d}$$

$\downarrow$ 4492

$$\frac{3}{5}\left(\frac{1}{3}\left(-\frac{2(a-b)\sqrt{a+b}\left(a^2+3b^2\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}E\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{b^2d}-\right.\right.$$
$$\frac{2\tan(c+dx)(a+b\sec(c+dx))^{3/2}}{5d}$$

input
```
Int[Sec[c + d*x]^2*(a + b*Sec[c + d*x])^(3/2),x]
```

output
```
(2*(a + b*Sec[c + d*x])^(3/2)*Tan[c + d*x])/(5*d) + (3*(((-2*(a - b)*Sqrt[
a + b]*(a^2 + 3*b^2)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]
]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt
[-((b*(1 + Sec[c + d*x]))/(a - b))])/(b^2*d) - (2*(a - 3*b)*(a - b)*Sqrt[a
 + b]*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]],
 (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[
c + d*x]))/(a - b))])/(b*d))/3 + (2*a*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x
])/(3*d)))/5
```

3.539. $\int \sec^2(c+dx)(a+b\sec(c+dx))^{3/2}\,dx$

3.539.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4319
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4322
```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(-Cot[e + f*x])*((a + b*Csc[e + f*x])^m/(f*(m + 1))), x]
+ Simp[m/(m + 1)   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(b + a*Csc
[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m,
 0]
```

rule 4490
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*((
a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Simp[1/(m + 1)   Int[Csc[e + f*x]*
(a + b*Csc[e + f*x])^(m - 1)*Simp[b*B*m + a*A*(m + 1) + (a*B*m + A*b*(m + 1
))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*
B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

rule 4492
```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

3.539. $\int \sec^2(c+dx)(a+b\sec(c+dx))^{3/2}\,dx$

rule 4493
```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(A - B)   Int[Csc[e
 + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Simp[B   Int[Csc[e + f*x]*((1 +
Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A, B}
, x] && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

3.539.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1975 vs. $2(252) = 504$.

Time = 12.06 (sec) , antiderivative size = 1976, normalized size of antiderivative = 7.01

method	result	size
default	Expression too large to display	1976

input
```
int(sec(d*x+c)^2*(a+b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/5/d/b*(a+b*sec(d*x+c))^(1/2)/(b+a*cos(d*x+c))/(cos(d*x+c)+1)*(3*a*b^2*si
n(d*x+c)+3*a^2*b*sin(d*x+c)+EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^
(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*
x+c)+1))^(1/2)*a^2*b+3*EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2)
)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+
1))^(1/2)*a*b^2+(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c
)/(cos(d*x+c)+1))^(1/2)*EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2
))*a^3*cos(d*x+c)^2+3*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos
(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b)
)^(1/2))*b^3*cos(d*x+c)^2-EllipticF(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1
/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+
c)+1))^(1/2)*a^2*b-4*EllipticF(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*
(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1)
)^(1/2)*a*b^2-6*EllipticF(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*(1/(a
+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/
2)*b^3*cos(d*x+c)+2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d
*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^
(1/2))*a^3*cos(d*x+c)-3*EllipticF(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2
))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)
+1))^(1/2)*b^3*cos(d*x+c)^2+6*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))...
```

3.539.5 Fricas [F]

$$\int \sec^2(c+dx)(a+b\sec(c+dx))^{3/2}\,dx = \int (b\sec(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^2\,dx$$

input
```
integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
integral((b*sec(d*x + c)^3 + a*sec(d*x + c)^2)*sqrt(b*sec(d*x + c) + a), x
)
```

3.539.6 Sympy [F]

$$\int \sec^2(c+dx)(a+b\sec(c+dx))^{3/2}\,dx = \int (a+b\sec(c+dx))^{\frac{3}{2}}\sec^2(c+dx)\,dx$$

input
```
integrate(sec(d*x+c)**2*(a+b*sec(d*x+c))**(3/2),x)
```

output
```
Integral((a + b*sec(c + d*x))**(3/2)*sec(c + d*x)**2, x)
```

3.539.7 Maxima [F]

$$\int \sec^2(c+dx)(a+b\sec(c+dx))^{3/2}\,dx = \int (b\sec(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^2\,dx$$

input
```
integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^(3/2)*sec(d*x + c)^2, x)
```

3.539.8 Giac [F]

$$\int \sec^2(c+dx)(a+b\sec(c+dx))^{3/2}\,dx = \int (b\sec(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^2\,dx$$

input
```
integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^(3/2)*sec(d*x + c)^2, x)
```

3.539.9 Mupad [F(-1)]

Timed out.

$$\int \sec^2(c+dx)(a+b\sec(c+dx))^{3/2}\,dx = \int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}}{\cos(c+d\,x)^2}\,dx$$

input
```
int((a + b/cos(c + d*x))^(3/2)/cos(c + d*x)^2,x)
```

output
```
int((a + b/cos(c + d*x))^(3/2)/cos(c + d*x)^2, x)
```

3.540 $\int \sec(c+dx)(a+b\sec(c+dx))^{3/2}\,dx$

3.540.1 Optimal result 4050
3.540.2 Mathematica [A] (verified) 4051
3.540.3 Rubi [A] (verified) 4051
3.540.4 Maple [B] (verified) 4054
3.540.5 Fricas [F] 4055
3.540.6 Sympy [F] 4055
3.540.7 Maxima [F] 4055
3.540.8 Giac [F] 4056
3.540.9 Mupad [F(-1)] 4056

3.540.1 Optimal result

Integrand size = 21, antiderivative size = 249

$$\int \sec(c+dx)(a+b\sec(c+dx))^{3/2}\,dx = -\frac{8a(a-b)\sqrt{a+b}\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{3bd} + \frac{2(a-b)(3a-b)\sqrt{a+b}\cot(c+dx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{3bd} + \frac{2b\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{3d}$$

output
```
-8/3*a*(a-b)*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b
)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+
c))/(a-b))^(1/2)/b/d+2/3*(a-b)*(3*a-b)*cot(d*x+c)*EllipticF((a+b*sec(d*x+c
))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a
+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/b/d+2/3*b*(a+b*sec(d*x+c))^(1/2
)*tan(d*x+c)/d
```

3.540.2 Mathematica [A] (verified)

Time = 7.49 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.22

$$\int \sec(c+dx)(a+b\sec(c+dx))^{3/2}\,dx = -\frac{2\sqrt{a+b\sec(c+dx)}\left(8a(a+b)\cos^2\left(\frac{1}{2}(c+dx)\right)\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\sqrt{\frac{b+a\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}E\left(\arcsin\left(\tan\left(\frac{1}{2}(c+dx)\right)\right.\right.\right.}{}$$

input
```
Integrate[Sec[c + d*x]*(a + b*Sec[c + d*x])^(3/2),x]
```

output
```
(-2*Sqrt[a + b*Sec[c + d*x]]*(8*a*(a + b)*Cos[(c + d*x)/2]^2*Sqrt[Cos[c +
d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d
*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] - 2*(3*a^2 + 4
*a*b + b^2)*Cos[(c + d*x)/2]^2*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[
(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c
 + d*x)/2]], (a - b)/(a + b)] - 5*a*b*Sin[c + d*x] - 2*a^2*Sin[2*(c + d*x)
] + 4*a*b*Cos[c + d*x]*Tan[(c + d*x)/2] + 4*a^2*Cos[c + d*x]^2*Tan[(c + d*
x)/2] - b^2*Tan[c + d*x]))/(3*d*(b + a*Cos[c + d*x]))
```

3.540.3 Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {3042, 4317, 27, 3042, 4493, 3042, 4319, 4492}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(c+dx)(a+b\sec(c+dx))^{3/2}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}dx$$

$$\Big\downarrow \; 4317$$

$$\frac{2}{3}\int \frac{\sec(c+dx)\left(3a^2+4b\sec(c+dx)a+b^2\right)}{2\sqrt{a+b\sec(c+dx)}}dx+\frac{2b\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{3}\int \frac{\sec(c+dx)\left(3a^2+4b\sec(c+dx)a+b^2\right)}{\sqrt{a+b\sec(c+dx)}}dx+\frac{2b\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(3a^2+4b\csc\left(c+dx+\frac{\pi}{2}\right)a+b^2\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2b\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}$$

$$\Big\downarrow \text{4493}$$

$$\frac{1}{3}\left((a-b)(3a-b)\int \frac{\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx+4ab\int \frac{\sec(c+dx)(\sec(c+dx)+1)}{\sqrt{a+b\sec(c+dx)}}dx\right)+\frac{2b\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\left((a-b)(3a-b)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+4ab\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)+\frac{2b\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}$$

$$\Big\downarrow \text{4319}$$

$$\frac{1}{3}\left(4ab\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2(a-b)(3a-b)\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\text{se}}{}}}{bd}\right.$$
$$\frac{2b\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}$$

$$\Big\downarrow \text{4492}$$

$$\frac{1}{3}\left(\frac{2(a-b)(3a-b)\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{bd}\right.$$
$$\frac{2b\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}$$

input
```
Int[Sec[c + d*x]*(a + b*Sec[c + d*x])^(3/2),x]
```

3.540. $\int \sec(c+dx)(a+b\sec(c+dx))^{3/2}\,dx$

output
```
((-8*a*(a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c
+ d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)
]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(b*d) + (2*(a - b)*(3*a - b)*Sq
rt[a + b]*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a +
b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 +
Sec[c + d*x]))/(a - b))])/(b*d))/3 + (2*b*Sqrt[a + b*Sec[c + d*x]]*Tan[c +
 d*x])/(3*d)
```

3.540.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4317
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[(-b)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m - 1)/(f*m)), x]
+ Simp[1/m   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)*(b^2*(m - 1) + a
^2*m + a*b*(2*m - 1)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x] && NeQ
[a^2 - b^2, 0] && GtQ[m, 1] && IntegerQ[2*m]
```

rule 4319
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4492
```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

rule 4493
```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(A - B)   Int[Csc[e
 + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Simp[B   Int[Csc[e + f*x]*((1 +
Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A, B}
, x] && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

3.540.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1382 vs. $2(223) = 446$.

Time = 11.09 (sec) , antiderivative size = 1383, normalized size of antiderivative = 5.55

method	result	size
default	Expression too large to display	1383

input
```
int(sec(d*x+c)*(a+b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3/d*(a+b*sec(d*x+c))^(1/2)/(b+a*cos(d*x+c))/(cos(d*x+c)+1)*(3*(cos(d*x+
c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*E
llipticF(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*a^2*cos(d*x+c)^2+4*Ell
ipticF(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c)
)/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*a*b*cos(d*x+c)^2
+EllipticF(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*(1/(a+b)*(b+a*cos(d*
x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*b^2*cos(d*x+
c)^2-4*EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*(1/(a+b)*(b+a*
cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*a^2*co
s(d*x+c)^2-4*EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*(1/(a+b)
*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*
a*b*cos(d*x+c)^2+6*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x
+c))/(cos(d*x+c)+1))^(1/2)*EllipticF(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(
1/2))*a^2*cos(d*x+c)+8*EllipticF(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2)
)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+
1))^(1/2)*a*b*cos(d*x+c)+2*EllipticF(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(
1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x
+c)+1))^(1/2)*b^2*cos(d*x+c)-8*EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b
))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos
(d*x+c)+1))^(1/2)*a^2*cos(d*x+c)-8*EllipticE(cot(d*x+c)-csc(d*x+c),((a-...
```

3.540.5 Fricas [F]

$$\int \sec(c+dx)(a+b\sec(c+dx))^{3/2}\,dx = \int (b\sec(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)\,dx$$

input
```
integrate(sec(d*x+c)*(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
integral((b*sec(d*x + c)^2 + a*sec(d*x + c))*sqrt(b*sec(d*x + c) + a), x)
```

3.540.6 Sympy [F]

$$\int \sec(c+dx)(a+b\sec(c+dx))^{3/2}\,dx = \int (a+b\sec(c+dx))^{\frac{3}{2}}\sec(c+dx)\,dx$$

input
```
integrate(sec(d*x+c)*(a+b*sec(d*x+c))**(3/2),x)
```

output
```
Integral((a + b*sec(c + d*x))**(3/2)*sec(c + d*x), x)
```

3.540.7 Maxima [F]

$$\int \sec(c+dx)(a+b\sec(c+dx))^{3/2}\,dx = \int (b\sec(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)\,dx$$

input
```
integrate(sec(d*x+c)*(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^(3/2)*sec(d*x + c), x)
```

3.540.8 Giac [F]

$$\int \sec(c+dx)(a+b\sec(c+dx))^{3/2}\,dx = \int (b\sec(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)\,dx$$

input
```
integrate(sec(d*x+c)*(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^(3/2)*sec(d*x + c), x)
```

3.540.9 Mupad [F(-1)]

Timed out.

$$\int \sec(c+dx)(a+b\sec(c+dx))^{3/2}\,dx = \int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}}{\cos(c+d\,x)}\,dx$$

input
```
int((a + b/cos(c + d*x))^(3/2)/cos(c + d*x),x)
```

output
```
int((a + b/cos(c + d*x))^(3/2)/cos(c + d*x), x)
```

3.541 $\int (a+b\sec(c+dx))^{3/2}\,dx$

3.541.1 Optimal result 4057
3.541.2 Mathematica [B] (verified) 4058
3.541.3 Rubi [A] (verified) 4059
3.541.4 Maple [B] (verified) 4062
3.541.5 Fricas [F] 4062
3.541.6 Sympy [F] 4063
3.541.7 Maxima [F] 4063
3.541.8 Giac [F] 4063
3.541.9 Mupad [F(-1)] 4064

3.541.1 Optimal result

Integrand size = 14, antiderivative size = 309

$$\int (a+b\sec(c+dx))^{3/2}\,dx = -\frac{2(a-b)\sqrt{a+b}\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{d} + \frac{2(2a-b)\sqrt{a+b}\cot(c+dx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{d} - \frac{2a\sqrt{a+b}\cot(c+dx)\,\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{d}$$

output
```
-2*(a-b)*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a
-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/
(a-b))^(1/2)/d+2*(2*a-b)*cot(d*x+c)*EllipticF((a+b*sec(d*x+c))^(1/2)/(a+b)
^(1/2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b
*(1+sec(d*x+c))/(a-b))^(1/2)/d-2*a*cot(d*x+c)*EllipticPi((a+b*sec(d*x+c))^
(1/2)/(a+b)^(1/2),(a+b)/a,((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c
))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/d
```

3.541.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 684 vs. $2(309) = 618$.

Time = 16.62 (sec) , antiderivative size = 684, normalized size of antiderivative = 2.21

$$\int (a+b\sec(c+dx))^{3/2}\,dx = \frac{2b\cos(c+dx)(a+b\sec(c+dx))^{3/2}\sin(c+dx)}{d(b+a\cos(c+dx))} + \frac{2(a+b\sec(c+dx))^{3/2}\left(ab\tan\left(\frac{1}{2}(c+dx)\right)+b^2\tan\left(\frac{1}{2}(c+dx)\right)-2ab\tan^3\left(\frac{1}{2}(c+dx)\right)+ab\tan^5\left(\frac{1}{2}(c+d\right.\right.}{}$$

input
```
Integrate[(a + b*Sec[c + d*x])^(3/2),x]
```

output
```
(2*b*Cos[c + d*x]*(a + b*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(d*(b + a*Cos[c
 + d*x])) + (2*(a + b*Sec[c + d*x])^(3/2)*(a*b*Tan[(c + d*x)/2] + b^2*Tan[
(c + d*x)/2] - 2*a*b*Tan[(c + d*x)/2]^3 + a*b*Tan[(c + d*x)/2]^5 - b^2*Tan
[(c + d*x)/2]^5 - 2*a^2*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (a - b)/(
a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 +
b*Tan[(c + d*x)/2]^2)/(a + b)] - 2*a^2*EllipticPi[-1, ArcSin[Tan[(c + d*x)
/2]], (a - b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqr
t[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + b*(a +
b)*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c +
d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b
*Tan[(c + d*x)/2]^2)/(a + b)] + (a^2 - 2*a*b - b^2)*EllipticF[ArcSin[Tan[(
c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c +
d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a +
 b)]))/(d*(b + a*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(3/2)*Sqrt[(1 - Tan[(c +
 d*x)/2]^2)^(-1)]*(-1 + Tan[(c + d*x)/2]^2)*(1 + Tan[(c + d*x)/2]^2)^(3/2)
*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c +
d*x)/2]^2)])
```

3.541.3 Rubi [A] (verified)

Time = 0.95 (sec) , antiderivative size = 309, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {3042, 4268, 3042, 4409, 3042, 4271, 4319, 4492}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\sec(c+dx))^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}dx$$

$$\Big\downarrow \text{4268}$$

$$\int \frac{a^2+(2a-b)b\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx+b^2\int \frac{\sec(c+dx)(\sec(c+dx)+1)}{\sqrt{a+b\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a^2+(2a-b)b\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+b^2\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4409}$$

$$a^2\int \frac{1}{\sqrt{a+b\sec(c+dx)}}dx+b^2\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+b(2a-b)\int \frac{\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$a^2\int \frac{1}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+b^2\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+b(2a-b)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4271}$$

$$b^2\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+b(2a-b)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2a\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{d}$$

$$\Big\downarrow \text{4319}$$

$$b^2\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+$$

$$\frac{2(2a-b)\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{d}-$$

$$\frac{2a\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{d}$$

$$\Big\downarrow \text{4492}$$

$$\frac{2(2a-b)\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{d}-$$

$$\frac{2(a-b)\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}E\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{d}-$$

$$\frac{2a\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{d}$$

input
```
Int[(a + b*Sec[c + d*x])^(3/2),x]
```

output
```
(-2*(a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d
*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*S
qrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/d + (2*(2*a - b)*Sqrt[a + b]*Cot[c
 + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a
 - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/
(a - b))])/d - (2*a*Sqrt[a + b]*Cot[c + d*x]*EllipticPi[(a + b)/a, ArcSin[
Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c
 + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/d
```

3.541. $\int (a+b\sec(c+dx))^{3/2}\,dx$

3.541.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4268
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(3/2), x_Symbol] :> Int[(a^2 + b
*(2*a - b)*Csc[c + d*x])/Sqrt[a + b*Csc[c + d*x]], x] + Simp[b^2   Int[Csc[
c + d*x]*((1 + Csc[c + d*x])/Sqrt[a + b*Csc[c + d*x]]), x], x] /; FreeQ[{a,
 b, c, d}, x] && NeQ[a^2 - b^2, 0]
```

rule 4271
```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*(Rt[a
 + b, 2]/(a*d*Cot[c + d*x]))*Sqrt[b*((1 - Csc[c + d*x])/(a + b))]*Sqrt[(-b)
*((1 + Csc[c + d*x])/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[
c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

rule 4319
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4409
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Simp[c   Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] +
Simp[d   Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

rule 4492
```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

3.541.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1604 vs. $2(282) = 564$.

Time = 8.51 (sec) , antiderivative size = 1605, normalized size of antiderivative = 5.19

method	result	size
default	Expression too large to display	1605

input
```
int((a+b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/d*(2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d
*x+c)+1))^(1/2)*EllipticPi(cot(d*x+c)-csc(d*x+c),-1,((a-b)/(a+b))^(1/2))*a
^2*cos(d*x+c)^2-EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*(1/(a
+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/
2)*a*b*cos(d*x+c)^2-(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*
x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^
(1/2))*b^2*cos(d*x+c)^2-(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*co
s(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF(cot(d*x+c)-csc(d*x+c),((a-b)/(a+
b))^(1/2))*a^2*cos(d*x+c)^2+2*EllipticF(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b)
)^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(
d*x+c)+1))^(1/2)*a*b*cos(d*x+c)^2+EllipticF(cot(d*x+c)-csc(d*x+c),((a-b)/(
a+b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(
cos(d*x+c)+1))^(1/2)*b^2*cos(d*x+c)^2+4*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*
(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi(cot(d*x+c)-csc(
d*x+c),-1,((a-b)/(a+b))^(1/2))*a^2*cos(d*x+c)-2*EllipticE(cot(d*x+c)-csc(d
*x+c),((a-b)/(a+b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)
*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*a*b*cos(d*x+c)-2*(cos(d*x+c)/(cos(d*x+c
)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE(cot(
d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*b^2*cos(d*x+c)-2*(cos(d*x+c)/(cos(d
*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ellipti...
```

3.541.5 Fricas [F]

$$\int (a + b\sec(c + dx))^{3/2}\,dx = \int (b\sec(dx + c) + a)^{\frac{3}{2}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
integral((b*sec(d*x + c) + a)^(3/2), x)
```

3.541.6 Sympy [F]

$$\int (a+b\sec(c+dx))^{3/2}\,dx=\int (a+b\sec(c+dx))^{\frac{3}{2}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))**(3/2),x)
```

output
```
Integral((a + b*sec(c + d*x))**(3/2), x)
```

3.541.7 Maxima [F]

$$\int (a+b\sec(c+dx))^{3/2}\,dx=\int (b\sec(dx+c)+a)^{\frac{3}{2}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^(3/2), x)
```

3.541.8 Giac [F]

$$\int (a+b\sec(c+dx))^{3/2}\,dx=\int (b\sec(dx+c)+a)^{\frac{3}{2}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^(3/2), x)
```

3.541.9 Mupad [F(-1)]

Timed out.

$$\int (a+b\sec(c+dx))^{3/2}\,dx=\int \left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)^{3/2}\,dx$$

input
```
int((a + b/cos(c + d*x))^(3/2),x)
```

output
```
int((a + b/cos(c + d*x))^(3/2), x)
```

3.542 $\int \cos(c+dx)(a+b\sec(c+dx))^{3/2}\,dx$

3.542.1 Optimal result 4065
3.542.2 Mathematica [A] (verified) 4066
3.542.3 Rubi [A] (verified) 4066
3.542.4 Maple [B] (verified) 4070
3.542.5 Fricas [F] 4071
3.542.6 Sympy [F] 4071
3.542.7 Maxima [F] 4071
3.542.8 Giac [F] 4072
3.542.9 Mupad [F(-1)] 4072

3.542.1 Optimal result

Integrand size = 21, antiderivative size = 334

$$\int \cos(c+dx)(a+b\sec(c+dx))^{3/2}\,dx = \frac{a(a-b)\sqrt{a+b}\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{bd} + \frac{\sqrt{a+b}(a+2b)\cot(c+dx)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{d} - \frac{3b\sqrt{a+b}\cot(c+dx)\operatorname{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{d} + \frac{a\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{d}$$

output
```
a*(a-b)*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-
b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(
a-b))^(1/2)/b/d+(a+2*b)*cot(d*x+c)*EllipticF((a+b*sec(d*x+c))^(1/2)/(a+b)^
(1/2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*
(1+sec(d*x+c))/(a-b))^(1/2)/d-3*b*cot(d*x+c)*EllipticPi((a+b*sec(d*x+c))^(
1/2)/(a+b)^(1/2),(a+b)/a,((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c)
)/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/d+a*sin(d*x+c)*(a+b*sec(d*x
+c))^(1/2)/d
```

3.542.2 Mathematica [A] (verified)

Time = 8.67 (sec) , antiderivative size = 332, normalized size of antiderivative = 0.99

$$\int \cos(c+dx)(a+b\sec(c+dx))^{3/2}\,dx = \frac{\cos^2\left(\frac{1}{2}(c+dx)\right)\cos(c+dx)(a+b\sec(c+dx))^{3/2}\left(2a(a+b)\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\sqrt{\frac{b+a\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}E\right.}{}$$

input
```
Integrate[Cos[c + d*x]*(a + b*Sec[c + d*x])^(3/2),x]
```

output
```
(Cos[(c + d*x)/2]^2*Cos[c + d*x]*(a + b*Sec[c + d*x])^(3/2)*(2*a*(a + b)*S
qrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1
 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] -
4*(2*a - b)*b*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*
x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a -
 b)/(a + b)] + 12*a*b*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Co
s[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticPi[-1, ArcSin[Tan[(c + d
*x)/2]], (a - b)/(a + b)] + a*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d
*x)/2]^2*Tan[(c + d*x)/2]))/(d*(b + a*Cos[c + d*x])^2)
```

3.542.3 Rubi [A] (verified)

Time = 1.19 (sec) , antiderivative size = 341, normalized size of antiderivative = 1.02, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.524$, Rules used = {3042, 4351, 25, 3042, 4546, 3042, 4409, 3042, 4271, 4319, 4492}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(c+dx)(a+b\sec(c+dx))^{3/2}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}{\csc\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \; 4351$$

$$\frac{a\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{d}-\frac{1}{2}\int -\frac{2\sec(c+dx)b^2-a\sec^2(c+dx)b+3ab}{\sqrt{a+b\sec(c+dx)}}dx$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\int \frac{2\sec(c+dx)b^2-a\sec^2(c+dx)b+3ab}{\sqrt{a+b\sec(c+dx)}}dx+\frac{a\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\int \frac{2\csc\left(c+dx+\frac{\pi}{2}\right)b^2-a\csc\left(c+dx+\frac{\pi}{2}\right)^2b+3ab}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{a\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{d}$$

$$\Big\downarrow \text{4546}$$

$$\frac{1}{2}\left(\int \frac{3ab+\left(2b^2+ab\right)\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx-ab\int \frac{\sec(c+dx)(\sec(c+dx)+1)}{\sqrt{a+b\sec(c+dx)}}dx\right)+\frac{a\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\left(\int \frac{3ab+\left(2b^2+ab\right)\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-ab\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)+\frac{a\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{d}$$

$$\Big\downarrow \text{4409}$$

$$\frac{1}{2}\left(-ab\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+3ab\int \frac{1}{\sqrt{a+b\sec(c+dx)}}dx+b(a+2b)\int \frac{\sec(c+d}{\sqrt{a+b\sec(c}}\right.$$
$$\frac{a\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\left(3ab\int \frac{1}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+b(a+2b)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-ab\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c\right.\right.}{\sqrt{a+b\csc\left(c+d\right.}}\right.$$
$$\frac{a\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{d}$$

$$\Big\downarrow \text{4271}$$

3.542. $\int \cos(c+dx)(a+b\sec(c+dx))^{3/2}\,dx$

$$\frac{1}{2}\left(b(a+2b)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-ab\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{6b\sqrt{a+b}\cot(c-}{}\right.$$

$$\frac{a\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{d}$$

$$\Big\downarrow \; 4319$$

$$\frac{1}{2}\left(-ab\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sqrt{a+b}(a+2b)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx}{a-b}}}{d}\right.$$

$$\frac{a\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{d}$$

$$\Big\downarrow \; 4492$$

$$\frac{1}{2}\left(\frac{2\sqrt{a+b}(a+2b)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{d}+\frac{2a(a}{}\right.$$

$$\frac{a\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{d}$$

input
```
Int[Cos[c + d*x]*(a + b*Sec[c + d*x])^(3/2),x]
```

output
```
((2*a*(a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c +
 d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]
*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(b*d) + (2*Sqrt[a + b]*(a + 2*b)
*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a +
 b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d
*x]))/(a - b))])/d - (6*b*Sqrt[a + b]*Cot[c + d*x]*EllipticPi[(a + b)/a, A
rcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 -
 Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/d)/2 + (
a*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/d
```

3.542. $\int \cos(c+dx)(a+b\sec(c+dx))^{3/2}\,dx$

3.542.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4271
```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*(Rt[a
 + b, 2]/(a*d*Cot[c + d*x]))*Sqrt[b*((1 - Csc[c + d*x])/(a + b))]*Sqrt[(-b)
*((1 + Csc[c + d*x])/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[
c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

rule 4319
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4351
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(3/2), x_Symbol] :> Simp[a*Cot[e + f*x]*Sqrt[a + b*Csc[e + f*x]]*((d*C
sc[e + f*x])^n/(f*n)), x] + Simp[1/(2*d*n)   Int[((d*Csc[e + f*x])^(n + 1)/
Sqrt[a + b*Csc[e + f*x]])*Simp[a*b*(2*n - 1) + 2*(b^2*n + a^2*(n + 1))*Csc[
e + f*x] + a*b*(2*n + 3)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f
}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1] && IntegersQ[2*n]
```

rule 4409
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Simp[c   Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] +
Simp[d   Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

rule 4492
```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

3.542. $\int \cos(c+dx)(a+b\sec(c+dx))^{3/2}\,dx$

rule 4546
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Simp[C   Int[Csc[e + f*x]*((
1 + Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A
, B, C}, x] && NeQ[a^2 - b^2, 0]
```

3.542.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1351 vs. $2(305) = 610$.

Time = 8.38 (sec) , antiderivative size = 1352, normalized size of antiderivative = 4.05

method	result	size
default	Expression too large to display	1352

input
```
int(cos(d*x+c)*(a+b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(4*EllipticF(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*(1/(a+b)*(b+a*
cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*a*b*co
s(d*x+c)^2-2*EllipticF(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*(1/(a+b)
*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*
b^2*cos(d*x+c)^2-EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*(1/(
a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1
/2)*a^2*cos(d*x+c)^2-EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*
(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1)
)^(1/2)*a*b*cos(d*x+c)^2-6*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a
*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi(cot(d*x+c)-csc(d*x+c),-1,((a
-b)/(a+b))^(1/2))*a*b*cos(d*x+c)^2+8*EllipticF(cot(d*x+c)-csc(d*x+c),((a-b
)/(a+b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c
)/(cos(d*x+c)+1))^(1/2)*a*b*cos(d*x+c)-4*EllipticF(cot(d*x+c)-csc(d*x+c),(
(a-b)/(a+b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d
*x+c)/(cos(d*x+c)+1))^(1/2)*b^2*cos(d*x+c)-2*EllipticE(cot(d*x+c)-csc(d*x+
c),((a-b)/(a+b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(c
os(d*x+c)/(cos(d*x+c)+1))^(1/2)*a^2*cos(d*x+c)-2*EllipticE(cot(d*x+c)-csc(
d*x+c),((a-b)/(a+b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2
)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*a*b*cos(d*x+c)-12*(cos(d*x+c)/(cos(d*x
+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticP...
```

3.542.5 Fricas [F]

$$\int \cos(c+dx)(a+b\sec(c+dx))^{3/2}\,dx = \int (b\sec(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)\,dx$$

input
```
integrate(cos(d*x+c)*(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
integral((b*cos(d*x + c)*sec(d*x + c) + a*cos(d*x + c))*sqrt(b*sec(d*x + c
) + a), x)
```

3.542.6 Sympy [F]

$$\int \cos(c+dx)(a+b\sec(c+dx))^{3/2}\,dx = \int (a+b\sec(c+dx))^{\frac{3}{2}}\cos(c+dx)\,dx$$

input
```
integrate(cos(d*x+c)*(a+b*sec(d*x+c))**(3/2),x)
```

output
```
Integral((a + b*sec(c + d*x))**(3/2)*cos(c + d*x), x)
```

3.542.7 Maxima [F]

$$\int \cos(c+dx)(a+b\sec(c+dx))^{3/2}\,dx = \int (b\sec(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)\,dx$$

input
```
integrate(cos(d*x+c)*(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^(3/2)*cos(d*x + c), x)
```

3.542.8 Giac [F]

$$\int \cos(c+dx)(a+b\sec(c+dx))^{3/2}\,dx = \int (b\sec(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)\,dx$$

input
```
integrate(cos(d*x+c)*(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^(3/2)*cos(d*x + c), x)
```

3.542.9 Mupad [F(-1)]

Timed out.

$$\int \cos(c+dx)(a+b\sec(c+dx))^{3/2}\,dx = \int \cos(c+d\,x)\,\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}\,dx$$

input
```
int(cos(c + d*x)*(a + b/cos(c + d*x))^(3/2),x)
```

output
```
int(cos(c + d*x)*(a + b/cos(c + d*x))^(3/2), x)
```

3.543 $\int \cos^2(c+dx)(a+b\sec(c+dx))^{3/2}\,dx$

3.543.1 Optimal result . . . 4073
3.543.2 Mathematica [B] (verified) . . . 4074
3.543.3 Rubi [A] (verified) . . . 4075
3.543.4 Maple [B] (verified) . . . 4080
3.543.5 Fricas [F] . . . 4080
3.543.6 Sympy [F(-1)] . . . 4081
3.543.7 Maxima [F] . . . 4081
3.543.8 Giac [F] . . . 4081
3.543.9 Mupad [F(-1)] . . . 4082

3.543.1 Optimal result

Integrand size = 23, antiderivative size = 390

$$\int \cos^2(c+dx)(a+b\sec(c+dx))^{3/2}\,dx = \frac{5(a-b)\sqrt{a+b}\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{4d} + \frac{\sqrt{a+b}(2a+5b)\cot(c+dx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{4d} - \frac{\sqrt{a+b}(4a^2+3b^2)\cot(c+dx)\,\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx)}{a-b}}}{4ad} + \frac{5b\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{4d} + \frac{a\cos(c+dx)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{2d}$$

output
```
5/4*(a-b)*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(
a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))
/(a-b))^(1/2)/d+1/4*(2*a+5*b)*cot(d*x+c)*EllipticF((a+b*sec(d*x+c))^(1/2)/
(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2
)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/d-1/4*(4*a^2+3*b^2)*cot(d*x+c)*EllipticP
i((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),(a+b)/a,((a+b)/(a-b))^(1/2))*(a+b)^(1
/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/a/d+5/4
*b*sin(d*x+c)*(a+b*sec(d*x+c))^(1/2)/d+1/2*a*cos(d*x+c)*sin(d*x+c)*(a+b*se
c(d*x+c))^(1/2)/d
```

3.543.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 891 vs. $2(390) = 780$.

Time = 17.21 (sec) , antiderivative size = 891, normalized size of antiderivative = 2.28

$$\int \cos^2(c+dx)(a+b\sec(c+dx))^{3/2}\,dx = \frac{a\cos(c+dx)(a+b\sec(c+dx))^{3/2}\sin(2(c+dx))}{4d(b+a\cos(c+dx))} - \frac{(a+b\sec(c+dx))^{3/2}\left(5ab\tan\left(\frac{1}{2}(c+dx)\right)+5b^2\tan\left(\frac{1}{2}(c+dx)\right)-10ab\tan^3\left(\frac{1}{2}(c+dx)\right)+5ab\tan^5\left(\frac{1}{2}(c\right.\right.}{}$$

input
```
Integrate[Cos[c + d*x]^2*(a + b*Sec[c + d*x])^(3/2),x]
```

output
```
(a*Cos[c + d*x]*(a + b*Sec[c + d*x])^(3/2)*Sin[2*(c + d*x)])/(4*d*(b + a*C
os[c + d*x])) - ((a + b*Sec[c + d*x])^(3/2)*(5*a*b*Tan[(c + d*x)/2] + 5*b^
2*Tan[(c + d*x)/2] - 10*a*b*Tan[(c + d*x)/2]^3 + 5*a*b*Tan[(c + d*x)/2]^5
- 5*b^2*Tan[(c + d*x)/2]^5 + 8*a^2*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]]
, (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d
*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + 6*b^2*EllipticPi[-1, ArcSin[Ta
n[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b
 - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + 8*a^2*EllipticP
i[-1, ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1
 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x
)/2]^2)/(a + b)] + 6*b^2*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (a - b)/
(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*T
an[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + 5*b*(a + b)*EllipticE
[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(
1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*
x)/2]^2)/(a + b)] - 2*(2*a^2 - a*b + 4*b^2)*EllipticF[ArcSin[Tan[(c + d*x)
/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^
2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)]))/(
4*d*(b + a*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(3/2)*Sqrt[(1 - Tan[(c + d*x)/
2]^2)^(-1)]*(-1 + Tan[(c + d*x)/2]^2)*(1 + Tan[(c + d*x)/2]^2)^(3/2)*Sq...
```

3.543.3 Rubi [A] (verified)

Time = 1.64 (sec) , antiderivative size = 394, normalized size of antiderivative = 1.01, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.609, Rules used = {3042, 4351, 25, 3042, 4592, 27, 3042, 4546, 3042, 4409, 3042, 4271, 4319, 4492}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(c+dx)(a+b\sec(c+dx))^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}{\csc\left(c+dx+\frac{\pi}{2}\right)^2}dx$$

$$\Big\downarrow \text{4351}$$

$$\frac{a\sin(c+dx)\cos(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}-\frac{1}{4}\int -\frac{\cos(c+dx)\left(ab\sec^2(c+dx)+2\left(a^2+2b^2\right)\sec(c+dx)+5ab\right)}{\sqrt{a+b\sec(c+dx)}}dx$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{4}\int \frac{\cos(c+dx)\left(ab\sec^2(c+dx)+2\left(a^2+2b^2\right)\sec(c+dx)+5ab\right)}{\sqrt{a+b\sec(c+dx)}}dx+\frac{a\sin(c+dx)\cos(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}\int \frac{ab\csc\left(c+dx+\frac{\pi}{2}\right)^2+2\left(a^2+2b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)+5ab}{\csc\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{a\sin(c+dx)\cos(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}$$

$$\Big\downarrow \text{4592}$$

$$\frac{1}{4}\left(\frac{5b\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{d}-\frac{\int -\frac{2b\sec(c+dx)a^2-5b^2\sec^2(c+dx)a+\left(4a^2+3b^2\right)a}{2\sqrt{a+b\sec(c+dx)}}dx}{a}\right)+\frac{a\sin(c+dx)\cos(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{4}\left(\frac{\int \frac{2b\sec(c+dx)a^2-5b^2\sec^2(c+dx)a+(4a^2+3b^2)a}{\sqrt{a+b\sec(c+dx)}}dx}{2a}+\frac{5b\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{d}\right)+\frac{a\sin(c+dx)\cos(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}$$

$\Big\downarrow$ 3042

$$\frac{1}{4}\left(\frac{\int \frac{2b\csc(c+dx+\frac{\pi}{2})a^2-5b^2\csc(c+dx+\frac{\pi}{2})^2a+(4a^2+3b^2)a}{\sqrt{a+b\csc(c+dx+\frac{\pi}{2})}}dx}{2a}+\frac{5b\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{d}\right)+\frac{a\sin(c+dx)\cos(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}$$

$\Big\downarrow$ 4546

$$\frac{1}{4}\left(\frac{\int \frac{a(4a^2+3b^2)+(2ba^2+5b^2a)\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx-5ab^2\int \frac{\sec(c+dx)(\sec(c+dx)+1)}{\sqrt{a+b\sec(c+dx)}}dx}{2a}+\frac{5b\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{d}\right)+\frac{a\sin(c+dx)\cos(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}$$

$\Big\downarrow$ 3042

$$\frac{1}{4}\left(\frac{\int \frac{a(4a^2+3b^2)+(2ba^2+5b^2a)\csc(c+dx+\frac{\pi}{2})}{\sqrt{a+b\csc(c+dx+\frac{\pi}{2})}}dx-5ab^2\int \frac{\csc(c+dx+\frac{\pi}{2})(\csc(c+dx+\frac{\pi}{2})+1)}{\sqrt{a+b\csc(c+dx+\frac{\pi}{2})}}dx}{2a}+\frac{5b\sin(c+dx)\sqrt{a+b\sec(c+}}{d}\right.$$

$$\frac{a\sin(c+dx)\cos(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}$$

$\Big\downarrow$ 4409

$$\frac{1}{4}\left(\frac{a(4a^2+3b^2)\int \frac{1}{\sqrt{a+b\sec(c+dx)}}dx-5ab^2\int \frac{\csc(c+dx+\frac{\pi}{2})(\csc(c+dx+\frac{\pi}{2})+1)}{\sqrt{a+b\csc(c+dx+\frac{\pi}{2})}}dx+ab(2a+5b)\int \frac{\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx}{2a}+\frac{5b}{}\right.$$

$$\frac{a\sin(c+dx)\cos(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}$$

$\Big\downarrow$ 3042

3.543. $\int \cos^2(c+dx)(a+b\sec(c+dx))^{3/2}\,dx$

$$\frac{1}{4}\left(\frac{a(4a^2+3b^2)\int \frac{1}{\sqrt{a+b\csc(c+dx+\frac{\pi}{2})}}dx-5ab^2\int \frac{\csc(c+dx+\frac{\pi}{2})(\csc(c+dx+\frac{\pi}{2})+1)}{\sqrt{a+b\csc(c+dx+\frac{\pi}{2})}}dx+ab(2a+5b)\int \frac{\csc(c+dx+\frac{\pi}{2})}{\sqrt{a+b\csc(c+dx+\frac{\pi}{2})}}dx}{2a}\right.$$

$$\frac{a\sin(c+dx)\cos(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}$$

$\Big\downarrow$ 4271

$$\frac{1}{4}\left(\frac{-5ab^2\int \frac{\csc(c+dx+\frac{\pi}{2})(\csc(c+dx+\frac{\pi}{2})+1)}{\sqrt{a+b\csc(c+dx+\frac{\pi}{2})}}dx+ab(2a+5b)\int \frac{\csc(c+dx+\frac{\pi}{2})}{\sqrt{a+b\csc(c+dx+\frac{\pi}{2})}}dx-\frac{2\sqrt{a+b}(4a^2+3b^2)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+}{a+b}}}{}}{2a}\right.$$

$$\frac{a\sin(c+dx)\cos(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}$$

$\Big\downarrow$ 4319

$$\frac{1}{4}\left(\frac{-5ab^2\int \frac{\csc(c+dx+\frac{\pi}{2})(\csc(c+dx+\frac{\pi}{2})+1)}{\sqrt{a+b\csc(c+dx+\frac{\pi}{2})}}dx-\frac{2\sqrt{a+b}(4a^2+3b^2)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\operatorname{EllipticPi}\left(\frac{a+b}{a},\arcsin\right.}{d}}{2a}\right.$$

$$\frac{a\sin(c+dx)\cos(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}$$

$\Big\downarrow$ 4492

$$\frac{1}{4}\left(\frac{-\frac{2\sqrt{a+b}(4a^2+3b^2)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\operatorname{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{d}+\frac{2a\sqrt{a+b}(2a+5b)\cot(}{}}{}\right.$$

$$\frac{a\sin(c+dx)\cos(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}$$

input
```
Int[Cos[c + d*x]^2*(a + b*Sec[c + d*x])^(3/2),x]
```

3.543. $\int \cos^2(c+dx)(a+b\sec(c+dx))^{3/2}\,dx$

output
```
(a*Cos[c + d*x]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(2*d) + (((10*a*(a
- b)*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sq
rt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((
b*(1 + Sec[c + d*x]))/(a - b))])/d + (2*a*Sqrt[a + b]*(2*a + 5*b)*Cot[c +
d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a -
b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a
- b))])/d - (2*Sqrt[a + b]*(4*a^2 + 3*b^2)*Cot[c + d*x]*EllipticPi[(a + b)
/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b
*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/d)/
(2*a) + (5*b*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/d)/4
```

3.543.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4271
```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*(Rt[a
 + b, 2]/(a*d*Cot[c + d*x]))*Sqrt[b*((1 - Csc[c + d*x])/(a + b))]*Sqrt[(-b)
*((1 + Csc[c + d*x])/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[
c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

rule 4319
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4351
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(3/2), x_Symbol] :> Simp[a*Cot[e + f*x]*Sqrt[a + b*Csc[e + f*x]]*((d*C
sc[e + f*x])^n/(f*n)), x] + Simp[1/(2*d*n)   Int[((d*Csc[e + f*x])^(n + 1)/
Sqrt[a + b*Csc[e + f*x]])*Simp[a*b*(2*n - 1) + 2*(b^2*n + a^2*(n + 1))*Csc[
e + f*x] + a*b*(2*n + 3)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f
}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1] && IntegersQ[2*n]
```

rule 4409
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Simp[c   Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] +
Simp[d   Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

rule 4492
```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

rule 4546
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Simp[C   Int[Csc[e + f*x]*((
1 + Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A
, B, C}, x] && NeQ[a^2 - b^2, 0]
```

rule 4592
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Simp[1/(a*d*n)   Int[(a + b*Csc[e + f*x])^m
*(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*
Csc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

3.543. $\int \cos^2(c+dx)(a+b\sec(c+dx))^{3/2}\,dx$

3.543.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1928 vs. 2(349) = 698.

Time = 4.76 (sec) , antiderivative size = 1929, normalized size of antiderivative = 4.95

method	result	size
default	Expression too large to display	1929

input
```
int(cos(d*x+c)^2*(a+b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/4/d*(2*a^2*cos(d*x+c)^3*sin(d*x+c)+2*a^2*cos(d*x+c)^2*sin(d*x+c)+5*b^2*c
os(d*x+c)*sin(d*x+c)+8*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ell
ipticF(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+
1))^(1/2)*b^2+2*cos(d*x+c)*sin(d*x+c)*a*b+7*a*b*cos(d*x+c)^2*sin(d*x+c)-8*
(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1)
)^(1/2)*EllipticPi(cot(d*x+c)-csc(d*x+c),-1,((a-b)/(a+b))^(1/2))*a^2-6*(co
s(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(
1/2)*EllipticPi(cot(d*x+c)-csc(d*x+c),-1,((a-b)/(a+b))^(1/2))*b^2-5*(cos(d
*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2
)*EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*b^2+4*(cos(d*x+c)/(
cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ellip
ticF(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*a^2-2*EllipticF(cot(d*x+c)
-csc(d*x+c),((a-b)/(a+b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))
^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*a*b*cos(d*x+c)^2-5*EllipticE(cot(
d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+
c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*a*b*cos(d*x+c)^2-4*Elliptic
F(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(co
s(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*a*b*cos(d*x+c)-10*Ell
ipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c)
)/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*a*b*cos(d*x+c...
```

3.543.5 Fricas [F]

$$\int \cos^2(c+dx)(a+b\sec(c+dx))^{3/2}\,dx = \int (b\sec(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^2\,dx$$

input
```
integrate(cos(d*x+c)^2*(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
integral((b*cos(d*x + c)^2*sec(d*x + c) + a*cos(d*x + c)^2)*sqrt(b*sec(d*x
 + c) + a), x)
```

3.543.6 Sympy [F(-1)]

Timed out.

$$\int \cos^2(c+dx)(a+b\sec(c+dx))^{3/2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**2*(a+b*sec(d*x+c))**(3/2),x)
```

output
```
Timed out
```

3.543.7 Maxima [F]

$$\int \cos^2(c+dx)(a+b\sec(c+dx))^{3/2}\,dx = \int (b\sec(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^2\,dx$$

input
```
integrate(cos(d*x+c)^2*(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^(3/2)*cos(d*x + c)^2, x)
```

3.543.8 Giac [F]

$$\int \cos^2(c+dx)(a+b\sec(c+dx))^{3/2}\,dx = \int (b\sec(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^2\,dx$$

input
```
integrate(cos(d*x+c)^2*(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^(3/2)*cos(d*x + c)^2, x)
```

3.543.9 Mupad [F(-1)]

Timed out.

$$\int \cos^2(c+dx)(a+b\sec(c+dx))^{3/2}\,dx = \int \cos{(c+d\,x)}^2\,{\left(a+\frac{b}{\cos{(c+d\,x)}}\right)}^{3/2}\,dx$$

input
```
int(cos(c + d*x)^2*(a + b/cos(c + d*x))^(3/2),x)
```

output
```
int(cos(c + d*x)^2*(a + b/cos(c + d*x))^(3/2), x)
```

3.544 $\int \sec^4(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$

3.544.1 Optimal result 4083
3.544.2 Mathematica [A] (warning: unable to verify) 4084
3.544.3 Rubi [A] (verified) 4085
3.544.4 Maple [B] (verified) 4091
3.544.5 Fricas [F] 4091
3.544.6 Sympy [F(-1)] 4092
3.544.7 Maxima [F(-1)] 4092
3.544.8 Giac [F] 4092
3.544.9 Mupad [F(-1)] 4093

3.544.1 Optimal result

Integrand size = 23, antiderivative size = 463

$$\int \sec^4(c+dx)(a+b\sec(c+dx))^{5/2}\,dx =$$

$$-\frac{2a(a-b)\sqrt{a+b}(8a^4+51a^2b^2+741b^4)\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1-}{}}}{693b^4d}$$

$$-\frac{2(a-b)\sqrt{a+b}(8a^4+6a^3b+57a^2b^2-606ab^3+135b^4)\cot(c+dx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right.}{693b^3d}$$

$$+\frac{2(8a^4+57a^2b^2+135b^4)\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{693b^2d}$$

$$+\frac{2a(8a^2+67b^2)(a+b\sec(c+dx))^{3/2}\tan(c+dx)}{693b^2d}$$

$$+\frac{2(8a^2+81b^2)(a+b\sec(c+dx))^{5/2}\tan(c+dx)}{693b^2d}$$

$$-\frac{8a(a+b\sec(c+dx))^{7/2}\tan(c+dx)}{99b^2d}+\frac{2\sec(c+dx)(a+b\sec(c+dx))^{7/2}\tan(c+dx)}{11bd}$$

output
```
-2/693*a*(a-b)*(8*a^4+51*a^2*b^2+741*b^4)*cot(d*x+c)*EllipticE((a+b*sec(d*
x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))
/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/b^4/d-2/693*(a-b)*(8*a^4+6*a
^3*b+57*a^2*b^2-606*a*b^3+135*b^4)*cot(d*x+c)*EllipticF((a+b*sec(d*x+c))^(
1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))
^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/b^3/d+2/693*a*(8*a^2+67*b^2)*(a+b*s
ec(d*x+c))^(3/2)*tan(d*x+c)/b^2/d+2/693*(8*a^2+81*b^2)*(a+b*sec(d*x+c))^(5
/2)*tan(d*x+c)/b^2/d-8/99*a*(a+b*sec(d*x+c))^(7/2)*tan(d*x+c)/b^2/d+2/11*s
ec(d*x+c)*(a+b*sec(d*x+c))^(7/2)*tan(d*x+c)/b/d+2/693*(8*a^4+57*a^2*b^2+13
5*b^4)*(a+b*sec(d*x+c))^(1/2)*tan(d*x+c)/b^2/d
```

3.544.2 Mathematica [A] (warning: unable to verify)

Time = 14.13 (sec) , antiderivative size = 615, normalized size of antiderivative = 1.33

$$\int \sec^4(c+dx)(a+b\sec(c+dx))^{5/2}\,dx =$$

$$-\frac{2\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)}(a+b\sec(c+dx))^{5/2}\left(2a(8a^5+8a^4b+51a^3b^2+51a^2b^3+741ab^4+741b^5\right.}{}$$

$$+\frac{\cos^2(c+dx)(a+b\sec(c+dx))^{5/2}\left(\frac{2a(8a^4+51a^2b^2+741b^4)\sin(c+dx)}{693b^3}+\frac{2}{693}\sec^3(c+dx)\left(113a^2\sin(c+dx)+81b^2\right.\right.}{}$$

input
```
Integrate[Sec[c + d*x]^4*(a + b*Sec[c + d*x])^(5/2),x]
```

output
```
(-2*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(a + b*Sec[c + d*x])^(5/2)*(2*a*
(8*a^5 + 8*a^4*b + 51*a^3*b^2 + 51*a^2*b^3 + 741*a*b^4 + 741*b^5)*Sqrt[Cos
[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[
c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] - 2*b*(8*
a^5 + 2*a^4*b + 51*a^3*b^2 + 663*a^2*b^3 + 741*a*b^4 + 135*b^5)*Sqrt[Cos[c
 + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c
+ d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + a*(8*a^4
+ 51*a^2*b^2 + 741*b^4)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]
^2*Tan[(c + d*x)/2]))/(693*b^3*d*(b + a*Cos[c + d*x])^3*Sqrt[Sec[(c + d*x)
/2]^2]*Sec[c + d*x]^(5/2)) + (Cos[c + d*x]^2*(a + b*Sec[c + d*x])^(5/2)*((
2*a*(8*a^4 + 51*a^2*b^2 + 741*b^4)*Sin[c + d*x])/(693*b^3) + (2*Sec[c + d*
x]^3*(113*a^2*Sin[c + d*x] + 81*b^2*Sin[c + d*x]))/693 + (2*Sec[c + d*x]^2
*(3*a^3*Sin[c + d*x] + 229*a*b^2*Sin[c + d*x]))/(693*b) + (2*Sec[c + d*x]*
(-4*a^4*Sin[c + d*x] + 205*a^2*b^2*Sin[c + d*x] + 135*b^4*Sin[c + d*x]))/(
693*b^2) + (46*a*b*Sec[c + d*x]^3*Tan[c + d*x])/99 + (2*b^2*Sec[c + d*x]^4
*Tan[c + d*x])/11))/(d*(b + a*Cos[c + d*x])^2)
```

3.544.3 Rubi [A] (verified)

Time = 2.16 (sec) , antiderivative size = 481, normalized size of antiderivative = 1.04, number of steps used = 20, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.870$, Rules used = {3042, 4352, 27, 3042, 4570, 27, 3042, 4490, 27, 3042, 4490, 27, 3042, 4490, 27, 3042, 4493, 3042, 4319, 4492}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^4(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^4\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}dx$$

$$\Big\downarrow \; 4352$$

$$\frac{2\int \frac{1}{2}\sec(c+dx)(a+b\sec(c+dx))^{5/2}\left(-4a\sec^2(c+dx)+9b\sec(c+dx)+2a\right)dx}{11b}+\frac{2\tan(c+dx)\sec(c+dx)(a+b\sec(c+dx))^{7/2}}{11bd}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \sec(c+dx)(a+b\sec(c+dx))^{5/2}\left(-4a\sec^2(c+dx)+9b\sec(c+dx)+2a\right)dx}{11b}+\frac{2\tan(c+dx)\sec(c+dx)(a+b\sec(c+dx))^{7/2}}{11bd}$$

$\Big\downarrow$ 3042

$$\frac{\int \csc\left(c+dx+\frac{\pi}{2}\right)\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}\left(-4a\csc\left(c+dx+\frac{\pi}{2}\right)^2+9b\csc\left(c+dx+\frac{\pi}{2}\right)+2a\right)dx}{11b}+\frac{2\tan(c+dx)\sec(c+dx)(a+b\sec(c+dx))^{7/2}}{11bd}$$

$\Big\downarrow$ 4570

$$\frac{\frac{2\int -\frac{1}{2}\sec(c+dx)(a+b\sec(c+dx))^{5/2}\left(10ab-\left(8a^2+81b^2\right)\sec(c+dx)\right)dx}{9b}-\frac{8a\tan(c+dx)(a+b\sec(c+dx))^{7/2}}{9bd}}{11b}+\frac{2\tan(c+dx)\sec(c+dx)(a+b\sec(c+dx))^{7/2}}{11bd}$$

$\Big\downarrow$ 27

$$\frac{-\frac{\int \sec(c+dx)(a+b\sec(c+dx))^{5/2}\left(10ab-\left(8a^2+81b^2\right)\sec(c+dx)\right)dx}{9b}-\frac{8a\tan(c+dx)(a+b\sec(c+dx))^{7/2}}{9bd}}{11b}+\frac{2\tan(c+dx)\sec(c+dx)(a+b\sec(c+dx))^{7/2}}{11bd}$$

$\Big\downarrow$ 3042

$$\frac{-\frac{\int \csc\left(c+dx+\frac{\pi}{2}\right)\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}\left(10ab+\left(-8a^2-81b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)\right)dx}{9b}-\frac{8a\tan(c+dx)(a+b\sec(c+dx))^{7/2}}{9bd}}{11b}+\frac{2\tan(c+dx)\sec(c+dx)(a+b\sec(c+dx))^{7/2}}{11bd}$$

$\Big\downarrow$ 4490

$$\frac{-\frac{\frac{2}{7}\int \frac{5}{2}\sec(c+dx)(a+b\sec(c+dx))^{3/2}\left(3b\left(2a^2-27b^2\right)-a\left(8a^2+67b^2\right)\sec(c+dx)\right)dx-\frac{2\left(8a^2+81b^2\right)\tan(c+dx)(a+b\sec(c+dx))^{5/2}}{7d}}{9b}-\frac{8a\tan(c+dx)(a+}{9}}{11b}$$
$$\frac{2\tan(c+dx)\sec(c+dx)(a+b\sec(c+dx))^{7/2}}{11bd}$$

$\Big\downarrow$ 27

$$\frac{-\frac{\frac{5}{7}\int \sec(c+dx)(a+b\sec(c+dx))^{3/2}\left(3b\left(2a^2-27b^2\right)-a\left(8a^2+67b^2\right)\sec(c+dx)\right)dx-\frac{2\left(8a^2+81b^2\right)\tan(c+dx)(a+b\sec(c+dx))^{5/2}}{7d}}{9b}-\frac{8a\tan(c+dx)(a+b}{9bd}}{11b}$$
$$\frac{2\tan(c+dx)\sec(c+dx)(a+b\sec(c+dx))^{7/2}}{11bd}$$

3.544. $\int \sec^4(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$

$$\Big\downarrow \; 3042$$

$$\frac{-\frac{\frac{5}{7}\int \csc\left(c+dx+\frac{\pi}{2}\right)\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}\left(3b\left(2a^2-27b^2\right)-a\left(8a^2+67b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)\right)dx-\frac{2\left(8a^2+81b^2\right)\tan(c+dx)(a+b\sec(c+dx))^{5/2}}{7d}}{9b}-8a\tan}{11b}$$

$$\frac{2\tan(c+dx)\sec(c+dx)(a+b\sec(c+dx))^{7/2}}{11bd}$$

$$\Big\downarrow \; 4490$$

$$\frac{-\frac{\frac{5}{7}\left(\frac{2}{5}\int \frac{3}{2}\sec(c+dx)\sqrt{a+b\sec(c+dx)}\left(2ab\left(a^2-101b^2\right)-\left(8a^4+57b^2a^2+135b^4\right)\sec(c+dx)\right)dx-\frac{2a\left(8a^2+67b^2\right)\tan(c+dx)(a+b\sec(c+dx))^{3/2}}{5d}\right)}{9b}-\frac{2\left(8a^2\right.}{}}{11b}$$

$$\frac{2\tan(c+dx)\sec(c+dx)(a+b\sec(c+dx))^{7/2}}{11bd}$$

$$\Big\downarrow \; 27$$

$$\frac{-\frac{\frac{5}{7}\left(\frac{3}{5}\int \sec(c+dx)\sqrt{a+b\sec(c+dx)}\left(2ab\left(a^2-101b^2\right)-\left(8a^4+57b^2a^2+135b^4\right)\sec(c+dx)\right)dx-\frac{2a\left(8a^2+67b^2\right)\tan(c+dx)(a+b\sec(c+dx))^{3/2}}{5d}\right)}{9b}-\frac{2\left(8a^2+\right.}{}}{11b}$$

$$\frac{2\tan(c+dx)\sec(c+dx)(a+b\sec(c+dx))^{7/2}}{11bd}$$

$$\Big\downarrow \; 3042$$

$$\frac{-\frac{\frac{5}{7}\left(\frac{3}{5}\int \csc\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}\left(2ab\left(a^2-101b^2\right)+\left(-8a^4-57b^2a^2-135b^4\right)\csc\left(c+dx+\frac{\pi}{2}\right)\right)dx-\frac{2a\left(8a^2+67b^2\right)\tan(c+dx)(a+b\sec(c+dx))}{5d}\right)}{9b}}{11b}$$

$$\frac{2\tan(c+dx)\sec(c+dx)(a+b\sec(c+dx))^{7/2}}{11bd}$$

$$\Big\downarrow \; 4490$$

$$\frac{-\frac{\frac{5}{7}\left(\frac{3}{5}\left(\frac{2}{3}\int -\frac{\sec(c+dx)\left(b\left(2a^4+663b^2a^2+135b^4\right)+a\left(8a^4+51b^2a^2+741b^4\right)\sec(c+dx)\right)}{2\sqrt{a+b\sec(c+dx)}}dx-\frac{2\left(8a^4+57a^2b^2+135b^4\right)\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}\right)\right.}{9b}-\frac{2a\left(8a^2+67\right.}{}}{11b}$$

$$\frac{2\tan(c+dx)\sec(c+dx)(a+b\sec(c+dx))^{7/2}}{11bd}$$

$$\Big\downarrow \; 27$$

3.544. $\int \sec^4(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$

$$-\frac{\frac{5}{7}\left(\frac{3}{5}\left(-\frac{1}{3}\int \frac{\sec(c+dx)\left(b\left(2a^4+663b^2a^2+135b^4\right)+a\left(8a^4+51b^2a^2+741b^4\right)\sec(c+dx)\right)}{\sqrt{a+b\sec(c+dx)}}dx-\frac{2\left(8a^4+57a^2b^2+135b^4\right)\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}\right)-\frac{2a\left(8a^2+67\right.}{}\right.}{9b}}{11b}$$

$$\frac{2\tan(c+dx)\sec(c+dx)(a+b\sec(c+dx))^{7/2}}{11bd}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\frac{5}{7}\left(\frac{3}{5}\left(-\frac{1}{3}\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(b\left(2a^4+663b^2a^2+135b^4\right)+a\left(8a^4+51b^2a^2+741b^4\right)\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2\left(8a^4+57a^2b^2+135b^4\right)\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}\right)-\frac{2}{}\right.}{9b}}{11b}$$

$$\frac{2\tan(c+dx)\sec(c+dx)(a+b\sec(c+dx))^{7/2}}{11bd}$$

$$\Big\downarrow \; 4493$$

$$-\frac{\frac{5}{7}\left(\frac{3}{5}\left(\frac{1}{3}\left((a-b)\left(8a^4+6a^3b+57a^2b^2-606ab^3+135b^4\right)\int \frac{\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx-a\left(8a^4+51a^2b^2+741b^4\right)\int \frac{\sec(c+dx)(\sec(c+dx)+1)}{\sqrt{a+b\sec(c+dx)}}dx\right)-\frac{2\left(8a^4+57a^2b^2\right.}{}\right.}{9b}}{}$$

$$\frac{2\tan(c+dx)\sec(c+dx)(a+b\sec(c+dx))^{7/2}}{11bd}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\frac{5}{7}\left(\frac{3}{5}\left(\frac{1}{3}\left((a-b)\left(8a^4+6a^3b+57a^2b^2-606ab^3+135b^4\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-a\left(8a^4+51a^2b^2+741b^4\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)-\right.}{9b}}{}$$

$$\frac{2\tan(c+dx)\sec(c+dx)(a+b\sec(c+dx))^{7/2}}{11bd}$$

$$\Big\downarrow \; 4319$$

$$-\frac{\frac{5}{7}\left(\frac{3}{5}\left(\frac{1}{3}\left(\frac{2(a-b)\sqrt{a+b}\left(8a^4+6a^3b+57a^2b^2-606ab^3+135b^4\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{bd}-a\right.\right.}{}}{}$$

$$\frac{2\tan(c+dx)\sec(c+dx)(a+b\sec(c+dx))^{7/2}}{11bd}$$

$$\Big\downarrow \; 4492$$

3.544. $\int \sec^4(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$

$$-\frac{\frac{5}{7}\left(\frac{3}{5}\left(\frac{1}{3}\left(\frac{2a(a-b)\sqrt{a+b}\left(8a^4+51a^2b^2+741b^4\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}E\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{b^2d}+\frac{2(a-b)\sqrt{a+b}\left(8a^4+6a\right.}{}\right.\right.\right.}{}$$

$$\frac{2\tan(c+dx)\sec(c+dx)(a+b\sec(c+dx))^{7/2}}{11bd}$$

input
```
Int[Sec[c + d*x]^4*(a + b*Sec[c + d*x])^(5/2),x]
```

output
```
(2*Sec[c + d*x]*(a + b*Sec[c + d*x])^(7/2)*Tan[c + d*x])/(11*b*d) + ((-8*a
*(a + b*Sec[c + d*x])^(7/2)*Tan[c + d*x])/(9*b*d) - ((-2*(8*a^2 + 81*b^2)*
(a + b*Sec[c + d*x])^(5/2)*Tan[c + d*x])/(7*d) + (5*((-2*a*(8*a^2 + 67*b^2
)*(a + b*Sec[c + d*x])^(3/2)*Tan[c + d*x])/(5*d) + (3*(((2*a*(a - b)*Sqrt[
a + b]*(8*a^4 + 51*a^2*b^2 + 741*b^4)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a
 + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x
]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(b^2*d) + (2*(a - b)
*Sqrt[a + b]*(8*a^4 + 6*a^3*b + 57*a^2*b^2 - 606*a*b^3 + 135*b^4)*Cot[c +
d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a -
b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a
- b))])/(b*d))/3 - (2*(8*a^4 + 57*a^2*b^2 + 135*b^4)*Sqrt[a + b*Sec[c + d*
x]]*Tan[c + d*x])/(3*d)))/5))/7)/(9*b))/(11*b)
```

3.544.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4319
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4352
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-d^3)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m +
1)*((d*Csc[e + f*x])^(n - 3)/(b*f*(m + n - 1))), x] + Simp[d^3/(b*(m + n -
1))   Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 3)*Simp[a*(n - 3) +
b*(m + n - 2)*Csc[e + f*x] - a*(n - 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[
{a, b, d, e, f, m}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 3] && (IntegerQ[n] ||
 IntegersQ[2*m, 2*n]) &&  !IGtQ[m, 2]
```

rule 4490
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*((
a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Simp[1/(m + 1)   Int[Csc[e + f*x]*
(a + b*Csc[e + f*x])^(m - 1)*Simp[b*B*m + a*A*(m + 1) + (a*B*m + A*b*(m + 1
))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*
B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

rule 4492
```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

rule 4493
```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(A - B)   Int[Csc[e
 + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Simp[B   Int[Csc[e + f*x]*((1 +
Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A, B}
, x] && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

rule 4570
```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> Simp[(-C)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2)
)), x] + Simp[1/(b*(m + 2))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[
b*A*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /;
 FreeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

3.544. $\int \sec^4(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$

3.544.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3642 vs. $2(421) = 842$.

Time = 166.32 (sec) , antiderivative size = 3643, normalized size of antiderivative = 7.87

method	result	size
default	Expression too large to display	3643

input
```
int(sec(d*x+c)^4*(a+b*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/693/d/b^3*(a+b*sec(d*x+c))^(1/2)/(b+a*cos(d*x+c))/(cos(d*x+c)+1)*(224*a*
b^5*tan(d*x+c)*sec(d*x+c)^3-4*a^5*b*cos(d*x+c)*sin(d*x+c)+51*a^4*b^2*cos(d
*x+c)*sin(d*x+c)+205*a^3*b^3*cos(d*x+c)*sin(d*x+c)+741*a^2*b^4*cos(d*x+c)*
sin(d*x+c)+8*EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x
+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*
a^6+135*a*b^5*cos(d*x+c)*sin(d*x+c)+116*a^3*b^3*tan(d*x+c)*sec(d*x+c)+274*
a^2*b^4*tan(d*x+c)*sec(d*x+c)+310*a*b^5*tan(d*x+c)*sec(d*x+c)+274*a^2*b^4*
tan(d*x+c)*sec(d*x+c)^2+224*a*b^5*tan(d*x+c)*sec(d*x+c)^2-135*EllipticF(co
t(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)
*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*b^6+8*(cos(d*x+c)/(cos(d*
x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE(c
ot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*a^5*b*cos(d*x+c)^2+51*(cos(d*x+c
)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*El
lipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*a^4*b^2*cos(d*x+c)^2+51
*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1
))^(1/2)*EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b^3*cos(
d*x+c)^2+741*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(
cos(d*x+c)+1))^(1/2)*EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*
a^2*b^4*cos(d*x+c)^2+741*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*c
os(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)...
```

3.544.5 Fricas [F]

$$\int \sec^4(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \int (b\sec(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^4\,dx$$

input
```
integrate(sec(d*x+c)^4*(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
integral((b^2*sec(d*x + c)^6 + 2*a*b*sec(d*x + c)^5 + a^2*sec(d*x + c)^4)*
sqrt(b*sec(d*x + c) + a), x)
```

3.544.6 Sympy [F(-1)]

Timed out.

$$\int \sec^4(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**4*(a+b*sec(d*x+c))**(5/2),x)
```

output
```
Timed out
```

3.544.7 Maxima [F(-1)]

Timed out.

$$\int \sec^4(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)^4*(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
Timed out
```

3.544.8 Giac [F]

$$\int \sec^4(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \int (b\sec(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^4\,dx$$

input
```
integrate(sec(d*x+c)^4*(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^(5/2)*sec(d*x + c)^4, x)
```

3.544.9 Mupad [F(-1)]

Timed out.

$$\int \sec^4(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2}}{\cos\left(c+d\,x\right)^4}\,dx$$

input
```
int((a + b/cos(c + d*x))^(5/2)/cos(c + d*x)^4,x)
```

output
```
int((a + b/cos(c + d*x))^(5/2)/cos(c + d*x)^4, x)
```

3.545 $\int \sec^3(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$

3.545.1 Optimal result 4094
3.545.2 Mathematica [A] (warning: unable to verify) 4095
3.545.3 Rubi [A] (verified) 4095
3.545.4 Maple [B] (verified) 4100
3.545.5 Fricas [F] 4101
3.545.6 Sympy [F] 4102
3.545.7 Maxima [F] 4102
3.545.8 Giac [F] 4102
3.545.9 Mupad [F(-1)] 4103

3.545.1 Optimal result

Integrand size = 23, antiderivative size = 399

$$\int \sec^3(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \frac{2(a-b)\sqrt{a+b}(10a^4-279a^2b^2-147b^4)\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec}{a-}}}{315b^3d} + \frac{2(a-b)\sqrt{a+b}(10a^3+165a^2b-114ab^2+147b^3)\cot(c+dx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(}{}}}{315b^2d} - \frac{4a(5a^2-57b^2)\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{315bd} - \frac{2(10a^2-49b^2)(a+b\sec(c+dx))^{3/2}\tan(c+dx)}{315bd} - \frac{4a(a+b\sec(c+dx))^{5/2}\tan(c+dx)}{63bd} + \frac{2(a+b\sec(c+dx))^{7/2}\tan(c+dx)}{9bd}$$

output
```
2/315*(a-b)*(10*a^4-279*a^2*b^2-147*b^4)*cot(d*x+c)*EllipticE((a+b*sec(d*x
+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/
(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/b^3/d+2/315*(a-b)*(10*a^3+165
*a^2*b-114*a*b^2+147*b^3)*cot(d*x+c)*EllipticF((a+b*sec(d*x+c))^(1/2)/(a+b
)^(1/2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-
b*(1+sec(d*x+c))/(a-b))^(1/2)/b^2/d-2/315*(10*a^2-49*b^2)*(a+b*sec(d*x+c))
^(3/2)*tan(d*x+c)/b/d-4/63*a*(a+b*sec(d*x+c))^(5/2)*tan(d*x+c)/b/d+2/9*(a+
b*sec(d*x+c))^(7/2)*tan(d*x+c)/b/d-4/315*a*(5*a^2-57*b^2)*(a+b*sec(d*x+c))
^(1/2)*tan(d*x+c)/b/d
```

3.545.2 Mathematica [A] (warning: unable to verify)

Time = 13.33 (sec) , antiderivative size = 552, normalized size of antiderivative = 1.38

$$\int \sec^3(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \frac{2\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)}(a+b\sec(c+dx))^{5/2}\left(2(10a^5+10a^4b-279a^3b^2-279a^2b^3-\right.}{}$$

$$+\frac{\cos^2(c+dx)(a+b\sec(c+dx))^{5/2}\left(\frac{2(-10a^4+279a^2b^2+147b^4)\sin(c+dx)}{315b^2}+\frac{2}{315}\sec^2(c+dx)\left(75a^2\sin(c+dx)+49b\right.\right.}{d(b+}$$

input
```
Integrate[Sec[c + d*x]^3*(a + b*Sec[c + d*x])^(5/2),x]
```

output
```
(2*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(a + b*Sec[c + d*x])^(5/2)*(2*(10
*a^5 + 10*a^4*b - 279*a^3*b^2 - 279*a^2*b^3 - 147*a*b^4 - 147*b^5)*Sqrt[Co
s[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos
[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + 2*b*(-
10*a^4 + 155*a^3*b + 279*a^2*b^2 + 261*a*b^3 + 147*b^4)*Sqrt[Cos[c + d*x]/
(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))
]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + (10*a^4 - 279*a^2
*b^2 - 147*b^4)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(
c + d*x)/2]))/(315*b^2*d*(b + a*Cos[c + d*x])^3*Sqrt[Sec[(c + d*x)/2]^2]*S
ec[c + d*x]^(5/2)) + (Cos[c + d*x]^2*(a + b*Sec[c + d*x])^(5/2)*((2*(-10*a
^4 + 279*a^2*b^2 + 147*b^4)*Sin[c + d*x])/(315*b^2) + (2*Sec[c + d*x]^2*(7
5*a^2*Sin[c + d*x] + 49*b^2*Sin[c + d*x]))/315 + (2*Sec[c + d*x]*(5*a^3*Si
n[c + d*x] + 163*a*b^2*Sin[c + d*x]))/(315*b) + (38*a*b*Sec[c + d*x]^2*Tan
[c + d*x])/63 + (2*b^2*Sec[c + d*x]^3*Tan[c + d*x])/9))/(d*(b + a*Cos[c +
d*x])^2)
```

3.545.3 Rubi [A] (verified)

Time = 1.70 (sec) , antiderivative size = 409, normalized size of antiderivative = 1.03, number of steps used = 17, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.739$, Rules used = {3042, 4327, 27, 3042, 4490, 27, 3042, 4490, 27, 3042, 4490, 27, 3042, 4493, 3042, 4319, 4492}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^3(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^3\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}dx$$

$$\Big\downarrow \; 4327$$

$$\frac{2\int \frac{1}{2}\sec(c+dx)(7b-2a\sec(c+dx))(a+b\sec(c+dx))^{5/2}dx}{9b}+\frac{2\tan(c+dx)(a+b\sec(c+dx))^{7/2}}{9bd}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \sec(c+dx)(7b-2a\sec(c+dx))(a+b\sec(c+dx))^{5/2}dx}{9b}+\frac{2\tan(c+dx)(a+b\sec(c+dx))^{7/2}}{9bd}$$

$$\Big\downarrow \; 3042$$

$$\frac{\int \csc\left(c+dx+\frac{\pi}{2}\right)\left(7b-2a\csc\left(c+dx+\frac{\pi}{2}\right)\right)\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}dx}{9b}+\frac{2\tan(c+dx)(a+b\sec(c+dx))^{7/2}}{9bd}$$

$$\Big\downarrow \; 4490$$

$$\frac{\frac{2}{7}\int \frac{1}{2}\sec(c+dx)(a+b\sec(c+dx))^{3/2}\left(39ab-\left(10a^2-49b^2\right)\sec(c+dx)\right)dx-\frac{4a\tan(c+dx)(a+b\sec(c+dx))^{5/2}}{7d}}{9b}+\frac{2\tan(c+dx)(a+b\sec(c+dx))^{7/2}}{9bd}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{1}{7}\int \sec(c+dx)(a+b\sec(c+dx))^{3/2}\left(39ab-\left(10a^2-49b^2\right)\sec(c+dx)\right)dx-\frac{4a\tan(c+dx)(a+b\sec(c+dx))^{5/2}}{7d}}{9b}+\frac{2\tan(c+dx)(a+b\sec(c+dx))^{7/2}}{9bd}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{1}{7}\int \csc\left(c+dx+\frac{\pi}{2}\right)\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}\left(39ab+\left(49b^2-10a^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)\right)dx-\frac{4a\tan(c+dx)(a+b\sec(c}{7d}}{9b}$$
$$\frac{2\tan(c+dx)(a+b\sec(c+dx))^{7/2}}{9bd}$$

$$\Big\downarrow \; 4490$$

3.545. $\int \sec^3(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$

$$\frac{\frac{1}{7}\left(\frac{2}{5}\int \frac{3}{2}\sec(c+dx)\sqrt{a+b\sec(c+dx)}\left(b\left(55a^2+49b^2\right)-2a\left(5a^2-57b^2\right)\sec(c+dx)\right)dx-\frac{2\left(10a^2-49b^2\right)\tan(c+dx)(a}{5d}\right.}{9b}$$

$$\frac{2\tan(c+dx)(a+b\sec(c+dx))^{7/2}}{9bd}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{1}{7}\left(\frac{3}{5}\int \sec(c+dx)\sqrt{a+b\sec(c+dx)}\left(b\left(55a^2+49b^2\right)-2a\left(5a^2-57b^2\right)\sec(c+dx)\right)dx-\frac{2\left(10a^2-49b^2\right)\tan(c+dx)(a+}{5d}\right.}{9b}$$

$$\frac{2\tan(c+dx)(a+b\sec(c+dx))^{7/2}}{9bd}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{1}{7}\left(\frac{3}{5}\int \csc\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}\left(b\left(55a^2+49b^2\right)-2a\left(5a^2-57b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)\right)dx-\frac{2\left(10a^2-\right.}{}\right.}{9b}$$

$$\frac{2\tan(c+dx)(a+b\sec(c+dx))^{7/2}}{9bd}$$

$$\Big\downarrow \; 4490$$

$$\frac{\frac{1}{7}\left(\frac{3}{5}\left(\frac{2}{3}\int \frac{\sec(c+dx)\left(ab\left(155a^2+261b^2\right)-\left(10a^4-279b^2a^2-147b^4\right)\sec(c+dx)\right)}{2\sqrt{a+b\sec(c+dx)}}dx-\frac{4a\left(5a^2-57b^2\right)\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}\right)-\frac{2\left(10a^2-\right.}{}\right.}{9b}$$

$$\frac{2\tan(c+dx)(a+b\sec(c+dx))^{7/2}}{9bd}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{1}{7}\left(\frac{3}{5}\left(\frac{1}{3}\int \frac{\sec(c+dx)\left(ab\left(155a^2+261b^2\right)-\left(10a^4-279b^2a^2-147b^4\right)\sec(c+dx)\right)}{\sqrt{a+b\sec(c+dx)}}dx-\frac{4a\left(5a^2-57b^2\right)\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}\right)-\frac{2\left(10a^2-\right.}{}\right.}{9b}$$

$$\frac{2\tan(c+dx)(a+b\sec(c+dx))^{7/2}}{9bd}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{1}{7}\left(\frac{3}{5}\left(\frac{1}{3}\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(ab\left(155a^2+261b^2\right)+\left(-10a^4+279b^2a^2+147b^4\right)\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{4a\left(5a^2-57b^2\right)\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}\right)-\right.}{9b}$$

$$\frac{2\tan(c+dx)(a+b\sec(c+dx))^{7/2}}{9bd}$$

$$\Big\downarrow \; 4493$$

3.545. $\int \sec^3(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$

$$\frac{\frac{1}{7}\left(\frac{3}{5}\left(\frac{1}{3}\left((a-b)\left(10a^3+165a^2b-114ab^2+147b^3\right)\int\frac{\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx-\left(10a^4-279a^2b^2-147b^4\right)\int\frac{\sec(c+dx)(\sec(c}{\sqrt{a+b\sec(c}}\right.\right.\right.}{\frac{2\tan(c+dx)(a+b\sec(c+dx))^{7/2}}{9bd}}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{1}{7}\left(\frac{3}{5}\left(\frac{1}{3}\left((a-b)\left(10a^3+165a^2b-114ab^2+147b^3\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-\left(10a^4-279a^2b^2-147b^4\right)\int\frac{\csc(c+dx+}{\sqrt{a-}}\right.\right.\right.}{\frac{2\tan(c+dx)(a+b\sec(c+dx))^{7/2}}{9bd}}$$

$$\Big\downarrow \; 4319$$

$$\frac{\frac{1}{7}\left(\frac{3}{5}\left(\frac{1}{3}\left(\frac{2(a-b)\sqrt{a+b}\left(10a^3+165a^2b-114ab^2+147b^3\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right.}{bd}\right.\right.\right.}{\frac{2\tan(c+dx)(a+b\sec(c+dx))^{7/2}}{9bd}}$$

$$\Big\downarrow \; 4492$$

$$\frac{\frac{1}{7}\left(\frac{3}{5}\left(\frac{1}{3}\left(\frac{2(a-b)\sqrt{a+b}\left(10a^4-279a^2b^2-147b^4\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}E\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{b^2d}+\frac{2(a-b)\sqrt{}}{}\right.\right.\right.}{\frac{2\tan(c+dx)(a+b\sec(c+dx))^{7/2}}{9bd}}$$

input
```
Int[Sec[c + d*x]^3*(a + b*Sec[c + d*x])^(5/2),x]
```

3.545. $\int \sec^3(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$

output
```
(2*(a + b*Sec[c + d*x])^(7/2)*Tan[c + d*x])/(9*b*d) + ((-4*a*(a + b*Sec[c
+ d*x])^(5/2)*Tan[c + d*x])/(7*d) + ((-2*(10*a^2 - 49*b^2)*(a + b*Sec[c +
d*x])^(3/2)*Tan[c + d*x])/(5*d) + (3*(((2*(a - b)*Sqrt[a + b]*(10*a^4 - 27
9*a^2*b^2 - 147*b^4)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]
]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt
[-((b*(1 + Sec[c + d*x]))/(a - b))])/(b^2*d) + (2*(a - b)*Sqrt[a + b]*(10*
a^3 + 165*a^2*b - 114*a*b^2 + 147*b^3)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[
a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*
x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(b*d))/3 - (4*a*(5*
a^2 - 57*b^2)*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(3*d)))/5)/7)/(9*b)
```

3.545.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4319
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4327
```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(-Cot[e + f*x])*((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2
))), x] + Simp[1/(b*(m + 2))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(b*(
m + 1) - a*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 -
b^2, 0] &&  !LtQ[m, -1]
```

rule 4490
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*((
a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Simp[1/(m + 1)   Int[Csc[e + f*x]*
(a + b*Csc[e + f*x])^(m - 1)*Simp[b*B*m + a*A*(m + 1) + (a*B*m + A*b*(m + 1
))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*
B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

3.545. $\int \sec^3(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$

rule 4492
```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

rule 4493
```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(A - B)   Int[Csc[e
 + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Simp[B   Int[Csc[e + f*x]*((1 +
Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A, B}
, x] && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

3.545.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3238 vs. $2(361) = 722$.

Time = 65.48 (sec) , antiderivative size = 3239, normalized size of antiderivative = 8.12

method	result	size
default	Expression too large to display	3239

input
```
int(sec(d*x+c)^3*(a+b*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/315/d/b^2*(a+b*sec(d*x+c))^(1/2)/(b+a*cos(d*x+c))/(cos(d*x+c)+1)*(147*b^
5*sin(d*x+c)+442*a^2*b^3*sin(d*x+c)+49*b^5*tan(d*x+c)+80*a^3*b^2*sin(d*x+c
)-279*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x
+c)+1))^(1/2)*EllipticF(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^3
*cos(d*x+c)^2-261*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x
+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1
/2))*a*b^4*cos(d*x+c)^2-20*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)
*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/
(a+b))^(1/2))*a^4*b*cos(d*x+c)+558*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1
))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(cot(d*x+c)-csc(d*x+c)
,((a-b)/(a+b))^(1/2))*a^3*b^2*cos(d*x+c)+558*(1/(a+b)*(b+a*cos(d*x+c))/(co
s(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(cot(d*x+c)-
csc(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^3*cos(d*x+c)-10*(1/(a+b)*(b+a*cos(d*
x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(co
t(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*a^4*b*cos(d*x+c)^2+294*(1/(a+b)*(
b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*El
lipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*a*b^4*cos(d*x+c)+20*(1/
(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(
1/2)*EllipticF(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*a^4*b*cos(d*x+c)
-310*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d...
```

3.545.5 Fricas [F]

$$\int \sec^3(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \int (b\sec(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^3\,dx$$

input
```
integrate(sec(d*x+c)^3*(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
integral((b^2*sec(d*x + c)^5 + 2*a*b*sec(d*x + c)^4 + a^2*sec(d*x + c)^3)*
sqrt(b*sec(d*x + c) + a), x)
```

3.545.6 Sympy [F]

$$\int \sec^3(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \int (a+b\sec(c+dx))^{\frac{5}{2}}\sec^3(c+dx)\,dx$$

input
```
integrate(sec(d*x+c)**3*(a+b*sec(d*x+c))**(5/2),x)
```

output
```
Integral((a + b*sec(c + d*x))**(5/2)*sec(c + d*x)**3, x)
```

3.545.7 Maxima [F]

$$\int \sec^3(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \int (b\sec(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^3\,dx$$

input
```
integrate(sec(d*x+c)^3*(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^(5/2)*sec(d*x + c)^3, x)
```

3.545.8 Giac [F]

$$\int \sec^3(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \int (b\sec(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^3\,dx$$

input
```
integrate(sec(d*x+c)^3*(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^(5/2)*sec(d*x + c)^3, x)
```

3.545.9 Mupad [F(-1)]

Timed out.

$$\int \sec^3(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2}}{\cos\left(c+d\,x\right)^3}\,dx$$

input
```
int((a + b/cos(c + d*x))^(5/2)/cos(c + d*x)^3,x)
```

output
```
int((a + b/cos(c + d*x))^(5/2)/cos(c + d*x)^3, x)
```

3.546 $\int \sec^2(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$

3.546.1 Optimal result 4104
3.546.2 Mathematica [A] (warning: unable to verify) 4105
3.546.3 Rubi [A] (verified) 4105
3.546.4 Maple [B] (verified) 4109
3.546.5 Fricas [F] 4110
3.546.6 Sympy [F] 4111
3.546.7 Maxima [F] 4111
3.546.8 Giac [F] 4111
3.546.9 Mupad [F(-1)] 4112

3.546.1 Optimal result

Integrand size = 23, antiderivative size = 333

$$\int \sec^2(c+dx)(a+b\sec(c+dx))^{5/2}\,dx =$$
$$-\frac{2a(a-b)\sqrt{a+b}(3a^2+29b^2)\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{21b^2d}$$
$$-\frac{2(a-b)\sqrt{a+b}(3a^2-24ab+5b^2)\cot(c+dx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-}}{21bd}$$
$$+\frac{2(3a^2+5b^2)\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{21d}$$
$$+\frac{2a(a+b\sec(c+dx))^{3/2}\tan(c+dx)}{7d}+\frac{2(a+b\sec(c+dx))^{5/2}\tan(c+dx)}{7d}$$

output
```
-2/21*a*(a-b)*(3*a^2+29*b^2)*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(
a+b)^(1/2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)
*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/b^2/d-2/21*(a-b)*(3*a^2-24*a*b+5*b^2)*cot
(d*x+c)*EllipticF((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*
(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)
/b/d+2/7*a*(a+b*sec(d*x+c))^(3/2)*tan(d*x+c)/d+2/7*(a+b*sec(d*x+c))^(5/2)*
tan(d*x+c)/d+2/21*(3*a^2+5*b^2)*(a+b*sec(d*x+c))^(1/2)*tan(d*x+c)/d
```

3.546.2 Mathematica [A] (warning: unable to verify)

Time = 11.88 (sec) , antiderivative size = 474, normalized size of antiderivative = 1.42

$$\int \sec^2(c+dx)(a+b\sec(c+dx))^{5/2}\,dx =$$
$$-\frac{2\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)}(a+b\sec(c+dx))^{5/2}\left(2a(3a^3+3a^2b+29ab^2+29b^3)\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\sqrt{\frac{b+}{(a+b)}}\right.}{}$$
$$+\frac{\cos^2(c+dx)(a+b\sec(c+dx))^{5/2}\left(\frac{2a(3a^2+29b^2)\sin(c+dx)}{21b}+\frac{2}{21}\sec(c+dx)\left(9a^2\sin(c+dx)+5b^2\sin(c+dx)\right)\right.}{d(b+a\cos(c+dx))^2}$$

input
```
Integrate[Sec[c + d*x]^2*(a + b*Sec[c + d*x])^(5/2),x]
```

output
```
(-2*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(a + b*Sec[c + d*x])^(5/2)*(2*a*
(3*a^3 + 3*a^2*b + 29*a*b^2 + 29*b^3)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])
]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin
[Tan[(c + d*x)/2]], (a - b)/(a + b)] - 2*b*(3*a^3 + 27*a^2*b + 29*a*b^2 +
5*b^3)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a
 + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a
+ b)] + a*(3*a^2 + 29*b^2)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)
/2]^2*Tan[(c + d*x)/2]))/(21*b*d*(b + a*Cos[c + d*x])^3*Sqrt[Sec[(c + d*x)
/2]^2]*Sec[c + d*x]^(5/2)) + (Cos[c + d*x]^2*(a + b*Sec[c + d*x])^(5/2)*((
2*a*(3*a^2 + 29*b^2)*Sin[c + d*x])/(21*b) + (2*Sec[c + d*x]*(9*a^2*Sin[c +
 d*x] + 5*b^2*Sin[c + d*x]))/21 + (6*a*b*Sec[c + d*x]*Tan[c + d*x])/7 + (2
*b^2*Sec[c + d*x]^2*Tan[c + d*x])/7))/(d*(b + a*Cos[c + d*x])^2)
```

3.546.3 Rubi [A] (verified)

Time = 1.35 (sec) , antiderivative size = 344, normalized size of antiderivative = 1.03, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.565$, Rules used = {3042, 4322, 3042, 4490, 27, 3042, 4490, 27, 3042, 4493, 3042, 4319, 4492}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^2\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}dx$$

$$\Big\downarrow \; 4322$$

$$\frac{5}{7}\int \sec(c+dx)(b+a\sec(c+dx))(a+b\sec(c+dx))^{3/2}dx+\frac{2\tan(c+dx)(a+b\sec(c+dx))^{5/2}}{7d}$$

$$\Big\downarrow \; 3042$$

$$\frac{5}{7}\int \csc\left(c+dx+\frac{\pi}{2}\right)\left(b+a\csc\left(c+dx+\frac{\pi}{2}\right)\right)\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}dx+\frac{2\tan(c+dx)(a+b\sec(c+dx))^{5/2}}{7d}$$

$$\Big\downarrow \; 4490$$

$$\frac{5}{7}\left(\frac{2}{5}\int \frac{1}{2}\sec(c+dx)\sqrt{a+b\sec(c+dx)}\left(8ab+\left(3a^2+5b^2\right)\sec(c+dx)\right)dx+\frac{2a\tan(c+dx)(a+b\sec(c+dx))^{3/}}{5d}\right.$$
$$\frac{2\tan(c+dx)(a+b\sec(c+dx))^{5/2}}{7d}$$

$$\Big\downarrow \; 27$$

$$\frac{5}{7}\left(\frac{1}{5}\int \sec(c+dx)\sqrt{a+b\sec(c+dx)}\left(8ab+\left(3a^2+5b^2\right)\sec(c+dx)\right)dx+\frac{2a\tan(c+dx)(a+b\sec(c+dx))^{3/2}}{5d}\right.$$
$$\frac{2\tan(c+dx)(a+b\sec(c+dx))^{5/2}}{7d}$$

$$\Big\downarrow \; 3042$$

$$\frac{5}{7}\left(\frac{1}{5}\int \csc\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}\left(8ab+\left(3a^2+5b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)\right)dx+\frac{2a\tan(c+dx)(a}{}\right.$$
$$\frac{2\tan(c+dx)(a+b\sec(c+dx))^{5/2}}{7d}$$

$$\Big\downarrow \; 4490$$

$$\frac{5}{7}\left(\frac{1}{5}\left(\frac{2}{3}\int \frac{\sec(c+dx)\left(b\left(27a^2+5b^2\right)+a\left(3a^2+29b^2\right)\sec(c+dx)\right)}{2\sqrt{a+b\sec(c+dx)}}dx+\frac{2\left(3a^2+5b^2\right)\tan(c+dx)\sqrt{a+b\sec(c-}}{3d}\right.\right.$$
$$\frac{2\tan(c+dx)(a+b\sec(c+dx))^{5/2}}{7d}$$

$$\Big\downarrow \; 27$$

3.546. $\int \sec^2(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$

$$\frac{5}{7}\left(\frac{1}{5}\left(\frac{1}{3}\int \frac{\sec(c+dx)\left(b\left(27a^2+5b^2\right)+a\left(3a^2+29b^2\right)\sec(c+dx)\right)}{\sqrt{a+b\sec(c+dx)}}dx+\frac{2\left(3a^2+5b^2\right)\tan(c+dx)\sqrt{a+b\sec(c+}}{3d}\right.\right.$$

$$\frac{2\tan(c+dx)(a+b\sec(c+dx))^{5/2}}{7d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5}{7}\left(\frac{1}{5}\left(\frac{1}{3}\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(b\left(27a^2+5b^2\right)+a\left(3a^2+29b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\left(3a^2+5b^2\right)\tan(c+dx)\sqrt{a}}{3d}\right.\right.$$

$$\frac{2\tan(c+dx)(a+b\sec(c+dx))^{5/2}}{7d}$$

$$\Big\downarrow \text{4493}$$

$$\frac{5}{7}\left(\frac{1}{5}\left(\frac{1}{3}\left(a\left(3a^2+29b^2\right)\int \frac{\sec(c+dx)(\sec(c+dx)+1)}{\sqrt{a+b\sec(c+dx)}}dx-(a-b)\left(3a^2-24ab+5b^2\right)\int \frac{\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx\right.\right.\right.$$

$$\frac{2\tan(c+dx)(a+b\sec(c+dx))^{5/2}}{7d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5}{7}\left(\frac{1}{5}\left(\frac{1}{3}\left(a\left(3a^2+29b^2\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-(a-b)\left(3a^2-24ab+5b^2\right)\int \frac{\csc(c}{\sqrt{a+b\,\mathrm{cs}}}\right.\right.\right.$$

$$\frac{2\tan(c+dx)(a+b\sec(c+dx))^{5/2}}{7d}$$

$$\Big\downarrow \text{4319}$$

$$\frac{5}{7}\left(\frac{1}{5}\left(\frac{1}{3}\left(a\left(3a^2+29b^2\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2(a-b)\sqrt{a+b}\left(3a^2-24ab+5b^2\right)\cot(c}{}\right.\right.\right.$$

$$\frac{2\tan(c+dx)(a+b\sec(c+dx))^{5/2}}{7d}$$

$$\Big\downarrow \text{4492}$$

$$\frac{5}{7}\left(\frac{1}{5}\left(\frac{1}{3}\left(-\frac{2(a-b)\sqrt{a+b}\left(3a^2-24ab+5b^2\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,\mathrm{EllipticF}\left(\arcsin\left(\frac{\sqrt{c}}{}\right.\right.}{bd}\right.\right.\right.$$

$$\frac{2\tan(c+dx)(a+b\sec(c+dx))^{5/2}}{7d}$$

3.546. $\int \sec^2(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$

input
```
Int[Sec[c + d*x]^2*(a + b*Sec[c + d*x])^(5/2),x]
```

output
```
(2*(a + b*Sec[c + d*x])^(5/2)*Tan[c + d*x])/(7*d) + (5*((2*a*(a + b*Sec[c
+ d*x])^(3/2)*Tan[c + d*x])/(5*d) + (((-2*a*(a - b)*Sqrt[a + b]*(3*a^2 + 2
9*b^2)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]]
, (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec
[c + d*x]))/(a - b))])/(b^2*d) - (2*(a - b)*Sqrt[a + b]*(3*a^2 - 24*a*b +
5*b^2)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]]
, (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec
[c + d*x]))/(a - b))])/(b*d))/3 + (2*(3*a^2 + 5*b^2)*Sqrt[a + b*Sec[c + d*
x]]*Tan[c + d*x])/(3*d))/5))/7
```

3.546.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4319
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4322
```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(-Cot[e + f*x])*((a + b*Csc[e + f*x])^m/(f*(m + 1))), x]
+ Simp[m/(m + 1)   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(b + a*Csc
[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m,
 0]
```

rule 4490
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*((
a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Simp[1/(m + 1)   Int[Csc[e + f*x]*
(a + b*Csc[e + f*x])^(m - 1)*Simp[b*B*m + a*A*(m + 1) + (a*B*m + A*b*(m + 1
))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*
B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

rule 4492
```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

rule 4493
```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(A - B)   Int[Csc[e
 + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Simp[B   Int[Csc[e + f*x]*((1 +
Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A, B}
, x] && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

3.546.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2342 vs. $2(299) = 598$.

Time = 24.00 (sec) , antiderivative size = 2343, normalized size of antiderivative = 7.04

method	result	size
default	Expression too large to display	2343

input
```
int(sec(d*x+c)^2*(a+b*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/21/d/b*(a+b*sec(d*x+c))^(1/2)/(b+a*cos(d*x+c))/(cos(d*x+c)+1)*(12*a^3*b*
sin(d*x+c)+34*a*b^3*sin(d*x+c)+5*b^4*sin(d*x+c)+18*a^2*b^2*sin(d*x+c)+3*(1
/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE(cot(d*x+c)-csc(d*x
+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*a^3*b+29*(1/(a+
b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE(cot(d*x+c)-csc(d*x+c),
((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*a^2*b^2+29*(1/(a+b)
*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE(cot(d*x+c)-csc(d*x+c),((
a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*a*b^3-5*EllipticF(cot
(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x
+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*b^4*cos(d*x+c)^2+3*(1/(a+b
)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)
*EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*a^4*cos(d*x+c)^2-10*
EllipticF(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*(1/(a+b)*(b+a*cos(d*x
+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*b^4*cos(d*x+c
)+6*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c
)+1))^(1/2)*EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*a^4*cos(d
*x+c)-3*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF(cot(d*x+
c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*a^3*b
-27*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF(cot(d*x+c)-c
sc(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*a^2*b^...
```

3.546.5 Fricas [F]

$$\int \sec^2(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \int (b\sec(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^2\,dx$$

input
```
integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
integral((b^2*sec(d*x + c)^4 + 2*a*b*sec(d*x + c)^3 + a^2*sec(d*x + c)^2)*
sqrt(b*sec(d*x + c) + a), x)
```

3.546.6 Sympy [F]

$$\int \sec^2(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \int (a+b\sec(c+dx))^{\frac{5}{2}}\sec^2(c+dx)\,dx$$

input
```
integrate(sec(d*x+c)**2*(a+b*sec(d*x+c))**(5/2),x)
```

output
```
Integral((a + b*sec(c + d*x))**(5/2)*sec(c + d*x)**2, x)
```

3.546.7 Maxima [F]

$$\int \sec^2(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \int (b\sec(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^2\,dx$$

input
```
integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^(5/2)*sec(d*x + c)^2, x)
```

3.546.8 Giac [F]

$$\int \sec^2(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \int (b\sec(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^2\,dx$$

input
```
integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^(5/2)*sec(d*x + c)^2, x)
```

3.546.9 Mupad [F(-1)]

Timed out.

$$\int \sec^2(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2}}{\cos\left(c+d\,x\right)^2}\,dx$$

input
```
int((a + b/cos(c + d*x))^(5/2)/cos(c + d*x)^2,x)
```

output
```
int((a + b/cos(c + d*x))^(5/2)/cos(c + d*x)^2, x)
```

3.547 $\int \sec(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$

3.547.1 Optimal result 4113
3.547.2 Mathematica [A] (warning: unable to verify) 4114
3.547.3 Rubi [A] (verified) 4114
3.547.4 Maple [B] (verified) 4118
3.547.5 Fricas [F] 4119
3.547.6 Sympy [F] 4119
3.547.7 Maxima [F] 4119
3.547.8 Giac [F] 4120
3.547.9 Mupad [F(-1)] 4120

3.547.1 Optimal result

Integrand size = 21, antiderivative size = 296

$$\int \sec(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = -\frac{2(a-b)\sqrt{a+b}(23a^2+9b^2)\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{15bd} + \frac{2(a-b)\sqrt{a+b}(15a^2-8ab+9b^2)\cot(c+dx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\cdots}}{15bd} + \frac{16ab\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{15d} + \frac{2b(a+b\sec(c+dx))^{3/2}\tan(c+dx)}{5d}$$

output
```
-2/15*(a-b)*(23*a^2+9*b^2)*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(a+
b)^(1/2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(
-b*(1+sec(d*x+c))/(a-b))^(1/2)/b/d+2/15*(a-b)*(15*a^2-8*a*b+9*b^2)*cot(d*x
+c)*EllipticF((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(a+b
)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/b/d
+2/5*b*(a+b*sec(d*x+c))^(3/2)*tan(d*x+c)/d+16/15*a*b*(a+b*sec(d*x+c))^(1/2
)*tan(d*x+c)/d
```

3.547.2 Mathematica [A] (warning: unable to verify)

Time = 13.45 (sec) , antiderivative size = 440, normalized size of antiderivative = 1.49

$$\int \sec(c+dx)(a+b\sec(c+dx))^{5/2}\,dx =$$

$$-\frac{2(a+b\sec(c+dx))^{5/2}\left(-2(23a^3+23a^2b+9ab^2+9b^3)\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\sqrt{\frac{b+a\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}E\left(\arcsin\left(\tan\left(\frac{1}{2}(c\right.\right.\right.}{15d(b+}$$

$$+\frac{\cos^2(c+dx)(a+b\sec(c+dx))^{5/2}\left(\frac{2}{15}(23a^2+9b^2)\sin(c+dx)+\frac{22}{15}ab\tan(c+dx)+\frac{2}{5}b^2\sec(c+dx)\tan(c\right.}{d(b+a\cos(c+dx))^2}$$

input
```
Integrate[Sec[c + d*x]*(a + b*Sec[c + d*x])^(5/2),x]
```

output
```
(-2*(a + b*Sec[c + d*x])^(5/2)*(-2*(23*a^3 + 23*a^2*b + 9*a*b^2 + 9*b^3)*S
qrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1
 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] +
2*(15*a^3 + 23*a^2*b + 17*a*b^2 + 9*b^3)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*
x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[Arc
Sin[Tan[(c + d*x)/2]], (a - b)/(a + b)] - (23*a^2 + 9*b^2)*Cos[c + d*x]*(b
 + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(15*d*(b + a*Cos[
c + d*x])^3*Sqrt[Sec[(c + d*x)/2]^2]*Sec[c + d*x]^(5/2)*Sqrt[Cos[(c + d*x)
/2]^2*Sec[c + d*x]]*(-1 + Tan[(c + d*x)/2]^2)) + (Cos[c + d*x]^2*(a + b*Se
c[c + d*x])^(5/2)*((2*(23*a^2 + 9*b^2)*Sin[c + d*x])/15 + (22*a*b*Tan[c +
d*x])/15 + (2*b^2*Sec[c + d*x]*Tan[c + d*x])/5))/(d*(b + a*Cos[c + d*x])^2
)
```

3.547.3 Rubi [A] (verified)

Time = 1.08 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.02, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.524$, Rules used = {3042, 4317, 27, 3042, 4490, 27, 3042, 4493, 3042, 4319, 4492}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}dx$$

$$\Big\downarrow \text{4317}$$

$$\frac{2}{5}\int \frac{1}{2}\sec(c+dx)\sqrt{a+b\sec(c+dx)}\left(5a^2+8b\sec(c+dx)a+3b^2\right)dx+\frac{2b\tan(c+dx)(a+b\sec(c+dx))^{3/2}}{5d}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{5}\int \sec(c+dx)\sqrt{a+b\sec(c+dx)}\left(5a^2+8b\sec(c+dx)a+3b^2\right)dx+\frac{2b\tan(c+dx)(a+b\sec(c+dx))^{3/2}}{5d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}\int \csc\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}\left(5a^2+8b\csc\left(c+dx+\frac{\pi}{2}\right)a+3b^2\right)dx+\frac{2b\tan(c+dx)(a+b\sec(c+dx))^{3/2}}{5d}$$

$$\Big\downarrow \text{4490}$$

$$\frac{1}{5}\left(\frac{2}{3}\int \frac{\sec(c+dx)\left(a\left(15a^2+17b^2\right)+b\left(23a^2+9b^2\right)\sec(c+dx)\right)}{2\sqrt{a+b\sec(c+dx)}}dx+\frac{16ab\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}\right)+\frac{2b\tan(c+dx)(a+b\sec(c+dx))^{3/2}}{5d}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{5}\left(\frac{1}{3}\int \frac{\sec(c+dx)\left(a\left(15a^2+17b^2\right)+b\left(23a^2+9b^2\right)\sec(c+dx)\right)}{\sqrt{a+b\sec(c+dx)}}dx+\frac{16ab\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}\right)+\frac{2b\tan(c+dx)(a+b\sec(c+dx))^{3/2}}{5d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}\left(\frac{1}{3}\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(a\left(15a^2+17b^2\right)+b\left(23a^2+9b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{16ab\tan(c+dx)\sqrt{a+b\sec(c+}}{3d}\right.$$

$$\frac{2b\tan(c+dx)(a+b\sec(c+dx))^{3/2}}{5d}$$

$$\Big\downarrow \text{4493}$$

3.547. $\int \sec(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$

$$\frac{1}{5}\left(\frac{1}{3}\left((a-b)\left(15a^2-8ab+9b^2\right)\int\frac{\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx+b\left(23a^2+9b^2\right)\int\frac{\sec(c+dx)(\sec(c+dx)+1)}{\sqrt{a+b\sec(c+dx)}}dx\right)-\frac{2b\tan(c+dx)(a+b\sec(c+dx))^{3/2}}{5d}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{5}\left(\frac{1}{3}\left((a-b)\left(15a^2-8ab+9b^2\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+b\left(23a^2+9b^2\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx\right.\right.}{\sqrt{a+b\csc\left(c+dx+\right.}}\right.\right.\frac{2b\tan(c+dx)(a+b\sec(c+dx))^{3/2}}{5d}$$

$$\Big\downarrow \; 4319$$

$$\frac{1}{5}\left(\frac{1}{3}\left(b\left(23a^2+9b^2\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2(a-b)\sqrt{a+b}\left(15a^2-8ab+9b^2\right)\cot(c+d}{}\right.\right.\frac{2b\tan(c+dx)(a+b\sec(c+dx))^{3/2}}{5d}$$

$$\Big\downarrow \; 4492$$

$$\frac{1}{5}\left(\frac{1}{3}\left(\frac{2(a-b)\sqrt{a+b}\left(15a^2-8ab+9b^2\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec}}{\sqrt{a+}}\right.\right.}{bd}\right.\right.\frac{2b\tan(c+dx)(a+b\sec(c+dx))^{3/2}}{5d}$$

input
```
Int[Sec[c + d*x]*(a + b*Sec[c + d*x])^(5/2),x]
```

output
```
(2*b*(a + b*Sec[c + d*x])^(3/2)*Tan[c + d*x])/(5*d) + (((-2*(a - b)*Sqrt[a
 + b]*(23*a^2 + 9*b^2)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*
x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sq
rt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(b*d) + (2*(a - b)*Sqrt[a + b]*(15*
a^2 - 8*a*b + 9*b^2)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]
]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt
[-((b*(1 + Sec[c + d*x]))/(a - b))])/(b*d))/3 + (16*a*b*Sqrt[a + b*Sec[c +
 d*x]]*Tan[c + d*x])/(3*d))/5
```

3.547. $\int \sec(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$

3.547.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4317
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[(-b)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m - 1)/(f*m)), x]
+ Simp[1/m   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)*(b^2*(m - 1) + a
^2*m + a*b*(2*m - 1)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x] && NeQ
[a^2 - b^2, 0] && GtQ[m, 1] && IntegerQ[2*m]
```

rule 4319
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4490
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*((
a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Simp[1/(m + 1)   Int[Csc[e + f*x]*
(a + b*Csc[e + f*x])^(m - 1)*Simp[b*B*m + a*A*(m + 1) + (a*B*m + A*b*(m + 1
))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*
B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

rule 4492
```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

3.547. $\int \sec(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$

rule 4493
```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(A - B)   Int[Csc[e
 + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Simp[B   Int[Csc[e + f*x]*((1 +
Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A, B}
, x] && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

3.547.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2236 vs. $2(266) = 532$.

Time = 14.61 (sec) , antiderivative size = 2237, normalized size of antiderivative = 7.56

method	result	size
default	Expression too large to display	2237

input
```
int(sec(d*x+c)*(a+b*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/15/d*(a+b*sec(d*x+c))^(1/2)/(b+a*cos(d*x+c))/(cos(d*x+c)+1)*(14*a*b^2*si
n(d*x+c)+34*a^2*b*sin(d*x+c)-15*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)
*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF(cot(d*x+c)-csc(d*x+c),((
a-b)/(a+b))^(1/2))*a^3*cos(d*x+c)^2+23*EllipticE(cot(d*x+c)-csc(d*x+c),((a
-b)/(a+b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x
+c)/(cos(d*x+c)+1))^(1/2)*a^2*b+9*EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(
a+b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(
cos(d*x+c)+1))^(1/2)*a*b^2+23*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1
/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(cot(d*x+c)-csc(d*x+c),((a-
b)/(a+b))^(1/2))*a^3*cos(d*x+c)^2+9*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+
1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(cot(d*x+c)-csc(d*x+c
),((a-b)/(a+b))^(1/2))*b^3*cos(d*x+c)^2-23*EllipticF(cot(d*x+c)-csc(d*x+c)
,((a-b)/(a+b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos
(d*x+c)/(cos(d*x+c)+1))^(1/2)*a^2*b-17*EllipticF(cot(d*x+c)-csc(d*x+c),((a
-b)/(a+b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x
+c)/(cos(d*x+c)+1))^(1/2)*a*b^2-18*EllipticF(cot(d*x+c)-csc(d*x+c),((a-b)/
(a+b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/
(cos(d*x+c)+1))^(1/2)*b^3*cos(d*x+c)+46*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x
+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(cot(d*x+c)-csc(d
*x+c),((a-b)/(a+b))^(1/2))*a^3*cos(d*x+c)-9*EllipticF(cot(d*x+c)-csc(d*...
```

3.547.5 Fricas [F]

$$\int \sec(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \int (b\sec(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)\,dx$$

input
```
integrate(sec(d*x+c)*(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
integral((b^2*sec(d*x + c)^3 + 2*a*b*sec(d*x + c)^2 + a^2*sec(d*x + c))*sq
rt(b*sec(d*x + c) + a), x)
```

3.547.6 Sympy [F]

$$\int \sec(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \int (a+b\sec(c+dx))^{\frac{5}{2}}\sec(c+dx)\,dx$$

input
```
integrate(sec(d*x+c)*(a+b*sec(d*x+c))**(5/2),x)
```

output
```
Integral((a + b*sec(c + d*x))**(5/2)*sec(c + d*x), x)
```

3.547.7 Maxima [F]

$$\int \sec(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \int (b\sec(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)\,dx$$

input
```
integrate(sec(d*x+c)*(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^(5/2)*sec(d*x + c), x)
```

3.547.8 Giac [F]

$$\int \sec(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \int (b\sec(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)\,dx$$

input
```
integrate(sec(d*x+c)*(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^(5/2)*sec(d*x + c), x)
```

3.547.9 Mupad [F(-1)]

Timed out.

$$\int \sec(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2}}{\cos(c+d\,x)}\,dx$$

input
```
int((a + b/cos(c + d*x))^(5/2)/cos(c + d*x),x)
```

output
```
int((a + b/cos(c + d*x))^(5/2)/cos(c + d*x), x)
```

3.548 $\int (a+b\sec(c+dx))^{5/2}\,dx$

3.548.1 Optimal result 4121
3.548.2 Mathematica [A] (verified) 4122
3.548.3 Rubi [A] (verified) 4122
3.548.4 Maple [B] (verified) 4126
3.548.5 Fricas [F] 4127
3.548.6 Sympy [F] 4127
3.548.7 Maxima [F] 4127
3.548.8 Giac [F] 4128
3.548.9 Mupad [F(-1)] 4128

3.548.1 Optimal result

Integrand size = 14, antiderivative size = 352

$$\int (a+b\sec(c+dx))^{5/2}\,dx = -\frac{14a(a-b)\sqrt{a+b}\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{3d} + \frac{2\sqrt{a+b}(9a^2-7ab+b^2)\cot(c+dx)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{3d} - \frac{2a^2\sqrt{a+b}\cot(c+dx)\operatorname{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{d} + \frac{2b^2\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{3d}$$

output
```
-14/3*a*(a-b)*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+
b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x
+c))/(a-b))^(1/2)/d+2/3*(9*a^2-7*a*b+b^2)*cot(d*x+c)*EllipticF((a+b*sec(d*
x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))
/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/d-2*a^2*cot(d*x+c)*EllipticP
i((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),(a+b)/a,((a+b)/(a-b))^(1/2))*(a+b)^(1
/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/d+2/3*b
^2*(a+b*sec(d*x+c))^(1/2)*tan(d*x+c)/d
```

3.548.2 Mathematica [A] (verified)

Time = 10.93 (sec) , antiderivative size = 421, normalized size of antiderivative = 1.20

$$\int (a+b\sec(c+dx))^{5/2}\,dx = -\frac{\cos^2\left(\frac{1}{2}(c+dx)\right)\cos^2(c+dx)(a+b\sec(c+dx))^{5/2}\left(28ab(a+b)\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\sqrt{\frac{b+a\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}E\left(\arcsin\left(\right.\right.\right.}{} + \frac{\cos^2(c+dx)(a+b\sec(c+dx))^{5/2}\left(\frac{14}{3}ab\sin(c+dx)+\frac{2}{3}b^2\tan(c+dx)\right)}{d(b+a\cos(c+dx))^2}$$

input
```
Integrate[(a + b*Sec[c + d*x])^(5/2),x]
```

output
```
-1/3*(Cos[(c + d*x)/2]^2*Cos[c + d*x]^2*(a + b*Sec[c + d*x])^(5/2)*(28*a*b
*(a + b)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/(
(a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(
a + b)] + 4*(3*a^3 - 9*a^2*b - 7*a*b^2 - b^3)*Sqrt[Cos[c + d*x]/(1 + Cos[c
 + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Elliptic
F[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] - 24*a^3*Sqrt[Cos[c + d*x]/(1
 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*
EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + 14*a*b*Cos[c +
 d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(d*(b + a
*Cos[c + d*x])^3) + (Cos[c + d*x]^2*(a + b*Sec[c + d*x])^(5/2)*((14*a*b*Si
n[c + d*x])/3 + (2*b^2*Tan[c + d*x])/3))/(d*(b + a*Cos[c + d*x])^2)
```

3.548.3 Rubi [A] (verified)

Time = 1.22 (sec) , antiderivative size = 353, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.786$, Rules used = {3042, 4269, 27, 3042, 4546, 3042, 4409, 3042, 4271, 4319, 4492}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\sec(c+dx))^{5/2}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}dx$$

$$\Big\downarrow \text{4269}$$

$$\frac{2}{3}\int \frac{3a^3+7b^2\sec^2(c+dx)a+b\left(9a^2+b^2\right)\sec(c+dx)}{2\sqrt{a+b\sec(c+dx)}}dx+\frac{2b^2\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\int \frac{3a^3+7b^2\sec^2(c+dx)a+b\left(9a^2+b^2\right)\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx+\frac{2b^2\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\int \frac{3a^3+7b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2a+b\left(9a^2+b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2b^2\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}$$

$$\Big\downarrow \text{4546}$$

$$\frac{1}{3}\left(\int \frac{3a^3+\left(b\left(9a^2+b^2\right)-7ab^2\right)\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx+7ab^2\int \frac{\sec(c+dx)(\sec(c+dx)+1)}{\sqrt{a+b\sec(c+dx)}}dx\right)+\frac{2b^2\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\left(\int \frac{3a^3+\left(b\left(9a^2+b^2\right)-7ab^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+7ab^2\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)+\frac{2b^2\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}$$

$$\Big\downarrow \text{4409}$$

$$\frac{1}{3}\left(3a^3\int \frac{1}{\sqrt{a+b\sec(c+dx)}}dx+b\left(9a^2-7ab+b^2\right)\int \frac{\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx+7ab^2\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c\right.\right.}{\sqrt{a+b\csc\left(c+\right.}}\right.$$
$$\frac{2b^2\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\left(3a^3\int \frac{1}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+b\left(9a^2-7ab+b^2\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+7ab^2\int \frac{\csc\left(c+dx+\right.}{\sqrt{a+}}\right.$$
$$\frac{2b^2\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}$$

3.548. $\int (a+b\sec(c+dx))^{5/2}\,dx$

$$\Big\downarrow \; 4271$$

$$\frac{1}{3}\left(b(9a^2-7ab+b^2)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+7ab^2\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{6a^2\sqrt{}}{}\right.$$
$$\frac{2b^2\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}$$

$$\Big\downarrow \; 4319$$

$$\frac{1}{3}\left(7ab^2\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sqrt{a+b}(9a^2-7ab+b^2)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-}}{d}\right.$$
$$\frac{2b^2\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}$$

$$\Big\downarrow \; 4492$$

$$\frac{1}{3}\left(\frac{2\sqrt{a+b}(9a^2-7ab+b^2)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{d}\right.$$
$$\frac{2b^2\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}$$

input
```
Int[(a + b*Sec[c + d*x])^(5/2),x]
```

output
```
((-14*a*(a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c
 + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b
)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/d + (2*Sqrt[a + b]*(9*a^2 - 7*
a*b + b^2)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a +
 b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 +
 Sec[c + d*x]))/(a - b))])/d - (6*a^2*Sqrt[a + b]*Cot[c + d*x]*EllipticPi[
(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*
Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b)
)])/d)/3 + (2*b^2*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(3*d)
```

3.548. $\int (a+b\sec(c+dx))^{5/2}\,dx$

3.548.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4269
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(-b^2)*C
ot[c + d*x]*((a + b*Csc[c + d*x])^(n - 2)/(d*(n - 1))), x] + Simp[1/(n - 1)
   Int[(a + b*Csc[c + d*x])^(n - 3)*Simp[a^3*(n - 1) + (b*(b^2*(n - 2) + 3*
a^2*(n - 1)))*Csc[c + d*x] + (a*b^2*(3*n - 4))*Csc[c + d*x]^2, x], x], x] /
; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 2] && IntegerQ[2*n]
```

rule 4271
```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*(Rt[a
 + b, 2]/(a*d*Cot[c + d*x]))*Sqrt[b*((1 - Csc[c + d*x])/(a + b))]*Sqrt[(-b)
*((1 + Csc[c + d*x])/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[
c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

rule 4319
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4409
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Simp[c   Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] +
Simp[d   Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

rule 4492
```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

3.548. $\int (a+b\sec(c+dx))^{5/2}\,dx$

rule 4546
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Simp[C   Int[Csc[e + f*x]*((
1 + Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A
, B, C}, x] && NeQ[a^2 - b^2, 0]
```

3.548.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1921 vs. $2(317) = 634$.

Time = 11.48 (sec) , antiderivative size = 1922, normalized size of antiderivative = 5.46

method	result	size
default	Expression too large to display	1922

input
```
int((a+b*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/3/d*(a+b*sec(d*x+c))^(1/2)/(b+a*cos(d*x+c))/(cos(d*x+c)+1)*(8*a*b^2*sin(
d*x+c)+3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(
d*x+c)+1))^(1/2)*EllipticF(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*a^3*
cos(d*x+c)^2-6*EllipticPi(cot(d*x+c)-csc(d*x+c),-1,((a-b)/(a+b))^(1/2))*(c
os(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^
(1/2)*a^3*cos(d*x+c)^2-12*EllipticPi(cot(d*x+c)-csc(d*x+c),-1,((a-b)/(a+b)
)^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(
d*x+c)+1))^(1/2)*a^3*cos(d*x+c)+7*EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(
a+b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(
cos(d*x+c)+1))^(1/2)*a^2*b+7*EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))
^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d
*x+c)+1))^(1/2)*a*b^2-9*EllipticF(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2
))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)
+1))^(1/2)*a^2*b-7*EllipticF(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*(1
/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^
(1/2)*a*b^2-2*EllipticF(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*(1/(a+b
)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)
*b^3*cos(d*x+c)-EllipticF(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*(1/(a
+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/
2)*b^3*cos(d*x+c)^2+b^3*sin(d*x+c)-6*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*...
```

3.548.5 Fricas [F]

$$\int (a+b\sec(c+dx))^{5/2}\,dx=\int (b\sec(dx+c)+a)^{\frac{5}{2}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
integral((b^2*sec(d*x + c)^2 + 2*a*b*sec(d*x + c) + a^2)*sqrt(b*sec(d*x +
c) + a), x)
```

3.548.6 Sympy [F]

$$\int (a+b\sec(c+dx))^{5/2}\,dx=\int (a+b\sec(c+dx))^{\frac{5}{2}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))**(5/2),x)
```

output
```
Integral((a + b*sec(c + d*x))**(5/2), x)
```

3.548.7 Maxima [F]

$$\int (a+b\sec(c+dx))^{5/2}\,dx=\int (b\sec(dx+c)+a)^{\frac{5}{2}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^(5/2), x)
```

3.548.8 Giac [F]

$$\int (a+b\sec(c+dx))^{5/2}\,dx=\int (b\sec(dx+c)+a)^{\frac{5}{2}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^(5/2), x)
```

3.548.9 Mupad [F(-1)]

Timed out.

$$\int (a+b\sec(c+dx))^{5/2}\,dx=\int \left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2}\,dx$$

input
```
int((a + b/cos(c + d*x))^(5/2),x)
```

output
```
int((a + b/cos(c + d*x))^(5/2), x)
```

3.549 $\int \cos(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$

3.549.1 Optimal result 4129
3.549.2 Mathematica [A] (verified) 4130
3.549.3 Rubi [A] (verified) 4130
3.549.4 Maple [B] (verified) 4134
3.549.5 Fricas [F] 4135
3.549.6 Sympy [F(-1)] 4136
3.549.7 Maxima [F] 4136
3.549.8 Giac [F] 4136
3.549.9 Mupad [F(-1)] 4137

3.549.1 Optimal result

Integrand size = 21, antiderivative size = 353

$$\int \cos(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \frac{(a-b)\sqrt{a+b}(a^2-2b^2)\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{bd} + \frac{\sqrt{a+b}(a^2+6ab-2b^2)\cot(c+dx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{d} - \frac{5ab\sqrt{a+b}\cot(c+dx)\,\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{d} + \frac{a^2\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{d}$$

output
```
(a-b)*(a^2-2*b^2)*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),
((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec
(d*x+c))/(a-b))^(1/2)/b/d+(a^2+6*a*b-2*b^2)*cot(d*x+c)*EllipticF((a+b*sec(
d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c
))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/d-5*a*b*cot(d*x+c)*Ellipti
cPi((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),(a+b)/a,((a+b)/(a-b))^(1/2))*(a+b)^
(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/d+a^2
*sin(d*x+c)*(a+b*sec(d*x+c))^(1/2)/d
```

3.549.2 Mathematica [A] (verified)

Time = 17.11 (sec) , antiderivative size = 454, normalized size of antiderivative = 1.29

$$\int \cos(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \frac{2b^2\cos^2(c+dx)(a+b\sec(c+dx))^{5/2}\sin(c+dx)}{d(b+a\cos(c+dx))^2} + \frac{\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)}(a+b\sec(c+dx))^{5/2}\left(2(a^3+a^2b-2ab^2-2b^3)\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\sqrt{\frac{b+a\cos(c+d}{(a+b)(1+\cos(c+}}\right.}{}$$

input
```
Integrate[Cos[c + d*x]*(a + b*Sec[c + d*x])^(5/2),x]
```

output
```
(2*b^2*Cos[c + d*x]^2*(a + b*Sec[c + d*x])^(5/2)*Sin[c + d*x])/(d*(b + a*C
os[c + d*x])^2) + (Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(a + b*Sec[c + d*
x])^(5/2)*(2*(a^3 + a^2*b - 2*a*b^2 - 2*b^3)*Sqrt[Cos[c + d*x]/(1 + Cos[c
+ d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE
[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + 4*b*(-3*a^2 + 3*a*b + b^2)*S
qrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1
 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] +
20*a^2*b*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/(
(a + b)*(1 + Cos[c + d*x]))]*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (a -
 b)/(a + b)] + ((a^2 - 2*b^2)*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^3*(-Si
n[(c + d*x)/2] + Sin[(3*(c + d*x))/2]))/2))/(d*(b + a*Cos[c + d*x])^3*Sqrt
[Sec[(c + d*x)/2]^2]*Sec[c + d*x]^(5/2))
```

3.549.3 Rubi [A] (verified)

Time = 1.30 (sec) , antiderivative size = 360, normalized size of antiderivative = 1.02, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.524$, Rules used = {3042, 4328, 27, 3042, 4546, 3042, 4409, 3042, 4271, 4319, 4492}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}{\csc\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{4328}$$

$$\int \frac{5ba^2+6b^2\sec(c+dx)a-b\left(a^2-2b^2\right)\sec^2(c+dx)}{2\sqrt{a+b\sec(c+dx)}}dx+\frac{a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\int \frac{5ba^2+6b^2\sec(c+dx)a-b\left(a^2-2b^2\right)\sec^2(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx+\frac{a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\int \frac{5ba^2+6b^2\csc\left(c+dx+\frac{\pi}{2}\right)a-b\left(a^2-2b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{d}$$

$$\Big\downarrow \text{4546}$$

$$\frac{1}{2}\left(\int \frac{5ba^2+\left(6ab^2+\left(a^2-2b^2\right)b\right)\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx-b\left(a^2-2b^2\right)\int \frac{\sec(c+dx)(\sec(c+dx)+1)}{\sqrt{a+b\sec(c+dx)}}dx\right)+\frac{a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\left(\int \frac{5ba^2+\left(6ab^2+\left(a^2-2b^2\right)b\right)\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-b\left(a^2-2b^2\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}d\right.$$
$$\frac{a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{d}$$

$$\Big\downarrow \text{4409}$$

$$\frac{1}{2}\left(-b\left(a^2-2b^2\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+b\left(a^2+6ab-2b^2\right)\int \frac{\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx+5a\right.$$
$$\frac{a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{d}$$

$$\Big\downarrow \text{3042}$$

3.549. $\int \cos(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$

$$\frac{1}{2}\left(b(a^2+6ab-2b^2)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-b(a^2-2b^2)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-\right.$$

$$\frac{a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{d}$$

$$\Big\downarrow \text{4271}$$

$$\frac{1}{2}\left(b(a^2+6ab-2b^2)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-b(a^2-2b^2)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-\right.$$

$$\frac{a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{d}$$

$$\Big\downarrow \text{4319}$$

$$\frac{1}{2}\left(-b(a^2-2b^2)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sqrt{a+b}(a^2+6ab-2b^2)\cot(c+dx)\sqrt{\frac{b(1-\sec(c-}{a+b}}}{}\right.$$

$$\frac{a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{d}$$

$$\Big\downarrow \text{4492}$$

$$\frac{1}{2}\left(\frac{2\sqrt{a+b}(a^2+6ab-2b^2)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{d}\right.$$

$$\frac{a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{d}$$

input
```
Int[Cos[c + d*x]*(a + b*Sec[c + d*x])^(5/2),x]
```

output
```
((2*(a - b)*Sqrt[a + b]*(a^2 - 2*b^2)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a
 + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x
]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(b*d) + (2*Sqrt[a +
b]*(a^2 + 6*a*b - 2*b^2)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c +
d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*
Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/d - (10*a*b*Sqrt[a + b]*Cot[c + d
*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a
 + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c +
 d*x]))/(a - b))])/d)/2 + (a^2*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/d
```

3.549. $\int \cos(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$

3.549.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4271
```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*(Rt[a
 + b, 2]/(a*d*Cot[c + d*x]))*Sqrt[b*((1 - Csc[c + d*x])/(a + b))]*Sqrt[(-b)
*((1 + Csc[c + d*x])/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[
c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

rule 4319
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4328
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)*
((d*Csc[e + f*x])^n/(f*n)), x] - Simp[1/(d*n)   Int[(a + b*Csc[e + f*x])^(m
 - 3)*(d*Csc[e + f*x])^(n + 1)*Simp[a^2*b*(m - 2*n - 2) - a*(3*b^2*n + a^2*
(n + 1))*Csc[e + f*x] - b*(b^2*n + a^2*(m + n - 1))*Csc[e + f*x]^2, x], x],
 x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && ((Int
egerQ[m] && LtQ[n, -1]) || (IntegersQ[m + 1/2, 2*n] && LeQ[n, -1]))
```

rule 4409
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Simp[c   Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] +
Simp[d   Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

3.549. $\int \cos(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$

rule 4492
```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

rule 4546
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Simp[C   Int[Csc[e + f*x]*((
1 + Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A
, B, C}, x] && NeQ[a^2 - b^2, 0]
```

3.549.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2177 vs. $2(324) = 648$.

Time = 17.65 (sec) , antiderivative size = 2178, normalized size of antiderivative = 6.17

method	result	size
default	Expression too large to display	2178

input
```
int(cos(d*x+c)*(a+b*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^3*cos(d*x+c)^2*sin(d*x+c)-EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a
+b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(c
os(d*x+c)+1))^(1/2)*a^2*b+2*EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^
(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*
x+c)+1))^(1/2)*a*b^2-(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(
d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))
^(1/2))*a^3*cos(d*x+c)^2+2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)
*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/
(a+b))^(1/2))*b^3*cos(d*x+c)^2+6*EllipticF(cot(d*x+c)-csc(d*x+c),((a-b)/(a
+b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(c
os(d*x+c)+1))^(1/2)*a^2*b-6*EllipticF(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^
(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*
x+c)+1))^(1/2)*a*b^2-4*EllipticF(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2)
)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+
1))^(1/2)*b^3*cos(d*x+c)-2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)
*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/
(a+b))^(1/2))*a^3*cos(d*x+c)-2*EllipticF(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b
))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos
(d*x+c)+1))^(1/2)*b^3*cos(d*x+c)^2+4*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)
+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(cot(d*x+c)-csc(d...
```

3.549.5 Fricas [F]

$$\int \cos(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \int (b\sec(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)\,dx$$

input
```
integrate(cos(d*x+c)*(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
integral((b^2*cos(d*x + c)*sec(d*x + c)^2 + 2*a*b*cos(d*x + c)*sec(d*x + c
) + a^2*cos(d*x + c))*sqrt(b*sec(d*x + c) + a), x)
```

3.549.6 Sympy [F(-1)]

Timed out.

$$\int \cos(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)*(a+b*sec(d*x+c))**(5/2),x)
```

output
```
Timed out
```

3.549.7 Maxima [F]

$$\int \cos(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \int (b\sec(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)\,dx$$

input
```
integrate(cos(d*x+c)*(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^(5/2)*cos(d*x + c), x)
```

3.549.8 Giac [F]

$$\int \cos(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \int (b\sec(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)\,dx$$

input
```
integrate(cos(d*x+c)*(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^(5/2)*cos(d*x + c), x)
```

3.549.9 Mupad [F(-1)]

Timed out.

$$\int \cos(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \int \cos\left(c+d\,x\right)\,\left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)^{5/2}\,dx$$

input
```
int(cos(c + d*x)*(a + b/cos(c + d*x))^(5/2),x)
```

output
```
int(cos(c + d*x)*(a + b/cos(c + d*x))^(5/2), x)
```

3.550 $\int \cos^2(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$

3.550.1 Optimal result 4138
3.550.2 Mathematica [A] (verified) 4139
3.550.3 Rubi [A] (verified) 4139
3.550.4 Maple [B] (verified) 4144
3.550.5 Fricas [F] 4145
3.550.6 Sympy [F(-1)] 4145
3.550.7 Maxima [F] 4145
3.550.8 Giac [F] 4146
3.550.9 Mupad [F(-1)] 4146

3.550.1 Optimal result

Integrand size = 23, antiderivative size = 399

$$\int \cos^2(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \frac{9a(a-b)\sqrt{a+b}\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{4d} + \frac{\sqrt{a+b}(2a^2+9ab+8b^2)\cot(c+dx)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{4d} - \frac{\sqrt{a+b}(4a^2+15b^2)\cot(c+dx)\operatorname{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{4d} + \frac{9ab\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{4d} + \frac{a^2\cos(c+dx)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{2d}$$

output
```
9/4*a*(a-b)*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)
/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c
))/(a-b))^(1/2)/d+1/4*(2*a^2+9*a*b+8*b^2)*cot(d*x+c)*EllipticF((a+b*sec(d*
x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))
/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/d-1/4*(4*a^2+15*b^2)*cot(d*x
+c)*EllipticPi((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),(a+b)/a,((a+b)/(a-b))^(1
/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^
(1/2)/d+9/4*a*b*sin(d*x+c)*(a+b*sec(d*x+c))^(1/2)/d+1/2*a^2*cos(d*x+c)*sin
(d*x+c)*(a+b*sec(d*x+c))^(1/2)/d
```

3.550.2 Mathematica [A] (verified)

Time = 13.99 (sec) , antiderivative size = 463, normalized size of antiderivative = 1.16

$$\int \cos^2(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \frac{a^2\cos^2(c+dx)(a+b\sec(c+dx))^{5/2}\sin(2(c+dx))}{4d(b+a\cos(c+dx))^2} + \frac{(a+b\sec(c+dx))^{5/2}\left(-18ab(a+b)\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\sqrt{\frac{b+a\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}E\left(\arcsin\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{a-b}{a+b}\right)+4(\right.}{}$$

input
```
Integrate[Cos[c + d*x]^2*(a + b*Sec[c + d*x])^(5/2),x]
```

output
```
(a^2*Cos[c + d*x]^2*(a + b*Sec[c + d*x])^(5/2)*Sin[2*(c + d*x)])/(4*d*(b +
 a*Cos[c + d*x])^2) + ((a + b*Sec[c + d*x])^(5/2)*(-18*a*b*(a + b)*Sqrt[Co
s[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos
[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + 4*(2*a
^3 - a^2*b + 12*a*b^2 - 4*b^3)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[
(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c
 + d*x)/2]], (a - b)/(a + b)] - 4*a*(4*a^2 + 15*b^2)*Sqrt[Cos[c + d*x]/(1
+ Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*E
llipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] - 9*a*b*Cos[c + d
*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(4*d*(b + a
*Cos[c + d*x])^3*Sqrt[Sec[(c + d*x)/2]^2]*Sec[c + d*x]^(5/2)*Sqrt[Cos[(c +
 d*x)/2]^2*Sec[c + d*x]]*(-1 + Tan[(c + d*x)/2]^2))
```

3.550.3 Rubi [A] (verified)

Time = 1.68 (sec) , antiderivative size = 408, normalized size of antiderivative = 1.02, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.609$, Rules used = {3042, 4328, 27, 3042, 4592, 27, 3042, 4546, 3042, 4409, 3042, 4271, 4319, 4492}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}{\csc\left(c+dx+\frac{\pi}{2}\right)^2}dx$$

$$\Big\downarrow \text{4328}$$

$$\frac{1}{2}\int \frac{\cos(c+dx)\left(9ba^2+2\left(a^2+6b^2\right)\sec(c+dx)a+b\left(a^2+4b^2\right)\sec^2(c+dx)\right)}{2\sqrt{a+b\sec(c+dx)}}dx+\frac{a^2\sin(c+dx)\cos(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{4}\int \frac{\cos(c+dx)\left(9ba^2+2\left(a^2+6b^2\right)\sec(c+dx)a+b\left(a^2+4b^2\right)\sec^2(c+dx)\right)}{\sqrt{a+b\sec(c+dx)}}dx+\frac{a^2\sin(c+dx)\cos(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}\int \frac{9ba^2+2\left(a^2+6b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)a+b\left(a^2+4b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{a^2\sin(c+dx)\cos(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}$$

$$\Big\downarrow \text{4592}$$

$$\frac{1}{4}\left(\frac{9ab\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{d}-\frac{\int -\frac{-9b^2\sec^2(c+dx)a^2+\left(4a^2+15b^2\right)a^2+2b\left(a^2+4b^2\right)\sec(c+dx)a}{2\sqrt{a+b\sec(c+dx)}}dx}{a}\right)+\frac{a^2\sin(c+dx)\cos(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{4}\left(\frac{\int \frac{-9b^2\sec^2(c+dx)a^2+\left(4a^2+15b^2\right)a^2+2b\left(a^2+4b^2\right)\sec(c+dx)a}{\sqrt{a+b\sec(c+dx)}}dx}{2a}+\frac{9ab\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{d}\right)+\frac{a^2\sin(c+dx)\cos(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}\left(\frac{\int \frac{-9b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+\left(4a^2+15b^2\right)a^2+2b\left(a^2+4b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)a}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{2a}+\frac{9ab\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{d}\right)+\frac{a^2\sin(c+dx)\cos(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}$$

$$\Big\downarrow \text{4546}$$

3.550. $\int \cos^2(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$

$$\frac{1}{4}\left(\frac{\int \frac{(4a^2+15b^2)a^2+(9a^2b^2+2a(a^2+4b^2)b)\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx-9a^2b^2\int \frac{\sec(c+dx)(\sec(c+dx)+1)}{\sqrt{a+b\sec(c+dx)}}dx}{2a}+\frac{9ab\sin(c+dx)\sqrt{a+b\sec(c+}}{d}\right.$$

$$\frac{a^2\sin(c+dx)\cos(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}$$

$\downarrow$ 3042

$$\frac{1}{4}\left(\frac{\int \frac{(4a^2+15b^2)a^2+(9a^2b^2+2a(a^2+4b^2)b)\csc(c+dx+\frac{\pi}{2})}{\sqrt{a+b\csc(c+dx+\frac{\pi}{2})}}dx-9a^2b^2\int \frac{\csc(c+dx+\frac{\pi}{2})(\csc(c+dx+\frac{\pi}{2})+1)}{\sqrt{a+b\csc(c+dx+\frac{\pi}{2})}}dx}{2a}+\frac{9ab\sin(c+dx)\sqrt{a}}{d}\right.$$

$$\frac{a^2\sin(c+dx)\cos(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}$$

$\downarrow$ 4409

$$\frac{1}{4}\left(\frac{-9a^2b^2\int \frac{\csc(c+dx+\frac{\pi}{2})(\csc(c+dx+\frac{\pi}{2})+1)}{\sqrt{a+b\csc(c+dx+\frac{\pi}{2})}}dx+a^2(4a^2+15b^2)\int \frac{1}{\sqrt{a+b\sec(c+dx)}}dx+ab(2a^2+9ab+8b^2)\int \frac{\sec(c+}{\sqrt{a+b\sec(}}}{2a}\right.$$

$$\frac{a^2\sin(c+dx)\cos(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}$$

$\downarrow$ 3042

$$\frac{1}{4}\left(\frac{a^2(4a^2+15b^2)\int \frac{1}{\sqrt{a+b\csc(c+dx+\frac{\pi}{2})}}dx-9a^2b^2\int \frac{\csc(c+dx+\frac{\pi}{2})(\csc(c+dx+\frac{\pi}{2})+1)}{\sqrt{a+b\csc(c+dx+\frac{\pi}{2})}}dx+ab(2a^2+9ab+8b^2)\int \frac{\csc(c}{\sqrt{a+b\mathrm{cs}}}}{2a}\right.$$

$$\frac{a^2\sin(c+dx)\cos(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}$$

$\downarrow$ 4271

$$\frac{1}{4}\left(\frac{-9a^2b^2\int \frac{\csc(c+dx+\frac{\pi}{2})(\csc(c+dx+\frac{\pi}{2})+1)}{\sqrt{a+b\csc(c+dx+\frac{\pi}{2})}}dx+ab(2a^2+9ab+8b^2)\int \frac{\csc(c+dx+\frac{\pi}{2})}{\sqrt{a+b\csc(c+dx+\frac{\pi}{2})}}dx-\frac{2a\sqrt{a+b}(4a^2+15b^2)\cot(c+d}{}}{2a}\right.$$

$$\frac{a^2\sin(c+dx)\cos(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}$$

$\downarrow$ 4319

3.550. $\int \cos^2(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$

$$\frac{1}{4}\left(\frac{-9a^2b^2\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a\sqrt{a+b}\left(2a^2+9ab+8b^2\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,\text{EllipticF}\left(\text{arcsi}\right.}{d}}{}\right.$$

$$\frac{a^2\sin(c+dx)\cos(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}$$

$$\Big\downarrow \; 4492$$

$$\frac{1}{4}\left(\frac{\frac{2a\sqrt{a+b}\left(2a^2+9ab+8b^2\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{d}-\frac{2a\sqrt{a+b}\left(4a^2+15b^2\right)\text{co}}{}}{}\right.$$

$$\frac{a^2\sin(c+dx)\cos(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}$$

input
```
Int[Cos[c + d*x]^2*(a + b*Sec[c + d*x])^(5/2),x]
```

output
```
(a^2*Cos[c + d*x]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(2*d) + (((18*a^2
*(a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]
]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt
[-((b*(1 + Sec[c + d*x]))/(a - b))])/d + (2*a*Sqrt[a + b]*(2*a^2 + 9*a*b +
 8*b^2)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]
], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Se
c[c + d*x]))/(a - b))])/d - (2*a*Sqrt[a + b]*(4*a^2 + 15*b^2)*Cot[c + d*x]
*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a +
b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*
x]))/(a - b))])/d)/(2*a) + (9*a*b*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/d
)/4
```

3.550.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4271
```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*(Rt[a
 + b, 2]/(a*d*Cot[c + d*x]))*Sqrt[b*((1 - Csc[c + d*x])/(a + b))]*Sqrt[(-b)
*((1 + Csc[c + d*x])/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[
c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

rule 4319
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4328
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)*
((d*Csc[e + f*x])^n/(f*n)), x] - Simp[1/(d*n)   Int[(a + b*Csc[e + f*x])^(m
 - 3)*(d*Csc[e + f*x])^(n + 1)*Simp[a^2*b*(m - 2*n - 2) - a*(3*b^2*n + a^2*
(n + 1))*Csc[e + f*x] - b*(b^2*n + a^2*(m + n - 1))*Csc[e + f*x]^2, x], x],
 x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && ((Int
egerQ[m] && LtQ[n, -1]) || (IntegersQ[m + 1/2, 2*n] && LeQ[n, -1]))
```

rule 4409
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Simp[c   Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] +
Simp[d   Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

rule 4492
```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

rule 4546
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Simp[C   Int[Csc[e + f*x]*((
1 + Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A
, B, C}, x] && NeQ[a^2 - b^2, 0]
```

3.550. $\int \cos^2(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$

rule 4592
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Simp[1/(a*d*n)   Int[(a + b*Csc[e + f*x])^m
*(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*
Csc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

3.550.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2211 vs. $2(358) = 716$.

Time = 42.59 (sec) , antiderivative size = 2212, normalized size of antiderivative = 5.54

method	result	size
default	Expression too large to display	2212

input
```
int(cos(d*x+c)^2*(a+b*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/4/d*(2*a^3*cos(d*x+c)^3*sin(d*x+c)+4*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(
1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF(cot(d*x+c)-csc(d*
x+c),((a-b)/(a+b))^(1/2))*a^3*cos(d*x+c)^2+2*a^3*cos(d*x+c)^2*sin(d*x+c)-3
0*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+
1))^(1/2)*EllipticPi(cot(d*x+c)-csc(d*x+c),-1,((a-b)/(a+b))^(1/2))*a*b^2*c
os(d*x+c)^2-60*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))
/(cos(d*x+c)+1))^(1/2)*EllipticPi(cot(d*x+c)-csc(d*x+c),-1,((a-b)/(a+b))^(
1/2))*a*b^2*cos(d*x+c)-30*EllipticPi(cot(d*x+c)-csc(d*x+c),-1,((a-b)/(a+b)
)^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(
d*x+c)+1))^(1/2)*a*b^2-8*EllipticPi(cot(d*x+c)-csc(d*x+c),-1,((a-b)/(a+b))
^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d
*x+c)+1))^(1/2)*a^3*cos(d*x+c)^2-16*EllipticPi(cot(d*x+c)-csc(d*x+c),-1,((
a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x
+c))/(cos(d*x+c)+1))^(1/2)*a^3*cos(d*x+c)-9*EllipticE(cot(d*x+c)-csc(d*x+c
),((a-b)/(a+b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(co
s(d*x+c)/(cos(d*x+c)+1))^(1/2)*a^2*b-9*EllipticE(cot(d*x+c)-csc(d*x+c),((a
-b)/(a+b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x
+c)/(cos(d*x+c)+1))^(1/2)*a*b^2-2*EllipticF(cot(d*x+c)-csc(d*x+c),((a-b)/(
a+b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(
cos(d*x+c)+1))^(1/2)*a^2*b+24*EllipticF(cot(d*x+c)-csc(d*x+c),((a-b)/(a...
```

3.550.5 Fricas [F]

$$\int \cos^2(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \int (b\sec(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^2\,dx$$

input
```
integrate(cos(d*x+c)^2*(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
integral((b^2*cos(d*x + c)^2*sec(d*x + c)^2 + 2*a*b*cos(d*x + c)^2*sec(d*x
 + c) + a^2*cos(d*x + c)^2)*sqrt(b*sec(d*x + c) + a), x)
```

3.550.6 Sympy [F(-1)]

Timed out.

$$\int \cos^2(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**2*(a+b*sec(d*x+c))**(5/2),x)
```

output
```
Timed out
```

3.550.7 Maxima [F]

$$\int \cos^2(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \int (b\sec(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^2\,dx$$

input
```
integrate(cos(d*x+c)^2*(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^2, x)
```

3.550.8 Giac [F]

$$\int \cos^2(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \int (b\sec(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^2\,dx$$

input
```
integrate(cos(d*x+c)^2*(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^2, x)
```

3.550.9 Mupad [F(-1)]

Timed out.

$$\int \cos^2(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \int \cos(c+d\,x)^2\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2}dx$$

input
```
int(cos(c + d*x)^2*(a + b/cos(c + d*x))^(5/2),x)
```

output
```
int(cos(c + d*x)^2*(a + b/cos(c + d*x))^(5/2), x)
```

3.551 $\int \cos^3(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$

3.551.1 Optimal result 4147
3.551.2 Mathematica [B] (warning: unable to verify) 4148
3.551.3 Rubi [A] (verified) 4149
3.551.4 Maple [B] (verified) 4155
3.551.5 Fricas [F(-1)] 4156
3.551.6 Sympy [F(-1)] 4156
3.551.7 Maxima [F] 4156
3.551.8 Giac [F] 4157
3.551.9 Mupad [F(-1)] 4157

3.551.1 Optimal result

Integrand size = 23, antiderivative size = 460

$$\int \cos^3(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \frac{(a-b)\sqrt{a+b}(16a^2+33b^2)\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1}{}}}{24bd}$$
$$+\frac{\sqrt{a+b}(16a^2+26ab+33b^2)\cot(c+dx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(}{a-}}}{24d}$$
$$-\frac{5b\sqrt{a+b}(4a^2+b^2)\cot(c+dx)\,\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+d}{a-b}}}{8ad}$$
$$+\frac{(16a^2+33b^2)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{24d}$$
$$+\frac{13ab\cos(c+dx)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{12d}$$
$$+\frac{a^2\cos^2(c+dx)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{3d}$$

output
```
1/24*(a-b)*(16*a^2+33*b^2)*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(a+
b)^(1/2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(
-b*(1+sec(d*x+c))/(a-b))^(1/2)/b/d+1/24*(16*a^2+26*a*b+33*b^2)*cot(d*x+c)*
EllipticF((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(a+b)^(1
/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/d-5/8*b
*(4*a^2+b^2)*cot(d*x+c)*EllipticPi((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),(a+b
)/a,((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1
+sec(d*x+c))/(a-b))^(1/2)/a/d+1/24*(16*a^2+33*b^2)*sin(d*x+c)*(a+b*sec(d*x
+c))^(1/2)/d+13/12*a*b*cos(d*x+c)*sin(d*x+c)*(a+b*sec(d*x+c))^(1/2)/d+1/3*
a^2*cos(d*x+c)^2*sin(d*x+c)*(a+b*sec(d*x+c))^(1/2)/d
```

3.551.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 1018 vs. $2(460) = 920$.

Time = 15.26 (sec) , antiderivative size = 1018, normalized size of antiderivative = 2.21

$$\int \cos^3(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \frac{\cos^2(c+dx)(a+b\sec(c+dx))^{5/2}\left(\frac{1}{12}a^2\sin(c+dx)+\frac{13}{24}ab\sin(2(c+dx))+\frac{1}{12}a^2\sin(3(c+dx\right.}{d(b+a\cos(c+dx))^2}$$
$$+\frac{(a+b\sec(c+dx))^{5/2}\sqrt{\frac{1}{1-\tan^2\left(\frac{1}{2}(c+dx)\right)}}\left(16a^3\tan\left(\frac{1}{2}(c+dx)\right)+16a^2b\tan\left(\frac{1}{2}(c+dx)\right)+33ab^2\tan\left(\frac{1}{2}(c+d\right.\right.}{}$$

input
```
Integrate[Cos[c + d*x]^3*(a + b*Sec[c + d*x])^(5/2),x]
```

output
```
(Cos[c + d*x]^2*(a + b*Sec[c + d*x])^(5/2)*((a^2*Sin[c + d*x])/12 + (13*a*
b*Sin[2*(c + d*x)])/24 + (a^2*Sin[3*(c + d*x)])/12))/(d*(b + a*Cos[c + d*x
])^2) + ((a + b*Sec[c + d*x])^(5/2)*Sqrt[(1 - Tan[(c + d*x)/2]^2)^(-1)]*(1
6*a^3*Tan[(c + d*x)/2] + 16*a^2*b*Tan[(c + d*x)/2] + 33*a*b^2*Tan[(c + d*x
)/2] + 33*b^3*Tan[(c + d*x)/2] - 32*a^3*Tan[(c + d*x)/2]^3 - 66*a*b^2*Tan[
(c + d*x)/2]^3 + 16*a^3*Tan[(c + d*x)/2]^5 - 16*a^2*b*Tan[(c + d*x)/2]^5 +
 33*a*b^2*Tan[(c + d*x)/2]^5 - 33*b^3*Tan[(c + d*x)/2]^5 + 120*a^2*b*Ellip
ticPi[-1, ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x
)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)
] + 30*b^3*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[
1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*
x)/2]^2)/(a + b)] + 120*a^2*b*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (a
- b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b
- a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + 30*b^3*EllipticP
i[-1, ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1
 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x
)/2]^2)/(a + b)] + (16*a^3 + 16*a^2*b + 33*a*b^2 + 33*b^3)*EllipticE[ArcSi
n[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Ta
n[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^
2)/(a + b)] - 2*b*(38*a^2 - 13*a*b + 24*b^2)*EllipticF[ArcSin[Tan[(c + ...
```

3.551.3 Rubi [A] (verified)

Time = 2.27 (sec) , antiderivative size = 477, normalized size of antiderivative = 1.04, number of steps used = 17, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.739$, Rules used = {3042, 4328, 27, 3042, 4592, 27, 3042, 4592, 27, 3042, 4546, 3042, 4409, 3042, 4271, 4319, 4492}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^3(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}{\csc\left(c+dx+\frac{\pi}{2}\right)^3}dx$$

$$\Big\downarrow \; 4328$$

$$\frac{1}{3}\int \frac{\cos^2(c+dx)\left(13ba^2+2\left(2a^2+9b^2\right)\sec(c+dx)a+3b\left(a^2+2b^2\right)\sec^2(c+dx)\right)}{2\sqrt{a+b\sec(c+dx)}}dx+\frac{a^2\sin(c+dx)\cos^2(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{6}\int \frac{\cos^2(c+dx)\left(13ba^2+2\left(2a^2+9b^2\right)\sec(c+dx)a+3b\left(a^2+2b^2\right)\sec^2(c+dx)\right)}{\sqrt{a+b\sec(c+dx)}}dx+\frac{a^2\sin(c+dx)\cos^2(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{6}\int \frac{13ba^2+2\left(2a^2+9b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)a+3b\left(a^2+2b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^2\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{a^2\sin(c+dx)\cos^2(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}$$

$$\Big\downarrow \; 4592$$

$$\frac{1}{6}\left(\frac{13ab\sin(c+dx)\cos(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}-\frac{\int -\frac{\cos(c+dx)\left(13b^2\sec^2(c+dx)a^2+\left(16a^2+33b^2\right)a^2+2b\left(19a^2+12b^2\right)\sec(c+\right.}{2\sqrt{a+b\sec(c+dx)}}}{2a}\right.$$

$$\frac{a^2\sin(c+dx)\cos^2(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{6}\left(\frac{\int \frac{\cos(c+dx)\left(13b^2\sec^2(c+dx)a^2+\left(16a^2+33b^2\right)a^2+2b\left(19a^2+12b^2\right)\sec(c+dx)a\right)}{\sqrt{a+b\sec(c+dx)}}dx}{4a}+\frac{13ab\sin(c+dx)\cos(c+dx)\sqrt{a+b\sec(c}}{2d}\right.$$

$$\frac{a^2\sin(c+dx)\cos^2(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{6}\left(\frac{\int \frac{13b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+\left(16a^2+33b^2\right)a^2+2b\left(19a^2+12b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)a}{\csc\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{4a}+\frac{13ab\sin(c+dx)\cos(c+dx)\sqrt{a+b\sec(c+}}{2d}\right.$$

$$\frac{a^2\sin(c+dx)\cos^2(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}$$

$$\Big\downarrow \; 4592$$

3.551. $\int \cos^3(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$

$$\frac{1}{6}\left(\frac{\frac{a\left(16a^2+33b^2\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{d}-\frac{\int -\frac{26b^2\sec(c+dx)a^3-b\left(16a^2+33b^2\right)\sec^2(c+dx)a^2+15b\left(4a^2+b^2\right)a^2}{2\sqrt{a+b\sec(c+dx)}}dx}{a}}{4a}+\frac{13ab\sin(c+dx)\cos}{}\right.$$

$$\frac{a^2\sin(c+dx)\cos^2(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}$$

$\downarrow$ 27

$$\frac{1}{6}\left(\frac{\frac{\int\frac{26b^2\sec(c+dx)a^3-b\left(16a^2+33b^2\right)\sec^2(c+dx)a^2+15b\left(4a^2+b^2\right)a^2}{\sqrt{a+b\sec(c+dx)}}dx}{2a}+\frac{a\left(16a^2+33b^2\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{d}}{4a}+\frac{13ab\sin(c+dx)\cos}{}\right.$$

$$\frac{a^2\sin(c+dx)\cos^2(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}$$

$\downarrow$ 3042

$$\frac{1}{6}\left(\frac{\frac{\int\frac{26b^2\csc\left(c+dx+\frac{\pi}{2}\right)a^3-b\left(16a^2+33b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+15b\left(4a^2+b^2\right)a^2}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{2a}+\frac{a\left(16a^2+33b^2\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{d}}{4a}+\frac{13ab\sin(c+}{}\right.$$

$$\frac{a^2\sin(c+dx)\cos^2(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}$$

$\downarrow$ 4546

$$\frac{1}{6}\left(\frac{\frac{\int\frac{15b\left(4a^2+b^2\right)a^2+\left(26b^2a^3+b\left(16a^2+33b^2\right)a^2\right)\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx-a^2b\left(16a^2+33b^2\right)\int\frac{\sec(c+dx)(\sec(c+dx)+1)}{\sqrt{a+b\sec(c+dx)}}dx}{2a}+\frac{a\left(16a^2+33b^2\right)\sin(c+dx)\sqrt{a+b\sec}}{d}}{4a}\right.$$

$$\frac{a^2\sin(c+dx)\cos^2(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}$$

$\downarrow$ 3042

$$\frac{1}{6}\left(\frac{\frac{\int\frac{15b\left(4a^2+b^2\right)a^2+\left(26b^2a^3+b\left(16a^2+33b^2\right)a^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-a^2b\left(16a^2+33b^2\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{2a}+\frac{a\left(16a^2+33b^2\right)\sin(c+}{}}{4a}\right.$$

$$\frac{a^2\sin(c+dx)\cos^2(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}$$

3.551. $\int \cos^3(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$

$$\Big\downarrow \; 4409$$

$$\frac{1}{6}\left(\frac{\frac{a^2(-b)(16a^2+33b^2)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+15a^2b(4a^2+b^2)\int \frac{1}{\sqrt{a+b\sec(c+dx)}}dx+a^2b(16a^2+26ab+33b^2)\int \frac{\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx}{2a}}{4a}\right.$$

$$\frac{a^2\sin(c+dx)\cos^2(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{6}\left(\frac{\frac{15a^2b(4a^2+b^2)\int \frac{1}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+a^2b(16a^2+26ab+33b^2)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-a^2b(16a^2+33b^2)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}}{2a}}{4a}\right.$$

$$\frac{a^2\sin(c+dx)\cos^2(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}$$

$$\Big\downarrow \; 4271$$

$$\frac{1}{6}\left(\frac{\frac{a^2b(16a^2+26ab+33b^2)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-a^2b(16a^2+33b^2)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{30ab\sqrt{a+b}\left(4a^2+b^2\right)\cot(c+dx)\sqrt{\frac{b(1-\mathrm{se}}{a}}}{}}{2a}}{4a}\right.$$

$$\frac{a^2\sin(c+dx)\cos^2(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}$$

$$\Big\downarrow \; 4319$$

$$\frac{1}{6}\left(\frac{\frac{a^2(-b)(16a^2+33b^2)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a^2\sqrt{a+b}\left(16a^2+26ab+33b^2\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,\mathrm{EllipticF}\left(\mathrm{ar}\right.}{d}}{2a}}{}\right.$$

$$\frac{a^2\sin(c+dx)\cos^2(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}$$

$$\Big\downarrow \; 4492$$

3.551. $\int \cos^3(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$

$$\frac{1}{6}\left(\frac{\frac{2a^2\sqrt{a+b}\left(16a^2+26ab+33b^2\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{d}+\frac{2a^2(a-b)\sqrt{a+b}\left(16a^2+33b^2\right)\text{co}}{}}{}\right.$$

$$\frac{a^2\sin(c+dx)\cos^2(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}$$

input
```
Int[Cos[c + d*x]^3*(a + b*Sec[c + d*x])^(5/2),x]
```

output
```
(a^2*Cos[c + d*x]^2*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(3*d) + ((13*a*
b*Cos[c + d*x]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(2*d) + (((2*a^2*(a
- b)*Sqrt[a + b]*(16*a^2 + 33*b^2)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a +
b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))
/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(b*d) + (2*a^2*Sqrt[a +
 b]*(16*a^2 + 26*a*b + 33*b^2)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Se
c[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a
+ b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/d - (30*a*b*Sqrt[a + b]*(4*
a^2 + b^2)*Cot[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*
x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sq
rt[-((b*(1 + Sec[c + d*x]))/(a - b))])/d)/(2*a) + (a*(16*a^2 + 33*b^2)*Sqr
t[a + b*Sec[c + d*x]]*Sin[c + d*x])/d)/(4*a))/6
```

3.551.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4271
```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*(Rt[a
 + b, 2]/(a*d*Cot[c + d*x]))*Sqrt[b*((1 - Csc[c + d*x])/(a + b))]*Sqrt[(-b)
*((1 + Csc[c + d*x])/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[
c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

rule 4319
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4328
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)*
((d*Csc[e + f*x])^n/(f*n)), x] - Simp[1/(d*n)   Int[(a + b*Csc[e + f*x])^(m
 - 3)*(d*Csc[e + f*x])^(n + 1)*Simp[a^2*b*(m - 2*n - 2) - a*(3*b^2*n + a^2*
(n + 1))*Csc[e + f*x] - b*(b^2*n + a^2*(m + n - 1))*Csc[e + f*x]^2, x], x],
 x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && ((Int
egerQ[m] && LtQ[n, -1]) || (IntegersQ[m + 1/2, 2*n] && LeQ[n, -1]))
```

rule 4409
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Simp[c   Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] +
Simp[d   Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

rule 4492
```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

rule 4546
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Simp[C   Int[Csc[e + f*x]*((
1 + Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A
, B, C}, x] && NeQ[a^2 - b^2, 0]
```

3.551. $\int \cos^3(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$

rule 4592
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Simp[1/(a*d*n)   Int[(a + b*Csc[e + f*x])^m
*(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*
Csc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

3.551.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2544 vs. $2(415) = 830$.

Time = 142.08 (sec) , antiderivative size = 2545, normalized size of antiderivative = 5.53

method	result	size
default	Expression too large to display	2545

input
```
int(cos(d*x+c)^3*(a+b*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/24/d*(-8*a^3*cos(d*x+c)^3*sin(d*x+c)-8*a^3*cos(d*x+c)^4*sin(d*x+c)-33*b
^3*cos(d*x+c)*sin(d*x+c)-16*a^3*cos(d*x+c)^2*sin(d*x+c)+16*EllipticE(cot(d
*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c
)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*a^2*b+33*EllipticE(cot(d*x+c
)-csc(d*x+c),((a-b)/(a+b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1)
)^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*a*b^2+16*(1/(a+b)*(b+a*cos(d*x+c
))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(cot(d
*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*a^3*cos(d*x+c)^2+33*(1/(a+b)*(b+a*co
s(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*Elliptic
E(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*b^3*cos(d*x+c)^2-76*EllipticF
(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos
(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*a^2*b+26*EllipticF(cot
(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x
+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*a*b^2-96*EllipticF(cot(d*x
+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+
1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*b^3*cos(d*x+c)+32*(1/(a+b)*(b+
a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*Elli
pticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*a^3*cos(d*x+c)-48*Ellipti
cF(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(c
os(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*b^3*cos(d*x+c)^2+...
```

3.551.5 Fricas [F(-1)]

Timed out.

$$\int \cos^3(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)^3*(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.551.6 Sympy [F(-1)]

Timed out.

$$\int \cos^3(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**3*(a+b*sec(d*x+c))**(5/2),x)
```

output
```
Timed out
```

3.551.7 Maxima [F]

$$\int \cos^3(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \int (b\sec(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^3\,dx$$

input
```
integrate(cos(d*x+c)^3*(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^3, x)
```

3.551.8 Giac [F]

$$\int \cos^3(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \int (b\sec(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^3\,dx$$

input
```
integrate(cos(d*x+c)^3*(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^3, x)
```

3.551.9 Mupad [F(-1)]

Timed out.

$$\int \cos^3(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \int \cos(c+d\,x)^3\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2}dx$$

input
```
int(cos(c + d*x)^3*(a + b/cos(c + d*x))^(5/2),x)
```

output
```
int(cos(c + d*x)^3*(a + b/cos(c + d*x))^(5/2), x)
```

3.552 $\int \cos^4(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$

3.552.1 Optimal result 4158
3.552.2 Mathematica [B] (warning: unable to verify) 4159
3.552.3 Rubi [A] (verified) 4160
3.552.4 Maple [B] (verified) 4167
3.552.5 Fricas [F] 4167
3.552.6 Sympy [F(-1)] 4168
3.552.7 Maxima [F] 4168
3.552.8 Giac [F] 4168
3.552.9 Mupad [F(-1)] 4169

3.552.1 Optimal result

Integrand size = 23, antiderivative size = 530

$$\int \cos^4(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \frac{(a-b)\sqrt{a+b}(284a^2+15b^2)\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(}{}}}{192ad}$$
$$+\frac{\sqrt{a+b}(72a^3+284a^2b+118ab^2+15b^3)\cot(c+dx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx)}{a+b}}}{192ad}$$
$$-\frac{\sqrt{a+b}(48a^4+120a^2b^2-5b^4)\cot(c+dx)\,\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-}}{64a^2d}$$
$$+\frac{b(284a^2+15b^2)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{192ad}$$
$$+\frac{(36a^2+59b^2)\cos(c+dx)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{96d}$$
$$+\frac{17ab\cos^2(c+dx)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{24d}$$
$$+\frac{a^2\cos^3(c+dx)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{4d}$$

output
```
1/192*(a-b)*(284*a^2+15*b^2)*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(
a+b)^(1/2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)
*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/a/d+1/192*(72*a^3+284*a^2*b+118*a*b^2+15*
b^3)*cot(d*x+c)*EllipticF((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))
^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b
))^(1/2)/a/d-1/64*(48*a^4+120*a^2*b^2-5*b^4)*cot(d*x+c)*EllipticPi((a+b*se
c(d*x+c))^(1/2)/(a+b)^(1/2),(a+b)/a,((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1
-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/a^2/d+1/192*b*(2
84*a^2+15*b^2)*sin(d*x+c)*(a+b*sec(d*x+c))^(1/2)/a/d+1/96*(36*a^2+59*b^2)*
cos(d*x+c)*sin(d*x+c)*(a+b*sec(d*x+c))^(1/2)/d+17/24*a*b*cos(d*x+c)^2*sin(
d*x+c)*(a+b*sec(d*x+c))^(1/2)/d+1/4*a^2*cos(d*x+c)^3*sin(d*x+c)*(a+b*sec(d
*x+c))^(1/2)/d
```

3.552.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 1274 vs. $2(530) = 1060$.

Time = 14.49 (sec) , antiderivative size = 1274, normalized size of antiderivative = 2.40

$$\int \cos^4(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \text{Too large to display}$$

input
```
Integrate[Cos[c + d*x]^4*(a + b*Sec[c + d*x])^(5/2),x]
```

output
```
(Cos[c + d*x]^2*(a + b*Sec[c + d*x])^(5/2)*((17*a*b*Sin[c + d*x])/96 + ((4
8*a^2 + 59*b^2)*Sin[2*(c + d*x)])/192 + (17*a*b*Sin[3*(c + d*x)])/96 + (a^
2*Sin[4*(c + d*x)])/32))/(d*(b + a*Cos[c + d*x])^2) + ((a + b*Sec[c + d*x]
)^(5/2)*(-284*a^3*b*Tan[(c + d*x)/2] - 284*a^2*b^2*Tan[(c + d*x)/2] - 15*a
*b^3*Tan[(c + d*x)/2] - 15*b^4*Tan[(c + d*x)/2] + 568*a^3*b*Tan[(c + d*x)/
2]^3 + 30*a*b^3*Tan[(c + d*x)/2]^3 - 284*a^3*b*Tan[(c + d*x)/2]^5 + 284*a^
2*b^2*Tan[(c + d*x)/2]^5 - 15*a*b^3*Tan[(c + d*x)/2]^5 + 15*b^4*Tan[(c + d
*x)/2]^5 - 288*a^4*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b
)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan
[(c + d*x)/2]^2)/(a + b)] - 720*a^2*b^2*EllipticPi[-1, ArcSin[Tan[(c + d*x
)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(
c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + 30*b^4*EllipticPi[-1, Arc
Sin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[
(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - 288*a^4*E
llipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Tan[(c + d*x)/2]^
2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[
(c + d*x)/2]^2)/(a + b)] - 720*a^2*b^2*EllipticPi[-1, ArcSin[Tan[(c + d*x)
/2]], (a - b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqr
t[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + 30*b^4*
EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Tan[(c + d*x)...
```

3.552.3 Rubi [A] (verified)

Time = 2.88 (sec) , antiderivative size = 547, normalized size of antiderivative = 1.03, number of steps used = 20, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.870$, Rules used = {3042, 4328, 27, 3042, 4592, 27, 3042, 4592, 27, 3042, 4592, 27, 3042, 4546, 3042, 4409, 3042, 4271, 4319, 4492}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^4(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}{\csc\left(c+dx+\frac{\pi}{2}\right)^4}dx$$

$$\Big\downarrow \; 4328$$

$$\frac{1}{4}\int \frac{\cos^3(c+dx)\left(17ba^2+6\left(a^2+4b^2\right)\sec(c+dx)a+b\left(5a^2+8b^2\right)\sec^2(c+dx)\right)}{2\sqrt{a+b\sec(c+dx)}}dx+\frac{a^2\sin(c+dx)\cos^3(c+dx)\sqrt{a+b\sec(c+dx)}}{4d}$$

$\Big\downarrow$ 27

$$\frac{1}{8}\int \frac{\cos^3(c+dx)\left(17ba^2+6\left(a^2+4b^2\right)\sec(c+dx)a+b\left(5a^2+8b^2\right)\sec^2(c+dx)\right)}{\sqrt{a+b\sec(c+dx)}}dx+\frac{a^2\sin(c+dx)\cos^3(c+dx)\sqrt{a+b\sec(c+dx)}}{4d}$$

$\Big\downarrow$ 3042

$$\frac{1}{8}\int \frac{17ba^2+6\left(a^2+4b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)a+b\left(5a^2+8b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^3\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{a^2\sin(c+dx)\cos^3(c+dx)\sqrt{a+b\sec(c+dx)}}{4d}$$

$\Big\downarrow$ 4592

$$\frac{1}{8}\left(\frac{17ab\sin(c+dx)\cos^2(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}-\frac{\int -\frac{\cos^2(c+dx)\left(51b^2\sec^2(c+dx)a^2+\left(36a^2+59b^2\right)a^2+2b\left(49a^2+24b^2\right)\sec}{2\sqrt{a+b\sec(c+dx)}}}{3a}\right.$$
$$\frac{a^2\sin(c+dx)\cos^3(c+dx)\sqrt{a+b\sec(c+dx)}}{4d}$$

$\Big\downarrow$ 27

$$\frac{1}{8}\left(\frac{\int \frac{\cos^2(c+dx)\left(51b^2\sec^2(c+dx)a^2+\left(36a^2+59b^2\right)a^2+2b\left(49a^2+24b^2\right)\sec(c+dx)a\right)}{\sqrt{a+b\sec(c+dx)}}dx}{6a}+\frac{17ab\sin(c+dx)\cos^2(c+dx)\sqrt{a+b\sec}}{3d}\right.$$
$$\frac{a^2\sin(c+dx)\cos^3(c+dx)\sqrt{a+b\sec(c+dx)}}{4d}$$

$\Big\downarrow$ 3042

$$\frac{1}{8}\left(\frac{\int \frac{51b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+\left(36a^2+59b^2\right)a^2+2b\left(49a^2+24b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)a}{\csc\left(c+dx+\frac{\pi}{2}\right)^2\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{6a}+\frac{17ab\sin(c+dx)\cos^2(c+dx)\sqrt{a+b\sec(c+}}{3d}\right.$$
$$\frac{a^2\sin(c+dx)\cos^3(c+dx)\sqrt{a+b\sec(c+dx)}}{4d}$$

$\Big\downarrow$ 4592

3.552. $\int \cos^4(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$

$$\frac{1}{8}\left(\frac{\frac{a\left(36a^2+59b^2\right)\sin(c+dx)\cos(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}-\frac{\int -\frac{\cos(c+dx)\left(2\left(36a^2+161b^2\right)\sec(c+dx)a^3+b\left(36a^2+59b^2\right)\sec^2(c+dx)a^2+b\left(284a^2+15b^2\right)a}{2\sqrt{a+b\sec(c+dx)}}}{2a}}{6a}\right.$$

$$\frac{a^2\sin(c+dx)\cos^3(c+dx)\sqrt{a+b\sec(c+dx)}}{4d}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{8}\left(\frac{\frac{\int\frac{\cos(c+dx)\left(2\left(36a^2+161b^2\right)\sec(c+dx)a^3+b\left(36a^2+59b^2\right)\sec^2(c+dx)a^2+b\left(284a^2+15b^2\right)a^2\right)}{\sqrt{a+b\sec(c+dx)}}dx}{4a}+\frac{a\left(36a^2+59b^2\right)\sin(c+dx)\cos(c+dx)\sqrt{a+b\sec(c+}}{2d}}{6a}\right.$$

$$\frac{a^2\sin(c+dx)\cos^3(c+dx)\sqrt{a+b\sec(c+dx)}}{4d}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{8}\left(\frac{\frac{\int\frac{2\left(36a^2+161b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)a^3+b\left(36a^2+59b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+b\left(284a^2+15b^2\right)a^2}{\csc\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{4a}+\frac{a\left(36a^2+59b^2\right)\sin(c+dx)\cos(c+dx)\sqrt{a+b\sec(c+dx}}{2d}}{6a}\right.$$

$$\frac{a^2\sin(c+dx)\cos^3(c+dx)\sqrt{a+b\sec(c+dx)}}{4d}$$

$$\Big\downarrow \; 4592$$

$$\frac{1}{8}\left(\frac{\frac{\frac{ab\left(284a^2+15b^2\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{d}-\frac{\int -\frac{2b\left(36a^2+59b^2\right)\sec(c+dx)a^3-b^2\left(284a^2+15b^2\right)\sec^2(c+dx)a^2+3\left(48a^4+120b^2a^2-5b^4\right)a^2}{2\sqrt{a+b\sec(c+dx)}}dx}{a}}{4a}+\frac{a\left(36a^2-\right.}{}}{6a}\right.$$

$$\frac{a^2\sin(c+dx)\cos^3(c+dx)\sqrt{a+b\sec(c+dx)}}{4d}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{8}\left(\frac{\frac{\frac{\int\frac{2b\left(36a^2+59b^2\right)\sec(c+dx)a^3-b^2\left(284a^2+15b^2\right)\sec^2(c+dx)a^2+3\left(48a^4+120b^2a^2-5b^4\right)a^2}{\sqrt{a+b\sec(c+dx)}}dx}{2a}+\frac{ab\left(284a^2+15b^2\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{d}}{4a}+\frac{a\left(36a^2+\right.}{}}{6a}\right.$$

$$\frac{a^2\sin(c+dx)\cos^3(c+dx)\sqrt{a+b\sec(c+dx)}}{4d}$$

3.552. $\int \cos^4(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$

$$\Big\downarrow \; 3042$$

$$\frac{1}{8}\left(\frac{\frac{\frac{\int \frac{2b\left(36a^2+59b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)a^3-b^2\left(284a^2+15b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+3\left(48a^4+120b^2a^2-5b^4\right)a^2}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{2a}+\frac{ab\left(284a^2+15b^2\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{d}}{4a}+\frac{a}{}}{6a}\right.$$

$$\frac{a^2\sin(c+dx)\cos^3(c+dx)\sqrt{a+b\sec(c+dx)}}{4d}$$

$$\Big\downarrow \; 4546$$

$$\frac{1}{8}\left(\frac{\frac{\frac{\int \frac{3\left(48a^4+120b^2a^2-5b^4\right)a^2+\left(2b\left(36a^2+59b^2\right)a^3+b^2\left(284a^2+15b^2\right)a^2\right)\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx-a^2b^2\left(284a^2+15b^2\right)\int \frac{\sec(c+dx)(\sec(c+dx)+1)}{\sqrt{a+b\sec(c+dx)}}dx}{2a}+\frac{ab\left(284a^2+15b^2\right)}{}}{4a}}{6a}\right.$$

$$\frac{a^2\sin(c+dx)\cos^3(c+dx)\sqrt{a+b\sec(c+dx)}}{4d}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{8}\left(\frac{\frac{\frac{\int \frac{3\left(48a^4+120b^2a^2-5b^4\right)a^2+\left(2b\left(36a^2+59b^2\right)a^3+b^2\left(284a^2+15b^2\right)a^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-a^2b^2\left(284a^2+15b^2\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{2a}+\frac{ab}{}}{4a}}{6a}\right.$$

$$\frac{a^2\sin(c+dx)\cos^3(c+dx)\sqrt{a+b\sec(c+dx)}}{4d}$$

$$\Big\downarrow \; 4409$$

$$\frac{1}{8}\left(\frac{\frac{\frac{a^2\left(-b^2\right)\left(284a^2+15b^2\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+3a^2\left(48a^4+120a^2b^2-5b^4\right)\int \frac{1}{\sqrt{a+b\sec(c+dx)}}dx+a^2b\left(72a^3+284a^2b+118ab^2+15b^3\right)\int \frac{s}{\sqrt{a-}}}{2a}}{4a}}{6a}\right.$$

$$\frac{a^2\sin(c+dx)\cos^3(c+dx)\sqrt{a+b\sec(c+dx)}}{4d}$$

$$\Big\downarrow \; 3042$$

3.552. $\int \cos^4(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$

$$\frac{1}{8}\left(\frac{\frac{\frac{-a^2b^2\left(284a^2+15b^2\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+3a^2\left(48a^4+120a^2b^2-5b^4\right)\int\frac{1}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+a^2b\left(72a^3+284a^2b+118ab^2+15b^3\right)\int -}{2a}}{4a}}{6a}\right.$$

$$\frac{a^2\sin(c+dx)\cos^3(c+dx)\sqrt{a+b\sec(c+dx)}}{4d}$$

$\downarrow$ 4271

$$\frac{1}{8}\left(\frac{\frac{\frac{-a^2b^2\left(284a^2+15b^2\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+a^2b\left(72a^3+284a^2b+118ab^2+15b^3\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{6a\sqrt{a+b}\left(48a^4+120a^2b^2-5b^4\right)}{}}{2a}}{4a}}{}\right.$$

$$\frac{a^2\sin(c+dx)\cos^3(c+dx)\sqrt{a+b\sec(c+dx)}}{4d}$$

$\downarrow$ 4319

$$\frac{1}{8}\left(\frac{\frac{a^2\left(-b^2\right)\left(284a^2+15b^2\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{6a\sqrt{a+b}\left(48a^4+120a^2b^2-5b^4\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,\text{EllipticPi}(}{d}}{}}{}\right.$$

$$\frac{a^2\sin(c+dx)\cos^3(c+dx)\sqrt{a+b\sec(c+dx)}}{4d}$$

$\downarrow$ 4492

$$\frac{a^2\sin(c+dx)\cos^3(c+dx)\sqrt{a+b\sec(c+dx)}}{4d}+$$

$$\frac{1}{8}\left(\frac{a\left(36a^2+59b^2\right)\sin(c+dx)\cos(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}+\frac{\frac{ab\left(284a^2+15b^2\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{d}+\frac{2a^2(a-b)\sqrt{a+b}\left(284a^2+15b^2\right)\cot(c+dx)\sqrt{\frac{b(1}{}}}{}}{}\right.$$

input
```
Int[Cos[c + d*x]^4*(a + b*Sec[c + d*x])^(5/2),x]
```

3.552. $\int \cos^4(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$

output
```
(a^2*Cos[c + d*x]^3*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(4*d) + ((17*a*
b*Cos[c + d*x]^2*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(3*d) + ((a*(36*a^
2 + 59*b^2)*Cos[c + d*x]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(2*d) + ((
(2*a^2*(a - b)*Sqrt[a + b]*(284*a^2 + 15*b^2)*Cot[c + d*x]*EllipticE[ArcSi
n[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec
[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/d + (2*a^2*S
qrt[a + b]*(72*a^3 + 284*a^2*b + 118*a*b^2 + 15*b^3)*Cot[c + d*x]*Elliptic
F[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(
1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/d - (
6*a*Sqrt[a + b]*(48*a^4 + 120*a^2*b^2 - 5*b^4)*Cot[c + d*x]*EllipticPi[(a
+ b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqr
t[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])
/d)/(2*a) + (a*b*(284*a^2 + 15*b^2)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])
/d)/(4*a))/(6*a))/8
```

3.552.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4271
```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*(Rt[a
 + b, 2]/(a*d*Cot[c + d*x]))*Sqrt[b*((1 - Csc[c + d*x])/(a + b))]*Sqrt[(-b)
*((1 + Csc[c + d*x])/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[
c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

rule 4319
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4328
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)*
((d*Csc[e + f*x])^n/(f*n)), x] - Simp[1/(d*n)   Int[(a + b*Csc[e + f*x])^(m
 - 3)*(d*Csc[e + f*x])^(n + 1)*Simp[a^2*b*(m - 2*n - 2) - a*(3*b^2*n + a^2*
(n + 1))*Csc[e + f*x] - b*(b^2*n + a^2*(m + n - 1))*Csc[e + f*x]^2, x], x],
 x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && ((Int
egerQ[m] && LtQ[n, -1]) || (IntegersQ[m + 1/2, 2*n] && LeQ[n, -1]))
```

rule 4409
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Simp[c   Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] +
Simp[d   Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

rule 4492
```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

rule 4546
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Simp[C   Int[Csc[e + f*x]*((
1 + Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A
, B, C}, x] && NeQ[a^2 - b^2, 0]
```

rule 4592
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Simp[1/(a*d*n)   Int[(a + b*Csc[e + f*x])^m
*(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*
Csc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

3.552. $\int \cos^4(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$

3.552.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3191 vs. $2(481) = 962$.

Time = 410.77 (sec) , antiderivative size = 3192, normalized size of antiderivative = 6.02

method	result	size
default	Expression too large to display	3192

input
```
int(cos(d*x+c)^4*(a+b*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/192/d/a*(-1440*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c
))/(cos(d*x+c)+1))^(1/2)*EllipticPi(cot(d*x+c)-csc(d*x+c),-1,((a-b)/(a+b))
^(1/2))*a^2*b^2*cos(d*x+c)-720*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*
(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi(cot(d*x+c)-csc(d*x+c),-1
,((a-b)/(a+b))^(1/2))*a^2*b^2*cos(d*x+c)^2-720*EllipticPi(cot(d*x+c)-csc(d
*x+c),-1,((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(
b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^2*b^2+60*(cos(d*x+c)/(cos(d*x+c)+1
))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi(cot(d*
x+c)-csc(d*x+c),-1,((a-b)/(a+b))^(1/2))*b^4*cos(d*x+c)-30*(cos(d*x+c)/(cos
(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Elliptic
E(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*b^4*cos(d*x+c)+144*(cos(d*x+c
)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*El
lipticF(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*a^4*cos(d*x+c)^2-288*(c
os(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^
(1/2)*EllipticPi(cot(d*x+c)-csc(d*x+c),-1,((a-b)/(a+b))^(1/2))*a^4*cos(d*x
+c)^2+30*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(
d*x+c)+1))^(1/2)*EllipticPi(cot(d*x+c)-csc(d*x+c),-1,((a-b)/(a+b))^(1/2))*
b^4*cos(d*x+c)^2-15*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*
x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^
(1/2))*b^4*cos(d*x+c)^2+288*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*...
```

3.552.5 Fricas [F]

$$\int \cos^4(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \int (b\sec(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^4\,dx$$

input
```
integrate(cos(d*x+c)^4*(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
integral((b^2*cos(d*x + c)^4*sec(d*x + c)^2 + 2*a*b*cos(d*x + c)^4*sec(d*x
 + c) + a^2*cos(d*x + c)^4)*sqrt(b*sec(d*x + c) + a), x)
```

3.552.6 Sympy [F(-1)]

Timed out.

$$\int \cos^4(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**4*(a+b*sec(d*x+c))**(5/2),x)
```

output
```
Timed out
```

3.552.7 Maxima [F]

$$\int \cos^4(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \int (b\sec(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^4\,dx$$

input
```
integrate(cos(d*x+c)^4*(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^4, x)
```

3.552.8 Giac [F]

$$\int \cos^4(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \int (b\sec(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^4\,dx$$

input
```
integrate(cos(d*x+c)^4*(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^4, x)
```

3.552.9 Mupad [F(-1)]

Timed out.

$$\int \cos^4(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \int \cos(c+d\,x)^4\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2}dx$$

input
```
int(cos(c + d*x)^4*(a + b/cos(c + d*x))^(5/2),x)
```

output
```
int(cos(c + d*x)^4*(a + b/cos(c + d*x))^(5/2), x)
```

3.553 $\int (a+b\sec(c+dx))^{7/2}\,dx$

3.553.1 Optimal result . . . 4170
3.553.2 Mathematica [B] (warning: unable to verify) . . . 4171
3.553.3 Rubi [A] (verified) . . . 4172
3.553.4 Maple [B] (verified) . . . 4176
3.553.5 Fricas [F] . . . 4177
3.553.6 Sympy [F(-1)] . . . 4178
3.553.7 Maxima [F] . . . 4178
3.553.8 Giac [F] . . . 4178
3.553.9 Mupad [F(-1)] . . . 4179

3.553.1 Optimal result

Integrand size = 14, antiderivative size = 403

$$\int (a+b\sec(c+dx))^{7/2}\,dx = -\frac{2(a-b)\sqrt{a+b}(58a^2+9b^2)\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{15d} + \frac{2\sqrt{a+b}(60a^3-58a^2b+22ab^2-9b^3)\cot(c+dx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{}}{15d} - \frac{2a^3\sqrt{a+b}\cot(c+dx)\,\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{d} + \frac{26ab^2\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{15d} + \frac{2b^2(a+b\sec(c+dx))^{3/2}\tan(c+dx)}{5d}$$

output
```
-2/15*(a-b)*(58*a^2+9*b^2)*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(a+
b)^(1/2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(
-b*(1+sec(d*x+c))/(a-b))^(1/2)/d+2/15*(60*a^3-58*a^2*b+22*a*b^2-9*b^3)*cot
(d*x+c)*EllipticF((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*
(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)
/d-2*a^3*cot(d*x+c)*EllipticPi((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),(a+b)/a,
((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec
(d*x+c))/(a-b))^(1/2)/d+2/5*b^2*(a+b*sec(d*x+c))^(3/2)*tan(d*x+c)/d+26/15*
a*b^2*(a+b*sec(d*x+c))^(1/2)*tan(d*x+c)/d
```

3.553.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 873 vs. $2(403) = 806$.

Time = 13.63 (sec) , antiderivative size = 873, normalized size of antiderivative = 2.17

$$\int (a+b\sec(c+dx))^{7/2}\,dx = \frac{2(a+b\sec(c+dx))^{7/2}\left(58a^3b\tan\left(\frac{1}{2}(c+dx)\right)+58a^2b^2\tan\left(\frac{1}{2}(c+dx)\right)+9ab^3\tan\left(\frac{1}{2}(c+dx\right.\right.}{}$$

$$+\frac{\cos^3(c+dx)(a+b\sec(c+dx))^{7/2}\left(\frac{2}{15}b(58a^2+9b^2)\sin(c+dx)+\frac{32}{15}ab^2\tan(c+dx)+\frac{2}{5}b^3\sec(c+dx)\tan(\right.}{d(b+a\cos(c+dx))^3}$$

input
```
Integrate[(a + b*Sec[c + d*x])^(7/2),x]
```

output
```
(2*(a + b*Sec[c + d*x])^(7/2)*(58*a^3*b*Tan[(c + d*x)/2] + 58*a^2*b^2*Tan[
(c + d*x)/2] + 9*a*b^3*Tan[(c + d*x)/2] + 9*b^4*Tan[(c + d*x)/2] - 116*a^3
*b*Tan[(c + d*x)/2]^3 - 18*a*b^3*Tan[(c + d*x)/2]^3 + 58*a^3*b*Tan[(c + d*
x)/2]^5 - 58*a^2*b^2*Tan[(c + d*x)/2]^5 + 9*a*b^3*Tan[(c + d*x)/2]^5 - 9*b
^4*Tan[(c + d*x)/2]^5 - 30*a^4*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (a
 - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/
2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - 30*a^4*EllipticPi[-1, ArcSin[Tan[(
c + d*x)/2]], (a - b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2
]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] +
 b*(58*a^3 + 58*a^2*b + 9*a*b^2 + 9*b^3)*EllipticE[ArcSin[Tan[(c + d*x)/2]
], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*
Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + (15*
a^4 - 60*a^3*b - 58*a^2*b^2 - 22*a*b^3 - 9*b^4)*EllipticF[ArcSin[Tan[(c +
d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)
/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)]
))/(15*d*(b + a*Cos[c + d*x])^(7/2)*Sec[c + d*x]^(7/2)*Sqrt[(1 - Tan[(c +
d*x)/2]^2)^(-1)]*(-1 + Tan[(c + d*x)/2]^2)*(1 + Tan[(c + d*x)/2]^2)^(3/2)*
Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d
*x)/2]^2)]) + (Cos[c + d*x]^3*(a + b*Sec[c + d*x])^(7/2)*((2*b*(58*a^2 + 9
*b^2)*Sin[c + d*x])/15 + (32*a*b^2*Tan[c + d*x])/15 + (2*b^3*Sec[c + d*...
```

3.553.3 Rubi [A] (verified)

Time = 1.57 (sec) , antiderivative size = 409, normalized size of antiderivative = 1.01, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {3042, 4269, 27, 3042, 4544, 27, 3042, 4546, 3042, 4409, 3042, 4271, 4319, 4492}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\sec(c+dx))^{7/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{7/2}dx$$

$$\Big\downarrow \text{4269}$$

$$\frac{2}{5}\int \frac{1}{2}\sqrt{a+b\sec(c+dx)}\left(5a^3+13b^2\sec^2(c+dx)a+3b\left(5a^2+b^2\right)\sec(c+dx)\right)dx+\frac{2b^2\tan(c+dx)(a+b\sec(c+dx))^{3/2}}{5d}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{5}\int \sqrt{a+b\sec(c+dx)}\left(5a^3+13b^2\sec^2(c+dx)a+3b\left(5a^2+b^2\right)\sec(c+dx)\right)dx+\frac{2b^2\tan(c+dx)(a+b\sec(c+dx))^{3/2}}{5d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}\int \sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}\left(5a^3+13b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2a+3b\left(5a^2+b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)\right)dx+\frac{2b^2\tan(c+dx)(a+b\sec(c+dx))^{3/2}}{5d}$$

$$\Big\downarrow \text{4544}$$

$$\frac{1}{5}\left(\frac{2}{3}\int \frac{15a^4+2b\left(30a^2+11b^2\right)\sec(c+dx)a+b^2\left(58a^2+9b^2\right)\sec^2(c+dx)}{2\sqrt{a+b\sec(c+dx)}}dx+\frac{26ab^2\tan(c+dx)\sqrt{a+b\sec(c+}}{3d}\right.+\frac{2b^2\tan(c+dx)(a+b\sec(c+dx))^{3/2}}{5d}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{5}\left(\frac{1}{3}\int \frac{15a^4+2b(30a^2+11b^2)\sec(c+dx)a+b^2(58a^2+9b^2)\sec^2(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx+\frac{26ab^2\tan(c+dx)\sqrt{a+b\sec(c+}}{3d}\right.$$
$$\frac{2b^2\tan(c+dx)(a+b\sec(c+dx))^{3/2}}{5d}$$

$\downarrow$ 3042

$$\frac{1}{5}\left(\frac{1}{3}\int \frac{15a^4+2b(30a^2+11b^2)\csc\left(c+dx+\frac{\pi}{2}\right)a+b^2(58a^2+9b^2)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{26ab^2\tan(c+dx)\sqrt{a}}{3d}\right.$$
$$\frac{2b^2\tan(c+dx)(a+b\sec(c+dx))^{3/2}}{5d}$$

$\downarrow$ 4546

$$\frac{1}{5}\left(\frac{1}{3}\left(b^2(58a^2+9b^2)\int \frac{\sec(c+dx)(\sec(c+dx)+1)}{\sqrt{a+b\sec(c+dx)}}dx+\int \frac{15a^4+\left(2ab(30a^2+11b^2)-b^2(58a^2+9b^2)\right)\sec(c+}{\sqrt{a+b\sec(c+dx)}}\right.\right.$$
$$\frac{2b^2\tan(c+dx)(a+b\sec(c+dx))^{3/2}}{5d}$$

$\downarrow$ 3042

$$\frac{1}{5}\left(\frac{1}{3}\left(b^2(58a^2+9b^2)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\int \frac{15a^4+\left(2ab(30a^2+11b^2)-b^2(58a^2+\right.}{\sqrt{a+b\csc\left(c+dx+\right.}}\right.\right.$$
$$\frac{2b^2\tan(c+dx)(a+b\sec(c+dx))^{3/2}}{5d}$$

$\downarrow$ 4409

$$\frac{1}{5}\left(\frac{1}{3}\left(15a^4\int \frac{1}{\sqrt{a+b\sec(c+dx)}}dx+b^2(58a^2+9b^2)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+b(60a^3-\right.\right.$$
$$\frac{2b^2\tan(c+dx)(a+b\sec(c+dx))^{3/2}}{5d}$$

$\downarrow$ 3042

$$\frac{1}{5}\left(\frac{1}{3}\left(15a^4\int \frac{1}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+b^2(58a^2+9b^2)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+b(60\right.\right.$$
$$\frac{2b^2\tan(c+dx)(a+b\sec(c+dx))^{3/2}}{5d}$$

$\downarrow$ 4271

3.553. $\int (a+b\sec(c+dx))^{7/2}\,dx$

$$\frac{1}{5}\left(\frac{1}{3}\left(b^2(58a^2+9b^2)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+b(60a^3-58a^2b+22ab^2-9b^3)\int\frac{\csc(}{\sqrt{a+b}}\right.\right.$$

$$\frac{2b^2\tan(c+dx)(a+b\sec(c+dx))^{3/2}}{5d}$$

$$\Big\downarrow \; 4319$$

$$\frac{1}{5}\left(\frac{1}{3}\left(b^2(58a^2+9b^2)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{30a^3\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-}}{}\right.\right.$$

$$\frac{2b^2\tan(c+dx)(a+b\sec(c+dx))^{3/2}}{5d}$$

$$\Big\downarrow \; 4492$$

$$\frac{1}{5}\left(\frac{1}{3}\left(-\frac{30a^3\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{d}-\right.\right.$$

$$\frac{2b^2\tan(c+dx)(a+b\sec(c+dx))^{3/2}}{5d}$$

input
```
Int[(a + b*Sec[c + d*x])^(7/2),x]
```

output
```
(2*b^2*(a + b*Sec[c + d*x])^(3/2)*Tan[c + d*x])/(5*d) + (((-2*(a - b)*Sqrt
[a + b]*(58*a^2 + 9*b^2)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c +
d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*
Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/d + (2*Sqrt[a + b]*(60*a^3 - 58*a
^2*b + 22*a*b^2 - 9*b^3)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c +
d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*
Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/d - (30*a^3*Sqrt[a + b]*Cot[c + d
*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a
 + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c +
 d*x]))/(a - b))])/d)/3 + (26*a*b^2*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])
/(3*d))/5
```

3.553. $\int (a+b\sec(c+dx))^{7/2}\,dx$

3.553.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4269
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(-b^2)*C
ot[c + d*x]*((a + b*Csc[c + d*x])^(n - 2)/(d*(n - 1))), x] + Simp[1/(n - 1)
   Int[(a + b*Csc[c + d*x])^(n - 3)*Simp[a^3*(n - 1) + (b*(b^2*(n - 2) + 3*
a^2*(n - 1)))*Csc[c + d*x] + (a*b^2*(3*n - 4))*Csc[c + d*x]^2, x], x], x] /
; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 2] && IntegerQ[2*n]
```

rule 4271
```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*(Rt[a
 + b, 2]/(a*d*Cot[c + d*x]))*Sqrt[b*((1 - Csc[c + d*x])/(a + b))]*Sqrt[(-b)
*((1 + Csc[c + d*x])/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[
c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

rule 4319
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4409
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Simp[c   Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] +
Simp[d   Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

rule 4492
```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

3.553. $\int (a+b\sec(c+dx))^{7/2}\,dx$

rule 4544
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.), x_Symbol] :> Simp[(-C)*Cot
[e + f*x]*((a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Simp[1/(m + 1)   Int[(
a + b*Csc[e + f*x])^(m - 1)*Simp[a*A*(m + 1) + ((A*b + a*B)*(m + 1) + b*C*m
)*Csc[e + f*x] + (b*B*(m + 1) + a*C*m)*Csc[e + f*x]^2, x], x], x] /; FreeQ[
{a, b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && IGtQ[2*m, 0]
```

rule 4546
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Simp[C   Int[Csc[e + f*x]*((
1 + Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A
, B, C}, x] && NeQ[a^2 - b^2, 0]
```

3.553.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2776 vs. $2(364) = 728$.

Time = 13.71 (sec) , antiderivative size = 2777, normalized size of antiderivative = 6.89

method	result	size
default	Expression too large to display	2777

input
```
int((a+b*sec(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

output
```
2/15/d*(a+b*sec(d*x+c))^(1/2)/(b+a*cos(d*x+c))/(cos(d*x+c)+1)*(18*(cos(d*x
+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*
EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*b^4*cos(d*x+c)+15*(co
s(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(
1/2)*EllipticF(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*a^4*cos(d*x+c)^2
-30*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c
)+1))^(1/2)*EllipticPi(cot(d*x+c)-csc(d*x+c),-1,((a-b)/(a+b))^(1/2))*a^4*c
os(d*x+c)^2+9*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/
(cos(d*x+c)+1))^(1/2)*EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))
*b^4*cos(d*x+c)^2+30*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d
*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))
^(1/2))*a^4*cos(d*x+c)-60*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*
cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi(cot(d*x+c)-csc(d*x+c),-1,((a-
b)/(a+b))^(1/2))*a^4*cos(d*x+c)+19*a*b^3*sin(d*x+c)+9*b^4*sin(d*x+c)+15*El
lipticF(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)
+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^4-30*Elliptic
Pi(cot(d*x+c)-csc(d*x+c),-1,((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1
))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^4+9*EllipticE(c
ot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2
)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*b^4+74*a^2*b^2*sin(d*...
```

3.553.5 Fricas [F]

$$\int (a+b\sec(c+dx))^{7/2}\,dx=\int (b\sec(dx+c)+a)^{\frac{7}{2}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(7/2),x, algorithm="fricas")
```

output
```
integral((b^3*sec(d*x + c)^3 + 3*a*b^2*sec(d*x + c)^2 + 3*a^2*b*sec(d*x +
c) + a^3)*sqrt(b*sec(d*x + c) + a), x)
```

3.553.6 Sympy [F(-1)]

Timed out.

$$\int (a+b\sec(c+dx))^{7/2}\,dx = \text{Timed out}$$

input
```
integrate((a+b*sec(d*x+c))**(7/2),x)
```

output
```
Timed out
```

3.553.7 Maxima [F]

$$\int (a+b\sec(c+dx))^{7/2}\,dx = \int (b\sec(dx+c)+a)^{\frac{7}{2}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(7/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^(7/2), x)
```

3.553.8 Giac [F]

$$\int (a+b\sec(c+dx))^{7/2}\,dx = \int (b\sec(dx+c)+a)^{\frac{7}{2}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(7/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^(7/2), x)
```

3.553.9 Mupad [F(-1)]

Timed out.

$$\int (a+b\sec(c+dx))^{7/2}\,dx=\int \left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)^{7/2}\,dx$$

input
```
int((a + b/cos(c + d*x))^(7/2),x)
```

output
```
int((a + b/cos(c + d*x))^(7/2), x)
```

3.554 $\int \frac{\sec^5(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$

3.554.1 Optimal result 4180
3.554.2 Mathematica [A] (warning: unable to verify) 4181
3.554.3 Rubi [A] (verified) 4181
3.554.4 Maple [B] (verified) 4186
3.554.5 Fricas [F] 4186
3.554.6 Sympy [F] 4187
3.554.7 Maxima [F] 4187
3.554.8 Giac [F] 4187
3.554.9 Mupad [F(-1)] 4188

3.554.1 Optimal result

Integrand size = 23, antiderivative size = 359

$$\begin{aligned}
&\int \frac{\sec^5(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx \\
&= \frac{8a(a-b)\sqrt{a+b}(12a^2+11b^2)\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{105b^5d} \\
&\quad+\frac{2\sqrt{a+b}(48a^3-12a^2b+44ab^2+25b^3)\cot(c+dx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+d}{a+b}}}{105b^4d} \\
&\quad+\frac{2(24a^2+25b^2)\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{105b^3d} \\
&\quad-\frac{12a\sec(c+dx)\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{35b^2d} \\
&\quad+\frac{2\sec^2(c+dx)\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{7bd}
\end{aligned}$$

output
```
8/105*a*(a-b)*(12*a^2+11*b^2)*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/
(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2
)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/b^5/d+2/105*(48*a^3-12*a^2*b+44*a*b^2+25
*b^3)*cot(d*x+c)*EllipticF((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b)
)^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-
b))^(1/2)/b^4/d+2/105*(24*a^2+25*b^2)*(a+b*sec(d*x+c))^(1/2)*tan(d*x+c)/b^
3/d-12/35*a*sec(d*x+c)*(a+b*sec(d*x+c))^(1/2)*tan(d*x+c)/b^2/d+2/7*sec(d*x
+c)^2*(a+b*sec(d*x+c))^(1/2)*tan(d*x+c)/b/d
```

3.554.2 Mathematica [A] (warning: unable to verify)

Time = 12.02 (sec) , antiderivative size = 463, normalized size of antiderivative = 1.29

$$\int \frac{\sec^5(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$$
$$= \frac{4\sqrt{\sec(c+dx)}\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)}\left(4a(12a^3+12a^2b+11ab^2+11b^3)\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\sqrt{\frac{b+a\cos(c}{(a+b)(1+\cos}}\right.}{}$$
$$+\frac{(b+a\cos(c+dx))\sec(c+dx)\left(-\frac{8a(12a^2+11b^2)\sin(c+dx)}{105b^4}+\frac{2\sec(c+dx)(24a^2\sin(c+dx)+25b^2\sin(c+dx))}{105b^3}-\frac{12a\sec(c+d}{35}\right.}{d\sqrt{a+b\sec(c+dx)}}$$

input
```
Integrate[Sec[c + d*x]^5/Sqrt[a + b*Sec[c + d*x]],x]
```

output
```
(4*Sqrt[Sec[c + d*x]]*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(4*a*(12*a^3 +
 12*a^2*b + 11*a*b^2 + 11*b^3)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[
(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c
 + d*x)/2]], (a - b)/(a + b)] + b*(-48*a^3 - 12*a^2*b - 44*a*b^2 + 25*b^3)
*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*
(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]
+ 2*a*(12*a^2 + 11*b^2)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]
^2*Tan[(c + d*x)/2]))/(105*b^4*d*Sqrt[Sec[(c + d*x)/2]^2]*Sqrt[a + b*Sec[c
 + d*x]]) + ((b + a*Cos[c + d*x])*Sec[c + d*x]*((-8*a*(12*a^2 + 11*b^2)*Si
n[c + d*x])/(105*b^4) + (2*Sec[c + d*x]*(24*a^2*Sin[c + d*x] + 25*b^2*Sin[
c + d*x]))/(105*b^3) - (12*a*Sec[c + d*x]*Tan[c + d*x])/(35*b^2) + (2*Sec[
c + d*x]^2*Tan[c + d*x])/(7*b)))/(d*Sqrt[a + b*Sec[c + d*x]])
```

3.554.3 Rubi [A] (verified)

Time = 1.51 (sec) , antiderivative size = 379, normalized size of antiderivative = 1.06, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.565$, Rules used = {3042, 4347, 3042, 4580, 27, 3042, 4570, 27, 3042, 4493, 3042, 4319, 4492}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^5(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$$
$$\Big\downarrow \; 3042$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^5}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \; 4347$$

$$\frac{\int \frac{\sec^2(c+dx)\left(-6a\sec^2(c+dx)+5b\sec(c+dx)+4a\right)}{\sqrt{a+b\sec(c+dx)}}dx}{7b}+\frac{2\tan(c+dx)\sec^2(c+dx)\sqrt{a+b\sec(c+dx)}}{7bd}$$

$$\Big\downarrow \; 3042$$

$$\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2\left(-6a\csc\left(c+dx+\frac{\pi}{2}\right)^2+5b\csc\left(c+dx+\frac{\pi}{2}\right)+4a\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{7b}+\frac{2\tan(c+dx)\sec^2(c+dx)\sqrt{a+b\sec(c+dx)}}{7bd}$$

$$\Big\downarrow \; 4580$$

$$\frac{\frac{2\int -\frac{\sec(c+dx)\left(12a^2-2b\sec(c+dx)a-\left(24a^2+25b^2\right)\sec^2(c+dx)\right)}{2\sqrt{a+b\sec(c+dx)}}dx}{5b}-\frac{12a\tan(c+dx)\sec(c+dx)\sqrt{a+b\sec(c+dx)}}{5bd}}{7b}+\frac{2\tan(c+dx)\sec^2(c+dx)\sqrt{a+b\sec(c+dx)}}{7bd}$$

$$\Big\downarrow \; 27$$

$$\frac{-\frac{\int \frac{\sec(c+dx)\left(12a^2-2b\sec(c+dx)a-\left(24a^2+25b^2\right)\sec^2(c+dx)\right)}{\sqrt{a+b\sec(c+dx)}}dx}{5b}-\frac{12a\tan(c+dx)\sec(c+dx)\sqrt{a+b\sec(c+dx)}}{5bd}}{7b}+\frac{2\tan(c+dx)\sec^2(c+dx)\sqrt{a+b\sec(c+dx)}}{7bd}$$

$$\Big\downarrow \; 3042$$

$$\frac{-\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(12a^2-2b\csc\left(c+dx+\frac{\pi}{2}\right)a+\left(-24a^2-25b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{5b}-\frac{12a\tan(c+dx)\sec(c+dx)\sqrt{a+b\sec(c+dx)}}{5bd}}{7b}+\frac{2\tan(c+dx)\sec^2(c+dx)\sqrt{a+b\sec(c+dx)}}{7bd}$$

$$\Big\downarrow \; 4570$$

$$\frac{-\frac{\frac{2\int \frac{\sec(c+dx)\left(b\left(12a^2-25b^2\right)+4a\left(12a^2+11b^2\right)\sec(c+dx)\right)}{2\sqrt{a+b\sec(c+dx)}}dx}{3b}-\frac{2\left(24a^2+25b^2\right)\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{3bd}}{5b}-\frac{12a\tan(c+dx)\sec(c+dx)\sqrt{a+b\sec(c+dx)}}{5bd}}{7b}+\frac{2\tan(c+dx)\sec^2(c+dx)\sqrt{a+b\sec(c+dx)}}{7bd}$$

$$\Big\downarrow \; 27$$

3.554. $\int \frac{\sec^5(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx$

$$-\frac{\frac{\int \frac{\sec(c+dx)\left(b\left(12a^2-25b^2\right)+4a\left(12a^2+11b^2\right)\sec(c+dx)\right)}{\sqrt{a+b\sec(c+dx)}}dx}{3b}-\frac{2\left(24a^2+25b^2\right)\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{3bd}}{5b}-\frac{12a\tan(c+dx)\sec(c+dx)\sqrt{a+b\sec(c+dx)}}{5bd}-$$

$$\frac{2\tan(c+dx)\sec^2(c+dx)\sqrt{a+b\sec(c+dx)}}{7bd}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(b\left(12a^2-25b^2\right)+4a\left(12a^2+11b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{3b}-\frac{2\left(24a^2+25b^2\right)\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{3bd}}{5b}-\frac{12a\tan(c+dx)\sec(c+dx)\sqrt{a+b\sec(}}{5bd}$$

$$\frac{2\tan(c+dx)\sec^2(c+dx)\sqrt{a+b\sec(c+dx)}}{7bd}$$

$$\Big\downarrow \; 4493$$

$$-\frac{\frac{4a\left(12a^2+11b^2\right)\int \frac{\sec(c+dx)(\sec(c+dx)+1)}{\sqrt{a+b\sec(c+dx)}}dx-\left(48a^3-12a^2b+44ab^2+25b^3\right)\int \frac{\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx}{3b}-\frac{2\left(24a^2+25b^2\right)\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{3bd}}{5b}-\frac{12a\tan(c-}{}$$

$$\frac{2\tan(c+dx)\sec^2(c+dx)\sqrt{a+b\sec(c+dx)}}{7bd}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\frac{4a\left(12a^2+11b^2\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-\left(48a^3-12a^2b+44ab^2+25b^3\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{3b}-\frac{2\left(24a^2+25b^2\right)\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{3bd}}{5b}$$

$$\frac{2\tan(c+dx)\sec^2(c+dx)\sqrt{a+b\sec(c+dx)}}{7bd}$$

$$\Big\downarrow \; 4319$$

$$-\frac{\frac{4a\left(12a^2+11b^2\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2\sqrt{a+b}\left(48a^3-12a^2b+44ab^2+25b^3\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,\text{EllipticF}\left(\arcsin\right.}{bd}}{3b}}{5b}$$

$$\frac{2\tan(c+dx)\sec^2(c+dx)\sqrt{a+b\sec(c+dx)}}{7bd}$$

$$\Big\downarrow \; 4492$$

3.554. $\int \frac{\sec^5(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$

$$-\frac{-\frac{-\frac{8a(a-b)\sqrt{a+b}\left(12a^2+11b^2\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}E\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{b^2d}-\frac{2\sqrt{a+b}\left(48a^3-12a^2b+44ab^2+25b^3\right)\cot(c}{3b}}{5b}}{}$$

$$\frac{2\tan(c+dx)\sec^2(c+dx)\sqrt{a+b\sec(c+dx)}}{7bd}$$

input
```
Int[Sec[c + d*x]^5/Sqrt[a + b*Sec[c + d*x]],x]
```

output
```
(2*Sec[c + d*x]^2*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(7*b*d) + ((-12*a
*Sec[c + d*x]*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(5*b*d) - (((-8*a*(a
- b)*Sqrt[a + b]*(12*a^2 + 11*b^2)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a +
b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))
/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(b^2*d) - (2*Sqrt[a + b
]*(48*a^3 - 12*a^2*b + 44*a*b^2 + 25*b^3)*Cot[c + d*x]*EllipticF[ArcSin[Sq
rt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c +
 d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(b*d))/(3*b) - (
2*(24*a^2 + 25*b^2)*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(3*b*d))/(5*b))
/(7*b)
```

3.554.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4319
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4347
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[-2*d^2*Cos[e + f*x]*(d*Csc[e + f*x])^(n - 2)*(S
qrt[a + b*Csc[e + f*x]]/(b*f*(2*n - 3))), x] + Simp[d^3/(b*(2*n - 3))   Int
[((d*Csc[e + f*x])^(n - 3)/Sqrt[a + b*Csc[e + f*x]])*Simp[2*a*(n - 3) + b*(
2*n - 5)*Csc[e + f*x] - 2*a*(n - 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 2] && IntegerQ[2*n]
```

rule 4492
```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

rule 4493
```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(A - B)   Int[Csc[e
 + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Simp[B   Int[Csc[e + f*x]*((1 +
Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A, B}
, x] && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

rule 4570
```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> Simp[(-C)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2)
)), x] + Simp[1/(b*(m + 2))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[
b*A*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /;
 FreeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

rule 4580
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[
(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x
_Symbol] :> Simp[(-C)*Csc[e + f*x]*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m +
1)/(b*f*(m + 3))), x] + Simp[1/(b*(m + 3))   Int[Csc[e + f*x]*(a + b*Csc[e
+ f*x])^m*Simp[a*C + b*(C*(m + 2) + A*(m + 3))*Csc[e + f*x] - (2*a*C - b*B*
(m + 3))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &
& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

3.554. $\int \frac{\sec^5(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$

3.554.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2342 vs. 2(325) = 650.

Time = 17.16 (sec) , antiderivative size = 2343, normalized size of antiderivative = 6.53

method	result	size
default	Expression too large to display	2343

input
```
int(sec(d*x+c)^5/(a+b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/105/d/b^4*(a+b*sec(d*x+c))^(1/2)/(b+a*cos(d*x+c))/(cos(d*x+c)+1)*(24*a^
3*b*sin(d*x+c)+19*a*b^3*sin(d*x+c)-25*b^4*sin(d*x+c)-6*a^2*b^2*sin(d*x+c)+
48*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE(cot(d*x+c)-cs
c(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*a^3*b+44*(
1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE(cot(d*x+c)-csc(d*
x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*a^2*b^2+44*(1/
(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE(cot(d*x+c)-csc(d*x+
c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*a*b^3+25*Ellipti
cF(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(c
os(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*b^4*cos(d*x+c)^2+48*
(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1)
)^(1/2)*EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*a^4*cos(d*x+c
)^2+50*EllipticF(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*(1/(a+b)*(b+a*
cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*b^4*co
s(d*x+c)+96*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(c
os(d*x+c)+1))^(1/2)*EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*a
^4*cos(d*x+c)-48*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF
(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1
/2)*a^3*b-12*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF(cot
(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/...
```

3.554.5 Fricas [F]

$$\int \frac{\sec^5(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\sec\left(dx+c\right)^5}{\sqrt{b\sec\left(dx+c\right)+a}}\,dx$$

input
```
integrate(sec(d*x+c)^5/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
integral(sec(d*x + c)^5/sqrt(b*sec(d*x + c) + a), x)
```

3.554.6 Sympy [F]

$$\int \frac{\sec^5(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\sec^5(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$$

input
```
integrate(sec(d*x+c)**5/(a+b*sec(d*x+c))**(1/2),x)
```

output
```
Integral(sec(c + d*x)**5/sqrt(a + b*sec(c + d*x)), x)
```

3.554.7 Maxima [F]

$$\int \frac{\sec^5(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\sec(dx+c)^5}{\sqrt{b\sec(dx+c)+a}}\,dx$$

input
```
integrate(sec(d*x+c)^5/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)^5/sqrt(b*sec(d*x + c) + a), x)
```

3.554.8 Giac [F]

$$\int \frac{\sec^5(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\sec(dx+c)^5}{\sqrt{b\sec(dx+c)+a}}\,dx$$

input
```
integrate(sec(d*x+c)^5/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^5/sqrt(b*sec(d*x + c) + a), x)
```

3.554.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^5(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{1}{\cos\left(c+d\,x\right)^5\,\sqrt{a+\frac{b}{\cos\left(c+d\,x\right)}}}\,dx$$

input
```
int(1/(cos(c + d*x)^5*(a + b/cos(c + d*x))^(1/2)),x)
```

output
```
int(1/(cos(c + d*x)^5*(a + b/cos(c + d*x))^(1/2)), x)
```

3.555 $\int \frac{\sec^4(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$

3.555.1 Optimal result . . . 4189
3.555.2 Mathematica [A] (verified) . . . 4190
3.555.3 Rubi [A] (verified) . . . 4190
3.555.4 Maple [B] (verified) . . . 4194
3.555.5 Fricas [F] . . . 4194
3.555.6 Sympy [F] . . . 4195
3.555.7 Maxima [F] . . . 4195
3.555.8 Giac [F] . . . 4195
3.555.9 Mupad [F(-1)] . . . 4196

3.555.1 Optimal result

Integrand size = 23, antiderivative size = 301

$$\int \frac{\sec^4(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = -\frac{2(a-b)\sqrt{a+b}(8a^2+9b^2)\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{15b^4d} - \frac{2\sqrt{a+b}(8a^2-2ab+9b^2)\cot(c+dx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec}{a}}}{15b^3d} - \frac{8a\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{15b^2d} + \frac{2\sec(c+dx)\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{5bd}$$

output
```
-2/15*(a-b)*(8*a^2+9*b^2)*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(a+b
)^(1/2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-
b*(1+sec(d*x+c))/(a-b))^(1/2)/b^4/d-2/15*(8*a^2-2*a*b+9*b^2)*cot(d*x+c)*El
lipticF((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2
)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/b^3/d-8/1
5*a*(a+b*sec(d*x+c))^(1/2)*tan(d*x+c)/b^2/d+2/5*sec(d*x+c)*(a+b*sec(d*x+c)
)^(1/2)*tan(d*x+c)/b/d
```

3.555.2 Mathematica [A] (verified)

Time = 10.05 (sec) , antiderivative size = 365, normalized size of antiderivative = 1.21

$$\int \frac{\sec^4(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \frac{2\sqrt{\sec(c+dx)}\left(-\frac{\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)}\left(2\left(8a^3+8a^2b+9ab^2+9b^3\right)E\left(\arcsin\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{a-b}{a+b}\right)\sqrt{\frac{1}{1+\sec(c+dx)}}\sqrt{\frac{a+b\sec(c+dx}{(a+b)(1+\sec(c+}}}{}\right.\right.}{}$$

input
```
Integrate[Sec[c + d*x]^4/Sqrt[a + b*Sec[c + d*x]],x]
```

output
```
(2*Sqrt[Sec[c + d*x]]*(-((Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(2*(8*a^3
+ 8*a^2*b + 9*a*b^2 + 9*b^3)*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(
a + b)]*Sqrt[(1 + Sec[c + d*x])^(-1)]*Sqrt[(a + b*Sec[c + d*x])/((a + b)*(
1 + Sec[c + d*x]))] - 2*b*(8*a^2 + 2*a*b + 9*b^2)*EllipticF[ArcSin[Tan[(c
+ d*x)/2]], (a - b)/(a + b)]*Sqrt[(1 + Sec[c + d*x])^(-1)]*Sqrt[(a + b*Sec
[c + d*x])/((a + b)*(1 + Sec[c + d*x]))] + (8*a^2 + 9*b^2)*Cos[c + d*x]*(b
 + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/Sqrt[Sec[(c + d*x
)/2]^2]) + (b + a*Cos[c + d*x])*Sqrt[Sec[c + d*x]]*((8*a^2 + 9*b^2)*Sin[c
+ d*x] + b*(-4*a + 3*b*Sec[c + d*x])*Tan[c + d*x])))/(15*b^3*d*Sqrt[a + b*
Sec[c + d*x]])
```

3.555.3 Rubi [A] (verified)

Time = 1.07 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.04, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {3042, 4347, 3042, 4570, 27, 3042, 4493, 3042, 4319, 4492}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^4(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^4}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4347}$$

$$\frac{\int \frac{\sec(c+dx)\left(-4a\sec^2(c+dx)+3b\sec(c+dx)+2a\right)}{\sqrt{a+b\sec(c+dx)}}dx}{5b}+\frac{2\tan(c+dx)\sec(c+dx)\sqrt{a+b\sec(c+dx)}}{5bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(-4a\csc\left(c+dx+\frac{\pi}{2}\right)^2+3b\csc\left(c+dx+\frac{\pi}{2}\right)+2a\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{5b}+\frac{2\tan(c+dx)\sec(c+dx)\sqrt{a+b\sec(c+dx)}}{5bd}$$

$$\Big\downarrow \text{4570}$$

$$\frac{\frac{2\int \frac{\sec(c+dx)\left(2ab+\left(8a^2+9b^2\right)\sec(c+dx)\right)}{2\sqrt{a+b\sec(c+dx)}}dx}{3b}-\frac{8a\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{3bd}}{5b}+\frac{2\tan(c+dx)\sec(c+dx)\sqrt{a+b\sec(c+dx)}}{5bd}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int \frac{\sec(c+dx)\left(2ab+\left(8a^2+9b^2\right)\sec(c+dx)\right)}{\sqrt{a+b\sec(c+dx)}}dx}{3b}-\frac{8a\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{3bd}}{5b}+\frac{2\tan(c+dx)\sec(c+dx)\sqrt{a+b\sec(c+dx)}}{5bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(2ab+\left(8a^2+9b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{3b}-\frac{8a\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{3bd}}{5b}+\frac{2\tan(c+dx)\sec(c+dx)\sqrt{a+b\sec(c+dx)}}{5bd}$$

$$\Big\downarrow \text{4493}$$

$$\frac{\frac{\left(8a^2+9b^2\right)\int \frac{\sec(c+dx)(\sec(c+dx)+1)}{\sqrt{a+b\sec(c+dx)}}dx-\left(8a^2-2ab+9b^2\right)\int \frac{\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx}{3b}-\frac{8a\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{3bd}}{5b}+\frac{2\tan(c+dx)\sec(c+dx)\sqrt{a+b\sec(c+dx)}}{5bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\left(8a^2+9b^2\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-\left(8a^2-2ab+9b^2\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{3b}-\frac{8a\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{3bd}}{5b}+\frac{2\tan(c+dx)\sec(c+dx)\sqrt{a+b\sec(c+dx)}}{5bd}$$

3.555. $\int \frac{\sec^4(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx$

$$\Big\downarrow \; 4319$$

$$\frac{\frac{\frac{(8a^2+9b^2)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2\sqrt{a+b}\left(8a^2-2ab+9b^2\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right.\right.}{bd}}{3b}}{5b}}{\frac{2\tan(c+dx)\sec(c+dx)\sqrt{a+b\sec(c+dx)}}{5bd}}$$

$$\Big\downarrow \; 4492$$

$$\frac{\frac{-\frac{\frac{2\sqrt{a+b}\left(8a^2-2ab+9b^2\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{bd}-\frac{2(a-b)\sqrt{a+b}\left(8a^2+9b^2\right)\cot(c+dx)\sqrt{\frac{b(1-}{}}}{}}{3b}}{5b}}{\frac{2\tan(c+dx)\sec(c+dx)\sqrt{a+b\sec(c+dx)}}{5bd}}$$

input
```
Int[Sec[c + d*x]^4/Sqrt[a + b*Sec[c + d*x]],x]
```

output
```
(2*Sec[c + d*x]*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(5*b*d) + (((-2*(a
- b)*Sqrt[a + b]*(8*a^2 + 9*b^2)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*
Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(
a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(b^2*d) - (2*Sqrt[a + b]*
(8*a^2 - 2*a*b + 9*b^2)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d
*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*S
qrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(b*d))/(3*b) - (8*a*Sqrt[a + b*Sec
[c + d*x]]*Tan[c + d*x])/(3*b*d))/(5*b)
```

3.555.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4319
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4347
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[-2*d^2*Cos[e + f*x]*(d*Csc[e + f*x])^(n - 2)*(S
qrt[a + b*Csc[e + f*x]]/(b*f*(2*n - 3))), x] + Simp[d^3/(b*(2*n - 3))   Int
[((d*Csc[e + f*x])^(n - 3)/Sqrt[a + b*Csc[e + f*x]])*Simp[2*a*(n - 3) + b*(
2*n - 5)*Csc[e + f*x] - 2*a*(n - 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 2] && IntegerQ[2*n]
```

rule 4492
```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

rule 4493
```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(A - B)   Int[Csc[e
 + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Simp[B   Int[Csc[e + f*x]*((1 +
Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A, B}
, x] && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

rule 4570
```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> Simp[(-C)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2)
)), x] + Simp[1/(b*(m + 2))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[
b*A*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /;
 FreeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

3.555. $\int \frac{\sec^4(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$

3.555.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1982 vs. $2(271) = 542$.

Time = 13.36 (sec) , antiderivative size = 1983, normalized size of antiderivative = 6.59

method	result	size
default	Expression too large to display	1983

input
```
int(sec(d*x+c)^4/(a+b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/15/d/b^3*(a+b*sec(d*x+c))^(1/2)/(b+a*cos(d*x+c))/(cos(d*x+c)+1)*(-a*b^2*
sin(d*x+c)+4*a^2*b*sin(d*x+c)+8*EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+
b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(co
s(d*x+c)+1))^(1/2)*a^2*b+9*EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(
1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x
+c)+1))^(1/2)*a*b^2+8*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos
(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b)
)^(1/2))*a^3*cos(d*x+c)^2+9*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2
)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)
/(a+b))^(1/2))*b^3*cos(d*x+c)^2-8*EllipticF(cot(d*x+c)-csc(d*x+c),((a-b)/(
a+b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(
cos(d*x+c)+1))^(1/2)*a^2*b-2*EllipticF(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))
^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d
*x+c)+1))^(1/2)*a*b^2-18*EllipticF(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/
2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c
)+1))^(1/2)*b^3*cos(d*x+c)+16*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1
/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(cot(d*x+c)-csc(d*x+c),((a-
b)/(a+b))^(1/2))*a^3*cos(d*x+c)-9*EllipticF(cot(d*x+c)-csc(d*x+c),((a-b)/(
a+b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(
cos(d*x+c)+1))^(1/2)*b^3*cos(d*x+c)^2+18*(1/(a+b)*(b+a*cos(d*x+c))/(cos...
```

3.555.5 Fricas [F]

$$\int \frac{\sec^4(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\sec{(dx+c)}^4}{\sqrt{b\sec{(dx+c)}+a}}\,dx$$

input
```
integrate(sec(d*x+c)^4/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
integral(sec(d*x + c)^4/sqrt(b*sec(d*x + c) + a), x)
```

3.555.6 Sympy [F]

$$\int \frac{\sec^4(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\sec^4(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$$

input
```
integrate(sec(d*x+c)**4/(a+b*sec(d*x+c))**(1/2),x)
```

output
```
Integral(sec(c + d*x)**4/sqrt(a + b*sec(c + d*x)), x)
```

3.555.7 Maxima [F]

$$\int \frac{\sec^4(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\sec(dx+c)^4}{\sqrt{b\sec(dx+c)+a}}\,dx$$

input
```
integrate(sec(d*x+c)^4/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)^4/sqrt(b*sec(d*x + c) + a), x)
```

3.555.8 Giac [F]

$$\int \frac{\sec^4(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\sec(dx+c)^4}{\sqrt{b\sec(dx+c)+a}}\,dx$$

input
```
integrate(sec(d*x+c)^4/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^4/sqrt(b*sec(d*x + c) + a), x)
```

3.555.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^4(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{1}{\cos\left(c+d\,x\right)^4\,\sqrt{a+\frac{b}{\cos\left(c+d\,x\right)}}}\,dx$$

input
```
int(1/(cos(c + d*x)^4*(a + b/cos(c + d*x))^(1/2)),x)
```

output
```
int(1/(cos(c + d*x)^4*(a + b/cos(c + d*x))^(1/2)), x)
```

3.556 $\int \frac{\sec^3(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$

3.556.1 Optimal result 4197
3.556.2 Mathematica [A] (warning: unable to verify) 4198
3.556.3 Rubi [A] (verified) 4198
3.556.4 Maple [B] (verified) 4201
3.556.5 Fricas [F] 4202
3.556.6 Sympy [F] 4202
3.556.7 Maxima [F] 4202
3.556.8 Giac [F] 4203
3.556.9 Mupad [F(-1)] 4203

3.556.1 Optimal result

Integrand size = 23, antiderivative size = 244

$$\int \frac{\sec^3(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \frac{4a(a-b)\sqrt{a+b}\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{3b^3d} + \frac{2\sqrt{a+b}(2a+b)\cot(c+dx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{3b^2d} + \frac{2\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{3bd}$$

output
```
4/3*a*(a-b)*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)
/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c
))/(a-b))^(1/2)/b^3/d+2/3*(2*a+b)*cot(d*x+c)*EllipticF((a+b*sec(d*x+c))^(1
/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^
(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/b^2/d+2/3*(a+b*sec(d*x+c))^(1/2)*tan
(d*x+c)/b/d
```

3.556.2 Mathematica [A] (warning: unable to verify)

Time = 11.51 (sec) , antiderivative size = 341, normalized size of antiderivative = 1.40

$$\int \frac{\sec^3(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$$
$$= \frac{4\sqrt{\sec(c+dx)}\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)}\left(2a(a+b)\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\sqrt{\frac{b+a\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}E\left(\arcsin\left(\tan\left(\frac{1}{2}(\right.\right.\right.}{}$$
$$+\frac{(b+a\cos(c+dx))\sec(c+dx)\left(-\frac{4a\sin(c+dx)}{3b^2}+\frac{2\tan(c+dx)}{3b}\right)}{d\sqrt{a+b\sec(c+dx)}}$$

input
```
Integrate[Sec[c + d*x]^3/Sqrt[a + b*Sec[c + d*x]],x]
```

output
```
(4*Sqrt[Sec[c + d*x]]*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(2*a*(a + b)*S
qrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1
 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] -
(2*a - b)*b*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x]
)/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b
)/(a + b)] + a*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c
 + d*x)/2]))/(3*b^2*d*Sqrt[Sec[(c + d*x)/2]^2]*Sqrt[a + b*Sec[c + d*x]]) +
 ((b + a*Cos[c + d*x])*Sec[c + d*x]*((-4*a*Sin[c + d*x])/(3*b^2) + (2*Tan[
c + d*x])/(3*b)))/(d*Sqrt[a + b*Sec[c + d*x]])
```

3.556.3 Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.02, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {3042, 4327, 27, 3042, 4493, 3042, 4319, 4492}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^3(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^3}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \; 4327$$

$$\frac{2\int \frac{\sec(c+dx)(b-2a\sec(c+dx))}{2\sqrt{a+b\sec(c+dx)}}dx}{3b}+\frac{2\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{3bd}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{\sec(c+dx)(b-2a\sec(c+dx))}{\sqrt{a+b\sec(c+dx)}}dx}{3b}+\frac{2\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{3bd}$$

$$\Big\downarrow \; 3042$$

$$\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(b-2a\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{3b}+\frac{2\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{3bd}$$

$$\Big\downarrow \; 4493$$

$$\frac{(2a+b)\int \frac{\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx-2a\int \frac{\sec(c+dx)(\sec(c+dx)+1)}{\sqrt{a+b\sec(c+dx)}}dx}{3b}+\frac{2\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{3bd}$$

$$\Big\downarrow \; 3042$$

$$\frac{(2a+b)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-2a\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{3b}+\frac{2\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{3bd}$$

$$\Big\downarrow \; 4319$$

$$\frac{\frac{2\sqrt{a+b}(2a+b)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{bd}-2a\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+\right.}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}}{3b}+\frac{2\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{3bd}$$

$$\Big\downarrow \; 4492$$

$$\frac{\frac{4a(a-b)\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,E\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{b^2d}+\frac{2\sqrt{a+b}(2a+b)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-}}{}}{3b}+\frac{2\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{3bd}$$

input
```
Int[Sec[c + d*x]^3/Sqrt[a + b*Sec[c + d*x]],x]
```

3.556. $\int \frac{\sec^3(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx$

output
```
((4*a*(a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c +
 d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]
*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(b^2*d) + (2*Sqrt[a + b]*(2*a +
b)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a
 + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c +
 d*x]))/(a - b))])/(b*d))/(3*b) + (2*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x]
)/(3*b*d)
```

3.556.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4319
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4327
```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(-Cot[e + f*x])*((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2
))), x] + Simp[1/(b*(m + 2))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(b*(
m + 1) - a*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 -
b^2, 0] &&  !LtQ[m, -1]
```

rule 4492
```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

3.556. $\int \frac{\sec^3(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$

rule 4493
```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(A - B)   Int[Csc[e
 + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Simp[B   Int[Csc[e + f*x]*((1 +
Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A, B}
, x] && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

3.556.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1127 vs. $2(218) = 436$.

Time = 12.05 (sec) , antiderivative size = 1128, normalized size of antiderivative = 4.62

method	result	size
default	Expression too large to display	1128

input
```
int(sec(d*x+c)^3/(a+b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3/d/b^2*(a+b*sec(d*x+c))^(1/2)/(b+a*cos(d*x+c))/(cos(d*x+c)+1)*(2*Ellip
ticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/
(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*a^2*cos(d*x+c)^2+2
*EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*(1/(a+b)*(b+a*cos(d*
x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*a*b*cos(d*x+
c)^2-2*EllipticF(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*(1/(a+b)*(b+a*
cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*a*b*co
s(d*x+c)^2+EllipticF(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*(1/(a+b)*(
b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*b^
2*cos(d*x+c)^2+4*EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*(1/(
a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1
/2)*a^2*cos(d*x+c)+4*EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*
(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1)
)^(1/2)*a*b*cos(d*x+c)-4*EllipticF(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/
2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c
)+1))^(1/2)*a*b*cos(d*x+c)+2*EllipticF(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))
^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d
*x+c)+1))^(1/2)*b^2*cos(d*x+c)+2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))
^(1/2)*EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(c
os(d*x+c)+1))^(1/2)*a^2+2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/...
```

3.556.5 Fricas [F]

$$\int \frac{\sec^3(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\sec{(dx+c)}^3}{\sqrt{b\sec{(dx+c)}+a}}\,dx$$

input
```
integrate(sec(d*x+c)^3/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
integral(sec(d*x + c)^3/sqrt(b*sec(d*x + c) + a), x)
```

3.556.6 Sympy [F]

$$\int \frac{\sec^3(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\sec^3{(c+dx)}}{\sqrt{a+b\sec{(c+dx)}}}\,dx$$

input
```
integrate(sec(d*x+c)**3/(a+b*sec(d*x+c))**(1/2),x)
```

output
```
Integral(sec(c + d*x)**3/sqrt(a + b*sec(c + d*x)), x)
```

3.556.7 Maxima [F]

$$\int \frac{\sec^3(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\sec{(dx+c)}^3}{\sqrt{b\sec{(dx+c)}+a}}\,dx$$

input
```
integrate(sec(d*x+c)^3/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)^3/sqrt(b*sec(d*x + c) + a), x)
```

3.556.8 Giac [F]

$$\int \frac{\sec^3(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\sec(dx+c)^3}{\sqrt{b\sec(dx+c)+a}}\,dx$$

input
```
integrate(sec(d*x+c)^3/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^3/sqrt(b*sec(d*x + c) + a), x)
```

3.556.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^3(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{1}{\cos(c+d\,x)^3\,\sqrt{a+\frac{b}{\cos(c+d\,x)}}}\,dx$$

input
```
int(1/(cos(c + d*x)^3*(a + b/cos(c + d*x))^(1/2)),x)
```

output
```
int(1/(cos(c + d*x)^3*(a + b/cos(c + d*x))^(1/2)), x)
```

3.557 $\int \frac{\sec^2(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$

3.557.1 Optimal result 4204
3.557.2 Mathematica [A] (verified) 4205
3.557.3 Rubi [A] (verified) 4205
3.557.4 Maple [B] (verified) 4207
3.557.5 Fricas [F] 4208
3.557.6 Sympy [F] 4209
3.557.7 Maxima [F] 4209
3.557.8 Giac [F] 4209
3.557.9 Mupad [F(-1)] 4210

3.557.1 Optimal result

Integrand size = 23, antiderivative size = 204

$$\int \frac{\sec^2(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = -\frac{2(a-b)\sqrt{a+b}\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{b^2d} - \frac{2\sqrt{a+b}\cot(c+dx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{bd}$$

output
```
-2*(a-b)*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a
-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/
(a-b))^(1/2)/b^2/d-2*cot(d*x+c)*EllipticF((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/
2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+
sec(d*x+c))/(a-b))^(1/2)/b/d
```

3.557.2 Mathematica [A] (verified)

Time = 13.36 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.17

$$\int \frac{\sec^2(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \frac{(1+\cos(c+dx))\left(-2(a+b)\sqrt{\frac{b+a\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}E\left(\arcsin\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{a-b}{a+b}\right)\sec(c+dx)\sqrt{\frac{1}{1+\sec(c+dx)}}\right.}{}$$

input
```
Integrate[Sec[c + d*x]^2/Sqrt[a + b*Sec[c + d*x]],x]
```

output
```
((1 + Cos[c + d*x])*(-2*(a + b)*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Co
s[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sec[c +
 d*x]*Sqrt[(1 + Sec[c + d*x])^(-1)] + 2*b*EllipticF[ArcSin[Tan[(c + d*x)/2
]], (a - b)/(a + b)]*Sec[c + d*x]*Sqrt[(1 + Sec[c + d*x])^(-1)]*Sqrt[(a +
b*Sec[c + d*x])/((a + b)*(1 + Sec[c + d*x]))] - (b + a*Cos[c + d*x])*Sec[(
c + d*x)/2]^2*Tan[(c + d*x)/2]) + 2*(b + a*Cos[c + d*x])*Tan[c + d*x])/(b*
d*Sqrt[a + b*Sec[c + d*x]])
```

3.557.3 Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 4324, 3042, 4319, 4492}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \; 4324$$

$$\int \frac{\sec(c+dx)(\sec(c+dx)+1)}{\sqrt{a+b\sec(c+dx)}}dx - \int \frac{\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4319}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{bd}$$

$$\Big\downarrow \text{4492}$$

$$-\frac{2(a-b)\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}E\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{b^2d}-\frac{2\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{bd}$$

input
```
Int[Sec[c + d*x]^2/Sqrt[a + b*Sec[c + d*x]],x]
```

output
```
(-2*(a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d
*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*S
qrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(b^2*d) - (2*Sqrt[a + b]*Cot[c + d
*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b
)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a -
 b))])/(b*d)
```

3.557.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4319
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4324
```
Int[csc[(e_.) + (f_.)*(x_)]^2/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x
_Symbol] :> -Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x] + Int[Csc[e + f*
x]*((1 + Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, e, f},
 x] && NeQ[a^2 - b^2, 0]
```

rule 4492
```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

3.557.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 814 vs. $2(186) = 372$.

Time = 8.61 (sec) , antiderivative size = 815, normalized size of antiderivative = 4.00

method	result	size
default	Expression too large to display	815

input
```
int(sec(d*x+c)^2/(a+b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/d/b*(EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(c
os(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a*cos(
d*x+c)^2+EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/
(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*b*co
s(d*x+c)^2-EllipticF(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c
)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*b*
cos(d*x+c)^2+2*EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d
*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2
)*a*cos(d*x+c)+2*EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*(cos
(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1
/2)*b*cos(d*x+c)-2*EllipticF(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*(c
os(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^
(1/2)*b*cos(d*x+c)+EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*(c
os(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^
(1/2)*a+EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(
cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*b-Ell
ipticF(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+
1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*b+a*sin(d*x+c)*c
os(d*x+c)+sin(d*x+c)*b)*(a+b*sec(d*x+c))^(1/2)/(b+a*cos(d*x+c))/(cos(d*x+c
)+1)
```

3.557.5 Fricas [F]

$$\int \frac{\sec^2(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\sec{(dx+c)}^2}{\sqrt{b\sec{(dx+c)}+a}}\,dx$$

input
```
integrate(sec(d*x+c)^2/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
integral(sec(d*x + c)^2/sqrt(b*sec(d*x + c) + a), x)
```

3.557.6 Sympy [F]

$$\int \frac{\sec^2(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\sec^2(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$$

input
```
integrate(sec(d*x+c)**2/(a+b*sec(d*x+c))**(1/2),x)
```

output
```
Integral(sec(c + d*x)**2/sqrt(a + b*sec(c + d*x)), x)
```

3.557.7 Maxima [F]

$$\int \frac{\sec^2(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\sec(dx+c)^2}{\sqrt{b\sec(dx+c)+a}}\,dx$$

input
```
integrate(sec(d*x+c)^2/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)^2/sqrt(b*sec(d*x + c) + a), x)
```

3.557.8 Giac [F]

$$\int \frac{\sec^2(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\sec(dx+c)^2}{\sqrt{b\sec(dx+c)+a}}\,dx$$

input
```
integrate(sec(d*x+c)^2/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^2/sqrt(b*sec(d*x + c) + a), x)
```

3.557.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^2(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{1}{\cos\left(c+d\,x\right)^2\,\sqrt{a+\frac{b}{\cos\left(c+d\,x\right)}}}\,dx$$

input
```
int(1/(cos(c + d*x)^2*(a + b/cos(c + d*x))^(1/2)),x)
```

output
```
int(1/(cos(c + d*x)^2*(a + b/cos(c + d*x))^(1/2)), x)
```

3.558 $\int \frac{\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$

3.558.1 Optimal result 4211
3.558.2 Mathematica [A] (verified) 4211
3.558.3 Rubi [A] (verified) 4212
3.558.4 Maple [A] (verified) 4213
3.558.5 Fricas [F] 4213
3.558.6 Sympy [F] 4213
3.558.7 Maxima [F] 4214
3.558.8 Giac [F] 4214
3.558.9 Mupad [F(-1)] 4214

3.558.1 Optimal result

Integrand size = 21, antiderivative size = 99

$$\int \frac{\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \frac{2\sqrt{a+b}\cot(c+dx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{bd}$$

output
```
2*cot(d*x+c)*EllipticF((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1
/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^
(1/2)/b/d
```

3.558.2 Mathematica [A] (verified)

Time = 1.95 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.94

$$\int \frac{\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \frac{2\sqrt{\frac{b+a\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}\,\text{EllipticF}\left(\arcsin\left(\tan\left(\frac{1}{2}(c+dx)\right)\right),\frac{a-b}{a+b}\right)}{d\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\sqrt{a+b\sec(c+dx)}}$$

input
```
Integrate[Sec[c + d*x]/Sqrt[a + b*Sec[c + d*x]],x]
```

output
```
(2*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSi
n[Tan[(c + d*x)/2]], (a - b)/(a + b)])/(d*Sqrt[Cos[c + d*x]/(1 + Cos[c + d
*x])]*Sqrt[a + b*Sec[c + d*x]])
```

3.558.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3042, 4319}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{ 4319}$$

$$\frac{2\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{bd}$$

input
```
Int[Sec[c + d*x]/Sqrt[a + b*Sec[c + d*x]],x]
```

output
```
(2*Sqrt[a + b]*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt
[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*
(1 + Sec[c + d*x]))/(a - b))])/(b*d)
```

3.558.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4319
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

3.558.4 Maple [A] (verified)

Time = 7.24 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.15

method	result	size
default	$-\frac{2(\cos(dx+c)+1)\sqrt{\frac{b+a\cos(dx+c)}{(a+b)(\cos(dx+c)+1)}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\mathrm{EllipticF}\left(\cot(dx+c)-\csc(dx+c),\sqrt{\frac{a-b}{a+b}}\right)\sqrt{a+b\sec(dx+c)}}{d(b+a\cos(dx+c))}$	114

input
```
int(sec(d*x+c)/(a+b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/d*(cos(d*x+c)+1)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d
*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^
(1/2))*(a+b*sec(d*x+c))^(1/2)/(b+a*cos(d*x+c))
```

3.558.5 Fricas [F]

$$\int \frac{\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\sec(dx+c)}{\sqrt{b\sec(dx+c)+a}}\,dx$$

input
```
integrate(sec(d*x+c)/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
integral(sec(d*x + c)/sqrt(b*sec(d*x + c) + a), x)
```

3.558.6 Sympy [F]

$$\int \frac{\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$$

input
```
integrate(sec(d*x+c)/(a+b*sec(d*x+c))**(1/2),x)
```

output
```
Integral(sec(c + d*x)/sqrt(a + b*sec(c + d*x)), x)
```

3.558.7 Maxima [F]

$$\int \frac{\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\sec(dx+c)}{\sqrt{b\sec(dx+c)+a}}\,dx$$

input
```
integrate(sec(d*x+c)/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)/sqrt(b*sec(d*x + c) + a), x)
```

3.558.8 Giac [F]

$$\int \frac{\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\sec(dx+c)}{\sqrt{b\sec(dx+c)+a}}\,dx$$

input
```
integrate(sec(d*x+c)/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)/sqrt(b*sec(d*x + c) + a), x)
```

3.558.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{1}{\cos(c+d\,x)\,\sqrt{a+\frac{b}{\cos(c+d\,x)}}}\,dx$$

input
```
int(1/(cos(c + d*x)*(a + b/cos(c + d*x))^(1/2)),x)
```

output
```
int(1/(cos(c + d*x)*(a + b/cos(c + d*x))^(1/2)), x)
```

3.559 $\int \frac{1}{\sqrt{a+b\sec(c+dx)}}\,dx$

3.559.1 Optimal result 4215
3.559.2 Mathematica [A] (verified) 4215
3.559.3 Rubi [A] (verified) 4216
3.559.4 Maple [A] (verified) 4217
3.559.5 Fricas [F] 4217
3.559.6 Sympy [F] 4218
3.559.7 Maxima [F] 4218
3.559.8 Giac [F] 4218
3.559.9 Mupad [F(-1)] 4219

3.559.1 Optimal result

Integrand size = 14, antiderivative size = 106

$$\int \frac{1}{\sqrt{a+b\sec(c+dx)}}\,dx = -\frac{2\sqrt{a+b}\cot(c+dx)\,\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{ad}$$

output
```
-2*cot(d*x+c)*EllipticPi((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),(a+b)/a,((a+b)
/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c
))/(a-b))^(1/2)/a/d
```

3.559.2 Mathematica [A] (verified)

Time = 2.11 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.30

$$\int \frac{1}{\sqrt{a+b\sec(c+dx)}}\,dx = -\frac{4\cos^2\left(\frac{1}{2}(c+dx)\right)\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\sqrt{\frac{b+a\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}\left(\text{EllipticF}\left(\arcsin\left(\tan\left(\frac{1}{2}(c+dx)\right)\right),\frac{a-b}{a+b}\right)-2\,\text{Ellipti}\right.}{d\sqrt{a+b\sec(c+dx)}}$$

input
```
Integrate[1/Sqrt[a + b*Sec[c + d*x]],x]
```

output
```
(-4*Cos[(c + d*x)/2]^2*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*C
os[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*(EllipticF[ArcSin[Tan[(c + d*x)
/2]], (a - b)/(a + b)] - 2*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (a - b
)/(a + b)])*Sec[c + d*x])/(d*Sqrt[a + b*Sec[c + d*x]])
```

3.559.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3042, 4271}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+b\sec(c+dx)}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{ 4271}$$

$$-\frac{2\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{ad}$$

input
```
Int[1/Sqrt[a + b*Sec[c + d*x]],x]
```

output
```
(-2*Sqrt[a + b]*Cot[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c
 + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b
)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(a*d)
```

3.559.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4271
```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*(Rt[a
 + b, 2]/(a*d*Cot[c + d*x]))*Sqrt[b*((1 - Csc[c + d*x])/(a + b))]*Sqrt[(-b)
*((1 + Csc[c + d*x])/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[
c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

3.559.4 Maple [A] (verified)

Time = 4.92 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.39

method	result
default	$\frac{2(\cos(dx+c)+1)\left(\operatorname{EllipticF}\left(\cot(dx+c)-\csc(dx+c),\sqrt{\frac{a-b}{a+b}}\right)-2\operatorname{EllipticPi}\left(\cot(dx+c)-\csc(dx+c),-1,\sqrt{\frac{a-b}{a+b}}\right)\right)\sqrt{\frac{b+a\cos(dx+c)}{(a+b)(\cos(dx+c)+1)}}}{d(b+a\cos(dx+c))}$

input
```
int(1/(a+b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/d*(cos(d*x+c)+1)*(EllipticF(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))-2
*EllipticPi(cot(d*x+c)-csc(d*x+c),-1,((a-b)/(a+b))^(1/2)))*(1/(a+b)*(b+a*c
os(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(a+b*se
c(d*x+c))^(1/2)/(b+a*cos(d*x+c))
```

3.559.5 Fricas [F]

$$\int \frac{1}{\sqrt{a+b\sec(c+dx)}}\,dx=\int \frac{1}{\sqrt{b\sec{(dx+c)}+a}}\,dx$$

input
```
integrate(1/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
integral(1/sqrt(b*sec(d*x + c) + a), x)
```

3.559.6 Sympy [F]

$$\int \frac{1}{\sqrt{a+b\sec(c+dx)}}\,dx=\int \frac{1}{\sqrt{a+b\sec{(c+dx)}}}\,dx$$

input
```
integrate(1/(a+b*sec(d*x+c))**(1/2),x)
```

output
```
Integral(1/sqrt(a + b*sec(c + d*x)), x)
```

3.559.7 Maxima [F]

$$\int \frac{1}{\sqrt{a+b\sec(c+dx)}}\,dx=\int \frac{1}{\sqrt{b\sec{(dx+c)}+a}}\,dx$$

input
```
integrate(1/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(b*sec(d*x + c) + a), x)
```

3.559.8 Giac [F]

$$\int \frac{1}{\sqrt{a+b\sec(c+dx)}}\,dx=\int \frac{1}{\sqrt{b\sec{(dx+c)}+a}}\,dx$$

input
```
integrate(1/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(b*sec(d*x + c) + a), x)
```

3.559.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+b\sec(c+dx)}}\,dx=\int \frac{1}{\sqrt{a+\frac{b}{\cos(c+d\,x)}}}\,dx$$

input
```
int(1/(a + b/cos(c + d*x))^(1/2),x)
```

output
```
int(1/(a + b/cos(c + d*x))^(1/2), x)
```

3.560 $\int \frac{\cos(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$

3.560.1 Optimal result 4220
3.560.2 Mathematica [A] (warning: unable to verify) 4221
3.560.3 Rubi [A] (verified) 4221
3.560.4 Maple [B] (verified) 4224
3.560.5 Fricas [F] 4225
3.560.6 Sympy [F] 4226
3.560.7 Maxima [F] 4226
3.560.8 Giac [F] 4226
3.560.9 Mupad [F(-1)] 4227

3.560.1 Optimal result

Integrand size = 21, antiderivative size = 338

$$\begin{aligned}
&\int \frac{\cos(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx \\
&= \frac{(a-b)\sqrt{a+b}\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{abd} \\
&\quad+\frac{\sqrt{a+b}\cot(c+dx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{ad} \\
&\quad+\frac{b\sqrt{a+b}\cot(c+dx)\,\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{a^2d} \\
&\quad+\frac{\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{ad}
\end{aligned}$$

output
```
(a-b)*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b)
)^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-
b))^(1/2)/a/b/d+cot(d*x+c)*EllipticF((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((
a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d
*x+c))/(a-b))^(1/2)/a/d+b*cot(d*x+c)*EllipticPi((a+b*sec(d*x+c))^(1/2)/(a+
b)^(1/2),(a+b)/a,((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))
^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/a^2/d+sin(d*x+c)*(a+b*sec(d*x+c))^(
1/2)/a/d
```

3.560.2 Mathematica [A] (warning: unable to verify)

Time = 5.89 (sec) , antiderivative size = 239, normalized size of antiderivative = 0.71

$$\int \frac{\cos(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$$
$$= \frac{2\sqrt{\frac{\cos(c+dx)}{(1+\cos(c+dx))^2}}\left(\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\right)^{3/2}\left((a+b)E\left(\arcsin\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{a-b}{a+b}\right)\sqrt{\frac{(b+a\cos(c+dx}{a}}\right.}{}$$

input
```
Integrate[Cos[c + d*x]/Sqrt[a + b*Sec[c + d*x]],x]
```

output
```
(2*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])^2]*(Cos[(c + d*x)/2]^2*Sec[c + d*x
])^(3/2)*((a + b)*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqr
t[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] - 2*b*EllipticPi[-1,
ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[((b + a*Cos[c + d*x])*Sec[
(c + d*x)/2]^2)/(a + b)] + (b + a*Cos[c + d*x])*Sqrt[Cos[c + d*x]*Sec[(c +
 d*x)/2]^2]*Tan[(c + d*x)/2]))/(a*d*Sqrt[Sec[c + d*x]]*Sqrt[a + b*Sec[c +
d*x]])
```

3.560.3 Rubi [A] (verified)

Time = 1.12 (sec) , antiderivative size = 346, normalized size of antiderivative = 1.02, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {3042, 4348, 3042, 4547, 3042, 4409, 3042, 4271, 4319, 4492}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$
$$\Big\downarrow \; 4348$$
$$\frac{\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{ad}-\frac{b\int\frac{\sec^2(c+dx)+1}{\sqrt{a+b\sec(c+dx)}}dx}{2a}$$

$$\Big\downarrow \; 3042$$

$$\frac{\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{ad}-\frac{b\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{2a}$$

$$\Big\downarrow \; 4547$$

$$\frac{\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{ad}-\frac{b\left(\int \frac{1-\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx+\int \frac{\sec(c+dx)(\sec(c+dx)+1)}{\sqrt{a+b\sec(c+dx)}}dx\right)}{2a}$$

$$\Big\downarrow \; 3042$$

$$\frac{\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{ad}-\frac{b\left(\int \frac{1-\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)}{2a}$$

$$\Big\downarrow \; 4409$$

$$\frac{\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{ad}-\frac{b\left(\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\int \frac{1}{\sqrt{a+b\sec(c+dx)}}dx-\int \frac{\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx\right)}{2a}$$

$$\Big\downarrow \; 3042$$

$$\frac{\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{ad}-\frac{b\left(\int \frac{1}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)}{2a}$$

$$\Big\downarrow \; 4271$$

$$\frac{\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{ad}-\frac{b\left(-\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,\text{EllipticP}}{ad}\right.}{2a}$$

$$\Big\downarrow \; 4319$$

$$\frac{\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{ad}-\frac{b\left(\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{bd}\right.}{2a}$$

3.560. $\int \frac{\cos(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$

$$\Big\downarrow \text{4492}$$

$$\frac{\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{ad} - \frac{b\left(-\frac{2(a-b)\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}E\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{b^2d} - \frac{2\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\cdots}}{\cdots}\right)}{\cdots}$$

input
```
Int[Cos[c + d*x]/Sqrt[a + b*Sec[c + d*x]],x]
```

output
```
-1/2*(b*((-2*(a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*
Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(
a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(b^2*d) - (2*Sqrt[a + b]*
Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a +
b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*
x]))/(a - b))])/(b*d) - (2*Sqrt[a + b]*Cot[c + d*x]*EllipticPi[(a + b)/a,
ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1
- Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(a*d)))
/a + (Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(a*d)
```

3.560.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4271
```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*(Rt[a
 + b, 2]/(a*d*Cot[c + d*x]))*Sqrt[b*((1 - Csc[c + d*x])/(a + b))]*Sqrt[(-b)
*((1 + Csc[c + d*x])/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[
c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

rule 4319
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4348
```
Int[1/(csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]),
 x_Symbol] :> Simp[(-Cos[e + f*x])*(Sqrt[a + b*Csc[e + f*x]]/(a*f)), x] - S
imp[b/(2*a)   Int[(1 + Csc[e + f*x]^2)/Sqrt[a + b*Csc[e + f*x]], x], x] /;
FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4409
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Simp[c   Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] +
Simp[d   Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

rule 4492
```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

rule 4547
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))/Sqrt[csc[(e_.) + (f_.)*(x_)]*
(b_.) + (a_)], x_Symbol] :> Int[(A - C*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x
]], x] + Simp[C   Int[Csc[e + f*x]*((1 + Csc[e + f*x])/Sqrt[a + b*Csc[e + f
*x]]), x], x] /; FreeQ[{a, b, e, f, A, C}, x] && NeQ[a^2 - b^2, 0]
```

3.560.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 828 vs. $2(309) = 618$.

Time = 7.38 (sec) , antiderivative size = 829, normalized size of antiderivative = 2.45

method	result	size
default	Expression too large to display	829

input
```
int(cos(d*x+c)/(a+b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/d/a*(2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(
d*x+c)+1))^(1/2)*EllipticPi(cot(d*x+c)-csc(d*x+c),-1,((a-b)/(a+b))^(1/2))*
b*cos(d*x+c)^2-EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d
*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2
)*a*cos(d*x+c)^2-EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*(cos
(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1
/2)*b*cos(d*x+c)^2+4*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(
d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticPi(cot(d*x+c)-csc(d*x+c),-1,((a-b)/(a
+b))^(1/2))*b*cos(d*x+c)-2*EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(
1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x
+c)+1))^(1/2)*a*cos(d*x+c)-2*EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))
^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d
*x+c)+1))^(1/2)*b*cos(d*x+c)+2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(
1/2)*EllipticPi(cot(d*x+c)-csc(d*x+c),-1,((a-b)/(a+b))^(1/2))*(cos(d*x+c)/
(cos(d*x+c)+1))^(1/2)*b-EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2
))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)
+1))^(1/2)*a-EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x
+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*
b+cos(d*x+c)^2*sin(d*x+c)*a+cos(d*x+c)*sin(d*x+c)*b)*(a+b*sec(d*x+c))^(1/2
)/(b+a*cos(d*x+c))/(cos(d*x+c)+1)
```

3.560.5 Fricas [F]

$$\int \frac{\cos(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\cos\left(dx+c\right)}{\sqrt{b\sec\left(dx+c\right)+a}}\,dx$$

input
```
integrate(cos(d*x+c)/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
integral(cos(d*x + c)/sqrt(b*sec(d*x + c) + a), x)
```

3.560.6 Sympy [F]

$$\int \frac{\cos(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\cos{(c+dx)}}{\sqrt{a+b\sec{(c+dx)}}}\,dx$$

input
```
integrate(cos(d*x+c)/(a+b*sec(d*x+c))**(1/2),x)
```

output
```
Integral(cos(c + d*x)/sqrt(a + b*sec(c + d*x)), x)
```

3.560.7 Maxima [F]

$$\int \frac{\cos(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\cos{(dx+c)}}{\sqrt{b\sec{(dx+c)}+a}}\,dx$$

input
```
integrate(cos(d*x+c)/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(cos(d*x + c)/sqrt(b*sec(d*x + c) + a), x)
```

3.560.8 Giac [F]

$$\int \frac{\cos(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\cos{(dx+c)}}{\sqrt{b\sec{(dx+c)}+a}}\,dx$$

input
```
integrate(cos(d*x+c)/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(cos(d*x + c)/sqrt(b*sec(d*x + c) + a), x)
```

3.560.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\cos\left(c+d\,x\right)}{\sqrt{a+\frac{b}{\cos(c+d\,x)}}}\,dx$$

input
```
int(cos(c + d*x)/(a + b/cos(c + d*x))^(1/2),x)
```

output
```
int(cos(c + d*x)/(a + b/cos(c + d*x))^(1/2), x)
```

3.561 $\int \frac{\cos^2(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$

3.561.1 Optimal result 4228
3.561.2 Mathematica [A] (verified) 4229
3.561.3 Rubi [A] (verified) 4229
3.561.4 Maple [B] (verified) 4234
3.561.5 Fricas [F] 4234
3.561.6 Sympy [F] 4235
3.561.7 Maxima [F] 4235
3.561.8 Giac [F] 4235
3.561.9 Mupad [F(-1)] 4236

3.561.1 Optimal result

Integrand size = 23, antiderivative size = 401

$$\int \frac{\cos^2(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = -\frac{3(a-b)\sqrt{a+b}\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{4a^2d} + \frac{(2a-3b)\sqrt{a+b}\cot(c+dx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{4a^2d} - \frac{\sqrt{a+b}(4a^2+3b^2)\cot(c+dx)\,\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+}{a-b}}}{4a^3d} - \frac{3b\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{4a^2d} + \frac{\cos(c+dx)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{2ad}$$

output
```
-3/4*(a-b)*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/
(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c)
)/(a-b))^(1/2)/a^2/d+1/4*(2*a-3*b)*cot(d*x+c)*EllipticF((a+b*sec(d*x+c))^(
1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))
^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/a^2/d-1/4*(4*a^2+3*b^2)*cot(d*x+c)*
EllipticPi((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),(a+b)/a,((a+b)/(a-b))^(1/2))
*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2
)/a^3/d-3/4*b*sin(d*x+c)*(a+b*sec(d*x+c))^(1/2)/a^2/d+1/2*cos(d*x+c)*sin(d
*x+c)*(a+b*sec(d*x+c))^(1/2)/a/d
```

3.561.2 Mathematica [A] (verified)

Time = 12.73 (sec) , antiderivative size = 682, normalized size of antiderivative = 1.70

$$\int \frac{\cos^2(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \frac{(b+a\cos(c+dx))\sec(c+dx)\sin(2(c+dx))}{4ad\sqrt{a+b\sec(c+dx)}} - \frac{\sec\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\left(24b(a+b)\cos^3\left(\frac{1}{2}(c+dx)\right)\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\sqrt{\frac{b+a\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}E\left(\arcsin\left(\tan\left(\right.\right.\right.\right.}{}$$

input
```
Integrate[Cos[c + d*x]^2/Sqrt[a + b*Sec[c + d*x]],x]
```

output
```
((b + a*Cos[c + d*x])*Sec[c + d*x]*Sin[2*(c + d*x)])/(4*a*d*Sqrt[a + b*Sec
[c + d*x]]) - (Sec[(c + d*x)/2]*Sec[c + d*x]*(24*b*(a + b)*Cos[(c + d*x)/2
]^3*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a +
b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b
)] + 16*a*(2*a - b)*Cos[(c + d*x)/2]^3*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x]
)]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSi
n[Tan[(c + d*x)/2]], (a - b)/(a + b)] - 48*a^2*Cos[(c + d*x)/2]*EllipticPi
[-1, ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[(1 + Sec[c + d*x])^(-
1)]*Sqrt[(a + b*Sec[c + d*x])/((a + b)*(1 + Sec[c + d*x]))] - 36*b^2*Cos[(
c + d*x)/2]*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt
[(1 + Sec[c + d*x])^(-1)]*Sqrt[(a + b*Sec[c + d*x])/((a + b)*(1 + Sec[c +
d*x]))] - 16*a^2*Cos[(3*(c + d*x))/2]*EllipticPi[-1, ArcSin[Tan[(c + d*x)/
2]], (a - b)/(a + b)]*Sqrt[(1 + Sec[c + d*x])^(-1)]*Sqrt[(a + b*Sec[c + d*
x])/((a + b)*(1 + Sec[c + d*x]))] - 12*b^2*Cos[(3*(c + d*x))/2]*EllipticPi
[-1, ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[(1 + Sec[c + d*x])^(-
1)]*Sqrt[(a + b*Sec[c + d*x])/((a + b)*(1 + Sec[c + d*x]))] + 6*a*b*Sin[(c
 + d*x)/2] - 6*b^2*Sin[(c + d*x)/2] - 3*a*b*Sin[(3*(c + d*x))/2] + 6*b^2*S
in[(3*(c + d*x))/2] + 3*a*b*Sin[(5*(c + d*x))/2]))/(16*a^2*d*Sqrt[a + b*Se
c[c + d*x]])
```

3.561.3 Rubi [A] (verified)

Time = 1.61 (sec) , antiderivative size = 403, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.565$, Rules used = {3042, 4350, 3042, 4592, 27, 3042, 4546, 3042, 4409, 3042, 4271, 4319, 4492}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^2(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^2\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \; 4350$$

$$\frac{\sin(c+dx)\cos(c+dx)\sqrt{a+b\sec(c+dx)}}{2ad}-\frac{\int \frac{\cos(c+dx)\left(-b\sec^2(c+dx)-2a\sec(c+dx)+3b\right)}{\sqrt{a+b\sec(c+dx)}}dx}{4a}$$

$$\Big\downarrow \; 3042$$

$$\frac{\sin(c+dx)\cos(c+dx)\sqrt{a+b\sec(c+dx)}}{2ad}-\frac{\int \frac{-b\csc\left(c+dx+\frac{\pi}{2}\right)^2-2a\csc\left(c+dx+\frac{\pi}{2}\right)+3b}{\csc\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{4a}$$

$$\Big\downarrow \; 4592$$

$$\frac{\sin(c+dx)\cos(c+dx)\sqrt{a+b\sec(c+dx)}}{2ad}-\frac{\frac{3b\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{ad}-\frac{\int \frac{4a^2+2b\sec(c+dx)a+3b^2+3b^2\sec^2(c+dx)}{2\sqrt{a+b\sec(c+dx)}}dx}{a}}{4a}$$

$$\Big\downarrow \; 27$$

$$\frac{\sin(c+dx)\cos(c+dx)\sqrt{a+b\sec(c+dx)}}{2ad}-\frac{\frac{3b\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{ad}-\frac{\int \frac{4a^2+2b\sec(c+dx)a+3b^2+3b^2\sec^2(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx}{2a}}{4a}$$

$$\Big\downarrow \; 3042$$

$$\frac{\sin(c+dx)\cos(c+dx)\sqrt{a+b\sec(c+dx)}}{2ad}-\frac{\frac{3b\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{ad}-\frac{\int \frac{4a^2+2b\csc\left(c+dx+\frac{\pi}{2}\right)a+3b^2+3b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{2a}}{4a}$$

$$\Big\downarrow \; 4546$$

$$\frac{\sin(c+dx)\cos(c+dx)\sqrt{a+b\sec(c+dx)}}{2ad}-\frac{\frac{3b\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{ad}-\frac{\int \frac{4a^2+3b^2+\left(2ab-3b^2\right)\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx+3b^2\int \frac{\sec(c+dx)(\sec(c+dx)+1)}{\sqrt{a+b\sec(c+dx)}}dx}{2a}}{4a}$$

3.561. $\int \frac{\cos^2(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$

$$\Big\downarrow \text{3042}$$

$$\frac{\sin(c+dx)\cos(c+dx)\sqrt{a+b\sec(c+dx)}}{2ad}-\frac{\frac{3b\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{ad}-\frac{\int\frac{4a^2+3b^2+\left(2ab-3b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+3b^2\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{2a}}{4a}$$

$$\Big\downarrow \text{4409}$$

$$\frac{\sin(c+dx)\cos(c+dx)\sqrt{a+b\sec(c+dx)}}{2ad}-\frac{\frac{3b\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{ad}-\frac{\left(4a^2+3b^2\right)\int\frac{1}{\sqrt{a+b\sec(c+dx)}}dx+3b^2\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+b(2a-3b)\int\frac{\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx}{2a}}{4a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sin(c+dx)\cos(c+dx)\sqrt{a+b\sec(c+dx)}}{2ad}-\frac{\frac{3b\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{ad}-\frac{\left(4a^2+3b^2\right)\int\frac{1}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+3b^2\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+b(2a-3b)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}d}{2a}}{4a}$$

$$\Big\downarrow \text{4271}$$

$$\frac{\sin(c+dx)\cos(c+dx)\sqrt{a+b\sec(c+dx)}}{2ad}-\frac{\frac{3b\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{ad}-\frac{3b^2\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+b(2a-3b)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2\sqrt{a+b}\left(4a^2+3b^2\right)\cot(c+dx)\sqrt{\frac{b(1-\sec}{}}}{}}{2a}}{4a}$$

$$\Big\downarrow \text{4319}$$

$$\frac{\sin(c+dx)\cos(c+dx)\sqrt{a+b\sec(c+dx)}}{2ad}-\frac{\frac{3b\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{ad}-\frac{3b^2\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2\sqrt{a+b}\left(4a^2+3b^2\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,\text{EllipticPi}}{ad}}{2a}}{4a}$$

$$\Big\downarrow \text{4492}$$

$$\frac{\sin(c+dx)\cos(c+dx)\sqrt{a+b\sec(c+dx)}}{2ad}-\frac{\frac{3b\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{ad}-\frac{-\frac{2\sqrt{a+b}\left(4a^2+3b^2\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{ad}+\frac{2(}{}}{}}{}$$

3.561. $\int \frac{\cos^2(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$

input
```
Int[Cos[c + d*x]^2/Sqrt[a + b*Sec[c + d*x]],x]
```

output
```
(Cos[c + d*x]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(2*a*d) - (-1/2*((-6*
(a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]
/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[
-((b*(1 + Sec[c + d*x]))/(a - b))])/d + (2*(2*a - 3*b)*Sqrt[a + b]*Cot[c +
 d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a -
 b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a
 - b))])/d - (2*Sqrt[a + b]*(4*a^2 + 3*b^2)*Cot[c + d*x]*EllipticPi[(a + b
)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(
b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(a
*d))/a + (3*b*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(a*d))/(4*a)
```

3.561.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4271
```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*(Rt[a
 + b, 2]/(a*d*Cot[c + d*x]))*Sqrt[b*((1 - Csc[c + d*x])/(a + b))]*Sqrt[(-b)
*((1 + Csc[c + d*x])/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[
c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

rule 4319
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4350
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[Cos[e + f*x]*(d*Csc[e + f*x])^(n + 1)*(Sqrt[a +
 b*Csc[e + f*x]]/(a*d*f*n)), x] + Simp[1/(2*a*d*n)   Int[((d*Csc[e + f*x])^
(n + 1)/Sqrt[a + b*Csc[e + f*x]])*Simp[(-b)*(2*n + 1) + 2*a*(n + 1)*Csc[e +
 f*x] + b*(2*n + 3)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x]
 && NeQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

rule 4409
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Simp[c   Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] +
Simp[d   Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

rule 4492
```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

rule 4546
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Simp[C   Int[Csc[e + f*x]*((
1 + Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A
, B, C}, x] && NeQ[a^2 - b^2, 0]
```

rule 4592
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Simp[1/(a*d*n)   Int[(a + b*Csc[e + f*x])^m
*(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*
Csc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

3.561. $\int \frac{\cos^2(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$

3.561.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1674 vs. 2(360) = 720.

Time = 7.24 (sec) , antiderivative size = 1675, normalized size of antiderivative = 4.18

method	result	size
default	Expression too large to display	1675

input
```
int(cos(d*x+c)^2/(a+b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4/d/a^2*(4*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(
cos(d*x+c)+1))^(1/2)*EllipticF(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*
a^2*cos(d*x+c)^2-2*EllipticF(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*(1
/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^
(1/2)*a*b*cos(d*x+c)^2+3*EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/
2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c
)+1))^(1/2)*a*b*cos(d*x+c)^2+3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*
(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE(cot(d*x+c)-csc(d*x+c),((a
-b)/(a+b))^(1/2))*b^2*cos(d*x+c)^2-8*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/
(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi(cot(d*x+c)-csc(d*x
+c),-1,((a-b)/(a+b))^(1/2))*a^2*cos(d*x+c)^2-6*(cos(d*x+c)/(cos(d*x+c)+1))
^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi(cot(d*x+
c)-csc(d*x+c),-1,((a-b)/(a+b))^(1/2))*b^2*cos(d*x+c)^2+8*(cos(d*x+c)/(cos(
d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF
(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*a^2*cos(d*x+c)-4*EllipticF(cot
(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x
+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*a*b*cos(d*x+c)+6*EllipticE
(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos
(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*a*b*cos(d*x+c)+6*(cos(
d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^...
```

3.561.5 Fricas [F]

$$\int \frac{\cos^2(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\cos\left(dx+c\right)^2}{\sqrt{b\sec\left(dx+c\right)+a}}\,dx$$

input
```
integrate(cos(d*x+c)^2/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
integral(cos(d*x + c)^2/sqrt(b*sec(d*x + c) + a), x)
```

3.561.6 Sympy [F]

$$\int \frac{\cos^2(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\cos^2(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$$

input
```
integrate(cos(d*x+c)**2/(a+b*sec(d*x+c))**(1/2),x)
```

output
```
Integral(cos(c + d*x)**2/sqrt(a + b*sec(c + d*x)), x)
```

3.561.7 Maxima [F]

$$\int \frac{\cos^2(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\cos(dx+c)^2}{\sqrt{b\sec(dx+c)+a}}\,dx$$

input
```
integrate(cos(d*x+c)^2/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(cos(d*x + c)^2/sqrt(b*sec(d*x + c) + a), x)
```

3.561.8 Giac [F]

$$\int \frac{\cos^2(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\cos(dx+c)^2}{\sqrt{b\sec(dx+c)+a}}\,dx$$

input
```
integrate(cos(d*x+c)^2/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(cos(d*x + c)^2/sqrt(b*sec(d*x + c) + a), x)
```

3.561.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^2(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\cos\left(c+d\,x\right)^2}{\sqrt{a+\frac{b}{\cos(c+d\,x)}}}\,dx$$

input
```
int(cos(c + d*x)^2/(a + b/cos(c + d*x))^(1/2),x)
```

output
```
int(cos(c + d*x)^2/(a + b/cos(c + d*x))^(1/2), x)
```

3.562 $\int \frac{\sec^5(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$

3.562.1 Optimal result 4237
3.562.2 Mathematica [A] (warning: unable to verify) 4238
3.562.3 Rubi [A] (verified) 4238
3.562.4 Maple [B] (verified) 4243
3.562.5 Fricas [F] 4244
3.562.6 Sympy [F] 4244
3.562.7 Maxima [F(-1)] 4244
3.562.8 Giac [F] 4245
3.562.9 Mupad [F(-1)] 4245

3.562.1 Optimal result

Integrand size = 23, antiderivative size = 399

$$\int \frac{\sec^5(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = -\frac{2(16a^4-8a^2b^2-3b^4)\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{5b^5\sqrt{a+b}d} - \frac{2(4a+3b)(4a^2+b^2)\cot(c+dx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{5b^4\sqrt{a+b}d} - \frac{2a^2\sec^2(c+dx)\tan(c+dx)}{b(a^2-b^2)d\sqrt{a+b\sec(c+dx)}} - \frac{2a(8a^2-3b^2)\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{5b^3(a^2-b^2)d} + \frac{2(6a^2-b^2)\sec(c+dx)\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{5b^2(a^2-b^2)d}$$

output
```
-2/5*(16*a^4-8*a^2*b^2-3*b^4)*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/
(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec
(d*x+c))/(a-b))^(1/2)/b^5/d/(a+b)^(1/2)-2/5*(4*a+3*b)*(4*a^2+b^2)*cot(d*x+
c)*EllipticF((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(b*(1
-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/b^4/d/(a+b)^(1/2
)-2*a^2*sec(d*x+c)^2*tan(d*x+c)/b/(a^2-b^2)/d/(a+b*sec(d*x+c))^(1/2)-2/5*a
*(8*a^2-3*b^2)*(a+b*sec(d*x+c))^(1/2)*tan(d*x+c)/b^3/(a^2-b^2)/d+2/5*(6*a^
2-b^2)*sec(d*x+c)*(a+b*sec(d*x+c))^(1/2)*tan(d*x+c)/b^2/(a^2-b^2)/d
```

3.562.2 Mathematica [A] (warning: unable to verify)

Time = 11.78 (sec) , antiderivative size = 455, normalized size of antiderivative = 1.14

$$\int \frac{\sec^5(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \frac{(b+a\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\left(\frac{2(4a^2+b^2)\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)}\left(2(4a^3+4a^2b-3ab^{\cdots}\right.}{\cdots}\right.}{\cdots}$$

input
```
Integrate[Sec[c + d*x]^5/(a + b*Sec[c + d*x])^(3/2),x]
```

output
```
((b + a*Cos[c + d*x])*Sec[c + d*x]^(3/2)*((2*(4*a^2 + b^2)*Sqrt[Cos[(c + d
*x)/2]^2*Sec[c + d*x]]*(2*(4*a^3 + 4*a^2*b - 3*a*b^2 - 3*b^3)*EllipticE[Ar
cSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[(1 + Sec[c + d*x])^(-1)]*Sqr
t[(a + b*Sec[c + d*x])/((a + b)*(1 + Sec[c + d*x]))] + 2*b*(-4*a^2 - a*b +
 3*b^2)*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[(1 + Sec
[c + d*x])^(-1)]*Sqrt[(a + b*Sec[c + d*x])/((a + b)*(1 + Sec[c + d*x]))] +
 (4*a^2 - 3*b^2)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[
(c + d*x)/2]))/(b^4*(-a^2 + b^2)*Sqrt[Sec[(c + d*x)/2]^2]) + (2*Sqrt[Sec[c
 + d*x]]*((-8*a^4*b + 5*a^2*b^3 + 3*b^5)*Sin[c + d*x] + (-8*a^5 + 4*a^3*b^
2 + (3*a*b^4)/2)*Sin[2*(c + d*x)] + b^2*(-a^2 + b^2)*(b - 2*a*Cos[c + d*x]
)*Sec[c + d*x]*Tan[c + d*x]))/(-(a^2*b^4) + b^6)))/(5*d*(a + b*Sec[c + d*x
])^(3/2))
```

3.562.3 Rubi [A] (verified)

Time = 1.75 (sec) , antiderivative size = 411, normalized size of antiderivative = 1.03, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.609$, Rules used = {3042, 4332, 27, 3042, 4580, 27, 3042, 4570, 27, 3042, 4493, 3042, 4319, 4492}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^5(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^5}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \; 4332$$

$$-\frac{2\int \frac{\sec^2(c+dx)\left(4a^2-b\sec(c+dx)a-\left(6a^2-b^2\right)\sec^2(c+dx)\right)}{2\sqrt{a+b\sec(c+dx)}}dx}{b\left(a^2-b^2\right)}-\frac{2a^2\tan(c+dx)\sec^2(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}$$

$\Big\downarrow$ 27

$$-\frac{\int \frac{\sec^2(c+dx)\left(4a^2-b\sec(c+dx)a-\left(6a^2-b^2\right)\sec^2(c+dx)\right)}{\sqrt{a+b\sec(c+dx)}}dx}{b\left(a^2-b^2\right)}-\frac{2a^2\tan(c+dx)\sec^2(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}$$

$\Big\downarrow$ 3042

$$-\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2\left(4a^2-b\csc\left(c+dx+\frac{\pi}{2}\right)a+\left(b^2-6a^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{b\left(a^2-b^2\right)}-\frac{2a^2\tan(c+dx)\sec^2(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}$$

$\Big\downarrow$ 4580

$$-\frac{\frac{2\int -\frac{\sec(c+dx)\left(-3a\left(8a^2-3b^2\right)\sec^2(c+dx)-b\left(2a^2+3b^2\right)\sec(c+dx)+2a\left(6a^2-b^2\right)\right)}{2\sqrt{a+b\sec(c+dx)}}dx}{5b}-\frac{2\left(6a^2-b^2\right)\tan(c+dx)\sec(c+dx)\sqrt{a+b\sec(c+dx)}}{5bd}}{b\left(a^2-b^2\right)}-\frac{2a^2\tan(c+dx)\sec^2(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}$$

$\Big\downarrow$ 27

$$-\frac{-\frac{\int \frac{\sec(c+dx)\left(-3a\left(8a^2-3b^2\right)\sec^2(c+dx)-b\left(2a^2+3b^2\right)\sec(c+dx)+2a\left(6a^2-b^2\right)\right)}{\sqrt{a+b\sec(c+dx)}}dx}{5b}-\frac{2\left(6a^2-b^2\right)\tan(c+dx)\sec(c+dx)\sqrt{a+b\sec(c+dx)}}{5bd}}{b\left(a^2-b^2\right)}-\frac{2a^2\tan(c+dx)\sec^2(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}$$

$\Big\downarrow$ 3042

$$-\frac{-\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(-3a\left(8a^2-3b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2-b\left(2a^2+3b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)+2a\left(6a^2-b^2\right)\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{5b}-\frac{2\left(6a^2-b^2\right)\tan(c+dx)\sec(c+dx)\sqrt{a+b\sec(c+dx)}}{5bd}}{b\left(a^2-b^2\right)}-\frac{2a^2\tan(c+dx)\sec^2(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}$$

$\Big\downarrow$ 4570

3.562. $\int \frac{\sec^5(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$

$$-\frac{-\frac{\frac{2\int \frac{3\sec(c+dx)\left(ab\left(4a^2+b^2\right)+\left(16a^4-8b^2a^2-3b^4\right)\sec(c+dx)\right)}{2\sqrt{a+b\sec(c+dx)}}dx}{3b}-\frac{2a\left(8a^2-3b^2\right)\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{bd}}{5b}-\frac{2\left(6a^2-b^2\right)\tan(c+dx)\sec(c+dx)\sqrt{a+}}{5bd}}{b\left(a^2-b^2\right)}$$

$$\frac{2a^2\tan(c+dx)\sec^2(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}$$

$\downarrow$ 27

$$-\frac{-\frac{\frac{\int \frac{\sec(c+dx)\left(ab\left(4a^2+b^2\right)+\left(16a^4-8b^2a^2-3b^4\right)\sec(c+dx)\right)}{\sqrt{a+b\sec(c+dx)}}dx}{b}-\frac{2a\left(8a^2-3b^2\right)\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{bd}}{5b}-\frac{2\left(6a^2-b^2\right)\tan(c+dx)\sec(c+dx)\sqrt{a+b\sec}}{5bd}}{b\left(a^2-b^2\right)}$$

$$\frac{2a^2\tan(c+dx)\sec^2(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}$$

$\downarrow$ 3042

$$-\frac{-\frac{\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(ab\left(4a^2+b^2\right)+\left(16a^4-8b^2a^2-3b^4\right)\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\frac{2a\left(8a^2-3b^2\right)\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{bd}}{5b}-\frac{2\left(6a^2-b^2\right)\tan(c+dx)\sec(c+dx)}{5bd}}{b\left(a^2-b^2\right)}$$

$$\frac{2a^2\tan(c+dx)\sec^2(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}$$

$\downarrow$ 4493

$$-\frac{-\frac{\frac{\left(16a^4-8a^2b^2-3b^4\right)\int \frac{\sec(c+dx)(\sec(c+dx)+1)}{\sqrt{a+b\sec(c+dx)}}dx-(a-b)(4a+3b)\left(4a^2+b^2\right)\int \frac{\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx}{b}-\frac{2a\left(8a^2-3b^2\right)\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{bd}}{5b}-\frac{2\left(6a^2-b\right.}{}}{b\left(a^2-b^2\right)}$$

$$\frac{2a^2\tan(c+dx)\sec^2(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}$$

$\downarrow$ 3042

$$-\frac{-\frac{\frac{\left(16a^4-8a^2b^2-3b^4\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-(a-b)(4a+3b)\left(4a^2+b^2\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\frac{2a\left(8a^2-3b^2\right)\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{bd}}{5b}}{b\left(a^2-b^2\right)}$$

$$\frac{2a^2\tan(c+dx)\sec^2(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}$$

$\downarrow$ 4319

3.562. $\int \frac{\sec^5(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$

$$-\frac{-\frac{\frac{\left(16a^4-8a^2b^2-3b^4\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2(a-b)\sqrt{a+b}(4a+3b)\left(4a^2+b^2\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,\text{EllipticF}\left(\text{arc}\right.}{bd}}{b}}{5b}}{b\left(a^2-b^2\right)}$$

$$\frac{2a^2\tan(c+dx)\sec^2(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}$$

$$\Big\downarrow \; 4492$$

$$-\frac{2a^2\tan(c+dx)\sec^2(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}-$$

$$-\frac{2\left(6a^2-b^2\right)\tan(c+dx)\sec(c+dx)\sqrt{a+b\sec(c+dx)}}{5bd}-\frac{-\frac{\frac{2(a-b)\sqrt{a+b}(4a+3b)\left(4a^2+b^2\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,\text{EllipticF}\left(\arcsin\right.}{bd}}{}}{}}{}$$

input
```
Int[Sec[c + d*x]^5/(a + b*Sec[c + d*x])^(3/2),x]
```

output
```
(-2*a^2*Sec[c + d*x]^2*Tan[c + d*x])/(b*(a^2 - b^2)*d*Sqrt[a + b*Sec[c + d
*x]]) - ((-2*(6*a^2 - b^2)*Sec[c + d*x]*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d
*x])/(5*b*d) - (((-2*(a - b)*Sqrt[a + b]*(16*a^4 - 8*a^2*b^2 - 3*b^4)*Cot[
c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(
a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))
/(a - b))])/(b^2*d) - (2*(a - b)*Sqrt[a + b]*(4*a + 3*b)*(4*a^2 + b^2)*Cot
[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/
(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x])
)/(a - b))])/(b*d))/b - (2*a*(8*a^2 - 3*b^2)*Sqrt[a + b*Sec[c + d*x]]*Tan[
c + d*x])/(b*d))/(5*b))/(b*(a^2 - b^2))
```

3.562.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4319
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4332
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-a^2)*d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(
m + 1)*((d*Csc[e + f*x])^(n - 3)/(b*f*(m + 1)*(a^2 - b^2))), x] + Simp[d^3/
(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])
^(n - 3)*Simp[a^2*(n - 3) + a*b*(m + 1)*Csc[e + f*x] - (a^2*(n - 2) + b^2*(
m + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2
- b^2, 0] && LtQ[m, -1] && (IGtQ[n, 3] || (IntegersQ[n + 1/2, 2*m] && GtQ[n
, 2]))
```

rule 4492
```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

rule 4493
```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(A - B)   Int[Csc[e
 + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Simp[B   Int[Csc[e + f*x]*((1 +
Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A, B}
, x] && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

rule 4570
```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> Simp[(-C)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2)
)), x] + Simp[1/(b*(m + 2))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[
b*A*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /;
 FreeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

3.562. $\int \frac{\sec^5(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$

rule 4580
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[
(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x
_Symbol] :> Simp[(-C)*Csc[e + f*x]*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m +
1)/(b*f*(m + 3))), x] + Simp[1/(b*(m + 3))   Int[Csc[e + f*x]*(a + b*Csc[e
+ f*x])^m*Simp[a*C + b*(C*(m + 2) + A*(m + 3))*Csc[e + f*x] - (2*a*C - b*B*
(m + 3))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &
& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

3.562.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3170 vs. $2(367) = 734$.

Time = 15.94 (sec) , antiderivative size = 3171, normalized size of antiderivative = 7.95

method	result	size
default	Expression too large to display	3171

input
```
int(sec(d*x+c)^5/(a+b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/5/d/(a-b)/(a+b)/b^4*(a+b*sec(d*x+c))^(1/2)/(b+a*cos(d*x+c))/(cos(d*x+c)
+1)*(3*b^5*sin(d*x+c)+5*a^2*b^3*sin(d*x+c)+b^5*tan(d*x+c)+2*a^3*b^2*sin(d*
x+c)-8*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*
x+c)+1))^(1/2)*EllipticF(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^
3*cos(d*x+c)^2+(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)
/(cos(d*x+c)+1))^(1/2)*EllipticF(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2)
)*a*b^4*cos(d*x+c)^2-32*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(c
os(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+
b))^(1/2))*a^4*b*cos(d*x+c)+16*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(
1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(cot(d*x+c)-csc(d*x+c),((a
-b)/(a+b))^(1/2))*a^3*b^2*cos(d*x+c)+16*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x
+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(cot(d*x+c)-csc(d
*x+c),((a-b)/(a+b))^(1/2))*a^2*b^3*cos(d*x+c)-16*(1/(a+b)*(b+a*cos(d*x+c))
/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(cot(d*x
+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*a^4*b*cos(d*x+c)^2+6*(1/(a+b)*(b+a*cos
(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE
(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*a*b^4*cos(d*x+c)+32*(1/(a+b)*(
b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*El
lipticF(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*a^4*b*cos(d*x+c)+8*(1/(
a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))...
```

3.562.5 Fricas [F]

$$\int \frac{\sec^5(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\sec\left(dx+c\right)^5}{\left(b\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^5/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^5/(b^2*sec(d*x + c)^2 + 2*a
*b*sec(d*x + c) + a^2), x)
```

3.562.6 Sympy [F]

$$\int \frac{\sec^5(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\sec^5\left(c+dx\right)}{\left(a+b\sec\left(c+dx\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)**5/(a+b*sec(d*x+c))**(3/2),x)
```

output
```
Integral(sec(c + d*x)**5/(a + b*sec(c + d*x))**(3/2), x)
```

3.562.7 Maxima [F(-1)]

Timed out.

$$\int \frac{\sec^5(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)^5/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
Timed out
```

3.562.8 Giac [F]

$$\int \frac{\sec^5(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\sec{(dx+c)}^5}{(b\sec{(dx+c)}+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^5/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^5/(b*sec(d*x + c) + a)^(3/2), x)
```

3.562.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^5(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{1}{\cos{(c+d\,x)}^5\,\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

input
```
int(1/(cos(c + d*x)^5*(a + b/cos(c + d*x))^(3/2)),x)
```

output
```
int(1/(cos(c + d*x)^5*(a + b/cos(c + d*x))^(3/2)), x)
```

3.563 $\int \frac{\sec^4(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$

3.563.1 Optimal result 4246
3.563.2 Mathematica [A] (warning: unable to verify) 4247
3.563.3 Rubi [A] (verified) 4247
3.563.4 Maple [B] (verified) 4251
3.563.5 Fricas [F] 4252
3.563.6 Sympy [F] 4253
3.563.7 Maxima [F] 4253
3.563.8 Giac [F] 4253
3.563.9 Mupad [F(-1)] 4254

3.563.1 Optimal result

Integrand size = 23, antiderivative size = 325

$$\int \frac{\sec^4(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \frac{2a(8a^2-5b^2)\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-}}{3b^4\sqrt{a+b}d}$$
$$+\frac{2(2a+b)(4a+b)\cot(c+dx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{3b^3\sqrt{a+b}d}$$
$$-\frac{2a^2\sec(c+dx)\tan(c+dx)}{b\,(a^2-b^2)\,d\sqrt{a+b\sec(c+dx)}}+\frac{2(4a^2-b^2)\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{3b^2\,(a^2-b^2)\,d}$$

output
```
2/3*a*(8*a^2-5*b^2)*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2
),((a+b)/(a-b))^(1/2))*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(
a-b))^(1/2)/b^4/d/(a+b)^(1/2)+2/3*(2*a+b)*(4*a+b)*cot(d*x+c)*EllipticF((a+
b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(b*(1-sec(d*x+c))/(a+
b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/b^3/d/(a+b)^(1/2)-2*a^2*sec(d*x+
c)*tan(d*x+c)/b/(a^2-b^2)/d/(a+b*sec(d*x+c))^(1/2)+2/3*(4*a^2-b^2)*(a+b*se
c(d*x+c))^(1/2)*tan(d*x+c)/b^2/(a^2-b^2)/d
```

3.563.2 Mathematica [A] (warning: unable to verify)

Time = 11.67 (sec) , antiderivative size = 470, normalized size of antiderivative = 1.45

$$\int \frac{\sec^4(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = -\frac{2(b+a\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)}\left(2a(8a^3+8a^2b-5ab^2-5b^3)\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx}}\right.}{} + \frac{(b+a\cos(c+dx))^2\sec^2(c+dx)\left(-\frac{2a(-8a^2+5b^2)\sin(c+dx)}{3b^3(-a^2+b^2)}-\frac{2a^3\sin(c+dx)}{b^2(-a^2+b^2)(b+a\cos(c+dx))}+\frac{2\tan(c+dx)}{3b^2}\right)}{d(a+b\sec(c+dx))^{3/2}}$$

input
```
Integrate[Sec[c + d*x]^4/(a + b*Sec[c + d*x])^(3/2),x]
```

output
```
(-2*(b + a*Cos[c + d*x])*Sec[c + d*x]^(3/2)*Sqrt[Cos[(c + d*x)/2]^2*Sec[c
+ d*x]]*(2*a*(8*a^3 + 8*a^2*b - 5*a*b^2 - 5*b^3)*Sqrt[Cos[c + d*x]/(1 + Co
s[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Ellip
ticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] - 2*b*(8*a^3 + 2*a^2*b - 5
*a*b^2 + b^3)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*
x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a -
 b)/(a + b)] + a*(8*a^2 - 5*b^2)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c
+ d*x)/2]^2*Tan[(c + d*x)/2]))/(3*b^3*(-a^2 + b^2)*d*Sqrt[Sec[(c + d*x)/2]
^2]*(a + b*Sec[c + d*x])^(3/2)) + ((b + a*Cos[c + d*x])^2*Sec[c + d*x]^2*(
(-2*a*(-8*a^2 + 5*b^2)*Sin[c + d*x])/(3*b^3*(-a^2 + b^2)) - (2*a^3*Sin[c +
 d*x])/(b^2*(-a^2 + b^2)*(b + a*Cos[c + d*x])) + (2*Tan[c + d*x])/(3*b^2))
)/(d*(a + b*Sec[c + d*x])^(3/2))
```

3.563.3 Rubi [A] (verified)

Time = 1.25 (sec) , antiderivative size = 345, normalized size of antiderivative = 1.06, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules used = {3042, 4332, 27, 3042, 4570, 27, 3042, 4493, 3042, 4319, 4492}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^4(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^4}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \; 4332$$

$$-\frac{2\int \frac{\sec(c+dx)\left(2a^2-b\sec(c+dx)a-\left(4a^2-b^2\right)\sec^2(c+dx)\right)}{2\sqrt{a+b\sec(c+dx)}}dx}{b\left(a^2-b^2\right)}-\frac{2a^2\tan(c+dx)\sec(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}$$

$$\Big\downarrow \; 27$$

$$-\frac{\int \frac{\sec(c+dx)\left(2a^2-b\sec(c+dx)a-\left(4a^2-b^2\right)\sec^2(c+dx)\right)}{\sqrt{a+b\sec(c+dx)}}dx}{b\left(a^2-b^2\right)}-\frac{2a^2\tan(c+dx)\sec(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(2a^2-b\csc\left(c+dx+\frac{\pi}{2}\right)a+\left(b^2-4a^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{b\left(a^2-b^2\right)}-\frac{2a^2\tan(c+dx)\sec(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}$$

$$\Big\downarrow \; 4570$$

$$-\frac{\frac{2\int \frac{\sec(c+dx)\left(b\left(2a^2+b^2\right)+a\left(8a^2-5b^2\right)\sec(c+dx)\right)}{2\sqrt{a+b\sec(c+dx)}}dx}{3b}-\frac{2\left(4a^2-b^2\right)\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{3bd}}{b\left(a^2-b^2\right)}-\frac{2a^2\tan(c+dx)\sec(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}$$

$$\Big\downarrow \; 27$$

$$-\frac{\frac{\int \frac{\sec(c+dx)\left(b\left(2a^2+b^2\right)+a\left(8a^2-5b^2\right)\sec(c+dx)\right)}{\sqrt{a+b\sec(c+dx)}}dx}{3b}-\frac{2\left(4a^2-b^2\right)\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{3bd}}{b\left(a^2-b^2\right)}-\frac{2a^2\tan(c+dx)\sec(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(b\left(2a^2+b^2\right)+a\left(8a^2-5b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{3b}-\frac{2\left(4a^2-b^2\right)\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{3bd}}{b\left(a^2-b^2\right)}-\frac{2a^2\tan(c+dx)\sec(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}$$

$$\Big\downarrow \; 4493$$

3.563. $\int \frac{\sec^4(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$

$$-\frac{\frac{a(8a^2-5b^2)\int \frac{\sec(c+dx)(\sec(c+dx)+1)}{\sqrt{a+b\sec(c+dx)}}dx-(a-b)(2a+b)(4a+b)\int \frac{\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx}{3b}-\frac{2(4a^2-b^2)\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{3bd}}{b(a^2-b^2)}-\frac{2a^2\tan(c+dx)\sec(c+dx)}{bd(a^2-b^2)\sqrt{a+b\sec(c+dx)}}$$

$\downarrow$ 3042

$$-\frac{\frac{a(8a^2-5b^2)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-(a-b)(2a+b)(4a+b)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{3b}-\frac{2(4a^2-b^2)\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{3bd}}{b(a^2-b^2)}-\frac{2a^2\tan(c+dx)\sec(c+dx)}{bd(a^2-b^2)\sqrt{a+b\sec(c+dx)}}$$

$\downarrow$ 4319

$$-\frac{\frac{a(8a^2-5b^2)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2(a-b)\sqrt{a+b}(2a+b)(4a+b)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec}}{\sqrt{a}}\right.\right.}{bd}}{3b}}{b(a^2-b^2)}-\frac{2a^2\tan(c+dx)\sec(c+dx)}{bd(a^2-b^2)\sqrt{a+b\sec(c+dx)}}$$

$\downarrow$ 4492

$$-\frac{\frac{-\frac{2a(a-b)\sqrt{a+b}\left(8a^2-5b^2\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}E\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{b^2d}-\frac{2(a-b)\sqrt{a+b}(2a+b)(4a+b)\cot(c+dx)\sqrt{\frac{b(1-}{}}}{}}{3b}}{b(a^2-b^2)}-\frac{2a^2\tan(c+dx)\sec(c+dx)}{bd(a^2-b^2)\sqrt{a+b\sec(c+dx)}}$$

input
```
Int[Sec[c + d*x]^4/(a + b*Sec[c + d*x])^(3/2),x]
```

3.563. $\int \frac{\sec^4(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$

output
```
(-2*a^2*Sec[c + d*x]*Tan[c + d*x])/(b*(a^2 - b^2)*d*Sqrt[a + b*Sec[c + d*x
]]) - (((-2*a*(a - b)*Sqrt[a + b]*(8*a^2 - 5*b^2)*Cot[c + d*x]*EllipticE[A
rcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 -
 Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(b^2*d)
- (2*(a - b)*Sqrt[a + b]*(2*a + b)*(4*a + b)*Cot[c + d*x]*EllipticF[ArcSin
[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[
c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(b*d))/(3*b)
- (2*(4*a^2 - b^2)*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(3*b*d))/(b*(a^2
 - b^2))
```

3.563.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4319
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4332
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-a^2)*d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(
m + 1)*((d*Csc[e + f*x])^(n - 3)/(b*f*(m + 1)*(a^2 - b^2))), x] + Simp[d^3/
(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])
^(n - 3)*Simp[a^2*(n - 3) + a*b*(m + 1)*Csc[e + f*x] - (a^2*(n - 2) + b^2*(
m + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2
- b^2, 0] && LtQ[m, -1] && (IGtQ[n, 3] || (IntegersQ[n + 1/2, 2*m] && GtQ[n
, 2]))
```

rule 4492
```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

rule 4493
```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(A - B)   Int[Csc[e
 + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Simp[B   Int[Csc[e + f*x]*((1 +
Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A, B}
, x] && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

rule 4570
```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> Simp[(-C)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2)
)), x] + Simp[1/(b*(m + 2))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[
b*A*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /;
 FreeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

3.563.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2285 vs. $2(297) = 594$.

Time = 12.33 (sec) , antiderivative size = 2286, normalized size of antiderivative = 7.03

method	result	size
default	Expression too large to display	2286

input
```
int(sec(d*x+c)^4/(a+b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3/d/(a-b)/(a+b)/b^3*(a+b*sec(d*x+c))^(1/2)/(b+a*cos(d*x+c))/(cos(d*x+c)
+1)*(4*a^3*b*sin(d*x+c)-4*a*b^3*sin(d*x+c)+b^4*sin(d*x+c)-a^2*b^2*sin(d*x+
c)+8*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE(cot(d*x+c)-
csc(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*a^3*b-5*
(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE(cot(d*x+c)-csc(d
*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*a^2*b^2-5*(1/
(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE(cot(d*x+c)-csc(d*x+
c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*a*b^3-EllipticF(
cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(
d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*b^4*cos(d*x+c)^2+8*(1/(
a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1
/2)*EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*a^4*cos(d*x+c)^2-
2*EllipticF(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*(1/(a+b)*(b+a*cos(d
*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*b^4*cos(d*x
+c)+16*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*
x+c)+1))^(1/2)*EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*a^4*co
s(d*x+c)-8*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF(cot(d
*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*a^
3*b-2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF(cot(d*x+c)
-csc(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*a^2*...
```

3.563.5 Fricas [F]

$$\int \frac{\sec^4(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\sec\left(dx+c\right)^4}{\left(b\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^4/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^4/(b^2*sec(d*x + c)^2 + 2*a
*b*sec(d*x + c) + a^2), x)
```

3.563.6 Sympy [F]

$$\int \frac{\sec^4(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\sec^4(c+dx)}{(a+b\sec(c+dx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)**4/(a+b*sec(d*x+c))**(3/2),x)
```

output
```
Integral(sec(c + d*x)**4/(a + b*sec(c + d*x))**(3/2), x)
```

3.563.7 Maxima [F]

$$\int \frac{\sec^4(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\sec(dx+c)^4}{(b\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^4/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)^4/(b*sec(d*x + c) + a)^(3/2), x)
```

3.563.8 Giac [F]

$$\int \frac{\sec^4(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\sec(dx+c)^4}{(b\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^4/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^4/(b*sec(d*x + c) + a)^(3/2), x)
```

3.563.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^4(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{1}{\cos\left(c+d\,x\right)^4\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

input
```
int(1/(cos(c + d*x)^4*(a + b/cos(c + d*x))^(3/2)),x)
```

output
```
int(1/(cos(c + d*x)^4*(a + b/cos(c + d*x))^(3/2)), x)
```

3.564 $\int \frac{\sec^3(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$

3.564.1 Optimal result . . . 4255
3.564.2 Mathematica [A] (verified) . . . 4256
3.564.3 Rubi [A] (verified) . . . 4256
3.564.4 Maple [B] (verified) . . . 4259
3.564.5 Fricas [F] . . . 4260
3.564.6 Sympy [F] . . . 4261
3.564.7 Maxima [F] . . . 4261
3.564.8 Giac [F] . . . 4261
3.564.9 Mupad [F(-1)] . . . 4262

3.564.1 Optimal result

Integrand size = 23, antiderivative size = 257

$$\int \frac{\sec^3(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = -\frac{2(2a^2-b^2)\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{b^3\sqrt{a+b}d} - \frac{2(2a+b)\cot(c+dx)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{b^2\sqrt{a+b}d} - \frac{2a^2\tan(c+dx)}{b\,(a^2-b^2)\,d\sqrt{a+b\sec(c+dx)}}$$

output
```
-2*(2*a^2-b^2)*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a
+b)/(a-b))^(1/2))*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))
^(1/2)/b^3/d/(a+b)^(1/2)-2*(2*a+b)*cot(d*x+c)*EllipticF((a+b*sec(d*x+c))^(
1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(
1+sec(d*x+c))/(a-b))^(1/2)/b^2/d/(a+b)^(1/2)-2*a^2*tan(d*x+c)/b/(a^2-b^2)/
d/(a+b*sec(d*x+c))^(1/2)
```

3.564.2 Mathematica [A] (verified)

Time = 13.68 (sec) , antiderivative size = 440, normalized size of antiderivative = 1.71

$$\int \frac{\sec^3(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \frac{(b+a\cos(c+dx))^2\sec^2(c+dx)\left(\frac{2(-2a^2+b^2)\sin(c+dx)}{b^2(-a^2+b^2)}+\frac{2a^2\sin(c+dx)}{b(-a^2+b^2)(b+a\cos(c+dx))}\right)}{d(a+b\sec(c+dx))^{3/2}} + \frac{2(b+a\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)}\left(2(2a^3+2a^2b-ab^2-b^3)\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\sqrt{\right.}}{}$$

input
```
Integrate[Sec[c + d*x]^3/(a + b*Sec[c + d*x])^(3/2),x]
```

output
```
((b + a*Cos[c + d*x])^2*Sec[c + d*x]^2*((2*(-2*a^2 + b^2)*Sin[c + d*x])/(b
^2*(-a^2 + b^2)) + (2*a^2*Sin[c + d*x])/(b*(-a^2 + b^2)*(b + a*Cos[c + d*x
]))))/(d*(a + b*Sec[c + d*x])^(3/2)) + (2*(b + a*Cos[c + d*x])*Sec[c + d*x
]^(3/2)*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(2*(2*a^3 + 2*a^2*b - a*b^2
- b^3)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a
 + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a
+ b)] + 2*b*(-2*a^2 - a*b + b^2)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqr
t[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[
(c + d*x)/2]], (a - b)/(a + b)] + (2*a^2 - b^2)*Cos[c + d*x]*(b + a*Cos[c
+ d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(b^2*(-a^2 + b^2)*d*Sqrt[Sec
[(c + d*x)/2]^2]*(a + b*Sec[c + d*x])^(3/2))
```

3.564.3 Rubi [A] (verified)

Time = 0.87 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.10, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {3042, 4326, 27, 3042, 4493, 3042, 4319, 4492}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^3(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^3}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{4326}$$

$$-\frac{2\int -\frac{\sec(c+dx)\left(ab+\left(2a^2-b^2\right)\sec(c+dx)\right)}{2\sqrt{a+b\sec(c+dx)}}dx}{b\left(a^2-b^2\right)}-\frac{2a^2\tan(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\sec(c+dx)\left(ab+\left(2a^2-b^2\right)\sec(c+dx)\right)}{\sqrt{a+b\sec(c+dx)}}dx}{b\left(a^2-b^2\right)}-\frac{2a^2\tan(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(ab+\left(2a^2-b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{b\left(a^2-b^2\right)}-\frac{2a^2\tan(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}$$

$$\Big\downarrow \text{4493}$$

$$\frac{\left(2a^2-b^2\right)\int \frac{\sec(c+dx)(\sec(c+dx)+1)}{\sqrt{a+b\sec(c+dx)}}dx-(a-b)(2a+b)\int \frac{\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx}{b\left(a^2-b^2\right)}-\frac{2a^2\tan(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\left(2a^2-b^2\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-(a-b)(2a+b)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{b\left(a^2-b^2\right)}-\frac{2a^2\tan(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}$$

$$\Big\downarrow \text{4319}$$

$$\frac{\left(2a^2-b^2\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2(a-b)\sqrt{a+b}(2a+b)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\operatorname{EllipticF}\left(\arcsin\left(\sqrt{\right.\right.}{bd}}{b\left(a^2-b^2\right)}-\frac{2a^2\tan(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}$$

$$\Big\downarrow \text{4492}$$

$$\frac{-\frac{2(a-b)\sqrt{a+b}\left(2a^2-b^2\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}E\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{b^2d}-\frac{2(a-b)\sqrt{a+b}(2a+b)\cot(c+dx)\sqrt{\frac{b}{}}}{}}{b\left(a^2-b^2\right)}-\frac{2a^2\tan(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}$$

3.564. $\int \frac{\sec^3(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$

input
```
Int[Sec[c + d*x]^3/(a + b*Sec[c + d*x])^(3/2),x]
```

output
```
((-2*(a - b)*Sqrt[a + b]*(2*a^2 - b^2)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[
a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*
x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(b^2*d) - (2*(a - b
)*Sqrt[a + b]*(2*a + b)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d
*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*S
qrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(b*d))/(b*(a^2 - b^2)) - (2*a^2*Ta
n[c + d*x])/(b*(a^2 - b^2)*d*Sqrt[a + b*Sec[c + d*x]])
```

3.564.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4319
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4326
```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(-a^2)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m
 + 1)*(a^2 - b^2))), x] + Simp[1/(b*(m + 1)*(a^2 - b^2))   Int[Csc[e + f*x]
*(a + b*Csc[e + f*x])^(m + 1)*Simp[a*b*(m + 1) - (a^2 + b^2*(m + 1))*Csc[e
+ f*x], x], x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m,
 -1]
```

rule 4492
```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

rule 4493
```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(A - B)   Int[Csc[e
 + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Simp[B   Int[Csc[e + f*x]*((1 +
Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A, B}
, x] && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

3.564.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1939 vs. $2(237) = 474$.

Time = 11.22 (sec) , antiderivative size = 1940, normalized size of antiderivative = 7.55

method	result	size
default	Expression too large to display	1940

input
```
int(sec(d*x+c)^3/(a+b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/d/b^2/(a+b)/(a-b)*(a^2*b*sin(d*x+c)+2*EllipticE(cot(d*x+c)-csc(d*x+c),((
a-b)/(a+b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*
x+c)/(cos(d*x+c)+1))^(1/2)*a^2*b-EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a
+b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(c
os(d*x+c)+1))^(1/2)*a*b^2+2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2
)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)
/(a+b))^(1/2))*a^3*cos(d*x+c)^2-(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^
(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(cot(d*x+c)-csc(d*x+c),((
a-b)/(a+b))^(1/2))*b^3*cos(d*x+c)^2-2*EllipticF(cot(d*x+c)-csc(d*x+c),((a-
b)/(a+b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+
c)/(cos(d*x+c)+1))^(1/2)*a^2*b-EllipticF(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b
))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos
(d*x+c)+1))^(1/2)*a*b^2+2*EllipticF(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1
/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+
c)+1))^(1/2)*b^3*cos(d*x+c)+4*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1
/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(cot(d*x+c)-csc(d*x+c),((a-
b)/(a+b))^(1/2))*a^3*cos(d*x+c)+EllipticF(cot(d*x+c)-csc(d*x+c),((a-b)/(a+
b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(co
s(d*x+c)+1))^(1/2)*b^3*cos(d*x+c)^2-2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c
)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(cot(d*x+c)-csc(...
```

3.564.5 Fricas [F]

$$\int \frac{\sec^3(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\sec{(dx+c)}^3}{(b\sec{(dx+c)}+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^3/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^3/(b^2*sec(d*x + c)^2 + 2*a
*b*sec(d*x + c) + a^2), x)
```

3.564.6 Sympy [F]

$$\int \frac{\sec^3(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\sec^3(c+dx)}{(a+b\sec(c+dx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)**3/(a+b*sec(d*x+c))**(3/2),x)
```

output
```
Integral(sec(c + d*x)**3/(a + b*sec(c + d*x))**(3/2), x)
```

3.564.7 Maxima [F]

$$\int \frac{\sec^3(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\sec(dx+c)^3}{(b\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^3/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)^3/(b*sec(d*x + c) + a)^(3/2), x)
```

3.564.8 Giac [F]

$$\int \frac{\sec^3(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\sec(dx+c)^3}{(b\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^3/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^3/(b*sec(d*x + c) + a)^(3/2), x)
```

3.564.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^3(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{1}{\cos\left(c+d\,x\right)^3\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

input
```
int(1/(cos(c + d*x)^3*(a + b/cos(c + d*x))^(3/2)),x)
```

output
```
int(1/(cos(c + d*x)^3*(a + b/cos(c + d*x))^(3/2)), x)
```

3.565 $\int \frac{\sec^2(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$

3.565.1 Optimal result 4263
3.565.2 Mathematica [A] (verified) 4264
3.565.3 Rubi [A] (verified) 4264
3.565.4 Maple [B] (verified) 4267
3.565.5 Fricas [F] 4267
3.565.6 Sympy [F] 4268
3.565.7 Maxima [F] 4268
3.565.8 Giac [F] 4268
3.565.9 Mupad [F(-1)] 4269

3.565.1 Optimal result

Integrand size = 23, antiderivative size = 237

$$\int \frac{\sec^2(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \frac{2a\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{b^2\sqrt{a+b}d} + \frac{2\cot(c+dx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{b\sqrt{a+b}d} + \frac{2a\tan(c+dx)}{(a^2-b^2)\,d\sqrt{a+b\sec(c+dx)}}$$

output
```
2*a*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^
(1/2))*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/b^2/
d/(a+b)^(1/2)+2*cot(d*x+c)*EllipticF((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((
a+b)/(a-b))^(1/2))*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b)
)^(1/2)/b/d/(a+b)^(1/2)+2*a*tan(d*x+c)/(a^2-b^2)/d/(a+b*sec(d*x+c))^(1/2)
```

3.565.2 Mathematica [A] (verified)

Time = 7.98 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.05

$$\int \frac{\sec^2(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \frac{\sec\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\left(4a(a+b)\cos^3\left(\frac{1}{2}(c+dx)\right)\sqrt{\frac{b+a\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}E(\right.}{}$$

input
```
Integrate[Sec[c + d*x]^2/(a + b*Sec[c + d*x])^(3/2),x]
```

output
```
(Sec[(c + d*x)/2]*Sec[c + d*x]*(4*a*(a + b)*Cos[(c + d*x)/2]^3*Sqrt[(b + a
*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x
)/2]], (a - b)/(a + b)]*Sqrt[(1 + Sec[c + d*x])^(-1)] - 4*b*(a + b)*Cos[(c
 + d*x)/2]^3*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Ellip
ticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[(1 + Sec[c + d*x])^(-
1)] + a*(a - b)*(Sin[(c + d*x)/2] - Sin[(3*(c + d*x))/2])))/(b*(a^2 - b^2)
*d*Sqrt[a + b*Sec[c + d*x]])
```

3.565.3 Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.10, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {3042, 4323, 27, 3042, 4493, 3042, 4319, 4492}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \; 4323$$

$$\frac{2\int -\frac{\sec(c+dx)(b+a\sec(c+dx))}{2\sqrt{a+b\sec(c+dx)}}dx}{a^2-b^2}+\frac{2a\tan(c+dx)}{d\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}$$

$$\Big\downarrow \; 27$$

$$\frac{2a\tan(c+dx)}{d(a^2-b^2)\sqrt{a+b\sec(c+dx)}}-\frac{\int\frac{\sec(c+dx)(b+a\sec(c+dx))}{\sqrt{a+b\sec(c+dx)}}dx}{a^2-b^2}$$

$\Big\downarrow$ 3042

$$\frac{2a\tan(c+dx)}{d(a^2-b^2)\sqrt{a+b\sec(c+dx)}}-\frac{\int\frac{\csc(c+dx+\frac{\pi}{2})(b+a\csc(c+dx+\frac{\pi}{2}))}{\sqrt{a+b\csc(c+dx+\frac{\pi}{2})}}dx}{a^2-b^2}$$

$\Big\downarrow$ 4493

$$\frac{2a\tan(c+dx)}{d(a^2-b^2)\sqrt{a+b\sec(c+dx)}}-\frac{a\int\frac{\sec(c+dx)(\sec(c+dx)+1)}{\sqrt{a+b\sec(c+dx)}}dx-(a-b)\int\frac{\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx}{a^2-b^2}$$

$\Big\downarrow$ 3042

$$\frac{2a\tan(c+dx)}{d(a^2-b^2)\sqrt{a+b\sec(c+dx)}}-\frac{a\int\frac{\csc(c+dx+\frac{\pi}{2})(\csc(c+dx+\frac{\pi}{2})+1)}{\sqrt{a+b\csc(c+dx+\frac{\pi}{2})}}dx-(a-b)\int\frac{\csc(c+dx+\frac{\pi}{2})}{\sqrt{a+b\csc(c+dx+\frac{\pi}{2})}}dx}{a^2-b^2}$$

$\Big\downarrow$ 4319

$$\frac{2a\tan(c+dx)}{d(a^2-b^2)\sqrt{a+b\sec(c+dx)}}-\frac{a\int\frac{\csc(c+dx+\frac{\pi}{2})(\csc(c+dx+\frac{\pi}{2})+1)}{\sqrt{a+b\csc(c+dx+\frac{\pi}{2})}}dx-\frac{2(a-b)\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+}{a-}\right.}{bd}}{a^2-b^2}$$

$\Big\downarrow$ 4492

$$\frac{2a\tan(c+dx)}{d(a^2-b^2)\sqrt{a+b\sec(c+dx)}}-\frac{-\frac{2a(a-b)\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}E\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{b^2d}-\frac{2(a-b)\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{}}{}}{a^2-b^2}$$

input
```
Int[Sec[c + d*x]^2/(a + b*Sec[c + d*x])^(3/2),x]
```

output
```
-(((-2*a*(a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[
c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a +
b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(b^2*d) - (2*(a - b)*Sqrt[a +
 b]*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (
a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c
+ d*x]))/(a - b))])/(b*d))/(a^2 - b^2)) + (2*a*Tan[c + d*x])/((a^2 - b^2)*
d*Sqrt[a + b*Sec[c + d*x]])
```

3.565. $\int\frac{\sec^2(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$

3.565.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4319
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4323
```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[a*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(f*(m + 1)*(
a^2 - b^2))), x] - Simp[1/((m + 1)*(a^2 - b^2))   Int[Csc[e + f*x]*(a + b*C
sc[e + f*x])^(m + 1)*(b*(m + 1) - a*(m + 2)*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

rule 4492
```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

rule 4493
```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(A - B)   Int[Csc[e
 + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Simp[B   Int[Csc[e + f*x]*((1 +
Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A, B}
, x] && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

3.565. $\int \frac{\sec^2(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$

3.565.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1105 vs. $2(217) = 434$.

Time = 7.38 (sec) , antiderivative size = 1106, normalized size of antiderivative = 4.67

method	result	size
default	Expression too large to display	1106

input
```
int(sec(d*x+c)^2/(a+b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/d/b/(a+b)/(a-b)*(EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*(
1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))
^(1/2)*a^2*cos(d*x+c)^2+EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2
))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)
+1))^(1/2)*a*b*cos(d*x+c)^2-EllipticF(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^
(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*
x+c)+1))^(1/2)*a*b*cos(d*x+c)^2-EllipticF(cot(d*x+c)-csc(d*x+c),((a-b)/(a+
b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(co
s(d*x+c)+1))^(1/2)*b^2*cos(d*x+c)^2+2*EllipticE(cot(d*x+c)-csc(d*x+c),((a-
b)/(a+b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+
c)/(cos(d*x+c)+1))^(1/2)*a^2*cos(d*x+c)+2*EllipticE(cot(d*x+c)-csc(d*x+c),
((a-b)/(a+b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(
d*x+c)/(cos(d*x+c)+1))^(1/2)*a*b*cos(d*x+c)-2*EllipticF(cot(d*x+c)-csc(d*x
+c),((a-b)/(a+b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(
cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*a*b*cos(d*x+c)-2*EllipticF(cot(d*x+c)-csc
(d*x+c),((a-b)/(a+b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/
2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*b^2*cos(d*x+c)+(1/(a+b)*(b+a*cos(d*x+
c))/(cos(d*x+c)+1))^(1/2)*EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1
/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*a^2+(1/(a+b)*(b+a*cos(d*x+c))/(cos(
d*x+c)+1))^(1/2)*EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*(...
```

3.565.5 Fricas [F]

$$\int \frac{\sec^2(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\sec\left(dx+c\right)^2}{\left(b\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^2/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^2/(b^2*sec(d*x + c)^2 + 2*a
*b*sec(d*x + c) + a^2), x)
```

3.565.6 Sympy [F]

$$\int \frac{\sec^2(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\sec^2(c+dx)}{(a+b\sec(c+dx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)**2/(a+b*sec(d*x+c))**(3/2),x)
```

output
```
Integral(sec(c + d*x)**2/(a + b*sec(c + d*x))**(3/2), x)
```

3.565.7 Maxima [F]

$$\int \frac{\sec^2(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\sec(dx+c)^2}{(b\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^2/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)^2/(b*sec(d*x + c) + a)^(3/2), x)
```

3.565.8 Giac [F]

$$\int \frac{\sec^2(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\sec(dx+c)^2}{(b\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^2/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^2/(b*sec(d*x + c) + a)^(3/2), x)
```

3.565.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^2(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{1}{\cos\left(c+d\,x\right)^2\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

input
```
int(1/(cos(c + d*x)^2*(a + b/cos(c + d*x))^(3/2)),x)
```

output
```
int(1/(cos(c + d*x)^2*(a + b/cos(c + d*x))^(3/2)), x)
```

3.566 $\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$

3.566.1 Optimal result 4270
3.566.2 Mathematica [A] (verified) 4271
3.566.3 Rubi [A] (verified) 4271
3.566.4 Maple [B] (verified) 4274
3.566.5 Fricas [F] 4275
3.566.6 Sympy [F] 4275
3.566.7 Maxima [F] 4275
3.566.8 Giac [F] 4276
3.566.9 Mupad [F(-1)] 4276

3.566.1 Optimal result

Integrand size = 21, antiderivative size = 236

$$\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = -\frac{2\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{b\sqrt{a+b}d} + \frac{2\cot(c+dx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{b\sqrt{a+b}d} - \frac{2b\tan(c+dx)}{(a^2-b^2)\,d\sqrt{a+b\sec(c+dx)}}$$

output
```
-2*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(
1/2))*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/b/d/(
a+b)^(1/2)+2*cot(d*x+c)*EllipticF((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b
)/(a-b))^(1/2))*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(
1/2)/b/d/(a+b)^(1/2)-2*b*tan(d*x+c)/(a^2-b^2)/d/(a+b*sec(d*x+c))^(1/2)
```

3.566.2 Mathematica [A] (verified)

Time = 7.38 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.03

$$\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = -\frac{\sec\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\left(4(a+b)\cos^3\left(\frac{1}{2}(c+dx)\right)\sqrt{\frac{b+a\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}E\left(\arcsin\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{a-b}{a+b}\right)\right.}{}$$

input
```
Integrate[Sec[c + d*x]/(a + b*Sec[c + d*x])^(3/2),x]
```

output
```
-((Sec[(c + d*x)/2]*Sec[c + d*x]*(4*(a + b)*Cos[(c + d*x)/2]^3*Sqrt[(b + a
*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x
)/2]], (a - b)/(a + b)]*Sqrt[(1 + Sec[c + d*x])^(-1)] - 4*(a + b)*Cos[(c +
 d*x)/2]^3*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Ellipti
cF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[(1 + Sec[c + d*x])^(-1)
] + (a - b)*(Sin[(c + d*x)/2] - Sin[(3*(c + d*x))/2])))/((a^2 - b^2)*d*Sqr
t[a + b*Sec[c + d*x]]))
```

3.566.3 Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.10, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {3042, 4320, 27, 3042, 4316, 3042, 4319, 4492}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \; 4320$$

$$-\frac{2\int -\frac{1}{2}\sec(c+dx)\sqrt{a+b\sec(c+dx)}dx}{a^2-b^2}-\frac{2b\tan(c+dx)}{d\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \sec(c+dx)\sqrt{a+b\sec(c+dx)}dx}{a^2-b^2}-\frac{2b\tan(c+dx)}{d\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}$$

$\Big\downarrow$ 3042

$$\frac{\int \csc\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2-b^2}-\frac{2b\tan(c+dx)}{d\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}$$

$\Big\downarrow$ 4316

$$\frac{(a-b)\int \frac{\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx+b\int \frac{\sec(c+dx)(\sec(c+dx)+1)}{\sqrt{a+b\sec(c+dx)}}dx}{a^2-b^2}-\frac{2b\tan(c+dx)}{d\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}$$

$\Big\downarrow$ 3042

$$\frac{(a-b)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+b\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2-b^2}-\frac{2b\tan(c+dx)}{d\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}$$

$\Big\downarrow$ 4319

$$\frac{b\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2(a-b)\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right.}{bd}}{a^2-b^2}$$

$$-\frac{2b\tan(c+dx)}{d\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}$$

$\Big\downarrow$ 4492

$$\frac{\frac{2(a-b)\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{bd}-\frac{2(a-b)\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx}{a+b}}}{}}{a^2-b^2}$$

$$-\frac{2b\tan(c+dx)}{d\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}$$

input
```
Int[Sec[c + d*x]/(a + b*Sec[c + d*x])^(3/2),x]
```

3.566. $\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$

output
```
((-2*(a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c +
d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*
Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(b*d) + (2*(a - b)*Sqrt[a + b]*Co
t[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)
/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]
))/(a - b))])/(b*d))/(a^2 - b^2) - (2*b*Tan[c + d*x])/((a^2 - b^2)*d*Sqrt[
a + b*Sec[c + d*x]])
```

3.566.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4316
```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(a - b)   Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x]
+ Simp[b   Int[Csc[e + f*x]*((1 + Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]),
x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4319
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4320
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[(-b)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(f*(m + 1)*
(a^2 - b^2))), x] + Simp[1/((m + 1)*(a^2 - b^2))   Int[Csc[e + f*x]*(a + b*
Csc[e + f*x])^(m + 1)*(a*(m + 1) - b*(m + 2)*Csc[e + f*x]), x], x] /; FreeQ
[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

rule 4492
```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

3.566.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1081 vs. $2(216) = 432$.

Time = 3.96 (sec) , antiderivative size = 1082, normalized size of antiderivative = 4.58

method	result	size
default	Expression too large to display	1082

input
```
int(sec(d*x+c)/(a+b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/d/(a-b)/(a+b)*(EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*(cos
(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1
/2)*a*cos(d*x+c)^2+EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*(c
os(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^
(1/2)*b*cos(d*x+c)^2-EllipticF(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*
(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1)
)^(1/2)*a*cos(d*x+c)^2-EllipticF(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2)
)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+
1))^(1/2)*b*cos(d*x+c)^2+2*EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(
1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x
+c)+1))^(1/2)*a*cos(d*x+c)+2*EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))
^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d
*x+c)+1))^(1/2)*b*cos(d*x+c)-2*EllipticF(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b
))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos
(d*x+c)+1))^(1/2)*a*cos(d*x+c)-2*EllipticF(cot(d*x+c)-csc(d*x+c),((a-b)/(a
+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(c
os(d*x+c)+1))^(1/2)*b*cos(d*x+c)+EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a
+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(c
os(d*x+c)+1))^(1/2)*a+EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))
*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c...
```

3.566.5 Fricas [F]

$$\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\sec(dx+c)}{(b\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)/(b^2*sec(d*x + c)^2 + 2*a*b
*sec(d*x + c) + a^2), x)
```

3.566.6 Sympy [F]

$$\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)/(a+b*sec(d*x+c))**(3/2),x)
```

output
```
Integral(sec(c + d*x)/(a + b*sec(c + d*x))**(3/2), x)
```

3.566.7 Maxima [F]

$$\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\sec(dx+c)}{(b\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)/(b*sec(d*x + c) + a)^(3/2), x)
```

3.566.8 Giac [F]

$$\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\sec(dx+c)}{(b\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)/(b*sec(d*x + c) + a)^(3/2), x)
```

3.566.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{1}{\cos(c+d\,x)\,\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

input
```
int(1/(cos(c + d*x)*(a + b/cos(c + d*x))^(3/2)),x)
```

output
```
int(1/(cos(c + d*x)*(a + b/cos(c + d*x))^(3/2)), x)
```

3.567 $\int \frac{1}{(a+b\sec(c+dx))^{3/2}}\,dx$

3.567.1 Optimal result 4277
3.567.2 Mathematica [B] (verified) 4278
3.567.3 Rubi [A] (verified) 4279
3.567.4 Maple [B] (warning: unable to verify) 4282
3.567.5 Fricas [F] 4283
3.567.6 Sympy [F] 4284
3.567.7 Maxima [F] 4284
3.567.8 Giac [F] 4284
3.567.9 Mupad [F(-1)] 4285

3.567.1 Optimal result

Integrand size = 14, antiderivative size = 347

$$\int \frac{1}{(a+b\sec(c+dx))^{3/2}}\,dx = \frac{2\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{a\sqrt{a+b}d} - \frac{2\cot(c+dx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{a\sqrt{a+b}d} - \frac{2\sqrt{a+b}\cot(c+dx)\,\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{a^2d} + \frac{2b^2\tan(c+dx)}{a\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}}$$

output
```
2*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1
/2))*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/a/d/(a
+b)^(1/2)-2*cot(d*x+c)*EllipticF((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)
/(a-b))^(1/2))*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1
/2)/a/d/(a+b)^(1/2)-2*cot(d*x+c)*EllipticPi((a+b*sec(d*x+c))^(1/2)/(a+b)^(
1/2),(a+b)/a,((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/
2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/a^2/d+2*b^2*tan(d*x+c)/a/(a^2-b^2)/d/(a
+b*sec(d*x+c))^(1/2)
```

3.567.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 972 vs. $2(347) = 694$.

Time = 16.92 (sec) , antiderivative size = 972, normalized size of antiderivative = 2.80

$$\int \frac{1}{(a+b\sec(c+dx))^{3/2}}\,dx = \frac{(b+a\cos(c+dx))^2\sec^2(c+dx)\left(\frac{2b\sin(c+dx)}{a(-a^2+b^2)}+\frac{2b^2\sin(c+dx)}{a(a^2-b^2)(b+a\cos(c+dx))}\right)}{d(a+b\sec(c+dx))^{3/2}} + \frac{2(b+a\cos(c+dx))^{3/2}\sec^{\frac{3}{2}}(c+dx)\sqrt{\frac{a+b-a\tan^2\left(\frac{1}{2}(c+dx)\right)+b\tan^2\left(\frac{1}{2}(c+dx)\right)}{1+\tan^2\left(\frac{1}{2}(c+dx)\right)}}\left(ab\tan\left(\frac{1}{2}(c+dx)\right)+b^2\tan\left(\frac{1}{2}(c+\right.\right.}{}$$

input
```
Integrate[(a + b*Sec[c + d*x])^(-3/2),x]
```

output
```
((b + a*Cos[c + d*x])^2*Sec[c + d*x]^2*((2*b*Sin[c + d*x])/(a*(-a^2 + b^2)
) + (2*b^2*Sin[c + d*x])/(a*(a^2 - b^2)*(b + a*Cos[c + d*x]))))/(d*(a + b*
Sec[c + d*x])^(3/2)) + (2*(b + a*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(3/2)*Sq
rt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x
)/2]^2)]*(a*b*Tan[(c + d*x)/2] + b^2*Tan[(c + d*x)/2] - 2*a*b*Tan[(c + d*x
)/2]^3 + a*b*Tan[(c + d*x)/2]^5 - b^2*Tan[(c + d*x)/2]^5 + 2*a^2*EllipticP
i[-1, ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]
^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] -
2*b^2*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - T
an[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]
^2)/(a + b)] + 2*a^2*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (a - b)/(a +
 b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(
c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - 2*b^2*EllipticPi[-1, ArcS
in[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c
+ d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a
 + b)] + b*(a + b)*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sq
rt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c
 + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - a*(a + b)*EllipticF[ArcSin
[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan
[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2...
```

3.567.3 Rubi [A] (verified)

Time = 1.29 (sec) , antiderivative size = 376, normalized size of antiderivative = 1.08, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.786, Rules used = {3042, 4272, 27, 3042, 4546, 3042, 4409, 3042, 4271, 4319, 4492}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+b\sec(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{4272}$$

$$\frac{2b^2\tan(c+dx)}{ad\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}-\frac{2\int -\frac{a^2-b\sec(c+dx)a-b^2-b^2\sec^2(c+dx)}{2\sqrt{a+b\sec(c+dx)}}dx}{a\left(a^2-b^2\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{a^2-b\sec(c+dx)a-b^2-b^2\sec^2(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx}{a\left(a^2-b^2\right)}+\frac{2b^2\tan(c+dx)}{ad\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{a^2-b\csc\left(c+dx+\frac{\pi}{2}\right)a-b^2-b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a\left(a^2-b^2\right)}+\frac{2b^2\tan(c+dx)}{ad\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}$$

$$\Big\downarrow \text{4546}$$

$$\frac{\int \frac{a^2-b^2+\left(b^2-ab\right)\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx-b^2\int \frac{\sec(c+dx)(\sec(c+dx)+1)}{\sqrt{a+b\sec(c+dx)}}dx}{a\left(a^2-b^2\right)}+\frac{2b^2\tan(c+dx)}{ad\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{a^2-b^2+\left(b^2-ab\right)\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-b^2\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a\left(a^2-b^2\right)}+\frac{2b^2\tan(c+dx)}{ad\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}$$

$$\Big\downarrow \text{4409}$$

$$\frac{(a^2-b^2)\int \frac{1}{\sqrt{a+b\sec(c+dx)}}dx+b^2\left(-\int \frac{\csc(c+dx+\frac{\pi}{2})(\csc(c+dx+\frac{\pi}{2})+1)}{\sqrt{a+b\csc(c+dx+\frac{\pi}{2})}}dx\right)-b(a-b)\int \frac{\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx}{a(a^2-b^2)}+\frac{2b^2\tan(c+dx)}{ad(a^2-b^2)\sqrt{a+b\sec(c+dx)}}$$

$\downarrow$ 3042

$$\frac{(a^2-b^2)\int \frac{1}{\sqrt{a+b\csc(c+dx+\frac{\pi}{2})}}dx+b^2\left(-\int \frac{\csc(c+dx+\frac{\pi}{2})(\csc(c+dx+\frac{\pi}{2})+1)}{\sqrt{a+b\csc(c+dx+\frac{\pi}{2})}}dx\right)-b(a-b)\int \frac{\csc(c+dx+\frac{\pi}{2})}{\sqrt{a+b\csc(c+dx+\frac{\pi}{2})}}dx}{a(a^2-b^2)}+\frac{2b^2\tan(c+dx)}{ad(a^2-b^2)\sqrt{a+b\sec(c+dx)}}$$

$\downarrow$ 4271

$$\frac{b^2\left(-\int \frac{\csc(c+dx+\frac{\pi}{2})(\csc(c+dx+\frac{\pi}{2})+1)}{\sqrt{a+b\csc(c+dx+\frac{\pi}{2})}}dx\right)-b(a-b)\int \frac{\csc(c+dx+\frac{\pi}{2})}{\sqrt{a+b\csc(c+dx+\frac{\pi}{2})}}dx-\frac{2\sqrt{a+b}(a^2-b^2)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-b}}{}}{a(a^2-b^2)}$$
$$\frac{2b^2\tan(c+dx)}{ad(a^2-b^2)\sqrt{a+b\sec(c+dx)}}$$

$\downarrow$ 4319

$$\frac{b^2\left(-\int \frac{\csc(c+dx+\frac{\pi}{2})(\csc(c+dx+\frac{\pi}{2})+1)}{\sqrt{a+b\csc(c+dx+\frac{\pi}{2})}}dx\right)-\frac{2\sqrt{a+b}(a^2-b^2)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+}}{}\right.\right.}{ad}}{a(a^2-b^2)}$$
$$\frac{2b^2\tan(c+dx)}{ad(a^2-b^2)\sqrt{a+b\sec(c+dx)}}$$

$\downarrow$ 4492

$$\frac{-\frac{2\sqrt{a+b}(a^2-b^2)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{ad}-\frac{2(a-b)\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(}{}}}{}}{}$$
$$\frac{2b^2\tan(c+dx)}{ad(a^2-b^2)\sqrt{a+b\sec(c+dx)}}$$

input
```
Int[(a + b*Sec[c + d*x])^(-3/2),x]
```

3.567. $\int \frac{1}{(a+b\sec(c+dx))^{3/2}}\,dx$

output
```
((2*(a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d
*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*S
qrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/d - (2*(a - b)*Sqrt[a + b]*Cot[c +
 d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a -
 b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a
 - b))])/d - (2*Sqrt[a + b]*(a^2 - b^2)*Cot[c + d*x]*EllipticPi[(a + b)/a,
 ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1
 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(a*d))
/(a*(a^2 - b^2)) + (2*b^2*Tan[c + d*x])/(a*(a^2 - b^2)*d*Sqrt[a + b*Sec[c
+ d*x]])
```

3.567.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4271
```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*(Rt[a
 + b, 2]/(a*d*Cot[c + d*x]))*Sqrt[b*((1 - Csc[c + d*x])/(a + b))]*Sqrt[(-b)
*((1 + Csc[c + d*x])/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[
c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

rule 4272
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[b^2*Cot[
c + d*x]*((a + b*Csc[c + d*x])^(n + 1)/(a*d*(n + 1)*(a^2 - b^2))), x] + Sim
p[1/(a*(n + 1)*(a^2 - b^2))   Int[(a + b*Csc[c + d*x])^(n + 1)*Simp[(a^2 -
b^2)*(n + 1) - a*b*(n + 1)*Csc[c + d*x] + b^2*(n + 2)*Csc[c + d*x]^2, x],
], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && LtQ[n, -1] && Integ
erQ[2*n]
```

rule 4319
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4409
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Simp[c   Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] +
Simp[d   Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

rule 4492
```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

rule 4546
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Simp[C   Int[Csc[e + f*x]*((
1 + Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A
, B, C}, x] && NeQ[a^2 - b^2, 0]
```

3.567.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 1798 vs. $2(318) = 636$.

Time = 6.36 (sec) , antiderivative size = 1799, normalized size of antiderivative = 5.18

method	result	size
default	Expression too large to display	1799

input
```
int(1/(a+b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/d/a/(a+b)/(a-b)*(-((a*(1-cos(d*x+c))^2*csc(d*x+c)^2-b*(1-cos(d*x+c))^2*c
sc(d*x+c)^2-a-b)*((1-cos(d*x+c))^2*csc(d*x+c)^2-1))^(1/2)*(-(1-cos(d*x+c))
^2*csc(d*x+c)^2+1)^(1/2)*(-(a*(1-cos(d*x+c))^2*csc(d*x+c)^2-b*(1-cos(d*x+c
))^2*csc(d*x+c)^2-a-b)/(a+b))^(1/2)*EllipticF(cot(d*x+c)-csc(d*x+c),((a-b)
/(a+b))^(1/2))*a^2-((a*(1-cos(d*x+c))^2*csc(d*x+c)^2-b*(1-cos(d*x+c))^2*cs
c(d*x+c)^2-a-b)*((1-cos(d*x+c))^2*csc(d*x+c)^2-1))^(1/2)*(-(1-cos(d*x+c))^
2*csc(d*x+c)^2+1)^(1/2)*(-(a*(1-cos(d*x+c))^2*csc(d*x+c)^2-b*(1-cos(d*x+c)
)^2*csc(d*x+c)^2-a-b)/(a+b))^(1/2)*EllipticF(cot(d*x+c)-csc(d*x+c),((a-b)/
(a+b))^(1/2))*a*b+((a*(1-cos(d*x+c))^2*csc(d*x+c)^2-b*(1-cos(d*x+c))^2*csc
(d*x+c)^2-a-b)*((1-cos(d*x+c))^2*csc(d*x+c)^2-1))^(1/2)*(-(1-cos(d*x+c))^2
*csc(d*x+c)^2+1)^(1/2)*(-(a*(1-cos(d*x+c))^2*csc(d*x+c)^2-b*(1-cos(d*x+c))
^2*csc(d*x+c)^2-a-b)/(a+b))^(1/2)*EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(
a+b))^(1/2))*a*b+((a*(1-cos(d*x+c))^2*csc(d*x+c)^2-b*(1-cos(d*x+c))^2*csc(
d*x+c)^2-a-b)*((1-cos(d*x+c))^2*csc(d*x+c)^2-1))^(1/2)*(-(1-cos(d*x+c))^2*
csc(d*x+c)^2+1)^(1/2)*(-(a*(1-cos(d*x+c))^2*csc(d*x+c)^2-b*(1-cos(d*x+c))^
2*csc(d*x+c)^2-a-b)/(a+b))^(1/2)*EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a
+b))^(1/2))*b^2+2*((a*(1-cos(d*x+c))^2*csc(d*x+c)^2-b*(1-cos(d*x+c))^2*csc
(d*x+c)^2-a-b)*((1-cos(d*x+c))^2*csc(d*x+c)^2-1))^(1/2)*(-(1-cos(d*x+c))^2
*csc(d*x+c)^2+1)^(1/2)*(-(a*(1-cos(d*x+c))^2*csc(d*x+c)^2-b*(1-cos(d*x+c))
^2*csc(d*x+c)^2-a-b)/(a+b))^(1/2)*EllipticPi(cot(d*x+c)-csc(d*x+c),-1,(...
```

3.567.5 Fricas [F]

$$\int \frac{1}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{1}{(b\sec{(dx+c)}+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sec(d*x + c) + a)/(b^2*sec(d*x + c)^2 + 2*a*b*sec(d*x + c)
 + a^2), x)
```

3.567.6 Sympy [F]

$$\int \frac{1}{(a+b\sec(c+dx))^{3/2}}\,dx=\int \frac{1}{(a+b\sec{(c+dx)})^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b*sec(d*x+c))**(3/2),x)
```

output
```
Integral((a + b*sec(c + d*x))**(-3/2), x)
```

3.567.7 Maxima [F]

$$\int \frac{1}{(a+b\sec(c+dx))^{3/2}}\,dx=\int \frac{1}{(b\sec{(dx+c)}+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^(-3/2), x)
```

3.567.8 Giac [F]

$$\int \frac{1}{(a+b\sec(c+dx))^{3/2}}\,dx=\int \frac{1}{(b\sec{(dx+c)}+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^(-3/2), x)
```

3.567.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\sec(c+dx))^{3/2}}\,dx=\int \frac{1}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

input
```
int(1/(a + b/cos(c + d*x))^(3/2),x)
```

output
```
int(1/(a + b/cos(c + d*x))^(3/2), x)
```

3.568 $\int \frac{\cos(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$

3.568.1 Optimal result 4286
3.568.2 Mathematica [B] (verified) 4287
3.568.3 Rubi [A] (verified) 4287
3.568.4 Maple [B] (verified) 4292
3.568.5 Fricas [F] 4293
3.568.6 Sympy [F] 4293
3.568.7 Maxima [F] 4293
3.568.8 Giac [F] 4294
3.568.9 Mupad [F(-1)] 4294

3.568.1 Optimal result

Integrand size = 21, antiderivative size = 396

$$\int \frac{\cos(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \frac{(a^2-3b^2)\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+s}{}}}{a^2b\sqrt{a+b}d}$$
$$+\frac{(a+3b)\cot(c+dx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{a^2\sqrt{a+b}d}$$
$$+\frac{3b\sqrt{a+b}\cot(c+dx)\,\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{a^3d}$$
$$+\frac{\sin(c+dx)}{ad\sqrt{a+b\sec(c+dx)}}+\frac{b(a^2-3b^2)\tan(c+dx)}{a^2(a^2-b^2)d\sqrt{a+b\sec(c+dx)}}$$

output
```
(a^2-3*b^2)*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)
/(a-b))^(1/2))*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1
/2)/a^2/b/d/(a+b)^(1/2)+(a+3*b)*cot(d*x+c)*EllipticF((a+b*sec(d*x+c))^(1/2
)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+s
ec(d*x+c))/(a-b))^(1/2)/a^2/d/(a+b)^(1/2)+3*b*cot(d*x+c)*EllipticPi((a+b*s
ec(d*x+c))^(1/2)/(a+b)^(1/2),(a+b)/a,((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(
1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/a^3/d+sin(d*x+c
)/a/d/(a+b*sec(d*x+c))^(1/2)+b*(a^2-3*b^2)*tan(d*x+c)/a^2/(a^2-b^2)/d/(a+b
*sec(d*x+c))^(1/2)
```

3.568.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1069 vs. $2(396) = 792$.

Time = 13.57 (sec) , antiderivative size = 1069, normalized size of antiderivative = 2.70

$$\int \frac{\cos(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
Integrate[Cos[c + d*x]/(a + b*Sec[c + d*x])^(3/2),x]
```

output
```
((b + a*Cos[c + d*x])^2*Sec[c + d*x]^2*((-2*b^2*Sin[c + d*x])/(a^2*(-a^2 +
 b^2)) - (2*b^3*Sin[c + d*x])/(a^2*(a^2 - b^2)*(b + a*Cos[c + d*x]))))/(d*
(a + b*Sec[c + d*x])^(3/2)) - ((b + a*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(3/
2)*Sqrt[(1 - Tan[(c + d*x)/2]^2)^(-1)]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2
+ b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2)]*(a^3*Tan[(c + d*x)/2] +
a^2*b*Tan[(c + d*x)/2] - 3*a*b^2*Tan[(c + d*x)/2] - 3*b^3*Tan[(c + d*x)/2]
 - 2*a^3*Tan[(c + d*x)/2]^3 + 6*a*b^2*Tan[(c + d*x)/2]^3 + a^3*Tan[(c + d*
x)/2]^5 - a^2*b*Tan[(c + d*x)/2]^5 - 3*a*b^2*Tan[(c + d*x)/2]^5 + 3*b^3*Ta
n[(c + d*x)/2]^5 - 6*a^2*b*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (a - b
)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2
 + b*Tan[(c + d*x)/2]^2)/(a + b)] + 6*b^3*EllipticPi[-1, ArcSin[Tan[(c + d
*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan
[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - 6*a^2*b*EllipticPi[-1,
ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan
[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2
)/(a + b)] + 6*b^3*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b
)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c
+ d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + (a^3 + a^2*b - 3*a*b^2 - 3*
b^3)*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c
+ d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^...
```

3.568.3 Rubi [A] (verified)

Time = 1.66 (sec) , antiderivative size = 436, normalized size of antiderivative = 1.10, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {3042, 4333, 27, 3042, 4549, 27, 3042, 4546, 3042, 4409, 3042, 4271, 4319, 4492}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \; 4333$$

$$\frac{\int -\frac{3b-b\sec^2(c+dx)}{2(a+b\sec(c+dx))^{3/2}}dx}{a}+\frac{\sin(c+dx)}{ad\sqrt{a+b\sec(c+dx)}}$$

$$\Big\downarrow \; 27$$

$$\frac{\sin(c+dx)}{ad\sqrt{a+b\sec(c+dx)}}-\frac{\int \frac{3b-b\sec^2(c+dx)}{(a+b\sec(c+dx))^{3/2}}dx}{2a}$$

$$\Big\downarrow \; 3042$$

$$\frac{\sin(c+dx)}{ad\sqrt{a+b\sec(c+dx)}}-\frac{\int \frac{3b-b\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{2a}$$

$$\Big\downarrow \; 4549$$

$$\frac{\sin(c+dx)}{ad\sqrt{a+b\sec(c+dx)}}-\frac{-\frac{2\int -\frac{-2a\sec(c+dx)b^2+\left(a^2-3b^2\right)\sec^2(c+dx)b+3\left(a^2-b^2\right)b}{2\sqrt{a+b\sec(c+dx)}}dx}{a(a^2-b^2)}-\frac{2b\left(a^2-3b^2\right)\tan(c+dx)}{ad(a^2-b^2)\sqrt{a+b\sec(c+dx)}}}{2a}$$

$$\Big\downarrow \; 27$$

$$\frac{\sin(c+dx)}{ad\sqrt{a+b\sec(c+dx)}}-\frac{\frac{\int \frac{-2a\sec(c+dx)b^2+\left(a^2-3b^2\right)\sec^2(c+dx)b+3\left(a^2-b^2\right)b}{\sqrt{a+b\sec(c+dx)}}dx}{a(a^2-b^2)}-\frac{2b\left(a^2-3b^2\right)\tan(c+dx)}{ad(a^2-b^2)\sqrt{a+b\sec(c+dx)}}}{2a}$$

$$\Big\downarrow \; 3042$$

$$\frac{\sin(c+dx)}{ad\sqrt{a+b\sec(c+dx)}}-\frac{\frac{\int \frac{-2a\csc\left(c+dx+\frac{\pi}{2}\right)b^2+\left(a^2-3b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2b+3\left(a^2-b^2\right)b}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a(a^2-b^2)}-\frac{2b\left(a^2-3b^2\right)\tan(c+dx)}{ad(a^2-b^2)\sqrt{a+b\sec(c+dx)}}}{2a}$$

$$\Big\downarrow \; 4546$$

3.568. $\int \frac{\cos(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$

$$\frac{\sin(c+dx)}{ad\sqrt{a+b\sec(c+dx)}}-\frac{\frac{b(a^2-3b^2)\int\frac{\sec(c+dx)(\sec(c+dx)+1)}{\sqrt{a+b\sec(c+dx)}}dx+\int\frac{3b(a^2-b^2)+(-2ab^2-(a^2-3b^2)b)\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx}{a(a^2-b^2)}-\frac{2b(a^2-3b^2)\tan(c+dx)}{ad(a^2-b^2)\sqrt{a+b\sec(c+dx)}}}{2a}$$

$\Big\downarrow$ 3042

$$\frac{\sin(c+dx)}{ad\sqrt{a+b\sec(c+dx)}}-\frac{\frac{b(a^2-3b^2)\int\frac{\csc(c+dx+\frac{\pi}{2})(\csc(c+dx+\frac{\pi}{2})+1)}{\sqrt{a+b\csc(c+dx+\frac{\pi}{2})}}dx+\int\frac{3b(a^2-b^2)+(-2ab^2-(a^2-3b^2)b)\csc(c+dx+\frac{\pi}{2})}{\sqrt{a+b\csc(c+dx+\frac{\pi}{2})}}dx}{a(a^2-b^2)}-\frac{2b(a^2-3b^2)\tan(c+dx)}{ad(a^2-b^2)\sqrt{a+b\sec(c+dx)}}}{2a}$$

$\Big\downarrow$ 4409

$$\frac{\sin(c+dx)}{ad\sqrt{a+b\sec(c+dx)}}-\frac{\frac{b(a^2-3b^2)\int\frac{\csc(c+dx+\frac{\pi}{2})(\csc(c+dx+\frac{\pi}{2})+1)}{\sqrt{a+b\csc(c+dx+\frac{\pi}{2})}}dx+3b(a^2-b^2)\int\frac{1}{\sqrt{a+b\sec(c+dx)}}dx-b(a-b)(a+3b)\int\frac{\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx}{a(a^2-b^2)}-\frac{2b(a^2-3b^2)\tan(c+dx)}{ad(a^2-b^2)\sqrt{a+b\sec(c+}}}{2a}$$

$\Big\downarrow$ 3042

$$\frac{\sin(c+dx)}{ad\sqrt{a+b\sec(c+dx)}}-\frac{\frac{3b(a^2-b^2)\int\frac{1}{\sqrt{a+b\csc(c+dx+\frac{\pi}{2})}}dx+b(a^2-3b^2)\int\frac{\csc(c+dx+\frac{\pi}{2})(\csc(c+dx+\frac{\pi}{2})+1)}{\sqrt{a+b\csc(c+dx+\frac{\pi}{2})}}dx-b(a-b)(a+3b)\int\frac{\csc(c+dx+\frac{\pi}{2})}{\sqrt{a+b\csc(c+dx+\frac{\pi}{2})}}dx}{a(a^2-b^2)}-\frac{2b(a^2-3b^2)\text{ta}}{ad(a^2-b^2)\sqrt{a+b}}}{2a}$$

$\Big\downarrow$ 4271

$$\frac{\sin(c+dx)}{ad\sqrt{a+b\sec(c+dx)}}-\frac{\frac{b(a^2-3b^2)\int\frac{\csc(c+dx+\frac{\pi}{2})(\csc(c+dx+\frac{\pi}{2})+1)}{\sqrt{a+b\csc(c+dx+\frac{\pi}{2})}}dx-b(a-b)(a+3b)\int\frac{\csc(c+dx+\frac{\pi}{2})}{\sqrt{a+b\csc(c+dx+\frac{\pi}{2})}}dx-\frac{6b\sqrt{a+b}(a^2-b^2)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)}{a-b}}}{ad}}{a(a^2-b^2)}}{2a}$$

$\Big\downarrow$ 4319

$$\frac{\sin(c+dx)}{ad\sqrt{a+b\sec(c+dx)}}-\frac{\frac{b(a^2-3b^2)\int\frac{\csc(c+dx+\frac{\pi}{2})(\csc(c+dx+\frac{\pi}{2})+1)}{\sqrt{a+b\csc(c+dx+\frac{\pi}{2})}}dx-\frac{6b\sqrt{a+b}(a^2-b^2)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right.\right.}{ad}}{a(a^2-b^2)}}{2a}$$

3.568. $\int \frac{\cos(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$

$$\Big\downarrow \; 4492$$

$$\frac{\sin(c+dx)}{ad\sqrt{a+b\sec(c+dx)}} - \frac{-\frac{2(a-b)\sqrt{a+b}\left(a^2-3b^2\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}E\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{bd} - \frac{6b\sqrt{a+b}\left(a^2-b^2\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{}}{}}{}$$

input
```
Int[Cos[c + d*x]/(a + b*Sec[c + d*x])^(3/2),x]
```

output
```
Sin[c + d*x]/(a*d*Sqrt[a + b*Sec[c + d*x]]) - (((-2*(a - b)*Sqrt[a + b]*(a
^2 - 3*b^2)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a
+ b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1
+ Sec[c + d*x]))/(a - b))])/(b*d) - (2*(a - b)*Sqrt[a + b]*(a + 3*b)*Cot[c
 + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a
 - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/
(a - b))])/d - (6*b*Sqrt[a + b]*(a^2 - b^2)*Cot[c + d*x]*EllipticPi[(a + b
)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(
b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(a
*d))/(a*(a^2 - b^2)) - (2*b*(a^2 - 3*b^2)*Tan[c + d*x])/(a*(a^2 - b^2)*d*S
qrt[a + b*Sec[c + d*x]]))/(2*a)
```

3.568.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4271
```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*(Rt[a
 + b, 2]/(a*d*Cot[c + d*x]))*Sqrt[b*((1 - Csc[c + d*x])/(a + b))]*Sqrt[(-b)
*((1 + Csc[c + d*x])/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[
c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

rule 4319
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4333
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d*
Csc[e + f*x])^n/(a*f*n)), x] - Simp[1/(a*d*n)   Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[b*(m + n + 1) - a*(n + 1)*Csc[e + f*x] - b*(m
 + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^
2 - b^2, 0] && ILtQ[m + 1/2, 0] && ILtQ[n, 0]
```

rule 4409
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Simp[c   Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] +
Simp[d   Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

rule 4492
```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

rule 4546
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Simp[C   Int[Csc[e + f*x]*((
1 + Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A
, B, C}, x] && NeQ[a^2 - b^2, 0]
```

3.568. $\int \frac{\cos(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$

rule 4549
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_))^(m_), x_Symbol] :> Simp[(A*b^2 + a^2*C)*Cot[e + f*x]*((a + b*Csc[
e + f*x])^(m + 1)/(a*f*(m + 1)*(a^2 - b^2))), x] + Simp[1/(a*(m + 1)*(a^2 -
 b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*Simp[A*(a^2 - b^2)*(m + 1) - a*b*
(A + C)*(m + 1)*Csc[e + f*x] + (A*b^2 + a^2*C)*(m + 2)*Csc[e + f*x]^2, x],
x], x] /; FreeQ[{a, b, e, f, A, C}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[2*m
] && LtQ[m, -1]
```

3.568.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2219 vs. $2(365) = 730$.

Time = 8.41 (sec) , antiderivative size = 2220, normalized size of antiderivative = 5.61

method	result	size
default	Expression too large to display	2220

input
```
int(cos(d*x+c)/(a+b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/d/a^2/(a+b)/(a-b)*(3*b^3*cos(d*x+c)*sin(d*x+c)-a^3*cos(d*x+c)^2*sin(d*x
+c)+EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*(1/(a+b)*(b+a*cos
(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*a^2*b-3*E
llipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+
c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*a*b^2+(1/(a+b)
*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*
EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*a^3*cos(d*x+c)^2-3*(1
/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^
(1/2)*EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*b^3*cos(d*x+c)^
2+2*EllipticF(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*(1/(a+b)*(b+a*cos
(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*a^2*b+2*E
llipticF(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+
c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*a*b^2+2*(1/(a+
b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2
)*EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*a^3*cos(d*x+c)-6*(1
/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^
(1/2)*EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*b^3*cos(d*x+c)-
6*EllipticPi(cot(d*x+c)-csc(d*x+c),-1,((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(co
s(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^2*b*c
os(d*x+c)^2-12*EllipticPi(cot(d*x+c)-csc(d*x+c),-1,((a-b)/(a+b))^(1/2))...
```

3.568.5 Fricas [F]

$$\int \frac{\cos(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\cos(dx+c)}{(b\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sec(d*x + c) + a)*cos(d*x + c)/(b^2*sec(d*x + c)^2 + 2*a*b
*sec(d*x + c) + a^2), x)
```

3.568.6 Sympy [F]

$$\int \frac{\cos(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\cos(c+dx)}{(a+b\sec(c+dx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)/(a+b*sec(d*x+c))**(3/2),x)
```

output
```
Integral(cos(c + d*x)/(a + b*sec(c + d*x))**(3/2), x)
```

3.568.7 Maxima [F]

$$\int \frac{\cos(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\cos(dx+c)}{(b\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate(cos(d*x + c)/(b*sec(d*x + c) + a)^(3/2), x)
```

3.568.8 Giac [F]

$$\int \frac{\cos(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\cos(dx+c)}{(b\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate(cos(d*x + c)/(b*sec(d*x + c) + a)^(3/2), x)
```

3.568.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\cos(c+d\,x)}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

input
```
int(cos(c + d*x)/(a + b/cos(c + d*x))^(3/2),x)
```

output
```
int(cos(c + d*x)/(a + b/cos(c + d*x))^(3/2), x)
```

3.569 $\int \frac{\cos^2(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$

3.569.1 Optimal result 4295
3.569.2 Mathematica [B] (verified) 4296
3.569.3 Rubi [A] (verified) 4296
3.569.4 Maple [B] (verified) 4302
3.569.5 Fricas [F] 4303
3.569.6 Sympy [F] 4303
3.569.7 Maxima [F] 4303
3.569.8 Giac [F] 4304
3.569.9 Mupad [F(-1)] 4304

3.569.1 Optimal result

Integrand size = 23, antiderivative size = 470

$$\int \frac{\cos^2(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = -\frac{(7a^2-15b^2)\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{4a^3\sqrt{a+b}d} + \frac{(2a^2-5ab-15b^2)\cot(c+dx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{4a^3\sqrt{a+b}d} - \frac{\sqrt{a+b}(4a^2+15b^2)\cot(c+dx)\,\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+d}{a-b}}}{4a^4d} - \frac{5b\sin(c+dx)}{4a^2d\sqrt{a+b\sec(c+dx)}} + \frac{\cos(c+dx)\sin(c+dx)}{2ad\sqrt{a+b\sec(c+dx)}} - \frac{b^2(7a^2-15b^2)\tan(c+dx)}{4a^3(a^2-b^2)d\sqrt{a+b\sec(c+dx)}}$$

output
```
-1/4*(7*a^2-15*b^2)*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2
),((a+b)/(a-b))^(1/2))*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(
a-b))^(1/2)/a^3/d/(a+b)^(1/2)+1/4*(2*a^2-5*a*b-15*b^2)*cot(d*x+c)*Elliptic
F((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(b*(1-sec(d*x+c)
)/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/a^3/d/(a+b)^(1/2)-1/4*(4*a^
2+15*b^2)*cot(d*x+c)*EllipticPi((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),(a+b)/a
,((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+se
c(d*x+c))/(a-b))^(1/2)/a^4/d-5/4*b*sin(d*x+c)/a^2/d/(a+b*sec(d*x+c))^(1/2)
+1/2*cos(d*x+c)*sin(d*x+c)/a/d/(a+b*sec(d*x+c))^(1/2)-1/4*b^2*(7*a^2-15*b^
2)*tan(d*x+c)/a^3/(a^2-b^2)/d/(a+b*sec(d*x+c))^(1/2)
```

3.569.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1330 vs. $2(470) = 940$.

Time = 12.99 (sec) , antiderivative size = 1330, normalized size of antiderivative = 2.83

$$\int \frac{\cos^2(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
Integrate[Cos[c + d*x]^2/(a + b*Sec[c + d*x])^(3/2),x]
```

output
```
((b + a*Cos[c + d*x])^2*Sec[c + d*x]^2*((2*b^3*Sin[c + d*x])/(a^3*(-a^2 +
b^2)) + (2*b^4*Sin[c + d*x])/(a^3*(a^2 - b^2)*(b + a*Cos[c + d*x])) + Sin[
2*(c + d*x)]/(4*a^2)))/(d*(a + b*Sec[c + d*x])^(3/2)) + ((b + a*Cos[c + d*
x])^(3/2)*Sec[c + d*x]^(3/2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c
 + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2)]*(-7*a^3*b*Tan[(c + d*x)/2] - 7*a^2
*b^2*Tan[(c + d*x)/2] + 15*a*b^3*Tan[(c + d*x)/2] + 15*b^4*Tan[(c + d*x)/2
] + 14*a^3*b*Tan[(c + d*x)/2]^3 - 30*a*b^3*Tan[(c + d*x)/2]^3 - 7*a^3*b*Ta
n[(c + d*x)/2]^5 + 7*a^2*b^2*Tan[(c + d*x)/2]^5 + 15*a*b^3*Tan[(c + d*x)/2
]^5 - 15*b^4*Tan[(c + d*x)/2]^5 + 8*a^4*EllipticPi[-1, ArcSin[Tan[(c + d*x
)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(
c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + 22*a^2*b^2*EllipticPi[-1,
 ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*S
qrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - 30*b^
4*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(
c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/
(a + b)] + 8*a^4*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]
*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c +
d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + 22*a^2*b^2*EllipticPi[-1, Arc
Sin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c
 + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2...
```

3.569.3 Rubi [A] (verified)

Time = 2.14 (sec) , antiderivative size = 504, normalized size of antiderivative = 1.07, number of steps used = 17, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.739$, Rules used = {3042, 4333, 27, 3042, 4592, 27, 3042, 4548, 27, 3042, 4546, 3042, 4409, 3042, 4271, 4319, 4492}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^2(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^2\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \; 4333$$

$$\frac{\int -\frac{\cos(c+dx)\left(-3b\sec^2(c+dx)-2a\sec(c+dx)+5b\right)}{2(a+b\sec(c+dx))^{3/2}}dx}{2a}+\frac{\sin(c+dx)\cos(c+dx)}{2ad\sqrt{a+b\sec(c+dx)}}$$

$$\Big\downarrow \; 27$$

$$\frac{\sin(c+dx)\cos(c+dx)}{2ad\sqrt{a+b\sec(c+dx)}}-\frac{\int \frac{\cos(c+dx)\left(-3b\sec^2(c+dx)-2a\sec(c+dx)+5b\right)}{(a+b\sec(c+dx))^{3/2}}dx}{4a}$$

$$\Big\downarrow \; 3042$$

$$\frac{\sin(c+dx)\cos(c+dx)}{2ad\sqrt{a+b\sec(c+dx)}}-\frac{\int \frac{-3b\csc\left(c+dx+\frac{\pi}{2}\right)^2-2a\csc\left(c+dx+\frac{\pi}{2}\right)+5b}{\csc\left(c+dx+\frac{\pi}{2}\right)\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{4a}$$

$$\Big\downarrow \; 4592$$

$$\frac{\sin(c+dx)\cos(c+dx)}{2ad\sqrt{a+b\sec(c+dx)}}-\frac{\frac{5b\sin(c+dx)}{ad\sqrt{a+b\sec(c+dx)}}-\frac{\int \frac{4a^2+6b\sec(c+dx)a+15b^2-5b^2\sec^2(c+dx)}{2(a+b\sec(c+dx))^{3/2}}dx}{a}}{4a}$$

$$\Big\downarrow \; 27$$

$$\frac{\sin(c+dx)\cos(c+dx)}{2ad\sqrt{a+b\sec(c+dx)}}-\frac{\frac{5b\sin(c+dx)}{ad\sqrt{a+b\sec(c+dx)}}-\frac{\int \frac{4a^2+6b\sec(c+dx)a+15b^2-5b^2\sec^2(c+dx)}{(a+b\sec(c+dx))^{3/2}}dx}{2a}}{4a}$$

$$\Big\downarrow \; 3042$$

$$\frac{\sin(c+dx)\cos(c+dx)}{2ad\sqrt{a+b\sec(c+dx)}}-\frac{\frac{5b\sin(c+dx)}{ad\sqrt{a+b\sec(c+dx)}}-\frac{\int \frac{4a^2+6b\csc\left(c+dx+\frac{\pi}{2}\right)a+15b^2-5b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{2a}}{4a}$$

$$\Big\downarrow \; 4548$$

3.569. $\int \frac{\cos^2(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$

$$\frac{\frac{5b\sin(c+dx)}{ad\sqrt{a+b\sec(c+dx)}}-\frac{-\frac{2\int -\frac{b^2\left(7a^2-15b^2\right)\sec^2(c+dx)+2ab\left(a^2-5b^2\right)\sec(c+dx)+\left(a^2-b^2\right)\left(4a^2+15b^2\right)}{2\sqrt{a+b\sec(c+dx)}}dx}{a\left(a^2-b^2\right)}-\frac{2b^2\left(7a^2-15b^2\right)\tan(c+dx)}{ad\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}}{2a}}{4a}-\frac{\sin(c+dx)\cos(c+dx)}{2ad\sqrt{a+b\sec(c+dx)}}$$

$\downarrow$ 27

$$\frac{\frac{5b\sin(c+dx)}{ad\sqrt{a+b\sec(c+dx)}}-\frac{\frac{\int\frac{4a^4+11b^2a^2+2b\left(a^2-5b^2\right)\sec(c+dx)a-15b^4+b^2\left(7a^2-15b^2\right)\sec^2(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx}{a\left(a^2-b^2\right)}-\frac{2b^2\left(7a^2-15b^2\right)\tan(c+dx)}{ad\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}}{2a}}{4a}-\frac{\sin(c+dx)\cos(c+dx)}{2ad\sqrt{a+b\sec(c+dx)}}$$

$\downarrow$ 3042

$$\frac{\frac{5b\sin(c+dx)}{ad\sqrt{a+b\sec(c+dx)}}-\frac{\frac{\int\frac{4a^4+11b^2a^2+2b\left(a^2-5b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)a-15b^4+b^2\left(7a^2-15b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a\left(a^2-b^2\right)}-\frac{2b^2\left(7a^2-15b^2\right)\tan(c+dx)}{ad\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}}{2a}}{4a}-\frac{\sin(c+dx)\cos(c+dx)}{2ad\sqrt{a+b\sec(c+dx)}}$$

$\downarrow$ 4546

$$\frac{\frac{5b\sin(c+dx)}{ad\sqrt{a+b\sec(c+dx)}}-\frac{\frac{b^2\left(7a^2-15b^2\right)\int\frac{\sec(c+dx)(\sec(c+dx)+1)}{\sqrt{a+b\sec(c+dx)}}dx+\int\frac{4a^4+11b^2a^2-15b^4+\left(2ab\left(a^2-5b^2\right)-b^2\left(7a^2-15b^2\right)\right)\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx}{a\left(a^2-b^2\right)}-\frac{2b^2\left(7a^2-15b^2\right)}{ad\left(a^2-b^2\right)\sqrt{a+b}}}{2a}}{4a}-\frac{\sin(c+dx)\cos(c+dx)}{2ad\sqrt{a+b\sec(c+dx)}}$$

$\downarrow$ 3042

$$\frac{\frac{5b\sin(c+dx)}{ad\sqrt{a+b\sec(c+dx)}}-\frac{\frac{b^2\left(7a^2-15b^2\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\int\frac{4a^4+11b^2a^2-15b^4+\left(2ab\left(a^2-5b^2\right)-b^2\left(7a^2-15b^2\right)\right)\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a\left(a^2-b^2\right)}-\frac{2b}{ad}}{2a}}{4a}-\frac{\sin(c+dx)\cos(c+dx)}{2ad\sqrt{a+b\sec(c+dx)}}$$

$\downarrow$ 4409

3.569. $\int \frac{\cos^2(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$

$$\frac{\sin(c+dx)\cos(c+dx)}{2ad\sqrt{a+b\sec(c+dx)}}-\frac{\frac{5b\sin(c+dx)}{ad\sqrt{a+b\sec(c+dx)}}-\frac{\frac{b^2\left(7a^2-15b^2\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+b(a-b)\left(2a^2-5ab-15b^2\right)\int\frac{\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx+\left(4a^4+11a^2b^2-15b^4\right)\int\frac{}{\sqrt{a+b}}}{a\left(a^2-b^2\right)}}{2a}}{4a}$$

$\downarrow$ 3042

$$\frac{\sin(c+dx)\cos(c+dx)}{2ad\sqrt{a+b\sec(c+dx)}}-\frac{\frac{5b\sin(c+dx)}{ad\sqrt{a+b\sec(c+dx)}}-\frac{\frac{b^2\left(7a^2-15b^2\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+b(a-b)\left(2a^2-5ab-15b^2\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\left(4a^4+11a^2b^2-15b^4\right)\int\frac{}{\sqrt{}}}{a\left(a^2-b^2\right)}}{2a}}{4a}$$

$\downarrow$ 4271

$$\frac{\sin(c+dx)\cos(c+dx)}{2ad\sqrt{a+b\sec(c+dx)}}-\frac{\frac{5b\sin(c+dx)}{ad\sqrt{a+b\sec(c+dx)}}-\frac{\frac{b^2\left(7a^2-15b^2\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+b(a-b)\left(2a^2-5ab-15b^2\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2\sqrt{a+b}\left(4a^4+11a^2b^2-15b\right.}{}}{a\left(a^2-b^2\right)}}{2a}}{4a}$$

$\downarrow$ 4319

$$\frac{\sin(c+dx)\cos(c+dx)}{2ad\sqrt{a+b\sec(c+dx)}}-\frac{\frac{5b\sin(c+dx)}{ad\sqrt{a+b\sec(c+dx)}}-\frac{\frac{b^2\left(7a^2-15b^2\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2(a-b)\sqrt{a+b}\left(2a^2-5ab-15b^2\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1}{a-b}}}{d}}{}}{}}{}$$

$\downarrow$ 4492

$$\frac{\sin(c+dx)\cos(c+dx)}{2ad\sqrt{a+b\sec(c+dx)}}-\frac{\frac{5b\sin(c+dx)}{ad\sqrt{a+b\sec(c+dx)}}-\frac{\frac{2(a-b)\sqrt{a+b}\left(2a^2-5ab-15b^2\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{d}-\frac{2(a-b)\sqrt{}}{}}{}}{}$$

input
```
Int[Cos[c + d*x]^2/(a + b*Sec[c + d*x])^(3/2),x]
```

3.569. $\int \frac{\cos^2(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$

output
```
(Cos[c + d*x]*Sin[c + d*x])/(2*a*d*Sqrt[a + b*Sec[c + d*x]]) - ((5*b*Sin[c
 + d*x])/(a*d*Sqrt[a + b*Sec[c + d*x]]) - (((-2*(a - b)*Sqrt[a + b]*(7*a^2
 - 15*b^2)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a +
 b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 +
 Sec[c + d*x]))/(a - b))])/d + (2*(a - b)*Sqrt[a + b]*(2*a^2 - 5*a*b - 15*
b^2)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]],
(a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c
 + d*x]))/(a - b))])/d - (2*Sqrt[a + b]*(4*a^4 + 11*a^2*b^2 - 15*b^4)*Cot[
c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]
], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Se
c[c + d*x]))/(a - b))])/(a*d))/(a*(a^2 - b^2)) - (2*b^2*(7*a^2 - 15*b^2)*T
an[c + d*x])/(a*(a^2 - b^2)*d*Sqrt[a + b*Sec[c + d*x]]))/(2*a))/(4*a)
```

3.569.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4271
```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*(Rt[a
 + b, 2]/(a*d*Cot[c + d*x]))*Sqrt[b*((1 - Csc[c + d*x])/(a + b))]*Sqrt[(-b)
*((1 + Csc[c + d*x])/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[
c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

rule 4319
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4333
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d*
Csc[e + f*x])^n/(a*f*n)), x] - Simp[1/(a*d*n)   Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[b*(m + n + 1) - a*(n + 1)*Csc[e + f*x] - b*(m
 + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^
2 - b^2, 0] && ILtQ[m + 1/2, 0] && ILtQ[n, 0]
```

rule 4409
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Simp[c   Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] +
Simp[d   Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

rule 4492
```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

rule 4546
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Simp[C   Int[Csc[e + f*x]*((
1 + Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A
, B, C}, x] && NeQ[a^2 - b^2, 0]
```

rule 4548
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(A*b^2 -
a*b*B + a^2*C)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(a*f*(m + 1)*(a^2
 - b^2))), x] + Simp[1/(a*(m + 1)*(a^2 - b^2))   Int[(a + b*Csc[e + f*x])^(
m + 1)*Simp[A*(a^2 - b^2)*(m + 1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x
] + (A*b^2 - a*b*B + a^2*C)*(m + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

3.569. $\int \frac{\cos^2(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$

rule 4592
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Simp[1/(a*d*n)   Int[(a + b*Csc[e + f*x])^m
*(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*
Csc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

3.569.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3125 vs. $2(425) = 850$.

Time = 8.34 (sec) , antiderivative size = 3126, normalized size of antiderivative = 6.65

method	result	size
default	Expression too large to display	3126

input
```
int(cos(d*x+c)^2/(a+b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/4/d/a^3/(a+b)/(a-b)*(-44*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a
*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi(cot(d*x+c)-csc(d*x+c),-1,((a
-b)/(a+b))^(1/2))*a^2*b^2*cos(d*x+c)-22*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*
(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi(cot(d*x+c)-csc(
d*x+c),-1,((a-b)/(a+b))^(1/2))*a^2*b^2*cos(d*x+c)^2-22*EllipticPi(cot(d*x+
c)-csc(d*x+c),-1,((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1
/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^2*b^2+60*(cos(d*x+c)/(cos(
d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticP
i(cot(d*x+c)-csc(d*x+c),-1,((a-b)/(a+b))^(1/2))*b^4*cos(d*x+c)-30*(cos(d*x
+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*
EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*b^4*cos(d*x+c)+4*(cos
(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1
/2)*EllipticF(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*a^4*cos(d*x+c)^2-
8*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+
1))^(1/2)*EllipticPi(cot(d*x+c)-csc(d*x+c),-1,((a-b)/(a+b))^(1/2))*a^4*cos
(d*x+c)^2+30*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(
cos(d*x+c)+1))^(1/2)*EllipticPi(cot(d*x+c)-csc(d*x+c),-1,((a-b)/(a+b))^(1/
2))*b^4*cos(d*x+c)^2-15*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*co
s(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+
b))^(1/2))*b^4*cos(d*x+c)^2+8*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b...
```

3.569.5 Fricas [F]

$$\int \frac{\cos^2(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\cos{(dx+c)}^2}{(b\sec{(dx+c)}+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^2/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sec(d*x + c) + a)*cos(d*x + c)^2/(b^2*sec(d*x + c)^2 + 2*a
*b*sec(d*x + c) + a^2), x)
```

3.569.6 Sympy [F]

$$\int \frac{\cos^2(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\cos^2{(c+dx)}}{(a+b\sec{(c+dx)})^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)**2/(a+b*sec(d*x+c))**(3/2),x)
```

output
```
Integral(cos(c + d*x)**2/(a + b*sec(c + d*x))**(3/2), x)
```

3.569.7 Maxima [F]

$$\int \frac{\cos^2(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\cos{(dx+c)}^2}{(b\sec{(dx+c)}+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^2/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate(cos(d*x + c)^2/(b*sec(d*x + c) + a)^(3/2), x)
```

3.569.8 Giac [F]

$$\int \frac{\cos^2(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\cos{(dx+c)}^2}{(b\sec{(dx+c)}+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^2/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate(cos(d*x + c)^2/(b*sec(d*x + c) + a)^(3/2), x)
```

3.569.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^2(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\cos{(c+d\,x)}^2}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

input
```
int(cos(c + d*x)^2/(a + b/cos(c + d*x))^(3/2),x)
```

output
```
int(cos(c + d*x)^2/(a + b/cos(c + d*x))^(3/2), x)
```

3.570 $\int \frac{\sec^5(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$

3.570.1 Optimal result 4305
3.570.2 Mathematica [A] (warning: unable to verify) 4306
3.570.3 Rubi [A] (verified) 4306
3.570.4 Maple [B] (warning: unable to verify) 4311
3.570.5 Fricas [F] 4311
3.570.6 Sympy [F] 4312
3.570.7 Maxima [F(-1)] 4312
3.570.8 Giac [F] 4312
3.570.9 Mupad [F(-1)] 4313

3.570.1 Optimal result

Integrand size = 23, antiderivative size = 427

$$\int \frac{\sec^5(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \frac{8a(4a^4-7a^2b^2+2b^4)\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+}{a+b}}}{3(a-b)b^5(a+b)^{3/2}d}$$
$$+\frac{2(16a^4+12a^3b-16a^2b^2-9ab^3-b^4)\cot(c+dx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{}}{3(a-b)b^4(a+b)^{3/2}d}$$
$$-\frac{2a^2\sec^2(c+dx)\tan(c+dx)}{3b\,(a^2-b^2)\,d(a+b\sec(c+dx))^{3/2}}+\frac{4a^3(3a^2-5b^2)\tan(c+dx)}{3b^3\,(a^2-b^2)^2\,d\sqrt{a+b\sec(c+dx)}}$$
$$+\frac{2(2a^2-b^2)\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{3b^3\,(a^2-b^2)\,d}$$

output
```
8/3*a*(4*a^4-7*a^2*b^2+2*b^4)*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/
(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec
(d*x+c))/(a-b))^(1/2)/(a-b)/b^5/(a+b)^(3/2)/d+2/3*(16*a^4+12*a^3*b-16*a^2*
b^2-9*a*b^3-b^4)*cot(d*x+c)*EllipticF((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),(
(a+b)/(a-b))^(1/2))*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b
))^(1/2)/(a-b)/b^4/(a+b)^(3/2)/d-2/3*a^2*sec(d*x+c)^2*tan(d*x+c)/b/(a^2-b^
2)/d/(a+b*sec(d*x+c))^(3/2)+4/3*a^3*(3*a^2-5*b^2)*tan(d*x+c)/b^3/(a^2-b^2)
^2/d/(a+b*sec(d*x+c))^(1/2)+2/3*(2*a^2-b^2)*(a+b*sec(d*x+c))^(1/2)*tan(d*x
+c)/b^3/(a^2-b^2)/d
```

3.570.2 Mathematica [A] (warning: unable to verify)

Time = 13.95 (sec) , antiderivative size = 578, normalized size of antiderivative = 1.35

$$\int \frac{\sec^5(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \frac{4(b+a\cos(c+dx))^2\sec^{\frac{5}{2}}(c+dx)\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)}\Big(4a(4a^5+4}{}$$

$$+\frac{(b+a\cos(c+dx))^3\sec^3(c+dx)\left(-\frac{8a(4a^4-7a^2b^2+2b^4)\sin(c+dx)}{3b^4(-a^2+b^2)^2}-\frac{2a^3\sin(c+dx)}{3b^2(-a^2+b^2)(b+a\cos(c+dx))^2}-\frac{2(-7a^5\sin(c+dx)+11a}{3b^3(-a^2+b^2)^2(b+a}\right.}{d(a+b\sec(c+dx))^{5/2}}$$

input
```
Integrate[Sec[c + d*x]^5/(a + b*Sec[c + d*x])^(5/2),x]
```

output
```
(4*(b + a*Cos[c + d*x])^2*Sec[c + d*x]^(5/2)*Sqrt[Cos[(c + d*x)/2]^2*Sec[c
 + d*x]]*(4*a*(4*a^5 + 4*a^4*b - 7*a^3*b^2 - 7*a^2*b^3 + 2*a*b^4 + 2*b^5)*
Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(
1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] +
 b*(-16*a^5 - 4*a^4*b + 28*a^3*b^2 + 7*a^2*b^3 - 8*a*b^4 + b^5)*Sqrt[Cos[c
 + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c
+ d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + 2*a*(4*a^
4 - 7*a^2*b^2 + 2*b^4)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^
2*Tan[(c + d*x)/2]))/(3*b^4*(a^2 - b^2)^2*d*Sqrt[Sec[(c + d*x)/2]^2]*(a +
b*Sec[c + d*x])^(5/2)) + ((b + a*Cos[c + d*x])^3*Sec[c + d*x]^3*((-8*a*(4*
a^4 - 7*a^2*b^2 + 2*b^4)*Sin[c + d*x])/(3*b^4*(-a^2 + b^2)^2) - (2*a^3*Sin
[c + d*x])/(3*b^2*(-a^2 + b^2)*(b + a*Cos[c + d*x])^2) - (2*(-7*a^5*Sin[c
+ d*x] + 11*a^3*b^2*Sin[c + d*x]))/(3*b^3*(-a^2 + b^2)^2*(b + a*Cos[c + d*
x])) + (2*Tan[c + d*x])/(3*b^3)))/(d*(a + b*Sec[c + d*x])^(5/2))
```

3.570.3 Rubi [A] (verified)

Time = 1.91 (sec) , antiderivative size = 447, normalized size of antiderivative = 1.05, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.609$, Rules used = {3042, 4332, 27, 3042, 4578, 27, 3042, 4570, 27, 3042, 4493, 3042, 4319, 4492}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^5(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^5}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \; 4332$$

$$-\frac{2\int \frac{\sec^2(c+dx)\left(4a^2-3b\sec(c+dx)a-3\left(2a^2-b^2\right)\sec^2(c+dx)\right)}{2(a+b\sec(c+dx))^{3/2}}dx}{3b\left(a^2-b^2\right)}-\frac{2a^2\tan(c+dx)\sec^2(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \; 27$$

$$-\frac{\int \frac{\sec^2(c+dx)\left(4a^2-3b\sec(c+dx)a-3\left(2a^2-b^2\right)\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^{3/2}}dx}{3b\left(a^2-b^2\right)}-\frac{2a^2\tan(c+dx)\sec^2(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2\left(4a^2-3b\csc\left(c+dx+\frac{\pi}{2}\right)a-3\left(2a^2-b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{3b\left(a^2-b^2\right)}-\frac{2a^2\tan(c+dx)\sec^2(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \; 4578$$

$$-\frac{\frac{2\int \frac{\sec(c+dx)\left(2b\left(3a^2-5b^2\right)a^2+2\left(6a^4-11b^2a^2+3b^4\right)\sec(c+dx)a-3b\left(a^2-b^2\right)\left(2a^2-b^2\right)\sec^2(c+dx)\right)}{2\sqrt{a+b\sec(c+dx)}}dx}{b^2\left(a^2-b^2\right)}-\frac{4a^3\left(3a^2-5b^2\right)\tan(c+dx)}{b^2d\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}}{3b\left(a^2-b^2\right)}-\frac{2a^2\tan(c+dx)\sec^2(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \; 27$$

$$-\frac{\frac{\int \frac{\sec(c+dx)\left(2b\left(3a^2-5b^2\right)a^2+2\left(6a^4-11b^2a^2+3b^4\right)\sec(c+dx)a-3b\left(a^2-b^2\right)\left(2a^2-b^2\right)\sec^2(c+dx)\right)}{\sqrt{a+b\sec(c+dx)}}dx}{b^2\left(a^2-b^2\right)}-\frac{4a^3\left(3a^2-5b^2\right)\tan(c+dx)}{b^2d\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}}{3b\left(a^2-b^2\right)}-\frac{2a^2\tan(c+dx)\sec^2(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(2b\left(3a^2-5b^2\right)a^2+2\left(6a^4-11b^2a^2+3b^4\right)\csc\left(c+dx+\frac{\pi}{2}\right)a-3b\left(a^2-b^2\right)\left(2a^2-b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{b^2\left(a^2-b^2\right)}-\frac{4a^3\left(3a^2-5b^2\right)\tan(c+dx)}{b^2d\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}}{3b\left(a^2-b^2\right)}-\frac{2a^2\tan(c+dx)\sec^2(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \; 4570$$

3.570. $\int \frac{\sec^5(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$

$$-\frac{\frac{\frac{2\int \frac{3\sec(c+dx)\left(\left(4a^4-7b^2a^2-b^4\right)b^2+4a\left(4a^4-7b^2a^2+2b^4\right)\sec(c+dx)b\right)}{2\sqrt{a+b\sec(c+dx)}}dx}{3b}-\frac{2\left(a^2-b^2\right)\left(2a^2-b^2\right)\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{d}}{b^2(a^2-b^2)}-\frac{4a^3\left(3a^2-5b^2\right)\tan(c+d}{b^2d(a^2-b^2)\sqrt{a+b\sec(c+}}}{3b\left(a^2-b^2\right)}$$

$$\frac{2a^2\tan(c+dx)\sec^2(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \; 27$$

$$-\frac{\frac{\frac{\int \frac{\sec(c+dx)\left(\left(4a^4-7b^2a^2-b^4\right)b^2+4a\left(4a^4-7b^2a^2+2b^4\right)\sec(c+dx)b\right)}{\sqrt{a+b\sec(c+dx)}}dx}{b}-\frac{2\left(a^2-b^2\right)\left(2a^2-b^2\right)\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{d}}{b^2(a^2-b^2)}-\frac{4a^3\left(3a^2-5b^2\right)\tan(c+dx)}{b^2d(a^2-b^2)\sqrt{a+b\sec(c+dx}}}{3b\left(a^2-b^2\right)}$$

$$\frac{2a^2\tan(c+dx)\sec^2(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\frac{\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\left(4a^4-7b^2a^2-b^4\right)b^2+4a\left(4a^4-7b^2a^2+2b^4\right)\csc\left(c+dx+\frac{\pi}{2}\right)b\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\frac{2\left(a^2-b^2\right)\left(2a^2-b^2\right)\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{d}}{b^2(a^2-b^2)}-\frac{4a^3\left(3a^2-5b^2\right)\tan}{b^2d(a^2-b^2)\sqrt{a+b\,\mathrm{s}}}}{3b\left(a^2-b^2\right)}$$

$$\frac{2a^2\tan(c+dx)\sec^2(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \; 4493$$

$$-\frac{\frac{\frac{4ab\left(4a^4-7a^2b^2+2b^4\right)\int \frac{\sec(c+dx)(\sec(c+dx)+1)}{\sqrt{a+b\sec(c+dx)}}dx-b(a-b)\left(16a^4+12a^3b-16a^2b^2-9ab^3-b^4\right)\int \frac{\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx}{b}-\frac{2\left(a^2-b^2\right)\left(2a^2-b^2\right)\tan(c+dx)\sqrt{a+b\,\mathrm{se}}}{d}}{b^2(a^2-b^2)}}{3b\left(a^2-b^2\right)}$$

$$\frac{2a^2\tan(c+dx)\sec^2(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\frac{\frac{4ab\left(4a^4-7a^2b^2+2b^4\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-b(a-b)\left(16a^4+12a^3b-16a^2b^2-9ab^3-b^4\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\frac{2\left(a^2-b^2\right)\left(2a^2-b^2\right)\tan}{d}}{b^2(a^2-b^2)}}{3b\left(a^2-b^2\right)}$$

$$\frac{2a^2\tan(c+dx)\sec^2(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \; 4319$$

3.570. $\int \frac{\sec^5(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$

$$-\frac{\frac{\frac{4ab\left(4a^4-7a^2b^2+2b^4\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2(a-b)\sqrt{a+b}\left(16a^4+12a^3b-16a^2b^2-9ab^3-b^4\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1}{a-b}}}{d}}{b}}{b^2(a^2-b^2)}}{3b\left(a^2-b^2\right)}$$

$$\frac{2a^2\tan(c+dx)\sec^2(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \; 4492$$

$$-\frac{2a^2\tan(c+dx)\sec^2(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}-$$

$$-\frac{\frac{\frac{8a(a-b)\sqrt{a+b}\left(4a^4-7a^2b^2+2b^4\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}E\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{bd}-\frac{2(a-b)\sqrt{a+b}\left(16a^4+12a^3b-16a^2b^2-9\right.}{b}}{b^2(a^2-b^2}}{}$$

input
```
Int[Sec[c + d*x]^5/(a + b*Sec[c + d*x])^(5/2),x]
```

output
```
(-2*a^2*Sec[c + d*x]^2*Tan[c + d*x])/(3*b*(a^2 - b^2)*d*(a + b*Sec[c + d*x
])^(3/2)) - ((-4*a^3*(3*a^2 - 5*b^2)*Tan[c + d*x])/(b^2*(a^2 - b^2)*d*Sqrt
[a + b*Sec[c + d*x]]) + (((-8*a*(a - b)*Sqrt[a + b]*(4*a^4 - 7*a^2*b^2 + 2
*b^4)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]],
 (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[
c + d*x]))/(a - b))])/(b*d) - (2*(a - b)*Sqrt[a + b]*(16*a^4 + 12*a^3*b -
16*a^2*b^2 - 9*a*b^3 - b^4)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c
 + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b
)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/d)/b - (2*(a^2 - b^2)*(2*a^2 -
 b^2)*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/d)/(b^2*(a^2 - b^2)))/(3*b*(a
^2 - b^2))
```

3.570.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4319
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4332
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-a^2)*d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(
m + 1)*((d*Csc[e + f*x])^(n - 3)/(b*f*(m + 1)*(a^2 - b^2))), x] + Simp[d^3/
(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])
^(n - 3)*Simp[a^2*(n - 3) + a*b*(m + 1)*Csc[e + f*x] - (a^2*(n - 2) + b^2*(
m + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2
- b^2, 0] && LtQ[m, -1] && (IGtQ[n, 3] || (IntegersQ[n + 1/2, 2*m] && GtQ[n
, 2]))
```

rule 4492
```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

rule 4493
```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(A - B)   Int[Csc[e
 + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Simp[B   Int[Csc[e + f*x]*((1 +
Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A, B}
, x] && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

rule 4570
```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> Simp[(-C)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2)
)), x] + Simp[1/(b*(m + 2))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[
b*A*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /;
 FreeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

3.570. $\int \frac{\sec^5(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$

rule 4578
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[
(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x
_Symbol] :> Simp[a*(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*((a + b*Csc[e + f*x
])^(m + 1)/(b^2*f*(m + 1)*(a^2 - b^2))), x] - Simp[1/(b^2*(m + 1)*(a^2 - b^
2))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[b*(m + 1)*((-a)*(b
*B - a*C) + A*b^2) + (b*B*(a^2 + b^2*(m + 1)) - a*(A*b^2*(m + 2) + C*(a^2 +
 b^2*(m + 1))))*Csc[e + f*x] - b*C*(m + 1)*(a^2 - b^2)*Csc[e + f*x]^2, x],
x], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1
]
```

3.570.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 5025 vs. $2(393) = 786$.

Time = 14.62 (sec) , antiderivative size = 5026, normalized size of antiderivative = 11.77

method	result	size
default	Expression too large to display	5026

input
```
int(sec(d*x+c)^5/(a+b*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.570.5 Fricas [F]

$$\int \frac{\sec^5(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{\sec\left(dx+c\right)^5}{\left(b\sec\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^5/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^5/(b^3*sec(d*x + c)^3 + 3*a
*b^2*sec(d*x + c)^2 + 3*a^2*b*sec(d*x + c) + a^3), x)
```

3.570.6 Sympy [F]

$$\int \frac{\sec^5(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{\sec^5(c+dx)}{(a+b\sec(c+dx))^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)**5/(a+b*sec(d*x+c))**(5/2),x)
```

output
```
Integral(sec(c + d*x)**5/(a + b*sec(c + d*x))**(5/2), x)
```

3.570.7 Maxima [F(-1)]

Timed out.

$$\int \frac{\sec^5(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)^5/(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
Timed out
```

3.570.8 Giac [F]

$$\int \frac{\sec^5(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{\sec(dx+c)^5}{(b\sec(dx+c)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^5/(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^5/(b*sec(d*x + c) + a)^(5/2), x)
```

3.570.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^5(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{1}{\cos\left(c+d\,x\right)^5\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

input
```
int(1/(cos(c + d*x)^5*(a + b/cos(c + d*x))^(5/2)),x)
```

output
```
int(1/(cos(c + d*x)^5*(a + b/cos(c + d*x))^(5/2)), x)
```

3.571 $\int \frac{\sec^4(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$

3.571.1 Optimal result 4314
3.571.2 Mathematica [A] (warning: unable to verify) 4315
3.571.3 Rubi [A] (verified) 4315
3.571.4 Maple [B] (verified) 4319
3.571.5 Fricas [F] 4320
3.571.6 Sympy [F] 4321
3.571.7 Maxima [F(-1)] 4321
3.571.8 Giac [F] 4321
3.571.9 Mupad [F(-1)] 4322

3.571.1 Optimal result

Integrand size = 23, antiderivative size = 362

$$\int \frac{\sec^4(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = -\frac{2(8a^4-15a^2b^2+3b^4)\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{3(a-b)b^4(a+b)^{3/2}d} - \frac{2(8a^3+6a^2b-9ab^2-3b^3)\cot(c+dx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c}{a-b}}}{3(a-b)b^3(a+b)^{3/2}d} - \frac{2a^2\sec(c+dx)\tan(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} - \frac{8a^2(a^2-2b^2)\tan(c+dx)}{3b^2\left(a^2-b^2\right)^2d\sqrt{a+b\sec(c+dx)}}$$

output
```
-2/3*(8*a^4-15*a^2*b^2+3*b^4)*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/
(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec
(d*x+c))/(a-b))^(1/2)/(a-b)/b^4/(a+b)^(3/2)/d-2/3*(8*a^3+6*a^2*b-9*a*b^2-3
*b^3)*cot(d*x+c)*EllipticF((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b)
)^(1/2))*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/(a
-b)/b^3/(a+b)^(3/2)/d-2/3*a^2*sec(d*x+c)*tan(d*x+c)/b/(a^2-b^2)/d/(a+b*sec
(d*x+c))^(3/2)-8/3*a^2*(a^2-2*b^2)*tan(d*x+c)/b^2/(a^2-b^2)^2/d/(a+b*sec(d
*x+c))^(1/2)
```

3.571.2 Mathematica [A] (warning: unable to verify)

Time = 14.47 (sec) , antiderivative size = 556, normalized size of antiderivative = 1.54

$$\int \frac{\sec^4(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \frac{(b+a\cos(c+dx))^3\sec^3(c+dx)\left(\frac{2(8a^4-15a^2b^2+3b^4)\sin(c+dx)}{3b^3(-a^2+b^2)^2}+\frac{2a^2\sin(c+dx)}{3b(-a^2+b^2)(b+a\cos(c}\right.}{d(a+b\sec(c+dx))^{5/2}}$$
$$-\frac{2(b+a\cos(c+dx))^2\sec^{\frac{5}{2}}(c+dx)\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)}\left(2(8a^5+8a^4b-15a^3b^2-15a^2b^3+3ab^4\right.}{}$$

input
```
Integrate[Sec[c + d*x]^4/(a + b*Sec[c + d*x])^(5/2),x]
```

output
```
((b + a*Cos[c + d*x])^3*Sec[c + d*x]^3*((2*(8*a^4 - 15*a^2*b^2 + 3*b^4)*Si
n[c + d*x])/(3*b^3*(-a^2 + b^2)^2) + (2*a^2*Sin[c + d*x])/(3*b*(-a^2 + b^2
)*(b + a*Cos[c + d*x])^2) + (8*(-(a^4*Sin[c + d*x]) + 2*a^2*b^2*Sin[c + d*
x]))/(3*b^2*(-a^2 + b^2)^2*(b + a*Cos[c + d*x]))))/(d*(a + b*Sec[c + d*x])
^(5/2)) - (2*(b + a*Cos[c + d*x])^2*Sec[c + d*x]^(5/2)*Sqrt[Cos[(c + d*x)/
2]^2*Sec[c + d*x]]*(2*(8*a^5 + 8*a^4*b - 15*a^3*b^2 - 15*a^2*b^3 + 3*a*b^4
 + 3*b^5)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/
((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/
(a + b)] - 2*b*(8*a^4 + 2*a^3*b - 15*a^2*b^2 - 6*a*b^3 + 3*b^4)*Sqrt[Cos[c
 + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c
+ d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + (8*a^4 -
15*a^2*b^2 + 3*b^4)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*T
an[(c + d*x)/2]))/(3*b^3*(a^2 - b^2)^2*d*Sqrt[Sec[(c + d*x)/2]^2]*(a + b*S
ec[c + d*x])^(5/2))
```

3.571.3 Rubi [A] (verified)

Time = 1.31 (sec) , antiderivative size = 388, normalized size of antiderivative = 1.07, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules used = {3042, 4332, 27, 3042, 4568, 27, 3042, 4493, 3042, 4319, 4492}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^4(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$$
$$\Big\downarrow \; 3042$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^4}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \; 4332$$

$$-\frac{2\int \frac{\sec(c+dx)\left(2a^2-3b\sec(c+dx)a-\left(4a^2-3b^2\right)\sec^2(c+dx)\right)}{2(a+b\sec(c+dx))^{3/2}}dx}{3b\left(a^2-b^2\right)}-\frac{2a^2\tan(c+dx)\sec(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \; 27$$

$$-\frac{\int \frac{\sec(c+dx)\left(2a^2-3b\sec(c+dx)a-\left(4a^2-3b^2\right)\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^{3/2}}dx}{3b\left(a^2-b^2\right)}-\frac{2a^2\tan(c+dx)\sec(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(2a^2-3b\csc\left(c+dx+\frac{\pi}{2}\right)a+\left(3b^2-4a^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{3b\left(a^2-b^2\right)}-\frac{2a^2\tan(c+dx)\sec(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \; 4568$$

$$-\frac{\frac{8a^2\left(a^2-2b^2\right)\tan(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}-\frac{2\int \frac{\sec(c+dx)\left(2ab\left(a^2-3b^2\right)+\left(8a^4-15b^2a^2+3b^4\right)\sec(c+dx)\right)}{2\sqrt{a+b\sec(c+dx)}}dx}{b\left(a^2-b^2\right)}}{3b\left(a^2-b^2\right)}-\frac{2a^2\tan(c+dx)\sec(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \; 27$$

$$-\frac{\frac{8a^2\left(a^2-2b^2\right)\tan(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}-\frac{\int \frac{\sec(c+dx)\left(2ab\left(a^2-3b^2\right)+\left(8a^4-15b^2a^2+3b^4\right)\sec(c+dx)\right)}{\sqrt{a+b\sec(c+dx)}}dx}{b\left(a^2-b^2\right)}}{3b\left(a^2-b^2\right)}-\frac{2a^2\tan(c+dx)\sec(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\frac{8a^2\left(a^2-2b^2\right)\tan(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}-\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(2ab\left(a^2-3b^2\right)+\left(8a^4-15b^2a^2+3b^4\right)\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{b\left(a^2-b^2\right)}}{3b\left(a^2-b^2\right)}-\frac{2a^2\tan(c+dx)\sec(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \; 4493$$

3.571. $\int \frac{\sec^4(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$

$$-\frac{\frac{8a^2(a^2-2b^2)\tan(c+dx)}{bd(a^2-b^2)\sqrt{a+b\sec(c+dx)}}-\frac{(8a^4-15a^2b^2+3b^4)\int\frac{\sec(c+dx)(\sec(c+dx)+1)}{\sqrt{a+b\sec(c+dx)}}dx-(a-b)(8a^3+6a^2b-9ab^2-3b^3)\int\frac{\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx}{b(a^2-b^2)}}{3b(a^2-b^2)}-\frac{2a^2\tan(c+dx)\sec(c+dx)}{3bd(a^2-b^2)(a+b\sec(c+dx))^{3/2}}$$

$\downarrow$ 3042

$$-\frac{\frac{8a^2(a^2-2b^2)\tan(c+dx)}{bd(a^2-b^2)\sqrt{a+b\sec(c+dx)}}-\frac{(8a^4-15a^2b^2+3b^4)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-(a-b)(8a^3+6a^2b-9ab^2-3b^3)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{b(a^2-b^2)}}{3b(a^2-b^2)}-\frac{2a^2\tan(c+dx)\sec(c+dx)}{3bd(a^2-b^2)(a+b\sec(c+dx))^{3/2}}$$

$\downarrow$ 4319

$$-\frac{\frac{8a^2(a^2-2b^2)\tan(c+dx)}{bd(a^2-b^2)\sqrt{a+b\sec(c+dx)}}-\frac{(8a^4-15a^2b^2+3b^4)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2(a-b)\sqrt{a+b}\left(8a^3+6a^2b-9ab^2-3b^3\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c}{a+b}}}{}}{b(a^2-b^2)}}{3b(a^2-b^2)}-\frac{2a^2\tan(c+dx)\sec(c+dx)}{3bd(a^2-b^2)(a+b\sec(c+dx))^{3/2}}$$

$\downarrow$ 4492

$$-\frac{\frac{8a^2(a^2-2b^2)\tan(c+dx)}{bd(a^2-b^2)\sqrt{a+b\sec(c+dx)}}-\frac{-\frac{2(a-b)\sqrt{a+b}\left(8a^4-15a^2b^2+3b^4\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}E\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{b^2d}-\frac{2(a}{}}{b(a^2}}{3b(a^2-b^2)}$$

input
```
Int[Sec[c + d*x]^4/(a + b*Sec[c + d*x])^(5/2),x]
```

3.571. $\int \frac{\sec^4(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$

output
```
(-2*a^2*Sec[c + d*x]*Tan[c + d*x])/(3*b*(a^2 - b^2)*d*(a + b*Sec[c + d*x])
^(3/2)) - (-(((-2*(a - b)*Sqrt[a + b]*(8*a^4 - 15*a^2*b^2 + 3*b^4)*Cot[c +
 d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a -
 b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a
 - b))])/(b^2*d) - (2*(a - b)*Sqrt[a + b]*(8*a^3 + 6*a^2*b - 9*a*b^2 - 3*b
^3)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (
a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c
+ d*x]))/(a - b))])/(b*d))/(b*(a^2 - b^2))) + (8*a^2*(a^2 - 2*b^2)*Tan[c +
 d*x])/(b*(a^2 - b^2)*d*Sqrt[a + b*Sec[c + d*x]]))/(3*b*(a^2 - b^2))
```

3.571.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4319
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4332
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-a^2)*d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(
m + 1)*((d*Csc[e + f*x])^(n - 3)/(b*f*(m + 1)*(a^2 - b^2))), x] + Simp[d^3/
(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])
^(n - 3)*Simp[a^2*(n - 3) + a*b*(m + 1)*Csc[e + f*x] - (a^2*(n - 2) + b^2*(
m + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2
- b^2, 0] && LtQ[m, -1] && (IGtQ[n, 3] || (IntegersQ[n + 1/2, 2*m] && GtQ[n
, 2]))
```

3.571. $\int \frac{\sec^4(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$

rule 4492
```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

rule 4493
```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(A - B)   Int[Csc[e
 + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Simp[B   Int[Csc[e + f*x]*((1 +
Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A, B}
, x] && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

rule 4568
```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cot[e + f*x]*((a + b*Csc[e + f*x]
)^(m + 1)/(b*f*(m + 1)*(a^2 - b^2))), x] + Simp[1/(b*(m + 1)*(a^2 - b^2))
 Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m
+ 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b - a*B + b*C)*(m + 1))*Csc[e + f*x],
x], x], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^
2, 0]
```

3.571.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 4449 vs. $2(332) = 664$.

Time = 11.24 (sec) , antiderivative size = 4450, normalized size of antiderivative = 12.29

method	result	size
default	Expression too large to display	4450

input
```
int(sec(d*x+c)^4/(a+b*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/3/d/(a-b)^2/(a+b)^2/b^3*(8*EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))
^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d
*x+c)+1))^(1/2)*a^5*b*cos(d*x+c)^3-15*EllipticE(cot(d*x+c)-csc(d*x+c),((a-
b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c
))/(cos(d*x+c)+1))^(1/2)*a^4*b^2*cos(d*x+c)^3-15*EllipticE(cot(d*x+c)-csc(
d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+
a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^3*b^3*cos(d*x+c)^3+3*EllipticE(cot(d
*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1
/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^2*b^4*cos(d*x+c)^3+3*Ellip
ticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1)
)^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a*b^5*cos(d*x+c)^3
-8*EllipticF(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d
*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^5*b*cos(
d*x+c)^3-2*EllipticF(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c
)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^
4*b^2*cos(d*x+c)^3+15*EllipticF(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))
*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1
))^(1/2)*a^3*b^3*cos(d*x+c)^3+6*EllipticF(cot(d*x+c)-csc(d*x+c),((a-b)/(a+
b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(co
s(d*x+c)+1))^(1/2)*a^2*b^4*cos(d*x+c)^3-3*EllipticF(cot(d*x+c)-csc(d*x+...
```

3.571.5 Fricas [F]

$$\int \frac{\sec^4(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{\sec{(dx+c)}^4}{(b\sec{(dx+c)}+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^4/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^4/(b^3*sec(d*x + c)^3 + 3*a
*b^2*sec(d*x + c)^2 + 3*a^2*b*sec(d*x + c) + a^3), x)
```

3.571.6 Sympy [F]

$$\int \frac{\sec^4(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{\sec^4(c+dx)}{(a+b\sec(c+dx))^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)**4/(a+b*sec(d*x+c))**(5/2),x)
```

output
```
Integral(sec(c + d*x)**4/(a + b*sec(c + d*x))**(5/2), x)
```

3.571.7 Maxima [F(-1)]

Timed out.

$$\int \frac{\sec^4(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)^4/(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
Timed out
```

3.571.8 Giac [F]

$$\int \frac{\sec^4(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{\sec(dx+c)^4}{(b\sec(dx+c)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^4/(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^4/(b*sec(d*x + c) + a)^(5/2), x)
```

3.571.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^4(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{1}{\cos\left(c+d\,x\right)^4\,\left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)^{5/2}}\,dx$$

input
```
int(1/(cos(c + d*x)^4*(a + b/cos(c + d*x))^(5/2)),x)
```

output
```
int(1/(cos(c + d*x)^4*(a + b/cos(c + d*x))^(5/2)), x)
```

3.572 $\int \frac{\sec^3(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$

3.572.1 Optimal result 4323
3.572.2 Mathematica [A] (warning: unable to verify) 4324
3.572.3 Rubi [A] (verified) 4324
3.572.4 Maple [B] (verified) 4328
3.572.5 Fricas [F] 4329
3.572.6 Sympy [F] 4330
3.572.7 Maxima [F(-1)] 4330
3.572.8 Giac [F] 4330
3.572.9 Mupad [F(-1)] 4331

3.572.1 Optimal result

Integrand size = 23, antiderivative size = 337

$$\int \frac{\sec^3(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \frac{4a(a^2-3b^2)\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(}{}}}{3(a-b)b^3(a+b)^{3/2}d}$$
$$+\frac{2(2a^2+3ab-3b^2)\cot(c+dx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{3(a-b)b^2(a+b)^{3/2}d}$$
$$-\frac{2a^2\tan(c+dx)}{3b\,(a^2-b^2)\,d(a+b\sec(c+dx))^{3/2}}+\frac{4a(a^2-3b^2)\tan(c+dx)}{3b\,(a^2-b^2)^2\,d\sqrt{a+b\sec(c+dx)}}$$

output
```
4/3*a*(a^2-3*b^2)*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),
((a+b)/(a-b))^(1/2))*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-
b))^(1/2)/(a-b)/b^3/(a+b)^(3/2)/d+2/3*(2*a^2+3*a*b-3*b^2)*cot(d*x+c)*Ellip
ticF((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(b*(1-sec(d*x
+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/(a-b)/b^2/(a+b)^(3/2)/d-
2/3*a^2*tan(d*x+c)/b/(a^2-b^2)/d/(a+b*sec(d*x+c))^(3/2)+4/3*a*(a^2-3*b^2)*
tan(d*x+c)/b/(a^2-b^2)^2/d/(a+b*sec(d*x+c))^(1/2)
```

3.572.2 Mathematica [A] (warning: unable to verify)

Time = 11.58 (sec) , antiderivative size = 503, normalized size of antiderivative = 1.49

$$\int \frac{\sec^3(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \frac{(b+a\cos(c+dx))^3\sec^3(c+dx)\left(\frac{4a(-a^2+3b^2)\sin(c+dx)}{3b^2(-a^2+b^2)^2}-\frac{2a\sin(c+dx)}{3(-a^2+b^2)(b+a\cos(c+dx))^2}\right.}{d(a+b\sec(c+dx))^{5/2}}$$
$$+\frac{4(b+a\cos(c+dx))^2\sec^{\frac{5}{2}}(c+dx)\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)}\left(2a(a^3+a^2b-3ab^2-3b^3)\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\right.}{}$$

input
```
Integrate[Sec[c + d*x]^3/(a + b*Sec[c + d*x])^(5/2),x]
```

output
```
((b + a*Cos[c + d*x])^3*Sec[c + d*x]^3*((4*a*(-a^2 + 3*b^2)*Sin[c + d*x])/
(3*b^2*(-a^2 + b^2)^2) - (2*a*Sin[c + d*x])/(3*(-a^2 + b^2)*(b + a*Cos[c +
 d*x])^2) - (2*(-(a^3*Sin[c + d*x]) + 5*a*b^2*Sin[c + d*x]))/(3*b*(-a^2 +
b^2)^2*(b + a*Cos[c + d*x]))))/(d*(a + b*Sec[c + d*x])^(5/2)) + (4*(b + a*
Cos[c + d*x])^2*Sec[c + d*x]^(5/2)*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(
2*a*(a^3 + a^2*b - 3*a*b^2 - 3*b^3)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*
Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[T
an[(c + d*x)/2]], (a - b)/(a + b)] + b*(-2*a^3 + a^2*b + 6*a*b^2 + 3*b^3)*
Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(
1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] +
 a*(a^2 - 3*b^2)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[
(c + d*x)/2]))/(3*(-(a^2*b) + b^3)^2*d*Sqrt[Sec[(c + d*x)/2]^2]*(a + b*Sec
[c + d*x])^(5/2))
```

3.572.3 Rubi [A] (verified)

Time = 1.21 (sec) , antiderivative size = 356, normalized size of antiderivative = 1.06, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules used = {3042, 4326, 27, 3042, 4491, 27, 3042, 4493, 3042, 4319, 4492}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^3(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$$
$$\Big\downarrow \; 3042$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^3}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \; 4326$$

$$-\frac{2\int -\frac{\sec(c+dx)\left(3ab+\left(2a^2-3b^2\right)\sec(c+dx)\right)}{2(a+b\sec(c+dx))^{3/2}}dx}{3b\left(a^2-b^2\right)}-\frac{2a^2\tan(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{\sec(c+dx)\left(3ab+\left(2a^2-3b^2\right)\sec(c+dx)\right)}{(a+b\sec(c+dx))^{3/2}}dx}{3b\left(a^2-b^2\right)}-\frac{2a^2\tan(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \; 3042$$

$$\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(3ab+\left(2a^2-3b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{3b\left(a^2-b^2\right)}-\frac{2a^2\tan(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \; 4491$$

$$\frac{\frac{4a\left(a^2-3b^2\right)\tan(c+dx)}{d\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}-\frac{2\int -\frac{\sec(c+dx)\left(b\left(a^2+3b^2\right)-2a\left(a^2-3b^2\right)\sec(c+dx)\right)}{2\sqrt{a+b\sec(c+dx)}}dx}{a^2-b^2}}{3b\left(a^2-b^2\right)}-\frac{2a^2\tan(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\int \frac{\sec(c+dx)\left(b\left(a^2+3b^2\right)-2a\left(a^2-3b^2\right)\sec(c+dx)\right)}{\sqrt{a+b\sec(c+dx)}}dx}{a^2-b^2}+\frac{4a\left(a^2-3b^2\right)\tan(c+dx)}{d\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}}{3b\left(a^2-b^2\right)}-\frac{2a^2\tan(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(b\left(a^2+3b^2\right)-2a\left(a^2-3b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2-b^2}+\frac{4a\left(a^2-3b^2\right)\tan(c+dx)}{d\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}}{3b\left(a^2-b^2\right)}-\frac{2a^2\tan(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \; 4493$$

3.572. $\int \frac{\sec^3(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$

$$\frac{\frac{(a-b)(2a^2+3ab-3b^2)\int \frac{\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx-2a(a^2-3b^2)\int \frac{\sec(c+dx)(\sec(c+dx)+1)}{\sqrt{a+b\sec(c+dx)}}dx}{a^2-b^2}+\frac{4a(a^2-3b^2)\tan(c+dx)}{d(a^2-b^2)\sqrt{a+b\sec(c+dx)}}}{3b(a^2-b^2)}-\frac{2a^2\tan(c+dx)}{3bd(a^2-b^2)(a+b\sec(c+dx))^{3/2}}$$

$\downarrow$ 3042

$$\frac{\frac{(a-b)(2a^2+3ab-3b^2)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-2a(a^2-3b^2)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2-b^2}+\frac{4a(a^2-3b^2)\tan(c+dx)}{d(a^2-b^2)\sqrt{a+b\sec(c+dx)}}}{3b(a^2-b^2)}-\frac{2a^2\tan(c+dx)}{3bd(a^2-b^2)(a+b\sec(c+dx))^{3/2}}$$

$\downarrow$ 4319

$$\frac{\frac{\frac{2(a-b)\sqrt{a+b}\left(2a^2+3ab-3b^2\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{bd}-2a(a^2-3b^2)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c\right.\right.}{\sqrt{a+b\csc\left(c+\right.}}}{a^2-b^2}}{3b(a^2-b^2)}-\frac{2a^2\tan(c+dx)}{3bd(a^2-b^2)(a+b\sec(c+dx))^{3/2}}$$

$\downarrow$ 4492

$$\frac{\frac{\frac{2(a-b)\sqrt{a+b}\left(2a^2+3ab-3b^2\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{bd}+\frac{4a(a-b)\sqrt{a+b}\left(a^2-3b^2\right)\cot(c+dx)\sqrt{}}{}}{a^2-b^2}}{3b(a^2-b^2)}-\frac{2a^2\tan(c+dx)}{3bd(a^2-b^2)(a+b\sec(c+dx))^{3/2}}$$

input
```
Int[Sec[c + d*x]^3/(a + b*Sec[c + d*x])^(5/2),x]
```

3.572. $\int \frac{\sec^3(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$

output
```
(-2*a^2*Tan[c + d*x])/(3*b*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^(3/2)) + (((
4*a*(a - b)*Sqrt[a + b]*(a^2 - 3*b^2)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a
 + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x
]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(b^2*d) + (2*(a - b)
*Sqrt[a + b]*(2*a^2 + 3*a*b - 3*b^2)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a
+ b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]
))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(b*d))/(a^2 - b^2) +
(4*a*(a^2 - 3*b^2)*Tan[c + d*x])/((a^2 - b^2)*d*Sqrt[a + b*Sec[c + d*x]]))
/(3*b*(a^2 - b^2))
```

3.572.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4319
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4326
```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(-a^2)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m
 + 1)*(a^2 - b^2))), x] + Simp[1/(b*(m + 1)*(a^2 - b^2))   Int[Csc[e + f*x]
*(a + b*Csc[e + f*x])^(m + 1)*Simp[a*b*(m + 1) - (a^2 + b^2*(m + 1))*Csc[e
+ f*x], x], x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m,
 -1]
```

rule 4491
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b - a*B))*Cot[e
 + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Simp[1
/((m + 1)*(a^2 - b^2))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp
[(a*A - b*B)*(m + 1) - (A*b - a*B)*(m + 2)*Csc[e + f*x], x], x], x] /; Free
Q[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m
, -1]
```

rule 4492
```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

rule 4493
```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(A - B)   Int[Csc[e
 + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Simp[B   Int[Csc[e + f*x]*((1 +
Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A, B}
, x] && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

3.572.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3302 vs. $2(307) = 614$.

Time = 9.45 (sec) , antiderivative size = 3303, normalized size of antiderivative = 9.80

method	result	size
default	Expression too large to display	3303

input
```
int(sec(d*x+c)^3/(a+b*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3/d/(a-b)^2/(a+b)^2/b^2*(6*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(
b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF(cot(d*x+c)-csc(d*x+c),((a-
b)/(a+b))^(1/2))*a^2*b^3*cos(d*x+c)^3+2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*
(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE(cot(d*x+c)-csc(d
*x+c),((a-b)/(a+b))^(1/2))*a^4*b*cos(d*x+c)^3-6*(cos(d*x+c)/(cos(d*x+c)+1)
)^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE(cot(d*x+
c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b^2*cos(d*x+c)^3-6*(cos(d*x+c)/(cos
(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Elliptic
E(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^3*cos(d*x+c)^3-2*(cos(d
*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2
)*EllipticF(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*a^4*b*cos(d*x+c)^3+
(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1)
)^(1/2)*EllipticF(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b^2*cos(d
*x+c)^3+3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos
(d*x+c)+1))^(1/2)*EllipticF(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*a*b
^4*cos(d*x+c)^3+13*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*
x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(
1/2))*a^2*b^3*cos(d*x+c)^2+12*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1
/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(cot(d*x+c)-csc(d*x+c),((a-
b)/(a+b))^(1/2))*a*b^4*cos(d*x+c)^2+6*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*...
```

3.572.5 Fricas [F]

$$\int \frac{\sec^3(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{\sec{(dx+c)}^3}{(b\sec{(dx+c)}+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^3/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^3/(b^3*sec(d*x + c)^3 + 3*a
*b^2*sec(d*x + c)^2 + 3*a^2*b*sec(d*x + c) + a^3), x)
```

3.572.6 Sympy [F]

$$\int \frac{\sec^3(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{\sec^3(c+dx)}{(a+b\sec(c+dx))^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)**3/(a+b*sec(d*x+c))**(5/2),x)
```

output
```
Integral(sec(c + d*x)**3/(a + b*sec(c + d*x))**(5/2), x)
```

3.572.7 Maxima [F(-1)]

Timed out.

$$\int \frac{\sec^3(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)^3/(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
Timed out
```

3.572.8 Giac [F]

$$\int \frac{\sec^3(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{\sec(dx+c)^3}{(b\sec(dx+c)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^3/(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^3/(b*sec(d*x + c) + a)^(5/2), x)
```

3.572.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^3(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{1}{\cos\left(c+d\,x\right)^3\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

input
```
int(1/(cos(c + d*x)^3*(a + b/cos(c + d*x))^(5/2)),x)
```

output
```
int(1/(cos(c + d*x)^3*(a + b/cos(c + d*x))^(5/2)), x)
```

3.573 $\int \frac{\sec^2(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$

3.573.1 Optimal result . . . 4332
3.573.2 Mathematica [A] (warning: unable to verify) . . . 4333
3.573.3 Rubi [A] (verified) . . . 4333
3.573.4 Maple [B] (verified) . . . 4337
3.573.5 Fricas [F] . . . 4338
3.573.6 Sympy [F] . . . 4338
3.573.7 Maxima [F] . . . 4338
3.573.8 Giac [F] . . . 4339
3.573.9 Mupad [F(-1)] . . . 4339

3.573.1 Optimal result

Integrand size = 23, antiderivative size = 317

$$\int \frac{\sec^2(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \frac{2(a^2+3b^2)\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1-}{}}}{3(a-b)b^2(a+b)^{3/2}d}$$
$$+\frac{2(a-3b)\cot(c+dx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{3(a-b)b(a+b)^{3/2}d}$$
$$+\frac{2a\tan(c+dx)}{3\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}}+\frac{2(a^2+3b^2)\tan(c+dx)}{3\left(a^2-b^2\right)^2d\sqrt{a+b\sec(c+dx)}}$$

output
```
2/3*(a^2+3*b^2)*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((
a+b)/(a-b))^(1/2))*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b)
)^(1/2)/(a-b)/b^2/(a+b)^(3/2)/d+2/3*(a-3*b)*cot(d*x+c)*EllipticF((a+b*sec(
d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(b*(1-sec(d*x+c))/(a+b))^(1
/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/(a-b)/b/(a+b)^(3/2)/d+2/3*a*tan(d*x+c)
/(a^2-b^2)/d/(a+b*sec(d*x+c))^(3/2)+2/3*(a^2+3*b^2)*tan(d*x+c)/(a^2-b^2)^2
/d/(a+b*sec(d*x+c))^(1/2)
```

3.573.2 Mathematica [A] (warning: unable to verify)

Time = 11.18 (sec) , antiderivative size = 486, normalized size of antiderivative = 1.53

$$\int \frac{\sec^2(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \frac{(b+a\cos(c+dx))^3\sec^3(c+dx)\left(-\frac{2(a^2+3b^2)\sin(c+dx)}{3b(-a^2+b^2)^2}+\frac{2b\sin(c+dx)}{3(-a^2+b^2)(b+a\cos(c+dx))^2}\right.}{d(a+b\sec(c+dx))^{5/2}}$$
$$+\frac{2(b+a\cos(c+dx))^2\sec^{\frac{5}{2}}(c+dx)\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)}\left(2(a^3+a^2b+3ab^2+3b^3)\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\sqrt{\right.}}{}$$

input
```
Integrate[Sec[c + d*x]^2/(a + b*Sec[c + d*x])^(5/2),x]
```

output
```
((b + a*Cos[c + d*x])^3*Sec[c + d*x]^3*((-2*(a^2 + 3*b^2)*Sin[c + d*x])/(3
*b*(-a^2 + b^2)^2) + (2*b*Sin[c + d*x])/(3*(-a^2 + b^2)*(b + a*Cos[c + d*x
])^2) + (4*(a^2*Sin[c + d*x] + b^2*Sin[c + d*x]))/(3*(-a^2 + b^2)^2*(b + a
*Cos[c + d*x]))))/(d*(a + b*Sec[c + d*x])^(5/2)) + (2*(b + a*Cos[c + d*x])
^2*Sec[c + d*x]^(5/2)*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(2*(a^3 + a^2*
b + 3*a*b^2 + 3*b^3)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos
[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]
], (a - b)/(a + b)] - 2*b*(a^2 + 4*a*b + 3*b^2)*Sqrt[Cos[c + d*x]/(1 + Cos
[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Ellipt
icF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + (a^2 + 3*b^2)*Cos[c + d*x
]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(3*b*(a^2 - b
^2)^2*d*Sqrt[Sec[(c + d*x)/2]^2]*(a + b*Sec[c + d*x])^(5/2))
```

3.573.3 Rubi [A] (verified)

Time = 1.15 (sec) , antiderivative size = 336, normalized size of antiderivative = 1.06, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules used = {3042, 4323, 27, 3042, 4491, 27, 3042, 4493, 3042, 4319, 4492}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \text{4323}$$

$$\frac{2\int -\frac{\sec(c+dx)(3b-a\sec(c+dx))}{2(a+b\sec(c+dx))^{3/2}}dx}{3\left(a^2-b^2\right)}+\frac{2a\tan(c+dx)}{3d\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{2a\tan(c+dx)}{3d\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}-\frac{\int \frac{\sec(c+dx)(3b-a\sec(c+dx))}{(a+b\sec(c+dx))^{3/2}}dx}{3\left(a^2-b^2\right)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2a\tan(c+dx)}{3d\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}-\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(3b-a\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{3\left(a^2-b^2\right)}$$

$$\Big\downarrow \text{4491}$$

$$\frac{2a\tan(c+dx)}{3d\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}-\frac{-\frac{2\int -\frac{\sec(c+dx)\left(4ab+\left(a^2+3b^2\right)\sec(c+dx)\right)}{2\sqrt{a+b\sec(c+dx)}}dx}{a^2-b^2}-\frac{2\left(a^2+3b^2\right)\tan(c+dx)}{d\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}}{3\left(a^2-b^2\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{2a\tan(c+dx)}{3d\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}-\frac{\frac{\int \frac{\sec(c+dx)\left(4ab+\left(a^2+3b^2\right)\sec(c+dx)\right)}{\sqrt{a+b\sec(c+dx)}}dx}{a^2-b^2}-\frac{2\left(a^2+3b^2\right)\tan(c+dx)}{d\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}}{3\left(a^2-b^2\right)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2a\tan(c+dx)}{3d\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}-\frac{\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(4ab+\left(a^2+3b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2-b^2}-\frac{2\left(a^2+3b^2\right)\tan(c+dx)}{d\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}}{3\left(a^2-b^2\right)}$$

$$\Big\downarrow \text{4493}$$

$$\frac{2a\tan(c+dx)}{3d\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}-\frac{\frac{\left(a^2+3b^2\right)\int \frac{\sec(c+dx)(\sec(c+dx)+1)}{\sqrt{a+b\sec(c+dx)}}dx-(a-3b)(a-b)\int \frac{\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx}{a^2-b^2}-\frac{2\left(a^2+3b^2\right)\tan(c+dx)}{d\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}}{3\left(a^2-b^2\right)}$$

$$\Big\downarrow \text{3042}$$

3.573. $\int \frac{\sec^2(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$

$$\frac{\frac{2a\tan(c+dx)}{3d(a^2-b^2)(a+b\sec(c+dx))^{3/2}}-\frac{\frac{(a^2+3b^2)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-(a-3b)(a-b)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2-b^2}-\frac{2(a^2+3b^2)\tan(c+dx)}{d(a^2-b^2)\sqrt{a+b\sec(c+dx)}}}{3(a^2-b^2)}$$

$\Big\downarrow$ 4319

$$\frac{\frac{2a\tan(c+dx)}{3d(a^2-b^2)(a+b\sec(c+dx))^{3/2}}-\frac{\frac{(a^2+3b^2)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2(a-3b)(a-b)\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a}{a}\right.}{bd}}{a^2-b^2}}{3(a^2-b^2)}$$

$\Big\downarrow$ 4492

$$\frac{\frac{2a\tan(c+dx)}{3d(a^2-b^2)(a+b\sec(c+dx))^{3/2}}-\frac{-\frac{2(a-b)\sqrt{a+b}\left(a^2+3b^2\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}E\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{b^2d}-\frac{2(a-3b)(a-b)\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}}{}}{a^2-b^2}}{3(a^2-b^2)}$$

input
```
Int[Sec[c + d*x]^2/(a + b*Sec[c + d*x])^(5/2),x]
```

output
```
(2*a*Tan[c + d*x])/(3*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^(3/2)) - (((-2*(a
 - b)*Sqrt[a + b]*(a^2 + 3*b^2)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*S
ec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a
 + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(b^2*d) - (2*(a - 3*b)*(a
- b)*Sqrt[a + b]*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sq
rt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((
b*(1 + Sec[c + d*x]))/(a - b))])/(b*d))/(a^2 - b^2) - (2*(a^2 + 3*b^2)*Tan
[c + d*x])/((a^2 - b^2)*d*Sqrt[a + b*Sec[c + d*x]]))/(3*(a^2 - b^2))
```

3.573. $\int \frac{\sec^2(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$

3.573.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4319
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4323
```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[a*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(f*(m + 1)*(
a^2 - b^2))), x] - Simp[1/((m + 1)*(a^2 - b^2))   Int[Csc[e + f*x]*(a + b*C
sc[e + f*x])^(m + 1)*(b*(m + 1) - a*(m + 2)*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

rule 4491
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b - a*B))*Cot[e
 + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Simp[1
/((m + 1)*(a^2 - b^2))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp
[(a*A - b*B)*(m + 1) - (A*b - a*B)*(m + 2)*Csc[e + f*x], x], x], x] /; Free
Q[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m
, -1]
```

rule 4492
```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

rule 4493
```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(A - B)   Int[Csc[e
 + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Simp[B   Int[Csc[e + f*x]*((1 +
Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A, B}
, x] && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

3.573.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3000 vs. $2(287) = 574$.

Time = 4.99 (sec) , antiderivative size = 3001, normalized size of antiderivative = 9.47

method	result	size
default	Expression too large to display	3001

input
```
int(sec(d*x+c)^2/(a+b*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/3/d/(a-b)^2/(a+b)^2/b*((1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(
cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(cot(d*x+c)-csc(d*x+c),((a-b)/(a
+b))^(1/2))*a^3*b*cos(d*x+c)^3+4*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))
^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(cot(d*x+c)-csc(d*x+c),(
(a-b)/(a+b))^(1/2))*a^2*b^2*cos(d*x+c)^3+3*(1/(a+b)*(b+a*cos(d*x+c))/(cos(
d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(cot(d*x+c)-cs
c(d*x+c),((a-b)/(a+b))^(1/2))*a*b^3*cos(d*x+c)^3-(1/(a+b)*(b+a*cos(d*x+c))
/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(cot(d*x
+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b*cos(d*x+c)^3-3*(1/(a+b)*(b+a*cos
(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE
(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^2*cos(d*x+c)^3-3*(1/(a+b
)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)
*EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*a*b^3*cos(d*x+c)^3-6
*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1
))^(1/2)*EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*b^4*cos(d*x+
c)-3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+
c)+1))^(1/2)*EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*b^4*cos(
d*x+c)^2+2*a^3*b*cos(d*x+c)^2*sin(d*x+c)-3*a^2*b^2*cos(d*x+c)^2*sin(d*x+c)
+2*a*b^3*cos(d*x+c)^2*sin(d*x+c)-3*EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/
(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c...
```

3.573.5 Fricas [F]

$$\int \frac{\sec^2(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{\sec{(dx+c)}^2}{(b\sec{(dx+c)}+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^2/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^2/(b^3*sec(d*x + c)^3 + 3*a
*b^2*sec(d*x + c)^2 + 3*a^2*b*sec(d*x + c) + a^3), x)
```

3.573.6 Sympy [F]

$$\int \frac{\sec^2(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{\sec^2{(c+dx)}}{(a+b\sec{(c+dx)})^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)**2/(a+b*sec(d*x+c))**(5/2),x)
```

output
```
Integral(sec(c + d*x)**2/(a + b*sec(c + d*x))**(5/2), x)
```

3.573.7 Maxima [F]

$$\int \frac{\sec^2(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{\sec{(dx+c)}^2}{(b\sec{(dx+c)}+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^2/(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)^2/(b*sec(d*x + c) + a)^(5/2), x)
```

3.573.8 Giac [F]

$$\int \frac{\sec^2(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{\sec(dx+c)^2}{(b\sec(dx+c)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^2/(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^2/(b*sec(d*x + c) + a)^(5/2), x)
```

3.573.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^2(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{1}{\cos(c+d\,x)^2\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

input
```
int(1/(cos(c + d*x)^2*(a + b/cos(c + d*x))^(5/2)),x)
```

output
```
int(1/(cos(c + d*x)^2*(a + b/cos(c + d*x))^(5/2)), x)
```

3.574 $\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$

3.574.1 Optimal result 4340
3.574.2 Mathematica [A] (verified) 4341
3.574.3 Rubi [A] (verified) 4341
3.574.4 Maple [B] (verified) 4345
3.574.5 Fricas [F] 4345
3.574.6 Sympy [F] 4346
3.574.7 Maxima [F] 4346
3.574.8 Giac [F] 4346
3.574.9 Mupad [F(-1)] 4347

3.574.1 Optimal result

Integrand size = 21, antiderivative size = 304

$$\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = -\frac{8a\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{3(a-b)b(a+b)^{3/2}d} + \frac{2(3a-b)\cot(c+dx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{3(a-b)b(a+b)^{3/2}d} - \frac{2b\tan(c+dx)}{3\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} - \frac{8ab\tan(c+dx)}{3\left(a^2-b^2\right)^2 d\sqrt{a+b\sec(c+dx)}}$$

output
```
-8/3*a*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b
))^(1/2))*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/(
a-b)/b/(a+b)^(3/2)/d+2/3*(3*a-b)*cot(d*x+c)*EllipticF((a+b*sec(d*x+c))^(1/
2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+
sec(d*x+c))/(a-b))^(1/2)/(a-b)/b/(a+b)^(3/2)/d-2/3*b*tan(d*x+c)/(a^2-b^2)/
d/(a+b*sec(d*x+c))^(3/2)-8/3*a*b*tan(d*x+c)/(a^2-b^2)^2/d/(a+b*sec(d*x+c))
^(1/2)
```

3.574.2 Mathematica [A] (verified)

Time = 5.97 (sec) , antiderivative size = 360, normalized size of antiderivative = 1.18

$$\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx =$$
$$-\frac{2(b+a\cos(c+dx))\sec^3(c+dx)\left(b^2(-a^2+b^2)\sin(c+dx)-b(-5a^2+b^2)(b+a\cos(c+dx))\sin(c+dx)\right.}{}$$

input
```
Integrate[Sec[c + d*x]/(a + b*Sec[c + d*x])^(5/2),x]
```

output
```
(-2*(b + a*Cos[c + d*x])*Sec[c + d*x]^3*(b^2*(-a^2 + b^2)*Sin[c + d*x] - b
*(-5*a^2 + b^2)*(b + a*Cos[c + d*x])*Sin[c + d*x] - 4*a^2*(b + a*Cos[c + d
*x])^2*Sin[c + d*x] + 2*a*Cos[(c + d*x)/2]^2*(b + a*Cos[c + d*x])*(4*a*(a
+ b)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a +
 b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a +
b)] - (3*a^2 + 4*a*b + b^2)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b
+ a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c +
d*x)/2]], (a - b)/(a + b)] + 2*a*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c
+ d*x)/2]^2*Tan[(c + d*x)/2])))/(3*a*(a^2 - b^2)^2*d*(a + b*Sec[c + d*x])^
(5/2))
```

3.574.3 Rubi [A] (verified)

Time = 1.10 (sec) , antiderivative size = 323, normalized size of antiderivative = 1.06, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.524$, Rules used = {3042, 4320, 27, 3042, 4491, 27, 3042, 4493, 3042, 4319, 4492}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$
$$\Big\downarrow \; 4320$$

$$-\frac{2\int -\frac{\sec(c+dx)(3a-b\sec(c+dx))}{2(a+b\sec(c+dx))^{3/2}}dx}{3\left(a^2-b^2\right)}-\frac{2b\tan(c+dx)}{3d\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}$$

$\Big\downarrow$ 27

$$\frac{\int \frac{\sec(c+dx)(3a-b\sec(c+dx))}{(a+b\sec(c+dx))^{3/2}}dx}{3\left(a^2-b^2\right)}-\frac{2b\tan(c+dx)}{3d\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}$$

$\Big\downarrow$ 3042

$$\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(3a-b\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{3\left(a^2-b^2\right)}-\frac{2b\tan(c+dx)}{3d\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}$$

$\Big\downarrow$ 4491

$$\frac{-\frac{2\int -\frac{\sec(c+dx)\left(3a^2+4b\sec(c+dx)a+b^2\right)}{2\sqrt{a+b\sec(c+dx)}}dx}{a^2-b^2}-\frac{8ab\tan(c+dx)}{d(a^2-b^2)\sqrt{a+b\sec(c+dx)}}}{3\left(a^2-b^2\right)}-\frac{2b\tan(c+dx)}{3d\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}$$

$\Big\downarrow$ 27

$$\frac{\frac{\int \frac{\sec(c+dx)\left(3a^2+4b\sec(c+dx)a+b^2\right)}{\sqrt{a+b\sec(c+dx)}}dx}{a^2-b^2}-\frac{8ab\tan(c+dx)}{d(a^2-b^2)\sqrt{a+b\sec(c+dx)}}}{3\left(a^2-b^2\right)}-\frac{2b\tan(c+dx)}{3d\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(3a^2+4b\csc\left(c+dx+\frac{\pi}{2}\right)a+b^2\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2-b^2}-\frac{8ab\tan(c+dx)}{d(a^2-b^2)\sqrt{a+b\sec(c+dx)}}}{3\left(a^2-b^2\right)}-\frac{2b\tan(c+dx)}{3d\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}$$

$\Big\downarrow$ 4493

$$\frac{\frac{(a-b)(3a-b)\int \frac{\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx+4ab\int \frac{\sec(c+dx)(\sec(c+dx)+1)}{\sqrt{a+b\sec(c+dx)}}dx}{a^2-b^2}-\frac{8ab\tan(c+dx)}{d(a^2-b^2)\sqrt{a+b\sec(c+dx)}}}{3\left(a^2-b^2\right)}-\frac{2b\tan(c+dx)}{3d\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}$$

$\Big\downarrow$ 3042

$$\frac{\frac{(a-b)(3a-b)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+4ab\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2-b^2}-\frac{8ab\tan(c+dx)}{d(a^2-b^2)\sqrt{a+b\sec(c+dx)}}}{3\left(a^2-b^2\right)}-\frac{2b\tan(c+dx)}{3d\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}$$

3.574. $\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$

$$\Big\downarrow \; 4319$$

$$\frac{\frac{4ab\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2(a-b)(3a-b)\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{bd}}{a^2-b^2}-}{3\left(a^2-b^2\right)}$$

$$\frac{2b\tan(c+dx)}{3d\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \; 4492$$

$$\frac{\frac{\frac{2(a-b)(3a-b)\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{bd}-\frac{8a(a-b)\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-}}{}}{a^2-b^2}}{3\left(a^2-b^2\right)}$$

$$\frac{2b\tan(c+dx)}{3d\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}$$

input
```
Int[Sec[c + d*x]/(a + b*Sec[c + d*x])^(5/2),x]
```

output
```
(-2*b*Tan[c + d*x])/(3*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^(3/2)) + (((-8*a
*(a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]
]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt
[-((b*(1 + Sec[c + d*x]))/(a - b))])/(b*d) + (2*(a - b)*(3*a - b)*Sqrt[a +
 b]*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (
a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c
+ d*x]))/(a - b))])/(b*d))/(a^2 - b^2) - (8*a*b*Tan[c + d*x])/((a^2 - b^2)
*d*Sqrt[a + b*Sec[c + d*x]]))/(3*(a^2 - b^2))
```

3.574.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4319
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4320
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[(-b)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(f*(m + 1)*
(a^2 - b^2))), x] + Simp[1/((m + 1)*(a^2 - b^2))   Int[Csc[e + f*x]*(a + b*
Csc[e + f*x])^(m + 1)*(a*(m + 1) - b*(m + 2)*Csc[e + f*x]), x], x] /; FreeQ
[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

rule 4491
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b - a*B))*Cot[e
 + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Simp[1
/((m + 1)*(a^2 - b^2))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp
[(a*A - b*B)*(m + 1) - (A*b - a*B)*(m + 2)*Csc[e + f*x], x], x], x] /; Free
Q[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m
, -1]
```

rule 4492
```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

rule 4493
```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(A - B)   Int[Csc[e
 + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Simp[B   Int[Csc[e + f*x]*((1 +
Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A, B}
, x] && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

3.574. $\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$

3.574.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2258 vs. $2(274) = 548$.

Time = 6.04 (sec) , antiderivative size = 2259, normalized size of antiderivative = 7.43

method	result	size
default	Expression too large to display	2259

input
```
int(sec(d*x+c)/(a+b*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/3/d/(a-b)^2/(a+b)^2*(-4*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*
cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF(cot(d*x+c)-csc(d*x+c),((a-b)/(
a+b))^(1/2))*a^2*b*cos(d*x+c)^3-(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)
*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF(cot(d*x+c)-csc(d*x+c),((
a-b)/(a+b))^(1/2))*a*b^2*cos(d*x+c)^3+4*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*
(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE(cot(d*x+c)-csc(d
*x+c),((a-b)/(a+b))^(1/2))*a^2*b*cos(d*x+c)^3-6*(cos(d*x+c)/(cos(d*x+c)+1)
)^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF(cot(d*x+
c)-csc(d*x+c),((a-b)/(a+b))^(1/2))*a^3*cos(d*x+c)^2+b^3*cos(d*x+c)*sin(d*x
+c)+4*a^3*cos(d*x+c)^2*sin(d*x+c)+4*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(
a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE(cot(d*x+c)-csc(d*x+c
),((a-b)/(a+b))^(1/2))*a^3*cos(d*x+c)^3-3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2
)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF(cot(d*x+c)-csc
(d*x+c),((a-b)/(a+b))^(1/2))*a^3*cos(d*x+c)^3+4*EllipticE(cot(d*x+c)-csc(d
*x+c),((a-b)/(a+b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)
*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*a^2*b+4*EllipticE(cot(d*x+c)-csc(d*x+c)
,((a-b)/(a+b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos
(d*x+c)/(cos(d*x+c)+1))^(1/2)*a*b^2+8*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c
)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(cot(d*x+c)-csc(d*x
+c),((a-b)/(a+b))^(1/2))*a^3*cos(d*x+c)^2-3*EllipticF(cot(d*x+c)-csc(d*...
```

3.574.5 Fricas [F]

$$\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{\sec{(dx+c)}}{(b\sec{(dx+c)}+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)/(b^3*sec(d*x + c)^3 + 3*a*b
^2*sec(d*x + c)^2 + 3*a^2*b*sec(d*x + c) + a^3), x)
```

3.574.6 Sympy [F]

$$\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{\sec{(c+dx)}}{(a+b\sec{(c+dx)})^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)/(a+b*sec(d*x+c))**(5/2),x)
```

output
```
Integral(sec(c + d*x)/(a + b*sec(c + d*x))**(5/2), x)
```

3.574.7 Maxima [F]

$$\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{\sec{(dx+c)}}{(b\sec{(dx+c)}+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)/(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)/(b*sec(d*x + c) + a)^(5/2), x)
```

3.574.8 Giac [F]

$$\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{\sec{(dx+c)}}{(b\sec{(dx+c)}+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)/(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)/(b*sec(d*x + c) + a)^(5/2), x)
```

3.574.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx=\int \frac{1}{\cos\left(c+d\,x\right)\,\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

input
```
int(1/(cos(c + d*x)*(a + b/cos(c + d*x))^(5/2)),x)
```

output
```
int(1/(cos(c + d*x)*(a + b/cos(c + d*x))^(5/2)), x)
```

3.575 $\int \frac{1}{(a+b\sec(c+dx))^{5/2}}\,dx$

3.575.1 Optimal result 4348
3.575.2 Mathematica [B] (verified) 4349
3.575.3 Rubi [A] (verified) 4349
3.575.4 Maple [B] (verified) 4354
3.575.5 Fricas [F] 4355
3.575.6 Sympy [F] 4356
3.575.7 Maxima [F] 4356
3.575.8 Giac [F] 4356
3.575.9 Mupad [F(-1)] 4357

3.575.1 Optimal result

Integrand size = 14, antiderivative size = 448

$$\int \frac{1}{(a+b\sec(c+dx))^{5/2}}\,dx = \frac{2(7a^2-3b^2)\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(}{}}}{3a^2(a-b)(a+b)^{3/2}d}$$
$$-\frac{2(6a^2-ab-3b^2)\cot(c+dx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{3a^2(a-b)(a+b)^{3/2}d}$$
$$-\frac{2\sqrt{a+b}\cot(c+dx)\,\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{a^3d}$$
$$+\frac{2b^2\tan(c+dx)}{3a\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}}+\frac{2b^2(7a^2-3b^2)\tan(c+dx)}{3a^2\left(a^2-b^2\right)^2d\sqrt{a+b\sec(c+dx)}}$$

output
```
2/3*(7*a^2-3*b^2)*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),
((a+b)/(a-b))^(1/2))*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-
b))^(1/2)/a^2/(a-b)/(a+b)^(3/2)/d-2/3*(6*a^2-a*b-3*b^2)*cot(d*x+c)*Ellipti
cF((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(b*(1-sec(d*x+c
))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/a^2/(a-b)/(a+b)^(3/2)/d-2*
cot(d*x+c)*EllipticPi((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),(a+b)/a,((a+b)/(a
-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/
(a-b))^(1/2)/a^3/d+2/3*b^2*tan(d*x+c)/a/(a^2-b^2)/d/(a+b*sec(d*x+c))^(3/2)
+2/3*b^2*(7*a^2-3*b^2)*tan(d*x+c)/a^2/(a^2-b^2)^2/d/(a+b*sec(d*x+c))^(1/2)
```

3.575.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1382 vs. $2(448) = 896$.

Time = 12.61 (sec) , antiderivative size = 1382, normalized size of antiderivative = 3.08

$$\int \frac{1}{(a+b\sec(c+dx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
Integrate[(a + b*Sec[c + d*x])^(-5/2),x]
```

output
```
((b + a*Cos[c + d*x])^3*Sec[c + d*x]^3*((2*b*(-7*a^2 + 3*b^2)*Sin[c + d*x]
)/(3*a^2*(-a^2 + b^2)^2) - (2*b^3*Sin[c + d*x])/(3*a^2*(a^2 - b^2)*(b + a*
Cos[c + d*x])^2) - (8*(-2*a^2*b^2*Sin[c + d*x] + b^4*Sin[c + d*x]))/(3*a^2
*(a^2 - b^2)^2*(b + a*Cos[c + d*x]))))/(d*(a + b*Sec[c + d*x])^(5/2)) + (2
*(b + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(5/2)*Sqrt[(a + b - a*Tan[(c + d*
x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2)]*(7*a^3*b*Tan[(c
+ d*x)/2] + 7*a^2*b^2*Tan[(c + d*x)/2] - 3*a*b^3*Tan[(c + d*x)/2] - 3*b^4*
Tan[(c + d*x)/2] - 14*a^3*b*Tan[(c + d*x)/2]^3 + 6*a*b^3*Tan[(c + d*x)/2]^
3 + 7*a^3*b*Tan[(c + d*x)/2]^5 - 7*a^2*b^2*Tan[(c + d*x)/2]^5 - 3*a*b^3*Ta
n[(c + d*x)/2]^5 + 3*b^4*Tan[(c + d*x)/2]^5 + 6*a^4*EllipticPi[-1, ArcSin[
Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a +
 b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - 12*a^2*b^2*El
lipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c +
d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a +
 b)] + 6*b^4*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqr
t[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c +
d*x)/2]^2)/(a + b)] + 6*a^4*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (a -
b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b -
a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - 12*a^2*b^2*Ellipti
cPi[-1, ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Tan[(c + d*x)/2]^2*S...
```

3.575.3 Rubi [A] (verified)

Time = 1.83 (sec) , antiderivative size = 478, normalized size of antiderivative = 1.07, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {3042, 4272, 27, 3042, 4548, 27, 3042, 4546, 3042, 4409, 3042, 4271, 4319, 4492}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+b\sec(c+dx))^{5/2}}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \; 4272$$

$$\frac{2b^2\tan(c+dx)}{3ad\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}-\frac{2\int -\frac{b^2\sec^2(c+dx)-3ab\sec(c+dx)+3\left(a^2-b^2\right)}{2(a+b\sec(c+dx))^{3/2}}dx}{3a\left(a^2-b^2\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{b^2\sec^2(c+dx)-3ab\sec(c+dx)+3\left(a^2-b^2\right)}{(a+b\sec(c+dx))^{3/2}}dx}{3a\left(a^2-b^2\right)}+\frac{2b^2\tan(c+dx)}{3ad\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \; 3042$$

$$\frac{\int \frac{b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2-3ab\csc\left(c+dx+\frac{\pi}{2}\right)+3\left(a^2-b^2\right)}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{3a\left(a^2-b^2\right)}+\frac{2b^2\tan(c+dx)}{3ad\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \; 4548$$

$$\frac{\frac{2b^2\left(7a^2-3b^2\right)\tan(c+dx)}{ad\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}-\frac{2\int -\frac{3\left(a^2-b^2\right)^2-b^2\left(7a^2-3b^2\right)\sec^2(c+dx)-2ab\left(3a^2-b^2\right)\sec(c+dx)}{2\sqrt{a+b\sec(c+dx)}}dx}{a\left(a^2-b^2\right)}}{3a\left(a^2-b^2\right)}+\frac{2b^2\tan(c+dx)}{3ad\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\int \frac{3\left(a^2-b^2\right)^2-b^2\left(7a^2-3b^2\right)\sec^2(c+dx)-2ab\left(3a^2-b^2\right)\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx}{a\left(a^2-b^2\right)}+\frac{2b^2\left(7a^2-3b^2\right)\tan(c+dx)}{ad\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}}{3a\left(a^2-b^2\right)}+\frac{2b^2\tan(c+dx)}{3ad\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{\int \frac{3\left(a^2-b^2\right)^2-b^2\left(7a^2-3b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2-2ab\left(3a^2-b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a\left(a^2-b^2\right)}+\frac{2b^2\left(7a^2-3b^2\right)\tan(c+dx)}{ad\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}}{3a\left(a^2-b^2\right)}+\frac{2b^2\tan(c+dx)}{3ad\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}$$

3.575. $\int \frac{1}{(a+b\sec(c+dx))^{5/2}}\,dx$

$$\Big\downarrow \; 4546$$

$$\frac{\frac{\int \frac{3\left(a^2-b^2\right)^2+\left(b^2\left(7a^2-3b^2\right)-2ab\left(3a^2-b^2\right)\right)\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx-b^2\left(7a^2-3b^2\right)\int \frac{\sec(c+dx)(\sec(c+dx)+1)}{\sqrt{a+b\sec(c+dx)}}dx}{a(a^2-b^2)}+\frac{2b^2\left(7a^2-3b^2\right)\tan(c+dx)}{ad(a^2-b^2)\sqrt{a+b\sec(c+dx)}}}{3a\left(a^2-b^2\right)}+\frac{2b^2\tan(c+dx)}{3ad\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{\int \frac{3\left(a^2-b^2\right)^2+\left(b^2\left(7a^2-3b^2\right)-2ab\left(3a^2-b^2\right)\right)\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-b^2\left(7a^2-3b^2\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a(a^2-b^2)}+\frac{2b^2\left(7a^2-3b^2\right)\tan(c+dx)}{ad(a^2-b^2)\sqrt{a+b\sec(c+dx)}}}{3a\left(a^2-b^2\right)}+\frac{2b^2\tan(c+dx)}{3ad\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \; 4409$$

$$\frac{\frac{-\left(b^2\left(7a^2-3b^2\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)-b(a-b)\left(6a^2-ab-3b^2\right)\int \frac{\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx+3\left(a^2-b^2\right)^2\int \frac{1}{\sqrt{a+b\sec(c+dx)}}dx}{a(a^2-b^2)}+\frac{2b^2(7a^2}{ad(a^2-b}}{3a\left(a^2-b^2\right)}+\frac{2b^2\tan(c+dx)}{3ad\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{-\left(b^2\left(7a^2-3b^2\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)-b(a-b)\left(6a^2-ab-3b^2\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+3\left(a^2-b^2\right)^2\int \frac{1}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a(a^2-b^2)}+}{3a\left(a^2-b^2\right)}+\frac{2b^2\tan(c+dx)}{3ad\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \; 4271$$

$$\frac{\frac{-\left(b^2\left(7a^2-3b^2\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)-b(a-b)\left(6a^2-ab-3b^2\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{6\sqrt{a+b}\left(a^2-b^2\right)^2\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx}{a+b}}}{}}{a(a^2-b^2)}}{3a\left(a^2-b^2\right)}+\frac{2b^2\tan(c+dx)}{3ad\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}$$

3.575. $\int \frac{1}{(a+b\sec(c+dx))^{5/2}}\,dx$

$$\Big\downarrow \; 4319$$

$$\frac{-\left(b^2\left(7a^2-3b^2\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)-\frac{2(a-b)\sqrt{a+b}\left(6a^2-ab-3b^2\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\right.\right.}{d}}{a(a^2-b^2)}$$

$$\frac{2b^2\tan(c+dx)}{3ad\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \; 4492$$

$$\frac{-\frac{2(a-b)\sqrt{a+b}\left(6a^2-ab-3b^2\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{d}+\frac{2(a-b)\sqrt{a+b}\left(7a^2-3b^2\right)\cot(c+dx)\sqrt{}}{}}{}$$

$$\frac{2b^2\tan(c+dx)}{3ad\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}$$

input
```
Int[(a + b*Sec[c + d*x])^(-5/2),x]
```

output
```
(2*b^2*Tan[c + d*x])/(3*a*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^(3/2)) + (((2
*(a - b)*Sqrt[a + b]*(7*a^2 - 3*b^2)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a
+ b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]
))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/d - (2*(a - b)*Sqrt[a
 + b]*(6*a^2 - a*b - 3*b^2)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c
 + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b
)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/d - (6*Sqrt[a + b]*(a^2 - b^2)
^2*Cot[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt
[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*
(1 + Sec[c + d*x]))/(a - b))])/(a*d))/(a*(a^2 - b^2)) + (2*b^2*(7*a^2 - 3*
b^2)*Tan[c + d*x])/(a*(a^2 - b^2)*d*Sqrt[a + b*Sec[c + d*x]]))/(3*a*(a^2 -
 b^2))
```

3.575. $\int \frac{1}{(a+b\sec(c+dx))^{5/2}}\,dx$

3.575.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4271
```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*(Rt[a
 + b, 2]/(a*d*Cot[c + d*x]))*Sqrt[b*((1 - Csc[c + d*x])/(a + b))]*Sqrt[(-b)
*((1 + Csc[c + d*x])/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[
c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

rule 4272
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[b^2*Cot[
c + d*x]*((a + b*Csc[c + d*x])^(n + 1)/(a*d*(n + 1)*(a^2 - b^2))), x] + Sim
p[1/(a*(n + 1)*(a^2 - b^2))   Int[(a + b*Csc[c + d*x])^(n + 1)*Simp[(a^2 -
b^2)*(n + 1) - a*b*(n + 1)*Csc[c + d*x] + b^2*(n + 2)*Csc[c + d*x]^2, x], x
], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && LtQ[n, -1] && Integ
erQ[2*n]
```

rule 4319
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4409
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Simp[c   Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] +
Simp[d   Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

3.575. $\int \frac{1}{(a+b\sec(c+dx))^{5/2}}\,dx$

rule 4492
```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

rule 4546
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Simp[C   Int[Csc[e + f*x]*((
1 + Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A
, B, C}, x] && NeQ[a^2 - b^2, 0]
```

rule 4548
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(A*b^2 -
a*b*B + a^2*C)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(a*f*(m + 1)*(a^2
 - b^2))), x] + Simp[1/(a*(m + 1)*(a^2 - b^2))   Int[(a + b*Csc[e + f*x])^(
m + 1)*Simp[A*(a^2 - b^2)*(m + 1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x
] + (A*b^2 - a*b*B + a^2*C)*(m + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

3.575.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 4913 vs. $2(409) = 818$.

Time = 7.32 (sec) , antiderivative size = 4914, normalized size of antiderivative = 10.97

method	result	size
default	Expression too large to display	4914

input
```
int(1/(a+b*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3/d/(a-b)^2/(a+b)^2/a^2*(6*EllipticPi(cot(d*x+c)-csc(d*x+c),-1,((a-b)/(
a+b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(
cos(d*x+c)+1))^(1/2)*a^4*b*cos(d*x+c)^2-24*EllipticPi(cot(d*x+c)-csc(d*x+c
),-1,((a-b)/(a+b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*
(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*a^3*b^2*cos(d*x+c)^2-12*EllipticPi(cot(d
*x+c)-csc(d*x+c),-1,((a-b)/(a+b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*
x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*a^2*b^3*cos(d*x+c)^2+12*E
llipticPi(cot(d*x+c)-csc(d*x+c),-1,((a-b)/(a+b))^(1/2))*(1/(a+b)*(b+a*cos(
d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*a*b^4*cos(
d*x+c)^2+6*EllipticPi(cot(d*x+c)-csc(d*x+c),-1,((a-b)/(a+b))^(1/2))*(1/(a+
b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2
)*a^4*b-12*EllipticPi(cot(d*x+c)-csc(d*x+c),-1,((a-b)/(a+b))^(1/2))*(1/(a+
b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2
)*a^2*b^3+6*EllipticPi(cot(d*x+c)-csc(d*x+c),-1,((a-b)/(a+b))^(1/2))*(1/(a
+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/
2)*a^5*cos(d*x+c)^3-3*EllipticF(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/2))
*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1
))^(1/2)*a^5*cos(d*x+c)^3+12*EllipticPi(cot(d*x+c)-csc(d*x+c),-1,((a-b)/(a
+b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(c
os(d*x+c)+1))^(1/2)*a^5*cos(d*x+c)^2+6*EllipticPi(cot(d*x+c)-csc(d*x+c)...
```

3.575.5 Fricas [F]

$$\int \frac{1}{(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{1}{(b\sec(dx+c)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sec(d*x + c) + a)/(b^3*sec(d*x + c)^3 + 3*a*b^2*sec(d*x +
c)^2 + 3*a^2*b*sec(d*x + c) + a^3), x)
```

3.575.6 Sympy [F]

$$\int \frac{1}{(a+b\sec(c+dx))^{5/2}}\,dx=\int \frac{1}{(a+b\sec{(c+dx)})^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a+b*sec(d*x+c))**(5/2),x)
```

output
```
Integral((a + b*sec(c + d*x))**(-5/2), x)
```

3.575.7 Maxima [F]

$$\int \frac{1}{(a+b\sec(c+dx))^{5/2}}\,dx=\int \frac{1}{(b\sec{(dx+c)}+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^(-5/2), x)
```

3.575.8 Giac [F]

$$\int \frac{1}{(a+b\sec(c+dx))^{5/2}}\,dx=\int \frac{1}{(b\sec{(dx+c)}+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^(-5/2), x)
```

3.575.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\sec(c+dx))^{5/2}}\,dx=\int \frac{1}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

input
```
int(1/(a + b/cos(c + d*x))^(5/2),x)
```

output
```
int(1/(a + b/cos(c + d*x))^(5/2), x)
```

3.576 $\int \frac{\cos(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$

3.576.1 Optimal result 4358
3.576.2 Mathematica [B] (verified) 4359
3.576.3 Rubi [A] (verified) 4360
3.576.4 Maple [B] (verified) 4365
3.576.5 Fricas [F] 4366
3.576.6 Sympy [F] 4366
3.576.7 Maxima [F] 4366
3.576.8 Giac [F] 4367
3.576.9 Mupad [F(-1)] 4367

3.576.1 Optimal result

Integrand size = 21, antiderivative size = 510

$$\int \frac{\cos(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \frac{(3a^4-26a^2b^2+15b^4)\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+}{a+b}}}{3a^3(a-b)b(a+b)^{3/2}d}$$
$$+\frac{(3a^3+21a^2b-5ab^2-15b^3)\cot(c+dx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(}{a-}}}{3a^3(a-b)(a+b)^{3/2}d}$$
$$+\frac{5b\sqrt{a+b}\cot(c+dx)\,\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{a^4d}$$
$$+\frac{\sin(c+dx)}{ad(a+b\sec(c+dx))^{3/2}}+\frac{b(3a^2-5b^2)\tan(c+dx)}{3a^2\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}}$$
$$+\frac{b(3a^4-26a^2b^2+15b^4)\tan(c+dx)}{3a^3\left(a^2-b^2\right)^2d\sqrt{a+b\sec(c+dx)}}$$

output
```
sin(d*x+c)/a/d/(a+b*sec(d*x+c))^(3/2)+1/3*(3*a^4-26*a^2*b^2+15*b^4)*cot(d*
x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(b*
(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/a^3/(a-b)/b/(a
+b)^(3/2)/d+1/3*(3*a^3+21*a^2*b-5*a*b^2-15*b^3)*cot(d*x+c)*EllipticF((a+b*
sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(b*(1-sec(d*x+c))/(a+b)
)^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/a^3/(a-b)/(a+b)^(3/2)/d+5*b*cot(d*
x+c)*EllipticPi((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),(a+b)/a,((a+b)/(a-b))^(
1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))
^(1/2)/a^4/d+1/3*b*(3*a^2-5*b^2)*tan(d*x+c)/a^2/(a^2-b^2)/d/(a+b*sec(d*x+c
))^(3/2)+1/3*b*(3*a^4-26*a^2*b^2+15*b^4)*tan(d*x+c)/a^3/(a^2-b^2)^2/d/(a+b
*sec(d*x+c))^(1/2)
```

3.576.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1481 vs. $2(510) = 1020$.

Time = 14.82 (sec) , antiderivative size = 1481, normalized size of antiderivative = 2.90

$$\int \frac{\cos(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
Integrate[Cos[c + d*x]/(a + b*Sec[c + d*x])^(5/2),x]
```

output
```
((b + a*Cos[c + d*x])^3*Sec[c + d*x]^3*((-4*b^2*(-5*a^2 + 3*b^2)*Sin[c + d
*x])/(3*a^3*(-a^2 + b^2)^2) + (2*b^4*Sin[c + d*x])/(3*a^3*(a^2 - b^2)*(b +
 a*Cos[c + d*x])^2) + (2*(-11*a^2*b^3*Sin[c + d*x] + 7*b^5*Sin[c + d*x]))/
(3*a^3*(a^2 - b^2)^2*(b + a*Cos[c + d*x]))))/(d*(a + b*Sec[c + d*x])^(5/2)
) - ((b + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(5/2)*Sqrt[(1 - Tan[(c + d*x)
/2]^2)^(-1)]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(1
 + Tan[(c + d*x)/2]^2)]*(3*a^5*Tan[(c + d*x)/2] + 3*a^4*b*Tan[(c + d*x)/2]
 - 26*a^3*b^2*Tan[(c + d*x)/2] - 26*a^2*b^3*Tan[(c + d*x)/2] + 15*a*b^4*Ta
n[(c + d*x)/2] + 15*b^5*Tan[(c + d*x)/2] - 6*a^5*Tan[(c + d*x)/2]^3 + 52*a
^3*b^2*Tan[(c + d*x)/2]^3 - 30*a*b^4*Tan[(c + d*x)/2]^3 + 3*a^5*Tan[(c + d
*x)/2]^5 - 3*a^4*b*Tan[(c + d*x)/2]^5 - 26*a^3*b^2*Tan[(c + d*x)/2]^5 + 26
*a^2*b^3*Tan[(c + d*x)/2]^5 + 15*a*b^4*Tan[(c + d*x)/2]^5 - 15*b^5*Tan[(c
+ d*x)/2]^5 - 30*a^4*b*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (a - b)/(a
 + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b
*Tan[(c + d*x)/2]^2)/(a + b)] + 60*a^2*b^3*EllipticPi[-1, ArcSin[Tan[(c +
d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Ta
n[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - 30*b^5*EllipticPi[-1,
ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sq
rt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - 30*a^4
*b*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Tan[(c + d...
```

3.576.3 Rubi [A] (verified)

Time = 2.27 (sec) , antiderivative size = 548, normalized size of antiderivative = 1.07, number of steps used = 17, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.810$, Rules used = {3042, 4333, 27, 3042, 4549, 27, 3042, 4548, 27, 3042, 4546, 3042, 4409, 3042, 4271, 4319, 4492}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(c+dx)}{(a+b\sec(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \text{4333}$$

$$\frac{\int -\frac{5b-3b\sec^2(c+dx)}{2(a+b\sec(c+dx))^{5/2}}dx}{a}+\frac{\sin(c+dx)}{ad(a+b\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{\sin(c+dx)}{ad(a+b\sec(c+dx))^{3/2}}-\frac{\int \frac{5b-3b\sec^2(c+dx)}{(a+b\sec(c+dx))^{5/2}}dx}{2a}$$

$$\Big\downarrow \; 3042$$

$$\frac{\sin(c+dx)}{ad(a+b\sec(c+dx))^{3/2}}-\frac{\int \frac{5b-3b\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx}{2a}$$

$$\Big\downarrow \; 4549$$

$$\frac{\sin(c+dx)}{ad(a+b\sec(c+dx))^{3/2}}-\frac{-\frac{2\int -\frac{-6a\sec(c+dx)b^2-\left(3a^2-5b^2\right)\sec^2(c+dx)b+15\left(a^2-b^2\right)b}{2(a+b\sec(c+dx))^{3/2}}dx}{3a(a^2-b^2)}-\frac{2b\left(3a^2-5b^2\right)\tan(c+dx)}{3ad(a^2-b^2)(a+b\sec(c+dx))^{3/2}}}{2a}$$

$$\Big\downarrow \; 27$$

$$\frac{\sin(c+dx)}{ad(a+b\sec(c+dx))^{3/2}}-\frac{\frac{\int \frac{-6a\sec(c+dx)b^2-\left(3a^2-5b^2\right)\sec^2(c+dx)b+15\left(a^2-b^2\right)b}{(a+b\sec(c+dx))^{3/2}}dx}{3a(a^2-b^2)}-\frac{2b\left(3a^2-5b^2\right)\tan(c+dx)}{3ad(a^2-b^2)(a+b\sec(c+dx))^{3/2}}}{2a}$$

$$\Big\downarrow \; 3042$$

$$\frac{\sin(c+dx)}{ad(a+b\sec(c+dx))^{3/2}}-\frac{\frac{\int \frac{-6a\csc\left(c+dx+\frac{\pi}{2}\right)b^2-\left(3a^2-5b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2b+15\left(a^2-b^2\right)b}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{3a(a^2-b^2)}-\frac{2b\left(3a^2-5b^2\right)\tan(c+dx)}{3ad(a^2-b^2)(a+b\sec(c+dx))^{3/2}}}{2a}$$

$$\Big\downarrow \; 4548$$

$$\frac{\sin(c+dx)}{ad(a+b\sec(c+dx))^{3/2}}-\frac{\frac{-\frac{2\int -\frac{-2a\left(9a^2-5b^2\right)\sec(c+dx)b^2+15\left(a^2-b^2\right)^2b+\left(3a^4-26b^2a^2+15b^4\right)\sec^2(c+dx)b}{2\sqrt{a+b\sec(c+dx)}}dx}{a\left(a^2-b^2\right)}-\frac{2b\left(3a^4-26a^2b^2+15b^4\right)\tan(c+dx)}{ad\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}}{3a(a^2-b^2)}-\frac{2b\left(3a^2-5b^2\right)\tan(c+dx)}{3ad(a^2-b^2)(a+b\sec(c+dx)}}{2a}$$

$$\Big\downarrow \; 27$$

$$\frac{\sin(c+dx)}{ad(a+b\sec(c+dx))^{3/2}}-\frac{\frac{\frac{\int \frac{-2a\left(9a^2-5b^2\right)\sec(c+dx)b^2+15\left(a^2-b^2\right)^2b+\left(3a^4-26b^2a^2+15b^4\right)\sec^2(c+dx)b}{\sqrt{a+b\sec(c+dx)}}dx}{a\left(a^2-b^2\right)}-\frac{2b\left(3a^4-26a^2b^2+15b^4\right)\tan(c+dx)}{ad\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}}{3a(a^2-b^2)}-\frac{2b\left(3a^2-5b^2\right)\tan(c+dx)}{3ad(a^2-b^2)(a+b\sec(c+dx))^{3/2}}}{2a}$$

3.576. $\int \frac{\cos(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$

$$\Big\downarrow \; 3042$$

$$\frac{\sin(c+dx)}{ad(a+b\sec(c+dx))^{3/2}}-\frac{\frac{\frac{\int \frac{-2a\left(9a^2-5b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)b^2+15\left(a^2-b^2\right)^2b+\left(3a^4-26b^2a^2+15b^4\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2b}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a\left(a^2-b^2\right)}-\frac{2b\left(3a^4-26a^2b^2+15b^4\right)\tan(c+dx)}{ad\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}}{3a(a^2-b^2)}-\frac{2b(3a^2-5b^2)\tan(c+}{3ad(a^2-b^2)(a+b\sec(c+}}{2a}$$

$$\Big\downarrow \; 4546$$

$$\frac{\sin(c+dx)}{ad(a+b\sec(c+dx))^{3/2}}-\frac{\frac{\frac{b\left(3a^4-26a^2b^2+15b^4\right)\int \frac{\sec(c+dx)(\sec(c+dx)+1)}{\sqrt{a+b\sec(c+dx)}}dx+\int \frac{15b\left(a^2-b^2\right)^2+\left(-2a\left(9a^2-5b^2\right)b^2-\left(3a^4-26b^2a^2+15b^4\right)b\right)\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx}{a\left(a^2-b^2\right)}-\frac{2b\left(3a^4-26a^2b^2+15b^4\right)\tan(c+}{ad\left(a^2-b^2\right)\sqrt{a+b\sec(c+d}}}{3a(a^2-b^2)}}{2a}$$

$$\Big\downarrow \; 3042$$

$$\frac{\sin(c+dx)}{ad(a+b\sec(c+dx))^{3/2}}-\frac{\frac{\frac{b\left(3a^4-26a^2b^2+15b^4\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\int \frac{15b\left(a^2-b^2\right)^2+\left(-2a\left(9a^2-5b^2\right)b^2-\left(3a^4-26b^2a^2+15b^4\right)b\right)\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a\left(a^2-b^2\right)}-\frac{2b\left(3a^4-26a^2b}{ad\left(a^2-b^2\right)}}{3a(a^2-b^2)}}{2a}$$

$$\Big\downarrow \; 4409$$

$$\frac{\sin(c+dx)}{ad(a+b\sec(c+dx))^{3/2}}-\frac{\frac{\frac{15b\left(a^2-b^2\right)^2\int \frac{1}{\sqrt{a+b\sec(c+dx)}}dx+b\left(3a^4-26a^2b^2+15b^4\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-b(a-b)\left(3a^3+21a^2b-5ab^2-15b^3\right)\int \frac{\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx}{a\left(a^2-b^2\right)}-\frac{2b}{}}{3a(a^2-b^2)}}{2a}$$

$$\Big\downarrow \; 3042$$

$$\frac{\sin(c+dx)}{ad(a+b\sec(c+dx))^{3/2}}-\frac{\frac{\frac{15b\left(a^2-b^2\right)^2\int \frac{1}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+b\left(3a^4-26a^2b^2+15b^4\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-b(a-b)\left(3a^3+21a^2b-5ab^2-15b^3\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}}{a\left(a^2-b^2\right)}}{3a(a^2-b^2)}}{2a}$$

$$\Big\downarrow \; 4271$$

3.576. $\int \frac{\cos(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$

$$\frac{\sin(c+dx)}{ad(a+b\sec(c+dx))^{3/2}}-\frac{\frac{\frac{b\left(3a^4-26a^2b^2+15b^4\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-b(a-b)\left(3a^3+21a^2b-5ab^2-15b^3\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{30b\sqrt{a+b}\left(a^2-b^2\right)^2\cot(c+dx)\sqrt{\frac{b(1}{}}}{}}{a\left(a^2-b^2\right)}}{3a(a^2-b^2)}}{2a}$$

$\Big\downarrow$ 4319

$$\frac{\sin(c+dx)}{ad(a+b\sec(c+dx))^{3/2}}-\frac{\frac{\frac{b\left(3a^4-26a^2b^2+15b^4\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{30b\sqrt{a+b}\left(a^2-b^2\right)^2\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\operatorname{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a-}}{}\right.\right.}{ad}}{a\left(a^2-b\right.}}{}}{}}{

3.576.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4271
```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*(Rt[a
 + b, 2]/(a*d*Cot[c + d*x]))*Sqrt[b*((1 - Csc[c + d*x])/(a + b))]*Sqrt[(-b)
*((1 + Csc[c + d*x])/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[
c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

rule 4319
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4333
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d*
Csc[e + f*x])^n/(a*f*n)), x] - Simp[1/(a*d*n)   Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[b*(m + n + 1) - a*(n + 1)*Csc[e + f*x] - b*(m
 + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^
2 - b^2, 0] && ILtQ[m + 1/2, 0] && ILtQ[n, 0]
```

rule 4409
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Simp[c   Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] +
Simp[d   Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

rule 4492
```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

rule 4546
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Simp[C   Int[Csc[e + f*x]*((
1 + Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A
, B, C}, x] && NeQ[a^2 - b^2, 0]
```

rule 4548
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(A*b^2 -
a*b*B + a^2*C)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(a*f*(m + 1)*(a^2
 - b^2))), x] + Simp[1/(a*(m + 1)*(a^2 - b^2))   Int[(a + b*Csc[e + f*x])^(
m + 1)*Simp[A*(a^2 - b^2)*(m + 1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x
] + (A*b^2 - a*b*B + a^2*C)*(m + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

rule 4549
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_))^(m_), x_Symbol] :> Simp[(A*b^2 + a^2*C)*Cot[e + f*x]*((a + b*Csc[
e + f*x])^(m + 1)/(a*f*(m + 1)*(a^2 - b^2))), x] + Simp[1/(a*(m + 1)*(a^2 -
 b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*Simp[A*(a^2 - b^2)*(m + 1) - a*b*
(A + C)*(m + 1)*Csc[e + f*x] + (A*b^2 + a^2*C)*(m + 2)*Csc[e + f*x]^2, x],
x], x] /; FreeQ[{a, b, e, f, A, C}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[2*m
] && LtQ[m, -1]
```

3.576.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 5834 vs. $2(469) = 938$.

Time = 9.70 (sec) , antiderivative size = 5835, normalized size of antiderivative = 11.44

method	result	size
default	Expression too large to display	5835

input
```
int(cos(d*x+c)/(a+b*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.576.5 Fricas [F]

$$\int \frac{\cos(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{\cos(dx+c)}{(b\sec(dx+c)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sec(d*x + c) + a)*cos(d*x + c)/(b^3*sec(d*x + c)^3 + 3*a*b
^2*sec(d*x + c)^2 + 3*a^2*b*sec(d*x + c) + a^3), x)
```

3.576.6 Sympy [F]

$$\int \frac{\cos(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{\cos(c+dx)}{(a+b\sec(c+dx))^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)/(a+b*sec(d*x+c))**(5/2),x)
```

output
```
Integral(cos(c + d*x)/(a + b*sec(c + d*x))**(5/2), x)
```

3.576.7 Maxima [F]

$$\int \frac{\cos(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{\cos(dx+c)}{(b\sec(dx+c)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)/(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate(cos(d*x + c)/(b*sec(d*x + c) + a)^(5/2), x)
```

3.576.8 Giac [F]

$$\int \frac{\cos(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{\cos(dx+c)}{(b\sec(dx+c)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)/(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate(cos(d*x + c)/(b*sec(d*x + c) + a)^(5/2), x)
```

3.576.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{\cos(c+d\,x)}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

input
```
int(cos(c + d*x)/(a + b/cos(c + d*x))^(5/2),x)
```

output
```
int(cos(c + d*x)/(a + b/cos(c + d*x))^(5/2), x)
```

3.577 $\int \frac{\cos^2(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$

3.577.1 Optimal result . . . 4368
3.577.2 Mathematica [B] (verified) . . . 4369
3.577.3 Rubi [A] (verified) . . . 4370
3.577.4 Maple [B] (verified) . . . 4376
3.577.5 Fricas [F(-1)] . . . 4376
3.577.6 Sympy [F] . . . 4377
3.577.7 Maxima [F] . . . 4377
3.577.8 Giac [F] . . . 4377
3.577.9 Mupad [F(-1)] . . . 4378

3.577.1 Optimal result

Integrand size = 23, antiderivative size = 562

$$\int \frac{\cos^2(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx =$$
$$-\frac{(33a^4-170a^2b^2+105b^4)\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{12a^4(a-b)(a+b)^{3/2}d}$$
$$+\frac{(a+3b)(6a^3-45a^2b+35b^3)\cot(c+dx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\text{se}}{a}}}{12a^4(a-b)(a+b)^{3/2}d}$$
$$-\frac{\sqrt{a+b}(4a^2+35b^2)\cot(c+dx)\,\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+d}{a-b}}}{4a^5d}$$
$$-\frac{7b\sin(c+dx)}{4a^2d(a+b\sec(c+dx))^{3/2}}+\frac{\cos(c+dx)\sin(c+dx)}{2ad(a+b\sec(c+dx))^{3/2}}$$
$$-\frac{b^2(27a^2-35b^2)\tan(c+dx)}{12a^3(a^2-b^2)d(a+b\sec(c+dx))^{3/2}}-\frac{b^2(33a^4-170a^2b^2+105b^4)\tan(c+dx)}{12a^4(a^2-b^2)^2d\sqrt{a+b\sec(c+dx)}}$$

output
```
-7/4*b*sin(d*x+c)/a^2/d/(a+b*sec(d*x+c))^(3/2)+1/2*cos(d*x+c)*sin(d*x+c)/a
/d/(a+b*sec(d*x+c))^(3/2)-1/12*(33*a^4-170*a^2*b^2+105*b^4)*cot(d*x+c)*Ell
ipticE((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(b*(1-sec(d
*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/a^4/(a-b)/(a+b)^(3/2)/
d+1/12*(a+3*b)*(6*a^3-45*a^2*b+35*b^3)*cot(d*x+c)*EllipticF((a+b*sec(d*x+c
))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(
-b*(1+sec(d*x+c))/(a-b))^(1/2)/a^4/(a-b)/(a+b)^(3/2)/d-1/4*(4*a^2+35*b^2)*
cot(d*x+c)*EllipticPi((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),(a+b)/a,((a+b)/(a
-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/
(a-b))^(1/2)/a^5/d-1/12*b^2*(27*a^2-35*b^2)*tan(d*x+c)/a^3/(a^2-b^2)/d/(a+
b*sec(d*x+c))^(3/2)-1/12*b^2*(33*a^4-170*a^2*b^2+105*b^4)*tan(d*x+c)/a^4/(
a^2-b^2)^2/d/(a+b*sec(d*x+c))^(1/2)
```

3.577.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1730 vs. $2(562) = 1124$.

Time = 13.84 (sec) , antiderivative size = 1730, normalized size of antiderivative = 3.08

$$\int \frac{\cos^2(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
Integrate[Cos[c + d*x]^2/(a + b*Sec[c + d*x])^(5/2),x]
```

output
```
((b + a*Cos[c + d*x])^3*Sec[c + d*x]^3*((2*b^3*(-13*a^2 + 9*b^2)*Sin[c + d
*x])/(3*a^4*(-a^2 + b^2)^2) - (2*b^5*Sin[c + d*x])/(3*a^4*(a^2 - b^2)*(b +
 a*Cos[c + d*x])^2) - (4*(-7*a^2*b^4*Sin[c + d*x] + 5*b^6*Sin[c + d*x]))/(
3*a^4*(a^2 - b^2)^2*(b + a*Cos[c + d*x])) + Sin[2*(c + d*x)]/(4*a^3)))/(d*
(a + b*Sec[c + d*x])^(5/2)) + ((b + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(5/
2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c
+ d*x)/2]^2)]*(-33*a^5*b*Tan[(c + d*x)/2] - 33*a^4*b^2*Tan[(c + d*x)/2] +
170*a^3*b^3*Tan[(c + d*x)/2] + 170*a^2*b^4*Tan[(c + d*x)/2] - 105*a*b^5*Ta
n[(c + d*x)/2] - 105*b^6*Tan[(c + d*x)/2] + 66*a^5*b*Tan[(c + d*x)/2]^3 -
340*a^3*b^3*Tan[(c + d*x)/2]^3 + 210*a*b^5*Tan[(c + d*x)/2]^3 - 33*a^5*b*T
an[(c + d*x)/2]^5 + 33*a^4*b^2*Tan[(c + d*x)/2]^5 + 170*a^3*b^3*Tan[(c + d
*x)/2]^5 - 170*a^2*b^4*Tan[(c + d*x)/2]^5 - 105*a*b^5*Tan[(c + d*x)/2]^5 +
 105*b^6*Tan[(c + d*x)/2]^5 + 24*a^6*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2
]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c +
 d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + 162*a^4*b^2*EllipticPi[-1, A
rcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqr
t[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - 396*a^2
*b^4*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Ta
n[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^
2)/(a + b)] + 210*b^6*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (a - b)/...
```

3.577.3 Rubi [A] (verified)

Time = 2.84 (sec) , antiderivative size = 604, normalized size of antiderivative = 1.07, number of steps used = 20, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.870$, Rules used = {3042, 4333, 27, 3042, 4592, 27, 3042, 4548, 27, 3042, 4548, 27, 3042, 4546, 3042, 4409, 3042, 4271, 4319, 4492}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^2(c+dx)}{(a+b\sec(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^2\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \text{4333}$$

$$\frac{\int -\frac{\cos(c+dx)\left(-5b\sec^2(c+dx)-2a\sec(c+dx)+7b\right)}{2(a+b\sec(c+dx))^{5/2}}dx}{2a}+\frac{\sin(c+dx)\cos(c+dx)}{2ad(a+b\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{\sin(c+dx)\cos(c+dx)}{2ad(a+b\sec(c+dx))^{3/2}}-\frac{\int \frac{\cos(c+dx)\left(-5b\sec^2(c+dx)-2a\sec(c+dx)+7b\right)}{(a+b\sec(c+dx))^{5/2}}dx}{4a}$$

$$\Big\downarrow \; 3042$$

$$\frac{\sin(c+dx)\cos(c+dx)}{2ad(a+b\sec(c+dx))^{3/2}}-\frac{\int \frac{-5b\csc\left(c+dx+\frac{\pi}{2}\right)^2-2a\csc\left(c+dx+\frac{\pi}{2}\right)+7b}{\csc\left(c+dx+\frac{\pi}{2}\right)\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx}{4a}$$

$$\Big\downarrow \; 4592$$

$$\frac{\sin(c+dx)\cos(c+dx)}{2ad(a+b\sec(c+dx))^{3/2}}-\frac{\frac{7b\sin(c+dx)}{ad(a+b\sec(c+dx))^{3/2}}-\frac{\int \frac{4a^2+10b\sec(c+dx)a+35b^2-21b^2\sec^2(c+dx)}{2(a+b\sec(c+dx))^{5/2}}dx}{a}}{4a}$$

$$\Big\downarrow \; 27$$

$$\frac{\sin(c+dx)\cos(c+dx)}{2ad(a+b\sec(c+dx))^{3/2}}-\frac{\frac{7b\sin(c+dx)}{ad(a+b\sec(c+dx))^{3/2}}-\frac{\int \frac{4a^2+10b\sec(c+dx)a+35b^2-21b^2\sec^2(c+dx)}{(a+b\sec(c+dx))^{5/2}}dx}{2a}}{4a}$$

$$\Big\downarrow \; 3042$$

$$\frac{\sin(c+dx)\cos(c+dx)}{2ad(a+b\sec(c+dx))^{3/2}}-\frac{\frac{7b\sin(c+dx)}{ad(a+b\sec(c+dx))^{3/2}}-\frac{\int \frac{4a^2+10b\csc\left(c+dx+\frac{\pi}{2}\right)a+35b^2-21b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx}{2a}}{4a}$$

$$\Big\downarrow \; 4548$$

$$\frac{\sin(c+dx)\cos(c+dx)}{2ad(a+b\sec(c+dx))^{3/2}}-\frac{\frac{7b\sin(c+dx)}{ad(a+b\sec(c+dx))^{3/2}}-\frac{-\frac{2\int -\frac{-b^2\left(27a^2-35b^2\right)\sec^2(c+dx)+6ab\left(3a^2-7b^2\right)\sec(c+dx)+3\left(a^2-b^2\right)\left(4a^2+35b^2\right)}{2(a+b\sec(c+dx))^{3/2}}dx}{3a\left(a^2-b^2\right)}-\frac{2b^2\left(27a^2-35b^2\right)\tan(c+dx)}{3ad\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}}{2a}}{4a}$$

$$\Big\downarrow \; 27$$

$$\frac{\sin(c+dx)\cos(c+dx)}{2ad(a+b\sec(c+dx))^{3/2}}-\frac{\frac{7b\sin(c+dx)}{ad(a+b\sec(c+dx))^{3/2}}-\frac{\frac{\int \frac{-b^2\left(27a^2-35b^2\right)\sec^2(c+dx)+6ab\left(3a^2-7b^2\right)\sec(c+dx)+3\left(a^2-b^2\right)\left(4a^2+35b^2\right)}{(a+b\sec(c+dx))^{3/2}}dx}{3a\left(a^2-b^2\right)}-\frac{2b^2\left(27a^2-35b^2\right)\tan(c+dx)}{3ad\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}}{2a}}{4a}$$

$$\Big\downarrow \; 3042$$

3.577. $\int \frac{\cos^2(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$

$$\frac{\sin(c+dx)\cos(c+dx)}{2ad(a+b\sec(c+dx))^{3/2}}-\frac{\frac{7b\sin(c+dx)}{ad(a+b\sec(c+dx))^{3/2}}-\frac{\frac{\int\frac{-b^2\left(27a^2-35b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2+6ab\left(3a^2-7b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)+3\left(a^2-b^2\right)\left(4a^2+35b^2\right)}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{3a\left(a^2-b^2\right)}-\frac{2b^2\left(27a^2-35b^2\right)\tan(c+dx)}{3ad\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}}{2a}}{4a}$$

$\downarrow$ 4548

$$\frac{\sin(c+dx)\cos(c+dx)}{2ad(a+b\sec(c+dx))^{3/2}}-\frac{\frac{7b\sin(c+dx)}{ad(a+b\sec(c+dx))^{3/2}}-\frac{\frac{-\frac{2\int-\frac{3\left(4a^2+35b^2\right)\left(a^2-b^2\right)^2+b^2\left(33a^4-170b^2a^2+105b^4\right)\sec^2(c+dx)+2ab\left(3a^4-54b^2a^2+35b^4\right)\sec(c+dx)}{2\sqrt{a+b\sec(c+dx)}}dx}{a\left(a^2-b^2\right)}-\frac{2b^2\left(33a^4-170a^2b\right.}{ad\left(a^2-b^2\right)\sqrt{}}}{3a\left(a^2-b^2\right)}}{2a}}{4a}$$

$\downarrow$ 27

$$\frac{\sin(c+dx)\cos(c+dx)}{2ad(a+b\sec(c+dx))^{3/2}}-\frac{\frac{7b\sin(c+dx)}{ad(a+b\sec(c+dx))^{3/2}}-\frac{\frac{\frac{\int\frac{3\left(4a^2+35b^2\right)\left(a^2-b^2\right)^2+b^2\left(33a^4-170b^2a^2+105b^4\right)\sec^2(c+dx)+2ab\left(3a^4-54b^2a^2+35b^4\right)\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx}{a\left(a^2-b^2\right)}-\frac{2b^2\left(33a^4-170a^2b^2+10\right.}{ad\left(a^2-b^2\right)\sqrt{a+b}}}{3a\left(a^2-b^2\right)}}{2a}}{4a}$$

$\downarrow$ 3042

$$\frac{\sin(c+dx)\cos(c+dx)}{2ad(a+b\sec(c+dx))^{3/2}}-\frac{\frac{7b\sin(c+dx)}{ad(a+b\sec(c+dx))^{3/2}}-\frac{\frac{\frac{\int\frac{3\left(4a^2+35b^2\right)\left(a^2-b^2\right)^2+b^2\left(33a^4-170b^2a^2+105b^4\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2+2ab\left(3a^4-54b^2a^2+35b^4\right)\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a\left(a^2-b^2\right)}-\frac{2b^2\left(33a^4-17\right.}{ad\left(a^2-\right.}}{3a\left(a^2-b^2\right)}}{2a}}{4a}$$

$\downarrow$ 4546

$$\frac{\sin(c+dx)\cos(c+dx)}{2ad(a+b\sec(c+dx))^{3/2}}-\frac{\frac{7b\sin(c+dx)}{ad(a+b\sec(c+dx))^{3/2}}-\frac{\frac{\frac{b^2\left(33a^4-170a^2b^2+105b^4\right)\int\frac{\sec(c+dx)(\sec(c+dx)+1)}{\sqrt{a+b\sec(c+dx)}}dx+\int\frac{3\left(4a^2+35b^2\right)\left(a^2-b^2\right)^2+\left(2ab\left(3a^4-54b^2a^2+35b^4\right)-b^2\left(33a^4-170b^2a\right.\right.}{\sqrt{a+b\sec(c+dx)}}}{a\left(a^2-b^2\right)}}{3a\left(a^2-b^2\right)}}{2a}}{4a}$$

$\downarrow$ 3042

3.577. $\int \frac{\cos^2(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$

$$\frac{\sin(c+dx)\cos(c+dx)}{2ad(a+b\sec(c+dx))^{3/2}}-\frac{\frac{\frac{b^2\left(33a^4-170a^2b^2+105b^4\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\int\frac{3\left(4a^2+35b^2\right)\left(a^2-b^2\right)^2+\left(2ab\left(3a^4-54b^2a^2+35b^4\right)-b^2\left(33a\right.\right.}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}}{a\left(a^2-b^2\right)}}{3a\left(a^2-b^2\right)}}{2a}}{4a}-\frac{7b\sin(c+dx)}{ad(a+b\sec(c+dx))^{3/2}}$$

$\downarrow$ 4409

$$\frac{\sin(c+dx)\cos(c+dx)}{2ad(a+b\sec(c+dx))^{3/2}}-\frac{\frac{\frac{3\left(a^2-b^2\right)^2\left(4a^2+35b^2\right)\int\frac{1}{\sqrt{a+b\sec(c+dx)}}dx+b^2\left(33a^4-170a^2b^2+105b^4\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+b(a-b)(a+3b)\left(\right.}{a\left(a^2-b^2\right)}}{3a\left(a^2-b^2\right)}}{2a}}{4a}-\frac{7b\sin(c+dx)}{ad(a+b\sec(c+dx))^{3/2}}$$

$\downarrow$ 3042

$$\frac{\sin(c+dx)\cos(c+dx)}{2ad(a+b\sec(c+dx))^{3/2}}-\frac{\frac{\frac{3\left(a^2-b^2\right)^2\left(4a^2+35b^2\right)\int\frac{1}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+b^2\left(33a^4-170a^2b^2+105b^4\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+b(a-b)(a+}{a\left(a^2-b^2\right)}}{3a\left(a^2-b^2\right)}}{2a}}{4a}-\frac{7b\sin(c+dx)}{ad(a+b\sec(c+dx))^{3/2}}$$

$\downarrow$ 4271

$$\frac{\sin(c+dx)\cos(c+dx)}{2ad(a+b\sec(c+dx))^{3/2}}-\frac{\frac{b^2\left(33a^4-170a^2b^2+105b^4\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+b(a-b)(a+3b)\left(6a^3-45a^2b+35b^3\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-}{a\left(a^2-\right.}}{}-\frac{7b\sin(c+dx)}{ad(a+b\sec(c+dx))^{3/2}}$$

$\downarrow$ 4319

$$\frac{\sin(c+dx)\cos(c+dx)}{2ad(a+b\sec(c+dx))^{3/2}}-\frac{\frac{b^2\left(33a^4-170a^2b^2+105b^4\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{6\sqrt{a+b}\left(a^2-b^2\right)^2\left(4a^2+35b^2\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-}}{}}{}}{}-\frac{7b\sin(c+dx)}{ad(a+b\sec(c+dx))^{3/2}}$$

3.577. $\int\frac{\cos^2(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$

$$\Big\downarrow \text{4492}$$

$$\frac{\sin(c+dx)\cos(c+dx)}{2ad(a+b\sec(c+dx))^{3/2}} - \frac{\frac{7b\sin(c+dx)}{ad(a+b\sec(c+dx))^{3/2}} - \frac{\frac{-\frac{6\sqrt{a+b}\left(4a^2+35b^2\right)\left(a^2-b^2\right)^2\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\operatorname{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{ad}}{}}{}$$

input
```
Int[Cos[c + d*x]^2/(a + b*Sec[c + d*x])^(5/2),x]
```

output
```
(Cos[c + d*x]*Sin[c + d*x])/(2*a*d*(a + b*Sec[c + d*x])^(3/2)) - ((7*b*Sin
[c + d*x])/(a*d*(a + b*Sec[c + d*x])^(3/2)) - ((-2*b^2*(27*a^2 - 35*b^2)*T
an[c + d*x])/(3*a*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^(3/2)) + (((-2*(a - b
)*Sqrt[a + b]*(33*a^4 - 170*a^2*b^2 + 105*b^4)*Cot[c + d*x]*EllipticE[ArcS
in[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Se
c[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/d + (2*(a -
 b)*Sqrt[a + b]*(a + 3*b)*(6*a^3 - 45*a^2*b + 35*b^3)*Cot[c + d*x]*Ellipti
cF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*
(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/d -
(6*Sqrt[a + b]*(a^2 - b^2)^2*(4*a^2 + 35*b^2)*Cot[c + d*x]*EllipticPi[(a +
 b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt
[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/
(a*d))/(a*(a^2 - b^2)) - (2*b^2*(33*a^4 - 170*a^2*b^2 + 105*b^4)*Tan[c + d
*x])/(a*(a^2 - b^2)*d*Sqrt[a + b*Sec[c + d*x]]))/(3*a*(a^2 - b^2)))/(2*a))
/(4*a)
```

3.577.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4271
```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*(Rt[a
 + b, 2]/(a*d*Cot[c + d*x]))*Sqrt[b*((1 - Csc[c + d*x])/(a + b))]*Sqrt[(-b)
*((1 + Csc[c + d*x])/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[
c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

rule 4319
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4333
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d*
Csc[e + f*x])^n/(a*f*n)), x] - Simp[1/(a*d*n)   Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[b*(m + n + 1) - a*(n + 1)*Csc[e + f*x] - b*(m
 + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^
2 - b^2, 0] && ILtQ[m + 1/2, 0] && ILtQ[n, 0]
```

rule 4409
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Simp[c   Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] +
Simp[d   Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

rule 4492
```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

rule 4546
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Simp[C   Int[Csc[e + f*x]*((
1 + Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A
, B, C}, x] && NeQ[a^2 - b^2, 0]
```

3.577. $\int \frac{\cos^2(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$

rule 4548
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(A*b^2 -
a*b*B + a^2*C)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(a*f*(m + 1)*(a^2
 - b^2))), x] + Simp[1/(a*(m + 1)*(a^2 - b^2))   Int[(a + b*Csc[e + f*x])^(
m + 1)*Simp[A*(a^2 - b^2)*(m + 1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x
] + (A*b^2 - a*b*B + a^2*C)*(m + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

rule 4592
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Simp[1/(a*d*n)   Int[(a + b*Csc[e + f*x])^m
*(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*
Csc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

3.577.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 7166 vs. $2(513) = 1026$.

Time = 9.61 (sec) , antiderivative size = 7167, normalized size of antiderivative = 12.75

method	result	size
default	Expression too large to display	7167

input
```
int(cos(d*x+c)^2/(a+b*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.577.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\cos^2(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)^2/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.577.6 Sympy [F]

$$\int \frac{\cos^2(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{\cos^2(c+dx)}{(a+b\sec(c+dx))^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)**2/(a+b*sec(d*x+c))**(5/2),x)
```

output
```
Integral(cos(c + d*x)**2/(a + b*sec(c + d*x))**(5/2), x)
```

3.577.7 Maxima [F]

$$\int \frac{\cos^2(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{\cos(dx+c)^2}{(b\sec(dx+c)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^2/(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate(cos(d*x + c)^2/(b*sec(d*x + c) + a)^(5/2), x)
```

3.577.8 Giac [F]

$$\int \frac{\cos^2(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{\cos(dx+c)^2}{(b\sec(dx+c)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^2/(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate(cos(d*x + c)^2/(b*sec(d*x + c) + a)^(5/2), x)
```

3.577.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^2(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{\cos\left(c+d\,x\right)^2}{\left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)^{5/2}}\,dx$$

input
```
int(cos(c + d*x)^2/(a + b/cos(c + d*x))^(5/2),x)
```

output
```
int(cos(c + d*x)^2/(a + b/cos(c + d*x))^(5/2), x)
```

3.578 $\int \frac{1}{(a+b\sec(c+dx))^{7/2}}\,dx$

3.578.1 Optimal result 4379
3.578.2 Mathematica [B] (verified) 4380
3.578.3 Rubi [A] (verified) 4381
3.578.4 Maple [B] (verified) 4387
3.578.5 Fricas [F] 4387
3.578.6 Sympy [F] 4387
3.578.7 Maxima [F] 4388
3.578.8 Giac [F] 4388
3.578.9 Mupad [F(-1)] 4388

3.578.1 Optimal result

Integrand size = 14, antiderivative size = 535

$$\int \frac{1}{(a+b\sec(c+dx))^{7/2}}\,dx = \frac{2(58a^4-41a^2b^2+15b^4)\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(}{a+}}}{15a^3(a-b)^2(a+b)^{5/2}d}$$
$$-\frac{2(45a^4-13a^3b-36a^2b^2+5ab^3+15b^4)\cot(c+dx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx}{a+b}}}{15a^3(a-b)^2(a+b)^{5/2}d}$$
$$-\frac{2\sqrt{a+b}\cot(c+dx)\,\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{a^4d}$$
$$+\frac{2b^2\tan(c+dx)}{5a\left(a^2-b^2\right)d(a+b\sec(c+dx))^{5/2}}+\frac{2b^2(13a^2-5b^2)\tan(c+dx)}{15a^2\left(a^2-b^2\right)^2d(a+b\sec(c+dx))^{3/2}}$$
$$+\frac{2b^2(58a^4-41a^2b^2+15b^4)\tan(c+dx)}{15a^3\left(a^2-b^2\right)^3d\sqrt{a+b\sec(c+dx)}}$$

output
```
2/15*(58*a^4-41*a^2*b^2+15*b^4)*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2
)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+s
ec(d*x+c))/(a-b))^(1/2)/a^3/(a-b)^2/(a+b)^(5/2)/d-2/15*(45*a^4-13*a^3*b-36
*a^2*b^2+5*a*b^3+15*b^4)*cot(d*x+c)*EllipticF((a+b*sec(d*x+c))^(1/2)/(a+b)
^(1/2),((a+b)/(a-b))^(1/2))*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+
c))/(a-b))^(1/2)/a^3/(a-b)^2/(a+b)^(5/2)/d-2*cot(d*x+c)*EllipticPi((a+b*se
c(d*x+c))^(1/2)/(a+b)^(1/2),(a+b)/a,((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1
-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/a^4/d+2/5*b^2*ta
n(d*x+c)/a/(a^2-b^2)/d/(a+b*sec(d*x+c))^(5/2)+2/15*b^2*(13*a^2-5*b^2)*tan(
d*x+c)/a^2/(a^2-b^2)^2/d/(a+b*sec(d*x+c))^(3/2)+2/15*b^2*(58*a^4-41*a^2*b^
2+15*b^4)*tan(d*x+c)/a^3/(a^2-b^2)^3/d/(a+b*sec(d*x+c))^(1/2)
```

3.578.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1790 vs. $2(535) = 1070$.

Time = 13.38 (sec) , antiderivative size = 1790, normalized size of antiderivative = 3.35

$$\int \frac{1}{(a+b\sec(c+dx))^{7/2}}\,dx = \text{Too large to display}$$

input
```
Integrate[(a + b*Sec[c + d*x])^(-7/2),x]
```

output
```
((b + a*Cos[c + d*x])^4*Sec[c + d*x]^4*((2*b*(58*a^4 - 41*a^2*b^2 + 15*b^4
)*Sin[c + d*x])/(15*a^3*(-a^2 + b^2)^3) + (2*b^4*Sin[c + d*x])/(5*a^3*(a^2
 - b^2)*(b + a*Cos[c + d*x])^3) + (2*(-19*a^2*b^3*Sin[c + d*x] + 11*b^5*Si
n[c + d*x]))/(15*a^3*(a^2 - b^2)^2*(b + a*Cos[c + d*x])^2) + (2*(74*a^4*b^
2*Sin[c + d*x] - 65*a^2*b^4*Sin[c + d*x] + 23*b^6*Sin[c + d*x]))/(15*a^3*(
a^2 - b^2)^3*(b + a*Cos[c + d*x]))))/(d*(a + b*Sec[c + d*x])^(7/2)) + (2*(
b + a*Cos[c + d*x])^(7/2)*Sec[c + d*x]^(7/2)*Sqrt[(a + b - a*Tan[(c + d*x)
/2]^2 + b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2)]*(58*a^5*b*Tan[(c +
 d*x)/2] + 58*a^4*b^2*Tan[(c + d*x)/2] - 41*a^3*b^3*Tan[(c + d*x)/2] - 41*
a^2*b^4*Tan[(c + d*x)/2] + 15*a*b^5*Tan[(c + d*x)/2] + 15*b^6*Tan[(c + d*x
)/2] - 116*a^5*b*Tan[(c + d*x)/2]^3 + 82*a^3*b^3*Tan[(c + d*x)/2]^3 - 30*a
*b^5*Tan[(c + d*x)/2]^3 + 58*a^5*b*Tan[(c + d*x)/2]^5 - 58*a^4*b^2*Tan[(c
+ d*x)/2]^5 - 41*a^3*b^3*Tan[(c + d*x)/2]^5 + 41*a^2*b^4*Tan[(c + d*x)/2]^
5 + 15*a*b^5*Tan[(c + d*x)/2]^5 - 15*b^6*Tan[(c + d*x)/2]^5 + 30*a^6*Ellip
ticPi[-1, ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x
)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)
] - 90*a^4*b^2*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*S
qrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c
+ d*x)/2]^2)/(a + b)] + 90*a^2*b^4*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]]
, (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c ...
```

3.578.3 Rubi [A] (verified)

Time = 2.44 (sec) , antiderivative size = 584, normalized size of antiderivative = 1.09, number of steps used = 17, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 1.214$, Rules used = {3042, 4272, 27, 3042, 4548, 27, 3042, 4548, 27, 3042, 4546, 3042, 4409, 3042, 4271, 4319, 4492}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+b\sec(c+dx))^{7/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{7/2}}dx$$

$$\Big\downarrow \text{4272}$$

$$\frac{2b^2\tan(c+dx)}{5ad\left(a^2-b^2\right)(a+b\sec(c+dx))^{5/2}}-\frac{2\int -\frac{3b^2\sec^2(c+dx)-5ab\sec(c+dx)+5\left(a^2-b^2\right)}{2(a+b\sec(c+dx))^{5/2}}dx}{5a\left(a^2-b^2\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{3b^2\sec^2(c+dx)-5ab\sec(c+dx)+5(a^2-b^2)}{(a+b\sec(c+dx))^{5/2}}dx}{5a\left(a^2-b^2\right)}+\frac{2b^2\tan(c+dx)}{5ad\left(a^2-b^2\right)(a+b\sec(c+dx))^{5/2}}$$

$$\Big\downarrow \; 3042$$

$$\frac{\int \frac{3b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2-5ab\csc\left(c+dx+\frac{\pi}{2}\right)+5(a^2-b^2)}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx}{5a\left(a^2-b^2\right)}+\frac{2b^2\tan(c+dx)}{5ad\left(a^2-b^2\right)(a+b\sec(c+dx))^{5/2}}$$

$$\Big\downarrow \; 4548$$

$$\frac{\frac{2b^2\left(13a^2-5b^2\right)\tan(c+dx)}{3ad(a^2-b^2)(a+b\sec(c+dx))^{3/2}}-\frac{2\int -\frac{15\left(a^2-b^2\right)^2+b^2\left(13a^2-5b^2\right)\sec^2(c+dx)-6ab\left(5a^2-b^2\right)\sec(c+dx)}{2(a+b\sec(c+dx))^{3/2}}dx}{3a(a^2-b^2)}}{5a\left(a^2-b^2\right)}+\frac{2b^2\tan(c+dx)}{5ad\left(a^2-b^2\right)(a+b\sec(c+dx))^{5/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\int \frac{15\left(a^2-b^2\right)^2+b^2\left(13a^2-5b^2\right)\sec^2(c+dx)-6ab\left(5a^2-b^2\right)\sec(c+dx)}{(a+b\sec(c+dx))^{3/2}}dx}{3a(a^2-b^2)}+\frac{2b^2\left(13a^2-5b^2\right)\tan(c+dx)}{3ad(a^2-b^2)(a+b\sec(c+dx))^{3/2}}}{5a\left(a^2-b^2\right)}+\frac{2b^2\tan(c+dx)}{5ad\left(a^2-b^2\right)(a+b\sec(c+dx))^{5/2}}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{\int \frac{15\left(a^2-b^2\right)^2+b^2\left(13a^2-5b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2-6ab\left(5a^2-b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{3a(a^2-b^2)}+\frac{2b^2\left(13a^2-5b^2\right)\tan(c+dx)}{3ad(a^2-b^2)(a+b\sec(c+dx))^{3/2}}}{5a\left(a^2-b^2\right)}+\frac{2b^2\tan(c+dx)}{5ad\left(a^2-b^2\right)(a+b\sec(c+dx))^{5/2}}$$

$$\Big\downarrow \; 4548$$

$$\frac{\frac{\frac{2b^2\left(58a^4-41a^2b^2+15b^4\right)\tan(c+dx)}{ad\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}-\frac{2\int -\frac{15\left(a^2-b^2\right)^3-b^2\left(58a^4-41b^2a^2+15b^4\right)\sec^2(c+dx)-ab\left(45a^4-23b^2a^2+10b^4\right)\sec(c+dx)}{2\sqrt{a+b\sec(c+dx)}}dx}{a\left(a^2-b^2\right)}}{3a(a^2-b^2)}+\frac{2b^2\left(13a^2-5b^2\right)\mathrm{t}}{3ad(a^2-b^2)(a+b\mathrm{se}}}{5a\left(a^2-b^2\right)}+\frac{2b^2\tan(c+dx)}{5ad\left(a^2-b^2\right)(a+b\sec(c+dx))^{5/2}}$$

$$\Big\downarrow \; 27$$

3.578. $\int \frac{1}{(a+b\sec(c+dx))^{7/2}}\,dx$

$$\frac{\frac{\frac{\int \frac{15\left(a^2-b^2\right)^3-b^2\left(58a^4-41b^2a^2+15b^4\right)\sec^2(c+dx)-ab\left(45a^4-23b^2a^2+10b^4\right)\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx}{a\left(a^2-b^2\right)}+\frac{2b^2\left(58a^4-41a^2b^2+15b^4\right)\tan(c+dx)}{ad\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}}{3a(a^2-b^2)}+\frac{2b^2\left(13a^2-5b^2\right)\tan}{3ad(a^2-b^2)(a+b\sec(c}}{5a\left(a^2-b^2\right)}$$

$$\frac{2b^2\tan(c+dx)}{5ad\left(a^2-b^2\right)(a+b\sec(c+dx))^{5/2}}$$

$\downarrow$ 3042

$$\frac{\frac{\frac{\int \frac{15\left(a^2-b^2\right)^3-b^2\left(58a^4-41b^2a^2+15b^4\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2-ab\left(45a^4-23b^2a^2+10b^4\right)\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a\left(a^2-b^2\right)}+\frac{2b^2\left(58a^4-41a^2b^2+15b^4\right)\tan(c+dx)}{ad\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}}{3a(a^2-b^2)}+\frac{2b^2\left(13a^2-\right.}{3ad(a^2-b^2)(a}}{5a\left(a^2-b^2\right)}$$

$$\frac{2b^2\tan(c+dx)}{5ad\left(a^2-b^2\right)(a+b\sec(c+dx))^{5/2}}$$

$\downarrow$ 4546

$$\frac{\frac{\frac{\int \frac{15\left(a^2-b^2\right)^3+\left(b^2\left(58a^4-41b^2a^2+15b^4\right)-ab\left(45a^4-23b^2a^2+10b^4\right)\right)\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx-b^2\left(58a^4-41a^2b^2+15b^4\right)\int\frac{\sec(c+dx)(\sec(c+dx)+1)}{\sqrt{a+b\sec(c+dx)}}dx}{a\left(a^2-b^2\right)}+\frac{2b^2\left(58a^4-41a^2b^2\right.}{ad\left(a^2-b^2\right)\sqrt{a}}}{3a(a^2-b^2)}}{5a\left(a^2-b^2\right)}$$

$$\frac{2b^2\tan(c+dx)}{5ad\left(a^2-b^2\right)(a+b\sec(c+dx))^{5/2}}$$

$\downarrow$ 3042

$$\frac{\frac{\frac{\int \frac{15\left(a^2-b^2\right)^3+\left(b^2\left(58a^4-41b^2a^2+15b^4\right)-ab\left(45a^4-23b^2a^2+10b^4\right)\right)\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-b^2\left(58a^4-41a^2b^2+15b^4\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a\left(a^2-b^2\right)}+\frac{2b^2(}{}}{3a(a^2-b^2)}}{5a\left(a^2-b^2\right)}$$

$$\frac{2b^2\tan(c+dx)}{5ad\left(a^2-b^2\right)(a+b\sec(c+dx))^{5/2}}$$

$\downarrow$ 4409

$$\frac{\frac{\frac{15\left(a^2-b^2\right)^3\int\frac{1}{\sqrt{a+b\sec(c+dx)}}dx-b^2\left(58a^4-41a^2b^2+15b^4\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-b(a-b)\left(45a^4-13a^3b-36a^2b^2+5ab^3+15b^4\right)\int\frac{\sec(c+dx)}{\sqrt{a+b\sec(c+}}}{a\left(a^2-b^2\right)}}{3a(a^2-b^2)}}{5a\left(a^2-b^2\right)}$$

$$\frac{2b^2\tan(c+dx)}{5ad\left(a^2-b^2\right)(a+b\sec(c+dx))^{5/2}}$$

3.578. $\int \frac{1}{(a+b\sec(c+dx))^{7/2}}\,dx$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{\frac{15\left(a^2-b^2\right)^3\int \frac{1}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-b^2\left(58a^4-41a^2b^2+15b^4\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-b(a-b)\left(45a^4-13a^3b-36a^2b^2+5ab^3+15b^4\right)\int \frac{\csc\left(c\right.}{\sqrt{a+b\,\mathrm{cs}}}}{a\left(a^2-b^2\right)}}{3a(a^2-b^2)}}{5a\left(a^2-b^2\right)}$$

$$\frac{2b^2\tan(c+dx)}{5ad\left(a^2-b^2\right)(a+b\sec(c+dx))^{5/2}}$$

$$\Big\downarrow \; 4271$$

$$\frac{\frac{\frac{-b^2\left(58a^4-41a^2b^2+15b^4\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-b(a-b)\left(45a^4-13a^3b-36a^2b^2+5ab^3+15b^4\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{30\sqrt{a+b}\left(a^2-b^2\right)^3\cot}{}}{a\left(a^2-b^2\right)}}{3a(a^2-b^2)}}{5}$$

$$\frac{2b^2\tan(c+dx)}{5ad\left(a^2-b^2\right)(a+b\sec(c+dx))^{5/2}}$$

$$\Big\downarrow \; 4319$$

$$\frac{\frac{-b^2\left(58a^4-41a^2b^2+15b^4\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(\csc\left(c+dx+\frac{\pi}{2}\right)+1\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{30\sqrt{a+b}\left(a^2-b^2\right)^3\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,\mathrm{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\right.\right.}{ad}}{a}}{}$$

$$\frac{2b^2\tan(c+dx)}{5ad\left(a^2-b^2\right)(a+b\sec(c+dx))^{5/2}}$$

$$\Big\downarrow \; 4492$$

$$\frac{2b^2\tan(c+dx)}{5ad\left(a^2-b^2\right)(a+b\sec(c+dx))^{5/2}}+$$

$$\frac{2b^2\left(13a^2-5b^2\right)\tan(c+dx)}{3ad(a^2-b^2)(a+b\sec(c+dx))^{3/2}}+\frac{\frac{2b^2\left(58a^4-41a^2b^2+15b^4\right)\tan(c+dx)}{ad\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}+\frac{-\frac{30\sqrt{a+b}\left(a^2-b^2\right)^3\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,\mathrm{EllipticPi}\left(\right.}{ad}}{}}{}$$

input
```
Int[(a + b*Sec[c + d*x])^(-7/2),x]
```

3.578. $\int \frac{1}{(a+b\sec(c+dx))^{7/2}}\,dx$

output
```
(2*b^2*Tan[c + d*x])/(5*a*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^(5/2)) + ((2*
b^2*(13*a^2 - 5*b^2)*Tan[c + d*x])/(3*a*(a^2 - b^2)*d*(a + b*Sec[c + d*x])
^(3/2)) + (((2*(a - b)*Sqrt[a + b]*(58*a^4 - 41*a^2*b^2 + 15*b^4)*Cot[c +
d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a -
b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a
- b))])/d - (2*(a - b)*Sqrt[a + b]*(45*a^4 - 13*a^3*b - 36*a^2*b^2 + 5*a*b
^3 + 15*b^4)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a
 + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1
 + Sec[c + d*x]))/(a - b))])/d - (30*Sqrt[a + b]*(a^2 - b^2)^3*Cot[c + d*x
]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a +
 b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d
*x]))/(a - b))])/(a*d))/(a*(a^2 - b^2)) + (2*b^2*(58*a^4 - 41*a^2*b^2 + 15
*b^4)*Tan[c + d*x])/(a*(a^2 - b^2)*d*Sqrt[a + b*Sec[c + d*x]]))/(3*a*(a^2
- b^2)))/(5*a*(a^2 - b^2))
```

3.578.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4271
```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*(Rt[a
 + b, 2]/(a*d*Cot[c + d*x]))*Sqrt[b*((1 - Csc[c + d*x])/(a + b))]*Sqrt[(-b)
*((1 + Csc[c + d*x])/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[
c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

rule 4272
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[b^2*Cot[
c + d*x]*((a + b*Csc[c + d*x])^(n + 1)/(a*d*(n + 1)*(a^2 - b^2))), x] + Sim
p[1/(a*(n + 1)*(a^2 - b^2))   Int[(a + b*Csc[c + d*x])^(n + 1)*Simp[(a^2 -
b^2)*(n + 1) - a*b*(n + 1)*Csc[c + d*x] + b^2*(n + 2)*Csc[c + d*x]^2, x],
], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && LtQ[n, -1] && Integ
erQ[2*n]
```

rule 4319
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4409
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Simp[c   Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] +
Simp[d   Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

rule 4492
```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

rule 4546
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Simp[C   Int[Csc[e + f*x]*((
1 + Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A
, B, C}, x] && NeQ[a^2 - b^2, 0]
```

rule 4548
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(A*b^2 -
a*b*B + a^2*C)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(a*f*(m + 1)*(a^2
 - b^2))), x] + Simp[1/(a*(m + 1)*(a^2 - b^2))   Int[(a + b*Csc[e + f*x])^(
m + 1)*Simp[A*(a^2 - b^2)*(m + 1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x
] + (A*b^2 - a*b*B + a^2*C)*(m + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

3.578. $\int \frac{1}{(a+b\sec(c+dx))^{7/2}}\,dx$

3.578.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 9414 vs. $2(492) = 984$.

Time = 8.58 (sec) , antiderivative size = 9415, normalized size of antiderivative = 17.60

method	result	size
default	Expression too large to display	9415

input
```
int(1/(a+b*sec(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.578.5 Fricas [F]

$$\int \frac{1}{(a+b\sec(c+dx))^{7/2}}\,dx = \int \frac{1}{(b\sec(dx+c)+a)^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/(a+b*sec(d*x+c))^(7/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sec(d*x + c) + a)/(b^4*sec(d*x + c)^4 + 4*a*b^3*sec(d*x +
c)^3 + 6*a^2*b^2*sec(d*x + c)^2 + 4*a^3*b*sec(d*x + c) + a^4), x)
```

3.578.6 Sympy [F]

$$\int \frac{1}{(a+b\sec(c+dx))^{7/2}}\,dx = \int \frac{1}{(a+b\sec(c+dx))^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/(a+b*sec(d*x+c))**(7/2),x)
```

output
```
Integral((a + b*sec(c + d*x))**(-7/2), x)
```

3.578.7 Maxima [F]

$$\int \frac{1}{(a+b\sec(c+dx))^{7/2}}\,dx = \int \frac{1}{(b\sec(dx+c)+a)^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/(a+b*sec(d*x+c))^(7/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^(-7/2), x)
```

3.578.8 Giac [F]

$$\int \frac{1}{(a+b\sec(c+dx))^{7/2}}\,dx = \int \frac{1}{(b\sec(dx+c)+a)^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/(a+b*sec(d*x+c))^(7/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^(-7/2), x)
```

3.578.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\sec(c+dx))^{7/2}}\,dx = \int \frac{1}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{7/2}}\,dx$$

input
```
int(1/(a + b/cos(c + d*x))^(7/2),x)
```

output
```
int(1/(a + b/cos(c + d*x))^(7/2), x)
```

3.579 $\int \sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))\,dx$

3.579.1 Optimal result 4389
3.579.2 Mathematica [A] (verified) 4390
3.579.3 Rubi [A] (verified) 4390
3.579.4 Maple [B] (verified) 4393
3.579.5 Fricas [C] (verification not implemented) 4394
3.579.6 Sympy [F(-1)] 4395
3.579.7 Maxima [F] 4395
3.579.8 Giac [F] 4395
3.579.9 Mupad [F(-1)] 4396

3.579.1 Optimal result

Integrand size = 21, antiderivative size = 151

$$\begin{aligned}\int \sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))\,dx &= -\frac{6b\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} \\ &\quad+\frac{2a\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{3d}+\frac{6b\sqrt{\sec(c+dx)}\sin(c+dx)}{5d} \\ &\quad+\frac{2a\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d}+\frac{2b\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d}\end{aligned}$$

output
```
2/3*a*sec(d*x+c)^(3/2)*sin(d*x+c)/d+2/5*b*sec(d*x+c)^(5/2)*sin(d*x+c)/d+6/
5*b*sin(d*x+c)*sec(d*x+c)^(1/2)/d-6/5*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1
/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d
*x+c)^(1/2)/d+2/3*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellipt
icF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

3.579.2 Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.64

$$\int \sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))\,dx$$
$$=\frac{\sec^{\frac{5}{2}}(c+dx)\left(-36b\cos^{\frac{5}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)+20a\cos^{\frac{5}{2}}(c+dx)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+21b\sin(c\right.}{30d}$$

input
```
Integrate[Sec[c + d*x]^(5/2)*(a + b*Sec[c + d*x]),x]
```

output
```
(Sec[c + d*x]^(5/2)*(-36*b*Cos[c + d*x]^(5/2)*EllipticE[(c + d*x)/2, 2] +
20*a*Cos[c + d*x]^(5/2)*EllipticF[(c + d*x)/2, 2] + 21*b*Sin[c + d*x] + 10
*a*Sin[2*(c + d*x)] + 9*b*Sin[3*(c + d*x)]))/(30*d)
```

3.579.3 Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.01, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.524, Rules used = {3042, 4274, 3042, 4255, 3042, 4255, 3042, 4258, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)dx$$
$$\Big\downarrow \text{4274}$$
$$a\int \sec^{\frac{5}{2}}(c+dx)dx+b\int \sec^{\frac{7}{2}}(c+dx)dx$$
$$\Big\downarrow \text{3042}$$
$$a\int \csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}dx+b\int \csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}dx$$
$$\Big\downarrow \text{4255}$$

$$a\left(\frac{1}{3}\int\sqrt{\sec(c+dx)}dx+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}\right)+b\left(\frac{3}{5}\int\sec^{\frac{3}{2}}(c+dx)dx+\frac{2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}\right)$$

$\downarrow$ 3042

$$a\left(\frac{1}{3}\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}\right)+b\left(\frac{3}{5}\int\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}dx+\frac{2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}\right)$$

$\downarrow$ 4255

$$a\left(\frac{1}{3}\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}\right)+b\left(\frac{3}{5}\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\int\frac{1}{\sqrt{\sec(c+dx)}}dx\right)+\frac{2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}\right)$$

$\downarrow$ 3042

$$a\left(\frac{1}{3}\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}\right)+b\left(\frac{3}{5}\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\int\frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)+\frac{2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}\right)$$

$\downarrow$ 4258

$$a\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}\right)+b\left(\frac{3}{5}\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\cos(c+dx)}dx\right)+\frac{2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}\right.$$

$\downarrow$ 3042

$$a\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}\right)+b\left(\frac{3}{5}\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx\right)+\frac{2\sin(c+dx)\sec^{\frac{5}{2}}(c}{5d}\right.$$

$\downarrow$ 3119

3.579. $\int\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))\,dx$

$$a\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}\right)+$$
$$b\left(\frac{2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}+\frac{3}{5}\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)\right)$$

$$\Big\downarrow \text{3120}$$

$$a\left(\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}\right)+$$
$$b\left(\frac{2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}+\frac{3}{5}\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)\right)$$

input
```
Int[Sec[c + d*x]^(5/2)*(a + b*Sec[c + d*x]),x]
```

output
```
a*((2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*
d) + (2*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d)) + b*((2*Sec[c + d*x]^(5/2)
*Sin[c + d*x])/(5*d) + (3*((-2*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2
]*Sqrt[Sec[c + d*x]])/d + (2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d))/5)
```

3.579.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

3.579.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 501 vs. $2(179) = 358$.

Time = 27.29 (sec) , antiderivative size = 502, normalized size of antiderivative = 3.32

method	result
default	$-\frac{\sqrt{-\left(-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(2a\left(-\frac{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}}{6\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-\frac{1}{2}\right)^2}+\frac{\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1}}{3\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+s}}\right.\right.}{}$
parts	$-\frac{2a\left(-2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1}\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2+\sqrt{\frac{1}{2}-\frac{\cos(dx}{2}}\right.}{3\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right.}$

input
```
int(sec(d*x+c)^(5/2)*(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*a*(-1/6*cos(
1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1
/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/
2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellip
ticF(cos(1/2*d*x+1/2*c),2^(1/2)))+2/5*b/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2
*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)^2*(24*sin(1/2*d
*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(c
os(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/
2*c)^4-24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+12*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(
1/2)*sin(1/2*d*x+1/2*c)^2+8*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-3*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2
))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

3.579.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.25

$$\int \sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))\,dx$$

$$= \frac{-5i\sqrt{2}a\cos(dx+c)^2\,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c))+5i\sqrt{2}a\cos(dx+c)^2\,\text{wei}}{}$$

input
```
integrate(sec(d*x+c)^(5/2)*(a+b*sec(d*x+c)),x, algorithm="fricas")
```

output
```
1/15*(-5*I*sqrt(2)*a*cos(d*x + c)^2*weierstrassPInverse(-4, 0, cos(d*x + c
) + I*sin(d*x + c)) + 5*I*sqrt(2)*a*cos(d*x + c)^2*weierstrassPInverse(-4,
 0, cos(d*x + c) - I*sin(d*x + c)) - 9*I*sqrt(2)*b*cos(d*x + c)^2*weierstr
assZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)))
+ 9*I*sqrt(2)*b*cos(d*x + c)^2*weierstrassZeta(-4, 0, weierstrassPInverse(
-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*(9*b*cos(d*x + c)^2 + 5*a*cos(d
*x + c) + 3*b)*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c)^2)
```

3.579.6 Sympy [F(-1)]

Timed out.

$$\int \sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**(5/2)*(a+b*sec(d*x+c)),x)
```

output
```
Timed out
```

3.579.7 Maxima [F]

$$\int \sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))\,dx = \int (b\sec(dx+c)+a)\sec(dx+c)^{\frac{5}{2}}\,dx$$

input
```
integrate(sec(d*x+c)^(5/2)*(a+b*sec(d*x+c)),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)*sec(d*x + c)^(5/2), x)
```

3.579.8 Giac [F]

$$\int \sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))\,dx = \int (b\sec(dx+c)+a)\sec(dx+c)^{\frac{5}{2}}\,dx$$

input
```
integrate(sec(d*x+c)^(5/2)*(a+b*sec(d*x+c)),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)*sec(d*x + c)^(5/2), x)
```

3.579.9 Mupad [F(-1)]

Timed out.

$$\int \sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))\,dx = \int \left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)\,{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{5/2}\,dx$$

input
```
int((a + b/cos(c + d*x))*(1/cos(c + d*x))^(5/2),x)
```

output
```
int((a + b/cos(c + d*x))*(1/cos(c + d*x))^(5/2), x)
```

3.580 $\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))\,dx$

3.580.1 Optimal result 4397
3.580.2 Mathematica [A] (verified) 4398
3.580.3 Rubi [A] (verified) 4398
3.580.4 Maple [B] (verified) 4401
3.580.5 Fricas [C] (verification not implemented) 4401
3.580.6 Sympy [F] 4402
3.580.7 Maxima [F] 4402
3.580.8 Giac [F] 4402
3.580.9 Mupad [F(-1)] 4403

3.580.1 Optimal result

Integrand size = 21, antiderivative size = 123

$$\begin{aligned}\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))\,dx &= -\frac{2a\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} \\ &\quad+\frac{2b\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{3d} \\ &\quad+\frac{2a\sqrt{\sec(c+dx)}\sin(c+dx)}{d}+\frac{2b\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d}\end{aligned}$$

output
```
2/3*b*sec(d*x+c)^(3/2)*sin(d*x+c)/d+2*a*sin(d*x+c)*sec(d*x+c)^(1/2)/d-2*a*
(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*
c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+2/3*b*(cos(1/2*d*x+1/2*c)^
2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+
c)^(1/2)*sec(d*x+c)^(1/2)/d
```

3.580.2 Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.69

$$\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))\,dx$$
$$=\frac{\sec^{\frac{3}{2}}(c+dx)\left(-6a\cos^{\frac{3}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)+2b\cos^{\frac{3}{2}}(c+dx)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+2(b+3a\cos\right.}{3d}$$

input
```
Integrate[Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x]),x]
```

output
```
(Sec[c + d*x]^(3/2)*(-6*a*Cos[c + d*x]^(3/2)*EllipticE[(c + d*x)/2, 2] + 2
*b*Cos[c + d*x]^(3/2)*EllipticF[(c + d*x)/2, 2] + 2*(b + 3*a*Cos[c + d*x])
*Sin[c + d*x]))/(3*d)
```

3.580.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.02, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3042, 4274, 3042, 4255, 3042, 4258, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)dx$$
$$\Big\downarrow \text{4274}$$
$$a\int \sec^{\frac{3}{2}}(c+dx)dx+b\int \sec^{\frac{5}{2}}(c+dx)dx$$
$$\Big\downarrow \text{3042}$$
$$a\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}dx+b\int \csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}dx$$
$$\Big\downarrow \text{4255}$$

$$a\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\int\frac{1}{\sqrt{\sec(c+dx)}}dx\right)+b\left(\frac{1}{3}\int\sqrt{\sec(c+dx)}dx+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}\right)$$

$\downarrow$ 3042

$$a\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\int\frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)+b\left(\frac{1}{3}\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}\right)$$

$\downarrow$ 4258

$$a\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\cos(c+dx)}dx\right)+b\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}\right)$$

$\downarrow$ 3042

$$a\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx\right)+b\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}\right)$$

$\downarrow$ 3119

$$b\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}\right)+a\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)$$

$\downarrow$ 3120

$$a\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)+b\left(\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}\right)$$

input
```
Int[Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x]),x]
```

3.580. $\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))\,dx$

output
```
a*((-2*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d
+ (2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d) + b*((2*Sqrt[Cos[c + d*x]]*Ellipt
icF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (2*Sec[c + d*x]^(3/2)*Sin[
c + d*x])/(3*d))
```

3.580.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

3.580. $\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))\,dx$

3.580.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 395 vs. $2(159) = 318$.

Time = 18.20 (sec) , antiderivative size = 396, normalized size of antiderivative = 3.22

method	result
default	$-\frac{2\sqrt{-\left(-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\left(12\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}a-2\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1}\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sqrt{}\right.}{}$
parts	$-\frac{2a\left(-2\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1}\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\mathrm{si}}\right.}{\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1}\,d}$

input
```
int(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-2/3*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(4*sin(1/2*
d*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)/sin(1/2*d*x+1/2*c)^3*(12*cos(1/2*d*
x+1/2*c)*sin(1/2*d*x+1/2*c)^4*a-2*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipti
cF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/
2*c)^2*b-6*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2
^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2*a-6*cos(1/2*d*x+
1/2*c)*sin(1/2*d*x+1/2*c)^2*a-2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2*b+
(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*b+3*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(c
os(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*a)*(-2*sin(1/2*d*x
+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

3.580.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.36

$$\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))\,dx$$
$$= \frac{-i\sqrt{2}b\cos(dx+c)\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+i\sqrt{2}b\cos(dx+c)\,\mathrm{weierstr}}{}$$

input
```
integrate(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c)),x, algorithm="fricas")
```

output
```
1/3*(-I*sqrt(2)*b*cos(d*x + c)*weierstrassPInverse(-4, 0, cos(d*x + c) + I
*sin(d*x + c)) + I*sqrt(2)*b*cos(d*x + c)*weierstrassPInverse(-4, 0, cos(d
*x + c) - I*sin(d*x + c)) - 3*I*sqrt(2)*a*cos(d*x + c)*weierstrassZeta(-4,
 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 3*I*sqrt(
2)*a*cos(d*x + c)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*
x + c) - I*sin(d*x + c))) + 2*(3*a*cos(d*x + c) + b)*sin(d*x + c)/sqrt(cos
(d*x + c)))/(d*cos(d*x + c))
```

3.580.6 Sympy [F]

$$\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))\,dx = \int (a+b\sec(c+dx))\sec^{\frac{3}{2}}(c+dx)\,dx$$

input
```
integrate(sec(d*x+c)**(3/2)*(a+b*sec(d*x+c)),x)
```

output
```
Integral((a + b*sec(c + d*x))*sec(c + d*x)**(3/2), x)
```

3.580.7 Maxima [F]

$$\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))\,dx = \int (b\sec(dx+c)+a)\sec(dx+c)^{\frac{3}{2}}\,dx$$

input
```
integrate(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c)),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)*sec(d*x + c)^(3/2), x)
```

3.580.8 Giac [F]

$$\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))\,dx = \int (b\sec(dx+c)+a)\sec(dx+c)^{\frac{3}{2}}\,dx$$

input
```
integrate(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c)),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)*sec(d*x + c)^(3/2), x)
```

3.580.9 Mupad [F(-1)]

Timed out.

$$\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))\,dx = \int \left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)\,{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{3/2}\,dx$$

input
```
int((a + b/cos(c + d*x))*(1/cos(c + d*x))^(3/2),x)
```

output
```
int((a + b/cos(c + d*x))*(1/cos(c + d*x))^(3/2), x)
```

3.581 $\int \sqrt{\sec(c+dx)}(a+b\sec(c+dx))\,dx$

3.581.1 Optimal result 4404
3.581.2 Mathematica [A] (verified) 4404
3.581.3 Rubi [A] (verified) 4405
3.581.4 Maple [A] (verified) 4407
3.581.5 Fricas [C] (verification not implemented) 4408
3.581.6 Sympy [F] 4408
3.581.7 Maxima [F] 4408
3.581.8 Giac [F] 4409
3.581.9 Mupad [F(-1)] 4409

3.581.1 Optimal result

Integrand size = 21, antiderivative size = 97

$$\int \sqrt{\sec(c+dx)}(a+b\sec(c+dx))\,dx = -\frac{2b\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{2a\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{d} + \frac{2b\sqrt{\sec(c+dx)}\sin(c+dx)}{d}$$

output
```
2*b*sin(d*x+c)*sec(d*x+c)^(1/2)/d-2*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2
*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x
+c)^(1/2)/d+2*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(
sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

3.581.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.73

$$\int \sqrt{\sec(c+dx)}(a+b\sec(c+dx))\,dx = \frac{2\sqrt{\sec(c+dx)}\left(-b\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)+a\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+b\sin(c+d\right.}{d}$$

input
```
Integrate[Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x]),x]
```

output
```
(2*Sqrt[Sec[c + d*x]]*(-(b*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]) +
 a*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + b*Sin[c + d*x]))/d
```

3.581.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.01, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 4274, 3042, 4255, 3042, 4258, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\sec(c+dx)}(a+b\sec(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{4274}$$

$$a\int \sqrt{\sec(c+dx)}dx+b\int \sec^{\frac{3}{2}}(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$a\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+b\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}dx$$

$$\Big\downarrow \text{4255}$$

$$a\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+b\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\int \frac{1}{\sqrt{\sec(c+dx)}}dx\right)$$

$$\Big\downarrow \text{3042}$$

$$a\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+b\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)$$

$$\Big\downarrow \text{4258}$$

$$a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx+$$
$$b\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\cos(c+dx)}dx\right)$$

$\Big\downarrow$ 3042

$$a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+$$
$$b\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx\right)$$

$\Big\downarrow$ 3119

$$a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+$$
$$b\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)$$

$\Big\downarrow$ 3120

$$\frac{2a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+$$
$$b\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)$$

input
```
Int[Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x]),x]
```

output
```
(2*a*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d +
b*((-2*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d
+ (2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d)
```

3.581.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

3.581.4 Maple [A] (verified)

Time = 11.89 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.55

method	result
default	$\frac{4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2 b-2\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1}\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,a-2\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1}\,\mathrm{EllipticE}\left(\right.}{\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1}\,d}$
parts	$-\frac{2a\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1}\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1}\,d}-\frac{2b\left(-2\sqrt{-2\sin\left(\right.}\right.}{}$

input
```
int(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
2*(2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2*b-(2*sin(1/2*d*x+1/2*c)^2-1)^
(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*a
-(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*b)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1
)^(1/2)/d
```

3.581.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.28

$$\int \sqrt{\sec(c+dx)}(a+b\sec(c+dx))\,dx$$
$$= \frac{-i\sqrt{2}a\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+i\sqrt{2}a\text{weierstrassPInverse}(-4,0,\cos(d}{}$$

input
```
integrate(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c)),x, algorithm="fricas")
```

output
```
(-I*sqrt(2)*a*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) +
I*sqrt(2)*a*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - I*
sqrt(2)*b*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) +
 I*sin(d*x + c))) + I*sqrt(2)*b*weierstrassZeta(-4, 0, weierstrassPInverse
(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*b*sin(d*x + c)/sqrt(cos(d*x +
c)))/d
```

3.581.6 Sympy [F]

$$\int \sqrt{\sec(c+dx)}(a+b\sec(c+dx))\,dx = \int (a+b\sec(c+dx))\sqrt{\sec(c+dx)}\,dx$$

input
```
integrate(sec(d*x+c)**(1/2)*(a+b*sec(d*x+c)),x)
```

output
```
Integral((a + b*sec(c + d*x))*sqrt(sec(c + d*x)), x)
```

3.581.7 Maxima [F]

$$\int \sqrt{\sec(c+dx)}(a+b\sec(c+dx))\,dx = \int (b\sec(dx+c)+a)\sqrt{\sec(dx+c)}\,dx$$

input
```
integrate(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c)),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)*sqrt(sec(d*x + c)), x)
```

3.581.8 Giac [F]

$$\int \sqrt{\sec(c+dx)}(a+b\sec(c+dx))\,dx = \int (b\sec(dx+c)+a)\sqrt{\sec(dx+c)}\,dx$$

input
```
integrate(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c)),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)*sqrt(sec(d*x + c)), x)
```

3.581.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{\sec(c+dx)}(a+b\sec(c+dx))\,dx = \int \left(a+\frac{b}{\cos(c+d\,x)}\right)\sqrt{\frac{1}{\cos(c+d\,x)}}\,dx$$

input
```
int((a + b/cos(c + d*x))*(1/cos(c + d*x))^(1/2),x)
```

output
```
int((a + b/cos(c + d*x))*(1/cos(c + d*x))^(1/2), x)
```

3.582 $\int \frac{a+b\sec(c+dx)}{\sqrt{\sec(c+dx)}}\,dx$

3.582.1 Optimal result 4410
3.582.2 Mathematica [A] (verified) 4410
3.582.3 Rubi [A] (verified) 4411
3.582.4 Maple [A] (verified) 4413
3.582.5 Fricas [C] (verification not implemented) 4413
3.582.6 Sympy [F] 4414
3.582.7 Maxima [F] 4414
3.582.8 Giac [F] 4414
3.582.9 Mupad [F(-1)] 4415

3.582.1 Optimal result

Integrand size = 21, antiderivative size = 75

$$\int \frac{a+b\sec(c+dx)}{\sqrt{\sec(c+dx)}}\,dx = \frac{2a\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{2b\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{d}$$

output
```
2*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+
1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+2*b*(cos(1/2*d*x+1/2*c
)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*
x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

3.582.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.69

$$\int \frac{a+b\sec(c+dx)}{\sqrt{\sec(c+dx)}}\,dx = \frac{2\sqrt{\cos(c+dx)}\left(aE\left(\frac{1}{2}(c+dx)\middle|2\right)+b\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\right)\sqrt{\sec(c+dx)}}{d}$$

input
```
Integrate[(a + b*Sec[c + d*x])/Sqrt[Sec[c + d*x]],x]
```

output
```
(2*Sqrt[Cos[c + d*x]]*(a*EllipticE[(c + d*x)/2, 2] + b*EllipticF[(c + d*x)
/2, 2])*Sqrt[Sec[c + d*x]])/d
```

3.582.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 4274, 3042, 4258, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\sec(c+dx)}{\sqrt{\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4274}$$

$$a\int \frac{1}{\sqrt{\sec(c+dx)}}dx+b\int \sqrt{\sec(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$a\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+b\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{4258}$$

$$a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\cos(c+dx)}dx+b\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+ b\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{3119}$$

$$b\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

$$\Big\downarrow \; 3120$$

$$\frac{2a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2b\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}$$

input
```
Int[(a + b*Sec[c + d*x])/Sqrt[Sec[c + d*x]],x]
```

output
```
(2*a*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d +
(2*b*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d
```

3.582.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

3.582.4 Maple [A] (verified)

Time = 10.08 (sec) , antiderivative size = 152, normalized size of antiderivative = 2.03

method	result
default	$-\frac{2\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+1}\,\left(b\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)-a\,\mathrm{EllipticE}\left(\cos\left(\frac{dx}{2}\right.\right.\right.}{\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1}\,d}$
parts	$\frac{2a\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+1}\,\mathrm{EllipticE}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1}\,d}-\frac{2b\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}}{}$
risch	$-\frac{ia\sqrt{2}}{d\sqrt{\frac{e^{i(dx+c)}}{e^{2i(dx+c)}+1}}}-\frac{i\left(\frac{ib\sqrt{-i\left(e^{i(dx+c)}+i\right)}\,\sqrt{2}\,\sqrt{i\left(e^{i(dx+c)}-i\right)}\,\sqrt{ie^{i(dx+c)}}\,\mathrm{EllipticF}\left(\sqrt{-i\left(e^{i(dx+c)}+i\right)},\frac{\sqrt{2}}{2}\right)}{\sqrt{e^{3i(dx+c)}+e^{i(dx+c)}}}+a\left(-\frac{2\left(e^{2i(dx+c)}+\right.}{\sqrt{e^{i(dx+c)}\left(e^{2i(dx+\right.}}}\right.\right.}{}$

input
```
int((a+b*sec(d*x+c))/sec(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*(b*EllipticF(cos(1/2*d*x+1
/2*c),2^(1/2))-a*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c
)^2-1)^(1/2)/d
```

3.582.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.43

$$\int \frac{a+b\sec(c+dx)}{\sqrt{\sec(c+dx)}}\,dx$$
$$=\frac{-i\sqrt{2}b\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+i\sqrt{2}b\mathrm{weierstrassPInverse}(-4,0,\cos(d}{}$$

input
```
integrate((a+b*sec(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
(-I*sqrt(2)*b*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) +
I*sqrt(2)*b*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + I*
sqrt(2)*a*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) +
 I*sin(d*x + c))) - I*sqrt(2)*a*weierstrassZeta(-4, 0, weierstrassPInverse
(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/d
```

3.582.6 Sympy [F]

$$\int \frac{a+b\sec(c+dx)}{\sqrt{\sec(c+dx)}}\,dx=\int \frac{a+b\sec{(c+dx)}}{\sqrt{\sec{(c+dx)}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))/sec(d*x+c)**(1/2),x)
```

output
```
Integral((a + b*sec(c + d*x))/sqrt(sec(c + d*x)), x)
```

3.582.7 Maxima [F]

$$\int \frac{a+b\sec(c+dx)}{\sqrt{\sec(c+dx)}}\,dx=\int \frac{b\sec{(dx+c)}+a}{\sqrt{\sec{(dx+c)}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)/sqrt(sec(d*x + c)), x)
```

3.582.8 Giac [F]

$$\int \frac{a+b\sec(c+dx)}{\sqrt{\sec(c+dx)}}\,dx=\int \frac{b\sec{(dx+c)}+a}{\sqrt{\sec{(dx+c)}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)/sqrt(sec(d*x + c)), x)
```

3.582.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\sec(c+dx)}{\sqrt{\sec(c+dx)}}\,dx=\int \frac{a+\frac{b}{\cos(c+d\,x)}}{\sqrt{\frac{1}{\cos(c+d\,x)}}}\,dx$$

input
```
int((a + b/cos(c + d*x))/(1/cos(c + d*x))^(1/2),x)
```

output
```
int((a + b/cos(c + d*x))/(1/cos(c + d*x))^(1/2), x)
```

3.583 $\int \frac{a+b\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

3.583.1 Optimal result 4416
3.583.2 Mathematica [A] (verified) 4416
3.583.3 Rubi [A] (verified) 4417
3.583.4 Maple [A] (verified) 4419
3.583.5 Fricas [C] (verification not implemented) 4420
3.583.6 Sympy [F] 4420
3.583.7 Maxima [F] 4421
3.583.8 Giac [F] 4421
3.583.9 Mupad [F(-1)] 4421

3.583.1 Optimal result

Integrand size = 21, antiderivative size = 101

$$\int \frac{a+b\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \frac{2b\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{2a\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{3d} + \frac{2a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}$$

output
```
2/3*a*sin(d*x+c)/d/sec(d*x+c)^(1/2)+2*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1
/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d
*x+c)^(1/2)/d+2/3*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellipt
icF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

3.583.2 Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.75

$$\int \frac{a+b\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \frac{\sqrt{\sec(c+dx)}\left(6b\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)+a\left(2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+\sin(2(c+\right.\right.}{3d}$$

input
```
Integrate[(a + b*Sec[c + d*x])/Sec[c + d*x]^(3/2),x]
```

output
```
(Sqrt[Sec[c + d*x]]*(6*b*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + a*
(2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + Sin[2*(c + d*x)])))/(3*d
)
```

3.583.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.01, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 4274, 3042, 4256, 3042, 4258, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{4274}$$

$$a\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}dx+b\int \frac{1}{\sqrt{\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$a\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+b\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4256}$$

$$a\left(\frac{1}{3}\int \sqrt{\sec(c+dx)}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)+b\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{1}{3}\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)+b\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4258}$$

$$a\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)+b\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\cos(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)+b\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3119}$$

$$a\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)+\frac{2b\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

$$\Big\downarrow \text{3120}$$

$$a\left(\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}\right)+\frac{2b\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

input
```
Int[(a + b*Sec[c + d*x])/Sec[c + d*x]^(3/2),x]
```

output
```
(2*b*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d +
a*((2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*
d) + (2*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]]))
```

3.583.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

3.583.4 Maple [A] (verified)

Time = 12.69 (sec) , antiderivative size = 228, normalized size of antiderivative = 2.26

method	result
default	$-\frac{2\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4 a-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2 a+\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1}\ \mathrm{Ellipt}\right.}{3\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\ \sin\left(\frac{dx}{2}+\right.}$
parts	$-\frac{2a\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(4\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\ \sqrt{2\sin\left(\frac{dx}{2}+\right.}\right.}{3\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\ \sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1}\,d}$

input
```
int((a+b*sec(d*x+c))/sec(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4*cos(1/2*d*
x+1/2*c)*sin(1/2*d*x+1/2*c)^4*a-2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2*
a+(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*a-3*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE
(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*b)/(-2*sin(1/2*d
*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+
1/2*c)^2-1)^(1/2)/d
```

3.583.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.24

$$\int \frac{a+b\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)}\,dx$$
$$= \frac{2\,a\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)-i\,\sqrt{2}a\text{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right))+i\,\sqrt{2}a\text{we}}{}$$

input
```
integrate((a+b*sec(d*x+c))/sec(d*x+c)^(3/2),x, algorithm="fricas")
```

output
```
1/3*(2*a*sqrt(cos(d*x + c))*sin(d*x + c) - I*sqrt(2)*a*weierstrassPInverse
(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + I*sqrt(2)*a*weierstrassPInverse(-
4, 0, cos(d*x + c) - I*sin(d*x + c)) + 3*I*sqrt(2)*b*weierstrassZeta(-4, 0
, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 3*I*sqrt(2)
*b*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(
d*x + c))))/d
```

3.583.6 Sympy [F]

$$\int \frac{a+b\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{a+b\sec\left(c+dx\right)}{\sec^{\frac{3}{2}}\left(c+dx\right)}\,dx$$

input
```
integrate((a+b*sec(d*x+c))/sec(d*x+c)**(3/2),x)
```

output
```
Integral((a + b*sec(c + d*x))/sec(c + d*x)**(3/2), x)
```

3.583.7 Maxima [F]

$$\int \frac{a+b\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)}\,dx=\int \frac{b\sec(dx+c)+a}{\sec(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))/sec(d*x+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)/sec(d*x + c)^(3/2), x)
```

3.583.8 Giac [F]

$$\int \frac{a+b\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)}\,dx=\int \frac{b\sec(dx+c)+a}{\sec(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))/sec(d*x+c)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)/sec(d*x + c)^(3/2), x)
```

3.583.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)}\,dx=\int \frac{a+\frac{b}{\cos(c+d\,x)}}{\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

input
```
int((a + b/cos(c + d*x))/(1/cos(c + d*x))^(3/2),x)
```

output
```
int((a + b/cos(c + d*x))/(1/cos(c + d*x))^(3/2), x)
```

3.584 $\int \frac{a+b\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)}\,dx$

3.584.1 Optimal result 4422
3.584.2 Mathematica [A] (verified) 4422
3.584.3 Rubi [A] (verified) 4423
3.584.4 Maple [A] (verified) 4425
3.584.5 Fricas [C] (verification not implemented) 4426
3.584.6 Sympy [F] 4427
3.584.7 Maxima [F] 4427
3.584.8 Giac [F] 4427
3.584.9 Mupad [F(-1)] 4428

3.584.1 Optimal result

Integrand size = 21, antiderivative size = 127

$$\int \frac{a+b\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \frac{6a\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{2b\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{3d} + \frac{2a\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{2b\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}$$

output
```
2/5*a*sin(d*x+c)/d/sec(d*x+c)^(3/2)+2/3*b*sin(d*x+c)/d/sec(d*x+c)^(1/2)+6/
5*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+
1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+2/3*b*(cos(1/2*d*x+1/2
*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(
d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

3.584.2 Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.69

$$\int \frac{a+b\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \frac{\sqrt{\sec(c+dx)}\left(18a\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)+10b\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+(5b+3a\right.}{15d}$$

input
```
Integrate[(a + b*Sec[c + d*x])/Sec[c + d*x]^(5/2),x]
```

output
```
(Sqrt[Sec[c + d*x]]*(18*a*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 1
0*b*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + (5*b + 3*a*Cos[c + d*x]
)*Sin[2*(c + d*x)]))/(15*d)
```

3.584.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.02, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 4274, 3042, 4256, 3042, 4258, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx$$

$$\Big\downarrow \text{4274}$$

$$a\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)}dx+b\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$a\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+b\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{4256}$$

$$a\left(\frac{3}{5}\int \frac{1}{\sqrt{\sec(c+dx)}}dx+\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}\right)+b\left(\frac{1}{3}\int \sqrt{\sec(c+dx)}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{3}{5}\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}\right)+ b\left(\frac{1}{3}\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)$$

$$\Big\downarrow \text{4258}$$

$$a\left(\frac{3}{5}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\cos(c+dx)}dx+\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}\right)+b\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{3}{5}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}\right)+b\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)$$

$$\Big\downarrow \text{3119}$$

$$b\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)+a\left(\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{6\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}\right)$$

$$\Big\downarrow \text{3120}$$

$$a\left(\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{6\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}\right)+b\left(\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}\right)$$

input
```
Int[(a + b*Sec[c + d*x])/Sec[c + d*x]^(5/2),x]
```

output
```
a*((6*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*
d) + (2*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2))) + b*((2*Sqrt[Cos[c + d*x]]
*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (2*Sin[c + d*x])/(3
*d*Sqrt[Sec[c + d*x]]))
```

3.584. $\int \frac{a+b\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)}\,dx$

3.584.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

3.584.4 Maple [A] (verified)

Time = 16.45 (sec) , antiderivative size = 262, normalized size of antiderivative = 2.06

method	result
default	$-\frac{2\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\left(-24a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}+(24a+20b)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+(-6a-10b)\sin\left(\right.\right.}{15\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right.}}$
parts	$-\frac{2a\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\left(-8\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+8\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}{5\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)}}$

input
```
int((a+b*sec(d*x+c))/sec(d*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/15*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-24*a*cos(1
/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+(24*a+20*b)*sin(1/2*d*x+1/2*c)^4*cos(1/
2*d*x+1/2*c)+(-6*a-10*b)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+5*(2*sin(
1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*b-9*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*
d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*a)/(-2*sin(1/2*d*x+1/2*c)
^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-
1)^(1/2)/d
```

3.584.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.14

$$\int \frac{a+b\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)}\,dx$$
$$= \frac{-5i\sqrt{2}b\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+5i\sqrt{2}b\text{weierstrassPInverse}(-4,0,\cos}{}$$

input
```
integrate((a+b*sec(d*x+c))/sec(d*x+c)^(5/2),x, algorithm="fricas")
```

output
```
1/15*(-5*I*sqrt(2)*b*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x +
 c)) + 5*I*sqrt(2)*b*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x +
 c)) + 9*I*sqrt(2)*a*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos
(d*x + c) + I*sin(d*x + c))) - 9*I*sqrt(2)*a*weierstrassZeta(-4, 0, weiers
trassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*(3*a*cos(d*x + c)
^2 + 5*b*cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/d
```

3.584.6 Sympy [F]

$$\int \frac{a+b\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)}\,dx=\int \frac{a+b\sec{(c+dx)}}{\sec^{\frac{5}{2}}{(c+dx)}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))/sec(d*x+c)**(5/2),x)
```

output
```
Integral((a + b*sec(c + d*x))/sec(c + d*x)**(5/2), x)
```

3.584.7 Maxima [F]

$$\int \frac{a+b\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)}\,dx=\int \frac{b\sec{(dx+c)}+a}{\sec{(dx+c)}^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))/sec(d*x+c)^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)/sec(d*x + c)^(5/2), x)
```

3.584.8 Giac [F]

$$\int \frac{a+b\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)}\,dx=\int \frac{b\sec{(dx+c)}+a}{\sec{(dx+c)}^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))/sec(d*x+c)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)/sec(d*x + c)^(5/2), x)
```

3.584.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)}\,dx=\int \frac{a+\frac{b}{\cos(c+d\,x)}}{\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

input
```
int((a + b/cos(c + d*x))/(1/cos(c + d*x))^(5/2),x)
```

output
```
int((a + b/cos(c + d*x))/(1/cos(c + d*x))^(5/2), x)
```

3.585 $\int \frac{a+b\sec(c+dx)}{\sec^{\frac{7}{2}}(c+dx)}\,dx$

3.585.1 Optimal result 4429
3.585.2 Mathematica [A] (verified) 4429
3.585.3 Rubi [A] (verified) 4430
3.585.4 Maple [A] (verified) 4433
3.585.5 Fricas [C] (verification not implemented) 4433
3.585.6 Sympy [F] 4434
3.585.7 Maxima [F] 4434
3.585.8 Giac [F] 4435
3.585.9 Mupad [F(-1)] 4435

3.585.1 Optimal result

Integrand size = 21, antiderivative size = 151

$$\int \frac{a+b\sec(c+dx)}{\sec^{\frac{7}{2}}(c+dx)}\,dx = \frac{6b\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{10a\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{21d} + \frac{2a\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{2b\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{10a\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}$$

output
```
2/7*a*sin(d*x+c)/d/sec(d*x+c)^(5/2)+2/5*b*sin(d*x+c)/d/sec(d*x+c)^(3/2)+10
/21*a*sin(d*x+c)/d/sec(d*x+c)^(1/2)+6/5*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos
(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec
(d*x+c)^(1/2)/d+10/21*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*El
lipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

3.585.2 Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.66

$$\int \frac{a+b\sec(c+dx)}{\sec^{\frac{7}{2}}(c+dx)}\,dx = \frac{\sqrt{\sec(c+dx)}\left(252b\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)+100a\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+(65a+\right.}{210d}$$

input
```
Integrate[(a + b*Sec[c + d*x])/Sec[c + d*x]^(7/2),x]
```

output
```
(Sqrt[Sec[c + d*x]]*(252*b*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] +
100*a*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + (65*a + 42*b*Cos[c +
d*x] + 15*a*Cos[2*(c + d*x)])*Sin[2*(c + d*x)]))/(210*d)
```

3.585.3 Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.04, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.524$, Rules used = {3042, 4274, 3042, 4256, 3042, 4256, 3042, 4258, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\sec(c+dx)}{\sec^{\frac{7}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx$$

$$\Big\downarrow \text{4274}$$

$$a\int \frac{1}{\sec^{\frac{7}{2}}(c+dx)}dx+b\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$a\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx+b\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx$$

$$\Big\downarrow \text{4256}$$

$$a\left(\frac{5}{7}\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}dx+\frac{2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}\right)+b\left(\frac{3}{5}\int \frac{1}{\sqrt{\sec(c+dx)}}dx+\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}\right)$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{5}{7}\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+\frac{2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}\right)+ b\left(\frac{3}{5}\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}\right)$$

$$\Big\downarrow \text{4256}$$

$$a\left(\frac{5}{7}\left(\frac{1}{3}\int\sqrt{\sec(c+dx)}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)+\frac{2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}\right)+b\left(\frac{3}{5}\int\frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}\right)$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{5}{7}\left(\frac{1}{3}\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)+\frac{2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}\right)+b\left(\frac{3}{5}\int\frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}\right)$$

$$\Big\downarrow \text{4258}$$

$$a\left(\frac{5}{7}\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)+\frac{2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}\right)+b\left(\frac{3}{5}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\cos(c+dx)}dx+\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}\right)$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{5}{7}\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)+\frac{2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}\right)+b\left(\frac{3}{5}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}\right)$$

$$\Big\downarrow \text{3119}$$

$$a\left(\frac{5}{7}\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)+\frac{2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}\right)+b\left(\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{6\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}\right)$$

$$\Big\downarrow \text{3120}$$

3.585. $\int\frac{a+b\sec(c+dx)}{\sec^{\frac{7}{2}}(c+dx)}\,dx$

$$a\left(\frac{2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+\frac{5}{7}\left(\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}\right)\right)+ b\left(\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{6\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}\right)$$

input
```
Int[(a + b*Sec[c + d*x])/Sec[c + d*x]^(7/2),x]
```

output
```
b*((6*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*
d) + (2*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2))) + a*((2*Sin[c + d*x])/(7*d
*Sec[c + d*x]^(5/2)) + (5*((2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]
*Sqrt[Sec[c + d*x]])/(3*d) + (2*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])))/7
)
```

3.585.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

3.585.4 Maple [A] (verified)

Time = 18.15 (sec) , antiderivative size = 290, normalized size of antiderivative = 1.92

method	result
default	$-\frac{2\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\left(240a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{8}+(-360a-168b)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+(280a+168b\right.}{}$
parts	$-\frac{2a\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\left(48\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{9}-120\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}+128\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}-72\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}+5\sqrt{\frac{1}{2}-\frac{\cos(d}{2}}\right.}{21\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\cos\left(\frac{dx}{2}+\right.}}$

input
```
int((a+b*sec(d*x+c))/sec(d*x+c)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-2/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(240*a*cos(
1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+(-360*a-168*b)*sin(1/2*d*x+1/2*c)^6*co
s(1/2*d*x+1/2*c)+(280*a+168*b)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-8
0*a-42*b)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+25*(2*sin(1/2*d*x+1/2*c)
^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*a-63*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^
(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*b)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*
x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

3.585.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.03

$$\int \frac{a+b\sec(c+dx)}{\sec^{\frac{7}{2}}(c+dx)}\,dx$$

$$= \frac{-25i\sqrt{2}a\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+25i\sqrt{2}a\text{weierstrassPInverse}(-4,0,c}{}$$

input
```
integrate((a+b*sec(d*x+c))/sec(d*x+c)^(7/2),x, algorithm="fricas")
```

output
```
1/105*(-25*I*sqrt(2)*a*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x
 + c)) + 25*I*sqrt(2)*a*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*
x + c)) + 63*I*sqrt(2)*b*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0,
 cos(d*x + c) + I*sin(d*x + c))) - 63*I*sqrt(2)*b*weierstrassZeta(-4, 0, w
eierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*(15*a*cos(d*
x + c)^3 + 21*b*cos(d*x + c)^2 + 25*a*cos(d*x + c))*sin(d*x + c)/sqrt(cos(
d*x + c)))/d
```

3.585.6 Sympy [F]

$$\int \frac{a+b\sec(c+dx)}{\sec^{\frac{7}{2}}(c+dx)}\,dx=\int \frac{a+b\sec{(c+dx)}}{\sec^{\frac{7}{2}}{(c+dx)}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))/sec(d*x+c)**(7/2),x)
```

output
```
Integral((a + b*sec(c + d*x))/sec(c + d*x)**(7/2), x)
```

3.585.7 Maxima [F]

$$\int \frac{a+b\sec(c+dx)}{\sec^{\frac{7}{2}}(c+dx)}\,dx=\int \frac{b\sec{(dx+c)}+a}{\sec{(dx+c)}^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))/sec(d*x+c)^(7/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)/sec(d*x + c)^(7/2), x)
```

3.585.8 Giac [F]

$$\int \frac{a+b\sec(c+dx)}{\sec^{\frac{7}{2}}(c+dx)}\,dx=\int \frac{b\sec(dx+c)+a}{\sec(dx+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))/sec(d*x+c)^(7/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)/sec(d*x + c)^(7/2), x)
```

3.585.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\sec(c+dx)}{\sec^{\frac{7}{2}}(c+dx)}\,dx=\int \frac{a+\frac{b}{\cos(c+d\,x)}}{\left(\frac{1}{\cos(c+d\,x)}\right)^{7/2}}\,dx$$

input
```
int((a + b/cos(c + d*x))/(1/cos(c + d*x))^(7/2),x)
```

output
```
int((a + b/cos(c + d*x))/(1/cos(c + d*x))^(7/2), x)
```

3.586 $\int \sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^2\,dx$

3.586.1 Optimal result 4436
3.586.2 Mathematica [A] (verified) 4437
3.586.3 Rubi [A] (verified) 4437
3.586.4 Maple [B] (verified) 4441
3.586.5 Fricas [C] (verification not implemented) 4442
3.586.6 Sympy [F(-1)] 4443
3.586.7 Maxima [F] 4443
3.586.8 Giac [F] 4444
3.586.9 Mupad [F(-1)] 4444

3.586.1 Optimal result

Integrand size = 23, antiderivative size = 200

$$\begin{aligned}\int \sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^2\,dx &= -\frac{12ab\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} \\ &\quad+\frac{2(7a^2+5b^2)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{21d} \\ &\quad+\frac{12ab\sqrt{\sec(c+dx)}\sin(c+dx)}{5d}+\frac{2(7a^2+5b^2)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{21d} \\ &\quad+\frac{4ab\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d}+\frac{2b^2\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{7d}\end{aligned}$$

output
```
2/21*(7*a^2+5*b^2)*sec(d*x+c)^(3/2)*sin(d*x+c)/d+4/5*a*b*sec(d*x+c)^(5/2)*
sin(d*x+c)/d+2/7*b^2*sec(d*x+c)^(7/2)*sin(d*x+c)/d+12/5*a*b*sin(d*x+c)*sec
(d*x+c)^(1/2)/d-12/5*a*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*E
llipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+2
/21*(7*a^2+5*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elliptic
F(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

3.586.2 Mathematica [A] (verified)

Time = 0.92 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.70

$$\int \sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^2\,dx$$
$$= \frac{\sec^{\frac{7}{2}}(c+dx)\left(-504ab\cos^{\frac{7}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)+20(7a^2+5b^2)\cos^{\frac{7}{2}}(c+dx)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\right.}{210}$$

input
```
Integrate[Sec[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^2,x]
```

output
```
(Sec[c + d*x]^(7/2)*(-504*a*b*Cos[c + d*x]^(7/2)*EllipticE[(c + d*x)/2, 2]
 + 20*(7*a^2 + 5*b^2)*Cos[c + d*x]^(7/2)*EllipticF[(c + d*x)/2, 2] + 2*(35
*a^2 + 55*b^2 + 273*a*b*Cos[c + d*x] + 5*(7*a^2 + 5*b^2)*Cos[2*(c + d*x)]
+ 63*a*b*Cos[3*(c + d*x)])*Sin[c + d*x]))/(210*d)
```

3.586.3 Rubi [A] (verified)

Time = 1.06 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.97, number of steps used = 17, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.739$, Rules used = {3042, 4275, 3042, 4255, 3042, 4255, 3042, 4258, 3042, 3119, 4534, 3042, 4255, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^2\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2dx$$
$$\Big\downarrow \text{4275}$$
$$\int \sec^{\frac{5}{2}}(c+dx)\left(a^2+b^2\sec^2(c+dx)\right)dx+2ab\int \sec^{\frac{7}{2}}(c+dx)dx$$
$$\Big\downarrow \text{3042}$$
$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)dx+2ab\int \csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}dx$$
$$\Big\downarrow \text{4255}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)dx+$$
$$2ab\left(\frac{3}{5}\int \sec^{\frac{3}{2}}(c+dx)dx+\frac{2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}\right)$$

$\Big\downarrow$ 3042

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)dx+$$
$$2ab\left(\frac{3}{5}\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}dx+\frac{2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}\right)$$

$\Big\downarrow$ 4255

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)dx+$$
$$2ab\left(\frac{3}{5}\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\int\frac{1}{\sqrt{\sec(c+dx)}}dx\right)+\frac{2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}\right)$$

$\Big\downarrow$ 3042

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)dx+$$
$$2ab\left(\frac{3}{5}\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\int\frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)+\frac{2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}\right)$$

$\Big\downarrow$ 4258

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)dx+$$
$$2ab\left(\frac{3}{5}\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\cos(c+dx)}dx\right)+\frac{2\sin(c+dx)\sec^{\frac{5}{2}}(c+d}{5d}\right.$$

$\Big\downarrow$ 3042

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)dx+$$
$$2ab\left(\frac{3}{5}\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx\right)+\frac{2\sin(c+dx)\sec^{\frac{5}{2}}}{5d}\right.$$

$\Big\downarrow$ 3119

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)dx+$$
$$2ab\left(\frac{2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}+\frac{3}{5}\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right.\right.$$

3.586. $\int \sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^2\,dx$

$$\Big\downarrow \text{4534}$$

$$\frac{1}{7}(7a^2+5b^2)\int \sec^{\frac{5}{2}}(c+dx)dx+$$
$$2ab\left(\frac{2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}+\frac{3}{5}\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right.\right.$$
$$\frac{2b^2\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{7d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{7}(7a^2+5b^2)\int \csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}dx+$$
$$2ab\left(\frac{2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}+\frac{3}{5}\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right.\right.$$
$$\frac{2b^2\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{7d}$$

$$\Big\downarrow \text{4255}$$

$$\frac{1}{7}(7a^2+5b^2)\left(\frac{1}{3}\int \sqrt{\sec(c+dx)}dx+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}\right)+$$
$$2ab\left(\frac{2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}+\frac{3}{5}\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right.\right.$$
$$\frac{2b^2\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{7d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{7}(7a^2+5b^2)\left(\frac{1}{3}\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}\right)+$$
$$2ab\left(\frac{2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}+\frac{3}{5}\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right.\right.$$
$$\frac{2b^2\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{7d}$$

$$\Big\downarrow \text{4258}$$

$$\frac{1}{7}(7a^2+5b^2)\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}\right)+$$
$$2ab\left(\frac{2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}+\frac{3}{5}\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right.\right.$$
$$\frac{2b^2\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{7d}$$

$$\Big\downarrow \text{3042}$$

3.586. $\int \sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^2\,dx$

$$\frac{1}{7}(7a^2+5b^2)\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}\right)+$$
$$2ab\left(\frac{2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}+\frac{3}{5}\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right.\right.$$
$$\frac{2b^2\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{7d}$$

$\Big\downarrow$ 3120

$$\frac{1}{7}(7a^2+5b^2)\left(\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}\right)+$$
$$2ab\left(\frac{2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}+\frac{3}{5}\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right.\right.$$
$$\frac{2b^2\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{7d}$$

input
```
Int[Sec[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^2,x]
```

output
```
(2*b^2*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(7*d) + ((7*a^2 + 5*b^2)*((2*Sqrt[
Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (2*Sec
[c + d*x]^(3/2)*Sin[c + d*x])/(3*d)))/7 + 2*a*b*((2*Sec[c + d*x]^(5/2)*Sin
[c + d*x])/(5*d) + (3*((-2*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sq
rt[Sec[c + d*x]])/d + (2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d))/5)
```

3.586.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4275
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Simp[2*a*(b/d)   Int[(d*Csc[e + f*x])^(n + 1), x], x]
 + Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
 e, f, n}, x]
```

rule 4534
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Simp[(C*m + A*(m + 1))/(m + 1)   Int[(b*Csc[e + f*x])^m, x], x] /;
 FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

3.586.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 688 vs. $2(224) = 448$.

Time = 79.16 (sec) , antiderivative size = 689, normalized size of antiderivative = 3.44

method	result
default	$-\frac{\sqrt{-\left(-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(2a^2\left(-\frac{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}}{6\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-\frac{1}{2}\right)^2}+\frac{\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2+}}{3\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+}}\right.\right.}{}$
parts	Expression too large to display

input
```
int(sec(d*x+c)^(5/2)*(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
-(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*a^2*(-1/6*co
s(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos
(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+
1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ell
ipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+2*b^2*(-1/56*cos(1/2*d*x+1/2*c)*(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^
4-5/42*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(
1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+5/21*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*co
s(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+4/5*a*b/sin(1/2*d*x+1/2*c)^2
/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)
*(24*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*
sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+12*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x
+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2+8*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1
/2*c)^2-3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2
))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+
1/2*c)^2)^(1/2))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

3.586.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.18

$$\int \sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^2\,dx$$

$$= \frac{-126i\sqrt{2}ab\cos(dx+c)^3\,\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))}{}$$

input
```
integrate(sec(d*x+c)^(5/2)*(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/105*(-126*I*sqrt(2)*a*b*cos(d*x + c)^3*weierstrassZeta(-4, 0, weierstras
sPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 126*I*sqrt(2)*a*b*cos(d
*x + c)^3*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) -
 I*sin(d*x + c))) - 5*sqrt(2)*(7*I*a^2 + 5*I*b^2)*cos(d*x + c)^3*weierstra
ssPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) - 5*sqrt(2)*(-7*I*a^2 - 5
*I*b^2)*cos(d*x + c)^3*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x
 + c)) + 2*(126*a*b*cos(d*x + c)^3 + 42*a*b*cos(d*x + c) + 5*(7*a^2 + 5*b^
2)*cos(d*x + c)^2 + 15*b^2)*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x +
c)^3)
```

3.586.6 Sympy [F(-1)]

Timed out.

$$\int \sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^2\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**(5/2)*(a+b*sec(d*x+c))**2,x)
```

output
```
Timed out
```

3.586.7 Maxima [F]

$$\int \sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^2\,dx = \int (b\sec(dx+c)+a)^2\sec(dx+c)^{\frac{5}{2}}\,dx$$

input
```
integrate(sec(d*x+c)^(5/2)*(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^2*sec(d*x + c)^(5/2), x)
```

3.586.8 Giac [F]

$$\int \sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^2\,dx = \int (b\sec(dx+c)+a)^2\sec(dx+c)^{\frac{5}{2}}\,dx$$

input
```
integrate(sec(d*x+c)^(5/2)*(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^2*sec(d*x + c)^(5/2), x)
```

3.586.9 Mupad [F(-1)]

Timed out.

$$\int \sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^2\,dx = \int \left(a+\frac{b}{\cos(c+d\,x)}\right)^2\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}dx$$

input
```
int((a + b/cos(c + d*x))^2*(1/cos(c + d*x))^(5/2),x)
```

output
```
int((a + b/cos(c + d*x))^2*(1/cos(c + d*x))^(5/2), x)
```

3.587 $\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2\,dx$

3.587.1 Optimal result 4445
3.587.2 Mathematica [A] (verified) 4446
3.587.3 Rubi [A] (verified) 4446
3.587.4 Maple [B] (verified) 4450
3.587.5 Fricas [C] (verification not implemented) 4451
3.587.6 Sympy [F(-1)] 4452
3.587.7 Maxima [F] 4452
3.587.8 Giac [F] 4452
3.587.9 Mupad [F(-1)] 4453

3.587.1 Optimal result

Integrand size = 23, antiderivative size = 175

$$\begin{aligned}\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2\,dx &= -\frac{2(5a^2+3b^2)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} \\ &\quad+\frac{4ab\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{3d} \\ &\quad+\frac{2(5a^2+3b^2)\sqrt{\sec(c+dx)}\sin(c+dx)}{5d} \\ &\quad+\frac{4ab\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d}+\frac{2b^2\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d}\end{aligned}$$

output
```
4/3*a*b*sec(d*x+c)^(3/2)*sin(d*x+c)/d+2/5*b^2*sec(d*x+c)^(5/2)*sin(d*x+c)/
d+2/5*(5*a^2+3*b^2)*sin(d*x+c)*sec(d*x+c)^(1/2)/d-2/5*(5*a^2+3*b^2)*(cos(1
/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(
1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+4/3*a*b*(cos(1/2*d*x+1/2*c)^2)^(
1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(
1/2)*sec(d*x+c)^(1/2)/d
```

3.587.2 Mathematica [A] (verified)

Time = 1.18 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.72

$$\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2\,dx$$
$$=\frac{\sec^{\frac{5}{2}}(c+dx)\left(-12(5a^2+3b^2)\cos^{\frac{5}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)+40ab\cos^{\frac{5}{2}}(c+dx)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\right.}{30d}$$

input
```
Integrate[Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^2,x]
```

output
```
(Sec[c + d*x]^(5/2)*(-12*(5*a^2 + 3*b^2)*Cos[c + d*x]^(5/2)*EllipticE[(c +
 d*x)/2, 2] + 40*a*b*Cos[c + d*x]^(5/2)*EllipticF[(c + d*x)/2, 2] + 2*(15*
(a^2 + b^2) + 20*a*b*Cos[c + d*x] + 3*(5*a^2 + 3*b^2)*Cos[2*(c + d*x)])*Si
n[c + d*x]))/(30*d)
```

3.587.3 Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.95, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.652, Rules used = {3042, 4275, 3042, 4255, 3042, 4258, 3042, 3120, 4534, 3042, 4255, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2dx$$
$$\Big\downarrow \text{4275}$$
$$\int \sec^{\frac{3}{2}}(c+dx)\left(a^2+b^2\sec^2(c+dx)\right)dx+2ab\int \sec^{\frac{5}{2}}(c+dx)dx$$
$$\Big\downarrow \text{3042}$$
$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)dx+2ab\int \csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}dx$$
$$\Big\downarrow \text{4255}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)dx+$$
$$2ab\left(\frac{1}{3}\int\sqrt{\sec(c+dx)}dx+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}\right)$$

$\Big\downarrow$ 3042

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)dx+$$
$$2ab\left(\frac{1}{3}\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}\right)$$

$\Big\downarrow$ 4258

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)dx+$$
$$2ab\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}\right)$$

$\Big\downarrow$ 3042

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)dx+$$
$$2ab\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}\right)$$

$\Big\downarrow$ 3120

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)dx+$$
$$2ab\left(\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}\right)$$

$\Big\downarrow$ 4534

$$\frac{1}{5}\left(5a^2+3b^2\right)\int\sec^{\frac{3}{2}}(c+dx)dx+$$
$$2ab\left(\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}\right)+$$
$$\frac{2b^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}$$

$\Big\downarrow$ 3042

3.587. $\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2\,dx$

$$\frac{1}{5}(5a^2+3b^2)\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}dx+$$
$$2ab\left(\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}\right)+$$
$$\frac{2b^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}$$
$$\Big\downarrow \text{4255}$$
$$\frac{1}{5}(5a^2+3b^2)\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\int\frac{1}{\sqrt{\sec(c+dx)}}dx\right)+$$
$$2ab\left(\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}\right)+$$
$$\frac{2b^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}$$
$$\Big\downarrow \text{3042}$$
$$\frac{1}{5}(5a^2+3b^2)\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\int\frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)+$$
$$2ab\left(\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}\right)+$$
$$\frac{2b^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}$$
$$\Big\downarrow \text{4258}$$
$$\frac{1}{5}(5a^2+3b^2)\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\cos(c+dx)}dx\right)+$$
$$2ab\left(\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}\right)+$$
$$\frac{2b^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}$$
$$\Big\downarrow \text{3042}$$
$$\frac{1}{5}(5a^2+3b^2)\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx\right)+$$
$$2ab\left(\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}\right)+$$
$$\frac{2b^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}$$
$$\Big\downarrow \text{3119}$$

3.587. $\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2\,dx$

$$\frac{1}{5}(5a^2+3b^2)\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)+2ab\left(\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}\right)+\frac{2b^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}$$

input
```
Int[Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^2,x]
```

output
```
(2*b^2*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5*d) + ((5*a^2 + 3*b^2)*((-2*Sqrt
[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (2*Sqrt[S
ec[c + d*x]]*Sin[c + d*x])/d))/5 + 2*a*b*((2*Sqrt[Cos[c + d*x]]*EllipticF[
(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (2*Sec[c + d*x]^(3/2)*Sin[c +
d*x])/(3*d))
```

3.587.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4275
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Simp[2*a*(b/d)   Int[(d*Csc[e + f*x])^(n + 1), x], x]
 + Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
 e, f, n}, x]
```

rule 4534
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Simp[(C*m + A*(m + 1))/(m + 1)   Int[(b*Csc[e + f*x])^m, x], x] /;
 FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

3.587.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 632 vs. $2(203) = 406$.

Time = 33.47 (sec) , antiderivative size = 633, normalized size of antiderivative = 3.62

method	result
default	$-\frac{\sqrt{-\left(-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(\frac{2b^2\left(24\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^6\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-12\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\mathrm{EllipticE}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sqrt{2\sin\left(\frac{dx}{2}+\right.}\right.}{}\right.}{}$
parts	Expression too large to display

input
```
int(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
-(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2/5*b^2/(8*sin
(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/
2*d*x+1/2*c)^2*(24*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*
c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2
*c)+12*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*
(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2+8*cos(1/2*d*x+1/2*c)
*sin(1/2*d*x+1/2*c)^2-3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x
+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2))*(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a^2/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/
2*c)^2-1)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*cos(1/2*
d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos
(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2))+4*a*b*(-1/6*cos
(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(
1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1
/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Elli
pticF(cos(1/2*d*x+1/2*c),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*
c)^2-1)^(1/2)/d
```

3.587.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.27

$$\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2\,dx$$

$$= \frac{-10i\sqrt{2}ab\cos(dx+c)^2\,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+10i\sqrt{2}ab\cos(dx+c)}{}$$

input
```
integrate(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/15*(-10*I*sqrt(2)*a*b*cos(d*x + c)^2*weierstrassPInverse(-4, 0, cos(d*x
+ c) + I*sin(d*x + c)) + 10*I*sqrt(2)*a*b*cos(d*x + c)^2*weierstrassPInver
se(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 3*sqrt(2)*(5*I*a^2 + 3*I*b^2)*c
os(d*x + c)^2*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x +
c) + I*sin(d*x + c))) - 3*sqrt(2)*(-5*I*a^2 - 3*I*b^2)*cos(d*x + c)^2*weie
rstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c
))) + 2*(10*a*b*cos(d*x + c) + 3*(5*a^2 + 3*b^2)*cos(d*x + c)^2 + 3*b^2)*s
in(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c)^2)
```

3.587.6 Sympy [F(-1)]

Timed out.

$$\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**(3/2)*(a+b*sec(d*x+c))**2,x)
```

output
```
Timed out
```

3.587.7 Maxima [F]

$$\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2\,dx = \int (b\sec(dx+c)+a)^2\sec(dx+c)^{\frac{3}{2}}\,dx$$

input
```
integrate(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^2*sec(d*x + c)^(3/2), x)
```

3.587.8 Giac [F]

$$\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2\,dx = \int (b\sec(dx+c)+a)^2\sec(dx+c)^{\frac{3}{2}}\,dx$$

input
```
integrate(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^2*sec(d*x + c)^(3/2), x)
```

3.587.9 Mupad [F(-1)]

Timed out.

$$\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2\,dx=\int \left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)^2\left(\frac{1}{\cos\left(c+d\,x\right)}\right)^{3/2}dx$$

input
```
int((a + b/cos(c + d*x))^2*(1/cos(c + d*x))^(3/2),x)
```

output
```
int((a + b/cos(c + d*x))^2*(1/cos(c + d*x))^(3/2), x)
```

3.588 $\int \sqrt{\sec(c+dx)}(a+b\sec(c+dx))^2\,dx$

3.588.1 Optimal result 4454
3.588.2 Mathematica [A] (verified) 4455
3.588.3 Rubi [A] (verified) 4455
3.588.4 Maple [B] (verified) 4459
3.588.5 Fricas [C] (verification not implemented) 4460
3.588.6 Sympy [F] 4460
3.588.7 Maxima [F] 4461
3.588.8 Giac [F] 4461
3.588.9 Mupad [F(-1)] 4461

3.588.1 Optimal result

Integrand size = 23, antiderivative size = 135

$$\begin{aligned}\int \sqrt{\sec(c+dx)}(a+b\sec(c+dx))^2\,dx &= -\frac{4ab\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} \\ &\quad+\frac{2(3a^2+b^2)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{3d} \\ &\quad+\frac{4ab\sqrt{\sec(c+dx)}\sin(c+dx)}{d}+\frac{2b^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d}\end{aligned}$$

output
```
2/3*b^2*sec(d*x+c)^(3/2)*sin(d*x+c)/d+4*a*b*sin(d*x+c)*sec(d*x+c)^(1/2)/d-
4*a*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*
x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+2/3*(3*a^2+b^2)*(cos
(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2
^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

3.588.2 Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.69

$$\int \sqrt{\sec(c+dx)}(a+b\sec(c+dx))^2\,dx$$
$$=\frac{2\sec^{\frac{3}{2}}(c+dx)\left(-6ab\cos^{\frac{3}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)+(3a^2+b^2)\cos^{\frac{3}{2}}(c+dx)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+b\right.}{3d}$$

input
```
Integrate[Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^2,x]
```

output
```
(2*Sec[c + d*x]^(3/2)*(-6*a*b*Cos[c + d*x]^(3/2)*EllipticE[(c + d*x)/2, 2]
 + (3*a^2 + b^2)*Cos[c + d*x]^(3/2)*EllipticF[(c + d*x)/2, 2] + b*(b + 6*a
*Cos[c + d*x])*Sin[c + d*x]))/(3*d)
```

3.588.3 Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.01, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.565, Rules used = {3042, 4275, 3042, 4255, 3042, 4258, 3042, 3119, 4534, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\sec(c+dx)}(a+b\sec(c+dx))^2\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2dx$$
$$\Big\downarrow \text{4275}$$
$$\int \sqrt{\sec(c+dx)}\left(a^2+b^2\sec^2(c+dx)\right)dx+2ab\int \sec^{\frac{3}{2}}(c+dx)dx$$
$$\Big\downarrow \text{3042}$$
$$\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)dx+2ab\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}dx$$
$$\Big\downarrow \text{4255}$$

$$\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)dx+$$
$$2ab\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\int\frac{1}{\sqrt{\sec(c+dx)}}dx\right)$$

$\Big\downarrow$ 3042

$$\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)dx+$$
$$2ab\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\int\frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)$$

$\Big\downarrow$ 4258

$$\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)dx+$$
$$2ab\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\cos(c+dx)}dx\right)$$

$\Big\downarrow$ 3042

$$\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)dx+$$
$$2ab\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx\right)$$

$\Big\downarrow$ 3119

$$\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)dx+$$
$$2ab\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)$$

$\Big\downarrow$ 4534

$$\frac{1}{3}(3a^2+b^2)\int\sqrt{\sec(c+dx)}dx+$$
$$2ab\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)+$$
$$\frac{2b^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}$$

$\Big\downarrow$ 3042

3.588. $\int \sqrt{\sec(c+dx)}(a+b\sec(c+dx))^2\,dx$

$$\frac{1}{3}(3a^2+b^2)\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+2ab\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)+\frac{2b^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}$$

$\Big\downarrow$ 4258

$$\frac{1}{3}(3a^2+b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx+2ab\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)+\frac{2b^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}$$

$\Big\downarrow$ 3042

$$\frac{1}{3}(3a^2+b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+2ab\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)+\frac{2b^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}$$

$\Big\downarrow$ 3120

$$\frac{2(3a^2+b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+2ab\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)+\frac{2b^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}$$

input
```
Int[Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^2,x]
```

output
```
(2*(3*a^2 + b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c +
 d*x]])/(3*d) + (2*b^2*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d) + 2*a*b*((-2
*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (2*S
qrt[Sec[c + d*x]]*Sin[c + d*x])/d)
```

3.588. $\int\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^2\,dx$

3.588.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4275
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Simp[2*a*(b/d)   Int[(d*Csc[e + f*x])^(n + 1), x], x]
 + Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
 e, f, n}, x]
```

rule 4534
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Simp[(C*m + A*(m + 1))/(m + 1)   Int[(b*Csc[e + f*x])^m, x], x] /;
 FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

3.588.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 512 vs. $2(171) = 342$.

Time = 22.76 (sec) , antiderivative size = 513, normalized size of antiderivative = 3.80

method	result
default	$-\frac{2\sqrt{-\left(-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\left(24\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}ab-6\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1}\,\mathrm{EllipticF}\left(\cos\right.\right.}{}$
parts	$-\frac{2a^{2}\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+1}\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1}\,d}-\frac{2b^{2}\left(-2\sqrt{\frac{1}{2}-\frac{\mathrm{co}}{}}\right.}{}$

input
```
int(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
-2/3*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(4*sin(1/2*
d*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)/sin(1/2*d*x+1/2*c)^3*(24*cos(1/2*d*
x+1/2*c)*sin(1/2*d*x+1/2*c)^4*a*b-6*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/
2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+
1/2*c)^2*a^2-2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/
2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2*b^2-12*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2
*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2*a*b-12*cos(1/2*d*x+1/2*c)*sin(1/
2*d*x+1/2*c)^2*a*b-2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2*b^2+3*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d
*x+1/2*c),2^(1/2))*a^2+(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^
2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b^2+6*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^
(1/2))*a*b)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/
2*d*x+1/2*c)^2-1)^(1/2)/d
```

3.588.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.41

$$\int \sqrt{\sec(c+dx)}(a+b\sec(c+dx))^2\,dx$$
$$= \frac{-6i\sqrt{2}ab\cos(dx+c)\,\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))+}{}$$

input
```
integrate(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/3*(-6*I*sqrt(2)*a*b*cos(d*x + c)*weierstrassZeta(-4, 0, weierstrassPInve
rse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 6*I*sqrt(2)*a*b*cos(d*x + c)*
weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x
 + c))) + sqrt(2)*(-3*I*a^2 - I*b^2)*cos(d*x + c)*weierstrassPInverse(-4,
0, cos(d*x + c) + I*sin(d*x + c)) + sqrt(2)*(3*I*a^2 + I*b^2)*cos(d*x + c)
*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 2*(6*a*b*cos(
d*x + c) + b^2)*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c))
```

3.588.6 Sympy [F]

$$\int \sqrt{\sec(c+dx)}(a+b\sec(c+dx))^2\,dx = \int (a+b\sec(c+dx))^2\sqrt{\sec(c+dx)}\,dx$$

input
```
integrate(sec(d*x+c)**(1/2)*(a+b*sec(d*x+c))**2,x)
```

output
```
Integral((a + b*sec(c + d*x))**2*sqrt(sec(c + d*x)), x)
```

3.588.7 Maxima [F]

$$\int \sqrt{\sec(c+dx)}(a+b\sec(c+dx))^2\,dx = \int (b\sec(dx+c)+a)^2\sqrt{\sec(dx+c)}\,dx$$

input
```
integrate(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^2*sqrt(sec(d*x + c)), x)
```

3.588.8 Giac [F]

$$\int \sqrt{\sec(c+dx)}(a+b\sec(c+dx))^2\,dx = \int (b\sec(dx+c)+a)^2\sqrt{\sec(dx+c)}\,dx$$

input
```
integrate(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^2*sqrt(sec(d*x + c)), x)
```

3.588.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{\sec(c+dx)}(a+b\sec(c+dx))^2\,dx = \int \left(a+\frac{b}{\cos(c+d\,x)}\right)^2\sqrt{\frac{1}{\cos(c+d\,x)}}\,dx$$

input
```
int((a + b/cos(c + d*x))^2*(1/cos(c + d*x))^(1/2),x)
```

output
```
int((a + b/cos(c + d*x))^2*(1/cos(c + d*x))^(1/2), x)
```

3.589 $\int \frac{(a+b\sec(c+dx))^2}{\sqrt{\sec(c+dx)}}\,dx$

3.589.1 Optimal result 4462
3.589.2 Mathematica [A] (verified) 4462
3.589.3 Rubi [A] (verified) 4463
3.589.4 Maple [A] (verified) 4465
3.589.5 Fricas [C] (verification not implemented) 4466
3.589.6 Sympy [F] 4466
3.589.7 Maxima [F] 4467
3.589.8 Giac [F] 4467
3.589.9 Mupad [F(-1)] 4467

3.589.1 Optimal result

Integrand size = 23, antiderivative size = 108

$$\int \frac{(a+b\sec(c+dx))^2}{\sqrt{\sec(c+dx)}}\,dx = \frac{2(a^2-b^2)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{4ab\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{d} + \frac{2b^2\sqrt{\sec(c+dx)}\sin(c+dx)}{d}$$

output
```
2*b^2*sin(d*x+c)*sec(d*x+c)^(1/2)/d+2*(a^2-b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/
2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/
2)*sec(d*x+c)^(1/2)/d+4*a*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c
)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/
d
```

3.589.2 Mathematica [A] (verified)

Time = 0.75 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.76

$$\int \frac{(a+b\sec(c+dx))^2}{\sqrt{\sec(c+dx)}}\,dx = \frac{2\sqrt{\sec(c+dx)}\left((a^2-b^2)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)+b\left(2a\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+b\right.\right.}{d}$$

input
```
Integrate[(a + b*Sec[c + d*x])^2/Sqrt[Sec[c + d*x]],x]
```

output
```
(2*Sqrt[Sec[c + d*x]]*((a^2 - b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/
2, 2] + b*(2*a*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + b*Sin[c + d*
x])))/d
```

3.589.3 Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules used = {3042, 4275, 3042, 4258, 3042, 3120, 4534, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\sec(c+dx))^2}{\sqrt{\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4275}$$

$$\int \frac{a^2+b^2\sec^2(c+dx)}{\sqrt{\sec(c+dx)}}dx+2ab\int \sqrt{\sec(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+2ab\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{4258}$$

$$\int \frac{a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+2ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+2ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{3120}$$

$$\int \frac{a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{4ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}$$

$$\Big\downarrow \text{4534}$$

$$\left(a^2-b^2\right)\int \frac{1}{\sqrt{\sec(c+dx)}}dx+\frac{4ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+\frac{2b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}$$

$$\Big\downarrow \text{3042}$$

$$\left(a^2-b^2\right)\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{4ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+\frac{2b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}$$

$$\Big\downarrow \text{4258}$$

$$\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\cos(c+dx)}dx+\frac{4ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+\frac{2b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}$$

$$\Big\downarrow \text{3042}$$

$$\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{4ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+\frac{2b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}$$

$$\Big\downarrow \text{3119}$$

$$\frac{2\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{4ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+\frac{2b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}$$

input
```
Int[(a + b*Sec[c + d*x])^2/Sqrt[Sec[c + d*x]],x]
```

output
```
(2*(a^2 - b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d
*x]])/d + (4*a*b*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c +
 d*x]])/d + (2*b^2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d
```

3.589. $\int \frac{(a+b\sec(c+dx))^2}{\sqrt{\sec(c+dx)}}\,dx$

3.589.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4275
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Simp[2*a*(b/d)   Int[(d*Csc[e + f*x])^(n + 1), x], x]
 + Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
 e, f, n}, x]
```

rule 4534
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Simp[(C*m + A*(m + 1))/(m + 1)   Int[(b*Csc[e + f*x])^m, x], x] /;
 FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

3.589.4 Maple [A] (verified)

Time = 16.61 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.87

method	result
default	$\frac{4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2 b^2-4\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1}\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,ab+2\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1}\,\mathrm{Ellipticl}}{\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2}}$
parts	$\frac{2a^2\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1}\,\mathrm{EllipticE}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1}\,d}-\frac{2b^2\left(-2\sqrt{-2\sin(}\right.}{}$

input
```
int((a+b*sec(d*x+c))^2/sec(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*(2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2*b^2-2*(2*sin(1/2*d*x+1/2*c)^2
-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*a*b+(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/
2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2-(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elli
pticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*b^2)/sin(1/
2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

3.589.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.35

$$\int \frac{(a+b\sec(c+dx))^2}{\sqrt{\sec(c+dx)}}\,dx$$
$$= \frac{-2i\sqrt{2}ab\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+2i\sqrt{2}ab\text{weierstrassPInverse}(-4,0,c}{}$$

input
```
integrate((a+b*sec(d*x+c))^2/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
(-2*I*sqrt(2)*a*b*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)
) + 2*I*sqrt(2)*a*b*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x +
c)) + 2*b^2*sin(d*x + c)/sqrt(cos(d*x + c)) + sqrt(2)*(I*a^2 - I*b^2)*weie
rstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c
))) + sqrt(2)*(-I*a^2 + I*b^2)*weierstrassZeta(-4, 0, weierstrassPInverse(
-4, 0, cos(d*x + c) - I*sin(d*x + c))))/d
```

3.589.6 Sympy [F]

$$\int \frac{(a+b\sec(c+dx))^2}{\sqrt{\sec(c+dx)}}\,dx = \int \frac{(a+b\sec(c+dx))^2}{\sqrt{\sec(c+dx)}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))**2/sec(d*x+c)**(1/2),x)
```

output
```
Integral((a + b*sec(c + d*x))**2/sqrt(sec(c + d*x)), x)
```

3.589.7 Maxima [F]

$$\int \frac{(a+b\sec(c+dx))^2}{\sqrt{\sec(c+dx)}}\,dx = \int \frac{(b\sec(dx+c)+a)^2}{\sqrt{\sec(dx+c)}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^2/sec(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^2/sqrt(sec(d*x + c)), x)
```

3.589.8 Giac [F]

$$\int \frac{(a+b\sec(c+dx))^2}{\sqrt{\sec(c+dx)}}\,dx = \int \frac{(b\sec(dx+c)+a)^2}{\sqrt{\sec(dx+c)}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^2/sec(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^2/sqrt(sec(d*x + c)), x)
```

3.589.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(c+dx))^2}{\sqrt{\sec(c+dx)}}\,dx = \int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^2}{\sqrt{\frac{1}{\cos(c+d\,x)}}}\,dx$$

input
```
int((a + b/cos(c + d*x))^2/(1/cos(c + d*x))^(1/2),x)
```

output
```
int((a + b/cos(c + d*x))^2/(1/cos(c + d*x))^(1/2), x)
```

3.590 $\int \frac{(a+b\sec(c+dx))^2}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

3.590.1 Optimal result 4468
3.590.2 Mathematica [A] (verified) 4469
3.590.3 Rubi [A] (verified) 4469
3.590.4 Maple [A] (verified) 4472
3.590.5 Fricas [C] (verification not implemented) 4472
3.590.6 Sympy [F] 4473
3.590.7 Maxima [F] 4473
3.590.8 Giac [F] 4473
3.590.9 Mupad [F(-1)] 4474

3.590.1 Optimal result

Integrand size = 23, antiderivative size = 112

$$\int \frac{(a+b\sec(c+dx))^2}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \frac{4ab\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{2(a^2+3b^2)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{3d} + \frac{2a^2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}$$

output
```
2/3*a^2*sin(d*x+c)/d/sec(d*x+c)^(1/2)+4*a*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/c
os(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*s
ec(d*x+c)^(1/2)/d+2/3*(a^2+3*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x
+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^
(1/2)/d
```

3.590.2 Mathematica [A] (verified)

Time = 0.61 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.78

$$\int \frac{(a+b\sec(c+dx))^2}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \frac{\sqrt{\sec(c+dx)}\left(12ab\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)+2(a^2+3b^2)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+\right.}{3d}$$

input
```
Integrate[(a + b*Sec[c + d*x])^2/Sec[c + d*x]^(3/2),x]
```

output
```
(Sqrt[Sec[c + d*x]]*(12*a*b*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] +
 2*(a^2 + 3*b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + a^2*Sin[2*
(c + d*x)]))/(3*d)
```

3.590.3 Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules used = {3042, 4275, 3042, 4258, 3042, 3119, 4533, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\sec(c+dx))^2}{\sec^{\frac{3}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{4275}$$

$$\int \frac{a^2+b^2\sec^2(c+dx)}{\sec^{\frac{3}{2}}(c+dx)}dx+2ab\int \frac{1}{\sqrt{\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+2ab\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4258}$$

$$\int \frac{a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+2ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\cos(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+2ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3119}$$

$$\int \frac{a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+\frac{4ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

$$\Big\downarrow \text{4533}$$

$$\frac{1}{3}\left(a^2+3b^2\right)\int\sqrt{\sec(c+dx)}dx+\frac{2a^2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{4ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\left(a^2+3b^2\right)\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2a^2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{4ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

$$\Big\downarrow \text{4258}$$

$$\frac{1}{3}\left(a^2+3b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2a^2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{4ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\left(a^2+3b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a^2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{4ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

$$\Big\downarrow \text{3120}$$

$$\frac{2\left(a^2+3b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2a^2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{4ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

3.590. $\int \frac{(a+b\sec(c+dx))^2}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

input
```
Int[(a + b*Sec[c + d*x])^2/Sec[c + d*x]^(3/2),x]
```

output
```
(4*a*b*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d
+ (2*(a^2 + 3*b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c
 + d*x]])/(3*d) + (2*a^2*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])
```

3.590.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4275
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Simp[2*a*(b/d)   Int[(d*Csc[e + f*x])^(n + 1), x], x]
 + Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
 e, f, n}, x]
```

rule 4533
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] +
Simp[(C*m + A*(m + 1))/(b^2*m)   Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fr
eeQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

3.590. $\int \frac{(a+b\sec(c+dx))^2}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

3.590.4 Maple [A] (verified)

Time = 18.55 (sec) , antiderivative size = 283, normalized size of antiderivative = 2.53

method	result
default	$-\frac{2\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\left(4a^{2}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}-2a^{2}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\sin(}\right.}{3\sqrt{-2s}}$
parts	$-\frac{2a^{2}\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\left(4\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\sin\left(\frac{dx}{2}\right.}\right.}{3\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1}\,d}$

input
```
int((a+b*sec(d*x+c))^2/sec(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4*a^2*cos(1/
2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4-2*a^2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1
/2*c)+(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipt
icF(cos(1/2*d*x+1/2*c),2^(1/2))*a^2+3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(
1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b^2-6*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/
2*d*x+1/2*c),2^(1/2))*a*b)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^
(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

3.590.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.31

$$\int \frac{(a+b\sec(c+dx))^2}{\sec^{\frac{3}{2}}(c+dx)}\,dx$$
$$= \frac{2\,a^2\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)+6i\,\sqrt{2}ab\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+}{}$$

input
```
integrate((a+b*sec(d*x+c))^2/sec(d*x+c)^(3/2),x, algorithm="fricas")
```

output
```
1/3*(2*a^2*sqrt(cos(d*x + c))*sin(d*x + c) + 6*I*sqrt(2)*a*b*weierstrassZe
ta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 6*I
*sqrt(2)*a*b*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c
) - I*sin(d*x + c))) + sqrt(2)*(-I*a^2 - 3*I*b^2)*weierstrassPInverse(-4,
0, cos(d*x + c) + I*sin(d*x + c)) + sqrt(2)*(I*a^2 + 3*I*b^2)*weierstrassP
Inverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)))/d
```

3.590.6 Sympy [F]

$$\int \frac{(a+b\sec(c+dx))^2}{\sec^{\frac{3}{2}}(c+dx)}\,dx=\int \frac{(a+b\sec{(c+dx)})^2}{\sec^{\frac{3}{2}}{(c+dx)}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))**2/sec(d*x+c)**(3/2),x)
```

output
```
Integral((a + b*sec(c + d*x))**2/sec(c + d*x)**(3/2), x)
```

3.590.7 Maxima [F]

$$\int \frac{(a+b\sec(c+dx))^2}{\sec^{\frac{3}{2}}(c+dx)}\,dx=\int \frac{(b\sec{(dx+c)}+a)^2}{\sec{(dx+c)}^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^2/sec(d*x+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^2/sec(d*x + c)^(3/2), x)
```

3.590.8 Giac [F]

$$\int \frac{(a+b\sec(c+dx))^2}{\sec^{\frac{3}{2}}(c+dx)}\,dx=\int \frac{(b\sec{(dx+c)}+a)^2}{\sec{(dx+c)}^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^2/sec(d*x+c)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^2/sec(d*x + c)^(3/2), x)
```

3.590.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(c+dx))^2}{\sec^{\frac{3}{2}}(c+dx)}\,dx=\int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^2}{\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

input
```
int((a + b/cos(c + d*x))^2/(1/cos(c + d*x))^(3/2),x)
```

output
```
int((a + b/cos(c + d*x))^2/(1/cos(c + d*x))^(3/2), x)
```

3.591 $\int \frac{(a+b\sec(c+dx))^2}{\sec^{\frac{5}{2}}(c+dx)}\,dx$

3.591.1 Optimal result 4475
3.591.2 Mathematica [A] (verified) 4476
3.591.3 Rubi [A] (verified) 4476
3.591.4 Maple [B] (verified) 4479
3.591.5 Fricas [C] (verification not implemented) 4480
3.591.6 Sympy [F] 4481
3.591.7 Maxima [F] 4481
3.591.8 Giac [F] 4481
3.591.9 Mupad [F(-1)] 4482

3.591.1 Optimal result

Integrand size = 23, antiderivative size = 141

$$\int \frac{(a+b\sec(c+dx))^2}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \frac{2(3a^2+5b^2)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{4ab\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{3d} + \frac{2a^2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{4ab\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}$$

output
```
2/5*a^2*sin(d*x+c)/d/sec(d*x+c)^(3/2)+4/3*a*b*sin(d*x+c)/d/sec(d*x+c)^(1/2
)+2/5*(3*a^2+5*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellipt
icE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+4/3*a*
b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/
2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

3.591.2 Mathematica [A] (verified)

Time = 0.84 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.71

$$\int \frac{(a+b\sec(c+dx))^2}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \frac{\sqrt{\sec(c+dx)}\left(6(3a^2+5b^2)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)+20ab\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)-\right.}{15d}$$

input
```
Integrate[(a + b*Sec[c + d*x])^2/Sec[c + d*x]^(5/2),x]
```

output
```
(Sqrt[Sec[c + d*x]]*(6*(3*a^2 + 5*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d
*x)/2, 2] + 20*a*b*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + a*(10*b
+ 3*a*Cos[c + d*x])*Sin[2*(c + d*x)]))/(15*d)
```

3.591.3 Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.01, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.565$, Rules used = {3042, 4275, 3042, 4256, 3042, 4258, 3042, 3120, 4533, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\sec(c+dx))^2}{\sec^{\frac{5}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx$$

$$\Big\downarrow \text{4275}$$

$$\int \frac{a^2+b^2\sec^2(c+dx)}{\sec^{\frac{5}{2}}(c+dx)}dx+2ab\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+2ab\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{4256}$$

$$\int \frac{a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+2ab\left(\frac{1}{3}\int\sqrt{\sec(c+dx)}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)$$

$\Big\downarrow$ 3042

$$\int \frac{a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+2ab\left(\frac{1}{3}\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)$$

$\Big\downarrow$ 4258

$$\int \frac{a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+$$
$$2ab\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)$$

$\Big\downarrow$ 3042

$$\int \frac{a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+$$
$$2ab\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)$$

$\Big\downarrow$ 3120

$$\int \frac{a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+$$
$$2ab\left(\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}\right)$$

$\Big\downarrow$ 4533

$$\frac{1}{5}(3a^2+5b^2)\int\frac{1}{\sqrt{\sec(c+dx)}}dx+\frac{2a^2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+$$
$$2ab\left(\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}\right)$$

$\Big\downarrow$ 3042

$$\frac{1}{5}(3a^2+5b^2)\int\frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a^2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+$$
$$2ab\left(\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}\right)$$

$\Big\downarrow$ 4258

3.591. $\int \frac{(a+b\sec(c+dx))^2}{\sec^{\frac{5}{2}}(c+dx)}\,dx$

$$\frac{1}{5}(3a^2+5b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\cos(c+dx)}dx+\frac{2a^2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+2ab\left(\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}\right)$$

$\downarrow$ 3042

$$\frac{1}{5}(3a^2+5b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2a^2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+2ab\left(\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}\right)$$

$\downarrow$ 3119

$$\frac{2(3a^2+5b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a^2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+2ab\left(\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}\right)$$

input
```
Int[(a + b*Sec[c + d*x])^2/Sec[c + d*x]^(5/2),x]
```

output
```
(2*(3*a^2 + 5*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
 + d*x]])/(5*d) + (2*a^2*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)) + 2*a*b*((
2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) +
 (2*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]]))
```

3.591.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4275
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Simp[2*a*(b/d)   Int[(d*Csc[e + f*x])^(n + 1), x], x]
 + Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
 e, f, n}, x]
```

rule 4533
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] +
Simp[(C*m + A*(m + 1))/(b^2*m)   Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fr
eeQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

3.591.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 356 vs. $2(173) = 346$.

Time = 19.53 (sec) , antiderivative size = 357, normalized size of antiderivative = 2.53

method	result
default	$-\frac{2\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(-24a^2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^6+24a^2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+40\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\right.\right.}{}$
parts	$-\frac{2a^2\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(-8\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^6\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+8\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}{5\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right.}}$

input
```
int((a+b*sec(d*x+c))^2/sec(d*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/15*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-24*a^2*cos
(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+24*a^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x
+1/2*c)^4+40*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4*a*b-6*a^2*sin(1/2*d*x
+1/2*c)^2*cos(1/2*d*x+1/2*c)-20*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2*a*
b+10*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)
)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*a*b-9*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elli
pticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2-15*(2*s
in(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*b^2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^
(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

3.591.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.21

$$\int \frac{(a+b\sec(c+dx))^2}{\sec^{\frac{5}{2}}(c+dx)}\,dx$$
$$= \frac{-10i\sqrt{2}ab\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+10i\sqrt{2}ab\text{weierstrassPInverse}(-4,0}{}$$

input
```
integrate((a+b*sec(d*x+c))^2/sec(d*x+c)^(5/2),x, algorithm="fricas")
```

output
```
1/15*(-10*I*sqrt(2)*a*b*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*
x + c)) + 10*I*sqrt(2)*a*b*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin
(d*x + c)) - 3*sqrt(2)*(-3*I*a^2 - 5*I*b^2)*weierstrassZeta(-4, 0, weierst
rassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 3*sqrt(2)*(3*I*a^2 +
 5*I*b^2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) -
 I*sin(d*x + c))) + 2*(3*a^2*cos(d*x + c)^2 + 10*a*b*cos(d*x + c))*sin(d*x
 + c)/sqrt(cos(d*x + c)))/d
```

3.591.6 Sympy [F]

$$\int \frac{(a+b\sec(c+dx))^2}{\sec^{\frac{5}{2}}(c+dx)}\,dx=\int \frac{(a+b\sec{(c+dx)})^2}{\sec^{\frac{5}{2}}{(c+dx)}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))**2/sec(d*x+c)**(5/2),x)
```

output
```
Integral((a + b*sec(c + d*x))**2/sec(c + d*x)**(5/2), x)
```

3.591.7 Maxima [F]

$$\int \frac{(a+b\sec(c+dx))^2}{\sec^{\frac{5}{2}}(c+dx)}\,dx=\int \frac{(b\sec{(dx+c)}+a)^2}{\sec{(dx+c)}^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^2/sec(d*x+c)^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^2/sec(d*x + c)^(5/2), x)
```

3.591.8 Giac [F]

$$\int \frac{(a+b\sec(c+dx))^2}{\sec^{\frac{5}{2}}(c+dx)}\,dx=\int \frac{(b\sec{(dx+c)}+a)^2}{\sec{(dx+c)}^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^2/sec(d*x+c)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^2/sec(d*x + c)^(5/2), x)
```

3.591.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(c+dx))^2}{\sec^{\frac{5}{2}}(c+dx)}\,dx=\int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^2}{\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

input
```
int((a + b/cos(c + d*x))^2/(1/cos(c + d*x))^(5/2),x)
```

output
```
int((a + b/cos(c + d*x))^2/(1/cos(c + d*x))^(5/2), x)
```

3.592 $\int \frac{(a+b\sec(c+dx))^2}{\sec^{\frac{7}{2}}(c+dx)}\,dx$

3.592.1 Optimal result 4483
3.592.2 Mathematica [A] (verified) 4484
3.592.3 Rubi [A] (verified) 4484
3.592.4 Maple [A] (verified) 4488
3.592.5 Fricas [C] (verification not implemented) 4488
3.592.6 Sympy [F] 4489
3.592.7 Maxima [F] 4489
3.592.8 Giac [F] 4490
3.592.9 Mupad [F(-1)] 4490

3.592.1 Optimal result

Integrand size = 23, antiderivative size = 175

$$\begin{aligned}\int \frac{(a+b\sec(c+dx))^2}{\sec^{\frac{7}{2}}(c+dx)}\,dx &= \frac{12ab\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} \\ &\quad+\frac{2(5a^2+7b^2)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{21d} \\ &\quad+\frac{2a^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+\frac{4ab\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{2(5a^2+7b^2)\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}\end{aligned}$$

output
```
2/7*a^2*sin(d*x+c)/d/sec(d*x+c)^(5/2)+4/5*a*b*sin(d*x+c)/d/sec(d*x+c)^(3/2
)+2/21*(5*a^2+7*b^2)*sin(d*x+c)/d/sec(d*x+c)^(1/2)+12/5*a*b*(cos(1/2*d*x+1
/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*co
s(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+2/21*(5*a^2+7*b^2)*(cos(1/2*d*x+1/2*c)^2
)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c
)^(1/2)*sec(d*x+c)^(1/2)/d
```

3.592.2 Mathematica [A] (verified)

Time = 1.10 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.69

$$\int \frac{(a+b\sec(c+dx))^2}{\sec^{\frac{7}{2}}(c+dx)}\,dx = \frac{\sqrt{\sec(c+dx)}\left(504ab\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)+20(5a^2+7b^2)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right.\right.}{210d}$$

input
```
Integrate[(a + b*Sec[c + d*x])^2/Sec[c + d*x]^(7/2),x]
```

output
```
(Sqrt[Sec[c + d*x]]*(504*a*b*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]
+ 20*(5*a^2 + 7*b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + (65*a^
2 + 70*b^2 + 84*a*b*Cos[c + d*x] + 15*a^2*Cos[2*(c + d*x)])*Sin[2*(c + d*x
)]))/(210*d)
```

3.592.3 Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.97, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.652$, Rules used = {3042, 4275, 3042, 4256, 3042, 4258, 3042, 3119, 4533, 3042, 4256, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\sec(c+dx))^2}{\sec^{\frac{7}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx$$

$$\Big\downarrow \text{4275}$$

$$\int \frac{a^2+b^2\sec^2(c+dx)}{\sec^{\frac{7}{2}}(c+dx)}dx+2ab\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx+2ab\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx$$

$\Big\downarrow$ 4256

$$\int \frac{a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx+2ab\left(\frac{3}{5}\int \frac{1}{\sqrt{\sec(c+dx)}}dx+\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}\right)$$

$\Big\downarrow$ 3042

$$\int \frac{a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx+2ab\left(\frac{3}{5}\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}\right)$$

$\Big\downarrow$ 4258

$$\int \frac{a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx+$$
$$2ab\left(\frac{3}{5}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\cos(c+dx)}dx+\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}\right)$$

$\Big\downarrow$ 3042

$$\int \frac{a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx+$$
$$2ab\left(\frac{3}{5}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}\right)$$

$\Big\downarrow$ 3119

$$\int \frac{a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx+$$
$$2ab\left(\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{6\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}\right)$$

$\Big\downarrow$ 4533

$$\frac{1}{7}\left(5a^2+7b^2\right)\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}dx+\frac{2a^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+$$
$$2ab\left(\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{6\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}\right)$$

$\Big\downarrow$ 3042

3.592. $\int \frac{(a+b\sec(c+dx))^2}{\sec^{\frac{7}{2}}(c+dx)}\,dx$

$$\frac{1}{7}(5a^2+7b^2)\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+\frac{2a^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+2ab\left(\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{6\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}\right)$$

$\downarrow$ 4256

$$\frac{1}{7}(5a^2+7b^2)\left(\frac{1}{3}\int \sqrt{\sec(c+dx)}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)+\frac{2a^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+2ab\left(\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{6\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}\right)$$

$\downarrow$ 3042

$$\frac{1}{7}(5a^2+7b^2)\left(\frac{1}{3}\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)+\frac{2a^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+2ab\left(\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{6\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}\right)$$

$\downarrow$ 4258

$$\frac{1}{7}(5a^2+7b^2)\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)+\frac{2a^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+2ab\left(\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{6\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}\right)$$

$\downarrow$ 3042

$$\frac{1}{7}(5a^2+7b^2)\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)+\frac{2a^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+2ab\left(\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{6\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}\right)$$

$\downarrow$ 3120

$$\frac{1}{7}(5a^2+7b^2)\left(\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}\right)+\frac{2a^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+2ab\left(\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{6\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}\right)$$

input
```
Int[(a + b*Sec[c + d*x])^2/Sec[c + d*x]^(7/2),x]
```

3.592. $\int \frac{(a+b\sec(c+dx))^2}{\sec^{\frac{7}{2}}(c+dx)}\,dx$

output
```
(2*a^2*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/2)) + 2*a*b*((6*Sqrt[Cos[c + d*x
]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d) + (2*Sin[c + d*x])/
(5*d*Sec[c + d*x]^(3/2))) + ((5*a^2 + 7*b^2)*((2*Sqrt[Cos[c + d*x]]*Ellipt
icF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (2*Sin[c + d*x])/(3*d*Sqrt
[Sec[c + d*x]])))/7
```

3.592.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4275
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Simp[2*a*(b/d)   Int[(d*Csc[e + f*x])^(n + 1), x], x]
 + Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
 e, f, n}, x]
```

rule 4533
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] +
Simp[(C*m + A*(m + 1))/(b^2*m)   Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fr
eeQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

3.592. $\int \frac{(a+b\sec(c+dx))^2}{\sec^{\frac{7}{2}}(c+dx)}\,dx$

3.592.4 Maple [A] (verified)

Time = 25.87 (sec) , antiderivative size = 362, normalized size of antiderivative = 2.07

method	result
default	$-\frac{2\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\left(240a^{2}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{8}+(-360a^{2}-336ab)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+(280a^{2}+\right.}{}$
parts	$-\frac{2a^{2}\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\left(48\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{9}-120\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}+128\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}-72\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}+5\sqrt{\frac{1}{2}-\frac{\cos(}{}}\right.}{21\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\cos\left(\frac{dx}{2}+\right.}}$

input
```
int((a+b*sec(d*x+c))^2/sec(d*x+c)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-2/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(240*a^2*co
s(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+(-360*a^2-336*a*b)*sin(1/2*d*x+1/2*c
)^6*cos(1/2*d*x+1/2*c)+(280*a^2+336*a*b+140*b^2)*sin(1/2*d*x+1/2*c)^4*cos(
1/2*d*x+1/2*c)+(-80*a^2-84*a*b-70*b^2)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/
2*c)+25*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elli
pticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^2+35*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*s
in(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b^2-126
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(c
os(1/2*d*x+1/2*c),2^(1/2))*a*b)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

3.592.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.09

$$\int \frac{(a+b\sec(c+dx))^2}{\sec^{\frac{7}{2}}(c+dx)}\,dx$$

$$= \frac{126i\sqrt{2}ab\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))-126i\sqrt{2}ab\text{w}}{}$$

input
```
integrate((a+b*sec(d*x+c))^2/sec(d*x+c)^(7/2),x, algorithm="fricas")
```

output
```
1/105*(126*I*sqrt(2)*a*b*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0,
 cos(d*x + c) + I*sin(d*x + c))) - 126*I*sqrt(2)*a*b*weierstrassZeta(-4, 0
, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - 5*sqrt(2)*(
5*I*a^2 + 7*I*b^2)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c
)) - 5*sqrt(2)*(-5*I*a^2 - 7*I*b^2)*weierstrassPInverse(-4, 0, cos(d*x + c
) - I*sin(d*x + c)) + 2*(15*a^2*cos(d*x + c)^3 + 42*a*b*cos(d*x + c)^2 + 5
*(5*a^2 + 7*b^2)*cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/d
```

3.592.6 Sympy [F]

$$\int \frac{(a+b\sec(c+dx))^2}{\sec^{\frac{7}{2}}(c+dx)}\,dx = \int \frac{(a+b\sec{(c+dx)})^2}{\sec^{\frac{7}{2}}{(c+dx)}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))**2/sec(d*x+c)**(7/2),x)
```

output
```
Integral((a + b*sec(c + d*x))**2/sec(c + d*x)**(7/2), x)
```

3.592.7 Maxima [F]

$$\int \frac{(a+b\sec(c+dx))^2}{\sec^{\frac{7}{2}}(c+dx)}\,dx = \int \frac{(b\sec{(dx+c)}+a)^2}{\sec{(dx+c)}^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^2/sec(d*x+c)^(7/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^2/sec(d*x + c)^(7/2), x)
```

3.592.8 Giac [F]

$$\int \frac{(a+b\sec(c+dx))^2}{\sec^{\frac{7}{2}}(c+dx)}\,dx=\int \frac{(b\sec(dx+c)+a)^2}{\sec(dx+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^2/sec(d*x+c)^(7/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^2/sec(d*x + c)^(7/2), x)
```

3.592.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(c+dx))^2}{\sec^{\frac{7}{2}}(c+dx)}\,dx=\int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^2}{\left(\frac{1}{\cos(c+d\,x)}\right)^{7/2}}\,dx$$

input
```
int((a + b/cos(c + d*x))^2/(1/cos(c + d*x))^(7/2),x)
```

output
```
int((a + b/cos(c + d*x))^2/(1/cos(c + d*x))^(7/2), x)
```

3.593 $\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^3\,dx$

3.593.1 Optimal result 4491
3.593.2 Mathematica [A] (verified) 4492
3.593.3 Rubi [A] (verified) 4492
3.593.4 Maple [B] (verified) 4497
3.593.5 Fricas [C] (verification not implemented) 4498
3.593.6 Sympy [F(-1)] 4498
3.593.7 Maxima [F] 4499
3.593.8 Giac [F] 4499
3.593.9 Mupad [F(-1)] 4499

3.593.1 Optimal result

Integrand size = 23, antiderivative size = 234

$$\begin{aligned}\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^3\,dx &= -\frac{2a(5a^2+9b^2)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} \\ &\quad+\frac{2b(21a^2+5b^2)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{21d} \\ &\quad+\frac{2a(5a^2+9b^2)\sqrt{\sec(c+dx)}\sin(c+dx)}{5d}+\frac{2b(21a^2+5b^2)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{21d} \\ &\quad+\frac{32ab^2\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{35d}+\frac{2b^2\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))\sin(c+dx)}{7d}\end{aligned}$$

output
```
2/21*b*(21*a^2+5*b^2)*sec(d*x+c)^(3/2)*sin(d*x+c)/d+32/35*a*b^2*sec(d*x+c)
^(5/2)*sin(d*x+c)/d+2/7*b^2*sec(d*x+c)^(5/2)*(a+b*sec(d*x+c))*sin(d*x+c)/d
+2/5*a*(5*a^2+9*b^2)*sin(d*x+c)*sec(d*x+c)^(1/2)/d-2/5*a*(5*a^2+9*b^2)*(co
s(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),
2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+2/21*b*(21*a^2+5*b^2)*(cos(1/
2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1
/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

3.593.2 Mathematica [A] (verified)

Time = 2.91 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.76

$$\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^3\,dx$$
$$= \frac{\sec^{\frac{7}{2}}(c+dx)\left(-168a(5a^2+9b^2)\cos^{\frac{7}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)+40b(21a^2+5b^2)\cos^{\frac{7}{2}}(c+dx)\,\text{EllipticF}\,(\right.}{}$$

input
```
Integrate[Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^3,x]
```

output
```
(Sec[c + d*x]^(7/2)*(-168*a*(5*a^2 + 9*b^2)*Cos[c + d*x]^(7/2)*EllipticE[(
c + d*x)/2, 2] + 40*b*(21*a^2 + 5*b^2)*Cos[c + d*x]^(7/2)*EllipticF[(c + d
*x)/2, 2] + 2*(210*a^2*b + 110*b^3 + 63*a*(5*a^2 + 13*b^2)*Cos[c + d*x] +
10*(21*a^2*b + 5*b^3)*Cos[2*(c + d*x)] + 105*a^3*Cos[3*(c + d*x)] + 189*a*
b^2*Cos[3*(c + d*x)])*Sin[c + d*x]))/(420*d)
```

3.593.3 Rubi [A] (verified)

Time = 1.31 (sec) , antiderivative size = 218, normalized size of antiderivative = 0.93, number of steps used = 18, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.783$, Rules used = {3042, 4329, 27, 3042, 4535, 3042, 4255, 3042, 4258, 3042, 3120, 4534, 3042, 4255, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^3\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^3dx$$

$$\Big\downarrow \; 4329$$

$$\frac{2}{7}\int \frac{1}{2}\sec^{\frac{3}{2}}(c+dx)\left(16ab^2\sec^2(c+dx)+b\left(21a^2+5b^2\right)\sec(c+dx)+a\left(7a^2+3b^2\right)\right)dx+\frac{2b^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))}{7d}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{7}\int \sec^{\frac{3}{2}}(c+dx)\left(16ab^2\sec^2(c+dx)+b\left(21a^2+5b^2\right)\sec(c+dx)+a\left(7a^2+3b^2\right)\right)dx+\frac{2b^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))}{7d}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{7}\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(16ab^2\csc\left(c+dx+\frac{\pi}{2}\right)^2+b\left(21a^2+5b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)+a\left(7a^2+3b^2\right)\right)dx+\frac{2b^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))}{7d}$$

$$\Big\downarrow \; 4535$$

$$\frac{1}{7}\left(b\left(21a^2+5b^2\right)\int \sec^{\frac{5}{2}}(c+dx)dx+\int \sec^{\frac{3}{2}}(c+dx)\left(16ab^2\sec^2(c+dx)+a\left(7a^2+3b^2\right)\right)dx\right)+\frac{2b^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))}{7d}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{7}\left(b\left(21a^2+5b^2\right)\int \csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}dx+\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(16ab^2\csc\left(c+dx+\frac{\pi}{2}\right)^2+a\left(7a^2+3b^2\right)\right)\right.$$
$$\frac{2b^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))}{7d}$$

$$\Big\downarrow \; 4255$$

$$\frac{1}{7}\left(\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(16ab^2\csc\left(c+dx+\frac{\pi}{2}\right)^2+a\left(7a^2+3b^2\right)\right)dx+b\left(21a^2+5b^2\right)\left(\frac{1}{3}\int \sqrt{\sec(c+dx)}dx\right.\right.$$
$$\frac{2b^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))}{7d}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{7}\left(\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(16ab^2\csc\left(c+dx+\frac{\pi}{2}\right)^2+a\left(7a^2+3b^2\right)\right)dx+b\left(21a^2+5b^2\right)\left(\frac{1}{3}\int \sqrt{\csc\left(c+dx+\right.}\right.\right.$$
$$\frac{2b^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))}{7d}$$

$$\Big\downarrow \; 4258$$

$$\frac{1}{7}\left(\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(16ab^2\csc\left(c+dx+\frac{\pi}{2}\right)^2+a\left(7a^2+3b^2\right)\right)dx+b\left(21a^2+5b^2\right)\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec}\right.\right.$$
$$\frac{2b^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))}{7d}$$

3.593. $\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^3\,dx$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{7}\left(\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(16ab^2\csc\left(c+dx+\frac{\pi}{2}\right)^2+a(7a^2+3b^2)\right)dx+b(21a^2+5b^2)\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec}\right.\right.$$
$$\frac{2b^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))}{7d}$$

$$\Big\downarrow \text{3120}$$

$$\frac{1}{7}\left(\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(16ab^2\csc\left(c+dx+\frac{\pi}{2}\right)^2+a(7a^2+3b^2)\right)dx+b(21a^2+5b^2)\left(\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c\right.\right.$$
$$\frac{2b^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))}{7d}$$

$$\Big\downarrow \text{4534}$$

$$\frac{1}{7}\left(\frac{7}{5}a(5a^2+9b^2)\int \sec^{\frac{3}{2}}(c+dx)dx+b(21a^2+5b^2)\left(\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{3d}\right.\right.$$
$$\frac{2b^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))}{7d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{7}\left(\frac{7}{5}a(5a^2+9b^2)\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}dx+b(21a^2+5b^2)\left(\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec}}{}\right.\right.$$
$$\frac{2b^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))}{7d}$$

$$\Big\downarrow \text{4255}$$

$$\frac{1}{7}\left(\frac{7}{5}a(5a^2+9b^2)\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\int\frac{1}{\sqrt{\sec(c+dx)}}dx\right)+b(21a^2+5b^2)\left(\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+}{3d}\right.\right.$$
$$\frac{2b^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))}{7d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{7}\left(\frac{7}{5}a(5a^2+9b^2)\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\int\frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)+b(21a^2+5b^2)\left(\frac{2\sin(c+dx)\sec^{\frac{3}{2}}}{3d}\right.\right.$$
$$\frac{2b^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))}{7d}$$

3.593. $\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^3\,dx$

$$\Big\downarrow \text{4258}$$

$$\frac{1}{7}\left(\frac{7}{5}a(5a^2+9b^2)\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\cos(c+dx)}dx\right)+b(21a^2+5b\right.$$
$$\frac{2b^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))}{7d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{7}\left(\frac{7}{5}a(5a^2+9b^2)\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx\right)+b(21a\right.$$
$$\frac{2b^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))}{7d}$$

$$\Big\downarrow \text{3119}$$

$$\frac{1}{7}\left(b(21a^2+5b^2)\left(\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}\right)+\frac{7}{5}a(5a^2\right.$$
$$\frac{2b^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))}{7d}$$

input
```
Int[Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^3,x]
```

output
```
(2*b^2*Sec[c + d*x]^(5/2)*(a + b*Sec[c + d*x])*Sin[c + d*x])/(7*d) + ((32*
a*b^2*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5*d) + (7*a*(5*a^2 + 9*b^2)*((-2*S
qrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (2*Sqr
t[Sec[c + d*x]]*Sin[c + d*x])/d))/5 + b*(21*a^2 + 5*b^2)*((2*Sqrt[Cos[c +
d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (2*Sec[c + d*x
]^(3/2)*Sin[c + d*x])/(3*d)))/7
```

3.593.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4329
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
2)*((d*Csc[e + f*x])^n/(f*(m + n - 1))), x] + Simp[1/(d*(m + n - 1))   Int[
(a + b*Csc[e + f*x])^(m - 3)*(d*Csc[e + f*x])^n*Simp[a^3*d*(m + n - 1) + a*
b^2*d*n + b*(b^2*d*(m + n - 2) + 3*a^2*d*(m + n - 1))*Csc[e + f*x] + a*b^2*
d*(3*m + 2*n - 4)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, n}, x
] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
 &&  !(IGtQ[n, 2] &&  !IntegerQ[m])
```

rule 4534
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Simp[(C*m + A*(m + 1))/(m + 1)   Int[(b*Csc[e + f*x])^m, x], x] /;
 FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

rule 4535
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Simp[B/b   Int[(b*Cs
c[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2)
, x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

3.593. $\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^3\,dx$

3.593.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 819 vs. $2(258) = 516$.

Time = 45.50 (sec) , antiderivative size = 820, normalized size of antiderivative = 3.50

method	result	size
default	Expression too large to display	820
parts	Expression too large to display	1008

input
```
int(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
-(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*a^3/sin(1/2*
d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d
*x+1/2*c)^2)^(1/2)*(2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-(sin(1/2*d*x
+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*
c)^2-1)^(1/2))+2*b^3*(-1/56*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+si
n(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^4-5/42*cos(1/2*d*x+1/
2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2
*c)^2-1/2)^2+5/21*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)
^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(
1/2*d*x+1/2*c),2^(1/2)))+6*a^2*b*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+
1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x
+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)
))+6/5*a*b^2/sin(1/2*d*x+1/2*c)^2/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1
/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)*(24*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2
*c)-12*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*
(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c
)^4*cos(1/2*d*x+1/2*c)+12*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d
*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2+8
*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-3*(sin(1/2*d*x+1/2*c)^2)^(1/2)...
```

3.593.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.15

$$\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^3\,dx =$$

$$-\frac{5\sqrt{2}(21i\,a^2b+5i\,b^3)\cos(dx+c)^3\,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c))+5\sqrt{2}(-21}{}$$

input
```
integrate(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
-1/105*(5*sqrt(2)*(21*I*a^2*b + 5*I*b^3)*cos(d*x + c)^3*weierstrassPInvers
e(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 5*sqrt(2)*(-21*I*a^2*b - 5*I*b^3
)*cos(d*x + c)^3*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))
 + 21*sqrt(2)*(5*I*a^3 + 9*I*a*b^2)*cos(d*x + c)^3*weierstrassZeta(-4, 0,
weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 21*sqrt(2)*(-
5*I*a^3 - 9*I*a*b^2)*cos(d*x + c)^3*weierstrassZeta(-4, 0, weierstrassPInv
erse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - 2*(63*a*b^2*cos(d*x + c) + 2
1*(5*a^3 + 9*a*b^2)*cos(d*x + c)^3 + 15*b^3 + 5*(21*a^2*b + 5*b^3)*cos(d*x
 + c)^2)*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c)^3)
```

3.593.6 Sympy [F(-1)]

Timed out.

$$\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^3\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**(3/2)*(a+b*sec(d*x+c))**3,x)
```

output
```
Timed out
```

3.593.7 Maxima [F]

$$\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^3\,dx = \int (b\sec(dx+c)+a)^3\sec(dx+c)^{\frac{3}{2}}\,dx$$

input
```
integrate(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^3*sec(d*x + c)^(3/2), x)
```

3.593.8 Giac [F]

$$\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^3\,dx = \int (b\sec(dx+c)+a)^3\sec(dx+c)^{\frac{3}{2}}\,dx$$

input
```
integrate(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^3*sec(d*x + c)^(3/2), x)
```

3.593.9 Mupad [F(-1)]

Timed out.

$$\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^3\,dx = \int \left(a+\frac{b}{\cos(c+d\,x)}\right)^3\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}dx$$

input
```
int((a + b/cos(c + d*x))^3*(1/cos(c + d*x))^(3/2),x)
```

output
```
int((a + b/cos(c + d*x))^3*(1/cos(c + d*x))^(3/2), x)
```

3.594 $\int \sqrt{\sec(c+dx)}(a+b\sec(c+dx))^3\,dx$

3.594.1 Optimal result 4500
3.594.2 Mathematica [A] (verified) 4501
3.594.3 Rubi [A] (verified) 4501
3.594.4 Maple [B] (verified) 4505
3.594.5 Fricas [C] (verification not implemented) 4506
3.594.6 Sympy [F(-1)] 4507
3.594.7 Maxima [F] 4507
3.594.8 Giac [F] 4508
3.594.9 Mupad [F(-1)] 4508

3.594.1 Optimal result

Integrand size = 23, antiderivative size = 189

$$\begin{aligned}\int \sqrt{\sec(c+dx)}(a+b\sec(c+dx))^3\,dx &= -\frac{6b(5a^2+b^2)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} \\ &\quad+\frac{2a(a^2+b^2)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{d} \\ &\quad+\frac{6b(5a^2+b^2)\sqrt{\sec(c+dx)}\sin(c+dx)}{5d}+\frac{8ab^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d} \\ &\quad+\frac{2b^2\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))\sin(c+dx)}{5d}\end{aligned}$$

output
```
8/5*a*b^2*sec(d*x+c)^(3/2)*sin(d*x+c)/d+2/5*b^2*sec(d*x+c)^(3/2)*(a+b*sec(
d*x+c))*sin(d*x+c)/d+6/5*b*(5*a^2+b^2)*sin(d*x+c)*sec(d*x+c)^(1/2)/d-6/5*b
*(5*a^2+b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin
(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+2*a*(a^2+b^2)
*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2
*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

3.594.2 Mathematica [A] (verified)

Time = 1.68 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.71

$$\int \sqrt{\sec(c+dx)}(a+b\sec(c+dx))^3\,dx$$
$$= \frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-3b(5a^2+b^2)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)+5a(a^2+b^2)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+\frac{b(5}{}\right.}{5d}$$

input
```
Integrate[Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^3,x]
```

output
```
(2*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(-3*b*(5*a^2 + b^2)*EllipticE[(c
+ d*x)/2, 2] + 5*a*(a^2 + b^2)*EllipticF[(c + d*x)/2, 2] + (b*(5*(3*a^2 +
b^2) + 10*a*b*Cos[c + d*x] + 3*(5*a^2 + b^2)*Cos[2*(c + d*x)])*Sin[c + d*x
])/(2*Cos[c + d*x]^(5/2))))/(5*d)
```

3.594.3 Rubi [A] (verified)

Time = 1.11 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.96, number of steps used = 16, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.696$, Rules used = {3042, 4329, 27, 3042, 4535, 3042, 4255, 3042, 4258, 3042, 3119, 4534, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\sec(c+dx)}(a+b\sec(c+dx))^3\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^3dx$$
$$\Big\downarrow \text{4329}$$
$$\frac{2}{5}\int \frac{1}{2}\sqrt{\sec(c+dx)}\left(12ab^2\sec^2(c+dx)+3b\left(5a^2+b^2\right)\sec(c+dx)+a\left(5a^2+b^2\right)\right)dx+\frac{2b^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))}{5d}$$
$$\Big\downarrow \text{27}$$

$$\frac{1}{5}\int \sqrt{\sec(c+dx)}\left(12ab^2\sec^2(c+dx)+3b\left(5a^2+b^2\right)\sec(c+dx)+a\left(5a^2+b^2\right)\right)dx+\frac{2b^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))}{5d}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{5}\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(12ab^2\csc\left(c+dx+\frac{\pi}{2}\right)^2+3b\left(5a^2+b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)+a\left(5a^2+b^2\right)\right)dx+\frac{2b^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))}{5d}$$

$$\Big\downarrow \; 4535$$

$$\frac{1}{5}\left(3b\left(5a^2+b^2\right)\int \sec^{\frac{3}{2}}(c+dx)dx+\int \sqrt{\sec(c+dx)}\left(12ab^2\sec^2(c+dx)+a\left(5a^2+b^2\right)\right)dx\right)+\frac{2b^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))}{5d}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{5}\left(3b\left(5a^2+b^2\right)\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}dx+\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(12ab^2\csc\left(c+dx+\frac{\pi}{2}\right)^2+a\left(5a^2+b^2\right)\right)dx\right.$$
$$\frac{2b^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))}{5d}$$

$$\Big\downarrow \; 4255$$

$$\frac{1}{5}\left(\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(12ab^2\csc\left(c+dx+\frac{\pi}{2}\right)^2+a\left(5a^2+b^2\right)\right)dx+3b\left(5a^2+b^2\right)\left(\frac{2\sin(c+dx)\sqrt{\sec(c+d}}{d}\right.\right.$$
$$\frac{2b^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))}{5d}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{5}\left(\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(12ab^2\csc\left(c+dx+\frac{\pi}{2}\right)^2+a\left(5a^2+b^2\right)\right)dx+3b\left(5a^2+b^2\right)\left(\frac{2\sin(c+dx)\sqrt{\sec(c+d}}{d}\right.\right.$$
$$\frac{2b^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))}{5d}$$

$$\Big\downarrow \; 4258$$

3.594. $\int \sqrt{\sec(c+dx)}(a+b\sec(c+dx))^3\,dx$

$$\frac{1}{5}\left(\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(12ab^2\csc\left(c+dx+\frac{\pi}{2}\right)^2+a\left(5a^2+b^2\right)\right)dx+3b\left(5a^2+b^2\right)\left(\frac{2\sin(c+dx)\sqrt{\sec(c+d}}{d}\right.\right.$$

$$\frac{2b^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))}{5d}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{5}\left(\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(12ab^2\csc\left(c+dx+\frac{\pi}{2}\right)^2+a\left(5a^2+b^2\right)\right)dx+3b\left(5a^2+b^2\right)\left(\frac{2\sin(c+dx)\sqrt{\sec(c+d}}{d}\right.\right.$$

$$\frac{2b^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))}{5d}$$

$$\Big\downarrow \; 3119$$

$$\frac{1}{5}\left(\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(12ab^2\csc\left(c+dx+\frac{\pi}{2}\right)^2+a\left(5a^2+b^2\right)\right)dx+3b\left(5a^2+b^2\right)\left(\frac{2\sin(c+dx)\sqrt{\sec(c+d}}{d}\right.\right.$$

$$\frac{2b^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))}{5d}$$

$$\Big\downarrow \; 4534$$

$$\frac{1}{5}\left(5a\left(a^2+b^2\right)\int \sqrt{\sec(c+dx)}dx+3b\left(5a^2+b^2\right)\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E(}{d}\right.\right.$$

$$\frac{2b^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))}{5d}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{5}\left(5a\left(a^2+b^2\right)\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+3b\left(5a^2+b^2\right)\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+}}{d}\right.\right.$$

$$\frac{2b^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))}{5d}$$

$$\Big\downarrow \; 4258$$

$$\frac{1}{5}\left(5a\left(a^2+b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx+3b\left(5a^2+b^2\right)\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{}}{}\right.\right.$$

$$\frac{2b^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))}{5d}$$

$$\Big\downarrow \; 3042$$

3.594. $\int \sqrt{\sec(c+dx)}(a+b\sec(c+dx))^3\,dx$

$$\frac{1}{5}\left(5a(a^2+b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+3b(5a^2+b^2)\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}\right.\right.$$
$$\frac{2b^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))}{5d}$$

$\Big\downarrow$ 3120

$$\frac{1}{5}\left(\frac{10a(a^2+b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+3b(5a^2+b^2)\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}\right.\right.$$
$$\frac{2b^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))}{5d}$$

input
```
Int[Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^3,x]
```

output
```
(2*b^2*Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x])*Sin[c + d*x])/(5*d) + ((10*
a*(a^2 + b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*
x]])/d + (8*a*b^2*Sec[c + d*x]^(3/2)*Sin[c + d*x])/d + 3*b*(5*a^2 + b^2)*(
(-2*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (
2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d))/5
```

3.594.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4329
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
2)*((d*Csc[e + f*x])^n/(f*(m + n - 1))), x] + Simp[1/(d*(m + n - 1))   Int[
(a + b*Csc[e + f*x])^(m - 3)*(d*Csc[e + f*x])^n*Simp[a^3*d*(m + n - 1) + a*
b^2*d*n + b*(b^2*d*(m + n - 2) + 3*a^2*d*(m + n - 1))*Csc[e + f*x] + a*b^2*
d*(3*m + 2*n - 4)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, n}, x
] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
 &&  !(IGtQ[n, 2] &&  !IntegerQ[m])
```

rule 4534
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Simp[(C*m + A*(m + 1))/(m + 1)   Int[(b*Csc[e + f*x])^m, x], x] /;
 FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

rule 4535
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Simp[B/b   Int[(b*Cs
c[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2)
, x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

3.594.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 710 vs. $2(219) = 438$.

Time = 37.53 (sec) , antiderivative size = 711, normalized size of antiderivative = 3.76

method	result	size
default	Expression too large to display	711
parts	Expression too large to display	898

input
```
int(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
-(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*a^3*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+2/
5*b^3/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)
^2-1)/sin(1/2*d*x+1/2*c)^2*(24*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(
1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^4*cos
(1/2*d*x+1/2*c)+12*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*
c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2+8*cos(1/
2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE
(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2))*(-2*sin(1/2
*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+6*a*b^2*(-1/6*cos(1/2*d*x+1/2*c)
*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^
2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2
)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d
*x+1/2*c),2^(1/2)))+6*a^2*b/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1
)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*cos(1/2*d*x+1/2*
c)*sin(1/2*d*x+1/2*c)^2-(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x
+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)))/sin(1/2*d*x+1/2*c)/(2*
cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

3.594.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.29

$$\int \sqrt{\sec(c+dx)}(a+b\sec(c+dx))^3\,dx =$$

$$-\frac{5\sqrt{2}(i\,a^3+i\,ab^2)\cos(dx+c)^2\,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c))+5\sqrt{2}(-i\,a^3-}{}$$

input
```
integrate(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
-1/5*(5*sqrt(2)*(I*a^3 + I*a*b^2)*cos(d*x + c)^2*weierstrassPInverse(-4, 0
, cos(d*x + c) + I*sin(d*x + c)) + 5*sqrt(2)*(-I*a^3 - I*a*b^2)*cos(d*x +
c)^2*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 3*sqrt(2)
*(5*I*a^2*b + I*b^3)*cos(d*x + c)^2*weierstrassZeta(-4, 0, weierstrassPInv
erse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 3*sqrt(2)*(-5*I*a^2*b - I*b^
3)*cos(d*x + c)^2*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*
x + c) - I*sin(d*x + c))) - 2*(5*a*b^2*cos(d*x + c) + b^3 + 3*(5*a^2*b + b
^3)*cos(d*x + c)^2)*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c)^2)
```

3.594.6 Sympy [F(-1)]

Timed out.

$$\int \sqrt{\sec(c+dx)}(a+b\sec(c+dx))^3\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**(1/2)*(a+b*sec(d*x+c))**3,x)
```

output
```
Timed out
```

3.594.7 Maxima [F]

$$\int \sqrt{\sec(c+dx)}(a+b\sec(c+dx))^3\,dx = \int (b\sec(dx+c)+a)^3\sqrt{\sec(dx+c)}\,dx$$

input
```
integrate(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^3*sqrt(sec(d*x + c)), x)
```

3.594.8 Giac [F]

$$\int \sqrt{\sec(c+dx)}(a+b\sec(c+dx))^3\,dx = \int (b\sec(dx+c)+a)^3\sqrt{\sec(dx+c)}\,dx$$

input
```
integrate(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^3*sqrt(sec(d*x + c)), x)
```

3.594.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{\sec(c+dx)}(a+b\sec(c+dx))^3\,dx = \int \left(a+\frac{b}{\cos(c+d\,x)}\right)^3\sqrt{\frac{1}{\cos(c+d\,x)}}\,dx$$

input
```
int((a + b/cos(c + d*x))^3*(1/cos(c + d*x))^(1/2),x)
```

output
```
int((a + b/cos(c + d*x))^3*(1/cos(c + d*x))^(1/2), x)
```

3.595 $\int \frac{(a+b\sec(c+dx))^3}{\sqrt{\sec(c+dx)}}\,dx$

3.595.1 Optimal result . . . 4509
3.595.2 Mathematica [A] (verified) . . . 4510
3.595.3 Rubi [A] (verified) . . . 4510
3.595.4 Maple [B] (verified) . . . 4514
3.595.5 Fricas [C] (verification not implemented) . . . 4515
3.595.6 Sympy [F] . . . 4516
3.595.7 Maxima [F] . . . 4516
3.595.8 Giac [F] . . . 4516
3.595.9 Mupad [F(-1)] . . . 4517

3.595.1 Optimal result

Integrand size = 23, antiderivative size = 158

$$\begin{aligned}\int \frac{(a+b\sec(c+dx))^3}{\sqrt{\sec(c+dx)}}\,dx &= \frac{2a(a^2-3b^2)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} \\ &\quad+\frac{2b(9a^2+b^2)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{3d} \\ &\quad+\frac{16ab^2\sqrt{\sec(c+dx)}\sin(c+dx)}{3d}+\frac{2b^2\sqrt{\sec(c+dx)}(a+b\sec(c+dx))\sin(c+dx)}{3d}\end{aligned}$$

output
```
16/3*a*b^2*sin(d*x+c)*sec(d*x+c)^(1/2)/d+2/3*b^2*(a+b*sec(d*x+c))*sin(d*x+
c)*sec(d*x+c)^(1/2)/d+2*a*(a^2-3*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2
*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x
+c)^(1/2)/d+2/3*b*(9*a^2+b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2
*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2
)/d
```

3.595.2 Mathematica [A] (verified)

Time = 1.04 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.67

$$\int \frac{(a+b\sec(c+dx))^3}{\sqrt{\sec(c+dx)}}\,dx$$
$$= \frac{\sec^{\frac{3}{2}}(c+dx)\left(6a(a^2-3b^2)\cos^{\frac{3}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)+b\left(2(9a^2+b^2)\cos^{\frac{3}{2}}(c+dx)\,\text{EllipticF}\left(\frac{1}{2}(c+d\right.\right.}{3d}$$

input
```
Integrate[(a + b*Sec[c + d*x])^3/Sqrt[Sec[c + d*x]],x]
```

output
```
(Sec[c + d*x]^(3/2)*(6*a*(a^2 - 3*b^2)*Cos[c + d*x]^(3/2)*EllipticE[(c + d
*x)/2, 2] + b*(2*(9*a^2 + b^2)*Cos[c + d*x]^(3/2)*EllipticF[(c + d*x)/2, 2
] + 2*b*(b + 9*a*Cos[c + d*x])*Sin[c + d*x])))/(3*d)
```

3.595.3 Rubi [A] (verified)

Time = 0.96 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.01, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.609$, Rules used = {3042, 4329, 27, 3042, 4535, 3042, 4258, 3042, 3120, 4534, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\sec(c+dx))^3}{\sqrt{\sec(c+dx)}}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^3}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$
$$\Big\downarrow \; 4329$$
$$\frac{2}{3}\int \frac{8ab^2\sec^2(c+dx)+b\left(9a^2+b^2\right)\sec(c+dx)+a\left(3a^2-b^2\right)}{2\sqrt{\sec(c+dx)}}dx+\frac{2b^2\sin(c+dx)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}{3d}$$
$$\Big\downarrow \; 27$$

$$\frac{1}{3}\int \frac{8ab^2\sec^2(c+dx)+b(9a^2+b^2)\sec(c+dx)+a(3a^2-b^2)}{\sqrt{\sec(c+dx)}}dx+\frac{2b^2\sin(c+dx)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}{3d}$$

$\Big\downarrow$ 3042

$$\frac{1}{3}\int \frac{8ab^2\csc\left(c+dx+\frac{\pi}{2}\right)^2+b(9a^2+b^2)\csc\left(c+dx+\frac{\pi}{2}\right)+a(3a^2-b^2)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2b^2\sin(c+dx)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}{3d}$$

$\Big\downarrow$ 4535

$$\frac{1}{3}\left(\int \frac{8ab^2\sec^2(c+dx)+a(3a^2-b^2)}{\sqrt{\sec(c+dx)}}dx+b(9a^2+b^2)\int \sqrt{\sec(c+dx)}dx\right)+\frac{2b^2\sin(c+dx)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}{3d}$$

$\Big\downarrow$ 3042

$$\frac{1}{3}\left(b(9a^2+b^2)\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\int \frac{8ab^2\csc\left(c+dx+\frac{\pi}{2}\right)^2+a(3a^2-b^2)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)+\frac{2b^2\sin(c+dx)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}{3d}$$

$\Big\downarrow$ 4258

$$\frac{1}{3}\left(\int \frac{8ab^2\csc\left(c+dx+\frac{\pi}{2}\right)^2+a(3a^2-b^2)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+b(9a^2+b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx\right)+\frac{2b^2\sin(c+dx)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}{3d}$$

$\Big\downarrow$ 3042

$$\frac{1}{3}\left(\int \frac{8ab^2\csc\left(c+dx+\frac{\pi}{2}\right)^2+a(3a^2-b^2)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+b(9a^2+b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx\right.$$
$$\frac{2b^2\sin(c+dx)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}{3d}$$

$\Big\downarrow$ 3120

3.595. $\int \frac{(a+b\sec(c+dx))^3}{\sqrt{\sec(c+dx)}}\,dx$

$$\frac{1}{3}\left(\int \frac{8ab^2\csc\left(c+dx+\frac{\pi}{2}\right)^2+a\left(3a^2-b^2\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2b\left(9a^2+b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}\right.$$
$$\frac{2b^2\sin(c+dx)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}{3d}$$

$\Big\downarrow$ 4534

$$\frac{1}{3}\left(3a\left(a^2-3b^2\right)\int \frac{1}{\sqrt{\sec(c+dx)}}dx+\frac{2b\left(9a^2+b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+\frac{16ab^2}{}\right.$$
$$\frac{2b^2\sin(c+dx)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}{3d}$$

$\Big\downarrow$ 3042

$$\frac{1}{3}\left(3a\left(a^2-3b^2\right)\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2b\left(9a^2+b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+\frac{1}{}\right.$$
$$\frac{2b^2\sin(c+dx)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}{3d}$$

$\Big\downarrow$ 4258

$$\frac{1}{3}\left(3a\left(a^2-3b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\cos(c+dx)}dx+\frac{2b\left(9a^2+b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{Ellipt}}{d}\right.$$
$$\frac{2b^2\sin(c+dx)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}{3d}$$

$\Big\downarrow$ 3042

$$\frac{1}{3}\left(3a\left(a^2-3b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2b\left(9a^2+b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{d}\right.$$
$$\frac{2b^2\sin(c+dx)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}{3d}$$

$\Big\downarrow$ 3119

$$\frac{1}{3}\left(\frac{2b\left(9a^2+b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+\frac{6a\left(a^2-3b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E}{d}\right.$$
$$\frac{2b^2\sin(c+dx)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}{3d}$$

3.595. $\int \frac{(a+b\sec(c+dx))^3}{\sqrt{\sec(c+dx)}}\,dx$

input
```
Int[(a + b*Sec[c + d*x])^3/Sqrt[Sec[c + d*x]],x]
```

output
```
(2*b^2*Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])*Sin[c + d*x])/(3*d) + ((6*a
*(a^2 - 3*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d
*x]])/d + (2*b*(9*a^2 + b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*
Sqrt[Sec[c + d*x]])/d + (16*a*b^2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d)/3
```

3.595.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4329
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
2)*((d*Csc[e + f*x])^n/(f*(m + n - 1))), x] + Simp[1/(d*(m + n - 1))   Int[
(a + b*Csc[e + f*x])^(m - 3)*(d*Csc[e + f*x])^n*Simp[a^3*d*(m + n - 1) + a*
b^2*d*n + b*(b^2*d*(m + n - 2) + 3*a^2*d*(m + n - 1))*Csc[e + f*x] + a*b^2*
d*(3*m + 2*n - 4)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, n}, x
] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
 &&  !(IGtQ[n, 2] &&  !IntegerQ[m])
```

rule 4534
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Simp[(C*m + A*(m + 1))/(m + 1)   Int[(b*Csc[e + f*x])^m, x], x] /;
 FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

rule 4535
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Simp[B/b   Int[(b*Cs
c[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2)
, x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

3.595.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 629 vs. $2(192) = 384$.

Time = 25.32 (sec) , antiderivative size = 630, normalized size of antiderivative = 3.99

method	result
default	$-\frac{2\sqrt{-\left(-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(36\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4 a\,b^2-18\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sqrt{2}\right.}{}$
parts	$\frac{2a^3\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1}\,\mathrm{EllipticE}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1}\,d}-\frac{2b^3\left(-2\sqrt{\frac{1}{2}-\frac{\cos(c}{}}\right.}{}$

input
```
int((a+b*sec(d*x+c))^3/sec(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(4*sin(1/2*
d*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)/sin(1/2*d*x+1/2*c)^3*(36*cos(1/2*d*
x+1/2*c)*sin(1/2*d*x+1/2*c)^4*a*b^2-18*(sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipt
icF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d
*x+1/2*c)^2*a^2*b-2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2
*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2*b^3+6*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1
/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2*a^3-18*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(
1/2)*sin(1/2*d*x+1/2*c)^2*a*b^2-18*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2
*a*b^2-2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2*b^3+9*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)
^(1/2)*a^2*b+(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(
1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*b^3-3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*a^3
+9*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*s
in(1/2*d*x+1/2*c)^2-1)^(1/2)*a*b^2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1
/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

3.595.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.35

$$\int \frac{(a+b\sec(c+dx))^3}{\sqrt{\sec(c+dx)}}\,dx$$
$$= \frac{\sqrt{2}(-9i\,a^2b-i\,b^3)\cos(dx+c)\,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c))+\sqrt{2}(9i\,a^2b+i\,b}{}$$

input
```
integrate((a+b*sec(d*x+c))^3/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
1/3*(sqrt(2)*(-9*I*a^2*b - I*b^3)*cos(d*x + c)*weierstrassPInverse(-4, 0,
cos(d*x + c) + I*sin(d*x + c)) + sqrt(2)*(9*I*a^2*b + I*b^3)*cos(d*x + c)*
weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 3*sqrt(2)*(-I*
a^3 + 3*I*a*b^2)*cos(d*x + c)*weierstrassZeta(-4, 0, weierstrassPInverse(-
4, 0, cos(d*x + c) + I*sin(d*x + c))) - 3*sqrt(2)*(I*a^3 - 3*I*a*b^2)*cos(
d*x + c)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) -
I*sin(d*x + c))) + 2*(9*a*b^2*cos(d*x + c) + b^3)*sin(d*x + c)/sqrt(cos(d*
x + c)))/(d*cos(d*x + c))
```

3.595.6 Sympy [F]

$$\int \frac{(a+b\sec(c+dx))^3}{\sqrt{\sec(c+dx)}}\,dx=\int \frac{(a+b\sec{(c+dx)})^3}{\sqrt{\sec{(c+dx)}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))**3/sec(d*x+c)**(1/2),x)
```

output
```
Integral((a + b*sec(c + d*x))**3/sqrt(sec(c + d*x)), x)
```

3.595.7 Maxima [F]

$$\int \frac{(a+b\sec(c+dx))^3}{\sqrt{\sec(c+dx)}}\,dx=\int \frac{(b\sec{(dx+c)}+a)^3}{\sqrt{\sec{(dx+c)}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^3/sec(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^3/sqrt(sec(d*x + c)), x)
```

3.595.8 Giac [F]

$$\int \frac{(a+b\sec(c+dx))^3}{\sqrt{\sec(c+dx)}}\,dx=\int \frac{(b\sec{(dx+c)}+a)^3}{\sqrt{\sec{(dx+c)}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^3/sec(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^3/sqrt(sec(d*x + c)), x)
```

3.595.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(c+dx))^3}{\sqrt{\sec(c+dx)}}\,dx=\int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^3}{\sqrt{\frac{1}{\cos(c+d\,x)}}}\,dx$$

input
```
int((a + b/cos(c + d*x))^3/(1/cos(c + d*x))^(1/2),x)
```

output
```
int((a + b/cos(c + d*x))^3/(1/cos(c + d*x))^(1/2), x)
```

3.596 $\int \frac{(a+b\sec(c+dx))^3}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

3.596.1 Optimal result 4518
3.596.2 Mathematica [A] (verified) 4519
3.596.3 Rubi [A] (verified) 4519
3.596.4 Maple [A] (verified) 4523
3.596.5 Fricas [C] (verification not implemented) 4524
3.596.6 Sympy [F] 4524
3.596.7 Maxima [F] 4525
3.596.8 Giac [F] 4525
3.596.9 Mupad [F(-1)] 4525

3.596.1 Optimal result

Integrand size = 23, antiderivative size = 166

$$\begin{aligned}\int \frac{(a+b\sec(c+dx))^3}{\sec^{\frac{3}{2}}(c+dx)}\,dx &= \frac{2b(3a^2-b^2)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} \\ &\quad + \frac{2a(a^2+9b^2)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{3d} \\ &\quad - \frac{2b(a^2-3b^2)\sqrt{\sec(c+dx)}\sin(c+dx)}{3d} + \frac{2a^2(a+b\sec(c+dx))\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\end{aligned}$$

output
```
2/3*a^2*(a+b*sec(d*x+c))*sin(d*x+c)/d/sec(d*x+c)^(1/2)-2/3*b*(a^2-3*b^2)*s
in(d*x+c)*sec(d*x+c)^(1/2)/d+2*b*(3*a^2-b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/
cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*
sec(d*x+c)^(1/2)/d+2/3*a*(a^2+9*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*
d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+
c)^(1/2)/d
```

3.596.2 Mathematica [A] (verified)

Time = 0.94 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.65

$$\int \frac{(a+b\sec(c+dx))^3}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \frac{\sqrt{\sec(c+dx)}\left(-6b(-3a^2+b^2)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)+2a(a^2+9b^2)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(\right.\right.}{3d}$$

input
```
Integrate[(a + b*Sec[c + d*x])^3/Sec[c + d*x]^(3/2),x]
```

output
```
(Sqrt[Sec[c + d*x]]*(-6*b*(-3*a^2 + b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c +
 d*x)/2, 2] + 2*a*(a^2 + 9*b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2,
2] + 2*(3*b^3 + a^3*Cos[c + d*x])*Sin[c + d*x]))/(3*d)
```

3.596.3 Rubi [A] (verified)

Time = 0.95 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.01, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.609$, Rules used = {3042, 4328, 27, 3042, 4535, 3042, 4258, 3042, 3120, 4534, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\sec(c+dx))^3}{\sec^{\frac{3}{2}}(c+dx)}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^3}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx$$

$$\Big\downarrow \; 4328$$

$$\frac{2}{3}\int \frac{8ba^2+\left(a^2+9b^2\right)\sec(c+dx)a-b\left(a^2-3b^2\right)\sec^2(c+dx)}{2\sqrt{\sec(c+dx)}}dx+\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))}{3d\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{3}\int \frac{8ba^2+\left(a^2+9b^2\right)\sec(c+dx)a-b\left(a^2-3b^2\right)\sec^2(c+dx)}{\sqrt{\sec(c+dx)}}dx+\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))}{3d\sqrt{\sec(c+dx)}}$$

$\Big\downarrow$ 3042

$$\frac{1}{3}\int \frac{8ba^2+\left(a^2+9b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)a-b\left(a^2-3b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))}{3d\sqrt{\sec(c+dx)}}$$

$\Big\downarrow$ 4535

$$\frac{1}{3}\left(\int \frac{8a^2b-b\left(a^2-3b^2\right)\sec^2(c+dx)}{\sqrt{\sec(c+dx)}}dx+a\left(a^2+9b^2\right)\int \sqrt{\sec(c+dx)}dx\right)+\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))}{3d\sqrt{\sec(c+dx)}}$$

$\Big\downarrow$ 3042

$$\frac{1}{3}\left(a\left(a^2+9b^2\right)\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\int \frac{8a^2b-b\left(a^2-3b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)+\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))}{3d\sqrt{\sec(c+dx)}}$$

$\Big\downarrow$ 4258

$$\frac{1}{3}\left(\int \frac{8a^2b-b\left(a^2-3b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+a\left(a^2+9b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx\right)+\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))}{3d\sqrt{\sec(c+dx)}}$$

$\Big\downarrow$ 3042

$$\frac{1}{3}\left(\int \frac{8a^2b-b\left(a^2-3b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+a\left(a^2+9b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx\right.$$
$$\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))}{3d\sqrt{\sec(c+dx)}}$$

$\Big\downarrow$ 3120

3.596. $\int \frac{(a+b\sec(c+dx))^3}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

$$\frac{1}{3}\left(\int \frac{8a^2b-b(a^2-3b^2)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a(a^2+9b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}\right.$$
$$\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))}{3d\sqrt{\sec(c+dx)}}$$

$\downarrow$ 4534

$$\frac{1}{3}\left(3b(3a^2-b^2)\int \frac{1}{\sqrt{\sec(c+dx)}}dx-\frac{2b(a^2-3b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{d}+\frac{2a(a^2+9b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(}}{d}\right.$$
$$\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))}{3d\sqrt{\sec(c+dx)}}$$

$\downarrow$ 3042

$$\frac{1}{3}\left(3b(3a^2-b^2)\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2b(a^2-3b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{d}+\frac{2a(a^2+9b^2)\sqrt{\cos(c+dx)}}{}\right.$$
$$\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))}{3d\sqrt{\sec(c+dx)}}$$

$\downarrow$ 4258

$$\frac{1}{3}\left(3b(3a^2-b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\cos(c+dx)}dx-\frac{2b(a^2-3b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{d}+\frac{2a(a^2}{}\right.$$
$$\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))}{3d\sqrt{\sec(c+dx)}}$$

$\downarrow$ 3042

$$\frac{1}{3}\left(3b(3a^2-b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx-\frac{2b(a^2-3b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{d}+\right.$$
$$\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))}{3d\sqrt{\sec(c+dx)}}$$

$\downarrow$ 3119

$$\frac{1}{3}\left(-\frac{2b(a^2-3b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{d}+\frac{2a(a^2+9b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+\right.$$
$$\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))}{3d\sqrt{\sec(c+dx)}}$$

3.596. $\int \frac{(a+b\sec(c+dx))^3}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

input
```
Int[(a + b*Sec[c + d*x])^3/Sec[c + d*x]^(3/2),x]
```

output
```
(2*a^2*(a + b*Sec[c + d*x])*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]]) + ((6*b
*(3*a^2 - b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d
*x]])/d + (2*a*(a^2 + 9*b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*
Sqrt[Sec[c + d*x]])/d - (2*b*(a^2 - 3*b^2)*Sqrt[Sec[c + d*x]]*Sin[c + d*x]
)/d)/3
```

3.596.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4328
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)*
((d*Csc[e + f*x])^n/(f*n)), x] - Simp[1/(d*n)   Int[(a + b*Csc[e + f*x])^(m
 - 3)*(d*Csc[e + f*x])^(n + 1)*Simp[a^2*b*(m - 2*n - 2) - a*(3*b^2*n + a^2*
(n + 1))*Csc[e + f*x] - b*(b^2*n + a^2*(m + n - 1))*Csc[e + f*x]^2, x], x],
 x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && ((Int
egerQ[m] && LtQ[n, -1]) || (IntegersQ[m + 1/2, 2*n] && LeQ[n, -1]))
```

3.596. $\int \frac{(a+b\sec(c+dx))^3}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

rule 4534
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Simp[(C*m + A*(m + 1))/(m + 1)   Int[(b*Csc[e + f*x])^m, x], x] /;
 FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

rule 4535
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Simp[B/b   Int[(b*Cs
c[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2)
, x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

3.596.4 Maple [A] (verified)

Time = 24.09 (sec) , antiderivative size = 303, normalized size of antiderivative = 1.83

method	result
default	$-\frac{2\left(4a^3\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2a^3-6\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2b^3+a^3\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\sin\left(\frac{dx}{2}+\right.}\right.}{}$
parts	$-\frac{2a^3\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(4\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\sin\left(\frac{dx}{2}\right.}\right.}{3\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1}\,d}$

input
```
int((a+b*sec(d*x+c))^3/sec(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*(4*a^3*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4-2*cos(1/2*d*x+1/2*c)*s
in(1/2*d*x+1/2*c)^2*a^3-6*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2*b^3+a^3*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(co
s(1/2*d*x+1/2*c),2^(1/2))+9*a*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-9*a^2*b*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*
d*x+1/2*c),2^(1/2))+3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2
-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b^3)/sin(1/2*d*x+1/2*c)/(2
*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

3.596.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.10

$$\int \frac{(a+b\sec(c+dx))^3}{\sec^{\frac{3}{2}}(c+dx)}\,dx$$
$$= \frac{\sqrt{2}(-i\,a^3-9i\,ab^2)\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c))+\sqrt{2}(i\,a^3+9i\,ab^2)\text{weierstras}}{}$$

input
```
integrate((a+b*sec(d*x+c))^3/sec(d*x+c)^(3/2),x, algorithm="fricas")
```

output
```
1/3*(sqrt(2)*(-I*a^3 - 9*I*a*b^2)*weierstrassPInverse(-4, 0, cos(d*x + c)
+ I*sin(d*x + c)) + sqrt(2)*(I*a^3 + 9*I*a*b^2)*weierstrassPInverse(-4, 0,
 cos(d*x + c) - I*sin(d*x + c)) - 3*sqrt(2)*(-3*I*a^2*b + I*b^3)*weierstra
ssZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) -
 3*sqrt(2)*(3*I*a^2*b - I*b^3)*weierstrassZeta(-4, 0, weierstrassPInverse(
-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*(a^3*cos(d*x + c) + 3*b^3)*sin(
d*x + c)/sqrt(cos(d*x + c)))/d
```

3.596.6 Sympy [F]

$$\int \frac{(a+b\sec(c+dx))^3}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(a+b\sec(c+dx))^3}{\sec^{\frac{3}{2}}(c+dx)}\,dx$$

input
```
integrate((a+b*sec(d*x+c))**3/sec(d*x+c)**(3/2),x)
```

output
```
Integral((a + b*sec(c + d*x))**3/sec(c + d*x)**(3/2), x)
```

3.596.7 Maxima [F]

$$\int \frac{(a+b\sec(c+dx))^3}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(b\sec(dx+c)+a)^3}{\sec(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^3/sec(d*x+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^3/sec(d*x + c)^(3/2), x)
```

3.596.8 Giac [F]

$$\int \frac{(a+b\sec(c+dx))^3}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(b\sec(dx+c)+a)^3}{\sec(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^3/sec(d*x+c)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^3/sec(d*x + c)^(3/2), x)
```

3.596.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(c+dx))^3}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^3}{\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

input
```
int((a + b/cos(c + d*x))^3/(1/cos(c + d*x))^(3/2),x)
```

output
```
int((a + b/cos(c + d*x))^3/(1/cos(c + d*x))^(3/2), x)
```

3.597 $\int \frac{(a+b\sec(c+dx))^3}{\sec^{\frac{5}{2}}(c+dx)}\,dx$

3.597.1 Optimal result 4526
3.597.2 Mathematica [A] (verified) 4527
3.597.3 Rubi [A] (verified) 4527
3.597.4 Maple [B] (verified) 4531
3.597.5 Fricas [C] (verification not implemented) 4532
3.597.6 Sympy [F] 4532
3.597.7 Maxima [F] 4533
3.597.8 Giac [F] 4533
3.597.9 Mupad [F(-1)] 4533

3.597.1 Optimal result

Integrand size = 23, antiderivative size = 156

$$\begin{aligned}\int \frac{(a+b\sec(c+dx))^3}{\sec^{\frac{5}{2}}(c+dx)}\,dx &= \frac{6a(a^2+5b^2)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} \\ &\quad+\frac{2b(a^2+b^2)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{d} \\ &\quad+\frac{8a^2b\sin(c+dx)}{5d\sqrt{\sec(c+dx)}}+\frac{2a^2(a+b\sec(c+dx))\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}\end{aligned}$$

output
```
2/5*a^2*(a+b*sec(d*x+c))*sin(d*x+c)/d/sec(d*x+c)^(3/2)+8/5*a^2*b*sin(d*x+c
)/d/sec(d*x+c)^(1/2)+6/5*a*(a^2+5*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/
2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*
x+c)^(1/2)/d+2*b*(a^2+b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)
*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

3.597.2 Mathematica [A] (verified)

Time = 0.85 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.68

$$\int \frac{(a+b\sec(c+dx))^3}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \frac{\sqrt{\sec(c+dx)}\left(6a(a^2+5b^2)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)+10b(a^2+b^2)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+\right.\right.}{5d}$$

input
```
Integrate[(a + b*Sec[c + d*x])^3/Sec[c + d*x]^(5/2),x]
```

output
```
(Sqrt[Sec[c + d*x]]*(6*a*(a^2 + 5*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d
*x)/2, 2] + 10*b*(a^2 + b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]
+ a^2*(5*b + a*Cos[c + d*x])*Sin[2*(c + d*x)]))/(5*d)
```

3.597.3 Rubi [A] (verified)

Time = 0.96 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.01, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.609$, Rules used = {3042, 4328, 27, 3042, 4535, 3042, 4258, 3042, 3119, 4533, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\sec(c+dx))^3}{\sec^{\frac{5}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^3}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx$$

$$\Big\downarrow \text{4328}$$

$$\frac{2}{5}\int \frac{12ba^2+3\left(a^2+5b^2\right)\sec(c+dx)a+b\left(a^2+5b^2\right)\sec^2(c+dx)}{2\sec^{\frac{3}{2}}(c+dx)}dx+\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{5}\int \frac{12ba^2+3\left(a^2+5b^2\right)\sec(c+dx)a+b\left(a^2+5b^2\right)\sec^2(c+dx)}{\sec^{\frac{3}{2}}(c+dx)}dx+\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$\Big\downarrow$ 3042

$$\frac{1}{5}\int \frac{12ba^2+3\left(a^2+5b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)a+b\left(a^2+5b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$\Big\downarrow$ 4535

$$\frac{1}{5}\left(\int \frac{12ba^2+b\left(a^2+5b^2\right)\sec^2(c+dx)}{\sec^{\frac{3}{2}}(c+dx)}dx+3a\left(a^2+5b^2\right)\int \frac{1}{\sqrt{\sec(c+dx)}}dx\right)+\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$\Big\downarrow$ 3042

$$\frac{1}{5}\left(3a\left(a^2+5b^2\right)\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\int \frac{12ba^2+b\left(a^2+5b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx\right)+\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$\Big\downarrow$ 4258

$$\frac{1}{5}\left(\int \frac{12ba^2+b\left(a^2+5b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+3a\left(a^2+5b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\cos(c+dx)}dx\right)+\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$\Big\downarrow$ 3042

$$\frac{1}{5}\left(\int \frac{12ba^2+b\left(a^2+5b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+3a\left(a^2+5b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\right.$$

$$\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$\Big\downarrow$ 3119

3.597. $\int \frac{(a+b\sec(c+dx))^3}{\sec^{\frac{5}{2}}(c+dx)}\,dx$

$$\frac{1}{5}\left(\int \frac{12ba^2+b(a^2+5b^2)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+\frac{6a(a^2+5b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)+\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$\downarrow$ 4533

$$\frac{1}{5}\left(5b(a^2+b^2)\int\sqrt{\sec(c+dx)}dx+\frac{6a(a^2+5b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{8a^2b\sin(c+dx)}{d\sqrt{\sec(c+dx)}}\right.$$
$$\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$\downarrow$ 3042

$$\frac{1}{5}\left(5b(a^2+b^2)\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{6a(a^2+5b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{8a^2b\sin(c}{d\sqrt{\sec(c}}\right.$$
$$\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$\downarrow$ 4258

$$\frac{1}{5}\left(5b(a^2+b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{6a(a^2+5b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c\right.}{d}\right.$$
$$\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$\downarrow$ 3042

$$\frac{1}{5}\left(5b(a^2+b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{6a(a^2+5b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E}{d}\right.$$
$$\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$\downarrow$ 3120

$$\frac{1}{5}\left(\frac{10b(a^2+b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+\frac{6a(a^2+5b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E}{d}\right.$$
$$\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))}{5d\sec^{\frac{3}{2}}(c+dx)}$$

3.597. $\int \frac{(a+b\sec(c+dx))^3}{\sec^{\frac{5}{2}}(c+dx)}\,dx$

input
```
Int[(a + b*Sec[c + d*x])^3/Sec[c + d*x]^(5/2),x]
```

output
```
(2*a^2*(a + b*Sec[c + d*x])*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)) + ((6*a
*(a^2 + 5*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d
*x]])/d + (10*b*(a^2 + b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*S
qrt[Sec[c + d*x]])/d + (8*a^2*b*Sin[c + d*x])/(d*Sqrt[Sec[c + d*x]]))/5
```

3.597.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4328
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)*
((d*Csc[e + f*x])^n/(f*n)), x] - Simp[1/(d*n)   Int[(a + b*Csc[e + f*x])^(m
 - 3)*(d*Csc[e + f*x])^(n + 1)*Simp[a^2*b*(m - 2*n - 2) - a*(3*b^2*n + a^2*
(n + 1))*Csc[e + f*x] - b*(b^2*n + a^2*(m + n - 1))*Csc[e + f*x]^2, x], x],
 x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && ((Int
egerQ[m] && LtQ[n, -1]) || (IntegersQ[m + 1/2, 2*n] && LeQ[n, -1]))
```

3.597. $\int \frac{(a+b\sec(c+dx))^3}{\sec^{\frac{5}{2}}(c+dx)}\,dx$

rule 4533
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] +
Simp[(C*m + A*(m + 1))/(b^2*m)   Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fr
eeQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

rule 4535
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Simp[B/b   Int[(b*Cs
c[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2)
, x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

3.597.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 411 vs. $2(190) = 380$.

Time = 22.57 (sec) , antiderivative size = 412, normalized size of antiderivative = 2.64

method	result
default	$-\frac{2\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(-8a^3\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^6+8a^3\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+20a^2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4\cos\right.}{}$
parts	$-\frac{2a^3\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(-8\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^6\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+8\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}{5\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right.}}$

input
```
int((a+b*sec(d*x+c))^3/sec(d*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/5*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-8*a^3*cos(1
/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+8*a^3*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/
2*c)^4+20*a^2*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)*b-2*cos(1/2*d*x+1/2*
c)*sin(1/2*d*x+1/2*c)^2*a^3-10*a^2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)
*b+5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2
*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*a^2*b+5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipt
icF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*b^3-3*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2
*d*x+1/2*c)^2-1)^(1/2)*a^3-15*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1
/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*a*b^2)/(-2*sin(1/2
*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*
x+1/2*c)^2-1)^(1/2)/d
```

3.597.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.24

$$\int \frac{(a+b\sec(c+dx))^3}{\sec^{\frac{5}{2}}(c+dx)}\,dx =$$

$$-\frac{5\sqrt{2}(i\,a^2b+i\,b^3)\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c))+5\sqrt{2}(-i\,a^2b-i\,b^3)\text{weierst}}{}$$

input
```
integrate((a+b*sec(d*x+c))^3/sec(d*x+c)^(5/2),x, algorithm="fricas")
```

output
```
-1/5*(5*sqrt(2)*(I*a^2*b + I*b^3)*weierstrassPInverse(-4, 0, cos(d*x + c)
+ I*sin(d*x + c)) + 5*sqrt(2)*(-I*a^2*b - I*b^3)*weierstrassPInverse(-4, 0
, cos(d*x + c) - I*sin(d*x + c)) + 3*sqrt(2)*(-I*a^3 - 5*I*a*b^2)*weierstr
assZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)))
+ 3*sqrt(2)*(I*a^3 + 5*I*a*b^2)*weierstrassZeta(-4, 0, weierstrassPInverse
(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - 2*(a^3*cos(d*x + c)^2 + 5*a^2*b*
cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/d
```

3.597.6 Sympy [F]

$$\int \frac{(a+b\sec(c+dx))^3}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{(a+b\sec(c+dx))^3}{\sec^{\frac{5}{2}}(c+dx)}\,dx$$

input
```
integrate((a+b*sec(d*x+c))**3/sec(d*x+c)**(5/2),x)
```

output
```
Integral((a + b*sec(c + d*x))**3/sec(c + d*x)**(5/2), x)
```

3.597.7 Maxima [F]

$$\int \frac{(a+b\sec(c+dx))^3}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{(b\sec(dx+c)+a)^3}{\sec(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^3/sec(d*x+c)^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^3/sec(d*x + c)^(5/2), x)
```

3.597.8 Giac [F]

$$\int \frac{(a+b\sec(c+dx))^3}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{(b\sec(dx+c)+a)^3}{\sec(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^3/sec(d*x+c)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^3/sec(d*x + c)^(5/2), x)
```

3.597.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(c+dx))^3}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^3}{\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

input
```
int((a + b/cos(c + d*x))^3/(1/cos(c + d*x))^(5/2),x)
```

output
```
int((a + b/cos(c + d*x))^3/(1/cos(c + d*x))^(5/2), x)
```

3.598 $\int \frac{(a+b\sec(c+dx))^3}{\sec^{\frac{7}{2}}(c+dx)}\,dx$

3.598.1 Optimal result 4534
3.598.2 Mathematica [A] (verified) 4535
3.598.3 Rubi [A] (verified) 4535
3.598.4 Maple [A] (verified) 4539
3.598.5 Fricas [C] (verification not implemented) 4540
3.598.6 Sympy [F] 4541
3.598.7 Maxima [F] 4541
3.598.8 Giac [F] 4541
3.598.9 Mupad [F(-1)] 4542

3.598.1 Optimal result

Integrand size = 23, antiderivative size = 199

$$\begin{aligned}\int \frac{(a+b\sec(c+dx))^3}{\sec^{\frac{7}{2}}(c+dx)}\,dx &= \frac{2b(9a^2+5b^2)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} \\ &\quad + \frac{2a(5a^2+21b^2)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{21d} \\ &\quad + \frac{32a^2b\sin(c+dx)}{35d\sec^{\frac{3}{2}}(c+dx)} + \frac{2a(5a^2+21b^2)\sin(c+dx)}{21d\sqrt{\sec(c+dx)}} + \frac{2a^2(a+b\sec(c+dx))\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}\end{aligned}$$

output
```
32/35*a^2*b*sin(d*x+c)/d/sec(d*x+c)^(3/2)+2/7*a^2*(a+b*sec(d*x+c))*sin(d*x
+c)/d/sec(d*x+c)^(5/2)+2/21*a*(5*a^2+21*b^2)*sin(d*x+c)/d/sec(d*x+c)^(1/2)
+2/5*b*(9*a^2+5*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellip
ticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+2/21*
a*(5*a^2+21*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF
(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

3.598.2 Mathematica [A] (verified)

Time = 1.27 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.66

$$\int \frac{(a+b\sec(c+dx))^3}{\sec^{\frac{7}{2}}(c+dx)}\,dx$$
$$= \frac{\sqrt{\sec(c+dx)}\left(84b(9a^2+5b^2)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)+20a(5a^2+21b^2)\sqrt{\cos(c+dx)}\,\text{EllipticF}\,(\right.}{210d}$$

input
```
Integrate[(a + b*Sec[c + d*x])^3/Sec[c + d*x]^(7/2),x]
```

output
```
(Sqrt[Sec[c + d*x]]*(84*b*(9*a^2 + 5*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c
+ d*x)/2, 2] + 20*a*(5*a^2 + 21*b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x
)/2, 2] + a*(65*a^2 + 210*b^2 + 126*a*b*Cos[c + d*x] + 15*a^2*Cos[2*(c + d
*x)])*Sin[2*(c + d*x)]))/(210*d)
```

3.598.3 Rubi [A] (verified)

Time = 1.14 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.97, number of steps used = 16, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.696$, Rules used = {3042, 4328, 27, 3042, 4535, 3042, 4256, 3042, 4258, 3042, 3120, 4533, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\sec(c+dx))^3}{\sec^{\frac{7}{2}}(c+dx)}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^3}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx$$
$$\Big\downarrow \; 4328$$
$$\frac{2}{7}\int \frac{16ba^2+\left(5a^2+21b^2\right)\sec(c+dx)a+b\left(3a^2+7b^2\right)\sec^2(c+dx)}{2\sec^{\frac{5}{2}}(c+dx)}dx+\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))}{7d\sec^{\frac{5}{2}}(c+dx)}$$
$$\Big\downarrow \; 27$$

$$\frac{1}{7}\int \frac{16ba^2+\left(5a^2+21b^2\right)\sec(c+dx)a+b\left(3a^2+7b^2\right)\sec^2(c+dx)}{\sec^{\frac{5}{2}}(c+dx)}dx+\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$\Big\downarrow$ 3042

$$\frac{1}{7}\int \frac{16ba^2+\left(5a^2+21b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)a+b\left(3a^2+7b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$\Big\downarrow$ 4535

$$\frac{1}{7}\left(a\left(5a^2+21b^2\right)\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}dx+\int \frac{16ba^2+b\left(3a^2+7b^2\right)\sec^2(c+dx)}{\sec^{\frac{5}{2}}(c+dx)}dx\right)+\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$\Big\downarrow$ 3042

$$\frac{1}{7}\left(a\left(5a^2+21b^2\right)\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+\int \frac{16ba^2+b\left(3a^2+7b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx\right)+\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$\Big\downarrow$ 4256

$$\frac{1}{7}\left(\int \frac{16ba^2+b\left(3a^2+7b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+a\left(5a^2+21b^2\right)\left(\frac{1}{3}\int \sqrt{\sec(c+dx)}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)\right)+\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$\Big\downarrow$ 3042

$$\frac{1}{7}\left(\int \frac{16ba^2+b\left(3a^2+7b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+a\left(5a^2+21b^2\right)\left(\frac{1}{3}\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)\right)+\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$\Big\downarrow$ 4258

3.598. $\int \frac{(a+b\sec(c+dx))^3}{\sec^{\frac{7}{2}}(c+dx)}\,dx$

$$\frac{1}{7}\left(\int \frac{16ba^2+b(3a^2+7b^2)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+a(5a^2+21b^2)\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}\right.\right.$$
$$\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$\downarrow$ 3042

$$\frac{1}{7}\left(\int \frac{16ba^2+b(3a^2+7b^2)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+a(5a^2+21b^2)\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin(c+dx}}\right.\right.$$
$$\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$\downarrow$ 3120

$$\frac{1}{7}\left(\int \frac{16ba^2+b(3a^2+7b^2)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+a(5a^2+21b^2)\left(\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+d}}{3}\right.\right.$$
$$\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$\downarrow$ 4533

$$\frac{1}{7}\left(\frac{7}{5}b(9a^2+5b^2)\int\frac{1}{\sqrt{\sec(c+dx)}}dx+a(5a^2+21b^2)\left(\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{Elliptic}}{3d}\right.\right.$$
$$\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$\downarrow$ 3042

$$\frac{1}{7}\left(\frac{7}{5}b(9a^2+5b^2)\int\frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+a(5a^2+21b^2)\left(\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{El}}{3d}\right.\right.$$
$$\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$\downarrow$ 4258

$$\frac{1}{7}\left(\frac{7}{5}b(9a^2+5b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\cos(c+dx)}dx+a(5a^2+21b^2)\left(\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2\sqrt{\cos(c}}{}\right.\right.$$
$$\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))}{7d\sec^{\frac{5}{2}}(c+dx)}$$

3.598. $\int \frac{(a+b\sec(c+dx))^3}{\sec^{\frac{7}{2}}(c+dx)}\,dx$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{7}\left(\frac{7}{5}b\left(9a^2+5b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+a\left(5a^2+21b^2\right)\left(\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2\sqrt{}}{}\right.\right.$$
$$\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{3119}$$

$$\frac{1}{7}\left(\frac{14b\left(9a^2+5b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+a\left(5a^2+21b^2\right)\left(\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2\sqrt{\cos(c+}}{}\right.\right.$$
$$\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))}{7d\sec^{\frac{5}{2}}(c+dx)}$$

input
```
Int[(a + b*Sec[c + d*x])^3/Sec[c + d*x]^(7/2),x]
```

output
```
(2*a^2*(a + b*Sec[c + d*x])*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/2)) + ((14*
b*(9*a^2 + 5*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
+ d*x]])/(5*d) + (32*a^2*b*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)) + a*(5*a
^2 + 21*b^2)*((2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c +
 d*x]])/(3*d) + (2*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])))/7
```

3.598.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4328
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)*
((d*Csc[e + f*x])^n/(f*n)), x] - Simp[1/(d*n)   Int[(a + b*Csc[e + f*x])^(m
 - 3)*(d*Csc[e + f*x])^(n + 1)*Simp[a^2*b*(m - 2*n - 2) - a*(3*b^2*n + a^2*
(n + 1))*Csc[e + f*x] - b*(b^2*n + a^2*(m + n - 1))*Csc[e + f*x]^2, x], x],
 x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && ((Int
egerQ[m] && LtQ[n, -1]) || (IntegersQ[m + 1/2, 2*n] && LeQ[n, -1]))
```

rule 4533
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] +
Simp[(C*m + A*(m + 1))/(b^2*m)   Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fr
eeQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

rule 4535
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Simp[B/b   Int[(b*Cs
c[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2)
, x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

3.598.4 Maple [A] (verified)

Time = 28.98 (sec) , antiderivative size = 421, normalized size of antiderivative = 2.12

method	result
default	$-\frac{2\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(240a^3\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^8+(-360a^3-504a^2b)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^6\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+(280a^3}{}$
parts	Expression too large to display

input
```
int((a+b*sec(d*x+c))^3/sec(d*x+c)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-2/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(240*a^3*co
s(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+(-360*a^3-504*a^2*b)*sin(1/2*d*x+1/2
*c)^6*cos(1/2*d*x+1/2*c)+(280*a^3+504*a^2*b+420*a*b^2)*sin(1/2*d*x+1/2*c)^
4*cos(1/2*d*x+1/2*c)+(-80*a^3-126*a^2*b-210*a*b^2)*sin(1/2*d*x+1/2*c)^2*co
s(1/2*d*x+1/2*c)+25*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)
^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+105*a*b^2*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*
c),2^(1/2))-189*a^2*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2
-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-105*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/
2))*b^3)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+
1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

3.598.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.09

$$\int \frac{(a+b\sec(c+dx))^3}{\sec^{\frac{7}{2}}(c+dx)}\,dx = -\frac{5\sqrt{2}(5i\,a^3+21i\,ab^2)\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c))+5\sqrt{2}(-5i\,a^3-21i\,ab^2)}{}$$

input
```
integrate((a+b*sec(d*x+c))^3/sec(d*x+c)^(7/2),x, algorithm="fricas")
```

output
```
-1/105*(5*sqrt(2)*(5*I*a^3 + 21*I*a*b^2)*weierstrassPInverse(-4, 0, cos(d*
x + c) + I*sin(d*x + c)) + 5*sqrt(2)*(-5*I*a^3 - 21*I*a*b^2)*weierstrassPI
nverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 21*sqrt(2)*(-9*I*a^2*b - 5*
I*b^3)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*
sin(d*x + c))) + 21*sqrt(2)*(9*I*a^2*b + 5*I*b^3)*weierstrassZeta(-4, 0, w
eierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - 2*(15*a^3*cos(
d*x + c)^3 + 63*a^2*b*cos(d*x + c)^2 + 5*(5*a^3 + 21*a*b^2)*cos(d*x + c))*
sin(d*x + c)/sqrt(cos(d*x + c)))/d
```

3.598.6 Sympy [F]

$$\int \frac{(a+b\sec(c+dx))^3}{\sec^{\frac{7}{2}}(c+dx)}\,dx=\int \frac{(a+b\sec{(c+dx)})^3}{\sec^{\frac{7}{2}}{(c+dx)}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))**3/sec(d*x+c)**(7/2),x)
```

output
```
Integral((a + b*sec(c + d*x))**3/sec(c + d*x)**(7/2), x)
```

3.598.7 Maxima [F]

$$\int \frac{(a+b\sec(c+dx))^3}{\sec^{\frac{7}{2}}(c+dx)}\,dx=\int \frac{(b\sec{(dx+c)}+a)^3}{\sec{(dx+c)}^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^3/sec(d*x+c)^(7/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^3/sec(d*x + c)^(7/2), x)
```

3.598.8 Giac [F]

$$\int \frac{(a+b\sec(c+dx))^3}{\sec^{\frac{7}{2}}(c+dx)}\,dx=\int \frac{(b\sec{(dx+c)}+a)^3}{\sec{(dx+c)}^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^3/sec(d*x+c)^(7/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^3/sec(d*x + c)^(7/2), x)
```

3.598.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(c+dx))^3}{\sec^{\frac{7}{2}}(c+dx)}\,dx=\int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^3}{\left(\frac{1}{\cos(c+d\,x)}\right)^{7/2}}\,dx$$

input
```
int((a + b/cos(c + d*x))^3/(1/cos(c + d*x))^(7/2),x)
```

output
```
int((a + b/cos(c + d*x))^3/(1/cos(c + d*x))^(7/2), x)
```

3.599 $\int \frac{(a+b\sec(c+dx))^3}{\sec^{\frac{9}{2}}(c+dx)}\,dx$

3.599.1 Optimal result 4543
3.599.2 Mathematica [A] (verified) 4544
3.599.3 Rubi [A] (verified) 4544
3.599.4 Maple [A] (verified) 4549
3.599.5 Fricas [C] (verification not implemented) 4550
3.599.6 Sympy [F(-1)] 4550
3.599.7 Maxima [F] 4551
3.599.8 Giac [F] 4551
3.599.9 Mupad [F(-1)] 4551

3.599.1 Optimal result

Integrand size = 23, antiderivative size = 234

$$\begin{aligned}\int \frac{(a+b\sec(c+dx))^3}{\sec^{\frac{9}{2}}(c+dx)}\,dx &= \frac{2a(7a^2+27b^2)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{15d} \\ &\quad+\frac{2b(15a^2+7b^2)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{21d} \\ &\quad+\frac{40a^2b\sin(c+dx)}{63d\sec^{\frac{5}{2}}(c+dx)}+\frac{2a(7a^2+27b^2)\sin(c+dx)}{45d\sec^{\frac{3}{2}}(c+dx)} \\ &\quad+\frac{2b(15a^2+7b^2)\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}+\frac{2a^2(a+b\sec(c+dx))\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)}\end{aligned}$$

output
```
40/63*a^2*b*sin(d*x+c)/d/sec(d*x+c)^(5/2)+2/45*a*(7*a^2+27*b^2)*sin(d*x+c)
/d/sec(d*x+c)^(3/2)+2/9*a^2*(a+b*sec(d*x+c))*sin(d*x+c)/d/sec(d*x+c)^(7/2)
+2/21*b*(15*a^2+7*b^2)*sin(d*x+c)/d/sec(d*x+c)^(1/2)+2/15*a*(7*a^2+27*b^2)
*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2
*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+2/21*b*(15*a^2+7*b^2)*(co
s(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),
2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

3.599.2 Mathematica [A] (verified)

Time = 1.48 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.68

$$\int \frac{(a+b\sec(c+dx))^3}{\sec^{\frac{9}{2}}(c+dx)}\,dx$$
$$= \frac{\sqrt{\sec(c+dx)}\left(168a(7a^2+27b^2)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)+120b(15a^2+7b^2)\sqrt{\cos(c+dx)}\,\text{Elliptic}\right.}{}$$

input
```
Integrate[(a + b*Sec[c + d*x])^3/Sec[c + d*x]^(9/2),x]
```

output
```
(Sqrt[Sec[c + d*x]]*(168*a*(7*a^2 + 27*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(
c + d*x)/2, 2] + 120*b*(15*a^2 + 7*b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c +
d*x)/2, 2] + (7*a*(43*a^2 + 108*b^2)*Cos[c + d*x] + 5*(234*a^2*b + 84*b^3
+ 54*a^2*b*Cos[2*(c + d*x)] + 7*a^3*Cos[3*(c + d*x)]))*Sin[2*(c + d*x)]))/
(1260*d)
```

3.599.3 Rubi [A] (verified)

Time = 1.34 (sec) , antiderivative size = 222, normalized size of antiderivative = 0.95, number of steps used = 18, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.783$, Rules used = {3042, 4328, 27, 3042, 4535, 3042, 4256, 3042, 4258, 3042, 3119, 4533, 3042, 4256, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\sec(c+dx))^3}{\sec^{\frac{9}{2}}(c+dx)}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^3}{\csc\left(c+dx+\frac{\pi}{2}\right)^{9/2}}dx$$
$$\Big\downarrow \; 4328$$
$$\frac{2}{9}\int \frac{20ba^2+\left(7a^2+27b^2\right)\sec(c+dx)a+b\left(5a^2+9b^2\right)\sec^2(c+dx)}{2\sec^{\frac{7}{2}}(c+dx)}dx+\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))}{9d\sec^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{9}\int \frac{20ba^2+\left(7a^2+27b^2\right)\sec(c+dx)a+b\left(5a^2+9b^2\right)\sec^2(c+dx)}{\sec^{\frac{7}{2}}(c+dx)}dx+\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))}{9d\sec^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{9}\int \frac{20ba^2+\left(7a^2+27b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)a+b\left(5a^2+9b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx+\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))}{9d\sec^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \text{4535}$$

$$\frac{1}{9}\left(a\left(7a^2+27b^2\right)\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)}dx+\int \frac{20ba^2+b\left(5a^2+9b^2\right)\sec^2(c+dx)}{\sec^{\frac{7}{2}}(c+dx)}dx\right)+\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))}{9d\sec^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{9}\left(a\left(7a^2+27b^2\right)\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+\int \frac{20ba^2+b\left(5a^2+9b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx\right)+\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))}{9d\sec^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \text{4256}$$

$$\frac{1}{9}\left(\int \frac{20ba^2+b\left(5a^2+9b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx+a\left(7a^2+27b^2\right)\left(\frac{3}{5}\int \frac{1}{\sqrt{\sec(c+dx)}}dx+\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}\right)\right)+\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))}{9d\sec^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{9}\left(\int \frac{20ba^2+b\left(5a^2+9b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx+a\left(7a^2+27b^2\right)\left(\frac{3}{5}\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}\right.\right.\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))}{9d\sec^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \text{4258}$$

3.599. $\int \frac{(a+b\sec(c+dx))^3}{\sec^{\frac{9}{2}}(c+dx)}\,dx$

$$\frac{1}{9}\left(\int \frac{20ba^2+b(5a^2+9b^2)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx+a(7a^2+27b^2)\left(\frac{3}{5}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\cos(c+dx)}\right.\right.$$
$$\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))}{9d\sec^{\frac{7}{2}}(c+dx)}$$

$\downarrow$ 3042

$$\frac{1}{9}\left(\int \frac{20ba^2+b(5a^2+9b^2)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx+a(7a^2+27b^2)\left(\frac{3}{5}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx\right.}\right.\right.$$
$$\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))}{9d\sec^{\frac{7}{2}}(c+dx)}$$

$\downarrow$ 3119

$$\frac{1}{9}\left(\int \frac{20ba^2+b(5a^2+9b^2)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx+a(7a^2+27b^2)\left(\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{6\sqrt{\cos(c+dx)}\sqrt{\sec(c+d}}{5d}\right.\right.$$
$$\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))}{9d\sec^{\frac{7}{2}}(c+dx)}$$

$\downarrow$ 4533

$$\frac{1}{9}\left(\frac{9}{7}b(15a^2+7b^2)\int\frac{1}{\sec^{\frac{3}{2}}(c+dx)}dx+a(7a^2+27b^2)\left(\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{6\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+\right.}{5d}\right.\right.$$
$$\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))}{9d\sec^{\frac{7}{2}}(c+dx)}$$

$\downarrow$ 3042

$$\frac{1}{9}\left(\frac{9}{7}b(15a^2+7b^2)\int\frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+a(7a^2+27b^2)\left(\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{6\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E}{5d}\right.\right.$$
$$\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))}{9d\sec^{\frac{7}{2}}(c+dx)}$$

$\downarrow$ 4256

$$\frac{1}{9}\left(\frac{9}{7}b(15a^2+7b^2)\left(\frac{1}{3}\int\sqrt{\sec(c+dx)}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)+a(7a^2+27b^2)\left(\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{6\sqrt{\cos(c+}}{}\right.\right.$$
$$\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))}{9d\sec^{\frac{7}{2}}(c+dx)}$$

$\downarrow$ 3042

3.599. $\int \frac{(a+b\sec(c+dx))^3}{\sec^{\frac{9}{2}}(c+dx)}\,dx$

$$\frac{1}{9}\left(\frac{9}{7}b(15a^2+7b^2)\left(\frac{1}{3}\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)+a(7a^2+27b^2)\left(\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{6\sqrt{\text{co}}}{}\right.\right.$$
$$\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))}{9d\sec^{\frac{7}{2}}(c+dx)}$$

$\Big\downarrow$ 4258

$$\frac{1}{9}\left(\frac{9}{7}b(15a^2+7b^2)\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)+a(7a^2+27b^2)\left(\frac{2}{5d}\right.\right.$$
$$\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))}{9d\sec^{\frac{7}{2}}(c+dx)}$$

$\Big\downarrow$ 3042

$$\frac{1}{9}\left(\frac{9}{7}b(15a^2+7b^2)\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)+a(7a^2+27b^2\right.$$
$$\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))}{9d\sec^{\frac{7}{2}}(c+dx)}$$

$\Big\downarrow$ 3120

$$\frac{1}{9}\left(\frac{9}{7}b(15a^2+7b^2)\left(\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}\right)+a(7a^2+27b^2)\right.$$
$$\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))}{9d\sec^{\frac{7}{2}}(c+dx)}$$

input
```
Int[(a + b*Sec[c + d*x])^3/Sec[c + d*x]^(9/2),x]
```

output
```
(2*a^2*(a + b*Sec[c + d*x])*Sin[c + d*x])/(9*d*Sec[c + d*x]^(7/2)) + ((40*
a^2*b*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/2)) + a*(7*a^2 + 27*b^2)*((6*Sqrt
[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d) + (2*Si
n[c + d*x])/(5*d*Sec[c + d*x]^(3/2))) + (9*b*(15*a^2 + 7*b^2)*((2*Sqrt[Cos
[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (2*Sin[c
+ d*x])/(3*d*Sqrt[Sec[c + d*x]])))/7)/9
```

3.599. $\int \frac{(a+b\sec(c+dx))^3}{\sec^{\frac{9}{2}}(c+dx)}\,dx$

3.599.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4328
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)*
((d*Csc[e + f*x])^n/(f*n)), x] - Simp[1/(d*n)   Int[(a + b*Csc[e + f*x])^(m
 - 3)*(d*Csc[e + f*x])^(n + 1)*Simp[a^2*b*(m - 2*n - 2) - a*(3*b^2*n + a^2*
(n + 1))*Csc[e + f*x] - b*(b^2*n + a^2*(m + n - 1))*Csc[e + f*x]^2, x], x],
 x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && ((Int
egerQ[m] && LtQ[n, -1]) || (IntegersQ[m + 1/2, 2*n] && LeQ[n, -1]))
```

rule 4533
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] +
Simp[(C*m + A*(m + 1))/(b^2*m)   Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fr
eeQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

3.599. $\int \frac{(a+b\sec(c+dx))^3}{\sec^{\frac{9}{2}}(c+dx)}\,dx$

rule 4535
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Simp[B/b   Int[(b*Cs
c[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2)
, x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

3.599.4 Maple [A] (verified)

Time = 38.28 (sec) , antiderivative size = 470, normalized size of antiderivative = 2.01

method	result
default	$-\frac{2\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\left(-1120a^{3}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}+\left(2240a^{3}+2160a^{2}b\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{8}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+(-}{}$
parts	Expression too large to display

input
```
int((a+b*sec(d*x+c))^3/sec(d*x+c)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
-2/315*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-1120*a^3*
cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10+(2240*a^3+2160*a^2*b)*sin(1/2*d*x
+1/2*c)^8*cos(1/2*d*x+1/2*c)+(-2072*a^3-3240*a^2*b-1512*a*b^2)*sin(1/2*d*x
+1/2*c)^6*cos(1/2*d*x+1/2*c)+(952*a^3+2520*a^2*b+1512*a*b^2+420*b^3)*sin(1
/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-168*a^3-720*a^2*b-378*a*b^2-210*b^3)*
sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+225*(sin(1/2*d*x+1/2*c)^2)^(1/2)*E
llipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*a^2*
b+105*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(
2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*b^3-147*(sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellip
ticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*a^3-567*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(
1/2*d*x+1/2*c)^2-1)^(1/2)*a*b^2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*
c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

3.599.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.02

$$\int \frac{(a+b\sec(c+dx))^3}{\sec^{\frac{9}{2}}(c+dx)}\,dx =$$

$$-\frac{15\sqrt{2}(15i\,a^2b+7i\,b^3)\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c))+15\sqrt{2}(-15i\,a^2b-7i\,b}{}$$

input
```
integrate((a+b*sec(d*x+c))^3/sec(d*x+c)^(9/2),x, algorithm="fricas")
```

output
```
-1/315*(15*sqrt(2)*(15*I*a^2*b + 7*I*b^3)*weierstrassPInverse(-4, 0, cos(d
*x + c) + I*sin(d*x + c)) + 15*sqrt(2)*(-15*I*a^2*b - 7*I*b^3)*weierstrass
PInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 21*sqrt(2)*(-7*I*a^3 - 27
*I*a*b^2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) +
 I*sin(d*x + c))) + 21*sqrt(2)*(7*I*a^3 + 27*I*a*b^2)*weierstrassZeta(-4,
0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - 2*(35*a^3*
cos(d*x + c)^4 + 135*a^2*b*cos(d*x + c)^3 + 7*(7*a^3 + 27*a*b^2)*cos(d*x +
 c)^2 + 15*(15*a^2*b + 7*b^3)*cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)
))/d
```

3.599.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(c+dx))^3}{\sec^{\frac{9}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+b*sec(d*x+c))**3/sec(d*x+c)**(9/2),x)
```

output
```
Timed out
```

3.599.7 Maxima [F]

$$\int \frac{(a+b\sec(c+dx))^3}{\sec^{\frac{9}{2}}(c+dx)}\,dx=\int \frac{(b\sec(dx+c)+a)^3}{\sec(dx+c)^{\frac{9}{2}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^3/sec(d*x+c)^(9/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^3/sec(d*x + c)^(9/2), x)
```

3.599.8 Giac [F]

$$\int \frac{(a+b\sec(c+dx))^3}{\sec^{\frac{9}{2}}(c+dx)}\,dx=\int \frac{(b\sec(dx+c)+a)^3}{\sec(dx+c)^{\frac{9}{2}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^3/sec(d*x+c)^(9/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^3/sec(d*x + c)^(9/2), x)
```

3.599.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(c+dx))^3}{\sec^{\frac{9}{2}}(c+dx)}\,dx=\int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^3}{\left(\frac{1}{\cos(c+d\,x)}\right)^{9/2}}\,dx$$

input
```
int((a + b/cos(c + d*x))^3/(1/cos(c + d*x))^(9/2),x)
```

output
```
int((a + b/cos(c + d*x))^3/(1/cos(c + d*x))^(9/2), x)
```

3.600 $\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^4\,dx$

3.600.1 Optimal result 4552
3.600.2 Mathematica [A] (verified) 4553
3.600.3 Rubi [A] (verified) 4553
3.600.4 Maple [B] (verified) 4559
3.600.5 Fricas [C] (verification not implemented) 4560
3.600.6 Sympy [F(-1)] 4560
3.600.7 Maxima [F(-1)] 4561
3.600.8 Giac [F] 4561
3.600.9 Mupad [F(-1)] 4561

3.600.1 Optimal result

Integrand size = 23, antiderivative size = 287

$$\begin{aligned}\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^4\,dx &= -\frac{2(15a^4+54a^2b^2+7b^4)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{15d} \\ &\quad+\frac{8ab(7a^2+5b^2)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{21d} \\ &\quad+\frac{2(15a^4+54a^2b^2+7b^4)\sqrt{\sec(c+dx)}\sin(c+dx)}{15d} \\ &\quad+\frac{8ab(7a^2+5b^2)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{21d}+\frac{14b^2(7a^2+b^2)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{45d} \\ &\quad+\frac{44ab^3\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{63d}+\frac{2b^2\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^2\sin(c+dx)}{9d}\end{aligned}$$

output
```
8/21*a*b*(7*a^2+5*b^2)*sec(d*x+c)^(3/2)*sin(d*x+c)/d+14/45*b^2*(7*a^2+b^2)
*sec(d*x+c)^(5/2)*sin(d*x+c)/d+44/63*a*b^3*sec(d*x+c)^(7/2)*sin(d*x+c)/d+2
/9*b^2*sec(d*x+c)^(5/2)*(a+b*sec(d*x+c))^2*sin(d*x+c)/d+2/15*(15*a^4+54*a^
2*b^2+7*b^4)*sin(d*x+c)*sec(d*x+c)^(1/2)/d-2/15*(15*a^4+54*a^2*b^2+7*b^4)*
(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*
c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+8/21*a*b*(7*a^2+5*b^2)*(co
s(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),
2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

3.600.2 Mathematica [A] (verified)

Time = 2.61 (sec) , antiderivative size = 256, normalized size of antiderivative = 0.89

$$\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^4\,dx = -\frac{2(a+b\sec(c+dx))^4\left(21(15a^4+54a^2b^2+7b^4)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)-60ab(7a^2+5b^2)\sqrt{\cos}\right.}{}$$

input
```
Integrate[Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^4,x]
```

output
```
(-2*(a + b*Sec[c + d*x])^4*(21*(15*a^4 + 54*a^2*b^2 + 7*b^4)*Sqrt[Cos[c +
d*x]]*EllipticE[(c + d*x)/2, 2] - 60*a*b*(7*a^2 + 5*b^2)*Sqrt[Cos[c + d*x]
]*EllipticF[(c + d*x)/2, 2] - 315*a^4*Sin[c + d*x] - 1134*a^2*b^2*Sin[c +
d*x] - 147*b^4*Sin[c + d*x] - 420*a^3*b*Tan[c + d*x] - 300*a*b^3*Tan[c + d
*x] - 378*a^2*b^2*Sec[c + d*x]*Tan[c + d*x] - 49*b^4*Sec[c + d*x]*Tan[c +
d*x] - 180*a*b^3*Sec[c + d*x]^2*Tan[c + d*x] - 35*b^4*Sec[c + d*x]^3*Tan[c
 + d*x]))/(315*d*(b + a*Cos[c + d*x])^4*Sec[c + d*x]^(7/2))
```

3.600.3 Rubi [A] (verified)

Time = 1.70 (sec) , antiderivative size = 269, normalized size of antiderivative = 0.94, number of steps used = 21, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.913$, Rules used = {3042, 4329, 27, 3042, 4564, 27, 3042, 4535, 3042, 4255, 3042, 4258, 3042, 3120, 4534, 3042, 4255, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^4\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^4 dx$$

$$\Big\downarrow \; 4329$$

$$\frac{2}{9}\int \frac{1}{2}\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))\left(22ab^2\sec^2(c+dx)+b\left(27a^2+7b^2\right)\sec(c+dx)+3a\left(3a^2+b^2\right)\right)dx+\frac{2b^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^2}{9d}$$

$\Big\downarrow$ 27

$$\frac{1}{9}\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))\left(22ab^2\sec^2(c+dx)+b\left(27a^2+7b^2\right)\sec(c+dx)+3a\left(3a^2+b^2\right)\right)dx+\frac{2b^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^2}{9d}$$

$\Big\downarrow$ 3042

$$\frac{1}{9}\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)\left(22ab^2\csc\left(c+dx+\frac{\pi}{2}\right)^2+b\left(27a^2+7b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)\right.$$
$$\frac{2b^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^2}{9d}$$

$\Big\downarrow$ 4564

$$\frac{1}{9}\left(\frac{2}{7}\int \frac{1}{2}\sec^{\frac{3}{2}}(c+dx)\left(21\left(3a^2+b^2\right)a^2+36b\left(7a^2+5b^2\right)\sec(c+dx)a+49b^2\left(7a^2+b^2\right)\sec^2(c+dx)\right)dx+\frac{44ab^{3}}{}\right.$$
$$\frac{2b^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^2}{9d}$$

$\Big\downarrow$ 27

$$\frac{1}{9}\left(\frac{1}{7}\int \sec^{\frac{3}{2}}(c+dx)\left(21\left(3a^2+b^2\right)a^2+36b\left(7a^2+5b^2\right)\sec(c+dx)a+49b^2\left(7a^2+b^2\right)\sec^2(c+dx)\right)dx+\frac{44ab^3\,\mathrm{s}}{}\right.$$
$$\frac{2b^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^2}{9d}$$

$\Big\downarrow$ 3042

$$\frac{1}{9}\left(\frac{1}{7}\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(21\left(3a^2+b^2\right)a^2+36b\left(7a^2+5b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)a+49b^2\left(7a^2+b^2\right)\csc\left(c+dx\right.\right.\right.$$
$$\frac{2b^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^2}{9d}$$

$\Big\downarrow$ 4535

3.600. $\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^4\,dx$

$$\frac{1}{9}\left(\frac{1}{7}\left(36ab(7a^2+5b^2)\int \sec^{\frac{5}{2}}(c+dx)dx+\int \sec^{\frac{3}{2}}(c+dx)\left(21(3a^2+b^2)a^2+49b^2(7a^2+b^2)\sec^2(c+dx)\right)dx\right)\right.$$
$$\frac{2b^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^2}{9d}$$
$$\Big\downarrow \text{3042}$$

$$\frac{1}{9}\left(\frac{1}{7}\left(36ab(7a^2+5b^2)\int \csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}dx+\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(21(3a^2+b^2)a^2+49b^2(7a^2+b^2)\text{cs}\right.\right.\right.$$
$$\frac{2b^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^2}{9d}$$
$$\Big\downarrow \text{4255}$$

$$\frac{1}{9}\left(\frac{1}{7}\left(\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(21(3a^2+b^2)a^2+49b^2(7a^2+b^2)\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)dx+36ab(7a^2+5b^2)\left(\frac{1}{3}\int\right.\right.\right.$$
$$\frac{2b^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^2}{9d}$$
$$\Big\downarrow \text{3042}$$

$$\frac{1}{9}\left(\frac{1}{7}\left(\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(21(3a^2+b^2)a^2+49b^2(7a^2+b^2)\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)dx+36ab(7a^2+5b^2)\left(\frac{1}{3}\int\right.\right.\right.$$
$$\frac{2b^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^2}{9d}$$
$$\Big\downarrow \text{4258}$$

$$\frac{1}{9}\left(\frac{1}{7}\left(\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(21(3a^2+b^2)a^2+49b^2(7a^2+b^2)\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)dx+36ab(7a^2+5b^2)\left(\frac{1}{3}\sqrt{\right.\right.\right.$$
$$\frac{2b^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^2}{9d}$$
$$\Big\downarrow \text{3042}$$

$$\frac{1}{9}\left(\frac{1}{7}\left(\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(21(3a^2+b^2)a^2+49b^2(7a^2+b^2)\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)dx+36ab(7a^2+5b^2)\left(\frac{1}{3}\sqrt{\right.\right.\right.$$
$$\frac{2b^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^2}{9d}$$
$$\Big\downarrow \text{3120}$$

3.600. $\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^4\,dx$

$$\frac{1}{9}\left(\frac{1}{7}\left(\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(21\left(3a^2+b^2\right)a^2+49b^2\left(7a^2+b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)dx+36ab\left(7a^2+5b^2\right)\left(\frac{2\,\mathrm{si}}{}\right.\right.\right.$$

$$\frac{2b^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^2}{9d}$$

$$\Big\downarrow \; 4534$$

$$\frac{1}{9}\left(\frac{1}{7}\left(\frac{21}{5}\left(15a^4+54a^2b^2+7b^4\right)\int \sec^{\frac{3}{2}}(c+dx)dx+\frac{98b^2\left(7a^2+b^2\right)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}+36ab\left(7a^2+5b^2\right)\right.\right.$$

$$\frac{2b^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^2}{9d}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{9}\left(\frac{1}{7}\left(\frac{21}{5}\left(15a^4+54a^2b^2+7b^4\right)\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}dx+\frac{98b^2\left(7a^2+b^2\right)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}+36ab\left(7a^2\right.\right.\right.$$

$$\frac{2b^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^2}{9d}$$

$$\Big\downarrow \; 4255$$

$$\frac{1}{9}\left(\frac{1}{7}\left(\frac{21}{5}\left(15a^4+54a^2b^2+7b^4\right)\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\int\frac{1}{\sqrt{\sec(c+dx)}}dx\right)+\frac{98b^2\left(7a^2+b^2\right)\sin(c+d}{5d}\right.\right.$$

$$\frac{2b^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^2}{9d}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{9}\left(\frac{1}{7}\left(\frac{21}{5}\left(15a^4+54a^2b^2+7b^4\right)\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\int\frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)+\frac{98b^2\left(7a^2+b^2\right)\sin(}{}\right.\right.$$

$$\frac{2b^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^2}{9d}$$

$$\Big\downarrow \; 4258$$

$$\frac{1}{9}\left(\frac{1}{7}\left(\frac{21}{5}\left(15a^4+54a^2b^2+7b^4\right)\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\cos(c+dx)}dx\right)\right.\right.$$

$$\frac{2b^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^2}{9d}$$

$$\Big\downarrow \; 3042$$

3.600. $\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^4\,dx$

$$\frac{1}{9}\left(\frac{1}{7}\left(\frac{21}{5}(15a^4+54a^2b^2+7b^4)\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right.}\right.\right.\right.$$
$$\frac{2b^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^2}{9d}$$

$\Big\downarrow$ 3119

$$\frac{1}{9}\left(\frac{1}{7}\left(\frac{98b^2(7a^2+b^2)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}+36ab(7a^2+5b^2)\left(\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{2\sqrt{\cos(c+dx)}}{}\right.\right.\right.$$
$$\frac{2b^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^2}{9d}$$

input
```
Int[Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^4,x]
```

output
```
(2*b^2*Sec[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^2*Sin[c + d*x])/(9*d) + ((4
4*a*b^3*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(7*d) + ((98*b^2*(7*a^2 + b^2)*Se
c[c + d*x]^(5/2)*Sin[c + d*x])/(5*d) + (21*(15*a^4 + 54*a^2*b^2 + 7*b^4)*(
(-2*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (
2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d))/5 + 36*a*b*(7*a^2 + 5*b^2)*((2*Sqrt
[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (2*Se
c[c + d*x]^(3/2)*Sin[c + d*x])/(3*d)))/7)/9
```

3.600.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4329
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
2)*((d*Csc[e + f*x])^n/(f*(m + n - 1))), x] + Simp[1/(d*(m + n - 1))   Int[
(a + b*Csc[e + f*x])^(m - 3)*(d*Csc[e + f*x])^n*Simp[a^3*d*(m + n - 1) + a*
b^2*d*n + b*(b^2*d*(m + n - 2) + 3*a^2*d*(m + n - 1))*Csc[e + f*x] + a*b^2*
d*(3*m + 2*n - 4)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, n}, x
] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
 &&  !(IGtQ[n, 2] &&  !IntegerQ[m])
```

rule 4534
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Simp[(C*m + A*(m + 1))/(m + 1)   Int[(b*Csc[e + f*x])^m, x], x] /;
 FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

rule 4535
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Simp[B/b   Int[(b*Cs
c[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2)
, x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

rule 4564
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-b)*C*Csc[e + f*x]*Cot[e + f*x]*((d*Csc[e + f*x])^
n/(f*(n + 2))), x] + Simp[1/(n + 2)   Int[(d*Csc[e + f*x])^n*Simp[A*a*(n +
2) + (B*a*(n + 2) + b*(C*(n + 1) + A*(n + 2)))*Csc[e + f*x] + (a*C + B*b)*(
n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x]
&&  !LtQ[n, -1]
```

3.600. $\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^4\,dx$

3.600.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1146 vs. $2(307) = 614$.

Time = 64.92 (sec) , antiderivative size = 1147, normalized size of antiderivative = 4.00

method	result	size
default	Expression too large to display	1147
parts	Expression too large to display	1396

input
```
int(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
-(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*b^4*(-1/144*
cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(c
os(1/2*d*x+1/2*c)^2-1/2)^5-7/180*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)
^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^3-14/15*sin(1/2*
d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)/(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+
1/2*c)^2)^(1/2)+7/15*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2
+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(c
os(1/2*d*x+1/2*c),2^(1/2))-7/15*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d
*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*
(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2
))))+2*a^4/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)*(-2*sin(1/2*d*x
+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/
2*c)^2-(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*
(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2))+12/5*a^2*b^2/(8*sin(1/2*d*x+1/2*c)^6-12*
sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)^2*(24*si
n(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*(sin(1/2*d*x+1/2*c)^2)^(1/2)*Elli
pticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2
*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+12*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)
^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2+8*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c...
```

3.600.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 318, normalized size of antiderivative = 1.11

$$\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^4\,dx =$$

$$-\frac{60\sqrt{2}(7i\,a^3b+5i\,ab^3)\cos(dx+c)^4\,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c))+60\sqrt{2}(-}{}$$

input
```
integrate(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))^4,x, algorithm="fricas")
```

output
```
-1/315*(60*sqrt(2)*(7*I*a^3*b + 5*I*a*b^3)*cos(d*x + c)^4*weierstrassPInve
rse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 60*sqrt(2)*(-7*I*a^3*b - 5*I*a
*b^3)*cos(d*x + c)^4*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x +
 c)) + 21*sqrt(2)*(15*I*a^4 + 54*I*a^2*b^2 + 7*I*b^4)*cos(d*x + c)^4*weier
strassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)
)) + 21*sqrt(2)*(-15*I*a^4 - 54*I*a^2*b^2 - 7*I*b^4)*cos(d*x + c)^4*weiers
trassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))
) - 2*(180*a*b^3*cos(d*x + c) + 21*(15*a^4 + 54*a^2*b^2 + 7*b^4)*cos(d*x +
 c)^4 + 35*b^4 + 60*(7*a^3*b + 5*a*b^3)*cos(d*x + c)^3 + 7*(54*a^2*b^2 + 7
*b^4)*cos(d*x + c)^2)*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c)^4)
```

3.600.6 Sympy [F(-1)]

Timed out.

$$\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^4\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**(3/2)*(a+b*sec(d*x+c))**4,x)
```

output
```
Timed out
```

3.600.7 Maxima [F(-1)]

Timed out.

$$\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^4\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))^4,x, algorithm="maxima")
```

output
```
Timed out
```

3.600.8 Giac [F]

$$\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^4\,dx = \int (b\sec(dx+c)+a)^4\sec(dx+c)^{\frac{3}{2}}\,dx$$

input
```
integrate(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))^4,x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^4*sec(d*x + c)^(3/2), x)
```

3.600.9 Mupad [F(-1)]

Timed out.

$$\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^4\,dx = \int \left(a+\frac{b}{\cos(c+d\,x)}\right)^4\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}dx$$

input
```
int((a + b/cos(c + d*x))^4*(1/cos(c + d*x))^(3/2),x)
```

output
```
int((a + b/cos(c + d*x))^4*(1/cos(c + d*x))^(3/2), x)
```

3.601 $\int \sqrt{\sec(c+dx)}(a+b\sec(c+dx))^4\,dx$

3.601.1 Optimal result 4562
3.601.2 Mathematica [A] (verified) 4563
3.601.3 Rubi [A] (verified) 4563
3.601.4 Maple [B] (verified) 4568
3.601.5 Fricas [C] (verification not implemented) 4569
3.601.6 Sympy [F(-1)] 4570
3.601.7 Maxima [F] 4570
3.601.8 Giac [F] 4571
3.601.9 Mupad [F(-1)] 4571

3.601.1 Optimal result

Integrand size = 23, antiderivative size = 247

$$\begin{aligned}\int \sqrt{\sec(c+dx)}(a+b\sec(c+dx))^4\,dx &= -\frac{8ab(5a^2+3b^2)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} \\ &\quad+\frac{2(21a^4+42a^2b^2+5b^4)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{21d} \\ &\quad+\frac{8ab(5a^2+3b^2)\sqrt{\sec(c+dx)}\sin(c+dx)}{5d}+\frac{2b^2(39a^2+5b^2)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{21d} \\ &\quad+\frac{36ab^3\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{35d}+\frac{2b^2\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2\sin(c+dx)}{7d}\end{aligned}$$

output
```
2/21*b^2*(39*a^2+5*b^2)*sec(d*x+c)^(3/2)*sin(d*x+c)/d+36/35*a*b^3*sec(d*x+
c)^(5/2)*sin(d*x+c)/d+2/7*b^2*sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))^2*sin(d*x+
c)/d+8/5*a*b*(5*a^2+3*b^2)*sin(d*x+c)*sec(d*x+c)^(1/2)/d-8/5*a*b*(5*a^2+3*
b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x
+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+2/21*(21*a^4+42*a^2*b
^2+5*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/
2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

3.601.2 Mathematica [A] (verified)

Time = 1.97 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.68

$$\int \sqrt{\sec(c+dx)}(a+b\sec(c+dx))^4\,dx$$
$$= \frac{2\sec^{\frac{7}{2}}(c+dx)\left(-84ab(5a^2+3b^2)\cos^{\frac{7}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)+5(21a^4+42a^2b^2+5b^4)\cos^{\frac{7}{2}}(c+dx)\,\mathrm{El}\right.}{}$$

input
```
Integrate[Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^4,x]
```

output
```
(2*Sec[c + d*x]^(7/2)*(-84*a*b*(5*a^2 + 3*b^2)*Cos[c + d*x]^(7/2)*Elliptic
E[(c + d*x)/2, 2] + 5*(21*a^4 + 42*a^2*b^2 + 5*b^4)*Cos[c + d*x]^(7/2)*Ell
ipticF[(c + d*x)/2, 2] + b*(15*b^3 + 5*b*(42*a^2 + 5*b^2)*Cos[c + d*x]^2 +
 84*a*(5*a^2 + 3*b^2)*Cos[c + d*x]^3)*Sin[c + d*x] + 42*a*b^3*Sin[2*(c + d
*x)]))/(105*d)
```

3.601.3 Rubi [A] (verified)

Time = 1.57 (sec) , antiderivative size = 243, normalized size of antiderivative = 0.98, number of steps used = 19, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.826$, Rules used = {3042, 4329, 27, 3042, 4564, 27, 3042, 4535, 3042, 4255, 3042, 4258, 3042, 3119, 4534, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\sec(c+dx)}(a+b\sec(c+dx))^4\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^4 dx$$

$$\Big\downarrow \; 4329$$

$$\frac{2}{7}\int \frac{1}{2}\sqrt{\sec(c+dx)}(a+b\sec(c+dx))\left(18ab^2\sec^2(c+dx)+b\left(21a^2+5b^2\right)\sec(c+dx)+a\left(7a^2+b^2\right)\right)dx+\frac{2b^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2}{7d}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{7}\int \sqrt{\sec(c+dx)}(a+b\sec(c+dx))\left(18ab^2\sec^2(c+dx)+b(21a^2+5b^2)\sec(c+dx)+a(7a^2+b^2)\right)dx+\frac{2b^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2}{7d}$$

$\Big\downarrow$ 3042

$$\frac{1}{7}\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)\left(18ab^2\csc\left(c+dx+\frac{\pi}{2}\right)^2+b(21a^2+5b^2)\csc\left(c+dx+\frac{\pi}{2}\right)+\right.$$
$$\frac{2b^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2}{7d}$$

$\Big\downarrow$ 4564

$$\frac{1}{7}\left(\frac{2}{5}\int \frac{1}{2}\sqrt{\sec(c+dx)}\left(5(7a^2+b^2)a^2+28b(5a^2+3b^2)\sec(c+dx)a+5b^2(39a^2+5b^2)\sec^2(c+dx)\right)dx+\frac{36ab}{}\right.$$
$$\frac{2b^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2}{7d}$$

$\Big\downarrow$ 27

$$\frac{1}{7}\left(\frac{1}{5}\int \sqrt{\sec(c+dx)}\left(5(7a^2+b^2)a^2+28b(5a^2+3b^2)\sec(c+dx)a+5b^2(39a^2+5b^2)\sec^2(c+dx)\right)dx+\frac{36ab^3}{}\right.$$
$$\frac{2b^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2}{7d}$$

$\Big\downarrow$ 3042

$$\frac{1}{7}\left(\frac{1}{5}\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(5(7a^2+b^2)a^2+28b(5a^2+3b^2)\csc\left(c+dx+\frac{\pi}{2}\right)a+5b^2(39a^2+5b^2)\csc\left(c+dx\right.\right.\right.$$
$$\frac{2b^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2}{7d}$$

$\Big\downarrow$ 4535

$$\frac{1}{7}\left(\frac{1}{5}\left(28ab(5a^2+3b^2)\int \sec^{\frac{3}{2}}(c+dx)dx+\int \sqrt{\sec(c+dx)}\left(5(7a^2+b^2)a^2+5b^2(39a^2+5b^2)\sec^2(c+dx)\right)dx\right)\right.$$
$$\frac{2b^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2}{7d}$$

$\Big\downarrow$ 3042

3.601. $\int \sqrt{\sec(c+dx)}(a+b\sec(c+dx))^4\,dx$

$$\frac{1}{7}\left(\frac{1}{5}\left(28ab\left(5a^2+3b^2\right)\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}dx+\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(5\left(7a^2+b^2\right)a^2+5b^2\left(39a^2+5b^2\right)\csc\right.\right.\right.$$

$$\frac{2b^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2}{7d}$$

$$\Big\downarrow \; 4255$$

$$\frac{1}{7}\left(\frac{1}{5}\left(\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(5\left(7a^2+b^2\right)a^2+5b^2\left(39a^2+5b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)dx+28ab\left(5a^2+3b^2\right)\left(\frac{2\sin}{}\right.\right.\right.$$

$$\frac{2b^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2}{7d}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{7}\left(\frac{1}{5}\left(\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(5\left(7a^2+b^2\right)a^2+5b^2\left(39a^2+5b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)dx+28ab\left(5a^2+3b^2\right)\left(\frac{2\sin}{}\right.\right.\right.$$

$$\frac{2b^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2}{7d}$$

$$\Big\downarrow \; 4258$$

$$\frac{1}{7}\left(\frac{1}{5}\left(\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(5\left(7a^2+b^2\right)a^2+5b^2\left(39a^2+5b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)dx+28ab\left(5a^2+3b^2\right)\left(\frac{2\sin}{}\right.\right.\right.$$

$$\frac{2b^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2}{7d}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{7}\left(\frac{1}{5}\left(\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(5\left(7a^2+b^2\right)a^2+5b^2\left(39a^2+5b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)dx+28ab\left(5a^2+3b^2\right)\left(\frac{2\sin}{}\right.\right.\right.$$

$$\frac{2b^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2}{7d}$$

$$\Big\downarrow \; 3119$$

$$\frac{1}{7}\left(\frac{1}{5}\left(\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(5\left(7a^2+b^2\right)a^2+5b^2\left(39a^2+5b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)dx+28ab\left(5a^2+3b^2\right)\left(\frac{2\sin}{}\right.\right.\right.$$

$$\frac{2b^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2}{7d}$$

$$\Big\downarrow \; 4534$$

3.601. $\int \sqrt{\sec(c+dx)}(a+b\sec(c+dx))^4\,dx$

$$\frac{1}{7}\left(\frac{1}{5}\left(\frac{5}{3}(21a^4+42a^2b^2+5b^4)\int\sqrt{\sec(c+dx)}dx+\frac{10b^2(39a^2+5b^2)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+28ab(5a^2+3b^2\right.\right.$$
$$\frac{2b^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2}{7d}$$

$\downarrow$ 3042

$$\frac{1}{7}\left(\frac{1}{5}\left(\frac{5}{3}(21a^4+42a^2b^2+5b^4)\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{10b^2(39a^2+5b^2)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+28ab(5a^2\right.\right.$$
$$\frac{2b^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2}{7d}$$

$\downarrow$ 4258

$$\frac{1}{7}\left(\frac{1}{5}\left(\frac{5}{3}(21a^4+42a^2b^2+5b^4)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{10b^2(39a^2+5b^2)\sin(c+dx)\sec\right.\right.$$
$$\frac{2b^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2}{7d}$$

$\downarrow$ 3042

$$\frac{1}{7}\left(\frac{1}{5}\left(\frac{5}{3}(21a^4+42a^2b^2+5b^4)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{10b^2(39a^2+5b^2)\sin(c+d}{3d}\right.\right.$$
$$\frac{2b^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2}{7d}$$

$\downarrow$ 3120

$$\frac{1}{7}\left(\frac{1}{5}\left(\frac{10b^2(39a^2+5b^2)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+28ab(5a^2+3b^2)\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+d}}{}\right.\right.\right.$$
$$\frac{2b^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2}{7d}$$

input
```
Int[Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^4,x]
```

3.601. $\int\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^4\,dx$

output
```
(2*b^2*Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^2*Sin[c + d*x])/(7*d) + ((3
6*a*b^3*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5*d) + ((10*(21*a^4 + 42*a^2*b^2
 + 5*b^4)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])
/(3*d) + (10*b^2*(39*a^2 + 5*b^2)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d) +
 28*a*b*(5*a^2 + 3*b^2)*((-2*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*
Sqrt[Sec[c + d*x]])/d + (2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d))/5)/7
```

3.601.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4329
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
2)*((d*Csc[e + f*x])^n/(f*(m + n - 1))), x] + Simp[1/(d*(m + n - 1))   Int[
(a + b*Csc[e + f*x])^(m - 3)*(d*Csc[e + f*x])^n*Simp[a^3*d*(m + n - 1) + a*
b^2*d*n + b*(b^2*d*(m + n - 2) + 3*a^2*d*(m + n - 1))*Csc[e + f*x] + a*b^2*
d*(3*m + 2*n - 4)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, n}, x
] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
 &&  !(IGtQ[n, 2] &&  !IntegerQ[m])
```

rule 4534
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Simp[(C*m + A*(m + 1))/(m + 1)   Int[(b*Csc[e + f*x])^m, x], x] /;
 FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

rule 4535
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Simp[B/b   Int[(b*Cs
c[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2)
, x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

rule 4564
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-b)*C*Csc[e + f*x]*Cot[e + f*x]*((d*Csc[e + f*x])^
n/(f*(n + 2))), x] + Simp[1/(n + 2)   Int[(d*Csc[e + f*x])^n*Simp[A*a*(n +
2) + (B*a*(n + 2) + b*(C*(n + 1) + A*(n + 2)))*Csc[e + f*x] + (a*C + B*b)*(
n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x]
&&  !LtQ[n, -1]
```

3.601.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 897 vs. $2(271) = 542$.

Time = 50.84 (sec) , antiderivative size = 898, normalized size of antiderivative = 3.64

method	result	size
default	Expression too large to display	898
parts	Expression too large to display	1146

input
```
int(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
-(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*a^4*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+2*
b^4*(-1/56*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^
2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^4-5/42*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*
d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+5/21
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/
2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^
(1/2)))+12*a^2*b^2*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1
/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(
1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+8*a^3*b/sin
(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)*(-2*sin(1/2*d*x+1/2*c)^4+sin(
1/2*d*x+1/2*c)^2)^(1/2)*(2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-(sin(1/
2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x
+1/2*c)^2-1)^(1/2))+8/5*a*b^3/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c
)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)^2*(24*sin(1/2*d*x+1/2*c)^
6*cos(1/2*d*x+1/2*c)-12*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x
+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*
sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+12*(sin(1/2*d*x+1/2*c)^2)^(1/2)...
```

3.601.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.17

$$\int \sqrt{\sec(c+dx)}(a+b\sec(c+dx))^4\,dx =$$

$$-\frac{5\sqrt{2}(21i\,a^4+42i\,a^2b^2+5i\,b^4)\cos(dx+c)^3\,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c))+}{}$$

input
```
integrate(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^4,x, algorithm="fricas")
```

output
```
-1/105*(5*sqrt(2)*(21*I*a^4 + 42*I*a^2*b^2 + 5*I*b^4)*cos(d*x + c)^3*weier
strassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 5*sqrt(2)*(-21*I*a^
4 - 42*I*a^2*b^2 - 5*I*b^4)*cos(d*x + c)^3*weierstrassPInverse(-4, 0, cos(
d*x + c) - I*sin(d*x + c)) + 84*sqrt(2)*(5*I*a^3*b + 3*I*a*b^3)*cos(d*x +
c)^3*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*si
n(d*x + c))) + 84*sqrt(2)*(-5*I*a^3*b - 3*I*a*b^3)*cos(d*x + c)^3*weierstr
assZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)))
- 2*(84*a*b^3*cos(d*x + c) + 15*b^4 + 84*(5*a^3*b + 3*a*b^3)*cos(d*x + c)^
3 + 5*(42*a^2*b^2 + 5*b^4)*cos(d*x + c)^2)*sin(d*x + c)/sqrt(cos(d*x + c))
)/(d*cos(d*x + c)^3)
```

3.601.6 Sympy [F(-1)]

Timed out.

$$\int \sqrt{\sec(c+dx)}(a+b\sec(c+dx))^4\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**(1/2)*(a+b*sec(d*x+c))**4,x)
```

output
```
Timed out
```

3.601.7 Maxima [F]

$$\int \sqrt{\sec(c+dx)}(a+b\sec(c+dx))^4\,dx = \int (b\sec(dx+c)+a)^4\sqrt{\sec(dx+c)}\,dx$$

input
```
integrate(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^4,x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^4*sqrt(sec(d*x + c)), x)
```

3.601.8 Giac [F]

$$\int \sqrt{\sec(c+dx)}(a+b\sec(c+dx))^4\,dx = \int (b\sec(dx+c)+a)^4\sqrt{\sec(dx+c)}\,dx$$

input
```
integrate(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^4,x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^4*sqrt(sec(d*x + c)), x)
```

3.601.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{\sec(c+dx)}(a+b\sec(c+dx))^4\,dx = \int \left(a+\frac{b}{\cos(c+d\,x)}\right)^4\sqrt{\frac{1}{\cos(c+d\,x)}}\,dx$$

input
```
int((a + b/cos(c + d*x))^4*(1/cos(c + d*x))^(1/2),x)
```

output
```
int((a + b/cos(c + d*x))^4*(1/cos(c + d*x))^(1/2), x)
```

3.602 $\int \frac{(a+b\sec(c+dx))^4}{\sqrt{\sec(c+dx)}}\,dx$

3.602.1 Optimal result 4572
3.602.2 Mathematica [A] (verified) 4573
3.602.3 Rubi [A] (verified) 4573
3.602.4 Maple [B] (verified) 4578
3.602.5 Fricas [C] (verification not implemented) 4579
3.602.6 Sympy [F(-1)] 4579
3.602.7 Maxima [F] 4580
3.602.8 Giac [F] 4580
3.602.9 Mupad [F(-1)] 4580

3.602.1 Optimal result

Integrand size = 23, antiderivative size = 209

$$\begin{aligned}\int \frac{(a+b\sec(c+dx))^4}{\sqrt{\sec(c+dx)}}\,dx &= \frac{2(5a^4-30a^2b^2-3b^4)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} \\ &\quad+\frac{8ab(3a^2+b^2)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{3d} \\ &\quad+\frac{2b^2(29a^2+3b^2)\sqrt{\sec(c+dx)}\sin(c+dx)}{5d}+\frac{28ab^3\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15d} \\ &\quad+\frac{2b^2\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^2\sin(c+dx)}{5d}\end{aligned}$$

output
```
28/15*a*b^3*sec(d*x+c)^(3/2)*sin(d*x+c)/d+2/5*b^2*(29*a^2+3*b^2)*sin(d*x+c
)*sec(d*x+c)^(1/2)/d+2/5*b^2*(a+b*sec(d*x+c))^2*sin(d*x+c)*sec(d*x+c)^(1/2
)/d+2/5*(5*a^4-30*a^2*b^2-3*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+
1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(
1/2)/d+8/3*a*b*(3*a^2+b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)
*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

3.602.2 Mathematica [A] (verified)

Time = 2.53 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.70

$$\int \frac{(a+b\sec(c+dx))^4}{\sqrt{\sec(c+dx)}}\,dx$$
$$= \frac{\sec^{\frac{5}{2}}(c+dx)\left(12(5a^4-30a^2b^2-3b^4)\cos^{\frac{5}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)+b\left(80a(3a^2+b^2)\cos^{\frac{5}{2}}(c+dx)\,\text{Ellip}\right.\right.}{30d}$$

input
```
Integrate[(a + b*Sec[c + d*x])^4/Sqrt[Sec[c + d*x]],x]
```

output
```
(Sec[c + d*x]^(5/2)*(12*(5*a^4 - 30*a^2*b^2 - 3*b^4)*Cos[c + d*x]^(5/2)*El
lipticE[(c + d*x)/2, 2] + b*(80*a*(3*a^2 + b^2)*Cos[c + d*x]^(5/2)*Ellipti
cF[(c + d*x)/2, 2] + 2*b*(15*(6*a^2 + b^2) + 40*a*b*Cos[c + d*x] + 9*(10*a
^2 + b^2)*Cos[2*(c + d*x)])*Sin[c + d*x])))/(30*d)
```

3.602.3 Rubi [A] (verified)

Time = 1.35 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.02, number of steps used = 17, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.739$, Rules used = {3042, 4329, 27, 3042, 4564, 27, 3042, 4535, 3042, 4258, 3042, 3120, 4534, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\sec(c+dx))^4}{\sqrt{\sec(c+dx)}}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^4}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$
$$\Big\downarrow \; 4329$$
$$\frac{2}{5}\int \frac{(a+b\sec(c+dx))\left(14ab^2\sec^2(c+dx)+3b\left(5a^2+b^2\right)\sec(c+dx)+a\left(5a^2-b^2\right)\right)}{2\sqrt{\sec(c+dx)}}dx+\frac{2b^2\sin(c+dx)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^2}{5d}$$
$$\Big\downarrow \; 27$$

$$\frac{1}{5}\int \frac{(a+b\sec(c+dx))\left(14ab^2\sec^2(c+dx)+3b(5a^2+b^2)\sec(c+dx)+a(5a^2-b^2)\right)}{\sqrt{\sec(c+dx)}}dx+\frac{2b^2\sin(c+dx)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^2}{5d}$$

$\Big\downarrow$ 3042

$$\frac{1}{5}\int \frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)\left(14ab^2\csc\left(c+dx+\frac{\pi}{2}\right)^2+3b(5a^2+b^2)\csc\left(c+dx+\frac{\pi}{2}\right)+a(5a^2-b^2)\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2b^2\sin(c+dx)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^2}{5d}$$

$\Big\downarrow$ 4564

$$\frac{1}{5}\left(\frac{2}{3}\int \frac{3(5a^2-b^2)a^2+20b(3a^2+b^2)\sec(c+dx)a+3b^2(29a^2+3b^2)\sec^2(c+dx)}{2\sqrt{\sec(c+dx)}}dx+\frac{28ab^3\sin(c+dx)\sec^{\frac{3}{2}}(c}{3d}\right.$$
$$\frac{2b^2\sin(c+dx)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^2}{5d}$$

$\Big\downarrow$ 27

$$\frac{1}{5}\left(\frac{1}{3}\int \frac{3(5a^2-b^2)a^2+20b(3a^2+b^2)\sec(c+dx)a+3b^2(29a^2+3b^2)\sec^2(c+dx)}{\sqrt{\sec(c+dx)}}dx+\frac{28ab^3\sin(c+dx)\sec^{\frac{3}{2}}(c}{3d}\right.$$
$$\frac{2b^2\sin(c+dx)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^2}{5d}$$

$\Big\downarrow$ 3042

$$\frac{1}{5}\left(\frac{1}{3}\int \frac{3(5a^2-b^2)a^2+20b(3a^2+b^2)\csc\left(c+dx+\frac{\pi}{2}\right)a+3b^2(29a^2+3b^2)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{28ab^3\sin(c+}{}\right.$$
$$\frac{2b^2\sin(c+dx)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^2}{5d}$$

$\Big\downarrow$ 4535

$$\frac{1}{5}\left(\frac{1}{3}\left(\int \frac{3(5a^2-b^2)a^2+3b^2(29a^2+3b^2)\sec^2(c+dx)}{\sqrt{\sec(c+dx)}}dx+20ab(3a^2+b^2)\int\sqrt{\sec(c+dx)}dx\right)+\frac{28ab^3\sin(c+}{}\right.$$
$$\frac{2b^2\sin(c+dx)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^2}{5d}$$

$\Big\downarrow$ 3042

3.602. $\int \frac{(a+b\sec(c+dx))^4}{\sqrt{\sec(c+dx)}}\,dx$

$$\frac{1}{5}\left(\frac{1}{3}\left(20ab(3a^2+b^2)\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\int\frac{3(5a^2-b^2)a^2+3b^2(29a^2+3b^2)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)+\frac{2b^2\sin(c+dx)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^2}{5d}$$

$\downarrow$ 4258

$$\frac{1}{5}\left(\frac{1}{3}\left(\int\frac{3(5a^2-b^2)a^2+3b^2(29a^2+3b^2)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+20ab(3a^2+b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\right.\right.\frac{2b^2\sin(c+dx)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^2}{5d}$$

$\downarrow$ 3042

$$\frac{1}{5}\left(\frac{1}{3}\left(\int\frac{3(5a^2-b^2)a^2+3b^2(29a^2+3b^2)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+20ab(3a^2+b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\right.\right.\frac{2b^2\sin(c+dx)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^2}{5d}$$

$\downarrow$ 3120

$$\frac{1}{5}\left(\frac{1}{3}\left(\int\frac{3(5a^2-b^2)a^2+3b^2(29a^2+3b^2)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{40ab(3a^2+b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\mathrm{Ell}}{d}\right.\right.\frac{2b^2\sin(c+dx)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^2}{5d}$$

$\downarrow$ 4534

$$\frac{1}{5}\left(\frac{1}{3}\left(3(5a^4-30a^2b^2-3b^4)\int\frac{1}{\sqrt{\sec(c+dx)}}dx+\frac{6b^2(29a^2+3b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{d}+\frac{40ab(3a^2+b^2)\sqrt{}}{}\right.\right.\frac{2b^2\sin(c+dx)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^2}{5d}$$

$\downarrow$ 3042

$$\frac{1}{5}\left(\frac{1}{3}\left(3(5a^4-30a^2b^2-3b^4)\int\frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{6b^2(29a^2+3b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{d}+\frac{40ab(3a^2+}{}\right.\right.\frac{2b^2\sin(c+dx)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^2}{5d}$$

3.602. $\int\frac{(a+b\sec(c+dx))^4}{\sqrt{\sec(c+dx)}}\,dx$

$$\Big\downarrow \text{4258}$$

$$\frac{1}{5}\left(\frac{1}{3}\left(3\left(5a^4-30a^2b^2-3b^4\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\cos(c+dx)}dx+\frac{6b^2\left(29a^2+3b^2\right)\sin(c+dx)\sqrt{\sec(}}{d}\right.\right.$$
$$\frac{2b^2\sin(c+dx)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^2}{5d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}\left(\frac{1}{3}\left(3\left(5a^4-30a^2b^2-3b^4\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{6b^2\left(29a^2+3b^2\right)\sin(c+dx)}{d}\right.\right.$$
$$\frac{2b^2\sin(c+dx)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^2}{5d}$$

$$\Big\downarrow \text{3119}$$

$$\frac{1}{5}\left(\frac{1}{3}\left(\frac{6b^2\left(29a^2+3b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{d}+\frac{40ab\left(3a^2+b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+d\right.}{d}\right.\right.$$
$$\frac{2b^2\sin(c+dx)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^2}{5d}$$

input
```
Int[(a + b*Sec[c + d*x])^4/Sqrt[Sec[c + d*x]],x]
```

output
```
(2*b^2*Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^2*Sin[c + d*x])/(5*d) + ((2
8*a*b^3*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d) + ((6*(5*a^4 - 30*a^2*b^2 -
 3*b^4)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d
 + (40*a*b*(3*a^2 + b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt
[Sec[c + d*x]])/d + (6*b^2*(29*a^2 + 3*b^2)*Sqrt[Sec[c + d*x]]*Sin[c + d*x
])/d)/3)/5
```

3.602.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4329
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
2)*((d*Csc[e + f*x])^n/(f*(m + n - 1))), x] + Simp[1/(d*(m + n - 1))   Int[
(a + b*Csc[e + f*x])^(m - 3)*(d*Csc[e + f*x])^n*Simp[a^3*d*(m + n - 1) + a*
b^2*d*n + b*(b^2*d*(m + n - 2) + 3*a^2*d*(m + n - 1))*Csc[e + f*x] + a*b^2*
d*(3*m + 2*n - 4)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, n}, x
] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
 &&  !(IGtQ[n, 2] &&  !IntegerQ[m])
```

rule 4534
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Simp[(C*m + A*(m + 1))/(m + 1)   Int[(b*Csc[e + f*x])^m, x], x] /;
 FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

rule 4535
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Simp[B/b   Int[(b*Cs
c[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2)
, x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

rule 4564
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-b)*C*Csc[e + f*x]*Cot[e + f*x]*((d*Csc[e + f*x])^
n/(f*(n + 2))), x] + Simp[1/(n + 2)   Int[(d*Csc[e + f*x])^n*Simp[A*a*(n +
2) + (B*a*(n + 2) + b*(C*(n + 1) + A*(n + 2)))*Csc[e + f*x] + (a*C + B*b)*(
n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x]
&&  !LtQ[n, -1]
```

3.602. $\int \frac{(a+b\sec(c+dx))^4}{\sqrt{\sec(c+dx)}}\,dx$

3.602.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 879 vs. $2(237) = 474$.

Time = 41.95 (sec) , antiderivative size = 880, normalized size of antiderivative = 4.21

method	result	size
default	Expression too large to display	880
parts	Expression too large to display	1036

input
```
int((a+b*sec(d*x+c))^4/sec(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*a^4*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+8
*a^3*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*
sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2
*c),2^(1/2))+2*a^4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1
)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(co
s(1/2*d*x+1/2*c),2^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))+2/5*b^4/(
8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/s
in(1/2*d*x+1/2*c)^2*(24*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x
+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*
x+1/2*c)+12*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1
/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2+8*cos(1/2*d*x+1
/2*c)*sin(1/2*d*x+1/2*c)^2-3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/
2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2))*(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+12*a^2*b^2/sin(1/2*d*x+1/2*c)^2/(2*sin(
1/2*d*x+1/2*c)^2-1)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(
2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-(sin(1/2*d*x+1/2*c)^2)^(1/2)*Ell
ipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2))+8*...
```

3.602.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.26

$$\int \frac{(a+b\sec(c+dx))^4}{\sqrt{\sec(c+dx)}}\,dx =$$

$$-\frac{20\sqrt{2}(3i\,a^3b+i\,ab^3)\cos(dx+c)^2\,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c))+20\sqrt{2}(-3}{}$$

input
```
integrate((a+b*sec(d*x+c))^4/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
-1/15*(20*sqrt(2)*(3*I*a^3*b + I*a*b^3)*cos(d*x + c)^2*weierstrassPInverse
(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 20*sqrt(2)*(-3*I*a^3*b - I*a*b^3)
*cos(d*x + c)^2*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))
+ 3*sqrt(2)*(-5*I*a^4 + 30*I*a^2*b^2 + 3*I*b^4)*cos(d*x + c)^2*weierstrass
Zeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 3
*sqrt(2)*(5*I*a^4 - 30*I*a^2*b^2 - 3*I*b^4)*cos(d*x + c)^2*weierstrassZeta
(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - 2*(20
*a*b^3*cos(d*x + c) + 3*b^4 + 9*(10*a^2*b^2 + b^4)*cos(d*x + c)^2)*sin(d*x
 + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c)^2)
```

3.602.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(c+dx))^4}{\sqrt{\sec(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate((a+b*sec(d*x+c))**4/sec(d*x+c)**(1/2),x)
```

output
```
Timed out
```

3.602.7 Maxima [F]

$$\int \frac{(a+b\sec(c+dx))^4}{\sqrt{\sec(c+dx)}}\,dx = \int \frac{(b\sec(dx+c)+a)^4}{\sqrt{\sec(dx+c)}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^4/sec(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^4/sqrt(sec(d*x + c)), x)
```

3.602.8 Giac [F]

$$\int \frac{(a+b\sec(c+dx))^4}{\sqrt{\sec(c+dx)}}\,dx = \int \frac{(b\sec(dx+c)+a)^4}{\sqrt{\sec(dx+c)}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^4/sec(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^4/sqrt(sec(d*x + c)), x)
```

3.602.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(c+dx))^4}{\sqrt{\sec(c+dx)}}\,dx = \int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^4}{\sqrt{\frac{1}{\cos(c+d\,x)}}}\,dx$$

input
```
int((a + b/cos(c + d*x))^4/(1/cos(c + d*x))^(1/2),x)
```

output
```
int((a + b/cos(c + d*x))^4/(1/cos(c + d*x))^(1/2), x)
```

3.603 $\int \frac{(a+b\sec(c+dx))^4}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

3.603.1 Optimal result 4581
3.603.2 Mathematica [A] (verified) 4582
3.603.3 Rubi [A] (verified) 4582
3.603.4 Maple [B] (verified) 4587
3.603.5 Fricas [C] (verification not implemented) 4588
3.603.6 Sympy [F] 4588
3.603.7 Maxima [F] 4589
3.603.8 Giac [F] 4589
3.603.9 Mupad [F(-1)] 4589

3.603.1 Optimal result

Integrand size = 23, antiderivative size = 208

$$\begin{aligned}\int \frac{(a+b\sec(c+dx))^4}{\sec^{\frac{3}{2}}(c+dx)}\,dx &= \frac{8ab(a^2-b^2)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} \\ &\quad + \frac{2(a^4+18a^2b^2+b^4)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{3d} \\ &\quad - \frac{4ab(a^2-6b^2)\sqrt{\sec(c+dx)}\sin(c+dx)}{3d} \\ &\quad - \frac{2b^2(a^2-b^2)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d} + \frac{2a^2(a+b\sec(c+dx))^2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\end{aligned}$$

output
```
-2/3*b^2*(a^2-b^2)*sec(d*x+c)^(3/2)*sin(d*x+c)/d+2/3*a^2*(a+b*sec(d*x+c))^
2*sin(d*x+c)/d/sec(d*x+c)^(1/2)-4/3*a*b*(a^2-6*b^2)*sin(d*x+c)*sec(d*x+c)^
(1/2)/d+8*a*b*(a^2-b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*El
lipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+2/
3*(a^4+18*a^2*b^2+b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ell
ipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

3.603.2 Mathematica [A] (verified)

Time = 1.64 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.62

$$\int \frac{(a+b\sec(c+dx))^4}{\sec^{\frac{3}{2}}(c+dx)}\,dx$$
$$= \frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(24ab(a^2-b^2)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)+2(a^4+18a^2b^2+b^4)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right.\right.}{3d}$$

input
```
Integrate[(a + b*Sec[c + d*x])^4/Sec[c + d*x]^(3/2),x]
```

output
```
(Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(24*a*b*(a^2 - b^2)*EllipticE[(c +
d*x)/2, 2] + 2*(a^4 + 18*a^2*b^2 + b^4)*EllipticF[(c + d*x)/2, 2] + ((a^4
+ 2*b^4 + 24*a*b^3*Cos[c + d*x] + a^4*Cos[2*(c + d*x)])*Sin[c + d*x])/Cos[
c + d*x]^(3/2)))/(3*d)
```

3.603.3 Rubi [A] (verified)

Time = 1.32 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.739$, Rules used = {3042, 4328, 27, 3042, 4564, 27, 3042, 4535, 3042, 4258, 3042, 3120, 4534, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\sec(c+dx))^4}{\sec^{\frac{3}{2}}(c+dx)}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^4}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx$$
$$\Big\downarrow \; 4328$$
$$\frac{2}{3}\int \frac{(a+b\sec(c+dx))\left(10ba^2+\left(a^2+9b^2\right)\sec(c+dx)a-3b\left(a^2-b^2\right)\sec^2(c+dx)\right)}{2\sqrt{\sec(c+dx)}}dx+\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))^2}{3d\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{3}\int \frac{(a+b\sec(c+dx))\left(10ba^2+\left(a^2+9b^2\right)\sec(c+dx)a-3b\left(a^2-b^2\right)\sec^2(c+dx)\right)}{\sqrt{\sec(c+dx)}}dx+\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))^2}{3d\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{3}\int \frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)\left(10ba^2+\left(a^2+9b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)a-3b\left(a^2-b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))^2}{3d\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \; 4564$$

$$\frac{1}{3}\left(\frac{2}{3}\int \frac{3\left(10ba^3-2b\left(a^2-6b^2\right)\sec^2(c+dx)a+\left(a^4+18b^2a^2+b^4\right)\sec(c+dx)\right)}{2\sqrt{\sec(c+dx)}}dx-\frac{2b^2\left(a^2-b^2\right)\sin(c+dx)\sec}{d}\right.$$
$$\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))^2}{3d\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{3}\left(\int \frac{10ba^3-2b\left(a^2-6b^2\right)\sec^2(c+dx)a+\left(a^4+18b^2a^2+b^4\right)\sec(c+dx)}{\sqrt{\sec(c+dx)}}dx-\frac{2b^2\left(a^2-b^2\right)\sin(c+dx)\sec^{\frac{3}{2}}(c+}{d}\right.$$
$$\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))^2}{3d\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{3}\left(\int \frac{10ba^3-2b\left(a^2-6b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2a+\left(a^4+18b^2a^2+b^4\right)\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2b^2\left(a^2-b^2\right)\sin(c+d}{d}\right.$$
$$\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))^2}{3d\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \; 4535$$

$$\frac{1}{3}\left(\left(a^4+18a^2b^2+b^4\right)\int\sqrt{\sec(c+dx)}dx+\int\frac{10a^3b-2ab\left(a^2-6b^2\right)\sec^2(c+dx)}{\sqrt{\sec(c+dx)}}dx-\frac{2b^2\left(a^2-b^2\right)\sin(c+dx)s}{d}\right.$$
$$\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))^2}{3d\sqrt{\sec(c+dx)}}$$

3.603. $\int \frac{(a+b\sec(c+dx))^4}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\left(\left(a^4+18a^2b^2+b^4\right)\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\int\frac{10a^3b-2ab\left(a^2-6b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2b^2\left(a^2-b^2\right)s}{}\right.$$
$$\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))^2}{3d\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{4258}$$

$$\frac{1}{3}\left(\left(a^4+18a^2b^2+b^4\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx+\int\frac{10a^3b-2ab\left(a^2-6b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}\right.$$
$$\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))^2}{3d\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\left(\left(a^4+18a^2b^2+b^4\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\int\frac{10a^3b-2ab\left(a^2-6b^2\right)\csc\left(c+d\right.}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}\right.$$
$$\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))^2}{3d\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{3120}$$

$$\frac{1}{3}\left(\int\frac{10a^3b-2ab\left(a^2-6b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2b^2\left(a^2-b^2\right)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d}+\frac{2\left(a^4+18a^2b^2+b^4\right)}{}\right.$$
$$\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))^2}{3d\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{4534}$$

$$\frac{1}{3}\left(12ab\left(a^2-b^2\right)\int\frac{1}{\sqrt{\sec(c+dx)}}dx-\frac{2b^2\left(a^2-b^2\right)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d}-\frac{4ab\left(a^2-6b^2\right)\sin(c+dx)\sqrt{\sec(c}}{d}\right.$$
$$\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))^2}{3d\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

3.603. $\int\frac{(a+b\sec(c+dx))^4}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

$$\frac{1}{3}\left(12ab(a^2-b^2)\int\frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2b^2(a^2-b^2)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d}-\frac{4ab(a^2-6b^2)\sin(c+dx)\sqrt{}}{d}\right.$$

$$\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))^2}{3d\sqrt{\sec(c+dx)}}$$

$\Big\downarrow$ 4258

$$\frac{1}{3}\left(12ab(a^2-b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\cos(c+dx)}dx-\frac{2b^2(a^2-b^2)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d}-\frac{4ab(}{}\right.$$

$$\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))^2}{3d\sqrt{\sec(c+dx)}}$$

$\Big\downarrow$ 3042

$$\frac{1}{3}\left(12ab(a^2-b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx-\frac{2b^2(a^2-b^2)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d}-\right.$$

$$\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))^2}{3d\sqrt{\sec(c+dx)}}$$

$\Big\downarrow$ 3119

$$\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))^2}{3d\sqrt{\sec(c+dx)}}+$$

$$\frac{1}{3}\left(-\frac{2b^2(a^2-b^2)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d}-\frac{4ab(a^2-6b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{d}+\frac{24ab(a^2-b^2)\sqrt{\cos(c+}}{}\right.$$

input
```
Int[(a + b*Sec[c + d*x])^4/Sec[c + d*x]^(3/2),x]
```

output
```
(2*a^2*(a + b*Sec[c + d*x])^2*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]]) + ((2
4*a*b*(a^2 - b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
+ d*x]])/d + (2*(a^4 + 18*a^2*b^2 + b^4)*Sqrt[Cos[c + d*x]]*EllipticF[(c +
 d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d - (4*a*b*(a^2 - 6*b^2)*Sqrt[Sec[c + d*x]
]*Sin[c + d*x])/d - (2*b^2*(a^2 - b^2)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/d)
/3
```

3.603. $\int \frac{(a+b\sec(c+dx))^4}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

3.603.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4328
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)*
((d*Csc[e + f*x])^n/(f*n)), x] - Simp[1/(d*n)   Int[(a + b*Csc[e + f*x])^(m
 - 3)*(d*Csc[e + f*x])^(n + 1)*Simp[a^2*b*(m - 2*n - 2) - a*(3*b^2*n + a^2*
(n + 1))*Csc[e + f*x] - b*(b^2*n + a^2*(m + n - 1))*Csc[e + f*x]^2, x], x],
 x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && ((Int
egerQ[m] && LtQ[n, -1]) || (IntegersQ[m + 1/2, 2*n] && LeQ[n, -1]))
```

rule 4534
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Simp[(C*m + A*(m + 1))/(m + 1)   Int[(b*Csc[e + f*x])^m, x], x] /;
 FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

rule 4535
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Simp[B/b   Int[(b*Cs
c[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2)
, x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

3.603. $\int \frac{(a+b\sec(c+dx))^4}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

rule 4564
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-b)*C*Csc[e + f*x]*Cot[e + f*x]*((d*Csc[e + f*x])^
n/(f*(n + 2))), x] + Simp[1/(n + 2)   Int[(d*Csc[e + f*x])^n*Simp[A*a*(n +
2) + (B*a*(n + 2) + b*(C*(n + 1) + A*(n + 2)))*Csc[e + f*x] + (a*C + B*b)*(
n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x]
&&  !LtQ[n, -1]
```

3.603.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 776 vs. $2(238) = 476$.

Time = 30.84 (sec) , antiderivative size = 777, normalized size of antiderivative = 3.74

method	result	size
default	Expression too large to display	777
parts	Expression too large to display	857

input
```
int((a+b*sec(d*x+c))^4/sec(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*(-8*a^4*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*
d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+8*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2
*c)^2)^(1/2)*a*(a^3+6*b^3)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-2*(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(a^4+12*a*b^3+b^4)*sin(1/2
*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/
2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^4+18*EllipticF(cos(1/2*d*x+1/2*
c),2^(1/2))*a^2*b^2+EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b^4-12*EllipticE
(cos(1/2*d*x+1/2*c),2^(1/2))*a^3*b+12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)
)*a*b^3)*sin(1/2*d*x+1/2*c)^2+a^4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(-2*sin(1/2*d*
x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+18*a^2*b^2*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)
)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b^4-12*(-2*sin
(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^
3*b+12*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1...
```

3.603.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.16

$$\int \frac{(a+b\sec(c+dx))^4}{\sec^{\frac{3}{2}}(c+dx)}\,dx$$
$$= \frac{\sqrt{2}(-i\,a^4-18i\,a^2b^2-i\,b^4)\cos(dx+c)\,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c))+\sqrt{2}(i\,a}{}$$

input
```
integrate((a+b*sec(d*x+c))^4/sec(d*x+c)^(3/2),x, algorithm="fricas")
```

output
```
1/3*(sqrt(2)*(-I*a^4 - 18*I*a^2*b^2 - I*b^4)*cos(d*x + c)*weierstrassPInve
rse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + sqrt(2)*(I*a^4 + 18*I*a^2*b^2
+ I*b^4)*cos(d*x + c)*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x
+ c)) - 12*sqrt(2)*(-I*a^3*b + I*a*b^3)*cos(d*x + c)*weierstrassZeta(-4, 0
, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 12*sqrt(2)*
(I*a^3*b - I*a*b^3)*cos(d*x + c)*weierstrassZeta(-4, 0, weierstrassPInvers
e(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*(a^4*cos(d*x + c)^2 + 12*a*b^
3*cos(d*x + c) + b^4)*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c))
```

3.603.6 Sympy [F]

$$\int \frac{(a+b\sec(c+dx))^4}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(a+b\sec(c+dx))^4}{\sec^{\frac{3}{2}}(c+dx)}\,dx$$

input
```
integrate((a+b*sec(d*x+c))**4/sec(d*x+c)**(3/2),x)
```

output
```
Integral((a + b*sec(c + d*x))**4/sec(c + d*x)**(3/2), x)
```

3.603.7 Maxima [F]

$$\int \frac{(a+b\sec(c+dx))^4}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(b\sec(dx+c)+a)^4}{\sec(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^4/sec(d*x+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^4/sec(d*x + c)^(3/2), x)
```

3.603.8 Giac [F]

$$\int \frac{(a+b\sec(c+dx))^4}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(b\sec(dx+c)+a)^4}{\sec(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^4/sec(d*x+c)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^4/sec(d*x + c)^(3/2), x)
```

3.603.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(c+dx))^4}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^4}{\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

input
```
int((a + b/cos(c + d*x))^4/(1/cos(c + d*x))^(3/2),x)
```

output
```
int((a + b/cos(c + d*x))^4/(1/cos(c + d*x))^(3/2), x)
```

3.604 $\int \frac{(a+b\sec(c+dx))^4}{\sec^{\frac{5}{2}}(c+dx)}\,dx$

3.604.1 Optimal result 4590
3.604.2 Mathematica [A] (verified) 4591
3.604.3 Rubi [A] (verified) 4591
3.604.4 Maple [A] (verified) 4596
3.604.5 Fricas [C] (verification not implemented) 4597
3.604.6 Sympy [F] 4597
3.604.7 Maxima [F] 4598
3.604.8 Giac [F] 4598
3.604.9 Mupad [F(-1)] 4598

3.604.1 Optimal result

Integrand size = 23, antiderivative size = 207

$$\begin{aligned}\int \frac{(a+b\sec(c+dx))^4}{\sec^{\frac{5}{2}}(c+dx)}\,dx &= \frac{2(3a^4+30a^2b^2-5b^4)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} \\ &\quad+\frac{8ab(a^2+3b^2)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{3d}+\frac{28a^3b\sin(c+dx)}{15d\sqrt{\sec(c+dx)}} \\ &\quad-\frac{2b^2(a^2-5b^2)\sqrt{\sec(c+dx)}\sin(c+dx)}{5d}+\frac{2a^2(a+b\sec(c+dx))^2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}\end{aligned}$$

output
```
2/5*a^2*(a+b*sec(d*x+c))^2*sin(d*x+c)/d/sec(d*x+c)^(3/2)+28/15*a^3*b*sin(d
*x+c)/d/sec(d*x+c)^(1/2)-2/5*b^2*(a^2-5*b^2)*sin(d*x+c)*sec(d*x+c)^(1/2)/d
+2/5*(3*a^4+30*a^2*b^2-5*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2
*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2
)/d+8/3*a*b*(a^2+3*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*El
lipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

3.604.2 Mathematica [A] (verified)

Time = 1.23 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.67

$$\int \frac{(a+b\sec(c+dx))^4}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \frac{\sqrt{\sec(c+dx)}\left(12(3a^4+30a^2b^2-5b^4)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)+80ab(a^2+3b^2)\sqrt{\cos(c+dx)}\,\text{Elli}\right.}{30d}$$

input
```
Integrate[(a + b*Sec[c + d*x])^4/Sec[c + d*x]^(5/2),x]
```

output
```
(Sqrt[Sec[c + d*x]]*(12*(3*a^4 + 30*a^2*b^2 - 5*b^4)*Sqrt[Cos[c + d*x]]*El
lipticE[(c + d*x)/2, 2] + 80*a*b*(a^2 + 3*b^2)*Sqrt[Cos[c + d*x]]*Elliptic
F[(c + d*x)/2, 2] + 2*(3*a^4 + 30*b^4 + 40*a^3*b*Cos[c + d*x] + 3*a^4*Cos[
2*(c + d*x)])*Sin[c + d*x]))/(30*d)
```

3.604.3 Rubi [A] (verified)

Time = 1.35 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.02, number of steps used = 17, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.739$, Rules used = {3042, 4328, 27, 3042, 4562, 27, 3042, 4535, 3042, 4258, 3042, 3120, 4534, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\sec(c+dx))^4}{\sec^{\frac{5}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^4}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx$$

$$\Big\downarrow \text{4328}$$

$$\frac{2}{5}\int \frac{(a+b\sec(c+dx))\left(14ba^2+3\left(a^2+5b^2\right)\sec(c+dx)a-b\left(a^2-5b^2\right)\sec^2(c+dx)\right)}{2\sec^{\frac{3}{2}}(c+dx)}dx+\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))^2}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{5}\int \frac{(a+b\sec(c+dx))\left(14ba^2+3\left(a^2+5b^2\right)\sec(c+dx)a-b\left(a^2-5b^2\right)\sec^2(c+dx)\right)}{\sec^{\frac{3}{2}}(c+dx)}dx+\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))^2}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$\downarrow$ 3042

$$\frac{1}{5}\int \frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)\left(14ba^2+3\left(a^2+5b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)a-b\left(a^2-5b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))^2}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$\downarrow$ 4562

$$\frac{1}{5}\left(\frac{28a^3b\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}-\frac{2}{3}\int -\frac{3\left(3a^2+29b^2\right)a^2+20b\left(a^2+3b^2\right)\sec(c+dx)a-3b^2\left(a^2-5b^2\right)\sec^2(c+dx)}{2\sqrt{\sec(c+dx)}}dx\right)+\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))^2}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$\downarrow$ 27

$$\frac{1}{5}\left(\frac{1}{3}\int \frac{3\left(3a^2+29b^2\right)a^2+20b\left(a^2+3b^2\right)\sec(c+dx)a-3b^2\left(a^2-5b^2\right)\sec^2(c+dx)}{\sqrt{\sec(c+dx)}}dx+\frac{28a^3b\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)+\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))^2}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$\downarrow$ 3042

$$\frac{1}{5}\left(\frac{1}{3}\int \frac{3\left(3a^2+29b^2\right)a^2+20b\left(a^2+3b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)a-3b^2\left(a^2-5b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{28a^3b\sin(c+}{3d\sqrt{\sec(c+}}\right.\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))^2}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$\downarrow$ 4535

$$\frac{1}{5}\left(\frac{1}{3}\left(\int \frac{3a^2\left(3a^2+29b^2\right)-3b^2\left(a^2-5b^2\right)\sec^2(c+dx)}{\sqrt{\sec(c+dx)}}dx+20ab\left(a^2+3b^2\right)\int \sqrt{\sec(c+dx)}dx\right)+\frac{28a^3b\sin(c+}{3d\sqrt{\sec(c+}}\right.\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))^2}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$\downarrow$ 3042

3.604. $\int \frac{(a+b\sec(c+dx))^4}{\sec^{\frac{5}{2}}(c+dx)}\,dx$

$$\frac{1}{5}\left(\frac{1}{3}\left(20ab(a^2+3b^2)\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\int\frac{3a^2(3a^2+29b^2)-3b^2(a^2-5b^2)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)+\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))^2}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$\downarrow$ 4258

$$\frac{1}{5}\left(\frac{1}{3}\left(\int\frac{3a^2(3a^2+29b^2)-3b^2(a^2-5b^2)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+20ab(a^2+3b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\right.\right.\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))^2}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$\downarrow$ 3042

$$\frac{1}{5}\left(\frac{1}{3}\left(\int\frac{3a^2(3a^2+29b^2)-3b^2(a^2-5b^2)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+20ab(a^2+3b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\right.\right.\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))^2}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$\downarrow$ 3120

$$\frac{1}{5}\left(\frac{1}{3}\left(\int\frac{3a^2(3a^2+29b^2)-3b^2(a^2-5b^2)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{40ab(a^2+3b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\mathrm{Ell}}{d}\right.\right.\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))^2}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$\downarrow$ 4534

$$\frac{1}{5}\left(\frac{1}{3}\left(3(3a^4+30a^2b^2-5b^4)\int\frac{1}{\sqrt{\sec(c+dx)}}dx-\frac{6b^2(a^2-5b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{d}+\frac{40ab(a^2+3b^2)\sqrt{c}}{}\right.\right.\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))^2}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$\downarrow$ 3042

3.604. $\int\frac{(a+b\sec(c+dx))^4}{\sec^{\frac{5}{2}}(c+dx)}\,dx$

$$\frac{1}{5}\left(\frac{1}{3}\left(3\left(3a^4+30a^2b^2-5b^4\right)\int\frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{6b^2\left(a^2-5b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{d}+\frac{40ab\left(a^2+3b\right.}{}\right.\right.$$

$$\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))^2}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{4258}$$

$$\frac{1}{5}\left(\frac{1}{3}\left(3\left(3a^4+30a^2b^2-5b^4\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\cos(c+dx)}dx-\frac{6b^2\left(a^2-5b^2\right)\sin(c+dx)\sqrt{\sec(c\right.}{d}\right.\right.$$

$$\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))^2}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}\left(\frac{1}{3}\left(3\left(3a^4+30a^2b^2-5b^4\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx-\frac{6b^2\left(a^2-5b^2\right)\sin(c+dx)\sqrt{}}{d}\right.\right.$$

$$\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))^2}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{3119}$$

$$\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))^2}{5d\sec^{\frac{3}{2}}(c+dx)}+$$

$$\frac{1}{5}\left(\frac{28a^3b\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{1}{3}\left(-\frac{6b^2\left(a^2-5b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{d}+\frac{40ab\left(a^2+3b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx}}{d}\right.\right.$$

input
```
Int[(a + b*Sec[c + d*x])^4/Sec[c + d*x]^(5/2),x]
```

output
```
(2*a^2*(a + b*Sec[c + d*x])^2*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)) + ((2
8*a^3*b*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]]) + ((6*(3*a^4 + 30*a^2*b^2 -
 5*b^4)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d
 + (40*a*b*(a^2 + 3*b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt
[Sec[c + d*x]])/d - (6*b^2*(a^2 - 5*b^2)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/
d)/3)/5
```

3.604. $\int \frac{(a+b\sec(c+dx))^4}{\sec^{\frac{5}{2}}(c+dx)}\,dx$

3.604.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4328
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)*
((d*Csc[e + f*x])^n/(f*n)), x] - Simp[1/(d*n)   Int[(a + b*Csc[e + f*x])^(m
 - 3)*(d*Csc[e + f*x])^(n + 1)*Simp[a^2*b*(m - 2*n - 2) - a*(3*b^2*n + a^2*
(n + 1))*Csc[e + f*x] - b*(b^2*n + a^2*(m + n - 1))*Csc[e + f*x]^2, x], x],
 x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && ((Int
egerQ[m] && LtQ[n, -1]) || (IntegersQ[m + 1/2, 2*n] && LeQ[n, -1]))
```

rule 4534
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Simp[(C*m + A*(m + 1))/(m + 1)   Int[(b*Csc[e + f*x])^m, x], x] /;
 FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

rule 4535
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Simp[B/b   Int[(b*Cs
c[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2)
, x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

3.604. $\int \frac{(a+b\sec(c+dx))^4}{\sec^{\frac{5}{2}}(c+dx)}\,dx$

rule 4562
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_)), x_Symbol] :> Simp[A*a*Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*n)), x] + Si
mp[1/(d*n)   Int[(d*Csc[e + f*x])^(n + 1)*Simp[n*(B*a + A*b) + (n*(a*C + B*
b) + A*a*(n + 1))*Csc[e + f*x] + b*C*n*Csc[e + f*x]^2, x], x], x] /; FreeQ[
{a, b, d, e, f, A, B, C}, x] && LtQ[n, -1]
```

3.604.4 Maple [A] (verified)

Time = 29.95 (sec) , antiderivative size = 432, normalized size of antiderivative = 2.09

method	result
default	$\frac{\frac{16a^4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{5}-\frac{16a^4\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)}{5}-\frac{32a^3\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)b}{3}+\frac{4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2a^4}{5}+\frac{16\cos\left(\frac{dx}{2}\right.}{}}{}$
parts	Expression too large to display

input
```
int((a+b*sec(d*x+c))^4/sec(d*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/15*(24*a^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-24*a^4*sin(1/2*d*x+1/
2*c)^4*cos(1/2*d*x+1/2*c)-80*a^3*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)*b
+6*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2*a^4+40*cos(1/2*d*x+1/2*c)*sin(1
/2*d*x+1/2*c)^2*a^3*b+30*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2*b^4-20*a^
3*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elliptic
F(cos(1/2*d*x+1/2*c),2^(1/2))-60*a*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin
(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+9*a^4*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1
/2*d*x+1/2*c),2^(1/2))+90*a^2*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-15*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1
/2*c),2^(1/2))*b^4)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

3.604.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.04

$$\int \frac{(a+b\sec(c+dx))^4}{\sec^{\frac{5}{2}}(c+dx)}\,dx =$$

$$-\frac{20\sqrt{2}(i\,a^3b+3i\,ab^3)\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c))+20\sqrt{2}(-i\,a^3b-3i\,ab^3)}{}$$

input
```
integrate((a+b*sec(d*x+c))^4/sec(d*x+c)^(5/2),x, algorithm="fricas")
```

output
```
-1/15*(20*sqrt(2)*(I*a^3*b + 3*I*a*b^3)*weierstrassPInverse(-4, 0, cos(d*x
 + c) + I*sin(d*x + c)) + 20*sqrt(2)*(-I*a^3*b - 3*I*a*b^3)*weierstrassPIn
verse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 3*sqrt(2)*(-3*I*a^4 - 30*I*a
^2*b^2 + 5*I*b^4)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*
x + c) + I*sin(d*x + c))) + 3*sqrt(2)*(3*I*a^4 + 30*I*a^2*b^2 - 5*I*b^4)*w
eierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x
+ c))) - 2*(3*a^4*cos(d*x + c)^2 + 20*a^3*b*cos(d*x + c) + 15*b^4)*sin(d*x
 + c)/sqrt(cos(d*x + c)))/d
```

3.604.6 Sympy [F]

$$\int \frac{(a+b\sec(c+dx))^4}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{(a+b\sec(c+dx))^4}{\sec^{\frac{5}{2}}(c+dx)}\,dx$$

input
```
integrate((a+b*sec(d*x+c))**4/sec(d*x+c)**(5/2),x)
```

output
```
Integral((a + b*sec(c + d*x))**4/sec(c + d*x)**(5/2), x)
```

3.604.7 Maxima [F]

$$\int \frac{(a+b\sec(c+dx))^4}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{(b\sec(dx+c)+a)^4}{\sec(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^4/sec(d*x+c)^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^4/sec(d*x + c)^(5/2), x)
```

3.604.8 Giac [F]

$$\int \frac{(a+b\sec(c+dx))^4}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{(b\sec(dx+c)+a)^4}{\sec(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^4/sec(d*x+c)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^4/sec(d*x + c)^(5/2), x)
```

3.604.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(c+dx))^4}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^4}{\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

input
```
int((a + b/cos(c + d*x))^4/(1/cos(c + d*x))^(5/2),x)
```

output
```
int((a + b/cos(c + d*x))^4/(1/cos(c + d*x))^(5/2), x)
```

3.605 $\int \frac{(a+b\sec(c+dx))^4}{\sec^{\frac{7}{2}}(c+dx)}\,dx$

3.605.1 Optimal result 4599
3.605.2 Mathematica [A] (verified) 4600
3.605.3 Rubi [A] (verified) 4600
3.605.4 Maple [A] (verified) 4605
3.605.5 Fricas [C] (verification not implemented) 4605
3.605.6 Sympy [F] 4606
3.605.7 Maxima [F] 4606
3.605.8 Giac [F] 4607
3.605.9 Mupad [F(-1)] 4607

3.605.1 Optimal result

Integrand size = 23, antiderivative size = 211

$$\begin{aligned}\int \frac{(a+b\sec(c+dx))^4}{\sec^{\frac{7}{2}}(c+dx)}\,dx &= \frac{8ab(3a^2+5b^2)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} \\ &\quad+\frac{2(5a^4+42a^2b^2+21b^4)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{21d} \\ &\quad+\frac{36a^3b\sin(c+dx)}{35d\sec^{\frac{3}{2}}(c+dx)}+\frac{2a^2(5a^2+39b^2)\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}+\frac{2a^2(a+b\sec(c+dx))^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}\end{aligned}$$

output
```
36/35*a^3*b*sin(d*x+c)/d/sec(d*x+c)^(3/2)+2/7*a^2*(a+b*sec(d*x+c))^2*sin(d
*x+c)/d/sec(d*x+c)^(5/2)+2/21*a^2*(5*a^2+39*b^2)*sin(d*x+c)/d/sec(d*x+c)^(
1/2)+8/5*a*b*(3*a^2+5*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)
*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
+2/21*(5*a^4+42*a^2*b^2+21*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1
/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1
/2)/d
```

3.605.2 Mathematica [A] (verified)

Time = 1.50 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.67

$$\int \frac{(a+b\sec(c+dx))^4}{\sec^{\frac{7}{2}}(c+dx)}\,dx$$
$$= \frac{\sqrt{\sec(c+dx)}\left(336ab(3a^2+5b^2)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)+20(5a^4+42a^2b^2+21b^4)\sqrt{\cos(c+dx)}\right.}{210}$$

input
```
Integrate[(a + b*Sec[c + d*x])^4/Sec[c + d*x]^(7/2),x]
```

output
```
(Sqrt[Sec[c + d*x]]*(336*a*b*(3*a^2 + 5*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[
(c + d*x)/2, 2] + 20*(5*a^4 + 42*a^2*b^2 + 21*b^4)*Sqrt[Cos[c + d*x]]*Elli
pticF[(c + d*x)/2, 2] + a^2*(65*a^2 + 420*b^2 + 168*a*b*Cos[c + d*x] + 15*
a^2*Cos[2*(c + d*x)])*Sin[2*(c + d*x)]))/(210*d)
```

3.605.3 Rubi [A] (verified)

Time = 1.34 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.04, number of steps used = 17, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.739$, Rules used = {3042, 4328, 27, 3042, 4562, 27, 3042, 4535, 3042, 4258, 3042, 3119, 4533, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\sec(c+dx))^4}{\sec^{\frac{7}{2}}(c+dx)}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^4}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx$$
$$\Big\downarrow \; 4328$$
$$\frac{2}{7}\int \frac{(a+b\sec(c+dx))\left(18ba^2+\left(5a^2+21b^2\right)\sec(c+dx)a+b\left(a^2+7b^2\right)\sec^2(c+dx)\right)}{2\sec^{\frac{5}{2}}(c+dx)}dx+\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))^2}{7d\sec^{\frac{5}{2}}(c+dx)}$$
$$\Big\downarrow \; 27$$

$$\frac{1}{7}\int \frac{(a+b\sec(c+dx))\left(18ba^2+\left(5a^2+21b^2\right)\sec(c+dx)a+b\left(a^2+7b^2\right)\sec^2(c+dx)\right)}{\sec^{\frac{5}{2}}(c+dx)}dx+\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))^2}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$\Big\downarrow$ 3042

$$\frac{1}{7}\int \frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)\left(18ba^2+\left(5a^2+21b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)a+b\left(a^2+7b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))^2}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$\Big\downarrow$ 4562

$$\frac{1}{7}\left(\frac{36a^3b\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}-\frac{2}{5}\int -\frac{5\left(5a^2+39b^2\right)a^2+28b\left(3a^2+5b^2\right)\sec(c+dx)a+5b^2\left(a^2+7b^2\right)\sec^2(c+dx)}{2\sec^{\frac{3}{2}}(c+dx)}dx\right)+\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))^2}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$\Big\downarrow$ 27

$$\frac{1}{7}\left(\frac{1}{5}\int \frac{5\left(5a^2+39b^2\right)a^2+28b\left(3a^2+5b^2\right)\sec(c+dx)a+5b^2\left(a^2+7b^2\right)\sec^2(c+dx)}{\sec^{\frac{3}{2}}(c+dx)}dx+\frac{36a^3b\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}\right)+\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))^2}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$\Big\downarrow$ 3042

$$\frac{1}{7}\left(\frac{1}{5}\int \frac{5\left(5a^2+39b^2\right)a^2+28b\left(3a^2+5b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)a+5b^2\left(a^2+7b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+\frac{36a^3b\sin(c}{5d\sec^{\frac{3}{2}}(c}\right.$$
$$\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))^2}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$\Big\downarrow$ 4535

$$\frac{1}{7}\left(\frac{1}{5}\left(\int \frac{5\left(5a^2+39b^2\right)a^2+5b^2\left(a^2+7b^2\right)\sec^2(c+dx)}{\sec^{\frac{3}{2}}(c+dx)}dx+28ab\left(3a^2+5b^2\right)\int \frac{1}{\sqrt{\sec(c+dx)}}dx\right)+\frac{36a^3b\sin(c}{5d\sec^{\frac{3}{2}}(c}\right.$$
$$\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))^2}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$\Big\downarrow$ 3042

3.605. $\int \frac{(a+b\sec(c+dx))^4}{\sec^{\frac{7}{2}}(c+dx)}\,dx$

$$\frac{1}{7}\left(\frac{1}{5}\left(28ab\left(3a^2+5b^2\right)\int\frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\int\frac{5\left(5a^2+39b^2\right)a^2+5b^2\left(a^2+7b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx\right)+\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))^2}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$\downarrow$ 4258

$$\frac{1}{7}\left(\frac{1}{5}\left(\int\frac{5\left(5a^2+39b^2\right)a^2+5b^2\left(a^2+7b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+28ab\left(3a^2+5b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\right.\right.\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))^2}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$\downarrow$ 3042

$$\frac{1}{7}\left(\frac{1}{5}\left(\int\frac{5\left(5a^2+39b^2\right)a^2+5b^2\left(a^2+7b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+28ab\left(3a^2+5b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\right.\right.\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))^2}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$\downarrow$ 3119

$$\frac{1}{7}\left(\frac{1}{5}\left(\int\frac{5\left(5a^2+39b^2\right)a^2+5b^2\left(a^2+7b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+\frac{56ab\left(3a^2+5b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E}{d}\right.\right.\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))^2}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$\downarrow$ 4533

$$\frac{1}{7}\left(\frac{1}{5}\left(\frac{5}{3}\left(5a^4+42a^2b^2+21b^4\right)\int\sqrt{\sec(c+dx)}dx+\frac{10a^2\left(5a^2+39b^2\right)\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{56ab\left(3a^2+5b^2\right)\sqrt{\cos(c+d}}{}\right.\right.\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))^2}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$\downarrow$ 3042

$$\frac{1}{7}\left(\frac{1}{5}\left(\frac{5}{3}\left(5a^4+42a^2b^2+21b^4\right)\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{10a^2\left(5a^2+39b^2\right)\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{56ab\left(3a^2+5b^2\right)\sqrt{\cos}}{}\right.\right.\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))^2}{7d\sec^{\frac{5}{2}}(c+dx)}$$

3.605. $\int\frac{(a+b\sec(c+dx))^4}{\sec^{\frac{7}{2}}(c+dx)}\,dx$

$$\Big\downarrow \text{4258}$$

$$\frac{1}{7}\left(\frac{1}{5}\left(\frac{5}{3}\left(5a^4+42a^2b^2+21b^4\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{10a^2\left(5a^2+39b^2\right)\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\right.\right.$$
$$\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))^2}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{7}\left(\frac{1}{5}\left(\frac{5}{3}\left(5a^4+42a^2b^2+21b^4\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{10a^2\left(5a^2+39b^2\right)\sin(c+d}{3d\sqrt{\sec(c+dx)}}\right.\right.$$
$$\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))^2}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{3120}$$

$$\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))^2}{7d\sec^{\frac{5}{2}}(c+dx)}+$$
$$\frac{1}{7}\left(\frac{36a^3b\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{1}{5}\left(\frac{10a^2\left(5a^2+39b^2\right)\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{56ab\left(3a^2+5b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+\right.}{d}\right.\right.$$

input
```
Int[(a + b*Sec[c + d*x])^4/Sec[c + d*x]^(7/2),x]
```

output
```
(2*a^2*(a + b*Sec[c + d*x])^2*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/2)) + ((3
6*a^3*b*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)) + ((56*a*b*(3*a^2 + 5*b^2)*
Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (10*(
5*a^4 + 42*a^2*b^2 + 21*b^4)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*
Sqrt[Sec[c + d*x]])/(3*d) + (10*a^2*(5*a^2 + 39*b^2)*Sin[c + d*x])/(3*d*Sq
rt[Sec[c + d*x]]))/5)/7
```

3.605.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4328
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)*
((d*Csc[e + f*x])^n/(f*n)), x] - Simp[1/(d*n)   Int[(a + b*Csc[e + f*x])^(m
 - 3)*(d*Csc[e + f*x])^(n + 1)*Simp[a^2*b*(m - 2*n - 2) - a*(3*b^2*n + a^2*
(n + 1))*Csc[e + f*x] - b*(b^2*n + a^2*(m + n - 1))*Csc[e + f*x]^2, x], x],
 x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && ((Int
egerQ[m] && LtQ[n, -1]) || (IntegersQ[m + 1/2, 2*n] && LeQ[n, -1]))
```

rule 4533
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] +
Simp[(C*m + A*(m + 1))/(b^2*m)   Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fr
eeQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

rule 4535
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Simp[B/b   Int[(b*Cs
c[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2)
, x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

rule 4562
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_)), x_Symbol] :> Simp[A*a*Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*n)), x] + Si
mp[1/(d*n)   Int[(d*Csc[e + f*x])^(n + 1)*Simp[n*(B*a + A*b) + (n*(a*C + B*
b) + A*a*(n + 1))*Csc[e + f*x] + b*C*n*Csc[e + f*x]^2, x], x], x] /; FreeQ[
{a, b, d, e, f, A, B, C}, x] && LtQ[n, -1]
```

3.605. $\int \frac{(a+b\sec(c+dx))^4}{\sec^{\frac{7}{2}}(c+dx)}\,dx$

3.605.4 Maple [A] (verified)

Time = 30.49 (sec) , antiderivative size = 476, normalized size of antiderivative = 2.26

method	result
default	$-\frac{2\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\left(240a^{4}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{8}+\left(-360a^{4}-672a^{3}b\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+(280a^{4}\right.}{}$
parts	Expression too large to display

input
```
int((a+b*sec(d*x+c))^4/sec(d*x+c)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-2/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(240*a^4*co
s(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+(-360*a^4-672*a^3*b)*sin(1/2*d*x+1/2
*c)^6*cos(1/2*d*x+1/2*c)+(280*a^4+672*a^3*b+840*a^2*b^2)*sin(1/2*d*x+1/2*c
)^4*cos(1/2*d*x+1/2*c)+(-80*a^4-168*a^3*b-420*a^2*b^2)*sin(1/2*d*x+1/2*c)^
2*cos(1/2*d*x+1/2*c)+25*a^4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/
2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+210*a^2*b^2*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*
x+1/2*c),2^(1/2))+105*b^4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*
c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-252*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2
^(1/2))*a^3*b-420*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^
(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b^3)/(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1
)^(1/2)/d
```

3.605.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.11

$$\int \frac{(a+b\sec(c+dx))^4}{\sec^{\frac{7}{2}}(c+dx)}\,dx =$$

$$-\frac{5\sqrt{2}(5i\,a^4+42i\,a^2b^2+21i\,b^4)\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c))+5\sqrt{2}(-5i\,a^4}{}$$

input
```
integrate((a+b*sec(d*x+c))^4/sec(d*x+c)^(7/2),x, algorithm="fricas")
```

output
```
-1/105*(5*sqrt(2)*(5*I*a^4 + 42*I*a^2*b^2 + 21*I*b^4)*weierstrassPInverse(
-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 5*sqrt(2)*(-5*I*a^4 - 42*I*a^2*b^2
 - 21*I*b^4)*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 8
4*sqrt(2)*(-3*I*a^3*b - 5*I*a*b^3)*weierstrassZeta(-4, 0, weierstrassPInve
rse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 84*sqrt(2)*(3*I*a^3*b + 5*I*a
*b^3)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*s
in(d*x + c))) - 2*(15*a^4*cos(d*x + c)^3 + 84*a^3*b*cos(d*x + c)^2 + 5*(5*
a^4 + 42*a^2*b^2)*cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/d
```

3.605.6 Sympy [F]

$$\int \frac{(a+b\sec(c+dx))^4}{\sec^{\frac{7}{2}}(c+dx)}\,dx = \int \frac{(a+b\sec{(c+dx)})^4}{\sec^{\frac{7}{2}}{(c+dx)}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))**4/sec(d*x+c)**(7/2),x)
```

output
```
Integral((a + b*sec(c + d*x))**4/sec(c + d*x)**(7/2), x)
```

3.605.7 Maxima [F]

$$\int \frac{(a+b\sec(c+dx))^4}{\sec^{\frac{7}{2}}(c+dx)}\,dx = \int \frac{(b\sec{(dx+c)}+a)^4}{\sec{(dx+c)}^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^4/sec(d*x+c)^(7/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^4/sec(d*x + c)^(7/2), x)
```

3.605.8 Giac [F]

$$\int \frac{(a+b\sec(c+dx))^4}{\sec^{\frac{7}{2}}(c+dx)}\,dx=\int \frac{(b\sec(dx+c)+a)^4}{\sec(dx+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^4/sec(d*x+c)^(7/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^4/sec(d*x + c)^(7/2), x)
```

3.605.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(c+dx))^4}{\sec^{\frac{7}{2}}(c+dx)}\,dx=\int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^4}{\left(\frac{1}{\cos(c+d\,x)}\right)^{7/2}}\,dx$$

input
```
int((a + b/cos(c + d*x))^4/(1/cos(c + d*x))^(7/2),x)
```

output
```
int((a + b/cos(c + d*x))^4/(1/cos(c + d*x))^(7/2), x)
```

3.606 $\int \frac{(a+b\sec(c+dx))^4}{\sec^{\frac{9}{2}}(c+dx)}\,dx$

3.606.1 Optimal result 4608
3.606.2 Mathematica [A] (verified) 4609
3.606.3 Rubi [A] (verified) 4609
3.606.4 Maple [A] (verified) 4614
3.606.5 Fricas [C] (verification not implemented) 4615
3.606.6 Sympy [F(-1)] 4616
3.606.7 Maxima [F] 4616
3.606.8 Giac [F] 4616
3.606.9 Mupad [F(-1)] 4617

3.606.1 Optimal result

Integrand size = 23, antiderivative size = 245

$$\begin{aligned}\int \frac{(a+b\sec(c+dx))^4}{\sec^{\frac{9}{2}}(c+dx)}\,dx &= \frac{2(7a^4+54a^2b^2+15b^4)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{15d} \\ &\quad+\frac{8ab(5a^2+7b^2)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{21d} \\ &\quad+\frac{44a^3b\sin(c+dx)}{63d\sec^{\frac{5}{2}}(c+dx)}+\frac{14a^2(a^2+7b^2)\sin(c+dx)}{45d\sec^{\frac{3}{2}}(c+dx)} \\ &\quad+\frac{8ab(5a^2+7b^2)\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}+\frac{2a^2(a+b\sec(c+dx))^2\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)}\end{aligned}$$

output
```
44/63*a^3*b*sin(d*x+c)/d/sec(d*x+c)^(5/2)+14/45*a^2*(a^2+7*b^2)*sin(d*x+c)
/d/sec(d*x+c)^(3/2)+2/9*a^2*(a+b*sec(d*x+c))^2*sin(d*x+c)/d/sec(d*x+c)^(7/
2)+8/21*a*b*(5*a^2+7*b^2)*sin(d*x+c)/d/sec(d*x+c)^(1/2)+2/15*(7*a^4+54*a^2
*b^2+15*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin
(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+8/21*a*b*(5*a
^2+7*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/
2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

3.606.2 Mathematica [A] (verified)

Time = 1.96 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.69

$$\int \frac{(a+b\sec(c+dx))^4}{\sec^{\frac{9}{2}}(c+dx)}\,dx$$
$$= \frac{\sqrt{\sec(c+dx)}\left(168(7a^4+54a^2b^2+15b^4)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)+480ab(5a^2+7b^2)\sqrt{\cos(c+dx)}\right.}{}$$

input
```
Integrate[(a + b*Sec[c + d*x])^4/Sec[c + d*x]^(9/2),x]
```

output
```
(Sqrt[Sec[c + d*x]]*(168*(7*a^4 + 54*a^2*b^2 + 15*b^4)*Sqrt[Cos[c + d*x]]*
EllipticE[(c + d*x)/2, 2] + 480*a*b*(5*a^2 + 7*b^2)*Sqrt[Cos[c + d*x]]*Ell
ipticF[(c + d*x)/2, 2] + a*(7*a*(43*a^2 + 216*b^2)*Cos[c + d*x] + 5*(312*a
^2*b + 336*b^3 + 72*a^2*b*Cos[2*(c + d*x)] + 7*a^3*Cos[3*(c + d*x)]))*Sin[
2*(c + d*x)]))/(1260*d)
```

3.606.3 Rubi [A] (verified)

Time = 1.57 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.826$, Rules used = {3042, 4328, 27, 3042, 4562, 27, 3042, 4535, 3042, 4256, 3042, 4258, 3042, 3120, 4533, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\sec(c+dx))^4}{\sec^{\frac{9}{2}}(c+dx)}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^4}{\csc\left(c+dx+\frac{\pi}{2}\right)^{9/2}}dx$$
$$\Big\downarrow \; 4328$$
$$\frac{2}{9}\int \frac{(a+b\sec(c+dx))\left(22ba^2+\left(7a^2+27b^2\right)\sec(c+dx)a+3b\left(a^2+3b^2\right)\sec^2(c+dx)\right)}{2\sec^{\frac{7}{2}}(c+dx)}dx+\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))^2}{9d\sec^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{9}\int \frac{(a+b\sec(c+dx))\left(22ba^2+\left(7a^2+27b^2\right)\sec(c+dx)a+3b\left(a^2+3b^2\right)\sec^2(c+dx)\right)}{\sec^{\frac{7}{2}}(c+dx)}dx+\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))^2}{9d\sec^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{9}\int \frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)\left(22ba^2+\left(7a^2+27b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)a+3b\left(a^2+3b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx+\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))^2}{9d\sec^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \; 4562$$

$$\frac{1}{9}\left(\frac{44a^3b\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}-\frac{2}{7}\int -\frac{49\left(a^2+7b^2\right)a^2+36b\left(5a^2+7b^2\right)\sec(c+dx)a+21b^2\left(a^2+3b^2\right)\sec^2(c+dx)}{2\sec^{\frac{5}{2}}(c+dx)}dx\right)+\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))^2}{9d\sec^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{9}\left(\frac{1}{7}\int \frac{49\left(a^2+7b^2\right)a^2+36b\left(5a^2+7b^2\right)\sec(c+dx)a+21b^2\left(a^2+3b^2\right)\sec^2(c+dx)}{\sec^{\frac{5}{2}}(c+dx)}dx+\frac{44a^3b\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}\right)+\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))^2}{9d\sec^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{9}\left(\frac{1}{7}\int \frac{49\left(a^2+7b^2\right)a^2+36b\left(5a^2+7b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)a+21b^2\left(a^2+3b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+\frac{44a^3b\sin(c}{7d\sec^{\frac{5}{2}}(c}\right.$$
$$\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))^2}{9d\sec^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \; 4535$$

$$\frac{1}{9}\left(\frac{1}{7}\left(36ab\left(5a^2+7b^2\right)\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}dx+\int \frac{49\left(a^2+7b^2\right)a^2+21b^2\left(a^2+3b^2\right)\sec^2(c+dx)}{\sec^{\frac{5}{2}}(c+dx)}dx\right)+\frac{44a^3b\sin(c}{7d\sec^{\frac{5}{2}}(c+}\right.$$
$$\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))^2}{9d\sec^{\frac{7}{2}}(c+dx)}$$

3.606. $\int \frac{(a+b\sec(c+dx))^4}{\sec^{\frac{9}{2}}(c+dx)}\,dx$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{9}\left(\frac{1}{7}\left(36ab\left(5a^2+7b^2\right)\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+\int \frac{49\left(a^2+7b^2\right)a^2+21b^2\left(a^2+3b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx\right)+\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))^2}{9d\sec^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \text{4256}$$

$$\frac{1}{9}\left(\frac{1}{7}\left(\int \frac{49\left(a^2+7b^2\right)a^2+21b^2\left(a^2+3b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+36ab\left(5a^2+7b^2\right)\left(\frac{1}{3}\int \sqrt{\sec(c+dx)}dx+\frac{2\,\mathrm{s}}{3d\sqrt{}}\right.\right.\right.\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))^2}{9d\sec^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{9}\left(\frac{1}{7}\left(\int \frac{49\left(a^2+7b^2\right)a^2+21b^2\left(a^2+3b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+36ab\left(5a^2+7b^2\right)\left(\frac{1}{3}\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx\right.\right.\right.\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))^2}{9d\sec^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \text{4258}$$

$$\frac{1}{9}\left(\frac{1}{7}\left(\int \frac{49\left(a^2+7b^2\right)a^2+21b^2\left(a^2+3b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+36ab\left(5a^2+7b^2\right)\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx}\right.\right.\right.\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))^2}{9d\sec^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{9}\left(\frac{1}{7}\left(\int \frac{49\left(a^2+7b^2\right)a^2+21b^2\left(a^2+3b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+36ab\left(5a^2+7b^2\right)\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx}\right.\right.\right.\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))^2}{9d\sec^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \text{3120}$$

3.606. $\int \frac{(a+b\sec(c+dx))^4}{\sec^{\frac{9}{2}}(c+dx)}\,dx$

$$\frac{1}{9}\left(\frac{1}{7}\left(\int \frac{49\left(a^2+7b^2\right)a^2+21b^2\left(a^2+3b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+36ab\left(5a^2+7b^2\right)\left(\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2\sqrt{\cos(c}}{}\right.\right.\right.$$
$$\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))^2}{9d\sec^{\frac{7}{2}}(c+dx)}$$

$\downarrow$ 4533

$$\frac{1}{9}\left(\frac{1}{7}\left(\frac{21}{5}\left(7a^4+54a^2b^2+15b^4\right)\int\frac{1}{\sqrt{\sec(c+dx)}}dx+\frac{98a^2\left(a^2+7b^2\right)\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+36ab\left(5a^2+7b^2\right)\left(\frac{2\sin(c+}{3d\sqrt{\sec(c}}\right.\right.\right.$$
$$\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))^2}{9d\sec^{\frac{7}{2}}(c+dx)}$$

$\downarrow$ 3042

$$\frac{1}{9}\left(\frac{1}{7}\left(\frac{21}{5}\left(7a^4+54a^2b^2+15b^4\right)\int\frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{98a^2\left(a^2+7b^2\right)\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+36ab\left(5a^2+7b^2\right)\left(\frac{2s}{3d\sqrt{}}\right.\right.\right.$$
$$\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))^2}{9d\sec^{\frac{7}{2}}(c+dx)}$$

$\downarrow$ 4258

$$\frac{1}{9}\left(\frac{1}{7}\left(\frac{21}{5}\left(7a^4+54a^2b^2+15b^4\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\cos(c+dx)}dx+\frac{98a^2\left(a^2+7b^2\right)\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+3\right.\right.$$
$$\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))^2}{9d\sec^{\frac{7}{2}}(c+dx)}$$

$\downarrow$ 3042

$$\frac{1}{9}\left(\frac{1}{7}\left(\frac{21}{5}\left(7a^4+54a^2b^2+15b^4\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{98a^2\left(a^2+7b^2\right)\sin(c+d}{5d\sec^{\frac{3}{2}}(c+dx)}\right.\right.$$
$$\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))^2}{9d\sec^{\frac{7}{2}}(c+dx)}$$

$\downarrow$ 3119

$$\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))^2}{9d\sec^{\frac{7}{2}}(c+dx)}+$$
$$\frac{1}{9}\left(\frac{44a^3b\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+\frac{1}{7}\left(\frac{98a^2\left(a^2+7b^2\right)\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+36ab\left(5a^2+7b^2\right)\left(\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2\sqrt{\cos(c+dx)}\sqrt{}}{}\right.\right.\right.$$

3.606. $\int \frac{(a+b\sec(c+dx))^4}{\sec^{\frac{9}{2}}(c+dx)}\,dx$

input
```
Int[(a + b*Sec[c + d*x])^4/Sec[c + d*x]^(9/2),x]
```

output
```
(2*a^2*(a + b*Sec[c + d*x])^2*Sin[c + d*x])/(9*d*Sec[c + d*x]^(7/2)) + ((4
4*a^3*b*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/2)) + ((42*(7*a^4 + 54*a^2*b^2
+ 15*b^4)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])
/(5*d) + (98*a^2*(a^2 + 7*b^2)*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)) + 36
*a*b*(5*a^2 + 7*b^2)*((2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt
[Sec[c + d*x]])/(3*d) + (2*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])))/7)/9
```

3.606.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4328
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)*
((d*Csc[e + f*x])^n/(f*n)), x] - Simp[1/(d*n)   Int[(a + b*Csc[e + f*x])^(m
 - 3)*(d*Csc[e + f*x])^(n + 1)*Simp[a^2*b*(m - 2*n - 2) - a*(3*b^2*n + a^2*
(n + 1))*Csc[e + f*x] - b*(b^2*n + a^2*(m + n - 1))*Csc[e + f*x]^2, x], x],
 x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && ((Int
egerQ[m] && LtQ[n, -1]) || (IntegersQ[m + 1/2, 2*n] && LeQ[n, -1]))
```

rule 4533
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] +
Simp[(C*m + A*(m + 1))/(b^2*m)   Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fr
eeQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

rule 4535
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Simp[B/b   Int[(b*Cs
c[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2)
, x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

rule 4562
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_)), x_Symbol] :> Simp[A*a*Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*n)), x] + Si
mp[1/(d*n)   Int[(d*Csc[e + f*x])^(n + 1)*Simp[n*(B*a + A*b) + (n*(a*C + B*
b) + A*a*(n + 1))*Csc[e + f*x] + b*C*n*Csc[e + f*x]^2, x], x], x] /; FreeQ[
{a, b, d, e, f, A, B, C}, x] && LtQ[n, -1]
```

3.606.4 Maple [A] (verified)

Time = 36.14 (sec) , antiderivative size = 529, normalized size of antiderivative = 2.16

method	result
default	$-\frac{2\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\left(-1120a^{4}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}+\left(2240a^{4}+2880a^{3}b\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{8}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+(-\right.}{}$
parts	Expression too large to display

input
```
int((a+b*sec(d*x+c))^4/sec(d*x+c)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
-2/315*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-1120*a^4*
cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10+(2240*a^4+2880*a^3*b)*sin(1/2*d*x
+1/2*c)^8*cos(1/2*d*x+1/2*c)+(-2072*a^4-4320*a^3*b-3024*a^2*b^2)*sin(1/2*d
*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(952*a^4+3360*a^3*b+3024*a^2*b^2+1680*a*b^3
)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-168*a^4-960*a^3*b-756*a^2*b^2-
840*a*b^3)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+300*a^3*b*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2
*c),2^(1/2))+420*a*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^
2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-147*a^4*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),
2^(1/2))-1134*a^2*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2
-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-315*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/
2))*b^4)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+
1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

3.606.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.05

$$\int \frac{(a+b\sec(c+dx))^4}{\sec^{\frac{9}{2}}(c+dx)}\,dx =$$

$$-\frac{60\sqrt{2}(5i\,a^3b+7i\,ab^3)\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c))+60\sqrt{2}(-5i\,a^3b-7i\,ab}{}$$

input
```
integrate((a+b*sec(d*x+c))^4/sec(d*x+c)^(9/2),x, algorithm="fricas")
```

output
```
-1/315*(60*sqrt(2)*(5*I*a^3*b + 7*I*a*b^3)*weierstrassPInverse(-4, 0, cos(
d*x + c) + I*sin(d*x + c)) + 60*sqrt(2)*(-5*I*a^3*b - 7*I*a*b^3)*weierstra
ssPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 21*sqrt(2)*(-7*I*a^4 -
54*I*a^2*b^2 - 15*I*b^4)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0,
 cos(d*x + c) + I*sin(d*x + c))) + 21*sqrt(2)*(7*I*a^4 + 54*I*a^2*b^2 + 15
*I*b^4)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I
*sin(d*x + c))) - 2*(35*a^4*cos(d*x + c)^4 + 180*a^3*b*cos(d*x + c)^3 + 7*
(7*a^4 + 54*a^2*b^2)*cos(d*x + c)^2 + 60*(5*a^3*b + 7*a*b^3)*cos(d*x + c))
*sin(d*x + c)/sqrt(cos(d*x + c)))/d
```

3.606.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(c+dx))^4}{\sec^{\frac{9}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+b*sec(d*x+c))**4/sec(d*x+c)**(9/2),x)
```

output
```
Timed out
```

3.606.7 Maxima [F]

$$\int \frac{(a+b\sec(c+dx))^4}{\sec^{\frac{9}{2}}(c+dx)}\,dx = \int \frac{(b\sec(dx+c)+a)^4}{\sec(dx+c)^{\frac{9}{2}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^4/sec(d*x+c)^(9/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^4/sec(d*x + c)^(9/2), x)
```

3.606.8 Giac [F]

$$\int \frac{(a+b\sec(c+dx))^4}{\sec^{\frac{9}{2}}(c+dx)}\,dx = \int \frac{(b\sec(dx+c)+a)^4}{\sec(dx+c)^{\frac{9}{2}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^4/sec(d*x+c)^(9/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^4/sec(d*x + c)^(9/2), x)
```

3.606.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(c+dx))^4}{\sec^{\frac{9}{2}}(c+dx)}\,dx=\int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^4}{\left(\frac{1}{\cos(c+d\,x)}\right)^{9/2}}\,dx$$

input
```
int((a + b/cos(c + d*x))^4/(1/cos(c + d*x))^(9/2),x)
```

output
```
int((a + b/cos(c + d*x))^4/(1/cos(c + d*x))^(9/2), x)
```

3.607 $\int \frac{(a+b\sec(c+dx))^4}{\sec^{\frac{11}{2}}(c+dx)}\,dx$

3.607.1 Optimal result 4618
3.607.2 Mathematica [A] (verified) 4619
3.607.3 Rubi [A] (verified) 4619
3.607.4 Maple [A] (verified) 4625
3.607.5 Fricas [C] (verification not implemented) 4625
3.607.6 Sympy [F(-1)] 4626
3.607.7 Maxima [F] 4626
3.607.8 Giac [F] 4627
3.607.9 Mupad [F(-1)] 4627

3.607.1 Optimal result

Integrand size = 23, antiderivative size = 289

$$\begin{aligned}\int \frac{(a+b\sec(c+dx))^4}{\sec^{\frac{11}{2}}(c+dx)}\,dx &= \frac{8ab(7a^2+9b^2)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{15d} \\ &\quad+\frac{2(45a^4+330a^2b^2+77b^4)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{231d} \\ &\quad+\frac{52a^3b\sin(c+dx)}{99d\sec^{\frac{7}{2}}(c+dx)}+\frac{2a^2(9a^2+59b^2)\sin(c+dx)}{77d\sec^{\frac{5}{2}}(c+dx)}+\frac{8ab(7a^2+9b^2)\sin(c+dx)}{45d\sec^{\frac{3}{2}}(c+dx)} \\ &\quad+\frac{2(45a^4+330a^2b^2+77b^4)\sin(c+dx)}{231d\sqrt{\sec(c+dx)}}+\frac{2a^2(a+b\sec(c+dx))^2\sin(c+dx)}{11d\sec^{\frac{9}{2}}(c+dx)}\end{aligned}$$

output
```
52/99*a^3*b*sin(d*x+c)/d/sec(d*x+c)^(7/2)+2/77*a^2*(9*a^2+59*b^2)*sin(d*x+
c)/d/sec(d*x+c)^(5/2)+8/45*a*b*(7*a^2+9*b^2)*sin(d*x+c)/d/sec(d*x+c)^(3/2)
+2/11*a^2*(a+b*sec(d*x+c))^2*sin(d*x+c)/d/sec(d*x+c)^(9/2)+2/231*(45*a^4+3
30*a^2*b^2+77*b^4)*sin(d*x+c)/d/sec(d*x+c)^(1/2)+8/15*a*b*(7*a^2+9*b^2)*(c
os(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c)
,2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+2/231*(45*a^4+330*a^2*b^2+77
*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*
x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

3.607.2 Mathematica [A] (verified)

Time = 2.47 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.69

$$\int \frac{(a+b\sec(c+dx))^4}{\sec^{\frac{11}{2}}(c+dx)}\,dx$$
$$= \frac{\sqrt{\sec(c+dx)}\left(14784ab(7a^2+9b^2)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)+240(45a^4+330a^2b^2+77b^4)\sqrt{\cos(c+}\right.}{}$$

input
```
Integrate[(a + b*Sec[c + d*x])^4/Sec[c + d*x]^(11/2),x]
```

output
```
(Sqrt[Sec[c + d*x]]*(14784*a*b*(7*a^2 + 9*b^2)*Sqrt[Cos[c + d*x]]*Elliptic
E[(c + d*x)/2, 2] + 240*(45*a^4 + 330*a^2*b^2 + 77*b^4)*Sqrt[Cos[c + d*x]]
*EllipticF[(c + d*x)/2, 2] + (616*a*b*(43*a^2 + 36*b^2)*Cos[c + d*x] + 5*(
1593*a^4 + 10296*a^2*b^2 + 1848*b^4 + 72*(8*a^4 + 33*a^2*b^2)*Cos[2*(c + d
*x)] + 616*a^3*b*Cos[3*(c + d*x)] + 63*a^4*Cos[4*(c + d*x)]))*Sin[2*(c + d
*x)]))/(27720*d)
```

3.607.3 Rubi [A] (verified)

Time = 1.78 (sec) , antiderivative size = 275, normalized size of antiderivative = 0.95, number of steps used = 21, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.913$, Rules used = {3042, 4328, 27, 3042, 4562, 27, 3042, 4535, 3042, 4256, 3042, 4258, 3042, 3119, 4533, 3042, 4256, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\sec(c+dx))^4}{\sec^{\frac{11}{2}}(c+dx)}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^4}{\csc\left(c+dx+\frac{\pi}{2}\right)^{11/2}}dx$$
$$\Big\downarrow \; 4328$$

$$\frac{2}{11}\int \frac{(a+b\sec(c+dx))\left(26ba^2+3\left(3a^2+11b^2\right)\sec(c+dx)a+b\left(5a^2+11b^2\right)\sec^2(c+dx)\right)}{2\sec^{\frac{9}{2}}(c+dx)}dx+\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))^2}{11d\sec^{\frac{9}{2}}(c+dx)}$$

$\Big\downarrow$ 27

$$\frac{1}{11}\int \frac{(a+b\sec(c+dx))\left(26ba^2+3\left(3a^2+11b^2\right)\sec(c+dx)a+b\left(5a^2+11b^2\right)\sec^2(c+dx)\right)}{\sec^{\frac{9}{2}}(c+dx)}dx+\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))^2}{11d\sec^{\frac{9}{2}}(c+dx)}$$

$\Big\downarrow$ 3042

$$\frac{1}{11}\int \frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)\left(26ba^2+3\left(3a^2+11b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)a+b\left(5a^2+11b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{9/2}}dx+\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))^2}{11d\sec^{\frac{9}{2}}(c+dx)}$$

$\Big\downarrow$ 4562

$$\frac{1}{11}\left(\frac{52a^3b\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)}-\frac{2}{9}\int -\frac{9\left(9a^2+59b^2\right)a^2+44b\left(7a^2+9b^2\right)\sec(c+dx)a+9b^2\left(5a^2+11b^2\right)\sec^2(c+dx)}{2\sec^{\frac{7}{2}}(c+dx)}dx\right.+\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))^2}{11d\sec^{\frac{9}{2}}(c+dx)}$$

$\Big\downarrow$ 27

$$\frac{1}{11}\left(\frac{1}{9}\int \frac{9\left(9a^2+59b^2\right)a^2+44b\left(7a^2+9b^2\right)\sec(c+dx)a+9b^2\left(5a^2+11b^2\right)\sec^2(c+dx)}{\sec^{\frac{7}{2}}(c+dx)}dx+\frac{52a^3b\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)}\right)+\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))^2}{11d\sec^{\frac{9}{2}}(c+dx)}$$

$\Big\downarrow$ 3042

$$\frac{1}{11}\left(\frac{1}{9}\int \frac{9\left(9a^2+59b^2\right)a^2+44b\left(7a^2+9b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)a+9b^2\left(5a^2+11b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx+\frac{52a^3b\sin}{9d\sec^{\frac{7}{2}}}\right.+\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))^2}{11d\sec^{\frac{9}{2}}(c+dx)}$$

$\Big\downarrow$ 4535

3.607. $\int \frac{(a+b\sec(c+dx))^4}{\sec^{\frac{11}{2}}(c+dx)}\,dx$

$$\frac{1}{11}\left(\frac{1}{9}\left(44ab(7a^2+9b^2)\int\frac{1}{\sec^{\frac{5}{2}}(c+dx)}dx+\int\frac{9(9a^2+59b^2)a^2+9b^2(5a^2+11b^2)\sec^2(c+dx)}{\sec^{\frac{7}{2}}(c+dx)}dx\right)+\frac{52a^3b\sin}{9d\sec^{\frac{7}{2}}}\right.$$
$$\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))^2}{11d\sec^{\frac{9}{2}}(c+dx)}$$

$\downarrow$ 3042

$$\frac{1}{11}\left(\frac{1}{9}\left(44ab(7a^2+9b^2)\int\frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+\int\frac{9(9a^2+59b^2)a^2+9b^2(5a^2+11b^2)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx\right)\right.$$
$$\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))^2}{11d\sec^{\frac{9}{2}}(c+dx)}$$

$\downarrow$ 4256

$$\frac{1}{11}\left(\frac{1}{9}\left(\int\frac{9(9a^2+59b^2)a^2+9b^2(5a^2+11b^2)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx+44ab(7a^2+9b^2)\left(\frac{3}{5}\int\frac{1}{\sqrt{\sec(c+dx)}}dx+\right.\right.\right.$$
$$\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))^2}{11d\sec^{\frac{9}{2}}(c+dx)}$$

$\downarrow$ 3042

$$\frac{1}{11}\left(\frac{1}{9}\left(\int\frac{9(9a^2+59b^2)a^2+9b^2(5a^2+11b^2)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx+44ab(7a^2+9b^2)\left(\frac{3}{5}\int\frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}\right.\right.\right.$$
$$\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))^2}{11d\sec^{\frac{9}{2}}(c+dx)}$$

$\downarrow$ 4258

$$\frac{1}{11}\left(\frac{1}{9}\left(\int\frac{9(9a^2+59b^2)a^2+9b^2(5a^2+11b^2)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx+44ab(7a^2+9b^2)\left(\frac{3}{5}\sqrt{\cos(c+dx)}\sqrt{\sec(c+}\right.\right.\right.$$
$$\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))^2}{11d\sec^{\frac{9}{2}}(c+dx)}$$

$\downarrow$ 3042

$$\frac{1}{11}\left(\frac{1}{9}\left(\int\frac{9(9a^2+59b^2)a^2+9b^2(5a^2+11b^2)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx+44ab(7a^2+9b^2)\left(\frac{3}{5}\sqrt{\cos(c+dx)}\sqrt{\sec(c+}\right.\right.\right.$$
$$\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))^2}{11d\sec^{\frac{9}{2}}(c+dx)}$$

3.607. $\int\frac{(a+b\sec(c+dx))^4}{\sec^{\frac{11}{2}}(c+dx)}\,dx$

$$\Big\downarrow \; 3119$$

$$\frac{1}{11}\left(\frac{1}{9}\left(\int \frac{9\left(9a^2+59b^2\right)a^2+9b^2\left(5a^2+11b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx+44ab\left(7a^2+9b^2\right)\left(\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{6\sqrt{\mathrm{co}}}{}\right.\right.\right.$$
$$\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))^2}{11d\sec^{\frac{9}{2}}(c+dx)}$$

$$\Big\downarrow \; 4533$$

$$\frac{1}{11}\left(\frac{1}{9}\left(\frac{9}{7}\left(45a^4+330a^2b^2+77b^4\right)\int\frac{1}{\sec^{\frac{3}{2}}(c+dx)}dx+\frac{18a^2\left(9a^2+59b^2\right)\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+44ab\left(7a^2+9b^2\right)\left(\frac{2\sin}{5d\sec}\right.\right.\right.$$
$$\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))^2}{11d\sec^{\frac{9}{2}}(c+dx)}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{11}\left(\frac{1}{9}\left(\frac{9}{7}\left(45a^4+330a^2b^2+77b^4\right)\int\frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+\frac{18a^2\left(9a^2+59b^2\right)\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+44ab\left(7a^2+9b^2\right)\left(\right.\right.\right.$$
$$\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))^2}{11d\sec^{\frac{9}{2}}(c+dx)}$$

$$\Big\downarrow \; 4256$$

$$\frac{1}{11}\left(\frac{1}{9}\left(\frac{9}{7}\left(45a^4+330a^2b^2+77b^4\right)\left(\frac{1}{3}\int\sqrt{\sec(c+dx)}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)+\frac{18a^2\left(9a^2+59b^2\right)\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+\right.\right.$$
$$\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))^2}{11d\sec^{\frac{9}{2}}(c+dx)}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{11}\left(\frac{1}{9}\left(\frac{9}{7}\left(45a^4+330a^2b^2+77b^4\right)\left(\frac{1}{3}\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)+\frac{18a^2\left(9a^2+59b^2\right)\sin(c+}{7d\sec^{\frac{5}{2}}(c+dx)}\right.\right.$$
$$\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))^2}{11d\sec^{\frac{9}{2}}(c+dx)}$$

$$\Big\downarrow \; 4258$$

$$\frac{1}{11}\left(\frac{1}{9}\left(\frac{9}{7}\left(45a^4+330a^2b^2+77b^4\right)\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)+\frac{18a}{}\right.\right.$$
$$\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))^2}{11d\sec^{\frac{9}{2}}(c+dx)}$$

3.607. $\int \frac{(a+b\sec(c+dx))^4}{\sec^{\frac{11}{2}}(c+dx)}\,dx$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{11}\left(\frac{1}{9}\left(\frac{9}{7}(45a^4+330a^2b^2+77b^4)\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)\right.\right.$$
$$\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))^2}{11d\sec^{\frac{9}{2}}(c+dx)}$$

$$\Big\downarrow \text{3120}$$

$$\frac{2a^2\sin(c+dx)(a+b\sec(c+dx))^2}{11d\sec^{\frac{9}{2}}(c+dx)}+$$
$$\frac{1}{11}\left(\frac{52a^3b\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)}+\frac{1}{9}\left(\frac{18a^2(9a^2+59b^2)\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+44ab(7a^2+9b^2)\left(\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{6\sqrt{\cos(c+dx)}}{}\right.\right.\right.$$

input
```
Int[(a + b*Sec[c + d*x])^4/Sec[c + d*x]^(11/2),x]
```

output
```
(2*a^2*(a + b*Sec[c + d*x])^2*Sin[c + d*x])/(11*d*Sec[c + d*x]^(9/2)) + ((
52*a^3*b*Sin[c + d*x])/(9*d*Sec[c + d*x]^(7/2)) + ((18*a^2*(9*a^2 + 59*b^2
)*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/2)) + 44*a*b*(7*a^2 + 9*b^2)*((6*Sqrt
[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d) + (2*Si
n[c + d*x])/(5*d*Sec[c + d*x]^(3/2))) + (9*(45*a^4 + 330*a^2*b^2 + 77*b^4)
*((2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d
) + (2*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])))/7)/9)/11
```

3.607.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4328
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)*
((d*Csc[e + f*x])^n/(f*n)), x] - Simp[1/(d*n)   Int[(a + b*Csc[e + f*x])^(m
 - 3)*(d*Csc[e + f*x])^(n + 1)*Simp[a^2*b*(m - 2*n - 2) - a*(3*b^2*n + a^2*
(n + 1))*Csc[e + f*x] - b*(b^2*n + a^2*(m + n - 1))*Csc[e + f*x]^2, x], x],
 x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && ((Int
egerQ[m] && LtQ[n, -1]) || (IntegersQ[m + 1/2, 2*n] && LeQ[n, -1]))
```

rule 4533
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] +
Simp[(C*m + A*(m + 1))/(b^2*m)   Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fr
eeQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

rule 4535
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Simp[B/b   Int[(b*Cs
c[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2)
, x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

rule 4562
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_)), x_Symbol] :> Simp[A*a*Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*n)), x] + Si
mp[1/(d*n)   Int[(d*Csc[e + f*x])^(n + 1)*Simp[n*(B*a + A*b) + (n*(a*C + B*
b) + A*a*(n + 1))*Csc[e + f*x] + b*C*n*Csc[e + f*x]^2, x], x], x] /; FreeQ[
{a, b, d, e, f, A, B, C}, x] && LtQ[n, -1]
```

3.607. $\int \frac{(a+b\sec(c+dx))^4}{\sec^{\frac{11}{2}}(c+dx)}\,dx$

3.607.4 Maple [A] (verified)

Time = 46.60 (sec) , antiderivative size = 586, normalized size of antiderivative = 2.03

method	result
default	$-\frac{2\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\left(20160a^{4}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{12}+\left(-50400a^{4}-49280a^{3}b\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}{}$
parts	Expression too large to display

input
```
int((a+b*sec(d*x+c))^4/sec(d*x+c)^(11/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3465*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(20160*a^4
*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^12+(-50400*a^4-49280*a^3*b)*sin(1/2
*d*x+1/2*c)^10*cos(1/2*d*x+1/2*c)+(56880*a^4+98560*a^3*b+47520*a^2*b^2)*si
n(1/2*d*x+1/2*c)^8*cos(1/2*d*x+1/2*c)+(-34920*a^4-91168*a^3*b-71280*a^2*b^
2-22176*a*b^3)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(13860*a^4+41888*a^
3*b+55440*a^2*b^2+22176*a*b^3+4620*b^4)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1
/2*c)+(-2790*a^4-7392*a^3*b-15840*a^2*b^2-5544*a*b^3-2310*b^4)*sin(1/2*d*x
+1/2*c)^2*cos(1/2*d*x+1/2*c)+675*a^4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1
/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+4950*a^2*b^
2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(
cos(1/2*d*x+1/2*c),2^(1/2))+1155*b^4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1
/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-6468*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*
d*x+1/2*c),2^(1/2))*a^3*b-8316*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x
+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b^3)/(-2*sin(1/
2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d
*x+1/2*c)^2-1)^(1/2)/d
```

3.607.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.14 (sec) , antiderivative size = 286, normalized size of antiderivative = 0.99

$$\int \frac{(a+b\sec(c+dx))^4}{\sec^{\frac{11}{2}}(c+dx)}\,dx =$$

$$-\frac{15\sqrt{2}(45i\,a^4+330i\,a^2b^2+77i\,b^4)\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c))+15\sqrt{2}(-4}{}$$

input
```
integrate((a+b*sec(d*x+c))^4/sec(d*x+c)^(11/2),x, algorithm="fricas")
```

output
```
-1/3465*(15*sqrt(2)*(45*I*a^4 + 330*I*a^2*b^2 + 77*I*b^4)*weierstrassPInve
rse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 15*sqrt(2)*(-45*I*a^4 - 330*I*
a^2*b^2 - 77*I*b^4)*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x +
c)) + 924*sqrt(2)*(-7*I*a^3*b - 9*I*a*b^3)*weierstrassZeta(-4, 0, weierstr
assPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 924*sqrt(2)*(7*I*a^3*
b + 9*I*a*b^3)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x +
 c) - I*sin(d*x + c))) - 2*(315*a^4*cos(d*x + c)^5 + 1540*a^3*b*cos(d*x +
c)^4 + 135*(3*a^4 + 22*a^2*b^2)*cos(d*x + c)^3 + 308*(7*a^3*b + 9*a*b^3)*c
os(d*x + c)^2 + 15*(45*a^4 + 330*a^2*b^2 + 77*b^4)*cos(d*x + c))*sin(d*x +
 c)/sqrt(cos(d*x + c)))/d
```

3.607.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(c+dx))^4}{\sec^{\frac{11}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+b*sec(d*x+c))**4/sec(d*x+c)**(11/2),x)
```

output
```
Timed out
```

3.607.7 Maxima [F]

$$\int \frac{(a+b\sec(c+dx))^4}{\sec^{\frac{11}{2}}(c+dx)}\,dx = \int \frac{(b\sec(dx+c)+a)^4}{\sec(dx+c)^{\frac{11}{2}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^4/sec(d*x+c)^(11/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^4/sec(d*x + c)^(11/2), x)
```

3.607.8 Giac [F]

$$\int \frac{(a+b\sec(c+dx))^4}{\sec^{\frac{11}{2}}(c+dx)}\,dx=\int \frac{(b\sec(dx+c)+a)^4}{\sec(dx+c)^{\frac{11}{2}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^4/sec(d*x+c)^(11/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^4/sec(d*x + c)^(11/2), x)
```

3.607.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(c+dx))^4}{\sec^{\frac{11}{2}}(c+dx)}\,dx=\int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^4}{\left(\frac{1}{\cos(c+d\,x)}\right)^{11/2}}\,dx$$

input
```
int((a + b/cos(c + d*x))^4/(1/cos(c + d*x))^(11/2),x)
```

output
```
int((a + b/cos(c + d*x))^4/(1/cos(c + d*x))^(11/2), x)
```

3.608 $\int \frac{\sec^{\frac{7}{2}}(c+dx)}{a+b\sec(c+dx)}\,dx$

3.608.1 Optimal result 4628
3.608.2 Mathematica [A] (verified) 4629
3.608.3 Rubi [A] (verified) 4629
3.608.4 Maple [A] (verified) 4634
3.608.5 Fricas [F(-1)] 4635
3.608.6 Sympy [F] 4635
3.608.7 Maxima [F] 4636
3.608.8 Giac [F] 4636
3.608.9 Mupad [F(-1)] 4636

3.608.1 Optimal result

Integrand size = 23, antiderivative size = 188

$$\begin{aligned}\int \frac{\sec^{\frac{7}{2}}(c+dx)}{a+b\sec(c+dx)}\,dx &= \frac{2a\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{b^2d} \\ &+ \frac{2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{3bd} \\ &+ \frac{2a^2\sqrt{\cos(c+dx)}\,\text{EllipticPi}\left(\frac{2a}{a+b},\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{b^2(a+b)d} \\ &- \frac{2a\sqrt{\sec(c+dx)}\sin(c+dx)}{b^2d} + \frac{2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3bd}\end{aligned}$$

output
```
2/3*sec(d*x+c)^(3/2)*sin(d*x+c)/b/d-2*a*sin(d*x+c)*sec(d*x+c)^(1/2)/b^2/d+
2*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+
1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/b^2/d+2/3*(cos(1/2*d*x+1
/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*co
s(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/b/d+2*a^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos
(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*a/(a+b),2^(1/2))*cos(d*x+c
)^(1/2)*sec(d*x+c)^(1/2)/b^2/(a+b)/d
```

3.608.2 Mathematica [A] (verified)

Time = 35.47 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.88

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{a+b\sec(c+dx)}\,dx = -\frac{\cot(c+dx)\left(-b^2\sec^{\frac{5}{2}}(c+dx)+b^2\cos(2(c+dx))\sec^{\frac{5}{2}}(c+dx)+6abE\left(\arcsin\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)\right.}{}$$

input
```
Integrate[Sec[c + d*x]^(7/2)/(a + b*Sec[c + d*x]),x]
```

output
```
-1/3*(Cot[c + d*x]*(-(b^2*Sec[c + d*x]^(5/2)) + b^2*Cos[2*(c + d*x)]*Sec[c
 + d*x]^(5/2) + 6*a*b*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[
c + d*x]^2] - 2*(3*a^2 + 3*a*b + b^2)*EllipticF[ArcSin[Sqrt[Sec[c + d*x]]]
, -1]*Sqrt[-Tan[c + d*x]^2] + 6*a^2*EllipticPi[-(b/a), ArcSin[Sqrt[Sec[c +
 d*x]]], -1]*Sqrt[-Tan[c + d*x]^2]))/(b^3*d)
```

3.608.3 Rubi [A] (verified)

Time = 1.56 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.07, number of steps used = 18, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.783$, Rules used = {3042, 4338, 27, 3042, 4590, 27, 3042, 4594, 3042, 4274, 3042, 4258, 3042, 3119, 3120, 4336, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{a+b\sec(c+dx)}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \; 4338$$

$$\frac{2\int \frac{\sqrt{\sec(c+dx)}\left(-3a\sec^2(c+dx)+b\sec(c+dx)+a\right)}{2(a+b\sec(c+dx))}dx}{3b}+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3bd}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{\sqrt{\sec(c+dx)}\left(-3a\sec^2(c+dx)+b\sec(c+dx)+a\right)}{a+b\sec(c+dx)}dx}{3b}+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3bd}$$

$\Big\downarrow$ 3042

$$\frac{\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(-3a\csc\left(c+dx+\frac{\pi}{2}\right)^2+b\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{3b}+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3bd}$$

$\Big\downarrow$ 4590

$$\frac{\frac{2\int \frac{3a^2+4b\sec(c+dx)a+\left(3a^2+b^2\right)\sec^2(c+dx)}{2\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}dx}{b}-\frac{6a\sin(c+dx)\sqrt{\sec(c+dx)}}{bd}}{3b}+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3bd}$$

$\Big\downarrow$ 27

$$\frac{\frac{\int \frac{3a^2+4b\sec(c+dx)a+\left(3a^2+b^2\right)\sec^2(c+dx)}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}dx}{b}-\frac{6a\sin(c+dx)\sqrt{\sec(c+dx)}}{bd}}{3b}+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3bd}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\int \frac{3a^2+4b\csc\left(c+dx+\frac{\pi}{2}\right)a+\left(3a^2+b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{b}-\frac{6a\sin(c+dx)\sqrt{\sec(c+dx)}}{bd}}{3b}+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3bd}$$

$\Big\downarrow$ 4594

$$\frac{\frac{3a^2\int \frac{\sec^{\frac{3}{2}}(c+dx)}{a+b\sec(c+dx)}dx+\frac{\int \frac{3a^3+b\sec(c+dx)a^2}{\sqrt{\sec(c+dx)}}dx}{a^2}}{b}-\frac{6a\sin(c+dx)\sqrt{\sec(c+dx)}}{bd}}{3b}+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3bd}$$

$\Big\downarrow$ 3042

$$\frac{\frac{3a^2\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{\int \frac{3a^3+b\csc\left(c+dx+\frac{\pi}{2}\right)a^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2}}{b}-\frac{6a\sin(c+dx)\sqrt{\sec(c+dx)}}{bd}}{3b}+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3bd}$$

$\Big\downarrow$ 4274

$$\frac{\frac{3a^2\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{3a^3\int \frac{1}{\sqrt{\sec(c+dx)}}dx+a^2b\int \sqrt{\sec(c+dx)}dx}{a^2}}{b}-\frac{6a\sin(c+dx)\sqrt{\sec(c+dx)}}{bd}}{3b}+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3bd}$$

$\Big\downarrow$ 3042

3.608. $\int \frac{\sec^{\frac{7}{2}}(c+dx)}{a+b\sec(c+dx)}\,dx$

$$\frac{\frac{3a^2\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{3a^3\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+a^2b\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}}{b}-\frac{6a\sin(c+dx)\sqrt{\sec(c+dx)}}{bd}}{3b}+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3bd}$$

$\downarrow$ 4258

$$\frac{\frac{3a^2\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{3a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\cos(c+dx)}dx+a^2b\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{a^2}}{b}-\frac{6a\sin(c+dx)\sqrt{\sec(c+dx)}}{bd}}{3b}+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3bd}$$

$\downarrow$ 3042

$$\frac{\frac{3a^2\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{3a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+a^2b\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2}}{b}-\frac{6a\sin(c+dx)\sqrt{\sec(c+}}{bd}}{3b}$$

$$\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3bd}$$

$\downarrow$ 3119

$$\frac{\frac{3a^2\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{a^2b\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{6a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}}{a^2}}{b}-\frac{6a\sin(c+dx)\sqrt{\sec(c+dx)}}{bd}}{3b}$$

$$\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3bd}$$

$\downarrow$ 3120

$$\frac{\frac{3a^2\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{\frac{6a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a^2b\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}}{a^2}}{b}-\frac{6a\sin(c+dx)\sqrt{\sec(c+dx)}}{bd}}{3b}$$

$$\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3bd}$$

$\downarrow$ 4336

3.608. $\int \frac{\sec^{\frac{7}{2}}(c+dx)}{a+b\sec(c+dx)}\,dx$

$$\frac{\frac{3a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}(b+a\cos(c+dx))}dx+\frac{\frac{6a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a^2b\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx\right.}{d}}{a^2}}{b}}{3b}$$

$$\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3bd}$$

$\Big\downarrow$ 3042

$$\frac{\frac{3a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(b+a\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx+\frac{\frac{6a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a^2b\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}}{d}}{a^2}}{b}}{3b}$$

$$\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3bd}$$

$\Big\downarrow$ 3284

$$\frac{\frac{\frac{6a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticPi}\left(\frac{2a}{a+b},\frac{1}{2}(c+dx),2\right)}{d(a+b)}+\frac{\frac{6a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a^2b\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}}{a^2}}{b}-}{3b}$$

$$\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3bd}$$

input
```
Int[Sec[c + d*x]^(7/2)/(a + b*Sec[c + d*x]),x]
```

output
```
(2*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*b*d) + ((((6*a^3*Sqrt[Cos[c + d*x]]
*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (2*a^2*b*Sqrt[Cos[c + d
*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d)/a^2 + (6*a^2*Sqrt[Co
s[c + d*x]]*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/
((a + b)*d))/b - (6*a*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(b*d))/(3*b)
```

3.608.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4336
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]]   Int[
1/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4338
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-d^3)*Cot[e + f*x]*((d*Csc[e + f*x])^(n - 3)/(b*f*
(n - 2))), x] + Simp[d^3/(b*(n - 2))   Int[(d*Csc[e + f*x])^(n - 3)*(Simp[a
*(n - 3) + b*(n - 3)*Csc[e + f*x] - a*(n - 2)*Csc[e + f*x]^2, x]/(a + b*Csc
[e + f*x])), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && Gt
Q[n, 3]
```

3.608. $\int \frac{\sec^{\frac{7}{2}}(c+dx)}{a+b\sec(c+dx)}\,dx$

rule 4590
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(-C)*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
)*((d*Csc[e + f*x])^(n - 1)/(b*f*(m + n + 1))), x] + Simp[d/(b*(m + n + 1))
   Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (
A*b*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2
 - b^2, 0] && GtQ[n, 0]
```

rule 4594
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Simp[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2)   Int[(d*Csc[e +
f*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Simp[1/a^2   Int[(a*A - (A*b - a
*B)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A,
B, C}, x] && NeQ[a^2 - b^2, 0]
```

3.608.4 Maple [A] (verified)

Time = 19.59 (sec) , antiderivative size = 423, normalized size of antiderivative = 2.25

method	result
default	$-\frac{\sqrt{-\left(-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(\frac{-\frac{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}}{3\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-\frac{1}{2}\right)^2}+\frac{2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1}\ \mathrm{Ell}}{3\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\sin\left(\frac{d}{}\right.}}}{b}\right.}{}$

input
```
int(sec(d*x+c)^(7/2)/(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2/b*(-1/6*cos(
1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1
/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/
2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellip
ticF(cos(1/2*d*x+1/2*c),2^(1/2)))-2*a/b^2/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*
d*x+1/2*c)^2-1)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*co
s(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-(sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipti
cE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2))-2*a^3/b^2
/(a^2-a*b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/
(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*
x+1/2*c),2*a/(a-b),2^(1/2)))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)
^(1/2)/d
```

3.608.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{a+b\sec(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)^(7/2)/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

output
```
Timed out
```

3.608.6 Sympy [F]

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{a+b\sec(c+dx)}\,dx = \int \frac{\sec^{\frac{7}{2}}(c+dx)}{a+b\sec(c+dx)}\,dx$$

input
```
integrate(sec(d*x+c)**(7/2)/(a+b*sec(d*x+c)),x)
```

output
```
Integral(sec(c + d*x)**(7/2)/(a + b*sec(c + d*x)), x)
```

3.608.7 Maxima [F]

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{a+b\sec(c+dx)}\,dx = \int \frac{\sec(dx+c)^{\frac{7}{2}}}{b\sec(dx+c)+a}\,dx$$

input
```
integrate(sec(d*x+c)^(7/2)/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)^(7/2)/(b*sec(d*x + c) + a), x)
```

3.608.8 Giac [F]

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{a+b\sec(c+dx)}\,dx = \int \frac{\sec(dx+c)^{\frac{7}{2}}}{b\sec(dx+c)+a}\,dx$$

input
```
integrate(sec(d*x+c)^(7/2)/(a+b*sec(d*x+c)),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^(7/2)/(b*sec(d*x + c) + a), x)
```

3.608.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{a+b\sec(c+dx)}\,dx = \int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{7/2}}{a+\frac{b}{\cos(c+d\,x)}}\,dx$$

input
```
int((1/cos(c + d*x))^(7/2)/(a + b/cos(c + d*x)),x)
```

output
```
int((1/cos(c + d*x))^(7/2)/(a + b/cos(c + d*x)), x)
```

3.609 $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{a+b\sec(c+dx)}\,dx$

3.609.1 Optimal result 4637
3.609.2 Mathematica [A] (verified) 4637
3.609.3 Rubi [A] (verified) 4638
3.609.4 Maple [B] (verified) 4641
3.609.5 Fricas [F(-1)] 4641
3.609.6 Sympy [F] 4642
3.609.7 Maxima [F] 4642
3.609.8 Giac [F] 4642
3.609.9 Mupad [F(-1)] 4643

3.609.1 Optimal result

Integrand size = 23, antiderivative size = 117

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{a+b\sec(c+dx)}\,dx = -\frac{2\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{bd} - \frac{2a\sqrt{\cos(c+dx)}\,\text{EllipticPi}\left(\frac{2a}{a+b},\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{b(a+b)d} + \frac{2\sqrt{\sec(c+dx)}\sin(c+dx)}{bd}$$

output
```
2*sin(d*x+c)*sec(d*x+c)^(1/2)/b/d-2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d
*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c
)^(1/2)/b/d-2*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi
(sin(1/2*d*x+1/2*c),2*a/(a+b),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/b
/(a+b)/d
```

3.609.2 Mathematica [A] (verified)

Time = 15.42 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.71

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{a+b\sec(c+dx)}\,dx = \frac{2\cot(c+dx)\left(bE\left(\arcsin\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)-(a+b)\,\text{EllipticF}\left(\arcsin\left(\sqrt{\sec(c+dx)}\right),-1\right)+a\,\text{E}}{b^2d}$$

input
```
Integrate[Sec[c + d*x]^(5/2)/(a + b*Sec[c + d*x]),x]
```

output
```
(2*Cot[c + d*x]*(b*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1] - (a + b)*Ell
ipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1] + a*EllipticPi[-(b/a), ArcSin[Sqrt[
Sec[c + d*x]]], -1])*Sqrt[-Tan[c + d*x]^2])/(b^2*d)
```

3.609.3 Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.99, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules used = {3042, 4337, 3042, 4255, 3042, 4258, 3042, 3119, 4336, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{a+b\sec(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{4337}$$

$$\frac{\int \sec^{\frac{3}{2}}(c+dx)dx}{b}-\frac{a\int \frac{\sec^{\frac{3}{2}}(c+dx)}{a+b\sec(c+dx)}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}dx}{b}-\frac{a\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b}$$

$$\Big\downarrow \text{4255}$$

$$\frac{\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\int \frac{1}{\sqrt{\sec(c+dx)}}dx}{b}-\frac{a\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\frac{a\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b}$$

$$\Big\downarrow \text{4258}$$

$$\frac{\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\cos(c+dx)}dx}{b}-\frac{a\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b}-\frac{a\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b}$$

$$\Big\downarrow \; 3119$$

$$\frac{\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|2\right)}{d}}{b}-\frac{a\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b}$$

$$\Big\downarrow \; 4336$$

$$\frac{\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|2\right)}{d}}{b}-\frac{a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}(b+a\cos(c+dx))}dx}{b}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|2\right)}{d}}{b}-\frac{a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(b+a\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{b}$$

$$\Big\downarrow \; 3284$$

$$\frac{\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|2\right)}{d}}{b}-\frac{2a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticPi}\left(\frac{2a}{a+b},\frac{1}{2}(c+dx),2\right)}{bd(a+b)}$$

input
```
Int[Sec[c + d*x]^(5/2)/(a + b*Sec[c + d*x]),x]
```

output
```
(-2*a*Sqrt[Cos[c + d*x]]*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2]*Sqrt[Se
c[c + d*x]])/(b*(a + b)*d) + ((-2*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2
, 2]*Sqrt[Sec[c + d*x]])/d + (2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d)/b
```

3.609. $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{a+b\sec(c+dx)}\,dx$

3.609.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4336
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]]   Int[
1/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4337
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(5/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[d/b   Int[(d*Csc[e + f*x])^(3/2), x], x] - Simp[a*
(d/b)   Int[(d*Csc[e + f*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a
, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

3.609. $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{a+b\sec(c+dx)}\,dx$

3.609.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 352 vs. $2(159) = 318$.

Time = 13.00 (sec) , antiderivative size = 353, normalized size of antiderivative = 3.02

method	result
default	$-\frac{2\left(-2\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,(a-b)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+a\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1}\,\mathrm{EllipticPi}\left(\cos\left(\right.\right.}{}$

input
```
int(sec(d*x+c)^(5/2)/(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-2*(-2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(a-b)*cos(1/2*
d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))*(-2
*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+(-2*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1
/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a-(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/
2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b)/b/(-2*sin
(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(a-b)/sin(1/2*d*x+1/2*c)/(2*
cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

3.609.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{a+b\sec(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)^(5/2)/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

output
```
Timed out
```

3.609.6 Sympy [F]

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{a+b\sec(c+dx)}\,dx=\int \frac{\sec^{\frac{5}{2}}(c+dx)}{a+b\sec(c+dx)}\,dx$$

input
```
integrate(sec(d*x+c)**(5/2)/(a+b*sec(d*x+c)),x)
```

output
```
Integral(sec(c + d*x)**(5/2)/(a + b*sec(c + d*x)), x)
```

3.609.7 Maxima [F]

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{a+b\sec(c+dx)}\,dx=\int \frac{\sec(dx+c)^{\frac{5}{2}}}{b\sec(dx+c)+a}\,dx$$

input
```
integrate(sec(d*x+c)^(5/2)/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)^(5/2)/(b*sec(d*x + c) + a), x)
```

3.609.8 Giac [F]

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{a+b\sec(c+dx)}\,dx=\int \frac{\sec(dx+c)^{\frac{5}{2}}}{b\sec(dx+c)+a}\,dx$$

input
```
integrate(sec(d*x+c)^(5/2)/(a+b*sec(d*x+c)),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^(5/2)/(b*sec(d*x + c) + a), x)
```

3.609.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{a+b\sec(c+dx)}\,dx=\int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}}{a+\frac{b}{\cos(c+d\,x)}}\,dx$$

input
```
int((1/cos(c + d*x))^(5/2)/(a + b/cos(c + d*x)),x)
```

output
```
int((1/cos(c + d*x))^(5/2)/(a + b/cos(c + d*x)), x)
```

3.610 $\int \frac{\sec^{\frac{3}{2}}(c+dx)}{a+b\sec(c+dx)}\,dx$

3.610.1 Optimal result 4644
3.610.2 Mathematica [A] (verified) 4644
3.610.3 Rubi [A] (verified) 4645
3.610.4 Maple [B] (verified) 4646
3.610.5 Fricas [F(-1)] 4647
3.610.6 Sympy [F] 4647
3.610.7 Maxima [F] 4647
3.610.8 Giac [F] 4648
3.610.9 Mupad [F(-1)] 4648

3.610.1 Optimal result

Integrand size = 23, antiderivative size = 49

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{a+b\sec(c+dx)}\,dx=\frac{2\sqrt{\cos(c+dx)}\,\text{EllipticPi}\left(\frac{2a}{a+b},\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{(a+b)d}$$

output
```
2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1
/2*c),2*a/(a+b),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/(a+b)/d
```

3.610.2 Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.29

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{a+b\sec(c+dx)}\,dx$$
$$=\frac{2\cot(c+dx)\left(\text{EllipticF}\left(\arcsin\left(\sqrt{\sec(c+dx)}\right),-1\right)-\text{EllipticPi}\left(-\frac{b}{a},\arcsin\left(\sqrt{\sec(c+dx)}\right),-1\right)\right)}{bd}$$

input
```
Integrate[Sec[c + d*x]^(3/2)/(a + b*Sec[c + d*x]),x]
```

output
```
(2*Cot[c + d*x]*(EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1] - EllipticPi[-(
b/a), ArcSin[Sqrt[Sec[c + d*x]]], -1])*Sqrt[-Tan[c + d*x]^2])/(b*d)
```

3.610.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3042, 4336, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{a+b\sec(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{4336}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}(b+a\cos(c+dx))}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(b+a\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx$$

$$\Big\downarrow \text{3284}$$

$$\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticPi}\left(\frac{2a}{a+b},\frac{1}{2}(c+dx),2\right)}{d(a+b)}$$

input
```
Int[Sec[c + d*x]^(3/2)/(a + b*Sec[c + d*x]),x]
```

output
```
(2*Sqrt[Cos[c + d*x]]*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c
 + d*x]])/((a + b)*d)
```

3.610.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 4336
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]]   Int[
1/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

3.610.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 149 vs. $2(71) = 142$.

Time = 3.61 (sec) , antiderivative size = 150, normalized size of antiderivative = 3.06

method	result	size
default	$\frac{2\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+1}\,\operatorname{EllipticPi}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\frac{2a}{a-b},\sqrt{2}\right)}{(a-b)\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1}\,d}$	150

input
```
int(sec(d*x+c)^(3/2)/(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
2*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*
c),2*a/(a-b),2^(1/2))/(a-b)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

3.610.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{a+b\sec(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)^(3/2)/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

output
```
Timed out
```

3.610.6 Sympy [F]

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{a+b\sec(c+dx)}\,dx = \int \frac{\sec^{\frac{3}{2}}(c+dx)}{a+b\sec(c+dx)}\,dx$$

input
```
integrate(sec(d*x+c)**(3/2)/(a+b*sec(d*x+c)),x)
```

output
```
Integral(sec(c + d*x)**(3/2)/(a + b*sec(c + d*x)), x)
```

3.610.7 Maxima [F]

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{a+b\sec(c+dx)}\,dx = \int \frac{\sec(dx+c)^{\frac{3}{2}}}{b\sec(dx+c)+a}\,dx$$

input
```
integrate(sec(d*x+c)^(3/2)/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)^(3/2)/(b*sec(d*x + c) + a), x)
```

3.610.8 Giac [F]

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{a+b\sec(c+dx)}\,dx = \int \frac{\sec(dx+c)^{\frac{3}{2}}}{b\sec(dx+c)+a}\,dx$$

input
```
integrate(sec(d*x+c)^(3/2)/(a+b*sec(d*x+c)),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^(3/2)/(b*sec(d*x + c) + a), x)
```

3.610.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{a+b\sec(c+dx)}\,dx = \int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}{a+\frac{b}{\cos(c+d\,x)}}\,dx$$

input
```
int((1/cos(c + d*x))^(3/2)/(a + b/cos(c + d*x)),x)
```

output
```
int((1/cos(c + d*x))^(3/2)/(a + b/cos(c + d*x)), x)
```

3.611 $\int \frac{\sqrt{\sec(c+dx)}}{a+b\sec(c+dx)}\,dx$

3.611.1 Optimal result 4649
3.611.2 Mathematica [A] (verified) 4649
3.611.3 Rubi [A] (verified) 4650
3.611.4 Maple [A] (verified) 4652
3.611.5 Fricas [F(-1)] 4652
3.611.6 Sympy [F] 4653
3.611.7 Maxima [F] 4653
3.611.8 Giac [F] 4653
3.611.9 Mupad [F(-1)] 4654

3.611.1 Optimal result

Integrand size = 23, antiderivative size = 93

$$\int \frac{\sqrt{\sec(c+dx)}}{a+b\sec(c+dx)}\,dx = \frac{2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{ad} - \frac{2b\sqrt{\cos(c+dx)}\,\text{EllipticPi}\left(\frac{2a}{a+b},\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{a(a+b)d}$$

output
```
2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/
2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a/d-2*b*(cos(1/2*d*x+1/2*c
)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*a/(a+b),2^(1
/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a/(a+b)/d
```

3.611.2 Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.51

$$\int \frac{\sqrt{\sec(c+dx)}}{a+b\sec(c+dx)}\,dx = \frac{2\cot(c+dx)\,\text{EllipticPi}\left(-\frac{b}{a},\arcsin\left(\sqrt{\sec(c+dx)}\right),-1\right)\sqrt{-\tan^2(c+dx)}}{ad}$$

input
```
Integrate[Sqrt[Sec[c + d*x]]/(a + b*Sec[c + d*x]),x]
```

output
```
(2*Cot[c + d*x]*EllipticPi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-T
an[c + d*x]^2])/(a*d)
```

3.611.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.80, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3042, 4335, 3042, 3282, 3042, 3120, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\sec(c+dx)}}{a+b\sec(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{4335}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sqrt{\cos(c+dx)}}{b+a\cos(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3282}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{a}-\frac{b\int \frac{1}{\sqrt{\cos(c+dx)}(b+a\cos(c+dx))}dx}{a}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}-\frac{b\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(b+a\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a}\right)$$

$$\Big\downarrow \text{3120}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{ad}-\frac{b\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(b+a\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a}\right)$$

$$\Big\downarrow \; 3284$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{ad}-\frac{2b\,\text{EllipticPi}\left(\frac{2a}{a+b},\frac{1}{2}(c+dx),2\right)}{ad(a+b)}\right)$$

input
```
Int[Sqrt[Sec[c + d*x]]/(a + b*Sec[c + d*x]),x]
```

output
```
Sqrt[Cos[c + d*x]]*((2*EllipticF[(c + d*x)/2, 2])/(a*d) - (2*b*EllipticPi[
(2*a)/(a + b), (c + d*x)/2, 2])/(a*(a + b)*d))*Sqrt[Sec[c + d*x]]
```

3.611.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3282
```
Int[Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]/((a_.) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[d/b   Int[1/Sqrt[c + d*Sin[e + f*x]], x], x
] + Simp[(b*c - a*d)/b   Int[1/((a + b*Sin[e + f*x])*Sqrt[c + d*Sin[e + f*x
]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2
 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 4335
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_)), x_Symbol] :> Simp[Sqrt[d*Sin[e + f*x]]*(Sqrt[d*Csc[e + f*x]]/d)   Int[
Sqrt[d*Sin[e + f*x]]/(b + a*Sin[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f},
 x] && NeQ[a^2 - b^2, 0]
```

3.611.4 Maple [A] (verified)

Time = 4.90 (sec) , antiderivative size = 187, normalized size of antiderivative = 2.01

method	result
default	$-\frac{2\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+1}\left(\operatorname{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)a-b\operatorname{EllipticF}\left(\cos\left(\frac{dx}{2}\right.\right.\right.}{a(a-b)\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1}\,d}$

input
```
int(sec(d*x+c)^(1/2)/(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-2*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*(EllipticF(cos(1/2*d*x+1/2
*c),2^(1/2))*a-b*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+b*EllipticPi(cos(1/
2*d*x+1/2*c),2*a/(a-b),2^(1/2)))/a/(a-b)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*
d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

3.611.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{\sec(c+dx)}}{a+b\sec(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)^(1/2)/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

output
```
Timed out
```

3.611.6 Sympy [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{a+b\sec(c+dx)}\,dx=\int \frac{\sqrt{\sec{(c+dx)}}}{a+b\sec{(c+dx)}}\,dx$$

input
```
integrate(sec(d*x+c)**(1/2)/(a+b*sec(d*x+c)),x)
```

output
```
Integral(sqrt(sec(c + d*x))/(a + b*sec(c + d*x)), x)
```

3.611.7 Maxima [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{a+b\sec(c+dx)}\,dx=\int \frac{\sqrt{\sec{(dx+c)}}}{b\sec{(dx+c)}+a}\,dx$$

input
```
integrate(sec(d*x+c)^(1/2)/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

output
```
integrate(sqrt(sec(d*x + c))/(b*sec(d*x + c) + a), x)
```

3.611.8 Giac [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{a+b\sec(c+dx)}\,dx=\int \frac{\sqrt{\sec{(dx+c)}}}{b\sec{(dx+c)}+a}\,dx$$

input
```
integrate(sec(d*x+c)^(1/2)/(a+b*sec(d*x+c)),x, algorithm="giac")
```

output
```
integrate(sqrt(sec(d*x + c))/(b*sec(d*x + c) + a), x)
```

3.611.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\sec(c+dx)}}{a+b\sec(c+dx)}\,dx=\int \frac{\sqrt{\frac{1}{\cos(c+d\,x)}}}{a+\frac{b}{\cos(c+d\,x)}}\,dx$$

input
```
int((1/cos(c + d*x))^(1/2)/(a + b/cos(c + d*x)),x)
```

output
```
int((1/cos(c + d*x))^(1/2)/(a + b/cos(c + d*x)), x)
```

3.612 $\int \frac{1}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}\,dx$

3.612.1 Optimal result . . . 4655
3.612.2 Mathematica [A] (verified) . . . 4656
3.612.3 Rubi [A] (verified) . . . 4656
3.612.4 Maple [A] (verified) . . . 4659
3.612.5 Fricas [F(-1)] . . . 4660
3.612.6 Sympy [F] . . . 4660
3.612.7 Maxima [F] . . . 4661
3.612.8 Giac [F] . . . 4661
3.612.9 Mupad [F(-1)] . . . 4661

3.612.1 Optimal result

Integrand size = 23, antiderivative size = 135

$$\begin{aligned}\int \frac{1}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}\,dx &= \frac{2\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{ad} \\ &\quad - \frac{2b\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{a^2d} \\ &\quad + \frac{2b^2\sqrt{\cos(c+dx)}\,\text{EllipticPi}\left(\frac{2a}{a+b},\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{a^2(a+b)d}\end{aligned}$$

output
```
2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/
2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a/d-2*b*(cos(1/2*d*x+1/2*c
)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*
x+c)^(1/2)*sec(d*x+c)^(1/2)/a^2/d+2*b^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1
/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*a/(a+b),2^(1/2))*cos(d*x+c)^
(1/2)*sec(d*x+c)^(1/2)/a^2/(a+b)/d
```

3.612.2 Mathematica [A] (verified)

Time = 18.31 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.30

$$\int \frac{1}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}\,dx$$
$$= \frac{\cot(c+dx)\left(-a\sec^{\frac{3}{2}}(c+dx)-a\cos(2(c+dx))\sec^{\frac{3}{2}}(c+dx)+a\sec^{\frac{7}{2}}(c+dx)+a\cos(2(c+dx))\sec^{\frac{7}{2}}(c\right.}{}$$

input
```
Integrate[1/(Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])),x]
```

output
```
(Cot[c + d*x]*(-(a*Sec[c + d*x]^(3/2)) - a*Cos[2*(c + d*x)]*Sec[c + d*x]^(
3/2) + a*Sec[c + d*x]^(7/2) + a*Cos[2*(c + d*x)]*Sec[c + d*x]^(7/2) - 2*a*
EllipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[c + d*x]^2] + 2*a*Elli
pticF[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[c + d*x]^2] - 2*b*Elliptic
Pi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[c + d*x]^2]))/(a^2*d)
```

3.612.3 Rubi [A] (verified)

Time = 0.88 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.522$, Rules used = {3042, 4339, 3042, 4274, 3042, 4258, 3042, 3119, 3120, 4336, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)}dx$$
$$\Big\downarrow \; 4339$$
$$\frac{b^2\int \frac{\sec^{\frac{3}{2}}(c+dx)}{a+b\sec(c+dx)}dx}{a^2}+\frac{\int \frac{a-b\sec(c+dx)}{\sqrt{\sec(c+dx)}}dx}{a^2}$$
$$\Big\downarrow \; 3042$$

$$\frac{b^2\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}+\frac{\int \frac{a-b\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2}$$

$\Big\downarrow$ 4274

$$\frac{b^2\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}+\frac{a\int \frac{1}{\sqrt{\sec(c+dx)}}dx-b\int \sqrt{\sec(c+dx)}dx}{a^2}$$

$\Big\downarrow$ 3042

$$\frac{b^2\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}+\frac{a\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-b\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}$$

$\Big\downarrow$ 4258

$$\frac{b^2\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}+\frac{a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\cos(c+dx)}dx-b\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{a^2}$$

$\Big\downarrow$ 3042

$$\frac{b^2\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}+\frac{a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx-b\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2}$$

$\Big\downarrow$ 3119

$$\frac{b^2\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}+\frac{\frac{2a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|2\right)}{d}-b\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2}$$

$\Big\downarrow$ 3120

$$\frac{b^2\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}+\frac{\frac{2a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|2\right)}{d}-\frac{2b\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}}{a^2}$$

$\Big\downarrow$ 4336

3.612. $\int \frac{1}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}dx$

$$\frac{b^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}(b+a\cos(c+dx))}dx}{a^2}+\frac{\frac{2a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|2\right)}{d}-\frac{2b\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}}{a^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{b^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(b+a\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2}+\frac{\frac{2a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|2\right)}{d}-\frac{2b\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}}{a^2}$$

$$\Big\downarrow \; 3284$$

$$\frac{2b^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticPi}\left(\frac{2a}{a+b},\frac{1}{2}(c+dx),2\right)}{a^2d(a+b)}+\frac{\frac{2a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|2\right)}{d}-\frac{2b\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}}{a^2}$$

input
```
Int[1/(Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])),x]
```

output
```
((2*a*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d -
 (2*b*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d)/
a^2 + (2*b^2*Sqrt[Cos[c + d*x]]*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2]*
Sqrt[Sec[c + d*x]])/(a^2*(a + b)*d)
```

3.612.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4336
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]]   Int[
1/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4339
```
Int[1/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_))), x_Symbol] :> Simp[b^2/(a^2*d^2)   Int[(d*Csc[e + f*x])^(3/2)/(a +
b*Csc[e + f*x]), x], x] + Simp[1/a^2   Int[(a - b*Csc[e + f*x])/Sqrt[d*Csc[
e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

3.612.4 Maple [A] (verified)

Time = 7.37 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.67

method	result
default	$\frac{2\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1}\left(\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)ab-\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\right.\right.\right.}{a^2(a-b)\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\sin\left(\right.}$

input
```
int(1/(a+b*sec(d*x+c))/sec(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*
c),2^(1/2))*a*b-EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b^2+EllipticE(cos(1/
2*d*x+1/2*c),2^(1/2))*a^2-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b+b^2*El
lipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2)))/a^2/(a-b)/(-2*sin(1/2*d*x+
1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2
*c)^2-1)^(1/2)/d
```

3.612.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+b*sec(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.612.6 Sympy [F]

$$\int \frac{1}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}\,dx = \int \frac{1}{(a+b\sec(c+dx))\sqrt{\sec(c+dx)}}\,dx$$

input
```
integrate(1/(a+b*sec(d*x+c))/sec(d*x+c)**(1/2),x)
```

output
```
Integral(1/((a + b*sec(c + d*x))*sqrt(sec(c + d*x))), x)
```

3.612.7 Maxima [F]

$$\int \frac{1}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}\,dx = \int \frac{1}{(b\sec(dx+c)+a)\sqrt{\sec(dx+c)}}\,dx$$

input
```
integrate(1/(a+b*sec(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*sec(d*x + c) + a)*sqrt(sec(d*x + c))), x)
```

3.612.8 Giac [F]

$$\int \frac{1}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}\,dx = \int \frac{1}{(b\sec(dx+c)+a)\sqrt{\sec(dx+c)}}\,dx$$

input
```
integrate(1/(a+b*sec(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/((b*sec(d*x + c) + a)*sqrt(sec(d*x + c))), x)
```

3.612.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}\,dx = \int \frac{1}{\left(a+\frac{b}{\cos(c+d\,x)}\right)\,\sqrt{\frac{1}{\cos(c+d\,x)}}}\,dx$$

input
```
int(1/((a + b/cos(c + d*x))*(1/cos(c + d*x))^(1/2)),x)
```

output
```
int(1/((a + b/cos(c + d*x))*(1/cos(c + d*x))^(1/2)), x)
```

3.613 $\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))}\,dx$

3.613.1 Optimal result 4662
3.613.2 Mathematica [A] (verified) 4663
3.613.3 Rubi [A] (verified) 4663
3.613.4 Maple [B] (verified) 4667
3.613.5 Fricas [F(-1)] 4668
3.613.6 Sympy [F] 4668
3.613.7 Maxima [F] 4669
3.613.8 Giac [F] 4669
3.613.9 Mupad [F(-1)] 4669

3.613.1 Optimal result

Integrand size = 23, antiderivative size = 172

$$\begin{aligned}\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))}\,dx &= -\frac{2b\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^2d} \\ &\quad+\frac{2(a^2+3b^2)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{3a^3d} \\ &\quad-\frac{2b^3\sqrt{\cos(c+dx)}\,\text{EllipticPi}\left(\frac{2a}{a+b},\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{a^3(a+b)d}+\frac{2\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}\end{aligned}$$

output
```
2/3*sin(d*x+c)/a/d/sec(d*x+c)^(1/2)-2*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1
/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d
*x+c)^(1/2)/a^2/d+2/3*(a^2+3*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x
+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^
(1/2)/a^3/d-2*b^3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elliptic
Pi(sin(1/2*d*x+1/2*c),2*a/(a+b),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)
/a^3/(a+b)/d
```

3.613.2 Mathematica [A] (verified)

Time = 21.44 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.13

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))}\,dx = -\frac{\cot(c+dx)\left(-a^2\sqrt{\sec(c+dx)}+6ab\sec^{\frac{3}{2}}(c+dx)-6ab\cos(2(c+dx))\sec^{\frac{3}{2}}(c+dx)+a^2\cos(3(c+dx}{}$$

input
```
Integrate[1/(Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x])),x]
```

output
```
-1/6*(Cot[c + d*x]*(-(a^2*Sqrt[Sec[c + d*x]]) + 6*a*b*Sec[c + d*x]^(3/2) -
 6*a*b*Cos[2*(c + d*x)]*Sec[c + d*x]^(3/2) + a^2*Cos[3*(c + d*x)]*Sec[c +
d*x]^(3/2) - 12*a*b*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[c
+ d*x]^2] - 4*a*(a - 3*b)*EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-
Tan[c + d*x]^2] - 12*b^2*EllipticPi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]], -1
]*Sqrt[-Tan[c + d*x]^2]))/(a^3*d)
```

3.613.3 Rubi [A] (verified)

Time = 1.21 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.03, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.652$, Rules used = {3042, 4340, 27, 3042, 4594, 3042, 4274, 3042, 4258, 3042, 3119, 3120, 4336, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)}dx$$

$$\Big\downarrow \; 4340$$

$$\frac{2\int -\frac{-b\sec^2(c+dx)-a\sec(c+dx)+3b}{2\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}dx}{3a}+\frac{2\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \; 27$$

$$\frac{2\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{\int\frac{-b\sec^2(c+dx)-a\sec(c+dx)+3b}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}dx}{3a}$$

$\Big\downarrow$ 3042

$$\frac{2\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{\int\frac{-b\csc\left(c+dx+\frac{\pi}{2}\right)^2-a\csc\left(c+dx+\frac{\pi}{2}\right)+3b}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{3a}$$

$\Big\downarrow$ 4594

$$\frac{2\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{3b^3\int\frac{\sec^{\frac{3}{2}}(c+dx)}{a+b\sec(c+dx)}dx}{a^2}+\frac{\int\frac{3ab-\left(a^2+3b^2\right)\sec(c+dx)}{\sqrt{\sec(c+dx)}}dx}{a^2}}{3a}$$

$\Big\downarrow$ 3042

$$\frac{2\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{3b^3\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}+\frac{\int\frac{3ab+\left(-a^2-3b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2}}{3a}$$

$\Big\downarrow$ 4274

$$\frac{2\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{3b^3\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}+\frac{3ab\int\frac{1}{\sqrt{\sec(c+dx)}}dx-\left(a^2+3b^2\right)\int\sqrt{\sec(c+dx)}dx}{a^2}}{3a}$$

$\Big\downarrow$ 3042

$$\frac{2\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{3b^3\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}+\frac{3ab\int\frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-\left(a^2+3b^2\right)\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}}{3a}$$

$\Big\downarrow$ 4258

$$\frac{2\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{3b^3\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}+\frac{3ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\cos(c+dx)}dx-\left(a^2+3b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx}{a^2}}{3a}$$

$\Big\downarrow$ 3042

$$\frac{2\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{3b^3\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}+\frac{3ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx-\left(a^2+3b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2}}{3a}$$

3.613. $\int\frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))}dx$

$$\Big\downarrow \; 3119$$

$$\frac{2\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{3b^3\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}+\frac{\frac{6ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-\left(a^2+3b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2}}{3a}$$

$$\Big\downarrow \; 3120$$

$$\frac{2\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{3b^3\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}+\frac{\frac{6ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-\frac{2\left(a^2+3b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}}{a^2}}{3a}$$

$$\Big\downarrow \; 4336$$

$$\frac{2\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{3b^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}(b+a\cos(c+dx))}dx}{a^2}+\frac{\frac{6ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-\frac{2\left(a^2+3b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}}{d}}{a^2}}{3a}$$

$$\Big\downarrow \; 3042$$

$$\frac{2\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{3b^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(b+a\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2}+\frac{\frac{6ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-\frac{2\left(a^2+3b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{d}}{a^2}}{3a}$$

$$\Big\downarrow \; 3284$$

$$\frac{2\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{6b^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticPi}\left(\frac{2a}{a+b},\frac{1}{2}(c+dx),2\right)}{a^2d(a+b)}+\frac{\frac{6ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-\frac{2\left(a^2+3b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\right.}{d}}{a^2}}{3a}$$

input
```
Int[1/(Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x])),x]
```

3.613. $\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))}\,dx$

output
```
-1/3*(((6*a*b*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*
x]])/d - (2*(a^2 + 3*b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqr
t[Sec[c + d*x]])/d)/a^2 + (6*b^3*Sqrt[Cos[c + d*x]]*EllipticPi[(2*a)/(a +
b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a^2*(a + b)*d))/a + (2*Sin[c + d*
x])/(3*a*d*Sqrt[Sec[c + d*x]])
```

3.613.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

3.613. $\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))}\,dx$

rule 4336
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]]   Int[
1/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4340
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[Cot[e + f*x]*((d*Csc[e + f*x])^n/(a*f*n)), x] - Sim
p[1/(a*d*n)   Int[((d*Csc[e + f*x])^(n + 1)/(a + b*Csc[e + f*x]))*Simp[b*n
- a*(n + 1)*Csc[e + f*x] - b*(n + 1)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a
, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1] && IntegerQ[2*n]
```

rule 4594
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Simp[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2)   Int[(d*Csc[e +
f*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Simp[1/a^2   Int[(a*A - (A*b - a
*B)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A,
B, C}, x] && NeQ[a^2 - b^2, 0]
```

3.613.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 551 vs. $2(232) = 464$.

Time = 7.53 (sec) , antiderivative size = 552, normalized size of antiderivative = 3.21

method	result
default	$-\frac{2\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(4a^3\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4-4a^2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)b-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\right.}{}$

input
```
int(1/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-2/3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4*a^3*cos(1/
2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4-4*a^2*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1
/2*c)*b-2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2*a^3+2*a^2*sin(1/2*d*x+1/
2*c)^2*cos(1/2*d*x+1/2*c)*b+a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*
x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-(sin(1/2*d*x+1/2
*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2
-1)^(1/2)*a^2*b+3*a*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)
^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1
/2)*b^3+3*a^2*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1
/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*E
llipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*a*b^
2+3*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elli
pticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2)))/a^3/(a-b)/(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c
)^2-1)^(1/2)/d
```

3.613.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))}\,dx = \text{Timed out}$$

input
```
integrate(1/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

output
```
Timed out
```

3.613.6 Sympy [F]

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))}\,dx = \int \frac{1}{(a+b\sec(c+dx))\sec^{\frac{3}{2}}(c+dx)}\,dx$$

input
```
integrate(1/sec(d*x+c)**(3/2)/(a+b*sec(d*x+c)),x)
```

output
```
Integral(1/((a + b*sec(c + d*x))*sec(c + d*x)**(3/2)), x)
```

3.613.7 Maxima [F]

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))}\,dx = \int \frac{1}{(b\sec(dx+c)+a)\sec(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

output
```
integrate(1/((b*sec(d*x + c) + a)*sec(d*x + c)^(3/2)), x)
```

3.613.8 Giac [F]

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))}\,dx = \int \frac{1}{(b\sec(dx+c)+a)\sec(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c)),x, algorithm="giac")
```

output
```
integrate(1/((b*sec(d*x + c) + a)*sec(d*x + c)^(3/2)), x)
```

3.613.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))}\,dx = \int \frac{1}{\left(a+\frac{b}{\cos(c+d\,x)}\right)\,\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

input
```
int(1/((a + b/cos(c + d*x))*(1/cos(c + d*x))^(3/2)),x)
```

output
```
int(1/((a + b/cos(c + d*x))*(1/cos(c + d*x))^(3/2)), x)
```

3.614 $\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

3.614.1 Optimal result 4670
3.614.2 Mathematica [A] (verified) 4671
3.614.3 Rubi [A] (verified) 4671
3.614.4 Maple [B] (verified) 4678
3.614.5 Fricas [F(-1)] 4678
3.614.6 Sympy [F(-1)] 4679
3.614.7 Maxima [F(-1)] 4679
3.614.8 Giac [F] 4679
3.614.9 Mupad [F(-1)] 4680

3.614.1 Optimal result

Integrand size = 23, antiderivative size = 342

$$\begin{aligned}\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a+b\sec(c+dx))^2}\,dx &= \frac{a(5a^2-4b^2)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{b^3(a^2-b^2)d} \\ &\quad+\frac{(5a^2-2b^2)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{3b^2(a^2-b^2)d} \\ &\quad+\frac{a^2(5a^2-7b^2)\sqrt{\cos(c+dx)}\,\text{EllipticPi}\left(\frac{2a}{a+b},\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{(a-b)b^3(a+b)^2d} \\ &\quad-\frac{a(5a^2-4b^2)\sqrt{\sec(c+dx)}\sin(c+dx)}{b^3(a^2-b^2)d} \\ &\quad+\frac{(5a^2-2b^2)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3b^2(a^2-b^2)d}-\frac{a^2\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{b(a^2-b^2)d(a+b\sec(c+dx))}\end{aligned}$$

output
```
1/3*(5*a^2-2*b^2)*sec(d*x+c)^(3/2)*sin(d*x+c)/b^2/(a^2-b^2)/d-a^2*sec(d*x+
c)^(5/2)*sin(d*x+c)/b/(a^2-b^2)/d/(a+b*sec(d*x+c))-a*(5*a^2-4*b^2)*sin(d*x
+c)*sec(d*x+c)^(1/2)/b^3/(a^2-b^2)/d+a*(5*a^2-4*b^2)*(cos(1/2*d*x+1/2*c)^2
)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c
)^(1/2)*sec(d*x+c)^(1/2)/b^3/(a^2-b^2)/d+1/3*(5*a^2-2*b^2)*(cos(1/2*d*x+1/
2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos
(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/b^2/(a^2-b^2)/d+a^2*(5*a^2-7*b^2)*(cos(1/2*
d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*a/(
a+b),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/(a-b)/b^3/(a+b)^2/d
```

3.614.2 Mathematica [A] (verified)

Time = 5.10 (sec) , antiderivative size = 294, normalized size of antiderivative = 0.86

$$\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \frac{\frac{2b\left(-\frac{3a^2\left(5a^2-4b^2\right)\sin(c+dx)}{a^2-b^2}+2b(-5a+b\sec(c+dx))\tan(c+dx)\right)}{(b+a\cos(c+dx))\sqrt{\sec(c+dx)}}+\frac{\cot(c+dx)\left(-6ab\left(5a^2-4b^2\right)E\left(\arcsin\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)\sqrt{-\tan^2(c+dx)}\right.}{}}{}$$

input
```
Integrate[Sec[c + d*x]^(9/2)/(a + b*Sec[c + d*x])^2,x]
```

output
```
((2*b*((-3*a^2*(5*a^2 - 4*b^2)*Sin[c + d*x])/(a^2 - b^2) + 2*b*(-5*a + b*S
ec[c + d*x])*Tan[c + d*x]))/((b + a*Cos[c + d*x])*Sqrt[Sec[c + d*x]]) + (C
ot[c + d*x]*(-6*a*b*(5*a^2 - 4*b^2)*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]],
-1]*Sqrt[-Tan[c + d*x]^2] + 2*(15*a^4 + 15*a^3*b - 16*a^2*b^2 - 12*a*b^3 -
 2*b^4)*EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[c + d*x]^2] -
6*a*(b*(-5*a^2 + 4*b^2)*Sec[c + d*x]^(3/2)*Sin[c + d*x]^2 + a*(5*a^2 - 7*b
^2)*EllipticPi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[c + d*x]^
2])))/((a - b)*(a + b)))/(6*b^4*d)
```

3.614.3 Rubi [A] (verified)

Time = 2.49 (sec) , antiderivative size = 325, normalized size of antiderivative = 0.95, number of steps used = 21, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.913$, Rules used = {3042, 4332, 27, 3042, 4590, 27, 3042, 4590, 27, 3042, 4594, 3042, 4274, 3042, 4258, 3042, 3119, 3120, 4336, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{9/2}}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \; 4332$$

$$-\frac{\int \frac{\sec^{\frac{3}{2}}(c+dx)\left(3a^2-2b\sec(c+dx)a-\left(5a^2-2b^2\right)\sec^2(c+dx)\right)}{2(a+b\sec(c+dx))}dx}{b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{bd\left(a^2-b^2\right)(a+b\sec(c+dx))}$$

$\Big\downarrow$ 27

$$-\frac{\int \frac{\sec^{\frac{3}{2}}(c+dx)\left(3a^2-2b\sec(c+dx)a-\left(5a^2-2b^2\right)\sec^2(c+dx)\right)}{a+b\sec(c+dx)}dx}{2b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{bd\left(a^2-b^2\right)(a+b\sec(c+dx))}$$

$\Big\downarrow$ 3042

$$-\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(3a^2-2b\csc\left(c+dx+\frac{\pi}{2}\right)a+\left(2b^2-5a^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{2b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{bd\left(a^2-b^2\right)(a+b\sec(c+dx))}$$

$\Big\downarrow$ 4590

$$-\frac{\frac{2\int -\frac{\sqrt{\sec(c+dx)}\left(-3a\left(5a^2-4b^2\right)\sec^2(c+dx)-2b\left(2a^2+b^2\right)\sec(c+dx)+a\left(5a^2-2b^2\right)\right)}{2(a+b\sec(c+dx))}dx}{3b}-\frac{2\left(5a^2-2b^2\right)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3bd}}{2b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{bd\left(a^2-b^2\right)(a+b\sec(c+dx))}$$

$\Big\downarrow$ 27

$$-\frac{-\frac{\int \frac{\sqrt{\sec(c+dx)}\left(-3a\left(5a^2-4b^2\right)\sec^2(c+dx)-2b\left(2a^2+b^2\right)\sec(c+dx)+a\left(5a^2-2b^2\right)\right)}{a+b\sec(c+dx)}dx}{3b}-\frac{2\left(5a^2-2b^2\right)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3bd}}{2b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{bd\left(a^2-b^2\right)(a+b\sec(c+dx))}$$

$\Big\downarrow$ 3042

$$-\frac{-\frac{\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(-3a\left(5a^2-4b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2-2b\left(2a^2+b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)+a\left(5a^2-2b^2\right)\right)}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{3b}-\frac{2\left(5a^2-2b^2\right)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3bd}}{2b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{bd\left(a^2-b^2\right)(a+b\sec(c+dx))}$$

$\Big\downarrow$ 4590

$$-\frac{-\frac{\frac{2\int \frac{3\left(5a^2-4b^2\right)a^2+2b\left(10a^2-7b^2\right)\sec(c+dx)a+\left(15a^4-16b^2a^2-2b^4\right)\sec^2(c+dx)}{2\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}dx}{b}-\frac{6a\left(5a^2-4b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd}}{3b}-\frac{2\left(5a^2-2b^2\right)\sin(c+dx)\sec}{3bd}}{2b\left(a^2-b^2\right)}$$

$$-\frac{a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{bd\left(a^2-b^2\right)(a+b\sec(c+dx))}$$

3.614. $\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

$$\Big\downarrow \; 27$$

$$-\frac{-\frac{\frac{\int \frac{3\left(5a^2-4b^2\right)a^2+2b\left(10a^2-7b^2\right)\sec(c+dx)a+\left(15a^4-16b^2a^2-2b^4\right)\sec^2(c+dx)}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}dx}{b}-\frac{6a\left(5a^2-4b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd}}{3b}-\frac{2\left(5a^2-2b^2\right)\sin(c+dx)\sec^{\frac{5}{2}}}{3bd}}{2b\left(a^2-b^2\right)}$$

$$\frac{a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{bd\left(a^2-b^2\right)(a+b\sec(c+dx))}$$

$$\Big\downarrow \; 3042$$

$$-\frac{-\frac{\frac{\int \frac{3\left(5a^2-4b^2\right)a^2+2b\left(10a^2-7b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)a+\left(15a^4-16b^2a^2-2b^4\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{b}-\frac{6a\left(5a^2-4b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd}}{3b}-\frac{2\left(5a^2-2b^2\right)\sin(c+}{3b}}{2b\left(a^2-b^2\right)}$$

$$\frac{a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{bd\left(a^2-b^2\right)(a+b\sec(c+dx))}$$

$$\Big\downarrow \; 4594$$

$$-\frac{-\frac{\frac{3a^2\left(5a^2-7b^2\right)\int \frac{\sec^{\frac{3}{2}}(c+dx)}{a+b\sec(c+dx)}dx+\frac{\int \frac{3\left(5a^2-4b^2\right)a^3+b\left(5a^2-2b^2\right)\sec(c+dx)a^2}{\sqrt{\sec(c+dx)}}dx}{a^2}}{b}-\frac{6a\left(5a^2-4b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd}}{3b}-\frac{2\left(5a^2-2b^2\right)\sin(c+dx)\sec}{3bd}}{2b\left(a^2-b^2\right)}$$

$$\frac{a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{bd\left(a^2-b^2\right)(a+b\sec(c+dx))}$$

$$\Big\downarrow \; 3042$$

$$-\frac{-\frac{\frac{3a^2\left(5a^2-7b^2\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{\int \frac{3\left(5a^2-4b^2\right)a^3+b\left(5a^2-2b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)a^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2}}{b}-\frac{6a\left(5a^2-4b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd}}{3b}-\frac{2\left(5a^2-2b^2\right)\sin(c}{3}}{2b\left(a^2-b^2\right)}$$

$$\frac{a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{bd\left(a^2-b^2\right)(a+b\sec(c+dx))}$$

$$\Big\downarrow \; 4274$$

$$-\frac{-\frac{\frac{3a^2\left(5a^2-7b^2\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{a^2b\left(5a^2-2b^2\right)\int \sqrt{\sec(c+dx)}dx+3a^3\left(5a^2-4b^2\right)\int \frac{1}{\sqrt{\sec(c+dx)}}dx}{a^2}}{b}-\frac{6a\left(5a^2-4b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd}}{3b}-\frac{2(5a}{}}{2b\left(a^2-b^2\right)}$$

$$\frac{a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{bd\left(a^2-b^2\right)(a+b\sec(c+dx))}$$

3.614. $\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

$$\Big\downarrow \; 3042$$

$$-\frac{\frac{3a^2\left(5a^2-7b^2\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{a^2b\left(5a^2-2b^2\right)\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+3a^3\left(5a^2-4b^2\right)\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2}}{b}-\frac{6a\left(5a^2-4b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd}}{3b}}{2b\left(a^2-b^2\right)}$$

$$\frac{a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{bd\left(a^2-b^2\right)(a+b\sec(c+dx))}$$

$$\Big\downarrow \; 4258$$

$$-\frac{\frac{3a^2\left(5a^2-7b^2\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{a^2b\left(5a^2-2b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx+3a^3\left(5a^2-4b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\cos(c+dx)}dx}{a^2}}{b}}{3b}}{2b\left(a^2-b^2\right)}$$

$$\frac{a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{bd\left(a^2-b^2\right)(a+b\sec(c+dx))}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\frac{3a^2\left(5a^2-7b^2\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{a^2b\left(5a^2-2b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+3a^3\left(5a^2-4b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\sin\left(c+dx\right.}}{a^2}}{b}}{3b}}{2b\left(a^2-b^2\right)}$$

$$\frac{a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{bd\left(a^2-b^2\right)(a+b\sec(c+dx))}$$

$$\Big\downarrow \; 3119$$

$$-\frac{\frac{3a^2\left(5a^2-7b^2\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{a^2b\left(5a^2-2b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{6a^3\left(5a^2-4b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\right.}{d}}{a^2}}{b}}{3b}}{2b\left(a^2-b^2\right)}$$

$$\frac{a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{bd\left(a^2-b^2\right)(a+b\sec(c+dx))}$$

$$\Big\downarrow \; 3120$$

3.614. $\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

$$-\frac{-\frac{3a^2\left(5a^2-7b^2\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{\frac{2a^2b\left(5a^2-2b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+\frac{6a^3\left(5a^2-4b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+d\right.}{d}}{a^2}}{b}}{3b}}{2b\left(a^2-b^2\right)}$$

$$\frac{a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{bd\left(a^2-b^2\right)(a+b\sec(c+dx))}$$

$\downarrow$ 4336

$$-\frac{-\frac{3a^2\left(5a^2-7b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}(b+a\cos(c+dx))}dx+\frac{\frac{2a^2b\left(5a^2-2b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+\frac{6a^3\left(5a^2-4b^2\right)\sqrt{}}{}}{a^2}}{b}}{3b}}{2b\left(a^2-b^2\right)}$$

$$\frac{a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{bd\left(a^2-b^2\right)(a+b\sec(c+dx))}$$

$\downarrow$ 3042

$$-\frac{-\frac{3a^2\left(5a^2-7b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(b+a\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx+\frac{\frac{2a^2b\left(5a^2-2b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+\frac{6a^3\left(5\right.}{}}{a^2}}{b}}{3b}}{2b\left(a^2-b^2\right)}$$

$$\frac{a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{bd\left(a^2-b^2\right)(a+b\sec(c+dx))}$$

$\downarrow$ 3284

$$-\frac{a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{bd\left(a^2-b^2\right)(a+b\sec(c+dx))}-$$

$$-\frac{\frac{2\left(5a^2-2b^2\right)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3bd}-\frac{\frac{\frac{6a^2\left(5a^2-7b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticPi}\left(\frac{2a}{a+b},\frac{1}{2}(c+dx),2\right)}{d(a+b)}+\frac{\frac{2a^2b\left(5a^2-2b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{Elli}}{d}}{b}}{3}}{2b\left(a^2-b^2\right)}$$

input
```
Int[Sec[c + d*x]^(9/2)/(a + b*Sec[c + d*x])^2,x]
```

3.614. $\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

output
```
-((a^2*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(b*(a^2 - b^2)*d*(a + b*Sec[c + d*
x]))) - ((-2*(5*a^2 - 2*b^2)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*b*d) - ((
((6*a^3*(5*a^2 - 4*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[
Sec[c + d*x]])/d + (2*a^2*b*(5*a^2 - 2*b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(
c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d)/a^2 + (6*a^2*(5*a^2 - 7*b^2)*Sqrt[Co
s[c + d*x]]*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/
((a + b)*d))/b - (6*a*(5*a^2 - 4*b^2)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(b*
d))/(3*b))/(2*b*(a^2 - b^2))
```

3.614.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

3.614. $\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4332
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-a^2)*d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(
m + 1)*((d*Csc[e + f*x])^(n - 3)/(b*f*(m + 1)*(a^2 - b^2))), x] + Simp[d^3/
(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])
^(n - 3)*Simp[a^2*(n - 3) + a*b*(m + 1)*Csc[e + f*x] - (a^2*(n - 2) + b^2*(
m + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2
- b^2, 0] && LtQ[m, -1] && (IGtQ[n, 3] || (IntegersQ[n + 1/2, 2*m] && GtQ[n
, 2]))
```

rule 4336
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]]   Int[
1/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4590
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(-C)*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
)*((d*Csc[e + f*x])^(n - 1)/(b*f*(m + n + 1))), x] + Simp[d/(b*(m + n + 1))
   Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (
A*b*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2
 - b^2, 0] && GtQ[n, 0]
```

rule 4594
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Simp[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2)   Int[(d*Csc[e +
f*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Simp[1/a^2   Int[(a*A - (A*b - a
*B)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A,
B, C}, x] && NeQ[a^2 - b^2, 0]
```

3.614. $\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

3.614.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 974 vs. $2(398) = 796$.

Time = 120.42 (sec) , antiderivative size = 975, normalized size of antiderivative = 2.85

method	result	size
default	Expression too large to display	975

input
```
int(sec(d*x+c)^(9/2)/(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
-(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2/b^2*(-1/6*co
s(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos
(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+
1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ell
ipticF(cos(1/2*d*x+1/2*c),2^(1/2)))-4/b^3*a/sin(1/2*d*x+1/2*c)^2/(2*sin(1/
2*d*x+1/2*c)^2-1)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*
cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-(sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellip
ticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2))+2*a^2/b
^2*(a^2/b/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*
x+1/2*c)^2)^(1/2)/(2*a*cos(1/2*d*x+1/2*c)^2-a+b)-1/2/(a+b)/b*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2*a/b
/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/
(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x
+1/2*c),2^(1/2))-1/2*a/b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/
2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/
2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-1/2/b/(a^2-b^2)/(a^2-a*b)*a^3*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*
x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(
a-b),2^(1/2))+3/2*b/(a^2-b^2)/(a^2-a*b)*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*...
```

3.614.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)^(9/2)/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
Timed out
```

3.614.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**(9/2)/(a+b*sec(d*x+c))**2,x)
```

output
```
Timed out
```

3.614.7 Maxima [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)^(9/2)/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
Timed out
```

3.614.8 Giac [F]

$$\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \int \frac{\sec\left(dx+c\right)^{\frac{9}{2}}}{\left(b\sec\left(dx+c\right)+a\right)^2}\,dx$$

input
```
integrate(sec(d*x+c)^(9/2)/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^(9/2)/(b*sec(d*x + c) + a)^2, x)
```

3.614.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{9/2}}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^2}\,dx$$

input
```
int((1/cos(c + d*x))^(9/2)/(a + b/cos(c + d*x))^2,x)
```

output
```
int((1/cos(c + d*x))^(9/2)/(a + b/cos(c + d*x))^2, x)
```

3.615 $\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

3.615.1 Optimal result 4681
3.615.2 Mathematica [A] (verified) 4682
3.615.3 Rubi [A] (verified) 4682
3.615.4 Maple [B] (verified) 4687
3.615.5 Fricas [F(-1)] 4688
3.615.6 Sympy [F(-1)] 4689
3.615.7 Maxima [F(-1)] 4689
3.615.8 Giac [F] 4689
3.615.9 Mupad [F(-1)] 4690

3.615.1 Optimal result

Integrand size = 23, antiderivative size = 279

$$\begin{aligned}\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+b\sec(c+dx))^2}\,dx &= -\frac{(3a^2-2b^2)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{b^2(a^2-b^2)d} \\ &\quad -\frac{a\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{b(a^2-b^2)d} \\ &\quad -\frac{a(3a^2-5b^2)\sqrt{\cos(c+dx)}\,\text{EllipticPi}\left(\frac{2a}{a+b},\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{(a-b)b^2(a+b)^2d} \\ &\quad +\frac{(3a^2-2b^2)\sqrt{\sec(c+dx)}\sin(c+dx)}{b^2(a^2-b^2)d}-\frac{a^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{b(a^2-b^2)d(a+b\sec(c+dx))}\end{aligned}$$

output
```
-a^2*sec(d*x+c)^(3/2)*sin(d*x+c)/b/(a^2-b^2)/d/(a+b*sec(d*x+c))+(3*a^2-2*b
^2)*sin(d*x+c)*sec(d*x+c)^(1/2)/b^2/(a^2-b^2)/d-(3*a^2-2*b^2)*(cos(1/2*d*x
+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*
cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/b^2/(a^2-b^2)/d-a*(cos(1/2*d*x+1/2*c)^2)
^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)
^(1/2)*sec(d*x+c)^(1/2)/b/(a^2-b^2)/d-a*(3*a^2-5*b^2)*(cos(1/2*d*x+1/2*c)^
2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*a/(a+b),2^(1/2
))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/(a-b)/b^2/(a+b)^2/d
```

3.615.2 Mathematica [A] (verified)

Time = 3.96 (sec) , antiderivative size = 351, normalized size of antiderivative = 1.26

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$
$$= \frac{\frac{2b(-3a^3+2ab^2+2b(-a^2+b^2)\sec(c+dx))\sin(c+dx)}{(-a^2+b^2)(b+a\cos(c+dx))\sqrt{\sec(c+dx)}} + \frac{\cot(c+dx)\left(-3a^2b\sec^{\frac{3}{2}}(c+dx)+2b^3\sec^{\frac{3}{2}}(c+dx)+3a^2b\cos(2(c+dx))\sec^{\frac{3}{2}}(c+dx)-2b^3\,\mathrm{co}\right.}{}}{}$$

input
```
Integrate[Sec[c + d*x]^(7/2)/(a + b*Sec[c + d*x])^2,x]
```

output
```
((2*b*(-3*a^3 + 2*a*b^2 + 2*b*(-a^2 + b^2)*Sec[c + d*x])*Sin[c + d*x])/((-
a^2 + b^2)*(b + a*Cos[c + d*x])*Sqrt[Sec[c + d*x]]) + (Cot[c + d*x]*(-3*a^
2*b*Sec[c + d*x]^(3/2) + 2*b^3*Sec[c + d*x]^(3/2) + 3*a^2*b*Cos[2*(c + d*x
)]*Sec[c + d*x]^(3/2) - 2*b^3*Cos[2*(c + d*x)]*Sec[c + d*x]^(3/2) + 2*b*(3
*a^2 - 2*b^2)*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[c + d*x]
^2] - 2*(3*a^3 + 3*a^2*b - 4*a*b^2 - 2*b^3)*EllipticF[ArcSin[Sqrt[Sec[c +
d*x]]], -1]*Sqrt[-Tan[c + d*x]^2] + 6*a^3*EllipticPi[-(b/a), ArcSin[Sqrt[S
ec[c + d*x]]], -1]*Sqrt[-Tan[c + d*x]^2] - 10*a*b^2*EllipticPi[-(b/a), Arc
Sin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[c + d*x]^2]))/((a - b)*(a + b)))/(2
*b^3*d)
```

3.615.3 Rubi [A] (verified)

Time = 1.92 (sec) , antiderivative size = 266, normalized size of antiderivative = 0.95, number of steps used = 18, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.783$, Rules used = {3042, 4332, 27, 3042, 4590, 27, 3042, 4594, 3042, 4274, 3042, 4258, 3042, 3119, 3120, 4336, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx$$
$$\Big\downarrow \; 4332$$

$$-\frac{\int \frac{\sqrt{\sec(c+dx)}\left(a^2-2b\sec(c+dx)a-\left(3a^2-2b^2\right)\sec^2(c+dx)\right)}{2(a+b\sec(c+dx))}dx}{b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{bd\left(a^2-b^2\right)(a+b\sec(c+dx))}$$

$\downarrow$ 27

$$-\frac{\int \frac{\sqrt{\sec(c+dx)}\left(a^2-2b\sec(c+dx)a-\left(3a^2-2b^2\right)\sec^2(c+dx)\right)}{a+b\sec(c+dx)}dx}{2b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{bd\left(a^2-b^2\right)(a+b\sec(c+dx))}$$

$\downarrow$ 3042

$$-\frac{\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a^2-2b\csc\left(c+dx+\frac{\pi}{2}\right)a+\left(2b^2-3a^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{2b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{bd\left(a^2-b^2\right)(a+b\sec(c+dx))}$$

$\downarrow$ 4590

$$-\frac{\frac{2\int \frac{a\left(3a^2-4b^2\right)\sec^2(c+dx)+2b\left(2a^2-b^2\right)\sec(c+dx)+a\left(3a^2-2b^2\right)}{2\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}dx}{b}-\frac{2\left(3a^2-2b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd}}{2b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{bd\left(a^2-b^2\right)(a+b\sec(c+dx))}$$

$\downarrow$ 27

$$-\frac{\frac{\int \frac{a\left(3a^2-4b^2\right)\sec^2(c+dx)+2b\left(2a^2-b^2\right)\sec(c+dx)+a\left(3a^2-2b^2\right)}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}dx}{b}-\frac{2\left(3a^2-2b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd}}{2b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{bd\left(a^2-b^2\right)(a+b\sec(c+dx))}$$

$\downarrow$ 3042

$$-\frac{\frac{\int \frac{a\left(3a^2-4b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2+2b\left(2a^2-b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)+a\left(3a^2-2b^2\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{b}-\frac{2\left(3a^2-2b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd}}{2b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{bd\left(a^2-b^2\right)(a+b\sec(c+dx))}$$

$\downarrow$ 4594

$$-\frac{\frac{a\left(3a^2-5b^2\right)\int \frac{\sec^{\frac{3}{2}}(c+dx)}{a+b\sec(c+dx)}dx+\frac{\int \frac{b\sec(c+dx)a^3+\left(3a^2-2b^2\right)a^2}{\sqrt{\sec(c+dx)}}dx}{a^2}}{b}-\frac{2\left(3a^2-2b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd}}{2b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{bd\left(a^2-b^2\right)(a+b\sec(c+dx))}$$

$\downarrow$ 3042

3.615. $\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

$$-\frac{\frac{a(3a^2-5b^2)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{\int \frac{b\csc\left(c+dx+\frac{\pi}{2}\right)a^3+\left(3a^2-2b^2\right)a^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2}}{b}-\frac{2(3a^2-2b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd}}{2b(a^2-b^2)}-\frac{a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{bd(a^2-b^2)(a+b\sec(c+dx))}$$

$\downarrow$ 4274

$$-\frac{\frac{a(3a^2-5b^2)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{a^3b\int \sqrt{\sec(c+dx)}dx+a^2\left(3a^2-2b^2\right)\int \frac{1}{\sqrt{\sec(c+dx)}}dx}{a^2}}{b}-\frac{2(3a^2-2b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd}}{2b(a^2-b^2)}-\frac{a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{bd(a^2-b^2)(a+b\sec(c+dx))}$$

$\downarrow$ 3042

$$-\frac{\frac{a(3a^2-5b^2)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{a^3b\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+a^2\left(3a^2-2b^2\right)\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2}}{b}-\frac{2(3a^2-2b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd}}{2b(a^2-b^2)}-\frac{a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{bd(a^2-b^2)(a+b\sec(c+dx))}$$

$\downarrow$ 4258

$$-\frac{\frac{a(3a^2-5b^2)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{a^3b\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx+a^2\left(3a^2-2b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\cos(c+dx)}dx}{a^2}}{b}-\frac{2(3a^2-2b^{}}{}}{2b(a^2-b^2)}-\frac{a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{bd(a^2-b^2)(a+b\sec(c+dx))}$$

$\downarrow$ 3042

$$-\frac{\frac{a(3a^2-5b^2)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{a^3b\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+a^2\left(3a^2-2b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}}{b}-\frac{2(}{}}{2b(a^2-b^2)}-\frac{a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{bd(a^2-b^2)(a+b\sec(c+dx))}$$

$\downarrow$ 3119

3.615. $\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

$$-\frac{\frac{a(3a^2-5b^2)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{a^3b\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a^2\left(3a^2-2b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}}{a^2}}{b}-\frac{2(3a^2}{}}{2b\left(a^2-b^2\right)}$$

$$\frac{a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{bd\left(a^2-b^2\right)(a+b\sec(c+dx))}$$

$\downarrow$ 3120

$$-\frac{\frac{a(3a^2-5b^2)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{\frac{2a^3b\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+\frac{2a^2\left(3a^2-2b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}}{a^2}}{b}-\frac{2(3a}{}}{2b\left(a^2-b^2\right)}$$

$$\frac{a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{bd\left(a^2-b^2\right)(a+b\sec(c+dx))}$$

$\downarrow$ 4336

$$-\frac{\frac{a(3a^2-5b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}(b+a\cos(c+dx))}dx+\frac{\frac{2a^3b\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+\frac{2a^2\left(3a^2-2b^2\right)\sqrt{\cos(c+dx)}}{}}{a^2}}{b}}{2b\left(a^2-b^2\right)}$$

$$\frac{a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{bd\left(a^2-b^2\right)(a+b\sec(c+dx))}$$

$\downarrow$ 3042

$$-\frac{\frac{a(3a^2-5b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(b+a\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx+\frac{\frac{2a^3b\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+\frac{2a^2\left(3a^2-2b^2\right)\sqrt{}}{}}{a^2}}{b}}{2b\left(a^2-b^2\right)}$$

$$\frac{a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{bd\left(a^2-b^2\right)(a+b\sec(c+dx))}$$

$\downarrow$ 3284

$$-\frac{a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{bd\left(a^2-b^2\right)(a+b\sec(c+dx))}-\frac{\frac{\frac{2a\left(3a^2-5b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticPi}\left(\frac{2a}{a+b},\frac{1}{2}(c+dx),2\right)}{d(a+b)}+\frac{\frac{2a^3b\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+\frac{2a^2\left(3a^2-2b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx}}{d}}{a^2}}{b}}{2b\left(a^2-b^2\right)}$$

input
```
Int[Sec[c + d*x]^(7/2)/(a + b*Sec[c + d*x])^2,x]
```

3.615. $\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

output
```
-((a^2*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(b*(a^2 - b^2)*d*(a + b*Sec[c + d*
x]))) - ((((2*a^2*(3*a^2 - 2*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2
, 2]*Sqrt[Sec[c + d*x]])/d + (2*a^3*b*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*
x)/2, 2]*Sqrt[Sec[c + d*x]])/d)/a^2 + (2*a*(3*a^2 - 5*b^2)*Sqrt[Cos[c + d*
x]]*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/((a + b)
*d))/b - (2*(3*a^2 - 2*b^2)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(b*d))/(2*b*(
a^2 - b^2))
```

3.615.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

3.615. $\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

rule 4332
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-a^2)*d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(
m + 1)*((d*Csc[e + f*x])^(n - 3)/(b*f*(m + 1)*(a^2 - b^2))), x] + Simp[d^3/
(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])
^(n - 3)*Simp[a^2*(n - 3) + a*b*(m + 1)*Csc[e + f*x] - (a^2*(n - 2) + b^2*(
m + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2
- b^2, 0] && LtQ[m, -1] && (IGtQ[n, 3] || (IntegersQ[n + 1/2, 2*m] && GtQ[n
, 2]))
```

rule 4336
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]]   Int[
1/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4590
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(-C)*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
)*((d*Csc[e + f*x])^(n - 1)/(b*f*(m + n + 1))), x] + Simp[d/(b*(m + n + 1))
   Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (
A*b*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2
 - b^2, 0] && GtQ[n, 0]
```

rule 4594
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Simp[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2)   Int[(d*Csc[e +
f*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Simp[1/a^2   Int[(a*A - (A*b - a
*B)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A,
B, C}, x] && NeQ[a^2 - b^2, 0]
```

3.615.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 840 vs. $2(341) = 682$.

Time = 111.12 (sec) , antiderivative size = 841, normalized size of antiderivative = 3.01

method	result	size
default	Expression too large to display	841

input
```
int(sec(d*x+c)^(7/2)/(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
-(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2/b^2/sin(1/2*
d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d
*x+1/2*c)^2)^(1/2)*(2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-(sin(1/2*d*x
+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*
c)^2-1)^(1/2))-2*a/b*(a^2/b/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*a*cos(1/2*d*x+1/2*c)^2-a+b)-1/2/(a+
b)/b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c
),2^(1/2))+1/2*a/b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+
1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ell
ipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2*a/b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2
*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-1/2/b/(a^2-b^2)
/(a^2-a*b)*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1
/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/
2*d*x+1/2*c),2*a/(a-b),2^(1/2))+3/2*b/(a^2-b^2)/(a^2-a*b)*a*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+
sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2
)))+2*a^2/b^2/(a^2-a*b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c
)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellip...
```

3.615.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)^(7/2)/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
Timed out
```

3.615.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**(7/2)/(a+b*sec(d*x+c))**2,x)
```

output
```
Timed out
```

3.615.7 Maxima [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)^(7/2)/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
Timed out
```

3.615.8 Giac [F]

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \int \frac{\sec\left(dx+c\right)^{\frac{7}{2}}}{\left(b\sec\left(dx+c\right)+a\right)^2}\,dx$$

input
```
integrate(sec(d*x+c)^(7/2)/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^(7/2)/(b*sec(d*x + c) + a)^2, x)
```

3.615.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{7/2}}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^2}\,dx$$

input
```
int((1/cos(c + d*x))^(7/2)/(a + b/cos(c + d*x))^2,x)
```

output
```
int((1/cos(c + d*x))^(7/2)/(a + b/cos(c + d*x))^2, x)
```

3.616 $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

3.616.1 Optimal result 4691
3.616.2 Mathematica [B] (warning: unable to verify) 4692
3.616.3 Rubi [A] (verified) 4692
3.616.4 Maple [B] (verified) 4697
3.616.5 Fricas [F(-1)] 4698
3.616.6 Sympy [F] 4698
3.616.7 Maxima [F(-1)] 4698
3.616.8 Giac [F] 4699
3.616.9 Mupad [F(-1)] 4699

3.616.1 Optimal result

Integrand size = 23, antiderivative size = 214

$$\begin{aligned}\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\sec(c+dx))^2}\,dx &= \frac{a\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{b\left(a^2-b^2\right)d} \\ &\quad+\frac{\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{\left(a^2-b^2\right)d} \\ &\quad+\frac{\left(a^2-3b^2\right)\sqrt{\cos(c+dx)}\,\text{EllipticPi}\left(\frac{2a}{a+b},\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{(a-b)b(a+b)^2d} \\ &\quad-\frac{a^2\sqrt{\sec(c+dx)}\sin(c+dx)}{b\left(a^2-b^2\right)d(a+b\sec(c+dx))}\end{aligned}$$

output
```
-a^2*sin(d*x+c)*sec(d*x+c)^(1/2)/b/(a^2-b^2)/d/(a+b*sec(d*x+c))+a*(cos(1/2
*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/
2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/b/(a^2-b^2)/d+(cos(1/2*d*x+1/2*c)^2)
^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)
^(1/2)*sec(d*x+c)^(1/2)/(a^2-b^2)/d+(a^2-3*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/
2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*a/(a+b),2^(1/2))*cos
(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/(a-b)/b/(a+b)^2/d
```

3.616.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 582 vs. $2(214) = 428$.

Time = 6.48 (sec) , antiderivative size = 582, normalized size of antiderivative = 2.72

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \frac{\sqrt{\sec(c+dx)}\left(\frac{a\sin(c+dx)}{b(-a^2+b^2)}+\frac{a\sin(c+dx)}{(a^2-b^2)(b+a\cos(c+dx))}\right)}{d} + \frac{\frac{2(3a^2-4b^2)\cos^2(c+dx)\left(\text{EllipticF}\left(\arcsin\left(\sqrt{\sec(c+dx)}\right),-1\right)-\text{EllipticPi}\left(-\frac{b}{a},\arcsin\left(\sqrt{\sec(c+dx)}\right),-1\right)\right)(a+b\sec(c+dx))\sqrt{1-\sec^2(c+dx)}}{b(b+a\cos(c+dx))(1-\cos^2(c+dx))}}{}$$

input
```
Integrate[Sec[c + d*x]^(5/2)/(a + b*Sec[c + d*x])^2,x]
```

output
```
(Sqrt[Sec[c + d*x]]*((a*Sin[c + d*x])/(b*(-a^2 + b^2)) + (a*Sin[c + d*x])/
((a^2 - b^2)*(b + a*Cos[c + d*x]))))/d + ((2*(3*a^2 - 4*b^2)*Cos[c + d*x]^
2*(EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1] - EllipticPi[-(b/a), ArcSin[S
qrt[Sec[c + d*x]]], -1])*(a + b*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin
[c + d*x])/(b*(b + a*Cos[c + d*x])*(1 - Cos[c + d*x]^2)) + (8*b*Cos[c + d*
x]^2*EllipticPi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]], -1]*(a + b*Sec[c + d*x
])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x])/((b + a*Cos[c + d*x])*(1 - Cos[c
 + d*x]^2)) + (Cos[2*(c + d*x)]*(a + b*Sec[c + d*x])*(-4*a*b + 4*a*b*Sec[c
 + d*x]^2 - 4*a*b*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d
*x]]*Sqrt[1 - Sec[c + d*x]^2] - 2*a*(a - 2*b)*EllipticF[ArcSin[Sqrt[Sec[c
+ d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] + 2*a^2*Elliptic
Pi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec
[c + d*x]^2] - 4*b^2*EllipticPi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sq
rt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2])*Sin[c + d*x])/(b*(b + a*Cos[c +
 d*x])*(1 - Cos[c + d*x]^2)*Sqrt[Sec[c + d*x]]*(2 - Sec[c + d*x]^2)))/(4*(
a - b)*b*(a + b)*d)
```

3.616.3 Rubi [A] (verified)

Time = 1.37 (sec) , antiderivative size = 212, normalized size of antiderivative = 0.99, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.652$, Rules used = {3042, 4332, 27, 3042, 4594, 3042, 4274, 3042, 4258, 3042, 3119, 3120, 4336, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \; 4332$$

$$-\frac{\int -\frac{a^2+2b\sec(c+dx)a+\left(a^2-2b^2\right)\sec^2(c+dx)}{2\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}dx}{b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{bd\left(a^2-b^2\right)(a+b\sec(c+dx))}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{a^2+2b\sec(c+dx)a+\left(a^2-2b^2\right)\sec^2(c+dx)}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}dx}{2b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{bd\left(a^2-b^2\right)(a+b\sec(c+dx))}$$

$$\Big\downarrow \; 3042$$

$$\frac{\int \frac{a^2+2b\csc\left(c+dx+\frac{\pi}{2}\right)a+\left(a^2-2b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{2b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{bd\left(a^2-b^2\right)(a+b\sec(c+dx))}$$

$$\Big\downarrow \; 4594$$

$$\frac{\left(a^2-3b^2\right)\int \frac{\sec^{\frac{3}{2}}(c+dx)}{a+b\sec(c+dx)}dx+\frac{\int \frac{a^3+b\sec(c+dx)a^2}{\sqrt{\sec(c+dx)}}dx}{a^2}}{2b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{bd\left(a^2-b^2\right)(a+b\sec(c+dx))}$$

$$\Big\downarrow \; 3042$$

$$\frac{\left(a^2-3b^2\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{\int \frac{a^3+b\csc\left(c+dx+\frac{\pi}{2}\right)a^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2}}{2b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{bd\left(a^2-b^2\right)(a+b\sec(c+dx))}$$

$$\Big\downarrow \; 4274$$

$$\frac{\left(a^2-3b^2\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{a^3\int \frac{1}{\sqrt{\sec(c+dx)}}dx+a^2b\int \sqrt{\sec(c+dx)}dx}{a^2}}{2b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{bd\left(a^2-b^2\right)(a+b\sec(c+dx))}$$

$$\Big\downarrow \; 3042$$

3.616. $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

$$\frac{\left(a^2-3b^2\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{a^3\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+a^2b\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}}{2b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{bd\left(a^2-b^2\right)(a+b\sec(c+dx))}$$

$\downarrow$ 4258

$$\frac{\left(a^2-3b^2\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\cos(c+dx)}dx+a^2b\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{a^2}}{2b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{bd\left(a^2-b^2\right)(a+b\sec(c+dx))}$$

$\downarrow$ 3042

$$\frac{\left(a^2-3b^2\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+a^2b\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2}}{2b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{bd\left(a^2-b^2\right)(a+b\sec(c+dx))}$$

$\downarrow$ 3119

$$\frac{\left(a^2-3b^2\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{a^2b\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}}{a^2}}{2b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{bd\left(a^2-b^2\right)(a+b\sec(c+dx))}$$

$\downarrow$ 3120

$$\frac{\left(a^2-3b^2\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{\frac{2a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a^2b\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}}{a^2}}{2b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{bd\left(a^2-b^2\right)(a+b\sec(c+dx))}$$

$\downarrow$ 4336

3.616. $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

$$\frac{\left(a^2-3b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}(b+a\cos(c+dx))}dx+\frac{\frac{2a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a^2b\sqrt{\cos(c+dx}}{a^2}}{2b\left(a^2-b^2\right)}}{\frac{a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{bd\left(a^2-b^2\right)(a+b\sec(c+dx))}}$$

$\Big\downarrow$ 3042

$$\frac{\left(a^2-3b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(b+a\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx+\frac{\frac{2a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a^2b\sqrt{c}}{a^2}}{2b\left(a^2-b^2\right)}}{\frac{a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{bd\left(a^2-b^2\right)(a+b\sec(c+dx))}}$$

$\Big\downarrow$ 3284

$$\frac{\frac{2\left(a^2-3b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticPi}\left(\frac{2a}{a+b},\frac{1}{2}(c+dx),2\right)}{d(a+b)}+\frac{\frac{2a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a^2b\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}(}{d}}{a^2}}{2b\left(a^2-b^2\right)}}{\frac{a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{bd\left(a^2-b^2\right)(a+b\sec(c+dx))}}$$

input
```
Int[Sec[c + d*x]^(5/2)/(a + b*Sec[c + d*x])^2,x]
```

output
```
(((2*a^3*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/
d + (2*a^2*b*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x
]])/d)/a^2 + (2*(a^2 - 3*b^2)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*a)/(a + b),
 (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/((a + b)*d))/(2*b*(a^2 - b^2)) - (a^2
*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(b*(a^2 - b^2)*d*(a + b*Sec[c + d*x]))
```

3.616.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4332
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-a^2)*d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(
m + 1)*((d*Csc[e + f*x])^(n - 3)/(b*f*(m + 1)*(a^2 - b^2))), x] + Simp[d^3/
(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])
^(n - 3)*Simp[a^2*(n - 3) + a*b*(m + 1)*Csc[e + f*x] - (a^2*(n - 2) + b^2*(
m + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2
- b^2, 0] && LtQ[m, -1] && (IGtQ[n, 3] || (IntegersQ[n + 1/2, 2*m] && GtQ[n
, 2]))
```

rule 4336
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]]   Int[
1/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

3.616. $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

rule 4594
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Simp[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2)   Int[(d*Csc[e +
f*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Simp[1/a^2   Int[(a*A - (A*b - a
*B)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A,
B, C}, x] && NeQ[a^2 - b^2, 0]
```

3.616.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 607 vs. $2(278) = 556$.

Time = 104.88 (sec) , antiderivative size = 608, normalized size of antiderivative = 2.84

method	result
default	$-\frac{\sqrt{-\left(-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(\frac{2a^2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}}{b\left(a^2-b^2\right)\left(2a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-a+b\right)}-\frac{\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1}\ \mathrm{El}}{(a+b)b\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\mathrm{s}}}\right.}{}$

input
```
int(sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
-(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*a^2/b/(a^2-b
^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2
)/(2*a*cos(1/2*d*x+1/2*c)^2-a+b)-1/(a+b)/b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-
2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+a/b/(a^2-b^2)*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)
^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-a/b/(
a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-
2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1
/2*c),2^(1/2))-1/b/(a^2-b^2)/(a^2-a*b)*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-
2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))+3*b/(a^2-b^2)/
(a^2-a*b)*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)
/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d
*x+1/2*c),2*a/(a-b),2^(1/2)))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1
)^(1/2)/d
```

3.616.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
Timed out
```

3.616.6 Sympy [F]

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$

input
```
integrate(sec(d*x+c)**(5/2)/(a+b*sec(d*x+c))**2,x)
```

output
```
Integral(sec(c + d*x)**(5/2)/(a + b*sec(c + d*x))**2, x)
```

3.616.7 Maxima [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
Timed out
```

3.616.8 Giac [F]

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \int \frac{\sec{(dx+c)}^{\frac{5}{2}}}{(b\sec{(dx+c)}+a)^2}\,dx$$

input
```
integrate(sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^(5/2)/(b*sec(d*x + c) + a)^2, x)
```

3.616.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^2}\,dx$$

input
```
int((1/cos(c + d*x))^(5/2)/(a + b/cos(c + d*x))^2,x)
```

output
```
int((1/cos(c + d*x))^(5/2)/(a + b/cos(c + d*x))^2, x)
```

3.617 $\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

3.617.1 Optimal result 4700
3.617.2 Mathematica [A] (verified) 4701
3.617.3 Rubi [A] (verified) 4701
3.617.4 Maple [B] (verified) 4705
3.617.5 Fricas [F(-1)] 4706
3.617.6 Sympy [F] 4707
3.617.7 Maxima [F(-1)] 4707
3.617.8 Giac [F] 4707
3.617.9 Mupad [F(-1)] 4708

3.617.1 Optimal result

Integrand size = 23, antiderivative size = 208

$$\begin{aligned}\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^2}\,dx &= -\frac{\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{(a^2-b^2)\,d} \\ &\quad -\frac{b\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{a\,(a^2-b^2)\,d} \\ &\quad +\frac{(a^2+b^2)\sqrt{\cos(c+dx)}\,\text{EllipticPi}\left(\frac{2a}{a+b},\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{a(a-b)(a+b)^2d} \\ &\quad +\frac{a\sqrt{\sec(c+dx)}\sin(c+dx)}{(a^2-b^2)\,d(a+b\sec(c+dx))}\end{aligned}$$

output
```
a*sin(d*x+c)*sec(d*x+c)^(1/2)/(a^2-b^2)/d/(a+b*sec(d*x+c))-(cos(1/2*d*x+1/
2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos
(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/(a^2-b^2)/d-b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/
cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*
sec(d*x+c)^(1/2)/a/(a^2-b^2)/d+(a^2+b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(
1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*a/(a+b),2^(1/2))*cos(d*x+c)
^(1/2)*sec(d*x+c)^(1/2)/a/(a-b)/(a+b)^2/d
```

3.617.2 Mathematica [A] (verified)

Time = 6.02 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.39

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$
$$= \frac{\frac{4a\sin(c+dx)}{(a^2-b^2)(b+a\cos(c+dx))\sqrt{\sec(c+dx)}} + \frac{2\cot(c+dx)\left(ab\sec^{\frac{3}{2}}(c+dx)+ab\cos(2(c+dx))\sec^{\frac{3}{2}}(c+dx)-ab\sec^{\frac{7}{2}}(c+dx)-ab\cos(2(c+dx))\sec^{\frac{7}{2}}(c+d\right.}{}}{}$$

input
```
Integrate[Sec[c + d*x]^(3/2)/(a + b*Sec[c + d*x])^2,x]
```

output
```
((4*a*Sin[c + d*x])/((a^2 - b^2)*(b + a*Cos[c + d*x])*Sqrt[Sec[c + d*x]])
+ (2*Cot[c + d*x]*(a*b*Sec[c + d*x]^(3/2) + a*b*Cos[2*(c + d*x)]*Sec[c + d
*x]^(3/2) - a*b*Sec[c + d*x]^(7/2) - a*b*Cos[2*(c + d*x)]*Sec[c + d*x]^(7/
2) + 2*a*b*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[c + d*x]^2]
 + 2*a*(a - b)*EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[c + d*x
]^2] - 2*a^2*EllipticPi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[
c + d*x]^2] - 2*b^2*EllipticPi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqr
t[-Tan[c + d*x]^2]))/(a*(a - b)*b*(a + b)))/(4*d)
```

3.617.3 Rubi [A] (verified)

Time = 1.28 (sec) , antiderivative size = 202, normalized size of antiderivative = 0.97, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.652$, Rules used = {3042, 4331, 27, 3042, 4594, 3042, 4274, 3042, 4258, 3042, 3119, 3120, 4336, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx$$
$$\Big\downarrow \; 4331$$

$$\frac{\int -\frac{-a\sec^2(c+dx)+2b\sec(c+dx)+a}{2\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}dx}{a^2-b^2}+\frac{a\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a^2-b^2)(a+b\sec(c+dx))}$$

$\downarrow$ 27

$$\frac{a\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a^2-b^2)(a+b\sec(c+dx))}-\frac{\int \frac{-a\sec^2(c+dx)+2b\sec(c+dx)+a}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}dx}{2(a^2-b^2)}$$

$\downarrow$ 3042

$$\frac{a\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a^2-b^2)(a+b\sec(c+dx))}-\frac{\int \frac{-a\csc\left(c+dx+\frac{\pi}{2}\right)^2+2b\csc\left(c+dx+\frac{\pi}{2}\right)+a}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{2(a^2-b^2)}$$

$\downarrow$ 4594

$$\frac{a\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a^2-b^2)(a+b\sec(c+dx))}-\frac{\frac{\int \frac{a^2+b\sec(c+dx)a}{\sqrt{\sec(c+dx)}}dx}{a^2}-\frac{(a^2+b^2)\int \frac{\sec^{\frac{3}{2}}(c+dx)}{a+b\sec(c+dx)}dx}{a}}{2(a^2-b^2)}$$

$\downarrow$ 3042

$$\frac{a\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a^2-b^2)(a+b\sec(c+dx))}-\frac{\frac{\int \frac{a^2+b\csc\left(c+dx+\frac{\pi}{2}\right)a}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2}-\frac{(a^2+b^2)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a}}{2(a^2-b^2)}$$

$\downarrow$ 4274

$$\frac{a\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a^2-b^2)(a+b\sec(c+dx))}-\frac{\frac{a^2\int \frac{1}{\sqrt{\sec(c+dx)}}dx+ab\int \sqrt{\sec(c+dx)}dx}{a^2}-\frac{(a^2+b^2)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a}}{2(a^2-b^2)}$$

$\downarrow$ 3042

$$\frac{a\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a^2-b^2)(a+b\sec(c+dx))}-\frac{\frac{a^2\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+ab\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}-\frac{(a^2+b^2)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a}}{2(a^2-b^2)}$$

$\downarrow$ 4258

$$\frac{a\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a^2-b^2)(a+b\sec(c+dx))}-$$
$$\frac{\frac{a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\cos(c+dx)}dx+ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{a^2}-\frac{(a^2+b^2)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a}}{2(a^2-b^2)}$$

$\downarrow$ 3042

3.617. $\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

$$\frac{a\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)}-\frac{\frac{a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2}-\frac{\left(a^2+b^2\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a}}{2\left(a^2-b^2\right)}$$

$\downarrow$ 3119

$$\frac{a\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)}-\frac{\frac{ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}}{a^2}-\frac{\left(a^2+b^2\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a}}{2\left(a^2-b^2\right)}$$

$\downarrow$ 3120

$$\frac{a\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)}-\frac{\frac{\frac{2a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}}{a^2}-\frac{\left(a^2+b^2\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a}}{2\left(a^2-b^2\right)}$$

$\downarrow$ 4336

$$\frac{a\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)}-\frac{\frac{\frac{2a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}}{a^2}-\frac{\left(a^2+b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}(b+a\cos}}{a}}{2\left(a^2-b^2\right)}$$

$\downarrow$ 3042

$$\frac{a\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)}-\frac{\frac{\frac{2a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}}{a^2}-\frac{\left(a^2+b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}(b+}}{a}}{2\left(a^2-b^2\right)}$$

$\downarrow$ 3284

$$\frac{a\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)}-\frac{\frac{\frac{2a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}}{a^2}-\frac{2\left(a^2+b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticPi}\left(\frac{2a}{a+b},\frac{1}{2}(c\right.}{ad(a+b)}}{2\left(a^2-b^2\right)}$$

3.617. $\int\frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

input
```
Int[Sec[c + d*x]^(3/2)/(a + b*Sec[c + d*x])^2,x]
```

output
```
-1/2*(((2*a^2*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*
x]])/d + (2*a*b*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c +
d*x]])/d)/a^2 - (2*(a^2 + b^2)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*a)/(a + b)
, (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a*(a + b)*d))/(a^2 - b^2) + (a*Sqrt
[Sec[c + d*x]]*Sin[c + d*x])/((a^2 - b^2)*d*(a + b*Sec[c + d*x]))
```

3.617.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4331
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[a*d^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
)*((d*Csc[e + f*x])^(n - 2)/(f*(m + 1)*(a^2 - b^2))), x] - Simp[d^2/((m + 1
)*(a^2 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*
(a*(n - 2) + b*(m + 1)*Csc[e + f*x] - a*(m + n)*Csc[e + f*x]^2), x], x] /;
FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && LtQ[1, n, 2
] && IntegersQ[2*m, 2*n]
```

rule 4336
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]]   Int[
1/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4594
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Simp[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2)   Int[(d*Csc[e +
f*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Simp[1/a^2   Int[(a*A - (A*b - a
*B)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A,
B, C}, x] && NeQ[a^2 - b^2, 0]
```

3.617.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 706 vs. $2(272) = 544$.

Time = 11.98 (sec) , antiderivative size = 707, normalized size of antiderivative = 3.40

method	result
default	$-\frac{\sqrt{-\left(-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(-\frac{2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1}\,\mathrm{EllipticPi}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\frac{2a}{a-b},\sqrt{2}\right)}{\left(a^2-ab\right)\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}}-\frac{2b\left(\frac{a^2\cos\left(\frac{dx}{2}+\right.}{b\left(a^2\right.}\right.}{}\right.}{}$

input
```
int(sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
-(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2/(a^2-a*b)*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*
d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a
/(a-b),2^(1/2))-2*b/a*(a^2/b/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+
1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*a*cos(1/2*d*x+1/2*c)^2-a+b)-1/2/(a
+b)/b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*
c),2^(1/2))+1/2*a/b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x
+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*El
lipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2*a/b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/
2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-1/2/b/(a^2-b^2
)/(a^2-a*b)*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(
1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1
/2*d*x+1/2*c),2*a/(a-b),2^(1/2))+3/2*b/(a^2-b^2)/(a^2-a*b)*a*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/
2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

3.617.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
Timed out
```

3.617.6 Sympy [F]

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$

input
```
integrate(sec(d*x+c)**(3/2)/(a+b*sec(d*x+c))**2,x)
```

output
```
Integral(sec(c + d*x)**(3/2)/(a + b*sec(c + d*x))**2, x)
```

3.617.7 Maxima [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
Timed out
```

3.617.8 Giac [F]

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \int \frac{\sec(dx+c)^{\frac{3}{2}}}{(b\sec(dx+c)+a)^2}\,dx$$

input
```
integrate(sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^(3/2)/(b*sec(d*x + c) + a)^2, x)
```

3.617.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^2}\,dx$$

input
```
int((1/cos(c + d*x))^(3/2)/(a + b/cos(c + d*x))^2,x)
```

output
```
int((1/cos(c + d*x))^(3/2)/(a + b/cos(c + d*x))^2, x)
```

3.618 $\int \frac{\sqrt{\sec(c+dx)}}{(a+b\sec(c+dx))^2}\,dx$

3.618.1 Optimal result . . . 4709
3.618.2 Mathematica [A] (verified) . . . 4710
3.618.3 Rubi [A] (verified) . . . 4710
3.618.4 Maple [B] (verified) . . . 4714
3.618.5 Fricas [F(-1)] . . . 4715
3.618.6 Sympy [F] . . . 4716
3.618.7 Maxima [F] . . . 4716
3.618.8 Giac [F] . . . 4716
3.618.9 Mupad [F(-1)] . . . 4717

3.618.1 Optimal result

Integrand size = 23, antiderivative size = 227

$$\begin{aligned}\int \frac{\sqrt{\sec(c+dx)}}{(a+b\sec(c+dx))^2}\,dx &= \frac{b\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a\left(a^2-b^2\right)d} \\ &\quad+\frac{\left(2a^2-b^2\right)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{a^2\left(a^2-b^2\right)d} \\ &\quad-\frac{b(3a^2-b^2)\sqrt{\cos(c+dx)}\,\text{EllipticPi}\left(\frac{2a}{a+b},\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{a^2(a-b)(a+b)^2d} \\ &\quad-\frac{b\sqrt{\sec(c+dx)}\sin(c+dx)}{\left(a^2-b^2\right)d(a+b\sec(c+dx))}\end{aligned}$$

output
```
-b*sin(d*x+c)*sec(d*x+c)^(1/2)/(a^2-b^2)/d/(a+b*sec(d*x+c))+b*(cos(1/2*d*x
+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*
cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a/(a^2-b^2)/d+(2*a^2-b^2)*(cos(1/2*d*x+1
/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*co
s(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^2/(a^2-b^2)/d-b*(3*a^2-b^2)*(cos(1/2*d*x
+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*a/(a+b
),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^2/(a-b)/(a+b)^2/d
```

3.618.2 Mathematica [A] (verified)

Time = 3.05 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.11

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+b\sec(c+dx))^2}\,dx$$
$$= \frac{\cos(2(c+dx))\csc(c+dx)\sqrt{\sec(c+dx)}\left(a(a-b)(b+a\cos(c+dx))\,\text{EllipticF}\left(\arcsin\left(\sqrt{\sec(c+dx)}\right),\right.\right.}{}$$

input
```
Integrate[Sqrt[Sec[c + d*x]]/(a + b*Sec[c + d*x])^2,x]
```

output
```
(Cos[2*(c + d*x)]*Csc[c + d*x]*Sqrt[Sec[c + d*x]]*(a*(a - b)*(b + a*Cos[c
+ d*x])*EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[
-Tan[c + d*x]^2] - (3*a^2 - b^2)*(b + a*Cos[c + d*x])*EllipticPi[-(b/a), A
rcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[-Tan[c + d*x]^2] +
a*b*(-(b*Tan[c + d*x]^2) + (b + a*Cos[c + d*x])*EllipticE[ArcSin[Sqrt[Sec[
c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[-Tan[c + d*x]^2])))/(a^2*(a - b)*(
a + b)*d*(b + a*Cos[c + d*x])*(-2 + Sec[c + d*x]^2))
```

3.618.3 Rubi [A] (verified)

Time = 1.39 (sec) , antiderivative size = 216, normalized size of antiderivative = 0.95, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.652$, Rules used = {3042, 4330, 27, 3042, 4594, 3042, 4274, 3042, 4258, 3042, 3119, 3120, 4336, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+b\sec(c+dx))^2}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx$$
$$\Big\downarrow \; 4330$$
$$-\frac{\int -\frac{-b\sec^2(c+dx)+2a\sec(c+dx)+b}{2\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}dx}{a^2-b^2} - \frac{b\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)(a+b\sec(c+dx))}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{-b\sec^2(c+dx)+2a\sec(c+dx)+b}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}dx}{2\left(a^2-b^2\right)}-\frac{b\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)(a+b\sec(c+dx))}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{-b\csc\left(c+dx+\frac{\pi}{2}\right)^2+2a\csc\left(c+dx+\frac{\pi}{2}\right)+b}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{2\left(a^2-b^2\right)}-\frac{b\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)(a+b\sec(c+dx))}$$

$$\Big\downarrow \text{4594}$$

$$\frac{\frac{\int \frac{ab+\left(2a^2-b^2\right)\sec(c+dx)}{\sqrt{\sec(c+dx)}}dx}{a^2}-\frac{b\left(3a^2-b^2\right)\int \frac{\sec^{\frac{3}{2}}(c+dx)}{a+b\sec(c+dx)}dx}{a^2}}{2\left(a^2-b^2\right)}-\frac{b\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)(a+b\sec(c+dx))}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\int \frac{ab+\left(2a^2-b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2}-\frac{b\left(3a^2-b^2\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}}{2\left(a^2-b^2\right)}-\frac{b\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)(a+b\sec(c+dx))}$$

$$\Big\downarrow \text{4274}$$

$$\frac{\frac{\left(2a^2-b^2\right)\int \sqrt{\sec(c+dx)}dx+ab\int \frac{1}{\sqrt{\sec(c+dx)}}dx}{a^2}-\frac{b\left(3a^2-b^2\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}}{2\left(a^2-b^2\right)}-\frac{b\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)(a+b\sec(c+dx))}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\left(2a^2-b^2\right)\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+ab\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2}-\frac{b\left(3a^2-b^2\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}}{2\left(a^2-b^2\right)}-\frac{b\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)(a+b\sec(c+dx))}$$

$$\Big\downarrow \text{4258}$$

$$\frac{\frac{\left(2a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx+ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\cos(c+dx)}dx}{a^2}-\frac{b\left(3a^2-b^2\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}}{2\left(a^2-b^2\right)}-\frac{b\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)(a+b\sec(c+dx))}$$

$$\Big\downarrow \text{3042}$$

3.618. $\int \frac{\sqrt{\sec(c+dx)}}{(a+b\sec(c+dx))^2}\,dx$

$$\frac{\frac{(2a^2-b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}-\frac{b(3a^2-b^2)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}}{\frac{2(a^2-b^2)}{\frac{b\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a^2-b^2)(a+b\sec(c+dx))}}}$$

$\downarrow$ 3119

$$\frac{\frac{(2a^2-b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}}{a^2}-\frac{b(3a^2-b^2)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}}{\frac{2(a^2-b^2)}{\frac{b\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a^2-b^2)(a+b\sec(c+dx))}}}$$

$\downarrow$ 3120

$$\frac{\frac{\frac{2\left(2a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+\frac{2ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}}{a^2}-\frac{b(3a^2-b^2)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}}{\frac{2(a^2-b^2)}{\frac{b\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a^2-b^2)(a+b\sec(c+dx))}}}$$

$\downarrow$ 4336

$$\frac{\frac{\frac{2\left(2a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+\frac{2ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}}{a^2}-\frac{b(3a^2-b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{}{\sqrt{\cos(c+d}}}{a^2}}{\frac{2(a^2-b^2)}{\frac{b\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a^2-b^2)(a+b\sec(c+dx))}}}$$

$\downarrow$ 3042

$$\frac{\frac{\frac{2\left(2a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+\frac{2ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}}{a^2}-\frac{b(3a^2-b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{}{\sqrt{\sin\left(c+d\right.}}}{a^2}}{\frac{2(a^2-b^2)}{\frac{b\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a^2-b^2)(a+b\sec(c+dx))}}}$$

$\downarrow$ 3284

3.618. $\int\frac{\sqrt{\sec(c+dx)}}{(a+b\sec(c+dx))^2}\,dx$

$$\frac{\frac{\frac{2\left(2a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+\frac{2ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}}{a^2}-\frac{2b\left(3a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticPi}}{a^2d(a+b)}}{2\left(a^2-b^2\right)}$$

$$\frac{b\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)(a+b\sec(c+dx))}$$

input
```
Int[Sqrt[Sec[c + d*x]]/(a + b*Sec[c + d*x])^2,x]
```

output
```
(((2*a*b*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/
d + (2*(2*a^2 - b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec
[c + d*x]])/d)/a^2 - (2*b*(3*a^2 - b^2)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*a
)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a^2*(a + b)*d))/(2*(a^2 -
b^2)) - (b*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/((a^2 - b^2)*d*(a + b*Sec[c +
d*x]))
```

3.618.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4330
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b)*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m +
1)*((d*Csc[e + f*x])^(n - 1)/(f*(m + 1)*(a^2 - b^2))), x] + Simp[1/((m + 1)
*(a^2 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1)*S
imp[b*d*(n - 1) + a*d*(m + 1)*Csc[e + f*x] - b*d*(m + n + 1)*Csc[e + f*x]^2
, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1
] && LtQ[0, n, 1] && IntegersQ[2*m, 2*n]
```

rule 4336
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]]   Int[
1/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4594
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Simp[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2)   Int[(d*Csc[e +
f*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Simp[1/a^2   Int[(a*A - (A*b - a
*B)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A,
B, C}, x] && NeQ[a^2 - b^2, 0]
```

3.618.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 787 vs. $2(291) = 582$.

Time = 14.59 (sec) , antiderivative size = 788, normalized size of antiderivative = 3.47

method	result	size
default	Expression too large to display	788

input
```
int(sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
-(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2/a^2*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+2/
a^2*b^2*(a^2/b/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1
/2*d*x+1/2*c)^2)^(1/2)/(2*a*cos(1/2*d*x+1/2*c)^2-a+b)-1/2/(a+b)/b*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/
2*a/b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(
1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/
2*d*x+1/2*c),2^(1/2))-1/2*a/b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*c
os(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2
)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-1/2/b/(a^2-b^2)/(a^2-a*b)*a^
3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1
/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),
2*a/(a-b),2^(1/2))+3/2*b/(a^2-b^2)/(a^2-a*b)*a*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1
/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2)))+4/a*b/(a^
2-a*b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*
sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/
2*c),2*a/(a-b),2^(1/2)))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^...
```

3.618.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+b\sec(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
Timed out
```

3.618.6 Sympy [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+b\sec(c+dx))^2}\,dx = \int \frac{\sqrt{\sec{(c+dx)}}}{(a+b\sec{(c+dx)})^2}\,dx$$

input
```
integrate(sec(d*x+c)**(1/2)/(a+b*sec(d*x+c))**2,x)
```

output
```
Integral(sqrt(sec(c + d*x))/(a + b*sec(c + d*x))**2, x)
```

3.618.7 Maxima [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+b\sec(c+dx))^2}\,dx = \int \frac{\sqrt{\sec{(dx+c)}}}{(b\sec{(dx+c)}+a)^2}\,dx$$

input
```
integrate(sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(sec(d*x + c))/(b*sec(d*x + c) + a)^2, x)
```

3.618.8 Giac [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+b\sec(c+dx))^2}\,dx = \int \frac{\sqrt{\sec{(dx+c)}}}{(b\sec{(dx+c)}+a)^2}\,dx$$

input
```
integrate(sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate(sqrt(sec(d*x + c))/(b*sec(d*x + c) + a)^2, x)
```

3.618.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+b\sec(c+dx))^2}\,dx = \int \frac{\sqrt{\frac{1}{\cos(c+d\,x)}}}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^2}\,dx$$

input
```
int((1/cos(c + d*x))^(1/2)/(a + b/cos(c + d*x))^2,x)
```

output
```
int((1/cos(c + d*x))^(1/2)/(a + b/cos(c + d*x))^2, x)
```

3.619 $\int \frac{1}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^2}\,dx$

3.619.1 Optimal result 4718
3.619.2 Mathematica [A] (verified) 4719
3.619.3 Rubi [A] (verified) 4719
3.619.4 Maple [B] (verified) 4724
3.619.5 Fricas [F(-1)] 4724
3.619.6 Sympy [F] 4725
3.619.7 Maxima [F] 4725
3.619.8 Giac [F] 4725
3.619.9 Mupad [F(-1)] 4726

3.619.1 Optimal result

Integrand size = 23, antiderivative size = 244

$$\begin{aligned}\int \frac{1}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^2}\,dx &= \frac{(2a^2-3b^2)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^2(a^2-b^2)d} \\ &\quad - \frac{b(4a^2-3b^2)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{a^3(a^2-b^2)d} \\ &\quad + \frac{b^2(5a^2-3b^2)\sqrt{\cos(c+dx)}\,\text{EllipticPi}\left(\frac{2a}{a+b},\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{a^3(a-b)(a+b)^2d} \\ &\quad + \frac{b^2\sqrt{\sec(c+dx)}\sin(c+dx)}{a(a^2-b^2)d(a+b\sec(c+dx))}\end{aligned}$$

output
```
b^2*sin(d*x+c)*sec(d*x+c)^(1/2)/a/(a^2-b^2)/d/(a+b*sec(d*x+c))+(2*a^2-3*b^
2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1
/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^2/(a^2-b^2)/d-b*(4*a^2-
3*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d
*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^3/(a^2-b^2)/d+b^2*(
5*a^2-3*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(si
n(1/2*d*x+1/2*c),2*a/(a+b),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^3/
(a-b)/(a+b)^2/d
```

3.619.2 Mathematica [A] (verified)

Time = 4.61 (sec) , antiderivative size = 319, normalized size of antiderivative = 1.31

$$\int \frac{1}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^2}\,dx$$
$$= \frac{\frac{4b^2\sin(c+dx)}{a(a^2-b^2)(b+a\cos(c+dx))\sqrt{\sec(c+dx)}} + \frac{2\cot(c+dx)\left(2a^3\sec^{\frac{3}{2}}(c+dx)-3ab^2\sec^{\frac{3}{2}}(c+dx)-2a^3\cos(2(c+dx))\sec^{\frac{3}{2}}(c+dx)+3ab^2\cos(2(c+dx))\right.}{}}{}$$

input
```
Integrate[1/(Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^2),x]
```

output
```
((4*b^2*Sin[c + d*x])/(a*(a^2 - b^2)*(b + a*Cos[c + d*x])*Sqrt[Sec[c + d*x
]]) + (2*Cot[c + d*x]*(2*a^3*Sec[c + d*x]^(3/2) - 3*a*b^2*Sec[c + d*x]^(3/
2) - 2*a^3*Cos[2*(c + d*x)]*Sec[c + d*x]^(3/2) + 3*a*b^2*Cos[2*(c + d*x)]*
Sec[c + d*x]^(3/2) - 2*a*(2*a^2 - 3*b^2)*EllipticE[ArcSin[Sqrt[Sec[c + d*x
]]], -1]*Sqrt[-Tan[c + d*x]^2] + 2*a*(2*a^2 + a*b - 3*b^2)*EllipticF[ArcSi
n[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[c + d*x]^2] - 10*a^2*b*EllipticPi[-(b
/a), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[c + d*x]^2] + 6*b^3*Ellipti
cPi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[c + d*x]^2]))/(a^3*(
a - b)*(a + b)))/(4*d)
```

3.619.3 Rubi [A] (verified)

Time = 1.45 (sec) , antiderivative size = 232, normalized size of antiderivative = 0.95, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.652$, Rules used = {3042, 4334, 27, 3042, 4594, 3042, 4274, 3042, 4258, 3042, 3119, 3120, 4336, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^2}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx$$
$$\Big\downarrow \; 4334$$

$$\frac{b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{ad\left(a^2-b^2\right)(a+b\sec(c+dx))}-\frac{\int -\frac{2a^2-2b\sec(c+dx)a-3b^2+b^2\sec^2(c+dx)}{2\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}dx}{a\left(a^2-b^2\right)}$$

$\Big\downarrow$ 27

$$\frac{\int \frac{2a^2-2b\sec(c+dx)a-3b^2+b^2\sec^2(c+dx)}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}dx}{2a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{ad\left(a^2-b^2\right)(a+b\sec(c+dx))}$$

$\Big\downarrow$ 3042

$$\frac{\int \frac{2a^2-2b\csc\left(c+dx+\frac{\pi}{2}\right)a-3b^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{2a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{ad\left(a^2-b^2\right)(a+b\sec(c+dx))}$$

$\Big\downarrow$ 4594

$$\frac{b^2\left(5-\frac{3b^2}{a^2}\right)\int \frac{\sec^{\frac{3}{2}}(c+dx)}{a+b\sec(c+dx)}dx+\frac{\int \frac{a\left(2a^2-3b^2\right)-b\left(4a^2-3b^2\right)\sec(c+dx)}{\sqrt{\sec(c+dx)}}dx}{a^2}}{2a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{ad\left(a^2-b^2\right)(a+b\sec(c+dx))}$$

$\Big\downarrow$ 3042

$$\frac{b^2\left(5-\frac{3b^2}{a^2}\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{\int \frac{a\left(2a^2-3b^2\right)-b\left(4a^2-3b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2}}{2a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{ad\left(a^2-b^2\right)(a+b\sec(c+dx))}$$

$\Big\downarrow$ 4274

$$\frac{b^2\left(5-\frac{3b^2}{a^2}\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{a\left(2a^2-3b^2\right)\int \frac{1}{\sqrt{\sec(c+dx)}}dx-b\left(4a^2-3b^2\right)\int \sqrt{\sec(c+dx)}dx}{a^2}}{2a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{ad\left(a^2-b^2\right)(a+b\sec(c+dx))}$$

$\Big\downarrow$ 3042

$$\frac{b^2\left(5-\frac{3b^2}{a^2}\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{a\left(2a^2-3b^2\right)\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-b\left(4a^2-3b^2\right)\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}}{2a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{ad\left(a^2-b^2\right)(a+b\sec(c+dx))}$$

$\Big\downarrow$ 4258

3.619. $\int \frac{1}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^2}\,dx$

$$\frac{b^2\left(5-\frac{3b^2}{a^2}\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{a\left(2a^2-3b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\cos(c+dx)}dx-b\left(4a^2-3b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{}{\sqrt{\cos}}}{a^2}}{2a\left(a^2-b^2\right)}$$
$$\frac{b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{ad\left(a^2-b^2\right)(a+b\sec(c+dx))}$$

$\downarrow$ 3042

$$\frac{b^2\left(5-\frac{3b^2}{a^2}\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{a\left(2a^2-3b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx-b\left(4a^2-3b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int}{a^2}}{2a\left(a^2-b^2\right)}$$
$$\frac{b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{ad\left(a^2-b^2\right)(a+b\sec(c+dx))}$$

$\downarrow$ 3119

$$\frac{b^2\left(5-\frac{3b^2}{a^2}\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{\frac{2a\left(2a^2-3b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-b\left(4a^2-3b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx\right.}}}{a^2}}{2a\left(a^2-b^2\right)}$$
$$\frac{b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{ad\left(a^2-b^2\right)(a+b\sec(c+dx))}$$

$\downarrow$ 3120

$$\frac{b^2\left(5-\frac{3b^2}{a^2}\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{\frac{2a\left(2a^2-3b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-\frac{2b\left(4a^2-3b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx\right.}{d}}{a^2}}{2a\left(a^2-b^2\right)}$$
$$\frac{b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{ad\left(a^2-b^2\right)(a+b\sec(c+dx))}$$

$\downarrow$ 4336

$$\frac{b^2\left(5-\frac{3b^2}{a^2}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}(b+a\cos(c+dx))}dx+\frac{\frac{2a\left(2a^2-3b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-\frac{2b\left(\right.}{a^2}}{2a\left(a^2-b^2\right)}$$
$$\frac{b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{ad\left(a^2-b^2\right)(a+b\sec(c+dx))}$$

$\downarrow$ 3042

3.619. $\int \frac{1}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^2}\,dx$

$$\frac{b^2\left(5-\frac{3b^2}{a^2}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(b+a\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx+\frac{\frac{2a\left(2a^2-3b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\right|}{d}}{}}{2a\left(a^2-b^2\right)}$$

$$\frac{b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{ad\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)}$$

$\Big\downarrow$ 3284

$$\frac{b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{ad\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)}+\frac{\frac{2b^2\left(5-\frac{3b^2}{a^2}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticPi}\left(\frac{2a}{a+b},\frac{1}{2}(c+dx),2\right)}{d(a+b)}+\frac{\frac{2a\left(2a^2-3b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\right|2\right)}{d}-\frac{2b\left(4a^2-3b^2\right)\sqrt{\cos(c+dx)}\sqrt{}}{a^2}}{}}{2a\left(a^2-b^2\right)}$$

input
```
Int[1/(Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^2),x]
```

output
```
(((2*a*(2*a^2 - 3*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[S
ec[c + d*x]])/d - (2*b*(4*a^2 - 3*b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d
*x)/2, 2]*Sqrt[Sec[c + d*x]])/d)/a^2 + (2*b^2*(5 - (3*b^2)/a^2)*Sqrt[Cos[c
 + d*x]]*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/((a
 + b)*d))/(2*a*(a^2 - b^2)) + (b^2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(a*(a^
2 - b^2)*d*(a + b*Sec[c + d*x]))
```

3.619.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4334
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*
((d*Csc[e + f*x])^n/(a*f*(m + 1)*(a^2 - b^2))), x] + Simp[1/(a*(m + 1)*(a^2
 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a^2*(m + 1)
 - b^2*(m + n + 1) - a*b*(m + 1)*Csc[e + f*x] + b^2*(m + n + 2)*Csc[e + f*x
]^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0] && LtQ[m,
 -1] && IntegersQ[2*m, 2*n]
```

rule 4336
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]]   Int[
1/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4594
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Simp[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2)   Int[(d*Csc[e +
f*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Simp[1/a^2   Int[(a*A - (A*b - a
*B)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A,
B, C}, x] && NeQ[a^2 - b^2, 0]
```

3.619. $\int \frac{1}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^2}\,dx$

3.619.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 808 vs. $2(308) = 616$.

Time = 14.77 (sec) , antiderivative size = 809, normalized size of antiderivative = 3.32

method	result	size
default	Expression too large to display	809

input
```
int(1/(a+b*sec(d*x+c))^2/sec(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2/a^3/(-2*sin
(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(2*b*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2
))+a*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))-2/a^3*b^3*(a^2/b/(a^2-b^2)*cos
(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*a*
cos(1/2*d*x+1/2*c)^2-a+b)-1/2/(a+b)/b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos
(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^
(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2*a/b/(a^2-b^2)*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2*a/
b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)
/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*
x+1/2*c),2^(1/2))-1/2/b/(a^2-b^2)/(a^2-a*b)*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+
1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))+3/2*b/(a^
2-b^2)/(a^2-a*b)*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1
)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(co
s(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2)))-6/a^2*b^2/(a^2-a*b)*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin
(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2...
```

3.619.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+b*sec(d*x+c))^2/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.619.6 Sympy [F]

$$\int \frac{1}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^2}\,dx = \int \frac{1}{(a+b\sec(c+dx))^2\sqrt{\sec(c+dx)}}\,dx$$

input
```
integrate(1/(a+b*sec(d*x+c))**2/sec(d*x+c)**(1/2),x)
```

output
```
Integral(1/((a + b*sec(c + d*x))**2*sqrt(sec(c + d*x))), x)
```

3.619.7 Maxima [F]

$$\int \frac{1}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^2}\,dx = \int \frac{1}{(b\sec(dx+c)+a)^2\sqrt{\sec(dx+c)}}\,dx$$

input
```
integrate(1/(a+b*sec(d*x+c))^2/sec(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*sec(d*x + c) + a)^2*sqrt(sec(d*x + c))), x)
```

3.619.8 Giac [F]

$$\int \frac{1}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^2}\,dx = \int \frac{1}{(b\sec(dx+c)+a)^2\sqrt{\sec(dx+c)}}\,dx$$

input
```
integrate(1/(a+b*sec(d*x+c))^2/sec(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/((b*sec(d*x + c) + a)^2*sqrt(sec(d*x + c))), x)
```

3.619.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^2}\,dx=\int \frac{1}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^2\,\sqrt{\frac{1}{\cos(c+d\,x)}}}\,dx$$

input
```
int(1/((a + b/cos(c + d*x))^2*(1/cos(c + d*x))^(1/2)),x)
```

output
```
int(1/((a + b/cos(c + d*x))^2*(1/cos(c + d*x))^(1/2)), x)
```

3.620 $\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2}\,dx$

3.620.1 Optimal result 4727
3.620.2 Mathematica [A] (verified) 4728
3.620.3 Rubi [A] (verified) 4728
3.620.4 Maple [B] (verified) 4733
3.620.5 Fricas [F(-1)] 4734
3.620.6 Sympy [F] 4735
3.620.7 Maxima [F] 4735
3.620.8 Giac [F] 4735
3.620.9 Mupad [F(-1)] 4736

3.620.1 Optimal result

Integrand size = 23, antiderivative size = 304

$$\begin{aligned}\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2}\,dx &= -\frac{b(4a^2-5b^2)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^3(a^2-b^2)d} \\ &\quad+\frac{(2a^4+16a^2b^2-15b^4)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{3a^4(a^2-b^2)d} \\ &\quad-\frac{b^3(7a^2-5b^2)\sqrt{\cos(c+dx)}\,\text{EllipticPi}\left(\frac{2a}{a+b},\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{a^4(a-b)(a+b)^2d} \\ &\quad+\frac{(2a^2-5b^2)\sin(c+dx)}{3a^2(a^2-b^2)d\sqrt{\sec(c+dx)}}+\frac{b^2\sin(c+dx)}{a(a^2-b^2)d\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}\end{aligned}$$

output
```
1/3*(2*a^2-5*b^2)*sin(d*x+c)/a^2/(a^2-b^2)/d/sec(d*x+c)^(1/2)+b^2*sin(d*x+
c)/a/(a^2-b^2)/d/(a+b*sec(d*x+c))/sec(d*x+c)^(1/2)-b*(4*a^2-5*b^2)*(cos(1/
2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1
/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^3/(a^2-b^2)/d+1/3*(2*a^4+16*a^2*b
^2-15*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1
/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^4/(a^2-b^2)/d-b
^3*(7*a^2-5*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticP
i(sin(1/2*d*x+1/2*c),2*a/(a+b),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/
a^4/(a-b)/(a+b)^2/d
```

3.620.2 Mathematica [A] (verified)

Time = 5.51 (sec) , antiderivative size = 279, normalized size of antiderivative = 0.92

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2}\,dx$$
$$= \frac{\frac{a^2(2a^2b-5b^3+2a(a^2-b^2)\cos(c+dx))\sin(c+dx)}{(a^2-b^2)(b+a\cos(c+dx))\sqrt{\sec(c+dx)}} + \frac{\cot(c+dx)\left(3ab(4a^2-5b^2)E\left(\arcsin\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)\sqrt{-\tan^2(c+dx)}+a(2a^3-12a^2b-5a}{}}{}$$

input
```
Integrate[1/(Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^2),x]
```

output
```
((a^2*(2*a^2*b - 5*b^3 + 2*a*(a^2 - b^2)*Cos[c + d*x])*Sin[c + d*x])/((a^2
 - b^2)*(b + a*Cos[c + d*x])*Sqrt[Sec[c + d*x]]) + (Cot[c + d*x]*(3*a*b*(4
*a^2 - 5*b^2)*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[c + d*x]
^2] + a*(2*a^3 - 12*a^2*b - 5*a*b^2 + 15*b^3)*EllipticF[ArcSin[Sqrt[Sec[c
+ d*x]]], -1]*Sqrt[-Tan[c + d*x]^2] - 3*b*(a*(4*a^2 - 5*b^2)*Sec[c + d*x]^
(3/2)*Sin[c + d*x]^2 + b*(-7*a^2 + 5*b^2)*EllipticPi[-(b/a), ArcSin[Sqrt[S
ec[c + d*x]]], -1]*Sqrt[-Tan[c + d*x]^2])))/((a - b)*(a + b)))/(3*a^4*d)
```

3.620.3 Rubi [A] (verified)

Time = 2.06 (sec) , antiderivative size = 289, normalized size of antiderivative = 0.95, number of steps used = 18, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.783$, Rules used = {3042, 4334, 27, 3042, 4592, 27, 3042, 4594, 3042, 4274, 3042, 4258, 3042, 3119, 3120, 4336, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx$$
$$\Big\downarrow \text{4334}$$
$$\frac{b^2\sin(c+dx)}{ad\left(a^2-b^2\right)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))} - \frac{\int -\frac{2a^2-2b\sec(c+dx)a-5b^2+3b^2\sec^2(c+dx)}{2\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))}dx}{a\left(a^2-b^2\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{2a^2-2b\sec(c+dx)a-5b^2+3b^2\sec^2(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))}dx}{2a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)}{ad\left(a^2-b^2\right)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}$$

$$\Big\downarrow \; 3042$$

$$\frac{\int \frac{2a^2-2b\csc\left(c+dx+\frac{\pi}{2}\right)a-5b^2+3b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{2a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)}{ad\left(a^2-b^2\right)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}$$

$$\Big\downarrow \; 4592$$

$$\frac{\frac{2\left(2a^2-5b^2\right)\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{2\int \frac{-b\left(2a^2-5b^2\right)\sec^2(c+dx)-2a\left(a^2+2b^2\right)\sec(c+dx)+3b\left(4a^2-5b^2\right)}{2\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}dx}{3a}}{2a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)}{ad\left(a^2-b^2\right)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{2\left(2a^2-5b^2\right)\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{\int \frac{-b\left(2a^2-5b^2\right)\sec^2(c+dx)-2a\left(a^2+2b^2\right)\sec(c+dx)+3b\left(4a^2-5b^2\right)}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}dx}{3a}}{2a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)}{ad\left(a^2-b^2\right)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{2\left(2a^2-5b^2\right)\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{\int \frac{-b\left(2a^2-5b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2-2a\left(a^2+2b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)+3b\left(4a^2-5b^2\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{3a}}{2a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)}{ad\left(a^2-b^2\right)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}$$

$$\Big\downarrow \; 4594$$

$$\frac{\frac{2\left(2a^2-5b^2\right)\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{3b^3\left(7a^2-5b^2\right)\int \frac{\sec^{\frac{3}{2}}(c+dx)}{a+b\sec(c+dx)}dx}{a^2}+\frac{\int \frac{3ab\left(4a^2-5b^2\right)-\left(2a^4+16b^2a^2-15b^4\right)\sec(c+dx)}{\sqrt{\sec(c+dx)}}dx}{a^2}}{3a}}{2a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)}{ad\left(a^2-b^2\right)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}$$

$$\Big\downarrow \; 3042$$

3.620. $\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2}dx$

$$\frac{\frac{2(2a^2-5b^2)\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{3b^3(7a^2-5b^2)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}+\frac{\int\frac{3ab(4a^2-5b^2)+(-2a^4-16b^2a^2+15b^4)\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2}}{3a}}{2a(a^2-b^2)}+\frac{b^2\sin(c+dx)}{ad(a^2-b^2)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}$$

$\downarrow$ 4274

$$\frac{\frac{2(2a^2-5b^2)\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{3b^3(7a^2-5b^2)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}+\frac{3ab(4a^2-5b^2)\int\frac{1}{\sqrt{\sec(c+dx)}}dx-(2a^4+16a^2b^2-15b^4)\int\sqrt{\sec(c+dx)}dx}{a^2}}{3a}}{2a(a^2-b^2)}+\frac{b^2\sin(c+dx)}{ad(a^2-b^2)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}$$

$\downarrow$ 3042

$$\frac{\frac{2(2a^2-5b^2)\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{3b^3(7a^2-5b^2)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}+\frac{3ab(4a^2-5b^2)\int\frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-(2a^4+16a^2b^2-15b^4)\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}}{3a}}{2a(a^2-b^2)}+\frac{b^2\sin(c+dx)}{ad(a^2-b^2)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}$$

$\downarrow$ 4258

$$\frac{\frac{2(2a^2-5b^2)\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{3b^3(7a^2-5b^2)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}+\frac{3ab(4a^2-5b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\cos(c+dx)}dx-(2a^4+16a^2b^2-15b^4)\sqrt{\cos(c+dx}}{a^2}}{3a}}{2a(a^2-b^2)}$$

$$\frac{b^2\sin(c+dx)}{ad(a^2-b^2)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}$$

$\downarrow$ 3042

$$\frac{\frac{2(2a^2-5b^2)\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{3b^3(7a^2-5b^2)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}+\frac{3ab(4a^2-5b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx-(2a^4+16a^2b^2-15b^4)\sqrt{\cos(c}}{a^2}}{3a}}{2a(a^2-b^2)}$$

$$\frac{b^2\sin(c+dx)}{ad(a^2-b^2)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}$$

$\downarrow$ 3119

3.620. $\int\frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2}dx$

$$\frac{\frac{2(2a^2-5b^2)\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{3b^3(7a^2-5b^2)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}+\frac{\frac{6ab(4a^2-5b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-\left(2a^4+16a^2b^2-15b^4\right)\sqrt{\cos(c+dx}}{a^2}}{3a}}{2a(a^2-b^2)}$$

$$\frac{b^2\sin(c+dx)}{ad(a^2-b^2)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}$$

$\downarrow$ 3120

$$\frac{\frac{2(2a^2-5b^2)\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{3b^3(7a^2-5b^2)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}+\frac{\frac{6ab(4a^2-5b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-\frac{2\left(2a^4+16a^2b^2-15b^4\right)\sqrt{\cos(c+d}}{a^2}}{3a}}{2a(a^2-b^2)}$$

$$\frac{b^2\sin(c+dx)}{ad(a^2-b^2)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}$$

$\downarrow$ 4336

$$\frac{\frac{2(2a^2-5b^2)\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{3b^3(7a^2-5b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}(b+a\cos(c+dx))}dx}{a^2}+\frac{\frac{6ab(4a^2-5b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}}{3a}}{2a(a^2-b^2)}$$

$$\frac{b^2\sin(c+dx)}{ad(a^2-b^2)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}$$

$\downarrow$ 3042

$$\frac{\frac{2(2a^2-5b^2)\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{3b^3(7a^2-5b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(b+a\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2}+\frac{\frac{6ab(4a^2-5b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c\right.}{d}}{3a}}{2a(a^2-b^2)}$$

$$\frac{b^2\sin(c+dx)}{ad(a^2-b^2)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}$$

$\downarrow$ 3284

$$\frac{b^2\sin(c+dx)}{ad(a^2-b^2)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}+$$

$$\frac{\frac{2(2a^2-5b^2)\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{6b^3(7a^2-5b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticPi}\left(\frac{2a}{a+b},\frac{1}{2}(c+dx),2\right)}{a^2d(a+b)}+\frac{\frac{6ab(4a^2-5b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-\frac{2(2a}{}}{3a}}{2a(a^2-b^2)}$$

input
```
Int[1/(Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^2),x]
```

3.620. $\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2}\,dx$

output
```
(b^2*Sin[c + d*x])/(a*(a^2 - b^2)*d*Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x]
)) + (-1/3*(((6*a*b*(4*a^2 - 5*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)
/2, 2]*Sqrt[Sec[c + d*x]])/d - (2*(2*a^4 + 16*a^2*b^2 - 15*b^4)*Sqrt[Cos[c
 + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d)/a^2 + (6*b^3*(7*
a^2 - 5*b^2)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2]*
Sqrt[Sec[c + d*x]])/(a^2*(a + b)*d))/a + (2*(2*a^2 - 5*b^2)*Sin[c + d*x])/
(3*a*d*Sqrt[Sec[c + d*x]]))/(2*a*(a^2 - b^2))
```

3.620.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4334
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*
((d*Csc[e + f*x])^n/(a*f*(m + 1)*(a^2 - b^2))), x] + Simp[1/(a*(m + 1)*(a^2
 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a^2*(m + 1)
 - b^2*(m + n + 1) - a*b*(m + 1)*Csc[e + f*x] + b^2*(m + n + 2)*Csc[e + f*x
]^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0] && LtQ[m,
 -1] && IntegersQ[2*m, 2*n]
```

rule 4336
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]]   Int[
1/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4592
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Simp[1/(a*d*n)   Int[(a + b*Csc[e + f*x])^m
*(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*
Csc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

rule 4594
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Simp[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2)   Int[(d*Csc[e +
f*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Simp[1/a^2   Int[(a*A - (A*b - a
*B)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A,
B, C}, x] && NeQ[a^2 - b^2, 0]
```

3.620.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1063 vs. $2(362) = 724$.

Time = 16.94 (sec) , antiderivative size = 1064, normalized size of antiderivative = 3.50

method	result	size
default	Expression too large to display	1064

input
```
int(1/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
-(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4/3/a^2*(2*sin
(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)
^2+2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipti
cF(cos(1/2*d*x+1/2*c),2^(1/2))-3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(co
s(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2))/(-2*sin(1/2*d*
x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)-4/a^3*(a+b)*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d
*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-EllipticE(cos(1/
2*d*x+1/2*c),2^(1/2)))+2/a^4*b^4*(a^2/b/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*a*cos(1/2*d*x+1/2*c)^2-
a+b)-1/2/(a+b)/b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^
(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1
/2*d*x+1/2*c),2^(1/2))+1/2*a/b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*
cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^
2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2*a/b/(a^2-b^2)*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*
c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-1/2
/b/(a^2-b^2)/(a^2-a*b)*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/
2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellip
ticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))+3/2*b/(a^2-b^2)/(a^2-a*b)*a...
```

3.620.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(1/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
Timed out
```

3.620.6 Sympy [F]

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2}\,dx = \int \frac{1}{(a+b\sec(c+dx))^2\sec^{\frac{3}{2}}(c+dx)}\,dx$$

input
```
integrate(1/sec(d*x+c)**(3/2)/(a+b*sec(d*x+c))**2,x)
```

output
```
Integral(1/((a + b*sec(c + d*x))**2*sec(c + d*x)**(3/2)), x)
```

3.620.7 Maxima [F]

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2}\,dx = \int \frac{1}{(b\sec(dx+c)+a)^2\sec(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
integrate(1/((b*sec(d*x + c) + a)^2*sec(d*x + c)^(3/2)), x)
```

3.620.8 Giac [F]

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2}\,dx = \int \frac{1}{(b\sec(dx+c)+a)^2\sec(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate(1/((b*sec(d*x + c) + a)^2*sec(d*x + c)^(3/2)), x)
```

3.620.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2}\,dx=\int \frac{1}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^2\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

input
```
int(1/((a + b/cos(c + d*x))^2*(1/cos(c + d*x))^(3/2)),x)
```

output
```
int(1/((a + b/cos(c + d*x))^2*(1/cos(c + d*x))^(3/2)), x)
```

3.621 $\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a+b\sec(c+dx))^3}\,dx$

3.621.1 Optimal result 4737
3.621.2 Mathematica [A] (warning: unable to verify) 4738
3.621.3 Rubi [A] (verified) 4739
3.621.4 Maple [B] (verified) 4746
3.621.5 Fricas [F(-1)] 4747
3.621.6 Sympy [F(-1)] 4747
3.621.7 Maxima [F(-1)] 4747
3.621.8 Giac [F] 4748
3.621.9 Mupad [F(-1)] 4748

3.621.1 Optimal result

Integrand size = 23, antiderivative size = 388

$$\begin{aligned}
&\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a+b\sec(c+dx))^3}\,dx \\
&= -\frac{(15a^4-29a^2b^2+8b^4)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{4b^3\left(a^2-b^2\right)^2 d} \\
&\quad -\frac{a(5a^2-11b^2)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{4b^2\left(a^2-b^2\right)^2 d} \\
&\quad -\frac{a(15a^4-38a^2b^2+35b^4)\sqrt{\cos(c+dx)}\,\text{EllipticPi}\left(\frac{2a}{a+b},\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{4(a-b)^2b^3(a+b)^3d} \\
&\quad +\frac{(15a^4-29a^2b^2+8b^4)\sqrt{\sec(c+dx)}\sin(c+dx)}{4b^3\left(a^2-b^2\right)^2 d} \\
&\quad -\frac{a^2\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\sec(c+dx))^2}-\frac{a^2(5a^2-11b^2)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4b^2\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))}
\end{aligned}$$

output
```
-1/2*a^2*sec(d*x+c)^(5/2)*sin(d*x+c)/b/(a^2-b^2)/d/(a+b*sec(d*x+c))^2-1/4*
a^2*(5*a^2-11*b^2)*sec(d*x+c)^(3/2)*sin(d*x+c)/b^2/(a^2-b^2)^2/d/(a+b*sec(
d*x+c))+1/4*(15*a^4-29*a^2*b^2+8*b^4)*sin(d*x+c)*sec(d*x+c)^(1/2)/b^3/(a^2
-b^2)^2/d-1/4*(15*a^4-29*a^2*b^2+8*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1
/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d
*x+c)^(1/2)/b^3/(a^2-b^2)^2/d-1/4*a*(5*a^2-11*b^2)*(cos(1/2*d*x+1/2*c)^2)^
(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^
(1/2)*sec(d*x+c)^(1/2)/b^2/(a^2-b^2)^2/d-1/4*a*(15*a^4-38*a^2*b^2+35*b^4)*
(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2
*c),2*a/(a+b),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/(a-b)^2/b^3/(a+b)
^3/d
```

3.621.2 Mathematica [A] (warning: unable to verify)

Time = 6.64 (sec) , antiderivative size = 721, normalized size of antiderivative = 1.86

$$\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a+b\sec(c+dx))^3}\,dx = -\frac{\frac{2(45a^5-95a^3b^2+56ab^4)\cos^2(c+dx)\left(\text{EllipticF}\left(\arcsin\left(\sqrt{\sec(c+dx)}\right),-1\right)-\text{EllipticPi}\left(-\frac{b}{a},\arcsin\left(\sqrt{\sec(c+dx)}\right),-1\right)\right)(a+b\sec(c+dx))\sqrt{1-}}{b(b+a\cos(c+dx))(1-\cos^2(c+dx))} + \frac{\sqrt{\sec(c+dx)}\left(\frac{(15a^4-29a^2b^2+8b^4)\sin(c+dx)}{4b^3(-a^2+b^2)^2}+\frac{a^2\sin(c+dx)}{2b(-a^2+b^2)(b+a\cos(c+dx))^2}+\frac{-5a^4\sin(c+dx)+11a^2b^2\sin(c+dx)}{4b^2(-a^2+b^2)^2(b+a\cos(c+dx))}\right)}{d}$$

input
```
Integrate[Sec[c + d*x]^(9/2)/(a + b*Sec[c + d*x])^3,x]
```

output
```
-1/16*((2*(45*a^5 - 95*a^3*b^2 + 56*a*b^4)*Cos[c + d*x]^2*(EllipticF[ArcSi
n[Sqrt[Sec[c + d*x]]], -1] - EllipticPi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]]
, -1])*(a + b*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x])/(b*(b +
 a*Cos[c + d*x])*(1 - Cos[c + d*x]^2)) + (2*(40*a^4*b - 80*a^2*b^3 + 16*b^
5)*Cos[c + d*x]^2*EllipticPi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]], -1]*(a +
b*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x])/(a*(b + a*Cos[c + d
*x])*(1 - Cos[c + d*x]^2)) + ((15*a^5 - 29*a^3*b^2 + 8*a*b^4)*Cos[2*(c + d
*x)]*(a + b*Sec[c + d*x])*(-4*a*b + 4*a*b*Sec[c + d*x]^2 - 4*a*b*EllipticE
[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^
2] - 2*a*(a - 2*b)*EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c +
d*x]]*Sqrt[1 - Sec[c + d*x]^2] + 2*a^2*EllipticPi[-(b/a), ArcSin[Sqrt[Sec[
c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] - 4*b^2*Ellipt
icPi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - S
ec[c + d*x]^2])*Sin[c + d*x])/(a^2*b*(b + a*Cos[c + d*x])*(1 - Cos[c + d*x
]^2)*Sqrt[Sec[c + d*x]]*(2 - Sec[c + d*x]^2)))/((a - b)^2*b^3*(a + b)^2*d)
 + (Sqrt[Sec[c + d*x]]*(((15*a^4 - 29*a^2*b^2 + 8*b^4)*Sin[c + d*x])/(4*b^
3*(-a^2 + b^2)^2) + (a^2*Sin[c + d*x])/(2*b*(-a^2 + b^2)*(b + a*Cos[c + d*
x])^2) + (-5*a^4*Sin[c + d*x] + 11*a^2*b^2*Sin[c + d*x])/(4*b^2*(-a^2 + b^
2)^2*(b + a*Cos[c + d*x]))))/d
```

3.621.3 Rubi [A] (verified)

Time = 2.79 (sec) , antiderivative size = 382, normalized size of antiderivative = 0.98, number of steps used = 21, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.913$, Rules used = {3042, 4332, 27, 3042, 4586, 27, 3042, 4590, 27, 3042, 4594, 3042, 4274, 3042, 4258, 3042, 3119, 3120, 4336, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a+b\sec(c+dx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{9/2}}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \text{4332}$$

$$-\frac{\int \frac{\sec^{\frac{3}{2}}(c+dx)\left(3a^2-4b\sec(c+dx)a-\left(5a^2-4b^2\right)\sec^2(c+dx)\right)}{2(a+b\sec(c+dx))^2}dx}{2b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$$\Big\downarrow \; 27$$

$$-\frac{\int \frac{\sec^{\frac{3}{2}}(c+dx)\left(3a^2-4b\sec(c+dx)a-\left(5a^2-4b^2\right)\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^2}dx}{4b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(3a^2-4b\csc\left(c+dx+\frac{\pi}{2}\right)a+\left(4b^2-5a^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx}{4b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$$\Big\downarrow \; 4586$$

$$-\frac{\frac{a^2\left(5a^2-11b^2\right)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{bd(a^2-b^2)(a+b\sec(c+dx))}-\frac{\int -\frac{\sqrt{\sec(c+dx)}\left(\left(5a^2-11b^2\right)a^2-4b\left(a^2-4b^2\right)\sec(c+dx)a-\left(15a^4-29b^2a^2+8b^4\right)\sec^2(c+dx)\right)}{2(a+b\sec(c+dx))}dx}{b(a^2-b^2)}}{4b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$$\Big\downarrow \; 27$$

$$-\frac{\frac{\int \frac{\sqrt{\sec(c+dx)}\left(\left(5a^2-11b^2\right)a^2-4b\left(a^2-4b^2\right)\sec(c+dx)a-\left(15a^4-29b^2a^2+8b^4\right)\sec^2(c+dx)\right)}{a+b\sec(c+dx)}dx}{2b(a^2-b^2)}+\frac{a^2\left(5a^2-11b^2\right)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{bd(a^2-b^2)(a+b\sec(c+dx))}}{4b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\frac{\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(\left(5a^2-11b^2\right)a^2-4b\left(a^2-4b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)a+\left(-15a^4+29b^2a^2-8b^4\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{2b(a^2-b^2)}+\frac{a^2\left(5a^2-11b^2\right)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{bd(a^2-b^2)(a+b\sec(c+dx))}}{4b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$$\Big\downarrow \; 4590$$

$$-\frac{\frac{\frac{2\int \frac{3a\left(5a^4-11b^2a^2+8b^4\right)\sec^2(c+dx)+4b\left(5a^4-10b^2a^2+2b^4\right)\sec(c+dx)+a\left(15a^4-29b^2a^2+8b^4\right)}{2\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}dx}{b}-\frac{2\left(15a^4-29a^2b^2+8b^4\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd}}{2b(a^2-b^2)}+\frac{a^2(5a^{\cdots}}{b^{\cdots}}}{4b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$$\Big\downarrow \; 27$$

3.621. $\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a+b\sec(c+dx))^3}\,dx$

$$-\frac{\frac{\frac{\int \frac{3a\left(5a^4-11b^2a^2+8b^4\right)\sec^2(c+dx)+4b\left(5a^4-10b^2a^2+2b^4\right)\sec(c+dx)+a\left(15a^4-29b^2a^2+8b^4\right)}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}dx}{b}-\frac{2\left(15a^4-29a^2b^2+8b^4\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd}}{2b(a^2-b^2)}+\frac{a^2(5a^2-}{bd}}{4b\left(a^2-b^2\right)}$$

$$\frac{a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$\downarrow$ 3042

$$-\frac{\frac{\frac{\int \frac{3a\left(5a^4-11b^2a^2+8b^4\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2+4b\left(5a^4-10b^2a^2+2b^4\right)\csc\left(c+dx+\frac{\pi}{2}\right)+a\left(15a^4-29b^2a^2+8b^4\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{b}-\frac{2\left(15a^4-29a^2b^2+8b^4\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd}}{2b(a^2-b^2)}+}{4b\left(a^2-b^2\right)}$$

$$\frac{a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$\downarrow$ 4594

$$-\frac{\frac{\frac{a\left(15a^4-38a^2b^2+35b^4\right)\int \frac{\sec^{\frac{3}{2}}(c+dx)}{a+b\sec(c+dx)}dx+\frac{\int \frac{b\left(5a^2-11b^2\right)\sec(c+dx)a^3+\left(15a^4-29b^2a^2+8b^4\right)a^2}{\sqrt{\sec(c+dx)}}dx}{a^2}}{b}-\frac{2\left(15a^4-29a^2b^2+8b^4\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd}}{2b(a^2-b^2)}+\frac{a^2(}{}}{4b\left(a^2-b^2\right)}$$

$$\frac{a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$\downarrow$ 3042

$$-\frac{\frac{\frac{a\left(15a^4-38a^2b^2+35b^4\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{\int \frac{b\left(5a^2-11b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)a^3+\left(15a^4-29b^2a^2+8b^4\right)a^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2}}{b}-\frac{2\left(15a^4-29a^2b^2+8b^4\right)\sin(c+dx)\sqrt{\sec(c+dx}}{bd}}{2b(a^2-b^2)}}{4b\left(a^2-b^2\right)}$$

$$\frac{a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$\downarrow$ 4274

$$-\frac{\frac{\frac{a\left(15a^4-38a^2b^2+35b^4\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{a^2\left(15a^4-29a^2b^2+8b^4\right)\int \frac{1}{\sqrt{\sec(c+dx)}}dx+a^3b\left(5a^2-11b^2\right)\int \sqrt{\sec(c+dx)}dx}{a^2}}{b}-\frac{2\left(15a^4-29a^2b^2+8b^4\right)\sin(c+}{bd}}{2b(a^2-b^2)}}{4b\left(a^2-b^2\right)}$$

$$\frac{a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

3.621. $\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a+b\sec(c+dx))^3}\,dx$

$$\Big\downarrow \; 3042$$

$$-\frac{\frac{\frac{a\left(15a^4-38a^2b^2+35b^4\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{a^2\left(15a^4-29a^2b^2+8b^4\right)\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+a^3b\left(5a^2-11b^2\right)\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}}{b}-\frac{2\left(15a^4-29a^2b^2+8b^4\right.}{}}{2b(a^2-b^2)}}{4b\left(a^2-b^2\right)}$$

$$\frac{a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$$\Big\downarrow \; 4258$$

$$-\frac{\frac{\frac{a\left(15a^4-38a^2b^2+35b^4\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{a^2\left(15a^4-29a^2b^2+8b^4\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\cos(c+dx)}dx+a^3b\left(5a^2-11b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int}{a^2}}{b}}{2b(a^2-b^2)}}{4b\left(a^2-b^2\right)}$$

$$\frac{a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\frac{\frac{a\left(15a^4-38a^2b^2+35b^4\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{a^2\left(15a^4-29a^2b^2+8b^4\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+a^3b\left(5a^2-11b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+d}}{a^2}}{b}}{2b(a^2-b^2)}}{4b\left(a^2-b^2\right)}$$

$$\frac{a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$$\Big\downarrow \; 3119$$

$$-\frac{\frac{\frac{a\left(15a^4-38a^2b^2+35b^4\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{a^3b\left(5a^2-11b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a^2\left(15a^4-29a^2b^2+8b^4\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c}}{d}}{a^2}}{b}}{2b(a^2-b^2)}}{4b\left(a^2-b^2\right)}$$

$$\frac{a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$$\Big\downarrow \; 3120$$

3.621. $\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a+b\sec(c+dx))^3}\,dx$

$$-\frac{\frac{\frac{a\left(15a^4-38a^2b^2+35b^4\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{\frac{2a^2\left(15a^4-29a^2b^2+8b^4\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a^3b\left(5a^2-11b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{d}}{a^2}}{b}}{2b(a^2-b^2)}}{4b\left(a^2-b^2\right)}$$

$$\frac{a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$\downarrow$ 4336

$$-\frac{\frac{\frac{a\left(15a^4-38a^2b^2+35b^4\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}(b+a\cos(c+dx))}dx+\frac{\frac{2a^2\left(15a^4-29a^2b^2+8b^4\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a^3b\left(5a\right.}{}}{a^2}}{b}}{2b(a^2-b^2)}}{4b\left(a^2-\right.}$$

$$\frac{a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$\downarrow$ 3042

$$-\frac{\frac{\frac{a\left(15a^4-38a^2b^2+35b^4\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(b+a\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx+\frac{\frac{2a^2\left(15a^4-29a^2b^2+8b^4\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}}{a}}{b}}{2b(a^2-b^2)}}{4b\left(a^2\right.}$$

$$\frac{a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$\downarrow$ 3284

$$-\frac{a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^2}-$$

$$\frac{\frac{a^2\left(5a^2-11b^2\right)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{bd(a^2-b^2)(a+b\sec(c+dx))}+\frac{\frac{\frac{2a\left(15a^4-38a^2b^2+35b^4\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticPi}\left(\frac{2a}{a+b},\frac{1}{2}(c+dx),2\right)}{d(a+b)}+\frac{\frac{2a^2\left(15a^4-29a^2b^2+8b^4\right)\sqrt{\cos(c+}}{d}}{}}{b}}{}}{4b\left(a^2-b^2\right)}$$

input
```
Int[Sec[c + d*x]^(9/2)/(a + b*Sec[c + d*x])^3,x]
```

3.621. $\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a+b\sec(c+dx))^3}\,dx$

output
```
-1/2*(a^2*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(b*(a^2 - b^2)*d*(a + b*Sec[c +
 d*x])^2) - ((a^2*(5*a^2 - 11*b^2)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(b*(a^
2 - b^2)*d*(a + b*Sec[c + d*x])) + ((((2*a^2*(15*a^4 - 29*a^2*b^2 + 8*b^4)
*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (2*a
^3*b*(5*a^2 - 11*b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Se
c[c + d*x]])/d)/a^2 + (2*a*(15*a^4 - 38*a^2*b^2 + 35*b^4)*Sqrt[Cos[c + d*x
]]*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/((a + b)*
d))/b - (2*(15*a^4 - 29*a^2*b^2 + 8*b^4)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/
(b*d))/(2*b*(a^2 - b^2)))/(4*b*(a^2 - b^2))
```

3.621.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

3.621. $\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a+b\sec(c+dx))^3}\,dx$

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4332
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-a^2)*d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(
m + 1)*((d*Csc[e + f*x])^(n - 3)/(b*f*(m + 1)*(a^2 - b^2))), x] + Simp[d^3/
(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])
^(n - 3)*Simp[a^2*(n - 3) + a*b*(m + 1)*Csc[e + f*x] - (a^2*(n - 2) + b^2*(
m + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2
- b^2, 0] && LtQ[m, -1] && (IGtQ[n, 3] || (IntegersQ[n + 1/2, 2*m] && GtQ[n
, 2]))
```

rule 4336
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]]   Int[
1/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4586
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(-d)*(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a +
b*Csc[e + f*x])^(m + 1)*((d*Csc[e + f*x])^(n - 1)/(b*f*(a^2 - b^2)*(m + 1))
), x] + Simp[d/(b*(a^2 - b^2)*(m + 1))   Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*b^2*(n - 1) - a*(b*B - a*C)*(n - 1) + b*(a*A
 - b*B + a*C)*(m + 1)*Csc[e + f*x] - (b*(A*b - a*B)*(m + n + 1) + C*(a^2*n
+ b^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C
}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n, 0]
```

rule 4590
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(-C)*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
)*((d*Csc[e + f*x])^(n - 1)/(b*f*(m + n + 1))), x] + Simp[d/(b*(m + n + 1))
   Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (
A*b*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2
 - b^2, 0] && GtQ[n, 0]
```

3.621. $\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a+b\sec(c+dx))^3}\,dx$

rule 4594
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Simp[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2)   Int[(d*Csc[e +
f*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Simp[1/a^2   Int[(a*A - (A*b - a
*B)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A,
B, C}, x] && NeQ[a^2 - b^2, 0]
```

3.621.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1986 vs. $2(436) = 872$.

Time = 488.59 (sec) , antiderivative size = 1987, normalized size of antiderivative = 5.12

method	result	size
default	Expression too large to display	1987

input
```
int(sec(d*x+c)^(9/2)/(a+b*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
-(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2/b^3/sin(1/2*
d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d
*x+1/2*c)^2)^(1/2)*(2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-(sin(1/2*d*x
+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*
c)^2-1)^(1/2))-2*a/b*(1/2*a^2/b/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d
*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*a*cos(1/2*d*x+1/2*c)^2-a+b)^2+3
/4*a^2*(a^2-3*b^2)/b^2/(a^2-b^2)^2*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*
c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*a*cos(1/2*d*x+1/2*c)^2-a+b)-3/8/(a+b)/
(a^2-b^2)/b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/
2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*
d*x+1/2*c),2^(1/2))*a^2-1/4/(a+b)/(a^2-b^2)/b*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/
2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a+7/8/(a+b)/(a^2-b^2)*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2
*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(
1/2))+3/8*a^3/b^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x
+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*El
lipticF(cos(1/2*d*x+1/2*c),2^(1/2))-9/8*a/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/
2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3/8*a^3/b^2...
```

3.621.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)^(9/2)/(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
Timed out
```

3.621.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**(9/2)/(a+b*sec(d*x+c))**3,x)
```

output
```
Timed out
```

3.621.7 Maxima [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)^(9/2)/(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
Timed out
```

3.621.8 Giac [F]

$$\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \int \frac{\sec(dx+c)^{\frac{9}{2}}}{(b\sec(dx+c)+a)^3}\,dx$$

input
```
integrate(sec(d*x+c)^(9/2)/(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^(9/2)/(b*sec(d*x + c) + a)^3, x)
```

3.621.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{9/2}}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^3}\,dx$$

input
```
int((1/cos(c + d*x))^(9/2)/(a + b/cos(c + d*x))^3,x)
```

output
```
int((1/cos(c + d*x))^(9/2)/(a + b/cos(c + d*x))^3, x)
```

3.622 $\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+b\sec(c+dx))^3}\,dx$

3.622.1 Optimal result 4749
3.622.2 Mathematica [A] (verified) 4750
3.622.3 Rubi [A] (verified) 4750
3.622.4 Maple [B] (verified) 4756
3.622.5 Fricas [F(-1)] 4756
3.622.6 Sympy [F(-1)] 4757
3.622.7 Maxima [F(-1)] 4757
3.622.8 Giac [F] 4757
3.622.9 Mupad [F(-1)] 4758

3.622.1 Optimal result

Integrand size = 23, antiderivative size = 315

$$\begin{aligned}
&\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+b\sec(c+dx))^3}\,dx \\
&= \frac{3a(a^2-3b^2)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{4b^2(a^2-b^2)^2d} \\
&\quad + \frac{(a^2-7b^2)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{4b(a^2-b^2)^2d} \\
&\quad + \frac{3(a^4-2a^2b^2+5b^4)\sqrt{\cos(c+dx)}\,\text{EllipticPi}\left(\frac{2a}{a+b},\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{4(a-b)^2b^2(a+b)^3d} \\
&\quad - \frac{a^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2b(a^2-b^2)d(a+b\sec(c+dx))^2} - \frac{3a^2(a^2-3b^2)\sqrt{\sec(c+dx)}\sin(c+dx)}{4b^2(a^2-b^2)^2d(a+b\sec(c+dx))}
\end{aligned}$$

output
```
-1/2*a^2*sec(d*x+c)^(3/2)*sin(d*x+c)/b/(a^2-b^2)/d/(a+b*sec(d*x+c))^2-3/4*
a^2*(a^2-3*b^2)*sin(d*x+c)*sec(d*x+c)^(1/2)/b^2/(a^2-b^2)^2/d/(a+b*sec(d*x
+c))+3/4*a*(a^2-3*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ell
ipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/b^2/(
a^2-b^2)^2/d+1/4*(a^2-7*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*
c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)
/b/(a^2-b^2)^2/d+3/4*(a^4-2*a^2*b^2+5*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/co
s(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*a/(a+b),2^(1/2))*cos(d*x+
c)^(1/2)*sec(d*x+c)^(1/2)/(a-b)^2/b^2/(a+b)^3/d
```

3.622.2 Mathematica [A] (verified)

Time = 5.64 (sec) , antiderivative size = 335, normalized size of antiderivative = 1.06

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+b\sec(c+dx))^3}\,dx$$
$$= \frac{\frac{4a^2b(-5a^2b+11b^3-3a(a^2-3b^2)\cos(c+dx))\sin(c+dx)}{(a^2-b^2)^2} + \frac{4\cos(c+dx)(b+a\cos(c+dx))\cot(c+dx)(a+b\sec(c+dx))\left(3ab(a^2-3b^2)\tan^2(c+dx)-3\right.}{}}{}$$

input
```
Integrate[Sec[c + d*x]^(7/2)/(a + b*Sec[c + d*x])^3,x]
```

output
```
((4*a^2*b*(-5*a^2*b + 11*b^3 - 3*a*(a^2 - 3*b^2)*Cos[c + d*x])*Sin[c + d*x
])/(a^2 - b^2)^2 + (4*Cos[c + d*x]*(b + a*Cos[c + d*x])*Cot[c + d*x]*(a +
b*Sec[c + d*x])*(3*a*b*(a^2 - 3*b^2)*Tan[c + d*x]^2 - 3*a*b*(a^2 - 3*b^2)*
EllipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[-Tan[c +
 d*x]^2] + (3*a^4 + 3*a^3*b - 5*a^2*b^2 - 9*a*b^3 + 8*b^4)*EllipticF[ArcSi
n[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[-Tan[c + d*x]^2] - 3*(a
^4 - 2*a^2*b^2 + 5*b^4)*EllipticPi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]], -1]
*Sqrt[Sec[c + d*x]]*Sqrt[-Tan[c + d*x]^2]))/((a - b)^2*(a + b)^2))/(16*b^3
*d*(b + a*Cos[c + d*x])^2*Sqrt[Sec[c + d*x]])
```

3.622.3 Rubi [A] (verified)

Time = 2.09 (sec) , antiderivative size = 318, normalized size of antiderivative = 1.01, number of steps used = 18, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.783$, Rules used = {3042, 4332, 27, 3042, 4586, 27, 3042, 4594, 3042, 4274, 3042, 4258, 3042, 3119, 3120, 4336, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+b\sec(c+dx))^3}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^3}dx$$
$$\Big\downarrow \; 4332$$

$$-\frac{\int \frac{\sqrt{\sec(c+dx)}\left(a^2-4b\sec(c+dx)a-\left(3a^2-4b^2\right)\sec^2(c+dx)\right)}{2(a+b\sec(c+dx))^2}dx}{2b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$\Big\downarrow$ 27

$$-\frac{\int \frac{\sqrt{\sec(c+dx)}\left(a^2-4b\sec(c+dx)a-\left(3a^2-4b^2\right)\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^2}dx}{4b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$\Big\downarrow$ 3042

$$-\frac{\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a^2-4b\csc\left(c+dx+\frac{\pi}{2}\right)a+\left(4b^2-3a^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx}{4b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$\Big\downarrow$ 4586

$$-\frac{\frac{3a^2\left(a^2-3b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd(a^2-b^2)(a+b\sec(c+dx))}-\frac{\int \frac{3\left(a^2-3b^2\right)a^2+4b\left(a^2-4b^2\right)\sec(c+dx)a+\left(3a^4-5b^2a^2+8b^4\right)\sec^2(c+dx)}{2\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}dx}{b(a^2-b^2)}}{4b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$\Big\downarrow$ 27

$$-\frac{\frac{3a^2\left(a^2-3b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd(a^2-b^2)(a+b\sec(c+dx))}-\frac{\int \frac{3\left(a^2-3b^2\right)a^2+4b\left(a^2-4b^2\right)\sec(c+dx)a+\left(3a^4-5b^2a^2+8b^4\right)\sec^2(c+dx)}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}dx}{2b(a^2-b^2)}}{4b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$\Big\downarrow$ 3042

$$-\frac{\frac{3a^2\left(a^2-3b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd(a^2-b^2)(a+b\sec(c+dx))}-\frac{\int \frac{3\left(a^2-3b^2\right)a^2+4b\left(a^2-4b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)a+\left(3a^4-5b^2a^2+8b^4\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{2b(a^2-b^2)}}{4b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$\Big\downarrow$ 4594

$$-\frac{\frac{3a^2\left(a^2-3b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd(a^2-b^2)(a+b\sec(c+dx))}-\frac{3\left(a^4-2a^2b^2+5b^4\right)\int \frac{\sec^{\frac{3}{2}}(c+dx)}{a+b\sec(c+dx)}dx+\frac{\int \frac{3\left(a^2-3b^2\right)a^3+b\left(a^2-7b^2\right)\sec(c+dx)a^2}{\sqrt{\sec(c+dx)}}dx}{a^2}}{2b(a^2-b^2)}}{4b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$\Big\downarrow$ 3042

3.622. $\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+b\sec(c+dx))^3}\,dx$

$$-\frac{\frac{3a^2\left(a^2-3b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd(a^2-b^2)(a+b\sec(c+dx))}-\frac{3\left(a^4-2a^2b^2+5b^4\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{\int\frac{3\left(a^2-3b^2\right)a^3+b\left(a^2-7b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)a^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2}}{2b(a^2-b^2)}}{4b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$\downarrow$ 4274

$$-\frac{\frac{3a^2\left(a^2-3b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd(a^2-b^2)(a+b\sec(c+dx))}-\frac{3\left(a^4-2a^2b^2+5b^4\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{a^2b\left(a^2-7b^2\right)\int\sqrt{\sec(c+dx)}dx+3a^3\left(a^2-3b^2\right)\int\frac{1}{\sqrt{\sec(c+dx)}}dx}{a^2}}{2b(a^2-b^2)}}{4b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$\downarrow$ 3042

$$-\frac{\frac{3a^2\left(a^2-3b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd(a^2-b^2)(a+b\sec(c+dx))}-\frac{3\left(a^4-2a^2b^2+5b^4\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{a^2b\left(a^2-7b^2\right)\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+3a^3\left(a^2-3b^2\right)\int\frac{1}{\sqrt{\csc\left(c+dx+\right.}}}{a^2}}{2b(a^2-b^2)}}{4b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$\downarrow$ 4258

$$-\frac{\frac{3a^2\left(a^2-3b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd(a^2-b^2)(a+b\sec(c+dx))}-\frac{3\left(a^4-2a^2b^2+5b^4\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{a^2b\left(a^2-7b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx+3a^3\left(a^2-\right.}{a^2}}{2b(a^2-b^2)}}{4b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$\downarrow$ 3042

$$-\frac{\frac{3a^2\left(a^2-3b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd(a^2-b^2)(a+b\sec(c+dx))}-\frac{3\left(a^4-2a^2b^2+5b^4\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{a^2b\left(a^2-7b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+3a^3\left(\right.}{a^2}}{2b(a^2-b^2)}}{4b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$\downarrow$ 3119

3.622. $\int\frac{\sec^{\frac{7}{2}}(c+dx)}{(a+b\sec(c+dx))^3}\,dx$

$$-\frac{\frac{3a^2(a^2-3b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd(a^2-b^2)(a+b\sec(c+dx))}-\frac{3(a^4-2a^2b^2+5b^4)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{\frac{a^2b(a^2-7b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{6a^3}{}}{a^2}}{2b(a^2-b^2)}}{4b(a^2-b^2)}$$

$$\frac{a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2bd(a^2-b^2)(a+b\sec(c+dx))^2}$$

$$\Big\downarrow \; 3120$$

$$-\frac{\frac{3a^2(a^2-3b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd(a^2-b^2)(a+b\sec(c+dx))}-\frac{3(a^4-2a^2b^2+5b^4)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{\frac{2a^2b(a^2-7b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+\frac{6a}{}}{a^2}}{2b(a^2-b^2)}}{4b(a^2-b^2)}$$

$$\frac{a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2bd(a^2-b^2)(a+b\sec(c+dx))^2}$$

$$\Big\downarrow \; 4336$$

$$-\frac{\frac{3a^2(a^2-3b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd(a^2-b^2)(a+b\sec(c+dx))}-\frac{3(a^4-2a^2b^2+5b^4)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}(b+a\cos(c+dx))}dx+\frac{\frac{2a^2b(a^2-7b^2)\sqrt{\cos(c+dx)}\sqrt{}}{}}{}}{2b(a^2-b^2)}}{4b(a^2-b^2)}$$

$$\frac{a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2bd(a^2-b^2)(a+b\sec(c+dx))^2}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\frac{3a^2(a^2-3b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd(a^2-b^2)(a+b\sec(c+dx))}-\frac{3(a^4-2a^2b^2+5b^4)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(b+a\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx+\frac{\frac{2a^2b(a^2-7b^2)\sqrt{\cos}}{}}{}}{2b(a^2-b^2)}}{4b(a^2-b^2)}$$

$$\frac{a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2bd(a^2-b^2)(a+b\sec(c+dx))^2}$$

$$\Big\downarrow \; 3284$$

$$-\frac{a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2bd(a^2-b^2)(a+b\sec(c+dx))^2}-$$

$$\frac{\frac{3a^2(a^2-3b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd(a^2-b^2)(a+b\sec(c+dx))}-\frac{\frac{6(a^4-2a^2b^2+5b^4)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticPi}\left(\frac{2a}{a+b},\frac{1}{2}(c+dx),2\right)}{d(a+b)}+\frac{\frac{2a^2b(a^2-7b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{El}}{d}}{}}{2b(a^2-b^2)}}{4b(a^2-b^2)}$$

input
```
Int[Sec[c + d*x]^(7/2)/(a + b*Sec[c + d*x])^3,x]
```

3.622. $\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+b\sec(c+dx))^3}\,dx$

output
```
-1/2*(a^2*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(b*(a^2 - b^2)*d*(a + b*Sec[c +
 d*x])^2) - (-1/2*(((6*a^3*(a^2 - 3*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c +
 d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (2*a^2*b*(a^2 - 7*b^2)*Sqrt[Cos[c + d*
x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d)/a^2 + (6*(a^4 - 2*a^2
*b^2 + 5*b^4)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2]
*Sqrt[Sec[c + d*x]])/((a + b)*d))/(b*(a^2 - b^2)) + (3*a^2*(a^2 - 3*b^2)*S
qrt[Sec[c + d*x]]*Sin[c + d*x])/(b*(a^2 - b^2)*d*(a + b*Sec[c + d*x])))/(4
*b*(a^2 - b^2))
```

3.622.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

3.622. $\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+b\sec(c+dx))^3}\,dx$

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4332
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-a^2)*d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(
m + 1)*((d*Csc[e + f*x])^(n - 3)/(b*f*(m + 1)*(a^2 - b^2))), x] + Simp[d^3/
(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])
^(n - 3)*Simp[a^2*(n - 3) + a*b*(m + 1)*Csc[e + f*x] - (a^2*(n - 2) + b^2*(
m + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2
- b^2, 0] && LtQ[m, -1] && (IGtQ[n, 3] || (IntegersQ[n + 1/2, 2*m] && GtQ[n
, 2]))
```

rule 4336
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]]   Int[
1/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4586
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(-d)*(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a +
b*Csc[e + f*x])^(m + 1)*((d*Csc[e + f*x])^(n - 1)/(b*f*(a^2 - b^2)*(m + 1))
), x] + Simp[d/(b*(a^2 - b^2)*(m + 1))   Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*b^2*(n - 1) - a*(b*B - a*C)*(n - 1) + b*(a*A
 - b*B + a*C)*(m + 1)*Csc[e + f*x] - (b*(A*b - a*B)*(m + n + 1) + C*(a^2*n
+ b^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C
}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n, 0]
```

rule 4594
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Simp[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2)   Int[(d*Csc[e +
f*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Simp[1/a^2   Int[(a*A - (A*b - a
*B)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A,
B, C}, x] && NeQ[a^2 - b^2, 0]
```

3.622. $\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+b\sec(c+dx))^3}\,dx$

3.622.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1202 vs. 2(367) = 734.

Time = 195.45 (sec) , antiderivative size = 1203, normalized size of antiderivative = 3.82

method	result	size
default	Expression too large to display	1203

input
```
int(sec(d*x+c)^(7/2)/(a+b*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
-(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(a^2/b/(a^2-b^2
)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/
(2*a*cos(1/2*d*x+1/2*c)^2-a+b)^2+3/2*a^2*(a^2-3*b^2)/b^2/(a^2-b^2)^2*cos(1
/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*a*co
s(1/2*d*x+1/2*c)^2-a+b)-3/4/(a+b)/(a^2-b^2)/b^2*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+
1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^2-1/2/(a+b)/(a^2-b
^2)/b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*
c),2^(1/2))*a+7/4/(a+b)/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2
*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2
)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3/4*a^3/b^2/(a^2-b^2)^2*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-9/4*
a/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/
2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*
d*x+1/2*c),2^(1/2))-3/4*a^3/b^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*
c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+9/4*a/(a^2-b^2)^2*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*...
```

3.622.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)^(7/2)/(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
Timed out
```

3.622.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**(7/2)/(a+b*sec(d*x+c))**3,x)
```

output
```
Timed out
```

3.622.7 Maxima [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)^(7/2)/(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
Timed out
```

3.622.8 Giac [F]

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \int \frac{\sec\left(dx+c\right)^{\frac{7}{2}}}{\left(b\sec\left(dx+c\right)+a\right)^3}\,dx$$

input
```
integrate(sec(d*x+c)^(7/2)/(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^(7/2)/(b*sec(d*x + c) + a)^3, x)
```

3.622.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{7/2}}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^3}\,dx$$

input
```
int((1/cos(c + d*x))^(7/2)/(a + b/cos(c + d*x))^3,x)
```

output
```
int((1/cos(c + d*x))^(7/2)/(a + b/cos(c + d*x))^3, x)
```

3.623 $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\sec(c+dx))^3}\,dx$

3.623.1 Optimal result 4759
3.623.2 Mathematica [A] (verified) 4760
3.623.3 Rubi [A] (verified) 4760
3.623.4 Maple [B] (verified) 4766
3.623.5 Fricas [F(-1)] 4767
3.623.6 Sympy [F] 4767
3.623.7 Maxima [F(-1)] 4767
3.623.8 Giac [F] 4768
3.623.9 Mupad [F(-1)] 4768

3.623.1 Optimal result

Integrand size = 23, antiderivative size = 313

$$\begin{aligned}\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\sec(c+dx))^3}\,dx &= \frac{(a^2+5b^2)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{4b\,(a^2-b^2)^2\,d} \\ &\quad+\frac{3(a^2+b^2)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{4a\,(a^2-b^2)^2\,d} \\ &\quad+\frac{(a^4-10a^2b^2-3b^4)\sqrt{\cos(c+dx)}\,\text{EllipticPi}\left(\frac{2a}{a+b},\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{4a(a-b)^2b(a+b)^3d} \\ &\quad-\frac{a^2\sqrt{\sec(c+dx)}\sin(c+dx)}{2b\,(a^2-b^2)\,d(a+b\sec(c+dx))^2}+\frac{a(a^2-7b^2)\sqrt{\sec(c+dx)}\sin(c+dx)}{4b\,(a^2-b^2)^2\,d(a+b\sec(c+dx))}\end{aligned}$$

output
```
-1/2*a^2*sin(d*x+c)*sec(d*x+c)^(1/2)/b/(a^2-b^2)/d/(a+b*sec(d*x+c))^2+1/4*
a*(a^2-7*b^2)*sin(d*x+c)*sec(d*x+c)^(1/2)/b/(a^2-b^2)^2/d/(a+b*sec(d*x+c))
+1/4*(a^2+5*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE
(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/b/(a^2-b^2)
^2/d+3/4*(a^2+b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellipti
cF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a/(a^2-b^
2)^2/d+1/4*(a^4-10*a^2*b^2-3*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x
+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*a/(a+b),2^(1/2))*cos(d*x+c)^(1/2)*
sec(d*x+c)^(1/2)/a/(a-b)^2/b/(a+b)^3/d
```

3.623.2 Mathematica [A] (verified)

Time = 6.26 (sec) , antiderivative size = 428, normalized size of antiderivative = 1.37

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\sec(c+dx))^3}\,dx$$
$$= \frac{-\frac{4ab(-a^2b+7b^3+a(a^2+5b^2)\cos(c+dx))\sin(c+dx)}{(a^2-b^2)^2}+\frac{4\cos(c+dx)(b+a\cos(c+dx))\cot(c+dx)(a+b\sec(c+dx))\left(-a^3b-5ab^3+a^3b\sec^2(c+dx)+\right.}{}}{}$$

input
```
Integrate[Sec[c + d*x]^(5/2)/(a + b*Sec[c + d*x])^3,x]
```

output
```
((-4*a*b*(-(a^2*b) + 7*b^3 + a*(a^2 + 5*b^2)*Cos[c + d*x])*Sin[c + d*x])/(
a^2 - b^2)^2 + (4*Cos[c + d*x]*(b + a*Cos[c + d*x])*Cot[c + d*x]*(a + b*Se
c[c + d*x])*(-(a^3*b) - 5*a*b^3 + a^3*b*Sec[c + d*x]^2 + 5*a*b^3*Sec[c + d
*x]^2 - a*b*(a^2 + 5*b^2)*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[S
ec[c + d*x]]*Sqrt[-Tan[c + d*x]^2] + a*(a^3 + a^2*b - 7*a*b^2 + 5*b^3)*Ell
ipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[-Tan[c + d*
x]^2] - a^4*EllipticPi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c
+ d*x]]*Sqrt[-Tan[c + d*x]^2] + 10*a^2*b^2*EllipticPi[-(b/a), ArcSin[Sqrt[
Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[-Tan[c + d*x]^2] + 3*b^4*Ellip
ticPi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[-Tan
[c + d*x]^2]))/(a*(a - b)^2*(a + b)^2))/(16*b^2*d*(b + a*Cos[c + d*x])^2*S
qrt[Sec[c + d*x]])
```

3.623.3 Rubi [A] (verified)

Time = 2.09 (sec) , antiderivative size = 310, normalized size of antiderivative = 0.99, number of steps used = 18, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.783$, Rules used = {3042, 4332, 27, 3042, 4588, 27, 3042, 4594, 3042, 4274, 3042, 4258, 3042, 3119, 3120, 4336, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\sec(c+dx))^3}\,dx$$
$$\Big\downarrow \; 3042$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^3}dx$$

$\Big\downarrow$ 4332

$$-\frac{\int -\frac{a^2+4b\sec(c+dx)a+\left(a^2-4b^2\right)\sec^2(c+dx)}{2\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^2}dx}{2b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$\Big\downarrow$ 27

$$\frac{\int \frac{a^2+4b\sec(c+dx)a+\left(a^2-4b^2\right)\sec^2(c+dx)}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^2}dx}{4b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$\Big\downarrow$ 3042

$$\frac{\int \frac{a^2+4b\csc\left(c+dx+\frac{\pi}{2}\right)a+\left(a^2-4b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx}{4b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$\Big\downarrow$ 4588

$$\frac{\frac{a\left(a^2-7b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a^2-b^2)(a+b\sec(c+dx))}-\frac{\int -\frac{\left(a^2-7b^2\right)\sec^2(c+dx)a^2+\left(a^2+5b^2\right)a^2+4b\left(a^2+2b^2\right)\sec(c+dx)a}{2\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}dx}{a(a^2-b^2)}}{4b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$\Big\downarrow$ 27

$$\frac{\frac{\int \frac{\left(a^2-7b^2\right)\sec^2(c+dx)a^2+\left(a^2+5b^2\right)a^2+4b\left(a^2+2b^2\right)\sec(c+dx)a}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}dx}{2a(a^2-b^2)}+\frac{a\left(a^2-7b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a^2-b^2)(a+b\sec(c+dx))}}{4b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\int \frac{\left(a^2-7b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+\left(a^2+5b^2\right)a^2+4b\left(a^2+2b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)a}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{2a(a^2-b^2)}+\frac{a\left(a^2-7b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a^2-b^2)(a+b\sec(c+dx))}}{4b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$\Big\downarrow$ 4594

3.623. $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\sec(c+dx))^3}\,dx$

$$\frac{\frac{\left(a^4-10a^2b^2-3b^4\right)\int \frac{\sec^{\frac{3}{2}}(c+dx)}{a+b\sec(c+dx)}dx+\frac{\int \frac{\left(a^2+5b^2\right)a^3+3b\left(a^2+b^2\right)\sec(c+dx)a^2}{\sqrt{\sec(c+dx)}}dx}{a^2}}{2a(a^2-b^2)}+\frac{a\left(a^2-7b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a^2-b^2)(a+b\sec(c+dx))}}{4b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\left(a^4-10a^2b^2-3b^4\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{\int \frac{\left(a^2+5b^2\right)a^3+3b\left(a^2+b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)a^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2}}{2a(a^2-b^2)}+\frac{a\left(a^2-7b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a^2-b^2)(a+b\sec(c+dx))}}{4b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$\Big\downarrow$ 4274

$$\frac{\frac{\left(a^4-10a^2b^2-3b^4\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{3a^2b\left(a^2+b^2\right)\int \sqrt{\sec(c+dx)}dx+a^3\left(a^2+5b^2\right)\int \frac{1}{\sqrt{\sec(c+dx)}}dx}{a^2}}{2a(a^2-b^2)}+\frac{a\left(a^2-7b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a^2-b^2)(a+b\sec(c+dx))}}{4b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\left(a^4-10a^2b^2-3b^4\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{3a^2b\left(a^2+b^2\right)\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+a^3\left(a^2+5b^2\right)\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2}}{2a(a^2-b^2)}+\frac{a\left(a^2-7b^2\right)\sin(c+dx)\sqrt{\sec(c+dx}}{d(a^2-b^2)(a+b\sec(c+dx))}}{4b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$\Big\downarrow$ 4258

$$\frac{\frac{\left(a^4-10a^2b^2-3b^4\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{3a^2b\left(a^2+b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx+a^3\left(a^2+5b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\cos(c+dx)}dx}{a^2}}{2a(a^2-b^2)}+}{4b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$\Big\downarrow$ 3042

3.623. $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\sec(c+dx))^3}\,dx$

$$\frac{\frac{\left(a^4-10a^2b^2-3b^4\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{3a^2b\left(a^2+b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+a^3\left(a^2+5b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\sin\left(c+dx+\right.}{a^2}}{2a(a^2-b^2)}}{4b\left(a^2-b^2\right)}$$

$$\frac{a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$\downarrow$ 3119

$$\frac{\frac{\left(a^4-10a^2b^2-3b^4\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{3a^2b\left(a^2+b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a^3\left(a^2+5b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\right|2}{d}}{a^2}}{2a(a^2-b^2)}}{4b\left(a^2-b^2\right)}$$

$$\frac{a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$\downarrow$ 3120

$$\frac{\frac{\left(a^4-10a^2b^2-3b^4\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{\frac{6a^2b\left(a^2+b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+\frac{2a^3\left(a^2+5b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\right|}{d}}{a^2}}{2a(a^2-b^2)}}{4b\left(a^2-b^2\right)}$$

$$\frac{a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$\downarrow$ 4336

$$\frac{\frac{\left(a^4-10a^2b^2-3b^4\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}(b+a\cos(c+dx))}dx+\frac{\frac{6a^2b\left(a^2+b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+\frac{2a^3\left(a^2+5b^2\right)}{}}{a^2}}{2a(a^2-b^2)}}{4b\left(a^2-b^2\right)}$$

$$\frac{a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$\downarrow$ 3042

$$\frac{\frac{\left(a^4-10a^2b^2-3b^4\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(b+a\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx+\frac{\frac{6a^2b\left(a^2+b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+\frac{2a^3}{}}{a^2}}{2a(a^2-b^2)}}{4b\left(a^2-b^2\right)}$$

$$\frac{a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$\downarrow$ 3284

3.623. $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\sec(c+dx))^3}\,dx$

$$\frac{\frac{a(a^2-7b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a^2-b^2)(a+b\sec(c+dx))}+\frac{\frac{2\left(a^4-10a^2b^2-3b^4\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticPi}\left(\frac{2a}{a+b},\frac{1}{2}(c+dx),2\right)}{d(a+b)}+\frac{\frac{6a^2b\left(a^2+b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{Ellip}}{d}}{2a(a^2-b^2)}}{4b\left(a^2-b^2\right)}}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

input
```
Int[Sec[c + d*x]^(5/2)/(a + b*Sec[c + d*x])^3,x]
```

output
```
-1/2*(a^2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(b*(a^2 - b^2)*d*(a + b*Sec[c +
 d*x])^2) + ((((2*a^3*(a^2 + 5*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)
/2, 2]*Sqrt[Sec[c + d*x]])/d + (6*a^2*b*(a^2 + b^2)*Sqrt[Cos[c + d*x]]*Ell
ipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d)/a^2 + (2*(a^4 - 10*a^2*b^2 -
 3*b^4)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2]*Sqrt[
Sec[c + d*x]])/((a + b)*d))/(2*a*(a^2 - b^2)) + (a*(a^2 - 7*b^2)*Sqrt[Sec[
c + d*x]]*Sin[c + d*x])/((a^2 - b^2)*d*(a + b*Sec[c + d*x])))/(4*b*(a^2 -
b^2))
```

3.623.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4332
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-a^2)*d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(
m + 1)*((d*Csc[e + f*x])^(n - 3)/(b*f*(m + 1)*(a^2 - b^2))), x] + Simp[d^3/
(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])
^(n - 3)*Simp[a^2*(n - 3) + a*b*(m + 1)*Csc[e + f*x] - (a^2*(n - 2) + b^2*(
m + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2
- b^2, 0] && LtQ[m, -1] && (IGtQ[n, 3] || (IntegersQ[n + 1/2, 2*m] && GtQ[n
, 2]))
```

rule 4336
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]]   Int[
1/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4588
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc
[e + f*x])^(m + 1)*((d*Csc[e + f*x])^n/(a*f*(m + 1)*(a^2 - b^2))), x] + Sim
p[1/(a*(m + 1)*(a^2 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f
*x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n +
1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m
+ n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x
] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

3.623. $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\sec(c+dx))^3}\,dx$

rule 4594
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Simp[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2)   Int[(d*Csc[e +
f*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Simp[1/a^2   Int[(a*A - (A*b - a
*B)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A,
B, C}, x] && NeQ[a^2 - b^2, 0]
```

3.623.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1759 vs. $2(365) = 730$.

Time = 372.30 (sec) , antiderivative size = 1760, normalized size of antiderivative = 5.62

method	result	size
default	Expression too large to display	1760

input
```
int(sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
-(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2/a*(a^2/b/(a^
2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(
1/2)/(2*a*cos(1/2*d*x+1/2*c)^2-a+b)-1/2/(a+b)/b*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+
1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2*a/b/(a^2-b^2)*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d
*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/
2))-1/2*a/b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^
2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(
cos(1/2*d*x+1/2*c),2^(1/2))-1/2/b/(a^2-b^2)/(a^2-a*b)*a^3*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+si
n(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))
+3/2*b/(a^2-b^2)/(a^2-a*b)*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+
1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ell
ipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2)))-2*b/a*(1/2*a^2/b/(a^2-b^2)*
cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2
*a*cos(1/2*d*x+1/2*c)^2-a+b)^2+3/4*a^2*(a^2-3*b^2)/b^2/(a^2-b^2)^2*cos(1/2
*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*a*cos(
1/2*d*x+1/2*c)^2-a+b)-3/8/(a+b)/(a^2-b^2)/b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x...
```

3.623.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
Timed out
```

3.623.6 Sympy [F]

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\sec(c+dx))^3}\,dx$$

input
```
integrate(sec(d*x+c)**(5/2)/(a+b*sec(d*x+c))**3,x)
```

output
```
Integral(sec(c + d*x)**(5/2)/(a + b*sec(c + d*x))**3, x)
```

3.623.7 Maxima [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
Timed out
```

3.623.8 Giac [F]

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \int \frac{\sec{(dx+c)}^{\frac{5}{2}}}{(b\sec{(dx+c)}+a)^3}\,dx$$

input
```
integrate(sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^(5/2)/(b*sec(d*x + c) + a)^3, x)
```

3.623.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^3}\,dx$$

input
```
int((1/cos(c + d*x))^(5/2)/(a + b/cos(c + d*x))^3,x)
```

output
```
int((1/cos(c + d*x))^(5/2)/(a + b/cos(c + d*x))^3, x)
```

3.624 $\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^3}\,dx$

3.624.1 Optimal result 4769
3.624.2 Mathematica [A] (verified) 4770
3.624.3 Rubi [A] (verified) 4770
3.624.4 Maple [B] (verified) 4776
3.624.5 Fricas [F(-1)] 4776
3.624.6 Sympy [F] 4777
3.624.7 Maxima [F(-1)] 4777
3.624.8 Giac [F] 4777
3.624.9 Mupad [F(-1)] 4778

3.624.1 Optimal result

Integrand size = 23, antiderivative size = 306

$$\begin{aligned}\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^3}\,dx &= -\frac{(5a^2+b^2)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{4a\left(a^2-b^2\right)^2 d} \\ &\quad -\frac{b(7a^2-b^2)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{4a^2\left(a^2-b^2\right)^2 d} \\ &\quad +\frac{(3a^4+10a^2b^2-b^4)\sqrt{\cos(c+dx)}\,\text{EllipticPi}\left(\frac{2a}{a+b},\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{4a^2(a-b)^2(a+b)^3 d} \\ &\quad +\frac{a\sqrt{\sec(c+dx)}\sin(c+dx)}{2\left(a^2-b^2\right)d(a+b\sec(c+dx))^2}+\frac{3(a^2+b^2)\sqrt{\sec(c+dx)}\sin(c+dx)}{4\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))}\end{aligned}$$

output
```
1/2*a*sin(d*x+c)*sec(d*x+c)^(1/2)/(a^2-b^2)/d/(a+b*sec(d*x+c))^2+3/4*(a^2+
b^2)*sin(d*x+c)*sec(d*x+c)^(1/2)/(a^2-b^2)^2/d/(a+b*sec(d*x+c))-1/4*(5*a^2
+b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*
x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a/(a^2-b^2)^2/d-1/4*b*
(7*a^2-b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(
1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^2/(a^2-b^2)^2/
d+1/4*(3*a^4+10*a^2*b^2-b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*
c)*EllipticPi(sin(1/2*d*x+1/2*c),2*a/(a+b),2^(1/2))*cos(d*x+c)^(1/2)*sec(d
*x+c)^(1/2)/a^2/(a-b)^2/(a+b)^3/d
```

3.624.2 Mathematica [A] (verified)

Time = 5.71 (sec) , antiderivative size = 429, normalized size of antiderivative = 1.40

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^3}\,dx$$
$$= \frac{\frac{4a^2(3b(a^2+b^2)+a(5a^2+b^2)\cos(c+dx))\sin(c+dx)}{(a^2-b^2)^2} + \frac{4\cos(c+dx)(b+a\cos(c+dx))\cot(c+dx)(a+b\sec(c+dx))\left(5a^3b+ab^3-5a^3b\sec^2(c+dx)-ab\right.}{}}{}$$

input
```
Integrate[Sec[c + d*x]^(3/2)/(a + b*Sec[c + d*x])^3,x]
```

output
```
((4*a^2*(3*b*(a^2 + b^2) + a*(5*a^2 + b^2)*Cos[c + d*x])*Sin[c + d*x])/(a^
2 - b^2)^2 + (4*Cos[c + d*x]*(b + a*Cos[c + d*x])*Cot[c + d*x]*(a + b*Sec[
c + d*x])*(5*a^3*b + a*b^3 - 5*a^3*b*Sec[c + d*x]^2 - a*b^3*Sec[c + d*x]^2
 + a*b*(5*a^2 + b^2)*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c
+ d*x]]*Sqrt[-Tan[c + d*x]^2] + a*(3*a^3 - 5*a^2*b + 3*a*b^2 - b^3)*Ellipt
icF[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[-Tan[c + d*x]^
2] - 3*a^4*EllipticPi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c +
 d*x]]*Sqrt[-Tan[c + d*x]^2] - 10*a^2*b^2*EllipticPi[-(b/a), ArcSin[Sqrt[S
ec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[-Tan[c + d*x]^2] + b^4*Elliptic
Pi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[-Tan[c
+ d*x]^2]))/((a - b)^2*b*(a + b)^2))/(16*a^2*d*(b + a*Cos[c + d*x])^2*Sqrt
[Sec[c + d*x]])
```

3.624.3 Rubi [A] (verified)

Time = 2.01 (sec) , antiderivative size = 307, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.783$, Rules used = {3042, 4331, 27, 3042, 4588, 27, 3042, 4594, 3042, 4274, 3042, 4258, 3042, 3119, 3120, 4336, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^3}\,dx$$
$$\Big\downarrow \; 3042$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \; 4331$$

$$\frac{\int -\frac{-3a\sec^2(c+dx)+4b\sec(c+dx)+a}{2\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^2}dx}{2\left(a^2-b^2\right)}+\frac{a\sin(c+dx)\sqrt{\sec(c+dx)}}{2d\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$$\Big\downarrow \; 27$$

$$\frac{a\sin(c+dx)\sqrt{\sec(c+dx)}}{2d\left(a^2-b^2\right)(a+b\sec(c+dx))^2}-\frac{\int \frac{-3a\sec^2(c+dx)+4b\sec(c+dx)+a}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^2}dx}{4\left(a^2-b^2\right)}$$

$$\Big\downarrow \; 3042$$

$$\frac{a\sin(c+dx)\sqrt{\sec(c+dx)}}{2d\left(a^2-b^2\right)(a+b\sec(c+dx))^2}-\frac{\int \frac{-3a\csc\left(c+dx+\frac{\pi}{2}\right)^2+4b\csc\left(c+dx+\frac{\pi}{2}\right)+a}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx}{4\left(a^2-b^2\right)}$$

$$\Big\downarrow \; 4588$$

$$\frac{a\sin(c+dx)\sqrt{\sec(c+dx)}}{2d\left(a^2-b^2\right)(a+b\sec(c+dx))^2}-\frac{-\frac{\int -\frac{12b\sec(c+dx)a^2-3\left(a^2+b^2\right)\sec^2(c+dx)a+\left(5a^2+b^2\right)a}{2\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}dx}{a(a^2-b^2)}-\frac{3\left(a^2+b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a^2-b^2)(a+b\sec(c+dx))}}{4\left(a^2-b^2\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{a\sin(c+dx)\sqrt{\sec(c+dx)}}{2d\left(a^2-b^2\right)(a+b\sec(c+dx))^2}-\frac{\frac{\int \frac{12b\sec(c+dx)a^2-3\left(a^2+b^2\right)\sec^2(c+dx)a+\left(5a^2+b^2\right)a}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}dx}{2a(a^2-b^2)}-\frac{3\left(a^2+b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a^2-b^2)(a+b\sec(c+dx))}}{4\left(a^2-b^2\right)}$$

$$\Big\downarrow \; 3042$$

$$\frac{a\sin(c+dx)\sqrt{\sec(c+dx)}}{2d\left(a^2-b^2\right)(a+b\sec(c+dx))^2}-\frac{\frac{\int \frac{12b\csc\left(c+dx+\frac{\pi}{2}\right)a^2-3\left(a^2+b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2a+\left(5a^2+b^2\right)a}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{2a(a^2-b^2)}-\frac{3\left(a^2+b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a^2-b^2)(a+b\sec(c+dx))}}{4\left(a^2-b^2\right)}$$

$$\Big\downarrow \; 4594$$

3.624. $\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^3}\,dx$

$$\frac{a\sin(c+dx)\sqrt{\sec(c+dx)}}{2d\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)^2}-\frac{\frac{\frac{\int\frac{\left(5a^2+b^2\right)a^2+b\left(7a^2-b^2\right)\sec(c+dx)a}{\sqrt{\sec(c+dx)}}dx}{a^2}-\frac{\left(3a^4+10a^2b^2-b^4\right)\int\frac{\sec^{\frac{3}{2}}(c+dx)}{a+b\sec(c+dx)}dx}{a}}{2a(a^2-b^2)}-\frac{3(a^2+b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a^2-b^2)(a+b\sec(c+dx))}}{4\left(a^2-b^2\right)}$$

$\downarrow$ 3042

$$\frac{a\sin(c+dx)\sqrt{\sec(c+dx)}}{2d\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)^2}-\frac{\frac{\frac{\int\frac{\left(5a^2+b^2\right)a^2+b\left(7a^2-b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)a}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2}-\frac{\left(3a^4+10a^2b^2-b^4\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a}}{2a(a^2-b^2)}-\frac{3(a^2+b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a^2-b^2)(a+b\sec(c+dx))}}{4\left(a^2-b^2\right)}$$

$\downarrow$ 4274

$$\frac{a\sin(c+dx)\sqrt{\sec(c+dx)}}{2d\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)^2}-\frac{\frac{\frac{a^2\left(5a^2+b^2\right)\int\frac{1}{\sqrt{\sec(c+dx)}}dx+ab\left(7a^2-b^2\right)\int\sqrt{\sec(c+dx)}dx}{a^2}-\frac{\left(3a^4+10a^2b^2-b^4\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a}}{2a(a^2-b^2)}-\frac{3(a^2+b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a^2-b^2)(a+b\sec(c+dx))}}{4\left(a^2-b^2\right)}$$

$\downarrow$ 3042

$$\frac{a\sin(c+dx)\sqrt{\sec(c+dx)}}{2d\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)^2}-\frac{\frac{\frac{a^2\left(5a^2+b^2\right)\int\frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+ab\left(7a^2-b^2\right)\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}-\frac{\left(3a^4+10a^2b^2-b^4\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a}}{2a(a^2-b^2)}-\frac{3(a^2+b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a^2-b^2)(a+b\sec(c+dx))}}{4\left(a^2-b^2\right)}$$

$\downarrow$ 4258

$$\frac{a\sin(c+dx)\sqrt{\sec(c+dx)}}{2d\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)^2}-\frac{\frac{\frac{a^2\left(5a^2+b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\cos(c+dx)}dx+ab\left(7a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx}{a^2}-\frac{\left(3a^4+10a^2b^2-b^4\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a}}{2a(a^2-b^2)}-}{4\left(a^2-b^2\right)}$$

$\downarrow$ 3042

3.624. $\int\frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^3}\,dx$

$$\frac{a\sin(c+dx)\sqrt{\sec(c+dx)}}{2d\left(a^2-b^2\right)(a+b\sec(c+dx))^2}-\frac{\frac{\frac{a^2\left(5a^2+b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+ab\left(7a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2}-\frac{\left(3a^4+10a^2b^2-b^4\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}{a}}{2a\left(a^2-b^2\right)}}{4\left(a^2-b^2\right)}$$

$\downarrow$ 3119

$$\frac{a\sin(c+dx)\sqrt{\sec(c+dx)}}{2d\left(a^2-b^2\right)(a+b\sec(c+dx))^2}-\frac{\frac{\frac{ab\left(7a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a^2\left(5a^2+b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}}{a^2}-\frac{\left(3a^4+10a^2b^2-b^4\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a}}{2a\left(a^2-b^2\right)}}{4\left(a^2-b^2\right)}$$

$\downarrow$ 3120

$$\frac{a\sin(c+dx)\sqrt{\sec(c+dx)}}{2d\left(a^2-b^2\right)(a+b\sec(c+dx))^2}-\frac{\frac{\frac{\frac{2ab\left(7a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+\frac{2a^2\left(5a^2+b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}}{a^2}-\frac{\left(3a^4+10a^2b^2-b^4\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}d}{a}}{2a\left(a^2-b^2\right)}}{4\left(a^2-b^2\right)}$$

$\downarrow$ 4336

$$\frac{a\sin(c+dx)\sqrt{\sec(c+dx)}}{2d\left(a^2-b^2\right)(a+b\sec(c+dx))^2}-\frac{\frac{\frac{\frac{2ab\left(7a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+\frac{2a^2\left(5a^2+b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}}{a^2}-\frac{\left(3a^4+10a^2b^2-b^4\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx}}{a}}{2a\left(a^2-b^2\right)}}{4\left(a^2-b^2\right)}$$

$\downarrow$ 3042

$$\frac{a\sin(c+dx)\sqrt{\sec(c+dx)}}{2d\left(a^2-b^2\right)(a+b\sec(c+dx))^2}-\frac{\frac{\frac{\frac{2ab\left(7a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+\frac{2a^2\left(5a^2+b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}}{a^2}-\frac{\left(3a^4+10a^2b^2-b^4\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx}}{}}{2a\left(a^2-b^2\right)}}{4\left(a^2-b^2\right)}$$

$\downarrow$ 3284

3.624. $\int\frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^3}\,dx$

$$\frac{\frac{a\sin(c+dx)\sqrt{\sec(c+dx)}}{2d\left(a^2-b^2\right)(a+b\sec(c+dx))^2}-\frac{\frac{\frac{2ab\left(7a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+\frac{2a^2\left(5a^2+b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}}{a^2}-\frac{2\left(3a^4+10a^2b^2-b^4\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+d}}{ad(a+b)}}{2a(a^2-b^2)}}{4\left(a^2-b^2\right)}$$

input
```
Int[Sec[c + d*x]^(3/2)/(a + b*Sec[c + d*x])^3,x]
```

output
```
(a*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(2*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^
2) - ((((2*a^2*(5*a^2 + b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*
Sqrt[Sec[c + d*x]])/d + (2*a*b*(7*a^2 - b^2)*Sqrt[Cos[c + d*x]]*EllipticF[
(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d)/a^2 - (2*(3*a^4 + 10*a^2*b^2 - b^4)
*Sqrt[Cos[c + d*x]]*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c +
 d*x]])/(a*(a + b)*d))/(2*a*(a^2 - b^2)) - (3*(a^2 + b^2)*Sqrt[Sec[c + d*x
]]*Sin[c + d*x])/((a^2 - b^2)*d*(a + b*Sec[c + d*x])))/(4*(a^2 - b^2))
```

3.624.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

3.624. $\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^3}\,dx$

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4331
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[a*d^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
)*((d*Csc[e + f*x])^(n - 2)/(f*(m + 1)*(a^2 - b^2))), x] - Simp[d^2/((m + 1
)*(a^2 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*
(a*(n - 2) + b*(m + 1)*Csc[e + f*x] - a*(m + n)*Csc[e + f*x]^2), x], x] /;
FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && LtQ[1, n, 2
] && IntegersQ[2*m, 2*n]
```

rule 4336
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]]   Int[
1/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4588
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc
[e + f*x])^(m + 1)*((d*Csc[e + f*x])^n/(a*f*(m + 1)*(a^2 - b^2))), x] + Sim
p[1/(a*(m + 1)*(a^2 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f
*x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n +
1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m
+ n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x
] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

rule 4594
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Simp[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2)   Int[(d*Csc[e +
f*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Simp[1/a^2   Int[(a*A - (A*b - a
*B)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A,
B, C}, x] && NeQ[a^2 - b^2, 0]
```

3.624. $\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^3}\,dx$

3.624.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1857 vs. 2(358) = 716.

Time = 26.06 (sec) , antiderivative size = 1858, normalized size of antiderivative = 6.07

method	result	size
default	Expression too large to display	1858

input
```
int(sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
-(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2/a/(a^2-a*b)
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/
2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2
*a/(a-b),2^(1/2))+2/a^2*b^2*(1/2*a^2/b/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*a*cos(1/2*d*x+1/2*c)^2-a
+b)^2+3/4*a^2*(a^2-3*b^2)/b^2/(a^2-b^2)^2*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d
*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*a*cos(1/2*d*x+1/2*c)^2-a+b)-3/8
/(a+b)/(a^2-b^2)/b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2
+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(c
os(1/2*d*x+1/2*c),2^(1/2))*a^2-1/4/(a+b)/(a^2-b^2)/b*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2
*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a+7/8/(a+b)/(a^
2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*
sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2
*c),2^(1/2))+3/8*a^3/b^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(
1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(
1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-9/8*a/(a^2-b^2)^2*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3/8*a^3
/b^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+...
```

3.624.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
Timed out
```

3.624.6 Sympy [F]

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^3}\,dx$$

input
```
integrate(sec(d*x+c)**(3/2)/(a+b*sec(d*x+c))**3,x)
```

output
```
Integral(sec(c + d*x)**(3/2)/(a + b*sec(c + d*x))**3, x)
```

3.624.7 Maxima [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
Timed out
```

3.624.8 Giac [F]

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \int \frac{\sec(dx+c)^{\frac{3}{2}}}{(b\sec(dx+c)+a)^3}\,dx$$

input
```
integrate(sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^(3/2)/(b*sec(d*x + c) + a)^3, x)
```

3.624.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^3}\,dx$$

input
```
int((1/cos(c + d*x))^(3/2)/(a + b/cos(c + d*x))^3,x)
```

output
```
int((1/cos(c + d*x))^(3/2)/(a + b/cos(c + d*x))^3, x)
```

3.625 $\int \frac{\sqrt{\sec(c+dx)}}{(a+b\sec(c+dx))^3}\,dx$

3.625.1 Optimal result . . . 4779
3.625.2 Mathematica [A] (verified) . . . 4780
3.625.3 Rubi [A] (verified) . . . 4780
3.625.4 Maple [B] (verified) . . . 4786
3.625.5 Fricas [F(-1)] . . . 4786
3.625.6 Sympy [F] . . . 4787
3.625.7 Maxima [F(-1)] . . . 4787
3.625.8 Giac [F] . . . 4787
3.625.9 Mupad [F(-1)] . . . 4788

3.625.1 Optimal result

Integrand size = 23, antiderivative size = 323

$$\begin{aligned}\int \frac{\sqrt{\sec(c+dx)}}{(a+b\sec(c+dx))^3}\,dx &= \frac{3b(3a^2-b^2)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{4a^2\left(a^2-b^2\right)^2 d} \\ &\quad+\frac{(8a^4-5a^2b^2+3b^4)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{4a^3\left(a^2-b^2\right)^2 d} \\ &\quad-\frac{3b(5a^4-2a^2b^2+b^4)\sqrt{\cos(c+dx)}\,\text{EllipticPi}\left(\frac{2a}{a+b},\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{4a^3(a-b)^2(a+b)^3 d} \\ &\quad-\frac{b\sqrt{\sec(c+dx)}\sin(c+dx)}{2\left(a^2-b^2\right)d(a+b\sec(c+dx))^2}-\frac{b(7a^2-b^2)\sqrt{\sec(c+dx)}\sin(c+dx)}{4a\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))}\end{aligned}$$

output
```
-1/2*b*sin(d*x+c)*sec(d*x+c)^(1/2)/(a^2-b^2)/d/(a+b*sec(d*x+c))^2-1/4*b*(7
*a^2-b^2)*sin(d*x+c)*sec(d*x+c)^(1/2)/a/(a^2-b^2)^2/d/(a+b*sec(d*x+c))+3/4
*b*(3*a^2-b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(s
in(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^2/(a^2-b^2)
^2/d+1/4*(8*a^4-5*a^2*b^2+3*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+
1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(
1/2)/a^3/(a^2-b^2)^2/d-3/4*b*(5*a^4-2*a^2*b^2+b^4)*(cos(1/2*d*x+1/2*c)^2)^
(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*a/(a+b),2^(1/2))*
cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^3/(a-b)^2/(a+b)^3/d
```

3.625.2 Mathematica [A] (verified)

Time = 4.80 (sec) , antiderivative size = 286, normalized size of antiderivative = 0.89

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+b\sec(c+dx))^3}\,dx$$
$$= \frac{\frac{4b(-7a^2b+b^3+(-9a^3+3ab^2)\cos(c+dx))\sin(c+dx)}{a(a^2-b^2)^2(b+a\cos(c+dx))^2\sqrt{\sec(c+dx)}} + \frac{2\cot(c+dx)\left(6ab(3a^2-b^2)\sec^{\frac{3}{2}}(c+dx)\sin^2(c+dx)-6ab(3a^2-b^2)E\left(\arcsin\left(\sqrt{\sec(c+dx}\right.\right.\right.}{}}{}$$

input
```
Integrate[Sqrt[Sec[c + d*x]]/(a + b*Sec[c + d*x])^3,x]
```

output
```
((4*b*(-7*a^2*b + b^3 + (-9*a^3 + 3*a*b^2)*Cos[c + d*x])*Sin[c + d*x])/(a*
(a^2 - b^2)^2*(b + a*Cos[c + d*x])^2*Sqrt[Sec[c + d*x]]) + (2*Cot[c + d*x]
*(6*a*b*(3*a^2 - b^2)*Sec[c + d*x]^(3/2)*Sin[c + d*x]^2 - 6*a*b*(3*a^2 - b
^2)*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[c + d*x]^2] - 2*a*
(7*a^3 - 9*a^2*b - a*b^2 + 3*b^3)*EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1
]*Sqrt[-Tan[c + d*x]^2] + 6*(5*a^4 - 2*a^2*b^2 + b^4)*EllipticPi[-(b/a), A
rcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[c + d*x]^2]))/(a^3*(a - b)^2*(a +
 b)^2))/(16*d)
```

3.625.3 Rubi [A] (verified)

Time = 2.09 (sec) , antiderivative size = 321, normalized size of antiderivative = 0.99, number of steps used = 18, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.783$, Rules used = {3042, 4330, 27, 3042, 4588, 27, 3042, 4594, 3042, 4274, 3042, 4258, 3042, 3119, 3120, 4336, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+b\sec(c+dx))^3}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^3}dx$$
$$\Big\downarrow \; 4330$$

$$-\frac{\int -\frac{-3b\sec^2(c+dx)+4a\sec(c+dx)+b}{2\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^2}dx}{2(a^2-b^2)}-\frac{b\sin(c+dx)\sqrt{\sec(c+dx)}}{2d(a^2-b^2)(a+b\sec(c+dx))^2}$$

$\Big\downarrow$ 27

$$\frac{\int \frac{-3b\sec^2(c+dx)+4a\sec(c+dx)+b}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^2}dx}{4(a^2-b^2)}-\frac{b\sin(c+dx)\sqrt{\sec(c+dx)}}{2d(a^2-b^2)(a+b\sec(c+dx))^2}$$

$\Big\downarrow$ 3042

$$\frac{\int \frac{-3b\csc\left(c+dx+\frac{\pi}{2}\right)^2+4a\csc\left(c+dx+\frac{\pi}{2}\right)+b}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx}{4(a^2-b^2)}-\frac{b\sin(c+dx)\sqrt{\sec(c+dx)}}{2d(a^2-b^2)(a+b\sec(c+dx))^2}$$

$\Big\downarrow$ 4588

$$\frac{-\frac{\int -\frac{-b\left(7a^2-b^2\right)\sec^2(c+dx)+4a\left(2a^2+b^2\right)\sec(c+dx)+3b\left(3a^2-b^2\right)}{2\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}dx}{a(a^2-b^2)}-\frac{b(7a^2-b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad(a^2-b^2)(a+b\sec(c+dx))}}{4(a^2-b^2)}-\frac{b\sin(c+dx)\sqrt{\sec(c+dx)}}{2d(a^2-b^2)(a+b\sec(c+dx))^2}$$

$\Big\downarrow$ 27

$$\frac{\frac{\int \frac{-b\left(7a^2-b^2\right)\sec^2(c+dx)+4a\left(2a^2+b^2\right)\sec(c+dx)+3b\left(3a^2-b^2\right)}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}dx}{2a(a^2-b^2)}-\frac{b(7a^2-b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad(a^2-b^2)(a+b\sec(c+dx))}}{4(a^2-b^2)}-\frac{b\sin(c+dx)\sqrt{\sec(c+dx)}}{2d(a^2-b^2)(a+b\sec(c+dx))^2}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\int \frac{-b\left(7a^2-b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2+4a\left(2a^2+b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)+3b\left(3a^2-b^2\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{2a(a^2-b^2)}-\frac{b(7a^2-b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad(a^2-b^2)(a+b\sec(c+dx))}}{4(a^2-b^2)}-\frac{b\sin(c+dx)\sqrt{\sec(c+dx)}}{2d(a^2-b^2)(a+b\sec(c+dx))^2}$$

$\Big\downarrow$ 4594

$$\frac{\frac{\frac{\int \frac{3ab\left(3a^2-b^2\right)+\left(8a^4-5b^2a^2+3b^4\right)\sec(c+dx)}{\sqrt{\sec(c+dx)}}dx}{a^2}-\frac{3b\left(5a^4-2a^2b^2+b^4\right)\int \frac{\sec^{\frac{3}{2}}(c+dx)}{a+b\sec(c+dx)}dx}{a^2}}{2a(a^2-b^2)}-\frac{b(7a^2-b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad(a^2-b^2)(a+b\sec(c+dx))}}{4(a^2-b^2)}-\frac{b\sin(c+dx)\sqrt{\sec(c+dx)}}{2d(a^2-b^2)(a+b\sec(c+dx))^2}$$

$\Big\downarrow$ 3042

3.625. $\int \frac{\sqrt{\sec(c+dx)}}{(a+b\sec(c+dx))^3}\,dx$

$$\frac{\frac{\frac{\int \frac{3ab\left(3a^2-b^2\right)+\left(8a^4-5b^2a^2+3b^4\right)\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2}-\frac{3b\left(5a^4-2a^2b^2+b^4\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}}{2a(a^2-b^2)}-\frac{b(7a^2-b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad(a^2-b^2)(a+b\sec(c+dx))}}{4\left(a^2-b^2\right)}-\frac{b\sin(c+dx)\sqrt{\sec(c+dx)}}{2d\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$\downarrow$ 4274

$$\frac{\frac{\frac{3ab\left(3a^2-b^2\right)\int \frac{1}{\sqrt{\sec(c+dx)}}dx+\left(8a^4-5a^2b^2+3b^4\right)\int \sqrt{\sec(c+dx)}dx}{a^2}-\frac{3b\left(5a^4-2a^2b^2+b^4\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}}{2a(a^2-b^2)}-\frac{b(7a^2-b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad(a^2-b^2)(a+b\sec(c+dx))}}{4\left(a^2-b^2\right)}-\frac{b\sin(c+dx)\sqrt{\sec(c+dx)}}{2d\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$\downarrow$ 3042

$$\frac{\frac{\frac{3ab\left(3a^2-b^2\right)\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\left(8a^4-5a^2b^2+3b^4\right)\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}-\frac{3b\left(5a^4-2a^2b^2+b^4\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}}{2a(a^2-b^2)}-\frac{b(7a^2-b^2)\sin(c+dx)\sqrt{\sec(c}}{ad(a^2-b^2)(a+b\sec(c+dx}}{4\left(a^2-b^2\right)}-\frac{b\sin(c+dx)\sqrt{\sec(c+dx)}}{2d\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$\downarrow$ 4258

$$\frac{\frac{\frac{3ab\left(3a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\cos(c+dx)}dx+\left(8a^4-5a^2b^2+3b^4\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{a^2}-\frac{3b\left(5a^4-2a^2b^2+b^4\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}{a^2}}{2a(a^2-b^2)}}{4\left(a^2-b^2\right)}-\frac{b\sin(c+dx)\sqrt{\sec(c+dx)}}{2d\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$\downarrow$ 3042

$$\frac{\frac{\frac{3ab\left(3a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\left(8a^4-5a^2b^2+3b^4\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2}-\frac{3b\left(5a^4-2a^2b^2+b^4\right)\int \frac{\csc\left(c+dx\right.}{a+b\csc\left(c\right.}}{a^2}}{2a(a^2-b^2)}}{4\left(a^2-b^2\right)}-\frac{b\sin(c+dx)\sqrt{\sec(c+dx)}}{2d\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$\downarrow$ 3119

3.625. $\int \frac{\sqrt{\sec(c+dx)}}{(a+b\sec(c+dx))^3}\,dx$

$$\frac{\frac{\frac{\left(8a^4-5a^2b^2+3b^4\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{6ab\left(3a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}}{a^2}-\frac{3b\left(5a^4-2a^2b^2+b^4\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{a+b\csc\left(c+dx+\right.}}{a^2}}{2a(a^2-b^2)}}{4\left(a^2-b^2\right)}$$

$$\frac{b\sin(c+dx)\sqrt{\sec(c+dx)}}{2d\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$\downarrow$ 3120

$$\frac{\frac{\frac{6ab\left(3a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2\left(8a^4-5a^2b^2+3b^4\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}}{a^2}-\frac{3b\left(5a^4-2a^2b^2+b^4\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{a+b\csc\left(c+dx\right.}}{a^2}}{2a(a^2-b^2)}}{4\left(a^2-b^2\right)}$$

$$\frac{b\sin(c+dx)\sqrt{\sec(c+dx)}}{2d\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$\downarrow$ 4336

$$\frac{\frac{\frac{6ab\left(3a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2\left(8a^4-5a^2b^2+3b^4\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}}{a^2}-\frac{3b\left(5a^4-2a^2b^2+b^4\right)\sqrt{\cos(c+dx)}\sqrt{\text{se}}}{}}{2a(a^2-b^2)}}{4\left(a^2-b^2\right)}$$

$$\frac{b\sin(c+dx)\sqrt{\sec(c+dx)}}{2d\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$\downarrow$ 3042

$$\frac{\frac{\frac{6ab\left(3a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2\left(8a^4-5a^2b^2+3b^4\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}}{a^2}-\frac{3b\left(5a^4-2a^2b^2+b^4\right)\sqrt{\cos(c+dx)}\sqrt{\text{se}}}{}}{2a(a^2-b^2)}}{4\left(a^2-b^2\right)}$$

$$\frac{b\sin(c+dx)\sqrt{\sec(c+dx)}}{2d\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$\downarrow$ 3284

$$\frac{\frac{\frac{6ab\left(3a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2\left(8a^4-5a^2b^2+3b^4\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}}{a^2}-\frac{6b\left(5a^4-2a^2b^2+b^4\right)\sqrt{\cos(c+dx)}\sqrt{\text{se}}}{a^2d}}{2a(a^2-b^2)}}{4\left(a^2-b^2\right)}$$

$$\frac{b\sin(c+dx)\sqrt{\sec(c+dx)}}{2d\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

input
```
Int[Sqrt[Sec[c + d*x]]/(a + b*Sec[c + d*x])^3,x]
```

3.625. $\int \frac{\sqrt{\sec(c+dx)}}{(a+b\sec(c+dx))^3}\,dx$

output
```
-1/2*(b*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/((a^2 - b^2)*d*(a + b*Sec[c + d*x
])^2) + ((((6*a*b*(3*a^2 - b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2,
2]*Sqrt[Sec[c + d*x]])/d + (2*(8*a^4 - 5*a^2*b^2 + 3*b^4)*Sqrt[Cos[c + d*x
]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d)/a^2 - (6*b*(5*a^4 - 2*
a^2*b^2 + b^4)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2
]*Sqrt[Sec[c + d*x]])/(a^2*(a + b)*d))/(2*a*(a^2 - b^2)) - (b*(7*a^2 - b^2
)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(a*(a^2 - b^2)*d*(a + b*Sec[c + d*x])))
/(4*(a^2 - b^2))
```

3.625.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4330
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b)*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m +
1)*((d*Csc[e + f*x])^(n - 1)/(f*(m + 1)*(a^2 - b^2))), x] + Simp[1/((m + 1)
*(a^2 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1)*S
imp[b*d*(n - 1) + a*d*(m + 1)*Csc[e + f*x] - b*d*(m + n + 1)*Csc[e + f*x]^2
, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1
] && LtQ[0, n, 1] && IntegersQ[2*m, 2*n]
```

rule 4336
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]]   Int[
1/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4588
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc
[e + f*x])^(m + 1)*((d*Csc[e + f*x])^n/(a*f*(m + 1)*(a^2 - b^2))), x] + Sim
p[1/(a*(m + 1)*(a^2 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f
*x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n +
1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m
+ n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x
] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

rule 4594
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Simp[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2)   Int[(d*Csc[e +
f*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Simp[1/a^2   Int[(a*A - (A*b - a
*B)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A,
B, C}, x] && NeQ[a^2 - b^2, 0]
```

3.625. $\int \frac{\sqrt{\sec(c+dx)}}{(a+b\sec(c+dx))^3}\,dx$

3.625.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1935 vs. 2(375) = 750.

Time = 28.16 (sec) , antiderivative size = 1936, normalized size of antiderivative = 5.99

method	result	size
default	Expression too large to display	1936

input
```
int(sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
-(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2/a^3*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-2/
a^3*b^3*(1/2*a^2/b/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+s
in(1/2*d*x+1/2*c)^2)^(1/2)/(2*a*cos(1/2*d*x+1/2*c)^2-a+b)^2+3/4*a^2*(a^2-3
*b^2)/b^2/(a^2-b^2)^2*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*
d*x+1/2*c)^2)^(1/2)/(2*a*cos(1/2*d*x+1/2*c)^2-a+b)-3/8/(a+b)/(a^2-b^2)/b^2
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/
2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^
(1/2))*a^2-1/4/(a+b)/(a^2-b^2)/b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*
d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)
*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a+7/8/(a+b)/(a^2-b^2)*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3/8*a^3
/b^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^
(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1
/2*d*x+1/2*c),2^(1/2))-9/8*a/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*
cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^
2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3/8*a^3/b^2/(a^2-b^2)^2*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2...
```

3.625.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+b\sec(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
Timed out
```

3.625.6 Sympy [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+b\sec(c+dx))^3}\,dx = \int \frac{\sqrt{\sec{(c+dx)}}}{(a+b\sec{(c+dx)})^3}\,dx$$

input
```
integrate(sec(d*x+c)**(1/2)/(a+b*sec(d*x+c))**3,x)
```

output
```
Integral(sqrt(sec(c + d*x))/(a + b*sec(c + d*x))**3, x)
```

3.625.7 Maxima [F(-1)]

Timed out.

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+b\sec(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
Timed out
```

3.625.8 Giac [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+b\sec(c+dx))^3}\,dx = \int \frac{\sqrt{\sec{(dx+c)}}}{(b\sec{(dx+c)}+a)^3}\,dx$$

input
```
integrate(sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate(sqrt(sec(d*x + c))/(b*sec(d*x + c) + a)^3, x)
```

3.625.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+b\sec(c+dx))^3}\,dx = \int \frac{\sqrt{\frac{1}{\cos(c+d\,x)}}}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^3}\,dx$$

input
```
int((1/cos(c + d*x))^(1/2)/(a + b/cos(c + d*x))^3,x)
```

output
```
int((1/cos(c + d*x))^(1/2)/(a + b/cos(c + d*x))^3, x)
```

3.626 $\int \frac{1}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^3}\,dx$

3.626.1 Optimal result 4789
3.626.2 Mathematica [B] (warning: unable to verify) 4790
3.626.3 Rubi [A] (verified) 4791
3.626.4 Maple [B] (verified) 4796
3.626.5 Fricas [F(-1)] 4797
3.626.6 Sympy [F] 4798
3.626.7 Maxima [F] 4798
3.626.8 Giac [F] 4798
3.626.9 Mupad [F(-1)] 4799

3.626.1 Optimal result

Integrand size = 23, antiderivative size = 342

$$\begin{aligned}
&\int \frac{1}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^3}\,dx \\
&= \frac{(8a^4-29a^2b^2+15b^4)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{4a^3(a^2-b^2)^2d} \\
&\quad - \frac{3b(8a^4-11a^2b^2+5b^4)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{4a^4(a^2-b^2)^2d} \\
&\quad + \frac{b^2(35a^4-38a^2b^2+15b^4)\sqrt{\cos(c+dx)}\,\text{EllipticPi}\left(\frac{2a}{a+b},\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{4a^4(a-b)^2(a+b)^3d} \\
&\quad + \frac{b^2\sqrt{\sec(c+dx)}\sin(c+dx)}{2a(a^2-b^2)d(a+b\sec(c+dx))^2} + \frac{b^2(11a^2-5b^2)\sqrt{\sec(c+dx)}\sin(c+dx)}{4a^2(a^2-b^2)^2d(a+b\sec(c+dx))}
\end{aligned}$$

output
```
1/2*b^2*sin(d*x+c)*sec(d*x+c)^(1/2)/a/(a^2-b^2)/d/(a+b*sec(d*x+c))^2+1/4*b
^2*(11*a^2-5*b^2)*sin(d*x+c)*sec(d*x+c)^(1/2)/a^2/(a^2-b^2)^2/d/(a+b*sec(d
*x+c))+1/4*(8*a^4-29*a^2*b^2+15*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*
d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+
c)^(1/2)/a^3/(a^2-b^2)^2/d-3/4*b*(8*a^4-11*a^2*b^2+5*b^4)*(cos(1/2*d*x+1/2
*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(
d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^4/(a^2-b^2)^2/d+1/4*b^2*(35*a^4-38*a^2*b^2
+15*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/
2*d*x+1/2*c),2*a/(a+b),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^4/(a-b
)^2/(a+b)^3/d
```

3.626.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 707 vs. $2(342) = 684$.

Time = 6.67 (sec) , antiderivative size = 707, normalized size of antiderivative = 2.07

$$\int \frac{1}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^3}\,dx$$
$$= \frac{\frac{2(8a^4-7a^2b^2+5b^4)\cos^2(c+dx)\left(\text{EllipticF}\left(\arcsin\left(\sqrt{\sec(c+dx)}\right),-1\right)-\text{EllipticPi}\left(-\frac{b}{a},\arcsin\left(\sqrt{\sec(c+dx)}\right),-1\right)\right)(a+b\sec(c+dx))\sqrt{1-\sec^2(c+}}{b(b+a\cos(c+dx))(1-\cos^2(c+dx))}}{}$$
$$+\frac{\sqrt{\sec(c+dx)}\left(-\frac{b^2(-13a^2+7b^2)\sin(c+dx)}{4a^3(-a^2+b^2)^2}+\frac{b^4\sin(c+dx)}{2a^3(a^2-b^2)(b+a\cos(c+dx))^2}+\frac{3(-5a^2b^3\sin(c+dx)+3b^5\sin(c+dx))}{4a^3(a^2-b^2)^2(b+a\cos(c+dx))}\right)}{d}$$

input
```
Integrate[1/(Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^3),x]
```

output
```
((2*(8*a^4 - 7*a^2*b^2 + 5*b^4)*Cos[c + d*x]^2*(EllipticF[ArcSin[Sqrt[Sec[
c + d*x]]], -1] - EllipticPi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]], -1])*(a +
 b*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x])/(b*(b + a*Cos[c +
d*x])*(1 - Cos[c + d*x]^2)) + (2*(-32*a^3*b + 8*a*b^3)*Cos[c + d*x]^2*Elli
pticPi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]], -1]*(a + b*Sec[c + d*x])*Sqrt[1
 - Sec[c + d*x]^2]*Sin[c + d*x])/(a*(b + a*Cos[c + d*x])*(1 - Cos[c + d*x]
^2)) + ((8*a^4 - 29*a^2*b^2 + 15*b^4)*Cos[2*(c + d*x)]*(a + b*Sec[c + d*x]
)*(-4*a*b + 4*a*b*Sec[c + d*x]^2 - 4*a*b*EllipticE[ArcSin[Sqrt[Sec[c + d*x
]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] - 2*a*(a - 2*b)*Ellip
ticF[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d
*x]^2] + 2*a^2*EllipticPi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec
[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] - 4*b^2*EllipticPi[-(b/a), ArcSin[Sqrt
[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2])*Sin[c +
d*x])/(a^2*b*(b + a*Cos[c + d*x])*(1 - Cos[c + d*x]^2)*Sqrt[Sec[c + d*x]]*
(2 - Sec[c + d*x]^2)))/(16*a^2*(a - b)^2*(a + b)^2*d) + (Sqrt[Sec[c + d*x]
]*(-1/4*(b^2*(-13*a^2 + 7*b^2)*Sin[c + d*x])/(a^3*(-a^2 + b^2)^2) + (b^4*S
in[c + d*x])/(2*a^3*(a^2 - b^2)*(b + a*Cos[c + d*x])^2) + (3*(-5*a^2*b^3*S
in[c + d*x] + 3*b^5*Sin[c + d*x]))/(4*a^3*(a^2 - b^2)^2*(b + a*Cos[c + d*x
]))))/d
```

3.626.3 Rubi [A] (verified)

Time = 2.28 (sec) , antiderivative size = 342, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.783$, Rules used = {3042, 4334, 27, 3042, 4588, 27, 3042, 4594, 3042, 4274, 3042, 4258, 3042, 3119, 3120, 4336, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \text{4334}$$

$$\frac{b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{2ad\left(a^2-b^2\right)(a+b\sec(c+dx))^2}-\frac{\int -\frac{4a^2-4b\sec(c+dx)a-5b^2+3b^2\sec^2(c+dx)}{2\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^2}dx}{2a\left(a^2-b^2\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{4a^2-4b\sec(c+dx)a-5b^2+3b^2\sec^2(c+dx)}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^2}dx}{4a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{2ad\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{4a^2-4b\csc\left(c+dx+\frac{\pi}{2}\right)a-5b^2+3b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx}{4a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{2ad\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$$\Big\downarrow \text{4588}$$

$$\frac{\frac{b^2\left(11a^2-5b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad(a^2-b^2)(a+b\sec(c+dx))}-\frac{\int -\frac{8a^4-29b^2a^2-4b\left(4a^2-b^2\right)\sec(c+dx)a+15b^4+b^2\left(11a^2-5b^2\right)\sec^2(c+dx)}{2\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}dx}{a(a^2-b^2)}}{4a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{2ad\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int \frac{8a^4-29b^2a^2-4b\left(4a^2-b^2\right)\sec(c+dx)a+15b^4+b^2\left(11a^2-5b^2\right)\sec^2(c+dx)}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}dx}{2a(a^2-b^2)}+\frac{b^2(11a^2-5b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad(a^2-b^2)(a+b\sec(c+dx))}}{4a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{2ad\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$\downarrow$ 3042

$$\frac{\frac{\int \frac{8a^4-29b^2a^2-4b\left(4a^2-b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)a+15b^4+b^2\left(11a^2-5b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{2a(a^2-b^2)}+\frac{b^2(11a^2-5b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad(a^2-b^2)(a+b\sec(c+dx))}}{4a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{2ad\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$\downarrow$ 4594

$$\frac{\frac{\frac{b^2\left(35a^4-38a^2b^2+15b^4\right)\int \frac{\sec^{\frac{3}{2}}(c+dx)}{a+b\sec(c+dx)}dx}{a^2}+\frac{\int \frac{a\left(8a^4-29b^2a^2+15b^4\right)-3b\left(8a^4-11b^2a^2+5b^4\right)\sec(c+dx)}{\sqrt{\sec(c+dx)}}dx}{a^2}}{2a(a^2-b^2)}+\frac{b^2(11a^2-5b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad(a^2-b^2)(a+b\sec(c+dx))}}{4a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{2ad\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$\downarrow$ 3042

$$\frac{\frac{\frac{b^2\left(35a^4-38a^2b^2+15b^4\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}+\frac{\int \frac{a\left(8a^4-29b^2a^2+15b^4\right)-3b\left(8a^4-11b^2a^2+5b^4\right)\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2}}{2a(a^2-b^2)}+\frac{b^2(11a^2-5b^2)\sin(c+dx)\sqrt{\sec(c+}}{ad(a^2-b^2)(a+b\sec(c+dx))}}{4a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{2ad\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$\downarrow$ 4274

$$\frac{\frac{\frac{b^2\left(35a^4-38a^2b^2+15b^4\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}+\frac{a\left(8a^4-29a^2b^2+15b^4\right)\int \frac{1}{\sqrt{\sec(c+dx)}}dx-3b\left(8a^4-11a^2b^2+5b^4\right)\int \sqrt{\sec(c+dx)}dx}{a^2}}{2a(a^2-b^2)}+\frac{b^2(11a^2-5b^2)\sin(c+}{ad(a^2-b^2)(a+b}}{4a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{2ad\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$\downarrow$ 3042

3.626. $\int \frac{1}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^3}\,dx$

$$\frac{\frac{\frac{b^2\left(35a^4-38a^2b^2+15b^4\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}+\frac{a\left(8a^4-29a^2b^2+15b^4\right)\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-3b\left(8a^4-11a^2b^2+5b^4\right)\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}}{2a(a^2-b^2)}+\frac{b^2(11a^2-5b^2}{ad(a^2-}}{4a\left(a^2-b^2\right)}$$

$$\frac{b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{2ad\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$\downarrow$ 4258

$$\frac{\frac{\frac{b^2\left(35a^4-38a^2b^2+15b^4\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}+\frac{a\left(8a^4-29a^2b^2+15b^4\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\cos(c+dx)}dx-3b\left(8a^4-11a^2b^2+5b^4\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+d}}{a^2}}{2a(a^2-b^2)}}{4a\left(a^2-b^2\right)}$$

$$\frac{b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{2ad\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$\downarrow$ 3042

$$\frac{\frac{\frac{b^2\left(35a^4-38a^2b^2+15b^4\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}+\frac{a\left(8a^4-29a^2b^2+15b^4\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx-3b\left(8a^4-11a^2b^2+5b^4\right)\sqrt{\cos(c+dx)}\sqrt{\sec(}}{a^2}}{2a(a^2-b^2)}}{4a\left(a^2-b^2\right)}$$

$$\frac{b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{2ad\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$\downarrow$ 3119

$$\frac{\frac{\frac{b^2\left(35a^4-38a^2b^2+15b^4\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}+\frac{\frac{2a\left(8a^4-29a^2b^2+15b^4\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-3b\left(8a^4-11a^2b^2+5b^4\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+}}{a^2}}{2a(a^2-b^2)}}{4a\left(a^2-b^2\right)}$$

$$\frac{b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{2ad\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$\downarrow$ 3120

$$\frac{\frac{\frac{b^2\left(35a^4-38a^2b^2+15b^4\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}+\frac{\frac{2a\left(8a^4-29a^2b^2+15b^4\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-\frac{6b\left(8a^4-11a^2b^2+5b^4\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+}}{d}}{a^2}}{2a(a^2-b^2)}}{4a\left(a^2-b^2\right)}$$

$$\frac{b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{2ad\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$\downarrow$ 4336

3.626. $\int \frac{1}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^3}\,dx$

$$\frac{\frac{\frac{b^2\left(35a^4-38a^2b^2+15b^4\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}(b+a\cos(c+dx))}dx}{a^2}+\frac{\frac{2a\left(8a^4-29a^2b^2+15b^4\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-\frac{6b\left(8a^4-1\right.}{a^2}}{2a(a^2-b^2)}}{4a\left(a^2-b^2\right)}$$

$$\frac{b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{2ad\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\frac{b^2\left(35a^4-38a^2b^2+15b^4\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(b+a\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2}+\frac{\frac{2a\left(8a^4-29a^2b^2+15b^4\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-}{2a(a^2-b^2)}}{4a\left(a^2-b^2\right)}$$

$$\frac{b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{2ad\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

$\Big\downarrow$ 3284

$$\frac{b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{2ad\left(a^2-b^2\right)(a+b\sec(c+dx))^2}+$$

$$\frac{\frac{b^2\left(11a^2-5b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad(a^2-b^2)(a+b\sec(c+dx))}+\frac{\frac{2b^2\left(35a^4-38a^2b^2+15b^4\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\operatorname{EllipticPi}\left(\frac{2a}{a+b},\frac{1}{2}(c+dx),2\right)}{a^2d(a+b)}+\frac{\frac{2a\left(8a^4-29a^2b^2+15b^4\right)\sqrt{\cos(c+d}}{d}}{2a(a^2-b^2)}}{4a\left(a^2-b^2\right)}$$

input
```
Int[1/(Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^3),x]
```

output
```
(b^2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(2*a*(a^2 - b^2)*d*(a + b*Sec[c + d*
x])^2) + ((((2*a*(8*a^4 - 29*a^2*b^2 + 15*b^4)*Sqrt[Cos[c + d*x]]*Elliptic
E[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d - (6*b*(8*a^4 - 11*a^2*b^2 + 5*b^4
)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d)/a^2
+ (2*b^2*(35*a^4 - 38*a^2*b^2 + 15*b^4)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*a
)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a^2*(a + b)*d))/(2*a*(a^2
- b^2)) + (b^2*(11*a^2 - 5*b^2)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(a*(a^2 -
 b^2)*d*(a + b*Sec[c + d*x])))/(4*a*(a^2 - b^2))
```

3.626. $\int \frac{1}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^3}\,dx$

3.626.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4334
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*
((d*Csc[e + f*x])^n/(a*f*(m + 1)*(a^2 - b^2))), x] + Simp[1/(a*(m + 1)*(a^2
 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a^2*(m + 1)
 - b^2*(m + n + 1) - a*b*(m + 1)*Csc[e + f*x] + b^2*(m + n + 2)*Csc[e + f*x
]^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0] && LtQ[m,
 -1] && IntegersQ[2*m, 2*n]
```

3.626. $\int \frac{1}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^3}\,dx$

rule 4336
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]]   Int[
1/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4588
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc
[e + f*x])^(m + 1)*((d*Csc[e + f*x])^n/(a*f*(m + 1)*(a^2 - b^2))), x] + Sim
p[1/(a*(m + 1)*(a^2 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f
*x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n +
1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m
+ n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x
] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

rule 4594
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Simp[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2)   Int[(d*Csc[e +
f*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Simp[1/a^2   Int[(a*A - (A*b - a
*B)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A,
B, C}, x] && NeQ[a^2 - b^2, 0]
```

3.626.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1956 vs. $2(394) = 788$.

Time = 29.91 (sec) , antiderivative size = 1957, normalized size of antiderivative = 5.72

method	result	size
default	Expression too large to display	1957

input
```
int(1/(a+b*sec(d*x+c))^3/sec(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2/a^4/(-2*sin
(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(
1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(3*b*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2
))+a*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))+2/a^4*b^4*(1/2*a^2/b/(a^2-b^2)
*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(
2*a*cos(1/2*d*x+1/2*c)^2-a+b)^2+3/4*a^2*(a^2-3*b^2)/b^2/(a^2-b^2)^2*cos(1/
2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*a*cos
(1/2*d*x+1/2*c)^2-a+b)-3/8/(a+b)/(a^2-b^2)/b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1
/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^2-1/4/(a+b)/(a^2-b^
2)/b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c
),2^(1/2))*a+7/8/(a+b)/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*
d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)
*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3/8*a^3/b^2/(a^2-b^2)^2*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)
^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-9/8*a
/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2
)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d
*x+1/2*c),2^(1/2))-3/8*a^3/b^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)...
```

3.626.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+b*sec(d*x+c))^3/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.626.6 Sympy [F]

$$\int \frac{1}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^3}\,dx = \int \frac{1}{(a+b\sec(c+dx))^3\sqrt{\sec(c+dx)}}\,dx$$

input
```
integrate(1/(a+b*sec(d*x+c))**3/sec(d*x+c)**(1/2),x)
```

output
```
Integral(1/((a + b*sec(c + d*x))**3*sqrt(sec(c + d*x))), x)
```

3.626.7 Maxima [F]

$$\int \frac{1}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^3}\,dx = \int \frac{1}{(b\sec(dx+c)+a)^3\sqrt{\sec(dx+c)}}\,dx$$

input
```
integrate(1/(a+b*sec(d*x+c))^3/sec(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*sec(d*x + c) + a)^3*sqrt(sec(d*x + c))), x)
```

3.626.8 Giac [F]

$$\int \frac{1}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^3}\,dx = \int \frac{1}{(b\sec(dx+c)+a)^3\sqrt{\sec(dx+c)}}\,dx$$

input
```
integrate(1/(a+b*sec(d*x+c))^3/sec(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/((b*sec(d*x + c) + a)^3*sqrt(sec(d*x + c))), x)
```

3.626.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^3}\,dx=\int \frac{1}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^3\,\sqrt{\frac{1}{\cos(c+d\,x)}}}\,dx$$

input
```
int(1/((a + b/cos(c + d*x))^3*(1/cos(c + d*x))^(1/2)),x)
```

output
```
int(1/((a + b/cos(c + d*x))^3*(1/cos(c + d*x))^(1/2)), x)
```

3.627 $\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^3}\,dx$

3.627.1 Optimal result 4800
3.627.2 Mathematica [A] (warning: unable to verify) 4801
3.627.3 Rubi [A] (verified) 4802
3.627.4 Maple [B] (verified) 4809
3.627.5 Fricas [F] 4810
3.627.6 Sympy [F] 4810
3.627.7 Maxima [F(-2)] 4810
3.627.8 Giac [F] 4811
3.627.9 Mupad [F(-1)] 4811

3.627.1 Optimal result

Integrand size = 23, antiderivative size = 406

$$\begin{aligned}
&\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^3}\,dx \\
&= -\frac{b(24a^4-65a^2b^2+35b^4)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{4a^4\left(a^2-b^2\right)^2 d} \\
&\quad + \frac{(8a^6+128a^4b^2-223a^2b^4+105b^6)\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{12a^5\left(a^2-b^2\right)^2 d} \\
&\quad - \frac{b^3(63a^4-86a^2b^2+35b^4)\sqrt{\cos(c+dx)}\,\text{EllipticPi}\left(\frac{2a}{a+b},\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{4a^5(a-b)^2(a+b)^3 d} \\
&\quad + \frac{(8a^4-61a^2b^2+35b^4)\sin(c+dx)}{12a^3\left(a^2-b^2\right)^2 d\sqrt{\sec(c+dx)}} + \frac{b^2\sin(c+dx)}{2a\left(a^2-b^2\right)d\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^2} \\
&\quad + \frac{b^2(13a^2-7b^2)\sin(c+dx)}{4a^2\left(a^2-b^2\right)^2 d\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}
\end{aligned}$$

output
```
1/12*(8*a^4-61*a^2*b^2+35*b^4)*sin(d*x+c)/a^3/(a^2-b^2)^2/d/sec(d*x+c)^(1/
2)+1/2*b^2*sin(d*x+c)/a/(a^2-b^2)/d/(a+b*sec(d*x+c))^2/sec(d*x+c)^(1/2)+1/
4*b^2*(13*a^2-7*b^2)*sin(d*x+c)/a^2/(a^2-b^2)^2/d/(a+b*sec(d*x+c))/sec(d*x
+c)^(1/2)-1/4*b*(24*a^4-65*a^2*b^2+35*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/co
s(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*se
c(d*x+c)^(1/2)/a^4/(a^2-b^2)^2/d+1/12*(8*a^6+128*a^4*b^2-223*a^2*b^4+105*b
^6)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+
1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^5/(a^2-b^2)^2/d-1/4*b^
3*(63*a^4-86*a^2*b^2+35*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*
c)*EllipticPi(sin(1/2*d*x+1/2*c),2*a/(a+b),2^(1/2))*cos(d*x+c)^(1/2)*sec(d
*x+c)^(1/2)/a^5/(a-b)^2/(a+b)^3/d
```

3.627.2 Mathematica [A] (warning: unable to verify)

Time = 6.86 (sec) , antiderivative size = 731, normalized size of antiderivative = 1.80

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^3}\,dx$$
$$= \frac{\frac{2(-56a^4b+73a^2b^3-35b^5)\cos^2(c+dx)\left(\text{EllipticF}\left(\arcsin\left(\sqrt{\sec(c+dx)}\right),-1\right)-\text{EllipticPi}\left(-\frac{b}{a},\arcsin\left(\sqrt{\sec(c+dx)}\right),-1\right)\right)(a+b\sec(c+dx))\sqrt{1-s}}{b(b+a\cos(c+dx))(1-\cos^2(c+dx))}}{}$$
$$+\frac{\sqrt{\sec(c+dx)}\left(\frac{b^3(-17a^2+11b^2)\sin(c+dx)}{4a^4(-a^2+b^2)^2}-\frac{b^5\sin(c+dx)}{2a^4(a^2-b^2)(b+a\cos(c+dx))^2}+\frac{19a^2b^4\sin(c+dx)-13b^6\sin(c+dx)}{4a^4(a^2-b^2)^2(b+a\cos(c+dx))}+\frac{\sin(2(c+dx))}{3a^3}\right)}{d}$$

input
```
Integrate[1/(Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^3),x]
```

output
```
((2*(-56*a^4*b + 73*a^2*b^3 - 35*b^5)*Cos[c + d*x]^2*(EllipticF[ArcSin[Sqr
t[Sec[c + d*x]]], -1] - EllipticPi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]], -1]
)*(a + b*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x])/(b*(b + a*Co
s[c + d*x])*(1 - Cos[c + d*x]^2)) + (2*(16*a^5 + 112*a^3*b^2 - 56*a*b^4)*C
os[c + d*x]^2*EllipticPi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]], -1]*(a + b*Se
c[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x])/(a*(b + a*Cos[c + d*x])
*(1 - Cos[c + d*x]^2)) + ((-72*a^4*b + 195*a^2*b^3 - 105*b^5)*Cos[2*(c + d
*x)]*(a + b*Sec[c + d*x])*(-4*a*b + 4*a*b*Sec[c + d*x]^2 - 4*a*b*EllipticE
[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^
2] - 2*a*(a - 2*b)*EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c +
d*x]]*Sqrt[1 - Sec[c + d*x]^2] + 2*a^2*EllipticPi[-(b/a), ArcSin[Sqrt[Sec[
c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] - 4*b^2*Ellipt
icPi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - S
ec[c + d*x]^2])*Sin[c + d*x])/(a^2*b*(b + a*Cos[c + d*x])*(1 - Cos[c + d*x
]^2)*Sqrt[Sec[c + d*x]]*(2 - Sec[c + d*x]^2)))/(48*a^3*(a - b)^2*(a + b)^2
*d) + (Sqrt[Sec[c + d*x]]*((b^3*(-17*a^2 + 11*b^2)*Sin[c + d*x])/(4*a^4*(-
a^2 + b^2)^2) - (b^5*Sin[c + d*x])/(2*a^4*(a^2 - b^2)*(b + a*Cos[c + d*x])
^2) + (19*a^2*b^4*Sin[c + d*x] - 13*b^6*Sin[c + d*x])/(4*a^4*(a^2 - b^2)^2
*(b + a*Cos[c + d*x])) + Sin[2*(c + d*x)]/(3*a^3)))/d
```

3.627.3 Rubi [A] (verified)

Time = 2.91 (sec) , antiderivative size = 403, normalized size of antiderivative = 0.99, number of steps used = 21, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.913$, Rules used = {3042, 4334, 27, 3042, 4588, 27, 3042, 4592, 27, 3042, 4594, 3042, 4274, 3042, 4258, 3042, 3119, 3120, 4336, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \text{4334}$$

$$\frac{b^2\sin(c+dx)}{2ad\left(a^2-b^2\right)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^2}-\frac{\int -\frac{4a^2-4b\sec(c+dx)a-7b^2+5b^2\sec^2(c+dx)}{2\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2}dx}{2a\left(a^2-b^2\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{4a^2-4b\sec(c+dx)a-7b^2+5b^2\sec^2(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2}dx}{4a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)}{2ad\left(a^2-b^2\right)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{\int \frac{4a^2-4b\csc\left(c+dx+\frac{\pi}{2}\right)a-7b^2+5b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx}{4a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)}{2ad\left(a^2-b^2\right)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^2}$$

$$\Big\downarrow \; 4588$$

$$\frac{\frac{b^2\left(13a^2-7b^2\right)\sin(c+dx)}{ad(a^2-b^2)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}-\frac{\int -\frac{8a^4-61b^2a^2-4b\left(4a^2-b^2\right)\sec(c+dx)a+35b^4+3b^2\left(13a^2-7b^2\right)\sec^2(c+dx)}{2\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))}dx}{a(a^2-b^2)}}{4a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)}{2ad\left(a^2-b^2\right)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^2}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\int \frac{8a^4-61b^2a^2-4b\left(4a^2-b^2\right)\sec(c+dx)a+35b^4+3b^2\left(13a^2-7b^2\right)\sec^2(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))}dx}{2a(a^2-b^2)}+\frac{b^2\left(13a^2-7b^2\right)\sin(c+dx)}{ad(a^2-b^2)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}}{4a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)}{2ad\left(a^2-b^2\right)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{\int \frac{8a^4-61b^2a^2-4b\left(4a^2-b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)a+35b^4+3b^2\left(13a^2-7b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{2a(a^2-b^2)}+\frac{b^2\left(13a^2-7b^2\right)\sin(c+dx)}{ad(a^2-b^2)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}}{4a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)}{2ad\left(a^2-b^2\right)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^2}$$

$$\Big\downarrow \; 4592$$

$$\frac{\frac{\frac{2\left(8a^4-61a^2b^2+35b^4\right)\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{2\int \frac{-b\left(8a^4-61b^2a^2+35b^4\right)\sec^2(c+dx)-4a\left(2a^4+14b^2a^2-7b^4\right)\sec(c+dx)+3b\left(24a^4-65b^2a^2+35b^4\right)}{2\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}dx}{3a}}{2a(a^2-b^2)}+\frac{b^2(13a^2}{ad(a^2-b^2)\sqrt{\sec}}}{4a\left(a^2-b^2\right)}$$

$$\frac{b^2\sin(c+dx)}{2ad\left(a^2-b^2\right)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^2}$$

$$\Big\downarrow \; 27$$

3.627. $\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^3}\,dx$

$$\frac{\frac{2\left(8a^4-61a^2b^2+35b^4\right)\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{\int\frac{-b\left(8a^4-61b^2a^2+35b^4\right)\sec^2(c+dx)-4a\left(2a^4+14b^2a^2-7b^4\right)\sec(c+dx)+3b\left(24a^4-65b^2a^2+35b^4\right)}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}dx}{3a}}{2a(a^2-b^2)}+\frac{b^2(13a^2-}{ad(a^2-b^2)\sqrt{\sec(}}$$
$$4a\left(a^2-b^2\right)$$
$$\frac{b^2\sin(c+dx)}{2ad\left(a^2-b^2\right)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^2}$$

$\downarrow$ 3042

$$\frac{\frac{2\left(8a^4-61a^2b^2+35b^4\right)\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{\int\frac{-b\left(8a^4-61b^2a^2+35b^4\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2-4a\left(2a^4+14b^2a^2-7b^4\right)\csc\left(c+dx+\frac{\pi}{2}\right)+3b\left(24a^4-65b^2a^2+35b^4\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{3a}}{2a(a^2-b^2)}+\frac{b^2}{ad(a^2-b^2}$$
$$4a\left(a^2-b^2\right)$$
$$\frac{b^2\sin(c+dx)}{2ad\left(a^2-b^2\right)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^2}$$

$\downarrow$ 4594

$$\frac{\frac{2\left(8a^4-61a^2b^2+35b^4\right)\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{3b^3\left(63a^4-86a^2b^2+35b^4\right)\int\frac{\sec^{\frac{3}{2}}(c+dx)}{a+b\sec(c+dx)}dx}{a^2}+\frac{\int\frac{3ab\left(24a^4-65b^2a^2+35b^4\right)-\left(8a^6+128b^2a^4-223b^4a^2+105b^6\right)\sec(c+dx)}{\sqrt{\sec(c+dx)}}dx}{a^2}}{3a}}{2a(a^2-b^2)}$$
$$4a\left(a^2-b^2\right)$$
$$\frac{b^2\sin(c+dx)}{2ad\left(a^2-b^2\right)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^2}$$

$\downarrow$ 3042

$$\frac{\frac{2\left(8a^4-61a^2b^2+35b^4\right)\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{3b^3\left(63a^4-86a^2b^2+35b^4\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}+\frac{\int\frac{3ab\left(24a^4-65b^2a^2+35b^4\right)+\left(-8a^6-128b^2a^4+223b^4a^2-105b^6\right)\csc(c+}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}}{a^2}}{3a}}{2a(a^2-b^2)}$$
$$4a\left(a^2-b^2\right)$$
$$\frac{b^2\sin(c+dx)}{2ad\left(a^2-b^2\right)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^2}$$

$\downarrow$ 4274

$$\frac{\frac{2\left(8a^4-61a^2b^2+35b^4\right)\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{3b^3\left(63a^4-86a^2b^2+35b^4\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}+\frac{3ab\left(24a^4-65a^2b^2+35b^4\right)\int\frac{1}{\sqrt{\sec(c+dx)}}dx-\left(8a^6+128a^4b^2-223a^2b^4+1\right.}{a^2}}{3a}}{2a(a^2-b^2)}$$
$$4a\left(a^2-b^2\right)$$
$$\frac{b^2\sin(c+dx)}{2ad\left(a^2-b^2\right)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^2}$$

3.627. $\int\frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^3}dx$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{\frac{2\left(8a^4-61a^2b^2+35b^4\right)\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{3b^3\left(63a^4-86a^2b^2+35b^4\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}+\frac{3ab\left(24a^4-65a^2b^2+35b^4\right)\int\frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-\left(8a^6+128a^4b^2-223a^2b\right.}{a^2}}{3a}}{2a(a^2-b^2)}}{4a\left(a^2-b^2\right)}$$

$$\frac{b^2\sin(c+dx)}{2ad\left(a^2-b^2\right)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^2}$$

$$\Big\downarrow \; 4258$$

$$\frac{\frac{\frac{2\left(8a^4-61a^2b^2+35b^4\right)\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{3b^3\left(63a^4-86a^2b^2+35b^4\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}+\frac{3ab\left(24a^4-65a^2b^2+35b^4\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\cos(c+dx)}dx-\left(8a^6\right.}{a^2}}{3a}}{2a(a^2-b^2)}}{4a\left(a^2-b^2\right)}$$

$$\frac{b^2\sin(c+dx)}{2ad\left(a^2-b^2\right)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{\frac{2\left(8a^4-61a^2b^2+35b^4\right)\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{3b^3\left(63a^4-86a^2b^2+35b^4\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}+\frac{3ab\left(24a^4-65a^2b^2+35b^4\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx-}{}}{3a}}{2a(a^2-b^2)}}{4a\left(a^2-b^2\right)}$$

$$\frac{b^2\sin(c+dx)}{2ad\left(a^2-b^2\right)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^2}$$

$$\Big\downarrow \; 3119$$

$$\frac{\frac{\frac{2\left(8a^4-61a^2b^2+35b^4\right)\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{3b^3\left(63a^4-86a^2b^2+35b^4\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}+\frac{\frac{6ab\left(24a^4-65a^2b^2+35b^4\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-\left(8a^6\right.}{}}{3a}}{2a(a^2-b^2)}}{4a\left(a^2-b^2\right)}$$

$$\frac{b^2\sin(c+dx)}{2ad\left(a^2-b^2\right)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^2}$$

$$\Big\downarrow \; 3120$$

3.627. $\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^3}\,dx$

$$\frac{\frac{\frac{2\left(8a^4-61a^2b^2+35b^4\right)\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{3b^3\left(63a^4-86a^2b^2+35b^4\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}+\frac{\frac{6ab\left(24a^4-65a^2b^2+35b^4\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-\frac{2(8a}{}}{3a}}{2a(a^2-b^2)}}{4a\left(a^2-b^2\right)}$$

$$\frac{b^2\sin(c+dx)}{2ad\left(a^2-b^2\right)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^2}$$

$$\Big\downarrow \; 4336$$

$$\frac{\frac{\frac{2\left(8a^4-61a^2b^2+35b^4\right)\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{3b^3\left(63a^4-86a^2b^2+35b^4\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}(b+a\cos(c+dx))}dx}{a^2}+\frac{\frac{6ab\left(24a^4-65a^2b^2+35b^4\right)\sqrt{\cos(c+dx)}\sqrt{}}{d}}{3a}}{2a(a^2-b^2)}}{4a\left(a^2\right.}$$

$$\frac{b^2\sin(c+dx)}{2ad\left(a^2-b^2\right)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{\frac{2\left(8a^4-61a^2b^2+35b^4\right)\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{3b^3\left(63a^4-86a^2b^2+35b^4\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(b+a\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2}+\frac{\frac{6ab\left(24a^4-65a^2b^2+35b^4\right)\sqrt{\mathrm{co}}}{}}{3a}}{2a(a^2-b^2)}}{4a\,(}$$

$$\frac{b^2\sin(c+dx)}{2ad\left(a^2-b^2\right)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^2}$$

$$\Big\downarrow \; 3284$$

$$\frac{b^2\sin(c+dx)}{2ad\left(a^2-b^2\right)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^2}+$$

$$\frac{\frac{b^2\left(13a^2-7b^2\right)\sin(c+dx)}{ad(a^2-b^2)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}+\frac{\frac{2\left(8a^4-61a^2b^2+35b^4\right)\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{6b^3\left(63a^4-86a^2b^2+35b^4\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\mathrm{EllipticPi}\left(\frac{2a}{a+b},\frac{1}{2}(c+dx),2\right.}{a^2d(a+b)}}{}}{}}{4a\,(a^2-}$$

input
```
Int[1/(Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^3),x]
```

3.627. $\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^3}\,dx$

output
```
(b^2*Sin[c + d*x])/(2*a*(a^2 - b^2)*d*Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*
x])^2) + ((b^2*(13*a^2 - 7*b^2)*Sin[c + d*x])/(a*(a^2 - b^2)*d*Sqrt[Sec[c
+ d*x]]*(a + b*Sec[c + d*x])) + (-1/3*(((6*a*b*(24*a^4 - 65*a^2*b^2 + 35*b
^4)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d - (
2*(8*a^6 + 128*a^4*b^2 - 223*a^2*b^4 + 105*b^6)*Sqrt[Cos[c + d*x]]*Ellipti
cF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d)/a^2 + (6*b^3*(63*a^4 - 86*a^2*b^
2 + 35*b^4)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2]*S
qrt[Sec[c + d*x]])/(a^2*(a + b)*d))/a + (2*(8*a^4 - 61*a^2*b^2 + 35*b^4)*S
in[c + d*x])/(3*a*d*Sqrt[Sec[c + d*x]]))/(2*a*(a^2 - b^2)))/(4*a*(a^2 - b^
2))
```

3.627.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

3.627. $\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^3}\,dx$

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4334
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*
((d*Csc[e + f*x])^n/(a*f*(m + 1)*(a^2 - b^2))), x] + Simp[1/(a*(m + 1)*(a^2
 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a^2*(m + 1)
 - b^2*(m + n + 1) - a*b*(m + 1)*Csc[e + f*x] + b^2*(m + n + 2)*Csc[e + f*x
]^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0] && LtQ[m,
 -1] && IntegersQ[2*m, 2*n]
```

rule 4336
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]]   Int[
1/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4588
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc
[e + f*x])^(m + 1)*((d*Csc[e + f*x])^n/(a*f*(m + 1)*(a^2 - b^2))), x] + Sim
p[1/(a*(m + 1)*(a^2 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f
*x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n +
1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m
+ n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x
] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

rule 4592
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Simp[1/(a*d*n)   Int[(a + b*Csc[e + f*x])^m
*(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*
Csc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

3.627. $\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^3}\,dx$

rule 4594
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Simp[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2)   Int[(d*Csc[e +
f*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Simp[1/a^2   Int[(a*A - (A*b - a
*B)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A,
B, C}, x] && NeQ[a^2 - b^2, 0]
```

3.627.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2215 vs. $2(454) = 908$.

Time = 31.01 (sec) , antiderivative size = 2216, normalized size of antiderivative = 5.46

method	result	size
default	Expression too large to display	2216

input
```
int(1/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
-(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4/3/a^3*(2*sin
(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)
^2+2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipti
cF(cos(1/2*d*x+1/2*c),2^(1/2))-3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(co
s(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2))/(-2*sin(1/2*d*
x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)-2/a^4*(2*a+3*b)*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1
/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-EllipticE(co
s(1/2*d*x+1/2*c),2^(1/2)))+2*(a^2+3*a*b+6*b^2)/a^5*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d
*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-2/a^5*b^5*(1/2*a^
2/b/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*
c)^2)^(1/2)/(2*a*cos(1/2*d*x+1/2*c)^2-a+b)^2+3/4*a^2*(a^2-3*b^2)/b^2/(a^2-
b^2)^2*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(
1/2)/(2*a*cos(1/2*d*x+1/2*c)^2-a+b)-3/8/(a+b)/(a^2-b^2)/b^2*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+
sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^2-1/4/
(a+b)/(a^2-b^2)/b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)
^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(
1/2*d*x+1/2*c),2^(1/2))*a+7/8/(a+b)/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1...
```

3.627.5 Fricas [F]

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^3}\,dx = \int \frac{1}{(b\sec(dx+c)+a)^3\sec(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
integral(sqrt(sec(d*x + c))/(b^3*sec(d*x + c)^5 + 3*a*b^2*sec(d*x + c)^4 +
 3*a^2*b*sec(d*x + c)^3 + a^3*sec(d*x + c)^2), x)
```

3.627.6 Sympy [F]

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^3}\,dx = \int \frac{1}{(a+b\sec(c+dx))^3\sec^{\frac{3}{2}}(c+dx)}\,dx$$

input
```
integrate(1/sec(d*x+c)**(3/2)/(a+b*sec(d*x+c))**3,x)
```

output
```
Integral(1/((a + b*sec(c + d*x))**3*sec(c + d*x)**(3/2)), x)
```

3.627.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^3}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

3.627.8 Giac [F]

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^3}\,dx = \int \frac{1}{(b\sec(dx+c)+a)^3\sec(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate(1/((b*sec(d*x + c) + a)^3*sec(d*x + c)^(3/2)), x)
```

3.627.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^3}\,dx = \int \frac{1}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^3\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

input
```
int(1/((a + b/cos(c + d*x))^3*(1/cos(c + d*x))^(3/2)),x)
```

output
```
int(1/((a + b/cos(c + d*x))^3*(1/cos(c + d*x))^(3/2)), x)
```

3.628 $\int \sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$

3.628.1 Optimal result 4812
3.628.2 Mathematica [C] (verified) 4813
3.628.3 Rubi [A] (verified) 4813
3.628.4 Maple [C] (verified) 4820
3.628.5 Fricas [F(-1)] 4821
3.628.6 Sympy [F] 4822
3.628.7 Maxima [F] 4822
3.628.8 Giac [F] 4822
3.628.9 Mupad [F(-1)] 4823

3.628.1 Optimal result

Integrand size = 25, antiderivative size = 237

$$\begin{aligned}\int \sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx &= \frac{b\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{d\sqrt{a+b\sec(c+dx)}} \\ &\quad + \frac{a\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{d\sqrt{a+b\sec(c+dx)}} \\ &\quad - \frac{E\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+dx)}}{d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\sqrt{\sec(c+dx)}} + \frac{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{d}\end{aligned}$$

output
```
b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/
2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)*sec(d*x+c)^(1
/2)/d/(a+b*sec(d*x+c))^(1/2)+a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/
2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2,2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*
x+c))/(a+b))^(1/2)*sec(d*x+c)^(1/2)/d/(a+b*sec(d*x+c))^(1/2)-(cos(1/2*d*x+
1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a
/(a+b))^(1/2))*(a+b*sec(d*x+c))^(1/2)/d/((b+a*cos(d*x+c))/(a+b))^(1/2)/sec
(d*x+c)^(1/2)+sin(d*x+c)*sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2)/d
```

3.628.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 25.12 (sec) , antiderivative size = 321, normalized size of antiderivative = 1.35

$$\int \sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$$
$$= \frac{\sqrt{a+b\sec(c+dx)}\left(\frac{2a\,\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{(a+b)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}} - \frac{2i\sqrt{-\frac{a(-1+\cos(c+dx))}{a+b}}\sqrt{\frac{a(1+\cos(c+dx))}{a-b}}\csc(c+dx)\left(-2b(a+b)E\left(i\text{arcsinh}\left(\sqrt{\right.\right.\right.}{}\right.}{}$$

input
```
Integrate[Sec[c + d*x]^(3/2)*Sqrt[a + b*Sec[c + d*x]],x]
```

output
```
(Sqrt[a + b*Sec[c + d*x]]*((2*a*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)])
/((a + b)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]) - ((2*I)*Sqrt[-((a*(-1 + Cos
[c + d*x]))/(a + b))]*Sqrt[(a*(1 + Cos[c + d*x]))/(a - b)]*Csc[c + d*x]*(-
2*b*(a + b)*EllipticE[I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]
]], (-a + b)/(a + b)] + a*(2*b*EllipticF[I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt
[b + a*Cos[c + d*x]]], (-a + b)/(a + b)] + a*EllipticPi[1 - a/b, I*ArcSinh
[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b)])))/(a*Sqr
t[(a - b)^(-1)]*b*Sqrt[b + a*Cos[c + d*x]]) + 4*Tan[c + d*x]))/(4*d*Sqrt[S
ec[c + d*x]])
```

3.628.3 Rubi [A] (verified)

Time = 2.31 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.07, number of steps used = 23, number of rules used = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.920$, Rules used = {3042, 4342, 25, 3042, 4597, 3042, 4346, 3042, 3286, 3042, 3284, 4349, 3042, 4343, 3042, 3134, 3042, 3132, 4345, 3042, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{4342}$$

$$\frac{1}{2}\int -\frac{a-a\sec^2(c+dx)}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx+\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}$$

$$\Big\downarrow \text{25}$$

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{1}{2}\int \frac{a-a\sec^2(c+dx)}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{1}{2}\int \frac{a-a\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4597}$$

$$\frac{1}{2}\left(a\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx-a\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx\right)+\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\left(a\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-a\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)+\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}$$

$$\Big\downarrow \text{4346}$$

$$\frac{1}{2}\left(\frac{a\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int \frac{\sec(c+dx)}{\sqrt{b+a\cos(c+dx)}}dx}{\sqrt{a+b\sec(c+dx)}}-a\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\left(\frac{a\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{\sqrt{a+b\sec(c+dx)}}-a\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+\right.}}\right.$$

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}$$

$$\Big\downarrow \text{3286}$$

3.628. $\int \sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$

$$\frac{1}{2}\left(\frac{a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{\sec(c+dx)}{\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}-a\int\frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)+\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}$$

$\downarrow$ 3042

$$\frac{1}{2}\left(\frac{a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}-a\int\frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx\right.}}\right.$$

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}$$

$\downarrow$ 3284

$$\frac{1}{2}\left(\frac{2a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}-a\int\frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\right.}}\right.$$

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}$$

$\downarrow$ 4349

$$\frac{1}{2}\left(\frac{2a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}-a\left(\frac{\int\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}dx}{a}-\frac{b\int\frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}dx}{a}\right)\right.$$

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}$$

$\downarrow$ 3042

$$\frac{1}{2}\left(\frac{2a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}-a\left(\frac{\int\frac{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}-\frac{b\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right.}}{\sqrt{a+b\csc(c+dx+}}}{a}\right.\right.$$

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}$$

$\downarrow$ 4343

3.628. $\int \sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$

$$\frac{1}{2}\left(\frac{2a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}-a\left(\frac{\sqrt{a+b\sec(c+dx)}\int\sqrt{b+a\cos(c+dx)}dx}{a\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}\right.\right.$$

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}$$

$\downarrow$ 3042

$$\frac{1}{2}\left(\frac{2a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}-a\left(\frac{\sqrt{a+b\sec(c+dx)}\int\sqrt{b+a\sin\left(c+dx+\right.}}{a\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}\right.\right.$$

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}$$

$\downarrow$ 3134

$$\frac{1}{2}\left(\frac{2a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}-a\left(\frac{\sqrt{a+b\sec(c+dx)}\int\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}dx}{a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}\right.\right.$$

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}$$

$\downarrow$ 3042

$$\frac{1}{2}\left(\frac{2a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}-a\left(\frac{\sqrt{a+b\sec(c+dx)}\int\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}{a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}\right.\right.$$

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}$$

$\downarrow$ 3132

3.628. $\int \sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$

$$\frac{1}{2}\left(\frac{2a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}-a\left(\frac{2\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}\right.\right.$$

$\downarrow$ 4345

$$\frac{1}{2}\left(\frac{2a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}-a\left(\frac{2\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}\right.\right.$$

$\downarrow$ 3042

$$\frac{1}{2}\left(\frac{2a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}-a\left(\frac{2\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}\right.\right.$$

$\downarrow$ 3142

$$\frac{1}{2}\left(\frac{2a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}-a\left(\frac{2\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}\right.\right.$$

$\downarrow$ 3042

$$\frac{1}{2}\left(\frac{2a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}-a\left(\frac{2\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}\right.\right.$$

$\downarrow$ 3140

3.628. $\int \sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}+\frac{1}{2}\left(\frac{2a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\operatorname{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}-a\left(\frac{2\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\right.\right.$$

input
```
Int[Sec[c + d*x]^(3/2)*Sqrt[a + b*Sec[c + d*x]],x]
```

output
```
((2*a*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/
(a + b)]*Sqrt[Sec[c + d*x]])/(d*Sqrt[a + b*Sec[c + d*x]]) - a*((-2*b*Sqrt[
(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[S
ec[c + d*x]])/(a*d*Sqrt[a + b*Sec[c + d*x]]) + (2*EllipticE[(c + d*x)/2, (
2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(a*d*Sqrt[(b + a*Cos[c + d*x])/(a
+ b)]*Sqrt[Sec[c + d*x]])))/2 + (Sqrt[Sec[c + d*x]]*Sqrt[a + b*Sec[c + d*x
]]*Sin[c + d*x])/d
```

3.628.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3286
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]]   Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
 d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

rule 4342
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[-2*d*Cos[e + f*x]*Sqrt[a + b*Csc[e + f*x]]*((d*
Csc[e + f*x])^(n - 1)/(f*(2*n - 1))), x] + Simp[d^2/(2*n - 1)   Int[(d*Csc[
e + f*x])^(n - 2)*(Simp[2*a*(n - 2) + b*(2*n - 3)*Csc[e + f*x] + a*Csc[e +
f*x]^2, x]/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, d, e, f}, x] &
& NeQ[a^2 - b^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

rule 4343
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]])   Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a
, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4345
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]])   Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[
{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

3.628. $\int \sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$

rule 4346
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Simp[d*Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x
]]/Sqrt[a + b*Csc[e + f*x]])   Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4349
```
Int[1/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)]), x_Symbol] :> Simp[1/a   Int[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Cs
c[e + f*x]], x], x] - Simp[b/(a*d)   Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Cs
c[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4597
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))/(Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Simp[C/d^
2   Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Csc[e + f*x]], x], x] + Simp[A
Int[1/(Sqrt[d*Csc[e + f*x]]*Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a,
b, d, e, f, A, C}, x] && NeQ[a^2 - b^2, 0]
```

3.628.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 7.58 (sec) , antiderivative size = 1009, normalized size of antiderivative = 4.26

method	result	size
default	Expression too large to display	1009

input
```
int(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/d/((a-b)/(a+b))^(1/2)*sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))^(1/2)/(b+a*cos(d
*x+c))/(cos(d*x+c)+1)*((1/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/
(cos(d*x+c)+1))^(1/2)*EllipticE(((a-b)/(a+b))^(1/2)*(cot(d*x+c)-csc(d*x+c)
),(-(a+b)/(a-b))^(1/2))*a*cos(d*x+c)^4-(1/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(
b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE(((a-b)/(a+b))^(1/2)*(cot(d
*x+c)-csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*b*cos(d*x+c)^4-2*(1/(cos(d*x+c)+1)
)^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi(((a-b)/
(a+b))^(1/2)*(cot(d*x+c)-csc(d*x+c)),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*a*
cos(d*x+c)^4+2*(1/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x
+c)+1))^(1/2)*EllipticE(((a-b)/(a+b))^(1/2)*(cot(d*x+c)-csc(d*x+c)),(-(a+b
)/(a-b))^(1/2))*a*cos(d*x+c)^3-2*(1/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*co
s(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE(((a-b)/(a+b))^(1/2)*(cot(d*x+c)-
csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*b*cos(d*x+c)^3-4*(1/(cos(d*x+c)+1))^(1/2
)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi(((a-b)/(a+b))
^(1/2)*(cot(d*x+c)-csc(d*x+c)),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*a*cos(d*
x+c)^3+(1/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^
(1/2)*EllipticE(((a-b)/(a+b))^(1/2)*(cot(d*x+c)-csc(d*x+c)),(-(a+b)/(a-b))
^(1/2))*a*cos(d*x+c)^2-(1/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/
(cos(d*x+c)+1))^(1/2)*EllipticE(((a-b)/(a+b))^(1/2)*(cot(d*x+c)-csc(d*x+c)
),(-(a+b)/(a-b))^(1/2))*b*cos(d*x+c)^2-2*(1/(cos(d*x+c)+1))^(1/2)*(1/(a...
```

3.628.5 Fricas [F(-1)]

Timed out.

$$\int \sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.628.6 Sympy [F]

$$\int \sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = \int \sqrt{a+b\sec(c+dx)}\sec^{\frac{3}{2}}(c+dx)\,dx$$

input
```
integrate(sec(d*x+c)**(3/2)*(a+b*sec(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(a + b*sec(c + d*x))*sec(c + d*x)**(3/2), x)
```

3.628.7 Maxima [F]

$$\int \sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = \int \sqrt{b\sec(dx+c)+a}\sec(dx+c)^{\frac{3}{2}}\,dx$$

input
```
integrate(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(3/2), x)
```

3.628.8 Giac [F]

$$\int \sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = \int \sqrt{b\sec(dx+c)+a}\sec(dx+c)^{\frac{3}{2}}\,dx$$

input
```
integrate(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(3/2), x)
```

3.628.9 Mupad [F(-1)]

Timed out.

$$\int \sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx=\int \sqrt{a+\frac{b}{\cos(c+d\,x)}}\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}dx$$

input
```
int((a + b/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(3/2),x)
```

output
```
int((a + b/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(3/2), x)
```

3.629 $\int \sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\,dx$

3.629.1 Optimal result 4824
3.629.2 Mathematica [A] (verified) 4825
3.629.3 Rubi [A] (verified) 4825
3.629.4 Maple [C] (verified) 4829
3.629.5 Fricas [F(-1)] 4829
3.629.6 Sympy [F] 4829
3.629.7 Maxima [F] 4830
3.629.8 Giac [F] 4830
3.629.9 Mupad [F(-1)] 4830

3.629.1 Optimal result

Integrand size = 25, antiderivative size = 138

$$\int \sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\,dx = \frac{2a\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{d\sqrt{a+b\sec(c+dx)}} + \frac{2b\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{d\sqrt{a+b\sec(c+dx)}}$$

output
```
2*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+
1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)*sec(d*x+c)^
(1/2)/d/(a+b*sec(d*x+c))^(1/2)+2*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*
x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2,2^(1/2)*(a/(a+b))^(1/2))*((b+a*co
s(d*x+c))/(a+b))^(1/2)*sec(d*x+c)^(1/2)/d/(a+b*sec(d*x+c))^(1/2)
```

3.629.2 Mathematica [A] (verified)

Time = 13.10 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.70

$$\int \sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\,dx = \frac{2\left(a\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)+b\,\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)\right)\sqrt{a+b\sec(c+dx)}}{(a+b)d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\sqrt{\sec(c+dx)}}$$

input
```
Integrate[Sqrt[Sec[c + d*x]]*Sqrt[a + b*Sec[c + d*x]],x]
```

output
```
(2*(a*EllipticF[(c + d*x)/2, (2*a)/(a + b)] + b*EllipticPi[2, (c + d*x)/2,
 (2*a)/(a + b)])*Sqrt[a + b*Sec[c + d*x]])/((a + b)*d*Sqrt[(b + a*Cos[c +
d*x])/(a + b)]*Sqrt[Sec[c + d*x]])
```

3.629.3 Rubi [A] (verified)

Time = 1.11 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.520$, Rules used = {3042, 4341, 3042, 4345, 3042, 3142, 3042, 3140, 4346, 3042, 3286, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{4341}$$

$$b\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx+a\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$a\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+b\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4345}$$

$$b\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{a\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int \frac{1}{\sqrt{b+a\cos(c+dx)}}dx}{\sqrt{a+b\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$b\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{a\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int \frac{1}{\sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{\sqrt{a+b\sec(c+dx)}}$$

$$\Big\downarrow \text{3142}$$

$$b\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int \frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$b\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int \frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}$$

$$\Big\downarrow \text{3140}$$

$$b\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}$$

$$\Big\downarrow \text{4346}$$

$$\frac{b\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int \frac{\sec(c+dx)}{\sqrt{b+a\cos(c+dx)}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{2a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{2a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}$$

$$\Big\downarrow \text{3286}$$

3.629. $\int \sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\,dx$

$$\frac{b\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{\sec(c+dx)}{\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{2a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}$$

$\Big\downarrow$ 3042

$$\frac{b\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{2a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}$$

$\Big\downarrow$ 3284

$$\frac{2a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}+\frac{2b\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}$$

input
```
Int[Sqrt[Sec[c + d*x]]*Sqrt[a + b*Sec[c + d*x]],x]
```

output
```
(2*a*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a +
b)]*Sqrt[Sec[c + d*x]])/(d*Sqrt[a + b*Sec[c + d*x]]) + (2*b*Sqrt[(b + a*Co
s[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c
+ d*x]])/(d*Sqrt[a + b*Sec[c + d*x]])
```

3.629.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3286
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]]   Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
 d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

rule 4341
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[a   Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e +
f*x]], x], x] + Simp[b/d   Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Csc[e + f*
x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4345
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]])   Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[
{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4346
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Simp[d*Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x
]]/Sqrt[a + b*Csc[e + f*x]])   Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

3.629. $\int \sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\,dx$

3.629.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 6.46 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.82

method	result
default	$\frac{2\sqrt{\sec(dx+c)}\,\sqrt{a+b\sec(dx+c)}\,\cos(dx+c)\sqrt{\frac{b+a\cos(dx+c)}{(a+b)(\cos(dx+c)+1)}}\left(\mathrm{EllipticF}\left(\sqrt{\frac{a-b}{a+b}}\,(-\cot(dx+c)+\csc(dx+c)),\sqrt{-\frac{a+b}{a-b}}\right)a-\mathrm{Elliptic}\right.}{d\sqrt{\frac{a-b}{a+b}}\,(b+a\cos(dx+c))\sqrt{\frac{}{c}}}$

input
```
int(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/d/((a-b)/(a+b))^(1/2)*sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2)*cos(d*x+c)
*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(EllipticF(((a-b)/(a+b))^
(1/2)*(-cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*a-EllipticF(((a-b)/(a
+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*b+2*EllipticPi((
(a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(a+b)/(a-b),I/((a-b)/(a+b))^(1
/2))*b)/(b+a*cos(d*x+c))/(1/(cos(d*x+c)+1))^(1/2)
```

3.629.5 Fricas [F(-1)]

Timed out.

$$\int \sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.629.6 Sympy [F]

$$\int \sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\,dx = \int \sqrt{a+b\sec(c+dx)}\sqrt{\sec(c+dx)}\,dx$$

input
```
integrate(sec(d*x+c)**(1/2)*(a+b*sec(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(a + b*sec(c + d*x))*sqrt(sec(c + d*x)), x)
```

3.629.7 Maxima [F]

$$\int \sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\,dx = \int \sqrt{b\sec(dx+c)+a}\sqrt{\sec(dx+c)}\,dx$$

input
```
integrate(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sec(d*x + c) + a)*sqrt(sec(d*x + c)), x)
```

3.629.8 Giac [F]

$$\int \sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\,dx = \int \sqrt{b\sec(dx+c)+a}\sqrt{\sec(dx+c)}\,dx$$

input
```
integrate(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*sec(d*x + c) + a)*sqrt(sec(d*x + c)), x)
```

3.629.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\,dx = \int \sqrt{a+\frac{b}{\cos(c+d\,x)}}\,\sqrt{\frac{1}{\cos(c+d\,x)}}\,dx$$

input
```
int((a + b/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(1/2),x)
```

output
```
int((a + b/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(1/2), x)
```

3.630 $\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}\,dx$

3.630.1 Optimal result 4831
3.630.2 Mathematica [A] (verified) 4831
3.630.3 Rubi [A] (verified) 4832
3.630.4 Maple [B] (warning: unable to verify) 4833
3.630.5 Fricas [C] (verification not implemented) 4834
3.630.6 Sympy [F] 4835
3.630.7 Maxima [F] 4835
3.630.8 Giac [F] 4836
3.630.9 Mupad [F(-1)] 4836

3.630.1 Optimal result

Integrand size = 25, antiderivative size = 67

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}\,dx=\frac{2E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+dx)}}{d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\sqrt{\sec(c+dx)}}$$

output
```
2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/
2*c),2^(1/2)*(a/(a+b))^(1/2))*(a+b*sec(d*x+c))^(1/2)/d/((b+a*cos(d*x+c))/(
a+b))^(1/2)/sec(d*x+c)^(1/2)
```

3.630.2 Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}\,dx=\frac{2E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+dx)}}{d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\sqrt{\sec(c+dx)}}$$

input
```
Integrate[Sqrt[a + b*Sec[c + d*x]]/Sqrt[Sec[c + d*x]],x]
```

output
```
(2*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(d*Sqrt
[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]])
```

3.630.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {3042, 4343, 3042, 3134, 3042, 3132}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4343}$$

$$\frac{\sqrt{a+b\sec(c+dx)}\int \sqrt{b+a\cos(c+dx)}dx}{\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{a+b\sec(c+dx)}\int \sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}$$

$$\Big\downarrow \text{3134}$$

$$\frac{\sqrt{a+b\sec(c+dx)}\int \sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}dx}{\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{a+b\sec(c+dx)}\int \sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}dx}{\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}$$

$$\Big\downarrow \text{3132}$$

$$\frac{2\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}$$

input
```
Int[Sqrt[a + b*Sec[c + d*x]]/Sqrt[Sec[c + d*x]],x]
```

output
```
(2*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(d*Sqrt
[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]])
```

3.630.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 4343
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]])   Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a
, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

3.630.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 868 vs. $2(90) = 180$.

Time = 3.37 (sec) , antiderivative size = 869, normalized size of antiderivative = 12.97

method	result
default	$\frac{2\left(\sqrt{\frac{a-b}{a+b}}\,a(1-\cos(dx+c))^3\csc(dx+c)^3-\sqrt{\frac{a-b}{a+b}}\,b(1-\cos(dx+c))^3\csc(dx+c)^3+\sqrt{-\frac{a(1-\cos(dx+c))^2\csc(dx+c)^2-b(1-\cos(dx+c))^2\csc(d}{a+b}}\right.}{}$
risch	Expression too large to display

input
```
int((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/d/((a-b)/(a+b))^(1/2)*(((a-b)/(a+b))^(1/2)*a*(1-cos(d*x+c))^3*csc(d*x+c)
^3-((a-b)/(a+b))^(1/2)*b*(1-cos(d*x+c))^3*csc(d*x+c)^3+(-(a*(1-cos(d*x+c))
^2*csc(d*x+c)^2-b*(1-cos(d*x+c))^2*csc(d*x+c)^2-a-b)/(a+b))^(1/2)*((1-cos(
d*x+c))^2*csc(d*x+c)^2+1)^(1/2)*EllipticF(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)
+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*a-(-(a*(1-cos(d*x+c))^2*csc(d*x+c)^2-b*
(1-cos(d*x+c))^2*csc(d*x+c)^2-a-b)/(a+b))^(1/2)*((1-cos(d*x+c))^2*csc(d*x+
c)^2+1)^(1/2)*EllipticF(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(-(a+
b)/(a-b))^(1/2))*b-(-(a*(1-cos(d*x+c))^2*csc(d*x+c)^2-b*(1-cos(d*x+c))^2*c
sc(d*x+c)^2-a-b)/(a+b))^(1/2)*((1-cos(d*x+c))^2*csc(d*x+c)^2+1)^(1/2)*Elli
pticE(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*a
+(-(a*(1-cos(d*x+c))^2*csc(d*x+c)^2-b*(1-cos(d*x+c))^2*csc(d*x+c)^2-a-b)/(
a+b))^(1/2)*((1-cos(d*x+c))^2*csc(d*x+c)^2+1)^(1/2)*EllipticE(((a-b)/(a+b)
)^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*b-((a-b)/(a+b))^(1/
2)*a*(-cot(d*x+c)+csc(d*x+c))-((a-b)/(a+b))^(1/2)*b*(-cot(d*x+c)+csc(d*x+c
)))*((a*(1-cos(d*x+c))^2*csc(d*x+c)^2-b*(1-cos(d*x+c))^2*csc(d*x+c)^2-a-b)
/((1-cos(d*x+c))^2*csc(d*x+c)^2-1))^(1/2)/(a*(1-cos(d*x+c))^2*csc(d*x+c)^2
-b*(1-cos(d*x+c))^2*csc(d*x+c)^2-a-b)/(-((1-cos(d*x+c))^2*csc(d*x+c)^2+1)/
((1-cos(d*x+c))^2*csc(d*x+c)^2-1))^(1/2)
```

3.630.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 355, normalized size of antiderivative = 5.30

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}\,dx$$
$$= \frac{-i\sqrt{2}\sqrt{a}b\text{weierstrassPInverse}\left(-\frac{4\,(3\,a^2-4\,b^2)}{3\,a^2},\frac{8\,(9\,a^2b-8\,b^3)}{27\,a^3},\frac{3\,a\cos(dx+c)+3i\,a\sin(dx+c)+2\,b}{3\,a}\right)+i\sqrt{2}\sqrt{a}b\text{weierstras}}{}$$

input
```
integrate((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
1/3*(-I*sqrt(2)*sqrt(a)*b*weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/
27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x + c) + 2*b
)/a) + I*sqrt(2)*sqrt(a)*b*weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8
/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*
b)/a) + 3*I*sqrt(2)*a^(3/2)*weierstrassZeta(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27
*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27
*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x + c) + 2*b)/
a)) - 3*I*sqrt(2)*a^(3/2)*weierstrassZeta(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(
9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(
9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a)
))/(a*d)
```

3.630.6 Sympy [F]

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}\,dx=\int \frac{\sqrt{a+b\sec{(c+dx)}}}{\sqrt{\sec{(c+dx)}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))**(1/2)/sec(d*x+c)**(1/2),x)
```

output
```
Integral(sqrt(a + b*sec(c + d*x))/sqrt(sec(c + d*x)), x)
```

3.630.7 Maxima [F]

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}\,dx=\int \frac{\sqrt{b\sec{(dx+c)}+a}}{\sqrt{\sec{(dx+c)}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sec(d*x + c) + a)/sqrt(sec(d*x + c)), x)
```

3.630.8 Giac [F]

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}\,dx = \int \frac{\sqrt{b\sec(dx+c)+a}}{\sqrt{\sec(dx+c)}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*sec(d*x + c) + a)/sqrt(sec(d*x + c)), x)
```

3.630.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}\,dx = \int \frac{\sqrt{a+\frac{b}{\cos(c+d\,x)}}}{\sqrt{\frac{1}{\cos(c+d\,x)}}}\,dx$$

input
```
int((a + b/cos(c + d*x))^(1/2)/(1/cos(c + d*x))^(1/2),x)
```

output
```
int((a + b/cos(c + d*x))^(1/2)/(1/cos(c + d*x))^(1/2), x)
```

3.631 $\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

3.631.1 Optimal result 4837
3.631.2 Mathematica [A] (verified) 4838
3.631.3 Rubi [A] (verified) 4838
3.631.4 Maple [B] (verified) 4843
3.631.5 Fricas [C] (verification not implemented) 4843
3.631.6 Sympy [F] 4844
3.631.7 Maxima [F] 4844
3.631.8 Giac [F] 4845
3.631.9 Mupad [F(-1)] 4845

3.631.1 Optimal result

Integrand size = 25, antiderivative size = 192

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \frac{2(a^2-b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{3ad\sqrt{a+b\sec(c+dx)}} + \frac{2bE\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\sqrt{\sec(c+dx)}} + \frac{2\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}$$

output
```
2/3*(a^2-b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(si
n(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)*s
ec(d*x+c)^(1/2)/a/d/(a+b*sec(d*x+c))^(1/2)+2/3*sin(d*x+c)*(a+b*sec(d*x+c))
^(1/2)/d/sec(d*x+c)^(1/2)+2/3*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1
/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*(a+b*sec(d*x+c
))^(1/2)/a/d/((b+a*cos(d*x+c))/(a+b))^(1/2)/sec(d*x+c)^(1/2)
```

3.631.2 Mathematica [A] (verified)

Time = 0.63 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.81

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \frac{2\sqrt{a+b\sec(c+dx)}\left(b(a+b)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)+(a^2-b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+\right.\right.}{3ad(b+a\cos(c+dx))\sqrt{\sec(c+dx)}}$$

input
```
Integrate[Sqrt[a + b*Sec[c + d*x]]/Sec[c + d*x]^(3/2),x]
```

output
```
(2*Sqrt[a + b*Sec[c + d*x]]*(b*(a + b)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*
EllipticE[(c + d*x)/2, (2*a)/(a + b)] + (a^2 - b^2)*Sqrt[(b + a*Cos[c + d*
x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)] + a*(b + a*Cos[c + d*x]
)*Sin[c + d*x]))/(3*a*d*(b + a*Cos[c + d*x])*Sqrt[Sec[c + d*x]])
```

3.631.3 Rubi [A] (verified)

Time = 1.37 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.01, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {3042, 4344, 3042, 4523, 3042, 4343, 3042, 3134, 3042, 3132, 4345, 3042, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{3}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{4344}$$

$$\frac{1}{3}\int \frac{b+a\sec(c+dx)}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx+\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3d\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\int \frac{b+a\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3d\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{4523}$$

$$\frac{1}{3}\left(\frac{\left(a^2-b^2\right)\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}dx}{a}+\frac{b\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}dx}{a}\right)+\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3d\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\left(\frac{\left(a^2-b^2\right)\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}+\frac{b\int \frac{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}\right)+\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3d\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{4343}$$

$$\frac{1}{3}\left(\frac{\left(a^2-b^2\right)\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}+\frac{b\sqrt{a+b\sec(c+dx)}\int \sqrt{b+a\cos(c+dx)}dx}{a\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}\right)+\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3d\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\left(\frac{\left(a^2-b^2\right)\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}+\frac{b\sqrt{a+b\sec(c+dx)}\int \sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{a\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}\right)+\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3d\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{3134}$$

$$\frac{1}{3}\left(\frac{\left(a^2-b^2\right)\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}+\frac{b\sqrt{a+b\sec(c+dx)}\int \sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}dx}{a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}\right)+\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3d\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

3.631. $\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{3}{2}}(c+dx)}dx$

$$\frac{1}{3}\left(\frac{(a^2-b^2)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}+\frac{b\sqrt{a+b\sec(c+dx)}\int\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}dx}{a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}\right)+\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3d\sqrt{\sec(c+dx)}}$$

$\downarrow$ 3132

$$\frac{1}{3}\left(\frac{(a^2-b^2)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}+\frac{2b\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}\right)+\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3d\sqrt{\sec(c+dx)}}$$

$\downarrow$ 4345

$$\frac{1}{3}\left(\frac{(a^2-b^2)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{1}{\sqrt{b+a\cos(c+dx)}}dx}{a\sqrt{a+b\sec(c+dx)}}+\frac{2b\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}\right)+\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3d\sqrt{\sec(c+dx)}}$$

$\downarrow$ 3042

$$\frac{1}{3}\left(\frac{(a^2-b^2)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{1}{\sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{a\sqrt{a+b\sec(c+dx)}}+\frac{2b\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}\right)+\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3d\sqrt{\sec(c+dx)}}$$

$\downarrow$ 3142

$$\frac{1}{3}\left(\frac{(a^2-b^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}}dx}{a\sqrt{a+b\sec(c+dx)}}+\frac{2b\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}\right)+\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3d\sqrt{\sec(c+dx)}}$$

$\downarrow$ 3042

3.631. $\int\frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{3}{2}}(c+dx)}dx$

$$\frac{1}{3}\left(\frac{(a^2-b^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{a\sqrt{a+b\sec(c+dx)}}+\frac{2b\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}\right)+\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3d\sqrt{\sec(c+dx)}}$$

$\Big\downarrow$ 3140

$$\frac{1}{3}\left(\frac{2(a^2-b^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{ad\sqrt{a+b\sec(c+dx)}}+\frac{2b\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}\right)+\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3d\sqrt{\sec(c+dx)}}$$

input
```
Int[Sqrt[a + b*Sec[c + d*x]]/Sec[c + d*x]^(3/2),x]
```

output
```
((2*(a^2 - b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2,
(2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(a*d*Sqrt[a + b*Sec[c + d*x]]) + (2*b*E
llipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(a*d*Sqrt[(
b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]))/3 + (2*Sqrt[a + b*Sec[c
+ d*x]]*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])
```

3.631.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 4343
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]])   Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a
, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4344
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[Cot[e + f*x]*Sqrt[a + b*Csc[e + f*x]]*((d*Csc[e
 + f*x])^n/(f*n)), x] - Simp[1/(2*d*n)   Int[(d*Csc[e + f*x])^(n + 1)*(Simp
[b - 2*a*(n + 1)*Csc[e + f*x] - b*(2*n + 3)*Csc[e + f*x]^2, x]/Sqrt[a + b*C
sc[e + f*x]]), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] &&
LeQ[n, -1] && IntegerQ[2*n]
```

rule 4345
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]])   Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[
{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4523
```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Simp[A/a   I
nt[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Simp[(A*b - a*B)
/(a*d)   Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ
[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

3.631. $\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

3.631.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1290 vs. $2(228) = 456$.

Time = 5.28 (sec) , antiderivative size = 1291, normalized size of antiderivative = 6.72

method	result	size
default	Expression too large to display	1291

input
```
int((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/3/d/a/((a-b)/(a+b))^(1/2)*(a+b*sec(d*x+c))^(1/2)/(b+a*cos(d*x+c))/sec(d*
x+c)^(3/2)/(cos(d*x+c)+1)*((1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)
*(1/(cos(d*x+c)+1))^(1/2)*EllipticE(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d
*x+c)),(-(a+b)/(a-b))^(1/2))*a*b*cos(d*x+c)-(1/(a+b)*(b+a*cos(d*x+c))/(cos
(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE(((a-b)/(a+b))^(1/2)*(
-cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*b^2*cos(d*x+c)+(1/(a+b)*(b+a
*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF(((a-
b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*a^2*cos(d*x
+c)-(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/
2)*EllipticF(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(
1/2))*a*b*cos(d*x+c)+2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ell
ipticE(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*
(1/(cos(d*x+c)+1))^(1/2)*a*b-2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(
1/2)*EllipticE(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))
^(1/2))*(1/(cos(d*x+c)+1))^(1/2)*b^2+2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+
c)+1))^(1/2)*EllipticF(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(-(a+b
)/(a-b))^(1/2))*(1/(cos(d*x+c)+1))^(1/2)*a^2-2*(1/(a+b)*(b+a*cos(d*x+c))/(
cos(d*x+c)+1))^(1/2)*EllipticF(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)
),(-(a+b)/(a-b))^(1/2))*(1/(cos(d*x+c)+1))^(1/2)*a*b+((a-b)/(a+b))^(1/2)*a
^2*cos(d*x+c)*sin(d*x+c)+(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2...
```

3.631.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 415, normalized size of antiderivative = 2.16

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{3}{2}}(c+dx)}\,dx$$
$$= \frac{6\,a^2\sqrt{\frac{a\cos(dx+c)+b}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)+3i\sqrt{2}a^{\frac{3}{2}}b\text{weierstrassZeta}\left(-\frac{4\,(3\,a^2-4\,b^2)}{3\,a^2},\frac{8\,(9\,a^2b-8\,b^3)}{27\,a^3},\text{weierst}\right.}{}$$

input
```
integrate((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(3/2),x, algorithm="fricas")
```

output
```
1/9*(6*a^2*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(
d*x + c) + 3*I*sqrt(2)*a^(3/2)*b*weierstrassZeta(-4/3*(3*a^2 - 4*b^2)/a^2,
 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2,
 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x + c) +
2*b)/a)) - 3*I*sqrt(2)*a^(3/2)*b*weierstrassZeta(-4/3*(3*a^2 - 4*b^2)/a^2,
 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2,
 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c) +
2*b)/a)) + sqrt(2)*(-3*I*a^2 + 2*I*b^2)*sqrt(a)*weierstrassPInverse(-4/3*(
3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*
I*a*sin(d*x + c) + 2*b)/a) + sqrt(2)*(3*I*a^2 - 2*I*b^2)*sqrt(a)*weierstra
ssPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*
cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a))/(a^2*d)
```

3.631.6 Sympy [F]

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{3}{2}}(c+dx)}\,dx=\int \frac{\sqrt{a+b\sec{(c+dx)}}}{\sec^{\frac{3}{2}}{(c+dx)}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))**(1/2)/sec(d*x+c)**(3/2),x)
```

output
```
Integral(sqrt(a + b*sec(c + d*x))/sec(c + d*x)**(3/2), x)
```

3.631.7 Maxima [F]

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{3}{2}}(c+dx)}\,dx=\int \frac{\sqrt{b\sec{(dx+c)}+a}}{\sec{(dx+c)}^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sec(d*x + c) + a)/sec(d*x + c)^(3/2), x)
```

3.631.8 Giac [F]

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{3}{2}}(c+dx)}\,dx=\int \frac{\sqrt{b\sec(dx+c)+a}}{\sec(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(3/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*sec(d*x + c) + a)/sec(d*x + c)^(3/2), x)
```

3.631.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{3}{2}}(c+dx)}\,dx=\int \frac{\sqrt{a+\frac{b}{\cos(c+d\,x)}}}{\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

input
```
int((a + b/cos(c + d*x))^(1/2)/(1/cos(c + d*x))^(3/2),x)
```

output
```
int((a + b/cos(c + d*x))^(1/2)/(1/cos(c + d*x))^(3/2), x)
```

3.632 $\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{5}{2}}(c+dx)}\,dx$

3.632.1 Optimal result . . . 4846
3.632.2 Mathematica [A] (verified) . . . 4847
3.632.3 Rubi [A] (verified) . . . 4847
3.632.4 Maple [B] (verified) . . . 4853
3.632.5 Fricas [C] (verification not implemented) . . . 4854
3.632.6 Sympy [F] . . . 4855
3.632.7 Maxima [F] . . . 4855
3.632.8 Giac [F] . . . 4856
3.632.9 Mupad [F(-1)] . . . 4856

3.632.1 Optimal result

Integrand size = 25, antiderivative size = 244

$$\begin{aligned}\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{5}{2}}(c+dx)}\,dx &= -\frac{4b(a^2-b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{15a^2d\sqrt{a+b\sec(c+dx)}} \\ &\quad+\frac{2(9a^2-2b^2)\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+dx)}}{15a^2d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\sqrt{\sec(c+dx)}} \\ &\quad+\frac{2\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{2b\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{15ad\sqrt{\sec(c+dx)}}\end{aligned}$$

output
```
-4/15*b*(a^2-b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elliptic
F(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/
2)*sec(d*x+c)^(1/2)/a^2/d/(a+b*sec(d*x+c))^(1/2)+2/5*sin(d*x+c)*(a+b*sec(d
*x+c))^(1/2)/d/sec(d*x+c)^(3/2)+2/15*b*sin(d*x+c)*(a+b*sec(d*x+c))^(1/2)/a
/d/sec(d*x+c)^(1/2)+2/15*(9*a^2-2*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/
2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*(a+b*se
c(d*x+c))^(1/2)/a^2/d/((b+a*cos(d*x+c))/(a+b))^(1/2)/sec(d*x+c)^(1/2)
```

3.632.2 Mathematica [A] (verified)

Time = 0.91 (sec) , antiderivative size = 203, normalized size of antiderivative = 0.83

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{5}{2}}(c+dx)}\,dx$$
$$= \frac{\sqrt{a+b\sec(c+dx)}\left(4(9a^3+9a^2b-2ab^2-2b^3)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}E\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)+8b(-a^2+b^2)\sqrt{\frac{b+a\cos(}{a+}}\right.}{30a^2d(b+a\cos(c+d}$$

input
```
Integrate[Sqrt[a + b*Sec[c + d*x]]/Sec[c + d*x]^(5/2),x]
```

output
```
(Sqrt[a + b*Sec[c + d*x]]*(4*(9*a^3 + 9*a^2*b - 2*a*b^2 - 2*b^3)*Sqrt[(b +
 a*Cos[c + d*x])/(a + b)]*EllipticE[(c + d*x)/2, (2*a)/(a + b)] + 8*b*(-a^
2 + b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(
a + b)] + 2*a*(3*a^2 + 2*b^2 + 8*a*b*Cos[c + d*x] + 3*a^2*Cos[2*(c + d*x)]
)*Sin[c + d*x]))/(30*a^2*d*(b + a*Cos[c + d*x])*Sqrt[Sec[c + d*x]])
```

3.632.3 Rubi [A] (verified)

Time = 1.84 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.04, number of steps used = 18, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.720$, Rules used = {3042, 4344, 3042, 4592, 27, 3042, 4523, 3042, 4343, 3042, 3134, 3042, 3132, 4345, 3042, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{5}{2}}(c+dx)}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx$$
$$\Big\downarrow \text{4344}$$
$$\frac{1}{5}\int \frac{2b\sec^2(c+dx)+3a\sec(c+dx)+b}{\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}dx+\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{5}\int \frac{2b\csc\left(c+dx+\frac{\pi}{2}\right)^2+3a\csc\left(c+dx+\frac{\pi}{2}\right)+b}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \; 4592$$

$$\frac{1}{5}\left(\frac{2b\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}-\frac{2\int -\frac{9a^2+7b\sec(c+dx)a-2b^2}{2\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{3a}\right)+\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{5}\left(\frac{\int \frac{9a^2+7b\sec(c+dx)a-2b^2}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{3a}+\frac{2b\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}\right)+\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{5}\left(\frac{\int \frac{9a^2+7b\csc\left(c+dx+\frac{\pi}{2}\right)a-2b^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{3a}+\frac{2b\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}\right)+\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \; 4523$$

$$\frac{1}{5}\left(\frac{\frac{\left(9a^2-2b^2\right)\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}dx}{a}-\frac{2b\left(a^2-b^2\right)\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}dx}{a}}{3a}+\frac{2b\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}\right)+\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \; 3042$$

3.632. $\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{5}{2}}(c+dx)}\,dx$

$$\frac{1}{5}\left(\frac{\frac{(9a^2-2b^2)\int \frac{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}-\frac{2b(a^2-b^2)\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}}{3a}+\frac{2b\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}\right)+\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$\Big\downarrow$ 4343

$$\frac{1}{5}\left(\frac{\frac{(9a^2-2b^2)\sqrt{a+b\sec(c+dx)}\int \sqrt{b+a\cos(c+dx)}dx}{a\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}-\frac{2b(a^2-b^2)\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}}{3a}+\frac{2b\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}\right)+\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$\Big\downarrow$ 3042

$$\frac{1}{5}\left(\frac{\frac{(9a^2-2b^2)\sqrt{a+b\sec(c+dx)}\int \sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{a\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}-\frac{2b(a^2-b^2)\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}}{3a}+\frac{2b\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}\right)+\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$\Big\downarrow$ 3134

$$\frac{1}{5}\left(\frac{\frac{(9a^2-2b^2)\sqrt{a+b\sec(c+dx)}\int \sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}dx}{a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{2b(a^2-b^2)\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}}{3a}+\frac{2b\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}\right)+\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$\Big\downarrow$ 3042

3.632. $\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{5}{2}}(c+dx)}\,dx$

$$\frac{1}{5}\left(\frac{\frac{(9a^2-2b^2)\sqrt{a+b\sec(c+dx)}\int\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}dx}{a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{2b(a^2-b^2)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}}{3a}+\frac{2b\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}\right.$$

$$\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$\downarrow$ 3132

$$\frac{1}{5}\left(\frac{\frac{2(9a^2-2b^2)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{2b(a^2-b^2)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}}{3a}+\frac{2b\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}\right)+$$

$$\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$\downarrow$ 4345

$$\frac{1}{5}\left(\frac{\frac{2(9a^2-2b^2)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{2b(a^2-b^2)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{1}{\sqrt{b+a\cos(c+dx)}}dx}{a\sqrt{a+b\sec(c+dx)}}}{3a}+\frac{2b\sin(c+dx)\sqrt{a}}{3ad\sqrt{\sec(}}\right.$$

$$\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$\downarrow$ 3042

$$\frac{1}{5}\left(\frac{\frac{2(9a^2-2b^2)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{2b(a^2-b^2)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{1}{\sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{a\sqrt{a+b\sec(c+dx)}}}{3a}+\frac{2b\sin(c+dx)\sqrt{}}{3ad\sqrt{\text{s}}}\right.$$

$$\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$\downarrow$ 3142

3.632. $\int\frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{5}{2}}(c+dx)}dx$

$$\frac{1}{5}\left(\frac{\frac{2(9a^2-2b^2)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{2b(a^2-b^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}}dx}{a\sqrt{a+b\sec(c+dx)}}}{3a}+\frac{2b\sin(c+dx)\sqrt{}}{3ad\sqrt{\text{se}}}\right.$$

$$\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$\downarrow$ 3042

$$\frac{1}{5}\left(\frac{\frac{2(9a^2-2b^2)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{2b(a^2-b^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{a\sqrt{a+b\sec(c+dx)}}}{3a}+\frac{2b\sin(c+dx}{3ad\sqrt{}}\right.$$

$$\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$\downarrow$ 3140

$$\frac{1}{5}\left(\frac{\frac{2(9a^2-2b^2)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{4b(a^2-b^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{ad\sqrt{a+b\sec(c+dx)}}}{3a}+\frac{2b\sin(c+dx)\sqrt{}}{3ad\sqrt{\text{s}}}\right.$$

$$\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5d\sec^{\frac{3}{2}}(c+dx)}$$

input
```
Int[Sqrt[a + b*Sec[c + d*x]]/Sec[c + d*x]^(5/2),x]
```

output
```
(2*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)) + (((-4
*b*(a^2 - b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (
2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(a*d*Sqrt[a + b*Sec[c + d*x]]) + (2*(9*a
^2 - 2*b^2)*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]]
)/(a*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]))/(3*a) + (2*
b*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(3*a*d*Sqrt[Sec[c + d*x]]))/5
```

3.632. $\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{5}{2}}(c+dx)}\,dx$

3.632.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 4343
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]])   Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a
, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

3.632. $\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{5}{2}}(c+dx)}\,dx$

rule 4344
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[Cot[e + f*x]*Sqrt[a + b*Csc[e + f*x]]*((d*Csc[e
 + f*x])^n/(f*n)), x] - Simp[1/(2*d*n)   Int[(d*Csc[e + f*x])^(n + 1)*(Simp
[b - 2*a*(n + 1)*Csc[e + f*x] - b*(2*n + 3)*Csc[e + f*x]^2, x]/Sqrt[a + b*C
sc[e + f*x]]), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] &&
LeQ[n, -1] && IntegerQ[2*n]
```

rule 4345
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]])   Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[
{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4523
```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Simp[A/a   I
nt[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Simp[(A*b - a*B)
/(a*d)   Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ
[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

rule 4592
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Simp[1/(a*d*n)   Int[(a + b*Csc[e + f*x])^m
*(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*
Csc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

3.632.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2287 vs. $2(274) = 548$.

Time = 6.52 (sec) , antiderivative size = 2288, normalized size of antiderivative = 9.38

method	result	size
default	Expression too large to display	2288

input
```
int((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/15/d/a^2/((a-b)/(a+b))^(1/2)*(a+b*sec(d*x+c))^(1/2)/(b+a*cos(d*x+c))/sec
(d*x+c)^(5/2)/(cos(d*x+c)+1)*(-14*(1/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*c
os(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF(((a-b)/(a+b))^(1/2)*(cot(d*x+c)
-csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*a^2*b*sec(d*x+c)-4*(1/(cos(d*x+c)+1))^(
1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF(((a-b)/(a+b
))^(1/2)*(cot(d*x+c)-csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*a*b^2*sec(d*x+c)+18
*(1/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*
EllipticE(((a-b)/(a+b))^(1/2)*(cot(d*x+c)-csc(d*x+c)),(-(a+b)/(a-b))^(1/2)
)*a^2*b*sec(d*x+c)+4*(1/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(c
os(d*x+c)+1))^(1/2)*EllipticE(((a-b)/(a+b))^(1/2)*(cot(d*x+c)-csc(d*x+c)),
(-(a+b)/(a-b))^(1/2))*a*b^2*sec(d*x+c)-7*(1/(cos(d*x+c)+1))^(1/2)*(1/(a+b)
*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF(((a-b)/(a+b))^(1/2)*(cot
(d*x+c)-csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*a^2*b*sec(d*x+c)^2-2*(1/(cos(d*x
+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF(((
a-b)/(a+b))^(1/2)*(cot(d*x+c)-csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*a*b^2*sec(
d*x+c)^2+9*(1/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+
1))^(1/2)*EllipticE(((a-b)/(a+b))^(1/2)*(cot(d*x+c)-csc(d*x+c)),(-(a+b)/(a
-b))^(1/2))*a^2*b*sec(d*x+c)^2+2*(1/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*co
s(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE(((a-b)/(a+b))^(1/2)*(cot(d*x+c)-
csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*a*b^2*sec(d*x+c)^2+3*((a-b)/(a+b))^(1...
```

3.632.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 461, normalized size of antiderivative = 1.89

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{5}{2}}(c+dx)}\,dx$$
$$= \frac{\sqrt{2}(-3i\,a^2b-4i\,b^3)\sqrt{a}\text{weierstrassPInverse}\left(-\frac{4\,(3\,a^2-4\,b^2)}{3\,a^2},\frac{8\,(9\,a^2b-8\,b^3)}{27\,a^3},\frac{3\,a\cos(dx+c)+3i\,a\sin(dx+c)+2\,b}{3\,a}\right)+\sqrt{2}(3i}{}$$

input
```
integrate((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(5/2),x, algorithm="fricas")
```

output
```
1/45*(sqrt(2)*(-3*I*a^2*b - 4*I*b^3)*sqrt(a)*weierstrassPInverse(-4/3*(3*a
^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a
*sin(d*x + c) + 2*b)/a) + sqrt(2)*(3*I*a^2*b + 4*I*b^3)*sqrt(a)*weierstras
sPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*c
os(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a) - 3*sqrt(2)*(-9*I*a^3 + 2*I*a*b
^2)*sqrt(a)*weierstrassZeta(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^
3)/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^
3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x + c) + 2*b)/a)) - 3*sqrt(2)*
(9*I*a^3 - 2*I*a*b^2)*sqrt(a)*weierstrassZeta(-4/3*(3*a^2 - 4*b^2)/a^2, 8/
27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/
27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*b
)/a)) + 6*(3*a^3*cos(d*x + c)^2 + a^2*b*cos(d*x + c))*sqrt((a*cos(d*x + c)
 + b)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^3*d)
```

3.632.6 Sympy [F]

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{5}{2}}(c+dx)}\,dx=\int \frac{\sqrt{a+b\sec{(c+dx)}}}{\sec^{\frac{5}{2}}{(c+dx)}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))**(1/2)/sec(d*x+c)**(5/2),x)
```

output
```
Integral(sqrt(a + b*sec(c + d*x))/sec(c + d*x)**(5/2), x)
```

3.632.7 Maxima [F]

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{5}{2}}(c+dx)}\,dx=\int \frac{\sqrt{b\sec{(dx+c)}+a}}{\sec{(dx+c)}^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(5/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sec(d*x + c) + a)/sec(d*x + c)^(5/2), x)
```

3.632.8 Giac [F]

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{5}{2}}(c+dx)}\,dx=\int \frac{\sqrt{b\sec(dx+c)+a}}{\sec(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(5/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*sec(d*x + c) + a)/sec(d*x + c)^(5/2), x)
```

3.632.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{5}{2}}(c+dx)}\,dx=\int \frac{\sqrt{a+\frac{b}{\cos(c+d\,x)}}}{\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

input
```
int((a + b/cos(c + d*x))^(1/2)/(1/cos(c + d*x))^(5/2),x)
```

output
```
int((a + b/cos(c + d*x))^(1/2)/(1/cos(c + d*x))^(5/2), x)
```

3.633 $\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{7}{2}}(c+dx)}\,dx$

3.633.1 Optimal result 4857
3.633.2 Mathematica [A] (verified) 4858
3.633.3 Rubi [A] (verified) 4858
3.633.4 Maple [B] (verified) 4866
3.633.5 Fricas [C] (verification not implemented) 4867
3.633.6 Sympy [F(-1)] 4867
3.633.7 Maxima [F] 4868
3.633.8 Giac [F] 4868
3.633.9 Mupad [F(-1)] 4868

3.633.1 Optimal result

Integrand size = 25, antiderivative size = 305

$$\begin{aligned}
&\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{7}{2}}(c+dx)}\,dx \\
&= \frac{2(25a^4-17a^2b^2-8b^4)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{105a^3d\sqrt{a+b\sec(c+dx)}} \\
&\quad+\frac{2b(19a^2+8b^2)\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+dx)}}{105a^3d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\sqrt{\sec(c+dx)}} \\
&\quad+\frac{2\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+\frac{2b\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{35ad\sec^{\frac{3}{2}}(c+dx)} \\
&\quad+\frac{2(25a^2-4b^2)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{105a^2d\sqrt{\sec(c+dx)}}
\end{aligned}$$

output
```
2/105*(25*a^4-17*a^2*b^2-8*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1
/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+
c))/(a+b))^(1/2)*sec(d*x+c)^(1/2)/a^3/d/(a+b*sec(d*x+c))^(1/2)+2/7*sin(d*x
+c)*(a+b*sec(d*x+c))^(1/2)/d/sec(d*x+c)^(5/2)+2/35*b*sin(d*x+c)*(a+b*sec(d
*x+c))^(1/2)/a/d/sec(d*x+c)^(3/2)+2/105*(25*a^2-4*b^2)*sin(d*x+c)*(a+b*sec
(d*x+c))^(1/2)/a^2/d/sec(d*x+c)^(1/2)+2/105*b*(19*a^2+8*b^2)*(cos(1/2*d*x+
1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a
/(a+b))^(1/2))*(a+b*sec(d*x+c))^(1/2)/a^3/d/((b+a*cos(d*x+c))/(a+b))^(1/2)
/sec(d*x+c)^(1/2)
```

3.633.2 Mathematica [A] (verified)

Time = 1.19 (sec) , antiderivative size = 237, normalized size of antiderivative = 0.78

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{7}{2}}(c+dx)}\,dx = \frac{\sqrt{a+b\sec(c+dx)}\left(8b(19a^3+19a^2b+8ab^2+8b^3)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)+8(25a^4-17a^2b^2-\right.}{}$$

input
```
Integrate[Sqrt[a + b*Sec[c + d*x]]/Sec[c + d*x]^(7/2),x]
```

output
```
(Sqrt[a + b*Sec[c + d*x]]*(8*b*(19*a^3 + 19*a^2*b + 8*a*b^2 + 8*b^3)*Sqrt[
(b + a*Cos[c + d*x])/(a + b)]*EllipticE[(c + d*x)/2, (2*a)/(a + b)] + 8*(2
5*a^4 - 17*a^2*b^2 - 8*b^4)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(
c + d*x)/2, (2*a)/(a + b)] + 2*a*(136*a^2*b - 16*b^3 + a*(145*a^2 - 4*b^2)
*Cos[c + d*x] + 36*a^2*b*Cos[2*(c + d*x)] + 15*a^3*Cos[3*(c + d*x)])*Sin[c
 + d*x]))/(420*a^3*d*(b + a*Cos[c + d*x])*Sqrt[Sec[c + d*x]])
```

3.633.3 Rubi [A] (verified)

Time = 2.43 (sec) , antiderivative size = 322, normalized size of antiderivative = 1.06, number of steps used = 21, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.840$, Rules used = {3042, 4344, 3042, 4592, 27, 3042, 4592, 27, 3042, 4523, 3042, 4343, 3042, 3134, 3042, 3132, 4345, 3042, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{7}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx$$

$$\Big\downarrow \text{4344}$$

$$\frac{1}{7}\int \frac{4b\sec^2(c+dx)+5a\sec(c+dx)+b}{\sec^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}dx+\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{7}\int \frac{4b\csc\left(c+dx+\frac{\pi}{2}\right)^2+5a\csc\left(c+dx+\frac{\pi}{2}\right)+b}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \; 4592$$

$$\frac{1}{7}\left(\frac{2b\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{2\int -\frac{25a^2+23b\sec(c+dx)a-4b^2+2b^2\sec^2(c+dx)}{2\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}dx}{5a}\right)+\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{7}\left(\frac{\int \frac{25a^2+23b\sec(c+dx)a-4b^2+2b^2\sec^2(c+dx)}{\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}dx}{5a}+\frac{2b\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5ad\sec^{\frac{3}{2}}(c+dx)}\right)+\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{7}\left(\frac{\int \frac{25a^2+23b\csc\left(c+dx+\frac{\pi}{2}\right)a-4b^2+2b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{5a}+\frac{2b\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5ad\sec^{\frac{3}{2}}(c+dx)}\right)+\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \; 4592$$

$$\frac{1}{7}\left(\frac{\frac{2\left(25a^2-4b^2\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}-\frac{2\int -\frac{b\left(19a^2+8b^2\right)+a\left(25a^2+2b^2\right)\sec(c+dx)}{2\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{3a}}{5a}+\frac{2b\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5ad\sec^{\frac{3}{2}}(c+dx)}\right)+\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \; 27$$

3.633. $\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{7}{2}}(c+dx)}dx$

$$\frac{1}{7}\left(\frac{\frac{\int \frac{b\left(19a^2+8b^2\right)+a\left(25a^2+2b^2\right)\sec(c+dx)}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{3a}+\frac{2\left(25a^2-4b^2\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}}{5a}+\frac{2b\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5ad\sec^{\frac{3}{2}}(c+dx)}\right)+\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$\Big\downarrow$ 3042

$$\frac{1}{7}\left(\frac{\frac{\int \frac{b\left(19a^2+8b^2\right)+a\left(25a^2+2b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{3a}+\frac{2\left(25a^2-4b^2\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}}{5a}+\frac{2b\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5ad\sec^{\frac{3}{2}}(c+dx)}\right)-\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$\Big\downarrow$ 4523

$$\frac{1}{7}\left(\frac{\frac{\frac{b\left(19a^2+8b^2\right)\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}dx}{a}+\frac{\left(25a^4-17a^2b^2-8b^4\right)\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}dx}{a}}{3a}+\frac{2\left(25a^2-4b^2\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}}{5a}+\frac{2b\sin(c+dx)}{5ad\,\mathrm{s}}\right.$$

$$\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$\Big\downarrow$ 3042

$$\frac{1}{7}\left(\frac{\frac{\frac{b\left(19a^2+8b^2\right)\int \frac{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}+\frac{\left(25a^4-17a^2b^2-8b^4\right)\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}}{3a}+\frac{2\left(25a^2-4b^2\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}}{5a}+\frac{2b\sin(c}{}\right.$$

$$\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$\Big\downarrow$ 4343

3.633. $\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{7}{2}}(c+dx)}\,dx$

$$\frac{1}{7}\left(\frac{\frac{\frac{b\left(19a^2+8b^2\right)\sqrt{a+b\sec(c+dx)}\int\sqrt{b+a\cos(c+dx)}dx}{a\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}+\frac{\left(25a^4-17a^2b^2-8b^4\right)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}}{3a}+\frac{2\left(25a^2-4b^2\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}}{5a}+\right.$$

$$\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$\downarrow$ 3042

$$\frac{1}{7}\left(\frac{\frac{\frac{b\left(19a^2+8b^2\right)\sqrt{a+b\sec(c+dx)}\int\sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{a\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}+\frac{\left(25a^4-17a^2b^2-8b^4\right)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}}{3a}+\frac{2\left(25a^2-4b^2\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}}{5a}\right.$$

$$\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$\downarrow$ 3134

$$\frac{1}{7}\left(\frac{\frac{\frac{b\left(19a^2+8b^2\right)\sqrt{a+b\sec(c+dx)}\int\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}dx}{a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{\left(25a^4-17a^2b^2-8b^4\right)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}}{3a}+\frac{2\left(25a^2-4b^2\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}}{5a}\right.$$

$$\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$\downarrow$ 3042

3.633. $\int\frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{7}{2}}(c+dx)}dx$

$$\frac{1}{7}\left(\frac{\frac{\frac{b\left(19a^2+8b^2\right)\sqrt{a+b\sec(c+dx)}\int\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}dx}{a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{\left(25a^4-17a^2b^2-8b^4\right)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}}{3a}+\frac{2\left(25a^2-4b^2\right)\sin(c+dx)\sqrt{a+b\sec(c+}}{3ad\sqrt{\sec(c+dx)}}}{5a}\right.$$

$$\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$\downarrow$ 3132

$$\frac{1}{7}\left(\frac{\frac{\frac{\left(25a^4-17a^2b^2-8b^4\right)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}+\frac{2b\left(19a^2+8b^2\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{3a}+\frac{2\left(25a^2-4b^2\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}}{5a}+\right.$$

$$\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$\downarrow$ 4345

$$\frac{1}{7}\left(\frac{\frac{\frac{\left(25a^4-17a^2b^2-8b^4\right)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{1}{\sqrt{b+a\cos(c+dx)}}dx}{a\sqrt{a+b\sec(c+dx)}}+\frac{2b\left(19a^2+8b^2\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{3a}+\frac{2\left(25a^2-4b^2\right)\sin(c+dx)}{3ad\sqrt{\sec(c+}}}{5a}\right.$$

$$\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$\downarrow$ 3042

3.633. $\int\frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{7}{2}}(c+dx)}dx$

$$\frac{1}{7}\left(\frac{\frac{\frac{\left(25a^4-17a^2b^2-8b^4\right)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{1}{\sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{a\sqrt{a+b\sec(c+dx)}}+\frac{2b\left(19a^2+8b^2\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{3a}+\frac{2\left(25a^2-4b^2\right)\sin(c+}{3ad\sqrt{se}}}{5a}\right.$$

$$\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \; 3142$$

$$\frac{1}{7}\left(\frac{\frac{\frac{\left(25a^4-17a^2b^2-8b^4\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}}dx}{a\sqrt{a+b\sec(c+dx)}}+\frac{2b\left(19a^2+8b^2\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{3a}+\frac{2\left(25a^2-4b^2\right)\sin(c+}{3ad\sqrt{se}}}{5a}\right.$$

$$\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{7}\left(\frac{\frac{\frac{\left(25a^4-17a^2b^2-8b^4\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{a\sqrt{a+b\sec(c+dx)}}+\frac{2b\left(19a^2+8b^2\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{3a}+\frac{2\left(25a^2-4b^2\right)\sin}{3ad}}{5a}\right.$$

$$\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \; 3140$$

3.633. $\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{7}{2}}(c+dx)}\,dx$

$$\frac{1}{7}\left(\frac{\frac{2\left(25a^2-4b^2\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}+\frac{\frac{2b\left(19a^2+8b^2\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{2\left(25a^4-17a^2b^2-8b^4\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\mathrm{E}}{ad\sqrt{a+b\sec(c+dx)}}}{3a}}{5a}\right.$$

$$\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{7d\sec^{\frac{5}{2}}(c+dx)}$$

input
```
Int[Sqrt[a + b*Sec[c + d*x]]/Sec[c + d*x]^(7/2),x]
```

output
```
(2*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/2)) + ((2*b
*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(5*a*d*Sec[c + d*x]^(3/2)) + (((2*
(25*a^4 - 17*a^2*b^2 - 8*b^4)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF
[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(a*d*Sqrt[a + b*Sec[c + d
*x]]) + (2*b*(19*a^2 + 8*b^2)*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a
 + b*Sec[c + d*x]])/(a*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d
*x]]))/(3*a) + (2*(25*a^2 - 4*b^2)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/
(3*a*d*Sqrt[Sec[c + d*x]]))/(5*a))/7
```

3.633.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 4343
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]])   Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a
, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4344
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[Cot[e + f*x]*Sqrt[a + b*Csc[e + f*x]]*((d*Csc[e
 + f*x])^n/(f*n)), x] - Simp[1/(2*d*n)   Int[(d*Csc[e + f*x])^(n + 1)*(Simp
[b - 2*a*(n + 1)*Csc[e + f*x] - b*(2*n + 3)*Csc[e + f*x]^2, x]/Sqrt[a + b*C
sc[e + f*x]]), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] &&
LeQ[n, -1] && IntegerQ[2*n]
```

rule 4345
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]])   Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[
{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4523
```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Simp[A/a   I
nt[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Simp[(A*b - a*B)
/(a*d)   Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ
[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

3.633. $\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{7}{2}}(c+dx)}\,dx$

rule 4592
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Simp[1/(a*d*n)   Int[(a + b*Csc[e + f*x])^m
*(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*
Csc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

3.633.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2763 vs. $2(329) = 658$.

Time = 7.64 (sec) , antiderivative size = 2764, normalized size of antiderivative = 9.06

method	result	size
default	Expression too large to display	2764

input
```
int((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-2/105/d/a^3/((a-b)/(a+b))^(1/2)*(-19*EllipticF(((a-b)/(a+b))^(1/2)*(cot(d
*x+c)-csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x
+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a^3*b*cos(d*x+c)^2-16*EllipticE(((a
-b)/(a+b))^(1/2)*(cot(d*x+c)-csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b
+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*b^4*cos(d*x+
c)+2*EllipticF(((a-b)/(a+b))^(1/2)*(cot(d*x+c)-csc(d*x+c)),(-(a+b)/(a-b))^
(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^
(1/2)*a^2*b^2*cos(d*x+c)^2-8*((a-b)/(a+b))^(1/2)*b^4*sin(d*x+c)-8*Elliptic
F(((a-b)/(a+b))^(1/2)*(cot(d*x+c)-csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*(1/(a+
b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a*b^3*c
os(d*x+c)^2+19*EllipticE(((a-b)/(a+b))^(1/2)*(cot(d*x+c)-csc(d*x+c)),(-(a+
b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d
*x+c)+1))^(1/2)*a^3*b*cos(d*x+c)^2-19*EllipticE(((a-b)/(a+b))^(1/2)*(cot(d
*x+c)-csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x
+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a^2*b^2*cos(d*x+c)^2+8*EllipticE(((
a-b)/(a+b))^(1/2)*(cot(d*x+c)-csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(
b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a*b^3*cos(d
*x+c)^2-38*EllipticF(((a-b)/(a+b))^(1/2)*(cot(d*x+c)-csc(d*x+c)),(-(a+b)/(
a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c
)+1))^(1/2)*a^3*b*cos(d*x+c)+4*EllipticF(((a-b)/(a+b))^(1/2)*(cot(d*x+c...
```

3.633.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.13 (sec) , antiderivative size = 501, normalized size of antiderivative = 1.64

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{7}{2}}(c+dx)}\,dx$$
$$= \frac{\sqrt{2}(-75i\,a^4+32i\,a^2b^2+16i\,b^4)\sqrt{a}\text{weierstrassPInverse}\left(-\frac{4\,(3\,a^2-4\,b^2)}{3\,a^2},\frac{8\,(9\,a^2b-8\,b^3)}{27\,a^3},\frac{3\,a\cos(dx+c)+3i\,a\sin(dx+c)+2}{3\,a}\right.}{}$$

input
```
integrate((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(7/2),x, algorithm="fricas")
```

output
```
1/315*(sqrt(2)*(-75*I*a^4 + 32*I*a^2*b^2 + 16*I*b^4)*sqrt(a)*weierstrassPI
nverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(
d*x + c) + 3*I*a*sin(d*x + c) + 2*b)/a) + sqrt(2)*(75*I*a^4 - 32*I*a^2*b^2
 - 16*I*b^4)*sqrt(a)*weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9
*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a)
- 3*sqrt(2)*(-19*I*a^3*b - 8*I*a*b^3)*sqrt(a)*weierstrassZeta(-4/3*(3*a^2
- 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a^2
- 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*si
n(d*x + c) + 2*b)/a)) - 3*sqrt(2)*(19*I*a^3*b + 8*I*a*b^3)*sqrt(a)*weierst
rassZeta(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrass
PInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*co
s(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a)) + 6*(15*a^4*cos(d*x + c)^3 + 3*
a^3*b*cos(d*x + c)^2 + (25*a^4 - 4*a^2*b^2)*cos(d*x + c))*sqrt((a*cos(d*x
+ c) + b)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^4*d)
```

3.633.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{7}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+b*sec(d*x+c))**(1/2)/sec(d*x+c)**(7/2),x)
```

output
```
Timed out
```

3.633.7 Maxima [F]

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{7}{2}}(c+dx)}\,dx=\int \frac{\sqrt{b\sec(dx+c)+a}}{\sec(dx+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(7/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sec(d*x + c) + a)/sec(d*x + c)^(7/2), x)
```

3.633.8 Giac [F]

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{7}{2}}(c+dx)}\,dx=\int \frac{\sqrt{b\sec(dx+c)+a}}{\sec(dx+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(7/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*sec(d*x + c) + a)/sec(d*x + c)^(7/2), x)
```

3.633.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{7}{2}}(c+dx)}\,dx=\int \frac{\sqrt{a+\frac{b}{\cos(c+d\,x)}}}{\left(\frac{1}{\cos(c+d\,x)}\right)^{7/2}}\,dx$$

input
```
int((a + b/cos(c + d*x))^(1/2)/(1/cos(c + d*x))^(7/2),x)
```

output
```
int((a + b/cos(c + d*x))^(1/2)/(1/cos(c + d*x))^(7/2), x)
```

3.634 $\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx$

3.634.1 Optimal result 4869
3.634.2 Mathematica [C] (verified) 4870
3.634.3 Rubi [A] (verified) 4871
3.634.4 Maple [C] (verified) 4879
3.634.5 Fricas [F(-1)] 4880
3.634.6 Sympy [F(-1)] 4881
3.634.7 Maxima [F] 4881
3.634.8 Giac [F] 4881
3.634.9 Mupad [F(-1)] 4882

3.634.1 Optimal result

Integrand size = 25, antiderivative size = 299

$$\begin{aligned}\int \sec^{\frac{3}{2}}(c+dx)(a&+b\sec(c+dx))^{3/2}\,dx=\frac{7ab\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{4d\sqrt{a+b\sec(c+dx)}}\\&+\frac{(3a^2+4b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{4d\sqrt{a+b\sec(c+dx)}}\\&-\frac{5aE\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+dx)}}{4d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\sqrt{\sec(c+dx)}}\\&+\frac{5a\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{4d}\\&+\frac{b\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{2d}\end{aligned}$$

output
```
7/4*a*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*
d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)*sec(d*x
+c)^(1/2)/d/(a+b*sec(d*x+c))^(1/2)+1/4*(3*a^2+4*b^2)*(cos(1/2*d*x+1/2*c)^2
)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2,2^(1/2)*(a/(a+b
))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)*sec(d*x+c)^(1/2)/d/(a+b*sec(d*x+c
))^(1/2)+1/2*b*sec(d*x+c)^(3/2)*sin(d*x+c)*(a+b*sec(d*x+c))^(1/2)/d-5/4*a*
(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*
c),2^(1/2)*(a/(a+b))^(1/2))*(a+b*sec(d*x+c))^(1/2)/d/((b+a*cos(d*x+c))/(a+
b))^(1/2)/sec(d*x+c)^(1/2)+5/4*a*sin(d*x+c)*sec(d*x+c)^(1/2)*(a+b*sec(d*x+
c))^(1/2)/d
```

3.634.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.77 (sec) , antiderivative size = 411, normalized size of antiderivative = 1.37

$$\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx = \frac{(a+b\sec(c+dx))^{3/2}\left(\frac{4ab\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{(b+a\cos(c+dx))^2}+\frac{(a^2+8b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(c+d\right.}{(b+a\cos(c+dx))^2}\right.}{}$$

input
```
Integrate[Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^(3/2),x]
```

output
```
((a + b*Sec[c + d*x])^(3/2)*((4*a*b*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Ell
ipticF[(c + d*x)/2, (2*a)/(a + b)])/(b + a*Cos[c + d*x])^2 + ((a^2 + 8*b^2
)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a +
 b)])/(b + a*Cos[c + d*x])^2 - ((5*I)*Sqrt[-((a*(-1 + Cos[c + d*x]))/(a +
b))]*Sqrt[(a*(1 + Cos[c + d*x]))/(a - b)]*Csc[c + d*x]*(-2*b*(a + b)*Ellip
ticE[I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a +
 b)] + a*(2*b*EllipticF[I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*
x]]], (-a + b)/(a + b)] + a*EllipticPi[1 - a/b, I*ArcSinh[Sqrt[(a - b)^(-1
)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b)])))/(Sqrt[(a - b)^(-1)]*b*(
b + a*Cos[c + d*x])^(3/2)) + (2*(5*a + 2*b*Sec[c + d*x])*Tan[c + d*x])/(b
+ a*Cos[c + d*x])))/(8*d*Sec[c + d*x]^(3/2))
```

3.634.3 Rubi [A] (verified)

Time = 3.15 (sec) , antiderivative size = 308, normalized size of antiderivative = 1.03, number of steps used = 26, number of rules used = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.040, Rules used = {3042, 4353, 27, 3042, 4590, 27, 3042, 4596, 3042, 4346, 3042, 3286, 3042, 3284, 4523, 3042, 4343, 3042, 3134, 3042, 3132, 4345, 3042, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}dx$$

$$\Big\downarrow \text{4353}$$

$$\frac{1}{2}\int \frac{\sqrt{\sec(c+dx)}\left(5ab\sec^2(c+dx)+2\left(2a^2+b^2\right)\sec(c+dx)+ab\right)}{2\sqrt{a+b\sec(c+dx)}}dx+\frac{b\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{4}\int \frac{\sqrt{\sec(c+dx)}\left(5ab\sec^2(c+dx)+2\left(2a^2+b^2\right)\sec(c+dx)+ab\right)}{\sqrt{a+b\sec(c+dx)}}dx+\frac{b\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(5ab\csc\left(c+dx+\frac{\pi}{2}\right)^2+2\left(2a^2+b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)+ab\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{b\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}$$

$$\Big\downarrow \text{4590}$$

$$\frac{1}{4}\left(\frac{\int -\frac{5ba^2-2b^2\sec(c+dx)a-b\left(3a^2+4b^2\right)\sec^2(c+dx)}{2\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{b}+\frac{5a\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}\right)+\frac{b\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{4}\left(\frac{5a\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{\int\frac{5ba^2-2b^2\sec(c+dx)a-b\left(3a^2+4b^2\right)\sec^2(c+dx)}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{2b}\right)+\frac{b\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{4}\left(\frac{5a\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{\int\frac{5ba^2-2b^2\csc\left(c+dx+\frac{\pi}{2}\right)a-b\left(3a^2+4b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{2b}\right)+\frac{b\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}$$

$$\Big\downarrow \; 4596$$

$$\frac{1}{4}\left(\frac{5a\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{\int\frac{5a^2b-2ab^2\sec(c+dx)}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx-b\left(3a^2+4b^2\right)\int\frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}}{2b}\right.$$

$$\frac{b\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{4}\left(\frac{5a\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{\int\frac{5a^2b-2ab^2\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-b\left(3a^2+4b^2\right)\int\frac{\csc(c+}{\sqrt{a+b\,\mathrm{cs}}}}{2b}\right.$$

$$\frac{b\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}$$

$$\Big\downarrow \; 4346$$

$$\frac{1}{4}\left(\frac{5a\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{\int\frac{5a^2b-2ab^2\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{b\left(3a^2+4b^2\right)\sqrt{\sec(c+dx)}\sqrt{a}}{\sqrt{a+}}}{2b}\right.$$

$$\frac{b\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}$$

3.634. $\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx$

$$\Big\downarrow \; 3042$$

$$\frac{1}{4}\left(\frac{5a\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{\int\frac{5a^2b-2ab^2\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{b(3a^2+4b^2)\sqrt{\sec(c+dx)}\sqrt{a}}{}}{2b}\right.$$

$$\frac{b\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}$$

$$\Big\downarrow \; 3286$$

$$\frac{1}{4}\left(\frac{5a\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{\int\frac{5a^2b-2ab^2\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{b(3a^2+4b^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a}{}}}{\sqrt{a-}}}{2b}\right.$$

$$\frac{b\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{4}\left(\frac{5a\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{\int\frac{5a^2b-2ab^2\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{b(3a^2+4b^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a}{}}}{}}{2b}\right.$$

$$\frac{b\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}$$

$$\Big\downarrow \; 3284$$

$$\frac{1}{4}\left(\frac{5a\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{\int\frac{5a^2b-2ab^2\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2b(3a^2+4b^2)\sqrt{\sec(c+dx)}\sqrt{}}{d\sqrt{}}}{2b}\right.$$

$$\frac{b\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}$$

$$\Big\downarrow \; 4523$$

3.634. $\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx$

$$\frac{1}{4}\left(\frac{5a\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{-7ab^2\int\frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}dx+5ab\int\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}dx-\frac{2b(3a^2+}{}}{2b}\right.$$

$$\frac{b\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}$$

$\downarrow$ 3042

$$\frac{1}{4}\left(\frac{5a\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{-7ab^2\int\frac{\sqrt{\csc(c+dx+\frac{\pi}{2})}}{\sqrt{a+b\csc(c+dx+\frac{\pi}{2})}}dx+5ab\int\frac{\sqrt{a+b\csc(c+dx+\frac{\pi}{2})}}{\sqrt{\csc(c+dx+\frac{\pi}{2})}}dx-}{2b}\right.$$

$$\frac{b\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}$$

$\downarrow$ 4343

$$\frac{1}{4}\left(\frac{5a\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{-7ab^2\int\frac{\sqrt{\csc(c+dx+\frac{\pi}{2})}}{\sqrt{a+b\csc(c+dx+\frac{\pi}{2})}}dx+\frac{5ab\sqrt{a+b\sec(c+dx)}\int\sqrt{b+a\cos(c+}}{\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}}{}\right.$$

$$\frac{b\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}$$

$\downarrow$ 3042

$$\frac{1}{4}\left(\frac{5a\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{-7ab^2\int\frac{\sqrt{\csc(c+dx+\frac{\pi}{2})}}{\sqrt{a+b\csc(c+dx+\frac{\pi}{2})}}dx+\frac{5ab\sqrt{a+b\sec(c+dx)}\int\sqrt{b+a\sin(c+}}{\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+}}}{}\right.$$

$$\frac{b\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}$$

$\downarrow$ 3134

$$\frac{1}{4}\left(\frac{5a\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{-7ab^2\int\frac{\sqrt{\csc(c+dx+\frac{\pi}{2})}}{\sqrt{a+b\csc(c+dx+\frac{\pi}{2})}}dx+\frac{5ab\sqrt{a+b\sec(c+dx)}\int\sqrt{\frac{b}{a+b}+\frac{a\cos(}{a}}}{\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{}\right.$$

$$\frac{b\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}$$

3.634. $\int\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx$

$$\Big\downarrow \; 3042$$

$$\frac{1}{4}\left(\frac{5a\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{-7ab^2\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{5ab\sqrt{a+b\sec(c+dx)}\int\sqrt{\frac{b}{a+b}+\frac{a\sin(}{}}}{\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)}{a+b}}}}{}\right.$$

$$\frac{b\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}$$

$$\Big\downarrow \; 3132$$

$$\frac{1}{4}\left(\frac{5a\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{-7ab^2\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2b\left(3a^2+4b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+}{a+b}}}{d\sqrt{a+b\sec}}}{}\right.$$

$$\frac{b\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}$$

$$\Big\downarrow \; 4345$$

$$\frac{1}{4}\left(\frac{5a\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{-\frac{7ab^2\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{1}{\sqrt{b+a\cos(c+dx)}}dx}{\sqrt{a+b\sec(c+dx)}}-\frac{2b\left(3a^2+4b^2\right)\sqrt{}}{}}{}\right.$$

$$\frac{b\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{4}\left(\frac{5a\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{-\frac{7ab^2\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{1}{\sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{\sqrt{a+b\sec(c+dx)}}-\frac{2b\left(3a^2+4b\right.}{}}{}\right.$$

$$\frac{b\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}$$

$$\Big\downarrow \; 3142$$

3.634. $\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx$

$$\frac{1}{4}\left(\frac{5a\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{-\frac{7ab^2\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}-\frac{2b(3a^2+4b^2}{}}{}\right.$$

$$\frac{b\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}$$

$\downarrow$ 3042

$$\frac{1}{4}\left(\frac{5a\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{-\frac{7ab^2\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}-\frac{2b(3a^2+}{}}{}\right.$$

$$\frac{b\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}$$

$\downarrow$ 3140

$$\frac{1}{4}\left(\frac{5a\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{-\frac{2b(3a^2+4b^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\operatorname{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}-}{}\right.$$

$$\frac{b\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}$$

input
```
Int[Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^(3/2),x]
```

output
```
(b*Sec[c + d*x]^(3/2)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(2*d) + (-1/2
*((-14*a*b^2*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*
a)/(a + b)]*Sqrt[Sec[c + d*x]])/(d*Sqrt[a + b*Sec[c + d*x]]) - (2*b*(3*a^2
 + 4*b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2
*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(d*Sqrt[a + b*Sec[c + d*x]]) + (10*a*b*El
lipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(d*Sqrt[(b +
 a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]))/b + (5*a*Sqrt[Sec[c + d*x]]
*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/d)/4
```

3.634. $\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx$

3.634.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3286
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]]   Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
 d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

rule 4343
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]])   Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a
, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4345
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]])   Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[
{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4346
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Simp[d*Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x
]]/Sqrt[a + b*Csc[e + f*x]])   Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4353
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b)*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
1)*((d*Csc[e + f*x])^(n - 1)/(f*(m + n - 1))), x] + Simp[d/(m + n - 1)   In
t[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^(n - 1)*Simp[a*b*(n - 1) +
(b^2*(m + n - 2) + a^2*(m + n - 1))*Csc[e + f*x] + a*b*(2*m + n - 2)*Csc[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && L
tQ[0, m, 2] && LtQ[0, n, 3] && NeQ[m + n - 1, 0] && (IntegerQ[m] || Integer
sQ[2*m, 2*n])
```

rule 4523
```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Simp[A/a   I
nt[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Simp[(A*b - a*B)
/(a*d)   Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ
[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

3.634. $\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx$

rule 4590
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(-C)*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
)*((d*Csc[e + f*x])^(n - 1)/(b*f*(m + n + 1))), x] + Simp[d/(b*(m + n + 1))
   Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (
A*b*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2
 - b^2, 0] && GtQ[n, 0]
```

rule 4596
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Simp[C/d^2   Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*C
sc[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[
a + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
 b^2, 0]
```

3.634.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 9.78 (sec) , antiderivative size = 2317, normalized size of antiderivative = 7.75

method	result	size
default	Expression too large to display	2317

input
```
int(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/4/d/((a-b)/(a+b))^(1/2)*(5*(1/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*
x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc
(d*x+c)),(-(a+b)/(a-b))^(1/2))*a*b*cos(d*x+c)^4+2*(1/(cos(d*x+c)+1))^(1/2)
*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF(((a-b)/(a+b))^(
1/2)*(-cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*a*b*cos(d*x+c)^4+10*(1
/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ell
ipticE(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*
a*b*cos(d*x+c)^3+4*(1/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos
(d*x+c)+1))^(1/2)*EllipticF(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(
-(a+b)/(a-b))^(1/2))*a*b*cos(d*x+c)^3+5*(1/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*
(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE(((a-b)/(a+b))^(1/2)*(-cot
(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*a*b*cos(d*x+c)^2+2*(1/(cos(d*x+c
)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF(((a-
b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*a*b*cos(d*x
+c)^2+2*((a-b)/(a+b))^(1/2)*b^2*sin(d*x+c)+5*((a-b)/(a+b))^(1/2)*a^2*cos(d
*x+c)^2*sin(d*x+c)+2*((a-b)/(a+b))^(1/2)*b^2*cos(d*x+c)*sin(d*x+c)+8*(1/(c
os(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ellipt
icPi(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(a+b)/(a-b),I/((a-b)/(a+
b))^(1/2))*b^2*cos(d*x+c)^2+2*((a-b)/(a+b))^(1/2)*a*b*cos(d*x+c)^2*sin(d*x
+c)+7*((a-b)/(a+b))^(1/2)*a*b*cos(d*x+c)*sin(d*x+c)-5*(1/(cos(d*x+c)+1)...
```

3.634.5 Fricas [F(-1)]

Timed out.

$$\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.634.6 Sympy [F(-1)]

Timed out.

$$\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**(3/2)*(a+b*sec(d*x+c))**(3/2),x)
```

output
```
Timed out
```

3.634.7 Maxima [F]

$$\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx = \int (b\sec(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{3}{2}}\,dx$$

input
```
integrate(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^(3/2)*sec(d*x + c)^(3/2), x)
```

3.634.8 Giac [F]

$$\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx = \int (b\sec(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{3}{2}}\,dx$$

input
```
integrate(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^(3/2)*sec(d*x + c)^(3/2), x)
```

3.634.9 Mupad [F(-1)]

Timed out.

$$\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx=\int \left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)^{3/2}\left(\frac{1}{\cos\left(c+d\,x\right)}\right)^{3/2}dx$$

input
```
int((a + b/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(3/2),x)
```

output
```
int((a + b/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(3/2), x)
```

3.635 $\int \sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}\,dx$

3.635.1 Optimal result 4883
3.635.2 Mathematica [C] (verified) 4884
3.635.3 Rubi [A] (verified) 4884
3.635.4 Maple [C] (verified) 4891
3.635.5 Fricas [F(-1)] 4892
3.635.6 Sympy [F] 4893
3.635.7 Maxima [F] 4893
3.635.8 Giac [F] 4893
3.635.9 Mupad [F(-1)] 4894

3.635.1 Optimal result

Integrand size = 25, antiderivative size = 249

$$\begin{aligned}\int \sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}\,dx &= \frac{(2a^2+b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{d\sqrt{a+b\sec(c+dx)}}\\ &+\frac{3ab\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{d\sqrt{a+b\sec(c+dx)}}\\ &-\frac{bE\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+dx)}}{d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\sqrt{\sec(c+dx)}}\\ &+\frac{b\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{d}\end{aligned}$$

output
```
(2*a^2+b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(
1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)*sec
(d*x+c)^(1/2)/d/(a+b*sec(d*x+c))^(1/2)+3*a*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/
cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2,2^(1/2)*(a/(a+b))^(1/2)
)*((b+a*cos(d*x+c))/(a+b))^(1/2)*sec(d*x+c)^(1/2)/d/(a+b*sec(d*x+c))^(1/2)
-b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1
/2*c),2^(1/2)*(a/(a+b))^(1/2))*(a+b*sec(d*x+c))^(1/2)/d/((b+a*cos(d*x+c))/
(a+b))^(1/2)/sec(d*x+c)^(1/2)+b*sin(d*x+c)*sec(d*x+c)^(1/2)*(a+b*sec(d*x+c
))^(1/2)/d
```

3.635.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 14.33 (sec) , antiderivative size = 394, normalized size of antiderivative = 1.58

$$\int \sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}\,dx = \frac{(a+b\sec(c+dx))^{3/2}\left(\frac{8a^2\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{(b+a\cos(c+dx))^2}+\frac{10ab\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2}{a}\right.}{(b+a\cos(c+dx))^2}\right.}{}$$

input
```
Integrate[Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^(3/2),x]
```

output
```
((a + b*Sec[c + d*x])^(3/2)*((8*a^2*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Ell
ipticF[(c + d*x)/2, (2*a)/(a + b)])/(b + a*Cos[c + d*x])^2 + (10*a*b*Sqrt[
(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)])/(
b + a*Cos[c + d*x])^2 - ((2*I)*Sqrt[-((a*(-1 + Cos[c + d*x]))/(a + b))]*Sq
rt[(a*(1 + Cos[c + d*x]))/(a - b)]*Csc[c + d*x]*(-2*b*(a + b)*EllipticE[I*
ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b)] +
a*(2*b*EllipticF[I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (
-a + b)/(a + b)] + a*EllipticPi[1 - a/b, I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt
[b + a*Cos[c + d*x]]], (-a + b)/(a + b)])))/(a*Sqrt[(a - b)^(-1)]*(b + a*C
os[c + d*x])^(3/2)) + (4*b*Tan[c + d*x])/(b + a*Cos[c + d*x])))/(4*d*Sec[c
 + d*x]^(3/2))
```

3.635.3 Rubi [A] (verified)

Time = 2.51 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.02, number of steps used = 23, number of rules used = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.920$, Rules used = {3042, 4353, 27, 3042, 4596, 3042, 4346, 3042, 3286, 3042, 3284, 4523, 3042, 4343, 3042, 3134, 3042, 3132, 4345, 3042, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}dx$$

$$\Big\downarrow \text{4353}$$

$$\int -\frac{-2\sec(c+dx)a^2-3b\sec^2(c+dx)a+ba}{2\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx+\frac{b\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{b\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{1}{2}\int \frac{-2\sec(c+dx)a^2-3b\sec^2(c+dx)a+ba}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{b\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{1}{2}\int \frac{-2\csc\left(c+dx+\frac{\pi}{2}\right)a^2-3b\csc\left(c+dx+\frac{\pi}{2}\right)^2a+ba}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4596}$$

$$\frac{1}{2}\left(3ab\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx-\int \frac{ab-2a^2\sec(c+dx)}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx\right)+\frac{b\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\left(3ab\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-\int \frac{ab-2a^2\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)+\frac{b\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}$$

$$\Big\downarrow \text{4346}$$

$$\frac{1}{2}\left(\frac{3ab\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int \frac{\sec(c+dx)}{\sqrt{b+a\cos(c+dx)}}dx}{\sqrt{a+b\sec(c+dx)}}-\int \frac{ab-2a^2\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)\frac{b\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}$$

$$\Big\downarrow \text{3042}$$

3.635. $\int \sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}\,dx$

$$\frac{1}{2}\left(\frac{3ab\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{\sqrt{a+b\sec(c+dx)}}-\int\frac{ab-2a^2\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+\right.}}\right.$$

$$\frac{b\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}$$

$\downarrow$ 3286

$$\frac{1}{2}\left(\frac{3ab\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{\sec(c+dx)}{\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}-\int\frac{ab-2a^2\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)+$$

$$\frac{b\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}$$

$\downarrow$ 3042

$$\frac{1}{2}\left(\frac{3ab\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}-\int\frac{ab-2a^2\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx\right.}}\right.$$

$$\frac{b\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}$$

$\downarrow$ 3284

$$\frac{1}{2}\left(\frac{6ab\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}-\int\frac{ab-2a^2\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right.}}\right.$$

$$\frac{b\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}$$

$\downarrow$ 4523

$$\frac{1}{2}\left(\left(2a^2+b^2\right)\int\frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}dx-b\int\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}dx+\frac{6ab\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\text{EllipticPi}}{d\sqrt{a+b\sec(c+dx)}}\right.$$

$$\frac{b\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}$$

$\downarrow$ 3042

3.635. $\int\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}\,dx$

$$\frac{1}{2}\left(\left(2a^2+b^2\right)\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-b\int \frac{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{6ab\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}{d\sqrt{a+b\,\mathrm{se}}}\right.$$

$$\frac{b\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}$$

$\downarrow$ 4343

$$\frac{1}{2}\left(\left(2a^2+b^2\right)\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{b\sqrt{a+b\sec(c+dx)}\int\sqrt{b+a\cos(c+dx)}dx}{\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}+\frac{6ab\sqrt{\sec(c+dx)}}{}\right.$$

$$\frac{b\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}$$

$\downarrow$ 3042

$$\frac{1}{2}\left(\left(2a^2+b^2\right)\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{b\sqrt{a+b\sec(c+dx)}\int\sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}+\frac{6ab\sqrt{\sec(c+}}{}\right.$$

$$\frac{b\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}$$

$\downarrow$ 3134

$$\frac{1}{2}\left(\left(2a^2+b^2\right)\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{b\sqrt{a+b\sec(c+dx)}\int\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}dx}{\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{6ab\sqrt{\sec(c+dx)}}{}\right.$$

$$\frac{b\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}$$

$\downarrow$ 3042

$$\frac{1}{2}\left(\left(2a^2+b^2\right)\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{b\sqrt{a+b\sec(c+dx)}\int\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}dx}{\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{6ab\sqrt{\sec(c+d}}{}\right.$$

$$\frac{b\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}$$

$\downarrow$ 3132

3.635. $\int \sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}\,dx$

$$\frac{1}{2}\left(\left(2a^2+b^2\right)\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2b\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{6ab\sqrt{\sec(c+dx)}\sqrt{\frac{a\,\mathrm{co}}{}}}{d}\right.$$

$$\frac{b\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}$$

$\downarrow$ 4345

$$\frac{1}{2}\left(\frac{\left(2a^2+b^2\right)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{1}{\sqrt{b+a\cos(c+dx)}}dx}{\sqrt{a+b\sec(c+dx)}}-\frac{2b\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\right.$$

$$\frac{b\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}$$

$\downarrow$ 3042

$$\frac{1}{2}\left(\frac{\left(2a^2+b^2\right)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{1}{\sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{\sqrt{a+b\sec(c+dx)}}-\frac{2b\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}\right.$$

$$\frac{b\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}$$

$\downarrow$ 3142

$$\frac{1}{2}\left(\frac{\left(2a^2+b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}-\frac{2b\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{6ab}{}\right.$$

$$\frac{b\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}$$

$\downarrow$ 3042

$$\frac{1}{2}\left(\frac{\left(2a^2+b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}-\frac{2b\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\right.$$

$$\frac{b\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}$$

$\downarrow$ 3140

3.635. $\int \sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}\,dx$

$$\frac{1}{2}\left(\frac{2(2a^2+b^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}-\frac{2b\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}\right.$$
$$\frac{b\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}$$

input
```
Int[Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^(3/2),x]
```

output
```
((2*(2*a^2 + b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2
, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(d*Sqrt[a + b*Sec[c + d*x]]) + (6*a*b
*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a +
b)]*Sqrt[Sec[c + d*x]])/(d*Sqrt[a + b*Sec[c + d*x]]) - (2*b*EllipticE[(c +
 d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(d*Sqrt[(b + a*Cos[c + d
*x])/(a + b)]*Sqrt[Sec[c + d*x]]))/2 + (b*Sqrt[Sec[c + d*x]]*Sqrt[a + b*Se
c[c + d*x]]*Sin[c + d*x])/d
```

3.635.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3286
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]]   Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
 d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

rule 4343
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]])   Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a
, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4345
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]])   Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[
{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4346
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Simp[d*Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x
]]/Sqrt[a + b*Csc[e + f*x]])   Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

3.635. $\int \sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}\,dx$

rule 4353
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b)*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
1)*((d*Csc[e + f*x])^(n - 1)/(f*(m + n - 1))), x] + Simp[d/(m + n - 1)   In
t[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^(n - 1)*Simp[a*b*(n - 1) +
(b^2*(m + n - 2) + a^2*(m + n - 1))*Csc[e + f*x] + a*b*(2*m + n - 2)*Csc[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && L
tQ[0, m, 2] && LtQ[0, n, 3] && NeQ[m + n - 1, 0] && (IntegerQ[m] || Integer
sQ[2*m, 2*n])
```

rule 4523
```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Simp[A/a   I
nt[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Simp[(A*b - a*B)
/(a*d)   Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ
[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

rule 4596
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Simp[C/d^2   Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*C
sc[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[
a + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
 b^2, 0]
```

3.635.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 8.89 (sec) , antiderivative size = 1585, normalized size of antiderivative = 6.37

method	result	size
default	Expression too large to display	1585

input
```
int(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/d/((a-b)/(a+b))^(1/2)*(2*(1/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+
c))/(cos(d*x+c)+1))^(1/2)*EllipticF(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d
*x+c)),(-(a+b)/(a-b))^(1/2))*a^2*cos(d*x+c)^3-2*(1/(cos(d*x+c)+1))^(1/2)*(
1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF(((a-b)/(a+b))^(1/
2)*(-cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*a*b*cos(d*x+c)^3+6*(1/(c
os(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ellipt
icPi(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(a+b)/(a-b),I/((a-b)/(a+
b))^(1/2))*a*b*cos(d*x+c)^3-(1/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x
+c))/(cos(d*x+c)+1))^(1/2)*EllipticE(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(
d*x+c)),(-(a+b)/(a-b))^(1/2))*a*b*cos(d*x+c)^3+(1/(cos(d*x+c)+1))^(1/2)*(1
/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE(((a-b)/(a+b))^(1/2
)*(-cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*b^2*cos(d*x+c)^3+4*(1/(co
s(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ellipti
cF(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*a^2*
cos(d*x+c)^2-4*(1/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x
+c)+1))^(1/2)*EllipticF(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(-(a+
b)/(a-b))^(1/2))*a*b*cos(d*x+c)^2+12*(1/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+
a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi(((a-b)/(a+b))^(1/2)*(-cot(d
*x+c)+csc(d*x+c)),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*a*b*cos(d*x+c)^2-2*(1
/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*...
```

3.635.5 Fricas [F(-1)]

Timed out.

$$\int \sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.635.6 Sympy [F]

$$\int \sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}\,dx = \int (a+b\sec(c+dx))^{\frac{3}{2}}\sqrt{\sec(c+dx)}\,dx$$

input
```
integrate(sec(d*x+c)**(1/2)*(a+b*sec(d*x+c))**(3/2),x)
```

output
```
Integral((a + b*sec(c + d*x))**(3/2)*sqrt(sec(c + d*x)), x)
```

3.635.7 Maxima [F]

$$\int \sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}\,dx = \int (b\sec(dx+c)+a)^{\frac{3}{2}}\sqrt{\sec(dx+c)}\,dx$$

input
```
integrate(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^(3/2)*sqrt(sec(d*x + c)), x)
```

3.635.8 Giac [F]

$$\int \sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}\,dx = \int (b\sec(dx+c)+a)^{\frac{3}{2}}\sqrt{\sec(dx+c)}\,dx$$

input
```
integrate(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^(3/2)*sqrt(sec(d*x + c)), x)
```

3.635.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}\,dx=\int \left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)^{3/2}\sqrt{\frac{1}{\cos\left(c+d\,x\right)}}\,dx$$

input
```
int((a + b/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(1/2),x)
```

output
```
int((a + b/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(1/2), x)
```

3.636 $\int \frac{(a+b\sec(c+dx))^{3/2}}{\sqrt{\sec(c+dx)}}\,dx$

3.636.1 Optimal result 4895
3.636.2 Mathematica [A] (verified) 4896
3.636.3 Rubi [A] (verified) 4896
3.636.4 Maple [C] (warning: unable to verify) 4902
3.636.5 Fricas [F] 4903
3.636.6 Sympy [F] 4904
3.636.7 Maxima [F] 4904
3.636.8 Giac [F] 4904
3.636.9 Mupad [F(-1)] 4905

3.636.1 Optimal result

Integrand size = 25, antiderivative size = 209

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sqrt{\sec(c+dx)}}\,dx = \frac{2ab\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{d\sqrt{a+b\sec(c+dx)}} + \frac{2b^2\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{d\sqrt{a+b\sec(c+dx)}} + \frac{2aE\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+dx)}}{d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\sqrt{\sec(c+dx)}}$$

output
```
2*a*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*
x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)*sec(d*x+c
)^(1/2)/d/(a+b*sec(d*x+c))^(1/2)+2*b^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/
2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2,2^(1/2)*(a/(a+b))^(1/2))*((b+
a*cos(d*x+c))/(a+b))^(1/2)*sec(d*x+c)^(1/2)/d/(a+b*sec(d*x+c))^(1/2)+2*a*(
cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c
),2^(1/2)*(a/(a+b))^(1/2))*(a+b*sec(d*x+c))^(1/2)/d/((b+a*cos(d*x+c))/(a+b
))^(1/2)/sec(d*x+c)^(1/2)
```

3.636.2 Mathematica [A] (verified)

Time = 27.26 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.62

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sqrt{\sec(c+dx)}}\,dx = \frac{2\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\left(a(a+b)E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)+b\left(a\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)+\right.\right.}{d(b+a\cos(c+dx))^2\sec^{\frac{3}{2}}(c}$$

input
```
Integrate[(a + b*Sec[c + d*x])^(3/2)/Sqrt[Sec[c + d*x]],x]
```

output
```
(2*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*(a*(a + b)*EllipticE[(c + d*x)/2, (2
*a)/(a + b)] + b*(a*EllipticF[(c + d*x)/2, (2*a)/(a + b)] + b*EllipticPi[2
, (c + d*x)/2, (2*a)/(a + b)]))*(a + b*Sec[c + d*x])^(3/2))/(d*(b + a*Cos[
c + d*x])^2*Sec[c + d*x]^(3/2))
```

3.636.3 Rubi [A] (verified)

Time = 2.01 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.800$, Rules used = {3042, 4355, 3042, 4341, 3042, 4343, 3042, 3134, 3042, 3132, 4345, 3042, 3142, 3042, 3140, 4346, 3042, 3286, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sqrt{\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4355}$$

$$a\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}dx+b\int \sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$a\int \frac{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+b\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{4341}$$

$$a\int \frac{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+b\left(b\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx+a\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}dx\right)$$

$$\Big\downarrow \text{3042}$$

$$b\left(a\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+b\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)+a\int \frac{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4343}$$

$$b\left(a\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+b\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)+\frac{a\sqrt{a+b\sec(c+dx)}\int \sqrt{b+a\cos(c+dx)}dx}{\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}$$

$$\Big\downarrow \text{3042}$$

$$b\left(a\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+b\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)+\frac{a\sqrt{a+b\sec(c+dx)}\int \sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}$$

$$\Big\downarrow \text{3134}$$

$$b\left(a\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+b\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)+\frac{a\sqrt{a+b\sec(c+dx)}\int \sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}dx}{\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}$$

$$\Big\downarrow \text{3042}$$

3.636. $\int \frac{(a+b\sec(c+dx))^{3/2}}{\sqrt{\sec(c+dx)}}\,dx$

$$b\left(a\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+b\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)+\frac{a\sqrt{a+b\sec(c+dx)}\int \sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}dx}{\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}$$

$\downarrow$ 3132

$$b\left(a\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+b\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)+\frac{2a\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}$$

$\downarrow$ 4345

$$b\left(b\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{a\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int \frac{1}{\sqrt{b+a\cos(c+dx)}}dx}{\sqrt{a+b\sec(c+dx)}}\right)+\frac{2a\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}$$

$\downarrow$ 3042

$$b\left(b\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{a\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int \frac{1}{\sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{\sqrt{a+b\sec(c+dx)}}\right)+\frac{2a\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}$$

$\downarrow$ 3142

$$b\left(b\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int \frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}\right)+\frac{2a\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}$$

$\downarrow$ 3042

3.636. $\int \frac{(a+b\sec(c+dx))^{3/2}}{\sqrt{\sec(c+dx)}}\,dx$

$$b\left(b\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int \frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}\right)+\frac{2a\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}$$

$\downarrow$ 3140

$$b\left(b\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}\right)+\frac{2a\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}$$

$\downarrow$ 4346

$$b\left(\frac{b\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int \frac{\sec(c+dx)}{\sqrt{b+a\cos(c+dx)}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{2a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2}{a}\right.}{d\sqrt{a+b\sec(c+dx)}}\right.$$
$$\frac{2a\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}$$

$\downarrow$ 3042

$$b\left(\frac{b\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{2a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}}{d\sqrt{a+b\sec(c+dx)}}\right.$$
$$\frac{2a\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}$$

$\downarrow$ 3286

$$b\left(\frac{b\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int \frac{\sec(c+dx)}{\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{2a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}\right.$$
$$\frac{2a\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}$$

3.636. $\int \frac{(a+b\sec(c+dx))^{3/2}}{\sqrt{\sec(c+dx)}}\,dx$

$$\Big\downarrow \; 3042$$

$$b\left(\frac{b\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{2a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}\right.}{d\sqrt{a+b\sec(c+dx)}}\right.$$

$$\frac{2a\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}$$

$$\Big\downarrow \; 3284$$

$$\frac{2a\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+$$

$$b\left(\frac{2a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}+\frac{2b\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticPi}\left(2,\frac{1}{2}(c+\right.}{d\sqrt{a+b\sec(c+dx)}}\right.$$

input
```
Int[(a + b*Sec[c + d*x])^(3/2)/Sqrt[Sec[c + d*x]],x]
```

output
```
(2*a*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(d*Sq
rt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]) + b*((2*a*Sqrt[(b + a
*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c +
 d*x]])/(d*Sqrt[a + b*Sec[c + d*x]]) + (2*b*Sqrt[(b + a*Cos[c + d*x])/(a +
 b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(d*Sqrt
[a + b*Sec[c + d*x]]))
```

3.636.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3286
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]]   Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
 d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

rule 4341
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[a   Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e +
f*x]]], x], x] + Simp[b/d   Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Csc[e + f*
x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4343
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]])   Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a
, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

3.636. $\int \frac{(a+b\sec(c+dx))^{3/2}}{\sqrt{\sec(c+dx)}}\,dx$

rule 4345
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]])   Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[
{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4346
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Simp[d*Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x
]]/Sqrt[a + b*Csc[e + f*x]])   Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4355
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_
)]*(d_.)], x_Symbol] :> Simp[a   Int[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e
+ f*x]], x], x] + Simp[b/d   Int[Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*
x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

3.636.4 Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 7.58 (sec) , antiderivative size = 1163, normalized size of antiderivative = 5.56

method	result	size
default	Expression too large to display	1163

input
```
int((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/d/((a-b)/(a+b))^(1/2)*(((a-b)/(a+b))^(1/2)*a^2*(1-cos(d*x+c))^3*csc(d*x+
c)^3-((a-b)/(a+b))^(1/2)*a*b*(1-cos(d*x+c))^3*csc(d*x+c)^3+(-(a*(1-cos(d*x
+c))^2*csc(d*x+c)^2-b*(1-cos(d*x+c))^2*csc(d*x+c)^2-a-b)/(a+b))^(1/2)*((1-
cos(d*x+c))^2*csc(d*x+c)^2+1)^(1/2)*EllipticF(((a-b)/(a+b))^(1/2)*(-cot(d*
x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*a^2-2*(-(a*(1-cos(d*x+c))^2*csc(d*x
+c)^2-b*(1-cos(d*x+c))^2*csc(d*x+c)^2-a-b)/(a+b))^(1/2)*((1-cos(d*x+c))^2*
csc(d*x+c)^2+1)^(1/2)*EllipticF(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c
)),(-(a+b)/(a-b))^(1/2))*a*b+(-(a*(1-cos(d*x+c))^2*csc(d*x+c)^2-b*(1-cos(d
*x+c))^2*csc(d*x+c)^2-a-b)/(a+b))^(1/2)*((1-cos(d*x+c))^2*csc(d*x+c)^2+1)^
(1/2)*EllipticF(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b)
)^(1/2))*b^2-(-(a*(1-cos(d*x+c))^2*csc(d*x+c)^2-b*(1-cos(d*x+c))^2*csc(d*x
+c)^2-a-b)/(a+b))^(1/2)*((1-cos(d*x+c))^2*csc(d*x+c)^2+1)^(1/2)*EllipticE(
((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*a^2+(-(
a*(1-cos(d*x+c))^2*csc(d*x+c)^2-b*(1-cos(d*x+c))^2*csc(d*x+c)^2-a-b)/(a+b)
)^(1/2)*((1-cos(d*x+c))^2*csc(d*x+c)^2+1)^(1/2)*EllipticE(((a-b)/(a+b))^(1
/2)*(-cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*a*b-2*(-(a*(1-cos(d*x+c
))^2*csc(d*x+c)^2-b*(1-cos(d*x+c))^2*csc(d*x+c)^2-a-b)/(a+b))^(1/2)*((1-co
s(d*x+c))^2*csc(d*x+c)^2+1)^(1/2)*EllipticPi(((a-b)/(a+b))^(1/2)*(-cot(d*x
+c)+csc(d*x+c)),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*b^2-((a-b)/(a+b))^(1/2)
*a^2*(-cot(d*x+c)+csc(d*x+c))-((a-b)/(a+b))^(1/2)*a*b*(-cot(d*x+c)+csc(...
```

3.636.5 Fricas [F]

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sqrt{\sec(c+dx)}}\,dx=\int \frac{(b\sec{(dx+c)}+a)^{\frac{3}{2}}}{\sqrt{\sec{(dx+c)}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
integral((b*sec(d*x + c) + a)^(3/2)/sqrt(sec(d*x + c)), x)
```

3.636.6 Sympy [F]

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sqrt{\sec(c+dx)}}\,dx=\int \frac{(a+b\sec{(c+dx)})^{\frac{3}{2}}}{\sqrt{\sec{(c+dx)}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))**(3/2)/sec(d*x+c)**(1/2),x)
```

output
```
Integral((a + b*sec(c + d*x))**(3/2)/sqrt(sec(c + d*x)), x)
```

3.636.7 Maxima [F]

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sqrt{\sec(c+dx)}}\,dx=\int \frac{(b\sec{(dx+c)}+a)^{\frac{3}{2}}}{\sqrt{\sec{(dx+c)}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^(3/2)/sqrt(sec(d*x + c)), x)
```

3.636.8 Giac [F]

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sqrt{\sec(c+dx)}}\,dx=\int \frac{(b\sec{(dx+c)}+a)^{\frac{3}{2}}}{\sqrt{\sec{(dx+c)}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^(3/2)/sqrt(sec(d*x + c)), x)
```

3.636.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sqrt{\sec(c+dx)}}\,dx=\int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}}{\sqrt{\frac{1}{\cos(c+d\,x)}}}\,dx$$

input
```
int((a + b/cos(c + d*x))^(3/2)/(1/cos(c + d*x))^(1/2),x)
```

output
```
int((a + b/cos(c + d*x))^(3/2)/(1/cos(c + d*x))^(1/2), x)
```

3.637 $\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

3.637.1 Optimal result 4906
3.637.2 Mathematica [A] (verified) 4907
3.637.3 Rubi [A] (verified) 4907
3.637.4 Maple [B] (verified) 4912
3.637.5 Fricas [C] (verification not implemented) 4912
3.637.6 Sympy [F] 4913
3.637.7 Maxima [F] 4913
3.637.8 Giac [F] 4914
3.637.9 Mupad [F(-1)] 4914

3.637.1 Optimal result

Integrand size = 25, antiderivative size = 187

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \frac{2(a^2-b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{3d\sqrt{a+b\sec(c+dx)}} + \frac{8bE\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+dx)}}{3d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\sqrt{\sec(c+dx)}} + \frac{2a\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}$$

output
```
2/3*(a^2-b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(si
n(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)*s
ec(d*x+c)^(1/2)/d/(a+b*sec(d*x+c))^(1/2)+2/3*a*sin(d*x+c)*(a+b*sec(d*x+c))
^(1/2)/d/sec(d*x+c)^(1/2)+8/3*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1
/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*(a+b*sec(d*x+c
))^(1/2)/d/((b+a*cos(d*x+c))/(a+b))^(1/2)/sec(d*x+c)^(1/2)
```

3.637.2 Mathematica [A] (verified)

Time = 0.67 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.83

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \frac{(a+b\sec(c+dx))^{3/2}\left(8b(a+b)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)+2(a^2-b^2)\right.}{3d(b+a\cos(c+}$$

input
```
Integrate[(a + b*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(3/2),x]
```

output
```
((a + b*Sec[c + d*x])^(3/2)*(8*b*(a + b)*Sqrt[(b + a*Cos[c + d*x])/(a + b)
]*EllipticE[(c + d*x)/2, (2*a)/(a + b)] + 2*(a^2 - b^2)*Sqrt[(b + a*Cos[c
+ d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)] + 2*a*(b + a*Cos[c
+ d*x])*Sin[c + d*x]))/(3*d*(b + a*Cos[c + d*x])^2*Sec[c + d*x]^(3/2))
```

3.637.3 Rubi [A] (verified)

Time = 1.42 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.01, number of steps used = 16, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.640$, Rules used = {3042, 4351, 25, 3042, 4523, 3042, 4343, 3042, 3134, 3042, 3132, 4345, 3042, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{3}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{4351}$$

$$\frac{2a\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3d\sqrt{\sec(c+dx)}}-\frac{1}{3}\int -\frac{4ab+\left(a^2+3b^2\right)\sec(c+dx)}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{3}\int \frac{4ab+\left(a^2+3b^2\right)\sec(c+dx)}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx+\frac{2a\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3d\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\int \frac{4ab+\left(a^2+3b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3d\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{4523}$$

$$\frac{1}{3}\left(\left(a^2-b^2\right)\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}dx+4b\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}dx\right)+\frac{2a\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3d\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\left(\left(a^2-b^2\right)\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+4b\int \frac{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)+\frac{2a\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3d\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{4343}$$

$$\frac{1}{3}\left(\left(a^2-b^2\right)\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{4b\sqrt{a+b\sec(c+dx)}\int \sqrt{b+a\cos(c+dx)}dx}{\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}\right)+\frac{2a\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3d\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\left(\left(a^2-b^2\right)\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{4b\sqrt{a+b\sec(c+dx)}\int \sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}\right)+\frac{2a\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3d\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{3134}$$

$$\frac{1}{3}\left(\left(a^2-b^2\right)\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{4b\sqrt{a+b\sec(c+dx)}\int \sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}dx}{\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}\right)+\frac{2a\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3d\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

3.637. $\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

$$\frac{1}{3}\left(\left(a^2-b^2\right)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{4b\sqrt{a+b\sec(c+dx)}\int\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}dx}{\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}\right)+\frac{2a\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3d\sqrt{\sec(c+dx)}}$$

$\Big\downarrow$ 3132

$$\frac{1}{3}\left(\left(a^2-b^2\right)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{8b\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}\right)+\frac{2a\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3d\sqrt{\sec(c+dx)}}$$

$\Big\downarrow$ 4345

$$\frac{1}{3}\left(\frac{\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{1}{\sqrt{b+a\cos(c+dx)}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{8b\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}\right)+\frac{2a\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3d\sqrt{\sec(c+dx)}}$$

$\Big\downarrow$ 3042

$$\frac{1}{3}\left(\frac{\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{1}{\sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{8b\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}\right)\frac{2a\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3d\sqrt{\sec(c+dx)}}$$

$\Big\downarrow$ 3142

$$\frac{1}{3}\left(\frac{\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{8b\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}\right)+\frac{2a\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3d\sqrt{\sec(c+dx)}}$$

$\Big\downarrow$ 3042

3.637. $\int\frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

$$\frac{1}{3}\left(\frac{(a^2-b^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{8b\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}\right)+\frac{2a\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3d\sqrt{\sec(c+dx)}}$$

$\Big\downarrow$ 3140

$$\frac{1}{3}\left(\frac{2(a^2-b^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}+\frac{8b\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}\right)+\frac{2a\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3d\sqrt{\sec(c+dx)}}$$

input
```
Int[(a + b*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(3/2),x]
```

output
```
((2*(a^2 - b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2,
(2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(d*Sqrt[a + b*Sec[c + d*x]]) + (8*b*Ell
ipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(d*Sqrt[(b +
a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]))/3 + (2*a*Sqrt[a + b*Sec[c +
d*x]]*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])
```

3.637.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 4343
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]])   Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a
, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4345
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]])   Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[
{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4351
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(3/2), x_Symbol] :> Simp[a*Cot[e + f*x]*Sqrt[a + b*Csc[e + f*x]]*((d*C
sc[e + f*x])^n/(f*n)), x] + Simp[1/(2*d*n)   Int[((d*Csc[e + f*x])^(n + 1)/
Sqrt[a + b*Csc[e + f*x]])*Simp[a*b*(2*n - 1) + 2*(b^2*n + a^2*(n + 1))*Csc[
e + f*x] + a*b*(2*n + 3)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f
}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1] && IntegersQ[2*n]
```

rule 4523
```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Simp[A/a   I
nt[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Simp[(A*b - a*B)
/(a*d)   Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ
[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

3.637. $\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

3.637.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1569 vs. $2(223) = 446$.

Time = 5.30 (sec) , antiderivative size = 1570, normalized size of antiderivative = 8.40

method	result	size
default	Expression too large to display	1570

input
```
int((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/3/d/((a-b)/(a+b))^(1/2)*(a+b*sec(d*x+c))^(1/2)/(b+a*cos(d*x+c))/sec(d*x+
c)^(3/2)/(cos(d*x+c)+1)*((1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(
1/(cos(d*x+c)+1))^(1/2)*EllipticF(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x
+c)),(-(a+b)/(a-b))^(1/2))*a^2*cos(d*x+c)-4*(1/(a+b)*(b+a*cos(d*x+c))/(cos
(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF(((a-b)/(a+b))^(1/2)*(
-cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*a*b*cos(d*x+c)+3*(1/(cos(d*x
+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF(((
a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*b^2*cos(d
*x+c)+4*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))
^(1/2)*EllipticE(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b
))^(1/2))*a*b*cos(d*x+c)-4*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)
*(1/(cos(d*x+c)+1))^(1/2)*EllipticE(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d
*x+c)),(-(a+b)/(a-b))^(1/2))*b^2*cos(d*x+c)+2*(1/(a+b)*(b+a*cos(d*x+c))/(c
os(d*x+c)+1))^(1/2)*EllipticF(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c))
,(-(a+b)/(a-b))^(1/2))*(1/(cos(d*x+c)+1))^(1/2)*a^2-8*(1/(a+b)*(b+a*cos(d*
x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc
(d*x+c)),(-(a+b)/(a-b))^(1/2))*(1/(cos(d*x+c)+1))^(1/2)*a*b+6*(1/(cos(d*x+
c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF(((a
-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*b^2+8*(1/(
a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE(((a-b)/(a+b))^(1/...
```

3.637.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 415, normalized size of antiderivative = 2.22

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \frac{6\,a^2\sqrt{\frac{a\cos(dx+c)+b}{\cos(dx+c)}}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)+12i\,\sqrt{2}a^{\frac{3}{2}}b\text{weierstrassZeta}\left(-\frac{4}{}\right.}{}$$

input
```
integrate((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(3/2),x, algorithm="fricas")
```

output
```
1/9*(6*a^2*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(
d*x + c) + 12*I*sqrt(2)*a^(3/2)*b*weierstrassZeta(-4/3*(3*a^2 - 4*b^2)/a^2
, 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2
, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x + c) +
 2*b)/a)) - 12*I*sqrt(2)*a^(3/2)*b*weierstrassZeta(-4/3*(3*a^2 - 4*b^2)/a^
2, 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^
2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c)
+ 2*b)/a)) + sqrt(2)*(-3*I*a^2 - I*b^2)*sqrt(a)*weierstrassPInverse(-4/3*(
3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*
I*a*sin(d*x + c) + 2*b)/a) + sqrt(2)*(3*I*a^2 + I*b^2)*sqrt(a)*weierstrass
PInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*co
s(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a))/(a*d)
```

3.637.6 Sympy [F]

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx=\int \frac{(a+b\sec{(c+dx)})^{\frac{3}{2}}}{\sec^{\frac{3}{2}}{(c+dx)}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))**(3/2)/sec(d*x+c)**(3/2),x)
```

output
```
Integral((a + b*sec(c + d*x))**(3/2)/sec(c + d*x)**(3/2), x)
```

3.637.7 Maxima [F]

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx=\int \frac{(b\sec{(dx+c)}+a)^{\frac{3}{2}}}{\sec{(dx+c)}^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^(3/2)/sec(d*x + c)^(3/2), x)
```

3.637.8 Giac [F]

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx=\int \frac{(b\sec(dx+c)+a)^{\frac{3}{2}}}{\sec(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^(3/2)/sec(d*x + c)^(3/2), x)
```

3.637.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx=\int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}}{\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

input
```
int((a + b/cos(c + d*x))^(3/2)/(1/cos(c + d*x))^(3/2),x)
```

output
```
int((a + b/cos(c + d*x))^(3/2)/(1/cos(c + d*x))^(3/2), x)
```

3.638 $\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{5}{2}}(c+dx)}\,dx$

3.638.1 Optimal result 4915
3.638.2 Mathematica [A] (verified) 4916
3.638.3 Rubi [A] (verified) 4916
3.638.4 Maple [B] (verified) 4922
3.638.5 Fricas [C] (verification not implemented) 4923
3.638.6 Sympy [F] 4924
3.638.7 Maxima [F] 4924
3.638.8 Giac [F] 4925
3.638.9 Mupad [F(-1)] 4925

3.638.1 Optimal result

Integrand size = 25, antiderivative size = 240

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \frac{2b(a^2-b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{5ad\sqrt{a+b\sec(c+dx)}} + \frac{2(3a^2+b^2)\,E\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+dx)}}{5ad\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\sqrt{\sec(c+dx)}} + \frac{2a\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{4b\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{5d\sqrt{\sec(c+dx)}}$$

output
```
2/5*b*(a^2-b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(
sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)
*sec(d*x+c)^(1/2)/a/d/(a+b*sec(d*x+c))^(1/2)+2/5*a*sin(d*x+c)*(a+b*sec(d*x
+c))^(1/2)/d/sec(d*x+c)^(3/2)+4/5*b*sin(d*x+c)*(a+b*sec(d*x+c))^(1/2)/d/se
c(d*x+c)^(1/2)+2/5*(3*a^2+b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/
2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*(a+b*sec(d*x+c)
)^(1/2)/a/d/((b+a*cos(d*x+c))/(a+b))^(1/2)/sec(d*x+c)^(1/2)
```

3.638.2 Mathematica [A] (verified)

Time = 1.03 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.82

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \frac{(a+b\sec(c+dx))^{3/2}\left(4(3a^3+3a^2b+ab^2+b^3)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}E\left(\frac{1}{2}(c+dx)\middle|\frac{2}{a}\right.\right.}{}$$

input
```
Integrate[(a + b*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(5/2),x]
```

output
```
((a + b*Sec[c + d*x])^(3/2)*(4*(3*a^3 + 3*a^2*b + a*b^2 + b^3)*Sqrt[(b + a
*Cos[c + d*x])/(a + b)]*EllipticE[(c + d*x)/2, (2*a)/(a + b)] + 4*b*(a^2 -
 b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a +
 b)] + 2*a*(a^2 + 4*b^2 + 6*a*b*Cos[c + d*x] + a^2*Cos[2*(c + d*x)])*Sin[c
 + d*x]))/(10*a*d*(b + a*Cos[c + d*x])^2*Sec[c + d*x]^(3/2))
```

3.638.3 Rubi [A] (verified)

Time = 1.83 (sec) , antiderivative size = 238, normalized size of antiderivative = 0.99, number of steps used = 19, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.760$, Rules used = {3042, 4351, 25, 3042, 4592, 27, 3042, 4523, 3042, 4343, 3042, 3134, 3042, 3132, 4345, 3042, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{5}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx$$

$$\Big\downarrow \text{4351}$$

$$\frac{2a\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5d\sec^{\frac{3}{2}}(c+dx)}-\frac{1}{5}\int -\frac{2ab\sec^2(c+dx)+\left(3a^2+5b^2\right)\sec(c+dx)+6ab}{\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}dx$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{5}\int \frac{2ab\sec^2(c+dx)+\left(3a^2+5b^2\right)\sec(c+dx)+6ab}{\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}dx+\frac{2a\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}\int \frac{2ab\csc\left(c+dx+\frac{\pi}{2}\right)^2+\left(3a^2+5b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)+6ab}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{4592}$$

$$\frac{1}{5}\left(\frac{4b\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{d\sqrt{\sec(c+dx)}}-\frac{2\int -\frac{3\left(4b\sec(c+dx)a^2+\left(3a^2+b^2\right)a\right)}{2\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{3a}\right)+\frac{2a\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{5}\left(\frac{\int \frac{4b\sec(c+dx)a^2+\left(3a^2+b^2\right)a}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{a}+\frac{4b\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{d\sqrt{\sec(c+dx)}}\right)+\frac{2a\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}\left(\frac{\int \frac{4b\csc\left(c+dx+\frac{\pi}{2}\right)a^2+\left(3a^2+b^2\right)a}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}+\frac{4b\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{d\sqrt{\sec(c+dx)}}\right)+\frac{2a\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{4523}$$

$$\frac{1}{5}\left(\frac{b\left(a^2-b^2\right)\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}dx+\left(3a^2+b^2\right)\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}dx}{a}+\frac{4b\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{d\sqrt{\sec(c+dx)}}\right)+\frac{2a\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

3.638. $\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{5}{2}}(c+dx)}\,dx$

$$\frac{1}{5}\left(\frac{b(a^2-b^2)\int \frac{\sqrt{\csc(c+dx+\frac{\pi}{2})}}{\sqrt{a+b\csc(c+dx+\frac{\pi}{2})}}dx+(3a^2+b^2)\int \frac{\sqrt{a+b\csc(c+dx+\frac{\pi}{2})}}{\sqrt{\csc(c+dx+\frac{\pi}{2})}}dx}{a}+\frac{4b\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{d\sqrt{\sec(c+dx)}}\right)+\frac{2a\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$\Big\downarrow$ 4343

$$\frac{1}{5}\left(\frac{b(a^2-b^2)\int \frac{\sqrt{\csc(c+dx+\frac{\pi}{2})}}{\sqrt{a+b\csc(c+dx+\frac{\pi}{2})}}dx+\frac{(3a^2+b^2)\sqrt{a+b\sec(c+dx)}\int \sqrt{b+a\cos(c+dx)}dx}{\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}}{a}+\frac{4b\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{d\sqrt{\sec(c+dx)}}\right)+\frac{2a\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$\Big\downarrow$ 3042

$$\frac{1}{5}\left(\frac{b(a^2-b^2)\int \frac{\sqrt{\csc(c+dx+\frac{\pi}{2})}}{\sqrt{a+b\csc(c+dx+\frac{\pi}{2})}}dx+\frac{(3a^2+b^2)\sqrt{a+b\sec(c+dx)}\int \sqrt{b+a\sin(c+dx+\frac{\pi}{2})}dx}{\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}}{a}+\frac{4b\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{d\sqrt{\sec(c+dx)}}\right)+\frac{2a\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$\Big\downarrow$ 3134

$$\frac{1}{5}\left(\frac{b(a^2-b^2)\int \frac{\sqrt{\csc(c+dx+\frac{\pi}{2})}}{\sqrt{a+b\csc(c+dx+\frac{\pi}{2})}}dx+\frac{(3a^2+b^2)\sqrt{a+b\sec(c+dx)}\int \sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}dx}{\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{a}+\frac{4b\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{d\sqrt{\sec(c+dx)}}\right)+\frac{2a\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$\Big\downarrow$ 3042

3.638. $\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{5}{2}}(c+dx)}\,dx$

$$\frac{1}{5}\left(\frac{b\left(a^2-b^2\right)\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{\left(3a^2+b^2\right)\sqrt{a+b\sec(c+dx)}\int \sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}dx}{\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{a}+\frac{4b\sin(c+dx)\sqrt{a+b\sec(c+}}{d\sqrt{\sec(c+dx)}}\right.$$

$$\frac{2a\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$\downarrow$ 3132

$$\frac{1}{5}\left(\frac{b\left(a^2-b^2\right)\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\left(3a^2+b^2\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{a}+\frac{4b\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{d\sqrt{\sec(c+dx)}}\right)$$

$$\frac{2a\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$\downarrow$ 4345

$$\frac{1}{5}\left(\frac{\frac{b\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int \frac{1}{\sqrt{b+a\cos(c+dx)}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{2\left(3a^2+b^2\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{a}+\frac{4b\sin(c+dx)\sqrt{a+}}{d\sqrt{\sec(c+}}\right.$$

$$\frac{2a\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$\downarrow$ 3042

$$\frac{1}{5}\left(\frac{\frac{b\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int \frac{1}{\sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{2\left(3a^2+b^2\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{a}+\frac{4b\sin(c+dx)\sqrt{a}}{d\sqrt{\sec(}}\right.$$

$$\frac{2a\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$\downarrow$ 3142

3.638. $\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{5}{2}}(c+dx)}\,dx$

$$\frac{1}{5}\left(\frac{\frac{b(a^2-b^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{2(3a^2+b^2)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{a}+\frac{4b\sin(c+dx)\sqrt{a}}{d\sqrt{\sec(c}}\right.$$

$$\frac{2a\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$\downarrow$ 3042

$$\frac{1}{5}\left(\frac{\frac{b(a^2-b^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{2(3a^2+b^2)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{a}+\frac{4b\sin(c+dx)\sqrt{}}{d\sqrt{\text{se}}}\right.$$

$$\frac{2a\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$\downarrow$ 3140

$$\frac{1}{5}\left(\frac{\frac{2b(a^2-b^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}+\frac{2(3a^2+b^2)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{a}+\frac{4b\sin(c+dx)\sqrt{}}{d\sqrt{\text{sec}}}\right.$$

$$\frac{2a\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5d\sec^{\frac{3}{2}}(c+dx)}$$

input
```
Int[(a + b*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(5/2),x]
```

output
```
(2*a*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)) + (((
2*b*(a^2 - b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2,
(2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(d*Sqrt[a + b*Sec[c + d*x]]) + (2*(3*a^
2 + b^2)*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(
d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]))/a + (4*b*Sqrt[a
+ b*Sec[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Sec[c + d*x]]))/5
```

3.638. $\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{5}{2}}(c+dx)}\,dx$

3.638.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 4343
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]])   Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a
, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

3.638. $\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{5}{2}}(c+dx)}\,dx$

rule 4345
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]])   Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[
{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4351
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(3/2), x_Symbol] :> Simp[a*Cot[e + f*x]*Sqrt[a + b*Csc[e + f*x]]*((d*C
sc[e + f*x])^n/(f*n)), x] + Simp[1/(2*d*n)   Int[((d*Csc[e + f*x])^(n + 1)/
Sqrt[a + b*Csc[e + f*x]])*Simp[a*b*(2*n - 1) + 2*(b^2*n + a^2*(n + 1))*Csc[
e + f*x] + a*b*(2*n + 3)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f
}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1] && IntegersQ[2*n]
```

rule 4523
```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Simp[A/a   I
nt[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Simp[(A*b - a*B)
/(a*d)   Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ
[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

rule 4592
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Simp[1/(a*d*n)   Int[(a + b*Csc[e + f*x])^m
*(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*
Csc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

3.638.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2281 vs. $2(270) = 540$.

Time = 6.45 (sec) , antiderivative size = 2282, normalized size of antiderivative = 9.51

method	result	size
default	Expression too large to display	2282

input
```
int((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/5/d/((a-b)/(a+b))^(1/2)/a*(a+b*sec(d*x+c))^(1/2)/(b+a*cos(d*x+c))/sec(d*
x+c)^(5/2)/(cos(d*x+c)+1)*(-8*(1/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d
*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF(((a-b)/(a+b))^(1/2)*(cot(d*x+c)-csc
(d*x+c)),(-(a+b)/(a-b))^(1/2))*a^2*b*sec(d*x+c)+2*(1/(cos(d*x+c)+1))^(1/2)
*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF(((a-b)/(a+b))^(
1/2)*(cot(d*x+c)-csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*a*b^2*sec(d*x+c)+6*(1/(
cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ellip
ticE(((a-b)/(a+b))^(1/2)*(cot(d*x+c)-csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*a^2
*b*sec(d*x+c)-2*(1/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*
x+c)+1))^(1/2)*EllipticE(((a-b)/(a+b))^(1/2)*(cot(d*x+c)-csc(d*x+c)),(-(a+
b)/(a-b))^(1/2))*a*b^2*sec(d*x+c)-4*(1/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a
*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF(((a-b)/(a+b))^(1/2)*(cot(d*x+
c)-csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*a^2*b*sec(d*x+c)^2+(1/(cos(d*x+c)+1))
^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF(((a-b)/(a
+b))^(1/2)*(cot(d*x+c)-csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*a*b^2*sec(d*x+c)^
2+3*(1/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/
2)*EllipticE(((a-b)/(a+b))^(1/2)*(cot(d*x+c)-csc(d*x+c)),(-(a+b)/(a-b))^(1
/2))*a^2*b*sec(d*x+c)^2-(1/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))
/(cos(d*x+c)+1))^(1/2)*EllipticE(((a-b)/(a+b))^(1/2)*(cot(d*x+c)-csc(d*x+c
)),(-(a+b)/(a-b))^(1/2))*a*b^2*sec(d*x+c)^2+((a-b)/(a+b))^(1/2)*a^3*sin...
```

3.638.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 463, normalized size of antiderivative = 1.93

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{5}{2}}(c+dx)}\,dx =$$

$$-\frac{2\sqrt{2}(3i\,a^2b-i\,b^3)\sqrt{a}\text{weierstrassPInverse}\left(-\frac{4\,(3\,a^2-4\,b^2)}{3\,a^2},\frac{8\,(9\,a^2b-8\,b^3)}{27\,a^3},\frac{3\,a\cos(dx+c)+3i\,a\sin(dx+c)+2\,b}{3\,a}\right)+2\sqrt{2}(-3}{}$$

input
```
integrate((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(5/2),x, algorithm="fricas")
```

output
```
-1/15*(2*sqrt(2)*(3*I*a^2*b - I*b^3)*sqrt(a)*weierstrassPInverse(-4/3*(3*a
^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a
*sin(d*x + c) + 2*b)/a) + 2*sqrt(2)*(-3*I*a^2*b + I*b^3)*sqrt(a)*weierstra
ssPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*
cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a) + 3*sqrt(2)*(-3*I*a^3 - I*a*b^
2)*sqrt(a)*weierstrassZeta(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3
)/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3
)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x + c) + 2*b)/a)) + 3*sqrt(2)*(
3*I*a^3 + I*a*b^2)*sqrt(a)*weierstrassZeta(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*
(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*
(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a
)) - 6*(a^3*cos(d*x + c)^2 + 2*a^2*b*cos(d*x + c))*sqrt((a*cos(d*x + c) +
b)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^2*d)
```

3.638.6 Sympy [F]

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{5}{2}}(c+dx)}\,dx=\int \frac{(a+b\sec{(c+dx)})^{\frac{3}{2}}}{\sec^{\frac{5}{2}}{(c+dx)}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))**(3/2)/sec(d*x+c)**(5/2),x)
```

output
```
Integral((a + b*sec(c + d*x))**(3/2)/sec(c + d*x)**(5/2), x)
```

3.638.7 Maxima [F]

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{5}{2}}(c+dx)}\,dx=\int \frac{(b\sec{(dx+c)}+a)^{\frac{3}{2}}}{\sec{(dx+c)}^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^(3/2)/sec(d*x + c)^(5/2), x)
```

3.638.8 Giac [F]

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{5}{2}}(c+dx)}\,dx=\int \frac{(b\sec(dx+c)+a)^{\frac{3}{2}}}{\sec(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^(3/2)/sec(d*x + c)^(5/2), x)
```

3.638.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{5}{2}}(c+dx)}\,dx=\int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}}{\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

input
```
int((a + b/cos(c + d*x))^(3/2)/(1/cos(c + d*x))^(5/2),x)
```

output
```
int((a + b/cos(c + d*x))^(3/2)/(1/cos(c + d*x))^(5/2), x)
```

3.639 $\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{7}{2}}(c+dx)}\,dx$

3.639.1 Optimal result 4926
3.639.2 Mathematica [A] (verified) 4927
3.639.3 Rubi [A] (verified) 4927
3.639.4 Maple [B] (verified) 4935
3.639.5 Fricas [C] (verification not implemented) 4935
3.639.6 Sympy [F(-1)] 4936
3.639.7 Maxima [F] 4936
3.639.8 Giac [F] 4937
3.639.9 Mupad [F(-1)] 4937

3.639.1 Optimal result

Integrand size = 25, antiderivative size = 303

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{7}{2}}(c+dx)}\,dx = \frac{2(25a^4-31a^2b^2+6b^4)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{105a^2d\sqrt{a+b\sec(c+dx)}} + \frac{4b(41a^2-3b^2)\,E\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+dx)}}{105a^2d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\sqrt{\sec(c+dx)}} + \frac{2a\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{16b\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{35d\sec^{\frac{3}{2}}(c+dx)} + \frac{2(25a^2+3b^2)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{105ad\sqrt{\sec(c+dx)}}$$

output
```
2/105*(25*a^4-31*a^2*b^2+6*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1
/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+
c))/(a+b))^(1/2)*sec(d*x+c)^(1/2)/a^2/d/(a+b*sec(d*x+c))^(1/2)+2/7*a*sin(d
*x+c)*(a+b*sec(d*x+c))^(1/2)/d/sec(d*x+c)^(5/2)+16/35*b*sin(d*x+c)*(a+b*se
c(d*x+c))^(1/2)/d/sec(d*x+c)^(3/2)+2/105*(25*a^2+3*b^2)*sin(d*x+c)*(a+b*se
c(d*x+c))^(1/2)/a/d/sec(d*x+c)^(1/2)+4/105*b*(41*a^2-3*b^2)*(cos(1/2*d*x+1
/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/
(a+b))^(1/2))*(a+b*sec(d*x+c))^(1/2)/a^2/d/((b+a*cos(d*x+c))/(a+b))^(1/2)/
sec(d*x+c)^(1/2)
```

3.639.2 Mathematica [A] (verified)

Time = 1.43 (sec) , antiderivative size = 237, normalized size of antiderivative = 0.78

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{7}{2}}(c+dx)}\,dx = \frac{(a+b\sec(c+dx))^{3/2}\left(16b(41a^3+41a^2b-3ab^2-3b^3)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}E\left(\frac{1}{2}(c-\right.\right.}{}$$

input
```
Integrate[(a + b*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(7/2),x]
```

output
```
((a + b*Sec[c + d*x])^(3/2)*(16*b*(41*a^3 + 41*a^2*b - 3*a*b^2 - 3*b^3)*Sq
rt[(b + a*Cos[c + d*x])/(a + b)]*EllipticE[(c + d*x)/2, (2*a)/(a + b)] + 8
*(25*a^4 - 31*a^2*b^2 + 6*b^4)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Elliptic
F[(c + d*x)/2, (2*a)/(a + b)] + 2*a*(178*a^2*b + 12*b^3 + a*(145*a^2 + 108
*b^2)*Cos[c + d*x] + 78*a^2*b*Cos[2*(c + d*x)] + 15*a^3*Cos[3*(c + d*x)])*
Sin[c + d*x]))/(420*a^2*d*(b + a*Cos[c + d*x])^2*Sec[c + d*x]^(3/2))
```

3.639.3 Rubi [A] (verified)

Time = 2.43 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.03, number of steps used = 22, number of rules used = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.880$, Rules used = {3042, 4351, 25, 3042, 4592, 27, 3042, 4592, 27, 3042, 4523, 3042, 4343, 3042, 3134, 3042, 3132, 4345, 3042, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{7}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx$$

$$\Big\downarrow \text{4351}$$

$$\frac{2a\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{7d\sec^{\frac{5}{2}}(c+dx)}-\frac{1}{7}\int -\frac{4ab\sec^2(c+dx)+\left(5a^2+7b^2\right)\sec(c+dx)+8ab}{\sec^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}dx$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{7}\int \frac{4ab\sec^2(c+dx)+\left(5a^2+7b^2\right)\sec(c+dx)+8ab}{\sec^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}dx+\frac{2a\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{7}\int \frac{4ab\csc\left(c+dx+\frac{\pi}{2}\right)^2+\left(5a^2+7b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)+8ab}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \; 4592$$

$$\frac{1}{7}\left(\frac{16b\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5d\sec^{\frac{3}{2}}(c+dx)}-\frac{2\int -\frac{44b\sec(c+dx)a^2+16b^2\sec^2(c+dx)a+\left(25a^2+3b^2\right)a}{2\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}dx}{5a}\right)+\frac{2a\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{7}\left(\frac{\int \frac{44b\sec(c+dx)a^2+16b^2\sec^2(c+dx)a+\left(25a^2+3b^2\right)a}{\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}dx}{5a}+\frac{16b\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5d\sec^{\frac{3}{2}}(c+dx)}\right)+\frac{2a\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{7}\left(\frac{\int \frac{44b\csc\left(c+dx+\frac{\pi}{2}\right)a^2+16b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2a+\left(25a^2+3b^2\right)a}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{5a}+\frac{16b\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5d\sec^{\frac{3}{2}}(c+dx)}\right)+\frac{2a\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \; 4592$$

$$\frac{1}{7}\left(\frac{\frac{2\left(25a^2+3b^2\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3d\sqrt{\sec(c+dx)}}-\frac{2\int -\frac{\left(25a^2+51b^2\right)\sec(c+dx)a^2+2b\left(41a^2-3b^2\right)a}{2\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{3a}}{5a}+\frac{16b\sin(c+dx)\sqrt{a+b\sec(c+dx}}{5d\sec^{\frac{3}{2}}(c+dx)}\right.$$

$$\frac{2a\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{7d\sec^{\frac{5}{2}}(c+dx)}$$

3.639. $\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{7}{2}}(c+dx)}\,dx$

$$\Big\downarrow \; 27$$

$$\frac{1}{7}\left(\frac{\frac{\int \frac{\left(25a^2+51b^2\right)\sec(c+dx)a^2+2b\left(41a^2-3b^2\right)a}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{3a}+\frac{2\left(25a^2+3b^2\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3d\sqrt{\sec(c+dx)}}}{5a}+\frac{16b\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5d\sec^{\frac{3}{2}}(c+dx)}\right)$$

$$\frac{2a\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{7}\left(\frac{\frac{\int \frac{\left(25a^2+51b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)a^2+2b\left(41a^2-3b^2\right)a}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{3a}+\frac{2\left(25a^2+3b^2\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3d\sqrt{\sec(c+dx)}}}{5a}+\frac{16b\sin(c+dx)\sqrt{a+b\sec(c+dx}}{5d\sec^{\frac{3}{2}}(c+dx)}\right.$$

$$\frac{2a\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \; 4523$$

$$\frac{1}{7}\left(\frac{\frac{2b\left(41a^2-3b^2\right)\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}dx+\left(25a^4-31a^2b^2+6b^4\right)\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}dx}{3a}+\frac{2\left(25a^2+3b^2\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3d\sqrt{\sec(c+dx)}}}{5a}+\frac{16b\sin(c+}{5}\right.$$

$$\frac{2a\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{7}\left(\frac{\frac{2b\left(41a^2-3b^2\right)\int \frac{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\left(25a^4-31a^2b^2+6b^4\right)\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{3a}+\frac{2\left(25a^2+3b^2\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3d\sqrt{\sec(c+dx)}}}{5a}+\frac{16b}{}\right.$$

$$\frac{2a\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \; 4343$$

3.639. $\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{7}{2}}(c+dx)}\,dx$

$$\frac{1}{7}\left(\frac{\frac{\frac{2b\left(41a^2-3b^2\right)\sqrt{a+b\sec(c+dx)}\int\sqrt{b+a\cos(c+dx)}dx}{\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}+\left(25a^4-31a^2b^2+6b^4\right)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{3a}+\frac{2\left(25a^2+3b^2\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3d\sqrt{\sec(c+dx)}}}{5a}\right.$$

$$\frac{2a\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{7}\left(\frac{\frac{\frac{2b\left(41a^2-3b^2\right)\sqrt{a+b\sec(c+dx)}\int\sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}+\left(25a^4-31a^2b^2+6b^4\right)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{3a}+\frac{2\left(25a^2+3b^2\right)\sin(c+dx)\sqrt{a+b\sec(c+}}{3d\sqrt{\sec(c+dx)}}}{5a}\right.$$

$$\frac{2a\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \; 3134$$

$$\frac{1}{7}\left(\frac{\frac{\frac{2b\left(41a^2-3b^2\right)\sqrt{a+b\sec(c+dx)}\int\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}dx}{\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\left(25a^4-31a^2b^2+6b^4\right)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{3a}+\frac{2\left(25a^2+3b^2\right)\sin(c+dx)\sqrt{a+b\sec(c+}}{3d\sqrt{\sec(c+dx)}}}{5a}\right.$$

$$\frac{2a\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{7}\left(\frac{\frac{\frac{2b\left(41a^2-3b^2\right)\sqrt{a+b\sec(c+dx)}\int\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}dx}{\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\left(25a^4-31a^2b^2+6b^4\right)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{3a}+\frac{2\left(25a^2+3b^2\right)\sin(c+dx)\sqrt{a+b\sec}}{3d\sqrt{\sec(c+dx)}}}{5a}\right.$$

$$\frac{2a\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{7d\sec^{\frac{5}{2}}(c+dx)}$$

3.639. $\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{7}{2}}(c+dx)}\,dx$

$$\Big\downarrow \; 3132$$

$$\frac{1}{7}\left(\frac{\frac{\left(25a^4-31a^2b^2+6b^4\right)\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{4b\left(41a^2-3b^2\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{3a}+\frac{2\left(25a^2+3b^2\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3d\sqrt{\sec(c+dx)}}}{5a}+\right.$$

$$\frac{2a\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \; 4345$$

$$\frac{1}{7}\left(\frac{\frac{\left(25a^4-31a^2b^2+6b^4\right)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int \frac{1}{\sqrt{b+a\cos(c+dx)}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{4b\left(41a^2-3b^2\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{3a}+\frac{2\left(25a^2+3b^2\right)\sin(c+dx)}{3d\sqrt{\sec(c+}}}{5a}\right.$$

$$\frac{2a\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{7}\left(\frac{\frac{\left(25a^4-31a^2b^2+6b^4\right)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int \frac{1}{\sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{4b\left(41a^2-3b^2\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{3a}+\frac{2\left(25a^2+3b^2\right)\sin(c+}{3d\sqrt{\sec}}}{5a}\right.$$

$$\frac{2a\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \; 3142$$

3.639. $\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{7}{2}}(c+dx)}\,dx$

$$\frac{1}{7}\left(\frac{\frac{\frac{\left(25a^4-31a^2b^2+6b^4\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{4b\left(41a^2-3b^2\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{3a}+\frac{2\left(25a^2+3b^2\right)\sin(c+}{3d\sqrt{\sec}}}{5a}\right.$$

$$\frac{2a\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{7}\left(\frac{\frac{\frac{\left(25a^4-31a^2b^2+6b^4\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{4b\left(41a^2-3b^2\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{3a}+\frac{2\left(25a^2+3b^2\right)\sin}{3d}}{5a}\right.$$

$$\frac{2a\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \; 3140$$

$$\frac{1}{7}\left(\frac{\frac{2\left(25a^2+3b^2\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3d\sqrt{\sec(c+dx)}}+\frac{\frac{4b\left(41a^2-3b^2\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{2\left(25a^4-31a^2b^2+6b^4\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\mathrm{E}}{d\sqrt{a+b\sec(c+dx)}}}{3a}}{5a}\right.$$

$$\frac{2a\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{7d\sec^{\frac{5}{2}}(c+dx)}$$

input
```
Int[(a + b*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(7/2),x]
```

3.639. $\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{7}{2}}(c+dx)}\,dx$

output
```
(2*a*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/2)) + ((1
6*b*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)) + (((2
*(25*a^4 - 31*a^2*b^2 + 6*b^4)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Elliptic
F[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(d*Sqrt[a + b*Sec[c + d*
x]]) + (4*b*(41*a^2 - 3*b^2)*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a
+ b*Sec[c + d*x]])/(d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]
]))/(3*a) + (2*(25*a^2 + 3*b^2)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(3*
d*Sqrt[Sec[c + d*x]]))/(5*a))/7
```

3.639.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

3.639. $\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{7}{2}}(c+dx)}\,dx$

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 4343
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]])   Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a
, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4345
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]])   Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[
{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4351
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(3/2), x_Symbol] :> Simp[a*Cot[e + f*x]*Sqrt[a + b*Csc[e + f*x]]*((d*C
sc[e + f*x])^n/(f*n)), x] + Simp[1/(2*d*n)   Int[((d*Csc[e + f*x])^(n + 1)/
Sqrt[a + b*Csc[e + f*x]])*Simp[a*b*(2*n - 1) + 2*(b^2*n + a^2*(n + 1))*Csc[
e + f*x] + a*b*(2*n + 3)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f
}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1] && IntegersQ[2*n]
```

rule 4523
```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Simp[A/a   I
nt[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Simp[(A*b - a*B)
/(a*d)   Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ
[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

rule 4592
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Simp[1/(a*d*n)   Int[(a + b*Csc[e + f*x])^m
*(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*
Csc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

3.639. $\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{7}{2}}(c+dx)}\,dx$

3.639.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2765 vs. $2(327) = 654$.

Time = 7.75 (sec) , antiderivative size = 2766, normalized size of antiderivative = 9.13

method	result	size
default	Expression too large to display	2766

input
```
int((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
2/105/d/((a-b)/(a+b))^(1/2)/a^2*(-6*(1/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a
*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE(((a-b)/(a+b))^(1/2)*(-cot(d*x
+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*a*b^3*cos(d*x+c)^2+6*(1/(cos(d*x+c)+
1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE(((a-b)
/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*b^4+25*(1/(co
s(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ellipti
cF(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*a^4-
6*((a-b)/(a+b))^(1/2)*b^4*sin(d*x+c)-82*(1/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*
(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF(((a-b)/(a+b))^(1/2)*(-cot
(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*a^3*b*cos(d*x+c)^2+51*(1/(cos(d*
x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((
(a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*a^2*b^2*
cos(d*x+c)^2+6*(1/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x
+c)+1))^(1/2)*EllipticF(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(-(a+
b)/(a-b))^(1/2))*a*b^3*cos(d*x+c)^2+164*(1/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*
(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE(((a-b)/(a+b))^(1/2)*(-cot
(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*a^3*b*cos(d*x+c)-164*(1/(cos(d*x
+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE(((
a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*a^2*b^2*c
os(d*x+c)-12*(1/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*...
```

3.639.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.13 (sec) , antiderivative size = 501, normalized size of antiderivative = 1.65

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{7}{2}}(c+dx)}\,dx = \frac{\sqrt{2}(-75i\,a^4+11i\,a^2b^2-12i\,b^4)\sqrt{a}\text{weierstrassPInverse}\left(-\frac{4\,(3\,a^2-4\,b^2)}{3\,a^2},\frac{8\,(9\,a^2b-8}{27\,a^3}\right.}{}$$

input
```
integrate((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(7/2),x, algorithm="fricas")
```

output
```
1/315*(sqrt(2)*(-75*I*a^4 + 11*I*a^2*b^2 - 12*I*b^4)*sqrt(a)*weierstrassPI
nverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(
d*x + c) + 3*I*a*sin(d*x + c) + 2*b)/a) + sqrt(2)*(75*I*a^4 - 11*I*a^2*b^2
 + 12*I*b^4)*sqrt(a)*weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9
*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a)
- 6*sqrt(2)*(-41*I*a^3*b + 3*I*a*b^3)*sqrt(a)*weierstrassZeta(-4/3*(3*a^2
- 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a^2
- 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*si
n(d*x + c) + 2*b)/a)) - 6*sqrt(2)*(41*I*a^3*b - 3*I*a*b^3)*sqrt(a)*weierst
rassZeta(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrass
PInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*co
s(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a)) + 6*(15*a^4*cos(d*x + c)^3 + 24
*a^3*b*cos(d*x + c)^2 + (25*a^4 + 3*a^2*b^2)*cos(d*x + c))*sqrt((a*cos(d*x
 + c) + b)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^3*d)
```

3.639.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{7}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+b*sec(d*x+c))**(3/2)/sec(d*x+c)**(7/2),x)
```

output
```
Timed out
```

3.639.7 Maxima [F]

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{7}{2}}(c+dx)}\,dx = \int \frac{(b\sec(dx+c)+a)^{\frac{3}{2}}}{\sec(dx+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(7/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^(3/2)/sec(d*x + c)^(7/2), x)
```

3.639.8 Giac [F]

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{7}{2}}(c+dx)}\,dx=\int \frac{(b\sec(dx+c)+a)^{\frac{3}{2}}}{\sec(dx+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(7/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^(3/2)/sec(d*x + c)^(7/2), x)
```

3.639.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{7}{2}}(c+dx)}\,dx=\int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}}{\left(\frac{1}{\cos(c+d\,x)}\right)^{7/2}}\,dx$$

input
```
int((a + b/cos(c + d*x))^(3/2)/(1/cos(c + d*x))^(7/2),x)
```

output
```
int((a + b/cos(c + d*x))^(3/2)/(1/cos(c + d*x))^(7/2), x)
```

3.640 $\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$

3.640.1 Optimal result 4938
3.640.2 Mathematica [C] (verified) 4939
3.640.3 Rubi [A] (verified) 4940
3.640.4 Maple [C] (verified) 4950
3.640.5 Fricas [F(-1)] 4950
3.640.6 Sympy [F(-1)] 4951
3.640.7 Maxima [F] 4951
3.640.8 Giac [F] 4951
3.640.9 Mupad [F(-1)] 4952

3.640.1 Optimal result

Integrand size = 25, antiderivative size = 369

$$\begin{aligned}\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c&\\ +dx))^{5/2}\,dx &= \frac{b(59a^2+16b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{24d\sqrt{a+b\sec(c+dx)}}\\ &+\frac{5a(a^2+4b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{8d\sqrt{a+b\sec(c+dx)}}\\ &-\frac{(33a^2+16b^2)\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+dx)}}{24d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\sqrt{\sec(c+dx)}}\\ &+\frac{(33a^2+16b^2)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{24d}\\ &+\frac{13ab\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{12d}\\ &+\frac{b^2\sec^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{3d}\end{aligned}$$

output
```
1/24*b*(59*a^2+16*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ell
ipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b)
)^(1/2)*sec(d*x+c)^(1/2)/d/(a+b*sec(d*x+c))^(1/2)+5/8*a*(a^2+4*b^2)*(cos(1
/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2,
2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)*sec(d*x+c)^(1/2)/d
/(a+b*sec(d*x+c))^(1/2)+13/12*a*b*sec(d*x+c)^(3/2)*sin(d*x+c)*(a+b*sec(d*x
+c))^(1/2)/d+1/3*b^2*sec(d*x+c)^(5/2)*sin(d*x+c)*(a+b*sec(d*x+c))^(1/2)/d-
1/24*(33*a^2+16*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellip
ticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*(a+b*sec(d*x+c))^(1/2)/d/
((b+a*cos(d*x+c))/(a+b))^(1/2)/sec(d*x+c)^(1/2)+1/24*(33*a^2+16*b^2)*sin(d
*x+c)*sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2)/d
```

3.640.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.59 (sec) , antiderivative size = 602, normalized size of antiderivative = 1.63

$$\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx =$$

$$-\frac{a(a+b\sec(c+dx))^{5/2}\left(-\frac{104ab\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{\sqrt{b+a\cos(c+dx)}}+\frac{2(3a^2-104b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2}{a}\right.}{\sqrt{b+a\cos(c+dx)}}\right.}{}$$

$$+\frac{(a+b\sec(c+dx))^{5/2}\left(\frac{1}{24}\sec(c+dx)\left(33a^2\sin(c+dx)+16b^2\sin(c+dx)\right)+\frac{13}{12}ab\sec(c+dx)\tan(c+dx)\right.}{d(b+a\cos(c+dx))^2\sec^{\frac{5}{2}}(c+dx)}$$

input
```
Integrate[Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^(5/2),x]
```

output
```
-1/96*(a*(a + b*Sec[c + d*x])^(5/2)*((-104*a*b*Sqrt[(b + a*Cos[c + d*x])/(
a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)])/Sqrt[b + a*Cos[c + d*x]] +
(2*(3*a^2 - 104*b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c +
 d*x)/2, (2*a)/(a + b)])/Sqrt[b + a*Cos[c + d*x]] + ((2*I)*(33*a^2 + 16*b^
2)*Sqrt[(a - a*Cos[c + d*x])/(a + b)]*Sqrt[(a + a*Cos[c + d*x])/(a - b)]*C
os[2*(c + d*x)]*(-2*b*(a + b)*EllipticE[I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[
b + a*Cos[c + d*x]]], (-a + b)/(a + b)] + a*(2*b*EllipticF[I*ArcSinh[Sqrt[
(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b)] + a*EllipticPi[
1 - a/b, I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/
(a + b)]))*Sin[c + d*x])/(Sqrt[(a - b)^(-1)]*b*Sqrt[1 - Cos[c + d*x]^2]*Sq
rt[(a^2 - a^2*Cos[c + d*x]^2)/a^2]*(-a^2 + 2*b^2 - 4*b*(b + a*Cos[c + d*x]
) + 2*(b + a*Cos[c + d*x])^2))))/(d*(b + a*Cos[c + d*x])^(5/2)*Sec[c + d*x
]^(5/2)) + ((a + b*Sec[c + d*x])^(5/2)*((Sec[c + d*x]*(33*a^2*Sin[c + d*x]
 + 16*b^2*Sin[c + d*x]))/24 + (13*a*b*Sec[c + d*x]*Tan[c + d*x])/12 + (b^2
*Sec[c + d*x]^2*Tan[c + d*x])/3))/(d*(b + a*Cos[c + d*x])^2*Sec[c + d*x]^(
5/2))
```

3.640.3 Rubi [A] (verified)

Time = 4.07 (sec) , antiderivative size = 390, normalized size of antiderivative = 1.06, number of steps used = 29, number of rules used = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 1.160$, Rules used = {3042, 4329, 27, 3042, 4590, 27, 3042, 4590, 27, 3042, 4596, 3042, 4346, 3042, 3286, 3042, 3284, 4523, 3042, 4343, 3042, 3134, 3042, 3132, 4345, 3042, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}dx$$

$$\Big\downarrow \; 4329$$

$$\frac{1}{3}\int \frac{\sec^{\frac{3}{2}}(c+dx)\left(13ab^2\sec^2(c+dx)+2b\left(9a^2+2b^2\right)\sec(c+dx)+3a\left(2a^2+b^2\right)\right)}{2\sqrt{a+b\sec(c+dx)}}dx+\frac{b^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{6}\int \frac{\sec^{\frac{3}{2}}(c+dx)\left(13ab^2\sec^2(c+dx)+2b\left(9a^2+2b^2\right)\sec(c+dx)+3a\left(2a^2+b^2\right)\right)}{\sqrt{a+b\sec(c+dx)}}dx+\frac{b^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}$$

$\Big\downarrow$ 3042

$$\frac{1}{6}\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(13ab^2\csc\left(c+dx+\frac{\pi}{2}\right)^2+2b\left(9a^2+2b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)+3a\left(2a^2+b^2\right)\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{b^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}$$

$\Big\downarrow$ 4590

$$\frac{1}{6}\left(\frac{\int \frac{\sqrt{\sec(c+dx)}\left(13a^2b^2+\left(33a^2+16b^2\right)\sec^2(c+dx)b^2+2a\left(12a^2+19b^2\right)\sec(c+dx)b\right)}{2\sqrt{a+b\sec(c+dx)}}dx}{2b}+\frac{13ab\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\mathrm{se}}}{2d}\right.$$

$$\frac{b^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}$$

$\Big\downarrow$ 27

$$\frac{1}{6}\left(\frac{\int \frac{\sqrt{\sec(c+dx)}\left(13a^2b^2+\left(33a^2+16b^2\right)\sec^2(c+dx)b^2+2a\left(12a^2+19b^2\right)\sec(c+dx)b\right)}{\sqrt{a+b\sec(c+dx)}}dx}{4b}+\frac{13ab\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\mathrm{se}}}{2d}\right.$$

$$\frac{b^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}$$

$\Big\downarrow$ 3042

$$\frac{1}{6}\left(\frac{\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(13a^2b^2+\left(33a^2+16b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2b^2+2a\left(12a^2+19b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)b\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{4b}+\frac{13ab\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2d}\right.$$

$$\frac{b^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}$$

$\Big\downarrow$ 4590

3.640. $\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$

$$\frac{1}{6}\left(\frac{\frac{\int -\frac{-26a^2\sec(c+dx)b^3-15a\left(a^2+4b^2\right)\sec^2(c+dx)b^2+a\left(33a^2+16b^2\right)b^2}{2\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{b}+\frac{b\left(33a^2+16b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}}{4b}+\frac{13ab\sin}{}\right.$$

$$\frac{b^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}$$

$\downarrow$ 27

$$\frac{1}{6}\left(\frac{\frac{b\left(33a^2+16b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{\int\frac{-26a^2\sec(c+dx)b^3-15a\left(a^2+4b^2\right)\sec^2(c+dx)b^2+a\left(33a^2+16b^2\right)b^2}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{2b}}{4b}+\frac{13ab\sin(}{}\right.$$

$$\frac{b^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}$$

$\downarrow$ 3042

$$\frac{1}{6}\left(\frac{\frac{b\left(33a^2+16b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{\int\frac{-26a^2\csc\left(c+dx+\frac{\pi}{2}\right)b^3-15a\left(a^2+4b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2b^2+a\left(33a^2+16b^2\right)b^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{2b}}{4b}+\frac{13}{}\right.$$

$$\frac{b^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}$$

$\downarrow$ 4596

$$\frac{1}{6}\left(\frac{\frac{b\left(33a^2+16b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{\int\frac{ab^2\left(33a^2+16b^2\right)-26a^2b^3\sec(c+dx)}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx-15ab^2\left(a^2+4b^2\right)\int\frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx}{2b}}{4b}+\frac{13}{}\right.$$

$$\frac{b^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}$$

$\downarrow$ 3042

$$\frac{1}{6}\left(\frac{\frac{b\left(33a^2+16b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{\int\frac{ab^2\left(33a^2+16b^2\right)-26a^2b^3\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-15ab^2\left(a^2+4b^2\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}d}{2b}}{4b}\right.$$

$$\frac{b^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}$$

3.640. $\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$

$$\Big\downarrow \; 4346$$

$$\frac{1}{6}\left(\frac{\frac{b\left(33a^2+16b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{\int\frac{ab^2\left(33a^2+16b^2\right)-26a^2b^3\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{15ab^2\left(a^2+4b^2\right)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)\cdots}}{\sqrt{a+b\sec(c+dx)}}}{2b}}{4b}\right.$$

$$\frac{b^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{6}\left(\frac{\frac{b\left(33a^2+16b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{\int\frac{ab^2\left(33a^2+16b^2\right)-26a^2b^3\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{15ab^2\left(a^2+4b^2\right)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)\cdots}}{\sqrt{a+b\,\mathrm{s}\cdots}}}{2b}}{4b}\right.$$

$$\frac{b^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}$$

$$\Big\downarrow \; 3286$$

$$\frac{1}{6}\left(\frac{\frac{b\left(33a^2+16b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{\int\frac{ab^2\left(33a^2+16b^2\right)-26a^2b^3\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{15ab^2\left(a^2+4b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)}{a+b}}}{\sqrt{a+b\sec(c+d\cdots}}}{2b}}{4b}\right.$$

$$\frac{b^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{6}\left(\frac{\frac{b\left(33a^2+16b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{\int\frac{ab^2\left(33a^2+16b^2\right)-26a^2b^3\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{15ab^2\left(a^2+4b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)}{a+b}}}{\sqrt{a+\cdots}}}{2b}}{4b}\right.$$

$$\frac{b^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}$$

3.640. $\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$

$$\Big\downarrow \; 3284$$

$$\frac{1}{6}\left(\frac{\frac{b(33a^2+16b^2)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{\int\frac{ab^2(33a^2+16b^2)-26a^2b^3\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{30ab^2(a^2+4b^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)}{a+b}}}{d\sqrt{a+b\sec(c+}}}{2b}}{4b}\right.$$

$$\frac{b^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}$$

$$\Big\downarrow \; 4523$$

$$\frac{1}{6}\left(\frac{\frac{b(33a^2+16b^2)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{b^2(33a^2+16b^2)\int\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}dx-\left(b^3(59a^2+16b^2)\int\frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}dx\right)-\frac{30a}{}}{2b}}{4b}\right.$$

$$\frac{b^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{6}\left(\frac{\frac{b(33a^2+16b^2)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{b^2(33a^2+16b^2)\int\frac{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-\left(b^3(59a^2+16b^2)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}d\right.}{2b}}{4b}\right.$$

$$\frac{b^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}$$

$$\Big\downarrow \; 4343$$

$$\frac{1}{6}\left(\frac{\frac{b(33a^2+16b^2)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{\frac{b^2(33a^2+16b^2)\sqrt{a+b\sec(c+dx)}\int\sqrt{b+a\cos(c+dx)}dx}{\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}-\left(b^3(59a^2+16b^2)\int\frac{\sqrt{\csc(c+d}}{\sqrt{a+b\csc(c+}}\right.}{2b}}{4b}\right.$$

$$\frac{b^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}$$

$$\Big\downarrow \; 3042$$

3.640. $\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$

$$\frac{1}{6}\left(\frac{\frac{b(33a^2+16b^2)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{\frac{b^2(33a^2+16b^2)\sqrt{a+b\sec(c+dx)}\int\sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}-\left(b^3(59a^2+16b^2)\int\frac{\sqrt{\csc(}}{\sqrt{a+b\,\mathrm{cs}}}\right.}{}}{4b}\right.$$

$$\frac{b^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}$$

$\downarrow$ 3134

$$\frac{1}{6}\left(\frac{\frac{b(33a^2+16b^2)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{\frac{b^2(33a^2+16b^2)\sqrt{a+b\sec(c+dx)}\int\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}dx}{\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\left(b^3(59a^2+16b^2)\int\frac{\sqrt{\csc(}}{\sqrt{a+b\,\mathrm{cs}}}\right.}{}}{4b}\right.$$

$$\frac{b^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}$$

$\downarrow$ 3042

$$\frac{1}{6}\left(\frac{\frac{b(33a^2+16b^2)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{\frac{b^2(33a^2+16b^2)\sqrt{a+b\sec(c+dx)}\int\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}dx}{\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\left(b^3(59a^2+16b^2)\int\frac{\sqrt{}}{\sqrt{a}}\right.}{}}{4b}\right.$$

$$\frac{b^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}$$

$\downarrow$ 3132

$$\frac{1}{6}\left(\frac{\frac{b(33a^2+16b^2)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{-\left(b^3(59a^2+16b^2)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)+\frac{2b^2(33a^2+16b^2)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}\right.}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)}{a+b}}}}{2}}{4b}\right.$$

$$\frac{b^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}$$

$\downarrow$ 4345

3.640. $\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$

$$\frac{1}{6}\left(\frac{\frac{b(33a^2+16b^2)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{-\frac{b^3(59a^2+16b^2)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{1}{\sqrt{b+a\cos(c+dx)}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{2b^2(33a^2+16b^2)\sqrt{a}}{d\sqrt{\sec(c+}}}{4b}}{}\right.$$

$$\frac{b^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}$$

$\downarrow$ 3042

$$\frac{1}{6}\left(\frac{\frac{b(33a^2+16b^2)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{-\frac{b^3(59a^2+16b^2)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{1}{\sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{2b^2(33a^2+16b^2}{d\sqrt{\sec}}}{4b}}{}\right.$$

$$\frac{b^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}$$

$\downarrow$ 3142

$$\frac{1}{6}\left(\frac{\frac{b(33a^2+16b^2)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{-\frac{b^3(59a^2+16b^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{2b^2(33a^2+16b}{d\sqrt{s}}}{4b}}{}\right.$$

$$\frac{b^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}$$

$\downarrow$ 3042

$$\frac{1}{6}\left(\frac{\frac{b(33a^2+16b^2)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{-\frac{b^3(59a^2+16b^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{2b^2(33a^2-}{}}{4b}}{}\right.$$

$$\frac{b^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}$$

$\downarrow$ 3140

3.640. $\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$

$$\frac{1}{6}\left(\frac{\frac{b(33a^2+16b^2)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{\frac{2b^2(33a^2+16b^2)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{30ab^2(a^2+4b^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(}{}}}{d\sqrt{a+b}}}{4b}}{}\right.$$

$$\frac{b^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}$$

input
```
Int[Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^(5/2),x]
```

output
```
(b^2*Sec[c + d*x]^(5/2)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(3*d) + ((1
3*a*b*Sec[c + d*x]^(3/2)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(2*d) + (-
1/2*((-2*b^3*(59*a^2 + 16*b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Elliptic
F[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(d*Sqrt[a + b*Sec[c + d*
x]]) - (30*a*b^2*(a^2 + 4*b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Elliptic
Pi[2, (c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(d*Sqrt[a + b*Sec[c
+ d*x]]) + (2*b^2*(33*a^2 + 16*b^2)*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*
Sqrt[a + b*Sec[c + d*x]])/(d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c
 + d*x]]))/b + (b*(33*a^2 + 16*b^2)*Sqrt[Sec[c + d*x]]*Sqrt[a + b*Sec[c +
d*x]]*Sin[c + d*x])/d)/(4*b))/6
```

3.640.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3286
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]]   Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
 d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

rule 4329
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
2)*((d*Csc[e + f*x])^n/(f*(m + n - 1))), x] + Simp[1/(d*(m + n - 1))   Int[
(a + b*Csc[e + f*x])^(m - 3)*(d*Csc[e + f*x])^n*Simp[a^3*d*(m + n - 1) + a*
b^2*d*n + b*(b^2*d*(m + n - 2) + 3*a^2*d*(m + n - 1))*Csc[e + f*x] + a*b^2*
d*(3*m + 2*n - 4)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, n}, x
] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
 &&  !(IGtQ[n, 2] &&  !IntegerQ[m])
```

3.640. $\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$

rule 4343
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]])   Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a
, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4345
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]])   Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[
{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4346
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Simp[d*Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x
]]/Sqrt[a + b*Csc[e + f*x]])   Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4523
```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Simp[A/a   I
nt[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Simp[(A*b - a*B)
/(a*d)   Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ
[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

rule 4590
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(-C)*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
)*((d*Csc[e + f*x])^(n - 1)/(b*f*(m + n + 1))), x] + Simp[d/(b*(m + n + 1))
   Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (
A*b*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2
 - b^2, 0] && GtQ[n, 0]
```

rule 4596
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Simp[C/d^2   Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*C
sc[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[
a + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
 b^2, 0]
```

3.640. $\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$

3.640.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 12.22 (sec) , antiderivative size = 3037, normalized size of antiderivative = 8.23

method	result	size
default	Expression too large to display	3037

input
```
int(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/24/d/((a-b)/(a+b))^(1/2)*(a+b*sec(d*x+c))^(1/2)*sec(d*x+c)^(3/2)/(b+a*co
s(d*x+c))/(cos(d*x+c)+1)*(-33*EllipticE(((a-b)/(a+b))^(1/2)*(cot(d*x+c)-cs
c(d*x+c)),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^
(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a^2*b*cos(d*x+c)^4+16*EllipticE(((a-b)/(a+b
))^(1/2)*(cot(d*x+c)-csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d
*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a*b^2*cos(d*x+c)^4-2
6*EllipticF(((a-b)/(a+b))^(1/2)*(cot(d*x+c)-csc(d*x+c)),(-(a+b)/(a-b))^(1/
2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/
2)*a^2*b*cos(d*x+c)^4+44*EllipticF(((a-b)/(a+b))^(1/2)*(cot(d*x+c)-csc(d*x
+c)),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)
*(1/(cos(d*x+c)+1))^(1/2)*a*b^2*cos(d*x+c)^4-120*EllipticPi(((a-b)/(a+b))^
(1/2)*(cot(d*x+c)-csc(d*x+c)),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*(1/(a+b)*
(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a*b^2*cos(
d*x+c)^4-66*EllipticE(((a-b)/(a+b))^(1/2)*(cot(d*x+c)-csc(d*x+c)),(-(a+b)/
(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+
c)+1))^(1/2)*a^2*b*cos(d*x+c)^3+32*EllipticE(((a-b)/(a+b))^(1/2)*(cot(d*x+
c)-csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)
+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a*b^2*cos(d*x+c)^3-52*EllipticF(((a-b)
/(a+b))^(1/2)*(cot(d*x+c)-csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*
cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a^2*b*cos(d*...
```

3.640.5 Fricas [F(-1)]

Timed out.

$$\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.640.6 Sympy [F(-1)]

Timed out.

$$\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**(3/2)*(a+b*sec(d*x+c))**(5/2),x)
```

output
```
Timed out
```

3.640.7 Maxima [F]

$$\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \int (b\sec(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^{\frac{3}{2}}\,dx$$

input
```
integrate(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^(5/2)*sec(d*x + c)^(3/2), x)
```

3.640.8 Giac [F]

$$\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \int (b\sec(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^{\frac{3}{2}}\,dx$$

input
```
integrate(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^(5/2)*sec(d*x + c)^(3/2), x)
```

3.640.9 Mupad [F(-1)]

Timed out.

$$\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx=\int \left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)^{5/2}\left(\frac{1}{\cos\left(c+d\,x\right)}\right)^{3/2}dx$$

input
```
int((a + b/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(3/2),x)
```

output
```
int((a + b/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(3/2), x)
```

3.641 $\int \sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{5/2}\,dx$

3.641.1 Optimal result 4953
3.641.2 Mathematica [C] (verified) 4954
3.641.3 Rubi [A] (verified) 4955
3.641.4 Maple [C] (verified) 4964
3.641.5 Fricas [F(-1)] 4965
3.641.6 Sympy [F(-1)] 4966
3.641.7 Maxima [F] 4966
3.641.8 Giac [F] 4966
3.641.9 Mupad [F(-1)] 4967

3.641.1 Optimal result

Integrand size = 25, antiderivative size = 314

$$\begin{aligned}\int \sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{5/2}\,dx &= \frac{a(8a^2+11b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{4d\sqrt{a+b\sec(c+dx)}}\\ &+\frac{b(15a^2+4b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{4d\sqrt{a+b\sec(c+dx)}}\\ &-\frac{9abE\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+dx)}}{4d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\sqrt{\sec(c+dx)}}\\ &+\frac{9ab\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{4d}\\ &+\frac{b^2\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{2d}\end{aligned}$$

output
```
1/4*a*(8*a^2+11*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellip
ticF(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^
(1/2)*sec(d*x+c)^(1/2)/d/(a+b*sec(d*x+c))^(1/2)+1/4*b*(15*a^2+4*b^2)*(cos(
1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2
,2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)*sec(d*x+c)^(1/2)/
d/(a+b*sec(d*x+c))^(1/2)+1/2*b^2*sec(d*x+c)^(3/2)*sin(d*x+c)*(a+b*sec(d*x+
c))^(1/2)/d-9/4*a*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellipt
icE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*(a+b*sec(d*x+c))^(1/2)/d/(
(b+a*cos(d*x+c))/(a+b))^(1/2)/sec(d*x+c)^(1/2)+9/4*a*b*sin(d*x+c)*sec(d*x+
c)^(1/2)*(a+b*sec(d*x+c))^(1/2)/d
```

3.641.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.53 (sec) , antiderivative size = 560, normalized size of antiderivative = 1.78

$$\int \sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{5/2}\,dx = \frac{(a+b\sec(c+dx))^{5/2}\left(\frac{2(16a^3+4ab^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{\sqrt{b+a\cos(c+dx)}}+\frac{2(21a^2b+8b^3)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{Ell}}{\sqrt{b+a\cos(c+}}\right.}{}$$

$$+\frac{(a+b\sec(c+dx))^{5/2}\left(\frac{9}{4}ab\tan(c+dx)+\frac{1}{2}b^2\sec(c+dx)\tan(c+dx)\right)}{d(b+a\cos(c+dx))^2\sec^{\frac{5}{2}}(c+dx)}$$

input
```
Integrate[Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^(5/2),x]
```

output
```
((a + b*Sec[c + d*x])^(5/2)*((2*(16*a^3 + 4*a*b^2)*Sqrt[(b + a*Cos[c + d*x
])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)])/Sqrt[b + a*Cos[c + d*x]
] + (2*(21*a^2*b + 8*b^3)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2,
 (c + d*x)/2, (2*a)/(a + b)])/Sqrt[b + a*Cos[c + d*x]] - ((18*I)*a^2*Sqrt[
(a - a*Cos[c + d*x])/(a + b)]*Sqrt[(a + a*Cos[c + d*x])/(a - b)]*Cos[2*(c
+ d*x)]*(-2*b*(a + b)*EllipticE[I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Co
s[c + d*x]]], (-a + b)/(a + b)] + a*(2*b*EllipticF[I*ArcSinh[Sqrt[(a - b)^
(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b)] + a*EllipticPi[1 - a/b,
 I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b)]
))*Sin[c + d*x])/(Sqrt[(a - b)^(-1)]*Sqrt[1 - Cos[c + d*x]^2]*Sqrt[(a^2 -
a^2*Cos[c + d*x]^2)/a^2]*(-a^2 + 2*b^2 - 4*b*(b + a*Cos[c + d*x]) + 2*(b +
 a*Cos[c + d*x])^2))))/(16*d*(b + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(5/2)
) + ((a + b*Sec[c + d*x])^(5/2)*((9*a*b*Tan[c + d*x])/4 + (b^2*Sec[c + d*x
]*Tan[c + d*x])/2))/(d*(b + a*Cos[c + d*x])^2*Sec[c + d*x]^(5/2))
```

3.641.3 Rubi [A] (verified)

Time = 3.45 (sec) , antiderivative size = 324, normalized size of antiderivative = 1.03, number of steps used = 26, number of rules used = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 1.040$, Rules used = {3042, 4329, 27, 3042, 4590, 27, 3042, 4596, 3042, 4346, 3042, 3286, 3042, 3284, 4523, 3042, 4343, 3042, 3134, 3042, 3132, 4345, 3042, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{5/2}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}dx$$

$$\Big\downarrow \; 4329$$

$$\frac{1}{2}\int \frac{\sqrt{\sec(c+dx)}\left(9ab^2\sec^2(c+dx)+2b\left(6a^2+b^2\right)\sec(c+dx)+a\left(4a^2+b^2\right)\right)}{2\sqrt{a+b\sec(c+dx)}}dx+\frac{b^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{4}\int \frac{\sqrt{\sec(c+dx)}\left(9ab^2\sec^2(c+dx)+2b\left(6a^2+b^2\right)\sec(c+dx)+a\left(4a^2+b^2\right)\right)}{\sqrt{a+b\sec(c+dx)}}dx+\frac{b^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}$$

$\downarrow$ 3042

$$\frac{1}{4}\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(9ab^2\csc\left(c+dx+\frac{\pi}{2}\right)^2+2b\left(6a^2+b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)+a\left(4a^2+b^2\right)\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{b^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}$$

$\downarrow$ 4590

$$\frac{1}{4}\left(\frac{\int -\frac{9a^2b^2-\left(15a^2+4b^2\right)\sec^2(c+dx)b^2-2a\left(4a^2+b^2\right)\sec(c+dx)b}{2\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{b}+\frac{9ab\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}\right)+\frac{b^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}$$

$\downarrow$ 27

$$\frac{1}{4}\left(\frac{9ab\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{\int \frac{9a^2b^2-\left(15a^2+4b^2\right)\sec^2(c+dx)b^2-2a\left(4a^2+b^2\right)\sec(c+dx)b}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{2b}\right)+\frac{b^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}$$

$\downarrow$ 3042

$$\frac{1}{4}\left(\frac{9ab\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{\int \frac{9a^2b^2-\left(15a^2+4b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2b^2-2a\left(4a^2+b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)b}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{2b}\right)+\frac{b^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}$$

$\downarrow$ 4596

3.641. $\int \sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{5/2}\,dx$

$$\frac{1}{4}\left(\frac{9ab\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{\int\frac{9a^2b^2-2ab(4a^2+b^2)\sec(c+dx)}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx-b^2(15a^2+4b^2)\int\frac{\sec^{\frac{3}{2}}(c+}{\sqrt{a+b\sec(}}}{2b}\right.$$

$$\frac{b^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{4}\left(\frac{9ab\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{\int\frac{9a^2b^2-2ab(4a^2+b^2)\csc(c+dx+\frac{\pi}{2})}{\sqrt{\csc(c+dx+\frac{\pi}{2})}\sqrt{a+b\csc(c+dx+\frac{\pi}{2})}}dx-b^2(15a^2+4b^2)\int\frac{\csc}{\sqrt{a+}}}{2b}\right.$$

$$\frac{b^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}$$

$$\Big\downarrow \; 4346$$

$$\frac{1}{4}\left(\frac{9ab\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{\int\frac{9a^2b^2-2ab(4a^2+b^2)\csc(c+dx+\frac{\pi}{2})}{\sqrt{\csc(c+dx+\frac{\pi}{2})}\sqrt{a+b\csc(c+dx+\frac{\pi}{2})}}dx-\frac{b^2(15a^2+4b^2)\sqrt{\sec(c+dx)}}{\sqrt{a}}}{2b}\right.$$

$$\frac{b^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{4}\left(\frac{9ab\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{\int\frac{9a^2b^2-2ab(4a^2+b^2)\csc(c+dx+\frac{\pi}{2})}{\sqrt{\csc(c+dx+\frac{\pi}{2})}\sqrt{a+b\csc(c+dx+\frac{\pi}{2})}}dx-\frac{b^2(15a^2+4b^2)\sqrt{\sec(c+dx)}}{}}{2b}\right.$$

$$\frac{b^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}$$

$$\Big\downarrow \; 3286$$

3.641. $\int \sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{5/2}\,dx$

$$\frac{1}{4}\left(\frac{9ab\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{\int\frac{9a^2b^2-2ab(4a^2+b^2)\csc(c+dx+\frac{\pi}{2})}{\sqrt{\csc(c+dx+\frac{\pi}{2})}\sqrt{a+b\csc(c+dx+\frac{\pi}{2})}}dx-\frac{b^2(15a^2+4b^2)\sqrt{\sec(c+dx)}}{\sqrt{}}}{2b}\right.$$

$$\frac{b^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}$$

$\downarrow$ 3042

$$\frac{1}{4}\left(\frac{9ab\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{\int\frac{9a^2b^2-2ab(4a^2+b^2)\csc(c+dx+\frac{\pi}{2})}{\sqrt{\csc(c+dx+\frac{\pi}{2})}\sqrt{a+b\csc(c+dx+\frac{\pi}{2})}}dx-\frac{b^2(15a^2+4b^2)\sqrt{\sec(c+dx)}}{}}{2b}\right.$$

$$\frac{b^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}$$

$\downarrow$ 3284

$$\frac{1}{4}\left(\frac{9ab\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{\int\frac{9a^2b^2-2ab(4a^2+b^2)\csc(c+dx+\frac{\pi}{2})}{\sqrt{\csc(c+dx+\frac{\pi}{2})}\sqrt{a+b\csc(c+dx+\frac{\pi}{2})}}dx-\frac{2b^2(15a^2+4b^2)\sqrt{\sec(c+dx}}{}}{2b}\right.$$

$$\frac{b^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}$$

$\downarrow$ 4523

$$\frac{1}{4}\left(\frac{9ab\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{-ab(8a^2+11b^2)\int\frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}dx+9ab^2\int\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}}{}\right.$$

$$\frac{b^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}$$

$\downarrow$ 3042

3.641. $\int\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{5/2}\,dx$

$$\frac{1}{4}\left(\frac{9ab\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{-ab\left(8a^2+11b^2\right)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+9ab^2\int\frac{\sqrt{a+b\csc(c}}{\sqrt{\csc(c+a}}}{}\right.$$

$$\frac{b^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}$$

$$\Big\downarrow \; 4343$$

$$\frac{1}{4}\left(\frac{9ab\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{-ab\left(8a^2+11b^2\right)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{9ab^2\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}\sqrt{}}}{}\right.$$

$$\frac{b^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{4}\left(\frac{9ab\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{-ab\left(8a^2+11b^2\right)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{9ab^2\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}}{}\right.$$

$$\frac{b^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}$$

$$\Big\downarrow \; 3134$$

$$\frac{1}{4}\left(\frac{9ab\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{-ab\left(8a^2+11b^2\right)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{9ab^2\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}}{}\right.$$

$$\frac{b^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}$$

$$\Big\downarrow \; 3042$$

3.641. $\int \sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{5/2}\,dx$

$$\frac{1}{4}\left(\frac{9ab\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{-ab\left(8a^2+11b^2\right)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{9ab^2\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+d}}}{}\right.$$

$$\frac{b^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}$$

$$\Big\downarrow \; 3132$$

$$\frac{1}{4}\left(\frac{9ab\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{-ab\left(8a^2+11b^2\right)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2b^2\left(15a^2+4b^2\right)\sqrt{\sec(}}{}\right.$$

$$\frac{b^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}$$

$$\Big\downarrow \; 4345$$

$$\frac{1}{4}\left(\frac{9ab\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{-\frac{ab\left(8a^2+11b^2\right)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{1}{\sqrt{b+a\cos(c+dx)}}dx}{\sqrt{a+b\sec(c+dx)}}-\frac{2b^2}{}}{}\right.$$

$$\frac{b^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{4}\left(\frac{9ab\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{-\frac{ab\left(8a^2+11b^2\right)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{1}{\sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{\sqrt{a+b\sec(c+dx)}}-}{}\right.$$

$$\frac{b^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}$$

$$\Big\downarrow \; 3142$$

3.641. $\int \sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{5/2}\,dx$

$$\frac{1}{4}\left(\frac{9ab\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{-\frac{ab(8a^2+11b^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}-\frac{2}{}}{}\right.$$

$$\frac{b^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}$$

$\downarrow$ 3042

$$\frac{1}{4}\left(\frac{9ab\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{-\frac{ab(8a^2+11b^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}}{}\right.$$

$$\frac{b^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}$$

$\downarrow$ 3140

$$\frac{1}{4}\left(\frac{9ab\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{-\frac{2ab(8a^2+11b^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}-}{}\right.$$

$$\frac{b^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}$$

input
```
Int[Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^(5/2),x]
```

output
```
(b^2*Sec[c + d*x]^(3/2)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(2*d) + (-1
/2*((-2*a*b*(8*a^2 + 11*b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[
(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(d*Sqrt[a + b*Sec[c + d*x]
]) - (2*b^2*(15*a^2 + 4*b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi
[2, (c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(d*Sqrt[a + b*Sec[c +
d*x]]) + (18*a*b^2*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c
+ d*x]])/(d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]))/b + (9
*a*b*Sqrt[Sec[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/d)/4
```

3.641. $\int \sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{5/2}\,dx$

3.641.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3286
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]]   Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
 d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

rule 4329
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
2)*((d*Csc[e + f*x])^n/(f*(m + n - 1))), x] + Simp[1/(d*(m + n - 1))   Int[
(a + b*Csc[e + f*x])^(m - 3)*(d*Csc[e + f*x])^n*Simp[a^3*d*(m + n - 1) + a*
b^2*d*n + b*(b^2*d*(m + n - 2) + 3*a^2*d*(m + n - 1))*Csc[e + f*x] + a*b^2*
d*(3*m + 2*n - 4)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, n}, x
] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
 &&  !(IGtQ[n, 2] &&  !IntegerQ[m])
```

rule 4343
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]])   Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a
, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4345
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]])   Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[
{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4346
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Simp[d*Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x
]]/Sqrt[a + b*Csc[e + f*x]])   Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4523
```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Simp[A/a   I
nt[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Simp[(A*b - a*B)
/(a*d)   Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ
[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

3.641. $\int \sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{5/2}\,dx$

rule 4590
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(-C)*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
)*((d*Csc[e + f*x])^(n - 1)/(b*f*(m + n + 1))), x] + Simp[d/(b*(m + n + 1))
   Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (
A*b*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2
 - b^2, 0] && GtQ[n, 0]
```

rule 4596
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Simp[C/d^2   Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*C
sc[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[
a + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
 b^2, 0]
```

3.641.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 10.88 (sec) , antiderivative size = 2602, normalized size of antiderivative = 8.29

method	result	size
default	Expression too large to display	2602

input
```
int(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4/d/((a-b)/(a+b))^(1/2)*(a+b*sec(d*x+c))^(1/2)*sec(d*x+c)^(1/2)/(b+a*co
s(d*x+c))/(cos(d*x+c)+1)*(-9*(1/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*
x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE(((a-b)/(a+b))^(1/2)*(cot(d*x+c)-csc(
d*x+c)),(-(a+b)/(a-b))^(1/2))*a^2*b*cos(d*x+c)+9*(1/(cos(d*x+c)+1))^(1/2)*
(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE(((a-b)/(a+b))^(1
/2)*(cot(d*x+c)-csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*a*b^2*cos(d*x+c)-6*(1/(c
os(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ellipt
icF(((a-b)/(a+b))^(1/2)*(cot(d*x+c)-csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*a^2*
b*cos(d*x+c)+2*(1/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x
+c)+1))^(1/2)*EllipticF(((a-b)/(a+b))^(1/2)*(cot(d*x+c)-csc(d*x+c)),(-(a+b
)/(a-b))^(1/2))*a*b^2*cos(d*x+c)+30*(1/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a
*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi(((a-b)/(a+b))^(1/2)*(cot(d*x
+c)-csc(d*x+c)),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*a^2*b*cos(d*x+c)+60*(1/
(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Elli
pticPi(((a-b)/(a+b))^(1/2)*(cot(d*x+c)-csc(d*x+c)),(a+b)/(a-b),I/((a-b)/(a
+b))^(1/2))*a^2*b*cos(d*x+c)^2+30*(1/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*c
os(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi(((a-b)/(a+b))^(1/2)*(cot(d*x+c
)-csc(d*x+c)),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*a^2*b*cos(d*x+c)^3+16*(1/
(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Elli
pticPi(((a-b)/(a+b))^(1/2)*(cot(d*x+c)-csc(d*x+c)),(a+b)/(a-b),I/((a-b)...
```

3.641.5 Fricas [F(-1)]

Timed out.

$$\int \sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.641.6 Sympy [F(-1)]

Timed out.

$$\int \sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**(1/2)*(a+b*sec(d*x+c))**(5/2),x)
```

output
```
Timed out
```

3.641.7 Maxima [F]

$$\int \sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{5/2}\,dx = \int (b\sec(dx+c)+a)^{\frac{5}{2}}\sqrt{\sec(dx+c)}\,dx$$

input
```
integrate(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^(5/2)*sqrt(sec(d*x + c)), x)
```

3.641.8 Giac [F]

$$\int \sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{5/2}\,dx = \int (b\sec(dx+c)+a)^{\frac{5}{2}}\sqrt{\sec(dx+c)}\,dx$$

input
```
integrate(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^(5/2)*sqrt(sec(d*x + c)), x)
```

3.641.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{5/2}\,dx=\int \left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)^{5/2}\sqrt{\frac{1}{\cos\left(c+d\,x\right)}}\,dx$$

input
```
int((a + b/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(1/2),x)
```

output
```
int((a + b/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(1/2), x)
```

3.642 $\int \frac{(a+b\sec(c+dx))^{5/2}}{\sqrt{\sec(c+dx)}}\,dx$

3.642.1 Optimal result 4968
3.642.2 Mathematica [C] (verified) 4969
3.642.3 Rubi [A] (verified) 4969
3.642.4 Maple [C] (verified) 4976
3.642.5 Fricas [F] 4977
3.642.6 Sympy [F(-1)] 4978
3.642.7 Maxima [F] 4978
3.642.8 Giac [F] 4978
3.642.9 Mupad [F(-1)] 4979

3.642.1 Optimal result

Integrand size = 25, antiderivative size = 263

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sqrt{\sec(c+dx)}}\,dx = \frac{b(4a^2+b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{d\sqrt{a+b\sec(c+dx)}} + \frac{5ab^2\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{d\sqrt{a+b\sec(c+dx)}} + \frac{(2a^2-b^2)\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+dx)}}{d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\sqrt{\sec(c+dx)}} + \frac{b^2\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{d}$$

output
```
b*(4*a^2+b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(si
n(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)*s
ec(d*x+c)^(1/2)/d/(a+b*sec(d*x+c))^(1/2)+5*a*b^2*(cos(1/2*d*x+1/2*c)^2)^(1
/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2,2^(1/2)*(a/(a+b))^(
1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)*sec(d*x+c)^(1/2)/d/(a+b*sec(d*x+c))^(
1/2)+(2*a^2-b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE
(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*(a+b*sec(d*x+c))^(1/2)/d/((b+
a*cos(d*x+c))/(a+b))^(1/2)/sec(d*x+c)^(1/2)+b^2*sin(d*x+c)*sec(d*x+c)^(1/2
)*(a+b*sec(d*x+c))^(1/2)/d
```

3.642.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 5.97 (sec) , antiderivative size = 421, normalized size of antiderivative = 1.60

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sqrt{\sec(c+dx)}}\,dx = \frac{(a+b\sec(c+dx))^{5/2}\left(\frac{24a^2b\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{(b+a\cos(c+dx))^3}+\frac{2a(2a^2+9b^2)\sqrt{\frac{b+a\,c}{}}}{(}\right.}{}$$

input
```
Integrate[(a + b*Sec[c + d*x])^(5/2)/Sqrt[Sec[c + d*x]],x]
```

output
```
((a + b*Sec[c + d*x])^(5/2)*((24*a^2*b*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*
EllipticF[(c + d*x)/2, (2*a)/(a + b)])/(b + a*Cos[c + d*x])^3 + (2*a*(2*a^
2 + 9*b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (
2*a)/(a + b)])/(b + a*Cos[c + d*x])^3 + ((2*I)*(2*a^2 - b^2)*Sqrt[-((a*(-1
 + Cos[c + d*x]))/(a + b))]*Sqrt[(a*(1 + Cos[c + d*x]))/(a - b)]*Csc[c + d
*x]*(-2*b*(a + b)*EllipticE[I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c
+ d*x]]], (-a + b)/(a + b)] + a*(2*b*EllipticF[I*ArcSinh[Sqrt[(a - b)^(-1)
]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b)] + a*EllipticPi[1 - a/b, I*A
rcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b)])))/
(a*Sqrt[(a - b)^(-1)]*b*(b + a*Cos[c + d*x])^(5/2)) + (4*b^2*Tan[c + d*x])
/(b + a*Cos[c + d*x])^2))/(4*d*Sec[c + d*x]^(5/2))
```

3.642.3 Rubi [A] (verified)

Time = 2.66 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.03, number of steps used = 23, number of rules used = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.920$, Rules used = {3042, 4329, 27, 3042, 4596, 3042, 4346, 3042, 3286, 3042, 3284, 4523, 3042, 4343, 3042, 3134, 3042, 3132, 4345, 3042, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sqrt{\sec(c+dx)}}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \; 4329$$

$$\int \frac{6b\sec(c+dx)a^2+5b^2\sec^2(c+dx)a+\left(2a^2-b^2\right)a}{2\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx+\frac{b^2\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{2}\int \frac{6b\sec(c+dx)a^2+5b^2\sec^2(c+dx)a+\left(2a^2-b^2\right)a}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx+\frac{b^2\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{2}\int \frac{6b\csc\left(c+dx+\frac{\pi}{2}\right)a^2+5b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2a+\left(2a^2-b^2\right)a}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{b^2\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}$$

$$\Big\downarrow \; 4596$$

$$\frac{1}{2}\left(\int \frac{6b\sec(c+dx)a^2+\left(2a^2-b^2\right)a}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx+5ab^2\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx\right)+\frac{b^2\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{2}\left(\int \frac{6b\csc\left(c+dx+\frac{\pi}{2}\right)a^2+\left(2a^2-b^2\right)a}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+5ab^2\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)+\frac{b^2\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}$$

$$\Big\downarrow \; 4346$$

$$\frac{1}{2}\left(\int \frac{6b\csc\left(c+dx+\frac{\pi}{2}\right)a^2+\left(2a^2-b^2\right)a}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{5ab^2\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int \frac{\sec(c+dx)}{\sqrt{b+a\cos(c+dx)}}dx}{\sqrt{a+b\sec(c+dx)}}\right)+\frac{b^2\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}$$

$$\Big\downarrow \; 3042$$

3.642. $\int \frac{(a+b\sec(c+dx))^{5/2}}{\sqrt{\sec(c+dx)}}\,dx$

$$\frac{1}{2}\left(\int \frac{6b\csc\left(c+dx+\frac{\pi}{2}\right)a^2+\left(2a^2-b^2\right)a}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{5ab^2\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{b+a\,\mathrm{si}}}}{\sqrt{a+b\sec(c+dx)}}\right.$$

$$\frac{b^2\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}$$

$\downarrow$ 3286

$$\frac{1}{2}\left(\int \frac{6b\csc\left(c+dx+\frac{\pi}{2}\right)a^2+\left(2a^2-b^2\right)a}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{5ab^2\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{\sec(c+dx)}{\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}\right)+$$

$$\frac{b^2\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}$$

$\downarrow$ 3042

$$\frac{1}{2}\left(\int \frac{6b\csc\left(c+dx+\frac{\pi}{2}\right)a^2+\left(2a^2-b^2\right)a}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{5ab^2\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{\frac{b}{a+b}+\frac{a\sin(c}{c}}}}{\sqrt{a+b\sec(c+dx)}}\right.$$

$$\frac{b^2\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}$$

$\downarrow$ 3284

$$\frac{1}{2}\left(\int \frac{6b\csc\left(c+dx+\frac{\pi}{2}\right)a^2+\left(2a^2-b^2\right)a}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{10ab^2\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\mathrm{EllipticPi}\left(2,\frac{1}{2}(c+dx)\right.}{d\sqrt{a+b\sec(c+dx)}}\right.$$

$$\frac{b^2\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}$$

$\downarrow$ 4523

$$\frac{1}{2}\left(b\left(4a^2+b^2\right)\int\frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}dx+\left(2a^2-b^2\right)\int\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}dx+\frac{10ab^2\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)}{a+b}}}{d\sqrt{a+b}}\right.$$

$$\frac{b^2\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}$$

$\downarrow$ 3042

3.642. $\int \frac{(a+b\sec(c+dx))^{5/2}}{\sqrt{\sec(c+dx)}}\,dx$

$$\frac{1}{2}\left(b(4a^2+b^2)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+(2a^2-b^2)\int\frac{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{10ab^2\sqrt{\sec(c+dx)}\sqrt{}}{}\right.$$

$$\frac{b^2\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}$$

$$\Big\downarrow \text{4343}$$

$$\frac{1}{2}\left(b(4a^2+b^2)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{(2a^2-b^2)\sqrt{a+b\sec(c+dx)}\int\sqrt{b+a\cos(c+dx)}dx}{\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}+\frac{10ab^2\sqrt{}}{}\right.$$

$$\frac{b^2\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\left(b(4a^2+b^2)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{(2a^2-b^2)\sqrt{a+b\sec(c+dx)}\int\sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}+\frac{10}{}\right.$$

$$\frac{b^2\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}$$

$$\Big\downarrow \text{3134}$$

$$\frac{1}{2}\left(b(4a^2+b^2)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{(2a^2-b^2)\sqrt{a+b\sec(c+dx)}\int\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}dx}{\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{10ab^2\sqrt{}}{}\right.$$

$$\frac{b^2\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\left(b(4a^2+b^2)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{(2a^2-b^2)\sqrt{a+b\sec(c+dx)}\int\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}dx}{\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{10ab}{}\right.$$

$$\frac{b^2\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}$$

$$\Big\downarrow \text{3132}$$

3.642. $\int\frac{(a+b\sec(c+dx))^{5/2}}{\sqrt{\sec(c+dx)}}\,dx$

$$\frac{1}{2}\left(b(4a^2+b^2)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2(2a^2-b^2)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{10ab^2\sqrt{\sec}}{}\right.$$

$$\frac{b^2\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}$$

$\downarrow$ 4345

$$\frac{1}{2}\left(\frac{b(4a^2+b^2)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{1}{\sqrt{b+a\cos(c+dx)}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{2(2a^2-b^2)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+d\right.}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}\right.$$

$$\frac{b^2\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}$$

$\downarrow$ 3042

$$\frac{1}{2}\left(\frac{b(4a^2+b^2)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{1}{\sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{2(2a^2-b^2)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+\right.}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}\right.$$

$$\frac{b^2\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}$$

$\downarrow$ 3142

$$\frac{1}{2}\left(\frac{b(4a^2+b^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{2(2a^2-b^2)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\right.}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}\right.$$

$$\frac{b^2\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}$$

$\downarrow$ 3042

$$\frac{1}{2}\left(\frac{b(4a^2+b^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{2(2a^2-b^2)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+d\right.}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}\right.$$

$$\frac{b^2\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}$$

$\downarrow$ 3140

3.642. $\int\frac{(a+b\sec(c+dx))^{5/2}}{\sqrt{\sec(c+dx)}}\,dx$

$$\frac{1}{2}\left(\frac{2b(4a^2+b^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}+\frac{2(2a^2-b^2)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c\right.}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+}{a+b}}}\right.$$
$$\frac{b^2\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}$$

input
```
Int[(a + b*Sec[c + d*x])^(5/2)/Sqrt[Sec[c + d*x]],x]
```

output
```
((2*b*(4*a^2 + b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)
/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(d*Sqrt[a + b*Sec[c + d*x]]) + (10*
a*b^2*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/
(a + b)]*Sqrt[Sec[c + d*x]])/(d*Sqrt[a + b*Sec[c + d*x]]) + (2*(2*a^2 - b^
2)*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(d*Sqrt
[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]))/2 + (b^2*Sqrt[Sec[c +
d*x]]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/d
```

3.642.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3286
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]]   Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
 d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

rule 4329
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
2)*((d*Csc[e + f*x])^n/(f*(m + n - 1))), x] + Simp[1/(d*(m + n - 1))   Int[
(a + b*Csc[e + f*x])^(m - 3)*(d*Csc[e + f*x])^n*Simp[a^3*d*(m + n - 1) + a*
b^2*d*n + b*(b^2*d*(m + n - 2) + 3*a^2*d*(m + n - 1))*Csc[e + f*x] + a*b^2*
d*(3*m + 2*n - 4)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, n}, x
] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
 &&  !(IGtQ[n, 2] &&  !IntegerQ[m])
```

rule 4343
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]])   Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a
, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

3.642. $\int \frac{(a+b\sec(c+dx))^{5/2}}{\sqrt{\sec(c+dx)}}\,dx$

rule 4345
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]])   Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[
{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4346
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Simp[d*Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x
]]/Sqrt[a + b*Csc[e + f*x]])   Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4523
```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Simp[A/a   I
nt[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Simp[(A*b - a*B)
/(a*d)   Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ
[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

rule 4596
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Simp[C/d^2   Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*C
sc[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[
a + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
 b^2, 0]
```

3.642.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 9.71 (sec) , antiderivative size = 2463, normalized size of antiderivative = 9.37

method	result	size
default	Expression too large to display	2463

input
```
int((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/d/((a-b)/(a+b))^(1/2)*(a+b*sec(d*x+c))^(1/2)/(b+a*cos(d*x+c))/sec(d*x+c)
^(1/2)/(cos(d*x+c)+1)*(4*(1/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c)
)/(cos(d*x+c)+1))^(1/2)*EllipticE(((a-b)/(a+b))^(1/2)*(cot(d*x+c)-csc(d*x+
c)),(-(a+b)/(a-b))^(1/2))*a^2*b*cos(d*x+c)+2*(1/(cos(d*x+c)+1))^(1/2)*(1/(
a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE(((a-b)/(a+b))^(1/2)*
(cot(d*x+c)-csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*a*b^2*cos(d*x+c)-12*(1/(cos(
d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF
(((a-b)/(a+b))^(1/2)*(cot(d*x+c)-csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*a^2*b*c
os(d*x+c)+8*(1/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)
+1))^(1/2)*EllipticF(((a-b)/(a+b))^(1/2)*(cot(d*x+c)-csc(d*x+c)),(-(a+b)/(
a-b))^(1/2))*a*b^2*cos(d*x+c)-10*EllipticPi(((a-b)/(a+b))^(1/2)*(cot(d*x+c
)-csc(d*x+c)),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*(1/(cos(d*x+c)+1))^(1/2)*
(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a*b^2-4*EllipticE(((a-b)/(
a+b))^(1/2)*(cot(d*x+c)-csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*(1/(cos(d*x+c)+1
))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^3*cos(d*x+c)-2*
EllipticE(((a-b)/(a+b))^(1/2)*(cot(d*x+c)-csc(d*x+c)),(-(a+b)/(a-b))^(1/2)
)*(1/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)
*b^3*cos(d*x+c)-20*EllipticPi(((a-b)/(a+b))^(1/2)*(cot(d*x+c)-csc(d*x+c)),
(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*(1/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*
cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a*b^2*cos(d*x+c)-6*EllipticF(((a-b)/(...
```

3.642.5 Fricas [F]

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sqrt{\sec(c+dx)}}\,dx = \int \frac{(b\sec(dx+c)+a)^{\frac{5}{2}}}{\sqrt{\sec(dx+c)}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
integral((b^2*sec(d*x + c)^2 + 2*a*b*sec(d*x + c) + a^2)*sqrt(b*sec(d*x +
c) + a)/sqrt(sec(d*x + c)), x)
```

3.642.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sqrt{\sec(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate((a+b*sec(d*x+c))**(5/2)/sec(d*x+c)**(1/2),x)
```

output
```
Timed out
```

3.642.7 Maxima [F]

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sqrt{\sec(c+dx)}}\,dx = \int \frac{(b\sec(dx+c)+a)^{\frac{5}{2}}}{\sqrt{\sec(dx+c)}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^(5/2)/sqrt(sec(d*x + c)), x)
```

3.642.8 Giac [F]

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sqrt{\sec(c+dx)}}\,dx = \int \frac{(b\sec(dx+c)+a)^{\frac{5}{2}}}{\sqrt{\sec(dx+c)}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^(5/2)/sqrt(sec(d*x + c)), x)
```

3.642.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sqrt{\sec(c+dx)}}\,dx=\int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2}}{\sqrt{\frac{1}{\cos(c+d\,x)}}}\,dx$$

input
```
int((a + b/cos(c + d*x))^(5/2)/(1/cos(c + d*x))^(1/2),x)
```

output
```
int((a + b/cos(c + d*x))^(5/2)/(1/cos(c + d*x))^(1/2), x)
```

3.643 $\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

3.643.1 Optimal result 4980
3.643.2 Mathematica [C] (verified) 4981
3.643.3 Rubi [A] (verified) 4981
3.643.4 Maple [C] (verified) 4988
3.643.5 Fricas [F] 4989
3.643.6 Sympy [F(-1)] 4990
3.643.7 Maxima [F] 4990
3.643.8 Giac [F] 4990
3.643.9 Mupad [F(-1)] 4991

3.643.1 Optimal result

Integrand size = 25, antiderivative size = 262

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \frac{2a(a^2+2b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{3d\sqrt{a+b\sec(c+dx)}} + \frac{2b^3\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{d\sqrt{a+b\sec(c+dx)}} + \frac{14abE\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+dx)}}{3d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\sqrt{\sec(c+dx)}} + \frac{2a^2\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}$$

output
```
2/3*a*(a^2+2*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elliptic
F(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/
2)*sec(d*x+c)^(1/2)/d/(a+b*sec(d*x+c))^(1/2)+2*b^3*(cos(1/2*d*x+1/2*c)^2)^
(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2,2^(1/2)*(a/(a+b))
^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)*sec(d*x+c)^(1/2)/d/(a+b*sec(d*x+c))
^(1/2)+2/3*a^2*sin(d*x+c)*(a+b*sec(d*x+c))^(1/2)/d/sec(d*x+c)^(1/2)+14/3*a
*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1
/2*c),2^(1/2)*(a/(a+b))^(1/2))*(a+b*sec(d*x+c))^(1/2)/d/((b+a*cos(d*x+c))/
(a+b))^(1/2)/sec(d*x+c)^(1/2)
```

3.643.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.27 (sec) , antiderivative size = 409, normalized size of antiderivative = 1.56

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \frac{(a+b\sec(c+dx))^{5/2}\left(\frac{2a(a^2+9b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{(b+a\cos(c+dx))^3}+\frac{b(7a^2+6b^2)\sqrt{}}{}\right.}{}$$

input
```
Integrate[(a + b*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(3/2),x]
```

output
```
((a + b*Sec[c + d*x])^(5/2)*((2*a*(a^2 + 9*b^2)*Sqrt[(b + a*Cos[c + d*x])/
(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)])/(b + a*Cos[c + d*x])^3 + (
b*(7*a^2 + 6*b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*
x)/2, (2*a)/(a + b)])/(b + a*Cos[c + d*x])^3 + ((7*I)*Sqrt[-((a*(-1 + Cos[
c + d*x]))/(a + b))]*Sqrt[(a*(1 + Cos[c + d*x]))/(a - b)]*Csc[c + d*x]*(-2
*b*(a + b)*EllipticE[I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]
], (-a + b)/(a + b)] + a*(2*b*EllipticF[I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[
b + a*Cos[c + d*x]]], (-a + b)/(a + b)] + a*EllipticPi[1 - a/b, I*ArcSinh[
Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b)])))/(Sqrt[(
a - b)^(-1)]*(b + a*Cos[c + d*x])^(5/2)) + (2*a^2*Sin[c + d*x])/(b + a*Cos
[c + d*x])^2))/(3*d*Sec[c + d*x]^(5/2))
```

3.643.3 Rubi [A] (verified)

Time = 2.66 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.920$, Rules used = {3042, 4328, 27, 3042, 4596, 3042, 4346, 3042, 3286, 3042, 3284, 4523, 3042, 4343, 3042, 3134, 3042, 3132, 4345, 3042, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{4328}$$

$$\frac{2}{3}\int \frac{3\sec^2(c+dx)b^3+7a^2b+a\left(a^2+9b^2\right)\sec(c+dx)}{2\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx+\frac{2a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3d\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\int \frac{3\sec^2(c+dx)b^3+7a^2b+a\left(a^2+9b^2\right)\sec(c+dx)}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx+\frac{2a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3d\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\int \frac{3\csc\left(c+dx+\frac{\pi}{2}\right)^2b^3+7a^2b+a\left(a^2+9b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3d\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{4596}$$

$$\frac{1}{3}\left(\int \frac{7ba^2+\left(a^2+9b^2\right)\sec(c+dx)a}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx+3b^3\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx\right)+\frac{2a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3d\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\left(\int \frac{7ba^2+\left(a^2+9b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)a}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+3b^3\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)+\frac{2a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3d\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{4346}$$

$$\frac{1}{3}\left(\int \frac{7ba^2+\left(a^2+9b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)a}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{3b^3\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int \frac{\sec(c+dx)}{\sqrt{b+a\cos(c+dx)}}dx}{\sqrt{a+b\sec(c+dx)}}\right)+\frac{2a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3d\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

3.643. $\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

$$\frac{1}{3}\left(\int \frac{7ba^2+\left(a^2+9b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)a}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{3b^3\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{b+a\sin}}}{\sqrt{a+b\sec(c+dx)}}\right.$$
$$\frac{2a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3d\sqrt{\sec(c+dx)}}$$

$\downarrow$ 3286

$$\frac{1}{3}\left(\int \frac{7ba^2+\left(a^2+9b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)a}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{3b^3\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{\sec(c+dx)}{\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}\right)+$$
$$\frac{2a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3d\sqrt{\sec(c+dx)}}$$

$\downarrow$ 3042

$$\frac{1}{3}\left(\int \frac{7ba^2+\left(a^2+9b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)a}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{3b^3\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+\right.}{a+}}}}{\sqrt{a+b\sec(c+dx)}}\right.$$
$$\frac{2a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3d\sqrt{\sec(c+dx)}}$$

$\downarrow$ 3284

$$\frac{1}{3}\left(\int \frac{7ba^2+\left(a^2+9b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)a}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{6b^3\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2}{a}\right.}{d\sqrt{a+b\sec(c+dx)}}\right.$$
$$\frac{2a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3d\sqrt{\sec(c+dx)}}$$

$\downarrow$ 4523

$$\frac{1}{3}\left(a\left(a^2+2b^2\right)\int\frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}dx+7ab\int\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}dx+\frac{6b^3\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\text{Ellipti}}{d\sqrt{a+b\sec(c+}}\right.$$
$$\frac{2a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3d\sqrt{\sec(c+dx)}}$$

$\downarrow$ 3042

3.643. $\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

$$\frac{1}{3}\left(a(a^2+2b^2)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+7ab\int\frac{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{6b^3\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)}{a+b}}}{d\sqrt{a+}}\right.$$
$$\frac{2a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3d\sqrt{\sec(c+dx)}}$$

$\downarrow$ 4343

$$\frac{1}{3}\left(a(a^2+2b^2)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{7ab\sqrt{a+b\sec(c+dx)}\int\sqrt{b+a\cos(c+dx)}dx}{\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}+\frac{6b^3\sqrt{\sec(c+d}}{}\right.$$
$$\frac{2a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3d\sqrt{\sec(c+dx)}}$$

$\downarrow$ 3042

$$\frac{1}{3}\left(a(a^2+2b^2)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{7ab\sqrt{a+b\sec(c+dx)}\int\sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}+\frac{6b^3\sqrt{\sec(}}{}\right.$$
$$\frac{2a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3d\sqrt{\sec(c+dx)}}$$

$\downarrow$ 3134

$$\frac{1}{3}\left(a(a^2+2b^2)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{7ab\sqrt{a+b\sec(c+dx)}\int\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}dx}{\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{6b^3\sqrt{\sec(c+d}}{}\right.$$
$$\frac{2a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3d\sqrt{\sec(c+dx)}}$$

$\downarrow$ 3042

$$\frac{1}{3}\left(a(a^2+2b^2)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{7ab\sqrt{a+b\sec(c+dx)}\int\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}dx}{\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{6b^3\sqrt{\sec(c}}{}\right.$$
$$\frac{2a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3d\sqrt{\sec(c+dx)}}$$

$\downarrow$ 3132

3.643. $\int\frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

$$\frac{1}{3}\left(a(a^2+2b^2)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{6b^3\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}+\frac{2a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3d\sqrt{\sec(c+dx)}}\right.$$

$\downarrow$ 4345

$$\frac{1}{3}\left(\frac{a(a^2+2b^2)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{1}{\sqrt{b+a\cos(c+dx)}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{6b^3\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\text{EllipticPi}\,(}{d\sqrt{a+b\sec(c+dx)}}\right.$$
$$\frac{2a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3d\sqrt{\sec(c+dx)}}$$

$\downarrow$ 3042

$$\frac{1}{3}\left(\frac{a(a^2+2b^2)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{1}{\sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{6b^3\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\text{Elliptic}}{d\sqrt{a+b\sec(c+dx}}\right.$$
$$\frac{2a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3d\sqrt{\sec(c+dx)}}$$

$\downarrow$ 3142

$$\frac{1}{3}\left(\frac{a(a^2+2b^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{6b^3\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\text{EllipticPi}\left(2,\right.}{d\sqrt{a+b\sec(c+dx)}}\right.$$
$$\frac{2a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3d\sqrt{\sec(c+dx)}}$$

$\downarrow$ 3042

$$\frac{1}{3}\left(\frac{a(a^2+2b^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{6b^3\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\text{EllipticPi}\,(}{d\sqrt{a+b\sec(c+dx)}}\right.$$
$$\frac{2a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3d\sqrt{\sec(c+dx)}}$$

$\downarrow$ 3140

3.643. $\int\frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

$$\frac{1}{3}\left(\frac{2a(a^2+2b^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}+\frac{6b^3\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\text{Ellipti}}{d\sqrt{a+b\sec(c+}}\right.$$

$$\frac{2a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3d\sqrt{\sec(c+dx)}}$$

input
```
Int[(a + b*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(3/2),x]
```

output
```
((2*a*(a^2 + 2*b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)
/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(d*Sqrt[a + b*Sec[c + d*x]]) + (6*b
^3*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a
+ b)]*Sqrt[Sec[c + d*x]])/(d*Sqrt[a + b*Sec[c + d*x]]) + (14*a*b*EllipticE
[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(d*Sqrt[(b + a*Cos[
c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]))/3 + (2*a^2*Sqrt[a + b*Sec[c + d*x]
]*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])
```

3.643.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3286
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]]   Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
 d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

rule 4328
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)*
((d*Csc[e + f*x])^n/(f*n)), x] - Simp[1/(d*n)   Int[(a + b*Csc[e + f*x])^(m
 - 3)*(d*Csc[e + f*x])^(n + 1)*Simp[a^2*b*(m - 2*n - 2) - a*(3*b^2*n + a^2*
(n + 1))*Csc[e + f*x] - b*(b^2*n + a^2*(m + n - 1))*Csc[e + f*x]^2, x], x],
 x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && ((Int
egerQ[m] && LtQ[n, -1]) || (IntegersQ[m + 1/2, 2*n] && LeQ[n, -1]))
```

rule 4343
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]])   Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a
, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

3.643. $\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

rule 4345
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]])   Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[
{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4346
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Simp[d*Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x
]]/Sqrt[a + b*Csc[e + f*x]])   Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4523
```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Simp[A/a   I
nt[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Simp[(A*b - a*B)
/(a*d)   Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ
[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

rule 4596
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Simp[C/d^2   Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*C
sc[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[
a + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
 b^2, 0]
```

3.643.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 8.46 (sec) , antiderivative size = 2190, normalized size of antiderivative = 8.36

method	result	size
default	Expression too large to display	2190

input
```
int((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/3/d/((a-b)/(a+b))^(1/2)*(a+b*sec(d*x+c))^(1/2)/(b+a*cos(d*x+c))/sec(d*x+
c)^(3/2)/(cos(d*x+c)+1)*(-7*EllipticF(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc
(d*x+c)),(-(a+b)/(a-b))^(1/2))*(1/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(
d*x+c))/(cos(d*x+c)+1))^(1/2)*a^2*b*cos(d*x+c)+9*EllipticF(((a-b)/(a+b))^(
1/2)*(-cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*(1/(cos(d*x+c)+1))^(1/
2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a*b^2*cos(d*x+c)+7*Elli
pticE(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*(
1/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^
2*b*cos(d*x+c)-7*EllipticE(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(-
(a+b)/(a-b))^(1/2))*(1/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(co
s(d*x+c)+1))^(1/2)*a*b^2*cos(d*x+c)-7*EllipticF(((a-b)/(a+b))^(1/2)*(-cot(
d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*(1/(cos(d*x+c)+1))^(1/2)*(1/(a+b)
*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^2*b*sec(d*x+c)+9*EllipticF(((a-b
)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*(1/(cos(d*x+
c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a*b^2*sec(d*x
+c)+7*EllipticE(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b)
)^(1/2))*(1/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1)
)^(1/2)*a^2*b*sec(d*x+c)-7*EllipticE(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(
d*x+c)),(-(a+b)/(a-b))^(1/2))*(1/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d
*x+c))/(cos(d*x+c)+1))^(1/2)*a*b^2*sec(d*x+c)+((a-b)/(a+b))^(1/2)*a^3*s...
```

3.643.5 Fricas [F]

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx=\int \frac{(b\sec{(dx+c)}+a)^{\frac{5}{2}}}{\sec{(dx+c)}^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(3/2),x, algorithm="fricas")
```

output
```
integral((b^2*sec(d*x + c)^2 + 2*a*b*sec(d*x + c) + a^2)*sqrt(b*sec(d*x +
c) + a)/sec(d*x + c)^(3/2), x)
```

3.643.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+b*sec(d*x+c))**(5/2)/sec(d*x+c)**(3/2),x)
```

output
```
Timed out
```

3.643.7 Maxima [F]

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(b\sec(dx+c)+a)^{\frac{5}{2}}}{\sec(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^(5/2)/sec(d*x + c)^(3/2), x)
```

3.643.8 Giac [F]

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(b\sec(dx+c)+a)^{\frac{5}{2}}}{\sec(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^(5/2)/sec(d*x + c)^(3/2), x)
```

3.643.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx=\int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2}}{\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

input
```
int((a + b/cos(c + d*x))^(5/2)/(1/cos(c + d*x))^(3/2),x)
```

output
```
int((a + b/cos(c + d*x))^(5/2)/(1/cos(c + d*x))^(3/2), x)
```

3.644 $\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{5}{2}}(c+dx)}\,dx$

3.644.1 Optimal result 4992
3.644.2 Mathematica [A] (verified) 4993
3.644.3 Rubi [A] (verified) 4993
3.644.4 Maple [B] (verified) 4999
3.644.5 Fricas [C] (verification not implemented) 5000
3.644.6 Sympy [F(-1)] 5001
3.644.7 Maxima [F] 5001
3.644.8 Giac [F] 5002
3.644.9 Mupad [F(-1)] 5002

3.644.1 Optimal result

Integrand size = 25, antiderivative size = 239

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \frac{16b(a^2-b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{15d\sqrt{a+b\sec(c+dx)}} + \frac{2(9a^2+23b^2)\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+dx)}}{15d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\sqrt{\sec(c+dx)}} + \frac{2a^2\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{22ab\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{15d\sqrt{\sec(c+dx)}}$$

output
```
16/15*b*(a^2-b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elliptic
F(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/
2)*sec(d*x+c)^(1/2)/d/(a+b*sec(d*x+c))^(1/2)+2/5*a^2*sin(d*x+c)*(a+b*sec(d
*x+c))^(1/2)/d/sec(d*x+c)^(3/2)+22/15*a*b*sin(d*x+c)*(a+b*sec(d*x+c))^(1/2
)/d/sec(d*x+c)^(1/2)+2/15*(9*a^2+23*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(
1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*(a+b*
sec(d*x+c))^(1/2)/d/((b+a*cos(d*x+c))/(a+b))^(1/2)/sec(d*x+c)^(1/2)
```

3.644.2 Mathematica [A] (verified)

Time = 1.16 (sec) , antiderivative size = 200, normalized size of antiderivative = 0.84

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \frac{(a+b\sec(c+dx))^{5/2}\left(4(9a^3+9a^2b+23ab^2+23b^3)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}E\left(\frac{1}{2}(c+d\right.\right.}{}$$

input
```
Integrate[(a + b*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(5/2),x]
```

output
```
((a + b*Sec[c + d*x])^(5/2)*(4*(9*a^3 + 9*a^2*b + 23*a*b^2 + 23*b^3)*Sqrt[
(b + a*Cos[c + d*x])/(a + b)]*EllipticE[(c + d*x)/2, (2*a)/(a + b)] + 32*b
*(a^2 - b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*
a)/(a + b)] + 2*a*(3*a^2 + 22*b^2 + 28*a*b*Cos[c + d*x] + 3*a^2*Cos[2*(c +
 d*x)])*Sin[c + d*x]))/(30*d*(b + a*Cos[c + d*x])^3*Sec[c + d*x]^(5/2))
```

3.644.3 Rubi [A] (verified)

Time = 1.99 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.05, number of steps used = 19, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.760$, Rules used = {3042, 4328, 27, 3042, 4592, 27, 3042, 4523, 3042, 4343, 3042, 3134, 3042, 3132, 4345, 3042, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{5}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx$$

$$\Big\downarrow \text{4328}$$

$$\frac{2}{5}\int \frac{11ba^2+3\left(a^2+5b^2\right)\sec(c+dx)a+b\left(2a^2+5b^2\right)\sec^2(c+dx)}{2\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}dx+\frac{2a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{5}\int \frac{11ba^2+3\left(a^2+5b^2\right)\sec(c+dx)a+b\left(2a^2+5b^2\right)\sec^2(c+dx)}{\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}dx+\frac{2a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$\Big\downarrow$ 3042

$$\frac{1}{5}\int \frac{11ba^2+3\left(a^2+5b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)a+b\left(2a^2+5b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$\Big\downarrow$ 4592

$$\frac{1}{5}\left(\frac{22ab\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3d\sqrt{\sec(c+dx)}}-\frac{2\int -\frac{\left(9a^2+23b^2\right)a^2+b\left(17a^2+15b^2\right)\sec(c+dx)a}{2\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{3a}\right)+\frac{2a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$\Big\downarrow$ 27

$$\frac{1}{5}\left(\frac{\int \frac{\left(9a^2+23b^2\right)a^2+b\left(17a^2+15b^2\right)\sec(c+dx)a}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{3a}+\frac{22ab\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3d\sqrt{\sec(c+dx)}}\right)+\frac{2a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$\Big\downarrow$ 3042

$$\frac{1}{5}\left(\frac{\int \frac{\left(9a^2+23b^2\right)a^2+b\left(17a^2+15b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)a}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{3a}+\frac{22ab\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3d\sqrt{\sec(c+dx)}}\right)+\frac{2a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$\Big\downarrow$ 4523

$$\frac{1}{5}\left(\frac{8ab\left(a^2-b^2\right)\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}dx+a\left(9a^2+23b^2\right)\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}dx}{3a}+\frac{22ab\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3d\sqrt{\sec(c+dx)}}\right)+\frac{2a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5d\sec^{\frac{3}{2}}(c+dx)}$$

3.644. $\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{5}{2}}(c+dx)}\,dx$

$$\Big\downarrow \; 3042$$

$$\frac{1}{5}\left(\frac{8ab(a^2-b^2)\int \frac{\sqrt{\csc(c+dx+\frac{\pi}{2})}}{\sqrt{a+b\csc(c+dx+\frac{\pi}{2})}}dx+a(9a^2+23b^2)\int \frac{\sqrt{a+b\csc(c+dx+\frac{\pi}{2})}}{\sqrt{\csc(c+dx+\frac{\pi}{2})}}dx}{3a}+\frac{22ab\sin(c+dx)\sqrt{a+b\sec(c+d}}{3d\sqrt{\sec(c+dx)}}\right.$$

$$\frac{2a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \; 4343$$

$$\frac{1}{5}\left(\frac{8ab(a^2-b^2)\int \frac{\sqrt{\csc(c+dx+\frac{\pi}{2})}}{\sqrt{a+b\csc(c+dx+\frac{\pi}{2})}}dx+\frac{a(9a^2+23b^2)\sqrt{a+b\sec(c+dx)}\int \sqrt{b+a\cos(c+dx)}dx}{\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}}{3a}+\frac{22ab\sin(c+dx)\sqrt{a+b\sec(}}{3d\sqrt{\sec(c+dx)}}\right.$$

$$\frac{2a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{5}\left(\frac{8ab(a^2-b^2)\int \frac{\sqrt{\csc(c+dx+\frac{\pi}{2})}}{\sqrt{a+b\csc(c+dx+\frac{\pi}{2})}}dx+\frac{a(9a^2+23b^2)\sqrt{a+b\sec(c+dx)}\int \sqrt{b+a\sin(c+dx+\frac{\pi}{2})}dx}{\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}}{3a}+\frac{22ab\sin(c+dx)\sqrt{a+b\s}}{3d\sqrt{\sec(c+dx}}\right.$$

$$\frac{2a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \; 3134$$

$$\frac{1}{5}\left(\frac{8ab(a^2-b^2)\int \frac{\sqrt{\csc(c+dx+\frac{\pi}{2})}}{\sqrt{a+b\csc(c+dx+\frac{\pi}{2})}}dx+\frac{a(9a^2+23b^2)\sqrt{a+b\sec(c+dx)}\int \sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}dx}{\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{3a}+\frac{22ab\sin(c+dx)\sqrt{a+b\se}}{3d\sqrt{\sec(c+dx)}}\right.$$

$$\frac{2a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \; 3042$$

3.644. $\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{5}{2}}(c+dx)}\,dx$

$$\frac{1}{5}\left(\frac{8ab(a^2-b^2)\int\frac{\sqrt{\csc(c+dx+\frac{\pi}{2})}}{\sqrt{a+b\csc(c+dx+\frac{\pi}{2})}}dx+\frac{a(9a^2+23b^2)\sqrt{a+b\sec(c+dx)}\int\sqrt{\frac{b}{a+b}+\frac{a\sin(c+dx+\frac{\pi}{2})}{a+b}}dx}{\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{3a}+\frac{22ab\sin(c+dx)\sqrt{a+}}{3d\sqrt{\sec(c+}}\right.$$

$$\frac{2a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$\downarrow$ 3132

$$\frac{1}{5}\left(\frac{8ab(a^2-b^2)\int\frac{\sqrt{\csc(c+dx+\frac{\pi}{2})}}{\sqrt{a+b\csc(c+dx+\frac{\pi}{2})}}dx+\frac{2a(9a^2+23b^2)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{3a}+\frac{22ab\sin(c+dx)\sqrt{a+b\sec(c}}{3d\sqrt{\sec(c+dx)}}\right.$$

$$\frac{2a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$\downarrow$ 4345

$$\frac{1}{5}\left(\frac{\frac{8ab(a^2-b^2)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{1}{\sqrt{b+a\cos(c+dx)}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{2a(9a^2+23b^2)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{3a}+\frac{22ab\sin(c+dx}{3d\sqrt{s}}\right.$$

$$\frac{2a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$\downarrow$ 3042

$$\frac{1}{5}\left(\frac{\frac{8ab(a^2-b^2)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{1}{\sqrt{b+a\sin(c+dx+\frac{\pi}{2})}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{2a(9a^2+23b^2)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{3a}+\frac{22ab\sin(c+}{3d}\right.$$

$$\frac{2a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$\downarrow$ 3142

3.644. $\int\frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{5}{2}}(c+dx)}\,dx$

$$\frac{1}{5}\left(\frac{\frac{8ab(a^2-b^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{2a(9a^2+23b^2)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{3a}+\frac{22ab\sin(c+d}{3d\sqrt{}}\right.$$

$$\frac{2a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{5}\left(\frac{\frac{8ab(a^2-b^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{2a(9a^2+23b^2)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{3a}+\frac{22ab\sin(c}{}\right.$$

$$\frac{2a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \; 3140$$

$$\frac{1}{5}\left(\frac{\frac{16ab(a^2-b^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}+\frac{2a(9a^2+23b^2)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{3a}+\frac{22ab\sin(c}{3}\right.$$

$$\frac{2a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5d\sec^{\frac{3}{2}}(c+dx)}$$

input
```
Int[(a + b*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(5/2),x]
```

output
```
(2*a^2*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)) + (
((16*a*b*(a^2 - b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x
)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(d*Sqrt[a + b*Sec[c + d*x]]) + (2*
a*(9*a^2 + 23*b^2)*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c
+ d*x]])/(d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]))/(3*a)
+ (22*a*b*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]]))
/5
```

3.644. $\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{5}{2}}(c+dx)}\,dx$

3.644.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 4328
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)*
((d*Csc[e + f*x])^n/(f*n)), x] - Simp[1/(d*n)   Int[(a + b*Csc[e + f*x])^(m
 - 3)*(d*Csc[e + f*x])^(n + 1)*Simp[a^2*b*(m - 2*n - 2) - a*(3*b^2*n + a^2*
(n + 1))*Csc[e + f*x] - b*(b^2*n + a^2*(m + n - 1))*Csc[e + f*x]^2, x], x],
 x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && ((Int
egerQ[m] && LtQ[n, -1]) || (IntegersQ[m + 1/2, 2*n] && LeQ[n, -1]))
```

3.644. $\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{5}{2}}(c+dx)}\,dx$

rule 4343
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]])   Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a
, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4345
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]])   Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[
{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4523
```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Simp[A/a   I
nt[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Simp[(A*b - a*B)
/(a*d)   Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ
[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

rule 4592
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Simp[1/(a*d*n)   Int[(a + b*Csc[e + f*x])^m
*(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*
Csc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

3.644.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2566 vs. $2(269) = 538$.

Time = 6.50 (sec) , antiderivative size = 2567, normalized size of antiderivative = 10.74

method	result	size
default	Expression too large to display	2567

input
```
int((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/15/d/((a-b)/(a+b))^(1/2)*(a+b*sec(d*x+c))^(1/2)/(b+a*cos(d*x+c))/sec(d*x
+c)^(5/2)/(cos(d*x+c)+1)*(-34*(1/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d
*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF(((a-b)/(a+b))^(1/2)*(cot(d*x+c)-csc
(d*x+c)),(-(a+b)/(a-b))^(1/2))*a^2*b*sec(d*x+c)-30*sec(d*x+c)*EllipticF(((
a-b)/(a+b))^(1/2)*(cot(d*x+c)-csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*(1/(cos(d*
x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*b^3+46*(1/(
cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ellip
ticF(((a-b)/(a+b))^(1/2)*(cot(d*x+c)-csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*a*b
^2*sec(d*x+c)+18*(1/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d
*x+c)+1))^(1/2)*EllipticE(((a-b)/(a+b))^(1/2)*(cot(d*x+c)-csc(d*x+c)),(-(a
+b)/(a-b))^(1/2))*a^2*b*sec(d*x+c)-46*(1/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b
+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE(((a-b)/(a+b))^(1/2)*(cot(d*
x+c)-csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*a*b^2*sec(d*x+c)-17*(1/(cos(d*x+c)+
1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF(((a-b)
/(a+b))^(1/2)*(cot(d*x+c)-csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*a^2*b*sec(d*x+
c)^2+23*(1/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))
^(1/2)*EllipticF(((a-b)/(a+b))^(1/2)*(cot(d*x+c)-csc(d*x+c)),(-(a+b)/(a-b)
)^(1/2))*a*b^2*sec(d*x+c)^2+9*(1/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d
*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE(((a-b)/(a+b))^(1/2)*(cot(d*x+c)-csc
(d*x+c)),(-(a+b)/(a-b))^(1/2))*a^2*b*sec(d*x+c)^2-23*(1/(cos(d*x+c)+1))...
```

3.644.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 462, normalized size of antiderivative = 1.93

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \frac{\sqrt{2}(-33i\,a^2b+i\,b^3)\sqrt{a}\text{weierstrassPInverse}\left(-\frac{4\,(3\,a^2-4\,b^2)}{3\,a^2},\frac{8\,(9\,a^2b-8\,b^3)}{27\,a^3},\frac{3\,a\cos(dx}{}\right.}{}$$

input
```
integrate((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(5/2),x, algorithm="fricas")
```

output
```
1/45*(sqrt(2)*(-33*I*a^2*b + I*b^3)*sqrt(a)*weierstrassPInverse(-4/3*(3*a^
2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*
sin(d*x + c) + 2*b)/a) + sqrt(2)*(33*I*a^2*b - I*b^3)*sqrt(a)*weierstrassP
Inverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos
(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a) - 3*sqrt(2)*(-9*I*a^3 - 23*I*a*b^
2)*sqrt(a)*weierstrassZeta(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3
)/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3
)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x + c) + 2*b)/a)) - 3*sqrt(2)*(
9*I*a^3 + 23*I*a*b^2)*sqrt(a)*weierstrassZeta(-4/3*(3*a^2 - 4*b^2)/a^2, 8/
27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/
27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*b
)/a)) + 6*(3*a^3*cos(d*x + c)^2 + 11*a^2*b*cos(d*x + c))*sqrt((a*cos(d*x +
 c) + b)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a*d)
```

3.644.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+b*sec(d*x+c))**(5/2)/sec(d*x+c)**(5/2),x)
```

output
```
Timed out
```

3.644.7 Maxima [F]

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{(b\sec(dx+c)+a)^{\frac{5}{2}}}{\sec(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^(5/2)/sec(d*x + c)^(5/2), x)
```

3.644.8 Giac [F]

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{5}{2}}(c+dx)}\,dx=\int \frac{(b\sec(dx+c)+a)^{\frac{5}{2}}}{\sec(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^(5/2)/sec(d*x + c)^(5/2), x)
```

3.644.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{5}{2}}(c+dx)}\,dx=\int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2}}{\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

input
```
int((a + b/cos(c + d*x))^(5/2)/(1/cos(c + d*x))^(5/2),x)
```

output
```
int((a + b/cos(c + d*x))^(5/2)/(1/cos(c + d*x))^(5/2), x)
```

3.645 $\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{7}{2}}(c+dx)}\,dx$

3.645.1 Optimal result 5003
3.645.2 Mathematica [A] (verified) 5004
3.645.3 Rubi [A] (verified) 5004
3.645.4 Maple [B] (verified) 5012
3.645.5 Fricas [C] (verification not implemented) 5012
3.645.6 Sympy [F(-1)] 5013
3.645.7 Maxima [F] 5013
3.645.8 Giac [F] 5014
3.645.9 Mupad [F(-1)] 5014

3.645.1 Optimal result

Integrand size = 25, antiderivative size = 303

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{7}{2}}(c+dx)}\,dx = \frac{2(5a^4-2a^2b^2-3b^4)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{21ad\sqrt{a+b\sec(c+dx)}} + \frac{2b(29a^2+3b^2)\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+dx)}}{21ad\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\sqrt{\sec(c+dx)}} + \frac{2a^2\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{6ab\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{7d\sec^{\frac{3}{2}}(c+dx)} + \frac{2(5a^2+9b^2)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}$$

output
```
2/21*(5*a^4-2*a^2*b^2-3*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*
c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))
/(a+b))^(1/2)*sec(d*x+c)^(1/2)/a/d/(a+b*sec(d*x+c))^(1/2)+2/7*a^2*sin(d*x+
c)*(a+b*sec(d*x+c))^(1/2)/d/sec(d*x+c)^(5/2)+6/7*a*b*sin(d*x+c)*(a+b*sec(d
*x+c))^(1/2)/d/sec(d*x+c)^(3/2)+2/21*(5*a^2+9*b^2)*sin(d*x+c)*(a+b*sec(d*x
+c))^(1/2)/d/sec(d*x+c)^(1/2)+2/21*b*(29*a^2+3*b^2)*(cos(1/2*d*x+1/2*c)^2)
^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(
1/2))*(a+b*sec(d*x+c))^(1/2)/a/d/((b+a*cos(d*x+c))/(a+b))^(1/2)/sec(d*x+c)
^(1/2)
```

3.645.2 Mathematica [A] (verified)

Time = 1.62 (sec) , antiderivative size = 237, normalized size of antiderivative = 0.78

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{7}{2}}(c+dx)}\,dx = \frac{(a+b\sec(c+dx))^{5/2}\left(8b(29a^3+29a^2b+3ab^2+3b^3)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}E\left(\frac{1}{2}(c+\right.\right.}{}$$

input
```
Integrate[(a + b*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(7/2),x]
```

output
```
((a + b*Sec[c + d*x])^(5/2)*(8*b*(29*a^3 + 29*a^2*b + 3*a*b^2 + 3*b^3)*Sqr
t[(b + a*Cos[c + d*x])/(a + b)]*EllipticE[(c + d*x)/2, (2*a)/(a + b)] + 8*
(5*a^4 - 2*a^2*b^2 - 3*b^4)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(
c + d*x)/2, (2*a)/(a + b)] + 2*a*(44*a^2*b + 36*b^3 + a*(29*a^2 + 72*b^2)*
Cos[c + d*x] + 24*a^2*b*Cos[2*(c + d*x)] + 3*a^3*Cos[3*(c + d*x)])*Sin[c +
 d*x]))/(84*a*d*(b + a*Cos[c + d*x])^3*Sec[c + d*x]^(5/2))
```

3.645.3 Rubi [A] (verified)

Time = 2.54 (sec) , antiderivative size = 312, normalized size of antiderivative = 1.03, number of steps used = 22, number of rules used = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.880$, Rules used = {3042, 4328, 27, 3042, 4592, 27, 3042, 4592, 27, 3042, 4523, 3042, 4343, 3042, 3134, 3042, 3132, 4345, 3042, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{7}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx$$

$$\Big\downarrow \text{4328}$$

$$\frac{2}{7}\int \frac{15ba^2+\left(5a^2+21b^2\right)\sec(c+dx)a+b\left(4a^2+7b^2\right)\sec^2(c+dx)}{2\sec^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}dx+\frac{\frac{2a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{7d\sec^{\frac{5}{2}}(c+dx)}}{}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{7}\int \frac{15ba^2+\left(5a^2+21b^2\right)\sec(c+dx)a+b\left(4a^2+7b^2\right)\sec^2(c+dx)}{\sec^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}dx+\frac{2a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{7}\int \frac{15ba^2+\left(5a^2+21b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)a+b\left(4a^2+7b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \; 4592$$

$$\frac{1}{7}\left(\frac{6ab\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{d\sec^{\frac{3}{2}}(c+dx)}-\frac{2\int -\frac{5\left(6b^2\sec^2(c+dx)a^2+\left(5a^2+9b^2\right)a^2+b\left(13a^2+7b^2\right)\sec(c+dx)a\right)}{2\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}dx}{5a}\right)+\frac{2a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{7}\left(\frac{\int \frac{6b^2\sec^2(c+dx)a^2+\left(5a^2+9b^2\right)a^2+b\left(13a^2+7b^2\right)\sec(c+dx)a}{\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}dx}{a}+\frac{6ab\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{d\sec^{\frac{3}{2}}(c+dx)}\right)+\frac{2a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{7}\left(\frac{\int \frac{6b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+\left(5a^2+9b^2\right)a^2+b\left(13a^2+7b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)a}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}+\frac{6ab\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{d\sec^{\frac{3}{2}}(c+dx)}\right)+\frac{2a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \; 4592$$

3.645. $\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{7}{2}}(c+dx)}\,dx$

$$\frac{1}{7}\left(\frac{\frac{2a\left(5a^2+9b^2\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3d\sqrt{\sec(c+dx)}}-\frac{2\int -\frac{\left(5a^2+27b^2\right)\sec(c+dx)a^3+b\left(29a^2+3b^2\right)a^2}{2\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{3a}}{a}+\frac{6ab\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{d\sec^{\frac{3}{2}}(c+dx)}\right.$$

$$\frac{2a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$\Big\downarrow$ 27

$$\frac{1}{7}\left(\frac{\frac{\int\frac{\left(5a^2+27b^2\right)\sec(c+dx)a^3+b\left(29a^2+3b^2\right)a^2}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{3a}+\frac{2a\left(5a^2+9b^2\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3d\sqrt{\sec(c+dx)}}}{a}+\frac{6ab\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{d\sec^{\frac{3}{2}}(c+dx)}\right)$$

$$\frac{2a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$\Big\downarrow$ 3042

$$\frac{1}{7}\left(\frac{\frac{\int\frac{\left(5a^2+27b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)a^3+b\left(29a^2+3b^2\right)a^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{3a}+\frac{2a\left(5a^2+9b^2\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3d\sqrt{\sec(c+dx)}}}{a}+\frac{6ab\sin(c+dx)\sqrt{a+b\sec(c+dx}}{d\sec^{\frac{3}{2}}(c+dx)}\right.$$

$$\frac{2a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$\Big\downarrow$ 4523

$$\frac{1}{7}\left(\frac{\frac{ab\left(29a^2+3b^2\right)\int\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}dx+a\left(5a^4-2a^2b^2-3b^4\right)\int\frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}dx}{3a}+\frac{2a\left(5a^2+9b^2\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3d\sqrt{\sec(c+dx)}}}{a}+\frac{6ab\sin(c+}{d}\right.$$

$$\frac{2a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$\Big\downarrow$ 3042

3.645. $\int\frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{7}{2}}(c+dx)}\,dx$

$$\frac{1}{7}\left(\frac{\frac{ab\left(29a^2+3b^2\right)\int\frac{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+a\left(5a^4-2a^2b^2-3b^4\right)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{3a}+\frac{2a\left(5a^2+9b^2\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3d\sqrt{\sec(c+dx)}}}{a}+\frac{6ab\,\mathrm{s}}{}\right.$$

$$\frac{2a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$\downarrow$ 4343

$$\frac{1}{7}\left(\frac{\frac{\frac{ab\left(29a^2+3b^2\right)\sqrt{a+b\sec(c+dx)}\int\sqrt{b+a\cos(c+dx)}dx}{\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}+a\left(5a^4-2a^2b^2-3b^4\right)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{3a}+\frac{2a\left(5a^2+9b^2\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3d\sqrt{\sec(c+dx)}}}{a}\right.$$

$$\frac{2a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$\downarrow$ 3042

$$\frac{1}{7}\left(\frac{\frac{\frac{ab\left(29a^2+3b^2\right)\sqrt{a+b\sec(c+dx)}\int\sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}+a\left(5a^4-2a^2b^2-3b^4\right)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{3a}+\frac{2a\left(5a^2+9b^2\right)\sin(c+dx)\sqrt{a+b\sec(c+d}}{3d\sqrt{\sec(c+dx)}}}{a}\right.$$

$$\frac{2a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$\downarrow$ 3134

$$\frac{1}{7}\left(\frac{\frac{\frac{ab\left(29a^2+3b^2\right)\sqrt{a+b\sec(c+dx)}\int\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}dx}{\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+a\left(5a^4-2a^2b^2-3b^4\right)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{3a}+\frac{2a\left(5a^2+9b^2\right)\sin(c+dx)\sqrt{a+b\sec(c+d}}{3d\sqrt{\sec(c+dx)}}}{a}\right.$$

$$\frac{2a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$\downarrow$ 3042

3.645. $\int\frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{7}{2}}(c+dx)}\,dx$

$$\frac{1}{7}\left(\frac{\frac{\frac{ab\left(29a^2+3b^2\right)\sqrt{a+b\sec(c+dx)}\int\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}dx}{\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+a\left(5a^4-2a^2b^2-3b^4\right)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{3a}+\frac{2a\left(5a^2+9b^2\right)\sin(c+dx)\sqrt{a+b\sec(c}}{3d\sqrt{\sec(c+dx)}}}{a}\right.$$

$$\frac{2a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$\downarrow$ 3132

$$\frac{1}{7}\left(\frac{\frac{\frac{a\left(5a^4-2a^2b^2-3b^4\right)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2ab\left(29a^2+3b^2\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{3a}+\frac{2a\left(5a^2+9b^2\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3d\sqrt{\sec(c+dx)}}}{a}+\right.$$

$$\frac{2a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$\downarrow$ 4345

$$\frac{1}{7}\left(\frac{\frac{\frac{a\left(5a^4-2a^2b^2-3b^4\right)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{1}{\sqrt{b+a\cos(c+dx)}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{2ab\left(29a^2+3b^2\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{3a}+\frac{2a\left(5a^2+9b^2\right)\sin(c+dx)}{3d\sqrt{\sec(c+}}}{a}\right.$$

$$\frac{2a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$\downarrow$ 3042

$$\frac{1}{7}\left(\frac{\frac{\frac{a\left(5a^4-2a^2b^2-3b^4\right)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{1}{\sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{2ab\left(29a^2+3b^2\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{3a}+\frac{2a\left(5a^2+9b^2\right)\sin(c+}{3d\sqrt{\sec}}}{a}\right.$$

$$\frac{2a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{7d\sec^{\frac{5}{2}}(c+dx)}$$

3.645. $\int\frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{7}{2}}(c+dx)}\,dx$

$$\Big\downarrow \; 3142$$

$$\frac{1}{7}\left(\frac{\frac{\frac{a\left(5a^4-2a^2b^2-3b^4\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{2ab\left(29a^2+3b^2\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{3a}+\frac{2a\left(5a^2+9b^2\right)\sin(c-}{3d\sqrt{\mathrm{se}}}}{a}\right.$$

$$\frac{2a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{7}\left(\frac{\frac{\frac{a\left(5a^4-2a^2b^2-3b^4\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{2ab\left(29a^2+3b^2\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{3a}+\frac{2a\left(5a^2+9b^2\right)\mathrm{si}}{3d}}{a}\right.$$

$$\frac{2a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \; 3140$$

$$\frac{2a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{7d\sec^{\frac{5}{2}}(c+dx)}+$$

$$\frac{1}{7}\left(\frac{\frac{2a\left(5a^2+9b^2\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3d\sqrt{\sec(c+dx)}}+\frac{\frac{2ab\left(29a^2+3b^2\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{2a\left(5a^4-2a^2b^2-3b^4\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\mathrm{E}}{d\sqrt{a+b\sec(c+dx)}}}{3a}}{a}\right.$$

input
```
Int[(a + b*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(7/2),x]
```

3.645. $\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{7}{2}}(c+dx)}\,dx$

output
```
(2*a^2*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/2)) + (
(6*a*b*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(d*Sec[c + d*x]^(3/2)) + (((
2*a*(5*a^4 - 2*a^2*b^2 - 3*b^4)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Ellipti
cF[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(d*Sqrt[a + b*Sec[c + d
*x]]) + (2*a*b*(29*a^2 + 3*b^2)*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt
[a + b*Sec[c + d*x]])/(d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d
*x]]))/(3*a) + (2*a*(5*a^2 + 9*b^2)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])
/(3*d*Sqrt[Sec[c + d*x]]))/a)/7
```

3.645.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 4328
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)*
((d*Csc[e + f*x])^n/(f*n)), x] - Simp[1/(d*n)   Int[(a + b*Csc[e + f*x])^(m
 - 3)*(d*Csc[e + f*x])^(n + 1)*Simp[a^2*b*(m - 2*n - 2) - a*(3*b^2*n + a^2*
(n + 1))*Csc[e + f*x] - b*(b^2*n + a^2*(m + n - 1))*Csc[e + f*x]^2, x], x],
 x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && ((Int
egerQ[m] && LtQ[n, -1]) || (IntegersQ[m + 1/2, 2*n] && LeQ[n, -1]))
```

rule 4343
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]])   Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a
, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4345
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]])   Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[
{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4523
```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Simp[A/a   I
nt[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Simp[(A*b - a*B)
/(a*d)   Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ
[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

rule 4592
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Simp[1/(a*d*n)   Int[(a + b*Csc[e + f*x])^m
*(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*
Csc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

3.645. $\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{7}{2}}(c+dx)}\,dx$

3.645.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2765 vs. $2(327) = 654$.

Time = 7.77 (sec) , antiderivative size = 2766, normalized size of antiderivative = 9.13

method	result	size
default	Expression too large to display	2766

input
```
int((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
2/21/d/((a-b)/(a+b))^(1/2)/a*(3*(1/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos
(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+
csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*a*b^3*cos(d*x+c)^2-3*(1/(cos(d*x+c)+1))^
(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE(((a-b)/(a+
b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*b^4+5*(1/(cos(d*x
+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF(((
a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*a^4+3*((a
-b)/(a+b))^(1/2)*b^4*sin(d*x+c)-29*(1/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*
cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF(((a-b)/(a+b))^(1/2)*(-cot(d*x+
c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*a^3*b*cos(d*x+c)^2+27*(1/(cos(d*x+c)+
1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF(((a-b)
/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*a^2*b^2*cos(d
*x+c)^2-3*(1/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1
))^(1/2)*EllipticF(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a
-b))^(1/2))*a*b^3*cos(d*x+c)^2+58*(1/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*c
os(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE(((a-b)/(a+b))^(1/2)*(-cot(d*x+c
)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*a^3*b*cos(d*x+c)-58*(1/(cos(d*x+c)+1))
^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE(((a-b)/(a
+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*a^2*b^2*cos(d*x+
c)+6*(1/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))...
```

3.645.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.13 (sec) , antiderivative size = 501, normalized size of antiderivative = 1.65

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{7}{2}}(c+dx)}\,dx = \frac{\sqrt{2}(-15i\,a^4-23i\,a^2b^2+6i\,b^4)\sqrt{a}\text{weierstrassPInverse}\left(-\frac{4\,(3\,a^2-4\,b^2)}{3\,a^2},\frac{8\,(9\,a^2b-8\,b}{27\,a^3}\right.}{}$$

input
```
integrate((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(7/2),x, algorithm="fricas")
```

output
```
1/63*(sqrt(2)*(-15*I*a^4 - 23*I*a^2*b^2 + 6*I*b^4)*sqrt(a)*weierstrassPInv
erse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*
x + c) + 3*I*a*sin(d*x + c) + 2*b)/a) + sqrt(2)*(15*I*a^4 + 23*I*a^2*b^2 -
 6*I*b^4)*sqrt(a)*weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^
2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a) - 3
*sqrt(2)*(-29*I*a^3*b - 3*I*a*b^3)*sqrt(a)*weierstrassZeta(-4/3*(3*a^2 - 4
*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4
*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d
*x + c) + 2*b)/a)) - 3*sqrt(2)*(29*I*a^3*b + 3*I*a*b^3)*sqrt(a)*weierstras
sZeta(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPIn
verse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d
*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a)) + 6*(3*a^4*cos(d*x + c)^3 + 9*a^3*
b*cos(d*x + c)^2 + (5*a^4 + 9*a^2*b^2)*cos(d*x + c))*sqrt((a*cos(d*x + c)
+ b)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^2*d)
```

3.645.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{7}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+b*sec(d*x+c))**(5/2)/sec(d*x+c)**(7/2),x)
```

output
```
Timed out
```

3.645.7 Maxima [F]

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{7}{2}}(c+dx)}\,dx = \int \frac{(b\sec(dx+c)+a)^{\frac{5}{2}}}{\sec(dx+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(7/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^(5/2)/sec(d*x + c)^(7/2), x)
```

3.645.8 Giac [F]

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{7}{2}}(c+dx)}\,dx=\int \frac{(b\sec(dx+c)+a)^{\frac{5}{2}}}{\sec(dx+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(7/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^(5/2)/sec(d*x + c)^(7/2), x)
```

3.645.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{7}{2}}(c+dx)}\,dx=\int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2}}{\left(\frac{1}{\cos(c+d\,x)}\right)^{7/2}}\,dx$$

input
```
int((a + b/cos(c + d*x))^(5/2)/(1/cos(c + d*x))^(7/2),x)
```

output
```
int((a + b/cos(c + d*x))^(5/2)/(1/cos(c + d*x))^(7/2), x)
```

3.646 $\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{9}{2}}(c+dx)}\,dx$

3.646.1 Optimal result 5015
3.646.2 Mathematica [A] (verified) 5016
3.646.3 Rubi [A] (verified) 5016
3.646.4 Maple [B] (warning: unable to verify) 5025
3.646.5 Fricas [C] (verification not implemented) 5026
3.646.6 Sympy [F(-1)] 5027
3.646.7 Maxima [F] 5027
3.646.8 Giac [F] 5028
3.646.9 Mupad [F(-1)] 5028

3.646.1 Optimal result

Integrand size = 25, antiderivative size = 363

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{9}{2}}(c+dx)}\,dx = \frac{4b(57a^4-62a^2b^2+5b^4)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{315a^2d\sqrt{a+b\sec(c+dx)}}$$
$$+\frac{2(147a^4+279a^2b^2-10b^4)\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+dx)}}{315a^2d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\sqrt{\sec(c+dx)}}$$
$$+\frac{2a^2\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)}+\frac{38ab\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{63d\sec^{\frac{5}{2}}(c+dx)}$$
$$+\frac{2(49a^2+75b^2)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{315d\sec^{\frac{3}{2}}(c+dx)}$$
$$+\frac{2b(163a^2+5b^2)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{315ad\sqrt{\sec(c+dx)}}$$

output
```
4/315*b*(57*a^4-62*a^2*b^2+5*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x
+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*
x+c))/(a+b))^(1/2)*sec(d*x+c)^(1/2)/a^2/d/(a+b*sec(d*x+c))^(1/2)+2/9*a^2*s
in(d*x+c)*(a+b*sec(d*x+c))^(1/2)/d/sec(d*x+c)^(7/2)+38/63*a*b*sin(d*x+c)*(
a+b*sec(d*x+c))^(1/2)/d/sec(d*x+c)^(5/2)+2/315*(49*a^2+75*b^2)*sin(d*x+c)*
(a+b*sec(d*x+c))^(1/2)/d/sec(d*x+c)^(3/2)+2/315*b*(163*a^2+5*b^2)*sin(d*x+
c)*(a+b*sec(d*x+c))^(1/2)/a/d/sec(d*x+c)^(1/2)+2/315*(147*a^4+279*a^2*b^2-
10*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*
d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*(a+b*sec(d*x+c))^(1/2)/a^2/d/((b+a*cos
(d*x+c))/(a+b))^(1/2)/sec(d*x+c)^(1/2)
```

3.646.2 Mathematica [A] (verified)

Time = 2.21 (sec) , antiderivative size = 286, normalized size of antiderivative = 0.79

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{9}{2}}(c+dx)}\,dx = \frac{(a+b\sec(c+dx))^{5/2}\left(16(147a^5+147a^4b+279a^3b^2+279a^2b^3-10ab^4-10\right.}{}$$

input
```
Integrate[(a + b*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(9/2),x]
```

output
```
((a + b*Sec[c + d*x])^(5/2)*(16*(147*a^5 + 147*a^4*b + 279*a^3*b^2 + 279*a
^2*b^3 - 10*a*b^4 - 10*b^5)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticE[(
c + d*x)/2, (2*a)/(a + b)] + 32*b*(57*a^4 - 62*a^2*b^2 + 5*b^4)*Sqrt[(b +
a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)] + 2*a*(301*
a^4 + 1984*a^2*b^2 + 40*b^4 + 4*a*b*(619*a^2 + 160*b^2)*Cos[c + d*x] + 8*(
42*a^4 + 85*a^2*b^2)*Cos[2*(c + d*x)] + 260*a^3*b*Cos[3*(c + d*x)] + 35*a^
4*Cos[4*(c + d*x)])*Sin[c + d*x]))/(2520*a^2*d*(b + a*Cos[c + d*x])^3*Sec[
c + d*x]^(5/2))
```

3.646.3 Rubi [A] (verified)

Time = 3.30 (sec) , antiderivative size = 378, normalized size of antiderivative = 1.04, number of steps used = 25, number of rules used = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {3042, 4328, 27, 3042, 4592, 27, 3042, 4592, 27, 3042, 4592, 27, 3042, 4523, 3042, 4343, 3042, 3134, 3042, 3132, 4345, 3042, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{9}{2}}(c+dx)}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}{\csc\left(c+dx+\frac{\pi}{2}\right)^{9/2}}dx$$

$$\Big\downarrow \; 4328$$

$$\frac{2}{9}\int \frac{19ba^2+\left(7a^2+27b^2\right)\sec(c+dx)a+3b\left(2a^2+3b^2\right)\sec^2(c+dx)}{2\sec^{\frac{7}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}dx+\frac{2a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{9d\sec^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{9}\int \frac{19ba^2+\left(7a^2+27b^2\right)\sec(c+dx)a+3b\left(2a^2+3b^2\right)\sec^2(c+dx)}{\sec^{\frac{7}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}dx+\frac{2a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{9d\sec^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{9}\int \frac{19ba^2+\left(7a^2+27b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)a+3b\left(2a^2+3b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{9d\sec^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \; 4592$$

$$\frac{1}{9}\left(\frac{38ab\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{7d\sec^{\frac{5}{2}}(c+dx)}-\frac{2\int -\frac{76b^2\sec^2(c+dx)a^2+\left(49a^2+75b^2\right)a^2+b\left(137a^2+63b^2\right)\sec(c+dx)a}{2\sec^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}dx}{7a}\right)+\frac{2a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{9d\sec^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{9}\left(\frac{\int \frac{76b^2\sec^2(c+dx)a^2+\left(49a^2+75b^2\right)a^2+b\left(137a^2+63b^2\right)\sec(c+dx)a}{\sec^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}dx}{7a}+\frac{38ab\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{7d\sec^{\frac{5}{2}}(c+dx)}\right)+\frac{2a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{9d\sec^{\frac{7}{2}}(c+dx)}$$

3.646. $\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{9}{2}}(c+dx)}\,dx$

$$\Big\downarrow \; 3042$$

$$\frac{1}{9}\left(\frac{\int \frac{76b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+\left(49a^2+75b^2\right)a^2+b\left(137a^2+63b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)a}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{7a}+\frac{38ab\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{7d\sec^{\frac{5}{2}}(c+dx)}\right)+\frac{2a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{9d\sec^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \; 4592$$

$$\frac{1}{9}\left(\frac{\frac{2a\left(49a^2+75b^2\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5d\sec^{\frac{3}{2}}(c+dx)}-\frac{2\int -\frac{\left(147a^2+605b^2\right)\sec(c+dx)a^3+2b\left(49a^2+75b^2\right)\sec^2(c+dx)a^2+3b\left(163a^2+5b^2\right)a^2}{2\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}dx}{5a}}{7a}+\frac{38ab\,\mathrm{s}}{}\right.$$

$$\frac{2a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{9d\sec^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{9}\left(\frac{\frac{\int \frac{\left(147a^2+605b^2\right)\sec(c+dx)a^3+2b\left(49a^2+75b^2\right)\sec^2(c+dx)a^2+3b\left(163a^2+5b^2\right)a^2}{\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}dx}{5a}+\frac{2a\left(49a^2+75b^2\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5d\sec^{\frac{3}{2}}(c+dx)}}{7a}+\frac{38ab\sin(}{}\right.$$

$$\frac{2a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{9d\sec^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{9}\left(\frac{\frac{\int \frac{\left(147a^2+605b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)a^3+2b\left(49a^2+75b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+3b\left(163a^2+5b^2\right)a^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{5a}+\frac{2a\left(49a^2+75b^2\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5d\sec^{\frac{3}{2}}(c+dx)}}{7a}+\frac{38}{}\right.$$

$$\frac{2a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{9d\sec^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \; 4592$$

3.646. $\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{9}{2}}(c+dx)}\,dx$

$$\frac{1}{9}\left(\frac{\frac{\frac{2ab\left(163a^2+5b^2\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{d\sqrt{\sec(c+dx)}}-\frac{2\int -\frac{3\left(b\left(261a^2+155b^2\right)\sec(c+dx)a^3+\left(147a^4+279b^2a^2-10b^4\right)a^2\right)}{2\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{3a}}{5a}+\frac{2a\left(49a^2+75b^2\right)\sin(c+dx)\sqrt{a-}}{5d\sec^{\frac{3}{2}}(c+dx)}}{7a}\right.$$

$$\frac{2a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{9d\sec^{\frac{7}{2}}(c+dx)}$$

$\downarrow$ 27

$$\frac{1}{9}\left(\frac{\frac{\frac{\int\frac{b\left(261a^2+155b^2\right)\sec(c+dx)a^3+\left(147a^4+279b^2a^2-10b^4\right)a^2}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{a}+\frac{2ab\left(163a^2+5b^2\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{d\sqrt{\sec(c+dx)}}}{5a}+\frac{2a\left(49a^2+75b^2\right)\sin(c+dx)\sqrt{a+b\sec(}}{5d\sec^{\frac{3}{2}}(c+dx)}}{7a}\right.$$

$$\frac{2a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{9d\sec^{\frac{7}{2}}(c+dx)}$$

$\downarrow$ 3042

$$\frac{1}{9}\left(\frac{\frac{\frac{\int\frac{b\left(261a^2+155b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)a^3+\left(147a^4+279b^2a^2-10b^4\right)a^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}+\frac{2ab\left(163a^2+5b^2\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{d\sqrt{\sec(c+dx)}}}{5a}+\frac{2a\left(49a^2+75b^2\right)\sin(c+dx)\sqrt{a+b}}{5d\sec^{\frac{3}{2}}(c+dx)}}{7a}\right.$$

$$\frac{2a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{9d\sec^{\frac{7}{2}}(c+dx)}$$

$\downarrow$ 4523

$$\frac{1}{9}\left(\frac{\frac{\frac{2ab\left(57a^4-62a^2b^2+5b^4\right)\int\frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}dx+a\left(147a^4+279a^2b^2-10b^4\right)\int\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}dx}{a}+\frac{2ab\left(163a^2+5b^2\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{d\sqrt{\sec(c+dx)}}}{5a}+\frac{2a(49a^2}{}}{7a}\right.$$

$$\frac{2a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{9d\sec^{\frac{7}{2}}(c+dx)}$$

$\downarrow$ 3042

3.646. $\int\frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{9}{2}}(c+dx)}\,dx$

$$\frac{1}{9}\left(\frac{\frac{\frac{2ab\left(57a^4-62a^2b^2+5b^4\right)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+a\left(147a^4+279a^2b^2-10b^4\right)\int\frac{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}+\frac{2ab\left(163a^2+5b^2\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{d\sqrt{\sec(c+dx)}}}{5a}+\right.}{7a}$$

$$\frac{2a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{9d\sec^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \; 4343$$

$$\frac{1}{9}\left(\frac{\frac{\frac{2ab\left(57a^4-62a^2b^2+5b^4\right)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{a\left(147a^4+279a^2b^2-10b^4\right)\sqrt{a+b\sec(c+dx)}\int\sqrt{b+a\cos(c+dx)}dx}{\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}}{a}+\frac{2ab\left(163a^2+5b^2\right)\sin(c+dx)\sqrt{a+b\sec}}{d\sqrt{\sec(c+dx)}}}{5a}}{7a}\right.$$

$$\frac{2a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{9d\sec^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{9}\left(\frac{\frac{\frac{2ab\left(57a^4-62a^2b^2+5b^4\right)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{a\left(147a^4+279a^2b^2-10b^4\right)\sqrt{a+b\sec(c+dx)}\int\sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}}{a}+\frac{2ab\left(163a^2+5b^2\right)\sin(c+dx)\sqrt{a+}}{d\sqrt{\sec(c+dx)}}}{5a}}{7a}\right.$$

$$\frac{2a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{9d\sec^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \; 3134$$

3.646. $\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{9}{2}}(c+dx)}\,dx$

$$\frac{1}{9}\left(\frac{\frac{\frac{2ab\left(57a^4-62a^2b^2+5b^4\right)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{a\left(147a^4+279a^2b^2-10b^4\right)\sqrt{a+b\sec(c+dx)}\int\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}dx}{\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{a}+\frac{2ab\left(163a^2+5b^2\right)\sin(c+dx)\sqrt{a+}}{d\sqrt{\sec(c+dx)}}}{5a}}{7a}\right.$$

$$\frac{2a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{9d\sec^{\frac{7}{2}}(c+dx)}$$

$\downarrow$ 3042

$$\frac{1}{9}\left(\frac{\frac{\frac{2ab\left(57a^4-62a^2b^2+5b^4\right)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{a\left(147a^4+279a^2b^2-10b^4\right)\sqrt{a+b\sec(c+dx)}\int\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}dx}{\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{a}+\frac{2ab\left(163a^2+5b^2\right)\sin(c+dx)}{d\sqrt{\sec(c+dx}}}{5a}}{7a}\right.$$

$$\frac{2a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{9d\sec^{\frac{7}{2}}(c+dx)}$$

$\downarrow$ 3132

$$\frac{1}{9}\left(\frac{\frac{\frac{2ab\left(57a^4-62a^2b^2+5b^4\right)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a\left(147a^4+279a^2b^2-10b^4\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{a}+\frac{2ab\left(163a^2+5b^2\right)\sin(c+dx)\sqrt{a+b\sec(}}{d\sqrt{\sec(c+dx)}}}{5a}}{7a}\right.$$

$$\frac{2a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{9d\sec^{\frac{7}{2}}(c+dx)}$$

$\downarrow$ 4345

3.646. $\int\frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{9}{2}}(c+dx)}\,dx$

$$\frac{1}{9}\left(\frac{\frac{\frac{\frac{2ab\left(57a^4-62a^2b^2+5b^4\right)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{1}{\sqrt{b+a\cos(c+dx)}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{2a\left(147a^4+279a^2b^2-10b^4\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{a}+\frac{2ab\left(163a^2+5\right.}{}}{5a}}{7a}\right.$$

$$\frac{2a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{9d\sec^{\frac{7}{2}}(c+dx)}$$

$\downarrow$ 3042

$$\frac{1}{9}\left(\frac{\frac{\frac{\frac{2ab\left(57a^4-62a^2b^2+5b^4\right)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{1}{\sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{2a\left(147a^4+279a^2b^2-10b^4\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{a}+\frac{2ab\left(163\right.}{}}{5a}}{7a}\right.$$

$$\frac{2a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{9d\sec^{\frac{7}{2}}(c+dx)}$$

$\downarrow$ 3142

$$\frac{1}{9}\left(\frac{\frac{\frac{\frac{2ab\left(57a^4-62a^2b^2+5b^4\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{2a\left(147a^4+279a^2b^2-10b^4\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{a}+\frac{2ab\left(163\right.}{}}{5a}}{7a}\right.$$

$$\frac{2a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{9d\sec^{\frac{7}{2}}(c+dx)}$$

$\downarrow$ 3042

3.646. $\int\frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{9}{2}}(c+dx)}\,dx$

$$\frac{1}{9}\left(\frac{\frac{\frac{\frac{2ab\left(57a^4-62a^2b^2+5b^4\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{2a\left(147a^4+279a^2b^2-10b^4\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{a}+\frac{2ab}{}}{5a}}{7a}\right.$$

$$\frac{2a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{9d\sec^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \; 3140$$

$$\frac{2a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{9d\sec^{\frac{7}{2}}(c+dx)}+$$

$$\frac{1}{9}\left(\frac{\frac{2a\left(49a^2+75b^2\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{\frac{2ab\left(163a^2+5b^2\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{d\sqrt{\sec(c+dx)}}+\frac{\frac{4ab\left(57a^4-62a^2b^2+5b^4\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\mathrm{El}}{d\sqrt{a+b\sec(c+dx)}}}{5a}}{7a}\right.$$

input
```
Int[(a + b*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(9/2),x]
```

output
```
(2*a^2*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(9*d*Sec[c + d*x]^(7/2)) + (
(38*a*b*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/2)) +
((2*a*(49*a^2 + 75*b^2)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(5*d*Sec[c
+ d*x]^(3/2)) + (((4*a*b*(57*a^4 - 62*a^2*b^2 + 5*b^4)*Sqrt[(b + a*Cos[c +
 d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/
(d*Sqrt[a + b*Sec[c + d*x]]) + (2*a*(147*a^4 + 279*a^2*b^2 - 10*b^4)*Ellip
ticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(d*Sqrt[(b + a*
Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]))/a + (2*a*b*(163*a^2 + 5*b^2)*S
qrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Sec[c + d*x]]))/(5*a))/(7*a)
)/9
```

3.646. $\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{9}{2}}(c+dx)}\,dx$

3.646.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 4328
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)*
((d*Csc[e + f*x])^n/(f*n)), x] - Simp[1/(d*n)   Int[(a + b*Csc[e + f*x])^(m
 - 3)*(d*Csc[e + f*x])^(n + 1)*Simp[a^2*b*(m - 2*n - 2) - a*(3*b^2*n + a^2*
(n + 1))*Csc[e + f*x] - b*(b^2*n + a^2*(m + n - 1))*Csc[e + f*x]^2, x], x],
 x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && ((Int
egerQ[m] && LtQ[n, -1]) || (IntegersQ[m + 1/2, 2*n] && LeQ[n, -1]))
```

3.646. $\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{9}{2}}(c+dx)}\,dx$

rule 4343
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]])   Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a
, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4345
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]])   Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[
{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4523
```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Simp[A/a   I
nt[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Simp[(A*b - a*B)
/(a*d)   Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ
[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

rule 4592
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Simp[1/(a*d*n)   Int[(a + b*Csc[e + f*x])^m
*(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*
Csc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

3.646.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 3821 vs. $2(381) = 762$.

Time = 8.94 (sec) , antiderivative size = 3822, normalized size of antiderivative = 10.53

method	result	size
default	Expression too large to display	3822

input
```
int((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
2/315/d/((a-b)/(a+b))^(1/2)/a^2*(-147*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c
)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE(((a-b)/(a+b))^(1/2)*(-cot(d
*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*a^4*b*cos(d*x+c)^2-10*((a-b)/(a+b)
)^(1/2)*b^5*sin(d*x+c)+155*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)
*(1/(cos(d*x+c)+1))^(1/2)*EllipticF(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d
*x+c)),(-(a+b)/(a-b))^(1/2))*a^2*b^3*cos(d*x+c)^2+10*(1/(a+b)*(b+a*cos(d*x
+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF(((a-b)/(a+b)
)^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*a*b^4*cos(d*x+c)^2-
294*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/
2)*EllipticE(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(
1/2))*a^4*b*cos(d*x+c)+558*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)
*(1/(cos(d*x+c)+1))^(1/2)*EllipticE(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d
*x+c)),(-(a+b)/(a-b))^(1/2))*a^3*b^2*cos(d*x+c)-558*(1/(a+b)*(b+a*cos(d*x+
c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE(((a-b)/(a+b))
^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*a^2*b^3*cos(d*x+c)-2
0*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)
*EllipticE(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/
2))*a*b^4*cos(d*x+c)+522*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(
1/(cos(d*x+c)+1))^(1/2)*EllipticF(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x
+c)),(-(a+b)/(a-b))^(1/2))*a^4*b*cos(d*x+c)-558*(1/(a+b)*(b+a*cos(d*x+c...
```

3.646.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.15 (sec) , antiderivative size = 541, normalized size of antiderivative = 1.49

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{9}{2}}(c+dx)}\,dx = \frac{\sqrt{2}(-489i\,a^4b+93i\,a^2b^3-20i\,b^5)\sqrt{a}\text{weierstrassPInverse}\left(-\frac{4\,(3\,a^2-4\,b^2)}{3\,a^2},\frac{8\,(9\,a^2}{27}\right.}{}$$

input
```
integrate((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(9/2),x, algorithm="fricas")
```

output
```
1/945*(sqrt(2)*(-489*I*a^4*b + 93*I*a^2*b^3 - 20*I*b^5)*sqrt(a)*weierstras
sPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*c
os(d*x + c) + 3*I*a*sin(d*x + c) + 2*b)/a) + sqrt(2)*(489*I*a^4*b - 93*I*a
^2*b^3 + 20*I*b^5)*sqrt(a)*weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8
/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*
b)/a) - 3*sqrt(2)*(-147*I*a^5 - 279*I*a^3*b^2 + 10*I*a*b^4)*sqrt(a)*weiers
trassZeta(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, weierstras
sPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*c
os(d*x + c) + 3*I*a*sin(d*x + c) + 2*b)/a)) - 3*sqrt(2)*(147*I*a^5 + 279*I
*a^3*b^2 - 10*I*a*b^4)*sqrt(a)*weierstrassZeta(-4/3*(3*a^2 - 4*b^2)/a^2, 8
/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8
/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*
b)/a)) + 6*(35*a^5*cos(d*x + c)^4 + 95*a^4*b*cos(d*x + c)^3 + (49*a^5 + 75
*a^3*b^2)*cos(d*x + c)^2 + (163*a^4*b + 5*a^2*b^3)*cos(d*x + c))*sqrt((a*c
os(d*x + c) + b)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^3*d)
```

3.646.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{9}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+b*sec(d*x+c))**(5/2)/sec(d*x+c)**(9/2),x)
```

output
```
Timed out
```

3.646.7 Maxima [F]

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{9}{2}}(c+dx)}\,dx = \int \frac{(b\sec(dx+c)+a)^{\frac{5}{2}}}{\sec(dx+c)^{\frac{9}{2}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(9/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^(5/2)/sec(d*x + c)^(9/2), x)
```

3.646.8 Giac [F]

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{9}{2}}(c+dx)}\,dx=\int \frac{(b\sec(dx+c)+a)^{\frac{5}{2}}}{\sec(dx+c)^{\frac{9}{2}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(9/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^(5/2)/sec(d*x + c)^(9/2), x)
```

3.646.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{9}{2}}(c+dx)}\,dx=\int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2}}{\left(\frac{1}{\cos(c+d\,x)}\right)^{9/2}}\,dx$$

input
```
int((a + b/cos(c + d*x))^(5/2)/(1/cos(c + d*x))^(9/2),x)
```

output
```
int((a + b/cos(c + d*x))^(5/2)/(1/cos(c + d*x))^(9/2), x)
```

3.647 $\int \frac{\sec^{\frac{7}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$

3.647.1 Optimal result 5029
3.647.2 Mathematica [C] (verified) 5030
3.647.3 Rubi [A] (verified) 5031
3.647.4 Maple [C] (verified) 5038
3.647.5 Fricas [F(-1)] 5039
3.647.6 Sympy [F(-1)] 5040
3.647.7 Maxima [F] 5040
3.647.8 Giac [F] 5040
3.647.9 Mupad [F(-1)] 5041

3.647.1 Optimal result

Integrand size = 25, antiderivative size = 312

$$\begin{aligned}
&\int \frac{\sec^{\frac{7}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx \\
&= -\frac{a\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{4bd\sqrt{a+b\sec(c+dx)}} \\
&\quad + \frac{(3a^2+4b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{4b^2d\sqrt{a+b\sec(c+dx)}} \\
&\quad + \frac{3aE\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+dx)}}{4b^2d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\sqrt{\sec(c+dx)}} \\
&\quad - \frac{3a\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{4b^2d} \\
&\quad + \frac{\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{2bd}
\end{aligned}$$

output
```
-1/4*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d
*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)*sec(d*x+
c)^(1/2)/b/d/(a+b*sec(d*x+c))^(1/2)+1/4*(3*a^2+4*b^2)*(cos(1/2*d*x+1/2*c)^
2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2,2^(1/2)*(a/(a+
b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)*sec(d*x+c)^(1/2)/b^2/d/(a+b*sec(
d*x+c))^(1/2)+1/2*sec(d*x+c)^(3/2)*sin(d*x+c)*(a+b*sec(d*x+c))^(1/2)/b/d+3
/4*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x
+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*(a+b*sec(d*x+c))^(1/2)/b^2/d/((b+a*cos(d*
x+c))/(a+b))^(1/2)/sec(d*x+c)^(1/2)-3/4*a*sin(d*x+c)*sec(d*x+c)^(1/2)*(a+b
*sec(d*x+c))^(1/2)/b^2/d
```

3.647.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.63 (sec) , antiderivative size = 397, normalized size of antiderivative = 1.27

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \frac{\sqrt{\sec(c+dx)}\left(4ab^2\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)+b(9a^2+8b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(\right.\right.}{}$$

input
```
Integrate[Sec[c + d*x]^(7/2)/Sqrt[a + b*Sec[c + d*x]],x]
```

output
```
(Sqrt[Sec[c + d*x]]*(4*a*b^2*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[
(c + d*x)/2, (2*a)/(a + b)] + b*(9*a^2 + 8*b^2)*Sqrt[(b + a*Cos[c + d*x])/
(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)] + ((3*I)*Sqrt[-((a*(-1
+ Cos[c + d*x]))/(a + b))]*Sqrt[(a*(1 + Cos[c + d*x]))/(a - b)]*Sqrt[b + a
*Cos[c + d*x]]*Csc[c + d*x]*(-2*b*(a + b)*EllipticE[I*ArcSinh[Sqrt[(a - b)
^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b)] + a*(2*b*EllipticF[I*A
rcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b)] + a
*EllipticPi[1 - a/b, I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]
], (-a + b)/(a + b)])))/Sqrt[(a - b)^(-1)] + 2*b*(2*b - 3*a*Cos[c + d*x])*
(b + a*Cos[c + d*x])*Sec[c + d*x]*Tan[c + d*x]))/(8*b^3*d*Sqrt[a + b*Sec[c
 + d*x]])
```

3.647.3 Rubi [A] (verified)

Time = 3.07 (sec) , antiderivative size = 312, normalized size of antiderivative = 1.00, number of steps used = 25, number of rules used = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {3042, 4347, 3042, 4590, 27, 3042, 4596, 3042, 4346, 3042, 3286, 3042, 3284, 4523, 3042, 4343, 3042, 3134, 3042, 3132, 4345, 3042, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4347}$$

$$\frac{\int \frac{\sqrt{\sec(c+dx)}\left(-3a\sec^2(c+dx)+2b\sec(c+dx)+a\right)}{\sqrt{a+b\sec(c+dx)}}dx}{4b}+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{2bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(-3a\csc\left(c+dx+\frac{\pi}{2}\right)^2+2b\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{4b}+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{2bd}$$

$$\Big\downarrow \text{4590}$$

$$\frac{\frac{\int \frac{3a^2+2b\sec(c+dx)a+\left(3a^2+4b^2\right)\sec^2(c+dx)}{2\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{b}-\frac{3a\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{bd}}{4b}+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{2bd}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int \frac{3a^2+2b\sec(c+dx)a+\left(3a^2+4b^2\right)\sec^2(c+dx)}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{2b}-\frac{3a\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{bd}}{4b}+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{2bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\int \frac{3a^2+2b\csc\left(c+dx+\frac{\pi}{2}\right)a+\left(3a^2+4b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{2b}-\frac{3a\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{bd}}{4b}+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{2bd}$$

$\downarrow$ 4596

$$\frac{\frac{\left(3a^2+4b^2\right)\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx+\int \frac{3a^2+2b\sec(c+dx)a}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{2b}-\frac{3a\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{bd}}{4b}+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{2bd}$$

$\downarrow$ 3042

$$\frac{\frac{\left(3a^2+4b^2\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\int \frac{3a^2+2b\csc\left(c+dx+\frac{\pi}{2}\right)a}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{2b}-\frac{3a\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{bd}}{4b}+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{2bd}$$

$\downarrow$ 4346

$$\frac{\frac{\frac{\left(3a^2+4b^2\right)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int \frac{\sec(c+dx)}{\sqrt{b+a\cos(c+dx)}}dx}{\sqrt{a+b\sec(c+dx)}}+\int \frac{3a^2+2b\csc\left(c+dx+\frac{\pi}{2}\right)a}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{2b}-\frac{3a\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{bd}}{4b}+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{2bd}$$

$\downarrow$ 3042

$$\frac{\frac{\frac{\left(3a^2+4b^2\right)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{\sqrt{a+b\sec(c+dx)}}+\int \frac{3a^2+2b\csc\left(c+dx+\frac{\pi}{2}\right)a}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{2b}-\frac{3a\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{}}{bd}}{4b}+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{2bd}$$

$\downarrow$ 3286

3.647. $\int \frac{\sec^{\frac{7}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx$

$$\frac{\frac{\left(3a^2+4b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{\sec(c+dx)}{\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}+\int\frac{3a^2+2b\csc\left(c+dx+\frac{\pi}{2}\right)a}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{2b}-\frac{3a\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+d}}{bd}$$

$$\frac{4b}{\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{2bd}}$$

$\downarrow$ 3042

$$\frac{\frac{\left(3a^2+4b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}+\int\frac{3a^2+2b\csc\left(c+dx+\frac{\pi}{2}\right)a}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{2b}-\frac{3a\sin(c+dx)\sqrt{\sec(c+d}}{bd}$$

$$\frac{4b}{\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{2bd}}$$

$\downarrow$ 3284

$$\frac{\int\frac{3a^2+2b\csc\left(c+dx+\frac{\pi}{2}\right)a}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\left(3a^2+4b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\operatorname{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}}{2b}-\frac{3a\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c}}{bd}$$

$$\frac{4b}{\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{2bd}}$$

$\downarrow$ 4523

$$\frac{\frac{-ab\int\frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}dx+3a\int\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}dx+\frac{2\left(3a^2+4b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\operatorname{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}}{2b}-\frac{3a\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{}}{bd}}{\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{2bd}}$$

$\downarrow$ 3042

$$\frac{\frac{-ab\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+3a\int\frac{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\left(3a^2+4b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\operatorname{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}}{2b}-\frac{3a\sin(c+dx)\sqrt{\sec}}{}}{\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{2bd}}$$

$\downarrow$ 4343

3.647. $\int\frac{\sec^{\frac{7}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx$

$$\frac{\frac{-ab\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{3a\sqrt{a+b\sec(c+dx)}\int\sqrt{b+a\cos(c+dx)}dx}{\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}+\frac{2\left(3a^2+4b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\operatorname{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}}{2b}-\frac{3a\sin(c+}{}}{\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{2bd}}$$

$\downarrow$ 3042

$$\frac{\frac{-ab\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{3a\sqrt{a+b\sec(c+dx)}\int\sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}+\frac{2\left(3a^2+4b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\operatorname{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}}{2b}-\frac{3a\sin}{}}{\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{2bd}}$$

$\downarrow$ 3134

$$\frac{\frac{-ab\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{3a\sqrt{a+b\sec(c+dx)}\int\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}dx}{\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{2\left(3a^2+4b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\operatorname{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}}{2b}-\frac{3a\sin}{}}{\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{2bd}}$$

$\downarrow$ 3042

$$\frac{\frac{-ab\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{3a\sqrt{a+b\sec(c+dx)}\int\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}dx}{\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{2\left(3a^2+4b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\operatorname{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}}{2b}-\frac{3}{}}{\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{2bd}}$$

$\downarrow$ 3132

$$\frac{\frac{-ab\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\left(3a^2+4b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\operatorname{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}+\frac{6a\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{2b}-\frac{3a\sin(c+d}{}}{\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{2bd}}$$

$\downarrow$ 4345

3.647. $\int \frac{\sec^{\frac{7}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$

$$\frac{-\frac{ab\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{1}{\sqrt{b+a\cos(c+dx)}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{2\left(3a^2+4b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\operatorname{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}+\frac{6a\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\right.}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)}{a+b}}}}{2b}}{4b}$$

$$\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{2bd}$$

$\downarrow$ 3042

$$\frac{-\frac{ab\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{1}{\sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{2\left(3a^2+4b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\operatorname{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}+\frac{6a\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+\right.}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+}{a+}}}}{2b}}{4b}$$

$$\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{2bd}$$

$\downarrow$ 3142

$$\frac{-\frac{ab\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{2\left(3a^2+4b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\operatorname{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}+\frac{6a\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c\right.}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c}{a}}}}{2b}}{4b}$$

$$\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{2bd}$$

$\downarrow$ 3042

$$\frac{-\frac{ab\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{2\left(3a^2+4b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\operatorname{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}+\frac{6a\sqrt{a+b\sec(c+dx)}E\left(\right.}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\,c}{}}}}{2b}}{4b}$$

$$\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{2bd}$$

$\downarrow$ 3140

$$\frac{\frac{2\left(3a^2+4b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\operatorname{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}-\frac{2ab\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}+\frac{6a\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+\right.}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+}{a+}}}}{2b}}{4b}$$

$$\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{2bd}$$

3.647. $\int\frac{\sec^{\frac{7}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx$

input
```
Int[Sec[c + d*x]^(7/2)/Sqrt[a + b*Sec[c + d*x]],x]
```

output
```
(Sec[c + d*x]^(3/2)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(2*b*d) + (((-2
*a*b*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a +
b)]*Sqrt[Sec[c + d*x]])/(d*Sqrt[a + b*Sec[c + d*x]]) + (2*(3*a^2 + 4*b^2)*
Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b
)]*Sqrt[Sec[c + d*x]])/(d*Sqrt[a + b*Sec[c + d*x]]) + (6*a*EllipticE[(c +
d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(d*Sqrt[(b + a*Cos[c + d*
x])/(a + b)]*Sqrt[Sec[c + d*x]]))/(2*b) - (3*a*Sqrt[Sec[c + d*x]]*Sqrt[a +
 b*Sec[c + d*x]]*Sin[c + d*x])/(b*d))/(4*b)
```

3.647.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3286
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]]   Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
 d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

rule 4343
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]])   Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a
, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4345
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]])   Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[
{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4346
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Simp[d*Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x
]]/Sqrt[a + b*Csc[e + f*x]])   Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

3.647. $\int \frac{\sec^{\frac{7}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$

rule 4347
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[-2*d^2*Cos[e + f*x]*(d*Csc[e + f*x])^(n - 2)*(S
qrt[a + b*Csc[e + f*x]]/(b*f*(2*n - 3))), x] + Simp[d^3/(b*(2*n - 3))   Int
[((d*Csc[e + f*x])^(n - 3)/Sqrt[a + b*Csc[e + f*x]])*Simp[2*a*(n - 3) + b*(
2*n - 5)*Csc[e + f*x] - 2*a*(n - 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 2] && IntegerQ[2*n]
```

rule 4523
```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Simp[A/a   I
nt[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Simp[(A*b - a*B)
/(a*d)   Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ
[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

rule 4590
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(-C)*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
)*((d*Csc[e + f*x])^(n - 1)/(b*f*(m + n + 1))), x] + Simp[d/(b*(m + n + 1))
   Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (
A*b*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2
 - b^2, 0] && GtQ[n, 0]
```

rule 4596
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Simp[C/d^2   Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*C
sc[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[
a + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
 b^2, 0]
```

3.647.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 11.01 (sec) , antiderivative size = 2331, normalized size of antiderivative = 7.47

method	result	size
default	Expression too large to display	2331

input
```
int(sec(d*x+c)^(7/2)/(a+b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4/d/b^2/((a-b)/(a+b))^(1/2)*(a+b*sec(d*x+c))^(1/2)*sec(d*x+c)^(7/2)/(b+
a*cos(d*x+c))/(cos(d*x+c)+1)*(3*(1/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos
(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+
csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*a*b*cos(d*x+c)^4-2*(1/(cos(d*x+c)+1))^(1
/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF(((a-b)/(a+b)
)^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*a*b*cos(d*x+c)^4+3*
((a-b)/(a+b))^(1/2)*a^2*cos(d*x+c)^4*sin(d*x+c)-2*((a-b)/(a+b))^(1/2)*b^2*
cos(d*x+c)^3*sin(d*x+c)-2*((a-b)/(a+b))^(1/2)*b^2*sin(d*x+c)*cos(d*x+c)^2-
2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)
*EllipticF(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/
2))*a*b*cos(d*x+c)^6+3*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/
(cos(d*x+c)+1))^(1/2)*EllipticE(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c
)),(-(a+b)/(a-b))^(1/2))*a*b*cos(d*x+c)^6-4*(1/(a+b)*(b+a*cos(d*x+c))/(cos
(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF(((a-b)/(a+b))^(1/2)*(
-cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*a*b*cos(d*x+c)^5+6*(1/(a+b)*
(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE(
((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*a*b*cos
(d*x+c)^5-16*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c
)+1))^(1/2)*EllipticPi(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(a+b)/
(a-b),I/((a-b)/(a+b))^(1/2))*b^2*cos(d*x+c)^5-6*(1/(a+b)*(b+a*cos(d*x+c...
```

3.647.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)^(7/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.647.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**(7/2)/(a+b*sec(d*x+c))**(1/2),x)
```

output
```
Timed out
```

3.647.7 Maxima [F]

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\sec(dx+c)^{\frac{7}{2}}}{\sqrt{b\sec(dx+c)+a}}\,dx$$

input
```
integrate(sec(d*x+c)^(7/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)^(7/2)/sqrt(b*sec(d*x + c) + a), x)
```

3.647.8 Giac [F]

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\sec(dx+c)^{\frac{7}{2}}}{\sqrt{b\sec(dx+c)+a}}\,dx$$

input
```
integrate(sec(d*x+c)^(7/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^(7/2)/sqrt(b*sec(d*x + c) + a), x)
```

3.647.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{7/2}}{\sqrt{a+\frac{b}{\cos(c+d\,x)}}}\,dx$$

input
```
int((1/cos(c + d*x))^(7/2)/(a + b/cos(c + d*x))^(1/2),x)
```

output
```
int((1/cos(c + d*x))^(7/2)/(a + b/cos(c + d*x))^(1/2), x)
```

3.648 $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$

3.648.1 Optimal result 5042
3.648.2 Mathematica [C] (verified) 5043
3.648.3 Rubi [A] (verified) 5043
3.648.4 Maple [C] (verified) 5050
3.648.5 Fricas [F(-1)] 5051
3.648.6 Sympy [F(-1)] 5052
3.648.7 Maxima [F] 5052
3.648.8 Giac [F] 5052
3.648.9 Mupad [F(-1)] 5053

3.648.1 Optimal result

Integrand size = 25, antiderivative size = 246

$$\begin{aligned}\int \frac{\sec^{\frac{5}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = &\frac{\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{d\sqrt{a+b\sec(c+dx)}}\\ &-\frac{a\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{bd\sqrt{a+b\sec(c+dx)}}\\ &-\frac{E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+dx)}}{bd\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\sqrt{\sec(c+dx)}}\\ &+\frac{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{bd}\end{aligned}$$

output
```
(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*
c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)*sec(d*x+c)^(1/2
)/d/(a+b*sec(d*x+c))^(1/2)-a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*
c)*EllipticPi(sin(1/2*d*x+1/2*c),2,2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+
c))/(a+b))^(1/2)*sec(d*x+c)^(1/2)/b/d/(a+b*sec(d*x+c))^(1/2)-(cos(1/2*d*x+
1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a
/(a+b))^(1/2))*(a+b*sec(d*x+c))^(1/2)/b/d/((b+a*cos(d*x+c))/(a+b))^(1/2)/s
ec(d*x+c)^(1/2)+sin(d*x+c)*sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2)/b/d
```

3.648.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 15.64 (sec) , antiderivative size = 329, normalized size of antiderivative = 1.34

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \frac{\sqrt{\sec(c+dx)}\left(-6a\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)-\frac{2i\sqrt{-\frac{a(-1+\cos(c+dx))}{a+b}}\sqrt{\frac{a(1+\cos(c+dx))}{a-b}}\sqrt{b+a\cos(c+}}{}\right.}{}$$

input
```
Integrate[Sec[c + d*x]^(5/2)/Sqrt[a + b*Sec[c + d*x]],x]
```

output
```
(Sqrt[Sec[c + d*x]]*(-6*a*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2,
 (c + d*x)/2, (2*a)/(a + b)] - ((2*I)*Sqrt[-((a*(-1 + Cos[c + d*x]))/(a +
b))]*Sqrt[(a*(1 + Cos[c + d*x]))/(a - b)]*Sqrt[b + a*Cos[c + d*x]]*Csc[c +
 d*x]*(-2*b*(a + b)*EllipticE[I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[
c + d*x]]], (-a + b)/(a + b)] + a*(2*b*EllipticF[I*ArcSinh[Sqrt[(a - b)^(-
1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b)] + a*EllipticPi[1 - a/b, I
*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b)]))
)/(a*Sqrt[(a - b)^(-1)]*b) + 4*(b + a*Cos[c + d*x])*Tan[c + d*x]))/(4*b*d*
Sqrt[a + b*Sec[c + d*x]])
```

3.648.3 Rubi [A] (verified)

Time = 2.28 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.05, number of steps used = 23, number of rules used = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.920$, Rules used = {3042, 4347, 25, 3042, 4597, 3042, 4346, 3042, 3286, 3042, 3284, 4349, 3042, 4343, 3042, 3134, 3042, 3132, 4345, 3042, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \; 4347$$

$$\frac{\int -\frac{a\sec^2(c+dx)+a}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{2b}+\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{bd}$$

$$\Big\downarrow \; 25$$

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{bd}-\frac{\int \frac{a\sec^2(c+dx)+a}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{2b}$$

$$\Big\downarrow \; 3042$$

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{bd}-\frac{\int \frac{a\csc\left(c+dx+\frac{\pi}{2}\right)^2+a}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{2b}$$

$$\Big\downarrow \; 4597$$

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{bd}-\frac{a\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx+a\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{2b}$$

$$\Big\downarrow \; 3042$$

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{bd}-\frac{a\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+a\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{2b}$$

$$\Big\downarrow \; 4346$$

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{bd}-\frac{a\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{a\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int \frac{\sec(c+dx)}{\sqrt{b+a\cos(c+dx)}}dx}{\sqrt{a+b\sec(c+dx)}}}{2b}$$

$$\Big\downarrow \; 3042$$

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{bd}-\frac{a\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{a\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{\sqrt{a+b\sec(c+dx)}}}{2b}$$

3.648. $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$

$$\Big\downarrow \text{3286}$$

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{bd}-\frac{a\int\frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{\sec(c+dx)}{\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}}{2b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{bd}-\frac{a\int\frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}}{2b}$$

$$\Big\downarrow \text{3284}$$

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{bd}-\frac{a\int\frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\operatorname{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}}{2b}$$

$$\Big\downarrow \text{4349}$$

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{bd}-\frac{a\left(\frac{\int\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}dx}{a}-\frac{b\int\frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}dx}{a}\right)+\frac{2a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\operatorname{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}}{2b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{bd}-\frac{a\left(\frac{\int\frac{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}-\frac{b\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}\right)+\frac{2a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\operatorname{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}}{2b}$$

$$\Big\downarrow \text{4343}$$

3.648. $\int\frac{\sec^{\frac{5}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx$

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{bd}-\frac{a\left(\frac{\sqrt{a+b\sec(c+dx)}\int\sqrt{b+a\cos(c+dx)}dx}{a\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}-\frac{b\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}\right)+\frac{2a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\operatorname{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}}{2b}$$

$\downarrow$ 3042

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{bd}-\frac{a\left(\frac{\sqrt{a+b\sec(c+dx)}\int\sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{a\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}-\frac{b\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}\right)+\frac{2a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\operatorname{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}}{2b}$$

$\downarrow$ 3134

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{bd}-\frac{a\left(\frac{\sqrt{a+b\sec(c+dx)}\int\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}dx}{a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{b\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}\right)+\frac{2a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\operatorname{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}}{2b}$$

$\downarrow$ 3042

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{bd}-\frac{a\left(\frac{\sqrt{a+b\sec(c+dx)}\int\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}dx}{a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{b\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}\right)+\frac{2a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\operatorname{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}}{2b}$$

$\downarrow$ 3132

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{bd}-\frac{a\left(\frac{2\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{b\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}\right)+\frac{2a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\operatorname{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}}{2b}$$

3.648. $\int\frac{\sec^{\frac{5}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx$

$$\Big\downarrow \text{4345}$$

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{bd}-\frac{a\left(\frac{2\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{b\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{1}{\sqrt{b+a\cos(c+dx)}}dx}{a\sqrt{a+b\sec(c+dx)}}\right)+\frac{2a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\text{EllipticPi}\left(\right.}{d\sqrt{a+b\sec(c+dx)}}}{2b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{bd}-\frac{a\left(\frac{2\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{b\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{1}{\sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{a\sqrt{a+b\sec(c+dx)}}\right)+\frac{2a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\text{Elliptic}}{d\sqrt{a+b\sec(c+dx)}}}{2b}$$

$$\Big\downarrow \text{3142}$$

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{bd}-\frac{a\left(\frac{2\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{b\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}}dx}{a\sqrt{a+b\sec(c+dx)}}\right)+\frac{2a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\text{Elliptic}}{d\sqrt{a+b\sec(c+dx)}}}{2b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{bd}-\frac{a\left(\frac{2\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{b\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{a\sqrt{a+b\sec(c+dx)}}\right)+\frac{2a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\text{Ellip}}{d\sqrt{a+b\sec(c+}}}{2b}$$

$$\Big\downarrow \text{3140}$$

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{bd}-\frac{\frac{2a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}+a\left(\frac{2\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{2b\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\text{Ellipt}}{ad\sqrt{a+b\sec(c+d}}\right.}{2b}$$

input
```
Int[Sec[c + d*x]^(5/2)/Sqrt[a + b*Sec[c + d*x]],x]
```

3.648. $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$

output
```
-1/2*((2*a*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (
2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(d*Sqrt[a + b*Sec[c + d*x]]) + a*((-2*b*
Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]*S
qrt[Sec[c + d*x]])/(a*d*Sqrt[a + b*Sec[c + d*x]]) + (2*EllipticE[(c + d*x)
/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(a*d*Sqrt[(b + a*Cos[c + d*x]
)/(a + b)]*Sqrt[Sec[c + d*x]])))/b + (Sqrt[Sec[c + d*x]]*Sqrt[a + b*Sec[c
+ d*x]]*Sin[c + d*x])/(b*d)
```

3.648.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

3.648. $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3286
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]]   Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
 d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

rule 4343
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]])   Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a
, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4345
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]])   Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[
{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4346
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Simp[d*Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x
]]/Sqrt[a + b*Csc[e + f*x]])   Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4347
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[-2*d^2*Cos[e + f*x]*(d*Csc[e + f*x])^(n - 2)*(S
qrt[a + b*Csc[e + f*x]]/(b*f*(2*n - 3))), x] + Simp[d^3/(b*(2*n - 3))   Int
[((d*Csc[e + f*x])^(n - 3)/Sqrt[a + b*Csc[e + f*x]])*Simp[2*a*(n - 3) + b*(
2*n - 5)*Csc[e + f*x] - 2*a*(n - 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 2] && IntegerQ[2*n]
```

3.648. $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$

rule 4349
```
Int[1/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)]), x_Symbol] :> Simp[1/a   Int[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Cs
c[e + f*x]], x], x] - Simp[b/(a*d)   Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Cs
c[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4597
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))/(Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Simp[C/d^
2   Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Csc[e + f*x]], x], x] + Simp[A
Int[1/(Sqrt[d*Csc[e + f*x]]*Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a,
b, d, e, f, A, C}, x] && NeQ[a^2 - b^2, 0]
```

3.648.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 8.69 (sec) , antiderivative size = 1299, normalized size of antiderivative = 5.28

method	result	size
default	Expression too large to display	1299

input
```
int(sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/d/((a-b)/(a+b))^(1/2)/b*sec(d*x+c)^(5/2)*(a+b*sec(d*x+c))^(1/2)/(b+a*cos
(d*x+c))/(cos(d*x+c)+1)*(EllipticE(((a-b)/(a+b))^(1/2)*(cot(d*x+c)-csc(d*x
+c)),(-(a+b)/(a-b))^(1/2))*(1/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+
c))/(cos(d*x+c)+1))^(1/2)*a*cos(d*x+c)^5-EllipticE(((a-b)/(a+b))^(1/2)*(co
t(d*x+c)-csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*(1/(cos(d*x+c)+1))^(1/2)*(1/(a+
b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*b*cos(d*x+c)^5-2*EllipticF(((a-b
)/(a+b))^(1/2)*(cot(d*x+c)-csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*(1/(cos(d*x+c
)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a*cos(d*x+c)^5
+2*EllipticPi(((a-b)/(a+b))^(1/2)*(cot(d*x+c)-csc(d*x+c)),(a+b)/(a-b),I/((
a-b)/(a+b))^(1/2))*(1/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos
(d*x+c)+1))^(1/2)*a*cos(d*x+c)^5+2*(1/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*
cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE(((a-b)/(a+b))^(1/2)*(cot(d*x+c
)-csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*a*cos(d*x+c)^4-2*(1/(cos(d*x+c)+1))^(1
/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE(((a-b)/(a+b)
)^(1/2)*(cot(d*x+c)-csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*b*cos(d*x+c)^4-4*Ell
ipticF(((a-b)/(a+b))^(1/2)*(cot(d*x+c)-csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*(
1/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a*
cos(d*x+c)^4+4*(1/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x
+c)+1))^(1/2)*EllipticPi(((a-b)/(a+b))^(1/2)*(cot(d*x+c)-csc(d*x+c)),(a+b)
/(a-b),I/((a-b)/(a+b))^(1/2))*a*cos(d*x+c)^4+(1/(cos(d*x+c)+1))^(1/2)*(...
```

3.648.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.648.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**(5/2)/(a+b*sec(d*x+c))**(1/2),x)
```

output
```
Timed out
```

3.648.7 Maxima [F]

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\sec\left(dx+c\right)^{\frac{5}{2}}}{\sqrt{b\sec\left(dx+c\right)+a}}\,dx$$

input
```
integrate(sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)^(5/2)/sqrt(b*sec(d*x + c) + a), x)
```

3.648.8 Giac [F]

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\sec\left(dx+c\right)^{\frac{5}{2}}}{\sqrt{b\sec\left(dx+c\right)+a}}\,dx$$

input
```
integrate(sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^(5/2)/sqrt(b*sec(d*x + c) + a), x)
```

3.648.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx=\int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}}{\sqrt{a+\frac{b}{\cos(c+d\,x)}}}\,dx$$

input
```
int((1/cos(c + d*x))^(5/2)/(a + b/cos(c + d*x))^(1/2),x)
```

output
```
int((1/cos(c + d*x))^(5/2)/(a + b/cos(c + d*x))^(1/2), x)
```

3.649 $\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$

3.649.1 Optimal result 5054
3.649.2 Mathematica [A] (verified) 5054
3.649.3 Rubi [A] (verified) 5055
3.649.4 Maple [C] (verified) 5056
3.649.5 Fricas [F(-1)] 5057
3.649.6 Sympy [F] 5057
3.649.7 Maxima [F] 5058
3.649.8 Giac [F] 5058
3.649.9 Mupad [F(-1)] 5058

3.649.1 Optimal result

Integrand size = 25, antiderivative size = 68

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx=\frac{2\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{d\sqrt{a+b\sec(c+dx)}}$$

output
```
2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1
/2*c),2,2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)*sec(d*x+c)
^(1/2)/d/(a+b*sec(d*x+c))^(1/2)
```

3.649.2 Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx=\frac{2\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{d\sqrt{a+b\sec(c+dx)}}$$

input
```
Integrate[Sec[c + d*x]^(3/2)/Sqrt[a + b*Sec[c + d*x]],x]
```

output
```
(2*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a
+ b)]*Sqrt[Sec[c + d*x]])/(d*Sqrt[a + b*Sec[c + d*x]])
```

3.649.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {3042, 4346, 3042, 3286, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4346}$$

$$\frac{\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int \frac{\sec(c+dx)}{\sqrt{b+a\cos(c+dx)}}dx}{\sqrt{a+b\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{\sqrt{a+b\sec(c+dx)}}$$

$$\Big\downarrow \text{3286}$$

$$\frac{\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int \frac{\sec(c+dx)}{\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}$$

$$\Big\downarrow \text{3284}$$

$$\frac{2\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}$$

input
```
Int[Sec[c + d*x]^(3/2)/Sqrt[a + b*Sec[c + d*x]],x]
```

output
```
(2*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a
+ b)]*Sqrt[Sec[c + d*x]])/(d*Sqrt[a + b*Sec[c + d*x]])
```

3.649.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3286
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]]   Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
 d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

rule 4346
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Simp[d*Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x
]]/Sqrt[a + b*Csc[e + f*x]])   Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

3.649.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 6.31 (sec) , antiderivative size = 205, normalized size of antiderivative = 3.01

method	result
default	$\frac{2\sqrt{\frac{b+a\cos(dx+c)}{(a+b)(\cos(dx+c)+1)}}\,\cos(dx+c)^2\left(2\,\mathrm{EllipticPi}\left(\sqrt{\frac{a-b}{a+b}}\,(-\cot(dx+c)+\csc(dx+c)),\frac{a+b}{a-b},\frac{i}{\sqrt{\frac{a-b}{a+b}}}\right)-\mathrm{EllipticF}\left(\sqrt{\frac{a-b}{a+b}}\,(-\cot(dx+c)-\right.\right.}{d\sqrt{\frac{a-b}{a+b}}\,(b+a\cos(dx+c))\sqrt{\frac{1}{\cos(dx+c)+1}}}$

input
```
int(sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/d/((a-b)/(a+b))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*co
s(d*x+c)^2*(2*EllipticPi(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(a+b
)/(a-b),I/((a-b)/(a+b))^(1/2))-EllipticF(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+
csc(d*x+c)),(-(a+b)/(a-b))^(1/2)))*(a+b*sec(d*x+c))^(1/2)*sec(d*x+c)^(3/2)
/(b+a*cos(d*x+c))/(1/(cos(d*x+c)+1))^(1/2)
```

3.649.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.649.6 Sympy [F]

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$$

input
```
integrate(sec(d*x+c)**(3/2)/(a+b*sec(d*x+c))**(1/2),x)
```

output
```
Integral(sec(c + d*x)**(3/2)/sqrt(a + b*sec(c + d*x)), x)
```

3.649.7 Maxima [F]

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\sec{(dx+c)}^{\frac{3}{2}}}{\sqrt{b\sec{(dx+c)}+a}}\,dx$$

input
```
integrate(sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)^(3/2)/sqrt(b*sec(d*x + c) + a), x)
```

3.649.8 Giac [F]

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\sec{(dx+c)}^{\frac{3}{2}}}{\sqrt{b\sec{(dx+c)}+a}}\,dx$$

input
```
integrate(sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^(3/2)/sqrt(b*sec(d*x + c) + a), x)
```

3.649.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}{\sqrt{a+\frac{b}{\cos(c+d\,x)}}}\,dx$$

input
```
int((1/cos(c + d*x))^(3/2)/(a + b/cos(c + d*x))^(1/2),x)
```

output
```
int((1/cos(c + d*x))^(3/2)/(a + b/cos(c + d*x))^(1/2), x)
```

3.650 $\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}\,dx$

3.650.1 Optimal result 5059
3.650.2 Mathematica [A] (verified) 5059
3.650.3 Rubi [A] (verified) 5060
3.650.4 Maple [A] (verified) 5061
3.650.5 Fricas [C] (verification not implemented) 5062
3.650.6 Sympy [F] 5062
3.650.7 Maxima [F] 5063
3.650.8 Giac [F] 5063
3.650.9 Mupad [F(-1)] 5063

3.650.1 Optimal result

Integrand size = 25, antiderivative size = 67

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}\,dx=\frac{2\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{d\sqrt{a+b\sec(c+dx)}}$$

output
```
2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/
2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)*sec(d*x+c)^(1
/2)/d/(a+b*sec(d*x+c))^(1/2)
```

3.650.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}\,dx=\frac{2\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{d\sqrt{a+b\sec(c+dx)}}$$

input
```
Integrate[Sqrt[Sec[c + d*x]]/Sqrt[a + b*Sec[c + d*x]],x]
```

output
```
(2*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)
]*Sqrt[Sec[c + d*x]])/(d*Sqrt[a + b*Sec[c + d*x]])
```

3.650.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {3042, 4345, 3042, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4345}$$

$$\frac{\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int \frac{1}{\sqrt{b+a\cos(c+dx)}}dx}{\sqrt{a+b\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int \frac{1}{\sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{\sqrt{a+b\sec(c+dx)}}$$

$$\Big\downarrow \text{3142}$$

$$\frac{\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int \frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int \frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}$$

$$\Big\downarrow \text{3140}$$

$$\frac{2\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}$$

input
```
Int[Sqrt[Sec[c + d*x]]/Sqrt[a + b*Sec[c + d*x]],x]
```

output
```
(2*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)
]*Sqrt[Sec[c + d*x]])/(d*Sqrt[a + b*Sec[c + d*x]])
```

3.650.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 4345
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]])   Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[
{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

3.650.4 Maple [A] (verified)

Time = 5.25 (sec) , antiderivative size = 141, normalized size of antiderivative = 2.10

method	result	size
default	$\frac{2\sqrt{\sec(dx+c)}\,\sqrt{a+b\sec(dx+c)}\,\cos(dx+c)\sqrt{\frac{b+a\cos(dx+c)}{(a+b)(\cos(dx+c)+1)}}\,\operatorname{EllipticF}\left(\sqrt{\frac{a-b}{a+b}}\,(-\cot(dx+c)+\csc(dx+c)),\sqrt{-\frac{a+b}{a-b}}\right)}{d\sqrt{\frac{a-b}{a+b}}\,(b+a\cos(dx+c))\sqrt{\frac{1}{\cos(dx+c)+1}}}$	141

input
```
int(sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/d/((a-b)/(a+b))^(1/2)*sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2)*cos(d*x+c)
*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF(((a-b)/(a+b))^(
1/2)*(-cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))/(b+a*cos(d*x+c))/(1/(c
os(d*x+c)+1))^(1/2)
```

3.650.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 146, normalized size of antiderivative = 2.18

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}\,dx = \frac{-i\sqrt{2}\sqrt{a}\text{weierstrassPInverse}\left(-\frac{4\,(3\,a^2-4\,b^2)}{3\,a^2},\frac{8\,(9\,a^2b-8\,b^3)}{27\,a^3},\frac{3\,a\cos(dx+c)+3i\,a\sin(dx+c)+2\,b}{3\,a}\right)+i\sqrt{2}\sqrt{a}\text{weierstrassP}}{ad}$$

input
```
integrate(sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
(-I*sqrt(2)*sqrt(a)*weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*
a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x + c) + 2*b)/a) +
 I*sqrt(2)*sqrt(a)*weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a
^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a))/(
a*d)
```

3.650.6 Sympy [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}\,dx$$

input
```
integrate(sec(d*x+c)**(1/2)/(a+b*sec(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(sec(c + d*x))/sqrt(a + b*sec(c + d*x)), x)
```

3.650.7 Maxima [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\sqrt{\sec(dx+c)}}{\sqrt{b\sec(dx+c)+a}}\,dx$$

input
```
integrate(sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(sec(d*x + c))/sqrt(b*sec(d*x + c) + a), x)
```

3.650.8 Giac [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\sqrt{\sec(dx+c)}}{\sqrt{b\sec(dx+c)+a}}\,dx$$

input
```
integrate(sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(sec(d*x + c))/sqrt(b*sec(d*x + c) + a), x)
```

3.650.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\sqrt{\frac{1}{\cos(c+d\,x)}}}{\sqrt{a+\frac{b}{\cos(c+d\,x)}}}\,dx$$

input
```
int((1/cos(c + d*x))^(1/2)/(a + b/cos(c + d*x))^(1/2),x)
```

output
```
int((1/cos(c + d*x))^(1/2)/(a + b/cos(c + d*x))^(1/2), x)
```

3.651 $\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}\,dx$

3.651.1 Optimal result . . . 5064
3.651.2 Mathematica [A] (verified) . . . 5065
3.651.3 Rubi [A] (verified) . . . 5065
3.651.4 Maple [B] (warning: unable to verify) . . . 5068
3.651.5 Fricas [C] (verification not implemented) . . . 5070
3.651.6 Sympy [F] . . . 5070
3.651.7 Maxima [F] . . . 5071
3.651.8 Giac [F] . . . 5071
3.651.9 Mupad [F(-1)] . . . 5071

3.651.1 Optimal result

Integrand size = 25, antiderivative size = 142

$$\begin{aligned}\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}\,dx &= -\frac{2b\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{ad\sqrt{a+b\sec(c+dx)}} \\ &\quad + \frac{2E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+dx)}}{ad\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\sqrt{\sec(c+dx)}}\end{aligned}$$

output
```
-2*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x
+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)*sec(d*x+c)
^(1/2)/a/d/(a+b*sec(d*x+c))^(1/2)+2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d
*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*(a+b*sec(d
*x+c))^(1/2)/a/d/((b+a*cos(d*x+c))/(a+b))^(1/2)/sec(d*x+c)^(1/2)
```

3.651.2 Mathematica [A] (verified)

Time = 3.31 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.68

$$\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}\,dx = \frac{2\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\left((a+b)E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)-b\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)\right)\sqrt{\sec(c+dx)}}{ad\sqrt{a+b\sec(c+dx)}}$$

input
```
Integrate[1/(Sqrt[Sec[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]),x]
```

output
```
(2*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*((a + b)*EllipticE[(c + d*x)/2, (2*a
)/(a + b)] - b*EllipticF[(c + d*x)/2, (2*a)/(a + b)])*Sqrt[Sec[c + d*x]])/
(a*d*Sqrt[a + b*Sec[c + d*x]])
```

3.651.3 Rubi [A] (verified)

Time = 1.04 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.520$, Rules used = {3042, 4349, 3042, 4343, 3042, 3134, 3042, 3132, 4345, 3042, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4349}$$

$$\frac{\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}dx}{a}-\frac{b\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}-\frac{b\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}$$

$$\Big\downarrow \text{4343}$$

$$\frac{\sqrt{a+b\sec(c+dx)}\int\sqrt{b+a\cos(c+dx)}dx}{a\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}-\frac{b\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{a+b\sec(c+dx)}\int\sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{a\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}-\frac{b\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}$$

$$\Big\downarrow \text{3134}$$

$$\frac{\sqrt{a+b\sec(c+dx)}\int\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}dx}{a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{b\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{a+b\sec(c+dx)}\int\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}dx}{a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{b\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}$$

$$\Big\downarrow \text{3132}$$

$$\frac{2\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{b\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}$$

$$\Big\downarrow \text{4345}$$

$$\frac{2\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{b\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{1}{\sqrt{b+a\cos(c+dx)}}dx}{a\sqrt{a+b\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{b\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{1}{\sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{a\sqrt{a+b\sec(c+dx)}}$$

$$\Big\downarrow \text{3142}$$

3.651. $\int\frac{1}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx$

$$\frac{2\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{b\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}}dx}{a\sqrt{a+b\sec(c+dx)}}$$

$\Big\downarrow$ 3042

$$\frac{2\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{b\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{a\sqrt{a+b\sec(c+dx)}}$$

$\Big\downarrow$ 3140

$$\frac{2\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{2b\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{ad\sqrt{a+b\sec(c+dx)}}$$

input
```
Int[1/(Sqrt[Sec[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]),x]
```

output
```
(-2*b*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a +
 b)]*Sqrt[Sec[c + d*x]])/(a*d*Sqrt[a + b*Sec[c + d*x]]) + (2*EllipticE[(c
+ d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(a*d*Sqrt[(b + a*Cos[c
+ d*x])/(a + b)]*Sqrt[Sec[c + d*x]])
```

3.651.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 4343
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]])   Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a
, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4345
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]])   Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[
{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4349
```
Int[1/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)]), x_Symbol] :> Simp[1/a   Int[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Cs
c[e + f*x]], x], x] - Simp[b/(a*d)   Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Cs
c[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

3.651.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 738 vs. $2(188) = 376$.

Time = 6.53 (sec) , antiderivative size = 739, normalized size of antiderivative = 5.20

method	result
default	$\frac{2\left(\sqrt{\frac{a-b}{a+b}}\,a(1-\cos(dx+c))^3\csc(dx+c)^3-\sqrt{\frac{a-b}{a+b}}\,b(1-\cos(dx+c))^3\csc(dx+c)^3+\sqrt{-\frac{a(1-\cos(dx+c))^2\csc(dx+c)^2-b(1-\cos(dx+c))^2\csc(d}{a+b}}\right.}{}$
risch	$-\frac{i\left(a\,e^{2i(dx+c)}+2b\,e^{i(dx+c)}+a\right)\sqrt{2}}{ad\sqrt{\frac{e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\left(e^{2i(dx+c)}+1\right)\sqrt{\frac{a\,e^{2i(dx+c)}+2b\,e^{i(dx+c)}+a}{e^{2i(dx+c)}+1}}}-\frac{i\left(-\frac{2\left(a\,e^{2i(dx+c)}+2b\,e^{i(dx+c)}+a\right)}{a\sqrt{e^{i(dx+c)}\left(a\,e^{2i(dx+c)}+2b\,e^{i(dx+c)}+a\right)}}+\frac{2\left(b+\sqrt{-a^2+b^2}\right)\sqrt{(}}{}\right.}{}$

input
```
int(1/sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/d/((a-b)/(a+b))^(1/2)/a*(((a-b)/(a+b))^(1/2)*a*(1-cos(d*x+c))^3*csc(d*x+
c)^3-((a-b)/(a+b))^(1/2)*b*(1-cos(d*x+c))^3*csc(d*x+c)^3+(-(a*(1-cos(d*x+c
))^2*csc(d*x+c)^2-b*(1-cos(d*x+c))^2*csc(d*x+c)^2-a-b)/(a+b))^(1/2)*((1-co
s(d*x+c))^2*csc(d*x+c)^2+1)^(1/2)*EllipticF(((a-b)/(a+b))^(1/2)*(-cot(d*x+
c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*a-(-(a*(1-cos(d*x+c))^2*csc(d*x+c)^2-
b*(1-cos(d*x+c))^2*csc(d*x+c)^2-a-b)/(a+b))^(1/2)*((1-cos(d*x+c))^2*csc(d*
x+c)^2+1)^(1/2)*EllipticE(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(-(
a+b)/(a-b))^(1/2))*a+(-(a*(1-cos(d*x+c))^2*csc(d*x+c)^2-b*(1-cos(d*x+c))^2
*csc(d*x+c)^2-a-b)/(a+b))^(1/2)*((1-cos(d*x+c))^2*csc(d*x+c)^2+1)^(1/2)*El
lipticE(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))
*b-((a-b)/(a+b))^(1/2)*a*(-cot(d*x+c)+csc(d*x+c))-((a-b)/(a+b))^(1/2)*b*(-
cot(d*x+c)+csc(d*x+c)))*((a*(1-cos(d*x+c))^2*csc(d*x+c)^2-b*(1-cos(d*x+c))
^2*csc(d*x+c)^2-a-b)/((1-cos(d*x+c))^2*csc(d*x+c)^2-1))^(1/2)/(a*(1-cos(d*
x+c))^2*csc(d*x+c)^2-b*(1-cos(d*x+c))^2*csc(d*x+c)^2-a-b)/(-((1-cos(d*x+c)
)^2*csc(d*x+c)^2+1)/((1-cos(d*x+c))^2*csc(d*x+c)^2-1))^(1/2)
```

3.651. $\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}\,dx$

3.651.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 355, normalized size of antiderivative = 2.50

$$\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}\,dx = \frac{2i\sqrt{2}\sqrt{a}b\text{weierstrassPInverse}\left(-\frac{4(3a^2-4b^2)}{3a^2}, \frac{8(9a^2b-8b^3)}{27a^3}, \frac{3a\cos(dx+c)+3ia\sin(dx+c)+2b}{3a}\right) - 2i\sqrt{2}\sqrt{a}b\text{weierstras}}{}$$

input
```
integrate(1/sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
1/3*(2*I*sqrt(2)*sqrt(a)*b*weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8
/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x + c) + 2*
b)/a) - 2*I*sqrt(2)*sqrt(a)*b*weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2
, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c) +
 2*b)/a) + 3*I*sqrt(2)*a^(3/2)*weierstrassZeta(-4/3*(3*a^2 - 4*b^2)/a^2, 8
/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8
/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x + c) + 2*
b)/a)) - 3*I*sqrt(2)*a^(3/2)*weierstrassZeta(-4/3*(3*a^2 - 4*b^2)/a^2, 8/2
7*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/2
7*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)
/a)))/(a^2*d)
```

3.651.6 Sympy [F]

$$\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{a+b\sec(c+dx)}\sqrt{\sec(c+dx)}}\,dx$$

input
```
integrate(1/sec(d*x+c)**(1/2)/(a+b*sec(d*x+c))**(1/2),x)
```

output
```
Integral(1/(sqrt(a + b*sec(c + d*x))*sqrt(sec(c + d*x))), x)
```

3.651.7 Maxima [F]

$$\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{b\sec(dx+c)+a}\sqrt{\sec(dx+c)}}\,dx$$

input
```
integrate(1/sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(b*sec(d*x + c) + a)*sqrt(sec(d*x + c))), x)
```

3.651.8 Giac [F]

$$\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{b\sec(dx+c)+a}\sqrt{\sec(dx+c)}}\,dx$$

input
```
integrate(1/sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(b*sec(d*x + c) + a)*sqrt(sec(d*x + c))), x)
```

3.651.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{a+\frac{b}{\cos(c+d\,x)}}\,\sqrt{\frac{1}{\cos(c+d\,x)}}}\,dx$$

input
```
int(1/((a + b/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(1/2)),x)
```

output
```
int(1/((a + b/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(1/2)), x)
```

3.652 $\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx$

3.652.1 Optimal result . . . 5072
3.652.2 Mathematica [A] (verified) . . . 5073
3.652.3 Rubi [A] (verified) . . . 5073
3.652.4 Maple [B] (verified) . . . 5077
3.652.5 Fricas [C] (verification not implemented) . . . 5078
3.652.6 Sympy [F] . . . 5079
3.652.7 Maxima [F] . . . 5079
3.652.8 Giac [F] . . . 5080
3.652.9 Mupad [F(-1)] . . . 5080

3.652.1 Optimal result

Integrand size = 25, antiderivative size = 195

$$\begin{aligned}\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx &= \frac{2(a^2+2b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{3a^2d\sqrt{a+b\sec(c+dx)}} \\ &\quad -\frac{4bE\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+dx)}}{3a^2d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\sqrt{\sec(c+dx)}}+\frac{2\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}\end{aligned}$$

output
```
2/3*(a^2+2*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(
sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)
*sec(d*x+c)^(1/2)/a^2/d/(a+b*sec(d*x+c))^(1/2)+2/3*sin(d*x+c)*(a+b*sec(d*x
+c))^(1/2)/a/d/sec(d*x+c)^(1/2)-4/3*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2
*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*(a+b*sec
(d*x+c))^(1/2)/a^2/d/((b+a*cos(d*x+c))/(a+b))^(1/2)/sec(d*x+c)^(1/2)
```

3.652.2 Mathematica [A] (verified)

Time = 0.70 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.75

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx = \frac{\sqrt{\sec(c+dx)}\left(-4b(a+b)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)+2(a^2+2b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+d\right.\right.}{3a^2d\sqrt{a+b\sec(c+dx)}}$$

input
```
Integrate[1/(Sec[c + d*x]^(3/2)*Sqrt[a + b*Sec[c + d*x]]),x]
```

output
```
(Sqrt[Sec[c + d*x]]*(-4*b*(a + b)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Ellip
ticE[(c + d*x)/2, (2*a)/(a + b)] + 2*(a^2 + 2*b^2)*Sqrt[(b + a*Cos[c + d*x
])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)] + 2*a*(b + a*Cos[c + d*x
])*Sin[c + d*x]))/(3*a^2*d*Sqrt[a + b*Sec[c + d*x]])
```

3.652.3 Rubi [A] (verified)

Time = 1.43 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.02, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {3042, 4350, 3042, 4523, 3042, 4343, 3042, 3134, 3042, 3132, 4345, 3042, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4350}$$

$$\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}-\frac{\int \frac{2b-a\sec(c+dx)}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{3a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}-\frac{\int\frac{2b-a\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{3a}$$

$\Big\downarrow$ 4523

$$\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{2b\int\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}dx}{a}-\frac{\left(a^2+2b^2\right)\int\frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}dx}{a}}{3a}$$

$\Big\downarrow$ 3042

$$\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{2b\int\frac{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}-\frac{\left(a^2+2b^2\right)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}}{3a}$$

$\Big\downarrow$ 4343

$$\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{2b\sqrt{a+b\sec(c+dx)}\int\sqrt{b+a\cos(c+dx)}dx}{a\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}-\frac{\left(a^2+2b^2\right)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}}{3a}$$

$\Big\downarrow$ 3042

$$\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{2b\sqrt{a+b\sec(c+dx)}\int\sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{a\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}-\frac{\left(a^2+2b^2\right)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}}{3a}$$

$\Big\downarrow$ 3134

$$\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{2b\sqrt{a+b\sec(c+dx)}\int\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}dx}{a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{\left(a^2+2b^2\right)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}}{3a}$$

$\Big\downarrow$ 3042

$$\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{2b\sqrt{a+b\sec(c+dx)}\int\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}dx}{a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{\left(a^2+2b^2\right)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}}{3a}$$

3.652. $\int\frac{1}{\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}dx$

$$\Big\downarrow \; 3132$$

$$\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{4b\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{(a^2+2b^2)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}}{3a}$$

$$\Big\downarrow \; 4345$$

$$\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{4b\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{(a^2+2b^2)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{1}{\sqrt{b+a\cos(c+dx)}}dx}{a\sqrt{a+b\sec(c+dx)}}}{3a}$$

$$\Big\downarrow \; 3042$$

$$\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{4b\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{(a^2+2b^2)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{1}{\sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{a\sqrt{a+b\sec(c+dx)}}}{3a}$$

$$\Big\downarrow \; 3142$$

$$\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{4b\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{(a^2+2b^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}}dx}{a\sqrt{a+b\sec(c+dx)}}}{3a}$$

$$\Big\downarrow \; 3042$$

$$\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{4b\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{(a^2+2b^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{a\sqrt{a+b\sec(c+dx)}}}{3a}$$

$$\Big\downarrow \; 3140$$

3.652. $\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx$

$$\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{4b\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{2(a^2+2b^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{ad\sqrt{a+b\sec(c+dx)}}}{3a}$$

input
```
Int[1/(Sec[c + d*x]^(3/2)*Sqrt[a + b*Sec[c + d*x]]),x]
```

output
```
-1/3*((-2*(a^2 + 2*b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c +
d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(a*d*Sqrt[a + b*Sec[c + d*x]])
+ (4*b*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(a*
d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]))/a + (2*Sqrt[a +
b*Sec[c + d*x]]*Sin[c + d*x])/(3*a*d*Sqrt[Sec[c + d*x]])
```

3.652.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 4343
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]])   Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a
, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4345
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]])   Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[
{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4350
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[Cos[e + f*x]*(d*Csc[e + f*x])^(n + 1)*(Sqrt[a +
 b*Csc[e + f*x]]/(a*d*f*n)), x] + Simp[1/(2*a*d*n)   Int[((d*Csc[e + f*x])^
(n + 1)/Sqrt[a + b*Csc[e + f*x]])*Simp[(-b)*(2*n + 1) + 2*a*(n + 1)*Csc[e +
 f*x] + b*(2*n + 3)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x]
 && NeQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

rule 4523
```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Simp[A/a   I
nt[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Simp[(A*b - a*B)
/(a*d)   Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ
[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

3.652.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1297 vs. $2(231) = 462$.

Time = 7.63 (sec) , antiderivative size = 1298, normalized size of antiderivative = 6.66

method	result	size
default	Expression too large to display	1298

input
```
int(1/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3/d/a^2/((a-b)/(a+b))^(1/2)*(a+b*sec(d*x+c))^(1/2)/(b+a*cos(d*x+c))/sec
(d*x+c)^(3/2)/(cos(d*x+c)+1)*(2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^
(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+
csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*a*b*cos(d*x+c)-2*(1/(a+b)*(b+a*cos(d*x+c
))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE(((a-b)/(a+b))^
(1/2)*(-cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*b^2*cos(d*x+c)-(1/(a+
b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*Ellipti
cF(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*a^2*
cos(d*x+c)-2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c
)+1))^(1/2)*EllipticF(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(-(a+b)
/(a-b))^(1/2))*a*b*cos(d*x+c)+4*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^
(1/2)*EllipticE(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b)
)^(1/2))*(1/(cos(d*x+c)+1))^(1/2)*a*b-4*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x
+c)+1))^(1/2)*EllipticE(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(-(a+
b)/(a-b))^(1/2))*(1/(cos(d*x+c)+1))^(1/2)*b^2-2*(1/(a+b)*(b+a*cos(d*x+c))/
(cos(d*x+c)+1))^(1/2)*EllipticF(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c
)),(-(a+b)/(a-b))^(1/2))*(1/(cos(d*x+c)+1))^(1/2)*a^2-4*(1/(a+b)*(b+a*cos(
d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+c
sc(d*x+c)),(-(a+b)/(a-b))^(1/2))*(1/(cos(d*x+c)+1))^(1/2)*a*b-((a-b)/(a+b)
)^(1/2)*a^2*cos(d*x+c)*sin(d*x+c)+2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+...
```

3.652.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 415, normalized size of antiderivative = 2.13

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx$$
$$= \frac{6\,a^2\sqrt{\frac{a\cos(dx+c)+b}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c) - 6i\sqrt{2}a^{\frac{3}{2}}b\text{weierstrassZeta}\left(-\frac{4\,(3\,a^2-4\,b^2)}{3\,a^2}, \frac{8\,(9\,a^2b-8\,b^3)}{27\,a^3}, \text{weierst}\right.}{}$$

input
```
integrate(1/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
1/9*(6*a^2*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(
d*x + c) - 6*I*sqrt(2)*a^(3/2)*b*weierstrassZeta(-4/3*(3*a^2 - 4*b^2)/a^2,
 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2,
 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x + c) +
2*b)/a)) + 6*I*sqrt(2)*a^(3/2)*b*weierstrassZeta(-4/3*(3*a^2 - 4*b^2)/a^2,
 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2,
 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c) +
2*b)/a)) + sqrt(2)*(-3*I*a^2 - 4*I*b^2)*sqrt(a)*weierstrassPInverse(-4/3*(
3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*
I*a*sin(d*x + c) + 2*b)/a) + sqrt(2)*(3*I*a^2 + 4*I*b^2)*sqrt(a)*weierstra
ssPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*
cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a))/(a^3*d)
```

3.652.6 Sympy [F]

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx=\int \frac{1}{\sqrt{a+b\sec\left(c+dx\right)}\sec^{\frac{3}{2}}\left(c+dx\right)}\,dx$$

input
```
integrate(1/sec(d*x+c)**(3/2)/(a+b*sec(d*x+c))**(1/2),x)
```

output
```
Integral(1/(sqrt(a + b*sec(c + d*x))*sec(c + d*x)**(3/2)), x)
```

3.652.7 Maxima [F]

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx=\int \frac{1}{\sqrt{b\sec\left(dx+c\right)+a}\sec\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(3/2)), x)
```

3.652.8 Giac [F]

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{b\sec(dx+c)+a}\sec(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(3/2)), x)
```

3.652.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{a+\frac{b}{\cos(c+d\,x)}}\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

input
```
int(1/((a + b/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(3/2)),x)
```

output
```
int(1/((a + b/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(3/2)), x)
```

3.653 $\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx$

3.653.1 Optimal result 5081
3.653.2 Mathematica [A] (verified) 5082
3.653.3 Rubi [A] (verified) 5082
3.653.4 Maple [B] (verified) 5088
3.653.5 Fricas [C] (verification not implemented) 5088
3.653.6 Sympy [F] 5089
3.653.7 Maxima [F] 5089
3.653.8 Giac [F] 5090
3.653.9 Mupad [F(-1)] 5090

3.653.1 Optimal result

Integrand size = 25, antiderivative size = 249

$$\begin{aligned}\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx &= -\frac{2b(7a^2+8b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{15a^3d\sqrt{a+b\sec(c+dx)}} \\ &\quad + \frac{2(9a^2+8b^2)\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+dx)}}{15a^3d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\sqrt{\sec(c+dx)}} \\ &\quad + \frac{2\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{5ad\sec^{\frac{3}{2}}(c+dx)} - \frac{8b\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{15a^2d\sqrt{\sec(c+dx)}}\end{aligned}$$

output
```
-2/15*b*(7*a^2+8*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elli
pticF(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))
^(1/2)*sec(d*x+c)^(1/2)/a^3/d/(a+b*sec(d*x+c))^(1/2)+2/5*sin(d*x+c)*(a+b*s
ec(d*x+c))^(1/2)/a/d/sec(d*x+c)^(3/2)-8/15*b*sin(d*x+c)*(a+b*sec(d*x+c))^(
1/2)/a^2/d/sec(d*x+c)^(1/2)+2/15*(9*a^2+8*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2
)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))
*(a+b*sec(d*x+c))^(1/2)/a^3/d/((b+a*cos(d*x+c))/(a+b))^(1/2)/sec(d*x+c)^(1
/2)
```

3.653.2 Mathematica [A] (verified)

Time = 0.81 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.78

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx$$
$$= \frac{\sqrt{\sec(c+dx)}\left(4(9a^3+9a^2b+8ab^2+8b^3)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}E\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)-4b(7a^2+8b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\right.}{30a^3d\sqrt{a+b\sec(}}$$

input
```
Integrate[1/(Sec[c + d*x]^(5/2)*Sqrt[a + b*Sec[c + d*x]]),x]
```

output
```
(Sqrt[Sec[c + d*x]]*(4*(9*a^3 + 9*a^2*b + 8*a*b^2 + 8*b^3)*Sqrt[(b + a*Cos
[c + d*x])/(a + b)]*EllipticE[(c + d*x)/2, (2*a)/(a + b)] - 4*b*(7*a^2 + 8
*b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a +
 b)] + 2*a*(3*a^2 - 8*b^2 - 2*a*b*Cos[c + d*x] + 3*a^2*Cos[2*(c + d*x)])*S
in[c + d*x]))/(30*a^3*d*Sqrt[a + b*Sec[c + d*x]])
```

3.653.3 Rubi [A] (verified)

Time = 1.95 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.05, number of steps used = 18, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.720$, Rules used = {3042, 4350, 3042, 4592, 27, 3042, 4523, 3042, 4343, 3042, 3134, 3042, 3132, 4345, 3042, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$
$$\Big\downarrow \text{4350}$$
$$\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{\int \frac{-2b\sec^2(c+dx)-3a\sec(c+dx)+4b}{\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}dx}{5a}$$

$$\Big\downarrow \; 3042$$

$$\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{\int \frac{-2b\csc\left(c+dx+\frac{\pi}{2}\right)^2-3a\csc\left(c+dx+\frac{\pi}{2}\right)+4b}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{5a}$$

$$\Big\downarrow \; 4592$$

$$\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{\frac{8b\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}-\frac{2\int \frac{9a^2+2b\sec(c+dx)a+8b^2}{2\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{3a}}{5a}$$

$$\Big\downarrow \; 27$$

$$\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{\frac{8b\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}-\frac{\int \frac{9a^2+2b\sec(c+dx)a+8b^2}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{3a}}{5a}$$

$$\Big\downarrow \; 3042$$

$$\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{\frac{8b\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}-\frac{\int \frac{9a^2+2b\csc\left(c+dx+\frac{\pi}{2}\right)a+8b^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{3a}}{5a}$$

$$\Big\downarrow \; 4523$$

$$\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{\frac{8b\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{\left(9a^2+8b^2\right)\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}dx}{a}-\frac{b\left(7a^2+8b^2\right)\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}dx}{a}}{3a}}{5a}$$

$$\Big\downarrow \; 3042$$

$$\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{\frac{8b\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{\left(9a^2+8b^2\right)\int \frac{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}-\frac{b\left(7a^2+8b^2\right)\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}}{3a}}{5a}$$

$$\Big\downarrow \; 4343$$

3.653. $\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx$

$$\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{\frac{8b\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{\left(9a^2+8b^2\right)\sqrt{a+b\sec(c+dx)}\int\sqrt{b+a\cos(c+dx)}dx}{a\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}-\frac{b\left(7a^2+8b^2\right)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}}{3a}}{5a}$$

$\downarrow$ 3042

$$\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{\frac{8b\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{\left(9a^2+8b^2\right)\sqrt{a+b\sec(c+dx)}\int\sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{a\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}-\frac{b\left(7a^2+8b^2\right)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}}{3a}}{5a}$$

$\downarrow$ 3134

$$\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{\frac{8b\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{\left(9a^2+8b^2\right)\sqrt{a+b\sec(c+dx)}\int\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}dx}{a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{b\left(7a^2+8b^2\right)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}}{3a}}{5a}$$

$\downarrow$ 3042

$$\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{\frac{8b\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{\left(9a^2+8b^2\right)\sqrt{a+b\sec(c+dx)}\int\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}dx}{a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{b\left(7a^2+8b^2\right)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}}{3a}}{5a}$$

$\downarrow$ 3132

$$\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{\frac{8b\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{2\left(9a^2+8b^2\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{b\left(7a^2+8b^2\right)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}}{3a}}{5a}$$

$\downarrow$ 4345

3.653. $\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx$

$$\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{\frac{8b\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{2\left(9a^2+8b^2\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{b\left(7a^2+8b^2\right)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{1}{\sqrt{b+a\cos(c+dx)}}dx}{a\sqrt{a+b\sec(c+dx)}}}{3a}}{5a}$$

$\downarrow$ 3042

$$\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{\frac{8b\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{2\left(9a^2+8b^2\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{b\left(7a^2+8b^2\right)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{1}{\sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{a\sqrt{a+b\sec(c+dx)}}}{3a}}{5a}$$

$\downarrow$ 3142

$$\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{\frac{8b\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{2\left(9a^2+8b^2\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{b\left(7a^2+8b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}}dx}{a\sqrt{a+b\sec(c+dx)}}}{3a}}{5a}$$

$\downarrow$ 3042

$$\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{\frac{8b\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{2\left(9a^2+8b^2\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{b\left(7a^2+8b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{a\sqrt{a+b\sec(c+dx)}}}{3a}}{5a}$$

$\downarrow$ 3140

$$\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{\frac{8b\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{2\left(9a^2+8b^2\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{2b\left(7a^2+8b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{ad\sqrt{a+b\sec(c+dx)}}}{3a}}{5a}$$

input
```
Int[1/(Sec[c + d*x]^(5/2)*Sqrt[a + b*Sec[c + d*x]]),x]
```

3.653. $\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx$

output
```
(2*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(5*a*d*Sec[c + d*x]^(3/2)) - (-1
/3*((-2*b*(7*a^2 + 8*b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c
+ d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(a*d*Sqrt[a + b*Sec[c + d*x]]
) + (2*(9*a^2 + 8*b^2)*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Se
c[c + d*x]])/(a*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]))/
a + (8*b*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(3*a*d*Sqrt[Sec[c + d*x]])
)/(5*a)
```

3.653.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

3.653. $\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx$

rule 4343
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]])   Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a
, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4345
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]])   Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[
{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4350
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[Cos[e + f*x]*(d*Csc[e + f*x])^(n + 1)*(Sqrt[a +
 b*Csc[e + f*x]]/(a*d*f*n)), x] + Simp[1/(2*a*d*n)   Int[((d*Csc[e + f*x])^
(n + 1)/Sqrt[a + b*Csc[e + f*x]])*Simp[(-b)*(2*n + 1) + 2*a*(n + 1)*Csc[e +
 f*x] + b*(2*n + 3)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x]
 && NeQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

rule 4523
```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Simp[A/a   I
nt[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Simp[(A*b - a*B)
/(a*d)   Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ
[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

rule 4592
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Simp[1/(a*d*n)   Int[(a + b*Csc[e + f*x])^m
*(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*
Csc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

3.653. $\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx$

3.653.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2288 vs. $2(279) = 558$.

Time = 8.75 (sec) , antiderivative size = 2289, normalized size of antiderivative = 9.19

method	result	size
default	Expression too large to display	2289

input
```
int(1/sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/15/d/a^3/((a-b)/(a+b))^(1/2)*(a+b*sec(d*x+c))^(1/2)/(b+a*cos(d*x+c))/sec
(d*x+c)^(5/2)/(cos(d*x+c)+1)*(-4*(1/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*co
s(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF(((a-b)/(a+b))^(1/2)*(cot(d*x+c)-
csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*a^2*b*sec(d*x+c)+16*(1/(cos(d*x+c)+1))^(
1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF(((a-b)/(a+b
))^(1/2)*(cot(d*x+c)-csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*a*b^2*sec(d*x+c)+18
*(1/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*
EllipticE(((a-b)/(a+b))^(1/2)*(cot(d*x+c)-csc(d*x+c)),(-(a+b)/(a-b))^(1/2)
)*a^2*b*sec(d*x+c)-16*(1/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(
cos(d*x+c)+1))^(1/2)*EllipticE(((a-b)/(a+b))^(1/2)*(cot(d*x+c)-csc(d*x+c))
,(-(a+b)/(a-b))^(1/2))*a*b^2*sec(d*x+c)-2*(1/(cos(d*x+c)+1))^(1/2)*(1/(a+b
)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF(((a-b)/(a+b))^(1/2)*(co
t(d*x+c)-csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*a^2*b*sec(d*x+c)^2+8*(1/(cos(d*
x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((
(a-b)/(a+b))^(1/2)*(cot(d*x+c)-csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*a*b^2*sec
(d*x+c)^2+9*(1/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)
+1))^(1/2)*EllipticE(((a-b)/(a+b))^(1/2)*(cot(d*x+c)-csc(d*x+c)),(-(a+b)/(
a-b))^(1/2))*a^2*b*sec(d*x+c)^2-8*(1/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*c
os(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE(((a-b)/(a+b))^(1/2)*(cot(d*x+c)
-csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*a*b^2*sec(d*x+c)^2+3*((a-b)/(a+b))^(...
```

3.653.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 464, normalized size of antiderivative = 1.86

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx =$$

$$-\frac{4\sqrt{2}(-3i\,a^2b-4i\,b^3)\sqrt{a}\text{weierstrassPInverse}\left(-\frac{4\,(3\,a^2-4\,b^2)}{3\,a^2},\frac{8\,(9\,a^2b-8\,b^3)}{27\,a^3},\frac{3\,a\cos(dx+c)+3i\,a\sin(dx+c)+2\,b}{3\,a}\right)+4\,\cdot}{}$$

input
```
integrate(1/sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
-1/45*(4*sqrt(2)*(-3*I*a^2*b - 4*I*b^3)*sqrt(a)*weierstrassPInverse(-4/3*(
3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*
I*a*sin(d*x + c) + 2*b)/a) + 4*sqrt(2)*(3*I*a^2*b + 4*I*b^3)*sqrt(a)*weier
strassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(
3*a*cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a) + 3*sqrt(2)*(-9*I*a^3 - 8*
I*a*b^2)*sqrt(a)*weierstrassZeta(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b -
 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b -
 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x + c) + 2*b)/a)) + 3*sqr
t(2)*(9*I*a^3 + 8*I*a*b^2)*sqrt(a)*weierstrassZeta(-4/3*(3*a^2 - 4*b^2)/a^
2, 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^
2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c)
+ 2*b)/a)) - 6*(3*a^3*cos(d*x + c)^2 - 4*a^2*b*cos(d*x + c))*sqrt((a*cos(d
*x + c) + b)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^4*d)
```

3.653.6 Sympy [F]

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{a+b\sec(c+dx)}\sec^{\frac{5}{2}}(c+dx)}\,dx$$

input
```
integrate(1/sec(d*x+c)**(5/2)/(a+b*sec(d*x+c))**(1/2),x)
```

output
```
Integral(1/(sqrt(a + b*sec(c + d*x))*sec(c + d*x)**(5/2)), x)
```

3.653.7 Maxima [F]

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{b\sec(dx+c)+a}\sec(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(5/2)), x)
```

3.653.8 Giac [F]

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{b\sec(dx+c)+a}\sec(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(5/2)), x)
```

3.653.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{a+\frac{b}{\cos(c+d\,x)}}\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

input
```
int(1/((a + b/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(5/2)),x)
```

output
```
int(1/((a + b/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(5/2)), x)
```

3.654 $\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$

3.654.1 Optimal result 5091
3.654.2 Mathematica [C] (verified) 5092
3.654.3 Rubi [A] (verified) 5092
3.654.4 Maple [C] (warning: unable to verify) 5100
3.654.5 Fricas [F(-1)] 5101
3.654.6 Sympy [F(-1)] 5102
3.654.7 Maxima [F] 5102
3.654.8 Giac [F] 5102
3.654.9 Mupad [F(-1)] 5103

3.654.1 Optimal result

Integrand size = 25, antiderivative size = 345

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \frac{\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{bd\sqrt{a+b\sec(c+dx)}} - \frac{3a\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{b^2d\sqrt{a+b\sec(c+dx)}} - \frac{(3a^2-b^2)\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+dx)}}{b^2(a^2-b^2)\,d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\sqrt{\sec(c+dx)}} - \frac{2a^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{b(a^2-b^2)\,d\sqrt{a+b\sec(c+dx)}} + \frac{(3a^2-b^2)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{b^2(a^2-b^2)\,d}$$

output
```
-2*a^2*sec(d*x+c)^(3/2)*sin(d*x+c)/b/(a^2-b^2)/d/(a+b*sec(d*x+c))^(1/2)+(c
os(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c)
,2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)*sec(d*x+c)^(1/2)/
b/d/(a+b*sec(d*x+c))^(1/2)-3*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/
2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2,2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*
x+c))/(a+b))^(1/2)*sec(d*x+c)^(1/2)/b^2/d/(a+b*sec(d*x+c))^(1/2)-(3*a^2-b^
2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1
/2*c),2^(1/2)*(a/(a+b))^(1/2))*(a+b*sec(d*x+c))^(1/2)/b^2/(a^2-b^2)/d/((b+
a*cos(d*x+c))/(a+b))^(1/2)/sec(d*x+c)^(1/2)+(3*a^2-b^2)*sin(d*x+c)*sec(d*x
+c)^(1/2)*(a+b*sec(d*x+c))^(1/2)/b^2/(a^2-b^2)/d
```

3.654.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.17 (sec) , antiderivative size = 478, normalized size of antiderivative = 1.39

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \frac{\sec^{\frac{3}{2}}(c+dx)\left(-\frac{a(b+a\cos(c+dx))^{3/2}\left(\frac{8ab\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{\sqrt{b+a\cos(c+dx)}}+\frac{2\left(9a^2-7b^2\right)\sqrt{\frac{b}{}}}{}\right.}{}\right.}{}$$

input
```
Integrate[Sec[c + d*x]^(7/2)/(a + b*Sec[c + d*x])^(3/2),x]
```

output
```
(Sec[c + d*x]^(3/2)*(-((a*(b + a*Cos[c + d*x])^(3/2)*((8*a*b*Sqrt[(b + a*C
os[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)])/Sqrt[b + a*Co
s[c + d*x]] + (2*(9*a^2 - 7*b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Ellipt
icPi[2, (c + d*x)/2, (2*a)/(a + b)])/Sqrt[b + a*Cos[c + d*x]] + ((2*I)*(3*
a^2 - b^2)*Sqrt[-((a*(-1 + Cos[c + d*x]))/(a + b))]*Sqrt[(a*(1 + Cos[c + d
*x]))/(a - b)]*Csc[c + d*x]*(-2*b*(a + b)*EllipticE[I*ArcSinh[Sqrt[(a - b)
^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b)] + a*(2*b*EllipticF[I*A
rcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b)] + a
*EllipticPi[1 - a/b, I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]
], (-a + b)/(a + b)])))/(a^2*Sqrt[(a - b)^(-1)]*b)))/((a - b)*b^2*(a + b))
) + (4*(b + a*Cos[c + d*x])*(-(a^2*b) + b^3 + (-3*a^3 + a*b^2)*Cos[c + d*x
])*Tan[c + d*x])/(-(a^2*b^2) + b^4)))/(4*d*(a + b*Sec[c + d*x])^(3/2))
```

3.654.3 Rubi [A] (verified)

Time = 3.53 (sec) , antiderivative size = 360, normalized size of antiderivative = 1.04, number of steps used = 26, number of rules used = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 1.040$, Rules used = {3042, 4332, 27, 3042, 4590, 27, 3042, 4596, 3042, 4346, 3042, 3286, 3042, 3284, 4523, 3042, 4343, 3042, 3134, 3042, 3132, 4345, 3042, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \; 4332$$

$$-\frac{2\int \frac{\sqrt{\sec(c+dx)}\left(a^2-b\sec(c+dx)a-\left(3a^2-b^2\right)\sec^2(c+dx)\right)}{2\sqrt{a+b\sec(c+dx)}}dx}{b\left(a^2-b^2\right)}-\frac{2a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}$$

$$\Big\downarrow \; 27$$

$$-\frac{\int \frac{\sqrt{\sec(c+dx)}\left(a^2-b\sec(c+dx)a-\left(3a^2-b^2\right)\sec^2(c+dx)\right)}{\sqrt{a+b\sec(c+dx)}}dx}{b\left(a^2-b^2\right)}-\frac{2a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a^2-b\csc\left(c+dx+\frac{\pi}{2}\right)a+\left(b^2-3a^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{b\left(a^2-b^2\right)}-\frac{2a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}$$

$$\Big\downarrow \; 4590$$

$$-\frac{\frac{\int \frac{2b\sec(c+dx)a^2+3\left(a^2-b^2\right)\sec^2(c+dx)a+\left(3a^2-b^2\right)a}{2\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{b}-\frac{\left(3a^2-b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{bd}}{b\left(a^2-b^2\right)}-\frac{2a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}$$

$$\Big\downarrow \; 27$$

$$-\frac{\frac{\int \frac{2b\sec(c+dx)a^2+3\left(a^2-b^2\right)\sec^2(c+dx)a+\left(3a^2-b^2\right)a}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{2b}-\frac{\left(3a^2-b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{bd}}{b\left(a^2-b^2\right)}-\frac{2a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\frac{\int \frac{2b\csc\left(c+dx+\frac{\pi}{2}\right)a^2+3\left(a^2-b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2a+\left(3a^2-b^2\right)a}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{2b}-\frac{\left(3a^2-b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{bd}}{b\left(a^2-b^2\right)}-\frac{2a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}$$

$$\Big\downarrow \; 4596$$

3.654. $\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$

$$-\frac{\frac{3a\left(a^2-b^2\right)\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx+\int \frac{2b\sec(c+dx)a^2+\left(3a^2-b^2\right)a}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{2b}-\frac{\left(3a^2-b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{bd}}{b\left(a^2-b^2\right)}-\frac{2a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}$$

$\downarrow$ 3042

$$-\frac{\frac{3a\left(a^2-b^2\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\int \frac{2b\csc\left(c+dx+\frac{\pi}{2}\right)a^2+\left(3a^2-b^2\right)a}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{2b}-\frac{\left(3a^2-b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{bd}}{b\left(a^2-b^2\right)}-\frac{2a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}$$

$\downarrow$ 4346

$$-\frac{\frac{\int \frac{2b\csc\left(c+dx+\frac{\pi}{2}\right)a^2+\left(3a^2-b^2\right)a}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{3a\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int \frac{\sec(c+dx)}{\sqrt{b+a\cos(c+dx)}}dx}{\sqrt{a+b\sec(c+dx)}}}{2b}-\frac{\left(3a^2-b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\mathrm{se}}}{bd}}{b\left(a^2-b^2\right)}-\frac{2a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}$$

$\downarrow$ 3042

$$-\frac{\frac{\int \frac{2b\csc\left(c+dx+\frac{\pi}{2}\right)a^2+\left(3a^2-b^2\right)a}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{3a\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{\sqrt{a+b\sec(c+dx)}}}{2b}-\frac{\left(3a^2-b^2\right)\sin(c+dx)\sqrt{\mathrm{sec}}}{b}}{b\left(a^2-b^2\right)}-\frac{2a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}$$

$\downarrow$ 3286

$$-\frac{\frac{\int \frac{2b\csc\left(c+dx+\frac{\pi}{2}\right)a^2+\left(3a^2-b^2\right)a}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{3a\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int \frac{\sec(c+dx)}{\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}}{2b}-\frac{\left(3a^2-b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a-}}{bd}}{b\left(a^2-b^2\right)}-\frac{2a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}$$

$\downarrow$ 3042

3.654. $\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$

$$-\frac{\frac{\int \frac{2b\csc\left(c+dx+\frac{\pi}{2}\right)a^2+\left(3a^2-b^2\right)a}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{3a\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}}{2b}}{b\left(a^2-b^2\right)}-\frac{\left(3a^2-b^2\right)\sin(c+dx)}{}$$

$$\frac{2a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}$$

$\downarrow$ 3284

$$-\frac{\frac{\int \frac{2b\csc\left(c+dx+\frac{\pi}{2}\right)a^2+\left(3a^2-b^2\right)a}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{6a\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\operatorname{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}}{2b}}{b\left(a^2-b^2\right)}-\frac{\left(3a^2-b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{}}{bd}$$

$$\frac{2a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}$$

$\downarrow$ 4523

$$-\frac{\frac{-b\left(a^2-b^2\right)\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}dx+\left(3a^2-b^2\right)\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}dx+\frac{6a\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\operatorname{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}}{2b}}{b\left(a^2-b^2\right)}-\frac{\left(3a^2-b^2\right)s}{}$$

$$\frac{2a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}$$

$\downarrow$ 3042

$$-\frac{\frac{-b\left(a^2-b^2\right)\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\left(3a^2-b^2\right)\int \frac{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{6a\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\operatorname{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}}{2b}}{b\left(a^2-b^2\right)}-\frac{(3}{}$$

$$\frac{2a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}$$

$\downarrow$ 4343

$$-\frac{\frac{-b\left(a^2-b^2\right)\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{\left(3a^2-b^2\right)\sqrt{a+b\sec(c+dx)}\int \sqrt{b+a\cos(c+dx)}dx}{\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}+\frac{6a\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\operatorname{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+}\right.}{d\sqrt{a+b\sec(c+dx)}}}{2b}}{b\left(a^2-b^2\right)}$$

$$\frac{2a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}$$

$\downarrow$ 3042

3.654. $\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$

$$-\frac{\frac{-b(a^2-b^2)\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{\left(3a^2-b^2\right)\sqrt{a+b\sec(c+dx)}\int \sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}+\frac{6a\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(c+dx)\right.}{d\sqrt{a+b\sec(c+dx)}}}{2b}}{b\left(a^2-b^2\right)}$$

$$\frac{2a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}$$

$\downarrow$ 3134

$$-\frac{\frac{-b(a^2-b^2)\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{\left(3a^2-b^2\right)\sqrt{a+b\sec(c+dx)}\int \sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}dx}{\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{6a\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(c+dx)\right.}{d\sqrt{a+b\sec(c+dx)}}}{2b}}{b\left(a^2-b^2\right)}$$

$$\frac{2a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}$$

$\downarrow$ 3042

$$-\frac{\frac{-b(a^2-b^2)\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{\left(3a^2-b^2\right)\sqrt{a+b\sec(c+dx)}\int \sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}dx}{\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{6a\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(c\right.}{d\sqrt{a+b\sec(c+dx)}}}{2b}}{b\left(a^2-b^2\right)}$$

$$\frac{2a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}$$

$\downarrow$ 3132

$$-\frac{\frac{-b(a^2-b^2)\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\left(3a^2-b^2\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{6a\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right.}{d\sqrt{a+b\sec(c+dx)}}}{2b}}{b\left(a^2-b^2\right)}$$

$$\frac{2a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}$$

$\downarrow$ 4345

$$-\frac{\frac{-\frac{b\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int \frac{1}{\sqrt{b+a\cos(c+dx)}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{2\left(3a^2-b^2\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{6a\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\text{Elli}}{d\sqrt{a+b\sec(c+dx)}}}{2b}}{b\left(a^2-b^2\right)}$$

$$\frac{2a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}$$

$\downarrow$ 3042

3.654. $\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$

$$-\frac{-\frac{b\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{1}{\sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{2\left(3a^2-b^2\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{6a\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}{d\sqrt{a+b\sec(c+}}}{2b}}{b\left(a^2-b^2\right)}$$

$$\frac{2a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}$$

$\downarrow$ 3142

$$-\frac{-\frac{b\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{2\left(3a^2-b^2\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{6a\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+}{a+b}}}{d\sqrt{a+b\sec(c+}}}{2b}}{b\left(a^2-b^2\right)}$$

$$\frac{2a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}$$

$\downarrow$ 3042

$$-\frac{-\frac{b\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{2\left(3a^2-b^2\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{6a\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+}{a+b}}}{d\sqrt{a+b\,\mathrm{se}}}}{2b}}{b\left(a^2-b^2\right)}$$

$$\frac{2a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}$$

$\downarrow$ 3140

$$-\frac{2a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}-$$

$$-\frac{-\frac{2b\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\mathrm{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}+\frac{2\left(3a^2-b^2\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{6a\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}{d\sqrt{a+b\sec(c+d}}}{2b}}{b\left(a^2-b^2\right)}$$

input
```
Int[Sec[c + d*x]^(7/2)/(a + b*Sec[c + d*x])^(3/2),x]
```

3.654. $\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$

output
```
(-2*a^2*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(b*(a^2 - b^2)*d*Sqrt[a + b*Sec[c
 + d*x]]) - (((-2*b*(a^2 - b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Ellipti
cF[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(d*Sqrt[a + b*Sec[c + d
*x]]) + (6*a*(a^2 - b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2,
(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(d*Sqrt[a + b*Sec[c + d*x]
]) + (2*(3*a^2 - b^2)*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec
[c + d*x]])/(d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]))/(2*
b) - ((3*a^2 - b^2)*Sqrt[Sec[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*
x])/(b*d))/(b*(a^2 - b^2))
```

3.654.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

3.654. $\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3286
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]]   Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
 d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

rule 4332
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-a^2)*d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(
m + 1)*((d*Csc[e + f*x])^(n - 3)/(b*f*(m + 1)*(a^2 - b^2))), x] + Simp[d^3/
(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])
^(n - 3)*Simp[a^2*(n - 3) + a*b*(m + 1)*Csc[e + f*x] - (a^2*(n - 2) + b^2*(
m + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2
- b^2, 0] && LtQ[m, -1] && (IGtQ[n, 3] || (IntegersQ[n + 1/2, 2*m] && GtQ[n
, 2]))
```

rule 4343
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]])   Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a
, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4345
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]])   Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[
{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4346
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Simp[d*Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x
]]/Sqrt[a + b*Csc[e + f*x]])   Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

3.654. $\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$

rule 4523
```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Simp[A/a   I
nt[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Simp[(A*b - a*B)
/(a*d)   Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ
[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

rule 4590
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(-C)*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
)*((d*Csc[e + f*x])^(n - 1)/(b*f*(m + n + 1))), x] + Simp[d/(b*(m + n + 1))
   Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (
A*b*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2
 - b^2, 0] && GtQ[n, 0]
```

rule 4596
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Simp[C/d^2   Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*C
sc[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[
a + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
 b^2, 0]
```

3.654.4 Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 9.75 (sec) , antiderivative size = 2300, normalized size of antiderivative = 6.67

method	result	size
default	Expression too large to display	2300

input
```
int(sec(d*x+c)^(7/2)/(a+b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/d/((a-b)/(a+b))^(1/2)/(a+b)/b^2*(-((1-cos(d*x+c))^2*csc(d*x+c)^2+1)/((1
-cos(d*x+c))^2*csc(d*x+c)^2-1))^(7/2)*((1-cos(d*x+c))^2*csc(d*x+c)^2-1)^3*
((a*(1-cos(d*x+c))^2*csc(d*x+c)^2-b*(1-cos(d*x+c))^2*csc(d*x+c)^2-a-b)/((1
-cos(d*x+c))^2*csc(d*x+c)^2-1))^(1/2)*(3*((a-b)/(a+b))^(1/2)*a^2*(1-cos(d*
x+c))^5*csc(d*x+c)^5-((a-b)/(a+b))^(1/2)*b^2*(1-cos(d*x+c))^5*csc(d*x+c)^5
+6*(-(a*(1-cos(d*x+c))^2*csc(d*x+c)^2-b*(1-cos(d*x+c))^2*csc(d*x+c)^2-a-b)
/(a+b))^(1/2)*((1-cos(d*x+c))^2*csc(d*x+c)^2+1)^(1/2)*EllipticF(((a-b)/(a+
b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*a^2*(1-cos(d*x+c)
)^2*csc(d*x+c)^2+4*(-(a*(1-cos(d*x+c))^2*csc(d*x+c)^2-b*(1-cos(d*x+c))^2*c
sc(d*x+c)^2-a-b)/(a+b))^(1/2)*((1-cos(d*x+c))^2*csc(d*x+c)^2+1)^(1/2)*Elli
pticF(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*a
*b*(1-cos(d*x+c))^2*csc(d*x+c)^2-3*(-(a*(1-cos(d*x+c))^2*csc(d*x+c)^2-b*(1
-cos(d*x+c))^2*csc(d*x+c)^2-a-b)/(a+b))^(1/2)*((1-cos(d*x+c))^2*csc(d*x+c)
^2+1)^(1/2)*EllipticE(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(-(a+b)
/(a-b))^(1/2))*a^2*(1-cos(d*x+c))^2*csc(d*x+c)^2+(-(a*(1-cos(d*x+c))^2*csc
(d*x+c)^2-b*(1-cos(d*x+c))^2*csc(d*x+c)^2-a-b)/(a+b))^(1/2)*((1-cos(d*x+c)
)^2*csc(d*x+c)^2+1)^(1/2)*EllipticE(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d
*x+c)),(-(a+b)/(a-b))^(1/2))*b^2*(1-cos(d*x+c))^2*csc(d*x+c)^2-6*(-(a*(1-c
os(d*x+c))^2*csc(d*x+c)^2-b*(1-cos(d*x+c))^2*csc(d*x+c)^2-a-b)/(a+b))^(1/2
)*((1-cos(d*x+c))^2*csc(d*x+c)^2+1)^(1/2)*EllipticPi(((a-b)/(a+b))^(1/2...
```

3.654.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)^(7/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.654.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**(7/2)/(a+b*sec(d*x+c))**(3/2),x)
```

output
```
Timed out
```

3.654.7 Maxima [F]

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\sec(dx+c)^{\frac{7}{2}}}{(b\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^(7/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)^(7/2)/(b*sec(d*x + c) + a)^(3/2), x)
```

3.654.8 Giac [F]

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\sec(dx+c)^{\frac{7}{2}}}{(b\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^(7/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^(7/2)/(b*sec(d*x + c) + a)^(3/2), x)
```

3.654.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx=\int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{7/2}}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

input
```
int((1/cos(c + d*x))^(7/2)/(a + b/cos(c + d*x))^(3/2),x)
```

output
```
int((1/cos(c + d*x))^(7/2)/(a + b/cos(c + d*x))^(3/2), x)
```

3.655 $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$

3.655.1 Optimal result 5104
3.655.2 Mathematica [C] (verified) 5105
3.655.3 Rubi [A] (verified) 5105
3.655.4 Maple [C] (warning: unable to verify) 5110
3.655.5 Fricas [F(-1)] 5111
3.655.6 Sympy [F(-1)] 5112
3.655.7 Maxima [F] 5112
3.655.8 Giac [F] 5112
3.655.9 Mupad [F(-1)] 5113

3.655.1 Optimal result

Integrand size = 25, antiderivative size = 206

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \frac{2\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{bd\sqrt{a+b\sec(c+dx)}} + \frac{2aE\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+dx)}}{b\left(a^2-b^2\right)d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\sqrt{\sec(c+dx)}} - \frac{2a^2\sqrt{\sec(c+dx)}\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}}$$

output
```
-2*a^2*sin(d*x+c)*sec(d*x+c)^(1/2)/b/(a^2-b^2)/d/(a+b*sec(d*x+c))^(1/2)+2*
(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2
*c),2,2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)*sec(d*x+c)^(
1/2)/b/d/(a+b*sec(d*x+c))^(1/2)+2*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d
*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*(a+b*sec(d
*x+c))^(1/2)/b/(a^2-b^2)/d/((b+a*cos(d*x+c))/(a+b))^(1/2)/sec(d*x+c)^(1/2)
```

3.655.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.83 (sec) , antiderivative size = 434, normalized size of antiderivative = 2.11

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \frac{\sec^{\frac{3}{2}}(c+dx)\left(\frac{(b+a\cos(c+dx))^{3/2}\left(\frac{4ab\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{\sqrt{b+a\cos(c+dx)}}+\frac{2\left(3a^2-2b^2\right)\sqrt{\frac{b+a\,c}{}}}{}\right.}{}\right.}{}$$

input
```
Integrate[Sec[c + d*x]^(5/2)/(a + b*Sec[c + d*x])^(3/2),x]
```

output
```
(Sec[c + d*x]^(3/2)*(((b + a*Cos[c + d*x])^(3/2)*((4*a*b*Sqrt[(b + a*Cos[c
 + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)])/Sqrt[b + a*Cos[c
+ d*x]] + (2*(3*a^2 - 2*b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi
[2, (c + d*x)/2, (2*a)/(a + b)])/Sqrt[b + a*Cos[c + d*x]] + ((2*I)*Sqrt[-(
(a*(-1 + Cos[c + d*x]))/(a + b))]*Sqrt[(a*(1 + Cos[c + d*x]))/(a - b)]*Csc
[c + d*x]*(-2*b*(a + b)*EllipticE[I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*
Cos[c + d*x]]], (-a + b)/(a + b)] + a*(2*b*EllipticF[I*ArcSinh[Sqrt[(a - b
)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b)] + a*EllipticPi[1 - a/
b, I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b
)]))/(Sqrt[(a - b)^(-1)]*b)))/((a - b)*(a + b)) + (4*a^2*(b + a*Cos[c + d
*x])*Sin[c + d*x])/(-a^2 + b^2)))/(2*b*d*(a + b*Sec[c + d*x])^(3/2))
```

3.655.3 Rubi [A] (verified)

Time = 1.65 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.04, number of steps used = 17, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.680$, Rules used = {3042, 4332, 27, 3042, 4596, 2011, 3042, 4343, 3042, 3134, 3042, 3132, 4346, 3042, 3286, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$\Big\downarrow$ 4332

$$-\frac{2\int -\frac{a^2+b\sec(c+dx)a+(a^2-b^2)\sec^2(c+dx)}{2\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{b\left(a^2-b^2\right)}-\frac{2a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{bd\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}$$

$\Big\downarrow$ 27

$$\frac{\int \frac{a^2+b\sec(c+dx)a+(a^2-b^2)\sec^2(c+dx)}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{b\left(a^2-b^2\right)}-\frac{2a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{bd\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}$$

$\Big\downarrow$ 3042

$$\frac{\int \frac{a^2+b\csc\left(c+dx+\frac{\pi}{2}\right)a+(a^2-b^2)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{b\left(a^2-b^2\right)}-\frac{2a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{bd\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}$$

$\Big\downarrow$ 4596

$$\frac{\left(a^2-b^2\right)\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx+\int \frac{a^2+b\sec(c+dx)a}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{b\left(a^2-b^2\right)}-\frac{2a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{bd\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}$$

$\Big\downarrow$ 2011

$$\frac{\left(a^2-b^2\right)\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx+a\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}dx}{b\left(a^2-b^2\right)}-\frac{2a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{bd\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}$$

$\Big\downarrow$ 3042

$$\frac{\left(a^2-b^2\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+a\int \frac{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{b\left(a^2-b^2\right)}-\frac{2a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{bd\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}$$

$\Big\downarrow$ 4343

$$\frac{\left(a^2-b^2\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{a\sqrt{a+b\sec(c+dx)}\int \sqrt{b+a\cos(c+dx)}dx}{\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}}{b\left(a^2-b^2\right)}-\frac{2a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{bd\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}$$

$\Big\downarrow$ 3042

3.655. $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$

$$\frac{(a^2-b^2)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{a\sqrt{a+b\sec(c+dx)}\int\sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}}{b(a^2-b^2)}-\frac{2a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{bd(a^2-b^2)\sqrt{a+b\sec(c+dx)}}$$

$\downarrow$ 3134

$$\frac{(a^2-b^2)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{a\sqrt{a+b\sec(c+dx)}\int\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}dx}{\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{b(a^2-b^2)}-\frac{2a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{bd(a^2-b^2)\sqrt{a+b\sec(c+dx)}}$$

$\downarrow$ 3042

$$\frac{(a^2-b^2)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{a\sqrt{a+b\sec(c+dx)}\int\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}dx}{\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{b(a^2-b^2)}-\frac{2a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{bd(a^2-b^2)\sqrt{a+b\sec(c+dx)}}$$

$\downarrow$ 3132

$$\frac{(a^2-b^2)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{b(a^2-b^2)}-\frac{2a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{bd(a^2-b^2)\sqrt{a+b\sec(c+dx)}}$$

$\downarrow$ 4346

$$\frac{\frac{(a^2-b^2)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{\sec(c+dx)}{\sqrt{b+a\cos(c+dx)}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{2a\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{b(a^2-b^2)}-\frac{2a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{bd(a^2-b^2)\sqrt{a+b\sec(c+dx)}}$$

$\downarrow$ 3042

$$\frac{\frac{(a^2-b^2)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{2a\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{b(a^2-b^2)}-\frac{2a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{bd(a^2-b^2)\sqrt{a+b\sec(c+dx)}}$$

$\downarrow$ 3286

3.655. $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$

$$\frac{\frac{(a^2-b^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{\sec(c+dx)}{\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{2a\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{b(a^2-b^2)}-\frac{2a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{bd(a^2-b^2)\sqrt{a+b\sec(c+dx)}}$$

$\Big\downarrow$ 3042

$$\frac{\frac{(a^2-b^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{2a\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{b(a^2-b^2)}-\frac{2a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{bd(a^2-b^2)\sqrt{a+b\sec(c+dx)}}$$

$\Big\downarrow$ 3284

$$\frac{\frac{2(a^2-b^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}+\frac{2a\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{b(a^2-b^2)}-\frac{2a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{bd(a^2-b^2)\sqrt{a+b\sec(c+dx)}}$$

input
```
Int[Sec[c + d*x]^(5/2)/(a + b*Sec[c + d*x])^(3/2),x]
```

output
```
((2*(a^2 - b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)
/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(d*Sqrt[a + b*Sec[c + d*x]]) + (2*a
*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(d*Sqrt[(
b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]))/(b*(a^2 - b^2)) - (2*a^2
*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(b*(a^2 - b^2)*d*Sqrt[a + b*Sec[c + d*x]
])
```

3.655. $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$

3.655.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2011
```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x
] && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x
, a + b*x])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3286
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]]   Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
 d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

3.655. $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$

rule 4332
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-a^2)*d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(
m + 1)*((d*Csc[e + f*x])^(n - 3)/(b*f*(m + 1)*(a^2 - b^2))), x] + Simp[d^3/
(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])
^(n - 3)*Simp[a^2*(n - 3) + a*b*(m + 1)*Csc[e + f*x] - (a^2*(n - 2) + b^2*(
m + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2
- b^2, 0] && LtQ[m, -1] && (IGtQ[n, 3] || (IntegersQ[n + 1/2, 2*m] && GtQ[n
, 2]))
```

rule 4343
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]])   Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a
, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4346
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Simp[d*Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x
]]/Sqrt[a + b*Csc[e + f*x]])   Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4596
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Simp[C/d^2   Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*C
sc[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[
a + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
 b^2, 0]
```

3.655.4 Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 7.57 (sec) , antiderivative size = 1019, normalized size of antiderivative = 4.95

method	result	size
default	Expression too large to display	1019

input
```
int(sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/d/((a-b)/(a+b))^(1/2)/b/(a+b)*(-((1-cos(d*x+c))^2*csc(d*x+c)^2+1)/((1-c
os(d*x+c))^2*csc(d*x+c)^2-1))^(5/2)*((1-cos(d*x+c))^2*csc(d*x+c)^2-1)^3*((
a*(1-cos(d*x+c))^2*csc(d*x+c)^2-b*(1-cos(d*x+c))^2*csc(d*x+c)^2-a-b)/((1-c
os(d*x+c))^2*csc(d*x+c)^2-1))^(1/2)*(((a-b)/(a+b))^(1/2)*a*(1-cos(d*x+c))^
3*csc(d*x+c)^3+2*(-(a*(1-cos(d*x+c))^2*csc(d*x+c)^2-b*(1-cos(d*x+c))^2*csc
(d*x+c)^2-a-b)/(a+b))^(1/2)*((1-cos(d*x+c))^2*csc(d*x+c)^2+1)^(1/2)*Ellipt
icF(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*a+(
-(a*(1-cos(d*x+c))^2*csc(d*x+c)^2-b*(1-cos(d*x+c))^2*csc(d*x+c)^2-a-b)/(a+
b))^(1/2)*((1-cos(d*x+c))^2*csc(d*x+c)^2+1)^(1/2)*EllipticF(((a-b)/(a+b))^
(1/2)*(-cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*b-(-(a*(1-cos(d*x+c))
^2*csc(d*x+c)^2-b*(1-cos(d*x+c))^2*csc(d*x+c)^2-a-b)/(a+b))^(1/2)*((1-cos(
d*x+c))^2*csc(d*x+c)^2+1)^(1/2)*EllipticE(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)
+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*a-2*(-(a*(1-cos(d*x+c))^2*csc(d*x+c)^2-
b*(1-cos(d*x+c))^2*csc(d*x+c)^2-a-b)/(a+b))^(1/2)*((1-cos(d*x+c))^2*csc(d*
x+c)^2+1)^(1/2)*EllipticPi(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(a
+b)/(a-b),I/((a-b)/(a+b))^(1/2))*a-2*(-(a*(1-cos(d*x+c))^2*csc(d*x+c)^2-b*
(1-cos(d*x+c))^2*csc(d*x+c)^2-a-b)/(a+b))^(1/2)*((1-cos(d*x+c))^2*csc(d*x+
c)^2+1)^(1/2)*EllipticPi(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(a+b
)/(a-b),I/((a-b)/(a+b))^(1/2))*b+((a-b)/(a+b))^(1/2)*a*(-cot(d*x+c)+csc(d*
x+c)))/((1-cos(d*x+c))^2*csc(d*x+c)^2+1)^3/(a*(1-cos(d*x+c))^2*csc(d*x+...
```

3.655.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.655.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**(5/2)/(a+b*sec(d*x+c))**(3/2),x)
```

output
```
Timed out
```

3.655.7 Maxima [F]

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\sec(dx+c)^{\frac{5}{2}}}{(b\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)^(5/2)/(b*sec(d*x + c) + a)^(3/2), x)
```

3.655.8 Giac [F]

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\sec(dx+c)^{\frac{5}{2}}}{(b\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^(5/2)/(b*sec(d*x + c) + a)^(3/2), x)
```

3.655.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx=\int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

input
```
int((1/cos(c + d*x))^(5/2)/(a + b/cos(c + d*x))^(3/2),x)
```

output
```
int((1/cos(c + d*x))^(5/2)/(a + b/cos(c + d*x))^(3/2), x)
```

3.656 $\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$

3.656.1 Optimal result . . . 5114
3.656.2 Mathematica [A] (verified) . . . 5114
3.656.3 Rubi [A] (verified) . . . 5115
3.656.4 Maple [B] (warning: unable to verify) . . . 5117
3.656.5 Fricas [C] (verification not implemented) . . . 5118
3.656.6 Sympy [F] . . . 5119
3.656.7 Maxima [F] . . . 5119
3.656.8 Giac [F] . . . 5119
3.656.9 Mupad [F(-1)] . . . 5120

3.656.1 Optimal result

Integrand size = 25, antiderivative size = 126

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = -\frac{2E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+dx)}}{(a^2-b^2)\,d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\sqrt{\sec(c+dx)}} + \frac{2a\sqrt{\sec(c+dx)}\sin(c+dx)}{(a^2-b^2)\,d\sqrt{a+b\sec(c+dx)}}$$

output
```
2*a*sin(d*x+c)*sec(d*x+c)^(1/2)/(a^2-b^2)/d/(a+b*sec(d*x+c))^(1/2)-2*(cos(
1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^
(1/2)*(a/(a+b))^(1/2))*(a+b*sec(d*x+c))^(1/2)/(a^2-b^2)/d/((b+a*cos(d*x+c)
)/(a+b))^(1/2)/sec(d*x+c)^(1/2)
```

3.656.2 Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.82

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = -\frac{2(b+a\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\left((a+b)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)-a\sin(c+dx)\right)}{(a-b)(a+b)d(a+b\sec(c+dx))^{3/2}}$$

input
```
Integrate[Sec[c + d*x]^(3/2)/(a + b*Sec[c + d*x])^(3/2),x]
```

output
```
(-2*(b + a*Cos[c + d*x])*Sec[c + d*x]^(3/2)*((a + b)*Sqrt[(b + a*Cos[c + d
*x])/(a + b)]*EllipticE[(c + d*x)/2, (2*a)/(a + b)] - a*Sin[c + d*x]))/((a
 - b)*(a + b)*d*(a + b*Sec[c + d*x])^(3/2))
```

3.656.3 Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {3042, 4331, 27, 3042, 4343, 3042, 3134, 3042, 3132}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{4331}$$

$$\frac{2\int -\frac{\sqrt{a+b\sec(c+dx)}}{2\sqrt{\sec(c+dx)}}dx}{a^2-b^2}+\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}$$

$$\Big\downarrow \text{27}$$

$$\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}-\frac{\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}dx}{a^2-b^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}-\frac{\int \frac{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2-b^2}$$

$$\Big\downarrow \text{4343}$$

$$\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}-\frac{\sqrt{a+b\sec(c+dx)}\int \sqrt{b+a\cos(c+dx)}dx}{\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}-\frac{\sqrt{a+b\sec(c+dx)}\int\sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}$$

$$\Big\downarrow \text{3134}$$

$$\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}-\frac{\sqrt{a+b\sec(c+dx)}\int\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}dx}{\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}-\frac{\sqrt{a+b\sec(c+dx)}\int\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}dx}{\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}$$

$$\Big\downarrow \text{3132}$$

$$\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}-\frac{2\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}$$

input
```
Int[Sec[c + d*x]^(3/2)/(a + b*Sec[c + d*x])^(3/2),x]
```

output
```
(-2*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/((a^2
- b^2)*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]) + (2*a*Sqr
t[Sec[c + d*x]]*Sin[c + d*x])/((a^2 - b^2)*d*Sqrt[a + b*Sec[c + d*x]])
```

3.656.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

3.656. $\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 4331
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[a*d^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
)*((d*Csc[e + f*x])^(n - 2)/(f*(m + 1)*(a^2 - b^2))), x] - Simp[d^2/((m + 1
)*(a^2 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*
(a*(n - 2) + b*(m + 1)*Csc[e + f*x] - a*(m + n)*Csc[e + f*x]^2), x], x] /;
FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && LtQ[1, n, 2
] && IntegersQ[2*m, 2*n]
```

rule 4343
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]])   Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a
, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

3.656.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 586 vs. $2(145) = 290$.

Time = 3.24 (sec) , antiderivative size = 587, normalized size of antiderivative = 4.66

method	result
default	$-\frac{2\left(-\frac{(1-\cos(dx+c))^2\csc(dx+c)^2+1}{(1-\cos(dx+c))^2\csc(dx+c)^2-1}\right)^{\frac{3}{2}}\left((1-\cos(dx+c))^2\csc(dx+c)^2-1\right)^2\sqrt{\frac{a(1-\cos(dx+c))^2\csc(dx+c)^2-b(1-\cos(dx+c))^2\csc(dx+c)^2-}{(1-\cos(dx+c))^2\csc(dx+c)^2-1}}}{}$

input
```
int(sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/d/(a+b)/((a-b)/(a+b))^(1/2)*(-((1-cos(d*x+c))^2*csc(d*x+c)^2+1)/((1-cos
(d*x+c))^2*csc(d*x+c)^2-1))^(3/2)*((1-cos(d*x+c))^2*csc(d*x+c)^2-1)^2*((a*
(1-cos(d*x+c))^2*csc(d*x+c)^2-b*(1-cos(d*x+c))^2*csc(d*x+c)^2-a-b)/((1-cos
(d*x+c))^2*csc(d*x+c)^2-1))^(1/2)*(((a-b)/(a+b))^(1/2)*(1-cos(d*x+c))^3*cs
c(d*x+c)^3+(-(a*(1-cos(d*x+c))^2*csc(d*x+c)^2-b*(1-cos(d*x+c))^2*csc(d*x+c
)^2-a-b)/(a+b))^(1/2)*((1-cos(d*x+c))^2*csc(d*x+c)^2+1)^(1/2)*EllipticF(((
a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))-(-(a*(1-c
os(d*x+c))^2*csc(d*x+c)^2-b*(1-cos(d*x+c))^2*csc(d*x+c)^2-a-b)/(a+b))^(1/2
)*((1-cos(d*x+c))^2*csc(d*x+c)^2+1)^(1/2)*EllipticE(((a-b)/(a+b))^(1/2)*(-
cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))+((a-b)/(a+b))^(1/2)*(-cot(d*x
+c)+csc(d*x+c)))/((1-cos(d*x+c))^2*csc(d*x+c)^2+1)^2/(a*(1-cos(d*x+c))^2*c
sc(d*x+c)^2-b*(1-cos(d*x+c))^2*csc(d*x+c)^2-a-b)
```

3.656.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 488, normalized size of antiderivative = 3.87

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \frac{6\,a^2\sqrt{\frac{a\cos(dx+c)+b}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)-\sqrt{2}(-i\,ab\cos(dx+c)-i\,b^2)\sqrt{}}{}$$

input
```
integrate(sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
1/3*(6*a^2*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(
d*x + c) - sqrt(2)*(-I*a*b*cos(d*x + c) - I*b^2)*sqrt(a)*weierstrassPInver
se(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x
+ c) + 3*I*a*sin(d*x + c) + 2*b)/a) - sqrt(2)*(I*a*b*cos(d*x + c) + I*b^2)
*sqrt(a)*weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b
^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a) + 3*sqrt(2)*
(-I*a^2*cos(d*x + c) - I*a*b)*sqrt(a)*weierstrassZeta(-4/3*(3*a^2 - 4*b^2)
/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)
/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x +
c) + 2*b)/a)) + 3*sqrt(2)*(I*a^2*cos(d*x + c) + I*a*b)*sqrt(a)*weierstrass
Zeta(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInv
erse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*
x + c) - 3*I*a*sin(d*x + c) + 2*b)/a)))/((a^4 - a^2*b^2)*d*cos(d*x + c) +
(a^3*b - a*b^3)*d)
```

3.656.6 Sympy [F]

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)**(3/2)/(a+b*sec(d*x+c))**(3/2),x)
```

output
```
Integral(sec(c + d*x)**(3/2)/(a + b*sec(c + d*x))**(3/2), x)
```

3.656.7 Maxima [F]

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\sec(dx+c)^{\frac{3}{2}}}{(b\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)^(3/2)/(b*sec(d*x + c) + a)^(3/2), x)
```

3.656.8 Giac [F]

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\sec(dx+c)^{\frac{3}{2}}}{(b\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^(3/2)/(b*sec(d*x + c) + a)^(3/2), x)
```

3.656.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx=\int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

input
```
int((1/cos(c + d*x))^(3/2)/(a + b/cos(c + d*x))^(3/2),x)
```

output
```
int((1/cos(c + d*x))^(3/2)/(a + b/cos(c + d*x))^(3/2), x)
```

3.657 $\int \frac{\sqrt{\sec(c+dx)}}{(a+b\sec(c+dx))^{3/2}}\,dx$

3.657.1 Optimal result 5121
3.657.2 Mathematica [A] (verified) 5121
3.657.3 Rubi [A] (verified) 5122
3.657.4 Maple [B] (warning: unable to verify) 5126
3.657.5 Fricas [C] (verification not implemented) 5127
3.657.6 Sympy [F] 5128
3.657.7 Maxima [F] 5128
3.657.8 Giac [F] 5129
3.657.9 Mupad [F(-1)] 5129

3.657.1 Optimal result

Integrand size = 25, antiderivative size = 200

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+b\sec(c+dx))^{3/2}}\,dx = \frac{2\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{ad\sqrt{a+b\sec(c+dx)}} + \frac{2bE\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+dx)}}{a\left(a^2-b^2\right)d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\sqrt{\sec(c+dx)}} - \frac{2b\sqrt{\sec(c+dx)}\sin(c+dx)}{\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}}$$

output
```
-2*b*sin(d*x+c)*sec(d*x+c)^(1/2)/(a^2-b^2)/d/(a+b*sec(d*x+c))^(1/2)+2*(cos
(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2
^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)*sec(d*x+c)^(1/2)/a/
d/(a+b*sec(d*x+c))^(1/2)+2*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*
c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*(a+b*sec(d*x+c))^
(1/2)/a/(a^2-b^2)/d/((b+a*cos(d*x+c))/(a+b))^(1/2)/sec(d*x+c)^(1/2)
```

3.657.2 Mathematica [A] (verified)

Time = 0.65 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.78

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+b\sec(c+dx))^{3/2}}\,dx = \frac{2(b+a\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\left(b(a+b)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)+\right.}{a(a-b)(a+b)d(a+b}$$

input
```
Integrate[Sqrt[Sec[c + d*x]]/(a + b*Sec[c + d*x])^(3/2),x]
```

output
```
(2*(b + a*Cos[c + d*x])*Sec[c + d*x]^(3/2)*(b*(a + b)*Sqrt[(b + a*Cos[c +
d*x])/(a + b)]*EllipticE[(c + d*x)/2, (2*a)/(a + b)] + (a^2 - b^2)*Sqrt[(b
 + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)] - a*b*Si
n[c + d*x]))/(a*(a - b)*(a + b)*d*(a + b*Sec[c + d*x])^(3/2))
```

3.657.3 Rubi [A] (verified)

Time = 1.46 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.06, number of steps used = 16, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.640$, Rules used = {3042, 4330, 27, 3042, 4523, 3042, 4343, 3042, 3134, 3042, 3132, 4345, 3042, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+b\sec(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{4330}$$

$$-\frac{2\int -\frac{b+a\sec(c+dx)}{2\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{a^2-b^2}-\frac{2b\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{b+a\sec(c+dx)}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{a^2-b^2}-\frac{2b\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{b+a\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2-b^2}-\frac{2b\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}$$

$$\Big\downarrow \text{4523}$$

$$\frac{\frac{(a^2-b^2)\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}dx}{a}+\frac{b\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}dx}{a}}{a^2-b^2}-\frac{2b\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}$$

$\downarrow$ 3042

$$\frac{\frac{(a^2-b^2)\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}+\frac{b\int \frac{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}}{a^2-b^2}-\frac{2b\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}$$

$\downarrow$ 4343

$$\frac{\frac{(a^2-b^2)\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}+\frac{b\sqrt{a+b\sec(c+dx)}\int \sqrt{b+a\cos(c+dx)}dx}{a\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}}{a^2-b^2}-\frac{2b\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}$$

$\downarrow$ 3042

$$\frac{\frac{(a^2-b^2)\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}+\frac{b\sqrt{a+b\sec(c+dx)}\int \sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{a\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}}{a^2-b^2}-\frac{2b\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}$$

$\downarrow$ 3134

$$\frac{\frac{(a^2-b^2)\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}+\frac{b\sqrt{a+b\sec(c+dx)}\int \sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}dx}{a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{a^2-b^2}-\frac{2b\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}$$

$\downarrow$ 3042

$$\frac{\frac{(a^2-b^2)\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}+\frac{b\sqrt{a+b\sec(c+dx)}\int \sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}dx}{a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{a^2-b^2}-\frac{2b\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}$$

$\downarrow$ 3132

$$\frac{\frac{(a^2-b^2)\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}+\frac{2b\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{a^2-b^2}-\frac{2b\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}$$

$\downarrow$ 4345

3.657. $\int \frac{\sqrt{\sec(c+dx)}}{(a+b\sec(c+dx))^{3/2}}\,dx$

$$\frac{\frac{(a^2-b^2)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{1}{\sqrt{b+a\cos(c+dx)}}dx}{a\sqrt{a+b\sec(c+dx)}}+\frac{2b\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{a^2-b^2}-\frac{2b\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a^2-b^2)\sqrt{a+b\sec(c+dx)}}$$

$\Big\downarrow$ 3042

$$\frac{\frac{(a^2-b^2)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{1}{\sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{a\sqrt{a+b\sec(c+dx)}}+\frac{2b\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{a^2-b^2}-\frac{2b\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a^2-b^2)\sqrt{a+b\sec(c+dx)}}$$

$\Big\downarrow$ 3142

$$\frac{\frac{(a^2-b^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}}dx}{a\sqrt{a+b\sec(c+dx)}}+\frac{2b\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{a^2-b^2}-\frac{2b\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a^2-b^2)\sqrt{a+b\sec(c+dx)}}$$

$\Big\downarrow$ 3042

$$\frac{\frac{(a^2-b^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{a\sqrt{a+b\sec(c+dx)}}+\frac{2b\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{a^2-b^2}-\frac{2b\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a^2-b^2)\sqrt{a+b\sec(c+dx)}}$$

$\Big\downarrow$ 3140

$$\frac{\frac{2(a^2-b^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{ad\sqrt{a+b\sec(c+dx)}}+\frac{2b\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{a^2-b^2}-\frac{2b\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a^2-b^2)\sqrt{a+b\sec(c+dx)}}$$

input
```
Int[Sqrt[Sec[c + d*x]]/(a + b*Sec[c + d*x])^(3/2),x]
```

3.657. $\int \frac{\sqrt{\sec(c+dx)}}{(a+b\sec(c+dx))^{3/2}}\,dx$

output
```
((2*(a^2 - b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2,
(2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(a*d*Sqrt[a + b*Sec[c + d*x]]) + (2*b*E
llipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(a*d*Sqrt[(
b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]))/(a^2 - b^2) - (2*b*Sqrt[
Sec[c + d*x]]*Sin[c + d*x])/((a^2 - b^2)*d*Sqrt[a + b*Sec[c + d*x]])
```

3.657.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

3.657. $\int \frac{\sqrt{\sec(c+dx)}}{(a+b\sec(c+dx))^{3/2}}\,dx$

rule 4330
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b)*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m +
1)*((d*Csc[e + f*x])^(n - 1)/(f*(m + 1)*(a^2 - b^2))), x] + Simp[1/((m + 1)
*(a^2 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1)*S
imp[b*d*(n - 1) + a*d*(m + 1)*Csc[e + f*x] - b*d*(m + n + 1)*Csc[e + f*x]^2
, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1
] && LtQ[0, n, 1] && IntegersQ[2*m, 2*n]
```

rule 4343
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]])   Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a
, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4345
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]])   Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[
{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4523
```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Simp[A/a   I
nt[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Simp[(A*b - a*B)
/(a*d)   Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ
[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

3.657.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 592 vs. $2(242) = 484$.

Time = 6.54 (sec) , antiderivative size = 593, normalized size of antiderivative = 2.96

method	result
default	$-\frac{2\sqrt{-\frac{(1-\cos(dx+c))^2\csc(dx+c)^2+1}{(1-\cos(dx+c))^2\csc(dx+c)^2-1}}\left((1-\cos(dx+c))^2\csc(dx+c)^2-1\right)\sqrt{\frac{a(1-\cos(dx+c))^2\csc(dx+c)^2-b(1-\cos(dx+c))^2\csc(dx+c)^2-a-b}{(1-\cos(dx+c))^2\csc(dx+c)^2-1}}}{}$

input
```
int(sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/d/((a-b)/(a+b))^(1/2)/(a+b)/a*(-((1-cos(d*x+c))^2*csc(d*x+c)^2+1)/((1-c
os(d*x+c))^2*csc(d*x+c)^2-1))^(1/2)*((1-cos(d*x+c))^2*csc(d*x+c)^2-1)*((a*
(1-cos(d*x+c))^2*csc(d*x+c)^2-b*(1-cos(d*x+c))^2*csc(d*x+c)^2-a-b)/((1-cos
(d*x+c))^2*csc(d*x+c)^2-1))^(1/2)*(((a-b)/(a+b))^(1/2)*b*(1-cos(d*x+c))^3*
csc(d*x+c)^3-(-(a*(1-cos(d*x+c))^2*csc(d*x+c)^2-b*(1-cos(d*x+c))^2*csc(d*x
+c)^2-a-b)/(a+b))^(1/2)*((1-cos(d*x+c))^2*csc(d*x+c)^2+1)^(1/2)*EllipticF(
((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*a-(-(a*
(1-cos(d*x+c))^2*csc(d*x+c)^2-b*(1-cos(d*x+c))^2*csc(d*x+c)^2-a-b)/(a+b))^
(1/2)*((1-cos(d*x+c))^2*csc(d*x+c)^2+1)^(1/2)*EllipticE(((a-b)/(a+b))^(1/2
)*(-cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*b+((a-b)/(a+b))^(1/2)*b*(
-cot(d*x+c)+csc(d*x+c)))/((1-cos(d*x+c))^2*csc(d*x+c)^2+1)/(a*(1-cos(d*x+c
))^2*csc(d*x+c)^2-b*(1-cos(d*x+c))^2*csc(d*x+c)^2-a-b)
```

3.657.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 525, normalized size of antiderivative = 2.62

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+b\sec(c+dx))^{3/2}}\,dx =$$
$$-\frac{6\,a^2b\sqrt{\frac{a\cos(dx+c)+b}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)+\sqrt{2}(3i\,a^2b-2i\,b^3+(3i\,a^3-2i\,ab^2)\cos(dx+c))\sqrt{a}\text{weiers}}{}$$

input
```
integrate(sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
-1/3*(6*a^2*b*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*sqrt(cos(d*x + c))*s
in(d*x + c) + sqrt(2)*(3*I*a^2*b - 2*I*b^3 + (3*I*a^3 - 2*I*a*b^2)*cos(d*x
 + c))*sqrt(a)*weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b
 - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x + c) + 2*b)/a) + sqrt
(2)*(-3*I*a^2*b + 2*I*b^3 + (-3*I*a^3 + 2*I*a*b^2)*cos(d*x + c))*sqrt(a)*w
eierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1
/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a) - 3*sqrt(2)*(I*a^2*b*c
os(d*x + c) + I*a*b^2)*sqrt(a)*weierstrassZeta(-4/3*(3*a^2 - 4*b^2)/a^2, 8
/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8
/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x + c) + 2*
b)/a)) - 3*sqrt(2)*(-I*a^2*b*cos(d*x + c) - I*a*b^2)*sqrt(a)*weierstrassZe
ta(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInver
se(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x
+ c) - 3*I*a*sin(d*x + c) + 2*b)/a)))/((a^5 - a^3*b^2)*d*cos(d*x + c) + (a
^4*b - a^2*b^3)*d)
```

3.657.6 Sympy [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\sqrt{\sec{(c+dx)}}}{(a+b\sec{(c+dx)})^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)**(1/2)/(a+b*sec(d*x+c))**(3/2),x)
```

output
```
Integral(sqrt(sec(c + d*x))/(a + b*sec(c + d*x))**(3/2), x)
```

3.657.7 Maxima [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\sqrt{\sec{(dx+c)}}}{(b\sec{(dx+c)}+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(sec(d*x + c))/(b*sec(d*x + c) + a)^(3/2), x)
```

3.657.8 Giac [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\sqrt{\sec(dx+c)}}{(b\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate(sqrt(sec(d*x + c))/(b*sec(d*x + c) + a)^(3/2), x)
```

3.657.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\sqrt{\frac{1}{\cos(c+d\,x)}}}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

input
```
int((1/cos(c + d*x))^(1/2)/(a + b/cos(c + d*x))^(3/2),x)
```

output
```
int((1/cos(c + d*x))^(1/2)/(a + b/cos(c + d*x))^(3/2), x)
```

3.658 $\int \frac{1}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}}\,dx$

3.658.1 Optimal result 5130
3.658.2 Mathematica [A] (verified) 5131
3.658.3 Rubi [A] (verified) 5131
3.658.4 Maple [B] (verified) 5136
3.658.5 Fricas [C] (verification not implemented) 5137
3.658.6 Sympy [F] 5137
3.658.7 Maxima [F] 5138
3.658.8 Giac [F] 5138
3.658.9 Mupad [F(-1)] 5138

3.658.1 Optimal result

Integrand size = 25, antiderivative size = 214

$$\int \frac{1}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}}\,dx = -\frac{4b\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{a^2d\sqrt{a+b\sec(c+dx)}} + \frac{2(a^2-2b^2)\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+dx)}}{a^2\left(a^2-b^2\right)d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\sqrt{\sec(c+dx)}} + \frac{2b^2\sqrt{\sec(c+dx)}\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}}$$

output
```
2*b^2*sin(d*x+c)*sec(d*x+c)^(1/2)/a/(a^2-b^2)/d/(a+b*sec(d*x+c))^(1/2)-4*b
*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2
*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)*sec(d*x+c)^(1/
2)/a^2/d/(a+b*sec(d*x+c))^(1/2)+2*(a^2-2*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)
/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*
(a+b*sec(d*x+c))^(1/2)/a^2/(a^2-b^2)/d/((b+a*cos(d*x+c))/(a+b))^(1/2)/sec(
d*x+c)^(1/2)
```

3.658.2 Mathematica [A] (verified)

Time = 0.73 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.77

$$\int \frac{1}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}}\,dx = \frac{2\sqrt{\sec(c+dx)}\left((a^3+a^2b-2ab^2-2b^3)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}E\left(\frac{1}{2}(c+\right.\right.}{a^2(a-}$$

input
```
Integrate[1/(Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^(3/2)),x]
```

output
```
(2*Sqrt[Sec[c + d*x]]*((a^3 + a^2*b - 2*a*b^2 - 2*b^3)*Sqrt[(b + a*Cos[c +
 d*x])/(a + b)]*EllipticE[(c + d*x)/2, (2*a)/(a + b)] + b*(-2*(a^2 - b^2)*
Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)] +
 a*b*Sin[c + d*x])))/(a^2*(a - b)*(a + b)*d*Sqrt[a + b*Sec[c + d*x]])
```

3.658.3 Rubi [A] (verified)

Time = 1.57 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.07, number of steps used = 16, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.640$, Rules used = {3042, 4334, 27, 3042, 4523, 3042, 4343, 3042, 3134, 3042, 3132, 4345, 3042, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{4334}$$

$$\frac{2b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{ad\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}-\frac{2\int -\frac{a^2-b\sec(c+dx)a-2b^2}{2\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{a\left(a^2-b^2\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{a^2-b\sec(c+dx)a-2b^2}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{a\left(a^2-b^2\right)}+\frac{2b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{ad\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{a^2-b\csc\left(c+dx+\frac{\pi}{2}\right)a-2b^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a\left(a^2-b^2\right)}+\frac{2b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{ad\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}$$

$$\Big\downarrow \text{4523}$$

$$\frac{\frac{\left(a^2-2b^2\right)\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}dx}{a}-\frac{2b\left(a^2-b^2\right)\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}dx}{a}}{a\left(a^2-b^2\right)}+\frac{2b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{ad\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\left(a^2-2b^2\right)\int \frac{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}-\frac{2b\left(a^2-b^2\right)\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}}{a\left(a^2-b^2\right)}+\frac{2b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{ad\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}$$

$$\Big\downarrow \text{4343}$$

$$\frac{\frac{\left(a^2-2b^2\right)\sqrt{a+b\sec(c+dx)}\int \sqrt{b+a\cos(c+dx)}dx}{a\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}-\frac{2b\left(a^2-b^2\right)\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}}{a\left(a^2-b^2\right)}+\frac{2b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{ad\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\left(a^2-2b^2\right)\sqrt{a+b\sec(c+dx)}\int \sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{a\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}-\frac{2b\left(a^2-b^2\right)\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}}{a\left(a^2-b^2\right)}+\frac{2b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{ad\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}$$

$$\Big\downarrow \text{3134}$$

$$\frac{\frac{\left(a^2-2b^2\right)\sqrt{a+b\sec(c+dx)}\int \sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}dx}{a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{2b\left(a^2-b^2\right)\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}}{a\left(a^2-b^2\right)}+\frac{2b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{ad\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

3.658. $\int \frac{1}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}}\,dx$

$$\frac{\frac{(a^2-2b^2)\sqrt{a+b\sec(c+dx)}\int\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}dx}{a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{2b(a^2-b^2)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}}{a(a^2-b^2)}+\frac{2b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{ad(a^2-b^2)\sqrt{a+b\sec(c+dx)}}$$

$\downarrow$ 3132

$$\frac{\frac{2(a^2-2b^2)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{2b(a^2-b^2)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}}{a(a^2-b^2)}+\frac{2b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{ad(a^2-b^2)\sqrt{a+b\sec(c+dx)}}$$

$\downarrow$ 4345

$$\frac{\frac{2(a^2-2b^2)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{2b(a^2-b^2)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{1}{\sqrt{b+a\cos(c+dx)}}dx}{a\sqrt{a+b\sec(c+dx)}}}{a(a^2-b^2)}+\frac{2b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{ad(a^2-b^2)\sqrt{a+b\sec(c+dx)}}$$

$\downarrow$ 3042

$$\frac{\frac{2(a^2-2b^2)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{2b(a^2-b^2)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{1}{\sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{a\sqrt{a+b\sec(c+dx)}}}{a(a^2-b^2)}+\frac{2b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{ad(a^2-b^2)\sqrt{a+b\sec(c+dx)}}$$

$\downarrow$ 3142

$$\frac{\frac{2(a^2-2b^2)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{2b(a^2-b^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}}dx}{a\sqrt{a+b\sec(c+dx)}}}{a(a^2-b^2)}+\frac{2b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{ad(a^2-b^2)\sqrt{a+b\sec(c+dx)}}$$

$\downarrow$ 3042

3.658. $\int\frac{1}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}}dx$

$$\frac{\frac{2(a^2-2b^2)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{2b(a^2-b^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{a\sqrt{a+b\sec(c+dx)}}}{a(a^2-b^2)}+\frac{2b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{ad(a^2-b^2)\sqrt{a+b\sec(c+dx)}}$$

$$\Big\downarrow \; 3140$$

$$\frac{2b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{ad(a^2-b^2)\sqrt{a+b\sec(c+dx)}}+\frac{\frac{2(a^2-2b^2)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{4b(a^2-b^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{ad\sqrt{a+b\sec(c+dx)}}}{a(a^2-b^2)}$$

input
```
Int[1/(Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^(3/2)),x]
```

output
```
((-4*b*(a^2 - b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/
2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(a*d*Sqrt[a + b*Sec[c + d*x]]) + (2*
(a^2 - 2*b^2)*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x
]])/(a*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]))/(a*(a^2 -
 b^2)) + (2*b^2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(a*(a^2 - b^2)*d*Sqrt[a +
 b*Sec[c + d*x]])
```

3.658.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 4334
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*
((d*Csc[e + f*x])^n/(a*f*(m + 1)*(a^2 - b^2))), x] + Simp[1/(a*(m + 1)*(a^2
 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a^2*(m + 1)
 - b^2*(m + n + 1) - a*b*(m + 1)*Csc[e + f*x] + b^2*(m + n + 2)*Csc[e + f*x
]^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0] && LtQ[m,
 -1] && IntegersQ[2*m, 2*n]
```

rule 4343
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]])   Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a
, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4345
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]])   Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[
{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

3.658. $\int \frac{1}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}}\,dx$

rule 4523
```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Simp[A/a   I
nt[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Simp[(A*b - a*B)
/(a*d)   Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ
[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

3.658.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 924 vs. $2(256) = 512$.

Time = 7.88 (sec) , antiderivative size = 925, normalized size of antiderivative = 4.32

method	result
default	$\frac{2\left(\sqrt{\frac{a-b}{a+b}}\,a^2(1-\cos(dx+c))^3\csc(dx+c)^3-2\sqrt{\frac{a-b}{a+b}}\,b^2(1-\cos(dx+c))^3\csc(dx+c)^3+\sqrt{-\frac{a(1-\cos(dx+c))^2\csc(dx+c)^2-b(1-\cos(dx+c))^2\,\mathrm{cs}}{a+b}}\right.}{}$

input
```
int(1/sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/d/((a-b)/(a+b))^(1/2)/(a+b)/a^2*(((a-b)/(a+b))^(1/2)*a^2*(1-cos(d*x+c))^
3*csc(d*x+c)^3-2*((a-b)/(a+b))^(1/2)*b^2*(1-cos(d*x+c))^3*csc(d*x+c)^3+(-(
a*(1-cos(d*x+c))^2*csc(d*x+c)^2-b*(1-cos(d*x+c))^2*csc(d*x+c)^2-a-b)/(a+b)
)^(1/2)*((1-cos(d*x+c))^2*csc(d*x+c)^2+1)^(1/2)*EllipticF(((a-b)/(a+b))^(1
/2)*(-cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*a^2+2*(-(a*(1-cos(d*x+c
))^2*csc(d*x+c)^2-b*(1-cos(d*x+c))^2*csc(d*x+c)^2-a-b)/(a+b))^(1/2)*((1-co
s(d*x+c))^2*csc(d*x+c)^2+1)^(1/2)*EllipticF(((a-b)/(a+b))^(1/2)*(-cot(d*x+
c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*a*b-(-(a*(1-cos(d*x+c))^2*csc(d*x+c)^
2-b*(1-cos(d*x+c))^2*csc(d*x+c)^2-a-b)/(a+b))^(1/2)*((1-cos(d*x+c))^2*csc(
d*x+c)^2+1)^(1/2)*EllipticE(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(
-(a+b)/(a-b))^(1/2))*a^2+2*(-(a*(1-cos(d*x+c))^2*csc(d*x+c)^2-b*(1-cos(d*x
+c))^2*csc(d*x+c)^2-a-b)/(a+b))^(1/2)*((1-cos(d*x+c))^2*csc(d*x+c)^2+1)^(1
/2)*EllipticE(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^
(1/2))*b^2-((a-b)/(a+b))^(1/2)*a^2*(-cot(d*x+c)+csc(d*x+c))-2*((a-b)/(a+b)
)^(1/2)*a*b*(-cot(d*x+c)+csc(d*x+c))-2*((a-b)/(a+b))^(1/2)*b^2*(-cot(d*x+c
)+csc(d*x+c)))*((a*(1-cos(d*x+c))^2*csc(d*x+c)^2-b*(1-cos(d*x+c))^2*csc(d*
x+c)^2-a-b)/((1-cos(d*x+c))^2*csc(d*x+c)^2-1))^(1/2)/(a*(1-cos(d*x+c))^2*c
sc(d*x+c)^2-b*(1-cos(d*x+c))^2*csc(d*x+c)^2-a-b)/(-((1-cos(d*x+c))^2*csc(d
*x+c)^2+1)/((1-cos(d*x+c))^2*csc(d*x+c)^2-1))^(1/2)
```

3.658.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.13 (sec) , antiderivative size = 565, normalized size of antiderivative = 2.64

$$\int \frac{1}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}}\,dx = \frac{6\,a^2b^2\sqrt{\frac{a\cos(dx+c)+b}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c) - \sqrt{2}(-5i\,a^2b^2 +}{}$$

input
```
integrate(1/sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
1/3*(6*a^2*b^2*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*sqrt(cos(d*x + c))*
sin(d*x + c) - sqrt(2)*(-5*I*a^2*b^2 + 4*I*b^4 + (-5*I*a^3*b + 4*I*a*b^3)*
cos(d*x + c))*sqrt(a)*weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(
9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x + c) + 2*b)/a)
 - sqrt(2)*(5*I*a^2*b^2 - 4*I*b^4 + (5*I*a^3*b - 4*I*a*b^3)*cos(d*x + c))*
sqrt(a)*weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^
3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a) + 3*sqrt(2)*(
I*a^3*b - 2*I*a*b^3 + (I*a^4 - 2*I*a^2*b^2)*cos(d*x + c))*sqrt(a)*weierstr
assZeta(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassP
Inverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos
(d*x + c) + 3*I*a*sin(d*x + c) + 2*b)/a)) + 3*sqrt(2)*(-I*a^3*b + 2*I*a*b^
3 + (-I*a^4 + 2*I*a^2*b^2)*cos(d*x + c))*sqrt(a)*weierstrassZeta(-4/3*(3*a
^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a
^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a
*sin(d*x + c) + 2*b)/a)))/((a^6 - a^4*b^2)*d*cos(d*x + c) + (a^5*b - a^3*b
^3)*d)
```

3.658.6 Sympy [F]

$$\int \frac{1}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{1}{(a+b\sec(c+dx))^{\frac{3}{2}}\sqrt{\sec(c+dx)}}\,dx$$

input
```
integrate(1/sec(d*x+c)**(1/2)/(a+b*sec(d*x+c))**(3/2),x)
```

output
```
Integral(1/((a + b*sec(c + d*x))**(3/2)*sqrt(sec(c + d*x))), x)
```

3.658.7 Maxima [F]

$$\int \frac{1}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{1}{(b\sec(dx+c)+a)^{\frac{3}{2}}\sqrt{\sec(dx+c)}}\,dx$$

input
```
integrate(1/sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*sec(d*x + c) + a)^(3/2)*sqrt(sec(d*x + c))), x)
```

3.658.8 Giac [F]

$$\int \frac{1}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{1}{(b\sec(dx+c)+a)^{\frac{3}{2}}\sqrt{\sec(dx+c)}}\,dx$$

input
```
integrate(1/sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((b*sec(d*x + c) + a)^(3/2)*sqrt(sec(d*x + c))), x)
```

3.658.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{1}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}\sqrt{\frac{1}{\cos(c+d\,x)}}}\,dx$$

input
```
int(1/((a + b/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(1/2)),x)
```

output
```
int(1/((a + b/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(1/2)), x)
```

3.659 $\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx$

3.659.1 Optimal result 5139
3.659.2 Mathematica [A] (verified) 5140
3.659.3 Rubi [A] (verified) 5140
3.659.4 Maple [B] (verified) 5146
3.659.5 Fricas [C] (verification not implemented) 5147
3.659.6 Sympy [F] 5148
3.659.7 Maxima [F] 5148
3.659.8 Giac [F] 5149
3.659.9 Mupad [F(-1)] 5149

3.659.1 Optimal result

Integrand size = 25, antiderivative size = 289

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx = \frac{2(a^2+8b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{3a^3d\sqrt{a+b\sec(c+dx)}}$$
$$-\frac{2b(5a^2-8b^2)\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+dx)}}{3a^3\left(a^2-b^2\right)d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\sqrt{\sec(c+dx)}}$$
$$+\frac{2b^2\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{2(a^2-4b^2)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{3a^2\left(a^2-b^2\right)d\sqrt{\sec(c+dx)}}$$

output
```
2*b^2*sin(d*x+c)/a/(a^2-b^2)/d/sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)+2/3
*(a^2+8*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin
(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)*se
c(d*x+c)^(1/2)/a^3/d/(a+b*sec(d*x+c))^(1/2)+2/3*(a^2-4*b^2)*sin(d*x+c)*(a+
b*sec(d*x+c))^(1/2)/a^2/(a^2-b^2)/d/sec(d*x+c)^(1/2)-2/3*b*(5*a^2-8*b^2)*(
cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c
),2^(1/2)*(a/(a+b))^(1/2))*(a+b*sec(d*x+c))^(1/2)/a^3/(a^2-b^2)/d/((b+a*co
s(d*x+c))/(a+b))^(1/2)/sec(d*x+c)^(1/2)
```

3.659.2 Mathematica [A] (verified)

Time = 0.95 (sec) , antiderivative size = 203, normalized size of antiderivative = 0.70

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx = \frac{2\sqrt{\sec(c+dx)}\left(b(-5a^3-5a^2b+8ab^2+8b^3)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}E\left(\frac{1}{2}\right.\right.}{}$$

input
```
Integrate[1/(Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^(3/2)),x]
```

output
```
(2*Sqrt[Sec[c + d*x]]*(b*(-5*a^3 - 5*a^2*b + 8*a*b^2 + 8*b^3)*Sqrt[(b + a*
Cos[c + d*x])/(a + b)]*EllipticE[(c + d*x)/2, (2*a)/(a + b)] + (a^4 + 7*a^
2*b^2 - 8*b^4)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (
2*a)/(a + b)] + a*(b*(a^2 - 4*b^2) + a*(a^2 - b^2)*Cos[c + d*x])*Sin[c + d
*x]))/(3*a^3*(a - b)*(a + b)*d*Sqrt[a + b*Sec[c + d*x]])
```

3.659.3 Rubi [A] (verified)

Time = 2.16 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.02, number of steps used = 19, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.760$, Rules used = {3042, 4334, 27, 3042, 4592, 27, 3042, 4523, 3042, 4343, 3042, 3134, 3042, 3132, 4345, 3042, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \; 4334$$

$$\frac{2b^2\sin(c+dx)}{ad\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}} - \frac{2\int -\frac{a^2-b\sec(c+dx)a-4b^2+2b^2\sec^2(c+dx)}{2\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}dx}{a\left(a^2-b^2\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{a^2-b\sec(c+dx)a-4b^2+2b^2\sec^2(c+dx)}{\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}dx}{a\left(a^2-b^2\right)}+\frac{2b^2\sin(c+dx)}{ad\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}$$

$\Big\downarrow$ 3042

$$\frac{\int \frac{a^2-b\csc\left(c+dx+\frac{\pi}{2}\right)a-4b^2+2b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a\left(a^2-b^2\right)}+\frac{2b^2\sin(c+dx)}{ad\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}$$

$\Big\downarrow$ 4592

$$\frac{\frac{2\left(a^2-4b^2\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}-\frac{2\int \frac{b\left(5a^2-8b^2\right)-a\left(a^2+2b^2\right)\sec(c+dx)}{2\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{3a}}{a\left(a^2-b^2\right)}+\frac{2b^2\sin(c+dx)}{ad\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}$$

$\Big\downarrow$ 27

$$\frac{\frac{2\left(a^2-4b^2\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}-\frac{\int \frac{b\left(5a^2-8b^2\right)-a\left(a^2+2b^2\right)\sec(c+dx)}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{3a}}{a\left(a^2-b^2\right)}+\frac{2b^2\sin(c+dx)}{ad\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}$$

$\Big\downarrow$ 3042

$$\frac{\frac{2\left(a^2-4b^2\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}-\frac{\int \frac{b\left(5a^2-8b^2\right)-a\left(a^2+2b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{3a}}{a\left(a^2-b^2\right)}+\frac{2b^2\sin(c+dx)}{ad\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}$$

$\Big\downarrow$ 4523

$$\frac{\frac{2\left(a^2-4b^2\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{b\left(5a^2-8b^2\right)\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}dx}{a}-\frac{\left(a^4+7a^2b^2-8b^4\right)\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}dx}{a}}{3a}}{a\left(a^2-b^2\right)}+\frac{2b^2\sin(c+dx)}{ad\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}$$

$\Big\downarrow$ 3042

3.659. $\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}dx$

$$\frac{\frac{2\left(a^2-4b^2\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{b\left(5a^2-8b^2\right)\int\frac{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}-\frac{\left(a^4+7a^2b^2-8b^4\right)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}}{3a}}{a\left(a^2-b^2\right)}+\frac{2b^2\sin(c+dx)}{ad\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}$$

$\downarrow$ 4343

$$\frac{\frac{2\left(a^2-4b^2\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{b\left(5a^2-8b^2\right)\sqrt{a+b\sec(c+dx)}\int\sqrt{b+a\cos(c+dx)}dx}{a\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}-\frac{\left(a^4+7a^2b^2-8b^4\right)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}}{3a}}{a\left(a^2-b^2\right)}+\frac{2b^2\sin(c+dx)}{ad\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}$$

$\downarrow$ 3042

$$\frac{\frac{2\left(a^2-4b^2\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{b\left(5a^2-8b^2\right)\sqrt{a+b\sec(c+dx)}\int\sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{a\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}-\frac{\left(a^4+7a^2b^2-8b^4\right)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}}{3a}}{a\left(a^2-b^2\right)}+\frac{2b^2\sin(c+dx)}{ad\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}$$

$\downarrow$ 3134

$$\frac{\frac{2\left(a^2-4b^2\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{b\left(5a^2-8b^2\right)\sqrt{a+b\sec(c+dx)}\int\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}dx}{a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{\left(a^4+7a^2b^2-8b^4\right)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}}{3a}}{a\left(a^2-b^2\right)}+\frac{2b^2\sin(c+dx)}{ad\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}$$

$\downarrow$ 3042

3.659. $\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx$

$$\frac{\frac{2\left(a^2-4b^2\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{b\left(5a^2-8b^2\right)\sqrt{a+b\sec(c+dx)}\int\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}dx}{a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{\left(a^4+7a^2b^2-8b^4\right)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}}{3a}}{a\left(a^2-b^2\right)}+\frac{2b^2\sin(c+dx)}{ad\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}$$

$\downarrow$ 3132

$$\frac{\frac{2\left(a^2-4b^2\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{2b\left(5a^2-8b^2\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{\left(a^4+7a^2b^2-8b^4\right)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}}{3a}}{a\left(a^2-b^2\right)}+\frac{2b^2\sin(c+dx)}{ad\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}$$

$\downarrow$ 4345

$$\frac{\frac{2\left(a^2-4b^2\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{2b\left(5a^2-8b^2\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{\left(a^4+7a^2b^2-8b^4\right)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{1}{\sqrt{b+a\cos(c+d}}}{a\sqrt{a+b\sec(c+dx)}}}{3a}}{a\left(a^2-b^2\right)}$$

$$\frac{2b^2\sin(c+dx)}{ad\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}$$

$\downarrow$ 3042

$$\frac{\frac{2\left(a^2-4b^2\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{2b\left(5a^2-8b^2\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{\left(a^4+7a^2b^2-8b^4\right)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{1}{\sqrt{b+a\sin\left(c+d}}}{a\sqrt{a+b\sec(c+dx)}}}{3a}}{a\left(a^2-b^2\right)}$$

$$\frac{2b^2\sin(c+dx)}{ad\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}$$

$\downarrow$ 3142

3.659. $\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx$

$$\frac{\frac{2(a^2-4b^2)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{2b(5a^2-8b^2)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{(a^4+7a^2b^2-8b^4)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\cos}{}}}}{a\sqrt{a+b\sec(c+dx)}}}{3a}}{a(a^2-b^2)}$$

$$\frac{2b^2\sin(c+dx)}{ad(a^2-b^2)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}$$

$\downarrow$ 3042

$$\frac{\frac{2(a^2-4b^2)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{2b(5a^2-8b^2)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{(a^4+7a^2b^2-8b^4)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\sin}{}}}}{a\sqrt{a+b\sec(c+dx)}}}{3a}}{a(a^2-b^2)}$$

$$\frac{2b^2\sin(c+dx)}{ad(a^2-b^2)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}$$

$\downarrow$ 3140

$$\frac{2b^2\sin(c+dx)}{ad(a^2-b^2)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}+$$

$$\frac{\frac{2(a^2-4b^2)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{2b(5a^2-8b^2)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{2(a^4+7a^2b^2-8b^4)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\mathrm{EllipticF}\left(\frac{1}{2}(c\right.}{ad\sqrt{a+b\sec(c+dx)}}}{3a}}{a(a^2-b^2)}$$

input
```
Int[1/(Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^(3/2)),x]
```

output
```
(2*b^2*Sin[c + d*x])/(a*(a^2 - b^2)*d*Sqrt[Sec[c + d*x]]*Sqrt[a + b*Sec[c
+ d*x]]) + (-1/3*((-2*(a^4 + 7*a^2*b^2 - 8*b^4)*Sqrt[(b + a*Cos[c + d*x])/
(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(a*d*Sq
rt[a + b*Sec[c + d*x]]) + (2*b*(5*a^2 - 8*b^2)*EllipticE[(c + d*x)/2, (2*a
)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(a*d*Sqrt[(b + a*Cos[c + d*x])/(a + b
)]*Sqrt[Sec[c + d*x]]))/a + (2*(a^2 - 4*b^2)*Sqrt[a + b*Sec[c + d*x]]*Sin[
c + d*x])/(3*a*d*Sqrt[Sec[c + d*x]]))/(a*(a^2 - b^2))
```

3.659. $\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx$

3.659.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 4334
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*
((d*Csc[e + f*x])^n/(a*f*(m + 1)*(a^2 - b^2))), x] + Simp[1/(a*(m + 1)*(a^2
 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a^2*(m + 1)
 - b^2*(m + n + 1) - a*b*(m + 1)*Csc[e + f*x] + b^2*(m + n + 2)*Csc[e + f*x
]^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0] && LtQ[m,
 -1] && IntegersQ[2*m, 2*n]
```

3.659. $\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx$

rule 4343
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]])   Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a
, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4345
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]])   Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[
{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4523
```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Simp[A/a   I
nt[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Simp[(A*b - a*B)
/(a*d)   Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ
[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

rule 4592
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Simp[1/(a*d*n)   Int[(a + b*Csc[e + f*x])^m
*(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*
Csc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

3.659.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1678 vs. $2(321) = 642$.

Time = 9.09 (sec) , antiderivative size = 1679, normalized size of antiderivative = 5.81

method	result	size
default	Expression too large to display	1679

input
```
int(1/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/3/d/a^3/(a+b)/((a-b)/(a+b))^(1/2)*(a+b*sec(d*x+c))^(1/2)/(b+a*cos(d*x+c)
)/sec(d*x+c)^(3/2)/(cos(d*x+c)+1)*(5*(1/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+
a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE(((a-b)/(a+b))^(1/2)*(cot(d*x
+c)-csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*a^2*b*cos(d*x+c)-8*EllipticE(((a-b)/
(a+b))^(1/2)*(cot(d*x+c)-csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*(1/(cos(d*x+c)+
1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*b^3*cos(d*x+c)-(
1/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*El
lipticF(((a-b)/(a+b))^(1/2)*(cot(d*x+c)-csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*
a^3*cos(d*x+c)-6*(1/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d
*x+c)+1))^(1/2)*EllipticF(((a-b)/(a+b))^(1/2)*(cot(d*x+c)-csc(d*x+c)),(-(a
+b)/(a-b))^(1/2))*a^2*b*cos(d*x+c)-8*(1/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+
a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF(((a-b)/(a+b))^(1/2)*(cot(d*x
+c)-csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*a*b^2*cos(d*x+c)+10*(1/(cos(d*x+c)+1
))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE(((a-b)/
(a+b))^(1/2)*(cot(d*x+c)-csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*a^2*b-16*(1/(co
s(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ellipti
cE(((a-b)/(a+b))^(1/2)*(cot(d*x+c)-csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*b^3-2
*(1/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*
EllipticF(((a-b)/(a+b))^(1/2)*(cot(d*x+c)-csc(d*x+c)),(-(a+b)/(a-b))^(1/2)
)*a^3-12*(1/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)...
```

3.659.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.14 (sec) , antiderivative size = 632, normalized size of antiderivative = 2.19

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx =$$

$$-\frac{\sqrt{2}(3i\,a^4b+16i\,a^2b^3-16i\,b^5+(3i\,a^5+16i\,a^3b^2-16i\,ab^4)\cos(dx+c))\sqrt{a}\text{weierstrassPInverse}\left(-\frac{4\,(3\,a^2-4}{3\,a^2}\right.}{}$$

input
```
integrate(1/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
-1/9*(sqrt(2)*(3*I*a^4*b + 16*I*a^2*b^3 - 16*I*b^5 + (3*I*a^5 + 16*I*a^3*b
^2 - 16*I*a*b^4)*cos(d*x + c))*sqrt(a)*weierstrassPInverse(-4/3*(3*a^2 - 4
*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d
*x + c) + 2*b)/a) + sqrt(2)*(-3*I*a^4*b - 16*I*a^2*b^3 + 16*I*b^5 + (-3*I*
a^5 - 16*I*a^3*b^2 + 16*I*a*b^4)*cos(d*x + c))*sqrt(a)*weierstrassPInverse
(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x +
c) - 3*I*a*sin(d*x + c) + 2*b)/a) - 3*sqrt(2)*(-5*I*a^3*b^2 + 8*I*a*b^4 +
(-5*I*a^4*b + 8*I*a^2*b^3)*cos(d*x + c))*sqrt(a)*weierstrassZeta(-4/3*(3*a
^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a
^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a
*sin(d*x + c) + 2*b)/a)) - 3*sqrt(2)*(5*I*a^3*b^2 - 8*I*a*b^4 + (5*I*a^4*b
 - 8*I*a^2*b^3)*cos(d*x + c))*sqrt(a)*weierstrassZeta(-4/3*(3*a^2 - 4*b^2)
/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)
/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x +
c) + 2*b)/a)) - 6*((a^5 - a^3*b^2)*cos(d*x + c)^2 + (a^4*b - 4*a^2*b^3)*co
s(d*x + c))*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(
d*x + c)))/((a^7 - a^5*b^2)*d*cos(d*x + c) + (a^6*b - a^4*b^3)*d)
```

3.659.6 Sympy [F]

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{1}{(a+b\sec(c+dx))^{\frac{3}{2}}\sec^{\frac{3}{2}}(c+dx)}\,dx$$

input
```
integrate(1/sec(d*x+c)**(3/2)/(a+b*sec(d*x+c))**(3/2),x)
```

output
```
Integral(1/((a + b*sec(c + d*x))**(3/2)*sec(c + d*x)**(3/2)), x)
```

3.659.7 Maxima [F]

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{1}{(b\sec(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*sec(d*x + c) + a)^(3/2)*sec(d*x + c)^(3/2)), x)
```

3.659.8 Giac [F]

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{1}{(b\sec(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((b*sec(d*x + c) + a)^(3/2)*sec(d*x + c)^(3/2)), x)
```

3.659.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{1}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

input
```
int(1/((a + b/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(3/2)),x)
```

output
```
int(1/((a + b/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(3/2)), x)
```

3.660 $\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx$

3.660.1 Optimal result . . . 5150
3.660.2 Mathematica [A] (verified) . . . 5151
3.660.3 Rubi [A] (verified) . . . 5151
3.660.4 Maple [B] (verified) . . . 5158
3.660.5 Fricas [C] (verification not implemented) . . . 5159
3.660.6 Sympy [F(-1)] . . . 5160
3.660.7 Maxima [F] . . . 5160
3.660.8 Giac [F] . . . 5161
3.660.9 Mupad [F(-1)] . . . 5161

3.660.1 Optimal result

Integrand size = 25, antiderivative size = 360

$$\begin{aligned}\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx &= \\ &-\frac{8b(a^2+4b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{5a^4d\sqrt{a+b\sec(c+dx)}} \\ &+\frac{2(3a^4+8a^2b^2-16b^4)\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+dx)}}{5a^4\,(a^2-b^2)\,d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\sqrt{\sec(c+dx)}} \\ &+\frac{2b^2\sin(c+dx)}{a\,(a^2-b^2)\,d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}} \\ &+\frac{2(a^2-6b^2)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{5a^2\,(a^2-b^2)\,d\sec^{\frac{3}{2}}(c+dx)} \\ &-\frac{2b(3a^2-8b^2)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{5a^3\,(a^2-b^2)\,d\sqrt{\sec(c+dx)}}\end{aligned}$$

output
```
2*b^2*sin(d*x+c)/a/(a^2-b^2)/d/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(1/2)-8/5
*b*(a^2+4*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(s
in(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)*
sec(d*x+c)^(1/2)/a^4/d/(a+b*sec(d*x+c))^(1/2)+2/5*(a^2-6*b^2)*sin(d*x+c)*(
a+b*sec(d*x+c))^(1/2)/a^2/(a^2-b^2)/d/sec(d*x+c)^(3/2)-2/5*b*(3*a^2-8*b^2)
*sin(d*x+c)*(a+b*sec(d*x+c))^(1/2)/a^3/(a^2-b^2)/d/sec(d*x+c)^(1/2)+2/5*(3
*a^4+8*a^2*b^2-16*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ell
ipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*(a+b*sec(d*x+c))^(1/2)/
a^4/(a^2-b^2)/d/((b+a*cos(d*x+c))/(a+b))^(1/2)/sec(d*x+c)^(1/2)
```

3.660.2 Mathematica [A] (verified)

Time = 1.37 (sec) , antiderivative size = 250, normalized size of antiderivative = 0.69

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx = \frac{\sqrt{\sec(c+dx)}\left(4(3a^5+3a^4b+8a^3b^2+8a^2b^3-16ab^4-16b^5)\sqrt{\right.}}{}$$

input
```
Integrate[1/(Sec[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^(3/2)),x]
```

output
```
(Sqrt[Sec[c + d*x]]*(4*(3*a^5 + 3*a^4*b + 8*a^3*b^2 + 8*a^2*b^3 - 16*a*b^4
 - 16*b^5)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticE[(c + d*x)/2, (2*a)
/(a + b)] - 16*b*(a^4 + 3*a^2*b^2 - 4*b^4)*Sqrt[(b + a*Cos[c + d*x])/(a +
b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)] + 2*a*(a^4 - 7*a^2*b^2 + 16*b^4
- 4*a*b*(a^2 - b^2)*Cos[c + d*x] + (a^4 - a^2*b^2)*Cos[2*(c + d*x)])*Sin[c
 + d*x]))/(10*a^4*(a - b)*(a + b)*d*Sqrt[a + b*Sec[c + d*x]])
```

3.660.3 Rubi [A] (verified)

Time = 2.77 (sec) , antiderivative size = 359, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.880$, Rules used = {3042, 4334, 27, 3042, 4592, 27, 3042, 4592, 27, 3042, 4523, 3042, 4343, 3042, 3134, 3042, 3132, 4345, 3042, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \; 4334$$

$$\frac{2b^2\sin(c+dx)}{ad\left(a^2-b^2\right)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}-\frac{2\int -\frac{a^2-b\sec(c+dx)a-6b^2+4b^2\sec^2(c+dx)}{2\sec^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}dx}{a\left(a^2-b^2\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{a^2-b\sec(c+dx)a-6b^2+4b^2\sec^2(c+dx)}{\sec^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}dx}{a\left(a^2-b^2\right)}+\frac{2b^2\sin(c+dx)}{ad\left(a^2-b^2\right)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}$$

$$\Big\downarrow \; 3042$$

$$\frac{\int \frac{a^2-b\csc\left(c+dx+\frac{\pi}{2}\right)a-6b^2+4b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a\left(a^2-b^2\right)}+\frac{2b^2\sin(c+dx)}{ad\left(a^2-b^2\right)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}$$

$$\Big\downarrow \; 4592$$

$$\frac{\frac{2\left(a^2-6b^2\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{2\int \frac{-2b\left(a^2-6b^2\right)\sec^2(c+dx)-a\left(3a^2+2b^2\right)\sec(c+dx)+3b\left(3a^2-8b^2\right)}{2\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}dx}{5a}}{a\left(a^2-b^2\right)}+\frac{2b^2\sin(c+dx)}{ad\left(a^2-b^2\right)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{2\left(a^2-6b^2\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{\int \frac{-2b\left(a^2-6b^2\right)\sec^2(c+dx)-a\left(3a^2+2b^2\right)\sec(c+dx)+3b\left(3a^2-8b^2\right)}{\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}dx}{5a}}{a\left(a^2-b^2\right)}+\frac{2b^2\sin(c+dx)}{ad\left(a^2-b^2\right)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}$$

$$\Big\downarrow \; 3042$$

3.660. $\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx$

$$\frac{\frac{2\left(a^2-6b^2\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{\int\frac{-2b\left(a^2-6b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2-a\left(3a^2+2b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)+3b\left(3a^2-8b^2\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{5a}}{a\left(a^2-b^2\right)}+\frac{2b^2\sin(c+dx)}{ad\left(a^2-b^2\right)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}$$

$\downarrow$ 4592

$$\frac{\frac{2\left(a^2-6b^2\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{\frac{2b\left(3a^2-8b^2\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{ad\sqrt{\sec(c+dx)}}-\frac{2\int\frac{3\left(3a^4+8b^2a^2-b\left(a^2+4b^2\right)\sec(c+dx)a-16b^4\right)}{2\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{3a}}{5a}}{a\left(a^2-b^2\right)}+\frac{2b^2\sin(c+dx)}{ad\left(a^2-b^2\right)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}$$

$\downarrow$ 27

$$\frac{\frac{2\left(a^2-6b^2\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{\frac{2b\left(3a^2-8b^2\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{ad\sqrt{\sec(c+dx)}}-\frac{\int\frac{3a^4+8b^2a^2-b\left(a^2+4b^2\right)\sec(c+dx)a-16b^4}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{a}}{5a}}{a\left(a^2-b^2\right)}+\frac{2b^2\sin(c+dx)}{ad\left(a^2-b^2\right)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}$$

$\downarrow$ 3042

$$\frac{\frac{2\left(a^2-6b^2\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{\frac{2b\left(3a^2-8b^2\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{ad\sqrt{\sec(c+dx)}}-\frac{\int\frac{3a^4+8b^2a^2-b\left(a^2+4b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)a-16b^4}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}}{5a}}{a\left(a^2-b^2\right)}+\frac{2b^2\sin(c+dx)}{ad\left(a^2-b^2\right)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}$$

$\downarrow$ 4523

$$\frac{\frac{2\left(a^2-6b^2\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{\frac{2b\left(3a^2-8b^2\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{ad\sqrt{\sec(c+dx)}}-\frac{\frac{\left(3a^4+8a^2b^2-16b^4\right)\int\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}dx}{a}-\frac{4b\left(a^4+3a^2b^2-4b^4\right)\int\frac{\sqrt{\ldots}}{\sqrt{a}\ldots}}{a}}{a}}{5a}}{a\left(a^2-b^2\right)}+\frac{2b^2\sin(c+dx)}{ad\left(a^2-b^2\right)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}$$

$\downarrow$ 3042

3.660. $\int\frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}dx$

$$\frac{\frac{2\left(a^2-6b^2\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{\frac{2b\left(3a^2-8b^2\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{ad\sqrt{\sec(c+dx)}}-\frac{\frac{\left(3a^4+8a^2b^2-16b^4\right)\int\frac{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}-\frac{4b\left(a^4+3a^2b^2-4b^4\right)\int}{a}}{5a}}{a\left(a^2-b^2\right)}$$

$$\frac{2b^2\sin(c+dx)}{ad\left(a^2-b^2\right)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}$$

$\downarrow$ 4343

$$\frac{\frac{2\left(a^2-6b^2\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{\frac{2b\left(3a^2-8b^2\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{ad\sqrt{\sec(c+dx)}}-\frac{\frac{\left(3a^4+8a^2b^2-16b^4\right)\sqrt{a+b\sec(c+dx)}\int\sqrt{b+a\cos(c+dx)}dx}{a\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}-\frac{4b\left(a^4+3a^2\right.}{a}}{5a}}{a\left(a^2-b^2\right)}$$

$$\frac{2b^2\sin(c+dx)}{ad\left(a^2-b^2\right)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}$$

$\downarrow$ 3042

$$\frac{\frac{2\left(a^2-6b^2\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{\frac{2b\left(3a^2-8b^2\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{ad\sqrt{\sec(c+dx)}}-\frac{\frac{\left(3a^4+8a^2b^2-16b^4\right)\sqrt{a+b\sec(c+dx)}\int\sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{a\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}-\frac{4b\left(a^4\right.}{a}}{5a}}{a\left(a^2-b^2\right)}$$

$$\frac{2b^2\sin(c+dx)}{ad\left(a^2-b^2\right)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}$$

$\downarrow$ 3134

$$\frac{\frac{2\left(a^2-6b^2\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{\frac{2b\left(3a^2-8b^2\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{ad\sqrt{\sec(c+dx)}}-\frac{\frac{\left(3a^4+8a^2b^2-16b^4\right)\sqrt{a+b\sec(c+dx)}\int\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}dx}{a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{4b\left(a^4\right.}{a}}{5a}}{a\left(a^2-b^2\right)}$$

$$\frac{2b^2\sin(c+dx)}{ad\left(a^2-b^2\right)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}$$

$\downarrow$ 3042

3.660. $\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx$

$$\frac{\frac{2\left(a^2-6b^2\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{\frac{2b\left(3a^2-8b^2\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{ad\sqrt{\sec(c+dx)}}-\frac{\frac{\left(3a^4+8a^2b^2-16b^4\right)\sqrt{a+b\sec(c+dx)}\int\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}dx}{a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{4b}{}}{a}}{5a}}{a\left(a^2-b^2\right)}$$

$$\frac{2b^2\sin(c+dx)}{ad\left(a^2-b^2\right)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}$$

$\downarrow$ 3132

$$\frac{\frac{2\left(a^2-6b^2\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{\frac{2b\left(3a^2-8b^2\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{ad\sqrt{\sec(c+dx)}}-\frac{\frac{2\left(3a^4+8a^2b^2-16b^4\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{4b\left(a^4+3a^2b\right.}{}}{a}}{5a}}{a\left(a^2-b^2\right)}$$

$$\frac{2b^2\sin(c+dx)}{ad\left(a^2-b^2\right)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}$$

$\downarrow$ 4345

$$\frac{\frac{2\left(a^2-6b^2\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{\frac{2b\left(3a^2-8b^2\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{ad\sqrt{\sec(c+dx)}}-\frac{\frac{2\left(3a^4+8a^2b^2-16b^4\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{4b\left(a^4+3a^2b\right.}{}}{a}}{5a}}{a\left(a^2-b^2\right)}$$

$$\frac{2b^2\sin(c+dx)}{ad\left(a^2-b^2\right)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}$$

$\downarrow$ 3042

$$\frac{\frac{2\left(a^2-6b^2\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{\frac{2b\left(3a^2-8b^2\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{ad\sqrt{\sec(c+dx)}}-\frac{\frac{2\left(3a^4+8a^2b^2-16b^4\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{4b\left(a^4+3a^2b\right.}{}}{a}}{5a}}{a\left(a^2-b^2\right)}$$

$$\frac{2b^2\sin(c+dx)}{ad\left(a^2-b^2\right)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}$$

$\downarrow$ 3142

3.660. $\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx$

$$\frac{\frac{2(a^2-6b^2)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{\frac{2b(3a^2-8b^2)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{ad\sqrt{\sec(c+dx)}}-\frac{\frac{2(3a^4+8a^2b^2-16b^4)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{4b(a^4+3a^2b}{a}}{5a}}{a(a^2-b^2)}$$

$$\frac{2b^2\sin(c+dx)}{ad(a^2-b^2)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}$$

$\downarrow$ 3042

$$\frac{\frac{2(a^2-6b^2)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{\frac{2b(3a^2-8b^2)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{ad\sqrt{\sec(c+dx)}}-\frac{\frac{2(3a^4+8a^2b^2-16b^4)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{4b(a^4+3a^2b}{a}}{5a}}{a(a^2-b^2)}$$

$$\frac{2b^2\sin(c+dx)}{ad(a^2-b^2)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}$$

$\downarrow$ 3140

$$\frac{2b^2\sin(c+dx)}{ad(a^2-b^2)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}+$$

$$\frac{\frac{2(a^2-6b^2)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{\frac{2b(3a^2-8b^2)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{ad\sqrt{\sec(c+dx)}}-\frac{\frac{2(3a^4+8a^2b^2-16b^4)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{8b(a^4+3a^2b}{a}}{5a}}{a(a^2-b^2)}$$

input
```
Int[1/(Sec[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^(3/2)),x]
```

output
```
(2*b^2*Sin[c + d*x])/(a*(a^2 - b^2)*d*Sec[c + d*x]^(3/2)*Sqrt[a + b*Sec[c
+ d*x]]) + ((2*(a^2 - 6*b^2)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(5*a*d
*Sec[c + d*x]^(3/2)) - (-(((-8*b*(a^4 + 3*a^2*b^2 - 4*b^4)*Sqrt[(b + a*Cos
[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x
]])/(a*d*Sqrt[a + b*Sec[c + d*x]]) + (2*(3*a^4 + 8*a^2*b^2 - 16*b^4)*Ellip
ticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(a*d*Sqrt[(b +
a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]))/a) + (2*b*(3*a^2 - 8*b^2)*Sq
rt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(a*d*Sqrt[Sec[c + d*x]]))/(5*a))/(a*(
a^2 - b^2))
```

3.660. $\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx$

3.660.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 4334
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*
((d*Csc[e + f*x])^n/(a*f*(m + 1)*(a^2 - b^2))), x] + Simp[1/(a*(m + 1)*(a^2
 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a^2*(m + 1)
 - b^2*(m + n + 1) - a*b*(m + 1)*Csc[e + f*x] + b^2*(m + n + 2)*Csc[e + f*x
]^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0] && LtQ[m,
 -1] && IntegersQ[2*m, 2*n]
```

3.660. $\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx$

rule 4343
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]])   Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a
, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4345
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]])   Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[
{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4523
```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Simp[A/a   I
nt[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Simp[(A*b - a*B)
/(a*d)   Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ
[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

rule 4592
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Simp[1/(a*d*n)   Int[(a + b*Csc[e + f*x])^m
*(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*
Csc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

3.660.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2386 vs. $2(386) = 772$.

Time = 10.07 (sec) , antiderivative size = 2387, normalized size of antiderivative = 6.63

method	result	size
default	Expression too large to display	2387

input
```
int(1/sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/5/d/a^4/(a+b)/((a-b)/(a+b))^(1/2)*(a+b*sec(d*x+c))^(1/2)/(b+a*cos(d*x+c)
)/sec(d*x+c)^(5/2)/(cos(d*x+c)+1)*(16*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c
)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE(((a-b)/(a+b))^(1/2)*(-cot(d
*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*a^2*b^2*sec(d*x+c)-16*(1/(cos(d*x+
c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE(((a
-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*b^4-3*(1/(
cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ellip
ticF(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*a^
4+((a-b)/(a+b))^(1/2)*a^3*b*tan(d*x+c)+6*((a-b)/(a+b))^(1/2)*a^2*b^2*tan(d
*x+c)+8*((a-b)/(a+b))^(1/2)*a*b^3*tan(d*x+c)+16*((a-b)/(a+b))^(1/2)*b^4*ta
n(d*x+c)*sec(d*x+c)+3*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(
cos(d*x+c)+1))^(1/2)*EllipticE(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)
),(-(a+b)/(a-b))^(1/2))*a^4+3*((a-b)/(a+b))^(1/2)*a^4*tan(d*x+c)+((a-b)/(a
+b))^(1/2)*a^4*sin(d*x+c)-8*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2
)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(
d*x+c)),(-(a+b)/(a-b))^(1/2))*a^3*b*sec(d*x+c)-24*(1/(a+b)*(b+a*cos(d*x+c)
)/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF(((a-b)/(a+b))^(
1/2)*(-cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*a^2*b^2*sec(d*x+c)+8*(
1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*El
lipticE(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/...
```

3.660.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.16 (sec) , antiderivative size = 692, normalized size of antiderivative = 1.92

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx =$$

$$-\frac{\sqrt{2}(-9i\,a^4b^2-28i\,a^2b^4+32i\,b^6+(-9i\,a^5b-28i\,a^3b^3+32i\,ab^5)\cos(dx+c))\sqrt{a}\text{weierstrassPInverse}\left(-\frac{4}{}\right.}{}$$

input
```
integrate(1/sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
-1/15*(sqrt(2)*(-9*I*a^4*b^2 - 28*I*a^2*b^4 + 32*I*b^6 + (-9*I*a^5*b - 28*
I*a^3*b^3 + 32*I*a*b^5)*cos(d*x + c))*sqrt(a)*weierstrassPInverse(-4/3*(3*
a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*
a*sin(d*x + c) + 2*b)/a) + sqrt(2)*(9*I*a^4*b^2 + 28*I*a^2*b^4 - 32*I*b^6
+ (9*I*a^5*b + 28*I*a^3*b^3 - 32*I*a*b^5)*cos(d*x + c))*sqrt(a)*weierstras
sPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*c
os(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a) - 3*sqrt(2)*(3*I*a^5*b + 8*I*a^
3*b^3 - 16*I*a*b^5 + (3*I*a^6 + 8*I*a^4*b^2 - 16*I*a^2*b^4)*cos(d*x + c))*
sqrt(a)*weierstrassZeta(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a
^3, weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a
^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x + c) + 2*b)/a)) - 3*sqrt(2)*(-3*
I*a^5*b - 8*I*a^3*b^3 + 16*I*a*b^5 + (-3*I*a^6 - 8*I*a^4*b^2 + 16*I*a^2*b^
4)*cos(d*x + c))*sqrt(a)*weierstrassZeta(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9
*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9
*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a))
 - 6*((a^6 - a^4*b^2)*cos(d*x + c)^3 - 2*(a^5*b - a^3*b^3)*cos(d*x + c)^2
- (3*a^4*b^2 - 8*a^2*b^4)*cos(d*x + c))*sqrt((a*cos(d*x + c) + b)/cos(d*x
+ c))*sin(d*x + c)/sqrt(cos(d*x + c)))/((a^8 - a^6*b^2)*d*cos(d*x + c) + (
a^7*b - a^5*b^3)*d)
```

3.660.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/sec(d*x+c)**(5/2)/(a+b*sec(d*x+c))**(3/2),x)
```

output
```
Timed out
```

3.660.7 Maxima [F]

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{1}{(b\sec(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*sec(d*x + c) + a)^(3/2)*sec(d*x + c)^(5/2)), x)
```

3.660.8 Giac [F]

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{1}{(b\sec(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((b*sec(d*x + c) + a)^(3/2)*sec(d*x + c)^(5/2)), x)
```

3.660.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{1}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

input
```
int(1/((a + b/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(5/2)),x)
```

output
```
int(1/((a + b/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(5/2)), x)
```

3.661 $\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$

3.661.1 Optimal result . . . 5162
3.661.2 Mathematica [C] (verified) . . . 5163
3.661.3 Rubi [A] (verified) . . . 5164
3.661.4 Maple [C] (verified) . . . 5173
3.661.5 Fricas [F(-1)] . . . 5174
3.661.6 Sympy [F(-1)] . . . 5174
3.661.7 Maxima [F] . . . 5174
3.661.8 Giac [F] . . . 5175
3.661.9 Mupad [F(-1)] . . . 5175

3.661.1 Optimal result

Integrand size = 25, antiderivative size = 458

$$\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \frac{(5a^2-3b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{3b^2(a^2-b^2)\,d\sqrt{a+b\sec(c+dx)}}$$
$$-\frac{5a\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{b^3d\sqrt{a+b\sec(c+dx)}}$$
$$-\frac{(15a^4-26a^2b^2+3b^4)\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+dx)}}{3b^3(a^2-b^2)^2\,d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\sqrt{\sec(c+dx)}}$$
$$-\frac{2a^2\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{3b(a^2-b^2)\,d(a+b\sec(c+dx))^{3/2}}-\frac{2a^2(5a^2-9b^2)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3b^2(a^2-b^2)^2\,d\sqrt{a+b\sec(c+dx)}}$$
$$+\frac{(15a^4-26a^2b^2+3b^4)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{3b^3(a^2-b^2)^2\,d}$$

output
```
-2/3*a^2*sec(d*x+c)^(5/2)*sin(d*x+c)/b/(a^2-b^2)/d/(a+b*sec(d*x+c))^(3/2)-
2/3*a^2*(5*a^2-9*b^2)*sec(d*x+c)^(3/2)*sin(d*x+c)/b^2/(a^2-b^2)^2/d/(a+b*s
ec(d*x+c))^(1/2)+1/3*(5*a^2-3*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*
x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d
*x+c))/(a+b))^(1/2)*sec(d*x+c)^(1/2)/b^2/(a^2-b^2)/d/(a+b*sec(d*x+c))^(1/2
)-5*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d
*x+1/2*c),2,2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)*sec(d*
x+c)^(1/2)/b^3/d/(a+b*sec(d*x+c))^(1/2)-1/3*(15*a^4-26*a^2*b^2+3*b^4)*(cos
(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2
^(1/2)*(a/(a+b))^(1/2))*(a+b*sec(d*x+c))^(1/2)/b^3/(a^2-b^2)^2/d/((b+a*cos
(d*x+c))/(a+b))^(1/2)/sec(d*x+c)^(1/2)+1/3*(15*a^4-26*a^2*b^2+3*b^4)*sin(d
*x+c)*sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2)/b^3/(a^2-b^2)^2/d
```

3.661.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 5.78 (sec) , antiderivative size = 561, normalized size of antiderivative = 1.22

$$\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \frac{\sec^{\frac{5}{2}}(c+dx)\left(-\frac{a(b+a\cos(c+dx))^{5/2}\left(\frac{8ab\left(5a^2-9b^2\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{\sqrt{b+a\cos(c+dx)}}+\frac{2\left(45\right.}{}\right.}{}\right.}{}$$

input
```
Integrate[Sec[c + d*x]^(9/2)/(a + b*Sec[c + d*x])^(5/2),x]
```

output
```
(Sec[c + d*x]^(5/2)*(-((a*(b + a*Cos[c + d*x])^(5/2)*((8*a*b*(5*a^2 - 9*b^
2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)
])/Sqrt[b + a*Cos[c + d*x]] + (2*(45*a^4 - 86*a^2*b^2 + 33*b^4)*Sqrt[(b +
a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)])/Sqrt[b
 + a*Cos[c + d*x]] + ((2*I)*(15*a^4 - 26*a^2*b^2 + 3*b^4)*Sqrt[-((a*(-1 +
Cos[c + d*x]))/(a + b))]*Sqrt[(a*(1 + Cos[c + d*x]))/(a - b)]*Csc[c + d*x]
*(-2*b*(a + b)*EllipticE[I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d
*x]]], (-a + b)/(a + b)] + a*(2*b*EllipticF[I*ArcSinh[Sqrt[(a - b)^(-1)]*S
qrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b)] + a*EllipticPi[1 - a/b, I*ArcS
inh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b)])))/(a^
2*Sqrt[(a - b)^(-1)]*b)))/((a - b)^2*(a + b)^2)) + (2*(b + a*Cos[c + d*x])
*(15*a^6 - 20*a^4*b^2 - 9*a^2*b^4 + 6*b^6 + 4*a*b*(10*a^4 - 17*a^2*b^2 + 3
*b^4)*Cos[c + d*x] + (15*a^6 - 26*a^4*b^2 + 3*a^2*b^4)*Cos[2*(c + d*x)])*T
an[c + d*x])/(a^2 - b^2)^2))/(12*b^3*d*(a + b*Sec[c + d*x])^(5/2))
```

3.661.3 Rubi [A] (verified)

Time = 4.74 (sec) , antiderivative size = 471, normalized size of antiderivative = 1.03, number of steps used = 29, number of rules used = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 1.160$, Rules used = {3042, 4332, 27, 3042, 4586, 27, 3042, 4590, 27, 3042, 4596, 3042, 4346, 3042, 3286, 3042, 3284, 4523, 3042, 4343, 3042, 3134, 3042, 3132, 4345, 3042, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{9/2}}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \; 4332$$

$$-\frac{2\int \frac{\sec^{\frac{3}{2}}(c+dx)\left(3a^2-3b\sec(c+dx)a-\left(5a^2-3b^2\right)\sec^2(c+dx)\right)}{2(a+b\sec(c+dx))^{3/2}}dx}{3b\left(a^2-b^2\right)}-\frac{2a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \; 27$$

$$-\frac{\int \frac{\sec^{\frac{3}{2}}(c+dx)\left(3a^2-3b\sec(c+dx)a-\left(5a^2-3b^2\right)\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^{3/2}}dx}{3b\left(a^2-b^2\right)}-\frac{2a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(3a^2-3b\csc\left(c+dx+\frac{\pi}{2}\right)a+\left(3b^2-5a^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{3b\left(a^2-b^2\right)}-\frac{2a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \; 4586$$

$$-\frac{\frac{2a^2\left(5a^2-9b^2\right)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{bd(a^2-b^2)\sqrt{a+b\sec(c+dx)}}-\frac{2\int -\frac{\sqrt{\sec(c+dx)}\left(\left(5a^2-9b^2\right)a^2-2b\left(a^2-3b^2\right)\sec(c+dx)a-\left(15a^4-26b^2a^2+3b^4\right)\sec^2(c+dx)\right)}{2\sqrt{a+b\sec(c+dx)}}dx}{b(a^2-b^2)}}{3b\left(a^2-b^2\right)}-\frac{2a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \; 27$$

$$-\frac{\frac{\int \frac{\sqrt{\sec(c+dx)}\left(\left(5a^2-9b^2\right)a^2-2b\left(a^2-3b^2\right)\sec(c+dx)a-\left(15a^4-26b^2a^2+3b^4\right)\sec^2(c+dx)\right)}{\sqrt{a+b\sec(c+dx)}}dx}{b(a^2-b^2)}+\frac{2a^2\left(5a^2-9b^2\right)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{bd(a^2-b^2)\sqrt{a+b\sec(c+dx)}}}{3b\left(a^2-b^2\right)}-\frac{2a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\frac{\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(\left(5a^2-9b^2\right)a^2-2b\left(a^2-3b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)a+\left(-15a^4+26b^2a^2-3b^4\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{b(a^2-b^2)}+\frac{2a^2\left(5a^2-9b^2\right)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{bd(a^2-b^2)\sqrt{a+b\sec(c+dx)}}}{3b\left(a^2-b^2\right)}-\frac{2a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \; 4590$$

$$-\frac{\frac{\frac{\int \frac{2b\left(5a^2-9b^2\right)\sec(c+dx)a^2+15\left(a^2-b^2\right)^2\sec^2(c+dx)a+\left(15a^4-26b^2a^2+3b^4\right)a}{2\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{b}-\frac{\left(15a^4-26a^2b^2+3b^4\right)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{bd}}{b(a^2-b^2)}+\frac{2a^2(5a}{bd}}{3b\left(a^2-b^2\right)}-\frac{2a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \; 27$$

3.661. $\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$

$$-\frac{\frac{\frac{\int \frac{2b\left(5a^2-9b^2\right)\sec(c+dx)a^2+15\left(a^2-b^2\right)^2\sec^2(c+dx)a+\left(15a^4-26b^2a^2+3b^4\right)a}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{2b}-\frac{\left(15a^4-26a^2b^2+3b^4\right)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{bd}}{b(a^2-b^2)}+\frac{2a^2(5a}{bd}}{3b\left(a^2-b^2\right)}$$

$$\frac{2a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}$$

$\downarrow$ 3042

$$-\frac{\frac{\frac{\int \frac{2b\left(5a^2-9b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)a^2+15\left(a^2-b^2\right)^2\csc\left(c+dx+\frac{\pi}{2}\right)^2a+\left(15a^4-26b^2a^2+3b^4\right)a}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{2b}-\frac{\left(15a^4-26a^2b^2+3b^4\right)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{bd}}{b(a^2-b^2)}+}{3b\left(a^2-b^2\right)}$$

$$\frac{2a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}$$

$\downarrow$ 4596

$$-\frac{\frac{\frac{15a\left(a^2-b^2\right)^2\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx+\int \frac{2b\left(5a^2-9b^2\right)\sec(c+dx)a^2+\left(15a^4-26b^2a^2+3b^4\right)a}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{2b}-\frac{\left(15a^4-26a^2b^2+3b^4\right)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{bd}}{b(a^2-b^2)}+}{3b\left(a^2-b^2\right)}$$

$$\frac{2a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}$$

$\downarrow$ 3042

$$-\frac{\frac{\frac{15a\left(a^2-b^2\right)^2\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\int \frac{2b\left(5a^2-9b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)a^2+\left(15a^4-26b^2a^2+3b^4\right)a}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{2b}-\frac{\left(15a^4-26a^2b^2+3b^4\right)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(}}{bd}}{b(a^2-b^2)}}{3b\left(a^2-b^2\right)}$$

$$\frac{2a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}$$

$\downarrow$ 4346

$$-\frac{\frac{\frac{15a\left(a^2-b^2\right)^2\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int \frac{\sec(c+dx)}{\sqrt{b+a\cos(c+dx)}}dx}{\sqrt{a+b\sec(c+dx)}}+\int \frac{2b\left(5a^2-9b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)a^2+\left(15a^4-26b^2a^2+3b^4\right)a}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{2b}-\frac{\left(15a^4-26a^2b^2+3b^4\right)\sin(c+d}{bd}}{b(a^2-b^2)}}{3b\left(a^2-b^2\right)}$$

$$\frac{2a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}$$

3.661. $\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$

$$\Big\downarrow \; 3042$$

$$-\frac{\frac{\frac{15a\left(a^2-b^2\right)^2\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{\sqrt{a+b\sec(c+dx)}}+\int\frac{2b\left(5a^2-9b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)a^2+\left(15a^4-26b^2a^2+3b^4\right)a}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{2b}-\frac{\left(15a^4-26\right.}{b(a^2-b^2)}}{3b\left(a^2-b^2\right)}$$

$$\frac{2a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \; 3286$$

$$-\frac{\frac{\frac{15a\left(a^2-b^2\right)^2\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{\sec(c+dx)}{\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}+\int\frac{2b\left(5a^2-9b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)a^2+\left(15a^4-26b^2a^2+3b^4\right)a}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{2b}-\frac{\left(15a^4-26a^2b^2+3b^4\right)\sin}{b(a^2-b^2)}}{3b\left(a^2-b^2\right)}$$

$$\frac{2a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\frac{\frac{15a\left(a^2-b^2\right)^2\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}+\int\frac{2b\left(5a^2-9b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)a^2+\left(15a^4-26b^2a^2+3b^4\right)a}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{2b}-\frac{\left(15a\right.}{b(a^2-b^2)}}{3b\left(a^2-b^2\right)}$$

$$\frac{2a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \; 3284$$

$$-\frac{\frac{\int\frac{2b\left(5a^2-9b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)a^2+\left(15a^4-26b^2a^2+3b^4\right)a}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{30a\left(a^2-b^2\right)^2\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\operatorname{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}}{2b}-\frac{\left(15a^4-26a^2b^2+3b^4\right)}{b(a^2-b^2)}}{3b\left(a^2-b^2\right)}$$

$$\frac{2a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \; 4523$$

3.661. $\int\frac{\sec^{\frac{9}{2}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$

$$-\frac{\frac{\frac{-b\left(5a^4-8a^2b^2+3b^4\right)\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}dx+\left(15a^4-26a^2b^2+3b^4\right)\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}dx+\frac{30a\left(a^2-b^2\right)^2\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+}\right.}{d\sqrt{a+b\sec(c+dx)}}}{2b}}{b(a^2-b^2)}}{3b\left(a^2-b^2\right)}$$

$$\frac{2a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}$$

$\downarrow$ 3042

$$-\frac{\frac{\frac{-b\left(5a^4-8a^2b^2+3b^4\right)\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\left(15a^4-26a^2b^2+3b^4\right)\int \frac{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{30a\left(a^2-b^2\right)^2\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(\right.}{d\sqrt{a+b\sec(c+dx)}}}{2b}}{b(a^2-b^2)}}{3b\left(a^2-b^2\right)}$$

$$\frac{2a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}$$

$\downarrow$ 4343

$$-\frac{\frac{\frac{-b\left(5a^4-8a^2b^2+3b^4\right)\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{\left(15a^4-26a^2b^2+3b^4\right)\sqrt{a+b\sec(c+dx)}\int \sqrt{b+a\cos(c+dx)}dx}{\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}+\frac{30a\left(a^2-b^2\right)^2\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\text{Elli}}{d\sqrt{a+b\sec(c+dx)}}}{2b}}{b(a^2-b^2)}}{3b\left(a^2-b^2\right)}$$

$$\frac{2a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}$$

$\downarrow$ 3042

$$-\frac{\frac{\frac{-b\left(5a^4-8a^2b^2+3b^4\right)\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{\left(15a^4-26a^2b^2+3b^4\right)\sqrt{a+b\sec(c+dx)}\int \sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}+\frac{30a\left(a^2-b^2\right)^2\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}{d\sqrt{a+b\sec(c+d}}}{2b}}{b(a^2-b^2)}}{3b\left(a^2-b^2\right)}$$

$$\frac{2a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}$$

$\downarrow$ 3134

$$-\frac{\frac{\frac{-b\left(5a^4-8a^2b^2+3b^4\right)\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{\left(15a^4-26a^2b^2+3b^4\right)\sqrt{a+b\sec(c+dx)}\int \sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}dx}{\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{30a\left(a^2-b^2\right)^2\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}{d\sqrt{a+b\sec(c+d}}}{2b}}{b(a^2-b^2)}}{3b\left(a^2-b^2\right)}$$

$$\frac{2a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}$$

3.661. $\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$

$$\Big\downarrow \; 3042$$

$$-\frac{\frac{\frac{-b\left(5a^4-8a^2b^2+3b^4\right)\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{\left(15a^4-26a^2b^2+3b^4\right)\sqrt{a+b\sec(c+dx)}\int \sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}dx}{\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{30a\left(a^2-b^2\right)^2\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}{d\sqrt{a+b\sec(}}}{2b}}{b(a^2-b^2)}}{3b\left(a^2-b^2\right)}$$

$$\frac{2a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \; 3132$$

$$-\frac{\frac{\frac{-b\left(5a^4-8a^2b^2+3b^4\right)\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{30a\left(a^2-b^2\right)^2\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\operatorname{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}+\frac{2\left(15a^4-26a^2b^2+3b^4\right)\sqrt{a+b\sec(c-}}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(}{}}}}{2b}}{b(a^2-b^2)}}{3b\left(a^2-b^2\right)}$$

$$\frac{2a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \; 4345$$

$$-\frac{\frac{\frac{-\frac{b\left(5a^4-8a^2b^2+3b^4\right)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int \frac{1}{\sqrt{b+a\cos(c+dx)}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{30a\left(a^2-b^2\right)^2\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\operatorname{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}+\frac{2\left(15a^4-2\right.}{}}{2b}}{b(a^2-b^2)}}{3b\left(a^2\right.}$$

$$\frac{2a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\frac{\frac{-\frac{b\left(5a^4-8a^2b^2+3b^4\right)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int \frac{1}{\sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{30a\left(a^2-b^2\right)^2\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\operatorname{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}+\frac{2\left(15a\right.}{}}{2b}}{b(a^2-b^2)}}{3b\left(a\right.}$$

$$\frac{2a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \; 3142$$

3.661. $\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$

$$-\frac{\frac{-\frac{b\left(5a^4-8a^2b^2+3b^4\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{30a\left(a^2-b^2\right)^2\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\operatorname{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}+\frac{2\left(15\right.}{}}{2b}}{b(a^2-b^2)}}{3b\,(a}$$

$$\frac{2a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\frac{-\frac{b\left(5a^4-8a^2b^2+3b^4\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{30a\left(a^2-b^2\right)^2\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\operatorname{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}+}{2b}}{b(a^2-b^2)}}{3b\,(}$$

$$\frac{2a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \; 3140$$

$$-\frac{2a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}-\frac{\frac{2a^2\left(5a^2-9b^2\right)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{bd(a^2-b^2)\sqrt{a+b\sec(c+dx)}}+\frac{\frac{30a\left(a^2-b^2\right)^2\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\operatorname{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}-\frac{2b\left(5a^4-8a^2b^2+3b^4\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos}{}}}{d\sqrt{a+b\sec(}}}{2b}}{3b\,(a^2}$$

input
```
Int[Sec[c + d*x]^(9/2)/(a + b*Sec[c + d*x])^(5/2),x]
```

output
```
(-2*a^2*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(3*b*(a^2 - b^2)*d*(a + b*Sec[c +
 d*x])^(3/2)) - ((2*a^2*(5*a^2 - 9*b^2)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(
b*(a^2 - b^2)*d*Sqrt[a + b*Sec[c + d*x]]) + (((-2*b*(5*a^4 - 8*a^2*b^2 + 3
*b^4)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a +
 b)]*Sqrt[Sec[c + d*x]])/(d*Sqrt[a + b*Sec[c + d*x]]) + (30*a*(a^2 - b^2)^
2*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a +
 b)]*Sqrt[Sec[c + d*x]])/(d*Sqrt[a + b*Sec[c + d*x]]) + (2*(15*a^4 - 26*a^
2*b^2 + 3*b^4)*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*
x]])/(d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]))/(2*b) - ((
15*a^4 - 26*a^2*b^2 + 3*b^4)*Sqrt[Sec[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]*S
in[c + d*x])/(b*d))/(b*(a^2 - b^2)))/(3*b*(a^2 - b^2))
```

3.661. $\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$

3.661.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

3.661. $\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$

rule 3286
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]]   Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
 d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

rule 4332
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-a^2)*d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(
m + 1)*((d*Csc[e + f*x])^(n - 3)/(b*f*(m + 1)*(a^2 - b^2))), x] + Simp[d^3/
(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])
^(n - 3)*Simp[a^2*(n - 3) + a*b*(m + 1)*Csc[e + f*x] - (a^2*(n - 2) + b^2*(
m + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2
- b^2, 0] && LtQ[m, -1] && (IGtQ[n, 3] || (IntegersQ[n + 1/2, 2*m] && GtQ[n
, 2]))
```

rule 4343
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]])   Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a
, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4345
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]])   Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[
{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4346
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Simp[d*Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x
]]/Sqrt[a + b*Csc[e + f*x]])   Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4523
```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Simp[A/a   I
nt[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Simp[(A*b - a*B)
/(a*d)   Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ
[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

3.661. $\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$

rule 4586
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(-d)*(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a +
b*Csc[e + f*x])^(m + 1)*((d*Csc[e + f*x])^(n - 1)/(b*f*(a^2 - b^2)*(m + 1))
), x] + Simp[d/(b*(a^2 - b^2)*(m + 1))   Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*b^2*(n - 1) - a*(b*B - a*C)*(n - 1) + b*(a*A
 - b*B + a*C)*(m + 1)*Csc[e + f*x] - (b*(A*b - a*B)*(m + n + 1) + C*(a^2*n
+ b^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C
}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n, 0]
```

rule 4590
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(-C)*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
)*((d*Csc[e + f*x])^(n - 1)/(b*f*(m + n + 1))), x] + Simp[d/(b*(m + n + 1))
   Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (
A*b*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2
 - b^2, 0] && GtQ[n, 0]
```

rule 4596
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Simp[C/d^2   Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*C
sc[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[
a + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
 b^2, 0]
```

3.661.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 11.48 (sec) , antiderivative size = 5987, normalized size of antiderivative = 13.07

method	result	size
default	Expression too large to display	5987

input
```
int(sec(d*x+c)^(9/2)/(a+b*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.661.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)^(9/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.661.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**(9/2)/(a+b*sec(d*x+c))**(5/2),x)
```

output
```
Timed out
```

3.661.7 Maxima [F]

$$\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{\sec(dx+c)^{\frac{9}{2}}}{(b\sec(dx+c)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^(9/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)^(9/2)/(b*sec(d*x + c) + a)^(5/2), x)
```

3.661.8 Giac [F]

$$\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{\sec(dx+c)^{\frac{9}{2}}}{(b\sec(dx+c)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^(9/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^(9/2)/(b*sec(d*x + c) + a)^(5/2), x)
```

3.661.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{9/2}}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

input
```
int((1/cos(c + d*x))^(9/2)/(a + b/cos(c + d*x))^(5/2),x)
```

output
```
int((1/cos(c + d*x))^(9/2)/(a + b/cos(c + d*x))^(5/2), x)
```

3.662 $\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$

3.662.1 Optimal result 5176
3.662.2 Mathematica [C] (verified) 5177
3.662.3 Rubi [A] (verified) 5178
3.662.4 Maple [C] (warning: unable to verify) 5186
3.662.5 Fricas [F(-1)] 5187
3.662.6 Sympy [F(-1)] 5188
3.662.7 Maxima [F] 5188
3.662.8 Giac [F] 5188
3.662.9 Mupad [F(-1)] 5189

3.662.1 Optimal result

Integrand size = 25, antiderivative size = 370

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = -\frac{2a\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{3b\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} + \frac{2\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{b^2d\sqrt{a+b\sec(c+dx)}} + \frac{2a(3a^2-7b^2)\,E\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+dx)}}{3b^2\left(a^2-b^2\right)^2d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\sqrt{\sec(c+dx)}} - \frac{2a^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} - \frac{2a^2(3a^2-7b^2)\sqrt{\sec(c+dx)}\sin(c+dx)}{3b^2\left(a^2-b^2\right)^2d\sqrt{a+b\sec(c+dx)}}$$

output
```
-2/3*a^2*sec(d*x+c)^(3/2)*sin(d*x+c)/b/(a^2-b^2)/d/(a+b*sec(d*x+c))^(3/2)-
2/3*a^2*(3*a^2-7*b^2)*sin(d*x+c)*sec(d*x+c)^(1/2)/b^2/(a^2-b^2)^2/d/(a+b*s
ec(d*x+c))^(1/2)-2/3*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ell
ipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b)
)^(1/2)*sec(d*x+c)^(1/2)/b/(a^2-b^2)/d/(a+b*sec(d*x+c))^(1/2)+2*(cos(1/2*d
*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2,2^(1
/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)*sec(d*x+c)^(1/2)/b^2/d
/(a+b*sec(d*x+c))^(1/2)+2/3*a*(3*a^2-7*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/c
os(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*(a
+b*sec(d*x+c))^(1/2)/b^2/(a^2-b^2)^2/d/((b+a*cos(d*x+c))/(a+b))^(1/2)/sec(
d*x+c)^(1/2)
```

3.662.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.17 (sec) , antiderivative size = 487, normalized size of antiderivative = 1.32

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \frac{\sec^{\frac{5}{2}}(c+dx)\left(\frac{4ab^2(a^2-3b^2)\left(\frac{b+a\cos(c+dx)}{a+b}\right)^{5/2}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{(a-b)^2}+\frac{b(9a^4-19a^2b^2+6b^4)}{}\right.}{}$$

input
```
Integrate[Sec[c + d*x]^(7/2)/(a + b*Sec[c + d*x])^(5/2),x]
```

output
```
(Sec[c + d*x]^(5/2)*((4*a*b^2*(a^2 - 3*b^2)*((b + a*Cos[c + d*x])/(a + b))
^(5/2)*EllipticF[(c + d*x)/2, (2*a)/(a + b)])/(a - b)^2 + (b*(9*a^4 - 19*a
^2*b^2 + 6*b^4)*((b + a*Cos[c + d*x])/(a + b))^(5/2)*EllipticPi[2, (c + d*
x)/2, (2*a)/(a + b)])/(a - b)^2 + (I*((a - b)^(-1))^(3/2)*(3*a^2 - 7*b^2)*
Sqrt[-((a*(-1 + Cos[c + d*x]))/(a + b))]*Sqrt[(a*(1 + Cos[c + d*x]))/(a -
b)]*(b + a*Cos[c + d*x])^(5/2)*Csc[c + d*x]*(-2*b*(a + b)*EllipticE[I*ArcS
inh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b)] + a*(2
*b*EllipticF[I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a +
 b)/(a + b)] + a*EllipticPi[1 - a/b, I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b +
 a*Cos[c + d*x]]], (-a + b)/(a + b)])))/(a + b)^2 + (2*a^2*b^2*(b + a*Cos[
c + d*x])*Sin[c + d*x])/(-a^2 + b^2) + (2*a^2*b*(-3*a^2 + 7*b^2)*(b + a*Co
s[c + d*x])^2*Sin[c + d*x])/(a^2 - b^2)^2))/(3*b^3*d*(a + b*Sec[c + d*x])^
(5/2))
```

3.662.3 Rubi [A] (verified)

Time = 3.88 (sec) , antiderivative size = 390, normalized size of antiderivative = 1.05, number of steps used = 26, number of rules used = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.040, Rules used = {3042, 4332, 27, 3042, 4586, 27, 3042, 4596, 3042, 4346, 3042, 3286, 3042, 3284, 4523, 3042, 4343, 3042, 3134, 3042, 3132, 4345, 3042, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \text{4332}$$

$$-\frac{2\int \frac{\sqrt{\sec(c+dx)}\left(a^2-3b\sec(c+dx)a-3\left(a^2-b^2\right)\sec^2(c+dx)\right)}{2(a+b\sec(c+dx))^{3/2}}dx}{3b\left(a^2-b^2\right)}-\frac{2a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{\sqrt{\sec(c+dx)}\left(a^2-3b\sec(c+dx)a-3\left(a^2-b^2\right)\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^{3/2}}dx}{3b\left(a^2-b^2\right)}-\frac{2a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a^2-3b\csc\left(c+dx+\frac{\pi}{2}\right)a-3\left(a^2-b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{3b\left(a^2-b^2\right)}-\frac{2a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \text{4586}$$

$$-\frac{\frac{2a^2\left(3a^2-7b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd(a^2-b^2)\sqrt{a+b\sec(c+dx)}}-\frac{2\int \frac{\left(3a^2-7b^2\right)a^2+2b\left(a^2-3b^2\right)\sec(c+dx)a+3\left(a^2-b^2\right)^2\sec^2(c+dx)}{2\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{b(a^2-b^2)}}{3b\left(a^2-b^2\right)}-\frac{2a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\frac{2a^2(3a^2-7b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd(a^2-b^2)\sqrt{a+b\sec(c+dx)}}-\frac{\int\frac{(3a^2-7b^2)a^2+2b(a^2-3b^2)\sec(c+dx)a+3(a^2-b^2)^2\sec^2(c+dx)}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{b(a^2-b^2)}}{3b(a^2-b^2)}-\frac{2a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3bd(a^2-b^2)(a+b\sec(c+dx))^{3/2}}$$

$\downarrow$ 3042

$$-\frac{\frac{2a^2(3a^2-7b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd(a^2-b^2)\sqrt{a+b\sec(c+dx)}}-\frac{\int\frac{(3a^2-7b^2)a^2+2b(a^2-3b^2)\csc\left(c+dx+\frac{\pi}{2}\right)a+3(a^2-b^2)^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{b(a^2-b^2)}}{3b(a^2-b^2)}-\frac{2a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3bd(a^2-b^2)(a+b\sec(c+dx))^{3/2}}$$

$\downarrow$ 4596

$$-\frac{\frac{2a^2(3a^2-7b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd(a^2-b^2)\sqrt{a+b\sec(c+dx)}}-\frac{3(a^2-b^2)^2\int\frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx+\int\frac{(3a^2-7b^2)a^2+2b(a^2-3b^2)\sec(c+dx)a}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{b(a^2-b^2)}}{3b(a^2-b^2)}-\frac{2a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3bd(a^2-b^2)(a+b\sec(c+dx))^{3/2}}$$

$\downarrow$ 3042

$$-\frac{\frac{2a^2(3a^2-7b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd(a^2-b^2)\sqrt{a+b\sec(c+dx)}}-\frac{3(a^2-b^2)^2\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\int\frac{(3a^2-7b^2)a^2+2b(a^2-3b^2)\csc\left(c+dx+\frac{\pi}{2}\right)a}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{b(a^2-b^2)}}{3b(a^2-b^2)}-\frac{2a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3bd(a^2-b^2)(a+b\sec(c+dx))^{3/2}}$$

$\downarrow$ 4346

$$-\frac{\frac{2a^2(3a^2-7b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd(a^2-b^2)\sqrt{a+b\sec(c+dx)}}-\frac{\int\frac{(3a^2-7b^2)a^2+2b(a^2-3b^2)\csc\left(c+dx+\frac{\pi}{2}\right)a}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{3(a^2-b^2)^2\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{\sec(c+dx)}{\sqrt{b+a\cos(c+dx)}}d}{\sqrt{a+b\sec(c+dx)}}}{b(a^2-b^2)}}{3b(a^2-b^2)}-\frac{2a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3bd(a^2-b^2)(a+b\sec(c+dx))^{3/2}}$$

$\downarrow$ 3042

3.662. $\int\frac{\sec^{\frac{7}{2}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$

$$-\frac{\frac{2a^2(3a^2-7b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd(a^2-b^2)\sqrt{a+b\sec(c+dx)}}-\frac{\int\frac{(3a^2-7b^2)a^2+2b(a^2-3b^2)\csc(c+dx+\frac{\pi}{2})a}{\sqrt{\csc(c+dx+\frac{\pi}{2})}\sqrt{a+b\csc(c+dx+\frac{\pi}{2})}}dx+\frac{3(a^2-b^2)^2\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{1}{\sin(c+dx+\frac{\pi}{2})\sqrt{b}}}{\sqrt{a+b\sec(c+dx)}}}{b(a^2-b^2)}}{3b(a^2-b^2)}$$

$$\frac{2a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3bd(a^2-b^2)(a+b\sec(c+dx))^{3/2}}$$

$\downarrow$ 3286

$$-\frac{\frac{2a^2(3a^2-7b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd(a^2-b^2)\sqrt{a+b\sec(c+dx)}}-\frac{\int\frac{(3a^2-7b^2)a^2+2b(a^2-3b^2)\csc(c+dx+\frac{\pi}{2})a}{\sqrt{\csc(c+dx+\frac{\pi}{2})}\sqrt{a+b\csc(c+dx+\frac{\pi}{2})}}dx+\frac{3(a^2-b^2)^2\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{\sec(c+dx)}{\sqrt{\frac{b}{a+b}+\frac{a\cos(c+}{a+b}}}}{\sqrt{a+b\sec(c+dx)}}}{b(a^2-b^2)}}{3b(a^2-b^2)}$$

$$\frac{2a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3bd(a^2-b^2)(a+b\sec(c+dx))^{3/2}}$$

$\downarrow$ 3042

$$-\frac{\frac{2a^2(3a^2-7b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd(a^2-b^2)\sqrt{a+b\sec(c+dx)}}-\frac{\int\frac{(3a^2-7b^2)a^2+2b(a^2-3b^2)\csc(c+dx+\frac{\pi}{2})a}{\sqrt{\csc(c+dx+\frac{\pi}{2})}\sqrt{a+b\csc(c+dx+\frac{\pi}{2})}}dx+\frac{3(a^2-b^2)^2\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sin(c+dx+\frac{\pi}{2})\sqrt{}}}{\sqrt{a+b\sec(c+dx)}}}{b(a^2-b^2)}}{3b(a^2-b^2)}$$

$$\frac{2a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3bd(a^2-b^2)(a+b\sec(c+dx))^{3/2}}$$

$\downarrow$ 3284

$$-\frac{\frac{2a^2(3a^2-7b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd(a^2-b^2)\sqrt{a+b\sec(c+dx)}}-\frac{\int\frac{(3a^2-7b^2)a^2+2b(a^2-3b^2)\csc(c+dx+\frac{\pi}{2})a}{\sqrt{\csc(c+dx+\frac{\pi}{2})}\sqrt{a+b\csc(c+dx+\frac{\pi}{2})}}dx+\frac{6(a^2-b^2)^2\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\operatorname{EllipticPi}(2,\frac{1}{2}(c+d}{d\sqrt{a+b\sec(c+dx)}}}{b(a^2-b^2)}}{3b(a^2-b^2)}$$

$$\frac{2a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3bd(a^2-b^2)(a+b\sec(c+dx))^{3/2}}$$

$\downarrow$ 4523

$$-\frac{\frac{2a^2(3a^2-7b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd(a^2-b^2)\sqrt{a+b\sec(c+dx)}}-\frac{-ab(a^2-b^2)\int\frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}dx+a(3a^2-7b^2)\int\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}dx+\frac{6(a^2-b^2)^2\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+d}{a+b}}}{d\sqrt{a+b\sec}}}{b(a^2-b^2)}}{3b(a^2-b^2)}$$

$$\frac{2a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3bd(a^2-b^2)(a+b\sec(c+dx))^{3/2}}$$

3.662. $\int\frac{\sec^{\frac{7}{2}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$

$$\Big\downarrow \; 3042$$

$$-\frac{\frac{2a^2\left(3a^2-7b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd(a^2-b^2)\sqrt{a+b\sec(c+dx)}}-\frac{-ab\left(a^2-b^2\right)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+a\left(3a^2-7b^2\right)\int\frac{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{6\left(a^2-b^2\right)^2\sqrt{\sec(c+dx)}\sqrt{\cdots}}{\cdots}}{b(a^2-b^2)}}{3b\left(a^2-b^2\right)}$$

$$\frac{2a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \; 4343$$

$$-\frac{\frac{2a^2\left(3a^2-7b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd(a^2-b^2)\sqrt{a+b\sec(c+dx)}}-\frac{-ab\left(a^2-b^2\right)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{a\left(3a^2-7b^2\right)\sqrt{a+b\sec(c+dx)}\int\sqrt{b+a\cos(c+dx)}dx}{\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}+\frac{6\left(a^2-b^2\right)^2\sqrt{\sec\cdots}}{\cdots}}{b(a^2-b^2)}}{3b\left(a^2-b^2\right)}$$

$$\frac{2a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\frac{2a^2\left(3a^2-7b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd(a^2-b^2)\sqrt{a+b\sec(c+dx)}}-\frac{-ab\left(a^2-b^2\right)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{a\left(3a^2-7b^2\right)\sqrt{a+b\sec(c+dx)}\int\sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}+\frac{6\left(a^2-b^2\right)^2\cdots}{\cdots}}{b(a^2-b^2)}}{3b\left(a^2-b^2\right)}$$

$$\frac{2a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \; 3134$$

$$-\frac{\frac{2a^2\left(3a^2-7b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd(a^2-b^2)\sqrt{a+b\sec(c+dx)}}-\frac{-ab\left(a^2-b^2\right)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{a\left(3a^2-7b^2\right)\sqrt{a+b\sec(c+dx)}\int\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}dx}{\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{6\left(a^2-b^2\right)^2\cdots}{\cdots}}{b(a^2-b^2)}}{3b\left(a^2-b^2\right)}$$

$$\frac{2a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \; 3042$$

3.662. $\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$

$$-\frac{\frac{2a^2(3a^2-7b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd(a^2-b^2)\sqrt{a+b\sec(c+dx)}}-\frac{-ab(a^2-b^2)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{a\left(3a^2-7b^2\right)\sqrt{a+b\sec(c+dx)}\int\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}dx}{\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{6\left(a^2-b\right.}{}}{b(a^2-b^2)}}{3b\left(a^2-b^2\right)}$$

$$\frac{2a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}$$

$\downarrow$ 3132

$$-\frac{\frac{2a^2(3a^2-7b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd(a^2-b^2)\sqrt{a+b\sec(c+dx)}}-\frac{-ab(a^2-b^2)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a\left(3a^2-7b^2\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{6\left(a^2-b^2\right)^2\sqrt{\sec(}}{}}{b(a^2-b^2)}}{3b\left(a^2-b^2\right)}$$

$$\frac{2a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}$$

$\downarrow$ 4345

$$-\frac{\frac{2a^2(3a^2-7b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd(a^2-b^2)\sqrt{a+b\sec(c+dx)}}-\frac{-\frac{ab\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{1}{\sqrt{b+a\cos(c+dx)}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{2a\left(3a^2-7b^2\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+}\right.}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{b(a^2-b^2)}}{3b\left(a^2-b^2\right)}$$

$$\frac{2a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}$$

$\downarrow$ 3042

$$-\frac{\frac{2a^2(3a^2-7b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd(a^2-b^2)\sqrt{a+b\sec(c+dx)}}-\frac{-\frac{ab\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{1}{\sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{2a\left(3a^2-7b^2\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx\right.}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{b(a^2-b^2)}}{3b\left(a^2-b^2\right)}$$

$$\frac{2a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}$$

$\downarrow$ 3142

$$-\frac{\frac{2a^2(3a^2-7b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd(a^2-b^2)\sqrt{a+b\sec(c+dx)}}-\frac{-\frac{ab\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{2a\left(3a^2-7b^2\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx\right.}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{b(a^2-b^2)}}{3b\left(a^2-b^2\right)}$$

$$\frac{2a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}$$

3.662. $\int\frac{\sec^{\frac{7}{2}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$

$$\Big\downarrow \; 3042$$

$$-\frac{\frac{2a^2(3a^2-7b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd(a^2-b^2)\sqrt{a+b\sec(c+dx)}}-\frac{-\frac{ab\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{2a\left(3a^2-7b^2\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(\right.}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)}{a+b}}}}{b(a^2-b^2)}}{3b\left(a^2-b^2\right)}$$

$$\frac{2a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \; 3140$$

$$-\frac{2a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}-\frac{\frac{2a^2(3a^2-7b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd(a^2-b^2)\sqrt{a+b\sec(c+dx)}}-\frac{-\frac{2ab\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}+\frac{2a\left(3a^2-7b^2\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\right.}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{b(a^2-b^2)}}{3b\left(a^2-b^2\right)}$$

input
```
Int[Sec[c + d*x]^(7/2)/(a + b*Sec[c + d*x])^(5/2),x]
```

output
```
(-2*a^2*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*b*(a^2 - b^2)*d*(a + b*Sec[c +
 d*x])^(3/2)) - (-(((-2*a*b*(a^2 - b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]
*EllipticF[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(d*Sqrt[a + b*S
ec[c + d*x]]) + (6*(a^2 - b^2)^2*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Ellipt
icPi[2, (c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(d*Sqrt[a + b*Sec[
c + d*x]]) + (2*a*(3*a^2 - 7*b^2)*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sq
rt[a + b*Sec[c + d*x]])/(d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c +
 d*x]]))/(b*(a^2 - b^2))) + (2*a^2*(3*a^2 - 7*b^2)*Sqrt[Sec[c + d*x]]*Sin[
c + d*x])/(b*(a^2 - b^2)*d*Sqrt[a + b*Sec[c + d*x]]))/(3*b*(a^2 - b^2))
```

3.662.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3286
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]]   Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
 d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

3.662. $\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$

rule 4332
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-a^2)*d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(
m + 1)*((d*Csc[e + f*x])^(n - 3)/(b*f*(m + 1)*(a^2 - b^2))), x] + Simp[d^3/
(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])
^(n - 3)*Simp[a^2*(n - 3) + a*b*(m + 1)*Csc[e + f*x] - (a^2*(n - 2) + b^2*(
m + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2
- b^2, 0] && LtQ[m, -1] && (IGtQ[n, 3] || (IntegersQ[n + 1/2, 2*m] && GtQ[n
, 2]))
```

rule 4343
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]])   Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a
, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4345
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]])   Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[
{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4346
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Simp[d*Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x
]]/Sqrt[a + b*Csc[e + f*x]])   Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4523
```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Simp[A/a   I
nt[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Simp[(A*b - a*B)
/(a*d)   Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ
[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

3.662. $\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$

rule 4586
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(-d)*(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a +
b*Csc[e + f*x])^(m + 1)*((d*Csc[e + f*x])^(n - 1)/(b*f*(a^2 - b^2)*(m + 1))
), x] + Simp[d/(b*(a^2 - b^2)*(m + 1))   Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*b^2*(n - 1) - a*(b*B - a*C)*(n - 1) + b*(a*A
 - b*B + a*C)*(m + 1)*Csc[e + f*x] - (b*(A*b - a*B)*(m + n + 1) + C*(a^2*n
+ b^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C
}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n, 0]
```

rule 4596
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Simp[C/d^2   Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*C
sc[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[
a + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
 b^2, 0]
```

3.662.4 Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 8.97 (sec) , antiderivative size = 4415, normalized size of antiderivative = 11.93

method	result	size
default	Expression too large to display	4415

input
```
int(sec(d*x+c)^(7/2)/(a+b*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/3/d/((a-b)/(a+b))^(1/2)/(a+b)^2/(a-b)/b^2*(-((1-cos(d*x+c))^2*csc(d*x+c)
^2+1)/((1-cos(d*x+c))^2*csc(d*x+c)^2-1))^(7/2)*((1-cos(d*x+c))^2*csc(d*x+c
)^2-1)^4*((a*(1-cos(d*x+c))^2*csc(d*x+c)^2-b*(1-cos(d*x+c))^2*csc(d*x+c)^2
-a-b)/((1-cos(d*x+c))^2*csc(d*x+c)^2-1))^(1/2)*(-3*(-(a*(1-cos(d*x+c))^2*c
sc(d*x+c)^2-b*(1-cos(d*x+c))^2*csc(d*x+c)^2-a-b)/(a+b))^(1/2)*((1-cos(d*x+
c))^2*csc(d*x+c)^2+1)^(1/2)*EllipticE(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc
(d*x+c)),(-(a+b)/(a-b))^(1/2))*a^4*(1-cos(d*x+c))^2*csc(d*x+c)^2-3*((a-b)/
(a+b))^(1/2)*a^4*(-cot(d*x+c)+csc(d*x+c))-13*(-(a*(1-cos(d*x+c))^2*csc(d*x
+c)^2-b*(1-cos(d*x+c))^2*csc(d*x+c)^2-a-b)/(a+b))^(1/2)*((1-cos(d*x+c))^2*
csc(d*x+c)^2+1)^(1/2)*EllipticF(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c
)),(-(a+b)/(a-b))^(1/2))*a^2*b^2*(1-cos(d*x+c))^2*csc(d*x+c)^2+6*(-(a*(1-c
os(d*x+c))^2*csc(d*x+c)^2-b*(1-cos(d*x+c))^2*csc(d*x+c)^2-a-b)/(a+b))^(1/2
)*((1-cos(d*x+c))^2*csc(d*x+c)^2+1)^(1/2)*EllipticF(((a-b)/(a+b))^(1/2)*(-
cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*a*b^3*(1-cos(d*x+c))^2*csc(d*
x+c)^2+3*(-(a*(1-cos(d*x+c))^2*csc(d*x+c)^2-b*(1-cos(d*x+c))^2*csc(d*x+c)^
2-a-b)/(a+b))^(1/2)*((1-cos(d*x+c))^2*csc(d*x+c)^2+1)^(1/2)*EllipticE(((a-
b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*a^3*b*(1-co
s(d*x+c))^2*csc(d*x+c)^2+7*(-(a*(1-cos(d*x+c))^2*csc(d*x+c)^2-b*(1-cos(d*x
+c))^2*csc(d*x+c)^2-a-b)/(a+b))^(1/2)*((1-cos(d*x+c))^2*csc(d*x+c)^2+1)^(1
/2)*EllipticE(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b...
```

3.662.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)^(7/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.662.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**(7/2)/(a+b*sec(d*x+c))**(5/2),x)
```

output
```
Timed out
```

3.662.7 Maxima [F]

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{\sec(dx+c)^{\frac{7}{2}}}{(b\sec(dx+c)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^(7/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)^(7/2)/(b*sec(d*x + c) + a)^(5/2), x)
```

3.662.8 Giac [F]

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{\sec(dx+c)^{\frac{7}{2}}}{(b\sec(dx+c)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^(7/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^(7/2)/(b*sec(d*x + c) + a)^(5/2), x)
```

3.662.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx=\int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{7/2}}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

input
```
int((1/cos(c + d*x))^(7/2)/(a + b/cos(c + d*x))^(5/2),x)
```

output
```
int((1/cos(c + d*x))^(7/2)/(a + b/cos(c + d*x))^(5/2), x)
```

3.663 $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$

3.663.1 Optimal result 5190
3.663.2 Mathematica [A] (verified) 5191
3.663.3 Rubi [A] (verified) 5191
3.663.4 Maple [B] (warning: unable to verify) 5197
3.663.5 Fricas [C] (verification not implemented) 5197
3.663.6 Sympy [F(-1)] 5198
3.663.7 Maxima [F] 5199
3.663.8 Giac [F] 5199
3.663.9 Mupad [F(-1)] 5199

3.663.1 Optimal result

Integrand size = 25, antiderivative size = 277

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \frac{2\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{3\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} + \frac{8bE\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+dx)}}{3\left(a^2-b^2\right)^2 d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\sqrt{\sec(c+dx)}} - \frac{2a^2\sqrt{\sec(c+dx)}\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} + \frac{2a(a^2-5b^2)\sqrt{\sec(c+dx)}\sin(c+dx)}{3b\left(a^2-b^2\right)^2 d\sqrt{a+b\sec(c+dx)}}$$

output
```
-2/3*a^2*sin(d*x+c)*sec(d*x+c)^(1/2)/b/(a^2-b^2)/d/(a+b*sec(d*x+c))^(3/2)+
2/3*a*(a^2-5*b^2)*sin(d*x+c)*sec(d*x+c)^(1/2)/b/(a^2-b^2)^2/d/(a+b*sec(d*x
+c))^(1/2)+2/3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(s
in(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)*
sec(d*x+c)^(1/2)/(a^2-b^2)/d/(a+b*sec(d*x+c))^(1/2)+8/3*b*(cos(1/2*d*x+1/2
*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a
+b))^(1/2))*(a+b*sec(d*x+c))^(1/2)/(a^2-b^2)^2/d/((b+a*cos(d*x+c))/(a+b))^
(1/2)/sec(d*x+c)^(1/2)
```

3.663.2 Mathematica [A] (verified)

Time = 0.90 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.61

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx =$$
$$-\frac{2\sec^{\frac{3}{2}}(c+dx)\left(-4b(a+b)^2\left(\frac{b+a\cos(c+dx)}{a+b}\right)^{3/2}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)-(a-b)(a+b)^2\left(\frac{b+a\cos(c+dx)}{a+b}\right)^{3/2}\text{Ellipt}\right.}{3(a-b)^2(a+b)^2d(a+b\sec(c+dx))^{3/}}$$

input
```
Integrate[Sec[c + d*x]^(5/2)/(a + b*Sec[c + d*x])^(5/2),x]
```

output
```
(-2*Sec[c + d*x]^(3/2)*(-4*b*(a + b)^2*((b + a*Cos[c + d*x])/(a + b))^(3/2
)*EllipticE[(c + d*x)/2, (2*a)/(a + b)] - (a - b)*(a + b)^2*((b + a*Cos[c
+ d*x])/(a + b))^(3/2)*EllipticF[(c + d*x)/2, (2*a)/(a + b)] + a*(-a^2 + 5
*b^2 + 4*a*b*Cos[c + d*x])*Sin[c + d*x]))/(3*(a - b)^2*(a + b)^2*d*(a + b*
Sec[c + d*x])^(3/2))
```

3.663.3 Rubi [A] (verified)

Time = 2.14 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.06, number of steps used = 19, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.760$, Rules used = {3042, 4332, 27, 3042, 4588, 27, 3042, 4523, 3042, 4343, 3042, 3134, 3042, 3132, 4345, 3042, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$
$$\Big\downarrow \; 4332$$
$$-\frac{2\int -\frac{a^2+3b\sec(c+dx)a+(a^2-3b^2)\sec^2(c+dx)}{2\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}}dx}{3b\left(a^2-b^2\right)}-\frac{2a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}$$
$$\Big\downarrow \; 27$$

$$\frac{\int \frac{a^2+3b\sec(c+dx)a+(a^2-3b^2)\sec^2(c+dx)}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}}dx}{3b(a^2-b^2)}-\frac{2a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{3bd(a^2-b^2)(a+b\sec(c+dx))^{3/2}}$$

$\downarrow$ 3042

$$\frac{\int \frac{a^2+3b\csc(c+dx+\frac{\pi}{2})a+(a^2-3b^2)\csc(c+dx+\frac{\pi}{2})^2}{\sqrt{\csc(c+dx+\frac{\pi}{2})}(a+b\csc(c+dx+\frac{\pi}{2}))^{3/2}}dx}{3b(a^2-b^2)}-\frac{2a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{3bd(a^2-b^2)(a+b\sec(c+dx))^{3/2}}$$

$\downarrow$ 4588

$$\frac{\frac{2a(a^2-5b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a^2-b^2)\sqrt{a+b\sec(c+dx)}}-\frac{2\int -\frac{4a^2b^2+a(a^2+3b^2)\sec(c+dx)b}{2\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{a(a^2-b^2)}}{3b(a^2-b^2)}-\frac{2a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{3bd(a^2-b^2)(a+b\sec(c+dx))^{3/2}}$$

$\downarrow$ 27

$$\frac{\frac{\int \frac{4a^2b^2+a(a^2+3b^2)\sec(c+dx)b}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{a(a^2-b^2)}+\frac{2a(a^2-5b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a^2-b^2)\sqrt{a+b\sec(c+dx)}}}{3b(a^2-b^2)}-\frac{2a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{3bd(a^2-b^2)(a+b\sec(c+dx))^{3/2}}$$

$\downarrow$ 3042

$$\frac{\frac{\int \frac{4a^2b^2+a(a^2+3b^2)\csc(c+dx+\frac{\pi}{2})b}{\sqrt{\csc(c+dx+\frac{\pi}{2})}\sqrt{a+b\csc(c+dx+\frac{\pi}{2})}}dx}{a(a^2-b^2)}+\frac{2a(a^2-5b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a^2-b^2)\sqrt{a+b\sec(c+dx)}}}{3b(a^2-b^2)}-\frac{2a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{3bd(a^2-b^2)(a+b\sec(c+dx))^{3/2}}$$

$\downarrow$ 4523

$$\frac{\frac{ab(a^2-b^2)\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}dx+4ab^2\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}dx}{a(a^2-b^2)}+\frac{2a(a^2-5b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a^2-b^2)\sqrt{a+b\sec(c+dx)}}}{3b(a^2-b^2)}-\frac{2a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{3bd(a^2-b^2)(a+b\sec(c+dx))^{3/2}}$$

$\downarrow$ 3042

$$\frac{\frac{ab(a^2-b^2)\int \frac{\sqrt{\csc(c+dx+\frac{\pi}{2})}}{\sqrt{a+b\csc(c+dx+\frac{\pi}{2})}}dx+4ab^2\int \frac{\sqrt{a+b\csc(c+dx+\frac{\pi}{2})}}{\sqrt{\csc(c+dx+\frac{\pi}{2})}}dx}{a(a^2-b^2)}+\frac{2a(a^2-5b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a^2-b^2)\sqrt{a+b\sec(c+dx)}}}{3b(a^2-b^2)}-\frac{2a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{3bd(a^2-b^2)(a+b\sec(c+dx))^{3/2}}$$

$\downarrow$ 4343

3.663. $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$

$$\frac{\frac{ab(a^2-b^2)\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{4ab^2\sqrt{a+b\sec(c+dx)}\int\sqrt{b+a\cos(c+dx)}dx}{\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}}{a(a^2-b^2)}+\frac{2a(a^2-5b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a^2-b^2)\sqrt{a+b\sec(c+dx)}}}{3b(a^2-b^2)}-\frac{2a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{3bd(a^2-b^2)(a+b\sec(c+dx))^{3/2}}$$

$\downarrow$ 3042

$$\frac{\frac{ab(a^2-b^2)\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{4ab^2\sqrt{a+b\sec(c+dx)}\int\sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}}{a(a^2-b^2)}+\frac{2a(a^2-5b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a^2-b^2)\sqrt{a+b\sec(c+dx)}}}{3b(a^2-b^2)}-\frac{2a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{3bd(a^2-b^2)(a+b\sec(c+dx))^{3/2}}$$

$\downarrow$ 3134

$$\frac{\frac{ab(a^2-b^2)\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{4ab^2\sqrt{a+b\sec(c+dx)}\int\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}dx}{\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{a(a^2-b^2)}+\frac{2a(a^2-5b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a^2-b^2)\sqrt{a+b\sec(c+dx)}}}{3b(a^2-b^2)}-\frac{2a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{3bd(a^2-b^2)(a+b\sec(c+dx))^{3/2}}$$

$\downarrow$ 3042

$$\frac{\frac{ab(a^2-b^2)\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{4ab^2\sqrt{a+b\sec(c+dx)}\int\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}dx}{\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{a(a^2-b^2)}+\frac{2a(a^2-5b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a^2-b^2)\sqrt{a+b\sec(c+dx)}}}{3b(a^2-b^2)}-\frac{2a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{3bd(a^2-b^2)(a+b\sec(c+dx))^{3/2}}$$

$\downarrow$ 3132

$$\frac{\frac{ab(a^2-b^2)\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{8ab^2\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{a(a^2-b^2)}+\frac{2a(a^2-5b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a^2-b^2)\sqrt{a+b\sec(c+dx)}}}{3b(a^2-b^2)}-\frac{2a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{3bd(a^2-b^2)(a+b\sec(c+dx))^{3/2}}$$

$\downarrow$ 4345

3.663. $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$

$$\frac{\frac{\frac{ab\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{1}{\sqrt{b+a\cos(c+dx)}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{8ab^2\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{a(a^2-b^2)}+\frac{2a\left(a^2-5b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a^2-b^2)\sqrt{a+b\sec(c+dx)}}}{3b\left(a^2-b^2\right)}-\frac{2a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}$$

$\downarrow$ 3042

$$\frac{\frac{\frac{ab\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{1}{\sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{8ab^2\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{a(a^2-b^2)}+\frac{2a\left(a^2-5b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a^2-b^2)\sqrt{a+b\sec(c+dx)}}}{3b\left(a^2-b^2\right)}-\frac{2a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}$$

$\downarrow$ 3142

$$\frac{\frac{\frac{ab\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{8ab^2\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{a(a^2-b^2)}+\frac{2a\left(a^2-5b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a^2-b^2)\sqrt{a+b\sec(c+dx)}}}{3b\left(a^2-b^2\right)}-\frac{2a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}$$

$\downarrow$ 3042

$$\frac{\frac{\frac{ab\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{8ab^2\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{a(a^2-b^2)}+\frac{2a\left(a^2-5b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a^2-b^2)\sqrt{a+b\sec(c+dx)}}}{3b\left(a^2-b^2\right)}-\frac{2a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}$$

$\downarrow$ 3140

$$\frac{\frac{2a\left(a^2-5b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a^2-b^2)\sqrt{a+b\sec(c+dx)}}+\frac{\frac{2ab\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}+\frac{8ab^2\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{a(a^2-b^2)}}{3b\left(a^2-b^2\right)}-\frac{2a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}$$

3.663. $\int\frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$

input
```
Int[Sec[c + d*x]^(5/2)/(a + b*Sec[c + d*x])^(5/2),x]
```

output
```
(-2*a^2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*b*(a^2 - b^2)*d*(a + b*Sec[c +
 d*x])^(3/2)) + (((2*a*b*(a^2 - b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*El
lipticF[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(d*Sqrt[a + b*Sec[
c + d*x]]) + (8*a*b^2*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec
[c + d*x]])/(d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]))/(a*
(a^2 - b^2)) + (2*a*(a^2 - 5*b^2)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/((a^2 -
 b^2)*d*Sqrt[a + b*Sec[c + d*x]]))/(3*b*(a^2 - b^2))
```

3.663.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

3.663. $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$

rule 4332
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-a^2)*d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(
m + 1)*((d*Csc[e + f*x])^(n - 3)/(b*f*(m + 1)*(a^2 - b^2))), x] + Simp[d^3/
(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])
^(n - 3)*Simp[a^2*(n - 3) + a*b*(m + 1)*Csc[e + f*x] - (a^2*(n - 2) + b^2*(
m + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2
- b^2, 0] && LtQ[m, -1] && (IGtQ[n, 3] || (IntegersQ[n + 1/2, 2*m] && GtQ[n
, 2]))
```

rule 4343
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]])   Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a
, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4345
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]])   Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[
{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4523
```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Simp[A/a   I
nt[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Simp[(A*b - a*B)
/(a*d)   Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ
[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

rule 4588
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc
[e + f*x])^(m + 1)*((d*Csc[e + f*x])^n/(a*f*(m + 1)*(a^2 - b^2))), x] + Sim
p[1/(a*(m + 1)*(a^2 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f
*x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n +
1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m
+ n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x
] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

3.663. $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$

3.663.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 1998 vs. 2(307) = 614.

Time = 5.80 (sec) , antiderivative size = 1999, normalized size of antiderivative = 7.22

method	result	size
default	Expression too large to display	1999

input
```
int(sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3/d/(a-b)/(a+b)^2/((a-b)/(a+b))^(1/2)*(-((1-cos(d*x+c))^2*csc(d*x+c)^2+
1)/((1-cos(d*x+c))^2*csc(d*x+c)^2-1))^(5/2)*((1-cos(d*x+c))^2*csc(d*x+c)^2
-1)^3*((a*(1-cos(d*x+c))^2*csc(d*x+c)^2-b*(1-cos(d*x+c))^2*csc(d*x+c)^2-a-
b)/((1-cos(d*x+c))^2*csc(d*x+c)^2-1))^(1/2)*(4*((a-b)/(a+b))^(1/2)*a*b*(1-
cos(d*x+c))^5*csc(d*x+c)^5-4*((a-b)/(a+b))^(1/2)*b^2*(1-cos(d*x+c))^5*csc(
d*x+c)^5-(-(a*(1-cos(d*x+c))^2*csc(d*x+c)^2-b*(1-cos(d*x+c))^2*csc(d*x+c)^
2-a-b)/(a+b))^(1/2)*((1-cos(d*x+c))^2*csc(d*x+c)^2+1)^(1/2)*EllipticF(((a-
b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*a^2*(1-cos(
d*x+c))^2*csc(d*x+c)^2+4*(-(a*(1-cos(d*x+c))^2*csc(d*x+c)^2-b*(1-cos(d*x+c
))^2*csc(d*x+c)^2-a-b)/(a+b))^(1/2)*((1-cos(d*x+c))^2*csc(d*x+c)^2+1)^(1/2
)*EllipticF(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1
/2))*a*b*(1-cos(d*x+c))^2*csc(d*x+c)^2-3*(-(a*(1-cos(d*x+c))^2*csc(d*x+c)^
2-b*(1-cos(d*x+c))^2*csc(d*x+c)^2-a-b)/(a+b))^(1/2)*((1-cos(d*x+c))^2*csc(
d*x+c)^2+1)^(1/2)*EllipticF(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(
-(a+b)/(a-b))^(1/2))*b^2*(1-cos(d*x+c))^2*csc(d*x+c)^2-4*(-(a*(1-cos(d*x+c
))^2*csc(d*x+c)^2-b*(1-cos(d*x+c))^2*csc(d*x+c)^2-a-b)/(a+b))^(1/2)*((1-co
s(d*x+c))^2*csc(d*x+c)^2+1)^(1/2)*EllipticE(((a-b)/(a+b))^(1/2)*(-cot(d*x+
c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*a*b*(1-cos(d*x+c))^2*csc(d*x+c)^2+4*(
-(a*(1-cos(d*x+c))^2*csc(d*x+c)^2-b*(1-cos(d*x+c))^2*csc(d*x+c)^2-a-b)/(a+
b))^(1/2)*((1-cos(d*x+c))^2*csc(d*x+c)^2+1)^(1/2)*EllipticE(((a-b)/(a+b...
```

3.663.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.14 (sec) , antiderivative size = 688, normalized size of antiderivative = 2.48

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \frac{\sqrt{2}\left(-3i\,a^2b^2 - i\,b^4 + \left(-3i\,a^4 - i\,a^2b^2\right)\cos\left(dx+c\right)^2 - 2\left(3i\,a^3b + i\,ab^3\right)\cos\left(d\right.\right.}{}$$

input
```
integrate(sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
1/9*(sqrt(2)*(-3*I*a^2*b^2 - I*b^4 + (-3*I*a^4 - I*a^2*b^2)*cos(d*x + c)^2
 - 2*(3*I*a^3*b + I*a*b^3)*cos(d*x + c))*sqrt(a)*weierstrassPInverse(-4/3*
(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3
*I*a*sin(d*x + c) + 2*b)/a) + sqrt(2)*(3*I*a^2*b^2 + I*b^4 + (3*I*a^4 + I*
a^2*b^2)*cos(d*x + c)^2 - 2*(-3*I*a^3*b - I*a*b^3)*cos(d*x + c))*sqrt(a)*w
eierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1
/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a) - 12*sqrt(2)*(-I*a^3*b
*cos(d*x + c)^2 - 2*I*a^2*b^2*cos(d*x + c) - I*a*b^3)*sqrt(a)*weierstrassZ
eta(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInve
rse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x
 + c) + 3*I*a*sin(d*x + c) + 2*b)/a)) - 12*sqrt(2)*(I*a^3*b*cos(d*x + c)^2
 + 2*I*a^2*b^2*cos(d*x + c) + I*a*b^3)*sqrt(a)*weierstrassZeta(-4/3*(3*a^2
 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a^2
 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*s
in(d*x + c) + 2*b)/a)) - 6*(4*a^3*b*cos(d*x + c)^2 - (a^4 - 5*a^2*b^2)*cos
(d*x + c))*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d
*x + c)))/((a^7 - 2*a^5*b^2 + a^3*b^4)*d*cos(d*x + c)^2 + 2*(a^6*b - 2*a^4
*b^3 + a^2*b^5)*d*cos(d*x + c) + (a^5*b^2 - 2*a^3*b^4 + a*b^6)*d)
```

3.663.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**(5/2)/(a+b*sec(d*x+c))**(5/2),x)
```

output
```
Timed out
```

3.663.7 Maxima [F]

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{\sec{(dx+c)}^{\frac{5}{2}}}{(b\sec{(dx+c)}+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)^(5/2)/(b*sec(d*x + c) + a)^(5/2), x)
```

3.663.8 Giac [F]

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{\sec{(dx+c)}^{\frac{5}{2}}}{(b\sec{(dx+c)}+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^(5/2)/(b*sec(d*x + c) + a)^(5/2), x)
```

3.663.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

input
```
int((1/cos(c + d*x))^(5/2)/(a + b/cos(c + d*x))^(5/2),x)
```

output
```
int((1/cos(c + d*x))^(5/2)/(a + b/cos(c + d*x))^(5/2), x)
```

3.664 $\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$

3.664.1 Optimal result 5200
3.664.2 Mathematica [A] (verified) 5201
3.664.3 Rubi [A] (verified) 5201
3.664.4 Maple [B] (verified) 5207
3.664.5 Fricas [C] (verification not implemented) 5207
3.664.6 Sympy [F] 5208
3.664.7 Maxima [F] 5209
3.664.8 Giac [F] 5209
3.664.9 Mupad [F(-1)] 5209

3.664.1 Optimal result

Integrand size = 25, antiderivative size = 281

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = -\frac{2b\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{3a\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} - \frac{2(3a^2+b^2)\,E\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+dx)}}{3a\left(a^2-b^2\right)^2 d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\sqrt{\sec(c+dx)}} + \frac{2a\sqrt{\sec(c+dx)}\sin(c+dx)}{3\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} + \frac{4(a^2+b^2)\sqrt{\sec(c+dx)}\sin(c+dx)}{3\left(a^2-b^2\right)^2 d\sqrt{a+b\sec(c+dx)}}$$

output
```
2/3*a*sin(d*x+c)*sec(d*x+c)^(1/2)/(a^2-b^2)/d/(a+b*sec(d*x+c))^(3/2)+4/3*(
a^2+b^2)*sin(d*x+c)*sec(d*x+c)^(1/2)/(a^2-b^2)^2/d/(a+b*sec(d*x+c))^(1/2)-
2/3*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*
x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)*sec(d*x+c
)^(1/2)/a/(a^2-b^2)/d/(a+b*sec(d*x+c))^(1/2)-2/3*(3*a^2+b^2)*(cos(1/2*d*x+
1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a
/(a+b))^(1/2))*(a+b*sec(d*x+c))^(1/2)/a/(a^2-b^2)^2/d/((b+a*cos(d*x+c))/(a
+b))^(1/2)/sec(d*x+c)^(1/2)
```

3.664.2 Mathematica [A] (verified)

Time = 1.00 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.63

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \frac{\sec^{\frac{5}{2}}(c+dx)\left(-\frac{2(a+b)\left(\frac{b+a\cos(c+dx)}{a+b}\right)^{5/2}\left((3a^2+b^2)E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)+(a-b)b\,\text{EllipticF}\left(\frac{1}{2}(c+d\right.\right.}{a(a-b)^2}\right.}{3d(a+b\sec(c+dx)}$$

input
```
Integrate[Sec[c + d*x]^(3/2)/(a + b*Sec[c + d*x])^(5/2),x]
```

output
```
(Sec[c + d*x]^(5/2)*((-2*(a + b)*((b + a*Cos[c + d*x])/(a + b))^(5/2)*((3*
a^2 + b^2)*EllipticE[(c + d*x)/2, (2*a)/(a + b)] + (a - b)*b*EllipticF[(c
+ d*x)/2, (2*a)/(a + b)]))/(a*(a - b)^2) + (2*(b + a*Cos[c + d*x])*(2*b*(a
^2 + b^2) + a*(3*a^2 + b^2)*Cos[c + d*x])*Sin[c + d*x])/(a^2 - b^2)^2))/(3
*d*(a + b*Sec[c + d*x])^(5/2))
```

3.664.3 Rubi [A] (verified)

Time = 2.00 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.02, number of steps used = 19, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.760$, Rules used = {3042, 4331, 27, 3042, 4588, 27, 3042, 4523, 3042, 4343, 3042, 3134, 3042, 3132, 4345, 3042, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \; 4331$$

$$\frac{2\int -\frac{-2a\sec^2(c+dx)+3b\sec(c+dx)+a}{2\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}}dx}{3\left(a^2-b^2\right)}+\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}-\frac{\int\frac{-2a\sec^2(c+dx)+3b\sec(c+dx)+a}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}}dx}{3\left(a^2-b^2\right)}$$

$\Big\downarrow$ 3042

$$\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}-\frac{\int\frac{-2a\csc\left(c+dx+\frac{\pi}{2}\right)^2+3b\csc\left(c+dx+\frac{\pi}{2}\right)+a}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{3\left(a^2-b^2\right)}$$

$\Big\downarrow$ 4588

$$\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}-\frac{-\frac{2\int-\frac{4b\sec(c+dx)a^2+\left(3a^2+b^2\right)a}{2\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{a\left(a^2-b^2\right)}-\frac{4\left(a^2+b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}}{3\left(a^2-b^2\right)}$$

$\Big\downarrow$ 27

$$\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}-\frac{\frac{\int\frac{4b\sec(c+dx)a^2+\left(3a^2+b^2\right)a}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{a\left(a^2-b^2\right)}-\frac{4\left(a^2+b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}}{3\left(a^2-b^2\right)}$$

$\Big\downarrow$ 3042

$$\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}-\frac{\frac{\int\frac{4b\csc\left(c+dx+\frac{\pi}{2}\right)a^2+\left(3a^2+b^2\right)a}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a\left(a^2-b^2\right)}-\frac{4\left(a^2+b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}}{3\left(a^2-b^2\right)}$$

$\Big\downarrow$ 4523

$$\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}-\frac{\frac{b\left(a^2-b^2\right)\int\frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}dx+\left(3a^2+b^2\right)\int\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}dx}{a\left(a^2-b^2\right)}-\frac{4\left(a^2+b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}}{3\left(a^2-b^2\right)}$$

$\Big\downarrow$ 3042

$$\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}-\frac{\frac{b\left(a^2-b^2\right)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\left(3a^2+b^2\right)\int\frac{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a\left(a^2-b^2\right)}-\frac{4\left(a^2+b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}}{3\left(a^2-b^2\right)}$$

$\Big\downarrow$ 4343

3.664. $\int\frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$

$$\frac{\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}-\frac{b\left(a^2-b^2\right)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{\left(3a^2+b^2\right)\sqrt{a+b\sec(c+dx)}\int\sqrt{b+a\cos(c+dx)}dx}{\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}}{a\left(a^2-b^2\right)}-\frac{4\left(a^2+b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}}{3\left(a^2-b^2\right)}$$

$\downarrow$ 3042

$$\frac{\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}-\frac{b\left(a^2-b^2\right)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{\left(3a^2+b^2\right)\sqrt{a+b\sec(c+dx)}\int\sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}}{a\left(a^2-b^2\right)}-\frac{4\left(a^2+b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}}{3\left(a^2-b^2\right)}$$

$\downarrow$ 3134

$$\frac{\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}-\frac{b\left(a^2-b^2\right)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{\left(3a^2+b^2\right)\sqrt{a+b\sec(c+dx)}\int\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}dx}{\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{a\left(a^2-b^2\right)}-\frac{4\left(a^2+b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}}{3\left(a^2-b^2\right)}$$

$\downarrow$ 3042

$$\frac{\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}-\frac{b\left(a^2-b^2\right)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{\left(3a^2+b^2\right)\sqrt{a+b\sec(c+dx)}\int\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}dx}{\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{a\left(a^2-b^2\right)}-\frac{4\left(a^2+b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}}{3\left(a^2-b^2\right)}$$

$\downarrow$ 3132

$$\frac{\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}-\frac{b\left(a^2-b^2\right)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\left(3a^2+b^2\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{a\left(a^2-b^2\right)}-\frac{4\left(a^2+b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}}{3\left(a^2-b^2\right)}$$

$\downarrow$ 4345

3.664. $\int\frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$

$$\frac{\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}-\frac{\frac{b\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{1}{\sqrt{b+a\cos(c+dx)}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{2\left(3a^2+b^2\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{a(a^2-b^2)}-\frac{4\left(a^2+b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a^2-b^2)\sqrt{a+b\sec(c+dx)}}}{3\left(a^2-b^2\right)}$$

$\Big\downarrow$ 3042

$$\frac{\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}-\frac{\frac{b\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{1}{\sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{2\left(3a^2+b^2\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{a(a^2-b^2)}-\frac{4\left(a^2+b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a^2-b^2)\sqrt{a+b\sec(c+dx)}}}{3\left(a^2-b^2\right)}$$

$\Big\downarrow$ 3142

$$\frac{\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}-\frac{\frac{b\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{2\left(3a^2+b^2\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{a(a^2-b^2)}-\frac{4\left(a^2+b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a^2-b^2)\sqrt{a+b\sec(c+dx)}}}{3\left(a^2-b^2\right)}$$

$\Big\downarrow$ 3042

$$\frac{\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}-\frac{\frac{b\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{2\left(3a^2+b^2\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{a(a^2-b^2)}-\frac{4\left(a^2+b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a^2-b^2)\sqrt{a+b\sec(c+dx)}}}{3\left(a^2-b^2\right)}$$

$\Big\downarrow$ 3140

$$\frac{\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}-\frac{\frac{2b\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}+\frac{2\left(3a^2+b^2\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{a(a^2-b^2)}-\frac{4\left(a^2+b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a^2-b^2)\sqrt{a+b\sec(c+dx)}}}{3\left(a^2-b^2\right)}$$

input
```
Int[Sec[c + d*x]^(3/2)/(a + b*Sec[c + d*x])^(5/2),x]
```

3.664. $\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$

output
```
(2*a*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*(a^2 - b^2)*d*(a + b*Sec[c + d*x]
)^(3/2)) - (((2*b*(a^2 - b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF
[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(d*Sqrt[a + b*Sec[c + d*x
]]) + (2*(3*a^2 + b^2)*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Se
c[c + d*x]])/(d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]))/(a
*(a^2 - b^2)) - (4*(a^2 + b^2)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/((a^2 - b^
2)*d*Sqrt[a + b*Sec[c + d*x]]))/(3*(a^2 - b^2))
```

3.664.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

3.664. $\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$

rule 4331
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[a*d^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
)*((d*Csc[e + f*x])^(n - 2)/(f*(m + 1)*(a^2 - b^2))), x] - Simp[d^2/((m + 1
)*(a^2 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*
(a*(n - 2) + b*(m + 1)*Csc[e + f*x] - a*(m + n)*Csc[e + f*x]^2), x], x] /;
FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && LtQ[1, n, 2
] && IntegersQ[2*m, 2*n]
```

rule 4343
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]])   Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a
, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4345
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]])   Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[
{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4523
```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Simp[A/a   I
nt[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Simp[(A*b - a*B)
/(a*d)   Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ
[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

rule 4588
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc
[e + f*x])^(m + 1)*((d*Csc[e + f*x])^n/(a*f*(m + 1)*(a^2 - b^2))), x] + Sim
p[1/(a*(m + 1)*(a^2 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f
*x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n +
1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m
+ n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x
] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

3.664. $\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$

3.664.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2311 vs. $2(311) = 622$.

Time = 5.32 (sec) , antiderivative size = 2312, normalized size of antiderivative = 8.23

method	result	size
default	Expression too large to display	2312

input
```
int(sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3/d/(a-b)/(a+b)^2/a/((a-b)/(a+b))^(1/2)*(-5*EllipticF(((a-b)/(a+b))^(1/
2)*(-cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*(1/(cos(d*x+c)+1))^(1/2)
*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^2*b*cos(d*x+c)+2*Ellipt
icF(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*(1/
(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a*b^
2*cos(d*x+c)+6*EllipticE(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(-(a
+b)/(a-b))^(1/2))*(1/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(
d*x+c)+1))^(1/2)*a^2*b*cos(d*x+c)+EllipticE(((a-b)/(a+b))^(1/2)*(-cot(d*x+
c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*(1/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+
a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a*b^2*cos(d*x+c)+2*EllipticE(((a-b)/(a
+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*co
s(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a*b^2*cos(d*x+c)^
2-EllipticF(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1
/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1
/2)*a^2*b*cos(d*x+c)^2+EllipticF(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+
c)),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*
(1/(cos(d*x+c)+1))^(1/2)*a*b^2*cos(d*x+c)^2+EllipticE(((a-b)/(a+b))^(1/2)*
(-cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(
cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a*b^2*cos(d*x+c)^3+EllipticF
(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*(1/...
```

3.664.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.14 (sec) , antiderivative size = 759, normalized size of antiderivative = 2.70

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx =$$

$$-\frac{2\sqrt{2}\left(-3i\,a^2b^3+i\,b^5+\left(-3i\,a^4b+i\,a^2b^3\right)\cos\left(dx+c\right)^2+2\left(-3i\,a^3b^2+i\,ab^4\right)\cos\left(dx+c\right)\right)\sqrt{a}\text{weierstrassP}}{}$$

input
```
integrate(sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
-1/9*(2*sqrt(2)*(-3*I*a^2*b^3 + I*b^5 + (-3*I*a^4*b + I*a^2*b^3)*cos(d*x +
 c)^2 + 2*(-3*I*a^3*b^2 + I*a*b^4)*cos(d*x + c))*sqrt(a)*weierstrassPInver
se(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x
+ c) + 3*I*a*sin(d*x + c) + 2*b)/a) + 2*sqrt(2)*(3*I*a^2*b^3 - I*b^5 + (3*
I*a^4*b - I*a^2*b^3)*cos(d*x + c)^2 + 2*(3*I*a^3*b^2 - I*a*b^4)*cos(d*x +
c))*sqrt(a)*weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b -
8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a) + 3*sqrt(
2)*(3*I*a^3*b^2 + I*a*b^4 + (3*I*a^5 + I*a^3*b^2)*cos(d*x + c)^2 + 2*(3*I*
a^4*b + I*a^2*b^3)*cos(d*x + c))*sqrt(a)*weierstrassZeta(-4/3*(3*a^2 - 4*b
^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4*b
^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x
 + c) + 2*b)/a)) + 3*sqrt(2)*(-3*I*a^3*b^2 - I*a*b^4 + (-3*I*a^5 - I*a^3*b
^2)*cos(d*x + c)^2 + 2*(-3*I*a^4*b - I*a^2*b^3)*cos(d*x + c))*sqrt(a)*weie
rstrassZeta(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, weierstr
assPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a
*cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a)) - 6*((3*a^5 + a^3*b^2)*cos(d
*x + c)^2 + 2*(a^4*b + a^2*b^3)*cos(d*x + c))*sqrt((a*cos(d*x + c) + b)/co
s(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/((a^8 - 2*a^6*b^2 + a^4*b^4)*
d*cos(d*x + c)^2 + 2*(a^7*b - 2*a^5*b^3 + a^3*b^5)*d*cos(d*x + c) + (a^6*b
^2 - 2*a^4*b^4 + a^2*b^6)*d)
```

3.664.6 Sympy [F]

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)**(3/2)/(a+b*sec(d*x+c))**(5/2),x)
```

output
```
Integral(sec(c + d*x)**(3/2)/(a + b*sec(c + d*x))**(5/2), x)
```

3.664.7 Maxima [F]

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{\sec{(dx+c)}^{\frac{3}{2}}}{(b\sec{(dx+c)}+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)^(3/2)/(b*sec(d*x + c) + a)^(5/2), x)
```

3.664.8 Giac [F]

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{\sec{(dx+c)}^{\frac{3}{2}}}{(b\sec{(dx+c)}+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^(3/2)/(b*sec(d*x + c) + a)^(5/2), x)
```

3.664.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

input
```
int((1/cos(c + d*x))^(3/2)/(a + b/cos(c + d*x))^(5/2),x)
```

output
```
int((1/cos(c + d*x))^(3/2)/(a + b/cos(c + d*x))^(5/2), x)
```

3.665 $\int \frac{\sqrt{\sec(c+dx)}}{(a+b\sec(c+dx))^{5/2}}\,dx$

3.665.1 Optimal result . . . 5210
3.665.2 Mathematica [A] (verified) . . . 5211
3.665.3 Rubi [A] (verified) . . . 5211
3.665.4 Maple [B] (warning: unable to verify) . . . 5217
3.665.5 Fricas [C] (verification not implemented) . . . 5218
3.665.6 Sympy [F] . . . 5219
3.665.7 Maxima [F] . . . 5220
3.665.8 Giac [F] . . . 5220
3.665.9 Mupad [F(-1)] . . . 5220

3.665.1 Optimal result

Integrand size = 25, antiderivative size = 302

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+b\sec(c+dx))^{5/2}}\,dx = \frac{2(3a^2-2b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{3a^2(a^2-b^2)\,d\sqrt{a+b\sec(c+dx)}} + \frac{4b(3a^2-b^2)\,E\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+dx)}}{3a^2(a^2-b^2)^2\,d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\sqrt{\sec(c+dx)}} - \frac{2b\sqrt{\sec(c+dx)}\sin(c+dx)}{3(a^2-b^2)\,d(a+b\sec(c+dx))^{3/2}} - \frac{2b(5a^2-b^2)\sqrt{\sec(c+dx)}\sin(c+dx)}{3a(a^2-b^2)^2\,d\sqrt{a+b\sec(c+dx)}}$$

output
```
-2/3*b*sin(d*x+c)*sec(d*x+c)^(1/2)/(a^2-b^2)/d/(a+b*sec(d*x+c))^(3/2)-2/3*
b*(5*a^2-b^2)*sin(d*x+c)*sec(d*x+c)^(1/2)/a/(a^2-b^2)^2/d/(a+b*sec(d*x+c))
^(1/2)+2/3*(3*a^2-2*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*E
llipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+
b))^(1/2)*sec(d*x+c)^(1/2)/a^2/(a^2-b^2)/d/(a+b*sec(d*x+c))^(1/2)+4/3*b*(3
*a^2-b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/
2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*(a+b*sec(d*x+c))^(1/2)/a^2/(a^2-b^2)
^2/d/((b+a*cos(d*x+c))/(a+b))^(1/2)/sec(d*x+c)^(1/2)
```

3.665.2 Mathematica [A] (verified)

Time = 1.15 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.65

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+b\sec(c+dx))^{5/2}}\,dx = \frac{2(b+a\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\left(\frac{\left(\frac{b+a\cos(c+dx)}{a+b}\right)^{3/2}\left((6a^2b-2b^3)E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)+(3a\right.}{(a-b)^2}\right.}{3a^2d(a+b\,\mathrm{se}}$$

input
```
Integrate[Sqrt[Sec[c + d*x]]/(a + b*Sec[c + d*x])^(5/2),x]
```

output
```
(2*(b + a*Cos[c + d*x])*Sec[c + d*x]^(5/2)*((((b + a*Cos[c + d*x])/(a + b)
)^(3/2)*((6*a^2*b - 2*b^3)*EllipticE[(c + d*x)/2, (2*a)/(a + b)] + (3*a^3
- 3*a^2*b - 2*a*b^2 + 2*b^3)*EllipticF[(c + d*x)/2, (2*a)/(a + b)]))/(a -
b)^2 + (a*b*(-5*a^2*b + b^3 + (-6*a^3 + 2*a*b^2)*Cos[c + d*x])*Sin[c + d*x
])/(a^2 - b^2)^2))/(3*a^2*d*(a + b*Sec[c + d*x])^(5/2))
```

3.665.3 Rubi [A] (verified)

Time = 2.15 (sec) , antiderivative size = 314, normalized size of antiderivative = 1.04, number of steps used = 19, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.760$, Rules used = {3042, 4330, 27, 3042, 4588, 27, 3042, 4523, 3042, 4343, 3042, 3134, 3042, 3132, 4345, 3042, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+b\sec(c+dx))^{5/2}}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \; 4330$$

$$-\frac{2\int -\frac{-2b\sec^2(c+dx)+3a\sec(c+dx)+b}{2\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}}dx}{3\left(a^2-b^2\right)}-\frac{2b\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{-2b\sec^2(c+dx)+3a\sec(c+dx)+b}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}}dx}{3(a^2-b^2)}-\frac{2b\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a^2-b^2)(a+b\sec(c+dx))^{3/2}}$$

$\Big\downarrow$ 3042

$$\frac{\int \frac{-2b\csc\left(c+dx+\frac{\pi}{2}\right)^2+3a\csc\left(c+dx+\frac{\pi}{2}\right)+b}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{3(a^2-b^2)}-\frac{2b\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a^2-b^2)(a+b\sec(c+dx))^{3/2}}$$

$\Big\downarrow$ 4588

$$\frac{-\frac{2\int -\frac{2b\left(3a^2-b^2\right)+a\left(3a^2+b^2\right)\sec(c+dx)}{2\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{a(a^2-b^2)}-\frac{2b\left(5a^2-b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad(a^2-b^2)\sqrt{a+b\sec(c+dx)}}}{3(a^2-b^2)}-\frac{2b\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a^2-b^2)(a+b\sec(c+dx))^{3/2}}$$

$\Big\downarrow$ 27

$$\frac{\frac{\int \frac{2b\left(3a^2-b^2\right)+a\left(3a^2+b^2\right)\sec(c+dx)}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{a(a^2-b^2)}-\frac{2b\left(5a^2-b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad(a^2-b^2)\sqrt{a+b\sec(c+dx)}}}{3(a^2-b^2)}-\frac{2b\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a^2-b^2)(a+b\sec(c+dx))^{3/2}}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\int \frac{2b\left(3a^2-b^2\right)+a\left(3a^2+b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a(a^2-b^2)}-\frac{2b\left(5a^2-b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad(a^2-b^2)\sqrt{a+b\sec(c+dx)}}}{3(a^2-b^2)}-\frac{2b\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a^2-b^2)(a+b\sec(c+dx))^{3/2}}$$

$\Big\downarrow$ 4523

$$\frac{\frac{\frac{2b\left(3a^2-b^2\right)\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}dx}{a}+\frac{\left(3a^4-5a^2b^2+2b^4\right)\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}dx}{a}}{a(a^2-b^2)}-\frac{2b\left(5a^2-b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad(a^2-b^2)\sqrt{a+b\sec(c+dx)}}}{3(a^2-b^2)}-\frac{2b\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a^2-b^2)(a+b\sec(c+dx))^{3/2}}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\frac{2b\left(3a^2-b^2\right)\int \frac{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}+\frac{\left(3a^4-5a^2b^2+2b^4\right)\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}}{a(a^2-b^2)}-\frac{2b\left(5a^2-b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad(a^2-b^2)\sqrt{a+b\sec(c+dx)}}}{3(a^2-b^2)}-\frac{2b\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a^2-b^2)(a+b\sec(c+dx))^{3/2}}$$

3.665. $\int \frac{\sqrt{\sec(c+dx)}}{(a+b\sec(c+dx))^{5/2}}\,dx$

$$\Big\downarrow \text{4343}$$

$$\frac{\frac{\frac{2b\left(3a^2-b^2\right)\sqrt{a+b\sec(c+dx)}\int\sqrt{b+a\cos(c+dx)}dx}{a\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}+\frac{\left(3a^4-5a^2b^2+2b^4\right)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}}{a(a^2-b^2)}-\frac{2b(5a^2-b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad(a^2-b^2)\sqrt{a+b\sec(c+dx)}}}{3\left(a^2-b^2\right)}-\frac{2b\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\frac{2b\left(3a^2-b^2\right)\sqrt{a+b\sec(c+dx)}\int\sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{a\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}+\frac{\left(3a^4-5a^2b^2+2b^4\right)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}}{a(a^2-b^2)}-\frac{2b(5a^2-b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad(a^2-b^2)\sqrt{a+b\sec(c+dx)}}}{3\left(a^2-b^2\right)}-\frac{2b\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3134}$$

$$\frac{\frac{\frac{2b\left(3a^2-b^2\right)\sqrt{a+b\sec(c+dx)}\int\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}dx}{a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{\left(3a^4-5a^2b^2+2b^4\right)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}}{a(a^2-b^2)}-\frac{2b(5a^2-b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad(a^2-b^2)\sqrt{a+b\sec(c+dx)}}}{3\left(a^2-b^2\right)}-\frac{2b\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\frac{2b\left(3a^2-b^2\right)\sqrt{a+b\sec(c+dx)}\int\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}dx}{a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{\left(3a^4-5a^2b^2+2b^4\right)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}}{a(a^2-b^2)}-\frac{2b(5a^2-b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad(a^2-b^2)\sqrt{a+b\sec(c+dx)}}}{3\left(a^2-b^2\right)}-\frac{2b\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3132}$$

3.665. $\int \frac{\sqrt{\sec(c+dx)}}{(a+b\sec(c+dx))^{5/2}}\,dx$

$$\frac{\frac{\frac{\left(3a^4-5a^2b^2+2b^4\right)\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}+\frac{4b\left(3a^2-b^2\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{a(a^2-b^2)}-\frac{2b\left(5a^2-b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad(a^2-b^2)\sqrt{a+b\sec(c+dx)}}}{3\left(a^2-b^2\right)}-\frac{2b\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}$$

$\downarrow$ 4345

$$\frac{\frac{\frac{\left(3a^4-5a^2b^2+2b^4\right)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int \frac{1}{\sqrt{b+a\cos(c+dx)}}dx}{a\sqrt{a+b\sec(c+dx)}}+\frac{4b\left(3a^2-b^2\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{a(a^2-b^2)}-\frac{2b\left(5a^2-b^2\right)\sin(c+dx)\sqrt{\sec(c+d}}{ad(a^2-b^2)\sqrt{a+b\sec(c+dx)}}}{3\left(a^2-b^2\right)}-\frac{2b\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}$$

$\downarrow$ 3042

$$\frac{\frac{\frac{\left(3a^4-5a^2b^2+2b^4\right)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int \frac{1}{\sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{a\sqrt{a+b\sec(c+dx)}}+\frac{4b\left(3a^2-b^2\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{a(a^2-b^2)}-\frac{2b\left(5a^2-b^2\right)\sin(c+dx)\sqrt{\sec(}}{ad(a^2-b^2)\sqrt{a+b\sec(c+d}}}{3\left(a^2-b^2\right)}-\frac{2b\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}$$

$\downarrow$ 3142

$$\frac{\frac{\frac{\left(3a^4-5a^2b^2+2b^4\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int \frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}}dx}{a\sqrt{a+b\sec(c+dx)}}+\frac{4b\left(3a^2-b^2\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{a(a^2-b^2)}-\frac{2b\left(5a^2-b^2\right)\sin(c+dx)\sqrt{\sec(}}{ad(a^2-b^2)\sqrt{a+b\sec(c+}}}{3\left(a^2-b^2\right)}-\frac{2b\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}$$

$\downarrow$ 3042

3.665. $\int \frac{\sqrt{\sec(c+dx)}}{(a+b\sec(c+dx))^{5/2}}\,dx$

$$\frac{\frac{\frac{\left(3a^4-5a^2b^2+2b^4\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{a\sqrt{a+b\sec(c+dx)}}+\frac{4b\left(3a^2-b^2\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{a(a^2-b^2)}-\frac{2b(5a^2-b^2)\sin(c+dx)\sqrt{}}{ad(a^2-b^2)\sqrt{a+b\sec}}}{\frac{2b\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a^2-b^2)(a+b\sec(c+dx))^{3/2}}}\,3(a^2-b^2)$$

$$\Big\downarrow \; 3140$$

$$\frac{\frac{\frac{4b\left(3a^2-b^2\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{2\left(3a^4-5a^2b^2+2b^4\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{ad\sqrt{a+b\sec(c+dx)}}}{a(a^2-b^2)}-\frac{2b(5a^2-b^2)\sin(c+dx)\sqrt{\sec}}{ad(a^2-b^2)\sqrt{a+b\sec(c+}}}{\frac{2b\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a^2-b^2)(a+b\sec(c+dx))^{3/2}}}\,3(a^2-b^2)$$

input
```
Int[Sqrt[Sec[c + d*x]]/(a + b*Sec[c + d*x])^(5/2),x]
```

output
```
(-2*b*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*(a^2 - b^2)*d*(a + b*Sec[c + d*x
])^(3/2)) + (((2*(3*a^4 - 5*a^2*b^2 + 2*b^4)*Sqrt[(b + a*Cos[c + d*x])/(a
+ b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(a*d*Sqrt[
a + b*Sec[c + d*x]]) + (4*b*(3*a^2 - b^2)*EllipticE[(c + d*x)/2, (2*a)/(a
+ b)]*Sqrt[a + b*Sec[c + d*x]])/(a*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sq
rt[Sec[c + d*x]]))/(a*(a^2 - b^2)) - (2*b*(5*a^2 - b^2)*Sqrt[Sec[c + d*x]]
*Sin[c + d*x])/(a*(a^2 - b^2)*d*Sqrt[a + b*Sec[c + d*x]]))/(3*(a^2 - b^2))
```

3.665.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 4330
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b)*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m +
1)*((d*Csc[e + f*x])^(n - 1)/(f*(m + 1)*(a^2 - b^2))), x] + Simp[1/((m + 1)
*(a^2 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1)*S
imp[b*d*(n - 1) + a*d*(m + 1)*Csc[e + f*x] - b*d*(m + n + 1)*Csc[e + f*x]^2
, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1
] && LtQ[0, n, 1] && IntegersQ[2*m, 2*n]
```

rule 4343
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]])   Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a
, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4345
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]])   Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[
{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

3.665. $\int \frac{\sqrt{\sec(c+dx)}}{(a+b\sec(c+dx))^{5/2}}\,dx$

rule 4523
```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Simp[A/a   I
nt[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Simp[(A*b - a*B)
/(a*d)   Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ
[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

rule 4588
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc
[e + f*x])^(m + 1)*((d*Csc[e + f*x])^n/(a*f*(m + 1)*(a^2 - b^2))), x] + Sim
p[1/(a*(m + 1)*(a^2 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f
*x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n +
1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m
+ n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x
] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

3.665.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 2884 vs. $2(332) = 664$.

Time = 7.60 (sec) , antiderivative size = 2885, normalized size of antiderivative = 9.55

method	result	size
default	Expression too large to display	2885

input
```
int(sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3/d/((a-b)/(a+b))^(1/2)/(a+b)^2/(a-b)/a^2*(-((1-cos(d*x+c))^2*csc(d*x+c
)^2+1)/((1-cos(d*x+c))^2*csc(d*x+c)^2-1))^(1/2)*((1-cos(d*x+c))^2*csc(d*x+
c)^2-1)*((a*(1-cos(d*x+c))^2*csc(d*x+c)^2-b*(1-cos(d*x+c))^2*csc(d*x+c)^2-
a-b)/((1-cos(d*x+c))^2*csc(d*x+c)^2-1))^(1/2)*(2*((a-b)/(a+b))^(1/2)*b^4*(
-cot(d*x+c)+csc(d*x+c))-2*(-(a*(1-cos(d*x+c))^2*csc(d*x+c)^2-b*(1-cos(d*x+
c))^2*csc(d*x+c)^2-a-b)/(a+b))^(1/2)*((1-cos(d*x+c))^2*csc(d*x+c)^2+1)^(1/
2)*EllipticE(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(
1/2))*b^4*(1-cos(d*x+c))^2*csc(d*x+c)^2-(-(a*(1-cos(d*x+c))^2*csc(d*x+c)^2
-b*(1-cos(d*x+c))^2*csc(d*x+c)^2-a-b)/(a+b))^(1/2)*((1-cos(d*x+c))^2*csc(d
*x+c)^2+1)^(1/2)*EllipticF(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(-
(a+b)/(a-b))^(1/2))*a^2*b^2*(1-cos(d*x+c))^2*csc(d*x+c)^2-2*(-(a*(1-cos(d*
x+c))^2*csc(d*x+c)^2-b*(1-cos(d*x+c))^2*csc(d*x+c)^2-a-b)/(a+b))^(1/2)*((1
-cos(d*x+c))^2*csc(d*x+c)^2+1)^(1/2)*EllipticF(((a-b)/(a+b))^(1/2)*(-cot(d
*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*a*b^3*(1-cos(d*x+c))^2*csc(d*x+c)^
2-6*(-(a*(1-cos(d*x+c))^2*csc(d*x+c)^2-b*(1-cos(d*x+c))^2*csc(d*x+c)^2-a-b
)/(a+b))^(1/2)*((1-cos(d*x+c))^2*csc(d*x+c)^2+1)^(1/2)*EllipticE(((a-b)/(a
+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*a^3*b*(1-cos(d*x
+c))^2*csc(d*x+c)^2+6*(-(a*(1-cos(d*x+c))^2*csc(d*x+c)^2-b*(1-cos(d*x+c))^
2*csc(d*x+c)^2-a-b)/(a+b))^(1/2)*((1-cos(d*x+c))^2*csc(d*x+c)^2+1)^(1/2)*E
llipticE(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1...
```

3.665.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.15 (sec) , antiderivative size = 811, normalized size of antiderivative = 2.69

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+b\sec(c+dx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
1/9*(sqrt(2)*(-9*I*a^4*b^2 + 9*I*a^2*b^4 - 4*I*b^6 + (-9*I*a^6 + 9*I*a^4*b
^2 - 4*I*a^2*b^4)*cos(d*x + c)^2 - 2*(9*I*a^5*b - 9*I*a^3*b^3 + 4*I*a*b^5)
*cos(d*x + c))*sqrt(a)*weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*
(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x + c) + 2*b)/a
) + sqrt(2)*(9*I*a^4*b^2 - 9*I*a^2*b^4 + 4*I*b^6 + (9*I*a^6 - 9*I*a^4*b^2
+ 4*I*a^2*b^4)*cos(d*x + c)^2 - 2*(-9*I*a^5*b + 9*I*a^3*b^3 - 4*I*a*b^5)*c
os(d*x + c))*sqrt(a)*weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9
*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a)
- 6*sqrt(2)*(-3*I*a^3*b^3 + I*a*b^5 + (-3*I*a^5*b + I*a^3*b^3)*cos(d*x + c
)^2 + 2*(-3*I*a^4*b^2 + I*a^2*b^4)*cos(d*x + c))*sqrt(a)*weierstrassZeta(-
4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-
4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c)
 + 3*I*a*sin(d*x + c) + 2*b)/a)) - 6*sqrt(2)*(3*I*a^3*b^3 - I*a*b^5 + (3*I
*a^5*b - I*a^3*b^3)*cos(d*x + c)^2 + 2*(3*I*a^4*b^2 - I*a^2*b^4)*cos(d*x +
 c))*sqrt(a)*weierstrassZeta(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b
^3)/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b
^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a)) - 6*(2*(3*a
^5*b - a^3*b^3)*cos(d*x + c)^2 + (5*a^4*b^2 - a^2*b^4)*cos(d*x + c))*sqrt(
(a*cos(d*x + c) + b)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/((a^9
- 2*a^7*b^2 + a^5*b^4)*d*cos(d*x + c)^2 + 2*(a^8*b - 2*a^6*b^3 + a^4*b^...
```

3.665.6 Sympy [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{\sqrt{\sec{(c+dx)}}}{(a+b\sec{(c+dx)})^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)**(1/2)/(a+b*sec(d*x+c))**(5/2),x)
```

output
```
Integral(sqrt(sec(c + d*x))/(a + b*sec(c + d*x))**(5/2), x)
```

3.665.7 Maxima [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{\sqrt{\sec(dx+c)}}{(b\sec(dx+c)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(sec(d*x + c))/(b*sec(d*x + c) + a)^(5/2), x)
```

3.665.8 Giac [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{\sqrt{\sec(dx+c)}}{(b\sec(dx+c)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate(sqrt(sec(d*x + c))/(b*sec(d*x + c) + a)^(5/2), x)
```

3.665.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{\sqrt{\frac{1}{\cos(c+d\,x)}}}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

input
```
int((1/cos(c + d*x))^(1/2)/(a + b/cos(c + d*x))^(5/2),x)
```

output
```
int((1/cos(c + d*x))^(1/2)/(a + b/cos(c + d*x))^(5/2), x)
```

3.666 $\int \frac{1}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{5/2}}\,dx$

3.666.1 Optimal result 5221
3.666.2 Mathematica [A] (verified) 5222
3.666.3 Rubi [A] (verified) 5222
3.666.4 Maple [B] (verified) 5228
3.666.5 Fricas [C] (verification not implemented) 5229
3.666.6 Sympy [F] 5230
3.666.7 Maxima [F] 5231
3.666.8 Giac [F] 5231
3.666.9 Mupad [F(-1)] 5231

3.666.1 Optimal result

Integrand size = 25, antiderivative size = 317

$$\int \frac{1}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{5/2}}\,dx = -\frac{2b(9a^2-8b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{3a^3(a^2-b^2)\,d\sqrt{a+b\sec(c+dx)}} + \frac{2(3a^4-15a^2b^2+8b^4)\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+dx)}}{3a^3(a^2-b^2)^2\,d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\sqrt{\sec(c+dx)}} + \frac{2b^2\sqrt{\sec(c+dx)}\sin(c+dx)}{3a(a^2-b^2)\,d(a+b\sec(c+dx))^{3/2}} + \frac{8b^2(2a^2-b^2)\sqrt{\sec(c+dx)}\sin(c+dx)}{3a^2(a^2-b^2)^2\,d\sqrt{a+b\sec(c+dx)}}$$

output
```
2/3*b^2*sin(d*x+c)*sec(d*x+c)^(1/2)/a/(a^2-b^2)/d/(a+b*sec(d*x+c))^(3/2)+8
/3*b^2*(2*a^2-b^2)*sin(d*x+c)*sec(d*x+c)^(1/2)/a^2/(a^2-b^2)^2/d/(a+b*sec(
d*x+c))^(1/2)-2/3*b*(9*a^2-8*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x
+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*
x+c))/(a+b))^(1/2)*sec(d*x+c)^(1/2)/a^3/(a^2-b^2)/d/(a+b*sec(d*x+c))^(1/2)
+2/3*(3*a^4-15*a^2*b^2+8*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2
*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*(a+b*sec(d*x+c))
^(1/2)/a^3/(a^2-b^2)^2/d/((b+a*cos(d*x+c))/(a+b))^(1/2)/sec(d*x+c)^(1/2)
```

3.666.2 Mathematica [A] (verified)

Time = 1.32 (sec) , antiderivative size = 208, normalized size of antiderivative = 0.66

$$\int \frac{1}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{5/2}}\,dx = \frac{2(b+a\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\left(\frac{\left(\frac{b+a\cos(c+dx)}{a+b}\right)^{3/2}\left((3a^4-15a^2b^2+8b\right.}{}\right.}{}$$

input
```
Integrate[1/(Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^(5/2)),x]
```

output
```
(2*(b + a*Cos[c + d*x])*Sec[c + d*x]^(5/2)*((((b + a*Cos[c + d*x])/(a + b)
)^(3/2)*((3*a^4 - 15*a^2*b^2 + 8*b^4)*EllipticE[(c + d*x)/2, (2*a)/(a + b)
] + b*(-9*a^3 + 9*a^2*b + 8*a*b^2 - 8*b^3)*EllipticF[(c + d*x)/2, (2*a)/(a
 + b)]))/(a - b)^2 + (a*b^2*(8*a^2*b - 4*b^3 + a*(9*a^2 - 5*b^2)*Cos[c + d
*x])*Sin[c + d*x])/(a^2 - b^2)^2))/(3*a^3*d*(a + b*Sec[c + d*x])^(5/2))
```

3.666.3 Rubi [A] (verified)

Time = 2.33 (sec) , antiderivative size = 332, normalized size of antiderivative = 1.05, number of steps used = 19, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.760$, Rules used = {3042, 4334, 27, 3042, 4588, 27, 3042, 4523, 3042, 4343, 3042, 3134, 3042, 3132, 4345, 3042, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{5/2}}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \; 4334$$

$$\frac{2b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{3ad\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}} - \frac{2\int -\frac{3a^2-3b\sec(c+dx)a-4b^2+2b^2\sec^2(c+dx)}{2\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}}dx}{3a\left(a^2-b^2\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{3a^2-3b\sec(c+dx)a-4b^2+2b^2\sec^2(c+dx)}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}}dx}{3a\left(a^2-b^2\right)}+\frac{2b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{3ad\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}$$

$\Big\downarrow$ 3042

$$\frac{\int \frac{3a^2-3b\csc\left(c+dx+\frac{\pi}{2}\right)a-4b^2+2b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{3a\left(a^2-b^2\right)}+\frac{2b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{3ad\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}$$

$\Big\downarrow$ 4588

$$\frac{\frac{8b^2\left(2a^2-b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad(a^2-b^2)\sqrt{a+b\sec(c+dx)}}-\frac{2\int -\frac{3a^4-15b^2a^2-2b\left(3a^2-b^2\right)\sec(c+dx)a+8b^4}{2\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{a(a^2-b^2)}}{3a\left(a^2-b^2\right)}+\frac{2b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{3ad\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}$$

$\Big\downarrow$ 27

$$\frac{\frac{\int \frac{3a^4-15b^2a^2-2b\left(3a^2-b^2\right)\sec(c+dx)a+8b^4}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{a(a^2-b^2)}+\frac{8b^2\left(2a^2-b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad(a^2-b^2)\sqrt{a+b\sec(c+dx)}}}{3a\left(a^2-b^2\right)}+\frac{2b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{3ad\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\int \frac{3a^4-15b^2a^2-2b\left(3a^2-b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)a+8b^4}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a(a^2-b^2)}+\frac{8b^2\left(2a^2-b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad(a^2-b^2)\sqrt{a+b\sec(c+dx)}}}{3a\left(a^2-b^2\right)}+\frac{2b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{3ad\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}$$

$\Big\downarrow$ 4523

$$\frac{\frac{\frac{\left(3a^4-15a^2b^2+8b^4\right)\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}dx}{a}-\frac{b\left(9a^4-17a^2b^2+8b^4\right)\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}dx}{a}}{a(a^2-b^2)}+\frac{8b^2\left(2a^2-b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad(a^2-b^2)\sqrt{a+b\sec(c+dx)}}}{3a\left(a^2-b^2\right)}+\frac{2b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{3ad\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}$$

$\Big\downarrow$ 3042

3.666. $\int \frac{1}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{5/2}}\,dx$

$$\frac{\frac{\frac{\left(3a^4-15a^2b^2+8b^4\right)\int \frac{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}-\frac{b\left(9a^4-17a^2b^2+8b^4\right)\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}}{a(a^2-b^2)}+\frac{8b^2\left(2a^2-b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad(a^2-b^2)\sqrt{a+b\sec(c+dx)}}}{3a\left(a^2-b^2\right)}+\frac{2b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{3ad\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}$$

$\downarrow$ 4343

$$\frac{\frac{\frac{\left(3a^4-15a^2b^2+8b^4\right)\sqrt{a+b\sec(c+dx)}\int \sqrt{b+a\cos(c+dx)}dx}{a\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}-\frac{b\left(9a^4-17a^2b^2+8b^4\right)\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}}{a(a^2-b^2)}+\frac{8b^2\left(2a^2-b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad(a^2-b^2)\sqrt{a+b\sec(c+dx)}}}{3a\left(a^2-b^2\right)}+\frac{2b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{3ad\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}$$

$\downarrow$ 3042

$$\frac{\frac{\frac{\left(3a^4-15a^2b^2+8b^4\right)\sqrt{a+b\sec(c+dx)}\int \sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{a\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}-\frac{b\left(9a^4-17a^2b^2+8b^4\right)\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}}{a(a^2-b^2)}+\frac{8b^2\left(2a^2-b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad(a^2-b^2)\sqrt{a+b\sec(c+dx)}}}{3a\left(a^2-b^2\right)}+\frac{2b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{3ad\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}$$

$\downarrow$ 3134

$$\frac{\frac{\frac{\left(3a^4-15a^2b^2+8b^4\right)\sqrt{a+b\sec(c+dx)}\int \sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}dx}{a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{b\left(9a^4-17a^2b^2+8b^4\right)\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}}{a(a^2-b^2)}+\frac{8b^2\left(2a^2-b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad(a^2-b^2)\sqrt{a+b\sec(c+dx)}}}{3a\left(a^2-b^2\right)}+\frac{2b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{3ad\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}$$

$\downarrow$ 3042

3.666. $\int \frac{1}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{5/2}}\,dx$

$$\frac{\frac{\frac{\left(3a^4-15a^2b^2+8b^4\right)\sqrt{a+b\sec(c+dx)}\int\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}dx}{a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{b\left(9a^4-17a^2b^2+8b^4\right)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}}{a(a^2-b^2)}+\frac{8b^2\left(2a^2-b^2\right)\sin(c+dx)\sqrt{\sec(c+dx}}{ad(a^2-b^2)\sqrt{a+b\sec(c+dx)}}}{3a\left(a^2-b^2\right)}$$

$$\frac{2b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{3ad\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}$$

$\downarrow$ 3132

$$\frac{\frac{\frac{2\left(3a^4-15a^2b^2+8b^4\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{b\left(9a^4-17a^2b^2+8b^4\right)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}}{a(a^2-b^2)}+\frac{8b^2\left(2a^2-b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad(a^2-b^2)\sqrt{a+b\sec(c+dx)}}}{3a\left(a^2-b^2\right)}+$$

$$\frac{2b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{3ad\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}$$

$\downarrow$ 4345

$$\frac{\frac{\frac{2\left(3a^4-15a^2b^2+8b^4\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{b\left(9a^4-17a^2b^2+8b^4\right)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{1}{\sqrt{b+a\cos(c+dx)}}dx}{a\sqrt{a+b\sec(c+dx)}}}{a(a^2-b^2)}+\frac{8b^2\left(2a^2-b^2\right)\sin(c+dx}{ad(a^2-b^2)\sqrt{a+b\mathrm{se}}}}{3a\left(a^2-b^2\right)}$$

$$\frac{2b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{3ad\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}$$

$\downarrow$ 3042

$$\frac{\frac{\frac{2\left(3a^4-15a^2b^2+8b^4\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{b\left(9a^4-17a^2b^2+8b^4\right)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{1}{\sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{a\sqrt{a+b\sec(c+dx)}}}{a(a^2-b^2)}+\frac{8b^2\left(2a^2-b^2\right)\sin(c-}{ad(a^2-b^2)\sqrt{a-}}}{3a\left(a^2-b^2\right)}$$

$$\frac{2b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{3ad\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}$$

$\downarrow$ 3142

3.666. $\int\frac{1}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{5/2}}\,dx$

$$\frac{\frac{\frac{2\left(3a^4-15a^2b^2+8b^4\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{b\left(9a^4-17a^2b^2+8b^4\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}}dx}{a\sqrt{a+b\sec(c+dx)}}}{a(a^2-b^2)}+\frac{8b^2\left(2a^2-b^2\right)\sin(c}{ad(a^2-b^2)\sqrt{a}}}{3a\left(a^2-b^2\right)}$$

$$\frac{2b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{3ad\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{\frac{2\left(3a^4-15a^2b^2+8b^4\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{b\left(9a^4-17a^2b^2+8b^4\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{a\sqrt{a+b\sec(c+dx)}}}{a(a^2-b^2)}+\frac{8b^2\left(2a^2-b^2\right)\mathrm{s}}{ad(a^2-b^2)}}{3a\left(a^2-b^2\right)}$$

$$\frac{2b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{3ad\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \; 3140$$

$$\frac{2b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{3ad\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}+$$

$$\frac{\frac{8b^2\left(2a^2-b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad(a^2-b^2)\sqrt{a+b\sec(c+dx)}}+\frac{\frac{\frac{2\left(3a^4-15a^2b^2+8b^4\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{2b\left(9a^4-17a^2b^2+8b^4\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\mathrm{Ellipt}}{ad\sqrt{a+b\sec(c+dx)}}}{a(a^2-b^2)}}{3a\left(a^2-b^2\right)}$$

input
```
Int[1/(Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^(5/2)),x]
```

output
```
(2*b^2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*a*(a^2 - b^2)*d*(a + b*Sec[c +
d*x])^(3/2)) + (((-2*b*(9*a^4 - 17*a^2*b^2 + 8*b^4)*Sqrt[(b + a*Cos[c + d*
x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(a*
d*Sqrt[a + b*Sec[c + d*x]]) + (2*(3*a^4 - 15*a^2*b^2 + 8*b^4)*EllipticE[(c
 + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(a*d*Sqrt[(b + a*Cos[c
 + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]))/(a*(a^2 - b^2)) + (8*b^2*(2*a^2 - b
^2)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(a*(a^2 - b^2)*d*Sqrt[a + b*Sec[c + d
*x]]))/(3*a*(a^2 - b^2))
```

3.666. $\int \frac{1}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{5/2}}\,dx$

3.666.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 4334
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*
((d*Csc[e + f*x])^n/(a*f*(m + 1)*(a^2 - b^2))), x] + Simp[1/(a*(m + 1)*(a^2
 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a^2*(m + 1)
 - b^2*(m + n + 1) - a*b*(m + 1)*Csc[e + f*x] + b^2*(m + n + 2)*Csc[e + f*x
]^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0] && LtQ[m,
 -1] && IntegersQ[2*m, 2*n]
```

3.666. $\int \frac{1}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{5/2}}\,dx$

rule 4343
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]])   Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a
, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4345
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]])   Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[
{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4523
```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Simp[A/a   I
nt[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Simp[(A*b - a*B)
/(a*d)   Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ
[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

rule 4588
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc
[e + f*x])^(m + 1)*((d*Csc[e + f*x])^n/(a*f*(m + 1)*(a^2 - b^2))), x] + Sim
p[1/(a*(m + 1)*(a^2 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f
*x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n +
1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m
+ n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x
] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

3.666.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 4119 vs. $2(347) = 694$.

Time = 8.94 (sec) , antiderivative size = 4120, normalized size of antiderivative = 13.00

method	result	size
default	Expression too large to display	4120

input
```
int(1/sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/3/d/a^3/(a-b)/(a+b)^2/((a-b)/(a+b))^(1/2)*(3*(-(a*(1-cos(d*x+c))^2*csc(d
*x+c)^2-b*(1-cos(d*x+c))^2*csc(d*x+c)^2-a-b)/(a+b))^(1/2)*((1-cos(d*x+c))^
2*csc(d*x+c)^2+1)^(1/2)*EllipticF(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x
+c)),(-(a+b)/(a-b))^(1/2))*a^5*(1-cos(d*x+c))^2*csc(d*x+c)^2+3*(-(a*(1-cos
(d*x+c))^2*csc(d*x+c)^2-b*(1-cos(d*x+c))^2*csc(d*x+c)^2-a-b)/(a+b))^(1/2)*
((1-cos(d*x+c))^2*csc(d*x+c)^2+1)^(1/2)*EllipticE(((a-b)/(a+b))^(1/2)*(-co
t(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*a^4*b*(1-cos(d*x+c))^2*csc(d*x+
c)^2+15*(-(a*(1-cos(d*x+c))^2*csc(d*x+c)^2-b*(1-cos(d*x+c))^2*csc(d*x+c)^2
-a-b)/(a+b))^(1/2)*((1-cos(d*x+c))^2*csc(d*x+c)^2+1)^(1/2)*EllipticE(((a-b
)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*a^3*b^2*(1-c
os(d*x+c))^2*csc(d*x+c)^2-15*(-(a*(1-cos(d*x+c))^2*csc(d*x+c)^2-b*(1-cos(d
*x+c))^2*csc(d*x+c)^2-a-b)/(a+b))^(1/2)*((1-cos(d*x+c))^2*csc(d*x+c)^2+1)^
(1/2)*EllipticE(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b)
)^(1/2))*a^2*b^3*(1-cos(d*x+c))^2*csc(d*x+c)^2-8*(-(a*(1-cos(d*x+c))^2*csc
(d*x+c)^2-b*(1-cos(d*x+c))^2*csc(d*x+c)^2-a-b)/(a+b))^(1/2)*((1-cos(d*x+c)
)^2*csc(d*x+c)^2+1)^(1/2)*EllipticE(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d
*x+c)),(-(a+b)/(a-b))^(1/2))*a*b^4*(1-cos(d*x+c))^2*csc(d*x+c)^2-15*((a-b)
/(a+b))^(1/2)*a^3*b^2*(1-cos(d*x+c))^5*csc(d*x+c)^5+15*((a-b)/(a+b))^(1/2)
*a^2*b^3*(1-cos(d*x+c))^5*csc(d*x+c)^5+8*((a-b)/(a+b))^(1/2)*a*b^4*(1-cos(
d*x+c))^5*csc(d*x+c)^5-6*((a-b)/(a+b))^(1/2)*a^4*b*(1-cos(d*x+c))^3*csc...
```

3.666.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.16 (sec) , antiderivative size = 863, normalized size of antiderivative = 2.72

$$\int \frac{1}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
-1/9*(4*sqrt(2)*(-6*I*a^4*b^3 + 9*I*a^2*b^5 - 4*I*b^7 + (-6*I*a^6*b + 9*I*
a^4*b^3 - 4*I*a^2*b^5)*cos(d*x + c)^2 + 2*(-6*I*a^5*b^2 + 9*I*a^3*b^4 - 4*
I*a*b^6)*cos(d*x + c))*sqrt(a)*weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^
2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x + c)
+ 2*b)/a) + 4*sqrt(2)*(6*I*a^4*b^3 - 9*I*a^2*b^5 + 4*I*b^7 + (6*I*a^6*b -
9*I*a^4*b^3 + 4*I*a^2*b^5)*cos(d*x + c)^2 + 2*(6*I*a^5*b^2 - 9*I*a^3*b^4 +
 4*I*a*b^6)*cos(d*x + c))*sqrt(a)*weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)
/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x +
c) + 2*b)/a) + 3*sqrt(2)*(-3*I*a^5*b^2 + 15*I*a^3*b^4 - 8*I*a*b^6 + (-3*I*
a^7 + 15*I*a^5*b^2 - 8*I*a^3*b^4)*cos(d*x + c)^2 + 2*(-3*I*a^6*b + 15*I*a^
4*b^3 - 8*I*a^2*b^5)*cos(d*x + c))*sqrt(a)*weierstrassZeta(-4/3*(3*a^2 - 4
*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4
*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d
*x + c) + 2*b)/a)) + 3*sqrt(2)*(3*I*a^5*b^2 - 15*I*a^3*b^4 + 8*I*a*b^6 + (
3*I*a^7 - 15*I*a^5*b^2 + 8*I*a^3*b^4)*cos(d*x + c)^2 + 2*(3*I*a^6*b - 15*I
*a^4*b^3 + 8*I*a^2*b^5)*cos(d*x + c))*sqrt(a)*weierstrassZeta(-4/3*(3*a^2
- 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a^2
- 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*si
n(d*x + c) + 2*b)/a)) - 6*((9*a^5*b^2 - 5*a^3*b^4)*cos(d*x + c)^2 + 4*(2*a
^4*b^3 - a^2*b^5)*cos(d*x + c))*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))...
```

3.666.6 Sympy [F]

$$\int \frac{1}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{1}{(a+b\sec(c+dx))^{\frac{5}{2}}\sqrt{\sec(c+dx)}}\,dx$$

input
```
integrate(1/sec(d*x+c)**(1/2)/(a+b*sec(d*x+c))**(5/2),x)
```

output
```
Integral(1/((a + b*sec(c + d*x))**(5/2)*sqrt(sec(c + d*x))), x)
```

3.666.7 Maxima [F]

$$\int \frac{1}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{1}{(b\sec(dx+c)+a)^{\frac{5}{2}}\sqrt{\sec(dx+c)}}\,dx$$

input
```
integrate(1/sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*sec(d*x + c) + a)^(5/2)*sqrt(sec(d*x + c))), x)
```

3.666.8 Giac [F]

$$\int \frac{1}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{1}{(b\sec(dx+c)+a)^{\frac{5}{2}}\sqrt{\sec(dx+c)}}\,dx$$

input
```
integrate(1/sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate(1/((b*sec(d*x + c) + a)^(5/2)*sqrt(sec(d*x + c))), x)
```

3.666.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{1}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2}\sqrt{\frac{1}{\cos(c+d\,x)}}}\,dx$$

input
```
int(1/((a + b/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(1/2)),x)
```

output
```
int(1/((a + b/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(1/2)), x)
```

3.667 $\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx$

3.667.1 Optimal result 5232
3.667.2 Mathematica [A] (verified) 5233
3.667.3 Rubi [A] (verified) 5233
3.667.4 Maple [B] (verified) 5240
3.667.5 Fricas [C] (verification not implemented) 5241
3.667.6 Sympy [F(-1)] 5242
3.667.7 Maxima [F] 5243
3.667.8 Giac [F] 5243
3.667.9 Mupad [F(-1)] 5243

3.667.1 Optimal result

Integrand size = 25, antiderivative size = 391

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx = \frac{2(a^4+16a^2b^2-16b^4)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)\sqrt{}}{3a^4(a^2-b^2)\,d\sqrt{a+b\sec(c+dx)}}$$
$$-\frac{8b(2a^4-7a^2b^2+4b^4)\,E\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+dx)}}{3a^4(a^2-b^2)^2\,d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\sqrt{\sec(c+dx)}}$$
$$+\frac{2b^2\sin(c+dx)}{3a(a^2-b^2)\,d\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}}$$
$$+\frac{4b^2(5a^2-3b^2)\sin(c+dx)}{3a^2(a^2-b^2)^2\,d\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}$$
$$+\frac{2(a^4-13a^2b^2+8b^4)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{3a^3(a^2-b^2)^2\,d\sqrt{\sec(c+dx)}}$$

output
```
2/3*b^2*sin(d*x+c)/a/(a^2-b^2)/d/(a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(1/2)+4
/3*b^2*(5*a^2-3*b^2)*sin(d*x+c)/a^2/(a^2-b^2)^2/d/sec(d*x+c)^(1/2)/(a+b*se
c(d*x+c))^(1/2)+2/3*(a^4+16*a^2*b^2-16*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/c
os(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((
b+a*cos(d*x+c))/(a+b))^(1/2)*sec(d*x+c)^(1/2)/a^4/(a^2-b^2)/d/(a+b*sec(d*x
+c))^(1/2)+2/3*(a^4-13*a^2*b^2+8*b^4)*sin(d*x+c)*(a+b*sec(d*x+c))^(1/2)/a^
3/(a^2-b^2)^2/d/sec(d*x+c)^(1/2)-8/3*b*(2*a^4-7*a^2*b^2+4*b^4)*(cos(1/2*d*
x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*
(a/(a+b))^(1/2))*(a+b*sec(d*x+c))^(1/2)/a^4/(a^2-b^2)^2/d/((b+a*cos(d*x+c)
)/(a+b))^(1/2)/sec(d*x+c)^(1/2)
```

3.667.2 Mathematica [A] (verified)

Time = 1.72 (sec) , antiderivative size = 257, normalized size of antiderivative = 0.66

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx = \frac{2(b+a\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\left(\frac{\left(\frac{b+a\cos(c+dx)}{a+b}\right)^{3/2}\left(-4(2a^4b-7a^2b^3+\right.}{}\right.}{}$$

input
```
Integrate[1/(Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^(5/2)),x]
```

output
```
(2*(b + a*Cos[c + d*x])*Sec[c + d*x]^(5/2)*((((b + a*Cos[c + d*x])/(a + b)
)^(3/2)*(-4*(2*a^4*b - 7*a^2*b^3 + 4*b^5)*EllipticE[(c + d*x)/2, (2*a)/(a
+ b)] + (a^5 - a^4*b + 16*a^3*b^2 - 16*a^2*b^3 - 16*a*b^4 + 16*b^5)*Ellipt
icF[(c + d*x)/2, (2*a)/(a + b)]))/(a - b)^2 + (a*(a^6 - 25*a^2*b^4 + 16*b^
6 + 4*a*b*(a^4 - 8*a^2*b^2 + 5*b^4)*Cos[c + d*x] + (a^3 - a*b^2)^2*Cos[2*(
c + d*x)])*Sin[c + d*x])/(2*(a^2 - b^2)^2)))/(3*a^4*d*(a + b*Sec[c + d*x])
^(5/2))
```

3.667.3 Rubi [A] (verified)

Time = 2.92 (sec) , antiderivative size = 399, normalized size of antiderivative = 1.02, number of steps used = 22, number of rules used = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.880$, Rules used = {3042, 4334, 27, 3042, 4588, 27, 3042, 4592, 27, 3042, 4523, 3042, 4343, 3042, 3134, 3042, 3132, 4345, 3042, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \text{4334}$$

$$\frac{2b^2\sin(c+dx)}{3ad\left(a^2-b^2\right)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}} - \frac{2\int -\frac{4b^2\sec^2(c+dx)-3ab\sec(c+dx)+3\left(a^2-2b^2\right)}{2\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}dx}{3a\left(a^2-b^2\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{4b^2\sec^2(c+dx)-3ab\sec(c+dx)+3(a^2-2b^2)}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}dx}{3a(a^2-b^2)}+\frac{2b^2\sin(c+dx)}{3ad(a^2-b^2)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}}$$

$\Big\downarrow$ 3042

$$\frac{\int \frac{4b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2-3ab\csc\left(c+dx+\frac{\pi}{2}\right)+3(a^2-2b^2)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{3a(a^2-b^2)}+\frac{2b^2\sin(c+dx)}{3ad(a^2-b^2)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}}$$

$\Big\downarrow$ 4588

$$\frac{\frac{4b^2(5a^2-3b^2)\sin(c+dx)}{ad(a^2-b^2)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}-\frac{2\int -\frac{4b^2\left(5a^2-3b^2\right)\sec^2(c+dx)-2ab\left(3a^2-b^2\right)\sec(c+dx)+3\left(a^4-13b^2a^2+8b^4\right)}{2\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}dx}{a(a^2-b^2)}}{3a(a^2-b^2)}+\frac{2b^2\sin(c+dx)}{3ad(a^2-b^2)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}}$$

$\Big\downarrow$ 27

$$\frac{\frac{\int \frac{4b^2\left(5a^2-3b^2\right)\sec^2(c+dx)-2ab\left(3a^2-b^2\right)\sec(c+dx)+3\left(a^4-13b^2a^2+8b^4\right)}{\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}dx}{a(a^2-b^2)}+\frac{4b^2(5a^2-3b^2)\sin(c+dx)}{ad(a^2-b^2)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}}{3a(a^2-b^2)}+\frac{2b^2\sin(c+dx)}{3ad(a^2-b^2)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\int \frac{4b^2\left(5a^2-3b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2-2ab\left(3a^2-b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)+3\left(a^4-13b^2a^2+8b^4\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a(a^2-b^2)}+\frac{4b^2(5a^2-3b^2)\sin(c+dx)}{ad(a^2-b^2)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}}{3a(a^2-b^2)}+\frac{2b^2\sin(c+dx)}{3ad(a^2-b^2)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}}$$

$\Big\downarrow$ 4592

$$\frac{\frac{2\left(a^4-13a^2b^2+8b^4\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{ad\sqrt{\sec(c+dx)}}-\frac{2\int \frac{3\left(4b\left(2a^4-7b^2a^2+4b^4\right)-a\left(a^4+7b^2a^2-4b^4\right)\sec(c+dx)\right)}{2\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{3a}}{a(a^2-b^2)}+\frac{4b^2(5a^2-3b^2)\sin(c+dx)}{ad(a^2-b^2)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}}{3a(a^2-b^2)}+\frac{2b^2\sin(c+dx)}{3ad(a^2-b^2)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}}$$

$\Big\downarrow$ 27

3.667. $\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}dx$

$$\frac{\frac{\frac{2\left(a^4-13a^2b^2+8b^4\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{ad\sqrt{\sec(c+dx)}}-\frac{\int\frac{4b\left(2a^4-7b^2a^2+4b^4\right)-a\left(a^4+7b^2a^2-4b^4\right)\sec(c+dx)}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{a}}{a(a^2-b^2)}+\frac{4b^2\left(5a^2-3b^2\right)\sin(c+dx)}{ad(a^2-b^2)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}}{3a\left(a^2-b^2\right)}+\frac{2b^2\sin(c+dx)}{3ad\left(a^2-b^2\right)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}}$$

$\downarrow$ 3042

$$\frac{\frac{\frac{2\left(a^4-13a^2b^2+8b^4\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{ad\sqrt{\sec(c+dx)}}-\frac{\int\frac{4b\left(2a^4-7b^2a^2+4b^4\right)-a\left(a^4+7b^2a^2-4b^4\right)\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}}{a(a^2-b^2)}+\frac{4b^2\left(5a^2-3b^2\right)\sin(c+dx)}{ad(a^2-b^2)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}}{3a\left(a^2-b^2\right)}+\frac{2b^2\sin(c+dx)}{3ad\left(a^2-b^2\right)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}}$$

$\downarrow$ 4523

$$\frac{\frac{\frac{2\left(a^4-13a^2b^2+8b^4\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{ad\sqrt{\sec(c+dx)}}-\frac{\frac{4b\left(2a^4-7a^2b^2+4b^4\right)\int\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}dx}{a}-\frac{\left(a^6+15a^4b^2-32a^2b^4+16b^6\right)\int\frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}dx}{a}}{a}}{a(a^2-b^2)}+\frac{4}{ad(a^2-b}}{3a\left(a^2-b^2\right)}+\frac{2b^2\sin(c+dx)}{3ad\left(a^2-b^2\right)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}}$$

$\downarrow$ 3042

$$\frac{\frac{\frac{2\left(a^4-13a^2b^2+8b^4\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{ad\sqrt{\sec(c+dx)}}-\frac{\frac{4b\left(2a^4-7a^2b^2+4b^4\right)\int\frac{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}-\frac{\left(a^6+15a^4b^2-32a^2b^4+16b^6\right)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}}{a}}{a(a^2-b^2)}+}{3a\left(a^2-b^2\right)}+\frac{2b^2\sin(c+dx)}{3ad\left(a^2-b^2\right)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}}$$

$\downarrow$ 4343

3.667. $\int\frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}dx$

$$\frac{\frac{\frac{2\left(a^4-13a^2b^2+8b^4\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{ad\sqrt{\sec(c+dx)}}-\frac{\frac{4b\left(2a^4-7a^2b^2+4b^4\right)\sqrt{a+b\sec(c+dx)}\int\sqrt{b+a\cos(c+dx)}dx}{a\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}-\frac{\left(a^6+15a^4b^2-32a^2b^4+16b^6\right)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}}{a}}{a}}{a(a^2-b^2)}}{3a\left(a^2-b^2\right)}$$

$$\frac{2b^2\sin(c+dx)}{3ad\left(a^2-b^2\right)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}}$$

$\downarrow$ 3042

$$\frac{\frac{\frac{2\left(a^4-13a^2b^2+8b^4\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{ad\sqrt{\sec(c+dx)}}-\frac{\frac{4b\left(2a^4-7a^2b^2+4b^4\right)\sqrt{a+b\sec(c+dx)}\int\sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{a\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}-\frac{\left(a^6+15a^4b^2-32a^2b^4+16b^6\right)\int\frac{\sqrt{\csc\left(c+dx-\right.}}{\sqrt{a+b\csc\left(c+d\right.}}}{a}}{a}}{a(a^2-b^2)}}{3a\left(a^2-b^2\right)}$$

$$\frac{2b^2\sin(c+dx)}{3ad\left(a^2-b^2\right)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}}$$

$\downarrow$ 3134

$$\frac{\frac{\frac{2\left(a^4-13a^2b^2+8b^4\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{ad\sqrt{\sec(c+dx)}}-\frac{\frac{4b\left(2a^4-7a^2b^2+4b^4\right)\sqrt{a+b\sec(c+dx)}\int\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}dx}{a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{\left(a^6+15a^4b^2-32a^2b^4+16b^6\right)\int\frac{\sqrt{\csc\left(c+dx-\right.}}{\sqrt{a+b\csc\left(c+d\right.}}}{a}}{a}}{a(a^2-b^2)}}{3a\left(a^2-b^2\right)}$$

$$\frac{2b^2\sin(c+dx)}{3ad\left(a^2-b^2\right)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}}$$

$\downarrow$ 3042

$$\frac{\frac{\frac{2\left(a^4-13a^2b^2+8b^4\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{ad\sqrt{\sec(c+dx)}}-\frac{\frac{4b\left(2a^4-7a^2b^2+4b^4\right)\sqrt{a+b\sec(c+dx)}\int\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}dx}{a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{\left(a^6+15a^4b^2-32a^2b^4+16b^6\right)\int\frac{\sqrt{\csc\left(c-\right.}}{\sqrt{a+b\csc\left(\right.}}}{a}}{a}}{a(a^2-b^2)}}{3a\left(a^2-b^2\right)}$$

$$\frac{2b^2\sin(c+dx)}{3ad\left(a^2-b^2\right)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}}$$

$\downarrow$ 3132

3.667. $\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx$

$$\frac{\frac{\frac{2\left(a^4-13a^2b^2+8b^4\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{ad\sqrt{\sec(c+dx)}}-\frac{\frac{8b\left(2a^4-7a^2b^2+4b^4\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{\left(a^6+15a^4b^2-32a^2b^4+16b^6\right)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}}{a}}{a}}{a(a^2-b^2)}}{3a\left(a^2-b^2\right)}$$

$$\frac{2b^2\sin(c+dx)}{3ad\left(a^2-b^2\right)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}}$$

$\Big\downarrow$ 4345

$$\frac{\frac{\frac{2\left(a^4-13a^2b^2+8b^4\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{ad\sqrt{\sec(c+dx)}}-\frac{\frac{8b\left(2a^4-7a^2b^2+4b^4\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{\left(a^6+15a^4b^2-32a^2b^4+16b^6\right)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+d}}{a\sqrt{a+b\sec(c+dx)}}}{a}}{a(a^2-b^2)}}{3a\left(a^2-b^2\right)}$$

$$\frac{2b^2\sin(c+dx)}{3ad\left(a^2-b^2\right)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\frac{2\left(a^4-13a^2b^2+8b^4\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{ad\sqrt{\sec(c+dx)}}-\frac{\frac{8b\left(2a^4-7a^2b^2+4b^4\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{\left(a^6+15a^4b^2-32a^2b^4+16b^6\right)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+d}}{a\sqrt{a+b\sec(c+dx)}}}{a}}{a(a^2-b^2)}}{3a\left(a^2-b^2\right)}$$

$$\frac{2b^2\sin(c+dx)}{3ad\left(a^2-b^2\right)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}}$$

$\Big\downarrow$ 3142

$$\frac{\frac{\frac{2\left(a^4-13a^2b^2+8b^4\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{ad\sqrt{\sec(c+dx)}}-\frac{\frac{8b\left(2a^4-7a^2b^2+4b^4\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{\left(a^6+15a^4b^2-32a^2b^4+16b^6\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+d}{a+b}}}{a\sqrt{a+b\sec(c+dx)}}}{a}}{a(a^2-b^2)}}{3a\left(a^2-b^2\right)}$$

$$\frac{2b^2\sin(c+dx)}{3ad\left(a^2-b^2\right)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}}$$

$\Big\downarrow$ 3042

3.667. $\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx$

$$\frac{\frac{\frac{2\left(a^4-13a^2b^2+8b^4\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{ad\sqrt{\sec(c+dx)}}-\frac{\frac{8b\left(2a^4-7a^2b^2+4b^4\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{\left(a^6+15a^4b^2-32a^2b^4+16b^6\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+d}{a+b}}}{a\sqrt{a+b\sec(c+dx)}}}{a}}{a(a^2-b^2)}}{3a\left(a^2-b^2\right)}$$

$$\frac{2b^2\sin(c+dx)}{3ad\left(a^2-b^2\right)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}}$$

$\Big\downarrow$ 3140

$$\frac{2b^2\sin(c+dx)}{3ad\left(a^2-b^2\right)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}}+$$

$$\frac{\frac{4b^2\left(5a^2-3b^2\right)\sin(c+dx)}{ad(a^2-b^2)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{\frac{2\left(a^4-13a^2b^2+8b^4\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{ad\sqrt{\sec(c+dx)}}-\frac{\frac{8b\left(2a^4-7a^2b^2+4b^4\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{2\left(a\right.}{}}{a(a^2-b^2)}}{3a\left(a^2-b^2\right)}$$

input
```
Int[1/(Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^(5/2)),x]
```

output
```
(2*b^2*Sin[c + d*x])/(3*a*(a^2 - b^2)*d*Sqrt[Sec[c + d*x]]*(a + b*Sec[c +
d*x])^(3/2)) + ((4*b^2*(5*a^2 - 3*b^2)*Sin[c + d*x])/(a*(a^2 - b^2)*d*Sqrt
[Sec[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + (-(((-2*(a^6 + 15*a^4*b^2 - 32*
a^2*b^4 + 16*b^6)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2
, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(a*d*Sqrt[a + b*Sec[c + d*x]]) + (8*b
*(2*a^4 - 7*a^2*b^2 + 4*b^4)*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a
+ b*Sec[c + d*x]])/(a*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*
x]]))/a) + (2*(a^4 - 13*a^2*b^2 + 8*b^4)*Sqrt[a + b*Sec[c + d*x]]*Sin[c +
d*x])/(a*d*Sqrt[Sec[c + d*x]]))/(a*(a^2 - b^2)))/(3*a*(a^2 - b^2))
```

3.667.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 4334
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*
((d*Csc[e + f*x])^n/(a*f*(m + 1)*(a^2 - b^2))), x] + Simp[1/(a*(m + 1)*(a^2
 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a^2*(m + 1)
 - b^2*(m + n + 1) - a*b*(m + 1)*Csc[e + f*x] + b^2*(m + n + 2)*Csc[e + f*x
]^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0] && LtQ[m,
 -1] && IntegersQ[2*m, 2*n]
```

rule 4343
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]])   Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a
, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4345
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]])   Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[
{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

3.667. $\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx$

rule 4523
```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Simp[A/a   I
nt[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Simp[(A*b - a*B)
/(a*d)   Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ
[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

rule 4588
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc
[e + f*x])^(m + 1)*((d*Csc[e + f*x])^n/(a*f*(m + 1)*(a^2 - b^2))), x] + Sim
p[1/(a*(m + 1)*(a^2 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f
*x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n +
1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m
+ n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x
] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

rule 4592
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Simp[1/(a*d*n)   Int[(a + b*Csc[e + f*x])^m
*(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*
Csc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

3.667.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 4448 vs. $2(415) = 830$.

Time = 9.96 (sec) , antiderivative size = 4449, normalized size of antiderivative = 11.38

method	result	size
default	Expression too large to display	4449

input
```
int(1/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/3/d/a^4/(a-b)/(a+b)^2/((a-b)/(a+b))^(1/2)*(a+b*sec(d*x+c))^(1/2)/(b+a*co
s(d*x+c))^2/sec(d*x+c)^(3/2)/(cos(d*x+c)+1)*(8*(1/(a+b)*(b+a*cos(d*x+c))/(
cos(d*x+c)+1))^(1/2)*EllipticE(((a-b)/(a+b))^(1/2)*(cot(d*x+c)-csc(d*x+c))
,(-(a+b)/(a-b))^(1/2))*(1/(cos(d*x+c)+1))^(1/2)*a^4*b^2*sec(d*x+c)+16*((a-
b)/(a+b))^(1/2)*b^6*tan(d*x+c)+((a-b)/(a+b))^(1/2)*a^6*cos(d*x+c)*sin(d*x+
c)+a^4*((a-b)/(a+b))^(1/2)*b^2*tan(d*x+c)-7*((a-b)/(a+b))^(1/2)*a^3*b^3*ta
n(d*x+c)-20*a^2*((a-b)/(a+b))^(1/2)*b^4*tan(d*x+c)+8*((a-b)/(a+b))^(1/2)*a
*b^5*tan(d*x+c)+2*((a-b)/(a+b))^(1/2)*a^5*b*sin(d*x+c)-13*((a-b)/(a+b))^(1
/2)*a^4*b^2*sin(d*x+c)-29*((a-b)/(a+b))^(1/2)*a^3*b^3*sin(d*x+c)+14*((a-b)
/(a+b))^(1/2)*a^2*b^4*sin(d*x+c)+24*((a-b)/(a+b))^(1/2)*a*b^5*sin(d*x+c)+(
(a-b)/(a+b))^(1/2)*a^6*cos(d*x+c)^2*sin(d*x+c)+32*(1/(a+b)*(b+a*cos(d*x+c)
)/(cos(d*x+c)+1))^(1/2)*EllipticE(((a-b)/(a+b))^(1/2)*(cot(d*x+c)-csc(d*x+
c)),(-(a+b)/(a-b))^(1/2))*(1/(cos(d*x+c)+1))^(1/2)*b^6-(1/(a+b)*(b+a*cos(d
*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF(((a-b)/(a+b))^(1/2)*(cot(d*x+c)-csc
(d*x+c)),(-(a+b)/(a-b))^(1/2))*(1/(cos(d*x+c)+1))^(1/2)*a^6-28*(1/(a+b)*(b
+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE(((a-b)/(a+b))^(1/2)*(cot(d*
x+c)-csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*(1/(cos(d*x+c)+1))^(1/2)*a^2*b^4*se
c(d*x+c)-(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF(((a-b)/
(a+b))^(1/2)*(cot(d*x+c)-csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*(1/(cos(d*x+c)+
1))^(1/2)*a^5*b*sec(d*x+c)-9*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^...
```

3.667.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.18 (sec) , antiderivative size = 949, normalized size of antiderivative = 2.43

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
1/9*(sqrt(2)*(-3*I*a^6*b^2 - 37*I*a^4*b^4 + 68*I*a^2*b^6 - 32*I*b^8 + (-3*
I*a^8 - 37*I*a^6*b^2 + 68*I*a^4*b^4 - 32*I*a^2*b^6)*cos(d*x + c)^2 - 2*(3*
I*a^7*b + 37*I*a^5*b^3 - 68*I*a^3*b^5 + 32*I*a*b^7)*cos(d*x + c))*sqrt(a)*
weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3,
1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x + c) + 2*b)/a) + sqrt(2)*(3*I*a^6*b^
2 + 37*I*a^4*b^4 - 68*I*a^2*b^6 + 32*I*b^8 + (3*I*a^8 + 37*I*a^6*b^2 - 68*
I*a^4*b^4 + 32*I*a^2*b^6)*cos(d*x + c)^2 - 2*(-3*I*a^7*b - 37*I*a^5*b^3 +
68*I*a^3*b^5 - 32*I*a*b^7)*cos(d*x + c))*sqrt(a)*weierstrassPInverse(-4/3*
(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3
*I*a*sin(d*x + c) + 2*b)/a) - 12*sqrt(2)*(2*I*a^5*b^3 - 7*I*a^3*b^5 + 4*I*
a*b^7 + (2*I*a^7*b - 7*I*a^5*b^3 + 4*I*a^3*b^5)*cos(d*x + c)^2 + 2*(2*I*a^
6*b^2 - 7*I*a^4*b^4 + 4*I*a^2*b^6)*cos(d*x + c))*sqrt(a)*weierstrassZeta(-
4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-
4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c)
 + 3*I*a*sin(d*x + c) + 2*b)/a)) - 12*sqrt(2)*(-2*I*a^5*b^3 + 7*I*a^3*b^5
- 4*I*a*b^7 + (-2*I*a^7*b + 7*I*a^5*b^3 - 4*I*a^3*b^5)*cos(d*x + c)^2 + 2*
(-2*I*a^6*b^2 + 7*I*a^4*b^4 - 4*I*a^2*b^6)*cos(d*x + c))*sqrt(a)*weierstra
ssZeta(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPI
nverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(
d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a)) + 6*((a^8 - 2*a^6*b^2 + a^4*b^...
```

3.667.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/sec(d*x+c)**(3/2)/(a+b*sec(d*x+c))**(5/2),x)
```

output
```
Timed out
```

3.667.7 Maxima [F]

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{1}{(b\sec(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*sec(d*x + c) + a)^(5/2)*sec(d*x + c)^(3/2)), x)
```

3.667.8 Giac [F]

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{1}{(b\sec(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate(1/((b*sec(d*x + c) + a)^(5/2)*sec(d*x + c)^(3/2)), x)
```

3.667.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{1}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2}\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

input
```
int(1/((a + b/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(3/2)),x)
```

output
```
int(1/((a + b/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(3/2)), x)
```

3.668 $\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx$

3.668.1 Optimal result 5244
3.668.2 Mathematica [A] (verified) 5245
3.668.3 Rubi [A] (verified) 5246
3.668.4 Maple [B] (verified) 5254
3.668.5 Fricas [C] (verification not implemented) 5255
3.668.6 Sympy [F(-1)] 5255
3.668.7 Maxima [F] 5256
3.668.8 Giac [F] 5256
3.668.9 Mupad [F(-1)] 5256

3.668.1 Optimal result

Integrand size = 25, antiderivative size = 474

$$\begin{aligned}
&\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx = \\
&-\frac{2b(17a^4+116a^2b^2-128b^4)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{15a^5\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} \\
&+\frac{2(9a^6+55a^4b^2-212a^2b^4+128b^6)\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+dx)}}{15a^5\left(a^2-b^2\right)^2 d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\sqrt{\sec(c+dx)}} \\
&+\frac{2b^2\sin(c+dx)}{3a\left(a^2-b^2\right)d\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}} \\
&+\frac{8b^2(3a^2-2b^2)\sin(c+dx)}{3a^2\left(a^2-b^2\right)^2 d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}} \\
&+\frac{2(3a^4-71a^2b^2+48b^4)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{15a^3\left(a^2-b^2\right)^2 d\sec^{\frac{3}{2}}(c+dx)} \\
&-\frac{4b(7a^4-49a^2b^2+32b^4)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{15a^4\left(a^2-b^2\right)^2 d\sqrt{\sec(c+dx)}}
\end{aligned}$$

output
```
2/3*b^2*sin(d*x+c)/a/(a^2-b^2)/d/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(3/2)+8
/3*b^2*(3*a^2-2*b^2)*sin(d*x+c)/a^2/(a^2-b^2)^2/d/sec(d*x+c)^(3/2)/(a+b*se
c(d*x+c))^(1/2)-2/15*b*(17*a^4+116*a^2*b^2-128*b^4)*(cos(1/2*d*x+1/2*c)^2)
^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(
1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)*sec(d*x+c)^(1/2)/a^5/(a^2-b^2)/d/(a+b
*sec(d*x+c))^(1/2)+2/15*(3*a^4-71*a^2*b^2+48*b^4)*sin(d*x+c)*(a+b*sec(d*x+
c))^(1/2)/a^3/(a^2-b^2)^2/d/sec(d*x+c)^(3/2)-4/15*b*(7*a^4-49*a^2*b^2+32*b
^4)*sin(d*x+c)*(a+b*sec(d*x+c))^(1/2)/a^4/(a^2-b^2)^2/d/sec(d*x+c)^(1/2)+2
/15*(9*a^6+55*a^4*b^2-212*a^2*b^4+128*b^6)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/co
s(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*(a+
b*sec(d*x+c))^(1/2)/a^5/(a^2-b^2)^2/d/((b+a*cos(d*x+c))/(a+b))^(1/2)/sec(d
*x+c)^(1/2)
```

3.668.2 Mathematica [A] (verified)

Time = 1.87 (sec) , antiderivative size = 292, normalized size of antiderivative = 0.62

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx = \frac{(b+a\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\left(\frac{2\left(\frac{b+a\cos(c+dx)}{a+b}\right)^{3/2}\left((9a^6+55a^4b^2-212\right.}{}\right.}{}$$

input
```
Integrate[1/(Sec[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^(5/2)),x]
```

output
```
((b + a*Cos[c + d*x])*Sec[c + d*x]^(5/2)*((2*((b + a*Cos[c + d*x])/(a + b)
)^(3/2)*((9*a^6 + 55*a^4*b^2 - 212*a^2*b^4 + 128*b^6)*EllipticE[(c + d*x)/
2, (2*a)/(a + b)] + b*(-17*a^5 + 17*a^4*b - 116*a^3*b^2 + 116*a^2*b^3 + 12
8*a*b^4 - 128*b^5)*EllipticF[(c + d*x)/2, (2*a)/(a + b)]))/(a - b)^2 + a*(
(10*b^5*Sin[c + d*x])/(-a^2 + b^2) - (10*b^4*(-15*a^2 + 11*b^2)*(b + a*Cos
[c + d*x])*Sin[c + d*x])/(a^2 - b^2)^2 - 28*b*(b + a*Cos[c + d*x])^2*Sin[c
 + d*x] + 3*a*(b + a*Cos[c + d*x])^2*Sin[2*(c + d*x)])))/(15*a^5*d*(a + b*
Sec[c + d*x])^(5/2))
```

3.668.3 Rubi [A] (verified)

Time = 3.78 (sec) , antiderivative size = 479, normalized size of antiderivative = 1.01, number of steps used = 25, number of rules used = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {3042, 4334, 27, 3042, 4588, 27, 3042, 4592, 27, 3042, 4592, 27, 3042, 4523, 3042, 4343, 3042, 3134, 3042, 3132, 4345, 3042, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \text{4334}$$

$$\frac{2b^2\sin(c+dx)}{3ad\left(a^2-b^2\right)\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}-\frac{2\int -\frac{3a^2-3b\sec(c+dx)a-8b^2+6b^2\sec^2(c+dx)}{2\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}dx}{3a\left(a^2-b^2\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{3a^2-3b\sec(c+dx)a-8b^2+6b^2\sec^2(c+dx)}{\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}dx}{3a\left(a^2-b^2\right)}+\frac{2b^2\sin(c+dx)}{3ad\left(a^2-b^2\right)\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{3a^2-3b\csc\left(c+dx+\frac{\pi}{2}\right)a-8b^2+6b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{3a\left(a^2-b^2\right)}+\frac{2b^2\sin(c+dx)}{3ad\left(a^2-b^2\right)\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \text{4588}$$

$$\frac{\frac{8b^2\left(3a^2-2b^2\right)\sin(c+dx)}{ad\left(a^2-b^2\right)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}-\frac{2\int -\frac{3a^4-71b^2a^2-2b\left(3a^2-b^2\right)\sec(c+dx)a+48b^4+16b^2\left(3a^2-2b^2\right)\sec^2(c+dx)}{2\sec^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}dx}{a\left(a^2-b^2\right)}}{3a\left(a^2-b^2\right)}+\frac{2b^2\sin(c+dx)}{3ad\left(a^2-b^2\right)\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int \frac{3a^4-71b^2a^2-2b\left(3a^2-b^2\right)\sec(c+dx)a+48b^4+16b^2\left(3a^2-2b^2\right)\sec^2(c+dx)}{\sec^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}dx}{a(a^2-b^2)}+\frac{8b^2\left(3a^2-2b^2\right)\sin(c+dx)}{ad(a^2-b^2)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}}{3a\left(a^2-b^2\right)}+\frac{2b^2\sin(c+dx)}{3ad\left(a^2-b^2\right)\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}$$

$\downarrow$ 3042

$$\frac{\frac{\int \frac{3a^4-71b^2a^2-2b\left(3a^2-b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)a+48b^4+16b^2\left(3a^2-2b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a(a^2-b^2)}+\frac{8b^2\left(3a^2-2b^2\right)\sin(c+dx)}{ad(a^2-b^2)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}}{3a\left(a^2-b^2\right)}+\frac{2b^2\sin(c+dx)}{3ad\left(a^2-b^2\right)\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}$$

$\downarrow$ 4592

$$\frac{\frac{\frac{2\left(3a^4-71a^2b^2+48b^4\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{2\int \frac{-2b\left(3a^4-71b^2a^2+48b^4\right)\sec^2(c+dx)-a\left(9a^4+27b^2a^2-16b^4\right)\sec(c+dx)+6b\left(7a^4-49b^2a^2+32b^4\right)}{2\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}dx}{5a}}{a(a^2-b^2)}+\frac{}{a}}{3a\left(a^2-b^2\right)}$$

$$\frac{2b^2\sin(c+dx)}{3ad\left(a^2-b^2\right)\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}$$

$\downarrow$ 27

$$\frac{\frac{\frac{2\left(3a^4-71a^2b^2+48b^4\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{\int \frac{-2b\left(3a^4-71b^2a^2+48b^4\right)\sec^2(c+dx)-a\left(9a^4+27b^2a^2-16b^4\right)\sec(c+dx)+6b\left(7a^4-49b^2a^2+32b^4\right)}{\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}dx}{5a}}{a(a^2-b^2)}+\frac{}{ad}}{3a\left(a^2-b^2\right)}$$

$$\frac{2b^2\sin(c+dx)}{3ad\left(a^2-b^2\right)\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}$$

$\downarrow$ 3042

$$\frac{\frac{\frac{2\left(3a^4-71a^2b^2+48b^4\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{\int \frac{-2b\left(3a^4-71b^2a^2+48b^4\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2-a\left(9a^4+27b^2a^2-16b^4\right)\csc\left(c+dx+\frac{\pi}{2}\right)+6b\left(7a^4-49b^2a^2+32b^4\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}d}{5a}}{a(a^2-b^2)}}{3a\left(a^2-b^2\right)}$$

$$\frac{2b^2\sin(c+dx)}{3ad\left(a^2-b^2\right)\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}$$

$\downarrow$ 4592

3.668. $\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx$

$$\frac{\frac{\frac{2\left(3a^4-71a^2b^2+48b^4\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{\frac{4b\left(7a^4-49a^2b^2+32b^4\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{ad\sqrt{\sec(c+dx)}}-\frac{2\int\frac{3\left(9a^6+55b^2a^4-212b^4a^2-4b\left(2a^4+11b^2a^2-8b^4\right)\sec(}{2\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}}{3a}}{5a}}{a(a^2-b^2)}}{3a\left(a^2-b^2\right)}$$

$$\frac{2b^2\sin(c+dx)}{3ad\left(a^2-b^2\right)\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}$$

$\downarrow$ 27

$$\frac{\frac{\frac{2\left(3a^4-71a^2b^2+48b^4\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{\frac{4b\left(7a^4-49a^2b^2+32b^4\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{ad\sqrt{\sec(c+dx)}}-\frac{\int\frac{9a^6+55b^2a^4-212b^4a^2-4b\left(2a^4+11b^2a^2-8b^4\right)\sec(c+d}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}}{a}}{5a}}{a(a^2-b^2)}}{3a\left(a^2-b^2\right)}$$

$$\frac{2b^2\sin(c+dx)}{3ad\left(a^2-b^2\right)\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}$$

$\downarrow$ 3042

$$\frac{\frac{\frac{2\left(3a^4-71a^2b^2+48b^4\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{\frac{4b\left(7a^4-49a^2b^2+32b^4\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{ad\sqrt{\sec(c+dx)}}-\frac{\int\frac{9a^6+55b^2a^4-212b^4a^2-4b\left(2a^4+11b^2a^2-8b^4\right)\csc\left(c+\right.}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}}{a}}{5a}}{a(a^2-b^2)}}{3a\left(a^2-b^2\right)}$$

$$\frac{2b^2\sin(c+dx)}{3ad\left(a^2-b^2\right)\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}$$

$\downarrow$ 4523

$$\frac{\frac{\frac{2\left(3a^4-71a^2b^2+48b^4\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{\frac{4b\left(7a^4-49a^2b^2+32b^4\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{ad\sqrt{\sec(c+dx)}}-\frac{\frac{\left(9a^6+55a^4b^2-212a^2b^4+128b^6\right)\int\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}dx}{a}-}{}}{5a}}{a(a^2-b^2)}}{3a\left(a^2-b^2\right)}$$

$$\frac{2b^2\sin(c+dx)}{3ad\left(a^2-b^2\right)\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}$$

$\downarrow$ 3042

3.668. $\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx$

$$\frac{\frac{\frac{2\left(3a^4-71a^2b^2+48b^4\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{\frac{4b\left(7a^4-49a^2b^2+32b^4\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{ad\sqrt{\sec(c+dx)}}-\frac{\left(9a^6+55a^4b^2-212a^2b^4+128b^6\right)\int\frac{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}}{a}}{5a}}{a(a^2-b^2)}}{3a\left(a^2-b^2\right)}$$

$$\frac{2b^2\sin(c+dx)}{3ad\left(a^2-b^2\right)\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}$$

$\downarrow$ 4343

$$\frac{\frac{\frac{2\left(3a^4-71a^2b^2+48b^4\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{\frac{4b\left(7a^4-49a^2b^2+32b^4\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{ad\sqrt{\sec(c+dx)}}-\frac{\left(9a^6+55a^4b^2-212a^2b^4+128b^6\right)\sqrt{a+b\sec(c+dx)}\int\sqrt{b+c}}{a\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}}{5a}}{a(a^2-b^2)}}{3a\left(a^2-b^2\right)}$$

$$\frac{2b^2\sin(c+dx)}{3ad\left(a^2-b^2\right)\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}$$

$\downarrow$ 3042

$$\frac{\frac{\frac{2\left(3a^4-71a^2b^2+48b^4\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{\frac{4b\left(7a^4-49a^2b^2+32b^4\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{ad\sqrt{\sec(c+dx)}}-\frac{\left(9a^6+55a^4b^2-212a^2b^4+128b^6\right)\sqrt{a+b\sec(c+dx)}\int\sqrt{b+c}}{a\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}}{5a}}{a(a^2-b^2)}}{3a\left(a^2-b^2\right)}$$

$$\frac{2b^2\sin(c+dx)}{3ad\left(a^2-b^2\right)\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}$$

$\downarrow$ 3134

$$\frac{\frac{\frac{2\left(3a^4-71a^2b^2+48b^4\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{\frac{4b\left(7a^4-49a^2b^2+32b^4\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{ad\sqrt{\sec(c+dx)}}-\frac{\left(9a^6+55a^4b^2-212a^2b^4+128b^6\right)\sqrt{a+b\sec(c+dx)}\int\sqrt{\frac{b}{a+}}}{a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{5a}}{a(a^2-b^2)}}{3a\left(a^2-b^2\right)}$$

$$\frac{2b^2\sin(c+dx)}{3ad\left(a^2-b^2\right)\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}$$

$\downarrow$ 3042

3.668. $\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx$

$$\frac{\frac{\frac{2\left(3a^4-71a^2b^2+48b^4\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{\frac{4b\left(7a^4-49a^2b^2+32b^4\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{ad\sqrt{\sec(c+dx)}}-\frac{\frac{\left(9a^6+55a^4b^2-212a^2b^4+128b^6\right)\sqrt{a+b\sec(c+dx)}\int\sqrt{\frac{b}{a+}}}{a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{5a}}{a(a^2-b^2)}}{3a\left(a^2-b^2\right)}$$

$$\frac{2b^2\sin(c+dx)}{3ad\left(a^2-b^2\right)\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}$$

$\downarrow$ 3132

$$\frac{\frac{\frac{2\left(3a^4-71a^2b^2+48b^4\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{\frac{4b\left(7a^4-49a^2b^2+32b^4\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{ad\sqrt{\sec(c+dx)}}-\frac{\frac{2\left(9a^6+55a^4b^2-212a^2b^4+128b^6\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c\right.}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{5a}}{a(a^2-b^2)}}{3a\left(a^2-b^2\right)}$$

$$\frac{2b^2\sin(c+dx)}{3ad\left(a^2-b^2\right)\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}$$

$\downarrow$ 4345

$$\frac{\frac{\frac{2\left(3a^4-71a^2b^2+48b^4\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{\frac{4b\left(7a^4-49a^2b^2+32b^4\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{ad\sqrt{\sec(c+dx)}}-\frac{\frac{2\left(9a^6+55a^4b^2-212a^2b^4+128b^6\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c\right.}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{5a}}{a(a^2-b^2)}}{3a\left(a^2-b\right.}$$

$$\frac{2b^2\sin(c+dx)}{3ad\left(a^2-b^2\right)\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}$$

$\downarrow$ 3042

$$\frac{\frac{\frac{2\left(3a^4-71a^2b^2+48b^4\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{\frac{4b\left(7a^4-49a^2b^2+32b^4\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{ad\sqrt{\sec(c+dx)}}-\frac{\frac{2\left(9a^6+55a^4b^2-212a^2b^4+128b^6\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c\right.}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{5a}}{a(a^2-b^2)}}{3a\left(a^2-\right.}$$

$$\frac{2b^2\sin(c+dx)}{3ad\left(a^2-b^2\right)\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}$$

$\downarrow$ 3142

3.668. $\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx$

$$\frac{\frac{\frac{2\left(3a^4-71a^2b^2+48b^4\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{\frac{4b\left(7a^4-49a^2b^2+32b^4\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{ad\sqrt{\sec(c+dx)}}-\frac{\frac{2\left(9a^6+55a^4b^2-212a^2b^2+128b^6\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c\right.}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{5a}}{a(a^2-b^2)}}{3a\,(a^2-}$$

$$\frac{2b^2\sin(c+dx)}{3ad\,(a^2-b^2)\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{\frac{2\left(3a^4-71a^2b^2+48b^4\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{\frac{4b\left(7a^4-49a^2b^2+32b^4\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{ad\sqrt{\sec(c+dx)}}-\frac{\frac{2\left(9a^6+55a^4b^2-212a^2b^2+128b^6\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c\right.}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{5}}{a(a^2-b^2)}}{3a\,(a^2}$$

$$\frac{2b^2\sin(c+dx)}{3ad\,(a^2-b^2)\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \; 3140$$

$$\frac{2b^2\sin(c+dx)}{3ad\,(a^2-b^2)\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}+$$

$$\frac{\frac{8b^2\left(3a^2-2b^2\right)\sin(c+dx)}{ad(a^2-b^2)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}+\frac{\frac{2\left(3a^4-71a^2b^2+48b^4\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{\frac{4b\left(7a^4-49a^2b^2+32b^4\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{ad\sqrt{\sec(c+dx)}}-\frac{2\left(9a^6+\right.}{}}{}}{3a\,(a^2-}$$

input
```
Int[1/(Sec[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^(5/2)),x]
```

3.668. $\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx$

output
```
(2*b^2*Sin[c + d*x])/(3*a*(a^2 - b^2)*d*Sec[c + d*x]^(3/2)*(a + b*Sec[c +
d*x])^(3/2)) + ((8*b^2*(3*a^2 - 2*b^2)*Sin[c + d*x])/(a*(a^2 - b^2)*d*Sec[
c + d*x]^(3/2)*Sqrt[a + b*Sec[c + d*x]]) + ((2*(3*a^4 - 71*a^2*b^2 + 48*b^
4)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(5*a*d*Sec[c + d*x]^(3/2)) - (-(
((-2*b*(17*a^6 + 99*a^4*b^2 - 244*a^2*b^4 + 128*b^6)*Sqrt[(b + a*Cos[c + d
*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(a
*d*Sqrt[a + b*Sec[c + d*x]]) + (2*(9*a^6 + 55*a^4*b^2 - 212*a^2*b^4 + 128*
b^6)*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(a*d*
Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]))/a) + (4*b*(7*a^4 -
 49*a^2*b^2 + 32*b^4)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(a*d*Sqrt[Sec
[c + d*x]]))/(5*a))/(a*(a^2 - b^2)))/(3*a*(a^2 - b^2))
```

3.668.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

3.668. $\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx$

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 4334
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*
((d*Csc[e + f*x])^n/(a*f*(m + 1)*(a^2 - b^2))), x] + Simp[1/(a*(m + 1)*(a^2
 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a^2*(m + 1)
 - b^2*(m + n + 1) - a*b*(m + 1)*Csc[e + f*x] + b^2*(m + n + 2)*Csc[e + f*x
]^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0] && LtQ[m,
 -1] && IntegersQ[2*m, 2*n]
```

rule 4343
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]])   Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a
, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4345
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]])   Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[
{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4523
```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Simp[A/a   I
nt[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Simp[(A*b - a*B)
/(a*d)   Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ
[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

3.668. $\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx$

rule 4588
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc
[e + f*x])^(m + 1)*((d*Csc[e + f*x])^n/(a*f*(m + 1)*(a^2 - b^2))), x] + Sim
p[1/(a*(m + 1)*(a^2 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f
*x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n +
1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m
+ n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x
] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

rule 4592
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Simp[1/(a*d*n)   Int[(a + b*Csc[e + f*x])^m
*(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*
Csc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

3.668.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 5655 vs. $2(492) = 984$.

Time = 11.09 (sec) , antiderivative size = 5656, normalized size of antiderivative = 11.93

method	result	size
default	Expression too large to display	5656

input
```
int(1/sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.668.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.21 (sec) , antiderivative size = 1036, normalized size of antiderivative = 2.19

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
-1/45*(2*sqrt(2)*(-21*I*a^6*b^3 - 121*I*a^4*b^5 + 260*I*a^2*b^7 - 128*I*b^
9 + (-21*I*a^8*b - 121*I*a^6*b^3 + 260*I*a^4*b^5 - 128*I*a^2*b^7)*cos(d*x
+ c)^2 + 2*(-21*I*a^7*b^2 - 121*I*a^5*b^4 + 260*I*a^3*b^6 - 128*I*a*b^8)*c
os(d*x + c))*sqrt(a)*weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9
*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x + c) + 2*b)/a)
+ 2*sqrt(2)*(21*I*a^6*b^3 + 121*I*a^4*b^5 - 260*I*a^2*b^7 + 128*I*b^9 + (2
1*I*a^8*b + 121*I*a^6*b^3 - 260*I*a^4*b^5 + 128*I*a^2*b^7)*cos(d*x + c)^2
+ 2*(21*I*a^7*b^2 + 121*I*a^5*b^4 - 260*I*a^3*b^6 + 128*I*a*b^8)*cos(d*x +
 c))*sqrt(a)*weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b -
 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a) + 3*sqrt
(2)*(-9*I*a^7*b^2 - 55*I*a^5*b^4 + 212*I*a^3*b^6 - 128*I*a*b^8 + (-9*I*a^9
 - 55*I*a^7*b^2 + 212*I*a^5*b^4 - 128*I*a^3*b^6)*cos(d*x + c)^2 + 2*(-9*I*
a^8*b - 55*I*a^6*b^3 + 212*I*a^4*b^5 - 128*I*a^2*b^7)*cos(d*x + c))*sqrt(a
)*weierstrassZeta(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, we
ierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/
3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x + c) + 2*b)/a)) + 3*sqrt(2)*(9*I*a^7*b
^2 + 55*I*a^5*b^4 - 212*I*a^3*b^6 + 128*I*a*b^8 + (9*I*a^9 + 55*I*a^7*b^2
- 212*I*a^5*b^4 + 128*I*a^3*b^6)*cos(d*x + c)^2 + 2*(9*I*a^8*b + 55*I*a^6*
b^3 - 212*I*a^4*b^5 + 128*I*a^2*b^7)*cos(d*x + c))*sqrt(a)*weierstrassZeta
(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInve...
```

3.668.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/sec(d*x+c)**(5/2)/(a+b*sec(d*x+c))**(5/2),x)
```

output
```
Timed out
```

3.668.7 Maxima [F]

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{1}{(b\sec(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*sec(d*x + c) + a)^(5/2)*sec(d*x + c)^(5/2)), x)
```

3.668.8 Giac [F]

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{1}{(b\sec(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate(1/((b*sec(d*x + c) + a)^(5/2)*sec(d*x + c)^(5/2)), x)
```

3.668.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{1}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2}\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

input
```
int(1/((a + b/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(5/2)),x)
```

output
```
int(1/((a + b/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(5/2)), x)
```

3.669 $\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{2+3\sec(c+dx)}}\,dx$

3.669.1 Optimal result 5257
3.669.2 Mathematica [A] (verified) 5257
3.669.3 Rubi [A] (verified) 5258
3.669.4 Maple [C] (verified) 5260
3.669.5 Fricas [C] (verification not implemented) 5261
3.669.6 Sympy [F] 5261
3.669.7 Maxima [F] 5262
3.669.8 Giac [F] 5262
3.669.9 Mupad [F(-1)] 5262

3.669.1 Optimal result

Integrand size = 25, antiderivative size = 122

$$\begin{aligned}\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{2+3\sec(c+dx)}}\,dx &= -\frac{3\sqrt{3+2\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{4}{5}\right)\sqrt{\sec(c+dx)}}{\sqrt{5}d\sqrt{2+3\sec(c+dx)}} \\ &\quad + \frac{\sqrt{5}E\left(\frac{1}{2}(c+dx)|\frac{4}{5}\right)\sqrt{2+3\sec(c+dx)}}{d\sqrt{3+2\cos(c+dx)}\sqrt{\sec(c+dx)}}\end{aligned}$$

output
```
-3/5*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x
+1/2*c),2/5*5^(1/2))*(3+2*cos(d*x+c))^(1/2)*sec(d*x+c)^(1/2)/d*5^(1/2)/(2+
3*sec(d*x+c))^(1/2)+(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellipt
icE(sin(1/2*d*x+1/2*c),2/5*5^(1/2))*5^(1/2)*(2+3*sec(d*x+c))^(1/2)/d/(3+2*
cos(d*x+c))^(1/2)/sec(d*x+c)^(1/2)
```

3.669.2 Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.64

$$\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{2+3\sec(c+dx)}}\,dx = \frac{\sqrt{3+2\cos(c+dx)}\left(5E\left(\frac{1}{2}(c+dx)|\frac{4}{5}\right)-3\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{4}{5}\right)\right)\sqrt{\sec(c+dx)}}{\sqrt{5}d\sqrt{2+3\sec(c+dx)}}$$

input
```
Integrate[1/(Sqrt[Sec[c + d*x]]*Sqrt[2 + 3*Sec[c + d*x]]),x]
```

output
```
(Sqrt[3 + 2*Cos[c + d*x]]*(5*EllipticE[(c + d*x)/2, 4/5] - 3*EllipticF[(c
+ d*x)/2, 4/5])*Sqrt[Sec[c + d*x]])/(Sqrt[5]*d*Sqrt[2 + 3*Sec[c + d*x]])
```

3.669.3 Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {3042, 4349, 3042, 4343, 3042, 3132, 4345, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{3\sec(c+dx)+2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{3\csc\left(c+dx+\frac{\pi}{2}\right)+2}}dx$$

$$\Big\downarrow \text{4349}$$

$$\frac{1}{2}\int \frac{\sqrt{3\sec(c+dx)+2}}{\sqrt{\sec(c+dx)}}dx-\frac{3}{2}\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{3\sec(c+dx)+2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\int \frac{\sqrt{3\csc\left(c+dx+\frac{\pi}{2}\right)+2}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{3}{2}\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{3\csc\left(c+dx+\frac{\pi}{2}\right)+2}}dx$$

$$\Big\downarrow \text{4343}$$

$$\frac{\sqrt{3\sec(c+dx)+2}\int \sqrt{2\cos(c+dx)+3}dx}{2\sqrt{2\cos(c+dx)+3}\sqrt{\sec(c+dx)}}-\frac{3}{2}\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{3\csc\left(c+dx+\frac{\pi}{2}\right)+2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{3\sec(c+dx)+2}\int \sqrt{2\sin\left(c+dx+\frac{\pi}{2}\right)+3}dx}{2\sqrt{2\cos(c+dx)+3}\sqrt{\sec(c+dx)}}-\frac{3}{2}\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{3\csc\left(c+dx+\frac{\pi}{2}\right)+2}}dx$$

$$\Big\downarrow \text{3132}$$

$$\frac{\sqrt{5}\sqrt{3\sec(c+dx)+2}E\left(\frac{1}{2}(c+dx)|\frac{4}{5}\right)}{d\sqrt{2\cos(c+dx)+3}\sqrt{\sec(c+dx)}}-\frac{3}{2}\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{3\csc\left(c+dx+\frac{\pi}{2}\right)+2}}dx$$

$$\Big\downarrow \text{4345}$$

$$\frac{\sqrt{5}\sqrt{3\sec(c+dx)+2}E\left(\frac{1}{2}(c+dx)|\frac{4}{5}\right)}{d\sqrt{2\cos(c+dx)+3}\sqrt{\sec(c+dx)}}-\frac{3\sqrt{2\cos(c+dx)+3}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{2\cos(c+dx)+3}}dx}{2\sqrt{3\sec(c+dx)+2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{5}\sqrt{3\sec(c+dx)+2}E\left(\frac{1}{2}(c+dx)|\frac{4}{5}\right)}{d\sqrt{2\cos(c+dx)+3}\sqrt{\sec(c+dx)}}-\frac{3\sqrt{2\cos(c+dx)+3}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{2\sin\left(c+dx+\frac{\pi}{2}\right)+3}}dx}{2\sqrt{3\sec(c+dx)+2}}$$

$$\Big\downarrow \text{3140}$$

$$\frac{\sqrt{5}\sqrt{3\sec(c+dx)+2}E\left(\frac{1}{2}(c+dx)|\frac{4}{5}\right)}{d\sqrt{2\cos(c+dx)+3}\sqrt{\sec(c+dx)}}-\frac{3\sqrt{2\cos(c+dx)+3}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{4}{5}\right)}{\sqrt{5}d\sqrt{3\sec(c+dx)+2}}$$

input
```
Int[1/(Sqrt[Sec[c + d*x]]*Sqrt[2 + 3*Sec[c + d*x]]),x]
```

output
```
(-3*Sqrt[3 + 2*Cos[c + d*x]]*EllipticF[(c + d*x)/2, 4/5]*Sqrt[Sec[c + d*x]
])/(Sqrt[5]*d*Sqrt[2 + 3*Sec[c + d*x]]) + (Sqrt[5]*EllipticE[(c + d*x)/2,
4/5]*Sqrt[2 + 3*Sec[c + d*x]])/(d*Sqrt[3 + 2*Cos[c + d*x]]*Sqrt[Sec[c + d*
x]])
```

3.669.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 4343
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]])   Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a
, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4345
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]])   Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[
{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4349
```
Int[1/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)]), x_Symbol] :> Simp[1/a   Int[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Cs
c[e + f*x]], x], x] - Simp[b/(a*d)   Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Cs
c[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

3.669.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 7.10 (sec) , antiderivative size = 338, normalized size of antiderivative = 2.77

method	result
default	$\frac{\sqrt{-\frac{(1-\cos(dx+c))^2\csc(dx+c)^2+5}{(1-\cos(dx+c))^2\csc(dx+c)^2-1}}\left(2i\sqrt{5}\,\mathrm{EllipticF}\left(\frac{i\sqrt{5}\,(-\cot(dx+c)+\csc(dx+c))}{5},\sqrt{5}\right)\sqrt{5(1-\cos(dx+c))^2\csc(dx+c)^2+25}\,\sqrt{(1-\cos(dx}\right.}{}$
risch	$-\frac{i\left(e^{2i(dx+c)}+3\,e^{i(dx+c)}+1\right)}{d\sqrt{\frac{e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\left(e^{2i(dx+c)}+1\right)\sqrt{\frac{e^{2i(dx+c)}+3\,e^{i(dx+c)}+1}{e^{2i(dx+c)}+1}}}-\frac{i\left(-\frac{2\left(e^{2i(dx+c)}+3\,e^{i(dx+c)}+1\right)}{\sqrt{\left(e^{2i(dx+c)}+3\,e^{i(dx+c)}+1\right)e^{i(dx+c)}}}+\frac{2\left(\frac{3}{2}+\frac{\sqrt{5}}{2}\right)\sqrt{\frac{e^{i(dx+c)}+\frac{3}{2}+}{\frac{3}{2}+\frac{\sqrt{5}}{2}}}}{}\right.}{}$

input
```
int(1/sec(d*x+c)^(1/2)/(2+3*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/5/d*(-((1-cos(d*x+c))^2*csc(d*x+c)^2+5)/((1-cos(d*x+c))^2*csc(d*x+c)^2-1
))^(1/2)*(2*I*5^(1/2)*EllipticF(1/5*I*5^(1/2)*(-cot(d*x+c)+csc(d*x+c)),5^(
1/2))*(5*(1-cos(d*x+c))^2*csc(d*x+c)^2+25)^(1/2)*((1-cos(d*x+c))^2*csc(d*x
+c)^2+1)^(1/2)+I*5^(1/2)*EllipticE(1/5*I*5^(1/2)*(-cot(d*x+c)+csc(d*x+c)),
5^(1/2))*(5*(1-cos(d*x+c))^2*csc(d*x+c)^2+25)^(1/2)*((1-cos(d*x+c))^2*csc(
d*x+c)^2+1)^(1/2)+5*(1-cos(d*x+c))^3*csc(d*x+c)^3-25*cot(d*x+c)+25*csc(d*x
+c))/((1-cos(d*x+c))^2*csc(d*x+c)^2+5)/(-((1-cos(d*x+c))^2*csc(d*x+c)^2+1)
/((1-cos(d*x+c))^2*csc(d*x+c)^2-1))^(1/2)
```

3.669.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.78

$$\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{2+3\sec(c+dx)}}\,dx$$
$$= \frac{i\,\text{weierstrassPInverse}(8,-4,\cos(dx+c)+i\,\sin(dx+c)+1)-i\,\text{weierstrassPInverse}(8,-4,\cos(dx+c)}{}$$

input
```
integrate(1/sec(d*x+c)^(1/2)/(2+3*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
(I*weierstrassPInverse(8, -4, cos(d*x + c) + I*sin(d*x + c) + 1) - I*weier
strassPInverse(8, -4, cos(d*x + c) - I*sin(d*x + c) + 1) + I*weierstrassZe
ta(8, -4, weierstrassPInverse(8, -4, cos(d*x + c) + I*sin(d*x + c) + 1)) -
 I*weierstrassZeta(8, -4, weierstrassPInverse(8, -4, cos(d*x + c) - I*sin(
d*x + c) + 1)))/d
```

3.669.6 Sympy [F]

$$\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{2+3\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{3\sec(c+dx)+2}\sqrt{\sec(c+dx)}}\,dx$$

input
```
integrate(1/sec(d*x+c)**(1/2)/(2+3*sec(d*x+c))**(1/2),x)
```

output
```
Integral(1/(sqrt(3*sec(c + d*x) + 2)*sqrt(sec(c + d*x))), x)
```

3.669.7 Maxima [F]

$$\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{2+3\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{3\,\sec(dx+c)+2}\sqrt{\sec(dx+c)}}\,dx$$

input
```
integrate(1/sec(d*x+c)^(1/2)/(2+3*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(3*sec(d*x + c) + 2)*sqrt(sec(d*x + c))), x)
```

3.669.8 Giac [F]

$$\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{2+3\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{3\,\sec(dx+c)+2}\sqrt{\sec(dx+c)}}\,dx$$

input
```
integrate(1/sec(d*x+c)^(1/2)/(2+3*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(3*sec(d*x + c) + 2)*sqrt(sec(d*x + c))), x)
```

3.669.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{2+3\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{\frac{3}{\cos(c+d\,x)}+2}\,\sqrt{\frac{1}{\cos(c+d\,x)}}}\,dx$$

input
```
int(1/((3/cos(c + d*x) + 2)^(1/2)*(1/cos(c + d*x))^(1/2)),x)
```

output
```
int(1/((3/cos(c + d*x) + 2)^(1/2)*(1/cos(c + d*x))^(1/2)), x)
```

3.670 $\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{-2+3\sec(c+dx)}}\,dx$

3.670.1 Optimal result 5263
3.670.2 Mathematica [A] (verified) 5263
3.670.3 Rubi [A] (verified) 5264
3.670.4 Maple [B] (verified) 5266
3.670.5 Fricas [C] (verification not implemented) 5267
3.670.6 Sympy [F] 5267
3.670.7 Maxima [F] 5268
3.670.8 Giac [F] 5268
3.670.9 Mupad [F(-1)] 5268

3.670.1 Optimal result

Integrand size = 25, antiderivative size = 109

$$\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{-2+3\sec(c+dx)}}\,dx = \frac{3\sqrt{3-2\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),-4\right)\sqrt{\sec(c+dx)}}{d\sqrt{-2+3\sec(c+dx)}} - \frac{E\left(\frac{1}{2}(c+dx)\middle|-4\right)\sqrt{-2+3\sec(c+dx)}}{d\sqrt{3-2\cos(c+dx)}\sqrt{\sec(c+dx)}}$$

output
```
3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/
2*c),2*I)*(3-2*cos(d*x+c))^(1/2)*sec(d*x+c)^(1/2)/d/(-2+3*sec(d*x+c))^(1/2
)-(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/
2*c),2*I)*(-2+3*sec(d*x+c))^(1/2)/d/(3-2*cos(d*x+c))^(1/2)/sec(d*x+c)^(1/2
)
```

3.670.2 Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.62

$$\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{-2+3\sec(c+dx)}}\,dx = -\frac{\sqrt{3-2\cos(c+dx)}\left(E\left(\frac{1}{2}(c+dx)\middle|-4\right)-3\,\text{EllipticF}\left(\frac{1}{2}(c+dx),-4\right)\right)\sqrt{\sec(c+dx)}}{d\sqrt{-2+3\sec(c+dx)}}$$

input
```
Integrate[1/(Sqrt[Sec[c + d*x]]*Sqrt[-2 + 3*Sec[c + d*x]]),x]
```

output
```
-((Sqrt[3 - 2*Cos[c + d*x]]*(EllipticE[(c + d*x)/2, -4] - 3*EllipticF[(c +
 d*x)/2, -4])*Sqrt[Sec[c + d*x]])/(d*Sqrt[-2 + 3*Sec[c + d*x]]))
```

3.670.3 Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {3042, 4349, 3042, 4343, 3042, 3132, 4345, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{3\sec(c+dx)-2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{3\csc\left(c+dx+\frac{\pi}{2}\right)-2}}dx$$

$$\Big\downarrow \text{4349}$$

$$\frac{3}{2}\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{3\sec(c+dx)-2}}dx-\frac{1}{2}\int \frac{\sqrt{3\sec(c+dx)-2}}{\sqrt{\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{3}{2}\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{3\csc\left(c+dx+\frac{\pi}{2}\right)-2}}dx-\frac{1}{2}\int \frac{\sqrt{3\csc\left(c+dx+\frac{\pi}{2}\right)-2}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4343}$$

$$\frac{3}{2}\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{3\csc\left(c+dx+\frac{\pi}{2}\right)-2}}dx-\frac{\sqrt{3\sec(c+dx)-2}\int \sqrt{3-2\cos(c+dx)}dx}{2\sqrt{3-2\cos(c+dx)}\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3}{2}\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{3\csc\left(c+dx+\frac{\pi}{2}\right)-2}}dx-\frac{\sqrt{3\sec(c+dx)-2}\int \sqrt{3-2\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{2\sqrt{3-2\cos(c+dx)}\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{3132}$$

$$\frac{3}{2}\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{3\csc\left(c+dx+\frac{\pi}{2}\right)-2}}dx-\frac{\sqrt{3\sec(c+dx)-2}E\left(\frac{1}{2}(c+dx)\middle|-4\right)}{d\sqrt{3-2\cos(c+dx)}\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \; 4345$$

$$\frac{3\sqrt{3-2\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{3-2\cos(c+dx)}}dx}{2\sqrt{3\sec(c+dx)-2}}-\frac{\sqrt{3\sec(c+dx)-2}E\left(\frac{1}{2}(c+dx)\middle|-4\right)}{d\sqrt{3-2\cos(c+dx)}\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \; 3042$$

$$\frac{3\sqrt{3-2\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{3-2\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{2\sqrt{3\sec(c+dx)-2}}-\frac{\sqrt{3\sec(c+dx)-2}E\left(\frac{1}{2}(c+dx)\middle|-4\right)}{d\sqrt{3-2\cos(c+dx)}\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \; 3140$$

$$\frac{3\sqrt{3-2\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),-4\right)}{d\sqrt{3\sec(c+dx)-2}}-\frac{\sqrt{3\sec(c+dx)-2}E\left(\frac{1}{2}(c+dx)\middle|-4\right)}{d\sqrt{3-2\cos(c+dx)}\sqrt{\sec(c+dx)}}$$

input
```
Int[1/(Sqrt[Sec[c + d*x]]*Sqrt[-2 + 3*Sec[c + d*x]]),x]
```

output
```
(3*Sqrt[3 - 2*Cos[c + d*x]]*EllipticF[(c + d*x)/2, -4]*Sqrt[Sec[c + d*x]])
/(d*Sqrt[-2 + 3*Sec[c + d*x]]) - (EllipticE[(c + d*x)/2, -4]*Sqrt[-2 + 3*S
ec[c + d*x]])/(d*Sqrt[3 - 2*Cos[c + d*x]]*Sqrt[Sec[c + d*x]])
```

3.670.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 4343
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]])   Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a
, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4345
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]])   Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[
{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4349
```
Int[1/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)]), x_Symbol] :> Simp[1/a   Int[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Cs
c[e + f*x]], x], x] - Simp[b/(a*d)   Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Cs
c[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

3.670.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 325 vs. $2(147) = 294$.

Time = 7.48 (sec) , antiderivative size = 326, normalized size of antiderivative = 2.99

method	result
default	$-\frac{\sqrt{-\frac{5(1-\cos(dx+c))^2\csc(dx+c)^2+1}{(1-\cos(dx+c))^2\csc(dx+c)^2-1}}\left(2i\,\mathrm{EllipticF}\left(i(-\cot(dx+c)+\csc(dx+c)),\sqrt{5}\right)\sqrt{(1-\cos(dx+c))^2\csc(dx+c)^2+1}\,\sqrt{5(1-\cos(dx-}\right.}{d\left(5\right.}$
risch	$-\frac{i\left(e^{2i(dx+c)}-3\,e^{i(dx+c)}+1\right)\sqrt{2}}{d\sqrt{\frac{e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\left(e^{2i(dx+c)}+1\right)\sqrt{-\frac{2\left(e^{2i(dx+c)}-3\,e^{i(dx+c)}+1\right)}{e^{2i(dx+c)}+1}}}-\frac{i\left(\frac{-2\,e^{2i(dx+c)}+6\,e^{i(dx+c)}-2}{\sqrt{e^{i(dx+c)}\left(-2\,e^{2i(dx+c)}+6\,e^{i(dx+c)}-2\right)}}+\frac{2\left(\frac{\sqrt{5}}{2}-\frac{3}{2}\right)\sqrt{-\frac{e^{i(dx}}{}}}{}\right.}{}$

input
```
int(1/sec(d*x+c)^(1/2)/(-2+3*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/d*(-(5*(1-cos(d*x+c))^2*csc(d*x+c)^2+1)/((1-cos(d*x+c))^2*csc(d*x+c)^2-
1))^(1/2)*(2*I*EllipticF(I*(-cot(d*x+c)+csc(d*x+c)),5^(1/2))*((1-cos(d*x+c
))^2*csc(d*x+c)^2+1)^(1/2)*(5*(1-cos(d*x+c))^2*csc(d*x+c)^2+1)^(1/2)+I*Ell
ipticE(I*(-cot(d*x+c)+csc(d*x+c)),5^(1/2))*((1-cos(d*x+c))^2*csc(d*x+c)^2+
1)^(1/2)*(5*(1-cos(d*x+c))^2*csc(d*x+c)^2+1)^(1/2)+5*(1-cos(d*x+c))^3*csc(
d*x+c)^3-cot(d*x+c)+csc(d*x+c))/(5*(1-cos(d*x+c))^2*csc(d*x+c)^2+1)/(-((1-
cos(d*x+c))^2*csc(d*x+c)^2+1)/((1-cos(d*x+c))^2*csc(d*x+c)^2-1))^(1/2)
```

3.670.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.84

$$\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{-2+3\sec(c+dx)}}\,dx = -\frac{\text{weierstrassPInverse}(8,4,\cos(dx+c)+i\sin(dx+c)-1)+\text{weierstrassPInverse}(8,4,\cos(dx+c)-i}{}$$

input
```
integrate(1/sec(d*x+c)^(1/2)/(-2+3*sec(d*x+c))^(1/2),x, algorithm="fricas"
)
```

output
```
-(weierstrassPInverse(8, 4, cos(d*x + c) + I*sin(d*x + c) - 1) + weierstra
ssPInverse(8, 4, cos(d*x + c) - I*sin(d*x + c) - 1) - weierstrassZeta(8, 4
, weierstrassPInverse(8, 4, cos(d*x + c) + I*sin(d*x + c) - 1)) - weierstr
assZeta(8, 4, weierstrassPInverse(8, 4, cos(d*x + c) - I*sin(d*x + c) - 1)
))/d
```

3.670.6 Sympy [F]

$$\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{-2+3\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{3\sec(c+dx)-2}\sqrt{\sec(c+dx)}}\,dx$$

input
```
integrate(1/sec(d*x+c)**(1/2)/(-2+3*sec(d*x+c))**(1/2),x)
```

output
```
Integral(1/(sqrt(3*sec(c + d*x) - 2)*sqrt(sec(c + d*x))), x)
```

3.670.7 Maxima [F]

$$\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{-2+3\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{3\,\sec\left(dx+c\right)-2}\sqrt{\sec\left(dx+c\right)}}\,dx$$

input
```
integrate(1/sec(d*x+c)^(1/2)/(-2+3*sec(d*x+c))^(1/2),x, algorithm="maxima"
)
```

output
```
integrate(1/(sqrt(3*sec(d*x + c) - 2)*sqrt(sec(d*x + c))), x)
```

3.670.8 Giac [F]

$$\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{-2+3\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{3\,\sec\left(dx+c\right)-2}\sqrt{\sec\left(dx+c\right)}}\,dx$$

input
```
integrate(1/sec(d*x+c)^(1/2)/(-2+3*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(3*sec(d*x + c) - 2)*sqrt(sec(d*x + c))), x)
```

3.670.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{-2+3\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{\frac{3}{\cos(c+d\,x)}-2}\,\sqrt{\frac{1}{\cos(c+d\,x)}}}\,dx$$

input
```
int(1/((3/cos(c + d*x) - 2)^(1/2)*(1/cos(c + d*x))^(1/2)),x)
```

output
```
int(1/((3/cos(c + d*x) - 2)^(1/2)*(1/cos(c + d*x))^(1/2)), x)
```

3.671 $\int \frac{1}{\sqrt{2-3\sec(c+dx)}\sqrt{\sec(c+dx)}}\,dx$

3.671.1 Optimal result 5269
3.671.2 Mathematica [A] (verified) 5269
3.671.3 Rubi [A] (verified) 5270
3.671.4 Maple [B] (verified) 5273
3.671.5 Fricas [C] (verification not implemented) 5274
3.671.6 Sympy [F] 5274
3.671.7 Maxima [F] 5274
3.671.8 Giac [F] 5275
3.671.9 Mupad [F(-1)] 5275

3.671.1 Optimal result

Integrand size = 25, antiderivative size = 108

$$\int \frac{1}{\sqrt{2-3\sec(c+dx)}\sqrt{\sec(c+dx)}}\,dx = \frac{E\left(\frac{1}{2}(c+dx)\middle|-4\right)\sqrt{2-3\sec(c+dx)}}{d\sqrt{3-2\cos(c+dx)}\sqrt{\sec(c+dx)}} + \frac{3\sqrt{3-2\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),-4\right)\sqrt{\sec(c+dx)}}{d\sqrt{2-3\sec(c+dx)}}$$

output
```
(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*
c),2*I)*(2-3*sec(d*x+c))^(1/2)/d/(3-2*cos(d*x+c))^(1/2)/sec(d*x+c)^(1/2)+3
*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2
*c),2*I)*(3-2*cos(d*x+c))^(1/2)*sec(d*x+c)^(1/2)/d/(2-3*sec(d*x+c))^(1/2)
```

3.671.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.63

$$\int \frac{1}{\sqrt{2-3\sec(c+dx)}\sqrt{\sec(c+dx)}}\,dx = -\frac{\sqrt{3-2\cos(c+dx)}\left(E\left(\frac{1}{2}(c+dx)\middle|-4\right)-3\,\text{EllipticF}\left(\frac{1}{2}(c+dx),-4\right)\right)\sqrt{\sec(c+dx)}}{d\sqrt{2-3\sec(c+dx)}}$$

input
```
Integrate[1/(Sqrt[2 - 3*Sec[c + d*x]]*Sqrt[Sec[c + d*x]]),x]
```

output
```
-((Sqrt[3 - 2*Cos[c + d*x]]*(EllipticE[(c + d*x)/2, -4] - 3*EllipticF[(c +
 d*x)/2, -4])*Sqrt[Sec[c + d*x]])/(d*Sqrt[2 - 3*Sec[c + d*x]]))
```

3.671.3 Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.520$, Rules used = {3042, 4349, 3042, 4343, 3042, 3134, 3042, 3132, 4345, 3042, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{2-3\sec(c+dx)}\sqrt{\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{2-3\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4349}$$

$$\frac{1}{2}\int \frac{\sqrt{2-3\sec(c+dx)}}{\sqrt{\sec(c+dx)}}dx+\frac{3}{2}\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{2-3\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\int \frac{\sqrt{2-3\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{3}{2}\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{2-3\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4343}$$

$$\frac{3}{2}\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{2-3\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{\sqrt{2-3\sec(c+dx)}\int \sqrt{2\cos(c+dx)-3}dx}{2\sqrt{2\cos(c+dx)-3}\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3}{2}\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{2-3\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{\sqrt{2-3\sec(c+dx)}\int \sqrt{2\sin\left(c+dx+\frac{\pi}{2}\right)-3}dx}{2\sqrt{2\cos(c+dx)-3}\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{3134}$$

$$\frac{3}{2}\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{2-3\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{\sqrt{2-3\sec(c+dx)}\int\sqrt{3-2\cos(c+dx)}dx}{2\sqrt{3-2\cos(c+dx)}\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \; 3042$$

$$\frac{3}{2}\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{2-3\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{\sqrt{2-3\sec(c+dx)}\int\sqrt{3-2\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{2\sqrt{3-2\cos(c+dx)}\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \; 3132$$

$$\frac{3}{2}\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{2-3\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{\sqrt{2-3\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|-4\right)}{d\sqrt{3-2\cos(c+dx)}\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \; 4345$$

$$\frac{3\sqrt{2\cos(c+dx)-3}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{2\cos(c+dx)-3}}dx}{2\sqrt{2-3\sec(c+dx)}}+\frac{\sqrt{2-3\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|-4\right)}{d\sqrt{3-2\cos(c+dx)}\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \; 3042$$

$$\frac{3\sqrt{2\cos(c+dx)-3}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{2\sin\left(c+dx+\frac{\pi}{2}\right)-3}}dx}{2\sqrt{2-3\sec(c+dx)}}+\frac{\sqrt{2-3\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|-4\right)}{d\sqrt{3-2\cos(c+dx)}\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \; 3142$$

$$\frac{3\sqrt{3-2\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{3-2\cos(c+dx)}}dx}{2\sqrt{2-3\sec(c+dx)}}+\frac{\sqrt{2-3\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|-4\right)}{d\sqrt{3-2\cos(c+dx)}\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \; 3042$$

$$\frac{3\sqrt{3-2\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{3-2\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{2\sqrt{2-3\sec(c+dx)}}+\frac{\sqrt{2-3\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|-4\right)}{d\sqrt{3-2\cos(c+dx)}\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \; 3140$$

$$\frac{3\sqrt{3-2\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),-4\right)}{d\sqrt{2-3\sec(c+dx)}}+\frac{\sqrt{2-3\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|-4\right)}{d\sqrt{3-2\cos(c+dx)}\sqrt{\sec(c+dx)}}$$

input
```
Int[1/(Sqrt[2 - 3*Sec[c + d*x]]*Sqrt[Sec[c + d*x]]),x]
```

3.671. $\int \frac{1}{\sqrt{2-3\sec(c+dx)}\sqrt{\sec(c+dx)}}\,dx$

output
```
(EllipticE[(c + d*x)/2, -4]*Sqrt[2 - 3*Sec[c + d*x]])/(d*Sqrt[3 - 2*Cos[c
+ d*x]]*Sqrt[Sec[c + d*x]]) + (3*Sqrt[3 - 2*Cos[c + d*x]]*EllipticF[(c + d
*x)/2, -4]*Sqrt[Sec[c + d*x]])/(d*Sqrt[2 - 3*Sec[c + d*x]])
```

3.671.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 4343
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]])   Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a
, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4345
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]])   Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[
{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4349
```
Int[1/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)]), x_Symbol] :> Simp[1/a   Int[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Cs
c[e + f*x]], x], x] - Simp[b/(a*d)   Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Cs
c[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

3.671.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 342 vs. $2(146) = 292$.

Time = 7.29 (sec) , antiderivative size = 343, normalized size of antiderivative = 3.18

method	result
default	$-\frac{\sqrt{\frac{5(1-\cos(dx+c))^2\csc(dx+c)^2+1}{(1-\cos(dx+c))^2\csc(dx+c)^2-1}}\left(2i\sqrt{5}\,\mathrm{EllipticF}\left(i\sqrt{5}\,(-\cot(dx+c)+\csc(dx+c)),\frac{\sqrt{5}}{5}\right)\sqrt{5(1-\cos(dx+c))^2\csc(dx+c)^2+1}\,\sqrt{(1-\mathrm{co}}\right.}{}$
risch	$-\frac{i\left(e^{2i(dx+c)}-3e^{i(dx+c)}+1\right)}{d\sqrt{\frac{e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\left(e^{2i(dx+c)}+1\right)\sqrt{\frac{e^{2i(dx+c)}-3e^{i(dx+c)}+1}{e^{2i(dx+c)}+1}}}-\frac{i\left(-\frac{2\left(e^{2i(dx+c)}-3e^{i(dx+c)}+1\right)}{\sqrt{e^{i(dx+c)}\left(e^{2i(dx+c)}-3e^{i(dx+c)}+1\right)}}+\frac{2\left(\frac{\sqrt{5}}{2}-\frac{3}{2}\right)\sqrt{-\frac{e^{i(dx+c)}-\frac{3}{2}}{\frac{3}{2}-\frac{\sqrt{5}}{2}}}}{}\right.}{}$

input
```
int(1/(2-3*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/5/d*((5*(1-cos(d*x+c))^2*csc(d*x+c)^2+1)/((1-cos(d*x+c))^2*csc(d*x+c)^2
-1))^(1/2)*(2*I*5^(1/2)*EllipticF(I*5^(1/2)*(-cot(d*x+c)+csc(d*x+c)),1/5*5
^(1/2))*(5*(1-cos(d*x+c))^2*csc(d*x+c)^2+1)^(1/2)*((1-cos(d*x+c))^2*csc(d*
x+c)^2+1)^(1/2)-5*I*5^(1/2)*EllipticE(I*5^(1/2)*(-cot(d*x+c)+csc(d*x+c)),1
/5*5^(1/2))*(5*(1-cos(d*x+c))^2*csc(d*x+c)^2+1)^(1/2)*((1-cos(d*x+c))^2*cs
c(d*x+c)^2+1)^(1/2)-25*(1-cos(d*x+c))^3*csc(d*x+c)^3+5*cot(d*x+c)-5*csc(d*
x+c))/(5*(1-cos(d*x+c))^2*csc(d*x+c)^2+1)/(-((1-cos(d*x+c))^2*csc(d*x+c)^2
+1)/((1-cos(d*x+c))^2*csc(d*x+c)^2-1))^(1/2)
```

3.671.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.88

$$\int \frac{1}{\sqrt{2-3\sec(c+dx)}\sqrt{\sec(c+dx)}}\,dx$$
$$= \frac{-i\,\text{weierstrassPInverse}(8,4,\cos(dx+c)+i\,\sin(dx+c)-1)+i\,\text{weierstrassPInverse}(8,4,\cos(dx+c)-}{}$$

input
```
integrate(1/(2-3*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
(-I*weierstrassPInverse(8, 4, cos(d*x + c) + I*sin(d*x + c) - 1) + I*weier
strassPInverse(8, 4, cos(d*x + c) - I*sin(d*x + c) - 1) + I*weierstrassZet
a(8, 4, weierstrassPInverse(8, 4, cos(d*x + c) + I*sin(d*x + c) - 1)) - I*
weierstrassZeta(8, 4, weierstrassPInverse(8, 4, cos(d*x + c) - I*sin(d*x +
 c) - 1)))/d
```

3.671.6 Sympy [F]

$$\int \frac{1}{\sqrt{2-3\sec(c+dx)}\sqrt{\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{2-3\sec(c+dx)}\sqrt{\sec(c+dx)}}\,dx$$

input
```
integrate(1/(2-3*sec(d*x+c))**(1/2)/sec(d*x+c)**(1/2),x)
```

output
```
Integral(1/(sqrt(2 - 3*sec(c + d*x))*sqrt(sec(c + d*x))), x)
```

3.671.7 Maxima [F]

$$\int \frac{1}{\sqrt{2-3\sec(c+dx)}\sqrt{\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{-3\,\sec(dx+c)+2}\sqrt{\sec(dx+c)}}\,dx$$

input
```
integrate(1/(2-3*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(-3*sec(d*x + c) + 2)*sqrt(sec(d*x + c))), x)
```

3.671.8 Giac [F]

$$\int \frac{1}{\sqrt{2-3\sec(c+dx)}\sqrt{\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{-3\,\sec(dx+c)+2}\sqrt{\sec(dx+c)}}\,dx$$

input
```
integrate(1/(2-3*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(-3*sec(d*x + c) + 2)*sqrt(sec(d*x + c))), x)
```

3.671.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{2-3\sec(c+dx)}\sqrt{\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{2-\frac{3}{\cos(c+d\,x)}}\,\sqrt{\frac{1}{\cos(c+d\,x)}}}\,dx$$

input
```
int(1/((2 - 3/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(1/2)),x)
```

output
```
int(1/((2 - 3/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(1/2)), x)
```

3.672 $\int \frac{1}{\sqrt{-2-3\sec(c+dx)}\sqrt{\sec(c+dx)}}\,dx$

3.672.1 Optimal result . . . 5276
3.672.2 Mathematica [A] (verified) . . . 5276
3.672.3 Rubi [A] (verified) . . . 5277
3.672.4 Maple [C] (verified) . . . 5280
3.672.5 Fricas [C] (verification not implemented) . . . 5281
3.672.6 Sympy [F] . . . 5281
3.672.7 Maxima [F] . . . 5282
3.672.8 Giac [F] . . . 5282
3.672.9 Mupad [F(-1)] . . . 5282

3.672.1 Optimal result

Integrand size = 25, antiderivative size = 123

$$\begin{aligned}\int \frac{1}{\sqrt{-2-3\sec(c+dx)}\sqrt{\sec(c+dx)}}\,dx &= -\frac{\sqrt{5}E\left(\frac{1}{2}(c+dx)|\frac{4}{5}\right)\sqrt{-2-3\sec(c+dx)}}{d\sqrt{3+2\cos(c+dx)}\sqrt{\sec(c+dx)}} \\ &\quad -\frac{3\sqrt{3+2\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{4}{5}\right)\sqrt{\sec(c+dx)}}{\sqrt{5}d\sqrt{-2-3\sec(c+dx)}}\end{aligned}$$

output
```
-(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2
*c),2/5*5^(1/2))*5^(1/2)*(-2-3*sec(d*x+c))^(1/2)/d/(3+2*cos(d*x+c))^(1/2)/
sec(d*x+c)^(1/2)-3/5*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellip
ticF(sin(1/2*d*x+1/2*c),2/5*5^(1/2))*(3+2*cos(d*x+c))^(1/2)*sec(d*x+c)^(1/
2)/d*5^(1/2)/(-2-3*sec(d*x+c))^(1/2)
```

3.672.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.63

$$\int \frac{1}{\sqrt{-2-3\sec(c+dx)}\sqrt{\sec(c+dx)}}\,dx = \frac{\sqrt{3+2\cos(c+dx)}\left(5E\left(\frac{1}{2}(c+dx)|\frac{4}{5}\right)-3\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{4}{5}\right)\right)\sqrt{\sec(c+dx)}}{\sqrt{5}d\sqrt{-2-3\sec(c+dx)}}$$

input
```
Integrate[1/(Sqrt[-2 - 3*Sec[c + d*x]]*Sqrt[Sec[c + d*x]]),x]
```

output
```
(Sqrt[3 + 2*Cos[c + d*x]]*(5*EllipticE[(c + d*x)/2, 4/5] - 3*EllipticF[(c
+ d*x)/2, 4/5])*Sqrt[Sec[c + d*x]])/(Sqrt[5]*d*Sqrt[-2 - 3*Sec[c + d*x]])
```

3.672.3 Rubi [A] (verified)

Time = 0.93 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.520$, Rules used = {3042, 4349, 3042, 4343, 3042, 3134, 3042, 3132, 4345, 3042, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{-3\sec(c+dx)-2}\sqrt{\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{-3\csc\left(c+dx+\frac{\pi}{2}\right)-2}\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4349}$$

$$-\frac{1}{2}\int \frac{\sqrt{-3\sec(c+dx)-2}}{\sqrt{\sec(c+dx)}}dx-\frac{3}{2}\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{-3\sec(c+dx)-2}}dx$$

$$\Big\downarrow \text{3042}$$

$$-\frac{1}{2}\int \frac{\sqrt{-3\csc\left(c+dx+\frac{\pi}{2}\right)-2}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{3}{2}\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{-3\csc\left(c+dx+\frac{\pi}{2}\right)-2}}dx$$

$$\Big\downarrow \text{4343}$$

$$-\frac{3}{2}\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{-3\csc\left(c+dx+\frac{\pi}{2}\right)-2}}dx-\frac{\sqrt{-3\sec(c+dx)-2}\int \sqrt{-2\cos(c+dx)-3}dx}{2\sqrt{-2\cos(c+dx)-3}\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{3}{2}\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{-3\csc\left(c+dx+\frac{\pi}{2}\right)-2}}dx-\frac{\sqrt{-3\sec(c+dx)-2}\int \sqrt{-2\sin\left(c+dx+\frac{\pi}{2}\right)-3}dx}{2\sqrt{-2\cos(c+dx)-3}\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{3134}$$

$$-\frac{3}{2}\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{-3\csc\left(c+dx+\frac{\pi}{2}\right)-2}}dx-\frac{\sqrt{5}\sqrt{-3\sec(c+dx)-2}\int \sqrt{\frac{2}{5}\cos(c+dx)+\frac{3}{5}}dx}{2\sqrt{2\cos(c+dx)+3}\sqrt{\sec(c+dx)}}$$

$\downarrow$ 3042

$$-\frac{3}{2}\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{-3\csc\left(c+dx+\frac{\pi}{2}\right)-2}}dx-\frac{\sqrt{5}\sqrt{-3\sec(c+dx)-2}\int \sqrt{\frac{2}{5}\sin\left(c+dx+\frac{\pi}{2}\right)+\frac{3}{5}}dx}{2\sqrt{2\cos(c+dx)+3}\sqrt{\sec(c+dx)}}$$

$\downarrow$ 3132

$$-\frac{3}{2}\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{-3\csc\left(c+dx+\frac{\pi}{2}\right)-2}}dx-\frac{\sqrt{5}\sqrt{-3\sec(c+dx)-2}E\left(\frac{1}{2}(c+dx)|\frac{4}{5}\right)}{d\sqrt{2\cos(c+dx)+3}\sqrt{\sec(c+dx)}}$$

$\downarrow$ 4345

$$-\frac{3\sqrt{-2\cos(c+dx)-3}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{-2\cos(c+dx)-3}}dx}{2\sqrt{-3\sec(c+dx)-2}}-\frac{\sqrt{5}\sqrt{-3\sec(c+dx)-2}E\left(\frac{1}{2}(c+dx)|\frac{4}{5}\right)}{d\sqrt{2\cos(c+dx)+3}\sqrt{\sec(c+dx)}}$$

$\downarrow$ 3042

$$-\frac{3\sqrt{-2\cos(c+dx)-3}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{-2\sin\left(c+dx+\frac{\pi}{2}\right)-3}}dx}{2\sqrt{-3\sec(c+dx)-2}}-\frac{\sqrt{5}\sqrt{-3\sec(c+dx)-2}E\left(\frac{1}{2}(c+dx)|\frac{4}{5}\right)}{d\sqrt{2\cos(c+dx)+3}\sqrt{\sec(c+dx)}}$$

$\downarrow$ 3142

$$-\frac{3\sqrt{2\cos(c+dx)+3}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\frac{2}{5}\cos(c+dx)+\frac{3}{5}}}dx}{2\sqrt{5}\sqrt{-3\sec(c+dx)-2}}-\frac{\sqrt{5}\sqrt{-3\sec(c+dx)-2}E\left(\frac{1}{2}(c+dx)|\frac{4}{5}\right)}{d\sqrt{2\cos(c+dx)+3}\sqrt{\sec(c+dx)}}$$

$\downarrow$ 3042

$$-\frac{3\sqrt{2\cos(c+dx)+3}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\frac{2}{5}\sin\left(c+dx+\frac{\pi}{2}\right)+\frac{3}{5}}}dx}{2\sqrt{5}\sqrt{-3\sec(c+dx)-2}}-\frac{\sqrt{5}\sqrt{-3\sec(c+dx)-2}E\left(\frac{1}{2}(c+dx)|\frac{4}{5}\right)}{d\sqrt{2\cos(c+dx)+3}\sqrt{\sec(c+dx)}}$$

$\downarrow$ 3140

3.672. $\int \frac{1}{\sqrt{-2-3\sec(c+dx)}\sqrt{\sec(c+dx)}}\,dx$

$$-\frac{3\sqrt{2\cos(c+dx)+3}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{4}{5}\right)}{\sqrt{5}d\sqrt{-3\sec(c+dx)-2}}-\frac{\sqrt{5}\sqrt{-3\sec(c+dx)-2}E\left(\frac{1}{2}(c+dx)|\frac{4}{5}\right)}{d\sqrt{2\cos(c+dx)+3}\sqrt{\sec(c+dx)}}$$

input
```
Int[1/(Sqrt[-2 - 3*Sec[c + d*x]]*Sqrt[Sec[c + d*x]]),x]
```

output
```
-((Sqrt[5]*EllipticE[(c + d*x)/2, 4/5]*Sqrt[-2 - 3*Sec[c + d*x]])/(d*Sqrt[
3 + 2*Cos[c + d*x]]*Sqrt[Sec[c + d*x]])) - (3*Sqrt[3 + 2*Cos[c + d*x]]*Ell
ipticF[(c + d*x)/2, 4/5]*Sqrt[Sec[c + d*x]])/(Sqrt[5]*d*Sqrt[-2 - 3*Sec[c
+ d*x]])
```

3.672.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 4343
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]])   Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a
, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4345
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]])   Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[
{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4349
```
Int[1/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)]), x_Symbol] :> Simp[1/a   Int[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Cs
c[e + f*x]], x], x] - Simp[b/(a*d)   Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Cs
c[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

3.672.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 7.49 (sec) , antiderivative size = 329, normalized size of antiderivative = 2.67

method	result
default	$\frac{\sqrt{\frac{(1-\cos(dx+c))^2\csc(dx+c)^2+5}{(1-\cos(dx+c))^2\csc(dx+c)^2-1}}\left(2i\,\mathrm{EllipticF}\left(i(-\cot(dx+c)+\csc(dx+c)),\frac{\sqrt{5}}{5}\right)\sqrt{(1-\cos(dx+c))^2\csc(dx+c)^2+1}\,\sqrt{5(1-\cos(dx+c))^2}\right.}{5d}$
risch	$-\frac{i\left(e^{2i(dx+c)}+3e^{i(dx+c)}+1\right)\sqrt{2}}{d\sqrt{\frac{e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\left(e^{2i(dx+c)}+1\right)\sqrt{-\frac{2\left(e^{2i(dx+c)}+3e^{i(dx+c)}+1\right)}{e^{2i(dx+c)}+1}}}-\frac{i\left(\frac{-2e^{2i(dx+c)}-6e^{i(dx+c)}-2}{\sqrt{e^{i(dx+c)}\left(-2e^{2i(dx+c)}-6e^{i(dx+c)}-2\right)}}+\frac{2\left(\frac{3}{2}+\frac{\sqrt{5}}{2}\right)\sqrt{\frac{e^{i(dx+c)}}{\frac{3}{2}}}}{}\right.}{}$

input
```
int(1/(-2-3*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/5/d*(((1-cos(d*x+c))^2*csc(d*x+c)^2+5)/((1-cos(d*x+c))^2*csc(d*x+c)^2-1)
)^(1/2)*(2*I*EllipticF(I*(-cot(d*x+c)+csc(d*x+c)),1/5*5^(1/2))*((1-cos(d*x
+c))^2*csc(d*x+c)^2+1)^(1/2)*(5*(1-cos(d*x+c))^2*csc(d*x+c)^2+25)^(1/2)-5*
I*EllipticE(I*(-cot(d*x+c)+csc(d*x+c)),1/5*5^(1/2))*((1-cos(d*x+c))^2*csc(
d*x+c)^2+1)^(1/2)*(5*(1-cos(d*x+c))^2*csc(d*x+c)^2+25)^(1/2)-5*(1-cos(d*x+
c))^3*csc(d*x+c)^3+25*cot(d*x+c)-25*csc(d*x+c))/((1-cos(d*x+c))^2*csc(d*x+
c)^2+5)/(-((1-cos(d*x+c))^2*csc(d*x+c)^2+1)/((1-cos(d*x+c))^2*csc(d*x+c)^2
-1))^(1/2)
```

3.672.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.71

$$\int \frac{1}{\sqrt{-2-3\sec(c+dx)}\sqrt{\sec(c+dx)}}\,dx$$
$$= \frac{\text{weierstrassPInverse}(8,-4,\cos(dx+c)+i\sin(dx+c)+1)+\text{weierstrassPInverse}(8,-4,\cos(dx+c)-}{}$$

input
```
integrate(1/(-2-3*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x, algorithm="fricas"
)
```

output
```
(weierstrassPInverse(8, -4, cos(d*x + c) + I*sin(d*x + c) + 1) + weierstra
ssPInverse(8, -4, cos(d*x + c) - I*sin(d*x + c) + 1) + weierstrassZeta(8,
-4, weierstrassPInverse(8, -4, cos(d*x + c) + I*sin(d*x + c) + 1)) + weier
strassZeta(8, -4, weierstrassPInverse(8, -4, cos(d*x + c) - I*sin(d*x + c)
 + 1)))/d
```

3.672.6 Sympy [F]

$$\int \frac{1}{\sqrt{-2-3\sec(c+dx)}\sqrt{\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{-3\sec(c+dx)-2}\sqrt{\sec(c+dx)}}\,dx$$

input
```
integrate(1/(-2-3*sec(d*x+c))**(1/2)/sec(d*x+c)**(1/2),x)
```

output
```
Integral(1/(sqrt(-3*sec(c + d*x) - 2)*sqrt(sec(c + d*x))), x)
```

3.672.7 Maxima [F]

$$\int \frac{1}{\sqrt{-2-3\sec(c+dx)}\sqrt{\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{-3\,\sec(dx+c)-2}\sqrt{\sec(dx+c)}}\,dx$$

input
```
integrate(1/(-2-3*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x, algorithm="maxima"
)
```

output
```
integrate(1/(sqrt(-3*sec(d*x + c) - 2)*sqrt(sec(d*x + c))), x)
```

3.672.8 Giac [F]

$$\int \frac{1}{\sqrt{-2-3\sec(c+dx)}\sqrt{\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{-3\,\sec(dx+c)-2}\sqrt{\sec(dx+c)}}\,dx$$

input
```
integrate(1/(-2-3*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(-3*sec(d*x + c) - 2)*sqrt(sec(d*x + c))), x)
```

3.672.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{-2-3\sec(c+dx)}\sqrt{\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{-\frac{3}{\cos(c+d\,x)}-2}\,\sqrt{\frac{1}{\cos(c+d\,x)}}}\,dx$$

input
```
int(1/((- 3/cos(c + d*x) - 2)^(1/2)*(1/cos(c + d*x))^(1/2)),x)
```

output
```
int(1/((- 3/cos(c + d*x) - 2)^(1/2)*(1/cos(c + d*x))^(1/2)), x)
```

3.673 $\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{3+2\sec(c+dx)}}\,dx$

3.673.1 Optimal result 5283
3.673.2 Mathematica [A] (verified) 5283
3.673.3 Rubi [A] (verified) 5284
3.673.4 Maple [C] (verified) 5286
3.673.5 Fricas [C] (verification not implemented) 5287
3.673.6 Sympy [F] 5288
3.673.7 Maxima [F] 5288
3.673.8 Giac [F] 5288
3.673.9 Mupad [F(-1)] 5289

3.673.1 Optimal result

Integrand size = 25, antiderivative size = 127

$$\begin{aligned}\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{3+2\sec(c+dx)}}\,dx &= -\frac{4\sqrt{2+3\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{6}{5}\right)\sqrt{\sec(c+dx)}}{3\sqrt{5}d\sqrt{3+2\sec(c+dx)}} \\ &\quad + \frac{2\sqrt{5}E\left(\frac{1}{2}(c+dx)|\frac{6}{5}\right)\sqrt{3+2\sec(c+dx)}}{3d\sqrt{2+3\cos(c+dx)}\sqrt{\sec(c+dx)}}\end{aligned}$$

output
```
-4/15*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*
x+1/2*c),1/5*30^(1/2))*(2+3*cos(d*x+c))^(1/2)*sec(d*x+c)^(1/2)/d*5^(1/2)/(
3+2*sec(d*x+c))^(1/2)+2/3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*
EllipticE(sin(1/2*d*x+1/2*c),1/5*30^(1/2))*5^(1/2)*(3+2*sec(d*x+c))^(1/2)/
d/(2+3*cos(d*x+c))^(1/2)/sec(d*x+c)^(1/2)
```

3.673.2 Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.64

$$\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{3+2\sec(c+dx)}}\,dx = \frac{2\sqrt{2+3\cos(c+dx)}\left(5E\left(\frac{1}{2}(c+dx)|\frac{6}{5}\right)-2\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{6}{5}\right)\right)\sqrt{\sec(c+dx)}}{3\sqrt{5}d\sqrt{3+2\sec(c+dx)}}$$

input
```
Integrate[1/(Sqrt[Sec[c + d*x]]*Sqrt[3 + 2*Sec[c + d*x]]),x]
```

output
```
(2*Sqrt[2 + 3*Cos[c + d*x]]*(5*EllipticE[(c + d*x)/2, 6/5] - 2*EllipticF[(
c + d*x)/2, 6/5])*Sqrt[Sec[c + d*x]])/(3*Sqrt[5]*d*Sqrt[3 + 2*Sec[c + d*x]
])
```

3.673.3 Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {3042, 4349, 3042, 4343, 3042, 3132, 4345, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{2\sec(c+dx)+3}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{2\csc\left(c+dx+\frac{\pi}{2}\right)+3}}dx$$

$$\Big\downarrow \text{4349}$$

$$\frac{1}{3}\int \frac{\sqrt{2\sec(c+dx)+3}}{\sqrt{\sec(c+dx)}}dx-\frac{2}{3}\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{2\sec(c+dx)+3}}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\int \frac{\sqrt{2\csc\left(c+dx+\frac{\pi}{2}\right)+3}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2}{3}\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{2\csc\left(c+dx+\frac{\pi}{2}\right)+3}}dx$$

$$\Big\downarrow \text{4343}$$

$$\frac{\sqrt{2\sec(c+dx)+3}\int \sqrt{3\cos(c+dx)+2}dx}{3\sqrt{3\cos(c+dx)+2}\sqrt{\sec(c+dx)}}-\frac{2}{3}\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{2\csc\left(c+dx+\frac{\pi}{2}\right)+3}}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{2\sec(c+dx)+3}\int \sqrt{3\sin\left(c+dx+\frac{\pi}{2}\right)+2}dx}{3\sqrt{3\cos(c+dx)+2}\sqrt{\sec(c+dx)}}-\frac{2}{3}\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{2\csc\left(c+dx+\frac{\pi}{2}\right)+3}}dx$$

$$\Big\downarrow \text{3132}$$

$$\frac{2\sqrt{5}\sqrt{2\sec(c+dx)+3}E\left(\frac{1}{2}(c+dx)|\frac{6}{5}\right)}{3d\sqrt{3\cos(c+dx)+2}\sqrt{\sec(c+dx)}}-\frac{2}{3}\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{2\csc\left(c+dx+\frac{\pi}{2}\right)+3}}dx$$

$$\Big\downarrow \text{4345}$$

$$\frac{2\sqrt{5}\sqrt{2\sec(c+dx)+3}E\left(\frac{1}{2}(c+dx)|\frac{6}{5}\right)}{3d\sqrt{3\cos(c+dx)+2}\sqrt{\sec(c+dx)}}-\frac{2\sqrt{3\cos(c+dx)+2}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{3\cos(c+dx)+2}}dx}{3\sqrt{2\sec(c+dx)+3}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\sqrt{5}\sqrt{2\sec(c+dx)+3}E\left(\frac{1}{2}(c+dx)|\frac{6}{5}\right)}{3d\sqrt{3\cos(c+dx)+2}\sqrt{\sec(c+dx)}}-\frac{2\sqrt{3\cos(c+dx)+2}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{3\sin\left(c+dx+\frac{\pi}{2}\right)+2}}dx}{3\sqrt{2\sec(c+dx)+3}}$$

$$\Big\downarrow \text{3140}$$

$$\frac{2\sqrt{5}\sqrt{2\sec(c+dx)+3}E\left(\frac{1}{2}(c+dx)|\frac{6}{5}\right)}{3d\sqrt{3\cos(c+dx)+2}\sqrt{\sec(c+dx)}}-\frac{4\sqrt{3\cos(c+dx)+2}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{6}{5}\right)}{3\sqrt{5}d\sqrt{2\sec(c+dx)+3}}$$

input
```
Int[1/(Sqrt[Sec[c + d*x]]*Sqrt[3 + 2*Sec[c + d*x]]),x]
```

output
```
(-4*Sqrt[2 + 3*Cos[c + d*x]]*EllipticF[(c + d*x)/2, 6/5]*Sqrt[Sec[c + d*x]
])/(3*Sqrt[5]*d*Sqrt[3 + 2*Sec[c + d*x]]) + (2*Sqrt[5]*EllipticE[(c + d*x)
/2, 6/5]*Sqrt[3 + 2*Sec[c + d*x]])/(3*d*Sqrt[2 + 3*Cos[c + d*x]]*Sqrt[Sec[
c + d*x]])
```

3.673.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 4343
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]])   Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a
, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4345
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]])   Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[
{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4349
```
Int[1/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)]), x_Symbol] :> Simp[1/a   Int[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Cs
c[e + f*x]], x], x] - Simp[b/(a*d)   Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Cs
c[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

3.673.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 7.48 (sec) , antiderivative size = 339, normalized size of antiderivative = 2.67

method	result
default	$\frac{2\sqrt{\frac{(1-\cos(dx+c))^2\csc(dx+c)^2-5}{(1-\cos(dx+c))^2\csc(dx+c)^2-1}}\left(3\,\mathrm{EllipticF}\left(\frac{\sqrt{5}\,(-\cot(dx+c)+\csc(dx+c))}{5},i\sqrt{5}\right)\sqrt{5}\,\sqrt{-5(1-\cos(dx+c))^2\csc(dx+c)^2+25}\,\sqrt{(1-\cos(d}$
risch	$-\frac{i\left(3\,\mathrm{e}^{2i(dx+c)}+4\,\mathrm{e}^{i(dx+c)}+3\right)\sqrt{2}}{3d\sqrt{\frac{\mathrm{e}^{i(dx+c)}}{\mathrm{e}^{2i(dx+c)}+1}}\left(\mathrm{e}^{2i(dx+c)}+1\right)\sqrt{\frac{3\,\mathrm{e}^{2i(dx+c)}+4\,\mathrm{e}^{i(dx+c)}+3}{\mathrm{e}^{2i(dx+c)}+1}}}-\frac{i\left(-\frac{2\left(3\,\mathrm{e}^{2i(dx+c)}+4\,\mathrm{e}^{i(dx+c)}+3\right)}{3\sqrt{\mathrm{e}^{i(dx+c)}\left(3\,\mathrm{e}^{2i(dx+c)}+4\,\mathrm{e}^{i(dx+c)}+3\right)}}+\frac{\left(\frac{2}{3}+\frac{i\sqrt{5}}{3}\right)\sqrt{\frac{\mathrm{e}^{i(dx+c)}}{\frac{2}{3}+}}}{}\right.}{}$

input
```
int(1/sec(d*x+c)^(1/2)/(3+2*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/15/d*(((1-cos(d*x+c))^2*csc(d*x+c)^2-5)/((1-cos(d*x+c))^2*csc(d*x+c)^2-1
))^(1/2)*(3*EllipticF(1/5*5^(1/2)*(-cot(d*x+c)+csc(d*x+c)),I*5^(1/2))*5^(1
/2)*(-5*(1-cos(d*x+c))^2*csc(d*x+c)^2+25)^(1/2)*((1-cos(d*x+c))^2*csc(d*x+
c)^2+1)^(1/2)-EllipticE(1/5*5^(1/2)*(-cot(d*x+c)+csc(d*x+c)),I*5^(1/2))*5^
(1/2)*(-5*(1-cos(d*x+c))^2*csc(d*x+c)^2+25)^(1/2)*((1-cos(d*x+c))^2*csc(d*
x+c)^2+1)^(1/2)+5*(1-cos(d*x+c))^3*csc(d*x+c)^3+25*cot(d*x+c)-25*csc(d*x+c
))/((1-cos(d*x+c))^2*csc(d*x+c)^2-5)/(-((1-cos(d*x+c))^2*csc(d*x+c)^2+1)/(
(1-cos(d*x+c))^2*csc(d*x+c)^2-1))^(1/2)
```

3.673.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.85

$$\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{3+2\sec(c+dx)}}\,dx$$
$$= \frac{4i\sqrt{6}\mathrm{weierstrassPInverse}\left(-\frac{44}{27},\frac{784}{729},\cos(dx+c)+i\sin(dx+c)+\frac{4}{9}\right)-4i\sqrt{6}\mathrm{weierstrassPInverse}\left(-\frac{44}{27},\frac{7}{7}\right.}{}$$

input
```
integrate(1/sec(d*x+c)^(1/2)/(3+2*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
1/27*(4*I*sqrt(6)*weierstrassPInverse(-44/27, 784/729, cos(d*x + c) + I*si
n(d*x + c) + 4/9) - 4*I*sqrt(6)*weierstrassPInverse(-44/27, 784/729, cos(d
*x + c) - I*sin(d*x + c) + 4/9) + 9*I*sqrt(6)*weierstrassZeta(-44/27, 784/
729, weierstrassPInverse(-44/27, 784/729, cos(d*x + c) + I*sin(d*x + c) +
4/9)) - 9*I*sqrt(6)*weierstrassZeta(-44/27, 784/729, weierstrassPInverse(-
44/27, 784/729, cos(d*x + c) - I*sin(d*x + c) + 4/9)))/d
```

3.673.6 Sympy [F]

$$\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{3+2\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{2\sec(c+dx)+3}\sqrt{\sec(c+dx)}}\,dx$$

input
```
integrate(1/sec(d*x+c)**(1/2)/(3+2*sec(d*x+c))**(1/2),x)
```

output
```
Integral(1/(sqrt(2*sec(c + d*x) + 3)*sqrt(sec(c + d*x))), x)
```

3.673.7 Maxima [F]

$$\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{3+2\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{2\sec(dx+c)+3}\sqrt{\sec(dx+c)}}\,dx$$

input
```
integrate(1/sec(d*x+c)^(1/2)/(3+2*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(2*sec(d*x + c) + 3)*sqrt(sec(d*x + c))), x)
```

3.673.8 Giac [F]

$$\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{3+2\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{2\sec(dx+c)+3}\sqrt{\sec(dx+c)}}\,dx$$

input
```
integrate(1/sec(d*x+c)^(1/2)/(3+2*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(2*sec(d*x + c) + 3)*sqrt(sec(d*x + c))), x)
```

3.673.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{3+2\sec(c+dx)}}\,dx=\int \frac{1}{\sqrt{\frac{2}{\cos(c+d\,x)}+3}\,\sqrt{\frac{1}{\cos(c+d\,x)}}}\,dx$$

input
```
int(1/((2/cos(c + d*x) + 3)^(1/2)*(1/cos(c + d*x))^(1/2)),x)
```

output
```
int(1/((2/cos(c + d*x) + 3)^(1/2)*(1/cos(c + d*x))^(1/2)), x)
```

3.674 $\int \frac{1}{\sqrt{3-2\sec(c+dx)}\sqrt{\sec(c+dx)}}\,dx$

3.674.1 Optimal result 5290
3.674.2 Mathematica [A] (verified) 5290
3.674.3 Rubi [A] (verified) 5291
3.674.4 Maple [C] (verified) 5293
3.674.5 Fricas [C] (verification not implemented) 5294
3.674.6 Sympy [F] 5294
3.674.7 Maxima [F] 5295
3.674.8 Giac [F] 5295
3.674.9 Mupad [F(-1)] 5295

3.674.1 Optimal result

Integrand size = 25, antiderivative size = 113

$$\begin{aligned}\int \frac{1}{\sqrt{3-2\sec(c+dx)}\sqrt{\sec(c+dx)}}\,dx &= \frac{2E\left(\frac{1}{2}(c+dx)\middle|6\right)\sqrt{3-2\sec(c+dx)}}{3d\sqrt{-2+3\cos(c+dx)}\sqrt{\sec(c+dx)}} \\ &\quad+\frac{4\sqrt{-2+3\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),6\right)\sqrt{\sec(c+dx)}}{3d\sqrt{3-2\sec(c+dx)}}\end{aligned}$$

output
```
2/3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+
1/2*c),6^(1/2))*(3-2*sec(d*x+c))^(1/2)/d/(-2+3*cos(d*x+c))^(1/2)/sec(d*x+c
)^(1/2)+4/3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(
1/2*d*x+1/2*c),6^(1/2))*(-2+3*cos(d*x+c))^(1/2)*sec(d*x+c)^(1/2)/d/(3-2*se
c(d*x+c))^(1/2)
```

3.674.2 Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.64

$$\int \frac{1}{\sqrt{3-2\sec(c+dx)}\sqrt{\sec(c+dx)}}\,dx = \frac{\sqrt{-2+3\cos(c+dx)}\left(2E\left(\frac{1}{2}(c+dx)\middle|6\right)+4\,\text{EllipticF}\left(\frac{1}{2}(c+dx),6\right)\right)\sqrt{\sec(c+dx)}}{3d\sqrt{3-2\sec(c+dx)}}$$

input
```
Integrate[1/(Sqrt[3 - 2*Sec[c + d*x]]*Sqrt[Sec[c + d*x]]),x]
```

output
```
(Sqrt[-2 + 3*Cos[c + d*x]]*(2*EllipticE[(c + d*x)/2, 6] + 4*EllipticF[(c +
 d*x)/2, 6])*Sqrt[Sec[c + d*x]])/(3*d*Sqrt[3 - 2*Sec[c + d*x]])
```

3.674.3 Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {3042, 4349, 3042, 4343, 3042, 3132, 4345, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{3-2\sec(c+dx)}\sqrt{\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{3-2\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4349}$$

$$\frac{1}{3}\int \frac{\sqrt{3-2\sec(c+dx)}}{\sqrt{\sec(c+dx)}}dx+\frac{2}{3}\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{3-2\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\int \frac{\sqrt{3-2\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2}{3}\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{3-2\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4343}$$

$$\frac{2}{3}\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{3-2\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{\sqrt{3-2\sec(c+dx)}\int \sqrt{3\cos(c+dx)-2}dx}{3\sqrt{3\cos(c+dx)-2}\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{3}\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{3-2\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{\sqrt{3-2\sec(c+dx)}\int \sqrt{3\sin\left(c+dx+\frac{\pi}{2}\right)-2}dx}{3\sqrt{3\cos(c+dx)-2}\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{3132}$$

$$\frac{2}{3}\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{3-2\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sqrt{3-2\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|6\right)}{3d\sqrt{3\cos(c+dx)-2}\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \; 4345$$

$$\frac{2\sqrt{3\cos(c+dx)-2}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{3\cos(c+dx)-2}}dx}{3\sqrt{3-2\sec(c+dx)}}+\frac{2\sqrt{3-2\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|6\right)}{3d\sqrt{3\cos(c+dx)-2}\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \; 3042$$

$$\frac{2\sqrt{3\cos(c+dx)-2}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{3\sin\left(c+dx+\frac{\pi}{2}\right)-2}}dx}{3\sqrt{3-2\sec(c+dx)}}+\frac{2\sqrt{3-2\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|6\right)}{3d\sqrt{3\cos(c+dx)-2}\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \; 3140$$

$$\frac{4\sqrt{3\cos(c+dx)-2}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),6\right)}{3d\sqrt{3-2\sec(c+dx)}}+\frac{2\sqrt{3-2\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|6\right)}{3d\sqrt{3\cos(c+dx)-2}\sqrt{\sec(c+dx)}}$$

input
```
Int[1/(Sqrt[3 - 2*Sec[c + d*x]]*Sqrt[Sec[c + d*x]]),x]
```

output
```
(2*EllipticE[(c + d*x)/2, 6]*Sqrt[3 - 2*Sec[c + d*x]])/(3*d*Sqrt[-2 + 3*Co
s[c + d*x]]*Sqrt[Sec[c + d*x]]) + (4*Sqrt[-2 + 3*Cos[c + d*x]]*EllipticF[(
c + d*x)/2, 6]*Sqrt[Sec[c + d*x]])/(3*d*Sqrt[3 - 2*Sec[c + d*x]])
```

3.674.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 4343
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]])   Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a
, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4345
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]])   Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[
{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4349
```
Int[1/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)]), x_Symbol] :> Simp[1/a   Int[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Cs
c[e + f*x]], x], x] - Simp[b/(a*d)   Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Cs
c[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

3.674.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 7.72 (sec) , antiderivative size = 339, normalized size of antiderivative = 3.00

method	result
default	$\frac{2\sqrt{\frac{5(1-\cos(dx+c))^2\csc(dx+c)^2-1}{(1-\cos(dx+c))^2\csc(dx+c)^2-1}}\left(3\sqrt{5}\,\mathrm{EllipticF}\left(\sqrt{5}\,(-\cot(dx+c)+\csc(dx+c)),\frac{i\sqrt{5}}{5}\right)\sqrt{-5(1-\cos(dx+c))^2\csc(dx+c)^2+1}\,\sqrt{(1-\mathrm{co}}{}$
risch	$-\frac{i(3\,\mathrm{e}^{2i(dx+c)}-4\,\mathrm{e}^{i(dx+c)}+3)\sqrt{2}}{3d\sqrt{\frac{\mathrm{e}^{i(dx+c)}}{\mathrm{e}^{2i(dx+c)}+1}}\,(\mathrm{e}^{2i(dx+c)}+1)\sqrt{\frac{3\,\mathrm{e}^{2i(dx+c)}-4\,\mathrm{e}^{i(dx+c)}+3}{\mathrm{e}^{2i(dx+c)}+1}}}-\frac{i\left(-\frac{2\left(3\,\mathrm{e}^{2i(dx+c)}-4\,\mathrm{e}^{i(dx+c)}+3\right)}{3\sqrt{\mathrm{e}^{i(dx+c)}\left(3\,\mathrm{e}^{2i(dx+c)}-4\,\mathrm{e}^{i(dx+c)}+3\right)}}+\frac{\left(-\frac{2}{3}+\frac{i\sqrt{5}}{3}\right)\sqrt{\frac{\mathrm{e}^{i(dx+}}{-}}}{}\right.}{}$

input
```
int(1/(3-2*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/15/d*((5*(1-cos(d*x+c))^2*csc(d*x+c)^2-1)/((1-cos(d*x+c))^2*csc(d*x+c)^2
-1))^(1/2)*(3*5^(1/2)*EllipticF(5^(1/2)*(-cot(d*x+c)+csc(d*x+c)),1/5*I*5^(
1/2))*(-5*(1-cos(d*x+c))^2*csc(d*x+c)^2+1)^(1/2)*((1-cos(d*x+c))^2*csc(d*x
+c)^2+1)^(1/2)-5*5^(1/2)*EllipticE(5^(1/2)*(-cot(d*x+c)+csc(d*x+c)),1/5*I*
5^(1/2))*(-5*(1-cos(d*x+c))^2*csc(d*x+c)^2+1)^(1/2)*((1-cos(d*x+c))^2*csc(
d*x+c)^2+1)^(1/2)+25*(1-cos(d*x+c))^3*csc(d*x+c)^3+5*cot(d*x+c)-5*csc(d*x+
c))/(5*(1-cos(d*x+c))^2*csc(d*x+c)^2-1)/(-((1-cos(d*x+c))^2*csc(d*x+c)^2+1
)/((1-cos(d*x+c))^2*csc(d*x+c)^2-1))^(1/2)
```

3.674.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.96

$$\int \frac{1}{\sqrt{3-2\sec(c+dx)}\sqrt{\sec(c+dx)}}\,dx$$
$$= \frac{-4i\sqrt{6}\text{weierstrassPInverse}\left(-\frac{44}{27},-\frac{784}{729},\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)-\frac{4}{9}\right)+4i\sqrt{6}\text{weierstrassPInverse}\left(-\right.}{}$$

input
```
integrate(1/(3-2*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
1/27*(-4*I*sqrt(6)*weierstrassPInverse(-44/27, -784/729, cos(d*x + c) + I*
sin(d*x + c) - 4/9) + 4*I*sqrt(6)*weierstrassPInverse(-44/27, -784/729, co
s(d*x + c) - I*sin(d*x + c) - 4/9) + 9*I*sqrt(6)*weierstrassZeta(-44/27, -
784/729, weierstrassPInverse(-44/27, -784/729, cos(d*x + c) + I*sin(d*x +
c) - 4/9)) - 9*I*sqrt(6)*weierstrassZeta(-44/27, -784/729, weierstrassPInv
erse(-44/27, -784/729, cos(d*x + c) - I*sin(d*x + c) - 4/9)))/d
```

3.674.6 Sympy [F]

$$\int \frac{1}{\sqrt{3-2\sec(c+dx)}\sqrt{\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{3-2\sec\left(c+dx\right)}\sqrt{\sec\left(c+dx\right)}}\,dx$$

input
```
integrate(1/(3-2*sec(d*x+c))**(1/2)/sec(d*x+c)**(1/2),x)
```

output
```
Integral(1/(sqrt(3 - 2*sec(c + d*x))*sqrt(sec(c + d*x))), x)
```

3.674.7 Maxima [F]

$$\int \frac{1}{\sqrt{3-2\sec(c+dx)}\sqrt{\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{-2\,\sec(dx+c)+3}\sqrt{\sec(dx+c)}}\,dx$$

input
```
integrate(1/(3-2*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(-2*sec(d*x + c) + 3)*sqrt(sec(d*x + c))), x)
```

3.674.8 Giac [F]

$$\int \frac{1}{\sqrt{3-2\sec(c+dx)}\sqrt{\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{-2\,\sec(dx+c)+3}\sqrt{\sec(dx+c)}}\,dx$$

input
```
integrate(1/(3-2*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(-2*sec(d*x + c) + 3)*sqrt(sec(d*x + c))), x)
```

3.674.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{3-2\sec(c+dx)}\sqrt{\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{3-\frac{2}{\cos(c+d\,x)}}\,\sqrt{\frac{1}{\cos(c+d\,x)}}}\,dx$$

input
```
int(1/((3 - 2/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(1/2)),x)
```

output
```
int(1/((3 - 2/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(1/2)), x)
```

3.675 $\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{-3+2\sec(c+dx)}}\,dx$

3.675.1 Optimal result 5296
3.675.2 Mathematica [A] (verified) 5296
3.675.3 Rubi [A] (verified) 5297
3.675.4 Maple [C] (verified) 5299
3.675.5 Fricas [C] (verification not implemented) 5300
3.675.6 Sympy [F] 5301
3.675.7 Maxima [F] 5301
3.675.8 Giac [F] 5301
3.675.9 Mupad [F(-1)] 5302

3.675.1 Optimal result

Integrand size = 25, antiderivative size = 129

$$\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{-3+2\sec(c+dx)}}\,dx = \frac{4\sqrt{2-3\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+\pi+dx),\frac{6}{5}\right)\sqrt{\sec(c+dx)}}{3\sqrt{5}d\sqrt{-3+2\sec(c+dx)}} - \frac{2\sqrt{5}E\left(\frac{1}{2}(c+\pi+dx)|\frac{6}{5}\right)\sqrt{-3+2\sec(c+dx)}}{3d\sqrt{2-3\cos(c+dx)}\sqrt{\sec(c+dx)}}$$

output
```
-4/15*(sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)*EllipticF(cos(1/2*d*
x+1/2*c),1/5*30^(1/2))*(2-3*cos(d*x+c))^(1/2)*sec(d*x+c)^(1/2)/d*5^(1/2)/(
-3+2*sec(d*x+c))^(1/2)+2/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)
*EllipticE(cos(1/2*d*x+1/2*c),1/5*30^(1/2))*5^(1/2)*(-3+2*sec(d*x+c))^(1/2
)/d/(2-3*cos(d*x+c))^(1/2)/sec(d*x+c)^(1/2)
```

3.675.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.56

$$\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{-3+2\sec(c+dx)}}\,dx = \frac{\sqrt{-2+3\cos(c+dx)}\left(2E\left(\frac{1}{2}(c+dx)\middle|6\right)+4\,\text{EllipticF}\left(\frac{1}{2}(c+dx),6\right)\right)\sqrt{\sec(c+dx)}}{3d\sqrt{-3+2\sec(c+dx)}}$$

input
```
Integrate[1/(Sqrt[Sec[c + d*x]]*Sqrt[-3 + 2*Sec[c + d*x]]),x]
```

output
```
(Sqrt[-2 + 3*Cos[c + d*x]]*(2*EllipticE[(c + d*x)/2, 6] + 4*EllipticF[(c +
 d*x)/2, 6])*Sqrt[Sec[c + d*x]])/(3*d*Sqrt[-3 + 2*Sec[c + d*x]])
```

3.675.3 Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {3042, 4349, 3042, 4343, 3042, 3133, 4345, 3042, 3141}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{2\sec(c+dx)-3}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{2\csc\left(c+dx+\frac{\pi}{2}\right)-3}}dx$$

$$\Big\downarrow \text{4349}$$

$$\frac{2}{3}\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{2\sec(c+dx)-3}}dx-\frac{1}{3}\int \frac{\sqrt{2\sec(c+dx)-3}}{\sqrt{\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{3}\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{2\csc\left(c+dx+\frac{\pi}{2}\right)-3}}dx-\frac{1}{3}\int \frac{\sqrt{2\csc\left(c+dx+\frac{\pi}{2}\right)-3}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4343}$$

$$\frac{2}{3}\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{2\csc\left(c+dx+\frac{\pi}{2}\right)-3}}dx-\frac{\sqrt{2\sec(c+dx)-3}\int \sqrt{2-3\cos(c+dx)}dx}{3\sqrt{2-3\cos(c+dx)}\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{3}\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{2\csc\left(c+dx+\frac{\pi}{2}\right)-3}}dx-\frac{\sqrt{2\sec(c+dx)-3}\int \sqrt{2-3\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{3\sqrt{2-3\cos(c+dx)}\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{3133}$$

$$\frac{2}{3}\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{2\csc\left(c+dx+\frac{\pi}{2}\right)-3}}dx-\frac{2\sqrt{5}\sqrt{2\sec(c+dx)-3}E\left(\frac{1}{2}(c+dx+\pi)|\frac{6}{5}\right)}{3d\sqrt{2-3\cos(c+dx)}\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{4345}$$

$$\frac{2\sqrt{2-3\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{2-3\cos(c+dx)}}dx}{3\sqrt{2\sec(c+dx)-3}}-\frac{2\sqrt{5}\sqrt{2\sec(c+dx)-3}E\left(\frac{1}{2}(c+dx+\pi)|\frac{6}{5}\right)}{3d\sqrt{2-3\cos(c+dx)}\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\sqrt{2-3\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{2-3\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3\sqrt{2\sec(c+dx)-3}}-\frac{2\sqrt{5}\sqrt{2\sec(c+dx)-3}E\left(\frac{1}{2}(c+dx+\pi)|\frac{6}{5}\right)}{3d\sqrt{2-3\cos(c+dx)}\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{3141}$$

$$\frac{4\sqrt{2-3\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx+\pi),\frac{6}{5}\right)}{3\sqrt{5}d\sqrt{2\sec(c+dx)-3}}-\frac{2\sqrt{5}\sqrt{2\sec(c+dx)-3}E\left(\frac{1}{2}(c+dx+\pi)|\frac{6}{5}\right)}{3d\sqrt{2-3\cos(c+dx)}\sqrt{\sec(c+dx)}}$$

input
```
Int[1/(Sqrt[Sec[c + d*x]]*Sqrt[-3 + 2*Sec[c + d*x]]),x]
```

output
```
(4*Sqrt[2 - 3*Cos[c + d*x]]*EllipticF[(c + Pi + d*x)/2, 6/5]*Sqrt[Sec[c +
d*x]])/(3*Sqrt[5]*d*Sqrt[-3 + 2*Sec[c + d*x]]) - (2*Sqrt[5]*EllipticE[(c +
 Pi + d*x)/2, 6/5]*Sqrt[-3 + 2*Sec[c + d*x]])/(3*d*Sqrt[2 - 3*Cos[c + d*x]
]*Sqrt[Sec[c + d*x]])
```

3.675.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3133
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 - b]/d)*EllipticE[(1/2)*(c + Pi/2 + d*x), -2*(b/(a - b))], x] /; FreeQ[{a,
 b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a - b, 0]
```

rule 3141
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a - b]))*EllipticF[(1/2)*(c + Pi/2 + d*x), -2*(b/(a - b))], x] /; FreeQ
[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a - b, 0]
```

rule 4343
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]])   Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a
, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4345
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]])   Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[
{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4349
```
Int[1/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)]), x_Symbol] :> Simp[1/a   Int[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Cs
c[e + f*x]], x], x] - Simp[b/(a*d)   Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Cs
c[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

3.675.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 7.57 (sec) , antiderivative size = 332, normalized size of antiderivative = 2.57

method	result
default	$-\frac{2\sqrt{-\frac{5(1-\cos(dx+c))^2\csc(dx+c)^2-1}{(1-\cos(dx+c))^2\csc(dx+c)^2-1}}\left(3i\,\mathrm{EllipticF}\left(i(-\cot(dx+c)+\csc(dx+c)),i\sqrt{5}\right)\sqrt{(1-\cos(dx+c))^2\csc(dx+c)^2+1}\,\sqrt{-5(1-\cos}\right.}{}$
risch	$-\frac{i\left(3\,\mathrm{e}^{2i(dx+c)}-4\,\mathrm{e}^{i(dx+c)}+3\right)\sqrt{2}}{3d\sqrt{\frac{\mathrm{e}^{i(dx+c)}}{\mathrm{e}^{2i(dx+c)}+1}}\left(\mathrm{e}^{2i(dx+c)}+1\right)\sqrt{-\frac{3\,\mathrm{e}^{2i(dx+c)}-4\,\mathrm{e}^{i(dx+c)}+3}{\mathrm{e}^{2i(dx+c)}+1}}}-\frac{i\left(\frac{-2\,\mathrm{e}^{2i(dx+c)}+\frac{8\,\mathrm{e}^{i(dx+c)}}{3}-2}{\sqrt{\left(-3\,\mathrm{e}^{2i(dx+c)}+4\,\mathrm{e}^{i(dx+c)}-3\right)\mathrm{e}^{i(dx+c)}}}+\frac{\left(-\frac{2}{3}+\frac{i\sqrt{5}}{3}\right)\sqrt{\frac{\mathrm{e}^{i(dx+}}{-}}}{}\right.}{}$

input
```
int(1/sec(d*x+c)^(1/2)/(-3+2*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3/d*(-(5*(1-cos(d*x+c))^2*csc(d*x+c)^2-1)/((1-cos(d*x+c))^2*csc(d*x+c)^
2-1))^(1/2)*(3*I*EllipticF(I*(-cot(d*x+c)+csc(d*x+c)),I*5^(1/2))*((1-cos(d
*x+c))^2*csc(d*x+c)^2+1)^(1/2)*(-5*(1-cos(d*x+c))^2*csc(d*x+c)^2+1)^(1/2)-
I*EllipticE(I*(-cot(d*x+c)+csc(d*x+c)),I*5^(1/2))*((1-cos(d*x+c))^2*csc(d*
x+c)^2+1)^(1/2)*(-5*(1-cos(d*x+c))^2*csc(d*x+c)^2+1)^(1/2)+5*(1-cos(d*x+c)
)^3*csc(d*x+c)^3+cot(d*x+c)-csc(d*x+c))/(5*(1-cos(d*x+c))^2*csc(d*x+c)^2-1
)/(-((1-cos(d*x+c))^2*csc(d*x+c)^2+1)/((1-cos(d*x+c))^2*csc(d*x+c)^2-1))^(
1/2)
```

3.675.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.84

$$\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{-3+2\sec(c+dx)}}\,dx = -\frac{4\sqrt{6}\mathrm{weierstrassPInverse}\left(-\frac{44}{27},-\frac{784}{729},\cos(dx+c)+i\sin(dx+c)-\frac{4}{9}\right)+4\sqrt{6}\mathrm{weierstrassPInverse}\left(-\frac{44}{27}\right.}{}$$

input
```
integrate(1/sec(d*x+c)^(1/2)/(-3+2*sec(d*x+c))^(1/2),x, algorithm="fricas"
)
```

output
```
-1/27*(4*sqrt(6)*weierstrassPInverse(-44/27, -784/729, cos(d*x + c) + I*si
n(d*x + c) - 4/9) + 4*sqrt(6)*weierstrassPInverse(-44/27, -784/729, cos(d*
x + c) - I*sin(d*x + c) - 4/9) - 9*sqrt(6)*weierstrassZeta(-44/27, -784/72
9, weierstrassPInverse(-44/27, -784/729, cos(d*x + c) + I*sin(d*x + c) - 4
/9)) - 9*sqrt(6)*weierstrassZeta(-44/27, -784/729, weierstrassPInverse(-44
/27, -784/729, cos(d*x + c) - I*sin(d*x + c) - 4/9)))/d
```

3.675.6 Sympy [F]

$$\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{-3+2\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{2\sec(c+dx)-3}\sqrt{\sec(c+dx)}}\,dx$$

input
```
integrate(1/sec(d*x+c)**(1/2)/(-3+2*sec(d*x+c))**(1/2),x)
```

output
```
Integral(1/(sqrt(2*sec(c + d*x) - 3)*sqrt(sec(c + d*x))), x)
```

3.675.7 Maxima [F]

$$\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{-3+2\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{2\sec(dx+c)-3}\sqrt{\sec(dx+c)}}\,dx$$

input
```
integrate(1/sec(d*x+c)^(1/2)/(-3+2*sec(d*x+c))^(1/2),x, algorithm="maxima"
)
```

output
```
integrate(1/(sqrt(2*sec(d*x + c) - 3)*sqrt(sec(d*x + c))), x)
```

3.675.8 Giac [F]

$$\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{-3+2\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{2\sec(dx+c)-3}\sqrt{\sec(dx+c)}}\,dx$$

input
```
integrate(1/sec(d*x+c)^(1/2)/(-3+2*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(2*sec(d*x + c) - 3)*sqrt(sec(d*x + c))), x)
```

3.675.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{-3+2\sec(c+dx)}}\,dx=\int \frac{1}{\sqrt{\frac{2}{\cos(c+d\,x)}-3}\,\sqrt{\frac{1}{\cos(c+d\,x)}}}\,dx$$

input
```
int(1/((2/cos(c + d*x) - 3)^(1/2)*(1/cos(c + d*x))^(1/2)),x)
```

output
```
int(1/((2/cos(c + d*x) - 3)^(1/2)*(1/cos(c + d*x))^(1/2)), x)
```

3.676 $\int \frac{1}{\sqrt{-3-2\sec(c+dx)}\sqrt{\sec(c+dx)}}\,dx$

3.676.1 Optimal result . . . 5303
3.676.2 Mathematica [A] (verified) . . . 5303
3.676.3 Rubi [A] (verified) . . . 5304
3.676.4 Maple [C] (verified) . . . 5306
3.676.5 Fricas [C] (verification not implemented) . . . 5307
3.676.6 Sympy [F] . . . 5308
3.676.7 Maxima [F] . . . 5308
3.676.8 Giac [F] . . . 5308
3.676.9 Mupad [F(-1)] . . . 5309

3.676.1 Optimal result

Integrand size = 25, antiderivative size = 115

$$\int \frac{1}{\sqrt{-3-2\sec(c+dx)}\sqrt{\sec(c+dx)}}\,dx = -\frac{2E\left(\frac{1}{2}(c+\pi+dx)\middle|6\right)\sqrt{-3-2\sec(c+dx)}}{3d\sqrt{-2-3\cos(c+dx)}\sqrt{\sec(c+dx)}} - \frac{4\sqrt{-2-3\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+\pi+dx),6\right)\sqrt{\sec(c+dx)}}{3d\sqrt{-3-2\sec(c+dx)}}$$

output
```
2/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)*EllipticE(cos(1/2*d*x+
1/2*c),6^(1/2))*(-3-2*sec(d*x+c))^(1/2)/d/(-2-3*cos(d*x+c))^(1/2)/sec(d*x+
c)^(1/2)+4/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)*EllipticF(cos
(1/2*d*x+1/2*c),6^(1/2))*(-2-3*cos(d*x+c))^(1/2)*sec(d*x+c)^(1/2)/d/(-3-2*
sec(d*x+c))^(1/2)
```

3.676.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.70

$$\int \frac{1}{\sqrt{-3-2\sec(c+dx)}\sqrt{\sec(c+dx)}}\,dx = \frac{2\sqrt{2+3\cos(c+dx)}\left(5E\left(\frac{1}{2}(c+dx)\middle|\frac{6}{5}\right)-2\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{6}{5}\right)\right)\sqrt{\sec(c+dx)}}{3\sqrt{5}d\sqrt{-3-2\sec(c+dx)}}$$

input
```
Integrate[1/(Sqrt[-3 - 2*Sec[c + d*x]]*Sqrt[Sec[c + d*x]]),x]
```

output
```
(2*Sqrt[2 + 3*Cos[c + d*x]]*(5*EllipticE[(c + d*x)/2, 6/5] - 2*EllipticF[(
c + d*x)/2, 6/5])*Sqrt[Sec[c + d*x]])/(3*Sqrt[5]*d*Sqrt[-3 - 2*Sec[c + d*x
]])
```

3.676.3 Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {3042, 4349, 3042, 4343, 3042, 3133, 4345, 3042, 3141}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{-2\sec(c+dx)-3}\sqrt{\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{-2\csc\left(c+dx+\frac{\pi}{2}\right)-3}\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4349}$$

$$-\frac{1}{3}\int \frac{\sqrt{-2\sec(c+dx)-3}}{\sqrt{\sec(c+dx)}}dx-\frac{2}{3}\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{-2\sec(c+dx)-3}}dx$$

$$\Big\downarrow \text{3042}$$

$$-\frac{1}{3}\int \frac{\sqrt{-2\csc\left(c+dx+\frac{\pi}{2}\right)-3}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2}{3}\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{-2\csc\left(c+dx+\frac{\pi}{2}\right)-3}}dx$$

$$\Big\downarrow \text{4343}$$

$$-\frac{2}{3}\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{-2\csc\left(c+dx+\frac{\pi}{2}\right)-3}}dx-\frac{\sqrt{-2\sec(c+dx)-3}\int \sqrt{-3\cos(c+dx)-2}dx}{3\sqrt{-3\cos(c+dx)-2}\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2}{3}\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{-2\csc\left(c+dx+\frac{\pi}{2}\right)-3}}dx-\frac{\sqrt{-2\sec(c+dx)-3}\int \sqrt{-3\sin\left(c+dx+\frac{\pi}{2}\right)-2}dx}{3\sqrt{-3\cos(c+dx)-2}\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{3133}$$

$$-\frac{2}{3}\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{-2\csc\left(c+dx+\frac{\pi}{2}\right)-3}}dx-\frac{2\sqrt{-2\sec(c+dx)-3}E\left(\frac{1}{2}(c+dx+\pi)\middle|6\right)}{3d\sqrt{-3\cos(c+dx)-2}\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{4345}$$

$$-\frac{2\sqrt{-3\cos(c+dx)-2}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{-3\cos(c+dx)-2}}dx}{3\sqrt{-2\sec(c+dx)-3}}-\frac{2\sqrt{-2\sec(c+dx)-3}E\left(\frac{1}{2}(c+dx+\pi)\middle|6\right)}{3d\sqrt{-3\cos(c+dx)-2}\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2\sqrt{-3\cos(c+dx)-2}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{-3\sin\left(c+dx+\frac{\pi}{2}\right)-2}}dx}{3\sqrt{-2\sec(c+dx)-3}}-\frac{2\sqrt{-2\sec(c+dx)-3}E\left(\frac{1}{2}(c+dx+\pi)\middle|6\right)}{3d\sqrt{-3\cos(c+dx)-2}\sqrt{\sec(c+dx)}}$$

$$\Big\downarrow \text{3141}$$

$$-\frac{4\sqrt{-3\cos(c+dx)-2}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx+\pi),6\right)}{3d\sqrt{-2\sec(c+dx)-3}}-\frac{2\sqrt{-2\sec(c+dx)-3}E\left(\frac{1}{2}(c+dx+\pi)\middle|6\right)}{3d\sqrt{-3\cos(c+dx)-2}\sqrt{\sec(c+dx)}}$$

input
```
Int[1/(Sqrt[-3 - 2*Sec[c + d*x]]*Sqrt[Sec[c + d*x]]),x]
```

output
```
(-2*EllipticE[(c + Pi + d*x)/2, 6]*Sqrt[-3 - 2*Sec[c + d*x]])/(3*d*Sqrt[-2
 - 3*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (4*Sqrt[-2 - 3*Cos[c + d*x]]*Elli
pticF[(c + Pi + d*x)/2, 6]*Sqrt[Sec[c + d*x]])/(3*d*Sqrt[-3 - 2*Sec[c + d*
x]])
```

3.676. $\int \frac{1}{\sqrt{-3-2\sec(c+dx)}\sqrt{\sec(c+dx)}}\,dx$

3.676.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3133
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 - b]/d)*EllipticE[(1/2)*(c + Pi/2 + d*x), -2*(b/(a - b))], x] /; FreeQ[{a,
 b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a - b, 0]
```

rule 3141
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a - b]))*EllipticF[(1/2)*(c + Pi/2 + d*x), -2*(b/(a - b))], x] /; FreeQ
[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a - b, 0]
```

rule 4343
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]])   Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a
, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4345
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]])   Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[
{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4349
```
Int[1/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)]), x_Symbol] :> Simp[1/a   Int[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Cs
c[e + f*x]], x], x] - Simp[b/(a*d)   Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Cs
c[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

3.676.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 7.53 (sec) , antiderivative size = 332, normalized size of antiderivative = 2.89

method	result
default	$-\frac{2\sqrt{-\frac{(1-\cos(dx+c))^2\csc(dx+c)^2-5}{(1-\cos(dx+c))^2\csc(dx+c)^2-1}}\left(3i\sqrt{(1-\cos(dx+c))^2\csc(dx+c)^2+1}\sqrt{-5(1-\cos(dx+c))^2\csc(dx+c)^2+25}\,\text{EllipticF}\left(i(-\cot(d\right.\right.}{}$
risch	$-\frac{i\left(3\,\mathrm{e}^{2i(dx+c)}+4\,\mathrm{e}^{i(dx+c)}+3\right)\sqrt{2}}{3d\sqrt{\frac{\mathrm{e}^{i(dx+c)}}{\mathrm{e}^{2i(dx+c)}+1}}\left(\mathrm{e}^{2i(dx+c)}+1\right)\sqrt{-\frac{3\,\mathrm{e}^{2i(dx+c)}+4\,\mathrm{e}^{i(dx+c)}+3}{\mathrm{e}^{2i(dx+c)}+1}}}-\frac{i\left(\frac{-2\,\mathrm{e}^{2i(dx+c)}-\frac{8\,\mathrm{e}^{i(dx+c)}}{3}-2}{\sqrt{\left(-3\,\mathrm{e}^{2i(dx+c)}-4\,\mathrm{e}^{i(dx+c)}-3\right)\mathrm{e}^{i(dx+c)}}}+\frac{\left(\frac{2}{3}+\frac{i\sqrt{5}}{3}\right)\sqrt{\frac{\mathrm{e}^{i(dx+c)}}{\frac{2}{3}+}}}{}\right.}{}$

input
```
int(1/(-3-2*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/15/d*(-((1-cos(d*x+c))^2*csc(d*x+c)^2-5)/((1-cos(d*x+c))^2*csc(d*x+c)^2
-1))^(1/2)*(3*I*((1-cos(d*x+c))^2*csc(d*x+c)^2+1)^(1/2)*(-5*(1-cos(d*x+c))
^2*csc(d*x+c)^2+25)^(1/2)*EllipticF(I*(-cot(d*x+c)+csc(d*x+c)),1/5*I*5^(1/
2))-5*I*EllipticE(I*(-cot(d*x+c)+csc(d*x+c)),1/5*I*5^(1/2))*((1-cos(d*x+c)
)^2*csc(d*x+c)^2+1)^(1/2)*(-5*(1-cos(d*x+c))^2*csc(d*x+c)^2+25)^(1/2)+5*(1
-cos(d*x+c))^3*csc(d*x+c)^3+25*cot(d*x+c)-25*csc(d*x+c))/((1-cos(d*x+c))^2
*csc(d*x+c)^2-5)/(-((1-cos(d*x+c))^2*csc(d*x+c)^2+1)/((1-cos(d*x+c))^2*csc
(d*x+c)^2-1))^(1/2)
```

3.676.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.94

$$\int \frac{1}{\sqrt{-3-2\sec(c+dx)}\sqrt{\sec(c+dx)}}\,dx$$
$$= \frac{4\sqrt{6}\text{weierstrassPInverse}\left(-\frac{44}{27},\frac{784}{729},\cos(dx+c)+i\sin(dx+c)+\frac{4}{9}\right)+4\sqrt{6}\text{weierstrassPInverse}\left(-\frac{44}{27},\frac{784}{729}\right.}{}$$

input
```
integrate(1/(-3-2*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x, algorithm="fricas"
)
```

output
```
1/27*(4*sqrt(6)*weierstrassPInverse(-44/27, 784/729, cos(d*x + c) + I*sin(
d*x + c) + 4/9) + 4*sqrt(6)*weierstrassPInverse(-44/27, 784/729, cos(d*x +
 c) - I*sin(d*x + c) + 4/9) + 9*sqrt(6)*weierstrassZeta(-44/27, 784/729, w
eierstrassPInverse(-44/27, 784/729, cos(d*x + c) + I*sin(d*x + c) + 4/9))
+ 9*sqrt(6)*weierstrassZeta(-44/27, 784/729, weierstrassPInverse(-44/27, 7
84/729, cos(d*x + c) - I*sin(d*x + c) + 4/9)))/d
```

3.676.6 Sympy [F]

$$\int \frac{1}{\sqrt{-3-2\sec(c+dx)}\sqrt{\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{-2\sec(c+dx)-3}\sqrt{\sec(c+dx)}}\,dx$$

input
```
integrate(1/(-3-2*sec(d*x+c))**(1/2)/sec(d*x+c)**(1/2),x)
```

output
```
Integral(1/(sqrt(-2*sec(c + d*x) - 3)*sqrt(sec(c + d*x))), x)
```

3.676.7 Maxima [F]

$$\int \frac{1}{\sqrt{-3-2\sec(c+dx)}\sqrt{\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{-2\sec(dx+c)-3}\sqrt{\sec(dx+c)}}\,dx$$

input
```
integrate(1/(-3-2*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x, algorithm="maxima"
)
```

output
```
integrate(1/(sqrt(-2*sec(d*x + c) - 3)*sqrt(sec(d*x + c))), x)
```

3.676.8 Giac [F]

$$\int \frac{1}{\sqrt{-3-2\sec(c+dx)}\sqrt{\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{-2\sec(dx+c)-3}\sqrt{\sec(dx+c)}}\,dx$$

input
```
integrate(1/(-3-2*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(-2*sec(d*x + c) - 3)*sqrt(sec(d*x + c))), x)
```

3.676.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{-3-2\sec(c+dx)}\sqrt{\sec(c+dx)}}\,dx=\int \frac{1}{\sqrt{-\frac{2}{\cos(c+d\,x)}-3}\,\sqrt{\frac{1}{\cos(c+d\,x)}}}\,dx$$

input
```
int(1/((- 2/cos(c + d*x) - 3)^(1/2)*(1/cos(c + d*x))^(1/2)),x)
```

output
```
int(1/((- 2/cos(c + d*x) - 3)^(1/2)*(1/cos(c + d*x))^(1/2)), x)
```

3.677 $\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{2+3\sec(c+dx)}}\,dx$

3.677.1 Optimal result 5310
3.677.2 Mathematica [A] (verified) 5310
3.677.3 Rubi [A] (verified) 5311
3.677.4 Maple [C] (verified) 5312
3.677.5 Fricas [C] (verification not implemented) 5313
3.677.6 Sympy [F] 5313
3.677.7 Maxima [F] 5313
3.677.8 Giac [F] 5314
3.677.9 Mupad [F(-1)] 5314

3.677.1 Optimal result

Integrand size = 25, antiderivative size = 61

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{2+3\sec(c+dx)}}\,dx=\frac{2\sqrt{3+2\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{4}{5}\right)\sqrt{\sec(c+dx)}}{\sqrt{5}d\sqrt{2+3\sec(c+dx)}}$$

output
```
2/5*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+
1/2*c),2/5*5^(1/2))*(3+2*cos(d*x+c))^(1/2)*sec(d*x+c)^(1/2)/d*5^(1/2)/(2+3
*sec(d*x+c))^(1/2)
```

3.677.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{2+3\sec(c+dx)}}\,dx=\frac{2\sqrt{3+2\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{4}{5}\right)\sqrt{\sec(c+dx)}}{\sqrt{5}d\sqrt{2+3\sec(c+dx)}}$$

input
```
Integrate[Sqrt[Sec[c + d*x]]/Sqrt[2 + 3*Sec[c + d*x]],x]
```

output
```
(2*Sqrt[3 + 2*Cos[c + d*x]]*EllipticF[(c + d*x)/2, 4/5]*Sqrt[Sec[c + d*x]]
)/(Sqrt[5]*d*Sqrt[2 + 3*Sec[c + d*x]])
```

3.677.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {3042, 4345, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{3\sec(c+dx)+2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{3\csc\left(c+dx+\frac{\pi}{2}\right)+2}}dx$$

$$\Big\downarrow \text{4345}$$

$$\frac{\sqrt{2\cos(c+dx)+3}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{2\cos(c+dx)+3}}dx}{\sqrt{3\sec(c+dx)+2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{2\cos(c+dx)+3}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{2\sin\left(c+dx+\frac{\pi}{2}\right)+3}}dx}{\sqrt{3\sec(c+dx)+2}}$$

$$\Big\downarrow \text{3140}$$

$$\frac{2\sqrt{2\cos(c+dx)+3}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{4}{5}\right)}{\sqrt{5}d\sqrt{3\sec(c+dx)+2}}$$

input
```
Int[Sqrt[Sec[c + d*x]]/Sqrt[2 + 3*Sec[c + d*x]],x]
```

output
```
(2*Sqrt[3 + 2*Cos[c + d*x]]*EllipticF[(c + d*x)/2, 4/5]*Sqrt[Sec[c + d*x]]
)/(Sqrt[5]*d*Sqrt[2 + 3*Sec[c + d*x]])
```

3.677.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 4345
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]])   Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[
{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

3.677.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 4.80 (sec) , antiderivative size = 123, normalized size of antiderivative = 2.02

method	result
default	$\frac{i\sqrt{5}\,\sqrt{\sec(dx+c)}\,\sqrt{2+3\sec(dx+c)}\,\sqrt{10}\,\sqrt{\frac{3+2\cos(dx+c)}{\cos(dx+c)+1}}\,\operatorname{EllipticF}\left(\frac{i\sqrt{5}\,(\cot(dx+c)-\csc(dx+c))}{5},\sqrt{5}\right)\sqrt{2}\,\sqrt{\frac{1}{\cos(dx+c)+1}}\,\left(\cos(dx+c)^2+c\right.}{5d(3+2\cos(dx+c))}$

input
```
int(sec(d*x+c)^(1/2)/(2+3*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/5*I/d*5^(1/2)*sec(d*x+c)^(1/2)*(2+3*sec(d*x+c))^(1/2)*10^(1/2)*((3+2*cos
(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF(1/5*I*5^(1/2)*(cot(d*x+c)-csc(d*x
+c)),5^(1/2))*2^(1/2)*(1/(cos(d*x+c)+1))^(1/2)/(3+2*cos(d*x+c))*(cos(d*x+c
)^2+cos(d*x+c))
```

3.677.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.77

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{2+3\sec(c+dx)}}\,dx = \frac{-i\,\text{weierstrassPInverse}(8,-4,\cos(dx+c)+i\,\sin(dx+c)+1)+i\,\text{weierstrassPInverse}(8,-4,\cos(dx+c}{d}$$

input
```
integrate(sec(d*x+c)^(1/2)/(2+3*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
(-I*weierstrassPInverse(8, -4, cos(d*x + c) + I*sin(d*x + c) + 1) + I*weie
rstrassPInverse(8, -4, cos(d*x + c) - I*sin(d*x + c) + 1))/d
```

3.677.6 Sympy [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{2+3\sec(c+dx)}}\,dx = \int \frac{\sqrt{\sec(c+dx)}}{\sqrt{3\sec(c+dx)+2}}\,dx$$

input
```
integrate(sec(d*x+c)**(1/2)/(2+3*sec(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(sec(c + d*x))/sqrt(3*sec(c + d*x) + 2), x)
```

3.677.7 Maxima [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{2+3\sec(c+dx)}}\,dx = \int \frac{\sqrt{\sec(dx+c)}}{\sqrt{3\sec(dx+c)+2}}\,dx$$

input
```
integrate(sec(d*x+c)^(1/2)/(2+3*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(sec(d*x + c))/sqrt(3*sec(d*x + c) + 2), x)
```

3.677.8 Giac [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{2+3\sec(c+dx)}}\,dx = \int \frac{\sqrt{\sec(dx+c)}}{\sqrt{3\,\sec(dx+c)+2}}\,dx$$

input
```
integrate(sec(d*x+c)^(1/2)/(2+3*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(sec(d*x + c))/sqrt(3*sec(d*x + c) + 2), x)
```

3.677.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{2+3\sec(c+dx)}}\,dx = \int \frac{\sqrt{\frac{1}{\cos(c+d\,x)}}}{\sqrt{\frac{3}{\cos(c+d\,x)}+2}}\,dx$$

input
```
int((1/cos(c + d*x))^(1/2)/(3/cos(c + d*x) + 2)^(1/2),x)
```

output
```
int((1/cos(c + d*x))^(1/2)/(3/cos(c + d*x) + 2)^(1/2), x)
```

3.678 $\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{-2+3\sec(c+dx)}}\,dx$

3.678.1 Optimal result 5315
3.678.2 Mathematica [A] (verified) 5315
3.678.3 Rubi [A] (verified) 5316
3.678.4 Maple [A] (verified) 5317
3.678.5 Fricas [C] (verification not implemented) 5317
3.678.6 Sympy [F] 5318
3.678.7 Maxima [F] 5318
3.678.8 Giac [F] 5318
3.678.9 Mupad [F(-1)] 5319

3.678.1 Optimal result

Integrand size = 25, antiderivative size = 54

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{-2+3\sec(c+dx)}}\,dx=\frac{2\sqrt{3-2\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),-4\right)\sqrt{\sec(c+dx)}}{d\sqrt{-2+3\sec(c+dx)}}$$

output
```
2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/
2*c),2*I)*(3-2*cos(d*x+c))^(1/2)*sec(d*x+c)^(1/2)/d/(-2+3*sec(d*x+c))^(1/2
)
```

3.678.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{-2+3\sec(c+dx)}}\,dx=\frac{2\sqrt{3-2\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),-4\right)\sqrt{\sec(c+dx)}}{d\sqrt{-2+3\sec(c+dx)}}$$

input
```
Integrate[Sqrt[Sec[c + d*x]]/Sqrt[-2 + 3*Sec[c + d*x]],x]
```

output
```
(2*Sqrt[3 - 2*Cos[c + d*x]]*EllipticF[(c + d*x)/2, -4]*Sqrt[Sec[c + d*x]])
/(d*Sqrt[-2 + 3*Sec[c + d*x]])
```

3.678.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {3042, 4345, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{3\sec(c+dx)-2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{3\csc\left(c+dx+\frac{\pi}{2}\right)-2}}dx$$

$$\Big\downarrow \text{ 4345}$$

$$\frac{\sqrt{3-2\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{3-2\cos(c+dx)}}dx}{\sqrt{3\sec(c+dx)-2}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\sqrt{3-2\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{3-2\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{\sqrt{3\sec(c+dx)-2}}$$

$$\Big\downarrow \text{ 3140}$$

$$\frac{2\sqrt{3-2\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),-4\right)}{d\sqrt{3\sec(c+dx)-2}}$$

input
```
Int[Sqrt[Sec[c + d*x]]/Sqrt[-2 + 3*Sec[c + d*x]],x]
```

output
```
(2*Sqrt[3 - 2*Cos[c + d*x]]*EllipticF[(c + d*x)/2, -4]*Sqrt[Sec[c + d*x]])
/(d*Sqrt[-2 + 3*Sec[c + d*x]])
```

3.678.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 4345
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]])   Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[
{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

3.678.4 Maple [A] (verified)

Time = 5.72 (sec) , antiderivative size = 115, normalized size of antiderivative = 2.13

method	result
default	$\frac{i\sqrt{\sec(dx+c)}\,\sqrt{-2+3\sec(dx+c)}\,\sqrt{-\frac{2(2\cos(dx+c)-3)}{\cos(dx+c)+1}}\,\mathrm{EllipticF}\left(i(-\cot(dx+c)+\csc(dx+c)),\sqrt{5}\right)\sqrt{2}\,\sqrt{\frac{1}{\cos(dx+c)+1}}\left(\cos(dx+c)^2+c\right.}{d(2\cos(dx+c)-3)}$

input
```
int(sec(d*x+c)^(1/2)/(-2+3*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
I/d*sec(d*x+c)^(1/2)*(-2+3*sec(d*x+c))^(1/2)*(-2*(2*cos(d*x+c)-3)/(cos(d*x
+c)+1))^(1/2)*EllipticF(I*(-cot(d*x+c)+csc(d*x+c)),5^(1/2))*2^(1/2)*(1/(co
s(d*x+c)+1))^(1/2)/(2*cos(d*x+c)-3)*(cos(d*x+c)^2+cos(d*x+c))
```

3.678.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.81

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{-2+3\sec(c+dx)}}\,dx = -\frac{\text{weierstrassPInverse}(8,4,\cos(dx+c)+i\,\sin(dx+c)-1)+\text{weierstrassPInverse}(8,4,\cos(dx+c)-i}{d}$$

input
```
integrate(sec(d*x+c)^(1/2)/(-2+3*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
-(weierstrassPInverse(8, 4, cos(d*x + c) + I*sin(d*x + c) - 1) + weierstra
ssPInverse(8, 4, cos(d*x + c) - I*sin(d*x + c) - 1))/d
```

3.678.6 Sympy [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{-2+3\sec(c+dx)}}\,dx = \int \frac{\sqrt{\sec(c+dx)}}{\sqrt{3\sec(c+dx)-2}}\,dx$$

input
```
integrate(sec(d*x+c)**(1/2)/(-2+3*sec(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(sec(c + d*x))/sqrt(3*sec(c + d*x) - 2), x)
```

3.678.7 Maxima [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{-2+3\sec(c+dx)}}\,dx = \int \frac{\sqrt{\sec(dx+c)}}{\sqrt{3\,\sec(dx+c)-2}}\,dx$$

input
```
integrate(sec(d*x+c)^(1/2)/(-2+3*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(sec(d*x + c))/sqrt(3*sec(d*x + c) - 2), x)
```

3.678.8 Giac [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{-2+3\sec(c+dx)}}\,dx = \int \frac{\sqrt{\sec(dx+c)}}{\sqrt{3\,\sec(dx+c)-2}}\,dx$$

input
```
integrate(sec(d*x+c)^(1/2)/(-2+3*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(sec(d*x + c))/sqrt(3*sec(d*x + c) - 2), x)
```

3.678.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{-2+3\sec(c+dx)}}\,dx = \int \frac{\sqrt{\frac{1}{\cos(c+d\,x)}}}{\sqrt{\frac{3}{\cos(c+d\,x)}-2}}\,dx$$

input
```
int((1/cos(c + d*x))^(1/2)/(3/cos(c + d*x) - 2)^(1/2),x)
```

output
```
int((1/cos(c + d*x))^(1/2)/(3/cos(c + d*x) - 2)^(1/2), x)
```

3.679 $\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{2-3\sec(c+dx)}}\,dx$

3.679.1 Optimal result 5320
3.679.2 Mathematica [A] (verified) 5320
3.679.3 Rubi [A] (verified) 5321
3.679.4 Maple [A] (verified) 5322
3.679.5 Fricas [C] (verification not implemented) 5323
3.679.6 Sympy [F] 5323
3.679.7 Maxima [F] 5324
3.679.8 Giac [F] 5324
3.679.9 Mupad [F(-1)] 5324

3.679.1 Optimal result

Integrand size = 25, antiderivative size = 54

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{2-3\sec(c+dx)}}\,dx=\frac{2\sqrt{3-2\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),-4\right)\sqrt{\sec(c+dx)}}{d\sqrt{2-3\sec(c+dx)}}$$

output
```
2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/
2*c),2*I)*(3-2*cos(d*x+c))^(1/2)*sec(d*x+c)^(1/2)/d/(2-3*sec(d*x+c))^(1/2)
```

3.679.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{2-3\sec(c+dx)}}\,dx=\frac{2\sqrt{3-2\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),-4\right)\sqrt{\sec(c+dx)}}{d\sqrt{2-3\sec(c+dx)}}$$

input
```
Integrate[Sqrt[Sec[c + d*x]]/Sqrt[2 - 3*Sec[c + d*x]],x]
```

output
```
(2*Sqrt[3 - 2*Cos[c + d*x]]*EllipticF[(c + d*x)/2, -4]*Sqrt[Sec[c + d*x]])
/(d*Sqrt[2 - 3*Sec[c + d*x]])
```

3.679.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {3042, 4345, 3042, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{2-3\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{2-3\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4345}$$

$$\frac{\sqrt{2\cos(c+dx)-3}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{2\cos(c+dx)-3}}dx}{\sqrt{2-3\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{2\cos(c+dx)-3}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{2\sin\left(c+dx+\frac{\pi}{2}\right)-3}}dx}{\sqrt{2-3\sec(c+dx)}}$$

$$\Big\downarrow \text{3142}$$

$$\frac{\sqrt{3-2\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{3-2\cos(c+dx)}}dx}{\sqrt{2-3\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{3-2\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{3-2\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{\sqrt{2-3\sec(c+dx)}}$$

$$\Big\downarrow \text{3140}$$

$$\frac{2\sqrt{3-2\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),-4\right)}{d\sqrt{2-3\sec(c+dx)}}$$

input
```
Int[Sqrt[Sec[c + d*x]]/Sqrt[2 - 3*Sec[c + d*x]],x]
```

output
```
(2*Sqrt[3 - 2*Cos[c + d*x]]*EllipticF[(c + d*x)/2, -4]*Sqrt[Sec[c + d*x]])
/(d*Sqrt[2 - 3*Sec[c + d*x]])
```

3.679.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 4345
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]])   Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[
{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

3.679.4 Maple [A] (verified)

Time = 5.52 (sec) , antiderivative size = 123, normalized size of antiderivative = 2.28

method	result
default	$-\frac{i\sqrt{5}\,\sqrt{-\frac{2(2\cos(dx+c)-3)}{\cos(dx+c)+1}}\,\sqrt{\sec(dx+c)}\,\sqrt{2-3\sec(dx+c)}\,\operatorname{EllipticF}\left(i\sqrt{5}\,(-\cot(dx+c)+\csc(dx+c)),\frac{\sqrt{5}}{5}\right)\sqrt{2}\,\sqrt{\frac{1}{\cos(dx+c)+1}}\,\left(\cos(dx\right.}{5d(2\cos(dx+c)-3)}$

input
```
int(sec(d*x+c)^(1/2)/(2-3*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/5*I/d*5^(1/2)*(-2*(2*cos(d*x+c)-3)/(cos(d*x+c)+1))^(1/2)*sec(d*x+c)^(1/
2)*(2-3*sec(d*x+c))^(1/2)*EllipticF(I*5^(1/2)*(-cot(d*x+c)+csc(d*x+c)),1/5
*5^(1/2))*2^(1/2)*(1/(cos(d*x+c)+1))^(1/2)/(2*cos(d*x+c)-3)*(cos(d*x+c)^2+
cos(d*x+c))
```

3.679.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.87

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{2-3\sec(c+dx)}}\,dx = \frac{-i\,\text{weierstrassPInverse}(8,4,\cos(dx+c)+i\,\sin(dx+c)-1)+i\,\text{weierstrassPInverse}(8,4,\cos(dx+c)-}{d}$$

input
```
integrate(sec(d*x+c)^(1/2)/(2-3*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
(-I*weierstrassPInverse(8, 4, cos(d*x + c) + I*sin(d*x + c) - 1) + I*weier
strassPInverse(8, 4, cos(d*x + c) - I*sin(d*x + c) - 1))/d
```

3.679.6 Sympy [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{2-3\sec(c+dx)}}\,dx = \int \frac{\sqrt{\sec(c+dx)}}{\sqrt{2-3\sec(c+dx)}}\,dx$$

input
```
integrate(sec(d*x+c)**(1/2)/(2-3*sec(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(sec(c + d*x))/sqrt(2 - 3*sec(c + d*x)), x)
```

3.679.7 Maxima [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{2-3\sec(c+dx)}}\,dx = \int \frac{\sqrt{\sec(dx+c)}}{\sqrt{-3\,\sec(dx+c)+2}}\,dx$$

input
```
integrate(sec(d*x+c)^(1/2)/(2-3*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(sec(d*x + c))/sqrt(-3*sec(d*x + c) + 2), x)
```

3.679.8 Giac [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{2-3\sec(c+dx)}}\,dx = \int \frac{\sqrt{\sec(dx+c)}}{\sqrt{-3\,\sec(dx+c)+2}}\,dx$$

input
```
integrate(sec(d*x+c)^(1/2)/(2-3*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(sec(d*x + c))/sqrt(-3*sec(d*x + c) + 2), x)
```

3.679.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{2-3\sec(c+dx)}}\,dx = \int \frac{\sqrt{\frac{1}{\cos(c+d\,x)}}}{\sqrt{2-\frac{3}{\cos(c+d\,x)}}}\,dx$$

input
```
int((1/cos(c + d*x))^(1/2)/(2 - 3/cos(c + d*x))^(1/2),x)
```

output
```
int((1/cos(c + d*x))^(1/2)/(2 - 3/cos(c + d*x))^(1/2), x)
```

3.680 $\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{-2-3\sec(c+dx)}}\,dx$

3.680.1 Optimal result 5325
3.680.2 Mathematica [A] (verified) 5325
3.680.3 Rubi [A] (verified) 5326
3.680.4 Maple [C] (verified) 5327
3.680.5 Fricas [C] (verification not implemented) 5328
3.680.6 Sympy [F] 5328
3.680.7 Maxima [F] 5329
3.680.8 Giac [F] 5329
3.680.9 Mupad [F(-1)] 5329

3.680.1 Optimal result

Integrand size = 25, antiderivative size = 61

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{-2-3\sec(c+dx)}}\,dx=\frac{2\sqrt{3+2\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{4}{5}\right)\sqrt{\sec(c+dx)}}{\sqrt{5}d\sqrt{-2-3\sec(c+dx)}}$$

output
```
2/5*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+
1/2*c),2/5*5^(1/2))*(3+2*cos(d*x+c))^(1/2)*sec(d*x+c)^(1/2)/d*5^(1/2)/(-2-
3*sec(d*x+c))^(1/2)
```

3.680.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{-2-3\sec(c+dx)}}\,dx=\frac{2\sqrt{3+2\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{4}{5}\right)\sqrt{\sec(c+dx)}}{\sqrt{5}d\sqrt{-2-3\sec(c+dx)}}$$

input
```
Integrate[Sqrt[Sec[c + d*x]]/Sqrt[-2 - 3*Sec[c + d*x]],x]
```

output
```
(2*Sqrt[3 + 2*Cos[c + d*x]]*EllipticF[(c + d*x)/2, 4/5]*Sqrt[Sec[c + d*x]]
)/(Sqrt[5]*d*Sqrt[-2 - 3*Sec[c + d*x]])
```

3.680.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {3042, 4345, 3042, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{-3\sec(c+dx)-2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{-3\csc\left(c+dx+\frac{\pi}{2}\right)-2}}dx$$

$$\Big\downarrow \text{ 4345}$$

$$\frac{\sqrt{-2\cos(c+dx)-3}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{-2\cos(c+dx)-3}}dx}{\sqrt{-3\sec(c+dx)-2}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\sqrt{-2\cos(c+dx)-3}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{-2\sin\left(c+dx+\frac{\pi}{2}\right)-3}}dx}{\sqrt{-3\sec(c+dx)-2}}$$

$$\Big\downarrow \text{ 3142}$$

$$\frac{\sqrt{2\cos(c+dx)+3}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\frac{2}{5}\cos(c+dx)+\frac{3}{5}}}dx}{\sqrt{5}\sqrt{-3\sec(c+dx)-2}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\sqrt{2\cos(c+dx)+3}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\frac{2}{5}\sin\left(c+dx+\frac{\pi}{2}\right)+\frac{3}{5}}}dx}{\sqrt{5}\sqrt{-3\sec(c+dx)-2}}$$

$$\Big\downarrow \text{ 3140}$$

$$\frac{2\sqrt{2\cos(c+dx)+3}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{4}{5}\right)}{\sqrt{5}d\sqrt{-3\sec(c+dx)-2}}$$

input
```
Int[Sqrt[Sec[c + d*x]]/Sqrt[-2 - 3*Sec[c + d*x]],x]
```

output
```
(2*Sqrt[3 + 2*Cos[c + d*x]]*EllipticF[(c + d*x)/2, 4/5]*Sqrt[Sec[c + d*x]]
)/(Sqrt[5]*d*Sqrt[-2 - 3*Sec[c + d*x]])
```

3.680.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 4345
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]])   Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[
{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

3.680.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 5.56 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.95

method	result
default	$-\frac{i\sqrt{-2-3\sec(dx+c)}\,\sqrt{\sec(dx+c)}\,\sqrt{2}\,\sqrt{\frac{1}{\cos(dx+c)+1}}\,\sqrt{10}\,\sqrt{\frac{3+2\cos(dx+c)}{\cos(dx+c)+1}}\,\operatorname{EllipticF}\left(i(\cot(dx+c)-\csc(dx+c)),\frac{\sqrt{5}}{5}\right)\left(\cos(dx+c)^2+\right.}{5d(3+2\cos(dx+c))}$

input
```
int(sec(d*x+c)^(1/2)/(-2-3*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/5*I/d*(-2-3*sec(d*x+c))^(1/2)*sec(d*x+c)^(1/2)*2^(1/2)*(1/(cos(d*x+c)+1
))^(1/2)*10^(1/2)*((3+2*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(cot
(d*x+c)-csc(d*x+c)),1/5*5^(1/2))/(3+2*cos(d*x+c))*(cos(d*x+c)^2+cos(d*x+c)
)
```

3.680.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.72

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{-2-3\sec(c+dx)}}\,dx = -\frac{\text{weierstrassPInverse}(8,-4,\cos(dx+c)+i\sin(dx+c)+1)+\text{weierstrassPInverse}(8,-4,\cos(dx+c)}{d}$$

input
```
integrate(sec(d*x+c)^(1/2)/(-2-3*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
-(weierstrassPInverse(8, -4, cos(d*x + c) + I*sin(d*x + c) + 1) + weierstr
assPInverse(8, -4, cos(d*x + c) - I*sin(d*x + c) + 1))/d
```

3.680.6 Sympy [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{-2-3\sec(c+dx)}}\,dx = \int \frac{\sqrt{\sec(c+dx)}}{\sqrt{-3\sec(c+dx)-2}}\,dx$$

input
```
integrate(sec(d*x+c)**(1/2)/(-2-3*sec(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(sec(c + d*x))/sqrt(-3*sec(c + d*x) - 2), x)
```

3.680.7 Maxima [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{-2-3\sec(c+dx)}}\,dx = \int \frac{\sqrt{\sec{(dx+c)}}}{\sqrt{-3\,\sec{(dx+c)}-2}}\,dx$$

input
```
integrate(sec(d*x+c)^(1/2)/(-2-3*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(sec(d*x + c))/sqrt(-3*sec(d*x + c) - 2), x)
```

3.680.8 Giac [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{-2-3\sec(c+dx)}}\,dx = \int \frac{\sqrt{\sec{(dx+c)}}}{\sqrt{-3\,\sec{(dx+c)}-2}}\,dx$$

input
```
integrate(sec(d*x+c)^(1/2)/(-2-3*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(sec(d*x + c))/sqrt(-3*sec(d*x + c) - 2), x)
```

3.680.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{-2-3\sec(c+dx)}}\,dx = \int \frac{\sqrt{\frac{1}{\cos(c+d\,x)}}}{\sqrt{-\frac{3}{\cos(c+d\,x)}-2}}\,dx$$

input
```
int((1/cos(c + d*x))^(1/2)/(- 3/cos(c + d*x) - 2)^(1/2),x)
```

output
```
int((1/cos(c + d*x))^(1/2)/(- 3/cos(c + d*x) - 2)^(1/2), x)
```

3.681 $\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{3+2\sec(c+dx)}}\,dx$

3.681.1 Optimal result 5330
3.681.2 Mathematica [A] (verified) 5330
3.681.3 Rubi [A] (verified) 5331
3.681.4 Maple [C] (verified) 5332
3.681.5 Fricas [C] (verification not implemented) 5333
3.681.6 Sympy [F] 5333
3.681.7 Maxima [F] 5333
3.681.8 Giac [F] 5334
3.681.9 Mupad [F(-1)] 5334

3.681.1 Optimal result

Integrand size = 25, antiderivative size = 61

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{3+2\sec(c+dx)}}\,dx=\frac{2\sqrt{2+3\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{6}{5}\right)\sqrt{\sec(c+dx)}}{\sqrt{5}d\sqrt{3+2\sec(c+dx)}}$$

output
```
2/5*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+
1/2*c),1/5*30^(1/2))*(2+3*cos(d*x+c))^(1/2)*sec(d*x+c)^(1/2)/d*5^(1/2)/(3+
2*sec(d*x+c))^(1/2)
```

3.681.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{3+2\sec(c+dx)}}\,dx=\frac{2\sqrt{2+3\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{6}{5}\right)\sqrt{\sec(c+dx)}}{\sqrt{5}d\sqrt{3+2\sec(c+dx)}}$$

input
```
Integrate[Sqrt[Sec[c + d*x]]/Sqrt[3 + 2*Sec[c + d*x]],x]
```

output
```
(2*Sqrt[2 + 3*Cos[c + d*x]]*EllipticF[(c + d*x)/2, 6/5]*Sqrt[Sec[c + d*x]]
)/(Sqrt[5]*d*Sqrt[3 + 2*Sec[c + d*x]])
```

3.681.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {3042, 4345, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{2\sec(c+dx)+3}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{2\csc\left(c+dx+\frac{\pi}{2}\right)+3}}dx$$

$$\Big\downarrow \text{4345}$$

$$\frac{\sqrt{3\cos(c+dx)+2}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{3\cos(c+dx)+2}}dx}{\sqrt{2\sec(c+dx)+3}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{3\cos(c+dx)+2}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{3\sin\left(c+dx+\frac{\pi}{2}\right)+2}}dx}{\sqrt{2\sec(c+dx)+3}}$$

$$\Big\downarrow \text{3140}$$

$$\frac{2\sqrt{3\cos(c+dx)+2}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{6}{5}\right)}{\sqrt{5}d\sqrt{2\sec(c+dx)+3}}$$

input
```
Int[Sqrt[Sec[c + d*x]]/Sqrt[3 + 2*Sec[c + d*x]],x]
```

output
```
(2*Sqrt[2 + 3*Cos[c + d*x]]*EllipticF[(c + d*x)/2, 6/5]*Sqrt[Sec[c + d*x]]
)/(Sqrt[5]*d*Sqrt[3 + 2*Sec[c + d*x]])
```

3.681.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 4345
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]])   Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[
{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

3.681.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 6.11 (sec) , antiderivative size = 124, normalized size of antiderivative = 2.03

method	result
default	$-\frac{\sqrt{5}\,\sqrt{\sec(dx+c)}\,\sqrt{3+2\sec(dx+c)}\,\sqrt{2}\,\sqrt{\frac{1}{\cos(dx+c)+1}}\,\operatorname{EllipticF}\left(\frac{\sqrt{5}\,(\cot(dx+c)-\csc(dx+c))}{5},i\sqrt{5}\right)\sqrt{10}\,\sqrt{\frac{2+3\cos(dx+c)}{\cos(dx+c)+1}}\,\left(\cos(dx+c)^2+\right.}{5d(2+3\cos(dx+c))}$

input
```
int(sec(d*x+c)^(1/2)/(3+2*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/5/d*5^(1/2)*sec(d*x+c)^(1/2)*(3+2*sec(d*x+c))^(1/2)*2^(1/2)*(1/(cos(d*x
+c)+1))^(1/2)*EllipticF(1/5*5^(1/2)*(cot(d*x+c)-csc(d*x+c)),I*5^(1/2))*10^
(1/2)*((2+3*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)/(2+3*cos(d*x+c))*(cos(d*x+c)
^2+cos(d*x+c))
```

3.681.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.89

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{3+2\sec(c+dx)}}\,dx = \frac{-i\sqrt{6}\text{weierstrassPInverse}\left(-\frac{44}{27},\frac{784}{729},\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)+\frac{4}{9}\right)+i\sqrt{6}\text{weierstrassPInverse}\left(-\frac{44}{27},\frac{78}{72}\right.}{3\,d}$$

input
```
integrate(sec(d*x+c)^(1/2)/(3+2*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
1/3*(-I*sqrt(6)*weierstrassPInverse(-44/27, 784/729, cos(d*x + c) + I*sin(
d*x + c) + 4/9) + I*sqrt(6)*weierstrassPInverse(-44/27, 784/729, cos(d*x +
 c) - I*sin(d*x + c) + 4/9))/d
```

3.681.6 Sympy [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{3+2\sec(c+dx)}}\,dx = \int \frac{\sqrt{\sec\left(c+dx\right)}}{\sqrt{2\sec\left(c+dx\right)+3}}\,dx$$

input
```
integrate(sec(d*x+c)**(1/2)/(3+2*sec(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(sec(c + d*x))/sqrt(2*sec(c + d*x) + 3), x)
```

3.681.7 Maxima [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{3+2\sec(c+dx)}}\,dx = \int \frac{\sqrt{\sec\left(dx+c\right)}}{\sqrt{2\,\sec\left(dx+c\right)+3}}\,dx$$

input
```
integrate(sec(d*x+c)^(1/2)/(3+2*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(sec(d*x + c))/sqrt(2*sec(d*x + c) + 3), x)
```

3.681.8 Giac [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{3+2\sec(c+dx)}}\,dx = \int \frac{\sqrt{\sec(dx+c)}}{\sqrt{2\sec(dx+c)+3}}\,dx$$

input
```
integrate(sec(d*x+c)^(1/2)/(3+2*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(sec(d*x + c))/sqrt(2*sec(d*x + c) + 3), x)
```

3.681.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{3+2\sec(c+dx)}}\,dx = \int \frac{\sqrt{\frac{1}{\cos(c+d\,x)}}}{\sqrt{\frac{2}{\cos(c+d\,x)}+3}}\,dx$$

input
```
int((1/cos(c + d*x))^(1/2)/(2/cos(c + d*x) + 3)^(1/2),x)
```

output
```
int((1/cos(c + d*x))^(1/2)/(2/cos(c + d*x) + 3)^(1/2), x)
```

3.682 $\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{3-2\sec(c+dx)}}\,dx$

3.682.1 Optimal result 5335
3.682.2 Mathematica [A] (verified) 5335
3.682.3 Rubi [A] (verified) 5336
3.682.4 Maple [C] (verified) 5337
3.682.5 Fricas [C] (verification not implemented) 5338
3.682.6 Sympy [F] 5338
3.682.7 Maxima [F] 5338
3.682.8 Giac [F] 5339
3.682.9 Mupad [F(-1)] 5339

3.682.1 Optimal result

Integrand size = 25, antiderivative size = 54

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{3-2\sec(c+dx)}}\,dx=\frac{2\sqrt{-2+3\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),6\right)\sqrt{\sec(c+dx)}}{d\sqrt{3-2\sec(c+dx)}}$$

output
```
2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/
2*c),6^(1/2))*(-2+3*cos(d*x+c))^(1/2)*sec(d*x+c)^(1/2)/d/(3-2*sec(d*x+c))^
(1/2)
```

3.682.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{3-2\sec(c+dx)}}\,dx=\frac{2\sqrt{-2+3\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),6\right)\sqrt{\sec(c+dx)}}{d\sqrt{3-2\sec(c+dx)}}$$

input
```
Integrate[Sqrt[Sec[c + d*x]]/Sqrt[3 - 2*Sec[c + d*x]],x]
```

output
```
(2*Sqrt[-2 + 3*Cos[c + d*x]]*EllipticF[(c + d*x)/2, 6]*Sqrt[Sec[c + d*x]])
/(d*Sqrt[3 - 2*Sec[c + d*x]])
```

3.682.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3042, 4345, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{3-2\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{3-2\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4345}$$

$$\frac{\sqrt{3\cos(c+dx)-2}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{3\cos(c+dx)-2}}dx}{\sqrt{3-2\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{3\cos(c+dx)-2}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{3\sin\left(c+dx+\frac{\pi}{2}\right)-2}}dx}{\sqrt{3-2\sec(c+dx)}}$$

$$\Big\downarrow \text{3140}$$

$$\frac{2\sqrt{3\cos(c+dx)-2}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),6\right)}{d\sqrt{3-2\sec(c+dx)}}$$

input
```
Int[Sqrt[Sec[c + d*x]]/Sqrt[3 - 2*Sec[c + d*x]],x]
```

output
```
(2*Sqrt[-2 + 3*Cos[c + d*x]]*EllipticF[(c + d*x)/2, 6]*Sqrt[Sec[c + d*x]])
/(d*Sqrt[3 - 2*Sec[c + d*x]])
```

3.682.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 4345
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]])   Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[
{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

3.682.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 5.53 (sec) , antiderivative size = 117, normalized size of antiderivative = 2.17

method	result
default	$\frac{2\sqrt{5}\,\sqrt{\sec(dx+c)}\,\sqrt{3-2\sec(dx+c)}\,\operatorname{EllipticF}\left(\sqrt{5}\,(-\cot(dx+c)+\csc(dx+c)),\frac{i\sqrt{5}}{5}\right)\sqrt{\frac{1}{\cos(dx+c)+1}}\,\sqrt{\frac{-2+3\cos(dx+c)}{\cos(dx+c)+1}}\,\left(\cos(dx+c)^2+\mathrm{co}\right.}{5d(-2+3\cos(dx+c))}$

input
```
int(sec(d*x+c)^(1/2)/(3-2*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/5/d*5^(1/2)*sec(d*x+c)^(1/2)*(3-2*sec(d*x+c))^(1/2)*EllipticF(5^(1/2)*(-
cot(d*x+c)+csc(d*x+c)),1/5*I*5^(1/2))*(1/(cos(d*x+c)+1))^(1/2)*((-2+3*cos(
d*x+c))/(cos(d*x+c)+1))^(1/2)/(-2+3*cos(d*x+c))*(cos(d*x+c)^2+cos(d*x+c))
```

3.682.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{3-2\sec(c+dx)}}\,dx = \frac{-i\sqrt{6}\text{weierstrassPInverse}\left(-\frac{44}{27},-\frac{784}{729},\cos(dx+c)+i\sin(dx+c)-\frac{4}{9}\right)+i\sqrt{6}\text{weierstrassPInverse}\left(-\frac{44}{27},\right.}{3d}$$

input
```
integrate(sec(d*x+c)^(1/2)/(3-2*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
1/3*(-I*sqrt(6)*weierstrassPInverse(-44/27, -784/729, cos(d*x + c) + I*sin
(d*x + c) - 4/9) + I*sqrt(6)*weierstrassPInverse(-44/27, -784/729, cos(d*x
 + c) - I*sin(d*x + c) - 4/9))/d
```

3.682.6 Sympy [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{3-2\sec(c+dx)}}\,dx = \int \frac{\sqrt{\sec(c+dx)}}{\sqrt{3-2\sec(c+dx)}}\,dx$$

input
```
integrate(sec(d*x+c)**(1/2)/(3-2*sec(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(sec(c + d*x))/sqrt(3 - 2*sec(c + d*x)), x)
```

3.682.7 Maxima [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{3-2\sec(c+dx)}}\,dx = \int \frac{\sqrt{\sec(dx+c)}}{\sqrt{-2\sec(dx+c)+3}}\,dx$$

input
```
integrate(sec(d*x+c)^(1/2)/(3-2*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(sec(d*x + c))/sqrt(-2*sec(d*x + c) + 3), x)
```

3.682.8 Giac [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{3-2\sec(c+dx)}}\,dx = \int \frac{\sqrt{\sec(dx+c)}}{\sqrt{-2\sec(dx+c)+3}}\,dx$$

input
```
integrate(sec(d*x+c)^(1/2)/(3-2*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(sec(d*x + c))/sqrt(-2*sec(d*x + c) + 3), x)
```

3.682.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{3-2\sec(c+dx)}}\,dx = \int \frac{\sqrt{\frac{1}{\cos(c+d\,x)}}}{\sqrt{3-\frac{2}{\cos(c+d\,x)}}}\,dx$$

input
```
int((1/cos(c + d*x))^(1/2)/(3 - 2/cos(c + d*x))^(1/2),x)
```

output
```
int((1/cos(c + d*x))^(1/2)/(3 - 2/cos(c + d*x))^(1/2), x)
```

3.683 $\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{-3+2\sec(c+dx)}}\,dx$

3.683.1 Optimal result 5340
3.683.2 Mathematica [A] (verified) 5340
3.683.3 Rubi [A] (verified) 5341
3.683.4 Maple [C] (verified) 5342
3.683.5 Fricas [C] (verification not implemented) 5343
3.683.6 Sympy [F] 5343
3.683.7 Maxima [F] 5343
3.683.8 Giac [F] 5344
3.683.9 Mupad [F(-1)] 5344

3.683.1 Optimal result

Integrand size = 25, antiderivative size = 62

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{-3+2\sec(c+dx)}}\,dx = \frac{2\sqrt{2-3\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+\pi+dx),\frac{6}{5}\right)\sqrt{\sec(c+dx)}}{\sqrt{5}d\sqrt{-3+2\sec(c+dx)}}$$

output
```
-2/5*(sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)*EllipticF(cos(1/2*d*x
+1/2*c),1/5*30^(1/2))*(2-3*cos(d*x+c))^(1/2)*sec(d*x+c)^(1/2)/d*5^(1/2)/(-
3+2*sec(d*x+c))^(1/2)
```

3.683.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.87

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{-3+2\sec(c+dx)}}\,dx = \frac{2\sqrt{-2+3\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),6\right)\sqrt{\sec(c+dx)}}{d\sqrt{-3+2\sec(c+dx)}}$$

input
```
Integrate[Sqrt[Sec[c + d*x]]/Sqrt[-3 + 2*Sec[c + d*x]],x]
```

output
```
(2*Sqrt[-2 + 3*Cos[c + d*x]]*EllipticF[(c + d*x)/2, 6]*Sqrt[Sec[c + d*x]])
/(d*Sqrt[-3 + 2*Sec[c + d*x]])
```

3.683.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {3042, 4345, 3042, 3141}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{2\sec(c+dx)-3}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{2\csc\left(c+dx+\frac{\pi}{2}\right)-3}}dx$$

$$\Big\downarrow \text{ 4345}$$

$$\frac{\sqrt{2-3\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{2-3\cos(c+dx)}}dx}{\sqrt{2\sec(c+dx)-3}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\sqrt{2-3\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{2-3\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{\sqrt{2\sec(c+dx)-3}}$$

$$\Big\downarrow \text{ 3141}$$

$$\frac{2\sqrt{2-3\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx+\pi),\frac{6}{5}\right)}{\sqrt{5}d\sqrt{2\sec(c+dx)-3}}$$

input
```
Int[Sqrt[Sec[c + d*x]]/Sqrt[-3 + 2*Sec[c + d*x]],x]
```

output
```
(2*Sqrt[2 - 3*Cos[c + d*x]]*EllipticF[(c + Pi + d*x)/2, 6/5]*Sqrt[Sec[c +
d*x]])/(Sqrt[5]*d*Sqrt[-3 + 2*Sec[c + d*x]])
```

3.683.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3141
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a - b]))*EllipticF[(1/2)*(c + Pi/2 + d*x), -2*(b/(a - b))], x] /; FreeQ
[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a - b, 0]
```

rule 4345
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]])   Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[
{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

3.683.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 5.66 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.84

method	result
default	$-\frac{2i\sqrt{-3+2\sec(dx+c)}\,\sqrt{\sec(dx+c)}\,\operatorname{EllipticF}\left(i(\cot(dx+c)-\csc(dx+c)),i\sqrt{5}\right)\sqrt{\frac{1}{\cos(dx+c)+1}}\,\sqrt{\frac{-2+3\cos(dx+c)}{\cos(dx+c)+1}}\,\left(\cos(dx+c)^2+\cos(d\right.}{d(-2+3\cos(dx+c))}$

input
```
int(sec(d*x+c)^(1/2)/(-3+2*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2*I/d*(-3+2*sec(d*x+c))^(1/2)*sec(d*x+c)^(1/2)*EllipticF(I*(cot(d*x+c)-cs
c(d*x+c)),I*5^(1/2))*(1/(cos(d*x+c)+1))^(1/2)*((-2+3*cos(d*x+c))/(cos(d*x+
c)+1))^(1/2)/(-2+3*cos(d*x+c))*(cos(d*x+c)^2+cos(d*x+c))
```

3.683.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.84

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{-3+2\sec(c+dx)}}\,dx = -\frac{\sqrt{6}\text{weierstrassPInverse}\left(-\frac{44}{27},-\frac{784}{729},\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)-\frac{4}{9}\right)+\sqrt{6}\text{weierstrassPInverse}\left(-\frac{44}{27},-\right.}{3\,d}$$

input
```
integrate(sec(d*x+c)^(1/2)/(-3+2*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
-1/3*(sqrt(6)*weierstrassPInverse(-44/27, -784/729, cos(d*x + c) + I*sin(d
*x + c) - 4/9) + sqrt(6)*weierstrassPInverse(-44/27, -784/729, cos(d*x + c
) - I*sin(d*x + c) - 4/9))/d
```

3.683.6 Sympy [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{-3+2\sec(c+dx)}}\,dx = \int \frac{\sqrt{\sec\left(c+dx\right)}}{\sqrt{2\sec\left(c+dx\right)-3}}\,dx$$

input
```
integrate(sec(d*x+c)**(1/2)/(-3+2*sec(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(sec(c + d*x))/sqrt(2*sec(c + d*x) - 3), x)
```

3.683.7 Maxima [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{-3+2\sec(c+dx)}}\,dx = \int \frac{\sqrt{\sec\left(dx+c\right)}}{\sqrt{2\,\sec\left(dx+c\right)-3}}\,dx$$

input
```
integrate(sec(d*x+c)^(1/2)/(-3+2*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(sec(d*x + c))/sqrt(2*sec(d*x + c) - 3), x)
```

3.683.8 Giac [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{-3+2\sec(c+dx)}}\,dx = \int \frac{\sqrt{\sec(dx+c)}}{\sqrt{2\sec(dx+c)-3}}\,dx$$

input
```
integrate(sec(d*x+c)^(1/2)/(-3+2*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(sec(d*x + c))/sqrt(2*sec(d*x + c) - 3), x)
```

3.683.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{-3+2\sec(c+dx)}}\,dx = \int \frac{\sqrt{\frac{1}{\cos(c+d\,x)}}}{\sqrt{\frac{2}{\cos(c+d\,x)}-3}}\,dx$$

input
```
int((1/cos(c + d*x))^(1/2)/(2/cos(c + d*x) - 3)^(1/2),x)
```

output
```
int((1/cos(c + d*x))^(1/2)/(2/cos(c + d*x) - 3)^(1/2), x)
```

3.684 $\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{-3-2\sec(c+dx)}}\,dx$

3.684.1 Optimal result 5345
3.684.2 Mathematica [A] (verified) 5345
3.684.3 Rubi [A] (verified) 5346
3.684.4 Maple [C] (verified) 5347
3.684.5 Fricas [C] (verification not implemented) 5348
3.684.6 Sympy [F] 5348
3.684.7 Maxima [F] 5348
3.684.8 Giac [F] 5349
3.684.9 Mupad [F(-1)] 5349

3.684.1 Optimal result

Integrand size = 25, antiderivative size = 55

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{-3-2\sec(c+dx)}}\,dx = \frac{2\sqrt{-2-3\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+\pi+dx),6\right)\sqrt{\sec(c+dx)}}{d\sqrt{-3-2\sec(c+dx)}}$$

output
```
-2*(sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)*EllipticF(cos(1/2*d*x+1
/2*c),6^(1/2))*(-2-3*cos(d*x+c))^(1/2)*sec(d*x+c)^(1/2)/d/(-3-2*sec(d*x+c)
)^(1/2)
```

3.684.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.11

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{-3-2\sec(c+dx)}}\,dx = \frac{2\sqrt{2+3\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{6}{5}\right)\sqrt{\sec(c+dx)}}{\sqrt{5}d\sqrt{-3-2\sec(c+dx)}}$$

input
```
Integrate[Sqrt[Sec[c + d*x]]/Sqrt[-3 - 2*Sec[c + d*x]],x]
```

output
```
(2*Sqrt[2 + 3*Cos[c + d*x]]*EllipticF[(c + d*x)/2, 6/5]*Sqrt[Sec[c + d*x]]
)/(Sqrt[5]*d*Sqrt[-3 - 2*Sec[c + d*x]])
```

3.684.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {3042, 4345, 3042, 3141}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{-2\sec(c+dx)-3}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{-2\csc\left(c+dx+\frac{\pi}{2}\right)-3}}dx$$

$$\Big\downarrow \text{4345}$$

$$\frac{\sqrt{-3\cos(c+dx)-2}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{-3\cos(c+dx)-2}}dx}{\sqrt{-2\sec(c+dx)-3}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{-3\cos(c+dx)-2}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{-3\sin\left(c+dx+\frac{\pi}{2}\right)-2}}dx}{\sqrt{-2\sec(c+dx)-3}}$$

$$\Big\downarrow \text{3141}$$

$$\frac{2\sqrt{-3\cos(c+dx)-2}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx+\pi),6\right)}{d\sqrt{-2\sec(c+dx)-3}}$$

input
```
Int[Sqrt[Sec[c + d*x]]/Sqrt[-3 - 2*Sec[c + d*x]],x]
```

output
```
(2*Sqrt[-2 - 3*Cos[c + d*x]]*EllipticF[(c + Pi + d*x)/2, 6]*Sqrt[Sec[c + d
*x]])/(d*Sqrt[-3 - 2*Sec[c + d*x]])
```

3.684.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3141
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a - b]))*EllipticF[(1/2)*(c + Pi/2 + d*x), -2*(b/(a - b))], x] /; FreeQ
[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a - b, 0]
```

rule 4345
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]])   Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[
{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

3.684.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 5.83 (sec) , antiderivative size = 120, normalized size of antiderivative = 2.18

method	result
default	$\frac{i\sqrt{-3-2\sec(dx+c)}\,\sqrt{\sec(dx+c)}\,\operatorname{EllipticF}\left(i(-\cot(dx+c)+\csc(dx+c)),\frac{i\sqrt{5}}{5}\right)\sqrt{10}\,\sqrt{\frac{2+3\cos(dx+c)}{\cos(dx+c)+1}}\,\sqrt{2}\,\sqrt{\frac{1}{\cos(dx+c)+1}}\,\left(\cos(dx+c)^2+\right.}{5d(2+3\cos(dx+c))}$

input
```
int(sec(d*x+c)^(1/2)/(-3-2*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/5*I/d*(-3-2*sec(d*x+c))^(1/2)*sec(d*x+c)^(1/2)*EllipticF(I*(-cot(d*x+c)+
csc(d*x+c)),1/5*I*5^(1/2))*10^(1/2)*((2+3*cos(d*x+c))/(cos(d*x+c)+1))^(1/2
)*2^(1/2)*(1/(cos(d*x+c)+1))^(1/2)/(2+3*cos(d*x+c))*(cos(d*x+c)^2+cos(d*x+
c))
```

3.684.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.95

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{-3-2\sec(c+dx)}}\,dx = -\frac{\sqrt{6}\text{weierstrassPInverse}\left(-\frac{44}{27},\frac{784}{729},\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)+\frac{4}{9}\right)+\sqrt{6}\text{weierstrassPInverse}\left(-\frac{44}{27},\frac{784}{729}\right.}{3\,d}$$

input
```
integrate(sec(d*x+c)^(1/2)/(-3-2*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
-1/3*(sqrt(6)*weierstrassPInverse(-44/27, 784/729, cos(d*x + c) + I*sin(d*
x + c) + 4/9) + sqrt(6)*weierstrassPInverse(-44/27, 784/729, cos(d*x + c)
- I*sin(d*x + c) + 4/9))/d
```

3.684.6 Sympy [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{-3-2\sec(c+dx)}}\,dx = \int \frac{\sqrt{\sec\left(c+dx\right)}}{\sqrt{-2\sec\left(c+dx\right)-3}}\,dx$$

input
```
integrate(sec(d*x+c)**(1/2)/(-3-2*sec(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(sec(c + d*x))/sqrt(-2*sec(c + d*x) - 3), x)
```

3.684.7 Maxima [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{-3-2\sec(c+dx)}}\,dx = \int \frac{\sqrt{\sec\left(dx+c\right)}}{\sqrt{-2\,\sec\left(dx+c\right)-3}}\,dx$$

input
```
integrate(sec(d*x+c)^(1/2)/(-3-2*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(sec(d*x + c))/sqrt(-2*sec(d*x + c) - 3), x)
```

3.684.8 Giac [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{-3-2\sec(c+dx)}}\,dx = \int \frac{\sqrt{\sec(dx+c)}}{\sqrt{-2\sec(dx+c)-3}}\,dx$$

input
```
integrate(sec(d*x+c)^(1/2)/(-3-2*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(sec(d*x + c))/sqrt(-2*sec(d*x + c) - 3), x)
```

3.684.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{-3-2\sec(c+dx)}}\,dx = \int \frac{\sqrt{\frac{1}{\cos(c+d\,x)}}}{\sqrt{-\frac{2}{\cos(c+d\,x)}-3}}\,dx$$

input
```
int((1/cos(c + d*x))^(1/2)/(- 2/cos(c + d*x) - 3)^(1/2),x)
```

output
```
int((1/cos(c + d*x))^(1/2)/(- 2/cos(c + d*x) - 3)^(1/2), x)
```

3.685 $\int \sec(c+dx)\sqrt[3]{a+b\sec(c+dx)}\,dx$

3.685.1 Optimal result 5350
3.685.2 Mathematica [B] (warning: unable to verify) 5350
3.685.3 Rubi [A] (verified) 5351
3.685.4 Maple [F] 5352
3.685.5 Fricas [F] 5353
3.685.6 Sympy [F] 5353
3.685.7 Maxima [F] 5353
3.685.8 Giac [F] 5354
3.685.9 Mupad [F(-1)] 5354

3.685.1 Optimal result

Integrand size = 21, antiderivative size = 105

$$\int \sec(c+dx)\sqrt[3]{a+b\sec(c+dx)}\,dx = \frac{\sqrt{2}\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},-\frac{1}{3},\frac{3}{2},\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)\sqrt[3]{a+b\sec(c+dx)}\tan(c+dx)}{d\sqrt{1+\sec(c+dx)}\sqrt[3]{\frac{a+b\sec(c+dx)}{a+b}}}$$

output
```
AppellF1(1/2,-1/3,1/2,3/2,b*(1-sec(d*x+c))/(a+b),1/2-1/2*sec(d*x+c))*(a+b*
sec(d*x+c))^(1/3)*2^(1/2)*tan(d*x+c)/d/((a+b*sec(d*x+c))/(a+b))^(1/3)/(1+s
ec(d*x+c))^(1/2)
```

3.685.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 7160 vs. 2(105) = 210.

Time = 49.64 (sec) , antiderivative size = 7160, normalized size of antiderivative = 68.19

$$\int \sec(c+dx)\sqrt[3]{a+b\sec(c+dx)}\,dx = \text{Result too large to show}$$

input
```
Integrate[Sec[c + d*x]*(a + b*Sec[c + d*x])^(1/3),x]
```

output
```
Result too large to show
```

3.685.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3042, 4321, 156, 155}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(c+dx)\sqrt[3]{a+b\sec(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)\sqrt[3]{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{4321}$$

$$-\frac{\tan(c+dx)\int \frac{\sqrt[3]{a+b\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}}d\sec(c+dx)}{d\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}}$$

$$\Big\downarrow \text{156}$$

$$-\frac{\tan(c+dx)\sqrt[3]{a+b\sec(c+dx)}\int \frac{\sqrt[3]{\frac{a}{a+b}+\frac{b\sec(c+dx)}{a+b}}}{\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}}d\sec(c+dx)}{d\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}\sqrt[3]{\frac{a+b\sec(c+dx)}{a+b}}}$$

$$\Big\downarrow \text{155}$$

$$\frac{\sqrt{2}\tan(c+dx)\sqrt[3]{a+b\sec(c+dx)}\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},-\frac{1}{3},\frac{3}{2},\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)}{d\sqrt{\sec(c+dx)+1}\sqrt[3]{\frac{a+b\sec(c+dx)}{a+b}}}$$

input
```
Int[Sec[c + d*x]*(a + b*Sec[c + d*x])^(1/3),x]
```

output
```
(Sqrt[2]*AppellF1[1/2, 1/2, -1/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[c
 + d*x]))/(a + b)]*(a + b*Sec[c + d*x])^(1/3)*Tan[c + d*x])/(d*Sqrt[1 + Se
c[c + d*x]]*((a + b*Sec[c + d*x])/(a + b))^(1/3))
```

3.685.3.1 Defintions of rubi rules used

rule 155
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*Simplify[b/(b*c - a*d)]^n*
Simplify[b/(b*e - a*f)]^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/
(b*c - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f,
 m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[Sim
plify[b/(b*c - a*d)], 0] && GtQ[Simplify[b/(b*e - a*f)], 0] && !(GtQ[Simpl
ify[d/(d*a - c*b)], 0] && GtQ[Simplify[d/(d*e - c*f)], 0] && SimplerQ[c + d
*x, a + b*x]) && !(GtQ[Simplify[f/(f*a - e*b)], 0] && GtQ[Simplify[f/(f*c
- e*d)], 0] && SimplerQ[e + f*x, a + b*x])
```

rule 156
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(e + f*x)^FracPart[p]/(Simplify[b/(b*e - a*f)]^IntPart[p
]*(b*((e + f*x)/(b*e - a*f)))^FracPart[p])   Int[(a + b*x)^m*(c + d*x)^n*Si
mp[b*(e/(b*e - a*f)) + b*f*(x/(b*e - a*f)), x]^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] &
& GtQ[Simplify[b/(b*c - a*d)], 0] && !GtQ[Simplify[b/(b*e - a*f)], 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4321
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[Cot[e + f*x]/(f*Sqrt[1 + Csc[e + f*x]]*Sqrt[1 - Csc[e + f*x
]])   Subst[Int[(a + b*x)^m/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Csc[e + f*x]]
, x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0] && !IntegerQ[2*m]
```

3.685.4 Maple [F]

$$\int \sec(dx+c)\,(a+b\sec(dx+c))^{\frac{1}{3}}\,dx$$

input
```
int(sec(d*x+c)*(a+b*sec(d*x+c))^(1/3),x)
```

output
```
int(sec(d*x+c)*(a+b*sec(d*x+c))^(1/3),x)
```

3.685.5 Fricas [F]

$$\int \sec(c+dx)\sqrt[3]{a+b\sec(c+dx)}\,dx=\int (b\sec(dx+c)+a)^{\frac{1}{3}}\sec(dx+c)\,dx$$

input
```
integrate(sec(d*x+c)*(a+b*sec(d*x+c))^(1/3),x, algorithm="fricas")
```

output
```
integral((b*sec(d*x + c) + a)^(1/3)*sec(d*x + c), x)
```

3.685.6 Sympy [F]

$$\int \sec(c+dx)\sqrt[3]{a+b\sec(c+dx)}\,dx=\int \sqrt[3]{a+b\sec(c+dx)}\sec(c+dx)\,dx$$

input
```
integrate(sec(d*x+c)*(a+b*sec(d*x+c))**(1/3),x)
```

output
```
Integral((a + b*sec(c + d*x))**(1/3)*sec(c + d*x), x)
```

3.685.7 Maxima [F]

$$\int \sec(c+dx)\sqrt[3]{a+b\sec(c+dx)}\,dx=\int (b\sec(dx+c)+a)^{\frac{1}{3}}\sec(dx+c)\,dx$$

input
```
integrate(sec(d*x+c)*(a+b*sec(d*x+c))^(1/3),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^(1/3)*sec(d*x + c), x)
```

3.685.8 Giac [F]

$$\int \sec(c+dx)\sqrt[3]{a+b\sec(c+dx)}\,dx = \int (b\sec(dx+c)+a)^{\frac{1}{3}}\sec(dx+c)\,dx$$

input
```
integrate(sec(d*x+c)*(a+b*sec(d*x+c))^(1/3),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^(1/3)*sec(d*x + c), x)
```

3.685.9 Mupad [F(-1)]

Timed out.

$$\int \sec(c+dx)\sqrt[3]{a+b\sec(c+dx)}\,dx = \int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{1/3}}{\cos(c+d\,x)}\,dx$$

input
```
int((a + b/cos(c + d*x))^(1/3)/cos(c + d*x),x)
```

output
```
int((a + b/cos(c + d*x))^(1/3)/cos(c + d*x), x)
```

3.686 $\int \sqrt[3]{a+b\sec(c+dx)}\,dx$

3.686.1 Optimal result . . . 5355
3.686.2 Mathematica [N/A] . . . 5355
3.686.3 Rubi [N/A] . . . 5356
3.686.4 Maple [N/A] (verified) . . . 5357
3.686.5 Fricas [F(-1)] . . . 5357
3.686.6 Sympy [N/A] . . . 5357
3.686.7 Maxima [N/A] . . . 5358
3.686.8 Giac [N/A] . . . 5358
3.686.9 Mupad [N/A] . . . 5358

3.686.1 Optimal result

Integrand size = 14, antiderivative size = 14

$$\int \sqrt[3]{a+b\sec(c+dx)}\,dx = \text{Int}\left(\sqrt[3]{a+b\sec(c+dx)},x\right)$$

output
```
Unintegrable((a+b*sec(d*x+c))^(1/3),x)
```

3.686.2 Mathematica [N/A]

Not integrable

Time = 18.05 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \sqrt[3]{a+b\sec(c+dx)}\,dx = \int \sqrt[3]{a+b\sec(c+dx)}\,dx$$

input
```
Integrate[(a + b*Sec[c + d*x])^(1/3),x]
```

output
```
Integrate[(a + b*Sec[c + d*x])^(1/3), x]
```

3.686.3 Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 4273}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt[3]{a+b\sec(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt[3]{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}\,dx$$

$$\Big\downarrow \text{4273}$$

$$\int \sqrt[3]{a+b\sec(c+dx)}dx$$

input
```
Int[(a + b*Sec[c + d*x])^(1/3),x]
```

output
```
$Aborted
```

3.686.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4273
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Unintegrable[
(a + b*Csc[c + d*x])^n, x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[a^2 - b^2, 0
] &&  !IntegerQ[2*n]
```

3.686.4 Maple [N/A] (verified)

Not integrable

Time = 0.37 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int (a+b\sec(dx+c))^{\frac{1}{3}}\,dx$$

input
```
int((a+b*sec(d*x+c))^(1/3),x)
```

output
```
int((a+b*sec(d*x+c))^(1/3),x)
```

3.686.5 Fricas [F(-1)]

Timed out.

$$\int \sqrt[3]{a+b\sec(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+b*sec(d*x+c))^(1/3),x, algorithm="fricas")
```

output
```
Timed out
```

3.686.6 Sympy [N/A]

Not integrable

Time = 0.49 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \sqrt[3]{a+b\sec(c+dx)}\,dx = \int \sqrt[3]{a+b\sec(c+dx)}\,dx$$

input
```
integrate((a+b*sec(d*x+c))**(1/3),x)
```

output
```
Integral((a + b*sec(c + d*x))**(1/3), x)
```

3.686.7 Maxima [N/A]

Not integrable

Time = 0.58 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \sqrt[3]{a+b\sec(c+dx)}\,dx = \int (b\sec(dx+c)+a)^{\frac{1}{3}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(1/3),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^(1/3), x)
```

3.686.8 Giac [N/A]

Not integrable

Time = 0.42 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \sqrt[3]{a+b\sec(c+dx)}\,dx = \int (b\sec(dx+c)+a)^{\frac{1}{3}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(1/3),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^(1/3), x)
```

3.686.9 Mupad [N/A]

Not integrable

Time = 13.71 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \sqrt[3]{a+b\sec(c+dx)}\,dx = \int \left(a+\frac{b}{\cos(c+d\,x)}\right)^{1/3}\,dx$$

input
```
int((a + b/cos(c + d*x))^(1/3),x)
```

output
```
int((a + b/cos(c + d*x))^(1/3), x)
```

3.687 $\int \sec^4(c+dx)(a+b\sec(c+dx))^{2/3}\,dx$

3.687.1 Optimal result . . . 5359
3.687.2 Mathematica [B] (warning: unable to verify) . . . 5360
3.687.3 Rubi [A] (verified) . . . 5360
3.687.4 Maple [F] . . . 5365
3.687.5 Fricas [F] . . . 5365
3.687.6 Sympy [F] . . . 5366
3.687.7 Maxima [F] . . . 5366
3.687.8 Giac [F] . . . 5366
3.687.9 Mupad [F(-1)] . . . 5367

3.687.1 Optimal result

Integrand size = 23, antiderivative size = 362

$$\int \sec^4(c+dx)(a+b\sec(c+dx))^{2/3}\,dx = \frac{3(9a^2+32b^2)\,(a+b\sec(c+dx))^{2/3}\tan(c+dx)}{220b^2d} - \frac{9a(a+b\sec(c+dx))^{5/3}\tan(c+dx)}{44b^2d} + \frac{3\sec(c+dx)(a+b\sec(c+dx))^{5/3}\tan(c+dx)}{11bd} + \frac{a(18a^2+49b^2)\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},-\frac{2}{3},\frac{3}{2},\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)(a+b\sec(c+dx))^{2/3}\tan(c+dx)}{110\sqrt{2}b^3d\sqrt{1+\sec(c+dx)}\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{2/3}} - \frac{(9a^4+23a^2b^2-32b^4)\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},\frac{1}{3},\frac{3}{2},\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)\sqrt[3]{\frac{a+b\sec(c+dx)}{a+b}}\tan(c+}{55\sqrt{2}b^3d\sqrt{1+\sec(c+dx)}\sqrt[3]{a+b\sec(c+dx)}}$$

output
```
3/220*(9*a^2+32*b^2)*(a+b*sec(d*x+c))^(2/3)*tan(d*x+c)/b^2/d-9/44*a*(a+b*s
ec(d*x+c))^(5/3)*tan(d*x+c)/b^2/d+3/11*sec(d*x+c)*(a+b*sec(d*x+c))^(5/3)*t
an(d*x+c)/b/d+1/220*a*(18*a^2+49*b^2)*AppellF1(1/2,-2/3,1/2,3/2,b*(1-sec(d
*x+c))/(a+b),1/2-1/2*sec(d*x+c))*(a+b*sec(d*x+c))^(2/3)*tan(d*x+c)/b^3/d/(
(a+b*sec(d*x+c))/(a+b))^(2/3)*2^(1/2)/(1+sec(d*x+c))^(1/2)-1/110*(9*a^4+23
*a^2*b^2-32*b^4)*AppellF1(1/2,1/3,1/2,3/2,b*(1-sec(d*x+c))/(a+b),1/2-1/2*s
ec(d*x+c))*((a+b*sec(d*x+c))/(a+b))^(1/3)*tan(d*x+c)/b^3/d/(a+b*sec(d*x+c)
)^(1/3)*2^(1/2)/(1+sec(d*x+c))^(1/2)
```

3.687.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 21877 vs. $2(362) = 724$.

Time = 46.27 (sec) , antiderivative size = 21877, normalized size of antiderivative = 60.43

$$\int \sec^4(c+dx)(a+b\sec(c+dx))^{2/3}\,dx = \text{Result too large to show}$$

input
```
Integrate[Sec[c + d*x]^4*(a + b*Sec[c + d*x])^(2/3),x]
```

output
```
Result too large to show
```

3.687.3 Rubi [A] (verified)

Time = 1.34 (sec) , antiderivative size = 376, normalized size of antiderivative = 1.04, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.652$, Rules used = {3042, 4352, 27, 3042, 4570, 27, 3042, 4490, 27, 3042, 4495, 3042, 4321, 156, 155}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^4(c+dx)(a+b\sec(c+dx))^{2/3}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^4\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{2/3}dx$$

$$\Big\downarrow \; 4352$$

$$\frac{3\int \frac{1}{3}\sec(c+dx)(a+b\sec(c+dx))^{2/3}\left(-6a\sec^2(c+dx)+8b\sec(c+dx)+3a\right)dx}{11b}+\frac{3\tan(c+dx)\sec(c+dx)(a+b\sec(c+dx))^{5/3}}{11bd}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \sec(c+dx)(a+b\sec(c+dx))^{2/3}\left(-6a\sec^2(c+dx)+8b\sec(c+dx)+3a\right)dx}{11b}+\frac{3\tan(c+dx)\sec(c+dx)(a+b\sec(c+dx))^{5/3}}{11bd}$$

$$\Big\downarrow \; 3042$$

$$\frac{\int \csc\left(c+dx+\frac{\pi}{2}\right)\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{2/3}\left(-6a\csc\left(c+dx+\frac{\pi}{2}\right)^2+8b\csc\left(c+dx+\frac{\pi}{2}\right)+3a\right)dx}{11b}+\frac{3\tan(c+dx)\sec(c+dx)(a+b\sec(c+dx))^{5/3}}{11bd}$$

$\downarrow$ 4570

$$\frac{\frac{3\int -\frac{2}{3}\sec(c+dx)(a+b\sec(c+dx))^{2/3}\left(3ab-\left(9a^2+32b^2\right)\sec(c+dx)\right)dx}{8b}-\frac{9a\tan(c+dx)(a+b\sec(c+dx))^{5/3}}{4bd}}{11b}+\frac{3\tan(c+dx)\sec(c+dx)(a+b\sec(c+dx))^{5/3}}{11bd}$$

$\downarrow$ 27

$$\frac{-\frac{\int \sec(c+dx)(a+b\sec(c+dx))^{2/3}\left(3ab-\left(9a^2+32b^2\right)\sec(c+dx)\right)dx}{4b}-\frac{9a\tan(c+dx)(a+b\sec(c+dx))^{5/3}}{4bd}}{11b}+\frac{3\tan(c+dx)\sec(c+dx)(a+b\sec(c+dx))^{5/3}}{11bd}$$

$\downarrow$ 3042

$$\frac{-\frac{\int \csc\left(c+dx+\frac{\pi}{2}\right)\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{2/3}\left(3ab+\left(-9a^2-32b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)\right)dx}{4b}-\frac{9a\tan(c+dx)(a+b\sec(c+dx))^{5/3}}{4bd}}{11b}+\frac{3\tan(c+dx)\sec(c+dx)(a+b\sec(c+dx))^{5/3}}{11bd}$$

$\downarrow$ 4490

$$\frac{-\frac{\frac{3}{5}\int -\frac{\sec(c+dx)\left(b\left(3a^2+64b^2\right)+a\left(18a^2+49b^2\right)\sec(c+dx)\right)}{3\sqrt[3]{a+b\sec(c+dx)}}dx-\frac{3\left(9a^2+32b^2\right)\tan(c+dx)(a+b\sec(c+dx))^{2/3}}{5d}}{4b}-\frac{9a\tan(c+dx)(a+b\sec(c+dx))^{5/3}}{4bd}}{11b}+\frac{3\tan(c+dx)\sec(c+dx)(a+b\sec(c+dx))^{5/3}}{11bd}$$

$\downarrow$ 27

$$\frac{-\frac{-\frac{1}{5}\int \frac{\sec(c+dx)\left(b\left(3a^2+64b^2\right)+a\left(18a^2+49b^2\right)\sec(c+dx)\right)}{\sqrt[3]{a+b\sec(c+dx)}}dx-\frac{3\left(9a^2+32b^2\right)\tan(c+dx)(a+b\sec(c+dx))^{2/3}}{5d}}{4b}-\frac{9a\tan(c+dx)(a+b\sec(c+dx))^{5/3}}{4bd}}{11b}+\frac{3\tan(c+dx)\sec(c+dx)(a+b\sec(c+dx))^{5/3}}{11bd}$$

$\downarrow$ 3042

3.687. $\int \sec^4(c+dx)(a+b\sec(c+dx))^{2/3}\,dx$

$$-\frac{-\frac{\frac{1}{5}\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(b\left(3a^2+64b^2\right)+a\left(18a^2+49b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt[3]{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{3\left(9a^2+32b^2\right)\tan(c+dx)(a+b\sec(c+dx))^{2/3}}{5d}}{4b}-\frac{9a\tan(c+dx)(a+b\sec(c+dx))^{5/}}{4bd}}{11b}$$

$$\frac{3\tan(c+dx)\sec(c+dx)(a+b\sec(c+dx))^{5/3}}{11bd}$$

$$\Big\downarrow \; 4495$$

$$-\frac{-\frac{\frac{1}{5}\left(\frac{2\left(9a^4+23a^2b^2-32b^4\right)\int \frac{\sec(c+dx)}{\sqrt[3]{a+b\sec(c+dx)}}dx}{b}-\frac{a\left(18a^2+49b^2\right)\int \sec(c+dx)(a+b\sec(c+dx))^{2/3}dx}{b}\right)-\frac{3\left(9a^2+32b^2\right)\tan(c+dx)(a+b\sec(c+dx))^{2/}}{5d}}{4b}}{11b}$$

$$\frac{3\tan(c+dx)\sec(c+dx)(a+b\sec(c+dx))^{5/3}}{11bd}$$

$$\Big\downarrow \; 3042$$

$$-\frac{-\frac{\frac{1}{5}\left(\frac{2\left(9a^4+23a^2b^2-32b^4\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt[3]{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\frac{a\left(18a^2+49b^2\right)\int \csc\left(c+dx+\frac{\pi}{2}\right)\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{2/3}dx}{b}\right)-\frac{3\left(9a^2+32b^2\right)\tan(c+dx}{5d}}{4b}}{11b}$$

$$\frac{3\tan(c+dx)\sec(c+dx)(a+b\sec(c+dx))^{5/3}}{11bd}$$

$$\Big\downarrow \; 4321$$

$$-\frac{-\frac{\frac{1}{5}\left(\frac{a\left(18a^2+49b^2\right)\tan(c+dx)\int \frac{(a+b\sec(c+dx))^{2/3}}{\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}}d\sec(c+dx)}{bd\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}}-\frac{2\left(9a^4+23a^2b^2-32b^4\right)\tan(c+dx)\int \frac{1}{\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}\sqrt[3]{a+b\sec(}}}{bd\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}}\right)}{4b}}{11b}$$

$$\frac{3\tan(c+dx)\sec(c+dx)(a+b\sec(c+dx))^{5/3}}{11bd}$$

$$\Big\downarrow \; 156$$

3.687. $\int \sec^4(c+dx)(a+b\sec(c+dx))^{2/3}\,dx$

$$-\frac{\frac{1}{5}\left(\frac{a\left(18a^2+49b^2\right)\tan(c+dx)(a+b\sec(c+dx))^{2/3}\int\frac{\left(\frac{a}{a+b}+\frac{b\sec(c+dx)}{a+b}\right)^{2/3}}{\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}}d\sec(c+dx)}{bd\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{2/3}}-\frac{2\left(9a^4+23a^2b^2-32b^4\right)\tan(c+dx)\sqrt[3]{\frac{a+b\sec(c+dx)}{a+b}}}{bd\sqrt{1-\sec(c+dx)}\sqrt{\sec}}\right)}{4b}}{}$$

$$\frac{3\tan(c+dx)\sec(c+dx)(a+b\sec(c+dx))^{5/3}}{11bd}$$

$$\Big\downarrow \; 155$$

$$-\frac{\frac{1}{5}\left(\frac{2\sqrt{2}\left(9a^4+23a^2b^2-32b^4\right)\tan(c+dx)\sqrt[3]{\frac{a+b\sec(c+dx)}{a+b}}\operatorname{AppellF1}\left(\frac{1}{2},\frac{1}{2},\frac{1}{3},\frac{3}{2},\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)}{bd\sqrt{\sec(c+dx)+1}\sqrt[3]{a+b\sec(c+dx)}}-\frac{\sqrt{2}a\left(18a^2+49b^2\right)\tan(c+dx)(a}{}\right)}{4b}}{}$$

$$\frac{3\tan(c+dx)\sec(c+dx)(a+b\sec(c+dx))^{5/3}}{11bd}$$

input
```
Int[Sec[c + d*x]^4*(a + b*Sec[c + d*x])^(2/3),x]
```

output
```
(3*Sec[c + d*x]*(a + b*Sec[c + d*x])^(5/3)*Tan[c + d*x])/(11*b*d) + ((-9*a
*(a + b*Sec[c + d*x])^(5/3)*Tan[c + d*x])/(4*b*d) - ((-3*(9*a^2 + 32*b^2)*
(a + b*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(5*d) + (-((Sqrt[2]*a*(18*a^2 + 4
9*b^2)*AppellF1[1/2, 1/2, -2/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[c +
 d*x]))/(a + b)]*(a + b*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(b*d*Sqrt[1 + Se
c[c + d*x]]*((a + b*Sec[c + d*x])/(a + b))^(2/3))) + (2*Sqrt[2]*(9*a^4 + 2
3*a^2*b^2 - 32*b^4)*AppellF1[1/2, 1/2, 1/3, 3/2, (1 - Sec[c + d*x])/2, (b*
(1 - Sec[c + d*x]))/(a + b)]*((a + b*Sec[c + d*x])/(a + b))^(1/3)*Tan[c +
d*x])/(b*d*Sqrt[1 + Sec[c + d*x]]*(a + b*Sec[c + d*x])^(1/3)))/5)/(4*b))/(
11*b)
```

3.687. $\int \sec^4(c+dx)(a+b\sec(c+dx))^{2/3}\,dx$

3.687.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 155
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*Simplify[b/(b*c - a*d)]^n*
Simplify[b/(b*e - a*f)]^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/
(b*c - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f,
 m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[Sim
plify[b/(b*c - a*d)], 0] && GtQ[Simplify[b/(b*e - a*f)], 0] &&  !(GtQ[Simpl
ify[d/(d*a - c*b)], 0] && GtQ[Simplify[d/(d*e - c*f)], 0] && SimplerQ[c + d
*x, a + b*x]) &&  !(GtQ[Simplify[f/(f*a - e*b)], 0] && GtQ[Simplify[f/(f*c
- e*d)], 0] && SimplerQ[e + f*x, a + b*x])
```

rule 156
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(e + f*x)^FracPart[p]/(Simplify[b/(b*e - a*f)]^IntPart[p
]*(b*((e + f*x)/(b*e - a*f)))^FracPart[p])   Int[(a + b*x)^m*(c + d*x)^n*Si
mp[b*(e/(b*e - a*f)) + b*f*(x/(b*e - a*f)), x]^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] &
& GtQ[Simplify[b/(b*c - a*d)], 0] &&  !GtQ[Simplify[b/(b*e - a*f)], 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4321
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[Cot[e + f*x]/(f*Sqrt[1 + Csc[e + f*x]]*Sqrt[1 - Csc[e + f*x
]])   Subst[Int[(a + b*x)^m/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Csc[e + f*x]]
, x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*m]
```

rule 4352
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-d^3)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m +
1)*((d*Csc[e + f*x])^(n - 3)/(b*f*(m + n - 1))), x] + Simp[d^3/(b*(m + n -
1))   Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 3)*Simp[a*(n - 3) +
b*(m + n - 2)*Csc[e + f*x] - a*(n - 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[
{a, b, d, e, f, m}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 3] && (IntegerQ[n] ||
 IntegersQ[2*m, 2*n]) &&  !IGtQ[m, 2]
```

3.687. $\int \sec^4(c+dx)(a+b\sec(c+dx))^{2/3}\,dx$

rule 4490
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*((
a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Simp[1/(m + 1)   Int[Csc[e + f*x]*
(a + b*Csc[e + f*x])^(m - 1)*Simp[b*B*m + a*A*(m + 1) + (a*B*m + A*b*(m + 1
))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*
B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

rule 4495
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b - a*B)/b   Int[
Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] + Simp[B/b   Int[Csc[e + f*x]*(
a + b*Csc[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, A, B, e, f, m}, x] && N
eQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

rule 4570
```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> Simp[(-C)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2)
)), x] + Simp[1/(b*(m + 2))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[
b*A*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /;
 FreeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

3.687.4 Maple [F]

$$\int \sec (dx+c)^4 (a+b\sec (dx+c))^{\frac{2}{3}} \, dx$$

input
```
int(sec(d*x+c)^4*(a+b*sec(d*x+c))^(2/3),x)
```

output
```
int(sec(d*x+c)^4*(a+b*sec(d*x+c))^(2/3),x)
```

3.687.5 Fricas [F]

$$\int \sec^4(c+dx)(a+b\sec(c+dx))^{2/3}\,dx = \int (b\sec (dx+c)+a)^{\frac{2}{3}} \sec (dx+c)^4 \, dx$$

input
```
integrate(sec(d*x+c)^4*(a+b*sec(d*x+c))^(2/3),x, algorithm="fricas")
```

output
```
integral((b*sec(d*x + c) + a)^(2/3)*sec(d*x + c)^4, x)
```

3.687.6 Sympy [F]

$$\int \sec^4(c+dx)(a+b\sec(c+dx))^{2/3}\,dx = \int (a+b\sec(c+dx))^{\frac{2}{3}}\sec^4(c+dx)\,dx$$

input
```
integrate(sec(d*x+c)**4*(a+b*sec(d*x+c))**(2/3),x)
```

output
```
Integral((a + b*sec(c + d*x))**(2/3)*sec(c + d*x)**4, x)
```

3.687.7 Maxima [F]

$$\int \sec^4(c+dx)(a+b\sec(c+dx))^{2/3}\,dx = \int (b\sec(dx+c)+a)^{\frac{2}{3}}\sec(dx+c)^4\,dx$$

input
```
integrate(sec(d*x+c)^4*(a+b*sec(d*x+c))^(2/3),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^(2/3)*sec(d*x + c)^4, x)
```

3.687.8 Giac [F]

$$\int \sec^4(c+dx)(a+b\sec(c+dx))^{2/3}\,dx = \int (b\sec(dx+c)+a)^{\frac{2}{3}}\sec(dx+c)^4\,dx$$

input
```
integrate(sec(d*x+c)^4*(a+b*sec(d*x+c))^(2/3),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^(2/3)*sec(d*x + c)^4, x)
```

3.687.9 Mupad [F(-1)]

Timed out.

$$\int \sec^4(c+dx)(a+b\sec(c+dx))^{2/3}\,dx = \int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{2/3}}{\cos\left(c+d\,x\right)^4}\,dx$$

input
```
int((a + b/cos(c + d*x))^(2/3)/cos(c + d*x)^4,x)
```

output
```
int((a + b/cos(c + d*x))^(2/3)/cos(c + d*x)^4, x)
```

3.688 $\int \sec^3(c+dx)(a+b\sec(c+dx))^{2/3}\,dx$

3.688.1 Optimal result . . . 5368
3.688.2 Mathematica [B] (warning: unable to verify) . . . 5369
3.688.3 Rubi [A] (verified) . . . 5369
3.688.4 Maple [F] . . . 5373
3.688.5 Fricas [F] . . . 5373
3.688.6 Sympy [F] . . . 5374
3.688.7 Maxima [F] . . . 5374
3.688.8 Giac [F] . . . 5374
3.688.9 Mupad [F(-1)] . . . 5375

3.688.1 Optimal result

Integrand size = 23, antiderivative size = 305

$$\int \sec^3(c+dx)(a+b\sec(c+dx))^{2/3}\,dx = -\frac{9a(a+b\sec(c+dx))^{2/3}\tan(c+dx)}{40bd}+\frac{3(a+b\sec(c+dx))^{5/3}\tan(c+dx)}{8bd} -\frac{(6a^2-25b^2)\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},-\frac{2}{3},\frac{3}{2},\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)(a+b\sec(c+dx))^{2/3}\tan(c+dx)}{20\sqrt{2}b^2d\sqrt{1+\sec(c+dx)}\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{2/3}} +\frac{3a(a^2-b^2)\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},\frac{1}{3},\frac{3}{2},\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)\sqrt[3]{\frac{a+b\sec(c+dx)}{a+b}}\tan(c+dx)}{10\sqrt{2}b^2d\sqrt{1+\sec(c+dx)}\sqrt[3]{a+b\sec(c+dx)}}$$

output
```
-9/40*a*(a+b*sec(d*x+c))^(2/3)*tan(d*x+c)/b/d+3/8*(a+b*sec(d*x+c))^(5/3)*t
an(d*x+c)/b/d-1/40*(6*a^2-25*b^2)*AppellF1(1/2,-2/3,1/2,3/2,b*(1-sec(d*x+c
))/(a+b),1/2-1/2*sec(d*x+c))*(a+b*sec(d*x+c))^(2/3)*tan(d*x+c)/b^2/d/((a+b
*sec(d*x+c))/(a+b))^(2/3)*2^(1/2)/(1+sec(d*x+c))^(1/2)+3/20*a*(a^2-b^2)*Ap
pellF1(1/2,1/3,1/2,3/2,b*(1-sec(d*x+c))/(a+b),1/2-1/2*sec(d*x+c))*((a+b*se
c(d*x+c))/(a+b))^(1/3)*tan(d*x+c)/b^2/d/(a+b*sec(d*x+c))^(1/3)*2^(1/2)/(1+
sec(d*x+c))^(1/2)
```

3.688.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 7783 vs. $2(305) = 610$.

Time = 45.20 (sec) , antiderivative size = 7783, normalized size of antiderivative = 25.52

$$\int \sec^3(c+dx)(a+b\sec(c+dx))^{2/3}\,dx = \text{Result too large to show}$$

input
```
Integrate[Sec[c + d*x]^3*(a + b*Sec[c + d*x])^(2/3),x]
```

output
```
Result too large to show
```

3.688.3 Rubi [A] (verified)

Time = 0.93 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.02, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.522$, Rules used = {3042, 4327, 27, 3042, 4490, 27, 3042, 4495, 3042, 4321, 156, 155}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^3(c+dx)(a+b\sec(c+dx))^{2/3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^3\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{2/3}dx$$

$$\Big\downarrow \text{4327}$$

$$\frac{3\int \frac{1}{3}\sec(c+dx)(5b-3a\sec(c+dx))(a+b\sec(c+dx))^{2/3}dx}{8b}+\frac{3\tan(c+dx)(a+b\sec(c+dx))^{5/3}}{8bd}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \sec(c+dx)(5b-3a\sec(c+dx))(a+b\sec(c+dx))^{2/3}dx}{8b}+\frac{3\tan(c+dx)(a+b\sec(c+dx))^{5/3}}{8bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \csc\left(c+dx+\frac{\pi}{2}\right)\left(5b-3a\csc\left(c+dx+\frac{\pi}{2}\right)\right)\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{2/3}dx}{8b}+\frac{3\tan(c+dx)(a+b\sec(c+dx))^{5/3}}{8bd}$$

$$\Big\downarrow \; 4490$$

$$\frac{\frac{3}{5}\int \frac{\sec(c+dx)\left(19ab-\left(6a^2-25b^2\right)\sec(c+dx)\right)}{3\sqrt[3]{a+b\sec(c+dx)}}dx-\frac{9a\tan(c+dx)(a+b\sec(c+dx))^{2/3}}{5d}}{8b}+\frac{3\tan(c+dx)(a+b\sec(c+dx))^{5/3}}{8bd}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{1}{5}\int \frac{\sec(c+dx)\left(19ab-\left(6a^2-25b^2\right)\sec(c+dx)\right)}{\sqrt[3]{a+b\sec(c+dx)}}dx-\frac{9a\tan(c+dx)(a+b\sec(c+dx))^{2/3}}{5d}}{8b}+\frac{3\tan(c+dx)(a+b\sec(c+dx))^{5/3}}{8bd}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{1}{5}\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(19ab+\left(25b^2-6a^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt[3]{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{9a\tan(c+dx)(a+b\sec(c+dx))^{2/3}}{5d}}{8b}+\frac{3\tan(c+dx)(a+b\sec(c+dx))^{5/3}}{8bd}$$

$$\Big\downarrow \; 4495$$

$$\frac{\frac{1}{5}\left(\frac{6a\left(a^2-b^2\right)\int \frac{\sec(c+dx)}{\sqrt[3]{a+b\sec(c+dx)}}dx}{b}-\frac{\left(6a^2-25b^2\right)\int \sec(c+dx)(a+b\sec(c+dx))^{2/3}dx}{b}\right)-\frac{9a\tan(c+dx)(a+b\sec(c+dx))^{2/3}}{5d}}{8b}+\frac{3\tan(c+dx)(a+b\sec(c+dx))^{5/3}}{8bd}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{1}{5}\left(\frac{6a\left(a^2-b^2\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt[3]{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\frac{\left(6a^2-25b^2\right)\int \csc\left(c+dx+\frac{\pi}{2}\right)\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{2/3}dx}{b}\right)-\frac{9a\tan(c+dx)(a+b\sec(c+}{5d}}{8b}$$

$$\frac{3\tan(c+dx)(a+b\sec(c+dx))^{5/3}}{8bd}$$

3.688. $\int \sec^3(c+dx)(a+b\sec(c+dx))^{2/3}\,dx$

$$\Big\downarrow \; 4321$$

$$\frac{\frac{1}{5}\left(\frac{(6a^2-25b^2)\tan(c+dx)\int \frac{(a+b\sec(c+dx))^{2/3}}{\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}}d\sec(c+dx)}{bd\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}} - \frac{6a(a^2-b^2)\tan(c+dx)\int \frac{1}{\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}\sqrt[3]{a+b\sec(c+}}}{bd\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}}\right.}{8b}}{\frac{3\tan(c+dx)(a+b\sec(c+dx))^{5/3}}{8bd}}$$

$$\Big\downarrow \; 156$$

$$\frac{\frac{1}{5}\left(\frac{(6a^2-25b^2)\tan(c+dx)(a+b\sec(c+dx))^{2/3}\int \frac{\left(\frac{a}{a+b}+\frac{b\sec(c+dx)}{a+b}\right)^{2/3}}{\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}}d\sec(c+dx)}{bd\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{2/3}} - \frac{6a(a^2-b^2)\tan(c+dx)\sqrt[3]{\frac{a+b\sec(c+dx)}{a+b}}\int -}{bd\sqrt{1-\sec(c+dx)}\sqrt{s}}\right.}{8b}}{\frac{3\tan(c+dx)(a+b\sec(c+dx))^{5/3}}{8bd}}$$

$$\Big\downarrow \; 155$$

$$\frac{\frac{1}{5}\left(\frac{6\sqrt{2}a(a^2-b^2)\tan(c+dx)\sqrt[3]{\frac{a+b\sec(c+dx)}{a+b}}\operatorname{AppellF1}\left(\frac{1}{2},\frac{1}{2},\frac{1}{3},\frac{3}{2},\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)}{bd\sqrt{\sec(c+dx)+1}\sqrt[3]{a+b\sec(c+dx)}} - \frac{\sqrt{2}(6a^2-25b^2)\tan(c+dx)(a+b}{}\right.}{8b}}{\frac{3\tan(c+dx)(a+b\sec(c+dx))^{5/3}}{8bd}}$$

input
```
Int[Sec[c + d*x]^3*(a + b*Sec[c + d*x])^(2/3),x]
```

output
```
(3*(a + b*Sec[c + d*x])^(5/3)*Tan[c + d*x])/(8*b*d) + ((-9*a*(a + b*Sec[c
+ d*x])^(2/3)*Tan[c + d*x])/(5*d) + (-((Sqrt[2]*(6*a^2 - 25*b^2)*AppellF1[
1/2, 1/2, -2/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[c + d*x]))/(a + b)]
*(a + b*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(b*d*Sqrt[1 + Sec[c + d*x]]*((a
+ b*Sec[c + d*x])/(a + b))^(2/3))) + (6*Sqrt[2]*a*(a^2 - b^2)*AppellF1[1/2
, 1/2, 1/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[c + d*x]))/(a + b)]*((a
 + b*Sec[c + d*x])/(a + b))^(1/3)*Tan[c + d*x])/(b*d*Sqrt[1 + Sec[c + d*x]
]*(a + b*Sec[c + d*x])^(1/3)))/5)/(8*b)
```

3.688. $\int \sec^3(c+dx)(a+b\sec(c+dx))^{2/3}\,dx$

3.688.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 155
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*Simplify[b/(b*c - a*d)]^n*
Simplify[b/(b*e - a*f)]^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/
(b*c - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f,
 m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[Sim
plify[b/(b*c - a*d)], 0] && GtQ[Simplify[b/(b*e - a*f)], 0] &&  !(GtQ[Simpl
ify[d/(d*a - c*b)], 0] && GtQ[Simplify[d/(d*e - c*f)], 0] && SimplerQ[c + d
*x, a + b*x]) &&  !(GtQ[Simplify[f/(f*a - e*b)], 0] && GtQ[Simplify[f/(f*c
- e*d)], 0] && SimplerQ[e + f*x, a + b*x])
```

rule 156
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(e + f*x)^FracPart[p]/(Simplify[b/(b*e - a*f)]^IntPart[p
]*(b*((e + f*x)/(b*e - a*f)))^FracPart[p])   Int[(a + b*x)^m*(c + d*x)^n*Si
mp[b*(e/(b*e - a*f)) + b*f*(x/(b*e - a*f)), x]^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] &
& GtQ[Simplify[b/(b*c - a*d)], 0] &&  !GtQ[Simplify[b/(b*e - a*f)], 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4321
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[Cot[e + f*x]/(f*Sqrt[1 + Csc[e + f*x]]*Sqrt[1 - Csc[e + f*x
]])   Subst[Int[(a + b*x)^m/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Csc[e + f*x]]
, x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*m]
```

rule 4327
```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(-Cot[e + f*x])*((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2
))), x] + Simp[1/(b*(m + 2))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(b*(
m + 1) - a*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 -
b^2, 0] &&  !LtQ[m, -1]
```

3.688. $\int \sec^3(c+dx)(a+b\sec(c+dx))^{2/3}\,dx$

rule 4490
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*((
a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Simp[1/(m + 1)   Int[Csc[e + f*x]*
(a + b*Csc[e + f*x])^(m - 1)*Simp[b*B*m + a*A*(m + 1) + (a*B*m + A*b*(m + 1
))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*
B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

rule 4495
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b - a*B)/b   Int[
Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] + Simp[B/b   Int[Csc[e + f*x]*(
a + b*Csc[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, A, B, e, f, m}, x] && N
eQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

3.688.4 Maple [F]

$$\int \sec (dx+c)^3 (a+b\sec (dx+c))^{\frac{2}{3}} dx$$

input
```
int(sec(d*x+c)^3*(a+b*sec(d*x+c))^(2/3),x)
```

output
```
int(sec(d*x+c)^3*(a+b*sec(d*x+c))^(2/3),x)
```

3.688.5 Fricas [F]

$$\int \sec^3(c+dx)(a+b\sec(c+dx))^{2/3}\,dx = \int (b\sec (dx+c)+a)^{\frac{2}{3}} \sec (dx+c)^3\,dx$$

input
```
integrate(sec(d*x+c)^3*(a+b*sec(d*x+c))^(2/3),x, algorithm="fricas")
```

output
```
integral((b*sec(d*x + c) + a)^(2/3)*sec(d*x + c)^3, x)
```

3.688.6 Sympy [F]

$$\int \sec^3(c+dx)(a+b\sec(c+dx))^{2/3}\,dx = \int (a+b\sec(c+dx))^{\frac{2}{3}}\sec^3(c+dx)\,dx$$

input
```
integrate(sec(d*x+c)**3*(a+b*sec(d*x+c))**(2/3),x)
```

output
```
Integral((a + b*sec(c + d*x))**(2/3)*sec(c + d*x)**3, x)
```

3.688.7 Maxima [F]

$$\int \sec^3(c+dx)(a+b\sec(c+dx))^{2/3}\,dx = \int (b\sec(dx+c)+a)^{\frac{2}{3}}\sec(dx+c)^3\,dx$$

input
```
integrate(sec(d*x+c)^3*(a+b*sec(d*x+c))^(2/3),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^(2/3)*sec(d*x + c)^3, x)
```

3.688.8 Giac [F]

$$\int \sec^3(c+dx)(a+b\sec(c+dx))^{2/3}\,dx = \int (b\sec(dx+c)+a)^{\frac{2}{3}}\sec(dx+c)^3\,dx$$

input
```
integrate(sec(d*x+c)^3*(a+b*sec(d*x+c))^(2/3),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^(2/3)*sec(d*x + c)^3, x)
```

3.688.9 Mupad [F(-1)]

Timed out.

$$\int \sec^3(c+dx)(a+b\sec(c+dx))^{2/3}\,dx = \int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{2/3}}{\cos\left(c+d\,x\right)^3}\,dx$$

input
```
int((a + b/cos(c + d*x))^(2/3)/cos(c + d*x)^3,x)
```

output
```
int((a + b/cos(c + d*x))^(2/3)/cos(c + d*x)^3, x)
```

3.689 $\int \sec^2(c+dx)(a+b\sec(c+dx))^{2/3}\,dx$

3.689.1 Optimal result . . . 5376
3.689.2 Mathematica [B] (warning: unable to verify) . . . 5377
3.689.3 Rubi [A] (verified) . . . 5377
3.689.4 Maple [F] . . . 5380
3.689.5 Fricas [F] . . . 5380
3.689.6 Sympy [F] . . . 5381
3.689.7 Maxima [F] . . . 5381
3.689.8 Giac [F] . . . 5381
3.689.9 Mupad [F(-1)] . . . 5382

3.689.1 Optimal result

Integrand size = 23, antiderivative size = 260

$$\int \sec^2(c+dx)(a+b\sec(c+dx))^{2/3}\,dx = \frac{3(a+b\sec(c+dx))^{2/3}\tan(c+dx)}{5d} + \frac{2\sqrt{2}a\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},-\frac{2}{3},\frac{3}{2},\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)(a+b\sec(c+dx))^{2/3}\tan(c+dx)}{5bd\sqrt{1+\sec(c+dx)}\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{2/3}} - \frac{2\sqrt{2}(a^2-b^2)\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},\frac{1}{3},\frac{3}{2},\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)\sqrt[3]{\frac{a+b\sec(c+dx)}{a+b}}\tan(c+dx)}{5bd\sqrt{1+\sec(c+dx)}\sqrt[3]{a+b\sec(c+dx)}}$$

output
```
3/5*(a+b*sec(d*x+c))^(2/3)*tan(d*x+c)/d+2/5*a*AppellF1(1/2,-2/3,1/2,3/2,b*
(1-sec(d*x+c))/(a+b),1/2-1/2*sec(d*x+c))*(a+b*sec(d*x+c))^(2/3)*2^(1/2)*ta
n(d*x+c)/b/d/((a+b*sec(d*x+c))/(a+b))^(2/3)/(1+sec(d*x+c))^(1/2)-2/5*(a^2-
b^2)*AppellF1(1/2,1/3,1/2,3/2,b*(1-sec(d*x+c))/(a+b),1/2-1/2*sec(d*x+c))*(
(a+b*sec(d*x+c))/(a+b))^(1/3)*2^(1/2)*tan(d*x+c)/b/d/(a+b*sec(d*x+c))^(1/3
)/(1+sec(d*x+c))^(1/2)
```

3.689.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 10909 vs. $2(260) = 520$.

Time = 42.67 (sec) , antiderivative size = 10909, normalized size of antiderivative = 41.96

$$\int \sec^2(c+dx)(a+b\sec(c+dx))^{2/3}\,dx = \text{Result too large to show}$$

input
```
Integrate[Sec[c + d*x]^2*(a + b*Sec[c + d*x])^(2/3),x]
```

output
```
Result too large to show
```

3.689.3 Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {3042, 4322, 3042, 4495, 3042, 4321, 156, 155}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(c+dx)(a+b\sec(c+dx))^{2/3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^2\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{2/3}dx$$

$$\Big\downarrow \text{4322}$$

$$\frac{2}{5}\int \frac{\sec(c+dx)(b+a\sec(c+dx))}{\sqrt[3]{a+b\sec(c+dx)}}dx+\frac{3\tan(c+dx)(a+b\sec(c+dx))^{2/3}}{5d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{5}\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(b+a\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt[3]{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{3\tan(c+dx)(a+b\sec(c+dx))^{2/3}}{5d}$$

$$\Big\downarrow \text{4495}$$

$$\frac{2}{5}\left(\frac{a\int \sec(c+dx)(a+b\sec(c+dx))^{2/3}dx}{b}-\frac{(a^2-b^2)\int \frac{\sec(c+dx)}{\sqrt[3]{a+b\sec(c+dx)}}dx}{b}\right)+\frac{3\tan(c+dx)(a+b\sec(c+dx))^{2/3}}{5d}$$

$\Big\downarrow$ 3042

$$\frac{2}{5}\left(\frac{a\int \csc\left(c+dx+\frac{\pi}{2}\right)\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{2/3}dx}{b}-\frac{(a^2-b^2)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt[3]{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}\right)+\frac{3\tan(c+dx)(a+b\sec(c+dx))^{2/3}}{5d}$$

$\Big\downarrow$ 4321

$$\frac{2}{5}\left(\frac{(a^2-b^2)\tan(c+dx)\int \frac{1}{\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}\sqrt[3]{a+b\sec(c+dx)}}d\sec(c+dx)}{bd\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}}-\frac{a\tan(c+dx)\int \frac{(a+b\,\mathrm{s}}{\sqrt{1-\sec(c+}}}{bd\sqrt{1-\sec(c+d}}\right.$$

$$\frac{3\tan(c+dx)(a+b\sec(c+dx))^{2/3}}{5d}$$

$\Big\downarrow$ 156

$$\frac{2}{5}\left(\frac{(a^2-b^2)\tan(c+dx)\sqrt[3]{\frac{a+b\sec(c+dx)}{a+b}}\int \frac{1}{\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}\sqrt[3]{\frac{a}{a+b}+\frac{b\sec(c+dx)}{a+b}}}d\sec(c+dx)}{bd\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}\sqrt[3]{a+b\sec(c+dx)}}-\right.$$

$$\frac{3\tan(c+dx)(a+b\sec(c+dx))^{2/3}}{5d}$$

$\Big\downarrow$ 155

$$\frac{2}{5}\left(\frac{\sqrt{2}a\tan(c+dx)(a+b\sec(c+dx))^{2/3}\operatorname{AppellF1}\left(\frac{1}{2},\frac{1}{2},-\frac{2}{3},\frac{3}{2},\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)}{bd\sqrt{\sec(c+dx)+1}\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{2/3}}-\frac{\sqrt{2}(a^2-b^2}{}\right.$$

$$\frac{3\tan(c+dx)(a+b\sec(c+dx))^{2/3}}{5d}$$

3.689. $\int \sec^2(c+dx)(a+b\sec(c+dx))^{2/3}\,dx$

input
```
Int[Sec[c + d*x]^2*(a + b*Sec[c + d*x])^(2/3),x]
```

output
```
(3*(a + b*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(5*d) + (2*((Sqrt[2]*a*AppellF
1[1/2, 1/2, -2/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[c + d*x]))/(a + b
)]*(a + b*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(b*d*Sqrt[1 + Sec[c + d*x]]*((
a + b*Sec[c + d*x])/(a + b))^(2/3)) - (Sqrt[2]*(a^2 - b^2)*AppellF1[1/2, 1
/2, 1/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[c + d*x]))/(a + b)]*((a +
b*Sec[c + d*x])/(a + b))^(1/3)*Tan[c + d*x])/(b*d*Sqrt[1 + Sec[c + d*x]]*(
a + b*Sec[c + d*x])^(1/3))))/5
```

3.689.3.1 Defintions of rubi rules used

rule 155
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*Simplify[b/(b*c - a*d)]^n*
Simplify[b/(b*e - a*f)]^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/
(b*c - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f,
 m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[Sim
plify[b/(b*c - a*d)], 0] && GtQ[Simplify[b/(b*e - a*f)], 0] && !(GtQ[Simpl
ify[d/(d*a - c*b)], 0] && GtQ[Simplify[d/(d*e - c*f)], 0] && SimplerQ[c + d
*x, a + b*x]) && !(GtQ[Simplify[f/(f*a - e*b)], 0] && GtQ[Simplify[f/(f*c
- e*d)], 0] && SimplerQ[e + f*x, a + b*x])
```

rule 156
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(e + f*x)^FracPart[p]/(Simplify[b/(b*e - a*f)]^IntPart[p
]*(b*((e + f*x)/(b*e - a*f)))^FracPart[p])   Int[(a + b*x)^m*(c + d*x)^n*Si
mp[b*(e/(b*e - a*f)) + b*f*(x/(b*e - a*f)), x]^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] &
& GtQ[Simplify[b/(b*c - a*d)], 0] && !GtQ[Simplify[b/(b*e - a*f)], 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4321
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[Cot[e + f*x]/(f*Sqrt[1 + Csc[e + f*x]]*Sqrt[1 - Csc[e + f*x
]])   Subst[Int[(a + b*x)^m/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Csc[e + f*x]]
, x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0] && !IntegerQ[2*m]
```

rule 4322
```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(-Cot[e + f*x])*((a + b*Csc[e + f*x])^m/(f*(m + 1))), x]
+ Simp[m/(m + 1)   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(b + a*Csc
[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m,
 0]
```

rule 4495
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b - a*B)/b   Int[
Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] + Simp[B/b   Int[Csc[e + f*x]*(
a + b*Csc[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, A, B, e, f, m}, x] && N
eQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

3.689.4 Maple [F]

$$\int \sec (dx+c)^2 (a+b\sec (dx+c))^{\frac{2}{3}} \, dx$$

input
```
int(sec(d*x+c)^2*(a+b*sec(d*x+c))^(2/3),x)
```

output
```
int(sec(d*x+c)^2*(a+b*sec(d*x+c))^(2/3),x)
```

3.689.5 Fricas [F]

$$\int \sec^2(c+dx)(a+b\sec(c+dx))^{2/3}\,dx = \int (b\sec (dx+c)+a)^{\frac{2}{3}} \sec (dx+c)^2 \, dx$$

input
```
integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^(2/3),x, algorithm="fricas")
```

output
```
integral((b*sec(d*x + c) + a)^(2/3)*sec(d*x + c)^2, x)
```

3.689.6 Sympy [F]

$$\int \sec^2(c+dx)(a+b\sec(c+dx))^{2/3}\,dx = \int (a+b\sec(c+dx))^{\frac{2}{3}}\sec^2(c+dx)\,dx$$

input
```
integrate(sec(d*x+c)**2*(a+b*sec(d*x+c))**(2/3),x)
```

output
```
Integral((a + b*sec(c + d*x))**(2/3)*sec(c + d*x)**2, x)
```

3.689.7 Maxima [F]

$$\int \sec^2(c+dx)(a+b\sec(c+dx))^{2/3}\,dx = \int (b\sec(dx+c)+a)^{\frac{2}{3}}\sec(dx+c)^2\,dx$$

input
```
integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^(2/3),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^(2/3)*sec(d*x + c)^2, x)
```

3.689.8 Giac [F]

$$\int \sec^2(c+dx)(a+b\sec(c+dx))^{2/3}\,dx = \int (b\sec(dx+c)+a)^{\frac{2}{3}}\sec(dx+c)^2\,dx$$

input
```
integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^(2/3),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^(2/3)*sec(d*x + c)^2, x)
```

3.689.9 Mupad [F(-1)]

Timed out.

$$\int \sec^2(c+dx)(a+b\sec(c+dx))^{2/3}\,dx = \int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{2/3}}{\cos\left(c+d\,x\right)^2}\,dx$$

input
```
int((a + b/cos(c + d*x))^(2/3)/cos(c + d*x)^2,x)
```

output
```
int((a + b/cos(c + d*x))^(2/3)/cos(c + d*x)^2, x)
```

3.690 $\int \sec(c+dx)(a+b\sec(c+dx))^{2/3}\,dx$

3.690.1 Optimal result 5383
3.690.2 Mathematica [B] (warning: unable to verify) 5383
3.690.3 Rubi [A] (verified) 5384
3.690.4 Maple [F] 5385
3.690.5 Fricas [F] 5386
3.690.6 Sympy [F] 5386
3.690.7 Maxima [F] 5386
3.690.8 Giac [F] 5387
3.690.9 Mupad [F(-1)] 5387

3.690.1 Optimal result

Integrand size = 21, antiderivative size = 105

$$\int \sec(c+dx)(a+b\sec(c+dx))^{2/3}\,dx=\frac{\sqrt{2}\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},-\frac{2}{3},\frac{3}{2},\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)(a+b\sec(c+dx))^{2/3}\tan(c+dx)}{d\sqrt{1+\sec(c+dx)}\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{2/3}}$$

output
```
AppellF1(1/2,-2/3,1/2,3/2,b*(1-sec(d*x+c))/(a+b),1/2-1/2*sec(d*x+c))*(a+b*
sec(d*x+c))^(2/3)*2^(1/2)*tan(d*x+c)/d/((a+b*sec(d*x+c))/(a+b))^(2/3)/(1+s
ec(d*x+c))^(1/2)
```

3.690.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 7142 vs. 2(105) = 210.

Time = 45.82 (sec) , antiderivative size = 7142, normalized size of antiderivative = 68.02

$$\int \sec(c+dx)(a+b\sec(c+dx))^{2/3}\,dx=\text{Result too large to show}$$

input
```
Integrate[Sec[c + d*x]*(a + b*Sec[c + d*x])^(2/3),x]
```

output
```
Result too large to show
```

3.690.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3042, 4321, 156, 155}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(c+dx)(a+b\sec(c+dx))^{2/3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{2/3}dx$$

$$\Big\downarrow \text{4321}$$

$$-\frac{\tan(c+dx)\int \frac{(a+b\sec(c+dx))^{2/3}}{\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}}d\sec(c+dx)}{d\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}}$$

$$\Big\downarrow \text{156}$$

$$-\frac{\tan(c+dx)(a+b\sec(c+dx))^{2/3}\int \frac{\left(\frac{a}{a+b}+\frac{b\sec(c+dx)}{a+b}\right)^{2/3}}{\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}}d\sec(c+dx)}{d\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{2/3}}$$

$$\Big\downarrow \text{155}$$

$$\frac{\sqrt{2}\tan(c+dx)(a+b\sec(c+dx))^{2/3}\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},-\frac{2}{3},\frac{3}{2},\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)}{d\sqrt{\sec(c+dx)+1}\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{2/3}}$$

input
```
Int[Sec[c + d*x]*(a + b*Sec[c + d*x])^(2/3),x]
```

output
```
(Sqrt[2]*AppellF1[1/2, 1/2, -2/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[c
 + d*x]))/(a + b)]*(a + b*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(d*Sqrt[1 + Se
c[c + d*x]]*((a + b*Sec[c + d*x])/(a + b))^(2/3))
```

3.690.3.1 Defintions of rubi rules used

rule 155
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*Simplify[b/(b*c - a*d)]^n*
Simplify[b/(b*e - a*f)]^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/
(b*c - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f,
 m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[Sim
plify[b/(b*c - a*d)], 0] && GtQ[Simplify[b/(b*e - a*f)], 0] && !(GtQ[Simpl
ify[d/(d*a - c*b)], 0] && GtQ[Simplify[d/(d*e - c*f)], 0] && SimplerQ[c + d
*x, a + b*x]) && !(GtQ[Simplify[f/(f*a - e*b)], 0] && GtQ[Simplify[f/(f*c
- e*d)], 0] && SimplerQ[e + f*x, a + b*x])
```

rule 156
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(e + f*x)^FracPart[p]/(Simplify[b/(b*e - a*f)]^IntPart[p
]*(b*((e + f*x)/(b*e - a*f)))^FracPart[p])   Int[(a + b*x)^m*(c + d*x)^n*Si
mp[b*(e/(b*e - a*f)) + b*f*(x/(b*e - a*f)), x]^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] &
& GtQ[Simplify[b/(b*c - a*d)], 0] && !GtQ[Simplify[b/(b*e - a*f)], 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4321
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[Cot[e + f*x]/(f*Sqrt[1 + Csc[e + f*x]]*Sqrt[1 - Csc[e + f*x
]])   Subst[Int[(a + b*x)^m/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Csc[e + f*x]]
, x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0] && !IntegerQ[2*m]
```

3.690.4 Maple [F]

$$\int \sec(dx+c)\,(a+b\sec(dx+c))^{\frac{2}{3}}\,dx$$

input
```
int(sec(d*x+c)*(a+b*sec(d*x+c))^(2/3),x)
```

output
```
int(sec(d*x+c)*(a+b*sec(d*x+c))^(2/3),x)
```

3.690.5 Fricas [F]

$$\int \sec(c+dx)(a+b\sec(c+dx))^{2/3}\,dx = \int (b\sec(dx+c)+a)^{\frac{2}{3}}\sec(dx+c)\,dx$$

input
```
integrate(sec(d*x+c)*(a+b*sec(d*x+c))^(2/3),x, algorithm="fricas")
```

output
```
integral((b*sec(d*x + c) + a)^(2/3)*sec(d*x + c), x)
```

3.690.6 Sympy [F]

$$\int \sec(c+dx)(a+b\sec(c+dx))^{2/3}\,dx = \int (a+b\sec(c+dx))^{\frac{2}{3}}\sec(c+dx)\,dx$$

input
```
integrate(sec(d*x+c)*(a+b*sec(d*x+c))**(2/3),x)
```

output
```
Integral((a + b*sec(c + d*x))**(2/3)*sec(c + d*x), x)
```

3.690.7 Maxima [F]

$$\int \sec(c+dx)(a+b\sec(c+dx))^{2/3}\,dx = \int (b\sec(dx+c)+a)^{\frac{2}{3}}\sec(dx+c)\,dx$$

input
```
integrate(sec(d*x+c)*(a+b*sec(d*x+c))^(2/3),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^(2/3)*sec(d*x + c), x)
```

3.690.8 Giac [F]

$$\int \sec(c+dx)(a+b\sec(c+dx))^{2/3}\,dx = \int (b\sec(dx+c)+a)^{\frac{2}{3}}\sec(dx+c)\,dx$$

input
```
integrate(sec(d*x+c)*(a+b*sec(d*x+c))^(2/3),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^(2/3)*sec(d*x + c), x)
```

3.690.9 Mupad [F(-1)]

Timed out.

$$\int \sec(c+dx)(a+b\sec(c+dx))^{2/3}\,dx = \int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{2/3}}{\cos(c+d\,x)}\,dx$$

input
```
int((a + b/cos(c + d*x))^(2/3)/cos(c + d*x),x)
```

output
```
int((a + b/cos(c + d*x))^(2/3)/cos(c + d*x), x)
```

3.691 $\int (a+b\sec(c+dx))^{2/3}\,dx$

3.691.1 Optimal result . . . 5388
3.691.2 Mathematica [N/A] . . . 5388
3.691.3 Rubi [N/A] . . . 5389
3.691.4 Maple [N/A] (verified) . . . 5390
3.691.5 Fricas [F(-1)] . . . 5390
3.691.6 Sympy [N/A] . . . 5390
3.691.7 Maxima [N/A] . . . 5391
3.691.8 Giac [N/A] . . . 5391
3.691.9 Mupad [N/A] . . . 5391

3.691.1 Optimal result

Integrand size = 14, antiderivative size = 14

$$\int (a+b\sec(c+dx))^{2/3}\,dx = \text{Int}\left((a+b\sec(c+dx))^{2/3},x\right)$$

output
```
Unintegrable((a+b*sec(d*x+c))^(2/3),x)
```

3.691.2 Mathematica [N/A]

Not integrable

Time = 18.73 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int (a+b\sec(c+dx))^{2/3}\,dx = \int (a+b\sec(c+dx))^{2/3}\,dx$$

input
```
Integrate[(a + b*Sec[c + d*x])^(2/3),x]
```

output
```
Integrate[(a + b*Sec[c + d*x])^(2/3), x]
```

3.691.3 Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 4273}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\sec(c+dx))^{2/3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{2/3}dx$$

$$\Big\downarrow \text{4273}$$

$$\int (a+b\sec(c+dx))^{2/3}dx$$

input
```
Int[(a + b*Sec[c + d*x])^(2/3),x]
```

output
```
$Aborted
```

3.691.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4273
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Unintegrable[
(a + b*Csc[c + d*x])^n, x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[a^2 - b^2, 0
] &&  !IntegerQ[2*n]
```

3.691.4 Maple [N/A] (verified)

Not integrable

Time = 0.31 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int (a+b\sec(dx+c))^{\frac{2}{3}}\,dx$$

input
```
int((a+b*sec(d*x+c))^(2/3),x)
```

output
```
int((a+b*sec(d*x+c))^(2/3),x)
```

3.691.5 Fricas [F(-1)]

Timed out.

$$\int (a+b\sec(c+dx))^{2/3}\,dx = \text{Timed out}$$

input
```
integrate((a+b*sec(d*x+c))^(2/3),x, algorithm="fricas")
```

output
```
Timed out
```

3.691.6 Sympy [N/A]

Not integrable

Time = 0.81 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int (a+b\sec(c+dx))^{2/3}\,dx = \int (a+b\sec(c+dx))^{\frac{2}{3}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))**(2/3),x)
```

output
```
Integral((a + b*sec(c + d*x))**(2/3), x)
```

3.691.7 Maxima [N/A]

Not integrable

Time = 0.54 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int (a+b\sec(c+dx))^{2/3}\,dx=\int (b\sec(dx+c)+a)^{\frac{2}{3}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(2/3),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^(2/3), x)
```

3.691.8 Giac [N/A]

Not integrable

Time = 0.44 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int (a+b\sec(c+dx))^{2/3}\,dx=\int (b\sec(dx+c)+a)^{\frac{2}{3}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(2/3),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^(2/3), x)
```

3.691.9 Mupad [N/A]

Not integrable

Time = 14.18 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int (a+b\sec(c+dx))^{2/3}\,dx=\int \left(a+\frac{b}{\cos(c+d\,x)}\right)^{2/3}\,dx$$

input
```
int((a + b/cos(c + d*x))^(2/3),x)
```

output
```
int((a + b/cos(c + d*x))^(2/3), x)
```

3.692 $\int \sec(c+dx)(a+b\sec(c+dx))^{4/3}\,dx$

3.692.1 Optimal result 5392
3.692.2 Mathematica [B] (warning: unable to verify) 5392
3.692.3 Rubi [A] (verified) 5393
3.692.4 Maple [F] 5394
3.692.5 Fricas [F] 5395
3.692.6 Sympy [F] 5395
3.692.7 Maxima [F] 5395
3.692.8 Giac [F] 5396
3.692.9 Mupad [F(-1)] 5396

3.692.1 Optimal result

Integrand size = 21, antiderivative size = 108

$$\int \sec(c+dx)(a+b\sec(c+dx))^{4/3}\,dx = \frac{\sqrt{2}(a+b)\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},-\frac{4}{3},\frac{3}{2},\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)\sqrt[3]{a+b\sec(c+dx)}\tan(c+dx)}{d\sqrt{1+\sec(c+dx)}\sqrt[3]{\frac{a+b\sec(c+dx)}{a+b}}}$$

output
```
(a+b)*AppellF1(1/2,-4/3,1/2,3/2,b*(1-sec(d*x+c))/(a+b),1/2-1/2*sec(d*x+c))
*(a+b*sec(d*x+c))^(1/3)*2^(1/2)*tan(d*x+c)/d/((a+b*sec(d*x+c))/(a+b))^(1/3
)/(1+sec(d*x+c))^(1/2)
```

3.692.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 7313 vs. 2(108) = 216.

Time = 48.68 (sec) , antiderivative size = 7313, normalized size of antiderivative = 67.71

$$\int \sec(c+dx)(a+b\sec(c+dx))^{4/3}\,dx = \text{Result too large to show}$$

input
```
Integrate[Sec[c + d*x]*(a + b*Sec[c + d*x])^(4/3),x]
```

output
```
Result too large to show
```

3.692.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3042, 4321, 156, 155}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(c+dx)(a+b\sec(c+dx))^{4/3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{4/3}dx$$

$$\Big\downarrow \text{4321}$$

$$-\frac{\tan(c+dx)\int \frac{(a+b\sec(c+dx))^{4/3}}{\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}}d\sec(c+dx)}{d\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}}$$

$$\Big\downarrow \text{156}$$

$$-\frac{(a+b)\tan(c+dx)\sqrt[3]{a+b\sec(c+dx)}\int \frac{\left(\frac{a}{a+b}+\frac{b\sec(c+dx)}{a+b}\right)^{4/3}}{\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}}d\sec(c+dx)}{d\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}\sqrt[3]{\frac{a+b\sec(c+dx)}{a+b}}}$$

$$\Big\downarrow \text{155}$$

$$\frac{\sqrt{2}(a+b)\tan(c+dx)\sqrt[3]{a+b\sec(c+dx)}\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},-\frac{4}{3},\frac{3}{2},\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)}{d\sqrt{\sec(c+dx)+1}\sqrt[3]{\frac{a+b\sec(c+dx)}{a+b}}}$$

input
```
Int[Sec[c + d*x]*(a + b*Sec[c + d*x])^(4/3),x]
```

output
```
(Sqrt[2]*(a + b)*AppellF1[1/2, 1/2, -4/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1
 - Sec[c + d*x]))/(a + b)]*(a + b*Sec[c + d*x])^(1/3)*Tan[c + d*x])/(d*Sqr
t[1 + Sec[c + d*x]]*((a + b*Sec[c + d*x])/(a + b))^(1/3))
```

3.692.3.1 Defintions of rubi rules used

rule 155
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*Simplify[b/(b*c - a*d)]^n*
Simplify[b/(b*e - a*f)]^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/
(b*c - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f,
 m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[Sim
plify[b/(b*c - a*d)], 0] && GtQ[Simplify[b/(b*e - a*f)], 0] && !(GtQ[Simpl
ify[d/(d*a - c*b)], 0] && GtQ[Simplify[d/(d*e - c*f)], 0] && SimplerQ[c + d
*x, a + b*x]) && !(GtQ[Simplify[f/(f*a - e*b)], 0] && GtQ[Simplify[f/(f*c
- e*d)], 0] && SimplerQ[e + f*x, a + b*x])
```

rule 156
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(e + f*x)^FracPart[p]/(Simplify[b/(b*e - a*f)]^IntPart[p
]*(b*((e + f*x)/(b*e - a*f)))^FracPart[p])   Int[(a + b*x)^m*(c + d*x)^n*Si
mp[b*(e/(b*e - a*f)) + b*f*(x/(b*e - a*f)), x]^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] &
& GtQ[Simplify[b/(b*c - a*d)], 0] && !GtQ[Simplify[b/(b*e - a*f)], 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4321
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[Cot[e + f*x]/(f*Sqrt[1 + Csc[e + f*x]]*Sqrt[1 - Csc[e + f*x
]])   Subst[Int[(a + b*x)^m/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Csc[e + f*x]]
, x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0] && !IntegerQ[2*m]
```

3.692.4 Maple [F]

$$\int \sec(dx+c)\,(a+b\sec(dx+c))^{\frac{4}{3}}\,dx$$

input
```
int(sec(d*x+c)*(a+b*sec(d*x+c))^(4/3),x)
```

output
```
int(sec(d*x+c)*(a+b*sec(d*x+c))^(4/3),x)
```

3.692.5 Fricas [F]

$$\int \sec(c+dx)(a+b\sec(c+dx))^{4/3}\,dx = \int (b\sec(dx+c)+a)^{\frac{4}{3}}\sec(dx+c)\,dx$$

input
```
integrate(sec(d*x+c)*(a+b*sec(d*x+c))^(4/3),x, algorithm="fricas")
```

output
```
integral((b*sec(d*x + c)^2 + a*sec(d*x + c))*(b*sec(d*x + c) + a)^(1/3), x
)
```

3.692.6 Sympy [F]

$$\int \sec(c+dx)(a+b\sec(c+dx))^{4/3}\,dx = \int (a+b\sec(c+dx))^{\frac{4}{3}}\sec(c+dx)\,dx$$

input
```
integrate(sec(d*x+c)*(a+b*sec(d*x+c))**(4/3),x)
```

output
```
Integral((a + b*sec(c + d*x))**(4/3)*sec(c + d*x), x)
```

3.692.7 Maxima [F]

$$\int \sec(c+dx)(a+b\sec(c+dx))^{4/3}\,dx = \int (b\sec(dx+c)+a)^{\frac{4}{3}}\sec(dx+c)\,dx$$

input
```
integrate(sec(d*x+c)*(a+b*sec(d*x+c))^(4/3),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^(4/3)*sec(d*x + c), x)
```

3.692.8 Giac [F]

$$\int \sec(c+dx)(a+b\sec(c+dx))^{4/3}\,dx = \int (b\sec(dx+c)+a)^{\frac{4}{3}}\sec(dx+c)\,dx$$

input
```
integrate(sec(d*x+c)*(a+b*sec(d*x+c))^(4/3),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^(4/3)*sec(d*x + c), x)
```

3.692.9 Mupad [F(-1)]

Timed out.

$$\int \sec(c+dx)(a+b\sec(c+dx))^{4/3}\,dx = \int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{4/3}}{\cos(c+d\,x)}\,dx$$

input
```
int((a + b/cos(c + d*x))^(4/3)/cos(c + d*x),x)
```

output
```
int((a + b/cos(c + d*x))^(4/3)/cos(c + d*x), x)
```

3.693 $\int (a+b\sec(c+dx))^{4/3}\,dx$

3.693.1 Optimal result . . . 5397
3.693.2 Mathematica [N/A] . . . 5397
3.693.3 Rubi [N/A] . . . 5398
3.693.4 Maple [N/A] (verified) . . . 5399
3.693.5 Fricas [F(-1)] . . . 5399
3.693.6 Sympy [N/A] . . . 5399
3.693.7 Maxima [N/A] . . . 5400
3.693.8 Giac [N/A] . . . 5400
3.693.9 Mupad [N/A] . . . 5400

3.693.1 Optimal result

Integrand size = 14, antiderivative size = 14

$$\int (a+b\sec(c+dx))^{4/3}\,dx = \text{Int}\left((a+b\sec(c+dx))^{4/3},x\right)$$

output
```
Unintegrable((a+b*sec(d*x+c))^(4/3),x)
```

3.693.2 Mathematica [N/A]

Not integrable

Time = 100.65 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int (a+b\sec(c+dx))^{4/3}\,dx = \int (a+b\sec(c+dx))^{4/3}\,dx$$

input
```
Integrate[(a + b*Sec[c + d*x])^(4/3),x]
```

output
```
Integrate[(a + b*Sec[c + d*x])^(4/3), x]
```

3.693.3 Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 4273}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\sec(c+dx))^{4/3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{4/3}dx$$

$$\Big\downarrow \text{4273}$$

$$\int (a+b\sec(c+dx))^{4/3}dx$$

input
```
Int[(a + b*Sec[c + d*x])^(4/3),x]
```

output
```
$Aborted
```

3.693.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4273
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Unintegrable[
(a + b*Csc[c + d*x])^n, x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[a^2 - b^2, 0
] &&  !IntegerQ[2*n]
```

3.693.4 Maple [N/A] (verified)

Not integrable

Time = 0.31 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int (a+b\sec(dx+c))^{\frac{4}{3}}\,dx$$

input
```
int((a+b*sec(d*x+c))^(4/3),x)
```

output
```
int((a+b*sec(d*x+c))^(4/3),x)
```

3.693.5 Fricas [F(-1)]

Timed out.

$$\int (a+b\sec(c+dx))^{4/3}\,dx = \text{Timed out}$$

input
```
integrate((a+b*sec(d*x+c))^(4/3),x, algorithm="fricas")
```

output
```
Timed out
```

3.693.6 Sympy [N/A]

Not integrable

Time = 13.31 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int (a+b\sec(c+dx))^{4/3}\,dx = \int (a+b\sec(c+dx))^{\frac{4}{3}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))**(4/3),x)
```

output
```
Integral((a + b*sec(c + d*x))**(4/3), x)
```

3.693.7 Maxima [N/A]

Not integrable

Time = 0.54 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int (a+b\sec(c+dx))^{4/3}\,dx=\int (b\sec(dx+c)+a)^{\frac{4}{3}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(4/3),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^(4/3), x)
```

3.693.8 Giac [N/A]

Not integrable

Time = 1.14 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int (a+b\sec(c+dx))^{4/3}\,dx=\int (b\sec(dx+c)+a)^{\frac{4}{3}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(4/3),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^(4/3), x)
```

3.693.9 Mupad [N/A]

Not integrable

Time = 15.30 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int (a+b\sec(c+dx))^{4/3}\,dx=\int \left(a+\frac{b}{\cos(c+d\,x)}\right)^{4/3}\,dx$$

input
```
int((a + b/cos(c + d*x))^(4/3),x)
```

output
```
int((a + b/cos(c + d*x))^(4/3), x)
```

3.694 $\int \sec^4(c+dx)(a+b\sec(c+dx))^{5/3}\,dx$

3.694.1 Optimal result . . . 5401
3.694.2 Mathematica [B] (warning: unable to verify) . . . 5402
3.694.3 Rubi [A] (verified) . . . 5402
3.694.4 Maple [F] . . . 5408
3.694.5 Fricas [F] . . . 5408
3.694.6 Sympy [F(-1)] . . . 5408
3.694.7 Maxima [F] . . . 5409
3.694.8 Giac [F] . . . 5409
3.694.9 Mupad [F(-1)] . . . 5409

3.694.1 Optimal result

Integrand size = 23, antiderivative size = 412

$$\int \sec^4(c+dx)(a+b\sec(c+dx))^{5/3}\,dx = \frac{3a(18a^2+97b^2)\,(a+b\sec(c+dx))^{2/3}\tan(c+dx)}{1232b^2d} + \frac{3(18a^2+121b^2)\,(a+b\sec(c+dx))^{5/3}\tan(c+dx)}{1232b^2d} - \frac{9a(a+b\sec(c+dx))^{8/3}\tan(c+dx)}{77b^2d} + \frac{3\sec(c+dx)(a+b\sec(c+dx))^{8/3}\tan(c+dx)}{14bd} + \frac{(36a^4+164a^2b^2+605b^4)\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},-\frac{2}{3},\frac{3}{2},\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)(a+b\sec(c+dx))^{2/3}\,\text{ta}}{616\sqrt{2}b^3d\sqrt{1+\sec(c+dx)}\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{2/3}} - \frac{a(18a^4+79a^2b^2-97b^4)\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},\frac{1}{3},\frac{3}{2},\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)\sqrt[3]{\frac{a+b\sec(c+dx)}{a+b}}\tan(c}{308\sqrt{2}b^3d\sqrt{1+\sec(c+dx)}\sqrt[3]{a+b\sec(c+dx)}}$$

output
```
3/1232*a*(18*a^2+97*b^2)*(a+b*sec(d*x+c))^(2/3)*tan(d*x+c)/b^2/d+3/1232*(1
8*a^2+121*b^2)*(a+b*sec(d*x+c))^(5/3)*tan(d*x+c)/b^2/d-9/77*a*(a+b*sec(d*x
+c))^(8/3)*tan(d*x+c)/b^2/d+3/14*sec(d*x+c)*(a+b*sec(d*x+c))^(8/3)*tan(d*x
+c)/b/d+1/1232*(36*a^4+164*a^2*b^2+605*b^4)*AppellF1(1/2,-2/3,1/2,3/2,b*(1
-sec(d*x+c))/(a+b),1/2-1/2*sec(d*x+c))*(a+b*sec(d*x+c))^(2/3)*tan(d*x+c)/b
^3/d/((a+b*sec(d*x+c))/(a+b))^(2/3)*2^(1/2)/(1+sec(d*x+c))^(1/2)-1/616*a*(
18*a^4+79*a^2*b^2-97*b^4)*AppellF1(1/2,1/3,1/2,3/2,b*(1-sec(d*x+c))/(a+b),
1/2-1/2*sec(d*x+c))*((a+b*sec(d*x+c))/(a+b))^(1/3)*tan(d*x+c)/b^3/d/(a+b*s
ec(d*x+c))^(1/3)*2^(1/2)/(1+sec(d*x+c))^(1/2)
```

3.694.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 28057 vs. $2(412) = 824$.

Time = 45.80 (sec) , antiderivative size = 28057, normalized size of antiderivative = 68.10

$$\int \sec^4(c+dx)(a+b\sec(c+dx))^{5/3}\,dx = \text{Result too large to show}$$

input
```
Integrate[Sec[c + d*x]^4*(a + b*Sec[c + d*x])^(5/3),x]
```

output
```
Result too large to show
```

3.694.3 Rubi [A] (verified)

Time = 1.64 (sec) , antiderivative size = 428, normalized size of antiderivative = 1.04, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.783$, Rules used = {3042, 4352, 27, 3042, 4570, 27, 3042, 4490, 27, 3042, 4490, 27, 3042, 4495, 3042, 4321, 156, 155}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^4(c+dx)(a+b\sec(c+dx))^{5/3}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^4\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/3}dx$$

$$\Big\downarrow \; 4352$$

$$\frac{3\int \frac{1}{3}\sec(c+dx)(a+b\sec(c+dx))^{5/3}\left(-6a\sec^2(c+dx)+11b\sec(c+dx)+3a\right)dx}{14b}+\frac{3\tan(c+dx)\sec(c+dx)(a+b\sec(c+dx))^{8/3}}{14bd}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \sec(c+dx)(a+b\sec(c+dx))^{5/3}\left(-6a\sec^2(c+dx)+11b\sec(c+dx)+3a\right)dx}{14b}+\frac{3\tan(c+dx)\sec(c+dx)(a+b\sec(c+dx))^{8/3}}{14bd}$$

$$\Big\downarrow \; 3042$$

$$\frac{\int \csc\left(c+dx+\frac{\pi}{2}\right)\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/3}\left(-6a\csc\left(c+dx+\frac{\pi}{2}\right)^2+11b\csc\left(c+dx+\frac{\pi}{2}\right)+3a\right)dx}{14b}+\frac{3\tan(c+dx)\sec(c+dx)(a+b\sec(c+dx))^{8/3}}{14bd}$$

$\Big\downarrow$ 4570

$$\frac{\frac{3\int -\frac{1}{3}\sec(c+dx)(a+b\sec(c+dx))^{5/3}\left(15ab-\left(18a^2+121b^2\right)\sec(c+dx)\right)dx}{11b}-\frac{18a\tan(c+dx)(a+b\sec(c+dx))^{8/3}}{11bd}}{14b}+\frac{3\tan(c+dx)\sec(c+dx)(a+b\sec(c+dx))^{8/3}}{14bd}$$

$\Big\downarrow$ 27

$$\frac{-\frac{\int \sec(c+dx)(a+b\sec(c+dx))^{5/3}\left(15ab-\left(18a^2+121b^2\right)\sec(c+dx)\right)dx}{11b}-\frac{18a\tan(c+dx)(a+b\sec(c+dx))^{8/3}}{11bd}}{14b}+\frac{3\tan(c+dx)\sec(c+dx)(a+b\sec(c+dx))^{8/3}}{14bd}$$

$\Big\downarrow$ 3042

$$\frac{-\frac{\int \csc\left(c+dx+\frac{\pi}{2}\right)\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/3}\left(15ab+\left(-18a^2-121b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)\right)dx}{11b}-\frac{18a\tan(c+dx)(a+b\sec(c+dx))^{8/3}}{11bd}}{14b}+\frac{3\tan(c+dx)\sec(c+dx)(a+b\sec(c+dx))^{8/3}}{14bd}$$

$\Big\downarrow$ 4490

$$\frac{-\frac{\frac{3}{8}\int \frac{5}{3}\sec(c+dx)(a+b\sec(c+dx))^{2/3}\left(b\left(6a^2-121b^2\right)-a\left(18a^2+97b^2\right)\sec(c+dx)\right)dx-\frac{3\left(18a^2+121b^2\right)\tan(c+dx)(a+b\sec(c+dx))^{5/3}}{8d}}{11b}-\frac{18a\tan(c+dx}{}}{14b}$$
$$+\frac{3\tan(c+dx)\sec(c+dx)(a+b\sec(c+dx))^{8/3}}{14bd}$$

$\Big\downarrow$ 27

$$\frac{-\frac{\frac{5}{8}\int \sec(c+dx)(a+b\sec(c+dx))^{2/3}\left(b\left(6a^2-121b^2\right)-a\left(18a^2+97b^2\right)\sec(c+dx)\right)dx-\frac{3\left(18a^2+121b^2\right)\tan(c+dx)(a+b\sec(c+dx))^{5/3}}{8d}}{11b}-\frac{18a\tan(c+dx)(}{}}{14b}$$
$$+\frac{3\tan(c+dx)\sec(c+dx)(a+b\sec(c+dx))^{8/3}}{14bd}$$

$\Big\downarrow$ 3042

3.694. $\int \sec^4(c+dx)(a+b\sec(c+dx))^{5/3}\,dx$

$$-\frac{\frac{\frac{5}{8}\int \csc\left(c+dx+\frac{\pi}{2}\right)\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{2/3}\left(b\left(6a^2-121b^2\right)-a\left(18a^2+97b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)\right)dx-\frac{3\left(18a^2+121b^2\right)\tan(c+dx)(a+b\sec(c+dx))^{5/3}}{8d}}{11b}-18}{14b}$$

$$\frac{3\tan(c+dx)\sec(c+dx)(a+b\sec(c+dx))^{8/3}}{14bd}$$

$\downarrow$ 4490

$$-\frac{\frac{\frac{5}{8}\left(\frac{3}{5}\int -\frac{\sec(c+dx)\left(ab\left(6a^2+799b^2\right)+\left(36a^4+164b^2a^2+605b^4\right)\sec(c+dx)\right)}{3\sqrt[3]{a+b\sec(c+dx)}}dx-\frac{3a\left(18a^2+97b^2\right)\tan(c+dx)(a+b\sec(c+dx))^{2/3}}{5d}\right)-\frac{3\left(18a^2+121b^2\right)\tan(c+dx)}{8d}}{11b}}{14b}$$

$$\frac{3\tan(c+dx)\sec(c+dx)(a+b\sec(c+dx))^{8/3}}{14bd}$$

$\downarrow$ 27

$$-\frac{\frac{\frac{5}{8}\left(-\frac{1}{5}\int \frac{\sec(c+dx)\left(ab\left(6a^2+799b^2\right)+\left(36a^4+164b^2a^2+605b^4\right)\sec(c+dx)\right)}{\sqrt[3]{a+b\sec(c+dx)}}dx-\frac{3a\left(18a^2+97b^2\right)\tan(c+dx)(a+b\sec(c+dx))^{2/3}}{5d}\right)-\frac{3\left(18a^2+121b^2\right)\tan(c+dx)}{8d}}{11b}}{14b}$$

$$\frac{3\tan(c+dx)\sec(c+dx)(a+b\sec(c+dx))^{8/3}}{14bd}$$

$\downarrow$ 3042

$$-\frac{\frac{\frac{5}{8}\left(-\frac{1}{5}\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(ab\left(6a^2+799b^2\right)+\left(36a^4+164b^2a^2+605b^4\right)\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt[3]{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{3a\left(18a^2+97b^2\right)\tan(c+dx)(a+b\sec(c+dx))^{2/3}}{5d}\right)-\frac{3\left(18a^2+121b^2\right)t}{}}{11b}}{14b}$$

$$\frac{3\tan(c+dx)\sec(c+dx)(a+b\sec(c+dx))^{8/3}}{14bd}$$

$\downarrow$ 4495

$$-\frac{\frac{\frac{5}{8}\left(\frac{1}{5}\left(\frac{2a\left(18a^4+79a^2b^2-97b^4\right)\int \frac{\sec(c+dx)}{\sqrt[3]{a+b\sec(c+dx)}}dx}{b}-\frac{\left(36a^4+164a^2b^2+605b^4\right)\int \sec(c+dx)(a+b\sec(c+dx))^{2/3}dx}{b}\right)-\frac{3a\left(18a^2+97b^2\right)\tan(c+dx}{5d}}{11b}}{14b}$$

$$\frac{3\tan(c+dx)\sec(c+dx)(a+b\sec(c+dx))^{8/3}}{14bd}$$

$\downarrow$ 3042

3.694. $\int \sec^4(c+dx)(a+b\sec(c+dx))^{5/3}\,dx$

$$-\frac{\frac{5}{8}\left(\frac{1}{5}\left(\frac{2a\left(18a^4+79a^2b^2-97b^4\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt[3]{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\frac{\left(36a^4+164a^2b^2+605b^4\right)\int\csc\left(c+dx+\frac{\pi}{2}\right)\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{2/3}dx}{b}\right)-\frac{3a\left(18a\right.}{}\right)}{11b}}{14b}$$

$$\frac{3\tan(c+dx)\sec(c+dx)(a+b\sec(c+dx))^{8/3}}{14bd}$$

$\downarrow$ 4321

$$-\frac{\frac{5}{8}\left(\frac{1}{5}\left(\frac{\left(36a^4+164a^2b^2+605b^4\right)\tan(c+dx)\int\frac{(a+b\sec(c+dx))^{2/3}}{\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}}d\sec(c+dx)}{bd\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}}-\frac{2a\left(18a^4+79a^2b^2-97b^4\right)\tan(c+dx)\int\frac{1}{\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}}}{bd\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+}}\right.\right.}{11b}}$$

$$\frac{3\tan(c+dx)\sec(c+dx)(a+b\sec(c+dx))^{8/3}}{14bd}$$

$\downarrow$ 156

$$-\frac{\frac{5}{8}\left(\frac{1}{5}\left(\frac{\left(36a^4+164a^2b^2+605b^4\right)\tan(c+dx)(a+b\sec(c+dx))^{2/3}\int\frac{\left(\frac{a}{a+b}+\frac{b\sec(c+dx)}{a+b}\right)^{2/3}}{\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}}d\sec(c+dx)}{bd\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{2/3}}-\frac{2a\left(18a^4+79a^2b^2-97b^4\right)\tan(c+dx)\sqrt[3]{\frac{a+b\mathrm{s}}{}}}{bd\sqrt{1-}}\right.\right.}{}}$$

$$\frac{3\tan(c+dx)\sec(c+dx)(a+b\sec(c+dx))^{8/3}}{14bd}$$

$\downarrow$ 155

$$-\frac{\frac{5}{8}\left(\frac{1}{5}\left(\frac{2\sqrt{2}a\left(18a^4+79a^2b^2-97b^4\right)\tan(c+dx)\sqrt[3]{\frac{a+b\sec(c+dx)}{a+b}}\operatorname{AppellF1}\left(\frac{1}{2},\frac{1}{2},\frac{1}{3},\frac{3}{2},\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)}{bd\sqrt{\sec(c+dx)+1}\sqrt[3]{a+b\sec(c+dx)}}-\frac{\sqrt{2}\left(36a^4+164a^2b^2+605\right.}{}\right.\right.}{}}$$

$$\frac{3\tan(c+dx)\sec(c+dx)(a+b\sec(c+dx))^{8/3}}{14bd}$$

input
```
Int[Sec[c + d*x]^4*(a + b*Sec[c + d*x])^(5/3),x]
```

3.694. $\int \sec^4(c+dx)(a+b\sec(c+dx))^{5/3}\,dx$

output
```
(3*Sec[c + d*x]*(a + b*Sec[c + d*x])^(8/3)*Tan[c + d*x])/(14*b*d) + ((-18*
a*(a + b*Sec[c + d*x])^(8/3)*Tan[c + d*x])/(11*b*d) - ((-3*(18*a^2 + 121*b
^2)*(a + b*Sec[c + d*x])^(5/3)*Tan[c + d*x])/(8*d) + (5*((-3*a*(18*a^2 + 9
7*b^2)*(a + b*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(5*d) + (-((Sqrt[2]*(36*a^
4 + 164*a^2*b^2 + 605*b^4)*AppellF1[1/2, 1/2, -2/3, 3/2, (1 - Sec[c + d*x]
)/2, (b*(1 - Sec[c + d*x]))/(a + b)]*(a + b*Sec[c + d*x])^(2/3)*Tan[c + d*
x])/(b*d*Sqrt[1 + Sec[c + d*x]]*((a + b*Sec[c + d*x])/(a + b))^(2/3))) + (
2*Sqrt[2]*a*(18*a^4 + 79*a^2*b^2 - 97*b^4)*AppellF1[1/2, 1/2, 1/3, 3/2, (1
 - Sec[c + d*x])/2, (b*(1 - Sec[c + d*x]))/(a + b)]*((a + b*Sec[c + d*x])/
(a + b))^(1/3)*Tan[c + d*x])/(b*d*Sqrt[1 + Sec[c + d*x]]*(a + b*Sec[c + d*
x])^(1/3)))/5))/8)/(11*b))/(14*b)
```

3.694.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 155
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*Simplify[b/(b*c - a*d)]^n*
Simplify[b/(b*e - a*f)]^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/
(b*c - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f,
 m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[Sim
plify[b/(b*c - a*d)], 0] && GtQ[Simplify[b/(b*e - a*f)], 0] &&  !(GtQ[Simpl
ify[d/(d*a - c*b)], 0] && GtQ[Simplify[d/(d*e - c*f)], 0] && SimplerQ[c + d
*x, a + b*x]) &&  !(GtQ[Simplify[f/(f*a - e*b)], 0] && GtQ[Simplify[f/(f*c
- e*d)], 0] && SimplerQ[e + f*x, a + b*x])
```

rule 156
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(e + f*x)^FracPart[p]/(Simplify[b/(b*e - a*f)]^IntPart[p
]*(b*((e + f*x)/(b*e - a*f)))^FracPart[p])   Int[(a + b*x)^m*(c + d*x)^n*Si
mp[b*(e/(b*e - a*f)) + b*f*(x/(b*e - a*f)), x]^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] &
& GtQ[Simplify[b/(b*c - a*d)], 0] &&  !GtQ[Simplify[b/(b*e - a*f)], 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4321
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[Cot[e + f*x]/(f*Sqrt[1 + Csc[e + f*x]]*Sqrt[1 - Csc[e + f*x
]])   Subst[Int[(a + b*x)^m/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Csc[e + f*x]]
, x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*m]
```

rule 4352
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-d^3)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m +
1)*((d*Csc[e + f*x])^(n - 3)/(b*f*(m + n - 1))), x] + Simp[d^3/(b*(m + n -
1))   Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 3)*Simp[a*(n - 3) +
b*(m + n - 2)*Csc[e + f*x] - a*(n - 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[
{a, b, d, e, f, m}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 3] && (IntegerQ[n] ||
 IntegersQ[2*m, 2*n]) &&  !IGtQ[m, 2]
```

rule 4490
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*((
a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Simp[1/(m + 1)   Int[Csc[e + f*x]*
(a + b*Csc[e + f*x])^(m - 1)*Simp[b*B*m + a*A*(m + 1) + (a*B*m + A*b*(m + 1
))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*
B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

rule 4495
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b - a*B)/b   Int[
Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] + Simp[B/b   Int[Csc[e + f*x]*(
a + b*Csc[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, A, B, e, f, m}, x] && N
eQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

rule 4570
```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> Simp[(-C)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2)
)), x] + Simp[1/(b*(m + 2))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[
b*A*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /;
 FreeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

3.694. $\int \sec^4(c+dx)(a+b\sec(c+dx))^{5/3}\,dx$

3.694.4 Maple [F]

$$\int \sec(dx+c)^4 (a+b\sec(dx+c))^{\frac{5}{3}}\,dx$$

input
```
int(sec(d*x+c)^4*(a+b*sec(d*x+c))^(5/3),x)
```

output
```
int(sec(d*x+c)^4*(a+b*sec(d*x+c))^(5/3),x)
```

3.694.5 Fricas [F]

$$\int \sec^4(c+dx)(a+b\sec(c+dx))^{5/3}\,dx = \int (b\sec(dx+c)+a)^{\frac{5}{3}}\sec(dx+c)^4\,dx$$

input
```
integrate(sec(d*x+c)^4*(a+b*sec(d*x+c))^(5/3),x, algorithm="fricas")
```

output
```
integral((b*sec(d*x + c)^5 + a*sec(d*x + c)^4)*(b*sec(d*x + c) + a)^(2/3),
 x)
```

3.694.6 Sympy [F(-1)]

Timed out.

$$\int \sec^4(c+dx)(a+b\sec(c+dx))^{5/3}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**4*(a+b*sec(d*x+c))**(5/3),x)
```

output
```
Timed out
```

3.694.7 Maxima [F]

$$\int \sec^4(c+dx)(a+b\sec(c+dx))^{5/3}\,dx = \int (b\sec(dx+c)+a)^{\frac{5}{3}}\sec(dx+c)^4\,dx$$

input
```
integrate(sec(d*x+c)^4*(a+b*sec(d*x+c))^(5/3),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^(5/3)*sec(d*x + c)^4, x)
```

3.694.8 Giac [F]

$$\int \sec^4(c+dx)(a+b\sec(c+dx))^{5/3}\,dx = \int (b\sec(dx+c)+a)^{\frac{5}{3}}\sec(dx+c)^4\,dx$$

input
```
integrate(sec(d*x+c)^4*(a+b*sec(d*x+c))^(5/3),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^(5/3)*sec(d*x + c)^4, x)
```

3.694.9 Mupad [F(-1)]

Timed out.

$$\int \sec^4(c+dx)(a+b\sec(c+dx))^{5/3}\,dx = \int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/3}}{\cos(c+d\,x)^4}\,dx$$

input
```
int((a + b/cos(c + d*x))^(5/3)/cos(c + d*x)^4,x)
```

output
```
int((a + b/cos(c + d*x))^(5/3)/cos(c + d*x)^4, x)
```

3.695 $\int \sec^3(c+dx)(a+b\sec(c+dx))^{5/3}\,dx$

3.695.1 Optimal result . . . 5410
3.695.2 Mathematica [B] (warning: unable to verify) . . . 5411
3.695.3 Rubi [A] (verified) . . . 5411
3.695.4 Maple [F] . . . 5416
3.695.5 Fricas [F] . . . 5416
3.695.6 Sympy [F(-1)] . . . 5417
3.695.7 Maxima [F] . . . 5417
3.695.8 Giac [F] . . . 5417
3.695.9 Mupad [F(-1)] . . . 5418

3.695.1 Optimal result

Integrand size = 23, antiderivative size = 356

$$\int \sec^3(c+dx)(a+b\sec(c+dx))^{5/3}\,dx = -\frac{3(15a^2-64b^2)\,(a+b\sec(c+dx))^{2/3}\tan(c+dx)}{440bd} - \frac{9a(a+b\sec(c+dx))^{5/3}\tan(c+dx)}{88bd} + \frac{3(a+b\sec(c+dx))^{8/3}\tan(c+dx)}{11bd} - \frac{a(30a^2-373b^2)\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},-\frac{2}{3},\frac{3}{2},\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)(a+b\sec(c+dx))^{2/3}\tan(c+dx}{220\sqrt{2}b^2d\sqrt{1+\sec(c+dx)}\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{2/3}} + \frac{(15a^4-79a^2b^2+64b^4)\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},\frac{1}{3},\frac{3}{2},\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)\sqrt[3]{\frac{a+b\sec(c+dx)}{a+b}}\tan(c+}{110\sqrt{2}b^2d\sqrt{1+\sec(c+dx)}\sqrt[3]{a+b\sec(c+dx)}}$$

output
```
-3/440*(15*a^2-64*b^2)*(a+b*sec(d*x+c))^(2/3)*tan(d*x+c)/b/d-9/88*a*(a+b*s
ec(d*x+c))^(5/3)*tan(d*x+c)/b/d+3/11*(a+b*sec(d*x+c))^(8/3)*tan(d*x+c)/b/d
-1/440*a*(30*a^2-373*b^2)*AppellF1(1/2,-2/3,1/2,3/2,b*(1-sec(d*x+c))/(a+b)
,1/2-1/2*sec(d*x+c))*(a+b*sec(d*x+c))^(2/3)*tan(d*x+c)/b^2/d/((a+b*sec(d*x
+c))/(a+b))^(2/3)*2^(1/2)/(1+sec(d*x+c))^(1/2)+1/220*(15*a^4-79*a^2*b^2+64
*b^4)*AppellF1(1/2,1/3,1/2,3/2,b*(1-sec(d*x+c))/(a+b),1/2-1/2*sec(d*x+c))*
((a+b*sec(d*x+c))/(a+b))^(1/3)*tan(d*x+c)/b^2/d/(a+b*sec(d*x+c))^(1/3)*2^(
1/2)/(1+sec(d*x+c))^(1/2)
```

3.695.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 21890 vs. $2(356) = 712$.

Time = 46.60 (sec) , antiderivative size = 21890, normalized size of antiderivative = 61.49

$$\int \sec^3(c+dx)(a+b\sec(c+dx))^{5/3}\,dx = \text{Result too large to show}$$

input
```
Integrate[Sec[c + d*x]^3*(a + b*Sec[c + d*x])^(5/3),x]
```

output
```
Result too large to show
```

3.695.3 Rubi [A] (verified)

Time = 1.23 (sec) , antiderivative size = 364, normalized size of antiderivative = 1.02, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.652$, Rules used = {3042, 4327, 27, 3042, 4490, 27, 3042, 4490, 27, 3042, 4495, 3042, 4321, 156, 155}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^3(c+dx)(a+b\sec(c+dx))^{5/3}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^3\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/3}dx$$

$$\Big\downarrow \; 4327$$

$$\frac{3\int \frac{1}{3}\sec(c+dx)(8b-3a\sec(c+dx))(a+b\sec(c+dx))^{5/3}dx}{11b}+\frac{3\tan(c+dx)(a+b\sec(c+dx))^{8/3}}{11bd}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \sec(c+dx)(8b-3a\sec(c+dx))(a+b\sec(c+dx))^{5/3}dx}{11b}+\frac{3\tan(c+dx)(a+b\sec(c+dx))^{8/3}}{11bd}$$

$$\Big\downarrow \; 3042$$

$$\frac{\int \csc\left(c+dx+\frac{\pi}{2}\right)\left(8b-3a\csc\left(c+dx+\frac{\pi}{2}\right)\right)\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/3}dx}{11b}+\frac{3\tan(c+dx)(a+b\sec(c+dx))^{8/3}}{11bd}$$

$\Big\downarrow$ 4490

$$\frac{\frac{3}{8}\int \frac{1}{3}\sec(c+dx)(a+b\sec(c+dx))^{2/3}\left(49ab-\left(15a^2-64b^2\right)\sec(c+dx)\right)dx-\frac{9a\tan(c+dx)(a+b\sec(c+dx))^{5/3}}{8d}}{11b}+\frac{3\tan(c+dx)(a+b\sec(c+dx))^{8/3}}{11bd}$$

$\Big\downarrow$ 27

$$\frac{\frac{1}{8}\int \sec(c+dx)(a+b\sec(c+dx))^{2/3}\left(49ab-\left(15a^2-64b^2\right)\sec(c+dx)\right)dx-\frac{9a\tan(c+dx)(a+b\sec(c+dx))^{5/3}}{8d}}{11b}+\frac{3\tan(c+dx)(a+b\sec(c+dx))^{8/3}}{11bd}$$

$\Big\downarrow$ 3042

$$\frac{\frac{1}{8}\int \csc\left(c+dx+\frac{\pi}{2}\right)\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{2/3}\left(49ab+\left(64b^2-15a^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)\right)dx-\frac{9a\tan(c+dx)(a+b\sec(c}{8d}}{11b}$$
$$\frac{3\tan(c+dx)(a+b\sec(c+dx))^{8/3}}{11bd}$$

$\Big\downarrow$ 4490

$$\frac{\frac{1}{8}\left(\frac{3}{5}\int \frac{\sec(c+dx)\left(b\left(215a^2+128b^2\right)-a\left(30a^2-373b^2\right)\sec(c+dx)\right)}{3\sqrt[3]{a+b\sec(c+dx)}}dx-\frac{3\left(15a^2-64b^2\right)\tan(c+dx)(a+b\sec(c+dx))^{2/3}}{5d}\right)-\frac{9a\tan(c+dx)(a+b\sec}{8d}}{11b}$$
$$\frac{3\tan(c+dx)(a+b\sec(c+dx))^{8/3}}{11bd}$$

$\Big\downarrow$ 27

$$\frac{\frac{1}{8}\left(\frac{1}{5}\int \frac{\sec(c+dx)\left(b\left(215a^2+128b^2\right)-a\left(30a^2-373b^2\right)\sec(c+dx)\right)}{\sqrt[3]{a+b\sec(c+dx)}}dx-\frac{3\left(15a^2-64b^2\right)\tan(c+dx)(a+b\sec(c+dx))^{2/3}}{5d}\right)-\frac{9a\tan(c+dx)(a+b\sec}{8d}}{11b}$$
$$\frac{3\tan(c+dx)(a+b\sec(c+dx))^{8/3}}{11bd}$$

$\Big\downarrow$ 3042

3.695. $\int \sec^3(c+dx)(a+b\sec(c+dx))^{5/3}\,dx$

$$\frac{\frac{1}{8}\left(\frac{1}{5}\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(b\left(215a^2+128b^2\right)-a\left(30a^2-373b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt[3]{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{3\left(15a^2-64b^2\right)\tan(c+dx)(a+b\sec(c+dx))^{2/3}}{5d}\right)-\frac{9a\tan(c+d}{11b}}{\frac{3\tan(c+dx)(a+b\sec(c+dx))^{8/3}}{11bd}}$$

$$\Big\downarrow \; 4495$$

$$\frac{\frac{1}{8}\left(\frac{1}{5}\left(\frac{2\left(15a^4-79a^2b^2+64b^4\right)\int \frac{\sec(c+dx)}{\sqrt[3]{a+b\sec(c+dx)}}dx}{b}-\frac{a\left(30a^2-373b^2\right)\int \sec(c+dx)(a+b\sec(c+dx))^{2/3}dx}{b}\right)-\frac{3\left(15a^2-64b^2\right)\tan(c-}{11b}\right)}{\frac{3\tan(c+dx)(a+b\sec(c+dx))^{8/3}}{11bd}}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{1}{8}\left(\frac{1}{5}\left(\frac{2\left(15a^4-79a^2b^2+64b^4\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt[3]{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\frac{a\left(30a^2-373b^2\right)\int \csc\left(c+dx+\frac{\pi}{2}\right)\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{2/3}dx}{b}\right)-\frac{3(15}{11b}\right)}{\frac{3\tan(c+dx)(a+b\sec(c+dx))^{8/3}}{11bd}}$$

$$\Big\downarrow \; 4321$$

$$\frac{\frac{1}{8}\left(\frac{1}{5}\left(\frac{a\left(30a^2-373b^2\right)\tan(c+dx)\int \frac{(a+b\sec(c+dx))^{2/3}}{\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}}d\sec(c+dx)}{bd\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}}-\frac{2\left(15a^4-79a^2b^2+64b^4\right)\tan(c+dx)\int \frac{1}{\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}}}{bd\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)}}\right)}{11b}}{\frac{3\tan(c+dx)(a+b\sec(c+dx))^{8/3}}{11bd}}$$

$$\Big\downarrow \; 156$$

3.695. $\int \sec^3(c+dx)(a+b\sec(c+dx))^{5/3}\,dx$

$$\frac{\frac{1}{8}\left(\frac{1}{5}\left(\frac{a(30a^2-373b^2)\tan(c+dx)(a+b\sec(c+dx))^{2/3}\int\frac{\left(\frac{a}{a+b}+\frac{b\sec(c+dx)}{a+b}\right)^{2/3}}{\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}}d\sec(c+dx)}{bd\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{2/3}}-\frac{2(15a^4-79a^2b^2+64b^4)\tan(c+dx)\sqrt[3]{\frac{a+b}{}}}{bd\sqrt{1-}}\right.\right.}{\frac{3\tan(c+dx)(a+b\sec(c+dx))^{8/3}}{11bd}}$$

$$\Big\downarrow \; 155$$

$$\frac{\frac{1}{8}\left(\frac{1}{5}\left(\frac{2\sqrt{2}(15a^4-79a^2b^2+64b^4)\tan(c+dx)\sqrt[3]{\frac{a+b\sec(c+dx)}{a+b}}\operatorname{AppellF1}\left(\frac{1}{2},\frac{1}{2},\frac{1}{3},\frac{3}{2},\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)}{bd\sqrt{\sec(c+dx)+1}\sqrt[3]{a+b\sec(c+dx)}}-\frac{\sqrt{2}a(30a^2-373}{}\right.\right.}{\frac{3\tan(c+dx)(a+b\sec(c+dx))^{8/3}}{11bd}}$$

input
```
Int[Sec[c + d*x]^3*(a + b*Sec[c + d*x])^(5/3),x]
```

output
```
(3*(a + b*Sec[c + d*x])^(8/3)*Tan[c + d*x])/(11*b*d) + ((-9*a*(a + b*Sec[c
 + d*x])^(5/3)*Tan[c + d*x])/(8*d) + ((-3*(15*a^2 - 64*b^2)*(a + b*Sec[c +
 d*x])^(2/3)*Tan[c + d*x])/(5*d) + (-((Sqrt[2]*a*(30*a^2 - 373*b^2)*Appell
F1[1/2, 1/2, -2/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[c + d*x]))/(a +
b)]*(a + b*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(b*d*Sqrt[1 + Sec[c + d*x]]*(
(a + b*Sec[c + d*x])/(a + b))^(2/3))) + (2*Sqrt[2]*(15*a^4 - 79*a^2*b^2 +
64*b^4)*AppellF1[1/2, 1/2, 1/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[c +
 d*x]))/(a + b)]*((a + b*Sec[c + d*x])/(a + b))^(1/3)*Tan[c + d*x])/(b*d*S
qrt[1 + Sec[c + d*x]]*(a + b*Sec[c + d*x])^(1/3)))/5)/8)/(11*b)
```

3.695. $\int \sec^3(c+dx)(a+b\sec(c+dx))^{5/3}\,dx$

3.695.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 155
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*Simplify[b/(b*c - a*d)]^n*
Simplify[b/(b*e - a*f)]^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/
(b*c - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f,
 m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[Sim
plify[b/(b*c - a*d)], 0] && GtQ[Simplify[b/(b*e - a*f)], 0] &&  !(GtQ[Simpl
ify[d/(d*a - c*b)], 0] && GtQ[Simplify[d/(d*e - c*f)], 0] && SimplerQ[c + d
*x, a + b*x]) &&  !(GtQ[Simplify[f/(f*a - e*b)], 0] && GtQ[Simplify[f/(f*c
- e*d)], 0] && SimplerQ[e + f*x, a + b*x])
```

rule 156
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(e + f*x)^FracPart[p]/(Simplify[b/(b*e - a*f)]^IntPart[p
]*(b*((e + f*x)/(b*e - a*f)))^FracPart[p])   Int[(a + b*x)^m*(c + d*x)^n*Si
mp[b*(e/(b*e - a*f)) + b*f*(x/(b*e - a*f)), x]^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] &
& GtQ[Simplify[b/(b*c - a*d)], 0] &&  !GtQ[Simplify[b/(b*e - a*f)], 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4321
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[Cot[e + f*x]/(f*Sqrt[1 + Csc[e + f*x]]*Sqrt[1 - Csc[e + f*x
]])   Subst[Int[(a + b*x)^m/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Csc[e + f*x]]
, x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*m]
```

rule 4327
```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(-Cot[e + f*x])*((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2
))), x] + Simp[1/(b*(m + 2))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(b*(
m + 1) - a*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 -
b^2, 0] &&  !LtQ[m, -1]
```

3.695. $\int \sec^3(c+dx)(a+b\sec(c+dx))^{5/3}\,dx$

rule 4490
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*((
a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Simp[1/(m + 1)   Int[Csc[e + f*x]*
(a + b*Csc[e + f*x])^(m - 1)*Simp[b*B*m + a*A*(m + 1) + (a*B*m + A*b*(m + 1
))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*
B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

rule 4495
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b - a*B)/b   Int[
Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] + Simp[B/b   Int[Csc[e + f*x]*(
a + b*Csc[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, A, B, e, f, m}, x] && N
eQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

3.695.4 Maple [F]

$$\int \sec (dx+c)^3 (a+b\sec (dx+c))^{\frac{5}{3}} dx$$

input
```
int(sec(d*x+c)^3*(a+b*sec(d*x+c))^(5/3),x)
```

output
```
int(sec(d*x+c)^3*(a+b*sec(d*x+c))^(5/3),x)
```

3.695.5 Fricas [F]

$$\int \sec^3(c+dx)(a+b\sec(c+dx))^{5/3}\,dx = \int (b\sec (dx+c)+a)^{\frac{5}{3}} \sec (dx+c)^3 \, dx$$

input
```
integrate(sec(d*x+c)^3*(a+b*sec(d*x+c))^(5/3),x, algorithm="fricas")
```

output
```
integral((b*sec(d*x + c)^4 + a*sec(d*x + c)^3)*(b*sec(d*x + c) + a)^(2/3),
 x)
```

3.695.6 Sympy [F(-1)]

Timed out.

$$\int \sec^3(c+dx)(a+b\sec(c+dx))^{5/3}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**3*(a+b*sec(d*x+c))**(5/3),x)
```

output
```
Timed out
```

3.695.7 Maxima [F]

$$\int \sec^3(c+dx)(a+b\sec(c+dx))^{5/3}\,dx = \int (b\sec(dx+c)+a)^{\frac{5}{3}}\sec(dx+c)^3\,dx$$

input
```
integrate(sec(d*x+c)^3*(a+b*sec(d*x+c))^(5/3),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^(5/3)*sec(d*x + c)^3, x)
```

3.695.8 Giac [F]

$$\int \sec^3(c+dx)(a+b\sec(c+dx))^{5/3}\,dx = \int (b\sec(dx+c)+a)^{\frac{5}{3}}\sec(dx+c)^3\,dx$$

input
```
integrate(sec(d*x+c)^3*(a+b*sec(d*x+c))^(5/3),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^(5/3)*sec(d*x + c)^3, x)
```

3.695.9 Mupad [F(-1)]

Timed out.

$$\int \sec^3(c+dx)(a+b\sec(c+dx))^{5/3}\,dx = \int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/3}}{\cos\left(c+d\,x\right)^3}\,dx$$

input
```
int((a + b/cos(c + d*x))^(5/3)/cos(c + d*x)^3,x)
```

output
```
int((a + b/cos(c + d*x))^(5/3)/cos(c + d*x)^3, x)
```

3.696 $\int \sec^2(c+dx)(a+b\sec(c+dx))^{5/3}\,dx$

3.696.1 Optimal result 5419
3.696.2 Mathematica [B] (warning: unable to verify) 5420
3.696.3 Rubi [A] (verified) 5420
3.696.4 Maple [F] 5424
3.696.5 Fricas [F] 5424
3.696.6 Sympy [F] 5425
3.696.7 Maxima [F] 5425
3.696.8 Giac [F] 5425
3.696.9 Mupad [F(-1)] 5426

3.696.1 Optimal result

Integrand size = 23, antiderivative size = 299

$$\int \sec^2(c+dx)(a+b\sec(c+dx))^{5/3}\,dx = \frac{3a(a+b\sec(c+dx))^{2/3}\tan(c+dx)}{8d} + \frac{3(a+b\sec(c+dx))^{5/3}\tan(c+dx)}{8d} + \frac{(2a^2+5b^2)\operatorname{AppellF1}\left(\frac{1}{2},\frac{1}{2},-\frac{2}{3},\frac{3}{2},\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)(a+b\sec(c+dx))^{2/3}\tan(c+dx)}{4\sqrt{2}bd\sqrt{1+\sec(c+dx)}\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{2/3}} - \frac{a(a^2-b^2)\operatorname{AppellF1}\left(\frac{1}{2},\frac{1}{2},\frac{1}{3},\frac{3}{2},\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)\sqrt[3]{\frac{a+b\sec(c+dx)}{a+b}}\tan(c+dx)}{2\sqrt{2}bd\sqrt{1+\sec(c+dx)}\sqrt[3]{a+b\sec(c+dx)}}$$

output
```
3/8*a*(a+b*sec(d*x+c))^(2/3)*tan(d*x+c)/d+3/8*(a+b*sec(d*x+c))^(5/3)*tan(d
*x+c)/d+1/8*(2*a^2+5*b^2)*AppellF1(1/2,-2/3,1/2,3/2,b*(1-sec(d*x+c))/(a+b)
,1/2-1/2*sec(d*x+c))*(a+b*sec(d*x+c))^(2/3)*tan(d*x+c)/b/d/((a+b*sec(d*x+c
))/(a+b))^(2/3)*2^(1/2)/(1+sec(d*x+c))^(1/2)-1/4*a*(a^2-b^2)*AppellF1(1/2,
1/3,1/2,3/2,b*(1-sec(d*x+c))/(a+b),1/2-1/2*sec(d*x+c))*((a+b*sec(d*x+c))/(
a+b))^(1/3)*tan(d*x+c)/b/d/(a+b*sec(d*x+c))^(1/3)*2^(1/2)/(1+sec(d*x+c))^(
1/2)
```

3.696.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 7809 vs. $2(299) = 598$.

Time = 45.02 (sec) , antiderivative size = 7809, normalized size of antiderivative = 26.12

$$\int \sec^2(c+dx)(a+b\sec(c+dx))^{5/3}\,dx = \text{Result too large to show}$$

input
```
Integrate[Sec[c + d*x]^2*(a + b*Sec[c + d*x])^(5/3),x]
```

output
```
Result too large to show
```

3.696.3 Rubi [A] (verified)

Time = 0.92 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.02, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules used = {3042, 4322, 3042, 4490, 27, 3042, 4495, 3042, 4321, 156, 155}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(c+dx)(a+b\sec(c+dx))^{5/3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^2\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/3}dx$$

$$\Big\downarrow \text{4322}$$

$$\frac{5}{8}\int \sec(c+dx)(b+a\sec(c+dx))(a+b\sec(c+dx))^{2/3}dx+\frac{3\tan(c+dx)(a+b\sec(c+dx))^{5/3}}{8d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5}{8}\int \csc\left(c+dx+\frac{\pi}{2}\right)\left(b+a\csc\left(c+dx+\frac{\pi}{2}\right)\right)\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{2/3}dx+\frac{3\tan(c+dx)(a+b\sec(c+dx))^{5/3}}{8d}$$

$$\Big\downarrow \text{4490}$$

$$\frac{5}{8}\left(\frac{3}{5}\int \frac{\sec(c+dx)\left(7ab+\left(2a^2+5b^2\right)\sec(c+dx)\right)}{3\sqrt[3]{a+b\sec(c+dx)}}dx+\frac{3a\tan(c+dx)(a+b\sec(c+dx))^{2/3}}{5d}\right)+\frac{3\tan(c+dx)(a+b\sec(c+dx))^{5/3}}{8d}$$

$$\Big\downarrow \; 27$$

$$\frac{5}{8}\left(\frac{1}{5}\int \frac{\sec(c+dx)\left(7ab+\left(2a^2+5b^2\right)\sec(c+dx)\right)}{\sqrt[3]{a+b\sec(c+dx)}}dx+\frac{3a\tan(c+dx)(a+b\sec(c+dx))^{2/3}}{5d}\right)+\frac{3\tan(c+dx)(a+b\sec(c+dx))^{5/3}}{8d}$$

$$\Big\downarrow \; 3042$$

$$\frac{5}{8}\left(\frac{1}{5}\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(7ab+\left(2a^2+5b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt[3]{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{3a\tan(c+dx)(a+b\sec(c+dx))^{2/3}}{5d}\right)+\frac{3\tan(c+dx)(a+b\sec(c+dx))^{5/3}}{8d}$$

$$\Big\downarrow \; 4495$$

$$\frac{5}{8}\left(\frac{1}{5}\left(\frac{\left(2a^2+5b^2\right)\int \sec(c+dx)(a+b\sec(c+dx))^{2/3}dx}{b}-\frac{2a\left(a^2-b^2\right)\int \frac{\sec(c+dx)}{\sqrt[3]{a+b\sec(c+dx)}}dx}{b}\right)+\frac{3a\tan(c+\right.$$

$$\frac{3\tan(c+dx)(a+b\sec(c+dx))^{5/3}}{8d}$$

$$\Big\downarrow \; 3042$$

$$\frac{5}{8}\left(\frac{1}{5}\left(\frac{\left(2a^2+5b^2\right)\int \csc\left(c+dx+\frac{\pi}{2}\right)\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{2/3}dx}{b}-\frac{2a\left(a^2-b^2\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt[3]{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}d}{b}\right.\right.$$

$$\frac{3\tan(c+dx)(a+b\sec(c+dx))^{5/3}}{8d}$$

$$\Big\downarrow \; 4321$$

3.696. $\int \sec^2(c+dx)(a+b\sec(c+dx))^{5/3}\,dx$

$$\frac{5}{8}\left(\frac{1}{5}\left(\frac{2a(a^2-b^2)\tan(c+dx)\int\frac{1}{\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}\sqrt[3]{a+b\sec(c+dx)}}d\sec(c+dx)}{bd\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}}-\frac{(2a^2+5b^2)\tan(c+}{bd\sqrt{1}}\right.\right.$$

$$\frac{3\tan(c+dx)(a+b\sec(c+dx))^{5/3}}{8d}$$

$\downarrow$ 156

$$\frac{5}{8}\left(\frac{1}{5}\left(\frac{2a(a^2-b^2)\tan(c+dx)\sqrt[3]{\frac{a+b\sec(c+dx)}{a+b}}\int\frac{1}{\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}\sqrt[3]{\frac{a}{a+b}+\frac{b\sec(c+dx)}{a+b}}}d\sec(c+d}{bd\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}\sqrt[3]{a+b\sec(c+dx)}}\right.\right.$$

$$\frac{3\tan(c+dx)(a+b\sec(c+dx))^{5/3}}{8d}$$

$\downarrow$ 155

$$\frac{5}{8}\left(\frac{1}{5}\left(\frac{\sqrt{2}(2a^2+5b^2)\tan(c+dx)(a+b\sec(c+dx))^{2/3}\operatorname{AppellF1}\left(\frac{1}{2},\frac{1}{2},-\frac{2}{3},\frac{3}{2},\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)}{bd\sqrt{\sec(c+dx)+1}\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{2/3}}\right.\right.$$

$$\frac{3\tan(c+dx)(a+b\sec(c+dx))^{5/3}}{8d}$$

input
```
Int[Sec[c + d*x]^2*(a + b*Sec[c + d*x])^(5/3),x]
```

output
```
(3*(a + b*Sec[c + d*x])^(5/3)*Tan[c + d*x])/(8*d) + (5*((3*a*(a + b*Sec[c
+ d*x])^(2/3)*Tan[c + d*x])/(5*d) + ((Sqrt[2]*(2*a^2 + 5*b^2)*AppellF1[1/2
, 1/2, -2/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[c + d*x]))/(a + b)]*(a
 + b*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(b*d*Sqrt[1 + Sec[c + d*x]]*((a + b
*Sec[c + d*x])/(a + b))^(2/3)) - (2*Sqrt[2]*a*(a^2 - b^2)*AppellF1[1/2, 1/
2, 1/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[c + d*x]))/(a + b)]*((a + b
*Sec[c + d*x])/(a + b))^(1/3)*Tan[c + d*x])/(b*d*Sqrt[1 + Sec[c + d*x]]*(a
 + b*Sec[c + d*x])^(1/3)))/5))/8
```

3.696. $\int \sec^2(c+dx)(a+b\sec(c+dx))^{5/3}\,dx$

3.696.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 155
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*Simplify[b/(b*c - a*d)]^n*
Simplify[b/(b*e - a*f)]^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/
(b*c - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f,
 m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[Sim
plify[b/(b*c - a*d)], 0] && GtQ[Simplify[b/(b*e - a*f)], 0] &&  !(GtQ[Simpl
ify[d/(d*a - c*b)], 0] && GtQ[Simplify[d/(d*e - c*f)], 0] && SimplerQ[c + d
*x, a + b*x]) &&  !(GtQ[Simplify[f/(f*a - e*b)], 0] && GtQ[Simplify[f/(f*c
- e*d)], 0] && SimplerQ[e + f*x, a + b*x])
```

rule 156
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(e + f*x)^FracPart[p]/(Simplify[b/(b*e - a*f)]^IntPart[p
]*(b*((e + f*x)/(b*e - a*f)))^FracPart[p])   Int[(a + b*x)^m*(c + d*x)^n*Si
mp[b*(e/(b*e - a*f)) + b*f*(x/(b*e - a*f)), x]^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] &
& GtQ[Simplify[b/(b*c - a*d)], 0] &&  !GtQ[Simplify[b/(b*e - a*f)], 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4321
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[Cot[e + f*x]/(f*Sqrt[1 + Csc[e + f*x]]*Sqrt[1 - Csc[e + f*x
]])   Subst[Int[(a + b*x)^m/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Csc[e + f*x]]
, x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*m]
```

rule 4322
```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(-Cot[e + f*x])*((a + b*Csc[e + f*x])^m/(f*(m + 1))), x]
+ Simp[m/(m + 1)   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(b + a*Csc
[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m,
 0]
```

3.696. $\int \sec^2(c+dx)(a+b\sec(c+dx))^{5/3}\,dx$

rule 4490
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*((
a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Simp[1/(m + 1)   Int[Csc[e + f*x]*
(a + b*Csc[e + f*x])^(m - 1)*Simp[b*B*m + a*A*(m + 1) + (a*B*m + A*b*(m + 1
))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*
B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

rule 4495
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b - a*B)/b   Int[
Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] + Simp[B/b   Int[Csc[e + f*x]*(
a + b*Csc[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, A, B, e, f, m}, x] && N
eQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

3.696.4 Maple [F]

$$\int \sec (dx+c)^2 (a+b\sec (dx+c))^{\frac{5}{3}} dx$$

input
```
int(sec(d*x+c)^2*(a+b*sec(d*x+c))^(5/3),x)
```

output
```
int(sec(d*x+c)^2*(a+b*sec(d*x+c))^(5/3),x)
```

3.696.5 Fricas [F]

$$\int \sec^2(c+dx)(a+b\sec(c+dx))^{5/3}\,dx=\int (b\sec (dx+c)+a)^{\frac{5}{3}}\sec (dx+c)^2\,dx$$

input
```
integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^(5/3),x, algorithm="fricas")
```

output
```
integral((b*sec(d*x + c)^3 + a*sec(d*x + c)^2)*(b*sec(d*x + c) + a)^(2/3),
 x)
```

3.696.6 Sympy [F]

$$\int \sec^2(c+dx)(a+b\sec(c+dx))^{5/3}\,dx = \int (a+b\sec(c+dx))^{\frac{5}{3}}\sec^2(c+dx)\,dx$$

input
```
integrate(sec(d*x+c)**2*(a+b*sec(d*x+c))**(5/3),x)
```

output
```
Integral((a + b*sec(c + d*x))**(5/3)*sec(c + d*x)**2, x)
```

3.696.7 Maxima [F]

$$\int \sec^2(c+dx)(a+b\sec(c+dx))^{5/3}\,dx = \int (b\sec(dx+c)+a)^{\frac{5}{3}}\sec(dx+c)^2\,dx$$

input
```
integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^(5/3),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^(5/3)*sec(d*x + c)^2, x)
```

3.696.8 Giac [F]

$$\int \sec^2(c+dx)(a+b\sec(c+dx))^{5/3}\,dx = \int (b\sec(dx+c)+a)^{\frac{5}{3}}\sec(dx+c)^2\,dx$$

input
```
integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^(5/3),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^(5/3)*sec(d*x + c)^2, x)
```

3.696.9 Mupad [F(-1)]

Timed out.

$$\int \sec^2(c+dx)(a+b\sec(c+dx))^{5/3}\,dx = \int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/3}}{\cos\left(c+d\,x\right)^2}\,dx$$

input
```
int((a + b/cos(c + d*x))^(5/3)/cos(c + d*x)^2,x)
```

output
```
int((a + b/cos(c + d*x))^(5/3)/cos(c + d*x)^2, x)
```

3.697 $\int \sec(c+dx)(a+b\sec(c+dx))^{5/3}\,dx$

3.697.1 Optimal result 5427
3.697.2 Mathematica [B] (warning: unable to verify) 5427
3.697.3 Rubi [A] (verified) 5428
3.697.4 Maple [F] 5429
3.697.5 Fricas [F] 5430
3.697.6 Sympy [F] 5430
3.697.7 Maxima [F] 5430
3.697.8 Giac [F] 5431
3.697.9 Mupad [F(-1)] 5431

3.697.1 Optimal result

Integrand size = 21, antiderivative size = 108

$$\int \sec(c+dx)(a+b\sec(c+dx))^{5/3}\,dx=\frac{\sqrt{2}(a+b)\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},-\frac{5}{3},\frac{3}{2},\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)(a+b\sec(c+dx))^{2/3}\tan(}{d\sqrt{1+\sec(c+dx)}\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{2/3}}$$

output
```
(a+b)*AppellF1(1/2,-5/3,1/2,3/2,b*(1-sec(d*x+c))/(a+b),1/2-1/2*sec(d*x+c))
*(a+b*sec(d*x+c))^(2/3)*2^(1/2)*tan(d*x+c)/d/((a+b*sec(d*x+c))/(a+b))^(2/3
)/(1+sec(d*x+c))^(1/2)
```

3.697.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 7321 vs. 2(108) = 216.

Time = 44.95 (sec) , antiderivative size = 7321, normalized size of antiderivative = 67.79

$$\int \sec(c+dx)(a+b\sec(c+dx))^{5/3}\,dx=\text{Result too large to show}$$

input
```
Integrate[Sec[c + d*x]*(a + b*Sec[c + d*x])^(5/3),x]
```

output
```
Result too large to show
```

3.697.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3042, 4321, 156, 155}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(c+dx)(a+b\sec(c+dx))^{5/3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/3}dx$$

$$\Big\downarrow \text{4321}$$

$$-\frac{\tan(c+dx)\int \frac{(a+b\sec(c+dx))^{5/3}}{\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}}d\sec(c+dx)}{d\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}}$$

$$\Big\downarrow \text{156}$$

$$-\frac{(a+b)\tan(c+dx)(a+b\sec(c+dx))^{2/3}\int \frac{\left(\frac{a}{a+b}+\frac{b\sec(c+dx)}{a+b}\right)^{5/3}}{\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}}d\sec(c+dx)}{d\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{2/3}}$$

$$\Big\downarrow \text{155}$$

$$\frac{\sqrt{2}(a+b)\tan(c+dx)(a+b\sec(c+dx))^{2/3}\operatorname{AppellF1}\left(\frac{1}{2},\frac{1}{2},-\frac{5}{3},\frac{3}{2},\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)}{d\sqrt{\sec(c+dx)+1}\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{2/3}}$$

input
```
Int[Sec[c + d*x]*(a + b*Sec[c + d*x])^(5/3),x]
```

output
```
(Sqrt[2]*(a + b)*AppellF1[1/2, 1/2, -5/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1
 - Sec[c + d*x]))/(a + b)]*(a + b*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(d*Sqr
t[1 + Sec[c + d*x]]*((a + b*Sec[c + d*x])/(a + b))^(2/3))
```

3.697.3.1 Defintions of rubi rules used

rule 155
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*Simplify[b/(b*c - a*d)]^n*
Simplify[b/(b*e - a*f)]^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/
(b*c - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f,
 m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[Sim
plify[b/(b*c - a*d)], 0] && GtQ[Simplify[b/(b*e - a*f)], 0] && !(GtQ[Simpl
ify[d/(d*a - c*b)], 0] && GtQ[Simplify[d/(d*e - c*f)], 0] && SimplerQ[c + d
*x, a + b*x]) && !(GtQ[Simplify[f/(f*a - e*b)], 0] && GtQ[Simplify[f/(f*c
- e*d)], 0] && SimplerQ[e + f*x, a + b*x])
```

rule 156
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(e + f*x)^FracPart[p]/(Simplify[b/(b*e - a*f)]^IntPart[p
]*(b*((e + f*x)/(b*e - a*f)))^FracPart[p])   Int[(a + b*x)^m*(c + d*x)^n*Si
mp[b*(e/(b*e - a*f)) + b*f*(x/(b*e - a*f)), x]^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] &
& GtQ[Simplify[b/(b*c - a*d)], 0] && !GtQ[Simplify[b/(b*e - a*f)], 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4321
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[Cot[e + f*x]/(f*Sqrt[1 + Csc[e + f*x]]*Sqrt[1 - Csc[e + f*x
]])   Subst[Int[(a + b*x)^m/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Csc[e + f*x]]
, x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0] && !IntegerQ[2*m]
```

3.697.4 Maple [F]

$$\int \sec(dx+c)\,(a+b\sec(dx+c))^{\frac{5}{3}}\,dx$$

input
```
int(sec(d*x+c)*(a+b*sec(d*x+c))^(5/3),x)
```

output
```
int(sec(d*x+c)*(a+b*sec(d*x+c))^(5/3),x)
```

3.697.5 Fricas [F]

$$\int \sec(c+dx)(a+b\sec(c+dx))^{5/3}\,dx = \int (b\sec(dx+c)+a)^{\frac{5}{3}}\sec(dx+c)\,dx$$

input
```
integrate(sec(d*x+c)*(a+b*sec(d*x+c))^(5/3),x, algorithm="fricas")
```

output
```
integral((b*sec(d*x + c)^2 + a*sec(d*x + c))*(b*sec(d*x + c) + a)^(2/3), x
)
```

3.697.6 Sympy [F]

$$\int \sec(c+dx)(a+b\sec(c+dx))^{5/3}\,dx = \int (a+b\sec(c+dx))^{\frac{5}{3}}\sec(c+dx)\,dx$$

input
```
integrate(sec(d*x+c)*(a+b*sec(d*x+c))**(5/3),x)
```

output
```
Integral((a + b*sec(c + d*x))**(5/3)*sec(c + d*x), x)
```

3.697.7 Maxima [F]

$$\int \sec(c+dx)(a+b\sec(c+dx))^{5/3}\,dx = \int (b\sec(dx+c)+a)^{\frac{5}{3}}\sec(dx+c)\,dx$$

input
```
integrate(sec(d*x+c)*(a+b*sec(d*x+c))^(5/3),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^(5/3)*sec(d*x + c), x)
```

3.697.8 Giac [F]

$$\int \sec(c+dx)(a+b\sec(c+dx))^{5/3}\,dx = \int (b\sec(dx+c)+a)^{\frac{5}{3}}\sec(dx+c)\,dx$$

input
```
integrate(sec(d*x+c)*(a+b*sec(d*x+c))^(5/3),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^(5/3)*sec(d*x + c), x)
```

3.697.9 Mupad [F(-1)]

Timed out.

$$\int \sec(c+dx)(a+b\sec(c+dx))^{5/3}\,dx = \int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/3}}{\cos(c+d\,x)}\,dx$$

input
```
int((a + b/cos(c + d*x))^(5/3)/cos(c + d*x),x)
```

output
```
int((a + b/cos(c + d*x))^(5/3)/cos(c + d*x), x)
```

3.698 $\int (a+b\sec(c+dx))^{5/3}\,dx$

3.698.1 Optimal result . . . 5432
3.698.2 Mathematica [N/A] . . . 5432
3.698.3 Rubi [N/A] . . . 5433
3.698.4 Maple [N/A] (verified) . . . 5434
3.698.5 Fricas [F(-1)] . . . 5434
3.698.6 Sympy [N/A] . . . 5434
3.698.7 Maxima [N/A] . . . 5435
3.698.8 Giac [N/A] . . . 5435
3.698.9 Mupad [N/A] . . . 5435

3.698.1 Optimal result

Integrand size = 14, antiderivative size = 14

$$\int (a+b\sec(c+dx))^{5/3}\,dx = \text{Int}\left((a+b\sec(c+dx))^{5/3},x\right)$$

output
```
Unintegrable((a+b*sec(d*x+c))^(5/3),x)
```

3.698.2 Mathematica [N/A]

Not integrable

Time = 101.67 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int (a+b\sec(c+dx))^{5/3}\,dx = \int (a+b\sec(c+dx))^{5/3}\,dx$$

input
```
Integrate[(a + b*Sec[c + d*x])^(5/3),x]
```

output
```
Integrate[(a + b*Sec[c + d*x])^(5/3), x]
```

3.698.3 Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 4273}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\sec(c+dx))^{5/3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/3}dx$$

$$\Big\downarrow \text{4273}$$

$$\int (a+b\sec(c+dx))^{5/3}dx$$

input
```
Int[(a + b*Sec[c + d*x])^(5/3),x]
```

output
```
$Aborted
```

3.698.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4273
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Unintegrable[
(a + b*Csc[c + d*x])^n, x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[a^2 - b^2, 0
] &&  !IntegerQ[2*n]
```

3.698.4 Maple [N/A] (verified)

Not integrable

Time = 0.34 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int (a+b\sec(dx+c))^{\frac{5}{3}}\,dx$$

input
```
int((a+b*sec(d*x+c))^(5/3),x)
```

output
```
int((a+b*sec(d*x+c))^(5/3),x)
```

3.698.5 Fricas [F(-1)]

Timed out.

$$\int (a+b\sec(c+dx))^{5/3}\,dx = \text{Timed out}$$

input
```
integrate((a+b*sec(d*x+c))^(5/3),x, algorithm="fricas")
```

output
```
Timed out
```

3.698.6 Sympy [N/A]

Not integrable

Time = 42.66 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int (a+b\sec(c+dx))^{5/3}\,dx = \int (a+b\sec(c+dx))^{\frac{5}{3}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))**(5/3),x)
```

output
```
Integral((a + b*sec(c + d*x))**(5/3), x)
```

3.698.7 Maxima [N/A]

Not integrable

Time = 0.54 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int (a+b\sec(c+dx))^{5/3}\,dx=\int (b\sec(dx+c)+a)^{\frac{5}{3}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(5/3),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^(5/3), x)
```

3.698.8 Giac [N/A]

Not integrable

Time = 1.55 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int (a+b\sec(c+dx))^{5/3}\,dx=\int (b\sec(dx+c)+a)^{\frac{5}{3}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(5/3),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^(5/3), x)
```

3.698.9 Mupad [N/A]

Not integrable

Time = 14.67 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int (a+b\sec(c+dx))^{5/3}\,dx=\int \left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/3}\,dx$$

input
```
int((a + b/cos(c + d*x))^(5/3),x)
```

output
```
int((a + b/cos(c + d*x))^(5/3), x)
```

3.699 $\int \frac{\sec^4(c+dx)}{\sqrt[3]{a+b\sec(c+dx)}}\,dx$

3.699.1 Optimal result 5436
3.699.2 Mathematica [B] (warning: unable to verify) 5437
3.699.3 Rubi [A] (verified) 5437
3.699.4 Maple [F] 5441
3.699.5 Fricas [F] 5442
3.699.6 Sympy [F] 5442
3.699.7 Maxima [F] 5442
3.699.8 Giac [F] 5443
3.699.9 Mupad [F(-1)] 5443

3.699.1 Optimal result

Integrand size = 23, antiderivative size = 313

$$\begin{aligned}
&\int \frac{\sec^4(c+dx)}{\sqrt[3]{a+b\sec(c+dx)}}\,dx \\
&= -\frac{9a(a+b\sec(c+dx))^{2/3}\tan(c+dx)}{20b^2d} + \frac{3\sec(c+dx)(a+b\sec(c+dx))^{2/3}\tan(c+dx)}{8bd} \\
&\quad + \frac{(18a^2+25b^2)\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},-\frac{2}{3},\frac{3}{2},\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)(a+b\sec(c+dx))^{2/3}\tan(c+dx)}{20\sqrt{2}b^3d\sqrt{1+\sec(c+dx)}\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{2/3}} \\
&\quad - \frac{a(9a^2+11b^2)\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},\frac{1}{3},\frac{3}{2},\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)\sqrt[3]{\frac{a+b\sec(c+dx)}{a+b}}\tan(c+dx)}{10\sqrt{2}b^3d\sqrt{1+\sec(c+dx)}\sqrt[3]{a+b\sec(c+dx)}}
\end{aligned}$$

output
```
-9/20*a*(a+b*sec(d*x+c))^(2/3)*tan(d*x+c)/b^2/d+3/8*sec(d*x+c)*(a+b*sec(d*
x+c))^(2/3)*tan(d*x+c)/b/d+1/40*(18*a^2+25*b^2)*AppellF1(1/2,-2/3,1/2,3/2,
b*(1-sec(d*x+c))/(a+b),1/2-1/2*sec(d*x+c))*(a+b*sec(d*x+c))^(2/3)*tan(d*x+
c)/b^3/d/((a+b*sec(d*x+c))/(a+b))^(2/3)*2^(1/2)/(1+sec(d*x+c))^(1/2)-1/20*
a*(9*a^2+11*b^2)*AppellF1(1/2,1/3,1/2,3/2,b*(1-sec(d*x+c))/(a+b),1/2-1/2*s
ec(d*x+c))*((a+b*sec(d*x+c))/(a+b))^(1/3)*tan(d*x+c)/b^3/d/(a+b*sec(d*x+c)
)^(1/3)*2^(1/2)/(1+sec(d*x+c))^(1/2)
```

3.699.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 19015 vs. $2(313) = 626$.

Time = 45.82 (sec) , antiderivative size = 19015, normalized size of antiderivative = 60.75

$$\int \frac{\sec^4(c+dx)}{\sqrt[3]{a+b\sec(c+dx)}}\,dx = \text{Result too large to show}$$

input
```
Integrate[Sec[c + d*x]^4/(a + b*Sec[c + d*x])^(1/3),x]
```

output
```
Result too large to show
```

3.699.3 Rubi [A] (verified)

Time = 1.00 (sec) , antiderivative size = 324, normalized size of antiderivative = 1.04, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.522$, Rules used = {3042, 4352, 27, 3042, 4570, 27, 3042, 4495, 3042, 4321, 156, 155}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^4(c+dx)}{\sqrt[3]{a+b\sec(c+dx)}}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^4}{\sqrt[3]{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \; 4352$$

$$\frac{3\int \frac{\sec(c+dx)\left(-6a\sec^2(c+dx)+5b\sec(c+dx)+3a\right)}{3\sqrt[3]{a+b\sec(c+dx)}}dx}{8b}+\frac{3\tan(c+dx)\sec(c+dx)(a+b\sec(c+dx))^{2/3}}{8bd}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{\sec(c+dx)\left(-6a\sec^2(c+dx)+5b\sec(c+dx)+3a\right)}{\sqrt[3]{a+b\sec(c+dx)}}dx}{8b}+\frac{3\tan(c+dx)\sec(c+dx)(a+b\sec(c+dx))^{2/3}}{8bd}$$

$$\Big\downarrow \; 3042$$

$$\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(-6a\csc\left(c+dx+\frac{\pi}{2}\right)^2+5b\csc\left(c+dx+\frac{\pi}{2}\right)+3a\right)}{\sqrt[3]{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{8b}+\frac{3\tan(c+dx)\sec(c+dx)(a+b\sec(c+dx))^{2/3}}{8bd}$$

$\downarrow$ 4570

$$\frac{\frac{3\int \frac{\sec(c+dx)\left(3ab+\left(18a^2+25b^2\right)\sec(c+dx)\right)}{3\sqrt[3]{a+b\sec(c+dx)}}dx}{5b}-\frac{18a\tan(c+dx)(a+b\sec(c+dx))^{2/3}}{5bd}}{8b}+\frac{3\tan(c+dx)\sec(c+dx)(a+b\sec(c+dx))^{2/3}}{8bd}$$

$\downarrow$ 27

$$\frac{\frac{\int \frac{\sec(c+dx)\left(3ab+\left(18a^2+25b^2\right)\sec(c+dx)\right)}{\sqrt[3]{a+b\sec(c+dx)}}dx}{5b}-\frac{18a\tan(c+dx)(a+b\sec(c+dx))^{2/3}}{5bd}}{8b}+\frac{3\tan(c+dx)\sec(c+dx)(a+b\sec(c+dx))^{2/3}}{8bd}$$

$\downarrow$ 3042

$$\frac{\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(3ab+\left(18a^2+25b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt[3]{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{5b}-\frac{18a\tan(c+dx)(a+b\sec(c+dx))^{2/3}}{5bd}}{8b}+\frac{3\tan(c+dx)\sec(c+dx)(a+b\sec(c+dx))^{2/3}}{8bd}$$

$\downarrow$ 4495

$$\frac{\frac{\frac{\left(18a^2+25b^2\right)\int \sec(c+dx)(a+b\sec(c+dx))^{2/3}dx}{b}-\frac{2a\left(9a^2+11b^2\right)\int \frac{\sec(c+dx)}{\sqrt[3]{a+b\sec(c+dx)}}dx}{b}}{5b}-\frac{18a\tan(c+dx)(a+b\sec(c+dx))^{2/3}}{5bd}}{8b}+\frac{3\tan(c+dx)\sec(c+dx)(a+b\sec(c+dx))^{2/3}}{8bd}$$

$\downarrow$ 3042

3.699. $\int \frac{\sec^4(c+dx)}{\sqrt[3]{a+b\sec(c+dx)}}dx$

$$\frac{\frac{\frac{\left(18a^2+25b^2\right)\int \csc\left(c+dx+\frac{\pi}{2}\right)\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{2/3}dx}{b}-\frac{2a\left(9a^2+11b^2\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt[3]{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}}{5b}-\frac{18a\tan(c+dx)(a+b\sec(c+dx))^{2/3}}{5bd}}{8b}$$

$$\frac{3\tan(c+dx)\sec(c+dx)(a+b\sec(c+dx))^{2/3}}{8bd}$$

$\downarrow$ 4321

$$\frac{\frac{\frac{2a\left(9a^2+11b^2\right)\tan(c+dx)\int \frac{1}{\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}\sqrt[3]{a+b\sec(c+dx)}}d\sec(c+dx)}{bd\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}}-\frac{\left(18a^2+25b^2\right)\tan(c+dx)\int \frac{(a+b\sec(c+dx))^{2/3}}{\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}}d\sec}{bd\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}}}{5b}}{8b}$$

$$\frac{3\tan(c+dx)\sec(c+dx)(a+b\sec(c+dx))^{2/3}}{8bd}$$

$\downarrow$ 156

$$\frac{\frac{\frac{2a\left(9a^2+11b^2\right)\tan(c+dx)\sqrt[3]{\frac{a+b\sec(c+dx)}{a+b}}\int \frac{1}{\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}\sqrt[3]{\frac{a}{a+b}+\frac{b\sec(c+dx)}{a+b}}}d\sec(c+dx)}{bd\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}\sqrt[3]{a+b\sec(c+dx)}}-\frac{\left(18a^2+25b^2\right)\tan(c+dx)(}{bd\sqrt{1-s}}}{5b}}{8b}$$

$$\frac{3\tan(c+dx)\sec(c+dx)(a+b\sec(c+dx))^{2/3}}{8bd}$$

$\downarrow$ 155

$$\frac{\frac{\frac{\sqrt{2}\left(18a^2+25b^2\right)\tan(c+dx)(a+b\sec(c+dx))^{2/3}\operatorname{AppellF1}\left(\frac{1}{2},\frac{1}{2},-\frac{2}{3},\frac{3}{2},\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)}{bd\sqrt{\sec(c+dx)+1}\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{2/3}}-\frac{2\sqrt{2}a\left(9a^2+11b^2\right)\tan(c+dx)\sqrt[3]{\frac{a+b\sec(c+d}{a+b}}}{bd\sqrt{\sec(c+dx)+1}\sqrt[3]{}}}{5b}}{8b}$$

$$\frac{3\tan(c+dx)\sec(c+dx)(a+b\sec(c+dx))^{2/3}}{8bd}$$

input
```
Int[Sec[c + d*x]^4/(a + b*Sec[c + d*x])^(1/3),x]
```

3.699. $\int \frac{\sec^4(c+dx)}{\sqrt[3]{a+b\sec(c+dx)}}\,dx$

output
```
(3*Sec[c + d*x]*(a + b*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(8*b*d) + ((-18*a
*(a + b*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(5*b*d) + ((Sqrt[2]*(18*a^2 + 25
*b^2)*AppellF1[1/2, 1/2, -2/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[c +
d*x]))/(a + b)]*(a + b*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(b*d*Sqrt[1 + Sec
[c + d*x]]*((a + b*Sec[c + d*x])/(a + b))^(2/3)) - (2*Sqrt[2]*a*(9*a^2 + 1
1*b^2)*AppellF1[1/2, 1/2, 1/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[c +
d*x]))/(a + b)]*((a + b*Sec[c + d*x])/(a + b))^(1/3)*Tan[c + d*x])/(b*d*Sq
rt[1 + Sec[c + d*x]]*(a + b*Sec[c + d*x])^(1/3)))/(5*b))/(8*b)
```

3.699.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 155
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*Simplify[b/(b*c - a*d)]^n*
Simplify[b/(b*e - a*f)]^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/
(b*c - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f,
 m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[Sim
plify[b/(b*c - a*d)], 0] && GtQ[Simplify[b/(b*e - a*f)], 0] &&  !(GtQ[Simpl
ify[d/(d*a - c*b)], 0] && GtQ[Simplify[d/(d*e - c*f)], 0] && SimplerQ[c + d
*x, a + b*x]) &&  !(GtQ[Simplify[f/(f*a - e*b)], 0] && GtQ[Simplify[f/(f*c
- e*d)], 0] && SimplerQ[e + f*x, a + b*x])
```

rule 156
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(e + f*x)^FracPart[p]/(Simplify[b/(b*e - a*f)]^IntPart[p
]*(b*((e + f*x)/(b*e - a*f)))^FracPart[p])   Int[(a + b*x)^m*(c + d*x)^n*Si
mp[b*(e/(b*e - a*f)) + b*f*(x/(b*e - a*f)), x]^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] &
& GtQ[Simplify[b/(b*c - a*d)], 0] &&  !GtQ[Simplify[b/(b*e - a*f)], 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.699. $\int \frac{\sec^4(c+dx)}{\sqrt[3]{a+b\sec(c+dx)}}\,dx$

rule 4321
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[Cot[e + f*x]/(f*Sqrt[1 + Csc[e + f*x]]*Sqrt[1 - Csc[e + f*x
]])   Subst[Int[(a + b*x)^m/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Csc[e + f*x]]
, x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*m]
```

rule 4352
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-d^3)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m +
1)*((d*Csc[e + f*x])^(n - 3)/(b*f*(m + n - 1))), x] + Simp[d^3/(b*(m + n -
1))   Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 3)*Simp[a*(n - 3) +
b*(m + n - 2)*Csc[e + f*x] - a*(n - 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[
{a, b, d, e, f, m}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 3] && (IntegerQ[n] ||
 IntegersQ[2*m, 2*n]) &&  !IGtQ[m, 2]
```

rule 4495
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b - a*B)/b   Int[
Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] + Simp[B/b   Int[Csc[e + f*x]*(
a + b*Csc[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, A, B, e, f, m}, x] && N
eQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

rule 4570
```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> Simp[(-C)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2)
)), x] + Simp[1/(b*(m + 2))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[
b*A*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /;
 FreeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

3.699.4 Maple [F]

$$\int \frac{\sec(dx+c)^4}{(a+b\sec(dx+c))^{\frac{1}{3}}}dx$$

input
```
int(sec(d*x+c)^4/(a+b*sec(d*x+c))^(1/3),x)
```

output
```
int(sec(d*x+c)^4/(a+b*sec(d*x+c))^(1/3),x)
```

3.699.5 Fricas [F]

$$\int \frac{\sec^4(c+dx)}{\sqrt[3]{a+b\sec(c+dx)}}\,dx = \int \frac{\sec\left(dx+c\right)^4}{\left(b\sec\left(dx+c\right)+a\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate(sec(d*x+c)^4/(a+b*sec(d*x+c))^(1/3),x, algorithm="fricas")
```

output
```
integral(sec(d*x + c)^4/(b*sec(d*x + c) + a)^(1/3), x)
```

3.699.6 Sympy [F]

$$\int \frac{\sec^4(c+dx)}{\sqrt[3]{a+b\sec(c+dx)}}\,dx = \int \frac{\sec^4\left(c+dx\right)}{\sqrt[3]{a+b\sec\left(c+dx\right)}}\,dx$$

input
```
integrate(sec(d*x+c)**4/(a+b*sec(d*x+c))**(1/3),x)
```

output
```
Integral(sec(c + d*x)**4/(a + b*sec(c + d*x))**(1/3), x)
```

3.699.7 Maxima [F]

$$\int \frac{\sec^4(c+dx)}{\sqrt[3]{a+b\sec(c+dx)}}\,dx = \int \frac{\sec\left(dx+c\right)^4}{\left(b\sec\left(dx+c\right)+a\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate(sec(d*x+c)^4/(a+b*sec(d*x+c))^(1/3),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)^4/(b*sec(d*x + c) + a)^(1/3), x)
```

3.699.8 Giac [F]

$$\int \frac{\sec^4(c+dx)}{\sqrt[3]{a+b\sec(c+dx)}}\,dx = \int \frac{\sec(dx+c)^4}{(b\sec(dx+c)+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate(sec(d*x+c)^4/(a+b*sec(d*x+c))^(1/3),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^4/(b*sec(d*x + c) + a)^(1/3), x)
```

3.699.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^4(c+dx)}{\sqrt[3]{a+b\sec(c+dx)}}\,dx = \int \frac{1}{\cos(c+d\,x)^4\left(a+\frac{b}{\cos(c+d\,x)}\right)^{1/3}}\,dx$$

input
```
int(1/(cos(c + d*x)^4*(a + b/cos(c + d*x))^(1/3)),x)
```

output
```
int(1/(cos(c + d*x)^4*(a + b/cos(c + d*x))^(1/3)), x)
```

3.700 $\int \frac{\sec^3(c+dx)}{\sqrt[3]{a+b\sec(c+dx)}}\,dx$

3.700.1 Optimal result 5444
3.700.2 Mathematica [B] (warning: unable to verify) 5445
3.700.3 Rubi [A] (verified) 5445
3.700.4 Maple [F] 5448
3.700.5 Fricas [F] 5448
3.700.6 Sympy [F] 5449
3.700.7 Maxima [F] 5449
3.700.8 Giac [F] 5449
3.700.9 Mupad [F(-1)] 5450

3.700.1 Optimal result

Integrand size = 23, antiderivative size = 265

$$\int \frac{\sec^3(c+dx)}{\sqrt[3]{a+b\sec(c+dx)}}\,dx = \frac{3(a+b\sec(c+dx))^{2/3}\tan(c+dx)}{5bd} - \frac{3\sqrt{2}a\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},-\frac{2}{3},\frac{3}{2},\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)(a+b\sec(c+dx))^{2/3}\tan(c+dx)}{5b^2d\sqrt{1+\sec(c+dx)}\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{2/3}} + \frac{\sqrt{2}(3a^2+2b^2)\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},\frac{1}{3},\frac{3}{2},\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)\sqrt[3]{\frac{a+b\sec(c+dx)}{a+b}}\tan(c+dx)}{5b^2d\sqrt{1+\sec(c+dx)}\sqrt[3]{a+b\sec(c+dx)}}$$

output
```
3/5*(a+b*sec(d*x+c))^(2/3)*tan(d*x+c)/b/d-3/5*a*AppellF1(1/2,-2/3,1/2,3/2,
b*(1-sec(d*x+c))/(a+b),1/2-1/2*sec(d*x+c))*(a+b*sec(d*x+c))^(2/3)*2^(1/2)*
tan(d*x+c)/b^2/d/((a+b*sec(d*x+c))/(a+b))^(2/3)/(1+sec(d*x+c))^(1/2)+1/5*(
3*a^2+2*b^2)*AppellF1(1/2,1/3,1/2,3/2,b*(1-sec(d*x+c))/(a+b),1/2-1/2*sec(d
*x+c))*((a+b*sec(d*x+c))/(a+b))^(1/3)*2^(1/2)*tan(d*x+c)/b^2/d/(a+b*sec(d*
x+c))^(1/3)/(1+sec(d*x+c))^(1/2)
```

3.700.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 7195 vs. $2(265) = 530$.

Time = 44.26 (sec) , antiderivative size = 7195, normalized size of antiderivative = 27.15

$$\int \frac{\sec^3(c+dx)}{\sqrt[3]{a+b\sec(c+dx)}}\,dx = \text{Result too large to show}$$

input
```
Integrate[Sec[c + d*x]^3/(a + b*Sec[c + d*x])^(1/3),x]
```

output
```
Result too large to show
```

3.700.3 Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.01, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {3042, 4327, 27, 3042, 4495, 3042, 4321, 156, 155}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^3(c+dx)}{\sqrt[3]{a+b\sec(c+dx)}}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^3}{\sqrt[3]{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \; 4327$$

$$\frac{3\int \frac{\sec(c+dx)(2b-3a\sec(c+dx))}{3\sqrt[3]{a+b\sec(c+dx)}}dx}{5b}+\frac{3\tan(c+dx)(a+b\sec(c+dx))^{2/3}}{5bd}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{\sec(c+dx)(2b-3a\sec(c+dx))}{\sqrt[3]{a+b\sec(c+dx)}}dx}{5b}+\frac{3\tan(c+dx)(a+b\sec(c+dx))^{2/3}}{5bd}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(2b-3a\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt[3]{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{5b}+\frac{3\tan(c+dx)(a+b\sec(c+dx))^{2/3}}{5bd}}{}$$

$\Big\downarrow$ 4495

$$\frac{\frac{\frac{(3a^2+2b^2)\int \frac{\sec(c+dx)}{\sqrt[3]{a+b\sec(c+dx)}}dx}{b}-\frac{3a\int \sec(c+dx)(a+b\sec(c+dx))^{2/3}dx}{b}}{5b}+\frac{3\tan(c+dx)(a+b\sec(c+dx))^{2/3}}{5bd}}{}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\frac{(3a^2+2b^2)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt[3]{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\frac{3a\int \csc\left(c+dx+\frac{\pi}{2}\right)\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{2/3}dx}{b}}{5b}+\frac{3\tan(c+dx)(a+b\sec(c+dx))^{2/3}}{5bd}}{}$$

$\Big\downarrow$ 4321

$$\frac{\frac{\frac{3a\tan(c+dx)\int \frac{(a+b\sec(c+dx))^{2/3}}{\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}}d\sec(c+dx)}{bd\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}}-\frac{(3a^2+2b^2)\tan(c+dx)\int \frac{1}{\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}\sqrt[3]{a+b\sec(c+dx)}}d\sec(c+d}{bd\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}}}{5b}+\frac{3\tan(c+dx)(a+b\sec(c+dx))^{2/3}}{5bd}}{}$$

$\Big\downarrow$ 156

$$\frac{\frac{\frac{3a\tan(c+dx)(a+b\sec(c+dx))^{2/3}\int \frac{\left(\frac{a}{a+b}+\frac{b\sec(c+dx)}{a+b}\right)^{2/3}}{\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}}d\sec(c+dx)}{bd\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{2/3}}-\frac{(3a^2+2b^2)\tan(c+dx)\sqrt[3]{\frac{a+b\sec(c+dx)}{a+b}}\int \frac{}{\sqrt{1-\sec(c+dx)}\sqrt{s}}}{bd\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}\sqrt[3]{}}}{5b}+\frac{3\tan(c+dx)(a+b\sec(c+dx))^{2/3}}{5bd}}{}$$

$\Big\downarrow$ 155

$$\frac{\frac{\frac{\sqrt{2}(3a^2+2b^2)\tan(c+dx)\sqrt[3]{\frac{a+b\sec(c+dx)}{a+b}}\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},\frac{1}{3},\frac{3}{2},\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)}{bd\sqrt{\sec(c+dx)+1}\sqrt[3]{a+b\sec(c+dx)}}-\frac{3\sqrt{2}a\tan(c+dx)(a+b\sec(c+dx))^{2/3}}{bd\sqrt{\sec(}}}{5b}+\frac{3\tan(c+dx)(a+b\sec(c+dx))^{2/3}}{5bd}}{}$$

3.700. $\int \frac{\sec^3(c+dx)}{\sqrt[3]{a+b\sec(c+dx)}}dx$

input
```
Int[Sec[c + d*x]^3/(a + b*Sec[c + d*x])^(1/3),x]
```

output
```
(3*(a + b*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(5*b*d) + ((-3*Sqrt[2]*a*Appel
lF1[1/2, 1/2, -2/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[c + d*x]))/(a +
 b)]*(a + b*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(b*d*Sqrt[1 + Sec[c + d*x]]*
((a + b*Sec[c + d*x])/(a + b))^(2/3)) + (Sqrt[2]*(3*a^2 + 2*b^2)*AppellF1[
1/2, 1/2, 1/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[c + d*x]))/(a + b)]*
((a + b*Sec[c + d*x])/(a + b))^(1/3)*Tan[c + d*x])/(b*d*Sqrt[1 + Sec[c + d
*x]]*(a + b*Sec[c + d*x])^(1/3)))/(5*b)
```

3.700.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 155
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*Simplify[b/(b*c - a*d)]^n*
Simplify[b/(b*e - a*f)]^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/
(b*c - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f,
 m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[Sim
plify[b/(b*c - a*d)], 0] && GtQ[Simplify[b/(b*e - a*f)], 0] &&  !(GtQ[Simpl
ify[d/(d*a - c*b)], 0] && GtQ[Simplify[d/(d*e - c*f)], 0] && SimplerQ[c + d
*x, a + b*x]) &&  !(GtQ[Simplify[f/(f*a - e*b)], 0] && GtQ[Simplify[f/(f*c
- e*d)], 0] && SimplerQ[e + f*x, a + b*x])
```

rule 156
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(e + f*x)^FracPart[p]/(Simplify[b/(b*e - a*f)]^IntPart[p
]*(b*((e + f*x)/(b*e - a*f)))^FracPart[p])   Int[(a + b*x)^m*(c + d*x)^n*Si
mp[b*(e/(b*e - a*f)) + b*f*(x/(b*e - a*f)), x]^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] &
& GtQ[Simplify[b/(b*c - a*d)], 0] &&  !GtQ[Simplify[b/(b*e - a*f)], 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.700. $\int \frac{\sec^3(c+dx)}{\sqrt[3]{a+b\sec(c+dx)}}\,dx$

rule 4321
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[Cot[e + f*x]/(f*Sqrt[1 + Csc[e + f*x]]*Sqrt[1 - Csc[e + f*x
]])   Subst[Int[(a + b*x)^m/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Csc[e + f*x]]
, x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*m]
```

rule 4327
```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(-Cot[e + f*x])*((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2
))), x] + Simp[1/(b*(m + 2))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(b*(
m + 1) - a*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 -
b^2, 0] &&  !LtQ[m, -1]
```

rule 4495
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b - a*B)/b   Int[
Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] + Simp[B/b   Int[Csc[e + f*x]*(
a + b*Csc[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, A, B, e, f, m}, x] && N
eQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

3.700.4 Maple [F]

$$\int \frac{\sec (dx+c)^3}{(a+b\sec (dx+c))^{\frac{1}{3}}}dx$$

input
```
int(sec(d*x+c)^3/(a+b*sec(d*x+c))^(1/3),x)
```

output
```
int(sec(d*x+c)^3/(a+b*sec(d*x+c))^(1/3),x)
```

3.700.5 Fricas [F]

$$\int \frac{\sec^3(c+dx)}{\sqrt[3]{a+b\sec(c+dx)}}\,dx=\int \frac{\sec (dx+c)^3}{(b\sec (dx+c)+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate(sec(d*x+c)^3/(a+b*sec(d*x+c))^(1/3),x, algorithm="fricas")
```

output
```
integral(sec(d*x + c)^3/(b*sec(d*x + c) + a)^(1/3), x)
```

3.700.6 Sympy [F]

$$\int \frac{\sec^3(c+dx)}{\sqrt[3]{a+b\sec(c+dx)}}\,dx = \int \frac{\sec^3(c+dx)}{\sqrt[3]{a+b\sec(c+dx)}}\,dx$$

input
```
integrate(sec(d*x+c)**3/(a+b*sec(d*x+c))**(1/3),x)
```

output
```
Integral(sec(c + d*x)**3/(a + b*sec(c + d*x))**(1/3), x)
```

3.700.7 Maxima [F]

$$\int \frac{\sec^3(c+dx)}{\sqrt[3]{a+b\sec(c+dx)}}\,dx = \int \frac{\sec(dx+c)^3}{(b\sec(dx+c)+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate(sec(d*x+c)^3/(a+b*sec(d*x+c))^(1/3),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)^3/(b*sec(d*x + c) + a)^(1/3), x)
```

3.700.8 Giac [F]

$$\int \frac{\sec^3(c+dx)}{\sqrt[3]{a+b\sec(c+dx)}}\,dx = \int \frac{\sec(dx+c)^3}{(b\sec(dx+c)+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate(sec(d*x+c)^3/(a+b*sec(d*x+c))^(1/3),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^3/(b*sec(d*x + c) + a)^(1/3), x)
```

3.700.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^3(c+dx)}{\sqrt[3]{a+b\sec(c+dx)}}\,dx = \int \frac{1}{\cos\left(c+d\,x\right)^3\,\left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)^{1/3}}\,dx$$

input
```
int(1/(cos(c + d*x)^3*(a + b/cos(c + d*x))^(1/3)),x)
```

output
```
int(1/(cos(c + d*x)^3*(a + b/cos(c + d*x))^(1/3)), x)
```

3.701 $\int \frac{\sec^2(c+dx)}{\sqrt[3]{a+b\sec(c+dx)}}\,dx$

3.701.1 Optimal result 5451
3.701.2 Mathematica [B] (warning: unable to verify) 5452
3.701.3 Rubi [A] (verified) 5452
3.701.4 Maple [F] 5455
3.701.5 Fricas [F] 5455
3.701.6 Sympy [F] 5455
3.701.7 Maxima [F] 5456
3.701.8 Giac [F] 5456
3.701.9 Mupad [F(-1)] 5456

3.701.1 Optimal result

Integrand size = 23, antiderivative size = 219

$$\int \frac{\sec^2(c+dx)}{\sqrt[3]{a+b\sec(c+dx)}}\,dx = \frac{\sqrt{2}\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},-\frac{2}{3},\frac{3}{2},\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)(a+b\sec(c+dx))^{2/3}\tan(c+dx)}{bd\sqrt{1+\sec(c+dx)}\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{2/3}} - \frac{\sqrt{2}a\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},\frac{1}{3},\frac{3}{2},\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)\sqrt[3]{\frac{a+b\sec(c+dx)}{a+b}}\tan(c+dx)}{bd\sqrt{1+\sec(c+dx)}\sqrt[3]{a+b\sec(c+dx)}}$$

output
```
AppellF1(1/2,-2/3,1/2,3/2,b*(1-sec(d*x+c))/(a+b),1/2-1/2*sec(d*x+c))*(a+b*
sec(d*x+c))^(2/3)*2^(1/2)*tan(d*x+c)/b/d/((a+b*sec(d*x+c))/(a+b))^(2/3)/(1
+sec(d*x+c))^(1/2)-a*AppellF1(1/2,1/3,1/2,3/2,b*(1-sec(d*x+c))/(a+b),1/2-1
/2*sec(d*x+c))*((a+b*sec(d*x+c))/(a+b))^(1/3)*2^(1/2)*tan(d*x+c)/b/d/(a+b*
sec(d*x+c))^(1/3)/(1+sec(d*x+c))^(1/2)
```

3.701.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 1748 vs. $2(219) = 438$.

Time = 23.82 (sec) , antiderivative size = 1748, normalized size of antiderivative = 7.98

$$\int \frac{\sec^2(c+dx)}{\sqrt[3]{a+b\sec(c+dx)}}\,dx = \text{Too large to display}$$

input
```
Integrate[Sec[c + d*x]^2/(a + b*Sec[c + d*x])^(1/3),x]
```

output
```
(3*(b + a*Cos[c + d*x])*Tan[c + d*x])/(2*b*d*(a + b*Sec[c + d*x])^(1/3)) -
 (3*(b + 3*a*Cos[c + d*x])*(a + b*Sec[c + d*x])^(2/3)*(5*(a^2 - b^2) + 3*b
*AppellF1[5/3, 3/2, 3/2, 8/3, (a + b*Sec[c + d*x])/(a - b), (a + b*Sec[c +
 d*x])/(a + b)]*Sec[c + d*x]*Sqrt[(b*(1 + Sec[c + d*x]))/(-a + b)]*Sqrt[(b
 - b*Sec[c + d*x])/(a + b)]*(a + b*Sec[c + d*x])))/(10*b*(-a^2 + b^2)*d*(b
 + a*Cos[c + d*x])^(1/3)*Sqrt[1 - Cos[c + d*x]^2]*Sec[c + d*x]^(7/3)*((3*b
*(5*(a^2 - b^2) + 3*b*AppellF1[5/3, 3/2, 3/2, 8/3, (a + b*Sec[c + d*x])/(a
 - b), (a + b*Sec[c + d*x])/(a + b)]*Sec[c + d*x]*Sqrt[(b*(1 + Sec[c + d*x
]))/(-a + b)]*Sqrt[(b - b*Sec[c + d*x])/(a + b)]*(a + b*Sec[c + d*x]))*Sin
[c + d*x])/(5*(-a^2 + b^2)*(b + a*Cos[c + d*x])^(1/3)*Sqrt[1 - Cos[c + d*x
]^2]*Sec[c + d*x]^(1/3)) - (3*(a + b*Sec[c + d*x])*(5*(a^2 - b^2) + 3*b*Ap
pellF1[5/3, 3/2, 3/2, 8/3, (a + b*Sec[c + d*x])/(a - b), (a + b*Sec[c + d*
x])/(a + b)]*Sec[c + d*x]*Sqrt[(b*(1 + Sec[c + d*x]))/(-a + b)]*Sqrt[(b -
b*Sec[c + d*x])/(a + b)]*(a + b*Sec[c + d*x]))*Sin[c + d*x])/(5*(-a^2 + b^
2)*(b + a*Cos[c + d*x])^(1/3)*(1 - Cos[c + d*x]^2)^(3/2)*Sec[c + d*x]^(10/
3)) + (a*(a + b*Sec[c + d*x])*(5*(a^2 - b^2) + 3*b*AppellF1[5/3, 3/2, 3/2,
 8/3, (a + b*Sec[c + d*x])/(a - b), (a + b*Sec[c + d*x])/(a + b)]*Sec[c +
d*x]*Sqrt[(b*(1 + Sec[c + d*x]))/(-a + b)]*Sqrt[(b - b*Sec[c + d*x])/(a +
b)]*(a + b*Sec[c + d*x]))*Sin[c + d*x])/(5*(-a^2 + b^2)*(b + a*Cos[c + d*x
])^(4/3)*Sqrt[1 - Cos[c + d*x]^2]*Sec[c + d*x]^(7/3)) - (7*(a + b*Sec[c...
```

3.701.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3042, 4325, 3042, 4321, 156, 155}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(c+dx)}{\sqrt[3]{a+b\sec(c+dx)}}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt[3]{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \; 4325$$

$$\frac{\int \sec(c+dx)(a+b\sec(c+dx))^{2/3}dx}{b}-\frac{a\int \frac{\sec(c+dx)}{\sqrt[3]{a+b\sec(c+dx)}}dx}{b}$$

$$\Big\downarrow \; 3042$$

$$\frac{\int \csc\left(c+dx+\frac{\pi}{2}\right)\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{2/3}dx}{b}-\frac{a\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt[3]{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}$$

$$\Big\downarrow \; 4321$$

$$\frac{a\tan(c+dx)\int \frac{1}{\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}\sqrt[3]{a+b\sec(c+dx)}}d\sec(c+dx)}{bd\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}}-\frac{\tan(c+dx)\int \frac{(a+b\sec(c+dx))^{2/3}}{\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}}d\sec(c+dx)}{bd\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}}$$

$$\Big\downarrow \; 156$$

$$\frac{a\tan(c+dx)\sqrt[3]{\frac{a+b\sec(c+dx)}{a+b}}\int \frac{1}{\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}\sqrt[3]{\frac{a}{a+b}+\frac{b\sec(c+dx)}{a+b}}}d\sec(c+dx)}{bd\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}\sqrt[3]{a+b\sec(c+dx)}}-\frac{\tan(c+dx)(a+b\sec(c+dx))^{2/3}\int \frac{\left(\frac{a}{a+b}+\frac{b\sec(c+dx)}{a+b}\right)^{2/3}}{\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}}d\sec(c+dx)}{bd\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{2/3}}$$

$$\Big\downarrow \; 155$$

$$\frac{\sqrt{2}\tan(c+dx)(a+b\sec(c+dx))^{2/3}\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},-\frac{2}{3},\frac{3}{2},\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)}{bd\sqrt{\sec(c+dx)+1}\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{2/3}}-\frac{\sqrt{2}a\tan(c+dx)\sqrt[3]{\frac{a+b\sec(c+dx)}{a+b}}\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},\frac{1}{3},\frac{3}{2},\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)}{bd\sqrt{\sec(c+dx)+1}\sqrt[3]{a+b\sec(c+dx)}}$$

3.701. $\int \frac{\sec^2(c+dx)}{\sqrt[3]{a+b\sec(c+dx)}}\,dx$

input
```
Int[Sec[c + d*x]^2/(a + b*Sec[c + d*x])^(1/3),x]
```

output
```
(Sqrt[2]*AppellF1[1/2, 1/2, -2/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[c
 + d*x]))/(a + b)]*(a + b*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(b*d*Sqrt[1 +
Sec[c + d*x]]*((a + b*Sec[c + d*x])/(a + b))^(2/3)) - (Sqrt[2]*a*AppellF1[
1/2, 1/2, 1/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[c + d*x]))/(a + b)]*
((a + b*Sec[c + d*x])/(a + b))^(1/3)*Tan[c + d*x])/(b*d*Sqrt[1 + Sec[c + d
*x]]*(a + b*Sec[c + d*x])^(1/3))
```

3.701.3.1 Defintions of rubi rules used

rule 155
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*Simplify[b/(b*c - a*d)]^n*
Simplify[b/(b*e - a*f)]^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/
(b*c - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f,
 m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[Sim
plify[b/(b*c - a*d)], 0] && GtQ[Simplify[b/(b*e - a*f)], 0] && !(GtQ[Simpl
ify[d/(d*a - c*b)], 0] && GtQ[Simplify[d/(d*e - c*f)], 0] && SimplerQ[c + d
*x, a + b*x]) && !(GtQ[Simplify[f/(f*a - e*b)], 0] && GtQ[Simplify[f/(f*c
- e*d)], 0] && SimplerQ[e + f*x, a + b*x])
```

rule 156
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(e + f*x)^FracPart[p]/(Simplify[b/(b*e - a*f)]^IntPart[p
]*(b*((e + f*x)/(b*e - a*f)))^FracPart[p])   Int[(a + b*x)^m*(c + d*x)^n*Si
mp[b*(e/(b*e - a*f)) + b*f*(x/(b*e - a*f)), x]^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] &
& GtQ[Simplify[b/(b*c - a*d)], 0] && !GtQ[Simplify[b/(b*e - a*f)], 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4321
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[Cot[e + f*x]/(f*Sqrt[1 + Csc[e + f*x]]*Sqrt[1 - Csc[e + f*x
]])   Subst[Int[(a + b*x)^m/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Csc[e + f*x]]
, x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0] && !IntegerQ[2*m]
```

3.701. $\int \frac{\sec^2(c+dx)}{\sqrt[3]{a+b\sec(c+dx)}}\,dx$

rule 4325
```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[-a/b   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] +
Simp[1/b   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x], x] /; FreeQ[{
a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0]
```

3.701.4 Maple [F]

$$\int \frac{\sec (dx+c)^2}{(a+b\sec (dx+c))^{\frac{1}{3}}}dx$$

input
```
int(sec(d*x+c)^2/(a+b*sec(d*x+c))^(1/3),x)
```

output
```
int(sec(d*x+c)^2/(a+b*sec(d*x+c))^(1/3),x)
```

3.701.5 Fricas [F]

$$\int \frac{\sec^2(c+dx)}{\sqrt[3]{a+b\sec(c+dx)}}\,dx=\int \frac{\sec (dx+c)^2}{(b\sec (dx+c)+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate(sec(d*x+c)^2/(a+b*sec(d*x+c))^(1/3),x, algorithm="fricas")
```

output
```
integral(sec(d*x + c)^2/(b*sec(d*x + c) + a)^(1/3), x)
```

3.701.6 Sympy [F]

$$\int \frac{\sec^2(c+dx)}{\sqrt[3]{a+b\sec(c+dx)}}\,dx=\int \frac{\sec^2(c+dx)}{\sqrt[3]{a+b\sec(c+dx)}}\,dx$$

input
```
integrate(sec(d*x+c)**2/(a+b*sec(d*x+c))**(1/3),x)
```

output
```
Integral(sec(c + d*x)**2/(a + b*sec(c + d*x))**(1/3), x)
```

3.701.7 Maxima [F]

$$\int \frac{\sec^2(c+dx)}{\sqrt[3]{a+b\sec(c+dx)}}\,dx = \int \frac{\sec(dx+c)^2}{(b\sec(dx+c)+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate(sec(d*x+c)^2/(a+b*sec(d*x+c))^(1/3),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)^2/(b*sec(d*x + c) + a)^(1/3), x)
```

3.701.8 Giac [F]

$$\int \frac{\sec^2(c+dx)}{\sqrt[3]{a+b\sec(c+dx)}}\,dx = \int \frac{\sec(dx+c)^2}{(b\sec(dx+c)+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate(sec(d*x+c)^2/(a+b*sec(d*x+c))^(1/3),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^2/(b*sec(d*x + c) + a)^(1/3), x)
```

3.701.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^2(c+dx)}{\sqrt[3]{a+b\sec(c+dx)}}\,dx = \int \frac{1}{\cos(c+d\,x)^2\left(a+\frac{b}{\cos(c+d\,x)}\right)^{1/3}}\,dx$$

input
```
int(1/(cos(c + d*x)^2*(a + b/cos(c + d*x))^(1/3)),x)
```

output
```
int(1/(cos(c + d*x)^2*(a + b/cos(c + d*x))^(1/3)), x)
```

3.702 $\int \frac{\sec(c+dx)}{\sqrt[3]{a+b\sec(c+dx)}}\,dx$

3.702.1 Optimal result 5457
3.702.2 Mathematica [B] (warning: unable to verify) 5457
3.702.3 Rubi [A] (verified) 5458
3.702.4 Maple [F] 5460
3.702.5 Fricas [F] 5460
3.702.6 Sympy [F] 5460
3.702.7 Maxima [F] 5461
3.702.8 Giac [F] 5461
3.702.9 Mupad [F(-1)] 5461

3.702.1 Optimal result

Integrand size = 21, antiderivative size = 105

$$\int \frac{\sec(c+dx)}{\sqrt[3]{a+b\sec(c+dx)}}\,dx = \frac{\sqrt{2}\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},\frac{1}{3},\frac{3}{2},\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)\sqrt[3]{\frac{a+b\sec(c+dx)}{a+b}}\tan(c+dx)}{d\sqrt{1+\sec(c+dx)}\sqrt[3]{a+b\sec(c+dx)}}$$

output
```
AppellF1(1/2,1/3,1/2,3/2,b*(1-sec(d*x+c))/(a+b),1/2-1/2*sec(d*x+c))*((a+b*
sec(d*x+c))/(a+b))^(1/3)*2^(1/2)*tan(d*x+c)/d/(a+b*sec(d*x+c))^(1/3)/(1+se
c(d*x+c))^(1/2)
```

3.702.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 310 vs. 2(105) = 210.

Time = 14.39 (sec) , antiderivative size = 310, normalized size of antiderivative = 2.95

$$\int \frac{\sec(c+dx)}{\sqrt[3]{a+b\sec(c+dx)}}\,dx = \frac{15(a-b)^2(a+b)\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},\frac{1}{2},\frac{5}{3},\frac{a+b\sec(c+dx)}{a-b},\frac{a+b\sec(c}{a+b}\right.}{b^2(-a+b)d\left(3(a-b)\,\text{AppellF1}\left(\frac{5}{3},\frac{1}{2},\frac{3}{2},\frac{8}{3},\frac{a+b\sec(c+dx)}{a-b},\frac{a+b\sec(c+dx)}{a+b}\right)(b+a\cos(c+dx))+(a+b)\left(10(a-\right.\right.}$$

input
```
Integrate[Sec[c + d*x]/(a + b*Sec[c + d*x])^(1/3),x]
```

output
```
(15*(a - b)^2*(a + b)*AppellF1[2/3, 1/2, 1/2, 5/3, (a + b*Sec[c + d*x])/(a
 - b), (a + b*Sec[c + d*x])/(a + b)]*Cos[c + d*x]*Cot[c + d*x]^3*(1 + Sec[
c + d*x])*(b - b*Sec[c + d*x])*(a + b*Sec[c + d*x])^(2/3))/(b^2*(-a + b)*d
*(3*(a - b)*AppellF1[5/3, 1/2, 3/2, 8/3, (a + b*Sec[c + d*x])/(a - b), (a
+ b*Sec[c + d*x])/(a + b)]*(b + a*Cos[c + d*x]) + (a + b)*(10*(a - b)*Appe
llF1[2/3, 1/2, 1/2, 5/3, (a + b*Sec[c + d*x])/(a - b), (a + b*Sec[c + d*x]
)/(a + b)]*Cos[c + d*x] + 3*AppellF1[5/3, 3/2, 1/2, 8/3, (a + b*Sec[c + d*
x])/(a - b), (a + b*Sec[c + d*x])/(a + b)]*(b + a*Cos[c + d*x]))))
```

3.702.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3042, 4321, 156, 155}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(c+dx)}{\sqrt[3]{a+b\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt[3]{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4321}$$

$$-\frac{\tan(c+dx)\int \frac{1}{\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}\sqrt[3]{a+b\sec(c+dx)}}d\sec(c+dx)}{d\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}}$$

$$\Big\downarrow \text{156}$$

$$-\frac{\tan(c+dx)\sqrt[3]{\frac{a+b\sec(c+dx)}{a+b}}\int \frac{1}{\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}\sqrt[3]{\frac{a}{a+b}+\frac{b\sec(c+dx)}{a+b}}}d\sec(c+dx)}{d\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}\sqrt[3]{a+b\sec(c+dx)}}$$

$$\Big\downarrow \text{155}$$

$$\frac{\sqrt{2}\tan(c+dx)\sqrt[3]{\frac{a+b\sec(c+dx)}{a+b}}\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},\frac{1}{3},\frac{3}{2},\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)}{d\sqrt{\sec(c+dx)+1}\sqrt[3]{a+b\sec(c+dx)}}$$

input
```
Int[Sec[c + d*x]/(a + b*Sec[c + d*x])^(1/3),x]
```

output
```
(Sqrt[2]*AppellF1[1/2, 1/2, 1/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[c
+ d*x]))/(a + b)]*((a + b*Sec[c + d*x])/(a + b))^(1/3)*Tan[c + d*x])/(d*Sq
rt[1 + Sec[c + d*x]]*(a + b*Sec[c + d*x])^(1/3))
```

3.702.3.1 Defintions of rubi rules used

rule 155
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*Simplify[b/(b*c - a*d)]^n*
Simplify[b/(b*e - a*f)]^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/
(b*c - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f,
 m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[Sim
plify[b/(b*c - a*d)], 0] && GtQ[Simplify[b/(b*e - a*f)], 0] && !(GtQ[Simpl
ify[d/(d*a - c*b)], 0] && GtQ[Simplify[d/(d*e - c*f)], 0] && SimplerQ[c + d
*x, a + b*x]) && !(GtQ[Simplify[f/(f*a - e*b)], 0] && GtQ[Simplify[f/(f*c
- e*d)], 0] && SimplerQ[e + f*x, a + b*x])
```

rule 156
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(e + f*x)^FracPart[p]/(Simplify[b/(b*e - a*f)]^IntPart[p
]*(b*((e + f*x)/(b*e - a*f)))^FracPart[p])   Int[(a + b*x)^m*(c + d*x)^n*Si
mp[b*(e/(b*e - a*f)) + b*f*(x/(b*e - a*f)), x]^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] &
& GtQ[Simplify[b/(b*c - a*d)], 0] && !GtQ[Simplify[b/(b*e - a*f)], 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4321
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[Cot[e + f*x]/(f*Sqrt[1 + Csc[e + f*x]]*Sqrt[1 - Csc[e + f*x
]])   Subst[Int[(a + b*x)^m/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Csc[e + f*x]]
, x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0] && !IntegerQ[2*m]
```

3.702. $\int \frac{\sec(c+dx)}{\sqrt[3]{a+b\sec(c+dx)}}\,dx$

3.702.4 Maple [F]

$$\int \frac{\sec(dx+c)}{(a+b\sec(dx+c))^{\frac{1}{3}}}\,dx$$

input
```
int(sec(d*x+c)/(a+b*sec(d*x+c))^(1/3),x)
```

output
```
int(sec(d*x+c)/(a+b*sec(d*x+c))^(1/3),x)
```

3.702.5 Fricas [F]

$$\int \frac{\sec(c+dx)}{\sqrt[3]{a+b\sec(c+dx)}}\,dx = \int \frac{\sec(dx+c)}{(b\sec(dx+c)+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate(sec(d*x+c)/(a+b*sec(d*x+c))^(1/3),x, algorithm="fricas")
```

output
```
integral(sec(d*x + c)/(b*sec(d*x + c) + a)^(1/3), x)
```

3.702.6 Sympy [F]

$$\int \frac{\sec(c+dx)}{\sqrt[3]{a+b\sec(c+dx)}}\,dx = \int \frac{\sec(c+dx)}{\sqrt[3]{a+b\sec(c+dx)}}\,dx$$

input
```
integrate(sec(d*x+c)/(a+b*sec(d*x+c))**(1/3),x)
```

output
```
Integral(sec(c + d*x)/(a + b*sec(c + d*x))**(1/3), x)
```

3.702.7 Maxima [F]

$$\int \frac{\sec(c+dx)}{\sqrt[3]{a+b\sec(c+dx)}}\,dx = \int \frac{\sec(dx+c)}{(b\sec(dx+c)+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate(sec(d*x+c)/(a+b*sec(d*x+c))^(1/3),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)/(b*sec(d*x + c) + a)^(1/3), x)
```

3.702.8 Giac [F]

$$\int \frac{\sec(c+dx)}{\sqrt[3]{a+b\sec(c+dx)}}\,dx = \int \frac{\sec(dx+c)}{(b\sec(dx+c)+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate(sec(d*x+c)/(a+b*sec(d*x+c))^(1/3),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)/(b*sec(d*x + c) + a)^(1/3), x)
```

3.702.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(c+dx)}{\sqrt[3]{a+b\sec(c+dx)}}\,dx = \int \frac{1}{\cos(c+d\,x)\,\left(a+\frac{b}{\cos(c+d\,x)}\right)^{1/3}}\,dx$$

input
```
int(1/(cos(c + d*x)*(a + b/cos(c + d*x))^(1/3)),x)
```

output
```
int(1/(cos(c + d*x)*(a + b/cos(c + d*x))^(1/3)), x)
```

3.703 $\int \frac{1}{\sqrt[3]{a+b\sec(c+dx)}}\,dx$

3.703.1 Optimal result 5462
3.703.2 Mathematica [N/A] 5462
3.703.3 Rubi [N/A] 5463
3.703.4 Maple [N/A] (verified) 5464
3.703.5 Fricas [F(-1)] 5464
3.703.6 Sympy [N/A] 5464
3.703.7 Maxima [N/A] 5465
3.703.8 Giac [N/A] 5465
3.703.9 Mupad [N/A] 5465

3.703.1 Optimal result

Integrand size = 14, antiderivative size = 14

$$\int \frac{1}{\sqrt[3]{a+b\sec(c+dx)}}\,dx = \text{Int}\left(\frac{1}{\sqrt[3]{a+b\sec(c+dx)}},x\right)$$

output
```
Unintegrable(1/(a+b*sec(d*x+c))^(1/3),x)
```

3.703.2 Mathematica [N/A]

Not integrable

Time = 1.57 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{\sqrt[3]{a+b\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt[3]{a+b\sec(c+dx)}}\,dx$$

input
```
Integrate[(a + b*Sec[c + d*x])^(-1/3),x]
```

output
```
Integrate[(a + b*Sec[c + d*x])^(-1/3), x]
```

3.703.3 Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 4273}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[3]{a+b\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt[3]{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4273}$$

$$\int \frac{1}{\sqrt[3]{a+b\sec(c+dx)}}dx$$

input
```
Int[(a + b*Sec[c + d*x])^(-1/3),x]
```

output
```
$Aborted
```

3.703.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4273
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Unintegrable[
(a + b*Csc[c + d*x])^n, x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[a^2 - b^2, 0
] &&  !IntegerQ[2*n]
```

3.703.4 Maple [N/A] (verified)

Not integrable

Time = 0.32 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{1}{(a+b\sec(dx+c))^{\frac{1}{3}}}dx$$

input
```
int(1/(a+b*sec(d*x+c))^(1/3),x)
```

output
```
int(1/(a+b*sec(d*x+c))^(1/3),x)
```

3.703.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{a+b\sec(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+b*sec(d*x+c))^(1/3),x, algorithm="fricas")
```

output
```
Timed out
```

3.703.6 Sympy [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

$$\int \frac{1}{\sqrt[3]{a+b\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt[3]{a+b\sec(c+dx)}}\,dx$$

input
```
integrate(1/(a+b*sec(d*x+c))**(1/3),x)
```

output
```
Integral((a + b*sec(c + d*x))**(-1/3), x)
```

3.703.7 Maxima [N/A]

Not integrable

Time = 0.56 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt[3]{a+b\sec(c+dx)}}\,dx = \int \frac{1}{(b\sec(dx+c)+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/(a+b*sec(d*x+c))^(1/3),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^(-1/3), x)
```

3.703.8 Giac [N/A]

Not integrable

Time = 0.51 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt[3]{a+b\sec(c+dx)}}\,dx = \int \frac{1}{(b\sec(dx+c)+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/(a+b*sec(d*x+c))^(1/3),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^(-1/3), x)
```

3.703.9 Mupad [N/A]

Not integrable

Time = 14.27 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{\sqrt[3]{a+b\sec(c+dx)}}\,dx = \int \frac{1}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{1/3}}\,dx$$

input
```
int(1/(a + b/cos(c + d*x))^(1/3),x)
```

output
```
int(1/(a + b/cos(c + d*x))^(1/3), x)
```

3.704 $\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^{2/3}}\,dx$

3.704.1 Optimal result 5466
3.704.2 Mathematica [B] (warning: unable to verify) 5466
3.704.3 Rubi [A] (verified) 5467
3.704.4 Maple [F] 5469
3.704.5 Fricas [F] 5469
3.704.6 Sympy [F] 5469
3.704.7 Maxima [F] 5470
3.704.8 Giac [F] 5470
3.704.9 Mupad [F(-1)] 5470

3.704.1 Optimal result

Integrand size = 21, antiderivative size = 105

$$\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^{2/3}}\,dx = \frac{\sqrt{2}\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},\frac{2}{3},\frac{3}{2},\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{2/3}\tan}{d\sqrt{1+\sec(c+dx)}(a+b\sec(c+dx))^{2/3}}$$

output
```
AppellF1(1/2,2/3,1/2,3/2,b*(1-sec(d*x+c))/(a+b),1/2-1/2*sec(d*x+c))*((a+b*
sec(d*x+c))/(a+b))^(2/3)*2^(1/2)*tan(d*x+c)/d/(a+b*sec(d*x+c))^(2/3)/(1+se
c(d*x+c))^(1/2)
```

3.704.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 310 vs. 2(105) = 210.

Time = 15.34 (sec) , antiderivative size = 310, normalized size of antiderivative = 2.95

$$\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^{2/3}}\,dx = \frac{24(a-b)^2(a+b)\,\text{AppellF1}\left(\right.}{b^2(-a+b)d\left(3(a-b)\,\text{AppellF1}\left(\frac{4}{3},\frac{1}{2},\frac{3}{2},\frac{7}{3},\frac{a+b\sec(c+dx)}{a-b},\frac{a+b\sec(c+dx)}{a+b}\right)(b+a\,\text{co}\right.}$$

input
```
Integrate[Sec[c + d*x]/(a + b*Sec[c + d*x])^(2/3),x]
```

output
```
(24*(a - b)^2*(a + b)*AppellF1[1/3, 1/2, 1/2, 4/3, (a + b*Sec[c + d*x])/(a
 - b), (a + b*Sec[c + d*x])/(a + b)]*Cos[c + d*x]*Cot[c + d*x]^3*(1 + Sec[
c + d*x])*(b - b*Sec[c + d*x])*(a + b*Sec[c + d*x])^(1/3))/(b^2*(-a + b)*d
*(3*(a - b)*AppellF1[4/3, 1/2, 3/2, 7/3, (a + b*Sec[c + d*x])/(a - b), (a
+ b*Sec[c + d*x])/(a + b)]*(b + a*Cos[c + d*x]) + (a + b)*(8*(a - b)*Appel
lF1[1/3, 1/2, 1/2, 4/3, (a + b*Sec[c + d*x])/(a - b), (a + b*Sec[c + d*x])
/(a + b)]*Cos[c + d*x] + 3*AppellF1[4/3, 3/2, 1/2, 7/3, (a + b*Sec[c + d*x
])/(a - b), (a + b*Sec[c + d*x])/(a + b)]*(b + a*Cos[c + d*x]))))
```

3.704.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3042, 4321, 156, 155}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^{2/3}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{2/3}}dx$$

$$\Big\downarrow \text{4321}$$

$$-\frac{\tan(c+dx)\int \frac{1}{\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}(a+b\sec(c+dx))^{2/3}}d\sec(c+dx)}{d\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}}$$

$$\Big\downarrow \text{156}$$

$$-\frac{\tan(c+dx)\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{2/3}\int \frac{1}{\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}\left(\frac{a}{a+b}+\frac{b\sec(c+dx)}{a+b}\right)^{2/3}}d\sec(c+dx)}{d\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}(a+b\sec(c+dx))^{2/3}}$$

$$\Big\downarrow \text{155}$$

$$\frac{\sqrt{2}\tan(c+dx)\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{2/3}\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},\frac{2}{3},\frac{3}{2},\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)}{d\sqrt{\sec(c+dx)+1}(a+b\sec(c+dx))^{2/3}}$$

input
```
Int[Sec[c + d*x]/(a + b*Sec[c + d*x])^(2/3),x]
```

output
```
(Sqrt[2]*AppellF1[1/2, 1/2, 2/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[c
+ d*x]))/(a + b)]*((a + b*Sec[c + d*x])/(a + b))^(2/3)*Tan[c + d*x])/(d*Sq
rt[1 + Sec[c + d*x]]*(a + b*Sec[c + d*x])^(2/3))
```

3.704.3.1 Defintions of rubi rules used

rule 155
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*Simplify[b/(b*c - a*d)]^n*
Simplify[b/(b*e - a*f)]^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/
(b*c - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f,
 m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[Sim
plify[b/(b*c - a*d)], 0] && GtQ[Simplify[b/(b*e - a*f)], 0] && !(GtQ[Simpl
ify[d/(d*a - c*b)], 0] && GtQ[Simplify[d/(d*e - c*f)], 0] && SimplerQ[c + d
*x, a + b*x]) && !(GtQ[Simplify[f/(f*a - e*b)], 0] && GtQ[Simplify[f/(f*c
- e*d)], 0] && SimplerQ[e + f*x, a + b*x])
```

rule 156
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(e + f*x)^FracPart[p]/(Simplify[b/(b*e - a*f)]^IntPart[p
]*(b*((e + f*x)/(b*e - a*f)))^FracPart[p])   Int[(a + b*x)^m*(c + d*x)^n*Si
mp[b*(e/(b*e - a*f)) + b*f*(x/(b*e - a*f)), x]^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] &
& GtQ[Simplify[b/(b*c - a*d)], 0] && !GtQ[Simplify[b/(b*e - a*f)], 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4321
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[Cot[e + f*x]/(f*Sqrt[1 + Csc[e + f*x]]*Sqrt[1 - Csc[e + f*x
]])   Subst[Int[(a + b*x)^m/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Csc[e + f*x]]
, x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0] && !IntegerQ[2*m]
```

3.704.4 Maple [F]

$$\int \frac{\sec(dx+c)}{(a+b\sec(dx+c))^{\frac{2}{3}}}\,dx$$

input
```
int(sec(d*x+c)/(a+b*sec(d*x+c))^(2/3),x)
```

output
```
int(sec(d*x+c)/(a+b*sec(d*x+c))^(2/3),x)
```

3.704.5 Fricas [F]

$$\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^{2/3}}\,dx = \int \frac{\sec(dx+c)}{(b\sec(dx+c)+a)^{\frac{2}{3}}}\,dx$$

input
```
integrate(sec(d*x+c)/(a+b*sec(d*x+c))^(2/3),x, algorithm="fricas")
```

output
```
integral(sec(d*x + c)/(b*sec(d*x + c) + a)^(2/3), x)
```

3.704.6 Sympy [F]

$$\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^{2/3}}\,dx = \int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^{\frac{2}{3}}}\,dx$$

input
```
integrate(sec(d*x+c)/(a+b*sec(d*x+c))**(2/3),x)
```

output
```
Integral(sec(c + d*x)/(a + b*sec(c + d*x))**(2/3), x)
```

3.704.7 Maxima [F]

$$\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^{2/3}}\,dx = \int \frac{\sec(dx+c)}{(b\sec(dx+c)+a)^{\frac{2}{3}}}\,dx$$

input
```
integrate(sec(d*x+c)/(a+b*sec(d*x+c))^(2/3),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)/(b*sec(d*x + c) + a)^(2/3), x)
```

3.704.8 Giac [F]

$$\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^{2/3}}\,dx = \int \frac{\sec(dx+c)}{(b\sec(dx+c)+a)^{\frac{2}{3}}}\,dx$$

input
```
integrate(sec(d*x+c)/(a+b*sec(d*x+c))^(2/3),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)/(b*sec(d*x + c) + a)^(2/3), x)
```

3.704.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^{2/3}}\,dx = \int \frac{1}{\cos(c+d\,x)\,\left(a+\frac{b}{\cos(c+d\,x)}\right)^{2/3}}\,dx$$

input
```
int(1/(cos(c + d*x)*(a + b/cos(c + d*x))^(2/3)),x)
```

output
```
int(1/(cos(c + d*x)*(a + b/cos(c + d*x))^(2/3)), x)
```

3.705 $\int \frac{1}{(a+b\sec(c+dx))^{2/3}}\,dx$

3.705.1 Optimal result 5471
3.705.2 Mathematica [N/A] 5471
3.705.3 Rubi [N/A] 5472
3.705.4 Maple [N/A] (verified) 5473
3.705.5 Fricas [F(-1)] 5473
3.705.6 Sympy [N/A] 5473
3.705.7 Maxima [N/A] 5474
3.705.8 Giac [N/A] 5474
3.705.9 Mupad [N/A] 5474

3.705.1 Optimal result

Integrand size = 14, antiderivative size = 14

$$\int \frac{1}{(a+b\sec(c+dx))^{2/3}}\,dx=\text{Int}\left(\frac{1}{(a+b\sec(c+dx))^{2/3}},x\right)$$

output
```
Unintegrable(1/(a+b*sec(d*x+c))^(2/3),x)
```

3.705.2 Mathematica [N/A]

Not integrable

Time = 1.73 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{(a+b\sec(c+dx))^{2/3}}\,dx=\int \frac{1}{(a+b\sec(c+dx))^{2/3}}\,dx$$

input
```
Integrate[(a + b*Sec[c + d*x])^(-2/3),x]
```

output
```
Integrate[(a + b*Sec[c + d*x])^(-2/3), x]
```

3.705.3 Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 4273}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+b\sec(c+dx))^{2/3}}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{2/3}}dx$$

$$\Big\downarrow \text{ 4273}$$

$$\int \frac{1}{(a+b\sec(c+dx))^{2/3}}dx$$

input
```
Int[(a + b*Sec[c + d*x])^(-2/3),x]
```

output
```
$Aborted
```

3.705.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4273
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Unintegrable[
(a + b*Csc[c + d*x])^n, x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[a^2 - b^2, 0
] &&  !IntegerQ[2*n]
```

3.705.4 Maple [N/A] (verified)

Not integrable

Time = 0.34 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{1}{(a+b\sec(dx+c))^{\frac{2}{3}}}dx$$

input
```
int(1/(a+b*sec(d*x+c))^(2/3),x)
```

output
```
int(1/(a+b*sec(d*x+c))^(2/3),x)
```

3.705.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\sec(c+dx))^{2/3}}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+b*sec(d*x+c))^(2/3),x, algorithm="fricas")
```

output
```
Timed out
```

3.705.6 Sympy [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

$$\int \frac{1}{(a+b\sec(c+dx))^{2/3}}\,dx = \int \frac{1}{(a+b\sec(c+dx))^{\frac{2}{3}}}\,dx$$

input
```
integrate(1/(a+b*sec(d*x+c))**(2/3),x)
```

output
```
Integral((a + b*sec(c + d*x))**(-2/3), x)
```

3.705.7 Maxima [N/A]

Not integrable

Time = 0.56 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{(a+b\sec(c+dx))^{2/3}}\,dx = \int \frac{1}{(b\sec(dx+c)+a)^{\frac{2}{3}}}\,dx$$

input
```
integrate(1/(a+b*sec(d*x+c))^(2/3),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^(-2/3), x)
```

3.705.8 Giac [N/A]

Not integrable

Time = 0.91 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{(a+b\sec(c+dx))^{2/3}}\,dx = \int \frac{1}{(b\sec(dx+c)+a)^{\frac{2}{3}}}\,dx$$

input
```
integrate(1/(a+b*sec(d*x+c))^(2/3),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^(-2/3), x)
```

3.705.9 Mupad [N/A]

Not integrable

Time = 14.44 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{(a+b\sec(c+dx))^{2/3}}\,dx = \int \frac{1}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{2/3}}\,dx$$

input
```
int(1/(a + b/cos(c + d*x))^(2/3),x)
```

output
```
int(1/(a + b/cos(c + d*x))^(2/3), x)
```

3.706 $\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^{4/3}}\,dx$

3.706.1 Optimal result 5475
3.706.2 Mathematica [B] (warning: unable to verify) 5475
3.706.3 Rubi [A] (verified) 5476
3.706.4 Maple [F] 5477
3.706.5 Fricas [F] 5478
3.706.6 Sympy [F] 5478
3.706.7 Maxima [F] 5478
3.706.8 Giac [F] 5479
3.706.9 Mupad [F(-1)] 5479

3.706.1 Optimal result

Integrand size = 21, antiderivative size = 110

$$\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^{4/3}}\,dx=\frac{\sqrt{2}\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},\frac{4}{3},\frac{3}{2},\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)\sqrt[3]{\frac{a+b\sec(c+dx)}{a+b}}}{(a+b)d\sqrt{1+\sec(c+dx)}\sqrt[3]{a+b\sec(c+dx)}}$$

output
```
AppellF1(1/2,4/3,1/2,3/2,b*(1-sec(d*x+c))/(a+b),1/2-1/2*sec(d*x+c))*((a+b*
sec(d*x+c))/(a+b))^(1/3)*2^(1/2)*tan(d*x+c)/(a+b)/d/(a+b*sec(d*x+c))^(1/3)
/(1+sec(d*x+c))^(1/2)
```

3.706.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 10343 vs. 2(110) = 220.

Time = 47.20 (sec) , antiderivative size = 10343, normalized size of antiderivative = 94.03

$$\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^{4/3}}\,dx=\text{Result too large to show}$$

input
```
Integrate[Sec[c + d*x]/(a + b*Sec[c + d*x])^(4/3),x]
```

output
```
Result too large to show
```

3.706.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3042, 4321, 156, 155}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^{4/3}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{4/3}}dx$$

$$\Big\downarrow \text{4321}$$

$$-\frac{\tan(c+dx)\int \frac{1}{\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}(a+b\sec(c+dx))^{4/3}}d\sec(c+dx)}{d\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}}$$

$$\Big\downarrow \text{156}$$

$$-\frac{\tan(c+dx)\sqrt[3]{\frac{a+b\sec(c+dx)}{a+b}}\int \frac{1}{\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}\left(\frac{a}{a+b}+\frac{b\sec(c+dx)}{a+b}\right)^{4/3}}d\sec(c+dx)}{d(a+b)\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}\sqrt[3]{a+b\sec(c+dx)}}$$

$$\Big\downarrow \text{155}$$

$$\frac{\sqrt{2}\tan(c+dx)\sqrt[3]{\frac{a+b\sec(c+dx)}{a+b}}\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},\frac{4}{3},\frac{3}{2},\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)}{d(a+b)\sqrt{\sec(c+dx)+1}\sqrt[3]{a+b\sec(c+dx)}}$$

input
```
Int[Sec[c + d*x]/(a + b*Sec[c + d*x])^(4/3),x]
```

output
```
(Sqrt[2]*AppellF1[1/2, 1/2, 4/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[c
+ d*x]))/(a + b)]*((a + b*Sec[c + d*x])/(a + b))^(1/3)*Tan[c + d*x])/((a +
 b)*d*Sqrt[1 + Sec[c + d*x]]*(a + b*Sec[c + d*x])^(1/3))
```

3.706.3.1 Defintions of rubi rules used

rule 155
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*Simplify[b/(b*c - a*d)]^n*
Simplify[b/(b*e - a*f)]^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/
(b*c - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f,
 m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[Sim
plify[b/(b*c - a*d)], 0] && GtQ[Simplify[b/(b*e - a*f)], 0] && !(GtQ[Simpl
ify[d/(d*a - c*b)], 0] && GtQ[Simplify[d/(d*e - c*f)], 0] && SimplerQ[c + d
*x, a + b*x]) && !(GtQ[Simplify[f/(f*a - e*b)], 0] && GtQ[Simplify[f/(f*c
- e*d)], 0] && SimplerQ[e + f*x, a + b*x])
```

rule 156
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(e + f*x)^FracPart[p]/(Simplify[b/(b*e - a*f)]^IntPart[p
]*(b*((e + f*x)/(b*e - a*f)))^FracPart[p])   Int[(a + b*x)^m*(c + d*x)^n*Si
mp[b*(e/(b*e - a*f)) + b*f*(x/(b*e - a*f)), x]^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] &
& GtQ[Simplify[b/(b*c - a*d)], 0] && !GtQ[Simplify[b/(b*e - a*f)], 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4321
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[Cot[e + f*x]/(f*Sqrt[1 + Csc[e + f*x]]*Sqrt[1 - Csc[e + f*x
]])   Subst[Int[(a + b*x)^m/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Csc[e + f*x]]
, x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0] && !IntegerQ[2*m]
```

3.706.4 Maple [F]

$$\int \frac{\sec(dx+c)}{(a+b\sec(dx+c))^{\frac{4}{3}}}\,dx$$

input
```
int(sec(d*x+c)/(a+b*sec(d*x+c))^(4/3),x)
```

output
```
int(sec(d*x+c)/(a+b*sec(d*x+c))^(4/3),x)
```

3.706.5 Fricas [F]

$$\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^{4/3}}\,dx = \int \frac{\sec(dx+c)}{(b\sec(dx+c)+a)^{\frac{4}{3}}}\,dx$$

input
```
integrate(sec(d*x+c)/(a+b*sec(d*x+c))^(4/3),x, algorithm="fricas")
```

output
```
integral((b*sec(d*x + c) + a)^(2/3)*sec(d*x + c)/(b^2*sec(d*x + c)^2 + 2*a
*b*sec(d*x + c) + a^2), x)
```

3.706.6 Sympy [F]

$$\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^{4/3}}\,dx = \int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^{\frac{4}{3}}}\,dx$$

input
```
integrate(sec(d*x+c)/(a+b*sec(d*x+c))**(4/3),x)
```

output
```
Integral(sec(c + d*x)/(a + b*sec(c + d*x))**(4/3), x)
```

3.706.7 Maxima [F]

$$\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^{4/3}}\,dx = \int \frac{\sec(dx+c)}{(b\sec(dx+c)+a)^{\frac{4}{3}}}\,dx$$

input
```
integrate(sec(d*x+c)/(a+b*sec(d*x+c))^(4/3),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)/(b*sec(d*x + c) + a)^(4/3), x)
```

3.706.8 Giac [F]

$$\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^{4/3}}\,dx = \int \frac{\sec(dx+c)}{(b\sec(dx+c)+a)^{\frac{4}{3}}}\,dx$$

input
```
integrate(sec(d*x+c)/(a+b*sec(d*x+c))^(4/3),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)/(b*sec(d*x + c) + a)^(4/3), x)
```

3.706.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^{4/3}}\,dx = \int \frac{1}{\cos(c+d\,x)\,\left(a+\frac{b}{\cos(c+d\,x)}\right)^{4/3}}\,dx$$

input
```
int(1/(cos(c + d*x)*(a + b/cos(c + d*x))^(4/3)),x)
```

output
```
int(1/(cos(c + d*x)*(a + b/cos(c + d*x))^(4/3)), x)
```

3.707 $\int \frac{1}{(a+b\sec(c+dx))^{4/3}}\,dx$

3.707.1 Optimal result 5480
3.707.2 Mathematica [N/A] 5480
3.707.3 Rubi [N/A] 5481
3.707.4 Maple [N/A] (verified) 5482
3.707.5 Fricas [F(-1)] 5482
3.707.6 Sympy [N/A] 5482
3.707.7 Maxima [N/A] 5483
3.707.8 Giac [N/A] 5483
3.707.9 Mupad [N/A] 5483

3.707.1 Optimal result

Integrand size = 14, antiderivative size = 14

$$\int \frac{1}{(a+b\sec(c+dx))^{4/3}}\,dx=\text{Int}\left(\frac{1}{(a+b\sec(c+dx))^{4/3}},x\right)$$

output
```
Unintegrable(1/(a+b*sec(d*x+c))^(4/3),x)
```

3.707.2 Mathematica [N/A]

Not integrable

Time = 91.41 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{(a+b\sec(c+dx))^{4/3}}\,dx=\int \frac{1}{(a+b\sec(c+dx))^{4/3}}\,dx$$

input
```
Integrate[(a + b*Sec[c + d*x])^(-4/3),x]
```

output
```
Integrate[(a + b*Sec[c + d*x])^(-4/3), x]
```

3.707.3 Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 4273}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+b\sec(c+dx))^{4/3}}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{4/3}}dx$$

$$\Big\downarrow \text{ 4273}$$

$$\int \frac{1}{(a+b\sec(c+dx))^{4/3}}dx$$

input
```
Int[(a + b*Sec[c + d*x])^(-4/3),x]
```

output
```
$Aborted
```

3.707.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4273
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Unintegrable[
(a + b*Csc[c + d*x])^n, x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[a^2 - b^2, 0
] &&  !IntegerQ[2*n]
```

3.707.4 Maple [N/A] (verified)

Not integrable

Time = 0.34 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{1}{(a+b\sec(dx+c))^{\frac{4}{3}}}dx$$

input
```
int(1/(a+b*sec(d*x+c))^(4/3),x)
```

output
```
int(1/(a+b*sec(d*x+c))^(4/3),x)
```

3.707.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\sec(c+dx))^{4/3}}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+b*sec(d*x+c))^(4/3),x, algorithm="fricas")
```

output
```
Timed out
```

3.707.6 Sympy [N/A]

Not integrable

Time = 1.26 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

$$\int \frac{1}{(a+b\sec(c+dx))^{4/3}}\,dx = \int \frac{1}{(a+b\sec(c+dx))^{\frac{4}{3}}}\,dx$$

input
```
integrate(1/(a+b*sec(d*x+c))**(4/3),x)
```

output
```
Integral((a + b*sec(c + d*x))**(-4/3), x)
```

3.707.7 Maxima [N/A]

Not integrable

Time = 0.57 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{(a+b\sec(c+dx))^{4/3}}\,dx = \int \frac{1}{(b\sec(dx+c)+a)^{\frac{4}{3}}}\,dx$$

input
```
integrate(1/(a+b*sec(d*x+c))^(4/3),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^(-4/3), x)
```

3.707.8 Giac [N/A]

Not integrable

Time = 0.88 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{(a+b\sec(c+dx))^{4/3}}\,dx = \int \frac{1}{(b\sec(dx+c)+a)^{\frac{4}{3}}}\,dx$$

input
```
integrate(1/(a+b*sec(d*x+c))^(4/3),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^(-4/3), x)
```

3.707.9 Mupad [N/A]

Not integrable

Time = 15.53 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{(a+b\sec(c+dx))^{4/3}}\,dx = \int \frac{1}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{4/3}}\,dx$$

input
```
int(1/(a + b/cos(c + d*x))^(4/3),x)
```

output
```
int(1/(a + b/cos(c + d*x))^(4/3), x)
```

3.708 $\int \frac{\sec^4(c+dx)}{(a+b\sec(c+dx))^{5/3}}\,dx$

3.708.1 Optimal result . . . 5484
3.708.2 Mathematica [B] (warning: unable to verify) . . . 5485
3.708.3 Rubi [A] (verified) . . . 5485
3.708.4 Maple [F] . . . 5489
3.708.5 Fricas [F] . . . 5490
3.708.6 Sympy [F] . . . 5490
3.708.7 Maxima [F] . . . 5490
3.708.8 Giac [F] . . . 5491
3.708.9 Mupad [F(-1)] . . . 5491

3.708.1 Optimal result

Integrand size = 23, antiderivative size = 378

$$\int \frac{\sec^4(c+dx)}{(a+b\sec(c+dx))^{5/3}}\,dx = -\frac{3a^2\sec(c+dx)\tan(c+dx)}{2b\,(a^2-b^2)\,d(a+b\sec(c+dx))^{2/3}} + \frac{3(3a^2-b^2)\sqrt[3]{a+b\sec(c+dx)}\tan(c+dx)}{4b^2\,(a^2-b^2)\,d} - \frac{a(9a^2-7b^2)\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},-\frac{1}{3},\frac{3}{2},\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)\sqrt[3]{a+b\sec(c+dx)}\tan(c+dx)}{2\sqrt{2}b^3\,(a^2-b^2)\,d\sqrt{1+\sec(c+dx)}\sqrt[3]{\frac{a+b\sec(c+dx)}{a+b}}} + \frac{(9a^4-10a^2b^2-b^4)\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},\frac{2}{3},\frac{3}{2},\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{2/3}\tan(c+dx)}{2\sqrt{2}b^3\,(a^2-b^2)\,d\sqrt{1+\sec(c+dx)}(a+b\sec(c+dx))^{2/3}}$$

output
```
-3/2*a^2*sec(d*x+c)*tan(d*x+c)/b/(a^2-b^2)/d/(a+b*sec(d*x+c))^(2/3)+3/4*(3
*a^2-b^2)*(a+b*sec(d*x+c))^(1/3)*tan(d*x+c)/b^2/(a^2-b^2)/d-1/4*a*(9*a^2-7
*b^2)*AppellF1(1/2,-1/3,1/2,3/2,b*(1-sec(d*x+c))/(a+b),1/2-1/2*sec(d*x+c))
*(a+b*sec(d*x+c))^(1/3)*tan(d*x+c)/b^3/(a^2-b^2)/d/((a+b*sec(d*x+c))/(a+b)
)^(1/3)*2^(1/2)/(1+sec(d*x+c))^(1/2)+1/4*(9*a^4-10*a^2*b^2-b^4)*AppellF1(1
/2,2/3,1/2,3/2,b*(1-sec(d*x+c))/(a+b),1/2-1/2*sec(d*x+c))*((a+b*sec(d*x+c)
)/(a+b))^(2/3)*tan(d*x+c)/b^3/(a^2-b^2)/d/(a+b*sec(d*x+c))^(2/3)*2^(1/2)/(
1+sec(d*x+c))^(1/2)
```

3.708.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 21987 vs. $2(378) = 756$.

Time = 44.23 (sec) , antiderivative size = 21987, normalized size of antiderivative = 58.17

$$\int \frac{\sec^4(c+dx)}{(a+b\sec(c+dx))^{5/3}}\,dx = \text{Result too large to show}$$

input
```
Integrate[Sec[c + d*x]^4/(a + b*Sec[c + d*x])^(5/3),x]
```

output
```
Result too large to show
```

3.708.3 Rubi [A] (verified)

Time = 1.18 (sec) , antiderivative size = 367, normalized size of antiderivative = 0.97, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.522$, Rules used = {3042, 4332, 27, 3042, 4570, 27, 3042, 4495, 3042, 4321, 156, 155}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^4(c+dx)}{(a+b\sec(c+dx))^{5/3}}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^4}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/3}}dx$$

$$\Big\downarrow \; 4332$$

$$-\frac{3\int \frac{\sec(c+dx)\left(3a^2-2b\sec(c+dx)a-2\left(3a^2-b^2\right)\sec^2(c+dx)\right)}{3(a+b\sec(c+dx))^{2/3}}dx}{2b\left(a^2-b^2\right)}-\frac{3a^2\tan(c+dx)\sec(c+dx)}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^{2/3}}$$

$$\Big\downarrow \; 27$$

$$-\frac{\int \frac{\sec(c+dx)\left(3a^2-2b\sec(c+dx)a-2\left(3a^2-b^2\right)\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^{2/3}}dx}{2b\left(a^2-b^2\right)}-\frac{3a^2\tan(c+dx)\sec(c+dx)}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^{2/3}}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(3a^2-2b\csc\left(c+dx+\frac{\pi}{2}\right)a-2\left(3a^2-b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{2/3}}dx}{2b\left(a^2-b^2\right)}-\frac{3a^2\tan(c+dx)\sec(c+dx)}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^{2/3}}$$

$\downarrow$ 4570

$$-\frac{\frac{3\int \frac{2\sec(c+dx)\left(b\left(3a^2+b^2\right)+a\left(9a^2-7b^2\right)\sec(c+dx)\right)}{3(a+b\sec(c+dx))^{2/3}}dx}{4b}-\frac{3\left(3a^2-b^2\right)\tan(c+dx)\sqrt[3]{a+b\sec(c+dx)}}{2bd}}{2b\left(a^2-b^2\right)}-\frac{3a^2\tan(c+dx)\sec(c+dx)}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^{2/3}}$$

$\downarrow$ 27

$$-\frac{\frac{\int \frac{\sec(c+dx)\left(b\left(3a^2+b^2\right)+a\left(9a^2-7b^2\right)\sec(c+dx)\right)}{(a+b\sec(c+dx))^{2/3}}dx}{2b}-\frac{3\left(3a^2-b^2\right)\tan(c+dx)\sqrt[3]{a+b\sec(c+dx)}}{2bd}}{2b\left(a^2-b^2\right)}-\frac{3a^2\tan(c+dx)\sec(c+dx)}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^{2/3}}$$

$\downarrow$ 3042

$$-\frac{\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(b\left(3a^2+b^2\right)+a\left(9a^2-7b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{2/3}}dx}{2b}-\frac{3\left(3a^2-b^2\right)\tan(c+dx)\sqrt[3]{a+b\sec(c+dx)}}{2bd}}{2b\left(a^2-b^2\right)}-\frac{3a^2\tan(c+dx)\sec(c+dx)}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^{2/3}}$$

$\downarrow$ 4495

$$-\frac{\frac{a\left(9a^2-7b^2\right)\int \sec(c+dx)\sqrt[3]{a+b\sec(c+dx)}dx}{b}-\frac{\left(9a^4-10a^2b^2-b^4\right)\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^{2/3}}dx}{b}}{2b}-\frac{3\left(3a^2-b^2\right)\tan(c+dx)\sqrt[3]{a+b\sec(c+dx)}}{2bd}}{2b\left(a^2-b^2\right)}-\frac{3a^2\tan(c+dx)\sec(c+dx)}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^{2/3}}$$

$\downarrow$ 3042

$$-\frac{\frac{a\left(9a^2-7b^2\right)\int \csc\left(c+dx+\frac{\pi}{2}\right)\sqrt[3]{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b}-\frac{\left(9a^4-10a^2b^2-b^4\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{2/3}}dx}{b}}{2b}-\frac{3\left(3a^2-b^2\right)\tan(c+dx)\sqrt[3]{a+}}{2bd}}{2b\left(a^2-b^2\right)}-\frac{3a^2\tan(c+dx)\sec(c+dx)}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^{2/3}}$$

$\downarrow$ 4321

3.708. $\int \frac{\sec^4(c+dx)}{(a+b\sec(c+dx))^{5/3}}dx$

$$-\frac{\frac{\frac{\left(9a^4-10a^2b^2-b^4\right)\tan(c+dx)\int \frac{1}{\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}(a+b\sec(c+dx))^{2/3}}d\sec(c+dx)}{bd\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}}-\frac{a\left(9a^2-7b^2\right)\tan(c+dx)\int \frac{\sqrt[3]{a+b\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}}d\sec(c}{bd\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}}}{2b}}{2b\left(a^2-b^2\right)}$$

$$\frac{3a^2\tan(c+dx)\sec(c+dx)}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^{2/3}}$$

$$\Big\downarrow \; 156$$

$$-\frac{\frac{\frac{\left(9a^4-10a^2b^2-b^4\right)\tan(c+dx)\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{2/3}\int \frac{1}{\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}\left(\frac{a}{a+b}+\frac{b\sec(c+dx)}{a+b}\right)^{2/3}}d\sec(c+dx)}{bd\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}(a+b\sec(c+dx))^{2/3}}-\frac{a\left(9a^2-7b^2\right)\tan(c+dx)\sqrt[3]{a+b}}{bd\sqrt{1-\sec(c+d}}}{2b}}{2b\left(a^2-b^2\right)}$$

$$\frac{3a^2\tan(c+dx)\sec(c+dx)}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^{2/3}}$$

$$\Big\downarrow \; 155$$

$$-\frac{3a^2\tan(c+dx)\sec(c+dx)}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^{2/3}}-\frac{\frac{\frac{\sqrt{2}a\left(9a^2-7b^2\right)\tan(c+dx)\sqrt[3]{a+b\sec(c+dx)}\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},-\frac{1}{3},\frac{3}{2},\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)}{bd\sqrt{\sec(c+dx)+1}\sqrt[3]{\frac{a+b\sec(c+dx)}{a+b}}}-\frac{\sqrt{2}\left(9a^4-10a^2b^2-b^4\right)\tan(c+dx)\left(\frac{a+b\sec(c+d}{a+b}\right.}{bd\sqrt{\sec(c}}}{2b}}{2b\left(a^2-b^2\right)}$$

input
```
Int[Sec[c + d*x]^4/(a + b*Sec[c + d*x])^(5/3),x]
```

output
```
(-3*a^2*Sec[c + d*x]*Tan[c + d*x])/(2*b*(a^2 - b^2)*d*(a + b*Sec[c + d*x])
^(2/3)) - ((-3*(3*a^2 - b^2)*(a + b*Sec[c + d*x])^(1/3)*Tan[c + d*x])/(2*b
*d) + ((Sqrt[2]*a*(9*a^2 - 7*b^2)*AppellF1[1/2, 1/2, -1/3, 3/2, (1 - Sec[c
 + d*x])/2, (b*(1 - Sec[c + d*x]))/(a + b)]*(a + b*Sec[c + d*x])^(1/3)*Tan
[c + d*x])/(b*d*Sqrt[1 + Sec[c + d*x]]*((a + b*Sec[c + d*x])/(a + b))^(1/3
)) - (Sqrt[2]*(9*a^4 - 10*a^2*b^2 - b^4)*AppellF1[1/2, 1/2, 2/3, 3/2, (1 -
 Sec[c + d*x])/2, (b*(1 - Sec[c + d*x]))/(a + b)]*((a + b*Sec[c + d*x])/(a
 + b))^(2/3)*Tan[c + d*x])/(b*d*Sqrt[1 + Sec[c + d*x]]*(a + b*Sec[c + d*x]
)^(2/3)))/(2*b))/(2*b*(a^2 - b^2))
```

3.708. $\int \frac{\sec^4(c+dx)}{(a+b\sec(c+dx))^{5/3}}\,dx$

3.708.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 155
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*Simplify[b/(b*c - a*d)]^n*
Simplify[b/(b*e - a*f)]^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/
(b*c - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f,
 m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[Sim
plify[b/(b*c - a*d)], 0] && GtQ[Simplify[b/(b*e - a*f)], 0] &&  !(GtQ[Simpl
ify[d/(d*a - c*b)], 0] && GtQ[Simplify[d/(d*e - c*f)], 0] && SimplerQ[c + d
*x, a + b*x]) &&  !(GtQ[Simplify[f/(f*a - e*b)], 0] && GtQ[Simplify[f/(f*c
- e*d)], 0] && SimplerQ[e + f*x, a + b*x])
```

rule 156
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(e + f*x)^FracPart[p]/(Simplify[b/(b*e - a*f)]^IntPart[p
]*(b*((e + f*x)/(b*e - a*f)))^FracPart[p])   Int[(a + b*x)^m*(c + d*x)^n*Si
mp[b*(e/(b*e - a*f)) + b*f*(x/(b*e - a*f)), x]^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] &
& GtQ[Simplify[b/(b*c - a*d)], 0] &&  !GtQ[Simplify[b/(b*e - a*f)], 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4321
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[Cot[e + f*x]/(f*Sqrt[1 + Csc[e + f*x]]*Sqrt[1 - Csc[e + f*x
]])   Subst[Int[(a + b*x)^m/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Csc[e + f*x]]
, x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*m]
```

3.708. $\int \frac{\sec^4(c+dx)}{(a+b\sec(c+dx))^{5/3}}\,dx$

rule 4332
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-a^2)*d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(
m + 1)*((d*Csc[e + f*x])^(n - 3)/(b*f*(m + 1)*(a^2 - b^2))), x] + Simp[d^3/
(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])
^(n - 3)*Simp[a^2*(n - 3) + a*b*(m + 1)*Csc[e + f*x] - (a^2*(n - 2) + b^2*(
m + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2
- b^2, 0] && LtQ[m, -1] && (IGtQ[n, 3] || (IntegersQ[n + 1/2, 2*m] && GtQ[n
, 2]))
```

rule 4495
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b - a*B)/b   Int[
Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] + Simp[B/b   Int[Csc[e + f*x]*(
a + b*Csc[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, A, B, e, f, m}, x] && N
eQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

rule 4570
```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> Simp[(-C)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2)
)), x] + Simp[1/(b*(m + 2))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[
b*A*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /;
 FreeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

3.708.4 Maple [F]

$$\int \frac{\sec(dx+c)^4}{(a+b\sec(dx+c))^{\frac{5}{3}}}dx$$

input
```
int(sec(d*x+c)^4/(a+b*sec(d*x+c))^(5/3),x)
```

output
```
int(sec(d*x+c)^4/(a+b*sec(d*x+c))^(5/3),x)
```

3.708.5 Fricas [F]

$$\int \frac{\sec^4(c+dx)}{(a+b\sec(c+dx))^{5/3}}\,dx = \int \frac{\sec(dx+c)^4}{(b\sec(dx+c)+a)^{\frac{5}{3}}}\,dx$$

input
```
integrate(sec(d*x+c)^4/(a+b*sec(d*x+c))^(5/3),x, algorithm="fricas")
```

output
```
integral((b*sec(d*x + c) + a)^(1/3)*sec(d*x + c)^4/(b^2*sec(d*x + c)^2 + 2
*a*b*sec(d*x + c) + a^2), x)
```

3.708.6 Sympy [F]

$$\int \frac{\sec^4(c+dx)}{(a+b\sec(c+dx))^{5/3}}\,dx = \int \frac{\sec^4(c+dx)}{(a+b\sec(c+dx))^{\frac{5}{3}}}\,dx$$

input
```
integrate(sec(d*x+c)**4/(a+b*sec(d*x+c))**(5/3),x)
```

output
```
Integral(sec(c + d*x)**4/(a + b*sec(c + d*x))**(5/3), x)
```

3.708.7 Maxima [F]

$$\int \frac{\sec^4(c+dx)}{(a+b\sec(c+dx))^{5/3}}\,dx = \int \frac{\sec(dx+c)^4}{(b\sec(dx+c)+a)^{\frac{5}{3}}}\,dx$$

input
```
integrate(sec(d*x+c)^4/(a+b*sec(d*x+c))^(5/3),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)^4/(b*sec(d*x + c) + a)^(5/3), x)
```

3.708.8 Giac [F]

$$\int \frac{\sec^4(c+dx)}{(a+b\sec(c+dx))^{5/3}}\,dx = \int \frac{\sec(dx+c)^4}{(b\sec(dx+c)+a)^{\frac{5}{3}}}\,dx$$

input
```
integrate(sec(d*x+c)^4/(a+b*sec(d*x+c))^(5/3),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^4/(b*sec(d*x + c) + a)^(5/3), x)
```

3.708.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^4(c+dx)}{(a+b\sec(c+dx))^{5/3}}\,dx = \int \frac{1}{\cos(c+d\,x)^4\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/3}}\,dx$$

input
```
int(1/(cos(c + d*x)^4*(a + b/cos(c + d*x))^(5/3)),x)
```

output
```
int(1/(cos(c + d*x)^4*(a + b/cos(c + d*x))^(5/3)), x)
```

3.709 $\int \frac{\sec^3(c+dx)}{(a+b\sec(c+dx))^{5/3}}\,dx$

3.709.1 Optimal result . . . 5492
3.709.2 Mathematica [B] (warning: unable to verify) . . . 5493
3.709.3 Rubi [A] (verified) . . . 5493
3.709.4 Maple [F] . . . 5496
3.709.5 Fricas [F] . . . 5496
3.709.6 Sympy [F] . . . 5497
3.709.7 Maxima [F] . . . 5497
3.709.8 Giac [F] . . . 5497
3.709.9 Mupad [F(-1)] . . . 5498

3.709.1 Optimal result

Integrand size = 23, antiderivative size = 307

$$\int \frac{\sec^3(c+dx)}{(a+b\sec(c+dx))^{5/3}}\,dx = -\frac{3a^2\tan(c+dx)}{2b\,(a^2-b^2)\,d(a+b\sec(c+dx))^{2/3}} + \frac{(3a^2-2b^2)\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},-\frac{1}{3},\frac{3}{2},\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)\sqrt[3]{a+b\sec(c+dx)}\tan(c+dx)}{\sqrt{2}b^2\,(a^2-b^2)\,d\sqrt{1+\sec(c+dx)}\sqrt[3]{\frac{a+b\sec(c+dx)}{a+b}}} - \frac{a(3a^2-4b^2)\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},\frac{2}{3},\frac{3}{2},\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{2/3}\tan(c+dx)}{\sqrt{2}b^2\,(a^2-b^2)\,d\sqrt{1+\sec(c+dx)}(a+b\sec(c+dx))^{2/3}}$$

output
```
-3/2*a^2*tan(d*x+c)/b/(a^2-b^2)/d/(a+b*sec(d*x+c))^(2/3)+1/2*(3*a^2-2*b^2)
*AppellF1(1/2,-1/3,1/2,3/2,b*(1-sec(d*x+c))/(a+b),1/2-1/2*sec(d*x+c))*(a+b
*sec(d*x+c))^(1/3)*tan(d*x+c)/b^2/(a^2-b^2)/d/((a+b*sec(d*x+c))/(a+b))^(1/
3)*2^(1/2)/(1+sec(d*x+c))^(1/2)-1/2*a*(3*a^2-4*b^2)*AppellF1(1/2,2/3,1/2,3
/2,b*(1-sec(d*x+c))/(a+b),1/2-1/2*sec(d*x+c))*((a+b*sec(d*x+c))/(a+b))^(2/
3)*tan(d*x+c)/b^2/(a^2-b^2)/d/(a+b*sec(d*x+c))^(2/3)*2^(1/2)/(1+sec(d*x+c)
)^(1/2)
```

3.709.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 19126 vs. $2(307) = 614$.

Time = 44.10 (sec) , antiderivative size = 19126, normalized size of antiderivative = 62.30

$$\int \frac{\sec^3(c+dx)}{(a+b\sec(c+dx))^{5/3}}\,dx = \text{Result too large to show}$$

input
```
Integrate[Sec[c + d*x]^3/(a + b*Sec[c + d*x])^(5/3),x]
```

output
```
Result too large to show
```

3.709.3 Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 304, normalized size of antiderivative = 0.99, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {3042, 4326, 27, 3042, 4495, 3042, 4321, 156, 155}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^3(c+dx)}{(a+b\sec(c+dx))^{5/3}}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^3}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/3}}dx$$

$$\Big\downarrow \; 4326$$

$$-\frac{3\int -\frac{\sec(c+dx)\left(2ab+\left(3a^2-2b^2\right)\sec(c+dx)\right)}{3(a+b\sec(c+dx))^{2/3}}dx}{2b\left(a^2-b^2\right)}-\frac{3a^2\tan(c+dx)}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^{2/3}}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{\sec(c+dx)\left(2ab+\left(3a^2-2b^2\right)\sec(c+dx)\right)}{(a+b\sec(c+dx))^{2/3}}dx}{2b\left(a^2-b^2\right)}-\frac{3a^2\tan(c+dx)}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^{2/3}}$$

$$\Big\downarrow \; 3042$$

$$\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(2ab+\left(3a^2-2b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{2/3}}dx}{2b\left(a^2-b^2\right)}-\frac{3a^2\tan(c+dx)}{2bd\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)^{2/3}}$$

$\Big\downarrow$ 4495

$$\frac{\frac{\left(3a^2-2b^2\right)\int\sec(c+dx)\sqrt[3]{a+b\sec(c+dx)}dx}{b}-\frac{a\left(3a^2-4b^2\right)\int\frac{\sec(c+dx)}{(a+b\sec(c+dx))^{2/3}}dx}{b}}{2b\left(a^2-b^2\right)}-\frac{3a^2\tan(c+dx)}{2bd\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)^{2/3}}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\left(3a^2-2b^2\right)\int\csc\left(c+dx+\frac{\pi}{2}\right)\sqrt[3]{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b}-\frac{a\left(3a^2-4b^2\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{2/3}}dx}{b}}{2b\left(a^2-b^2\right)}-\frac{3a^2\tan(c+dx)}{2bd\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)^{2/3}}$$

$\Big\downarrow$ 4321

$$\frac{\frac{a\left(3a^2-4b^2\right)\tan(c+dx)\int\frac{1}{\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}(a+b\sec(c+dx))^{2/3}}d\sec(c+dx)}{bd\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}}-\frac{\left(3a^2-2b^2\right)\tan(c+dx)\int\frac{\sqrt[3]{a+b\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}}d\sec(c-}{bd\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}}}{2b\left(a^2-b^2\right)}$$
$$-\frac{3a^2\tan(c+dx)}{2bd\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)^{2/3}}$$

$\Big\downarrow$ 156

$$\frac{\frac{a\left(3a^2-4b^2\right)\tan(c+dx)\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{2/3}\int\frac{1}{\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}\left(\frac{a}{a+b}+\frac{b\sec(c+dx)}{a+b}\right)^{2/3}}d\sec(c+dx)}{bd\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}(a+b\sec(c+dx))^{2/3}}-\frac{\left(3a^2-2b^2\right)\tan(c+dx)\sqrt[3]{a+b\sec}}{bd\sqrt{1-\sec(c+dx)}}}{2b\left(a^2-b^2\right)}$$
$$-\frac{3a^2\tan(c+dx)}{2bd\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)^{2/3}}$$

$\Big\downarrow$ 155

$$\frac{\frac{\sqrt{2}\left(3a^2-2b^2\right)\tan(c+dx)\sqrt[3]{a+b\sec(c+dx)}\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},-\frac{1}{3},\frac{3}{2},\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)}{bd\sqrt{\sec(c+dx)+1}\sqrt[3]{\frac{a+b\sec(c+dx)}{a+b}}}-\frac{\sqrt{2}a\left(3a^2-4b^2\right)\tan(c+dx)\left(\frac{a+b\sec(}{a+}\right.}{bd\sqrt{}}}{2b\left(a^2-b^2\right)}$$
$$-\frac{3a^2\tan(c+dx)}{2bd\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)^{2/3}}$$

3.709. $\int\frac{\sec^3(c+dx)}{(a+b\sec(c+dx))^{5/3}}dx$

input
```
Int[Sec[c + d*x]^3/(a + b*Sec[c + d*x])^(5/3),x]
```

output
```
(-3*a^2*Tan[c + d*x])/(2*b*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^(2/3)) + ((S
qrt[2]*(3*a^2 - 2*b^2)*AppellF1[1/2, 1/2, -1/3, 3/2, (1 - Sec[c + d*x])/2,
 (b*(1 - Sec[c + d*x]))/(a + b)]*(a + b*Sec[c + d*x])^(1/3)*Tan[c + d*x])/
(b*d*Sqrt[1 + Sec[c + d*x]]*((a + b*Sec[c + d*x])/(a + b))^(1/3)) - (Sqrt[
2]*a*(3*a^2 - 4*b^2)*AppellF1[1/2, 1/2, 2/3, 3/2, (1 - Sec[c + d*x])/2, (b
*(1 - Sec[c + d*x]))/(a + b)]*((a + b*Sec[c + d*x])/(a + b))^(2/3)*Tan[c +
 d*x])/(b*d*Sqrt[1 + Sec[c + d*x]]*(a + b*Sec[c + d*x])^(2/3)))/(2*b*(a^2
- b^2))
```

3.709.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 155
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*Simplify[b/(b*c - a*d)]^n*
Simplify[b/(b*e - a*f)]^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/
(b*c - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f,
 m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[Sim
plify[b/(b*c - a*d)], 0] && GtQ[Simplify[b/(b*e - a*f)], 0] &&  !(GtQ[Simpl
ify[d/(d*a - c*b)], 0] && GtQ[Simplify[d/(d*e - c*f)], 0] && SimplerQ[c + d
*x, a + b*x]) &&  !(GtQ[Simplify[f/(f*a - e*b)], 0] && GtQ[Simplify[f/(f*c
- e*d)], 0] && SimplerQ[e + f*x, a + b*x])
```

rule 156
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(e + f*x)^FracPart[p]/(Simplify[b/(b*e - a*f)]^IntPart[p
]*(b*((e + f*x)/(b*e - a*f)))^FracPart[p])   Int[(a + b*x)^m*(c + d*x)^n*Si
mp[b*(e/(b*e - a*f)) + b*f*(x/(b*e - a*f)), x]^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] &
& GtQ[Simplify[b/(b*c - a*d)], 0] &&  !GtQ[Simplify[b/(b*e - a*f)], 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4321
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[Cot[e + f*x]/(f*Sqrt[1 + Csc[e + f*x]]*Sqrt[1 - Csc[e + f*x
]])   Subst[Int[(a + b*x)^m/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Csc[e + f*x]]
, x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*m]
```

rule 4326
```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(-a^2)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m
 + 1)*(a^2 - b^2))), x] + Simp[1/(b*(m + 1)*(a^2 - b^2))   Int[Csc[e + f*x]
*(a + b*Csc[e + f*x])^(m + 1)*Simp[a*b*(m + 1) - (a^2 + b^2*(m + 1))*Csc[e
+ f*x], x], x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m,
 -1]
```

rule 4495
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b - a*B)/b   Int[
Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] + Simp[B/b   Int[Csc[e + f*x]*(
a + b*Csc[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, A, B, e, f, m}, x] && N
eQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

3.709.4 Maple [F]

$$\int \frac{\sec (dx+c)^{3}}{(a+b\sec (dx+c))^{\frac{5}{3}}}dx$$

input
```
int(sec(d*x+c)^3/(a+b*sec(d*x+c))^(5/3),x)
```

output
```
int(sec(d*x+c)^3/(a+b*sec(d*x+c))^(5/3),x)
```

3.709.5 Fricas [F]

$$\int \frac{\sec^3(c+dx)}{(a+b\sec(c+dx))^{5/3}}\,dx=\int \frac{\sec (dx+c)^{3}}{(b\sec (dx+c)+a)^{\frac{5}{3}}}\,dx$$

input
```
integrate(sec(d*x+c)^3/(a+b*sec(d*x+c))^(5/3),x, algorithm="fricas")
```

output
```
integral((b*sec(d*x + c) + a)^(1/3)*sec(d*x + c)^3/(b^2*sec(d*x + c)^2 + 2
*a*b*sec(d*x + c) + a^2), x)
```

3.709.6 Sympy [F]

$$\int \frac{\sec^3(c+dx)}{(a+b\sec(c+dx))^{5/3}}\,dx = \int \frac{\sec^3(c+dx)}{(a+b\sec(c+dx))^{\frac{5}{3}}}\,dx$$

input
```
integrate(sec(d*x+c)**3/(a+b*sec(d*x+c))**(5/3),x)
```

output
```
Integral(sec(c + d*x)**3/(a + b*sec(c + d*x))**(5/3), x)
```

3.709.7 Maxima [F]

$$\int \frac{\sec^3(c+dx)}{(a+b\sec(c+dx))^{5/3}}\,dx = \int \frac{\sec(dx+c)^3}{(b\sec(dx+c)+a)^{\frac{5}{3}}}\,dx$$

input
```
integrate(sec(d*x+c)^3/(a+b*sec(d*x+c))^(5/3),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)^3/(b*sec(d*x + c) + a)^(5/3), x)
```

3.709.8 Giac [F]

$$\int \frac{\sec^3(c+dx)}{(a+b\sec(c+dx))^{5/3}}\,dx = \int \frac{\sec(dx+c)^3}{(b\sec(dx+c)+a)^{\frac{5}{3}}}\,dx$$

input
```
integrate(sec(d*x+c)^3/(a+b*sec(d*x+c))^(5/3),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^3/(b*sec(d*x + c) + a)^(5/3), x)
```

3.709.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^3(c+dx)}{(a+b\sec(c+dx))^{5/3}}\,dx = \int \frac{1}{\cos\left(c+d\,x\right)^3\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/3}}\,dx$$

input
```
int(1/(cos(c + d*x)^3*(a + b/cos(c + d*x))^(5/3)),x)
```

output
```
int(1/(cos(c + d*x)^3*(a + b/cos(c + d*x))^(5/3)), x)
```

3.710 $\int \frac{\sec^2(c+dx)}{(a+b\sec(c+dx))^{5/3}}\,dx$

3.710.1 Optimal result . . . 5499
3.710.2 Mathematica [B] (warning: unable to verify) . . . 5500
3.710.3 Rubi [A] (verified) . . . 5500
3.710.4 Maple [F] . . . 5503
3.710.5 Fricas [F] . . . 5503
3.710.6 Sympy [F] . . . 5504
3.710.7 Maxima [F] . . . 5504
3.710.8 Giac [F] . . . 5504
3.710.9 Mupad [F(-1)] . . . 5505

3.710.1 Optimal result

Integrand size = 23, antiderivative size = 289

$$\int \frac{\sec^2(c+dx)}{(a+b\sec(c+dx))^{5/3}}\,dx = \frac{3a\tan(c+dx)}{2\,(a^2-b^2)\,d(a+b\sec(c+dx))^{2/3}} - \frac{a\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},-\frac{1}{3},\frac{3}{2},\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)\sqrt[3]{a+b\sec(c+dx)}\tan(c+dx)}{\sqrt{2}b\,(a^2-b^2)\,d\sqrt{1+\sec(c+dx)}\sqrt[3]{\frac{a+b\sec(c+dx)}{a+b}}} + \frac{(a^2-2b^2)\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},\frac{2}{3},\frac{3}{2},\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{2/3}\tan(c+dx)}{\sqrt{2}b\,(a^2-b^2)\,d\sqrt{1+\sec(c+dx)}(a+b\sec(c+dx))^{2/3}}$$

output
```
3/2*a*tan(d*x+c)/(a^2-b^2)/d/(a+b*sec(d*x+c))^(2/3)-1/2*a*AppellF1(1/2,-1/
3,1/2,3/2,b*(1-sec(d*x+c))/(a+b),1/2-1/2*sec(d*x+c))*(a+b*sec(d*x+c))^(1/3
)*tan(d*x+c)/b/(a^2-b^2)/d/((a+b*sec(d*x+c))/(a+b))^(1/3)*2^(1/2)/(1+sec(d
*x+c))^(1/2)+1/2*(a^2-2*b^2)*AppellF1(1/2,2/3,1/2,3/2,b*(1-sec(d*x+c))/(a+
b),1/2-1/2*sec(d*x+c))*((a+b*sec(d*x+c))/(a+b))^(2/3)*tan(d*x+c)/b/(a^2-b^
2)/d/(a+b*sec(d*x+c))^(2/3)*2^(1/2)/(1+sec(d*x+c))^(1/2)
```

3.710.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 7325 vs. $2(289) = 578$.

Time = 46.96 (sec) , antiderivative size = 7325, normalized size of antiderivative = 25.35

$$\int \frac{\sec^2(c+dx)}{(a+b\sec(c+dx))^{5/3}}\,dx = \text{Result too large to show}$$

input
```
Integrate[Sec[c + d*x]^2/(a + b*Sec[c + d*x])^(5/3),x]
```

output
```
Result too large to show
```

3.710.3 Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 283, normalized size of antiderivative = 0.98, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {3042, 4323, 27, 3042, 4495, 3042, 4321, 156, 155}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(c+dx)}{(a+b\sec(c+dx))^{5/3}}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/3}}dx$$

$$\Big\downarrow \; 4323$$

$$\frac{3\int -\frac{\sec(c+dx)(2b+a\sec(c+dx))}{3(a+b\sec(c+dx))^{2/3}}dx}{2\left(a^2-b^2\right)}+\frac{3a\tan(c+dx)}{2d\left(a^2-b^2\right)(a+b\sec(c+dx))^{2/3}}$$

$$\Big\downarrow \; 27$$

$$\frac{3a\tan(c+dx)}{2d\left(a^2-b^2\right)(a+b\sec(c+dx))^{2/3}}-\frac{\int \frac{\sec(c+dx)(2b+a\sec(c+dx))}{(a+b\sec(c+dx))^{2/3}}dx}{2\left(a^2-b^2\right)}$$

$$\Big\downarrow \; 3042$$

$$\frac{3a\tan(c+dx)}{2d\left(a^2-b^2\right)(a+b\sec(c+dx))^{2/3}}-\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\left(2b+a\csc\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{2/3}}dx}{2\left(a^2-b^2\right)}$$

$$\Big\downarrow \; 4495$$

$$\frac{3a\tan(c+dx)}{2d\left(a^2-b^2\right)(a+b\sec(c+dx))^{2/3}}-\frac{\frac{a\int\sec(c+dx)\sqrt[3]{a+b\sec(c+dx)}dx}{b}-\frac{\left(a^2-2b^2\right)\int\frac{\sec(c+dx)}{(a+b\sec(c+dx))^{2/3}}dx}{b}}{2\left(a^2-b^2\right)}$$

$$\Big\downarrow \; 3042$$

$$\frac{3a\tan(c+dx)}{2d\left(a^2-b^2\right)(a+b\sec(c+dx))^{2/3}}-\frac{\frac{a\int\csc\left(c+dx+\frac{\pi}{2}\right)\sqrt[3]{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b}-\frac{\left(a^2-2b^2\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{2/3}}dx}{b}}{2\left(a^2-b^2\right)}$$

$$\Big\downarrow \; 4321$$

$$\frac{3a\tan(c+dx)}{2d\left(a^2-b^2\right)(a+b\sec(c+dx))^{2/3}}-\frac{\frac{\left(a^2-2b^2\right)\tan(c+dx)\int\frac{1}{\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}(a+b\sec(c+dx))^{2/3}}d\sec(c+dx)}{bd\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}}-\frac{a\tan(c+dx)\int\frac{\sqrt[3]{a+b\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}}d\sec(c+dx)}{bd\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}}}{2\left(a^2-b^2\right)}$$

$$\Big\downarrow \; 156$$

$$\frac{3a\tan(c+dx)}{2d\left(a^2-b^2\right)(a+b\sec(c+dx))^{2/3}}-\frac{\frac{\left(a^2-2b^2\right)\tan(c+dx)\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{2/3}\int\frac{1}{\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}\left(\frac{a}{a+b}+\frac{b\sec(c+dx)}{a+b}\right)^{2/3}}d\sec(c+dx)}{bd\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}(a+b\sec(c+dx))^{2/3}}-\frac{a\tan(c+dx)\sqrt[3]{a+b\sec(c+dx)}}{bd\sqrt{1-\sec(c+dx)}\sqrt{\sec(}}}{2\left(a^2-b^2\right)}$$

$$\Big\downarrow \; 155$$

$$\frac{3a\tan(c+dx)}{2d\left(a^2-b^2\right)(a+b\sec(c+dx))^{2/3}}-\frac{\frac{\sqrt{2}a\tan(c+dx)\sqrt[3]{a+b\sec(c+dx)}\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},-\frac{1}{3},\frac{3}{2},\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)}{bd\sqrt{\sec(c+dx)+1}\sqrt[3]{\frac{a+b\sec(c+dx)}{a+b}}}-\frac{\sqrt{2}\left(a^2-2b^2\right)\tan(c+dx)\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{2/3}A}{bd\sqrt{\sec(c+dx)}}}{2\left(a^2-b^2\right)}$$

input
```
Int[Sec[c + d*x]^2/(a + b*Sec[c + d*x])^(5/3),x]
```

3.710. $\int\frac{\sec^2(c+dx)}{(a+b\sec(c+dx))^{5/3}}dx$

output
```
(3*a*Tan[c + d*x])/(2*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^(2/3)) - ((Sqrt[2
]*a*AppellF1[1/2, 1/2, -1/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[c + d*
x]))/(a + b)]*(a + b*Sec[c + d*x])^(1/3)*Tan[c + d*x])/(b*d*Sqrt[1 + Sec[c
 + d*x]]*((a + b*Sec[c + d*x])/(a + b))^(1/3)) - (Sqrt[2]*(a^2 - 2*b^2)*Ap
pellF1[1/2, 1/2, 2/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[c + d*x]))/(a
 + b)]*((a + b*Sec[c + d*x])/(a + b))^(2/3)*Tan[c + d*x])/(b*d*Sqrt[1 + Se
c[c + d*x]]*(a + b*Sec[c + d*x])^(2/3)))/(2*(a^2 - b^2))
```

3.710.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 155
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*Simplify[b/(b*c - a*d)]^n*
Simplify[b/(b*e - a*f)]^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/
(b*c - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f,
 m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[Sim
plify[b/(b*c - a*d)], 0] && GtQ[Simplify[b/(b*e - a*f)], 0] &&  !(GtQ[Simpl
ify[d/(d*a - c*b)], 0] && GtQ[Simplify[d/(d*e - c*f)], 0] && SimplerQ[c + d
*x, a + b*x]) &&  !(GtQ[Simplify[f/(f*a - e*b)], 0] && GtQ[Simplify[f/(f*c
- e*d)], 0] && SimplerQ[e + f*x, a + b*x])
```

rule 156
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(e + f*x)^FracPart[p]/(Simplify[b/(b*e - a*f)]^IntPart[p
]*(b*((e + f*x)/(b*e - a*f)))^FracPart[p])   Int[(a + b*x)^m*(c + d*x)^n*Si
mp[b*(e/(b*e - a*f)) + b*f*(x/(b*e - a*f)), x]^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] &
& GtQ[Simplify[b/(b*c - a*d)], 0] &&  !GtQ[Simplify[b/(b*e - a*f)], 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4321
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[Cot[e + f*x]/(f*Sqrt[1 + Csc[e + f*x]]*Sqrt[1 - Csc[e + f*x
]])   Subst[Int[(a + b*x)^m/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Csc[e + f*x]]
, x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*m]
```

rule 4323
```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[a*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(f*(m + 1)*(
a^2 - b^2))), x] - Simp[1/((m + 1)*(a^2 - b^2))   Int[Csc[e + f*x]*(a + b*C
sc[e + f*x])^(m + 1)*(b*(m + 1) - a*(m + 2)*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

rule 4495
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b - a*B)/b   Int[
Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] + Simp[B/b   Int[Csc[e + f*x]*(
a + b*Csc[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, A, B, e, f, m}, x] && N
eQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

3.710.4 Maple [F]

$$\int \frac{\sec (dx+c)^2}{(a+b\sec (dx+c))^{\frac{5}{3}}}dx$$

input
```
int(sec(d*x+c)^2/(a+b*sec(d*x+c))^(5/3),x)
```

output
```
int(sec(d*x+c)^2/(a+b*sec(d*x+c))^(5/3),x)
```

3.710.5 Fricas [F]

$$\int \frac{\sec^2(c+dx)}{(a+b\sec(c+dx))^{5/3}}\,dx=\int \frac{\sec (dx+c)^2}{(b\sec (dx+c)+a)^{\frac{5}{3}}}\,dx$$

input
```
integrate(sec(d*x+c)^2/(a+b*sec(d*x+c))^(5/3),x, algorithm="fricas")
```

output
```
integral((b*sec(d*x + c) + a)^(1/3)*sec(d*x + c)^2/(b^2*sec(d*x + c)^2 + 2
*a*b*sec(d*x + c) + a^2), x)
```

3.710.6 Sympy [F]

$$\int \frac{\sec^2(c+dx)}{(a+b\sec(c+dx))^{5/3}}\,dx = \int \frac{\sec^2(c+dx)}{(a+b\sec(c+dx))^{\frac{5}{3}}}\,dx$$

input
```
integrate(sec(d*x+c)**2/(a+b*sec(d*x+c))**(5/3),x)
```

output
```
Integral(sec(c + d*x)**2/(a + b*sec(c + d*x))**(5/3), x)
```

3.710.7 Maxima [F]

$$\int \frac{\sec^2(c+dx)}{(a+b\sec(c+dx))^{5/3}}\,dx = \int \frac{\sec(dx+c)^2}{(b\sec(dx+c)+a)^{\frac{5}{3}}}\,dx$$

input
```
integrate(sec(d*x+c)^2/(a+b*sec(d*x+c))^(5/3),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)^2/(b*sec(d*x + c) + a)^(5/3), x)
```

3.710.8 Giac [F]

$$\int \frac{\sec^2(c+dx)}{(a+b\sec(c+dx))^{5/3}}\,dx = \int \frac{\sec(dx+c)^2}{(b\sec(dx+c)+a)^{\frac{5}{3}}}\,dx$$

input
```
integrate(sec(d*x+c)^2/(a+b*sec(d*x+c))^(5/3),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^2/(b*sec(d*x + c) + a)^(5/3), x)
```

3.710.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^2(c+dx)}{(a+b\sec(c+dx))^{5/3}}\,dx = \int \frac{1}{\cos\left(c+d\,x\right)^2\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/3}}\,dx$$

input
```
int(1/(cos(c + d*x)^2*(a + b/cos(c + d*x))^(5/3)),x)
```

output
```
int(1/(cos(c + d*x)^2*(a + b/cos(c + d*x))^(5/3)), x)
```

3.711 $\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^{5/3}}\,dx$

3.711.1 Optimal result 5506
3.711.2 Mathematica [B] (warning: unable to verify) 5506
3.711.3 Rubi [A] (verified) 5507
3.711.4 Maple [F] 5508
3.711.5 Fricas [F] 5509
3.711.6 Sympy [F] 5509
3.711.7 Maxima [F] 5509
3.711.8 Giac [F] 5510
3.711.9 Mupad [F(-1)] 5510

3.711.1 Optimal result

Integrand size = 21, antiderivative size = 110

$$\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^{5/3}}\,dx = \frac{\sqrt{2}\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},\frac{5}{3},\frac{3}{2},\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{2/3}\text{ta}}{(a+b)d\sqrt{1+\sec(c+dx)}(a+b\sec(c+dx))^{2/3}}$$

output
```
AppellF1(1/2,5/3,1/2,3/2,b*(1-sec(d*x+c))/(a+b),1/2-1/2*sec(d*x+c))*((a+b*
sec(d*x+c))/(a+b))^(2/3)*2^(1/2)*tan(d*x+c)/(a+b)/d/(a+b*sec(d*x+c))^(2/3)
/(1+sec(d*x+c))^(1/2)
```

3.711.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 10363 vs. 2(110) = 220.

Time = 49.84 (sec) , antiderivative size = 10363, normalized size of antiderivative = 94.21

$$\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^{5/3}}\,dx = \text{Result too large to show}$$

input
```
Integrate[Sec[c + d*x]/(a + b*Sec[c + d*x])^(5/3),x]
```

output
```
Result too large to show
```

3.711.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3042, 4321, 156, 155}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^{5/3}}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/3}}dx$$

$$\Big\downarrow \text{ 4321}$$

$$-\frac{\tan(c+dx)\int \frac{1}{\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}(a+b\sec(c+dx))^{5/3}}d\sec(c+dx)}{d\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}}$$

$$\Big\downarrow \text{ 156}$$

$$-\frac{\tan(c+dx)\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{2/3}\int \frac{1}{\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}\left(\frac{a}{a+b}+\frac{b\sec(c+dx)}{a+b}\right)^{5/3}}d\sec(c+dx)}{d(a+b)\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}(a+b\sec(c+dx))^{2/3}}$$

$$\Big\downarrow \text{ 155}$$

$$\frac{\sqrt{2}\tan(c+dx)\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{2/3}\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},\frac{5}{3},\frac{3}{2},\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)}{d(a+b)\sqrt{\sec(c+dx)+1}(a+b\sec(c+dx))^{2/3}}$$

input
```
Int[Sec[c + d*x]/(a + b*Sec[c + d*x])^(5/3),x]
```

output
```
(Sqrt[2]*AppellF1[1/2, 1/2, 5/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[c
+ d*x]))/(a + b)]*((a + b*Sec[c + d*x])/(a + b))^(2/3)*Tan[c + d*x])/((a +
 b)*d*Sqrt[1 + Sec[c + d*x]]*(a + b*Sec[c + d*x])^(2/3))
```

3.711.3.1 Defintions of rubi rules used

rule 155
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*Simplify[b/(b*c - a*d)]^n*
Simplify[b/(b*e - a*f)]^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/
(b*c - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f,
 m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[Sim
plify[b/(b*c - a*d)], 0] && GtQ[Simplify[b/(b*e - a*f)], 0] && !(GtQ[Simpl
ify[d/(d*a - c*b)], 0] && GtQ[Simplify[d/(d*e - c*f)], 0] && SimplerQ[c + d
*x, a + b*x]) && !(GtQ[Simplify[f/(f*a - e*b)], 0] && GtQ[Simplify[f/(f*c
- e*d)], 0] && SimplerQ[e + f*x, a + b*x])
```

rule 156
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(e + f*x)^FracPart[p]/(Simplify[b/(b*e - a*f)]^IntPart[p
]*(b*((e + f*x)/(b*e - a*f)))^FracPart[p])   Int[(a + b*x)^m*(c + d*x)^n*Si
mp[b*(e/(b*e - a*f)) + b*f*(x/(b*e - a*f)), x]^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] &
& GtQ[Simplify[b/(b*c - a*d)], 0] && !GtQ[Simplify[b/(b*e - a*f)], 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4321
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[Cot[e + f*x]/(f*Sqrt[1 + Csc[e + f*x]]*Sqrt[1 - Csc[e + f*x
]])   Subst[Int[(a + b*x)^m/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Csc[e + f*x]]
, x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0] && !IntegerQ[2*m]
```

3.711.4 Maple [F]

$$\int \frac{\sec(dx+c)}{(a+b\sec(dx+c))^{\frac{5}{3}}}\,dx$$

input
```
int(sec(d*x+c)/(a+b*sec(d*x+c))^(5/3),x)
```

output
```
int(sec(d*x+c)/(a+b*sec(d*x+c))^(5/3),x)
```

3.711.5 Fricas [F]

$$\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^{5/3}}\,dx = \int \frac{\sec(dx+c)}{(b\sec(dx+c)+a)^{\frac{5}{3}}}\,dx$$

input
```
integrate(sec(d*x+c)/(a+b*sec(d*x+c))^(5/3),x, algorithm="fricas")
```

output
```
integral((b*sec(d*x + c) + a)^(1/3)*sec(d*x + c)/(b^2*sec(d*x + c)^2 + 2*a
*b*sec(d*x + c) + a^2), x)
```

3.711.6 Sympy [F]

$$\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^{5/3}}\,dx = \int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^{\frac{5}{3}}}\,dx$$

input
```
integrate(sec(d*x+c)/(a+b*sec(d*x+c))**(5/3),x)
```

output
```
Integral(sec(c + d*x)/(a + b*sec(c + d*x))**(5/3), x)
```

3.711.7 Maxima [F]

$$\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^{5/3}}\,dx = \int \frac{\sec(dx+c)}{(b\sec(dx+c)+a)^{\frac{5}{3}}}\,dx$$

input
```
integrate(sec(d*x+c)/(a+b*sec(d*x+c))^(5/3),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)/(b*sec(d*x + c) + a)^(5/3), x)
```

3.711.8 Giac [F]

$$\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^{5/3}}\,dx = \int \frac{\sec(dx+c)}{(b\sec(dx+c)+a)^{\frac{5}{3}}}\,dx$$

input
```
integrate(sec(d*x+c)/(a+b*sec(d*x+c))^(5/3),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)/(b*sec(d*x + c) + a)^(5/3), x)
```

3.711.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^{5/3}}\,dx = \int \frac{1}{\cos(c+d\,x)\,\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/3}}\,dx$$

input
```
int(1/(cos(c + d*x)*(a + b/cos(c + d*x))^(5/3)),x)
```

output
```
int(1/(cos(c + d*x)*(a + b/cos(c + d*x))^(5/3)), x)
```

3.712 $\int \frac{1}{(a+b\sec(c+dx))^{5/3}}\,dx$

3.712.1 Optimal result 5511
3.712.2 Mathematica [N/A] 5511
3.712.3 Rubi [N/A] 5512
3.712.4 Maple [N/A] (verified) 5513
3.712.5 Fricas [F(-1)] 5513
3.712.6 Sympy [N/A] 5513
3.712.7 Maxima [N/A] 5514
3.712.8 Giac [N/A] 5514
3.712.9 Mupad [N/A] 5514

3.712.1 Optimal result

Integrand size = 14, antiderivative size = 14

$$\int \frac{1}{(a+b\sec(c+dx))^{5/3}}\,dx=\text{Int}\left(\frac{1}{(a+b\sec(c+dx))^{5/3}},x\right)$$

output
```
Unintegrable(1/(a+b*sec(d*x+c))^(5/3),x)
```

3.712.2 Mathematica [N/A]

Not integrable

Time = 95.81 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{(a+b\sec(c+dx))^{5/3}}\,dx=\int \frac{1}{(a+b\sec(c+dx))^{5/3}}\,dx$$

input
```
Integrate[(a + b*Sec[c + d*x])^(-5/3),x]
```

output
```
Integrate[(a + b*Sec[c + d*x])^(-5/3), x]
```

3.712.3 Rubi [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 4273}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+b\sec(c+dx))^{5/3}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/3}}dx$$

$$\Big\downarrow \text{ 4273}$$

$$\int \frac{1}{(a+b\sec(c+dx))^{5/3}}dx$$

input
```
Int[(a + b*Sec[c + d*x])^(-5/3),x]
```

output
```
$Aborted
```

3.712.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4273
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Unintegrable[
(a + b*Csc[c + d*x])^n, x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[a^2 - b^2, 0
] &&  !IntegerQ[2*n]
```

3.712.4 Maple [N/A] (verified)

Not integrable

Time = 0.35 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{1}{(a+b\sec(dx+c))^{\frac{5}{3}}}dx$$

input
```
int(1/(a+b*sec(d*x+c))^(5/3),x)
```

output
```
int(1/(a+b*sec(d*x+c))^(5/3),x)
```

3.712.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\sec(c+dx))^{5/3}}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+b*sec(d*x+c))^(5/3),x, algorithm="fricas")
```

output
```
Timed out
```

3.712.6 Sympy [N/A]

Not integrable

Time = 2.16 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

$$\int \frac{1}{(a+b\sec(c+dx))^{5/3}}\,dx = \int \frac{1}{(a+b\sec(c+dx))^{\frac{5}{3}}}\,dx$$

input
```
integrate(1/(a+b*sec(d*x+c))**(5/3),x)
```

output
```
Integral((a + b*sec(c + d*x))**(-5/3), x)
```

3.712.7 Maxima [N/A]

Not integrable

Time = 0.57 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{(a+b\sec(c+dx))^{5/3}}\,dx = \int \frac{1}{(b\sec(dx+c)+a)^{\frac{5}{3}}}\,dx$$

input
```
integrate(1/(a+b*sec(d*x+c))^(5/3),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^(-5/3), x)
```

3.712.8 Giac [N/A]

Not integrable

Time = 2.11 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{(a+b\sec(c+dx))^{5/3}}\,dx = \int \frac{1}{(b\sec(dx+c)+a)^{\frac{5}{3}}}\,dx$$

input
```
integrate(1/(a+b*sec(d*x+c))^(5/3),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^(-5/3), x)
```

3.712.9 Mupad [N/A]

Not integrable

Time = 15.50 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{(a+b\sec(c+dx))^{5/3}}\,dx = \int \frac{1}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/3}}\,dx$$

input
```
int(1/(a + b/cos(c + d*x))^(5/3),x)
```

output
```
int(1/(a + b/cos(c + d*x))^(5/3), x)
```

3.713 $\int \frac{\sec^{\frac{2}{3}}(c+dx)}{a+b\sec(c+dx)}\,dx$

3.713.1 Optimal result 5515
3.713.2 Mathematica [B] (warning: unable to verify) 5515
3.713.3 Rubi [A] (verified) 5516
3.713.4 Maple [F] 5519
3.713.5 Fricas [F(-1)] 5519
3.713.6 Sympy [F] 5519
3.713.7 Maxima [F] 5520
3.713.8 Giac [F] 5520
3.713.9 Mupad [F(-1)] 5520

3.713.1 Optimal result

Integrand size = 23, antiderivative size = 174

$$\int \frac{\sec^{\frac{2}{3}}(c+dx)}{a+b\sec(c+dx)}\,dx = \frac{a\,\text{AppellF1}\left(\frac{1}{2},-\frac{1}{6},1,\frac{3}{2},\sin^2(c+dx),\frac{a^2\sin^2(c+dx)}{a^2-b^2}\right)\sin(c+dx)}{(a^2-b^2)\,d\sqrt[6]{\cos^2(c+dx)}\sqrt[3]{\sec(c+dx)}} - \frac{b\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{3},1,\frac{3}{2},\sin^2(c+dx),\frac{a^2\sin^2(c+dx)}{a^2-b^2}\right)\sqrt[3]{\cos^2(c+dx)}\sec^{\frac{2}{3}}(c+dx)\sin(c+dx)}{(a^2-b^2)\,d}$$

output
```
a*AppellF1(1/2,-1/6,1,3/2,sin(d*x+c)^2,a^2*sin(d*x+c)^2/(a^2-b^2))*sin(d*x
+c)/(a^2-b^2)/d/(cos(d*x+c)^2)^(1/6)/sec(d*x+c)^(1/3)-b*AppellF1(1/2,1/3,1
,3/2,sin(d*x+c)^2,a^2*sin(d*x+c)^2/(a^2-b^2))*(cos(d*x+c)^2)^(1/3)*sec(d*x
+c)^(2/3)*sin(d*x+c)/(a^2-b^2)/d
```

3.713.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 4543 vs. 2(174) = 348.

Time = 38.30 (sec) , antiderivative size = 4543, normalized size of antiderivative = 26.11

$$\int \frac{\sec^{\frac{2}{3}}(c+dx)}{a+b\sec(c+dx)}\,dx = \text{Result too large to show}$$

input
```
Integrate[Sec[c + d*x]^(2/3)/(a + b*Sec[c + d*x]),x]
```

output
```
(9*(a^2 - b^2)*Sec[c + d*x]^(5/3)*Sin[c + d*x]*((b*AppellF1[1/2, 1/6, 1, 3
/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)]*Sqrt[Sec[c + d*x]^2
])/(9*(a^2 - b^2)*AppellF1[1/2, 1/6, 1, 3/2, -Tan[c + d*x]^2, (b^2*Tan[c +
 d*x]^2)/(a^2 - b^2)] + (6*b^2*AppellF1[3/2, 1/6, 2, 5/2, -Tan[c + d*x]^2,
 (b^2*Tan[c + d*x]^2)/(a^2 - b^2)] + (-a^2 + b^2)*AppellF1[3/2, 7/6, 1, 5/
2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)])*Tan[c + d*x]^2) + (
a*AppellF1[1/2, 2/3, 1, 3/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 -
b^2)])/(-9*(a^2 - b^2)*AppellF1[1/2, 2/3, 1, 3/2, -Tan[c + d*x]^2, (b^2*Ta
n[c + d*x]^2)/(a^2 - b^2)] - 2*(3*b^2*AppellF1[3/2, 2/3, 2, 5/2, -Tan[c +
d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)] + 2*(-a^2 + b^2)*AppellF1[3/2, 5
/3, 1, 5/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)])*Tan[c + d*
x]^2)))/(d*(Sec[c + d*x]^2)^(2/3)*(a + b*Sec[c + d*x])*(-a^2 + b^2*Sec[c +
 d*x]^2)*((9*(a^2 - b^2)*(Sec[c + d*x]^2)^(1/3)*((b*AppellF1[1/2, 1/6, 1,
3/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)]*Sqrt[Sec[c + d*x]^
2])/(9*(a^2 - b^2)*AppellF1[1/2, 1/6, 1, 3/2, -Tan[c + d*x]^2, (b^2*Tan[c
+ d*x]^2)/(a^2 - b^2)] + (6*b^2*AppellF1[3/2, 1/6, 2, 5/2, -Tan[c + d*x]^2
, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)] + (-a^2 + b^2)*AppellF1[3/2, 7/6, 1, 5
/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)])*Tan[c + d*x]^2) +
(a*AppellF1[1/2, 2/3, 1, 3/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 -
 b^2)])/(-9*(a^2 - b^2)*AppellF1[1/2, 2/3, 1, 3/2, -Tan[c + d*x]^2, (b^...
```

3.713.3 Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.12, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {3042, 4356, 3042, 3302, 3042, 3668, 25, 333}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{\frac{2}{3}}(c+dx)}{a+b\sec(c+dx)}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{2/3}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \; 4356$$

$$\cos^{\frac{2}{3}}(c+dx)\sec^{\frac{2}{3}}(c+dx)\int \frac{\sqrt[3]{\cos(c+dx)}}{b+a\cos(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\cos^{\frac{2}{3}}(c+dx)\sec^{\frac{2}{3}}(c+dx)\int \frac{\sqrt[3]{\sin\left(c+dx+\frac{\pi}{2}\right)}}{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3302}$$

$$\cos^{\frac{2}{3}}(c+dx)\sec^{\frac{2}{3}}(c+dx)\left(b\int \frac{\sqrt[3]{\cos(c+dx)}}{b^2-a^2\cos^2(c+dx)}dx-a\int \frac{\cos^{\frac{4}{3}}(c+dx)}{b^2-a^2\cos^2(c+dx)}dx\right)$$

$$\Big\downarrow \text{3042}$$

$$\cos^{\frac{2}{3}}(c+dx)\sec^{\frac{2}{3}}(c+dx)\left(b\int \frac{\sqrt[3]{\sin\left(c+dx+\frac{\pi}{2}\right)}}{b^2-a^2\sin\left(c+dx+\frac{\pi}{2}\right)^2}dx-a\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{4/3}}{b^2-a^2\sin\left(c+dx+\frac{\pi}{2}\right)^2}dx\right)$$

$$\Big\downarrow \text{3668}$$

$$\frac{\cos^{\frac{2}{3}}(c+dx)\sec^{\frac{2}{3}}(c+dx)\left(\frac{b\sqrt[3]{\cos^2(c+dx)}\int -\frac{1}{\sqrt[3]{1-\sin^2(c+dx)}(-\sin^2(c+dx)a^2+a^2-b^2)}d\sin(c+dx)}{d\cos^{\frac{2}{3}}(c+dx)}-\frac{a\sqrt[3]{\cos(c+dx)}\int -\frac{\sqrt[6]{1-\sin^2(}}{-\sin^2(c+dx)a^2}}{d\sqrt[6]{\cos^2(c+d}}\right.}{}$$

$$\Big\downarrow \text{25}$$

$$\frac{\cos^{\frac{2}{3}}(c+dx)\sec^{\frac{2}{3}}(c+dx)\left(\frac{a\sqrt[3]{\cos(c+dx)}\int \frac{\sqrt[6]{1-\sin^2(c+dx)}}{-\sin^2(c+dx)a^2+a^2-b^2}d\sin(c+dx)}{d\sqrt[6]{\cos^2(c+dx)}}-\frac{b\sqrt[3]{\cos^2(c+dx)}\int \frac{1}{\sqrt[3]{1-\sin^2(c+dx)}(-\sin^2(c+dx)a^2+a^2}}{d\cos^{\frac{2}{3}}(c+dx)}\right.}{}$$

$$\Big\downarrow \text{333}$$

$$\frac{\cos^{\frac{2}{3}}(c+dx)\sec^{\frac{2}{3}}(c+dx)\left(\frac{a\sin(c+dx)\sqrt[3]{\cos(c+dx)}\,\text{AppellF1}\left(\frac{1}{2},-\frac{1}{6},1,\frac{3}{2},\sin^2(c+dx),\frac{a^2\sin^2(c+dx)}{a^2-b^2}\right)}{d\left(a^2-b^2\right)\sqrt[6]{\cos^2(c+dx)}}-\frac{b\sin(c+dx)\sqrt[3]{\cos^2(c+dx)}}{d(}\right.}{}$$

input
```
Int[Sec[c + d*x]^(2/3)/(a + b*Sec[c + d*x]),x]
```

3.713. $\int \frac{\sec^{\frac{2}{3}}(c+dx)}{a+b\sec(c+dx)}\,dx$

output
```
Cos[c + d*x]^(2/3)*Sec[c + d*x]^(2/3)*((a*AppellF1[1/2, -1/6, 1, 3/2, Sin[
c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)]*Cos[c + d*x]^(1/3)*Sin[c + d
*x])/((a^2 - b^2)*d*(Cos[c + d*x]^2)^(1/6)) - (b*AppellF1[1/2, 1/3, 1, 3/2
, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)]*(Cos[c + d*x]^2)^(1/3)
*Sin[c + d*x])/((a^2 - b^2)*d*Cos[c + d*x]^(2/3)))
```

3.713.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 333
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^p*c^q*x*AppellF1[1/2, -p, -q, 3/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; F
reeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[p] || GtQ[a,
0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3302
```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[a   Int[(d*Sin[e + f*x])^n/(a^2 - b^2*Sin[e + f*x]
^2), x], x] - Simp[b/d   Int[(d*Sin[e + f*x])^(n + 1)/(a^2 - b^2*Sin[e + f*
x]^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0]
```

rule 3668
```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[(
-ff)*d^(2*IntPart[(m - 1)/2] + 1)*((d*Sin[e + f*x])^(2*FracPart[(m - 1)/2])
/(f*(Sin[e + f*x]^2)^FracPart[(m - 1)/2]))   Subst[Int[(1 - ff^2*x^2)^((m -
 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b,
 d, e, f, m, p}, x] &&  !IntegerQ[m]
```

rule 4356
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Simp[Sin[e + f*x]^n*(d*Csc[e + f*x])^n   Int[(b +
 a*Sin[e + f*x])^m/Sin[e + f*x]^(m + n), x], x] /; FreeQ[{a, b, d, e, f, n}
, x] && NeQ[a^2 - b^2, 0] && IntegerQ[m]
```

3.713.4 Maple [F]

$$\int \frac{\sec (dx+c)^{\frac{2}{3}}}{a+b\sec (dx+c)}dx$$

input
```
int(sec(d*x+c)^(2/3)/(a+b*sec(d*x+c)),x)
```

output
```
int(sec(d*x+c)^(2/3)/(a+b*sec(d*x+c)),x)
```

3.713.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{2}{3}}(c+dx)}{a+b\sec(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)^(2/3)/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

output
```
Timed out
```

3.713.6 Sympy [F]

$$\int \frac{\sec^{\frac{2}{3}}(c+dx)}{a+b\sec(c+dx)}\,dx = \int \frac{\sec^{\frac{2}{3}}(c+dx)}{a+b\sec(c+dx)}\,dx$$

input
```
integrate(sec(d*x+c)**(2/3)/(a+b*sec(d*x+c)),x)
```

output
```
Integral(sec(c + d*x)**(2/3)/(a + b*sec(c + d*x)), x)
```

3.713.7 Maxima [F]

$$\int \frac{\sec^{\frac{2}{3}}(c+dx)}{a+b\sec(c+dx)}\,dx = \int \frac{\sec(dx+c)^{\frac{2}{3}}}{b\sec(dx+c)+a}\,dx$$

input
```
integrate(sec(d*x+c)^(2/3)/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)^(2/3)/(b*sec(d*x + c) + a), x)
```

3.713.8 Giac [F]

$$\int \frac{\sec^{\frac{2}{3}}(c+dx)}{a+b\sec(c+dx)}\,dx = \int \frac{\sec(dx+c)^{\frac{2}{3}}}{b\sec(dx+c)+a}\,dx$$

input
```
integrate(sec(d*x+c)^(2/3)/(a+b*sec(d*x+c)),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^(2/3)/(b*sec(d*x + c) + a), x)
```

3.713.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{2}{3}}(c+dx)}{a+b\sec(c+dx)}\,dx = \int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{2/3}}{a+\frac{b}{\cos(c+d\,x)}}\,dx$$

input
```
int((1/cos(c + d*x))^(2/3)/(a + b/cos(c + d*x)),x)
```

output
```
int((1/cos(c + d*x))^(2/3)/(a + b/cos(c + d*x)), x)
```

3.714 $\int \frac{\sqrt[3]{\sec(c+dx)}}{a+b\sec(c+dx)}\,dx$

3.714.1 Optimal result 5521
3.714.2 Mathematica [B] (warning: unable to verify) 5521
3.714.3 Rubi [A] (verified) 5522
3.714.4 Maple [F] 5525
3.714.5 Fricas [F(-1)] 5525
3.714.6 Sympy [F] 5525
3.714.7 Maxima [F] 5526
3.714.8 Giac [F] 5526
3.714.9 Mupad [F(-1)] 5526

3.714.1 Optimal result

Integrand size = 23, antiderivative size = 174

$$\int \frac{\sqrt[3]{\sec(c+dx)}}{a+b\sec(c+dx)}\,dx = \frac{a\,\text{AppellF1}\left(\frac{1}{2},-\frac{1}{3},1,\frac{3}{2},\sin^2(c+dx),\frac{a^2\sin^2(c+dx)}{a^2-b^2}\right)\sin(c+dx)}{(a^2-b^2)\,d\sqrt[3]{\cos^2(c+dx)}\sec^{\frac{2}{3}}(c+dx)} - \frac{b\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{6},1,\frac{3}{2},\sin^2(c+dx),\frac{a^2\sin^2(c+dx)}{a^2-b^2}\right)\sqrt[6]{\cos^2(c+dx)}\sqrt[3]{\sec(c+dx)}\sin(c+dx)}{(a^2-b^2)\,d}$$

output
```
a*AppellF1(1/2,-1/3,1,3/2,sin(d*x+c)^2,a^2*sin(d*x+c)^2/(a^2-b^2))*sin(d*x
+c)/(a^2-b^2)/d/(cos(d*x+c)^2)^(1/3)/sec(d*x+c)^(2/3)-b*AppellF1(1/2,1/6,1
,3/2,sin(d*x+c)^2,a^2*sin(d*x+c)^2/(a^2-b^2))*(cos(d*x+c)^2)^(1/6)*sec(d*x
+c)^(1/3)*sin(d*x+c)/(a^2-b^2)/d
```

3.714.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 4544 vs. 2(174) = 348.

Time = 38.04 (sec) , antiderivative size = 4544, normalized size of antiderivative = 26.11

$$\int \frac{\sqrt[3]{\sec(c+dx)}}{a+b\sec(c+dx)}\,dx = \text{Result too large to show}$$

input
```
Integrate[Sec[c + d*x]^(1/3)/(a + b*Sec[c + d*x]),x]
```

output
```
(9*(a^2 - b^2)*Sec[c + d*x]^(4/3)*Sin[c + d*x]*((b*AppellF1[1/2, 1/3, 1, 3
/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)]*Sqrt[Sec[c + d*x]^2
])/(9*(a^2 - b^2)*AppellF1[1/2, 1/3, 1, 3/2, -Tan[c + d*x]^2, (b^2*Tan[c +
 d*x]^2)/(a^2 - b^2)] + 2*(3*b^2*AppellF1[3/2, 1/3, 2, 5/2, -Tan[c + d*x]^
2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)] + (-a^2 + b^2)*AppellF1[3/2, 4/3, 1,
5/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)])*Tan[c + d*x]^2) +
 (a*AppellF1[1/2, 5/6, 1, 3/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2
- b^2)])/(-9*(a^2 - b^2)*AppellF1[1/2, 5/6, 1, 3/2, -Tan[c + d*x]^2, (b^2*
Tan[c + d*x]^2)/(a^2 - b^2)] + (-6*b^2*AppellF1[3/2, 5/6, 2, 5/2, -Tan[c +
 d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)] + 5*(a^2 - b^2)*AppellF1[3/2, 1
1/6, 1, 5/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)])*Tan[c + d
*x]^2)))/(d*(Sec[c + d*x]^2)^(5/6)*(a + b*Sec[c + d*x])*(-a^2 + b^2*Sec[c
+ d*x]^2)*((9*(a^2 - b^2)*(Sec[c + d*x]^2)^(1/6)*((b*AppellF1[1/2, 1/3, 1,
 3/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)]*Sqrt[Sec[c + d*x]
^2])/(9*(a^2 - b^2)*AppellF1[1/2, 1/3, 1, 3/2, -Tan[c + d*x]^2, (b^2*Tan[c
 + d*x]^2)/(a^2 - b^2)] + 2*(3*b^2*AppellF1[3/2, 1/3, 2, 5/2, -Tan[c + d*x
]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)] + (-a^2 + b^2)*AppellF1[3/2, 4/3, 1
, 5/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)])*Tan[c + d*x]^2)
 + (a*AppellF1[1/2, 5/6, 1, 3/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^
2 - b^2)])/(-9*(a^2 - b^2)*AppellF1[1/2, 5/6, 1, 3/2, -Tan[c + d*x]^2, ...
```

3.714.3 Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.12, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {3042, 4356, 3042, 3302, 3042, 3668, 25, 333}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{\sec(c+dx)}}{a+b\sec(c+dx)}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\sqrt[3]{\csc\left(c+dx+\frac{\pi}{2}\right)}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \; 4356$$

$$\sqrt[3]{\cos(c+dx)}\sqrt[3]{\sec(c+dx)}\int \frac{\cos^{\frac{2}{3}}(c+dx)}{b+a\cos(c+dx)}dx$$

$$\Big\downarrow \; 3042$$

$$\sqrt[3]{\cos(c+dx)}\sqrt[3]{\sec(c+dx)}\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{2/3}}{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \; 3302$$

$$\sqrt[3]{\cos(c+dx)}\sqrt[3]{\sec(c+dx)}\left(b\int \frac{\cos^{\frac{2}{3}}(c+dx)}{b^2-a^2\cos^2(c+dx)}dx-a\int \frac{\cos^{\frac{5}{3}}(c+dx)}{b^2-a^2\cos^2(c+dx)}dx\right)$$

$$\Big\downarrow \; 3042$$

$$\sqrt[3]{\cos(c+dx)}\sqrt[3]{\sec(c+dx)}\left(b\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{2/3}}{b^2-a^2\sin\left(c+dx+\frac{\pi}{2}\right)^2}dx-a\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/3}}{b^2-a^2\sin\left(c+dx+\frac{\pi}{2}\right)^2}dx\right)$$

$$\Big\downarrow \; 3668$$

$$\sqrt[3]{\cos(c+dx)}\sqrt[3]{\sec(c+dx)}\left(\frac{b\sqrt[6]{\cos^2(c+dx)}\int -\frac{1}{\sqrt[6]{1-\sin^2(c+dx)}\left(-\sin^2(c+dx)a^2+a^2-b^2\right)}d\sin(c+dx)}{d\sqrt[3]{\cos(c+dx)}}-\frac{a\cos^{\frac{2}{3}}(c}{}\right.$$

$$\Big\downarrow \; 25$$

$$\sqrt[3]{\cos(c+dx)}\sqrt[3]{\sec(c+dx)}\left(\frac{a\cos^{\frac{2}{3}}(c+dx)\int \frac{\sqrt[3]{1-\sin^2(c+dx)}}{-\sin^2(c+dx)a^2+a^2-b^2}d\sin(c+dx)}{d\sqrt[3]{\cos^2(c+dx)}}-\frac{b\sqrt[6]{\cos^2(c+dx)}\int \frac{}{\sqrt[6]{1-\sin^2(c}}}{d\sqrt[3]{}}\right.$$

$$\Big\downarrow \; 333$$

$$\sqrt[3]{\cos(c+dx)}\sqrt[3]{\sec(c+dx)}\left(\frac{a\sin(c+dx)\cos^{\frac{2}{3}}(c+dx)\,\text{AppellF1}\left(\frac{1}{2},-\frac{1}{3},1,\frac{3}{2},\sin^2(c+dx),\frac{a^2\sin^2(c+dx)}{a^2-b^2}\right)}{d\left(a^2-b^2\right)\sqrt[3]{\cos^2(c+dx)}}-\frac{b\sin(}{}\right.$$

input
```
Int[Sec[c + d*x]^(1/3)/(a + b*Sec[c + d*x]),x]
```

output
```
Cos[c + d*x]^(1/3)*Sec[c + d*x]^(1/3)*((a*AppellF1[1/2, -1/3, 1, 3/2, Sin[
c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)]*Cos[c + d*x]^(2/3)*Sin[c + d
*x])/((a^2 - b^2)*d*(Cos[c + d*x]^2)^(1/3)) - (b*AppellF1[1/2, 1/6, 1, 3/2
, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)]*(Cos[c + d*x]^2)^(1/6)
*Sin[c + d*x])/((a^2 - b^2)*d*Cos[c + d*x]^(1/3)))
```

3.714. $\int \frac{\sqrt[3]{\sec(c+dx)}}{a+b\sec(c+dx)}\,dx$

3.714.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 333
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^p*c^q*x*AppellF1[1/2, -p, -q, 3/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; F
reeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[p] || GtQ[a,
0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3302
```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[a   Int[(d*Sin[e + f*x])^n/(a^2 - b^2*Sin[e + f*x]
^2), x], x] - Simp[b/d   Int[(d*Sin[e + f*x])^(n + 1)/(a^2 - b^2*Sin[e + f*
x]^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0]
```

rule 3668
```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[(
-ff)*d^(2*IntPart[(m - 1)/2] + 1)*((d*Sin[e + f*x])^(2*FracPart[(m - 1)/2])
/(f*(Sin[e + f*x]^2)^FracPart[(m - 1)/2]))   Subst[Int[(1 - ff^2*x^2)^((m -
 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b,
 d, e, f, m, p}, x] &&  !IntegerQ[m]
```

rule 4356
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Simp[Sin[e + f*x]^n*(d*Csc[e + f*x])^n   Int[(b +
 a*Sin[e + f*x])^m/Sin[e + f*x]^(m + n), x], x] /; FreeQ[{a, b, d, e, f, n}
, x] && NeQ[a^2 - b^2, 0] && IntegerQ[m]
```

3.714. $\int \frac{\sqrt[3]{\sec(c+dx)}}{a+b\sec(c+dx)}\,dx$

3.714.4 Maple [F]

$$\int \frac{\sec (dx+c)^{\frac{1}{3}}}{a+b\sec (dx+c)}dx$$

input
```
int(sec(d*x+c)^(1/3)/(a+b*sec(d*x+c)),x)
```

output
```
int(sec(d*x+c)^(1/3)/(a+b*sec(d*x+c)),x)
```

3.714.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{\sec(c+dx)}}{a+b\sec(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)^(1/3)/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

output
```
Timed out
```

3.714.6 Sympy [F]

$$\int \frac{\sqrt[3]{\sec(c+dx)}}{a+b\sec(c+dx)}\,dx = \int \frac{\sqrt[3]{\sec (c+dx)}}{a+b\sec (c+dx)}\,dx$$

input
```
integrate(sec(d*x+c)**(1/3)/(a+b*sec(d*x+c)),x)
```

output
```
Integral(sec(c + d*x)**(1/3)/(a + b*sec(c + d*x)), x)
```

3.714.7 Maxima [F]

$$\int \frac{\sqrt[3]{\sec(c+dx)}}{a+b\sec(c+dx)}\,dx = \int \frac{\sec(dx+c)^{\frac{1}{3}}}{b\sec(dx+c)+a}\,dx$$

input
```
integrate(sec(d*x+c)^(1/3)/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)^(1/3)/(b*sec(d*x + c) + a), x)
```

3.714.8 Giac [F]

$$\int \frac{\sqrt[3]{\sec(c+dx)}}{a+b\sec(c+dx)}\,dx = \int \frac{\sec(dx+c)^{\frac{1}{3}}}{b\sec(dx+c)+a}\,dx$$

input
```
integrate(sec(d*x+c)^(1/3)/(a+b*sec(d*x+c)),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^(1/3)/(b*sec(d*x + c) + a), x)
```

3.714.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{\sec(c+dx)}}{a+b\sec(c+dx)}\,dx = \int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{1/3}}{a+\frac{b}{\cos(c+d\,x)}}\,dx$$

input
```
int((1/cos(c + d*x))^(1/3)/(a + b/cos(c + d*x)),x)
```

output
```
int((1/cos(c + d*x))^(1/3)/(a + b/cos(c + d*x)), x)
```

3.715 $\int \frac{1}{\sqrt[3]{\sec(c+dx)}(a+b\sec(c+dx))}\,dx$

3.715.1 Optimal result 5527
3.715.2 Mathematica [B] (warning: unable to verify) 5528
3.715.3 Rubi [A] (verified) 5528
3.715.4 Maple [F] 5531
3.715.5 Fricas [F(-1)] 5531
3.715.6 Sympy [F] 5531
3.715.7 Maxima [F] 5532
3.715.8 Giac [F] 5532
3.715.9 Mupad [F(-1)] 5532

3.715.1 Optimal result

Integrand size = 23, antiderivative size = 174

$$\int \frac{1}{\sqrt[3]{\sec(c+dx)}(a+b\sec(c+dx))}\,dx = -\frac{b\,\text{AppellF1}\left(\frac{1}{2},-\frac{1}{6},1,\frac{3}{2},\sin^2(c+dx),\frac{a^2\sin^2(c+dx)}{a^2-b^2}\right)\sin(c+dx)}{(a^2-b^2)\,d\sqrt[6]{\cos^2(c+dx)}\sqrt[3]{\sec(c+dx)}} + \frac{a\,\text{AppellF1}\left(\frac{1}{2},-\frac{2}{3},1,\frac{3}{2},\sin^2(c+dx),\frac{a^2\sin^2(c+dx)}{a^2-b^2}\right)\sqrt[3]{\cos^2(c+dx)}\sec^{\frac{2}{3}}(c+dx)\sin(c+dx)}{(a^2-b^2)\,d}$$

output
```
-b*AppellF1(1/2,-1/6,1,3/2,sin(d*x+c)^2,a^2*sin(d*x+c)^2/(a^2-b^2))*sin(d*
x+c)/(a^2-b^2)/d/(cos(d*x+c)^2)^(1/6)/sec(d*x+c)^(1/3)+a*AppellF1(1/2,-2/3
,1,3/2,sin(d*x+c)^2,a^2*sin(d*x+c)^2/(a^2-b^2))*(cos(d*x+c)^2)^(1/3)*sec(d
*x+c)^(2/3)*sin(d*x+c)/(a^2-b^2)/d
```

3.715.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 7542 vs. $2(174) = 348$.

Time = 84.74 (sec) , antiderivative size = 7542, normalized size of antiderivative = 43.34

$$\int \frac{1}{\sqrt[3]{\sec(c+dx)}(a+b\sec(c+dx))}\,dx = \text{Result too large to show}$$

input
```
Integrate[1/(Sec[c + d*x]^(1/3)*(a + b*Sec[c + d*x])),x]
```

output
```
Result too large to show
```

3.715.3 Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.12, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {3042, 4356, 3042, 3302, 3042, 3668, 25, 333}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[3]{\sec(c+dx)}(a+b\sec(c+dx))}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt[3]{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)}dx$$

$$\Big\downarrow \text{4356}$$

$$\cos^{\frac{2}{3}}(c+dx)\sec^{\frac{2}{3}}(c+dx)\int \frac{\cos^{\frac{4}{3}}(c+dx)}{b+a\cos(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\cos^{\frac{2}{3}}(c+dx)\sec^{\frac{2}{3}}(c+dx)\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{4/3}}{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3302}$$

$$\cos^{\frac{2}{3}}(c+dx)\sec^{\frac{2}{3}}(c+dx)\left(b\int \frac{\cos^{\frac{4}{3}}(c+dx)}{b^2-a^2\cos^2(c+dx)}dx - a\int \frac{\cos^{\frac{7}{3}}(c+dx)}{b^2-a^2\cos^2(c+dx)}dx\right)$$

$$\Big\downarrow \text{3042}$$

$$\cos^{\frac{2}{3}}(c+dx)\sec^{\frac{2}{3}}(c+dx)\left(b\int\frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{4/3}}{b^2-a^2\sin\left(c+dx+\frac{\pi}{2}\right)^2}dx-a\int\frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{7/3}}{b^2-a^2\sin\left(c+dx+\frac{\pi}{2}\right)^2}dx\right)$$

$$\Big\downarrow \text{3668}$$

$$\cos^{\frac{2}{3}}(c+dx)\sec^{\frac{2}{3}}(c+dx)\left(\frac{b\sqrt[3]{\cos(c+dx)}\int-\frac{\sqrt[6]{1-\sin^2(c+dx)}}{-\sin^2(c+dx)a^2+a^2-b^2}d\sin(c+dx)}{d\sqrt[6]{\cos^2(c+dx)}}-\frac{a\cos^{\frac{4}{3}}(c+dx)\int-\frac{\left(1-\sin^2(c+dx)\right)^{2/3}}{-\sin^2(c+dx)a^2+a^2-b^2}d\sin(c+dx)}{d\cos^2(c+dx)^{2/3}}\right)$$

$$\Big\downarrow \text{25}$$

$$\cos^{\frac{2}{3}}(c+dx)\sec^{\frac{2}{3}}(c+dx)\left(\frac{a\cos^{\frac{4}{3}}(c+dx)\int\frac{\left(1-\sin^2(c+dx)\right)^{2/3}}{-\sin^2(c+dx)a^2+a^2-b^2}d\sin(c+dx)}{d\cos^2(c+dx)^{2/3}}-\frac{b\sqrt[3]{\cos(c+dx)}\int\frac{\sqrt[6]{1-\sin^2(c+dx)}}{-\sin^2(c+dx)a^2+a^2-b^2}d\sin(c+dx)}{d\sqrt[6]{\cos^2(c+dx)}}\right)$$

$$\Big\downarrow \text{333}$$

$$\cos^{\frac{2}{3}}(c+dx)\sec^{\frac{2}{3}}(c+dx)\left(\frac{a\sin(c+dx)\cos^{\frac{4}{3}}(c+dx)\,\text{AppellF1}\left(\frac{1}{2},-\frac{2}{3},1,\frac{3}{2},\sin^2(c+dx),\frac{a^2\sin^2(c+dx)}{a^2-b^2}\right)}{d\left(a^2-b^2\right)\cos^2(c+dx)^{2/3}}-\frac{b\sin(c+dx)\sqrt[3]{\cos(c+dx)}\,\text{A}}{d\,(a}\right.$$

input
```
Int[1/(Sec[c + d*x]^(1/3)*(a + b*Sec[c + d*x])),x]
```

output
```
Cos[c + d*x]^(2/3)*Sec[c + d*x]^(2/3)*((a*AppellF1[1/2, -2/3, 1, 3/2, Sin[
c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)]*Cos[c + d*x]^(4/3)*Sin[c + d
*x])/((a^2 - b^2)*d*(Cos[c + d*x]^2)^(2/3)) - (b*AppellF1[1/2, -1/6, 1, 3/
2, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)]*Cos[c + d*x]^(1/3)*Si
n[c + d*x])/((a^2 - b^2)*d*(Cos[c + d*x]^2)^(1/6)))
```

3.715. $\int \frac{1}{\sqrt[3]{\sec(c+dx)}(a+b\sec(c+dx))}\,dx$

3.715.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 333
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^p*c^q*x*AppellF1[1/2, -p, -q, 3/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; F
reeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[p] || GtQ[a,
0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3302
```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[a   Int[(d*Sin[e + f*x])^n/(a^2 - b^2*Sin[e + f*x]
^2), x], x] - Simp[b/d   Int[(d*Sin[e + f*x])^(n + 1)/(a^2 - b^2*Sin[e + f*
x]^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0]
```

rule 3668
```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[(
-ff)*d^(2*IntPart[(m - 1)/2] + 1)*((d*Sin[e + f*x])^(2*FracPart[(m - 1)/2])
/(f*(Sin[e + f*x]^2)^FracPart[(m - 1)/2]))   Subst[Int[(1 - ff^2*x^2)^((m -
 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b,
 d, e, f, m, p}, x] &&  !IntegerQ[m]
```

rule 4356
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Simp[Sin[e + f*x]^n*(d*Csc[e + f*x])^n   Int[(b +
 a*Sin[e + f*x])^m/Sin[e + f*x]^(m + n), x], x] /; FreeQ[{a, b, d, e, f, n}
, x] && NeQ[a^2 - b^2, 0] && IntegerQ[m]
```

3.715. $\int \frac{1}{\sqrt[3]{\sec(c+dx)}(a+b\sec(c+dx))}\,dx$

3.715.4 Maple [F]

$$\int \frac{1}{\sec(dx+c)^{\frac{1}{3}}(a+b\sec(dx+c))}dx$$

input
```
int(1/sec(d*x+c)^(1/3)/(a+b*sec(d*x+c)),x)
```

output
```
int(1/sec(d*x+c)^(1/3)/(a+b*sec(d*x+c)),x)
```

3.715.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{\sec(c+dx)}(a+b\sec(c+dx))}dx = \text{Timed out}$$

input
```
integrate(1/sec(d*x+c)^(1/3)/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

output
```
Timed out
```

3.715.6 Sympy [F]

$$\int \frac{1}{\sqrt[3]{\sec(c+dx)}(a+b\sec(c+dx))}dx = \int \frac{1}{(a+b\sec(c+dx))\sqrt[3]{\sec(c+dx)}}dx$$

input
```
integrate(1/sec(d*x+c)**(1/3)/(a+b*sec(d*x+c)),x)
```

output
```
Integral(1/((a + b*sec(c + d*x))*sec(c + d*x)**(1/3)), x)
```

3.715.7 Maxima [F]

$$\int \frac{1}{\sqrt[3]{\sec(c+dx)}(a+b\sec(c+dx))}\,dx = \int \frac{1}{(b\sec(dx+c)+a)\sec(dx+c)^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/sec(d*x+c)^(1/3)/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

output
```
integrate(1/((b*sec(d*x + c) + a)*sec(d*x + c)^(1/3)), x)
```

3.715.8 Giac [F]

$$\int \frac{1}{\sqrt[3]{\sec(c+dx)}(a+b\sec(c+dx))}\,dx = \int \frac{1}{(b\sec(dx+c)+a)\sec(dx+c)^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/sec(d*x+c)^(1/3)/(a+b*sec(d*x+c)),x, algorithm="giac")
```

output
```
integrate(1/((b*sec(d*x + c) + a)*sec(d*x + c)^(1/3)), x)
```

3.715.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{\sec(c+dx)}(a+b\sec(c+dx))}\,dx = \int \frac{1}{\left(a+\frac{b}{\cos(c+d\,x)}\right)\,\left(\frac{1}{\cos(c+d\,x)}\right)^{1/3}}\,dx$$

input
```
int(1/((a + b/cos(c + d*x))*(1/cos(c + d*x))^(1/3)),x)
```

output
```
int(1/((a + b/cos(c + d*x))*(1/cos(c + d*x))^(1/3)), x)
```

3.716 $\int \frac{1}{\sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))}\,dx$

3.716.1 Optimal result 5533
3.716.2 Mathematica [B] (warning: unable to verify) 5533
3.716.3 Rubi [A] (verified) 5534
3.716.4 Maple [F] 5536
3.716.5 Fricas [F(-1)] 5536
3.716.6 Sympy [F] 5537
3.716.7 Maxima [F] 5537
3.716.8 Giac [F] 5537
3.716.9 Mupad [F(-1)] 5538

3.716.1 Optimal result

Integrand size = 23, antiderivative size = 174

$$\int \frac{1}{\sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))}\,dx = -\frac{b\,\text{AppellF1}\left(\frac{1}{2},-\frac{1}{3},1,\frac{3}{2},\sin^2(c+dx),\frac{a^2\sin^2(c+dx)}{a^2-b^2}\right)\sin(c+dx)}{(a^2-b^2)\,d\sqrt[3]{\cos^2(c+dx)}\sec^{\frac{2}{3}}(c+dx)} + \frac{a\,\text{AppellF1}\left(\frac{1}{2},-\frac{5}{6},1,\frac{3}{2},\sin^2(c+dx),\frac{a^2\sin^2(c+dx)}{a^2-b^2}\right)\sqrt[6]{\cos^2(c+dx)}\sqrt[3]{\sec(c+dx)}\sin(c+dx)}{(a^2-b^2)\,d}$$

output
```
-b*AppellF1(1/2,-1/3,1,3/2,sin(d*x+c)^2,a^2*sin(d*x+c)^2/(a^2-b^2))*sin(d*
x+c)/(a^2-b^2)/d/(cos(d*x+c)^2)^(1/3)/sec(d*x+c)^(2/3)+a*AppellF1(1/2,-5/6
,1,3/2,sin(d*x+c)^2,a^2*sin(d*x+c)^2/(a^2-b^2))*(cos(d*x+c)^2)^(1/6)*sec(d
*x+c)^(1/3)*sin(d*x+c)/(a^2-b^2)/d
```

3.716.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 7588 vs. 2(174) = 348.

Time = 82.40 (sec) , antiderivative size = 7588, normalized size of antiderivative = 43.61

$$\int \frac{1}{\sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))}\,dx = \text{Result too large to show}$$

input
```
Integrate[1/(Sec[c + d*x]^(2/3)*(a + b*Sec[c + d*x])),x]
```

output
```
Result too large to show
```

3.716.3 Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.12, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {3042, 4356, 3042, 3302, 3042, 3668, 25, 333}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{2/3}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)}dx$$

$$\Big\downarrow \text{4356}$$

$$\sqrt[3]{\cos(c+dx)}\sqrt[3]{\sec(c+dx)}\int \frac{\cos^{\frac{5}{3}}(c+dx)}{b+a\cos(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt[3]{\cos(c+dx)}\sqrt[3]{\sec(c+dx)}\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/3}}{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3302}$$

$$\sqrt[3]{\cos(c+dx)}\sqrt[3]{\sec(c+dx)}\left(b\int \frac{\cos^{\frac{5}{3}}(c+dx)}{b^2-a^2\cos^2(c+dx)}dx-a\int \frac{\cos^{\frac{8}{3}}(c+dx)}{b^2-a^2\cos^2(c+dx)}dx\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt[3]{\cos(c+dx)}\sqrt[3]{\sec(c+dx)}\left(b\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/3}}{b^2-a^2\sin\left(c+dx+\frac{\pi}{2}\right)^2}dx-a\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{8/3}}{b^2-a^2\sin\left(c+dx+\frac{\pi}{2}\right)^2}dx\right)$$

$$\Big\downarrow \text{3668}$$

$$\sqrt[3]{\cos(c+dx)}\sqrt[3]{\sec(c+dx)}\left(\frac{b\cos^{\frac{2}{3}}(c+dx)\int -\frac{\sqrt[3]{1-\sin^2(c+dx)}}{-\sin^2(c+dx)a^2+a^2-b^2}d\sin(c+dx)}{d\sqrt[3]{\cos^2(c+dx)}}-\frac{a\cos^{\frac{5}{3}}(c+dx)\int -\frac{(1-\sin^2(c+}{-\sin^2(c+dx)}}{d\cos^2(c+d}\right.$$

$$\Big\downarrow \; 25$$

$$\sqrt[3]{\cos(c+dx)}\sqrt[3]{\sec(c+dx)}\left(\frac{a\cos^{\frac{5}{3}}(c+dx)\int \frac{(1-\sin^2(c+dx))^{5/6}}{-\sin^2(c+dx)a^2+a^2-b^2}d\sin(c+dx)}{d\cos^2(c+dx)^{5/6}}-\frac{b\cos^{\frac{2}{3}}(c+dx)\int \frac{\sqrt[3]{1-\sin^2(c+}}{-\sin^2(c+dx)a^2+a}}{d\sqrt[3]{\cos^2(c+d}}\right.$$

$$\Big\downarrow \; 333$$

$$\sqrt[3]{\cos(c+dx)}\sqrt[3]{\sec(c+dx)}\left(\frac{a\sin(c+dx)\cos^{\frac{5}{3}}(c+dx)\,\text{AppellF1}\left(\frac{1}{2},-\frac{5}{6},1,\frac{3}{2},\sin^2(c+dx),\frac{a^2\sin^2(c+dx)}{a^2-b^2}\right)}{d\,(a^2-b^2)\cos^2(c+dx)^{5/6}}-\frac{b\sin(}{}\right.$$

input
```
Int[1/(Sec[c + d*x]^(2/3)*(a + b*Sec[c + d*x])),x]
```

output
```
Cos[c + d*x]^(1/3)*Sec[c + d*x]^(1/3)*((a*AppellF1[1/2, -5/6, 1, 3/2, Sin[
c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)]*Cos[c + d*x]^(5/3)*Sin[c + d
*x])/((a^2 - b^2)*d*(Cos[c + d*x]^2)^(5/6)) - (b*AppellF1[1/2, -1/3, 1, 3/
2, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)]*Cos[c + d*x]^(2/3)*Si
n[c + d*x])/((a^2 - b^2)*d*(Cos[c + d*x]^2)^(1/3)))
```

3.716.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 333
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^p*c^q*x*AppellF1[1/2, -p, -q, 3/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; F
reeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[p] || GtQ[a,
0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3302
```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[a   Int[(d*Sin[e + f*x])^n/(a^2 - b^2*Sin[e + f*x]
^2), x], x] - Simp[b/d   Int[(d*Sin[e + f*x])^(n + 1)/(a^2 - b^2*Sin[e + f*
x]^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0]
```

rule 3668
```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[(
-ff)*d^(2*IntPart[(m - 1)/2] + 1)*((d*Sin[e + f*x])^(2*FracPart[(m - 1)/2])
/(f*(Sin[e + f*x]^2)^FracPart[(m - 1)/2]))   Subst[Int[(1 - ff^2*x^2)^((m -
 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b,
 d, e, f, m, p}, x] &&  !IntegerQ[m]
```

rule 4356
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Simp[Sin[e + f*x]^n*(d*Csc[e + f*x])^n   Int[(b +
 a*Sin[e + f*x])^m/Sin[e + f*x]^(m + n), x], x] /; FreeQ[{a, b, d, e, f, n}
, x] && NeQ[a^2 - b^2, 0] && IntegerQ[m]
```

3.716.4 Maple [F]

$$\int \frac{1}{\sec\left(dx+c\right)^{\frac{2}{3}}\left(a+b\sec\left(dx+c\right)\right)}dx$$

input
```
int(1/sec(d*x+c)^(2/3)/(a+b*sec(d*x+c)),x)
```

output
```
int(1/sec(d*x+c)^(2/3)/(a+b*sec(d*x+c)),x)
```

3.716.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))}\,dx = \text{Timed out}$$

input
```
integrate(1/sec(d*x+c)^(2/3)/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

output
```
Timed out
```

3.716.6 Sympy [F]

$$\int \frac{1}{\sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))}\,dx = \int \frac{1}{(a+b\sec(c+dx))\sec^{\frac{2}{3}}(c+dx)}\,dx$$

input
```
integrate(1/sec(d*x+c)**(2/3)/(a+b*sec(d*x+c)),x)
```

output
```
Integral(1/((a + b*sec(c + d*x))*sec(c + d*x)**(2/3)), x)
```

3.716.7 Maxima [F]

$$\int \frac{1}{\sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))}\,dx = \int \frac{1}{(b\sec(dx+c)+a)\sec(dx+c)^{\frac{2}{3}}}\,dx$$

input
```
integrate(1/sec(d*x+c)^(2/3)/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

output
```
integrate(1/((b*sec(d*x + c) + a)*sec(d*x + c)^(2/3)), x)
```

3.716.8 Giac [F]

$$\int \frac{1}{\sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))}\,dx = \int \frac{1}{(b\sec(dx+c)+a)\sec(dx+c)^{\frac{2}{3}}}\,dx$$

input
```
integrate(1/sec(d*x+c)^(2/3)/(a+b*sec(d*x+c)),x, algorithm="giac")
```

output
```
integrate(1/((b*sec(d*x + c) + a)*sec(d*x + c)^(2/3)), x)
```

3.716.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))}\,dx=\int \frac{1}{\left(a+\frac{b}{\cos(c+d\,x)}\right)\,{\left(\frac{1}{\cos(c+d\,x)}\right)}^{2/3}}\,dx$$

input
```
int(1/((a + b/cos(c + d*x))*(1/cos(c + d*x))^(2/3)),x)
```

output
```
int(1/((a + b/cos(c + d*x))*(1/cos(c + d*x))^(2/3)), x)
```

3.717 $\int \sec^{\frac{7}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$

3.717.1 Optimal result 5539
3.717.2 Mathematica [F(-1)] 5539
3.717.3 Rubi [N/A] 5540
3.717.4 Maple [N/A] (verified) 5541
3.717.5 Fricas [N/A] 5541
3.717.6 Sympy [F(-1)] 5541
3.717.7 Maxima [N/A] 5542
3.717.8 Giac [N/A] 5542
3.717.9 Mupad [N/A] 5542

3.717.1 Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \sec^{\frac{7}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = \text{Int}\left(\sec^{\frac{7}{3}}(c+dx)\sqrt{a+b\sec(c+dx)},x\right)$$

output
```
Unintegrable(sec(d*x+c)^(7/3)*(a+b*sec(d*x+c))^(1/2),x)
```

3.717.2 Mathematica [F(-1)]

Timed out.

$$\int \sec^{\frac{7}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = \$\text{Aborted}$$

input
```
Integrate[Sec[c + d*x]^(7/3)*Sqrt[a + b*Sec[c + d*x]],x]
```

output
```
$Aborted
```

3.717.3 Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 4357}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{\frac{7}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{7/3}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{ 4357}$$

$$\int \sec^{\frac{7}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}dx$$

input
```
Int[Sec[c + d*x]^(7/3)*Sqrt[a + b*Sec[c + d*x]],x]
```

output
```
$Aborted
```

3.717.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4357
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Unintegrable[(d*Csc[e + f*x])^n*(a + b*Csc[e + f*
x])^m, x] /; FreeQ[{a, b, d, e, f, m, n}, x]
```

3.717.4 Maple [N/A] (verified)

Not integrable

Time = 0.51 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \sec (dx+c)^{\frac{7}{3}} \sqrt{a+b\sec (dx+c)}dx$$

input
```
int(sec(d*x+c)^(7/3)*(a+b*sec(d*x+c))^(1/2),x)
```

output
```
int(sec(d*x+c)^(7/3)*(a+b*sec(d*x+c))^(1/2),x)
```

3.717.5 Fricas [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \sec^{\frac{7}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = \int \sqrt{b\sec (dx+c)+a}\sec (dx+c)^{\frac{7}{3}}\,dx$$

input
```
integrate(sec(d*x+c)^(7/3)*(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(7/3), x)
```

3.717.6 Sympy [F(-1)]

Timed out.

$$\int \sec^{\frac{7}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**(7/3)*(a+b*sec(d*x+c))**(1/2),x)
```

output
```
Timed out
```

3.717.7 Maxima [N/A]

Not integrable

Time = 0.78 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \sec^{\frac{7}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = \int \sqrt{b\sec(dx+c)+a}\sec(dx+c)^{\frac{7}{3}}\,dx$$

input
```
integrate(sec(d*x+c)^(7/3)*(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(7/3), x)
```

3.717.8 Giac [N/A]

Not integrable

Time = 1.18 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \sec^{\frac{7}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = \int \sqrt{b\sec(dx+c)+a}\sec(dx+c)^{\frac{7}{3}}\,dx$$

input
```
integrate(sec(d*x+c)^(7/3)*(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(7/3), x)
```

3.717.9 Mupad [N/A]

Not integrable

Time = 14.96 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \sec^{\frac{7}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = \int \sqrt{a+\frac{b}{\cos(c+d\,x)}}\left(\frac{1}{\cos(c+d\,x)}\right)^{7/3}dx$$

input
```
int((a + b/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(7/3),x)
```

output
```
int((a + b/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(7/3), x)
```

3.718 $\int \sec^{\frac{5}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$

3.718.1 Optimal result 5543
3.718.2 Mathematica [F(-1)] 5543
3.718.3 Rubi [N/A] 5544
3.718.4 Maple [N/A] (verified) 5545
3.718.5 Fricas [N/A] 5545
3.718.6 Sympy [F(-1)] 5545
3.718.7 Maxima [N/A] 5546
3.718.8 Giac [N/A] 5546
3.718.9 Mupad [N/A] 5546

3.718.1 Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \sec^{\frac{5}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = \text{Int}\left(\sec^{\frac{5}{3}}(c+dx)\sqrt{a+b\sec(c+dx)},x\right)$$

output
```
Unintegrable(sec(d*x+c)^(5/3)*(a+b*sec(d*x+c))^(1/2),x)
```

3.718.2 Mathematica [F(-1)]

Timed out.

$$\int \sec^{\frac{5}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = \$\text{Aborted}$$

input
```
Integrate[Sec[c + d*x]^(5/3)*Sqrt[a + b*Sec[c + d*x]],x]
```

output
```
$Aborted
```

3.718.3 Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 4357}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{\frac{5}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{5/3}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{ 4357}$$

$$\int \sec^{\frac{5}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}dx$$

input
```
Int[Sec[c + d*x]^(5/3)*Sqrt[a + b*Sec[c + d*x]],x]
```

output
```
$Aborted
```

3.718.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4357
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Unintegrable[(d*Csc[e + f*x])^n*(a + b*Csc[e + f*
x])^m, x] /; FreeQ[{a, b, d, e, f, m, n}, x]
```

3.718.4 Maple [N/A] (verified)

Not integrable

Time = 0.48 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \sec (dx+c)^{\frac{5}{3}} \sqrt{a+b\sec (dx+c)}dx$$

input
```
int(sec(d*x+c)^(5/3)*(a+b*sec(d*x+c))^(1/2),x)
```

output
```
int(sec(d*x+c)^(5/3)*(a+b*sec(d*x+c))^(1/2),x)
```

3.718.5 Fricas [N/A]

Not integrable

Time = 0.42 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \sec^{\frac{5}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = \int \sqrt{b\sec (dx+c)+a}\sec (dx+c)^{\frac{5}{3}}\,dx$$

input
```
integrate(sec(d*x+c)^(5/3)*(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(5/3), x)
```

3.718.6 Sympy [F(-1)]

Timed out.

$$\int \sec^{\frac{5}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**(5/3)*(a+b*sec(d*x+c))**(1/2),x)
```

output
```
Timed out
```

3.718.7 Maxima [N/A]

Not integrable

Time = 0.80 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \sec^{\frac{5}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = \int \sqrt{b\sec(dx+c)+a}\sec(dx+c)^{\frac{5}{3}}\,dx$$

input
```
integrate(sec(d*x+c)^(5/3)*(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(5/3), x)
```

3.718.8 Giac [N/A]

Not integrable

Time = 0.98 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \sec^{\frac{5}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = \int \sqrt{b\sec(dx+c)+a}\sec(dx+c)^{\frac{5}{3}}\,dx$$

input
```
integrate(sec(d*x+c)^(5/3)*(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(5/3), x)
```

3.718.9 Mupad [N/A]

Not integrable

Time = 14.78 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \sec^{\frac{5}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = \int \sqrt{a+\frac{b}{\cos(c+d\,x)}}\left(\frac{1}{\cos(c+d\,x)}\right)^{5/3}dx$$

input
```
int((a + b/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(5/3),x)
```

output
```
int((a + b/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(5/3), x)
```

3.719 $\int \sec^{\frac{4}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$

3.719.1 Optimal result 5547
3.719.2 Mathematica [N/A] 5547
3.719.3 Rubi [N/A] 5548
3.719.4 Maple [N/A] (verified) 5549
3.719.5 Fricas [N/A] 5549
3.719.6 Sympy [F(-1)] 5549
3.719.7 Maxima [N/A] 5550
3.719.8 Giac [N/A] 5550
3.719.9 Mupad [N/A] 5550

3.719.1 Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \sec^{\frac{4}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = \text{Int}\left(\sec^{\frac{4}{3}}(c+dx)\sqrt{a+b\sec(c+dx)},x\right)$$

output
```
Unintegrable(sec(d*x+c)^(4/3)*(a+b*sec(d*x+c))^(1/2),x)
```

3.719.2 Mathematica [N/A]

Not integrable

Time = 105.48 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \sec^{\frac{4}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = \int \sec^{\frac{4}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$$

input
```
Integrate[Sec[c + d*x]^(4/3)*Sqrt[a + b*Sec[c + d*x]],x]
```

output
```
Integrate[Sec[c + d*x]^(4/3)*Sqrt[a + b*Sec[c + d*x]], x]
```

3.719.3 Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 4357}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{\frac{4}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{4/3}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{ 4357}$$

$$\int \sec^{\frac{4}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}dx$$

input
```
Int[Sec[c + d*x]^(4/3)*Sqrt[a + b*Sec[c + d*x]],x]
```

output
```
$Aborted
```

3.719.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4357
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Unintegrable[(d*Csc[e + f*x])^n*(a + b*Csc[e + f*
x])^m, x] /; FreeQ[{a, b, d, e, f, m, n}, x]
```

3.719.4 Maple [N/A] (verified)

Not integrable

Time = 0.52 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \sec (dx+c)^{\frac{4}{3}} \sqrt{a+b\sec (dx+c)}dx$$

input
```
int(sec(d*x+c)^(4/3)*(a+b*sec(d*x+c))^(1/2),x)
```

output
```
int(sec(d*x+c)^(4/3)*(a+b*sec(d*x+c))^(1/2),x)
```

3.719.5 Fricas [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \sec^{\frac{4}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = \int \sqrt{b\sec (dx+c)+a}\sec (dx+c)^{\frac{4}{3}}\,dx$$

input
```
integrate(sec(d*x+c)^(4/3)*(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(4/3), x)
```

3.719.6 Sympy [F(-1)]

Timed out.

$$\int \sec^{\frac{4}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**(4/3)*(a+b*sec(d*x+c))**(1/2),x)
```

output
```
Timed out
```

3.719.7 Maxima [N/A]

Not integrable

Time = 0.78 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \sec^{\frac{4}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = \int \sqrt{b\sec(dx+c)+a}\sec(dx+c)^{\frac{4}{3}}\,dx$$

input
```
integrate(sec(d*x+c)^(4/3)*(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(4/3), x)
```

3.719.8 Giac [N/A]

Not integrable

Time = 0.77 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \sec^{\frac{4}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = \int \sqrt{b\sec(dx+c)+a}\sec(dx+c)^{\frac{4}{3}}\,dx$$

input
```
integrate(sec(d*x+c)^(4/3)*(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(4/3), x)
```

3.719.9 Mupad [N/A]

Not integrable

Time = 14.86 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \sec^{\frac{4}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = \int \sqrt{a+\frac{b}{\cos(c+d\,x)}}\left(\frac{1}{\cos(c+d\,x)}\right)^{4/3}dx$$

input
```
int((a + b/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(4/3),x)
```

output
```
int((a + b/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(4/3), x)
```

3.720 $\int \sec^{\frac{2}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$

3.720.1 Optimal result 5551
3.720.2 Mathematica [N/A] 5551
3.720.3 Rubi [N/A] 5552
3.720.4 Maple [N/A] (verified) 5553
3.720.5 Fricas [N/A] 5553
3.720.6 Sympy [N/A] 5553
3.720.7 Maxima [N/A] 5554
3.720.8 Giac [N/A] 5554
3.720.9 Mupad [N/A] 5554

3.720.1 Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \sec^{\frac{2}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = \text{Int}\left(\sec^{\frac{2}{3}}(c+dx)\sqrt{a+b\sec(c+dx)},x\right)$$

output
```
Unintegrable(sec(d*x+c)^(2/3)*(a+b*sec(d*x+c))^(1/2),x)
```

3.720.2 Mathematica [N/A]

Not integrable

Time = 135.59 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \sec^{\frac{2}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = \int \sec^{\frac{2}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$$

input
```
Integrate[Sec[c + d*x]^(2/3)*Sqrt[a + b*Sec[c + d*x]],x]
```

output
```
Integrate[Sec[c + d*x]^(2/3)*Sqrt[a + b*Sec[c + d*x]], x]
```

3.720.3 Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 4357}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{\frac{2}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{2/3}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{4357}$$

$$\int \sec^{\frac{2}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}dx$$

input
```
Int[Sec[c + d*x]^(2/3)*Sqrt[a + b*Sec[c + d*x]],x]
```

output
```
$Aborted
```

3.720.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4357
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Unintegrable[(d*Csc[e + f*x])^n*(a + b*Csc[e + f*
x])^m, x] /; FreeQ[{a, b, d, e, f, m, n}, x]
```

3.720.4 Maple [N/A] (verified)

Not integrable

Time = 0.47 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \sec (dx+c)^{\frac{2}{3}} \sqrt{a+b\sec (dx+c)}dx$$

input
```
int(sec(d*x+c)^(2/3)*(a+b*sec(d*x+c))^(1/2),x)
```

output
```
int(sec(d*x+c)^(2/3)*(a+b*sec(d*x+c))^(1/2),x)
```

3.720.5 Fricas [N/A]

Not integrable

Time = 1.12 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \sec^{\frac{2}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = \int \sqrt{b\sec (dx+c)+a}\sec (dx+c)^{\frac{2}{3}}\,dx$$

input
```
integrate(sec(d*x+c)^(2/3)*(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(2/3), x)
```

3.720.6 Sympy [N/A]

Not integrable

Time = 9.41 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int \sec^{\frac{2}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = \int \sqrt{a+b\sec (c+dx)}\sec^{\frac{2}{3}}(c+dx)\,dx$$

input
```
integrate(sec(d*x+c)**(2/3)*(a+b*sec(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(a + b*sec(c + d*x))*sec(c + d*x)**(2/3), x)
```

3.720.7 Maxima [N/A]

Not integrable

Time = 0.75 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \sec^{\frac{2}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = \int \sqrt{b\sec(dx+c)+a}\sec(dx+c)^{\frac{2}{3}}\,dx$$

input
```
integrate(sec(d*x+c)^(2/3)*(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(2/3), x)
```

3.720.8 Giac [N/A]

Not integrable

Time = 1.15 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \sec^{\frac{2}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = \int \sqrt{b\sec(dx+c)+a}\sec(dx+c)^{\frac{2}{3}}\,dx$$

input
```
integrate(sec(d*x+c)^(2/3)*(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(2/3), x)
```

3.720.9 Mupad [N/A]

Not integrable

Time = 14.51 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \sec^{\frac{2}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = \int \sqrt{a+\frac{b}{\cos(c+d\,x)}}\left(\frac{1}{\cos(c+d\,x)}\right)^{2/3}dx$$

input
```
int((a + b/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(2/3),x)
```

output
```
int((a + b/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(2/3), x)
```

3.721 $\int \sqrt[3]{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\,dx$

3.721.1 Optimal result 5555
3.721.2 Mathematica [N/A] 5555
3.721.3 Rubi [N/A] 5556
3.721.4 Maple [N/A] (verified) 5557
3.721.5 Fricas [N/A] 5557
3.721.6 Sympy [N/A] 5557
3.721.7 Maxima [N/A] 5558
3.721.8 Giac [N/A] 5558
3.721.9 Mupad [N/A] 5558

3.721.1 Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \sqrt[3]{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\,dx = \text{Int}\left(\sqrt[3]{\sec(c+dx)}\sqrt{a+b\sec(c+dx)},x\right)$$

output
```
Unintegrable(sec(d*x+c)^(1/3)*(a+b*sec(d*x+c))^(1/2),x)
```

3.721.2 Mathematica [N/A]

Not integrable

Time = 22.27 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \sqrt[3]{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\,dx = \int \sqrt[3]{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\,dx$$

input
```
Integrate[Sec[c + d*x]^(1/3)*Sqrt[a + b*Sec[c + d*x]],x]
```

output
```
Integrate[Sec[c + d*x]^(1/3)*Sqrt[a + b*Sec[c + d*x]], x]
```

3.721.3 Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 4357}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt[3]{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt[3]{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{4357}$$

$$\int \sqrt[3]{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}dx$$

input
```
Int[Sec[c + d*x]^(1/3)*Sqrt[a + b*Sec[c + d*x]],x]
```

output
```
$Aborted
```

3.721.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4357
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Unintegrable[(d*Csc[e + f*x])^n*(a + b*Csc[e + f*
x])^m, x] /; FreeQ[{a, b, d, e, f, m, n}, x]
```

3.721.4 Maple [N/A] (verified)

Not integrable

Time = 0.47 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \sec (dx+c)^{\frac{1}{3}} \sqrt{a+b\sec (dx+c)}dx$$

input
```
int(sec(d*x+c)^(1/3)*(a+b*sec(d*x+c))^(1/2),x)
```

output
```
int(sec(d*x+c)^(1/3)*(a+b*sec(d*x+c))^(1/2),x)
```

3.721.5 Fricas [N/A]

Not integrable

Time = 0.46 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \sqrt[3]{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\,dx = \int \sqrt{b\sec (dx+c)+a}\sec (dx+c)^{\frac{1}{3}}\,dx$$

input
```
integrate(sec(d*x+c)^(1/3)*(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(1/3), x)
```

3.721.6 Sympy [N/A]

Not integrable

Time = 1.10 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int \sqrt[3]{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\,dx = \int \sqrt{a+b\sec (c+dx)}\sqrt[3]{\sec (c+dx)}\,dx$$

input
```
integrate(sec(d*x+c)**(1/3)*(a+b*sec(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(a + b*sec(c + d*x))*sec(c + d*x)**(1/3), x)
```

3.721.7 Maxima [N/A]

Not integrable

Time = 0.74 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \sqrt[3]{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\,dx = \int \sqrt{b\sec(dx+c)+a}\sec(dx+c)^{\frac{1}{3}}\,dx$$

input
```
integrate(sec(d*x+c)^(1/3)*(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(1/3), x)
```

3.721.8 Giac [N/A]

Not integrable

Time = 0.86 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \sqrt[3]{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\,dx = \int \sqrt{b\sec(dx+c)+a}\sec(dx+c)^{\frac{1}{3}}\,dx$$

input
```
integrate(sec(d*x+c)^(1/3)*(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(1/3), x)
```

3.721.9 Mupad [N/A]

Not integrable

Time = 14.36 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \sqrt[3]{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\,dx = \int \sqrt{a+\frac{b}{\cos(c+d\,x)}}\left(\frac{1}{\cos(c+d\,x)}\right)^{1/3}dx$$

input
```
int((a + b/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(1/3),x)
```

output
```
int((a + b/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(1/3), x)
```

3.722 $\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt[3]{\sec(c+dx)}}\,dx$

3.722.1 Optimal result . . . 5559
3.722.2 Mathematica [N/A] . . . 5559
3.722.3 Rubi [N/A] . . . 5560
3.722.4 Maple [N/A] (verified) . . . 5561
3.722.5 Fricas [N/A] . . . 5561
3.722.6 Sympy [N/A] . . . 5561
3.722.7 Maxima [N/A] . . . 5562
3.722.8 Giac [N/A] . . . 5562
3.722.9 Mupad [N/A] . . . 5562

3.722.1 Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt[3]{\sec(c+dx)}}\,dx = \text{Int}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt[3]{\sec(c+dx)}},x\right)$$

output
```
Unintegrable((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/3),x)
```

3.722.2 Mathematica [N/A]

Not integrable

Time = 89.47 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt[3]{\sec(c+dx)}}\,dx = \int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt[3]{\sec(c+dx)}}\,dx$$

input
```
Integrate[Sqrt[a + b*Sec[c + d*x]]/Sec[c + d*x]^(1/3),x]
```

output
```
Integrate[Sqrt[a + b*Sec[c + d*x]]/Sec[c + d*x]^(1/3), x]
```

3.722.3 Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 4357}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt[3]{\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt[3]{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4357}$$

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt[3]{\sec(c+dx)}}dx$$

input
```
Int[Sqrt[a + b*Sec[c + d*x]]/Sec[c + d*x]^(1/3),x]
```

output
```
$Aborted
```

3.722.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4357
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Unintegrable[(d*Csc[e + f*x])^n*(a + b*Csc[e + f*
x])^m, x] /; FreeQ[{a, b, d, e, f, m, n}, x]
```

3.722.4 Maple [N/A] (verified)

Not integrable

Time = 0.50 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{\sqrt{a+b\sec(dx+c)}}{\sec(dx+c)^{\frac{1}{3}}}dx$$

input
```
int((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/3),x)
```

output
```
int((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/3),x)
```

3.722.5 Fricas [N/A]

Not integrable

Time = 1.06 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt[3]{\sec(c+dx)}}\,dx = \int \frac{\sqrt{b\sec(dx+c)+a}}{\sec(dx+c)^{\frac{1}{3}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/3),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sec(d*x + c) + a)/sec(d*x + c)^(1/3), x)
```

3.722.6 Sympy [N/A]

Not integrable

Time = 0.67 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt[3]{\sec(c+dx)}}\,dx = \int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt[3]{\sec(c+dx)}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))**(1/2)/sec(d*x+c)**(1/3),x)
```

output
```
Integral(sqrt(a + b*sec(c + d*x))/sec(c + d*x)**(1/3), x)
```

3.722.7 Maxima [N/A]

Not integrable

Time = 0.76 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt[3]{\sec(c+dx)}}\,dx = \int \frac{\sqrt{b\sec(dx+c)+a}}{\sec(dx+c)^{\frac{1}{3}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/3),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sec(d*x + c) + a)/sec(d*x + c)^(1/3), x)
```

3.722.8 Giac [N/A]

Not integrable

Time = 1.21 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt[3]{\sec(c+dx)}}\,dx = \int \frac{\sqrt{b\sec(dx+c)+a}}{\sec(dx+c)^{\frac{1}{3}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/3),x, algorithm="giac")
```

output
```
integrate(sqrt(b*sec(d*x + c) + a)/sec(d*x + c)^(1/3), x)
```

3.722.9 Mupad [N/A]

Not integrable

Time = 14.92 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt[3]{\sec(c+dx)}}\,dx = \int \frac{\sqrt{a+\frac{b}{\cos(c+d\,x)}}}{\left(\frac{1}{\cos(c+d\,x)}\right)^{1/3}}\,dx$$

input
```
int((a + b/cos(c + d*x))^(1/2)/(1/cos(c + d*x))^(1/3),x)
```

output
```
int((a + b/cos(c + d*x))^(1/2)/(1/cos(c + d*x))^(1/3), x)
```

3.723 $\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{2}{3}}(c+dx)}\,dx$

3.723.1 Optimal result 5563
3.723.2 Mathematica [N/A] 5563
3.723.3 Rubi [N/A] 5564
3.723.4 Maple [N/A] (verified) 5565
3.723.5 Fricas [N/A] 5565
3.723.6 Sympy [N/A] 5565
3.723.7 Maxima [N/A] 5566
3.723.8 Giac [N/A] 5566
3.723.9 Mupad [N/A] 5566

3.723.1 Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{2}{3}}(c+dx)}\,dx = \text{Int}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{2}{3}}(c+dx)},x\right)$$

output
```
Unintegrable((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(2/3),x)
```

3.723.2 Mathematica [N/A]

Not integrable

Time = 100.39 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{2}{3}}(c+dx)}\,dx = \int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{2}{3}}(c+dx)}\,dx$$

input
```
Integrate[Sqrt[a + b*Sec[c + d*x]]/Sec[c + d*x]^(2/3),x]
```

output
```
Integrate[Sqrt[a + b*Sec[c + d*x]]/Sec[c + d*x]^(2/3), x]
```

3.723.3 Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 4357}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{2}{3}}(c+dx)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}{\csc\left(c+dx+\frac{\pi}{2}\right)^{2/3}}dx$$

$$\Big\downarrow \text{ 4357}$$

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{2}{3}}(c+dx)}dx$$

input
```
Int[Sqrt[a + b*Sec[c + d*x]]/Sec[c + d*x]^(2/3),x]
```

output
```
$Aborted
```

3.723.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4357
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Unintegrable[(d*Csc[e + f*x])^n*(a + b*Csc[e + f*
x])^m, x] /; FreeQ[{a, b, d, e, f, m, n}, x]
```

3.723.4 Maple [N/A] (verified)

Not integrable

Time = 0.49 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{\sqrt{a+b\sec(dx+c)}}{\sec(dx+c)^{\frac{2}{3}}}dx$$

input
```
int((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(2/3),x)
```

output
```
int((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(2/3),x)
```

3.723.5 Fricas [N/A]

Not integrable

Time = 0.67 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{2}{3}}(c+dx)}\,dx = \int \frac{\sqrt{b\sec(dx+c)+a}}{\sec(dx+c)^{\frac{2}{3}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(2/3),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sec(d*x + c) + a)/sec(d*x + c)^(2/3), x)
```

3.723.6 Sympy [N/A]

Not integrable

Time = 0.69 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{2}{3}}(c+dx)}\,dx = \int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{2}{3}}(c+dx)}\,dx$$

input
```
integrate((a+b*sec(d*x+c))**(1/2)/sec(d*x+c)**(2/3),x)
```

output
```
Integral(sqrt(a + b*sec(c + d*x))/sec(c + d*x)**(2/3), x)
```

3.723.7 Maxima [N/A]

Not integrable

Time = 0.76 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{2}{3}}(c+dx)}\,dx = \int \frac{\sqrt{b\sec(dx+c)+a}}{\sec(dx+c)^{\frac{2}{3}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(2/3),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sec(d*x + c) + a)/sec(d*x + c)^(2/3), x)
```

3.723.8 Giac [N/A]

Not integrable

Time = 1.59 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{2}{3}}(c+dx)}\,dx = \int \frac{\sqrt{b\sec(dx+c)+a}}{\sec(dx+c)^{\frac{2}{3}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(2/3),x, algorithm="giac")
```

output
```
integrate(sqrt(b*sec(d*x + c) + a)/sec(d*x + c)^(2/3), x)
```

3.723.9 Mupad [N/A]

Not integrable

Time = 13.98 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{2}{3}}(c+dx)}\,dx = \int \frac{\sqrt{a+\frac{b}{\cos(c+d\,x)}}}{\left(\frac{1}{\cos(c+d\,x)}\right)^{2/3}}\,dx$$

input
```
int((a + b/cos(c + d*x))^(1/2)/(1/cos(c + d*x))^(2/3),x)
```

output
```
int((a + b/cos(c + d*x))^(1/2)/(1/cos(c + d*x))^(2/3), x)
```

3.724 $\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{4}{3}}(c+dx)}\,dx$

3.724.1 Optimal result 5567
3.724.2 Mathematica [N/A] 5567
3.724.3 Rubi [N/A] 5568
3.724.4 Maple [N/A] (verified) 5569
3.724.5 Fricas [N/A] 5569
3.724.6 Sympy [N/A] 5569
3.724.7 Maxima [N/A] 5570
3.724.8 Giac [N/A] 5570
3.724.9 Mupad [N/A] 5570

3.724.1 Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{4}{3}}(c+dx)}\,dx = \text{Int}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{4}{3}}(c+dx)},x\right)$$

output
```
Unintegrable((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(4/3),x)
```

3.724.2 Mathematica [N/A]

Not integrable

Time = 88.29 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{4}{3}}(c+dx)}\,dx = \int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{4}{3}}(c+dx)}\,dx$$

input
```
Integrate[Sqrt[a + b*Sec[c + d*x]]/Sec[c + d*x]^(4/3),x]
```

output
```
Integrate[Sqrt[a + b*Sec[c + d*x]]/Sec[c + d*x]^(4/3), x]
```

3.724.3 Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 4357}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{4}{3}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}{\csc\left(c+dx+\frac{\pi}{2}\right)^{4/3}}dx$$

$$\Big\downarrow \text{4357}$$

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{4}{3}}(c+dx)}dx$$

input
```
Int[Sqrt[a + b*Sec[c + d*x]]/Sec[c + d*x]^(4/3),x]
```

output
```
$Aborted
```

3.724.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4357
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Unintegrable[(d*Csc[e + f*x])^n*(a + b*Csc[e + f*
x])^m, x] /; FreeQ[{a, b, d, e, f, m, n}, x]
```

3.724.4 Maple [N/A] (verified)

Not integrable

Time = 0.48 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{\sqrt{a+b\sec(dx+c)}}{\sec(dx+c)^{\frac{4}{3}}}dx$$

input
```
int((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(4/3),x)
```

output
```
int((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(4/3),x)
```

3.724.5 Fricas [N/A]

Not integrable

Time = 1.10 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{4}{3}}(c+dx)}\,dx = \int \frac{\sqrt{b\sec(dx+c)+a}}{\sec(dx+c)^{\frac{4}{3}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(4/3),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sec(d*x + c) + a)/sec(d*x + c)^(4/3), x)
```

3.724.6 Sympy [N/A]

Not integrable

Time = 3.98 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{4}{3}}(c+dx)}\,dx = \int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{4}{3}}(c+dx)}\,dx$$

input
```
integrate((a+b*sec(d*x+c))**(1/2)/sec(d*x+c)**(4/3),x)
```

output
```
Integral(sqrt(a + b*sec(c + d*x))/sec(c + d*x)**(4/3), x)
```

3.724.7 Maxima [N/A]

Not integrable

Time = 0.79 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{4}{3}}(c+dx)}\,dx = \int \frac{\sqrt{b\sec(dx+c)+a}}{\sec(dx+c)^{\frac{4}{3}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(4/3),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sec(d*x + c) + a)/sec(d*x + c)^(4/3), x)
```

3.724.8 Giac [N/A]

Not integrable

Time = 0.91 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{4}{3}}(c+dx)}\,dx = \int \frac{\sqrt{b\sec(dx+c)+a}}{\sec(dx+c)^{\frac{4}{3}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(4/3),x, algorithm="giac")
```

output
```
integrate(sqrt(b*sec(d*x + c) + a)/sec(d*x + c)^(4/3), x)
```

3.724.9 Mupad [N/A]

Not integrable

Time = 15.18 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{4}{3}}(c+dx)}\,dx = \int \frac{\sqrt{a+\frac{b}{\cos(c+d\,x)}}}{\left(\frac{1}{\cos(c+d\,x)}\right)^{4/3}}\,dx$$

input
```
int((a + b/cos(c + d*x))^(1/2)/(1/cos(c + d*x))^(4/3),x)
```

output
```
int((a + b/cos(c + d*x))^(1/2)/(1/cos(c + d*x))^(4/3), x)
```

3.725 $\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{5}{3}}(c+dx)}\,dx$

3.725.1 Optimal result 5571
3.725.2 Mathematica [N/A] 5571
3.725.3 Rubi [N/A] 5572
3.725.4 Maple [N/A] (verified) 5573
3.725.5 Fricas [N/A] 5573
3.725.6 Sympy [N/A] 5573
3.725.7 Maxima [N/A] 5574
3.725.8 Giac [N/A] 5574
3.725.9 Mupad [N/A] 5574

3.725.1 Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{5}{3}}(c+dx)}\,dx = \text{Int}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{5}{3}}(c+dx)},x\right)$$

output
```
Unintegrable((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(5/3),x)
```

3.725.2 Mathematica [N/A]

Not integrable

Time = 146.02 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{5}{3}}(c+dx)}\,dx = \int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{5}{3}}(c+dx)}\,dx$$

input
```
Integrate[Sqrt[a + b*Sec[c + d*x]]/Sec[c + d*x]^(5/3),x]
```

output
```
Integrate[Sqrt[a + b*Sec[c + d*x]]/Sec[c + d*x]^(5/3), x]
```

3.725.3 Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 4357}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{5}{3}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/3}}dx$$

$$\Big\downarrow \text{4357}$$

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{5}{3}}(c+dx)}dx$$

input
```
Int[Sqrt[a + b*Sec[c + d*x]]/Sec[c + d*x]^(5/3),x]
```

output
```
$Aborted
```

3.725.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4357
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Unintegrable[(d*Csc[e + f*x])^n*(a + b*Csc[e + f*
x])^m, x] /; FreeQ[{a, b, d, e, f, m, n}, x]
```

3.725.4 Maple [N/A] (verified)

Not integrable

Time = 0.48 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{\sqrt{a+b\sec(dx+c)}}{\sec(dx+c)^{\frac{5}{3}}}dx$$

input
```
int((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(5/3),x)
```

output
```
int((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(5/3),x)
```

3.725.5 Fricas [N/A]

Not integrable

Time = 0.73 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{5}{3}}(c+dx)}\,dx = \int \frac{\sqrt{b\sec(dx+c)+a}}{\sec(dx+c)^{\frac{5}{3}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(5/3),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sec(d*x + c) + a)/sec(d*x + c)^(5/3), x)
```

3.725.6 Sympy [N/A]

Not integrable

Time = 11.37 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{5}{3}}(c+dx)}\,dx = \int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{5}{3}}(c+dx)}\,dx$$

input
```
integrate((a+b*sec(d*x+c))**(1/2)/sec(d*x+c)**(5/3),x)
```

output
```
Integral(sqrt(a + b*sec(c + d*x))/sec(c + d*x)**(5/3), x)
```

3.725.7 Maxima [N/A]

Not integrable

Time = 0.79 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{5}{3}}(c+dx)}\,dx = \int \frac{\sqrt{b\sec(dx+c)+a}}{\sec(dx+c)^{\frac{5}{3}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(5/3),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sec(d*x + c) + a)/sec(d*x + c)^(5/3), x)
```

3.725.8 Giac [N/A]

Not integrable

Time = 1.17 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{5}{3}}(c+dx)}\,dx = \int \frac{\sqrt{b\sec(dx+c)+a}}{\sec(dx+c)^{\frac{5}{3}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(5/3),x, algorithm="giac")
```

output
```
integrate(sqrt(b*sec(d*x + c) + a)/sec(d*x + c)^(5/3), x)
```

3.725.9 Mupad [N/A]

Not integrable

Time = 14.32 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{5}{3}}(c+dx)}\,dx = \int \frac{\sqrt{a+\frac{b}{\cos(c+d\,x)}}}{\left(\frac{1}{\cos(c+d\,x)}\right)^{5/3}}\,dx$$

input
```
int((a + b/cos(c + d*x))^(1/2)/(1/cos(c + d*x))^(5/3),x)
```

output
```
int((a + b/cos(c + d*x))^(1/2)/(1/cos(c + d*x))^(5/3), x)
```

3.726 $\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{7}{3}}(c+dx)}\,dx$

3.726.1 Optimal result 5575
3.726.2 Mathematica [N/A] 5575
3.726.3 Rubi [N/A] 5576
3.726.4 Maple [N/A] (verified) 5577
3.726.5 Fricas [N/A] 5577
3.726.6 Sympy [F(-1)] 5577
3.726.7 Maxima [N/A] 5578
3.726.8 Giac [N/A] 5578
3.726.9 Mupad [N/A] 5578

3.726.1 Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{7}{3}}(c+dx)}\,dx = \text{Int}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{7}{3}}(c+dx)},x\right)$$

output
```
Unintegrable((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(7/3),x)
```

3.726.2 Mathematica [N/A]

Not integrable

Time = 162.74 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{7}{3}}(c+dx)}\,dx = \int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{7}{3}}(c+dx)}\,dx$$

input
```
Integrate[Sqrt[a + b*Sec[c + d*x]]/Sec[c + d*x]^(7/3),x]
```

output
```
Integrate[Sqrt[a + b*Sec[c + d*x]]/Sec[c + d*x]^(7/3), x]
```

3.726.3 Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 4357}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{7}{3}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/3}}dx$$

$$\Big\downarrow \text{4357}$$

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{7}{3}}(c+dx)}dx$$

input
```
Int[Sqrt[a + b*Sec[c + d*x]]/Sec[c + d*x]^(7/3),x]
```

output
```
$Aborted
```

3.726.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4357
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Unintegrable[(d*Csc[e + f*x])^n*(a + b*Csc[e + f*
x])^m, x] /; FreeQ[{a, b, d, e, f, m, n}, x]
```

3.726.4 Maple [N/A] (verified)

Not integrable

Time = 0.46 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{\sqrt{a+b\sec(dx+c)}}{\sec(dx+c)^{\frac{7}{3}}}dx$$

input
```
int((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(7/3),x)
```

output
```
int((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(7/3),x)
```

3.726.5 Fricas [N/A]

Not integrable

Time = 1.05 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{7}{3}}(c+dx)}\,dx = \int \frac{\sqrt{b\sec(dx+c)+a}}{\sec(dx+c)^{\frac{7}{3}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(7/3),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sec(d*x + c) + a)/sec(d*x + c)^(7/3), x)
```

3.726.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{7}{3}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+b*sec(d*x+c))**(1/2)/sec(d*x+c)**(7/3),x)
```

output
```
Timed out
```

3.726.7 Maxima [N/A]

Not integrable

Time = 0.81 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{7}{3}}(c+dx)}\,dx = \int \frac{\sqrt{b\sec(dx+c)+a}}{\sec(dx+c)^{\frac{7}{3}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(7/3),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sec(d*x + c) + a)/sec(d*x + c)^(7/3), x)
```

3.726.8 Giac [N/A]

Not integrable

Time = 1.24 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{7}{3}}(c+dx)}\,dx = \int \frac{\sqrt{b\sec(dx+c)+a}}{\sec(dx+c)^{\frac{7}{3}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(7/3),x, algorithm="giac")
```

output
```
integrate(sqrt(b*sec(d*x + c) + a)/sec(d*x + c)^(7/3), x)
```

3.726.9 Mupad [N/A]

Not integrable

Time = 16.62 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{7}{3}}(c+dx)}\,dx = \int \frac{\sqrt{a+\frac{b}{\cos(c+d\,x)}}}{\left(\frac{1}{\cos(c+d\,x)}\right)^{7/3}}\,dx$$

input
```
int((a + b/cos(c + d*x))^(1/2)/(1/cos(c + d*x))^(7/3),x)
```

output
```
int((a + b/cos(c + d*x))^(1/2)/(1/cos(c + d*x))^(7/3), x)
```

3.727 $\int \sec^{\frac{7}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx$

3.727.1 Optimal result 5579
3.727.2 Mathematica [F(-1)] 5579
3.727.3 Rubi [N/A] 5580
3.727.4 Maple [N/A] (verified) 5581
3.727.5 Fricas [N/A] 5581
3.727.6 Sympy [F(-1)] 5581
3.727.7 Maxima [N/A] 5582
3.727.8 Giac [N/A] 5582
3.727.9 Mupad [N/A] 5582

3.727.1 Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \sec^{\frac{7}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx = \text{Int}\left(\sec^{\frac{7}{3}}(c+dx)(a+b\sec(c+dx))^{3/2},x\right)$$

output
```
Unintegrable(sec(d*x+c)^(7/3)*(a+b*sec(d*x+c))^(3/2),x)
```

3.727.2 Mathematica [F(-1)]

Timed out.

$$\int \sec^{\frac{7}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx = \$\text{Aborted}$$

input
```
Integrate[Sec[c + d*x]^(7/3)*(a + b*Sec[c + d*x])^(3/2),x]
```

output
```
$Aborted
```

3.727.3 Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 4357}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{\frac{7}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{7/3}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}dx$$

$$\Big\downarrow \text{4357}$$

$$\int \sec^{\frac{7}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}dx$$

input
```
Int[Sec[c + d*x]^(7/3)*(a + b*Sec[c + d*x])^(3/2),x]
```

output
```
$Aborted
```

3.727.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4357
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Unintegrable[(d*Csc[e + f*x])^n*(a + b*Csc[e + f*
x])^m, x] /; FreeQ[{a, b, d, e, f, m, n}, x]
```

3.727.4 Maple [N/A] (verified)

Not integrable

Time = 0.44 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \sec (dx+c)^{\frac{7}{3}} (a+b\sec (dx+c))^{\frac{3}{2}} \, dx$$

input
```
int(sec(d*x+c)^(7/3)*(a+b*sec(d*x+c))^(3/2),x)
```

output
```
int(sec(d*x+c)^(7/3)*(a+b*sec(d*x+c))^(3/2),x)
```

3.727.5 Fricas [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.76

$$\int \sec^{\frac{7}{3}}(c+dx)(a+b\sec(c+dx))^{3/2} \, dx = \int (b\sec (dx+c)+a)^{\frac{3}{2}} \sec (dx+c)^{\frac{7}{3}} \, dx$$

input
```
integrate(sec(d*x+c)^(7/3)*(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
integral((b*sec(d*x + c)^3 + a*sec(d*x + c)^2)*sqrt(b*sec(d*x + c) + a)*se
c(d*x + c)^(1/3), x)
```

3.727.6 Sympy [F(-1)]

Timed out.

$$\int \sec^{\frac{7}{3}}(c+dx)(a+b\sec(c+dx))^{3/2} \, dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**(7/3)*(a+b*sec(d*x+c))**(3/2),x)
```

output
```
Timed out
```

3.727.7 Maxima [N/A]

Not integrable

Time = 0.80 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \sec^{\frac{7}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx = \int (b\sec(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{7}{3}}\,dx$$

input
```
integrate(sec(d*x+c)^(7/3)*(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^(3/2)*sec(d*x + c)^(7/3), x)
```

3.727.8 Giac [N/A]

Not integrable

Time = 2.72 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \sec^{\frac{7}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx = \int (b\sec(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{7}{3}}\,dx$$

input
```
integrate(sec(d*x+c)^(7/3)*(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^(3/2)*sec(d*x + c)^(7/3), x)
```

3.727.9 Mupad [N/A]

Not integrable

Time = 16.28 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \sec^{\frac{7}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx = \int \left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}\left(\frac{1}{\cos(c+d\,x)}\right)^{7/3}dx$$

input
```
int((a + b/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(7/3),x)
```

output
```
int((a + b/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(7/3), x)
```

3.728 $\int \sec^{\frac{5}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx$

3.728.1 Optimal result 5583
3.728.2 Mathematica [N/A] 5583
3.728.3 Rubi [N/A] 5584
3.728.4 Maple [N/A] (verified) 5585
3.728.5 Fricas [N/A] 5585
3.728.6 Sympy [F(-1)] 5585
3.728.7 Maxima [N/A] 5586
3.728.8 Giac [N/A] 5586
3.728.9 Mupad [N/A] 5586

3.728.1 Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \sec^{\frac{5}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx = \text{Int}\left(\sec^{\frac{5}{3}}(c+dx)(a+b\sec(c+dx))^{3/2},x\right)$$

output
```
Unintegrable(sec(d*x+c)^(5/3)*(a+b*sec(d*x+c))^(3/2),x)
```

3.728.2 Mathematica [N/A]

Not integrable

Time = 137.53 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \sec^{\frac{5}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx = \int \sec^{\frac{5}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx$$

input
```
Integrate[Sec[c + d*x]^(5/3)*(a + b*Sec[c + d*x])^(3/2),x]
```

output
```
Integrate[Sec[c + d*x]^(5/3)*(a + b*Sec[c + d*x])^(3/2), x]
```

3.728.3 Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 4357}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{\frac{5}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{5/3}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}dx$$

$$\Big\downarrow \text{4357}$$

$$\int \sec^{\frac{5}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}dx$$

input
```
Int[Sec[c + d*x]^(5/3)*(a + b*Sec[c + d*x])^(3/2),x]
```

output
```
$Aborted
```

3.728.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4357
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Unintegrable[(d*Csc[e + f*x])^n*(a + b*Csc[e + f*
x])^m, x] /; FreeQ[{a, b, d, e, f, m, n}, x]
```

3.728.4 Maple [N/A] (verified)

Not integrable

Time = 0.44 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \sec (dx+c)^{\frac{5}{3}} (a+b\sec (dx+c))^{\frac{3}{2}} \, dx$$

input
```
int(sec(d*x+c)^(5/3)*(a+b*sec(d*x+c))^(3/2),x)
```

output
```
int(sec(d*x+c)^(5/3)*(a+b*sec(d*x+c))^(3/2),x)
```

3.728.5 Fricas [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.68

$$\int \sec^{\frac{5}{3}}(c+dx)(a+b\sec(c+dx))^{3/2} \, dx = \int (b\sec (dx+c)+a)^{\frac{3}{2}} \sec (dx+c)^{\frac{5}{3}} \, dx$$

input
```
integrate(sec(d*x+c)^(5/3)*(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
integral((b*sec(d*x + c)^2 + a*sec(d*x + c))*sqrt(b*sec(d*x + c) + a)*sec(
d*x + c)^(2/3), x)
```

3.728.6 Sympy [F(-1)]

Timed out.

$$\int \sec^{\frac{5}{3}}(c+dx)(a+b\sec(c+dx))^{3/2} \, dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**(5/3)*(a+b*sec(d*x+c))**(3/2),x)
```

output
```
Timed out
```

3.728.7 Maxima [N/A]

Not integrable

Time = 0.80 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \sec^{\frac{5}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx = \int (b\sec(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{5}{3}}\,dx$$

input
```
integrate(sec(d*x+c)^(5/3)*(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^(3/2)*sec(d*x + c)^(5/3), x)
```

3.728.8 Giac [N/A]

Not integrable

Time = 2.34 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \sec^{\frac{5}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx = \int (b\sec(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{5}{3}}\,dx$$

input
```
integrate(sec(d*x+c)^(5/3)*(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^(3/2)*sec(d*x + c)^(5/3), x)
```

3.728.9 Mupad [N/A]

Not integrable

Time = 15.68 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \sec^{\frac{5}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx = \int \left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}\left(\frac{1}{\cos(c+d\,x)}\right)^{5/3}dx$$

input
```
int((a + b/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(5/3),x)
```

output
```
int((a + b/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(5/3), x)
```

3.729 $\int \sec^{\frac{4}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx$

3.729.1 Optimal result 5587
3.729.2 Mathematica [N/A] 5587
3.729.3 Rubi [N/A] 5588
3.729.4 Maple [N/A] (verified) 5589
3.729.5 Fricas [N/A] 5589
3.729.6 Sympy [F(-1)] 5589
3.729.7 Maxima [N/A] 5590
3.729.8 Giac [N/A] 5590
3.729.9 Mupad [N/A] 5590

3.729.1 Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \sec^{\frac{4}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx = \text{Int}\left(\sec^{\frac{4}{3}}(c+dx)(a+b\sec(c+dx))^{3/2},x\right)$$

output
```
Unintegrable(sec(d*x+c)^(4/3)*(a+b*sec(d*x+c))^(3/2),x)
```

3.729.2 Mathematica [N/A]

Not integrable

Time = 104.90 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \sec^{\frac{4}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx = \int \sec^{\frac{4}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx$$

input
```
Integrate[Sec[c + d*x]^(4/3)*(a + b*Sec[c + d*x])^(3/2),x]
```

output
```
Integrate[Sec[c + d*x]^(4/3)*(a + b*Sec[c + d*x])^(3/2), x]
```

3.729.3 Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 4357}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{\frac{4}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{4/3}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}dx$$

$$\Big\downarrow \text{4357}$$

$$\int \sec^{\frac{4}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}dx$$

input
```
Int[Sec[c + d*x]^(4/3)*(a + b*Sec[c + d*x])^(3/2),x]
```

output
```
$Aborted
```

3.729.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4357
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Unintegrable[(d*Csc[e + f*x])^n*(a + b*Csc[e + f*
x])^m, x] /; FreeQ[{a, b, d, e, f, m, n}, x]
```

3.729.4 Maple [N/A] (verified)

Not integrable

Time = 0.43 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \sec (dx+c)^{\frac{4}{3}} (a+b\sec (dx+c))^{\frac{3}{2}} \, dx$$

input
```
int(sec(d*x+c)^(4/3)*(a+b*sec(d*x+c))^(3/2),x)
```

output
```
int(sec(d*x+c)^(4/3)*(a+b*sec(d*x+c))^(3/2),x)
```

3.729.5 Fricas [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.68

$$\int \sec^{\frac{4}{3}}(c+dx)(a+b\sec(c+dx))^{3/2} \, dx = \int (b\sec (dx+c)+a)^{\frac{3}{2}} \sec (dx+c)^{\frac{4}{3}} \, dx$$

input
```
integrate(sec(d*x+c)^(4/3)*(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
integral((b*sec(d*x + c)^2 + a*sec(d*x + c))*sqrt(b*sec(d*x + c) + a)*sec(
d*x + c)^(1/3), x)
```

3.729.6 Sympy [F(-1)]

Timed out.

$$\int \sec^{\frac{4}{3}}(c+dx)(a+b\sec(c+dx))^{3/2} \, dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**(4/3)*(a+b*sec(d*x+c))**(3/2),x)
```

output
```
Timed out
```

3.729.7 Maxima [N/A]

Not integrable

Time = 0.78 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \sec^{\frac{4}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx = \int (b\sec(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{4}{3}}\,dx$$

input
```
integrate(sec(d*x+c)^(4/3)*(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^(3/2)*sec(d*x + c)^(4/3), x)
```

3.729.8 Giac [N/A]

Not integrable

Time = 1.66 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \sec^{\frac{4}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx = \int (b\sec(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{4}{3}}\,dx$$

input
```
integrate(sec(d*x+c)^(4/3)*(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^(3/2)*sec(d*x + c)^(4/3), x)
```

3.729.9 Mupad [N/A]

Not integrable

Time = 15.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \sec^{\frac{4}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx = \int \left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}\left(\frac{1}{\cos(c+d\,x)}\right)^{4/3}dx$$

input
```
int((a + b/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(4/3),x)
```

output
```
int((a + b/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(4/3), x)
```

3.730 $\int \sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx$

3.730.1 Optimal result 5591
3.730.2 Mathematica [N/A] 5591
3.730.3 Rubi [N/A] 5592
3.730.4 Maple [N/A] (verified) 5593
3.730.5 Fricas [N/A] 5593
3.730.6 Sympy [F(-1)] 5593
3.730.7 Maxima [N/A] 5594
3.730.8 Giac [N/A] 5594
3.730.9 Mupad [N/A] 5594

3.730.1 Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx = \text{Int}\left(\sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))^{3/2},x\right)$$

output
```
Unintegrable(sec(d*x+c)^(2/3)*(a+b*sec(d*x+c))^(3/2),x)
```

3.730.2 Mathematica [N/A]

Not integrable

Time = 135.55 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx = \int \sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx$$

input
```
Integrate[Sec[c + d*x]^(2/3)*(a + b*Sec[c + d*x])^(3/2),x]
```

output
```
Integrate[Sec[c + d*x]^(2/3)*(a + b*Sec[c + d*x])^(3/2), x]
```

3.730.3 Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 4357}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{2/3}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}dx$$

$$\Big\downarrow \text{4357}$$

$$\int \sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}dx$$

input
```
Int[Sec[c + d*x]^(2/3)*(a + b*Sec[c + d*x])^(3/2),x]
```

output
```
$Aborted
```

3.730.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4357
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Unintegrable[(d*Csc[e + f*x])^n*(a + b*Csc[e + f*
x])^m, x] /; FreeQ[{a, b, d, e, f, m, n}, x]
```

3.730.4 Maple [N/A] (verified)

Not integrable

Time = 0.48 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \sec (dx+c)^{\frac{2}{3}} (a+b\sec (dx+c))^{\frac{3}{2}} \, dx$$

input
```
int(sec(d*x+c)^(2/3)*(a+b*sec(d*x+c))^(3/2),x)
```

output
```
int(sec(d*x+c)^(2/3)*(a+b*sec(d*x+c))^(3/2),x)
```

3.730.5 Fricas [N/A]

Not integrable

Time = 1.05 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))^{3/2} \, dx = \int (b\sec (dx+c)+a)^{\frac{3}{2}} \sec (dx+c)^{\frac{2}{3}} \, dx$$

input
```
integrate(sec(d*x+c)^(2/3)*(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
integral((b*sec(d*x + c) + a)^(3/2)*sec(d*x + c)^(2/3), x)
```

3.730.6 Sympy [F(-1)]

Timed out.

$$\int \sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))^{3/2} \, dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**(2/3)*(a+b*sec(d*x+c))**(3/2),x)
```

output
```
Timed out
```

3.730.7 Maxima [N/A]

Not integrable

Time = 0.74 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx = \int (b\sec(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{2}{3}}\,dx$$

input
```
integrate(sec(d*x+c)^(2/3)*(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^(3/2)*sec(d*x + c)^(2/3), x)
```

3.730.8 Giac [N/A]

Not integrable

Time = 2.31 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx = \int (b\sec(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{2}{3}}\,dx$$

input
```
integrate(sec(d*x+c)^(2/3)*(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^(3/2)*sec(d*x + c)^(2/3), x)
```

3.730.9 Mupad [N/A]

Not integrable

Time = 14.75 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx = \int \left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}\left(\frac{1}{\cos(c+d\,x)}\right)^{2/3}dx$$

input
```
int((a + b/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(2/3),x)
```

output
```
int((a + b/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(2/3), x)
```

3.731 $\int \sqrt[3]{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}\,dx$

3.731.1 Optimal result 5595
3.731.2 Mathematica [N/A] 5595
3.731.3 Rubi [N/A] 5596
3.731.4 Maple [N/A] (verified) 5597
3.731.5 Fricas [N/A] 5597
3.731.6 Sympy [F(-1)] 5597
3.731.7 Maxima [N/A] 5598
3.731.8 Giac [N/A] 5598
3.731.9 Mupad [N/A] 5598

3.731.1 Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \sqrt[3]{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}\,dx = \text{Int}\left(\sqrt[3]{\sec(c+dx)}(a+b\sec(c+dx))^{3/2},x\right)$$

output
```
Unintegrable(sec(d*x+c)^(1/3)*(a+b*sec(d*x+c))^(3/2),x)
```

3.731.2 Mathematica [N/A]

Not integrable

Time = 94.48 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \sqrt[3]{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}\,dx = \int \sqrt[3]{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}\,dx$$

input
```
Integrate[Sec[c + d*x]^(1/3)*(a + b*Sec[c + d*x])^(3/2),x]
```

output
```
Integrate[Sec[c + d*x]^(1/3)*(a + b*Sec[c + d*x])^(3/2), x]
```

3.731.3 Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 4357}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt[3]{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt[3]{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}dx$$

$$\Big\downarrow \text{4357}$$

$$\int \sqrt[3]{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}dx$$

input
```
Int[Sec[c + d*x]^(1/3)*(a + b*Sec[c + d*x])^(3/2),x]
```

output
```
$Aborted
```

3.731.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4357
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Unintegrable[(d*Csc[e + f*x])^n*(a + b*Csc[e + f*
x])^m, x] /; FreeQ[{a, b, d, e, f, m, n}, x]
```

3.731.4 Maple [N/A] (verified)

Not integrable

Time = 0.40 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \sec (dx+c)^{\frac{1}{3}} (a+b\sec (dx+c))^{\frac{3}{2}} \, dx$$

input
```
int(sec(d*x+c)^(1/3)*(a+b*sec(d*x+c))^(3/2),x)
```

output
```
int(sec(d*x+c)^(1/3)*(a+b*sec(d*x+c))^(3/2),x)
```

3.731.5 Fricas [N/A]

Not integrable

Time = 0.68 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \sqrt[3]{\sec(c+dx)}(a+b\sec(c+dx))^{3/2} \, dx = \int (b\sec (dx+c)+a)^{\frac{3}{2}} \sec (dx+c)^{\frac{1}{3}} \, dx$$

input
```
integrate(sec(d*x+c)^(1/3)*(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
integral((b*sec(d*x + c) + a)^(3/2)*sec(d*x + c)^(1/3), x)
```

3.731.6 Sympy [F(-1)]

Timed out.

$$\int \sqrt[3]{\sec(c+dx)}(a+b\sec(c+dx))^{3/2} \, dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**(1/3)*(a+b*sec(d*x+c))**(3/2),x)
```

output
```
Timed out
```

3.731.7 Maxima [N/A]

Not integrable

Time = 0.73 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \sqrt[3]{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}\,dx = \int (b\sec(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{1}{3}}\,dx$$

input
```
integrate(sec(d*x+c)^(1/3)*(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^(3/2)*sec(d*x + c)^(1/3), x)
```

3.731.8 Giac [N/A]

Not integrable

Time = 1.67 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \sqrt[3]{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}\,dx = \int (b\sec(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{1}{3}}\,dx$$

input
```
integrate(sec(d*x+c)^(1/3)*(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^(3/2)*sec(d*x + c)^(1/3), x)
```

3.731.9 Mupad [N/A]

Not integrable

Time = 15.09 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \sqrt[3]{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}\,dx = \int \left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}\left(\frac{1}{\cos(c+d\,x)}\right)^{1/3}dx$$

input
```
int((a + b/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(1/3),x)
```

output
```
int((a + b/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(1/3), x)
```

3.732 $\int \frac{(a+b\sec(c+dx))^{3/2}}{\sqrt[3]{\sec(c+dx)}}\,dx$

3.732.1 Optimal result 5599
3.732.2 Mathematica [N/A] 5599
3.732.3 Rubi [N/A] 5600
3.732.4 Maple [N/A] (verified) 5601
3.732.5 Fricas [N/A] 5601
3.732.6 Sympy [N/A] 5601
3.732.7 Maxima [N/A] 5602
3.732.8 Giac [N/A] 5602
3.732.9 Mupad [N/A] 5602

3.732.1 Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sqrt[3]{\sec(c+dx)}}\,dx = \text{Int}\left(\frac{(a+b\sec(c+dx))^{3/2}}{\sqrt[3]{\sec(c+dx)}},x\right)$$

output
```
Unintegrable((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(1/3),x)
```

3.732.2 Mathematica [N/A]

Not integrable

Time = 177.25 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sqrt[3]{\sec(c+dx)}}\,dx = \int \frac{(a+b\sec(c+dx))^{3/2}}{\sqrt[3]{\sec(c+dx)}}\,dx$$

input
```
Integrate[(a + b*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(1/3),x]
```

output
```
Integrate[(a + b*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(1/3), x]
```

3.732.3 Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 4357}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sqrt[3]{\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}{\sqrt[3]{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4357}$$

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sqrt[3]{\sec(c+dx)}}dx$$

input
```
Int[(a + b*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(1/3),x]
```

output
```
$Aborted
```

3.732.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4357
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Unintegrable[(d*Csc[e + f*x])^n*(a + b*Csc[e + f*
x])^m, x] /; FreeQ[{a, b, d, e, f, m, n}, x]
```

3.732.4 Maple [N/A] (verified)

Not integrable

Time = 0.40 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{(a+b\sec(dx+c))^{\frac{3}{2}}}{\sec(dx+c)^{\frac{1}{3}}}dx$$

input
```
int((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(1/3),x)
```

output
```
int((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(1/3),x)
```

3.732.5 Fricas [N/A]

Not integrable

Time = 1.05 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sqrt[3]{\sec(c+dx)}}\,dx = \int \frac{(b\sec(dx+c)+a)^{\frac{3}{2}}}{\sec(dx+c)^{\frac{1}{3}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(1/3),x, algorithm="fricas")
```

output
```
integral((b*sec(d*x + c) + a)^(3/2)/sec(d*x + c)^(1/3), x)
```

3.732.6 Sympy [N/A]

Not integrable

Time = 21.79 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sqrt[3]{\sec(c+dx)}}\,dx = \int \frac{(a+b\sec(c+dx))^{\frac{3}{2}}}{\sqrt[3]{\sec(c+dx)}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))**(3/2)/sec(d*x+c)**(1/3),x)
```

output
```
Integral((a + b*sec(c + d*x))**(3/2)/sec(c + d*x)**(1/3), x)
```

3.732.7 Maxima [N/A]

Not integrable

Time = 0.75 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sqrt[3]{\sec(c+dx)}}\,dx = \int \frac{(b\sec(dx+c)+a)^{\frac{3}{2}}}{\sec(dx+c)^{\frac{1}{3}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(1/3),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^(3/2)/sec(d*x + c)^(1/3), x)
```

3.732.8 Giac [N/A]

Not integrable

Time = 235.74 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sqrt[3]{\sec(c+dx)}}\,dx = \int \frac{(b\sec(dx+c)+a)^{\frac{3}{2}}}{\sec(dx+c)^{\frac{1}{3}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(1/3),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^(3/2)/sec(d*x + c)^(1/3), x)
```

3.732.9 Mupad [N/A]

Not integrable

Time = 15.60 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sqrt[3]{\sec(c+dx)}}\,dx = \int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}}{\left(\frac{1}{\cos(c+d\,x)}\right)^{1/3}}\,dx$$

input
```
int((a + b/cos(c + d*x))^(3/2)/(1/cos(c + d*x))^(1/3),x)
```

output
```
int((a + b/cos(c + d*x))^(3/2)/(1/cos(c + d*x))^(1/3), x)
```

3.733 $\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{2}{3}}(c+dx)}\,dx$

3.733.1 Optimal result 5603
3.733.2 Mathematica [N/A] 5603
3.733.3 Rubi [N/A] 5604
3.733.4 Maple [N/A] (verified) 5605
3.733.5 Fricas [N/A] 5605
3.733.6 Sympy [N/A] 5605
3.733.7 Maxima [N/A] 5606
3.733.8 Giac [N/A] 5606
3.733.9 Mupad [N/A] 5606

3.733.1 Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{2}{3}}(c+dx)}\,dx = \text{Int}\left(\frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{2}{3}}(c+dx)},x\right)$$

output
```
Unintegrable((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(2/3),x)
```

3.733.2 Mathematica [N/A]

Not integrable

Time = 105.91 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{2}{3}}(c+dx)}\,dx = \int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{2}{3}}(c+dx)}\,dx$$

input
```
Integrate[(a + b*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(2/3),x]
```

output
```
Integrate[(a + b*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(2/3), x]
```

3.733.3 Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 4357}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{2}{3}}(c+dx)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}{\csc\left(c+dx+\frac{\pi}{2}\right)^{2/3}}dx$$

$$\Big\downarrow \text{ 4357}$$

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{2}{3}}(c+dx)}dx$$

input
```
Int[(a + b*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(2/3),x]
```

output
```
$Aborted
```

3.733.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4357
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Unintegrable[(d*Csc[e + f*x])^n*(a + b*Csc[e + f*
x])^m, x] /; FreeQ[{a, b, d, e, f, m, n}, x]
```

3.733.4 Maple [N/A] (verified)

Not integrable

Time = 0.44 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{(a+b\sec(dx+c))^{\frac{3}{2}}}{\sec(dx+c)^{\frac{2}{3}}}dx$$

input
```
int((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(2/3),x)
```

output
```
int((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(2/3),x)
```

3.733.5 Fricas [N/A]

Not integrable

Time = 0.66 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{2}{3}}(c+dx)}\,dx = \int \frac{(b\sec(dx+c)+a)^{\frac{3}{2}}}{\sec(dx+c)^{\frac{2}{3}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(2/3),x, algorithm="fricas")
```

output
```
integral((b*sec(d*x + c) + a)^(3/2)/sec(d*x + c)^(2/3), x)
```

3.733.6 Sympy [N/A]

Not integrable

Time = 13.33 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{2}{3}}(c+dx)}\,dx = \int \frac{(a+b\sec(c+dx))^{\frac{3}{2}}}{\sec^{\frac{2}{3}}(c+dx)}\,dx$$

input
```
integrate((a+b*sec(d*x+c))**(3/2)/sec(d*x+c)**(2/3),x)
```

output
```
Integral((a + b*sec(c + d*x))**(3/2)/sec(c + d*x)**(2/3), x)
```

3.733.7 Maxima [N/A]

Not integrable

Time = 0.76 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{2}{3}}(c+dx)}\,dx=\int \frac{(b\sec(dx+c)+a)^{\frac{3}{2}}}{\sec(dx+c)^{\frac{2}{3}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(2/3),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^(3/2)/sec(d*x + c)^(2/3), x)
```

3.733.8 Giac [N/A]

Not integrable

Time = 157.94 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{2}{3}}(c+dx)}\,dx=\int \frac{(b\sec(dx+c)+a)^{\frac{3}{2}}}{\sec(dx+c)^{\frac{2}{3}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(2/3),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^(3/2)/sec(d*x + c)^(2/3), x)
```

3.733.9 Mupad [N/A]

Not integrable

Time = 13.36 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{2}{3}}(c+dx)}\,dx=\int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}}{\left(\frac{1}{\cos(c+d\,x)}\right)^{2/3}}\,dx$$

input
```
int((a + b/cos(c + d*x))^(3/2)/(1/cos(c + d*x))^(2/3),x)
```

output
```
int((a + b/cos(c + d*x))^(3/2)/(1/cos(c + d*x))^(2/3), x)
```

3.734 $\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{4}{3}}(c+dx)}\,dx$

3.734.1 Optimal result 5607
3.734.2 Mathematica [N/A] 5607
3.734.3 Rubi [N/A] 5608
3.734.4 Maple [N/A] (verified) 5609
3.734.5 Fricas [N/A] 5609
3.734.6 Sympy [N/A] 5609
3.734.7 Maxima [N/A] 5610
3.734.8 Giac [N/A] 5610
3.734.9 Mupad [N/A] 5610

3.734.1 Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{4}{3}}(c+dx)}\,dx=\text{Int}\left(\frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{4}{3}}(c+dx)},x\right)$$

output
```
Unintegrable((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(4/3),x)
```

3.734.2 Mathematica [N/A]

Not integrable

Time = 87.88 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{4}{3}}(c+dx)}\,dx=\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{4}{3}}(c+dx)}\,dx$$

input
```
Integrate[(a + b*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(4/3),x]
```

output
```
Integrate[(a + b*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(4/3), x]
```

3.734.3 Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 4357}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{4}{3}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}{\csc\left(c+dx+\frac{\pi}{2}\right)^{4/3}}dx$$

$$\Big\downarrow \text{4357}$$

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{4}{3}}(c+dx)}dx$$

input
```
Int[(a + b*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(4/3),x]
```

output
```
$Aborted
```

3.734.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4357
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Unintegrable[(d*Csc[e + f*x])^n*(a + b*Csc[e + f*
x])^m, x] /; FreeQ[{a, b, d, e, f, m, n}, x]
```

3.734.4 Maple [N/A] (verified)

Not integrable

Time = 0.39 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{(a+b\sec(dx+c))^{\frac{3}{2}}}{\sec(dx+c)^{\frac{4}{3}}}dx$$

input
```
int((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(4/3),x)
```

output
```
int((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(4/3),x)
```

3.734.5 Fricas [N/A]

Not integrable

Time = 1.11 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{4}{3}}(c+dx)}\,dx = \int \frac{(b\sec(dx+c)+a)^{\frac{3}{2}}}{\sec(dx+c)^{\frac{4}{3}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(4/3),x, algorithm="fricas")
```

output
```
integral((b*sec(d*x + c) + a)^(3/2)/sec(d*x + c)^(4/3), x)
```

3.734.6 Sympy [N/A]

Not integrable

Time = 32.53 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{4}{3}}(c+dx)}\,dx = \int \frac{(a+b\sec(c+dx))^{\frac{3}{2}}}{\sec^{\frac{4}{3}}(c+dx)}\,dx$$

input
```
integrate((a+b*sec(d*x+c))**(3/2)/sec(d*x+c)**(4/3),x)
```

output
```
Integral((a + b*sec(c + d*x))**(3/2)/sec(c + d*x)**(4/3), x)
```

3.734.7 Maxima [N/A]

Not integrable

Time = 0.80 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{4}{3}}(c+dx)}\,dx=\int \frac{(b\sec(dx+c)+a)^{\frac{3}{2}}}{\sec(dx+c)^{\frac{4}{3}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(4/3),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^(3/2)/sec(d*x + c)^(4/3), x)
```

3.734.8 Giac [N/A]

Not integrable

Time = 178.25 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{4}{3}}(c+dx)}\,dx=\int \frac{(b\sec(dx+c)+a)^{\frac{3}{2}}}{\sec(dx+c)^{\frac{4}{3}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(4/3),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^(3/2)/sec(d*x + c)^(4/3), x)
```

3.734.9 Mupad [N/A]

Not integrable

Time = 14.39 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{4}{3}}(c+dx)}\,dx=\int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}}{\left(\frac{1}{\cos(c+d\,x)}\right)^{4/3}}\,dx$$

input
```
int((a + b/cos(c + d*x))^(3/2)/(1/cos(c + d*x))^(4/3),x)
```

output
```
int((a + b/cos(c + d*x))^(3/2)/(1/cos(c + d*x))^(4/3), x)
```

3.735 $\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{5}{3}}(c+dx)}\,dx$

3.735.1 Optimal result 5611
3.735.2 Mathematica [N/A] 5611
3.735.3 Rubi [N/A] 5612
3.735.4 Maple [N/A] (verified) 5613
3.735.5 Fricas [N/A] 5613
3.735.6 Sympy [N/A] 5613
3.735.7 Maxima [N/A] 5614
3.735.8 Giac [N/A] 5614
3.735.9 Mupad [N/A] 5614

3.735.1 Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{5}{3}}(c+dx)}\,dx = \text{Int}\left(\frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{5}{3}}(c+dx)},x\right)$$

output
```
Unintegrable((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(5/3),x)
```

3.735.2 Mathematica [N/A]

Not integrable

Time = 172.17 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{5}{3}}(c+dx)}\,dx = \int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{5}{3}}(c+dx)}\,dx$$

input
```
Integrate[(a + b*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(5/3),x]
```

output
```
Integrate[(a + b*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(5/3), x]
```

3.735.3 Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 4357}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{5}{3}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/3}}dx$$

$$\Big\downarrow \text{4357}$$

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{5}{3}}(c+dx)}dx$$

input
```
Int[(a + b*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(5/3),x]
```

output
```
$Aborted
```

3.735.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4357
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Unintegrable[(d*Csc[e + f*x])^n*(a + b*Csc[e + f*
x])^m, x] /; FreeQ[{a, b, d, e, f, m, n}, x]
```

3.735.4 Maple [N/A] (verified)

Not integrable

Time = 0.44 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{(a+b\sec(dx+c))^{\frac{3}{2}}}{\sec(dx+c)^{\frac{5}{3}}}dx$$

input
```
int((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(5/3),x)
```

output
```
int((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(5/3),x)
```

3.735.5 Fricas [N/A]

Not integrable

Time = 0.70 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{5}{3}}(c+dx)}\,dx = \int \frac{(b\sec(dx+c)+a)^{\frac{3}{2}}}{\sec(dx+c)^{\frac{5}{3}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(5/3),x, algorithm="fricas")
```

output
```
integral((b*sec(d*x + c) + a)^(3/2)/sec(d*x + c)^(5/3), x)
```

3.735.6 Sympy [N/A]

Not integrable

Time = 78.97 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{5}{3}}(c+dx)}\,dx = \int \frac{(a+b\sec(c+dx))^{\frac{3}{2}}}{\sec^{\frac{5}{3}}(c+dx)}\,dx$$

input
```
integrate((a+b*sec(d*x+c))**(3/2)/sec(d*x+c)**(5/3),x)
```

output
```
Integral((a + b*sec(c + d*x))**(3/2)/sec(c + d*x)**(5/3), x)
```

3.735.7 Maxima [N/A]

Not integrable

Time = 0.78 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{5}{3}}(c+dx)}\,dx=\int \frac{(b\sec(dx+c)+a)^{\frac{3}{2}}}{\sec(dx+c)^{\frac{5}{3}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(5/3),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^(3/2)/sec(d*x + c)^(5/3), x)
```

3.735.8 Giac [N/A]

Not integrable

Time = 144.14 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{5}{3}}(c+dx)}\,dx=\int \frac{(b\sec(dx+c)+a)^{\frac{3}{2}}}{\sec(dx+c)^{\frac{5}{3}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(5/3),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^(3/2)/sec(d*x + c)^(5/3), x)
```

3.735.9 Mupad [N/A]

Not integrable

Time = 12.87 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{5}{3}}(c+dx)}\,dx=\int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}}{\left(\frac{1}{\cos(c+d\,x)}\right)^{5/3}}\,dx$$

input
```
int((a + b/cos(c + d*x))^(3/2)/(1/cos(c + d*x))^(5/3),x)
```

output
```
int((a + b/cos(c + d*x))^(3/2)/(1/cos(c + d*x))^(5/3), x)
```

3.736 $\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{7}{3}}(c+dx)}\,dx$

3.736.1 Optimal result 5615
3.736.2 Mathematica [F(-1)] 5615
3.736.3 Rubi [N/A] 5616
3.736.4 Maple [N/A] (verified) 5617
3.736.5 Fricas [N/A] 5617
3.736.6 Sympy [F(-1)] 5617
3.736.7 Maxima [N/A] 5618
3.736.8 Giac [N/A] 5618
3.736.9 Mupad [N/A] 5618

3.736.1 Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{7}{3}}(c+dx)}\,dx=\text{Int}\left(\frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{7}{3}}(c+dx)},x\right)$$

output
```
Unintegrable((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(7/3),x)
```

3.736.2 Mathematica [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{7}{3}}(c+dx)}\,dx=\$\text{Aborted}$$

input
```
Integrate[(a + b*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(7/3),x]
```

output
```
$Aborted
```

3.736.3 Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 4357}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{7}{3}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/3}}dx$$

$$\Big\downarrow \text{4357}$$

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{7}{3}}(c+dx)}dx$$

input
```
Int[(a + b*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(7/3),x]
```

output
```
$Aborted
```

3.736.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4357
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Unintegrable[(d*Csc[e + f*x])^n*(a + b*Csc[e + f*
x])^m, x] /; FreeQ[{a, b, d, e, f, m, n}, x]
```

3.736.4 Maple [N/A] (verified)

Not integrable

Time = 0.39 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{(a+b\sec(dx+c))^{\frac{3}{2}}}{\sec(dx+c)^{\frac{7}{3}}}dx$$

input
```
int((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(7/3),x)
```

output
```
int((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(7/3),x)
```

3.736.5 Fricas [N/A]

Not integrable

Time = 1.05 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{7}{3}}(c+dx)}\,dx = \int \frac{(b\sec(dx+c)+a)^{\frac{3}{2}}}{\sec(dx+c)^{\frac{7}{3}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(7/3),x, algorithm="fricas")
```

output
```
integral((b*sec(d*x + c) + a)^(3/2)/sec(d*x + c)^(7/3), x)
```

3.736.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{7}{3}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+b*sec(d*x+c))**(3/2)/sec(d*x+c)**(7/3),x)
```

output
```
Timed out
```

3.736.7 Maxima [N/A]

Not integrable

Time = 0.79 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{7}{3}}(c+dx)}\,dx=\int \frac{(b\sec(dx+c)+a)^{\frac{3}{2}}}{\sec(dx+c)^{\frac{7}{3}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(7/3),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^(3/2)/sec(d*x + c)^(7/3), x)
```

3.736.8 Giac [N/A]

Not integrable

Time = 147.93 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{7}{3}}(c+dx)}\,dx=\int \frac{(b\sec(dx+c)+a)^{\frac{3}{2}}}{\sec(dx+c)^{\frac{7}{3}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(7/3),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^(3/2)/sec(d*x + c)^(7/3), x)
```

3.736.9 Mupad [N/A]

Not integrable

Time = 15.19 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{7}{3}}(c+dx)}\,dx=\int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}}{\left(\frac{1}{\cos(c+d\,x)}\right)^{7/3}}\,dx$$

input
```
int((a + b/cos(c + d*x))^(3/2)/(1/cos(c + d*x))^(7/3),x)
```

output
```
int((a + b/cos(c + d*x))^(3/2)/(1/cos(c + d*x))^(7/3), x)
```

3.737 $\int \sec^{\frac{7}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$

3.737.1 Optimal result 5619
3.737.2 Mathematica [N/A] 5619
3.737.3 Rubi [N/A] 5620
3.737.4 Maple [N/A] (verified) 5621
3.737.5 Fricas [N/A] 5621
3.737.6 Sympy [F(-1)] 5621
3.737.7 Maxima [N/A] 5622
3.737.8 Giac [N/A] 5622
3.737.9 Mupad [N/A] 5622

3.737.1 Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \sec^{\frac{7}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \text{Int}\left(\sec^{\frac{7}{3}}(c+dx)(a+b\sec(c+dx))^{5/2},x\right)$$

output
```
Unintegrable(sec(d*x+c)^(7/3)*(a+b*sec(d*x+c))^(5/2),x)
```

3.737.2 Mathematica [N/A]

Not integrable

Time = 168.43 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \sec^{\frac{7}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \int \sec^{\frac{7}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$$

input
```
Integrate[Sec[c + d*x]^(7/3)*(a + b*Sec[c + d*x])^(5/2),x]
```

output
```
Integrate[Sec[c + d*x]^(7/3)*(a + b*Sec[c + d*x])^(5/2), x]
```

3.737.3 Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 4357}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{\frac{7}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{7/3}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}dx$$

$$\Big\downarrow \text{4357}$$

$$\int \sec^{\frac{7}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}dx$$

input
```
Int[Sec[c + d*x]^(7/3)*(a + b*Sec[c + d*x])^(5/2),x]
```

output
```
$Aborted
```

3.737.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4357
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Unintegrable[(d*Csc[e + f*x])^n*(a + b*Csc[e + f*
x])^m, x] /; FreeQ[{a, b, d, e, f, m, n}, x]
```

3.737.4 Maple [N/A] (verified)

Not integrable

Time = 0.43 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \sec (dx+c)^{\frac{7}{3}} (a+b\sec (dx+c))^{\frac{5}{2}} \, dx$$

input
```
int(sec(d*x+c)^(7/3)*(a+b*sec(d*x+c))^(5/2),x)
```

output
```
int(sec(d*x+c)^(7/3)*(a+b*sec(d*x+c))^(5/2),x)
```

3.737.5 Fricas [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 60, normalized size of antiderivative = 2.40

$$\int \sec^{\frac{7}{3}}(c+dx)(a+b\sec(c+dx))^{5/2} \, dx = \int (b\sec (dx+c)+a)^{\frac{5}{2}} \sec (dx+c)^{\frac{7}{3}} \, dx$$

input
```
integrate(sec(d*x+c)^(7/3)*(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
integral((b^2*sec(d*x + c)^4 + 2*a*b*sec(d*x + c)^3 + a^2*sec(d*x + c)^2)*
sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(1/3), x)
```

3.737.6 Sympy [F(-1)]

Timed out.

$$\int \sec^{\frac{7}{3}}(c+dx)(a+b\sec(c+dx))^{5/2} \, dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**(7/3)*(a+b*sec(d*x+c))**(5/2),x)
```

output
```
Timed out
```

3.737.7 Maxima [N/A]

Not integrable

Time = 2.48 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \sec^{\frac{7}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \int (b\sec(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^{\frac{7}{3}}\,dx$$

input
```
integrate(sec(d*x+c)^(7/3)*(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^(5/2)*sec(d*x + c)^(7/3), x)
```

3.737.8 Giac [N/A]

Not integrable

Time = 4.42 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \sec^{\frac{7}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \int (b\sec(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^{\frac{7}{3}}\,dx$$

input
```
integrate(sec(d*x+c)^(7/3)*(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^(5/2)*sec(d*x + c)^(7/3), x)
```

3.737.9 Mupad [N/A]

Not integrable

Time = 16.96 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \sec^{\frac{7}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \int \left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2}\left(\frac{1}{\cos(c+d\,x)}\right)^{7/3}dx$$

input
```
int((a + b/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(7/3),x)
```

output
```
int((a + b/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(7/3), x)
```

3.738 $\int \sec^{\frac{5}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$

3.738.1 Optimal result 5623
3.738.2 Mathematica [N/A] 5623
3.738.3 Rubi [N/A] 5624
3.738.4 Maple [N/A] (verified) 5625
3.738.5 Fricas [N/A] 5625
3.738.6 Sympy [F(-1)] 5625
3.738.7 Maxima [N/A] 5626
3.738.8 Giac [N/A] 5626
3.738.9 Mupad [N/A] 5626

3.738.1 Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \sec^{\frac{5}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \text{Int}\left(\sec^{\frac{5}{3}}(c+dx)(a+b\sec(c+dx))^{5/2},x\right)$$

output
```
Unintegrable(sec(d*x+c)^(5/3)*(a+b*sec(d*x+c))^(5/2),x)
```

3.738.2 Mathematica [N/A]

Not integrable

Time = 131.81 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \sec^{\frac{5}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \int \sec^{\frac{5}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$$

input
```
Integrate[Sec[c + d*x]^(5/3)*(a + b*Sec[c + d*x])^(5/2),x]
```

output
```
Integrate[Sec[c + d*x]^(5/3)*(a + b*Sec[c + d*x])^(5/2), x]
```

3.738.3 Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 4357}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{\frac{5}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{5/3}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}dx$$

$$\Big\downarrow \text{4357}$$

$$\int \sec^{\frac{5}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}dx$$

input
```
Int[Sec[c + d*x]^(5/3)*(a + b*Sec[c + d*x])^(5/2),x]
```

output
```
$Aborted
```

3.738.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4357
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Unintegrable[(d*Csc[e + f*x])^n*(a + b*Csc[e + f*
x])^m, x] /; FreeQ[{a, b, d, e, f, m, n}, x]
```

3.738.4 Maple [N/A] (verified)

Not integrable

Time = 0.48 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \sec (dx+c)^{\frac{5}{3}} (a+b\sec (dx+c))^{\frac{5}{2}} \, dx$$

input
```
int(sec(d*x+c)^(5/3)*(a+b*sec(d*x+c))^(5/2),x)
```

output
```
int(sec(d*x+c)^(5/3)*(a+b*sec(d*x+c))^(5/2),x)
```

3.738.5 Fricas [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 58, normalized size of antiderivative = 2.32

$$\int \sec^{\frac{5}{3}}(c+dx)(a+b\sec(c+dx))^{5/2} \, dx = \int (b\sec (dx+c)+a)^{\frac{5}{2}} \sec (dx+c)^{\frac{5}{3}} \, dx$$

input
```
integrate(sec(d*x+c)^(5/3)*(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
integral((b^2*sec(d*x + c)^3 + 2*a*b*sec(d*x + c)^2 + a^2*sec(d*x + c))*sq
rt(b*sec(d*x + c) + a)*sec(d*x + c)^(2/3), x)
```

3.738.6 Sympy [F(-1)]

Timed out.

$$\int \sec^{\frac{5}{3}}(c+dx)(a+b\sec(c+dx))^{5/2} \, dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**(5/3)*(a+b*sec(d*x+c))**(5/2),x)
```

output
```
Timed out
```

3.738.7 Maxima [N/A]

Not integrable

Time = 2.47 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \sec^{\frac{5}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \int (b\sec(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^{\frac{5}{3}}\,dx$$

input
```
integrate(sec(d*x+c)^(5/3)*(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^(5/2)*sec(d*x + c)^(5/3), x)
```

3.738.8 Giac [N/A]

Not integrable

Time = 4.27 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \sec^{\frac{5}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \int (b\sec(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^{\frac{5}{3}}\,dx$$

input
```
integrate(sec(d*x+c)^(5/3)*(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^(5/2)*sec(d*x + c)^(5/3), x)
```

3.738.9 Mupad [N/A]

Not integrable

Time = 16.40 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \sec^{\frac{5}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \int \left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2}\left(\frac{1}{\cos(c+d\,x)}\right)^{5/3}dx$$

input
```
int((a + b/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(5/3),x)
```

output
```
int((a + b/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(5/3), x)
```

3.739 $\int \sec^{\frac{4}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$

3.739.1 Optimal result 5627
3.739.2 Mathematica [N/A] 5627
3.739.3 Rubi [N/A] 5628
3.739.4 Maple [N/A] (verified) 5629
3.739.5 Fricas [N/A] 5629
3.739.6 Sympy [F(-1)] 5629
3.739.7 Maxima [N/A] 5630
3.739.8 Giac [N/A] 5630
3.739.9 Mupad [N/A] 5630

3.739.1 Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \sec^{\frac{4}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \text{Int}\left(\sec^{\frac{4}{3}}(c+dx)(a+b\sec(c+dx))^{5/2},x\right)$$

output
```
Unintegrable(sec(d*x+c)^(4/3)*(a+b*sec(d*x+c))^(5/2),x)
```

3.739.2 Mathematica [N/A]

Not integrable

Time = 125.06 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \sec^{\frac{4}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \int \sec^{\frac{4}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$$

input
```
Integrate[Sec[c + d*x]^(4/3)*(a + b*Sec[c + d*x])^(5/2),x]
```

output
```
Integrate[Sec[c + d*x]^(4/3)*(a + b*Sec[c + d*x])^(5/2), x]
```

3.739.3 Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 4357}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{\frac{4}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{4/3}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}dx$$

$$\Big\downarrow \text{4357}$$

$$\int \sec^{\frac{4}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}dx$$

input
```
Int[Sec[c + d*x]^(4/3)*(a + b*Sec[c + d*x])^(5/2),x]
```

output
```
$Aborted
```

3.739.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4357
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Unintegrable[(d*Csc[e + f*x])^n*(a + b*Csc[e + f*
x])^m, x] /; FreeQ[{a, b, d, e, f, m, n}, x]
```

3.739.4 Maple [N/A] (verified)

Not integrable

Time = 0.43 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \sec (dx+c)^{\frac{4}{3}} (a+b\sec (dx+c))^{\frac{5}{2}} \, dx$$

input
```
int(sec(d*x+c)^(4/3)*(a+b*sec(d*x+c))^(5/2),x)
```

output
```
int(sec(d*x+c)^(4/3)*(a+b*sec(d*x+c))^(5/2),x)
```

3.739.5 Fricas [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 58, normalized size of antiderivative = 2.32

$$\int \sec^{\frac{4}{3}}(c+dx)(a+b\sec(c+dx))^{5/2} \, dx = \int (b\sec (dx+c)+a)^{\frac{5}{2}} \sec (dx+c)^{\frac{4}{3}} \, dx$$

input
```
integrate(sec(d*x+c)^(4/3)*(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
integral((b^2*sec(d*x + c)^3 + 2*a*b*sec(d*x + c)^2 + a^2*sec(d*x + c))*sq
rt(b*sec(d*x + c) + a)*sec(d*x + c)^(1/3), x)
```

3.739.6 Sympy [F(-1)]

Timed out.

$$\int \sec^{\frac{4}{3}}(c+dx)(a+b\sec(c+dx))^{5/2} \, dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**(4/3)*(a+b*sec(d*x+c))**(5/2),x)
```

output
```
Timed out
```

3.739.7 Maxima [N/A]

Not integrable

Time = 2.20 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \sec^{\frac{4}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \int (b\sec(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^{\frac{4}{3}}\,dx$$

input
```
integrate(sec(d*x+c)^(4/3)*(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^(5/2)*sec(d*x + c)^(4/3), x)
```

3.739.8 Giac [N/A]

Not integrable

Time = 2.82 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \sec^{\frac{4}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \int (b\sec(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^{\frac{4}{3}}\,dx$$

input
```
integrate(sec(d*x+c)^(4/3)*(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^(5/2)*sec(d*x + c)^(4/3), x)
```

3.739.9 Mupad [N/A]

Not integrable

Time = 16.40 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \sec^{\frac{4}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \int \left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2}\left(\frac{1}{\cos(c+d\,x)}\right)^{4/3}dx$$

input
```
int((a + b/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(4/3),x)
```

output
```
int((a + b/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(4/3), x)
```

3.740 $\int \sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$

3.740.1 Optimal result 5631
3.740.2 Mathematica [N/A] 5631
3.740.3 Rubi [N/A] 5632
3.740.4 Maple [N/A] (verified) 5633
3.740.5 Fricas [N/A] 5633
3.740.6 Sympy [F(-1)] 5633
3.740.7 Maxima [N/A] 5634
3.740.8 Giac [N/A] 5634
3.740.9 Mupad [N/A] 5634

3.740.1 Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \text{Int}\left(\sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))^{5/2},x\right)$$

output
```
Unintegrable(sec(d*x+c)^(2/3)*(a+b*sec(d*x+c))^(5/2),x)
```

3.740.2 Mathematica [N/A]

Not integrable

Time = 127.24 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \int \sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$$

input
```
Integrate[Sec[c + d*x]^(2/3)*(a + b*Sec[c + d*x])^(5/2),x]
```

output
```
Integrate[Sec[c + d*x]^(2/3)*(a + b*Sec[c + d*x])^(5/2), x]
```

3.740.3 Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 4357}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(c+dx+\frac{\pi}{2}\right)^{2/3}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}dx$$

$$\Big\downarrow \text{4357}$$

$$\int \sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}dx$$

input
```
Int[Sec[c + d*x]^(2/3)*(a + b*Sec[c + d*x])^(5/2),x]
```

output
```
$Aborted
```

3.740.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4357
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Unintegrable[(d*Csc[e + f*x])^n*(a + b*Csc[e + f*
x])^m, x] /; FreeQ[{a, b, d, e, f, m, n}, x]
```

3.740.4 Maple [N/A] (verified)

Not integrable

Time = 0.47 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \sec (dx+c)^{\frac{2}{3}} (a+b\sec (dx+c))^{\frac{5}{2}} \, dx$$

input
```
int(sec(d*x+c)^(2/3)*(a+b*sec(d*x+c))^(5/2),x)
```

output
```
int(sec(d*x+c)^(2/3)*(a+b*sec(d*x+c))^(5/2),x)
```

3.740.5 Fricas [N/A]

Not integrable

Time = 1.07 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.96

$$\int \sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))^{5/2} \, dx = \int (b\sec (dx+c)+a)^{\frac{5}{2}} \sec (dx+c)^{\frac{2}{3}} \, dx$$

input
```
integrate(sec(d*x+c)^(2/3)*(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
integral((b^2*sec(d*x + c)^2 + 2*a*b*sec(d*x + c) + a^2)*sqrt(b*sec(d*x +
c) + a)*sec(d*x + c)^(2/3), x)
```

3.740.6 Sympy [F(-1)]

Timed out.

$$\int \sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))^{5/2} \, dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**(2/3)*(a+b*sec(d*x+c))**(5/2),x)
```

output
```
Timed out
```

3.740.7 Maxima [N/A]

Not integrable

Time = 2.14 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \int (b\sec(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^{\frac{2}{3}}\,dx$$

input
```
integrate(sec(d*x+c)^(2/3)*(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^(5/2)*sec(d*x + c)^(2/3), x)
```

3.740.8 Giac [N/A]

Not integrable

Time = 4.49 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \int (b\sec(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^{\frac{2}{3}}\,dx$$

input
```
integrate(sec(d*x+c)^(2/3)*(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^(5/2)*sec(d*x + c)^(2/3), x)
```

3.740.9 Mupad [N/A]

Not integrable

Time = 16.00 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \int \left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2}\left(\frac{1}{\cos(c+d\,x)}\right)^{2/3}dx$$

input
```
int((a + b/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(2/3),x)
```

output
```
int((a + b/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(2/3), x)
```

3.741 $\int \sqrt[3]{\sec(c+dx)}(a+b\sec(c+dx))^{5/2}\,dx$

3.741.1 Optimal result . . . 5635
3.741.2 Mathematica [N/A] . . . 5635
3.741.3 Rubi [N/A] . . . 5636
3.741.4 Maple [N/A] (verified) . . . 5637
3.741.5 Fricas [N/A] . . . 5637
3.741.6 Sympy [F(-1)] . . . 5637
3.741.7 Maxima [N/A] . . . 5638
3.741.8 Giac [N/A] . . . 5638
3.741.9 Mupad [N/A] . . . 5638

3.741.1 Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \sqrt[3]{\sec(c+dx)}(a+b\sec(c+dx))^{5/2}\,dx = \text{Int}\left(\sqrt[3]{\sec(c+dx)}(a+b\sec(c+dx))^{5/2},x\right)$$

output
```
Unintegrable(sec(d*x+c)^(1/3)*(a+b*sec(d*x+c))^(5/2),x)
```

3.741.2 Mathematica [N/A]

Not integrable

Time = 107.66 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \sqrt[3]{\sec(c+dx)}(a+b\sec(c+dx))^{5/2}\,dx = \int \sqrt[3]{\sec(c+dx)}(a+b\sec(c+dx))^{5/2}\,dx$$

input
```
Integrate[Sec[c + d*x]^(1/3)*(a + b*Sec[c + d*x])^(5/2),x]
```

output
```
Integrate[Sec[c + d*x]^(1/3)*(a + b*Sec[c + d*x])^(5/2), x]
```

3.741.3 Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 4357}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt[3]{\sec(c+dx)}(a+b\sec(c+dx))^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt[3]{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}dx$$

$$\Big\downarrow \text{4357}$$

$$\int \sqrt[3]{\sec(c+dx)}(a+b\sec(c+dx))^{5/2}dx$$

input
```
Int[Sec[c + d*x]^(1/3)*(a + b*Sec[c + d*x])^(5/2),x]
```

output
```
$Aborted
```

3.741.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4357
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Unintegrable[(d*Csc[e + f*x])^n*(a + b*Csc[e + f*
x])^m, x] /; FreeQ[{a, b, d, e, f, m, n}, x]
```

3.741.4 Maple [N/A] (verified)

Not integrable

Time = 0.49 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \sec (dx+c)^{\frac{1}{3}} (a+b\sec (dx+c))^{\frac{5}{2}} \, dx$$

input
```
int(sec(d*x+c)^(1/3)*(a+b*sec(d*x+c))^(5/2),x)
```

output
```
int(sec(d*x+c)^(1/3)*(a+b*sec(d*x+c))^(5/2),x)
```

3.741.5 Fricas [N/A]

Not integrable

Time = 0.71 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.96

$$\int \sqrt[3]{\sec(c+dx)}(a+b\sec(c+dx))^{5/2} \, dx = \int (b\sec (dx+c)+a)^{\frac{5}{2}} \sec (dx+c)^{\frac{1}{3}} \, dx$$

input
```
integrate(sec(d*x+c)^(1/3)*(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
integral((b^2*sec(d*x + c)^2 + 2*a*b*sec(d*x + c) + a^2)*sqrt(b*sec(d*x +
c) + a)*sec(d*x + c)^(1/3), x)
```

3.741.6 Sympy [F(-1)]

Timed out.

$$\int \sqrt[3]{\sec(c+dx)}(a+b\sec(c+dx))^{5/2} \, dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**(1/3)*(a+b*sec(d*x+c))**(5/2),x)
```

output
```
Timed out
```

3.741.7 Maxima [N/A]

Not integrable

Time = 2.00 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \sqrt[3]{\sec(c+dx)}(a+b\sec(c+dx))^{5/2}\,dx = \int (b\sec(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^{\frac{1}{3}}\,dx$$

input
```
integrate(sec(d*x+c)^(1/3)*(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^(5/2)*sec(d*x + c)^(1/3), x)
```

3.741.8 Giac [N/A]

Not integrable

Time = 2.86 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \sqrt[3]{\sec(c+dx)}(a+b\sec(c+dx))^{5/2}\,dx = \int (b\sec(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^{\frac{1}{3}}\,dx$$

input
```
integrate(sec(d*x+c)^(1/3)*(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^(5/2)*sec(d*x + c)^(1/3), x)
```

3.741.9 Mupad [N/A]

Not integrable

Time = 16.20 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \sqrt[3]{\sec(c+dx)}(a+b\sec(c+dx))^{5/2}\,dx = \int \left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2}\left(\frac{1}{\cos(c+d\,x)}\right)^{1/3}dx$$

input
```
int((a + b/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(1/3),x)
```

output
```
int((a + b/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(1/3), x)
```

3.742 $\int \frac{(a+b\sec(c+dx))^{5/2}}{\sqrt[3]{\sec(c+dx)}}\,dx$

3.742.1 Optimal result 5639
3.742.2 Mathematica [F(-1)] 5639
3.742.3 Rubi [N/A] 5640
3.742.4 Maple [N/A] (verified) 5641
3.742.5 Fricas [N/A] 5641
3.742.6 Sympy [F(-1)] 5641
3.742.7 Maxima [N/A] 5642
3.742.8 Giac [F(-1)] 5642
3.742.9 Mupad [N/A] 5642

3.742.1 Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sqrt[3]{\sec(c+dx)}}\,dx = \text{Int}\left(\frac{(a+b\sec(c+dx))^{5/2}}{\sqrt[3]{\sec(c+dx)}},x\right)$$

output
```
Unintegrable((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(1/3),x)
```

3.742.2 Mathematica [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sqrt[3]{\sec(c+dx)}}\,dx = \$\text{Aborted}$$

input
```
Integrate[(a + b*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(1/3),x]
```

output
```
$Aborted
```

3.742.3 Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 4357}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sqrt[3]{\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}{\sqrt[3]{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4357}$$

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sqrt[3]{\sec(c+dx)}}dx$$

input
```
Int[(a + b*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(1/3),x]
```

output
```
$Aborted
```

3.742.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4357
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Unintegrable[(d*Csc[e + f*x])^n*(a + b*Csc[e + f*
x])^m, x] /; FreeQ[{a, b, d, e, f, m, n}, x]
```

3.742.4 Maple [N/A] (verified)

Not integrable

Time = 0.43 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{(a+b\sec(dx+c))^{\frac{5}{2}}}{\sec(dx+c)^{\frac{1}{3}}}dx$$

input
```
int((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(1/3),x)
```

output
```
int((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(1/3),x)
```

3.742.5 Fricas [N/A]

Not integrable

Time = 1.08 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.96

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sqrt[3]{\sec(c+dx)}}\,dx = \int \frac{(b\sec(dx+c)+a)^{\frac{5}{2}}}{\sec(dx+c)^{\frac{1}{3}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(1/3),x, algorithm="fricas")
```

output
```
integral((b^2*sec(d*x + c)^2 + 2*a*b*sec(d*x + c) + a^2)*sqrt(b*sec(d*x +
c) + a)/sec(d*x + c)^(1/3), x)
```

3.742.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sqrt[3]{\sec(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate((a+b*sec(d*x+c))**(5/2)/sec(d*x+c)**(1/3),x)
```

output
```
Timed out
```

3.742.7 Maxima [N/A]

Not integrable

Time = 2.08 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sqrt[3]{\sec(c+dx)}}\,dx = \int \frac{(b\sec(dx+c)+a)^{\frac{5}{2}}}{\sec(dx+c)^{\frac{1}{3}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(1/3),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^(5/2)/sec(d*x + c)^(1/3), x)
```

3.742.8 Giac [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sqrt[3]{\sec(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(1/3),x, algorithm="giac")
```

output
```
Timed out
```

3.742.9 Mupad [N/A]

Not integrable

Time = 17.08 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sqrt[3]{\sec(c+dx)}}\,dx = \int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2}}{\left(\frac{1}{\cos(c+d\,x)}\right)^{1/3}}\,dx$$

input
```
int((a + b/cos(c + d*x))^(5/2)/(1/cos(c + d*x))^(1/3),x)
```

output
```
int((a + b/cos(c + d*x))^(5/2)/(1/cos(c + d*x))^(1/3), x)
```

3.743 $\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{2}{3}}(c+dx)}\,dx$

3.743.1 Optimal result 5643
3.743.2 Mathematica [N/A] 5643
3.743.3 Rubi [N/A] 5644
3.743.4 Maple [N/A] (verified) 5645
3.743.5 Fricas [N/A] 5645
3.743.6 Sympy [F(-1)] 5645
3.743.7 Maxima [N/A] 5646
3.743.8 Giac [F(-1)] 5646
3.743.9 Mupad [N/A] 5646

3.743.1 Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{2}{3}}(c+dx)}\,dx=\text{Int}\left(\frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{2}{3}}(c+dx)},x\right)$$

output
```
Unintegrable((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(2/3),x)
```

3.743.2 Mathematica [N/A]

Not integrable

Time = 159.67 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{2}{3}}(c+dx)}\,dx=\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{2}{3}}(c+dx)}\,dx$$

input
```
Integrate[(a + b*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(2/3),x]
```

output
```
Integrate[(a + b*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(2/3), x]
```

3.743.3 Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 4357}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{2}{3}}(c+dx)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}{\csc\left(c+dx+\frac{\pi}{2}\right)^{2/3}}dx$$

$$\Big\downarrow \text{ 4357}$$

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{2}{3}}(c+dx)}dx$$

input
```
Int[(a + b*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(2/3),x]
```

output
```
$Aborted
```

3.743.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4357
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Unintegrable[(d*Csc[e + f*x])^n*(a + b*Csc[e + f*
x])^m, x] /; FreeQ[{a, b, d, e, f, m, n}, x]
```

3.743.4 Maple [N/A] (verified)

Not integrable

Time = 0.42 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{(a+b\sec(dx+c))^{\frac{5}{2}}}{\sec(dx+c)^{\frac{2}{3}}}dx$$

input
```
int((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(2/3),x)
```

output
```
int((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(2/3),x)
```

3.743.5 Fricas [N/A]

Not integrable

Time = 0.76 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.96

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{2}{3}}(c+dx)}\,dx = \int \frac{(b\sec(dx+c)+a)^{\frac{5}{2}}}{\sec(dx+c)^{\frac{2}{3}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(2/3),x, algorithm="fricas")
```

output
```
integral((b^2*sec(d*x + c)^2 + 2*a*b*sec(d*x + c) + a^2)*sqrt(b*sec(d*x +
c) + a)/sec(d*x + c)^(2/3), x)
```

3.743.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{2}{3}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+b*sec(d*x+c))**(5/2)/sec(d*x+c)**(2/3),x)
```

output
```
Timed out
```

3.743.7 Maxima [N/A]

Not integrable

Time = 2.11 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{2}{3}}(c+dx)}\,dx = \int \frac{(b\sec(dx+c)+a)^{\frac{5}{2}}}{\sec(dx+c)^{\frac{2}{3}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(2/3),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^(5/2)/sec(d*x + c)^(2/3), x)
```

3.743.8 Giac [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{2}{3}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(2/3),x, algorithm="giac")
```

output
```
Timed out
```

3.743.9 Mupad [N/A]

Not integrable

Time = 14.21 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{2}{3}}(c+dx)}\,dx = \int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2}}{\left(\frac{1}{\cos(c+d\,x)}\right)^{2/3}}\,dx$$

input
```
int((a + b/cos(c + d*x))^(5/2)/(1/cos(c + d*x))^(2/3),x)
```

output
```
int((a + b/cos(c + d*x))^(5/2)/(1/cos(c + d*x))^(2/3), x)
```

3.744 $\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{4}{3}}(c+dx)}\,dx$

3.744.1 Optimal result 5647
3.744.2 Mathematica [N/A] 5647
3.744.3 Rubi [N/A] 5648
3.744.4 Maple [N/A] (verified) 5649
3.744.5 Fricas [N/A] 5649
3.744.6 Sympy [F(-1)] 5649
3.744.7 Maxima [N/A] 5650
3.744.8 Giac [F(-2)] 5650
3.744.9 Mupad [N/A] 5650

3.744.1 Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{4}{3}}(c+dx)}\,dx = \text{Int}\left(\frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{4}{3}}(c+dx)},x\right)$$

output
```
Unintegrable((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(4/3),x)
```

3.744.2 Mathematica [N/A]

Not integrable

Time = 167.85 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{4}{3}}(c+dx)}\,dx = \int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{4}{3}}(c+dx)}\,dx$$

input
```
Integrate[(a + b*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(4/3),x]
```

output
```
Integrate[(a + b*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(4/3), x]
```

3.744.3 Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 4357}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{4}{3}}(c+dx)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}{\csc\left(c+dx+\frac{\pi}{2}\right)^{4/3}}dx$$

$$\Big\downarrow \text{ 4357}$$

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{4}{3}}(c+dx)}dx$$

input
```
Int[(a + b*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(4/3),x]
```

output
```
$Aborted
```

3.744.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4357
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Unintegrable[(d*Csc[e + f*x])^n*(a + b*Csc[e + f*
x])^m, x] /; FreeQ[{a, b, d, e, f, m, n}, x]
```

3.744.4 Maple [N/A] (verified)

Not integrable

Time = 0.44 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{(a+b\sec(dx+c))^{\frac{5}{2}}}{\sec(dx+c)^{\frac{4}{3}}}dx$$

input
```
int((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(4/3),x)
```

output
```
int((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(4/3),x)
```

3.744.5 Fricas [N/A]

Not integrable

Time = 1.06 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.96

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{4}{3}}(c+dx)}\,dx = \int \frac{(b\sec(dx+c)+a)^{\frac{5}{2}}}{\sec(dx+c)^{\frac{4}{3}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(4/3),x, algorithm="fricas")
```

output
```
integral((b^2*sec(d*x + c)^2 + 2*a*b*sec(d*x + c) + a^2)*sqrt(b*sec(d*x +
c) + a)/sec(d*x + c)^(4/3), x)
```

3.744.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{4}{3}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+b*sec(d*x+c))**(5/2)/sec(d*x+c)**(4/3),x)
```

output
```
Timed out
```

3.744.7 Maxima [N/A]

Not integrable

Time = 2.10 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{4}{3}}(c+dx)}\,dx = \int \frac{(b\sec(dx+c)+a)^{\frac{5}{2}}}{\sec(dx+c)^{\frac{4}{3}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(4/3),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^(5/2)/sec(d*x + c)^(4/3), x)
```

3.744.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{4}{3}}(c+dx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(4/3),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:int()  Error: Bad Argument Value
```

3.744.9 Mupad [N/A]

Not integrable

Time = 17.02 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{4}{3}}(c+dx)}\,dx = \int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2}}{\left(\frac{1}{\cos(c+d\,x)}\right)^{4/3}}\,dx$$

input
```
int((a + b/cos(c + d*x))^(5/2)/(1/cos(c + d*x))^(4/3),x)
```

output
```
int((a + b/cos(c + d*x))^(5/2)/(1/cos(c + d*x))^(4/3), x)
```

3.745 $\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{5}{3}}(c+dx)}\,dx$

3.745.1 Optimal result 5651
3.745.2 Mathematica [F(-1)] 5651
3.745.3 Rubi [N/A] 5652
3.745.4 Maple [N/A] (verified) 5653
3.745.5 Fricas [N/A] 5653
3.745.6 Sympy [F(-1)] 5653
3.745.7 Maxima [N/A] 5654
3.745.8 Giac [F(-1)] 5654
3.745.9 Mupad [N/A] 5654

3.745.1 Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{5}{3}}(c+dx)}\,dx=\text{Int}\left(\frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{5}{3}}(c+dx)},x\right)$$

output
```
Unintegrable((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(5/3),x)
```

3.745.2 Mathematica [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{5}{3}}(c+dx)}\,dx=\$\text{Aborted}$$

input
```
Integrate[(a + b*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(5/3),x]
```

output
```
$Aborted
```

3.745.3 Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 4357}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{5}{3}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/3}}dx$$

$$\Big\downarrow \text{4357}$$

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{5}{3}}(c+dx)}dx$$

input
```
Int[(a + b*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(5/3),x]
```

output
```
$Aborted
```

3.745.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4357
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Unintegrable[(d*Csc[e + f*x])^n*(a + b*Csc[e + f*
x])^m, x] /; FreeQ[{a, b, d, e, f, m, n}, x]
```

3.745.4 Maple [N/A] (verified)

Not integrable

Time = 0.44 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{(a+b\sec(dx+c))^{\frac{5}{2}}}{\sec(dx+c)^{\frac{5}{3}}}dx$$

input
```
int((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(5/3),x)
```

output
```
int((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(5/3),x)
```

3.745.5 Fricas [N/A]

Not integrable

Time = 0.70 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.96

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{5}{3}}(c+dx)}\,dx = \int \frac{(b\sec(dx+c)+a)^{\frac{5}{2}}}{\sec(dx+c)^{\frac{5}{3}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(5/3),x, algorithm="fricas")
```

output
```
integral((b^2*sec(d*x + c)^2 + 2*a*b*sec(d*x + c) + a^2)*sqrt(b*sec(d*x +
c) + a)/sec(d*x + c)^(5/3), x)
```

3.745.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{5}{3}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+b*sec(d*x+c))**(5/2)/sec(d*x+c)**(5/3),x)
```

output
```
Timed out
```

3.745.7 Maxima [N/A]

Not integrable

Time = 2.11 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{5}{3}}(c+dx)}\,dx = \int \frac{(b\sec(dx+c)+a)^{\frac{5}{2}}}{\sec(dx+c)^{\frac{5}{3}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(5/3),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^(5/2)/sec(d*x + c)^(5/3), x)
```

3.745.8 Giac [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{5}{3}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(5/3),x, algorithm="giac")
```

output
```
Timed out
```

3.745.9 Mupad [N/A]

Not integrable

Time = 14.17 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{5}{3}}(c+dx)}\,dx = \int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2}}{\left(\frac{1}{\cos(c+d\,x)}\right)^{5/3}}\,dx$$

input
```
int((a + b/cos(c + d*x))^(5/2)/(1/cos(c + d*x))^(5/3),x)
```

output
```
int((a + b/cos(c + d*x))^(5/2)/(1/cos(c + d*x))^(5/3), x)
```

3.746 $\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{7}{3}}(c+dx)}\,dx$

3.746.1 Optimal result 5655
3.746.2 Mathematica [F(-1)] 5655
3.746.3 Rubi [N/A] 5656
3.746.4 Maple [N/A] (verified) 5657
3.746.5 Fricas [N/A] 5657
3.746.6 Sympy [F(-1)] 5657
3.746.7 Maxima [N/A] 5658
3.746.8 Giac [F(-2)] 5658
3.746.9 Mupad [N/A] 5658

3.746.1 Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{7}{3}}(c+dx)}\,dx=\text{Int}\left(\frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{7}{3}}(c+dx)},x\right)$$

output
```
Unintegrable((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(7/3),x)
```

3.746.2 Mathematica [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{7}{3}}(c+dx)}\,dx=\$\text{Aborted}$$

input
```
Integrate[(a + b*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(7/3),x]
```

output
```
$Aborted
```

3.746.3 Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 4357}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{7}{3}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/3}}dx$$

$$\Big\downarrow \text{4357}$$

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{7}{3}}(c+dx)}dx$$

input
```
Int[(a + b*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(7/3),x]
```

output
```
$Aborted
```

3.746.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4357
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Unintegrable[(d*Csc[e + f*x])^n*(a + b*Csc[e + f*
x])^m, x] /; FreeQ[{a, b, d, e, f, m, n}, x]
```

3.746.4 Maple [N/A] (verified)

Not integrable

Time = 0.45 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{(a+b\sec(dx+c))^{\frac{5}{2}}}{\sec(dx+c)^{\frac{7}{3}}}dx$$

input
```
int((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(7/3),x)
```

output
```
int((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(7/3),x)
```

3.746.5 Fricas [N/A]

Not integrable

Time = 1.14 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.96

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{7}{3}}(c+dx)}\,dx = \int \frac{(b\sec(dx+c)+a)^{\frac{5}{2}}}{\sec(dx+c)^{\frac{7}{3}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(7/3),x, algorithm="fricas")
```

output
```
integral((b^2*sec(d*x + c)^2 + 2*a*b*sec(d*x + c) + a^2)*sqrt(b*sec(d*x +
c) + a)/sec(d*x + c)^(7/3), x)
```

3.746.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{7}{3}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+b*sec(d*x+c))**(5/2)/sec(d*x+c)**(7/3),x)
```

output
```
Timed out
```

3.746.7 Maxima [N/A]

Not integrable

Time = 2.15 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{7}{3}}(c+dx)}\,dx = \int \frac{(b\sec(dx+c)+a)^{\frac{5}{2}}}{\sec(dx+c)^{\frac{7}{3}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(7/3),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^(5/2)/sec(d*x + c)^(7/3), x)
```

3.746.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{7}{3}}(c+dx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(7/3),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:int()  Error: Bad Argument Value
```

3.746.9 Mupad [N/A]

Not integrable

Time = 17.04 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{7}{3}}(c+dx)}\,dx = \int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2}}{\left(\frac{1}{\cos(c+d\,x)}\right)^{7/3}}\,dx$$

input
```
int((a + b/cos(c + d*x))^(5/2)/(1/cos(c + d*x))^(7/3),x)
```

output
```
int((a + b/cos(c + d*x))^(5/2)/(1/cos(c + d*x))^(7/3), x)
```

3.747 $\int \frac{\sec^{\frac{7}{3}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$

3.747.1 Optimal result 5659
3.747.2 Mathematica [N/A] 5659
3.747.3 Rubi [N/A] 5660
3.747.4 Maple [N/A] (verified) 5661
3.747.5 Fricas [N/A] 5661
3.747.6 Sympy [F(-1)] 5661
3.747.7 Maxima [N/A] 5662
3.747.8 Giac [F(-1)] 5662
3.747.9 Mupad [N/A] 5662

3.747.1 Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{\sec^{\frac{7}{3}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \text{Int}\left(\frac{\sec^{\frac{7}{3}}(c+dx)}{\sqrt{a+b\sec(c+dx)}},x\right)$$

output
```
Unintegrable(sec(d*x+c)^(7/3)/(a+b*sec(d*x+c))^(1/2),x)
```

3.747.2 Mathematica [N/A]

Not integrable

Time = 105.06 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{\sec^{\frac{7}{3}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\sec^{\frac{7}{3}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$$

input
```
Integrate[Sec[c + d*x]^(7/3)/Sqrt[a + b*Sec[c + d*x]],x]
```

output
```
Integrate[Sec[c + d*x]^(7/3)/Sqrt[a + b*Sec[c + d*x]], x]
```

3.747.3 Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 4357}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{\frac{7}{3}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/3}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{ 4357}$$

$$\int \frac{\sec^{\frac{7}{3}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx$$

input
```
Int[Sec[c + d*x]^(7/3)/Sqrt[a + b*Sec[c + d*x]],x]
```

output
```
$Aborted
```

3.747.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4357
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Unintegrable[(d*Csc[e + f*x])^n*(a + b*Csc[e + f*
x])^m, x] /; FreeQ[{a, b, d, e, f, m, n}, x]
```

3.747.4 Maple [N/A] (verified)

Not integrable

Time = 0.47 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{\sec (dx+c)^{\frac{7}{3}}}{\sqrt{a+b\sec (dx+c)}}dx$$

input
```
int(sec(d*x+c)^(7/3)/(a+b*sec(d*x+c))^(1/2),x)
```

output
```
int(sec(d*x+c)^(7/3)/(a+b*sec(d*x+c))^(1/2),x)
```

3.747.5 Fricas [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{\sec^{\frac{7}{3}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx=\int \frac{\sec (dx+c)^{\frac{7}{3}}}{\sqrt{b\sec (dx+c)+a}}\,dx$$

input
```
integrate(sec(d*x+c)^(7/3)/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
integral(sec(d*x + c)^(7/3)/sqrt(b*sec(d*x + c) + a), x)
```

3.747.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{7}{3}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx=\text{Timed out}$$

input
```
integrate(sec(d*x+c)**(7/3)/(a+b*sec(d*x+c))**(1/2),x)
```

output
```
Timed out
```

3.747.7 Maxima [N/A]

Not integrable

Time = 0.77 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{\sec^{\frac{7}{3}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\sec(dx+c)^{\frac{7}{3}}}{\sqrt{b\sec(dx+c)+a}}\,dx$$

input
```
integrate(sec(d*x+c)^(7/3)/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)^(7/3)/sqrt(b*sec(d*x + c) + a), x)
```

3.747.8 Giac [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{7}{3}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)^(7/3)/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
Timed out
```

3.747.9 Mupad [N/A]

Not integrable

Time = 15.59 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{\sec^{\frac{7}{3}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{7/3}}{\sqrt{a+\frac{b}{\cos(c+d\,x)}}}\,dx$$

input
```
int((1/cos(c + d*x))^(7/3)/(a + b/cos(c + d*x))^(1/2),x)
```

output
```
int((1/cos(c + d*x))^(7/3)/(a + b/cos(c + d*x))^(1/2), x)
```

3.748 $\int \frac{\sec^{\frac{5}{3}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$

3.748.1 Optimal result 5663
3.748.2 Mathematica [N/A] 5663
3.748.3 Rubi [N/A] 5664
3.748.4 Maple [N/A] (verified) 5665
3.748.5 Fricas [N/A] 5665
3.748.6 Sympy [F(-1)] 5665
3.748.7 Maxima [N/A] 5666
3.748.8 Giac [F(-1)] 5666
3.748.9 Mupad [N/A] 5666

3.748.1 Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{\sec^{\frac{5}{3}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \text{Int}\left(\frac{\sec^{\frac{5}{3}}(c+dx)}{\sqrt{a+b\sec(c+dx)}},x\right)$$

output
```
Unintegrable(sec(d*x+c)^(5/3)/(a+b*sec(d*x+c))^(1/2),x)
```

3.748.2 Mathematica [N/A]

Not integrable

Time = 124.66 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{\sec^{\frac{5}{3}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\sec^{\frac{5}{3}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$$

input
```
Integrate[Sec[c + d*x]^(5/3)/Sqrt[a + b*Sec[c + d*x]],x]
```

output
```
Integrate[Sec[c + d*x]^(5/3)/Sqrt[a + b*Sec[c + d*x]], x]
```

3.748.3 Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 4357}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{\frac{5}{3}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/3}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{ 4357}$$

$$\int \frac{\sec^{\frac{5}{3}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx$$

input
```
Int[Sec[c + d*x]^(5/3)/Sqrt[a + b*Sec[c + d*x]],x]
```

output
```
$Aborted
```

3.748.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4357
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Unintegrable[(d*Csc[e + f*x])^n*(a + b*Csc[e + f*
x])^m, x] /; FreeQ[{a, b, d, e, f, m, n}, x]
```

3.748.4 Maple [N/A] (verified)

Not integrable

Time = 0.41 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{\sec (dx+c)^{\frac{5}{3}}}{\sqrt{a+b\sec (dx+c)}}dx$$

input
```
int(sec(d*x+c)^(5/3)/(a+b*sec(d*x+c))^(1/2),x)
```

output
```
int(sec(d*x+c)^(5/3)/(a+b*sec(d*x+c))^(1/2),x)
```

3.748.5 Fricas [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{\sec^{\frac{5}{3}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx=\int \frac{\sec (dx+c)^{\frac{5}{3}}}{\sqrt{b\sec (dx+c)+a}}\,dx$$

input
```
integrate(sec(d*x+c)^(5/3)/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
integral(sec(d*x + c)^(5/3)/sqrt(b*sec(d*x + c) + a), x)
```

3.748.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{5}{3}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx=\text{Timed out}$$

input
```
integrate(sec(d*x+c)**(5/3)/(a+b*sec(d*x+c))**(1/2),x)
```

output
```
Timed out
```

3.748.7 Maxima [N/A]

Not integrable

Time = 0.78 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{\sec^{\frac{5}{3}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\sec(dx+c)^{\frac{5}{3}}}{\sqrt{b\sec(dx+c)+a}}\,dx$$

input
```
integrate(sec(d*x+c)^(5/3)/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)^(5/3)/sqrt(b*sec(d*x + c) + a), x)
```

3.748.8 Giac [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{5}{3}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)^(5/3)/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
Timed out
```

3.748.9 Mupad [N/A]

Not integrable

Time = 15.42 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{\sec^{\frac{5}{3}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{5/3}}{\sqrt{a+\frac{b}{\cos(c+d\,x)}}}\,dx$$

input
```
int((1/cos(c + d*x))^(5/3)/(a + b/cos(c + d*x))^(1/2),x)
```

output
```
int((1/cos(c + d*x))^(5/3)/(a + b/cos(c + d*x))^(1/2), x)
```

3.749 $\int \frac{\sec^{\frac{4}{3}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$

3.749.1 Optimal result 5667
3.749.2 Mathematica [N/A] 5667
3.749.3 Rubi [N/A] 5668
3.749.4 Maple [N/A] (verified) 5669
3.749.5 Fricas [N/A] 5669
3.749.6 Sympy [N/A] 5669
3.749.7 Maxima [N/A] 5670
3.749.8 Giac [F(-1)] 5670
3.749.9 Mupad [N/A] 5670

3.749.1 Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{\sec^{\frac{4}{3}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \text{Int}\left(\frac{\sec^{\frac{4}{3}}(c+dx)}{\sqrt{a+b\sec(c+dx)}},x\right)$$

output
```
Unintegrable(sec(d*x+c)^(4/3)/(a+b*sec(d*x+c))^(1/2),x)
```

3.749.2 Mathematica [N/A]

Not integrable

Time = 17.64 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{\sec^{\frac{4}{3}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\sec^{\frac{4}{3}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$$

input
```
Integrate[Sec[c + d*x]^(4/3)/Sqrt[a + b*Sec[c + d*x]],x]
```

output
```
Integrate[Sec[c + d*x]^(4/3)/Sqrt[a + b*Sec[c + d*x]], x]
```

3.749.3 Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 4357}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{\frac{4}{3}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{4/3}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{ 4357}$$

$$\int \frac{\sec^{\frac{4}{3}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx$$

input
```
Int[Sec[c + d*x]^(4/3)/Sqrt[a + b*Sec[c + d*x]],x]
```

output
```
$Aborted
```

3.749.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4357
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Unintegrable[(d*Csc[e + f*x])^n*(a + b*Csc[e + f*
x])^m, x] /; FreeQ[{a, b, d, e, f, m, n}, x]
```

3.749.4 Maple [N/A] (verified)

Not integrable

Time = 0.48 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{\sec (dx+c)^{\frac{4}{3}}}{\sqrt{a+b\sec (dx+c)}}dx$$

input
```
int(sec(d*x+c)^(4/3)/(a+b*sec(d*x+c))^(1/2),x)
```

output
```
int(sec(d*x+c)^(4/3)/(a+b*sec(d*x+c))^(1/2),x)
```

3.749.5 Fricas [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{\sec^{\frac{4}{3}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx=\int \frac{\sec (dx+c)^{\frac{4}{3}}}{\sqrt{b\sec (dx+c)+a}}\,dx$$

input
```
integrate(sec(d*x+c)^(4/3)/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
integral(sec(d*x + c)^(4/3)/sqrt(b*sec(d*x + c) + a), x)
```

3.749.6 Sympy [N/A]

Not integrable

Time = 123.11 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int \frac{\sec^{\frac{4}{3}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx=\int \frac{\sec^{\frac{4}{3}}(c+dx)}{\sqrt{a+b\sec (c+dx)}}\,dx$$

input
```
integrate(sec(d*x+c)**(4/3)/(a+b*sec(d*x+c))**(1/2),x)
```

output
```
Integral(sec(c + d*x)**(4/3)/sqrt(a + b*sec(c + d*x)), x)
```

3.749.7 Maxima [N/A]

Not integrable

Time = 0.80 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{\sec^{\frac{4}{3}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\sec(dx+c)^{\frac{4}{3}}}{\sqrt{b\sec(dx+c)+a}}\,dx$$

input
```
integrate(sec(d*x+c)^(4/3)/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)^(4/3)/sqrt(b*sec(d*x + c) + a), x)
```

3.749.8 Giac [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{4}{3}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)^(4/3)/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
Timed out
```

3.749.9 Mupad [N/A]

Not integrable

Time = 15.71 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{\sec^{\frac{4}{3}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{4/3}}{\sqrt{a+\frac{b}{\cos(c+d\,x)}}}\,dx$$

input
```
int((1/cos(c + d*x))^(4/3)/(a + b/cos(c + d*x))^(1/2),x)
```

output
```
int((1/cos(c + d*x))^(4/3)/(a + b/cos(c + d*x))^(1/2), x)
```

3.750 $\int \frac{\sec^{\frac{2}{3}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$

3.750.1 Optimal result 5671
3.750.2 Mathematica [N/A] 5671
3.750.3 Rubi [N/A] 5672
3.750.4 Maple [N/A] (verified) 5673
3.750.5 Fricas [N/A] 5673
3.750.6 Sympy [N/A] 5673
3.750.7 Maxima [N/A] 5674
3.750.8 Giac [F(-1)] 5674
3.750.9 Mupad [N/A] 5674

3.750.1 Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{\sec^{\frac{2}{3}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \text{Int}\left(\frac{\sec^{\frac{2}{3}}(c+dx)}{\sqrt{a+b\sec(c+dx)}},x\right)$$

output
```
Unintegrable(sec(d*x+c)^(2/3)/(a+b*sec(d*x+c))^(1/2),x)
```

3.750.2 Mathematica [N/A]

Not integrable

Time = 17.87 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{\sec^{\frac{2}{3}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\sec^{\frac{2}{3}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$$

input
```
Integrate[Sec[c + d*x]^(2/3)/Sqrt[a + b*Sec[c + d*x]],x]
```

output
```
Integrate[Sec[c + d*x]^(2/3)/Sqrt[a + b*Sec[c + d*x]], x]
```

3.750.3 Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 4357}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{\frac{2}{3}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{2/3}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{ 4357}$$

$$\int \frac{\sec^{\frac{2}{3}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx$$

input
```
Int[Sec[c + d*x]^(2/3)/Sqrt[a + b*Sec[c + d*x]],x]
```

output
```
$Aborted
```

3.750.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4357
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Unintegrable[(d*Csc[e + f*x])^n*(a + b*Csc[e + f*
x])^m, x] /; FreeQ[{a, b, d, e, f, m, n}, x]
```

3.750.4 Maple [N/A] (verified)

Not integrable

Time = 0.50 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{\sec (dx+c)^{\frac{2}{3}}}{\sqrt{a+b\sec (dx+c)}}dx$$

input
```
int(sec(d*x+c)^(2/3)/(a+b*sec(d*x+c))^(1/2),x)
```

output
```
int(sec(d*x+c)^(2/3)/(a+b*sec(d*x+c))^(1/2),x)
```

3.750.5 Fricas [N/A]

Not integrable

Time = 1.21 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{\sec^{\frac{2}{3}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx=\int \frac{\sec (dx+c)^{\frac{2}{3}}}{\sqrt{b\sec (dx+c)+a}}\,dx$$

input
```
integrate(sec(d*x+c)^(2/3)/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
integral(sec(d*x + c)^(2/3)/sqrt(b*sec(d*x + c) + a), x)
```

3.750.6 Sympy [N/A]

Not integrable

Time = 2.96 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int \frac{\sec^{\frac{2}{3}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx=\int \frac{\sec^{\frac{2}{3}}(c+dx)}{\sqrt{a+b\sec (c+dx)}}\,dx$$

input
```
integrate(sec(d*x+c)**(2/3)/(a+b*sec(d*x+c))**(1/2),x)
```

output
```
Integral(sec(c + d*x)**(2/3)/sqrt(a + b*sec(c + d*x)), x)
```

3.750.7 Maxima [N/A]

Not integrable

Time = 0.75 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{\sec^{\frac{2}{3}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\sec(dx+c)^{\frac{2}{3}}}{\sqrt{b\sec(dx+c)+a}}\,dx$$

input
```
integrate(sec(d*x+c)^(2/3)/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)^(2/3)/sqrt(b*sec(d*x + c) + a), x)
```

3.750.8 Giac [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{2}{3}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)^(2/3)/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
Timed out
```

3.750.9 Mupad [N/A]

Not integrable

Time = 15.61 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{\sec^{\frac{2}{3}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{2/3}}{\sqrt{a+\frac{b}{\cos(c+d\,x)}}}\,dx$$

input
```
int((1/cos(c + d*x))^(2/3)/(a + b/cos(c + d*x))^(1/2),x)
```

output
```
int((1/cos(c + d*x))^(2/3)/(a + b/cos(c + d*x))^(1/2), x)
```

3.751 $\int \frac{\sqrt[3]{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}\,dx$

3.751.1 Optimal result 5675
3.751.2 Mathematica [N/A] 5675
3.751.3 Rubi [N/A] 5676
3.751.4 Maple [N/A] (verified) 5677
3.751.5 Fricas [N/A] 5677
3.751.6 Sympy [N/A] 5677
3.751.7 Maxima [N/A] 5678
3.751.8 Giac [F(-1)] 5678
3.751.9 Mupad [N/A] 5678

3.751.1 Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{\sqrt[3]{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}\,dx = \text{Int}\left(\frac{\sqrt[3]{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}},x\right)$$

output
```
Unintegrable(sec(d*x+c)^(1/3)/(a+b*sec(d*x+c))^(1/2),x)
```

3.751.2 Mathematica [N/A]

Not integrable

Time = 2.33 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{\sqrt[3]{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\sqrt[3]{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}\,dx$$

input
```
Integrate[Sec[c + d*x]^(1/3)/Sqrt[a + b*Sec[c + d*x]],x]
```

output
```
Integrate[Sec[c + d*x]^(1/3)/Sqrt[a + b*Sec[c + d*x]], x]
```

3.751.3 Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 4357}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt[3]{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4357}$$

$$\int \frac{\sqrt[3]{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}dx$$

input
```
Int[Sec[c + d*x]^(1/3)/Sqrt[a + b*Sec[c + d*x]],x]
```

output
```
$Aborted
```

3.751.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4357
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Unintegrable[(d*Csc[e + f*x])^n*(a + b*Csc[e + f*
x])^m, x] /; FreeQ[{a, b, d, e, f, m, n}, x]
```

3.751.4 Maple [N/A] (verified)

Not integrable

Time = 0.41 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{\sec (dx+c)^{\frac{1}{3}}}{\sqrt{a+b\sec (dx+c)}}dx$$

input
```
int(sec(d*x+c)^(1/3)/(a+b*sec(d*x+c))^(1/2),x)
```

output
```
int(sec(d*x+c)^(1/3)/(a+b*sec(d*x+c))^(1/2),x)
```

3.751.5 Fricas [N/A]

Not integrable

Time = 0.47 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt[3]{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}\,dx=\int \frac{\sec (dx+c)^{\frac{1}{3}}}{\sqrt{b\sec (dx+c)+a}}\,dx$$

input
```
integrate(sec(d*x+c)^(1/3)/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
integral(sec(d*x + c)^(1/3)/sqrt(b*sec(d*x + c) + a), x)
```

3.751.6 Sympy [N/A]

Not integrable

Time = 1.14 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int \frac{\sqrt[3]{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}\,dx=\int \frac{\sqrt[3]{\sec (c+dx)}}{\sqrt{a+b\sec (c+dx)}}\,dx$$

input
```
integrate(sec(d*x+c)**(1/3)/(a+b*sec(d*x+c))**(1/2),x)
```

output
```
Integral(sec(c + d*x)**(1/3)/sqrt(a + b*sec(c + d*x)), x)
```

3.751.7 Maxima [N/A]

Not integrable

Time = 0.74 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt[3]{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\sec(dx+c)^{\frac{1}{3}}}{\sqrt{b\sec(dx+c)+a}}\,dx$$

input
```
integrate(sec(d*x+c)^(1/3)/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)^(1/3)/sqrt(b*sec(d*x + c) + a), x)
```

3.751.8 Giac [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)^(1/3)/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
Timed out
```

3.751.9 Mupad [N/A]

Not integrable

Time = 15.28 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{\sqrt[3]{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{1/3}}{\sqrt{a+\frac{b}{\cos(c+d\,x)}}}\,dx$$

input
```
int((1/cos(c + d*x))^(1/3)/(a + b/cos(c + d*x))^(1/2),x)
```

output
```
int((1/cos(c + d*x))^(1/3)/(a + b/cos(c + d*x))^(1/2), x)
```

3.752 $\int \frac{1}{\sqrt[3]{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}\,dx$

3.752.1 Optimal result 5679
3.752.2 Mathematica [N/A] 5679
3.752.3 Rubi [N/A] 5680
3.752.4 Maple [N/A] (verified) 5681
3.752.5 Fricas [N/A] 5681
3.752.6 Sympy [N/A] 5681
3.752.7 Maxima [N/A] 5682
3.752.8 Giac [F(-1)] 5682
3.752.9 Mupad [N/A] 5682

3.752.1 Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{1}{\sqrt[3]{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}\,dx = \text{Int}\left(\frac{1}{\sqrt[3]{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}},x\right)$$

output
```
Unintegrable(1/sec(d*x+c)^(1/3)/(a+b*sec(d*x+c))^(1/2),x)
```

3.752.2 Mathematica [N/A]

Not integrable

Time = 3.58 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{1}{\sqrt[3]{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt[3]{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}\,dx$$

input
```
Integrate[1/(Sec[c + d*x]^(1/3)*Sqrt[a + b*Sec[c + d*x]]),x]
```

output
```
Integrate[1/(Sec[c + d*x]^(1/3)*Sqrt[a + b*Sec[c + d*x]]), x]
```

3.752.3 Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 4357}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[3]{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt[3]{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4357}$$

$$\int \frac{1}{\sqrt[3]{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx$$

input
```
Int[1/(Sec[c + d*x]^(1/3)*Sqrt[a + b*Sec[c + d*x]]),x]
```

output
```
$Aborted
```

3.752.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4357
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Unintegrable[(d*Csc[e + f*x])^n*(a + b*Csc[e + f*
x])^m, x] /; FreeQ[{a, b, d, e, f, m, n}, x]
```

3.752.4 Maple [N/A] (verified)

Not integrable

Time = 0.47 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{1}{\sec(dx+c)^{\frac{1}{3}}\sqrt{a+b\sec(dx+c)}}dx$$

input
```
int(1/sec(d*x+c)^(1/3)/(a+b*sec(d*x+c))^(1/2),x)
```

output
```
int(1/sec(d*x+c)^(1/3)/(a+b*sec(d*x+c))^(1/2),x)
```

3.752.5 Fricas [N/A]

Not integrable

Time = 1.10 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.76

$$\int \frac{1}{\sqrt[3]{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{b\sec(dx+c)+a}\sec(dx+c)^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/sec(d*x+c)^(1/3)/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(2/3)/(b*sec(d*x + c)^2 + a
*sec(d*x + c)), x)
```

3.752.6 Sympy [N/A]

Not integrable

Time = 0.72 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.04

$$\int \frac{1}{\sqrt[3]{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{a+b\sec(c+dx)}\sqrt[3]{\sec(c+dx)}}\,dx$$

input
```
integrate(1/sec(d*x+c)**(1/3)/(a+b*sec(d*x+c))**(1/2),x)
```

output
```
Integral(1/(sqrt(a + b*sec(c + d*x))*sec(c + d*x)**(1/3)), x)
```

3.752.7 Maxima [N/A]

Not integrable

Time = 0.76 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{1}{\sqrt[3]{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{b\sec(dx+c)+a}\sec(dx+c)^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/sec(d*x+c)^(1/3)/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(1/3)), x)
```

3.752.8 Giac [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate(1/sec(d*x+c)^(1/3)/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
Timed out
```

3.752.9 Mupad [N/A]

Not integrable

Time = 16.20 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{1}{\sqrt[3]{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{a+\frac{b}{\cos(c+d\,x)}}\left(\frac{1}{\cos(c+d\,x)}\right)^{1/3}}\,dx$$

input
```
int(1/((a + b/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(1/3)),x)
```

output
```
int(1/((a + b/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(1/3)), x)
```

3.753 $\int \frac{1}{\sec^{\frac{2}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx$

3.753.1 Optimal result 5683
3.753.2 Mathematica [N/A] 5683
3.753.3 Rubi [N/A] 5684
3.753.4 Maple [N/A] (verified) 5685
3.753.5 Fricas [N/A] 5685
3.753.6 Sympy [N/A] 5685
3.753.7 Maxima [N/A] 5686
3.753.8 Giac [F(-1)] 5686
3.753.9 Mupad [N/A] 5686

3.753.1 Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{1}{\sec^{\frac{2}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx = \text{Int}\left(\frac{1}{\sec^{\frac{2}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}},x\right)$$

output
```
Unintegrable(1/sec(d*x+c)^(2/3)/(a+b*sec(d*x+c))^(1/2),x)
```

3.753.2 Mathematica [N/A]

Not integrable

Time = 103.36 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{1}{\sec^{\frac{2}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{1}{\sec^{\frac{2}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx$$

input
```
Integrate[1/(Sec[c + d*x]^(2/3)*Sqrt[a + b*Sec[c + d*x]]),x]
```

output
```
Integrate[1/(Sec[c + d*x]^(2/3)*Sqrt[a + b*Sec[c + d*x]]), x]
```

3.753.3 Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 4357}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sec^{\frac{2}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{2/3}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4357}$$

$$\int \frac{1}{\sec^{\frac{2}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}}dx$$

input
```
Int[1/(Sec[c + d*x]^(2/3)*Sqrt[a + b*Sec[c + d*x]]),x]
```

output
```
$Aborted
```

3.753.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4357
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Unintegrable[(d*Csc[e + f*x])^n*(a + b*Csc[e + f*
x])^m, x] /; FreeQ[{a, b, d, e, f, m, n}, x]
```

3.753.4 Maple [N/A] (verified)

Not integrable

Time = 0.45 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{1}{\sec(dx+c)^{\frac{2}{3}}\sqrt{a+b\sec(dx+c)}}dx$$

input
```
int(1/sec(d*x+c)^(2/3)/(a+b*sec(d*x+c))^(1/2),x)
```

output
```
int(1/sec(d*x+c)^(2/3)/(a+b*sec(d*x+c))^(1/2),x)
```

3.753.5 Fricas [N/A]

Not integrable

Time = 0.69 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.76

$$\int \frac{1}{\sec^{\frac{2}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{b\sec(dx+c)+a}\sec(dx+c)^{\frac{2}{3}}}\,dx$$

input
```
integrate(1/sec(d*x+c)^(2/3)/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(1/3)/(b*sec(d*x + c)^2 + a
*sec(d*x + c)), x)
```

3.753.6 Sympy [N/A]

Not integrable

Time = 1.54 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.04

$$\int \frac{1}{\sec^{\frac{2}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{a+b\sec(c+dx)}\sec^{\frac{2}{3}}(c+dx)}\,dx$$

input
```
integrate(1/sec(d*x+c)**(2/3)/(a+b*sec(d*x+c))**(1/2),x)
```

output
```
Integral(1/(sqrt(a + b*sec(c + d*x))*sec(c + d*x)**(2/3)), x)
```

3.753.7 Maxima [N/A]

Not integrable

Time = 0.76 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{1}{\sec^{\frac{2}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{b\sec(dx+c)+a}\sec(dx+c)^{\frac{2}{3}}}\,dx$$

input
```
integrate(1/sec(d*x+c)^(2/3)/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(2/3)), x)
```

3.753.8 Giac [F(-1)]

Timed out.

$$\int \frac{1}{\sec^{\frac{2}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate(1/sec(d*x+c)^(2/3)/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
Timed out
```

3.753.9 Mupad [N/A]

Not integrable

Time = 14.11 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{1}{\sec^{\frac{2}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{a+\frac{b}{\cos(c+d\,x)}}\left(\frac{1}{\cos(c+d\,x)}\right)^{2/3}}\,dx$$

input
```
int(1/((a + b/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(2/3)),x)
```

output
```
int(1/((a + b/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(2/3)), x)
```

3.754 $\int \frac{1}{\sec^{\frac{4}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx$

3.754.1 Optimal result 5687
3.754.2 Mathematica [N/A] 5687
3.754.3 Rubi [N/A] 5688
3.754.4 Maple [N/A] (verified) 5689
3.754.5 Fricas [N/A] 5689
3.754.6 Sympy [N/A] 5689
3.754.7 Maxima [N/A] 5690
3.754.8 Giac [F(-1)] 5690
3.754.9 Mupad [N/A] 5690

3.754.1 Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{1}{\sec^{\frac{4}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx = \text{Int}\left(\frac{1}{\sec^{\frac{4}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}},x\right)$$

output
```
Unintegrable(1/sec(d*x+c)^(4/3)/(a+b*sec(d*x+c))^(1/2),x)
```

3.754.2 Mathematica [N/A]

Not integrable

Time = 107.78 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{1}{\sec^{\frac{4}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{1}{\sec^{\frac{4}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx$$

input
```
Integrate[1/(Sec[c + d*x]^(4/3)*Sqrt[a + b*Sec[c + d*x]]),x]
```

output
```
Integrate[1/(Sec[c + d*x]^(4/3)*Sqrt[a + b*Sec[c + d*x]]), x]
```

3.754.3 Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 4357}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sec^{\frac{4}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{4/3}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4357}$$

$$\int \frac{1}{\sec^{\frac{4}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}}dx$$

input
```
Int[1/(Sec[c + d*x]^(4/3)*Sqrt[a + b*Sec[c + d*x]]),x]
```

output
```
$Aborted
```

3.754.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4357
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Unintegrable[(d*Csc[e + f*x])^n*(a + b*Csc[e + f*
x])^m, x] /; FreeQ[{a, b, d, e, f, m, n}, x]
```

3.754.4 Maple [N/A] (verified)

Not integrable

Time = 0.47 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{1}{\sec(dx+c)^{\frac{4}{3}}\sqrt{a+b\sec(dx+c)}}dx$$

input
```
int(1/sec(d*x+c)^(4/3)/(a+b*sec(d*x+c))^(1/2),x)
```

output
```
int(1/sec(d*x+c)^(4/3)/(a+b*sec(d*x+c))^(1/2),x)
```

3.754.5 Fricas [N/A]

Not integrable

Time = 1.05 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.84

$$\int \frac{1}{\sec^{\frac{4}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{b\sec(dx+c)+a}\sec(dx+c)^{\frac{4}{3}}}\,dx$$

input
```
integrate(1/sec(d*x+c)^(4/3)/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(2/3)/(b*sec(d*x + c)^3 + a
*sec(d*x + c)^2), x)
```

3.754.6 Sympy [N/A]

Not integrable

Time = 12.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.04

$$\int \frac{1}{\sec^{\frac{4}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{a+b\sec(c+dx)}\sec^{\frac{4}{3}}(c+dx)}\,dx$$

input
```
integrate(1/sec(d*x+c)**(4/3)/(a+b*sec(d*x+c))**(1/2),x)
```

output
```
Integral(1/(sqrt(a + b*sec(c + d*x))*sec(c + d*x)**(4/3)), x)
```

3.754.7 Maxima [N/A]

Not integrable

Time = 0.81 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{1}{\sec^{\frac{4}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{b\sec(dx+c)+a}\sec(dx+c)^{\frac{4}{3}}}\,dx$$

input
```
integrate(1/sec(d*x+c)^(4/3)/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(4/3)), x)
```

3.754.8 Giac [F(-1)]

Timed out.

$$\int \frac{1}{\sec^{\frac{4}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate(1/sec(d*x+c)^(4/3)/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
Timed out
```

3.754.9 Mupad [N/A]

Not integrable

Time = 17.42 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{1}{\sec^{\frac{4}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{a+\frac{b}{\cos(c+d\,x)}}\left(\frac{1}{\cos(c+d\,x)}\right)^{4/3}}\,dx$$

input
```
int(1/((a + b/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(4/3)),x)
```

output
```
int(1/((a + b/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(4/3)), x)
```

3.755 $\int \frac{1}{\sec^{\frac{5}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx$

3.755.1 Optimal result 5691
3.755.2 Mathematica [N/A] 5691
3.755.3 Rubi [N/A] 5692
3.755.4 Maple [N/A] (verified) 5693
3.755.5 Fricas [N/A] 5693
3.755.6 Sympy [N/A] 5693
3.755.7 Maxima [N/A] 5694
3.755.8 Giac [F(-1)] 5694
3.755.9 Mupad [N/A] 5694

3.755.1 Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{1}{\sec^{\frac{5}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx = \text{Int}\left(\frac{1}{\sec^{\frac{5}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}},x\right)$$

output
```
Unintegrable(1/sec(d*x+c)^(5/3)/(a+b*sec(d*x+c))^(1/2),x)
```

3.755.2 Mathematica [N/A]

Not integrable

Time = 122.16 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{1}{\sec^{\frac{5}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{1}{\sec^{\frac{5}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx$$

input
```
Integrate[1/(Sec[c + d*x]^(5/3)*Sqrt[a + b*Sec[c + d*x]]),x]
```

output
```
Integrate[1/(Sec[c + d*x]^(5/3)*Sqrt[a + b*Sec[c + d*x]]), x]
```

3.755.3 Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 4357}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sec^{\frac{5}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/3}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4357}$$

$$\int \frac{1}{\sec^{\frac{5}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}}dx$$

input
```
Int[1/(Sec[c + d*x]^(5/3)*Sqrt[a + b*Sec[c + d*x]]),x]
```

output
```
$Aborted
```

3.755.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4357
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Unintegrable[(d*Csc[e + f*x])^n*(a + b*Csc[e + f*
x])^m, x] /; FreeQ[{a, b, d, e, f, m, n}, x]
```

3.755.4 Maple [N/A] (verified)

Not integrable

Time = 0.56 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{1}{\sec(dx+c)^{\frac{5}{3}}\sqrt{a+b\sec(dx+c)}}dx$$

input
```
int(1/sec(d*x+c)^(5/3)/(a+b*sec(d*x+c))^(1/2),x)
```

output
```
int(1/sec(d*x+c)^(5/3)/(a+b*sec(d*x+c))^(1/2),x)
```

3.755.5 Fricas [N/A]

Not integrable

Time = 0.77 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.84

$$\int \frac{1}{\sec^{\frac{5}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{b\sec(dx+c)+a}\sec(dx+c)^{\frac{5}{3}}}\,dx$$

input
```
integrate(1/sec(d*x+c)^(5/3)/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(1/3)/(b*sec(d*x + c)^3 + a
*sec(d*x + c)^2), x)
```

3.755.6 Sympy [N/A]

Not integrable

Time = 28.32 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.04

$$\int \frac{1}{\sec^{\frac{5}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{a+b\sec(c+dx)}\sec^{\frac{5}{3}}(c+dx)}\,dx$$

input
```
integrate(1/sec(d*x+c)**(5/3)/(a+b*sec(d*x+c))**(1/2),x)
```

output
```
Integral(1/(sqrt(a + b*sec(c + d*x))*sec(c + d*x)**(5/3)), x)
```

3.755.7 Maxima [N/A]

Not integrable

Time = 1.30 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{1}{\sec^{\frac{5}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{b\sec(dx+c)+a}\sec(dx+c)^{\frac{5}{3}}}\,dx$$

input
```
integrate(1/sec(d*x+c)^(5/3)/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(5/3)), x)
```

3.755.8 Giac [F(-1)]

Timed out.

$$\int \frac{1}{\sec^{\frac{5}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate(1/sec(d*x+c)^(5/3)/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
Timed out
```

3.755.9 Mupad [N/A]

Not integrable

Time = 14.41 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{1}{\sec^{\frac{5}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{a+\frac{b}{\cos(c+d\,x)}}\left(\frac{1}{\cos(c+d\,x)}\right)^{5/3}}\,dx$$

input
```
int(1/((a + b/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(5/3)),x)
```

output
```
int(1/((a + b/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(5/3)), x)
```

3.756 $\int \frac{1}{\sec^{\frac{7}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx$

3.756.1 Optimal result 5695
3.756.2 Mathematica [N/A] 5695
3.756.3 Rubi [N/A] 5696
3.756.4 Maple [N/A] (verified) 5697
3.756.5 Fricas [N/A] 5697
3.756.6 Sympy [F(-1)] 5697
3.756.7 Maxima [N/A] 5698
3.756.8 Giac [F(-1)] 5698
3.756.9 Mupad [N/A] 5698

3.756.1 Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{1}{\sec^{\frac{7}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx = \text{Int}\left(\frac{1}{\sec^{\frac{7}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}},x\right)$$

output
```
Unintegrable(1/sec(d*x+c)^(7/3)/(a+b*sec(d*x+c))^(1/2),x)
```

3.756.2 Mathematica [N/A]

Not integrable

Time = 138.69 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{1}{\sec^{\frac{7}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{1}{\sec^{\frac{7}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx$$

input
```
Integrate[1/(Sec[c + d*x]^(7/3)*Sqrt[a + b*Sec[c + d*x]]),x]
```

output
```
Integrate[1/(Sec[c + d*x]^(7/3)*Sqrt[a + b*Sec[c + d*x]]), x]
```

3.756.3 Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 4357}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sec^{\frac{7}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/3}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4357}$$

$$\int \frac{1}{\sec^{\frac{7}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}}dx$$

input
```
Int[1/(Sec[c + d*x]^(7/3)*Sqrt[a + b*Sec[c + d*x]]),x]
```

output
```
$Aborted
```

3.756.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4357
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Unintegrable[(d*Csc[e + f*x])^n*(a + b*Csc[e + f*
x])^m, x] /; FreeQ[{a, b, d, e, f, m, n}, x]
```

3.756.4 Maple [N/A] (verified)

Not integrable

Time = 0.47 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{1}{\sec(dx+c)^{\frac{7}{3}}\sqrt{a+b\sec(dx+c)}}dx$$

input
```
int(1/sec(d*x+c)^(7/3)/(a+b*sec(d*x+c))^(1/2),x)
```

output
```
int(1/sec(d*x+c)^(7/3)/(a+b*sec(d*x+c))^(1/2),x)
```

3.756.5 Fricas [N/A]

Not integrable

Time = 1.12 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.84

$$\int \frac{1}{\sec^{\frac{7}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{b\sec(dx+c)+a}\sec(dx+c)^{\frac{7}{3}}}\,dx$$

input
```
integrate(1/sec(d*x+c)^(7/3)/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(2/3)/(b*sec(d*x + c)^4 + a
*sec(d*x + c)^3), x)
```

3.756.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\sec^{\frac{7}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate(1/sec(d*x+c)**(7/3)/(a+b*sec(d*x+c))**(1/2),x)
```

output
```
Timed out
```

3.756.7 Maxima [N/A]

Not integrable

Time = 1.31 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{1}{\sec^{\frac{7}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{b\sec(dx+c)+a}\sec(dx+c)^{\frac{7}{3}}}\,dx$$

input
```
integrate(1/sec(d*x+c)^(7/3)/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(7/3)), x)
```

3.756.8 Giac [F(-1)]

Timed out.

$$\int \frac{1}{\sec^{\frac{7}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate(1/sec(d*x+c)^(7/3)/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
Timed out
```

3.756.9 Mupad [N/A]

Not integrable

Time = 17.53 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{1}{\sec^{\frac{7}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{a+\frac{b}{\cos(c+d\,x)}}\left(\frac{1}{\cos(c+d\,x)}\right)^{7/3}}\,dx$$

input
```
int(1/((a + b/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(7/3)),x)
```

output
```
int(1/((a + b/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(7/3)), x)
```

3.757 $\int \frac{\sec^{\frac{7}{3}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$

3.757.1 Optimal result 5699
3.757.2 Mathematica [N/A] 5699
3.757.3 Rubi [N/A] 5700
3.757.4 Maple [N/A] (verified) 5701
3.757.5 Fricas [N/A] 5701
3.757.6 Sympy [F(-1)] 5701
3.757.7 Maxima [N/A] 5702
3.757.8 Giac [F(-1)] 5702
3.757.9 Mupad [N/A] 5702

3.757.1 Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{\sec^{\frac{7}{3}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \text{Int}\left(\frac{\sec^{\frac{7}{3}}(c+dx)}{(a+b\sec(c+dx))^{3/2}},x\right)$$

output
```
Unintegrable(sec(d*x+c)^(7/3)/(a+b*sec(d*x+c))^(3/2),x)
```

3.757.2 Mathematica [N/A]

Not integrable

Time = 135.06 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{\sec^{\frac{7}{3}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\sec^{\frac{7}{3}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$$

input
```
Integrate[Sec[c + d*x]^(7/3)/(a + b*Sec[c + d*x])^(3/2),x]
```

output
```
Integrate[Sec[c + d*x]^(7/3)/(a + b*Sec[c + d*x])^(3/2), x]
```

3.757.3 Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 4357}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{\frac{7}{3}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/3}}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 4357}$$

$$\int \frac{\sec^{\frac{7}{3}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}dx$$

input
```
Int[Sec[c + d*x]^(7/3)/(a + b*Sec[c + d*x])^(3/2),x]
```

output
```
$Aborted
```

3.757.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4357
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Unintegrable[(d*Csc[e + f*x])^n*(a + b*Csc[e + f*
x])^m, x] /; FreeQ[{a, b, d, e, f, m, n}, x]
```

3.757.4 Maple [N/A] (verified)

Not integrable

Time = 0.48 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{\sec (dx+c)^{\frac{7}{3}}}{(a+b\sec (dx+c))^{\frac{3}{2}}}dx$$

input
```
int(sec(d*x+c)^(7/3)/(a+b*sec(d*x+c))^(3/2),x)
```

output
```
int(sec(d*x+c)^(7/3)/(a+b*sec(d*x+c))^(3/2),x)
```

3.757.5 Fricas [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.04

$$\int \frac{\sec^{\frac{7}{3}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\sec (dx+c)^{\frac{7}{3}}}{(b\sec (dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^(7/3)/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(7/3)/(b^2*sec(d*x + c)^2 +
 2*a*b*sec(d*x + c) + a^2), x)
```

3.757.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{7}{3}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**(7/3)/(a+b*sec(d*x+c))**(3/2),x)
```

output
```
Timed out
```

3.757.7 Maxima [N/A]

Not integrable

Time = 0.76 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{\sec^{\frac{7}{3}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\sec(dx+c)^{\frac{7}{3}}}{(b\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^(7/3)/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)^(7/3)/(b*sec(d*x + c) + a)^(3/2), x)
```

3.757.8 Giac [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{7}{3}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)^(7/3)/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
Timed out
```

3.757.9 Mupad [N/A]

Not integrable

Time = 16.59 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{\sec^{\frac{7}{3}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{7/3}}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

input
```
int((1/cos(c + d*x))^(7/3)/(a + b/cos(c + d*x))^(3/2),x)
```

output
```
int((1/cos(c + d*x))^(7/3)/(a + b/cos(c + d*x))^(3/2), x)
```

3.758 $\int \frac{\sec^{\frac{5}{3}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$

3.758.1 Optimal result . . . 5703
3.758.2 Mathematica [N/A] . . . 5703
3.758.3 Rubi [N/A] . . . 5704
3.758.4 Maple [N/A] (verified) . . . 5705
3.758.5 Fricas [N/A] . . . 5705
3.758.6 Sympy [F(-1)] . . . 5705
3.758.7 Maxima [N/A] . . . 5706
3.758.8 Giac [F(-1)] . . . 5706
3.758.9 Mupad [N/A] . . . 5706

3.758.1 Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{\sec^{\frac{5}{3}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \text{Int}\left(\frac{\sec^{\frac{5}{3}}(c+dx)}{(a+b\sec(c+dx))^{3/2}},x\right)$$

output
```
Unintegrable(sec(d*x+c)^(5/3)/(a+b*sec(d*x+c))^(3/2),x)
```

3.758.2 Mathematica [N/A]

Not integrable

Time = 143.55 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{\sec^{\frac{5}{3}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\sec^{\frac{5}{3}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$$

input
```
Integrate[Sec[c + d*x]^(5/3)/(a + b*Sec[c + d*x])^(3/2),x]
```

output
```
Integrate[Sec[c + d*x]^(5/3)/(a + b*Sec[c + d*x])^(3/2), x]
```

3.758.3 Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 4357}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{\frac{5}{3}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/3}}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \; 4357$$

$$\int \frac{\sec^{\frac{5}{3}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}dx$$

input
```
Int[Sec[c + d*x]^(5/3)/(a + b*Sec[c + d*x])^(3/2),x]
```

output
```
$Aborted
```

3.758.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4357
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Unintegrable[(d*Csc[e + f*x])^n*(a + b*Csc[e + f*
x])^m, x] /; FreeQ[{a, b, d, e, f, m, n}, x]
```

3.758.4 Maple [N/A] (verified)

Not integrable

Time = 0.41 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{\sec (dx+c)^{\frac{5}{3}}}{(a+b\sec (dx+c))^{\frac{3}{2}}}dx$$

input
```
int(sec(d*x+c)^(5/3)/(a+b*sec(d*x+c))^(3/2),x)
```

output
```
int(sec(d*x+c)^(5/3)/(a+b*sec(d*x+c))^(3/2),x)
```

3.758.5 Fricas [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.04

$$\int \frac{\sec^{\frac{5}{3}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\sec (dx+c)^{\frac{5}{3}}}{(b\sec (dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^(5/3)/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(5/3)/(b^2*sec(d*x + c)^2 +
 2*a*b*sec(d*x + c) + a^2), x)
```

3.758.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{5}{3}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**(5/3)/(a+b*sec(d*x+c))**(3/2),x)
```

output
```
Timed out
```

3.758.7 Maxima [N/A]

Not integrable

Time = 0.77 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{\sec^{\frac{5}{3}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\sec(dx+c)^{\frac{5}{3}}}{(b\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^(5/3)/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)^(5/3)/(b*sec(d*x + c) + a)^(3/2), x)
```

3.758.8 Giac [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{5}{3}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)^(5/3)/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
Timed out
```

3.758.9 Mupad [N/A]

Not integrable

Time = 16.53 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{\sec^{\frac{5}{3}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{5/3}}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

input
```
int((1/cos(c + d*x))^(5/3)/(a + b/cos(c + d*x))^(3/2),x)
```

output
```
int((1/cos(c + d*x))^(5/3)/(a + b/cos(c + d*x))^(3/2), x)
```

3.759 $\int \frac{\sec^{\frac{4}{3}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$

3.759.1 Optimal result 5707
3.759.2 Mathematica [N/A] 5707
3.759.3 Rubi [N/A] 5708
3.759.4 Maple [N/A] (verified) 5709
3.759.5 Fricas [N/A] 5709
3.759.6 Sympy [N/A] 5709
3.759.7 Maxima [N/A] 5710
3.759.8 Giac [F(-1)] 5710
3.759.9 Mupad [N/A] 5710

3.759.1 Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{\sec^{\frac{4}{3}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \text{Int}\left(\frac{\sec^{\frac{4}{3}}(c+dx)}{(a+b\sec(c+dx))^{3/2}},x\right)$$

output
```
Unintegrable(sec(d*x+c)^(4/3)/(a+b*sec(d*x+c))^(3/2),x)
```

3.759.2 Mathematica [N/A]

Not integrable

Time = 125.27 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{\sec^{\frac{4}{3}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\sec^{\frac{4}{3}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$$

input
```
Integrate[Sec[c + d*x]^(4/3)/(a + b*Sec[c + d*x])^(3/2),x]
```

output
```
Integrate[Sec[c + d*x]^(4/3)/(a + b*Sec[c + d*x])^(3/2), x]
```

3.759.3 Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 4357}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{\frac{4}{3}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{4/3}}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \; 4357$$

$$\int \frac{\sec^{\frac{4}{3}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}dx$$

input
```
Int[Sec[c + d*x]^(4/3)/(a + b*Sec[c + d*x])^(3/2),x]
```

output
```
$Aborted
```

3.759.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4357
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Unintegrable[(d*Csc[e + f*x])^n*(a + b*Csc[e + f*
x])^m, x] /; FreeQ[{a, b, d, e, f, m, n}, x]
```

3.759.4 Maple [N/A] (verified)

Not integrable

Time = 0.45 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{\sec (dx+c)^{\frac{4}{3}}}{(a+b\sec (dx+c))^{\frac{3}{2}}}dx$$

input
```
int(sec(d*x+c)^(4/3)/(a+b*sec(d*x+c))^(3/2),x)
```

output
```
int(sec(d*x+c)^(4/3)/(a+b*sec(d*x+c))^(3/2),x)
```

3.759.5 Fricas [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.04

$$\int \frac{\sec^{\frac{4}{3}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx=\int \frac{\sec (dx+c)^{\frac{4}{3}}}{(b\sec (dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^(4/3)/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(4/3)/(b^2*sec(d*x + c)^2 +
 2*a*b*sec(d*x + c) + a^2), x)
```

3.759.6 Sympy [N/A]

Not integrable

Time = 137.51 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int \frac{\sec^{\frac{4}{3}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx=\int \frac{\sec^{\frac{4}{3}}(c+dx)}{(a+b\sec (c+dx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)**(4/3)/(a+b*sec(d*x+c))**(3/2),x)
```

output
```
Integral(sec(c + d*x)**(4/3)/(a + b*sec(c + d*x))**(3/2), x)
```

3.759.7 Maxima [N/A]

Not integrable

Time = 0.77 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{\sec^{\frac{4}{3}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\sec(dx+c)^{\frac{4}{3}}}{(b\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^(4/3)/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)^(4/3)/(b*sec(d*x + c) + a)^(3/2), x)
```

3.759.8 Giac [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{4}{3}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)^(4/3)/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
Timed out
```

3.759.9 Mupad [N/A]

Not integrable

Time = 16.49 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{\sec^{\frac{4}{3}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{4/3}}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

input
```
int((1/cos(c + d*x))^(4/3)/(a + b/cos(c + d*x))^(3/2),x)
```

output
```
int((1/cos(c + d*x))^(4/3)/(a + b/cos(c + d*x))^(3/2), x)
```

3.760 $\int \frac{\sec^{\frac{2}{3}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$

3.760.1 Optimal result 5711
3.760.2 Mathematica [N/A] 5711
3.760.3 Rubi [N/A] 5712
3.760.4 Maple [N/A] (verified) 5713
3.760.5 Fricas [N/A] 5713
3.760.6 Sympy [N/A] 5713
3.760.7 Maxima [N/A] 5714
3.760.8 Giac [F(-1)] 5714
3.760.9 Mupad [N/A] 5714

3.760.1 Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{\sec^{\frac{2}{3}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \text{Int}\left(\frac{\sec^{\frac{2}{3}}(c+dx)}{(a+b\sec(c+dx))^{3/2}},x\right)$$

output
```
Unintegrable(sec(d*x+c)^(2/3)/(a+b*sec(d*x+c))^(3/2),x)
```

3.760.2 Mathematica [N/A]

Not integrable

Time = 140.73 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{\sec^{\frac{2}{3}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\sec^{\frac{2}{3}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$$

input
```
Integrate[Sec[c + d*x]^(2/3)/(a + b*Sec[c + d*x])^(3/2),x]
```

output
```
Integrate[Sec[c + d*x]^(2/3)/(a + b*Sec[c + d*x])^(3/2), x]
```

3.760.3 Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 4357}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{\frac{2}{3}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{2/3}}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 4357}$$

$$\int \frac{\sec^{\frac{2}{3}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}dx$$

input
```
Int[Sec[c + d*x]^(2/3)/(a + b*Sec[c + d*x])^(3/2),x]
```

output
```
$Aborted
```

3.760.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4357
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Unintegrable[(d*Csc[e + f*x])^n*(a + b*Csc[e + f*
x])^m, x] /; FreeQ[{a, b, d, e, f, m, n}, x]
```

3.760.4 Maple [N/A] (verified)

Not integrable

Time = 0.47 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{\sec (dx+c)^{\frac{2}{3}}}{(a+b\sec (dx+c))^{\frac{3}{2}}}dx$$

input
```
int(sec(d*x+c)^(2/3)/(a+b*sec(d*x+c))^(3/2),x)
```

output
```
int(sec(d*x+c)^(2/3)/(a+b*sec(d*x+c))^(3/2),x)
```

3.760.5 Fricas [N/A]

Not integrable

Time = 1.13 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.04

$$\int \frac{\sec^{\frac{2}{3}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx=\int \frac{\sec (dx+c)^{\frac{2}{3}}}{(b\sec (dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^(2/3)/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(2/3)/(b^2*sec(d*x + c)^2 +
 2*a*b*sec(d*x + c) + a^2), x)
```

3.760.6 Sympy [N/A]

Not integrable

Time = 10.72 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int \frac{\sec^{\frac{2}{3}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx=\int \frac{\sec^{\frac{2}{3}}(c+dx)}{(a+b\sec (c+dx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)**(2/3)/(a+b*sec(d*x+c))**(3/2),x)
```

output
```
Integral(sec(c + d*x)**(2/3)/(a + b*sec(c + d*x))**(3/2), x)
```

3.760.7 Maxima [N/A]

Not integrable

Time = 0.74 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{\sec^{\frac{2}{3}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\sec(dx+c)^{\frac{2}{3}}}{(b\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^(2/3)/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)^(2/3)/(b*sec(d*x + c) + a)^(3/2), x)
```

3.760.8 Giac [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{2}{3}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)^(2/3)/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
Timed out
```

3.760.9 Mupad [N/A]

Not integrable

Time = 16.62 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{\sec^{\frac{2}{3}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{2/3}}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

input
```
int((1/cos(c + d*x))^(2/3)/(a + b/cos(c + d*x))^(3/2),x)
```

output
```
int((1/cos(c + d*x))^(2/3)/(a + b/cos(c + d*x))^(3/2), x)
```

3.761 $\int \frac{\sqrt[3]{\sec(c+dx)}}{(a+b\sec(c+dx))^{3/2}}\,dx$

3.761.1 Optimal result 5715
3.761.2 Mathematica [N/A] 5715
3.761.3 Rubi [N/A] 5716
3.761.4 Maple [N/A] (verified) 5717
3.761.5 Fricas [N/A] 5717
3.761.6 Sympy [N/A] 5717
3.761.7 Maxima [N/A] 5718
3.761.8 Giac [F(-1)] 5718
3.761.9 Mupad [N/A] 5718

3.761.1 Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{\sqrt[3]{\sec(c+dx)}}{(a+b\sec(c+dx))^{3/2}}\,dx = \text{Int}\left(\frac{\sqrt[3]{\sec(c+dx)}}{(a+b\sec(c+dx))^{3/2}},x\right)$$

output
```
Unintegrable(sec(d*x+c)^(1/3)/(a+b*sec(d*x+c))^(3/2),x)
```

3.761.2 Mathematica [N/A]

Not integrable

Time = 120.85 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{\sqrt[3]{\sec(c+dx)}}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\sqrt[3]{\sec(c+dx)}}{(a+b\sec(c+dx))^{3/2}}\,dx$$

input
```
Integrate[Sec[c + d*x]^(1/3)/(a + b*Sec[c + d*x])^(3/2),x]
```

output
```
Integrate[Sec[c + d*x]^(1/3)/(a + b*Sec[c + d*x])^(3/2), x]
```

3.761.3 Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 4357}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{\sec(c+dx)}}{(a+b\sec(c+dx))^{3/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt[3]{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{4357}$$

$$\int \frac{\sqrt[3]{\sec(c+dx)}}{(a+b\sec(c+dx))^{3/2}}dx$$

input
```
Int[Sec[c + d*x]^(1/3)/(a + b*Sec[c + d*x])^(3/2),x]
```

output
```
$Aborted
```

3.761.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4357
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Unintegrable[(d*Csc[e + f*x])^n*(a + b*Csc[e + f*
x])^m, x] /; FreeQ[{a, b, d, e, f, m, n}, x]
```

3.761.4 Maple [N/A] (verified)

Not integrable

Time = 0.44 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{\sec (dx+c)^{\frac{1}{3}}}{(a+b\sec (dx+c))^{\frac{3}{2}}}dx$$

input
```
int(sec(d*x+c)^(1/3)/(a+b*sec(d*x+c))^(3/2),x)
```

output
```
int(sec(d*x+c)^(1/3)/(a+b*sec(d*x+c))^(3/2),x)
```

3.761.5 Fricas [N/A]

Not integrable

Time = 0.75 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.04

$$\int \frac{\sqrt[3]{\sec(c+dx)}}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\sec (dx+c)^{\frac{1}{3}}}{(b\sec (dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^(1/3)/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(1/3)/(b^2*sec(d*x + c)^2 +
 2*a*b*sec(d*x + c) + a^2), x)
```

3.761.6 Sympy [N/A]

Not integrable

Time = 3.92 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int \frac{\sqrt[3]{\sec(c+dx)}}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\sqrt[3]{\sec (c+dx)}}{(a+b\sec (c+dx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)**(1/3)/(a+b*sec(d*x+c))**(3/2),x)
```

output
```
Integral(sec(c + d*x)**(1/3)/(a + b*sec(c + d*x))**(3/2), x)
```

3.761.7 Maxima [N/A]

Not integrable

Time = 0.74 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt[3]{\sec(c+dx)}}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\sec\left(dx+c\right)^{\frac{1}{3}}}{\left(b\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^(1/3)/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)^(1/3)/(b*sec(d*x + c) + a)^(3/2), x)
```

3.761.8 Giac [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{\sec(c+dx)}}{(a+b\sec(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)^(1/3)/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
Timed out
```

3.761.9 Mupad [N/A]

Not integrable

Time = 16.80 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{\sqrt[3]{\sec(c+dx)}}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{1/3}}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

input
```
int((1/cos(c + d*x))^(1/3)/(a + b/cos(c + d*x))^(3/2),x)
```

output
```
int((1/cos(c + d*x))^(1/3)/(a + b/cos(c + d*x))^(3/2), x)
```

3.762 $\int \frac{1}{\sqrt[3]{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}}\,dx$

3.762.1 Optimal result 5719
3.762.2 Mathematica [N/A] 5719
3.762.3 Rubi [N/A] 5720
3.762.4 Maple [N/A] (verified) 5721
3.762.5 Fricas [N/A] 5721
3.762.6 Sympy [N/A] 5721
3.762.7 Maxima [N/A] 5722
3.762.8 Giac [F(-1)] 5722
3.762.9 Mupad [N/A] 5722

3.762.1 Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{1}{\sqrt[3]{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}}\,dx=\text{Int}\left(\frac{1}{\sqrt[3]{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}},x\right)$$

output
```
Unintegrable(1/sec(d*x+c)^(1/3)/(a+b*sec(d*x+c))^(3/2),x)
```

3.762.2 Mathematica [N/A]

Not integrable

Time = 126.56 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{1}{\sqrt[3]{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}}\,dx=\int \frac{1}{\sqrt[3]{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}}\,dx$$

input
```
Integrate[1/(Sec[c + d*x]^(1/3)*(a + b*Sec[c + d*x])^(3/2)),x]
```

output
```
Integrate[1/(Sec[c + d*x]^(1/3)*(a + b*Sec[c + d*x])^(3/2)), x]
```

3.762.3 Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 4357}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[3]{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt[3]{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{4357}$$

$$\int \frac{1}{\sqrt[3]{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}}dx$$

input
```
Int[1/(Sec[c + d*x]^(1/3)*(a + b*Sec[c + d*x])^(3/2)),x]
```

output
```
$Aborted
```

3.762.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4357
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Unintegrable[(d*Csc[e + f*x])^n*(a + b*Csc[e + f*
x])^m, x] /; FreeQ[{a, b, d, e, f, m, n}, x]
```

3.762.4 Maple [N/A] (verified)

Not integrable

Time = 0.39 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{1}{\sec\left(dx+c\right)^{\frac{1}{3}}\left(a+b\sec\left(dx+c\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
int(1/sec(d*x+c)^(1/3)/(a+b*sec(d*x+c))^(3/2),x)
```

output
```
int(1/sec(d*x+c)^(1/3)/(a+b*sec(d*x+c))^(3/2),x)
```

3.762.5 Fricas [N/A]

Not integrable

Time = 1.11 (sec) , antiderivative size = 60, normalized size of antiderivative = 2.40

$$\int \frac{1}{\sqrt[3]{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}}\,dx=\int \frac{1}{\left(b\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}\sec\left(dx+c\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/sec(d*x+c)^(1/3)/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(2/3)/(b^2*sec(d*x + c)^3 +
 2*a*b*sec(d*x + c)^2 + a^2*sec(d*x + c)), x)
```

3.762.6 Sympy [N/A]

Not integrable

Time = 5.04 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.04

$$\int \frac{1}{\sqrt[3]{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}}\,dx=\int \frac{1}{\left(a+b\sec\left(c+dx\right)\right)^{\frac{3}{2}}\sqrt[3]{\sec\left(c+dx\right)}}\,dx$$

input
```
integrate(1/sec(d*x+c)**(1/3)/(a+b*sec(d*x+c))**(3/2),x)
```

output
```
Integral(1/((a + b*sec(c + d*x))**(3/2)*sec(c + d*x)**(1/3)), x)
```

3.762.7 Maxima [N/A]

Not integrable

Time = 0.76 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{1}{\sqrt[3]{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{1}{(b\sec(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/sec(d*x+c)^(1/3)/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*sec(d*x + c) + a)^(3/2)*sec(d*x + c)^(1/3)), x)
```

3.762.8 Giac [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/sec(d*x+c)^(1/3)/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
Timed out
```

3.762.9 Mupad [N/A]

Not integrable

Time = 17.47 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{1}{\sqrt[3]{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{1}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}\left(\frac{1}{\cos(c+d\,x)}\right)^{1/3}}\,dx$$

input
```
int(1/((a + b/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(1/3)),x)
```

output
```
int(1/((a + b/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(1/3)), x)
```

3.763 $\int \frac{1}{\sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx$

3.763.1 Optimal result 5723
3.763.2 Mathematica [N/A] 5723
3.763.3 Rubi [N/A] 5724
3.763.4 Maple [N/A] (verified) 5725
3.763.5 Fricas [N/A] 5725
3.763.6 Sympy [N/A] 5725
3.763.7 Maxima [N/A] 5726
3.763.8 Giac [F(-1)] 5726
3.763.9 Mupad [N/A] 5726

3.763.1 Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{1}{\sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx=\text{Int}\left(\frac{1}{\sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}},x\right)$$

output
```
Unintegrable(1/sec(d*x+c)^(2/3)/(a+b*sec(d*x+c))^(3/2),x)
```

3.763.2 Mathematica [N/A]

Not integrable

Time = 128.18 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{1}{\sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx=\int \frac{1}{\sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx$$

input
```
Integrate[1/(Sec[c + d*x]^(2/3)*(a + b*Sec[c + d*x])^(3/2)),x]
```

output
```
Integrate[1/(Sec[c + d*x]^(2/3)*(a + b*Sec[c + d*x])^(3/2)), x]
```

3.763.3 Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 4357}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{2/3}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{4357}$$

$$\int \frac{1}{\sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}}dx$$

input
```
Int[1/(Sec[c + d*x]^(2/3)*(a + b*Sec[c + d*x])^(3/2)),x]
```

output
```
$Aborted
```

3.763.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4357
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Unintegrable[(d*Csc[e + f*x])^n*(a + b*Csc[e + f*
x])^m, x] /; FreeQ[{a, b, d, e, f, m, n}, x]
```

3.763.4 Maple [N/A] (verified)

Not integrable

Time = 0.48 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{1}{\sec\left(dx+c\right)^{\frac{2}{3}}\left(a+b\sec\left(dx+c\right)\right)^{\frac{3}{2}}}dx$$

input
```
int(1/sec(d*x+c)^(2/3)/(a+b*sec(d*x+c))^(3/2),x)
```

output
```
int(1/sec(d*x+c)^(2/3)/(a+b*sec(d*x+c))^(3/2),x)
```

3.763.5 Fricas [N/A]

Not integrable

Time = 0.88 (sec) , antiderivative size = 60, normalized size of antiderivative = 2.40

$$\int \frac{1}{\sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx=\int \frac{1}{\left(b\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}\sec\left(dx+c\right)^{\frac{2}{3}}}\,dx$$

input
```
integrate(1/sec(d*x+c)^(2/3)/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(1/3)/(b^2*sec(d*x + c)^3 +
 2*a*b*sec(d*x + c)^2 + a^2*sec(d*x + c)), x)
```

3.763.6 Sympy [N/A]

Not integrable

Time = 12.66 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.04

$$\int \frac{1}{\sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx=\int \frac{1}{\left(a+b\sec\left(c+dx\right)\right)^{\frac{3}{2}}\sec^{\frac{2}{3}}\left(c+dx\right)}\,dx$$

input
```
integrate(1/sec(d*x+c)**(2/3)/(a+b*sec(d*x+c))**(3/2),x)
```

output
```
Integral(1/((a + b*sec(c + d*x))**(3/2)*sec(c + d*x)**(2/3)), x)
```

3.763.7 Maxima [N/A]

Not integrable

Time = 1.28 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{1}{\sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{1}{(b\sec(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{2}{3}}}\,dx$$

input
```
integrate(1/sec(d*x+c)^(2/3)/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*sec(d*x + c) + a)^(3/2)*sec(d*x + c)^(2/3)), x)
```

3.763.8 Giac [F(-1)]

Timed out.

$$\int \frac{1}{\sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/sec(d*x+c)^(2/3)/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
Timed out
```

3.763.9 Mupad [N/A]

Not integrable

Time = 14.64 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{1}{\sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{1}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}\left(\frac{1}{\cos(c+d\,x)}\right)^{2/3}}\,dx$$

input
```
int(1/((a + b/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(2/3)),x)
```

output
```
int(1/((a + b/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(2/3)), x)
```

3.764 $\int \frac{1}{\sec^{\frac{4}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx$

3.764.1 Optimal result 5727
3.764.2 Mathematica [N/A] 5727
3.764.3 Rubi [N/A] 5728
3.764.4 Maple [N/A] (verified) 5729
3.764.5 Fricas [N/A] 5729
3.764.6 Sympy [N/A] 5729
3.764.7 Maxima [N/A] 5730
3.764.8 Giac [F(-1)] 5730
3.764.9 Mupad [N/A] 5730

3.764.1 Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{1}{\sec^{\frac{4}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx=\text{Int}\left(\frac{1}{\sec^{\frac{4}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}},x\right)$$

output
```
Unintegrable(1/sec(d*x+c)^(4/3)/(a+b*sec(d*x+c))^(3/2),x)
```

3.764.2 Mathematica [N/A]

Not integrable

Time = 142.50 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{1}{\sec^{\frac{4}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx=\int \frac{1}{\sec^{\frac{4}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx$$

input
```
Integrate[1/(Sec[c + d*x]^(4/3)*(a + b*Sec[c + d*x])^(3/2)),x]
```

output
```
Integrate[1/(Sec[c + d*x]^(4/3)*(a + b*Sec[c + d*x])^(3/2)), x]
```

3.764.3 Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 4357}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sec^{\frac{4}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{4/3}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{4357}$$

$$\int \frac{1}{\sec^{\frac{4}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}}dx$$

input
```
Int[1/(Sec[c + d*x]^(4/3)*(a + b*Sec[c + d*x])^(3/2)),x]
```

output
```
$Aborted
```

3.764.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4357
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Unintegrable[(d*Csc[e + f*x])^n*(a + b*Csc[e + f*
x])^m, x] /; FreeQ[{a, b, d, e, f, m, n}, x]
```

3.764.4 Maple [N/A] (verified)

Not integrable

Time = 0.46 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{1}{\sec(dx+c)^{\frac{4}{3}}(a+b\sec(dx+c))^{\frac{3}{2}}}\,dx$$

input
```
int(1/sec(d*x+c)^(4/3)/(a+b*sec(d*x+c))^(3/2),x)
```

output
```
int(1/sec(d*x+c)^(4/3)/(a+b*sec(d*x+c))^(3/2),x)
```

3.764.5 Fricas [N/A]

Not integrable

Time = 1.17 (sec) , antiderivative size = 62, normalized size of antiderivative = 2.48

$$\int \frac{1}{\sec^{\frac{4}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{1}{(b\sec(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{4}{3}}}\,dx$$

input
```
integrate(1/sec(d*x+c)^(4/3)/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(2/3)/(b^2*sec(d*x + c)^4 +
 2*a*b*sec(d*x + c)^3 + a^2*sec(d*x + c)^2), x)
```

3.764.6 Sympy [N/A]

Not integrable

Time = 72.58 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.04

$$\int \frac{1}{\sec^{\frac{4}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{1}{(a+b\sec(c+dx))^{\frac{3}{2}}\sec^{\frac{4}{3}}(c+dx)}\,dx$$

input
```
integrate(1/sec(d*x+c)**(4/3)/(a+b*sec(d*x+c))**(3/2),x)
```

output
```
Integral(1/((a + b*sec(c + d*x))**(3/2)*sec(c + d*x)**(4/3)), x)
```

3.764.7 Maxima [N/A]

Not integrable

Time = 1.30 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{1}{\sec^{\frac{4}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{1}{(b\sec(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{4}{3}}}\,dx$$

input
```
integrate(1/sec(d*x+c)^(4/3)/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*sec(d*x + c) + a)^(3/2)*sec(d*x + c)^(4/3)), x)
```

3.764.8 Giac [F(-1)]

Timed out.

$$\int \frac{1}{\sec^{\frac{4}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/sec(d*x+c)^(4/3)/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
Timed out
```

3.764.9 Mupad [N/A]

Not integrable

Time = 16.74 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{1}{\sec^{\frac{4}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{1}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}\left(\frac{1}{\cos(c+d\,x)}\right)^{4/3}}\,dx$$

input
```
int(1/((a + b/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(4/3)),x)
```

output
```
int(1/((a + b/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(4/3)), x)
```

3.765 $\int \frac{1}{\sec^{\frac{5}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx$

3.765.1 Optimal result 5731
3.765.2 Mathematica [N/A] 5731
3.765.3 Rubi [N/A] 5732
3.765.4 Maple [N/A] (verified) 5733
3.765.5 Fricas [N/A] 5733
3.765.6 Sympy [N/A] 5733
3.765.7 Maxima [N/A] 5734
3.765.8 Giac [F(-1)] 5734
3.765.9 Mupad [N/A] 5734

3.765.1 Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{1}{\sec^{\frac{5}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx=\text{Int}\left(\frac{1}{\sec^{\frac{5}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}},x\right)$$

output
```
Unintegrable(1/sec(d*x+c)^(5/3)/(a+b*sec(d*x+c))^(3/2),x)
```

3.765.2 Mathematica [N/A]

Not integrable

Time = 110.66 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{1}{\sec^{\frac{5}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx=\int \frac{1}{\sec^{\frac{5}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx$$

input
```
Integrate[1/(Sec[c + d*x]^(5/3)*(a + b*Sec[c + d*x])^(3/2)),x]
```

output
```
Integrate[1/(Sec[c + d*x]^(5/3)*(a + b*Sec[c + d*x])^(3/2)), x]
```

3.765.3 Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 4357}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sec^{\frac{5}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/3}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{4357}$$

$$\int \frac{1}{\sec^{\frac{5}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}}dx$$

input
```
Int[1/(Sec[c + d*x]^(5/3)*(a + b*Sec[c + d*x])^(3/2)),x]
```

output
```
$Aborted
```

3.765.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4357
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Unintegrable[(d*Csc[e + f*x])^n*(a + b*Csc[e + f*
x])^m, x] /; FreeQ[{a, b, d, e, f, m, n}, x]
```

3.765.4 Maple [N/A] (verified)

Not integrable

Time = 0.47 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{1}{\sec(dx+c)^{\frac{5}{3}}(a+b\sec(dx+c))^{\frac{3}{2}}}\,dx$$

input
```
int(1/sec(d*x+c)^(5/3)/(a+b*sec(d*x+c))^(3/2),x)
```

output
```
int(1/sec(d*x+c)^(5/3)/(a+b*sec(d*x+c))^(3/2),x)
```

3.765.5 Fricas [N/A]

Not integrable

Time = 0.97 (sec) , antiderivative size = 62, normalized size of antiderivative = 2.48

$$\int \frac{1}{\sec^{\frac{5}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{1}{(b\sec(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{5}{3}}}\,dx$$

input
```
integrate(1/sec(d*x+c)^(5/3)/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(1/3)/(b^2*sec(d*x + c)^4 +
 2*a*b*sec(d*x + c)^3 + a^2*sec(d*x + c)^2), x)
```

3.765.6 Sympy [N/A]

Not integrable

Time = 166.15 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.04

$$\int \frac{1}{\sec^{\frac{5}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{1}{(a+b\sec(c+dx))^{\frac{3}{2}}\sec^{\frac{5}{3}}(c+dx)}\,dx$$

input
```
integrate(1/sec(d*x+c)**(5/3)/(a+b*sec(d*x+c))**(3/2),x)
```

output
```
Integral(1/((a + b*sec(c + d*x))**(3/2)*sec(c + d*x)**(5/3)), x)
```

3.765.7 Maxima [N/A]

Not integrable

Time = 1.31 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{1}{\sec^{\frac{5}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{1}{(b\sec(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{5}{3}}}\,dx$$

input
```
integrate(1/sec(d*x+c)^(5/3)/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*sec(d*x + c) + a)^(3/2)*sec(d*x + c)^(5/3)), x)
```

3.765.8 Giac [F(-1)]

Timed out.

$$\int \frac{1}{\sec^{\frac{5}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/sec(d*x+c)^(5/3)/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
Timed out
```

3.765.9 Mupad [N/A]

Not integrable

Time = 13.64 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{1}{\sec^{\frac{5}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{1}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}\left(\frac{1}{\cos(c+d\,x)}\right)^{5/3}}\,dx$$

input
```
int(1/((a + b/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(5/3)),x)
```

output
```
int(1/((a + b/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(5/3)), x)
```

3.766 $\int \frac{1}{\sec^{\frac{7}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx$

3.766.1 Optimal result 5735
3.766.2 Mathematica [N/A] 5735
3.766.3 Rubi [N/A] 5736
3.766.4 Maple [N/A] (verified) 5737
3.766.5 Fricas [N/A] 5737
3.766.6 Sympy [F(-1)] 5737
3.766.7 Maxima [N/A] 5738
3.766.8 Giac [F(-1)] 5738
3.766.9 Mupad [N/A] 5738

3.766.1 Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{1}{\sec^{\frac{7}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx=\text{Int}\left(\frac{1}{\sec^{\frac{7}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}},x\right)$$

output
```
Unintegrable(1/sec(d*x+c)^(7/3)/(a+b*sec(d*x+c))^(3/2),x)
```

3.766.2 Mathematica [N/A]

Not integrable

Time = 147.76 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{1}{\sec^{\frac{7}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx=\int \frac{1}{\sec^{\frac{7}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx$$

input
```
Integrate[1/(Sec[c + d*x]^(7/3)*(a + b*Sec[c + d*x])^(3/2)),x]
```

output
```
Integrate[1/(Sec[c + d*x]^(7/3)*(a + b*Sec[c + d*x])^(3/2)), x]
```

3.766.3 Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 4357}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sec^{\frac{7}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/3}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{4357}$$

$$\int \frac{1}{\sec^{\frac{7}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}}dx$$

input
```
Int[1/(Sec[c + d*x]^(7/3)*(a + b*Sec[c + d*x])^(3/2)),x]
```

output
```
$Aborted
```

3.766.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4357
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Unintegrable[(d*Csc[e + f*x])^n*(a + b*Csc[e + f*
x])^m, x] /; FreeQ[{a, b, d, e, f, m, n}, x]
```

3.766.4 Maple [N/A] (verified)

Not integrable

Time = 0.43 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{1}{\sec(dx+c)^{\frac{7}{3}}(a+b\sec(dx+c))^{\frac{3}{2}}}\,dx$$

input
```
int(1/sec(d*x+c)^(7/3)/(a+b*sec(d*x+c))^(3/2),x)
```

output
```
int(1/sec(d*x+c)^(7/3)/(a+b*sec(d*x+c))^(3/2),x)
```

3.766.5 Fricas [N/A]

Not integrable

Time = 1.11 (sec) , antiderivative size = 62, normalized size of antiderivative = 2.48

$$\int \frac{1}{\sec^{\frac{7}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{1}{(b\sec(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{7}{3}}}\,dx$$

input
```
integrate(1/sec(d*x+c)^(7/3)/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(2/3)/(b^2*sec(d*x + c)^5 +
 2*a*b*sec(d*x + c)^4 + a^2*sec(d*x + c)^3), x)
```

3.766.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\sec^{\frac{7}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/sec(d*x+c)**(7/3)/(a+b*sec(d*x+c))**(3/2),x)
```

output
```
Timed out
```

3.766.7 Maxima [N/A]

Not integrable

Time = 1.31 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{1}{\sec^{\frac{7}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{1}{(b\sec(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{7}{3}}}\,dx$$

input
```
integrate(1/sec(d*x+c)^(7/3)/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*sec(d*x + c) + a)^(3/2)*sec(d*x + c)^(7/3)), x)
```

3.766.8 Giac [F(-1)]

Timed out.

$$\int \frac{1}{\sec^{\frac{7}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/sec(d*x+c)^(7/3)/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
Timed out
```

3.766.9 Mupad [N/A]

Not integrable

Time = 18.37 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{1}{\sec^{\frac{7}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{1}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}\left(\frac{1}{\cos(c+d\,x)}\right)^{7/3}}\,dx$$

input
```
int(1/((a + b/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(7/3)),x)
```

output
```
int(1/((a + b/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(7/3)), x)
```

3.767 $\int \frac{\sec^{\frac{7}{3}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$

3.767.1 Optimal result . . . 5739
3.767.2 Mathematica [N/A] . . . 5739
3.767.3 Rubi [N/A] . . . 5740
3.767.4 Maple [N/A] (verified) . . . 5741
3.767.5 Fricas [N/A] . . . 5741
3.767.6 Sympy [F(-1)] . . . 5741
3.767.7 Maxima [N/A] . . . 5742
3.767.8 Giac [F(-1)] . . . 5742
3.767.9 Mupad [N/A] . . . 5742

3.767.1 Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{\sec^{\frac{7}{3}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \text{Int}\left(\frac{\sec^{\frac{7}{3}}(c+dx)}{(a+b\sec(c+dx))^{5/2}},x\right)$$

output
```
Unintegrable(sec(d*x+c)^(7/3)/(a+b*sec(d*x+c))^(5/2),x)
```

3.767.2 Mathematica [N/A]

Not integrable

Time = 129.79 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{\sec^{\frac{7}{3}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{\sec^{\frac{7}{3}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$$

input
```
Integrate[Sec[c + d*x]^(7/3)/(a + b*Sec[c + d*x])^(5/2),x]
```

output
```
Integrate[Sec[c + d*x]^(7/3)/(a + b*Sec[c + d*x])^(5/2), x]
```

3.767.3 Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 4357}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{\frac{7}{3}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/3}}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \text{ 4357}$$

$$\int \frac{\sec^{\frac{7}{3}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}dx$$

input
```
Int[Sec[c + d*x]^(7/3)/(a + b*Sec[c + d*x])^(5/2),x]
```

output
```
$Aborted
```

3.767.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4357
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Unintegrable[(d*Csc[e + f*x])^n*(a + b*Csc[e + f*
x])^m, x] /; FreeQ[{a, b, d, e, f, m, n}, x]
```

3.767.4 Maple [N/A] (verified)

Not integrable

Time = 0.48 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{\sec (dx+c)^{\frac{7}{3}}}{(a+b\sec (dx+c))^{\frac{5}{2}}}dx$$

input
```
int(sec(d*x+c)^(7/3)/(a+b*sec(d*x+c))^(5/2),x)
```

output
```
int(sec(d*x+c)^(7/3)/(a+b*sec(d*x+c))^(5/2),x)
```

3.767.5 Fricas [N/A]

Not integrable

Time = 0.45 (sec) , antiderivative size = 67, normalized size of antiderivative = 2.68

$$\int \frac{\sec^{\frac{7}{3}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{\sec (dx+c)^{\frac{7}{3}}}{(b\sec (dx+c)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^(7/3)/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(7/3)/(b^3*sec(d*x + c)^3 +
 3*a*b^2*sec(d*x + c)^2 + 3*a^2*b*sec(d*x + c) + a^3), x)
```

3.767.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{7}{3}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**(7/3)/(a+b*sec(d*x+c))**(5/2),x)
```

output
```
Timed out
```

3.767.7 Maxima [N/A]

Not integrable

Time = 0.76 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{\sec^{\frac{7}{3}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{\sec(dx+c)^{\frac{7}{3}}}{(b\sec(dx+c)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^(7/3)/(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)^(7/3)/(b*sec(d*x + c) + a)^(5/2), x)
```

3.767.8 Giac [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{7}{3}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)^(7/3)/(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
Timed out
```

3.767.9 Mupad [N/A]

Not integrable

Time = 17.15 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{\sec^{\frac{7}{3}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{7/3}}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

input
```
int((1/cos(c + d*x))^(7/3)/(a + b/cos(c + d*x))^(5/2),x)
```

output
```
int((1/cos(c + d*x))^(7/3)/(a + b/cos(c + d*x))^(5/2), x)
```

3.768 $\int \frac{\sec^{\frac{5}{3}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$

3.768.1 Optimal result 5743
3.768.2 Mathematica [N/A] 5743
3.768.3 Rubi [N/A] 5744
3.768.4 Maple [N/A] (verified) 5745
3.768.5 Fricas [N/A] 5745
3.768.6 Sympy [F(-1)] 5745
3.768.7 Maxima [N/A] 5746
3.768.8 Giac [F(-1)] 5746
3.768.9 Mupad [N/A] 5746

3.768.1 Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{\sec^{\frac{5}{3}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \text{Int}\left(\frac{\sec^{\frac{5}{3}}(c+dx)}{(a+b\sec(c+dx))^{5/2}},x\right)$$

output
```
Unintegrable(sec(d*x+c)^(5/3)/(a+b*sec(d*x+c))^(5/2),x)
```

3.768.2 Mathematica [N/A]

Not integrable

Time = 148.85 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{\sec^{\frac{5}{3}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{\sec^{\frac{5}{3}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$$

input
```
Integrate[Sec[c + d*x]^(5/3)/(a + b*Sec[c + d*x])^(5/2),x]
```

output
```
Integrate[Sec[c + d*x]^(5/3)/(a + b*Sec[c + d*x])^(5/2), x]
```

3.768.3 Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 4357}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{\frac{5}{3}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/3}}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \; 4357$$

$$\int \frac{\sec^{\frac{5}{3}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}dx$$

input
```
Int[Sec[c + d*x]^(5/3)/(a + b*Sec[c + d*x])^(5/2),x]
```

output
```
$Aborted
```

3.768.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4357
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Unintegrable[(d*Csc[e + f*x])^n*(a + b*Csc[e + f*
x])^m, x] /; FreeQ[{a, b, d, e, f, m, n}, x]
```

3.768.4 Maple [N/A] (verified)

Not integrable

Time = 0.46 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{\sec (dx+c)^{\frac{5}{3}}}{(a+b\sec (dx+c))^{\frac{5}{2}}}dx$$

input
```
int(sec(d*x+c)^(5/3)/(a+b*sec(d*x+c))^(5/2),x)
```

output
```
int(sec(d*x+c)^(5/3)/(a+b*sec(d*x+c))^(5/2),x)
```

3.768.5 Fricas [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 67, normalized size of antiderivative = 2.68

$$\int \frac{\sec^{\frac{5}{3}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{\sec (dx+c)^{\frac{5}{3}}}{(b\sec (dx+c)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^(5/3)/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(5/3)/(b^3*sec(d*x + c)^3 +
 3*a*b^2*sec(d*x + c)^2 + 3*a^2*b*sec(d*x + c) + a^3), x)
```

3.768.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{5}{3}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**(5/3)/(a+b*sec(d*x+c))**(5/2),x)
```

output
```
Timed out
```

3.768.7 Maxima [N/A]

Not integrable

Time = 0.78 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{\sec^{\frac{5}{3}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{\sec(dx+c)^{\frac{5}{3}}}{(b\sec(dx+c)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^(5/3)/(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)^(5/3)/(b*sec(d*x + c) + a)^(5/2), x)
```

3.768.8 Giac [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{5}{3}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)^(5/3)/(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
Timed out
```

3.768.9 Mupad [N/A]

Not integrable

Time = 18.06 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{\sec^{\frac{5}{3}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{5/3}}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

input
```
int((1/cos(c + d*x))^(5/3)/(a + b/cos(c + d*x))^(5/2),x)
```

output
```
int((1/cos(c + d*x))^(5/3)/(a + b/cos(c + d*x))^(5/2), x)
```

3.769 $\int \frac{\sec^{\frac{4}{3}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$

3.769.1 Optimal result 5747
3.769.2 Mathematica [N/A] 5747
3.769.3 Rubi [N/A] 5748
3.769.4 Maple [N/A] (verified) 5749
3.769.5 Fricas [N/A] 5749
3.769.6 Sympy [F(-1)] 5749
3.769.7 Maxima [N/A] 5750
3.769.8 Giac [F(-1)] 5750
3.769.9 Mupad [N/A] 5750

3.769.1 Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{\sec^{\frac{4}{3}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \text{Int}\left(\frac{\sec^{\frac{4}{3}}(c+dx)}{(a+b\sec(c+dx))^{5/2}},x\right)$$

output
```
Unintegrable(sec(d*x+c)^(4/3)/(a+b*sec(d*x+c))^(5/2),x)
```

3.769.2 Mathematica [N/A]

Not integrable

Time = 126.55 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{\sec^{\frac{4}{3}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{\sec^{\frac{4}{3}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$$

input
```
Integrate[Sec[c + d*x]^(4/3)/(a + b*Sec[c + d*x])^(5/2),x]
```

output
```
Integrate[Sec[c + d*x]^(4/3)/(a + b*Sec[c + d*x])^(5/2), x]
```

3.769.3 Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 4357}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{\frac{4}{3}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{4/3}}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \; 4357$$

$$\int \frac{\sec^{\frac{4}{3}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}dx$$

input
```
Int[Sec[c + d*x]^(4/3)/(a + b*Sec[c + d*x])^(5/2),x]
```

output
```
$Aborted
```

3.769.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4357
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Unintegrable[(d*Csc[e + f*x])^n*(a + b*Csc[e + f*
x])^m, x] /; FreeQ[{a, b, d, e, f, m, n}, x]
```

3.769.4 Maple [N/A] (verified)

Not integrable

Time = 0.40 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{\sec (dx+c)^{\frac{4}{3}}}{(a+b\sec (dx+c))^{\frac{5}{2}}}dx$$

input
```
int(sec(d*x+c)^(4/3)/(a+b*sec(d*x+c))^(5/2),x)
```

output
```
int(sec(d*x+c)^(4/3)/(a+b*sec(d*x+c))^(5/2),x)
```

3.769.5 Fricas [N/A]

Not integrable

Time = 0.44 (sec) , antiderivative size = 67, normalized size of antiderivative = 2.68

$$\int \frac{\sec^{\frac{4}{3}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{\sec (dx+c)^{\frac{4}{3}}}{(b\sec (dx+c)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^(4/3)/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(4/3)/(b^3*sec(d*x + c)^3 +
 3*a*b^2*sec(d*x + c)^2 + 3*a^2*b*sec(d*x + c) + a^3), x)
```

3.769.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{4}{3}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**(4/3)/(a+b*sec(d*x+c))**(5/2),x)
```

output
```
Timed out
```

3.769.7 Maxima [N/A]

Not integrable

Time = 0.80 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{\sec^{\frac{4}{3}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{\sec(dx+c)^{\frac{4}{3}}}{(b\sec(dx+c)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^(4/3)/(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)^(4/3)/(b*sec(d*x + c) + a)^(5/2), x)
```

3.769.8 Giac [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{4}{3}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)^(4/3)/(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
Timed out
```

3.769.9 Mupad [N/A]

Not integrable

Time = 17.97 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{\sec^{\frac{4}{3}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{4/3}}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

input
```
int((1/cos(c + d*x))^(4/3)/(a + b/cos(c + d*x))^(5/2),x)
```

output
```
int((1/cos(c + d*x))^(4/3)/(a + b/cos(c + d*x))^(5/2), x)
```

3.770 $\int \frac{\sec^{\frac{2}{3}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$

3.770.1 Optimal result 5751
3.770.2 Mathematica [N/A] 5751
3.770.3 Rubi [N/A] 5752
3.770.4 Maple [N/A] (verified) 5753
3.770.5 Fricas [N/A] 5753
3.770.6 Sympy [N/A] 5753
3.770.7 Maxima [N/A] 5754
3.770.8 Giac [F(-1)] 5754
3.770.9 Mupad [N/A] 5754

3.770.1 Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{\sec^{\frac{2}{3}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \text{Int}\left(\frac{\sec^{\frac{2}{3}}(c+dx)}{(a+b\sec(c+dx))^{5/2}},x\right)$$

output
```
Unintegrable(sec(d*x+c)^(2/3)/(a+b*sec(d*x+c))^(5/2),x)
```

3.770.2 Mathematica [N/A]

Not integrable

Time = 144.62 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{\sec^{\frac{2}{3}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{\sec^{\frac{2}{3}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$$

input
```
Integrate[Sec[c + d*x]^(2/3)/(a + b*Sec[c + d*x])^(5/2),x]
```

output
```
Integrate[Sec[c + d*x]^(2/3)/(a + b*Sec[c + d*x])^(5/2), x]
```

3.770.3 Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 4357}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{\frac{2}{3}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{2/3}}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \text{ 4357}$$

$$\int \frac{\sec^{\frac{2}{3}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}dx$$

input
```
Int[Sec[c + d*x]^(2/3)/(a + b*Sec[c + d*x])^(5/2),x]
```

output
```
$Aborted
```

3.770.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4357
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Unintegrable[(d*Csc[e + f*x])^n*(a + b*Csc[e + f*
x])^m, x] /; FreeQ[{a, b, d, e, f, m, n}, x]
```

3.770.4 Maple [N/A] (verified)

Not integrable

Time = 0.48 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{\sec (dx+c)^{\frac{2}{3}}}{(a+b\sec (dx+c))^{\frac{5}{2}}}dx$$

input
```
int(sec(d*x+c)^(2/3)/(a+b*sec(d*x+c))^(5/2),x)
```

output
```
int(sec(d*x+c)^(2/3)/(a+b*sec(d*x+c))^(5/2),x)
```

3.770.5 Fricas [N/A]

Not integrable

Time = 1.10 (sec) , antiderivative size = 67, normalized size of antiderivative = 2.68

$$\int \frac{\sec^{\frac{2}{3}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx=\int \frac{\sec (dx+c)^{\frac{2}{3}}}{(b\sec (dx+c)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^(2/3)/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(2/3)/(b^3*sec(d*x + c)^3 +
 3*a*b^2*sec(d*x + c)^2 + 3*a^2*b*sec(d*x + c) + a^3), x)
```

3.770.6 Sympy [N/A]

Not integrable

Time = 62.85 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int \frac{\sec^{\frac{2}{3}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx=\int \frac{\sec^{\frac{2}{3}}(c+dx)}{(a+b\sec (c+dx))^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)**(2/3)/(a+b*sec(d*x+c))**(5/2),x)
```

output
```
Integral(sec(c + d*x)**(2/3)/(a + b*sec(c + d*x))**(5/2), x)
```

3.770.7 Maxima [N/A]

Not integrable

Time = 0.75 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{\sec^{\frac{2}{3}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{\sec\left(dx+c\right)^{\frac{2}{3}}}{\left(b\sec\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^(2/3)/(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)^(2/3)/(b*sec(d*x + c) + a)^(5/2), x)
```

3.770.8 Giac [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{2}{3}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)^(2/3)/(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
Timed out
```

3.770.9 Mupad [N/A]

Not integrable

Time = 19.30 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{\sec^{\frac{2}{3}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{2/3}}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

input
```
int((1/cos(c + d*x))^(2/3)/(a + b/cos(c + d*x))^(5/2),x)
```

output
```
int((1/cos(c + d*x))^(2/3)/(a + b/cos(c + d*x))^(5/2), x)
```

3.771 $\int \frac{\sqrt[3]{\sec(c+dx)}}{(a+b\sec(c+dx))^{5/2}}\,dx$

3.771.1 Optimal result 5755
3.771.2 Mathematica [N/A] 5755
3.771.3 Rubi [N/A] 5756
3.771.4 Maple [N/A] (verified) 5757
3.771.5 Fricas [N/A] 5757
3.771.6 Sympy [N/A] 5757
3.771.7 Maxima [N/A] 5758
3.771.8 Giac [F(-1)] 5758
3.771.9 Mupad [N/A] 5758

3.771.1 Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{\sqrt[3]{\sec(c+dx)}}{(a+b\sec(c+dx))^{5/2}}\,dx = \text{Int}\left(\frac{\sqrt[3]{\sec(c+dx)}}{(a+b\sec(c+dx))^{5/2}},x\right)$$

output
```
Unintegrable(sec(d*x+c)^(1/3)/(a+b*sec(d*x+c))^(5/2),x)
```

3.771.2 Mathematica [N/A]

Not integrable

Time = 131.28 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{\sqrt[3]{\sec(c+dx)}}{(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{\sqrt[3]{\sec(c+dx)}}{(a+b\sec(c+dx))^{5/2}}\,dx$$

input
```
Integrate[Sec[c + d*x]^(1/3)/(a + b*Sec[c + d*x])^(5/2),x]
```

output
```
Integrate[Sec[c + d*x]^(1/3)/(a + b*Sec[c + d*x])^(5/2), x]
```

3.771.3 Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 4357}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{\sec(c+dx)}}{(a+b\sec(c+dx))^{5/2}}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\sqrt[3]{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \; 4357$$

$$\int \frac{\sqrt[3]{\sec(c+dx)}}{(a+b\sec(c+dx))^{5/2}}dx$$

input
```
Int[Sec[c + d*x]^(1/3)/(a + b*Sec[c + d*x])^(5/2),x]
```

output
```
$Aborted
```

3.771.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4357
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Unintegrable[(d*Csc[e + f*x])^n*(a + b*Csc[e + f*
x])^m, x] /; FreeQ[{a, b, d, e, f, m, n}, x]
```

3.771.4 Maple [N/A] (verified)

Not integrable

Time = 0.48 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{\sec(dx+c)^{\frac{1}{3}}}{(a+b\sec(dx+c))^{\frac{5}{2}}}dx$$

input
```
int(sec(d*x+c)^(1/3)/(a+b*sec(d*x+c))^(5/2),x)
```

output
```
int(sec(d*x+c)^(1/3)/(a+b*sec(d*x+c))^(5/2),x)
```

3.771.5 Fricas [N/A]

Not integrable

Time = 1.19 (sec) , antiderivative size = 67, normalized size of antiderivative = 2.68

$$\int \frac{\sqrt[3]{\sec(c+dx)}}{(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{\sec(dx+c)^{\frac{1}{3}}}{(b\sec(dx+c)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^(1/3)/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(1/3)/(b^3*sec(d*x + c)^3 +
 3*a*b^2*sec(d*x + c)^2 + 3*a^2*b*sec(d*x + c) + a^3), x)
```

3.771.6 Sympy [N/A]

Not integrable

Time = 25.26 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int \frac{\sqrt[3]{\sec(c+dx)}}{(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{\sqrt[3]{\sec(c+dx)}}{(a+b\sec(c+dx))^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)**(1/3)/(a+b*sec(d*x+c))**(5/2),x)
```

output
```
Integral(sec(c + d*x)**(1/3)/(a + b*sec(c + d*x))**(5/2), x)
```

3.771.7 Maxima [N/A]

Not integrable

Time = 1.28 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt[3]{\sec(c+dx)}}{(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{\sec(dx+c)^{\frac{1}{3}}}{(b\sec(dx+c)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^(1/3)/(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)^(1/3)/(b*sec(d*x + c) + a)^(5/2), x)
```

3.771.8 Giac [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{\sec(c+dx)}}{(a+b\sec(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)^(1/3)/(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
Timed out
```

3.771.9 Mupad [N/A]

Not integrable

Time = 19.34 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{\sqrt[3]{\sec(c+dx)}}{(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{1/3}}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

input
```
int((1/cos(c + d*x))^(1/3)/(a + b/cos(c + d*x))^(5/2),x)
```

output
```
int((1/cos(c + d*x))^(1/3)/(a + b/cos(c + d*x))^(5/2), x)
```

3.772 $\int \frac{1}{\sqrt[3]{\sec(c+dx)}(a+b\sec(c+dx))^{5/2}}\,dx$

3.772.1 Optimal result 5759
3.772.2 Mathematica [N/A] 5759
3.772.3 Rubi [N/A] 5760
3.772.4 Maple [N/A] (verified) 5761
3.772.5 Fricas [N/A] 5761
3.772.6 Sympy [N/A] 5761
3.772.7 Maxima [N/A] 5762
3.772.8 Giac [F(-1)] 5762
3.772.9 Mupad [N/A] 5762

3.772.1 Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{1}{\sqrt[3]{\sec(c+dx)}(a+b\sec(c+dx))^{5/2}}\,dx = \text{Int}\left(\frac{1}{\sqrt[3]{\sec(c+dx)}(a+b\sec(c+dx))^{5/2}},x\right)$$

output
```
Unintegrable(1/sec(d*x+c)^(1/3)/(a+b*sec(d*x+c))^(5/2),x)
```

3.772.2 Mathematica [N/A]

Not integrable

Time = 148.09 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{1}{\sqrt[3]{\sec(c+dx)}(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{1}{\sqrt[3]{\sec(c+dx)}(a+b\sec(c+dx))^{5/2}}\,dx$$

input
```
Integrate[1/(Sec[c + d*x]^(1/3)*(a + b*Sec[c + d*x])^(5/2)),x]
```

output
```
Integrate[1/(Sec[c + d*x]^(1/3)*(a + b*Sec[c + d*x])^(5/2)), x]
```

3.772.3 Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 4357}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[3]{\sec(c+dx)}(a+b\sec(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt[3]{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \text{4357}$$

$$\int \frac{1}{\sqrt[3]{\sec(c+dx)}(a+b\sec(c+dx))^{5/2}}dx$$

input
```
Int[1/(Sec[c + d*x]^(1/3)*(a + b*Sec[c + d*x])^(5/2)),x]
```

output
```
$Aborted
```

3.772.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4357
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Unintegrable[(d*Csc[e + f*x])^n*(a + b*Csc[e + f*
x])^m, x] /; FreeQ[{a, b, d, e, f, m, n}, x]
```

3.772.4 Maple [N/A] (verified)

Not integrable

Time = 0.40 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{1}{\sec\left(dx+c\right)^{\frac{1}{3}}\left(a+b\sec\left(dx+c\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
int(1/sec(d*x+c)^(1/3)/(a+b*sec(d*x+c))^(5/2),x)
```

output
```
int(1/sec(d*x+c)^(1/3)/(a+b*sec(d*x+c))^(5/2),x)
```

3.772.5 Fricas [N/A]

Not integrable

Time = 1.13 (sec) , antiderivative size = 76, normalized size of antiderivative = 3.04

$$\int \frac{1}{\sqrt[3]{\sec(c+dx)}(a+b\sec(c+dx))^{5/2}}\,dx=\int \frac{1}{\left(b\sec\left(dx+c\right)+a\right)^{\frac{5}{2}}\sec\left(dx+c\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/sec(d*x+c)^(1/3)/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(2/3)/(b^3*sec(d*x + c)^4 +
 3*a*b^2*sec(d*x + c)^3 + 3*a^2*b*sec(d*x + c)^2 + a^3*sec(d*x + c)), x)
```

3.772.6 Sympy [N/A]

Not integrable

Time = 78.16 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.04

$$\int \frac{1}{\sqrt[3]{\sec(c+dx)}(a+b\sec(c+dx))^{5/2}}\,dx=\int \frac{1}{\left(a+b\sec\left(c+dx\right)\right)^{\frac{5}{2}}\sqrt[3]{\sec\left(c+dx\right)}}\,dx$$

input
```
integrate(1/sec(d*x+c)**(1/3)/(a+b*sec(d*x+c))**(5/2),x)
```

output
```
Integral(1/((a + b*sec(c + d*x))**(5/2)*sec(c + d*x)**(1/3)), x)
```

3.772.7 Maxima [N/A]

Not integrable

Time = 1.28 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{1}{\sqrt[3]{\sec(c+dx)}(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{1}{(b\sec(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/sec(d*x+c)^(1/3)/(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*sec(d*x + c) + a)^(5/2)*sec(d*x + c)^(1/3)), x)
```

3.772.8 Giac [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{\sec(c+dx)}(a+b\sec(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/sec(d*x+c)^(1/3)/(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
Timed out
```

3.772.9 Mupad [N/A]

Not integrable

Time = 17.81 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{1}{\sqrt[3]{\sec(c+dx)}(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{1}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2}\left(\frac{1}{\cos(c+d\,x)}\right)^{1/3}}\,dx$$

input
```
int(1/((a + b/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(1/3)),x)
```

output
```
int(1/((a + b/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(1/3)), x)
```

3.773 $\int \frac{1}{\sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx$

3.773.1 Optimal result 5763
3.773.2 Mathematica [N/A] 5763
3.773.3 Rubi [N/A] 5764
3.773.4 Maple [N/A] (verified) 5765
3.773.5 Fricas [N/A] 5765
3.773.6 Sympy [N/A] 5765
3.773.7 Maxima [N/A] 5766
3.773.8 Giac [F(-1)] 5766
3.773.9 Mupad [N/A] 5766

3.773.1 Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{1}{\sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx=\text{Int}\left(\frac{1}{\sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}},x\right)$$

output
```
Unintegrable(1/sec(d*x+c)^(2/3)/(a+b*sec(d*x+c))^(5/2),x)
```

3.773.2 Mathematica [N/A]

Not integrable

Time = 135.39 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{1}{\sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx=\int \frac{1}{\sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx$$

input
```
Integrate[1/(Sec[c + d*x]^(2/3)*(a + b*Sec[c + d*x])^(5/2)),x]
```

output
```
Integrate[1/(Sec[c + d*x]^(2/3)*(a + b*Sec[c + d*x])^(5/2)), x]
```

3.773.3 Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 4357}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{2/3}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \text{4357}$$

$$\int \frac{1}{\sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}}dx$$

input
```
Int[1/(Sec[c + d*x]^(2/3)*(a + b*Sec[c + d*x])^(5/2)),x]
```

output
```
$Aborted
```

3.773.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4357
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Unintegrable[(d*Csc[e + f*x])^n*(a + b*Csc[e + f*
x])^m, x] /; FreeQ[{a, b, d, e, f, m, n}, x]
```

3.773.4 Maple [N/A] (verified)

Not integrable

Time = 0.48 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{1}{\sec\left(dx+c\right)^{\frac{2}{3}}\left(a+b\sec\left(dx+c\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
int(1/sec(d*x+c)^(2/3)/(a+b*sec(d*x+c))^(5/2),x)
```

output
```
int(1/sec(d*x+c)^(2/3)/(a+b*sec(d*x+c))^(5/2),x)
```

3.773.5 Fricas [N/A]

Not integrable

Time = 1.62 (sec) , antiderivative size = 76, normalized size of antiderivative = 3.04

$$\int \frac{1}{\sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx=\int \frac{1}{\left(b\sec\left(dx+c\right)+a\right)^{\frac{5}{2}}\sec\left(dx+c\right)^{\frac{2}{3}}}\,dx$$

input
```
integrate(1/sec(d*x+c)^(2/3)/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(1/3)/(b^3*sec(d*x + c)^4 +
 3*a*b^2*sec(d*x + c)^3 + 3*a^2*b*sec(d*x + c)^2 + a^3*sec(d*x + c)), x)
```

3.773.6 Sympy [N/A]

Not integrable

Time = 166.17 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.04

$$\int \frac{1}{\sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx=\int \frac{1}{\left(a+b\sec\left(c+dx\right)\right)^{\frac{5}{2}}\sec^{\frac{2}{3}}\left(c+dx\right)}\,dx$$

input
```
integrate(1/sec(d*x+c)**(2/3)/(a+b*sec(d*x+c))**(5/2),x)
```

output
```
Integral(1/((a + b*sec(c + d*x))**(5/2)*sec(c + d*x)**(2/3)), x)
```

3.773.7 Maxima [N/A]

Not integrable

Time = 1.29 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{1}{\sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{1}{(b\sec(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^{\frac{2}{3}}}\,dx$$

input
```
integrate(1/sec(d*x+c)^(2/3)/(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*sec(d*x + c) + a)^(5/2)*sec(d*x + c)^(2/3)), x)
```

3.773.8 Giac [F(-1)]

Timed out.

$$\int \frac{1}{\sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/sec(d*x+c)^(2/3)/(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
Timed out
```

3.773.9 Mupad [N/A]

Not integrable

Time = 14.45 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{1}{\sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{1}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2}\left(\frac{1}{\cos(c+d\,x)}\right)^{2/3}}\,dx$$

input
```
int(1/((a + b/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(2/3)),x)
```

output
```
int(1/((a + b/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(2/3)), x)
```

3.774 $\int \frac{1}{\sec^{\frac{4}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx$

3.774.1 Optimal result 5767
3.774.2 Mathematica [N/A] 5767
3.774.3 Rubi [N/A] 5768
3.774.4 Maple [N/A] (verified) 5769
3.774.5 Fricas [N/A] 5769
3.774.6 Sympy [F(-1)] 5769
3.774.7 Maxima [N/A] 5770
3.774.8 Giac [F(-1)] 5770
3.774.9 Mupad [N/A] 5770

3.774.1 Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{1}{\sec^{\frac{4}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx=\text{Int}\left(\frac{1}{\sec^{\frac{4}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}},x\right)$$

output
```
Unintegrable(1/sec(d*x+c)^(4/3)/(a+b*sec(d*x+c))^(5/2),x)
```

3.774.2 Mathematica [N/A]

Not integrable

Time = 153.89 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{1}{\sec^{\frac{4}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx=\int \frac{1}{\sec^{\frac{4}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx$$

input
```
Integrate[1/(Sec[c + d*x]^(4/3)*(a + b*Sec[c + d*x])^(5/2)),x]
```

output
```
Integrate[1/(Sec[c + d*x]^(4/3)*(a + b*Sec[c + d*x])^(5/2)), x]
```

3.774.3 Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 4357}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sec^{\frac{4}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{4/3}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \text{4357}$$

$$\int \frac{1}{\sec^{\frac{4}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}}dx$$

input
```
Int[1/(Sec[c + d*x]^(4/3)*(a + b*Sec[c + d*x])^(5/2)),x]
```

output
```
$Aborted
```

3.774.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4357
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Unintegrable[(d*Csc[e + f*x])^n*(a + b*Csc[e + f*
x])^m, x] /; FreeQ[{a, b, d, e, f, m, n}, x]
```

3.774.4 Maple [N/A] (verified)

Not integrable

Time = 0.48 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{1}{\sec(dx+c)^{\frac{4}{3}}(a+b\sec(dx+c))^{\frac{5}{2}}}\,dx$$

input
```
int(1/sec(d*x+c)^(4/3)/(a+b*sec(d*x+c))^(5/2),x)
```

output
```
int(1/sec(d*x+c)^(4/3)/(a+b*sec(d*x+c))^(5/2),x)
```

3.774.5 Fricas [N/A]

Not integrable

Time = 1.14 (sec) , antiderivative size = 78, normalized size of antiderivative = 3.12

$$\int \frac{1}{\sec^{\frac{4}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{1}{(b\sec(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^{\frac{4}{3}}}\,dx$$

input
```
integrate(1/sec(d*x+c)^(4/3)/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(2/3)/(b^3*sec(d*x + c)^5 +
 3*a*b^2*sec(d*x + c)^4 + 3*a^2*b*sec(d*x + c)^3 + a^3*sec(d*x + c)^2), x)
```

3.774.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\sec^{\frac{4}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/sec(d*x+c)**(4/3)/(a+b*sec(d*x+c))**(5/2),x)
```

output
```
Timed out
```

3.774.7 Maxima [N/A]

Not integrable

Time = 1.33 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{1}{\sec^{\frac{4}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{1}{(b\sec(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^{\frac{4}{3}}}\,dx$$

input
```
integrate(1/sec(d*x+c)^(4/3)/(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*sec(d*x + c) + a)^(5/2)*sec(d*x + c)^(4/3)), x)
```

3.774.8 Giac [F(-1)]

Timed out.

$$\int \frac{1}{\sec^{\frac{4}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/sec(d*x+c)^(4/3)/(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
Timed out
```

3.774.9 Mupad [N/A]

Not integrable

Time = 19.24 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{1}{\sec^{\frac{4}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{1}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2}\left(\frac{1}{\cos(c+d\,x)}\right)^{4/3}}\,dx$$

input
```
int(1/((a + b/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(4/3)),x)
```

output
```
int(1/((a + b/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(4/3)), x)
```

3.775 $\int \frac{1}{\sec^{\frac{5}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx$

3.775.1 Optimal result 5771
3.775.2 Mathematica [N/A] 5771
3.775.3 Rubi [N/A] 5772
3.775.4 Maple [N/A] (verified) 5773
3.775.5 Fricas [N/A] 5773
3.775.6 Sympy [F(-1)] 5773
3.775.7 Maxima [N/A] 5774
3.775.8 Giac [F(-1)] 5774
3.775.9 Mupad [N/A] 5774

3.775.1 Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{1}{\sec^{\frac{5}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx=\text{Int}\left(\frac{1}{\sec^{\frac{5}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}},x\right)$$

output
```
Unintegrable(1/sec(d*x+c)^(5/3)/(a+b*sec(d*x+c))^(5/2),x)
```

3.775.2 Mathematica [N/A]

Not integrable

Time = 114.96 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{1}{\sec^{\frac{5}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx=\int \frac{1}{\sec^{\frac{5}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx$$

input
```
Integrate[1/(Sec[c + d*x]^(5/3)*(a + b*Sec[c + d*x])^(5/2)),x]
```

output
```
Integrate[1/(Sec[c + d*x]^(5/3)*(a + b*Sec[c + d*x])^(5/2)), x]
```

3.775.3 Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 4357}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sec^{\frac{5}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/3}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \text{4357}$$

$$\int \frac{1}{\sec^{\frac{5}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}}dx$$

input
```
Int[1/(Sec[c + d*x]^(5/3)*(a + b*Sec[c + d*x])^(5/2)),x]
```

output
```
$Aborted
```

3.775.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4357
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Unintegrable[(d*Csc[e + f*x])^n*(a + b*Csc[e + f*
x])^m, x] /; FreeQ[{a, b, d, e, f, m, n}, x]
```

3.775.4 Maple [N/A] (verified)

Not integrable

Time = 0.39 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{1}{\sec(dx+c)^{\frac{5}{3}}(a+b\sec(dx+c))^{\frac{5}{2}}}\,dx$$

input
```
int(1/sec(d*x+c)^(5/3)/(a+b*sec(d*x+c))^(5/2),x)
```

output
```
int(1/sec(d*x+c)^(5/3)/(a+b*sec(d*x+c))^(5/2),x)
```

3.775.5 Fricas [N/A]

Not integrable

Time = 2.14 (sec) , antiderivative size = 78, normalized size of antiderivative = 3.12

$$\int \frac{1}{\sec^{\frac{5}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{1}{(b\sec(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^{\frac{5}{3}}}\,dx$$

input
```
integrate(1/sec(d*x+c)^(5/3)/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(1/3)/(b^3*sec(d*x + c)^5 +
 3*a*b^2*sec(d*x + c)^4 + 3*a^2*b*sec(d*x + c)^3 + a^3*sec(d*x + c)^2), x)
```

3.775.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\sec^{\frac{5}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/sec(d*x+c)**(5/3)/(a+b*sec(d*x+c))**(5/2),x)
```

output
```
Timed out
```

3.775.7 Maxima [N/A]

Not integrable

Time = 2.28 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{1}{\sec^{\frac{5}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{1}{(b\sec(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^{\frac{5}{3}}}\,dx$$

input
```
integrate(1/sec(d*x+c)^(5/3)/(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*sec(d*x + c) + a)^(5/2)*sec(d*x + c)^(5/3)), x)
```

3.775.8 Giac [F(-1)]

Timed out.

$$\int \frac{1}{\sec^{\frac{5}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/sec(d*x+c)^(5/3)/(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
Timed out
```

3.775.9 Mupad [N/A]

Not integrable

Time = 14.52 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{1}{\sec^{\frac{5}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{1}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2}\left(\frac{1}{\cos(c+d\,x)}\right)^{5/3}}\,dx$$

input
```
int(1/((a + b/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(5/3)),x)
```

output
```
int(1/((a + b/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(5/3)), x)
```

3.776 $\int \frac{1}{\sec^{\frac{7}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx$

3.776.1 Optimal result 5775
3.776.2 Mathematica [N/A] 5775
3.776.3 Rubi [N/A] 5776
3.776.4 Maple [N/A] (verified) 5777
3.776.5 Fricas [N/A] 5777
3.776.6 Sympy [F(-1)] 5777
3.776.7 Maxima [N/A] 5778
3.776.8 Giac [F(-1)] 5778
3.776.9 Mupad [N/A] 5778

3.776.1 Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{1}{\sec^{\frac{7}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx=\text{Int}\left(\frac{1}{\sec^{\frac{7}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}},x\right)$$

output
```
Unintegrable(1/sec(d*x+c)^(7/3)/(a+b*sec(d*x+c))^(5/2),x)
```

3.776.2 Mathematica [N/A]

Not integrable

Time = 160.99 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{1}{\sec^{\frac{7}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx=\int \frac{1}{\sec^{\frac{7}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx$$

input
```
Integrate[1/(Sec[c + d*x]^(7/3)*(a + b*Sec[c + d*x])^(5/2)),x]
```

output
```
Integrate[1/(Sec[c + d*x]^(7/3)*(a + b*Sec[c + d*x])^(5/2)), x]
```

3.776.3 Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 4357}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sec^{\frac{7}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/3}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \text{4357}$$

$$\int \frac{1}{\sec^{\frac{7}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}}dx$$

input
```
Int[1/(Sec[c + d*x]^(7/3)*(a + b*Sec[c + d*x])^(5/2)),x]
```

output
```
$Aborted
```

3.776.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4357
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Unintegrable[(d*Csc[e + f*x])^n*(a + b*Csc[e + f*
x])^m, x] /; FreeQ[{a, b, d, e, f, m, n}, x]
```

3.776.4 Maple [N/A] (verified)

Not integrable

Time = 0.41 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{1}{\sec(dx+c)^{\frac{7}{3}}(a+b\sec(dx+c))^{\frac{5}{2}}}\,dx$$

input
```
int(1/sec(d*x+c)^(7/3)/(a+b*sec(d*x+c))^(5/2),x)
```

output
```
int(1/sec(d*x+c)^(7/3)/(a+b*sec(d*x+c))^(5/2),x)
```

3.776.5 Fricas [N/A]

Not integrable

Time = 1.17 (sec) , antiderivative size = 78, normalized size of antiderivative = 3.12

$$\int \frac{1}{\sec^{\frac{7}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{1}{(b\sec(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^{\frac{7}{3}}}\,dx$$

input
```
integrate(1/sec(d*x+c)^(7/3)/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(2/3)/(b^3*sec(d*x + c)^6 +
 3*a*b^2*sec(d*x + c)^5 + 3*a^2*b*sec(d*x + c)^4 + a^3*sec(d*x + c)^3), x)
```

3.776.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\sec^{\frac{7}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/sec(d*x+c)**(7/3)/(a+b*sec(d*x+c))**(5/2),x)
```

output
```
Timed out
```

3.776.7 Maxima [N/A]

Not integrable

Time = 2.32 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{1}{\sec^{\frac{7}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{1}{(b\sec(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^{\frac{7}{3}}}\,dx$$

input
```
integrate(1/sec(d*x+c)^(7/3)/(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*sec(d*x + c) + a)^(5/2)*sec(d*x + c)^(7/3)), x)
```

3.776.8 Giac [F(-1)]

Timed out.

$$\int \frac{1}{\sec^{\frac{7}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/sec(d*x+c)^(7/3)/(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
Timed out
```

3.776.9 Mupad [N/A]

Not integrable

Time = 19.19 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{1}{\sec^{\frac{7}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{1}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2}\left(\frac{1}{\cos(c+d\,x)}\right)^{7/3}}\,dx$$

input
```
int(1/((a + b/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(7/3)),x)
```

output
```
int(1/((a + b/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(7/3)), x)
```

3.777 $\int (d\sec(e+fx))^n(a+b\sec(e+fx))^3\,dx$

3.777.1 Optimal result . . . 5779
3.777.2 Mathematica [A] (verified) . . . 5780
3.777.3 Rubi [A] (verified) . . . 5780
3.777.4 Maple [F] . . . 5784
3.777.5 Fricas [F] . . . 5784
3.777.6 Sympy [F] . . . 5784
3.777.7 Maxima [F] . . . 5785
3.777.8 Giac [F] . . . 5785
3.777.9 Mupad [F(-1)] . . . 5785

3.777.1 Optimal result

Integrand size = 23, antiderivative size = 251

$$\begin{aligned}\int (d\sec(e+fx))^n(a+b\sec(e+fx))^3\,dx = &-\frac{ad(3b^2n+a^2(1+n))\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1-n}{2},\frac{3-n}{2},\cos^2(e+fx)\right)(d\sec(e+fx))^{-1+n}\sin(e+fx)}{f\,(1-n^2)\sqrt{\sin^2(e+fx)}}\\ &+\frac{b(b^2(1+n)+3a^2(2+n))\,\text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{n}{2},\frac{2-n}{2},\cos^2(e+fx)\right)(d\sec(e+fx))^n\sin(e+fx)}{fn(2+n)\sqrt{\sin^2(e+fx)}}\\ &+\frac{ab^2(5+2n)(d\sec(e+fx))^n\tan(e+fx)}{f(1+n)(2+n)}\\ &+\frac{b^2(d\sec(e+fx))^n(a+b\sec(e+fx))\tan(e+fx)}{f(2+n)}\end{aligned}$$

output
```
-a*d*(3*b^2*n+a^2*(1+n))*hypergeom([1/2, 1/2-1/2*n],[3/2-1/2*n],cos(f*x+e)
^2)*(d*sec(f*x+e))^(-1+n)*sin(f*x+e)/f/(-n^2+1)/(sin(f*x+e)^2)^(1/2)+b*(b^
2*(1+n)+3*a^2*(2+n))*hypergeom([1/2, -1/2*n],[1-1/2*n],cos(f*x+e)^2)*(d*se
c(f*x+e))^n*sin(f*x+e)/f/n/(2+n)/(sin(f*x+e)^2)^(1/2)+a*b^2*(5+2*n)*(d*sec
(f*x+e))^n*tan(f*x+e)/f/(1+n)/(2+n)+b^2*(d*sec(f*x+e))^n*(a+b*sec(f*x+e))*
tan(f*x+e)/f/(2+n)
```

3.777.2 Mathematica [A] (verified)

Time = 0.93 (sec) , antiderivative size = 231, normalized size of antiderivative = 0.92

$$\int (d\sec(e+fx))^n(a+b\sec(e+fx))^3\,dx =$$
$$-\frac{\csc^3(e+fx)\left(a^3(6+11n+6n^2+n^3)\cos^3(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n}{2},\frac{2+n}{2},\sec^2(e+fx)\right)+bn(\right.}{}$$

input
```
Integrate[(d*Sec[e + f*x])^n*(a + b*Sec[e + f*x])^3,x]
```

output
```
-((Csc[e + f*x]^3*(a^3*(6 + 11*n + 6*n^2 + n^3)*Cos[e + f*x]^3*Hypergeomet
ric2F1[1/2, n/2, (2 + n)/2, Sec[e + f*x]^2] + b*n*(3*a^2*(6 + 5*n + n^2)*C
os[e + f*x]^2*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2, Sec[e + f*x]^2]
 + b*(1 + n)*(3*a*(3 + n)*Cos[e + f*x]*Hypergeometric2F1[1/2, (2 + n)/2, (
4 + n)/2, Sec[e + f*x]^2] + b*(2 + n)*Hypergeometric2F1[1/2, (3 + n)/2, (5
 + n)/2, Sec[e + f*x]^2])))*(d*Sec[e + f*x])^n*(-Tan[e + f*x]^2)^(3/2))/(f
*n*(1 + n)*(2 + n)*(3 + n)))
```

3.777.3 Rubi [A] (verified)

Time = 1.39 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.04, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.565$, Rules used = {3042, 4329, 3042, 4535, 3042, 4259, 3042, 3122, 4534, 3042, 4259, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\sec(e+fx))^3(d\sec(e+fx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a+b\csc\left(e+fx+\frac{\pi}{2}\right)\right)^3\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^n dx$$

$$\Big\downarrow \text{4329}$$

$$\frac{\int (d\sec(e+fx))^n\left(ab^2d(2n+5)\sec^2(e+fx)+bd\left(3(n+2)a^2+b^2(n+1)\right)\sec(e+fx)+ad\left((n+2)a^2+b^2n\right)\right)d}{d(n+2)}$$
$$\frac{b^2\tan(e+fx)(a+b\sec(e+fx))(d\sec(e+fx))^n}{f(n+2)}$$

$$\Big\downarrow \; 3042$$

$$\frac{\int \left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^n \left(ab^2d(2n+5)\csc\left(e+fx+\frac{\pi}{2}\right)^2+bd\left(3(n+2)a^2+b^2(n+1)\right)\csc\left(e+fx+\frac{\pi}{2}\right)+ad\left((n\right.\right.}{d(n+2)}$$
$$\frac{b^2\tan(e+fx)(a+b\sec(e+fx))(d\sec(e+fx))^n}{f(n+2)}$$

$$\Big\downarrow \; 4535$$

$$\frac{\int (d\sec(e+fx))^n\left(ab^2d(2n+5)\sec^2(e+fx)+ad\left((n+2)a^2+b^2n\right)\right)dx+b\left(3a^2(n+2)+b^2(n+1)\right)\int (d\sec(e+}{d(n+2)}$$
$$\frac{b^2\tan(e+fx)(a+b\sec(e+fx))(d\sec(e+fx))^n}{f(n+2)}$$

$$\Big\downarrow \; 3042$$

$$\frac{b\left(3a^2(n+2)+b^2(n+1)\right)\int \left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{n+1}dx+\int \left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^n\left(ab^2d(2n+5)\csc\left(e+fx+\frac{\pi}{2}\right.\right.}{d(n+2)}$$
$$\frac{b^2\tan(e+fx)(a+b\sec(e+fx))(d\sec(e+fx))^n}{f(n+2)}$$

$$\Big\downarrow \; 4259$$

$$\frac{\int \left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^n\left(ab^2d(2n+5)\csc\left(e+fx+\frac{\pi}{2}\right)^2+ad\left((n+2)a^2+b^2n\right)\right)dx+b\left(3a^2(n+2)+b^2(n+1)\right)}{d(n+2)}$$
$$\frac{b^2\tan(e+fx)(a+b\sec(e+fx))(d\sec(e+fx))^n}{f(n+2)}$$

$$\Big\downarrow \; 3042$$

$$\frac{\int \left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^n\left(ab^2d(2n+5)\csc\left(e+fx+\frac{\pi}{2}\right)^2+ad\left((n+2)a^2+b^2n\right)\right)dx+b\left(3a^2(n+2)+b^2(n+1)\right)}{d(n+2)}$$
$$\frac{b^2\tan(e+fx)(a+b\sec(e+fx))(d\sec(e+fx))^n}{f(n+2)}$$

$$\Big\downarrow \; 3122$$

$$\frac{\int \left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^n\left(ab^2d(2n+5)\csc\left(e+fx+\frac{\pi}{2}\right)^2+ad\left((n+2)a^2+b^2n\right)\right)dx+\frac{bd\left(3a^2(n+2)+b^2(n+1)\right)\sin(e+f}{}}{d(n+2)}$$
$$\frac{b^2\tan(e+fx)(a+b\sec(e+fx))(d\sec(e+fx))^n}{f(n+2)}$$

3.777. $\int (d\sec(e+fx))^n(a+b\sec(e+fx))^3\,dx$

$$\Big\downarrow \; 4534$$

$$\frac{\frac{ad(n+2)\left(a^2(n+1)+3b^2n\right)\int(d\sec(e+fx))^n dx}{n+1}+\frac{bd\left(3a^2(n+2)+b^2(n+1)\right)\sin(e+fx)(d\sec(e+fx))^n\,\text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{n}{2},\frac{2-n}{2},\cos^2(e+fx)\right.}{fn\sqrt{\sin^2(e+fx)}}}{d(n+2)}$$
$$\frac{b^2\tan(e+fx)(a+b\sec(e+fx))(d\sec(e+fx))^n}{f(n+2)}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{ad(n+2)\left(a^2(n+1)+3b^2n\right)\int\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^n dx}{n+1}+\frac{bd\left(3a^2(n+2)+b^2(n+1)\right)\sin(e+fx)(d\sec(e+fx))^n\,\text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{n}{2},\frac{2-n}{2},\cos^2(e-\right.}{fn\sqrt{\sin^2(e+fx)}}}{d(n+2)}$$
$$\frac{b^2\tan(e+fx)(a+b\sec(e+fx))(d\sec(e+fx))^n}{f(n+2)}$$

$$\Big\downarrow \; 4259$$

$$\frac{\frac{ad(n+2)\left(a^2(n+1)+3b^2n\right)\left(\frac{\cos(e+fx)}{d}\right)^n(d\sec(e+fx))^n\int\left(\frac{\cos(e+fx)}{d}\right)^{-n}dx}{n+1}+\frac{bd\left(3a^2(n+2)+b^2(n+1)\right)\sin(e+fx)(d\sec(e+fx))^n\,\text{Hypergeometr}}{fn\sqrt{\sin^2(e+fx)}}}{d(n+2)}$$
$$\frac{b^2\tan(e+fx)(a+b\sec(e+fx))(d\sec(e+fx))^n}{f(n+2)}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{ad(n+2)\left(a^2(n+1)+3b^2n\right)\left(\frac{\cos(e+fx)}{d}\right)^n(d\sec(e+fx))^n\int\left(\frac{\sin\left(e+fx+\frac{\pi}{2}\right)}{d}\right)^{-n}dx}{n+1}+\frac{bd\left(3a^2(n+2)+b^2(n+1)\right)\sin(e+fx)(d\sec(e+fx))^n\,\text{Hypergeo}}{fn\sqrt{\sin^2(e+fx)}}}{d(n+2)}$$
$$\frac{b^2\tan(e+fx)(a+b\sec(e+fx))(d\sec(e+fx))^n}{f(n+2)}$$

$$\Big\downarrow \; 3122$$

$$\frac{-\frac{ad^2(n+2)\left(a^2(n+1)+3b^2n\right)\sin(e+fx)(d\sec(e+fx))^{n-1}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1-n}{2},\frac{3-n}{2},\cos^2(e+fx)\right)}{f(1-n)(n+1)\sqrt{\sin^2(e+fx)}}+\frac{bd\left(3a^2(n+2)+b^2(n+1)\right)\sin(e+fx)(}{}}{d(n+2)}$$
$$\frac{b^2\tan(e+fx)(a+b\sec(e+fx))(d\sec(e+fx))^n}{f(n+2)}$$

input
```
Int[(d*Sec[e + f*x])^n*(a + b*Sec[e + f*x])^3,x]
```

3.777. $\int(d\sec(e+fx))^n(a+b\sec(e+fx))^3\,dx$

output
```
(b^2*(d*Sec[e + f*x])^n*(a + b*Sec[e + f*x])*Tan[e + f*x])/(f*(2 + n)) + (
-((a*d^2*(2 + n)*(3*b^2*n + a^2*(1 + n))*Hypergeometric2F1[1/2, (1 - n)/2,
 (3 - n)/2, Cos[e + f*x]^2]*(d*Sec[e + f*x])^(-1 + n)*Sin[e + f*x])/(f*(1
- n)*(1 + n)*Sqrt[Sin[e + f*x]^2])) + (b*d*(b^2*(1 + n) + 3*a^2*(2 + n))*H
ypergeometric2F1[1/2, -1/2*n, (2 - n)/2, Cos[e + f*x]^2]*(d*Sec[e + f*x])^
n*Sin[e + f*x])/(f*n*Sqrt[Sin[e + f*x]^2]) + (a*b^2*d*(5 + 2*n)*(d*Sec[e +
 f*x])^n*Tan[e + f*x])/(f*(1 + n)))/(d*(2 + n))
```

3.777.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

rule 4259
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)   Int[1/(Sin[c + d*x]/b)^n, x]), x] /;
FreeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

rule 4329
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
2)*((d*Csc[e + f*x])^n/(f*(m + n - 1))), x] + Simp[1/(d*(m + n - 1))   Int[
(a + b*Csc[e + f*x])^(m - 3)*(d*Csc[e + f*x])^n*Simp[a^3*d*(m + n - 1) + a*
b^2*d*n + b*(b^2*d*(m + n - 2) + 3*a^2*d*(m + n - 1))*Csc[e + f*x] + a*b^2*
d*(3*m + 2*n - 4)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, n}, x
] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
 && !(IGtQ[n, 2] && !IntegerQ[m])
```

rule 4534
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Simp[(C*m + A*(m + 1))/(m + 1)   Int[(b*Csc[e + f*x])^m, x], x] /;
 FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] && !LeQ[m, -1]
```

rule 4535
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Simp[B/b   Int[(b*Cs
c[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2)
, x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

3.777.4 Maple [F]

$$\int (d\sec(fx+e))^n (a+b\sec(fx+e))^3 \, dx$$

input
```
int((d*sec(f*x+e))^n*(a+b*sec(f*x+e))^3,x)
```

output
```
int((d*sec(f*x+e))^n*(a+b*sec(f*x+e))^3,x)
```

3.777.5 Fricas [F]

$$\int (d\sec(e+fx))^n(a+b\sec(e+fx))^3 \, dx = \int (b\sec(fx+e)+a)^3(d\sec(fx+e))^n \, dx$$

input
```
integrate((d*sec(f*x+e))^n*(a+b*sec(f*x+e))^3,x, algorithm="fricas")
```

output
```
integral((b^3*sec(f*x + e)^3 + 3*a*b^2*sec(f*x + e)^2 + 3*a^2*b*sec(f*x +
e) + a^3)*(d*sec(f*x + e))^n, x)
```

3.777.6 Sympy [F]

$$\int (d\sec(e+fx))^n(a+b\sec(e+fx))^3 \, dx = \int (d\sec(e+fx))^n (a+b\sec(e+fx))^3 \, dx$$

input
```
integrate((d*sec(f*x+e))**n*(a+b*sec(f*x+e))**3,x)
```

output
```
Integral((d*sec(e + f*x))**n*(a + b*sec(e + f*x))**3, x)
```

3.777.7 Maxima [F]

$$\int (d\sec(e+fx))^n(a+b\sec(e+fx))^3\,dx = \int (b\sec(fx+e)+a)^3(d\sec(fx+e))^n\,dx$$

input
```
integrate((d*sec(f*x+e))^n*(a+b*sec(f*x+e))^3,x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e) + a)^3*(d*sec(f*x + e))^n, x)
```

3.777.8 Giac [F]

$$\int (d\sec(e+fx))^n(a+b\sec(e+fx))^3\,dx = \int (b\sec(fx+e)+a)^3(d\sec(fx+e))^n\,dx$$

input
```
integrate((d*sec(f*x+e))^n*(a+b*sec(f*x+e))^3,x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e) + a)^3*(d*sec(f*x + e))^n, x)
```

3.777.9 Mupad [F(-1)]

Timed out.

$$\int (d\sec(e+fx))^n(a+b\sec(e+fx))^3\,dx = \int \left(a+\frac{b}{\cos(e+f\,x)}\right)^3\left(\frac{d}{\cos(e+f\,x)}\right)^n dx$$

input
```
int((a + b/cos(e + f*x))^3*(d/cos(e + f*x))^n,x)
```

output
```
int((a + b/cos(e + f*x))^3*(d/cos(e + f*x))^n, x)
```

3.778 $\int (d\sec(e+fx))^n(a+b\sec(e+fx))^2\,dx$

3.778.1 Optimal result 5786
3.778.2 Mathematica [A] (verified) 5787
3.778.3 Rubi [A] (verified) 5787
3.778.4 Maple [F] 5790
3.778.5 Fricas [F] 5790
3.778.6 Sympy [F] 5791
3.778.7 Maxima [F] 5791
3.778.8 Giac [F] 5791
3.778.9 Mupad [F(-1)] 5792

3.778.1 Optimal result

Integrand size = 23, antiderivative size = 181

$$\int (d\sec(e+fx))^n(a+b\sec(e+fx))^2\,dx = -\frac{d(b^2n+a^2(1+n))\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1-n}{2},\frac{3-n}{2},\cos^2(e+fx)\right)(d\sec(e+fx))^{-1+n}\sin(e+fx)}{f\,(1-n^2)\sqrt{\sin^2(e+fx)}} + \frac{2ab\,\text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{n}{2},\frac{2-n}{2},\cos^2(e+fx)\right)(d\sec(e+fx))^n\sin(e+fx)}{fn\sqrt{\sin^2(e+fx)}} + \frac{b^2(d\sec(e+fx))^n\tan(e+fx)}{f(1+n)}$$

output
```
-d*(b^2*n+a^2*(1+n))*hypergeom([1/2, 1/2-1/2*n],[3/2-1/2*n],cos(f*x+e)^2)*
(d*sec(f*x+e))^(-1+n)*sin(f*x+e)/f/(-n^2+1)/(sin(f*x+e)^2)^(1/2)+2*a*b*hyp
ergeom([1/2, -1/2*n],[1-1/2*n],cos(f*x+e)^2)*(d*sec(f*x+e))^n*sin(f*x+e)/f
/n/(sin(f*x+e)^2)^(1/2)+b^2*(d*sec(f*x+e))^n*tan(f*x+e)/f/(1+n)
```

3.778.2 Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.94

$$\int (d\sec(e+fx))^n(a+b\sec(e+fx))^2\,dx$$
$$= \frac{\csc(e+fx)\left(a^2(2+3n+n^2)\cos^2(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n}{2},\frac{2+n}{2},\sec^2(e+fx)\right)+bn\left(2a(2+n)\,c\right.\right.}{}$$

input
```
Integrate[(d*Sec[e + f*x])^n*(a + b*Sec[e + f*x])^2,x]
```

output
```
(Csc[e + f*x]*(a^2*(2 + 3*n + n^2)*Cos[e + f*x]^2*Hypergeometric2F1[1/2, n
/2, (2 + n)/2, Sec[e + f*x]^2] + b*n*(2*a*(2 + n)*Cos[e + f*x]*Hypergeomet
ric2F1[1/2, (1 + n)/2, (3 + n)/2, Sec[e + f*x]^2] + b*(1 + n)*Hypergeometr
ic2F1[1/2, (2 + n)/2, (4 + n)/2, Sec[e + f*x]^2]))*Sec[e + f*x]*(d*Sec[e +
 f*x])^n*Sqrt[-Tan[e + f*x]^2])/(f*n*(1 + n)*(2 + n))
```

3.778.3 Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.99, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules used = {3042, 4275, 3042, 4259, 3042, 3122, 4534, 3042, 4259, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\sec(e+fx))^2(d\sec(e+fx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a+b\csc\left(e+fx+\frac{\pi}{2}\right)\right)^2\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^n dx$$

$$\Big\downarrow \text{4275}$$

$$\int (d\sec(e+fx))^n\left(a^2+b^2\sec^2(e+fx)\right)dx+\frac{2ab\int (d\sec(e+fx))^{n+1}dx}{d}$$

$$\Big\downarrow \text{3042}$$

$$\int \left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^n\left(a^2+b^2\csc\left(e+fx+\frac{\pi}{2}\right)^2\right)dx+\frac{2ab\int \left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{n+1}dx}{d}$$

$$\Big\downarrow \text{4259}$$

$$\int \left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^n \left(a^2+b^2\csc\left(e+fx+\frac{\pi}{2}\right)^2\right)dx+\frac{2ab\left(\frac{\cos(e+fx)}{d}\right)^n (d\sec(e+fx))^n\int\left(\frac{\cos(e+fx)}{d}\right)^{-n-1}dx}{d}$$

$$\Big\downarrow \text{3042}$$

$$\int \left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^n \left(a^2+b^2\csc\left(e+fx+\frac{\pi}{2}\right)^2\right)dx+\frac{2ab\left(\frac{\cos(e+fx)}{d}\right)^n (d\sec(e+fx))^n\int\left(\frac{\sin\left(e+fx+\frac{\pi}{2}\right)}{d}\right)^{-n-1}dx}{d}$$

$$\Big\downarrow \text{3122}$$

$$\int \left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^n \left(a^2+b^2\csc\left(e+fx+\frac{\pi}{2}\right)^2\right)dx+\frac{2ab\sin(e+fx)(d\sec(e+fx))^n\,\text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{n}{2},\frac{2-n}{2},\cos^2(e+fx)\right)}{fn\sqrt{\sin^2(e+fx)}}$$

$$\Big\downarrow \text{4534}$$

$$\left(a^2+\frac{b^2n}{n+1}\right)\int(d\sec(e+fx))^n dx+\frac{2ab\sin(e+fx)(d\sec(e+fx))^n\,\text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{n}{2},\frac{2-n}{2},\cos^2(e+fx)\right)}{fn\sqrt{\sin^2(e+fx)}}+\frac{b^2\tan(e+fx)(d\sec(e+fx))^n}{f(n+1)}$$

$$\Big\downarrow \text{3042}$$

$$\left(a^2+\frac{b^2n}{n+1}\right)\int\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^n dx+\frac{2ab\sin(e+fx)(d\sec(e+fx))^n\,\text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{n}{2},\frac{2-n}{2},\cos^2(e+fx)\right)}{fn\sqrt{\sin^2(e+fx)}}+\frac{b^2\tan(e+fx)(d\sec(e+fx))^n}{f(n+1)}$$

$$\Big\downarrow \text{4259}$$

$$\left(a^2+\frac{b^2n}{n+1}\right)\left(\frac{\cos(e+fx)}{d}\right)^n(d\sec(e+fx))^n\int\left(\frac{\cos(e+fx)}{d}\right)^{-n}dx+\frac{2ab\sin(e+fx)(d\sec(e+fx))^n\,\text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{n}{2},\frac{2-n}{2},\cos^2(e+fx)\right)}{fn\sqrt{\sin^2(e+fx)}}+\frac{b^2\tan(e+fx)(d\sec(e+fx))^n}{f(n+1)}$$

3.778. $\int (d\sec(e+fx))^n(a+b\sec(e+fx))^2\,dx$

$$\Big\downarrow \text{3042}$$

$$\left(a^2+\frac{b^2n}{n+1}\right)\left(\frac{\cos(e+fx)}{d}\right)^n(d\sec(e+fx))^n\int\left(\frac{\sin\left(e+fx+\frac{\pi}{2}\right)}{d}\right)^{-n}dx+\frac{2ab\sin(e+fx)(d\sec(e+fx))^n\,\text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{n}{2},\frac{2-n}{2},\cos^2(e+fx)\right)}{fn\sqrt{\sin^2(e+fx)}}+\frac{b^2\tan(e+fx)(d\sec(e+fx))^n}{f(n+1)}$$

$$\Big\downarrow \text{3122}$$

$$-\frac{d\left(a^2+\frac{b^2n}{n+1}\right)\sin(e+fx)(d\sec(e+fx))^{n-1}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1-n}{2},\frac{3-n}{2},\cos^2(e+fx)\right)}{f(1-n)\sqrt{\sin^2(e+fx)}}+\frac{2ab\sin(e+fx)(d\sec(e+fx))^n\,\text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{n}{2},\frac{2-n}{2},\cos^2(e+fx)\right)}{fn\sqrt{\sin^2(e+fx)}}+\frac{b^2\tan(e+fx)(d\sec(e+fx))^n}{f(n+1)}$$

input
```
Int[(d*Sec[e + f*x])^n*(a + b*Sec[e + f*x])^2,x]
```

output
```
-((d*(a^2 + (b^2*n)/(1 + n))*Hypergeometric2F1[1/2, (1 - n)/2, (3 - n)/2,
Cos[e + f*x]^2]*(d*Sec[e + f*x])^(-1 + n)*Sin[e + f*x])/(f*(1 - n)*Sqrt[Si
n[e + f*x]^2])) + (2*a*b*Hypergeometric2F1[1/2, -1/2*n, (2 - n)/2, Cos[e +
 f*x]^2]*(d*Sec[e + f*x])^n*Sin[e + f*x])/(f*n*Sqrt[Sin[e + f*x]^2]) + (b^
2*(d*Sec[e + f*x])^n*Tan[e + f*x])/(f*(1 + n))
```

3.778.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

rule 4259
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)   Int[1/(Sin[c + d*x]/b)^n, x]), x] /;
FreeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

rule 4275
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Simp[2*a*(b/d)   Int[(d*Csc[e + f*x])^(n + 1), x], x]
 + Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
 e, f, n}, x]
```

rule 4534
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Simp[(C*m + A*(m + 1))/(m + 1)   Int[(b*Csc[e + f*x])^m, x], x] /;
 FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

3.778.4 Maple [F]

$$\int (d\sec(fx+e))^n (a+b\sec(fx+e))^2 \, dx$$

input
```
int((d*sec(f*x+e))^n*(a+b*sec(f*x+e))^2,x)
```

output
```
int((d*sec(f*x+e))^n*(a+b*sec(f*x+e))^2,x)
```

3.778.5 Fricas [F]

$$\int (d\sec(e+fx))^n(a+b\sec(e+fx))^2 \, dx = \int (b\sec(fx+e)+a)^2(d\sec(fx+e))^n \, dx$$

input
```
integrate((d*sec(f*x+e))^n*(a+b*sec(f*x+e))^2,x, algorithm="fricas")
```

output
```
integral((b^2*sec(f*x + e)^2 + 2*a*b*sec(f*x + e) + a^2)*(d*sec(f*x + e))^
n, x)
```

3.778.6 Sympy [F]

$$\int (d\sec(e+fx))^n(a+b\sec(e+fx))^2\,dx = \int (d\sec(e+fx))^n(a+b\sec(e+fx))^2\,dx$$

input
```
integrate((d*sec(f*x+e))**n*(a+b*sec(f*x+e))**2,x)
```

output
```
Integral((d*sec(e + f*x))**n*(a + b*sec(e + f*x))**2, x)
```

3.778.7 Maxima [F]

$$\int (d\sec(e+fx))^n(a+b\sec(e+fx))^2\,dx = \int (b\sec(fx+e)+a)^2(d\sec(fx+e))^n\,dx$$

input
```
integrate((d*sec(f*x+e))^n*(a+b*sec(f*x+e))^2,x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e) + a)^2*(d*sec(f*x + e))^n, x)
```

3.778.8 Giac [F]

$$\int (d\sec(e+fx))^n(a+b\sec(e+fx))^2\,dx = \int (b\sec(fx+e)+a)^2(d\sec(fx+e))^n\,dx$$

input
```
integrate((d*sec(f*x+e))^n*(a+b*sec(f*x+e))^2,x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e) + a)^2*(d*sec(f*x + e))^n, x)
```

3.778.9 Mupad [F(-1)]

Timed out.

$$\int (d\sec(e+fx))^n(a+b\sec(e+fx))^2\,dx=\int \left(a+\frac{b}{\cos\left(e+f\,x\right)}\right)^2\left(\frac{d}{\cos\left(e+f\,x\right)}\right)^n\,dx$$

input
```
int((a + b/cos(e + f*x))^2*(d/cos(e + f*x))^n,x)
```

output
```
int((a + b/cos(e + f*x))^2*(d/cos(e + f*x))^n, x)
```

3.779 $\int (d\sec(e+fx))^n(a+b\sec(e+fx))\,dx$

3.779.1 Optimal result 5793
3.779.2 Mathematica [A] (verified) 5793
3.779.3 Rubi [A] (verified) 5794
3.779.4 Maple [F] 5796
3.779.5 Fricas [F] 5796
3.779.6 Sympy [F] 5796
3.779.7 Maxima [F] 5797
3.779.8 Giac [F] 5797
3.779.9 Mupad [F(-1)] 5797

3.779.1 Optimal result

Integrand size = 21, antiderivative size = 137

$$\int (d\sec(e+fx))^n(a+b\sec(e+fx))\,dx = -\frac{ad\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1-n}{2},\frac{3-n}{2},\cos^2(e+fx)\right)(d\sec(e+fx))^{-1+n}\sin(e+fx)}{f(1-n)\sqrt{\sin^2(e+fx)}} + \frac{b\,\text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{n}{2},\frac{2-n}{2},\cos^2(e+fx)\right)(d\sec(e+fx))^n\sin(e+fx)}{fn\sqrt{\sin^2(e+fx)}}$$

output
```
-a*d*hypergeom([1/2, 1/2-1/2*n],[3/2-1/2*n],cos(f*x+e)^2)*(d*sec(f*x+e))^(
-1+n)*sin(f*x+e)/f/(1-n)/(sin(f*x+e)^2)^(1/2)+b*hypergeom([1/2, -1/2*n],[1
-1/2*n],cos(f*x+e)^2)*(d*sec(f*x+e))^n*sin(f*x+e)/f/n/(sin(f*x+e)^2)^(1/2)
```

3.779.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.78

$$\int (d\sec(e+fx))^n(a+b\sec(e+fx))\,dx = \frac{\csc(e+fx)\left(a(1+n)\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n}{2},\frac{2+n}{2},\sec^2(e+fx)\right)+bn\,\text{Hypergeometric2F1}\right.}{fn(1+n)}$$

input
```
Integrate[(d*Sec[e + f*x])^n*(a + b*Sec[e + f*x]),x]
```

output
```
(Csc[e + f*x]*(a*(1 + n)*Cos[e + f*x]*Hypergeometric2F1[1/2, n/2, (2 + n)/
2, Sec[e + f*x]^2] + b*n*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2, Sec[
e + f*x]^2])*(d*Sec[e + f*x])^n*Sqrt[-Tan[e + f*x]^2])/(f*n*(1 + n))
```

3.779.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 4274, 3042, 4259, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\sec(e+fx))(d\sec(e+fx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a+b\csc\left(e+fx+\frac{\pi}{2}\right)\right)\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^n dx$$

$$\Big\downarrow \text{4274}$$

$$a\int (d\sec(e+fx))^n dx+\frac{b\int (d\sec(e+fx))^{n+1}dx}{d}$$

$$\Big\downarrow \text{3042}$$

$$a\int \left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^n dx+\frac{b\int \left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{n+1}dx}{d}$$

$$\Big\downarrow \text{4259}$$

$$a\left(\frac{\cos(e+fx)}{d}\right)^n(d\sec(e+fx))^n\int\left(\frac{\cos(e+fx)}{d}\right)^{-n}dx+\frac{b\left(\frac{\cos(e+fx)}{d}\right)^n(d\sec(e+fx))^n\int\left(\frac{\cos(e+fx)}{d}\right)^{-n-1}dx}{d}$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{\cos(e+fx)}{d}\right)^n(d\sec(e+fx))^n\int\left(\frac{\sin\left(e+fx+\frac{\pi}{2}\right)}{d}\right)^{-n}dx+\frac{b\left(\frac{\cos(e+fx)}{d}\right)^n(d\sec(e+fx))^n\int\left(\frac{\sin\left(e+fx+\frac{\pi}{2}\right)}{d}\right)^{-n-1}dx}{d}$$

$$\Big\downarrow \text{3122}$$

$$\frac{b\sin(e+fx)(d\sec(e+fx))^n\,\text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{n}{2},\frac{2-n}{2},\cos^2(e+fx)\right)}{fn\sqrt{\sin^2(e+fx)}}-\frac{ad\sin(e+fx)(d\sec(e+fx))^{n-1}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1-n}{2},\frac{3-n}{2},\cos^2(e+fx)\right)}{f(1-n)\sqrt{\sin^2(e+fx)}}$$

input
```
Int[(d*Sec[e + f*x])^n*(a + b*Sec[e + f*x]),x]
```

output
```
-((a*d*Hypergeometric2F1[1/2, (1 - n)/2, (3 - n)/2, Cos[e + f*x]^2]*(d*Sec
[e + f*x])^(-1 + n)*Sin[e + f*x])/(f*(1 - n)*Sqrt[Sin[e + f*x]^2])) + (b*H
ypergeometric2F1[1/2, -1/2*n, (2 - n)/2, Cos[e + f*x]^2]*(d*Sec[e + f*x])^
n*Sin[e + f*x])/(f*n*Sqrt[Sin[e + f*x]^2])
```

3.779.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

rule 4259
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)   Int[1/(Sin[c + d*x]/b)^n, x]), x] /;
FreeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

3.779.4 Maple [F]

$$\int (d\sec(fx+e))^n (a+b\sec(fx+e))\,dx$$

input
```
int((d*sec(f*x+e))^n*(a+b*sec(f*x+e)),x)
```

output
```
int((d*sec(f*x+e))^n*(a+b*sec(f*x+e)),x)
```

3.779.5 Fricas [F]

$$\int (d\sec(e+fx))^n(a+b\sec(e+fx))\,dx = \int (b\sec(fx+e)+a)(d\sec(fx+e))^n\,dx$$

input
```
integrate((d*sec(f*x+e))^n*(a+b*sec(f*x+e)),x, algorithm="fricas")
```

output
```
integral((b*sec(f*x + e) + a)*(d*sec(f*x + e))^n, x)
```

3.779.6 Sympy [F]

$$\int (d\sec(e+fx))^n(a+b\sec(e+fx))\,dx = \int (d\sec(e+fx))^n(a+b\sec(e+fx))\,dx$$

input
```
integrate((d*sec(f*x+e))**n*(a+b*sec(f*x+e)),x)
```

output
```
Integral((d*sec(e + f*x))**n*(a + b*sec(e + f*x)), x)
```

3.779.7 Maxima [F]

$$\int (d\sec(e+fx))^n(a+b\sec(e+fx))\,dx = \int (b\sec(fx+e)+a)(d\sec(fx+e))^n\,dx$$

input
```
integrate((d*sec(f*x+e))^n*(a+b*sec(f*x+e)),x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e) + a)*(d*sec(f*x + e))^n, x)
```

3.779.8 Giac [F]

$$\int (d\sec(e+fx))^n(a+b\sec(e+fx))\,dx = \int (b\sec(fx+e)+a)(d\sec(fx+e))^n\,dx$$

input
```
integrate((d*sec(f*x+e))^n*(a+b*sec(f*x+e)),x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e) + a)*(d*sec(f*x + e))^n, x)
```

3.779.9 Mupad [F(-1)]

Timed out.

$$\int (d\sec(e+fx))^n(a+b\sec(e+fx))\,dx = \int \left(a+\frac{b}{\cos(e+f\,x)}\right)\left(\frac{d}{\cos(e+f\,x)}\right)^n dx$$

input
```
int((a + b/cos(e + f*x))*(d/cos(e + f*x))^n,x)
```

output
```
int((a + b/cos(e + f*x))*(d/cos(e + f*x))^n, x)
```

3.780 $\int \frac{(d\sec(e+fx))^n}{a+b\sec(e+fx)}\,dx$

3.780.1 Optimal result . . . 5798
3.780.2 Mathematica [B] (warning: unable to verify) . . . 5798
3.780.3 Rubi [A] (verified) . . . 5799
3.780.4 Maple [F] . . . 5801
3.780.5 Fricas [F] . . . 5801
3.780.6 Sympy [F] . . . 5802
3.780.7 Maxima [F] . . . 5802
3.780.8 Giac [F] . . . 5802
3.780.9 Mupad [F(-1)] . . . 5803

3.780.1 Optimal result

Integrand size = 23, antiderivative size = 192

$$\int \frac{(d\sec(e+fx))^n}{a+b\sec(e+fx)}\,dx = \frac{a\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2}(-1+n),1,\frac{3}{2},\sin^2(e+fx),\frac{a^2\sin^2(e+fx)}{a^2-b^2}\right)\cos(e+fx)\cos^2(e+fx)^{\frac{1}{2}(-1+n)}(d\sec(e+fx))}{(a^2-b^2)\,f} - \frac{b\,\text{AppellF1}\left(\frac{1}{2},\frac{n}{2},1,\frac{3}{2},\sin^2(e+fx),\frac{a^2\sin^2(e+fx)}{a^2-b^2}\right)\cos^2(e+fx)^{n/2}(d\sec(e+fx))^n\sin(e+fx)}{(a^2-b^2)\,f}$$

output
```
a*AppellF1(1/2,-1/2+1/2*n,1,3/2,sin(f*x+e)^2,a^2*sin(f*x+e)^2/(a^2-b^2))*c
os(f*x+e)*(cos(f*x+e)^2)^(-1/2+1/2*n)*(d*sec(f*x+e))^n*sin(f*x+e)/(a^2-b^2
)/f-b*AppellF1(1/2,1/2*n,1,3/2,sin(f*x+e)^2,a^2*sin(f*x+e)^2/(a^2-b^2))*(c
os(f*x+e)^2)^(1/2*n)*(d*sec(f*x+e))^n*sin(f*x+e)/(a^2-b^2)/f
```

3.780.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 5280 vs. 2(192) = 384.

Time = 35.15 (sec) , antiderivative size = 5280, normalized size of antiderivative = 27.50

$$\int \frac{(d\sec(e+fx))^n}{a+b\sec(e+fx)}\,dx = \text{Result too large to show}$$

input
```
Integrate[(d*Sec[e + f*x])^n/(a + b*Sec[e + f*x]),x]
```

output
```
Result too large to show
```

3.780.3 Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.08, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {3042, 4356, 3042, 3302, 3042, 3668, 25, 333}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d\sec(e+fx))^n}{a+b\sec(e+fx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^n}{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{4356}$$

$$\cos^n(e+fx)(d\sec(e+fx))^n\int \frac{\cos^{1-n}(e+fx)}{b+a\cos(e+fx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\cos^n(e+fx)(d\sec(e+fx))^n\int \frac{\sin\left(e+fx+\frac{\pi}{2}\right)^{1-n}}{b+a\sin\left(e+fx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3302}$$

$$\cos^n(e+fx)(d\sec(e+fx))^n\left(b\int \frac{\cos^{1-n}(e+fx)}{b^2-a^2\cos^2(e+fx)}dx-a\int \frac{\cos^{2-n}(e+fx)}{b^2-a^2\cos^2(e+fx)}dx\right)$$

$$\Big\downarrow \text{3042}$$

$$\cos^n(e+fx)(d\sec(e+fx))^n\left(b\int \frac{\sin\left(e+fx+\frac{\pi}{2}\right)^{1-n}}{b^2-a^2\sin\left(e+fx+\frac{\pi}{2}\right)^2}dx-a\int \frac{\sin\left(e+fx+\frac{\pi}{2}\right)^{2-n}}{b^2-a^2\sin\left(e+fx+\frac{\pi}{2}\right)^2}dx\right)$$

$$\Big\downarrow \text{3668}$$

$$\cos^n(e+fx)(d\sec(e+fx))^n\left(\frac{b\cos^{-n}(e+fx)\cos^2(e+fx)^{n/2}\int -\frac{(1-\sin^2(e+fx))^{-n/2}}{-\sin^2(e+fx)a^2+a^2-b^2}d\sin(e+fx)}{f}-\frac{a\cos^{1-n}(e+fx)\cos^2(e+fx)^{\frac{n-1}{2}}}{}\right.$$

$$\Big\downarrow \; 25$$

$$\cos^n(e+fx)(d\sec(e+fx))^n\left(\frac{a\cos^{1-n}(e+fx)\cos^2(e+fx)^{\frac{n-1}{2}}\int \frac{(1-\sin^2(e+fx))^{\frac{1-n}{2}}}{-\sin^2(e+fx)a^2+a^2-b^2}d\sin(e+fx)}{f}-\frac{b\cos^{-n}(e+fx)\cos^2(e+fx)^{n/2}\int}{}\right.$$

$$\Big\downarrow \; 333$$

$$\cos^n(e+fx)(d\sec(e+fx))^n\left(\frac{a\sin(e+fx)\cos^{1-n}(e+fx)\cos^2(e+fx)^{\frac{n-1}{2}}\text{AppellF1}\left(\frac{1}{2},\frac{n-1}{2},1,\frac{3}{2},\sin^2(e+fx),\frac{a^2\sin^2(e+fx)}{a^2-b^2}\right)}{f\left(a^2-b^2\right)}-\frac{b\sin(e}{}\right.$$

input
```
Int[(d*Sec[e + f*x])^n/(a + b*Sec[e + f*x]),x]
```

output
```
Cos[e + f*x]^n*(d*Sec[e + f*x])^n*((a*AppellF1[1/2, (-1 + n)/2, 1, 3/2, Si
n[e + f*x]^2, (a^2*Sin[e + f*x]^2)/(a^2 - b^2)]*Cos[e + f*x]^(1 - n)*(Cos[
e + f*x]^2)^((-1 + n)/2)*Sin[e + f*x])/((a^2 - b^2)*f) - (b*AppellF1[1/2,
n/2, 1, 3/2, Sin[e + f*x]^2, (a^2*Sin[e + f*x]^2)/(a^2 - b^2)]*(Cos[e + f*
x]^2)^(n/2)*Sin[e + f*x])/((a^2 - b^2)*f*Cos[e + f*x]^n))
```

3.780.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 333
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^p*c^q*x*AppellF1[1/2, -p, -q, 3/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; F
reeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[p] || GtQ[a,
0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3302
```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[a   Int[(d*Sin[e + f*x])^n/(a^2 - b^2*Sin[e + f*x]
^2), x], x] - Simp[b/d   Int[(d*Sin[e + f*x])^(n + 1)/(a^2 - b^2*Sin[e + f*
x]^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0]
```

rule 3668
```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[(
-ff)*d^(2*IntPart[(m - 1)/2] + 1)*((d*Sin[e + f*x])^(2*FracPart[(m - 1)/2])
/(f*(Sin[e + f*x]^2)^FracPart[(m - 1)/2]))   Subst[Int[(1 - ff^2*x^2)^((m -
 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b,
 d, e, f, m, p}, x] &&  !IntegerQ[m]
```

rule 4356
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Simp[Sin[e + f*x]^n*(d*Csc[e + f*x])^n   Int[(b +
 a*Sin[e + f*x])^m/Sin[e + f*x]^(m + n), x], x] /; FreeQ[{a, b, d, e, f, n}
, x] && NeQ[a^2 - b^2, 0] && IntegerQ[m]
```

3.780.4 Maple [F]

$$\int \frac{(d\sec(fx+e))^n}{a+b\sec(fx+e)}dx$$

input
```
int((d*sec(f*x+e))^n/(a+b*sec(f*x+e)),x)
```

output
```
int((d*sec(f*x+e))^n/(a+b*sec(f*x+e)),x)
```

3.780.5 Fricas [F]

$$\int \frac{(d\sec(e+fx))^n}{a+b\sec(e+fx)}\,dx = \int \frac{(d\sec(fx+e))^n}{b\sec(fx+e)+a}\,dx$$

input
```
integrate((d*sec(f*x+e))^n/(a+b*sec(f*x+e)),x, algorithm="fricas")
```

output
```
integral((d*sec(f*x + e))^n/(b*sec(f*x + e) + a), x)
```

3.780.6 Sympy [F]

$$\int \frac{(d\sec(e+fx))^n}{a+b\sec(e+fx)}\,dx=\int \frac{(d\sec(e+fx))^n}{a+b\sec(e+fx)}\,dx$$

input
```
integrate((d*sec(f*x+e))**n/(a+b*sec(f*x+e)),x)
```

output
```
Integral((d*sec(e + f*x))**n/(a + b*sec(e + f*x)), x)
```

3.780.7 Maxima [F]

$$\int \frac{(d\sec(e+fx))^n}{a+b\sec(e+fx)}\,dx=\int \frac{(d\sec(fx+e))^n}{b\sec(fx+e)+a}\,dx$$

input
```
integrate((d*sec(f*x+e))^n/(a+b*sec(f*x+e)),x, algorithm="maxima")
```

output
```
integrate((d*sec(f*x + e))^n/(b*sec(f*x + e) + a), x)
```

3.780.8 Giac [F]

$$\int \frac{(d\sec(e+fx))^n}{a+b\sec(e+fx)}\,dx=\int \frac{(d\sec(fx+e))^n}{b\sec(fx+e)+a}\,dx$$

input
```
integrate((d*sec(f*x+e))^n/(a+b*sec(f*x+e)),x, algorithm="giac")
```

output
```
integrate((d*sec(f*x + e))^n/(b*sec(f*x + e) + a), x)
```

3.780.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d\sec(e+fx))^n}{a+b\sec(e+fx)}\,dx=\int \frac{\left(\frac{d}{\cos(e+f\,x)}\right)^n}{a+\frac{b}{\cos(e+f\,x)}}\,dx$$

input
```
int((d/cos(e + f*x))^n/(a + b/cos(e + f*x)),x)
```

output
```
int((d/cos(e + f*x))^n/(a + b/cos(e + f*x)), x)
```

3.781 $\int \frac{(d\sec(e+fx))^n}{(a+b\sec(e+fx))^2}\,dx$

3.781.1 Optimal result 5804
3.781.2 Mathematica [B] (warning: unable to verify) 5805
3.781.3 Rubi [A] (verified) 5805
3.781.4 Maple [F] 5807
3.781.5 Fricas [F] 5807
3.781.6 Sympy [F] 5807
3.781.7 Maxima [F] 5808
3.781.8 Giac [F] 5808
3.781.9 Mupad [F(-1)] 5808

3.781.1 Optimal result

Integrand size = 23, antiderivative size = 299

$$\begin{aligned}
&\int \frac{(d\sec(e+fx))^n}{(a+b\sec(e+fx))^2}\,dx \\
&= \frac{a^2\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2}(-3+n),2,\frac{3}{2},\sin^2(e+fx),\frac{a^2\sin^2(e+fx)}{a^2-b^2}\right)\cos(e+fx)\cos^2(e+fx)^{\frac{1}{2}(-1+n)}(d\sec(e+fx)}{(a^2-b^2)^2 f} \\
&\quad + \frac{b^2\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2}(-1+n),2,\frac{3}{2},\sin^2(e+fx),\frac{a^2\sin^2(e+fx)}{a^2-b^2}\right)\cos(e+fx)\cos^2(e+fx)^{\frac{1}{2}(-1+n)}(d\sec(e+f}{(a^2-b^2)^2 f} \\
&\quad - \frac{2ab\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2}(-2+n),2,\frac{3}{2},\sin^2(e+fx),\frac{a^2\sin^2(e+fx)}{a^2-b^2}\right)\cos^2(e+fx)^{n/2}(d\sec(e+fx))^n\sin(e+fx}{(a^2-b^2)^2 f}
\end{aligned}$$

output
```
a^2*AppellF1(1/2,-3/2+1/2*n,2,3/2,sin(f*x+e)^2,a^2*sin(f*x+e)^2/(a^2-b^2))
*cos(f*x+e)*(cos(f*x+e)^2)^(-1/2+1/2*n)*(d*sec(f*x+e))^n*sin(f*x+e)/(a^2-b
^2)^2/f+b^2*AppellF1(1/2,-1/2+1/2*n,2,3/2,sin(f*x+e)^2,a^2*sin(f*x+e)^2/(a
^2-b^2))*cos(f*x+e)*(cos(f*x+e)^2)^(-1/2+1/2*n)*(d*sec(f*x+e))^n*sin(f*x+e
)/(a^2-b^2)^2/f-2*a*b*AppellF1(1/2,-1+1/2*n,2,3/2,sin(f*x+e)^2,a^2*sin(f*x
+e)^2/(a^2-b^2))*(cos(f*x+e)^2)^(1/2*n)*(d*sec(f*x+e))^n*sin(f*x+e)/(a^2-b
^2)^2/f
```

3.781.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 13816 vs. $2(299) = 598$.

Time = 52.09 (sec) , antiderivative size = 13816, normalized size of antiderivative = 46.21

$$\int \frac{(d\sec(e+fx))^n}{(a+b\sec(e+fx))^2}\,dx = \text{Result too large to show}$$

input
```
Integrate[(d*Sec[e + f*x])^n/(a + b*Sec[e + f*x])^2,x]
```

output
```
Result too large to show
```

3.781.3 Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 310, normalized size of antiderivative = 1.04, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 4356, 3042, 3303, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d\sec(e+fx))^n}{(a+b\sec(e+fx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^n}{\left(a+b\csc\left(e+fx+\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \text{4356}$$

$$\cos^n(e+fx)(d\sec(e+fx))^n\int \frac{\cos^{2-n}(e+fx)}{(b+a\cos(e+fx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\cos^n(e+fx)(d\sec(e+fx))^n\int \frac{\sin\left(e+fx+\frac{\pi}{2}\right)^{2-n}}{\left(b+a\sin\left(e+fx+\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \text{3303}$$

$$\cos^n(e+fx)(d\sec(e+fx))^n\int \left(\frac{b^2\cos^{2-n}(e+fx)}{(b^2-a^2\cos^2(e+fx))^2}-\frac{2ab\cos^{3-n}(e+fx)}{(b^2-a^2\cos^2(e+fx))^2}+\frac{a^2\cos^{4-n}(e+fx)}{(a^2\cos^2(e+fx)-b^2)^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\cos^n(e+fx)(d\sec(e+fx))^n\left(\frac{a^2\sin(e+fx)\cos^2(e+fx)^{\frac{n-1}{2}}\cos^{1-n}(e+fx)\operatorname{AppellF1}\left(\frac{1}{2},\frac{n-3}{2},2,\frac{3}{2},\sin^2(e+fx),\frac{a^2\sin^2(e+fx)}{a^2-b^2}\right)}{f\left(a^2-b^2\right)^2}+\frac{b^2\sin}{}\right.$$

input
```
Int[(d*Sec[e + f*x])^n/(a + b*Sec[e + f*x])^2,x]
```

output
```
Cos[e + f*x]^n*(d*Sec[e + f*x])^n*((a^2*AppellF1[1/2, (-3 + n)/2, 2, 3/2,
Sin[e + f*x]^2, (a^2*Sin[e + f*x]^2)/(a^2 - b^2)]*Cos[e + f*x]^(1 - n)*(Co
s[e + f*x]^2)^((-1 + n)/2)*Sin[e + f*x])/((a^2 - b^2)^2*f) + (b^2*AppellF1
[1/2, (-1 + n)/2, 2, 3/2, Sin[e + f*x]^2, (a^2*Sin[e + f*x]^2)/(a^2 - b^2)
]*Cos[e + f*x]^(1 - n)*(Cos[e + f*x]^2)^((-1 + n)/2)*Sin[e + f*x])/((a^2 -
 b^2)^2*f) - (2*a*b*AppellF1[1/2, (-2 + n)/2, 2, 3/2, Sin[e + f*x]^2, (a^2
*Sin[e + f*x]^2)/(a^2 - b^2)]*(Cos[e + f*x]^2)^(n/2)*Sin[e + f*x])/((a^2 -
 b^2)^2*f*Cos[e + f*x]^n))
```

3.781.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3303
```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(d*sin[e + f*x])^n*(1/((a - b*sin[
e + f*x])^m/(a^2 - b^2*sin[e + f*x]^2)^m)), x], x] /; FreeQ[{a, b, d, e, f,
 n}, x] && NeQ[a^2 - b^2, 0] && ILtQ[m, -1]
```

rule 4356
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Simp[Sin[e + f*x]^n*(d*Csc[e + f*x])^n   Int[(b +
 a*Sin[e + f*x])^m/Sin[e + f*x]^(m + n), x], x] /; FreeQ[{a, b, d, e, f, n}
, x] && NeQ[a^2 - b^2, 0] && IntegerQ[m]
```

3.781.4 Maple [F]

$$\int \frac{(d\sec(fx+e))^n}{(a+b\sec(fx+e))^2}dx$$

input
```
int((d*sec(f*x+e))^n/(a+b*sec(f*x+e))^2,x)
```

output
```
int((d*sec(f*x+e))^n/(a+b*sec(f*x+e))^2,x)
```

3.781.5 Fricas [F]

$$\int \frac{(d\sec(e+fx))^n}{(a+b\sec(e+fx))^2}\,dx = \int \frac{(d\sec(fx+e))^n}{(b\sec(fx+e)+a)^2}\,dx$$

input
```
integrate((d*sec(f*x+e))^n/(a+b*sec(f*x+e))^2,x, algorithm="fricas")
```

output
```
integral((d*sec(f*x + e))^n/(b^2*sec(f*x + e)^2 + 2*a*b*sec(f*x + e) + a^2
), x)
```

3.781.6 Sympy [F]

$$\int \frac{(d\sec(e+fx))^n}{(a+b\sec(e+fx))^2}\,dx = \int \frac{(d\sec(e+fx))^n}{(a+b\sec(e+fx))^2}\,dx$$

input
```
integrate((d*sec(f*x+e))**n/(a+b*sec(f*x+e))**2,x)
```

output
```
Integral((d*sec(e + f*x))**n/(a + b*sec(e + f*x))**2, x)
```

3.781.7 Maxima [F]

$$\int \frac{(d\sec(e+fx))^n}{(a+b\sec(e+fx))^2}\,dx = \int \frac{(d\sec(fx+e))^n}{(b\sec(fx+e)+a)^2}\,dx$$

input
```
integrate((d*sec(f*x+e))^n/(a+b*sec(f*x+e))^2,x, algorithm="maxima")
```

output
```
integrate((d*sec(f*x + e))^n/(b*sec(f*x + e) + a)^2, x)
```

3.781.8 Giac [F]

$$\int \frac{(d\sec(e+fx))^n}{(a+b\sec(e+fx))^2}\,dx = \int \frac{(d\sec(fx+e))^n}{(b\sec(fx+e)+a)^2}\,dx$$

input
```
integrate((d*sec(f*x+e))^n/(a+b*sec(f*x+e))^2,x, algorithm="giac")
```

output
```
integrate((d*sec(f*x + e))^n/(b*sec(f*x + e) + a)^2, x)
```

3.781.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d\sec(e+fx))^n}{(a+b\sec(e+fx))^2}\,dx = \int \frac{\left(\frac{d}{\cos(e+f\,x)}\right)^n}{\left(a+\frac{b}{\cos(e+f\,x)}\right)^2}\,dx$$

input
```
int((d/cos(e + f*x))^n/(a + b/cos(e + f*x))^2,x)
```

output
```
int((d/cos(e + f*x))^n/(a + b/cos(e + f*x))^2, x)
```

3.782 $\int (d\sec(e+fx))^n(a+b\sec(e+fx))^{3/2}\,dx$

3.782.1 Optimal result . . . 5809
3.782.2 Mathematica [N/A] . . . 5809
3.782.3 Rubi [N/A] . . . 5810
3.782.4 Maple [N/A] (verified) . . . 5811
3.782.5 Fricas [N/A] . . . 5811
3.782.6 Sympy [N/A] . . . 5811
3.782.7 Maxima [N/A] . . . 5812
3.782.8 Giac [N/A] . . . 5812
3.782.9 Mupad [N/A] . . . 5812

3.782.1 Optimal result

Integrand size = 25, antiderivative size = 25

$$\int (d\sec(e+fx))^n(a+b\sec(e+fx))^{3/2}\,dx=\text{Int}\left((d\sec(e+fx))^n(a+b\sec(e+fx))^{3/2},x\right)$$

output
```
Unintegrable((d*sec(f*x+e))^n*(a+b*sec(f*x+e))^(3/2),x)
```

3.782.2 Mathematica [N/A]

Not integrable

Time = 88.89 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int (d\sec(e+fx))^n(a+b\sec(e+fx))^{3/2}\,dx=\int (d\sec(e+fx))^n(a+b\sec(e+fx))^{3/2}\,dx$$

input
```
Integrate[(d*Sec[e + f*x])^n*(a + b*Sec[e + f*x])^(3/2),x]
```

output
```
Integrate[(d*Sec[e + f*x])^n*(a + b*Sec[e + f*x])^(3/2), x]
```

3.782.3 Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 4357}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\sec(e+fx))^{3/2}(d\sec(e+fx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a+b\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^n dx$$

$$\Big\downarrow \text{4357}$$

$$\int (a+b\sec(e+fx))^{3/2}(d\sec(e+fx))^n dx$$

input
```
Int[(d*Sec[e + f*x])^n*(a + b*Sec[e + f*x])^(3/2),x]
```

output
```
$Aborted
```

3.782.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4357
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Unintegrable[(d*Csc[e + f*x])^n*(a + b*Csc[e + f*
x])^m, x] /; FreeQ[{a, b, d, e, f, m, n}, x]
```

3.782.4 Maple [N/A] (verified)

Not integrable

Time = 0.68 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int (d\sec(fx+e))^n (a+b\sec(fx+e))^{\frac{3}{2}}\,dx$$

input
```
int((d*sec(f*x+e))^n*(a+b*sec(f*x+e))^(3/2),x)
```

output
```
int((d*sec(f*x+e))^n*(a+b*sec(f*x+e))^(3/2),x)
```

3.782.5 Fricas [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int (d\sec(e+fx))^n(a+b\sec(e+fx))^{3/2}\,dx = \int (b\sec(fx+e)+a)^{\frac{3}{2}}(d\sec(fx+e))^n\,dx$$

input
```
integrate((d*sec(f*x+e))^n*(a+b*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
integral((b*sec(f*x + e) + a)^(3/2)*(d*sec(f*x + e))^n, x)
```

3.782.6 Sympy [N/A]

Not integrable

Time = 26.36 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int (d\sec(e+fx))^n(a+b\sec(e+fx))^{3/2}\,dx = \int (d\sec(e+fx))^n (a+b\sec(e+fx))^{\frac{3}{2}}\,dx$$

input
```
integrate((d*sec(f*x+e))**n*(a+b*sec(f*x+e))**(3/2),x)
```

output
```
Integral((d*sec(e + f*x))**n*(a + b*sec(e + f*x))**(3/2), x)
```

3.782.7 Maxima [N/A]

Not integrable

Time = 1.07 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int (d\sec(e+fx))^n(a+b\sec(e+fx))^{3/2}\,dx = \int (b\sec(fx+e)+a)^{\frac{3}{2}}(d\sec(fx+e))^n\,dx$$

input
```
integrate((d*sec(f*x+e))^n*(a+b*sec(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e) + a)^(3/2)*(d*sec(f*x + e))^n, x)
```

3.782.8 Giac [N/A]

Not integrable

Time = 0.98 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int (d\sec(e+fx))^n(a+b\sec(e+fx))^{3/2}\,dx = \int (b\sec(fx+e)+a)^{\frac{3}{2}}(d\sec(fx+e))^n\,dx$$

input
```
integrate((d*sec(f*x+e))^n*(a+b*sec(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e) + a)^(3/2)*(d*sec(f*x + e))^n, x)
```

3.782.9 Mupad [N/A]

Not integrable

Time = 15.77 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.16

$$\int (d\sec(e+fx))^n(a+b\sec(e+fx))^{3/2}\,dx = \int \left(a+\frac{b}{\cos(e+f\,x)}\right)^{3/2}\left(\frac{d}{\cos(e+f\,x)}\right)^n dx$$

input
```
int((a + b/cos(e + f*x))^(3/2)*(d/cos(e + f*x))^n,x)
```

output
```
int((a + b/cos(e + f*x))^(3/2)*(d/cos(e + f*x))^n, x)
```

3.783 $\int (d\sec(e+fx))^n \sqrt{a+b\sec(e+fx)}\,dx$

3.783.1 Optimal result 5813
3.783.2 Mathematica [N/A] 5813
3.783.3 Rubi [N/A] 5814
3.783.4 Maple [N/A] (verified) 5815
3.783.5 Fricas [N/A] 5815
3.783.6 Sympy [N/A] 5815
3.783.7 Maxima [N/A] 5816
3.783.8 Giac [N/A] 5816
3.783.9 Mupad [N/A] 5816

3.783.1 Optimal result

Integrand size = 25, antiderivative size = 25

$$\int (d\sec(e+fx))^n \sqrt{a+b\sec(e+fx)}\,dx = \text{Int}\left((d\sec(e+fx))^n \sqrt{a+b\sec(e+fx)},x\right)$$

output
```
Unintegrable((d*sec(f*x+e))^n*(a+b*sec(f*x+e))^(1/2),x)
```

3.783.2 Mathematica [N/A]

Not integrable

Time = 0.93 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int (d\sec(e+fx))^n \sqrt{a+b\sec(e+fx)}\,dx = \int (d\sec(e+fx))^n \sqrt{a+b\sec(e+fx)}\,dx$$

input
```
Integrate[(d*Sec[e + f*x])^n*Sqrt[a + b*Sec[e + f*x]],x]
```

output
```
Integrate[(d*Sec[e + f*x])^n*Sqrt[a + b*Sec[e + f*x]], x]
```

3.783.3 Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 4357}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+b\sec(e+fx)}(d\sec(e+fx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^n dx$$

$$\Big\downarrow \text{4357}$$

$$\int \sqrt{a+b\sec(e+fx)}(d\sec(e+fx))^n dx$$

input
```
Int[(d*Sec[e + f*x])^n*Sqrt[a + b*Sec[e + f*x]],x]
```

output
```
$Aborted
```

3.783.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4357
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Unintegrable[(d*Csc[e + f*x])^n*(a + b*Csc[e + f*
x])^m, x] /; FreeQ[{a, b, d, e, f, m, n}, x]
```

3.783.4 Maple [N/A] (verified)

Not integrable

Time = 0.68 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int (d\sec(fx+e))^n \sqrt{a+b\sec(fx+e)}dx$$

input
```
int((d*sec(f*x+e))^n*(a+b*sec(f*x+e))^(1/2),x)
```

output
```
int((d*sec(f*x+e))^n*(a+b*sec(f*x+e))^(1/2),x)
```

3.783.5 Fricas [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int (d\sec(e+fx))^n \sqrt{a+b\sec(e+fx)}\,dx = \int \sqrt{b\sec(fx+e)+a}(d\sec(fx+e))^n\,dx$$

input
```
integrate((d*sec(f*x+e))^n*(a+b*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sec(f*x + e) + a)*(d*sec(f*x + e))^n, x)
```

3.783.6 Sympy [N/A]

Not integrable

Time = 0.60 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int (d\sec(e+fx))^n \sqrt{a+b\sec(e+fx)}\,dx = \int (d\sec(e+fx))^n \sqrt{a+b\sec(e+fx)}\,dx$$

input
```
integrate((d*sec(f*x+e))**n*(a+b*sec(f*x+e))**(1/2),x)
```

output
```
Integral((d*sec(e + f*x))**n*sqrt(a + b*sec(e + f*x)), x)
```

3.783.7 Maxima [N/A]

Not integrable

Time = 1.05 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int (d\sec(e+fx))^n\sqrt{a+b\sec(e+fx)}\,dx=\int \sqrt{b\sec{(fx+e)}+a}(d\sec{(fx+e)})^n\,dx$$

input
```
integrate((d*sec(f*x+e))^n*(a+b*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sec(f*x + e) + a)*(d*sec(f*x + e))^n, x)
```

3.783.8 Giac [N/A]

Not integrable

Time = 0.55 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int (d\sec(e+fx))^n\sqrt{a+b\sec(e+fx)}\,dx=\int \sqrt{b\sec{(fx+e)}+a}(d\sec{(fx+e)})^n\,dx$$

input
```
integrate((d*sec(f*x+e))^n*(a+b*sec(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*sec(f*x + e) + a)*(d*sec(f*x + e))^n, x)
```

3.783.9 Mupad [N/A]

Not integrable

Time = 14.91 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.16

$$\int (d\sec(e+fx))^n\sqrt{a+b\sec(e+fx)}\,dx=\int \sqrt{a+\frac{b}{\cos{(e+f\,x)}}}\,\left(\frac{d}{\cos{(e+f\,x)}}\right)^n\,dx$$

input
```
int((a + b/cos(e + f*x))^(1/2)*(d/cos(e + f*x))^n,x)
```

output
```
int((a + b/cos(e + f*x))^(1/2)*(d/cos(e + f*x))^n, x)
```

3.784 $\int \frac{(d\sec(e+fx))^n}{\sqrt{a+b\sec(e+fx)}}\,dx$

3.784.1 Optimal result 5817
3.784.2 Mathematica [N/A] 5817
3.784.3 Rubi [N/A] 5818
3.784.4 Maple [N/A] (verified) 5819
3.784.5 Fricas [N/A] 5819
3.784.6 Sympy [N/A] 5819
3.784.7 Maxima [N/A] 5820
3.784.8 Giac [N/A] 5820
3.784.9 Mupad [N/A] 5820

3.784.1 Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{(d\sec(e+fx))^n}{\sqrt{a+b\sec(e+fx)}}\,dx = \text{Int}\left(\frac{(d\sec(e+fx))^n}{\sqrt{a+b\sec(e+fx)}},x\right)$$

output
```
Unintegrable((d*sec(f*x+e))^n/(a+b*sec(f*x+e))^(1/2),x)
```

3.784.2 Mathematica [N/A]

Not integrable

Time = 22.05 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{(d\sec(e+fx))^n}{\sqrt{a+b\sec(e+fx)}}\,dx = \int \frac{(d\sec(e+fx))^n}{\sqrt{a+b\sec(e+fx)}}\,dx$$

input
```
Integrate[(d*Sec[e + f*x])^n/Sqrt[a + b*Sec[e + f*x]],x]
```

output
```
Integrate[(d*Sec[e + f*x])^n/Sqrt[a + b*Sec[e + f*x]], x]
```

3.784.3 Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 4357}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d\sec(e+fx))^n}{\sqrt{a+b\sec(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^n}{\sqrt{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4357}$$

$$\int \frac{(d\sec(e+fx))^n}{\sqrt{a+b\sec(e+fx)}}dx$$

input
```
Int[(d*Sec[e + f*x])^n/Sqrt[a + b*Sec[e + f*x]],x]
```

output
```
$Aborted
```

3.784.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4357
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Unintegrable[(d*Csc[e + f*x])^n*(a + b*Csc[e + f*
x])^m, x] /; FreeQ[{a, b, d, e, f, m, n}, x]
```

3.784.4 Maple [N/A] (verified)

Not integrable

Time = 0.65 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{(d\sec(fx+e))^n}{\sqrt{a+b\sec(fx+e)}}dx$$

input
```
int((d*sec(f*x+e))^n/(a+b*sec(f*x+e))^(1/2),x)
```

output
```
int((d*sec(f*x+e))^n/(a+b*sec(f*x+e))^(1/2),x)
```

3.784.5 Fricas [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{(d\sec(e+fx))^n}{\sqrt{a+b\sec(e+fx)}}\,dx = \int \frac{(d\sec(fx+e))^n}{\sqrt{b\sec(fx+e)+a}}\,dx$$

input
```
integrate((d*sec(f*x+e))^n/(a+b*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
integral((d*sec(f*x + e))^n/sqrt(b*sec(f*x + e) + a), x)
```

3.784.6 Sympy [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int \frac{(d\sec(e+fx))^n}{\sqrt{a+b\sec(e+fx)}}\,dx = \int \frac{(d\sec(e+fx))^n}{\sqrt{a+b\sec(e+fx)}}\,dx$$

input
```
integrate((d*sec(f*x+e))**n/(a+b*sec(f*x+e))**(1/2),x)
```

output
```
Integral((d*sec(e + f*x))**n/sqrt(a + b*sec(e + f*x)), x)
```

3.784.7 Maxima [N/A]

Not integrable

Time = 1.08 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{(d\sec(e+fx))^n}{\sqrt{a+b\sec(e+fx)}}\,dx = \int \frac{(d\sec(fx+e))^n}{\sqrt{b\sec(fx+e)+a}}\,dx$$

input
```
integrate((d*sec(f*x+e))^n/(a+b*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate((d*sec(f*x + e))^n/sqrt(b*sec(f*x + e) + a), x)
```

3.784.8 Giac [N/A]

Not integrable

Time = 0.69 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{(d\sec(e+fx))^n}{\sqrt{a+b\sec(e+fx)}}\,dx = \int \frac{(d\sec(fx+e))^n}{\sqrt{b\sec(fx+e)+a}}\,dx$$

input
```
integrate((d*sec(f*x+e))^n/(a+b*sec(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate((d*sec(f*x + e))^n/sqrt(b*sec(f*x + e) + a), x)
```

3.784.9 Mupad [N/A]

Not integrable

Time = 15.53 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.16

$$\int \frac{(d\sec(e+fx))^n}{\sqrt{a+b\sec(e+fx)}}\,dx = \int \frac{\left(\frac{d}{\cos(e+f\,x)}\right)^n}{\sqrt{a+\frac{b}{\cos(e+f\,x)}}}\,dx$$

input
```
int((d/cos(e + f*x))^n/(a + b/cos(e + f*x))^(1/2),x)
```

output
```
int((d/cos(e + f*x))^n/(a + b/cos(e + f*x))^(1/2), x)
```

3.785 $\int \frac{(d\sec(e+fx))^n}{(a+b\sec(e+fx))^{3/2}}\,dx$

3.785.1 Optimal result 5821
3.785.2 Mathematica [N/A] 5821
3.785.3 Rubi [N/A] 5822
3.785.4 Maple [N/A] (verified) 5823
3.785.5 Fricas [N/A] 5823
3.785.6 Sympy [N/A] 5823
3.785.7 Maxima [N/A] 5824
3.785.8 Giac [N/A] 5824
3.785.9 Mupad [N/A] 5824

3.785.1 Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{(d\sec(e+fx))^n}{(a+b\sec(e+fx))^{3/2}}\,dx = \text{Int}\left(\frac{(d\sec(e+fx))^n}{(a+b\sec(e+fx))^{3/2}},x\right)$$

output
```
Unintegrable((d*sec(f*x+e))^n/(a+b*sec(f*x+e))^(3/2),x)
```

3.785.2 Mathematica [N/A]

Not integrable

Time = 22.40 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{(d\sec(e+fx))^n}{(a+b\sec(e+fx))^{3/2}}\,dx = \int \frac{(d\sec(e+fx))^n}{(a+b\sec(e+fx))^{3/2}}\,dx$$

input
```
Integrate[(d*Sec[e + f*x])^n/(a + b*Sec[e + f*x])^(3/2),x]
```

output
```
Integrate[(d*Sec[e + f*x])^n/(a + b*Sec[e + f*x])^(3/2), x]
```

3.785.3 Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 4357}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d\sec(e+fx))^n}{(a+b\sec(e+fx))^{3/2}}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^n}{\left(a+b\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 4357}$$

$$\int \frac{(d\sec(e+fx))^n}{(a+b\sec(e+fx))^{3/2}}dx$$

input
```
Int[(d*Sec[e + f*x])^n/(a + b*Sec[e + f*x])^(3/2),x]
```

output
```
$Aborted
```

3.785.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4357
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Unintegrable[(d*Csc[e + f*x])^n*(a + b*Csc[e + f*
x])^m, x] /; FreeQ[{a, b, d, e, f, m, n}, x]
```

3.785.4 Maple [N/A] (verified)

Not integrable

Time = 0.56 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{(d\sec(fx+e))^n}{(a+b\sec(fx+e))^{\frac{3}{2}}}dx$$

input
```
int((d*sec(f*x+e))^n/(a+b*sec(f*x+e))^(3/2),x)
```

output
```
int((d*sec(f*x+e))^n/(a+b*sec(f*x+e))^(3/2),x)
```

3.785.5 Fricas [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 53, normalized size of antiderivative = 2.12

$$\int \frac{(d\sec(e+fx))^n}{(a+b\sec(e+fx))^{3/2}}\,dx = \int \frac{(d\sec(fx+e))^n}{(b\sec(fx+e)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*sec(f*x+e))^n/(a+b*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sec(f*x + e) + a)*(d*sec(f*x + e))^n/(b^2*sec(f*x + e)^2 +
 2*a*b*sec(f*x + e) + a^2), x)
```

3.785.6 Sympy [N/A]

Not integrable

Time = 1.64 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int \frac{(d\sec(e+fx))^n}{(a+b\sec(e+fx))^{3/2}}\,dx = \int \frac{(d\sec(e+fx))^n}{(a+b\sec(e+fx))^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*sec(f*x+e))**n/(a+b*sec(f*x+e))**(3/2),x)
```

output
```
Integral((d*sec(e + f*x))**n/(a + b*sec(e + f*x))**(3/2), x)
```

3.785.7 Maxima [N/A]

Not integrable

Time = 1.06 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{(d\sec(e+fx))^n}{(a+b\sec(e+fx))^{3/2}}\,dx = \int \frac{(d\sec(fx+e))^n}{(b\sec(fx+e)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*sec(f*x+e))^n/(a+b*sec(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
integrate((d*sec(f*x + e))^n/(b*sec(f*x + e) + a)^(3/2), x)
```

3.785.8 Giac [N/A]

Not integrable

Time = 1.10 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{(d\sec(e+fx))^n}{(a+b\sec(e+fx))^{3/2}}\,dx = \int \frac{(d\sec(fx+e))^n}{(b\sec(fx+e)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*sec(f*x+e))^n/(a+b*sec(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
integrate((d*sec(f*x + e))^n/(b*sec(f*x + e) + a)^(3/2), x)
```

3.785.9 Mupad [N/A]

Not integrable

Time = 17.78 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.16

$$\int \frac{(d\sec(e+fx))^n}{(a+b\sec(e+fx))^{3/2}}\,dx = \int \frac{\left(\frac{d}{\cos(e+f\,x)}\right)^n}{\left(a+\frac{b}{\cos(e+f\,x)}\right)^{3/2}}\,dx$$

input
```
int((d/cos(e + f*x))^n/(a + b/cos(e + f*x))^(3/2),x)
```

output
```
int((d/cos(e + f*x))^n/(a + b/cos(e + f*x))^(3/2), x)
```

3.786 $\int \sec^n(e+fx)(a+b\sec(e+fx))^m\,dx$

3.786.1 Optimal result 5825
3.786.2 Mathematica [N/A] 5825
3.786.3 Rubi [N/A] 5826
3.786.4 Maple [N/A] (verified) 5827
3.786.5 Fricas [N/A] 5827
3.786.6 Sympy [N/A] 5827
3.786.7 Maxima [N/A] 5828
3.786.8 Giac [N/A] 5828
3.786.9 Mupad [N/A] 5828

3.786.1 Optimal result

Integrand size = 21, antiderivative size = 21

$$\int \sec^n(e+fx)(a+b\sec(e+fx))^m\,dx=\text{Int}(\sec^n(e+fx)(a+b\sec(e+fx))^m,x)$$

output
```
Unintegrable(sec(f*x+e)^n*(a+b*sec(f*x+e))^m,x)
```

3.786.2 Mathematica [N/A]

Not integrable

Time = 8.39 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \sec^n(e+fx)(a+b\sec(e+fx))^m\,dx=\int \sec^n(e+fx)(a+b\sec(e+fx))^m\,dx$$

input
```
Integrate[Sec[e + f*x]^n*(a + b*Sec[e + f*x])^m,x]
```

output
```
Integrate[Sec[e + f*x]^n*(a + b*Sec[e + f*x])^m, x]
```

3.786.3 Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 4357}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^n(e+fx)(a+b\sec(e+fx))^m\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \csc\left(e+fx+\frac{\pi}{2}\right)^n\left(a+b\csc\left(e+fx+\frac{\pi}{2}\right)\right)^m dx$$

$$\Big\downarrow \text{ 4357}$$

$$\int \sec^n(e+fx)(a+b\sec(e+fx))^m dx$$

input
```
Int[Sec[e + f*x]^n*(a + b*Sec[e + f*x])^m,x]
```

output
```
$Aborted
```

3.786.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4357
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Unintegrable[(d*Csc[e + f*x])^n*(a + b*Csc[e + f*
x])^m, x] /; FreeQ[{a, b, d, e, f, m, n}, x]
```

3.786.4 Maple [N/A] (verified)

Not integrable

Time = 0.62 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \sec(fx+e)^n (a+b\sec(fx+e))^m \, dx$$

input
```
int(sec(f*x+e)^n*(a+b*sec(f*x+e))^m,x)
```

output
```
int(sec(f*x+e)^n*(a+b*sec(f*x+e))^m,x)
```

3.786.5 Fricas [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \sec^n(e+fx)(a+b\sec(e+fx))^m \, dx = \int (b\sec(fx+e)+a)^m \sec(fx+e)^n \, dx$$

input
```
integrate(sec(f*x+e)^n*(a+b*sec(f*x+e))^m,x, algorithm="fricas")
```

output
```
integral((b*sec(f*x + e) + a)^m*sec(f*x + e)^n, x)
```

3.786.6 Sympy [N/A]

Not integrable

Time = 5.62 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.95

$$\int \sec^n(e+fx)(a+b\sec(e+fx))^m \, dx = \int (a+b\sec(e+fx))^m \sec^n(e+fx) \, dx$$

input
```
integrate(sec(f*x+e)**n*(a+b*sec(f*x+e))**m,x)
```

output
```
Integral((a + b*sec(e + f*x))**m*sec(e + f*x)**n, x)
```

3.786.7 Maxima [N/A]

Not integrable

Time = 1.24 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \sec^n(e+fx)(a+b\sec(e+fx))^m\,dx = \int (b\sec(fx+e)+a)^m\sec(fx+e)^n\,dx$$

input
```
integrate(sec(f*x+e)^n*(a+b*sec(f*x+e))^m,x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e) + a)^m*sec(f*x + e)^n, x)
```

3.786.8 Giac [N/A]

Not integrable

Time = 0.51 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \sec^n(e+fx)(a+b\sec(e+fx))^m\,dx = \int (b\sec(fx+e)+a)^m\sec(fx+e)^n\,dx$$

input
```
integrate(sec(f*x+e)^n*(a+b*sec(f*x+e))^m,x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e) + a)^m*sec(f*x + e)^n, x)
```

3.786.9 Mupad [N/A]

Not integrable

Time = 15.41 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.29

$$\int \sec^n(e+fx)(a+b\sec(e+fx))^m\,dx = \int \left(a+\frac{b}{\cos(e+f\,x)}\right)^m\left(\frac{1}{\cos(e+f\,x)}\right)^n dx$$

input
```
int((a + b/cos(e + f*x))^m*(1/cos(e + f*x))^n,x)
```

output
```
int((a + b/cos(e + f*x))^m*(1/cos(e + f*x))^n, x)
```

3.787 $\int (d\sec(e+fx))^n(a+b\sec(e+fx))^m\,dx$

3.787.1 Optimal result . . . 5829
3.787.2 Mathematica [N/A] . . . 5829
3.787.3 Rubi [N/A] . . . 5830
3.787.4 Maple [N/A] (verified) . . . 5831
3.787.5 Fricas [N/A] . . . 5831
3.787.6 Sympy [N/A] . . . 5831
3.787.7 Maxima [N/A] . . . 5832
3.787.8 Giac [N/A] . . . 5832
3.787.9 Mupad [N/A] . . . 5832

3.787.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int (d\sec(e+fx))^n(a+b\sec(e+fx))^m\,dx=\text{Int}((d\sec(e+fx))^n(a+b\sec(e+fx))^m,x)$$

output
```
Unintegrable((d*sec(f*x+e))^n*(a+b*sec(f*x+e))^m,x)
```

3.787.2 Mathematica [N/A]

Not integrable

Time = 1.20 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int (d\sec(e+fx))^n(a+b\sec(e+fx))^m\,dx=\int (d\sec(e+fx))^n(a+b\sec(e+fx))^m\,dx$$

input
```
Integrate[(d*Sec[e + f*x])^n*(a + b*Sec[e + f*x])^m,x]
```

output
```
Integrate[(d*Sec[e + f*x])^n*(a + b*Sec[e + f*x])^m, x]
```

3.787.3 Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 4357}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d\sec(e+fx))^n(a+b\sec(e+fx))^m\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^n\left(a+b\csc\left(e+fx+\frac{\pi}{2}\right)\right)^m dx$$

$$\Big\downarrow \text{4357}$$

$$\int (d\sec(e+fx))^n(a+b\sec(e+fx))^m dx$$

input
```
Int[(d*Sec[e + f*x])^n*(a + b*Sec[e + f*x])^m,x]
```

output
```
$Aborted
```

3.787.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4357
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Unintegrable[(d*Csc[e + f*x])^n*(a + b*Csc[e + f*
x])^m, x] /; FreeQ[{a, b, d, e, f, m, n}, x]
```

3.787.4 Maple [N/A] (verified)

Not integrable

Time = 0.58 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int (d\sec(fx+e))^n (a+b\sec(fx+e))^m \, dx$$

input
```
int((d*sec(f*x+e))^n*(a+b*sec(f*x+e))^m,x)
```

output
```
int((d*sec(f*x+e))^n*(a+b*sec(f*x+e))^m,x)
```

3.787.5 Fricas [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int (d\sec(e+fx))^n(a+b\sec(e+fx))^m \, dx = \int (b\sec(fx+e)+a)^m(d\sec(fx+e))^n \, dx$$

input
```
integrate((d*sec(f*x+e))^n*(a+b*sec(f*x+e))^m,x, algorithm="fricas")
```

output
```
integral((b*sec(f*x + e) + a)^m*(d*sec(f*x + e))^n, x)
```

3.787.6 Sympy [N/A]

Not integrable

Time = 5.66 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int (d\sec(e+fx))^n(a+b\sec(e+fx))^m \, dx = \int (d\sec(e+fx))^n (a+b\sec(e+fx))^m \, dx$$

input
```
integrate((d*sec(f*x+e))**n*(a+b*sec(f*x+e))**m,x)
```

output
```
Integral((d*sec(e + f*x))**n*(a + b*sec(e + f*x))**m, x)
```

3.787.7 Maxima [N/A]

Not integrable

Time = 1.25 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int (d\sec(e+fx))^n(a+b\sec(e+fx))^m\,dx=\int (b\sec(fx+e)+a)^m(d\sec(fx+e))^n\,dx$$

input
```
integrate((d*sec(f*x+e))^n*(a+b*sec(f*x+e))^m,x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e) + a)^m*(d*sec(f*x + e))^n, x)
```

3.787.8 Giac [N/A]

Not integrable

Time = 0.54 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int (d\sec(e+fx))^n(a+b\sec(e+fx))^m\,dx=\int (b\sec(fx+e)+a)^m(d\sec(fx+e))^n\,dx$$

input
```
integrate((d*sec(f*x+e))^n*(a+b*sec(f*x+e))^m,x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e) + a)^m*(d*sec(f*x + e))^n, x)
```

3.787.9 Mupad [N/A]

Not integrable

Time = 15.37 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.26

$$\int (d\sec(e+fx))^n(a+b\sec(e+fx))^m\,dx=\int \left(a+\frac{b}{\cos(e+f\,x)}\right)^m\left(\frac{d}{\cos(e+f\,x)}\right)^n dx$$

input
```
int((a + b/cos(e + f*x))^m*(d/cos(e + f*x))^n,x)
```

output
```
int((a + b/cos(e + f*x))^m*(d/cos(e + f*x))^n, x)
```

3.788 $\int \sec^3(e+fx)(a+b\sec(e+fx))^m\,dx$

3.788.1 Optimal result . . . 5833
3.788.2 Mathematica [B] (warning: unable to verify) . . . 5834
3.788.3 Rubi [A] (verified) . . . 5834
3.788.4 Maple [F] . . . 5837
3.788.5 Fricas [F] . . . 5837
3.788.6 Sympy [F] . . . 5838
3.788.7 Maxima [F] . . . 5838
3.788.8 Giac [F] . . . 5838
3.788.9 Mupad [F(-1)] . . . 5839

3.788.1 Optimal result

Integrand size = 21, antiderivative size = 273

$$\int \sec^3(e+fx)(a+b\sec(e+fx))^m\,dx = \frac{(a+b\sec(e+fx))^{1+m}\tan(e+fx)}{bf(2+m)} - \frac{\sqrt{2}a(a+b)\operatorname{AppellF1}\left(\frac{1}{2},\frac{1}{2},-1-m,\frac{3}{2},\frac{1}{2}(1-\sec(e+fx)),\frac{b(1-\sec(e+fx))}{a+b}\right)(a+b\sec(e+fx))^m\left(\frac{a+b\sec(}{a+}\right.}{b^2f(2+m)\sqrt{1+\sec(e+fx)}} + \frac{\sqrt{2}(a^2+b^2(1+m))\operatorname{AppellF1}\left(\frac{1}{2},\frac{1}{2},-m,\frac{3}{2},\frac{1}{2}(1-\sec(e+fx)),\frac{b(1-\sec(e+fx))}{a+b}\right)(a+b\sec(e+fx))^m\left(\frac{a+}{}\right.}{b^2f(2+m)\sqrt{1+\sec(e+fx)}}$$

output
```
(a+b*sec(f*x+e))^(1+m)*tan(f*x+e)/b/f/(2+m)-a*(a+b)*AppellF1(1/2,-1-m,1/2,
3/2,b*(1-sec(f*x+e))/(a+b),1/2-1/2*sec(f*x+e))*(a+b*sec(f*x+e))^m*2^(1/2)*
tan(f*x+e)/b^2/f/(2+m)/(((a+b*sec(f*x+e))/(a+b))^m)/(1+sec(f*x+e))^(1/2)+(
a^2+b^2*(1+m))*AppellF1(1/2,-m,1/2,3/2,b*(1-sec(f*x+e))/(a+b),1/2-1/2*sec(
f*x+e))*(a+b*sec(f*x+e))^m*2^(1/2)*tan(f*x+e)/b^2/f/(2+m)/(((a+b*sec(f*x+e
))/(a+b))^m)/(1+sec(f*x+e))^(1/2)
```

3.788.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 8899 vs. $2(273) = 546$.

Time = 27.25 (sec) , antiderivative size = 8899, normalized size of antiderivative = 32.60

$$\int \sec^3(e+fx)(a+b\sec(e+fx))^m\,dx = \text{Result too large to show}$$

input
```
Integrate[Sec[e + f*x]^3*(a + b*Sec[e + f*x])^m,x]
```

output
```
Result too large to show
```

3.788.3 Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {3042, 4327, 3042, 4495, 3042, 4321, 156, 155}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^3(e+fx)(a+b\sec(e+fx))^m\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(e+fx+\frac{\pi}{2}\right)^3\left(a+b\csc\left(e+fx+\frac{\pi}{2}\right)\right)^m dx$$

$$\Big\downarrow \text{4327}$$

$$\frac{\int \sec(e+fx)(b(m+1)-a\sec(e+fx))(a+b\sec(e+fx))^m dx}{b(m+2)}+\frac{\tan(e+fx)(a+b\sec(e+fx))^{m+1}}{bf(m+2)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \csc\left(e+fx+\frac{\pi}{2}\right)\left(b(m+1)-a\csc\left(e+fx+\frac{\pi}{2}\right)\right)\left(a+b\csc\left(e+fx+\frac{\pi}{2}\right)\right)^m dx}{b(m+2)}+\frac{\tan(e+fx)(a+b\sec(e+fx))^{m+1}}{bf(m+2)}$$

$$\Big\downarrow \text{4495}$$

$$\frac{\frac{(a^2+b^2(m+1))\int \sec(e+fx)(a+b\sec(e+fx))^m dx}{b}-\frac{a\int \sec(e+fx)(a+b\sec(e+fx))^{m+1}dx}{b}}{b(m+2)}+\frac{\tan(e+fx)(a+b\sec(e+fx))^{m+1}}{bf(m+2)}$$

$\Big\downarrow$ 3042

$$\frac{\frac{(a^2+b^2(m+1))\int \csc(e+fx+\frac{\pi}{2})(a+b\csc(e+fx+\frac{\pi}{2}))^m dx}{b}-\frac{a\int \csc(e+fx+\frac{\pi}{2})(a+b\csc(e+fx+\frac{\pi}{2}))^{m+1}dx}{b}}{b(m+2)}+\frac{\tan(e+fx)(a+b\sec(e+fx))^{m+1}}{bf(m+2)}$$

$\Big\downarrow$ 4321

$$\frac{\frac{a\tan(e+fx)\int \frac{(a+b\sec(e+fx))^{m+1}}{\sqrt{1-\sec(e+fx)}\sqrt{\sec(e+fx)+1}}d\sec(e+fx)}{bf\sqrt{1-\sec(e+fx)}\sqrt{\sec(e+fx)+1}}-\frac{(a^2+b^2(m+1))\tan(e+fx)\int \frac{(a+b\sec(e+fx))^m}{\sqrt{1-\sec(e+fx)}\sqrt{\sec(e+fx)+1}}d\sec(e+fx)}{bf\sqrt{1-\sec(e+fx)}\sqrt{\sec(e+fx)+1}}}{b(m+2)}+\frac{\tan(e+fx)(a+b\sec(e+fx))^{m+1}}{bf(m+2)}$$

$\Big\downarrow$ 156

$$\frac{\frac{a(a+b)\tan(e+fx)(a+b\sec(e+fx))^m\left(\frac{a+b\sec(e+fx)}{a+b}\right)^{-m}\int \frac{\left(\frac{a}{a+b}+\frac{b\sec(e+fx)}{a+b}\right)^{m+1}}{\sqrt{1-\sec(e+fx)}\sqrt{\sec(e+fx)+1}}d\sec(e+fx)}{bf\sqrt{1-\sec(e+fx)}\sqrt{\sec(e+fx)+1}}-\frac{(a^2+b^2(m+1))\tan(e+fx)(a+b\sec(e+fx}{bf\sqrt{}}}{b(m+2)}+\frac{\tan(e+fx)(a+b\sec(e+fx))^{m+1}}{bf(m+2)}$$

$\Big\downarrow$ 155

$$\frac{\frac{\sqrt{2}(a^2+b^2(m+1))\tan(e+fx)(a+b\sec(e+fx))^m\left(\frac{a+b\sec(e+fx)}{a+b}\right)^{-m}\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},-m,\frac{3}{2},\frac{1}{2}(1-\sec(e+fx)),\frac{b(1-\sec(e+fx))}{a+b}\right)}{bf\sqrt{\sec(e+fx)+1}}-\frac{\sqrt{2}a(a+b)\tan(e}{}}{b(m+2)}+\frac{\tan(e+fx)(a+b\sec(e+fx))^{m+1}}{bf(m+2)}$$

input
```
Int[Sec[e + f*x]^3*(a + b*Sec[e + f*x])^m,x]
```

3.788. $\int \sec^3(e+fx)(a+b\sec(e+fx))^m\,dx$

output
```
((a + b*Sec[e + f*x])^(1 + m)*Tan[e + f*x])/(b*f*(2 + m)) + (-((Sqrt[2]*a*
(a + b)*AppellF1[1/2, 1/2, -1 - m, 3/2, (1 - Sec[e + f*x])/2, (b*(1 - Sec[
e + f*x]))/(a + b)]*(a + b*Sec[e + f*x])^m*Tan[e + f*x])/(b*f*Sqrt[1 + Sec
[e + f*x]]*((a + b*Sec[e + f*x])/(a + b))^m)) + (Sqrt[2]*(a^2 + b^2*(1 + m
))*AppellF1[1/2, 1/2, -m, 3/2, (1 - Sec[e + f*x])/2, (b*(1 - Sec[e + f*x])
)/(a + b)]*(a + b*Sec[e + f*x])^m*Tan[e + f*x])/(b*f*Sqrt[1 + Sec[e + f*x]
]*((a + b*Sec[e + f*x])/(a + b))^m))/(b*(2 + m))
```

3.788.3.1 Defintions of rubi rules used

rule 155
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*Simplify[b/(b*c - a*d)]^n*
Simplify[b/(b*e - a*f)]^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/
(b*c - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f,
 m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[Sim
plify[b/(b*c - a*d)], 0] && GtQ[Simplify[b/(b*e - a*f)], 0] &&  !(GtQ[Simpl
ify[d/(d*a - c*b)], 0] && GtQ[Simplify[d/(d*e - c*f)], 0] && SimplerQ[c + d
*x, a + b*x]) &&  !(GtQ[Simplify[f/(f*a - e*b)], 0] && GtQ[Simplify[f/(f*c
- e*d)], 0] && SimplerQ[e + f*x, a + b*x])
```

rule 156
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(e + f*x)^FracPart[p]/(Simplify[b/(b*e - a*f)]^IntPart[p
]*(b*((e + f*x)/(b*e - a*f)))^FracPart[p])   Int[(a + b*x)^m*(c + d*x)^n*Si
mp[b*(e/(b*e - a*f)) + b*f*(x/(b*e - a*f)), x]^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] &
& GtQ[Simplify[b/(b*c - a*d)], 0] &&  !GtQ[Simplify[b/(b*e - a*f)], 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4321
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[Cot[e + f*x]/(f*Sqrt[1 + Csc[e + f*x]]*Sqrt[1 - Csc[e + f*x
]])   Subst[Int[(a + b*x)^m/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Csc[e + f*x]]
, x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*m]
```

3.788. $\int \sec^3(e+fx)(a+b\sec(e+fx))^m\,dx$

rule 4327
```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(-Cot[e + f*x])*((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2
))), x] + Simp[1/(b*(m + 2))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(b*(
m + 1) - a*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 -
b^2, 0] &&  !LtQ[m, -1]
```

rule 4495
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b - a*B)/b   Int[
Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] + Simp[B/b   Int[Csc[e + f*x]*(
a + b*Csc[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, A, B, e, f, m}, x] && N
eQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

3.788.4 Maple [F]

$$\int \sec (fx+e)^3 (a+b\sec (fx+e))^m \, dx$$

input
```
int(sec(f*x+e)^3*(a+b*sec(f*x+e))^m,x)
```

output
```
int(sec(f*x+e)^3*(a+b*sec(f*x+e))^m,x)
```

3.788.5 Fricas [F]

$$\int \sec^3(e+fx)(a+b\sec(e+fx))^m \, dx = \int (b\sec (fx+e)+a)^m \sec (fx+e)^3 \, dx$$

input
```
integrate(sec(f*x+e)^3*(a+b*sec(f*x+e))^m,x, algorithm="fricas")
```

output
```
integral((b*sec(f*x + e) + a)^m*sec(f*x + e)^3, x)
```

3.788.6 Sympy [F]

$$\int \sec^3(e+fx)(a+b\sec(e+fx))^m\,dx=\int (a+b\sec{(e+fx)})^m\sec^3{(e+fx)}\,dx$$

input
```
integrate(sec(f*x+e)**3*(a+b*sec(f*x+e))**m,x)
```

output
```
Integral((a + b*sec(e + f*x))**m*sec(e + f*x)**3, x)
```

3.788.7 Maxima [F]

$$\int \sec^3(e+fx)(a+b\sec(e+fx))^m\,dx=\int (b\sec{(fx+e)}+a)^m\sec{(fx+e)}^3\,dx$$

input
```
integrate(sec(f*x+e)^3*(a+b*sec(f*x+e))^m,x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e) + a)^m*sec(f*x + e)^3, x)
```

3.788.8 Giac [F]

$$\int \sec^3(e+fx)(a+b\sec(e+fx))^m\,dx=\int (b\sec{(fx+e)}+a)^m\sec{(fx+e)}^3\,dx$$

input
```
integrate(sec(f*x+e)^3*(a+b*sec(f*x+e))^m,x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e) + a)^m*sec(f*x + e)^3, x)
```

3.788.9 Mupad [F(-1)]

Timed out.

$$\int \sec^3(e+fx)(a+b\sec(e+fx))^m\,dx=\int \frac{\left(a+\frac{b}{\cos(e+f\,x)}\right)^m}{\cos\left(e+f\,x\right)^3}\,dx$$

input
```
int((a + b/cos(e + f*x))^m/cos(e + f*x)^3,x)
```

output
```
int((a + b/cos(e + f*x))^m/cos(e + f*x)^3, x)
```

3.789 $\int \sec^2(e+fx)(a+b\sec(e+fx))^m\,dx$

3.789.1 Optimal result . . . 5840
3.789.2 Mathematica [B] (warning: unable to verify) . . . 5841
3.789.3 Rubi [A] (verified) . . . 5841
3.789.4 Maple [F] . . . 5844
3.789.5 Fricas [F] . . . 5844
3.789.6 Sympy [F] . . . 5844
3.789.7 Maxima [F] . . . 5845
3.789.8 Giac [F] . . . 5845
3.789.9 Mupad [F(-1)] . . . 5845

3.789.1 Optimal result

Integrand size = 21, antiderivative size = 220

$$\int \sec^2(e+fx)(a+b\sec(e+fx))^m\,dx$$
$$= \frac{\sqrt{2}(a+b)\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},-1-m,\frac{3}{2},\frac{1}{2}(1-\sec(e+fx)),\frac{b(1-\sec(e+fx))}{a+b}\right)(a+b\sec(e+fx))^m\left(\frac{a+b\sec(e+fx}{a+b}\right.}{bf\sqrt{1+\sec(e+fx)}}$$
$$-\frac{\sqrt{2}a\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},-m,\frac{3}{2},\frac{1}{2}(1-\sec(e+fx)),\frac{b(1-\sec(e+fx))}{a+b}\right)(a+b\sec(e+fx))^m\left(\frac{a+b\sec(e+fx)}{a+b}\right)^{-m}\text{ta}}{bf\sqrt{1+\sec(e+fx)}}$$

output
```
(a+b)*AppellF1(1/2,-1-m,1/2,3/2,b*(1-sec(f*x+e))/(a+b),1/2-1/2*sec(f*x+e))
*(a+b*sec(f*x+e))^m*2^(1/2)*tan(f*x+e)/b/f/(((a+b*sec(f*x+e))/(a+b))^m)/(1
+sec(f*x+e))^(1/2)-a*AppellF1(1/2,-m,1/2,3/2,b*(1-sec(f*x+e))/(a+b),1/2-1/
2*sec(f*x+e))*(a+b*sec(f*x+e))^m*2^(1/2)*tan(f*x+e)/b/f/(((a+b*sec(f*x+e))
/(a+b))^m)/(1+sec(f*x+e))^(1/2)
```

3.789.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 5564 vs. $2(220) = 440$.

Time = 23.09 (sec) , antiderivative size = 5564, normalized size of antiderivative = 25.29

$$\int \sec^2(e+fx)(a+b\sec(e+fx))^m\,dx = \text{Result too large to show}$$

input
```
Integrate[Sec[e + f*x]^2*(a + b*Sec[e + f*x])^m,x]
```

output
```
Result too large to show
```

3.789.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 4325, 3042, 4321, 156, 155}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(e+fx)(a+b\sec(e+fx))^m\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(e+fx+\frac{\pi}{2}\right)^2\left(a+b\csc\left(e+fx+\frac{\pi}{2}\right)\right)^m dx$$

$$\Big\downarrow \text{4325}$$

$$\frac{\int \sec(e+fx)(a+b\sec(e+fx))^{m+1}dx}{b}-\frac{a\int \sec(e+fx)(a+b\sec(e+fx))^m dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \csc\left(e+fx+\frac{\pi}{2}\right)\left(a+b\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{m+1}dx}{b}-\frac{a\int \csc\left(e+fx+\frac{\pi}{2}\right)\left(a+b\csc\left(e+fx+\frac{\pi}{2}\right)\right)^m dx}{b}$$

$$\Big\downarrow \text{4321}$$

$$\frac{a\tan(e+fx)\int \frac{(a+b\sec(e+fx))^m}{\sqrt{1-\sec(e+fx)}\sqrt{\sec(e+fx)+1}}d\sec(e+fx)}{bf\sqrt{1-\sec(e+fx)}\sqrt{\sec(e+fx)+1}}-\frac{\tan(e+fx)\int \frac{(a+b\sec(e+fx))^{m+1}}{\sqrt{1-\sec(e+fx)}\sqrt{\sec(e+fx)+1}}d\sec(e+fx)}{bf\sqrt{1-\sec(e+fx)}\sqrt{\sec(e+fx)+1}}$$

$\downarrow$ 156

$$\frac{a\tan(e+fx)(a+b\sec(e+fx))^m\left(\frac{a+b\sec(e+fx)}{a+b}\right)^{-m}\int \frac{\left(\frac{a}{a+b}+\frac{b\sec(e+fx)}{a+b}\right)^m}{\sqrt{1-\sec(e+fx)}\sqrt{\sec(e+fx)+1}}d\sec(e+fx)}{bf\sqrt{1-\sec(e+fx)}\sqrt{\sec(e+fx)+1}}-\frac{(a+b)\tan(e+fx)(a+b\sec(e+fx))^m\left(\frac{a+b\sec(e+fx)}{a+b}\right)^{-m}\int \frac{\left(\frac{a}{a+b}+\frac{b\sec(e+fx)}{a+b}\right)^{m+1}}{\sqrt{1-\sec(e+fx)}\sqrt{\sec(e+fx)+1}}d\sec(e+fx)}{bf\sqrt{1-\sec(e+fx)}\sqrt{\sec(e+fx)+1}}$$

$\downarrow$ 155

$$\frac{\sqrt{2}(a+b)\tan(e+fx)(a+b\sec(e+fx))^m\left(\frac{a+b\sec(e+fx)}{a+b}\right)^{-m}\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},-m-1,\frac{3}{2},\frac{1}{2}(1-\sec(e+fx)),\frac{b(1-}{}\right.}{bf\sqrt{\sec(e+fx)+1}}$$

$$\frac{\sqrt{2}a\tan(e+fx)(a+b\sec(e+fx))^m\left(\frac{a+b\sec(e+fx)}{a+b}\right)^{-m}\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},-m,\frac{3}{2},\frac{1}{2}(1-\sec(e+fx)),\frac{b(1-\sec(e+fx))}{a+b}\right.}{bf\sqrt{\sec(e+fx)+1}}$$

input
```
Int[Sec[e + f*x]^2*(a + b*Sec[e + f*x])^m,x]
```

output
```
(Sqrt[2]*(a + b)*AppellF1[1/2, 1/2, -1 - m, 3/2, (1 - Sec[e + f*x])/2, (b*
(1 - Sec[e + f*x]))/(a + b)]*(a + b*Sec[e + f*x])^m*Tan[e + f*x])/(b*f*Sqr
t[1 + Sec[e + f*x]]*((a + b*Sec[e + f*x])/(a + b))^m) - (Sqrt[2]*a*AppellF
1[1/2, 1/2, -m, 3/2, (1 - Sec[e + f*x])/2, (b*(1 - Sec[e + f*x]))/(a + b)]
*(a + b*Sec[e + f*x])^m*Tan[e + f*x])/(b*f*Sqrt[1 + Sec[e + f*x]]*((a + b*
Sec[e + f*x])/(a + b))^m)
```

3.789. $\int \sec^2(e+fx)(a+b\sec(e+fx))^m\,dx$

3.789.3.1 Defintions of rubi rules used

rule 155
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*Simplify[b/(b*c - a*d)]^n*
Simplify[b/(b*e - a*f)]^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/
(b*c - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f,
 m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[Sim
plify[b/(b*c - a*d)], 0] && GtQ[Simplify[b/(b*e - a*f)], 0] && !(GtQ[Simpl
ify[d/(d*a - c*b)], 0] && GtQ[Simplify[d/(d*e - c*f)], 0] && SimplerQ[c + d
*x, a + b*x]) && !(GtQ[Simplify[f/(f*a - e*b)], 0] && GtQ[Simplify[f/(f*c
- e*d)], 0] && SimplerQ[e + f*x, a + b*x])
```

rule 156
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(e + f*x)^FracPart[p]/(Simplify[b/(b*e - a*f)]^IntPart[p
]*(b*((e + f*x)/(b*e - a*f)))^FracPart[p])   Int[(a + b*x)^m*(c + d*x)^n*Si
mp[b*(e/(b*e - a*f)) + b*f*(x/(b*e - a*f)), x]^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] &
& GtQ[Simplify[b/(b*c - a*d)], 0] && !GtQ[Simplify[b/(b*e - a*f)], 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4321
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[Cot[e + f*x]/(f*Sqrt[1 + Csc[e + f*x]]*Sqrt[1 - Csc[e + f*x
]])   Subst[Int[(a + b*x)^m/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Csc[e + f*x]]
, x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0] && !IntegerQ[2*m]
```

rule 4325
```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[-a/b   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] +
Simp[1/b   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x], x] /; FreeQ[{
a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0]
```

3.789. $\int \sec^2(e+fx)(a+b\sec(e+fx))^m\,dx$

3.789.4 Maple [F]

$$\int \sec(fx+e)^2 (a+b\sec(fx+e))^m \, dx$$

input
```
int(sec(f*x+e)^2*(a+b*sec(f*x+e))^m,x)
```

output
```
int(sec(f*x+e)^2*(a+b*sec(f*x+e))^m,x)
```

3.789.5 Fricas [F]

$$\int \sec^2(e+fx)(a+b\sec(e+fx))^m \, dx = \int (b\sec(fx+e)+a)^m \sec(fx+e)^2 \, dx$$

input
```
integrate(sec(f*x+e)^2*(a+b*sec(f*x+e))^m,x, algorithm="fricas")
```

output
```
integral((b*sec(f*x + e) + a)^m*sec(f*x + e)^2, x)
```

3.789.6 Sympy [F]

$$\int \sec^2(e+fx)(a+b\sec(e+fx))^m \, dx = \int (a+b\sec(e+fx))^m \sec^2(e+fx) \, dx$$

input
```
integrate(sec(f*x+e)**2*(a+b*sec(f*x+e))**m,x)
```

output
```
Integral((a + b*sec(e + f*x))**m*sec(e + f*x)**2, x)
```

3.789.7 Maxima [F]

$$\int \sec^2(e+fx)(a+b\sec(e+fx))^m\,dx = \int (b\sec(fx+e)+a)^m\sec(fx+e)^2\,dx$$

input
```
integrate(sec(f*x+e)^2*(a+b*sec(f*x+e))^m,x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e) + a)^m*sec(f*x + e)^2, x)
```

3.789.8 Giac [F]

$$\int \sec^2(e+fx)(a+b\sec(e+fx))^m\,dx = \int (b\sec(fx+e)+a)^m\sec(fx+e)^2\,dx$$

input
```
integrate(sec(f*x+e)^2*(a+b*sec(f*x+e))^m,x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e) + a)^m*sec(f*x + e)^2, x)
```

3.789.9 Mupad [F(-1)]

Timed out.

$$\int \sec^2(e+fx)(a+b\sec(e+fx))^m\,dx = \int \frac{\left(a+\frac{b}{\cos(e+f\,x)}\right)^m}{\cos(e+f\,x)^2}\,dx$$

input
```
int((a + b/cos(e + f*x))^m/cos(e + f*x)^2,x)
```

output
```
int((a + b/cos(e + f*x))^m/cos(e + f*x)^2, x)
```

3.790 $\int \sec(e+fx)(a+b\sec(e+fx))^m\,dx$

3.790.1 Optimal result 5846
3.790.2 Mathematica [B] (warning: unable to verify) 5846
3.790.3 Rubi [A] (verified) 5847
3.790.4 Maple [F] 5849
3.790.5 Fricas [F] 5849
3.790.6 Sympy [F] 5849
3.790.7 Maxima [F] 5850
3.790.8 Giac [F] 5850
3.790.9 Mupad [F(-1)] 5850

3.790.1 Optimal result

Integrand size = 19, antiderivative size = 103

$$\int \sec(e+fx)(a+b\sec(e+fx))^m\,dx = \frac{\sqrt{2}\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},-m,\frac{3}{2},\frac{1}{2}(1-\sec(e+fx)),\frac{b(1-\sec(e+fx))}{a+b}\right)(a+b\sec(e+fx))^m\left(\frac{a+b\sec(e+fx)}{a+b}\right)^{-m}\tan(e}{f\sqrt{1+\sec(e+fx)}}$$

output
```
AppellF1(1/2,-m,1/2,3/2,b*(1-sec(f*x+e))/(a+b),1/2-1/2*sec(f*x+e))*(a+b*se
c(f*x+e))^m*2^(1/2)*tan(f*x+e)/f/(((a+b*sec(f*x+e))/(a+b))^m)/(1+sec(f*x+e
))^(1/2)
```

3.790.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 2828 vs. 2(103) = 206.

Time = 15.55 (sec) , antiderivative size = 2828, normalized size of antiderivative = 27.46

$$\int \sec(e+fx)(a+b\sec(e+fx))^m\,dx = \text{Result too large to show}$$

input
```
Integrate[Sec[e + f*x]*(a + b*Sec[e + f*x])^m,x]
```

output
```
(-6*(a + b)*AppellF1[1/2, 1 + m, -m, 3/2, Tan[(e + f*x)/2]^2, ((a - b)*Tan
[(e + f*x)/2]^2)/(a + b)]*(b + a*Cos[e + f*x])^m*Sec[e + f*x]^(1 + m)*(a +
 b*Sec[e + f*x])^m*Tan[(e + f*x)/2])/(f*(-1 + Tan[(e + f*x)/2]^2)*(3*(a +
b)*AppellF1[1/2, 1 + m, -m, 3/2, Tan[(e + f*x)/2]^2, ((a - b)*Tan[(e + f*x
)/2]^2)/(a + b)] + 2*(-((a - b)*m*AppellF1[3/2, 1 + m, 1 - m, 5/2, Tan[(e
+ f*x)/2]^2, ((a - b)*Tan[(e + f*x)/2]^2)/(a + b)]) + (a + b)*(1 + m)*Appe
llF1[3/2, 2 + m, -m, 5/2, Tan[(e + f*x)/2]^2, ((a - b)*Tan[(e + f*x)/2]^2)
/(a + b)])*Tan[(e + f*x)/2]^2)*((6*(a + b)*AppellF1[1/2, 1 + m, -m, 3/2, T
an[(e + f*x)/2]^2, ((a - b)*Tan[(e + f*x)/2]^2)/(a + b)]*(b + a*Cos[e + f*
x])^m*Sec[(e + f*x)/2]^2*Sec[e + f*x]^m*Tan[(e + f*x)/2]^2)/((-1 + Tan[(e
+ f*x)/2]^2)^2*(3*(a + b)*AppellF1[1/2, 1 + m, -m, 3/2, Tan[(e + f*x)/2]^2
, ((a - b)*Tan[(e + f*x)/2]^2)/(a + b)] + 2*(-((a - b)*m*AppellF1[3/2, 1 +
 m, 1 - m, 5/2, Tan[(e + f*x)/2]^2, ((a - b)*Tan[(e + f*x)/2]^2)/(a + b)])
 + (a + b)*(1 + m)*AppellF1[3/2, 2 + m, -m, 5/2, Tan[(e + f*x)/2]^2, ((a -
 b)*Tan[(e + f*x)/2]^2)/(a + b)])*Tan[(e + f*x)/2]^2)) - (3*(a + b)*Appell
F1[1/2, 1 + m, -m, 3/2, Tan[(e + f*x)/2]^2, ((a - b)*Tan[(e + f*x)/2]^2)/(
a + b)]*(b + a*Cos[e + f*x])^m*Sec[(e + f*x)/2]^2*Sec[e + f*x]^m)/((-1 + T
an[(e + f*x)/2]^2)*(3*(a + b)*AppellF1[1/2, 1 + m, -m, 3/2, Tan[(e + f*x)/
2]^2, ((a - b)*Tan[(e + f*x)/2]^2)/(a + b)] + 2*(-((a - b)*m*AppellF1[3/2,
 1 + m, 1 - m, 5/2, Tan[(e + f*x)/2]^2, ((a - b)*Tan[(e + f*x)/2]^2)/(a...
```

3.790.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {3042, 4321, 156, 155}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(e+fx)(a+b\sec(e+fx))^m\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \csc\left(e+fx+\frac{\pi}{2}\right)\left(a+b\csc\left(e+fx+\frac{\pi}{2}\right)\right)^m dx$$

$$\Big\downarrow \; 4321$$

$$-\frac{\tan(e+fx)\int \frac{(a+b\sec(e+fx))^m}{\sqrt{1-\sec(e+fx)}\sqrt{\sec(e+fx)+1}}d\sec(e+fx)}{f\sqrt{1-\sec(e+fx)}\sqrt{\sec(e+fx)+1}}$$

$$\Big\downarrow \; 156$$

$$-\frac{\tan(e+fx)(a+b\sec(e+fx))^m\left(\frac{a+b\sec(e+fx)}{a+b}\right)^{-m}\int\frac{\left(\frac{a}{a+b}+\frac{b\sec(e+fx)}{a+b}\right)^m}{\sqrt{1-\sec(e+fx)}\sqrt{\sec(e+fx)+1}}d\sec(e+fx)}{f\sqrt{1-\sec(e+fx)}\sqrt{\sec(e+fx)+1}}$$

$\Big\downarrow$ 155

$$\frac{\sqrt{2}\tan(e+fx)(a+b\sec(e+fx))^m\left(\frac{a+b\sec(e+fx)}{a+b}\right)^{-m}\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},-m,\frac{3}{2},\frac{1}{2}(1-\sec(e+fx)),\frac{b(1-\sec(e+fx))}{a+b}\right)}{f\sqrt{\sec(e+fx)+1}}$$

input
```
Int[Sec[e + f*x]*(a + b*Sec[e + f*x])^m,x]
```

output
```
(Sqrt[2]*AppellF1[1/2, 1/2, -m, 3/2, (1 - Sec[e + f*x])/2, (b*(1 - Sec[e +
 f*x]))/(a + b)]*(a + b*Sec[e + f*x])^m*Tan[e + f*x])/(f*Sqrt[1 + Sec[e +
f*x]]*((a + b*Sec[e + f*x])/(a + b))^m)
```

3.790.3.1 Defintions of rubi rules used

rule 155
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*Simplify[b/(b*c - a*d)]^n*
Simplify[b/(b*e - a*f)]^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/
(b*c - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f,
 m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[Sim
plify[b/(b*c - a*d)], 0] && GtQ[Simplify[b/(b*e - a*f)], 0] && !(GtQ[Simpl
ify[d/(d*a - c*b)], 0] && GtQ[Simplify[d/(d*e - c*f)], 0] && SimplerQ[c + d
*x, a + b*x]) && !(GtQ[Simplify[f/(f*a - e*b)], 0] && GtQ[Simplify[f/(f*c
- e*d)], 0] && SimplerQ[e + f*x, a + b*x])
```

rule 156
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(e + f*x)^FracPart[p]/(Simplify[b/(b*e - a*f)]^IntPart[p
]*(b*((e + f*x)/(b*e - a*f)))^FracPart[p])   Int[(a + b*x)^m*(c + d*x)^n*Si
mp[b*(e/(b*e - a*f)) + b*f*(x/(b*e - a*f)), x]^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] &
& GtQ[Simplify[b/(b*c - a*d)], 0] && !GtQ[Simplify[b/(b*e - a*f)], 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4321
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[Cot[e + f*x]/(f*Sqrt[1 + Csc[e + f*x]]*Sqrt[1 - Csc[e + f*x
]])   Subst[Int[(a + b*x)^m/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Csc[e + f*x]]
, x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*m]
```

3.790.4 Maple [F]

$$\int \sec(fx+e)(a+b\sec(fx+e))^m\,dx$$

input
```
int(sec(f*x+e)*(a+b*sec(f*x+e))^m,x)
```

output
```
int(sec(f*x+e)*(a+b*sec(f*x+e))^m,x)
```

3.790.5 Fricas [F]

$$\int \sec(e+fx)(a+b\sec(e+fx))^m\,dx=\int (b\sec(fx+e)+a)^m\sec(fx+e)\,dx$$

input
```
integrate(sec(f*x+e)*(a+b*sec(f*x+e))^m,x, algorithm="fricas")
```

output
```
integral((b*sec(f*x + e) + a)^m*sec(f*x + e), x)
```

3.790.6 Sympy [F]

$$\int \sec(e+fx)(a+b\sec(e+fx))^m\,dx=\int (a+b\sec(e+fx))^m\sec(e+fx)\,dx$$

input
```
integrate(sec(f*x+e)*(a+b*sec(f*x+e))**m,x)
```

output
```
Integral((a + b*sec(e + f*x))**m*sec(e + f*x), x)
```

3.790.7 Maxima [F]

$$\int \sec(e+fx)(a+b\sec(e+fx))^m\,dx = \int (b\sec(fx+e)+a)^m\sec(fx+e)\,dx$$

input
```
integrate(sec(f*x+e)*(a+b*sec(f*x+e))^m,x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e) + a)^m*sec(f*x + e), x)
```

3.790.8 Giac [F]

$$\int \sec(e+fx)(a+b\sec(e+fx))^m\,dx = \int (b\sec(fx+e)+a)^m\sec(fx+e)\,dx$$

input
```
integrate(sec(f*x+e)*(a+b*sec(f*x+e))^m,x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e) + a)^m*sec(f*x + e), x)
```

3.790.9 Mupad [F(-1)]

Timed out.

$$\int \sec(e+fx)(a+b\sec(e+fx))^m\,dx = \int \frac{\left(a+\frac{b}{\cos(e+f\,x)}\right)^m}{\cos(e+f\,x)}\,dx$$

input
```
int((a + b/cos(e + f*x))^m/cos(e + f*x),x)
```

output
```
int((a + b/cos(e + f*x))^m/cos(e + f*x), x)
```

3.791 $\int (a+b\sec(e+fx))^m\,dx$

3.791.1 Optimal result 5851
3.791.2 Mathematica [N/A] 5851
3.791.3 Rubi [N/A] 5852
3.791.4 Maple [N/A] (verified) 5853
3.791.5 Fricas [N/A] 5853
3.791.6 Sympy [N/A] 5853
3.791.7 Maxima [N/A] 5854
3.791.8 Giac [N/A] 5854
3.791.9 Mupad [N/A] 5854

3.791.1 Optimal result

Integrand size = 12, antiderivative size = 12

$$\int (a+b\sec(e+fx))^m\,dx=\text{Int}((a+b\sec(e+fx))^m,x)$$

output
```
Unintegrable((a+b*sec(f*x+e))^m,x)
```

3.791.2 Mathematica [N/A]

Not integrable

Time = 6.65 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int (a+b\sec(e+fx))^m\,dx=\int (a+b\sec(e+fx))^m\,dx$$

input
```
Integrate[(a + b*Sec[e + f*x])^m,x]
```

output
```
Integrate[(a + b*Sec[e + f*x])^m, x]
```

3.791.3 Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 4273}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\sec(e+fx))^m \, dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \left(a+b\csc\left(e+fx+\frac{\pi}{2}\right)\right)^m dx$$

$$\Big\downarrow \text{ 4273}$$

$$\int (a+b\sec(e+fx))^m dx$$

input
```
Int[(a + b*Sec[e + f*x])^m,x]
```

output
```
$Aborted
```

3.791.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4273
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Unintegrable[
(a + b*Csc[c + d*x])^n, x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[a^2 - b^2, 0
] &&  !IntegerQ[2*n]
```

3.791.4 Maple [N/A] (verified)

Not integrable

Time = 0.60 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int (a+b\sec(fx+e))^m\,dx$$

input
```
int((a+b*sec(f*x+e))^m,x)
```

output
```
int((a+b*sec(f*x+e))^m,x)
```

3.791.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int (a+b\sec(e+fx))^m\,dx=\int (b\sec(fx+e)+a)^m\,dx$$

input
```
integrate((a+b*sec(f*x+e))^m,x, algorithm="fricas")
```

output
```
integral((b*sec(f*x + e) + a)^m, x)
```

3.791.6 Sympy [N/A]

Not integrable

Time = 0.77 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int (a+b\sec(e+fx))^m\,dx=\int (a+b\sec(e+fx))^m\,dx$$

input
```
integrate((a+b*sec(f*x+e))**m,x)
```

output
```
Integral((a + b*sec(e + f*x))**m, x)
```

3.791.7 Maxima [N/A]

Not integrable

Time = 1.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int (a+b\sec(e+fx))^m\,dx=\int (b\sec{(fx+e)}+a)^m\,dx$$

input
```
integrate((a+b*sec(f*x+e))^m,x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e) + a)^m, x)
```

3.791.8 Giac [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int (a+b\sec(e+fx))^m\,dx=\int (b\sec{(fx+e)}+a)^m\,dx$$

input
```
integrate((a+b*sec(f*x+e))^m,x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e) + a)^m, x)
```

3.791.9 Mupad [N/A]

Not integrable

Time = 14.08 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.33

$$\int (a+b\sec(e+fx))^m\,dx=\int \left(a+\frac{b}{\cos{(e+f\,x)}}\right)^m\,dx$$

input
```
int((a + b/cos(e + f*x))^m,x)
```

output
```
int((a + b/cos(e + f*x))^m, x)
```

3.792 $\int \cos(e+fx)(a+b\sec(e+fx))^m\,dx$

3.792.1 Optimal result 5855
3.792.2 Mathematica [N/A] 5855
3.792.3 Rubi [N/A] 5856
3.792.4 Maple [N/A] (verified) 5857
3.792.5 Fricas [N/A] 5857
3.792.6 Sympy [N/A] 5857
3.792.7 Maxima [N/A] 5858
3.792.8 Giac [N/A] 5858
3.792.9 Mupad [N/A] 5858

3.792.1 Optimal result

Integrand size = 19, antiderivative size = 19

$$\int \cos(e+fx)(a+b\sec(e+fx))^m\,dx = \text{Int}(\cos(e+fx)(a+b\sec(e+fx))^m,x)$$

output
```
Unintegrable(cos(f*x+e)*(a+b*sec(f*x+e))^m,x)
```

3.792.2 Mathematica [N/A]

Not integrable

Time = 13.48 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \cos(e+fx)(a+b\sec(e+fx))^m\,dx = \int \cos(e+fx)(a+b\sec(e+fx))^m\,dx$$

input
```
Integrate[Cos[e + f*x]*(a + b*Sec[e + f*x])^m,x]
```

output
```
Integrate[Cos[e + f*x]*(a + b*Sec[e + f*x])^m, x]
```

3.792.3 Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 4357}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(e+fx)(a+b\sec(e+fx))^m \, dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\left(a+b\csc\left(e+fx+\frac{\pi}{2}\right)\right)^m}{\csc\left(e+fx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \; 4357$$

$$\int \cos(e+fx)(a+b\sec(e+fx))^m dx$$

input
```
Int[Cos[e + f*x]*(a + b*Sec[e + f*x])^m,x]
```

output
```
$Aborted
```

3.792.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4357
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Unintegrable[(d*Csc[e + f*x])^n*(a + b*Csc[e + f*
x])^m, x] /; FreeQ[{a, b, d, e, f, m, n}, x]
```

3.792.4 Maple [N/A] (verified)

Not integrable

Time = 0.56 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \cos(fx+e)(a+b\sec(fx+e))^m\,dx$$

input
```
int(cos(f*x+e)*(a+b*sec(f*x+e))^m,x)
```

output
```
int(cos(f*x+e)*(a+b*sec(f*x+e))^m,x)
```

3.792.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \cos(e+fx)(a+b\sec(e+fx))^m\,dx = \int (b\sec(fx+e)+a)^m\cos(fx+e)\,dx$$

input
```
integrate(cos(f*x+e)*(a+b*sec(f*x+e))^m,x, algorithm="fricas")
```

output
```
integral((b*sec(f*x + e) + a)^m*cos(f*x + e), x)
```

3.792.6 Sympy [N/A]

Not integrable

Time = 7.94 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \cos(e+fx)(a+b\sec(e+fx))^m\,dx = \int (a+b\sec(e+fx))^m\cos(e+fx)\,dx$$

input
```
integrate(cos(f*x+e)*(a+b*sec(f*x+e))**m,x)
```

output
```
Integral((a + b*sec(e + f*x))**m*cos(e + f*x), x)
```

3.792.7 Maxima [N/A]

Not integrable

Time = 1.56 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \cos(e+fx)(a+b\sec(e+fx))^m\,dx = \int (b\sec(fx+e)+a)^m\cos(fx+e)\,dx$$

input
```
integrate(cos(f*x+e)*(a+b*sec(f*x+e))^m,x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e) + a)^m*cos(f*x + e), x)
```

3.792.8 Giac [N/A]

Not integrable

Time = 0.47 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \cos(e+fx)(a+b\sec(e+fx))^m\,dx = \int (b\sec(fx+e)+a)^m\cos(fx+e)\,dx$$

input
```
integrate(cos(f*x+e)*(a+b*sec(f*x+e))^m,x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e) + a)^m*cos(f*x + e), x)
```

3.792.9 Mupad [N/A]

Not integrable

Time = 14.10 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.21

$$\int \cos(e+fx)(a+b\sec(e+fx))^m\,dx = \int \cos(e+f\,x)\,\left(a+\frac{b}{\cos(e+f\,x)}\right)^m\,dx$$

input
```
int(cos(e + f*x)*(a + b/cos(e + f*x))^m,x)
```

output
```
int(cos(e + f*x)*(a + b/cos(e + f*x))^m, x)
```

3.793 $\int \cos^2(e+fx)(a+b\sec(e+fx))^m\,dx$

3.793.1 Optimal result 5859
3.793.2 Mathematica [N/A] 5859
3.793.3 Rubi [N/A] 5860
3.793.4 Maple [N/A] (verified) 5861
3.793.5 Fricas [N/A] 5861
3.793.6 Sympy [N/A] 5861
3.793.7 Maxima [N/A] 5862
3.793.8 Giac [N/A] 5862
3.793.9 Mupad [N/A] 5862

3.793.1 Optimal result

Integrand size = 21, antiderivative size = 21

$$\int \cos^2(e+fx)(a+b\sec(e+fx))^m\,dx=\text{Int}\left(\cos^2(e+fx)(a+b\sec(e+fx))^m,x\right)$$

output
```
Unintegrable(cos(f*x+e)^2*(a+b*sec(f*x+e))^m,x)
```

3.793.2 Mathematica [N/A]

Not integrable

Time = 14.32 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \cos^2(e+fx)(a+b\sec(e+fx))^m\,dx=\int \cos^2(e+fx)(a+b\sec(e+fx))^m\,dx$$

input
```
Integrate[Cos[e + f*x]^2*(a + b*Sec[e + f*x])^m,x]
```

output
```
Integrate[Cos[e + f*x]^2*(a + b*Sec[e + f*x])^m, x]
```

3.793.3 Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 4357}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(e+fx)(a+b\sec(e+fx))^m\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\left(a+b\csc\left(e+fx+\frac{\pi}{2}\right)\right)^m}{\csc\left(e+fx+\frac{\pi}{2}\right)^2}dx$$

$$\Big\downarrow \text{ 4357}$$

$$\int \cos^2(e+fx)(a+b\sec(e+fx))^m dx$$

input
```
Int[Cos[e + f*x]^2*(a + b*Sec[e + f*x])^m,x]
```

output
```
$Aborted
```

3.793.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4357
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Unintegrable[(d*Csc[e + f*x])^n*(a + b*Csc[e + f*
x])^m, x] /; FreeQ[{a, b, d, e, f, m, n}, x]
```

3.793.4 Maple [N/A] (verified)

Not integrable

Time = 1.68 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \cos(fx+e)^2 (a+b\sec(fx+e))^m \, dx$$

input
```
int(cos(f*x+e)^2*(a+b*sec(f*x+e))^m,x)
```

output
```
int(cos(f*x+e)^2*(a+b*sec(f*x+e))^m,x)
```

3.793.5 Fricas [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \cos^2(e+fx)(a+b\sec(e+fx))^m \, dx = \int (b\sec(fx+e)+a)^m \cos(fx+e)^2 \, dx$$

input
```
integrate(cos(f*x+e)^2*(a+b*sec(f*x+e))^m,x, algorithm="fricas")
```

output
```
integral((b*sec(f*x + e) + a)^m*cos(f*x + e)^2, x)
```

3.793.6 Sympy [N/A]

Not integrable

Time = 37.77 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.95

$$\int \cos^2(e+fx)(a+b\sec(e+fx))^m \, dx = \int (a+b\sec(e+fx))^m \cos^2(e+fx) \, dx$$

input
```
integrate(cos(f*x+e)**2*(a+b*sec(f*x+e))**m,x)
```

output
```
Integral((a + b*sec(e + f*x))**m*cos(e + f*x)**2, x)
```

3.793.7 Maxima [N/A]

Not integrable

Time = 2.22 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \cos^2(e+fx)(a+b\sec(e+fx))^m\,dx = \int (b\sec(fx+e)+a)^m\cos(fx+e)^2\,dx$$

input
```
integrate(cos(f*x+e)^2*(a+b*sec(f*x+e))^m,x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e) + a)^m*cos(f*x + e)^2, x)
```

3.793.8 Giac [N/A]

Not integrable

Time = 0.49 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \cos^2(e+fx)(a+b\sec(e+fx))^m\,dx = \int (b\sec(fx+e)+a)^m\cos(fx+e)^2\,dx$$

input
```
integrate(cos(f*x+e)^2*(a+b*sec(f*x+e))^m,x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e) + a)^m*cos(f*x + e)^2, x)
```

3.793.9 Mupad [N/A]

Not integrable

Time = 14.46 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.19

$$\int \cos^2(e+fx)(a+b\sec(e+fx))^m\,dx = \int \cos(e+f\,x)^2\left(a+\frac{b}{\cos(e+f\,x)}\right)^m dx$$

input
```
int(cos(e + f*x)^2*(a + b/cos(e + f*x))^m,x)
```

output
```
int(cos(e + f*x)^2*(a + b/cos(e + f*x))^m, x)
```

3.794 $\int \cos^{\frac{9}{2}}(c+dx)(a+b\sec(c+dx))\,dx$

3.794.1 Optimal result 5863
3.794.2 Mathematica [A] (verified) 5864
3.794.3 Rubi [A] (verified) 5864
3.794.4 Maple [A] (verified) 5867
3.794.5 Fricas [C] (verification not implemented) 5867
3.794.6 Sympy [F(-1)] 5868
3.794.7 Maxima [F] 5868
3.794.8 Giac [F] 5868
3.794.9 Mupad [B] (verification not implemented) 5869

3.794.1 Optimal result

Integrand size = 21, antiderivative size = 135

$$\begin{aligned}\int \cos^{\frac{9}{2}}(c+dx)(a+b\sec(c+dx))\,dx = {} & \frac{14aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{10b\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d} \\ & +\frac{10b\sqrt{\cos(c+dx)}\sin(c+dx)}{21d} \\ & +\frac{14a\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{45d} \\ & +\frac{2b\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{7d} \\ & +\frac{2a\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{9d}\end{aligned}$$

output
```
14/15*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*
d*x+1/2*c),2^(1/2))/d+10/21*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2
*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+14/45*a*cos(d*x+c)^(3/2)*sin(d
*x+c)/d+2/7*b*cos(d*x+c)^(5/2)*sin(d*x+c)/d+2/9*a*cos(d*x+c)^(7/2)*sin(d*x
+c)/d+10/21*b*sin(d*x+c)*cos(d*x+c)^(1/2)/d
```

3.794.2 Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.67

$$\int \cos^{\frac{9}{2}}(c+dx)(a+b\sec(c+dx))\,dx$$
$$= \frac{1176aE\left(\frac{1}{2}(c+dx)\middle|2\right)+600b\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+\sqrt{\cos(c+dx)}(690b\sin(c+dx)+266a\sin(2(c+}{1260d}$$

input
```
Integrate[Cos[c + d*x]^(9/2)*(a + b*Sec[c + d*x]),x]
```

output
```
(1176*a*EllipticE[(c + d*x)/2, 2] + 600*b*EllipticF[(c + d*x)/2, 2] + Sqrt
[Cos[c + d*x]]*(690*b*Sin[c + d*x] + 266*a*Sin[2*(c + d*x)] + 90*b*Sin[3*(
c + d*x)] + 35*a*Sin[4*(c + d*x)]))/(1260*d)
```

3.794.3 Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.07, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.524$, Rules used = {3042, 4713, 3042, 3227, 3042, 3115, 3042, 3115, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^{\frac{9}{2}}(c+dx)(a+b\sec(c+dx))\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^{9/2}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)dx$$
$$\Big\downarrow \text{4713}$$
$$\int \cos^{\frac{7}{2}}(c+dx)(a\cos(c+dx)+b)dx$$
$$\Big\downarrow \text{3042}$$
$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^{7/2}\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+b\right)dx$$
$$\Big\downarrow \text{3227}$$
$$a\int \cos^{\frac{9}{2}}(c+dx)dx+b\int \cos^{\frac{7}{2}}(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$a\int \sin\left(c+dx+\frac{\pi}{2}\right)^{9/2}dx+b\int \sin\left(c+dx+\frac{\pi}{2}\right)^{7/2}dx$$

$$\Big\downarrow \text{3115}$$

$$a\left(\frac{7}{9}\int \cos^{\frac{5}{2}}(c+dx)dx+\frac{2\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{9d}\right)+b\left(\frac{5}{7}\int \cos^{\frac{3}{2}}(c+dx)dx+\frac{2\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d}\right)$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{7}{9}\int \sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}dx+\frac{2\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{9d}\right)+b\left(\frac{5}{7}\int \sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}dx+\frac{2\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d}\right)$$

$$\Big\downarrow \text{3115}$$

$$a\left(\frac{7}{9}\left(\frac{3}{5}\int \sqrt{\cos(c+dx)}dx+\frac{2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}\right)+\frac{2\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{9d}\right)+b\left(\frac{5}{7}\left(\frac{1}{3}\int \frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}\right)+\frac{2\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d}\right)$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{7}{9}\left(\frac{3}{5}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}\right)+\frac{2\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{9d}\right)+b\left(\frac{5}{7}\left(\frac{1}{3}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}\right)+\frac{2\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d}\right)$$

$$\Big\downarrow \text{3119}$$

$$b\left(\frac{5}{7}\left(\frac{1}{3}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}\right)+\frac{2\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d}\right)+a\left(\frac{2\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{9d}+\frac{7}{9}\left(\frac{6E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}\right)\right)$$

$$\Big\downarrow \text{3120}$$

3.794. $\int \cos^{\frac{9}{2}}(c+dx)(a+b\sec(c+dx))\,dx$

$$a\left(\frac{2\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{9d}+\frac{7}{9}\left(\frac{6E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}\right)\right)+b\left(\frac{2\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d}+\frac{5}{7}\left(\frac{2\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}\right)\right)$$

input
```
Int[Cos[c + d*x]^(9/2)*(a + b*Sec[c + d*x]),x]
```

output
```
b*((2*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(7*d) + (5*((2*EllipticF[(c + d*x)/
2, 2])/(3*d) + (2*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d)))/7) + a*((2*Cos[
c + d*x]^(7/2)*Sin[c + d*x])/(9*d) + (7*((6*EllipticE[(c + d*x)/2, 2])/(5*
d) + (2*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(5*d)))/9)
```

3.794.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 4713
```
Int[(csc[(a_.) + (b_.)*(x_)]*(B_.) + (A_))*(u_), x_Symbol] :> Int[ActivateT
rig[u]*((B + A*Sin[a + b*x])/Sin[a + b*x]), x] /; FreeQ[{a, b, A, B}, x] &&
 KnownSineIntegrandQ[u, x]
```

3.794.4 Maple [A] (verified)

Time = 12.22 (sec) , antiderivative size = 318, normalized size of antiderivative = 2.36

method	result
default	$-\frac{2\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\left(-1120a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}+(2240a+720b)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{8}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+(-2072a}{}$

input
```
int(cos(d*x+c)^(9/2)*(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-2/315*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-1120*a*co
s(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10+(2240*a+720*b)*sin(1/2*d*x+1/2*c)^8
*cos(1/2*d*x+1/2*c)+(-2072*a-1080*b)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*
c)+(952*a+840*b)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-168*a-240*b)*si
n(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+75*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*
EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*b-147*(
2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*a)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

3.794.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.18

$$\int \cos^{\frac{9}{2}}(c+dx)(a+b\sec(c+dx))\,dx$$
$$=\frac{2\left(35\,a\cos\left(dx+c\right)^{3}+45\,b\cos\left(dx+c\right)^{2}+49\,a\cos\left(dx+c\right)+75\,b\right)\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)-75i\,\sqrt{2}b\mathrm{w}}{}$$

input
```
integrate(cos(d*x+c)^(9/2)*(a+b*sec(d*x+c)),x, algorithm="fricas")
```

output
```
1/315*(2*(35*a*cos(d*x + c)^3 + 45*b*cos(d*x + c)^2 + 49*a*cos(d*x + c) +
75*b)*sqrt(cos(d*x + c))*sin(d*x + c) - 75*I*sqrt(2)*b*weierstrassPInverse
(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 75*I*sqrt(2)*b*weierstrassPInvers
e(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 147*I*sqrt(2)*a*weierstrassZeta(
-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 147*I*
sqrt(2)*a*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) -
 I*sin(d*x + c))))/d
```

3.794.6 Sympy [F(-1)]

Timed out.

$$\int \cos^{\frac{9}{2}}(c+dx)(a+b\sec(c+dx))\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(9/2)*(a+b*sec(d*x+c)),x)
```

output
```
Timed out
```

3.794.7 Maxima [F]

$$\int \cos^{\frac{9}{2}}(c+dx)(a+b\sec(c+dx))\,dx = \int (b\sec(dx+c)+a)\cos(dx+c)^{\frac{9}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(9/2)*(a+b*sec(d*x+c)),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)*cos(d*x + c)^(9/2), x)
```

3.794.8 Giac [F]

$$\int \cos^{\frac{9}{2}}(c+dx)(a+b\sec(c+dx))\,dx = \int (b\sec(dx+c)+a)\cos(dx+c)^{\frac{9}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(9/2)*(a+b*sec(d*x+c)),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)*cos(d*x + c)^(9/2), x)
```

3.794.9 Mupad [B] (verification not implemented)

Time = 14.09 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.64

$$\int \cos^{\frac{9}{2}}(c+dx)(a+b\sec(c+dx))\,dx$$
$$= -\frac{2\,a\cos\left(c+d\,x\right)^{11/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{11}{4};\ \frac{15}{4};\ \cos\left(c+d\,x\right)^2\right)}{11\,d\,\sqrt{\sin\left(c+d\,x\right)^2}} - \frac{2\,b\cos\left(c+d\,x\right)^{9/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{9}{4};\ \frac{13}{4};\ \cos\left(c+d\,x\right)^2\right)}{9\,d\,\sqrt{\sin\left(c+d\,x\right)^2}}$$

input
```
int(cos(c + d*x)^(9/2)*(a + b/cos(c + d*x)),x)
```

output
```
- (2*a*cos(c + d*x)^(11/2)*sin(c + d*x)*hypergeom([1/2, 11/4], 15/4, cos(c
 + d*x)^2))/(11*d*(sin(c + d*x)^2)^(1/2)) - (2*b*cos(c + d*x)^(9/2)*sin(c
+ d*x)*hypergeom([1/2, 9/4], 13/4, cos(c + d*x)^2))/(9*d*(sin(c + d*x)^2)^
(1/2))
```

3.795 $\int \cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))\,dx$

3.795.1 Optimal result 5870
3.795.2 Mathematica [A] (verified) 5871
3.795.3 Rubi [A] (verified) 5871
3.795.4 Maple [A] (verified) 5874
3.795.5 Fricas [C] (verification not implemented) 5874
3.795.6 Sympy [F(-1)] 5875
3.795.7 Maxima [F] 5875
3.795.8 Giac [F] 5875
3.795.9 Mupad [B] (verification not implemented) 5876

3.795.1 Optimal result

Integrand size = 21, antiderivative size = 111

$$\int \cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))\,dx = \frac{6bE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{10a\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d}+\frac{10a\sqrt{\cos(c+dx)}\sin(c+dx)}{21d}+\frac{2b\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d}+\frac{2a\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{7d}$$

output
```
6/5*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*
x+1/2*c),2^(1/2))/d+10/21*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c
)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/5*b*cos(d*x+c)^(3/2)*sin(d*x+c
)/d+2/7*a*cos(d*x+c)^(5/2)*sin(d*x+c)/d+10/21*a*sin(d*x+c)*cos(d*x+c)^(1/2
)/d
```

3.795.2 Mathematica [A] (verified)

Time = 0.61 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.69

$$\int \cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))\,dx$$
$$= \frac{126bE\left(\frac{1}{2}(c+dx)\middle|2\right)+50a\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+\sqrt{\cos(c+dx)}(65a+42b\cos(c+dx)+15a\cos(2(c}{105d}$$

input
```
Integrate[Cos[c + d*x]^(7/2)*(a + b*Sec[c + d*x]),x]
```

output
```
(126*b*EllipticE[(c + d*x)/2, 2] + 50*a*EllipticF[(c + d*x)/2, 2] + Sqrt[C
os[c + d*x]]*(65*a + 42*b*Cos[c + d*x] + 15*a*Cos[2*(c + d*x)])*Sin[c + d*
x])/(105*d)
```

3.795.3 Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.05, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.524$, Rules used = {3042, 4713, 3042, 3227, 3042, 3115, 3042, 3115, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^{7/2}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)dx$$
$$\Big\downarrow \text{4713}$$
$$\int \cos^{\frac{5}{2}}(c+dx)(a\cos(c+dx)+b)dx$$
$$\Big\downarrow \text{3042}$$
$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+b\right)dx$$
$$\Big\downarrow \text{3227}$$
$$a\int \cos^{\frac{7}{2}}(c+dx)dx+b\int \cos^{\frac{5}{2}}(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$a\int \sin\left(c+dx+\frac{\pi}{2}\right)^{7/2}dx+b\int \sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}dx$$

$$\Big\downarrow \text{3115}$$

$$a\left(\frac{5}{7}\int \cos^{\frac{3}{2}}(c+dx)dx+\frac{2\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d}\right)+b\left(\frac{3}{5}\int \sqrt{\cos(c+dx)}dx+\frac{2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}\right)$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{5}{7}\int \sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}dx+\frac{2\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d}\right)+b\left(\frac{3}{5}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}\right)$$

$$\Big\downarrow \text{3115}$$

$$a\left(\frac{5}{7}\left(\frac{1}{3}\int \frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}\right)+\frac{2\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d}\right)+b\left(\frac{3}{5}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}\right)$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{5}{7}\left(\frac{1}{3}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}\right)+\frac{2\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d}\right)+b\left(\frac{3}{5}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}\right)$$

$$\Big\downarrow \text{3119}$$

$$a\left(\frac{5}{7}\left(\frac{1}{3}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}\right)+\frac{2\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d}\right)+b\left(\frac{6E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}\right)$$

$$\Big\downarrow \text{3120}$$

3.795. $\int \cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))\,dx$

$$a\left(\frac{2\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d}+\frac{5}{7}\left(\frac{2\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}\right)\right)+b\left(\frac{6E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}\right)$$

input
```
Int[Cos[c + d*x]^(7/2)*(a + b*Sec[c + d*x]),x]
```

output
```
b*((6*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*Cos[c + d*x]^(3/2)*Sin[c + d*x
])/(5*d)) + a*((2*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(7*d) + (5*((2*Elliptic
F[(c + d*x)/2, 2])/(3*d) + (2*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d)))/7)
```

3.795.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 4713
```
Int[(csc[(a_.) + (b_.)*(x_)]*(B_.) + (A_))*(u_), x_Symbol] :> Int[ActivateT
rig[u]*((B + A*Sin[a + b*x])/Sin[a + b*x]), x] /; FreeQ[{a, b, A, B}, x] &&
 KnownSineIntegrandQ[u, x]
```

3.795. $\int \cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))\,dx$

3.795.4 Maple [A] (verified)

Time = 10.97 (sec) , antiderivative size = 290, normalized size of antiderivative = 2.61

method	result
default	$-\frac{2\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\left(240a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{8}+(-360a-168b)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+(280a+168b\right.}{}$

input
```
int(cos(d*x+c)^(7/2)*(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-2/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(240*a*cos(
1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+(-360*a-168*b)*sin(1/2*d*x+1/2*c)^6*co
s(1/2*d*x+1/2*c)+(280*a+168*b)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-8
0*a-42*b)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+25*(2*sin(1/2*d*x+1/2*c)
^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*a-63*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^
(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*b)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*
x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

3.795.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.33

$$\int \cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))\,dx$$
$$=\frac{2\left(15\,a\cos\left(dx+c\right)^{2}+21\,b\cos\left(dx+c\right)+25\,a\right)\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)-25i\,\sqrt{2}a\text{weierstrassPInverse}(}{}$$

input
```
integrate(cos(d*x+c)^(7/2)*(a+b*sec(d*x+c)),x, algorithm="fricas")
```

output
```
1/105*(2*(15*a*cos(d*x + c)^2 + 21*b*cos(d*x + c) + 25*a)*sqrt(cos(d*x + c
))*sin(d*x + c) - 25*I*sqrt(2)*a*weierstrassPInverse(-4, 0, cos(d*x + c) +
 I*sin(d*x + c)) + 25*I*sqrt(2)*a*weierstrassPInverse(-4, 0, cos(d*x + c)
- I*sin(d*x + c)) + 63*I*sqrt(2)*b*weierstrassZeta(-4, 0, weierstrassPInve
rse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 63*I*sqrt(2)*b*weierstrassZet
a(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/d
```

3.795.6 Sympy [F(-1)]

Timed out.

$$\int \cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(7/2)*(a+b*sec(d*x+c)),x)
```

output
```
Timed out
```

3.795.7 Maxima [F]

$$\int \cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))\,dx = \int (b\sec(dx+c)+a)\cos(dx+c)^{\frac{7}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(7/2)*(a+b*sec(d*x+c)),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)*cos(d*x + c)^(7/2), x)
```

3.795.8 Giac [F]

$$\int \cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))\,dx = \int (b\sec(dx+c)+a)\cos(dx+c)^{\frac{7}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(7/2)*(a+b*sec(d*x+c)),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)*cos(d*x + c)^(7/2), x)
```

3.795.9 Mupad [B] (verification not implemented)

Time = 14.37 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.78

$$\int \cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))\,dx = -\frac{2\,a\cos\left(c+d\,x\right)^{9/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{9}{4};\ \frac{13}{4};\ \cos\left(c+d\,x\right)^2\right)}{9\,d\,\sqrt{\sin\left(c+d\,x\right)^2}} - \frac{2\,b\cos\left(c+d\,x\right)^{7/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{7}{4};\ \frac{11}{4};\ \cos\left(c+d\,x\right)^2\right)}{7\,d\,\sqrt{\sin\left(c+d\,x\right)^2}}$$

input
```
int(cos(c + d*x)^(7/2)*(a + b/cos(c + d*x)),x)
```

output
```
- (2*a*cos(c + d*x)^(9/2)*sin(c + d*x)*hypergeom([1/2, 9/4], 13/4, cos(c +
 d*x)^2))/(9*d*(sin(c + d*x)^2)^(1/2)) - (2*b*cos(c + d*x)^(7/2)*sin(c + d
*x)*hypergeom([1/2, 7/4], 11/4, cos(c + d*x)^2))/(7*d*(sin(c + d*x)^2)^(1/
2))
```

3.796 $\int \cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))\,dx$

3.796.1 Optimal result 5877
3.796.2 Mathematica [A] (verified) 5877
3.796.3 Rubi [A] (verified) 5878
3.796.4 Maple [B] (verified) 5880
3.796.5 Fricas [C] (verification not implemented) 5881
3.796.6 Sympy [F(-1)] 5881
3.796.7 Maxima [F] 5881
3.796.8 Giac [F] 5882
3.796.9 Mupad [B] (verification not implemented) 5882

3.796.1 Optimal result

Integrand size = 21, antiderivative size = 87

$$\int \cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))\,dx = \frac{6aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2b\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2b\sqrt{\cos(c+dx)}\sin(c+dx)}{3d}+\frac{2a\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d}$$

output
```
6/5*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*
x+1/2*c),2^(1/2))/d+2/3*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*
EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/5*a*cos(d*x+c)^(3/2)*sin(d*x+c)/
d+2/3*b*sin(d*x+c)*cos(d*x+c)^(1/2)/d
```

3.796.2 Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.76

$$\int \cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))\,dx = \frac{2\left(9aE\left(\frac{1}{2}(c+dx)\middle|2\right)+5b\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+\sqrt{\cos(c+dx)}(5b+3a\cos(c+dx))\sin(c+dx)\right)}{15d}$$

input
```
Integrate[Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x]),x]
```

output
```
(2*(9*a*EllipticE[(c + d*x)/2, 2] + 5*b*EllipticF[(c + d*x)/2, 2] + Sqrt[C
os[c + d*x]]*(5*b + 3*a*Cos[c + d*x])*Sin[c + d*x]))/(15*d)
```

3.796.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.02, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 4713, 3042, 3227, 3042, 3115, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{4713}$$

$$\int \cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+b)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+b\right)dx$$

$$\Big\downarrow \text{3227}$$

$$a\int \cos^{\frac{5}{2}}(c+dx)dx+b\int \cos^{\frac{3}{2}}(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$a\int \sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}dx+b\int \sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}dx$$

$$\Big\downarrow \text{3115}$$

$$a\left(\frac{3}{5}\int \sqrt{\cos(c+dx)}dx+\frac{2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}\right)+b\left(\frac{1}{3}\int \frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}\right)$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{3}{5}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}\right)+b\left(\frac{1}{3}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}\right)$$

$$\Big\downarrow \text{3119}$$

$$b\left(\frac{1}{3}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}\right)+a\left(\frac{6E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}\right)$$

$$\Big\downarrow \text{3120}$$

$$a\left(\frac{6E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}\right)+b\left(\frac{2\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}\right)$$

input
```
Int[Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x]),x]
```

output
```
b*((2*EllipticF[(c + d*x)/2, 2])/(3*d) + (2*Sqrt[Cos[c + d*x]]*Sin[c + d*x
])/(3*d)) + a*((6*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*Cos[c + d*x]^(3/2)
*Sin[c + d*x])/(5*d))
```

3.796.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 4713
```
Int[(csc[(a_.) + (b_.)*(x_)]*(B_.) + (A_))*(u_), x_Symbol] :> Int[ActivateT
rig[u]*((B + A*Sin[a + b*x])/Sin[a + b*x]), x] /; FreeQ[{a, b, A, B}, x] &&
 KnownSineIntegrandQ[u, x]
```

3.796.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 261 vs. $2(127) = 254$.

Time = 9.10 (sec) , antiderivative size = 262, normalized size of antiderivative = 3.01

method	result
default	$-\frac{2\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\left(-24a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}+(24a+20b)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+(-6a-10b)\sin\left(\right.\right.}{15\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right.}}$

input
```
int(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-2/15*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-24*a*cos(1
/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+(24*a+20*b)*sin(1/2*d*x+1/2*c)^4*cos(1/
2*d*x+1/2*c)+(-6*a-10*b)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+5*(2*sin(
1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*b-9*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*
d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*a)/(-2*sin(1/2*d*x+1/2*c)
^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-
1)^(1/2)/d
```

3.796.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.57

$$\int \cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))\,dx$$
$$= \frac{2\,(3\,a\cos(dx+c)+5\,b)\sqrt{\cos(dx+c)}\sin(dx+c)-5i\sqrt{2}b\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\ \text{si}}{}$$

input
```
integrate(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c)),x, algorithm="fricas")
```

output
```
1/15*(2*(3*a*cos(d*x + c) + 5*b)*sqrt(cos(d*x + c))*sin(d*x + c) - 5*I*sqr
t(2)*b*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 5*I*sqr
t(2)*b*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 9*I*sqr
t(2)*a*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*
sin(d*x + c))) - 9*I*sqrt(2)*a*weierstrassZeta(-4, 0, weierstrassPInverse(
-4, 0, cos(d*x + c) - I*sin(d*x + c))))/d
```

3.796.6 Sympy [F(-1)]

Timed out.

$$\int \cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(5/2)*(a+b*sec(d*x+c)),x)
```

output
```
Timed out
```

3.796.7 Maxima [F]

$$\int \cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))\,dx = \int (b\sec(dx+c)+a)\cos(dx+c)^{\frac{5}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c)),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)*cos(d*x + c)^(5/2), x)
```

3.796.8 Giac [F]

$$\int \cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))\,dx = \int (b\sec(dx+c)+a)\cos(dx+c)^{\frac{5}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c)),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)*cos(d*x + c)^(5/2), x)
```

3.796.9 Mupad [B] (verification not implemented)

Time = 14.35 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.92

$$\int \cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))\,dx = \frac{2\,b\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{3\,d}+\frac{2\,b\,\sqrt{\cos(c+d\,x)}\,\sin(c+d\,x)}{3\,d}-\frac{2\,a\cos(c+d\,x)^{7/2}\,\sin(c+d\,x)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{7}{4};\ \frac{11}{4};\ \cos(c+d\,x)^2\right)}{7\,d\,\sqrt{\sin(c+d\,x)^2}}$$

input
```
int(cos(c + d*x)^(5/2)*(a + b/cos(c + d*x)),x)
```

output
```
(2*b*ellipticF(c/2 + (d*x)/2, 2))/(3*d) + (2*b*cos(c + d*x)^(1/2)*sin(c +
d*x))/(3*d) - (2*a*cos(c + d*x)^(7/2)*sin(c + d*x)*hypergeom([1/2, 7/4], 1
1/4, cos(c + d*x)^2))/(7*d*(sin(c + d*x)^2)^(1/2))
```

3.797 $\int \cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))\,dx$

3.797.1 Optimal result 5883
3.797.2 Mathematica [A] (verified) 5883
3.797.3 Rubi [A] (verified) 5884
3.797.4 Maple [B] (verified) 5886
3.797.5 Fricas [C] (verification not implemented) 5886
3.797.6 Sympy [F(-1)] 5887
3.797.7 Maxima [F] 5887
3.797.8 Giac [F] 5887
3.797.9 Mupad [B] (verification not implemented) 5888

3.797.1 Optimal result

Integrand size = 21, antiderivative size = 61

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))\,dx = \frac{2bE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2a\sqrt{\cos(c+dx)}\sin(c+dx)}{3d}$$

output
```
2*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+
1/2*c),2^(1/2))/d+2/3*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*El
lipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/3*a*sin(d*x+c)*cos(d*x+c)^(1/2)/d
```

3.797.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.87

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))\,dx = \frac{2\left(3bE\left(\frac{1}{2}(c+dx)\middle|2\right)+a\left(\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+\sqrt{\cos(c+dx)}\sin(c+dx)\right)\right)}{3d}$$

input
```
Integrate[Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x]),x]
```

output
```
(2*(3*b*EllipticE[(c + d*x)/2, 2] + a*(EllipticF[(c + d*x)/2, 2] + Sqrt[Co
s[c + d*x]]*Sin[c + d*x])))/(3*d)
```

3.797.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.02, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 4713, 3042, 3227, 3042, 3115, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{4713}$$

$$\int \sqrt{\cos(c+dx)}(a\cos(c+dx)+b)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+b\right)dx$$

$$\Big\downarrow \text{3227}$$

$$a\int \cos^{\frac{3}{2}}(c+dx)dx+b\int \sqrt{\cos(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$a\int \sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}dx+b\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3115}$$

$$a\left(\frac{1}{3}\int \frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}\right)+b\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{1}{3}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}\right)+b\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3119}$$

$$a\left(\frac{1}{3}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}\right)+\frac{2bE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

$$\Big\downarrow \text{3120}$$

$$a\left(\frac{2\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}\right)+\frac{2bE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

input
```
Int[Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x]),x]
```

output
```
(2*b*EllipticE[(c + d*x)/2, 2])/d + a*((2*EllipticF[(c + d*x)/2, 2])/(3*d)
 + (2*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d))
```

3.797.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 4713
```
Int[(csc[(a_.) + (b_.)*(x_)]*(B_.) + (A_))*(u_), x_Symbol] :> Int[ActivateT
rig[u]*((B + A*Sin[a + b*x])/Sin[a + b*x]), x] /; FreeQ[{a, b, A, B}, x] &&
 KnownSineIntegrandQ[u, x]
```

3.797.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 227 vs. $2(107) = 214$.

Time = 7.04 (sec) , antiderivative size = 228, normalized size of antiderivative = 3.74

method	result
default	$-\frac{2\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4a-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2a+\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1}\ \mathrm{Ellipt}\right.}{3\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\ \sin\left(\frac{dx}{2}+\right.}$

input
```
int(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-2/3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4*cos(1/2*d*
x+1/2*c)*sin(1/2*d*x+1/2*c)^4*a-2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2*
a+(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*a-3*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE
(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*b)/(-2*sin(1/2*d
*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+
1/2*c)^2-1)^(1/2)/d
```

3.797.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 125, normalized size of antiderivative = 2.05

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))\,dx$$
$$= \frac{2\,a\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)-i\,\sqrt{2}a\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right))+i\,\sqrt{2}a\mathrm{we}}{}$$

input
```
integrate(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c)),x, algorithm="fricas")
```

output
```
1/3*(2*a*sqrt(cos(d*x + c))*sin(d*x + c) - I*sqrt(2)*a*weierstrassPInverse
(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + I*sqrt(2)*a*weierstrassPInverse(-
4, 0, cos(d*x + c) - I*sin(d*x + c)) + 3*I*sqrt(2)*b*weierstrassZeta(-4, 0
, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 3*I*sqrt(2)
*b*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(
d*x + c))))/d
```

3.797.6 Sympy [F(-1)]

Timed out.

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(3/2)*(a+b*sec(d*x+c)),x)
```

output
```
Timed out
```

3.797.7 Maxima [F]

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))\,dx = \int (b\sec(dx+c)+a)\cos(dx+c)^{\frac{3}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c)),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)*cos(d*x + c)^(3/2), x)
```

3.797.8 Giac [F]

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))\,dx = \int (b\sec(dx+c)+a)\cos(dx+c)^{\frac{3}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c)),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)*cos(d*x + c)^(3/2), x)
```

3.797.9 Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.87

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))\,dx = \frac{2\,a\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{3\,d}+\frac{2\,b\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{2\,a\,\sqrt{\cos\left(c+d\,x\right)}\,\sin\left(c+d\,x\right)}{3\,d}$$

input
```
int(cos(c + d*x)^(3/2)*(a + b/cos(c + d*x)),x)
```

output
```
(2*a*ellipticF(c/2 + (d*x)/2, 2))/(3*d) + (2*b*ellipticE(c/2 + (d*x)/2, 2)
)/d + (2*a*cos(c + d*x)^(1/2)*sin(c + d*x))/(3*d)
```

3.798 $\int \sqrt{\cos(c+dx)}(a+b\sec(c+dx))\,dx$

3.798.1 Optimal result 5889
3.798.2 Mathematica [A] (verified) 5889
3.798.3 Rubi [A] (verified) 5890
3.798.4 Maple [A] (verified) 5891
3.798.5 Fricas [C] (verification not implemented) 5892
3.798.6 Sympy [F] 5892
3.798.7 Maxima [F] 5893
3.798.8 Giac [F] 5893
3.798.9 Mupad [B] (verification not implemented) 5893

3.798.1 Optimal result

Integrand size = 21, antiderivative size = 35

$$\int \sqrt{\cos(c+dx)}(a+b\sec(c+dx))\,dx=\frac{2aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2b\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}$$

output
```
2*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+
1/2*c),2^(1/2))/d+2*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elli
pticF(sin(1/2*d*x+1/2*c),2^(1/2))/d
```

3.798.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.91

$$\int \sqrt{\cos(c+dx)}(a+b\sec(c+dx))\,dx=\frac{2\left(aE\left(\frac{1}{2}(c+dx)\middle|2\right)+b\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\right)}{d}$$

input
```
Integrate[Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x]),x]
```

output
```
(2*(a*EllipticE[(c + d*x)/2, 2] + b*EllipticF[(c + d*x)/2, 2]))/d
```

3.798.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 4713, 3042, 3227, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\cos(c+dx)}(a+b\sec(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{4713}$$

$$\int \frac{a\cos(c+dx)+b}{\sqrt{\cos(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a\sin\left(c+dx+\frac{\pi}{2}\right)+b}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{3227}$$

$$a\int \sqrt{\cos(c+dx)}dx+b\int \frac{1}{\sqrt{\cos(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$a\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+b\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{3119}$$

$$b\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

$$\Big\downarrow \text{3120}$$

$$\frac{2aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2b\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}$$

input
```
Int[Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x]),x]
```

output
```
(2*a*EllipticE[(c + d*x)/2, 2])/d + (2*b*EllipticF[(c + d*x)/2, 2])/d
```

3.798.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 4713
```
Int[(csc[(a_.) + (b_.)*(x_)]*(B_.) + (A_))*(u_), x_Symbol] :> Int[ActivateT
rig[u]*((B + A*Sin[a + b*x])/Sin[a + b*x]), x] /; FreeQ[{a, b, A, B}, x] &&
 KnownSineIntegrandQ[u, x]
```

3.798.4 Maple [A] (verified)

Time = 4.54 (sec) , antiderivative size = 152, normalized size of antiderivative = 4.34

method	result
default	$-\frac{2\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1}\left(b\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)-a\,\mathrm{EllipticE}\left(\cos\left(\frac{dx}{2}\right.\right.\right.}{\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1}\,d}$
risch	$-\frac{ia\sqrt{2}\sqrt{\left(e^{2i(dx+c)}+1\right)e^{-i(dx+c)}}}{d}-\frac{i\left(\frac{ib\sqrt{-i\left(e^{i(dx+c)}+i\right)}\sqrt{2}\sqrt{i\left(e^{i(dx+c)}-i\right)}\sqrt{ie^{i(dx+c)}}\,\mathrm{EllipticF}\left(\sqrt{-i\left(e^{i(dx+c)}+i\right)},\frac{\sqrt{2}}{2}\right)}{\sqrt{e^{3i(dx+c)}+e^{i(dx+c)}}}+a\left(-\right.\right.}{}$

input
```
int(cos(d*x+c)^(1/2)*(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-2*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*(b*EllipticF(cos(1/2*d*x+1
/2*c),2^(1/2))-a*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c
)^2-1)^(1/2)/d
```

3.798.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 107, normalized size of antiderivative = 3.06

$$\int \sqrt{\cos(c+dx)}(a+b\sec(c+dx))\,dx$$
$$= \frac{-i\sqrt{2}b\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+i\sqrt{2}b\text{weierstrassPInverse}(-4,0,\cos(d}{}$$

input
```
integrate(cos(d*x+c)^(1/2)*(a+b*sec(d*x+c)),x, algorithm="fricas")
```

output
```
(-I*sqrt(2)*b*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) +
I*sqrt(2)*b*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + I*
sqrt(2)*a*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) +
 I*sin(d*x + c))) - I*sqrt(2)*a*weierstrassZeta(-4, 0, weierstrassPInverse
(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/d
```

3.798.6 Sympy [F]

$$\int \sqrt{\cos(c+dx)}(a+b\sec(c+dx))\,dx = \int (a+b\sec(c+dx))\sqrt{\cos(c+dx)}\,dx$$

input
```
integrate(cos(d*x+c)**(1/2)*(a+b*sec(d*x+c)),x)
```

output
```
Integral((a + b*sec(c + d*x))*sqrt(cos(c + d*x)), x)
```

3.798.7 Maxima [F]

$$\int \sqrt{\cos(c+dx)}(a+b\sec(c+dx))\,dx = \int (b\sec(dx+c)+a)\sqrt{\cos(dx+c)}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)*(a+b*sec(d*x+c)),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)*sqrt(cos(d*x + c)), x)
```

3.798.8 Giac [F]

$$\int \sqrt{\cos(c+dx)}(a+b\sec(c+dx))\,dx = \int (b\sec(dx+c)+a)\sqrt{\cos(dx+c)}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)*(a+b*sec(d*x+c)),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)*sqrt(cos(d*x + c)), x)
```

3.798.9 Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.94

$$\int \sqrt{\cos(c+dx)}(a+b\sec(c+dx))\,dx = \frac{2\,a\,\mathrm{E}\left(\frac{c}{2}+\frac{dx}{2}\middle|2\right)}{d}+\frac{2\,b\,\mathrm{F}\left(\frac{c}{2}+\frac{dx}{2}\middle|2\right)}{d}$$

input
```
int(cos(c + d*x)^(1/2)*(a + b/cos(c + d*x)),x)
```

output
```
(2*a*ellipticE(c/2 + (d*x)/2, 2))/d + (2*b*ellipticF(c/2 + (d*x)/2, 2))/d
```

3.799 $\int \frac{a+b\sec(c+dx)}{\sqrt{\cos(c+dx)}}\,dx$

3.799.1 Optimal result 5894
3.799.2 Mathematica [A] (verified) 5894
3.799.3 Rubi [A] (verified) 5895
3.799.4 Maple [A] (verified) 5897
3.799.5 Fricas [C] (verification not implemented) 5897
3.799.6 Sympy [F] 5898
3.799.7 Maxima [F] 5898
3.799.8 Giac [F] 5898
3.799.9 Mupad [B] (verification not implemented) 5899

3.799.1 Optimal result

Integrand size = 21, antiderivative size = 57

$$\int \frac{a+b\sec(c+dx)}{\sqrt{\cos(c+dx)}}\,dx=-\frac{2bE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+\frac{2b\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

output
```
-2*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x
+1/2*c),2^(1/2))/d+2*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ell
ipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2*b*sin(d*x+c)/d/cos(d*x+c)^(1/2)
```

3.799.2 Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.89

$$\int \frac{a+b\sec(c+dx)}{\sqrt{\cos(c+dx)}}\,dx=\frac{2\left(-bE\left(\frac{1}{2}(c+dx)\middle|2\right)+a\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+\frac{b\sin(c+dx)}{\sqrt{\cos(c+dx)}}\right)}{d}$$

input
```
Integrate[(a + b*Sec[c + d*x])/Sqrt[Cos[c + d*x]],x]
```

output
```
(2*(-(b*EllipticE[(c + d*x)/2, 2]) + a*EllipticF[(c + d*x)/2, 2] + (b*Sin[
c + d*x])/Sqrt[Cos[c + d*x]]))/d
```

3.799.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.02, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 4713, 3042, 3227, 3042, 3116, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\sec(c+dx)}{\sqrt{\cos(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4713}$$

$$\int \frac{a\cos(c+dx)+b}{\cos^{\frac{3}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a\sin\left(c+dx+\frac{\pi}{2}\right)+b}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{3227}$$

$$a\int \frac{1}{\sqrt{\cos(c+dx)}}dx+b\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$a\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+b\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{3116}$$

$$a\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+b\left(\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\int \sqrt{\cos(c+dx)}dx\right)$$

$$\Big\downarrow \text{3042}$$

$$a\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+b\left(\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx\right)$$

$$\Big\downarrow \text{3119}$$

$$a\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+b\left(\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)$$

$$\Big\downarrow \; 3120$$

$$\frac{2a\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+b\left(\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)$$

input
```
Int[(a + b*Sec[c + d*x])/Sqrt[Cos[c + d*x]],x]
```

output
```
(2*a*EllipticF[(c + d*x)/2, 2])/d + b*((-2*EllipticE[(c + d*x)/2, 2])/d +
(2*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]))
```

3.799.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 4713
```
Int[(csc[(a_.) + (b_.)*(x_)]*(B_.) + (A_))*(u_), x_Symbol] :> Int[ActivateT
rig[u]*((B + A*Sin[a + b*x])/Sin[a + b*x]), x] /; FreeQ[{a, b, A, B}, x] &&
 KnownSineIntegrandQ[u, x]
```

3.799.4 Maple [A] (verified)

Time = 5.72 (sec) , antiderivative size = 150, normalized size of antiderivative = 2.63

method	result
default	$\frac{4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}b-2\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1}\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,a-2\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1}\,\mathrm{EllipticE}\left(\right.}{\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1}\,d}$

input
```
int((a+b*sec(d*x+c))/cos(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*(2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2*b-(2*sin(1/2*d*x+1/2*c)^2-1)^
(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*a
-(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*b)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1
)^(1/2)/d
```

3.799.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 156, normalized size of antiderivative = 2.74

$$\int \frac{a+b\sec(c+dx)}{\sqrt{\cos(c+dx)}}\,dx$$
$$= \frac{-i\sqrt{2}a\cos(dx+c)\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+i\sqrt{2}a\cos(dx+c)\,\mathrm{weierst}}{}$$

input
```
integrate((a+b*sec(d*x+c))/cos(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
(-I*sqrt(2)*a*cos(d*x + c)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin
(d*x + c)) + I*sqrt(2)*a*cos(d*x + c)*weierstrassPInverse(-4, 0, cos(d*x +
 c) - I*sin(d*x + c)) - I*sqrt(2)*b*cos(d*x + c)*weierstrassZeta(-4, 0, we
ierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + I*sqrt(2)*b*cos
(d*x + c)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) -
 I*sin(d*x + c))) + 2*b*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c))
```

3.799.6 Sympy [F]

$$\int \frac{a+b\sec(c+dx)}{\sqrt{\cos(c+dx)}}\,dx=\int \frac{a+b\sec{(c+dx)}}{\sqrt{\cos{(c+dx)}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))/cos(d*x+c)**(1/2),x)
```

output
```
Integral((a + b*sec(c + d*x))/sqrt(cos(c + d*x)), x)
```

3.799.7 Maxima [F]

$$\int \frac{a+b\sec(c+dx)}{\sqrt{\cos(c+dx)}}\,dx=\int \frac{b\sec{(dx+c)}+a}{\sqrt{\cos{(dx+c)}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))/cos(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)/sqrt(cos(d*x + c)), x)
```

3.799.8 Giac [F]

$$\int \frac{a+b\sec(c+dx)}{\sqrt{\cos(c+dx)}}\,dx=\int \frac{b\sec{(dx+c)}+a}{\sqrt{\cos{(dx+c)}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))/cos(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)/sqrt(cos(d*x + c)), x)
```

3.799.9 Mupad [B] (verification not implemented)

Time = 14.58 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.05

$$\int \frac{a+b\sec(c+dx)}{\sqrt{\cos(c+dx)}}\,dx=\frac{2\,a\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{2\,b\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{1}{2};\ \frac{3}{4};\ \cos\left(c+d\,x\right)^2\right)}{d\,\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{\sin\left(c+d\,x\right)^2}}$$

input
```
int((a + b/cos(c + d*x))/cos(c + d*x)^(1/2),x)
```

output
```
(2*a*ellipticF(c/2 + (d*x)/2, 2))/d + (2*b*sin(c + d*x)*hypergeom([-1/4, 1
/2], 3/4, cos(c + d*x)^2))/(d*cos(c + d*x)^(1/2)*(sin(c + d*x)^2)^(1/2))
```

3.800 $\int \frac{a+b\sec(c+dx)}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

3.800.1 Optimal result 5900
3.800.2 Mathematica [A] (verified) 5900
3.800.3 Rubi [A] (verified) 5901
3.800.4 Maple [B] (verified) 5903
3.800.5 Fricas [C] (verification not implemented) 5904
3.800.6 Sympy [F] 5904
3.800.7 Maxima [F] 5905
3.800.8 Giac [F] 5905
3.800.9 Mupad [B] (verification not implemented) 5905

3.800.1 Optimal result

Integrand size = 21, antiderivative size = 83

$$\int \frac{a+b\sec(c+dx)}{\cos^{\frac{3}{2}}(c+dx)}\,dx = -\frac{2aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2b\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2b\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{2a\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

output
```
-2*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x
+1/2*c),2^(1/2))/d+2/3*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*E
llipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/3*b*sin(d*x+c)/d/cos(d*x+c)^(3/2)
+2*a*sin(d*x+c)/d/cos(d*x+c)^(1/2)
```

3.800.2 Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.78

$$\int \frac{a+b\sec(c+dx)}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{-6aE\left(\frac{1}{2}(c+dx)\middle|2\right)+2b\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+\frac{2(b+3a\cos(c+dx))\sin(c+dx)}{\cos^{\frac{3}{2}}(c+dx)}}{3d}$$

input
```
Integrate[(a + b*Sec[c + d*x])/Cos[c + d*x]^(3/2),x]
```

output
```
(-6*a*EllipticE[(c + d*x)/2, 2] + 2*b*EllipticF[(c + d*x)/2, 2] + (2*(b +
3*a*Cos[c + d*x])*Sin[c + d*x])/Cos[c + d*x]^(3/2))/(3*d)
```

3.800.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.02, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 4713, 3042, 3227, 3042, 3116, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\sec(c+dx)}{\cos^{\frac{3}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{4713}$$

$$\int \frac{a\cos(c+dx)+b}{\cos^{\frac{5}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a\sin\left(c+dx+\frac{\pi}{2}\right)+b}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx$$

$$\Big\downarrow \text{3227}$$

$$a\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)}dx+b\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$a\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+b\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx$$

$$\Big\downarrow \text{3116}$$

$$a\left(\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\int \sqrt{\cos(c+dx)}dx\right)+b\left(\frac{1}{3}\int \frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}\right)$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx\right)+b\left(\frac{1}{3}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}\right)$$

$\Big\downarrow$ 3119

$$b\left(\frac{1}{3}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}\right)+a\left(\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)$$

$\Big\downarrow$ 3120

$$a\left(\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)+b\left(\frac{2\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}\right)$$

input
```
Int[(a + b*Sec[c + d*x])/Cos[c + d*x]^(3/2),x]
```

output
```
b*((2*EllipticF[(c + d*x)/2, 2])/(3*d) + (2*Sin[c + d*x])/(3*d*Cos[c + d*x
]^(3/2))) + a*((-2*EllipticE[(c + d*x)/2, 2])/d + (2*Sin[c + d*x])/(d*Sqrt
[Cos[c + d*x]]))
```

3.800.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 4713
```
Int[(csc[(a_.) + (b_.)*(x_)]*(B_.) + (A_))*(u_), x_Symbol] :> Int[ActivateT
rig[u]*((B + A*Sin[a + b*x])/Sin[a + b*x]), x] /; FreeQ[{a, b, A, B}, x] &&
 KnownSineIntegrandQ[u, x]
```

3.800.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 395 vs. $2(127) = 254$.

Time = 11.24 (sec) , antiderivative size = 396, normalized size of antiderivative = 4.77

method	result
default	$-\frac{2\sqrt{-\left(-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(12\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4 a-2\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1}\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sqrt{}}{}$

input
```
int((a+b*sec(d*x+c))/cos(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(4*sin(1/2*
d*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)/sin(1/2*d*x+1/2*c)^3*(12*cos(1/2*d*
x+1/2*c)*sin(1/2*d*x+1/2*c)^4*a-2*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipti
cF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/
2*c)^2*b-6*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2
^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2*a-6*cos(1/2*d*x+
1/2*c)*sin(1/2*d*x+1/2*c)^2*a-2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2*b+
(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*b+3*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(c
os(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*a)*(-2*sin(1/2*d*x
+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

3.800.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 175, normalized size of antiderivative = 2.11

$$\int \frac{a+b\sec(c+dx)}{\cos^{\frac{3}{2}}(c+dx)}\,dx$$
$$= \frac{-i\sqrt{2}b\cos(dx+c)^2\,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+i\sqrt{2}b\cos(dx+c)^2\,\text{weiers}}{}$$

input
```
integrate((a+b*sec(d*x+c))/cos(d*x+c)^(3/2),x, algorithm="fricas")
```

output
```
1/3*(-I*sqrt(2)*b*cos(d*x + c)^2*weierstrassPInverse(-4, 0, cos(d*x + c) +
 I*sin(d*x + c)) + I*sqrt(2)*b*cos(d*x + c)^2*weierstrassPInverse(-4, 0, c
os(d*x + c) - I*sin(d*x + c)) - 3*I*sqrt(2)*a*cos(d*x + c)^2*weierstrassZe
ta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 3*I
*sqrt(2)*a*cos(d*x + c)^2*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0
, cos(d*x + c) - I*sin(d*x + c))) + 2*(3*a*cos(d*x + c) + b)*sqrt(cos(d*x
+ c))*sin(d*x + c))/(d*cos(d*x + c)^2)
```

3.800.6 Sympy [F]

$$\int \frac{a+b\sec(c+dx)}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{a+b\sec(c+dx)}{\cos^{\frac{3}{2}}(c+dx)}\,dx$$

input
```
integrate((a+b*sec(d*x+c))/cos(d*x+c)**(3/2),x)
```

output
```
Integral((a + b*sec(c + d*x))/cos(c + d*x)**(3/2), x)
```

3.800.7 Maxima [F]

$$\int \frac{a+b\sec(c+dx)}{\cos^{\frac{3}{2}}(c+dx)}\,dx=\int \frac{b\sec(dx+c)+a}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))/cos(d*x+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)/cos(d*x + c)^(3/2), x)
```

3.800.8 Giac [F]

$$\int \frac{a+b\sec(c+dx)}{\cos^{\frac{3}{2}}(c+dx)}\,dx=\int \frac{b\sec(dx+c)+a}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))/cos(d*x+c)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)/cos(d*x + c)^(3/2), x)
```

3.800.9 Mupad [B] (verification not implemented)

Time = 14.96 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.05

$$\int \frac{a+b\sec(c+dx)}{\cos^{\frac{3}{2}}(c+dx)}\,dx=\frac{2\,a\,\sin(c+d\,x)\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{1}{2};\ \frac{3}{4};\ \cos(c+d\,x)^2\right)}{d\,\sqrt{\cos(c+d\,x)}\,\sqrt{\sin(c+d\,x)^2}}+\frac{2\,b\,\sin(c+d\,x)\,{}_2\mathrm{F}_1\left(-\frac{3}{4},\frac{1}{2};\ \frac{1}{4};\ \cos(c+d\,x)^2\right)}{3\,d\cos(c+d\,x)^{3/2}\,\sqrt{\sin(c+d\,x)^2}}$$

input
```
int((a + b/cos(c + d*x))/cos(c + d*x)^(3/2),x)
```

output
```
(2*a*sin(c + d*x)*hypergeom([-1/4, 1/2], 3/4, cos(c + d*x)^2))/(d*cos(c +
d*x)^(1/2)*(sin(c + d*x)^2)^(1/2)) + (2*b*sin(c + d*x)*hypergeom([-3/4, 1/
2], 1/4, cos(c + d*x)^2))/(3*d*cos(c + d*x)^(3/2)*(sin(c + d*x)^2)^(1/2))
```

3.801 $\int \frac{a+b\sec(c+dx)}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

3.801.1 Optimal result 5906
3.801.2 Mathematica [A] (verified) 5906
3.801.3 Rubi [A] (verified) 5907
3.801.4 Maple [B] (verified) 5909
3.801.5 Fricas [C] (verification not implemented) 5910
3.801.6 Sympy [F(-1)] 5911
3.801.7 Maxima [F] 5911
3.801.8 Giac [F] 5911
3.801.9 Mupad [B] (verification not implemented) 5912

3.801.1 Optimal result

Integrand size = 21, antiderivative size = 111

$$\int \frac{a+b\sec(c+dx)}{\cos^{\frac{5}{2}}(c+dx)}\,dx = -\frac{6bE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2b\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{2a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{6b\sin(c+dx)}{5d\sqrt{\cos(c+dx)}}$$

output
```
-6/5*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d
*x+1/2*c),2^(1/2))/d+2/3*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)
*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/5*b*sin(d*x+c)/d/cos(d*x+c)^(5/
2)+2/3*a*sin(d*x+c)/d/cos(d*x+c)^(3/2)+6/5*b*sin(d*x+c)/d/cos(d*x+c)^(1/2)
```

3.801.2 Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.86

$$\int \frac{a+b\sec(c+dx)}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{-18b\cos^{\frac{3}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)+10a\cos^{\frac{3}{2}}(c+dx)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+10a\sin(c+dx)+9b\sin}{15d\cos^{\frac{3}{2}}(c+dx)}$$

input
```
Integrate[(a + b*Sec[c + d*x])/Cos[c + d*x]^(5/2),x]
```

output
```
(-18*b*Cos[c + d*x]^(3/2)*EllipticE[(c + d*x)/2, 2] + 10*a*Cos[c + d*x]^(3
/2)*EllipticF[(c + d*x)/2, 2] + 10*a*Sin[c + d*x] + 9*b*Sin[2*(c + d*x)] +
 6*b*Tan[c + d*x])/(15*d*Cos[c + d*x]^(3/2))
```

3.801.3 Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.02, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.524$, Rules used = {3042, 4713, 3042, 3227, 3042, 3116, 3042, 3116, 3042, 3119, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\sec(c+dx)}{\cos^{\frac{5}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx$$

$$\Big\downarrow \text{4713}$$

$$\int \frac{a\cos(c+dx)+b}{\cos^{\frac{7}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a\sin\left(c+dx+\frac{\pi}{2}\right)+b}{\sin\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx$$

$$\Big\downarrow \text{3227}$$

$$a\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)}dx+b\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$a\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+b\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx$$

$$\Big\downarrow \text{3116}$$

$$a\left(\frac{1}{3}\int \frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}\right)+b\left(\frac{3}{5}\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)}dx+\frac{2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}\right)$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{1}{3}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}\right)+b\left(\frac{3}{5}\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+\frac{2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}\right)$$

$\Big\downarrow$ 3116

$$a\left(\frac{1}{3}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}\right)+b\left(\frac{3}{5}\left(\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\int \sqrt{\cos(c+dx)}dx\right)+\frac{2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}\right)$$

$\Big\downarrow$ 3042

$$a\left(\frac{1}{3}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}\right)+b\left(\frac{3}{5}\left(\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx\right)+\frac{2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}\right)$$

$\Big\downarrow$ 3119

$$a\left(\frac{1}{3}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}\right)+b\left(\frac{2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{3}{5}\left(\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)\right)$$

$\Big\downarrow$ 3120

$$a\left(\frac{2\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}\right)+b\left(\frac{2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{3}{5}\left(\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)\right)$$

input
```
Int[(a + b*Sec[c + d*x])/Cos[c + d*x]^(5/2),x]
```

output
```
a*((2*EllipticF[(c + d*x)/2, 2])/(3*d) + (2*Sin[c + d*x])/(3*d*Cos[c + d*x
]^(3/2))) + b*((2*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2)) + (3*((-2*Ellipti
cE[(c + d*x)/2, 2])/d + (2*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])))/5)
```

3.801. $\int \frac{a+b\sec(c+dx)}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

3.801.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 4713
```
Int[(csc[(a_.) + (b_.)*(x_)]*(B_.) + (A_))*(u_), x_Symbol] :> Int[ActivateT
rig[u]*((B + A*Sin[a + b*x])/Sin[a + b*x]), x] /; FreeQ[{a, b, A, B}, x] &&
 KnownSineIntegrandQ[u, x]
```

3.801.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 501 vs. $2(147) = 294$.

Time = 15.05 (sec) , antiderivative size = 502, normalized size of antiderivative = 4.52

method	result
default	$-\frac{\sqrt{-\left(-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(2a\left(-\frac{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}}{6\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-\frac{1}{2}\right)^2}+\frac{\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1}}{3\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+s}}\right.\right.}{}$

input
```
int((a+b*sec(d*x+c))/cos(d*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*a*(-1/6*cos(
1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1
/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/
2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellip
ticF(cos(1/2*d*x+1/2*c),2^(1/2)))+2/5*b/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2
*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)^2*(24*sin(1/2*d
*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(c
os(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/
2*c)^4-24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+12*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(
1/2)*sin(1/2*d*x+1/2*c)^2+8*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-3*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2
))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

3.801.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.69

$$\int \frac{a+b\sec(c+dx)}{\cos^{\frac{5}{2}}(c+dx)}\,dx$$
$$= \frac{-5i\sqrt{2}a\cos(dx+c)^3\,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+5i\sqrt{2}a\cos(dx+c)^3\,\text{wei}}{}$$

input
```
integrate((a+b*sec(d*x+c))/cos(d*x+c)^(5/2),x, algorithm="fricas")
```

output
```
1/15*(-5*I*sqrt(2)*a*cos(d*x + c)^3*weierstrassPInverse(-4, 0, cos(d*x + c
) + I*sin(d*x + c)) + 5*I*sqrt(2)*a*cos(d*x + c)^3*weierstrassPInverse(-4,
 0, cos(d*x + c) - I*sin(d*x + c)) - 9*I*sqrt(2)*b*cos(d*x + c)^3*weierstr
assZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)))
+ 9*I*sqrt(2)*b*cos(d*x + c)^3*weierstrassZeta(-4, 0, weierstrassPInverse(
-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*(9*b*cos(d*x + c)^2 + 5*a*cos(d
*x + c) + 3*b)*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^3)
```

3.801.6 Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\sec(c+dx)}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+b*sec(d*x+c))/cos(d*x+c)**(5/2),x)
```

output
```
Timed out
```

3.801.7 Maxima [F]

$$\int \frac{a+b\sec(c+dx)}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{b\sec(dx+c)+a}{\cos(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))/cos(d*x+c)^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)/cos(d*x + c)^(5/2), x)
```

3.801.8 Giac [F]

$$\int \frac{a+b\sec(c+dx)}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{b\sec(dx+c)+a}{\cos(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))/cos(d*x+c)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)/cos(d*x + c)^(5/2), x)
```

3.801.9 Mupad [B] (verification not implemented)

Time = 15.04 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.78

$$\int \frac{a+b\sec(c+dx)}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{2\,a\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{3}{4},\frac{1}{2};\ \frac{1}{4};\ \cos\left(c+d\,x\right)^2\right)}{3\,d\cos\left(c+d\,x\right)^{3/2}\,\sqrt{\sin\left(c+d\,x\right)^2}} + \frac{2\,b\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{5}{4},\frac{1}{2};\ -\frac{1}{4};\ \cos\left(c+d\,x\right)^2\right)}{5\,d\cos\left(c+d\,x\right)^{5/2}\,\sqrt{\sin\left(c+d\,x\right)^2}}$$

input
```
int((a + b/cos(c + d*x))/cos(c + d*x)^(5/2),x)
```

output
```
(2*a*sin(c + d*x)*hypergeom([-3/4, 1/2], 1/4, cos(c + d*x)^2))/(3*d*cos(c
+ d*x)^(3/2)*(sin(c + d*x)^2)^(1/2)) + (2*b*sin(c + d*x)*hypergeom([-5/4,
1/2], -1/4, cos(c + d*x)^2))/(5*d*cos(c + d*x)^(5/2)*(sin(c + d*x)^2)^(1/2
))
```

3.802 $\int \cos^{\frac{9}{2}}(c+dx)(a+b\sec(c+dx))^2\,dx$

3.802.1 Optimal result 5913
3.802.2 Mathematica [A] (verified) 5914
3.802.3 Rubi [A] (verified) 5914
3.802.4 Maple [B] (verified) 5918
3.802.5 Fricas [C] (verification not implemented) 5919
3.802.6 Sympy [F(-1)] 5919
3.802.7 Maxima [F] 5920
3.802.8 Giac [F] 5920
3.802.9 Mupad [B] (verification not implemented) 5920

3.802.1 Optimal result

Integrand size = 23, antiderivative size = 160

$$\begin{aligned}\int \cos^{\frac{9}{2}}(c+dx)(a+b\sec(c+dx))^2\,dx = {} & \frac{2(7a^2+9b^2)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d} \\ & + \frac{20ab\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d} \\ & + \frac{20ab\sqrt{\cos(c+dx)}\sin(c+dx)}{21d} \\ & + \frac{2(7a^2+9b^2)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{45d} \\ & + \frac{4ab\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{7d} \\ & + \frac{2a^2\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{9d}\end{aligned}$$

output
```
2/15*(7*a^2+9*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellipti
cE(sin(1/2*d*x+1/2*c),2^(1/2))/d+20/21*a*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/co
s(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/45*(7*a^2+9*b^2
)*cos(d*x+c)^(3/2)*sin(d*x+c)/d+4/7*a*b*cos(d*x+c)^(5/2)*sin(d*x+c)/d+2/9*
a^2*cos(d*x+c)^(7/2)*sin(d*x+c)/d+20/21*a*b*sin(d*x+c)*cos(d*x+c)^(1/2)/d
```

3.802.2 Mathematica [A] (verified)

Time = 1.34 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.71

$$\int \cos^{\frac{9}{2}}(c+dx)(a+b\sec(c+dx))^2\,dx$$
$$= \frac{84(7a^2+9b^2)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)+600ab\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+\sqrt{\cos(c+dx)}(7(43a^2+36b^2)\cos(c+d}{630d}$$

input
```
Integrate[Cos[c + d*x]^(9/2)*(a + b*Sec[c + d*x])^2,x]
```

output
```
(84*(7*a^2 + 9*b^2)*EllipticE[(c + d*x)/2, 2] + 600*a*b*EllipticF[(c + d*x
)/2, 2] + Sqrt[Cos[c + d*x]]*(7*(43*a^2 + 36*b^2)*Cos[c + d*x] + 5*a*(156*
b + 36*b*Cos[2*(c + d*x)] + 7*a*Cos[3*(c + d*x)]))*Sin[c + d*x])/(630*d)
```

3.802.3 Rubi [A] (verified)

Time = 1.28 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.37, number of steps used = 19, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.826$, Rules used = {3042, 4752, 3042, 4275, 3042, 4256, 3042, 4256, 3042, 4258, 3042, 3120, 4533, 3042, 4256, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^{\frac{9}{2}}(c+dx)(a+b\sec(c+dx))^2\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^{9/2}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2dx$$
$$\Big\downarrow \text{4752}$$
$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{(a+b\sec(c+dx))^2}{\sec^{\frac{9}{2}}(c+dx)}dx$$
$$\Big\downarrow \text{3042}$$
$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{9/2}}dx$$
$$\Big\downarrow \text{4275}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \frac{a^2+b^2\sec^2(c+dx)}{\sec^{\frac{9}{2}}(c+dx)}dx+2ab\int \frac{1}{\sec^{\frac{7}{2}}(c+dx)}dx\right)$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \frac{a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{9/2}}dx+2ab\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx\right)$$

$\Big\downarrow$ 4256

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \frac{a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{9/2}}dx+2ab\left(\frac{5}{7}\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}dx+\frac{2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}\right)\right)$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \frac{a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{9/2}}dx+2ab\left(\frac{5}{7}\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+\frac{2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx}\right.\right.$$

$\Big\downarrow$ 4256

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \frac{a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{9/2}}dx+2ab\left(\frac{5}{7}\left(\frac{1}{3}\int \sqrt{\sec(c+dx)}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)\right.\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \frac{a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{9/2}}dx+2ab\left(\frac{5}{7}\left(\frac{1}{3}\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+}{3d\sqrt{\sec(c+}}\right.\right.\right.$$

$\Big\downarrow$ 4258

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \frac{a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{9/2}}dx+2ab\left(\frac{5}{7}\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+}}\right.\right.\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \frac{a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{9/2}}dx+2ab\left(\frac{5}{7}\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{}{\sqrt{\sin(c}}\right.\right.\right.$$

$\Big\downarrow$ 3120

3.802. $\int \cos^{\frac{9}{2}}(c+dx)(a+b\sec(c+dx))^2\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \frac{a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{9/2}}dx+2ab\left(\frac{2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+\frac{5}{7}\left(\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2\sqrt{}}{}\right.\right.\right.$$

$\Big\downarrow$ 4533

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{9}(7a^2+9b^2)\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)}dx+\frac{2a^2\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)}+2ab\left(\frac{2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+\frac{5}{7}\left(\frac{2}{3d}\right.\right.\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{9}(7a^2+9b^2)\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+\frac{2a^2\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)}+2ab\left(\frac{2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+\right.\right.$$

$\Big\downarrow$ 4256

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{9}(7a^2+9b^2)\left(\frac{3}{5}\int \frac{1}{\sqrt{\sec(c+dx)}}dx+\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}\right)+\frac{2a^2\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)}+2ab\left(\right.\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{9}(7a^2+9b^2)\left(\frac{3}{5}\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}\right)+\frac{2a^2\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)}+\right.$$

$\Big\downarrow$ 4258

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{9}(7a^2+9b^2)\left(\frac{3}{5}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\cos(c+dx)}dx+\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}\right)\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{9}(7a^2+9b^2)\left(\frac{3}{5}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+}{5d\sec^{\frac{3}{2}}(c+}\right.\right.$$

$\Big\downarrow$ 3119

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{9}(7a^2+9b^2)\left(\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{6\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}\right)+\right.$$

input
```
Int[Cos[c + d*x]^(9/2)*(a + b*Sec[c + d*x])^2,x]
```

3.802. $\int \cos^{\frac{9}{2}}(c+dx)(a+b\sec(c+dx))^2\,dx$

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*a^2*Sin[c + d*x])/(9*d*Sec[c + d
*x]^(7/2)) + ((7*a^2 + 9*b^2)*((6*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2
, 2]*Sqrt[Sec[c + d*x]])/(5*d) + (2*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2))
))/9 + 2*a*b*((2*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/2)) + (5*((2*Sqrt[Cos[
c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (2*Sin[c +
 d*x])/(3*d*Sqrt[Sec[c + d*x]])))/7))
```

3.802.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4275
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Simp[2*a*(b/d)   Int[(d*Csc[e + f*x])^(n + 1), x], x]
 + Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
 e, f, n}, x]
```

rule 4533
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] +
Simp[(C*m + A*(m + 1))/(b^2*m)   Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fr
eeQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

3.802.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 397 vs. $2(192) = 384$.

Time = 30.08 (sec) , antiderivative size = 398, normalized size of antiderivative = 2.49

method	result
default	$-\frac{2\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(-1120a^2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}+(2240a^2+1440ab)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^8\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+(-2}{}$

input
```
int(cos(d*x+c)^(9/2)*(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
-2/315*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-1120*a^2*
cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10+(2240*a^2+1440*a*b)*sin(1/2*d*x+1
/2*c)^8*cos(1/2*d*x+1/2*c)+(-2072*a^2-2160*a*b-504*b^2)*sin(1/2*d*x+1/2*c)
^6*cos(1/2*d*x+1/2*c)+(952*a^2+1680*a*b+504*b^2)*sin(1/2*d*x+1/2*c)^4*cos(
1/2*d*x+1/2*c)+(-168*a^2-480*a*b-126*b^2)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x
+1/2*c)+150*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),
2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*a*b-147*(2*sin(1/2*d*x+1/2*c)^2-1)^(
1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*a^
2-189*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2
))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*b^2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+
1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

3.802.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.22

$$\int \cos^{\frac{9}{2}}(c+dx)(a+b\sec(c+dx))^2\,dx$$
$$= \frac{-150i\sqrt{2}ab\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+150i\sqrt{2}ab\text{weierstrassPInverse}(-4}{}$$

input
```
integrate(cos(d*x+c)^(9/2)*(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/315*(-150*I*sqrt(2)*a*b*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(
d*x + c)) + 150*I*sqrt(2)*a*b*weierstrassPInverse(-4, 0, cos(d*x + c) - I*
sin(d*x + c)) + 2*(35*a^2*cos(d*x + c)^3 + 90*a*b*cos(d*x + c)^2 + 150*a*b
 + 7*(7*a^2 + 9*b^2)*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 21*sq
rt(2)*(-7*I*a^2 - 9*I*b^2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4,
0, cos(d*x + c) + I*sin(d*x + c))) - 21*sqrt(2)*(7*I*a^2 + 9*I*b^2)*weiers
trassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))
))/d
```

3.802.6 Sympy [F(-1)]

Timed out.

$$\int \cos^{\frac{9}{2}}(c+dx)(a+b\sec(c+dx))^2\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(9/2)*(a+b*sec(d*x+c))**2,x)
```

output
```
Timed out
```

3.802.7 Maxima [F]

$$\int \cos^{\frac{9}{2}}(c+dx)(a+b\sec(c+dx))^2\,dx = \int (b\sec(dx+c)+a)^2\cos(dx+c)^{\frac{9}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(9/2)*(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^2*cos(d*x + c)^(9/2), x)
```

3.802.8 Giac [F]

$$\int \cos^{\frac{9}{2}}(c+dx)(a+b\sec(c+dx))^2\,dx = \int (b\sec(dx+c)+a)^2\cos(dx+c)^{\frac{9}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(9/2)*(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^2*cos(d*x + c)^(9/2), x)
```

3.802.9 Mupad [B] (verification not implemented)

Time = 14.66 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.84

$$\begin{aligned}\int \cos^{\frac{9}{2}}(c+dx)(a+b\sec(c+dx))^2\,dx &= -\frac{2\,a^2\cos(c+d\,x)^{11/2}\,\sin(c+d\,x)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{11}{4};\ \frac{15}{4};\ \cos(c+d\,x)^2\right)}{11\,d\,\sqrt{\sin(c+d\,x)^2}} \\ &\quad -\frac{2\,b^2\cos(c+d\,x)^{7/2}\,\sin(c+d\,x)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{7}{4};\ \frac{11}{4};\ \cos(c+d\,x)^2\right)}{7\,d\,\sqrt{\sin(c+d\,x)^2}} \\ &\quad -\frac{4\,a\,b\cos(c+d\,x)^{9/2}\,\sin(c+d\,x)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{9}{4};\ \frac{13}{4};\ \cos(c+d\,x)^2\right)}{9\,d\,\sqrt{\sin(c+d\,x)^2}}\end{aligned}$$

input
```
int(cos(c + d*x)^(9/2)*(a + b/cos(c + d*x))^2,x)
```

output
```
- (2*a^2*cos(c + d*x)^(11/2)*sin(c + d*x)*hypergeom([1/2, 11/4], 15/4, cos
(c + d*x)^2))/(11*d*(sin(c + d*x)^2)^(1/2)) - (2*b^2*cos(c + d*x)^(7/2)*si
n(c + d*x)*hypergeom([1/2, 7/4], 11/4, cos(c + d*x)^2))/(7*d*(sin(c + d*x)
^2)^(1/2)) - (4*a*b*cos(c + d*x)^(9/2)*sin(c + d*x)*hypergeom([1/2, 9/4],
13/4, cos(c + d*x)^2))/(9*d*(sin(c + d*x)^2)^(1/2))
```

3.802. $\int \cos^{\frac{9}{2}}(c+dx)(a+b\sec(c+dx))^2\,dx$

3.803 $\int \cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^2\,dx$

3.803.1 Optimal result . . . 5922
3.803.2 Mathematica [A] (verified) . . . 5923
3.803.3 Rubi [A] (verified) . . . 5923
3.803.4 Maple [B] (verified) . . . 5927
3.803.5 Fricas [C] (verification not implemented) . . . 5927
3.803.6 Sympy [F(-1)] . . . 5928
3.803.7 Maxima [F] . . . 5928
3.803.8 Giac [F] . . . 5928
3.803.9 Mupad [B] (verification not implemented) . . . 5929

3.803.1 Optimal result

Integrand size = 23, antiderivative size = 135

$$\begin{aligned}\int \cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^2\,dx = {} & \frac{12abE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} \\ & + \frac{2(5a^2+7b^2)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d} \\ & + \frac{2(5a^2+7b^2)\sqrt{\cos(c+dx)}\sin(c+dx)}{21d} \\ & + \frac{4ab\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d} \\ & + \frac{2a^2\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{7d}\end{aligned}$$

output
```
12/5*a*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2
*d*x+1/2*c),2^(1/2))/d+2/21*(5*a^2+7*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos
(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+4/5*a*b*cos(d*x+c)
^(3/2)*sin(d*x+c)/d+2/7*a^2*cos(d*x+c)^(5/2)*sin(d*x+c)/d+2/21*(5*a^2+7*b^
2)*sin(d*x+c)*cos(d*x+c)^(1/2)/d
```

3.803.2 Mathematica [A] (verified)

Time = 1.03 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.73

$$\int \cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^2\,dx$$
$$=\frac{252abE\left(\frac{1}{2}(c+dx)\middle|2\right)+10(5a^2+7b^2)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+\sqrt{\cos(c+dx)}(65a^2+70b^2+84ab\cos(c}{105d}$$

input
```
Integrate[Cos[c + d*x]^(7/2)*(a + b*Sec[c + d*x])^2,x]
```

output
```
(252*a*b*EllipticE[(c + d*x)/2, 2] + 10*(5*a^2 + 7*b^2)*EllipticF[(c + d*x
)/2, 2] + Sqrt[Cos[c + d*x]]*(65*a^2 + 70*b^2 + 84*a*b*Cos[c + d*x] + 15*a
^2*Cos[2*(c + d*x)])*Sin[c + d*x])/(105*d)
```

3.803.3 Rubi [A] (verified)

Time = 1.18 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.41, number of steps used = 17, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.739, Rules used = {3042, 4752, 3042, 4275, 3042, 4256, 3042, 4258, 3042, 3119, 4533, 3042, 4256, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^2\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^{7/2}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2dx$$
$$\Big\downarrow \text{4752}$$
$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{(a+b\sec(c+dx))^2}{\sec^{\frac{7}{2}}(c+dx)}dx$$
$$\Big\downarrow \text{3042}$$
$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx$$
$$\Big\downarrow \text{4275}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \frac{a^2+b^2\sec^2(c+dx)}{\sec^{\frac{7}{2}}(c+dx)}dx+2ab\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)}dx\right)$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \frac{a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx+2ab\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx\right)$$

$\Big\downarrow$ 4256

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \frac{a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx+2ab\left(\frac{3}{5}\int \frac{1}{\sqrt{\sec(c+dx)}}dx+\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}\right)\right)$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \frac{a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx+2ab\left(\frac{3}{5}\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}\right.\right.$$

$\Big\downarrow$ 4258

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \frac{a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx+2ab\left(\frac{3}{5}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\cos(c+dx}\right.\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \frac{a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx+2ab\left(\frac{3}{5}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\sin\left(c+dx\right.}\right.\right.$$

$\Big\downarrow$ 3119

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \frac{a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx+2ab\left(\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{6\sqrt{\cos(c+dx)}\sqrt{\sec(c+d}}{5d}\right.\right.$$

$\Big\downarrow$ 4533

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}(5a^2+7b^2)\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}dx+\frac{2a^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+2ab\left(\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{6\sqrt{\text{co}}}{}\right.\right.$$

$\Big\downarrow$ 3042

3.803. $\int \cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^2\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}(5a^2+7b^2)\int\frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+\frac{2a^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+2ab\left(\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\right.\right.$$

$\Big\downarrow$ 4256

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}(5a^2+7b^2)\left(\frac{1}{3}\int\sqrt{\sec(c+dx)}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)+\frac{2a^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+2ab\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}(5a^2+7b^2)\left(\frac{1}{3}\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)+\frac{2a^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}-\right.$$

$\Big\downarrow$ 4258

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}(5a^2+7b^2)\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right.\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}(5a^2+7b^2)\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+\right.\right.$$

$\Big\downarrow$ 3120

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}(5a^2+7b^2)\left(\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx)\right.}{3d}\right.\right.$$

input
```
Int[Cos[c + d*x]^(7/2)*(a + b*Sec[c + d*x])^2,x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*a^2*Sin[c + d*x])/(7*d*Sec[c + d
*x]^(5/2)) + 2*a*b*((6*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[S
ec[c + d*x]])/(5*d) + (2*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2))) + ((5*a^2
 + 7*b^2)*((2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*
x]])/(3*d) + (2*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])))/7)
```

3.803. $\int \cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^2\,dx$

3.803.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4275
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Simp[2*a*(b/d)   Int[(d*Csc[e + f*x])^(n + 1), x], x]
 + Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
 e, f, n}, x]
```

rule 4533
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] +
Simp[(C*m + A*(m + 1))/(b^2*m)   Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fr
eeQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

3.803.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 361 vs. $2(171) = 342$.

Time = 32.79 (sec) , antiderivative size = 362, normalized size of antiderivative = 2.68

method	result
default	$-\frac{2\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\left(240a^{2}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{8}+(-360a^{2}-336ab)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+(280a^{2}+}{}$

input
```
int(cos(d*x+c)^(7/2)*(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
-2/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(240*a^2*co
s(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+(-360*a^2-336*a*b)*sin(1/2*d*x+1/2*c
)^6*cos(1/2*d*x+1/2*c)+(280*a^2+336*a*b+140*b^2)*sin(1/2*d*x+1/2*c)^4*cos(
1/2*d*x+1/2*c)+(-80*a^2-84*a*b-70*b^2)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/
2*c)+25*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elli
pticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^2+35*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*s
in(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b^2-126
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(c
os(1/2*d*x+1/2*c),2^(1/2))*a*b)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

3.803.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.33

$$\int \cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^2\,dx$$
$$= \frac{126i\sqrt{2}ab\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))-126i\sqrt{2}ab\text{w}}{}$$

input
```
integrate(cos(d*x+c)^(7/2)*(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/105*(126*I*sqrt(2)*a*b*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0,
 cos(d*x + c) + I*sin(d*x + c))) - 126*I*sqrt(2)*a*b*weierstrassZeta(-4, 0
, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*(15*a^2*c
os(d*x + c)^2 + 42*a*b*cos(d*x + c) + 25*a^2 + 35*b^2)*sqrt(cos(d*x + c))*
sin(d*x + c) - 5*sqrt(2)*(5*I*a^2 + 7*I*b^2)*weierstrassPInverse(-4, 0, co
s(d*x + c) + I*sin(d*x + c)) - 5*sqrt(2)*(-5*I*a^2 - 7*I*b^2)*weierstrassP
Inverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)))/d
```

3.803.6 Sympy [F(-1)]

Timed out.

$$\int \cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^2\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(7/2)*(a+b*sec(d*x+c))**2,x)
```

output
```
Timed out
```

3.803.7 Maxima [F]

$$\int \cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^2\,dx = \int (b\sec(dx+c)+a)^2\cos(dx+c)^{\frac{7}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(7/2)*(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^2*cos(d*x + c)^(7/2), x)
```

3.803.8 Giac [F]

$$\int \cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^2\,dx = \int (b\sec(dx+c)+a)^2\cos(dx+c)^{\frac{7}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(7/2)*(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^2*cos(d*x + c)^(7/2), x)
```

3.803.9 Mupad [B] (verification not implemented)

Time = 14.20 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.95

$$\int \cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^2\,dx$$
$$= \frac{2\left(b^2\,\mathrm{F}\left(\frac{c}{2}+\frac{dx}{2}\middle|2\right)+b^2\,\sqrt{\cos\left(c+d\,x\right)}\,\sin\left(c+d\,x\right)\right)}{3\,d} - \frac{2\,a^2\cos\left(c+d\,x\right)^{9/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{9}{4};\ \frac{13}{4};\ \cos\left(c+d\,x\right)^2\right)}{9\,d\,\sqrt{\sin\left(c+d\,x\right)^2}} - \frac{4\,a\,b\cos\left(c+d\,x\right)^{7/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{7}{4};\ \frac{11}{4};\ \cos\left(c+d\,x\right)^2\right)}{7\,d\,\sqrt{\sin\left(c+d\,x\right)^2}}$$

input
```
int(cos(c + d*x)^(7/2)*(a + b/cos(c + d*x))^2,x)
```

output
```
(2*(b^2*ellipticF(c/2 + (d*x)/2, 2) + b^2*cos(c + d*x)^(1/2)*sin(c + d*x))
)/(3*d) - (2*a^2*cos(c + d*x)^(9/2)*sin(c + d*x)*hypergeom([1/2, 9/4], 13/
4, cos(c + d*x)^2))/(9*d*(sin(c + d*x)^2)^(1/2)) - (4*a*b*cos(c + d*x)^(7/
2)*sin(c + d*x)*hypergeom([1/2, 7/4], 11/4, cos(c + d*x)^2))/(7*d*(sin(c +
 d*x)^2)^(1/2))
```

3.804 $\int \cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^2\,dx$

3.804.1 Optimal result 5930
3.804.2 Mathematica [A] (verified) 5930
3.804.3 Rubi [A] (verified) 5931
3.804.4 Maple [B] (verified) 5934
3.804.5 Fricas [C] (verification not implemented) 5935
3.804.6 Sympy [F(-1)] 5935
3.804.7 Maxima [F] 5936
3.804.8 Giac [F] 5936
3.804.9 Mupad [B] (verification not implemented) 5936

3.804.1 Optimal result

Integrand size = 23, antiderivative size = 101

$$\int \cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^2\,dx = \frac{2(3a^2+5b^2)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{4ab\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d} + \frac{4ab\sqrt{\cos(c+dx)}\sin(c+dx)}{3d} + \frac{2a^2\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d}$$

output
```
2/5*(3*a^2+5*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elliptic
E(sin(1/2*d*x+1/2*c),2^(1/2))/d+4/3*a*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1
/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/5*a^2*cos(d*x+c)^(
3/2)*sin(d*x+c)/d+4/3*a*b*sin(d*x+c)*cos(d*x+c)^(1/2)/d
```

3.804.2 Mathematica [A] (verified)

Time = 0.64 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.78

$$\int \cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^2\,dx = \frac{6(3a^2+5b^2)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)+20ab\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+2a\sqrt{\cos(c+dx)}(10b+3a\cos(c+dx))\,\text{sin}}{15d}$$

input
```
Integrate[Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^2,x]
```

output
```
(6*(3*a^2 + 5*b^2)*EllipticE[(c + d*x)/2, 2] + 20*a*b*EllipticF[(c + d*x)/
2, 2] + 2*a*Sqrt[Cos[c + d*x]]*(10*b + 3*a*Cos[c + d*x])*Sin[c + d*x])/(15
*d)
```

3.804.3 Rubi [A] (verified)

Time = 0.95 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.61, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.652$, Rules used = {3042, 4752, 3042, 4275, 3042, 4256, 3042, 4258, 3042, 3120, 4533, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{(a+b\sec(c+dx))^2}{\sec^{\frac{5}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx$$

$$\Big\downarrow \text{4275}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \frac{a^2+b^2\sec^2(c+dx)}{\sec^{\frac{5}{2}}(c+dx)}dx+2ab\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}dx\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \frac{a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+2ab\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx\right)$$

$$\Big\downarrow \text{4256}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \frac{a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+2ab\left(\frac{1}{3}\int\sqrt{\sec(c+dx)}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)\right)$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \frac{a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+2ab\left(\frac{1}{3}\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx}}\right.\right.$$

$\downarrow$ 4258

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \frac{a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+2ab\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx}}\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \frac{a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+2ab\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+d\right.}}\right.\right.$$

$\downarrow$ 3120

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \frac{a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+2ab\left(\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+}}{3}\right.\right.$$

$\downarrow$ 4533

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(3a^2+5b^2\right)\int\frac{1}{\sqrt{\sec(c+dx)}}dx+\frac{2a^2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+2ab\left(\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2\sqrt{c}}{}\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(3a^2+5b^2\right)\int\frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a^2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+2ab\left(\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\right.\right.$$

$\downarrow$ 4258

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(3a^2+5b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\cos(c+dx)}dx+\frac{2a^2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+2a\right.$$

$\downarrow$ 3042

3.804. $\int \cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^2\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}(3a^2+5b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2a^2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}\right.$$

$$\Big\downarrow \text{3119}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2(3a^2+5b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a^2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+2ab\right.$$

input
```
Int[Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^2,x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*(3*a^2 + 5*b^2)*Sqrt[Cos[c + d*x
]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d) + (2*a^2*Sin[c + d*
x])/(5*d*Sec[c + d*x]^(3/2)) + 2*a*b*((2*Sqrt[Cos[c + d*x]]*EllipticF[(c +
 d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (2*Sin[c + d*x])/(3*d*Sqrt[Sec[c +
 d*x]])))
```

3.804.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4275
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Simp[2*a*(b/d)   Int[(d*Csc[e + f*x])^(n + 1), x], x]
 + Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
 e, f, n}, x]
```

rule 4533
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] +
Simp[(C*m + A*(m + 1))/(b^2*m)   Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fr
eeQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

3.804.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 356 vs. $2(141) = 282$.

Time = 28.58 (sec) , antiderivative size = 357, normalized size of antiderivative = 3.53

method	result
default	$-\frac{2\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(-24a^2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^6+24a^2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+40\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\right.\right.}{}$

input
```
int(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
-2/15*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-24*a^2*cos
(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+24*a^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x
+1/2*c)^4+40*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4*a*b-6*a^2*sin(1/2*d*x
+1/2*c)^2*cos(1/2*d*x+1/2*c)-20*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2*a*
b+10*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)
)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*a*b-9*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elli
pticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2-15*(2*s
in(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*b^2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^
(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

3.804.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.60

$$\int \cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^2\,dx$$
$$= \frac{-10i\sqrt{2}ab\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+10i\sqrt{2}ab\text{weierstrassPInverse}(-4,0}{}$$

input
```
integrate(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/15*(-10*I*sqrt(2)*a*b*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*
x + c)) + 10*I*sqrt(2)*a*b*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin
(d*x + c)) + 2*(3*a^2*cos(d*x + c) + 10*a*b)*sqrt(cos(d*x + c))*sin(d*x +
c) - 3*sqrt(2)*(-3*I*a^2 - 5*I*b^2)*weierstrassZeta(-4, 0, weierstrassPInv
erse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 3*sqrt(2)*(3*I*a^2 + 5*I*b^2
)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d
*x + c))))/d
```

3.804.6 Sympy [F(-1)]

Timed out.

$$\int \cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^2\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(5/2)*(a+b*sec(d*x+c))**2,x)
```

output
```
Timed out
```

3.804.7 Maxima [F]

$$\int \cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^2\,dx = \int (b\sec(dx+c)+a)^2\cos(dx+c)^{\frac{5}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^2*cos(d*x + c)^(5/2), x)
```

3.804.8 Giac [F]

$$\int \cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^2\,dx = \int (b\sec(dx+c)+a)^2\cos(dx+c)^{\frac{5}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^2*cos(d*x + c)^(5/2), x)
```

3.804.9 Mupad [B] (verification not implemented)

Time = 14.00 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.01

$$\int \cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^2\,dx = \frac{2\,b^2\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{4\,a\,b\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{3\,d}+\frac{4\,a\,b\,\sqrt{\cos(c+d\,x)}\,\sin(c+d\,x)}{3\,d} - \frac{2\,a^2\cos(c+d\,x)^{7/2}\,\sin(c+d\,x)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{7}{4};\ \frac{11}{4};\ \cos(c+d\,x)^2\right)}{7\,d\,\sqrt{\sin(c+d\,x)^2}}$$

input
```
int(cos(c + d*x)^(5/2)*(a + b/cos(c + d*x))^2,x)
```

output
```
(2*b^2*ellipticE(c/2 + (d*x)/2, 2))/d + (4*a*b*ellipticF(c/2 + (d*x)/2, 2)
)/(3*d) + (4*a*b*cos(c + d*x)^(1/2)*sin(c + d*x))/(3*d) - (2*a^2*cos(c + d
*x)^(7/2)*sin(c + d*x)*hypergeom([1/2, 7/4], 11/4, cos(c + d*x)^2))/(7*d*(
sin(c + d*x)^2)^(1/2))
```

3.805 $\int \cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2\,dx$

3.805.1 Optimal result 5937
3.805.2 Mathematica [A] (verified) 5937
3.805.3 Rubi [A] (verified) 5938
3.805.4 Maple [B] (verified) 5941
3.805.5 Fricas [C] (verification not implemented) 5941
3.805.6 Sympy [F(-1)] 5942
3.805.7 Maxima [F] 5942
3.805.8 Giac [F] 5942
3.805.9 Mupad [B] (verification not implemented) 5943

3.805.1 Optimal result

Integrand size = 23, antiderivative size = 72

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2\,dx = \frac{4abE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2(a^2+3b^2)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d} + \frac{2a^2\sqrt{\cos(c+dx)}\sin(c+dx)}{3d}$$

output
```
4*a*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*
x+1/2*c),2^(1/2))/d+2/3*(a^2+3*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d
*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/3*a^2*sin(d*x+c)*cos(d
*x+c)^(1/2)/d
```

3.805.2 Mathematica [A] (verified)

Time = 1.22 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.89

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2\,dx = \frac{2\left(6abE\left(\frac{1}{2}(c+dx)\middle|2\right)+(a^2+3b^2)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+a^2\sqrt{\cos(c+dx)}\sin(c+dx)\right)}{3d}$$

input
```
Integrate[Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^2,x]
```

output
```
(2*(6*a*b*EllipticE[(c + d*x)/2, 2] + (a^2 + 3*b^2)*EllipticF[(c + d*x)/2,
 2] + a^2*Sqrt[Cos[c + d*x]]*Sin[c + d*x]))/(3*d)
```

3.805.3 Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.85, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.565$, Rules used = {3042, 4752, 3042, 4275, 3042, 4258, 3042, 3119, 4533, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{(a+b\sec(c+dx))^2}{\sec^{\frac{3}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{4275}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \frac{a^2+b^2\sec^2(c+dx)}{\sec^{\frac{3}{2}}(c+dx)}dx+2ab\int \frac{1}{\sqrt{\sec(c+dx)}}dx\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \frac{a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+2ab\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)$$

$$\Big\downarrow \text{4258}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \frac{a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+2ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\cos(c+dx)}dx\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \frac{a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+2ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\right.}\right.$$

$\Big\downarrow$ 3119

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \frac{a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+\frac{4ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\right)$$

$\Big\downarrow$ 4533

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}(a^2+3b^2)\int\sqrt{\sec(c+dx)}dx+\frac{2a^2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{4ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx}}{d}\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}(a^2+3b^2)\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2a^2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{4ab\sqrt{\cos(c+dx)}\sqrt{\sec(c}}{d}\right.$$

$\Big\downarrow$ 4258

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}(a^2+3b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2a^2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{4c}{}\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}(a^2+3b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a^2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right.$$

$\Big\downarrow$ 3120

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2(a^2+3b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2a^2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right.$$

input
```
Int[Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^2,x]
```

3.805. $\int \cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2\,dx$

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((4*a*b*Sqrt[Cos[c + d*x]]*EllipticE
[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (2*(a^2 + 3*b^2)*Sqrt[Cos[c + d*x
]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (2*a^2*Sin[c + d*
x])/(3*d*Sqrt[Sec[c + d*x]]))
```

3.805.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4275
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Simp[2*a*(b/d)   Int[(d*Csc[e + f*x])^(n + 1), x], x]
 + Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
 e, f, n}, x]
```

rule 4533
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] +
Simp[(C*m + A*(m + 1))/(b^2*m)   Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fr
eeQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

3.805. $\int \cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2\,dx$

3.805.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 282 vs. $2(118) = 236$.

Time = 7.92 (sec) , antiderivative size = 283, normalized size of antiderivative = 3.93

method	result
default	$-\frac{2\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(4a^2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4-2a^2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\sin(}\right.}{3\sqrt{-2}}$

input
```
int(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
-2/3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4*a^2*cos(1/
2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4-2*a^2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1
/2*c)+(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipt
icF(cos(1/2*d*x+1/2*c),2^(1/2))*a^2+3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(
1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b^2-6*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/
2*d*x+1/2*c),2^(1/2))*a*b)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^
(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

3.805.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 147, normalized size of antiderivative = 2.04

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2\,dx$$
$$= \frac{2\,a^2\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)+6i\,\sqrt{2}ab\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+}{}$$

input
```
integrate(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/3*(2*a^2*sqrt(cos(d*x + c))*sin(d*x + c) + 6*I*sqrt(2)*a*b*weierstrassZe
ta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 6*I
*sqrt(2)*a*b*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c
) - I*sin(d*x + c))) + sqrt(2)*(-I*a^2 - 3*I*b^2)*weierstrassPInverse(-4,
0, cos(d*x + c) + I*sin(d*x + c)) + sqrt(2)*(I*a^2 + 3*I*b^2)*weierstrassP
Inverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)))/d
```

3.805.6 Sympy [F(-1)]

Timed out.

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(3/2)*(a+b*sec(d*x+c))**2,x)
```

output
```
Timed out
```

3.805.7 Maxima [F]

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2\,dx = \int (b\sec(dx+c)+a)^2\cos(dx+c)^{\frac{3}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^2*cos(d*x + c)^(3/2), x)
```

3.805.8 Giac [F]

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2\,dx = \int (b\sec(dx+c)+a)^2\cos(dx+c)^{\frac{3}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^2*cos(d*x + c)^(3/2), x)
```

3.805.9 Mupad [B] (verification not implemented)

Time = 13.71 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.06

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2\,dx = \frac{2\,a^2\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{3\,d}+\frac{2\,b^2\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{2\,a^2\,\sqrt{\cos\left(c+d\,x\right)}\,\sin\left(c+d\,x\right)}{3\,d}+\frac{4\,a\,b\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}$$

input
```
int(cos(c + d*x)^(3/2)*(a + b/cos(c + d*x))^2,x)
```

output
```
(2*a^2*ellipticF(c/2 + (d*x)/2, 2))/(3*d) + (2*b^2*ellipticF(c/2 + (d*x)/2
, 2))/d + (2*a^2*cos(c + d*x)^(1/2)*sin(c + d*x))/(3*d) + (4*a*b*ellipticE
(c/2 + (d*x)/2, 2))/d
```

3.806 $\int \sqrt{\cos(c+dx)}(a+b\sec(c+dx))^2\,dx$

3.806.1 Optimal result 5944
3.806.2 Mathematica [A] (verified) 5944
3.806.3 Rubi [A] (verified) 5945
3.806.4 Maple [A] (verified) 5948
3.806.5 Fricas [C] (verification not implemented) 5948
3.806.6 Sympy [F] 5949
3.806.7 Maxima [F] 5949
3.806.8 Giac [F] 5949
3.806.9 Mupad [B] (verification not implemented) 5950

3.806.1 Optimal result

Integrand size = 23, antiderivative size = 68

$$\int \sqrt{\cos(c+dx)}(a+b\sec(c+dx))^2\,dx = \frac{2(a^2-b^2)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{4ab\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d} + \frac{2b^2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

output
```
2*(a^2-b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(
1/2*d*x+1/2*c),2^(1/2))/d+4*a*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1
/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2*b^2*sin(d*x+c)/d/cos(d*x+c
)^(1/2)
```

3.806.2 Mathematica [A] (verified)

Time = 0.86 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.91

$$\int \sqrt{\cos(c+dx)}(a+b\sec(c+dx))^2\,dx = \frac{2\left((a^2-b^2)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)+b\left(2a\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+\frac{b\sin(c+dx)}{\sqrt{\cos(c+dx)}}\right)\right)}{d}$$

input
```
Integrate[Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x])^2,x]
```

output
```
(2*((a^2 - b^2)*EllipticE[(c + d*x)/2, 2] + b*(2*a*EllipticF[(c + d*x)/2,
2] + (b*Sin[c + d*x])/Sqrt[Cos[c + d*x]])))/d
```

3.806.3 Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.90, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.565$, Rules used = {3042, 4752, 3042, 4275, 3042, 4258, 3042, 3120, 4534, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\cos(c+dx)}(a+b\sec(c+dx))^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{(a+b\sec(c+dx))^2}{\sqrt{\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4275}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \frac{a^2+b^2\sec^2(c+dx)}{\sqrt{\sec(c+dx)}}dx+2ab\int \sqrt{\sec(c+dx)}dx\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \frac{a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+2ab\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx\right)$$

$$\Big\downarrow \text{4258}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \frac{a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+2ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx\right.$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \frac{a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+2ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\right.}}\right.$$

$$\Big\downarrow \; 3120$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \frac{a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{4ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx\right.}{d}\right.$$

$$\Big\downarrow \; 4534$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\left(a^2-b^2\right)\int \frac{1}{\sqrt{\sec(c+dx)}}dx+\frac{4ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\right.}{d}\right.$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\left(a^2-b^2\right)\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{4ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+\right.}{d}\right.$$

$$\Big\downarrow \; 4258$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\cos(c+dx)}dx+\frac{4ab\sqrt{\cos(c+dx)}\sqrt{\sec(c}}{}\right.$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{4ab\sqrt{\cos(c+dx)}\sqrt{}}{}\right.$$

$$\Big\downarrow \; 3119$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{4ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+}}{}\right.$$

input
```
Int[Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x])^2,x]
```

3.806. $\int \sqrt{\cos(c+dx)}(a+b\sec(c+dx))^2\,dx$

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*(a^2 - b^2)*Sqrt[Cos[c + d*x]]*E
llipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (4*a*b*Sqrt[Cos[c + d*x]]
*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (2*b^2*Sqrt[Sec[c + d*x
]]*Sin[c + d*x])/d)
```

3.806.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4275
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Simp[2*a*(b/d)   Int[(d*Csc[e + f*x])^(n + 1), x], x]
 + Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
 e, f, n}, x]
```

rule 4534
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Simp[(C*m + A*(m + 1))/(m + 1)   Int[(b*Csc[e + f*x])^m, x], x] /;
 FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

3.806. $\int \sqrt{\cos(c+dx)}(a+b\sec(c+dx))^2\,dx$

3.806.4 Maple [A] (verified)

Time = 8.26 (sec) , antiderivative size = 202, normalized size of antiderivative = 2.97

method	result
default	$\frac{4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}b^{2}-4\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1}\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,ab+2\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1}\,\mathrm{Ellipti}}{\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)}}$

input
```
int(cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
2*(2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2*b^2-2*(2*sin(1/2*d*x+1/2*c)^2
-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*a*b+(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/
2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2-(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elli
pticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*b^2)/sin(1/
2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

3.806.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 178, normalized size of antiderivative = 2.62

$$\int \sqrt{\cos(c+dx)}(a+b\sec(c+dx))^2\,dx$$
$$= \frac{-2i\sqrt{2}ab\cos(dx+c)\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+2i\sqrt{2}ab\cos(dx+c)\,\mathrm{wei}}{}$$

input
```
integrate(cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
(-2*I*sqrt(2)*a*b*cos(d*x + c)*weierstrassPInverse(-4, 0, cos(d*x + c) + I
*sin(d*x + c)) + 2*I*sqrt(2)*a*b*cos(d*x + c)*weierstrassPInverse(-4, 0, c
os(d*x + c) - I*sin(d*x + c)) + 2*b^2*sqrt(cos(d*x + c))*sin(d*x + c) + sq
rt(2)*(I*a^2 - I*b^2)*cos(d*x + c)*weierstrassZeta(-4, 0, weierstrassPInve
rse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + sqrt(2)*(-I*a^2 + I*b^2)*cos(
d*x + c)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) -
I*sin(d*x + c))))/(d*cos(d*x + c))
```

3.806.6 Sympy [F]

$$\int \sqrt{\cos(c+dx)}(a+b\sec(c+dx))^2\,dx = \int (a+b\sec(c+dx))^2\sqrt{\cos(c+dx)}\,dx$$

input
```
integrate(cos(d*x+c)**(1/2)*(a+b*sec(d*x+c))**2,x)
```

output
```
Integral((a + b*sec(c + d*x))**2*sqrt(cos(c + d*x)), x)
```

3.806.7 Maxima [F]

$$\int \sqrt{\cos(c+dx)}(a+b\sec(c+dx))^2\,dx = \int (b\sec(dx+c)+a)^2\sqrt{\cos(dx+c)}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^2*sqrt(cos(d*x + c)), x)
```

3.806.8 Giac [F]

$$\int \sqrt{\cos(c+dx)}(a+b\sec(c+dx))^2\,dx = \int (b\sec(dx+c)+a)^2\sqrt{\cos(dx+c)}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^2*sqrt(cos(d*x + c)), x)
```

3.806.9 Mupad [B] (verification not implemented)

Time = 14.11 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.19

$$\int \sqrt{\cos(c+dx)}(a+b\sec(c+dx))^2\,dx = \frac{2\,a^2\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{4\,a\,b\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{2\,b^2\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{1}{2};\ \frac{3}{4};\ \cos\left(c+d\,x\right)^2\right)}{d\,\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{\sin\left(c+d\,x\right)^2}}$$

input
```
int(cos(c + d*x)^(1/2)*(a + b/cos(c + d*x))^2,x)
```

output
```
(2*a^2*ellipticE(c/2 + (d*x)/2, 2))/d + (4*a*b*ellipticF(c/2 + (d*x)/2, 2)
)/d + (2*b^2*sin(c + d*x)*hypergeom([-1/4, 1/2], 3/4, cos(c + d*x)^2))/(d*
cos(c + d*x)^(1/2)*(sin(c + d*x)^2)^(1/2))
```

3.807 $\int \frac{(a+b\sec(c+dx))^2}{\sqrt{\cos(c+dx)}}\,dx$

3.807.1 Optimal result 5951
3.807.2 Mathematica [A] (verified) 5951
3.807.3 Rubi [A] (verified) 5952
3.807.4 Maple [B] (verified) 5955
3.807.5 Fricas [C] (verification not implemented) 5956
3.807.6 Sympy [F] 5957
3.807.7 Maxima [F] 5957
3.807.8 Giac [F] 5957
3.807.9 Mupad [B] (verification not implemented) 5958

3.807.1 Optimal result

Integrand size = 23, antiderivative size = 95

$$\int \frac{(a+b\sec(c+dx))^2}{\sqrt{\cos(c+dx)}}\,dx = -\frac{4abE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2(3a^2+b^2)\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2b^2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{4ab\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

output
```
-4*a*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d
*x+1/2*c),2^(1/2))/d+2/3*(3*a^2+b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*
d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/3*b^2*sin(d*x+c)/d/co
s(d*x+c)^(3/2)+4*a*b*sin(d*x+c)/d/cos(d*x+c)^(1/2)
```

3.807.2 Mathematica [A] (verified)

Time = 1.06 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.77

$$\int \frac{(a+b\sec(c+dx))^2}{\sqrt{\cos(c+dx)}}\,dx = \frac{2\left(-6abE\left(\frac{1}{2}(c+dx)\middle|2\right)+(3a^2+b^2)\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+\frac{b(b+6a\cos(c+dx))\sin(c+dx)}{\cos^{\frac{3}{2}}(c+dx)}\right)}{3d}$$

input
```
Integrate[(a + b*Sec[c + d*x])^2/Sqrt[Cos[c + d*x]],x]
```

output
```
(2*(-6*a*b*EllipticE[(c + d*x)/2, 2] + (3*a^2 + b^2)*EllipticF[(c + d*x)/2
, 2] + (b*(b + 6*a*Cos[c + d*x])*Sin[c + d*x])/Cos[c + d*x]^(3/2)))/(3*d)
```

3.807.3 Rubi [A] (verified)

Time = 0.94 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.65, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.652$, Rules used = {3042, 4752, 3042, 4275, 3042, 4255, 3042, 4258, 3042, 3119, 4534, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\sec(c+dx))^2}{\sqrt{\cos(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\sec(c+dx)}(a+b\sec(c+dx))^2dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2dx$$

$$\Big\downarrow \text{4275}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \sqrt{\sec(c+dx)}\left(a^2+b^2\sec^2(c+dx)\right)dx+2ab\int \sec^{\frac{3}{2}}(c+dx)dx\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)dx+2ab\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\right.$$

$$\Big\downarrow \text{4255}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)dx+2ab\left(\frac{2\sin(c+dx)\sqrt{\sec(c}}{d}\right.\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)dx+2ab\left(\frac{2\sin(c+dx)\sqrt{\sec(c}}{d}\right.\right.$$

$$\Big\downarrow \text{4258}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)dx+2ab\left(\frac{2\sin(c+dx)\sqrt{\sec(c}}{d}\right.\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)dx+2ab\left(\frac{2\sin(c+dx)\sqrt{\sec(c}}{d}\right.\right.$$

$$\Big\downarrow \text{3119}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)dx+2ab\left(\frac{2\sin(c+dx)\sqrt{\sec(c}}{d}\right.\right.$$

$$\Big\downarrow \text{4534}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}(3a^2+b^2)\int\sqrt{\sec(c+dx)}dx+2ab\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}}{}\right.\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}(3a^2+b^2)\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+2ab\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c}}{}\right.\right.$$

$$\Big\downarrow \text{4258}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}(3a^2+b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx+2ab\left(\frac{2\sin(c+dx)\sqrt{s}}{d}\right.\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}(3a^2+b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+2ab\left(\frac{2\sin(c+d}{}\right.\right.$$

3.807. $\int \frac{(a+b\sec(c+dx))^2}{\sqrt{\cos(c+dx)}}\,dx$

$$\Big\downarrow \text{3120}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\left(3a^2+b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+2ab\left(\frac{2\sin(c+dx}{}\right.\right.$$

input
```
Int[(a + b*Sec[c + d*x])^2/Sqrt[Cos[c + d*x]],x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*(3*a^2 + b^2)*Sqrt[Cos[c + d*x]]
*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (2*b^2*Sec[c + d*x]
^(3/2)*Sin[c + d*x])/(3*d) + 2*a*b*((-2*Sqrt[Cos[c + d*x]]*EllipticE[(c +
d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d))
```

3.807.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4275
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Simp[2*a*(b/d)   Int[(d*Csc[e + f*x])^(n + 1), x], x]
 + Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
 e, f, n}, x]
```

rule 4534
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Simp[(C*m + A*(m + 1))/(m + 1)   Int[(b*Csc[e + f*x])^m, x], x] /;
 FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

3.807.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 512 vs. $2(139) = 278$.

Time = 12.44 (sec) , antiderivative size = 513, normalized size of antiderivative = 5.40

method	result
default	$-\frac{2\sqrt{-\left(-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(24\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4 ab-6\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1}\,\mathrm{EllipticF}\left(\cos\right.\right.}{}$

input
```
int((a+b*sec(d*x+c))^2/cos(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(4*sin(1/2*
d*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)/sin(1/2*d*x+1/2*c)^3*(24*cos(1/2*d*
x+1/2*c)*sin(1/2*d*x+1/2*c)^4*a*b-6*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/
2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+
1/2*c)^2*a^2-2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/
2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2*b^2-12*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2
*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2*a*b-12*cos(1/2*d*x+1/2*c)*sin(1/
2*d*x+1/2*c)^2*a*b-2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2*b^2+3*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d
*x+1/2*c),2^(1/2))*a^2+(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^
2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b^2+6*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^
(1/2))*a*b)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/
2*d*x+1/2*c)^2-1)^(1/2)/d
```

3.807.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 198, normalized size of antiderivative = 2.08

$$\int \frac{(a+b\sec(c+dx))^2}{\sqrt{\cos(c+dx)}}\,dx$$
$$= \frac{-6i\sqrt{2}ab\cos(dx+c)^2\,\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))+}{}$$

input
```
integrate((a+b*sec(d*x+c))^2/cos(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
1/3*(-6*I*sqrt(2)*a*b*cos(d*x + c)^2*weierstrassZeta(-4, 0, weierstrassPIn
verse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 6*I*sqrt(2)*a*b*cos(d*x + c
)^2*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin
(d*x + c))) + sqrt(2)*(-3*I*a^2 - I*b^2)*cos(d*x + c)^2*weierstrassPInvers
e(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + sqrt(2)*(3*I*a^2 + I*b^2)*cos(d*
x + c)^2*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 2*(6*
a*b*cos(d*x + c) + b^2)*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^2
)
```

3.807.6 Sympy [F]

$$\int \frac{(a+b\sec(c+dx))^2}{\sqrt{\cos(c+dx)}}\,dx=\int \frac{(a+b\sec{(c+dx)})^2}{\sqrt{\cos{(c+dx)}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))**2/cos(d*x+c)**(1/2),x)
```

output
```
Integral((a + b*sec(c + d*x))**2/sqrt(cos(c + d*x)), x)
```

3.807.7 Maxima [F]

$$\int \frac{(a+b\sec(c+dx))^2}{\sqrt{\cos(c+dx)}}\,dx=\int \frac{(b\sec{(dx+c)}+a)^2}{\sqrt{\cos{(dx+c)}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^2/cos(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^2/sqrt(cos(d*x + c)), x)
```

3.807.8 Giac [F]

$$\int \frac{(a+b\sec(c+dx))^2}{\sqrt{\cos(c+dx)}}\,dx=\int \frac{(b\sec{(dx+c)}+a)^2}{\sqrt{\cos{(dx+c)}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^2/cos(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^2/sqrt(cos(d*x + c)), x)
```

3.807.9 Mupad [B] (verification not implemented)

Time = 14.59 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.14

$$\int \frac{(a+b\sec(c+dx))^2}{\sqrt{\cos(c+dx)}}\,dx = \frac{2\,a^2\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d} + \frac{2\,b^2\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{3}{4},\frac{1}{2};\ \frac{1}{4};\ \cos\left(c+d\,x\right)^2\right)}{3\,d\cos\left(c+d\,x\right)^{3/2}\,\sqrt{\sin\left(c+d\,x\right)^2}} + \frac{4\,a\,b\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{1}{2};\ \frac{3}{4};\ \cos\left(c+d\,x\right)^2\right)}{d\,\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{\sin\left(c+d\,x\right)^2}}$$

input
```
int((a + b/cos(c + d*x))^2/cos(c + d*x)^(1/2),x)
```

output
```
(2*a^2*ellipticF(c/2 + (d*x)/2, 2))/d + (2*b^2*sin(c + d*x)*hypergeom([-3/
4, 1/2], 1/4, cos(c + d*x)^2))/(3*d*cos(c + d*x)^(3/2)*(sin(c + d*x)^2)^(1
/2)) + (4*a*b*sin(c + d*x)*hypergeom([-1/4, 1/2], 3/4, cos(c + d*x)^2))/(d
*cos(c + d*x)^(1/2)*(sin(c + d*x)^2)^(1/2))
```

3.808 $\int \frac{(a+b\sec(c+dx))^2}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

3.808.1 Optimal result 5959
3.808.2 Mathematica [A] (verified) 5959
3.808.3 Rubi [A] (verified) 5960
3.808.4 Maple [B] (verified) 5963
3.808.5 Fricas [C] (verification not implemented) 5964
3.808.6 Sympy [F] 5965
3.808.7 Maxima [F] 5965
3.808.8 Giac [F] 5965
3.808.9 Mupad [B] (verification not implemented) 5966

3.808.1 Optimal result

Integrand size = 23, antiderivative size = 135

$$\int \frac{(a+b\sec(c+dx))^2}{\cos^{\frac{3}{2}}(c+dx)}\,dx = -\frac{2(5a^2+3b^2)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{4ab\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2b^2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{4ab\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{2(5a^2+3b^2)\sin(c+dx)}{5d\sqrt{\cos(c+dx)}}$$

output
```
-2/5*(5*a^2+3*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellipti
cE(sin(1/2*d*x+1/2*c),2^(1/2))/d+4/3*a*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(
1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/5*b^2*sin(d*x+c)/
d/cos(d*x+c)^(5/2)+4/3*a*b*sin(d*x+c)/d/cos(d*x+c)^(3/2)+2/5*(5*a^2+3*b^2)
*sin(d*x+c)/d/cos(d*x+c)^(1/2)
```

3.808.2 Mathematica [A] (verified)

Time = 0.82 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.92

$$\int \frac{(a+b\sec(c+dx))^2}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{-6(5a^2+3b^2)\cos^{\frac{3}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)+20ab\cos^{\frac{3}{2}}(c+dx)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+20ab\sin(c+d}{15d\cos^{\frac{3}{2}}(c+dx)}$$

input
```
Integrate[(a + b*Sec[c + d*x])^2/Cos[c + d*x]^(3/2),x]
```

output
```
(-6*(5*a^2 + 3*b^2)*Cos[c + d*x]^(3/2)*EllipticE[(c + d*x)/2, 2] + 20*a*b*
Cos[c + d*x]^(3/2)*EllipticF[(c + d*x)/2, 2] + 20*a*b*Sin[c + d*x] + 15*a^
2*Sin[2*(c + d*x)] + 9*b^2*Sin[2*(c + d*x)] + 6*b^2*Tan[c + d*x])/(15*d*Co
s[c + d*x]^(3/2))
```

3.808.3 Rubi [A] (verified)

Time = 1.09 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.39, number of steps used = 17, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.739$, Rules used = {3042, 4752, 3042, 4275, 3042, 4255, 3042, 4258, 3042, 3120, 4534, 3042, 4255, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\sec(c+dx))^2}{\cos^{\frac{3}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2dx$$

$$\Big\downarrow \text{4275}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \sec^{\frac{3}{2}}(c+dx)\left(a^2+b^2\sec^2(c+dx)\right)dx+2ab\int \sec^{\frac{5}{2}}(c+dx)dx\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)dx+2ab\int \csc\left(c+dx+\frac{\pi}{2}\right)^{5/}\right.$$

$$\Big\downarrow \text{4255}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)dx+2ab\left(\frac{1}{3}\int\sqrt{\sec(c+dx)}d\right.\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)dx+2ab\left(\frac{1}{3}\int\sqrt{\csc\left(c+dx+\right.}\right.\right.$$

$\Big\downarrow$ 4258

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)dx+2ab\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec}\right.\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)dx+2ab\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec}\right.\right.$$

$\Big\downarrow$ 3120

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)dx+2ab\left(\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c}{3d}\right.\right.$$

$\Big\downarrow$ 4534

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}(5a^2+3b^2)\int\sec^{\frac{3}{2}}(c+dx)dx+2ab\left(\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{2\sqrt{\cos(c+dx)}}{}\right.\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}(5a^2+3b^2)\int\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}dx+2ab\left(\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{2\sqrt{\cos}}{}\right.\right.$$

$\Big\downarrow$ 4255

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}(5a^2+3b^2)\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\int\frac{1}{\sqrt{\sec(c+dx)}}dx\right)+2ab\left(\frac{2\sin(c+}{}\right.\right.$$

$\Big\downarrow$ 3042

3.808. $\int \frac{(a+b\sec(c+dx))^2}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}(5a^2+3b^2)\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\int\frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)+2ab\left(\frac{2\,\mathrm{si}}{}\right.\right.$$

$$\Big\downarrow \text{4258}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}(5a^2+3b^2)\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\cos(c}\right.\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}(5a^2+3b^2)\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c\right.}\right.\right.$$

$$\Big\downarrow \text{3119}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}(5a^2+3b^2)\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+\right.}{d}\right.\right.$$

input
```
Int[(a + b*Sec[c + d*x])^2/Cos[c + d*x]^(3/2),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*b^2*Sec[c + d*x]^(5/2)*Sin[c + d
*x])/(5*d) + ((5*a^2 + 3*b^2)*((-2*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/
2, 2]*Sqrt[Sec[c + d*x]])/d + (2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d))/5 +
2*a*b*((2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])
/(3*d) + (2*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d)))
```

3.808.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4275
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Simp[2*a*(b/d)   Int[(d*Csc[e + f*x])^(n + 1), x], x]
 + Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
 e, f, n}, x]
```

rule 4534
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Simp[(C*m + A*(m + 1))/(m + 1)   Int[(b*Csc[e + f*x])^m, x], x] /;
 FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

3.808.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 632 vs. $2(171) = 342$.

Time = 18.06 (sec) , antiderivative size = 633, normalized size of antiderivative = 4.69

method	result
default	$-\frac{\sqrt{-\left(-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(\frac{2b^2\left(24\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^6\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-12\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\mathrm{EllipticE}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sqrt{2\sin\left(\frac{dx}{2}+\right.}\right.}{}$

input
```
int((a+b*sec(d*x+c))^2/cos(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2/5*b^2/(8*sin
(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/
2*d*x+1/2*c)^2*(24*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*
c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2
*c)+12*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*
(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2+8*cos(1/2*d*x+1/2*c)
*sin(1/2*d*x+1/2*c)^2-3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x
+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2))*(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a^2/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/
2*c)^2-1)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*cos(1/2*
d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos
(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2))+4*a*b*(-1/6*cos
(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(
1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1
/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Elli
pticF(cos(1/2*d*x+1/2*c),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*
c)^2-1)^(1/2)/d
```

3.808.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.65

$$\int \frac{(a+b\sec(c+dx))^2}{\cos^{\frac{3}{2}}(c+dx)}\,dx$$
$$= \frac{-10i\sqrt{2}ab\cos(dx+c)^3\,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+10i\sqrt{2}ab\cos(dx+c)}{}$$

input
```
integrate((a+b*sec(d*x+c))^2/cos(d*x+c)^(3/2),x, algorithm="fricas")
```

output
```
1/15*(-10*I*sqrt(2)*a*b*cos(d*x + c)^3*weierstrassPInverse(-4, 0, cos(d*x
+ c) + I*sin(d*x + c)) + 10*I*sqrt(2)*a*b*cos(d*x + c)^3*weierstrassPInver
se(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 3*sqrt(2)*(5*I*a^2 + 3*I*b^2)*c
os(d*x + c)^3*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x +
c) + I*sin(d*x + c))) - 3*sqrt(2)*(-5*I*a^2 - 3*I*b^2)*cos(d*x + c)^3*weie
rstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c
))) + 2*(10*a*b*cos(d*x + c) + 3*(5*a^2 + 3*b^2)*cos(d*x + c)^2 + 3*b^2)*s
qrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^3)
```

3.808.6 Sympy [F]

$$\int \frac{(a+b\sec(c+dx))^2}{\cos^{\frac{3}{2}}(c+dx)}\,dx=\int \frac{(a+b\sec{(c+dx)})^2}{\cos^{\frac{3}{2}}{(c+dx)}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))**2/cos(d*x+c)**(3/2),x)
```

output
```
Integral((a + b*sec(c + d*x))**2/cos(c + d*x)**(3/2), x)
```

3.808.7 Maxima [F]

$$\int \frac{(a+b\sec(c+dx))^2}{\cos^{\frac{3}{2}}(c+dx)}\,dx=\int \frac{(b\sec{(dx+c)}+a)^2}{\cos{(dx+c)}^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^2/cos(d*x+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^2/cos(d*x + c)^(3/2), x)
```

3.808.8 Giac [F]

$$\int \frac{(a+b\sec(c+dx))^2}{\cos^{\frac{3}{2}}(c+dx)}\,dx=\int \frac{(b\sec{(dx+c)}+a)^2}{\cos{(dx+c)}^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^2/cos(d*x+c)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^2/cos(d*x + c)^(3/2), x)
```

3.808.9 Mupad [B] (verification not implemented)

Time = 14.55 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.84

$$\int \frac{(a+b\sec(c+dx))^2}{\cos^{\frac{3}{2}}(c+dx)}\,dx$$
$$= \frac{6\,b^2\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{5}{4},\frac{1}{2};\ -\frac{1}{4};\ \cos\left(c+d\,x\right)^2\right)+30\,a^2\cos\left(c+d\,x\right)^2\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{1}{2};\ \frac{3}{4};\ \cos\left(c\right.\right.}{15\,d\cos\left(c+d\,x\right)^{5/2}\,\sqrt{1-\cos\left(c+d\right.}}$$

input
```
int((a + b/cos(c + d*x))^2/cos(c + d*x)^(3/2),x)
```

output
```
(6*b^2*sin(c + d*x)*hypergeom([-5/4, 1/2], -1/4, cos(c + d*x)^2) + 30*a^2*
cos(c + d*x)^2*sin(c + d*x)*hypergeom([-1/4, 1/2], 3/4, cos(c + d*x)^2) +
20*a*b*cos(c + d*x)*sin(c + d*x)*hypergeom([-3/4, 1/2], 1/4, cos(c + d*x)^
2))/(15*d*cos(c + d*x)^(5/2)*(1 - cos(c + d*x)^2)^(1/2))
```

3.809 $\int \frac{(a+b\sec(c+dx))^2}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

3.809.1 Optimal result 5967
3.809.2 Mathematica [A] (verified) 5968
3.809.3 Rubi [A] (verified) 5968
3.809.4 Maple [B] (verified) 5972
3.809.5 Fricas [C] (verification not implemented) 5973
3.809.6 Sympy [F(-1)] 5974
3.809.7 Maxima [F] 5974
3.809.8 Giac [F] 5975
3.809.9 Mupad [B] (verification not implemented) 5975

3.809.1 Optimal result

Integrand size = 23, antiderivative size = 160

$$\int \frac{(a+b\sec(c+dx))^2}{\cos^{\frac{5}{2}}(c+dx)}\,dx = -\frac{12abE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2(7a^2+5b^2)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d}+\frac{2b^2\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)}+\frac{4ab\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{2(7a^2+5b^2)\sin(c+dx)}{21d\cos^{\frac{3}{2}}(c+dx)}+\frac{12ab\sin(c+dx)}{5d\sqrt{\cos(c+dx)}}$$

output
```
-12/5*a*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/
2*d*x+1/2*c),2^(1/2))/d+2/21*(7*a^2+5*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/co
s(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/7*b^2*sin(d*x+c
)/d/cos(d*x+c)^(7/2)+4/5*a*b*sin(d*x+c)/d/cos(d*x+c)^(5/2)+2/21*(7*a^2+5*b
^2)*sin(d*x+c)/d/cos(d*x+c)^(3/2)+12/5*a*b*sin(d*x+c)/d/cos(d*x+c)^(1/2)
```

3.809.2 Mathematica [A] (verified)

Time = 1.11 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.89

$$\int \frac{(a+b\sec(c+dx))^2}{\cos^{\frac{5}{2}}(c+dx)}\,dx$$
$$= \frac{-252ab\cos^{\frac{5}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)+10(7a^2+5b^2)\cos^{\frac{5}{2}}(c+dx)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+84ab\sin(c+}{105d\cos^{\frac{5}{2}}(}$$

input
```
Integrate[(a + b*Sec[c + d*x])^2/Cos[c + d*x]^(5/2),x]
```

output
```
(-252*a*b*Cos[c + d*x]^(5/2)*EllipticE[(c + d*x)/2, 2] + 10*(7*a^2 + 5*b^2
)*Cos[c + d*x]^(5/2)*EllipticF[(c + d*x)/2, 2] + 84*a*b*Sin[c + d*x] + 252
*a*b*Cos[c + d*x]^2*Sin[c + d*x] + 35*a^2*Sin[2*(c + d*x)] + 25*b^2*Sin[2*
(c + d*x)] + 30*b^2*Tan[c + d*x])/(105*d*Cos[c + d*x]^(5/2))
```

3.809.3 Rubi [A] (verified)

Time = 1.28 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.34, number of steps used = 19, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.826$, Rules used = {3042, 4752, 3042, 4275, 3042, 4255, 3042, 4255, 3042, 4258, 3042, 3119, 4534, 3042, 4255, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\sec(c+dx))^2}{\cos^{\frac{5}{2}}(c+dx)}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx$$
$$\Big\downarrow \; 4752$$
$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^2dx$$
$$\Big\downarrow \; 3042$$
$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2dx$$

$$\Big\downarrow \text{4275}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \sec^{\frac{5}{2}}(c+dx)\left(a^2+b^2\sec^2(c+dx)\right)dx+2ab\int \sec^{\frac{7}{2}}(c+dx)dx\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)dx+2ab\int \csc\left(c+dx+\frac{\pi}{2}\right)^{7/}\right.$$

$$\Big\downarrow \text{4255}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)dx+2ab\left(\frac{3}{5}\int \sec^{\frac{3}{2}}(c+dx)dx\right.\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)dx+2ab\left(\frac{3}{5}\int \csc\left(c+dx+\frac{\pi}{2}\right.\right.\right.$$

$$\Big\downarrow \text{4255}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)dx+2ab\left(\frac{3}{5}\left(\frac{2\sin(c+dx)\sqrt{\text{se}}}{d}\right.\right.\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)dx+2ab\left(\frac{3}{5}\left(\frac{2\sin(c+dx)\sqrt{\text{s}}}{d}\right.\right.\right.$$

$$\Big\downarrow \text{4258}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)dx+2ab\left(\frac{3}{5}\left(\frac{2\sin(c+dx)\sqrt{\text{se}}}{d}\right.\right.\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)dx+2ab\left(\frac{3}{5}\left(\frac{2\sin(c+dx)\sqrt{\text{se}}}{d}\right.\right.\right.$$

$$\Big\downarrow \text{3119}$$

3.809. $\int \frac{(a+b\sec(c+dx))^2}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)dx+2ab\left(\frac{2\sin(c+dx)\sec^{\frac{5}{2}}(c\right.\right.$$

$\downarrow$ 4534

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}(7a^2+5b^2)\int \sec^{\frac{5}{2}}(c+dx)dx+2ab\left(\frac{2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}+\frac{3}{5}\left(\frac{2\sin(c+dx}{}\right.\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}(7a^2+5b^2)\int \csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}dx+2ab\left(\frac{2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}+\frac{3}{5}\left(\frac{2\,\mathrm{si}}{}\right.\right.\right.$$

$\downarrow$ 4255

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}(7a^2+5b^2)\left(\frac{1}{3}\int \sqrt{\sec(c+dx)}dx+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}\right)+2ab\left(\frac{2\sin(c+}{}\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}(7a^2+5b^2)\left(\frac{1}{3}\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}\right)+2ab\left(\frac{2\,\mathrm{s}}{}\right.\right.$$

$\downarrow$ 4258

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}(7a^2+5b^2)\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2\sin(c+dx)\sec}{3d}\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}(7a^2+5b^2)\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+d}{}\right.\right.$$

$\downarrow$ 3120

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}(7a^2+5b^2)\left(\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\mathrm{EllipticF}\,(}{3d}\right.\right.$$

input
```
Int[(a + b*Sec[c + d*x])^2/Cos[c + d*x]^(5/2),x]
```

3.809. $\int \frac{(a+b\sec(c+dx))^2}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*b^2*Sec[c + d*x]^(7/2)*Sin[c + d
*x])/(7*d) + ((7*a^2 + 5*b^2)*((2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2
, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (2*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d)
))/7 + 2*a*b*((2*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5*d) + (3*((-2*Sqrt[Cos
[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (2*Sqrt[Sec[c
 + d*x]]*Sin[c + d*x])/d))/5))
```

3.809.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4275
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Simp[2*a*(b/d)   Int[(d*Csc[e + f*x])^(n + 1), x], x]
 + Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
 e, f, n}, x]
```

3.809. $\int \frac{(a+b\sec(c+dx))^2}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

rule 4534
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Simp[(C*m + A*(m + 1))/(m + 1)   Int[(b*Csc[e + f*x])^m, x], x] /;
 FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

3.809.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 688 vs. $2(192) = 384$.

Time = 22.42 (sec) , antiderivative size = 689, normalized size of antiderivative = 4.31

method	result
default	$-\frac{\sqrt{-\left(-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(2a^2\left(-\frac{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}}{6\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-\frac{1}{2}\right)^2}+\frac{\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2+}}{3\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+}}\right.\right.}{}$

input
```
int((a+b*sec(d*x+c))^2/cos(d*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*a^2*(-1/6*co
s(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos
(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+
1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ell
ipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+2*b^2*(-1/56*cos(1/2*d*x+1/2*c)*(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^
4-5/42*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(
1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+5/21*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*co
s(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+4/5*a*b/sin(1/2*d*x+1/2*c)^2
/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)
*(24*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*
sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+12*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x
+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2+8*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1
/2*c)^2-3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2
))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+
1/2*c)^2)^(1/2))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

3.809.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.47

$$\int \frac{(a+b\sec(c+dx))^2}{\cos^{\frac{5}{2}}(c+dx)}\,dx$$
$$= \frac{-126i\sqrt{2}ab\cos(dx+c)^4\,\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))}{}$$

input
```
integrate((a+b*sec(d*x+c))^2/cos(d*x+c)^(5/2),x, algorithm="fricas")
```

output
```
1/105*(-126*I*sqrt(2)*a*b*cos(d*x + c)^4*weierstrassZeta(-4, 0, weierstras
sPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 126*I*sqrt(2)*a*b*cos(d
*x + c)^4*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) -
 I*sin(d*x + c))) - 5*sqrt(2)*(7*I*a^2 + 5*I*b^2)*cos(d*x + c)^4*weierstra
ssPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) - 5*sqrt(2)*(-7*I*a^2 - 5
*I*b^2)*cos(d*x + c)^4*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x
 + c)) + 2*(126*a*b*cos(d*x + c)^3 + 42*a*b*cos(d*x + c) + 5*(7*a^2 + 5*b^
2)*cos(d*x + c)^2 + 15*b^2)*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x +
c)^4)
```

3.809.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(c+dx))^2}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+b*sec(d*x+c))**2/cos(d*x+c)**(5/2),x)
```

output
```
Timed out
```

3.809.7 Maxima [F]

$$\int \frac{(a+b\sec(c+dx))^2}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{(b\sec(dx+c)+a)^2}{\cos(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^2/cos(d*x+c)^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^2/cos(d*x + c)^(5/2), x)
```

3.809.8 Giac [F]

$$\int \frac{(a+b\sec(c+dx))^2}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{(b\sec(dx+c)+a)^2}{\cos(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^2/cos(d*x+c)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^2/cos(d*x + c)^(5/2), x)
```

3.809.9 Mupad [B] (verification not implemented)

Time = 14.71 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.71

$$\int \frac{(a+b\sec(c+dx))^2}{\cos^{\frac{5}{2}}(c+dx)}\,dx$$
$$= \frac{30\,b^2\,\sin(c+d\,x)\,{}_2\mathrm{F}_1\left(-\frac{7}{4},\frac{1}{2};\ -\frac{3}{4};\ \cos(c+d\,x)^2\right)+70\,a^2\cos(c+d\,x)^2\,\sin(c+d\,x)\,{}_2\mathrm{F}_1\left(-\frac{3}{4},\frac{1}{2};\ \frac{1}{4};\ \cos(c\right.}{105\,d\cos(c+d\,x)^{7/2}\,\sqrt{1-\cos(c+}}$$

input
```
int((a + b/cos(c + d*x))^2/cos(c + d*x)^(5/2),x)
```

output
```
(30*b^2*sin(c + d*x)*hypergeom([-7/4, 1/2], -3/4, cos(c + d*x)^2) + 70*a^2
*cos(c + d*x)^2*sin(c + d*x)*hypergeom([-3/4, 1/2], 1/4, cos(c + d*x)^2) +
 84*a*b*cos(c + d*x)*sin(c + d*x)*hypergeom([-5/4, 1/2], -1/4, cos(c + d*x
)^2))/(105*d*cos(c + d*x)^(7/2)*(1 - cos(c + d*x)^2)^(1/2))
```

3.810 $\int \cos^{\frac{9}{2}}(c+dx)(a+b\sec(c+dx))^3\,dx$

3.810.1 Optimal result . . . 5976
3.810.2 Mathematica [A] (verified) . . . 5977
3.810.3 Rubi [A] (verified) . . . 5977
3.810.4 Maple [B] (verified) . . . 5981
3.810.5 Fricas [C] (verification not implemented) . . . 5982
3.810.6 Sympy [F(-1)] . . . 5983
3.810.7 Maxima [F] . . . 5983
3.810.8 Giac [F] . . . 5983
3.810.9 Mupad [B] (verification not implemented) . . . 5984

3.810.1 Optimal result

Integrand size = 23, antiderivative size = 194

$$\begin{aligned}\int \cos^{\frac{9}{2}}(c+dx)(a+b\sec(c+dx))^3\,dx = {} & \frac{2a(7a^2+27b^2)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d} \\ & + \frac{2b(15a^2+7b^2)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d} \\ & + \frac{2b(15a^2+7b^2)\sqrt{\cos(c+dx)}\sin(c+dx)}{21d} \\ & + \frac{2a(7a^2+27b^2)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{45d} \\ & + \frac{40a^2b\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{63d} \\ & + \frac{2a^2\cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))\sin(c+dx)}{9d}\end{aligned}$$

output
```
2/15*a*(7*a^2+27*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elli
pticE(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/21*b*(15*a^2+7*b^2)*(cos(1/2*d*x+1/2
*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/
45*a*(7*a^2+27*b^2)*cos(d*x+c)^(3/2)*sin(d*x+c)/d+40/63*a^2*b*cos(d*x+c)^(
5/2)*sin(d*x+c)/d+2/9*a^2*cos(d*x+c)^(7/2)*(a+b*sec(d*x+c))*sin(d*x+c)/d+2
/21*b*(15*a^2+7*b^2)*sin(d*x+c)*cos(d*x+c)^(1/2)/d
```

3.810.2 Mathematica [A] (verified)

Time = 1.68 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.71

$$\int \cos^{\frac{9}{2}}(c+dx)(a+b\sec(c+dx))^3\,dx$$
$$=\frac{84(7a^3+27ab^2)\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)+60(15a^2b+7b^3)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+\sqrt{\cos(c+dx)}(7a(43a^2+1}{630d}$$

input
```
Integrate[Cos[c + d*x]^(9/2)*(a + b*Sec[c + d*x])^3,x]
```

output
```
(84*(7*a^3 + 27*a*b^2)*EllipticE[(c + d*x)/2, 2] + 60*(15*a^2*b + 7*b^3)*E
llipticF[(c + d*x)/2, 2] + Sqrt[Cos[c + d*x]]*(7*a*(43*a^2 + 108*b^2)*Cos[
c + d*x] + 5*(234*a^2*b + 84*b^3 + 54*a^2*b*Cos[2*(c + d*x)] + 7*a^3*Cos[3
*(c + d*x)]))*Sin[c + d*x])/(630*d)
```

3.810.3 Rubi [A] (verified)

Time = 1.49 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.25, number of steps used = 20, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.870, Rules used = {3042, 4752, 3042, 4328, 27, 3042, 4535, 3042, 4256, 3042, 4258, 3042, 3119, 4533, 3042, 4256, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^{\frac{9}{2}}(c+dx)(a+b\sec(c+dx))^3\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^{9/2}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^3dx$$
$$\Big\downarrow \text{4752}$$
$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{(a+b\sec(c+dx))^3}{\sec^{\frac{9}{2}}(c+dx)}dx$$
$$\Big\downarrow \text{3042}$$
$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^3}{\csc\left(c+dx+\frac{\pi}{2}\right)^{9/2}}dx$$

$$\Big\downarrow \text{4328}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2}{9}\int\frac{20ba^2+\left(7a^2+27b^2\right)\sec(c+dx)a+b\left(5a^2+9b^2\right)\sec^2(c+dx)}{2\sec^{\frac{7}{2}}(c+dx)}dx+\frac{2a^2\sin(c+}{9c}\right.$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{9}\int\frac{20ba^2+\left(7a^2+27b^2\right)\sec(c+dx)a+b\left(5a^2+9b^2\right)\sec^2(c+dx)}{\sec^{\frac{7}{2}}(c+dx)}dx+\frac{2a^2\sin(c+}{9c}\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{9}\int\frac{20ba^2+\left(7a^2+27b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)a+b\left(5a^2+9b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx+\right.$$

$$\Big\downarrow \text{4535}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{9}\left(a\left(7a^2+27b^2\right)\int\frac{1}{\sec^{\frac{5}{2}}(c+dx)}dx+\int\frac{20ba^2+b\left(5a^2+9b^2\right)\sec^2(c+dx)}{\sec^{\frac{7}{2}}(c+dx)}dx\right)+\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{9}\left(a\left(7a^2+27b^2\right)\int\frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+\int\frac{20ba^2+b\left(5a^2+9b^2\right)\csc\left(c+dx+\right.}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}\right.\right.$$

$$\Big\downarrow \text{4256}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{9}\left(\int\frac{20ba^2+b\left(5a^2+9b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx+a\left(7a^2+27b^2\right)\left(\frac{3}{5}\int\frac{1}{\sqrt{\sec(c+d}}\right.\right.\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{9}\left(\int\frac{20ba^2+b\left(5a^2+9b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx+a\left(7a^2+27b^2\right)\left(\frac{3}{5}\int\frac{1}{\sqrt{\csc\left(c+\right.}}\right.\right.\right.$$

$$\Big\downarrow \text{4258}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{9}\left(\int\frac{20ba^2+b\left(5a^2+9b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx+a\left(7a^2+27b^2\right)\left(\frac{3}{5}\sqrt{\cos(c+dx)}\sqrt{}\right.\right.\right.$$

$$\Big\downarrow \text{3042}$$

3.810. $\int\cos^{\frac{9}{2}}(c+dx)(a+b\sec(c+dx))^3\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{9}\left(\int\frac{20ba^2+b(5a^2+9b^2)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx+a(7a^2+27b^2)\left(\frac{3}{5}\sqrt{\cos(c+dx)}\sqrt{}\right.\right.\right.$$

$\downarrow$ 3119

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{9}\left(\int\frac{20ba^2+b(5a^2+9b^2)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx+a(7a^2+27b^2)\left(\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}\right.\right.\right.$$

$\downarrow$ 4533

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{9}\left(\frac{9}{7}b(15a^2+7b^2)\int\frac{1}{\sec^{\frac{3}{2}}(c+dx)}dx+a(7a^2+27b^2)\left(\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{6\sqrt{\cos(}}{}\right.\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{9}\left(\frac{9}{7}b(15a^2+7b^2)\int\frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+a(7a^2+27b^2)\left(\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{6}{}\right.\right.\right.$$

$\downarrow$ 4256

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{9}\left(\frac{9}{7}b(15a^2+7b^2)\left(\frac{1}{3}\int\sqrt{\sec(c+dx)}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)+a(7a^2+27b^2)\left(\frac{}{5}\right.\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{9}\left(\frac{9}{7}b(15a^2+7b^2)\left(\frac{1}{3}\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right)+a(7a^2+27b\right.\right.\right.$$

$\downarrow$ 4258

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{9}\left(\frac{9}{7}b(15a^2+7b^2)\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2\sin(c+}{3d\sqrt{\sec(c}}\right.\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{9}\left(\frac{9}{7}b(15a^2+7b^2)\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin}{3d\sqrt{}}\right.\right.\right.$$

$\downarrow$ 3120

3.810. $\int\cos^{\frac{9}{2}}(c+dx)(a+b\sec(c+dx))^3\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{9}\left(\frac{9}{7}b(15a^2+7b^2)\left(\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c\right.}{3d}\right.\right.\right.$$

input
```
Int[Cos[c + d*x]^(9/2)*(a + b*Sec[c + d*x])^3,x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*a^2*(a + b*Sec[c + d*x])*Sin[c +
 d*x])/(9*d*Sec[c + d*x]^(7/2)) + ((40*a^2*b*Sin[c + d*x])/(7*d*Sec[c + d*
x]^(5/2)) + a*(7*a^2 + 27*b^2)*((6*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/
2, 2]*Sqrt[Sec[c + d*x]])/(5*d) + (2*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)
)) + (9*b*(15*a^2 + 7*b^2)*((2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2
]*Sqrt[Sec[c + d*x]])/(3*d) + (2*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])))/
7)/9)
```

3.810.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4328
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)*
((d*Csc[e + f*x])^n/(f*n)), x] - Simp[1/(d*n)   Int[(a + b*Csc[e + f*x])^(m
 - 3)*(d*Csc[e + f*x])^(n + 1)*Simp[a^2*b*(m - 2*n - 2) - a*(3*b^2*n + a^2*
(n + 1))*Csc[e + f*x] - b*(b^2*n + a^2*(m + n - 1))*Csc[e + f*x]^2, x], x],
 x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && ((Int
egerQ[m] && LtQ[n, -1]) || (IntegersQ[m + 1/2, 2*n] && LeQ[n, -1]))
```

rule 4533
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] +
Simp[(C*m + A*(m + 1))/(b^2*m)   Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fr
eeQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

rule 4535
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Simp[B/b   Int[(b*Cs
c[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2)
, x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

3.810.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 469 vs. $2(226) = 452$.

Time = 165.94 (sec) , antiderivative size = 470, normalized size of antiderivative = 2.42

method	result
default	$-\frac{2\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(-1120a^3\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}+(2240a^3+2160a^2b)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^8\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+(-}{}$

input
```
int(cos(d*x+c)^(9/2)*(a+b*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
-2/315*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-1120*a^3*
cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10+(2240*a^3+2160*a^2*b)*sin(1/2*d*x
+1/2*c)^8*cos(1/2*d*x+1/2*c)+(-2072*a^3-3240*a^2*b-1512*a*b^2)*sin(1/2*d*x
+1/2*c)^6*cos(1/2*d*x+1/2*c)+(952*a^3+2520*a^2*b+1512*a*b^2+420*b^3)*sin(1
/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-168*a^3-720*a^2*b-378*a*b^2-210*b^3)*
sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+225*(sin(1/2*d*x+1/2*c)^2)^(1/2)*E
llipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*a^2*
b+105*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(
2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*b^3-147*(sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellip
ticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*a^3-567*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(
1/2*d*x+1/2*c)^2-1)^(1/2)*a*b^2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*
c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

3.810.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.17

$$\int \cos^{\frac{9}{2}}(c+dx)(a+b\sec(c+dx))^3\,dx$$
$$= \frac{2\left(35\,a^3\cos\left(dx+c\right)^3+135\,a^2b\cos\left(dx+c\right)^2+225\,a^2b+105\,b^3+7\left(7\,a^3+27\,ab^2\right)\cos\left(dx+c\right)\right)\sqrt{\cos\left(dx\right.}}{}$$

input
```
integrate(cos(d*x+c)^(9/2)*(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/315*(2*(35*a^3*cos(d*x + c)^3 + 135*a^2*b*cos(d*x + c)^2 + 225*a^2*b + 1
05*b^3 + 7*(7*a^3 + 27*a*b^2)*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c
) - 15*sqrt(2)*(15*I*a^2*b + 7*I*b^3)*weierstrassPInverse(-4, 0, cos(d*x +
 c) + I*sin(d*x + c)) - 15*sqrt(2)*(-15*I*a^2*b - 7*I*b^3)*weierstrassPInv
erse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 21*sqrt(2)*(-7*I*a^3 - 27*I*a
*b^2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*s
in(d*x + c))) - 21*sqrt(2)*(7*I*a^3 + 27*I*a*b^2)*weierstrassZeta(-4, 0, w
eierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/d
```

3.810.6 Sympy [F(-1)]

Timed out.

$$\int \cos^{\frac{9}{2}}(c+dx)(a+b\sec(c+dx))^3\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(9/2)*(a+b*sec(d*x+c))**3,x)
```

output
```
Timed out
```

3.810.7 Maxima [F]

$$\int \cos^{\frac{9}{2}}(c+dx)(a+b\sec(c+dx))^3\,dx = \int (b\sec(dx+c)+a)^3\cos(dx+c)^{\frac{9}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(9/2)*(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^3*cos(d*x + c)^(9/2), x)
```

3.810.8 Giac [F]

$$\int \cos^{\frac{9}{2}}(c+dx)(a+b\sec(c+dx))^3\,dx = \int (b\sec(dx+c)+a)^3\cos(dx+c)^{\frac{9}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(9/2)*(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^3*cos(d*x + c)^(9/2), x)
```

3.810.9 Mupad [B] (verification not implemented)

Time = 14.50 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.92

$$\int \cos^{\frac{9}{2}}(c+dx)(a+b\sec(c+dx))^3\,dx$$
$$= \frac{2\,b^3\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{3\,d}+\frac{2\,b^3\,\sqrt{\cos\left(c+d\,x\right)}\,\sin\left(c+d\,x\right)}{3\,d}$$
$$-\frac{2\,a^3\cos\left(c+d\,x\right)^{11/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{11}{4};\ \frac{15}{4};\ \cos\left(c+d\,x\right)^2\right)}{11\,d\,\sqrt{\sin\left(c+d\,x\right)^2}}$$
$$-\frac{6\,a\,b^2\cos\left(c+d\,x\right)^{7/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{7}{4};\ \frac{11}{4};\ \cos\left(c+d\,x\right)^2\right)}{7\,d\,\sqrt{\sin\left(c+d\,x\right)^2}}$$
$$-\frac{2\,a^2\,b\cos\left(c+d\,x\right)^{9/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{9}{4};\ \frac{13}{4};\ \cos\left(c+d\,x\right)^2\right)}{3\,d\,\sqrt{\sin\left(c+d\,x\right)^2}}$$

input
```
int(cos(c + d*x)^(9/2)*(a + b/cos(c + d*x))^3,x)
```

output
```
(2*b^3*ellipticF(c/2 + (d*x)/2, 2))/(3*d) + (2*b^3*cos(c + d*x)^(1/2)*sin(
c + d*x))/(3*d) - (2*a^3*cos(c + d*x)^(11/2)*sin(c + d*x)*hypergeom([1/2,
11/4], 15/4, cos(c + d*x)^2))/(11*d*(sin(c + d*x)^2)^(1/2)) - (6*a*b^2*cos
(c + d*x)^(7/2)*sin(c + d*x)*hypergeom([1/2, 7/4], 11/4, cos(c + d*x)^2))/
(7*d*(sin(c + d*x)^2)^(1/2)) - (2*a^2*b*cos(c + d*x)^(9/2)*sin(c + d*x)*hy
pergeom([1/2, 9/4], 13/4, cos(c + d*x)^2))/(3*d*(sin(c + d*x)^2)^(1/2))
```

3.811 $\int \cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^3\,dx$

3.811.1 Optimal result 5985
3.811.2 Mathematica [A] (verified) 5986
3.811.3 Rubi [A] (verified) 5986
3.811.4 Maple [B] (verified) 5990
3.811.5 Fricas [C] (verification not implemented) 5991
3.811.6 Sympy [F(-1)] 5992
3.811.7 Maxima [F] 5992
3.811.8 Giac [F] 5992
3.811.9 Mupad [B] (verification not implemented) 5993

3.811.1 Optimal result

Integrand size = 23, antiderivative size = 159

$$\begin{aligned}\int \cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^3\,dx = {} & \frac{2b(9a^2+5b^2)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} \\ & + \frac{2a(5a^2+21b^2)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d} \\ & + \frac{2a(5a^2+21b^2)\sqrt{\cos(c+dx)}\sin(c+dx)}{21d} \\ & + \frac{32a^2b\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{35d} \\ & + \frac{2a^2\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))\sin(c+dx)}{7d}\end{aligned}$$

output
```
2/5*b*(9*a^2+5*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellipt
icE(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/21*a*(5*a^2+21*b^2)*(cos(1/2*d*x+1/2*c
)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+32/3
5*a^2*b*cos(d*x+c)^(3/2)*sin(d*x+c)/d+2/7*a^2*cos(d*x+c)^(5/2)*(a+b*sec(d*
x+c))*sin(d*x+c)/d+2/21*a*(5*a^2+21*b^2)*sin(d*x+c)*cos(d*x+c)^(1/2)/d
```

3.811.2 Mathematica [A] (verified)

Time = 1.27 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.69

$$\int \cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^3\,dx$$
$$=\frac{42(9a^2b+5b^3)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)+10(5a^3+21ab^2)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+a\sqrt{\cos(c+dx)}(65a^2+210b}{105d}$$

input
```
Integrate[Cos[c + d*x]^(7/2)*(a + b*Sec[c + d*x])^3,x]
```

output
```
(42*(9*a^2*b + 5*b^3)*EllipticE[(c + d*x)/2, 2] + 10*(5*a^3 + 21*a*b^2)*El
lipticF[(c + d*x)/2, 2] + a*Sqrt[Cos[c + d*x]]*(65*a^2 + 210*b^2 + 126*a*b
*Cos[c + d*x] + 15*a^2*Cos[2*(c + d*x)])*Sin[c + d*x])/(105*d)
```

3.811.3 Rubi [A] (verified)

Time = 1.33 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.35, number of steps used = 18, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.783, Rules used = {3042, 4752, 3042, 4328, 27, 3042, 4535, 3042, 4256, 3042, 4258, 3042, 3120, 4533, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^3\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^{7/2}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^3dx$$
$$\Big\downarrow \text{4752}$$
$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{(a+b\sec(c+dx))^3}{\sec^{\frac{7}{2}}(c+dx)}dx$$
$$\Big\downarrow \text{3042}$$
$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^3}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx$$
$$\Big\downarrow \text{4328}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2}{7}\int\frac{16ba^2+(5a^2+21b^2)\sec(c+dx)a+b(3a^2+7b^2)\sec^2(c+dx)}{2\sec^{\frac{5}{2}}(c+dx)}dx+\frac{2a^2\sin(c+}{7}\right.$$

$\Big\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}\int\frac{16ba^2+(5a^2+21b^2)\sec(c+dx)a+b(3a^2+7b^2)\sec^2(c+dx)}{\sec^{\frac{5}{2}}(c+dx)}dx+\frac{2a^2\sin(c+}{7}\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}\int\frac{16ba^2+(5a^2+21b^2)\csc\left(c+dx+\frac{\pi}{2}\right)a+b(3a^2+7b^2)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+\right.$$

$\Big\downarrow$ 4535

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}\left(a(5a^2+21b^2)\int\frac{1}{\sec^{\frac{3}{2}}(c+dx)}dx+\int\frac{16ba^2+b(3a^2+7b^2)\sec^2(c+dx)}{\sec^{\frac{5}{2}}(c+dx)}dx\right)+\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}\left(a(5a^2+21b^2)\int\frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+\int\frac{16ba^2+b(3a^2+7b^2)\csc(c+dx+}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}\right.\right.$$

$\Big\downarrow$ 4256

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}\left(\int\frac{16ba^2+b(3a^2+7b^2)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+a(5a^2+21b^2)\left(\frac{1}{3}\int\sqrt{\sec(c+d}\right.\right.\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}\left(\int\frac{16ba^2+b(3a^2+7b^2)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+a(5a^2+21b^2)\left(\frac{1}{3}\int\sqrt{\csc\left(c+d\right.}\right.\right.\right.$$

$\Big\downarrow$ 4258

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}\left(\int\frac{16ba^2+b(3a^2+7b^2)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+a(5a^2+21b^2)\left(\frac{1}{3}\sqrt{\cos(c+dx)}\sqrt{}\right.\right.\right.$$

$\Big\downarrow$ 3042

3.811. $\int\cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^3\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}\left(\int\frac{16ba^2+b(3a^2+7b^2)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+a(5a^2+21b^2)\left(\frac{1}{3}\sqrt{\cos(c+dx)}\right.\right.\right.$$

$\downarrow$ 3120

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}\left(\int\frac{16ba^2+b(3a^2+7b^2)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx+a(5a^2+21b^2)\left(\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\right.\right.\right.$$

$\downarrow$ 4533

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}\left(\frac{7}{5}b(9a^2+5b^2)\int\frac{1}{\sqrt{\sec(c+dx)}}dx+a(5a^2+21b^2)\left(\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2\sqrt{\cos(}}{}\right.\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}\left(\frac{7}{5}b(9a^2+5b^2)\int\frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+a(5a^2+21b^2)\left(\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2}{}\right.\right.\right.$$

$\downarrow$ 4258

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}\left(\frac{7}{5}b(9a^2+5b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\cos(c+dx)}dx+a(5a^2+21b^2)\right.\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}\left(\frac{7}{5}b(9a^2+5b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+a(5a^2+2\right.\right.\right.$$

$\downarrow$ 3119

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}\left(\frac{14b(9a^2+5b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+a(5a^2+21b^2)\left(\frac{}{3}\right.\right.\right.$$

input
```
Int[Cos[c + d*x]^(7/2)*(a + b*Sec[c + d*x])^3,x]
```

3.811. $\int \cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^3\,dx$

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*a^2*(a + b*Sec[c + d*x])*Sin[c +
 d*x])/(7*d*Sec[c + d*x]^(5/2)) + ((14*b*(9*a^2 + 5*b^2)*Sqrt[Cos[c + d*x]
]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d) + (32*a^2*b*Sin[c +
d*x])/(5*d*Sec[c + d*x]^(3/2)) + a*(5*a^2 + 21*b^2)*((2*Sqrt[Cos[c + d*x]]
*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (2*Sin[c + d*x])/(3
*d*Sqrt[Sec[c + d*x]])))/7)
```

3.811.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4328
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)*
((d*Csc[e + f*x])^n/(f*n)), x] - Simp[1/(d*n)   Int[(a + b*Csc[e + f*x])^(m
 - 3)*(d*Csc[e + f*x])^(n + 1)*Simp[a^2*b*(m - 2*n - 2) - a*(3*b^2*n + a^2*
(n + 1))*Csc[e + f*x] - b*(b^2*n + a^2*(m + n - 1))*Csc[e + f*x]^2, x], x],
 x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && ((Int
egerQ[m] && LtQ[n, -1]) || (IntegersQ[m + 1/2, 2*n] && LeQ[n, -1]))
```

rule 4533
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] +
Simp[(C*m + A*(m + 1))/(b^2*m)   Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fr
eeQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

rule 4535
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Simp[B/b   Int[(b*Cs
c[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2)
, x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

3.811.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 420 vs. $2(195) = 390$.

Time = 186.95 (sec) , antiderivative size = 421, normalized size of antiderivative = 2.65

method	result
default	$-\frac{2\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(240a^3\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^8+(-360a^3-504a^2b)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^6\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+(280a^3}{}$

input
```
int(cos(d*x+c)^(7/2)*(a+b*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
-2/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(240*a^3*co
s(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+(-360*a^3-504*a^2*b)*sin(1/2*d*x+1/2
*c)^6*cos(1/2*d*x+1/2*c)+(280*a^3+504*a^2*b+420*a*b^2)*sin(1/2*d*x+1/2*c)^
4*cos(1/2*d*x+1/2*c)+(-80*a^3-126*a^2*b-210*a*b^2)*sin(1/2*d*x+1/2*c)^2*co
s(1/2*d*x+1/2*c)+25*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)
^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+105*a*b^2*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*
c),2^(1/2))-189*a^2*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2
-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-105*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/
2))*b^3)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+
1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

3.811.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.29

$$\int \cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^3\,dx$$
$$= \frac{2\left(15\,a^3\cos\left(dx+c\right)^2+63\,a^2b\cos\left(dx+c\right)+25\,a^3+105\,ab^2\right)\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)-5\,\sqrt{2}(5i\,a^3+2}{}$$

input
```
integrate(cos(d*x+c)^(7/2)*(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/105*(2*(15*a^3*cos(d*x + c)^2 + 63*a^2*b*cos(d*x + c) + 25*a^3 + 105*a*b
^2)*sqrt(cos(d*x + c))*sin(d*x + c) - 5*sqrt(2)*(5*I*a^3 + 21*I*a*b^2)*wei
erstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) - 5*sqrt(2)*(-5*I*a
^3 - 21*I*a*b^2)*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))
 - 21*sqrt(2)*(-9*I*a^2*b - 5*I*b^3)*weierstrassZeta(-4, 0, weierstrassPIn
verse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 21*sqrt(2)*(9*I*a^2*b + 5*I
*b^3)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*s
in(d*x + c))))/d
```

3.811.6 Sympy [F(-1)]

Timed out.

$$\int \cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^3\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(7/2)*(a+b*sec(d*x+c))**3,x)
```

output
```
Timed out
```

3.811.7 Maxima [F]

$$\int \cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^3\,dx = \int (b\sec(dx+c)+a)^3\cos(dx+c)^{\frac{7}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(7/2)*(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^3*cos(d*x + c)^(7/2), x)
```

3.811.8 Giac [F]

$$\int \cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^3\,dx = \int (b\sec(dx+c)+a)^3\cos(dx+c)^{\frac{7}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(7/2)*(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^3*cos(d*x + c)^(7/2), x)
```

3.811.9 Mupad [B] (verification not implemented)

Time = 14.17 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.92

$$\int \cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^3\,dx$$
$$= \frac{2\left(b^3\,\mathrm{E}\left(\frac{c}{2}+\frac{dx}{2}\middle|2\right)+a\,b^2\,\mathrm{F}\left(\frac{c}{2}+\frac{dx}{2}\middle|2\right)+a\,b^2\,\sqrt{\cos\left(c+d\,x\right)}\,\sin\left(c+d\,x\right)\right)}{d}$$
$$-\frac{2\,a^3\cos\left(c+d\,x\right)^{9/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{9}{4};\ \frac{13}{4};\ \cos\left(c+d\,x\right)^2\right)}{9\,d\,\sqrt{\sin\left(c+d\,x\right)^2}}$$
$$-\frac{6\,a^2\,b\cos\left(c+d\,x\right)^{7/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{7}{4};\ \frac{11}{4};\ \cos\left(c+d\,x\right)^2\right)}{7\,d\,\sqrt{\sin\left(c+d\,x\right)^2}}$$

input
```
int(cos(c + d*x)^(7/2)*(a + b/cos(c + d*x))^3,x)
```

output
```
(2*(b^3*ellipticE(c/2 + (d*x)/2, 2) + a*b^2*ellipticF(c/2 + (d*x)/2, 2) +
a*b^2*cos(c + d*x)^(1/2)*sin(c + d*x)))/d - (2*a^3*cos(c + d*x)^(9/2)*sin(
c + d*x)*hypergeom([1/2, 9/4], 13/4, cos(c + d*x)^2))/(9*d*(sin(c + d*x)^2
)^(1/2)) - (6*a^2*b*cos(c + d*x)^(7/2)*sin(c + d*x)*hypergeom([1/2, 7/4],
11/4, cos(c + d*x)^2))/(7*d*(sin(c + d*x)^2)^(1/2))
```

3.812 $\int \cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^3\,dx$

3.812.1 Optimal result 5994
3.812.2 Mathematica [A] (verified) 5995
3.812.3 Rubi [A] (verified) 5995
3.812.4 Maple [B] (verified) 5999
3.812.5 Fricas [C] (verification not implemented) 6000
3.812.6 Sympy [F(-1)] 6000
3.812.7 Maxima [F] 6001
3.812.8 Giac [F] 6001
3.812.9 Mupad [B] (verification not implemented) 6001

3.812.1 Optimal result

Integrand size = 23, antiderivative size = 116

$$\begin{aligned}\int \cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^3\,dx = &\frac{6a(a^2+5b^2)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}\\ &+\frac{2b(a^2+b^2)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}\\ &+\frac{8a^2b\sqrt{\cos(c+dx)}\sin(c+dx)}{5d}\\ &+\frac{2a^2\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))\sin(c+dx)}{5d}\end{aligned}$$

output
```
6/5*a*(a^2+5*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elliptic
E(sin(1/2*d*x+1/2*c),2^(1/2))/d+2*b*(a^2+b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)
/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/5*a^2*cos(d*
x+c)^(3/2)*(a+b*sec(d*x+c))*sin(d*x+c)/d+8/5*a^2*b*sin(d*x+c)*cos(d*x+c)^(
1/2)/d
```

3.812.2 Mathematica [A] (verified)

Time = 1.23 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.72

$$\int \cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^3\,dx$$
$$=\frac{2\left(3(a^3+5ab^2)\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)+5b(a^2+b^2)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+a^2\sqrt{\cos(c+dx)}(5b+a\cos(c+d\right.}{5d}$$

input
```
Integrate[Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^3,x]
```

output
```
(2*(3*(a^3 + 5*a*b^2)*EllipticE[(c + d*x)/2, 2] + 5*b*(a^2 + b^2)*Elliptic
F[(c + d*x)/2, 2] + a^2*Sqrt[Cos[c + d*x]]*(5*b + a*Cos[c + d*x])*Sin[c +
d*x]))/(5*d)
```

3.812.3 Rubi [A] (verified)

Time = 1.10 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.53, number of steps used = 16, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.696, Rules used = {3042, 4752, 3042, 4328, 27, 3042, 4535, 3042, 4258, 3042, 3119, 4533, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^3\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^3dx$$
$$\Big\downarrow \text{4752}$$
$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{(a+b\sec(c+dx))^3}{\sec^{\frac{5}{2}}(c+dx)}dx$$
$$\Big\downarrow \text{3042}$$
$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^3}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx$$
$$\Big\downarrow \text{4328}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2}{5}\int\frac{12ba^2+3(a^2+5b^2)\sec(c+dx)a+b(a^2+5b^2)\sec^2(c+dx)}{2\sec^{\frac{3}{2}}(c+dx)}dx+\frac{2a^2\sin(c+d}{5d\,\mathrm{s}}\right.$$

$\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\int\frac{12ba^2+3(a^2+5b^2)\sec(c+dx)a+b(a^2+5b^2)\sec^2(c+dx)}{\sec^{\frac{3}{2}}(c+dx)}dx+\frac{2a^2\sin(c+d}{5d\,\mathrm{s}}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\int\frac{12ba^2+3(a^2+5b^2)\csc\left(c+dx+\frac{\pi}{2}\right)a+b(a^2+5b^2)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+\frac{2a}{}\right.$$

$\downarrow$ 4535

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(\int\frac{12ba^2+b(a^2+5b^2)\sec^2(c+dx)}{\sec^{\frac{3}{2}}(c+dx)}dx+3a(a^2+5b^2)\int\frac{1}{\sqrt{\sec(c+dx)}}dx\right)+\frac{2a}{}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(3a(a^2+5b^2)\int\frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\int\frac{12ba^2+b(a^2+5b^2)\csc\left(c+dx+\frac{\pi}{2}\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}\right.\right.$$

$\downarrow$ 4258

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(\int\frac{12ba^2+b(a^2+5b^2)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+3a(a^2+5b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c}\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(\int\frac{12ba^2+b(a^2+5b^2)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+3a(a^2+5b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c}\right.\right.$$

$\downarrow$ 3119

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(\int\frac{12ba^2+b(a^2+5b^2)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx+\frac{6a(a^2+5b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c}}{d}\right.\right.$$

$\downarrow$ 4533

3.812. $\int\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^3\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(5b(a^2+b^2)\int\sqrt{\sec(c+dx)}dx+\frac{6a(a^2+5b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c\right.}{d}\right.\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(5b(a^2+b^2)\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{6a(a^2+5b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{d}\right.\right.$$

$$\Big\downarrow \text{4258}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(5b(a^2+b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{6a(a^2+5b^2)\sqrt{\cos}}{}\right.\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(5b(a^2+b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{6a(a^2+5b^2)}{}\right.\right.$$

$$\Big\downarrow \text{3120}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(\frac{10b(a^2+b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+\frac{6a(a^2+5b^2)}{}\right.\right.$$

input
```
Int[Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^3,x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*a^2*(a + b*Sec[c + d*x])*Sin[c +
 d*x])/(5*d*Sec[c + d*x]^(3/2)) + ((6*a*(a^2 + 5*b^2)*Sqrt[Cos[c + d*x]]*E
llipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (10*b*(a^2 + b^2)*Sqrt[Co
s[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (8*a^2*b*Sin
[c + d*x])/(d*Sqrt[Sec[c + d*x]]))/5)
```

3.812. $\int \cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^3\,dx$

3.812.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4328
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)*
((d*Csc[e + f*x])^n/(f*n)), x] - Simp[1/(d*n)   Int[(a + b*Csc[e + f*x])^(m
 - 3)*(d*Csc[e + f*x])^(n + 1)*Simp[a^2*b*(m - 2*n - 2) - a*(3*b^2*n + a^2*
(n + 1))*Csc[e + f*x] - b*(b^2*n + a^2*(m + n - 1))*Csc[e + f*x]^2, x], x],
 x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && ((Int
egerQ[m] && LtQ[n, -1]) || (IntegersQ[m + 1/2, 2*n] && LeQ[n, -1]))
```

rule 4533
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] +
Simp[(C*m + A*(m + 1))/(b^2*m)   Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fr
eeQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

rule 4535
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Simp[B/b   Int[(b*Cs
c[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2)
, x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

3.812. $\int \cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^3\,dx$

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

3.812.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 411 vs. $2(158) = 316$.

Time = 169.41 (sec) , antiderivative size = 412, normalized size of antiderivative = 3.55

method	result
default	$-\frac{2\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(-8a^3\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^6+8a^3\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+20a^2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4\cos(\right.}{}$

input
```
int(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
-2/5*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-8*a^3*cos(1
/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+8*a^3*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/
2*c)^4+20*a^2*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)*b-2*cos(1/2*d*x+1/2*
c)*sin(1/2*d*x+1/2*c)^2*a^3-10*a^2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)
*b+5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2
*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*a^2*b+5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipt
icF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*b^3-3*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2
*d*x+1/2*c)^2-1)^(1/2)*a^3-15*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1
/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*a*b^2)/(-2*sin(1/2
*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*
x+1/2*c)^2-1)^(1/2)/d
```

3.812.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.59

$$\int \cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^3\,dx$$
$$= \frac{2\,(a^3\cos(dx+c)+5\,a^2b)\sqrt{\cos(dx+c)}\sin(dx+c)-5\,\sqrt{2}(i\,a^2b+i\,b^3)\text{weierstrassPInverse}(-4,0,\cos(d}{}$$

input
```
integrate(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/5*(2*(a^3*cos(d*x + c) + 5*a^2*b)*sqrt(cos(d*x + c))*sin(d*x + c) - 5*sq
rt(2)*(I*a^2*b + I*b^3)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*
x + c)) - 5*sqrt(2)*(-I*a^2*b - I*b^3)*weierstrassPInverse(-4, 0, cos(d*x
+ c) - I*sin(d*x + c)) - 3*sqrt(2)*(-I*a^3 - 5*I*a*b^2)*weierstrassZeta(-4
, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 3*sqrt(2
)*(I*a^3 + 5*I*a*b^2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, co
s(d*x + c) - I*sin(d*x + c))))/d
```

3.812.6 Sympy [F(-1)]

Timed out.

$$\int \cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^3\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(5/2)*(a+b*sec(d*x+c))**3,x)
```

output
```
Timed out
```

3.812.7 Maxima [F]

$$\int \cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^3\,dx = \int (b\sec(dx+c)+a)^3\cos(dx+c)^{\frac{5}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^3*cos(d*x + c)^(5/2), x)
```

3.812.8 Giac [F]

$$\int \cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^3\,dx = \int (b\sec(dx+c)+a)^3\cos(dx+c)^{\frac{5}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^3*cos(d*x + c)^(5/2), x)
```

3.812.9 Mupad [B] (verification not implemented)

Time = 14.04 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.08

$$\begin{aligned}\int \cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^3\,dx &= \frac{2\,b^3\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{6\,a\,b^2\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}\\ &\quad+\frac{2\,a^2\,b\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{2\,a^2\,b\,\sqrt{\cos(c+d\,x)}\,\sin(c+d\,x)}{d}\\ &\quad-\frac{2\,a^3\cos(c+d\,x)^{7/2}\,\sin(c+d\,x)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{7}{4};\ \frac{11}{4};\ \cos(c+d\,x)^2\right)}{7\,d\,\sqrt{\sin(c+d\,x)^2}}\end{aligned}$$

input
```
int(cos(c + d*x)^(5/2)*(a + b/cos(c + d*x))^3,x)
```

output
```
(2*b^3*ellipticF(c/2 + (d*x)/2, 2))/d + (6*a*b^2*ellipticE(c/2 + (d*x)/2,
2))/d + (2*a^2*b*ellipticF(c/2 + (d*x)/2, 2))/d + (2*a^2*b*cos(c + d*x)^(1
/2)*sin(c + d*x))/d - (2*a^3*cos(c + d*x)^(7/2)*sin(c + d*x)*hypergeom([1/
2, 7/4], 11/4, cos(c + d*x)^2))/(7*d*(sin(c + d*x)^2)^(1/2))
```

3.813 $\int \cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^3\,dx$

3.813.1 Optimal result 6002
3.813.2 Mathematica [A] (verified) 6003
3.813.3 Rubi [A] (verified) 6003
3.813.4 Maple [A] (verified) 6007
3.813.5 Fricas [C] (verification not implemented) 6007
3.813.6 Sympy [F(-1)] 6008
3.813.7 Maxima [F] 6008
3.813.8 Giac [F] 6009
3.813.9 Mupad [B] (verification not implemented) 6009

3.813.1 Optimal result

Integrand size = 23, antiderivative size = 126

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^3\,dx = \frac{2b(3a^2-b^2)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2a(a^2+9b^2)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d} - \frac{2b(a^2-3b^2)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}} + \frac{2a^2\sqrt{\cos(c+dx)}(a+b\sec(c+dx))\sin(c+dx)}{3d}$$

output
```
2*b*(3*a^2-b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(
sin(1/2*d*x+1/2*c),2^(1/2))/d+2/3*a*(a^2+9*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/
2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d-2/3*b*(a^2-3
*b^2)*sin(d*x+c)/d/cos(d*x+c)^(1/2)+2/3*a^2*(a+b*sec(d*x+c))*sin(d*x+c)*co
s(d*x+c)^(1/2)/d
```

3.813.2 Mathematica [A] (verified)

Time = 1.23 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.69

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^3\,dx = \frac{2\left((9a^2b-3b^3)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)+(a^3+9ab^2)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+\frac{(3b^3+a^3\cos(c+dx))\sin(c+dx)}{\sqrt{\cos(c+dx)}}\right)}{3d}$$

input
```
Integrate[Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^3,x]
```

output
```
(2*((9*a^2*b - 3*b^3)*EllipticE[(c + d*x)/2, 2] + (a^3 + 9*a*b^2)*Elliptic
F[(c + d*x)/2, 2] + ((3*b^3 + a^3*Cos[c + d*x])*Sin[c + d*x])/Sqrt[Cos[c +
 d*x]]))/(3*d)
```

3.813.3 Rubi [A] (verified)

Time = 1.16 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.49, number of steps used = 16, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.696$, Rules used = {3042, 4752, 3042, 4328, 27, 3042, 4535, 3042, 4258, 3042, 3120, 4534, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^3dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{(a+b\sec(c+dx))^3}{\sec^{\frac{3}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^3}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{4328}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2}{3}\int\frac{8ba^2+(a^2+9b^2)\sec(c+dx)a-b(a^2-3b^2)\sec^2(c+dx)}{2\sqrt{\sec(c+dx)}}dx+\frac{2a^2\sin(c+dx)(}{3d\sqrt{se}}\right.$$

$$\Big\downarrow \; 27$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\int\frac{8ba^2+(a^2+9b^2)\sec(c+dx)a-b(a^2-3b^2)\sec^2(c+dx)}{\sqrt{\sec(c+dx)}}dx+\frac{2a^2\sin(c+dx)(}{3d\sqrt{se}}\right.$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\int\frac{8ba^2+(a^2+9b^2)\csc\left(c+dx+\frac{\pi}{2}\right)a-b(a^2-3b^2)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a^2s}{}\right.$$

$$\Big\downarrow \; 4535$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\left(\int\frac{8a^2b-b(a^2-3b^2)\sec^2(c+dx)}{\sqrt{\sec(c+dx)}}dx+a(a^2+9b^2)\int\sqrt{\sec(c+dx)}dx\right)+\frac{2a^2s}{}\right.$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\left(a(a^2+9b^2)\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\int\frac{8a^2b-b(a^2-3b^2)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}\right.\right.$$

$$\Big\downarrow \; 4258$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\left(\int\frac{8a^2b-b(a^2-3b^2)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+a(a^2+9b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+}\right.\right.$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\left(\int\frac{8a^2b-b(a^2-3b^2)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+a(a^2+9b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+}\right.\right.$$

$$\Big\downarrow \; 3120$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\left(\int\frac{8a^2b-b(a^2-3b^2)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a(a^2+9b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c}}{d}\right.\right.$$

3.813. $\int\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^3\,dx$

$$\Big\downarrow \; 4534$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\left(3b\left(3a^2-b^2\right)\int\frac{1}{\sqrt{\sec(c+dx)}}dx-\frac{2b\left(a^2-3b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{d}+\frac{2a(}{}\right.\right.$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\left(3b\left(3a^2-b^2\right)\int\frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2b\left(a^2-3b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\right.\right.$$

$$\Big\downarrow \; 4258$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\left(3b\left(3a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\cos(c+dx)}dx-\frac{2b\left(a^2-3b^2\right)\sin(c}{}\right.\right.$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\left(3b\left(3a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx-\frac{2b\left(a^2-3b^2\right.}{}\right.\right.$$

$$\Big\downarrow \; 3119$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\left(-\frac{2b\left(a^2-3b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{d}+\frac{2a\left(a^2+9b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+}}{d}\right.\right.$$

input
```
Int[Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^3,x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*a^2*(a + b*Sec[c + d*x])*Sin[c +
 d*x])/(3*d*Sqrt[Sec[c + d*x]]) + ((6*b*(3*a^2 - b^2)*Sqrt[Cos[c + d*x]]*E
llipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (2*a*(a^2 + 9*b^2)*Sqrt[C
os[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d - (2*b*(a^2 -
 3*b^2)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d)/3)
```

3.813. $\int \cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^3\,dx$

3.813.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4328
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)*
((d*Csc[e + f*x])^n/(f*n)), x] - Simp[1/(d*n)   Int[(a + b*Csc[e + f*x])^(m
 - 3)*(d*Csc[e + f*x])^(n + 1)*Simp[a^2*b*(m - 2*n - 2) - a*(3*b^2*n + a^2*
(n + 1))*Csc[e + f*x] - b*(b^2*n + a^2*(m + n - 1))*Csc[e + f*x]^2, x], x],
 x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && ((Int
egerQ[m] && LtQ[n, -1]) || (IntegersQ[m + 1/2, 2*n] && LeQ[n, -1]))
```

rule 4534
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Simp[(C*m + A*(m + 1))/(m + 1)   Int[(b*Csc[e + f*x])^m, x], x] /;
 FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

rule 4535
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Simp[B/b   Int[(b*Cs
c[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2)
, x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

3.813.4 Maple [A] (verified)

Time = 8.75 (sec) , antiderivative size = 303, normalized size of antiderivative = 2.40

method	result
default	$-\frac{2\left(4a^3\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2a^3-6\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2b^3+a^3\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\sin\left(\frac{dx}{2}+\right.}\right.}{}$

input
```
int(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
-2/3*(4*a^3*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4-2*cos(1/2*d*x+1/2*c)*s
in(1/2*d*x+1/2*c)^2*a^3-6*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2*b^3+a^3*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(co
s(1/2*d*x+1/2*c),2^(1/2))+9*a*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-9*a^2*b*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*
d*x+1/2*c),2^(1/2))+3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2
-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b^3)/sin(1/2*d*x+1/2*c)/(2
*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

3.813.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.70

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^3\,dx$$
$$= \frac{\sqrt{2}(-i\,a^3-9i\,ab^2)\cos(dx+c)\,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c))+\sqrt{2}(i\,a^3+9i\,ab}{}$$

input
```
integrate(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/3*(sqrt(2)*(-I*a^3 - 9*I*a*b^2)*cos(d*x + c)*weierstrassPInverse(-4, 0,
cos(d*x + c) + I*sin(d*x + c)) + sqrt(2)*(I*a^3 + 9*I*a*b^2)*cos(d*x + c)*
weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 3*sqrt(2)*(-3*
I*a^2*b + I*b^3)*cos(d*x + c)*weierstrassZeta(-4, 0, weierstrassPInverse(-
4, 0, cos(d*x + c) + I*sin(d*x + c))) - 3*sqrt(2)*(3*I*a^2*b - I*b^3)*cos(
d*x + c)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) -
I*sin(d*x + c))) + 2*(a^3*cos(d*x + c) + 3*b^3)*sqrt(cos(d*x + c))*sin(d*x
 + c))/(d*cos(d*x + c))
```

3.813.6 Sympy [F(-1)]

Timed out.

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^3\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(3/2)*(a+b*sec(d*x+c))**3,x)
```

output
```
Timed out
```

3.813.7 Maxima [F]

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^3\,dx = \int (b\sec(dx+c)+a)^3\cos(dx+c)^{\frac{3}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^3*cos(d*x + c)^(3/2), x)
```

3.813.8 Giac [F]

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^3\,dx = \int (b\sec(dx+c)+a)^3\cos(dx+c)^{\frac{3}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^3*cos(d*x + c)^(3/2), x)
```

3.813.9 Mupad [B] (verification not implemented)

Time = 13.56 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.98

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^3\,dx = \frac{2\,a^3\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{3\,d} + \frac{6\,a^2\,b\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d} + \frac{6\,a\,b^2\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d} + \frac{2\,a^3\,\sqrt{\cos(c+d\,x)}\,\sin(c+d\,x)}{3\,d} + \frac{2\,b^3\,\sin(c+d\,x)\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{1}{2};\ \frac{3}{4};\ \cos(c+d\,x)^2\right)}{d\,\sqrt{\cos(c+d\,x)}\,\sqrt{\sin(c+d\,x)^2}}$$

input
```
int(cos(c + d*x)^(3/2)*(a + b/cos(c + d*x))^3,x)
```

output
```
(2*a^3*ellipticF(c/2 + (d*x)/2, 2))/(3*d) + (6*a^2*b*ellipticE(c/2 + (d*x)
/2, 2))/d + (6*a*b^2*ellipticF(c/2 + (d*x)/2, 2))/d + (2*a^3*cos(c + d*x)^
(1/2)*sin(c + d*x))/(3*d) + (2*b^3*sin(c + d*x)*hypergeom([-1/4, 1/2], 3/4
, cos(c + d*x)^2))/(d*cos(c + d*x)^(1/2)*(sin(c + d*x)^2)^(1/2))
```

3.814 $\int \sqrt{\cos(c+dx)}(a+b\sec(c+dx))^3\,dx$

3.814.1 Optimal result 6010
3.814.2 Mathematica [A] (verified) 6011
3.814.3 Rubi [A] (verified) 6011
3.814.4 Maple [B] (verified) 6015
3.814.5 Fricas [C] (verification not implemented) 6016
3.814.6 Sympy [F] 6016
3.814.7 Maxima [F] 6017
3.814.8 Giac [F] 6017
3.814.9 Mupad [B] (verification not implemented) 6017

3.814.1 Optimal result

Integrand size = 23, antiderivative size = 118

$$\int \sqrt{\cos(c+dx)}(a+b\sec(c+dx))^3\,dx = \frac{2a(a^2-3b^2)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2b(9a^2+b^2)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d} + \frac{16ab^2\sin(c+dx)}{3d\sqrt{\cos(c+dx)}} + \frac{2b^2(a+b\sec(c+dx))\sin(c+dx)}{3d\sqrt{\cos(c+dx)}}$$

output
```
2*a*(a^2-3*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(
sin(1/2*d*x+1/2*c),2^(1/2))/d+2/3*b*(9*a^2+b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/
2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+16/3*a*b^2*s
in(d*x+c)/d/cos(d*x+c)^(1/2)+2/3*b^2*(a+b*sec(d*x+c))*sin(d*x+c)/d/cos(d*x
+c)^(1/2)
```

3.814.2 Mathematica [A] (verified)

Time = 1.91 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.71

$$\int \sqrt{\cos(c+dx)}(a+b\sec(c+dx))^3\,dx = \frac{2\left(3(a^3-3ab^2)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)+b\left((9a^2+b^2)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+\frac{b(b+9a\cos(c+dx))\sin(c+dx)}{\cos^{\frac{3}{2}}(c+dx)}\right)\right)}{3d}$$

input
```
Integrate[Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x])^3,x]
```

output
```
(2*(3*(a^3 - 3*a*b^2)*EllipticE[(c + d*x)/2, 2] + b*((9*a^2 + b^2)*Ellipti
cF[(c + d*x)/2, 2] + (b*(b + 9*a*Cos[c + d*x])*Sin[c + d*x])/Cos[c + d*x]^
(3/2))))/(3*d)
```

3.814.3 Rubi [A] (verified)

Time = 1.13 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.53, number of steps used = 16, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.696$, Rules used = {3042, 4752, 3042, 4329, 27, 3042, 4535, 3042, 4258, 3042, 3120, 4534, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\cos(c+dx)}(a+b\sec(c+dx))^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^3dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{(a+b\sec(c+dx))^3}{\sqrt{\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^3}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \; 4329$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2}{3}\int \frac{8ab^2\sec^2(c+dx)+b(9a^2+b^2)\sec(c+dx)+a(3a^2-b^2)}{2\sqrt{\sec(c+dx)}}dx+\frac{2b^2\sin(c+dx)}{}\right.$$

$$\Big\downarrow \; 27$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\int \frac{8ab^2\sec^2(c+dx)+b(9a^2+b^2)\sec(c+dx)+a(3a^2-b^2)}{\sqrt{\sec(c+dx)}}dx+\frac{2b^2\sin(c+dx)}{}\right.$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\int \frac{8ab^2\csc\left(c+dx+\frac{\pi}{2}\right)^2+b(9a^2+b^2)\csc\left(c+dx+\frac{\pi}{2}\right)+a(3a^2-b^2)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2b^2\,\mathrm{s}}{}\right.$$

$$\Big\downarrow \; 4535$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\left(\int \frac{8ab^2\sec^2(c+dx)+a(3a^2-b^2)}{\sqrt{\sec(c+dx)}}dx+b(9a^2+b^2)\int \sqrt{\sec(c+dx)}dx\right)+\frac{2b^2\,\mathrm{s}}{}\right.$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\left(b(9a^2+b^2)\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\int \frac{8ab^2\csc\left(c+dx+\frac{\pi}{2}\right)^2+a(3a^2-b^2)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}\right.\right.$$

$$\Big\downarrow \; 4258$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\left(\int \frac{8ab^2\csc\left(c+dx+\frac{\pi}{2}\right)^2+a(3a^2-b^2)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+b(9a^2+b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+}\right.\right.$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\left(\int \frac{8ab^2\csc\left(c+dx+\frac{\pi}{2}\right)^2+a(3a^2-b^2)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+b(9a^2+b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+}\right.\right.$$

$$\Big\downarrow \; 3120$$

3.814. $\int \sqrt{\cos(c+dx)}(a+b\sec(c+dx))^3\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\left(\int\frac{8ab^2\csc\left(c+dx+\frac{\pi}{2}\right)^2+a\left(3a^2-b^2\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2b\left(9a^2+b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c}}{d}\right.\right.$$

$\Big\downarrow$ 4534

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\left(3a\left(a^2-3b^2\right)\int\frac{1}{\sqrt{\sec(c+dx)}}dx+\frac{2b\left(9a^2+b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{Elli}}{d}\right.\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\left(3a\left(a^2-3b^2\right)\int\frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2b\left(9a^2+b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx}}{d}\right.\right.$$

$\Big\downarrow$ 4258

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\left(3a\left(a^2-3b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\cos(c+dx)}dx+\frac{2b\left(9a^2+b^2\right)\sqrt{\text{co}}}{}\right.\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\left(3a\left(a^2-3b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2b\left(9a^2+b^2\right.}{}\right.\right.$$

$\Big\downarrow$ 3119

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\left(\frac{2b\left(9a^2+b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+\frac{6a\left(a^2-3b^2\right)}{}\right.\right.$$

input
```
Int[Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x])^3,x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*b^2*Sqrt[Sec[c + d*x]]*(a + b*Se
c[c + d*x])*Sin[c + d*x])/(3*d) + ((6*a*(a^2 - 3*b^2)*Sqrt[Cos[c + d*x]]*E
llipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (2*b*(9*a^2 + b^2)*Sqrt[C
os[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (16*a*b^2*S
qrt[Sec[c + d*x]]*Sin[c + d*x])/d)/3)
```

3.814. $\int\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^3\,dx$

3.814.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4329
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
2)*((d*Csc[e + f*x])^n/(f*(m + n - 1))), x] + Simp[1/(d*(m + n - 1))   Int[
(a + b*Csc[e + f*x])^(m - 3)*(d*Csc[e + f*x])^n*Simp[a^3*d*(m + n - 1) + a*
b^2*d*n + b*(b^2*d*(m + n - 2) + 3*a^2*d*(m + n - 1))*Csc[e + f*x] + a*b^2*
d*(3*m + 2*n - 4)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, n}, x
] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
 &&  !(IGtQ[n, 2] &&  !IntegerQ[m])
```

rule 4534
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Simp[(C*m + A*(m + 1))/(m + 1)   Int[(b*Csc[e + f*x])^m, x], x] /;
 FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

rule 4535
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Simp[B/b   Int[(b*Cs
c[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2)
, x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

3.814. $\int \sqrt{\cos(c+dx)}(a+b\sec(c+dx))^3\,dx$

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

3.814.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 629 vs. $2(160) = 320$.

Time = 13.13 (sec) , antiderivative size = 630, normalized size of antiderivative = 5.34

method	result
default	$-\frac{2\sqrt{-\left(-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(36\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4 a\,b^2-18\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sqrt{2}\right.}{}$

input
```
int(cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
-2/3*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(4*sin(1/2*
d*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)/sin(1/2*d*x+1/2*c)^3*(36*cos(1/2*d*
x+1/2*c)*sin(1/2*d*x+1/2*c)^4*a*b^2-18*(sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipt
icF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d
*x+1/2*c)^2*a^2*b-2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2
*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2*b^3+6*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1
/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2*a^3-18*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(
1/2)*sin(1/2*d*x+1/2*c)^2*a*b^2-18*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2
*a*b^2-2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2*b^3+9*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)
^(1/2)*a^2*b+(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(
1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*b^3-3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*a^3
+9*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*s
in(1/2*d*x+1/2*c)^2-1)^(1/2)*a*b^2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1
/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

3.814.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.88

$$\int \sqrt{\cos(c+dx)}(a+b\sec(c+dx))^3\,dx$$
$$= \frac{\sqrt{2}(-9i\,a^2b-i\,b^3)\cos(dx+c)^2\,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c))+\sqrt{2}(9i\,a^2b+i}{}$$

input
```
integrate(cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/3*(sqrt(2)*(-9*I*a^2*b - I*b^3)*cos(d*x + c)^2*weierstrassPInverse(-4, 0
, cos(d*x + c) + I*sin(d*x + c)) + sqrt(2)*(9*I*a^2*b + I*b^3)*cos(d*x + c
)^2*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 3*sqrt(2)*
(-I*a^3 + 3*I*a*b^2)*cos(d*x + c)^2*weierstrassZeta(-4, 0, weierstrassPInv
erse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 3*sqrt(2)*(I*a^3 - 3*I*a*b^2
)*cos(d*x + c)^2*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x
 + c) - I*sin(d*x + c))) + 2*(9*a*b^2*cos(d*x + c) + b^3)*sqrt(cos(d*x + c
))*sin(d*x + c))/(d*cos(d*x + c)^2)
```

3.814.6 Sympy [F]

$$\int \sqrt{\cos(c+dx)}(a+b\sec(c+dx))^3\,dx = \int (a+b\sec(c+dx))^3\sqrt{\cos(c+dx)}\,dx$$

input
```
integrate(cos(d*x+c)**(1/2)*(a+b*sec(d*x+c))**3,x)
```

output
```
Integral((a + b*sec(c + d*x))**3*sqrt(cos(c + d*x)), x)
```

3.814.7 Maxima [F]

$$\int \sqrt{\cos(c+dx)}(a+b\sec(c+dx))^3\,dx = \int (b\sec(dx+c)+a)^3\sqrt{\cos(dx+c)}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^3*sqrt(cos(d*x + c)), x)
```

3.814.8 Giac [F]

$$\int \sqrt{\cos(c+dx)}(a+b\sec(c+dx))^3\,dx = \int (b\sec(dx+c)+a)^3\sqrt{\cos(dx+c)}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^3*sqrt(cos(d*x + c)), x)
```

3.814.9 Mupad [B] (verification not implemented)

Time = 15.23 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.08

$$\int \sqrt{\cos(c+dx)}(a+b\sec(c+dx))^3\,dx = \frac{2\left(\mathrm{E}\left(\frac{c}{2}+\frac{dx}{2}\middle|2\right)a^3+3\,b\,\mathrm{F}\left(\frac{c}{2}+\frac{dx}{2}\middle|2\right)a^2\right)}{d} + \frac{2\,b^3\sin(c+d\,x)\,{}_2\mathrm{F}_1\left(-\frac{3}{4},\frac{1}{2};\ \frac{1}{4};\ \cos(c+d\,x)^2\right)}{3\,d\cos(c+d\,x)^{3/2}\sqrt{\sin(c+d\,x)^2}} + \frac{6\,a\,b^2\sin(c+d\,x)\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{1}{2};\ \frac{3}{4};\ \cos(c+d\,x)^2\right)}{d\sqrt{\cos(c+d\,x)}\sqrt{\sin(c+d\,x)^2}}$$

input
```
int(cos(c + d*x)^(1/2)*(a + b/cos(c + d*x))^3,x)
```

output
```
(2*(a^3*ellipticE(c/2 + (d*x)/2, 2) + 3*a^2*b*ellipticF(c/2 + (d*x)/2, 2))
)/d + (2*b^3*sin(c + d*x)*hypergeom([-3/4, 1/2], 1/4, cos(c + d*x)^2))/(3*
d*cos(c + d*x)^(3/2)*(sin(c + d*x)^2)^(1/2)) + (6*a*b^2*sin(c + d*x)*hyper
geom([-1/4, 1/2], 3/4, cos(c + d*x)^2))/(d*cos(c + d*x)^(1/2)*(sin(c + d*x
)^2)^(1/2))
```

3.815 $\int \frac{(a+b\sec(c+dx))^3}{\sqrt{\cos(c+dx)}}\,dx$

3.815.1 Optimal result . . . 6018
3.815.2 Mathematica [A] (verified) . . . 6018
3.815.3 Rubi [A] (verified) . . . 6019
3.815.4 Maple [B] (verified) . . . 6023
3.815.5 Fricas [C] (verification not implemented) . . . 6024
3.815.6 Sympy [F] . . . 6025
3.815.7 Maxima [F] . . . 6025
3.815.8 Giac [F] . . . 6026
3.815.9 Mupad [B] (verification not implemented) . . . 6026

3.815.1 Optimal result

Integrand size = 23, antiderivative size = 149

$$\int \frac{(a+b\sec(c+dx))^3}{\sqrt{\cos(c+dx)}}\,dx = -\frac{6b(5a^2+b^2)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{2a(a^2+b^2)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d} + \frac{8ab^2\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)} + \frac{6b(5a^2+b^2)\sin(c+dx)}{5d\sqrt{\cos(c+dx)}} + \frac{2b^2(a+b\sec(c+dx))\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)}$$

output
```
-6/5*b*(5*a^2+b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellipti
cE(sin(1/2*d*x+1/2*c),2^(1/2))/d+2*a*(a^2+b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2
)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+8/5*a*b^2*sin
(d*x+c)/d/cos(d*x+c)^(3/2)+2/5*b^2*(a+b*sec(d*x+c))*sin(d*x+c)/d/cos(d*x+c
)^(3/2)+6/5*b*(5*a^2+b^2)*sin(d*x+c)/d/cos(d*x+c)^(1/2)
```

3.815.2 Mathematica [A] (verified)

Time = 1.54 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.84

$$\int \frac{(a+b\sec(c+dx))^3}{\sqrt{\cos(c+dx)}}\,dx = \frac{-6b(5a^2+b^2)\cos^{\frac{3}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right) + 10a(a^2+b^2)\cos^{\frac{3}{2}}(c+dx)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right) + 10ab^2}{5d\cos^{\frac{3}{2}}(c+dx)}$$

input
```
Integrate[(a + b*Sec[c + d*x])^3/Sqrt[Cos[c + d*x]],x]
```

output
```
(-6*b*(5*a^2 + b^2)*Cos[c + d*x]^(3/2)*EllipticE[(c + d*x)/2, 2] + 10*a*(a
^2 + b^2)*Cos[c + d*x]^(3/2)*EllipticF[(c + d*x)/2, 2] + 10*a*b^2*Sin[c +
d*x] + 3*(5*a^2*b + b^3)*Sin[2*(c + d*x)] + 2*b^3*Tan[c + d*x])/(5*d*Cos[c
 + d*x]^(3/2))
```

3.815.3 Rubi [A] (verified)

Time = 1.32 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.36, number of steps used = 18, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.783$, Rules used = {3042, 4752, 3042, 4329, 27, 3042, 4535, 3042, 4255, 3042, 4258, 3042, 3119, 4534, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\sec(c+dx))^3}{\sqrt{\cos(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^3}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\sec(c+dx)}(a+b\sec(c+dx))^3dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^3dx$$

$$\Big\downarrow \text{4329}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2}{5}\int \frac{1}{2}\sqrt{\sec(c+dx)}\left(12ab^2\sec^2(c+dx)+3b\left(5a^2+b^2\right)\sec(c+dx)+a\left(5a^2+b^2\right)\right)d\right.$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\int\sqrt{\sec(c+dx)}\left(12ab^2\sec^2(c+dx)+3b\left(5a^2+b^2\right)\sec(c+dx)+a\left(5a^2+b^2\right)\right)dx\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(12ab^2\csc\left(c+dx+\frac{\pi}{2}\right)^2+3b\left(5a^2+b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)\right.\right.$$

$\downarrow$ 4535

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(3b\left(5a^2+b^2\right)\int\sec^{\frac{3}{2}}(c+dx)dx+\int\sqrt{\sec(c+dx)}\left(12ab^2\sec^2(c+dx)+a\left(5a^2+\right.\right.\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(3b\left(5a^2+b^2\right)\int\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}dx+\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(12ab^2\csc\left(c+\right.\right.\right.\right.$$

$\downarrow$ 4255

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(12ab^2\csc\left(c+dx+\frac{\pi}{2}\right)^2+a\left(5a^2+b^2\right)\right)dx+3b\left(5a^2+\right.\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(12ab^2\csc\left(c+dx+\frac{\pi}{2}\right)^2+a\left(5a^2+b^2\right)\right)dx+3b\left(5a^2+\right.\right.\right.$$

$\downarrow$ 4258

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(12ab^2\csc\left(c+dx+\frac{\pi}{2}\right)^2+a\left(5a^2+b^2\right)\right)dx+3b\left(5a^2+\right.\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(12ab^2\csc\left(c+dx+\frac{\pi}{2}\right)^2+a\left(5a^2+b^2\right)\right)dx+3b\left(5a^2+\right.\right.\right.$$

$\downarrow$ 3119

3.815. $\int\frac{(a+b\sec(c+dx))^3}{\sqrt{\cos(c+dx)}}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(12ab^2\csc\left(c+dx+\frac{\pi}{2}\right)^2+a\left(5a^2+b^2\right)\right)dx+3b\left(5a^2+\right.\right.\right.$$

$$\Big\downarrow \; 4534$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(5a\left(a^2+b^2\right)\int\sqrt{\sec(c+dx)}dx+3b\left(5a^2+b^2\right)\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2}{}\right.\right.\right.$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(5a\left(a^2+b^2\right)\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+3b\left(5a^2+b^2\right)\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx}}{d}\right.\right.\right.$$

$$\Big\downarrow \; 4258$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(5a\left(a^2+b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx+3b\left(5a^2+b^2\right)\left(\frac{2\,\mathrm{si}}{}\right.\right.\right.$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(5a\left(a^2+b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+3b\left(5a^2+b^2\right)\right.\right.$$

$$\Big\downarrow \; 3120$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(\frac{10a\left(a^2+b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\mathrm{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+3b\left(5a^2+b^2\right)\right.\right.$$

input
```
Int[(a + b*Sec[c + d*x])^3/Sqrt[Cos[c + d*x]],x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*b^2*Sec[c + d*x]^(3/2)*(a + b*Se
c[c + d*x])*Sin[c + d*x])/(5*d) + ((10*a*(a^2 + b^2)*Sqrt[Cos[c + d*x]]*El
lipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (8*a*b^2*Sec[c + d*x]^(3/2
)*Sin[c + d*x])/d + 3*b*(5*a^2 + b^2)*((-2*Sqrt[Cos[c + d*x]]*EllipticE[(c
 + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/
d))/5)
```

3.815. $\int \frac{(a+b\sec(c+dx))^3}{\sqrt{\cos(c+dx)}}\,dx$

3.815.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4329
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
2)*((d*Csc[e + f*x])^n/(f*(m + n - 1))), x] + Simp[1/(d*(m + n - 1))   Int[
(a + b*Csc[e + f*x])^(m - 3)*(d*Csc[e + f*x])^n*Simp[a^3*d*(m + n - 1) + a*
b^2*d*n + b*(b^2*d*(m + n - 2) + 3*a^2*d*(m + n - 1))*Csc[e + f*x] + a*b^2*
d*(3*m + 2*n - 4)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, n}, x
] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
 &&  !(IGtQ[n, 2] &&  !IntegerQ[m])
```

rule 4534
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Simp[(C*m + A*(m + 1))/(m + 1)   Int[(b*Csc[e + f*x])^m, x], x] /;
 FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

3.815. $\int \frac{(a+b\sec(c+dx))^3}{\sqrt{\cos(c+dx)}}\,dx$

rule 4535
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Simp[B/b   Int[(b*Cs
c[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2)
, x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

3.815.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 710 vs. $2(187) = 374$.

Time = 17.43 (sec) , antiderivative size = 711, normalized size of antiderivative = 4.77

method	result	size
default	Expression too large to display	711

input
```
int((a+b*sec(d*x+c))^3/cos(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*a^3*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+2/
5*b^3/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)
^2-1)/sin(1/2*d*x+1/2*c)^2*(24*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(
1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^4*cos
(1/2*d*x+1/2*c)+12*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*
c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2+8*cos(1/
2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE
(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2))*(-2*sin(1/2
*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+6*a*b^2*(-1/6*cos(1/2*d*x+1/2*c)
*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^
2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2
)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d
*x+1/2*c),2^(1/2)))+6*a^2*b/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1
)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*cos(1/2*d*x+1/2*
c)*sin(1/2*d*x+1/2*c)^2-(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x
+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)))/sin(1/2*d*x+1/2*c)/(2*
cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

3.815.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.64

$$\int \frac{(a+b\sec(c+dx))^3}{\sqrt{\cos(c+dx)}}\,dx = -\frac{5\sqrt{2}(i\,a^3+i\,ab^2)\cos(dx+c)^3\,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c))+5\sqrt{2}(-i\,a^3-}{}$$

input
```
integrate((a+b*sec(d*x+c))^3/cos(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
-1/5*(5*sqrt(2)*(I*a^3 + I*a*b^2)*cos(d*x + c)^3*weierstrassPInverse(-4, 0
, cos(d*x + c) + I*sin(d*x + c)) + 5*sqrt(2)*(-I*a^3 - I*a*b^2)*cos(d*x +
c)^3*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 3*sqrt(2)
*(5*I*a^2*b + I*b^3)*cos(d*x + c)^3*weierstrassZeta(-4, 0, weierstrassPInv
erse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 3*sqrt(2)*(-5*I*a^2*b - I*b^
3)*cos(d*x + c)^3*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*
x + c) - I*sin(d*x + c))) - 2*(5*a*b^2*cos(d*x + c) + b^3 + 3*(5*a^2*b + b
^3)*cos(d*x + c)^2)*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^3)
```

3.815.6 Sympy [F]

$$\int \frac{(a+b\sec(c+dx))^3}{\sqrt{\cos(c+dx)}}\,dx = \int \frac{(a+b\sec{(c+dx)})^3}{\sqrt{\cos{(c+dx)}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))**3/cos(d*x+c)**(1/2),x)
```

output
```
Integral((a + b*sec(c + d*x))**3/sqrt(cos(c + d*x)), x)
```

3.815.7 Maxima [F]

$$\int \frac{(a+b\sec(c+dx))^3}{\sqrt{\cos(c+dx)}}\,dx = \int \frac{(b\sec{(dx+c)}+a)^3}{\sqrt{\cos{(dx+c)}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^3/cos(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^3/sqrt(cos(d*x + c)), x)
```

3.815.8 Giac [F]

$$\int \frac{(a+b\sec(c+dx))^3}{\sqrt{\cos(c+dx)}}\,dx = \int \frac{(b\sec(dx+c)+a)^3}{\sqrt{\cos(dx+c)}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^3/cos(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^3/sqrt(cos(d*x + c)), x)
```

3.815.9 Mupad [B] (verification not implemented)

Time = 14.90 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.05

$$\begin{aligned}\int \frac{(a+b\sec(c+dx))^3}{\sqrt{\cos(c+dx)}}\,dx = {} & \frac{2\,a^3\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d} \\ & + \frac{2\,b^3\,\sin(c+d\,x)\,{}_2\mathrm{F}_1\left(-\frac{5}{4},\frac{1}{2};\ -\frac{1}{4};\ \cos(c+d\,x)^2\right)}{5\,d\cos(c+d\,x)^{5/2}\,\sqrt{\sin(c+d\,x)^2}} \\ & + \frac{6\,a^2\,b\,\sin(c+d\,x)\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{1}{2};\ \frac{3}{4};\ \cos(c+d\,x)^2\right)}{d\,\sqrt{\cos(c+d\,x)}\,\sqrt{\sin(c+d\,x)^2}} \\ & + \frac{2\,a\,b^2\,\sin(c+d\,x)\,{}_2\mathrm{F}_1\left(-\frac{3}{4},\frac{1}{2};\ \frac{1}{4};\ \cos(c+d\,x)^2\right)}{d\cos(c+d\,x)^{3/2}\,\sqrt{\sin(c+d\,x)^2}}\end{aligned}$$

input
```
int((a + b/cos(c + d*x))^3/cos(c + d*x)^(1/2),x)
```

output
```
(2*a^3*ellipticF(c/2 + (d*x)/2, 2))/d + (2*b^3*sin(c + d*x)*hypergeom([-5/
4, 1/2], -1/4, cos(c + d*x)^2))/(5*d*cos(c + d*x)^(5/2)*(sin(c + d*x)^2)^(
1/2)) + (6*a^2*b*sin(c + d*x)*hypergeom([-1/4, 1/2], 3/4, cos(c + d*x)^2))
/(d*cos(c + d*x)^(1/2)*(sin(c + d*x)^2)^(1/2)) + (2*a*b^2*sin(c + d*x)*hyp
ergeom([-3/4, 1/2], 1/4, cos(c + d*x)^2))/(d*cos(c + d*x)^(3/2)*(sin(c + d
*x)^2)^(1/2))
```

3.816 $\int \frac{(a+b\sec(c+dx))^3}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

3.816.1 Optimal result 6027
3.816.2 Mathematica [A] (verified) 6028
3.816.3 Rubi [A] (verified) 6028
3.816.4 Maple [B] (verified) 6032
3.816.5 Fricas [C] (verification not implemented) 6033
3.816.6 Sympy [F] 6034
3.816.7 Maxima [F(-1)] 6034
3.816.8 Giac [F] 6035
3.816.9 Mupad [B] (verification not implemented) 6035

3.816.1 Optimal result

Integrand size = 23, antiderivative size = 194

$$\begin{aligned}\int \frac{(a+b\sec(c+dx))^3}{\cos^{\frac{3}{2}}(c+dx)}\,dx = &-\frac{2a(5a^2+9b^2)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}\\ &+\frac{2b(21a^2+5b^2)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d}+\frac{32ab^2\sin(c+dx)}{35d\cos^{\frac{5}{2}}(c+dx)}\\ &+\frac{2b(21a^2+5b^2)\sin(c+dx)}{21d\cos^{\frac{3}{2}}(c+dx)}+\frac{2a(5a^2+9b^2)\sin(c+dx)}{5d\sqrt{\cos(c+dx)}}\\ &+\frac{2b^2(a+b\sec(c+dx))\sin(c+dx)}{7d\cos^{\frac{5}{2}}(c+dx)}\end{aligned}$$

output
```
-2/5*a*(5*a^2+9*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellip
ticE(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/21*b*(21*a^2+5*b^2)*(cos(1/2*d*x+1/2*
c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+32/
35*a*b^2*sin(d*x+c)/d/cos(d*x+c)^(5/2)+2/21*b*(21*a^2+5*b^2)*sin(d*x+c)/d/
cos(d*x+c)^(3/2)+2/7*b^2*(a+b*sec(d*x+c))*sin(d*x+c)/d/cos(d*x+c)^(5/2)+2/
5*a*(5*a^2+9*b^2)*sin(d*x+c)/d/cos(d*x+c)^(1/2)
```

3.816.2 Mathematica [A] (verified)

Time = 1.46 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.91

$$\int \frac{(a+b\sec(c+dx))^3}{\cos^{\frac{3}{2}}(c+dx)}\,dx$$
$$= \frac{-42a(5a^2+9b^2)\cos^{\frac{5}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)+10b(21a^2+5b^2)\cos^{\frac{5}{2}}(c+dx)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+}{}$$

input
```
Integrate[(a + b*Sec[c + d*x])^3/Cos[c + d*x]^(3/2),x]
```

output
```
(-42*a*(5*a^2 + 9*b^2)*Cos[c + d*x]^(5/2)*EllipticE[(c + d*x)/2, 2] + 10*b
*(21*a^2 + 5*b^2)*Cos[c + d*x]^(5/2)*EllipticF[(c + d*x)/2, 2] + 126*a*b^2
*Sin[c + d*x] + 210*a^3*Cos[c + d*x]^2*Sin[c + d*x] + 378*a*b^2*Cos[c + d*
x]^2*Sin[c + d*x] + 105*a^2*b*Sin[2*(c + d*x)] + 25*b^3*Sin[2*(c + d*x)] +
 30*b^3*Tan[c + d*x])/(105*d*Cos[c + d*x]^(5/2))
```

3.816.3 Rubi [A] (verified)

Time = 1.52 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.23, number of steps used = 20, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.870$, Rules used = {3042, 4752, 3042, 4329, 27, 3042, 4535, 3042, 4255, 3042, 4258, 3042, 3120, 4534, 3042, 4255, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\sec(c+dx))^3}{\cos^{\frac{3}{2}}(c+dx)}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^3}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx$$
$$\Big\downarrow \; 4752$$
$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^3dx$$
$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^3 dx$$

$$\Big\downarrow \text{4329}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2}{7}\int \frac{1}{2}\sec^{\frac{3}{2}}(c+dx)\left(16ab^2\sec^2(c+dx)+b\left(21a^2+5b^2\right)\sec(c+dx)+a\left(7a^2+3b^2\right)\right)\right.$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}\int \sec^{\frac{3}{2}}(c+dx)\left(16ab^2\sec^2(c+dx)+b\left(21a^2+5b^2\right)\sec(c+dx)+a\left(7a^2+3b^2\right)\right)d\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(16ab^2\csc\left(c+dx+\frac{\pi}{2}\right)^2+b\left(21a^2+5b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right.\right.\right.$$

$$\Big\downarrow \text{4535}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}\left(b\left(21a^2+5b^2\right)\int \sec^{\frac{5}{2}}(c+dx)dx+\int \sec^{\frac{3}{2}}(c+dx)\left(16ab^2\sec^2(c+dx)+a\left(7a^2+\right.\right.\right.\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}\left(b\left(21a^2+5b^2\right)\int \csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}dx+\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(16ab^2\csc\left(c\right.\right.\right.\right.$$

$$\Big\downarrow \text{4255}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}\left(\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(16ab^2\csc\left(c+dx+\frac{\pi}{2}\right)^2+a\left(7a^2+3b^2\right)\right)dx+b\left(21a^2\right.\right.\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}\left(\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(16ab^2\csc\left(c+dx+\frac{\pi}{2}\right)^2+a\left(7a^2+3b^2\right)\right)dx+b\left(21a^2\right.\right.\right.$$

$$\Big\downarrow \text{4258}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}\left(\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(16ab^2\csc\left(c+dx+\frac{\pi}{2}\right)^2+a\left(7a^2+3b^2\right)\right)dx+b\left(21a^2\right.\right.\right.$$

3.816. $\int \frac{(a+b\sec(c+dx))^3}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}\left(\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(16ab^2\csc\left(c+dx+\frac{\pi}{2}\right)^2+a(7a^2+3b^2)\right)dx+b(21a^2\right.\right.$$

$$\Big\downarrow \text{3120}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}\left(\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(16ab^2\csc\left(c+dx+\frac{\pi}{2}\right)^2+a(7a^2+3b^2)\right)dx+b(21a^2\right.\right.$$

$$\Big\downarrow \text{4534}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}\left(\frac{7}{5}a(5a^2+9b^2)\int \sec^{\frac{3}{2}}(c+dx)dx+b(21a^2+5b^2)\left(\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\right.\right.\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}\left(\frac{7}{5}a(5a^2+9b^2)\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}dx+b(21a^2+5b^2)\left(\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c\right.\right.\right.$$

$$\Big\downarrow \text{4255}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}\left(\frac{7}{5}a(5a^2+9b^2)\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\int\frac{1}{\sqrt{\sec(c+dx)}}dx\right)+b(21a^2+\right.\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}\left(\frac{7}{5}a(5a^2+9b^2)\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\int\frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)+b(21\right.\right.$$

$$\Big\downarrow \text{4258}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}\left(\frac{7}{5}a(5a^2+9b^2)\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{c}\right.\right.\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}\left(\frac{7}{5}a(5a^2+9b^2)\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{s}\right.\right.\right.$$

3.816. $\int \frac{(a+b\sec(c+dx))^3}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

$$\Big\downarrow \text{3119}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}\left(b\left(21a^2+5b^2\right)\left(\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{Ellipti}}{3d}\right.\right.\right.$$

input
```
Int[(a + b*Sec[c + d*x])^3/Cos[c + d*x]^(3/2),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*b^2*Sec[c + d*x]^(5/2)*(a + b*Se
c[c + d*x])*Sin[c + d*x])/(7*d) + ((32*a*b^2*Sec[c + d*x]^(5/2)*Sin[c + d*
x])/(5*d) + (7*a*(5*a^2 + 9*b^2)*((-2*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*
x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d))/5
 + b*(21*a^2 + 5*b^2)*((2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqr
t[Sec[c + d*x]])/(3*d) + (2*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d)))/7)
```

3.816.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4329
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
2)*((d*Csc[e + f*x])^n/(f*(m + n - 1))), x] + Simp[1/(d*(m + n - 1))   Int[
(a + b*Csc[e + f*x])^(m - 3)*(d*Csc[e + f*x])^n*Simp[a^3*d*(m + n - 1) + a*
b^2*d*n + b*(b^2*d*(m + n - 2) + 3*a^2*d*(m + n - 1))*Csc[e + f*x] + a*b^2*
d*(3*m + 2*n - 4)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, n}, x
] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
 &&  !(IGtQ[n, 2] &&  !IntegerQ[m])
```

rule 4534
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Simp[(C*m + A*(m + 1))/(m + 1)   Int[(b*Csc[e + f*x])^m, x], x] /;
 FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

rule 4535
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Simp[B/b   Int[(b*Cs
c[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2)
, x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

3.816.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 819 vs. $2(226) = 452$.

Time = 22.42 (sec) , antiderivative size = 820, normalized size of antiderivative = 4.23

method	result	size
default	Expression too large to display	820

input
```
int((a+b*sec(d*x+c))^3/cos(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*a^3/sin(1/2*
d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d
*x+1/2*c)^2)^(1/2)*(2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-(sin(1/2*d*x
+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*
c)^2-1)^(1/2))+2*b^3*(-1/56*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+si
n(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^4-5/42*cos(1/2*d*x+1/
2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2
*c)^2-1/2)^2+5/21*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)
^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(
1/2*d*x+1/2*c),2^(1/2)))+6*a^2*b*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+
1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x
+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)
))+6/5*a*b^2/sin(1/2*d*x+1/2*c)^2/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1
/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)*(24*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2
*c)-12*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*
(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c
)^4*cos(1/2*d*x+1/2*c)+12*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d
*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2+8
*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-3*(sin(1/2*d*x+1/2*c)^2)^(1/2)...
```

3.816.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.39

$$\int \frac{(a+b\sec(c+dx))^3}{\cos^{\frac{3}{2}}(c+dx)}\,dx = -\frac{5\sqrt{2}(21i\,a^2b+5i\,b^3)\cos(dx+c)^4\,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c))+5\sqrt{2}(-21}{}$$

input
```
integrate((a+b*sec(d*x+c))^3/cos(d*x+c)^(3/2),x, algorithm="fricas")
```

output
```
-1/105*(5*sqrt(2)*(21*I*a^2*b + 5*I*b^3)*cos(d*x + c)^4*weierstrassPInvers
e(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 5*sqrt(2)*(-21*I*a^2*b - 5*I*b^3
)*cos(d*x + c)^4*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))
 + 21*sqrt(2)*(5*I*a^3 + 9*I*a*b^2)*cos(d*x + c)^4*weierstrassZeta(-4, 0,
weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 21*sqrt(2)*(-
5*I*a^3 - 9*I*a*b^2)*cos(d*x + c)^4*weierstrassZeta(-4, 0, weierstrassPInv
erse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - 2*(63*a*b^2*cos(d*x + c) + 2
1*(5*a^3 + 9*a*b^2)*cos(d*x + c)^3 + 15*b^3 + 5*(21*a^2*b + 5*b^3)*cos(d*x
 + c)^2)*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^4)
```

3.816.6 Sympy [F]

$$\int \frac{(a+b\sec(c+dx))^3}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(a+b\sec{(c+dx)})^3}{\cos^{\frac{3}{2}}{(c+dx)}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))**3/cos(d*x+c)**(3/2),x)
```

output
```
Integral((a + b*sec(c + d*x))**3/cos(c + d*x)**(3/2), x)
```

3.816.7 Maxima [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(c+dx))^3}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+b*sec(d*x+c))^3/cos(d*x+c)^(3/2),x, algorithm="maxima")
```

output
```
Timed out
```

3.816.8 Giac [F]

$$\int \frac{(a+b\sec(c+dx))^3}{\cos^{\frac{3}{2}}(c+dx)}\,dx=\int \frac{(b\sec(dx+c)+a)^3}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^3/cos(d*x+c)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^3/cos(d*x + c)^(3/2), x)
```

3.816.9 Mupad [B] (verification not implemented)

Time = 15.16 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.76

$$\int \frac{(a+b\sec(c+dx))^3}{\cos^{\frac{3}{2}}(c+dx)}\,dx$$
$$=\frac{\frac{2\,b^3\,\sin(c+d\,x)\,{}_2\mathrm{F}_1\left(-\frac{7}{4},\frac{1}{2};\ -\frac{3}{4};\ \cos(c+d\,x)^2\right)}{7}+2\,a^3\cos(c+d\,x)^3\,\sin(c+d\,x)\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{1}{2};\ \frac{3}{4};\ \cos(c+d\,x)^2\right)+\frac{6\,a\,b^2}{}}{d\cos(c+d\,x)^{7/2}\,\sqrt{}}$$

input
```
int((a + b/cos(c + d*x))^3/cos(c + d*x)^(3/2),x)
```

output
```
((2*b^3*sin(c + d*x)*hypergeom([-7/4, 1/2], -3/4, cos(c + d*x)^2))/7 + 2*a
^3*cos(c + d*x)^3*sin(c + d*x)*hypergeom([-1/4, 1/2], 3/4, cos(c + d*x)^2)
 + (6*a*b^2*cos(c + d*x)*sin(c + d*x)*hypergeom([-5/4, 1/2], -1/4, cos(c +
 d*x)^2))/5 + 2*a^2*b*cos(c + d*x)^2*sin(c + d*x)*hypergeom([-3/4, 1/2], 1
/4, cos(c + d*x)^2))/(d*cos(c + d*x)^(7/2)*(1 - cos(c + d*x)^2)^(1/2))
```

3.817 $\int \frac{\cos^{\frac{5}{2}}(c+dx)}{a+b\sec(c+dx)}\,dx$

3.817.1 Optimal result . . . 6036
3.817.2 Mathematica [A] (verified) . . . 6037
3.817.3 Rubi [A] (verified) . . . 6037
3.817.4 Maple [B] (verified) . . . 6043
3.817.5 Fricas [F(-1)] . . . 6044
3.817.6 Sympy [F(-1)] . . . 6044
3.817.7 Maxima [F] . . . 6044
3.817.8 Giac [F] . . . 6045
3.817.9 Mupad [F(-1)] . . . 6045

3.817.1 Optimal result

Integrand size = 23, antiderivative size = 152

$$\begin{aligned}\int \frac{\cos^{\frac{5}{2}}(c+dx)}{a+b\sec(c+dx)}\,dx = {} & \frac{2(3a^2+5b^2)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5a^3d} \\ & - \frac{2b(a^2+3b^2)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3a^4d} \\ & + \frac{2b^4\,\text{EllipticPi}\left(\frac{2a}{a+b},\frac{1}{2}(c+dx),2\right)}{a^4(a+b)d} \\ & - \frac{2b\sqrt{\cos(c+dx)}\sin(c+dx)}{3a^2d} + \frac{2\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5ad}\end{aligned}$$

output
```
2/5*(3*a^2+5*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elliptic
E(sin(1/2*d*x+1/2*c),2^(1/2))/a^3/d-2/3*b*(a^2+3*b^2)*(cos(1/2*d*x+1/2*c)^
2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a^4/d+2*
b^4*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x
+1/2*c),2*a/(a+b),2^(1/2))/a^4/(a+b)/d+2/5*cos(d*x+c)^(3/2)*sin(d*x+c)/a/d
-2/3*b*sin(d*x+c)*cos(d*x+c)^(1/2)/a^2/d
```

3.817.2 Mathematica [A] (verified)

Time = 12.07 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.49

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{a+b\sec(c+dx)}\,dx = \frac{\frac{2(9a^2+5b^2)\,\text{EllipticPi}\left(\frac{2a}{a+b},\frac{1}{2}(c+dx),2\right)}{a+b}+8b\left(2\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)-\frac{2b\,\text{EllipticPi}\left(\frac{2a}{a+b},\frac{1}{2}(c+dx),2\right)}{a+b}\right)+4\sqrt{\cos(c+d}}{}$$

input
```
Integrate[Cos[c + d*x]^(5/2)/(a + b*Sec[c + d*x]),x]
```

output
```
((2*(9*a^2 + 5*b^2)*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(a + b) + 8
*b*(2*EllipticF[(c + d*x)/2, 2] - (2*b*EllipticPi[(2*a)/(a + b), (c + d*x)
/2, 2])/(a + b)) + 4*Sqrt[Cos[c + d*x]]*(-5*b + 3*a*Cos[c + d*x])*Sin[c +
d*x] + (6*(3*a^2 + 5*b^2)*(-2*a*b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1
] + 2*b*(a + b)*EllipticF[ArcSin[Sqrt[Cos[c + d*x]]], -1] + (a^2 - 2*b^2)*
EllipticPi[-(a/b), ArcSin[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d*x])/(a^2*b*S
qrt[Sin[c + d*x]^2]))/(30*a^2*d)
```

3.817.3 Rubi [A] (verified)

Time = 1.87 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.61, number of steps used = 20, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.870$, Rules used = {3042, 4752, 3042, 4340, 27, 3042, 4592, 27, 3042, 4594, 3042, 4274, 3042, 4258, 3042, 3119, 3120, 4336, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{a+b\sec(c+dx)}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \; 4752$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)}dx$$

$$\Big\downarrow \text{4340}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\int -\frac{-3b\sec^2(c+dx)-3a\sec(c+dx)+5b}{2\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))}dx}{5a}+\frac{2\sin(c+dx)}{5ad\sec^{\frac{3}{2}}(c+dx)}\right)$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sin(c+dx)}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{\int \frac{-3b\sec^2(c+dx)-3a\sec(c+dx)+5b}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))}dx}{5a}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sin(c+dx)}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{\int \frac{-3b\csc\left(c+dx+\frac{\pi}{2}\right)^2-3a\csc\left(c+dx+\frac{\pi}{2}\right)+5b}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{5a}\right)$$

$$\Big\downarrow \text{4592}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sin(c+dx)}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{\frac{10b\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{2\int \frac{-5b^2\sec^2(c+dx)+4ab\sec(c+dx)+3\left(3a^2+5b^2\right)}{2\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}dx}{3a}}{5a}\right)$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sin(c+dx)}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{\frac{10b\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{\int \frac{-5b^2\sec^2(c+dx)+4ab\sec(c+dx)+3\left(3a^2+5b^2\right)}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}dx}{3a}}{5a}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sin(c+dx)}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{\frac{10b\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{\int \frac{-5b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2+4ab\csc\left(c+dx+\frac{\pi}{2}\right)+3\left(3a^2+5b^2\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{3a}}{5a}\right)$$

3.817. $\int \frac{\cos^{\frac{5}{2}}(c+dx)}{a+b\sec(c+dx)}\,dx$

$$\Big\downarrow \; 4594$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sin(c+dx)}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{\frac{10b\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{15b^4\int\frac{\sec^{\frac{3}{2}}(c+dx)}{a+b\sec(c+dx)}dx}{a^2}+\frac{\int\frac{3a\left(3a^2+5b^2\right)-5b\left(a^2+3b^2\right)\sec(c+dx)}{\sqrt{\sec(c+dx)}}}{a^2}}{3a}}{5a}\right.$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sin(c+dx)}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{\frac{10b\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{15b^4\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}+\frac{\int\frac{3a\left(3a^2+5b^2\right)-5b\left(a^2+3b^2\right)\csc\left(c\right.}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}}{a^2}}{3a}}{5a}\right.$$

$$\Big\downarrow \; 4274$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sin(c+dx)}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{\frac{10b\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{15b^4\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}+\frac{3a\left(3a^2+5b^2\right)\int\frac{1}{\sqrt{\sec(c+dx)}}dx-5b\left(\right.}{a^2}}{3a}}{5a}\right.$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sin(c+dx)}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{\frac{10b\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{15b^4\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}+\frac{3a\left(3a^2+5b^2\right)\int\frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2}}{3a}}{5a}\right.$$

$$\Big\downarrow \; 4258$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sin(c+dx)}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{\frac{10b\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{15b^4\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}+\frac{3a\left(3a^2+5b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+d}}{}}{5a}\right.$$

3.817. $\int \frac{\cos^{\frac{5}{2}}(c+dx)}{a+b\sec(c+dx)}\,dx$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sin(c+dx)}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{\frac{10b\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{15b^4\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}+\frac{3a\left(3a^2+5b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+d}}{}}{}}{5a}\right.$$

$$\Big\downarrow \; 3119$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sin(c+dx)}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{\frac{10b\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{15b^4\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}+\frac{\frac{6a\left(3a^2+5b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+d}}{d}}{}}{}}{5a}\right.$$

$$\Big\downarrow \; 3120$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sin(c+dx)}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{\frac{10b\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{15b^4\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}+\frac{\frac{6a\left(3a^2+5b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+d}}{d}}{}}{}}{5a}\right.$$

$$\Big\downarrow \; 4336$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sin(c+dx)}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{\frac{10b\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{15b^4\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}(b+a\cos(c+dx))}dx}{a^2}+\frac{6a}{}}{}}{}\right.$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sin(c+dx)}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{\frac{10b\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{15b^4\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(b+a\sin\left(c+dx+\frac{\pi}{2}\right)\right.}}{a^2}}{}}{}\right.$$

3.817. $\int \frac{\cos^{\frac{5}{2}}(c+dx)}{a+b\sec(c+dx)}\,dx$

$$\Big\downarrow \; 3284$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sin(c+dx)}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{\frac{10b\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{30b^4\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticPi}\left(\frac{2a}{a+b},\frac{1}{2}(c+dx),2\right)}{a^2d(a+b)}+\frac{6a\left(3a^2\right.}{}}{}\right.$$

input
```
Int[Cos[c + d*x]^(5/2)/(a + b*Sec[c + d*x]),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*Sin[c + d*x])/(5*a*d*Sec[c + d*x
]^(3/2)) - (-1/3*(((6*a*(3*a^2 + 5*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c +
d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d - (10*b*(a^2 + 3*b^2)*Sqrt[Cos[c + d*x]]*
EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d)/a^2 + (30*b^4*Sqrt[Cos[c
+ d*x]]*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a^2
*(a + b)*d))/a + (10*b*Sin[c + d*x])/(3*a*d*Sqrt[Sec[c + d*x]]))/(5*a))
```

3.817.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4336
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]]   Int[
1/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4340
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[Cot[e + f*x]*((d*Csc[e + f*x])^n/(a*f*n)), x] - Sim
p[1/(a*d*n)   Int[((d*Csc[e + f*x])^(n + 1)/(a + b*Csc[e + f*x]))*Simp[b*n
- a*(n + 1)*Csc[e + f*x] - b*(n + 1)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a
, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1] && IntegerQ[2*n]
```

rule 4592
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Simp[1/(a*d*n)   Int[(a + b*Csc[e + f*x])^m
*(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*
Csc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

rule 4594
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Simp[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2)   Int[(d*Csc[e +
f*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Simp[1/a^2   Int[(a*A - (A*b - a
*B)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A,
B, C}, x] && NeQ[a^2 - b^2, 0]
```

3.817. $\int \frac{\cos^{\frac{5}{2}}(c+dx)}{a+b\sec(c+dx)}\,dx$

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

3.817.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 667 vs. $2(218) = 436$.

Time = 10.12 (sec) , antiderivative size = 668, normalized size of antiderivative = 4.39

method	result
default	$-\frac{2\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left((-24a^4+24a^3b)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^6+(24a^4-44a^3b+20a^2b^2)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4\cos\left(\frac{d}{2}\right.\right.}{}$

input
```
int(cos(d*x+c)^(5/2)/(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-2/15*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*((-24*a^4+24
*a^3*b)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+(24*a^4-44*a^3*b+20*a^2*b^
2)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-6*a^4+16*a^3*b-10*a^2*b^2)*si
n(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-5*a^3*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+5*
a^2*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elli
pticF(cos(1/2*d*x+1/2*c),2^(1/2))-15*a*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2
*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+15*b^
4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(
cos(1/2*d*x+1/2*c),2^(1/2))-9*a^4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+9*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/
2*c),2^(1/2))*a^3*b-15*a^2*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x
+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+15*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*
c),2^(1/2))*a*b^3-15*b^4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c
)^2-1)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2)))/a^4/(a-b)/(
-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*
cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

3.817.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{a+b\sec(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)^(5/2)/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

output
```
Timed out
```

3.817.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{a+b\sec(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(5/2)/(a+b*sec(d*x+c)),x)
```

output
```
Timed out
```

3.817.7 Maxima [F]

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{a+b\sec(c+dx)}\,dx = \int \frac{\cos\left(dx+c\right)^{\frac{5}{2}}}{b\sec\left(dx+c\right)+a}\,dx$$

input
```
integrate(cos(d*x+c)^(5/2)/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

output
```
integrate(cos(d*x + c)^(5/2)/(b*sec(d*x + c) + a), x)
```

3.817.8 Giac [F]

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{a+b\sec(c+dx)}\,dx = \int \frac{\cos(dx+c)^{\frac{5}{2}}}{b\sec(dx+c)+a}\,dx$$

input
```
integrate(cos(d*x+c)^(5/2)/(a+b*sec(d*x+c)),x, algorithm="giac")
```

output
```
integrate(cos(d*x + c)^(5/2)/(b*sec(d*x + c) + a), x)
```

3.817.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{a+b\sec(c+dx)}\,dx = \int \frac{\cos(c+d\,x)^{5/2}}{a+\frac{b}{\cos(c+d\,x)}}\,dx$$

input
```
int(cos(c + d*x)^(5/2)/(a + b/cos(c + d*x)),x)
```

output
```
int(cos(c + d*x)^(5/2)/(a + b/cos(c + d*x)), x)
```

3.818 $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{a+b\sec(c+dx)}\,dx$

3.818.1 Optimal result . . . 6046
3.818.2 Mathematica [A] (verified) . . . 6046
3.818.3 Rubi [A] (verified) . . . 6047
3.818.4 Maple [B] (verified) . . . 6052
3.818.5 Fricas [F(-1)] . . . 6052
3.818.6 Sympy [F] . . . 6053
3.818.7 Maxima [F] . . . 6053
3.818.8 Giac [F] . . . 6053
3.818.9 Mupad [F(-1)] . . . 6054

3.818.1 Optimal result

Integrand size = 23, antiderivative size = 112

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{a+b\sec(c+dx)}\,dx = -\frac{2bE\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d} + \frac{2(a^2+3b^2)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3a^3d} - \frac{2b^3\,\text{EllipticPi}\left(\frac{2a}{a+b},\frac{1}{2}(c+dx),2\right)}{a^3(a+b)d} + \frac{2\sqrt{\cos(c+dx)}\sin(c+dx)}{3ad}$$

output
```
-2*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x
+1/2*c),2^(1/2))/a^2/d+2/3*(a^2+3*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/
2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a^3/d-2*b^3*(cos(1/2*d*
x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*a/(a+
b),2^(1/2))/a^3/(a+b)/d+2/3*sin(d*x+c)*cos(d*x+c)^(1/2)/a/d
```

3.818.2 Mathematica [A] (verified)

Time = 1.83 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.41

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{a+b\sec(c+dx)}\,dx = \frac{4\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right) - \frac{6b\,\text{EllipticPi}\left(\frac{2a}{a+b},\frac{1}{2}(c+dx),2\right)}{a+b} + 4\sqrt{\cos(c+dx)}\sin(c+dx) - \frac{6\left(-2abE\left(\arcsin\left(\sqrt{\cos(c+d}\right.\right.\right.}{}}{6ad}$$

input
```
Integrate[Cos[c + d*x]^(3/2)/(a + b*Sec[c + d*x]),x]
```

output
```
(4*EllipticF[(c + d*x)/2, 2] - (6*b*EllipticPi[(2*a)/(a + b), (c + d*x)/2,
 2])/(a + b) + 4*Sqrt[Cos[c + d*x]]*Sin[c + d*x] - (6*(-2*a*b*EllipticE[Ar
cSin[Sqrt[Cos[c + d*x]]], -1] + 2*b*(a + b)*EllipticF[ArcSin[Sqrt[Cos[c +
d*x]]], -1] + (a^2 - 2*b^2)*EllipticPi[-(a/b), ArcSin[Sqrt[Cos[c + d*x]]],
 -1])*Sin[c + d*x])/(a^2*Sqrt[Sin[c + d*x]^2]))/(6*a*d)
```

3.818.3 Rubi [A] (verified)

Time = 1.38 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.78, number of steps used = 17, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.739$, Rules used = {3042, 4752, 3042, 4340, 27, 3042, 4594, 3042, 4274, 3042, 4258, 3042, 3119, 3120, 4336, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{a+b\sec(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)}dx$$

$$\Big\downarrow \text{4340}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\int -\frac{-b\sec^2(c+dx)-a\sec(c+dx)+3b}{2\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}dx}{3a}+\frac{2\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}\right)$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{\int\frac{-b\sec^2(c+dx)-a\sec(c+dx)+3b}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}dx}{3a}\right)$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{\int\frac{-b\csc\left(c+dx+\frac{\pi}{2}\right)^2-a\csc\left(c+dx+\frac{\pi}{2}\right)+3b}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{3a}\right)$$

$\downarrow$ 4594

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{3b^3\int\frac{\sec^{\frac{3}{2}}(c+dx)}{a+b\sec(c+dx)}dx}{a^2}+\frac{\int\frac{3ab-\left(a^2+3b^2\right)\sec(c+dx)}{\sqrt{\sec(c+dx)}}dx}{a^2}}{3a}\right)$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{3b^3\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}+\frac{\int\frac{3ab+\left(-a^2-3b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2}}{3a}\right)$$

$\downarrow$ 4274

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{3b^3\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}+\frac{3ab\int\frac{1}{\sqrt{\sec(c+dx)}}dx-\left(a^2+3b^2\right)\int\sqrt{\sec(c+dx)}dx}{a^2}}{3a}\right)$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{3b^3\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}+\frac{3ab\int\frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-\left(a^2+3b^2\right)\int\sqrt{\csc(c+dx+}}{a^2}}{3a}\right)$$

$\downarrow$ 4258

3.818. $\int\frac{\cos^{\frac{3}{2}}(c+dx)}{a+b\sec(c+dx)}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{3b^3\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}+\frac{3ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\cos(c+dx)}dx-(a}{a^2}}{3a}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{3b^3\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}+\frac{3ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{}}{3a}\right.$$

$\downarrow$ 3119

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{3b^3\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}+\frac{\frac{6ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-(a^2+3b^2)}{a^2}}{3a}\right.$$

$\downarrow$ 3120

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{3b^3\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}+\frac{\frac{6ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-\frac{2\left(a^2+3b^2\right)}{a^2}}{}}{3a}\right.$$

$\downarrow$ 4336

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{3b^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}(b+a\cos(c+dx))}dx}{a^2}+\frac{\frac{6ab\sqrt{\cos(c+dx)}\sqrt{s}}{}}{}}{3a}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{3b^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(b+a\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2}+\frac{\frac{6ab\sqrt{\cos(}}{}}{}}{}\right.$$

$\downarrow$ 3284

3.818. $\int\frac{\cos^{\frac{3}{2}}(c+dx)}{a+b\sec(c+dx)}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{6b^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticPi}\left(\frac{2a}{a+b},\frac{1}{2}(c+dx),2\right)}{a^2d(a+b)}+\frac{\frac{6ab\sqrt{\cos(c+dx)}\sqrt{\sec(}}{}}{}}{3a}\right.$$

input
```
Int[Cos[c + d*x]^(3/2)/(a + b*Sec[c + d*x]),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(-1/3*(((6*a*b*Sqrt[Cos[c + d*x]]*El
lipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d - (2*(a^2 + 3*b^2)*Sqrt[Cos[
c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d)/a^2 + (6*b^3*Sq
rt[Cos[c + d*x]]*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*
x]])/(a^2*(a + b)*d))/a + (2*Sin[c + d*x])/(3*a*d*Sqrt[Sec[c + d*x]]))
```

3.818.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

3.818. $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{a+b\sec(c+dx)}\,dx$

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4336
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]]   Int[
1/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4340
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[Cot[e + f*x]*((d*Csc[e + f*x])^n/(a*f*n)), x] - Sim
p[1/(a*d*n)   Int[((d*Csc[e + f*x])^(n + 1)/(a + b*Csc[e + f*x]))*Simp[b*n
- a*(n + 1)*Csc[e + f*x] - b*(n + 1)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a
, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1] && IntegerQ[2*n]
```

rule 4594
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Simp[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2)   Int[(d*Csc[e +
f*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Simp[1/a^2   Int[(a*A - (A*b - a
*B)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A,
B, C}, x] && NeQ[a^2 - b^2, 0]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

3.818. $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{a+b\sec(c+dx)}\,dx$

3.818.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 551 vs. $2(184) = 368$.

Time = 7.42 (sec) , antiderivative size = 552, normalized size of antiderivative = 4.93

method	result
default	$-\frac{2\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(4a^3\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4-4a^2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)b-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\right.}{}$

input
```
int(cos(d*x+c)^(3/2)/(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-2/3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4*a^3*cos(1/
2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4-4*a^2*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1
/2*c)*b-2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2*a^3+2*a^2*sin(1/2*d*x+1/
2*c)^2*cos(1/2*d*x+1/2*c)*b+a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*
x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-(sin(1/2*d*x+1/2
*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2
-1)^(1/2)*a^2*b+3*a*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)
^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1
/2)*b^3+3*a^2*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1
/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*E
llipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*a*b^
2+3*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elli
pticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2)))/a^3/(a-b)/(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c
)^2-1)^(1/2)/d
```

3.818.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{a+b\sec(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)^(3/2)/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

output
```
Timed out
```

3.818.6 Sympy [F]

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{a+b\sec(c+dx)}\,dx=\int \frac{\cos^{\frac{3}{2}}(c+dx)}{a+b\sec(c+dx)}\,dx$$

input
```
integrate(cos(d*x+c)**(3/2)/(a+b*sec(d*x+c)),x)
```

output
```
Integral(cos(c + d*x)**(3/2)/(a + b*sec(c + d*x)), x)
```

3.818.7 Maxima [F]

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{a+b\sec(c+dx)}\,dx=\int \frac{\cos(dx+c)^{\frac{3}{2}}}{b\sec(dx+c)+a}\,dx$$

input
```
integrate(cos(d*x+c)^(3/2)/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

output
```
integrate(cos(d*x + c)^(3/2)/(b*sec(d*x + c) + a), x)
```

3.818.8 Giac [F]

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{a+b\sec(c+dx)}\,dx=\int \frac{\cos(dx+c)^{\frac{3}{2}}}{b\sec(dx+c)+a}\,dx$$

input
```
integrate(cos(d*x+c)^(3/2)/(a+b*sec(d*x+c)),x, algorithm="giac")
```

output
```
integrate(cos(d*x + c)^(3/2)/(b*sec(d*x + c) + a), x)
```

3.818.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{a+b\sec(c+dx)}\,dx=\int \frac{\cos\left(c+d\,x\right)^{3/2}}{a+\frac{b}{\cos\left(c+d\,x\right)}}\,dx$$

input
```
int(cos(c + d*x)^(3/2)/(a + b/cos(c + d*x)),x)
```

output
```
int(cos(c + d*x)^(3/2)/(a + b/cos(c + d*x)), x)
```

3.819 $\int \frac{\sqrt{\cos(c+dx)}}{a+b\sec(c+dx)}\,dx$

3.819.1 Optimal result 6055
3.819.2 Mathematica [A] (verified) 6055
3.819.3 Rubi [B] (verified) 6056
3.819.4 Maple [A] (verified) 6060
3.819.5 Fricas [F] 6060
3.819.6 Sympy [F] 6060
3.819.7 Maxima [F] 6061
3.819.8 Giac [F] 6061
3.819.9 Mupad [F(-1)] 6061

3.819.1 Optimal result

Integrand size = 23, antiderivative size = 75

$$\int \frac{\sqrt{\cos(c+dx)}}{a+b\sec(c+dx)}\,dx = \frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad} - \frac{2b\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{a^2d} + \frac{2b^2\,\text{EllipticPi}\left(\frac{2a}{a+b},\frac{1}{2}(c+dx),2\right)}{a^2(a+b)d}$$

output
```
2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/
2*c),2^(1/2))/a/d-2*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elli
pticF(sin(1/2*d*x+1/2*c),2^(1/2))/a^2/d+2*b^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)
/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*a/(a+b),2^(1/2))/a^2/(
a+b)/d
```

3.819.2 Mathematica [A] (verified)

Time = 10.68 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.08

$$\int \frac{\sqrt{\cos(c+dx)}}{a+b\sec(c+dx)}\,dx = -\frac{2\left(aE\left(\arcsin\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)-(a+b)\,\text{EllipticF}\left(\arcsin\left(\sqrt{\cos(c+dx)}\right),-1\right)+b\,\text{EllipticPi}\left(\right.\right.}{a^2d\sqrt{\sin^2(c+dx)}}$$

input
```
Integrate[Sqrt[Cos[c + d*x]]/(a + b*Sec[c + d*x]),x]
```

output
```
(-2*(a*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] - (a + b)*EllipticF[ArcSi
n[Sqrt[Cos[c + d*x]]], -1] + b*EllipticPi[-(a/b), ArcSin[Sqrt[Cos[c + d*x]
]], -1])*Sin[c + d*x])/(a^2*d*Sqrt[Sin[c + d*x]^2])
```

3.819.3 Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 156 vs. $2(75) = 150$.

Time = 1.03 (sec) , antiderivative size = 156, normalized size of antiderivative = 2.08, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.609$, Rules used = {3042, 4752, 3042, 4339, 3042, 4274, 3042, 4258, 3042, 3119, 3120, 4336, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\cos(c+dx)}}{a+b\sec(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)}dx$$

$$\Big\downarrow \text{4339}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{b^2\int \frac{\sec^{\frac{3}{2}}(c+dx)}{a+b\sec(c+dx)}dx}{a^2}+\frac{\int \frac{a-b\sec(c+dx)}{\sqrt{\sec(c+dx)}}dx}{a^2}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{b^2\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}+\frac{\int\frac{a-b\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2}\right)$$

$\Big\downarrow$ 4274

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{b^2\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}+\frac{a\int\frac{1}{\sqrt{\sec(c+dx)}}dx-b\int\sqrt{\sec(c+dx)}dx}{a^2}\right)$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{b^2\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}+\frac{a\int\frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-b\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}\right)$$

$\Big\downarrow$ 4258

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{b^2\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}+\frac{a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\cos(c+dx)}dx-b\sqrt{\cos(c}}{a^2}\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{b^2\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}+\frac{a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx-b\sqrt{}}{a^2}\right.$$

$\Big\downarrow$ 3119

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{b^2\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}+\frac{\frac{2a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|2\right)}{d}-b\sqrt{\cos(c+dx)}\sqrt{\sec(c}}{a^2}\right.$$

$\Big\downarrow$ 3120

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{b^2\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}+\frac{\frac{2a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|2\right)}{d}-\frac{2b\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{d}}{a^2}\right.$$

$\Big\downarrow$ 4336

3.819. $\int\frac{\sqrt{\cos(c+dx)}}{a+b\sec(c+dx)}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{b^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}(b+a\cos(c+dx))}dx}{a^2}+\frac{2a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E}{d}\right.$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{b^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(b+a\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2}+\frac{2a\sqrt{\cos(c+dx)}\sqrt{\sec(}}{}\right.$$

$$\Big\downarrow \; 3284$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2b^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticPi}\left(\frac{2a}{a+b},\frac{1}{2}(c+dx),2\right)}{a^2d(a+b)}+\frac{2a\sqrt{\cos(c+dx)}\sqrt{\sec(c+d}}{d}\right.$$

input
```
Int[Sqrt[Cos[c + d*x]]/(a + b*Sec[c + d*x]),x]
```

output
```
Sqrt[Cos[c + d*x]]*(((2*a*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqr
t[Sec[c + d*x]])/d - (2*b*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqr
t[Sec[c + d*x]])/d)/a^2 + (2*b^2*Sqrt[Cos[c + d*x]]*EllipticPi[(2*a)/(a +
b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a^2*(a + b)*d))*Sqrt[Sec[c + d*x]
]
```

3.819.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4336
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]]   Int[
1/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4339
```
Int[1/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_))), x_Symbol] :> Simp[b^2/(a^2*d^2)   Int[(d*Csc[e + f*x])^(3/2)/(a +
b*Csc[e + f*x]), x], x] + Simp[1/a^2   Int[(a - b*Csc[e + f*x])/Sqrt[d*Csc[
e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

3.819. $\int \frac{\sqrt{\cos(c+dx)}}{a+b\sec(c+dx)}\,dx$

3.819.4 Maple [A] (verified)

Time = 6.11 (sec) , antiderivative size = 226, normalized size of antiderivative = 3.01

method	result
default	$\frac{2\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+1}\left(\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)ab-\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\right.\right.\right.}{a^{2}(a-b)\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\sin\left(\right.}$

input
```
int(cos(d*x+c)^(1/2)/(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
2*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*
c),2^(1/2))*a*b-EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b^2+EllipticE(cos(1/
2*d*x+1/2*c),2^(1/2))*a^2-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b+b^2*El
lipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2)))/a^2/(a-b)/(-2*sin(1/2*d*x+
1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2
*c)^2-1)^(1/2)/d
```

3.819.5 Fricas [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{a+b\sec(c+dx)}\,dx=\int \frac{\sqrt{\cos{(dx+c)}}}{b\sec{(dx+c)}+a}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

output
```
integral(sqrt(cos(d*x + c))/(b*sec(d*x + c) + a), x)
```

3.819.6 Sympy [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{a+b\sec(c+dx)}\,dx=\int \frac{\sqrt{\cos{(c+dx)}}}{a+b\sec{(c+dx)}}\,dx$$

input
```
integrate(cos(d*x+c)**(1/2)/(a+b*sec(d*x+c)),x)
```

output
```
Integral(sqrt(cos(c + d*x))/(a + b*sec(c + d*x)), x)
```

3.819.7 Maxima [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{a+b\sec(c+dx)}\,dx = \int \frac{\sqrt{\cos(dx+c)}}{b\sec(dx+c)+a}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

output
```
integrate(sqrt(cos(d*x + c))/(b*sec(d*x + c) + a), x)
```

3.819.8 Giac [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{a+b\sec(c+dx)}\,dx = \int \frac{\sqrt{\cos(dx+c)}}{b\sec(dx+c)+a}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)/(a+b*sec(d*x+c)),x, algorithm="giac")
```

output
```
integrate(sqrt(cos(d*x + c))/(b*sec(d*x + c) + a), x)
```

3.819.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\cos(c+dx)}}{a+b\sec(c+dx)}\,dx = \int \frac{\sqrt{\cos(c+d\,x)}}{a+\frac{b}{\cos(c+d\,x)}}\,dx$$

input
```
int(cos(c + d*x)^(1/2)/(a + b/cos(c + d*x)),x)
```

output
```
int(cos(c + d*x)^(1/2)/(a + b/cos(c + d*x)), x)
```

3.820 $\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\sec(c+dx))}\,dx$

3.820.1 Optimal result . . . 6062
3.820.2 Mathematica [A] (verified) . . . 6062
3.820.3 Rubi [A] (verified) . . . 6063
3.820.4 Maple [A] (verified) . . . 6065
3.820.5 Fricas [F(-1)] . . . 6065
3.820.6 Sympy [F] . . . 6066
3.820.7 Maxima [F] . . . 6066
3.820.8 Giac [F] . . . 6066
3.820.9 Mupad [F(-1)] . . . 6067

3.820.1 Optimal result

Integrand size = 23, antiderivative size = 53

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\sec(c+dx))}\,dx = \frac{2\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{ad} - \frac{2b\,\text{EllipticPi}\left(\frac{2a}{a+b},\frac{1}{2}(c+dx),2\right)}{a(a+b)d}$$

output
```
2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/
2*c),2^(1/2))/a/d-2*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elli
pticPi(sin(1/2*d*x+1/2*c),2*a/(a+b),2^(1/2))/a/(a+b)/d
```

3.820.2 Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.91

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\sec(c+dx))}\,dx = \frac{2\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right) - \frac{2b\,\text{EllipticPi}\left(\frac{2a}{a+b},\frac{1}{2}(c+dx),2\right)}{a+b}}{ad}$$

input
```
Integrate[1/(Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x])),x]
```

output
```
(2*EllipticF[(c + d*x)/2, 2] - (2*b*EllipticPi[(2*a)/(a + b), (c + d*x)/2,
 2])/(a + b))/(a*d)
```

3.820.3 Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {3042, 4752, 3042, 4335, 3042, 3282, 3042, 3120, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\sec(c+dx))}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)}dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sqrt{\sec(c+dx)}}{a+b\sec(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{4335}$$

$$\int \frac{\sqrt{\cos(c+dx)}}{a\cos(c+dx)+b}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}{a\sin\left(c+dx+\frac{\pi}{2}\right)+b}dx$$

$$\Big\downarrow \text{3282}$$

$$\frac{\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{a}-\frac{b\int \frac{1}{\sqrt{\cos(c+dx)}(b+a\cos(c+dx))}dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}-\frac{b\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(b+a\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a}$$

$$\Big\downarrow \text{3120}$$

$$\frac{2\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{ad}-\frac{b\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(b+a\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a}$$

$$\Big\downarrow \text{3284}$$

$$\frac{2\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{ad}-\frac{2b\,\text{EllipticPi}\left(\frac{2a}{a+b},\frac{1}{2}(c+dx),2\right)}{ad(a+b)}$$

input
```
Int[1/(Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x])),x]
```

output
```
(2*EllipticF[(c + d*x)/2, 2])/(a*d) - (2*b*EllipticPi[(2*a)/(a + b), (c +
d*x)/2, 2])/(a*(a + b)*d)
```

3.820.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3282
```
Int[Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]/((a_.) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[d/b   Int[1/Sqrt[c + d*Sin[e + f*x]], x], x
] + Simp[(b*c - a*d)/b   Int[1/((a + b*Sin[e + f*x])*Sqrt[c + d*Sin[e + f*x
]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2
 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 4335
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_)), x_Symbol] :> Simp[Sqrt[d*Sin[e + f*x]]*(Sqrt[d*Csc[e + f*x]]/d)   Int[
Sqrt[d*Sin[e + f*x]]/(b + a*Sin[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f},
 x] && NeQ[a^2 - b^2, 0]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

3.820.4 Maple [A] (verified)

Time = 4.80 (sec) , antiderivative size = 187, normalized size of antiderivative = 3.53

method	result
default	$-\frac{2\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1}\left(\operatorname{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)a-b\operatorname{EllipticF}\left(\cos\left(\frac{dx}{2}\right.\right.\right.}{a(a-b)\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1}\,d}$

input
```
int(1/(a+b*sec(d*x+c))/cos(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*(EllipticF(cos(1/2*d*x+1/2
*c),2^(1/2))*a-b*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+b*EllipticPi(cos(1/
2*d*x+1/2*c),2*a/(a-b),2^(1/2)))/a/(a-b)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*
d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

3.820.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\sec(c+dx))}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+b*sec(d*x+c))/cos(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.820.6 Sympy [F]

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\sec(c+dx))}\,dx = \int \frac{1}{(a+b\sec(c+dx))\sqrt{\cos(c+dx)}}\,dx$$

input
```
integrate(1/(a+b*sec(d*x+c))/cos(d*x+c)**(1/2),x)
```

output
```
Integral(1/((a + b*sec(c + d*x))*sqrt(cos(c + d*x))), x)
```

3.820.7 Maxima [F]

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\sec(c+dx))}\,dx = \int \frac{1}{(b\sec(dx+c)+a)\sqrt{\cos(dx+c)}}\,dx$$

input
```
integrate(1/(a+b*sec(d*x+c))/cos(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*sec(d*x + c) + a)*sqrt(cos(d*x + c))), x)
```

3.820.8 Giac [F]

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\sec(c+dx))}\,dx = \int \frac{1}{(b\sec(dx+c)+a)\sqrt{\cos(dx+c)}}\,dx$$

input
```
integrate(1/(a+b*sec(d*x+c))/cos(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/((b*sec(d*x + c) + a)*sqrt(cos(d*x + c))), x)
```

3.820.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\sec(c+dx))}\,dx=\int \frac{1}{\sqrt{\cos\left(c+d\,x\right)}\,\left(a+\frac{b}{\cos(c+d\,x)}\right)}\,dx$$

input
```
int(1/(cos(c + d*x)^(1/2)*(a + b/cos(c + d*x))),x)
```

output
```
int(1/(cos(c + d*x)^(1/2)*(a + b/cos(c + d*x))), x)
```

3.821 $\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))}\,dx$

3.821.1 Optimal result 6068
3.821.2 Mathematica [A] (verified) 6068
3.821.3 Rubi [A] (verified) 6069
3.821.4 Maple [B] (verified) 6070
3.821.5 Fricas [F(-1)] 6071
3.821.6 Sympy [F] 6071
3.821.7 Maxima [F] 6071
3.821.8 Giac [F] 6072
3.821.9 Mupad [F(-1)] 6072

3.821.1 Optimal result

Integrand size = 23, antiderivative size = 29

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))}\,dx=\frac{2\,\text{EllipticPi}\left(\frac{2a}{a+b},\frac{1}{2}(c+dx),2\right)}{(a+b)d}$$

output
```
2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1
/2*c),2*a/(a+b),2^(1/2))/(a+b)/d
```

3.821.2 Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))}\,dx=\frac{2\,\text{EllipticPi}\left(\frac{2a}{a+b},\frac{1}{2}(c+dx),2\right)}{(a+b)d}$$

input
```
Integrate[1/(Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x])),x]
```

output
```
(2*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/((a + b)*d)
```

3.821.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3042, 4752, 3042, 4336, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)}dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sec^{\frac{3}{2}}(c+dx)}{a+b\sec(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{4336}$$

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a\cos(c+dx)+b)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+b\right)}dx$$

$$\Big\downarrow \text{3284}$$

$$\frac{2\,\text{EllipticPi}\left(\frac{2a}{a+b},\frac{1}{2}(c+dx),2\right)}{d(a+b)}$$

input
```
Int[1/(Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x])),x]
```

output
```
(2*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/((a + b)*d)
```

3.821.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 4336
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]]   Int[
1/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

3.821.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 149 vs. $2(55) = 110$.

Time = 3.99 (sec) , antiderivative size = 150, normalized size of antiderivative = 5.17

method	result	size
default	$\frac{2\sqrt{\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+1}\,\operatorname{EllipticPi}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\frac{2a}{a-b},\sqrt{2}\right)}{(a-b)\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1}\,d}$	150

input
```
int(1/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
2*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*
c),2*a/(a-b),2^(1/2))/(a-b)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

3.821.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))}\,dx = \text{Timed out}$$

input
```
integrate(1/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

output
```
Timed out
```

3.821.6 Sympy [F]

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))}\,dx = \int \frac{1}{(a+b\sec(c+dx))\cos^{\frac{3}{2}}(c+dx)}\,dx$$

input
```
integrate(1/cos(d*x+c)**(3/2)/(a+b*sec(d*x+c)),x)
```

output
```
Integral(1/((a + b*sec(c + d*x))*cos(c + d*x)**(3/2)), x)
```

3.821.7 Maxima [F]

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))}\,dx = \int \frac{1}{(b\sec(dx+c)+a)\cos(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

output
```
integrate(1/((b*sec(d*x + c) + a)*cos(d*x + c)^(3/2)), x)
```

3.821.8 Giac [F]

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))}\,dx = \int \frac{1}{(b\sec(dx+c)+a)\cos(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c)),x, algorithm="giac")
```

output
```
integrate(1/((b*sec(d*x + c) + a)*cos(d*x + c)^(3/2)), x)
```

3.821.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))}\,dx = \int \frac{1}{\cos(c+d\,x)^{3/2}\,\left(a+\frac{b}{\cos(c+d\,x)}\right)}\,dx$$

input
```
int(1/(cos(c + d*x)^(3/2)*(a + b/cos(c + d*x))),x)
```

output
```
int(1/(cos(c + d*x)^(3/2)*(a + b/cos(c + d*x))), x)
```

3.822 $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))}\,dx$

3.822.1 Optimal result 6073
3.822.2 Mathematica [B] (verified) 6073
3.822.3 Rubi [A] (verified) 6074
3.822.4 Maple [B] (verified) 6077
3.822.5 Fricas [F(-1)] 6078
3.822.6 Sympy [F(-1)] 6078
3.822.7 Maxima [F] 6079
3.822.8 Giac [F] 6079
3.822.9 Mupad [F(-1)] 6079

3.822.1 Optimal result

Integrand size = 23, antiderivative size = 77

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))}\,dx = -\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd} - \frac{2a\,\text{EllipticPi}\left(\frac{2a}{a+b},\frac{1}{2}(c+dx),2\right)}{b(a+b)d} + \frac{2\sin(c+dx)}{bd\sqrt{\cos(c+dx)}}$$

output
```
-2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1
/2*c),2^(1/2))/b/d-2*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ell
ipticPi(sin(1/2*d*x+1/2*c),2*a/(a+b),2^(1/2))/b/(a+b)/d+2*sin(d*x+c)/b/d/c
os(d*x+c)^(1/2)
```

3.822.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 195 vs. 2(77) = 154.

Time = 3.14 (sec) , antiderivative size = 195, normalized size of antiderivative = 2.53

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))}\,dx = -\frac{\frac{6a\,\text{EllipticPi}\left(\frac{2a}{a+b},\frac{1}{2}(c+dx),2\right)}{a+b} + \frac{2b\left(2\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)-\frac{2b\,\text{EllipticPi}\left(\frac{2a}{a+b},\frac{1}{2}(c+dx),2\right)}{a+b}\right)}{a} - \frac{4\sin(c+dx)}{\sqrt{\cos(c+dx)}} + \frac{2\left(-2abE\left(\arcsin\left(\sqrt{\cos}\right.\right.\right.}{}}{2bd}$$

input
```
Integrate[1/(Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])),x]
```

output
```
-1/2*((6*a*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(a + b) + (2*b*(2*El
lipticF[(c + d*x)/2, 2] - (2*b*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/
(a + b)))/a - (4*Sin[c + d*x])/Sqrt[Cos[c + d*x]] + (2*(-2*a*b*EllipticE[A
rcSin[Sqrt[Cos[c + d*x]]], -1] + 2*b*(a + b)*EllipticF[ArcSin[Sqrt[Cos[c +
 d*x]]], -1] + (a^2 - 2*b^2)*EllipticPi[-(a/b), ArcSin[Sqrt[Cos[c + d*x]]]
, -1])*Sin[c + d*x])/(a*b*Sqrt[Sin[c + d*x]^2]))/(b*d)
```

3.822.3 Rubi [A] (verified)

Time = 0.92 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.78, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.565$, Rules used = {3042, 4752, 3042, 4337, 3042, 4255, 3042, 4258, 3042, 3119, 4336, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)}dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sec^{\frac{5}{2}}(c+dx)}{a+b\sec(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{4337}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \sec^{\frac{3}{2}}(c+dx)dx}{b}-\frac{a\int \frac{\sec^{\frac{3}{2}}(c+dx)}{a+b\sec(c+dx)}dx}{b}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}dx}{b}-\frac{a\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b}\right)$$

$\Big\downarrow$ 4255

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\int \frac{1}{\sqrt{\sec(c+dx)}}dx}{b}-\frac{a\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b}\right)$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}-\frac{a\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b}\right)$$

$\Big\downarrow$ 4258

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\cos(c+dx)}dx}{b}-\frac{a\int \frac{\csc(c+dx}{a+b\csc(c+}}{b}\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b}-\frac{a\int \frac{\csc(}{a+b}}{}\right.$$

$\Big\downarrow$ 3119

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|2\right)}{d}}{b}-\frac{a\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{b}\right)$$

$\Big\downarrow$ 4336

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|2\right)}{d}}{b}-\frac{a\sqrt{\cos(c+dx)}\sqrt{\sec(c+}}{}\right.$$

$\Big\downarrow$ 3042

3.822. $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))}dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|2\right)}{d}}{b}-\frac{a\sqrt{\cos(c+dx)}\sqrt{\sec(c+}}{}\right.$$

$$\Big\downarrow \; 3284$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|2\right)}{d}}{b}-\frac{2a\sqrt{\cos(c+dx)}\sqrt{\sec(c+}}{}\right.$$

input
```
Int[1/(Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((-2*a*Sqrt[Cos[c + d*x]]*EllipticPi
[(2*a)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(b*(a + b)*d) + ((-2*S
qrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (2*Sqr
t[Sec[c + d*x]]*Sin[c + d*x])/d)/b)
```

3.822.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4336
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]]   Int[
1/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4337
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(5/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[d/b   Int[(d*Csc[e + f*x])^(3/2), x], x] - Simp[a*
(d/b)   Int[(d*Csc[e + f*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a
, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

3.822.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 352 vs. $2(127) = 254$.

Time = 6.53 (sec) , antiderivative size = 353, normalized size of antiderivative = 4.58

method	result
default	$-\frac{2\left(-2\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}(a-b)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2+a\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1}\,\mathrm{EllipticPi}\left(\cos\left(\right.\right.}{}$

input
```
int(1/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-2*(-2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(a-b)*cos(1/2*
d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))*(-2
*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+(-2*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1
/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a-(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/
2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b)/b/(-2*sin
(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(a-b)/sin(1/2*d*x+1/2*c)/(2*
cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

3.822.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))}\,dx = \text{Timed out}$$

input
```
integrate(1/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

output
```
Timed out
```

3.822.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))}\,dx = \text{Timed out}$$

input
```
integrate(1/cos(d*x+c)**(5/2)/(a+b*sec(d*x+c)),x)
```

output
```
Timed out
```

3.822.7 Maxima [F]

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))}\,dx = \int \frac{1}{(b\sec(dx+c)+a)\cos(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

output
```
integrate(1/((b*sec(d*x + c) + a)*cos(d*x + c)^(5/2)), x)
```

3.822.8 Giac [F]

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))}\,dx = \int \frac{1}{(b\sec(dx+c)+a)\cos(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c)),x, algorithm="giac")
```

output
```
integrate(1/((b*sec(d*x + c) + a)*cos(d*x + c)^(5/2)), x)
```

3.822.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))}\,dx = \int \frac{1}{\cos(c+d\,x)^{5/2}\,\left(a+\frac{b}{\cos(c+d\,x)}\right)}\,dx$$

input
```
int(1/(cos(c + d*x)^(5/2)*(a + b/cos(c + d*x))),x)
```

output
```
int(1/(cos(c + d*x)^(5/2)*(a + b/cos(c + d*x))), x)
```

3.823 $\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))}\,dx$

3.823.1 Optimal result 6080
3.823.2 Mathematica [A] (verified) 6081
3.823.3 Rubi [A] (verified) 6081
3.823.4 Maple [B] (verified) 6087
3.823.5 Fricas [F(-1)] 6088
3.823.6 Sympy [F(-1)] 6088
3.823.7 Maxima [F] 6088
3.823.8 Giac [F] 6089
3.823.9 Mupad [F(-1)] 6089

3.823.1 Optimal result

Integrand size = 23, antiderivative size = 128

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))}\,dx = \frac{2aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d} + \frac{2\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3bd} + \frac{2a^2\,\text{EllipticPi}\left(\frac{2a}{a+b},\frac{1}{2}(c+dx),2\right)}{b^2(a+b)d} + \frac{2\sin(c+dx)}{3bd\cos^{\frac{3}{2}}(c+dx)} - \frac{2a\sin(c+dx)}{b^2d\sqrt{\cos(c+dx)}}$$

output
```
2*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+
1/2*c),2^(1/2))/b^2/d+2/3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*
EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/b/d+2*a^2*(cos(1/2*d*x+1/2*c)^2)^(1/
2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*a/(a+b),2^(1/2))/b^2
/(a+b)/d+2/3*sin(d*x+c)/b/d/cos(d*x+c)^(3/2)-2*a*sin(d*x+c)/b^2/d/cos(d*x+
c)^(1/2)
```

3.823.2 Mathematica [A] (verified)

Time = 4.26 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.64

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))}\,dx = \frac{\frac{2(9a^2+2b^2)\,\text{EllipticPi}\left(\frac{2a}{a+b},\frac{1}{2}(c+dx),2\right)}{a+b}+8b\left(2\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)-\frac{2b\,\text{EllipticPi}\left(\frac{2a}{a+b},\frac{1}{2}(c+dx),2\right)}{a+b}\right)+\frac{4(b-3a\cos(c+dx}{\cos^{\frac{3}{2}}(c+}}{}$$

input
```
Integrate[1/(Cos[c + d*x]^(7/2)*(a + b*Sec[c + d*x])),x]
```

output
```
((2*(9*a^2 + 2*b^2)*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(a + b) + 8
*b*(2*EllipticF[(c + d*x)/2, 2] - (2*b*EllipticPi[(2*a)/(a + b), (c + d*x)
/2, 2])/(a + b)) + (4*(b - 3*a*Cos[c + d*x])*Sin[c + d*x])/Cos[c + d*x]^(3
/2) + (6*(-2*a*b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] + 2*b*(a + b)*E
llipticF[ArcSin[Sqrt[Cos[c + d*x]]], -1] + (a^2 - 2*b^2)*EllipticPi[-(a/b)
, ArcSin[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d*x])/(b*Sqrt[Sin[c + d*x]^2]))
/(6*b^2*d)
```

3.823.3 Rubi [A] (verified)

Time = 1.75 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.73, number of steps used = 20, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.870$, Rules used = {3042, 4752, 3042, 4338, 27, 3042, 4590, 27, 3042, 4594, 3042, 4274, 3042, 4258, 3042, 3119, 3120, 4336, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{7/2}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)}dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sec^{\frac{7}{2}}(c+dx)}{a+b\sec(c+dx)}dx$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \; 4338$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\int \frac{\sqrt{\sec(c+dx)}\left(-3a\sec^2(c+dx)+b\sec(c+dx)+a\right)}{2(a+b\sec(c+dx))}dx}{3b}+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3bd}\right)$$

$$\Big\downarrow \; 27$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{\sqrt{\sec(c+dx)}\left(-3a\sec^2(c+dx)+b\sec(c+dx)+a\right)}{a+b\sec(c+dx)}dx}{3b}+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3bd}\right)$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(-3a\csc\left(c+dx+\frac{\pi}{2}\right)^2+b\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{3b}+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3bd}\right)$$

$$\Big\downarrow \; 4590$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2\int \frac{3a^2+4b\sec(c+dx)a+\left(3a^2+b^2\right)\sec^2(c+dx)}{2\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}dx}{b}-\frac{6a\sin(c+dx)\sqrt{\sec(c+dx)}}{bd}}{3b}+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+}{3bd}\right.$$

$$\Big\downarrow \; 27$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\int \frac{3a^2+4b\sec(c+dx)a+\left(3a^2+b^2\right)\sec^2(c+dx)}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}dx}{b}-\frac{6a\sin(c+dx)\sqrt{\sec(c+dx)}}{bd}}{3b}+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+}{3bd}\right.$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\int \frac{3a^2+4b\csc\left(c+dx+\frac{\pi}{2}\right)a+\left(3a^2+b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{b}-\frac{6a\sin(c+dx)\sqrt{\sec(c+dx)}}{bd}}{3b}+\frac{2\sin(c+dx)\sec}{3bd}\right.$$

3.823. $\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))}dx$

$$\Big\downarrow \; 4594$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{3a^2\int \frac{\sec^{\frac{3}{2}}(c+dx)}{a+b\sec(c+dx)}dx+\frac{\int \frac{3a^3+b\sec(c+dx)a^2}{\sqrt{\sec(c+dx)}}dx}{a^2}}{b}-\frac{6a\sin(c+dx)\sqrt{\sec(c+dx)}}{bd}}{3b}+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+\right.$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{3a^2\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{\int \frac{3a^3+b\csc\left(c+dx+\frac{\pi}{2}\right)a^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2}}{b}-\frac{6a\sin(c+dx)\sqrt{\sec(c+dx)}}{bd}}{3b}+\frac{2\sin(c+dx)\sec}{3ba}\right.$$

$$\Big\downarrow \; 4274$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{3a^2\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{3a^3\int \frac{1}{\sqrt{\sec(c+dx)}}dx+a^2b\int \sqrt{\sec(c+dx)}dx}{a^2}}{b}-\frac{6a\sin(c+dx)\sqrt{\sec(c+dx)}}{bd}}{3b}+\frac{2\sin}{}\right.$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{3a^2\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{3a^3\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+a^2b\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}}{b}-\frac{6a\sin(c+dx)\sqrt{\sec(c+dx)}}{bd}}{3b}\right.$$

$$\Big\downarrow \; 4258$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{3a^2\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{3a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\cos(c+dx)}dx+a^2b\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}}{a^2}}{b}}{3b}\right.$$

$$\Big\downarrow \; 3042$$

3.823. $\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{3a^2\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{3a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+a^2b\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{}{\sqrt{\sin\left(c\right.}}}{a^2}}{b}}{3b}\right.$$

$\downarrow$ 3119

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{3a^2\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{a^2b\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{6a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+\right.}{d}}{a^2}}{b}}{3b}\right.$$

$\downarrow$ 3120

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{3a^2\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{\frac{6a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a^2b\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(\right.}{d}}{a^2}}{b}}{3b}\right.$$

$\downarrow$ 4336

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{3a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}(b+a\cos(c+dx))}dx+\frac{\frac{6a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a^2b}{}}{a^2}}{b}}{3b}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{3a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(b+a\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx+\frac{\frac{6a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\right.}{d}}{}}{b}}{3b}\right.$$

$\downarrow$ 3284

3.823. $\int\frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{6a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticPi}\left(\frac{2a}{a+b},\frac{1}{2}(c+dx),2\right)}{d(a+b)}+\frac{6a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a^2b\sqrt{\cos(c+d}}{a^2}}{b}}{3b}\right.$$

input
```
Int[1/(Cos[c + d*x]^(7/2)*(a + b*Sec[c + d*x])),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*Sec[c + d*x]^(3/2)*Sin[c + d*x])
/(3*b*d) + ((((6*a^3*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec
[c + d*x]])/d + (2*a^2*b*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt
[Sec[c + d*x]])/d)/a^2 + (6*a^2*Sqrt[Cos[c + d*x]]*EllipticPi[(2*a)/(a + b
), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/((a + b)*d))/b - (6*a*Sqrt[Sec[c +
d*x]]*Sin[c + d*x])/(b*d))/(3*b))
```

3.823.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4336
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]]   Int[
1/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4338
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-d^3)*Cot[e + f*x]*((d*Csc[e + f*x])^(n - 3)/(b*f*
(n - 2))), x] + Simp[d^3/(b*(n - 2))   Int[(d*Csc[e + f*x])^(n - 3)*(Simp[a
*(n - 3) + b*(n - 3)*Csc[e + f*x] - a*(n - 2)*Csc[e + f*x]^2, x]/(a + b*Csc
[e + f*x])), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && Gt
Q[n, 3]
```

rule 4590
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(-C)*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
)*((d*Csc[e + f*x])^(n - 1)/(b*f*(m + n + 1))), x] + Simp[d/(b*(m + n + 1))
   Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (
A*b*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2
 - b^2, 0] && GtQ[n, 0]
```

rule 4594
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Simp[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2)   Int[(d*Csc[e +
f*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Simp[1/a^2   Int[(a*A - (A*b - a
*B)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A,
B, C}, x] && NeQ[a^2 - b^2, 0]
```

3.823. $\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))}\,dx$

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

3.823.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 422 vs. $2(198) = 396$.

Time = 13.70 (sec) , antiderivative size = 423, normalized size of antiderivative = 3.30

method	result
default	$-\frac{\sqrt{-\left(-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(\frac{-\frac{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}}{3\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-\frac{1}{2}\right)^2}+\frac{2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1}\ \mathrm{Ell}}{3\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\sin\left(\frac{d}{2}\right.}}}{b}\right.}{}$

input
```
int(1/cos(d*x+c)^(7/2)/(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2/b*(-1/6*cos(
1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1
/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/
2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellip
ticF(cos(1/2*d*x+1/2*c),2^(1/2)))-2*a/b^2/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*
d*x+1/2*c)^2-1)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*co
s(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-(sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipti
cE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2))-2*a^3/b^2
/(a^2-a*b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/
(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*
x+1/2*c),2*a/(a-b),2^(1/2)))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)
^(1/2)/d
```

3.823.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))}\,dx = \text{Timed out}$$

input
```
integrate(1/cos(d*x+c)^(7/2)/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

output
```
Timed out
```

3.823.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))}\,dx = \text{Timed out}$$

input
```
integrate(1/cos(d*x+c)**(7/2)/(a+b*sec(d*x+c)),x)
```

output
```
Timed out
```

3.823.7 Maxima [F]

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))}\,dx = \int \frac{1}{(b\sec(dx+c)+a)\cos(dx+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(7/2)/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

output
```
integrate(1/((b*sec(d*x + c) + a)*cos(d*x + c)^(7/2)), x)
```

3.823.8 Giac [F]

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))}\,dx = \int \frac{1}{(b\sec(dx+c)+a)\cos(dx+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(7/2)/(a+b*sec(d*x+c)),x, algorithm="giac")
```

output
```
integrate(1/((b*sec(d*x + c) + a)*cos(d*x + c)^(7/2)), x)
```

3.823.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))}\,dx = \int \frac{1}{\cos(c+d\,x)^{7/2}\,\left(a+\frac{b}{\cos(c+d\,x)}\right)}\,dx$$

input
```
int(1/(cos(c + d*x)^(7/2)*(a + b/cos(c + d*x))),x)
```

output
```
int(1/(cos(c + d*x)^(7/2)*(a + b/cos(c + d*x))), x)
```

3.824 $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

3.824.1 Optimal result 6090
3.824.2 Mathematica [A] (verified) 6091
3.824.3 Rubi [A] (verified) 6091
3.824.4 Maple [B] (verified) 6097
3.824.5 Fricas [F(-1)] 6098
3.824.6 Sympy [F] 6099
3.824.7 Maxima [F] 6099
3.824.8 Giac [F] 6099
3.824.9 Mupad [F(-1)] 6100

3.824.1 Optimal result

Integrand size = 23, antiderivative size = 244

$$\begin{aligned}\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^2}\,dx = &-\frac{b(4a^2-5b^2)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^3\,(a^2-b^2)\,d}\\ &+\frac{(2a^4+16a^2b^2-15b^4)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3a^4\,(a^2-b^2)\,d}\\ &-\frac{b^3(7a^2-5b^2)\,\text{EllipticPi}\left(\frac{2a}{a+b},\frac{1}{2}(c+dx),2\right)}{a^4(a-b)(a+b)^2d}\\ &+\frac{(2a^2-5b^2)\,\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3a^2\,(a^2-b^2)\,d}\\ &+\frac{b^2\sqrt{\cos(c+dx)}\,\sin(c+dx)}{a\,(a^2-b^2)\,d(a+b\sec(c+dx))}\end{aligned}$$

output
```
-b*(4*a^2-5*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE
(sin(1/2*d*x+1/2*c),2^(1/2))/a^3/(a^2-b^2)/d+1/3*(2*a^4+16*a^2*b^2-15*b^4)
*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2
*c),2^(1/2))/a^4/(a^2-b^2)/d-b^3*(7*a^2-5*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2
)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*a/(a+b),2^(1/2))/a^4/
(a-b)/(a+b)^2/d+1/3*(2*a^2-5*b^2)*sin(d*x+c)*cos(d*x+c)^(1/2)/a^2/(a^2-b^2
)/d+b^2*sin(d*x+c)*cos(d*x+c)^(1/2)/a/(a^2-b^2)/d/(a+b*sec(d*x+c))
```

3.824.2 Mathematica [A] (verified)

Time = 2.11 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.09

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$
$$= \frac{4\sqrt{\cos(c+dx)}\left(2+\frac{3b^3}{(-a^2+b^2)(b+a\cos(c+dx))}\right)\sin(c+dx) - \frac{\frac{2\left(-8a^2b+5b^3\right)\text{EllipticPi}\left(\frac{2a}{a+b},\frac{1}{2}(c+dx),2\right)}{a+b}+\frac{8\left(a^2+2b^2\right)\left((a+b)\,\text{EllipticF}\right.}{}}{}}{}$$

input
```
Integrate[Cos[c + d*x]^(3/2)/(a + b*Sec[c + d*x])^2,x]
```

output
```
(4*Sqrt[Cos[c + d*x]]*(2 + (3*b^3)/((-a^2 + b^2)*(b + a*Cos[c + d*x])))*Si
n[c + d*x] - ((2*(-8*a^2*b + 5*b^3)*EllipticPi[(2*a)/(a + b), (c + d*x)/2,
 2])/(a + b) + (8*(a^2 + 2*b^2)*((a + b)*EllipticF[(c + d*x)/2, 2] - b*Ell
ipticPi[(2*a)/(a + b), (c + d*x)/2, 2]))/(a + b) - (6*(4*a^2 - 5*b^2)*(-2*
a*b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] + 2*b*(a + b)*EllipticF[ArcS
in[Sqrt[Cos[c + d*x]]], -1] + (a^2 - 2*b^2)*EllipticPi[-(a/b), ArcSin[Sqrt
[Cos[c + d*x]]], -1])*Sin[c + d*x])/(a^2*Sqrt[Sin[c + d*x]^2]))/((-a + b)*
(a + b)))/(12*a^2*d)
```

3.824.3 Rubi [A] (verified)

Time = 2.15 (sec) , antiderivative size = 310, normalized size of antiderivative = 1.27, number of steps used = 20, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.870$, Rules used = {3042, 4752, 3042, 4334, 27, 3042, 4592, 27, 3042, 4594, 3042, 4274, 3042, 4258, 3042, 3119, 3120, 4336, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx$$
$$\Big\downarrow \; 4752$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2}dx$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \; 4334$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{b^2\sin(c+dx)}{ad\left(a^2-b^2\right)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}-\frac{\int -\frac{2a^2-2b\sec(c+dx)a-5b^2+3b^2\sec^2(c+dx)}{2\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))}}{a\left(a^2-b^2\right)}\right.$$

$$\Big\downarrow \; 27$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{2a^2-2b\sec(c+dx)a-5b^2+3b^2\sec^2(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))}dx}{2a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)}{ad\left(a^2-b^2\right)\sqrt{\sec(c+dx)}(a+b\sec(c+dx)}\right.$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{2a^2-2b\csc\left(c+dx+\frac{\pi}{2}\right)a-5b^2+3b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{2a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)}{ad\left(a^2-b^2\right)\sqrt{\sec(c+dx)}(a+b\sec(c\right.}\right.$$

$$\Big\downarrow \; 4592$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2\left(2a^2-5b^2\right)\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{2\int \frac{-b\left(2a^2-5b^2\right)\sec^2(c+dx)-2a\left(a^2+2b^2\right)\sec(c+dx)+3b\left(4a^2-5b^2\right)}{2\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}dx}{3a}}{2a\left(a^2-b^2\right)}+\frac{}{ad\left(a^2-\right.}\right.$$

$$\Big\downarrow \; 27$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2\left(2a^2-5b^2\right)\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{\int \frac{-b\left(2a^2-5b^2\right)\sec^2(c+dx)-2a\left(a^2+2b^2\right)\sec(c+dx)+3b\left(4a^2-5b^2\right)}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}dx}{3a}}{2a\left(a^2-b^2\right)}+\frac{}{ad\left(a^2-\right.}\right.$$

$$\Big\downarrow \; 3042$$

3.824. $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2(2a^2-5b^2)\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{\int\frac{-b(2a^2-5b^2)\csc(c+dx+\frac{\pi}{2})^2-2a(a^2+2b^2)\csc(c+dx+\frac{\pi}{2})+3b(4a^2-5b^2)}{\sqrt{\csc(c+dx+\frac{\pi}{2})}(a+b\csc(c+dx+\frac{\pi}{2}))}dx}{3a}}{2a(a^2-b^2)}+\frac{}{ad}\right.$$

$\downarrow$ 4594

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2(2a^2-5b^2)\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{3b^3(7a^2-5b^2)\int\frac{\sec^{\frac{3}{2}}(c+dx)}{a+b\sec(c+dx)}dx}{a^2}+\frac{\int\frac{3ab(4a^2-5b^2)-(2a^4+16b^2a^2-15b^4)\sec(c+dx)}{\sqrt{\sec(c+dx)}}d}{a^2}}{3a}}{2a(a^2-b^2)}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2(2a^2-5b^2)\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{3b^3(7a^2-5b^2)\int\frac{\csc(c+dx+\frac{\pi}{2})^{3/2}}{a+b\csc(c+dx+\frac{\pi}{2})}dx}{a^2}+\frac{\int\frac{3ab(4a^2-5b^2)+(-2a^4-16b^2a^2+15b^4)\csc(c}{\sqrt{\csc(c+dx+\frac{\pi}{2})}}}{a^2}}{3a}}{2a(a^2-b^2)}\right.$$

$\downarrow$ 4274

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2(2a^2-5b^2)\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{3b^3(7a^2-5b^2)\int\frac{\csc(c+dx+\frac{\pi}{2})^{3/2}}{a+b\csc(c+dx+\frac{\pi}{2})}dx}{a^2}+\frac{3ab(4a^2-5b^2)\int\frac{1}{\sqrt{\sec(c+dx)}}dx-(2a^4+16a^2b^2}{a^2}}{3a}}{2a(a^2-b^2)}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2(2a^2-5b^2)\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{3b^3(7a^2-5b^2)\int\frac{\csc(c+dx+\frac{\pi}{2})^{3/2}}{a+b\csc(c+dx+\frac{\pi}{2})}dx}{a^2}+\frac{3ab(4a^2-5b^2)\int\frac{1}{\sqrt{\csc(c+dx+\frac{\pi}{2})}}dx-(2a^4+16}{a^2}}{3a}}{2a(a^2-b^2)}\right.$$

$\downarrow$ 4258

3.824. $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2(2a^2-5b^2)\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{3b^3(7a^2-5b^2)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}+\frac{3ab(4a^2-5b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\cos(}}{3a}}{2a(a^2-b^2)}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2(2a^2-5b^2)\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{3b^3(7a^2-5b^2)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}+\frac{3ab(4a^2-5b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin(}}{3a}}{2a(a^2-b^2)}\right.$$

$\downarrow$ 3119

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2(2a^2-5b^2)\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{3b^3(7a^2-5b^2)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}+\frac{\frac{6ab(4a^2-5b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+\right.}{d}}{3a}}{2a(a^2-b^2)}\right.$$

$\downarrow$ 3120

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2(2a^2-5b^2)\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{3b^3(7a^2-5b^2)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}+\frac{\frac{6ab(4a^2-5b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+\right.}{d}}{3a}}{2a(a^2-b^2)}\right.$$

$\downarrow$ 4336

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2(2a^2-5b^2)\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{3b^3(7a^2-5b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}(b+a\cos(c+dx))}dx}{a^2}+\frac{6ab(4a^2-5b^2}{}}{}}{2a(a^2-}\right.$$

$\downarrow$ 3042

3.824. $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2(2a^2-5b^2)\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{3b^3(7a^2-5b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(b+a\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2}+\frac{6ab(}{}}{}}{2a\,(a}\right.$$

$$\Big\downarrow \; 3284$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{b^2\sin(c+dx)}{ad\,(a^2-b^2)\,\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}+\frac{\frac{2(2a^2-5b^2)\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{6b^3(7a^2-5b^2)\sqrt{\cos(}}{}}{}}{}\right.$$

input
```
Int[Cos[c + d*x]^(3/2)/(a + b*Sec[c + d*x])^2,x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((b^2*Sin[c + d*x])/(a*(a^2 - b^2)*d
*Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])) + (-1/3*(((6*a*b*(4*a^2 - 5*b^2)
*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d - (2*(
2*a^4 + 16*a^2*b^2 - 15*b^4)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*
Sqrt[Sec[c + d*x]])/d)/a^2 + (6*b^3*(7*a^2 - 5*b^2)*Sqrt[Cos[c + d*x]]*Ell
ipticPi[(2*a)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a^2*(a + b)*d)
)/a + (2*(2*a^2 - 5*b^2)*Sin[c + d*x])/(3*a*d*Sqrt[Sec[c + d*x]]))/(2*a*(a
^2 - b^2)))
```

3.824.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4334
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*
((d*Csc[e + f*x])^n/(a*f*(m + 1)*(a^2 - b^2))), x] + Simp[1/(a*(m + 1)*(a^2
 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a^2*(m + 1)
 - b^2*(m + n + 1) - a*b*(m + 1)*Csc[e + f*x] + b^2*(m + n + 2)*Csc[e + f*x
]^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0] && LtQ[m,
 -1] && IntegersQ[2*m, 2*n]
```

rule 4336
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]]   Int[
1/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

3.824. $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

rule 4592
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Simp[1/(a*d*n)   Int[(a + b*Csc[e + f*x])^m
*(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*
Csc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

rule 4594
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Simp[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2)   Int[(d*Csc[e +
f*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Simp[1/a^2   Int[(a*A - (A*b - a
*B)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A,
B, C}, x] && NeQ[a^2 - b^2, 0]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

3.824.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1063 vs. $2(314) = 628$.

Time = 15.54 (sec) , antiderivative size = 1064, normalized size of antiderivative = 4.36

method	result	size
default	Expression too large to display	1064

input
```
int(cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
-(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4/3/a^2*(2*sin
(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)
^2+2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipti
cF(cos(1/2*d*x+1/2*c),2^(1/2))-3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(co
s(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2))/(-2*sin(1/2*d*
x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)-4/a^3*(a+b)*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d
*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-EllipticE(cos(1/
2*d*x+1/2*c),2^(1/2)))+2/a^4*b^4*(a^2/b/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*a*cos(1/2*d*x+1/2*c)^2-
a+b)-1/2/(a+b)/b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^
(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1
/2*d*x+1/2*c),2^(1/2))+1/2*a/b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*
cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^
2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2*a/b/(a^2-b^2)*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*
c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-1/2
/b/(a^2-b^2)/(a^2-a*b)*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/
2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellip
ticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))+3/2*b/(a^2-b^2)/(a^2-a*b)*a...
```

3.824.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
Timed out
```

3.824.6 Sympy [F]

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$

input
```
integrate(cos(d*x+c)**(3/2)/(a+b*sec(d*x+c))**2,x)
```

output
```
Integral(cos(c + d*x)**(3/2)/(a + b*sec(c + d*x))**2, x)
```

3.824.7 Maxima [F]

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \int \frac{\cos(dx+c)^{\frac{3}{2}}}{(b\sec(dx+c)+a)^2}\,dx$$

input
```
integrate(cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
integrate(cos(d*x + c)^(3/2)/(b*sec(d*x + c) + a)^2, x)
```

3.824.8 Giac [F]

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \int \frac{\cos(dx+c)^{\frac{3}{2}}}{(b\sec(dx+c)+a)^2}\,dx$$

input
```
integrate(cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate(cos(d*x + c)^(3/2)/(b*sec(d*x + c) + a)^2, x)
```

3.824.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^2}\,dx=\int \frac{\cos\left(c+d\,x\right)^{3/2}}{\left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)^2}\,dx$$

input
```
int(cos(c + d*x)^(3/2)/(a + b/cos(c + d*x))^2,x)
```

output
```
int(cos(c + d*x)^(3/2)/(a + b/cos(c + d*x))^2, x)
```

3.825 $\int \frac{\sqrt{\cos(c+dx)}}{(a+b\sec(c+dx))^2}\,dx$

3.825.1 Optimal result . . . 6101
3.825.2 Mathematica [A] (verified) . . . 6102
3.825.3 Rubi [A] (verified) . . . 6102
3.825.4 Maple [B] (verified) . . . 6107
3.825.5 Fricas [F] . . . 6108
3.825.6 Sympy [F] . . . 6109
3.825.7 Maxima [F] . . . 6109
3.825.8 Giac [F] . . . 6109
3.825.9 Mupad [F(-1)] . . . 6110

3.825.1 Optimal result

Integrand size = 23, antiderivative size = 184

$$\begin{aligned}\int \frac{\sqrt{\cos(c+dx)}}{(a+b\sec(c+dx))^2}\,dx = &\frac{(2a^2-3b^2)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2\,(a^2-b^2)\,d}\\ &-\frac{b(4a^2-3b^2)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{a^3\,(a^2-b^2)\,d}\\ &+\frac{b^2(5a^2-3b^2)\,\text{EllipticPi}\left(\frac{2a}{a+b},\frac{1}{2}(c+dx),2\right)}{a^3(a-b)(a+b)^2d}\\ &+\frac{b^2\sin(c+dx)}{a\,(a^2-b^2)\,d\sqrt{\cos(c+dx)}(a+b\sec(c+dx))}\end{aligned}$$

output
```
(2*a^2-3*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(si
n(1/2*d*x+1/2*c),2^(1/2))/a^2/(a^2-b^2)/d-b*(4*a^2-3*b^2)*(cos(1/2*d*x+1/2
*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a^3/
(a^2-b^2)/d+b^2*(5*a^2-3*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2
*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*a/(a+b),2^(1/2))/a^3/(a-b)/(a+b)^2/d+b
^2*sin(d*x+c)/a/(a^2-b^2)/d/(a+b*sec(d*x+c))/cos(d*x+c)^(1/2)
```

3.825.2 Mathematica [A] (verified)

Time = 2.06 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.37

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+b\sec(c+dx))^2}\,dx = \frac{\frac{4b^2\sqrt{\cos(c+dx)}\sin(c+dx)}{(a^2-b^2)(b+a\cos(c+dx))} + \frac{\frac{2(2a^2-b^2)\,\text{EllipticPi}\left(\frac{2a}{a+b},\frac{1}{2}(c+dx),2\right)}{a+b}+8b\left(-\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+\frac{b\,\text{EllipticPi}\left(\frac{2a}{a+b},\frac{1}{2}(c+dx),2\right)}{a+b}\right)+\frac{2(2a^2-3b^2)(-}{}}{}}{4ad}$$

input
```
Integrate[Sqrt[Cos[c + d*x]]/(a + b*Sec[c + d*x])^2,x]
```

output
```
((4*b^2*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/((a^2 - b^2)*(b + a*Cos[c + d*x])
) + ((2*(2*a^2 - b^2)*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(a + b) +
 8*b*(-EllipticF[(c + d*x)/2, 2] + (b*EllipticPi[(2*a)/(a + b), (c + d*x)/
2, 2])/(a + b)) + (2*(2*a^2 - 3*b^2)*(-2*a*b*EllipticE[ArcSin[Sqrt[Cos[c +
 d*x]]], -1] + 2*b*(a + b)*EllipticF[ArcSin[Sqrt[Cos[c + d*x]]], -1] + (a^
2 - 2*b^2)*EllipticPi[-(a/b), ArcSin[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d*x
])/(a^2*b*Sqrt[Sin[c + d*x]^2]))/((a - b)*(a + b)))/(4*a*d)
```

3.825.3 Rubi [A] (verified)

Time = 1.68 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.38, number of steps used = 17, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.739$, Rules used = {3042, 4752, 3042, 4334, 27, 3042, 4594, 3042, 4274, 3042, 4258, 3042, 3119, 3120, 4336, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+b\sec(c+dx))^2}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \; 4752$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^2}dx$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \; 4334$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{ad\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)}-\frac{\int -\frac{2a^2-2b\sec(c+dx)a-3b^2+b^2\sec^2(c+dx)}{2\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}dx}{a\left(a^2-b^2\right)}\right)$$

$$\Big\downarrow \; 27$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{2a^2-2b\sec(c+dx)a-3b^2+b^2\sec^2(c+dx)}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}dx}{2a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{ad\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)}\right)$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{2a^2-2b\csc\left(c+dx+\frac{\pi}{2}\right)a-3b^2+b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{2a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{ad\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)}\right)$$

$$\Big\downarrow \; 4594$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{b^2\left(5-\frac{3b^2}{a^2}\right)\int \frac{\sec^{\frac{3}{2}}(c+dx)}{a+b\sec(c+dx)}dx+\frac{\int \frac{a\left(2a^2-3b^2\right)-b\left(4a^2-3b^2\right)\sec(c+dx)}{\sqrt{\sec(c+dx)}}dx}{a^2}}{2a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)\sqrt{s}}{ad\left(a^2-b^2\right)(a+b}\right.$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{b^2\left(5-\frac{3b^2}{a^2}\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{\int \frac{a\left(2a^2-3b^2\right)-b\left(4a^2-3b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2}}{2a\left(a^2-b^2\right)}+\frac{b^2\sin(c+}{ad\left(a^2-b^2\right)}\right.$$

$$\Big\downarrow \; 4274$$

3.825. $\int \frac{\sqrt{\cos(c+dx)}}{(a+b\sec(c+dx))^2}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{b^2\left(5-\frac{3b^2}{a^2}\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{a\left(2a^2-3b^2\right)\int\frac{1}{\sqrt{\sec(c+dx)}}dx-b\left(4a^2-3b^2\right)\int\sqrt{\sec(c+dx)}dx}{a^2}}{2a\left(a^2-b^2\right)}+\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{b^2\left(5-\frac{3b^2}{a^2}\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{a\left(2a^2-3b^2\right)\int\frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-b\left(4a^2-3b^2\right)\int\sqrt{\csc(c+dx+}}{a^2}}{2a\left(a^2-b^2\right)}\right.$$

$\downarrow$ 4258

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{b^2\left(5-\frac{3b^2}{a^2}\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{a\left(2a^2-3b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\cos(c+dx)}dx-b\left(4a^2\right.}{a^2}}{2a\left(a^2-b^2\right)}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{b^2\left(5-\frac{3b^2}{a^2}\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{a\left(2a^2-3b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx-b\left(\right.}{a^2}}{2a\left(a^2-b^2\right)}\right.$$

$\downarrow$ 3119

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{b^2\left(5-\frac{3b^2}{a^2}\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{\frac{2a\left(2a^2-3b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-b\left(4a^2-3b^2\right)}{a^2}}{2a\left(a^2-b^2\right)}\right.$$

$\downarrow$ 3120

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{b^2\left(5-\frac{3b^2}{a^2}\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{\frac{2a\left(2a^2-3b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-\frac{2b\left(4a^2-3b^2\right)}{a^2}}{2a\left(a^2-b^2\right)}\right.$$

$\downarrow$ 4336

3.825. $\int\frac{\sqrt{\cos(c+dx)}}{(a+b\sec(c+dx))^2}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{b^2\left(5-\frac{3b^2}{a^2}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}(b+a\cos(c+dx))}dx+\frac{\frac{2a\left(2a^2-3b^2\right)\sqrt{\cos}}{}}{}}{2a\left(a^2-b^2\right)}\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{b^2\left(5-\frac{3b^2}{a^2}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(b+a\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx+\frac{\frac{2a\left(2a^2-\right.}{}}{}}{2a\left(a^2-b\right.}\right.$$

$\Big\downarrow$ 3284

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{ad\left(a^2-b^2\right)(a+b\sec(c+dx))}+\frac{\frac{2b^2\left(5-\frac{3b^2}{a^2}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticPi}\left(\frac{2a}{a+b},\frac{1}{2}(c+dx)\right.}{d(a+b)}}{}\right.$$

input
```
Int[Sqrt[Cos[c + d*x]]/(a + b*Sec[c + d*x])^2,x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((((2*a*(2*a^2 - 3*b^2)*Sqrt[Cos[c +
 d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d - (2*b*(4*a^2 - 3*b
^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d)/a^
2 + (2*b^2*(5 - (3*b^2)/a^2)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*a)/(a + b),
(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/((a + b)*d))/(2*a*(a^2 - b^2)) + (b^2*
Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(a*(a^2 - b^2)*d*(a + b*Sec[c + d*x])))
```

3.825.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4334
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*
((d*Csc[e + f*x])^n/(a*f*(m + 1)*(a^2 - b^2))), x] + Simp[1/(a*(m + 1)*(a^2
 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a^2*(m + 1)
 - b^2*(m + n + 1) - a*b*(m + 1)*Csc[e + f*x] + b^2*(m + n + 2)*Csc[e + f*x
]^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0] && LtQ[m,
 -1] && IntegersQ[2*m, 2*n]
```

rule 4336
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]]   Int[
1/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

3.825. $\int \frac{\sqrt{\cos(c+dx)}}{(a+b\sec(c+dx))^2}\,dx$

rule 4594
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Simp[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2)   Int[(d*Csc[e +
f*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Simp[1/a^2   Int[(a*A - (A*b - a
*B)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A,
B, C}, x] && NeQ[a^2 - b^2, 0]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

3.825.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 808 vs. $2(260) = 520$.

Time = 15.01 (sec) , antiderivative size = 809, normalized size of antiderivative = 4.40

method	result	size
default	Expression too large to display	809

input
```
int(cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
-(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2/a^3/(-2*sin
(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(2*b*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2
))+a*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))-2/a^3*b^3*(a^2/b/(a^2-b^2)*cos
(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*a*
cos(1/2*d*x+1/2*c)^2-a+b)-1/2/(a+b)/b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos
(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^
(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2*a/b/(a^2-b^2)*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2*a/
b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)
/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*
x+1/2*c),2^(1/2))-1/2/b/(a^2-b^2)/(a^2-a*b)*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+
1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))+3/2*b/(a^
2-b^2)/(a^2-a*b)*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1
)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(co
s(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2)))-6/a^2*b^2/(a^2-a*b)*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin
(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2...
```

3.825.5 Fricas [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+b\sec(c+dx))^2}\,dx = \int \frac{\sqrt{\cos{(dx+c)}}}{(b\sec{(dx+c)}+a)^2}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
integral(sqrt(cos(d*x + c))/(b^2*sec(d*x + c)^2 + 2*a*b*sec(d*x + c) + a^2
), x)
```

3.825.6 Sympy [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+b\sec(c+dx))^2}\,dx = \int \frac{\sqrt{\cos{(c+dx)}}}{(a+b\sec{(c+dx)})^2}\,dx$$

input
```
integrate(cos(d*x+c)**(1/2)/(a+b*sec(d*x+c))**2,x)
```

output
```
Integral(sqrt(cos(c + d*x))/(a + b*sec(c + d*x))**2, x)
```

3.825.7 Maxima [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+b\sec(c+dx))^2}\,dx = \int \frac{\sqrt{\cos{(dx+c)}}}{(b\sec{(dx+c)}+a)^2}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(cos(d*x + c))/(b*sec(d*x + c) + a)^2, x)
```

3.825.8 Giac [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+b\sec(c+dx))^2}\,dx = \int \frac{\sqrt{\cos{(dx+c)}}}{(b\sec{(dx+c)}+a)^2}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate(sqrt(cos(d*x + c))/(b*sec(d*x + c) + a)^2, x)
```

3.825.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+b\sec(c+dx))^2}\,dx = \int \frac{\sqrt{\cos\left(c+d\,x\right)}}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^2}\,dx$$

input
```
int(cos(c + d*x)^(1/2)/(a + b/cos(c + d*x))^2,x)
```

output
```
int(cos(c + d*x)^(1/2)/(a + b/cos(c + d*x))^2, x)
```

3.826 $\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^2}\,dx$

3.826.1 Optimal result 6111
3.826.2 Mathematica [A] (verified) 6112
3.826.3 Rubi [A] (verified) 6112
3.826.4 Maple [B] (verified) 6117
3.826.5 Fricas [F(-1)] 6118
3.826.6 Sympy [F] 6118
3.826.7 Maxima [F] 6118
3.826.8 Giac [F] 6119
3.826.9 Mupad [F(-1)] 6119

3.826.1 Optimal result

Integrand size = 23, antiderivative size = 167

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^2}\,dx = \frac{bE\left(\frac{1}{2}(c+dx)\middle|2\right)}{a\left(a^2-b^2\right)d} + \frac{\left(2a^2-b^2\right)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{a^2\left(a^2-b^2\right)d} - \frac{b(3a^2-b^2)\,\text{EllipticPi}\left(\frac{2a}{a+b},\frac{1}{2}(c+dx),2\right)}{a^2(a-b)(a+b)^2d} - \frac{b\sin(c+dx)}{\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}(a+b\sec(c+dx))}$$

output
```
b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/
2*c),2^(1/2))/a/(a^2-b^2)/d+(2*a^2-b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1
/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a^2/(a^2-b^2)/d-b*(3*a
^2-b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2
*d*x+1/2*c),2*a/(a+b),2^(1/2))/a^2/(a-b)/(a+b)^2/d-b*sin(d*x+c)/(a^2-b^2)/
d/(a+b*sec(d*x+c))/cos(d*x+c)^(1/2)
```

3.826.2 Mathematica [A] (verified)

Time = 3.95 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.16

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^2}\,dx$$
$$= \frac{\frac{4b\sqrt{\cos(c+dx)}\sin(c+dx)}{(-a^2+b^2)(b+a\cos(c+dx))} - \frac{8\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right) - \frac{10b\,\text{EllipticPi}\left(\frac{2a}{a+b},\frac{1}{2}(c+dx),2\right)}{a+b} + \frac{2\left(-2abE\left(\arcsin\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+2b(a+b)\,\text{EllipticF}\left(\arcsin\right.\right.}{(-a+b)(a+b)}}{4d}$$

input
```
Integrate[1/(Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x])^2),x]
```

output
```
((4*b*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/((-a^2 + b^2)*(b + a*Cos[c + d*x]))
 - (8*EllipticF[(c + d*x)/2, 2] - (10*b*EllipticPi[(2*a)/(a + b), (c + d*x
)/2, 2])/(a + b) + (2*(-2*a*b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] +
2*b*(a + b)*EllipticF[ArcSin[Sqrt[Cos[c + d*x]]], -1] + (a^2 - 2*b^2)*Elli
pticPi[-(a/b), ArcSin[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d*x])/(a^2*Sqrt[Si
n[c + d*x]^2]))/((-a + b)*(a + b)))/(4*d)
```

3.826.3 Rubi [A] (verified)

Time = 1.52 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.42, number of steps used = 17, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.739$, Rules used = {3042, 4752, 3042, 4330, 27, 3042, 4594, 3042, 4274, 3042, 4258, 3042, 3119, 3120, 4336, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^2}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx$$
$$\Big\downarrow \; 4752$$
$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sqrt{\sec(c+dx)}}{(a+b\sec(c+dx))^2}dx$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \; 4330$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\int -\frac{-b\sec^2(c+dx)+2a\sec(c+dx)+b}{2\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}dx}{a^2-b^2}-\frac{b\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)}\right)$$

$$\Big\downarrow \; 27$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{-b\sec^2(c+dx)+2a\sec(c+dx)+b}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}dx}{2\left(a^2-b^2\right)}-\frac{b\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)}\right)$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{-b\csc\left(c+dx+\frac{\pi}{2}\right)^2+2a\csc\left(c+dx+\frac{\pi}{2}\right)+b}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{2\left(a^2-b^2\right)}-\frac{b\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)}\right)$$

$$\Big\downarrow \; 4594$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\int \frac{ab+\left(2a^2-b^2\right)\sec(c+dx)}{\sqrt{\sec(c+dx)}}dx}{a^2}-\frac{b\left(3a^2-b^2\right)\int \frac{\sec^{\frac{3}{2}}(c+dx)}{a+b\sec(c+dx)}dx}{a^2}}{2\left(a^2-b^2\right)}-\frac{b\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)}\right)$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\int \frac{ab+\left(2a^2-b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2}-\frac{b\left(3a^2-b^2\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}}{2\left(a^2-b^2\right)}-\frac{b\sin(c+dx)\sqrt{\sec(c+d}}{d\left(a^2-b^2\right)\left(a+b\sec(c+d\right.}\right.$$

$$\Big\downarrow \; 4274$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\left(2a^2-b^2\right)\int \sqrt{\sec(c+dx)}dx+ab\int \frac{1}{\sqrt{\sec(c+dx)}}dx}{a^2}-\frac{b\left(3a^2-b^2\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}}{2\left(a^2-b^2\right)}-\frac{b\sin(c+dx)}{d\left(a^2-b^2\right)(a}\right.$$

3.826. $\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^2}\,dx$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{(2a^2-b^2)\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+ab\int\frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2}-\frac{b(3a^2-b^2)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}}{2(a^2-b^2)}-\frac{b\sin(}{d(a^2-}\right.$$

$$\Big\downarrow \; 4258$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{(2a^2-b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx+ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\cos(c+dx)}dx}{a^2}-\frac{b(3a^2}{}}{2(a^2-b^2)}\right.$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{(2a^2-b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}}{2(a^2-b^2)}\right.$$

$$\Big\downarrow \; 3119$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{(2a^2-b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}}{a^2}-\frac{b(3a^2-}{}}{2(a^2-b^2)}\right.$$

$$\Big\downarrow \; 3120$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{2\left(2a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+\frac{2ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}}{a^2}-\frac{b(3a^2-b^2}{}}{2(a^2-b^2)}\right.$$

$$\Big\downarrow \; 4336$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{2\left(2a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+\frac{2ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}}{a^2}-\frac{b(3a^2-b^2}{}}{2(a^2-b^2)}\right.$$

3.826. $\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^2}\,dx$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{2\left(2a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+\frac{2ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}}{a^2}}{2\left(a^2-b^2\right)}-\frac{b\left(3a^2-b^2\right.}{}\right.$$

$$\Big\downarrow \; 3284$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{2\left(2a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+\frac{2ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}}{a^2}}{2\left(a^2-b^2\right)}-\frac{2b\left(3a^2-b\right.}{}\right.$$

input
```
Int[1/(Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x])^2),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((((2*a*b*Sqrt[Cos[c + d*x]]*Ellipti
cE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (2*(2*a^2 - b^2)*Sqrt[Cos[c + d
*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d)/a^2 - (2*b*(3*a^2 -
b^2)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec
[c + d*x]])/(a^2*(a + b)*d))/(2*(a^2 - b^2)) - (b*Sqrt[Sec[c + d*x]]*Sin[c
 + d*x])/((a^2 - b^2)*d*(a + b*Sec[c + d*x])))
```

3.826.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4330
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b)*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m +
1)*((d*Csc[e + f*x])^(n - 1)/(f*(m + 1)*(a^2 - b^2))), x] + Simp[1/((m + 1)
*(a^2 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1)*S
imp[b*d*(n - 1) + a*d*(m + 1)*Csc[e + f*x] - b*d*(m + n + 1)*Csc[e + f*x]^2
, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1
] && LtQ[0, n, 1] && IntegersQ[2*m, 2*n]
```

rule 4336
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]]   Int[
1/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4594
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Simp[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2)   Int[(d*Csc[e +
f*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Simp[1/a^2   Int[(a*A - (A*b - a
*B)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A,
B, C}, x] && NeQ[a^2 - b^2, 0]
```

3.826. $\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^2}\,dx$

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

3.826.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 787 vs. $2(243) = 486$.

Time = 13.51 (sec) , antiderivative size = 788, normalized size of antiderivative = 4.72

method	result	size
default	Expression too large to display	788

input
```
int(1/(a+b*sec(d*x+c))^2/cos(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2/a^2*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+2/
a^2*b^2*(a^2/b/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1
/2*d*x+1/2*c)^2)^(1/2)/(2*a*cos(1/2*d*x+1/2*c)^2-a+b)-1/2/(a+b)/b*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/
2*a/b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(
1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/
2*d*x+1/2*c),2^(1/2))-1/2*a/b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*c
os(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2
)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-1/2/b/(a^2-b^2)/(a^2-a*b)*a^
3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1
/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),
2*a/(a-b),2^(1/2))+3/2*b/(a^2-b^2)/(a^2-a*b)*a*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1
/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2)))+4/a*b/(a^
2-a*b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*
sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/
2*c),2*a/(a-b),2^(1/2)))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^...
```

3.826.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+b*sec(d*x+c))^2/cos(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.826.6 Sympy [F]

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^2}\,dx = \int \frac{1}{(a+b\sec(c+dx))^2\sqrt{\cos(c+dx)}}\,dx$$

input
```
integrate(1/(a+b*sec(d*x+c))**2/cos(d*x+c)**(1/2),x)
```

output
```
Integral(1/((a + b*sec(c + d*x))**2*sqrt(cos(c + d*x))), x)
```

3.826.7 Maxima [F]

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^2}\,dx = \int \frac{1}{(b\sec(dx+c)+a)^2\sqrt{\cos(dx+c)}}\,dx$$

input
```
integrate(1/(a+b*sec(d*x+c))^2/cos(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*sec(d*x + c) + a)^2*sqrt(cos(d*x + c))), x)
```

3.826.8 Giac [F]

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^2}\,dx = \int \frac{1}{(b\sec(dx+c)+a)^2\sqrt{\cos(dx+c)}}\,dx$$

input
```
integrate(1/(a+b*sec(d*x+c))^2/cos(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/((b*sec(d*x + c) + a)^2*sqrt(cos(d*x + c))), x)
```

3.826.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^2}\,dx = \int \frac{1}{\sqrt{\cos(c+d\,x)}\left(a+\frac{b}{\cos(c+d\,x)}\right)^2}\,dx$$

input
```
int(1/(cos(c + d*x)^(1/2)*(a + b/cos(c + d*x))^2),x)
```

output
```
int(1/(cos(c + d*x)^(1/2)*(a + b/cos(c + d*x))^2), x)
```

3.827 $\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2}\,dx$

3.827.1 Optimal result 6120
3.827.2 Mathematica [A] (verified) 6121
3.827.3 Rubi [A] (verified) 6121
3.827.4 Maple [B] (verified) 6126
3.827.5 Fricas [F(-1)] 6127
3.827.6 Sympy [F] 6127
3.827.7 Maxima [F(-1)] 6127
3.827.8 Giac [F] 6128
3.827.9 Mupad [F(-1)] 6128

3.827.1 Optimal result

Integrand size = 23, antiderivative size = 148

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2}\,dx = -\frac{E\left(\frac{1}{2}(c+dx)\middle|2\right)}{(a^2-b^2)\,d} - \frac{b\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{a\,(a^2-b^2)\,d} + \frac{(a^2+b^2)\,\text{EllipticPi}\left(\frac{2a}{a+b},\frac{1}{2}(c+dx),2\right)}{a(a-b)(a+b)^2d} + \frac{a\sin(c+dx)}{(a^2-b^2)\,d\sqrt{\cos(c+dx)}(a+b\sec(c+dx))}$$

output
```
-(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2
*c),2^(1/2))/(a^2-b^2)/d-b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)
*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a/(a^2-b^2)/d+(a^2+b^2)*(cos(1/2*d*
x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*a/(a+
b),2^(1/2))/a/(a-b)/(a+b)^2/d+a*sin(d*x+c)/(a^2-b^2)/d/(a+b*sec(d*x+c))/co
s(d*x+c)^(1/2)
```

3.827.2 Mathematica [A] (verified)

Time = 3.72 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.55

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2}\,dx$$
$$= \frac{\frac{4a\sqrt{\cos(c+dx)}\sin(c+dx)}{(a^2-b^2)(b+a\cos(c+dx))} - \frac{2\left(-\frac{a^2\,\text{EllipticPi}\left(\frac{2a}{a+b},\frac{1}{2}(c+dx),2\right)}{a+b}+2b\left(2\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)-\frac{2b\,\text{EllipticPi}\left(\frac{2a}{a+b},\frac{1}{2}(c+dx),2\right)}{a+b}\right)+\frac{\left(-2abE\left(\arcsin\left(\sqrt{\right.\right.\right.}{a(a-b}}{4d}$$

input
```
Integrate[1/(Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^2),x]
```

output
```
((4*a*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/((a^2 - b^2)*(b + a*Cos[c + d*x]))
- (2*(-((a^2*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(a + b)) + 2*b*(2*
EllipticF[(c + d*x)/2, 2] - (2*b*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2]
)/(a + b)) + ((-2*a*b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] + 2*b*(a +
 b)*EllipticF[ArcSin[Sqrt[Cos[c + d*x]]], -1] + (a^2 - 2*b^2)*EllipticPi[-
(a/b), ArcSin[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d*x])/(b*Sqrt[Sin[c + d*x]
^2])))/(a*(a - b)*(a + b)))/(4*d)
```

3.827.3 Rubi [A] (verified)

Time = 1.45 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.51, number of steps used = 17, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.739$, Rules used = {3042, 4752, 3042, 4331, 27, 3042, 4594, 3042, 4274, 3042, 4258, 3042, 3119, 3120, 4336, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx$$
$$\Big\downarrow \text{4752}$$
$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^2}dx$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \; 4331$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int -\frac{-a\sec^2(c+dx)+2b\sec(c+dx)+a}{2\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}dx}{a^2-b^2}+\frac{a\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)}\right)$$

$$\Big\downarrow \; 27$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{a\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)}-\frac{\int \frac{-a\sec^2(c+dx)+2b\sec(c+dx)+a}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}dx}{2\left(a^2-b^2\right)}\right)$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{a\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)}-\frac{\int \frac{-a\csc\left(c+dx+\frac{\pi}{2}\right)^2+2b\csc\left(c+dx+\frac{\pi}{2}\right)+a}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{2\left(a^2-b^2\right)}\right)$$

$$\Big\downarrow \; 4594$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{a\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)}-\frac{\frac{\int \frac{a^2+b\sec(c+dx)a}{\sqrt{\sec(c+dx)}}dx}{a^2}-\frac{\left(a^2+b^2\right)\int \frac{\sec^{\frac{3}{2}}(c+dx)}{a+b\sec(c+dx)}dx}{a}}{2\left(a^2-b^2\right)}\right)$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{a\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)}-\frac{\frac{\int \frac{a^2+b\csc\left(c+dx+\frac{\pi}{2}\right)a}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2}-\frac{\left(a^2+b^2\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a}}{2\left(a^2-b^2\right)}\right)$$

$$\Big\downarrow \; 4274$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{a\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)}-\frac{\frac{a^2\int \frac{1}{\sqrt{\sec(c+dx)}}dx+ab\int \sqrt{\sec(c+dx)}dx}{a^2}-\frac{\left(a^2+b^2\right)\int \frac{\csc\left(c+dx\right.}{a+b\csc\left(c\right.}}{a}}{2\left(a^2-b^2\right)}\right.$$

3.827. $\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2}\,dx$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{a\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)(a+b\sec(c+dx))}-\frac{\frac{a^2\int\frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+ab\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}-\frac{\left(a^2+b^2\right)\int\frac{\cdots}{a\cdots}}{\cdots}}{2\left(a^2-b^2\right)}\right.$$

$$\Big\downarrow \; 4258$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{a\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)(a+b\sec(c+dx))}-\frac{\frac{a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\cos(c+dx)}dx+ab\sqrt{\cos(c+dx)}\sqrt{\cdots}}{a^2}}{2\left(a^2-\cdots\right.}\right.$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{a\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)(a+b\sec(c+dx))}-\frac{\frac{a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+ab\sqrt{\cos(c+dx\cdots}}{a^2}}{2(a\cdots}\right.$$

$$\Big\downarrow \; 3119$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{a\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)(a+b\sec(c+dx))}-\frac{\frac{ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a^2\sqrt{\cos(c+dx)}\sqrt{\cdots}}{\cdots}}{a^2}}{2\left(a^2-\cdots\right.}\right.$$

$$\Big\downarrow \; 3120$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{a\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)(a+b\sec(c+dx))}-\frac{\frac{\frac{2a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx\cdots}}{d\cdots}}{a^2}}{2\left(a^2-b\cdots\right.}\right.$$

$$\Big\downarrow \; 4336$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{a\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)(a+b\sec(c+dx))}-\frac{\frac{\frac{2a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx\cdots}}{d\cdots}}{a^2}}{\cdots}\right.$$

3.827. $\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2}\,dx$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{a\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)(a+b\sec(c+dx))}-\frac{\frac{\frac{2a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{d}}{a^2}}{}\right.$$

$$\Big\downarrow \; 3284$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{a\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)(a+b\sec(c+dx))}-\frac{\frac{\frac{2a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{d}}{a^2}}{}\right.$$

input
```
Int[1/(Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^2),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(-1/2*(((2*a^2*Sqrt[Cos[c + d*x]]*El
lipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (2*a*b*Sqrt[Cos[c + d*x]]*
EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d)/a^2 - (2*(a^2 + b^2)*Sqrt
[Cos[c + d*x]]*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]
])/(a*(a + b)*d))/(a^2 - b^2) + (a*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/((a^2
- b^2)*d*(a + b*Sec[c + d*x])))
```

3.827.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4331
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[a*d^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
)*((d*Csc[e + f*x])^(n - 2)/(f*(m + 1)*(a^2 - b^2))), x] - Simp[d^2/((m + 1
)*(a^2 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*
(a*(n - 2) + b*(m + 1)*Csc[e + f*x] - a*(m + n)*Csc[e + f*x]^2), x], x] /;
FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && LtQ[1, n, 2
] && IntegersQ[2*m, 2*n]
```

rule 4336
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]]   Int[
1/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4594
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Simp[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2)   Int[(d*Csc[e +
f*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Simp[1/a^2   Int[(a*A - (A*b - a
*B)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A,
B, C}, x] && NeQ[a^2 - b^2, 0]
```

3.827. $\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2}\,dx$

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

3.827.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 706 vs. $2(224) = 448$.

Time = 11.99 (sec) , antiderivative size = 707, normalized size of antiderivative = 4.78

method	result
default	$-\frac{\sqrt{-\left(-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(-\frac{2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1}\,\mathrm{EllipticPi}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\frac{2a}{a-b},\sqrt{2}\right)}{\left(a^2-ab\right)\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}}-\frac{2b\left(\frac{a^2\cos\left(\frac{dx}{2}+\right.}{b\left(a^2\right.}\right.}{}\right.}{}$

input
```
int(1/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
-(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2/(a^2-a*b)*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*
d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a
/(a-b),2^(1/2))-2*b/a*(a^2/b/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+
1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*a*cos(1/2*d*x+1/2*c)^2-a+b)-1/2/(a
+b)/b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*
c),2^(1/2))+1/2*a/b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x
+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*El
lipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2*a/b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/
2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-1/2/b/(a^2-b^2
)/(a^2-a*b)*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(
1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1
/2*d*x+1/2*c),2*a/(a-b),2^(1/2))+3/2*b/(a^2-b^2)/(a^2-a*b)*a*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/
2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

3.827.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(1/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
Timed out
```

3.827.6 Sympy [F]

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2}\,dx = \int \frac{1}{(a+b\sec(c+dx))^2\cos^{\frac{3}{2}}(c+dx)}\,dx$$

input
```
integrate(1/cos(d*x+c)**(3/2)/(a+b*sec(d*x+c))**2,x)
```

output
```
Integral(1/((a + b*sec(c + d*x))**2*cos(c + d*x)**(3/2)), x)
```

3.827.7 Maxima [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(1/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
Timed out
```

3.827.8 Giac [F]

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2}\,dx = \int \frac{1}{(b\sec(dx+c)+a)^2\cos(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate(1/((b*sec(d*x + c) + a)^2*cos(d*x + c)^(3/2)), x)
```

3.827.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2}\,dx = \int \frac{1}{\cos(c+d\,x)^{3/2}\left(a+\frac{b}{\cos(c+d\,x)}\right)^2}\,dx$$

input
```
int(1/(cos(c + d*x)^(3/2)*(a + b/cos(c + d*x))^2),x)
```

output
```
int(1/(cos(c + d*x)^(3/2)*(a + b/cos(c + d*x))^2), x)
```

3.828 $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^2}\,dx$

3.828.1 Optimal result . . . 6129
3.828.2 Mathematica [A] (verified) . . . 6130
3.828.3 Rubi [A] (verified) . . . 6130
3.828.4 Maple [B] (verified) . . . 6135
3.828.5 Fricas [F(-1)] . . . 6136
3.828.6 Sympy [F(-1)] . . . 6136
3.828.7 Maxima [F(-1)] . . . 6137
3.828.8 Giac [F] . . . 6137
3.828.9 Mupad [F(-1)] . . . 6137

3.828.1 Optimal result

Integrand size = 23, antiderivative size = 154

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^2}\,dx = \frac{aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{b\left(a^2-b^2\right)d}+\frac{\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{\left(a^2-b^2\right)d}+\frac{\left(a^2-3b^2\right)\text{EllipticPi}\left(\frac{2a}{a+b},\frac{1}{2}(c+dx),2\right)}{(a-b)b(a+b)^2d}-\frac{a^2\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}(a+b\sec(c+dx))}$$

output
```
a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/
2*c),2^(1/2))/b/(a^2-b^2)/d+(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c
)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/(a^2-b^2)/d+(a^2-3*b^2)*(cos(1/2*d
*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*a/(a
+b),2^(1/2))/(a-b)/b/(a+b)^2/d-a^2*sin(d*x+c)/b/(a^2-b^2)/d/(a+b*sec(d*x+c
))/cos(d*x+c)^(1/2)
```

3.828.2 Mathematica [A] (verified)

Time = 3.81 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.55

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^2}\,dx$$
$$= \frac{\frac{4a^2\sqrt{\cos(c+dx)}\sin(c+dx)}{(-a^2+b^2)(b+a\cos(c+dx))} + \frac{\frac{2\left(3a^2-4b^2\right)\operatorname{EllipticPi}\left(\frac{2a}{a+b},\frac{1}{2}(c+dx),2\right)}{a+b}+4b\left(2\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)-\frac{2b\operatorname{EllipticPi}\left(\frac{2a}{a+b},\frac{1}{2}(c+dx),2\right)}{a+b}\right)+\frac{2\left(-2abE\left(\mathrm{ar}\right.\right.}{(a}}{}}{4bd}$$

input
```
Integrate[1/(Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^2),x]
```

output
```
((4*a^2*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/((-a^2 + b^2)*(b + a*Cos[c + d*x]
)) + ((2*(3*a^2 - 4*b^2)*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(a + b
) + 4*b*(2*EllipticF[(c + d*x)/2, 2] - (2*b*EllipticPi[(2*a)/(a + b), (c +
 d*x)/2, 2])/(a + b)) + (2*(-2*a*b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -
1] + 2*b*(a + b)*EllipticF[ArcSin[Sqrt[Cos[c + d*x]]], -1] + (a^2 - 2*b^2)
*EllipticPi[-(a/b), ArcSin[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d*x])/(b*Sqrt
[Sin[c + d*x]^2]))/((a - b)*(a + b)))/(4*b*d)
```

3.828.3 Rubi [A] (verified)

Time = 1.56 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.51, number of steps used = 17, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.739$, Rules used = {3042, 4752, 3042, 4332, 27, 3042, 4594, 3042, 4274, 3042, 4258, 3042, 3119, 3120, 4336, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^2}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx$$
$$\Big\downarrow \; 4752$$
$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\sec(c+dx))^2}dx$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \; 4332$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\int -\frac{a^2+2b\sec(c+dx)a+\left(a^2-2b^2\right)\sec^2(c+dx)}{2\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}dx}{b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{bd\left(a^2-b^2\right)(a+b\sec(c+dx))}\right)$$

$$\Big\downarrow \; 27$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{a^2+2b\sec(c+dx)a+\left(a^2-2b^2\right)\sec^2(c+dx)}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}dx}{2b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{bd\left(a^2-b^2\right)(a+b\sec(c+dx))}\right)$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{a^2+2b\csc\left(c+dx+\frac{\pi}{2}\right)a+\left(a^2-2b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{2b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{bd\left(a^2-b^2\right)(a+b\sec(c+dx))}\right)$$

$$\Big\downarrow \; 4594$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\left(a^2-3b^2\right)\int \frac{\sec^{\frac{3}{2}}(c+dx)}{a+b\sec(c+dx)}dx+\frac{\int \frac{a^3+b\sec(c+dx)a^2}{\sqrt{\sec(c+dx)}}dx}{a^2}}{2b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{bd\left(a^2-b^2\right)(a+b\sec(c+dx))}\right)$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\left(a^2-3b^2\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{\int \frac{a^3+b\csc\left(c+dx+\frac{\pi}{2}\right)a^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2}}{2b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\sqrt{\sec(c+d}}{bd\left(a^2-b^2\right)(a+b\sec(c+}\right.$$

$$\Big\downarrow \; 4274$$

3.828. $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^2}dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\left(a^2-3b^2\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{a^3\int\frac{1}{\sqrt{\sec(c+dx)}}dx+a^2b\int\sqrt{\sec(c+dx)}dx}{a^2}}{2b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx}{bd\left(a^2-b^2\right)(a}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\left(a^2-3b^2\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{a^3\int\frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+a^2b\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}}{2b\left(a^2-b^2\right)}-\frac{a^2\sin(}{bd\left(a^2-\right.}\right.$$

$\downarrow$ 4258

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\left(a^2-3b^2\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\cos(c+dx)}dx+a^2b\sqrt{\cos(c+dx)}\sqrt{}}{a^2}}{2b\left(a^2-b^2\right)}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\left(a^2-3b^2\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+a^2b\sqrt{\cos(c+d}}{a^2}}{2b\left(a^2-b^2\right)}\right.$$

$\downarrow$ 3119

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\left(a^2-3b^2\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{a^2b\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a^3\sqrt{\cos(c+dx}}{}}{a^2}}{2b\left(a^2-b^2\right)}\right.$$

$\downarrow$ 3120

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\left(a^2-3b^2\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{\frac{2a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a^2b\sqrt{\cos(c+dx)}\sqrt{\sec(c+}}{}}{a^2}}{2b\left(a^2-b^2\right)}\right.$$

$\downarrow$ 4336

3.828. $\int\frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^2}dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\left(a^2-3b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}(b+a\cos(c+dx))}dx+\frac{2a^3\sqrt{\cos(c+dx)}\sqrt{\sec}}{}}{2b\left(a^2-b^2\right)}\right.$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\left(a^2-3b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(b+a\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx+\frac{2a^3\sqrt{\cos(c}}{}}{2b\left(a^2-b^2\right)}\right.$$

$$\Big\downarrow \; 3284$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2\left(a^2-3b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticPi}\left(\frac{2a}{a+b},\frac{1}{2}(c+dx),2\right)}{d(a+b)}+\frac{2a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}}{2b\left(a^2-b^2\right)}\right.$$

input
```
Int[1/(Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^2),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((((2*a^3*Sqrt[Cos[c + d*x]]*Ellipti
cE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (2*a^2*b*Sqrt[Cos[c + d*x]]*Ell
ipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d)/a^2 + (2*(a^2 - 3*b^2)*Sqrt[
Cos[c + d*x]]*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]]
)/((a + b)*d))/(2*b*(a^2 - b^2)) - (a^2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(
b*(a^2 - b^2)*d*(a + b*Sec[c + d*x])))
```

3.828.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4332
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-a^2)*d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(
m + 1)*((d*Csc[e + f*x])^(n - 3)/(b*f*(m + 1)*(a^2 - b^2))), x] + Simp[d^3/
(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])
^(n - 3)*Simp[a^2*(n - 3) + a*b*(m + 1)*Csc[e + f*x] - (a^2*(n - 2) + b^2*(
m + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2
- b^2, 0] && LtQ[m, -1] && (IGtQ[n, 3] || (IntegersQ[n + 1/2, 2*m] && GtQ[n
, 2]))
```

rule 4336
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]]   Int[
1/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

3.828. $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^2}\,dx$

rule 4594
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Simp[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2)   Int[(d*Csc[e +
f*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Simp[1/a^2   Int[(a*A - (A*b - a
*B)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A,
B, C}, x] && NeQ[a^2 - b^2, 0]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

3.828.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 607 vs. $2(230) = 460$.

Time = 10.20 (sec) , antiderivative size = 608, normalized size of antiderivative = 3.95

method	result
default	$-\frac{\sqrt{-\left(-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\left(\frac{2a^2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2}}{b\left(a^2-b^2\right)\left(2a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2-a+b\right)}-\frac{\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1}\ \mathrm{El}}{(a+b)b\sqrt{-2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\mathrm{s}}}\right.}{}$

input
```
int(1/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
-(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*a^2/b/(a^2-b
^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2
)/(2*a*cos(1/2*d*x+1/2*c)^2-a+b)-1/(a+b)/b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-
2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+a/b/(a^2-b^2)*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)
^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-a/b/(
a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-
2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1
/2*c),2^(1/2))-1/b/(a^2-b^2)/(a^2-a*b)*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-
2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))+3*b/(a^2-b^2)/
(a^2-a*b)*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)
/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d
*x+1/2*c),2*a/(a-b),2^(1/2)))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1
)^(1/2)/d
```

3.828.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(1/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
Timed out
```

3.828.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(1/cos(d*x+c)**(5/2)/(a+b*sec(d*x+c))**2,x)
```

output
```
Timed out
```

3.828.7 Maxima [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(1/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
Timed out
```

3.828.8 Giac [F]

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^2}\,dx = \int \frac{1}{(b\sec(dx+c)+a)^2\cos(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate(1/((b*sec(d*x + c) + a)^2*cos(d*x + c)^(5/2)), x)
```

3.828.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^2}\,dx = \int \frac{1}{\cos(c+d\,x)^{5/2}\left(a+\frac{b}{\cos(c+d\,x)}\right)^2}\,dx$$

input
```
int(1/(cos(c + d*x)^(5/2)*(a + b/cos(c + d*x))^2),x)
```

output
```
int(1/(cos(c + d*x)^(5/2)*(a + b/cos(c + d*x))^2), x)
```

3.829 $\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^2}\,dx$

3.829.1 Optimal result . . . 6138
3.829.2 Mathematica [A] (verified) . . . 6139
3.829.3 Rubi [A] (verified) . . . 6139
3.829.4 Maple [B] (verified) . . . 6145
3.829.5 Fricas [F(-1)] . . . 6146
3.829.6 Sympy [F(-1)] . . . 6147
3.829.7 Maxima [F(-1)] . . . 6147
3.829.8 Giac [F] . . . 6147
3.829.9 Mupad [F(-1)] . . . 6148

3.829.1 Optimal result

Integrand size = 23, antiderivative size = 219

$$\begin{aligned}\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^2}\,dx = &-\frac{(3a^2-2b^2)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2\,(a^2-b^2)\,d}\\ &-\frac{a\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{b\,(a^2-b^2)\,d}\\ &-\frac{a(3a^2-5b^2)\,\text{EllipticPi}\left(\frac{2a}{a+b},\frac{1}{2}(c+dx),2\right)}{(a-b)b^2(a+b)^2d}\\ &+\frac{(3a^2-2b^2)\sin(c+dx)}{b^2\,(a^2-b^2)\,d\sqrt{\cos(c+dx)}}\\ &-\frac{a^2\sin(c+dx)}{b\,(a^2-b^2)\,d\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))}\end{aligned}$$

output
```
-(3*a^2-2*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(s
in(1/2*d*x+1/2*c),2^(1/2))/b^2/(a^2-b^2)/d-a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/
cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/b/(a^2-b^2)/d-a*(
3*a^2-5*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(si
n(1/2*d*x+1/2*c),2*a/(a+b),2^(1/2))/(a-b)/b^2/(a+b)^2/d-a^2*sin(d*x+c)/b/(
a^2-b^2)/d/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))+(3*a^2-2*b^2)*sin(d*x+c)/b^2/
(a^2-b^2)/d/cos(d*x+c)^(1/2)
```

3.829.2 Mathematica [A] (verified)

Time = 3.01 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.27

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^2}\,dx$$
$$= \frac{-\frac{\frac{2\left(9a^3-10ab^2\right)\operatorname{EllipticPi}\left(\frac{2a}{a+b},\frac{1}{2}(c+dx),2\right)}{a+b}+\frac{\left(8a^2b-4b^3\right)\left(2\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)-\frac{2b\operatorname{EllipticPi}\left(\frac{2a}{a+b},\frac{1}{2}(c+dx),2\right)}{a+b}\right)}{a}+\frac{2\left(3a^2-2b^2\right)\left(-2abE\left(\arcsin\left(\sqrt{\cos(}\right.\right.\right.}{}}{(a-b)(a+b)}}{}$$

input
```
Integrate[1/(Cos[c + d*x]^(7/2)*(a + b*Sec[c + d*x])^2),x]
```

output
```
(-(((2*(9*a^3 - 10*a*b^2)*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(a +
b) + ((8*a^2*b - 4*b^3)*(2*EllipticF[(c + d*x)/2, 2] - (2*b*EllipticPi[(2*
a)/(a + b), (c + d*x)/2, 2])/(a + b)))/a + (2*(3*a^2 - 2*b^2)*(-2*a*b*Elli
pticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] + 2*b*(a + b)*EllipticF[ArcSin[Sqrt[
Cos[c + d*x]]], -1] + (a^2 - 2*b^2)*EllipticPi[-(a/b), ArcSin[Sqrt[Cos[c +
 d*x]]], -1])*Sin[c + d*x])/(a*b*Sqrt[Sin[c + d*x]^2]))/((a - b)*(a + b)))
 + 4*Sqrt[Cos[c + d*x]]*((a^3*Sin[c + d*x])/((a^2 - b^2)*(b + a*Cos[c + d*
x])) + 2*Tan[c + d*x]))/(4*b^2*d)
```

3.829.3 Rubi [A] (verified)

Time = 2.08 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.31, number of steps used = 20, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.870$, Rules used = {3042, 4752, 3042, 4332, 27, 3042, 4590, 27, 3042, 4594, 3042, 4274, 3042, 4258, 3042, 3119, 3120, 4336, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^2}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{7/2}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx$$
$$\Big\downarrow \; 4752$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+b\sec(c+dx))^2}dx$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx$$

$\downarrow$ 4332

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\int \frac{\sqrt{\sec(c+dx)}\left(a^2-2b\sec(c+dx)a-\left(3a^2-2b^2\right)\sec^2(c+dx)\right)}{2(a+b\sec(c+dx))}dx}{b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{bd\left(a^2-b^2\right)(a+b\sec(c+dx}\right.$$

$\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\int \frac{\sqrt{\sec(c+dx)}\left(a^2-2b\sec(c+dx)a-\left(3a^2-2b^2\right)\sec^2(c+dx)\right)}{a+b\sec(c+dx)}dx}{2b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{bd\left(a^2-b^2\right)(a+b\sec(c+dx}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a^2-2b\csc\left(c+dx+\frac{\pi}{2}\right)a+\left(2b^2-3a^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{2b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\sec}{bd\left(a^2-b^2\right)(a+b\,s}\right.$$

$\downarrow$ 4590

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{2\int \frac{a\left(3a^2-4b^2\right)\sec^2(c+dx)+2b\left(2a^2-b^2\right)\sec(c+dx)+a\left(3a^2-2b^2\right)}{2\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}dx}{b}-\frac{2\left(3a^2-2b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd}}{2b\left(a^2-b^2\right)}-\right.$$

$\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{\int \frac{a\left(3a^2-4b^2\right)\sec^2(c+dx)+2b\left(2a^2-b^2\right)\sec(c+dx)+a\left(3a^2-2b^2\right)}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}dx}{b}-\frac{2\left(3a^2-2b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd}}{2b\left(a^2-b^2\right)}-\right.$$

$\downarrow$ 3042

3.829. $\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^2}dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{\int \frac{a\left(3a^2-4b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2+2b\left(2a^2-b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)+a\left(3a^2-2b^2\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{b}-\frac{2\left(3a^2-2b^2\right)\sin(c+dx)\sqrt{\sec(c+}}{bd}}{2b\left(a^2-b^2\right)}\right.$$

$\downarrow$ 4594

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{a\left(3a^2-5b^2\right)\int \frac{\sec^{\frac{3}{2}}(c+dx)}{a+b\sec(c+dx)}dx+\frac{\int \frac{b\sec(c+dx)a^3+\left(3a^2-2b^2\right)a^2}{\sqrt{\sec(c+dx)}}dx}{a^2}}{b}-\frac{2\left(3a^2-2b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd}}{2b\left(a^2-b^2\right)}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{a\left(3a^2-5b^2\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{\int \frac{b\csc\left(c+dx+\frac{\pi}{2}\right)a^3+\left(3a^2-2b^2\right)a^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2}}{b}-\frac{2\left(3a^2-2b^2\right)\sin(c+dx)\sqrt{\sec(}}{bd}}{2b\left(a^2-b^2\right)}\right.$$

$\downarrow$ 4274

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{a\left(3a^2-5b^2\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{a^3b\int \sqrt{\sec(c+dx)}dx+a^2\left(3a^2-2b^2\right)\int \frac{1}{\sqrt{\sec(c+dx)}}dx}{a^2}}{b}-\frac{2\left(3a^2-2b^2\right)\sin(}{}}{2b\left(a^2-b^2\right)}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{a\left(3a^2-5b^2\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{a^3b\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+a^2\left(3a^2-2b^2\right)\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2}}{b}-\frac{2(3a^2-}{}}{2b\left(a^2-b^2\right)}\right.$$

$\downarrow$ 4258

3.829. $\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^2}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{a(3a^2-5b^2)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{a^3b\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx+a^2\left(3a^2-2b^2\right)\sqrt{\cos(c+dx)}\sqrt{s}}{a^2}}{b}}{2b\left(a^2-b^2\right)}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{a(3a^2-5b^2)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{a^3b\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+a^2\left(3a^2-2b^2\right)\sqrt{\cos(c+dx}}{a^2}}{b}}{2b\left(a^2-b^2\right)}\right.$$

$\downarrow$ 3119

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{a(3a^2-5b^2)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{a^3b\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a^2\left(3a^2-2b^2\right)\sqrt{\cos(c+}}{}}{a^2}}{b}}{2b\left(a^2-b^2\right)}\right.$$

$\downarrow$ 3120

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{a(3a^2-5b^2)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{\frac{2a^3b\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+\frac{2a^2\left(3a^2-2b^2\right)\sqrt{\cos(c}}{}}{a^2}}{b}}{2b\left(a^2-b^2\right)}\right.$$

$\downarrow$ 4336

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{a(3a^2-5b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}(b+a\cos(c+dx))}dx+\frac{\frac{2a^3b\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}}{d}}{}}{b}}{2b\left(a^2\right.}\right.$$

$\downarrow$ 3042

3.829. $\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^2}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{a(3a^2-5b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(b+a\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx+\frac{\frac{2a^3b\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{d}}{}}{b}}{2b(}\right.$$

$$\Big\downarrow \; 3284$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{bd\left(a^2-b^2\right)(a+b\sec(c+dx))}-\frac{\frac{\frac{2a\left(3a^2-5b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticPi}\left(\frac{2a}{a+b},\frac{1}{2}(c+dx),2\right)}{d(a+b)}+}{}}{}\right.$$

input
```
Int[1/(Cos[c + d*x]^(7/2)*(a + b*Sec[c + d*x])^2),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(-((a^2*Sec[c + d*x]^(3/2)*Sin[c + d
*x])/(b*(a^2 - b^2)*d*(a + b*Sec[c + d*x]))) - ((((2*a^2*(3*a^2 - 2*b^2)*S
qrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (2*a^3
*b*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d)/a^2
 + (2*a*(3*a^2 - 5*b^2)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*a)/(a + b), (c +
d*x)/2, 2]*Sqrt[Sec[c + d*x]])/((a + b)*d))/b - (2*(3*a^2 - 2*b^2)*Sqrt[Se
c[c + d*x]]*Sin[c + d*x])/(b*d))/(2*b*(a^2 - b^2)))
```

3.829.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4332
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-a^2)*d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(
m + 1)*((d*Csc[e + f*x])^(n - 3)/(b*f*(m + 1)*(a^2 - b^2))), x] + Simp[d^3/
(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])
^(n - 3)*Simp[a^2*(n - 3) + a*b*(m + 1)*Csc[e + f*x] - (a^2*(n - 2) + b^2*(
m + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2
- b^2, 0] && LtQ[m, -1] && (IGtQ[n, 3] || (IntegersQ[n + 1/2, 2*m] && GtQ[n
, 2]))
```

rule 4336
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]]   Int[
1/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4590
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(-C)*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
)*((d*Csc[e + f*x])^(n - 1)/(b*f*(m + n + 1))), x] + Simp[d/(b*(m + n + 1))
   Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (
A*b*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2
 - b^2, 0] && GtQ[n, 0]
```

3.829. $\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^2}\,dx$

rule 4594
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Simp[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2)   Int[(d*Csc[e +
f*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Simp[1/a^2   Int[(a*A - (A*b - a
*B)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A,
B, C}, x] && NeQ[a^2 - b^2, 0]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

3.829.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 840 vs. $2(293) = 586$.

Time = 16.22 (sec) , antiderivative size = 841, normalized size of antiderivative = 3.84

method	result	size
default	Expression too large to display	841

input
```
int(1/cos(d*x+c)^(7/2)/(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
-(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2/b^2/sin(1/2*
d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d
*x+1/2*c)^2)^(1/2)*(2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-(sin(1/2*d*x
+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*
c)^2-1)^(1/2))-2*a/b*(a^2/b/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*a*cos(1/2*d*x+1/2*c)^2-a+b)-1/2/(a+
b)/b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c
),2^(1/2))+1/2*a/b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+
1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ell
ipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2*a/b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2
*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-1/2/b/(a^2-b^2)
/(a^2-a*b)*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1
/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/
2*d*x+1/2*c),2*a/(a-b),2^(1/2))+3/2*b/(a^2-b^2)/(a^2-a*b)*a*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+
sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2
)))+2*a^2/b^2/(a^2-a*b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c
)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellip...
```

3.829.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(1/cos(d*x+c)^(7/2)/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

output
```
Timed out
```

3.829.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(1/cos(d*x+c)**(7/2)/(a+b*sec(d*x+c))**2,x)
```

output
```
Timed out
```

3.829.7 Maxima [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(1/cos(d*x+c)^(7/2)/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

output
```
Timed out
```

3.829.8 Giac [F]

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^2}\,dx = \int \frac{1}{(b\sec(dx+c)+a)^2\cos(dx+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(7/2)/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate(1/((b*sec(d*x + c) + a)^2*cos(d*x + c)^(7/2)), x)
```

3.829.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^2}\,dx=\int \frac{1}{\cos\left(c+d\,x\right)^{7/2}\left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)^2}\,dx$$

input
```
int(1/(cos(c + d*x)^(7/2)*(a + b/cos(c + d*x))^2),x)
```

output
```
int(1/(cos(c + d*x)^(7/2)*(a + b/cos(c + d*x))^2), x)
```

3.830 $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^3}\,dx$

3.830.1 Optimal result 6149
3.830.2 Mathematica [A] (verified) 6150
3.830.3 Rubi [A] (verified) 6151
3.830.4 Maple [B] (verified) 6158
3.830.5 Fricas [F] 6159
3.830.6 Sympy [F] 6160
3.830.7 Maxima [F] 6160
3.830.8 Giac [F] 6160
3.830.9 Mupad [F(-1)] 6161

3.830.1 Optimal result

Integrand size = 23, antiderivative size = 346

$$\begin{aligned}\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^3}\,dx = &-\frac{b(24a^4-65a^2b^2+35b^4)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4a^4\left(a^2-b^2\right)^2 d}\\ &+\frac{(8a^6+128a^4b^2-223a^2b^4+105b^6)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{12a^5\left(a^2-b^2\right)^2 d}\\ &-\frac{b^3(63a^4-86a^2b^2+35b^4)\,\text{EllipticPi}\left(\frac{2a}{a+b},\frac{1}{2}(c+dx),2\right)}{4a^5(a-b)^2(a+b)^3 d}\\ &+\frac{(8a^4-61a^2b^2+35b^4)\sqrt{\cos(c+dx)}\sin(c+dx)}{12a^3\left(a^2-b^2\right)^2 d}\\ &+\frac{b^2\sqrt{\cos(c+dx)}\sin(c+dx)}{2a\left(a^2-b^2\right)d(a+b\sec(c+dx))^2}\\ &+\frac{b^2(13a^2-7b^2)\sqrt{\cos(c+dx)}\sin(c+dx)}{4a^2\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))}\end{aligned}$$

output
```
-1/4*b*(24*a^4-65*a^2*b^2+35*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x
+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/a^4/(a^2-b^2)^2/d+1/12*(8*a^
6+128*a^4*b^2-223*a^2*b^4+105*b^6)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*
x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a^5/(a^2-b^2)^2/d-1/4*b^3*(
63*a^4-86*a^2*b^2+35*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*
EllipticPi(sin(1/2*d*x+1/2*c),2*a/(a+b),2^(1/2))/a^5/(a-b)^2/(a+b)^3/d+1/1
2*(8*a^4-61*a^2*b^2+35*b^4)*sin(d*x+c)*cos(d*x+c)^(1/2)/a^3/(a^2-b^2)^2/d+
1/2*b^2*sin(d*x+c)*cos(d*x+c)^(1/2)/a/(a^2-b^2)/d/(a+b*sec(d*x+c))^2+1/4*b
^2*(13*a^2-7*b^2)*sin(d*x+c)*cos(d*x+c)^(1/2)/a^2/(a^2-b^2)^2/d/(a+b*sec(d
*x+c))
```

3.830.2 Mathematica [A] (verified)

Time = 4.18 (sec) , antiderivative size = 353, normalized size of antiderivative = 1.02

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^3}\,dx$$
$$= \frac{\frac{4\sqrt{\cos(c+dx)}\left(4a^6-57a^2b^4+35b^6+ab\left(16a^4-83a^2b^2+49b^4\right)\cos(c+dx)+4\left(a^3-ab^2\right)^2\cos(2(c+dx))\right)\sin(c+dx)}{(a^2-b^2)^2(b+a\cos(c+dx))^2} + \frac{-\frac{2\left(56a^4b-73a^2b^3+35b^5\right)\text{Ellipt}}{a+b}}{}}{}$$

input
```
Integrate[Cos[c + d*x]^(3/2)/(a + b*Sec[c + d*x])^3,x]
```

output
```
((4*Sqrt[Cos[c + d*x]]*(4*a^6 - 57*a^2*b^4 + 35*b^6 + a*b*(16*a^4 - 83*a^2
*b^2 + 49*b^4)*Cos[c + d*x] + 4*(a^3 - a*b^2)^2*Cos[2*(c + d*x)])*Sin[c +
d*x])/((a^2 - b^2)^2*(b + a*Cos[c + d*x])^2) + ((-2*(56*a^4*b - 73*a^2*b^3
 + 35*b^5)*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(a + b) + (16*(2*a^4
 + 14*a^2*b^2 - 7*b^4)*((a + b)*EllipticF[(c + d*x)/2, 2] - b*EllipticPi[(
2*a)/(a + b), (c + d*x)/2, 2]))/(a + b) - (6*(24*a^4 - 65*a^2*b^2 + 35*b^4
)*(-2*a*b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] + 2*b*(a + b)*Elliptic
F[ArcSin[Sqrt[Cos[c + d*x]]], -1] + (a^2 - 2*b^2)*EllipticPi[-(a/b), ArcSi
n[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d*x])/(a^2*Sqrt[Sin[c + d*x]^2]))/((a
- b)^2*(a + b)^2))/(48*a^3*d)
```

3.830.3 Rubi [A] (verified)

Time = 3.00 (sec) , antiderivative size = 424, normalized size of antiderivative = 1.23, number of steps used = 23, number of rules used = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {3042, 4752, 3042, 4334, 27, 3042, 4588, 27, 3042, 4592, 27, 3042, 4594, 3042, 4274, 3042, 4258, 3042, 3119, 3120, 4336, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \text{4334}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{b^2\sin(c+dx)}{2ad\left(a^2-b^2\right)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^2}-\frac{\int -\frac{4a^2-4b\sec(c+dx)a-7b^2+5b^2\sec^2(c+}{2\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2}}{2a\left(a^2-b^2\right)}\right.$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{4a^2-4b\sec(c+dx)a-7b^2+5b^2\sec^2(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2}dx}{4a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)}{2ad\left(a^2-b^2\right)\sqrt{\sec(c+dx)}(a+b\sec(c+d}\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{4a^2-4b\csc\left(c+dx+\frac{\pi}{2}\right)a-7b^2+5b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx}{4a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)}{2ad\left(a^2-b^2\right)\sqrt{\sec(c+dx)}(a+b\sec}\right.$$

$$\Big\downarrow \; 4588$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{b^2\left(13a^2-7b^2\right)\sin(c+dx)}{ad(a^2-b^2)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}-\frac{\int -\frac{8a^4-61b^2a^2-4b\left(4a^2-b^2\right)\sec(c+dx)a+35b^4+3b^2\left(13a^2-7b^2\right)\sec^2}{2\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))}}{a(a^2-b^2)}}{4a\left(a^2-b^2\right)}\right.$$

$$\Big\downarrow \; 27$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\int \frac{8a^4-61b^2a^2-4b\left(4a^2-b^2\right)\sec(c+dx)a+35b^4+3b^2\left(13a^2-7b^2\right)\sec^2(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))}dx}{2a(a^2-b^2)}+\frac{b^2\left(13a^2-7b^2\right)\sin(c+dx)}{ad(a^2-b^2)\sqrt{\sec(c+dx)}(a+b\sec}}{4a\left(a^2-b^2\right)}\right.$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\int \frac{8a^4-61b^2a^2-4b\left(4a^2-b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)a+35b^4+3b^2\left(13a^2-7b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{2a(a^2-b^2)}+\frac{b^2\left(13a^2-7b^2\right)\mathrm{si}}{ad(a^2-b^2)\sqrt{\sec(c+dx)}}}{4a\left(a^2-b^2\right)}\right.$$

$$\Big\downarrow \; 4592$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{2\left(8a^4-61a^2b^2+35b^4\right)\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{2\int \frac{-b\left(8a^4-61b^2a^2+35b^4\right)\sec^2(c+dx)-4a\left(2a^4+14b^2a^2-7b^4\right)\sec(c+dx)+3b\left(24a^4\right.}{2\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}}{3a}}{2a(a^2-b^2)}}{4a\left(a^2-b^2\right)}\right.$$

$$\Big\downarrow \; 27$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{2\left(8a^4-61a^2b^2+35b^4\right)\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{\int \frac{-b\left(8a^4-61b^2a^2+35b^4\right)\sec^2(c+dx)-4a\left(2a^4+14b^2a^2-7b^4\right)\sec(c+dx)+3b\left(24a^4-\right.}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}}{3a}}{2a(a^2-b^2)}}{4a\left(a^2-b^2\right)}\right.$$

$$\Big\downarrow \; 3042$$

3.830. $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^3}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{2\left(8a^4-61a^2b^2+35b^4\right)\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{\int\frac{-b\left(8a^4-61b^2a^2+35b^4\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2-4a\left(2a^4+14b^2a^2-7b^4\right)\csc\left(c+dx+\frac{\pi}{2}\right)+}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)}}{3a}}{2a(a^2-b^2)}}{4a\left(a^2-b^2\right)}\right.$$

$\downarrow$ 4594

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{2\left(8a^4-61a^2b^2+35b^4\right)\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{3b^3\left(63a^4-86a^2b^2+35b^4\right)\int\frac{\sec^{\frac{3}{2}}(c+dx)}{a+b\sec(c+dx)}dx}{a^2}+\frac{\int\frac{3ab\left(24a^4-65b^2a^2+35b^4\right)-\left(8a^6-\right.}{\sqrt{\mathrm{se}}}}{3a}}{2a(a^2-b^2)}}{4a\left(a^2-b^2\right)}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{2\left(8a^4-61a^2b^2+35b^4\right)\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{3b^3\left(63a^4-86a^2b^2+35b^4\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}+\frac{\int\frac{3ab\left(24a^4-65b^2a^2+35b^4\right)+(}{}}{3a}}{2a(a^2-b^2)}}{4a\left(a^2-b^2\right)}\right.$$

$\downarrow$ 4274

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{2\left(8a^4-61a^2b^2+35b^4\right)\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{3b^3\left(63a^4-86a^2b^2+35b^4\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}+\frac{3ab\left(24a^4-65a^2b^2+35b^4\right)\int\frac{}{\sqrt{}}}{3a}}{2a(a^2-b^2)}}{4a\left(a^2-\right.}\right.$$

$\downarrow$ 3042

3.830. $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^3}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{2\left(8a^4-61a^2b^2+35b^4\right)\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{3b^3\left(63a^4-86a^2b^2+35b^4\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}+\frac{3ab\left(24a^4-65a^2b^2+35b^4\right)\int\frac{}{\sqrt{}}}{3a}}{2a(a^2-b^2)}}{4a\,(a}\right.$$

$\downarrow$ 4258

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{2\left(8a^4-61a^2b^2+35b^4\right)\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{3b^3\left(63a^4-86a^2b^2+35b^4\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}+\frac{3ab\left(24a^4-65a^2b^2+35b^4\right)\sqrt{\cos}}{}}{}}{2a(a^2-b}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{2\left(8a^4-61a^2b^2+35b^4\right)\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{3b^3\left(63a^4-86a^2b^2+35b^4\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}+\frac{3ab\left(24a^4-65a^2b^2+35b^4\right)\sqrt{\cos}}{}}{}}{2a(a}\right.$$

$\downarrow$ 3119

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{2\left(8a^4-61a^2b^2+35b^4\right)\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{3b^3\left(63a^4-86a^2b^2+35b^4\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}+\frac{\frac{6ab\left(24a^4-65a^2b^2+35b^4\right)\sqrt{\cos}}{}}{}}{}}{2a(a^2-}\right.$$

$\downarrow$ 3120

3.830. $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^3}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2\left(8a^4-61a^2b^2+35b^4\right)\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{3b^3\left(63a^4-86a^2b^2+35b^4\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}+\frac{6ab\left(24a^4-65a^2b^2+35b^4\right)\sqrt{\cos}}{}}{2a(a^2}\right.$$

$\downarrow$ 4336

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2\left(8a^4-61a^2b^2+35b^4\right)\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{3b^3\left(63a^4-86a^2b^2+35b^4\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}(b+a\cos(c+dx))}dx}{a^2}}{}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2\left(8a^4-61a^2b^2+35b^4\right)\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{3b^3\left(63a^4-86a^2b^2+35b^4\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(b+a\sin\left(c+dx\right.\right.}}{a^2}}{}\right.$$

$\downarrow$ 3284

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{b^2\sin(c+dx)}{2ad\left(a^2-b^2\right)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^2}+\frac{b^2\left(13a^2-7b^2\right)\sin(c+dx)}{ad(a^2-b^2)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}+\right.$$

input
```
Int[Cos[c + d*x]^(3/2)/(a + b*Sec[c + d*x])^3,x]
```

3.830. $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^3}\,dx$

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((b^2*Sin[c + d*x])/(2*a*(a^2 - b^2)
*d*Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^2) + ((b^2*(13*a^2 - 7*b^2)*Sin
[c + d*x])/(a*(a^2 - b^2)*d*Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])) + (-1
/3*(((6*a*b*(24*a^4 - 65*a^2*b^2 + 35*b^4)*Sqrt[Cos[c + d*x]]*EllipticE[(c
 + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d - (2*(8*a^6 + 128*a^4*b^2 - 223*a^2*b^
4 + 105*b^6)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x
]])/d)/a^2 + (6*b^3*(63*a^4 - 86*a^2*b^2 + 35*b^4)*Sqrt[Cos[c + d*x]]*Elli
pticPi[(2*a)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a^2*(a + b)*d))
/a + (2*(8*a^4 - 61*a^2*b^2 + 35*b^4)*Sin[c + d*x])/(3*a*d*Sqrt[Sec[c + d*
x]]))/(2*a*(a^2 - b^2)))/(4*a*(a^2 - b^2)))
```

3.830.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

3.830. $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^3}\,dx$

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4334
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*
((d*Csc[e + f*x])^n/(a*f*(m + 1)*(a^2 - b^2))), x] + Simp[1/(a*(m + 1)*(a^2
 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a^2*(m + 1)
 - b^2*(m + n + 1) - a*b*(m + 1)*Csc[e + f*x] + b^2*(m + n + 2)*Csc[e + f*x
]^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0] && LtQ[m,
 -1] && IntegersQ[2*m, 2*n]
```

rule 4336
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]]   Int[
1/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4588
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc
[e + f*x])^(m + 1)*((d*Csc[e + f*x])^n/(a*f*(m + 1)*(a^2 - b^2))), x] + Sim
p[1/(a*(m + 1)*(a^2 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f
*x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n +
1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m
+ n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x
] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

rule 4592
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Simp[1/(a*d*n)   Int[(a + b*Csc[e + f*x])^m
*(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*
Csc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

3.830. $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^3}\,dx$

rule 4594
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Simp[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2)   Int[(d*Csc[e +
f*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Simp[1/a^2   Int[(a*A - (A*b - a
*B)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A,
B, C}, x] && NeQ[a^2 - b^2, 0]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

3.830.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2215 vs. $2(406) = 812$.

Time = 33.71 (sec) , antiderivative size = 2216, normalized size of antiderivative = 6.40

method	result	size
default	Expression too large to display	2216

input
```
int(cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
-(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4/3/a^3*(2*sin
(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)
^2+2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipti
cF(cos(1/2*d*x+1/2*c),2^(1/2))-3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(co
s(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2))/(-2*sin(1/2*d*
x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)-2/a^4*(2*a+3*b)*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1
/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-EllipticE(co
s(1/2*d*x+1/2*c),2^(1/2)))+2*(a^2+3*a*b+6*b^2)/a^5*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d
*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-2/a^5*b^5*(1/2*a^
2/b/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*
c)^2)^(1/2)/(2*a*cos(1/2*d*x+1/2*c)^2-a+b)^2+3/4*a^2*(a^2-3*b^2)/b^2/(a^2-
b^2)^2*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(
1/2)/(2*a*cos(1/2*d*x+1/2*c)^2-a+b)-3/8/(a+b)/(a^2-b^2)/b^2*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+
sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^2-1/4/
(a+b)/(a^2-b^2)/b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)
^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(
1/2*d*x+1/2*c),2^(1/2))*a+7/8/(a+b)/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1...
```

3.830.5 Fricas [F]

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \int \frac{\cos\left(dx+c\right)^{\frac{3}{2}}}{\left(b\sec\left(dx+c\right)+a\right)^3}\,dx$$

input
```
integrate(cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
integral(cos(d*x + c)^(3/2)/(b^3*sec(d*x + c)^3 + 3*a*b^2*sec(d*x + c)^2 +
 3*a^2*b*sec(d*x + c) + a^3), x)
```

3.830.6 Sympy [F]

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^3}\,dx$$

input
```
integrate(cos(d*x+c)**(3/2)/(a+b*sec(d*x+c))**3,x)
```

output
```
Integral(cos(c + d*x)**(3/2)/(a + b*sec(c + d*x))**3, x)
```

3.830.7 Maxima [F]

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \int \frac{\cos(dx+c)^{\frac{3}{2}}}{(b\sec(dx+c)+a)^3}\,dx$$

input
```
integrate(cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
integrate(cos(d*x + c)^(3/2)/(b*sec(d*x + c) + a)^3, x)
```

3.830.8 Giac [F]

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \int \frac{\cos(dx+c)^{\frac{3}{2}}}{(b\sec(dx+c)+a)^3}\,dx$$

input
```
integrate(cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate(cos(d*x + c)^(3/2)/(b*sec(d*x + c) + a)^3, x)
```

3.830.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^3}\,dx=\int \frac{\cos\left(c+d\,x\right)^{3/2}}{\left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)^3}\,dx$$

input
```
int(cos(c + d*x)^(3/2)/(a + b/cos(c + d*x))^3,x)
```

output
```
int(cos(c + d*x)^(3/2)/(a + b/cos(c + d*x))^3, x)
```

3.831 $\int \frac{\sqrt{\cos(c+dx)}}{(a+b\sec(c+dx))^3}\,dx$

3.831.1 Optimal result . . . 6162
3.831.2 Mathematica [A] (verified) . . . 6163
3.831.3 Rubi [A] (verified) . . . 6163
3.831.4 Maple [B] (verified) . . . 6169
3.831.5 Fricas [F(-1)] . . . 6170
3.831.6 Sympy [F] . . . 6171
3.831.7 Maxima [F] . . . 6171
3.831.8 Giac [F] . . . 6171
3.831.9 Mupad [F(-1)] . . . 6172

3.831.1 Optimal result

Integrand size = 23, antiderivative size = 282

$$\begin{aligned}\int \frac{\sqrt{\cos(c+dx)}}{(a+b\sec(c+dx))^3}\,dx = {} & \frac{(8a^4-29a^2b^2+15b^4)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4a^3\left(a^2-b^2\right)^2 d} \\ & - \frac{3b(8a^4-11a^2b^2+5b^4)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{4a^4\left(a^2-b^2\right)^2 d} \\ & + \frac{b^2(35a^4-38a^2b^2+15b^4)\,\text{EllipticPi}\left(\frac{2a}{a+b},\frac{1}{2}(c+dx),2\right)}{4a^4(a-b)^2(a+b)^3 d} \\ & + \frac{b^2\sin(c+dx)}{2a\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^2} \\ & + \frac{b^2(11a^2-5b^2)\sin(c+dx)}{4a^2\left(a^2-b^2\right)^2 d\sqrt{\cos(c+dx)}(a+b\sec(c+dx))}\end{aligned}$$

output
```
1/4*(8*a^4-29*a^2*b^2+15*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2
*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/a^3/(a^2-b^2)^2/d-3/4*b*(8*a^4-1
1*a^2*b^2+5*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF
(sin(1/2*d*x+1/2*c),2^(1/2))/a^4/(a^2-b^2)^2/d+1/4*b^2*(35*a^4-38*a^2*b^2+
15*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2
*d*x+1/2*c),2*a/(a+b),2^(1/2))/a^4/(a-b)^2/(a+b)^3/d+1/2*b^2*sin(d*x+c)/a/
(a^2-b^2)/d/(a+b*sec(d*x+c))^2/cos(d*x+c)^(1/2)+1/4*b^2*(11*a^2-5*b^2)*sin
(d*x+c)/a^2/(a^2-b^2)^2/d/(a+b*sec(d*x+c))/cos(d*x+c)^(1/2)
```

3.831.2 Mathematica [A] (verified)

Time = 3.22 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.10

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+b\sec(c+dx))^3}\,dx$$
$$= \frac{\frac{2b^2\sqrt{\cos(c+dx)}\left(11a^2b-5b^3+a\left(13a^2-7b^2\right)\cos(c+dx)\right)\sin(c+dx)}{\left(a^2-b^2\right)^2(b+a\cos(c+dx))^2} + \frac{\frac{\left(8a^4-7a^2b^2+5b^4\right)\text{EllipticPi}\left(\frac{2a}{a+b},\frac{1}{2}(c+dx),2\right)}{a+b} - \frac{8\left(4a^2b-b^3\right)\left((a+b)\,\text{EllipticF}\left(\frac{1}{2}\right.\right.}{}}{}$$

input
```
Integrate[Sqrt[Cos[c + d*x]]/(a + b*Sec[c + d*x])^3,x]
```

output
```
((2*b^2*Sqrt[Cos[c + d*x]]*(11*a^2*b - 5*b^3 + a*(13*a^2 - 7*b^2)*Cos[c +
d*x])*Sin[c + d*x])/((a^2 - b^2)^2*(b + a*Cos[c + d*x])^2) + (((8*a^4 - 7*
a^2*b^2 + 5*b^4)*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(a + b) - (8*(
4*a^2*b - b^3)*((a + b)*EllipticF[(c + d*x)/2, 2] - b*EllipticPi[(2*a)/(a
+ b), (c + d*x)/2, 2]))/(a + b) + ((8*a^4 - 29*a^2*b^2 + 15*b^4)*(-2*a*b*E
llipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] + 2*b*(a + b)*EllipticF[ArcSin[Sq
rt[Cos[c + d*x]]], -1] + (a^2 - 2*b^2)*EllipticPi[-(a/b), ArcSin[Sqrt[Cos[
c + d*x]]], -1])*Sin[c + d*x])/(a^2*b*Sqrt[Sin[c + d*x]^2]))/((a - b)^2*(a
 + b)^2))/(8*a^2*d)
```

3.831.3 Rubi [A] (verified)

Time = 2.50 (sec) , antiderivative size = 363, normalized size of antiderivative = 1.29, number of steps used = 20, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.870$, Rules used = {3042, 4752, 3042, 4334, 27, 3042, 4588, 27, 3042, 4594, 3042, 4274, 3042, 4258, 3042, 3119, 3120, 4336, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+b\sec(c+dx))^3}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^3}dx$$
$$\Big\downarrow \; 4752$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \text{4334}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{2ad\left(a^2-b^2\right)(a+b\sec(c+dx))^2}-\frac{\int -\frac{4a^2-4b\sec(c+dx)a-5b^2+3b^2\sec^2(c+dx)}{2\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^2}dx}{2a\left(a^2-b^2\right)}\right)$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{4a^2-4b\sec(c+dx)a-5b^2+3b^2\sec^2(c+dx)}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^2}dx}{4a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{2ad\left(a^2-b^2\right)(a+b\sec(c+dx))^2}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{4a^2-4b\csc\left(c+dx+\frac{\pi}{2}\right)a-5b^2+3b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx}{4a\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{2ad\left(a^2-b^2\right)(a+b\sec(c+dx))^2}\right)$$

$$\Big\downarrow \text{4588}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{b^2\left(11a^2-5b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad(a^2-b^2)(a+b\sec(c+dx))}-\frac{\int -\frac{8a^4-29b^2a^2-4b\left(4a^2-b^2\right)\sec(c+dx)a+15b^4+b^2\left(11a^2-5b^2\right)\sec^2(c+}{2\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}}{a(a^2-b^2)}}{4a\left(a^2-b^2\right)}\right.$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\int \frac{8a^4-29b^2a^2-4b\left(4a^2-b^2\right)\sec(c+dx)a+15b^4+b^2\left(11a^2-5b^2\right)\sec^2(c+dx)}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}dx}{2a(a^2-b^2)}+\frac{b^2\left(11a^2-5b^2\right)\sin(c+dx)\sqrt{\sec(c-}}{ad(a^2-b^2)(a+b\sec(c+dx))}}{4a\left(a^2-b^2\right)}\right.$$

$$\Big\downarrow \text{3042}$$

3.831. $\int \frac{\sqrt{\cos(c+dx)}}{(a+b\sec(c+dx))^3}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\int \frac{8a^4-29b^2a^2-4b\left(4a^2-b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)a+15b^4+b^2\left(11a^2-5b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{2a(a^2-b^2)}+\frac{b^2\left(11a^2-5b^2\right)\sin(c+dx)}{ad(a^2-b^2)(a+b\sec}}{4a\left(a^2-b^2\right)}\right.$$

$\downarrow$ 4594

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{b^2\left(35a^4-38a^2b^2+15b^4\right)\int \frac{\sec^{\frac{3}{2}}(c+dx)}{a+b\sec(c+dx)}dx}{a^2}+\frac{\int \frac{a\left(8a^4-29b^2a^2+15b^4\right)-3b\left(8a^4-11b^2a^2+5b^4\right)\sec(c+dx)}{\sqrt{\sec(c+dx)}}dx}{a^2}}{2a(a^2-b^2)}+\frac{b^2(1}{}}{4a\left(a^2-b^2\right)}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{b^2\left(35a^4-38a^2b^2+15b^4\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}+\frac{\int \frac{a\left(8a^4-29b^2a^2+15b^4\right)-3b\left(8a^4-11b^2a^2+5b^4\right)\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2}}{2a(a^2-b^2)}}{4a\left(a^2-b^2\right)}\right.$$

$\downarrow$ 4274

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{b^2\left(35a^4-38a^2b^2+15b^4\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}+\frac{a\left(8a^4-29a^2b^2+15b^4\right)\int \frac{1}{\sqrt{\sec(c+dx)}}dx-3b\left(8a^4-11a^2b^2+5b^4\right)\int \sqrt{}}{a^2}}{2a(a^2-b^2)}}{4a\left(a^2-b^2\right)}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{b^2\left(35a^4-38a^2b^2+15b^4\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}+\frac{a\left(8a^4-29a^2b^2+15b^4\right)\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-3b\left(8a^4-11a^2b^2+5b^4\right.}{a^2}}{2a(a^2-b^2)}}{4a\left(a^2-b^2\right)}\right.$$

$\downarrow$ 4258

3.831. $\int \frac{\sqrt{\cos(c+dx)}}{(a+b\sec(c+dx))^3}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{b^2\left(35a^4-38a^2b^2+15b^4\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}+\frac{a\left(8a^4-29a^2b^2+15b^4\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\cos(c+dx)}dx-3b}{a^2}}{2a(a^2-b^2)}}{4a\left(a^2-b\right.}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{b^2\left(35a^4-38a^2b^2+15b^4\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}+\frac{a\left(8a^4-29a^2b^2+15b^4\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{}}{2a(a^2-b^2)}}{4a\left(a^2\right.}\right.$$

$\downarrow$ 3119

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{b^2\left(35a^4-38a^2b^2+15b^4\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}+\frac{\frac{2a\left(8a^4-29a^2b^2+15b^4\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-3b}{a}}{2a(a^2-b^2)}}{4a\left(a^2-\right.}\right.$$

$\downarrow$ 3120

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{b^2\left(35a^4-38a^2b^2+15b^4\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}+\frac{\frac{2a\left(8a^4-29a^2b^2+15b^4\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-\frac{6}{}}{a}}{2a(a^2-b^2)}}{4a\left(a^2-\right.}\right.$$

$\downarrow$ 4336

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{b^2\left(35a^4-38a^2b^2+15b^4\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}(b+a\cos(c+dx))}dx}{a^2}+\frac{\frac{2a\left(8a^4-29a^2b^2+15b^4\right)\sqrt{\cos(c+d}}{d}}{}}{2a(a^2-b^2)}}{}\right.$$

$\downarrow$ 3042

3.831. $\int\frac{\sqrt{\cos(c+dx)}}{(a+b\sec(c+dx))^3}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{b^2\left(35a^4-38a^2b^2+15b^4\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(b+a\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2}+\frac{2a\left(8a^4-29a^2b^2+15b^4\right)}{}}{2a(a^2-b^2}}{}\right.$$

$$\Big\downarrow \; 3284$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{2ad\left(a^2-b^2\right)(a+b\sec(c+dx))^2}+\frac{\frac{b^2\left(11a^2-5b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad(a^2-b^2)(a+b\sec(c+dx))}+\frac{\frac{2b^2\left(35a^4-38a^2b^2+1\right.}{}}{}}{}\right.$$

input
```
Int[Sqrt[Cos[c + d*x]]/(a + b*Sec[c + d*x])^3,x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((b^2*Sqrt[Sec[c + d*x]]*Sin[c + d*x
])/(2*a*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^2) + ((((2*a*(8*a^4 - 29*a^2*b^
2 + 15*b^4)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]
])/d - (6*b*(8*a^4 - 11*a^2*b^2 + 5*b^4)*Sqrt[Cos[c + d*x]]*EllipticF[(c +
 d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d)/a^2 + (2*b^2*(35*a^4 - 38*a^2*b^2 + 15*
b^4)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec
[c + d*x]])/(a^2*(a + b)*d))/(2*a*(a^2 - b^2)) + (b^2*(11*a^2 - 5*b^2)*Sqr
t[Sec[c + d*x]]*Sin[c + d*x])/(a*(a^2 - b^2)*d*(a + b*Sec[c + d*x])))/(4*a
*(a^2 - b^2)))
```

3.831.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4334
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*
((d*Csc[e + f*x])^n/(a*f*(m + 1)*(a^2 - b^2))), x] + Simp[1/(a*(m + 1)*(a^2
 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a^2*(m + 1)
 - b^2*(m + n + 1) - a*b*(m + 1)*Csc[e + f*x] + b^2*(m + n + 2)*Csc[e + f*x
]^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0] && LtQ[m,
 -1] && IntegersQ[2*m, 2*n]
```

rule 4336
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]]   Int[
1/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

3.831. $\int \frac{\sqrt{\cos(c+dx)}}{(a+b\sec(c+dx))^3}\,dx$

rule 4588
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc
[e + f*x])^(m + 1)*((d*Csc[e + f*x])^n/(a*f*(m + 1)*(a^2 - b^2))), x] + Sim
p[1/(a*(m + 1)*(a^2 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f
*x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n +
1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m
+ n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x
] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

rule 4594
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Simp[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2)   Int[(d*Csc[e +
f*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Simp[1/a^2   Int[(a*A - (A*b - a
*B)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A,
B, C}, x] && NeQ[a^2 - b^2, 0]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

3.831.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1956 vs. $2(346) = 692$.

Time = 30.59 (sec) , antiderivative size = 1957, normalized size of antiderivative = 6.94

method	result	size
default	Expression too large to display	1957

input
```
int(cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
-(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2/a^4/(-2*sin
(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(
1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(3*b*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2
))+a*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))+2/a^4*b^4*(1/2*a^2/b/(a^2-b^2)
*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(
2*a*cos(1/2*d*x+1/2*c)^2-a+b)^2+3/4*a^2*(a^2-3*b^2)/b^2/(a^2-b^2)^2*cos(1/
2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*a*cos
(1/2*d*x+1/2*c)^2-a+b)-3/8/(a+b)/(a^2-b^2)/b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1
/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^2-1/4/(a+b)/(a^2-b^
2)/b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c
),2^(1/2))*a+7/8/(a+b)/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*
d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)
*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3/8*a^3/b^2/(a^2-b^2)^2*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)
^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-9/8*a
/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2
)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d
*x+1/2*c),2^(1/2))-3/8*a^3/b^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)...
```

3.831.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+b\sec(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
Timed out
```

3.831.6 Sympy [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+b\sec(c+dx))^3}\,dx = \int \frac{\sqrt{\cos{(c+dx)}}}{(a+b\sec{(c+dx)})^3}\,dx$$

input
```
integrate(cos(d*x+c)**(1/2)/(a+b*sec(d*x+c))**3,x)
```

output
```
Integral(sqrt(cos(c + d*x))/(a + b*sec(c + d*x))**3, x)
```

3.831.7 Maxima [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+b\sec(c+dx))^3}\,dx = \int \frac{\sqrt{\cos{(dx+c)}}}{(b\sec{(dx+c)}+a)^3}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
integrate(sqrt(cos(d*x + c))/(b*sec(d*x + c) + a)^3, x)
```

3.831.8 Giac [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+b\sec(c+dx))^3}\,dx = \int \frac{\sqrt{\cos{(dx+c)}}}{(b\sec{(dx+c)}+a)^3}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate(sqrt(cos(d*x + c))/(b*sec(d*x + c) + a)^3, x)
```

3.831.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+b\sec(c+dx))^3}\,dx=\int \frac{\sqrt{\cos\left(c+d\,x\right)}}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^3}\,dx$$

input
```
int(cos(c + d*x)^(1/2)/(a + b/cos(c + d*x))^3,x)
```

output
```
int(cos(c + d*x)^(1/2)/(a + b/cos(c + d*x))^3, x)
```

3.832 $\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^3}\,dx$

3.832.1 Optimal result 6173
3.832.2 Mathematica [A] (verified) 6174
3.832.3 Rubi [A] (verified) 6174
3.832.4 Maple [B] (verified) 6180
3.832.5 Fricas [F(-1)] 6181
3.832.6 Sympy [F] 6182
3.832.7 Maxima [F(-1)] 6182
3.832.8 Giac [F] 6182
3.832.9 Mupad [F(-1)] 6183

3.832.1 Optimal result

Integrand size = 23, antiderivative size = 263

$$\begin{aligned}\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^3}\,dx &= \frac{3b(3a^2-b^2)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4a^2\left(a^2-b^2\right)^2 d}+\frac{(8a^4-5a^2b^2+3b^4)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{4a^3\left(a^2-b^2\right)^2 d}\\ &\quad-\frac{3b(5a^4-2a^2b^2+b^4)\,\text{EllipticPi}\left(\frac{2a}{a+b},\frac{1}{2}(c+dx),2\right)}{4a^3(a-b)^2(a+b)^3 d}\\ &\quad-\frac{b\sin(c+dx)}{2\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^2}\\ &\quad-\frac{b(7a^2-b^2)\sin(c+dx)}{4a\left(a^2-b^2\right)^2 d\sqrt{\cos(c+dx)}(a+b\sec(c+dx))}\end{aligned}$$

output
```
3/4*b*(3*a^2-b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elliptic
E(sin(1/2*d*x+1/2*c),2^(1/2))/a^2/(a^2-b^2)^2/d+1/4*(8*a^4-5*a^2*b^2+3*b^4
)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/
2*c),2^(1/2))/a^3/(a^2-b^2)^2/d-3/4*b*(5*a^4-2*a^2*b^2+b^4)*(cos(1/2*d*x+1
/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*a/(a+b),
2^(1/2))/a^3/(a-b)^2/(a+b)^3/d-1/2*b*sin(d*x+c)/(a^2-b^2)/d/(a+b*sec(d*x+c
))^2/cos(d*x+c)^(1/2)-1/4*b*(7*a^2-b^2)*sin(d*x+c)/a/(a^2-b^2)^2/d/(a+b*se
c(d*x+c))/cos(d*x+c)^(1/2)
```

3.832.2 Mathematica [A] (verified)

Time = 3.62 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.09

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^3}\,dx$$
$$= \frac{\frac{4b\sqrt{\cos(c+dx)}\left(-7a^2b+b^3+\left(-9a^3+3ab^2\right)\cos(c+dx)\right)\sin(c+dx)}{\left(a^2-b^2\right)^2(b+a\cos(c+dx))^2} + \frac{-\frac{2\left(5a^2b+b^3\right)\text{EllipticPi}\left(\frac{2a}{a+b},\frac{1}{2}(c+dx),2\right)}{a+b}+\frac{16\left(2a^2+b^2\right)\left((a+b)\,\text{EllipticF}\left(\frac{1}{2}(c+dx\right.\right.}{a+}}{}$$

input
```
Integrate[1/(Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x])^3),x]
```

output
```
((4*b*Sqrt[Cos[c + d*x]]*(-7*a^2*b + b^3 + (-9*a^3 + 3*a*b^2)*Cos[c + d*x]
)*Sin[c + d*x])/((a^2 - b^2)^2*(b + a*Cos[c + d*x])^2) + ((-2*(5*a^2*b + b
^3)*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(a + b) + (16*(2*a^2 + b^2)
*((a + b)*EllipticF[(c + d*x)/2, 2] - b*EllipticPi[(2*a)/(a + b), (c + d*x
)/2, 2]))/(a + b) + (6*(3*a^2 - b^2)*(-2*a*b*EllipticE[ArcSin[Sqrt[Cos[c +
 d*x]]], -1] + 2*b*(a + b)*EllipticF[ArcSin[Sqrt[Cos[c + d*x]]], -1] + (a^
2 - 2*b^2)*EllipticPi[-(a/b), ArcSin[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d*x
])/(a^2*Sqrt[Sin[c + d*x]^2]))/((a - b)^2*(a + b)^2))/(16*a*d)
```

3.832.3 Rubi [A] (verified)

Time = 2.28 (sec) , antiderivative size = 342, normalized size of antiderivative = 1.30, number of steps used = 20, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.870$, Rules used = {3042, 4752, 3042, 4330, 27, 3042, 4588, 27, 3042, 4594, 3042, 4274, 3042, 4258, 3042, 3119, 3120, 4336, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^3}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^3}dx$$
$$\Big\downarrow \; 4752$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sqrt{\sec(c+dx)}}{(a+b\sec(c+dx))^3}dx$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \; 4330$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\int -\frac{-3b\sec^2(c+dx)+4a\sec(c+dx)+b}{2\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^2}dx}{2\left(a^2-b^2\right)}-\frac{b\sin(c+dx)\sqrt{\sec(c+dx)}}{2d\left(a^2-b^2\right)(a+b\sec(c+dx))^2}\right)$$

$$\Big\downarrow \; 27$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{-3b\sec^2(c+dx)+4a\sec(c+dx)+b}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^2}dx}{4\left(a^2-b^2\right)}-\frac{b\sin(c+dx)\sqrt{\sec(c+dx)}}{2d\left(a^2-b^2\right)(a+b\sec(c+dx))^2}\right)$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{-3b\csc\left(c+dx+\frac{\pi}{2}\right)^2+4a\csc\left(c+dx+\frac{\pi}{2}\right)+b}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx}{4\left(a^2-b^2\right)}-\frac{b\sin(c+dx)\sqrt{\sec(c+dx)}}{2d\left(a^2-b^2\right)(a+b\sec(c+dx))^2}\right)$$

$$\Big\downarrow \; 4588$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{-\frac{\int -\frac{-b\left(7a^2-b^2\right)\sec^2(c+dx)+4a\left(2a^2+b^2\right)\sec(c+dx)+3b\left(3a^2-b^2\right)}{2\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}dx}{a\left(a^2-b^2\right)}-\frac{b\left(7a^2-b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad\left(a^2-b^2\right)(a+b\sec(c+dx))}}{4\left(a^2-b^2\right)}-\right.$$

$$\Big\downarrow \; 27$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\int \frac{-b\left(7a^2-b^2\right)\sec^2(c+dx)+4a\left(2a^2+b^2\right)\sec(c+dx)+3b\left(3a^2-b^2\right)}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}dx}{2a\left(a^2-b^2\right)}-\frac{b\left(7a^2-b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad\left(a^2-b^2\right)(a+b\sec(c+dx))}}{4\left(a^2-b^2\right)}-\frac{}{2a}\right.$$

$$\Big\downarrow \; 3042$$

3.832. $\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^3}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\int \frac{-b\left(7a^2-b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2+4a\left(2a^2+b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)+3b\left(3a^2-b^2\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{2a(a^2-b^2)}-\frac{b(7a^2-b^2)\sin(c+dx)\sqrt{\sec(c+dx}}{ad(a^2-b^2)(a+b\sec(c+dx))}}{4\left(a^2-b^2\right)}\right.$$

$\downarrow$ 4594

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{\int \frac{3ab\left(3a^2-b^2\right)+\left(8a^4-5b^2a^2+3b^4\right)\sec(c+dx)}{\sqrt{\sec(c+dx)}}dx}{a^2}-\frac{3b\left(5a^4-2a^2b^2+b^4\right)\int \frac{\sec^{\frac{3}{2}}(c+dx)}{a+b\sec(c+dx)}dx}{a^2}}{2a(a^2-b^2)}-\frac{b(7a^2-b^2)\sin(c+dx}{ad(a^2-b^2)(a+b}}{4\left(a^2-b^2\right)}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{\int \frac{3ab\left(3a^2-b^2\right)+\left(8a^4-5b^2a^2+3b^4\right)\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2}-\frac{3b\left(5a^4-2a^2b^2+b^4\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}}{2a(a^2-b^2)}-\frac{b(7a^2-b^2)}{ad(a^2-}}{4\left(a^2-b^2\right)}\right.$$

$\downarrow$ 4274

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{3ab\left(3a^2-b^2\right)\int \frac{1}{\sqrt{\sec(c+dx)}}dx+\left(8a^4-5a^2b^2+3b^4\right)\int \sqrt{\sec(c+dx)}dx}{a^2}-\frac{3b\left(5a^4-2a^2b^2+b^4\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}}{2a(a^2-b^2)}-}{4\left(a^2-b^2\right)}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{3ab\left(3a^2-b^2\right)\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\left(8a^4-5a^2b^2+3b^4\right)\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2}-\frac{3b\left(5a^4-2a^2b^2+b^4\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3}}{a+b\csc\left(c+dx+\right.}}{a^2}}{2a(a^2-b^2)}}{4\left(a^2-b^2\right)}\right.$$

$\downarrow$ 4258

3.832. $\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^3}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{3ab\left(3a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\cos(c+dx)}dx+\left(8a^4-5a^2b^2+3b^4\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx}{a^2}}{2a(a^2-b^2)}-}{4\left(a^2-b^2\right)}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{3ab\left(3a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\left(8a^4-5a^2b^2+3b^4\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\right.}}}{a^2}}{2a(a^2-b^2)}}{4\left(a^2-b^2\right)}\right.$$

$\downarrow$ 3119

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{\left(8a^4-5a^2b^2+3b^4\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{6ab\left(3a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right.}{d}}{a^2}}{2a(a^2-b^2)}}{4\left(a^2-b^2\right)}\right.$$

$\downarrow$ 3120

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{\frac{6ab\left(3a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2\left(8a^4-5a^2b^2+3b^4\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx)\right.}{d}}{a^2}}{2a(a^2-b^2)}}{4\left(a^2-b^2\right)}\right.$$

$\downarrow$ 4336

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{\frac{6ab\left(3a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2\left(8a^4-5a^2b^2+3b^4\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx)\right.}{d}}{a^2}}{2a(a^2-b^2)}}{4\left(\right.}\right.$$

$\downarrow$ 3042

3.832. $\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^3}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{6ab\left(3a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2\left(8a^4-5a^2b^2+3b^4\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx)\right.}{d}}{a^2}}{2a(a^2-b^2)}\right.$$

$$\Big\downarrow \; 3284$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{6ab\left(3a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2\left(8a^4-5a^2b^2+3b^4\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx)\right.}{d}}{a^2}}{2a(a^2-b^2)}{4\,(a^2}\right.$$

input
```
Int[1/(Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x])^3),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(-1/2*(b*Sqrt[Sec[c + d*x]]*Sin[c +
d*x])/((a^2 - b^2)*d*(a + b*Sec[c + d*x])^2) + ((((6*a*b*(3*a^2 - b^2)*Sqr
t[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (2*(8*a^
4 - 5*a^2*b^2 + 3*b^4)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[S
ec[c + d*x]])/d)/a^2 - (6*b*(5*a^4 - 2*a^2*b^2 + b^4)*Sqrt[Cos[c + d*x]]*E
llipticPi[(2*a)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a^2*(a + b)*
d))/(2*a*(a^2 - b^2)) - (b*(7*a^2 - b^2)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/
(a*(a^2 - b^2)*d*(a + b*Sec[c + d*x])))/(4*(a^2 - b^2)))
```

3.832.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4330
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b)*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m +
1)*((d*Csc[e + f*x])^(n - 1)/(f*(m + 1)*(a^2 - b^2))), x] + Simp[1/((m + 1)
*(a^2 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1)*S
imp[b*d*(n - 1) + a*d*(m + 1)*Csc[e + f*x] - b*d*(m + n + 1)*Csc[e + f*x]^2
, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1
] && LtQ[0, n, 1] && IntegersQ[2*m, 2*n]
```

rule 4336
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]]   Int[
1/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

3.832. $\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^3}\,dx$

rule 4588
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc
[e + f*x])^(m + 1)*((d*Csc[e + f*x])^n/(a*f*(m + 1)*(a^2 - b^2))), x] + Sim
p[1/(a*(m + 1)*(a^2 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f
*x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n +
1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m
+ n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x
] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

rule 4594
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Simp[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2)   Int[(d*Csc[e +
f*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Simp[1/a^2   Int[(a*A - (A*b - a
*B)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A,
B, C}, x] && NeQ[a^2 - b^2, 0]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

3.832.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1935 vs. $2(327) = 654$.

Time = 26.37 (sec) , antiderivative size = 1936, normalized size of antiderivative = 7.36

method	result	size
default	Expression too large to display	1936

input
```
int(1/(a+b*sec(d*x+c))^3/cos(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2/a^3*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-2/
a^3*b^3*(1/2*a^2/b/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+s
in(1/2*d*x+1/2*c)^2)^(1/2)/(2*a*cos(1/2*d*x+1/2*c)^2-a+b)^2+3/4*a^2*(a^2-3
*b^2)/b^2/(a^2-b^2)^2*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*
d*x+1/2*c)^2)^(1/2)/(2*a*cos(1/2*d*x+1/2*c)^2-a+b)-3/8/(a+b)/(a^2-b^2)/b^2
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/
2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^
(1/2))*a^2-1/4/(a+b)/(a^2-b^2)/b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*
d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)
*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a+7/8/(a+b)/(a^2-b^2)*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3/8*a^3
/b^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^
(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1
/2*d*x+1/2*c),2^(1/2))-9/8*a/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*
cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^
2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3/8*a^3/b^2/(a^2-b^2)^2*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2...
```

3.832.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+b*sec(d*x+c))^3/cos(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.832.6 Sympy [F]

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^3}\,dx = \int \frac{1}{(a+b\sec(c+dx))^3\sqrt{\cos(c+dx)}}\,dx$$

input
```
integrate(1/(a+b*sec(d*x+c))**3/cos(d*x+c)**(1/2),x)
```

output
```
Integral(1/((a + b*sec(c + d*x))**3*sqrt(cos(c + d*x))), x)
```

3.832.7 Maxima [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+b*sec(d*x+c))^3/cos(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
Timed out
```

3.832.8 Giac [F]

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^3}\,dx = \int \frac{1}{(b\sec(dx+c)+a)^3\sqrt{\cos(dx+c)}}\,dx$$

input
```
integrate(1/(a+b*sec(d*x+c))^3/cos(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/((b*sec(d*x + c) + a)^3*sqrt(cos(d*x + c))), x)
```

3.832.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^3}\,dx = \int \frac{1}{\sqrt{\cos(c+d\,x)}\left(a+\frac{b}{\cos(c+d\,x)}\right)^3}\,dx$$

input
```
int(1/(cos(c + d*x)^(1/2)*(a + b/cos(c + d*x))^3),x)
```

output
```
int(1/(cos(c + d*x)^(1/2)*(a + b/cos(c + d*x))^3), x)
```

3.833 $\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^3}\,dx$

3.833.1 Optimal result 6184
3.833.2 Mathematica [A] (verified) 6185
3.833.3 Rubi [A] (verified) 6185
3.833.4 Maple [B] (verified) 6191
3.833.5 Fricas [F(-1)] 6192
3.833.6 Sympy [F(-1)] 6193
3.833.7 Maxima [F(-1)] 6193
3.833.8 Giac [F] 6193
3.833.9 Mupad [F(-1)] 6194

3.833.1 Optimal result

Integrand size = 23, antiderivative size = 246

$$\begin{aligned}\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^3}\,dx = &-\frac{(5a^2+b^2)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4a\,(a^2-b^2)^2\,d}\\ &-\frac{b(7a^2-b^2)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{4a^2\,(a^2-b^2)^2\,d}\\ &+\frac{(3a^4+10a^2b^2-b^4)\,\text{EllipticPi}\left(\frac{2a}{a+b},\frac{1}{2}(c+dx),2\right)}{4a^2(a-b)^2(a+b)^3d}\\ &+\frac{a\sin(c+dx)}{2\,(a^2-b^2)\,d\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^2}\\ &+\frac{3(a^2+b^2)\sin(c+dx)}{4\,(a^2-b^2)^2\,d\sqrt{\cos(c+dx)}(a+b\sec(c+dx))}\end{aligned}$$

output
```
-1/4*(5*a^2+b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE
(sin(1/2*d*x+1/2*c),2^(1/2))/a/(a^2-b^2)^2/d-1/4*b*(7*a^2-b^2)*(cos(1/2*d*
x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))
/a^2/(a^2-b^2)^2/d+1/4*(3*a^4+10*a^2*b^2-b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)
/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*a/(a+b),2^(1/2))/a^2/(
a-b)^2/(a+b)^3/d+1/2*a*sin(d*x+c)/(a^2-b^2)/d/(a+b*sec(d*x+c))^2/cos(d*x+c
)^(1/2)+3/4*(a^2+b^2)*sin(d*x+c)/(a^2-b^2)^2/d/(a+b*sec(d*x+c))/cos(d*x+c)
^(1/2)
```

3.833.2 Mathematica [A] (verified)

Time = 2.32 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.11

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^3}\,dx$$
$$= \frac{\frac{4\sqrt{\cos(c+dx)}\left(3b(a^2+b^2)+a(5a^2+b^2)\cos(c+dx)\right)\sin(c+dx)}{(a^2-b^2)^2(b+a\cos(c+dx))^2} - \frac{-\frac{2(a^2+5b^2)\,\text{EllipticPi}\left(\frac{2a}{a+b},\frac{1}{2}(c+dx),2\right)}{a+b}+24b\left(2\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)-\frac{2b\,\text{Ellipti}}{}\right.}{}}{}$$

input
```
Integrate[1/(Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^3),x]
```

output
```
((4*Sqrt[Cos[c + d*x]]*(3*b*(a^2 + b^2) + a*(5*a^2 + b^2)*Cos[c + d*x])*Si
n[c + d*x])/((a^2 - b^2)^2*(b + a*Cos[c + d*x])^2) - ((-2*(a^2 + 5*b^2)*El
lipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(a + b) + 24*b*(2*EllipticF[(c +
d*x)/2, 2] - (2*b*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(a + b)) + (2
*(5*a^2 + b^2)*(-2*a*b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] + 2*b*(a
+ b)*EllipticF[ArcSin[Sqrt[Cos[c + d*x]]], -1] + (a^2 - 2*b^2)*EllipticPi[
-(a/b), ArcSin[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d*x])/(a^2*b*Sqrt[Sin[c +
 d*x]^2]))/((a - b)^2*(a + b)^2))/(16*d)
```

3.833.3 Rubi [A] (verified)

Time = 2.19 (sec) , antiderivative size = 328, normalized size of antiderivative = 1.33, number of steps used = 20, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.870$, Rules used = {3042, 4752, 3042, 4331, 27, 3042, 4588, 27, 3042, 4594, 3042, 4274, 3042, 4258, 3042, 3119, 3120, 4336, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^3}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^3}dx$$
$$\Big\downarrow \; 4752$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^3}dx$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^3}dx$$

$\Big\downarrow$ 4331

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int -\frac{-3a\sec^2(c+dx)+4b\sec(c+dx)+a}{2\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^2}dx}{2\left(a^2-b^2\right)}+\frac{a\sin(c+dx)\sqrt{\sec(c+dx)}}{2d\left(a^2-b^2\right)(a+b\sec(c+dx))^2}\right)$$

$\Big\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{a\sin(c+dx)\sqrt{\sec(c+dx)}}{2d\left(a^2-b^2\right)(a+b\sec(c+dx))^2}-\frac{\int \frac{-3a\sec^2(c+dx)+4b\sec(c+dx)+a}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^2}dx}{4\left(a^2-b^2\right)}\right)$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{a\sin(c+dx)\sqrt{\sec(c+dx)}}{2d\left(a^2-b^2\right)(a+b\sec(c+dx))^2}-\frac{\int \frac{-3a\csc\left(c+dx+\frac{\pi}{2}\right)^2+4b\csc\left(c+dx+\frac{\pi}{2}\right)+a}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx}{4\left(a^2-b^2\right)}\right)$$

$\Big\downarrow$ 4588

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{a\sin(c+dx)\sqrt{\sec(c+dx)}}{2d\left(a^2-b^2\right)(a+b\sec(c+dx))^2}-\frac{-\frac{\int -\frac{12b\sec(c+dx)a^2-3\left(a^2+b^2\right)\sec^2(c+dx)a+\left(5a^2+b^2\right)a}{2\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}dx}{a\left(a^2-b^2\right)}-}{4\left(a^2-b^2\right)}\right.$$

$\Big\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{a\sin(c+dx)\sqrt{\sec(c+dx)}}{2d\left(a^2-b^2\right)(a+b\sec(c+dx))^2}-\frac{\frac{\int \frac{12b\sec(c+dx)a^2-3\left(a^2+b^2\right)\sec^2(c+dx)a+\left(5a^2+b^2\right)a}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}dx}{2a\left(a^2-b^2\right)}-\frac{3(a^2}{}}{4\left(a^2-b^2\right)}\right.$$

$\Big\downarrow$ 3042

3.833. $\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^3}dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{a\sin(c+dx)\sqrt{\sec(c+dx)}}{2d\left(a^2-b^2\right)(a+b\sec(c+dx))^2}-\frac{\frac{\int\frac{12b\csc\left(c+dx+\frac{\pi}{2}\right)a^2-3\left(a^2+b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2a+\left(5a^2+b^2\right)a}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{2a(a^2-b^2)}}{4\left(a^2-b^2\right)}\right.$$

$\downarrow$ 4594

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{a\sin(c+dx)\sqrt{\sec(c+dx)}}{2d\left(a^2-b^2\right)(a+b\sec(c+dx))^2}-\frac{\frac{\frac{\int\frac{\left(5a^2+b^2\right)a^2+b\left(7a^2-b^2\right)\sec(c+dx)a}{\sqrt{\sec(c+dx)}}dx}{a^2}-\frac{\left(3a^4+10a^2b^2-b^4\right)\int\frac{s}{a-}}{a}}{2a(a^2-b^2)}}{4\left(a^2-b^2\right.}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{a\sin(c+dx)\sqrt{\sec(c+dx)}}{2d\left(a^2-b^2\right)(a+b\sec(c+dx))^2}-\frac{\frac{\frac{\int\frac{\left(5a^2+b^2\right)a^2+b\left(7a^2-b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)a}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2}-\frac{\left(3a^4+10a^2b^2-b^4\right)}{}}{2a(a^2-b^2)}}{4\left(a^2-\right.}\right.$$

$\downarrow$ 4274

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{a\sin(c+dx)\sqrt{\sec(c+dx)}}{2d\left(a^2-b^2\right)(a+b\sec(c+dx))^2}-\frac{\frac{\frac{a^2\left(5a^2+b^2\right)\int\frac{1}{\sqrt{\sec(c+dx)}}dx+ab\left(7a^2-b^2\right)\int\sqrt{\sec(c+dx)}dx}{a^2}-\frac{\left(3a^4\right.}{}}{2a(a^2-b^2)}}{4}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{a\sin(c+dx)\sqrt{\sec(c+dx)}}{2d\left(a^2-b^2\right)(a+b\sec(c+dx))^2}-\frac{\frac{\frac{a^2\left(5a^2+b^2\right)\int\frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+ab\left(7a^2-b^2\right)\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}a}{a^2}}{2a(a^2-b^2)}}{}\right.$$

$\downarrow$ 4258

3.833. $\int\frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^3}dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{a\sin(c+dx)\sqrt{\sec(c+dx)}}{2d\left(a^2-b^2\right)(a+b\sec(c+dx))^2}-\frac{\frac{\frac{a^2\left(5a^2+b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\cos(c+dx)}dx+ab\left(7a^2-b^2\right)\cdots}{a^2}}{\cdots}}{\cdots}\right.$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{a\sin(c+dx)\sqrt{\sec(c+dx)}}{2d\left(a^2-b^2\right)(a+b\sec(c+dx))^2}-\frac{\frac{\frac{a^2\left(5a^2+b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+ab\left(7a^2-\cdots\right.}{a^2}}{\cdots}}{\cdots}\right.$$

$$\Big\downarrow \; 3119$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{a\sin(c+dx)\sqrt{\sec(c+dx)}}{2d\left(a^2-b^2\right)(a+b\sec(c+dx))^2}-\frac{\frac{\frac{ab\left(7a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a^2\left(5a^2\cdots\right.}{\cdots}}{a^2}}{\cdots}}{\cdots}\right.$$

$$\Big\downarrow \; 3120$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{a\sin(c+dx)\sqrt{\sec(c+dx)}}{2d\left(a^2-b^2\right)(a+b\sec(c+dx))^2}-\frac{\frac{\frac{\frac{2ab\left(7a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+\frac{2a^2\left(5\cdots\right.}{\cdots}}{a^2}}{\cdots}}{\cdots}\right.$$

$$\Big\downarrow \; 4336$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{a\sin(c+dx)\sqrt{\sec(c+dx)}}{2d\left(a^2-b^2\right)(a+b\sec(c+dx))^2}-\frac{\frac{\frac{\frac{2ab\left(7a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+\frac{2a^2\left(5\cdots\right.}{\cdots}}{a^2}}{\cdots}}{\cdots}\right.$$

$$\Big\downarrow \; 3042$$

3.833. $\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^3}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{a\sin(c+dx)\sqrt{\sec(c+dx)}}{2d\left(a^2-b^2\right)(a+b\sec(c+dx))^2}-\frac{\frac{\frac{2ab\left(7a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+\frac{2a^2\left(5\right.}{}}{a^2}}{}\right.$$

$$\Big\downarrow \; 3284$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{a\sin(c+dx)\sqrt{\sec(c+dx)}}{2d\left(a^2-b^2\right)(a+b\sec(c+dx))^2}-\frac{\frac{\frac{2ab\left(7a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+\frac{2a^2\left(5\right.}{}}{a^2}}{}\right.$$

input
```
Int[1/(Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^3),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((a*Sqrt[Sec[c + d*x]]*Sin[c + d*x])
/(2*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^2) - ((((2*a^2*(5*a^2 + b^2)*Sqrt[C
os[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (2*a*b*(7*a
^2 - b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])
/d)/a^2 - (2*(3*a^4 + 10*a^2*b^2 - b^4)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*a
)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a*(a + b)*d))/(2*a*(a^2 -
b^2)) - (3*(a^2 + b^2)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/((a^2 - b^2)*d*(a
+ b*Sec[c + d*x])))/(4*(a^2 - b^2)))
```

3.833.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4331
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[a*d^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
)*((d*Csc[e + f*x])^(n - 2)/(f*(m + 1)*(a^2 - b^2))), x] - Simp[d^2/((m + 1
)*(a^2 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*
(a*(n - 2) + b*(m + 1)*Csc[e + f*x] - a*(m + n)*Csc[e + f*x]^2), x], x] /;
FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && LtQ[1, n, 2
] && IntegersQ[2*m, 2*n]
```

rule 4336
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]]   Int[
1/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

3.833. $\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^3}\,dx$

rule 4588
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc
[e + f*x])^(m + 1)*((d*Csc[e + f*x])^n/(a*f*(m + 1)*(a^2 - b^2))), x] + Sim
p[1/(a*(m + 1)*(a^2 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f
*x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n +
1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m
+ n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x
] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

rule 4594
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Simp[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2)   Int[(d*Csc[e +
f*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Simp[1/a^2   Int[(a*A - (A*b - a
*B)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A,
B, C}, x] && NeQ[a^2 - b^2, 0]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

3.833.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1857 vs. $2(310) = 620$.

Time = 25.32 (sec) , antiderivative size = 1858, normalized size of antiderivative = 7.55

method	result	size
default	Expression too large to display	1858

input
```
int(1/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
-(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2/a/(a^2-a*b)
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/
2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2
*a/(a-b),2^(1/2))+2/a^2*b^2*(1/2*a^2/b/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*a*cos(1/2*d*x+1/2*c)^2-a
+b)^2+3/4*a^2*(a^2-3*b^2)/b^2/(a^2-b^2)^2*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d
*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*a*cos(1/2*d*x+1/2*c)^2-a+b)-3/8
/(a+b)/(a^2-b^2)/b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2
+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(c
os(1/2*d*x+1/2*c),2^(1/2))*a^2-1/4/(a+b)/(a^2-b^2)/b*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2
*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a+7/8/(a+b)/(a^
2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*
sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2
*c),2^(1/2))+3/8*a^3/b^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(
1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(
1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-9/8*a/(a^2-b^2)^2*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3/8*a^3
/b^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+...
```

3.833.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(1/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
Timed out
```

3.833.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(1/cos(d*x+c)**(3/2)/(a+b*sec(d*x+c))**3,x)
```

output
```
Timed out
```

3.833.7 Maxima [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(1/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
Timed out
```

3.833.8 Giac [F]

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^3}\,dx = \int \frac{1}{(b\sec(dx+c)+a)^3\cos(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate(1/((b*sec(d*x + c) + a)^3*cos(d*x + c)^(3/2)), x)
```

3.833.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^3}\,dx=\int \frac{1}{\cos\left(c+d\,x\right)^{3/2}\left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)^3}\,dx$$

input
```
int(1/(cos(c + d*x)^(3/2)*(a + b/cos(c + d*x))^3),x)
```

output
```
int(1/(cos(c + d*x)^(3/2)*(a + b/cos(c + d*x))^3), x)
```

3.834 $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^3}\,dx$

3.834.1 Optimal result . . . 6195
3.834.2 Mathematica [A] (verified) . . . 6196
3.834.3 Rubi [A] (verified) . . . 6196
3.834.4 Maple [B] (verified) . . . 6202
3.834.5 Fricas [F(-1)] . . . 6203
3.834.6 Sympy [F(-1)] . . . 6204
3.834.7 Maxima [F(-1)] . . . 6204
3.834.8 Giac [F] . . . 6204
3.834.9 Mupad [F(-1)] . . . 6205

3.834.1 Optimal result

Integrand size = 23, antiderivative size = 253

$$\begin{aligned}\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^3}\,dx = {} & \frac{(a^2+5b^2)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4b\,(a^2-b^2)^2\,d} \\ & + \frac{3(a^2+b^2)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{4a\,(a^2-b^2)^2\,d} \\ & + \frac{(a^4-10a^2b^2-3b^4)\,\text{EllipticPi}\left(\frac{2a}{a+b},\frac{1}{2}(c+dx),2\right)}{4a(a-b)^2b(a+b)^3d} \\ & - \frac{a^2\sin(c+dx)}{2b\,(a^2-b^2)\,d\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^2} \\ & + \frac{a(a^2-7b^2)\sin(c+dx)}{4b\,(a^2-b^2)^2\,d\sqrt{\cos(c+dx)}(a+b\sec(c+dx))}\end{aligned}$$

output
```
1/4*(a^2+5*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(
sin(1/2*d*x+1/2*c),2^(1/2))/b/(a^2-b^2)^2/d+3/4*(a^2+b^2)*(cos(1/2*d*x+1/2
*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a/(a
^2-b^2)^2/d+1/4*(a^4-10*a^2*b^2-3*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/
2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*a/(a+b),2^(1/2))/a/(a-b)^2/b/
(a+b)^3/d-1/2*a^2*sin(d*x+c)/b/(a^2-b^2)/d/(a+b*sec(d*x+c))^2/cos(d*x+c)^(
1/2)+1/4*a*(a^2-7*b^2)*sin(d*x+c)/b/(a^2-b^2)^2/d/(a+b*sec(d*x+c))/cos(d*x
+c)^(1/2)
```

3.834.2 Mathematica [A] (verified)

Time = 3.23 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.14

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^3}\,dx$$
$$= \frac{-\frac{4a\sqrt{\cos(c+dx)}\left(-a^2b+7b^3+a\left(a^2+5b^2\right)\cos(c+dx)\right)\sin(c+dx)}{\left(a^2-b^2\right)^2(b+a\cos(c+dx))^2}+\frac{\frac{6\left(a^3-3ab^2\right)\text{EllipticPi}\left(\frac{2a}{a+b},\frac{1}{2}(c+dx),2\right)}{a+b}+\frac{8b\left(a^2+2b^2\right)\left(2\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)-\right.}{a}}{}}{}$$

input
```
Integrate[1/(Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^3),x]
```

output
```
((-4*a*Sqrt[Cos[c + d*x]]*(-(a^2*b) + 7*b^3 + a*(a^2 + 5*b^2)*Cos[c + d*x]
)*Sin[c + d*x])/((a^2 - b^2)^2*(b + a*Cos[c + d*x])^2) + ((6*(a^3 - 3*a*b^
2)*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(a + b) + (8*b*(a^2 + 2*b^2)
*(2*EllipticF[(c + d*x)/2, 2] - (2*b*EllipticPi[(2*a)/(a + b), (c + d*x)/2
, 2])/(a + b)))/a + (2*(a^2 + 5*b^2)*(-2*a*b*EllipticE[ArcSin[Sqrt[Cos[c +
 d*x]]], -1] + 2*b*(a + b)*EllipticF[ArcSin[Sqrt[Cos[c + d*x]]], -1] + (a^
2 - 2*b^2)*EllipticPi[-(a/b), ArcSin[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d*x
])/(a*b*Sqrt[Sin[c + d*x]^2]))/((a - b)^2*(a + b)^2))/(16*b*d)
```

3.834.3 Rubi [A] (verified)

Time = 2.30 (sec) , antiderivative size = 331, normalized size of antiderivative = 1.31, number of steps used = 20, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.870$, Rules used = {3042, 4752, 3042, 4332, 27, 3042, 4588, 27, 3042, 4594, 3042, 4274, 3042, 4258, 3042, 3119, 3120, 4336, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^3}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^3}dx$$
$$\Big\downarrow \; 4752$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\sec(c+dx))^3}dx$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^3}dx$$

$\Big\downarrow$ 4332

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\int -\frac{a^2+4b\sec(c+dx)a+\left(a^2-4b^2\right)\sec^2(c+dx)}{2\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^2}dx}{2b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^2}\right)$$

$\Big\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{a^2+4b\sec(c+dx)a+\left(a^2-4b^2\right)\sec^2(c+dx)}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^2}dx}{4b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^2}\right)$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{a^2+4b\csc\left(c+dx+\frac{\pi}{2}\right)a+\left(a^2-4b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx}{4b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^2}\right)$$

$\Big\downarrow$ 4588

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{a\left(a^2-7b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a^2-b^2)(a+b\sec(c+dx))}-\frac{\int -\frac{\left(a^2-7b^2\right)\sec^2(c+dx)a^2+\left(a^2+5b^2\right)a^2+4b\left(a^2+2b^2\right)\sec(c+dx)a}{2\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}dx}{a\left(a^2-b^2\right)}}{4b\left(a^2-b^2\right)}-\right.$$

$\Big\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\int \frac{\left(a^2-7b^2\right)\sec^2(c+dx)a^2+\left(a^2+5b^2\right)a^2+4b\left(a^2+2b^2\right)\sec(c+dx)a}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}dx}{2a\left(a^2-b^2\right)}+\frac{a\left(a^2-7b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a^2-b^2)(a+b\sec(c+dx))}}{4b\left(a^2-b^2\right)}-\frac{}{2}\right.$$

$\Big\downarrow$ 3042

3.834. $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^3}dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\int \frac{\left(a^2-7b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2 a^2+\left(a^2+5b^2\right)a^2+4b\left(a^2+2b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)a}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{2a(a^2-b^2)}+\frac{a\left(a^2-7b^2\right)\sin(c+dx)\sqrt{\sec(c+d}}{d(a^2-b^2)(a+b\sec(c+dx))}}{4b\left(a^2-b^2\right)}\right.$$

$\downarrow$ 4594

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\left(a^4-10a^2b^2-3b^4\right)\int \frac{\sec^{\frac{3}{2}}(c+dx)}{a+b\sec(c+dx)}dx+\frac{\int \frac{\left(a^2+5b^2\right)a^3+3b\left(a^2+b^2\right)\sec(c+dx)a^2}{\sqrt{\sec(c+dx)}}dx}{a^2}}{2a(a^2-b^2)}+\frac{a\left(a^2-7b^2\right)\sin(c+dx)\sqrt{s}}{d(a^2-b^2)(a+b\sec(c+}}{4b\left(a^2-b^2\right)}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\left(a^4-10a^2b^2-3b^4\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{\int \frac{\left(a^2+5b^2\right)a^3+3b\left(a^2+b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)a^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2}}{2a(a^2-b^2)}+\frac{a\left(a^2-7b^2\right)\sin(c}{d(a^2-b^2)(a-}}{4b\left(a^2-b^2\right)}\right.$$

$\downarrow$ 4274

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\left(a^4-10a^2b^2-3b^4\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{3a^2b\left(a^2+b^2\right)\int \sqrt{\sec(c+dx)}dx+a^3\left(a^2+5b^2\right)\int \frac{1}{\sqrt{\sec(c+dx)}}dx}{a^2}}{2a(a^2-b^2)}+\frac{a(a}{}}{4b\left(a^2-b^2\right)}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\left(a^4-10a^2b^2-3b^4\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{3a^2b\left(a^2+b^2\right)\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}dx+a^3\left(a^2+5b^2\right)\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2}}{2a(a^2-b^2)}}{4b\left(a^2-b^2\right)}\right.$$

$\downarrow$ 4258

3.834. $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^3}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{(a^4-10a^2b^2-3b^4)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{3a^2b\left(a^2+b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx+a^3\left(a^2+5b^2\right)\sqrt{}}{a^2}}{2a(a^2-b^2)}}{4b\left(a^2-b^2\right)}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{(a^4-10a^2b^2-3b^4)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{3a^2b\left(a^2+b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+a^3\left(a^2+5b\right.}{a^2}}{2a(a^2-b^2)}}{4b\left(a^2-b^2\right)}\right.$$

$\downarrow$ 3119

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{(a^4-10a^2b^2-3b^4)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{3a^2b\left(a^2+b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a^3\left(a^2+\right.}{}}{a^2}}{2a(a^2-b^2)}}{4b\left(a^2-b^2\right)}\right.$$

$\downarrow$ 3120

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{(a^4-10a^2b^2-3b^4)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{\frac{6a^2b\left(a^2+b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+\frac{2a^3\left(a^2+\right.}{}}{a^2}}{2a(a^2-b^2)}}{4b\left(a^2-b^2\right)}\right.$$

$\downarrow$ 4336

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{(a^4-10a^2b^2-3b^4)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}(b+a\cos(c+dx))}dx+\frac{\frac{6a^2b\left(a^2+b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+}}{d}}{}}{2a(a^2-b^2)}}{4b(}\right.$$

$\downarrow$ 3042

3.834. $\int\frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^3}dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\left(a^4-10a^2b^2-3b^4\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(b+a\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx+\frac{6a^2b\left(a^2+b^2\right)\sqrt{\cos(c+dx}}{}}{2a(a^2-b^2)}}{}\right.$$

$$\Big\downarrow \; 3284$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{a\left(a^2-7b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a^2-b^2)(a+b\sec(c+dx))}+\frac{\frac{2\left(a^4-10a^2b^2-3b^4\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticPi}\left(\frac{2a}{a+b},\frac{1}{2}(c+dx),2\right)}{d(a+b)}+}{}}{4b\left(a^2-\right.}\right.$$

input
```
Int[1/(Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^3),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(-1/2*(a^2*Sqrt[Sec[c + d*x]]*Sin[c
+ d*x])/(b*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^2) + ((((2*a^3*(a^2 + 5*b^2)
*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (6*a
^2*b*(a^2 + b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c +
 d*x]])/d)/a^2 + (2*(a^4 - 10*a^2*b^2 - 3*b^4)*Sqrt[Cos[c + d*x]]*Elliptic
Pi[(2*a)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/((a + b)*d))/(2*a*(a
^2 - b^2)) + (a*(a^2 - 7*b^2)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/((a^2 - b^2
)*d*(a + b*Sec[c + d*x])))/(4*b*(a^2 - b^2)))
```

3.834.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4332
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-a^2)*d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(
m + 1)*((d*Csc[e + f*x])^(n - 3)/(b*f*(m + 1)*(a^2 - b^2))), x] + Simp[d^3/
(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])
^(n - 3)*Simp[a^2*(n - 3) + a*b*(m + 1)*Csc[e + f*x] - (a^2*(n - 2) + b^2*(
m + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2
- b^2, 0] && LtQ[m, -1] && (IGtQ[n, 3] || (IntegersQ[n + 1/2, 2*m] && GtQ[n
, 2]))
```

rule 4336
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]]   Int[
1/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

3.834. $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^3}\,dx$

rule 4588
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc
[e + f*x])^(m + 1)*((d*Csc[e + f*x])^n/(a*f*(m + 1)*(a^2 - b^2))), x] + Sim
p[1/(a*(m + 1)*(a^2 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f
*x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n +
1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m
+ n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x
] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

rule 4594
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Simp[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2)   Int[(d*Csc[e +
f*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Simp[1/a^2   Int[(a*A - (A*b - a
*B)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A,
B, C}, x] && NeQ[a^2 - b^2, 0]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

3.834.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1759 vs. $2(317) = 634$.

Time = 25.12 (sec) , antiderivative size = 1760, normalized size of antiderivative = 6.96

method	result	size
default	Expression too large to display	1760

input
```
int(1/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
-(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2/a*(a^2/b/(a^
2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(
1/2)/(2*a*cos(1/2*d*x+1/2*c)^2-a+b)-1/2/(a+b)/b*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+
1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2*a/b/(a^2-b^2)*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d
*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/
2))-1/2*a/b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^
2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(
cos(1/2*d*x+1/2*c),2^(1/2))-1/2/b/(a^2-b^2)/(a^2-a*b)*a^3*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+si
n(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))
+3/2*b/(a^2-b^2)/(a^2-a*b)*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+
1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ell
ipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2)))-2*b/a*(1/2*a^2/b/(a^2-b^2)*
cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2
*a*cos(1/2*d*x+1/2*c)^2-a+b)^2+3/4*a^2*(a^2-3*b^2)/b^2/(a^2-b^2)^2*cos(1/2
*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*a*cos(
1/2*d*x+1/2*c)^2-a+b)-3/8/(a+b)/(a^2-b^2)/b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x...
```

3.834.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(1/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
Timed out
```

3.834.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(1/cos(d*x+c)**(5/2)/(a+b*sec(d*x+c))**3,x)
```

output
```
Timed out
```

3.834.7 Maxima [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(1/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
Timed out
```

3.834.8 Giac [F]

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^3}\,dx = \int \frac{1}{(b\sec(dx+c)+a)^3\cos(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate(1/((b*sec(d*x + c) + a)^3*cos(d*x + c)^(5/2)), x)
```

3.834.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^3}\,dx=\int \frac{1}{\cos\left(c+d\,x\right)^{5/2}\left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)^3}\,dx$$

input
```
int(1/(cos(c + d*x)^(5/2)*(a + b/cos(c + d*x))^3),x)
```

output
```
int(1/(cos(c + d*x)^(5/2)*(a + b/cos(c + d*x))^3), x)
```

3.835 $\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^3}\,dx$

3.835.1 Optimal result . . . 6206
3.835.2 Mathematica [A] (verified) . . . 6207
3.835.3 Rubi [A] (verified) . . . 6207
3.835.4 Maple [B] (verified) . . . 6213
3.835.5 Fricas [F(-1)] . . . 6214
3.835.6 Sympy [F(-1)] . . . 6215
3.835.7 Maxima [F(-1)] . . . 6215
3.835.8 Giac [F] . . . 6215
3.835.9 Mupad [F(-1)] . . . 6216

3.835.1 Optimal result

Integrand size = 23, antiderivative size = 255

$$\begin{aligned}\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^3}\,dx = {} & \frac{3a(a^2-3b^2)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4b^2\left(a^2-b^2\right)^2 d} \\ & + \frac{(a^2-7b^2)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{4b\left(a^2-b^2\right)^2 d} \\ & + \frac{3(a^4-2a^2b^2+5b^4)\,\text{EllipticPi}\left(\frac{2a}{a+b},\frac{1}{2}(c+dx),2\right)}{4(a-b)^2b^2(a+b)^3 d} \\ & - \frac{a^2\sin(c+dx)}{2b\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2} \\ & - \frac{3a^2(a^2-3b^2)\sin(c+dx)}{4b^2\left(a^2-b^2\right)^2 d\sqrt{\cos(c+dx)}(a+b\sec(c+dx))}\end{aligned}$$

output
```
3/4*a*(a^2-3*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elliptic
E(sin(1/2*d*x+1/2*c),2^(1/2))/b^2/(a^2-b^2)^2/d+1/4*(a^2-7*b^2)*(cos(1/2*d
*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)
)/b/(a^2-b^2)^2/d+3/4*(a^4-2*a^2*b^2+5*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/c
os(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*a/(a+b),2^(1/2))/(a-b)^2
/b^2/(a+b)^3/d-1/2*a^2*sin(d*x+c)/b/(a^2-b^2)/d/cos(d*x+c)^(3/2)/(a+b*sec(
d*x+c))^2-3/4*a^2*(a^2-3*b^2)*sin(d*x+c)/b^2/(a^2-b^2)^2/d/(a+b*sec(d*x+c)
)/cos(d*x+c)^(1/2)
```

3.835.2 Mathematica [A] (verified)

Time = 3.15 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.16

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^3}\,dx$$
$$= \frac{-\frac{4a^2\sqrt{\cos(c+dx)}\left(5a^2b-11b^3+3a\left(a^2-3b^2\right)\cos(c+dx)\right)\sin(c+dx)}{\left(a^2-b^2\right)^2(b+a\cos(c+dx))^2}+\frac{\frac{2\left(9a^4-19a^2b^2+16b^4\right)\text{EllipticPi}\left(\frac{2a}{a+b},\frac{1}{2}(c+dx),2\right)}{a+b}+\frac{16b\left(a^2-4b^2\right)\left((a+b)\,\text{Ellipt}\right.}{}}{}}{}$$

input
```
Integrate[1/(Cos[c + d*x]^(7/2)*(a + b*Sec[c + d*x])^3),x]
```

output
```
((-4*a^2*Sqrt[Cos[c + d*x]]*(5*a^2*b - 11*b^3 + 3*a*(a^2 - 3*b^2)*Cos[c +
d*x])*Sin[c + d*x])/((a^2 - b^2)^2*(b + a*Cos[c + d*x])^2) + ((2*(9*a^4 -
19*a^2*b^2 + 16*b^4)*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(a + b) +
(16*b*(a^2 - 4*b^2)*((a + b)*EllipticF[(c + d*x)/2, 2] - b*EllipticPi[(2*a
)/(a + b), (c + d*x)/2, 2]))/(a + b) + (6*(a^2 - 3*b^2)*(-2*a*b*EllipticE[
ArcSin[Sqrt[Cos[c + d*x]]], -1] + 2*b*(a + b)*EllipticF[ArcSin[Sqrt[Cos[c
+ d*x]]], -1] + (a^2 - 2*b^2)*EllipticPi[-(a/b), ArcSin[Sqrt[Cos[c + d*x]]
], -1])*Sin[c + d*x])/(b*Sqrt[Sin[c + d*x]^2]))/((a - b)^2*(a + b)^2))/(16
*b^2*d)
```

3.835.3 Rubi [A] (verified)

Time = 2.31 (sec) , antiderivative size = 339, normalized size of antiderivative = 1.33, number of steps used = 20, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.870$, Rules used = {3042, 4752, 3042, 4332, 27, 3042, 4586, 27, 3042, 4594, 3042, 4274, 3042, 4258, 3042, 3119, 3120, 4336, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^3}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{7/2}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^3}dx$$
$$\Big\downarrow \; 4752$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+b\sec(c+dx))^3}dx$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \; 4332$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\int \frac{\sqrt{\sec(c+dx)}\left(a^2-4b\sec(c+dx)a-\left(3a^2-4b^2\right)\sec^2(c+dx)\right)}{2(a+b\sec(c+dx))^2}dx}{2b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx}{2bd\left(a^2-b^2\right)(a+b\sec(c+d}\right.$$

$$\Big\downarrow \; 27$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\int \frac{\sqrt{\sec(c+dx)}\left(a^2-4b\sec(c+dx)a-\left(3a^2-4b^2\right)\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^2}dx}{4b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx}{2bd\left(a^2-b^2\right)(a+b\sec(c+d}\right.$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a^2-4b\csc\left(c+dx+\frac{\pi}{2}\right)a+\left(4b^2-3a^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx}{4b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\,\mathrm{se}}{2bd\left(a^2-b^2\right)(a+b}\right.$$

$$\Big\downarrow \; 4586$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{3a^2\left(a^2-3b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd(a^2-b^2)(a+b\sec(c+dx))}-\frac{\int \frac{3\left(a^2-3b^2\right)a^2+4b\left(a^2-4b^2\right)\sec(c+dx)a+\left(3a^4-5b^2a^2+8b^4\right)\sec^2(c+d}{2\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}}{b(a^2-b^2)}}{4b\left(a^2-b^2\right)}\right.$$

$$\Big\downarrow \; 27$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{3a^2\left(a^2-3b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd(a^2-b^2)(a+b\sec(c+dx))}-\frac{\int \frac{3\left(a^2-3b^2\right)a^2+4b\left(a^2-4b^2\right)\sec(c+dx)a+\left(3a^4-5b^2a^2+8b^4\right)\sec^2(c+d}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}}{2b(a^2-b^2)}}{4b\left(a^2-b^2\right)}\right.$$

$$\Big\downarrow \; 3042$$

3.835. $\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^3}dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{3a^2(a^2-3b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd(a^2-b^2)(a+b\sec(c+dx))}-\frac{\int\frac{3(a^2-3b^2)a^2+4b(a^2-4b^2)\csc(c+dx+\frac{\pi}{2})a+(3a^4-5b^2a^2+8b^4)\csc(}{\sqrt{\csc(c+dx+\frac{\pi}{2})}(a+b\csc(c+dx+\frac{\pi}{2}))}}{2b(a^2-b^2)}}{4b(a^2-b^2)}\right.$$

$\downarrow$ 4594

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{3a^2(a^2-3b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd(a^2-b^2)(a+b\sec(c+dx))}-\frac{3(a^4-2a^2b^2+5b^4)\int\frac{\sec^{\frac{3}{2}}(c+dx)}{a+b\sec(c+dx)}dx+\frac{\int\frac{3(a^2-3b^2)a^3+b(a^2-7b^2}{\sqrt{\sec(c+dx)}}}{a^2}}{2b(a^2-b^2)}}{4b(a^2-b^2)}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{3a^2(a^2-3b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd(a^2-b^2)(a+b\sec(c+dx))}-\frac{3(a^4-2a^2b^2+5b^4)\int\frac{\csc(c+dx+\frac{\pi}{2})^{3/2}}{a+b\csc(c+dx+\frac{\pi}{2})}dx+\frac{\int\frac{3(a^2-3b^2)a^3+b(a^2-}{\sqrt{\csc(c-}}}{}}{2b(a^2-b^2)}}{4b(a^2-b^2)}\right.$$

$\downarrow$ 4274

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{3a^2(a^2-3b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd(a^2-b^2)(a+b\sec(c+dx))}-\frac{3(a^4-2a^2b^2+5b^4)\int\frac{\csc(c+dx+\frac{\pi}{2})^{3/2}}{a+b\csc(c+dx+\frac{\pi}{2})}dx+\frac{a^2b(a^2-7b^2)\int\sqrt{\sec(c-}}{}}{2b(a^2-b^2)}}{4b(a^2-b^2)}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{3a^2(a^2-3b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd(a^2-b^2)(a+b\sec(c+dx))}-\frac{3(a^4-2a^2b^2+5b^4)\int\frac{\csc(c+dx+\frac{\pi}{2})^{3/2}}{a+b\csc(c+dx+\frac{\pi}{2})}dx+\frac{a^2b(a^2-7b^2)\int\sqrt{\csc(c-}}{}}{2b(a^2-b^2)}}{4b(a^2-b^2)}\right.$$

$\downarrow$ 4258

3.835. $\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^3}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{3a^2(a^2-3b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd(a^2-b^2)(a+b\sec(c+dx))}-\frac{3(a^4-2a^2b^2+5b^4)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{a^2b(a^2-7b^2)\sqrt{\cos(c+d}}{}}{}}{4b(a^2-b^2)}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{3a^2(a^2-3b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd(a^2-b^2)(a+b\sec(c+dx))}-\frac{3(a^4-2a^2b^2+5b^4)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{a^2b(a^2-7b^2)\sqrt{\cos(c+d}}{}}{}}{4b(a^2-b^2)}\right.$$

$\downarrow$ 3119

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{3a^2(a^2-3b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd(a^2-b^2)(a+b\sec(c+dx))}-\frac{3(a^4-2a^2b^2+5b^4)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{a^2b(a^2-7b^2)\sqrt{\cos(c+d}}{}}{}}{4b(a^2-b^2)}\right.$$

$\downarrow$ 3120

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{3a^2(a^2-3b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd(a^2-b^2)(a+b\sec(c+dx))}-\frac{3(a^4-2a^2b^2+5b^4)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2a^2b(a^2-7b^2)\sqrt{\cos(c+}}{}}{}}{4b(a^2-b^2)}\right.$$

$\downarrow$ 4336

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{3a^2(a^2-3b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd(a^2-b^2)(a+b\sec(c+dx))}-\frac{3(a^4-2a^2b^2+5b^4)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}(b+a\cos}}{}}{}\right.$$

$\downarrow$ 3042

3.835. $\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^3}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{3a^2(a^2-3b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd(a^2-b^2)(a+b\sec(c+dx))}-\frac{3(a^4-2a^2b^2+5b^4)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}(b+}}{}}{}\right.$$

$\downarrow$ 3284

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2bd(a^2-b^2)(a+b\sec(c+dx))^2}-\frac{\frac{3a^2(a^2-3b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd(a^2-b^2)(a+b\sec(c+dx))}-\frac{6(a^4-2a^2b^2+5b^4)}{}}{}\right.$$

input
```
Int[1/(Cos[c + d*x]^(7/2)*(a + b*Sec[c + d*x])^3),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(-1/2*(a^2*Sec[c + d*x]^(3/2)*Sin[c
+ d*x])/(b*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^2) - (-1/2*(((6*a^3*(a^2 - 3
*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d +
 (2*a^2*b*(a^2 - 7*b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[
Sec[c + d*x]])/d)/a^2 + (6*(a^4 - 2*a^2*b^2 + 5*b^4)*Sqrt[Cos[c + d*x]]*El
lipticPi[(2*a)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/((a + b)*d))/(
b*(a^2 - b^2)) + (3*a^2*(a^2 - 3*b^2)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(b*
(a^2 - b^2)*d*(a + b*Sec[c + d*x])))/(4*b*(a^2 - b^2)))
```

3.835.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4332
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-a^2)*d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(
m + 1)*((d*Csc[e + f*x])^(n - 3)/(b*f*(m + 1)*(a^2 - b^2))), x] + Simp[d^3/
(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])
^(n - 3)*Simp[a^2*(n - 3) + a*b*(m + 1)*Csc[e + f*x] - (a^2*(n - 2) + b^2*(
m + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2
- b^2, 0] && LtQ[m, -1] && (IGtQ[n, 3] || (IntegersQ[n + 1/2, 2*m] && GtQ[n
, 2]))
```

rule 4336
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]]   Int[
1/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

3.835. $\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^3}\,dx$

rule 4586
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(-d)*(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a +
b*Csc[e + f*x])^(m + 1)*((d*Csc[e + f*x])^(n - 1)/(b*f*(a^2 - b^2)*(m + 1))
), x] + Simp[d/(b*(a^2 - b^2)*(m + 1))   Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*b^2*(n - 1) - a*(b*B - a*C)*(n - 1) + b*(a*A
 - b*B + a*C)*(m + 1)*Csc[e + f*x] - (b*(A*b - a*B)*(m + n + 1) + C*(a^2*n
+ b^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C
}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n, 0]
```

rule 4594
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Simp[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2)   Int[(d*Csc[e +
f*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Simp[1/a^2   Int[(a*A - (A*b - a
*B)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A,
B, C}, x] && NeQ[a^2 - b^2, 0]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

3.835.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1202 vs. $2(319) = 638$.

Time = 15.59 (sec) , antiderivative size = 1203, normalized size of antiderivative = 4.72

method	result	size
default	Expression too large to display	1203

input
```
int(1/cos(d*x+c)^(7/2)/(a+b*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
-(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(a^2/b/(a^2-b^2
)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/
(2*a*cos(1/2*d*x+1/2*c)^2-a+b)^2+3/2*a^2*(a^2-3*b^2)/b^2/(a^2-b^2)^2*cos(1
/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*a*co
s(1/2*d*x+1/2*c)^2-a+b)-3/4/(a+b)/(a^2-b^2)/b^2*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+
1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^2-1/2/(a+b)/(a^2-b
^2)/b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*
c),2^(1/2))*a+7/4/(a+b)/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2
*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2
)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3/4*a^3/b^2/(a^2-b^2)^2*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-9/4*
a/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/
2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*
d*x+1/2*c),2^(1/2))-3/4*a^3/b^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*
c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+9/4*a/(a^2-b^2)^2*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*...
```

3.835.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(1/cos(d*x+c)^(7/2)/(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
Timed out
```

3.835.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(1/cos(d*x+c)**(7/2)/(a+b*sec(d*x+c))**3,x)
```

output
```
Timed out
```

3.835.7 Maxima [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(1/cos(d*x+c)^(7/2)/(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
Timed out
```

3.835.8 Giac [F]

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^3}\,dx = \int \frac{1}{(b\sec(dx+c)+a)^3\cos(dx+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(7/2)/(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate(1/((b*sec(d*x + c) + a)^3*cos(d*x + c)^(7/2)), x)
```

3.835.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^3}\,dx=\int \frac{1}{\cos\left(c+d\,x\right)^{7/2}\left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)^3}\,dx$$

input
```
int(1/(cos(c + d*x)^(7/2)*(a + b/cos(c + d*x))^3),x)
```

output
```
int(1/(cos(c + d*x)^(7/2)*(a + b/cos(c + d*x))^3), x)
```

3.836 $\int \frac{1}{\cos^{\frac{9}{2}}(c+dx)(a+b\sec(c+dx))^3}\,dx$

3.836.1 Optimal result 6217
3.836.2 Mathematica [A] (verified) 6218
3.836.3 Rubi [A] (verified) 6218
3.836.4 Maple [B] (verified) 6226
3.836.5 Fricas [F(-1)] 6226
3.836.6 Sympy [F(-1)] 6227
3.836.7 Maxima [F(-1)] 6227
3.836.8 Giac [F] 6227
3.836.9 Mupad [F(-1)] 6228

3.836.1 Optimal result

Integrand size = 23, antiderivative size = 328

$$\begin{aligned}\int \frac{1}{\cos^{\frac{9}{2}}(c+dx)(a+b\sec(c+dx))^3}\,dx &= -\frac{(15a^4-29a^2b^2+8b^4)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4b^3\left(a^2-b^2\right)^2 d}-\frac{a(5a^2-11b^2)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{4b^2\left(a^2-b^2\right)^2 d}\\ &\quad -\frac{a(15a^4-38a^2b^2+35b^4)\,\text{EllipticPi}\left(\frac{2a}{a+b},\frac{1}{2}(c+dx),2\right)}{4(a-b)^2b^3(a+b)^3d}\\ &\quad +\frac{(15a^4-29a^2b^2+8b^4)\sin(c+dx)}{4b^3\left(a^2-b^2\right)^2 d\sqrt{\cos(c+dx)}}-\frac{a^2\sin(c+dx)}{2b\left(a^2-b^2\right)d\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^2}\\ &\quad -\frac{a^2(5a^2-11b^2)\sin(c+dx)}{4b^2\left(a^2-b^2\right)^2 d\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))}\end{aligned}$$

output
```
-1/4*(15*a^4-29*a^2*b^2+8*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/
2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/b^3/(a^2-b^2)^2/d-1/4*a*(5*a^2-
11*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*
d*x+1/2*c),2^(1/2))/b^2/(a^2-b^2)^2/d-1/4*a*(15*a^4-38*a^2*b^2+35*b^4)*(co
s(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c)
,2*a/(a+b),2^(1/2))/(a-b)^2/b^3/(a+b)^3/d-1/2*a^2*sin(d*x+c)/b/(a^2-b^2)/d
/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^2-1/4*a^2*(5*a^2-11*b^2)*sin(d*x+c)/b^2
/(a^2-b^2)^2/d/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))+1/4*(15*a^4-29*a^2*b^2+8*
b^4)*sin(d*x+c)/b^3/(a^2-b^2)^2/d/cos(d*x+c)^(1/2)
```

3.836.2 Mathematica [A] (verified)

Time = 4.03 (sec) , antiderivative size = 334, normalized size of antiderivative = 1.02

$$\int \frac{1}{\cos^{\frac{9}{2}}(c+dx)(a+b\sec(c+dx))^3}\,dx$$
$$= \frac{-\frac{\frac{2\left(45a^5-95a^3b^2+56ab^4\right)\text{EllipticPi}\left(\frac{2a}{a+b},\frac{1}{2}(c+dx),2\right)}{a+b}+\frac{8b\left(5a^4-10a^2b^2+2b^4\right)\left(2\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)-\frac{2b\,\text{EllipticPi}\left(\frac{2a}{a+b},\frac{1}{2}(c+dx),2\right)}{a+b}\right)}{a}+\frac{2\left(15a^4-29a^2b^2\right.}{}}{(a-b)^2(a}}{}$$

input
```
Integrate[1/(Cos[c + d*x]^(9/2)*(a + b*Sec[c + d*x])^3),x]
```

output
```
(-(((2*(45*a^5 - 95*a^3*b^2 + 56*a*b^4)*EllipticPi[(2*a)/(a + b), (c + d*x
)/2, 2])/(a + b) + (8*b*(5*a^4 - 10*a^2*b^2 + 2*b^4)*(2*EllipticF[(c + d*x
)/2, 2] - (2*b*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(a + b)))/a + (2
*(15*a^4 - 29*a^2*b^2 + 8*b^4)*(-2*a*b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]
], -1] + 2*b*(a + b)*EllipticF[ArcSin[Sqrt[Cos[c + d*x]]], -1] + (a^2 - 2*
b^2)*EllipticPi[-(a/b), ArcSin[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d*x])/(a*
b*Sqrt[Sin[c + d*x]^2]))/((a - b)^2*(a + b)^2)) + 4*Sqrt[Cos[c + d*x]]*((a
^3*(9*a^2*b - 15*b^3 + a*(7*a^2 - 13*b^2)*Cos[c + d*x])*Sin[c + d*x])/((a^
2 - b^2)^2*(b + a*Cos[c + d*x])^2) + 8*Tan[c + d*x]))/(16*b^3*d)
```

3.836.3 Rubi [A] (verified)

Time = 3.00 (sec) , antiderivative size = 403, normalized size of antiderivative = 1.23, number of steps used = 23, number of rules used = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {3042, 4752, 3042, 4332, 27, 3042, 4586, 27, 3042, 4590, 27, 3042, 4594, 3042, 4274, 3042, 4258, 3042, 3119, 3120, 4336, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\cos^{\frac{9}{2}}(c+dx)(a+b\sec(c+dx))^3}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{9/2}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^3}dx$$
$$\Big\downarrow \; 4752$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a+b\sec(c+dx))^3}dx$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{9/2}}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \; 4332$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\int \frac{\sec^{\frac{3}{2}}(c+dx)\left(3a^2-4b\sec(c+dx)a-\left(5a^2-4b^2\right)\sec^2(c+dx)\right)}{2(a+b\sec(c+dx))^2}dx}{2b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx}{2bd\left(a^2-b^2\right)(a+b\sec(c+d}\right.$$

$$\Big\downarrow \; 27$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\int \frac{\sec^{\frac{3}{2}}(c+dx)\left(3a^2-4b\sec(c+dx)a-\left(5a^2-4b^2\right)\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^2}dx}{4b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx}{2bd\left(a^2-b^2\right)(a+b\sec(c+d}\right.$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(3a^2-4b\csc\left(c+dx+\frac{\pi}{2}\right)a+\left(4b^2-5a^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx}{4b\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)}{2bd\left(a^2-b^2\right)(a+}\right.$$

$$\Big\downarrow \; 4586$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{a^2\left(5a^2-11b^2\right)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{bd(a^2-b^2)(a+b\sec(c+dx))}-\frac{\int -\frac{\sqrt{\sec(c+dx)}\left(\left(5a^2-11b^2\right)a^2-4b\left(a^2-4b^2\right)\sec(c+dx)a-\left(15a^4-29\right.}{2(a+b\sec(c+dx))}}{b(a^2-b^2)}}{4b\left(a^2-b^2\right)}\right.$$

$$\Big\downarrow \; 27$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{\int \frac{\sqrt{\sec(c+dx)}\left(\left(5a^2-11b^2\right)a^2-4b\left(a^2-4b^2\right)\sec(c+dx)a-\left(15a^4-29b^2a^2+8b^4\right)\sec^2(c+dx)\right)}{a+b\sec(c+dx)}dx}{2b(a^2-b^2)}+\frac{a^2\left(5a^2-11b^2\right)}{bd(a^2-b^2}}{4b\left(a^2-b^2\right)}\right.$$

$$\Big\downarrow \; 3042$$

3.836. $\int \frac{1}{\cos^{\frac{9}{2}}(c+dx)(a+b\sec(c+dx))^3}dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(\left(5a^2-11b^2\right)a^2-4b\left(a^2-4b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)a+\left(-15a^4+29b^2a^2-8b^4\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{2b(a^2-b^2)}+}{4b\left(a^2-b^2\right)}\right.$$

$\downarrow$ 4590

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{\frac{2\int \frac{3a\left(5a^4-11b^2a^2+8b^4\right)\sec^2(c+dx)+4b\left(5a^4-10b^2a^2+2b^4\right)\sec(c+dx)+a\left(15a^4-29b^2a^2+8b^4\right)}{2\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}dx}{b}-\frac{2\left(15a^4-29a^2b\right.}{}}{2b(a^2-b^2)}}{4b\left(a^2-b^2\right)}\right.$$

$\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{\frac{\int \frac{3a\left(5a^4-11b^2a^2+8b^4\right)\sec^2(c+dx)+4b\left(5a^4-10b^2a^2+2b^4\right)\sec(c+dx)+a\left(15a^4-29b^2a^2+8b^4\right)}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}dx}{b}-\frac{2\left(15a^4-29a^2b^2\right.}{}}{2b(a^2-b^2)}}{4b\left(a^2-b^2\right)}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{\frac{\int \frac{3a\left(5a^4-11b^2a^2+8b^4\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2+4b\left(5a^4-10b^2a^2+2b^4\right)\csc\left(c+dx+\frac{\pi}{2}\right)+a\left(15a^4-29b^2a^2+8b^4\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{b}-\frac{2\left(15a^4\right.}{}}{2b(a^2-b^2)}}{4b\left(a^2-b^2\right)}\right.$$

$\downarrow$ 4594

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{\frac{a\left(15a^4-38a^2b^2+35b^4\right)\int \frac{\sec^{\frac{3}{2}}(c+dx)}{a+b\sec(c+dx)}dx+\frac{\int \frac{b\left(5a^2-11b^2\right)\sec(c+dx)a^3+\left(15a^4-29b^2a^2+8b^4\right)a^2}{\sqrt{\sec(c+dx)}}dx}{a^2}}{b}-\frac{2\left(15a^4-29\right.}{}}{2b(a^2-b^2)}}{4b\left(a^2-b^2\right)}\right.$$

$\downarrow$ 3042

3.836. $\int \frac{1}{\cos^{\frac{9}{2}}(c+dx)(a+b\sec(c+dx))^3}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{\frac{a\left(15a^4-38a^2b^2+35b^4\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{\int\frac{b\left(5a^2-11b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)a^3+\left(15a^4-29b^2a^2+8b^4\right)a^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2}}{b}}{2b(a^2-b^2)}-\frac{2(}{}}{4b\left(a^2-b^2\right)}\right.$$

$\downarrow$ 4274

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{\frac{a\left(15a^4-38a^2b^2+35b^4\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{a^2\left(15a^4-29a^2b^2+8b^4\right)\int\frac{1}{\sqrt{\sec(c+dx)}}dx+a^3b\left(5a^2-11b^2\right)\int\sqrt{\sec}}{a^2}}{b}}{2b(a^2-b^2)}}{4b\left(a^2-b^2\right.}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{\frac{a\left(15a^4-38a^2b^2+35b^4\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{a^2\left(15a^4-29a^2b^2+8b^4\right)\int\frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+a^3b\left(5a^2-11b^2\right)\int}{a^2}}{b}}{2b(a^2-b^2)}}{4b\left(a^2-\right.}\right.$$

$\downarrow$ 4258

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{\frac{a\left(15a^4-38a^2b^2+35b^4\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{a^2\left(15a^4-29a^2b^2+8b^4\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\cos(c+dx)}dx+}{a^2}}{b}}{2b(a^2-b^2)}}{}\right.$$

$\downarrow$ 3042

3.836. $\int \frac{1}{\cos^{\frac{9}{2}}(c+dx)(a+b\sec(c+dx))^3}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{\frac{a\left(15a^4-38a^2b^2+35b^4\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{a^2\left(15a^4-29a^2b^2+8b^4\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}{a}}{b}}{2b(a^2-}\right.$$

$\downarrow$ 3119

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{\frac{a\left(15a^4-38a^2b^2+35b^4\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{a^3b\left(5a^2-11b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a}{}}{a^2}}{b}}{2b(a^2-b}\right.$$

$\downarrow$ 3120

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{\frac{a\left(15a^4-38a^2b^2+35b^4\right)\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2a^2\left(15a^4-29a^2b^2+8b^4\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+}{a^2}}{b}}{2b(a^2-}\right.$$

$\downarrow$ 4336

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{\frac{a\left(15a^4-38a^2b^2+35b^4\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}(b+a\cos(c+dx))}dx+\frac{2a^2\left(15a^4-29a^2b^2+8b^4\right)\sqrt{\cos(c-}}{d}}{}}{b}}{}\right.$$

$\downarrow$ 3042

3.836. $\int \frac{1}{\cos^{\frac{9}{2}}(c+dx)(a+b\sec(c+dx))^3}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{\frac{a\left(15a^4-38a^2b^2+35b^4\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(b+a\sin\left(c+dx+\frac{\pi}{2}\right)\right)}dx+\frac{2a^2\left(15a^4-29a^2b^2+8b^4\right.}{}}{}}{b}}{}\right.$$

$$\Big\downarrow \; 3284$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^2}-\frac{\frac{a^2\left(5a^2-11b^2\right)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{bd(a^2-b^2)(a+b\sec(c+dx))}+\frac{\frac{2a\left(15a^4-38a^2b^2-\right.}{}}{}}{}\right.$$

input
```
Int[1/(Cos[c + d*x]^(9/2)*(a + b*Sec[c + d*x])^3),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(-1/2*(a^2*Sec[c + d*x]^(5/2)*Sin[c
+ d*x])/(b*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^2) - ((a^2*(5*a^2 - 11*b^2)*
Sec[c + d*x]^(3/2)*Sin[c + d*x])/(b*(a^2 - b^2)*d*(a + b*Sec[c + d*x])) +
((((2*a^2*(15*a^4 - 29*a^2*b^2 + 8*b^4)*Sqrt[Cos[c + d*x]]*EllipticE[(c +
d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (2*a^3*b*(5*a^2 - 11*b^2)*Sqrt[Cos[c +
d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d)/a^2 + (2*a*(15*a^4
- 38*a^2*b^2 + 35*b^4)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*a)/(a + b), (c + d
*x)/2, 2]*Sqrt[Sec[c + d*x]])/((a + b)*d))/b - (2*(15*a^4 - 29*a^2*b^2 + 8
*b^4)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(b*d))/(2*b*(a^2 - b^2)))/(4*b*(a^2
 - b^2)))
```

3.836.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4274
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[a   Int[(d*Csc[e + f*x])^n, x], x] + Simp[b/d   In
t[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

rule 4332
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-a^2)*d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(
m + 1)*((d*Csc[e + f*x])^(n - 3)/(b*f*(m + 1)*(a^2 - b^2))), x] + Simp[d^3/
(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])
^(n - 3)*Simp[a^2*(n - 3) + a*b*(m + 1)*Csc[e + f*x] - (a^2*(n - 2) + b^2*(
m + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2
- b^2, 0] && LtQ[m, -1] && (IGtQ[n, 3] || (IntegersQ[n + 1/2, 2*m] && GtQ[n
, 2]))
```

rule 4336
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Simp[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]]   Int[
1/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

3.836. $\int \frac{1}{\cos^{\frac{9}{2}}(c+dx)(a+b\sec(c+dx))^3}\,dx$

rule 4586
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(-d)*(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a +
b*Csc[e + f*x])^(m + 1)*((d*Csc[e + f*x])^(n - 1)/(b*f*(a^2 - b^2)*(m + 1))
), x] + Simp[d/(b*(a^2 - b^2)*(m + 1))   Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*b^2*(n - 1) - a*(b*B - a*C)*(n - 1) + b*(a*A
 - b*B + a*C)*(m + 1)*Csc[e + f*x] - (b*(A*b - a*B)*(m + n + 1) + C*(a^2*n
+ b^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C
}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n, 0]
```

rule 4590
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(-C)*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
)*((d*Csc[e + f*x])^(n - 1)/(b*f*(m + n + 1))), x] + Simp[d/(b*(m + n + 1))
   Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (
A*b*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2
 - b^2, 0] && GtQ[n, 0]
```

rule 4594
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Simp[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2)   Int[(d*Csc[e +
f*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Simp[1/a^2   Int[(a*A - (A*b - a
*B)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A,
B, C}, x] && NeQ[a^2 - b^2, 0]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

3.836. $\int \frac{1}{\cos^{\frac{9}{2}}(c+dx)(a+b\sec(c+dx))^3}\,dx$

3.836.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1986 vs. 2(388) = 776.

Time = 32.90 (sec) , antiderivative size = 1987, normalized size of antiderivative = 6.06

method	result	size
default	Expression too large to display	1987

input
```
int(1/cos(d*x+c)^(9/2)/(a+b*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
-(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2/b^3/sin(1/2*
d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d
*x+1/2*c)^2)^(1/2)*(2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-(sin(1/2*d*x
+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*
c)^2-1)^(1/2))-2*a/b*(1/2*a^2/b/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d
*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*a*cos(1/2*d*x+1/2*c)^2-a+b)^2+3
/4*a^2*(a^2-3*b^2)/b^2/(a^2-b^2)^2*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*
c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*a*cos(1/2*d*x+1/2*c)^2-a+b)-3/8/(a+b)/
(a^2-b^2)/b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/
2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*
d*x+1/2*c),2^(1/2))*a^2-1/4/(a+b)/(a^2-b^2)/b*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/
2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a+7/8/(a+b)/(a^2-b^2)*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2
*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(
1/2))+3/8*a^3/b^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x
+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*El
lipticF(cos(1/2*d*x+1/2*c),2^(1/2))-9/8*a/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/
2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3/8*a^3/b^2...
```

3.836.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{9}{2}}(c+dx)(a+b\sec(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(1/cos(d*x+c)^(9/2)/(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

output
```
Timed out
```

3.836.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{9}{2}}(c+dx)(a+b\sec(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(1/cos(d*x+c)**(9/2)/(a+b*sec(d*x+c))**3,x)
```

output
```
Timed out
```

3.836.7 Maxima [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{9}{2}}(c+dx)(a+b\sec(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(1/cos(d*x+c)^(9/2)/(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

output
```
Timed out
```

3.836.8 Giac [F]

$$\int \frac{1}{\cos^{\frac{9}{2}}(c+dx)(a+b\sec(c+dx))^3}\,dx = \int \frac{1}{(b\sec(dx+c)+a)^3\cos(dx+c)^{\frac{9}{2}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(9/2)/(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate(1/((b*sec(d*x + c) + a)^3*cos(d*x + c)^(9/2)), x)
```

3.836.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{9}{2}}(c+dx)(a+b\sec(c+dx))^3}\,dx = \int \frac{1}{\cos{(c+d\,x)}^{9/2}\left(a+\frac{b}{\cos(c+d\,x)}\right)^3}\,dx$$

input
```
int(1/(cos(c + d*x)^(9/2)*(a + b/cos(c + d*x))^3),x)
```

output
```
int(1/(cos(c + d*x)^(9/2)*(a + b/cos(c + d*x))^3), x)
```

3.837 $\int \cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$

3.837.1 Optimal result 6229
3.837.2 Mathematica [C] (warning: unable to verify) 6230
3.837.3 Rubi [A] (verified) 6230
3.837.4 Maple [B] (verified) 6236
3.837.5 Fricas [C] (verification not implemented) 6237
3.837.6 Sympy [F(-1)] 6238
3.837.7 Maxima [F] 6238
3.837.8 Giac [F] 6239
3.837.9 Mupad [F(-1)] 6239

3.837.1 Optimal result

Integrand size = 25, antiderivative size = 244

$$\begin{aligned}\int \cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx &= -\frac{4b(a^2-b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{15a^2d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}\\ &\quad+\frac{2(9a^2-2b^2)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+dx)}}{15a^2d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}}\\ &\quad+\frac{2b\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{15ad}\\ &\quad+\frac{2\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{5d}\end{aligned}$$

output
```
-4/15*b*(a^2-b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elliptic
F(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/
2)/a^2/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)+2/5*cos(d*x+c)^(3/2)*sin(
d*x+c)*(a+b*sec(d*x+c))^(1/2)/d+2/15*b*sin(d*x+c)*cos(d*x+c)^(1/2)*(a+b*se
c(d*x+c))^(1/2)/a/d+2/15*(9*a^2-2*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/
2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*cos(d*x
+c)^(1/2)*(a+b*sec(d*x+c))^(1/2)/a^2/d/((b+a*cos(d*x+c))/(a+b))^(1/2)
```

3.837.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 10.21 (sec) , antiderivative size = 340, normalized size of antiderivative = 1.39

$$\int \cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$$
$$= \frac{2\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\left(a(b+3a\cos(c+dx))\sin(c+dx)-\frac{\left(\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\right)^{3/2}\left(-i(9a^3+9a\right.}{}\right.}{}$$

input
```
Integrate[Cos[c + d*x]^(5/2)*Sqrt[a + b*Sec[c + d*x]],x]
```

output
```
(2*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]*(a*(b + 3*a*Cos[c + d*x])*S
in[c + d*x] - ((Cos[(c + d*x)/2]^2*Sec[c + d*x])^(3/2)*((-I)*(9*a^3 + 9*a^
2*b - 2*a*b^2 - 2*b^3)*EllipticE[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a
+ b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a
 + b)] + I*a*(9*a^2 + 7*a*b - 2*b^2)*EllipticF[I*ArcSinh[Tan[(c + d*x)/2]]
, (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c +
 d*x)/2]^2)/(a + b)] - (9*a^2 - 2*b^2)*(b + a*Cos[c + d*x])*(Sec[(c + d*x)
/2]^2)^(3/2)*Tan[(c + d*x)/2]))/((b + a*Cos[c + d*x])*Sec[c + d*x]^(3/2)))
)/(15*a^2*d)
```

3.837.3 Rubi [A] (verified)

Time = 2.05 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.12, number of steps used = 20, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.800$, Rules used = {3042, 4752, 3042, 4344, 3042, 4592, 27, 3042, 4523, 3042, 4343, 3042, 3134, 3042, 3132, 4345, 3042, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{5}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx$$

$$\Big\downarrow \text{4344}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\int \frac{2b\sec^2(c+dx)+3a\sec(c+dx)+b}{\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}dx+\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5d\sec^{\frac{3}{2}}(c+dx)}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\int \frac{2b\csc\left(c+dx+\frac{\pi}{2}\right)^2+3a\csc\left(c+dx+\frac{\pi}{2}\right)+b}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)\sqrt{a+b\sec(c+}}{5d\sec^{\frac{3}{2}}(c+dx)}\right.$$

$$\Big\downarrow \text{4592}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(\frac{2b\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}-\frac{2\int -\frac{9a^2+7b\sec(c+dx)a-2b^2}{2\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{3a}\right)+\frac{2\sin(}{}\right.$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(\frac{\int \frac{9a^2+7b\sec(c+dx)a-2b^2}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{3a}+\frac{2b\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}\right)+\frac{2\sin(c+}{5}\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(\frac{\int \frac{9a^2+7b\csc\left(c+dx+\frac{\pi}{2}\right)a-2b^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{3a}+\frac{2b\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}\right)+\frac{2\sin}{}\right.$$

$$\Big\downarrow \text{4523}$$

3.837. $\int \cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(\frac{\frac{(9a^2-2b^2)\int\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}dx}{a}-\frac{2b(a^2-b^2)\int\frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}dx}{a}}{3a}+\frac{2b\sin(c+dx)\sqrt{a+b\sec}}{3ad\sqrt{\sec(c+dx)}}\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(\frac{\frac{(9a^2-2b^2)\int\frac{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}-\frac{2b(a^2-b^2)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}}{3a}+\frac{2b\sin(c+dx)\sqrt{a}}{3ad\sqrt{\sec(}}\right.\right.$$

$\downarrow$ 4343

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(\frac{\frac{(9a^2-2b^2)\sqrt{a+b\sec(c+dx)}\int\sqrt{b+a\cos(c+dx)}dx}{a\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}-\frac{2b(a^2-b^2)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}}{3a}+\frac{2b\sin(}{}\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(\frac{\frac{(9a^2-2b^2)\sqrt{a+b\sec(c+dx)}\int\sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{a\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}-\frac{2b(a^2-b^2)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}}{3a}+\frac{2b\,\mathrm{s}}{}\right.\right.$$

$\downarrow$ 3134

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(\frac{\frac{(9a^2-2b^2)\sqrt{a+b\sec(c+dx)}\int\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}dx}{a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{2b(a^2-b^2)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}}{3a}+\frac{2b\,\mathrm{si}}{}\right.\right.$$

$\downarrow$ 3042

3.837. $\int \cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(\frac{\frac{(9a^2-2b^2)\sqrt{a+b\sec(c+dx)}\int\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}dx}{a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{2b(a^2-b^2)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}}{3a}+\frac{2b}{}\right.\right.$$

$\downarrow$ 3132

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(\frac{\frac{2(9a^2-2b^2)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{2b(a^2-b^2)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}}{3a}+\frac{2b\sin(c-}{3}\right.\right.$$

$\downarrow$ 4345

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(\frac{\frac{2(9a^2-2b^2)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{2b(a^2-b^2)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{1}{\sqrt{b+a\,\mathrm{co}}}}{a\sqrt{a+b\sec(c+dx)}}}{3a}\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(\frac{\frac{2(9a^2-2b^2)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{2b(a^2-b^2)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{}{\sqrt{b+a\,\mathrm{sin}}}}{a\sqrt{a+b\sec(c+dx)}}}{3a}\right.\right.$$

$\downarrow$ 3142

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(\frac{\frac{2(9a^2-2b^2)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{2b(a^2-b^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{}{\sqrt{\frac{b}{a+b}+\frac{a}{}}}}{a\sqrt{a+b\sec(c+dx)}}}{3a}\right.\right.$$

$\downarrow$ 3042

3.837. $\int \cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(\frac{\frac{2(9a^2-2b^2)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{2b(a^2-b^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{}{\sqrt{\frac{b}{a+b}+\frac{a}{}}}}{a\sqrt{a+b\sec(c+dx)}}}{3a}\right.\right.$$

$\downarrow$ 3140

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(\frac{\frac{2(9a^2-2b^2)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{4b(a^2-b^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\text{EllipticF}\left(\frac{}{}\right.}{ad\sqrt{a+b\sec(c+dx)}}}{3a}\right.\right.$$

input
```
Int[Cos[c + d*x]^(5/2)*Sqrt[a + b*Sec[c + d*x]],x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*Sqrt[a + b*Sec[c + d*x]]*Sin[c +
 d*x])/(5*d*Sec[c + d*x]^(3/2)) + (((-4*b*(a^2 - b^2)*Sqrt[(b + a*Cos[c +
d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(
a*d*Sqrt[a + b*Sec[c + d*x]]) + (2*(9*a^2 - 2*b^2)*EllipticE[(c + d*x)/2,
(2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(a*d*Sqrt[(b + a*Cos[c + d*x])/(a
 + b)]*Sqrt[Sec[c + d*x]]))/(3*a) + (2*b*Sqrt[a + b*Sec[c + d*x]]*Sin[c +
d*x])/(3*a*d*Sqrt[Sec[c + d*x]]))/5)
```

3.837.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 4343
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]])   Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a
, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4344
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[Cot[e + f*x]*Sqrt[a + b*Csc[e + f*x]]*((d*Csc[e
 + f*x])^n/(f*n)), x] - Simp[1/(2*d*n)   Int[(d*Csc[e + f*x])^(n + 1)*(Simp
[b - 2*a*(n + 1)*Csc[e + f*x] - b*(2*n + 3)*Csc[e + f*x]^2, x]/Sqrt[a + b*C
sc[e + f*x]]), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] &&
LeQ[n, -1] && IntegerQ[2*n]
```

rule 4345
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]])   Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[
{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4523
```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Simp[A/a   I
nt[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Simp[(A*b - a*B)
/(a*d)   Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ
[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

3.837. $\int \cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$

rule 4592
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Simp[1/(a*d*n)   Int[(a + b*Csc[e + f*x])^m
*(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*
Csc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

3.837.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2305 vs. $2(274) = 548$.

Time = 6.67 (sec) , antiderivative size = 2306, normalized size of antiderivative = 9.45

method	result	size
default	Expression too large to display	2306

input
```
int(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/15/d/((a-b)/(a+b))^(1/2)/a^2*(14*EllipticF(((a-b)/(a+b))^(1/2)*(-cot(d*x
+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*(1/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b
+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^2*b*cos(d*x+c)+4*EllipticF(((a-b)/(
a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*(1/(cos(d*x+c)+
1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a*b^2*cos(d*x+c)
-18*EllipticE(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^
(1/2))*(1/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^
(1/2)*a^2*b*cos(d*x+c)-4*EllipticE(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*
x+c)),(-(a+b)/(a-b))^(1/2))*(1/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x
+c))/(cos(d*x+c)+1))^(1/2)*a*b^2*cos(d*x+c)-2*EllipticE(((a-b)/(a+b))^(1/2
)*(-cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))
/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a*b^2*cos(d*x+c)^2+7*Ellip
ticF(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*(1
/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a^2
*b*cos(d*x+c)^2+2*EllipticF(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(
-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(c
os(d*x+c)+1))^(1/2)*a*b^2*cos(d*x+c)^2-9*EllipticE(((a-b)/(a+b))^(1/2)*(-c
ot(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos
(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a^2*b*cos(d*x+c)^2+2*EllipticE(
((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*(1/(...
```

3.837.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.13 (sec) , antiderivative size = 453, normalized size of antiderivative = 1.86

$$\int \cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$$
$$= \frac{6\left(3\,a^3\cos\left(dx+c\right)+a^2b\right)\sqrt{\frac{a\cos(dx+c)+b}{\cos(dx+c)}}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)+\sqrt{2}(-3i\,a^2b-4i\,b^3)\sqrt{a}\text{weierstrassPI}}{}$$

input
```
integrate(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
1/45*(6*(3*a^3*cos(d*x + c) + a^2*b)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c
))*sqrt(cos(d*x + c))*sin(d*x + c) + sqrt(2)*(-3*I*a^2*b - 4*I*b^3)*sqrt(a
)*weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3
, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x + c) + 2*b)/a) + sqrt(2)*(3*I*a^2*
b + 4*I*b^3)*sqrt(a)*weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9
*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a)
- 3*sqrt(2)*(-9*I*a^3 + 2*I*a*b^2)*sqrt(a)*weierstrassZeta(-4/3*(3*a^2 - 4
*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4
*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d
*x + c) + 2*b)/a)) - 3*sqrt(2)*(9*I*a^3 - 2*I*a*b^2)*sqrt(a)*weierstrassZe
ta(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInver
se(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x
+ c) - 3*I*a*sin(d*x + c) + 2*b)/a)))/(a^3*d)
```

3.837.6 Sympy [F(-1)]

Timed out.

$$\int \cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(5/2)*(a+b*sec(d*x+c))**(1/2),x)
```

output
```
Timed out
```

3.837.7 Maxima [F]

$$\int \cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = \int \sqrt{b\sec(dx+c)+a}\cos(dx+c)^{\frac{5}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sec(d*x + c) + a)*cos(d*x + c)^(5/2), x)
```

3.837.8 Giac [F]

$$\int \cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = \int \sqrt{b\sec(dx+c)+a}\cos(dx+c)^{\frac{5}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*sec(d*x + c) + a)*cos(d*x + c)^(5/2), x)
```

3.837.9 Mupad [F(-1)]

Timed out.

$$\int \cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = \int \cos(c+d\,x)^{5/2}\sqrt{a+\frac{b}{\cos(c+d\,x)}}\,dx$$

input
```
int(cos(c + d*x)^(5/2)*(a + b/cos(c + d*x))^(1/2),x)
```

output
```
int(cos(c + d*x)^(5/2)*(a + b/cos(c + d*x))^(1/2), x)
```

3.838 $\int \cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$

3.838.1 Optimal result 6240
3.838.2 Mathematica [C] (verified) 6241
3.838.3 Rubi [A] (verified) 6241
3.838.4 Maple [B] (verified) 6246
3.838.5 Fricas [C] (verification not implemented) 6246
3.838.6 Sympy [F(-1)] 6247
3.838.7 Maxima [F] 6247
3.838.8 Giac [F] 6248
3.838.9 Mupad [F(-1)] 6248

3.838.1 Optimal result

Integrand size = 25, antiderivative size = 192

$$\begin{aligned}\int \cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx &= \frac{2(a^2-b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{3ad\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} \\ &\quad+\frac{2b\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\frac{b+a\cos(c+dx)}{a+b}}} \\ &\quad+\frac{2\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{3d}\end{aligned}$$

output
```
2/3*(a^2-b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(si
n(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)/a
/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)+2/3*sin(d*x+c)*cos(d*x+c)^(1/2)
*(a+b*sec(d*x+c))^(1/2)/d+2/3*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1
/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*cos(d*x+c)^(1/
2)*(a+b*sec(d*x+c))^(1/2)/a/d/((b+a*cos(d*x+c))/(a+b))^(1/2)
```

3.838.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.02 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.42

$$\int \cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$$
$$= \frac{2\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\left(ib(a+b)\sqrt{\frac{b+a\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}E\left(i\operatorname{arcsinh}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{-a+b}{a+b}\right)\sqrt{\sec(}\right.}{}$$

input
```
Integrate[Cos[c + d*x]^(3/2)*Sqrt[a + b*Sec[c + d*x]],x]
```

output
```
(2*Cos[c + d*x]^(3/2)*Sqrt[a + b*Sec[c + d*x]]*(I*b*(a + b)*Sqrt[(b + a*Co
s[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[I*ArcSinh[Tan[(c + d*x
)/2]], (-a + b)/(a + b)]*Sqrt[Sec[c + d*x]]*Sqrt[1 + Sec[c + d*x]] - I*a*(
a + b)*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[I
*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[Sec[c + d*x]]*Sqrt[1 +
Sec[c + d*x]] + a^2*Sin[c + d*x] + a*b*Tan[(c + d*x)/2] + b^2*Sec[c + d*x]
*Tan[(c + d*x)/2] + a*b*Tan[c + d*x]))/(3*a*d*(b + a*Cos[c + d*x]))
```

3.838.3 Rubi [A] (verified)

Time = 1.54 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.11, number of steps used = 17, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.680$, Rules used = {3042, 4752, 3042, 4344, 3042, 4523, 3042, 4343, 3042, 3134, 3042, 3132, 4345, 3042, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx$$
$$\Big\downarrow \; 4752$$
$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{3}{2}}(c+dx)}dx$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx$$

$$\Big\downarrow \; 4344$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\int \frac{b+a\sec(c+dx)}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx+\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3d\sqrt{\sec(c+dx)}}\right)$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\int \frac{b+a\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(c+dx)\sqrt{a+b\sec(c+d}}{3d\sqrt{\sec(c+dx)}}\right.$$

$$\Big\downarrow \; 4523$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\left(\frac{\left(a^2-b^2\right)\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}dx}{a}+\frac{b\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}dx}{a}\right)+\frac{2\sin(c+dx)\sqrt{a+b\sec}}{3d\sqrt{\sec(c+dx)}}\right.$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\left(\frac{\left(a^2-b^2\right)\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}+\frac{b\int \frac{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}\right)+\frac{2\sin(c+dx)\sqrt{a}}{3d\sqrt{\sec(c}}\right.$$

$$\Big\downarrow \; 4343$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\left(\frac{\left(a^2-b^2\right)\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}+\frac{b\sqrt{a+b\sec(c+dx)}\int \sqrt{b+a\cos(c+dx)}dx}{a\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}\right.\right.$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\left(\frac{\left(a^2-b^2\right)\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}+\frac{b\sqrt{a+b\sec(c+dx)}\int \sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right.}}{a\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}\right.\right.$$

3.838. $\int \cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$

$$\Big\downarrow \text{3134}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\left(\frac{(a^2-b^2)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}+\frac{b\sqrt{a+b\sec(c+dx)}\int\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}dx}{a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}\right)\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\left(\frac{(a^2-b^2)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}+\frac{b\sqrt{a+b\sec(c+dx)}\int\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}d}{a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}\right.\right.$$

$$\Big\downarrow \text{3132}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\left(\frac{(a^2-b^2)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}+\frac{2b\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}\right)+\right.$$

$$\Big\downarrow \text{4345}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\left(\frac{(a^2-b^2)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{1}{\sqrt{b+a\cos(c+dx)}}dx}{a\sqrt{a+b\sec(c+dx)}}+\frac{2b\sqrt{a+b\sec(c}}{ad\sqrt{\sec(c}}\right.\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\left(\frac{(a^2-b^2)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{1}{\sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{a\sqrt{a+b\sec(c+dx)}}+\frac{2b\sqrt{a+b\sec}}{ad\sqrt{\sec}}\right.\right.$$

$$\Big\downarrow \text{3142}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\left(\frac{(a^2-b^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}}dx}{a\sqrt{a+b\sec(c+dx)}}+\frac{2b\sqrt{a+b\sec(c+}}{ad\sqrt{\sec(c+}}\right.\right.$$

$$\Big\downarrow \text{3042}$$

3.838. $\int\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\left(\frac{(a^2-b^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{a\sqrt{a+b\sec(c+dx)}}+\frac{2b\sqrt{a+b\sec(c}}{ad\sqrt{\sec(c}}\right.\right.$$

$\downarrow$ 3140

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\left(\frac{2(a^2-b^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{ad\sqrt{a+b\sec(c+dx)}}+\frac{2b\sqrt{a+b\,s}}{ad\sqrt{s}}\right.\right.$$

input
```
Int[Cos[c + d*x]^(3/2)*Sqrt[a + b*Sec[c + d*x]],x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(((2*(a^2 - b^2)*Sqrt[(b + a*Cos[c +
 d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/
(a*d*Sqrt[a + b*Sec[c + d*x]]) + (2*b*EllipticE[(c + d*x)/2, (2*a)/(a + b)
]*Sqrt[a + b*Sec[c + d*x]])/(a*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[S
ec[c + d*x]]))/3 + (2*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(3*d*Sqrt[Sec
[c + d*x]]))
```

3.838.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 4343
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]])   Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a
, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4344
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[Cot[e + f*x]*Sqrt[a + b*Csc[e + f*x]]*((d*Csc[e
 + f*x])^n/(f*n)), x] - Simp[1/(2*d*n)   Int[(d*Csc[e + f*x])^(n + 1)*(Simp
[b - 2*a*(n + 1)*Csc[e + f*x] - b*(2*n + 3)*Csc[e + f*x]^2, x]/Sqrt[a + b*C
sc[e + f*x]]), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] &&
LeQ[n, -1] && IntegerQ[2*n]
```

rule 4345
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]])   Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[
{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4523
```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Simp[A/a   I
nt[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Simp[(A*b - a*B)
/(a*d)   Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ
[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

3.838. $\int \cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$

3.838.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1312 vs. $2(228) = 456$.

Time = 5.96 (sec) , antiderivative size = 1313, normalized size of antiderivative = 6.84

method	result	size
default	Expression too large to display	1313

input
```
int(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3/d/((a-b)/(a+b))^(1/2)/a*((1/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*
x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc
(d*x+c)),(-(a+b)/(a-b))^(1/2))*a*b*cos(d*x+c)^2-(1/(cos(d*x+c)+1))^(1/2)*(
1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE(((a-b)/(a+b))^(1/
2)*(-cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*b^2*cos(d*x+c)^2+(1/(cos
(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Elliptic
F(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*a^2*c
os(d*x+c)^2-(1/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)
+1))^(1/2)*EllipticF(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(-(a+b)/
(a-b))^(1/2))*a*b*cos(d*x+c)^2+2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))
^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)
+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*a*b*cos(d*x+c)-2*(1/(a+b)*(b+a*cos(d*x+
c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE(((a-b)/(a+b))
^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*b^2*cos(d*x+c)+2*(1/
(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*Elli
pticF(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*a
^2*cos(d*x+c)-2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*
x+c)+1))^(1/2)*EllipticF(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(-(a
+b)/(a-b))^(1/2))*a*b*cos(d*x+c)+((a-b)/(a+b))^(1/2)*a^2*cos(d*x+c)^2*sin(
d*x+c)+(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE(((a-b)...
```

3.838.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 415, normalized size of antiderivative = 2.16

$$\int \cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$$
$$= \frac{6\,a^2\sqrt{\frac{a\cos(dx+c)+b}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)+3i\sqrt{2}a^{\frac{3}{2}}b\text{weierstrassZeta}\left(-\frac{4\,(3\,a^2-4\,b^2)}{3\,a^2},\frac{8\,(9\,a^2b-8\,b^3)}{27\,a^3},\text{weierst}\right.}{}$$

input
```
integrate(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
1/9*(6*a^2*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(
d*x + c) + 3*I*sqrt(2)*a^(3/2)*b*weierstrassZeta(-4/3*(3*a^2 - 4*b^2)/a^2,
 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2,
 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x + c) +
2*b)/a)) - 3*I*sqrt(2)*a^(3/2)*b*weierstrassZeta(-4/3*(3*a^2 - 4*b^2)/a^2,
 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2,
 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c) +
2*b)/a)) + sqrt(2)*(-3*I*a^2 + 2*I*b^2)*sqrt(a)*weierstrassPInverse(-4/3*(
3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*
I*a*sin(d*x + c) + 2*b)/a) + sqrt(2)*(3*I*a^2 - 2*I*b^2)*sqrt(a)*weierstra
ssPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*
cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a))/(a^2*d)
```

3.838.6 Sympy [F(-1)]

Timed out.

$$\int \cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(3/2)*(a+b*sec(d*x+c))**(1/2),x)
```

output
```
Timed out
```

3.838.7 Maxima [F]

$$\int \cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = \int \sqrt{b\sec(dx+c)+a}\cos(dx+c)^{\frac{3}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sec(d*x + c) + a)*cos(d*x + c)^(3/2), x)
```

3.838.8 Giac [F]

$$\int \cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = \int \sqrt{b\sec(dx+c)+a}\cos(dx+c)^{\frac{3}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*sec(d*x + c) + a)*cos(d*x + c)^(3/2), x)
```

3.838.9 Mupad [F(-1)]

Timed out.

$$\int \cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = \int \cos(c+d\,x)^{3/2}\sqrt{a+\frac{b}{\cos(c+d\,x)}}\,dx$$

input
```
int(cos(c + d*x)^(3/2)*(a + b/cos(c + d*x))^(1/2),x)
```

output
```
int(cos(c + d*x)^(3/2)*(a + b/cos(c + d*x))^(1/2), x)
```

3.839 $\int \sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\,dx$

3.839.1 Optimal result 6249
3.839.2 Mathematica [C] (verified) 6249
3.839.3 Rubi [A] (verified) 6250
3.839.4 Maple [B] (warning: unable to verify) 6252
3.839.5 Fricas [C] (verification not implemented) 6253
3.839.6 Sympy [F] 6254
3.839.7 Maxima [F] 6254
3.839.8 Giac [F] 6255
3.839.9 Mupad [F(-1)] 6255

3.839.1 Optimal result

Integrand size = 25, antiderivative size = 67

$$\int \sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\,dx = \frac{2\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+dx)}}{d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}}$$

output
```
2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/
2*c),2^(1/2)*(a/(a+b))^(1/2))*cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2)/d/((
b+a*cos(d*x+c))/(a+b))^(1/2)
```

3.839.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.74 (sec) , antiderivative size = 244, normalized size of antiderivative = 3.64

$$\int \sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\,dx = \frac{2\sqrt{\cos(c+dx)}\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\left(i(a+b)E\left(i\operatorname{arcsinh}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)\right|\right.}{}$$

input
```
Integrate[Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]],x]
```

output
```
(2*Sqrt[Cos[c + d*x]]*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[a + b*Sec
[c + d*x]]*(I*(a + b)*EllipticE[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a +
 b)]*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] - I*(a + b)*E
llipticF[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[((b + a*Cos[c
 + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] + (b + a*Cos[c + d*x])*Sqrt[Sec[(c +
 d*x)/2]^2]*Tan[(c + d*x)/2]))/(d*(b + a*Cos[c + d*x])*Sqrt[Sec[c + d*x]])
```

3.839.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3042, 4752, 3042, 4343, 3042, 3134, 3042, 3132}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \; 4752$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}dx$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \; 4343$$

$$\frac{\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\int \sqrt{b+a\cos(c+dx)}dx}{\sqrt{a\cos(c+dx)+b}}$$

$$\Big\downarrow \; 3042$$

$$\frac{\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\int\sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{\sqrt{a\cos(c+dx)+b}}$$

$$\Big\downarrow \text{3134}$$

$$\frac{\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\int\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}dx}{\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\int\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}dx}{\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}$$

$$\Big\downarrow \text{3132}$$

$$\frac{2\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}$$

input
```
Int[Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]],x]
```

output
```
(2*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec
[c + d*x]])/(d*Sqrt[(b + a*Cos[c + d*x])/(a + b)])
```

3.839.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 4343
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]])   Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a
, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

3.839.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 868 vs. $2(90) = 180$.

Time = 3.81 (sec) , antiderivative size = 869, normalized size of antiderivative = 12.97

method	result
default	$\frac{2\sqrt{-\frac{(1-\cos(dx+c))^2\csc(dx+c)^2-1}{(1-\cos(dx+c))^2\csc(dx+c)^2+1}}\sqrt{\frac{a(1-\cos(dx+c))^2\csc(dx+c)^2-b(1-\cos(dx+c))^2\csc(dx+c)^2-a-b}{(1-\cos(dx+c))^2\csc(dx+c)^2-1}}\left(\sqrt{\frac{a-b}{a+b}}\,a(1-\cos(dx+c))^3\csc(dx+c\right.}{}$
risch	Expression too large to display

input
```
int(cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/d/((a-b)/(a+b))^(1/2)*(-((1-cos(d*x+c))^2*csc(d*x+c)^2-1)/((1-cos(d*x+c)
)^2*csc(d*x+c)^2+1))^(1/2)*((a*(1-cos(d*x+c))^2*csc(d*x+c)^2-b*(1-cos(d*x+
c))^2*csc(d*x+c)^2-a-b)/((1-cos(d*x+c))^2*csc(d*x+c)^2-1))^(1/2)*(((a-b)/(
a+b))^(1/2)*a*(1-cos(d*x+c))^3*csc(d*x+c)^3-((a-b)/(a+b))^(1/2)*b*(1-cos(d
*x+c))^3*csc(d*x+c)^3+(-(a*(1-cos(d*x+c))^2*csc(d*x+c)^2-b*(1-cos(d*x+c))^
2*csc(d*x+c)^2-a-b)/(a+b))^(1/2)*((1-cos(d*x+c))^2*csc(d*x+c)^2+1)^(1/2)*E
llipticF(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2)
)*a-(-(a*(1-cos(d*x+c))^2*csc(d*x+c)^2-b*(1-cos(d*x+c))^2*csc(d*x+c)^2-a-b
)/(a+b))^(1/2)*((1-cos(d*x+c))^2*csc(d*x+c)^2+1)^(1/2)*EllipticF(((a-b)/(a
+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*b-(-(a*(1-cos(d*
x+c))^2*csc(d*x+c)^2-b*(1-cos(d*x+c))^2*csc(d*x+c)^2-a-b)/(a+b))^(1/2)*((1
-cos(d*x+c))^2*csc(d*x+c)^2+1)^(1/2)*EllipticE(((a-b)/(a+b))^(1/2)*(-cot(d
*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*a+(-(a*(1-cos(d*x+c))^2*csc(d*x+c)
^2-b*(1-cos(d*x+c))^2*csc(d*x+c)^2-a-b)/(a+b))^(1/2)*((1-cos(d*x+c))^2*csc
(d*x+c)^2+1)^(1/2)*EllipticE(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),
(-(a+b)/(a-b))^(1/2))*b-((a-b)/(a+b))^(1/2)*a*(-cot(d*x+c)+csc(d*x+c))-((a
-b)/(a+b))^(1/2)*b*(-cot(d*x+c)+csc(d*x+c)))/(a*(1-cos(d*x+c))^2*csc(d*x+c
)^2-b*(1-cos(d*x+c))^2*csc(d*x+c)^2-a-b)
```

3.839.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 355, normalized size of antiderivative = 5.30

$$\int \sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\,dx$$
$$= \frac{-i\sqrt{2}\sqrt{a}b\text{weierstrassPInverse}\left(-\frac{4\,(3\,a^2-4\,b^2)}{3\,a^2},\frac{8\,(9\,a^2b-8\,b^3)}{27\,a^3},\frac{3\,a\cos(dx+c)+3i\,a\sin(dx+c)+2\,b}{3\,a}\right)+i\sqrt{2}\sqrt{a}b\text{weierstras}}{}$$

input
```
integrate(cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
1/3*(-I*sqrt(2)*sqrt(a)*b*weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/
27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x + c) + 2*b
)/a) + I*sqrt(2)*sqrt(a)*b*weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8
/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*
b)/a) + 3*I*sqrt(2)*a^(3/2)*weierstrassZeta(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27
*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27
*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x + c) + 2*b)/
a)) - 3*I*sqrt(2)*a^(3/2)*weierstrassZeta(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(
9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(
9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a)
))/(a*d)
```

3.839.6 Sympy [F]

$$\int \sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\,dx = \int \sqrt{a+b\sec{(c+dx)}}\sqrt{\cos{(c+dx)}}\,dx$$

input
```
integrate(cos(d*x+c)**(1/2)*(a+b*sec(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(a + b*sec(c + d*x))*sqrt(cos(c + d*x)), x)
```

3.839.7 Maxima [F]

$$\int \sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\,dx = \int \sqrt{b\sec{(dx+c)}+a}\sqrt{\cos{(dx+c)}}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sec(d*x + c) + a)*sqrt(cos(d*x + c)), x)
```

3.839.8 Giac [F]

$$\int \sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\,dx = \int \sqrt{b\sec(dx+c)+a}\sqrt{\cos(dx+c)}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*sec(d*x + c) + a)*sqrt(cos(d*x + c)), x)
```

3.839.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\,dx = \int \sqrt{\cos(c+d\,x)}\,\sqrt{a+\frac{b}{\cos(c+d\,x)}}\,dx$$

input
```
int(cos(c + d*x)^(1/2)*(a + b/cos(c + d*x))^(1/2),x)
```

output
```
int(cos(c + d*x)^(1/2)*(a + b/cos(c + d*x))^(1/2), x)
```

3.840 $\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\cos(c+dx)}}\,dx$

3.840.1 Optimal result 6256
3.840.2 Mathematica [C] (warning: unable to verify) 6256
3.840.3 Rubi [A] (verified) 6257
3.840.4 Maple [C] (verified) 6261
3.840.5 Fricas [F(-1)] 6261
3.840.6 Sympy [F] 6262
3.840.7 Maxima [F] 6262
3.840.8 Giac [F] 6262
3.840.9 Mupad [F(-1)] 6263

3.840.1 Optimal result

Integrand size = 25, antiderivative size = 138

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\cos(c+dx)}}\,dx = \frac{2a\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} + \frac{2b\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}$$

output
```
2*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+
1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)/d/cos(d*x+c
)^(1/2)/(a+b*sec(d*x+c))^(1/2)+2*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*
x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2,2^(1/2)*(a/(a+b))^(1/2))*((b+a*co
s(d*x+c))/(a+b))^(1/2)/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)
```

3.840.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 31.60 (sec) , antiderivative size = 14885, normalized size of antiderivative = 107.86

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\cos(c+dx)}}\,dx = \text{Result too large to show}$$

input
```
Integrate[Sqrt[a + b*Sec[c + d*x]]/Sqrt[Cos[c + d*x]],x]
```

output
```
Result too large to show
```

3.840.3 Rubi [A] (verified)

Time = 1.34 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.15, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {3042, 4752, 3042, 4341, 3042, 4345, 3042, 3142, 3042, 3140, 4346, 3042, 3286, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\cos(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{4341}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(b\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx+a\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}dx\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(a\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+b\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)$$

$$\Big\downarrow \text{4345}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(b\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{a\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{1}{\sqrt{b+a\cos(c+dx}}}{\sqrt{a+b\sec(c+dx)}}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(b\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{a\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{1}{\sqrt{b+a\sin(c+dx}}}{\sqrt{a+b\sec(c+dx)}}\right.$$

$\downarrow$ 3142

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(b\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(b\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}}{\sqrt{a+b\sec(c+dx)}}\right.$$

$\downarrow$ 3140

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(b\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+d\right.}{d\sqrt{a+b\sec(c+dx)}}\right.$$

$\downarrow$ 4346

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{b\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{\sec(c+dx)}{\sqrt{b+a\cos(c+dx)}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{2a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)}{a+b}}}{d\sqrt{a+b}}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{b\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{2a\sqrt{\sec(c+d}}{}\right.$$

3.840. $\int\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\cos(c+dx)}}\,dx$

$$\Big\downarrow \text{3286}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{b\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{\sec(c+dx)}{\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{2a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}{d\sqrt{a+b\sec}}\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{b\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{2a\sqrt{\sec(c+dx)}}{}\right.$$

$$\Big\downarrow \text{3284}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}+\frac{2b\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(}{}}}{d\sqrt{}}\right.$$

input
```
Int[Sqrt[a + b*Sec[c + d*x]]/Sqrt[Cos[c + d*x]],x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*a*Sqrt[(b + a*Cos[c + d*x])/(a +
 b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(d*Sqrt[a +
 b*Sec[c + d*x]]) + (2*b*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2,
(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(d*Sqrt[a + b*Sec[c + d*x]
]))
```

3.840.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3286
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]]   Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
 d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

rule 4341
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[a   Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e +
f*x]], x], x] + Simp[b/d   Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Csc[e + f*
x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4345
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]])   Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[
{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4346
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Simp[d*Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x
]]/Sqrt[a + b*Csc[e + f*x]])   Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

3.840. $\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\cos(c+dx)}}\,dx$

3.840.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 7.01 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.78

method	result
default	$\frac{2\left(\operatorname{EllipticF}\left(\sqrt{\frac{a-b}{a+b}}\,(-\cot(dx+c)+\csc(dx+c)),\sqrt{-\frac{a+b}{a-b}}\right)a-\operatorname{EllipticF}\left(\sqrt{\frac{a-b}{a+b}}\,(-\cot(dx+c)+\csc(dx+c)),\sqrt{-\frac{a+b}{a-b}}\right)b+2\operatorname{EllipticPi}\left(\right.\right.}{d\sqrt{\frac{a-b}{a+b}}\,(b+a\cos(dx+c))\sqrt{\frac{1}{\cos(dx+}}}$

input
```
int((a+b*sec(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/d/((a-b)/(a+b))^(1/2)*(EllipticF(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*
x+c)),(-(a+b)/(a-b))^(1/2))*a-EllipticF(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+c
sc(d*x+c)),(-(a+b)/(a-b))^(1/2))*b+2*EllipticPi(((a-b)/(a+b))^(1/2)*(-cot(
d*x+c)+csc(d*x+c)),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*b)*(a+b*sec(d*x+c))^
(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^(1/2)/(b+
a*cos(d*x+c))/(1/(cos(d*x+c)+1))^(1/2)
```

3.840.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\cos(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate((a+b*sec(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.840.6 Sympy [F]

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\cos(c+dx)}}\,dx=\int \frac{\sqrt{a+b\sec{(c+dx)}}}{\sqrt{\cos{(c+dx)}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))**(1/2)/cos(d*x+c)**(1/2),x)
```

output
```
Integral(sqrt(a + b*sec(c + d*x))/sqrt(cos(c + d*x)), x)
```

3.840.7 Maxima [F]

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\cos(c+dx)}}\,dx=\int \frac{\sqrt{b\sec{(dx+c)}+a}}{\sqrt{\cos{(dx+c)}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sec(d*x + c) + a)/sqrt(cos(d*x + c)), x)
```

3.840.8 Giac [F]

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\cos(c+dx)}}\,dx=\int \frac{\sqrt{b\sec{(dx+c)}+a}}{\sqrt{\cos{(dx+c)}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*sec(d*x + c) + a)/sqrt(cos(d*x + c)), x)
```

3.840.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\cos(c+dx)}}\,dx=\int \frac{\sqrt{a+\frac{b}{\cos(c+d\,x)}}}{\sqrt{\cos\left(c+d\,x\right)}}\,dx$$

input
```
int((a + b/cos(c + d*x))^(1/2)/cos(c + d*x)^(1/2),x)
```

output
```
int((a + b/cos(c + d*x))^(1/2)/cos(c + d*x)^(1/2), x)
```

3.841 $\int \frac{\sqrt{a+b\sec(c+dx)}}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

3.841.1 Optimal result 6264
3.841.2 Mathematica [C] (warning: unable to verify) 6265
3.841.3 Rubi [A] (verified) 6265
3.841.4 Maple [C] (verified) 6271
3.841.5 Fricas [F(-1)] 6272
3.841.6 Sympy [F] 6273
3.841.7 Maxima [F] 6273
3.841.8 Giac [F] 6273
3.841.9 Mupad [F(-1)] 6274

3.841.1 Optimal result

Integrand size = 25, antiderivative size = 237

$$\begin{aligned}\int \frac{\sqrt{a+b\sec(c+dx)}}{\cos^{\frac{3}{2}}(c+dx)}\,dx = {} & \frac{b\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} \\ & + \frac{a\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} \\ & - \frac{\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+dx)}}{d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}} \\ & + \frac{\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}}\end{aligned}$$

output
```
b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/
2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)/d/cos(d*x+c)^
(1/2)/(a+b*sec(d*x+c))^(1/2)+a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/
2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2,2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*
x+c))/(a+b))^(1/2)/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)+sin(d*x+c)*(a
+b*sec(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)-(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1
/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*cos(d*
x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2)/d/((b+a*cos(d*x+c))/(a+b))^(1/2)
```

3.841.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 32.38 (sec) , antiderivative size = 33693, normalized size of antiderivative = 142.16

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \text{Result too large to show}$$

input
```
Integrate[Sqrt[a + b*Sec[c + d*x]]/Cos[c + d*x]^(3/2),x]
```

output
```
Result too large to show
```

3.841.3 Rubi [A] (verified)

Time = 2.44 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.16, number of steps used = 25, number of rules used = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {3042, 4752, 3042, 4342, 25, 3042, 4597, 3042, 4346, 3042, 3286, 3042, 3284, 4349, 3042, 4343, 3042, 3134, 3042, 3132, 4345, 3042, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\cos^{\frac{3}{2}}(c+dx)}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx$$

$$\Big\downarrow \; 4752$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}dx$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \; 4342$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{2}\int -\frac{a-a\sec^2(c+dx)}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx+\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+\right.$$

$\Big\downarrow$ 25

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{1}{2}\int\frac{a-a\sec^2(c+dx)}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx}}\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{1}{2}\int\frac{a-a\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(\right.}}\right.$$

$\Big\downarrow$ 4597

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{2}\left(a\int\frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx-a\int\frac{1}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx\right)+\frac{\sin(c}{}\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{2}\left(a\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-a\int\frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\right.}}\right.\right.$$

$\Big\downarrow$ 4346

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{2}\left(\frac{a\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{\sec(c+dx)}{\sqrt{b+a\cos(c+dx)}}dx}{\sqrt{a+b\sec(c+dx)}}-a\int\frac{}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}\right.\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{2}\left(\frac{a\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{\sqrt{a+b\sec(c+dx)}}-a\int\frac{}{\sqrt{\csc}}\right.\right.$$

$\Big\downarrow$ 3286

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{2}\left(\frac{a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{\sec(c+dx)}{\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}-a\int\frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a}}\right.\right.$$

3.841. $\int\frac{\sqrt{a+b\sec(c+dx)}}{\cos^{\frac{3}{2}}(c+dx)}dx$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{2}\left(\frac{a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}-a\int\frac{}{\sqrt{\csc(c}}\right.\right.$$

$$\Big\downarrow \text{3284}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{2}\left(\frac{2a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}-a\int\frac{}{\sqrt{\csc(c+d}}\right.\right.$$

$$\Big\downarrow \text{4349}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{2}\left(\frac{2a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}-a\left(\frac{\int\frac{\sqrt{a+b\sec(c+}}{\sqrt{\sec(c+dx}}}{a}\right.\right.\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{2}\left(\frac{2a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}-a\left(\frac{\int\frac{\sqrt{a+b\csc(c+}}{\sqrt{\csc(c+dx}}}{a}\right.\right.\right.$$

$$\Big\downarrow \text{4343}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{2}\left(\frac{2a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}-a\left(\frac{\sqrt{a+b\sec(c}}{a\sqrt{\sec(c}}\right.\right.\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{2}\left(\frac{2a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}-a\left(\frac{\sqrt{a+b\sec(c}}{a\sqrt{\sec}}\right.\right.\right.$$

$$\Big\downarrow \text{3134}$$

3.841. $\int\frac{\sqrt{a+b\sec(c+dx)}}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{2}\left(\frac{2a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}-a\left(\frac{\sqrt{a+b\sec(c}}{a\sqrt{\sec}}\right.\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{2}\left(\frac{2a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}-a\left(\frac{\sqrt{a+b\sec(c}}{a\sqrt{s}}\right.\right.\right.$$

$\downarrow$ 3132

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{2}\left(\frac{2a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}-a\left(\frac{2\sqrt{a+b\sec(}}{ad\sqrt{\sec(}}\right.\right.\right.$$

$\downarrow$ 4345

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{2}\left(\frac{2a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}-a\left(\frac{2\sqrt{a+b\sec(}}{ad\sqrt{\sec(}}\right.\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{2}\left(\frac{2a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}-a\left(\frac{2\sqrt{a+b\sec(}}{ad\sqrt{\sec(}}\right.\right.\right.$$

$\downarrow$ 3142

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{2}\left(\frac{2a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}-a\left(\frac{2\sqrt{a+b\sec(}}{ad\sqrt{\sec(}}\right.\right.\right.$$

$\downarrow$ 3042

3.841. $\int \frac{\sqrt{a+b\sec(c+dx)}}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{2}\left(\frac{2a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}-a\left(\frac{2\sqrt{a+b\sec(}}{ad\sqrt{\sec(}}\right.\right.\right.$$

$\downarrow$ 3140

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}+\frac{1}{2}\left(\frac{2a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\text{E}}{d\sqrt{a+b\sec(}}\right.\right.$$

input
```
Int[Sqrt[a + b*Sec[c + d*x]]/Cos[c + d*x]^(3/2),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(((2*a*Sqrt[(b + a*Cos[c + d*x])/(a
+ b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(d*Sqr
t[a + b*Sec[c + d*x]]) - a*((-2*b*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Ellip
ticF[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(a*d*Sqrt[a + b*Sec[c
 + d*x]]) + (2*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*
x]])/(a*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]])))/2 + (Sq
rt[Sec[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/d)
```

3.841.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3286
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]]   Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
 d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

rule 4342
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[-2*d*Cos[e + f*x]*Sqrt[a + b*Csc[e + f*x]]*((d*
Csc[e + f*x])^(n - 1)/(f*(2*n - 1))), x] + Simp[d^2/(2*n - 1)   Int[(d*Csc[
e + f*x])^(n - 2)*(Simp[2*a*(n - 2) + b*(2*n - 3)*Csc[e + f*x] + a*Csc[e +
f*x]^2, x]/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, d, e, f}, x] &
& NeQ[a^2 - b^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

rule 4343
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]])   Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a
, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

3.841. $\int \frac{\sqrt{a+b\sec(c+dx)}}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

rule 4345
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]])   Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[
{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4346
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Simp[d*Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x
]]/Sqrt[a + b*Csc[e + f*x]])   Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4349
```
Int[1/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)]), x_Symbol] :> Simp[1/a   Int[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Cs
c[e + f*x]], x], x] - Simp[b/(a*d)   Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Cs
c[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4597
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))/(Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Simp[C/d^
2   Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Csc[e + f*x]], x], x] + Simp[A
Int[1/(Sqrt[d*Csc[e + f*x]]*Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a,
b, d, e, f, A, C}, x] && NeQ[a^2 - b^2, 0]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

3.841.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 8.00 (sec) , antiderivative size = 995, normalized size of antiderivative = 4.20

method	result	size
default	Expression too large to display	995

input
```
int((a+b*sec(d*x+c))^(1/2)/cos(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/d/((a-b)/(a+b))^(1/2)*(-EllipticE(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d
*x+c)),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/
2)*(1/(cos(d*x+c)+1))^(1/2)*a*cos(d*x+c)^3+EllipticE(((a-b)/(a+b))^(1/2)*(
-cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(c
os(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*b*cos(d*x+c)^3+2*EllipticPi((
(a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(a+b)/(a-b),I/((a-b)/(a+b))^(1
/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1
/2)*a*cos(d*x+c)^3-2*EllipticE(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)
),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1
/(cos(d*x+c)+1))^(1/2)*a*cos(d*x+c)^2+2*EllipticE(((a-b)/(a+b))^(1/2)*(-co
t(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(
d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*b*cos(d*x+c)^2+4*EllipticPi(((a-
b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2)
)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)
*a*cos(d*x+c)^2-EllipticE(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(-(
a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos
(d*x+c)+1))^(1/2)*a*cos(d*x+c)+EllipticE(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+
csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1)
)^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*b*cos(d*x+c)+2*EllipticPi(((a-b)/(a+b))^(
1/2)*(-cot(d*x+c)+csc(d*x+c)),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*(1/(a+...
```

3.841.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+b*sec(d*x+c))^(1/2)/cos(d*x+c)^(3/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.841.6 Sympy [F]

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\cos^{\frac{3}{2}}(c+dx)}\,dx=\int \frac{\sqrt{a+b\sec{(c+dx)}}}{\cos^{\frac{3}{2}}{(c+dx)}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))**(1/2)/cos(d*x+c)**(3/2),x)
```

output
```
Integral(sqrt(a + b*sec(c + d*x))/cos(c + d*x)**(3/2), x)
```

3.841.7 Maxima [F]

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\cos^{\frac{3}{2}}(c+dx)}\,dx=\int \frac{\sqrt{b\sec{(dx+c)}+a}}{\cos{(dx+c)}^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(1/2)/cos(d*x+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sec(d*x + c) + a)/cos(d*x + c)^(3/2), x)
```

3.841.8 Giac [F]

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\cos^{\frac{3}{2}}(c+dx)}\,dx=\int \frac{\sqrt{b\sec{(dx+c)}+a}}{\cos{(dx+c)}^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(1/2)/cos(d*x+c)^(3/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*sec(d*x + c) + a)/cos(d*x + c)^(3/2), x)
```

3.841.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\cos^{\frac{3}{2}}(c+dx)}\,dx=\int \frac{\sqrt{a+\frac{b}{\cos(c+d\,x)}}}{\cos\left(c+d\,x\right)^{3/2}}\,dx$$

input
```
int((a + b/cos(c + d*x))^(1/2)/cos(c + d*x)^(3/2),x)
```

output
```
int((a + b/cos(c + d*x))^(1/2)/cos(c + d*x)^(3/2), x)
```

3.842 $\int \cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx$

3.842.1 Optimal result 6275
3.842.2 Mathematica [C] (warning: unable to verify) 6276
3.842.3 Rubi [A] (verified) 6276
3.842.4 Maple [B] (verified) 6283
3.842.5 Fricas [C] (verification not implemented) 6284
3.842.6 Sympy [F(-1)] 6285
3.842.7 Maxima [F] 6285
3.842.8 Giac [F] 6286
3.842.9 Mupad [F(-1)] 6286

3.842.1 Optimal result

Integrand size = 25, antiderivative size = 303

$$\begin{aligned}\int \cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx &= \frac{2(25a^4-31a^2b^2+6b^4)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{105a^2d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}\\ &+\frac{4b(41a^2-3b^2)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+dx)}}{105a^2d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}}\\ &+\frac{2(25a^2+3b^2)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{105ad}\\ &+\frac{16b\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{35d}\\ &+\frac{2a\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{7d}\end{aligned}$$

output
```
2/105*(25*a^4-31*a^2*b^2+6*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1
/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+
c))/(a+b))^(1/2)/a^2/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)+16/35*b*cos
(d*x+c)^(3/2)*sin(d*x+c)*(a+b*sec(d*x+c))^(1/2)/d+2/7*a*cos(d*x+c)^(5/2)*s
in(d*x+c)*(a+b*sec(d*x+c))^(1/2)/d+2/105*(25*a^2+3*b^2)*sin(d*x+c)*cos(d*x
+c)^(1/2)*(a+b*sec(d*x+c))^(1/2)/a/d+4/105*b*(41*a^2-3*b^2)*(cos(1/2*d*x+1
/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/
(a+b))^(1/2))*cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2)/a^2/d/((b+a*cos(d*x+
c))/(a+b))^(1/2)
```

3.842.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 11.22 (sec) , antiderivative size = 383, normalized size of antiderivative = 1.26

$$\int \cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx = \frac{\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}\left(a(b+a\cos(c+dx))\left(65a^2+6b^2+48ab\cos(c+dx)+15\right.\right.}{}$$

input
```
Integrate[Cos[c + d*x]^(7/2)*(a + b*Sec[c + d*x])^(3/2),x]
```

output
```
(Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^(3/2)*(a*(b + a*Cos[c + d*x])*(65
*a^2 + 6*b^2 + 48*a*b*Cos[c + d*x] + 15*a^2*Cos[2*(c + d*x)])*Sin[c + d*x]
 - (2*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(3/2)*((2*I)*b*(-41*a^3 - 41*a^2*b
 + 3*a*b^2 + 3*b^3)*EllipticE[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b
)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a +
b)] + I*a*(25*a^3 + 82*a^2*b + 51*a*b^2 - 6*b^3)*EllipticF[I*ArcSinh[Tan[(
c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*
x])*Sec[(c + d*x)/2]^2)/(a + b)] + 2*b*(-41*a^2 + 3*b^2)*(b + a*Cos[c + d*
x])*(Sec[(c + d*x)/2]^2)^(3/2)*Tan[(c + d*x)/2]))/Sec[c + d*x]^(3/2)))/(10
5*a^2*d*(b + a*Cos[c + d*x])^2)
```

3.842.3 Rubi [A] (verified)

Time = 2.73 (sec) , antiderivative size = 332, normalized size of antiderivative = 1.10, number of steps used = 24, number of rules used = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.960$, Rules used = {3042, 4752, 3042, 4351, 25, 3042, 4592, 27, 3042, 4592, 27, 3042, 4523, 3042, 4343, 3042, 3134, 3042, 3132, 4345, 3042, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^{7/2}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}dx$$

$$\Big\downarrow \; 4752$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{7}{2}}(c+dx)}dx$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx$$

$$\Big\downarrow \; 4351$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2a\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{7d\sec^{\frac{5}{2}}(c+dx)}-\frac{1}{7}\int -\frac{4ab\sec^2(c+dx)+\left(5a^2+7b^2\right)\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}\right.$$

$$\Big\downarrow \; 25$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}\int \frac{4ab\sec^2(c+dx)+\left(5a^2+7b^2\right)\sec(c+dx)+8ab}{\sec^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}dx+\frac{2a\sin(c+dx)\sqrt{a+b\sec}}{7d\sec^{\frac{5}{2}}(c+dx)}\right.$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}\int \frac{4ab\csc\left(c+dx+\frac{\pi}{2}\right)^2+\left(5a^2+7b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)+8ab}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a\sin(c+dx)}{7d\sec}\right.$$

$$\Big\downarrow \; 4592$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}\left(\frac{16b\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5d\sec^{\frac{3}{2}}(c+dx)}-\frac{2\int -\frac{44b\sec(c+dx)a^2+16b^2\sec^2(c+dx)a+(25a^2+3b}{2\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}}{5a}\right.\right.$$

$$\Big\downarrow \; 27$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}\left(\frac{\int \frac{44b\sec(c+dx)a^2+16b^2\sec^2(c+dx)a+\left(25a^2+3b^2\right)a}{\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}dx}{5a}+\frac{16b\sin(c+dx)\sqrt{a+b\sec(c+d}}{5d\sec^{\frac{3}{2}}(c+dx)}\right.\right.$$

$$\Big\downarrow \; 3042$$

3.842. $\int \cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}\left(\frac{\int \frac{44b\csc\left(c+dx+\frac{\pi}{2}\right)a^2+16b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2a+\left(25a^2+3b^2\right)a}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{5a}+\frac{16b\sin(c+dx)\sqrt{a+b\sec}}{5d\sec^{\frac{3}{2}}(c+dx)}\right.\right.$$

$\downarrow$ 4592

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}\left(\frac{\frac{2\left(25a^2+3b^2\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3d\sqrt{\sec(c+dx)}}-\frac{2\int -\frac{\left(25a^2+51b^2\right)\sec(c+dx)a^2+2b\left(41a^2-3b^2\right)a}{2\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{3a}}{5a}+\frac{16b}{}\right.\right.$$

$\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}\left(\frac{\frac{\int \frac{\left(25a^2+51b^2\right)\sec(c+dx)a^2+2b\left(41a^2-3b^2\right)a}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{3a}+\frac{2\left(25a^2+3b^2\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3d\sqrt{\sec(c+dx)}}}{5a}+\frac{16b\sin}{}\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}\left(\frac{\frac{\int \frac{\left(25a^2+51b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)a^2+2b\left(41a^2-3b^2\right)a}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{3a}+\frac{2\left(25a^2+3b^2\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3d\sqrt{\sec(c+dx)}}}{5a}+\frac{16b}{}\right.\right.$$

$\downarrow$ 4523

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}\left(\frac{\frac{2b\left(41a^2-3b^2\right)\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}dx+\left(25a^4-31a^2b^2+6b^4\right)\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}dx}{3a}+\frac{2\left(25a^2+3b^2\right)\sin(c+c}{3d\sqrt{\sec}}}{5a}\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}\left(\frac{\frac{2b\left(41a^2-3b^2\right)\int \frac{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\left(25a^4-31a^2b^2+6b^4\right)\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{3a}+\frac{2\left(25a^2+3b^2\right)}{}}{5a}\right.\right.$$

3.842. $\int \cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx$

$$\Big\downarrow \text{4343}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}\left(\frac{\frac{\frac{2b\left(41a^2-3b^2\right)\sqrt{a+b\sec(c+dx)}\int\sqrt{b+a\cos(c+dx)}dx}{\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}+\left(25a^4-31a^2b^2+6b^4\right)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{3a}+\frac{2(2}{}}{5a}\right.\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}\left(\frac{\frac{\frac{2b\left(41a^2-3b^2\right)\sqrt{a+b\sec(c+dx)}\int\sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}+\left(25a^4-31a^2b^2+6b^4\right)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{3a}+}{5a}\right.\right.$$

$$\Big\downarrow \text{3134}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}\left(\frac{\frac{\frac{2b\left(41a^2-3b^2\right)\sqrt{a+b\sec(c+dx)}\int\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}dx}{\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\left(25a^4-31a^2b^2+6b^4\right)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{3a}+}{5a}\right.\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}\left(\frac{\frac{\frac{2b\left(41a^2-3b^2\right)\sqrt{a+b\sec(c+dx)}\int\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}dx}{\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\left(25a^4-31a^2b^2+6b^4\right)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{3a}}{5a}\right.\right.$$

$$\Big\downarrow \text{3132}$$

3.842. $\int \cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}\left(\frac{\frac{(25a^4-31a^2b^2+6b^4)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{4b\left(41a^2-3b^2\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{3a}+\frac{2(25a}{}}{5a}\right.\right.$$

$\downarrow$ 4345

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}\left(\frac{\frac{\left(25a^4-31a^2b^2+6b^4\right)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{1}{\sqrt{b+a\cos(c+dx)}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{4b\left(41a^2-3b^2\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+\right.}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{3a}}{5a}\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}\left(\frac{\frac{\left(25a^4-31a^2b^2+6b^4\right)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{1}{\sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{4b\left(41a^2-3b^2\right)\sqrt{a+b\sec(c+dx)}E\left(\right.}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+d}{a+b}}}}{3a}}{5a}\right.\right.$$

$\downarrow$ 3142

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}\left(\frac{\frac{\left(25a^4-31a^2b^2+6b^4\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{4b\left(41a^2-3b^2\right)\sqrt{a+b\sec(c+dx)}E\left(\right.}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+d}{a+b}}}}{3a}}{5a}\right.\right.$$

$\downarrow$ 3042

3.842. $\int \cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}\left(\frac{\frac{\frac{\left(25a^4-31a^2b^2+6b^4\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{4b\left(41a^2-3b^2\right)\sqrt{a+b\sec(c+dx}}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos}{}}}}{3a}}{5a}\right.\right.$$

$\downarrow$ 3140

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}\left(\frac{\frac{2\left(25a^2+3b^2\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3d\sqrt{\sec(c+dx)}}+\frac{\frac{4b\left(41a^2-3b^2\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{2\left(25a^4-31\right.}{}}{}}{5a}\right.\right.$$

input
```
Int[Cos[c + d*x]^(7/2)*(a + b*Sec[c + d*x])^(3/2),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*a*Sqrt[a + b*Sec[c + d*x]]*Sin[c
 + d*x])/(7*d*Sec[c + d*x]^(5/2)) + ((16*b*Sqrt[a + b*Sec[c + d*x]]*Sin[c
+ d*x])/(5*d*Sec[c + d*x]^(3/2)) + (((2*(25*a^4 - 31*a^2*b^2 + 6*b^4)*Sqrt
[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[
Sec[c + d*x]])/(d*Sqrt[a + b*Sec[c + d*x]]) + (4*b*(41*a^2 - 3*b^2)*Ellipt
icE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(d*Sqrt[(b + a*C
os[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]))/(3*a) + (2*(25*a^2 + 3*b^2)*Sqr
t[a + b*Sec[c + d*x]]*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]]))/(5*a))/7)
```

3.842.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 4343
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]])   Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a
, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4345
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]])   Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[
{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4351
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(3/2), x_Symbol] :> Simp[a*Cot[e + f*x]*Sqrt[a + b*Csc[e + f*x]]*((d*C
sc[e + f*x])^n/(f*n)), x] + Simp[1/(2*d*n)   Int[((d*Csc[e + f*x])^(n + 1)/
Sqrt[a + b*Csc[e + f*x]])*Simp[a*b*(2*n - 1) + 2*(b^2*n + a^2*(n + 1))*Csc[
e + f*x] + a*b*(2*n + 3)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f
}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1] && IntegersQ[2*n]
```

3.842. $\int \cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx$

rule 4523
```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Simp[A/a   I
nt[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Simp[(A*b - a*B)
/(a*d)   Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ
[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

rule 4592
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Simp[1/(a*d*n)   Int[(a + b*Csc[e + f*x])^m
*(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*
Csc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

3.842.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2765 vs. $2(327) = 654$.

Time = 8.02 (sec) , antiderivative size = 2766, normalized size of antiderivative = 9.13

method	result	size
default	Expression too large to display	2766

input
```
int(cos(d*x+c)^(7/2)*(a+b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/105/d/a^2/((a-b)/(a+b))^(1/2)*(-6*(1/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a
*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE(((a-b)/(a+b))^(1/2)*(-cot(d*x
+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*a*b^3*cos(d*x+c)^2+6*(1/(cos(d*x+c)+
1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE(((a-b)
/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*b^4+25*(1/(co
s(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ellipti
cF(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*a^4-
6*((a-b)/(a+b))^(1/2)*b^4*sin(d*x+c)-82*(1/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*
(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF(((a-b)/(a+b))^(1/2)*(-cot
(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*a^3*b*cos(d*x+c)^2+51*(1/(cos(d*
x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((
(a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*a^2*b^2*
cos(d*x+c)^2+6*(1/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x
+c)+1))^(1/2)*EllipticF(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(-(a+
b)/(a-b))^(1/2))*a*b^3*cos(d*x+c)^2+164*(1/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*
(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE(((a-b)/(a+b))^(1/2)*(-cot
(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*a^3*b*cos(d*x+c)-164*(1/(cos(d*x
+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE(((
a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*a^2*b^2*c
os(d*x+c)-12*(1/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*...
```

3.842.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 491, normalized size of antiderivative = 1.62

$$\int \cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx = \frac{6\left(15\,a^4\cos\left(dx+c\right)^2+24\,a^3b\cos\left(dx+c\right)+25\,a^4+3\,a^2b^2\right)\sqrt{\frac{a\cos(dx+c)+b}{\cos(dx+c)}}\sqrt{\cos\left(dx+c\right)}\sin\left(d\right.}{}$$

input
```
integrate(cos(d*x+c)^(7/2)*(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
1/315*(6*(15*a^4*cos(d*x + c)^2 + 24*a^3*b*cos(d*x + c) + 25*a^4 + 3*a^2*b
^2)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c
) + sqrt(2)*(-75*I*a^4 + 11*I*a^2*b^2 - 12*I*b^4)*sqrt(a)*weierstrassPInve
rse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x
 + c) + 3*I*a*sin(d*x + c) + 2*b)/a) + sqrt(2)*(75*I*a^4 - 11*I*a^2*b^2 +
12*I*b^4)*sqrt(a)*weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^
2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a) - 6
*sqrt(2)*(-41*I*a^3*b + 3*I*a*b^3)*sqrt(a)*weierstrassZeta(-4/3*(3*a^2 - 4
*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4
*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d
*x + c) + 2*b)/a)) - 6*sqrt(2)*(41*I*a^3*b - 3*I*a*b^3)*sqrt(a)*weierstras
sZeta(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPIn
verse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d
*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a)))/(a^3*d)
```

3.842.6 Sympy [F(-1)]

Timed out.

$$\int \cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(7/2)*(a+b*sec(d*x+c))**(3/2),x)
```

output
```
Timed out
```

3.842.7 Maxima [F]

$$\int \cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx = \int (b\sec(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^{\frac{7}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(7/2)*(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^(3/2)*cos(d*x + c)^(7/2), x)
```

3.842.8 Giac [F]

$$\int \cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx = \int (b\sec(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^{\frac{7}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(7/2)*(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^(3/2)*cos(d*x + c)^(7/2), x)
```

3.842.9 Mupad [F(-1)]

Timed out.

$$\int \cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx = \int \cos(c+d\,x)^{7/2}\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}dx$$

input
```
int(cos(c + d*x)^(7/2)*(a + b/cos(c + d*x))^(3/2),x)
```

output
```
int(cos(c + d*x)^(7/2)*(a + b/cos(c + d*x))^(3/2), x)
```

3.843 $\int \cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx$

3.843.1 Optimal result 6287
3.843.2 Mathematica [C] (verified) 6288
3.843.3 Rubi [A] (verified) 6288
3.843.4 Maple [B] (verified) 6294
3.843.5 Fricas [C] (verification not implemented) 6295
3.843.6 Sympy [F(-1)] 6296
3.843.7 Maxima [F] 6296
3.843.8 Giac [F] 6297
3.843.9 Mupad [F(-1)] 6297

3.843.1 Optimal result

Integrand size = 25, antiderivative size = 240

$$\begin{aligned}\int \cos^{\frac{5}{2}}(c+dx)(a&+b\sec(c+dx))^{3/2}\,dx=\frac{2b(a^2-b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{5ad\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}\\&+\frac{2(3a^2+b^2)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+dx)}}{5ad\sqrt{\frac{b+a\cos(c+dx)}{a+b}}}\\&+\frac{4b\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{5d}\\&+\frac{2a\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{5d}\end{aligned}$$

output
```
2/5*b*(a^2-b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(
sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)
/a/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)+2/5*a*cos(d*x+c)^(3/2)*sin(d*
x+c)*(a+b*sec(d*x+c))^(1/2)/d+4/5*b*sin(d*x+c)*cos(d*x+c)^(1/2)*(a+b*sec(d
*x+c))^(1/2)/d+2/5*(3*a^2+b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/
2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*cos(d*x+c)^(1/2
)*(a+b*sec(d*x+c))^(1/2)/a/d/((b+a*cos(d*x+c))/(a+b))^(1/2)
```

3.843.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 9.27 (sec) , antiderivative size = 344, normalized size of antiderivative = 1.43

$$\int \cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx = \frac{\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}\left(2(b+a\cos(c+dx))(2b+a\cos(c+dx))\sin(c+dx)-\frac{2(}{}\right.}{}$$

input
```
Integrate[Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^(3/2),x]
```

output
```
(Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^(3/2)*(2*(b + a*Cos[c + d*x])*(2*
b + a*Cos[c + d*x])*Sin[c + d*x] - (2*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(3
/2)*((-I)*(3*a^3 + 3*a^2*b + a*b^2 + b^3)*EllipticE[I*ArcSinh[Tan[(c + d*x
)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec
[(c + d*x)/2]^2)/(a + b)] + I*a*(3*a^2 + 4*a*b + b^2)*EllipticF[I*ArcSinh[
Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c
 + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] - (3*a^2 + b^2)*(b + a*Cos[c + d*x])
*(Sec[(c + d*x)/2]^2)^(3/2)*Tan[(c + d*x)/2]))/(a*Sec[c + d*x]^(3/2))))/(5
*d*(b + a*Cos[c + d*x])^2)
```

3.843.3 Rubi [A] (verified)

Time = 2.03 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.08, number of steps used = 21, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.840$, Rules used = {3042, 4752, 3042, 4351, 25, 3042, 4592, 27, 3042, 4523, 3042, 4343, 3042, 3134, 3042, 3132, 4345, 3042, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{5}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx$$

$$\Big\downarrow \text{4351}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2a\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5d\sec^{\frac{3}{2}}(c+dx)}-\frac{1}{5}\int -\frac{2ab\sec^2(c+dx)+\left(3a^2+5b^2\right)\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}\right.$$

$$\Big\downarrow \text{25}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\int \frac{2ab\sec^2(c+dx)+\left(3a^2+5b^2\right)\sec(c+dx)+6ab}{\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}dx+\frac{2a\sin(c+dx)\sqrt{a+b\sec}}{5d\sec^{\frac{3}{2}}(c+dx)}\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\int \frac{2ab\csc\left(c+dx+\frac{\pi}{2}\right)^2+\left(3a^2+5b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)+6ab}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a\sin(c+dx)}{5d\sec}\right.$$

$$\Big\downarrow \text{4592}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(\frac{4b\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{d\sqrt{\sec(c+dx)}}-\frac{2\int -\frac{3\left(4b\sec(c+dx)a^2+\left(3a^2+b^2\right)a\right)}{2\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{3a}\right)+\frac{2a\sin}{}\right.$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(\frac{\int \frac{4b\sec(c+dx)a^2+\left(3a^2+b^2\right)a}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{a}+\frac{4b\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{d\sqrt{\sec(c+dx)}}\right)+\frac{2a\sin(c+}{5}\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(\frac{\int \frac{4b\csc\left(c+dx+\frac{\pi}{2}\right)a^2+\left(3a^2+b^2\right)a}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}+\frac{4b\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{d\sqrt{\sec(c+dx)}}\right)+\frac{2a\,\mathrm{s}}{}\right.$$

3.843. $\int \cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx$

$$\Big\downarrow \text{4523}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(\frac{b(a^2-b^2)\int\frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}dx+(3a^2+b^2)\int\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}dx}{a}+\frac{4b\sin(c+dx)\sqrt{}}{d\sqrt{\sec}}\right.\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(\frac{b(a^2-b^2)\int\frac{\sqrt{\csc(c+dx+\frac{\pi}{2})}}{\sqrt{a+b\csc(c+dx+\frac{\pi}{2})}}dx+(3a^2+b^2)\int\frac{\sqrt{a+b\csc(c+dx+\frac{\pi}{2})}}{\sqrt{\csc(c+dx+\frac{\pi}{2})}}dx}{a}+\frac{4b\sin(c+}{}\right.\right.$$

$$\Big\downarrow \text{4343}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(\frac{b(a^2-b^2)\int\frac{\sqrt{\csc(c+dx+\frac{\pi}{2})}}{\sqrt{a+b\csc(c+dx+\frac{\pi}{2})}}dx+\frac{(3a^2+b^2)\sqrt{a+b\sec(c+dx)}\int\sqrt{b+a\cos(c+dx)}dx}{\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}}{a}+\frac{4b\,s}{}\right.\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(\frac{b(a^2-b^2)\int\frac{\sqrt{\csc(c+dx+\frac{\pi}{2})}}{\sqrt{a+b\csc(c+dx+\frac{\pi}{2})}}dx+\frac{(3a^2+b^2)\sqrt{a+b\sec(c+dx)}\int\sqrt{b+a\sin(c+dx+\frac{\pi}{2})}dx}{\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}}{a}+\right.\right.$$

$$\Big\downarrow \text{3134}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(\frac{b(a^2-b^2)\int\frac{\sqrt{\csc(c+dx+\frac{\pi}{2})}}{\sqrt{a+b\csc(c+dx+\frac{\pi}{2})}}dx+\frac{(3a^2+b^2)\sqrt{a+b\sec(c+dx)}\int\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}dx}{\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{a}+\frac{4}{}\right.\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(\frac{b(a^2-b^2)\int\frac{\sqrt{\csc(c+dx+\frac{\pi}{2})}}{\sqrt{a+b\csc(c+dx+\frac{\pi}{2})}}dx+\frac{(3a^2+b^2)\sqrt{a+b\sec(c+dx)}\int\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}dx}{\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{a}-\right.\right.$$

3.843. $\int\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx$

$$\Big\downarrow \; 3132$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(\frac{b\left(a^2-b^2\right)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\left(3a^2+b^2\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{a}+\frac{4b\sin}{}\right.\right.$$

$$\Big\downarrow \; 4345$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(\frac{\frac{b\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{1}{\sqrt{b+a\cos(c+dx)}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{2\left(3a^2+b^2\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx\right.}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{a}\right.\right.$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(\frac{\frac{b\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{1}{\sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{2\left(3a^2+b^2\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c\right.}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx}{a+b}}}}{a}\right.\right.$$

$$\Big\downarrow \; 3142$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(\frac{\frac{b\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{2\left(3a^2+b^2\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+\right.}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)}{a+b}}}}{a}\right.\right.$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(\frac{\frac{b\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{2\left(3a^2+b^2\right)\sqrt{a+b\sec(c+dx)}E\left(\right.}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+}{a+}}}}{a}\right.\right.$$

$$\Big\downarrow \; 3140$$

3.843. $\int \cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(\frac{\frac{2b(a^2-b^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}+\frac{2(3a^2+b^2)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}\right.}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+d}{a+b}}}}{a}\right.\right.$$

input
```
Int[Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^(3/2),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*a*Sqrt[a + b*Sec[c + d*x]]*Sin[c
 + d*x])/(5*d*Sec[c + d*x]^(3/2)) + (((2*b*(a^2 - b^2)*Sqrt[(b + a*Cos[c +
 d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/
(d*Sqrt[a + b*Sec[c + d*x]]) + (2*(3*a^2 + b^2)*EllipticE[(c + d*x)/2, (2*
a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(d*Sqrt[(b + a*Cos[c + d*x])/(a + b)
]*Sqrt[Sec[c + d*x]]))/a + (4*b*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(d*
Sqrt[Sec[c + d*x]]))/5)
```

3.843.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 4343
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]])   Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a
, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4345
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]])   Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[
{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4351
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(3/2), x_Symbol] :> Simp[a*Cot[e + f*x]*Sqrt[a + b*Csc[e + f*x]]*((d*C
sc[e + f*x])^n/(f*n)), x] + Simp[1/(2*d*n)   Int[((d*Csc[e + f*x])^(n + 1)/
Sqrt[a + b*Csc[e + f*x]])*Simp[a*b*(2*n - 1) + 2*(b^2*n + a^2*(n + 1))*Csc[
e + f*x] + a*b*(2*n + 3)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f
}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1] && IntegersQ[2*n]
```

rule 4523
```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Simp[A/a   I
nt[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Simp[(A*b - a*B)
/(a*d)   Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ
[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

3.843. $\int \cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx$

rule 4592
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Simp[1/(a*d*n)   Int[(a + b*Csc[e + f*x])^m
*(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*
Csc[e + f*x] + A*b*(m + n + 1)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

3.843.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2301 vs. $2(270) = 540$.

Time = 6.90 (sec) , antiderivative size = 2302, normalized size of antiderivative = 9.59

method	result	size
default	Expression too large to display	2302

input
```
int(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/5/d/a/((a-b)/(a+b))^(1/2)*(8*EllipticF(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+
csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*(1/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*c
os(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^2*b*cos(d*x+c)-2*EllipticF(((a-b)/(a+b)
)^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*(1/(cos(d*x+c)+1))^
(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a*b^2*cos(d*x+c)-6*E
llipticE(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2)
)*(1/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)
*a^2*b*cos(d*x+c)+2*EllipticE(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c))
,(-(a+b)/(a-b))^(1/2))*(1/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/
(cos(d*x+c)+1))^(1/2)*a*b^2*cos(d*x+c)+EllipticE(((a-b)/(a+b))^(1/2)*(-cot
(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d
*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a*b^2*cos(d*x+c)^2+4*EllipticF(((
a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*
(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a^2*b*cos(
d*x+c)^2-EllipticF(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a
-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)
+1))^(1/2)*a*b^2*cos(d*x+c)^2-3*EllipticE(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)
+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1
))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a^2*b*cos(d*x+c)^2-EllipticE(((a-b)/(a+b
))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*c...
```

3.843.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 455, normalized size of antiderivative = 1.90

$$\int \cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx = \frac{6\,(a^3\cos(dx+c)+2\,a^2b)\sqrt{\frac{a\cos(dx+c)+b}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)-2\sqrt{2}(3i\,a^2b-i\,b^3)\sqrt{a}\text{wei}}{}$$

input
```
integrate(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
1/15*(6*(a^3*cos(d*x + c) + 2*a^2*b)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c
))*sqrt(cos(d*x + c))*sin(d*x + c) - 2*sqrt(2)*(3*I*a^2*b - I*b^3)*sqrt(a)
*weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3,
 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x + c) + 2*b)/a) - 2*sqrt(2)*(-3*I*a^
2*b + I*b^3)*sqrt(a)*weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9
*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a)
- 3*sqrt(2)*(-3*I*a^3 - I*a*b^2)*sqrt(a)*weierstrassZeta(-4/3*(3*a^2 - 4*b
^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4*b
^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x
 + c) + 2*b)/a)) - 3*sqrt(2)*(3*I*a^3 + I*a*b^2)*sqrt(a)*weierstrassZeta(-
4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-
4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c)
 - 3*I*a*sin(d*x + c) + 2*b)/a)))/(a^2*d)
```

3.843.6 Sympy [F(-1)]

Timed out.

$$\int \cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(5/2)*(a+b*sec(d*x+c))**(3/2),x)
```

output
```
Timed out
```

3.843.7 Maxima [F]

$$\int \cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx = \int (b\sec(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^{\frac{5}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^(3/2)*cos(d*x + c)^(5/2), x)
```

3.843.8 Giac [F]

$$\int \cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx = \int (b\sec(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^{\frac{5}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^(3/2)*cos(d*x + c)^(5/2), x)
```

3.843.9 Mupad [F(-1)]

Timed out.

$$\int \cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx = \int \cos(c+d\,x)^{5/2}\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}dx$$

input
```
int(cos(c + d*x)^(5/2)*(a + b/cos(c + d*x))^(3/2),x)
```

output
```
int(cos(c + d*x)^(5/2)*(a + b/cos(c + d*x))^(3/2), x)
```

3.844 $\int \cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx$

3.844.1 Optimal result 6298
3.844.2 Mathematica [C] (verified) 6299
3.844.3 Rubi [A] (verified) 6299
3.844.4 Maple [B] (verified) 6304
3.844.5 Fricas [C] (verification not implemented) 6304
3.844.6 Sympy [F(-1)] 6305
3.844.7 Maxima [F] 6305
3.844.8 Giac [F] 6306
3.844.9 Mupad [F(-1)] 6306

3.844.1 Optimal result

Integrand size = 25, antiderivative size = 187

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx = \frac{2(a^2-b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{3d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} + \frac{8b\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+dx)}}{3d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}} + \frac{2a\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{3d}$$

output
```
2/3*(a^2-b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(si
n(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)/d
/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)+2/3*a*sin(d*x+c)*cos(d*x+c)^(1/2)
*(a+b*sec(d*x+c))^(1/2)/d+8/3*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1
/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*cos(d*x+c)^(1/
2)*(a+b*sec(d*x+c))^(1/2)/d/((b+a*cos(d*x+c))/(a+b))^(1/2)
```

3.844.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 7.72 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.52

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx = \frac{2\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^{3/2}\left(\frac{1}{2}a(b+a\cos(c+dx))\sin(2(c+dx))+\frac{\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec}}{}\right.}{}$$

input
```
Integrate[Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^(3/2),x]
```

output
```
(2*Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x])^(3/2)*((a*(b + a*Cos[c + d*x])*
Sin[2*(c + d*x)])/2 + (Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*((4*I)*b*(a +
 b)*EllipticE[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[((b + a*
Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] - I*(a^2 + 4*a*b + 3*b^2)*Ellip
ticF[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[((b + a*Cos[c + d
*x])*Sec[(c + d*x)/2]^2)/(a + b)] + 4*b*(b + a*Cos[c + d*x])*Sqrt[Sec[(c +
 d*x)/2]^2]*Tan[(c + d*x)/2]))/Sec[c + d*x]^(3/2)))/(3*d*(b + a*Cos[c + d*
x])^2)
```

3.844.3 Rubi [A] (verified)

Time = 1.60 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.12, number of steps used = 18, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.720$, Rules used = {3042, 4752, 3042, 4351, 25, 3042, 4523, 3042, 4343, 3042, 3134, 3042, 3132, 4345, 3042, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}dx$$

$$\Big\downarrow \; 4752$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{3}{2}}(c+dx)}dx$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx$$

$\Big\downarrow$ 4351

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2a\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3d\sqrt{\sec(c+dx)}}-\frac{1}{3}\int -\frac{4ab+\left(a^2+3b^2\right)\sec(c+dx)}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx\right)$$

$\Big\downarrow$ 25

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\int \frac{4ab+\left(a^2+3b^2\right)\sec(c+dx)}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx+\frac{2a\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3d\sqrt{\sec(c+dx)}}\right)$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\int \frac{4ab+\left(a^2+3b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a\sin(c+dx)\sqrt{a+b\sec(c+}}{3d\sqrt{\sec(c+dx)}}\right.$$

$\Big\downarrow$ 4523

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\left(\left(a^2-b^2\right)\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}dx+4b\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}dx\right)+\frac{2a\sin(c+}{3a}\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\left(\left(a^2-b^2\right)\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+4b\int \frac{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)+\right.$$

$\Big\downarrow$ 4343

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\left(\left(a^2-b^2\right)\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{4b\sqrt{a+b\sec(c+dx)}\int\sqrt{b+a\cos(c}}{\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)}+}\right.\right.$$

$\Big\downarrow$ 3042

3.844. $\int \cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\left(\left(a^2-b^2\right)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{4b\sqrt{a+b\sec(c+dx)}\int\sqrt{b+a\sin\left(c\right.}}{\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)}}\right.\right.$$

$\downarrow$ 3134

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\left(\left(a^2-b^2\right)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{4b\sqrt{a+b\sec(c+dx)}\int\sqrt{\frac{b}{a+b}+\frac{a\cos(}{a}}}{\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\left(\left(a^2-b^2\right)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{4b\sqrt{a+b\sec(c+dx)}\int\sqrt{\frac{b}{a+b}+\frac{a\sin(}{}}}{\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}\right.\right.$$

$\downarrow$ 3132

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\left(\left(a^2-b^2\right)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{8b\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2c}{a+}\right.}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}\right.\right.$$

$\downarrow$ 4345

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\left(\frac{\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{1}{\sqrt{b+a\cos(c+dx)}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{8b\sqrt{a+b\sec(c}}{d\sqrt{\sec(c}}\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\left(\frac{\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{1}{\sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{8b\sqrt{a+b\sec}}{d\sqrt{\sec}}\right.\right.$$

$\downarrow$ 3142

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\left(\frac{\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{8b\sqrt{a+b\sec(c+}}{d\sqrt{\sec(c+d}}\right.\right.$$

$\downarrow$ 3042

3.844. $\int\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\left(\frac{(a^2-b^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{8b\sqrt{a+b\sec(c}}{d\sqrt{\sec(c}}\right.\right.$$

$\downarrow$ 3140

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\left(\frac{2(a^2-b^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}+\frac{8b\sqrt{a+b\,\text{s}}}{d\sqrt{\text{se}}}\right.\right.$$

input
```
Int[Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^(3/2),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(((2*(a^2 - b^2)*Sqrt[(b + a*Cos[c +
 d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/
(d*Sqrt[a + b*Sec[c + d*x]]) + (8*b*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*
Sqrt[a + b*Sec[c + d*x]])/(d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c
 + d*x]]))/3 + (2*a*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(3*d*Sqrt[Sec[c
 + d*x]]))
```

3.844.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 4343
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]])   Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a
, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4345
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]])   Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[
{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4351
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(3/2), x_Symbol] :> Simp[a*Cot[e + f*x]*Sqrt[a + b*Csc[e + f*x]]*((d*C
sc[e + f*x])^n/(f*n)), x] + Simp[1/(2*d*n)   Int[((d*Csc[e + f*x])^(n + 1)/
Sqrt[a + b*Csc[e + f*x]])*Simp[a*b*(2*n - 1) + 2*(b^2*n + a^2*(n + 1))*Csc[
e + f*x] + a*b*(2*n + 3)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f
}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1] && IntegersQ[2*n]
```

rule 4523
```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Simp[A/a   I
nt[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Simp[(A*b - a*B)
/(a*d)   Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ
[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

3.844. $\int \cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx$

3.844.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1593 vs. $2(223) = 446$.

Time = 5.67 (sec) , antiderivative size = 1594, normalized size of antiderivative = 8.52

method	result	size
default	Expression too large to display	1594

input
```
int(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/3/d/((a-b)/(a+b))^(1/2)*(4*(1/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*
x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc
(d*x+c)),(-(a+b)/(a-b))^(1/2))*a*b*cos(d*x+c)^2-4*(1/(cos(d*x+c)+1))^(1/2)
*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE(((a-b)/(a+b))^(
1/2)*(-cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*b^2*cos(d*x+c)^2+(1/(c
os(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ellipt
icF(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*a^2
*cos(d*x+c)^2-4*(1/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*
x+c)+1))^(1/2)*EllipticF(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(-(a
+b)/(a-b))^(1/2))*a*b*cos(d*x+c)^2+3*(1/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+
a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF(((a-b)/(a+b))^(1/2)*(-cot(d*
x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*b^2*cos(d*x+c)^2+((a-b)/(a+b))^(1/2
)*a^2*cos(d*x+c)^2*sin(d*x+c)+8*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^
(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+
csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*a*b*cos(d*x+c)-8*(1/(a+b)*(b+a*cos(d*x+c
))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE(((a-b)/(a+b))^
(1/2)*(-cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*b^2*cos(d*x+c)+2*(1/(
a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*Ellip
ticF(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*a^
2*cos(d*x+c)-8*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(...
```

3.844.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 415, normalized size of antiderivative = 2.22

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx = \frac{6\,a^2\sqrt{\frac{a\cos(dx+c)+b}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)+12i\sqrt{2}a^{\frac{3}{2}}b\text{weierstrassZeta}\left(-\frac{4\,(3\,a^2-4\,b^2)}{3\,a^2},\frac{8\,(9\,a^2b}{27\,a}\right.}{}$$

input
```
integrate(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
1/9*(6*a^2*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(
d*x + c) + 12*I*sqrt(2)*a^(3/2)*b*weierstrassZeta(-4/3*(3*a^2 - 4*b^2)/a^2
, 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2
, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x + c) +
 2*b)/a)) - 12*I*sqrt(2)*a^(3/2)*b*weierstrassZeta(-4/3*(3*a^2 - 4*b^2)/a^
2, 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^
2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c)
+ 2*b)/a)) + sqrt(2)*(-3*I*a^2 - I*b^2)*sqrt(a)*weierstrassPInverse(-4/3*(
3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*
I*a*sin(d*x + c) + 2*b)/a) + sqrt(2)*(3*I*a^2 + I*b^2)*sqrt(a)*weierstrass
PInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*co
s(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a))/(a*d)
```

3.844.6 Sympy [F(-1)]

Timed out.

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(3/2)*(a+b*sec(d*x+c))**(3/2),x)
```

output
```
Timed out
```

3.844.7 Maxima [F]

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx = \int (b\sec(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^{\frac{3}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^(3/2)*cos(d*x + c)^(3/2), x)
```

3.844.8 Giac [F]

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx = \int (b\sec(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^{\frac{3}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^(3/2)*cos(d*x + c)^(3/2), x)
```

3.844.9 Mupad [F(-1)]

Timed out.

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx = \int \cos(c+d\,x)^{3/2}\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}dx$$

input
```
int(cos(c + d*x)^(3/2)*(a + b/cos(c + d*x))^(3/2),x)
```

output
```
int(cos(c + d*x)^(3/2)*(a + b/cos(c + d*x))^(3/2), x)
```

3.845 $\int \sqrt{\cos(c+dx)}(a+b\sec(c+dx))^{3/2}\,dx$

3.845.1 Optimal result 6307
3.845.2 Mathematica [C] (warning: unable to verify) 6308
3.845.3 Rubi [A] (verified) 6308
3.845.4 Maple [C] (warning: unable to verify) 6314
3.845.5 Fricas [F(-1)] 6314
3.845.6 Sympy [F(-1)] 6315
3.845.7 Maxima [F] 6315
3.845.8 Giac [F] 6315
3.845.9 Mupad [F(-1)] 6316

3.845.1 Optimal result

Integrand size = 25, antiderivative size = 209

$$\int \sqrt{\cos(c+dx)}(a+b\sec(c+dx))^{3/2}\,dx = \frac{2ab\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} + \frac{2b^2\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} + \frac{2a\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+dx)}}{d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}}$$

output
```
2*a*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*
x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)/d/cos(d*x
+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)+2*b^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/
2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2,2^(1/2)*(a/(a+b))^(1/2))*((b+
a*cos(d*x+c))/(a+b))^(1/2)/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)+2*a*(
cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c
),2^(1/2)*(a/(a+b))^(1/2))*cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2)/d/((b+a
*cos(d*x+c))/(a+b))^(1/2)
```

3.845.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 33.92 (sec) , antiderivative size = 25369, normalized size of antiderivative = 121.38

$$\int \sqrt{\cos(c+dx)}(a+b\sec(c+dx))^{3/2}\,dx = \text{Result too large to show}$$

input
```
Integrate[Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x])^(3/2),x]
```

output
```
Result too large to show
```

3.845.3 Rubi [A] (verified)

Time = 2.18 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.10, number of steps used = 22, number of rules used = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.880$, Rules used = {3042, 4752, 3042, 4355, 3042, 4341, 3042, 4343, 3042, 3134, 3042, 3132, 4345, 3042, 3142, 3042, 3140, 4346, 3042, 3286, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\cos(c+dx)}(a+b\sec(c+dx))^{3/2}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}dx$$

$$\Big\downarrow \; 4752$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{(a+b\sec(c+dx))^{3/2}}{\sqrt{\sec(c+dx)}}dx$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \; 4355$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(a\int\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}dx+b\int\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}dx\right)$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(a\int\frac{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+b\int\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}\right.$$

$\Big\downarrow$ 4341

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(a\int\frac{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+b\left(b\int\frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx+a\int\frac{\sqrt{\sec(c+}}{\sqrt{a+b\sec(c}}\right.\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(b\left(a\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+b\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)+a\int\frac{\sqrt{a+}}{\sqrt{}}\right.$$

$\Big\downarrow$ 4343

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(b\left(a\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+b\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)+\frac{a\sqrt{a+b}}{\sqrt{\text{se}}}\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(b\left(a\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+b\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)+\frac{a\sqrt{a+b}}{\sqrt{}}\right.$$

$\Big\downarrow$ 3134

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(b\left(a\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+b\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)+\frac{a\sqrt{a+b}}{\sqrt{}}\right.$$

$\Big\downarrow$ 3042

3.845. $\int\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^{3/2}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(b\left(a\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+b\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)+\frac{a\sqrt{a+b\s}}{}$$

$\downarrow$ 3132

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(b\left(a\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+b\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)+\frac{2a\sqrt{a+b}}{d\sqrt{\s}}$$

$\downarrow$ 4345

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(b\left(b\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{a\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{1}{\sqrt{b+a\cos(c}}}{\sqrt{a+b\sec(c+dx)}}\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(b\left(b\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{a\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{1}{\sqrt{b+a\sin(c}}}{\sqrt{a+b\sec(c+dx)}}\right.\right.$$

$\downarrow$ 3142

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(b\left(b\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx}{a+b}}}}{\sqrt{a+b\sec(c+dx)}}\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(b\left(b\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx\right.}{a+b}}}}{\sqrt{a+b\sec(c+dx)}}\right.\right.$$

$\downarrow$ 3140

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(b\left(b\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c-\right.}{d\sqrt{a+b\sec(c+dx)}}\right.\right.$$

$\downarrow$ 4346

3.845. $\int\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^{3/2}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(b\left(\frac{b\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{\sec(c+dx)}{\sqrt{b+a\cos(c+dx)}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{2a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+}{a+}}}{d\sqrt{a-}}\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(b\left(\frac{b\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{2a\sqrt{\sec(c-}}{}\right.\right.$$

$\downarrow$ 3286

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(b\left(\frac{b\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{\sec(c+dx)}{\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{2a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)}{a+b}}}{d\sqrt{a+b}}\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(b\left(\frac{b\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{2a\sqrt{\sec(c+d}}{}\right.\right.$$

$\downarrow$ 3284

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2a\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+b\left(\frac{2a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\text{Ellipti}}{d\sqrt{a+b\sec(c+d}}\right.\right.$$

input
```
Int[Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x])^(3/2),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*a*EllipticE[(c + d*x)/2, (2*a)/(
a + b)]*Sqrt[a + b*Sec[c + d*x]])/(d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sq
rt[Sec[c + d*x]]) + b*((2*a*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(
c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(d*Sqrt[a + b*Sec[c + d*x]]
) + (2*b*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*
a)/(a + b)]*Sqrt[Sec[c + d*x]])/(d*Sqrt[a + b*Sec[c + d*x]])))
```

3.845. $\int \sqrt{\cos(c+dx)}(a+b\sec(c+dx))^{3/2}\,dx$

3.845.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3286
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]]   Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
 d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

rule 4341
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[a   Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e +
f*x]], x], x] + Simp[b/d   Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Csc[e + f*
x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4343
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]])   Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a
, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4345
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]])   Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[
{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4346
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Simp[d*Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x
]]/Sqrt[a + b*Csc[e + f*x]])   Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4355
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_
)]*(d_.)], x_Symbol] :> Simp[a   Int[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e
+ f*x]], x], x] + Simp[b/d   Int[Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*
x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

3.845. $\int \sqrt{\cos(c+dx)}(a+b\sec(c+dx))^{3/2}\,dx$

3.845.4 Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 7.98 (sec) , antiderivative size = 1163, normalized size of antiderivative = 5.56

method	result	size
default	Expression too large to display	1163

input
```
int(cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/d/((a-b)/(a+b))^(1/2)*(-((1-cos(d*x+c))^2*csc(d*x+c)^2-1)/((1-cos(d*x+c)
)^2*csc(d*x+c)^2+1))^(1/2)*((a*(1-cos(d*x+c))^2*csc(d*x+c)^2-b*(1-cos(d*x+
c))^2*csc(d*x+c)^2-a-b)/((1-cos(d*x+c))^2*csc(d*x+c)^2-1))^(1/2)*(((a-b)/(
a+b))^(1/2)*a^2*(1-cos(d*x+c))^3*csc(d*x+c)^3-((a-b)/(a+b))^(1/2)*a*b*(1-c
os(d*x+c))^3*csc(d*x+c)^3+(-(a*(1-cos(d*x+c))^2*csc(d*x+c)^2-b*(1-cos(d*x+
c))^2*csc(d*x+c)^2-a-b)/(a+b))^(1/2)*((1-cos(d*x+c))^2*csc(d*x+c)^2+1)^(1/
2)*EllipticF(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(
1/2))*a^2-2*(-(a*(1-cos(d*x+c))^2*csc(d*x+c)^2-b*(1-cos(d*x+c))^2*csc(d*x+
c)^2-a-b)/(a+b))^(1/2)*((1-cos(d*x+c))^2*csc(d*x+c)^2+1)^(1/2)*EllipticF((
(a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*a*b+(-(a
*(1-cos(d*x+c))^2*csc(d*x+c)^2-b*(1-cos(d*x+c))^2*csc(d*x+c)^2-a-b)/(a+b))
^(1/2)*((1-cos(d*x+c))^2*csc(d*x+c)^2+1)^(1/2)*EllipticF(((a-b)/(a+b))^(1/
2)*(-cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*b^2-(-(a*(1-cos(d*x+c))^
2*csc(d*x+c)^2-b*(1-cos(d*x+c))^2*csc(d*x+c)^2-a-b)/(a+b))^(1/2)*((1-cos(d
*x+c))^2*csc(d*x+c)^2+1)^(1/2)*EllipticE(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+
csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*a^2+(-(a*(1-cos(d*x+c))^2*csc(d*x+c)^2-b
*(1-cos(d*x+c))^2*csc(d*x+c)^2-a-b)/(a+b))^(1/2)*((1-cos(d*x+c))^2*csc(d*x
+c)^2+1)^(1/2)*EllipticE(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(-(a
+b)/(a-b))^(1/2))*a*b-2*(-(a*(1-cos(d*x+c))^2*csc(d*x+c)^2-b*(1-cos(d*x+c)
)^2*csc(d*x+c)^2-a-b)/(a+b))^(1/2)*((1-cos(d*x+c))^2*csc(d*x+c)^2+1)^(1...
```

3.845.5 Fricas [F(-1)]

Timed out.

$$\int \sqrt{\cos(c+dx)}(a+b\sec(c+dx))^{3/2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.845.6 Sympy [F(-1)]

Timed out.

$$\int \sqrt{\cos(c+dx)}(a+b\sec(c+dx))^{3/2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(1/2)*(a+b*sec(d*x+c))**(3/2),x)
```

output
```
Timed out
```

3.845.7 Maxima [F]

$$\int \sqrt{\cos(c+dx)}(a+b\sec(c+dx))^{3/2}\,dx = \int (b\sec(dx+c)+a)^{\frac{3}{2}}\sqrt{\cos(dx+c)}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^(3/2)*sqrt(cos(d*x + c)), x)
```

3.845.8 Giac [F]

$$\int \sqrt{\cos(c+dx)}(a+b\sec(c+dx))^{3/2}\,dx = \int (b\sec(dx+c)+a)^{\frac{3}{2}}\sqrt{\cos(dx+c)}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^(3/2)*sqrt(cos(d*x + c)), x)
```

3.845.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{\cos(c+dx)}(a+b\sec(c+dx))^{3/2}\,dx=\int \sqrt{\cos\left(c+d\,x\right)}\,\left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)^{3/2}\,dx$$

input
```
int(cos(c + d*x)^(1/2)*(a + b/cos(c + d*x))^(3/2),x)
```

output
```
int(cos(c + d*x)^(1/2)*(a + b/cos(c + d*x))^(3/2), x)
```

3.846 $\int \frac{(a+b\sec(c+dx))^{3/2}}{\sqrt{\cos(c+dx)}}\,dx$

3.846.1 Optimal result 6317
3.846.2 Mathematica [C] (warning: unable to verify) 6318
3.846.3 Rubi [A] (verified) 6318
3.846.4 Maple [C] (verified) 6324
3.846.5 Fricas [F(-1)] 6325
3.846.6 Sympy [F] 6326
3.846.7 Maxima [F] 6326
3.846.8 Giac [F] 6326
3.846.9 Mupad [F(-1)] 6327

3.846.1 Optimal result

Integrand size = 25, antiderivative size = 249

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sqrt{\cos(c+dx)}}\,dx = \frac{(2a^2+b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} + \frac{3ab\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} - \frac{b\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+dx)}}{d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}} + \frac{b\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

output
```
(2*a^2+b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(
1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)/d/c
os(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)+3*a*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/
cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2,2^(1/2)*(a/(a+b))^(1/2)
)*((b+a*cos(d*x+c))/(a+b))^(1/2)/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)
+b*sin(d*x+c)*(a+b*sec(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)-b*(cos(1/2*d*x+1/2
*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a
+b))^(1/2))*cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2)/d/((b+a*cos(d*x+c))/(a
+b))^(1/2)
```

3.846.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 33.14 (sec) , antiderivative size = 35156, normalized size of antiderivative = 141.19

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sqrt{\cos(c+dx)}}\,dx = \text{Result too large to show}$$

input
```
Integrate[(a + b*Sec[c + d*x])^(3/2)/Sqrt[Cos[c + d*x]],x]
```

output
```
Result too large to show
```

3.846.3 Rubi [A] (verified)

Time = 2.72 (sec) , antiderivative size = 276, normalized size of antiderivative = 1.11, number of steps used = 25, number of rules used = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {3042, 4752, 3042, 4353, 27, 3042, 4596, 3042, 4346, 3042, 3286, 3042, 3284, 4523, 3042, 4343, 3042, 3134, 3042, 3132, 4345, 3042, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sqrt{\cos(c+dx)}}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \; 4752$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}dx$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}dx$$

$$\Big\downarrow \; 4353$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int -\frac{-2\sec(c+dx)a^2-3b\sec^2(c+dx)a+ba}{2\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx+\frac{b\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+}}{d}\right.$$

$\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{b\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{1}{2}\int\frac{-2\sec(c+dx)a^2-3b\sec^2(c+}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c}}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{b\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{1}{2}\int\frac{-2\csc\left(c+dx+\frac{\pi}{2}\right)a^2-3b\,\mathrm{cs}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+}}\right.$$

$\downarrow$ 4596

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{2}\left(3ab\int\frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx-\int\frac{ab-2a^2\sec(c+dx)}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx\right)+\frac{b\sin(}{}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{2}\left(3ab\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-\int\frac{ab-2a^2\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\right.}}\right.\right.$$

$\downarrow$ 4346

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{2}\left(\frac{3ab\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{\sec(c+dx)}{\sqrt{b+a\cos(c+dx)}}dx}{\sqrt{a+b\sec(c+dx)}}-\int\frac{ab-2a^2\,\mathrm{c}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{2}\left(\frac{3ab\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{\sqrt{a+b\sec(c+dx)}}-\int\frac{}{\sqrt{\mathrm{csc}}}\right.\right.$$

$\downarrow$ 3286

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{2}\left(\frac{3ab\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{\sec(c+dx)}{\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}-\int\frac{ab-2a^2\csc(}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{}}\right.\right.$$

3.846. $\int\frac{(a+b\sec(c+dx))^{3/2}}{\sqrt{\cos(c+dx)}}\,dx$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{2}\left(\frac{3ab\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}-\int\frac{}{\sqrt{\csc\left(c\right.}}\right.\right.$$

$$\Big\downarrow \; 3284$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{2}\left(\frac{6ab\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}-\int\frac{ab}{\sqrt{\csc\left(c+d\right.}}\right.\right.$$

$$\Big\downarrow \; 4523$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{2}\left(\left(2a^2+b^2\right)\int\frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}dx-b\int\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}dx+\frac{6ab\sqrt{\sec(c}}{}\right.\right.$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{2}\left(\left(2a^2+b^2\right)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-b\int\frac{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{6}{}\right.\right.$$

$$\Big\downarrow \; 4343$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{2}\left(\left(2a^2+b^2\right)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{b\sqrt{a+b\sec(c+dx)}\int\sqrt{b+a\cos(c}}{\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)}}\right.\right.$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{2}\left(\left(2a^2+b^2\right)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{b\sqrt{a+b\sec(c+dx)}\int\sqrt{b+a\sin\left(c\right.}}{\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx}}\right.\right.$$

$$\Big\downarrow \; 3134$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{2}\left(\left(2a^2+b^2\right)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{b\sqrt{a+b\sec(c+dx)}\int\sqrt{\frac{b}{a+b}+\frac{a\cos(}{a}}}{\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}\right.\right.$$

3.846. $\int\frac{(a+b\sec(c+dx))^{3/2}}{\sqrt{\cos(c+dx)}}\,dx$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{2}\left(\left(2a^2+b^2\right)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{b\sqrt{a+b\sec(c+dx)}\int\sqrt{\frac{b}{a+b}+\frac{a\sin(}{}}}{\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+}{a+b}}}\right.\right.$$

$$\Big\downarrow \; 3132$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{2}\left(\left(2a^2+b^2\right)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2b\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\right.}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}\right.\right.$$

$$\Big\downarrow \; 4345$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{2}\left(\frac{\left(2a^2+b^2\right)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{1}{\sqrt{b+a\cos(c+dx)}}dx}{\sqrt{a+b\sec(c+dx)}}-\frac{2b\sqrt{a+b\sec(}}{d\sqrt{\sec(c}}\right.\right.$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{2}\left(\frac{\left(2a^2+b^2\right)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{1}{\sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{\sqrt{a+b\sec(c+dx)}}-\frac{2b\sqrt{a+b\mathrm{s}}}{d\sqrt{\mathrm{se}}}\right.\right.$$

$$\Big\downarrow \; 3142$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{2}\left(\frac{\left(2a^2+b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}-\frac{2b\sqrt{a+b\sec(c+}}{d\sqrt{\sec(c+}}\right.\right.$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{2}\left(\frac{\left(2a^2+b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}-\frac{2b\sqrt{a+b\sec(}}{d\sqrt{\sec(c}}\right.\right.$$

$$\Big\downarrow \; 3140$$

3.846. $\int\frac{(a+b\sec(c+dx))^{3/2}}{\sqrt{\cos(c+dx)}}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{2}\left(\frac{2(2a^2+b^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}-\frac{2b\sqrt{a+b}}{d\sqrt{s}}\right.\right.$$

input
```
Int[(a + b*Sec[c + d*x])^(3/2)/Sqrt[Cos[c + d*x]],x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(((2*(2*a^2 + b^2)*Sqrt[(b + a*Cos[c
 + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]]
)/(d*Sqrt[a + b*Sec[c + d*x]]) + (6*a*b*Sqrt[(b + a*Cos[c + d*x])/(a + b)]
*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(d*Sqrt[a +
 b*Sec[c + d*x]]) - (2*b*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*
Sec[c + d*x]])/(d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]))/
2 + (b*Sqrt[Sec[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/d)
```

3.846.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

3.846. $\int \frac{(a+b\sec(c+dx))^{3/2}}{\sqrt{\cos(c+dx)}}\,dx$

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3286
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]]   Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
 d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

rule 4343
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]])   Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a
, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4345
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]])   Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[
{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4346
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Simp[d*Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x
]]/Sqrt[a + b*Csc[e + f*x]])   Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

3.846. $\int \frac{(a+b\sec(c+dx))^{3/2}}{\sqrt{\cos(c+dx)}}\,dx$

rule 4353
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b)*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
1)*((d*Csc[e + f*x])^(n - 1)/(f*(m + n - 1))), x] + Simp[d/(m + n - 1)   In
t[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^(n - 1)*Simp[a*b*(n - 1) +
(b^2*(m + n - 2) + a^2*(m + n - 1))*Csc[e + f*x] + a*b*(2*m + n - 2)*Csc[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && L
tQ[0, m, 2] && LtQ[0, n, 3] && NeQ[m + n - 1, 0] && (IntegerQ[m] || Integer
sQ[2*m, 2*n])
```

rule 4523
```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Simp[A/a   I
nt[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Simp[(A*b - a*B)
/(a*d)   Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ
[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

rule 4596
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Simp[C/d^2   Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*C
sc[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[
a + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
 b^2, 0]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

3.846.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 9.30 (sec) , antiderivative size = 1585, normalized size of antiderivative = 6.37

method	result	size
default	Expression too large to display	1585

input
```
int((a+b*sec(d*x+c))^(3/2)/cos(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/d/((a-b)/(a+b))^(1/2)*(2*(1/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+
c))/(cos(d*x+c)+1))^(1/2)*EllipticF(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d
*x+c)),(-(a+b)/(a-b))^(1/2))*a^2*cos(d*x+c)^3-2*(1/(cos(d*x+c)+1))^(1/2)*(
1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF(((a-b)/(a+b))^(1/
2)*(-cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*a*b*cos(d*x+c)^3+6*(1/(c
os(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ellipt
icPi(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(a+b)/(a-b),I/((a-b)/(a+
b))^(1/2))*a*b*cos(d*x+c)^3-(1/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x
+c))/(cos(d*x+c)+1))^(1/2)*EllipticE(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(
d*x+c)),(-(a+b)/(a-b))^(1/2))*a*b*cos(d*x+c)^3+(1/(cos(d*x+c)+1))^(1/2)*(1
/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE(((a-b)/(a+b))^(1/2
)*(-cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*b^2*cos(d*x+c)^3+4*(1/(co
s(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ellipti
cF(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*a^2*
cos(d*x+c)^2-4*(1/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x
+c)+1))^(1/2)*EllipticF(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(-(a+
b)/(a-b))^(1/2))*a*b*cos(d*x+c)^2+12*(1/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+
a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi(((a-b)/(a+b))^(1/2)*(-cot(d
*x+c)+csc(d*x+c)),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*a*b*cos(d*x+c)^2-2*(1
/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*...
```

3.846.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sqrt{\cos(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate((a+b*sec(d*x+c))^(3/2)/cos(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.846.6 Sympy [F]

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sqrt{\cos(c+dx)}}\,dx=\int \frac{(a+b\sec{(c+dx)})^{\frac{3}{2}}}{\sqrt{\cos{(c+dx)}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))**(3/2)/cos(d*x+c)**(1/2),x)
```

output
```
Integral((a + b*sec(c + d*x))**(3/2)/sqrt(cos(c + d*x)), x)
```

3.846.7 Maxima [F]

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sqrt{\cos(c+dx)}}\,dx=\int \frac{(b\sec{(dx+c)}+a)^{\frac{3}{2}}}{\sqrt{\cos{(dx+c)}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(3/2)/cos(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^(3/2)/sqrt(cos(d*x + c)), x)
```

3.846.8 Giac [F]

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sqrt{\cos(c+dx)}}\,dx=\int \frac{(b\sec{(dx+c)}+a)^{\frac{3}{2}}}{\sqrt{\cos{(dx+c)}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(3/2)/cos(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^(3/2)/sqrt(cos(d*x + c)), x)
```

3.846.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sqrt{\cos(c+dx)}}\,dx=\int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}}{\sqrt{\cos\left(c+d\,x\right)}}\,dx$$

input
```
int((a + b/cos(c + d*x))^(3/2)/cos(c + d*x)^(1/2),x)
```

output
```
int((a + b/cos(c + d*x))^(3/2)/cos(c + d*x)^(1/2), x)
```

3.847 $\int \frac{(a+b\sec(c+dx))^{3/2}}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

3.847.1 Optimal result 6328
3.847.2 Mathematica [C] (warning: unable to verify) 6329
3.847.3 Rubi [A] (verified) 6329
3.847.4 Maple [C] (verified) 6337
3.847.5 Fricas [F(-1)] 6338
3.847.6 Sympy [F] 6338
3.847.7 Maxima [F] 6338
3.847.8 Giac [F] 6339
3.847.9 Mupad [F(-1)] 6339

3.847.1 Optimal result

Integrand size = 25, antiderivative size = 299

$$\begin{aligned}\int \frac{(a+b\sec(c+dx))^{3/2}}{\cos^{\frac{3}{2}}(c+dx)}\,dx &= \frac{7ab\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{4d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}\\ &+\frac{(3a^2+4b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{4d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}\\ &-\frac{5a\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+dx)}}{4d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}}\\ &+\frac{b\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)}+\frac{5a\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{4d\sqrt{\cos(c+dx)}}\end{aligned}$$

output
```
7/4*a*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*
d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)/d/cos(d
*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)+1/4*(3*a^2+4*b^2)*(cos(1/2*d*x+1/2*c)^2
)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2,2^(1/2)*(a/(a+b
))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c
))^(1/2)+1/2*b*sin(d*x+c)*(a+b*sec(d*x+c))^(1/2)/d/cos(d*x+c)^(3/2)+5/4*a*
sin(d*x+c)*(a+b*sec(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)-5/4*a*(cos(1/2*d*x+1/
2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(
a+b))^(1/2))*cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2)/d/((b+a*cos(d*x+c))/(
a+b))^(1/2)
```

3.847.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 34.02 (sec) , antiderivative size = 51904, normalized size of antiderivative = 173.59

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \text{Result too large to show}$$

input
```
Integrate[(a + b*Sec[c + d*x])^(3/2)/Cos[c + d*x]^(3/2),x]
```

output
```
Result too large to show
```

3.847.3 Rubi [A] (verified)

Time = 3.42 (sec) , antiderivative size = 329, normalized size of antiderivative = 1.10, number of steps used = 28, number of rules used = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 1.120$, Rules used = {3042, 4752, 3042, 4353, 27, 3042, 4590, 27, 3042, 4596, 3042, 4346, 3042, 3286, 3042, 3284, 4523, 3042, 4343, 3042, 3134, 3042, 3132, 4345, 3042, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\cos^{\frac{3}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}dx$$

$$\Big\downarrow \text{4353}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{2}\int\frac{\sqrt{\sec(c+dx)}\left(5ab\sec^2(c+dx)+2\left(2a^2+b^2\right)\sec(c+dx)+ab\right)}{2\sqrt{a+b\sec(c+dx)}}dx+\frac{b\sin(c+d}{}\right.$$

$\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{4}\int\frac{\sqrt{\sec(c+dx)}\left(5ab\sec^2(c+dx)+2\left(2a^2+b^2\right)\sec(c+dx)+ab\right)}{\sqrt{a+b\sec(c+dx)}}dx+\frac{b\sin(c+d}{}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{4}\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(5ab\csc\left(c+dx+\frac{\pi}{2}\right)^2+2\left(2a^2+b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)+ab\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}\right.$$

$\downarrow$ 4590

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{4}\left(\frac{\int-\frac{5ba^2-2b^2\sec(c+dx)a-b\left(3a^2+4b^2\right)\sec^2(c+dx)}{2\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{b}+\frac{5a\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+}}{d}\right.\right.$$

$\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{4}\left(\frac{5a\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{\int\frac{5ba^2-2b^2\sec(c+dx)a-b\left(3a^2+4b^2\right)}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+}}}{2b}\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{4}\left(\frac{5a\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{\int\frac{5ba^2-2b^2\csc\left(c+dx+\frac{\pi}{2}\right)a-b\left(3a^2+\right.}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc}}}{2b}\right.\right.$$

$\downarrow$ 4596

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{4}\left(\frac{5a\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{\int\frac{5a^2b-2ab^2\sec(c+dx)}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx-}{}\right.\right.$$

$\downarrow$ 3042

3.847. $\int\frac{(a+b\sec(c+dx))^{3/2}}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{4}\left(\frac{5a\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{\int\frac{5a^2b-2ab^2\csc(c+dx+\frac{\pi}{2})}{\sqrt{\csc(c+dx+\frac{\pi}{2})}\sqrt{a+b\csc(c+dx+\frac{\pi}{2}}}}{}\right.\right.$$

$\downarrow$ 4346

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{4}\left(\frac{5a\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{\int\frac{5a^2b-2ab^2\csc(c+dx+\frac{\pi}{2})}{\sqrt{\csc(c+dx+\frac{\pi}{2})}\sqrt{a+b\csc(c+dx+\frac{\pi}{2}}}}{}\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{4}\left(\frac{5a\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{\int\frac{5a^2b-2ab^2\csc(c+dx+\frac{\pi}{2})}{\sqrt{\csc(c+dx+\frac{\pi}{2})}\sqrt{a+b\csc(c+dx+\frac{\pi}{2}}}}{}\right.\right.$$

$\downarrow$ 3286

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{4}\left(\frac{5a\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{\int\frac{5a^2b-2ab^2\csc(c+dx+\frac{\pi}{2})}{\sqrt{\csc(c+dx+\frac{\pi}{2})}\sqrt{a+b\csc(c+dx+\frac{\pi}{2}}}}{}\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{4}\left(\frac{5a\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{\int\frac{5a^2b-2ab^2\csc(c+dx+\frac{\pi}{2})}{\sqrt{\csc(c+dx+\frac{\pi}{2})}\sqrt{a+b\csc(c+dx+\frac{\pi}{2}}}}{}\right.\right.$$

$\downarrow$ 3284

3.847. $\int \frac{(a+b\sec(c+dx))^{3/2}}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{4}\left(\frac{5a\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{\int\frac{5a^2b-2ab^2\csc(c+dx+\frac{\pi}{2})}{\sqrt{\csc(c+dx+\frac{\pi}{2})}\sqrt{a+b\csc(c+dx+\frac{\pi}{2}}}}{}\right.\right.$$

$\downarrow$ 4523

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{4}\left(\frac{5a\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{-7ab^2\int\frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}dx+5a}{}\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{4}\left(\frac{5a\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{-7ab^2\int\frac{\sqrt{\csc(c+dx+\frac{\pi}{2})}}{\sqrt{a+b\csc(c+dx+\frac{\pi}{2})}}dx+}{}\right.\right.$$

$\downarrow$ 4343

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{4}\left(\frac{5a\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{-7ab^2\int\frac{\sqrt{\csc(c+dx+\frac{\pi}{2})}}{\sqrt{a+b\csc(c+dx+\frac{\pi}{2})}}dx+}{}\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{4}\left(\frac{5a\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{-7ab^2\int\frac{\sqrt{\csc(c+dx+\frac{\pi}{2})}}{\sqrt{a+b\csc(c+dx+\frac{\pi}{2})}}dx+}{}\right.\right.$$

$\downarrow$ 3134

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{4}\left(\frac{5a\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{-7ab^2\int\frac{\sqrt{\csc(c+dx+\frac{\pi}{2})}}{\sqrt{a+b\csc(c+dx+\frac{\pi}{2})}}dx+}{}\right.\right.$$

$\downarrow$ 3042

3.847. $\int \frac{(a+b\sec(c+dx))^{3/2}}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{4}\left(\frac{5a\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{-7ab^2\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+}{}\right.\right.$$

$\downarrow$ 3132

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{4}\left(\frac{5a\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{-7ab^2\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-}{}\right.\right.$$

$\downarrow$ 4345

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{4}\left(\frac{5a\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{-\frac{7ab^2\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int}{\sqrt{a+b\sec(c+dx)}}}{}\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{4}\left(\frac{5a\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{-\frac{7ab^2\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int}{\sqrt{a+b\sec(c+d}}}{}\right.\right.$$

$\downarrow$ 3142

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{4}\left(\frac{5a\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{-\frac{7ab^2\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int}{\sqrt{a+b\sec(c+dx}}}{}\right.\right.$$

$\downarrow$ 3042

3.847. $\int \frac{(a+b\sec(c+dx))^{3/2}}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{4}\left(\frac{5a\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{-\frac{7ab^2\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int}{\sqrt{a+b\sec(c+}}}{}\right.\right.$$

$\downarrow$ 3140

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{4}\left(\frac{5a\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{-\frac{2b(3a^2+4b^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+}{a+}}}{d\sqrt{a+b\,\mathrm{se}}}}{}\right.\right.$$

input
```
Int[(a + b*Sec[c + d*x])^(3/2)/Cos[c + d*x]^(3/2),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((b*Sec[c + d*x]^(3/2)*Sqrt[a + b*Se
c[c + d*x]]*Sin[c + d*x])/(2*d) + (-1/2*((-14*a*b^2*Sqrt[(b + a*Cos[c + d*
x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(d*
Sqrt[a + b*Sec[c + d*x]]) - (2*b*(3*a^2 + 4*b^2)*Sqrt[(b + a*Cos[c + d*x])
/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(d
*Sqrt[a + b*Sec[c + d*x]]) + (10*a*b*EllipticE[(c + d*x)/2, (2*a)/(a + b)]
*Sqrt[a + b*Sec[c + d*x]])/(d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[
c + d*x]]))/b + (5*a*Sqrt[Sec[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d
*x])/d)/4)
```

3.847.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3286
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]]   Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
 d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

rule 4343
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]])   Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a
, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4345
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]])   Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[
{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

3.847. $\int \frac{(a+b\sec(c+dx))^{3/2}}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

rule 4346
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Simp[d*Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x
]]/Sqrt[a + b*Csc[e + f*x]])   Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4353
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b)*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
1)*((d*Csc[e + f*x])^(n - 1)/(f*(m + n - 1))), x] + Simp[d/(m + n - 1)   In
t[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^(n - 1)*Simp[a*b*(n - 1) +
(b^2*(m + n - 2) + a^2*(m + n - 1))*Csc[e + f*x] + a*b*(2*m + n - 2)*Csc[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && L
tQ[0, m, 2] && LtQ[0, n, 3] && NeQ[m + n - 1, 0] && (IntegerQ[m] || Integer
sQ[2*m, 2*n])
```

rule 4523
```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Simp[A/a   I
nt[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Simp[(A*b - a*B)
/(a*d)   Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ
[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

rule 4590
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(-C)*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
)*((d*Csc[e + f*x])^(n - 1)/(b*f*(m + n + 1))), x] + Simp[d/(b*(m + n + 1))
   Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (
A*b*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2
 - b^2, 0] && GtQ[n, 0]
```

rule 4596
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Simp[C/d^2   Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*C
sc[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[
a + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
 b^2, 0]
```

3.847. $\int \frac{(a+b\sec(c+dx))^{3/2}}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

3.847.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 10.14 (sec) , antiderivative size = 2317, normalized size of antiderivative = 7.75

method	result	size
default	Expression too large to display	2317

input
```
int((a+b*sec(d*x+c))^(3/2)/cos(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4/d/((a-b)/(a+b))^(1/2)*(-5*(1/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(
d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+c
sc(d*x+c)),(-(a+b)/(a-b))^(1/2))*a*b*cos(d*x+c)^4-2*(1/(cos(d*x+c)+1))^(1/
2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF(((a-b)/(a+b))
^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*a*b*cos(d*x+c)^4-10*
(1/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*E
llipticE(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2)
)*a*b*cos(d*x+c)^3-4*(1/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(c
os(d*x+c)+1))^(1/2)*EllipticF(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c))
,(-(a+b)/(a-b))^(1/2))*a*b*cos(d*x+c)^3-5*(1/(cos(d*x+c)+1))^(1/2)*(1/(a+b
)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE(((a-b)/(a+b))^(1/2)*(-c
ot(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*a*b*cos(d*x+c)^2-2*(1/(cos(d*x
+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF(((
a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*a*b*cos(d
*x+c)^2-2*((a-b)/(a+b))^(1/2)*b^2*sin(d*x+c)-5*((a-b)/(a+b))^(1/2)*a^2*cos
(d*x+c)^2*sin(d*x+c)-2*((a-b)/(a+b))^(1/2)*b^2*cos(d*x+c)*sin(d*x+c)-8*(1/
(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Elli
pticPi(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(a+b)/(a-b),I/((a-b)/(
a+b))^(1/2))*b^2*cos(d*x+c)^2-2*((a-b)/(a+b))^(1/2)*a*b*cos(d*x+c)^2*sin(d
*x+c)-7*((a-b)/(a+b))^(1/2)*a*b*cos(d*x+c)*sin(d*x+c)+5*(1/(cos(d*x+c)+...
```

3.847.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+b*sec(d*x+c))^(3/2)/cos(d*x+c)^(3/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.847.6 Sympy [F]

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(a+b\sec(c+dx))^{\frac{3}{2}}}{\cos^{\frac{3}{2}}(c+dx)}\,dx$$

input
```
integrate((a+b*sec(d*x+c))**(3/2)/cos(d*x+c)**(3/2),x)
```

output
```
Integral((a + b*sec(c + d*x))**(3/2)/cos(c + d*x)**(3/2), x)
```

3.847.7 Maxima [F]

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(b\sec(dx+c)+a)^{\frac{3}{2}}}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(3/2)/cos(d*x+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^(3/2)/cos(d*x + c)^(3/2), x)
```

3.847.8 Giac [F]

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\cos^{\frac{3}{2}}(c+dx)}\,dx=\int \frac{(b\sec(dx+c)+a)^{\frac{3}{2}}}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(3/2)/cos(d*x+c)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^(3/2)/cos(d*x + c)^(3/2), x)
```

3.847.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\cos^{\frac{3}{2}}(c+dx)}\,dx=\int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}}{\cos(c+d\,x)^{3/2}}\,dx$$

input
```
int((a + b/cos(c + d*x))^(3/2)/cos(c + d*x)^(3/2),x)
```

output
```
int((a + b/cos(c + d*x))^(3/2)/cos(c + d*x)^(3/2), x)
```

3.848 $\int \cos^{\frac{9}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$

3.848.1 Optimal result 6340
3.848.2 Mathematica [C] (warning: unable to verify) 6341
3.848.3 Rubi [A] (verified) 6342
3.848.4 Maple [B] (warning: unable to verify) 6349
3.848.5 Fricas [C] (verification not implemented) 6350
3.848.6 Sympy [F(-1)] 6351
3.848.7 Maxima [F] 6351
3.848.8 Giac [F] 6352
3.848.9 Mupad [F(-1)] 6352

3.848.1 Optimal result

Integrand size = 25, antiderivative size = 363

$$\begin{aligned}\int \cos^{\frac{9}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx &= \frac{4b(57a^4-62a^2b^2+5b^4)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{315a^2d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}\\ &+\frac{2(147a^4+279a^2b^2-10b^4)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+dx)}}{315a^2d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}}\\ &+\frac{2b(163a^2+5b^2)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{315ad}\\ &+\frac{2(49a^2+75b^2)\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{315d}\\ &+\frac{38ab\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{63d}\\ &+\frac{2a^2\cos^{\frac{7}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{9d}\end{aligned}$$

output
```
4/315*b*(57*a^4-62*a^2*b^2+5*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x
+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*
x+c))/(a+b))^(1/2)/a^2/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)+2/315*(49
*a^2+75*b^2)*cos(d*x+c)^(3/2)*sin(d*x+c)*(a+b*sec(d*x+c))^(1/2)/d+38/63*a*
b*cos(d*x+c)^(5/2)*sin(d*x+c)*(a+b*sec(d*x+c))^(1/2)/d+2/9*a^2*cos(d*x+c)^
(7/2)*sin(d*x+c)*(a+b*sec(d*x+c))^(1/2)/d+2/315*b*(163*a^2+5*b^2)*sin(d*x+
c)*cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2)/a/d+2/315*(147*a^4+279*a^2*b^2-
10*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*
d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2
)/a^2/d/((b+a*cos(d*x+c))/(a+b))^(1/2)
```

3.848.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 15.61 (sec) , antiderivative size = 477, normalized size of antiderivative = 1.31

$$\int \cos^{\frac{9}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \frac{\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}\left(\frac{b(747a^2+20b^2)\sin(c+dx)}{630a}+\frac{1}{630}(133a^2+150b^2)\sin(2(c+dx))+\right.}{d(b+a\cos(c+dx))^2}$$
$$-\frac{2\cos^{\frac{3}{2}}(c+dx)\left(\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\right)^{3/2}(a+b\sec(c+dx))^{5/2}\left(-i(147a^5+147a^4b+279a^3b^2+2\right.}{}$$

input
```
Integrate[Cos[c + d*x]^(9/2)*(a + b*Sec[c + d*x])^(5/2),x]
```

output
```
(Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^(5/2)*((b*(747*a^2 + 20*b^2)*Sin[
c + d*x])/(630*a) + ((133*a^2 + 150*b^2)*Sin[2*(c + d*x)])/630 + (19*a*b*S
in[3*(c + d*x)])/126 + (a^2*Sin[4*(c + d*x)])/36))/(d*(b + a*Cos[c + d*x])
^2) - (2*Cos[c + d*x]^(3/2)*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(3/2)*(a + b
*Sec[c + d*x])^(5/2)*((-I)*(147*a^5 + 147*a^4*b + 279*a^3*b^2 + 279*a^2*b^
3 - 10*a*b^4 - 10*b^5)*EllipticE[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a
+ b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a
 + b)] + I*a*(147*a^4 + 261*a^3*b + 279*a^2*b^2 + 155*a*b^3 - 10*b^4)*Elli
pticF[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sq
rt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] - (147*a^4 + 279*a^2
*b^2 - 10*b^4)*(b + a*Cos[c + d*x])*(Sec[(c + d*x)/2]^2)^(3/2)*Tan[(c + d*
x)/2]))/(315*a^2*d*(b + a*Cos[c + d*x])^3*Sec[c + d*x]^(5/2))
```

3.848.3 Rubi [A] (verified)

Time = 3.48 (sec) , antiderivative size = 399, normalized size of antiderivative = 1.10, number of steps used = 27, number of rules used = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.080, Rules used = {3042, 4752, 3042, 4328, 27, 3042, 4592, 27, 3042, 4592, 27, 3042, 4592, 27, 3042, 4523, 3042, 4343, 3042, 3134, 3042, 3132, 4345, 3042, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^{\frac{9}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^{9/2}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{9}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}{\csc\left(c+dx+\frac{\pi}{2}\right)^{9/2}}dx$$

$$\Big\downarrow \text{4328}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2}{9}\int \frac{19ba^2+\left(7a^2+27b^2\right)\sec(c+dx)a+3b\left(2a^2+3b^2\right)\sec^2(c+dx)}{2\sec^{\frac{7}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}dx+\frac{2a^2\sin(c}{}\right.$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{9}\int \frac{19ba^2+\left(7a^2+27b^2\right)\sec(c+dx)a+3b\left(2a^2+3b^2\right)\sec^2(c+dx)}{\sec^{\frac{7}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}dx+\frac{2a^2\sin(c}{}\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{9}\int \frac{19ba^2+\left(7a^2+27b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)a+3b\left(2a^2+3b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\right.$$

$$\Big\downarrow \text{4592}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{9}\left(\frac{38ab\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{7d\sec^{\frac{5}{2}}(c+dx)}-\frac{2\int-\frac{76b^2\sec^2(c+dx)a^2+(49a^2+75b^2)a^2+b(137a^2}{2\sec^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+d}}}{7a}\right.\right.$$

$\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{9}\left(\frac{\int\frac{76b^2\sec^2(c+dx)a^2+(49a^2+75b^2)a^2+b(137a^2+63b^2)\sec(c+dx)a}{\sec^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}dx}{7a}+\frac{38ab\sin(c+dx)\sqrt{a}}{7d\sec^{\frac{5}{2}}(c+}\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{9}\left(\frac{\int\frac{76b^2\csc(c+dx+\frac{\pi}{2})^2a^2+(49a^2+75b^2)a^2+b(137a^2+63b^2)\csc(c+dx+\frac{\pi}{2})a}{\csc(c+dx+\frac{\pi}{2})^{5/2}\sqrt{a+b\csc(c+dx+\frac{\pi}{2})}}dx}{7a}+\frac{38ab\sin(c+dx}{7d\mathrm{se}}\right.\right.$$

$\downarrow$ 4592

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{9}\left(\frac{\frac{2a(49a^2+75b^2)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5d\sec^{\frac{3}{2}}(c+dx)}-\frac{2\int-\frac{(147a^2+605b^2)\sec(c+dx)a^3+2b(49a^2+75b^2)\sec^2(c+}{2\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}}{5a}}{7a}\right.\right.$$

$\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{9}\left(\frac{\frac{\int\frac{(147a^2+605b^2)\sec(c+dx)a^3+2b(49a^2+75b^2)\sec^2(c+dx)a^2+3b(163a^2+5b^2)a^2}{\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}dx}{5a}+\frac{2a(49a^2+75b^2)\sin(c}{5d\sec^{\frac{3}{2}}}}{7a}\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{9}\left(\frac{\frac{\int\frac{(147a^2+605b^2)\csc(c+dx+\frac{\pi}{2})a^3+2b(49a^2+75b^2)\csc(c+dx+\frac{\pi}{2})^2a^2+3b(163a^2+5b^2)a^2}{\csc(c+dx+\frac{\pi}{2})^{3/2}\sqrt{a+b\csc(c+dx+\frac{\pi}{2})}}dx}{5a}+\frac{2a(49a^2+75}{}}{7a}\right.\right.$$

$\downarrow$ 4592

3.848. $\int \cos^{\frac{9}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{9}\left(\frac{\frac{\frac{2ab\left(163a^2+5b^2\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{d\sqrt{\sec(c+dx)}}-\frac{2\int-\frac{3\left(b\left(261a^2+155b^2\right)\sec(c+dx)a^3+\left(147a^4+279b^2a^2-10b^4\right)a^2\right)}{2\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}}{3a}}{5a}}{7a}\right.\right.$$

$\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{9}\left(\frac{\frac{\frac{\int\frac{b\left(261a^2+155b^2\right)\sec(c+dx)a^3+\left(147a^4+279b^2a^2-10b^4\right)a^2}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{a}+\frac{2ab\left(163a^2+5b^2\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{d\sqrt{\sec(c+dx)}}}{5a}+}{7a}\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{9}\left(\frac{\frac{\frac{\int\frac{b\left(261a^2+155b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)a^3+\left(147a^4+279b^2a^2-10b^4\right)a^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}+\frac{2ab\left(163a^2+5b^2\right)\sin(c+dx)\sqrt{a+b\sec(c+dx}}{d\sqrt{\sec(c+dx)}}}{5a}}{7a}\right.\right.$$

$\downarrow$ 4523

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{9}\left(\frac{\frac{\frac{2ab\left(57a^4-62a^2b^2+5b^4\right)\int\frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}dx+a\left(147a^4+279a^2b^2-10b^4\right)\int\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}dx}{a}+\frac{2ab\left(163a^2+5b^2\right.}{}}{5a}}{7a}\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{9}\left(\frac{\frac{\frac{2ab\left(57a^4-62a^2b^2+5b^4\right)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+a\left(147a^4+279a^2b^2-10b^4\right)\int\frac{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}+\frac{2ab\left(1\right.}{}}{5a}}{7a}\right.\right.$$

3.848. $\int \cos^{\frac{9}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$

$$\Big\downarrow \; 4343$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{9}\left(\frac{\frac{\frac{2ab\left(57a^4-62a^2b^2+5b^4\right)\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{a\left(147a^4+279a^2b^2-10b^4\right)\sqrt{a+b\sec(c+dx)}\int \sqrt{b+a\cos(c+dx)}}{\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}}{a}}{5a}}{7a}\right.\right.$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{9}\left(\frac{\frac{\frac{2ab\left(57a^4-62a^2b^2+5b^4\right)\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{a\left(147a^4+279a^2b^2-10b^4\right)\sqrt{a+b\sec(c+dx)}\int \sqrt{b+a\sin\left(c+dx\right.}}{\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}}{a}}{5a}}{7a}\right.\right.$$

$$\Big\downarrow \; 3134$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{9}\left(\frac{\frac{\frac{2ab\left(57a^4-62a^2b^2+5b^4\right)\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{a\left(147a^4+279a^2b^2-10b^4\right)\sqrt{a+b\sec(c+dx)}\int \sqrt{\frac{b}{a+b}+\frac{a\cos(c}{a+b}}}{\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{a}}{5a}}{7a}\right.\right.$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{9}\left(\frac{\frac{\frac{2ab\left(57a^4-62a^2b^2+5b^4\right)\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{a\left(147a^4+279a^2b^2-10b^4\right)\sqrt{a+b\sec(c+dx)}\int \sqrt{\frac{b}{a+b}+\frac{a\sin\left(c\right.}{a}}}{\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{a}}{5a}}{7a}\right.\right.$$

$$\Big\downarrow \; 3132$$

3.848. $\int \cos^{\frac{9}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{9}\left(\frac{\frac{\frac{2ab\left(57a^4-62a^2b^2+5b^4\right)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a\left(147a^4+279a^2b^2-10b^4\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right.}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{a}}{5a}}{7a}\right.\right.$$

$\downarrow$ 4345

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{9}\left(\frac{\frac{\frac{2ab\left(57a^4-62a^2b^2+5b^4\right)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{1}{\sqrt{b+a\cos(c+dx)}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{2a\left(147a^4+279a^2b^2-10b^4\right)\sqrt{a+b\sec}}{d\sqrt{\sec(c+dx)}\sqrt{a\cos}}}{a}}{5a}\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{9}\left(\frac{\frac{\frac{2ab\left(57a^4-62a^2b^2+5b^4\right)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{1}{\sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{2a\left(147a^4+279a^2b^2-10b^4\right)\sqrt{a+}}{d\sqrt{\sec(c+dx)}\sqrt{}}}{a}}{5a}\right.\right.$$

$\downarrow$ 3142

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{9}\left(\frac{\frac{\frac{2ab\left(57a^4-62a^2b^2+5b^4\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{2a\left(147a^4+279a^2b^2-10b^4\right)\sqrt{a}}{d\sqrt{\sec(c+dx)}\sqrt{}}}{a}}{5a}\right.\right.$$

$\downarrow$ 3042

3.848. $\int \cos^{\frac{9}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{9}\left(\frac{\frac{\frac{\frac{\frac{2ab\left(57a^4-62a^2b^2+5b^4\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{2a\left(147a^4+279a^2b^2-10b^4\right)}{d\sqrt{\sec(c+d}}}{a}}{5a}}{}\right.\right.$$

$$\Big\downarrow \; 3140$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{9d\sec^{\frac{7}{2}}(c+dx)}+\frac{1}{9}\left(\frac{\frac{2a\left(49a^2+75b^2\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{2ab\left(16\right.}{}}{}\right.\right.$$

input
```
Int[Cos[c + d*x]^(9/2)*(a + b*Sec[c + d*x])^(5/2),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*a^2*Sqrt[a + b*Sec[c + d*x]]*Sin
[c + d*x])/(9*d*Sec[c + d*x]^(7/2)) + ((38*a*b*Sqrt[a + b*Sec[c + d*x]]*Si
n[c + d*x])/(7*d*Sec[c + d*x]^(5/2)) + ((2*a*(49*a^2 + 75*b^2)*Sqrt[a + b*
Sec[c + d*x]]*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)) + (((4*a*b*(57*a^4 -
62*a^2*b^2 + 5*b^4)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)
/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(d*Sqrt[a + b*Sec[c + d*x]]) + (2*a
*(147*a^4 + 279*a^2*b^2 - 10*b^4)*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sq
rt[a + b*Sec[c + d*x]])/(d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c +
 d*x]]))/a + (2*a*b*(163*a^2 + 5*b^2)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x
])/(d*Sqrt[Sec[c + d*x]]))/(5*a))/(7*a))/9)
```

3.848. $\int \cos^{\frac{9}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$

3.848.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 4328
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)*
((d*Csc[e + f*x])^n/(f*n)), x] - Simp[1/(d*n)   Int[(a + b*Csc[e + f*x])^(m
 - 3)*(d*Csc[e + f*x])^(n + 1)*Simp[a^2*b*(m - 2*n - 2) - a*(3*b^2*n + a^2*
(n + 1))*Csc[e + f*x] - b*(b^2*n + a^2*(m + n - 1))*Csc[e + f*x]^2, x], x],
 x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && ((Int
egerQ[m] && LtQ[n, -1]) || (IntegersQ[m + 1/2, 2*n] && LeQ[n, -1]))
```

rule 4343
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]])   Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a
, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4345
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]])   Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[
{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4523
```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Simp[A/a   I
nt[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Simp[(A*b - a*B)
/(a*d)   Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ
[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

rule 4592
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Simp[1/(a*d*n)   Int[(a + b*Csc[e + f*x])^m
*(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*
Csc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

3.848.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 3821 vs. $2(381) = 762$.

Time = 1492.86 (sec) , antiderivative size = 3822, normalized size of antiderivative = 10.53

method	result	size
default	Expression too large to display	3822

input
```
int(cos(d*x+c)^(9/2)*(a+b*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/315/d/a^2/((a-b)/(a+b))^(1/2)*(147*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)
+1))^(1/2)*EllipticE(((a-b)/(a+b))^(1/2)*(cot(d*x+c)-csc(d*x+c)),(-(a+b)/(
a-b))^(1/2))*(1/(cos(d*x+c)+1))^(1/2)*a^4*b-279*(1/(a+b)*(b+a*cos(d*x+c))/
(cos(d*x+c)+1))^(1/2)*EllipticE(((a-b)/(a+b))^(1/2)*(cot(d*x+c)-csc(d*x+c)
),(-(a+b)/(a-b))^(1/2))*(1/(cos(d*x+c)+1))^(1/2)*a^3*b^2+279*(1/(a+b)*(b+a
*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE(((a-b)/(a+b))^(1/2)*(cot(d*x+
c)-csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*(1/(cos(d*x+c)+1))^(1/2)*a^2*b^3+10*(
1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE(((a-b)/(a+b))^(1/
2)*(cot(d*x+c)-csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*(1/(cos(d*x+c)+1))^(1/2)*
a*b^4-261*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF(((a-b)
/(a+b))^(1/2)*(cot(d*x+c)-csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*(1/(cos(d*x+c)
+1))^(1/2)*a^4*b+279*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ellip
ticF(((a-b)/(a+b))^(1/2)*(cot(d*x+c)-csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*(1/
(cos(d*x+c)+1))^(1/2)*a^3*b^2-155*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1)
)^(1/2)*EllipticF(((a-b)/(a+b))^(1/2)*(cot(d*x+c)-csc(d*x+c)),(-(a+b)/(a-b
))^(1/2))*(1/(cos(d*x+c)+1))^(1/2)*a^2*b^3-10*(1/(a+b)*(b+a*cos(d*x+c))/(c
os(d*x+c)+1))^(1/2)*EllipticF(((a-b)/(a+b))^(1/2)*(cot(d*x+c)-csc(d*x+c)),
(-(a+b)/(a-b))^(1/2))*(1/(cos(d*x+c)+1))^(1/2)*a*b^4+294*EllipticF(((a-b)/
(a+b))^(1/2)*(cot(d*x+c)-csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*c
os(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a^5*cos(d*x+c...
```

3.848.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.13 (sec) , antiderivative size = 531, normalized size of antiderivative = 1.46

$$\int \cos^{\frac{9}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \frac{6\left(35\,a^5\cos\left(dx+c\right)^3+95\,a^4b\cos\left(dx+c\right)^2+163\,a^4b+5\,a^2b^3+\left(49\,a^5+75\,a^3b^2\right)\cos\left(dx+c\right)\right.}{}$$

input
```
integrate(cos(d*x+c)^(9/2)*(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
1/945*(6*(35*a^5*cos(d*x + c)^3 + 95*a^4*b*cos(d*x + c)^2 + 163*a^4*b + 5*
a^2*b^3 + (49*a^5 + 75*a^3*b^2)*cos(d*x + c))*sqrt((a*cos(d*x + c) + b)/co
s(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) + sqrt(2)*(-489*I*a^4*b + 93*I
*a^2*b^3 - 20*I*b^5)*sqrt(a)*weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2,
 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x + c) +
2*b)/a) + sqrt(2)*(489*I*a^4*b - 93*I*a^2*b^3 + 20*I*b^5)*sqrt(a)*weierstr
assPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a
*cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a) - 3*sqrt(2)*(-147*I*a^5 - 279
*I*a^3*b^2 + 10*I*a*b^4)*sqrt(a)*weierstrassZeta(-4/3*(3*a^2 - 4*b^2)/a^2,
 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2,
 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x + c) +
2*b)/a)) - 3*sqrt(2)*(147*I*a^5 + 279*I*a^3*b^2 - 10*I*a*b^4)*sqrt(a)*weie
rstrassZeta(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, weierstr
assPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a
*cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a)))/(a^3*d)
```

3.848.6 Sympy [F(-1)]

Timed out.

$$\int \cos^{\frac{9}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(9/2)*(a+b*sec(d*x+c))**(5/2),x)
```

output
```
Timed out
```

3.848.7 Maxima [F]

$$\int \cos^{\frac{9}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \int (b\sec(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^{\frac{9}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(9/2)*(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^(9/2), x)
```

3.848.8 Giac [F]

$$\int \cos^{\frac{9}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \int (b\sec(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^{\frac{9}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(9/2)*(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^(9/2), x)
```

3.848.9 Mupad [F(-1)]

Timed out.

$$\int \cos^{\frac{9}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \int \cos(c+d\,x)^{9/2}\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2}dx$$

input
```
int(cos(c + d*x)^(9/2)*(a + b/cos(c + d*x))^(5/2),x)
```

output
```
int(cos(c + d*x)^(9/2)*(a + b/cos(c + d*x))^(5/2), x)
```

3.849 $\int \cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$

3.849.1 Optimal result 6353
3.849.2 Mathematica [C] (warning: unable to verify) 6354
3.849.3 Rubi [A] (verified) 6354
3.849.4 Maple [B] (verified) 6361
3.849.5 Fricas [C] (verification not implemented) 6362
3.849.6 Sympy [F(-1)] 6363
3.849.7 Maxima [F] 6363
3.849.8 Giac [F] 6364
3.849.9 Mupad [F(-1)] 6364

3.849.1 Optimal result

Integrand size = 25, antiderivative size = 303

$$\begin{aligned}\int \cos^{\frac{7}{2}}(c+dx)(a&+b\sec(c+dx))^{5/2}\,dx=\frac{2(5a^4-2a^2b^2-3b^4)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{21ad\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}\\&+\frac{2b(29a^2+3b^2)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+dx)}}{21ad\sqrt{\frac{b+a\cos(c+dx)}{a+b}}}\\&+\frac{2(5a^2+9b^2)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{21d}\\&+\frac{6ab\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{7d}\\&+\frac{2a^2\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{7d}\end{aligned}$$

output
```
2/21*(5*a^4-2*a^2*b^2-3*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*
c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))
/(a+b))^(1/2)/a/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)+6/7*a*b*cos(d*x+
c)^(3/2)*sin(d*x+c)*(a+b*sec(d*x+c))^(1/2)/d+2/7*a^2*cos(d*x+c)^(5/2)*sin(
d*x+c)*(a+b*sec(d*x+c))^(1/2)/d+2/21*(5*a^2+9*b^2)*sin(d*x+c)*cos(d*x+c)^(
1/2)*(a+b*sec(d*x+c))^(1/2)/d+2/21*b*(29*a^2+3*b^2)*(cos(1/2*d*x+1/2*c)^2)
^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(
1/2))*cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2)/a/d/((b+a*cos(d*x+c))/(a+b))
^(1/2)
```

3.849.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 13.80 (sec) , antiderivative size = 419, normalized size of antiderivative = 1.38

$$\int \cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \frac{\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}\left(\frac{1}{42}(23a^2+36b^2)\sin(c+dx)+\frac{3}{7}ab\sin(2(c+dx))+\frac{1}{14}a^2\sin\right.}{d(b+a\cos(c+dx))^2} + \frac{2\cos^{\frac{3}{2}}(c+dx)\left(\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\right)^{3/2}(a+b\sec(c+dx))^{5/2}\left(ib(29a^3+29a^2b+3ab^2+3b^3)E(i\right.}{}$$

input
```
Integrate[Cos[c + d*x]^(7/2)*(a + b*Sec[c + d*x])^(5/2),x]
```

output
```
(Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^(5/2)*(((23*a^2 + 36*b^2)*Sin[c +
 d*x])/42 + (3*a*b*Sin[2*(c + d*x)])/7 + (a^2*Sin[3*(c + d*x)])/14))/(d*(b
 + a*Cos[c + d*x])^2) + (2*Cos[c + d*x]^(3/2)*(Cos[(c + d*x)/2]^2*Sec[c +
d*x])^(3/2)*(a + b*Sec[c + d*x])^(5/2)*(I*b*(29*a^3 + 29*a^2*b + 3*a*b^2 +
 3*b^3)*EllipticE[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c +
d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] - I*a*(5
*a^3 + 29*a^2*b + 27*a*b^2 + 3*b^3)*EllipticF[I*ArcSinh[Tan[(c + d*x)/2]],
 (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c +
d*x)/2]^2)/(a + b)] + b*(29*a^2 + 3*b^2)*(b + a*Cos[c + d*x])*(Sec[(c + d*
x)/2]^2)^(3/2)*Tan[(c + d*x)/2]))/(21*a*d*(b + a*Cos[c + d*x])^3*Sec[c + d
*x]^(5/2))
```

3.849.3 Rubi [A] (verified)

Time = 2.78 (sec) , antiderivative size = 333, normalized size of antiderivative = 1.10, number of steps used = 24, number of rules used = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.960$, Rules used = {3042, 4752, 3042, 4328, 27, 3042, 4592, 27, 3042, 4592, 27, 3042, 4523, 3042, 4343, 3042, 3134, 3042, 3132, 4345, 3042, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^{7/2}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}dx$$

$$\Big\downarrow \; 4752$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{7}{2}}(c+dx)}dx$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}dx$$

$$\Big\downarrow \; 4328$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2}{7}\int \frac{15ba^2+\left(5a^2+21b^2\right)\sec(c+dx)a+b\left(4a^2+7b^2\right)\sec^2(c+dx)}{2\sec^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}dx+\frac{2a^2\sin(c+}{7}\right.$$

$$\Big\downarrow \; 27$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}\int \frac{15ba^2+\left(5a^2+21b^2\right)\sec(c+dx)a+b\left(4a^2+7b^2\right)\sec^2(c+dx)}{\sec^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}dx+\frac{2a^2\sin(c+}{7}\right.$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}\int \frac{15ba^2+\left(5a^2+21b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)a+b\left(4a^2+7b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\right.$$

$$\Big\downarrow \; 4592$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}\left(\frac{6ab\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{d\sec^{\frac{3}{2}}(c+dx)}-\frac{2\int -\frac{5\left(6b^2\sec^2(c+dx)a^2+\left(5a^2+9b^2\right)a^2+b\left(13a^2+7b\right.\right.}{2\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}}{5a}\right.\right.$$

$$\Big\downarrow \; 27$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}\left(\frac{\int \frac{6b^2\sec^2(c+dx)a^2+\left(5a^2+9b^2\right)a^2+b\left(13a^2+7b^2\right)\sec(c+dx)a}{\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}dx}{a}+\frac{6ab\sin(c+dx)\sqrt{a+b\sec}}{d\sec^{\frac{3}{2}}(c+dx)}\right.\right.$$

$$\Big\downarrow \; 3042$$

3.849. $\int \cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}\left(\frac{\int \frac{6b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2a^2+\left(5a^2+9b^2\right)a^2+b\left(13a^2+7b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)a}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}+\frac{6ab\sin(c+dx)\sqrt{a}}{d\sec^{\frac{3}{2}}(c}\right.\right.$$

$\downarrow$ 4592

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}\left(\frac{\frac{2a\left(5a^2+9b^2\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3d\sqrt{\sec(c+dx)}}-\frac{2\int -\frac{\left(5a^2+27b^2\right)\sec(c+dx)a^3+b\left(29a^2+3b^2\right)a^2}{2\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{3a}}{a}+\frac{6ab\,s}{}\right.\right.$$

$\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}\left(\frac{\frac{\int \frac{\left(5a^2+27b^2\right)\sec(c+dx)a^3+b\left(29a^2+3b^2\right)a^2}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{3a}+\frac{2a\left(5a^2+9b^2\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3d\sqrt{\sec(c+dx)}}}{a}+\frac{6ab\sin}{}\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}\left(\frac{\frac{\int \frac{\left(5a^2+27b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)a^3+b\left(29a^2+3b^2\right)a^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{3a}+\frac{2a\left(5a^2+9b^2\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3d\sqrt{\sec(c+dx)}}}{a}+\frac{6ab}{}\right.\right.$$

$\downarrow$ 4523

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}\left(\frac{\frac{ab\left(29a^2+3b^2\right)\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}dx+a\left(5a^4-2a^2b^2-3b^4\right)\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}dx}{3a}+\frac{2a\left(5a^2+9b^2\right)\sin(c+d}{3d\sqrt{\sec(}}}{a}\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}\left(\frac{\frac{ab\left(29a^2+3b^2\right)\int \frac{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+a\left(5a^4-2a^2b^2-3b^4\right)\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{3a}+\frac{2a\left(5a^2+9b^2\right)}{}}{a}\right.\right.$$

3.849. $\int \cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$

$$\Big\downarrow \; 4343$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}\left(\frac{\frac{\frac{ab\left(29a^2+3b^2\right)\sqrt{a+b\sec(c+dx)}\int\sqrt{b+a\cos(c+dx)}dx}{\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}+a\left(5a^4-2a^2b^2-3b^4\right)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{3a}+\frac{2a(5}{}}{a}\right.\right.$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}\left(\frac{\frac{\frac{ab\left(29a^2+3b^2\right)\sqrt{a+b\sec(c+dx)}\int\sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}+a\left(5a^4-2a^2b^2-3b^4\right)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{3a}+\frac{2}{}}{a}\right.\right.$$

$$\Big\downarrow \; 3134$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}\left(\frac{\frac{\frac{ab\left(29a^2+3b^2\right)\sqrt{a+b\sec(c+dx)}\int\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}dx}{\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+a\left(5a^4-2a^2b^2-3b^4\right)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{3a}+\frac{2}{}}{a}\right.\right.$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}\left(\frac{\frac{\frac{ab\left(29a^2+3b^2\right)\sqrt{a+b\sec(c+dx)}\int\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}dx}{\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+a\left(5a^4-2a^2b^2-3b^4\right)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{3a}}{a}\right.\right.$$

$$\Big\downarrow \; 3132$$

3.849. $\int \cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}\left(\frac{\frac{a\left(5a^4-2a^2b^2-3b^4\right)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2ab\left(29a^2+3b^2\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{3a}+\frac{2a(5a}{}}{a}\right.\right.$$

$\downarrow$ 4345

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}\left(\frac{\frac{a\left(5a^4-2a^2b^2-3b^4\right)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{1}{\sqrt{b+a\cos(c+dx)}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{2ab\left(29a^2+3b^2\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+\right.}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{3a}}{a}\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}\left(\frac{\frac{a\left(5a^4-2a^2b^2-3b^4\right)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{1}{\sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{2ab\left(29a^2+3b^2\right)\sqrt{a+b\sec(c+dx)}E\left(\right.}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx}{a+b}}}}{3a}}{a}\right.\right.$$

$\downarrow$ 3142

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}\left(\frac{\frac{a\left(5a^4-2a^2b^2-3b^4\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{2ab\left(29a^2+3b^2\right)\sqrt{a+b\sec(c+dx)}E\left(\right.}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+d}{a+b}}}}{3a}}{a}\right.\right.$$

$\downarrow$ 3042

3.849. $\int \cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{7}\left(\frac{\frac{\frac{a\left(5a^4-2a^2b^2-3b^4\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{2ab\left(29a^2+3b^2\right)\sqrt{a+b\sec(c+dx)}}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos}{}}}}{3a}}{a}\right.\right.$$

$\downarrow$ 3140

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{7d\sec^{\frac{5}{2}}(c+dx)}+\frac{1}{7}\left(\frac{\frac{2a\left(5a^2+9b^2\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3d\sqrt{\sec(c+dx)}}+\frac{2ab\left(29a^2\right.}{}}{}\right.\right.$$

input
```
Int[Cos[c + d*x]^(7/2)*(a + b*Sec[c + d*x])^(5/2),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*a^2*Sqrt[a + b*Sec[c + d*x]]*Sin
[c + d*x])/(7*d*Sec[c + d*x]^(5/2)) + ((6*a*b*Sqrt[a + b*Sec[c + d*x]]*Sin
[c + d*x])/(d*Sec[c + d*x]^(3/2)) + (((2*a*(5*a^4 - 2*a^2*b^2 - 3*b^4)*Sqr
t[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]*Sqrt
[Sec[c + d*x]])/(d*Sqrt[a + b*Sec[c + d*x]]) + (2*a*b*(29*a^2 + 3*b^2)*Ell
ipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(d*Sqrt[(b +
a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]))/(3*a) + (2*a*(5*a^2 + 9*b^2)
*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]]))/a)/7)
```

3.849.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 4328
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)*
((d*Csc[e + f*x])^n/(f*n)), x] - Simp[1/(d*n)   Int[(a + b*Csc[e + f*x])^(m
 - 3)*(d*Csc[e + f*x])^(n + 1)*Simp[a^2*b*(m - 2*n - 2) - a*(3*b^2*n + a^2*
(n + 1))*Csc[e + f*x] - b*(b^2*n + a^2*(m + n - 1))*Csc[e + f*x]^2, x], x],
 x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && ((Int
egerQ[m] && LtQ[n, -1]) || (IntegersQ[m + 1/2, 2*n] && LeQ[n, -1]))
```

rule 4343
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]])   Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a
, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4345
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]])   Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[
{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

3.849. $\int \cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$

rule 4523
```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Simp[A/a   I
nt[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Simp[(A*b - a*B)
/(a*d)   Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ
[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

rule 4592
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Simp[1/(a*d*n)   Int[(a + b*Csc[e + f*x])^m
*(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*
Csc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

3.849.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2765 vs. $2(327) = 654$.

Time = 1253.64 (sec) , antiderivative size = 2766, normalized size of antiderivative = 9.13

method	result	size
default	Expression too large to display	2766

input
```
int(cos(d*x+c)^(7/2)*(a+b*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/21/d/a/((a-b)/(a+b))^(1/2)*(3*(1/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos
(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+
csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*a*b^3*cos(d*x+c)^2-3*(1/(cos(d*x+c)+1))^
(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE(((a-b)/(a+
b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*b^4+5*(1/(cos(d*x
+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF(((
a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*a^4+3*((a
-b)/(a+b))^(1/2)*b^4*sin(d*x+c)-29*(1/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*
cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF(((a-b)/(a+b))^(1/2)*(-cot(d*x+
c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*a^3*b*cos(d*x+c)^2+27*(1/(cos(d*x+c)+
1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF(((a-b)
/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*a^2*b^2*cos(d
*x+c)^2-3*(1/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1
))^(1/2)*EllipticF(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a
-b))^(1/2))*a*b^3*cos(d*x+c)^2+58*(1/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*c
os(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE(((a-b)/(a+b))^(1/2)*(-cot(d*x+c
)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*a^3*b*cos(d*x+c)-58*(1/(cos(d*x+c)+1))
^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE(((a-b)/(a
+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*a^2*b^2*cos(d*x+
c)+6*(1/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))...
```

3.849.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 491, normalized size of antiderivative = 1.62

$$\int \cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \frac{6\left(3\,a^4\cos\left(dx+c\right)^2+9\,a^3b\cos\left(dx+c\right)+5\,a^4+9\,a^2b^2\right)\sqrt{\frac{a\cos(dx+c)+b}{\cos(dx+c)}}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+\right.}{}$$

input
```
integrate(cos(d*x+c)^(7/2)*(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
1/63*(6*(3*a^4*cos(d*x + c)^2 + 9*a^3*b*cos(d*x + c) + 5*a^4 + 9*a^2*b^2)*
sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) +
sqrt(2)*(-15*I*a^4 - 23*I*a^2*b^2 + 6*I*b^4)*sqrt(a)*weierstrassPInverse(-
4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c)
 + 3*I*a*sin(d*x + c) + 2*b)/a) + sqrt(2)*(15*I*a^4 + 23*I*a^2*b^2 - 6*I*b
^4)*sqrt(a)*weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b -
8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a) - 3*sqrt(
2)*(-29*I*a^3*b - 3*I*a*b^3)*sqrt(a)*weierstrassZeta(-4/3*(3*a^2 - 4*b^2)/
a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/
a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x + c
) + 2*b)/a)) - 3*sqrt(2)*(29*I*a^3*b + 3*I*a*b^3)*sqrt(a)*weierstrassZeta(
-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(
-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c
) - 3*I*a*sin(d*x + c) + 2*b)/a)))/(a^2*d)
```

3.849.6 Sympy [F(-1)]

Timed out.

$$\int \cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(7/2)*(a+b*sec(d*x+c))**(5/2),x)
```

output
```
Timed out
```

3.849.7 Maxima [F]

$$\int \cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \int (b\sec(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^{\frac{7}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(7/2)*(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^(7/2), x)
```

3.849.8 Giac [F]

$$\int \cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \int (b\sec(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^{\frac{7}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(7/2)*(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^(7/2), x)
```

3.849.9 Mupad [F(-1)]

Timed out.

$$\int \cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \int \cos(c+d\,x)^{7/2}\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2}dx$$

input
```
int(cos(c + d*x)^(7/2)*(a + b/cos(c + d*x))^(5/2),x)
```

output
```
int(cos(c + d*x)^(7/2)*(a + b/cos(c + d*x))^(5/2), x)
```

3.850 $\int \cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$

3.850.1 Optimal result 6365
3.850.2 Mathematica [C] (warning: unable to verify) 6366
3.850.3 Rubi [A] (verified) 6366
3.850.4 Maple [B] (verified) 6372
3.850.5 Fricas [C] (verification not implemented) 6373
3.850.6 Sympy [F(-1)] 6374
3.850.7 Maxima [F] 6374
3.850.8 Giac [F] 6375
3.850.9 Mupad [F(-1)] 6375

3.850.1 Optimal result

Integrand size = 25, antiderivative size = 239

$$\begin{aligned}\int \cos^{\frac{5}{2}}(c+dx)(a&+b\sec(c+dx))^{5/2}\,dx=\frac{16b(a^2-b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{15d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}\\&+\frac{2(9a^2+23b^2)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+dx)}}{15d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}}\\&+\frac{22ab\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{15d}\\&+\frac{2a^2\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{5d}\end{aligned}$$

output
```
16/15*b*(a^2-b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elliptic
F(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/
2)/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)+2/5*a^2*cos(d*x+c)^(3/2)*sin(
d*x+c)*(a+b*sec(d*x+c))^(1/2)/d+22/15*a*b*sin(d*x+c)*cos(d*x+c)^(1/2)*(a+b
*sec(d*x+c))^(1/2)/d+2/15*(9*a^2+23*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(
1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*cos(d
*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2)/d/((b+a*cos(d*x+c))/(a+b))^(1/2)
```

3.850.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 13.17 (sec) , antiderivative size = 391, normalized size of antiderivative = 1.64

$$\int \cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \frac{\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}\left(\frac{22}{15}ab\sin(c+dx)+\frac{1}{5}a^2\sin(2(c+dx))\right)}{d(b+a\cos(c+dx))^2} - \frac{2\cos^{\frac{3}{2}}(c+dx)\left(\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\right)^{3/2}(a+b\sec(c+dx))^{5/2}\left(-i(9a^3+9a^2b+23ab^2+23b^3)E\right.}{}$$

input
```
Integrate[Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^(5/2),x]
```

output
```
(Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^(5/2)*((22*a*b*Sin[c + d*x])/15 +
 (a^2*Sin[2*(c + d*x)])/5))/(d*(b + a*Cos[c + d*x])^2) - (2*Cos[c + d*x]^(
3/2)*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(3/2)*(a + b*Sec[c + d*x])^(5/2)*((
-I)*(9*a^3 + 9*a^2*b + 23*a*b^2 + 23*b^3)*EllipticE[I*ArcSinh[Tan[(c + d*x
)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec
[(c + d*x)/2]^2)/(a + b)] + I*(9*a^3 + 17*a^2*b + 23*a*b^2 + 15*b^3)*Ellip
ticF[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqr
t[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] - (9*a^2 + 23*b^2)*(b
 + a*Cos[c + d*x])*(Sec[(c + d*x)/2]^2)^(3/2)*Tan[(c + d*x)/2]))/(15*d*(b
+ a*Cos[c + d*x])^3*Sec[c + d*x]^(5/2))
```

3.850.3 Rubi [A] (verified)

Time = 2.15 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.13, number of steps used = 21, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.840$, Rules used = {3042, 4752, 3042, 4328, 27, 3042, 4592, 27, 3042, 4523, 3042, 4343, 3042, 3134, 3042, 3132, 4345, 3042, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}dx$$

$$\Big\downarrow \; 4752$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{5}{2}}(c+dx)}dx$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}dx$$

$$\Big\downarrow \; 4328$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2}{5}\int \frac{11ba^2+3\left(a^2+5b^2\right)\sec(c+dx)a+b\left(2a^2+5b^2\right)\sec^2(c+dx)}{2\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}dx+\frac{2a^2\sin(c+\right.$$

$$\Big\downarrow \; 27$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\int \frac{11ba^2+3\left(a^2+5b^2\right)\sec(c+dx)a+b\left(2a^2+5b^2\right)\sec^2(c+dx)}{\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}dx+\frac{2a^2\sin(c+\right.$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\int \frac{11ba^2+3\left(a^2+5b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)a+b\left(2a^2+5b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\right.$$

$$\Big\downarrow \; 4592$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(\frac{22ab\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3d\sqrt{\sec(c+dx)}}-\frac{2\int -\frac{\left(9a^2+23b^2\right)a^2+b\left(17a^2+15b^2\right)\sec(c+dx)a}{2\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{3a}\right.\right.$$

$$\Big\downarrow \; 27$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(\frac{\int \frac{\left(9a^2+23b^2\right)a^2+b\left(17a^2+15b^2\right)\sec(c+dx)a}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{3a}+\frac{22ab\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3d\sqrt{\sec(c+dx)}}\right)\right.$$

$$\Big\downarrow \; 3042$$

3.850. $\int \cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(\frac{\int \frac{(9a^2+23b^2)a^2+b(17a^2+15b^2)\csc(c+dx+\frac{\pi}{2})a}{\sqrt{\csc(c+dx+\frac{\pi}{2})}\sqrt{a+b\csc(c+dx+\frac{\pi}{2})}}dx}{3a}+\frac{22ab\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3d\sqrt{\sec(c+dx)}}\right.\right.$$

$\downarrow$ 4523

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(\frac{8ab(a^2-b^2)\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}dx+a(9a^2+23b^2)\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}dx}{3a}+\frac{22ab\sin(c\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(\frac{8ab(a^2-b^2)\int \frac{\sqrt{\csc(c+dx+\frac{\pi}{2})}}{\sqrt{a+b\csc(c+dx+\frac{\pi}{2})}}dx+a(9a^2+23b^2)\int \frac{\sqrt{a+b\csc(c+dx+\frac{\pi}{2})}}{\sqrt{\csc(c+dx+\frac{\pi}{2})}}dx}{3a}+\frac{22a\right.\right.$$

$\downarrow$ 4343

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(\frac{8ab(a^2-b^2)\int \frac{\sqrt{\csc(c+dx+\frac{\pi}{2})}}{\sqrt{a+b\csc(c+dx+\frac{\pi}{2})}}dx+\frac{a(9a^2+23b^2)\sqrt{a+b\sec(c+dx)}\int \sqrt{b+a\cos(c+dx)}dx}{\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}}{3a}\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(\frac{8ab(a^2-b^2)\int \frac{\sqrt{\csc(c+dx+\frac{\pi}{2})}}{\sqrt{a+b\csc(c+dx+\frac{\pi}{2})}}dx+\frac{a(9a^2+23b^2)\sqrt{a+b\sec(c+dx)}\int \sqrt{b+a\sin(c+dx+\frac{\pi}{2})a}}{\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}}{3a}\right.\right.$$

$\downarrow$ 3134

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(\frac{8ab(a^2-b^2)\int \frac{\sqrt{\csc(c+dx+\frac{\pi}{2})}}{\sqrt{a+b\csc(c+dx+\frac{\pi}{2})}}dx+\frac{a(9a^2+23b^2)\sqrt{a+b\sec(c+dx)}\int \sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}dx}{\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{3a}\right.\right.$$

$\downarrow$ 3042

3.850. $\int \cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(\frac{8ab\left(a^2-b^2\right)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{a\left(9a^2+23b^2\right)\sqrt{a+b\sec(c+dx)}\int\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}{\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{3a}\right.\right.$$

$\downarrow$ 3132

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(\frac{8ab\left(a^2-b^2\right)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a\left(9a^2+23b^2\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{3a}+\right.\right.$$

$\downarrow$ 4345

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(\frac{\frac{8ab\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{1}{\sqrt{b+a\cos(c+dx)}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{2a\left(9a^2+23b^2\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}\right.}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx}{a+b}}}}{3a}\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(\frac{\frac{8ab\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{1}{\sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{2a\left(9a^2+23b^2\right)\sqrt{a+b\sec(c+dx)}E}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c}{a}}}}{3a}\right.\right.$$

$\downarrow$ 3142

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(\frac{\frac{8ab\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{2a\left(9a^2+23b^2\right)\sqrt{a+b\sec(c+dx)}E}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+}{a+}}}}{3a}\right.\right.$$

$\downarrow$ 3042

3.850. $\int\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(\frac{\frac{8ab(a^2-b^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{2a(9a^2+23b^2)\sqrt{a+b\sec(c+d}}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos}{}}}}{3a}\right.\right.$$

$$\Big\downarrow \; 3140$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{5}\left(\frac{\frac{16ab(a^2-b^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}+\frac{2a(9a^2+23b^2)\sqrt{a+b\sec(c+dx}}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos}{}}}}{3a}\right.\right.$$

input
```
Int[Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^(5/2),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*a^2*Sqrt[a + b*Sec[c + d*x]]*Sin
[c + d*x])/(5*d*Sec[c + d*x]^(3/2)) + (((16*a*b*(a^2 - b^2)*Sqrt[(b + a*Co
s[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*
x]])/(d*Sqrt[a + b*Sec[c + d*x]]) + (2*a*(9*a^2 + 23*b^2)*EllipticE[(c + d
*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(d*Sqrt[(b + a*Cos[c + d*x
])/(a + b)]*Sqrt[Sec[c + d*x]]))/(3*a) + (22*a*b*Sqrt[a + b*Sec[c + d*x]]*
Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]]))/5)
```

3.850.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 4328
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)*
((d*Csc[e + f*x])^n/(f*n)), x] - Simp[1/(d*n)   Int[(a + b*Csc[e + f*x])^(m
 - 3)*(d*Csc[e + f*x])^(n + 1)*Simp[a^2*b*(m - 2*n - 2) - a*(3*b^2*n + a^2*
(n + 1))*Csc[e + f*x] - b*(b^2*n + a^2*(m + n - 1))*Csc[e + f*x]^2, x], x],
 x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && ((Int
egerQ[m] && LtQ[n, -1]) || (IntegersQ[m + 1/2, 2*n] && LeQ[n, -1]))
```

rule 4343
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]])   Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a
, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4345
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]])   Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[
{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

3.850. $\int \cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$

rule 4523
```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Simp[A/a   I
nt[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Simp[(A*b - a*B)
/(a*d)   Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ
[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

rule 4592
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Simp[1/(a*d*n)   Int[(a + b*Csc[e + f*x])^m
*(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*
Csc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

3.850.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2584 vs. $2(269) = 538$.

Time = 1324.40 (sec) , antiderivative size = 2585, normalized size of antiderivative = 10.82

method	result	size
default	Expression too large to display	2585

input
```
int(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/15/d/((a-b)/(a+b))^(1/2)*(34*EllipticF(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+
csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*(1/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*c
os(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^2*b*cos(d*x+c)-46*EllipticF(((a-b)/(a+b
))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*(1/(cos(d*x+c)+1))
^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a*b^2*cos(d*x+c)-18
*EllipticE(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/
2))*(1/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/
2)*a^2*b*cos(d*x+c)+46*EllipticE(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+
c)),(-(a+b)/(a-b))^(1/2))*(1/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c
))/(cos(d*x+c)+1))^(1/2)*a*b^2*cos(d*x+c)+23*EllipticE(((a-b)/(a+b))^(1/2)
*(-cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/
(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a*b^2*cos(d*x+c)^2+17*Ellip
ticF(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*(1
/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a^2
*b*cos(d*x+c)^2-23*EllipticF(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),
(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(
cos(d*x+c)+1))^(1/2)*a*b^2*cos(d*x+c)^2-9*EllipticE(((a-b)/(a+b))^(1/2)*(-
cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(co
s(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a^2*b*cos(d*x+c)^2+15*Elliptic
F(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*(1...
```

3.850.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 454, normalized size of antiderivative = 1.90

$$\int \cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \frac{6\left(3\,a^3\cos\left(dx+c\right)+11\,a^2b\right)\sqrt{\frac{a\cos(dx+c)+b}{\cos(dx+c)}}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)+\sqrt{2}(-33i\,a^2b+i\,b^3)\sqrt{}}{}$$

input
```
integrate(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
1/45*(6*(3*a^3*cos(d*x + c) + 11*a^2*b)*sqrt((a*cos(d*x + c) + b)/cos(d*x
+ c))*sqrt(cos(d*x + c))*sin(d*x + c) + sqrt(2)*(-33*I*a^2*b + I*b^3)*sqrt
(a)*weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a
^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x + c) + 2*b)/a) + sqrt(2)*(33*I*a
^2*b - I*b^3)*sqrt(a)*weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(
9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a)
 - 3*sqrt(2)*(-9*I*a^3 - 23*I*a*b^2)*sqrt(a)*weierstrassZeta(-4/3*(3*a^2 -
 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a^2 -
 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin
(d*x + c) + 2*b)/a)) - 3*sqrt(2)*(9*I*a^3 + 23*I*a*b^2)*sqrt(a)*weierstras
sZeta(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPIn
verse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d
*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a)))/(a*d)
```

3.850.6 Sympy [F(-1)]

Timed out.

$$\int \cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(5/2)*(a+b*sec(d*x+c))**(5/2),x)
```

output
```
Timed out
```

3.850.7 Maxima [F]

$$\int \cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \int (b\sec(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^{\frac{5}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^(5/2), x)
```

3.850.8 Giac [F]

$$\int \cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \int (b\sec(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^{\frac{5}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^(5/2), x)
```

3.850.9 Mupad [F(-1)]

Timed out.

$$\int \cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \int \cos(c+d\,x)^{5/2}\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2}dx$$

input
```
int(cos(c + d*x)^(5/2)*(a + b/cos(c + d*x))^(5/2),x)
```

output
```
int(cos(c + d*x)^(5/2)*(a + b/cos(c + d*x))^(5/2), x)
```

3.851 $\int \cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$

3.851.1 Optimal result 6376
3.851.2 Mathematica [C] (warning: unable to verify) 6377
3.851.3 Rubi [A] (verified) 6377
3.851.4 Maple [C] (warning: unable to verify) 6383
3.851.5 Fricas [F] 6384
3.851.6 Sympy [F(-1)] 6385
3.851.7 Maxima [F] 6385
3.851.8 Giac [F] 6385
3.851.9 Mupad [F(-1)] 6386

3.851.1 Optimal result

Integrand size = 25, antiderivative size = 262

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \frac{2a(a^2+2b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{3d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} + \frac{2b^3\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} + \frac{14ab\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+dx)}}{3d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}} + \frac{2a^2\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{3d}$$

output
```
2/3*a*(a^2+2*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elliptic
F(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/
2)/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)+2*b^3*(cos(1/2*d*x+1/2*c)^2)^
(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2,2^(1/2)*(a/(a+b))
^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))
^(1/2)+2/3*a^2*sin(d*x+c)*cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2)/d+14/3*a
*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1
/2*c),2^(1/2)*(a/(a+b))^(1/2))*cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2)/d/(
(b+a*cos(d*x+c))/(a+b))^(1/2)
```

3.851.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 79.33 (sec) , antiderivative size = 60796, normalized size of antiderivative = 232.05

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \text{Result too large to show}$$

input
```
Integrate[Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^(5/2),x]
```

output
```
Result too large to show
```

3.851.3 Rubi [A] (verified)

Time = 2.92 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.08, number of steps used = 25, number of rules used = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {3042, 4752, 3042, 4328, 27, 3042, 4596, 3042, 4346, 3042, 3286, 3042, 3284, 4523, 3042, 4343, 3042, 3134, 3042, 3132, 4345, 3042, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{3}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{4328}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2}{3}\int\frac{3\sec^2(c+dx)b^3+7a^2b+a\left(a^2+9b^2\right)\sec(c+dx)}{2\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx+\frac{2a^2\sin(c+dx)\sqrt{a+b\,\mathrm{s}}}{3d\sqrt{\sec(c+d\mathrm{x}}}\right.$$

$\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\int\frac{3\sec^2(c+dx)b^3+7a^2b+a\left(a^2+9b^2\right)\sec(c+dx)}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx+\frac{2a^2\sin(c+dx)\sqrt{a+b\,\mathrm{s}}}{3d\sqrt{\sec(c+d\mathrm{x}}}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\int\frac{3\csc\left(c+dx+\frac{\pi}{2}\right)^2b^3+7a^2b+a\left(a^2+9b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a^2\sin(c+d}{3d\sqrt{}}\right.$$

$\downarrow$ 4596

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\left(\int\frac{7ba^2+\left(a^2+9b^2\right)\sec(c+dx)a}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx+3b^3\int\frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx\right)+\frac{2a^2\,\mathrm{si}}{}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\left(\int\frac{7ba^2+\left(a^2+9b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)a}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+3b^3\int\frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\sqrt{a+b\csc\left(c+dx+\right.}}\right.\right.$$

$\downarrow$ 4346

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\left(\int\frac{7ba^2+\left(a^2+9b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)a}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{3b^3\sqrt{\sec(c+dx)}\sqrt{a\cos(c+}}{\sqrt{a+b\,\mathrm{sec}}}\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\left(\int\frac{7ba^2+\left(a^2+9b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)a}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{3b^3\sqrt{\sec(c+dx)}\sqrt{a\cos(c+}}{\sqrt{a}}\right.\right.$$

$\downarrow$ 3286

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\left(\int\frac{7ba^2+\left(a^2+9b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)a}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{3b^3\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx}{a+b}}}{\sqrt{a+b\sec(}}\right.\right.$$

3.851. $\int\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\left(\int\frac{7ba^2+\left(a^2+9b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)a}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{3b^3\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx}{a+b}}}{\sqrt{a}}\right.\right.$$

$$\Big\downarrow \text{3284}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\left(\int\frac{7ba^2+\left(a^2+9b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)a}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{6b^3\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx}{a+b}}}{d\sqrt{a+}}\right.\right.$$

$$\Big\downarrow \text{4523}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\left(a\left(a^2+2b^2\right)\int\frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}dx+7ab\int\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}dx+\frac{6b^3\sqrt{\sec}}{}\right.\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\left(a\left(a^2+2b^2\right)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+7ab\int\frac{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right.\right.$$

$$\Big\downarrow \text{4343}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\left(a\left(a^2+2b^2\right)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{7ab\sqrt{a+b\sec(c+dx)}\int\sqrt{b+a\,\mathrm{co}}}{\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx}}\right.\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\left(a\left(a^2+2b^2\right)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{7ab\sqrt{a+b\sec(c+dx)}\int\sqrt{b+a\,\mathrm{sin}}}{\sqrt{\sec(c+dx)}\sqrt{a\cos(c+}}\right.\right.$$

$$\Big\downarrow \text{3134}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\left(a\left(a^2+2b^2\right)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{7ab\sqrt{a+b\sec(c+dx)}\int\sqrt{\frac{b}{a+b}+}}{\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)}{a+b}}}\right.\right.$$

3.851. $\int\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\left(a(a^2+2b^2)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{7ab\sqrt{a+b\sec(c+dx)}\int\sqrt{\frac{b}{a+b}+\frac{a}{}}}{\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)}{a+b}}}\right.\right.$$

$$\Big\downarrow \text{3132}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\left(a(a^2+2b^2)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{6b^3\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\text{Elli}}{d\sqrt{a+b\sec(c}}\right.\right.$$

$$\Big\downarrow \text{4345}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\left(\frac{a(a^2+2b^2)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{1}{\sqrt{b+a\cos(c+dx)}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{6b^3\sqrt{\sec(c+}}{}\right.\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\left(\frac{a(a^2+2b^2)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{1}{\sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{6b^3\sqrt{\sec(}}{}\right.\right.$$

$$\Big\downarrow \text{3142}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\left(\frac{a(a^2+2b^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{6b^3\sqrt{\sec(c+dx}}{}\right.\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\left(\frac{a(a^2+2b^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{6b^3\sqrt{\sec(c+}}{}\right.\right.$$

$$\Big\downarrow \text{3140}$$

3.851. $\int\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\left(\frac{2a(a^2+2b^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}+\frac{6b^3\sqrt{\sec}}{}\right.\right.$$

input
```
Int[Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^(5/2),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(((2*a*(a^2 + 2*b^2)*Sqrt[(b + a*Cos
[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x
]])/(d*Sqrt[a + b*Sec[c + d*x]]) + (6*b^3*Sqrt[(b + a*Cos[c + d*x])/(a + b
)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(d*Sqrt[a
 + b*Sec[c + d*x]]) + (14*a*b*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a
 + b*Sec[c + d*x]])/(d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x
]]))/3 + (2*a^2*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d
*x]]))
```

3.851.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3286
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]]   Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
 d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

rule 4328
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)*
((d*Csc[e + f*x])^n/(f*n)), x] - Simp[1/(d*n)   Int[(a + b*Csc[e + f*x])^(m
 - 3)*(d*Csc[e + f*x])^(n + 1)*Simp[a^2*b*(m - 2*n - 2) - a*(3*b^2*n + a^2*
(n + 1))*Csc[e + f*x] - b*(b^2*n + a^2*(m + n - 1))*Csc[e + f*x]^2, x], x],
 x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && ((Int
egerQ[m] && LtQ[n, -1]) || (IntegersQ[m + 1/2, 2*n] && LeQ[n, -1]))
```

rule 4343
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]])   Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a
, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4345
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]])   Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[
{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

3.851. $\int \cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$

rule 4346
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Simp[d*Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x
]]/Sqrt[a + b*Csc[e + f*x]])   Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4523
```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Simp[A/a   I
nt[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Simp[(A*b - a*B)
/(a*d)   Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ
[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

rule 4596
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Simp[C/d^2   Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*C
sc[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[
a + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
 b^2, 0]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

3.851.4 Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 9.83 (sec) , antiderivative size = 2588, normalized size of antiderivative = 9.88

method	result	size
default	Expression too large to display	2588

input
```
int(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/3/d/((a-b)/(a+b))^(1/2)*(-((1-cos(d*x+c))^2*csc(d*x+c)^2-1)/((1-cos(d*x+
c))^2*csc(d*x+c)^2+1))^(3/2)*((a*(1-cos(d*x+c))^2*csc(d*x+c)^2-b*(1-cos(d*
x+c))^2*csc(d*x+c)^2-a-b)/((1-cos(d*x+c))^2*csc(d*x+c)^2-1))^(1/2)*(-7*((a
-b)/(a+b))^(1/2)*a^2*b*(1-cos(d*x+c))^5*csc(d*x+c)^5+7*((a-b)/(a+b))^(1/2)
*a*b^2*(1-cos(d*x+c))^5*csc(d*x+c)^5+(-(a*(1-cos(d*x+c))^2*csc(d*x+c)^2-b*
(1-cos(d*x+c))^2*csc(d*x+c)^2-a-b)/(a+b))^(1/2)*((1-cos(d*x+c))^2*csc(d*x+
c)^2+1)^(1/2)*EllipticF(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(-(a+
b)/(a-b))^(1/2))*a^3*(1-cos(d*x+c))^2*csc(d*x+c)^2-7*(-(a*(1-cos(d*x+c))^2
*csc(d*x+c)^2-b*(1-cos(d*x+c))^2*csc(d*x+c)^2-a-b)/(a+b))^(1/2)*((1-cos(d*
x+c))^2*csc(d*x+c)^2+1)^(1/2)*EllipticF(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+c
sc(d*x+c)),(-(a+b)/(a-b))^(1/2))*a^2*b*(1-cos(d*x+c))^2*csc(d*x+c)^2+9*(-(
a*(1-cos(d*x+c))^2*csc(d*x+c)^2-b*(1-cos(d*x+c))^2*csc(d*x+c)^2-a-b)/(a+b)
)^(1/2)*((1-cos(d*x+c))^2*csc(d*x+c)^2+1)^(1/2)*EllipticF(((a-b)/(a+b))^(1
/2)*(-cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*a*b^2*(1-cos(d*x+c))^2*
csc(d*x+c)^2-3*(-(a*(1-cos(d*x+c))^2*csc(d*x+c)^2-b*(1-cos(d*x+c))^2*csc(d
*x+c)^2-a-b)/(a+b))^(1/2)*((1-cos(d*x+c))^2*csc(d*x+c)^2+1)^(1/2)*Elliptic
F(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*b^3*(
1-cos(d*x+c))^2*csc(d*x+c)^2+7*(-(a*(1-cos(d*x+c))^2*csc(d*x+c)^2-b*(1-cos
(d*x+c))^2*csc(d*x+c)^2-a-b)/(a+b))^(1/2)*((1-cos(d*x+c))^2*csc(d*x+c)^2+1
)^(1/2)*EllipticE(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(-(a+b)/...
```

3.851.5 Fricas [F]

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \int (b\sec(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^{\frac{3}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
integral((b^2*cos(d*x + c)*sec(d*x + c)^2 + 2*a*b*cos(d*x + c)*sec(d*x + c
) + a^2*cos(d*x + c))*sqrt(b*sec(d*x + c) + a)*sqrt(cos(d*x + c)), x)
```

3.851.6 Sympy [F(-1)]

Timed out.

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(3/2)*(a+b*sec(d*x+c))**(5/2),x)
```

output
```
Timed out
```

3.851.7 Maxima [F]

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \int (b\sec(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^{\frac{3}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^(3/2), x)
```

3.851.8 Giac [F]

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \int (b\sec(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^{\frac{3}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^(3/2), x)
```

3.851.9 Mupad [F(-1)]

Timed out.

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx=\int \cos{(c+d\,x)}^{3/2}\left(a+\frac{b}{\cos{(c+d\,x)}}\right)^{5/2}dx$$

input
```
int(cos(c + d*x)^(3/2)*(a + b/cos(c + d*x))^(5/2),x)
```

output
```
int(cos(c + d*x)^(3/2)*(a + b/cos(c + d*x))^(5/2), x)
```

3.852 $\int \sqrt{\cos(c+dx)}(a+b\sec(c+dx))^{5/2}\,dx$

3.852.1 Optimal result . . . 6387
3.852.2 Mathematica [C] (warning: unable to verify) . . . 6388
3.852.3 Rubi [A] (verified) . . . 6388
3.852.4 Maple [C] (verified) . . . 6394
3.852.5 Fricas [F(-1)] . . . 6395
3.852.6 Sympy [F(-1)] . . . 6396
3.852.7 Maxima [F] . . . 6396
3.852.8 Giac [F] . . . 6396
3.852.9 Mupad [F(-1)] . . . 6397

3.852.1 Optimal result

Integrand size = 25, antiderivative size = 263

$$\int \sqrt{\cos(c+dx)}(a+b\sec(c+dx))^{5/2}\,dx = \frac{b(4a^2+b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} + \frac{5ab^2\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} + \frac{(2a^2-b^2)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+dx)}}{d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}} + \frac{b^2\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

output
```
b*(4*a^2+b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(si
n(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)/d
/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)+5*a*b^2*(cos(1/2*d*x+1/2*c)^2)^(1
/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2,2^(1/2)*(a/(a+b))^(
1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(
1/2)+b^2*sin(d*x+c)*(a+b*sec(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)+(2*a^2-b^2)*
(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*
c),2^(1/2)*(a/(a+b))^(1/2))*cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2)/d/((b+
a*cos(d*x+c))/(a+b))^(1/2)
```

3.852.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 34.51 (sec) , antiderivative size = 44895, normalized size of antiderivative = 170.70

$$\int \sqrt{\cos(c+dx)}(a+b\sec(c+dx))^{5/2}\,dx = \text{Result too large to show}$$

input
```
Integrate[Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x])^(5/2),x]
```

output
```
Result too large to show
```

3.852.3 Rubi [A] (verified)

Time = 2.99 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.11, number of steps used = 25, number of rules used = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {3042, 4752, 3042, 4329, 27, 3042, 4596, 3042, 4346, 3042, 3286, 3042, 3284, 4523, 3042, 4343, 3042, 3134, 3042, 3132, 4345, 3042, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\cos(c+dx)}(a+b\sec(c+dx))^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{(a+b\sec(c+dx))^{5/2}}{\sqrt{\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4329}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\int \frac{6b\sec(c+dx)a^2+5b^2\sec^2(c+dx)a+\left(2a^2-b^2\right)a}{2\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx+\frac{b^2\sin(c+dx)\sqrt{\sec(c+}}{d}\right.$$

$\Big\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{2}\int \frac{6b\sec(c+dx)a^2+5b^2\sec^2(c+dx)a+\left(2a^2-b^2\right)a}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx+\frac{b^2\sin(c+dx)\sqrt{\sec(c-}}{}\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{2}\int \frac{6b\csc\left(c+dx+\frac{\pi}{2}\right)a^2+5b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2a+\left(2a^2-b^2\right)a}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{b^2\sin(c+}{}\right.$$

$\Big\downarrow$ 4596

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{2}\left(\int \frac{6b\sec(c+dx)a^2+\left(2a^2-b^2\right)a}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx+5ab^2\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx\right)+\frac{b^2\sin}{}\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{2}\left(\int \frac{6b\csc\left(c+dx+\frac{\pi}{2}\right)a^2+\left(2a^2-b^2\right)a}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+5ab^2\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/}}{\sqrt{a+b\csc\left(c+dx+\right.}}\right.\right.$$

$\Big\downarrow$ 4346

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{2}\left(\int \frac{6b\csc\left(c+dx+\frac{\pi}{2}\right)a^2+\left(2a^2-b^2\right)a}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{5ab^2\sqrt{\sec(c+dx)}\sqrt{a\cos(c-}}{\sqrt{a+b\mathrm{se}}}\right.\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{2}\left(\int \frac{6b\csc\left(c+dx+\frac{\pi}{2}\right)a^2+\left(2a^2-b^2\right)a}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{5ab^2\sqrt{\sec(c+dx)}\sqrt{a\cos(c-}}{\sqrt{}}\right.\right.$$

$\Big\downarrow$ 3286

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{2}\left(\int \frac{6b\csc\left(c+dx+\frac{\pi}{2}\right)a^2+\left(2a^2-b^2\right)a}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{5ab^2\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+d}{a+b}}}{\sqrt{a+b\sec}}\right.\right.$$

3.852. $\int \sqrt{\cos(c+dx)}(a+b\sec(c+dx))^{5/2}\,dx$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{2}\left(\int \frac{6b\csc\left(c+dx+\frac{\pi}{2}\right)a^2+\left(2a^2-b^2\right)a}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{5ab^2\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+d}{a+b}}}{\sqrt{a}}\right.\right.$$

$$\Big\downarrow \; 3284$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{2}\left(\int \frac{6b\csc\left(c+dx+\frac{\pi}{2}\right)a^2+\left(2a^2-b^2\right)a}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{10ab^2\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+}{a+}}}{d\sqrt{a+}}\right.\right.$$

$$\Big\downarrow \; 4523$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{2}\left(b\left(4a^2+b^2\right)\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}dx+\left(2a^2-b^2\right)\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}dx+\frac{1}{}\right.\right.$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{2}\left(b\left(4a^2+b^2\right)\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\left(2a^2-b^2\right)\int \frac{\sqrt{a+b\csc\left(c+dx+\right.}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}\right.\right.$$

$$\Big\downarrow \; 4343$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{2}\left(b\left(4a^2+b^2\right)\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{\left(2a^2-b^2\right)\sqrt{a+b\sec(c+dx)}\int\sqrt{}}{\sqrt{\sec(c+dx)}\sqrt{a\cos(c+}}\right.\right.$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{2}\left(b\left(4a^2+b^2\right)\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{\left(2a^2-b^2\right)\sqrt{a+b\sec(c+dx)}\int\sqrt{}}{\sqrt{\sec(c+dx)}\sqrt{a\cos(}}\right.\right.$$

$$\Big\downarrow \; 3134$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{2}\left(b\left(4a^2+b^2\right)\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{\left(2a^2-b^2\right)\sqrt{a+b\sec(c+dx)}\int\sqrt{}}{\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+}{a+}}}\right.\right.$$

3.852. $\int \sqrt{\cos(c+dx)}(a+b\sec(c+dx))^{5/2}\,dx$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{2}\left(b\left(4a^2+b^2\right)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{\left(2a^2-b^2\right)\sqrt{a+b\sec(c+dx)}\int\sqrt{}}{\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(}{}}}\right.\right.$$

$$\Big\downarrow \text{3132}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{2}\left(b\left(4a^2+b^2\right)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\left(2a^2-b^2\right)\sqrt{a+b\sec(c+dx)}E\left(\right.}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+d}{a+b}}}\right.\right.$$

$$\Big\downarrow \text{4345}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{2}\left(\frac{b\left(4a^2+b^2\right)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{1}{\sqrt{b+a\cos(c+dx)}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{2\left(2a^2-b^2\right)\sqrt{}}{d\sqrt{}}\right.\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{2}\left(\frac{b\left(4a^2+b^2\right)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{1}{\sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{2\left(2a^2-b^2\right.}{}\right.\right.$$

$$\Big\downarrow \text{3142}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{2}\left(\frac{b\left(4a^2+b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{2\left(2a^2-b^2\right)\sqrt{a}}{d\sqrt{\text{se}}}\right.\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{2}\left(\frac{b\left(4a^2+b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{2\left(2a^2-b^2\right)\sqrt{}}{d\sqrt{}}\right.\right.$$

$$\Big\downarrow \text{3140}$$

3.852. $\int\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^{5/2}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{2}\left(\frac{2b\left(4a^2+b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}+\frac{2\left(2a^2-\right.}{}\right.\right.$$

input
```
Int[Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x])^(5/2),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(((2*b*(4*a^2 + b^2)*Sqrt[(b + a*Cos
[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x
]])/(d*Sqrt[a + b*Sec[c + d*x]]) + (10*a*b^2*Sqrt[(b + a*Cos[c + d*x])/(a
+ b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(d*Sqr
t[a + b*Sec[c + d*x]]) + (2*(2*a^2 - b^2)*EllipticE[(c + d*x)/2, (2*a)/(a
+ b)]*Sqrt[a + b*Sec[c + d*x]])/(d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt
[Sec[c + d*x]]))/2 + (b^2*Sqrt[Sec[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]*Sin[
c + d*x])/d)
```

3.852.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3286
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]]   Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
 d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

rule 4329
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
2)*((d*Csc[e + f*x])^n/(f*(m + n - 1))), x] + Simp[1/(d*(m + n - 1))   Int[
(a + b*Csc[e + f*x])^(m - 3)*(d*Csc[e + f*x])^n*Simp[a^3*d*(m + n - 1) + a*
b^2*d*n + b*(b^2*d*(m + n - 2) + 3*a^2*d*(m + n - 1))*Csc[e + f*x] + a*b^2*
d*(3*m + 2*n - 4)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, n}, x
] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
 &&  !(IGtQ[n, 2] &&  !IntegerQ[m])
```

rule 4343
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]])   Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a
, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4345
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]])   Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[
{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

3.852. $\int \sqrt{\cos(c+dx)}(a+b\sec(c+dx))^{5/2}\,dx$

rule 4346
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Simp[d*Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x
]]/Sqrt[a + b*Csc[e + f*x]])   Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4523
```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Simp[A/a   I
nt[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Simp[(A*b - a*B)
/(a*d)   Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ
[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

rule 4596
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Simp[C/d^2   Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*C
sc[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[
a + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
 b^2, 0]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

3.852.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 10.74 (sec) , antiderivative size = 2544, normalized size of antiderivative = 9.67

method	result	size
default	Expression too large to display	2544

input
```
int(cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/d/((a-b)/(a+b))^(1/2)*(-6*EllipticF(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+cs
c(d*x+c)),(-(a+b)/(a-b))^(1/2))*(1/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos
(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^2*b*cos(d*x+c)+4*EllipticF(((a-b)/(a+b))^
(1/2)*(-cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*(1/(cos(d*x+c)+1))^(1
/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a*b^2*cos(d*x+c)+2*Ell
ipticE(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*
(1/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a
^2*b*cos(d*x+c)+EllipticE(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(-(
a+b)/(a-b))^(1/2))*(1/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos
(d*x+c)+1))^(1/2)*a*b^2*cos(d*x+c)+2*EllipticE(((a-b)/(a+b))^(1/2)*(-cot(d
*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x
+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a*b^2*cos(d*x+c)^2-12*EllipticF(((a
-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(
b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a^2*b*cos(d
*x+c)^2+8*EllipticF(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(-(a+b)/(
a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c
)+1))^(1/2)*a*b^2*cos(d*x+c)^2+EllipticE(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+
csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1)
)^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a*b^2*cos(d*x+c)^3-6*EllipticF(((a-b)/(a+
b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*...
```

3.852.5 Fricas [F(-1)]

Timed out.

$$\int \sqrt{\cos(c+dx)}(a+b\sec(c+dx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.852.6 Sympy [F(-1)]

Timed out.

$$\int \sqrt{\cos(c+dx)}(a+b\sec(c+dx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(1/2)*(a+b*sec(d*x+c))**(5/2),x)
```

output
```
Timed out
```

3.852.7 Maxima [F]

$$\int \sqrt{\cos(c+dx)}(a+b\sec(c+dx))^{5/2}\,dx = \int (b\sec(dx+c)+a)^{\frac{5}{2}}\sqrt{\cos(dx+c)}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^(5/2)*sqrt(cos(d*x + c)), x)
```

3.852.8 Giac [F]

$$\int \sqrt{\cos(c+dx)}(a+b\sec(c+dx))^{5/2}\,dx = \int (b\sec(dx+c)+a)^{\frac{5}{2}}\sqrt{\cos(dx+c)}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^(5/2)*sqrt(cos(d*x + c)), x)
```

3.852.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{\cos(c+dx)}(a+b\sec(c+dx))^{5/2}\,dx=\int \sqrt{\cos\left(c+d\,x\right)}\,\left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)^{5/2}\,dx$$

input
```
int(cos(c + d*x)^(1/2)*(a + b/cos(c + d*x))^(5/2),x)
```

output
```
int(cos(c + d*x)^(1/2)*(a + b/cos(c + d*x))^(5/2), x)
```

3.853 $\int \frac{(a+b\sec(c+dx))^{5/2}}{\sqrt{\cos(c+dx)}}\,dx$

3.853.1 Optimal result 6398
3.853.2 Mathematica [C] (warning: unable to verify) 6399
3.853.3 Rubi [A] (verified) 6399
3.853.4 Maple [C] (verified) 6407
3.853.5 Fricas [F(-1)] 6408
3.853.6 Sympy [F(-1)] 6408
3.853.7 Maxima [F] 6408
3.853.8 Giac [F] 6409
3.853.9 Mupad [F(-1)] 6409

3.853.1 Optimal result

Integrand size = 25, antiderivative size = 314

$$\begin{aligned}\int \frac{(a+b\sec(c+dx))^{5/2}}{\sqrt{\cos(c+dx)}}\,dx &= \frac{a(8a^2+11b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{4d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}\\ &+\frac{b(15a^2+4b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{4d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}\\ &-\frac{9ab\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+dx)}}{4d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}}\\ &+\frac{b^2\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)}+\frac{9ab\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{4d\sqrt{\cos(c+dx)}}\end{aligned}$$

output
```
1/4*a*(8*a^2+11*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellip
ticF(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^
(1/2)/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)+1/4*b*(15*a^2+4*b^2)*(cos(
1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2
,2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)/d/cos(d*x+c)^(1/2
)/(a+b*sec(d*x+c))^(1/2)+1/2*b^2*sin(d*x+c)*(a+b*sec(d*x+c))^(1/2)/d/cos(d
*x+c)^(3/2)+9/4*a*b*sin(d*x+c)*(a+b*sec(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)-9
/4*a*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d
*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2)
/d/((b+a*cos(d*x+c))/(a+b))^(1/2)
```

3.853.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 33.44 (sec) , antiderivative size = 53538, normalized size of antiderivative = 170.50

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sqrt{\cos(c+dx)}}\,dx = \text{Result too large to show}$$

input
```
Integrate[(a + b*Sec[c + d*x])^(5/2)/Sqrt[Cos[c + d*x]],x]
```

output
```
Result too large to show
```

3.853.3 Rubi [A] (verified)

Time = 3.72 (sec) , antiderivative size = 345, normalized size of antiderivative = 1.10, number of steps used = 28, number of rules used = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 1.120$, Rules used = {3042, 4752, 3042, 4329, 27, 3042, 4590, 27, 3042, 4596, 3042, 4346, 3042, 3286, 3042, 3284, 4523, 3042, 4343, 3042, 3134, 3042, 3132, 4345, 3042, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sqrt{\cos(c+dx)}}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \; 4752$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{5/2}dx$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}dx$$

$$\Big\downarrow \; 4329$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{2}\int\frac{\sqrt{\sec(c+dx)}\left(9ab^2\sec^2(c+dx)+2b\left(6a^2+b^2\right)\sec(c+dx)+a\left(4a^2+b^2\right)\right)}{2\sqrt{a+b\sec(c+dx)}}dx+\right.$$

$\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{4}\int\frac{\sqrt{\sec(c+dx)}\left(9ab^2\sec^2(c+dx)+2b\left(6a^2+b^2\right)\sec(c+dx)+a\left(4a^2+b^2\right)\right)}{\sqrt{a+b\sec(c+dx)}}dx+\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{4}\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(9ab^2\csc\left(c+dx+\frac{\pi}{2}\right)^2+2b\left(6a^2+b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)+a\right.}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}\right.$$

$\downarrow$ 4590

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{4}\left(\frac{\int-\frac{9a^2b^2-\left(15a^2+4b^2\right)\sec^2(c+dx)b^2-2a\left(4a^2+b^2\right)\sec(c+dx)b}{2\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{b}+\frac{9ab\sin(c+dx)\sqrt{\sec(c+\right.}{d}\right.\right.$$

$\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{4}\left(\frac{9ab\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{\int\frac{9a^2b^2-\left(15a^2+4b^2\right)\sec^2(c+dx)b^2-}{\sqrt{\sec(c+dx)}\sqrt{a+b}}}{2b}\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{4}\left(\frac{9ab\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{\int\frac{9a^2b^2-\left(15a^2+4b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{}}}{2}\right.\right.$$

$\downarrow$ 4596

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{4}\left(\frac{9ab\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{\int\frac{9a^2b^2-2ab\left(4a^2+b^2\right)\sec(c+dx)}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{}\right.\right.$$

$\downarrow$ 3042

3.853. $\int\frac{(a+b\sec(c+dx))^{5/2}}{\sqrt{\cos(c+dx)}}dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{4}\left(\frac{9ab\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{\int\frac{9a^2b^2-2ab(4a^2+b^2)\csc(c+dx+}{\sqrt{\csc(c+dx+\frac{\pi}{2})}\sqrt{a+b\csc(c+dx+}}}{}\right.\right.$$

$\Big\downarrow$ 4346

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{4}\left(\frac{9ab\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{\int\frac{9a^2b^2-2ab(4a^2+b^2)\csc(c+dx+}{\sqrt{\csc(c+dx+\frac{\pi}{2})}\sqrt{a+b\csc(c+dx+}}}{}\right.\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{4}\left(\frac{9ab\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{\int\frac{9a^2b^2-2ab(4a^2+b^2)\csc(c+dx+}{\sqrt{\csc(c+dx+\frac{\pi}{2})}\sqrt{a+b\csc(c+dx+}}}{}\right.\right.$$

$\Big\downarrow$ 3286

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{4}\left(\frac{9ab\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{\int\frac{9a^2b^2-2ab(4a^2+b^2)\csc(c+dx+}{\sqrt{\csc(c+dx+\frac{\pi}{2})}\sqrt{a+b\csc(c+dx+}}}{}\right.\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{4}\left(\frac{9ab\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{\int\frac{9a^2b^2-2ab(4a^2+b^2)\csc(c+dx+}{\sqrt{\csc(c+dx+\frac{\pi}{2})}\sqrt{a+b\csc(c+dx+}}}{}\right.\right.$$

$\Big\downarrow$ 3284

3.853. $\int \frac{(a+b\sec(c+dx))^{5/2}}{\sqrt{\cos(c+dx)}}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{4}\left(\frac{9ab\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{\int\frac{9a^2b^2-2ab(4a^2+b^2)\csc(c+dx+\frac{}{}}{\sqrt{\csc(c+dx+\frac{\pi}{2})}\sqrt{a+b\csc(c+dx+}}}{}\right.\right.$$

$\downarrow$ 4523

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{4}\left(\frac{9ab\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{-ab(8a^2+11b^2)\int\frac{\sqrt{\sec(c+}}{\sqrt{a+b\sec(c}}}{}\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{4}\left(\frac{9ab\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{-ab(8a^2+11b^2)\int\frac{\sqrt{\csc(c+}}{\sqrt{a+b\csc(c}}}{}\right.\right.$$

$\downarrow$ 4343

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{4}\left(\frac{9ab\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{-ab(8a^2+11b^2)\int\frac{\sqrt{\csc(c+}}{\sqrt{a+b\csc(c}}}{}\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{4}\left(\frac{9ab\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{-ab(8a^2+11b^2)\int\frac{\sqrt{\csc(c+}}{\sqrt{a+b\csc(c}}}{}\right.\right.$$

$\downarrow$ 3134

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{4}\left(\frac{9ab\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{-ab(8a^2+11b^2)\int\frac{\sqrt{\csc(c+}}{\sqrt{a+b\csc(c}}}{}\right.\right.$$

$\downarrow$ 3042

3.853. $\int \frac{(a+b\sec(c+dx))^{5/2}}{\sqrt{\cos(c+dx)}}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{4}\left(\frac{9ab\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{-ab\left(8a^2+11b^2\right)\int\frac{\sqrt{\csc(c+}}{\sqrt{a+b\csc(c}}}{}\right.\right.$$

$\downarrow$ 3132

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{4}\left(\frac{9ab\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{-ab\left(8a^2+11b^2\right)\int\frac{\sqrt{\csc(c+}}{\sqrt{a+b\csc(c}}}{}\right.\right.$$

$\downarrow$ 4345

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{4}\left(\frac{9ab\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{-\frac{ab\left(8a^2+11b^2\right)\sqrt{\sec(c+dx)}\sqrt{a\cos(}}{\sqrt{a+b\sec(}}}{}\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{4}\left(\frac{9ab\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{-\frac{ab\left(8a^2+11b^2\right)\sqrt{\sec(c+dx)}\sqrt{a\cos(}}{\sqrt{a+b\sec}}}{}\right.\right.$$

$\downarrow$ 3142

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{4}\left(\frac{9ab\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{-\frac{ab\left(8a^2+11b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(}{}}}{\sqrt{a+b\sec}}}{}\right.\right.$$

$\downarrow$ 3042

3.853. $\int \frac{(a+b\sec(c+dx))^{5/2}}{\sqrt{\cos(c+dx)}}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{4}\left(\frac{9ab\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{-\frac{ab(8a^2+11b^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(}{}}}{\sqrt{a+b\,s}}}{}\right.\right.$$

$$\Big\downarrow \; 3140$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{4}\left(\frac{9ab\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{-\frac{2ab(8a^2+11b^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos}{}}}{d\sqrt{a+b\,s}}}{}\right.\right.$$

input
```
Int[(a + b*Sec[c + d*x])^(5/2)/Sqrt[Cos[c + d*x]],x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((b^2*Sec[c + d*x]^(3/2)*Sqrt[a + b*
Sec[c + d*x]]*Sin[c + d*x])/(2*d) + (-1/2*((-2*a*b*(8*a^2 + 11*b^2)*Sqrt[(
b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Se
c[c + d*x]])/(d*Sqrt[a + b*Sec[c + d*x]]) - (2*b^2*(15*a^2 + 4*b^2)*Sqrt[(
b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)]*Sqr
t[Sec[c + d*x]])/(d*Sqrt[a + b*Sec[c + d*x]]) + (18*a*b^2*EllipticE[(c + d
*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(d*Sqrt[(b + a*Cos[c + d*x
])/(a + b)]*Sqrt[Sec[c + d*x]]))/b + (9*a*b*Sqrt[Sec[c + d*x]]*Sqrt[a + b*
Sec[c + d*x]]*Sin[c + d*x])/d)/4)
```

3.853.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3286
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]]   Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
 d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

rule 4329
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
2)*((d*Csc[e + f*x])^n/(f*(m + n - 1))), x] + Simp[1/(d*(m + n - 1))   Int[
(a + b*Csc[e + f*x])^(m - 3)*(d*Csc[e + f*x])^n*Simp[a^3*d*(m + n - 1) + a*
b^2*d*n + b*(b^2*d*(m + n - 2) + 3*a^2*d*(m + n - 1))*Csc[e + f*x] + a*b^2*
d*(3*m + 2*n - 4)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, n}, x
] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
 &&  !(IGtQ[n, 2] &&  !IntegerQ[m])
```

3.853. $\int \frac{(a+b\sec(c+dx))^{5/2}}{\sqrt{\cos(c+dx)}}\,dx$

rule 4343
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]])   Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a
, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4345
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]])   Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[
{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4346
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Simp[d*Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x
]]/Sqrt[a + b*Csc[e + f*x]])   Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4523
```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Simp[A/a   I
nt[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Simp[(A*b - a*B)
/(a*d)   Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ
[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

rule 4590
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(-C)*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
)*((d*Csc[e + f*x])^(n - 1)/(b*f*(m + n + 1))), x] + Simp[d/(b*(m + n + 1))
   Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (
A*b*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2
 - b^2, 0] && GtQ[n, 0]
```

rule 4596
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Simp[C/d^2   Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*C
sc[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[
a + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
 b^2, 0]
```

3.853. $\int \frac{(a+b\sec(c+dx))^{5/2}}{\sqrt{\cos(c+dx)}}\,dx$

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

3.853.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 12.01 (sec) , antiderivative size = 2634, normalized size of antiderivative = 8.39

method	result	size
default	Expression too large to display	2634

input
```
int((a+b*sec(d*x+c))^(5/2)/cos(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4/d/((a-b)/(a+b))^(1/2)*(60*(1/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d
*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+c
sc(d*x+c)),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*a^2*b*cos(d*x+c)^3+9*Ellipti
cE(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*(1/(
a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a*b^2
*cos(d*x+c)^2-6*EllipticF(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(-(
a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos
(d*x+c)+1))^(1/2)*a^2*b*cos(d*x+c)^2+2*EllipticF(((a-b)/(a+b))^(1/2)*(-cot
(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d
*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a*b^2*cos(d*x+c)^2+18*EllipticE((
(a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*(1/(a+b)
*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a*b^2*cos
(d*x+c)^3-12*EllipticF(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(-(a+b
)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*
x+c)+1))^(1/2)*a^2*b*cos(d*x+c)^3-9*EllipticE(((a-b)/(a+b))^(1/2)*(-cot(d*
x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+
c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a^2*b*cos(d*x+c)^2+4*EllipticF(((a-b
)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+
a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a*b^2*cos(d*x
+c)^3-18*EllipticE(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(-(a+b)...
```

3.853.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sqrt{\cos(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate((a+b*sec(d*x+c))^(5/2)/cos(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.853.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sqrt{\cos(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate((a+b*sec(d*x+c))**(5/2)/cos(d*x+c)**(1/2),x)
```

output
```
Timed out
```

3.853.7 Maxima [F]

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sqrt{\cos(c+dx)}}\,dx = \int \frac{(b\sec(dx+c)+a)^{\frac{5}{2}}}{\sqrt{\cos(dx+c)}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(5/2)/cos(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^(5/2)/sqrt(cos(d*x + c)), x)
```

3.853.8 Giac [F]

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sqrt{\cos(c+dx)}}\,dx=\int \frac{(b\sec(dx+c)+a)^{\frac{5}{2}}}{\sqrt{\cos(dx+c)}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(5/2)/cos(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^(5/2)/sqrt(cos(d*x + c)), x)
```

3.853.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sqrt{\cos(c+dx)}}\,dx=\int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2}}{\sqrt{\cos(c+d\,x)}}\,dx$$

input
```
int((a + b/cos(c + d*x))^(5/2)/cos(c + d*x)^(1/2),x)
```

output
```
int((a + b/cos(c + d*x))^(5/2)/cos(c + d*x)^(1/2), x)
```

3.854 $\int \frac{(a+b\sec(c+dx))^{5/2}}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

3.854.1 Optimal result 6410
3.854.2 Mathematica [C] (warning: unable to verify) 6411
3.854.3 Rubi [F] 6411
3.854.4 Maple [C] (verified) 6419
3.854.5 Fricas [F(-1)] 6420
3.854.6 Sympy [F(-1)] 6421
3.854.7 Maxima [F] 6421
3.854.8 Giac [F] 6421
3.854.9 Mupad [F(-1)] 6422

3.854.1 Optimal result

Integrand size = 25, antiderivative size = 369

$$\begin{aligned}\int \frac{(a+b\sec(c+dx))^{5/2}}{\cos^{\frac{3}{2}}(c+dx)}\,dx &= \frac{b(59a^2+16b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{24d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}\\ &+\frac{5a(a^2+4b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{8d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}\\ &-\frac{(33a^2+16b^2)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+dx)}}{24d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}}\\ &+\frac{b^2\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{3d\cos^{\frac{5}{2}}(c+dx)}+\frac{13ab\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{12d\cos^{\frac{3}{2}}(c+dx)}\\ &+\frac{(33a^2+16b^2)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{24d\sqrt{\cos(c+dx)}}\end{aligned}$$

output
```
1/24*b*(59*a^2+16*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ell
ipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b)
)^(1/2)/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)+5/8*a*(a^2+4*b^2)*(cos(1
/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2,
2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)/d/cos(d*x+c)^(1/2)
/(a+b*sec(d*x+c))^(1/2)+1/3*b^2*sin(d*x+c)*(a+b*sec(d*x+c))^(1/2)/d/cos(d*
x+c)^(5/2)+13/12*a*b*sin(d*x+c)*(a+b*sec(d*x+c))^(1/2)/d/cos(d*x+c)^(3/2)+
1/24*(33*a^2+16*b^2)*sin(d*x+c)*(a+b*sec(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)-
1/24*(33*a^2+16*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellip
ticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*cos(d*x+c)^(1/2)*(a+b*sec
(d*x+c))^(1/2)/d/((b+a*cos(d*x+c))/(a+b))^(1/2)
```

3.854.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 34.25 (sec) , antiderivative size = 62811, normalized size of antiderivative = 170.22

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \text{Result too large to show}$$

input
```
Integrate[(a + b*Sec[c + d*x])^(5/2)/Cos[c + d*x]^(3/2),x]
```

output
```
Result too large to show
```

3.854.3 Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\cos^{\frac{3}{2}}(c+dx)}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}dx$$

$$\Big\downarrow \; 4752$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}dx$$

$$\Big\downarrow \text{4329}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{3}\int \frac{\sec^{\frac{3}{2}}(c+dx)\left(13ab^2\sec^2(c+dx)+2b\left(9a^2+2b^2\right)\sec(c+dx)+3a\left(2a^2+b^2\right)\right)}{2\sqrt{a+b\sec(c+dx)}}d\right.$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{6}\int \frac{\sec^{\frac{3}{2}}(c+dx)\left(13ab^2\sec^2(c+dx)+2b\left(9a^2+2b^2\right)\sec(c+dx)+3a\left(2a^2+b^2\right)\right)}{\sqrt{a+b\sec(c+dx)}}d\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{6}\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(13ab^2\csc\left(c+dx+\frac{\pi}{2}\right)^2+2b\left(9a^2+2b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)\right.}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}\right.$$

$$\Big\downarrow \text{4590}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{6}\left(\frac{\int \frac{\sqrt{\sec(c+dx)}\left(13a^2b^2+\left(33a^2+16b^2\right)\sec^2(c+dx)b^2+2a\left(12a^2+19b^2\right)\sec(c+dx)b\right)}{2\sqrt{a+b\sec(c+dx)}}dx}{2b}+\frac{13ab\sin(c}{}\right.\right.$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{6}\left(\frac{\int \frac{\sqrt{\sec(c+dx)}\left(13a^2b^2+\left(33a^2+16b^2\right)\sec^2(c+dx)b^2+2a\left(12a^2+19b^2\right)\sec(c+dx)b\right)}{\sqrt{a+b\sec(c+dx)}}dx}{4b}+\frac{13ab\sin(c}{}\right.\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{6}\left(\frac{\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(13a^2b^2+\left(33a^2+16b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2b^2+2a\left(12a^2+19b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)b\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{4b}+\right.\right.$$

$$\Big\downarrow \text{4590}$$

3.854. $\int \frac{(a+b\sec(c+dx))^{5/2}}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{6}\left(\frac{\frac{\int -\frac{-26a^2\sec(c+dx)b^3-15a\left(a^2+4b^2\right)\sec^2(c+dx)b^2+a\left(33a^2+16b^2\right)b^2}{2\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{b}+\frac{b\left(33a^2+16b^2\right)\sin(c+dx)\sqrt{\sec(}}{d}}{4b}\right.\right.$$

$\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{6}\left(\frac{\frac{b\left(33a^2+16b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{\int\frac{-26a^2\sec(c+dx)b^3-15a\left(a^2+4b^2\right)\sec^2(c+dx)}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx}}}{2b}}{4b}\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{6}\left(\frac{\frac{b\left(33a^2+16b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{\int\frac{-26a^2\csc\left(c+dx+\frac{\pi}{2}\right)b^3-15a\left(a^2+4b^2\right)\csc\left(c+\right.}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc(}}}{2b}}{4b}\right.\right.$$

$\downarrow$ 4596

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{6}\left(\frac{\frac{b\left(33a^2+16b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{\int\frac{ab^2\left(33a^2+16b^2\right)-26a^2b^3\sec(c+dx)}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx-15ab}{2b}}{4b}\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{6}\left(\frac{\frac{b\left(33a^2+16b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{\int\frac{ab^2\left(33a^2+16b^2\right)-26a^2b^3\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-}{2b}}{4b}\right.\right.$$

$\downarrow$ 4346

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{6}\left(\frac{\frac{b\left(33a^2+16b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{\int\frac{ab^2\left(33a^2+16b^2\right)-26a^2b^3\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-}{}}{4b}\right.\right.$$

$\downarrow$ 3042

3.854. $\int \frac{(a+b\sec(c+dx))^{5/2}}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{6}\left(\frac{\frac{b(33a^2+16b^2)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{\int\frac{ab^2(33a^2+16b^2)-26a^2b^3\csc(c+dx+\frac{\pi}{2})}{\sqrt{\csc(c+dx+\frac{\pi}{2})}\sqrt{a+b\csc(c+dx+\frac{\pi}{2})}}dx-}{}}{4b}\right.\right.$$

$\downarrow$ 3286

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{6}\left(\frac{\frac{b(33a^2+16b^2)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{\int\frac{ab^2(33a^2+16b^2)-26a^2b^3\csc(c+dx+\frac{\pi}{2})}{\sqrt{\csc(c+dx+\frac{\pi}{2})}\sqrt{a+b\csc(c+dx+\frac{\pi}{2})}}dx-}{}}{4b}\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{6}\left(\frac{\frac{b(33a^2+16b^2)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{\int\frac{ab^2(33a^2+16b^2)-26a^2b^3\csc(c+dx+\frac{\pi}{2})}{\sqrt{\csc(c+dx+\frac{\pi}{2})}\sqrt{a+b\csc(c+dx+\frac{\pi}{2})}}dx-}{}}{4b}\right.\right.$$

$\downarrow$ 3284

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{6}\left(\frac{\frac{b(33a^2+16b^2)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{\int\frac{ab^2(33a^2+16b^2)-26a^2b^3\csc(c+dx+\frac{\pi}{2})}{\sqrt{\csc(c+dx+\frac{\pi}{2})}\sqrt{a+b\csc(c+dx+\frac{\pi}{2})}}dx-}{}}{4b}\right.\right.$$

$\downarrow$ 4523

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{6}\left(\frac{\frac{b(33a^2+16b^2)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{b^2(33a^2+16b^2)\int\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}dx-\left(b^3(59\right.}{}}{4b}\right.\right.$$

$\downarrow$ 3042

3.854. $\int \frac{(a+b\sec(c+dx))^{5/2}}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{6}\left(\frac{b(33a^2+16b^2)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{b^2(33a^2+16b^2)\int\frac{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-\left(\right.}{}\right.\right.$$

$\downarrow$ 4343

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{6}\left(\frac{b(33a^2+16b^2)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{b^2\left(33a^2+16b^2\right)\sqrt{a+b\sec(c+dx)}\int\sqrt{b+a\cos(c+d}}{\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{6}\left(\frac{b(33a^2+16b^2)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{b^2\left(33a^2+16b^2\right)\sqrt{a+b\sec(c+dx)}\int\sqrt{b+a\sin\left(c+d\right.}}{\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}\right.\right.$$

$\downarrow$ 3134

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{6}\left(\frac{b(33a^2+16b^2)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{b^2\left(33a^2+16b^2\right)\sqrt{a+b\sec(c+dx)}\int\sqrt{\frac{b}{a+b}+\frac{a\cos(}{a}}}{\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{6}\left(\frac{b(33a^2+16b^2)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{b^2\left(33a^2+16b^2\right)\sqrt{a+b\sec(c+dx)}\int\sqrt{\frac{b}{a+b}+\frac{a\sin(}{}}}{\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}\right.\right.$$

$\downarrow$ 3132

3.854. $\int\frac{(a+b\sec(c+dx))^{5/2}}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{6}\left(\frac{b\left(33a^2+16b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{-\left(b^3\left(59a^2+16b^2\right)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right.}{}\right.\right.$$

$\downarrow$ 4345

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{6}\left(\frac{b\left(33a^2+16b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{-\frac{b^3\left(59a^2+16b^2\right)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int}{\sqrt{a+b\sec(c+dx)}}}{}\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{6}\left(\frac{b\left(33a^2+16b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{-\frac{b^3\left(59a^2+16b^2\right)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int}{\sqrt{a+b\sec(c+dx)}}}{}\right.\right.$$

$\downarrow$ 3142

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{6}\left(\frac{b\left(33a^2+16b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{-\frac{b^3\left(59a^2+16b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int}{\sqrt{a+b\sec(c+dx)}}}{}\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{6}\left(\frac{b\left(33a^2+16b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{-\frac{b^3\left(59a^2+16b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int}{\sqrt{a+b\sec(c+dx)}}}{}\right.\right.$$

3.854. $\int \frac{(a+b\sec(c+dx))^{5/2}}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

input
```
Int[(a + b*Sec[c + d*x])^(5/2)/Cos[c + d*x]^(3/2),x]
```

output
```
$Aborted
```

3.854.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3286
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]]   Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
 d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

rule 4329
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
2)*((d*Csc[e + f*x])^n/(f*(m + n - 1))), x] + Simp[1/(d*(m + n - 1))   Int[
(a + b*Csc[e + f*x])^(m - 3)*(d*Csc[e + f*x])^n*Simp[a^3*d*(m + n - 1) + a*
b^2*d*n + b*(b^2*d*(m + n - 2) + 3*a^2*d*(m + n - 1))*Csc[e + f*x] + a*b^2*
d*(3*m + 2*n - 4)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, n}, x
] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
 &&  !(IGtQ[n, 2] &&  !IntegerQ[m])
```

rule 4343
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]])   Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a
, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4345
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]])   Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[
{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4346
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Simp[d*Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x
]]/Sqrt[a + b*Csc[e + f*x]])   Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4523
```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Simp[A/a   I
nt[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Simp[(A*b - a*B)
/(a*d)   Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ
[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

3.854. $\int \frac{(a+b\sec(c+dx))^{5/2}}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

rule 4590
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(-C)*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
)*((d*Csc[e + f*x])^(n - 1)/(b*f*(m + n + 1))), x] + Simp[d/(b*(m + n + 1))
   Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (
A*b*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2
 - b^2, 0] && GtQ[n, 0]
```

rule 4596
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Simp[C/d^2   Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*C
sc[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[
a + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
 b^2, 0]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

3.854.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 13.00 (sec) , antiderivative size = 3055, normalized size of antiderivative = 8.28

method	result	size
default	Expression too large to display	3055

input
```
int((a+b*sec(d*x+c))^(5/2)/cos(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/24/d/((a-b)/(a+b))^(1/2)*(16*((a-b)/(a+b))^(1/2)*b^3*cos(d*x+c)^2*sin(d*
x+c)-16*EllipticE(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a-
b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+
1))^(1/2)*a*b^2*cos(d*x+c)^3+26*EllipticF(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)
+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1
))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a^2*b*cos(d*x+c)^3-44*EllipticF(((a-b)/(
a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*c
os(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a*b^2*cos(d*x+c)
^3+120*EllipticPi(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(a+b)/(a-b)
,I/((a-b)/(a+b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1
/(cos(d*x+c)+1))^(1/2)*a*b^2*cos(d*x+c)^3+33*EllipticE(((a-b)/(a+b))^(1/2)
*(-cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/
(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a^2*b*cos(d*x+c)^3+16*Ellip
ticE(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*(1
/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*b^3
*cos(d*x+c)^3+33*((a-b)/(a+b))^(1/2)*a^3*cos(d*x+c)^3*sin(d*x+c)+36*(1/(co
s(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ellipti
cF(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*a^3*
cos(d*x+c)^4+52*(1/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*
x+c)+1))^(1/2)*EllipticF(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(...
```

3.854.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+b*sec(d*x+c))^(5/2)/cos(d*x+c)^(3/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.854.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+b*sec(d*x+c))**(5/2)/cos(d*x+c)**(3/2),x)
```

output
```
Timed out
```

3.854.7 Maxima [F]

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(b\sec(dx+c)+a)^{\frac{5}{2}}}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(5/2)/cos(d*x+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(d*x + c) + a)^(5/2)/cos(d*x + c)^(3/2), x)
```

3.854.8 Giac [F]

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(b\sec(dx+c)+a)^{\frac{5}{2}}}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(5/2)/cos(d*x+c)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*sec(d*x + c) + a)^(5/2)/cos(d*x + c)^(3/2), x)
```

3.854.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\cos^{\frac{3}{2}}(c+dx)}\,dx=\int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2}}{\cos\left(c+d\,x\right)^{3/2}}\,dx$$

input
```
int((a + b/cos(c + d*x))^(5/2)/cos(c + d*x)^(3/2),x)
```

output
```
int((a + b/cos(c + d*x))^(5/2)/cos(c + d*x)^(3/2), x)
```

3.855 $\int \frac{\cos^{\frac{5}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$

3.855.1 Optimal result 6423
3.855.2 Mathematica [C] (warning: unable to verify) 6424
3.855.3 Rubi [A] (verified) 6424
3.855.4 Maple [B] (verified) 6431
3.855.5 Fricas [C] (verification not implemented) 6431
3.855.6 Sympy [F(-1)] 6432
3.855.7 Maxima [F] 6432
3.855.8 Giac [F] 6433
3.855.9 Mupad [F(-1)] 6433

3.855.1 Optimal result

Integrand size = 25, antiderivative size = 249

$$\begin{aligned}\int \frac{\cos^{\frac{5}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx &= -\frac{2b(7a^2+8b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{15a^3d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} \\ &\quad+\frac{2(9a^2+8b^2)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+dx)}}{15a^3d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}} \\ &\quad-\frac{8b\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{15a^2d} \\ &\quad+\frac{2\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{5ad}\end{aligned}$$

output
```
-2/15*b*(7*a^2+8*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elli
pticF(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))
^(1/2)/a^3/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)+2/5*cos(d*x+c)^(3/2)*
sin(d*x+c)*(a+b*sec(d*x+c))^(1/2)/a/d-8/15*b*sin(d*x+c)*cos(d*x+c)^(1/2)*(
a+b*sec(d*x+c))^(1/2)/a^2/d+2/15*(9*a^2+8*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2
)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))
*cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2)/a^3/d/((b+a*cos(d*x+c))/(a+b))^(1
/2)
```

3.855.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 10.19 (sec) , antiderivative size = 340, normalized size of antiderivative = 1.37

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$$
$$= \frac{2a(b+a\cos(c+dx))(-4b+3a\cos(c+dx))\sin(c+dx)+\frac{2\left(\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\right)^{3/2}\left(i(9a^3+9a^2b+8ab^2+8b^3)E\left(i\right.\right.}{}}{}$$

input
```
Integrate[Cos[c + d*x]^(5/2)/Sqrt[a + b*Sec[c + d*x]],x]
```

output
```
(2*a*(b + a*Cos[c + d*x])*(-4*b + 3*a*Cos[c + d*x])*Sin[c + d*x] + (2*(Cos
[(c + d*x)/2]^2*Sec[c + d*x])^(3/2)*(I*(9*a^3 + 9*a^2*b + 8*a*b^2 + 8*b^3)
*EllipticE[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]
^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] - I*a*(9*a^2 +
2*a*b + 8*b^2)*EllipticF[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Se
c[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] +
 (9*a^2 + 8*b^2)*(b + a*Cos[c + d*x])*(Sec[(c + d*x)/2]^2)^(3/2)*Tan[(c +
d*x)/2]))/Sec[c + d*x]^(3/2))/(15*a^3*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[
c + d*x]])
```

3.855.3 Rubi [A] (verified)

Time = 2.15 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.13, number of steps used = 20, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.800$, Rules used = {3042, 4752, 3042, 4350, 3042, 4592, 27, 3042, 4523, 3042, 4343, 3042, 3134, 3042, 3132, 4345, 3042, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$$
$$\Big\downarrow \; 3042$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \; 4752$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}dx$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \; 4350$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{\int \frac{-2b\sec^2(c+dx)-3a\sec(c+dx)+4b}{\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}dx}{5a}\right)$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{\int \frac{-2b\csc\left(c+dx+\frac{\pi}{2}\right)^2-3a\csc\left(c+dx+\frac{\pi}{2}\right)+4b}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{5a}\right)$$

$$\Big\downarrow \; 4592$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{\frac{8b\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}-\frac{2\int \frac{9a^2+2b\sec(c+dx)a+8b^2}{2\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+d}}}{3a}}{5a}\right.$$

$$\Big\downarrow \; 27$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{\frac{8b\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}-\frac{\int \frac{9a^2+2b\sec(c+dx)a+8b^2}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}}{3a}}{5a}\right.$$

$$\Big\downarrow \; 3042$$

3.855. $\int \frac{\cos^{\frac{5}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{\frac{8b\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}-\frac{\int\frac{9a^2+2b\csc\left(c+dx+\frac{\pi}{2}\right)a\cdots}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc(c\cdots}}}{3a}}{5a}\right.$$

$\downarrow$ 4523

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{\frac{8b\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}-\frac{\left(9a^2+8b^2\right)\int\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}}{a}}{5a}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{\frac{8b\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}-\frac{\left(9a^2+8b^2\right)\int\frac{\sqrt{a+b\csc\left(c+dx+\cdots\right.}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\cdots\right.}}}{a}}{5a}\right.$$

$\downarrow$ 4343

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{\frac{8b\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}-\frac{\left(9a^2+8b^2\right)\sqrt{a+b\sec(c+dx)}\int\cdots}{a\sqrt{\sec(c+dx)}\sqrt{a\cos(\cdots}}}{5a}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{\frac{8b\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}-\frac{\left(9a^2+8b^2\right)\sqrt{a+b\sec(c+dx)}\int\cdots}{a\sqrt{\sec(c+dx)}\sqrt{a\,\mathrm{co}\cdots}}}{5a}\right.$$

3.855. $\int\frac{\cos^{\frac{5}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$

$$\Big\downarrow \; 3134$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{\frac{8b\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}-\frac{\left(9a^2+8b^2\right)\sqrt{a+b\sec(c+dx)}\int\sqrt{}}{a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos}{}}}}{5a}\right.$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{\frac{8b\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}-\frac{\left(9a^2+8b^2\right)\sqrt{a+b\sec(c+dx)}\int\sqrt{}}{a\sqrt{\sec(c+dx)}\sqrt{\frac{a}{}}}}{5}\right.$$

$$\Big\downarrow \; 3132$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{\frac{8b\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}-\frac{2\left(9a^2+8b^2\right)\sqrt{a+b\sec(c+dx)}E}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(}{}}}}{5a}\right.$$

$$\Big\downarrow \; 4345$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{\frac{8b\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}-\frac{2\left(9a^2+8b^2\right)\sqrt{a+b\sec(c+dx)}E}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(}{}}}}{}\right.$$

$$\Big\downarrow \; 3042$$

3.855. $\int \frac{\cos^{\frac{5}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{\frac{8b\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}-\frac{2\left(9a^2+8b^2\right)\sqrt{a+b\sec(c+dx)}E}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c}{a}}}}{}\right.$$

$\downarrow$ 3142

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{\frac{8b\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}-\frac{2\left(9a^2+8b^2\right)\sqrt{a+b\sec(c+dx)}E}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c}{a}}}}{}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{\frac{8b\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}-\frac{2\left(9a^2+8b^2\right)\sqrt{a+b\sec(c+dx)}E}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c}{a}}}}{}\right.$$

$\downarrow$ 3140

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{\frac{8b\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}-\frac{2\left(9a^2+8b^2\right)\sqrt{a+b\sec(c+dx)}E}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c}{a}}}}{}\right.$$

input
```
Int[Cos[c + d*x]^(5/2)/Sqrt[a + b*Sec[c + d*x]],x]
```

3.855. $\int \frac{\cos^{\frac{5}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*Sqrt[a + b*Sec[c + d*x]]*Sin[c +
 d*x])/(5*a*d*Sec[c + d*x]^(3/2)) - (-1/3*((-2*b*(7*a^2 + 8*b^2)*Sqrt[(b +
 a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c
 + d*x]])/(a*d*Sqrt[a + b*Sec[c + d*x]]) + (2*(9*a^2 + 8*b^2)*EllipticE[(c
 + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(a*d*Sqrt[(b + a*Cos[c
 + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]))/a + (8*b*Sqrt[a + b*Sec[c + d*x]]*S
in[c + d*x])/(3*a*d*Sqrt[Sec[c + d*x]]))/(5*a))
```

3.855.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

3.855. $\int \frac{\cos^{\frac{5}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$

rule 4343
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]])   Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a
, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4345
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]])   Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[
{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4350
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[Cos[e + f*x]*(d*Csc[e + f*x])^(n + 1)*(Sqrt[a +
 b*Csc[e + f*x]]/(a*d*f*n)), x] + Simp[1/(2*a*d*n)   Int[((d*Csc[e + f*x])^
(n + 1)/Sqrt[a + b*Csc[e + f*x]])*Simp[(-b)*(2*n + 1) + 2*a*(n + 1)*Csc[e +
 f*x] + b*(2*n + 3)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x]
 && NeQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

rule 4523
```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Simp[A/a   I
nt[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Simp[(A*b - a*B)
/(a*d)   Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ
[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

rule 4592
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Simp[1/(a*d*n)   Int[(a + b*Csc[e + f*x])^m
*(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*
Csc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

3.855. $\int \frac{\cos^{\frac{5}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$

3.855.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2306 vs. $2(279) = 558$.

Time = 9.74 (sec) , antiderivative size = 2307, normalized size of antiderivative = 9.27

method	result	size
default	Expression too large to display	2307

input
```
int(cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/15/d/((a-b)/(a+b))^(1/2)/a^3*(4*EllipticF(((a-b)/(a+b))^(1/2)*(-cot(d*x+
c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*(1/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+
a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^2*b*cos(d*x+c)-16*EllipticF(((a-b)/(
a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*(1/(cos(d*x+c)+
1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a*b^2*cos(d*x+c)
-18*EllipticE(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^
(1/2))*(1/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^
(1/2)*a^2*b*cos(d*x+c)+16*EllipticE(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d
*x+c)),(-(a+b)/(a-b))^(1/2))*(1/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*
x+c))/(cos(d*x+c)+1))^(1/2)*a*b^2*cos(d*x+c)+8*EllipticE(((a-b)/(a+b))^(1/
2)*(-cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c)
)/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a*b^2*cos(d*x+c)^2+2*Elli
pticF(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*(
1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a^
2*b*cos(d*x+c)^2-8*EllipticF(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),
(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(
cos(d*x+c)+1))^(1/2)*a*b^2*cos(d*x+c)^2-9*EllipticE(((a-b)/(a+b))^(1/2)*(-
cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(co
s(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a^2*b*cos(d*x+c)^2-8*EllipticE
(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*(1/...
```

3.855.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.13 (sec) , antiderivative size = 456, normalized size of antiderivative = 1.83

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$$
$$= \frac{6\,(3\,a^3\cos(dx+c) - 4\,a^2b)\sqrt{\frac{a\cos(dx+c)+b}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c) - 4\,\sqrt{2}(-3i\,a^2b - 4i\,b^3)\sqrt{a}\text{weierstras}}{}$$

input
```
integrate(cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
1/45*(6*(3*a^3*cos(d*x + c) - 4*a^2*b)*sqrt((a*cos(d*x + c) + b)/cos(d*x +
 c))*sqrt(cos(d*x + c))*sin(d*x + c) - 4*sqrt(2)*(-3*I*a^2*b - 4*I*b^3)*sq
rt(a)*weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)
/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x + c) + 2*b)/a) - 4*sqrt(2)*(3*
I*a^2*b + 4*I*b^3)*sqrt(a)*weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8
/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*
b)/a) - 3*sqrt(2)*(-9*I*a^3 - 8*I*a*b^2)*sqrt(a)*weierstrassZeta(-4/3*(3*a
^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a
^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a
*sin(d*x + c) + 2*b)/a)) - 3*sqrt(2)*(9*I*a^3 + 8*I*a*b^2)*sqrt(a)*weierst
rassZeta(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrass
PInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*co
s(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a)))/(a^4*d)
```

3.855.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(5/2)/(a+b*sec(d*x+c))**(1/2),x)
```

output
```
Timed out
```

3.855.7 Maxima [F]

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\cos\left(dx+c\right)^{\frac{5}{2}}}{\sqrt{b\sec\left(dx+c\right)+a}}\,dx$$

input
```
integrate(cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(cos(d*x + c)^(5/2)/sqrt(b*sec(d*x + c) + a), x)
```

3.855.8 Giac [F]

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\cos\left(dx+c\right)^{\frac{5}{2}}}{\sqrt{b\sec\left(dx+c\right)+a}}\,dx$$

input
```
integrate(cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(cos(d*x + c)^(5/2)/sqrt(b*sec(d*x + c) + a), x)
```

3.855.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\cos\left(c+d\,x\right)^{5/2}}{\sqrt{a+\frac{b}{\cos(c+d\,x)}}}\,dx$$

input
```
int(cos(c + d*x)^(5/2)/(a + b/cos(c + d*x))^(1/2),x)
```

output
```
int(cos(c + d*x)^(5/2)/(a + b/cos(c + d*x))^(1/2), x)
```

3.856 $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$

3.856.1 Optimal result 6434
3.856.2 Mathematica [C] (verified) 6435
3.856.3 Rubi [A] (verified) 6435
3.856.4 Maple [B] (verified) 6440
3.856.5 Fricas [C] (verification not implemented) 6441
3.856.6 Sympy [F] 6442
3.856.7 Maxima [F] 6442
3.856.8 Giac [F] 6443
3.856.9 Mupad [F(-1)] 6443

3.856.1 Optimal result

Integrand size = 25, antiderivative size = 195

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \frac{2(a^2+2b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{3a^2d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} - \frac{4b\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+dx)}}{3a^2d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}} + \frac{2\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{3ad}$$

output
```
2/3*(a^2+2*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(
sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)
/a^2/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)+2/3*sin(d*x+c)*cos(d*x+c)^(
1/2)*(a+b*sec(d*x+c))^(1/2)/a/d-4/3*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2
*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*cos(d*x+
c)^(1/2)*(a+b*sec(d*x+c))^(1/2)/a^2/d/((b+a*cos(d*x+c))/(a+b))^(1/2)
```

3.856.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 9.66 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.36

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$$
$$= \frac{2\sqrt{\cos(c+dx)}\left(-2ib(a+b)\sqrt{\frac{b+a\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}E\left(i\operatorname{arcsinh}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{-a+b}{a+b}\right)\sqrt{\sec(c+dx)}\sqrt{1+\text{se}}\right.}{}$$

input
```
Integrate[Cos[c + d*x]^(3/2)/Sqrt[a + b*Sec[c + d*x]],x]
```

output
```
(2*Sqrt[Cos[c + d*x]]*((-2*I)*b*(a + b)*Sqrt[(b + a*Cos[c + d*x])/((a + b)
*(1 + Cos[c + d*x]))]*EllipticE[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a +
 b)]*Sqrt[Sec[c + d*x]]*Sqrt[1 + Sec[c + d*x]] - I*a*(a - 2*b)*Sqrt[(b + a
*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[I*ArcSinh[Tan[(c +
d*x)/2]], (-a + b)/(a + b)]*Sqrt[Sec[c + d*x]]*Sqrt[1 + Sec[c + d*x]] + a^
2*Sin[c + d*x] - 2*a*b*Tan[(c + d*x)/2] - 2*b^2*Sec[c + d*x]*Tan[(c + d*x)
/2] + a*b*Tan[c + d*x]))/(3*a^2*d*Sqrt[a + b*Sec[c + d*x]])
```

3.856.3 Rubi [A] (verified)

Time = 1.60 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.13, number of steps used = 17, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.680$, Rules used = {3042, 4752, 3042, 4350, 3042, 4523, 3042, 4343, 3042, 3134, 3042, 3132, 4345, 3042, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$
$$\Big\downarrow \; 4752$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}dx$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$\Big\downarrow$ 4350

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}-\frac{\int \frac{2b-a\sec(c+dx)}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{3a}\right)$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}-\frac{\int \frac{2b-a\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{3a}\right)$$

$\Big\downarrow$ 4523

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{2b\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}dx}{a}-\frac{(a^2+2b^2)\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}dx}{a}}{3a}\right)$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{2b\int \frac{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}-\frac{(a^2+2b^2)\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}}{a}}{3a}\right.$$

$\Big\downarrow$ 4343

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{2b\sqrt{a+b\sec(c+dx)}\int \sqrt{b+a\cos(c+dx)}dx}{a\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}-\frac{(a^2+2b^2)\int \frac{\sqrt{}}{\sqrt{a+}}}{}}{3a}\right.$$

$\Big\downarrow$ 3042

3.856. $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{2b\sqrt{a+b\sec(c+dx)}\int\sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{a\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}-\frac{(a^2+2b^2)\int}{}}{3a}\right.$$

$\downarrow$ 3134

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{2b\sqrt{a+b\sec(c+dx)}\int\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}dx}{a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{(a^2+2b^2)\int}{\sqrt{}}}{3a}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{2b\sqrt{a+b\sec(c+dx)}\int\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}dx}{a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{(a^2+2b^2)}{}}{3a}\right.$$

$\downarrow$ 3132

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{4b\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{(a^2+2b^2)\int\frac{\sqrt{\csc(}}{\sqrt{a+b\,\mathrm{cs}}}}{a}}{3a}\right.$$

$\downarrow$ 4345

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{4b\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{(a^2+2b^2)\sqrt{\sec(c+d}}{}}{3a}\right.$$

$\downarrow$ 3042

3.856. $\int\frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{4b\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{(a^2+2b^2)\sqrt{\sec(c+d}}{}}{3a}\right.$$

$\downarrow$ 3142

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{4b\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{(a^2+2b^2)\sqrt{\sec(c+d}}{}}{3a}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{4b\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{(a^2+2b^2)\sqrt{\sec(c+d}}{}}{3a}\right.$$

$\downarrow$ 3140

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{4b\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{2(a^2+2b^2)\sqrt{\sec(c+}}{}}{3a}\right.$$

input
```
Int[Cos[c + d*x]^(3/2)/Sqrt[a + b*Sec[c + d*x]],x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(-1/3*((-2*(a^2 + 2*b^2)*Sqrt[(b + a
*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c +
 d*x]])/(a*d*Sqrt[a + b*Sec[c + d*x]]) + (4*b*EllipticE[(c + d*x)/2, (2*a)
/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(a*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)
]*Sqrt[Sec[c + d*x]]))/a + (2*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(3*a*
d*Sqrt[Sec[c + d*x]]))
```

3.856. $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$

3.856.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 4343
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]])   Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a
, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4345
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]])   Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[
{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

3.856. $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$

rule 4350
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[Cos[e + f*x]*(d*Csc[e + f*x])^(n + 1)*(Sqrt[a +
 b*Csc[e + f*x]]/(a*d*f*n)), x] + Simp[1/(2*a*d*n)   Int[((d*Csc[e + f*x])^
(n + 1)/Sqrt[a + b*Csc[e + f*x]])*Simp[(-b)*(2*n + 1) + 2*a*(n + 1)*Csc[e +
 f*x] + b*(2*n + 3)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x]
 && NeQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

rule 4523
```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Simp[A/a   I
nt[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Simp[(A*b - a*B)
/(a*d)   Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ
[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

3.856.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1315 vs. $2(231) = 462$.

Time = 8.18 (sec) , antiderivative size = 1316, normalized size of antiderivative = 6.75

method	result	size
default	Expression too large to display	1316

input
```
int(cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3/d/((a-b)/(a+b))^(1/2)/a^2*((1/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(
d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+c
sc(d*x+c)),(-(a+b)/(a-b))^(1/2))*a^2*cos(d*x+c)^2+2*(1/(cos(d*x+c)+1))^(1/
2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF(((a-b)/(a+b))
^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*a*b*cos(d*x+c)^2-2*(
1/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*El
lipticE(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))
*a*b*cos(d*x+c)^2+2*(1/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(co
s(d*x+c)+1))^(1/2)*EllipticE(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),
(-(a+b)/(a-b))^(1/2))*b^2*cos(d*x+c)^2+((a-b)/(a+b))^(1/2)*a^2*cos(d*x+c)^
2*sin(d*x+c)+2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x
+c)+1))^(1/2)*EllipticF(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(-(a+
b)/(a-b))^(1/2))*a^2*cos(d*x+c)+4*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1)
)^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF(((a-b)/(a+b))^(1/2)*(-cot(d*x+c
)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*a*b*cos(d*x+c)-4*(1/(a+b)*(b+a*cos(d*x
+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE(((a-b)/(a+b)
)^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*a*b*cos(d*x+c)+4*(1
/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*Ell
ipticE(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*
b^2*cos(d*x+c)+((a-b)/(a+b))^(1/2)*a^2*cos(d*x+c)*sin(d*x+c)-((a-b)/(a+...
```

3.856.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 415, normalized size of antiderivative = 2.13

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$$
$$= \frac{6\,a^2\sqrt{\frac{a\cos(dx+c)+b}{\cos(dx+c)}}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)-6i\,\sqrt{2}a^{\frac{3}{2}}b\mathrm{weierstrassZeta}\left(-\frac{4\,(3\,a^2-4\,b^2)}{3\,a^2},\frac{8\,(9\,a^2b-8\,b^3)}{27\,a^3},\mathrm{weierst}\right.}{}$$

input
```
integrate(cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
1/9*(6*a^2*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(
d*x + c) - 6*I*sqrt(2)*a^(3/2)*b*weierstrassZeta(-4/3*(3*a^2 - 4*b^2)/a^2,
 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2,
 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x + c) +
2*b)/a)) + 6*I*sqrt(2)*a^(3/2)*b*weierstrassZeta(-4/3*(3*a^2 - 4*b^2)/a^2,
 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2,
 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c) +
2*b)/a)) + sqrt(2)*(-3*I*a^2 - 4*I*b^2)*sqrt(a)*weierstrassPInverse(-4/3*(
3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*
I*a*sin(d*x + c) + 2*b)/a) + sqrt(2)*(3*I*a^2 + 4*I*b^2)*sqrt(a)*weierstra
ssPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*
cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a))/(a^3*d)
```

3.856.6 Sympy [F]

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$$

input
```
integrate(cos(d*x+c)**(3/2)/(a+b*sec(d*x+c))**(1/2),x)
```

output
```
Integral(cos(c + d*x)**(3/2)/sqrt(a + b*sec(c + d*x)), x)
```

3.856.7 Maxima [F]

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\cos(dx+c)^{\frac{3}{2}}}{\sqrt{b\sec(dx+c)+a}}\,dx$$

input
```
integrate(cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(cos(d*x + c)^(3/2)/sqrt(b*sec(d*x + c) + a), x)
```

3.856.8 Giac [F]

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\cos{(dx+c)}^{\frac{3}{2}}}{\sqrt{b\sec{(dx+c)}+a}}\,dx$$

input
```
integrate(cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(cos(d*x + c)^(3/2)/sqrt(b*sec(d*x + c) + a), x)
```

3.856.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\cos{(c+d\,x)}^{3/2}}{\sqrt{a+\frac{b}{\cos(c+d\,x)}}}\,dx$$

input
```
int(cos(c + d*x)^(3/2)/(a + b/cos(c + d*x))^(1/2),x)
```

output
```
int(cos(c + d*x)^(3/2)/(a + b/cos(c + d*x))^(1/2), x)
```

3.857 $\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+b\sec(c+dx)}}\,dx$

3.857.1 Optimal result 6444
3.857.2 Mathematica [C] (verified) 6444
3.857.3 Rubi [A] (verified) 6445
3.857.4 Maple [B] (verified) 6449
3.857.5 Fricas [C] (verification not implemented) 6450
3.857.6 Sympy [F] 6451
3.857.7 Maxima [F] 6451
3.857.8 Giac [F] 6451
3.857.9 Mupad [F(-1)] 6452

3.857.1 Optimal result

Integrand size = 25, antiderivative size = 142

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+b\sec(c+dx)}}\,dx = -\frac{2b\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{ad\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} + \frac{2\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+dx)}}{ad\sqrt{\frac{b+a\cos(c+dx)}{a+b}}}$$

output
```
-2*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x
+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)/a/d/cos(d*
x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)+2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d
*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*cos(d*x+c)
^(1/2)*(a+b*sec(d*x+c))^(1/2)/a/d/((b+a*cos(d*x+c))/(a+b))^(1/2)
```

3.857.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.94 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.52

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+b\sec(c+dx)}}\,dx = \frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sqrt{1+\sec(c+dx)}\left(i(a+b)\sqrt{\frac{b+a\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}E\left(i\text{arcsinh}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\right.\right.}{a}$$

input
```
Integrate[Sqrt[Cos[c + d*x]]/Sqrt[a + b*Sec[c + d*x]],x]
```

output
```
(2*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sqrt[1 + Sec[c + d*x]]*(I*(a + b)
*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[I*ArcSi
nh[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - I*a*Sqrt[(b + a*Cos[c + d*x])/((
a + b)*(1 + Cos[c + d*x]))]*EllipticF[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b
)/(a + b)] + Sqrt[(1 + Cos[c + d*x])^(-1)]*(b + a*Cos[c + d*x])*Tan[(c + d
*x)/2]))/(a*d*Sqrt[a + b*Sec[c + d*x]])
```

3.857.3 Rubi [A] (verified)

Time = 1.24 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.15, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {3042, 4752, 3042, 4349, 3042, 4343, 3042, 3134, 3042, 3132, 4345, 3042, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+b\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4349}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}dx}{a}-\frac{b\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}dx}{a}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}-\frac{b\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}\right)$$

$\downarrow$ 4343

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\sqrt{a+b\sec(c+dx)}\int \sqrt{b+a\cos(c+dx)}dx}{a\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}-\frac{b\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}\right)$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\sqrt{a+b\sec(c+dx)}\int \sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{a\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}-\frac{b\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}\right)$$

$\downarrow$ 3134

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\sqrt{a+b\sec(c+dx)}\int \sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}dx}{a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{b\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}\right)$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\sqrt{a+b\sec(c+dx)}\int \sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}dx}{a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{b\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}\right)$$

$\downarrow$ 3132

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{b\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}\right)$$

$\downarrow$ 4345

3.857. $\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+b\sec(c+dx)}}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{b\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{}{\sqrt{b+a}}}{a\sqrt{a+b\sec(c+dx)}}\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{b\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{}{\sqrt{b+a}}}{a\sqrt{a+b\sec(c+dx)}}\right.$$

$\Big\downarrow$ 3142

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{b\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\cos}{}}}}{a\sqrt{a+b\sec(c+dx)}}\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{b\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\sin}{}}}}{a\sqrt{a+b\sec(c+dx)}}\right.$$

$\Big\downarrow$ 3140

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{2b\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\text{EllipticF}\,(}{ad\sqrt{a+b\sec(c+dx)}}\right.$$

input
```
Int[Sqrt[Cos[c + d*x]]/Sqrt[a + b*Sec[c + d*x]],x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((-2*b*Sqrt[(b + a*Cos[c + d*x])/(a
+ b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(a*d*Sqrt[
a + b*Sec[c + d*x]]) + (2*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b
*Sec[c + d*x]])/(a*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]
))
```

3.857. $\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+b\sec(c+dx)}}\,dx$

3.857.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 4343
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]])   Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a
, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4345
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]])   Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[
{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

3.857. $\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+b\sec(c+dx)}}\,dx$

rule 4349
```
Int[1/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)]), x_Symbol] :> Simp[1/a   Int[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Cs
c[e + f*x]], x], x] - Simp[b/(a*d)   Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Cs
c[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

3.857.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 738 vs. $2(188) = 376$.

Time = 7.38 (sec) , antiderivative size = 739, normalized size of antiderivative = 5.20

method	result
default	$\frac{2\sqrt{-\frac{(1-\cos(dx+c))^2\csc(dx+c)^2-1}{(1-\cos(dx+c))^2\csc(dx+c)^2+1}}\sqrt{\frac{a(1-\cos(dx+c))^2\csc(dx+c)^2-b(1-\cos(dx+c))^2\csc(dx+c)^2-a-b}{(1-\cos(dx+c))^2\csc(dx+c)^2-1}}\left(\sqrt{\frac{a-b}{a+b}}\,a(1-\cos(dx+c))^3\csc(dx+c\right.}{}$
risch	$-\frac{i\left(a\,e^{2i(dx+c)}+2b\,e^{i(dx+c)}+a\right)\sqrt{2}\sqrt{\left(e^{2i(dx+c)}+1\right)e^{-i(dx+c)}}}{ad\sqrt{\frac{a\,e^{2i(dx+c)}+2b\,e^{i(dx+c)}+a}{e^{2i(dx+c)}+1}}\left(e^{2i(dx+c)}+1\right)}-\frac{i\left(-\frac{2\left(a\,e^{2i(dx+c)}+2b\,e^{i(dx+c)}+a\right)}{a\sqrt{e^{i(dx+c)}\left(a\,e^{2i(dx+c)}+2b\,e^{i(dx+c)}+a\right)}}+\frac{2\left(b+\sqrt{-a^2+b^2}\right)\sqrt{\left(\right.}}{}\right.}{}$

input
```
int(cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/d/a/((a-b)/(a+b))^(1/2)*(-((1-cos(d*x+c))^2*csc(d*x+c)^2-1)/((1-cos(d*x+
c))^2*csc(d*x+c)^2+1))^(1/2)*((a*(1-cos(d*x+c))^2*csc(d*x+c)^2-b*(1-cos(d*
x+c))^2*csc(d*x+c)^2-a-b)/((1-cos(d*x+c))^2*csc(d*x+c)^2-1))^(1/2)*(((a-b)
/(a+b))^(1/2)*a*(1-cos(d*x+c))^3*csc(d*x+c)^3-((a-b)/(a+b))^(1/2)*b*(1-cos
(d*x+c))^3*csc(d*x+c)^3+(-(a*(1-cos(d*x+c))^2*csc(d*x+c)^2-b*(1-cos(d*x+c)
)^2*csc(d*x+c)^2-a-b)/(a+b))^(1/2)*((1-cos(d*x+c))^2*csc(d*x+c)^2+1)^(1/2)
*EllipticF(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/
2))*a-(-(a*(1-cos(d*x+c))^2*csc(d*x+c)^2-b*(1-cos(d*x+c))^2*csc(d*x+c)^2-a
-b)/(a+b))^(1/2)*((1-cos(d*x+c))^2*csc(d*x+c)^2+1)^(1/2)*EllipticE(((a-b)/
(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*a+(-(a*(1-cos(
d*x+c))^2*csc(d*x+c)^2-b*(1-cos(d*x+c))^2*csc(d*x+c)^2-a-b)/(a+b))^(1/2)*(
(1-cos(d*x+c))^2*csc(d*x+c)^2+1)^(1/2)*EllipticE(((a-b)/(a+b))^(1/2)*(-cot
(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*b-((a-b)/(a+b))^(1/2)*a*(-cot(d*
x+c)+csc(d*x+c))-((a-b)/(a+b))^(1/2)*b*(-cot(d*x+c)+csc(d*x+c)))/(a*(1-cos
(d*x+c))^2*csc(d*x+c)^2-b*(1-cos(d*x+c))^2*csc(d*x+c)^2-a-b)
```

3.857.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 355, normalized size of antiderivative = 2.50

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+b\sec(c+dx)}}\,dx$$
$$= \frac{2i\sqrt{2}\sqrt{a}b\text{weierstrassPInverse}\left(-\frac{4\,(3\,a^2-4\,b^2)}{3\,a^2},\frac{8\,(9\,a^2b-8\,b^3)}{27\,a^3},\frac{3\,a\cos(dx+c)+3i\,a\sin(dx+c)+2\,b}{3\,a}\right)-2i\sqrt{2}\sqrt{a}b\text{weierstras}}{}$$

input
```
integrate(cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
1/3*(2*I*sqrt(2)*sqrt(a)*b*weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8
/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x + c) + 2*
b)/a) - 2*I*sqrt(2)*sqrt(a)*b*weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2
, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c) +
 2*b)/a) + 3*I*sqrt(2)*a^(3/2)*weierstrassZeta(-4/3*(3*a^2 - 4*b^2)/a^2, 8
/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8
/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x + c) + 2*
b)/a)) - 3*I*sqrt(2)*a^(3/2)*weierstrassZeta(-4/3*(3*a^2 - 4*b^2)/a^2, 8/2
7*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/2
7*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)
/a)))/(a^2*d)
```

3.857.6 Sympy [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\sqrt{\cos{(c+dx)}}}{\sqrt{a+b\sec{(c+dx)}}}\,dx$$

input
```
integrate(cos(d*x+c)**(1/2)/(a+b*sec(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(cos(c + d*x))/sqrt(a + b*sec(c + d*x)), x)
```

3.857.7 Maxima [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\sqrt{\cos{(dx+c)}}}{\sqrt{b\sec{(dx+c)}+a}}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(cos(d*x + c))/sqrt(b*sec(d*x + c) + a), x)
```

3.857.8 Giac [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\sqrt{\cos{(dx+c)}}}{\sqrt{b\sec{(dx+c)}+a}}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(cos(d*x + c))/sqrt(b*sec(d*x + c) + a), x)
```

3.857.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+b\sec(c+dx)}}\,dx=\int \frac{\sqrt{\cos\left(c+d\,x\right)}}{\sqrt{a+\frac{b}{\cos(c+d\,x)}}}\,dx$$

input
```
int(cos(c + d*x)^(1/2)/(a + b/cos(c + d*x))^(1/2),x)
```

output
```
int(cos(c + d*x)^(1/2)/(a + b/cos(c + d*x))^(1/2), x)
```

3.858 $\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}\,dx$

3.858.1 Optimal result 6453
3.858.2 Mathematica [C] (verified) 6453
3.858.3 Rubi [A] (verified) 6454
3.858.4 Maple [A] (verified) 6456
3.858.5 Fricas [C] (verification not implemented) 6456
3.858.6 Sympy [F] 6457
3.858.7 Maxima [F] 6457
3.858.8 Giac [F] 6457
3.858.9 Mupad [F(-1)] 6458

3.858.1 Optimal result

Integrand size = 25, antiderivative size = 67

$$\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}\,dx=\frac{2\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}$$

output
```
2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/
2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)/d/cos(d*x+c)^
(1/2)/(a+b*sec(d*x+c))^(1/2)
```

3.858.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.96 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.52

$$\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}\,dx$$
$$=-\frac{2i\sqrt{\frac{b+a\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}\,\text{EllipticF}\left(i\text{arcsinh}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right),\frac{-a+b}{a+b}\right)}{d\sqrt{\cos(c+dx)}\sqrt{\frac{1}{1+\cos(c+dx)}}\sqrt{a+b\sec(c+dx)}}$$

input
```
Integrate[1/(Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]),x]
```

output
```
((-2*I)*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[
I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)])/(d*Sqrt[Cos[c + d*x]]*Sqrt
[(1 + Cos[c + d*x])^(-1)]*Sqrt[a + b*Sec[c + d*x]])
```

3.858.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3042, 4752, 3042, 4345, 3042, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4345}$$

$$\frac{\sqrt{a\cos(c+dx)+b}\int \frac{1}{\sqrt{b+a\cos(c+dx)}}dx}{\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{a\cos(c+dx)+b}\int \frac{1}{\sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}$$

$$\Big\downarrow \text{3142}$$

$$\frac{\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}}dx}{\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}$$

$$\Big\downarrow \; 3042$$

$$\frac{\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}$$

$$\Big\downarrow \; 3140$$

$$\frac{2\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}$$

input
```
Int[1/(Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]),x]
```

output
```
(2*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)
])/(d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]])
```

3.858.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 4345
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]])   Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[
{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

3.858.4 Maple [A] (verified)

Time = 6.16 (sec) , antiderivative size = 135, normalized size of antiderivative = 2.01

method	result	size
default	$\frac{2\,\mathrm{EllipticF}\left(\sqrt{\frac{a-b}{a+b}}\,(-\cot(dx+c)+\csc(dx+c)),\sqrt{-\frac{a+b}{a-b}}\right)\sqrt{a+b\sec(dx+c)}\,\sqrt{\frac{b+a\cos(dx+c)}{(a+b)(\cos(dx+c)+1)}}\,\sqrt{\cos(dx+c)}}{d\sqrt{\frac{a-b}{a+b}}\,(b+a\cos(dx+c))\sqrt{\frac{1}{\cos(dx+c)+1}}}$	135

input
```
int(1/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/d/((a-b)/(a+b))^(1/2)*EllipticF(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x
+c)),(-(a+b)/(a-b))^(1/2))*(a+b*sec(d*x+c))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c)
)/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^(1/2)/(b+a*cos(d*x+c))/(1/(cos(d*x+c)+1
))^(1/2)
```

3.858.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 146, normalized size of antiderivative = 2.18

$$\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}\,dx$$
$$= \frac{-i\sqrt{2}\sqrt{a}\mathrm{weierstrassPInverse}\left(-\frac{4\,(3\,a^2-4\,b^2)}{3\,a^2},\frac{8\,(9\,a^2b-8\,b^3)}{27\,a^3},\frac{3\,a\cos(dx+c)+3i\,a\sin(dx+c)+2\,b}{3\,a}\right)+i\sqrt{2}\sqrt{a}\mathrm{weierstrassP}}{ad}$$

input
```
integrate(1/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
(-I*sqrt(2)*sqrt(a)*weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*
a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x + c) + 2*b)/a) +
 I*sqrt(2)*sqrt(a)*weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a
^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a))/(
a*d)
```

3.858.6 Sympy [F]

$$\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{a+b\sec(c+dx)}\sqrt{\cos(c+dx)}}\,dx$$

input
```
integrate(1/cos(d*x+c)**(1/2)/(a+b*sec(d*x+c))**(1/2),x)
```

output
```
Integral(1/(sqrt(a + b*sec(c + d*x))*sqrt(cos(c + d*x))), x)
```

3.858.7 Maxima [F]

$$\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{b\sec(dx+c)+a}\sqrt{\cos(dx+c)}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(b*sec(d*x + c) + a)*sqrt(cos(d*x + c))), x)
```

3.858.8 Giac [F]

$$\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{b\sec(dx+c)+a}\sqrt{\cos(dx+c)}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(b*sec(d*x + c) + a)*sqrt(cos(d*x + c))), x)
```

3.858.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}\,dx=\int \frac{1}{\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{a+\frac{b}{\cos(c+d\,x)}}}\,dx$$

input
```
int(1/(cos(c + d*x)^(1/2)*(a + b/cos(c + d*x))^(1/2)),x)
```

output
```
int(1/(cos(c + d*x)^(1/2)*(a + b/cos(c + d*x))^(1/2)), x)
```

3.859 $\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx$

3.859.1 Optimal result 6459
3.859.2 Mathematica [C] (warning: unable to verify) 6459
3.859.3 Rubi [A] (verified) 6460
3.859.4 Maple [C] (verified) 6462
3.859.5 Fricas [F(-1)] 6462
3.859.6 Sympy [F] 6463
3.859.7 Maxima [F] 6463
3.859.8 Giac [F] 6463
3.859.9 Mupad [F(-1)] 6464

3.859.1 Optimal result

Integrand size = 25, antiderivative size = 68

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx=\frac{2\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}$$

output
```
2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1
/2*c),2,2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)/d/cos(d*x+
c)^(1/2)/(a+b*sec(d*x+c))^(1/2)
```

3.859.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 6.90 (sec) , antiderivative size = 14986, normalized size of antiderivative = 220.38

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx=\text{Result too large to show}$$

input
```
Integrate[1/(Cos[c + d*x]^(3/2)*Sqrt[a + b*Sec[c + d*x]]),x]
```

output
```
Result too large to show
```

3.859.3 Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {3042, 4752, 3042, 4346, 3042, 3286, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4346}$$

$$\frac{\sqrt{a\cos(c+dx)+b}\int \frac{\sec(c+dx)}{\sqrt{b+a\cos(c+dx)}}dx}{\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{a\cos(c+dx)+b}\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}$$

$$\Big\downarrow \text{3286}$$

$$\frac{\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int \frac{\sec(c+dx)}{\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}}dx}{\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}$$

$$\Big\downarrow \; 3284$$

$$\frac{2\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}$$

input
```
Int[1/(Cos[c + d*x]^(3/2)*Sqrt[a + b*Sec[c + d*x]]),x]
```

output
```
(2*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a
+ b)])/(d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]])
```

3.859.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3286
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]]   Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
 d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

rule 4346
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Simp[d*Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x
]]/Sqrt[a + b*Csc[e + f*x]])   Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

3.859.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 7.57 (sec) , antiderivative size = 195, normalized size of antiderivative = 2.87

method	result
default	$-\frac{2\sqrt{\frac{b+a\cos(dx+c)}{(a+b)(\cos(dx+c)+1)}}\sqrt{\cos(dx+c)}\left(\operatorname{EllipticF}\left(\sqrt{\frac{a-b}{a+b}}\,(-\cot(dx+c)+\csc(dx+c)),\sqrt{-\frac{a+b}{a-b}}\right)-2\operatorname{EllipticPi}\left(\sqrt{\frac{a-b}{a+b}}\,(-\cot(dx+c)+\right.\right.}{d\sqrt{\frac{a-b}{a+b}}\,(b+a\cos(dx+c))\sqrt{\frac{1}{\cos(dx+c)+1}}}$

input
```
int(1/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/d/((a-b)/(a+b))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*c
os(d*x+c)^(1/2)*(EllipticF(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(-
(a+b)/(a-b))^(1/2))-2*EllipticPi(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+
c)),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2)))*(a+b*sec(d*x+c))^(1/2)/(b+a*cos(d*
x+c))/(1/(cos(d*x+c)+1))^(1/2)
```

3.859.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate(1/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.859.6 Sympy [F]

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{a+b\sec(c+dx)}\cos^{\frac{3}{2}}(c+dx)}\,dx$$

input
```
integrate(1/cos(d*x+c)**(3/2)/(a+b*sec(d*x+c))**(1/2),x)
```

output
```
Integral(1/(sqrt(a + b*sec(c + d*x))*cos(c + d*x)**(3/2)), x)
```

3.859.7 Maxima [F]

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{b\sec(dx+c)+a}\cos(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(b*sec(d*x + c) + a)*cos(d*x + c)^(3/2)), x)
```

3.859.8 Giac [F]

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{b\sec(dx+c)+a}\cos(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(b*sec(d*x + c) + a)*cos(d*x + c)^(3/2)), x)
```

3.859.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx=\int \frac{1}{\cos\left(c+d\,x\right)^{3/2}\,\sqrt{a+\frac{b}{\cos\left(c+d\,x\right)}}}\,dx$$

input
```
int(1/(cos(c + d*x)^(3/2)*(a + b/cos(c + d*x))^(1/2)),x)
```

output
```
int(1/(cos(c + d*x)^(3/2)*(a + b/cos(c + d*x))^(1/2)), x)
```

3.860 $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx$

3.860.1 Optimal result 6465
3.860.2 Mathematica [C] (warning: unable to verify) 6466
3.860.3 Rubi [A] (verified) 6466
3.860.4 Maple [C] (verified) 6474
3.860.5 Fricas [F(-1)] 6475
3.860.6 Sympy [F(-1)] 6475
3.860.7 Maxima [F] 6475
3.860.8 Giac [F] 6476
3.860.9 Mupad [F(-1)] 6476

3.860.1 Optimal result

Integrand size = 25, antiderivative size = 246

$$\begin{aligned}\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx &= \frac{\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} - \frac{a\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{bd\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} \\ &\quad - \frac{\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+dx)}}{bd\sqrt{\frac{b+a\cos(c+dx)}{a+b}}} + \frac{\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{bd\sqrt{\cos(c+dx)}}\end{aligned}$$

output
```
(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*
c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)/d/cos(d*x+c)^(1
/2)/(a+b*sec(d*x+c))^(1/2)-a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*
c)*EllipticPi(sin(1/2*d*x+1/2*c),2,2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+
c))/(a+b))^(1/2)/b/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)+sin(d*x+c)*(a
+b*sec(d*x+c))^(1/2)/b/d/cos(d*x+c)^(1/2)-(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos
(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*cos(
d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2)/b/d/((b+a*cos(d*x+c))/(a+b))^(1/2)
```

3.860.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 32.59 (sec) , antiderivative size = 30674, normalized size of antiderivative = 124.69

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx = \text{Result too large to show}$$

input
```
Integrate[1/(Cos[c + d*x]^(5/2)*Sqrt[a + b*Sec[c + d*x]]),x]
```

output
```
Result too large to show
```

3.860.3 Rubi [A] (verified)

Time = 2.47 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.14, number of steps used = 25, number of rules used = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {3042, 4752, 3042, 4347, 25, 3042, 4597, 3042, 4346, 3042, 3286, 3042, 3284, 4349, 3042, 4343, 3042, 3134, 3042, 3132, 4345, 3042, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \; 4752$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sec^{\frac{5}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \; 4347$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int -\frac{a\sec^2(c+dx)+a}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{2b}+\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{bd}\right)$$

$\downarrow$ 25

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{bd}-\frac{\int \frac{a\sec^2(c+dx)+a}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{2b}\right)$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{bd}-\frac{\int \frac{a\csc\left(c+dx+\frac{\pi}{2}\right)^2+a}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{2b}\right)$$

$\downarrow$ 4597

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{bd}-\frac{a\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx+a\int \frac{}{\sqrt{\sec(c+dx)}}}{2b}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{bd}-\frac{a\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{2b}\right.$$

$\downarrow$ 4346

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{bd}-\frac{a\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{}\right.$$

$\downarrow$ 3042

3.860. $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{bd}-\frac{a\int\frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{}\right.$$

$\downarrow$ 3286

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{bd}-\frac{a\int\frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{bd}-\frac{a\int\frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{}\right.$$

$\downarrow$ 3284

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{bd}-\frac{a\int\frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{}\right.$$

$\downarrow$ 4349

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{bd}-\frac{a\left(\frac{\int\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}dx}{a}-\frac{b\int\frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}}{a}\right.}{}\right.$$

$\downarrow$ 3042

3.860. $\int\frac{1}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{bd}-\frac{a\left(\frac{\int\frac{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}-\frac{b\int\frac{\sqrt{\csc\left(c+\right.}}{\sqrt{a+b\csc\left(\right.}}}{a}\right.}{}\right.$$

$\downarrow$ 4343

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{bd}-\frac{a\left(\frac{\sqrt{a+b\sec(c+dx)}\int\sqrt{b+a\cos(c+dx)}dx}{a\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}-\right.}{}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{bd}-\frac{a\left(\frac{\sqrt{a+b\sec(c+dx)}\int\sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{a\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}\right.}{}\right.$$

$\downarrow$ 3134

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{bd}-\frac{a\left(\frac{\sqrt{a+b\sec(c+dx)}\int\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}dx}{a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}\right.}{}\right.$$

3.860. $\int\frac{1}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}dx$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{bd}-\frac{a\left(\frac{\sqrt{a+b\sec(c+dx)}\int\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}{a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}\right.}{}\right.$$

$$\Big\downarrow \; 3132$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{bd}-\frac{a\left(\frac{2\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{b}{}\right.}{}\right.$$

$$\Big\downarrow \; 4345$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{bd}-\frac{a\left(\frac{2\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{b}{}\right.}{}\right.$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{bd}-\frac{a\left(\frac{2\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{b}{}\right.}{}\right.$$

$$\Big\downarrow \; 3142$$

3.860. $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{bd}-\frac{a\left(\frac{2\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\right.}{}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{bd}-\frac{a\left(\frac{2\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\right.}{}\right.$$

$\downarrow$ 3140

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{bd}-\frac{\frac{2a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(\right.}{d\sqrt{a+b\sec(c+dx)}}}{}\right.$$

input
```
Int[1/(Cos[c + d*x]^(5/2)*Sqrt[a + b*Sec[c + d*x]]),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(-1/2*((2*a*Sqrt[(b + a*Cos[c + d*x]
)/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(
d*Sqrt[a + b*Sec[c + d*x]]) + a*((-2*b*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*
EllipticF[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(a*d*Sqrt[a + b*
Sec[c + d*x]]) + (2*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c
 + d*x]])/(a*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]])))/b
+ (Sqrt[Sec[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(b*d))
```

3.860. $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx$

3.860.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3286
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]]   Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
 d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

3.860. $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx$

rule 4343
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]])   Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a
, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4345
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]])   Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[
{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4346
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Simp[d*Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x
]]/Sqrt[a + b*Csc[e + f*x]])   Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4347
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[-2*d^2*Cos[e + f*x]*(d*Csc[e + f*x])^(n - 2)*(S
qrt[a + b*Csc[e + f*x]]/(b*f*(2*n - 3))), x] + Simp[d^3/(b*(2*n - 3))   Int
[((d*Csc[e + f*x])^(n - 3)/Sqrt[a + b*Csc[e + f*x]])*Simp[2*a*(n - 3) + b*(
2*n - 5)*Csc[e + f*x] - 2*a*(n - 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 2] && IntegerQ[2*n]
```

rule 4349
```
Int[1/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)]), x_Symbol] :> Simp[1/a   Int[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Cs
c[e + f*x]], x], x] - Simp[b/(a*d)   Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Cs
c[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4597
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))/(Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Simp[C/d^
2   Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Csc[e + f*x]], x], x] + Simp[A
Int[1/(Sqrt[d*Csc[e + f*x]]*Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a,
b, d, e, f, A, C}, x] && NeQ[a^2 - b^2, 0]
```

3.860. $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx$

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

3.860.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 9.72 (sec) , antiderivative size = 1281, normalized size of antiderivative = 5.21

method	result	size
default	Expression too large to display	1281

input
```
int(1/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/d/((a-b)/(a+b))^(1/2)/b*(-2*EllipticPi(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+
csc(d*x+c)),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(
cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a*cos(d*x+c)^3-EllipticE(((a
-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(
b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a*cos(d*x+c
)^3+EllipticE(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^
(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^
(1/2)*b*cos(d*x+c)^3+2*EllipticF(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+
c)),(-(a+b)/(a-b))^(1/2))*(1/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c
))/(cos(d*x+c)+1))^(1/2)*a*cos(d*x+c)^3-4*EllipticPi(((a-b)/(a+b))^(1/2)*(
-cot(d*x+c)+csc(d*x+c)),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*(1/(a+b)*(b+a*c
os(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a*cos(d*x+c)^2-2
*EllipticE(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/
2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/
2)*a*cos(d*x+c)^2+2*EllipticE(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c))
,(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/
(cos(d*x+c)+1))^(1/2)*b*cos(d*x+c)^2+4*EllipticF(((a-b)/(a+b))^(1/2)*(-cot
(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*(1/(cos(d*x+c)+1))^(1/2)*(1/(a+b
)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a*cos(d*x+c)^2-2*EllipticPi(((a-b
)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(a+b)/(a-b),I/((a-b)/(a+b))^(1/...
```

3.860.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate(1/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.860.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate(1/cos(d*x+c)**(5/2)/(a+b*sec(d*x+c))**(1/2),x)
```

output
```
Timed out
```

3.860.7 Maxima [F]

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{b\sec(dx+c)+a}\cos(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(b*sec(d*x + c) + a)*cos(d*x + c)^(5/2)), x)
```

3.860.8 Giac [F]

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{b\sec(dx+c)+a}\cos(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(b*sec(d*x + c) + a)*cos(d*x + c)^(5/2)), x)
```

3.860.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{1}{\cos(c+d\,x)^{5/2}\,\sqrt{a+\frac{b}{\cos(c+d\,x)}}}\,dx$$

input
```
int(1/(cos(c + d*x)^(5/2)*(a + b/cos(c + d*x))^(1/2)),x)
```

output
```
int(1/(cos(c + d*x)^(5/2)*(a + b/cos(c + d*x))^(1/2)), x)
```

3.861 $\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx$

3.861.1 Optimal result 6477
3.861.2 Mathematica [C] (warning: unable to verify) 6478
3.861.3 Rubi [A] (verified) 6478
3.861.4 Maple [C] (verified) 6487
3.861.5 Fricas [F(-1)] 6487
3.861.6 Sympy [F(-1)] 6488
3.861.7 Maxima [F] 6488
3.861.8 Giac [F] 6488
3.861.9 Mupad [F(-1)] 6489

3.861.1 Optimal result

Integrand size = 25, antiderivative size = 312

$$\begin{aligned}
&\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx \\
&= -\frac{a\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{4bd\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} \\
&\quad + \frac{(3a^2+4b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{4b^2d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} \\
&\quad + \frac{3a\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+dx)}}{4b^2d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}} \\
&\quad + \frac{\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{2bd\cos^{\frac{3}{2}}(c+dx)} - \frac{3a\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{4b^2d\sqrt{\cos(c+dx)}}
\end{aligned}$$

output
```
-1/4*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d
*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)/b/d/cos(
d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)+1/4*(3*a^2+4*b^2)*(cos(1/2*d*x+1/2*c)^
2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2,2^(1/2)*(a/(a+
b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)/b^2/d/cos(d*x+c)^(1/2)/(a+b*sec(
d*x+c))^(1/2)+1/2*sin(d*x+c)*(a+b*sec(d*x+c))^(1/2)/b/d/cos(d*x+c)^(3/2)-3
/4*a*sin(d*x+c)*(a+b*sec(d*x+c))^(1/2)/b^2/d/cos(d*x+c)^(1/2)+3/4*a*(cos(1
/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(
1/2)*(a/(a+b))^(1/2))*cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2)/b^2/d/((b+a*
cos(d*x+c))/(a+b))^(1/2)
```

3.861.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 34.20 (sec) , antiderivative size = 51912, normalized size of antiderivative = 166.38

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx = \text{Result too large to show}$$

input
```
Integrate[1/(Cos[c + d*x]^(7/2)*Sqrt[a + b*Sec[c + d*x]]),x]
```

output
```
Result too large to show
```

3.861.3 Rubi [A] (verified)

Time = 3.36 (sec) , antiderivative size = 333, normalized size of antiderivative = 1.07, number of steps used = 27, number of rules used = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 1.080$, Rules used = {3042, 4752, 3042, 4347, 3042, 4590, 27, 3042, 4596, 3042, 4346, 3042, 3286, 3042, 3284, 4523, 3042, 4343, 3042, 3134, 3042, 3132, 4345, 3042, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{7/2}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \; 4752$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sec^{\frac{7}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \; 4347$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{\sqrt{\sec(c+dx)}\left(-3a\sec^2(c+dx)+2b\sec(c+dx)+a\right)}{\sqrt{a+b\sec(c+dx)}}dx}{4b}+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c}}{2bd}\right.$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(-3a\csc\left(c+dx+\frac{\pi}{2}\right)^2+2b\csc\left(c+dx+\frac{\pi}{2}\right)+a\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{4b}+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a}}{2bd}\right.$$

$$\Big\downarrow \; 4590$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\int \frac{3a^2+2b\sec(c+dx)a+\left(3a^2+4b^2\right)\sec^2(c+dx)}{2\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{b}-\frac{3a\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{bd}}{4b}+\frac{\sin(c+d}{}\right.$$

$$\Big\downarrow \; 27$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\int \frac{3a^2+2b\sec(c+dx)a+\left(3a^2+4b^2\right)\sec^2(c+dx)}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{2b}-\frac{3a\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{bd}}{4b}+\frac{\sin(c+d}{}\right.$$

$$\Big\downarrow \; 3042$$

3.861. $\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\int \frac{3a^2+2b\csc\left(c+dx+\frac{\pi}{2}\right)a+\left(3a^2+4b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{2b}-\frac{3a\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{bd}}{4b}+\sin\right.$$

$\downarrow$ 4596

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\left(3a^2+4b^2\right)\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx+\int \frac{3a^2+2b\sec(c+dx)a}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{2b}-\frac{3a\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+c}}{bd}}{4b}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\left(3a^2+4b^2\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\int \frac{3a^2+2b\csc\left(c+dx+\frac{\pi}{2}\right)a}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{2b}-\frac{3a\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{}}{bd}}{4b}\right.$$

$\downarrow$ 4346

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{\left(3a^2+4b^2\right)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int \frac{\sec(c+dx)}{\sqrt{b+a\cos(c+dx)}}dx}{\sqrt{a+b\sec(c+dx)}}+\int \frac{3a^2+2b\csc\left(c+dx+\frac{\pi}{2}\right)a}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{2b}-\frac{3a\sin(}{}}{4b}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{\left(3a^2+4b^2\right)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{\sqrt{a+b\sec(c+dx)}}+\int \frac{3a^2+2b\csc\left(c+dx+\frac{\pi}{2}\right)a}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+}}}{2b}}{4b}\right.$$

$\downarrow$ 3286

3.861. $\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{\left(3a^2+4b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{\sec(c+dx)}{\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}+\int\frac{3a^2+2b\csc\left(c+dx+\frac{\pi}{2}\right)a}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{2b}-3\right.}{4b}$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{\left(3a^2+4b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}+\int\frac{3a^2+2b\csc\left(c+dx+\frac{\pi}{2}\right.}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c\right.}}}{2b}}{4b}\right.$$

$\downarrow$ 3284

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\int\frac{3a^2+2b\csc\left(c+dx+\frac{\pi}{2}\right)a}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\left(3a^2+4b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\operatorname{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}}{2b}-}{4b}\right.$$

$\downarrow$ 4523

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{-ab\int\frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}dx+3a\int\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}dx+\frac{2\left(3a^2+4b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\operatorname{EllipticPi}\left(2,\frac{1}{2}(c+dx\right.}{d\sqrt{a+b\sec(c+dx)}}}{2b}}{4b}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{-ab\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+3a\int\frac{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\left(3a^2+4b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\operatorname{EllipticPi}\left(2\right.}{d\sqrt{a+b\sec(c+dx)}}}{2b}}{4b}\right.$$

$\downarrow$ 4343

3.861. $\int\frac{1}{\cos^{\frac{7}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{-ab\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{3a\sqrt{a+b\sec(c+dx)}\int\sqrt{b+a\cos(c+dx)}dx}{\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}+\frac{2\left(3a^2+4b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\mathrm{E}}{d\sqrt{a+b\sec(c+dx}}}{2b}}{4b}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{-ab\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{3a\sqrt{a+b\sec(c+dx)}\int\sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}+\frac{2\left(3a^2+4b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)}{a+b}}}{d\sqrt{a+b\sec(c}}}{2b}}{4b}\right.$$

$\downarrow$ 3134

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{-ab\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{3a\sqrt{a+b\sec(c+dx)}\int\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}dx}{\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{2\left(3a^2+4b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)}{a+b}}}{d\sqrt{a+b\sec(c}}}{2b}}{4b}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{-ab\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{3a\sqrt{a+b\sec(c+dx)}\int\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}dx}{\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{2\left(3a^2+4b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c-}{a-}}}{d\sqrt{a+b}}}{2b}}{4b}\right.$$

$\downarrow$ 3132

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{-ab\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\left(3a^2+4b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\mathrm{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}+\frac{6a\sqrt{a+b\sec(}}{d\sqrt{\sec(c+}}}{2b}}{4b}\right.$$

$\downarrow$ 4345

3.861. $\int\frac{1}{\cos^{\frac{7}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{-\frac{ab\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{1}{\sqrt{b+a\cos(c+dx)}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{2\left(3a^2+4b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(c+dx\right.}{d\sqrt{a+b\sec(c+dx)}}}{2b}\right.$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{-\frac{ab\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{1}{\sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{2\left(3a^2+4b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(\right.}{d\sqrt{a+b\sec(c+dx)}}}{2b}\right.$$

$$\Big\downarrow \; 3142$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{-\frac{ab\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{2\left(3a^2+4b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}\right.}{d\sqrt{a+b\sec(c+dx)}}}{2b}\right.$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{-\frac{ab\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{2\left(3a^2+4b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\text{EllipticPi}(}{d\sqrt{a+b\sec(c+dx)}}}{2b}\right.$$

$$\Big\downarrow \; 3140$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2\left(3a^2+4b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}-\frac{2ab\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c\right.}{d\sqrt{a+b\sec(c+dx)}}}{2b}\right.$$

3.861. $\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx$

input
```
Int[1/(Cos[c + d*x]^(7/2)*Sqrt[a + b*Sec[c + d*x]]),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((Sec[c + d*x]^(3/2)*Sqrt[a + b*Sec[
c + d*x]]*Sin[c + d*x])/(2*b*d) + (((-2*a*b*Sqrt[(b + a*Cos[c + d*x])/(a +
 b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(d*Sqrt[a +
 b*Sec[c + d*x]]) + (2*(3*a^2 + 4*b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*
EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(d*Sqrt[a +
b*Sec[c + d*x]]) + (6*a*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*S
ec[c + d*x]])/(d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]))/(
2*b) - (3*a*Sqrt[Sec[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(b*d
))/(4*b))
```

3.861.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

3.861. $\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx$

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3286
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]]   Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
 d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

rule 4343
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]])   Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a
, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4345
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]])   Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[
{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4346
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Simp[d*Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x
]]/Sqrt[a + b*Csc[e + f*x]])   Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

3.861. $\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx$

rule 4347
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[-2*d^2*Cos[e + f*x]*(d*Csc[e + f*x])^(n - 2)*(S
qrt[a + b*Csc[e + f*x]]/(b*f*(2*n - 3))), x] + Simp[d^3/(b*(2*n - 3))   Int
[((d*Csc[e + f*x])^(n - 3)/Sqrt[a + b*Csc[e + f*x]])*Simp[2*a*(n - 3) + b*(
2*n - 5)*Csc[e + f*x] - 2*a*(n - 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 2] && IntegerQ[2*n]
```

rule 4523
```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Simp[A/a   I
nt[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Simp[(A*b - a*B)
/(a*d)   Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ
[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

rule 4590
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(-C)*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
)*((d*Csc[e + f*x])^(n - 1)/(b*f*(m + n + 1))), x] + Simp[d/(b*(m + n + 1))
   Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (
A*b*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2
 - b^2, 0] && GtQ[n, 0]
```

rule 4596
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Simp[C/d^2   Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*C
sc[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[
a + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
 b^2, 0]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

3.861. $\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx$

3.861.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 12.11 (sec) , antiderivative size = 2319, normalized size of antiderivative = 7.43

method	result	size
default	Expression too large to display	2319

input
```
int(1/cos(d*x+c)^(7/2)/(a+b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4/d/((a-b)/(a+b))^(1/2)/b^2*(3*(1/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*c
os(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE(((a-b)/(a+b))^(1/2)*(-cot(d*x+c
)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*a*b*cos(d*x+c)^4-2*(1/(cos(d*x+c)+1))^
(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF(((a-b)/(a+
b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*a*b*cos(d*x+c)^4+
6*(1/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)
*EllipticE(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/
2))*a*b*cos(d*x+c)^3-4*(1/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/
(cos(d*x+c)+1))^(1/2)*EllipticF(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c
)),(-(a+b)/(a-b))^(1/2))*a*b*cos(d*x+c)^3+3*(1/(cos(d*x+c)+1))^(1/2)*(1/(a
+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE(((a-b)/(a+b))^(1/2)*(
-cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*a*b*cos(d*x+c)^2-2*(1/(cos(d
*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF(
((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*a*b*cos
(d*x+c)^2-2*((a-b)/(a+b))^(1/2)*b^2*sin(d*x+c)+3*((a-b)/(a+b))^(1/2)*a^2*c
os(d*x+c)^2*sin(d*x+c)-2*((a-b)/(a+b))^(1/2)*b^2*cos(d*x+c)*sin(d*x+c)-8*(
1/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*El
lipticPi(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(a+b)/(a-b),I/((a-b)
/(a+b))^(1/2))*b^2*cos(d*x+c)^2-2*((a-b)/(a+b))^(1/2)*a*b*cos(d*x+c)^2*sin
(d*x+c)+((a-b)/(a+b))^(1/2)*a*b*cos(d*x+c)*sin(d*x+c)-3*(1/(cos(d*x+c)+...
```

3.861.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate(1/cos(d*x+c)^(7/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.861.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate(1/cos(d*x+c)**(7/2)/(a+b*sec(d*x+c))**(1/2),x)
```

output
```
Timed out
```

3.861.7 Maxima [F]

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{b\sec(dx+c)+a}\cos(dx+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(7/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(b*sec(d*x + c) + a)*cos(d*x + c)^(7/2)), x)
```

3.861.8 Giac [F]

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt{b\sec(dx+c)+a}\cos(dx+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(7/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(b*sec(d*x + c) + a)*cos(d*x + c)^(7/2)), x)
```

3.861.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx=\int \frac{1}{\cos\left(c+d\,x\right)^{7/2}\,\sqrt{a+\frac{b}{\cos\left(c+d\,x\right)}}}\,dx$$

input
```
int(1/(cos(c + d*x)^(7/2)*(a + b/cos(c + d*x))^(1/2)),x)
```

output
```
int(1/(cos(c + d*x)^(7/2)*(a + b/cos(c + d*x))^(1/2)), x)
```

3.862 $\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$

3.862.1 Optimal result 6490
3.862.2 Mathematica [C] (verified) 6491
3.862.3 Rubi [A] (verified) 6492
3.862.4 Maple [B] (verified) 6499
3.862.5 Fricas [C] (verification not implemented) 6500
3.862.6 Sympy [F(-1)] 6501
3.862.7 Maxima [F] 6501
3.862.8 Giac [F] 6502
3.862.9 Mupad [F(-1)] 6502

3.862.1 Optimal result

Integrand size = 25, antiderivative size = 360

$$\begin{aligned}
&\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \\
&-\frac{8b(a^2+4b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{5a^4d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} \\
&+\frac{2(3a^4+8a^2b^2-16b^4)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+dx)}}{5a^4(a^2-b^2)d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}} \\
&+\frac{2b^2\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{a(a^2-b^2)d\sqrt{a+b\sec(c+dx)}} \\
&-\frac{2b(3a^2-8b^2)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{5a^3(a^2-b^2)d} \\
&+\frac{2(a^2-6b^2)\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{5a^2(a^2-b^2)d}
\end{aligned}$$

output
```
2*b^2*cos(d*x+c)^(3/2)*sin(d*x+c)/a/(a^2-b^2)/d/(a+b*sec(d*x+c))^(1/2)-8/5
*b*(a^2+4*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(s
in(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)/
a^4/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)+2/5*(a^2-6*b^2)*cos(d*x+c)^(
3/2)*sin(d*x+c)*(a+b*sec(d*x+c))^(1/2)/a^2/(a^2-b^2)/d-2/5*b*(3*a^2-8*b^2)
*sin(d*x+c)*cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2)/a^3/(a^2-b^2)/d+2/5*(3
*a^4+8*a^2*b^2-16*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ell
ipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*cos(d*x+c)^(1/2)*(a+b*s
ec(d*x+c))^(1/2)/a^4/(a^2-b^2)/d/((b+a*cos(d*x+c))/(a+b))^(1/2)
```

3.862.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 9.07 (sec) , antiderivative size = 419, normalized size of antiderivative = 1.16

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \frac{(b+a\cos(c+dx))\left(a\sec^{\frac{3}{2}}(c+dx)\left(10b^4\sin(c+dx)+6b(-a^2+b^2)\,(b+a\,\mathrm{co}\right.\right.}{}$$

input
```
Integrate[Cos[c + d*x]^(5/2)/(a + b*Sec[c + d*x])^(3/2),x]
```

output
```
((b + a*Cos[c + d*x])*(a*Sec[c + d*x]^(3/2)*(10*b^4*Sin[c + d*x] + 6*b*(-a
^2 + b^2)*(b + a*Cos[c + d*x])*Sin[c + d*x] + a*(a^2 - b^2)*(b + a*Cos[c +
 d*x])*Sin[2*(c + d*x)]) + 2*(a^2 + 4*b^2)*(Cos[(c + d*x)/2]^2*Sec[c + d*x
])^(3/2)*(I*(3*a^3 + 3*a^2*b - 4*a*b^2 - 4*b^3)*EllipticE[I*ArcSinh[Tan[(c
 + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x
])*Sec[(c + d*x)/2]^2)/(a + b)] - I*a*(3*a^2 - a*b - 4*b^2)*EllipticF[I*Ar
cSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a
*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] + (3*a^2 - 4*b^2)*(b + a*Cos[c
 + d*x])*(Sec[(c + d*x)/2]^2)^(3/2)*Tan[(c + d*x)/2])))/(5*a^4*(a^2 - b^2)
*d*Cos[c + d*x]^(3/2)*Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^(3/2))
```

3.862.3 Rubi [A] (verified)

Time = 3.02 (sec) , antiderivative size = 380, normalized size of antiderivative = 1.06, number of steps used = 24, number of rules used = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.960$, Rules used = {3042, 4752, 3042, 4334, 27, 3042, 4592, 27, 3042, 4592, 27, 3042, 4523, 3042, 4343, 3042, 3134, 3042, 3132, 4345, 3042, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{4334}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2b^2\sin(c+dx)}{ad\left(a^2-b^2\right)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}-\frac{2\int -\frac{a^2-b\sec(c+dx)a-6b^2+4b^2\sec^2(c+dx)}{2\sec^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}}{a\left(a^2-b^2\right)}\right.$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{a^2-b\sec(c+dx)a-6b^2+4b^2\sec^2(c+dx)}{\sec^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}dx}{a\left(a^2-b^2\right)}+\frac{2b^2\sin(c+dx)}{ad\left(a^2-b^2\right)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{a^2-b\csc\left(c+dx+\frac{\pi}{2}\right)a-6b^2+4b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a\left(a^2-b^2\right)}+\frac{2b^2\sin(c+dx)}{ad\left(a^2-b^2\right)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c}}\right.$$

$$\Big\downarrow \; 4592$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2\left(a^2-6b^2\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{2\int\frac{-2b\left(a^2-6b^2\right)\sec^2(c+dx)-a\left(3a^2+2b^2\right)\sec(c+dx)+3b\left(3a^2-8b^2\right)}{2\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}d}{5a}}{a\left(a^2-b^2\right)}\right.$$

$$\Big\downarrow \; 27$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2\left(a^2-6b^2\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{\int\frac{-2b\left(a^2-6b^2\right)\sec^2(c+dx)-a\left(3a^2+2b^2\right)\sec(c+dx)+3b\left(3a^2-8b^2\right)}{\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}dx}{5a}}{a\left(a^2-b^2\right)}\right.$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2\left(a^2-6b^2\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{\int\frac{-2b\left(a^2-6b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2-a\left(3a^2+2b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)+3b\left(3a^2-\right.}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}}{5a}}{a\left(a^2-b^2\right)}\right.$$

$$\Big\downarrow \; 4592$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2\left(a^2-6b^2\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{\frac{2b\left(3a^2-8b^2\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{ad\sqrt{\sec(c+dx)}}-\frac{2\int\frac{3\left(3a^4+8b^2a^2-b\left(a^2+4b^2\right.\right.}{2\sqrt{\sec(c+dx)}\sqrt{a-}}}{3a}}{5a}}{a\left(a^2-b^2\right)}\right.$$

$$\Big\downarrow \; 27$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2\left(a^2-6b^2\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{\frac{2b\left(3a^2-8b^2\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{ad\sqrt{\sec(c+dx)}}-\frac{\int\frac{3a^4+8b^2a^2-b\left(a^2+4b^2\right)\sec}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec}}}{a}}{5a}}{a\left(a^2-b^2\right)}\right.$$

$$\Big\downarrow \; 3042$$

3.862. $\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2\left(a^2-6b^2\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{\frac{2b\left(3a^2-8b^2\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{ad\sqrt{\sec(c+dx)}}-\frac{\int\frac{3a^4+8b^2a^2-b\left(a^2+4b^2\right)\mathrm{cs}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b}}}{a}}{5a}}{a\left(a^2-b^2\right)}\right.$$

$\downarrow$ 4523

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2\left(a^2-6b^2\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{\frac{2b\left(3a^2-8b^2\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{ad\sqrt{\sec(c+dx)}}-\frac{\left(3a^4+8a^2b^2-16b^4\right)\int\frac{\sqrt{a+b}}{\sqrt{\mathrm{se}}}}{a}}{5a}}{a\left(a^2-b^2\right)}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2\left(a^2-6b^2\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{\frac{2b\left(3a^2-8b^2\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{ad\sqrt{\sec(c+dx)}}-\frac{\left(3a^4+8a^2b^2-16b^4\right)\int\frac{\sqrt{a+b}}{\sqrt{\mathrm{cs}}}}{a}}{5a}}{a\left(a^2-b^2\right)}\right.$$

$\downarrow$ 4343

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2\left(a^2-6b^2\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{\frac{2b\left(3a^2-8b^2\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{ad\sqrt{\sec(c+dx)}}-\frac{\left(3a^4+8a^2b^2-16b^4\right)\sqrt{a+b\,\mathrm{se}}}{a\sqrt{\sec(c+dx)}\sqrt{}}}{}}{a\left(a^2-b^2\right)}\right.$$

$\downarrow$ 3042

3.862. $\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2(a^2-6b^2)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{\frac{2b(3a^2-8b^2)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{ad\sqrt{\sec(c+dx)}}-\frac{(3a^4+8a^2b^2-16b^4)\sqrt{a+b\,\mathrm{se}}}{a\sqrt{\sec(c+dx}}}{}}{a(a^2-b^2)}\right.$$

$\downarrow$ 3134

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2(a^2-6b^2)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{\frac{2b(3a^2-8b^2)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{ad\sqrt{\sec(c+dx)}}-\frac{(3a^4+8a^2b^2-16b^4)\sqrt{a+b\,\mathrm{se}}}{a\sqrt{\sec(c+dx}}}{}}{a(a^2-b^2)}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2(a^2-6b^2)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{\frac{2b(3a^2-8b^2)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{ad\sqrt{\sec(c+dx)}}-\frac{(3a^4+8a^2b^2-16b^4)\sqrt{a+b\,\mathrm{se}}}{a\sqrt{\sec(c+}}}{}}{a(a^2-b^2)}\right.$$

$\downarrow$ 3132

3.862. $\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2(a^2-6b^2)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{\frac{2b(3a^2-8b^2)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{ad\sqrt{\sec(c+dx)}}-\frac{2(3a^4+8a^2b^2-16b^4)\sqrt{a+b\,s}}{ad\sqrt{\sec(c+dx)}\sqrt{}}}{}}{a(a^2-b^2)}\right.$$

$\downarrow$ 4345

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2(a^2-6b^2)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{\frac{2b(3a^2-8b^2)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{ad\sqrt{\sec(c+dx)}}-\frac{2(3a^4+8a^2b^2-16b^4)\sqrt{a+b\,s}}{ad\sqrt{\sec(c+dx)}\sqrt{}}}{}}{a(a^2-b^2)}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2(a^2-6b^2)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{\frac{2b(3a^2-8b^2)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{ad\sqrt{\sec(c+dx)}}-\frac{2(3a^4+8a^2b^2-16b^4)\sqrt{a+b\,s}}{ad\sqrt{\sec(c+dx)}\sqrt{}}}{}}{a(a^2-b^2)}\right.$$

$\downarrow$ 3142

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2(a^2-6b^2)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{\frac{2b(3a^2-8b^2)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{ad\sqrt{\sec(c+dx)}}-\frac{2(3a^4+8a^2b^2-16b^4)\sqrt{a+b\,s}}{ad\sqrt{\sec(c+dx)}\sqrt{}}}{}}{a(a^2-b^2}\right.$$

3.862. $\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2(a^2-6b^2)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{\frac{2b(3a^2-8b^2)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{ad\sqrt{\sec(c+dx)}}-\frac{\frac{2(3a^4+8a^2b^2-16b^4)\sqrt{a+b\,s}}{ad\sqrt{\sec(c+dx)}\sqrt{}}}{}}{}}{a\,(a^2-b}\right.$$

$$\Big\downarrow \; 3140$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2b^2\sin(c+dx)}{ad\,(a^2-b^2)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}+\frac{\frac{2(a^2-6b^2)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{2b(}{}}{}\right.$$

input
```
Int[Cos[c + d*x]^(5/2)/(a + b*Sec[c + d*x])^(3/2),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*b^2*Sin[c + d*x])/(a*(a^2 - b^2)
*d*Sec[c + d*x]^(3/2)*Sqrt[a + b*Sec[c + d*x]]) + ((2*(a^2 - 6*b^2)*Sqrt[a
 + b*Sec[c + d*x]]*Sin[c + d*x])/(5*a*d*Sec[c + d*x]^(3/2)) - (-(((-8*b*(a
^4 + 3*a^2*b^2 - 4*b^4)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c +
d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(a*d*Sqrt[a + b*Sec[c + d*x]])
+ (2*(3*a^4 + 8*a^2*b^2 - 16*b^4)*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sq
rt[a + b*Sec[c + d*x]])/(a*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c
 + d*x]]))/a) + (2*b*(3*a^2 - 8*b^2)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x]
)/(a*d*Sqrt[Sec[c + d*x]]))/(5*a))/(a*(a^2 - b^2)))
```

3.862. $\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$

3.862.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 4334
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*
((d*Csc[e + f*x])^n/(a*f*(m + 1)*(a^2 - b^2))), x] + Simp[1/(a*(m + 1)*(a^2
 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a^2*(m + 1)
 - b^2*(m + n + 1) - a*b*(m + 1)*Csc[e + f*x] + b^2*(m + n + 2)*Csc[e + f*x
]^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0] && LtQ[m,
 -1] && IntegersQ[2*m, 2*n]
```

3.862. $\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$

rule 4343
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]])   Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a
, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4345
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]])   Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[
{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4523
```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Simp[A/a   I
nt[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Simp[(A*b - a*B)
/(a*d)   Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ
[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

rule 4592
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Simp[1/(a*d*n)   Int[(a + b*Csc[e + f*x])^m
*(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*
Csc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

3.862.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2420 vs. $2(386) = 772$.

Time = 10.72 (sec) , antiderivative size = 2421, normalized size of antiderivative = 6.72

method	result	size
default	Expression too large to display	2421

input
```
int(cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/5/d/((a-b)/(a+b))^(1/2)/(a+b)/a^4*(-16*(1/(cos(d*x+c)+1))^(1/2)*(1/(a+b)
*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE(((a-b)/(a+b))^(1/2)*(-co
t(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*b^4-3*(1/(cos(d*x+c)+1))^(1/2)*
(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF(((a-b)/(a+b))^(1
/2)*(-cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*a^4+3*(1/(a+b)*(b+a*cos
(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE(((a-b)/(
a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*a^4+16*((a-b)/(
a+b))^(1/2)*b^4*sin(d*x+c)-4*(1/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*
x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc
(d*x+c)),(-(a+b)/(a-b))^(1/2))*a^3*b*cos(d*x+c)^2-12*(1/(cos(d*x+c)+1))^(1
/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF(((a-b)/(a+b)
)^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*a^2*b^2*cos(d*x+c)^
2-16*(1/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1
/2)*EllipticF(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^
(1/2))*a*b^3*cos(d*x+c)^2+16*(1/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*
x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc
(d*x+c)),(-(a+b)/(a-b))^(1/2))*a^2*b^2*cos(d*x+c)-32*(1/(cos(d*x+c)+1))^(1
/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF(((a-b)/(a+b)
)^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*a*b^3*cos(d*x+c)+8*
(1/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2...
```

3.862.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.16 (sec) , antiderivative size = 702, normalized size of antiderivative = 1.95

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx =$$
$$-\frac{6\left(3\,a^4b^2-8\,a^2b^4-\left(a^6-a^4b^2\right)\cos\left(dx+c\right)^2+2\left(a^5b-a^3b^3\right)\cos\left(dx+c\right)\right)\sqrt{\frac{a\cos(dx+c)+b}{\cos(dx+c)}}\sqrt{\cos\left(dx+c\right)}\sin}{}$$

input
```
integrate(cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
-1/15*(6*(3*a^4*b^2 - 8*a^2*b^4 - (a^6 - a^4*b^2)*cos(d*x + c)^2 + 2*(a^5*
b - a^3*b^3)*cos(d*x + c))*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*sqrt(co
s(d*x + c))*sin(d*x + c) + (sqrt(2)*(-9*I*a^5*b - 28*I*a^3*b^3 + 32*I*a*b^
5)*cos(d*x + c) + sqrt(2)*(-9*I*a^4*b^2 - 28*I*a^2*b^4 + 32*I*b^6))*sqrt(a
)*weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3
, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x + c) + 2*b)/a) + (sqrt(2)*(9*I*a^5
*b + 28*I*a^3*b^3 - 32*I*a*b^5)*cos(d*x + c) + sqrt(2)*(9*I*a^4*b^2 + 28*I
*a^2*b^4 - 32*I*b^6))*sqrt(a)*weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2
, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c) +
 2*b)/a) - 3*(sqrt(2)*(3*I*a^6 + 8*I*a^4*b^2 - 16*I*a^2*b^4)*cos(d*x + c)
+ sqrt(2)*(3*I*a^5*b + 8*I*a^3*b^3 - 16*I*a*b^5))*sqrt(a)*weierstrassZeta(
-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(
-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c
) + 3*I*a*sin(d*x + c) + 2*b)/a)) - 3*(sqrt(2)*(-3*I*a^6 - 8*I*a^4*b^2 + 1
6*I*a^2*b^4)*cos(d*x + c) + sqrt(2)*(-3*I*a^5*b - 8*I*a^3*b^3 + 16*I*a*b^5
))*sqrt(a)*weierstrassZeta(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3
)/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3
)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a)))/((a^8 - a^6*
b^2)*d*cos(d*x + c) + (a^7*b - a^5*b^3)*d)
```

3.862.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(5/2)/(a+b*sec(d*x+c))**(3/2),x)
```

output
```
Timed out
```

3.862.7 Maxima [F]

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\cos\left(dx+c\right)^{\frac{5}{2}}}{\left(b\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate(cos(d*x + c)^(5/2)/(b*sec(d*x + c) + a)^(3/2), x)
```

3.862.8 Giac [F]

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\cos{(dx+c)}^{\frac{5}{2}}}{(b\sec{(dx+c)}+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate(cos(d*x + c)^(5/2)/(b*sec(d*x + c) + a)^(3/2), x)
```

3.862.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\cos{(c+d\,x)}^{5/2}}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

input
```
int(cos(c + d*x)^(5/2)/(a + b/cos(c + d*x))^(3/2),x)
```

output
```
int(cos(c + d*x)^(5/2)/(a + b/cos(c + d*x))^(3/2), x)
```

3.863 $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$

3.863.1 Optimal result 6503
3.863.2 Mathematica [C] (verified) 6504
3.863.3 Rubi [A] (verified) 6504
3.863.4 Maple [B] (verified) 6511
3.863.5 Fricas [C] (verification not implemented) 6511
3.863.6 Sympy [F] 6512
3.863.7 Maxima [F] 6513
3.863.8 Giac [F] 6513
3.863.9 Mupad [F(-1)] 6513

3.863.1 Optimal result

Integrand size = 25, antiderivative size = 289

$$\begin{aligned}\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx &= \frac{2(a^2+8b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{3a^3d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}\\ &- \frac{2b(5a^2-8b^2)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+dx)}}{3a^3(a^2-b^2)d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}}\\ &+ \frac{2b^2\sqrt{\cos(c+dx)}\sin(c+dx)}{a(a^2-b^2)d\sqrt{a+b\sec(c+dx)}}\\ &+ \frac{2(a^2-4b^2)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{3a^2(a^2-b^2)d}\end{aligned}$$

output
```
2*b^2*sin(d*x+c)*cos(d*x+c)^(1/2)/a/(a^2-b^2)/d/(a+b*sec(d*x+c))^(1/2)+2/3
*(a^2+8*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin
(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)/a^
3/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)+2/3*(a^2-4*b^2)*sin(d*x+c)*cos
(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2)/a^2/(a^2-b^2)/d-2/3*b*(5*a^2-8*b^2)*(
cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c
),2^(1/2)*(a/(a+b))^(1/2))*cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2)/a^3/(a^
2-b^2)/d/((b+a*cos(d*x+c))/(a+b))^(1/2)
```

3.863.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.63 (sec) , antiderivative size = 382, normalized size of antiderivative = 1.32

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx =$$
$$-\frac{2(b+a\cos(c+dx))\left(a(3b^3-(a^2-b^2)(b+a\cos(c+dx)))\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)-\left(\cos^2\left(\frac{1}{2}(c+dx)\right)\right.\right.}{}$$

input
```
Integrate[Cos[c + d*x]^(3/2)/(a + b*Sec[c + d*x])^(3/2),x]
```

output
```
(-2*(b + a*Cos[c + d*x])*(a*(3*b^3 - (a^2 - b^2)*(b + a*Cos[c + d*x]))*Sec
[c + d*x]^(3/2)*Sin[c + d*x] - (Cos[(c + d*x)/2]^2*Sec[c + d*x])^(3/2)*(I*
b*(-5*a^3 - 5*a^2*b + 8*a*b^2 + 8*b^3)*EllipticE[I*ArcSinh[Tan[(c + d*x)/2
]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c
 + d*x)/2]^2)/(a + b)] - I*a*(a^3 - 5*a^2*b + 2*a*b^2 + 8*b^3)*EllipticF[I
*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b
+ a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] + b*(-5*a^2 + 8*b^2)*(b + a
*Cos[c + d*x])*(Sec[(c + d*x)/2]^2)^(3/2)*Tan[(c + d*x)/2])))/(3*a^3*(a^2
- b^2)*d*Cos[c + d*x]^(3/2)*Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^(3/2))
```

3.863.3 Rubi [A] (verified)

Time = 2.26 (sec) , antiderivative size = 316, normalized size of antiderivative = 1.09, number of steps used = 21, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.840$, Rules used = {3042, 4752, 3042, 4334, 27, 3042, 4592, 27, 3042, 4523, 3042, 4343, 3042, 3134, 3042, 3132, 4345, 3042, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \; 4752$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}dx$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \; 4334$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2b^2\sin(c+dx)}{ad\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}-\frac{2\int -\frac{a^2-b\sec(c+dx)a-4b^2+2b^2\sec^2(c+dx}{2\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}}{a\left(a^2-b^2\right)}\right.$$

$$\Big\downarrow \; 27$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{a^2-b\sec(c+dx)a-4b^2+2b^2\sec^2(c+dx)}{\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}dx}{a\left(a^2-b^2\right)}+\frac{2b^2\sin(c+dx)}{ad\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}\right.$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{a^2-b\csc\left(c+dx+\frac{\pi}{2}\right)a-4b^2+2b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a\left(a^2-b^2\right)}+\frac{2b^2\sin(c+dx)}{ad\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c}}\right.$$

$$\Big\downarrow \; 4592$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2\left(a^2-4b^2\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}-\frac{2\int \frac{b\left(5a^2-8b^2\right)-a\left(a^2+2b^2\right)\sec(c+dx)}{2\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{3a}}{a\left(a^2-b^2\right)}+\frac{2}{ad\left(a^2-b^2\right)\sqrt{\text{se}}}\right.$$

$$\Big\downarrow \; 27$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2\left(a^2-4b^2\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}-\frac{\int \frac{b\left(5a^2-8b^2\right)-a\left(a^2+2b^2\right)\sec(c+dx)}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{3a}}{a\left(a^2-b^2\right)}+\frac{2b}{ad\left(a^2-b^2\right)\sqrt{\text{sec}}}\right.$$

$$\Big\downarrow \; 3042$$

3.863. $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2(a^2-4b^2)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}-\frac{\int\frac{b(5a^2-8b^2)-a(a^2+2b^2)\csc(c+dx+\frac{\pi}{2})}{\sqrt{\csc(c+dx+\frac{\pi}{2})}\sqrt{a+b\csc(c+dx+\frac{\pi}{2})}}dx}{3a}}{a(a^2-b^2)}+\frac{}{ad(a^2-b^2)\sqrt{}}\right.$$

$\downarrow$ 4523

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2(a^2-4b^2)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{b(5a^2-8b^2)\int\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}dx}{a}-\frac{(a^4+7a^2b^2-8b^4)\int\frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\sec(c+d}}}{a}}{3a}}{a(a^2-b^2)}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2(a^2-4b^2)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{b(5a^2-8b^2)\int\frac{\sqrt{a+b\csc(c+dx+\frac{\pi}{2})}}{\sqrt{\csc(c+dx+\frac{\pi}{2})}}dx}{a}-\frac{(a^4+7a^2b^2-8b^4)\int\frac{\sqrt{\csc(c}}{\sqrt{a+b\csc}}}{a}}{3a}}{a(a^2-b^2)}\right.$$

$\downarrow$ 4343

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2(a^2-4b^2)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{b(5a^2-8b^2)\sqrt{a+b\sec(c+dx)}\int\sqrt{b+a\cos(c+dx)}dx}{a\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}-\frac{(a^4+7a^2b^2-8b^4)}{}}{3a}}{a(a^2-b^2)}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2(a^2-4b^2)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{b(5a^2-8b^2)\sqrt{a+b\sec(c+dx)}\int\sqrt{b+a\sin(c+dx+\frac{\pi}{2})}dx}{a\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}-\frac{(a^4+7a^2b^2-8}{}}{3a}}{a(a^2-b^2)}\right.$$

3.863. $\int\frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}dx$

$$\Big\downarrow \; 3134$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2\left(a^2-4b^2\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{b\left(5a^2-8b^2\right)\sqrt{a+b\sec(c+dx)}\int\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}\,dx}{a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{\left(a^4+7a^2b^2-8\right.}{}}{3a}}{a\left(a^2-b^2\right)}\right.$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2\left(a^2-4b^2\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{b\left(5a^2-8b^2\right)\sqrt{a+b\sec(c+dx)}\int\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}\,dx}{a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{\left(a^4+7a^2b\right.}{}}{3a}}{a\left(a^2-b^2\right)}\right.$$

$$\Big\downarrow \; 3132$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2\left(a^2-4b^2\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{2b\left(5a^2-8b^2\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{\left(a^4+7a^2b^2-8b^4\right)\int}{}}{3a}}{a\left(a^2-b^2\right)}\right.$$

$$\Big\downarrow \; 4345$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2\left(a^2-4b^2\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{2b\left(5a^2-8b^2\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{\left(a^4+7a^2b^2-8b^4\right)\sqrt{}}{}}{3a}}{a\left(a^2-b^2\right)}\right.$$

$$\Big\downarrow \; 3042$$

3.863. $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2(a^2-4b^2)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{2b(5a^2-8b^2)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{(a^4+7a^2b^2-8b^4)\sqrt{\cdots}}{\cdots}}{3a}}{a(a^2-b^2)}\right.$$

$\downarrow$ 3142

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2(a^2-4b^2)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{2b(5a^2-8b^2)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{(a^4+7a^2b^2-8b^4)\sqrt{\cdots}}{\cdots}}{3a}}{a(a^2-b^2)}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2(a^2-4b^2)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}-\frac{\frac{2b(5a^2-8b^2)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{(a^4+7a^2b^2-8b^4)\sqrt{\cdots}}{\cdots}}{3a}}{a(a^2-b^2)}\right.$$

$\downarrow$ 3140

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2b^2\sin(c+dx)}{ad(a^2-b^2)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{\frac{2(a^2-4b^2)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}-\frac{2b}{\cdots}}{\cdots}\right.$$

input
```
Int[Cos[c + d*x]^(3/2)/(a + b*Sec[c + d*x])^(3/2),x]
```

3.863. $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*b^2*Sin[c + d*x])/(a*(a^2 - b^2)
*d*Sqrt[Sec[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + (-1/3*((-2*(a^4 + 7*a^2*
b^2 - 8*b^4)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*
a)/(a + b)]*Sqrt[Sec[c + d*x]])/(a*d*Sqrt[a + b*Sec[c + d*x]]) + (2*b*(5*a
^2 - 8*b^2)*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]]
)/(a*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]))/a + (2*(a^2
 - 4*b^2)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(3*a*d*Sqrt[Sec[c + d*x]]
))/(a*(a^2 - b^2)))
```

3.863.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

3.863. $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$

rule 4334
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*
((d*Csc[e + f*x])^n/(a*f*(m + 1)*(a^2 - b^2))), x] + Simp[1/(a*(m + 1)*(a^2
 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a^2*(m + 1)
 - b^2*(m + n + 1) - a*b*(m + 1)*Csc[e + f*x] + b^2*(m + n + 2)*Csc[e + f*x
]^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0] && LtQ[m,
 -1] && IntegersQ[2*m, 2*n]
```

rule 4343
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]])   Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a
, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4345
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]])   Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[
{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4523
```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Simp[A/a   I
nt[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Simp[(A*b - a*B)
/(a*d)   Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ
[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

rule 4592
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Simp[1/(a*d*n)   Int[(a + b*Csc[e + f*x])^m
*(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*
Csc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

3.863. $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$

3.863.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1710 vs. $2(321) = 642$.

Time = 10.27 (sec) , antiderivative size = 1711, normalized size of antiderivative = 5.92

method	result	size
default	Expression too large to display	1711

input
```
int(cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/3/d/((a-b)/(a+b))^(1/2)/(a+b)/a^3*(-EllipticF(((a-b)/(a+b))^(1/2)*(cot(d
*x+c)-csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x
+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a^3*cos(d*x+c)^2-6*EllipticF(((a-b)
/(a+b))^(1/2)*(cot(d*x+c)-csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*
cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a^2*b*cos(d*x+c
)^2-8*EllipticF(((a-b)/(a+b))^(1/2)*(cot(d*x+c)-csc(d*x+c)),(-(a+b)/(a-b))
^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))
^(1/2)*a*b^2*cos(d*x+c)^2+5*EllipticE(((a-b)/(a+b))^(1/2)*(cot(d*x+c)-csc(
d*x+c)),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1
/2)*(1/(cos(d*x+c)+1))^(1/2)*a^2*b*cos(d*x+c)^2-8*EllipticE(((a-b)/(a+b))^
(1/2)*(cot(d*x+c)-csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+
c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*b^3*cos(d*x+c)^2+((a-b)
/(a+b))^(1/2)*a^3*cos(d*x+c)^2*sin(d*x+c)+((a-b)/(a+b))^(1/2)*a^2*b*cos(d*
x+c)^2*sin(d*x+c)-2*(1/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(co
s(d*x+c)+1))^(1/2)*EllipticF(((a-b)/(a+b))^(1/2)*(cot(d*x+c)-csc(d*x+c)),(
-(a+b)/(a-b))^(1/2))*a^3*cos(d*x+c)-12*(1/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(
b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF(((a-b)/(a+b))^(1/2)*(cot(d
*x+c)-csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*a^2*b*cos(d*x+c)-16*(1/(cos(d*x+c)
+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF(((a-b
)/(a+b))^(1/2)*(cot(d*x+c)-csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*a*b^2*cos(...
```

3.863.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.15 (sec) , antiderivative size = 644, normalized size of antiderivative = 2.23

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \frac{6\,(a^4b-4\,a^2b^3+(a^5-a^3b^2)\cos(dx+c))\sqrt{\frac{a\cos(dx+c)+b}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx}{}$$

input
```
integrate(cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
1/9*(6*(a^4*b - 4*a^2*b^3 + (a^5 - a^3*b^2)*cos(d*x + c))*sqrt((a*cos(d*x
+ c) + b)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - (sqrt(2)*(3*I*a^
5 + 16*I*a^3*b^2 - 16*I*a*b^4)*cos(d*x + c) + sqrt(2)*(3*I*a^4*b + 16*I*a^
2*b^3 - 16*I*b^5))*sqrt(a)*weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8
/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x + c) + 2*
b)/a) - (sqrt(2)*(-3*I*a^5 - 16*I*a^3*b^2 + 16*I*a*b^4)*cos(d*x + c) + sqr
t(2)*(-3*I*a^4*b - 16*I*a^2*b^3 + 16*I*b^5))*sqrt(a)*weierstrassPInverse(-
4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c)
 - 3*I*a*sin(d*x + c) + 2*b)/a) + 3*(sqrt(2)*(-5*I*a^4*b + 8*I*a^2*b^3)*co
s(d*x + c) + sqrt(2)*(-5*I*a^3*b^2 + 8*I*a*b^4))*sqrt(a)*weierstrassZeta(-
4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-
4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c)
 + 3*I*a*sin(d*x + c) + 2*b)/a)) + 3*(sqrt(2)*(5*I*a^4*b - 8*I*a^2*b^3)*co
s(d*x + c) + sqrt(2)*(5*I*a^3*b^2 - 8*I*a*b^4))*sqrt(a)*weierstrassZeta(-4
/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4
/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c)
- 3*I*a*sin(d*x + c) + 2*b)/a)))/((a^7 - a^5*b^2)*d*cos(d*x + c) + (a^6*b
- a^4*b^3)*d)
```

3.863.6 Sympy [F]

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)**(3/2)/(a+b*sec(d*x+c))**(3/2),x)
```

output
```
Integral(cos(c + d*x)**(3/2)/(a + b*sec(c + d*x))**(3/2), x)
```

3.863.7 Maxima [F]

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\cos{(dx+c)}^{\frac{3}{2}}}{(b\sec{(dx+c)}+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate(cos(d*x + c)^(3/2)/(b*sec(d*x + c) + a)^(3/2), x)
```

3.863.8 Giac [F]

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\cos{(dx+c)}^{\frac{3}{2}}}{(b\sec{(dx+c)}+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate(cos(d*x + c)^(3/2)/(b*sec(d*x + c) + a)^(3/2), x)
```

3.863.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\cos{(c+d\,x)}^{3/2}}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

input
```
int(cos(c + d*x)^(3/2)/(a + b/cos(c + d*x))^(3/2),x)
```

output
```
int(cos(c + d*x)^(3/2)/(a + b/cos(c + d*x))^(3/2), x)
```

3.864 $\int \frac{\sqrt{\cos(c+dx)}}{(a+b\sec(c+dx))^{3/2}}\,dx$

3.864.1 Optimal result 6514
3.864.2 Mathematica [C] (verified) 6515
3.864.3 Rubi [A] (verified) 6515
3.864.4 Maple [B] (verified) 6520
3.864.5 Fricas [C] (verification not implemented) 6521
3.864.6 Sympy [F] 6522
3.864.7 Maxima [F] 6522
3.864.8 Giac [F] 6523
3.864.9 Mupad [F(-1)] 6523

3.864.1 Optimal result

Integrand size = 25, antiderivative size = 214

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+b\sec(c+dx))^{3/2}}\,dx = -\frac{4b\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{a^2d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} + \frac{2(a^2-2b^2)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+dx)}}{a^2(a^2-b^2)d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}} + \frac{2b^2\sin(c+dx)}{a(a^2-b^2)d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}$$

output
```
2*b^2*sin(d*x+c)/a/(a^2-b^2)/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)-4*b
*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2
*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)/a^2/d/cos(d*x+
c)^(1/2)/(a+b*sec(d*x+c))^(1/2)+2*(a^2-2*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)
/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*
cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2)/a^2/(a^2-b^2)/d/((b+a*cos(d*x+c))/
(a+b))^(1/2)
```

3.864.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 7.20 (sec) , antiderivative size = 330, normalized size of antiderivative = 1.54

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+b\sec(c+dx))^{3/2}}\,dx = \frac{2(b+a\cos(c+dx))\left(ab^2\sin(c+dx)+\frac{(\cos^2(\frac{1}{2}(c+dx))\sec(c+dx))^{3/2}\left(i(a^3+a^2b-2ab^2-\right.}{}\right.}{}$$

input
```
Integrate[Sqrt[Cos[c + d*x]]/(a + b*Sec[c + d*x])^(3/2),x]
```

output
```
(2*(b + a*Cos[c + d*x])*(a*b^2*Sin[c + d*x] + ((Cos[(c + d*x)/2]^2*Sec[c +
 d*x])^(3/2)*(I*(a^3 + a^2*b - 2*a*b^2 - 2*b^3)*EllipticE[I*ArcSinh[Tan[(c
 + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x
])*Sec[(c + d*x)/2]^2)/(a + b)] - I*a*(a^2 - a*b - 2*b^2)*EllipticF[I*ArcS
inh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*C
os[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] + (a^2 - 2*b^2)*(b + a*Cos[c + d
*x])*(Sec[(c + d*x)/2]^2)^(3/2)*Tan[(c + d*x)/2]))/Sec[c + d*x]^(3/2)))/(a
^2*(a^2 - b^2)*d*Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^(3/2))
```

3.864.3 Rubi [A] (verified)

Time = 1.77 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.16, number of steps used = 18, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.720$, Rules used = {3042, 4752, 3042, 4334, 27, 3042, 4523, 3042, 4343, 3042, 3134, 3042, 3132, 4345, 3042, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+b\sec(c+dx))^{3/2}}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \; 4752$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}}dx$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$\Big\downarrow$ 4334

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{ad\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}-\frac{2\int -\frac{a^2-b\sec(c+dx)a-2b^2}{2\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{a\left(a^2-b^2\right)}\right)$$

$\Big\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{a^2-b\sec(c+dx)a-2b^2}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{a\left(a^2-b^2\right)}+\frac{2b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{ad\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}\right)$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{a^2-b\csc\left(c+dx+\frac{\pi}{2}\right)a-2b^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a\left(a^2-b^2\right)}+\frac{2b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{ad\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}\right)$$

$\Big\downarrow$ 4523

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\left(a^2-2b^2\right)\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}dx}{a}-\frac{2b\left(a^2-b^2\right)\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}dx}{a}}{a\left(a^2-b^2\right)}+\frac{2b^2\sin(c+dx)\sqrt{\sec(c+dx}}{ad\left(a^2-b^2\right)\sqrt{a+b\sec(c+d}}\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\left(a^2-2b^2\right)\int \frac{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}-\frac{2b\left(a^2-b^2\right)\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}}{a\left(a^2-b^2\right)}+\frac{2b^2\sin(c+dx)\sqrt{\sec(}}{ad\left(a^2-b^2\right)\sqrt{a+b\sec}}\right.$$

$\Big\downarrow$ 4343

3.864. $\int \frac{\sqrt{\cos(c+dx)}}{(a+b\sec(c+dx))^{3/2}}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{(a^2-2b^2)\sqrt{a+b\sec(c+dx)}\int\sqrt{b+a\cos(c+dx)}dx}{a\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}-\frac{2b(a^2-b^2)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}}{a(a^2-b^2)}+\frac{2b^2\sin(c+}{ad(a^2-b^2)}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{(a^2-2b^2)\sqrt{a+b\sec(c+dx)}\int\sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{a\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}-\frac{2b(a^2-b^2)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}}{a(a^2-b^2)}+\frac{2b^2\sin(}{ad(a^2-}\right.$$

$\downarrow$ 3134

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{(a^2-2b^2)\sqrt{a+b\sec(c+dx)}\int\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}dx}{a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{2b(a^2-b^2)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}}{a(a^2-b^2)}+\frac{2b^2\sin(}{ad(a^2-b}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{(a^2-2b^2)\sqrt{a+b\sec(c+dx)}\int\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}dx}{a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{2b(a^2-b^2)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}}{a(a^2-b^2)}+\frac{2b^2\mathrm{si}}{ad(a^2}\right.$$

$\downarrow$ 3132

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2(a^2-2b^2)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{2b(a^2-b^2)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}}{a(a^2-b^2)}+\frac{2b^2\sin(c+}{ad(a^2-b^2)\sqrt{}}\right.$$

3.864. $\int\frac{\sqrt{\cos(c+dx)}}{(a+b\sec(c+dx))^{3/2}}\,dx$

$$\Big\downarrow \; 4345$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2(a^2-2b^2)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{2b(a^2-b^2)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{1}{\sqrt{b+a\cos(c+dx}}}{a\sqrt{a+b\sec(c+dx)}}}{a(a^2-b^2)}\right.$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2(a^2-2b^2)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{2b(a^2-b^2)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{1}{\sqrt{b+a\sin\left(c+dx\right.}}}{a\sqrt{a+b\sec(c+dx)}}}{a(a^2-b^2)}\right.$$

$$\Big\downarrow \; 3142$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2(a^2-2b^2)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{2b(a^2-b^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\cos(c+}{a+b}}}}{a\sqrt{a+b\sec(c+dx)}}}{a(a^2-b^2)}\right.$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2(a^2-2b^2)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{2b(a^2-b^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+\right.}{a}}}}{a\sqrt{a+b\sec(c+dx)}}}{a(a^2-b^2)}\right.$$

$$\Big\downarrow \; 3140$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{ad(a^2-b^2)\sqrt{a+b\sec(c+dx)}}+\frac{\frac{2(a^2-2b^2)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{4b(a^2-b^2)\sqrt{}}{}}{a(a^2-b^2}\right.$$

input
```
Int[Sqrt[Cos[c + d*x]]/(a + b*Sec[c + d*x])^(3/2),x]
```

3.864. $\int \frac{\sqrt{\cos(c+dx)}}{(a+b\sec(c+dx))^{3/2}}\,dx$

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(((-4*b*(a^2 - b^2)*Sqrt[(b + a*Cos[
c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]
])/(a*d*Sqrt[a + b*Sec[c + d*x]]) + (2*(a^2 - 2*b^2)*EllipticE[(c + d*x)/2
, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(a*d*Sqrt[(b + a*Cos[c + d*x])/
(a + b)]*Sqrt[Sec[c + d*x]]))/(a*(a^2 - b^2)) + (2*b^2*Sqrt[Sec[c + d*x]]*
Sin[c + d*x])/(a*(a^2 - b^2)*d*Sqrt[a + b*Sec[c + d*x]]))
```

3.864.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 4334
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*
((d*Csc[e + f*x])^n/(a*f*(m + 1)*(a^2 - b^2))), x] + Simp[1/(a*(m + 1)*(a^2
 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a^2*(m + 1)
 - b^2*(m + n + 1) - a*b*(m + 1)*Csc[e + f*x] + b^2*(m + n + 2)*Csc[e + f*x
]^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0] && LtQ[m,
 -1] && IntegersQ[2*m, 2*n]
```

rule 4343
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]])   Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a
, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4345
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]])   Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[
{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4523
```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Simp[A/a   I
nt[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Simp[(A*b - a*B)
/(a*d)   Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ
[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

3.864.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 924 vs. $2(256) = 512$.

Time = 8.86 (sec) , antiderivative size = 925, normalized size of antiderivative = 4.32

method	result
default	$\frac{2\sqrt{-\frac{(1-\cos(dx+c))^2\csc(dx+c)^2-1}{(1-\cos(dx+c))^2\csc(dx+c)^2+1}}\sqrt{\frac{a(1-\cos(dx+c))^2\csc(dx+c)^2-b(1-\cos(dx+c))^2\csc(dx+c)^2-a-b}{(1-\cos(dx+c))^2\csc(dx+c)^2-1}}\left(\sqrt{\frac{a-b}{a+b}}\,a^2(1-\cos(dx+c))^3\csc(dx+\right.}{}$

input
```
int(cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/d/a^2/(a+b)/((a-b)/(a+b))^(1/2)*(-((1-cos(d*x+c))^2*csc(d*x+c)^2-1)/((1-
cos(d*x+c))^2*csc(d*x+c)^2+1))^(1/2)*((a*(1-cos(d*x+c))^2*csc(d*x+c)^2-b*(
1-cos(d*x+c))^2*csc(d*x+c)^2-a-b)/((1-cos(d*x+c))^2*csc(d*x+c)^2-1))^(1/2)
*(((a-b)/(a+b))^(1/2)*a^2*(1-cos(d*x+c))^3*csc(d*x+c)^3-2*((a-b)/(a+b))^(1
/2)*b^2*(1-cos(d*x+c))^3*csc(d*x+c)^3+(-(a*(1-cos(d*x+c))^2*csc(d*x+c)^2-b
*(1-cos(d*x+c))^2*csc(d*x+c)^2-a-b)/(a+b))^(1/2)*((1-cos(d*x+c))^2*csc(d*x
+c)^2+1)^(1/2)*EllipticF(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(-(a
+b)/(a-b))^(1/2))*a^2+2*(-(a*(1-cos(d*x+c))^2*csc(d*x+c)^2-b*(1-cos(d*x+c)
)^2*csc(d*x+c)^2-a-b)/(a+b))^(1/2)*((1-cos(d*x+c))^2*csc(d*x+c)^2+1)^(1/2)
*EllipticF(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/
2))*a*b-(-(a*(1-cos(d*x+c))^2*csc(d*x+c)^2-b*(1-cos(d*x+c))^2*csc(d*x+c)^2
-a-b)/(a+b))^(1/2)*((1-cos(d*x+c))^2*csc(d*x+c)^2+1)^(1/2)*EllipticE(((a-b
)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*a^2+2*(-(a*(
1-cos(d*x+c))^2*csc(d*x+c)^2-b*(1-cos(d*x+c))^2*csc(d*x+c)^2-a-b)/(a+b))^(
1/2)*((1-cos(d*x+c))^2*csc(d*x+c)^2+1)^(1/2)*EllipticE(((a-b)/(a+b))^(1/2)
*(-cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*b^2-((a-b)/(a+b))^(1/2)*a^
2*(-cot(d*x+c)+csc(d*x+c))-2*((a-b)/(a+b))^(1/2)*a*b*(-cot(d*x+c)+csc(d*x+
c))-2*((a-b)/(a+b))^(1/2)*b^2*(-cot(d*x+c)+csc(d*x+c)))/(a*(1-cos(d*x+c))^
2*csc(d*x+c)^2-b*(1-cos(d*x+c))^2*csc(d*x+c)^2-a-b)
```

3.864.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 585, normalized size of antiderivative = 2.73

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+b\sec(c+dx))^{3/2}}\,dx = \frac{6\,a^2b^2\sqrt{\frac{a\cos(dx+c)+b}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c) - \left(\sqrt{2}(-5i\,a^3b+4i\,ab^3)\cos(d\right.}{}$$

input
```
integrate(cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
1/3*(6*a^2*b^2*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*sqrt(cos(d*x + c))*
sin(d*x + c) - (sqrt(2)*(-5*I*a^3*b + 4*I*a*b^3)*cos(d*x + c) + sqrt(2)*(-
5*I*a^2*b^2 + 4*I*b^4))*sqrt(a)*weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a
^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x + c)
 + 2*b)/a) - (sqrt(2)*(5*I*a^3*b - 4*I*a*b^3)*cos(d*x + c) + sqrt(2)*(5*I*
a^2*b^2 - 4*I*b^4))*sqrt(a)*weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2,
8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c) + 2
*b)/a) + 3*(sqrt(2)*(I*a^4 - 2*I*a^2*b^2)*cos(d*x + c) + sqrt(2)*(I*a^3*b
- 2*I*a*b^3))*sqrt(a)*weierstrassZeta(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^
2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^
2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x + c) + 2*b)/a)) +
3*(sqrt(2)*(-I*a^4 + 2*I*a^2*b^2)*cos(d*x + c) + sqrt(2)*(-I*a^3*b + 2*I*a
*b^3))*sqrt(a)*weierstrassZeta(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8
*b^3)/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8
*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a)))/((a^6 -
a^4*b^2)*d*cos(d*x + c) + (a^5*b - a^3*b^3)*d)
```

3.864.6 Sympy [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\sqrt{\cos{(c+dx)}}}{(a+b\sec{(c+dx)})^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)**(1/2)/(a+b*sec(d*x+c))**(3/2),x)
```

output
```
Integral(sqrt(cos(c + d*x))/(a + b*sec(c + d*x))**(3/2), x)
```

3.864.7 Maxima [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\sqrt{\cos{(dx+c)}}}{(b\sec{(dx+c)}+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(cos(d*x + c))/(b*sec(d*x + c) + a)^(3/2), x)
```

3.864.8 Giac [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\sqrt{\cos(dx+c)}}{(b\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate(sqrt(cos(d*x + c))/(b*sec(d*x + c) + a)^(3/2), x)
```

3.864.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\sqrt{\cos(c+d\,x)}}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

input
```
int(cos(c + d*x)^(1/2)/(a + b/cos(c + d*x))^(3/2),x)
```

output
```
int(cos(c + d*x)^(1/2)/(a + b/cos(c + d*x))^(3/2), x)
```

3.865 $\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^{3/2}}\,dx$

3.865.1 Optimal result 6524
3.865.2 Mathematica [C] (verified) 6525
3.865.3 Rubi [A] (verified) 6525
3.865.4 Maple [B] (verified) 6530
3.865.5 Fricas [C] (verification not implemented) 6531
3.865.6 Sympy [F] 6532
3.865.7 Maxima [F] 6532
3.865.8 Giac [F] 6533
3.865.9 Mupad [F(-1)] 6533

3.865.1 Optimal result

Integrand size = 25, antiderivative size = 200

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^{3/2}}\,dx = \frac{2\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{ad\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} + \frac{2b\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+dx)}}{a\left(a^2-b^2\right)d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}} - \frac{2b\sin(c+dx)}{\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}$$

output
```
-2*b*sin(d*x+c)/(a^2-b^2)/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)+2*(cos
(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2
^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)/a/d/cos(d*x+c)^(1/2
)/(a+b*sec(d*x+c))^(1/2)+2*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*
c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*cos(d*x+c)^(1/2)*
(a+b*sec(d*x+c))^(1/2)/a/(a^2-b^2)/d/((b+a*cos(d*x+c))/(a+b))^(1/2)
```

3.865.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.37 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.22

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^{3/2}}\,dx = \frac{2\sqrt{\cos(c+dx)}(b+a\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\left(ib(a+b)\sqrt{\frac{b}{(a+}}\right.}{}$$

input
```
Integrate[1/(Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x])^(3/2)),x]
```

output
```
(2*Sqrt[Cos[c + d*x]]*(b + a*Cos[c + d*x])*Sec[c + d*x]^(3/2)*(I*b*(a + b)
*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[I*ArcSi
nh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 + Sec[c + d*x]] - I*a*(a +
b)*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[I*Arc
Sinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 + Sec[c + d*x]] + b*(-a +
 b)*Sqrt[Sec[c + d*x]]*Tan[(c + d*x)/2]))/(a*(a^2 - b^2)*d*(a + b*Sec[c +
d*x])^(3/2))
```

3.865.3 Rubi [A] (verified)

Time = 1.61 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.16, number of steps used = 18, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.720$, Rules used = {3042, 4752, 3042, 4330, 27, 3042, 4523, 3042, 4343, 3042, 3134, 3042, 3132, 4345, 3042, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sqrt{\sec(c+dx)}}{(a+b\sec(c+dx))^{3/2}}dx$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \; 4330$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{2\int -\frac{b+a\sec(c+dx)}{2\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{a^2-b^2}-\frac{2b\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}\right)$$

$$\Big\downarrow \; 27$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{b+a\sec(c+dx)}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{a^2-b^2}-\frac{2b\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}\right)$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{b+a\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2-b^2}-\frac{2b\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}\right)$$

$$\Big\downarrow \; 4523$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\left(a^2-b^2\right)\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}dx}{a}+\frac{b\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}dx}{a}}{a^2-b^2}-\frac{2b\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}\right)$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\left(a^2-b^2\right)\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}+\frac{b\int \frac{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}}{a^2-b^2}-\frac{2b\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}\right)$$

$$\Big\downarrow \; 4343$$

3.865. $\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^{3/2}}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{(a^2-b^2)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}+\frac{b\sqrt{a+b\sec(c+dx)}\int\sqrt{b+a\cos(c+dx)}dx}{a\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}}{a^2-b^2}-\frac{2b\sin(c+dx)\sqrt{\sec}}{d\left(a^2-b^2\right)\sqrt{a+b\,\mathrm{s}}}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{(a^2-b^2)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}+\frac{b\sqrt{a+b\sec(c+dx)}\int\sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{a\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}}{a^2-b^2}-\frac{2b\sin(c+dx)\sqrt{}}{d\left(a^2-b^2\right)\sqrt{a+}}\right.$$

$\downarrow$ 3134

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{(a^2-b^2)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}+\frac{b\sqrt{a+b\sec(c+dx)}\int\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}dx}{a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{a^2-b^2}-\frac{2b\sin(c+dx)\sqrt{}}{d\left(a^2-b^2\right)\sqrt{a+b}}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{(a^2-b^2)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}+\frac{b\sqrt{a+b\sec(c+dx)}\int\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}dx}{a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{a^2-b^2}-\frac{2b\sin(c+dx)}{d\left(a^2-b^2\right)\sqrt{a}}\right.$$

$\downarrow$ 3132

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{(a^2-b^2)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}+\frac{2b\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{a^2-b^2}-\frac{2b\sin(c+dx)\sqrt{\sec(}}{d\left(a^2-b^2\right)\sqrt{a+b\sec}}\right.$$

3.865. $\int\frac{1}{\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^{3/2}}\,dx$

$$\Big\downarrow \text{4345}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{(a^2-b^2)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{1}{\sqrt{b+a\cos(c+dx)}}dx}{a\sqrt{a+b\sec(c+dx)}}+\frac{2b\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{a^2-b^2}-\frac{2}{d}\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{(a^2-b^2)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{1}{\sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{a\sqrt{a+b\sec(c+dx)}}+\frac{2b\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{a^2-b^2}-\right.$$

$$\Big\downarrow \text{3142}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{(a^2-b^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}}dx}{a\sqrt{a+b\sec(c+dx)}}+\frac{2b\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{a^2-b^2}-\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{(a^2-b^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{a\sqrt{a+b\sec(c+dx)}}+\frac{2b\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{a^2-b^2}\right.$$

$$\Big\downarrow \text{3140}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2(a^2-b^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{ad\sqrt{a+b\sec(c+dx)}}+\frac{2b\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{a^2-b^2}\right.$$

input
```
Int[1/(Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x])^(3/2)),x]
```

3.865. $\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^{3/2}}\,dx$

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(((2*(a^2 - b^2)*Sqrt[(b + a*Cos[c +
 d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/
(a*d*Sqrt[a + b*Sec[c + d*x]]) + (2*b*EllipticE[(c + d*x)/2, (2*a)/(a + b)
]*Sqrt[a + b*Sec[c + d*x]])/(a*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[S
ec[c + d*x]]))/(a^2 - b^2) - (2*b*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/((a^2 -
 b^2)*d*Sqrt[a + b*Sec[c + d*x]]))
```

3.865.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 4330
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b)*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m +
1)*((d*Csc[e + f*x])^(n - 1)/(f*(m + 1)*(a^2 - b^2))), x] + Simp[1/((m + 1)
*(a^2 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1)*S
imp[b*d*(n - 1) + a*d*(m + 1)*Csc[e + f*x] - b*d*(m + n + 1)*Csc[e + f*x]^2
, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1
] && LtQ[0, n, 1] && IntegersQ[2*m, 2*n]
```

rule 4343
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]])   Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a
, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4345
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]])   Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[
{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4523
```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Simp[A/a   I
nt[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Simp[(A*b - a*B)
/(a*d)   Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ
[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

3.865.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 592 vs. $2(242) = 484$.

Time = 7.52 (sec) , antiderivative size = 593, normalized size of antiderivative = 2.96

method	result
default	$-\frac{2\left((1-\cos(dx+c))^2\csc(dx+c)^2-1\right)\sqrt{\frac{a(1-\cos(dx+c))^2\csc(dx+c)^2-b(1-\cos(dx+c))^2\csc(dx+c)^2-a-b}{(1-\cos(dx+c))^2\csc(dx+c)^2-1}}\left(\sqrt{\frac{a-b}{a+b}}\,b(1-\cos(dx+c))^3\csc(d\ldots\right.}{\ldots}$

input
```
int(1/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/d/((a-b)/(a+b))^(1/2)/(a+b)/a*((1-cos(d*x+c))^2*csc(d*x+c)^2-1)*((a*(1-
cos(d*x+c))^2*csc(d*x+c)^2-b*(1-cos(d*x+c))^2*csc(d*x+c)^2-a-b)/((1-cos(d*
x+c))^2*csc(d*x+c)^2-1))^(1/2)*(((a-b)/(a+b))^(1/2)*b*(1-cos(d*x+c))^3*csc
(d*x+c)^3-(-(a*(1-cos(d*x+c))^2*csc(d*x+c)^2-b*(1-cos(d*x+c))^2*csc(d*x+c)
^2-a-b)/(a+b))^(1/2)*((1-cos(d*x+c))^2*csc(d*x+c)^2+1)^(1/2)*EllipticF(((a
-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*a-(-(a*(1-
cos(d*x+c))^2*csc(d*x+c)^2-b*(1-cos(d*x+c))^2*csc(d*x+c)^2-a-b)/(a+b))^(1/
2)*((1-cos(d*x+c))^2*csc(d*x+c)^2+1)^(1/2)*EllipticE(((a-b)/(a+b))^(1/2)*(
-cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*b+((a-b)/(a+b))^(1/2)*b*(-co
t(d*x+c)+csc(d*x+c)))/(-((1-cos(d*x+c))^2*csc(d*x+c)^2-1)/((1-cos(d*x+c))^
2*csc(d*x+c)^2+1))^(1/2)/((1-cos(d*x+c))^2*csc(d*x+c)^2+1)/(a*(1-cos(d*x+c
))^2*csc(d*x+c)^2-b*(1-cos(d*x+c))^2*csc(d*x+c)^2-a-b)
```

3.865.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 541, normalized size of antiderivative = 2.70

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^{3/2}}\,dx =$$
$$-\frac{6\,a^2b\sqrt{\frac{a\cos(dx+c)+b}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)+\left(\sqrt{2}(3i\,a^3-2i\,ab^2)\cos(dx+c)+\sqrt{2}(3i\,a^2b-2i\,b^3)\right)\sqrt{a}\mathrm{v}}{\ldots}$$

input
```
integrate(1/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
-1/3*(6*a^2*b*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*sqrt(cos(d*x + c))*s
in(d*x + c) + (sqrt(2)*(3*I*a^3 - 2*I*a*b^2)*cos(d*x + c) + sqrt(2)*(3*I*a
^2*b - 2*I*b^3))*sqrt(a)*weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/2
7*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x + c) + 2*b)
/a) + (sqrt(2)*(-3*I*a^3 + 2*I*a*b^2)*cos(d*x + c) + sqrt(2)*(-3*I*a^2*b +
 2*I*b^3))*sqrt(a)*weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a
^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a) -
3*(I*sqrt(2)*a^2*b*cos(d*x + c) + I*sqrt(2)*a*b^2)*sqrt(a)*weierstrassZeta
(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse
(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x +
c) + 3*I*a*sin(d*x + c) + 2*b)/a)) - 3*(-I*sqrt(2)*a^2*b*cos(d*x + c) - I*
sqrt(2)*a*b^2)*sqrt(a)*weierstrassZeta(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a
^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a
^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a)))/
((a^5 - a^3*b^2)*d*cos(d*x + c) + (a^4*b - a^2*b^3)*d)
```

3.865.6 Sympy [F]

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{1}{(a+b\sec(c+dx))^{\frac{3}{2}}\sqrt{\cos(c+dx)}}\,dx$$

input
```
integrate(1/cos(d*x+c)**(1/2)/(a+b*sec(d*x+c))**(3/2),x)
```

output
```
Integral(1/((a + b*sec(c + d*x))**(3/2)*sqrt(cos(c + d*x))), x)
```

3.865.7 Maxima [F]

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{1}{(b\sec(dx+c)+a)^{\frac{3}{2}}\sqrt{\cos(dx+c)}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*sec(d*x + c) + a)^(3/2)*sqrt(cos(d*x + c))), x)
```

3.865.8 Giac [F]

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{1}{(b\sec(dx+c)+a)^{\frac{3}{2}}\sqrt{\cos(dx+c)}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((b*sec(d*x + c) + a)^(3/2)*sqrt(cos(d*x + c))), x)
```

3.865.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{1}{\sqrt{\cos(c+d\,x)}\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

input
```
int(1/(cos(c + d*x)^(1/2)*(a + b/cos(c + d*x))^(3/2)),x)
```

output
```
int(1/(cos(c + d*x)^(1/2)*(a + b/cos(c + d*x))^(3/2)), x)
```

3.866 $\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx$

3.866.1 Optimal result 6534
3.866.2 Mathematica [C] (verified) 6534
3.866.3 Rubi [A] (verified) 6535
3.866.4 Maple [B] (verified) 6538
3.866.5 Fricas [C] (verification not implemented) 6539
3.866.6 Sympy [F(-1)] 6539
3.866.7 Maxima [F] 6540
3.866.8 Giac [F] 6540
3.866.9 Mupad [F(-1)] 6540

3.866.1 Optimal result

Integrand size = 25, antiderivative size = 126

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx = -\frac{2\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+dx)}}{(a^2-b^2)\,d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}} + \frac{2a\sin(c+dx)}{(a^2-b^2)\,d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}$$

output
```
2*a*sin(d*x+c)/(a^2-b^2)/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)-2*(cos(
1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^
(1/2)*(a/(a+b))^(1/2))*cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2)/(a^2-b^2)/d
/((b+a*cos(d*x+c))/(a+b))^(1/2)
```

3.866.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.08 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.90

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx = \frac{2\sqrt{\cos(c+dx)}(b+a\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\left(-i(a+b)\sqrt{\frac{b}{(a+}}\right.}{}$$

input
```
Integrate[1/(Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^(3/2)),x]
```

output
```
(2*Sqrt[Cos[c + d*x]]*(b + a*Cos[c + d*x])*Sec[c + d*x]^(3/2)*((-I)*(a + b
)*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[I*ArcS
inh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 + Sec[c + d*x]] + I*(a + b
)*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[I*ArcS
inh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 + Sec[c + d*x]] + (a - b)*
Sqrt[Sec[c + d*x]]*Tan[(c + d*x)/2]))/((a^2 - b^2)*d*(a + b*Sec[c + d*x])^
(3/2))
```

3.866.3 Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.17, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {3042, 4752, 3042, 4331, 27, 3042, 4343, 3042, 3134, 3042, 3132}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{4331}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\int -\frac{\sqrt{a+b\sec(c+dx)}}{2\sqrt{\sec(c+dx)}}dx}{a^2-b^2}+\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}\right)$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}-\frac{\int\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}dx}{a^2-b^2}\right)$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}-\frac{\int\frac{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2-b^2}\right)$$

$\Big\downarrow$ 4343

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}-\frac{\sqrt{a+b\sec(c+dx)}\int\sqrt{b+a\cos(c+dx)}dx}{\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}\right)$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}-\frac{\sqrt{a+b\sec(c+dx)}\int\sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}\right)$$

$\Big\downarrow$ 3134

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}-\frac{\sqrt{a+b\sec(c+dx)}\int\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}dx}{\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}\right)$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}-\frac{\sqrt{a+b\sec(c+dx)}\int\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}dx}{\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}\right)$$

$\Big\downarrow$ 3132

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}-\frac{2\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}\right)$$

input
```
Int[1/(Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^(3/2)),x]
```

3.866. $\int\frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}dx$

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((-2*EllipticE[(c + d*x)/2, (2*a)/(a
 + b)]*Sqrt[a + b*Sec[c + d*x]])/((a^2 - b^2)*d*Sqrt[(b + a*Cos[c + d*x])/
(a + b)]*Sqrt[Sec[c + d*x]]) + (2*a*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/((a^2
 - b^2)*d*Sqrt[a + b*Sec[c + d*x]]))
```

3.866.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 4331
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[a*d^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
)*((d*Csc[e + f*x])^(n - 2)/(f*(m + 1)*(a^2 - b^2))), x] - Simp[d^2/((m + 1
)*(a^2 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*
(a*(n - 2) + b*(m + 1)*Csc[e + f*x] - a*(m + n)*Csc[e + f*x]^2), x], x] /;
FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && LtQ[1, n, 2
] && IntegersQ[2*m, 2*n]
```

rule 4343
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]])   Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a
, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

3.866. $\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx$

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

3.866.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 586 vs. $2(145) = 290$.

Time = 4.34 (sec) , antiderivative size = 587, normalized size of antiderivative = 4.66

method	result
default	$-\frac{2\left((1-\cos(dx+c))^2\csc(dx+c)^2-1\right)^2\sqrt{\frac{a(1-\cos(dx+c))^2\csc(dx+c)^2-b(1-\cos(dx+c))^2\csc(dx+c)^2-a-b}{(1-\cos(dx+c))^2\csc(dx+c)^2-1}}\left(\sqrt{\frac{a-b}{a+b}}\,(1-\cos(dx+c))^3\csc(d\right.}{}$

input
```
int(1/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/d/(a+b)/((a-b)/(a+b))^(1/2)*((1-cos(d*x+c))^2*csc(d*x+c)^2-1)^2*((a*(1-
cos(d*x+c))^2*csc(d*x+c)^2-b*(1-cos(d*x+c))^2*csc(d*x+c)^2-a-b)/((1-cos(d*
x+c))^2*csc(d*x+c)^2-1))^(1/2)*(((a-b)/(a+b))^(1/2)*(1-cos(d*x+c))^3*csc(d
*x+c)^3+(-(a*(1-cos(d*x+c))^2*csc(d*x+c)^2-b*(1-cos(d*x+c))^2*csc(d*x+c)^2
-a-b)/(a+b))^(1/2)*((1-cos(d*x+c))^2*csc(d*x+c)^2+1)^(1/2)*EllipticF(((a-b
)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))-(-(a*(1-cos(
d*x+c))^2*csc(d*x+c)^2-b*(1-cos(d*x+c))^2*csc(d*x+c)^2-a-b)/(a+b))^(1/2)*(
(1-cos(d*x+c))^2*csc(d*x+c)^2+1)^(1/2)*EllipticE(((a-b)/(a+b))^(1/2)*(-cot
(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))+((a-b)/(a+b))^(1/2)*(-cot(d*x+c)
+csc(d*x+c)))/(-((1-cos(d*x+c))^2*csc(d*x+c)^2-1)/((1-cos(d*x+c))^2*csc(d*
x+c)^2+1))^(3/2)/((1-cos(d*x+c))^2*csc(d*x+c)^2+1)^2/(a*(1-cos(d*x+c))^2*c
sc(d*x+c)^2-b*(1-cos(d*x+c))^2*csc(d*x+c)^2-a-b)
```

3.866.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 500, normalized size of antiderivative = 3.97

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx = \frac{6\,a^2\sqrt{\frac{a\cos(dx+c)+b}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c) - (-i\sqrt{2}ab\cos(dx}{}$$

input
```
integrate(1/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
1/3*(6*a^2*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(
d*x + c) - (-I*sqrt(2)*a*b*cos(d*x + c) - I*sqrt(2)*b^2)*sqrt(a)*weierstra
ssPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*
cos(d*x + c) + 3*I*a*sin(d*x + c) + 2*b)/a) - (I*sqrt(2)*a*b*cos(d*x + c)
+ I*sqrt(2)*b^2)*sqrt(a)*weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/2
7*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)
/a) + 3*(-I*sqrt(2)*a^2*cos(d*x + c) - I*sqrt(2)*a*b)*sqrt(a)*weierstrassZ
eta(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInve
rse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x
 + c) + 3*I*a*sin(d*x + c) + 2*b)/a)) + 3*(I*sqrt(2)*a^2*cos(d*x + c) + I*
sqrt(2)*a*b)*sqrt(a)*weierstrassZeta(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2
*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2
*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a)))/((
a^4 - a^2*b^2)*d*cos(d*x + c) + (a^3*b - a*b^3)*d)
```

3.866.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/cos(d*x+c)**(3/2)/(a+b*sec(d*x+c))**(3/2),x)
```

output
```
Timed out
```

3.866.7 Maxima [F]

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{1}{(b\sec(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*sec(d*x + c) + a)^(3/2)*cos(d*x + c)^(3/2)), x)
```

3.866.8 Giac [F]

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{1}{(b\sec(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((b*sec(d*x + c) + a)^(3/2)*cos(d*x + c)^(3/2)), x)
```

3.866.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{1}{\cos(c+d\,x)^{3/2}\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

input
```
int(1/(cos(c + d*x)^(3/2)*(a + b/cos(c + d*x))^(3/2)),x)
```

output
```
int(1/(cos(c + d*x)^(3/2)*(a + b/cos(c + d*x))^(3/2)), x)
```

3.867 $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx$

3.867.1 Optimal result 6541
3.867.2 Mathematica [C] (warning: unable to verify) 6542
3.867.3 Rubi [A] (verified) 6542
3.867.4 Maple [C] (warning: unable to verify) 6548
3.867.5 Fricas [F(-1)] 6548
3.867.6 Sympy [F(-1)] 6549
3.867.7 Maxima [F] 6549
3.867.8 Giac [F] 6549
3.867.9 Mupad [F(-1)] 6550

3.867.1 Optimal result

Integrand size = 25, antiderivative size = 206

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx = \frac{2\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{bd\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} + \frac{2a\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+dx)}}{b\left(a^2-b^2\right)d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}} - \frac{2a^2\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}$$

output
```
-2*a^2*sin(d*x+c)/b/(a^2-b^2)/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)+2*
(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2
*c),2,2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)/b/d/cos(d*x+
c)^(1/2)/(a+b*sec(d*x+c))^(1/2)+2*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d
*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*cos(d*x+c)
^(1/2)*(a+b*sec(d*x+c))^(1/2)/b/(a^2-b^2)/d/((b+a*cos(d*x+c))/(a+b))^(1/2)
```

3.867.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 32.91 (sec) , antiderivative size = 48278, normalized size of antiderivative = 234.36

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx = \text{Result too large to show}$$

input
```
Integrate[1/(Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^(3/2)),x]
```

output
```
Result too large to show
```

3.867.3 Rubi [A] (verified)

Time = 1.83 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.14, number of steps used = 19, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.760$, Rules used = {3042, 4752, 3042, 4332, 27, 3042, 4596, 2011, 3042, 4343, 3042, 3134, 3042, 3132, 4346, 3042, 3286, 3042, 3284}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{4752}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{4332}$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{2\int -\frac{a^2+b\sec(c+dx)a+(a^2-b^2)\sec^2(c+dx)}{2\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{b(a^2-b^2)}-\frac{2a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{bd(a^2-b^2)\sqrt{a+b\sec(c+dx)}}\right)$$

$\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{a^2+b\sec(c+dx)a+(a^2-b^2)\sec^2(c+dx)}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{b(a^2-b^2)}-\frac{2a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{bd(a^2-b^2)\sqrt{a+b\sec(c+dx)}}\right)$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{a^2+b\csc(c+dx+\frac{\pi}{2})a+(a^2-b^2)\csc(c+dx+\frac{\pi}{2})^2}{\sqrt{\csc(c+dx+\frac{\pi}{2})}\sqrt{a+b\csc(c+dx+\frac{\pi}{2})}}dx}{b(a^2-b^2)}-\frac{2a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{bd(a^2-b^2)\sqrt{a+b\sec(c+dx)}}\right)$$

$\downarrow$ 4596

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{(a^2-b^2)\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx+\int \frac{a^2+b\sec(c+dx)a}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{b(a^2-b^2)}-\frac{2a^2\sin(c+dx)\sqrt{\sec}}{bd(a^2-b^2)\sqrt{a+b\,\mathrm{se}}}\right.$$

$\downarrow$ 2011

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{(a^2-b^2)\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx+a\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}dx}{b(a^2-b^2)}-\frac{2a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{bd(a^2-b^2)\sqrt{a+b\sec(c+dx}}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{(a^2-b^2)\int \frac{\csc(c+dx+\frac{\pi}{2})^{3/2}}{\sqrt{a+b\csc(c+dx+\frac{\pi}{2})}}dx+a\int \frac{\sqrt{a+b\csc(c+dx+\frac{\pi}{2})}}{\sqrt{\csc(c+dx+\frac{\pi}{2})}}dx}{b(a^2-b^2)}-\frac{2a^2\sin(c+dx)\sqrt{\sec(c}}{bd(a^2-b^2)\sqrt{a+b\sec(}}\right.$$

$\downarrow$ 4343

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{(a^2-b^2)\int \frac{\csc(c+dx+\frac{\pi}{2})^{3/2}}{\sqrt{a+b\csc(c+dx+\frac{\pi}{2})}}dx+\frac{a\sqrt{a+b\sec(c+dx)}\int \sqrt{b+a\cos(c+dx)}dx}{\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}}{b(a^2-b^2)}-\frac{2a^2\sin(c+d}{bd(a^2-b^2)\sqrt{}}\right.$$

3.867. $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\left(a^2-b^2\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{a\sqrt{a+b\sec(c+dx)}\int \sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}}{b\left(a^2-b^2\right)}-\frac{2a^2\sin(c+}{bd\left(a^2-b^2\right)}\right.$$

$$\Big\downarrow \; 3134$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\left(a^2-b^2\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{a\sqrt{a+b\sec(c+dx)}\int \sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}dx}{\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{b\left(a^2-b^2\right)}-\frac{2a^2\sin(c+}{bd\left(a^2-b^2\right)}\right.$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\left(a^2-b^2\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{a\sqrt{a+b\sec(c+dx)}\int \sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}dx}{\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{b\left(a^2-b^2\right)}-\frac{2a^2\sin(}{bd\left(a^2-b\right.}\right.$$

$$\Big\downarrow \; 3132$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\left(a^2-b^2\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{b\left(a^2-b^2\right)}-\frac{2a^2\sin(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a}}\right.$$

$$\Big\downarrow \; 4346$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int \frac{\sec(c+dx)}{\sqrt{b+a\cos(c+dx)}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{2a\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{b\left(a^2-b^2\right)}-\frac{2}{bd}\right.$$

$$\Big\downarrow \; 3042$$

3.867. $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{(a^2-b^2)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{2a\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+\right.}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+}{a+}}}}{b(a^2-b^2)}\right.$$

$\downarrow$ 3286

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{(a^2-b^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{\sec(c+dx)}{\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{2a\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{b(a^2-b^2)}-\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{(a^2-b^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{2a\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}\right.}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos}{}}}}{b(a^2-b^2)}\right.$$

$\downarrow$ 3284

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2(a^2-b^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}+\frac{2a\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right.}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{b(a^2-b^2)}\right.$$

input
```
Int[1/(Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^(3/2)),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(((2*(a^2 - b^2)*Sqrt[(b + a*Cos[c +
 d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x
]])/(d*Sqrt[a + b*Sec[c + d*x]]) + (2*a*EllipticE[(c + d*x)/2, (2*a)/(a +
b)]*Sqrt[a + b*Sec[c + d*x]])/(d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[S
ec[c + d*x]]))/(b*(a^2 - b^2)) - (2*a^2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(
b*(a^2 - b^2)*d*Sqrt[a + b*Sec[c + d*x]]))
```

3.867. $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx$

3.867.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2011
```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x
] && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x
, a + b*x])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3286
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]]   Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
 d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

3.867. $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx$

rule 4332
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-a^2)*d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(
m + 1)*((d*Csc[e + f*x])^(n - 3)/(b*f*(m + 1)*(a^2 - b^2))), x] + Simp[d^3/
(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])
^(n - 3)*Simp[a^2*(n - 3) + a*b*(m + 1)*Csc[e + f*x] - (a^2*(n - 2) + b^2*(
m + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2
- b^2, 0] && LtQ[m, -1] && (IGtQ[n, 3] || (IntegersQ[n + 1/2, 2*m] && GtQ[n
, 2]))
```

rule 4343
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]])   Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a
, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4346
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Simp[d*Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x
]]/Sqrt[a + b*Csc[e + f*x]])   Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4596
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Simp[C/d^2   Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*C
sc[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[
a + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
 b^2, 0]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

3.867. $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx$

3.867.4 Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 8.87 (sec) , antiderivative size = 1019, normalized size of antiderivative = 4.95

method	result	size
default	Expression too large to display	1019

input
```
int(1/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/d/((a-b)/(a+b))^(1/2)/b/(a+b)*((1-cos(d*x+c))^2*csc(d*x+c)^2-1)^3*((a*(
1-cos(d*x+c))^2*csc(d*x+c)^2-b*(1-cos(d*x+c))^2*csc(d*x+c)^2-a-b)/((1-cos(
d*x+c))^2*csc(d*x+c)^2-1))^(1/2)*(((a-b)/(a+b))^(1/2)*a*(1-cos(d*x+c))^3*c
sc(d*x+c)^3+2*(-(a*(1-cos(d*x+c))^2*csc(d*x+c)^2-b*(1-cos(d*x+c))^2*csc(d*
x+c)^2-a-b)/(a+b))^(1/2)*((1-cos(d*x+c))^2*csc(d*x+c)^2+1)^(1/2)*EllipticF
(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*a+(-(a
*(1-cos(d*x+c))^2*csc(d*x+c)^2-b*(1-cos(d*x+c))^2*csc(d*x+c)^2-a-b)/(a+b))
^(1/2)*((1-cos(d*x+c))^2*csc(d*x+c)^2+1)^(1/2)*EllipticF(((a-b)/(a+b))^(1/
2)*(-cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*b-(-(a*(1-cos(d*x+c))^2*
csc(d*x+c)^2-b*(1-cos(d*x+c))^2*csc(d*x+c)^2-a-b)/(a+b))^(1/2)*((1-cos(d*x
+c))^2*csc(d*x+c)^2+1)^(1/2)*EllipticE(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+cs
c(d*x+c)),(-(a+b)/(a-b))^(1/2))*a-2*(-(a*(1-cos(d*x+c))^2*csc(d*x+c)^2-b*(
1-cos(d*x+c))^2*csc(d*x+c)^2-a-b)/(a+b))^(1/2)*((1-cos(d*x+c))^2*csc(d*x+c
)^2+1)^(1/2)*EllipticPi(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(a+b)
/(a-b),I/((a-b)/(a+b))^(1/2))*a-2*(-(a*(1-cos(d*x+c))^2*csc(d*x+c)^2-b*(1-
cos(d*x+c))^2*csc(d*x+c)^2-a-b)/(a+b))^(1/2)*((1-cos(d*x+c))^2*csc(d*x+c)^
2+1)^(1/2)*EllipticPi(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(a+b)/(
a-b),I/((a-b)/(a+b))^(1/2))*b+((a-b)/(a+b))^(1/2)*a*(-cot(d*x+c)+csc(d*x+c
)))/(-((1-cos(d*x+c))^2*csc(d*x+c)^2-1)/((1-cos(d*x+c))^2*csc(d*x+c)^2+1))
^(5/2)/((1-cos(d*x+c))^2*csc(d*x+c)^2+1)^3/(a*(1-cos(d*x+c))^2*csc(d*x+...
```

3.867.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.867.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/cos(d*x+c)**(5/2)/(a+b*sec(d*x+c))**(3/2),x)
```

output
```
Timed out
```

3.867.7 Maxima [F]

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{1}{(b\sec(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*sec(d*x + c) + a)^(3/2)*cos(d*x + c)^(5/2)), x)
```

3.867.8 Giac [F]

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{1}{(b\sec(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((b*sec(d*x + c) + a)^(3/2)*cos(d*x + c)^(5/2)), x)
```

3.867.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx=\int \frac{1}{\cos\left(c+d\,x\right)^{5/2}\,{\left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)}^{3/2}}\,dx$$

input
```
int(1/(cos(c + d*x)^(5/2)*(a + b/cos(c + d*x))^(3/2)),x)
```

output
```
int(1/(cos(c + d*x)^(5/2)*(a + b/cos(c + d*x))^(3/2)), x)
```

3.868 $\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx$

3.868.1 Optimal result 6551
3.868.2 Mathematica [C] (warning: unable to verify) 6552
3.868.3 Rubi [A] (verified) 6552
3.868.4 Maple [C] (warning: unable to verify) 6561
3.868.5 Fricas [F(-1)] 6561
3.868.6 Sympy [F(-1)] 6562
3.868.7 Maxima [F] 6562
3.868.8 Giac [F] 6562
3.868.9 Mupad [F(-1)] 6563

3.868.1 Optimal result

Integrand size = 25, antiderivative size = 345

$$\begin{aligned}\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx &= \frac{\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{bd\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}\\ &-\frac{3a\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{b^2d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}\\ &-\frac{(3a^2-b^2)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+dx)}}{b^2(a^2-b^2)d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}}\\ &-\frac{2a^2\sin(c+dx)}{b(a^2-b^2)d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}\\ &+\frac{(3a^2-b^2)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{b^2(a^2-b^2)d\sqrt{\cos(c+dx)}}\end{aligned}$$

output
```
-2*a^2*sin(d*x+c)/b/(a^2-b^2)/d/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(1/2)+(c
os(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c)
,2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)/b/d/cos(d*x+c)^(1
/2)/(a+b*sec(d*x+c))^(1/2)-3*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/
2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2,2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*
x+c))/(a+b))^(1/2)/b^2/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)+(3*a^2-b^
2)*sin(d*x+c)*(a+b*sec(d*x+c))^(1/2)/b^2/(a^2-b^2)/d/cos(d*x+c)^(1/2)-(3*a
^2-b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*
d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2
)/b^2/(a^2-b^2)/d/((b+a*cos(d*x+c))/(a+b))^(1/2)
```

3.868.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 32.83 (sec) , antiderivative size = 52199, normalized size of antiderivative = 151.30

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx = \text{Result too large to show}$$

input
```
Integrate[1/(Cos[c + d*x]^(7/2)*(a + b*Sec[c + d*x])^(3/2)),x]
```

output
```
Result too large to show
```

3.868.3 Rubi [A] (verified)

Time = 3.76 (sec) , antiderivative size = 381, normalized size of antiderivative = 1.10, number of steps used = 28, number of rules used = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 1.120$, Rules used = {3042, 4752, 3042, 4332, 27, 3042, 4590, 27, 3042, 4596, 3042, 4346, 3042, 3286, 3042, 3284, 4523, 3042, 4343, 3042, 3134, 3042, 3132, 4345, 3042, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{7/2}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$\Big\downarrow$ 4752

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}dx$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$\Big\downarrow$ 4332

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{2\int \frac{\sqrt{\sec(c+dx)}\left(a^2-b\sec(c+dx)a-\left(3a^2-b^2\right)\sec^2(c+dx)\right)}{2\sqrt{a+b\sec(c+dx)}}dx}{b\left(a^2-b^2\right)}-\frac{2a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\sec(c+d}}\right.$$

$\Big\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\int \frac{\sqrt{\sec(c+dx)}\left(a^2-b\sec(c+dx)a-\left(3a^2-b^2\right)\sec^2(c+dx)\right)}{\sqrt{a+b\sec(c+dx)}}dx}{b\left(a^2-b^2\right)}-\frac{2a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a^2-b\csc\left(c+dx+\frac{\pi}{2}\right)a+\left(b^2-3a^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{b\left(a^2-b^2\right)}-\frac{2a^2\sin(c+dx)\sec^{\frac{3}{2}}}{bd\left(a^2-b^2\right)\sqrt{a+b\,\mathrm{s}}}\right.$$

$\Big\downarrow$ 4590

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{\int \frac{2b\sec(c+dx)a^2+3\left(a^2-b^2\right)\sec^2(c+dx)a+\left(3a^2-b^2\right)a}{2\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{b}-\frac{\left(3a^2-b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+d}}{bd}}{b\left(a^2-b^2\right)}\right.$$

$\Big\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{\int \frac{2b\sec(c+dx)a^2+3\left(a^2-b^2\right)\sec^2(c+dx)a+\left(3a^2-b^2\right)a}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{2b}-\frac{\left(3a^2-b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+d}}{bd}}{b\left(a^2-b^2\right)}\right.$$

3.868. $\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{\int \frac{2b\csc\left(c+dx+\frac{\pi}{2}\right)a^2+3\left(a^2-b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2a+\left(3a^2-b^2\right)a}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{2b}-\frac{\left(3a^2-b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b}}{bd}}{b\left(a^2-b^2\right)}\right.$$

$$\Big\downarrow \; 4596$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{3a\left(a^2-b^2\right)\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx+\int \frac{2b\sec(c+dx)a^2+\left(3a^2-b^2\right)a}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{2b}-\frac{\left(3a^2-b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a-}}{bd}}{b\left(a^2-b^2\right)}\right.$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{3a\left(a^2-b^2\right)\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\int \frac{2b\csc\left(c+dx+\frac{\pi}{2}\right)a^2+\left(3a^2-b^2\right)a}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{2b}-\frac{\left(3a^2-b^2\right)\sin(c+dx)\sqrt{\text{se}}}{b}}{b\left(a^2-b^2\right)}\right.$$

$$\Big\downarrow \; 4346$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{\int \frac{2b\csc\left(c+dx+\frac{\pi}{2}\right)a^2+\left(3a^2-b^2\right)a}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{3a\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int \frac{\sec(c+dx)}{\sqrt{b+a\cos(c+dx)}}dx}{\sqrt{a+b\sec(c+dx)}}}{2b}-\frac{(3a}{}}{b\left(a^2-b^2\right)}\right.$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{\int \frac{2b\csc\left(c+dx+\frac{\pi}{2}\right)a^2+\left(3a^2-b^2\right)a}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{3a\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{b+a\sin\left(c+d\right.}}}{\sqrt{a+b\sec(c+dx)}}}{2b}}{b\left(a^2-b^2\right)}\right.$$

$$\Big\downarrow \; 3286$$

3.868. $\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{\int \frac{2b\csc\left(c+dx+\frac{\pi}{2}\right)a^2+\left(3a^2-b^2\right)a}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{3a\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int \frac{\sec(c+dx)}{\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}}{2b}}{b\left(a^2-b^2\right)}-\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{\int \frac{2b\csc\left(c+dx+\frac{\pi}{2}\right)a^2+\left(3a^2-b^2\right)a}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{3a\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{\frac{b}{a+b}+\frac{a\sin}{}}}}{\sqrt{a+b\sec(c+dx)}}}{2b}}{b\left(a^2-b^2\right)}\right.$$

$\downarrow$ 3284

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{\int \frac{2b\csc\left(c+dx+\frac{\pi}{2}\right)a^2+\left(3a^2-b^2\right)a}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{6a\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\operatorname{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}}{2b}}{b\left(a^2-b^2\right)}\right.$$

$\downarrow$ 4523

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{-b\left(a^2-b^2\right)\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}dx+\left(3a^2-b^2\right)\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}dx+\frac{6a\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\operatorname{Ell}}{d\sqrt{a+b\sec(c+dx)}}}{2b}}{b\left(a^2-b^2\right)}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{-b\left(a^2-b^2\right)\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\left(3a^2-b^2\right)\int \frac{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{6a\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+}{a+}}}{d\sqrt{a+b\,\mathrm{se}}}}{2b}}{b\left(a^2-b^2\right)}\right.$$

$\downarrow$ 4343

3.868. $\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{-b(a^2-b^2)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{\left(3a^2-b^2\right)\sqrt{a+b\sec(c+dx)}\int\sqrt{b+a\cos(c+dx)}dx}{\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}+\frac{6a\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{}}{}}{2b}}{b\left(a^2-b^2\right.}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{-b(a^2-b^2)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{\left(3a^2-b^2\right)\sqrt{a+b\sec(c+dx)}\int\sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}+\frac{6a\left(a^2-b^2\right)\sqrt{\sec(c+}}{}}{2b}}{b\left(a^2-\right.}\right.$$

$\downarrow$ 3134

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{-b(a^2-b^2)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{\left(3a^2-b^2\right)\sqrt{a+b\sec(c+dx)}\int\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}dx}{\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{6a\left(a^2-b^2\right)\sqrt{\sec(c+}}{}}{2b}}{b\left(a^2-\right.}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{-b(a^2-b^2)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{\left(3a^2-b^2\right)\sqrt{a+b\sec(c+dx)}\int\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}dx}{\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{6a\left(a^2-b^2\right)\sqrt{\mathrm{se}}}{}}{2b}}{b\left(a^2-\right.}\right.$$

$\downarrow$ 3132

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{-b(a^2-b^2)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\left(3a^2-b^2\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{6a\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{}}{}}{2b}}{b\left(a^2-b^2\right.}\right.$$

$\downarrow$ 4345

3.868. $\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{-\frac{b\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{1}{\sqrt{b+a\cos(c+dx)}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{2\left(3a^2-b^2\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{6a}{}}{2b}}{b}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{-\frac{b\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{1}{\sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{2\left(3a^2-b^2\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{2b}\right.$$

$\downarrow$ 3142

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{-\frac{b\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{2\left(3a^2-b^2\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{2b}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{-\frac{b\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{2\left(3a^2-b^2\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{}{a}\right.}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{2b}\right.$$

$\downarrow$ 3140

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{2a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}-\frac{-\frac{2b\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right.}{d\sqrt{a+b\sec(c+dx)}}}{}\right.$$

3.868. $\int\frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}dx$

input
```
Int[1/(Cos[c + d*x]^(7/2)*(a + b*Sec[c + d*x])^(3/2)),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((-2*a^2*Sec[c + d*x]^(3/2)*Sin[c +
d*x])/(b*(a^2 - b^2)*d*Sqrt[a + b*Sec[c + d*x]]) - (((-2*b*(a^2 - b^2)*Sqr
t[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]*Sqrt
[Sec[c + d*x]])/(d*Sqrt[a + b*Sec[c + d*x]]) + (6*a*(a^2 - b^2)*Sqrt[(b +
a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)]*Sqrt[Se
c[c + d*x]])/(d*Sqrt[a + b*Sec[c + d*x]]) + (2*(3*a^2 - b^2)*EllipticE[(c
+ d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(d*Sqrt[(b + a*Cos[c +
d*x])/(a + b)]*Sqrt[Sec[c + d*x]]))/(2*b) - ((3*a^2 - b^2)*Sqrt[Sec[c + d*
x]]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(b*d))/(b*(a^2 - b^2)))
```

3.868.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

3.868. $\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx$

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

rule 3286
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]]   Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
 d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

rule 4332
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-a^2)*d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(
m + 1)*((d*Csc[e + f*x])^(n - 3)/(b*f*(m + 1)*(a^2 - b^2))), x] + Simp[d^3/
(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])
^(n - 3)*Simp[a^2*(n - 3) + a*b*(m + 1)*Csc[e + f*x] - (a^2*(n - 2) + b^2*(
m + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2
- b^2, 0] && LtQ[m, -1] && (IGtQ[n, 3] || (IntegersQ[n + 1/2, 2*m] && GtQ[n
, 2]))
```

rule 4343
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]])   Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a
, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4345
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]])   Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[
{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

3.868. $\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx$

rule 4346
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Simp[d*Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x
]]/Sqrt[a + b*Csc[e + f*x]])   Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4523
```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Simp[A/a   I
nt[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Simp[(A*b - a*B)
/(a*d)   Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ
[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

rule 4590
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(-C)*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
)*((d*Csc[e + f*x])^(n - 1)/(b*f*(m + n + 1))), x] + Simp[d/(b*(m + n + 1))
   Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (
A*b*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2
 - b^2, 0] && GtQ[n, 0]
```

rule 4596
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Simp[C/d^2   Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*C
sc[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[
a + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
 b^2, 0]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

3.868. $\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx$

3.868.4 Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 10.16 (sec) , antiderivative size = 2300, normalized size of antiderivative = 6.67

method	result	size
default	Expression too large to display	2300

input
```
int(1/cos(d*x+c)^(7/2)/(a+b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/d/((a-b)/(a+b))^(1/2)/(a+b)/b^2*((1-cos(d*x+c))^2*csc(d*x+c)^2-1)^3*((a
*(1-cos(d*x+c))^2*csc(d*x+c)^2-b*(1-cos(d*x+c))^2*csc(d*x+c)^2-a-b)/((1-co
s(d*x+c))^2*csc(d*x+c)^2-1))^(1/2)*(3*((a-b)/(a+b))^(1/2)*a^2*(1-cos(d*x+c
))^5*csc(d*x+c)^5-((a-b)/(a+b))^(1/2)*b^2*(1-cos(d*x+c))^5*csc(d*x+c)^5+6*
(-(a*(1-cos(d*x+c))^2*csc(d*x+c)^2-b*(1-cos(d*x+c))^2*csc(d*x+c)^2-a-b)/(a
+b))^(1/2)*((1-cos(d*x+c))^2*csc(d*x+c)^2+1)^(1/2)*EllipticF(((a-b)/(a+b))
^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*a^2*(1-cos(d*x+c))^2
*csc(d*x+c)^2+4*(-(a*(1-cos(d*x+c))^2*csc(d*x+c)^2-b*(1-cos(d*x+c))^2*csc(
d*x+c)^2-a-b)/(a+b))^(1/2)*((1-cos(d*x+c))^2*csc(d*x+c)^2+1)^(1/2)*Ellipti
cF(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*a*b*
(1-cos(d*x+c))^2*csc(d*x+c)^2-3*(-(a*(1-cos(d*x+c))^2*csc(d*x+c)^2-b*(1-co
s(d*x+c))^2*csc(d*x+c)^2-a-b)/(a+b))^(1/2)*((1-cos(d*x+c))^2*csc(d*x+c)^2+
1)^(1/2)*EllipticE(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a
-b))^(1/2))*a^2*(1-cos(d*x+c))^2*csc(d*x+c)^2+(-(a*(1-cos(d*x+c))^2*csc(d*
x+c)^2-b*(1-cos(d*x+c))^2*csc(d*x+c)^2-a-b)/(a+b))^(1/2)*((1-cos(d*x+c))^2
*csc(d*x+c)^2+1)^(1/2)*EllipticE(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+
c)),(-(a+b)/(a-b))^(1/2))*b^2*(1-cos(d*x+c))^2*csc(d*x+c)^2-6*(-(a*(1-cos(
d*x+c))^2*csc(d*x+c)^2-b*(1-cos(d*x+c))^2*csc(d*x+c)^2-a-b)/(a+b))^(1/2)*(
(1-cos(d*x+c))^2*csc(d*x+c)^2+1)^(1/2)*EllipticPi(((a-b)/(a+b))^(1/2)*(-co
t(d*x+c)+csc(d*x+c)),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*a^2*(1-cos(d*x+...
```

3.868.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/cos(d*x+c)^(7/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.868.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/cos(d*x+c)**(7/2)/(a+b*sec(d*x+c))**(3/2),x)
```

output
```
Timed out
```

3.868.7 Maxima [F]

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{1}{(b\sec(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(7/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*sec(d*x + c) + a)^(3/2)*cos(d*x + c)^(7/2)), x)
```

3.868.8 Giac [F]

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{1}{(b\sec(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(7/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((b*sec(d*x + c) + a)^(3/2)*cos(d*x + c)^(7/2)), x)
```

3.868.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx=\int \frac{1}{\cos\left(c+d\,x\right)^{7/2}\,\left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)^{3/2}}\,dx$$

input
```
int(1/(cos(c + d*x)^(7/2)*(a + b/cos(c + d*x))^(3/2)),x)
```

output
```
int(1/(cos(c + d*x)^(7/2)*(a + b/cos(c + d*x))^(3/2)), x)
```

3.869 $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$

3.869.1 Optimal result 6564
3.869.2 Mathematica [C] (warning: unable to verify) 6565
3.869.3 Rubi [A] (verified) 6565
3.869.4 Maple [B] (warning: unable to verify) 6573
3.869.5 Fricas [C] (verification not implemented) 6574
3.869.6 Sympy [F(-1)] 6575
3.869.7 Maxima [F] 6576
3.869.8 Giac [F] 6576
3.869.9 Mupad [F(-1)] 6576

3.869.1 Optimal result

Integrand size = 25, antiderivative size = 391

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \frac{2(a^4+16a^2b^2-16b^4)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{3a^4(a^2-b^2)\,d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} - \frac{8b(2a^4-7a^2b^2+4b^4)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+dx)}}{3a^4(a^2-b^2)^2\,d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}} + \frac{2b^2\sqrt{\cos(c+dx)}\sin(c+dx)}{3a(a^2-b^2)\,d(a+b\sec(c+dx))^{3/2}} + \frac{4b^2(5a^2-3b^2)\sqrt{\cos(c+dx)}\sin(c+dx)}{3a^2(a^2-b^2)^2\,d\sqrt{a+b\sec(c+dx)}} + \frac{2(a^4-13a^2b^2+8b^4)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{3a^3(a^2-b^2)^2\,d}$$

output
```
2/3*b^2*sin(d*x+c)*cos(d*x+c)^(1/2)/a/(a^2-b^2)/d/(a+b*sec(d*x+c))^(3/2)+4
/3*b^2*(5*a^2-3*b^2)*sin(d*x+c)*cos(d*x+c)^(1/2)/a^2/(a^2-b^2)^2/d/(a+b*se
c(d*x+c))^(1/2)+2/3*(a^4+16*a^2*b^2-16*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/c
os(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((
b+a*cos(d*x+c))/(a+b))^(1/2)/a^4/(a^2-b^2)/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x
+c))^(1/2)+2/3*(a^4-13*a^2*b^2+8*b^4)*sin(d*x+c)*cos(d*x+c)^(1/2)*(a+b*sec
(d*x+c))^(1/2)/a^3/(a^2-b^2)^2/d-8/3*b*(2*a^4-7*a^2*b^2+4*b^4)*(cos(1/2*d*
x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*
(a/(a+b))^(1/2))*cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2)/a^4/(a^2-b^2)^2/d
/((b+a*cos(d*x+c))/(a+b))^(1/2)
```

3.869.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 12.41 (sec) , antiderivative size = 527, normalized size of antiderivative = 1.35

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \frac{(b+a\cos(c+dx))^3\left(\frac{2\sin(c+dx)}{3a^3}+\frac{2b^4\sin(c+dx)}{3a^3(a^2-b^2)(b+a\cos(c+dx))^2}+\frac{8(-3a^2b^3\sin(c+dx)+2b^5}{3a^3(a^2-b^2)^2(b+a\cos}\right.}{d\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}$$
$$+\frac{2\cos^{\frac{3}{2}}(c+dx)(b+a\cos(c+dx))^2\sec^{\frac{5}{2}}(c+dx)\left(\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\right)^{3/2}\left(-4ib(2a^5+2a^4b-7a^3\right.}{}$$

input
```
Integrate[Cos[c + d*x]^(3/2)/(a + b*Sec[c + d*x])^(5/2),x]
```

output
```
((b + a*Cos[c + d*x])^3*((2*Sin[c + d*x])/(3*a^3) + (2*b^4*Sin[c + d*x])/(
3*a^3*(a^2 - b^2)*(b + a*Cos[c + d*x])^2) + (8*(-3*a^2*b^3*Sin[c + d*x] +
2*b^5*Sin[c + d*x]))/(3*a^3*(a^2 - b^2)^2*(b + a*Cos[c + d*x]))))/(d*Cos[c
 + d*x]^(5/2)*(a + b*Sec[c + d*x])^(5/2)) + (2*Cos[c + d*x]^(3/2)*(b + a*C
os[c + d*x])^2*Sec[c + d*x]^(5/2)*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(3/2)*
((-4*I)*b*(2*a^5 + 2*a^4*b - 7*a^3*b^2 - 7*a^2*b^3 + 4*a*b^4 + 4*b^5)*Elli
pticE[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sq
rt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] - I*a*(a^5 - 8*a^4*b
 + 7*a^3*b^2 + 28*a^2*b^3 - 4*a*b^4 - 16*b^5)*EllipticF[I*ArcSinh[Tan[(c +
 d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])
*Sec[(c + d*x)/2]^2)/(a + b)] - 4*b*(2*a^4 - 7*a^2*b^2 + 4*b^4)*(b + a*Cos
[c + d*x])*(Sec[(c + d*x)/2]^2)^(3/2)*Tan[(c + d*x)/2]))/(3*a^4*(a^2 - b^2
)^2*d*(a + b*Sec[c + d*x])^(5/2))
```

3.869.3 Rubi [A] (verified)

Time = 3.19 (sec) , antiderivative size = 420, normalized size of antiderivative = 1.07, number of steps used = 24, number of rules used = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.960$, Rules used = {3042, 4752, 3042, 4334, 27, 3042, 4588, 27, 3042, 4592, 27, 3042, 4523, 3042, 4343, 3042, 3134, 3042, 3132, 4345, 3042, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \; 4752$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}dx$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \; 4334$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2b^2\sin(c+dx)}{3ad\left(a^2-b^2\right)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}}-\frac{2\int -\frac{4b^2\sec^2(c+dx)-3ab\sec(c+dx)+3}{2\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx}}{3a\left(a^2-b^2\right)}\right.$$

$$\Big\downarrow \; 27$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{4b^2\sec^2(c+dx)-3ab\sec(c+dx)+3\left(a^2-2b^2\right)}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}dx}{3a\left(a^2-b^2\right)}+\frac{2b^2\sin(c+dx)}{3ad\left(a^2-b^2\right)\sqrt{\sec(c+dx)}(a+b\sec(c+\right.$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{4b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2-3ab\csc\left(c+dx+\frac{\pi}{2}\right)+3\left(a^2-2b^2\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{3a\left(a^2-b^2\right)}+\frac{2b^2\sin(c+dx)}{3ad\left(a^2-b^2\right)\sqrt{\sec(c+dx)}(a+b\mathrm{se}}\right.$$

$$\Big\downarrow \; 4588$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{4b^2\left(5a^2-3b^2\right)\sin(c+dx)}{ad(a^2-b^2)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}-\frac{2\int -\frac{4b^2\left(5a^2-3b^2\right)\sec^2(c+dx)-2ab\left(3a^2-b^2\right)\sec(c+dx)+3\left(a^4-13b\right.}{2\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}}{a(a^2-b^2)}}{3a\left(a^2-b^2\right)}\right.$$

$$\Big\downarrow \; 27$$

3.869. $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\int \frac{4b^2\left(5a^2-3b^2\right)\sec^2(c+dx)-2ab\left(3a^2-b^2\right)\sec(c+dx)+3\left(a^4-13b^2a^2+8b^4\right)}{\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}dx}{a(a^2-b^2)}+\frac{4b^2\left(5a^2-3b^2\right)\sin(c+dx}{ad(a^2-b^2)\sqrt{\sec(c+dx)}\sqrt{a+b\sec}}}{3a\left(a^2-b^2\right)}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\int \frac{4b^2\left(5a^2-3b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2-2ab\left(3a^2-b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)+3\left(a^4-13b^2a^2+8b^4\right)}{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a(a^2-b^2)}+\frac{4b^2\left(5a^2-3b^2\right)\sin}{ad(a^2-b^2)\sqrt{\sec(c+dx)}}}{3a\left(a^2-b^2\right)}\right.$$

$\downarrow$ 4592

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{2\left(a^4-13a^2b^2+8b^4\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{ad\sqrt{\sec(c+dx)}}-\frac{2\int\frac{3\left(4b\left(2a^4-7b^2a^2+4b^4\right)-a\left(a^4+7b^2a^2-4b^4\right)\sec(c+dx)\right)}{2\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{3a}}{a(a^2-b^2)}+\frac{}{a}}{3a\left(a^2-b^2\right)}\right.$$

$\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{2\left(a^4-13a^2b^2+8b^4\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{ad\sqrt{\sec(c+dx)}}-\frac{\int\frac{4b\left(2a^4-7b^2a^2+4b^4\right)-a\left(a^4+7b^2a^2-4b^4\right)\sec(c+dx)}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{a}}{a(a^2-b^2)}+\frac{}{ad(a^2}}{3a\left(a^2-b^2\right)}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{2\left(a^4-13a^2b^2+8b^4\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{ad\sqrt{\sec(c+dx)}}-\frac{\int\frac{4b\left(2a^4-7b^2a^2+4b^4\right)-a\left(a^4+7b^2a^2-4b^4\right)\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}}{a(a^2-b^2)}+\frac{}{ad}}{3a\left(a^2-b^2\right)}\right.$$

$\downarrow$ 4523

3.869. $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{2\left(a^4-13a^2b^2+8b^4\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{ad\sqrt{\sec(c+dx)}}-\frac{\frac{4b\left(2a^4-7a^2b^2+4b^4\right)\int\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}dx}{a}-\frac{\left(a^6+15a^4b^2-32a^2b^4+1\right.}{a}}{a(a^2-b^2)}}{3a\left(a^2-b^2\right)}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{2\left(a^4-13a^2b^2+8b^4\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{ad\sqrt{\sec(c+dx)}}-\frac{\frac{4b\left(2a^4-7a^2b^2+4b^4\right)\int\frac{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}-\frac{\left(a^6+15a^4b^2-32a^2\right.}{a}}{a(a^2-b^2)}}{3a\left(a^2-b^2\right)}\right.$$

$\downarrow$ 4343

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{2\left(a^4-13a^2b^2+8b^4\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{ad\sqrt{\sec(c+dx)}}-\frac{\frac{4b\left(2a^4-7a^2b^2+4b^4\right)\sqrt{a+b\sec(c+dx)}\int\sqrt{b+a\cos(c+dx)}dx}{a\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}-\frac{\left(a^6+15\right.}{a}}{a(a^2-b^2)}}{3a\left(a^2-b^2\right)}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{2\left(a^4-13a^2b^2+8b^4\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{ad\sqrt{\sec(c+dx)}}-\frac{\frac{4b\left(2a^4-7a^2b^2+4b^4\right)\sqrt{a+b\sec(c+dx)}\int\sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{a\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}-\frac{\left(a^6\right.}{a}}{a(a^2-b^2)}}{3a\left(a^2-b^2\right)}\right.$$

$\downarrow$ 3134

3.869. $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2\left(a^4-13a^2b^2+8b^4\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{ad\sqrt{\sec(c+dx)}}-\frac{\frac{4b\left(2a^4-7a^2b^2+4b^4\right)\sqrt{a+b\sec(c+dx)}\int\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}dx}{a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{\left(a^6\right.}{a}}{a(a^2-b^2)}}{3a\left(a^2-b^2\right)}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2\left(a^4-13a^2b^2+8b^4\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{ad\sqrt{\sec(c+dx)}}-\frac{\frac{4b\left(2a^4-7a^2b^2+4b^4\right)\sqrt{a+b\sec(c+dx)}\int\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}dx}{a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{}{a}}{a(a^2-b^2)}}{3a\left(a^2-b^2\right.}\right.$$

$\downarrow$ 3132

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2\left(a^4-13a^2b^2+8b^4\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{ad\sqrt{\sec(c+dx)}}-\frac{\frac{8b\left(2a^4-7a^2b^2+4b^4\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{\left(a^6+15a^4\right.}{a}}{a(a^2-b^2)}}{3a\left(a^2-b^2\right)}\right.$$

$\downarrow$ 4345

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2\left(a^4-13a^2b^2+8b^4\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{ad\sqrt{\sec(c+dx)}}-\frac{\frac{8b\left(2a^4-7a^2b^2+4b^4\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{\left(a^6+15a^4\right.}{a}}{a(a^2-b^2)}}{3a\left(a^2\right.}\right.$$

3.869. $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{2\left(a^4-13a^2b^2+8b^4\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{ad\sqrt{\sec(c+dx)}}-\frac{\frac{8b\left(2a^4-7a^2b^2+4b^4\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{\left(a^6+15a^4\right.}{}}{}}{a(a^2-b^2)}}{3a\,(a}\right.$$

$\downarrow$ 3142

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{2\left(a^4-13a^2b^2+8b^4\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{ad\sqrt{\sec(c+dx)}}-\frac{\frac{8b\left(2a^4-7a^2b^2+4b^4\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{\left(a^6+15a^4\right.}{}}{}}{a(a^2-b^2)}}{3a\,(a}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{2\left(a^4-13a^2b^2+8b^4\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{ad\sqrt{\sec(c+dx)}}-\frac{\frac{8b\left(2a^4-7a^2b^2+4b^4\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{\left(a^6+15a^4\right.}{}}{}}{a(a^2-b^2)}}{3a}\right.$$

$\downarrow$ 3140

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2b^2\sin(c+dx)}{3ad\,(a^2-b^2)\,\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}}+\frac{\frac{4b^2\left(5a^2-3b^2\right)\sin(c+dx)}{ad(a^2-b^2)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}}{}\right.$$

3.869. $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$

input
```
Int[Cos[c + d*x]^(3/2)/(a + b*Sec[c + d*x])^(5/2),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*b^2*Sin[c + d*x])/(3*a*(a^2 - b^
2)*d*Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^(3/2)) + ((4*b^2*(5*a^2 - 3*b
^2)*Sin[c + d*x])/(a*(a^2 - b^2)*d*Sqrt[Sec[c + d*x]]*Sqrt[a + b*Sec[c + d
*x]]) + (-(((-2*(a^6 + 15*a^4*b^2 - 32*a^2*b^4 + 16*b^6)*Sqrt[(b + a*Cos[c
 + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]]
)/(a*d*Sqrt[a + b*Sec[c + d*x]]) + (8*b*(2*a^4 - 7*a^2*b^2 + 4*b^4)*Ellipt
icE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(a*d*Sqrt[(b + a
*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]))/a) + (2*(a^4 - 13*a^2*b^2 + 8
*b^4)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(a*d*Sqrt[Sec[c + d*x]]))/(a*
(a^2 - b^2)))/(3*a*(a^2 - b^2)))
```

3.869.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

3.869. $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 4334
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*
((d*Csc[e + f*x])^n/(a*f*(m + 1)*(a^2 - b^2))), x] + Simp[1/(a*(m + 1)*(a^2
 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a^2*(m + 1)
 - b^2*(m + n + 1) - a*b*(m + 1)*Csc[e + f*x] + b^2*(m + n + 2)*Csc[e + f*x
]^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0] && LtQ[m,
 -1] && IntegersQ[2*m, 2*n]
```

rule 4343
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]])   Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a
, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4345
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]])   Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[
{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4523
```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Simp[A/a   I
nt[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Simp[(A*b - a*B)
/(a*d)   Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ
[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

3.869. $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$

rule 4588
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc
[e + f*x])^(m + 1)*((d*Csc[e + f*x])^n/(a*f*(m + 1)*(a^2 - b^2))), x] + Sim
p[1/(a*(m + 1)*(a^2 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f
*x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n +
1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m
+ n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x
] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

rule 4592
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Simp[1/(a*d*n)   Int[(a + b*Csc[e + f*x])^m
*(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*
Csc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

3.869.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 4533 vs. $2(415) = 830$.

Time = 10.85 (sec) , antiderivative size = 4534, normalized size of antiderivative = 11.60

method	result	size
default	Expression too large to display	4534

input
```
int(cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/3/d/((a-b)/(a+b))^(1/2)/(a+b)^2/(a-b)/a^4*(9*(1/(a+b)*(b+a*cos(d*x+c))/(
cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF(((a-b)/(a+b))^(1/2
)*(-cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*a^5*b*cos(d*x+c)^3+24*((a
-b)/(a+b))^(1/2)*a*b^5*cos(d*x+c)*sin(d*x+c)+(1/(a+b)*(b+a*cos(d*x+c))/(co
s(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF(((a-b)/(a+b))^(1/2)*
(-cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*a^6*cos(d*x+c)^3+2*(1/(a+b)
*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF
(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*a^6*co
s(d*x+c)^2-16*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+
c)+1))^(1/2)*EllipticE(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(-(a+b
)/(a-b))^(1/2))*b^6*cos(d*x+c)^2+(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))
^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)
+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*a^5*b+9*(1/(a+b)*(b+a*cos(d*x+c))/(cos(
d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF(((a-b)/(a+b))^(1/2)*(-
cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*a^4*b^2+16*(1/(a+b)*(b+a*cos(
d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF(((a-b)/(a
+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*a^3*b^3-12*(1/(a
+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*Ellipt
icF(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*a^2
*b^4-16*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+...
```

3.869.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.18 (sec) , antiderivative size = 971, normalized size of antiderivative = 2.48

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
1/9*(6*(a^6*b^2 - 13*a^4*b^4 + 8*a^2*b^6 + (a^8 - 2*a^6*b^2 + a^4*b^4)*cos
(d*x + c)^2 + 2*(a^7*b - 8*a^5*b^3 + 5*a^3*b^5)*cos(d*x + c))*sqrt((a*cos(
d*x + c) + b)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) + (sqrt(2)*(-3
*I*a^8 - 37*I*a^6*b^2 + 68*I*a^4*b^4 - 32*I*a^2*b^6)*cos(d*x + c)^2 - 2*sq
rt(2)*(3*I*a^7*b + 37*I*a^5*b^3 - 68*I*a^3*b^5 + 32*I*a*b^7)*cos(d*x + c)
+ sqrt(2)*(-3*I*a^6*b^2 - 37*I*a^4*b^4 + 68*I*a^2*b^6 - 32*I*b^8))*sqrt(a)
*weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3,
 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x + c) + 2*b)/a) + (sqrt(2)*(3*I*a^8
+ 37*I*a^6*b^2 - 68*I*a^4*b^4 + 32*I*a^2*b^6)*cos(d*x + c)^2 - 2*sqrt(2)*(
-3*I*a^7*b - 37*I*a^5*b^3 + 68*I*a^3*b^5 - 32*I*a*b^7)*cos(d*x + c) + sqrt
(2)*(3*I*a^6*b^2 + 37*I*a^4*b^4 - 68*I*a^2*b^6 + 32*I*b^8))*sqrt(a)*weiers
trassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3
*a*cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a) - 12*(sqrt(2)*(2*I*a^7*b -
7*I*a^5*b^3 + 4*I*a^3*b^5)*cos(d*x + c)^2 + 2*sqrt(2)*(2*I*a^6*b^2 - 7*I*a
^4*b^4 + 4*I*a^2*b^6)*cos(d*x + c) + sqrt(2)*(2*I*a^5*b^3 - 7*I*a^3*b^5 +
4*I*a*b^7))*sqrt(a)*weierstrassZeta(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*
b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*
b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x + c) + 2*b)/a)) - 12
*(sqrt(2)*(-2*I*a^7*b + 7*I*a^5*b^3 - 4*I*a^3*b^5)*cos(d*x + c)^2 + 2*sqrt
(2)*(-2*I*a^6*b^2 + 7*I*a^4*b^4 - 4*I*a^2*b^6)*cos(d*x + c) + sqrt(2)*(...
```

3.869.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(3/2)/(a+b*sec(d*x+c))**(5/2),x)
```

output
```
Timed out
```

3.869.7 Maxima [F]

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{\cos{(dx+c)}^{\frac{3}{2}}}{(b\sec{(dx+c)}+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate(cos(d*x + c)^(3/2)/(b*sec(d*x + c) + a)^(5/2), x)
```

3.869.8 Giac [F]

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{\cos{(dx+c)}^{\frac{3}{2}}}{(b\sec{(dx+c)}+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate(cos(d*x + c)^(3/2)/(b*sec(d*x + c) + a)^(5/2), x)
```

3.869.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{\cos{(c+d\,x)}^{3/2}}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

input
```
int(cos(c + d*x)^(3/2)/(a + b/cos(c + d*x))^(5/2),x)
```

output
```
int(cos(c + d*x)^(3/2)/(a + b/cos(c + d*x))^(5/2), x)
```

3.870 $\int \frac{\sqrt{\cos(c+dx)}}{(a+b\sec(c+dx))^{5/2}}\,dx$

3.870.1 Optimal result 6577
3.870.2 Mathematica [C] (warning: unable to verify) 6578
3.870.3 Rubi [A] (verified) 6578
3.870.4 Maple [B] (verified) 6585
3.870.5 Fricas [C] (verification not implemented) 6586
3.870.6 Sympy [F(-1)] 6587
3.870.7 Maxima [F] 6588
3.870.8 Giac [F] 6588
3.870.9 Mupad [F(-1)] 6588

3.870.1 Optimal result

Integrand size = 25, antiderivative size = 317

$$\begin{aligned}\int \frac{\sqrt{\cos(c+dx)}}{(a+b\sec(c+dx))^{5/2}}\,dx &= -\frac{2b(9a^2-8b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{3a^3\,(a^2-b^2)\,d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}\\ &+\frac{2(3a^4-15a^2b^2+8b^4)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+dx)}}{3a^3\,(a^2-b^2)^2\,d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}}\\ &+\frac{2b^2\sin(c+dx)}{3a\,(a^2-b^2)\,d\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^{3/2}}\\ &+\frac{8b^2(2a^2-b^2)\sin(c+dx)}{3a^2\,(a^2-b^2)^2\,d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}\end{aligned}$$

output
```
2/3*b^2*sin(d*x+c)/a/(a^2-b^2)/d/(a+b*sec(d*x+c))^(3/2)/cos(d*x+c)^(1/2)+8
/3*b^2*(2*a^2-b^2)*sin(d*x+c)/a^2/(a^2-b^2)^2/d/cos(d*x+c)^(1/2)/(a+b*sec(
d*x+c))^(1/2)-2/3*b*(9*a^2-8*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x
+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*
x+c))/(a+b))^(1/2)/a^3/(a^2-b^2)/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)
+2/3*(3*a^4-15*a^2*b^2+8*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2
*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*cos(d*x+c)^(1/2)
*(a+b*sec(d*x+c))^(1/2)/a^3/(a^2-b^2)^2/d/((b+a*cos(d*x+c))/(a+b))^(1/2)
```

3.870.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 18.25 (sec) , antiderivative size = 507, normalized size of antiderivative = 1.60

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+b\sec(c+dx))^{5/2}}\,dx = \frac{(b+a\cos(c+dx))^3\left(-\frac{2b^3\sin(c+dx)}{3a^2(a^2-b^2)(b+a\cos(c+dx))^2}-\frac{2(-9a^2b^2\sin(c+dx)+5b^4\sin(c+dx))}{3a^2(a^2-b^2)^2(b+a\cos(c+dx))}\right)}{d\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}$$
$$-\frac{2\cos^{\frac{3}{2}}(c+dx)(b+a\cos(c+dx))^2\sec^{\frac{5}{2}}(c+dx)\left(\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\right)^{3/2}\left(-i(3a^5+3a^4b-15a^3b\right.}{}$$

input
```
Integrate[Sqrt[Cos[c + d*x]]/(a + b*Sec[c + d*x])^(5/2),x]
```

output
```
((b + a*Cos[c + d*x])^3*((-2*b^3*Sin[c + d*x])/(3*a^2*(a^2 - b^2)*(b + a*C
os[c + d*x])^2) - (2*(-9*a^2*b^2*Sin[c + d*x] + 5*b^4*Sin[c + d*x]))/(3*a^
2*(a^2 - b^2)^2*(b + a*Cos[c + d*x]))))/(d*Cos[c + d*x]^(5/2)*(a + b*Sec[c
 + d*x])^(5/2)) - (2*Cos[c + d*x]^(3/2)*(b + a*Cos[c + d*x])^2*Sec[c + d*x
]^(5/2)*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(3/2)*((-I)*(3*a^5 + 3*a^4*b - 1
5*a^3*b^2 - 15*a^2*b^3 + 8*a*b^4 + 8*b^5)*EllipticE[I*ArcSinh[Tan[(c + d*x
)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec
[(c + d*x)/2]^2)/(a + b)] + I*a*(3*a^4 - 6*a^3*b - 15*a^2*b^2 + 2*a*b^3 +
8*b^4)*EllipticF[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d
*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] - (3*a^4
- 15*a^2*b^2 + 8*b^4)*(b + a*Cos[c + d*x])*(Sec[(c + d*x)/2]^2)^(3/2)*Tan[
(c + d*x)/2]))/(3*a*(a^3 - a*b^2)^2*d*(a + b*Sec[c + d*x])^(5/2))
```

3.870.3 Rubi [A] (verified)

Time = 2.54 (sec) , antiderivative size = 353, normalized size of antiderivative = 1.11, number of steps used = 21, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.840$, Rules used = {3042, 4752, 3042, 4334, 27, 3042, 4588, 27, 3042, 4523, 3042, 4343, 3042, 3134, 3042, 3132, 4345, 3042, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+b\sec(c+dx))^{5/2}}\,dx$$
$$\Big\downarrow \; 3042$$

$$\int \frac{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \; 4752$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{5/2}}dx$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{1}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \; 4334$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{3ad\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}-\frac{2\int -\frac{3a^2-3b\sec(c+dx)a-4b^2+2b^2\sec^2(c+dx)}{2\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}}dx}{3a\left(a^2-b^2\right)}\right)$$

$$\Big\downarrow \; 27$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{3a^2-3b\sec(c+dx)a-4b^2+2b^2\sec^2(c+dx)}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}}dx}{3a\left(a^2-b^2\right)}+\frac{2b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{3ad\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}\right)$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{3a^2-3b\csc\left(c+dx+\frac{\pi}{2}\right)a-4b^2+2b^2\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{3a\left(a^2-b^2\right)}+\frac{2b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{3ad\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}\right)$$

$$\Big\downarrow \; 4588$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{8b^2\left(2a^2-b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}-\frac{2\int -\frac{3a^4-15b^2a^2-2b\left(3a^2-b^2\right)\sec(c+dx)a+8b^4}{2\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{a\left(a^2-b^2\right)}}{3a\left(a^2-b^2\right)}+\frac{2b^2\sin(c}{3ad\left(a^2-b^2\right.}\right.$$

$$\Big\downarrow \; 27$$

3.870. $\int \frac{\sqrt{\cos(c+dx)}}{(a+b\sec(c+dx))^{5/2}}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\int \frac{3a^4-15b^2a^2-2b\left(3a^2-b^2\right)\sec(c+dx)a+8b^4}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{a(a^2-b^2)}+\frac{8b^2\left(2a^2-b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad(a^2-b^2)\sqrt{a+b\sec(c+dx)}}}{3a\left(a^2-b^2\right)}+\frac{2b^2\sin(c+}{3ad\left(a^2-b^2\right)(}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\int \frac{3a^4-15b^2a^2-2b\left(3a^2-b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)a+8b^4}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a(a^2-b^2)}+\frac{8b^2\left(2a^2-b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad(a^2-b^2)\sqrt{a+b\sec(c+dx)}}}{3a\left(a^2-b^2\right)}+\frac{2b^2\sin(}{3ad\left(a^2-b\right.}\right.$$

$\downarrow$ 4523

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{\left(3a^4-15a^2b^2+8b^4\right)\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}dx}{a}-\frac{b\left(9a^4-17a^2b^2+8b^4\right)\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}dx}{a}}{a(a^2-b^2)}+\frac{8b^2\left(2a^2-b^2\right)\sin(c+dx)}{ad(a^2-b^2)\sqrt{a+b\,\mathrm{se}}}}{3a\left(a^2-b^2\right)}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{\left(3a^4-15a^2b^2+8b^4\right)\int \frac{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}-\frac{b\left(9a^4-17a^2b^2+8b^4\right)\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}}{a(a^2-b^2)}+\frac{8b^2\left(2a^2-b^2\right)\mathrm{si}}{ad(a^2-b^2)}}{3a\left(a^2-b^2\right)}\right.$$

$\downarrow$ 4343

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{\left(3a^4-15a^2b^2+8b^4\right)\sqrt{a+b\sec(c+dx)}\int \sqrt{b+a\cos(c+dx)}dx}{a\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}-\frac{b\left(9a^4-17a^2b^2+8b^4\right)\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}}{a(a^2-b^2)}+\frac{8b^2(2}{a}}{3a\left(a^2-b^2\right)}\right.$$

$\downarrow$ 3042

3.870. $\int \frac{\sqrt{\cos(c+dx)}}{(a+b\sec(c+dx))^{5/2}}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{\left(3a^4-15a^2b^2+8b^4\right)\sqrt{a+b\sec(c+dx)}\int\sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{a\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}-\frac{b\left(9a^4-17a^2b^2+8b^4\right)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}}{a(a^2-b^2)}+\frac{8}{}}{3a\left(a^2-b^2\right)}\right.$$

$\downarrow$ 3134

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{\left(3a^4-15a^2b^2+8b^4\right)\sqrt{a+b\sec(c+dx)}\int\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}dx}{a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{b\left(9a^4-17a^2b^2+8b^4\right)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}}{a(a^2-b^2)}+\frac{8}{}}{3a\left(a^2-b^2\right)}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{\left(3a^4-15a^2b^2+8b^4\right)\sqrt{a+b\sec(c+dx)}\int\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}dx}{a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{b\left(9a^4-17a^2b^2+8b^4\right)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}}{a(a^2-b^2)}-}{3a\left(a^2-b^2\right)}\right.$$

$\downarrow$ 3132

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{2\left(3a^4-15a^2b^2+8b^4\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{b\left(9a^4-17a^2b^2+8b^4\right)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}}{a(a^2-b^2)}+\frac{8b^2(2a}{ad}}{3a\left(a^2-b^2\right)}\right.$$

$\downarrow$ 4345

3.870. $\int \frac{\sqrt{\cos(c+dx)}}{(a+b\sec(c+dx))^{5/2}}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{2\left(3a^4-15a^2b^2+8b^4\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{b\left(9a^4-17a^2b^2+8b^4\right)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{}{\sqrt{b+a\,\mathrm{co}}}}{a\sqrt{a+b\sec(c+dx)}}}{a(a^2-b^2)}}{3a\left(a^2-b^2\right)}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{2\left(3a^4-15a^2b^2+8b^4\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{b\left(9a^4-17a^2b^2+8b^4\right)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{}{\sqrt{b+a\,\mathrm{si}}}}{a\sqrt{a+b\sec(c+dx)}}}{a(a^2-b^2)}}{3a\left(a^2-b^2\right)}\right.$$

$\downarrow$ 3142

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{2\left(3a^4-15a^2b^2+8b^4\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{b\left(9a^4-17a^2b^2+8b^4\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{}{\sqrt{\frac{b}{a+b}}}}{a\sqrt{a+b\sec(c+dx)}}}{a(a^2-b^2)}}{3a\left(a^2-b^2\right)}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{2\left(3a^4-15a^2b^2+8b^4\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{b\left(9a^4-17a^2b^2+8b^4\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{}{\sqrt{\frac{b}{a+b}}}}{a\sqrt{a+b\sec(c+dx)}}}{a(a^2-b^2)}}{3a\left(a^2-b^2\right)}\right.$$

$\downarrow$ 3140

3.870. $\int \frac{\sqrt{\cos(c+dx)}}{(a+b\sec(c+dx))^{5/2}}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{3ad\left(a^2-b^2\right)\left(a+b\sec(c+dx)\right)^{3/2}}+\frac{\frac{8b^2\left(2a^2-b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad(a^2-b^2)\sqrt{a+b\sec(c+dx)}}+\frac{\frac{2\left(3a^4-15a^2b^2+8b^4\right.}{ad\sqrt{\sec}}}{}}{}\right.$$

input
```
Int[Sqrt[Cos[c + d*x]]/(a + b*Sec[c + d*x])^(5/2),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*b^2*Sqrt[Sec[c + d*x]]*Sin[c + d
*x])/(3*a*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^(3/2)) + (((-2*b*(9*a^4 - 17*
a^2*b^2 + 8*b^4)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2,
 (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(a*d*Sqrt[a + b*Sec[c + d*x]]) + (2*(3
*a^4 - 15*a^2*b^2 + 8*b^4)*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a +
b*Sec[c + d*x]])/(a*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]
]))/(a*(a^2 - b^2)) + (8*b^2*(2*a^2 - b^2)*Sqrt[Sec[c + d*x]]*Sin[c + d*x]
)/(a*(a^2 - b^2)*d*Sqrt[a + b*Sec[c + d*x]]))/(3*a*(a^2 - b^2)))
```

3.870.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 4334
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*
((d*Csc[e + f*x])^n/(a*f*(m + 1)*(a^2 - b^2))), x] + Simp[1/(a*(m + 1)*(a^2
 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a^2*(m + 1)
 - b^2*(m + n + 1) - a*b*(m + 1)*Csc[e + f*x] + b^2*(m + n + 2)*Csc[e + f*x
]^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0] && LtQ[m,
 -1] && IntegersQ[2*m, 2*n]
```

rule 4343
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]])   Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a
, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4345
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]])   Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[
{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4523
```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Simp[A/a   I
nt[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Simp[(A*b - a*B)
/(a*d)   Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ
[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

3.870. $\int \frac{\sqrt{\cos(c+dx)}}{(a+b\sec(c+dx))^{5/2}}\,dx$

rule 4588
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc
[e + f*x])^(m + 1)*((d*Csc[e + f*x])^n/(a*f*(m + 1)*(a^2 - b^2))), x] + Sim
p[1/(a*(m + 1)*(a^2 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f
*x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n +
1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m
+ n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x
] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

3.870.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 4119 vs. $2(347) = 694$.

Time = 9.19 (sec) , antiderivative size = 4120, normalized size of antiderivative = 13.00

method	result	size
default	Expression too large to display	4120

input
```
int(cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/3/d/((a-b)/(a+b))^(1/2)/(a+b)^2/(a-b)/a^3*(-((1-cos(d*x+c))^2*csc(d*x+c)
^2-1)/((1-cos(d*x+c))^2*csc(d*x+c)^2+1))^(1/2)*((a*(1-cos(d*x+c))^2*csc(d*
x+c)^2-b*(1-cos(d*x+c))^2*csc(d*x+c)^2-a-b)/((1-cos(d*x+c))^2*csc(d*x+c)^2
-1))^(1/2)*(3*(-(a*(1-cos(d*x+c))^2*csc(d*x+c)^2-b*(1-cos(d*x+c))^2*csc(d*
x+c)^2-a-b)/(a+b))^(1/2)*((1-cos(d*x+c))^2*csc(d*x+c)^2+1)^(1/2)*EllipticF
(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*a^5*(1
-cos(d*x+c))^2*csc(d*x+c)^2+3*(-(a*(1-cos(d*x+c))^2*csc(d*x+c)^2-b*(1-cos(
d*x+c))^2*csc(d*x+c)^2-a-b)/(a+b))^(1/2)*((1-cos(d*x+c))^2*csc(d*x+c)^2+1)
^(1/2)*EllipticE(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b
))^(1/2))*a^4*b*(1-cos(d*x+c))^2*csc(d*x+c)^2+15*(-(a*(1-cos(d*x+c))^2*csc
(d*x+c)^2-b*(1-cos(d*x+c))^2*csc(d*x+c)^2-a-b)/(a+b))^(1/2)*((1-cos(d*x+c)
)^2*csc(d*x+c)^2+1)^(1/2)*EllipticE(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d
*x+c)),(-(a+b)/(a-b))^(1/2))*a^3*b^2*(1-cos(d*x+c))^2*csc(d*x+c)^2-15*(-(a
*(1-cos(d*x+c))^2*csc(d*x+c)^2-b*(1-cos(d*x+c))^2*csc(d*x+c)^2-a-b)/(a+b))
^(1/2)*((1-cos(d*x+c))^2*csc(d*x+c)^2+1)^(1/2)*EllipticE(((a-b)/(a+b))^(1/
2)*(-cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*a^2*b^3*(1-cos(d*x+c))^2
*csc(d*x+c)^2-8*(-(a*(1-cos(d*x+c))^2*csc(d*x+c)^2-b*(1-cos(d*x+c))^2*csc(
d*x+c)^2-a-b)/(a+b))^(1/2)*((1-cos(d*x+c))^2*csc(d*x+c)^2+1)^(1/2)*Ellipti
cE(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*a*b^
4*(1-cos(d*x+c))^2*csc(d*x+c)^2-15*((a-b)/(a+b))^(1/2)*a^3*b^2*(1-cos(d...
```

3.870.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.16 (sec) , antiderivative size = 884, normalized size of antiderivative = 2.79

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+b\sec(c+dx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
1/9*(6*(8*a^4*b^3 - 4*a^2*b^5 + (9*a^5*b^2 - 5*a^3*b^4)*cos(d*x + c))*sqrt
((a*cos(d*x + c) + b)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 4*(s
qrt(2)*(-6*I*a^6*b + 9*I*a^4*b^3 - 4*I*a^2*b^5)*cos(d*x + c)^2 + 2*sqrt(2)
*(-6*I*a^5*b^2 + 9*I*a^3*b^4 - 4*I*a*b^6)*cos(d*x + c) + sqrt(2)*(-6*I*a^4
*b^3 + 9*I*a^2*b^5 - 4*I*b^7))*sqrt(a)*weierstrassPInverse(-4/3*(3*a^2 - 4
*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d
*x + c) + 2*b)/a) - 4*(sqrt(2)*(6*I*a^6*b - 9*I*a^4*b^3 + 4*I*a^2*b^5)*cos
(d*x + c)^2 + 2*sqrt(2)*(6*I*a^5*b^2 - 9*I*a^3*b^4 + 4*I*a*b^6)*cos(d*x +
c) + sqrt(2)*(6*I*a^4*b^3 - 9*I*a^2*b^5 + 4*I*b^7))*sqrt(a)*weierstrassPIn
verse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d
*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a) - 3*(sqrt(2)*(-3*I*a^7 + 15*I*a^5*b
^2 - 8*I*a^3*b^4)*cos(d*x + c)^2 + 2*sqrt(2)*(-3*I*a^6*b + 15*I*a^4*b^3 -
8*I*a^2*b^5)*cos(d*x + c) + sqrt(2)*(-3*I*a^5*b^2 + 15*I*a^3*b^4 - 8*I*a*b
^6))*sqrt(a)*weierstrassZeta(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b
^3)/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b
^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x + c) + 2*b)/a)) - 3*(sqrt(2
)*(3*I*a^7 - 15*I*a^5*b^2 + 8*I*a^3*b^4)*cos(d*x + c)^2 + 2*sqrt(2)*(3*I*a
^6*b - 15*I*a^4*b^3 + 8*I*a^2*b^5)*cos(d*x + c) + sqrt(2)*(3*I*a^5*b^2 - 1
5*I*a^3*b^4 + 8*I*a*b^6))*sqrt(a)*weierstrassZeta(-4/3*(3*a^2 - 4*b^2)/a^2
, 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/...
```

3.870.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+b\sec(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(1/2)/(a+b*sec(d*x+c))**(5/2),x)
```

output
```
Timed out
```

3.870.7 Maxima [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{\sqrt{\cos(dx+c)}}{(b\sec(dx+c)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(cos(d*x + c))/(b*sec(d*x + c) + a)^(5/2), x)
```

3.870.8 Giac [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{\sqrt{\cos(dx+c)}}{(b\sec(dx+c)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate(sqrt(cos(d*x + c))/(b*sec(d*x + c) + a)^(5/2), x)
```

3.870.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{\sqrt{\cos(c+d\,x)}}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

input
```
int(cos(c + d*x)^(1/2)/(a + b/cos(c + d*x))^(5/2),x)
```

output
```
int(cos(c + d*x)^(1/2)/(a + b/cos(c + d*x))^(5/2), x)
```

3.871 $\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^{5/2}}\,dx$

3.871.1 Optimal result . . . 6589
3.871.2 Mathematica [C] (verified) . . . 6590
3.871.3 Rubi [A] (verified) . . . 6590
3.871.4 Maple [B] (warning: unable to verify) . . . 6597
3.871.5 Fricas [C] (verification not implemented) . . . 6598
3.871.6 Sympy [F(-1)] . . . 6598
3.871.7 Maxima [F] . . . 6599
3.871.8 Giac [F] . . . 6599
3.871.9 Mupad [F(-1)] . . . 6599

3.871.1 Optimal result

Integrand size = 25, antiderivative size = 302

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^{5/2}}\,dx = \frac{2(3a^2-2b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{3a^2(a^2-b^2)d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} + \frac{4b(3a^2-b^2)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+dx)}}{3a^2(a^2-b^2)^2d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}} - \frac{2b\sin(c+dx)}{3(a^2-b^2)d\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^{3/2}} - \frac{2b(5a^2-b^2)\sin(c+dx)}{3a(a^2-b^2)^2d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}$$

output
```
-2/3*b*sin(d*x+c)/(a^2-b^2)/d/(a+b*sec(d*x+c))^(3/2)/cos(d*x+c)^(1/2)-2/3*
b*(5*a^2-b^2)*sin(d*x+c)/a/(a^2-b^2)^2/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))
^(1/2)+2/3*(3*a^2-2*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*E
llipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+
b))^(1/2)/a^2/(a^2-b^2)/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)+4/3*b*(3
*a^2-b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/
2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1
/2)/a^2/(a^2-b^2)^2/d/((b+a*cos(d*x+c))/(a+b))^(1/2)
```

3.871.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 8.73 (sec) , antiderivative size = 398, normalized size of antiderivative = 1.32

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^{5/2}}\,dx = \frac{(b+a\cos(c+dx))^2\left(\frac{2b(-5a^2b+b^3+(-6a^3+2ab^2)\cos(c+dx))\sin(c+dx)}{a(a^2-b^2)^2(b+a\cos(c+dx))} - \right.}{}$$

input
```
Integrate[1/(Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x])^(5/2)),x]
```

output
```
((b + a*Cos[c + d*x])^2*((2*b*(-5*a^2*b + b^3 + (-6*a^3 + 2*a*b^2)*Cos[c +
 d*x])*Sin[c + d*x])/(a*(a^2 - b^2)^2*(b + a*Cos[c + d*x])) - (2*(Cos[(c +
 d*x)/2]^2*Sec[c + d*x])^(3/2)*((2*I)*b*(-3*a^3 - 3*a^2*b + a*b^2 + b^3)*E
llipticE[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2
*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] + I*a*(3*a^3 + 6*
a^2*b + a*b^2 - 2*b^3)*EllipticF[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a
+ b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a
 + b)] + 2*b*(-3*a^2 + b^2)*(b + a*Cos[c + d*x])*(Sec[(c + d*x)/2]^2)^(3/2
)*Tan[(c + d*x)/2]))/((a^3 - a*b^2)^2*Sec[c + d*x]^(3/2))))/(3*d*Cos[c + d
*x]^(5/2)*(a + b*Sec[c + d*x])^(5/2))
```

3.871.3 Rubi [A] (verified)

Time = 2.38 (sec) , antiderivative size = 335, normalized size of antiderivative = 1.11, number of steps used = 21, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.840$, Rules used = {3042, 4752, 3042, 4330, 27, 3042, 4588, 27, 3042, 4523, 3042, 4343, 3042, 3134, 3042, 3132, 4345, 3042, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^{5/2}}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \; 4752$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sqrt{\sec(c+dx)}}{(a+b\sec(c+dx))^{5/2}}dx$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \; 4330$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{2\int -\frac{-2b\sec^2(c+dx)+3a\sec(c+dx)+b}{2\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}}dx}{3\left(a^2-b^2\right)}-\frac{2b\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}\right)$$

$$\Big\downarrow \; 27$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{-2b\sec^2(c+dx)+3a\sec(c+dx)+b}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}}dx}{3\left(a^2-b^2\right)}-\frac{2b\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}\right)$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{-2b\csc\left(c+dx+\frac{\pi}{2}\right)^2+3a\csc\left(c+dx+\frac{\pi}{2}\right)+b}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{3\left(a^2-b^2\right)}-\frac{2b\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}\right)$$

$$\Big\downarrow \; 4588$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{-\frac{2\int -\frac{2b\left(3a^2-b^2\right)+a\left(3a^2+b^2\right)\sec(c+dx)}{2\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{a\left(a^2-b^2\right)}-\frac{2b\left(5a^2-b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}}{3\left(a^2-b^2\right)}-\frac{2b\sin(c+dx}{3d\left(a^2-b^2\right)(a+}\right.$$

$$\Big\downarrow \; 27$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\int \frac{2b\left(3a^2-b^2\right)+a\left(3a^2+b^2\right)\sec(c+dx)}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{a\left(a^2-b^2\right)}-\frac{2b\left(5a^2-b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}}{3\left(a^2-b^2\right)}-\frac{2b\sin(c+dx)\sqrt{\mathrm{s}}}{3d\left(a^2-b^2\right)(a+b\mathrm{se}}\right.$$

$$\Big\downarrow \; 3042$$

3.871. $\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^{5/2}}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\int \frac{2b\left(3a^2-b^2\right)+a\left(3a^2+b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a(a^2-b^2)}-\frac{2b(5a^2-b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad(a^2-b^2)\sqrt{a+b\sec(c+dx)}}}{3\left(a^2-b^2\right)}-\frac{2b\sin(c+dx)}{3d\left(a^2-b^2\right)(a+}\right.$$

$\downarrow$ 4523

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{2b\left(3a^2-b^2\right)\int\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}dx}{a}+\frac{\left(3a^4-5a^2b^2+2b^4\right)\int\frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}dx}{a}}{a(a^2-b^2)}-\frac{2b(5a^2-b^2)\sin(c+dx)\sqrt{\sec(c+d}}{ad(a^2-b^2)\sqrt{a+b\sec(c+dx)}}}{3\left(a^2-b^2\right)}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{2b\left(3a^2-b^2\right)\int\frac{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}+\frac{\left(3a^4-5a^2b^2+2b^4\right)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}}{a(a^2-b^2)}-\frac{2b(5a^2-b^2)\sin(c+dx)\sqrt{}}{ad(a^2-b^2)\sqrt{a+b\sec}}}{3\left(a^2-b^2\right)}\right.$$

$\downarrow$ 4343

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{2b\left(3a^2-b^2\right)\sqrt{a+b\sec(c+dx)}\int\sqrt{b+a\cos(c+dx)}dx}{a\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}+\frac{\left(3a^4-5a^2b^2+2b^4\right)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}}{a(a^2-b^2)}-\frac{2b(5a^2-b^2)\sin}{ad(a^2-b^2)}}{3\left(a^2-b^2\right)}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{2b\left(3a^2-b^2\right)\sqrt{a+b\sec(c+dx)}\int\sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{a\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}+\frac{\left(3a^4-5a^2b^2+2b^4\right)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}}{a(a^2-b^2)}-\frac{2b(5a^2-b^2}{ad(a^2-}}{3\left(a^2-b^2\right)}\right.$$

3.871. $\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^{5/2}}\,dx$

$$\Big\downarrow \; 3134$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{2b\left(3a^2-b^2\right)\sqrt{a+b\sec(c+dx)}\int\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}dx}{a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{\left(3a^4-5a^2b^2+2b^4\right)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}}{a(a^2-b^2)}-\frac{2b(5a^2-b^2}{ad(a^2-}}{3\left(a^2-b^2\right)}\right.$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{2b\left(3a^2-b^2\right)\sqrt{a+b\sec(c+dx)}\int\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}dx}{a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{\left(3a^4-5a^2b^2+2b^4\right)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}}{a(a^2-b^2)}-\frac{2b(5a^2}{ad}}{3\left(a^2-b^2\right)}\right.$$

$$\Big\downarrow \; 3132$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{\left(3a^4-5a^2b^2+2b^4\right)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}+\frac{4b\left(3a^2-b^2\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{a(a^2-b^2)}-\frac{2b(5a^2-b^2)\sin(}{ad(a^2-b^2)\sqrt{}}}{3\left(a^2-b^2\right)}\right.$$

$$\Big\downarrow \; 4345$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{\left(3a^4-5a^2b^2+2b^4\right)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{1}{\sqrt{b+a\cos(c+dx)}}dx}{a\sqrt{a+b\sec(c+dx)}}+\frac{4b\left(3a^2-b^2\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{a(a^2-b^2)}}{3\left(a^2-b^2\right)}\right.$$

$$\Big\downarrow \; 3042$$

3.871. $\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^{5/2}}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{\left(3a^4-5a^2b^2+2b^4\right)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{1}{\sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{a\sqrt{a+b\sec(c+dx)}}+\frac{4b\left(3a^2-b^2\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{}{a}\right.}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{a(a^2-b^2)}}{3\left(a^2-b^2\right)}\right.$$

$\downarrow$ 3142

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{\left(3a^4-5a^2b^2+2b^4\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}}dx}{a\sqrt{a+b\sec(c+dx)}}+\frac{4b\left(3a^2-b^2\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\right.}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{a(a^2-b^2)}}{3\left(a^2-b^2\right)}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{\left(3a^4-5a^2b^2+2b^4\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{a\sqrt{a+b\sec(c+dx)}}+\frac{4b\left(3a^2-b^2\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+\right.}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+}{a+b}}}}{a(a^2-b^2)}}{3\left(a^2-b^2\right)}\right.$$

$\downarrow$ 3140

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{4b\left(3a^2-b^2\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{2\left(3a^4-5a^2b^2+2b^4\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\right.}{ad\sqrt{a+b\sec(c+dx)}}}{a(a^2-b^2)}}{3\left(a^2-b^2\right)}\right.$$

input
```
Int[1/(Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x])^(5/2)),x]
```

3.871. $\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^{5/2}}\,dx$

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((-2*b*Sqrt[Sec[c + d*x]]*Sin[c + d*
x])/(3*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^(3/2)) + (((2*(3*a^4 - 5*a^2*b^2
 + 2*b^4)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/
(a + b)]*Sqrt[Sec[c + d*x]])/(a*d*Sqrt[a + b*Sec[c + d*x]]) + (4*b*(3*a^2
- b^2)*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(a*
d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]))/(a*(a^2 - b^2))
- (2*b*(5*a^2 - b^2)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(a*(a^2 - b^2)*d*Sqr
t[a + b*Sec[c + d*x]]))/(3*(a^2 - b^2)))
```

3.871.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

3.871. $\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^{5/2}}\,dx$

rule 4330
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b)*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m +
1)*((d*Csc[e + f*x])^(n - 1)/(f*(m + 1)*(a^2 - b^2))), x] + Simp[1/((m + 1)
*(a^2 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1)*S
imp[b*d*(n - 1) + a*d*(m + 1)*Csc[e + f*x] - b*d*(m + n + 1)*Csc[e + f*x]^2
, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1
] && LtQ[0, n, 1] && IntegersQ[2*m, 2*n]
```

rule 4343
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]])   Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a
, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4345
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]])   Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[
{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4523
```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Simp[A/a   I
nt[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Simp[(A*b - a*B)
/(a*d)   Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ
[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

rule 4588
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc
[e + f*x])^(m + 1)*((d*Csc[e + f*x])^n/(a*f*(m + 1)*(a^2 - b^2))), x] + Sim
p[1/(a*(m + 1)*(a^2 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f
*x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n +
1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m
+ n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x
] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

3.871. $\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^{5/2}}\,dx$

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

3.871.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 2884 vs. $2(332) = 664$.

Time = 8.10 (sec) , antiderivative size = 2885, normalized size of antiderivative = 9.55

method	result	size
default	Expression too large to display	2885

input
```
int(1/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3/d/((a-b)/(a+b))^(1/2)/(a+b)^2/(a-b)/a^2*((1-cos(d*x+c))^2*csc(d*x+c)^
2-1)*((a*(1-cos(d*x+c))^2*csc(d*x+c)^2-b*(1-cos(d*x+c))^2*csc(d*x+c)^2-a-b
)/((1-cos(d*x+c))^2*csc(d*x+c)^2-1))^(1/2)*(2*((a-b)/(a+b))^(1/2)*b^4*(-co
t(d*x+c)+csc(d*x+c))-2*(-(a*(1-cos(d*x+c))^2*csc(d*x+c)^2-b*(1-cos(d*x+c))
^2*csc(d*x+c)^2-a-b)/(a+b))^(1/2)*((1-cos(d*x+c))^2*csc(d*x+c)^2+1)^(1/2)*
EllipticE(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2
))*b^4*(1-cos(d*x+c))^2*csc(d*x+c)^2-(-(a*(1-cos(d*x+c))^2*csc(d*x+c)^2-b*
(1-cos(d*x+c))^2*csc(d*x+c)^2-a-b)/(a+b))^(1/2)*((1-cos(d*x+c))^2*csc(d*x+
c)^2+1)^(1/2)*EllipticF(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(-(a+
b)/(a-b))^(1/2))*a^2*b^2*(1-cos(d*x+c))^2*csc(d*x+c)^2-2*(-(a*(1-cos(d*x+c
))^2*csc(d*x+c)^2-b*(1-cos(d*x+c))^2*csc(d*x+c)^2-a-b)/(a+b))^(1/2)*((1-co
s(d*x+c))^2*csc(d*x+c)^2+1)^(1/2)*EllipticF(((a-b)/(a+b))^(1/2)*(-cot(d*x+
c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*a*b^3*(1-cos(d*x+c))^2*csc(d*x+c)^2-6
*(-(a*(1-cos(d*x+c))^2*csc(d*x+c)^2-b*(1-cos(d*x+c))^2*csc(d*x+c)^2-a-b)/(
a+b))^(1/2)*((1-cos(d*x+c))^2*csc(d*x+c)^2+1)^(1/2)*EllipticE(((a-b)/(a+b)
)^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*a^3*b*(1-cos(d*x+c)
)^2*csc(d*x+c)^2+6*(-(a*(1-cos(d*x+c))^2*csc(d*x+c)^2-b*(1-cos(d*x+c))^2*c
sc(d*x+c)^2-a-b)/(a+b))^(1/2)*((1-cos(d*x+c))^2*csc(d*x+c)^2+1)^(1/2)*Elli
pticE(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*a
^2*b^2*(1-cos(d*x+c))^2*csc(d*x+c)^2+2*(-(a*(1-cos(d*x+c))^2*csc(d*x+c)...
```

3.871.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.14 (sec) , antiderivative size = 835, normalized size of antiderivative = 2.76

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
-1/9*(6*(5*a^4*b^2 - a^2*b^4 + 2*(3*a^5*b - a^3*b^3)*cos(d*x + c))*sqrt((a
*cos(d*x + c) + b)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - (sqrt(2
)*(-9*I*a^6 + 9*I*a^4*b^2 - 4*I*a^2*b^4)*cos(d*x + c)^2 - 2*sqrt(2)*(9*I*a
^5*b - 9*I*a^3*b^3 + 4*I*a*b^5)*cos(d*x + c) + sqrt(2)*(-9*I*a^4*b^2 + 9*I
*a^2*b^4 - 4*I*b^6))*sqrt(a)*weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2,
 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x + c) +
2*b)/a) - (sqrt(2)*(9*I*a^6 - 9*I*a^4*b^2 + 4*I*a^2*b^4)*cos(d*x + c)^2 -
2*sqrt(2)*(-9*I*a^5*b + 9*I*a^3*b^3 - 4*I*a*b^5)*cos(d*x + c) + sqrt(2)*(9
*I*a^4*b^2 - 9*I*a^2*b^4 + 4*I*b^6))*sqrt(a)*weierstrassPInverse(-4/3*(3*a
^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a
*sin(d*x + c) + 2*b)/a) + 6*(sqrt(2)*(-3*I*a^5*b + I*a^3*b^3)*cos(d*x + c)
^2 + 2*sqrt(2)*(-3*I*a^4*b^2 + I*a^2*b^4)*cos(d*x + c) + sqrt(2)*(-3*I*a^3
*b^3 + I*a*b^5))*sqrt(a)*weierstrassZeta(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9
*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9
*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x + c) + 2*b)/a))
 + 6*(sqrt(2)*(3*I*a^5*b - I*a^3*b^3)*cos(d*x + c)^2 + 2*sqrt(2)*(3*I*a^4*
b^2 - I*a^2*b^4)*cos(d*x + c) + sqrt(2)*(3*I*a^3*b^3 - I*a*b^5))*sqrt(a)*w
eierstrassZeta(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, weier
strassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(
3*a*cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a)))/((a^9 - 2*a^7*b^2 + a...
```

3.871.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/cos(d*x+c)**(1/2)/(a+b*sec(d*x+c))**(5/2),x)
```

output
```
Timed out
```

3.871.7 Maxima [F]

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{1}{(b\sec(dx+c)+a)^{\frac{5}{2}}\sqrt{\cos(dx+c)}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*sec(d*x + c) + a)^(5/2)*sqrt(cos(d*x + c))), x)
```

3.871.8 Giac [F]

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{1}{(b\sec(dx+c)+a)^{\frac{5}{2}}\sqrt{\cos(dx+c)}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate(1/((b*sec(d*x + c) + a)^(5/2)*sqrt(cos(d*x + c))), x)
```

3.871.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{1}{\sqrt{\cos(c+d\,x)}\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

input
```
int(1/(cos(c + d*x)^(1/2)*(a + b/cos(c + d*x))^(5/2)),x)
```

output
```
int(1/(cos(c + d*x)^(1/2)*(a + b/cos(c + d*x))^(5/2)), x)
```

3.872 $\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx$

3.872.1 Optimal result 6600
3.872.2 Mathematica [C] (warning: unable to verify) 6601
3.872.3 Rubi [A] (verified) 6601
3.872.4 Maple [B] (verified) 6608
3.872.5 Fricas [C] (verification not implemented) 6609
3.872.6 Sympy [F(-1)] 6609
3.872.7 Maxima [F] 6610
3.872.8 Giac [F] 6610
3.872.9 Mupad [F(-1)] 6610

3.872.1 Optimal result

Integrand size = 25, antiderivative size = 281

$$\begin{aligned}\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx &= \\ &-\frac{2b\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{3a\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} \\ &-\frac{2(3a^2+b^2)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+dx)}}{3a\left(a^2-b^2\right)^2 d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}} \\ &+\frac{2a\sin(c+dx)}{3\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^{3/2}} \\ &+\frac{4(a^2+b^2)\sin(c+dx)}{3\left(a^2-b^2\right)^2 d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}\end{aligned}$$

output
```
2/3*a*sin(d*x+c)/(a^2-b^2)/d/(a+b*sec(d*x+c))^(3/2)/cos(d*x+c)^(1/2)+4/3*(
a^2+b^2)*sin(d*x+c)/(a^2-b^2)^2/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)-
2/3*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*
x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)/a/(a^2-b^
2)/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)-2/3*(3*a^2+b^2)*(cos(1/2*d*x+
1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a
/(a+b))^(1/2))*cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2)/a/(a^2-b^2)^2/d/((b
+a*cos(d*x+c))/(a+b))^(1/2)
```

3.872.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 11.97 (sec) , antiderivative size = 447, normalized size of antiderivative = 1.59

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx = \frac{(b+a\cos(c+dx))^3\left(\frac{2b\sin(c+dx)}{3(-a^2+b^2)(b+a\cos(c+dx))^2}+\frac{2(3a^2\sin(c+dx)+b^2\sin(}{3(-a^2+b^2)^2(b+a\cos(c}\right.}{d\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}$$
$$+\frac{2\cos^{\frac{3}{2}}(c+dx)(b+a\cos(c+dx))^2\sec^{\frac{5}{2}}(c+dx)\left(\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\right)^{3/2}\left(-i(3a^3+3a^2b+ab^2+\right.}{}$$

input
```
Integrate[1/(Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^(5/2)),x]
```

output
```
((b + a*Cos[c + d*x])^3*((2*b*Sin[c + d*x])/(3*(-a^2 + b^2)*(b + a*Cos[c +
 d*x])^2) + (2*(3*a^2*Sin[c + d*x] + b^2*Sin[c + d*x]))/(3*(-a^2 + b^2)^2*
(b + a*Cos[c + d*x]))))/(d*Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^(5/2))
+ (2*Cos[c + d*x]^(3/2)*(b + a*Cos[c + d*x])^2*Sec[c + d*x]^(5/2)*(Cos[(c
+ d*x)/2]^2*Sec[c + d*x])^(3/2)*((-I)*(3*a^3 + 3*a^2*b + a*b^2 + b^3)*Elli
pticE[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sq
rt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] + I*a*(3*a^2 + 4*a*b
 + b^2)*EllipticF[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c +
d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] - (3*a^2
 + b^2)*(b + a*Cos[c + d*x])*(Sec[(c + d*x)/2]^2)^(3/2)*Tan[(c + d*x)/2]))
/(3*a*(a^2 - b^2)^2*d*(a + b*Sec[c + d*x])^(5/2))
```

3.872.3 Rubi [A] (verified)

Time = 2.19 (sec) , antiderivative size = 309, normalized size of antiderivative = 1.10, number of steps used = 21, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.840$, Rules used = {3042, 4752, 3042, 4331, 27, 3042, 4588, 27, 3042, 4523, 3042, 4343, 3042, 3134, 3042, 3132, 4345, 3042, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx$$
$$\Big\downarrow \; 3042$$

$$\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{3/2}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \; 4752$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}dx$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{3/2}}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \; 4331$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\int -\frac{-2a\sec^2(c+dx)+3b\sec(c+dx)+a}{2\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}}dx}{3\left(a^2-b^2\right)}+\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}\right)$$

$$\Big\downarrow \; 27$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}-\frac{\int \frac{-2a\sec^2(c+dx)+3b\sec(c+dx)+a}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}}dx}{3\left(a^2-b^2\right)}\right)$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}-\frac{\int \frac{-2a\csc\left(c+dx+\frac{\pi}{2}\right)^2+3b\csc\left(c+dx+\frac{\pi}{2}\right)+a}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{3\left(a^2-b^2\right)}\right)$$

$$\Big\downarrow \; 4588$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}-\frac{-\frac{2\int -\frac{4b\sec(c+dx)a^2+\left(3a^2+b^2\right)a}{2\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{a\left(a^2-b^2\right)}-\frac{4\left(a^2+b^2\right)\sin(c+d}{d\left(a^2-b^2\right)\sqrt{a+}}}{3\left(a^2-b^2\right)}\right)$$

$$\Big\downarrow \; 27$$

3.872. $\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}-\frac{\frac{\int\frac{4b\sec(c+dx)a^2+\left(3a^2+b^2\right)a}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{a(a^2-b^2)}-\frac{4\left(a^2+b^2\right)\sin(c+dx)\sqrt{\sec}}{d(a^2-b^2)\sqrt{a+b\sec(c}}}{3\left(a^2-b^2\right)}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}-\frac{\frac{\int\frac{4b\csc\left(c+dx+\frac{\pi}{2}\right)a^2+\left(3a^2+b^2\right)a}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a(a^2-b^2)}-\frac{4\left(a^2+b^2\right)\sin(c+}{d(a^2-b^2)\sqrt{a}}}{3\left(a^2-b^2\right)}\right.$$

$\downarrow$ 4523

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}-\frac{\frac{b\left(a^2-b^2\right)\int\frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}dx+\left(3a^2+b^2\right)\int\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}}{a(a^2-b^2)}}{3\left(a^2-b^2\right)}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}-\frac{\frac{b\left(a^2-b^2\right)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\left(3a^2+b^2\right)\int\frac{\sqrt{a+b\csc\left(c+}}{\sqrt{\csc\left(c+d}}}{a(a^2-b^2)}}{3\left(a^2-b^2\right)}\right.$$

$\downarrow$ 4343

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}-\frac{\frac{b\left(a^2-b^2\right)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{\left(3a^2+b^2\right)\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}\sqrt{a\,c}}}{a(a^2-b^2)}}{3\left(a^2-\right.}\right.$$

$\downarrow$ 3042

3.872. $\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}-\frac{\frac{b\left(a^2-b^2\right)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{\left(3a^2+b^2\right)\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}\sqrt{}}}{a\left(a^2-b^2\right)}}{3\left(a^2\right.}\right.$$

$\downarrow$ 3134

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}-\frac{\frac{b\left(a^2-b^2\right)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{\left(3a^2+b^2\right)\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}\sqrt{}}}{a\left(a^2-b^2\right)}}{3\left(a^2\right.}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}-\frac{\frac{b\left(a^2-b^2\right)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{\left(3a^2+b^2\right)\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}}{a\left(a^2-b^2\right)}}{3\left(a\right.}\right.$$

$\downarrow$ 3132

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}-\frac{\frac{b\left(a^2-b^2\right)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\left(3a^2+b^2\right)\sqrt{a+b\sec(c+dx}}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\,c}{}}}}{a\left(a^2-b^2\right)}}{3\left(a^2-\right.}\right.$$

$\downarrow$ 4345

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}-\frac{\frac{b\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{1}{\sqrt{b+a\cos(c+dx)}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{2(}{}}{a\left(a^2-b^2\right)}\right.$$

$\downarrow$ 3042

3.872. $\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}-\frac{\frac{b\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{1}{\sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{\sqrt{a+b\sec(c+dx)}}}{a(a^2-b^2)}\right.$$

$\downarrow$ 3142

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}-\frac{\frac{b\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}}{a(a^2-b^2)}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}-\frac{\frac{b\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}}{\sqrt{a+b\sec(c+dx)}}}{a(a^2-b^2)}\right.$$

$\downarrow$ 3140

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}-\frac{\frac{2b\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}}{a(a^2-b^2)}\right.$$

input
```
Int[1/(Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^(5/2)),x]
```

3.872. $\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx$

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*a*Sqrt[Sec[c + d*x]]*Sin[c + d*x
])/(3*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^(3/2)) - (((2*b*(a^2 - b^2)*Sqrt[
(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[S
ec[c + d*x]])/(d*Sqrt[a + b*Sec[c + d*x]]) + (2*(3*a^2 + b^2)*EllipticE[(c
 + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(d*Sqrt[(b + a*Cos[c +
 d*x])/(a + b)]*Sqrt[Sec[c + d*x]]))/(a*(a^2 - b^2)) - (4*(a^2 + b^2)*Sqrt
[Sec[c + d*x]]*Sin[c + d*x])/((a^2 - b^2)*d*Sqrt[a + b*Sec[c + d*x]]))/(3*
(a^2 - b^2)))
```

3.872.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

3.872. $\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx$

rule 4331
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[a*d^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
)*((d*Csc[e + f*x])^(n - 2)/(f*(m + 1)*(a^2 - b^2))), x] - Simp[d^2/((m + 1
)*(a^2 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*
(a*(n - 2) + b*(m + 1)*Csc[e + f*x] - a*(m + n)*Csc[e + f*x]^2), x], x] /;
FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && LtQ[1, n, 2
] && IntegersQ[2*m, 2*n]
```

rule 4343
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]])   Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a
, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4345
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]])   Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[
{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4523
```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Simp[A/a   I
nt[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Simp[(A*b - a*B)
/(a*d)   Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ
[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

rule 4588
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc
[e + f*x])^(m + 1)*((d*Csc[e + f*x])^n/(a*f*(m + 1)*(a^2 - b^2))), x] + Sim
p[1/(a*(m + 1)*(a^2 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f
*x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n +
1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m
+ n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x
] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

3.872. $\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx$

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

3.872.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2303 vs. $2(311) = 622$.

Time = 5.80 (sec) , antiderivative size = 2304, normalized size of antiderivative = 8.20

method	result	size
default	Expression too large to display	2304

input
```
int(1/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3/d/(a-b)/(a+b)^2/a/((a-b)/(a+b))^(1/2)*(-5*EllipticF(((a-b)/(a+b))^(1/
2)*(-cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*(1/(cos(d*x+c)+1))^(1/2)
*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^2*b*cos(d*x+c)+2*Ellipt
icF(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*(1/
(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a*b^
2*cos(d*x+c)+6*EllipticE(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(-(a
+b)/(a-b))^(1/2))*(1/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(
d*x+c)+1))^(1/2)*a^2*b*cos(d*x+c)+EllipticE(((a-b)/(a+b))^(1/2)*(-cot(d*x+
c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*(1/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+
a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a*b^2*cos(d*x+c)+2*EllipticE(((a-b)/(a
+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*co
s(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a*b^2*cos(d*x+c)^
2-EllipticF(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1
/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1
/2)*a^2*b*cos(d*x+c)^2+EllipticF(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+
c)),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*
(1/(cos(d*x+c)+1))^(1/2)*a*b^2*cos(d*x+c)^2+EllipticE(((a-b)/(a+b))^(1/2)*
(-cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(
cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a*b^2*cos(d*x+c)^3+EllipticF
(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*(1/...
```

3.872.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.14 (sec) , antiderivative size = 782, normalized size of antiderivative = 2.78

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx = \frac{6\left(2\,a^4b+2\,a^2b^3+\left(3\,a^5+a^3b^2\right)\cos\left(dx+c\right)\right)\sqrt{\frac{a\cos(dx+c)+b}{\cos(dx+c)}}\sqrt{\cos}}{}$$

input
```
integrate(1/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
1/9*(6*(2*a^4*b + 2*a^2*b^3 + (3*a^5 + a^3*b^2)*cos(d*x + c))*sqrt((a*cos(
d*x + c) + b)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 2*(sqrt(2)*(
-3*I*a^4*b + I*a^2*b^3)*cos(d*x + c)^2 + 2*sqrt(2)*(-3*I*a^3*b^2 + I*a*b^4
)*cos(d*x + c) + sqrt(2)*(-3*I*a^2*b^3 + I*b^5))*sqrt(a)*weierstrassPInver
se(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x
+ c) + 3*I*a*sin(d*x + c) + 2*b)/a) - 2*(sqrt(2)*(3*I*a^4*b - I*a^2*b^3)*c
os(d*x + c)^2 + 2*sqrt(2)*(3*I*a^3*b^2 - I*a*b^4)*cos(d*x + c) + sqrt(2)*(
3*I*a^2*b^3 - I*b^5))*sqrt(a)*weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2
, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c) +
 2*b)/a) - 3*(sqrt(2)*(3*I*a^5 + I*a^3*b^2)*cos(d*x + c)^2 + 2*sqrt(2)*(3*
I*a^4*b + I*a^2*b^3)*cos(d*x + c) + sqrt(2)*(3*I*a^3*b^2 + I*a*b^4))*sqrt(
a)*weierstrassZeta(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, w
eierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1
/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x + c) + 2*b)/a)) - 3*(sqrt(2)*(-3*I*a^
5 - I*a^3*b^2)*cos(d*x + c)^2 + 2*sqrt(2)*(-3*I*a^4*b - I*a^2*b^3)*cos(d*x
 + c) + sqrt(2)*(-3*I*a^3*b^2 - I*a*b^4))*sqrt(a)*weierstrassZeta(-4/3*(3*
a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*
a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*
a*sin(d*x + c) + 2*b)/a)))/((a^8 - 2*a^6*b^2 + a^4*b^4)*d*cos(d*x + c)^2 +
 2*(a^7*b - 2*a^5*b^3 + a^3*b^5)*d*cos(d*x + c) + (a^6*b^2 - 2*a^4*b^4 ...
```

3.872.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/cos(d*x+c)**(3/2)/(a+b*sec(d*x+c))**(5/2),x)
```

output
```
Timed out
```

3.872.7 Maxima [F]

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{1}{(b\sec(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^(3/2)), x)
```

3.872.8 Giac [F]

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{1}{(b\sec(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate(1/((b*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^(3/2)), x)
```

3.872.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{1}{\cos(c+d\,x)^{3/2}\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

input
```
int(1/(cos(c + d*x)^(3/2)*(a + b/cos(c + d*x))^(5/2)),x)
```

output
```
int(1/(cos(c + d*x)^(3/2)*(a + b/cos(c + d*x))^(5/2)), x)
```

3.873 $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx$

3.873.1 Optimal result 6611
3.873.2 Mathematica [C] (warning: unable to verify) 6612
3.873.3 Rubi [A] (verified) 6612
3.873.4 Maple [B] (warning: unable to verify) 6619
3.873.5 Fricas [C] (verification not implemented) 6619
3.873.6 Sympy [F(-1)] 6620
3.873.7 Maxima [F] 6621
3.873.8 Giac [F] 6621
3.873.9 Mupad [F(-1)] 6621

3.873.1 Optimal result

Integrand size = 25, antiderivative size = 277

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx = \frac{2\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{3\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} + \frac{8b\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+dx)}}{3\left(a^2-b^2\right)^2 d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}} - \frac{2a^2\sin(c+dx)}{3b\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^{3/2}} + \frac{2a(a^2-5b^2)\sin(c+dx)}{3b\left(a^2-b^2\right)^2 d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}$$

output
```
-2/3*a^2*sin(d*x+c)/b/(a^2-b^2)/d/(a+b*sec(d*x+c))^(3/2)/cos(d*x+c)^(1/2)+
2/3*a*(a^2-5*b^2)*sin(d*x+c)/b/(a^2-b^2)^2/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x
+c))^(1/2)+2/3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(s
in(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)/
(a^2-b^2)/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)+8/3*b*(cos(1/2*d*x+1/2
*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a
+b))^(1/2))*cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2)/(a^2-b^2)^2/d/((b+a*co
s(d*x+c))/(a+b))^(1/2)
```

3.873.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 8.71 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.12

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx = \frac{2(b+a\cos(c+dx))^2\left(\frac{a(a^2-5b^2-4ab\cos(c+dx))\sin(c+dx)}{b+a\cos(c+dx)}+\frac{\sqrt{\cos^2\left(\frac{1}{2}(c+\right.}}{}\right.}{}$$

input
```
Integrate[1/(Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^(5/2)),x]
```

output
```
(2*(b + a*Cos[c + d*x])^2*((a*(a^2 - 5*b^2 - 4*a*b*Cos[c + d*x])*Sin[c + d
*x])/(b + a*Cos[c + d*x]) + (Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*((4*I)*
b*(a + b)*EllipticE[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[((
b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] - I*(a^2 + 4*a*b + 3*b^2)
*EllipticF[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[((b + a*Cos
[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] + 4*b*(b + a*Cos[c + d*x])*Sqrt[Se
c[(c + d*x)/2]^2]*Tan[(c + d*x)/2]))/Sqrt[Sec[c + d*x]]))/(3*(a^2 - b^2)^2
*d*Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^(5/2))
```

3.873.3 Rubi [A] (verified)

Time = 2.29 (sec) , antiderivative size = 316, normalized size of antiderivative = 1.14, number of steps used = 21, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.840$, Rules used = {3042, 4752, 3042, 4332, 27, 3042, 4588, 27, 3042, 4523, 3042, 4343, 3042, 3134, 3042, 3132, 4345, 3042, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{5/2}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \; 4752$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}dx$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{5/2}}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \; 4332$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{2\int -\frac{a^2+3b\sec(c+dx)a+\left(a^2-3b^2\right)\sec^2(c+dx)}{2\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}}dx}{3b\left(a^2-b^2\right)}-\frac{2a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}\right)$$

$$\Big\downarrow \; 27$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{a^2+3b\sec(c+dx)a+\left(a^2-3b^2\right)\sec^2(c+dx)}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}}dx}{3b\left(a^2-b^2\right)}-\frac{2a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}\right)$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\int \frac{a^2+3b\csc\left(c+dx+\frac{\pi}{2}\right)a+\left(a^2-3b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{3b\left(a^2-b^2\right)}-\frac{2a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}\right)$$

$$\Big\downarrow \; 4588$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2a\left(a^2-5b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}-\frac{2\int -\frac{4a^2b^2+a\left(a^2+3b^2\right)\sec(c+dx)b}{2\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{a\left(a^2-b^2\right)}}{3b\left(a^2-b^2\right)}-\frac{2a^2\sin(c+dx)\sqrt{\mathrm{s}}}{3bd\left(a^2-b^2\right)(a+b\,\mathrm{se}}\right.$$

$$\Big\downarrow \; 27$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\int \frac{4a^2b^2+a\left(a^2+3b^2\right)\sec(c+dx)b}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{a\left(a^2-b^2\right)}+\frac{2a\left(a^2-5b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}}{3b\left(a^2-b^2\right)}-\frac{2a^2\sin(c+dx)\sqrt{\sec(}}{3bd\left(a^2-b^2\right)(a+b\sec(}\right.$$

$$\Big\downarrow \; 3042$$

3.873. $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\int \frac{4a^2b^2+a\left(a^2+3b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)b}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a(a^2-b^2)}+\frac{2a\left(a^2-5b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a^2-b^2)\sqrt{a+b\sec(c+dx)}}}{3b\left(a^2-b^2\right)}-\frac{2a^2\sin(c+dx)\sqrt{}}{3bd\left(a^2-b^2\right)(a+b}\right.$$

$\downarrow$ 4523

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{ab(a^2-b^2)\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}dx+4ab^2\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}dx}{a(a^2-b^2)}+\frac{2a\left(a^2-5b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a^2-b^2)\sqrt{a+b\sec(c+dx)}}}{3b\left(a^2-b^2\right)}-\frac{2}{3bd(}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{ab(a^2-b^2)\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+4ab^2\int \frac{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a(a^2-b^2)}+\frac{2a\left(a^2-5b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a^2-b^2)\sqrt{a+b\sec(c+dx)}}}{3b\left(a^2-b^2\right)}\right.$$

$\downarrow$ 4343

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{ab(a^2-b^2)\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{4ab^2\sqrt{a+b\sec(c+dx)}\int \sqrt{b+a\cos(c+dx)}dx}{\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}}{a(a^2-b^2)}+\frac{2a\left(a^2-5b^2\right)\sin(c+dx)\sqrt{\sec}}{d(a^2-b^2)\sqrt{a+b\sec(c+}}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{ab(a^2-b^2)\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{4ab^2\sqrt{a+b\sec(c+dx)}\int \sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}}}{a(a^2-b^2)}+\frac{2a\left(a^2-5b^2\right)\sin(c+dx)\sqrt{}}{d(a^2-b^2)\sqrt{a+b\sec}}\right.$$

$\downarrow$ 3134

3.873. $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{ab(a^2-b^2)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{4ab^2\sqrt{a+b\sec(c+dx)}\int\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}dx}{\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{a(a^2-b^2)}+\frac{2a(a^2-5b^2)\sin(c+dx)\sqrt{}}{d(a^2-b^2)\sqrt{a+b\sec}}}{3b(a^2-b^2)}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{ab(a^2-b^2)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{4ab^2\sqrt{a+b\sec(c+dx)}\int\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}dx}{\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{a(a^2-b^2)}+\frac{2a(a^2-5b^2)\sin(c+}{d(a^2-b^2)\sqrt{a+}}}{3b(a^2-b^2)}\right.$$

$\downarrow$ 3132

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{ab(a^2-b^2)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{8ab^2\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{a(a^2-b^2)}+\frac{2a(a^2-5b^2)\sin(c+dx)\sqrt{\sec(c}}{d(a^2-b^2)\sqrt{a+b\sec(c+dx}}}{3b(a^2-b^2)}\right.$$

$\downarrow$ 4345

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{ab\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{1}{\sqrt{b+a\cos(c+dx)}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{8ab^2\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{a(a^2-b^2)}+\frac{2a(a^2-5}{d(a^2}}{3b(a^2-b^2)}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{ab\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{1}{\sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{8ab^2\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{a(a^2-b^2)}+\frac{2a(a}{}}{3b(a^2-b^2)}\right.$$

3.873. $\int\frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx$

$$\Big\downarrow \; 3142$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{ab\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{8ab^2\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{a(a^2-b^2)}+\frac{2a(a}{}}{3b\left(a^2-b^2\right)}\right.$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{\frac{ab\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{8ab^2\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}}{a(a^2-b^2)}+}{3b\left(a^2-b^2\right)}\right.$$

$$\Big\downarrow \; 3140$$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\frac{2a\left(a^2-5b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a^2-b^2)\sqrt{a+b\sec(c+dx)}}+\frac{\frac{2ab\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}+\frac{8ab^2}{}}{a(a^2-b^2)}}{3b\left(a^2-b^2\right)}\right.$$

input
```
Int[1/(Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^(5/2)),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((-2*a^2*Sqrt[Sec[c + d*x]]*Sin[c +
d*x])/(3*b*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^(3/2)) + (((2*a*b*(a^2 - b^2
)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]
*Sqrt[Sec[c + d*x]])/(d*Sqrt[a + b*Sec[c + d*x]]) + (8*a*b^2*EllipticE[(c
+ d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(d*Sqrt[(b + a*Cos[c +
d*x])/(a + b)]*Sqrt[Sec[c + d*x]]))/(a*(a^2 - b^2)) + (2*a*(a^2 - 5*b^2)*S
qrt[Sec[c + d*x]]*Sin[c + d*x])/((a^2 - b^2)*d*Sqrt[a + b*Sec[c + d*x]]))/
(3*b*(a^2 - b^2)))
```

3.873. $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx$

3.873.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 4332
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-a^2)*d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(
m + 1)*((d*Csc[e + f*x])^(n - 3)/(b*f*(m + 1)*(a^2 - b^2))), x] + Simp[d^3/
(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])
^(n - 3)*Simp[a^2*(n - 3) + a*b*(m + 1)*Csc[e + f*x] - (a^2*(n - 2) + b^2*(
m + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2
- b^2, 0] && LtQ[m, -1] && (IGtQ[n, 3] || (IntegersQ[n + 1/2, 2*m] && GtQ[n
, 2]))
```

3.873. $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx$

rule 4343
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]])   Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a
, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4345
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]])   Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[
{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4523
```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Simp[A/a   I
nt[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Simp[(A*b - a*B)
/(a*d)   Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ
[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

rule 4588
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc
[e + f*x])^(m + 1)*((d*Csc[e + f*x])^n/(a*f*(m + 1)*(a^2 - b^2))), x] + Sim
p[1/(a*(m + 1)*(a^2 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f
*x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n +
1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m
+ n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x
] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

3.873. $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx$

3.873.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 1998 vs. $2(307) = 614$.

Time = 6.17 (sec) , antiderivative size = 1999, normalized size of antiderivative = 7.22

method	result	size
default	Expression too large to display	1999

input
```
int(1/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3/d/(a-b)/(a+b)^2/((a-b)/(a+b))^(1/2)*((1-cos(d*x+c))^2*csc(d*x+c)^2-1)
^3*((a*(1-cos(d*x+c))^2*csc(d*x+c)^2-b*(1-cos(d*x+c))^2*csc(d*x+c)^2-a-b)/
((1-cos(d*x+c))^2*csc(d*x+c)^2-1))^(1/2)*(4*((a-b)/(a+b))^(1/2)*a*b*(1-cos
(d*x+c))^5*csc(d*x+c)^5-4*((a-b)/(a+b))^(1/2)*b^2*(1-cos(d*x+c))^5*csc(d*x
+c)^5-(-(a*(1-cos(d*x+c))^2*csc(d*x+c)^2-b*(1-cos(d*x+c))^2*csc(d*x+c)^2-a
-b)/(a+b))^(1/2)*((1-cos(d*x+c))^2*csc(d*x+c)^2+1)^(1/2)*EllipticF(((a-b)/
(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*a^2*(1-cos(d*x
+c))^2*csc(d*x+c)^2+4*(-(a*(1-cos(d*x+c))^2*csc(d*x+c)^2-b*(1-cos(d*x+c))^
2*csc(d*x+c)^2-a-b)/(a+b))^(1/2)*((1-cos(d*x+c))^2*csc(d*x+c)^2+1)^(1/2)*E
llipticF(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2)
)*a*b*(1-cos(d*x+c))^2*csc(d*x+c)^2-3*(-(a*(1-cos(d*x+c))^2*csc(d*x+c)^2-b
*(1-cos(d*x+c))^2*csc(d*x+c)^2-a-b)/(a+b))^(1/2)*((1-cos(d*x+c))^2*csc(d*x
+c)^2+1)^(1/2)*EllipticF(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(-(a
+b)/(a-b))^(1/2))*b^2*(1-cos(d*x+c))^2*csc(d*x+c)^2-4*(-(a*(1-cos(d*x+c))^
2*csc(d*x+c)^2-b*(1-cos(d*x+c))^2*csc(d*x+c)^2-a-b)/(a+b))^(1/2)*((1-cos(d
*x+c))^2*csc(d*x+c)^2+1)^(1/2)*EllipticE(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+
csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*a*b*(1-cos(d*x+c))^2*csc(d*x+c)^2+4*(-(a
*(1-cos(d*x+c))^2*csc(d*x+c)^2-b*(1-cos(d*x+c))^2*csc(d*x+c)^2-a-b)/(a+b))
^(1/2)*((1-cos(d*x+c))^2*csc(d*x+c)^2+1)^(1/2)*EllipticE(((a-b)/(a+b))^(1/
2)*(-cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*b^2*(1-cos(d*x+c))^2*...
```

3.873.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 709, normalized size of antiderivative = 2.56

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx =$$

$$-\frac{6\,(4\,a^3b\cos(dx+c)-a^4+5\,a^2b^2)\sqrt{\frac{a\cos(dx+c)+b}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)-\left(\sqrt{2}(-3i\,a^4-i\,a^2b^2)\cos(dx\right.}{}$$

input
```
integrate(1/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
-1/9*(6*(4*a^3*b*cos(d*x + c) - a^4 + 5*a^2*b^2)*sqrt((a*cos(d*x + c) + b)
/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - (sqrt(2)*(-3*I*a^4 - I*a^
2*b^2)*cos(d*x + c)^2 - 2*sqrt(2)*(3*I*a^3*b + I*a*b^3)*cos(d*x + c) + sqr
t(2)*(-3*I*a^2*b^2 - I*b^4))*sqrt(a)*weierstrassPInverse(-4/3*(3*a^2 - 4*b
^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x
 + c) + 2*b)/a) - (sqrt(2)*(3*I*a^4 + I*a^2*b^2)*cos(d*x + c)^2 - 2*sqrt(2
)*(-3*I*a^3*b - I*a*b^3)*cos(d*x + c) + sqrt(2)*(3*I*a^2*b^2 + I*b^4))*sqr
t(a)*weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/
a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a) + 12*(-I*sqrt(2)
*a^3*b*cos(d*x + c)^2 - 2*I*sqrt(2)*a^2*b^2*cos(d*x + c) - I*sqrt(2)*a*b^3
)*sqrt(a)*weierstrassZeta(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)
/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)
/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x + c) + 2*b)/a)) + 12*(I*sqrt(2
)*a^3*b*cos(d*x + c)^2 + 2*I*sqrt(2)*a^2*b^2*cos(d*x + c) + I*sqrt(2)*a*b^
3)*sqrt(a)*weierstrassZeta(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3
)/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3
)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a)))/((a^7 - 2*a^
5*b^2 + a^3*b^4)*d*cos(d*x + c)^2 + 2*(a^6*b - 2*a^4*b^3 + a^2*b^5)*d*cos(
d*x + c) + (a^5*b^2 - 2*a^3*b^4 + a*b^6)*d)
```

3.873.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/cos(d*x+c)**(5/2)/(a+b*sec(d*x+c))**(5/2),x)
```

output
```
Timed out
```

3.873.7 Maxima [F]

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{1}{(b\sec(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^(5/2)), x)
```

3.873.8 Giac [F]

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{1}{(b\sec(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate(1/((b*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^(5/2)), x)
```

3.873.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{1}{\cos(c+d\,x)^{5/2}\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

input
```
int(1/(cos(c + d*x)^(5/2)*(a + b/cos(c + d*x))^(5/2)),x)
```

output
```
int(1/(cos(c + d*x)^(5/2)*(a + b/cos(c + d*x))^(5/2)), x)
```

3.874 $\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx$

3.874.1 Optimal result 6622
3.874.2 Mathematica [C] (warning: unable to verify) 6623
3.874.3 Rubi [A] (verified) 6623
3.874.4 Maple [C] (warning: unable to verify) 6632
3.874.5 Fricas [F(-1)] 6633
3.874.6 Sympy [F(-1)] 6634
3.874.7 Maxima [F] 6634
3.874.8 Giac [F] 6634
3.874.9 Mupad [F(-1)] 6635

3.874.1 Optimal result

Integrand size = 25, antiderivative size = 370

$$\begin{aligned}
&\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx = \\
&-\frac{2a\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{3b\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} \\
&+\frac{2\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{b^2d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} \\
&+\frac{2a(3a^2-7b^2)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+dx)}}{3b^2\left(a^2-b^2\right)^2d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}} \\
&-\frac{2a^2\sin(c+dx)}{3b\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}} \\
&-\frac{2a^2(3a^2-7b^2)\sin(c+dx)}{3b^2\left(a^2-b^2\right)^2d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}
\end{aligned}$$

output
```
-2/3*a^2*sin(d*x+c)/b/(a^2-b^2)/d/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(3/2)-
2/3*a^2*(3*a^2-7*b^2)*sin(d*x+c)/b^2/(a^2-b^2)^2/d/cos(d*x+c)^(1/2)/(a+b*s
ec(d*x+c))^(1/2)-2/3*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ell
ipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b)
)^(1/2)/b/(a^2-b^2)/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)+2*(cos(1/2*d
*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2,2^(1
/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)/b^2/d/cos(d*x+c)^(1/2)
/(a+b*sec(d*x+c))^(1/2)+2/3*a*(3*a^2-7*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/c
os(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*co
s(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2)/b^2/(a^2-b^2)^2/d/((b+a*cos(d*x+c))/
(a+b))^(1/2)
```

3.874.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 34.59 (sec) , antiderivative size = 93062, normalized size of antiderivative = 251.52

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx = \text{Result too large to show}$$

input
```
Integrate[1/(Cos[c + d*x]^(7/2)*(a + b*Sec[c + d*x])^(5/2)),x]
```

output
```
Result too large to show
```

3.874.3 Rubi [A] (verified)

Time = 3.98 (sec) , antiderivative size = 411, normalized size of antiderivative = 1.11, number of steps used = 28, number of rules used = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 1.120$, Rules used = {3042, 4752, 3042, 4332, 27, 3042, 4586, 27, 3042, 4596, 3042, 4346, 3042, 3286, 3042, 3284, 4523, 3042, 4343, 3042, 3134, 3042, 3132, 4345, 3042, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^{7/2}\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$\Big\downarrow$ 4752

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}dx$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)^{7/2}}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$\Big\downarrow$ 4332

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{2\int \frac{\sqrt{\sec(c+dx)}\left(a^2-3b\sec(c+dx)a-3\left(a^2-b^2\right)\sec^2(c+dx)\right)}{2(a+b\sec(c+dx))^{3/2}}dx}{3b\left(a^2-b^2\right)}-\frac{2a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+\right.$$

$\Big\downarrow$ 27

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\int \frac{\sqrt{\sec(c+dx)}\left(a^2-3b\sec(c+dx)a-3\left(a^2-b^2\right)\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^{3/2}}dx}{3b\left(a^2-b^2\right)}-\frac{2a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx}\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\int \frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}\left(a^2-3b\csc\left(c+dx+\frac{\pi}{2}\right)a-3\left(a^2-b^2\right)\csc\left(c+dx+\frac{\pi}{2}\right)^2\right)}{\left(a+b\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{3b\left(a^2-b^2\right)}-\frac{2a^2\sin(c+dx)\mathrm{s}}{3bd\left(a^2-b^2\right)(a+b}\right.$$

$\Big\downarrow$ 4586

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{2a^2\left(3a^2-7b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd(a^2-b^2)\sqrt{a+b\sec(c+dx)}}-\frac{2\int \frac{\left(3a^2-7b^2\right)a^2+2b\left(a^2-3b^2\right)\sec(c+dx)a+3\left(a^2-b^2\right)^2\sec^2(c+dx)}{2\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}d}{b(a^2-b^2)}}{3b\left(a^2-b^2\right)}\right.$$

$\Big\downarrow$ 27

3.874. $\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{2a^2(3a^2-7b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd(a^2-b^2)\sqrt{a+b\sec(c+dx)}}-\frac{\int\frac{(3a^2-7b^2)a^2+2b(a^2-3b^2)\sec(c+dx)a+3(a^2-b^2)^2\sec^2(c+dx)}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{b(a^2-b^2)}}{3b(a^2-b^2)}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{2a^2(3a^2-7b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd(a^2-b^2)\sqrt{a+b\sec(c+dx)}}-\frac{\int\frac{(3a^2-7b^2)a^2+2b(a^2-3b^2)\csc(c+dx+\frac{\pi}{2})a+3(a^2-b^2)^2\csc(c+dx}{\sqrt{\csc(c+dx+\frac{\pi}{2})}\sqrt{a+b\csc(c+dx+\frac{\pi}{2})}}}{b(a^2-b^2)}}{3b(a^2-b^2)}\right.$$

$\downarrow$ 4596

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{2a^2(3a^2-7b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd(a^2-b^2)\sqrt{a+b\sec(c+dx)}}-\frac{3(a^2-b^2)^2\int\frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}dx+\int\frac{(3a^2-7b^2)a^2+2b(a^2-3b^2)\sec}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+}}}{b(a^2-b^2)}}{3b(a^2-b^2)}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{2a^2(3a^2-7b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd(a^2-b^2)\sqrt{a+b\sec(c+dx)}}-\frac{3(a^2-b^2)^2\int\frac{\csc(c+dx+\frac{\pi}{2})^{3/2}}{\sqrt{a+b\csc(c+dx+\frac{\pi}{2})}}dx+\int\frac{(3a^2-7b^2)a^2+2b(a^2-3b^2}{\sqrt{\csc(c+dx+\frac{\pi}{2})}\sqrt{a+b}}}{b(a^2-b^2)}}{3b(a^2-b^2)}\right.$$

$\downarrow$ 4346

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{2a^2(3a^2-7b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd(a^2-b^2)\sqrt{a+b\sec(c+dx)}}-\frac{\int\frac{(3a^2-7b^2)a^2+2b(a^2-3b^2)\csc(c+dx+\frac{\pi}{2})a}{\sqrt{\csc(c+dx+\frac{\pi}{2})}\sqrt{a+b\csc(c+dx+\frac{\pi}{2})}}dx+\frac{3(a^2-b^2)^2\sqrt{\sec(}}{}}{b(a^2-b^2)}}{3b(a^2-b^2)}\right.$$

$\downarrow$ 3042

3.874. $\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{2a^2(3a^2-7b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd(a^2-b^2)\sqrt{a+b\sec(c+dx)}}-\frac{\int\frac{(3a^2-7b^2)a^2+2b(a^2-3b^2)\csc(c+dx+\frac{\pi}{2})a}{\sqrt{\csc(c+dx+\frac{\pi}{2})}\sqrt{a+b\csc(c+dx+\frac{\pi}{2})}}dx+\frac{3(a^2-b^2)^2\sqrt{\sec(c}}{}}{b(a^2-b^2}}{3b(a^2-b^2)}\right.$$

$\downarrow$ 3286

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{2a^2(3a^2-7b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd(a^2-b^2)\sqrt{a+b\sec(c+dx)}}-\frac{\int\frac{(3a^2-7b^2)a^2+2b(a^2-3b^2)\csc(c+dx+\frac{\pi}{2})a}{\sqrt{\csc(c+dx+\frac{\pi}{2})}\sqrt{a+b\csc(c+dx+\frac{\pi}{2})}}dx+\frac{3(a^2-b^2)^2\sqrt{\sec(c}}{}}{b(a^2-b^2)}}{3b(a^2-b^2)}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{2a^2(3a^2-7b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd(a^2-b^2)\sqrt{a+b\sec(c+dx)}}-\frac{\int\frac{(3a^2-7b^2)a^2+2b(a^2-3b^2)\csc(c+dx+\frac{\pi}{2})a}{\sqrt{\csc(c+dx+\frac{\pi}{2})}\sqrt{a+b\csc(c+dx+\frac{\pi}{2})}}dx+\frac{3(a^2-b^2)^2\sqrt{\sec(c}}{}}{b(a^2-}}{3b(a^2-b^2)}\right.$$

$\downarrow$ 3284

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{2a^2(3a^2-7b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd(a^2-b^2)\sqrt{a+b\sec(c+dx)}}-\frac{\int\frac{(3a^2-7b^2)a^2+2b(a^2-3b^2)\csc(c+dx+\frac{\pi}{2})a}{\sqrt{\csc(c+dx+\frac{\pi}{2})}\sqrt{a+b\csc(c+dx+\frac{\pi}{2})}}dx+\frac{6(a^2-b^2)^2\sqrt{\sec(c}}{}}{b(a^2-b^2)}}{3b(a^2-b^2)}\right.$$

$\downarrow$ 4523

3.874. $\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{2a^2(3a^2-7b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd(a^2-b^2)\sqrt{a+b\sec(c+dx)}}-\frac{-ab(a^2-b^2)\int\frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}dx+a(3a^2-7b^2)\int\frac{\sqrt{a+b\sec(c+dx}}{\sqrt{\sec(c+dx)}}}{b(}}{3b(a^2-b^2)}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{2a^2(3a^2-7b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd(a^2-b^2)\sqrt{a+b\sec(c+dx)}}-\frac{-ab(a^2-b^2)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+a(3a^2-7b^2)\int\frac{\sqrt{a+b\csc(}}{\sqrt{\csc(c+}}}{}}{3b(a^2-b^2)}\right.$$

$\downarrow$ 4343

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{2a^2(3a^2-7b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd(a^2-b^2)\sqrt{a+b\sec(c+dx)}}-\frac{-ab(a^2-b^2)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{a\left(3a^2-7b^2\right)\sqrt{a+b\sec(c+d}}{\sqrt{\sec(c+dx)}\sqrt{a}}}{}}{3b(a^2-b^2)}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{2a^2(3a^2-7b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd(a^2-b^2)\sqrt{a+b\sec(c+dx)}}-\frac{-ab(a^2-b^2)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{a\left(3a^2-7b^2\right)\sqrt{a+b\sec(c+d}}{\sqrt{\sec(c+dx)}\sqrt{}}}{}}{3b(a^2-b^2)}\right.$$

$\downarrow$ 3134

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{2a^2(3a^2-7b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd(a^2-b^2)\sqrt{a+b\sec(c+dx)}}-\frac{-ab(a^2-b^2)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{a\left(3a^2-7b^2\right)\sqrt{a+b\sec(c+d}}{\sqrt{\sec(c+dx)}\sqrt{}}}{}}{3b(a^2-b^2)}\right.$$

$\downarrow$ 3042

3.874. $\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{2a^2(3a^2-7b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd(a^2-b^2)\sqrt{a+b\sec(c+dx)}}-\frac{-ab(a^2-b^2)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{a\left(3a^2-7b^2\right)\sqrt{a+b\sec(c+d}}{\sqrt{\sec(c+dx)}}}{3b\left(a^2-b^2\right.}\right.$$

$\downarrow$ 3132

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{2a^2(3a^2-7b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd(a^2-b^2)\sqrt{a+b\sec(c+dx)}}-\frac{-ab(a^2-b^2)\int\frac{\sqrt{\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a\left(3a^2-7b^2\right)\sqrt{a+b\sec(c+}}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a}{}}}}{3b\left(a^2-b^2\right)}\right.$$

$\downarrow$ 4345

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{2a^2(3a^2-7b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd(a^2-b^2)\sqrt{a+b\sec(c+dx)}}-\frac{-\frac{ab\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{1}{\sqrt{b+a\cos(c+dx)}}dx}{\sqrt{a+b\sec(c+dx)}}+\frac{2a(}{}}{3b\left(a^2\right.}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{2a^2(3a^2-7b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd(a^2-b^2)\sqrt{a+b\sec(c+dx)}}-\frac{-\frac{ab\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+b}\int\frac{1}{\sqrt{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{\sqrt{a+b\sec(c+dx)}}+}{3b\left(\right.}\right.$$

$\downarrow$ 3142

3.874. $\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx$

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{2a^2(3a^2-7b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd(a^2-b^2)\sqrt{a+b\sec(c+dx)}}-\frac{-\frac{ab(a^2-b^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}}dx}{\sqrt{a+b\sec(c+dx)}}-}{}}{3b(}\right.$$

$\downarrow$ 3042

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{\frac{2a^2(3a^2-7b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd(a^2-b^2)\sqrt{a+b\sec(c+dx)}}-\frac{-\frac{ab(a^2-b^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\sin\left(c+dx+\frac{\pi}{2}\right)}{a+b}}}}{\sqrt{a+b\sec(c+dx)}}-}{}}{3b}\right.$$

$\downarrow$ 3140

$$\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{2a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3bd(a^2-b^2)(a+b\sec(c+dx))^{3/2}}-\frac{\frac{2a^2(3a^2-7b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd(a^2-b^2)\sqrt{a+b\sec(c+dx)}}-\frac{-\frac{2ab(a^2-b^2)\sqrt{}}{}}{}}{}\right.$$

input
```
Int[1/(Cos[c + d*x]^(7/2)*(a + b*Sec[c + d*x])^(5/2)),x]
```

output
```
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((-2*a^2*Sec[c + d*x]^(3/2)*Sin[c +
d*x])/(3*b*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^(3/2)) - (-(((-2*a*b*(a^2 -
b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a +
b)]*Sqrt[Sec[c + d*x]])/(d*Sqrt[a + b*Sec[c + d*x]]) + (6*(a^2 - b^2)^2*Sq
rt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)]
*Sqrt[Sec[c + d*x]])/(d*Sqrt[a + b*Sec[c + d*x]]) + (2*a*(3*a^2 - 7*b^2)*E
llipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(d*Sqrt[(b
+ a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]))/(b*(a^2 - b^2))) + (2*a^2*
(3*a^2 - 7*b^2)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(b*(a^2 - b^2)*d*Sqrt[a +
 b*Sec[c + d*x]]))/(3*b*(a^2 - b^2)))
```

3.874. $\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx$

3.874.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 3284
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

3.874. $\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx$

rule 3286
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]]   Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
 d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

rule 4332
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-a^2)*d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(
m + 1)*((d*Csc[e + f*x])^(n - 3)/(b*f*(m + 1)*(a^2 - b^2))), x] + Simp[d^3/
(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])
^(n - 3)*Simp[a^2*(n - 3) + a*b*(m + 1)*Csc[e + f*x] - (a^2*(n - 2) + b^2*(
m + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2
- b^2, 0] && LtQ[m, -1] && (IGtQ[n, 3] || (IntegersQ[n + 1/2, 2*m] && GtQ[n
, 2]))
```

rule 4343
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]])   Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a
, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4345
```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Simp[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]])   Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[
{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4346
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Simp[d*Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x
]]/Sqrt[a + b*Csc[e + f*x]])   Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4523
```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Simp[A/a   I
nt[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Simp[(A*b - a*B)
/(a*d)   Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ
[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

3.874. $\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx$

rule 4586
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(-d)*(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a +
b*Csc[e + f*x])^(m + 1)*((d*Csc[e + f*x])^(n - 1)/(b*f*(a^2 - b^2)*(m + 1))
), x] + Simp[d/(b*(a^2 - b^2)*(m + 1))   Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*b^2*(n - 1) - a*(b*B - a*C)*(n - 1) + b*(a*A
 - b*B + a*C)*(m + 1)*Csc[e + f*x] - (b*(A*b - a*B)*(m + n + 1) + C*(a^2*n
+ b^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C
}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n, 0]
```

rule 4596
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Simp[C/d^2   Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*C
sc[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[
a + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
 b^2, 0]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

3.874.4 Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 9.29 (sec) , antiderivative size = 4415, normalized size of antiderivative = 11.93

method	result	size
default	Expression too large to display	4415

input
```
int(1/cos(d*x+c)^(7/2)/(a+b*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/3/d/((a-b)/(a+b))^(1/2)/(a+b)^2/(a-b)/b^2*((1-cos(d*x+c))^2*csc(d*x+c)^2
-1)^4*((a*(1-cos(d*x+c))^2*csc(d*x+c)^2-b*(1-cos(d*x+c))^2*csc(d*x+c)^2-a-
b)/((1-cos(d*x+c))^2*csc(d*x+c)^2-1))^(1/2)*(-3*(-(a*(1-cos(d*x+c))^2*csc(
d*x+c)^2-b*(1-cos(d*x+c))^2*csc(d*x+c)^2-a-b)/(a+b))^(1/2)*((1-cos(d*x+c))
^2*csc(d*x+c)^2+1)^(1/2)*EllipticE(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*
x+c)),(-(a+b)/(a-b))^(1/2))*a^4*(1-cos(d*x+c))^2*csc(d*x+c)^2-3*((a-b)/(a+
b))^(1/2)*a^4*(-cot(d*x+c)+csc(d*x+c))-13*(-(a*(1-cos(d*x+c))^2*csc(d*x+c)
^2-b*(1-cos(d*x+c))^2*csc(d*x+c)^2-a-b)/(a+b))^(1/2)*((1-cos(d*x+c))^2*csc
(d*x+c)^2+1)^(1/2)*EllipticF(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),
(-(a+b)/(a-b))^(1/2))*a^2*b^2*(1-cos(d*x+c))^2*csc(d*x+c)^2+6*(-(a*(1-cos(
d*x+c))^2*csc(d*x+c)^2-b*(1-cos(d*x+c))^2*csc(d*x+c)^2-a-b)/(a+b))^(1/2)*(
(1-cos(d*x+c))^2*csc(d*x+c)^2+1)^(1/2)*EllipticF(((a-b)/(a+b))^(1/2)*(-cot
(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*a*b^3*(1-cos(d*x+c))^2*csc(d*x+c
)^2+3*(-(a*(1-cos(d*x+c))^2*csc(d*x+c)^2-b*(1-cos(d*x+c))^2*csc(d*x+c)^2-a
-b)/(a+b))^(1/2)*((1-cos(d*x+c))^2*csc(d*x+c)^2+1)^(1/2)*EllipticE(((a-b)/
(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/2))*a^3*b*(1-cos(d
*x+c))^2*csc(d*x+c)^2+7*(-(a*(1-cos(d*x+c))^2*csc(d*x+c)^2-b*(1-cos(d*x+c)
)^2*csc(d*x+c)^2-a-b)/(a+b))^(1/2)*((1-cos(d*x+c))^2*csc(d*x+c)^2+1)^(1/2)
*EllipticE(((a-b)/(a+b))^(1/2)*(-cot(d*x+c)+csc(d*x+c)),(-(a+b)/(a-b))^(1/
2))*a^2*b^2*(1-cos(d*x+c))^2*csc(d*x+c)^2-7*(-(a*(1-cos(d*x+c))^2*csc(d...
```

3.874.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/cos(d*x+c)^(7/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.874.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/cos(d*x+c)**(7/2)/(a+b*sec(d*x+c))**(5/2),x)
```

output
```
Timed out
```

3.874.7 Maxima [F]

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{1}{(b\sec(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(7/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^(7/2)), x)
```

3.874.8 Giac [F]

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{1}{(b\sec(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(7/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate(1/((b*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^(7/2)), x)
```

3.874.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx=\int \frac{1}{\cos\left(c+d\,x\right)^{7/2}\,\left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)^{5/2}}\,dx$$

input
```
int(1/(cos(c + d*x)^(7/2)*(a + b/cos(c + d*x))^(5/2)),x)
```

output
```
int(1/(cos(c + d*x)^(7/2)*(a + b/cos(c + d*x))^(5/2)), x)
```

3.875 $\int (d\cos(e+fx))^n(a+b\sec(e+fx))^3\,dx$

3.875.1 Optimal result 6636
3.875.2 Mathematica [A] (verified) 6637
3.875.3 Rubi [A] (verified) 6637
3.875.4 Maple [F] 6641
3.875.5 Fricas [F] 6641
3.875.6 Sympy [F] 6642
3.875.7 Maxima [F] 6642
3.875.8 Giac [F] 6642
3.875.9 Mupad [F(-1)] 6643

3.875.1 Optimal result

Integrand size = 23, antiderivative size = 266

$$\begin{aligned}\int (d\cos(e+fx))^n(a+b\sec(e+fx))^3\,dx = &-\frac{b(b^2(1-n)+3a^2(2-n))\,(d\cos(e+fx))^n\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n}{2},\frac{2+n}{2},\cos^2(e+fx)\right)\sin(e+fx)}{f(2-n)n\sqrt{\sin^2(e+fx)}}\\ &-\frac{a(a^2(1-n)-3b^2n)\cos(e+fx)(d\cos(e+fx))^n\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+n}{2},\frac{3+n}{2},\cos^2(e+fx)\right)\sin(e}{f(1-n)(1+n)\sqrt{\sin^2(e+fx)}}\\ &+\frac{ab^2(5-2n)(d\cos(e+fx))^n\tan(e+fx)}{f(1-n)(2-n)}\\ &+\frac{b^2(d\cos(e+fx))^n(a+b\sec(e+fx))\tan(e+fx)}{f(2-n)}\end{aligned}$$

output
```
-b*(b^2*(1-n)+3*a^2*(2-n))*(d*cos(f*x+e))^n*hypergeom([1/2, 1/2*n],[1+1/2*
n],cos(f*x+e)^2)*sin(f*x+e)/f/(2-n)/n/(sin(f*x+e)^2)^(1/2)-a*(a^2*(1-n)-3*
b^2*n)*cos(f*x+e)*(d*cos(f*x+e))^n*hypergeom([1/2, 1/2+1/2*n],[3/2+1/2*n],
cos(f*x+e)^2)*sin(f*x+e)/f/(-n^2+1)/(sin(f*x+e)^2)^(1/2)+a*b^2*(5-2*n)*(d*
cos(f*x+e))^n*tan(f*x+e)/f/(n^2-3*n+2)+b^2*(d*cos(f*x+e))^n*(a+b*sec(f*x+e
))*tan(f*x+e)/f/(2-n)
```

3.875.2 Mathematica [A] (verified)

Time = 0.65 (sec) , antiderivative size = 222, normalized size of antiderivative = 0.83

$$\int (d\cos(e+fx))^n(a+b\sec(e+fx))^3\,dx = -\frac{(d\cos(e+fx))^n\csc(e+fx)\left(b^3n(-1+n^2)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(-2+n),\frac{n}{2},\cos^2(e+fx)\right)+\frac{1}{2}a(}{}$$

input
```
Integrate[(d*Cos[e + f*x])^n*(a + b*Sec[e + f*x])^3,x]
```

output
```
-(((d*Cos[e + f*x])^n*Csc[e + f*x]*(b^3*n*(-1 + n^2)*Hypergeometric2F1[1/2
, (-2 + n)/2, n/2, Cos[e + f*x]^2] + (a*(-2 + n)*Cos[e + f*x]*(6*b^2*n*(1
+ n)*Hypergeometric2F1[1/2, (-1 + n)/2, (1 + n)/2, Cos[e + f*x]^2] + 2*a*(
-1 + n)*Cos[e + f*x]*(3*b*(1 + n)*Hypergeometric2F1[1/2, n/2, (2 + n)/2, C
os[e + f*x]^2] + a*n*Cos[e + f*x]*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n
)/2, Cos[e + f*x]^2])))/2)*Sec[e + f*x]^2*Sqrt[Sin[e + f*x]^2])/(f*(-2 + n
)*(-1 + n)*n*(1 + n)))
```

3.875.3 Rubi [A] (verified)

Time = 1.64 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.12, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.652$, Rules used = {3042, 4752, 3042, 4329, 3042, 4535, 3042, 4259, 3042, 3122, 4534, 3042, 4259, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\sec(e+fx))^3(d\cos(e+fx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a+b\csc\left(e+fx+\frac{\pi}{2}\right)\right)^3\left(d\sin\left(e+fx+\frac{\pi}{2}\right)\right)^n dx$$

$$\Big\downarrow \text{4752}$$

$$(d\cos(e+fx))^n(d\sec(e+fx))^n\int (d\sec(e+fx))^{-n}(a+b\sec(e+fx))^3dx$$

$$\Big\downarrow \text{3042}$$

$$(d\cos(e+fx))^n(d\sec(e+fx))^n\int\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{-n}\left(a+b\csc\left(e+fx+\frac{\pi}{2}\right)\right)^3dx$$

$\Big\downarrow$ 4329

$$(d\cos(e+fx))^n(d\sec(e+fx))^n\left(\frac{\int(d\sec(e+fx))^{-n}\left(ab^2d(5-2n)\sec^2(e+fx)+bd\left(3(2-n)a^2+b^2(1-n)\right)\sec(e+fx)+ad\left(a^2(2-n)\right.\right.}{d(2-n)}\right.$$

$\Big\downarrow$ 3042

$$(d\cos(e+fx))^n(d\sec(e+fx))^n\left(\frac{\int\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{-n}\left(ab^2d(5-2n)\csc\left(e+fx+\frac{\pi}{2}\right)^2+bd\left(3(2-n)a^2+b^2(1-n)\right)\csc\left(e+fx+\frac{\pi}{2}\right)\right.}{d(2-n)}\right.$$

$\Big\downarrow$ 4535

$$(d\cos(e+fx))^n(d\sec(e+fx))^n\left(\frac{\int(d\sec(e+fx))^{-n}\left(ab^2d(5-2n)\sec^2(e+fx)+ad\left(a^2(2-n)-b^2n\right)\right)dx+b\left(3a^2(2-n)+b^2(1-n)\right)\int(}{d(2-n)}\right.$$

$\Big\downarrow$ 3042

$$(d\cos(e+fx))^n(d\sec(e+fx))^n\left(\frac{b\left(3a^2(2-n)+b^2(1-n)\right)\int\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{1-n}dx+\int\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{-n}\left(ab^2d(5-2n)\csc(e\right.}{d(2-n)}\right.$$

$\Big\downarrow$ 4259

$$(d\cos(e+fx))^n(d\sec(e+fx))^n\left(\frac{\int\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{-n}\left(ab^2d(5-2n)\csc\left(e+fx+\frac{\pi}{2}\right)^2+ad\left(a^2(2-n)-b^2n\right)\right)dx+b\left(3a^2(2-n)+b\right.}{d(2-n)}\right.$$

$\Big\downarrow$ 3042

$$(d\cos(e+fx))^n(d\sec(e+fx))^n\left(\frac{\int\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{-n}\left(ab^2d(5-2n)\csc\left(e+fx+\frac{\pi}{2}\right)^2+ad\left(a^2(2-n)-b^2n\right)\right)dx+b\left(3a^2(2-n)+b\right.}{d(2-n)}\right.$$

$\Big\downarrow$ 3122

$$(d\cos(e+fx))^n(d\sec(e+fx))^n\left(\frac{\int\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{-n}\left(ab^2d(5-2n)\csc\left(e+fx+\frac{\pi}{2}\right)^2+ad\left(a^2(2-n)-b^2n\right)\right)dx-\frac{bd(3a^2(2-n)+b^2(1-}{}}{d(2-n)}\right.$$

3.875. $\int(d\cos(e+fx))^n(a+b\sec(e+fx))^3\,dx$

$$\Big\downarrow \; 4534$$

$$(d\cos(e+fx))^n(d\sec(e+fx))^n\left(\frac{\frac{ad(2-n)(a^2(1-n)-3b^2n)\int(d\sec(e+fx))^{-n}dx}{1-n}-\frac{bd(3a^2(2-n)+b^2(1-n))\sin(e+fx)(d\sec(e+fx))^{-n}\text{Hypergeometric2F1}(\frac{1}{2},\frac{n}{2},\frac{n+2}{2}}{fn\sqrt{\sin^2(e+fx)}}}{d(2-n)}\right.$$

$$\Big\downarrow \; 3042$$

$$(d\cos(e+fx))^n(d\sec(e+fx))^n\left(\frac{\frac{ad(2-n)(a^2(1-n)-3b^2n)\int(d\csc(e+fx+\frac{\pi}{2}))^{-n}dx}{1-n}-\frac{bd(3a^2(2-n)+b^2(1-n))\sin(e+fx)(d\sec(e+fx))^{-n}\text{Hypergeometric2F1}(\frac{1}{2},\frac{n}{2},}{fn\sqrt{\sin^2(e+fx)}}}{d(2-n)}\right.$$

$$\Big\downarrow \; 4259$$

$$(d\cos(e+fx))^n(d\sec(e+fx))^n\left(\frac{\frac{ad(2-n)(a^2(1-n)-3b^2n)\left(\frac{\cos(e+fx)}{d}\right)^{-n}(d\sec(e+fx))^{-n}\int\left(\frac{\cos(e+fx)}{d}\right)^n dx}{1-n}-\frac{bd(3a^2(2-n)+b^2(1-n))\sin(e+fx)(d\sec(e+fx))^{-n}\text{H}}{fn\sqrt{\sin^2(e+}}}{d(2-n)}\right.$$

$$\Big\downarrow \; 3042$$

$$(d\cos(e+fx))^n(d\sec(e+fx))^n\left(\frac{\frac{ad(2-n)(a^2(1-n)-3b^2n)\left(\frac{\cos(e+fx)}{d}\right)^{-n}(d\sec(e+fx))^{-n}\int\left(\frac{\sin\left(e+fx+\frac{\pi}{2}\right)}{d}\right)^n dx}{1-n}-\frac{bd(3a^2(2-n)+b^2(1-n))\sin(e+fx)(d\sec(e+fx)}{fn\sqrt{\sin^2}}}{d(2-n)}\right.$$

$$\Big\downarrow \; 3122$$

$$(d\cos(e+fx))^n(d\sec(e+fx))^n\left(\frac{-\frac{ad^2(2-n)(a^2(1-n)-3b^2n)\sin(e+fx)(d\sec(e+fx))^{-n-1}\text{Hypergeometric2F1}(\frac{1}{2},\frac{n+1}{2},\frac{n+3}{2},\cos^2(e+fx))}{f(1-n)(n+1)\sqrt{\sin^2(e+fx)}}-\frac{bd(3a^2(2-n)+b^2(1-n)}{}}{d(2-n)}\right.$$

input
```
Int[(d*Cos[e + f*x])^n*(a + b*Sec[e + f*x])^3,x]
```

3.875. $\int (d\cos(e+fx))^n(a+b\sec(e+fx))^3\,dx$

output
```
(d*Cos[e + f*x])^n*(d*Sec[e + f*x])^n*((b^2*(a + b*Sec[e + f*x])*Tan[e + f
*x])/(f*(2 - n)*(d*Sec[e + f*x])^n) + (-((a*d^2*(2 - n)*(a^2*(1 - n) - 3*b
^2*n)*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2, Cos[e + f*x]^2]*(d*Sec[
e + f*x])^(-1 - n)*Sin[e + f*x])/(f*(1 - n)*(1 + n)*Sqrt[Sin[e + f*x]^2]))
 - (b*d*(b^2*(1 - n) + 3*a^2*(2 - n))*Hypergeometric2F1[1/2, n/2, (2 + n)/
2, Cos[e + f*x]^2]*Sin[e + f*x])/(f*n*(d*Sec[e + f*x])^n*Sqrt[Sin[e + f*x]
^2]) + (a*b^2*d*(5 - 2*n)*Tan[e + f*x])/(f*(1 - n)*(d*Sec[e + f*x])^n))/(d
*(2 - n)))
```

3.875.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

rule 4259
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)   Int[1/(Sin[c + d*x]/b)^n, x]), x] /;
FreeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

rule 4329
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
2)*((d*Csc[e + f*x])^n/(f*(m + n - 1))), x] + Simp[1/(d*(m + n - 1))   Int[
(a + b*Csc[e + f*x])^(m - 3)*(d*Csc[e + f*x])^n*Simp[a^3*d*(m + n - 1) + a*
b^2*d*n + b*(b^2*d*(m + n - 2) + 3*a^2*d*(m + n - 1))*Csc[e + f*x] + a*b^2*
d*(3*m + 2*n - 4)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, n}, x
] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
 && !(IGtQ[n, 2] && !IntegerQ[m])
```

rule 4534
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Simp[(C*m + A*(m + 1))/(m + 1)   Int[(b*Csc[e + f*x])^m, x], x] /;
 FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] && !LeQ[m, -1]
```

rule 4535
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Simp[B/b   Int[(b*Cs
c[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2)
, x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

3.875.4 Maple [F]

$$\int (d\cos(fx+e))^n (a+b\sec(fx+e))^3 \, dx$$

input
```
int((d*cos(f*x+e))^n*(a+b*sec(f*x+e))^3,x)
```

output
```
int((d*cos(f*x+e))^n*(a+b*sec(f*x+e))^3,x)
```

3.875.5 Fricas [F]

$$\int (d\cos(e+fx))^n(a+b\sec(e+fx))^3 \, dx = \int (b\sec(fx+e)+a)^3(d\cos(fx+e))^n \, dx$$

input
```
integrate((d*cos(f*x+e))^n*(a+b*sec(f*x+e))^3,x, algorithm="fricas")
```

output
```
integral((b^3*sec(f*x + e)^3 + 3*a*b^2*sec(f*x + e)^2 + 3*a^2*b*sec(f*x +
e) + a^3)*(d*cos(f*x + e))^n, x)
```

3.875.6 Sympy [F]

$$\int (d\cos(e+fx))^n(a+b\sec(e+fx))^3\,dx = \int (d\cos(e+fx))^n(a+b\sec(e+fx))^3\,dx$$

input
```
integrate((d*cos(f*x+e))**n*(a+b*sec(f*x+e))**3,x)
```

output
```
Integral((d*cos(e + f*x))**n*(a + b*sec(e + f*x))**3, x)
```

3.875.7 Maxima [F]

$$\int (d\cos(e+fx))^n(a+b\sec(e+fx))^3\,dx = \int (b\sec(fx+e)+a)^3(d\cos(fx+e))^n\,dx$$

input
```
integrate((d*cos(f*x+e))^n*(a+b*sec(f*x+e))^3,x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e) + a)^3*(d*cos(f*x + e))^n, x)
```

3.875.8 Giac [F]

$$\int (d\cos(e+fx))^n(a+b\sec(e+fx))^3\,dx = \int (b\sec(fx+e)+a)^3(d\cos(fx+e))^n\,dx$$

input
```
integrate((d*cos(f*x+e))^n*(a+b*sec(f*x+e))^3,x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e) + a)^3*(d*cos(f*x + e))^n, x)
```

3.875.9 Mupad [F(-1)]

Timed out.

$$\int (d\cos(e+fx))^n(a+b\sec(e+fx))^3\,dx=\int (d\,\cos\left(e+f\,x\right))^n\,\left(a+\frac{b}{\cos\left(e+f\,x\right)}\right)^3\,dx$$

input
```
int((d*cos(e + f*x))^n*(a + b/cos(e + f*x))^3,x)
```

output
```
int((d*cos(e + f*x))^n*(a + b/cos(e + f*x))^3, x)
```

3.876 $\int (d\cos(e+fx))^n(a+b\sec(e+fx))^2\,dx$

3.876.1 Optimal result 6644
3.876.2 Mathematica [A] (verified) 6645
3.876.3 Rubi [A] (verified) 6645
3.876.4 Maple [F] 6648
3.876.5 Fricas [F] 6648
3.876.6 Sympy [F] 6649
3.876.7 Maxima [F] 6649
3.876.8 Giac [F] 6649
3.876.9 Mupad [F(-1)] 6650

3.876.1 Optimal result

Integrand size = 23, antiderivative size = 186

$$\begin{aligned}\int (d\cos(e+fx))^n(a+b\sec(e+fx))^2\,dx &= -\frac{2ab(d\cos(e+fx))^n\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n}{2},\frac{2+n}{2},\cos^2(e+fx)\right)\sin(e+fx)}{fn\sqrt{\sin^2(e+fx)}} \\ &\quad -\frac{(a^2(1-n)-b^2n)\cos(e+fx)(d\cos(e+fx))^n\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+n}{2},\frac{3+n}{2},\cos^2(e+fx)\right)\sin(e+}{f(1-n)(1+n)\sqrt{\sin^2(e+fx)}} \\ &\quad +\frac{b^2(d\cos(e+fx))^n\tan(e+fx)}{f(1-n)}\end{aligned}$$

output
```
-2*a*b*(d*cos(f*x+e))^n*hypergeom([1/2, 1/2*n],[1+1/2*n],cos(f*x+e)^2)*sin
(f*x+e)/f/n/(sin(f*x+e)^2)^(1/2)-(a^2*(1-n)-b^2*n)*cos(f*x+e)*(d*cos(f*x+e
))^n*hypergeom([1/2, 1/2+1/2*n],[3/2+1/2*n],cos(f*x+e)^2)*sin(f*x+e)/f/(-n
^2+1)/(sin(f*x+e)^2)^(1/2)+b^2*(d*cos(f*x+e))^n*tan(f*x+e)/f/(1-n)
```

3.876.2 Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.87

$$\int (d\cos(e+fx))^n(a+b\sec(e+fx))^2\,dx = -\frac{d(d\cos(e+fx))^{-1+n}\csc(e+fx)\left(b^2n(1+n)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(-1+n),\frac{1+n}{2},\cos^2(e+fx)\right)+\right.}{}$$

input
```
Integrate[(d*Cos[e + f*x])^n*(a + b*Sec[e + f*x])^2,x]
```

output
```
-((d*(d*Cos[e + f*x])^(-1 + n)*Csc[e + f*x]*(b^2*n*(1 + n)*Hypergeometric2
F1[1/2, (-1 + n)/2, (1 + n)/2, Cos[e + f*x]^2] + a*(-1 + n)*Cos[e + f*x]*(
2*b*(1 + n)*Hypergeometric2F1[1/2, n/2, (2 + n)/2, Cos[e + f*x]^2] + a*n*C
os[e + f*x]*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2, Cos[e + f*x]^2]))
*Sqrt[Sin[e + f*x]^2])/(f*(-1 + n)*n*(1 + n)))
```

3.876.3 Rubi [A] (verified)

Time = 0.97 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.10, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.565$, Rules used = {3042, 4752, 3042, 4275, 3042, 4259, 3042, 3122, 4534, 3042, 4259, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\sec(e+fx))^2(d\cos(e+fx))^n\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \left(a+b\csc\left(e+fx+\frac{\pi}{2}\right)\right)^2\left(d\sin\left(e+fx+\frac{\pi}{2}\right)\right)^n dx$$

$$\Big\downarrow \; 4752$$

$$(d\cos(e+fx))^n(d\sec(e+fx))^n\int (d\sec(e+fx))^{-n}(a+b\sec(e+fx))^2dx$$

$$\Big\downarrow \; 3042$$

$$(d\cos(e+fx))^n(d\sec(e+fx))^n\int \left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{-n}\left(a+b\csc\left(e+fx+\frac{\pi}{2}\right)\right)^2 dx$$

$$\Big\downarrow \; 4275$$

$$(d\cos(e+fx))^n(d\sec(e+fx))^n\left(\int(d\sec(e+fx))^{-n}\left(a^2+b^2\sec^2(e+fx)\right)dx+\frac{2ab\int(d\sec(e+fx))^{1-n}dx}{d}\right)$$

$\Big\downarrow$ 3042

$$(d\cos(e+fx))^n(d\sec(e+fx))^n\left(\int\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{-n}\left(a^2+b^2\csc\left(e+fx+\frac{\pi}{2}\right)^2\right)dx+\frac{2ab\int\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{1-n}dx}{d}\right)$$

$\Big\downarrow$ 4259

$$(d\cos(e+fx))^n(d\sec(e+fx))^n\left(\int\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{-n}\left(a^2+b^2\csc\left(e+fx+\frac{\pi}{2}\right)^2\right)dx+\frac{2ab\left(\frac{\cos(e+fx)}{d}\right)^{-n}(d\sec(e+fx))^{-n}\int\left(\cos\right.}{d}\right.$$

$\Big\downarrow$ 3042

$$(d\cos(e+fx))^n(d\sec(e+fx))^n\left(\int\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{-n}\left(a^2+b^2\csc\left(e+fx+\frac{\pi}{2}\right)^2\right)dx+\frac{2ab\left(\frac{\cos(e+fx)}{d}\right)^{-n}(d\sec(e+fx))^{-n}\int\left(\sin\right.}{d}\right.$$

$\Big\downarrow$ 3122

$$(d\cos(e+fx))^n(d\sec(e+fx))^n\left(\int\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{-n}\left(a^2+b^2\csc\left(e+fx+\frac{\pi}{2}\right)^2\right)dx-\frac{2ab\sin(e+fx)(d\sec(e+fx))^{-n}\text{Hyperge}}{fn\sqrt{\sin^2(e}}\right.$$

$\Big\downarrow$ 4534

$$(d\cos(e+fx))^n(d\sec(e+fx))^n\left(\left(a^2-\frac{b^2n}{1-n}\right)\int(d\sec(e+fx))^{-n}dx-\frac{2ab\sin(e+fx)(d\sec(e+fx))^{-n}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n}{2},\frac{n+2}{2},\right.}{fn\sqrt{\sin^2(e+fx)}}\right.$$

$\Big\downarrow$ 3042

$$(d\cos(e+fx))^n(d\sec(e+fx))^n\left(\left(a^2-\frac{b^2n}{1-n}\right)\int\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{-n}dx-\frac{2ab\sin(e+fx)(d\sec(e+fx))^{-n}\text{Hypergeometric2F1}\left(\frac{1}{2},\right.}{fn\sqrt{\sin^2(e+fx)}}\right.$$

$\Big\downarrow$ 4259

$$(d\cos(e+fx))^n(d\sec(e+fx))^n\left(\left(a^2-\frac{b^2n}{1-n}\right)\left(\frac{\cos(e+fx)}{d}\right)^{-n}(d\sec(e+fx))^{-n}\int\left(\frac{\cos(e+fx)}{d}\right)^n dx-\frac{2ab\sin(e+fx)(d\sec(e+fx}{}\right.$$

3.876. $\int(d\cos(e+fx))^n(a+b\sec(e+fx))^2\,dx$

$$\Big\downarrow \text{3042}$$

$$(d\cos(e+fx))^n(d\sec(e+fx))^n\left(\left(a^2-\frac{b^2n}{1-n}\right)\left(\frac{\cos(e+fx)}{d}\right)^{-n}(d\sec(e+fx))^{-n}\int\left(\frac{\sin\left(e+fx+\frac{\pi}{2}\right)}{d}\right)^n dx-\frac{2ab\sin(e+fx)(d\sec(e}{}\right.$$

$$\Big\downarrow \text{3122}$$

$$(d\cos(e+fx))^n(d\sec(e+fx))^n\left(-\frac{d\left(a^2-\frac{b^2n}{1-n}\right)\sin(e+fx)(d\sec(e+fx))^{-n-1}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n+1}{2},\frac{n+3}{2},\cos^2(e+fx)\right)}{f(n+1)\sqrt{\sin^2(e+fx)}}-\frac{2ab\sin}{}\right.$$

input
```
Int[(d*Cos[e + f*x])^n*(a + b*Sec[e + f*x])^2,x]
```

output
```
(d*Cos[e + f*x])^n*(d*Sec[e + f*x])^n*(-((d*(a^2 - (b^2*n)/(1 - n))*Hyperg
eometric2F1[1/2, (1 + n)/2, (3 + n)/2, Cos[e + f*x]^2]*(d*Sec[e + f*x])^(-
1 - n)*Sin[e + f*x])/(f*(1 + n)*Sqrt[Sin[e + f*x]^2])) - (2*a*b*Hypergeome
tric2F1[1/2, n/2, (2 + n)/2, Cos[e + f*x]^2]*Sin[e + f*x])/(f*n*(d*Sec[e +
 f*x])^n*Sqrt[Sin[e + f*x]^2]) + (b^2*Tan[e + f*x])/(f*(1 - n)*(d*Sec[e +
f*x])^n))
```

3.876.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

rule 4259
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)   Int[1/(Sin[c + d*x]/b)^n, x]), x] /;
FreeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

rule 4275
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Simp[2*a*(b/d)   Int[(d*Csc[e + f*x])^(n + 1), x], x]
 + Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
 e, f, n}, x]
```

rule 4534
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Simp[(C*m + A*(m + 1))/(m + 1)   Int[(b*Csc[e + f*x])^m, x], x] /;
 FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

3.876.4 Maple [F]

$$\int (d\cos(fx+e))^n (a+b\sec(fx+e))^2 \, dx$$

input
```
int((d*cos(f*x+e))^n*(a+b*sec(f*x+e))^2,x)
```

output
```
int((d*cos(f*x+e))^n*(a+b*sec(f*x+e))^2,x)
```

3.876.5 Fricas [F]

$$\int (d\cos(e+fx))^n (a+b\sec(e+fx))^2 \, dx = \int (b\sec(fx+e)+a)^2 (d\cos(fx+e))^n \, dx$$

input
```
integrate((d*cos(f*x+e))^n*(a+b*sec(f*x+e))^2,x, algorithm="fricas")
```

output
```
integral((b^2*sec(f*x + e)^2 + 2*a*b*sec(f*x + e) + a^2)*(d*cos(f*x + e))^
n, x)
```

3.876.6 Sympy [F]

$$\int (d\cos(e+fx))^n(a+b\sec(e+fx))^2\,dx = \int (d\cos(e+fx))^n(a+b\sec(e+fx))^2\,dx$$

input
```
integrate((d*cos(f*x+e))**n*(a+b*sec(f*x+e))**2,x)
```

output
```
Integral((d*cos(e + f*x))**n*(a + b*sec(e + f*x))**2, x)
```

3.876.7 Maxima [F]

$$\int (d\cos(e+fx))^n(a+b\sec(e+fx))^2\,dx = \int (b\sec(fx+e)+a)^2(d\cos(fx+e))^n\,dx$$

input
```
integrate((d*cos(f*x+e))^n*(a+b*sec(f*x+e))^2,x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e) + a)^2*(d*cos(f*x + e))^n, x)
```

3.876.8 Giac [F]

$$\int (d\cos(e+fx))^n(a+b\sec(e+fx))^2\,dx = \int (b\sec(fx+e)+a)^2(d\cos(fx+e))^n\,dx$$

input
```
integrate((d*cos(f*x+e))^n*(a+b*sec(f*x+e))^2,x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e) + a)^2*(d*cos(f*x + e))^n, x)
```

3.876.9 Mupad [F(-1)]

Timed out.

$$\int (d\cos(e+fx))^n(a+b\sec(e+fx))^2\,dx=\int (d\,\cos\left(e+f\,x\right))^n\,\left(a+\frac{b}{\cos\left(e+f\,x\right)}\right)^2\,dx$$

input
```
int((d*cos(e + f*x))^n*(a + b/cos(e + f*x))^2,x)
```

output
```
int((d*cos(e + f*x))^n*(a + b/cos(e + f*x))^2, x)
```

3.877 $\int (d\cos(e+fx))^n(a+b\sec(e+fx))\,dx$

3.877.1 Optimal result 6651
3.877.2 Mathematica [A] (verified) 6651
3.877.3 Rubi [A] (verified) 6652
3.877.4 Maple [F] 6654
3.877.5 Fricas [F] 6654
3.877.6 Sympy [F] 6654
3.877.7 Maxima [F] 6655
3.877.8 Giac [F] 6655
3.877.9 Mupad [F(-1)] 6655

3.877.1 Optimal result

Integrand size = 21, antiderivative size = 132

$$\int (d\cos(e+fx))^n(a+b\sec(e+fx))\,dx = -\frac{b(d\cos(e+fx))^n\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n}{2},\frac{2+n}{2},\cos^2(e+fx)\right)\sin(e+fx)}{fn\sqrt{\sin^2(e+fx)}} - \frac{a(d\cos(e+fx))^{1+n}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+n}{2},\frac{3+n}{2},\cos^2(e+fx)\right)\sin(e+fx)}{df(1+n)\sqrt{\sin^2(e+fx)}}$$

output
```
-b*(d*cos(f*x+e))^n*hypergeom([1/2, 1/2*n],[1+1/2*n],cos(f*x+e)^2)*sin(f*x
+e)/f/n/(sin(f*x+e)^2)^(1/2)-a*(d*cos(f*x+e))^(1+n)*hypergeom([1/2, 1/2+1/
2*n],[3/2+1/2*n],cos(f*x+e)^2)*sin(f*x+e)/d/f/(1+n)/(sin(f*x+e)^2)^(1/2)
```

3.877.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.80

$$\int (d\cos(e+fx))^n(a+b\sec(e+fx))\,dx = -\frac{(d\cos(e+fx))^n\csc(e+fx)\left(b(1+n)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n}{2},\frac{2+n}{2},\cos^2(e+fx)\right)+an\cos(e+fx)\right.}{fn(1+n)}$$

input
```
Integrate[(d*Cos[e + f*x])^n*(a + b*Sec[e + f*x]),x]
```

output
```
-(((d*Cos[e + f*x])^n*Csc[e + f*x]*(b*(1 + n)*Hypergeometric2F1[1/2, n/2,
(2 + n)/2, Cos[e + f*x]^2] + a*n*Cos[e + f*x]*Hypergeometric2F1[1/2, (1 +
n)/2, (3 + n)/2, Cos[e + f*x]^2])*Sqrt[Sin[e + f*x]^2])/(f*n*(1 + n)))
```

3.877.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.04, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 4713, 3042, 2030, 3227, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\sec(e+fx))(d\cos(e+fx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a+b\csc\left(e+fx+\frac{\pi}{2}\right)\right)\left(d\sin\left(e+fx+\frac{\pi}{2}\right)\right)^n dx$$

$$\Big\downarrow \text{4713}$$

$$\int \sec(e+fx)(a\cos(e+fx)+b)(d\cos(e+fx))^n dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\sin\left(e+fx+\frac{\pi}{2}\right)+b\right)\left(d\sin\left(e+fx+\frac{\pi}{2}\right)\right)^n}{\sin\left(e+fx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{2030}$$

$$d\int \left(d\sin\left(\frac{1}{2}(2e+\pi)+fx\right)\right)^{n-1}\left(b+a\sin\left(\frac{1}{2}(2e+\pi)+fx\right)\right)dx$$

$$\Big\downarrow \text{3227}$$

$$d\left(\frac{a\int (d\cos(e+fx))^n dx}{d}+b\int (d\cos(e+fx))^{n-1}dx\right)$$

$$\Big\downarrow \text{3042}$$

$$d\left(\frac{a\int \left(d\sin\left(e+fx+\frac{\pi}{2}\right)\right)^n dx}{d}+b\int \left(d\sin\left(e+fx+\frac{\pi}{2}\right)\right)^{n-1}dx\right)$$

$$\Big\downarrow \text{3122}$$

$$d\left(-\frac{a\sin(e+fx)(d\cos(e+fx))^{n+1}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n+1}{2},\frac{n+3}{2},\cos^2(e+fx)\right)}{d^2f(n+1)\sqrt{\sin^2(e+fx)}}-\frac{b\sin(e+fx)(d\cos(e+f\right.$$

input
```
Int[(d*Cos[e + f*x])^n*(a + b*Sec[e + f*x]),x]
```

output
```
d*(-((b*(d*Cos[e + f*x])^n*Hypergeometric2F1[1/2, n/2, (2 + n)/2, Cos[e +
f*x]^2]*Sin[e + f*x])/(d*f*n*Sqrt[Sin[e + f*x]^2])) - (a*(d*Cos[e + f*x])^
(1 + n)*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2, Cos[e + f*x]^2]*Sin[e
 + f*x])/(d^2*f*(1 + n)*Sqrt[Sin[e + f*x]^2]))
```

3.877.3.1 Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 4713
```
Int[(csc[(a_.) + (b_.)*(x_)]*(B_.) + (A_))*(u_), x_Symbol] :> Int[ActivateT
rig[u]*((B + A*Sin[a + b*x])/Sin[a + b*x]), x] /; FreeQ[{a, b, A, B}, x] &&
 KnownSineIntegrandQ[u, x]
```

3.877.4 Maple [F]

$$\int (d\cos(fx+e))^n (a+b\sec(fx+e))\,dx$$

input
```
int((d*cos(f*x+e))^n*(a+b*sec(f*x+e)),x)
```

output
```
int((d*cos(f*x+e))^n*(a+b*sec(f*x+e)),x)
```

3.877.5 Fricas [F]

$$\int (d\cos(e+fx))^n(a+b\sec(e+fx))\,dx = \int (b\sec(fx+e)+a)(d\cos(fx+e))^n\,dx$$

input
```
integrate((d*cos(f*x+e))^n*(a+b*sec(f*x+e)),x, algorithm="fricas")
```

output
```
integral((b*sec(f*x + e) + a)*(d*cos(f*x + e))^n, x)
```

3.877.6 Sympy [F]

$$\int (d\cos(e+fx))^n(a+b\sec(e+fx))\,dx = \int (d\cos(e+fx))^n (a+b\sec(e+fx))\,dx$$

input
```
integrate((d*cos(f*x+e))**n*(a+b*sec(f*x+e)),x)
```

output
```
Integral((d*cos(e + f*x))**n*(a + b*sec(e + f*x)), x)
```

3.877.7 Maxima [F]

$$\int (d\cos(e+fx))^n(a+b\sec(e+fx))\,dx = \int (b\sec(fx+e)+a)(d\cos(fx+e))^n\,dx$$

input
```
integrate((d*cos(f*x+e))^n*(a+b*sec(f*x+e)),x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e) + a)*(d*cos(f*x + e))^n, x)
```

3.877.8 Giac [F]

$$\int (d\cos(e+fx))^n(a+b\sec(e+fx))\,dx = \int (b\sec(fx+e)+a)(d\cos(fx+e))^n\,dx$$

input
```
integrate((d*cos(f*x+e))^n*(a+b*sec(f*x+e)),x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e) + a)*(d*cos(f*x + e))^n, x)
```

3.877.9 Mupad [F(-1)]

Timed out.

$$\int (d\cos(e+fx))^n(a+b\sec(e+fx))\,dx = \int (d\,\cos(e+f\,x))^n\left(a+\frac{b}{\cos(e+f\,x)}\right)\,dx$$

input
```
int((d*cos(e + f*x))^n*(a + b/cos(e + f*x)),x)
```

output
```
int((d*cos(e + f*x))^n*(a + b/cos(e + f*x)), x)
```

3.878 $\int \frac{(d\cos(e+fx))^n}{a+b\sec(e+fx)}\,dx$

3.878.1 Optimal result 6656
3.878.2 Mathematica [B] (warning: unable to verify) 6656
3.878.3 Rubi [A] (verified) 6657
3.878.4 Maple [F] 6659
3.878.5 Fricas [F] 6660
3.878.6 Sympy [F] 6660
3.878.7 Maxima [F] 6660
3.878.8 Giac [F] 6661
3.878.9 Mupad [F(-1)] 6661

3.878.1 Optimal result

Integrand size = 23, antiderivative size = 196

$$\int \frac{(d\cos(e+fx))^n}{a+b\sec(e+fx)}\,dx = \frac{a\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2}(-1-n),1,\frac{3}{2},\sin^2(e+fx),\frac{a^2\sin^2(e+fx)}{a^2-b^2}\right)\cos(e+fx)(d\cos(e+fx))^n\cos^2(e+fx)^{\frac{1}{2}(-1-n}}{(a^2-b^2)\,f} - \frac{b\,\text{AppellF1}\left(\frac{1}{2},-\frac{n}{2},1,\frac{3}{2},\sin^2(e+fx),\frac{a^2\sin^2(e+fx)}{a^2-b^2}\right)(d\cos(e+fx))^n\cos^2(e+fx)^{-n/2}\sin(e+fx)}{(a^2-b^2)\,f}$$

output
```
a*AppellF1(1/2,-1/2-1/2*n,1,3/2,sin(f*x+e)^2,a^2*sin(f*x+e)^2/(a^2-b^2))*c
os(f*x+e)*(d*cos(f*x+e))^n*(cos(f*x+e)^2)^(-1/2-1/2*n)*sin(f*x+e)/(a^2-b^2
)/f-b*AppellF1(1/2,-1/2*n,1,3/2,sin(f*x+e)^2,a^2*sin(f*x+e)^2/(a^2-b^2))*(
d*cos(f*x+e))^n*sin(f*x+e)/(a^2-b^2)/f/((cos(f*x+e)^2)^(1/2*n))
```

3.878.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 5216 vs. 2(196) = 392.

Time = 27.48 (sec) , antiderivative size = 5216, normalized size of antiderivative = 26.61

$$\int \frac{(d\cos(e+fx))^n}{a+b\sec(e+fx)}\,dx = \text{Result too large to show}$$

input
```
Integrate[(d*Cos[e + f*x])^n/(a + b*Sec[e + f*x]),x]
```

output
```
Result too large to show
```

3.878.3 Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.07, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {3042, 4752, 3042, 4356, 3042, 3302, 3042, 3668, 25, 333}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d\cos(e+fx))^n}{a+b\sec(e+fx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(d\sin\left(e+fx+\frac{\pi}{2}\right)\right)^n}{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{4752}$$

$$(d\cos(e+fx))^n(d\sec(e+fx))^n\int \frac{(d\sec(e+fx))^{-n}}{a+b\sec(e+fx)}dx$$

$$\Big\downarrow \text{3042}$$

$$(d\cos(e+fx))^n(d\sec(e+fx))^n\int \frac{\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{-n}}{a+b\csc\left(e+fx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{4356}$$

$$\cos^{-n}(e+fx)(d\cos(e+fx))^n\int \frac{\cos^{n+1}(e+fx)}{b+a\cos(e+fx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\cos^{-n}(e+fx)(d\cos(e+fx))^n\int \frac{\sin\left(e+fx+\frac{\pi}{2}\right)^{n+1}}{b+a\sin\left(e+fx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3302}$$

$$\cos^{-n}(e+fx)(d\cos(e+fx))^n\left(b\int \frac{\cos^{n+1}(e+fx)}{b^2-a^2\cos^2(e+fx)}dx-a\int \frac{\cos^{n+2}(e+fx)}{b^2-a^2\cos^2(e+fx)}dx\right)$$

$$\Big\downarrow \text{3042}$$

$$\cos^{-n}(e+fx)(d\cos(e+fx))^n\left(b\int \frac{\sin\left(e+fx+\frac{\pi}{2}\right)^{n+1}}{b^2-a^2\sin\left(e+fx+\frac{\pi}{2}\right)^2}dx-a\int \frac{\sin\left(e+fx+\frac{\pi}{2}\right)^{n+2}}{b^2-a^2\sin\left(e+fx+\frac{\pi}{2}\right)^2}dx\right)$$

$\Big\downarrow$ 3668

$$\cos^{-n}(e+fx)(d\cos(e+fx))^n\left(\frac{b\cos^n(e+fx)\cos^2(e+fx)^{-n/2}\int -\frac{(1-\sin^2(e+fx))^{n/2}}{-\sin^2(e+fx)a^2+a^2-b^2}d\sin(e+fx)}{f}-\frac{a\cos^{n+1}(e+fx)\cos^2(e+fx)^{\frac{1}{2}(-}}{}\right.$$

$\Big\downarrow$ 25

$$\cos^{-n}(e+fx)(d\cos(e+fx))^n\left(\frac{a\cos^{n+1}(e+fx)\cos^2(e+fx)^{\frac{1}{2}(-n-1)}\int \frac{(1-\sin^2(e+fx))^{\frac{n+1}{2}}}{-\sin^2(e+fx)a^2+a^2-b^2}d\sin(e+fx)}{f}-\frac{b\cos^n(e+fx)\cos^2(e+fx)^{-}}{}\right.$$

$\Big\downarrow$ 333

$$\cos^{-n}(e+fx)(d\cos(e+fx))^n\left(\frac{a\sin(e+fx)\cos^{n+1}(e+fx)\cos^2(e+fx)^{\frac{1}{2}(-n-1)}\text{AppellF1}\left(\frac{1}{2},\frac{1}{2}(-n-1),1,\frac{3}{2},\sin^2(e+fx),\frac{a^2\sin^2(e+fx)}{a^2-b^2}\right.}{f\left(a^2-b^2\right)}\right.$$

input
```
Int[(d*Cos[e + f*x])^n/(a + b*Sec[e + f*x]),x]
```

output
```
((d*Cos[e + f*x])^n*((a*AppellF1[1/2, (-1 - n)/2, 1, 3/2, Sin[e + f*x]^2,
(a^2*Sin[e + f*x]^2)/(a^2 - b^2)]*Cos[e + f*x]^(1 + n)*(Cos[e + f*x]^2)^((
-1 - n)/2)*Sin[e + f*x])/((a^2 - b^2)*f) - (b*AppellF1[1/2, -1/2*n, 1, 3/2
, Sin[e + f*x]^2, (a^2*Sin[e + f*x]^2)/(a^2 - b^2)]*Cos[e + f*x]^n*Sin[e +
 f*x])/((a^2 - b^2)*f*(Cos[e + f*x]^2)^(n/2))))/Cos[e + f*x]^n
```

3.878.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 333
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^p*c^q*x*AppellF1[1/2, -p, -q, 3/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; F
reeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[p] || GtQ[a,
0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3302
```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[a   Int[(d*Sin[e + f*x])^n/(a^2 - b^2*Sin[e + f*x]
^2), x], x] - Simp[b/d   Int[(d*Sin[e + f*x])^(n + 1)/(a^2 - b^2*Sin[e + f*
x]^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0]
```

rule 3668
```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[(
-ff)*d^(2*IntPart[(m - 1)/2] + 1)*((d*Sin[e + f*x])^(2*FracPart[(m - 1)/2])
/(f*(Sin[e + f*x]^2)^FracPart[(m - 1)/2]))   Subst[Int[(1 - ff^2*x^2)^((m -
 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b,
 d, e, f, m, p}, x] &&  !IntegerQ[m]
```

rule 4356
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Simp[Sin[e + f*x]^n*(d*Csc[e + f*x])^n   Int[(b +
 a*Sin[e + f*x])^m/Sin[e + f*x]^(m + n), x], x] /; FreeQ[{a, b, d, e, f, n}
, x] && NeQ[a^2 - b^2, 0] && IntegerQ[m]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

3.878.4 Maple [F]

$$\int \frac{(d\cos(fx+e))^n}{a+b\sec(fx+e)}dx$$

input
```
int((d*cos(f*x+e))^n/(a+b*sec(f*x+e)),x)
```

output
```
int((d*cos(f*x+e))^n/(a+b*sec(f*x+e)),x)
```

3.878.5 Fricas [F]

$$\int \frac{(d\cos(e+fx))^n}{a+b\sec(e+fx)}\,dx = \int \frac{(d\cos(fx+e))^n}{b\sec(fx+e)+a}\,dx$$

input
```
integrate((d*cos(f*x+e))^n/(a+b*sec(f*x+e)),x, algorithm="fricas")
```

output
```
integral((d*cos(f*x + e))^n/(b*sec(f*x + e) + a), x)
```

3.878.6 Sympy [F]

$$\int \frac{(d\cos(e+fx))^n}{a+b\sec(e+fx)}\,dx = \int \frac{(d\cos(e+fx))^n}{a+b\sec(e+fx)}\,dx$$

input
```
integrate((d*cos(f*x+e))**n/(a+b*sec(f*x+e)),x)
```

output
```
Integral((d*cos(e + f*x))**n/(a + b*sec(e + f*x)), x)
```

3.878.7 Maxima [F]

$$\int \frac{(d\cos(e+fx))^n}{a+b\sec(e+fx)}\,dx = \int \frac{(d\cos(fx+e))^n}{b\sec(fx+e)+a}\,dx$$

input
```
integrate((d*cos(f*x+e))^n/(a+b*sec(f*x+e)),x, algorithm="maxima")
```

output
```
integrate((d*cos(f*x + e))^n/(b*sec(f*x + e) + a), x)
```

3.878.8 Giac [F]

$$\int \frac{(d\cos(e+fx))^n}{a+b\sec(e+fx)}\,dx = \int \frac{(d\cos(fx+e))^n}{b\sec(fx+e)+a}\,dx$$

input
```
integrate((d*cos(f*x+e))^n/(a+b*sec(f*x+e)),x, algorithm="giac")
```

output
```
integrate((d*cos(f*x + e))^n/(b*sec(f*x + e) + a), x)
```

3.878.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d\cos(e+fx))^n}{a+b\sec(e+fx)}\,dx = \int \frac{(d\,\cos(e+f\,x))^n}{a+\frac{b}{\cos(e+f\,x)}}\,dx$$

input
```
int((d*cos(e + f*x))^n/(a + b/cos(e + f*x)),x)
```

output
```
int((d*cos(e + f*x))^n/(a + b/cos(e + f*x)), x)
```

3.879 $\int \frac{(d\cos(e+fx))^n}{(a+b\sec(e+fx))^2}\,dx$

3.879.1 Optimal result 6662
3.879.2 Mathematica [B] (warning: unable to verify) 6663
3.879.3 Rubi [A] (verified) 6663
3.879.4 Maple [F] 6665
3.879.5 Fricas [F] 6665
3.879.6 Sympy [F] 6666
3.879.7 Maxima [F] 6666
3.879.8 Giac [F] 6666
3.879.9 Mupad [F(-1)] 6667

3.879.1 Optimal result

Integrand size = 23, antiderivative size = 309

$$\int \frac{(d\cos(e+fx))^n}{(a+b\sec(e+fx))^2}\,dx$$
$$= \frac{a^2\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2}(-3-n),2,\frac{3}{2},\sin^2(e+fx),\frac{a^2\sin^2(e+fx)}{a^2-b^2}\right)\cos(e+fx)(d\cos(e+fx))^n\cos^2(e+fx)^{\frac{1}{2}(-1-}}{(a^2-b^2)^2 f}$$
$$+\frac{b^2\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2}(-1-n),2,\frac{3}{2},\sin^2(e+fx),\frac{a^2\sin^2(e+fx)}{a^2-b^2}\right)\cos(e+fx)(d\cos(e+fx))^n\cos^2(e+fx)^{\frac{1}{2}(-}}{(a^2-b^2)^2 f}$$
$$-\frac{2ab\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2}(-2-n),2,\frac{3}{2},\sin^2(e+fx),\frac{a^2\sin^2(e+fx)}{a^2-b^2}\right)(d\cos(e+fx))^n\cos^2(e+fx)^{-n/2}\sin(e+}{(a^2-b^2)^2 f}$$

output
```
a^2*AppellF1(1/2,-3/2-1/2*n,2,3/2,sin(f*x+e)^2,a^2*sin(f*x+e)^2/(a^2-b^2))
*cos(f*x+e)*(d*cos(f*x+e))^n*(cos(f*x+e)^2)^(-1/2-1/2*n)*sin(f*x+e)/(a^2-b
^2)^2/f+b^2*AppellF1(1/2,-1/2-1/2*n,2,3/2,sin(f*x+e)^2,a^2*sin(f*x+e)^2/(a
^2-b^2))*cos(f*x+e)*(d*cos(f*x+e))^n*(cos(f*x+e)^2)^(-1/2-1/2*n)*sin(f*x+e
)/(a^2-b^2)^2/f-2*a*b*AppellF1(1/2,-1-1/2*n,2,3/2,sin(f*x+e)^2,a^2*sin(f*x
+e)^2/(a^2-b^2))*(d*cos(f*x+e))^n*sin(f*x+e)/(a^2-b^2)^2/f/((cos(f*x+e)^2)
^(1/2*n))
```

3.879.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 13974 vs. $2(309) = 618$.

Time = 51.68 (sec) , antiderivative size = 13974, normalized size of antiderivative = 45.22

$$\int \frac{(d\cos(e+fx))^n}{(a+b\sec(e+fx))^2}\,dx = \text{Result too large to show}$$

input
```
Integrate[(d*Cos[e + f*x])^n/(a + b*Sec[e + f*x])^2,x]
```

output
```
Result too large to show
```

3.879.3 Rubi [A] (verified)

Time = 0.87 (sec) , antiderivative size = 316, normalized size of antiderivative = 1.02, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3042, 4752, 3042, 4356, 3042, 3303, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d\cos(e+fx))^n}{(a+b\sec(e+fx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(d\sin\left(e+fx+\frac{\pi}{2}\right)\right)^n}{\left(a+b\csc\left(e+fx+\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \text{4752}$$

$$(d\cos(e+fx))^n(d\sec(e+fx))^n\int \frac{(d\sec(e+fx))^{-n}}{(a+b\sec(e+fx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$(d\cos(e+fx))^n(d\sec(e+fx))^n\int \frac{\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{-n}}{\left(a+b\csc\left(e+fx+\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \text{4356}$$

$$\cos^{-n}(e+fx)(d\cos(e+fx))^n\int \frac{\cos^{n+2}(e+fx)}{(b+a\cos(e+fx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\cos^{-n}(e+fx)(d\cos(e+fx))^n\int \frac{\sin\left(e+fx+\frac{\pi}{2}\right)^{n+2}}{\left(b+a\sin\left(e+fx+\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \text{3303}$$

$$\cos^{-n}(e+fx)(d\cos(e+fx))^n\int \left(\frac{b^2\cos^{n+2}(e+fx)}{\left(b^2-a^2\cos^2(e+fx)\right)^2}-\frac{2ab\cos^{n+3}(e+fx)}{\left(b^2-a^2\cos^2(e+fx)\right)^2}+\frac{a^2\cos^{n+4}(e+fx)}{\left(a^2\cos^2(e+fx)-b^2\right)^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\cos^{-n}(e+fx)(d\cos(e+fx))^n\left(\frac{a^2\sin(e+fx)\cos^{n+1}(e+fx)\cos^2(e+fx)^{\frac{1}{2}(-n-1)}\text{AppellF1}\left(\frac{1}{2},\frac{1}{2}(-n-3),2,\frac{3}{2},\sin^2(e+fx),\frac{a^2\sin^2(e+fx}{a^2-b^2}\right.}{f\left(a^2-b^2\right)^2}\right.$$

input
```
Int[(d*Cos[e + f*x])^n/(a + b*Sec[e + f*x])^2,x]
```

output
```
((d*Cos[e + f*x])^n*((a^2*AppellF1[1/2, (-3 - n)/2, 2, 3/2, Sin[e + f*x]^2
, (a^2*Sin[e + f*x]^2)/(a^2 - b^2)]*Cos[e + f*x]^(1 + n)*(Cos[e + f*x]^2)^
((-1 - n)/2)*Sin[e + f*x])/((a^2 - b^2)^2*f) + (b^2*AppellF1[1/2, (-1 - n)
/2, 2, 3/2, Sin[e + f*x]^2, (a^2*Sin[e + f*x]^2)/(a^2 - b^2)]*Cos[e + f*x]
^(1 + n)*(Cos[e + f*x]^2)^((-1 - n)/2)*Sin[e + f*x])/((a^2 - b^2)^2*f) - (
2*a*b*AppellF1[1/2, (-2 - n)/2, 2, 3/2, Sin[e + f*x]^2, (a^2*Sin[e + f*x]^
2)/(a^2 - b^2)]*Cos[e + f*x]^n*Sin[e + f*x])/((a^2 - b^2)^2*f*(Cos[e + f*x
]^2)^(n/2))))/Cos[e + f*x]^n
```

3.879.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3303
```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(d*sin[e + f*x])^n*(1/((a - b*sin[
e + f*x])^m/(a^2 - b^2*sin[e + f*x]^2)^m)), x], x] /; FreeQ[{a, b, d, e, f,
 n}, x] && NeQ[a^2 - b^2, 0] && ILtQ[m, -1]
```

rule 4356
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Simp[Sin[e + f*x]^n*(d*Csc[e + f*x])^n   Int[(b +
 a*Sin[e + f*x])^m/Sin[e + f*x]^(m + n), x], x] /; FreeQ[{a, b, d, e, f, n}
, x] && NeQ[a^2 - b^2, 0] && IntegerQ[m]
```

rule 4752
```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Simp[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m   Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x
]
```

3.879.4 Maple [F]

$$\int \frac{(d\cos(fx+e))^n}{(a+b\sec(fx+e))^2}dx$$

input
```
int((d*cos(f*x+e))^n/(a+b*sec(f*x+e))^2,x)
```

output
```
int((d*cos(f*x+e))^n/(a+b*sec(f*x+e))^2,x)
```

3.879.5 Fricas [F]

$$\int \frac{(d\cos(e+fx))^n}{(a+b\sec(e+fx))^2}\,dx = \int \frac{(d\cos(fx+e))^n}{(b\sec(fx+e)+a)^2}\,dx$$

input
```
integrate((d*cos(f*x+e))^n/(a+b*sec(f*x+e))^2,x, algorithm="fricas")
```

output
```
integral((d*cos(f*x + e))^n/(b^2*sec(f*x + e)^2 + 2*a*b*sec(f*x + e) + a^2
), x)
```

3.879.6 Sympy [F]

$$\int \frac{(d\cos(e+fx))^n}{(a+b\sec(e+fx))^2}\,dx = \int \frac{(d\cos(e+fx))^n}{(a+b\sec(e+fx))^2}\,dx$$

input
```
integrate((d*cos(f*x+e))**n/(a+b*sec(f*x+e))**2,x)
```

output
```
Integral((d*cos(e + f*x))**n/(a + b*sec(e + f*x))**2, x)
```

3.879.7 Maxima [F]

$$\int \frac{(d\cos(e+fx))^n}{(a+b\sec(e+fx))^2}\,dx = \int \frac{(d\cos(fx+e))^n}{(b\sec(fx+e)+a)^2}\,dx$$

input
```
integrate((d*cos(f*x+e))^n/(a+b*sec(f*x+e))^2,x, algorithm="maxima")
```

output
```
integrate((d*cos(f*x + e))^n/(b*sec(f*x + e) + a)^2, x)
```

3.879.8 Giac [F]

$$\int \frac{(d\cos(e+fx))^n}{(a+b\sec(e+fx))^2}\,dx = \int \frac{(d\cos(fx+e))^n}{(b\sec(fx+e)+a)^2}\,dx$$

input
```
integrate((d*cos(f*x+e))^n/(a+b*sec(f*x+e))^2,x, algorithm="giac")
```

output
```
integrate((d*cos(f*x + e))^n/(b*sec(f*x + e) + a)^2, x)
```

3.879.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d\cos(e+fx))^n}{(a+b\sec(e+fx))^2}\,dx = \int \frac{(d\,\cos\left(e+f\,x\right))^n}{\left(a+\frac{b}{\cos\left(e+f\,x\right)}\right)^2}\,dx$$

input
```
int((d*cos(e + f*x))^n/(a + b/cos(e + f*x))^2,x)
```

output
```
int((d*cos(e + f*x))^n/(a + b/cos(e + f*x))^2, x)
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions 6668

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.1.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)



(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
```

```
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A"," "}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A"," "}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]
```

```
(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]]
```

```
ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.1.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
```

```
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;
```

```
if ExpnType_result<=ExpnType_optimal then
    if debug then
         print("ExpnType_result<=ExpnType_optimal");
    fi;
    if is_contains_complex(result) then
        if is_contains_complex(optimal) then
            if debug then
                    print("both result and optimal complex");
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                return "A"," ";
            else
               return "B",cat("Both result and optimal contain complex but leaf count of
                                convert(leaf_count_result,string)," vs. $2 (",
                                convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_
            end if
        else #result contains complex but optimal is not
            if debug then
                    print("result contains complex but optimal is not");
            fi;
            return "C","Result contains complex when optimal does not.";
        fi;
    else # result do not contain complex
         # this assumes optimal do not as well. No check is needed here.
        if debug then
               print("result do not contain complex, this assumes optimal do not as well"
        fi;
        if leaf_count_result<=2*leaf_count_optimal then
            if debug then
                print("leaf_count_result<=2*leaf_count_optimal");
            fi;
            return "A"," ";
        else
            if debug then
                print("leaf_count_result>2*leaf_count_optimal");
            fi;
            return "B",cat("Leaf count of result is larger than twice the leaf count of o
                                convert(leaf_count_result,string),"$ vs. $2(",
                                convert(leaf_count_optimal,string),")=",convert(2*leaf_cou
        fi;
    fi;
```

```
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
```

```
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
```

```
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.1.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
```

```
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
            return True
        else:
            return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
```

```
    return 1
  elif isinstance(expn,list):
    return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
  elif is_sqrt(expn):
    if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
      return 1
    else:
      return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
  elif isinstance(expn,Pow):   #type(expn,'`^`')
    if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
      return expnType(expn.args[0])   #ExpnType(op(1,expn))
    elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
      if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
        return 1
      else:
        return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    else:
      return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
  elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
    m1 = expnType(expn.args[0])
    m2 = expnType(list(expn.args[1:]))
    return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
    return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
  elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
    m1 = max(map(expnType, list(expn.args)))
    return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
  elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
    m1 = max(map(expnType, list(expn.args)))
    return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
  elif is_appell_function(expn.func):
    m1 = max(map(expnType, list(expn.args)))
    return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
  elif isinstance(expn,RootSum):
    m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
    return max(7,m1)
  elif str(expn).find("Integral") != -1:
    m1 = max(map(expnType, list(expn.args)))
    return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
  else:
    return 9

#main function
```

```
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is larg
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(le
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_


    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)
```

```
    return grade, grade_annotation
```

4.1.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
```

```
        return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
            'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
            'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
            'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
            'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m
```

```
def is_hypergeometric_function(func):
   return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
   return func.name() in ['hypergeometric']    #[appellf1] can't find this in sagemath

def is_atom(expn):

   #debug=False
   if debug:
       print ("Enter is_atom, expn=",expn)

   if not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).__name__ )
      return False


def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:    #isinstance(expn,list):
```

```
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif is_sqrt(expn):
        if type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinst
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
            #is checked before calling the grading function that is passed.
            #but kept it here.
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):
```

```
if debug:
    print ("Enter grade_antiderivative for sagemath")
    print("Enter grade_antiderivative, result=",result)
    print("Enter grade_antiderivative, optimal=",optimal)
    print("type(anti)=",type(result))
    print("type(optimal)=",type(optimal))

leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation =" "
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation =" "
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)
```

```
    return grade, grade_annotation
```